EĞRİLERİN KÜRESEL İZDÜŞÜMLERİ

Niyazi ÇELİK

Yüksek Lisans Tezi

Danışman: Yrd. Doç. Dr. Yılmaz TUNCER

Uşak

Haziran,2012

T.C.
UŞAK ÜNİVERSİTESİ
FEN BİLİMLERİ ENSTİTÜSÜ

MATEMATİK ANABİLİM DALI

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Niyazi ÇELİK tarafından hazırlanan **"Eğrilerin Küresel İzdüşümleri"** adlı bu tezin Yüksek Lisans tezi olarak uygun olduğunu onaylarım.

Yrd. Doç. Dr. Yılmaz TUNÇER ………………………………
Tez Danışmanı, Matematik Anabilim Dalı

Bu çalışma, jürimiz tarafından oy birliği ile Matematik Anabilim Dalında Yüksek Lisans tezi olarak kabul edilmiştir.

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Fen Bilimleri Enstitüsü Müdürü

TEZ BİLDİRİMİ

Tez içindeki bütün bilgilerin etik davranış ve akademik kurallar çerçevesinde elde edilerek sunulduğunu, ayrıca tez yazım kurallarına uygun olarak hazırlanan bu çalışmada bana ait olmayan her türlü ifade ve bilginin kaynağına eksiksiz atıf yapıldığını bildiririm.

Niyazi ÇELİK

EĞRİLERİN KÜRESEL İZDÜŞÜMLERİ
(Yüksek Lisans Tezi)

Niyazi ÇELİK

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ÖZET

Bu tez çalışması iki bölümden oluşmaktadır.

Tezin birinci bölümünde konuyla ilgili temel tanım ve teoremler verildi.

İkinci bölümde uzay eğrisinin yer vektörü kullanılarak küre üzerindeki izdüşüm eğrisinin eğrilikleri bulunmuştur. Uzay eğrisinin eğriliklerine göre izdüşüm eğrisi için elde edilen sonuçlar verilmiştir.

Bilim Kodu : 53A04
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SPHERİCAL PROJECTİONS OF THE CURVES
(M. Sc. Thesis)

Niyazi ÇELİK

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ABSTRACT

This thesis consists of two chapters.

In the first chapter, the basic definitions and theorems are given related to this subject.

In the second chapter, curvatures of the projection curve on the sphere found in by using a vector space curve. For a projection space it is obtained some results according to curvatures of the projection space.

Science Code : 53A04
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TEŞEKKÜR

Öncelikle, bu tezin hazırlanması esnasında büyük yardımlarını gördüğüm ve her an her konuda manevi desteğini benden esirgemeyen değerli danışman hocam Sayın Yrd. Doç. Dr. Yılmaz TUNÇER'e teşekkürlerimi ve şükranlarımı sunarım.

Ayrıca, çalışmalarım sırasında bana anlayış gösteren, destek olan, duydukları ve hissettirdikleri sonsuz güven için sevgili eşime, anneme ve babama teşekkürlerimi sunarım.

İÇİNDEKİLER

SİMGELERİN LİSTESİ

Bu çalışmada kullanılmış bazı simgeler açıklamaları ile birlikte aşağıda sunulmuştur.

Simgeler	**Açıklama**
$\lVert , \rVert$	Norm
$\wedge$	Vektörel çarpım
$\langle , \rangle$	İç çarpım
T_α	α eğrisinin teğet vektörü
N_α	α eğrisinin asli normal vektörü
B_α	α eğrisinin binormal vektörü
T_β	β eğrisinin teğet vektörü
N_β	β eğrisinin asli normal vektörü
B_β	β eğrisinin binormal vektörü
κ_α	α eğrisinin eğriliği
τ_α	α eğrisinin burulması
κ_β	β eğrisinin eğriliği
τ_β	β eğrisinin burulması

1. BÖLÜM

GİRİŞ

Eğrilerin küresel izdüşümleri ile ilgili bugüne kadar 2005 yılında Bang-Yen Chen ve Franki Dillen yaptıkları çalışmada konudan bahsedilmiştir.

Bu çalışmada ise, uzay eğrisinin yer vektörü kullanılarak, küre üzerindeki orjin merkezli izdüşüm eğrisinin Frenet vektörleri ve eğrilikleri hesaplanmıştır. Hesaplamalar bilgisayar yardımıyla düzenlenmiş ve en kısa şekliyle ifade edilmiştir. Uzay eğrisinin eğriliklerine göre izdüşüm eğrisinin küre üzerinde düzlemsel eğri veya küre üzerinde bir helis olması ile ilgili çeşitli sonuçlar elde edilmiştir.

1.1 Temel Tanım ve Teoremler

Tanım 1.1.1 V bir reel vektör uzayı olsun. V üzerinde bir ***iç çarpım*** diye aşağıdaki aksiyomları ile tanımlanan bir

$$\langle , \rangle : V \times V \to \mathbb{R}$$

dönüşümüne (reel değerli fonksiyon) denir ve değeri $u, v \in V$ olmak üzere $\langle u, v \rangle$ şeklinde gösterilir.

(i) Simetri aksiyomu:

$$\langle u, v \rangle = \langle v, u \rangle, \forall u, v \in V.$$

(ii)Bilineerlik aksiyomu:

$$\langle cu, v \rangle = c\langle u, v \rangle = \langle u, cv \rangle, \forall c \in \mathbb{R}, \forall u, v \in V,$$
$$\langle u_1 + u_2, v \rangle = \langle u_1, v \rangle + \langle u_2, v \rangle, \forall v \in V,$$
$$\langle u_1, v_1 + v_2 \rangle = \langle u, v_1 \rangle + \langle u, v_2 \rangle, \forall u \in V.$$

Bu aksiyom kısaca

$$\langle au_1 + bu_2, v \rangle = a\langle u_1, v \rangle + b\langle u_2, v \rangle$$

$$\langle u, av_1 + bv_2 \rangle = a\langle u, v_1 \rangle + b\langle u, v_2 \rangle$$

şeklinde de yazılabilir.

(iii) Pozitif tanımlılık aksiyomu:

$$\langle u, u \rangle \geq 0, \forall u \in V,$$

$$\langle u, u \rangle = 0 \Leftrightarrow u = \vec{0}[1].$$

Tanım 1.1.2 V bir kompleks vektör uzayı olsun. V vektör uzayı üzerinde bir ***iç çarpım*** diye aşağıdaki aksiyomları ile tanımlanan bir

$$\langle , \rangle : V \times V \to \mathbb{C}$$

dönüşüm (kompleks değerli fonksiyon)üne denir ve değeri , $u, v \in V$ olmak üzere $\langle u, v \rangle$ şeklinde gösterilir.

(i)' Hermit aksiyomu:

$$\langle u, v \rangle = \overline{\langle v, u \rangle}, \forall u, v \in V,$$

burada $\left(\overline{\ }\right)$ kompleks eşleniği göstermektedir.

(ii)' Bilineerlik aksiyomu:

$$\langle cu,v\rangle = c\langle u,v\rangle, c\in\mathbb{C};$$

$$\langle u,cv\rangle = \overline{c}\langle u,v\rangle,$$

$$\langle u_1+u_2,v\rangle = \langle u_1,v\rangle + \langle u_2,v\rangle,$$

$$\langle u,v_1+v_2\rangle = \langle u,v_1\rangle + \langle u,v_2\rangle.$$

(iii)' Pozitif tanımlılık aksiyomu:

$$\langle u,u\rangle \text{ reeldir ve } \geq 0$$

$$\langle u,u\rangle = 0 \Leftrightarrow u = \vec{0} \ [1].$$

Tanım 1.1.3 $\mathbb{R}^n$ de bir u vektörünün uzunluğu veya boyu $\|u\|$ ile gösterilir ve u ya karşılık gelen AB yönlü doğru parçasının uzunluğu olarak alınır. Eğer $AB = CD$ ise AB ve CD yönlü doğru parçalarının aynı uzunlukta oldukları açıktır. $\|u\| = \sqrt{\langle u,u\rangle}$ olarak tanımlanır ve bu değere u vektörünün ***normu*** da denir $[1]$.

Tanım 1.1.4 Normu 1 olan vektöre bir ***birim vektör*** denir.Eğer $x \neq 0$ ve $y \neq 0$ iken $\langle x,y\rangle = 0$ ise bu iki vektöre birbirlerine ***ortogonaldir***ler denir. Sıfırdan farklı vektörlerin bir S cümlesinde herhangi iki vektör birbirine dikse bu S cümlesine ***ortogonaldir*** denir. S ortogonal iken S deki her bir vektör birer birim vektör ise S ye ***ortonolmal*** denir $[1]$.

Tanım 1.1.5 Eğer

$$A: V \rightarrow W$$

lineer dönüşümü birebir ve üzerine (örten) ise ***lineer izomorfizm*** olarak adlandırılır $[1]$.

Tanım 1.1.6 K cismi üzerindeki vektör uzaylarından biri V olsun. V vektör uzayının elemanlarının bir $\{\alpha_1,\alpha_2,...,\alpha_k\}$ cümlesi için

$$\sum_{i=1}^{k} c_i\alpha_i = 0, (1 \leq i \leq k) \Rightarrow \forall c_i = 0$$

ise bu cümleye ***lineer bağımsız*** aksi halde ***lineer bağımlı***dır denir $[1]$. Diğer bir ifade ile eğer $\alpha_1,\alpha_2,...,\alpha_k$ lineer bağımlı ise

$$\sum_{i=1}^{k} c_i \alpha_i = 0$$

olacak şekilde hepsi birden sıfır olmayan $c_1, c_2, ..., c_k \in K$ vardır. V vektör uzayında sıfır olmayan p tane $x_1, x_2, ..., x_p$ vektörlerini alalım.

$$c_1 x_1 + c_2 x_2 + ... + c_p x_p = 0$$

olacak şekilde hepsi birden sıfır olmayan p tane $c_1, c_2, ..., c_p$ sayısını bulmak imkansız ise,bu vektörlerin p-inci dereceden ***lineer olarak bağımsız*** bir sistem meydana getirdikleri söylenir. Aksi halde,verilen p vektörden ibaret olan sisteme ***lineer olarak bağlı*** denir. Lineer olarak bağlı sistem ile lineer olarak bağımsız sistem ifadelerinin yerine kısaca bazen ***bağlı sistem*** ve ***serbest sistem*** ifadeleri kullanılır [1].

Tanım 1.1.7 Bir vektör uzayının bir S alt cümlesi aşağıdaki iki özelliğe sahipse V vektör uzayının bir ***bazı*** adını alır [1].

B1. S lineer bağımsızdır.

B2. $V = Sp\{S\}$, yani $\alpha \in V$ elemanı S deki sonlu sayıda elemanın bir lineer birleşimidir.

Bu ikinci aksiyoma baz için ***germe aksiyomu*** denir [1].

Tanım 1.1.8 Boş olmayan bir cümle A ve bir K cismi üzerinde bir vektör uzayı V olsun. Aşağıdaki önermeleri doğrulayan bir

$$f : A \times A \to V$$

fonksiyonu varsa A ya V ile birleştirilmiş bir ***afin uzay*** denir:

(A1). $\forall P, Q, R \in A$ için $f(P,Q) + f(Q,R) = f(P,R)$

(A2). $\forall P \in A$ ve $\forall \alpha \in V$ için $f(P,Q) = \alpha$ olacak biçimde bir tek $Q \in A$ noktası vardır [2].

Tanımdaki (A1) önermesinin anlamı açıktır. (A2) önermesinin biraz daha açıklanması gerekirse denilebilir ki "A da bir P noktası ve V de bir α vektörü verildiğinde, $f(P,Q) = \alpha$ olacak biçimde A cümlesinin en az bir Q noktası vardır."Bir başka deyişle, A da bir nokta tespit edildiğinde V vektör uzayının vektörleriyle A cümlesinin geri kalan noktaları arasında birebir bir eşleme gerçekleşmiş olur. $P, Q \in A$

için $f(P,Q)$ vektörü genellikle $f(P,Q)=\overrightarrow{PQ}$ biçiminde gösterilir ve P ye ***başlangıç***, Q ya da ***uç noktası*** denir [2].

Tanım 1.1.9 Bir reel afin uzay A ve A ile birleşen vektör uzayı da V olsun. V de bir iç çarpım işlemi olarak

$$\langle , \rangle : V \times V \to \mathbb{R}$$

$$(x,y) \to \langle x,y \rangle = \sum_{i=1}^{n} x_i y_i$$

Öklid iç çarpım tanımlanırsa bu işlem yardımı ile A da bir uzaklık va açı gibi metrik kavramlar tanımlanabilir. Burada $x=(x_1,...,x_n)$, $y=(y_1,...,y_n)$.Böylece A afin uzayı da yeni bir ad olarak ***Öklid uzayı*** adını alır. Örnek 1.1.1'de verildiği gibi $A=\mathbb{R}^n$ ve $V=\mathbb{R}^n$ olması hali esas alınır ve ayrıca $A=\mathbb{R}^n$ Öklid uzayı, standart Öklid uzayı anlamında diğerlerinden fark etmesi için, E^n ile gösterilir [2].

Örnek 1.1.1 1-Boyutlu Standart Öklid Uzayı.

Reel sayılar ekseni olarak şimdiye kadar duyulan sayı doğrusu ele alınırsa bu doğru , reel sayılar cismi (kendi üzerinde bir boyutlu reel vektör uzayıdır) ile birleştirilmiş $\mathbb{R}^1$ afin uzayıdır. Ayrıca bu vektör uzayında

$$\langle , \rangle : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$$

$$(\vec{x},\vec{y}) \to \langle \vec{x},\vec{y} \rangle = xy$$

şeklinde tanımlandığı için $\mathbb{R}^1$ afin uzayı 1-boyutlu Öklid uzayı olur ve E ile gösterilen bu uzaya ***Öklid doğrusu*** da denir.Burada $\vec{x}=(x), \vec{y}=(y)$ [2] .

Örnek 1.1.2 2-Boyutlu Standart Öklid Uzayı.

Reel düzlem olarak şimdiye kadar duyulan düzlemi ele alalım. Bu düzlem, 2-Boyutlu reel standart vektör uzayı ile birleştirilmiş $\mathbb{R}^2$ afin uzayıdır.Ayrıca bu vektör uzayında Öklid iç çarpımı da

$$\langle , \rangle : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$$

$$(x,y) \to \langle x,y \rangle = \sum_{i=1}^{2} x_i y_i$$

şeklinde tanımlandığından $\mathbb{R}^2$ afin uzayı 2-Boyutlu Öklid uzayı olur ve E^2 ile gösterilen bu uzaya ***Öklid düzlemi*** de denir.Burada $x=(x_1,x_2)$, $y=(y_1,y_2)$ [2].

Örnek 1.1.3 3-Boyutlu Standart Öklid Uzayı.

3-boyutlu standart $\mathbb{R}^3$ ile birleştirilmiş $\mathbb{R}^3$ afin uzayı ele alınsın.Bu $\mathbb{R}^3$ vektör uzayında Öklid iç çarpımı

$$\langle , \rangle : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$$

$$(x, y) \to \langle x, y \rangle = \sum_{i=1}^{3} x_i y_i$$

biçiminde tanımlanır. Burada $x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3)$ .Böylece $\mathbb{R}^3$ afin uzayı 3-boyutlu Öklid uzayı olur ve E^3 ile gösterilir. Bu uzay, şimdiye kadar duyduğunuz 3-boyutlu Öklid uzayının kendisidir [2].

Tanım 1.1.10

$$d : E^n \times E^n \to \mathbb{R}$$

$$(x, y) \to d(x, y) = \left\| \overrightarrow{xy} \right\| = \sqrt{\sum_{i=1}^{n} (y_i - x_i)^2}$$

olarak tanımlanan d fonksiyonuna E^n Öklid uzayında ***uzaklık fonksiyonu*** ve $d(x, y)$ reel sayısına da $x, y \in E^n$ noktaları arasındaki ***uzaklık*** denir[2].

Tanım 1.1.11

$$d : E^n \times E^n \to \mathbb{R}$$

$$(x, y) \to d(x, y) = \left\| \overrightarrow{xy} \right\|$$

biçiminde tanımlanan d fonksiyonuna E^n de ***Öklid metriği*** denir[2].

Tanım 1.1.12

$$x : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}^3$$

$$(\alpha, \beta) \to \alpha \times \beta = \psi(\alpha \wedge \beta)$$

şeklinde tanımlı x iç işlemine ***vektörel çarpım*** işlemi ve $\alpha \times \beta$ vektörüne de α ile β eğrilerinin ***vektörel çarpımı*** denir [2].

Tanım 1.1.13 $I, \mathbb{R}^n$ uzayının bir açık aralığı olmak üzere,

$$\alpha : I \to \mathbb{R}^n$$

biçiminde diferensiyellenebilir bir α dönüşümüne, $\mathbb{R}^n$ uzayı içinde bir ***eğri*** denir [3].

Tanım1.1.14 E^n de bir M eğrisinin (I, α) ve (J, β) iki koordinat komşuluğu verilsin.

$$h = \alpha^{-1} o \beta : J \to I$$

diferensiyellenebilir fonksiyonuna M eğrisinin bir ***parametre değişimi*** denir [2].

Tanım 1.1.15 E^n de bir M eğrisi (I,α) koordinat komşuluğuyla verilsin. $\alpha : I \to E^n$ fonksiyonunun Öklidiyen koordinat fonksiyonları $\alpha_1, \alpha_2, ..., \alpha_n$ olmak üzere

$$(\alpha_1, \alpha_2, ..., \alpha_n), \alpha(t) \in M$$

$$\alpha'(t) = \left(\left.\frac{d\alpha_1}{dt}\right|_t, ..., \left.\frac{d\alpha_n}{dt}\right|_t \right).$$

$(\alpha(t), \alpha'(t)) \in T_{E^n}(t)$ tanjant vektörüne, M eğrisinin $t \in I$ parametre değerine karşılık gelen $\alpha(t)$ noktasında (I,α) koordinat komşuluğuna göre ***hız vektörü*** denir [2].

Tanım 1.1.16 $M \subset E^n$ eğrisi verilsin. M eğrisinin $m \in M$ noktasındaki ***tanjant uzayı*** diye, $m \in M$ noktasında M eğrisinin hız vektörlerini içine alan $T_M(m) = V(m)$ vektör uzayına denir. $m \in M$ seçilmiş bir nokta olmak üzere, E^n uzayının $T_M(m)$ ile birleşen alt afin uzayına da, M eğrisinin $m \in M$ noktasındaki ***teğet doğrusu*** denir [2].

Tanım 1.1.17 $M \subset E^n$ eğrisi (I,α) koordinat komşuluğu ile verilsin.

$$\|\alpha'\| : I \to \mathbb{R}$$
$$t \to \|\alpha'\|(t) = \|\alpha'(t)\|$$

şeklinde tanımlı $\|\alpha'\|$ fonksiyonuna, M eğrisinin (I,α) koordinat komşuluğuna göre ***skalar hız fonksiyonu*** ve $\|\alpha'(t)\|$ reel sayısına da M eğrisinin (I,α) koordinat komşuluğuna göre $\alpha(t)$ noktasındaki ***skalar hızı*** denir [5].

Tanım 1.1.18 M eğrisi (I,α) koordinat komşuluğuyla verilsin.Eğer $\forall s \in I$ için, $\|\alpha'(s)\| = 1$ ise M eğrisi (I,α) ya göre ***birim hızlı eğri***dir denir. Bu durumda, eğrinin $s \in I$ parametresine ***yay-parametresi*** adı verilir [2].

Tanım 1.1.19 M eğrisi (I,α) koordinat komşuluğuyla verilmiş olsun. $a, b \in I$ olmak üzere , a dan b ye M eğrisinin ***yay uzunluğu*** diye, eğrinin $\alpha(a)$ ve $\alpha(b)$ noktaları arasındaki uzunluğuna karşılık gelen

$$\int_a^b \|\alpha'(t)\| \, dt \ , t \in I$$

reel sayısına denir. Kolayca görülebilir ki bu değer koordinat komşuluğundan bağımsızdır.

Tanım 1.1.20 Her noktasında hız vektörü sıfırdan farklı olan eğriye ***regüler eğri*** denir [2].

Tanım 1.1.21 $M \subset E^n$ eğrisi (I,α) koordinat komşuluğu ile verilsin.Bu durumda, $\psi = \left(\alpha', \alpha'', ..., \alpha^{(r)}\right)$ sistemi lineer bağımsız ve $\forall \alpha^{(k)}, k > r$ için $\alpha^{(k)} \in S_p\{\psi\}$ olmak üzere , ψ den elde edilen $\{V_1, ..., V_r\}$ ortonormal sistemine, M eğrisinin ***Serret-Frenet r-ayaklı alanı*** ve $m \in M$ için $\{V_1(m), ..., V_r(m)\}$ ye ise $m \in M$ noktasındaki ***Serret-Frenet r-ayaklısı*** denir.Her bir $V_i, 1 \le i \le r$ ye ***Serret-Frenet vektörü*** adı verilir.

$n = 3$ özel halinde, E^3 3- boyutlu Öklid uzayında Frenet iki ayaklısı ve Frenet 3-ayaklısı elde edilir. Bu özel halde Frenet 3-ayaklısının teşkili,vektörel çarpım(dış çarpım) ile basitleştirilebilir. Bu özel halde; M eğrisi (I,α) koordinat komşuluğu ile verilmiş ise $s \in I$ yay parametresi olmak üzere,

$$T = \alpha'$$

ve

$$N = \frac{1}{\|\alpha''\|}\alpha''$$

olsun.Bu durumda, $T(s) = V_1(s)$ ve $N(s) = V_2(s)$.Gerçekten, $s \in I$ yay parametresi olduğu için,

$$\|\alpha'(s)\| = 1$$

$$\Rightarrow \langle \alpha'(s), \alpha'(s) \rangle = 1$$

$$\Rightarrow \langle \alpha'(s), \alpha''(s) \rangle = 0$$

$$\Rightarrow V_2(s) \in S_p\{\alpha''(s)\} = S_p\{N(s)\}.$$

Bu ise yukarıda $V_1(s)$ ve $V_2(s)$ için verilen eşitlikleri doğrular.Buna göre

$$B = T \times N$$

tanımlanırsa, $B(s) = V_3(s)$ olduğu görülür.Böylece,

$$\{T(s), N(s), B(s)\}$$

sistemine, $\alpha(s)$ noktasında, M eğrisinin ***Frenet 3-ayaklısı*** denir [2].Burada $T(s)=V_1(s)$ eğrinin ***teğet vektör***ü, $N(s)=V_2(s)$ eğrinin ***asli normal vektör***ü, $B(s)=V_3(s)$ eğrinin ***binormal vektör***ü adını alır [6].

Tanım 1.1.22 $s\in I$ için, $\{T(s),N(s)\}$ kümesinin gerdiği düzleme, $\alpha(s)$ noktasındaki ***dokunum düzlemi*** veya ***oskülatör düzlem*** denir. $\{T(s),B(s)\}$ kümesinin gerdiği düzleme, $\alpha(s)$ noktasındaki ***doğrultma düzlemi*** veya ***rektifyen düzlem*** denir. $\{N(s),B(s)\}$ kümesinin gerdiği düzleme, $\alpha(s)$ noktasındaki ***dik düzlem*** veya ***normal düzlem*** denir.Frenet formüllerindeki katsayılar matrisi olan

$$\begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix}$$

matrisi ters simetrik bir matristir [7].

Tanım 1.1.23 $M\subset E^n$ eğrisi (I,α) koordinat komşuluğuyla verilsin. $s\in I$ ya karşılık gelen $\alpha(s)$ noktasındaki Frenet r-ayaklısı

$$\{V_1(s),...,V_r(s)\}$$

olsun.Buna göre,

$$k_i : I\to\mathbb{R}, 1\le i< r,$$
$$s\to k_i(s)=\left\langle V_i'(s),V_{i+1}(s)\right\rangle$$

biçiminde tanımlı k_i fonksiyonuna M eğrisinin ***i-inci eğrilik fonksiyonu*** ve $s\in I$ için $k_i(s)$ reel sayısına da $\alpha(s)$ noktasında M eğrisinin ***i-inci eğriliği*** denir [2].

Tanım 1.1.24 $n=3$ için E^3 de bir eğrinin iki tane eğriliğinden bahsedilebilir.Bunlar k_1 ve k_2 olmak üzere k_1 eğrinin **eğriliği**, k_2 de eğrinin **burulması**dır. k_1 eğriliği eğrinin teğetten ne kadar saptığının ölçüsüdür. k_2 eğriliği de eğrinin oskülatör düzlemden sapmasının ölçüsüdür [3].

Ayrıca $\{V_1(s),V_2(s),...,V_r(s)\}$ Frenet r-ayaklısının $V_i(s)$ Frenet vektörlerinin eğri boyunca kovaryant türevleri ile ilgili eşitlikler

$$\begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ . \\ . \\ . \\ V_{r-2}' \\ V_{r-1}' \\ V_r' \end{bmatrix} = \begin{bmatrix} 0 & k_1 & 0 & 0 & . & . & . & 0 & 0 & 0 \\ -k_1 & 0 & k_2 & 0 & . & . & . & 0 & 0 & 0 \\ 0 & -k_2 & 0 & 0 & . & . & . & 0 & 0 & 0 \\ . & . & . & 0 & . & . & . & . & . & . \\ . & . & . & 0 & . & . & . & . & . & . \\ . & . & . & 0 & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & . & . & . & 0 & k_{r-2} & 0 \\ 0 & 0 & 0 & 0 & . & . & . & -k_{r-2} & 0 & k_{r-1} \\ 0 & 0 & 0 & 0 & . & . & . & 0 & -k_{r-1} & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ . \\ . \\ . \\ V_{r-2} \\ V_{r-1} \\ V_r \end{bmatrix}$$

şeklinde yazılabilir.

Buna göre

1) $V_1'(s) = k_1(s)V_2(s)$

2) $V_i'(s) = -k_{i-1}(s)V_{i-1}(s) + k_i(s)V_{i+1}(s), 1 < i < r,$

3) $V_r'(s) = -k_{r-1}(s)V_{r-1}(s)$

eşitlikleri elde edilir. Bu eşitliklere ***Frenet formülleri*** denir. $r = 3$ halinde yukarıdaki matris eşitliği

$$\begin{bmatrix} V_1' \\ V_2' \\ V_3' \end{bmatrix} = \begin{bmatrix} 0 & k_1 & 0 \\ -k_1 & 0 & k_2 \\ 0 & -k_2 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \text{ veya } \begin{bmatrix} T' \\ N' \\ B' \end{bmatrix} = \begin{bmatrix} 0 & k_1 & 0 \\ -k_1 & 0 & k_2 \\ 0 & -k_2 & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$

şeklinde olur [2].

Teorem 1.1.1 Eğrilik fonksiyonları;

$$k_1(s) = \frac{\left\| \alpha'(s) \wedge \alpha''(s) \right\|}{\left\| \alpha'(s) \right\|^3}$$

$$k_2(s) = \frac{\det\left(\alpha'(s), \alpha''(s), \alpha'''(s)\right)}{\left\| \alpha'(s) \wedge \alpha''(s) \right\|^2} \text{ [4].}$$

Bu çalışmamız boyunca $k_1 = \kappa$, $k_2 = \tau$ alınacaktır.

Tanım1.1.25 $\alpha : I \to \mathbb{R}^3$ eğrisinin eğrilik fonksiyonu κ olmak üzere, $\frac{1}{\kappa}$ fonksiyonuna eğrinin ***eğrilik yarıçapı fonksiyonu*** denir [8].

Tanım 1.1.26 $M \subset E^n$ eğrisi (I,α) koordinat komşuluğu ile verilsin. $\forall s \in I$ parametresine karşılık gelen $\alpha(s) \in M$ noktasında M eğrisinin 1. ve 2. eğrilikleri $k_1(s)$ ve $k_2(s)$ ise

$$H : I \to \mathbb{R}$$
$$s \to H(s) = \frac{k_1(s)}{k_2(s)}$$

şeklinde tanımlı H fonksiyonuna, M eğrisinin s noktasındaki ***1-inci harmonik eğriliği*** denir. Eğer $\frac{k_1}{k_2} = sabit$ ise $\alpha(s)$ eğrisi helistir [4].

Tanım 1.1.27

$$\alpha : I \subset \mathbb{R} \to E^3$$
$$t \to \alpha(t) = (r\cos t, r\sin t, ht)$$

ile verilen bir α eğrisi için $r > 0$. α eğrisinde $h > 0$ ise ***sağ dairesel helis*** ve $h < 0$ ise ***sol dairesel helis*** denir. Dairesel sözü eğrinin (x, y) düzlemi üzerindeki dik izdüşümünün bir çember olmasından ileri gelmektedir [4].

2.EĞRİLERİN KÜRESEL İZDÜŞÜMLERİ

Bu bölümde yay parametreli bir $\alpha(s)$ eğrisinin yer vektörünü kullanarak, birim küre üzerine izdüşüm eğrisinin Frenet elemanlarını genel halde elde ettik. Daha sonra mümkün olan tüm durumları alt bölümler halinde inceledik.

$f,g,h\in C^{\infty}\left(E^3,\mathbb{R}\right)$ ve $s\in I\subset\mathbb{R}$ yay parametresi olmak üzere $\alpha(s)$ eğrisinin yer vektörü

$$\alpha(s)=f(s)T_\alpha(s)+g(s)N_\alpha(s)+h(s)B_\alpha(s) \tag{2.1}$$

şeklinde yazılabilir. Bu durumda f,g,h fonksiyonları

$$\begin{aligned} &f'(s)-g(s)\kappa_\alpha(s)=1\\ &g'(s)+f(s)\kappa_\alpha(s)-h(s)\tau_\alpha(s)=0\\ &h'(s)+g(s)\tau_\alpha(s)=0 \end{aligned} \tag{2.2}$$

denklemlerini sağlar. Burada $\kappa_\alpha(s)$ ve $\tau_\alpha(s)$ $\alpha(s)$ eğrisinin eğriliği ve burulmasıdır. (2.1) eşitliğini kullanarak $\alpha(s)$ eğrisinin küresel izdüşümü olan eğriyi

$$\beta(s^*)=\sigma(s)\alpha(s) \tag{2.3}$$

şeklinde tanımlayalım. s^* β eğrisinin yay parametresi olmak üzere

$$\frac{ds^*}{ds}=\rho(s) \tag{2.4}$$

olsun. Burada

$$\rho(s)=\frac{1}{\sigma}\left(\sigma'^2+2\sigma'\sigma^3 f+\sigma^3\right)^{1/2}$$

şeklindedir.

β eğrisinin eğriliğini $\kappa_\beta(s^*)$, burulmasını $\tau_\beta(s^*)$ ve Frenet vektörlerini $T_\beta, N_\beta, B_\beta$ ile gösterelim.

$$u_1=\frac{\sigma' f+\sigma}{\rho} \qquad u_2=\frac{\sigma' g}{\rho} \qquad u_3=\frac{\sigma' h}{\rho}$$

$$v_1=\frac{u_1'-u_2\kappa_\alpha}{\rho} \qquad v_2=\frac{u_2'+u_1\kappa_\alpha-u_3\tau_\alpha}{\rho} \qquad v_3=\frac{u_3'+u_2\tau_\alpha}{\rho}$$

$$n_1=\frac{v_1}{\sqrt{v_1^2+v_2^2+v_3^2}} \qquad n_2=\frac{v_2}{\sqrt{v_1^2+v_2^2+v_3^2}} \qquad n_3=\frac{v_3}{\sqrt{v_1^2+v_2^2+v_3^2}}$$

olmak üzere β eğrisinin Frenet vektörleri

$$T_\beta=u_1T_\alpha+u_2N_\alpha+u_3B_\alpha$$

$$N_\beta=n_1T_\alpha+n_2N_\alpha+n_3B_\alpha$$

$$B_\beta=\left(u_2n_3-u_3n_2\right)T_\alpha+\left(u_3n_1-u_1n_3\right)N_\alpha+\left(u_1n_2-u_2n_1\right)B_\alpha$$

şeklinde elde edilir. β eğrisinin kendi teğeti üzerinde bileşeni olmayacağından $\sigma(s)$ ile $f(s)$ arasında

$$\sigma'(s)+\sigma^3(s)f(s)=0$$

elde edilir. Dolayısıyla

$$\sigma(s)=\frac{1}{\sqrt{2\int fds+c_0}}$$

ve buradan da

$$\|\alpha\|=2\int fds+c_0$$

bulunur.Bu ise bizi şu sonuca götürür.

Sonuç 2.1: Bir $\alpha(s)$ normal eğrisinin küre üzerine izdüşüm eğrisi varsa $\alpha(s)$ eğrisinin kendisi küre üzerindedir.

Bu çalışmada, $f\neq 0$ olan eğrileri inceleyeceğiz.

Diğer taraftan

$$c=\frac{1}{\sigma^2}$$

ve

$$B=\sigma^2\left(1-\sigma^2 f\right)$$

olmak üzere β eğrisinin eğriliği, $f(s)$ in bir polinomu olarak

$$\kappa_\beta=\frac{1}{B^{9/4}c^{9/2}}\left(\sum_{i=0}^{10}r_i f^i\right)^{1/2} \qquad \textbf{(2.5)}$$

olup, burada

$$r_0=c^6B^{5/2}\left(\left(c^2+h^2g^2+g^4-2cg^2\right)\kappa_\alpha{}^2+\left(2g^3+2h^2g-2cg\right)\kappa_\alpha+h^2+g^2\right)$$

$$r_1=r_3=r_5=r_7=r_9=0$$

$$r_2 = c^4 \begin{pmatrix} \begin{pmatrix} 2h^2g^2B^{3/2} - 4cg^2B^{5/2} - 2cg^2B^{3/2} + 2g^4B^{3/2} + 4c^2g^2B^{7/2} \\ +g^2B^{3/2} + c^2g^2B^{5/2} + 2cg^2B^2 - 4c^2g^2B^3 \end{pmatrix} {\kappa_\alpha}^2 \\ + \begin{pmatrix} 6h^2gB^{3/2} - 6cg^3B^{5/2} - 6ch^2gB^{5/2} + 12c^2gB^{7/2} + 4cgB^2 \\ -14cgB^{5/2} - 4cgB^{3/2} + 6g^3B^{3/2} + 6c^2gB^{5/2} + 4gB^{3/2} - 6c^2gB^3 \end{pmatrix} \kappa_\alpha \\ +4g^2B^{3/2} - 6cg^2B^{5/2} - 6ch^2B^{5/2} + 4h^2B^{3/2} - 12cB^{5/2} + 9c^2B^{7/2} + 4B^{3/2} \end{pmatrix}$$

$$r_4 = c^2 \begin{pmatrix} \left(g^4\sqrt{B} + 4c^2g^2B^{5/2} - 2cg^2B^{3/2} + h^2g^2\sqrt{B} - 2c^2g^2B^2 \right) {\kappa_\alpha}^2 \\ + \begin{pmatrix} 4g^3\sqrt{B} + 4h^2g\sqrt{B} - 8c^2gB^2 - 10ch^2gB^{3/2} + 6c^3gB^3 \\ +28c^2gB^{5/2} - 12c^3gB^{7/2} + 4c^2gB^{3/2} - 10cg^3B^{3/2} - 12cgB^{3/2} \end{pmatrix} \kappa_\alpha \\ +4g^2\sqrt{B} - 16cB^{3/2} - 16cg^2B^{3/2} - 16ch^2B^{3/2} + 36c^2B^{5/2} \\ +9c^2h^2B^{5/2} + 9c^2g^2B^{5/2} - 18c^3B^{7/2} + 4h^2\sqrt{B} \end{pmatrix}$$

$$\begin{aligned} r_6 &= g^2{\kappa_\alpha}^2B^{3/2}c^2 + c\left(12cgB^{3/2} + 4c^2gB^2 - 4h^2g\sqrt{B} - 14c^2gB^{5/2} - 4g^3\sqrt{B}\right)\kappa_\alpha \\ &\quad + c\left(24cB^{3/2} + 12cg^2B^{3/2} - 36c^2B^{5/2} + 9c^3B^{7/2} - 8g^2\sqrt{B} + 12ch^2B^{3/2} - 8h^2\sqrt{B}\right) \end{aligned}$$

$$r_8 = 4\sqrt{B}\left(3c^2B^2 - 4cB - Bg\kappa_\alpha c + g^2 + h^2\right)$$

$$r_{10} = 4B^{3/2}$$

şeklinde elde edilir. β eğrisinin burulması da ;

$$t_1 = \frac{n_1' - n_2\kappa_\alpha}{\rho} \quad t_2 = \frac{n_2' + n_1\kappa_\alpha - n_3\tau_\alpha}{\rho} \quad t_3 = \frac{n_3' + n_2\tau_\alpha}{\rho}$$

$$m_1 = u_2n_3 - u_3n_2 \qquad m_2 = u_3n_1 - u_1n_3 \qquad m_3 = u_1n_2 - u_2n_1$$

olmak üzere

$$\tau_\beta = \left\langle N_\beta', B_\beta \right\rangle$$

$N_\beta' = \left\{ \left(n_1' - n_2\kappa_\alpha\right)T_\alpha + \left(n_2' + n_1\kappa_\alpha - n_3\tau_\alpha\right)N_\alpha + \left(n_3' + n_2\tau_\alpha\right)B_\alpha \right\} \frac{1}{\rho}$ şeklindedir. Buradan,

τ_β yine $f(s)$ in polinomu olarak,

$$\tau_\beta = \frac{\sum_{i=0}^{11} p_i f^i}{Bc^{3/2}\sum_{j=0}^{10} r_j f^j} \tag{2.6}$$

şeklinde elde edilir. Burada p_i değerleri aşağıdaki şekildedir.

$p_8 = p_{10} = 0$ olmak üzere

$$p_0 = c^8 B^{5/2}\left(\kappa_\alpha{}^2\tau c - g\tau\kappa_\alpha + \kappa_\alpha{}^3 h^2\tau g - g^2\kappa_\alpha{}^2\tau + h\kappa_\alpha{}'\right)$$

$$p_1 = -hc^6\kappa_\alpha\left(1+\kappa_\alpha g\right)\begin{pmatrix} 2B^{3/2}\kappa_\alpha g + 4gcB^3\kappa_\alpha - 4gcB^{5/2}\kappa_\alpha - 2B^2\kappa_\alpha g \\ +6cB^3 + 4B^{3/2} - 4B^2 - 9cB^{5/2} \end{pmatrix}$$

$$p_2 = c^6\begin{pmatrix} \left(ch^2 g\kappa_\alpha{}^3\tau_\alpha - 3cg\tau_\alpha\kappa_\alpha - ch^2\kappa_\alpha{}^2\tau_\alpha - 2cg^2\kappa_\alpha{}^2\tau_\alpha + 3ch\kappa_\alpha{}'\right)B^3 + \\ \left(-ch^2 g\kappa_\alpha{}^3\tau_\alpha + 2cg^2\tau_\alpha\kappa_\alpha{}^2 + 4cg\tau_\alpha\kappa_\alpha - \kappa_\alpha{}^2\tau_\alpha c^2 - 4ch\kappa_\alpha{}' + ch^2\kappa_\alpha{}^2\tau_\alpha\right)B^{5/2} \\ +\left(-2h\kappa_\alpha{}' + \kappa_\alpha{}^2\tau_\alpha g^2 + h^2\kappa_\alpha{}^2\tau_\alpha + 2\kappa_\alpha\tau_\alpha g\right)B^2 \\ +\left(2h\kappa_\alpha{}' - 2\kappa_\alpha\tau_\alpha g - h^2\kappa_\alpha{}^2\tau_\alpha - \kappa_\alpha{}^2\tau_\alpha g^2\right)B^{3/2} \end{pmatrix}$$

$$p_3 = c^4 h\begin{pmatrix} \begin{pmatrix} 2Bg^2 + 6cB^{3/2}g^2 - 2c^3B^{5/2} + 4cB^{5/2}g^2 - 4c^2B^{5/2}g^2 - 4c^2B^{7/2}g^2 \\ -2\sqrt{B}g^2 - B^{3/2}g^2 - c^2B^2 + 2c^3B^3 - 8cB^2g^2 + c^2B^{3/2} + 8c^2B^3g^2 \end{pmatrix}\kappa_\alpha{}^3 \\ +\begin{pmatrix} 14cB^{5/2}g + 30cB^{3/2}g + 38c^2B^3g + 8Bg - 26c^2B^{5/2}g - 37cB^2g \\ -12c^2B^{7/2}g - 4B^{3/2}g - 8\sqrt{B}g \end{pmatrix}\kappa_\alpha{}^2 \\ +\begin{pmatrix} 8B + 28cB^{3/2} + 12cB^{5/2} - 34cB^2 - 4B^{3/2} - 27c^2B^{5/2} + 33c^2B^3 \\ -9c^2B^{7/2} - 8\sqrt{B} \end{pmatrix}\kappa_\alpha \end{pmatrix}$$

$$p_4 = c^5\begin{pmatrix} \left(B^{3/2}g^2\tau_\alpha - h^2B^2\tau_\alpha - g^2B^2\tau_\alpha + h^2B^{3/2}\tau_\alpha\right)\kappa_\alpha{}^2 \\ +\left(3cB^3g\tau_\alpha - 3cB^{5/2}g\tau_\alpha + 4B^{3/2}g\tau_\alpha - 4B^2g\tau_\alpha\right)\kappa_\alpha \\ +\left(3cB^{5/2}h\kappa_\alpha{}' + 4B^2h\kappa_\alpha{}' - 3cB^3h\kappa_\alpha{}' - 4B^{3/2}h\kappa_\alpha{}'\right) \end{pmatrix}$$

$$p_5 = hc^3 \left(\begin{array}{l} \left(\begin{array}{l} -4cB^{3/2}g^2 + 8cB^2g^2 - 4cB^{5/2}g^2 + 2B^{3/2}g^2 - 4Bg^2 + 3\sqrt{B}g^2 \\ -c^2B^{3/2} + c^2B^2 \end{array} \right) \kappa_\alpha{}^3 \\ + \left(\begin{array}{l} 12c^2B^{7/2}g + 12B^{3/2}g + 12c^2B^{5/2}g - 24Bg + 20\sqrt{B}g - 24c^2B^3g \\ -38cB^{3/2}g - 28cB^{5/2}g + 59cB^2g \end{array} \right) \kappa_\alpha{}^2 \\ + \left(\begin{array}{l} 16B^{3/2} + 28\sqrt{B} - 56cB^{3/2} - 36cB^{5/2} + 18c^2B^{7/2} - 45c^2B^3 - 32B \\ +80cB^2 + 27c^2B^{5/2} \end{array} \right) \kappa_\alpha \end{array} \right)$$

$$p_6 = 2c^4B\left(\sqrt{B} - B\right)\left(h\kappa_\alpha{}' - \kappa_\alpha\tau_\alpha g\right)$$

$$p_7 = c^2h \left(\begin{array}{l} \left(2Bg^2 - \sqrt{B}g^2 - B^{3/2}g^2\right)\kappa_\alpha{}^3 \\ + \left(24Bg - 12B^{3/2}g + 14cB^{3/2}g + 14cB^{5/2}g - 28cB^2g - 16\sqrt{B}g\right)\kappa_\alpha{}^2 \\ + \left(\begin{array}{l} 36cB^{5/2} - 24B^{3/2} - 74cB^2 + 48B - 9c^2B^{5/2} + 44cB^{3/2} \\ -9c^2B^{7/2} + 18c^2B^3 - 36\sqrt{B} \end{array} \right) \kappa_\alpha \end{array} \right)$$

$$p_9 = 4ch\kappa_\alpha\left(\kappa_\alpha\sqrt{B}g + \kappa_\alpha B^{3/2}g - 2\kappa_\alpha Bg - 3B^{3/2}c - 8B + 5\sqrt{B} + 6cB^2 + 4B^{3/2} - 3cB^{5/2}\right)$$

$$p_{11} = 4\kappa_\alpha h\left(2B - \sqrt{B} - B^{3/2}\right)$$

şeklinde elde edilir. α eğrisinin κ_α ve τ_α değerlerine göre β eğrisinin eğriliğini ve burulmasını üç durumda inceleyebiliriz.

2.1 Sabit Olmayan Eğrilik ve Burulmaya Sahip Eğrilerin Küresel Resimleri

Bu durum (2.1) eşitliğindeki f, g, h değerlerine göre iki farklı durumda incelenebilir.

i. **$\alpha(s)$ eğrisinin bir rektifiyan eğri olması durumu**

$\alpha(s)$ eğrisi sabit olmayan eğriliğe ve burulmaya sahip bir rektifiyan eğri ise kendi yer vektörünün, eğrinin asli normali üzerinde bileşeni yoktur. Buna göre (2.1) eşitliği

$$\alpha(s) = f(s)T_\alpha + h(s)B_\alpha(s)$$

şeklinde yazılır ve $\alpha(s)$ rektifiyan eğrisinin Frenet vektörleri

$$T_\beta = u_1 T_\alpha + u_3 B_\alpha$$

$$N_\beta = n_1 T_\alpha + n_2 N_\alpha + n_3 B_\alpha$$

$$B_\beta = (u_2 n_3 - u_3 n_2) T_\alpha + (u_3 n_1 - u_1 n_3) N_\alpha + (u_1 n_2 - u_2 n_1) B_\alpha$$

olup burada $u_1, u_2, u_3, n_1, n_2, n_3, v_1, v_2, v_3$ aşağıdaki gibidir.

$$u_1 = \frac{\sigma' f + \sigma}{\rho} \qquad u_2 = 0 \qquad u_3 = \frac{\sigma' h}{\rho}$$

$$n_1 = \frac{v_1}{\sqrt{v_1^2 + v_2^2 + v_3^2}} \qquad n_2 = \frac{v_2}{\sqrt{v_1^2 + v_2^2 + v_3^2}} \qquad n_3 = \frac{v_3}{\sqrt{v_1^2 + v_2^2 + v_3^2}}$$

$$v_1 = \frac{u_1'}{\rho} \qquad v_2 = \frac{u_1 \kappa_\alpha - u_3 \tau_\alpha}{\rho} \qquad v_3 = \frac{u_3'}{\rho}$$

β eğrisinin eğriliği, $f(s)$ in bir polinomu olarak

$$\kappa_\beta = \frac{1}{B^2 c^{9/2}} \left(\sum_{i=0}^{10} r_i f^i \right)^{1/2} \tag{2.7}$$

şeklinde olup, burada $r_5 = r_7 = r_9 = 0$ ve diğerleri

$$r_0 = B^2 c^6 \left(c^2 \kappa_\alpha^{\ 2} + h^2 \right)$$

$$r_1 = 2\kappa_\alpha h \tau_\alpha B^2 c^7$$

$$r_2 = Bc^4 \left(9c^2 B^2 - 12cB + c^2 B h^2 \tau_\alpha^{\ 2} + 4 + 4h^2 - 2c^3 B \kappa^2 - 6cBh^2 \right)$$

$$r_3 = -2\kappa h \tau_\alpha B^2 c^6$$

$$r_4 = c^2 \left(\kappa^2 B^2 c^4 + 9h^2 B^2 c^2 - 16h^2 Bc - 16Bc + 4h^2 - 18B^3 c^3 + 36B^2 c^2 \right)$$

$$r_6 = c\left(12h^2 Bc - 36B^2 c^2 + 24Bc - 8h^2 + 9B^3 c^3 \right)$$

$$r_8 = 4\left(h^2 + 3B^2 c^2 - 4Bc \right)$$

$r_{10} = 4B$

şeklindedir. β eğrisinin burulması da,

$$p_0 = c^7 B^{5/2} \left(\kappa_\alpha{}^2 \tau_\alpha c^2 + ch\kappa_\alpha{}' + 2h^2 \tau_\alpha \right)$$

$$p_1 = -hc^6 \left(c^2 B^{5/2} \kappa_\alpha{}^3 - hcB^{5/2} \tau_\alpha{}' - 6cB^{5/2} \kappa_\alpha + 6cB^3 \kappa_\alpha - 4B^2 \kappa_\alpha + 4B^{3/2} \kappa_\alpha - 2c^2 B^{5/2} \kappa_\alpha \tau_\alpha{}^2 \right)$$

$$p_2 = c^5 \left(\begin{array}{l} c^2 B^{5/2} h^2 \tau_\alpha{}^3 + 3c^2 B^3 h\kappa_\alpha{}' - 2cB^2 h\kappa_\alpha{}' + 2cB^{3/2} h\kappa_\alpha{}' - 3c^3 B^{5/2} \kappa_\alpha{}^2 \tau_\alpha - 2h^2 B^2 \tau_\alpha \\ +3cB^3 h^2 \tau_\alpha + 2h^2 B^{3/2} \tau_\alpha - 5c^2 B^{5/2} h\kappa_\alpha{}' - 2c^2 B^{5/2} h^2 \kappa_\alpha{}^2 \tau_\alpha - 5cB^{5/2} h^2 \tau_\alpha \end{array} \right)$$

$$p_3 = c^4 h \left(\begin{array}{l} 8B\kappa_\alpha - h^2 c^2 B^{5/2} \tau_\alpha{}^2 \kappa - 4B^{3/2} \kappa_\alpha - 8\sqrt{B} \kappa_\alpha - 30cB^2 \kappa_\alpha + 27c^2 B^3 \kappa_\alpha \\ -4c^3 B^{5/2} \kappa_\alpha \tau_\alpha{}^2 - 4hc^2 B^{5/2} \tau_\alpha{}' + 2c^3 B^{5/2} \kappa_\alpha{}^3 - 18c^2 B^{5/2} \kappa_\alpha + 12cB^{5/2} \kappa_\alpha \\ -9c^2 B^{7/2} \kappa_\alpha + 24cB^{3/2} \kappa_\alpha + 3hc^2 B^3 \tau_\alpha{}' + 2hcB^{3/2} \tau_\alpha{}' - 2hcB^2 \tau_\alpha{}' \end{array} \right)$$

$$p_4 = c^3 \left(\begin{array}{l} \left(3c^4 B^{5/2} \tau_\alpha + 2c^3 B^{5/2} h^2 \tau_\alpha \right) \kappa_\alpha{}^2 \\ -c^3 B^{5/2} h^2 \tau_\alpha{}^3 - 6c^3 B^3 h\kappa_\alpha{}' + 6c^2 B^2 h\kappa_\alpha{}' - 4h^2 \sqrt{B} \tau_\alpha - 6c^2 hB^{3/2} \kappa_\alpha{}' \\ -4cB^{3/2} h^2 \tau_\alpha + 7c^3 hB^{5/2} \kappa_\alpha{}' + 6c^2 h^2 B^{5/2} \tau_\alpha - 6c^2 h^2 B^3 \tau_\alpha + 10ch^2 B^2 \tau_\alpha \end{array} \right)$$

$$p_5 = hc^3 \left(\begin{array}{l} -B^{5/2} \kappa_\alpha{}^3 c^3 - 4hcB^{3/2} \tau_\alpha{}' + 3hc^2 B^{5/2} \tau_\alpha{}' + 4hcB^2 \tau_\alpha{}' - 3hc^2 B^3 \tau_\alpha{}' \\ -\left(\begin{array}{l} 36c^2 B^3 - 66cB^2 + 36cB^{5/2} - 16B^{3/2} - 18c^2 B^{7/2} \\ +32B - 18c^2 B^{5/2} - 32\sqrt{B} - 2c^3 B^{5/2} \tau_\alpha{}^2 + 48cB^{3/2} \end{array} \right) \kappa_\alpha \end{array} \right)$$

$$p_6 = c^2 \left(\begin{array}{l} 3c^3 B^3 h\kappa_\alpha{}' - c^4 B^{5/2} \kappa_\alpha{}^2 \tau_\alpha - 6c^2 B^2 h\kappa_\alpha{}' - 3c^3 B^{5/2} h\kappa_\alpha{}' + 12h^2 \sqrt{B} \tau_\alpha \\ +3c^2 B^3 h^2 \tau_\alpha - 14cB^2 h^2 \tau_\alpha + 2cB^{5/2} h^2 \tau_\alpha + 6c^2 B^{3/2} h\kappa_\alpha{}' - 3c^2 B^{5/2} h^2 \tau_\alpha \end{array} \right)$$

$$p_7 = c^2 h \left(\begin{array}{l} 15c^2 B^3 \kappa_\alpha - 6c^2 B^{5/2} \kappa_\alpha - 9c^2 B^{7/2} \kappa_\alpha + 2hcB^{3/2} \tau_\alpha{}' - 2hcB^2 \tau_\alpha{}' \\ -58cB^2 \kappa_\alpha + 40cB^{3/2} \kappa_\alpha + 36cB^{5/2} \kappa_\alpha + 48B\kappa_\alpha - 24B^{3/2} \kappa_\alpha - 48\sqrt{B} \kappa_\alpha \end{array} \right)$$

$$p_8 = 2ch\left(c^2B^2\kappa_\alpha{}' + 3chB^2\tau_\alpha - 6h\sqrt{B}\tau_\alpha - c^2B^{3/2}\kappa_\alpha{}'\right)$$

$$p_9 = 2ch\kappa_\alpha\left(9cB^2 - 6B^{3/2}c - 16B + 16\sqrt{B} + 8B^{3/2} - 6cB^{5/2}\right)$$

$$p_{10} = 4h^2\tau_\alpha\sqrt{B}$$

$$p_{11} = 4\kappa_\alpha h\left(2B - 2\sqrt{B} - B^{3/2}\right)$$

olmak üzere, $f(s)$ ye göre düzenlenirse

$$\tau_\beta = \frac{\sum_{i=0}^{11} p_i f^i}{B^{3/2}c^{3/2}\sum_{j=0}^{10} r_j f^j} \qquad \textbf{(2.8)}$$

olarak elde edilir. p_{10} göz önüne alındığında şu sonucu verebiliriz.

Sonuç 2.2: Sabit olmayan eğrilik ve burulmaya sahip bir rektifiyan eğrinin küresel izdüşüm eğrisi, küre üzerinde bir düzlemsel eğri olamaz.

(2.7) ve (2.8) eşitliklerinden $\frac{\tau_\beta}{\kappa_\beta}$ oranının türevi hesaplanırsa,

$$\left(\frac{\tau_\beta}{\kappa_\beta}\right)' = \frac{D}{E} \qquad \textbf{(2.9)}$$

şeklinde elde edilir. Burada *D* ve *E* $h(s)$ e göre düzenlenirse,

$$D = \sum_{i=0}^{21} d_i h_i \qquad \textbf{(2.10)}$$

$$d_0 = A^2\tau_\alpha{}^3 f^{23}\left(\tau_\alpha{}'\kappa_\alpha - \kappa_\alpha{}'\tau_\alpha\right)$$

$$d_1 = 3A^2\tau_\alpha{}^2\kappa_\alpha f^{22}\left(\tau_\alpha{}'\kappa_\alpha - \kappa_\alpha{}'\tau_\alpha\right) - A^2\tau_\alpha{}^3 f^{21}\left(\kappa_\alpha{}^2 + \tau_\alpha{}^2\right)$$

$$d_2 = A^2\tau_\alpha f^{21}\left(10\tau_\alpha{}'\tau_\alpha{}^2\kappa_\alpha - 3\kappa_\alpha{}'\tau_\alpha\kappa_\alpha{}^2 - 10\kappa_\alpha{}'\tau_\alpha{}^3 + 3\kappa_\alpha{}^3\tau_\alpha{}'\right) - 3A^2\tau_\alpha{}^2\kappa_\alpha f^{20}\left(\kappa_\alpha{}^2 + \tau_\alpha{}^2\right)$$

$$\begin{aligned} d_3 &= A^2\kappa_\alpha f^{20}\left(30\tau_\alpha{}'\tau_\alpha{}^2\kappa_\alpha - 30\kappa_\alpha{}'\tau_\alpha{}^3 + \kappa_\alpha{}^3\tau_\alpha{}' - \kappa_\alpha{}'\kappa_\alpha{}^2\tau_\alpha\right) \\ &+ A^2\tau_\alpha f^{19}\left(\tau_\alpha{}''\tau_\alpha - 9\tau_\alpha{}^4 - 12\kappa_\alpha{}^2\tau_\alpha{}^2 - 3\tau_\alpha{}'^2 - 3\kappa_\alpha{}^4\right) \\ &- 10A^2\tau_\alpha{}'\tau_\alpha{}^2 f^{18} + A\tau_\alpha f^{17}\left(\tau_\alpha{}''\tau_\alpha - 15A\tau_\alpha{}^2 - 3\tau_\alpha{}'^2\right) \\ &- 12A\tau_\alpha{}'\tau_\alpha{}^2 f^{16} - 20A\tau_\alpha{}^3 f^{15} \end{aligned}$$

$$\begin{aligned} d_4 &= A^2 f^{19}\left(45\tau_\alpha{}'\tau_\alpha{}^3\kappa_\alpha + 30\tau_\alpha{}'\tau_\alpha\kappa_\alpha{}^3 - 45\tau_\alpha{}^4\kappa_\alpha{}' - 30\kappa_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2\right) \\ &+ A^2 f^{18}\left(\kappa_\alpha{}''\tau_\alpha{}^2 - 28\tau_\alpha{}^2\kappa_\alpha{}^3 - \kappa_\alpha{}^5 + 2\tau_\alpha{}''\tau_\alpha\kappa_\alpha - 3\tau_\alpha{}'^2\kappa_\alpha - 6\kappa_\alpha{}'\tau_\alpha{}'\tau_\alpha - 27\tau_\alpha{}^4\kappa_\alpha\right) \\ &- A^2 f^{17}\left(13\kappa_\alpha{}'\tau_\alpha{}^2 + 16\tau_\alpha{}'\tau_\alpha\kappa_\alpha\right) - Af^{14}\left(46\kappa_\alpha\tau_\alpha{}^2 + 4A\kappa_\alpha\tau_\alpha{}^2\right) \\ &+ Af^{16}\left(\kappa_\alpha{}''\tau_\alpha{}^2 - 42A\tau_\alpha{}^2\kappa_\alpha - 6\kappa_\alpha{}'\tau_\alpha{}'\tau_\alpha + 2\tau_\alpha{}''\tau_\alpha\kappa_\alpha - 3\tau_\alpha{}'^2\kappa_\alpha\right) \\ &+ Af^{15}\left(A\tau_\alpha{}'\kappa_\alpha\tau_\alpha - 2A\kappa_\alpha{}'\tau_\alpha{}^2 - 18\tau_\alpha{}'\kappa_\alpha\tau_\alpha - 14\kappa_\alpha{}'\tau_\alpha{}^2\right) \end{aligned}$$

$$\begin{aligned} d_5 &= A^2 f^{18}\left(10\tau_\alpha{}'\kappa_\alpha{}^4 + 135\tau_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2 - 135\tau_\alpha{}^3\kappa_\alpha{}'\kappa_\alpha - 10\kappa_\alpha{}'\tau_\alpha\kappa_\alpha{}^3\right) \\ &+ A^2 f^{17}\left(\begin{aligned} &-36\tau_\alpha{}^5 + 2\kappa_\alpha{}''\tau_\alpha\kappa_\alpha + 8\tau_\alpha{}''\tau_\alpha{}^2 - 63\tau_\alpha{}^3\kappa_\alpha{}^2 \\ &-6\kappa_\alpha{}'\tau_\alpha{}'\kappa_\alpha - 27\tau_\alpha\kappa_\alpha{}^4 - 3\kappa_\alpha{}'^2\tau_\alpha - 24\tau_\alpha{}'^2\tau_\alpha + \tau_\alpha{}''\kappa_\alpha{}^2 \end{aligned}\right) \\ &- A^2 f^{16}\left(22\kappa_\alpha{}'\tau_\alpha\kappa_\alpha + 6\tau_\alpha{}'\kappa_\alpha{}^2 + 73\tau_\alpha{}'\tau_\alpha{}^2\right) \\ &+ Af^{15}\left(\begin{aligned} &2\kappa_\alpha{}''\kappa_\alpha\tau_\alpha - 6\kappa_\alpha{}'\tau_\alpha{}'\kappa_\alpha - 40A\tau_\alpha\kappa_\alpha{}^2 + \tau_\alpha{}''\kappa_\alpha{}^2 - 99A\tau_\alpha{}^3 \\ &-18\tau_\alpha{}'^2\tau_\alpha + 6\tau_\alpha{}''\tau_\alpha{}^2 - 3\kappa_\alpha{}'\tau_\alpha \end{aligned}\right) \\ &+ Af^{14}\left(-22\kappa_\alpha{}'\tau_\alpha\kappa_\alpha + 3A\tau_\alpha{}'\tau_\alpha{}^2 - 3A\kappa_\alpha{}'\kappa_\alpha\tau_\alpha + A\tau_\alpha{}'\kappa_\alpha{}^2 - 62\tau_\alpha{}'\tau_\alpha{}^2 - 6\tau_\alpha{}'\kappa_\alpha{}^2\right) \\ &+ Af^{13}\left(11A\tau_\alpha{}^3 - 35\kappa_\alpha{}^2\tau_\alpha - 9A\kappa_\alpha{}^2\tau_\alpha - 83\tau_\alpha{}^3\right) \end{aligned}$$

$$d_6 = A^2 f^{17}\left(-120\tau_\alpha{}^4\kappa_\alpha{}' + 135\tau_\alpha{}'\tau_\alpha\kappa_\alpha{}^3 - 135\kappa_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2 + 120\tau_\alpha{}'\tau_\alpha{}^3\kappa_\alpha\right)$$

$$+ A^2 f^{16}\begin{pmatrix} \kappa_\alpha{}''\kappa_\alpha{}^2 - 24\tau_\alpha{}'^2\kappa_\alpha - 9\kappa_\alpha{}^5 - 3\kappa_\alpha{}'^2\kappa_\alpha + 16\tau_\alpha{}''\tau_\alpha\kappa_\alpha \\ +8\kappa_\alpha{}''\tau_\alpha{}^2 - 108\tau_\alpha{}^4\kappa_\alpha - 48\kappa_\alpha{}'\tau_\alpha{}'\tau_\alpha - 117\tau_\alpha{}^2\kappa_\alpha{}^3 \end{pmatrix}$$

$$- A^2 f^{15}\left(111\tau_\alpha{}'\tau_\alpha\kappa_\alpha + 9\kappa_\alpha{}'\kappa_\alpha{}^2 + 98\kappa_\alpha{}'\tau_\alpha{}^2\right)$$

$$+ Af^{14}\begin{pmatrix} -3\kappa_\alpha{}'\kappa_\alpha + 12\tau_\alpha{}''\kappa_\alpha\tau_\alpha - 36\kappa_\alpha{}'\tau_\alpha{}'\tau_\alpha - 13A\kappa_\alpha{}^3 - 18\tau_\alpha{}'^2\kappa_\alpha \\ +\kappa_\alpha{}''\kappa_\alpha{}^2 - 264A\tau_\alpha{}^2\kappa_\alpha + 6\kappa_\alpha{}''\tau_\alpha{}^2 \end{pmatrix}$$

$$+ Af^{13}\left(-8\kappa_\alpha{}'\kappa_\alpha{}^2 - 82\tau_\alpha{}'\tau_\alpha\kappa_\alpha - A\kappa_\alpha{}'\kappa_\alpha{}^2 - 8A\kappa_\alpha{}'\tau_\alpha{}^2 - 76\kappa_\alpha{}'\tau_\alpha{}^2 + 9A\tau_\alpha{}'\kappa_\alpha\tau_\alpha\right)$$

$$+ Af^{12}\left(4A\tau_\alpha{}^2\kappa_\alpha - 9\kappa_\alpha{}^3 - 5A\kappa_\alpha{}^3 - 159\kappa_\alpha\tau_\alpha{}^2\right)$$

$$d_7 = A^2 f^{16}\left(45\tau_\alpha{}'\kappa_\alpha{}^4 - 360\tau_\alpha{}^3\kappa_\alpha{}'\kappa_\alpha + 360\tau_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2 - 45\kappa_\alpha{}'\tau_\alpha\kappa_\alpha{}^3\right)$$

$$+ A^2 f^{15}\begin{pmatrix} -84\tau_\alpha{}^5 + 28\tau_\alpha{}''\tau_\alpha{}^2 - 192\tau_\alpha{}^3\kappa_\alpha{}^2 - 48\kappa_\alpha{}'\tau_\alpha{}'\kappa_\alpha \\ -108\tau_\alpha\kappa_\alpha{}^4 + 8\tau_\alpha{}''\kappa_\alpha{}^2 + 16\kappa_\alpha{}''\tau_\alpha\kappa_\alpha - 24\kappa_\alpha{}'^2\tau_\alpha - 84\tau_\alpha{}'^2\tau_\alpha \end{pmatrix}$$

$$- A^2 f^{14}\left(161\kappa_\alpha{}'\tau_\alpha\kappa_\alpha + 38\tau_\alpha{}'\kappa_\alpha{}^2 + 231\tau_\alpha{}'\tau_\alpha{}^2\right)$$

$$+ Af^{13}\begin{pmatrix} -283A\tau_\alpha{}^3 - 242A\tau_\alpha\kappa_\alpha{}^2 + 15\tau_\alpha{}''\tau_\alpha{}^2 - 18\kappa_\alpha{}'^2\tau_\alpha + A\tau_\alpha{}'' \\ +6\tau_\alpha{}''\kappa_\alpha{}^2 - 45\tau_\alpha{}'^2\tau_\alpha + 12\kappa_\alpha{}''\kappa_\alpha\tau_\alpha - 36\kappa_\alpha{}'\tau_\alpha{}'\kappa_\alpha \end{pmatrix}$$

$$+ Af^{12}\begin{pmatrix} 15A\tau_\alpha{}'\tau_\alpha{}^2 + 6A\tau_\alpha{}'\kappa_\alpha{}^2 - 130\tau_\alpha{}'\tau_\alpha{}^2 + 8A\tau_\alpha{}' - 20\tau_\alpha{}'\kappa_\alpha{}^2 \\ -110\kappa_\alpha{}'\tau_\alpha\kappa_\alpha - 13A\kappa_\alpha{}'\kappa_\alpha\tau_\alpha \end{pmatrix}$$

$$+ Af^{11}\left(-28A\kappa_\alpha{}^2\tau_\alpha + \tau_\alpha{}'' + 43A\tau_\alpha{}^3 + A\tau_\alpha{}'' - 93\kappa_\alpha{}^2\tau_\alpha + 12A\tau_\alpha - 133\tau_\alpha{}^3\right)$$

$$+ Af^{10}\left(9\tau_\alpha{}' + 11A\tau_\alpha{}'\right) + Af^9\left(21A\tau_\alpha + 16\tau_\alpha + \tau_\alpha{}''\right) + 12A\tau_\alpha{}'f^8 + 28A\tau_\alpha f^7$$

$$
\begin{aligned}
d_8 = {} & A^2 f^{15}\left(-360\kappa_\alpha'\tau_\alpha^{\,2}\kappa_\alpha^{\,2} + 210\tau_\alpha'\tau_\alpha^{\,3}\kappa_\alpha + 360\tau_\alpha'\tau_\alpha\kappa_\alpha^{\,3} - 210\tau_\alpha^{\,4}\kappa_\alpha'\right) \\
& + A^2 f^{14}\left(\begin{array}{l} -84\tau_\alpha'^2\kappa_\alpha - 36\kappa_\alpha^{\,5} - 168\kappa_\alpha'\tau_\alpha'\tau_\alpha + 56\tau_\alpha''\tau_\alpha\kappa_\alpha - 288\tau_\alpha^{\,2}\kappa_\alpha^{\,3} - 24\kappa_\alpha'^2\kappa_\alpha \\ +8\kappa_\alpha''\kappa_\alpha^{\,2} + 28\kappa_\alpha''\tau_\alpha^{\,2} - 252\tau_\alpha^{\,4}\kappa_\alpha \end{array}\right) \\
& - A^2 f^{13}\left(329\tau_\alpha'\tau_\alpha\kappa_\alpha + 63\kappa_\alpha'\kappa_\alpha^{\,2} + 322\kappa_\alpha'\tau_\alpha^{\,2}\right) + A\kappa_\alpha' f^9\left(8A+7\right) \\
& + Af^{12}\left(\begin{array}{l} -45\tau_\alpha'^2\kappa_\alpha + 6\kappa_\alpha''\kappa_\alpha^{\,2} + 15\kappa_\alpha''\tau_\alpha^{\,2} - 706A\tau_\alpha^{\,2}\kappa_\alpha + A\kappa_\alpha'' \\ +30\tau_\alpha''\kappa_\alpha\tau_\alpha - 77A\kappa_\alpha^{\,3} - 18\kappa_\alpha'\kappa_\alpha - 90\kappa_\alpha'\tau_\alpha'\tau_\alpha \end{array}\right) \\
& + Af^{11}\left(\begin{array}{l} -34\kappa_\alpha'\kappa_\alpha^{\,2} - 170A\kappa_\alpha'\tau_\alpha^{\,2} + 6A\kappa_\alpha' - 140\tau_\alpha'\tau_\alpha\kappa_\alpha \\ -10A\kappa_\alpha'\tau_\alpha^{\,2} - 5A\kappa_\alpha'\kappa_\alpha^{\,2} + 30A\tau_\alpha'\kappa_\alpha\tau_\alpha \end{array}\right) \\
& + Af^{10}\left(-21A\kappa_\alpha^{\,3} + \kappa_\alpha'' + A\kappa_\alpha'' - 17\kappa_\alpha^{\,3} + 52A\tau_\alpha^{\,2}\kappa_\alpha - 171\kappa_\alpha\tau_\alpha^{\,2} + 6A\kappa_\alpha\right) \\
& + Af^{8}\left(11A\kappa_\alpha + 9\kappa_\alpha + \kappa_\alpha''\right) + A\kappa_\alpha' f^7\left(10-A\right) + A\kappa_\alpha f^6\left(18-4A\right)
\end{aligned}
$$

$$
\begin{aligned}
d_9 = {} & A^2 f^{14}\left(120\tau_\alpha'\kappa_\alpha^{\,4} - 630\tau_\alpha^{\,3}\kappa_\alpha'\kappa_\alpha + 630\tau_\alpha'\tau_\alpha^{\,2}\kappa_\alpha^{\,2} - 120\kappa_\alpha'\tau_\alpha\kappa_\alpha^{\,3}\right) \\
& + A^2 f^{13}\left(\begin{array}{l} -126\tau_\alpha^{\,5} - 84\kappa_\alpha'^2\tau_\alpha - 168\kappa_\alpha'\tau_\alpha'\kappa_\alpha + 56\tau_\alpha''\tau_\alpha^{\,2} \\ -252\tau_\alpha\kappa_\alpha^{\,4} + 56\kappa_\alpha''\tau_\alpha\kappa_\alpha - 378\tau_\alpha^{\,3}\kappa_\alpha^{\,2} + 28\tau_\alpha''\kappa_\alpha^{\,2} - 168\tau_\alpha'^2\tau_\alpha \end{array}\right) \\
& - A^2 f^{12}\left(511\kappa_\alpha'\tau_\alpha\kappa_\alpha + 98\tau_\alpha'\kappa_\alpha^{\,2} + 413\tau_\alpha'\tau_\alpha^{\,2}\right) \\
& + Af^{11}\left(\begin{array}{l} -45\kappa_\alpha'^2\tau_\alpha + 15\tau_\alpha''\kappa_\alpha^{\,2} + 6A\tau_\alpha'' + 20\tau_\alpha''\tau_\alpha^{\,2} - 60\tau_\alpha'^2\tau_\alpha \\ -459A\tau_\alpha^{\,3} + 30\kappa_\alpha''\kappa_\alpha\tau_\alpha - 612A\tau_\alpha\kappa_\alpha^{\,2} - 90\kappa_\alpha'\tau_\alpha'\kappa_\alpha \end{array}\right) \\
& + Af^{10}\left(\begin{array}{l} -10\tau_\alpha'\kappa_\alpha^{\,2} + 43A\tau_\alpha' - 20A\kappa_\alpha'\kappa_\alpha\tau_\alpha + 15A\tau_\alpha'\kappa_\alpha^{\,2} \\ -140\tau_\alpha'\tau_\alpha^{\,2} + 30A\tau_\alpha'\tau_\alpha^{\,2} - 220\kappa_\alpha'\tau_\alpha\kappa_\alpha \end{array}\right) \\
& + Af^{9}\left(4A\tau_\alpha'' + 62A\tau_\alpha^{\,3} + 4\tau_\alpha'' + 58A\tau_\alpha - 25A\kappa_\alpha^{\,2}\tau_\alpha - 18\kappa_\alpha^{\,2}\tau_\alpha - 102\tau_\alpha^{\,3}\right) \\
& + Af^{8}\left(31\tau_\alpha' + 33A\tau_\alpha'\right) \\
& + Af^{7}\left(49A\tau_\alpha + 50\tau_\alpha + 2\tau_\alpha''\right) + 13A\tau_\alpha' f^6 + 9A\tau_\alpha f^5
\end{aligned}
$$

$$d_{10} = A^2 f^{13}\left(630\tau_\alpha{}'\tau_\alpha\kappa_\alpha{}^3 - 630\kappa_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2 + 252\tau_\alpha{}'\tau_\alpha{}^3\kappa_\alpha - 252\tau_\alpha{}^4\kappa_\alpha{}'\right)$$

$$+ A^2 f^{12}\begin{pmatrix} 112\tau_\alpha{}''\tau_\alpha\kappa_\alpha - 84\kappa_\alpha{}^5 + 56\kappa_\alpha{}''\tau_\alpha{}^2 - 84\kappa_\alpha{}'^2\kappa_\alpha - 168\tau_\alpha{}'^2\kappa_\alpha \\ -462\tau_\alpha{}^2\kappa_\alpha{}^3 - 378\tau_\alpha{}^4\kappa_\alpha - 336\kappa_\alpha{}'\tau_\alpha{}'\tau_\alpha + 28\kappa_\alpha{}''\kappa_\alpha{}^2 \end{pmatrix}$$

$$- A^2 f^{11}\left(539\tau_\alpha{}'\tau_\alpha\kappa_\alpha + 189\kappa_\alpha{}'\kappa_\alpha{}^2 + 602\kappa_\alpha{}'\tau_\alpha{}^2\right)$$

$$+ Af^{10}\begin{pmatrix} -120\kappa_\alpha{}'\tau_\alpha{}'\tau_\alpha - 45\kappa_\alpha{}'^2\kappa_\alpha + 40\tau_\alpha{}''\kappa_\alpha\tau_\alpha - 1044A\tau_\alpha{}^2\kappa_\alpha \\ -60\tau_\alpha{}'^2\kappa_\alpha + 15\kappa_\alpha{}''\kappa_\alpha{}^2 + 20\kappa_\alpha{}''\tau_\alpha{}^2 + 6A\kappa_\alpha{}'' - 189A\kappa_\alpha{}^3 \end{pmatrix}$$

$$+ Af^9\left(-50\kappa_\alpha{}'\kappa_\alpha{}^2 - 200\kappa_\alpha{}'\tau_\alpha{}^2 + 30A\kappa_\alpha{}' - 100\tau_\alpha{}'\tau_\alpha\kappa_\alpha + 50A\tau_\alpha{}'\kappa_\alpha\tau_\alpha - 10A\kappa_\alpha{}'\kappa_\alpha{}^2\right)$$

$$+ Af^8\left(88A\tau_\alpha{}^2\kappa_\alpha - 14\kappa_\alpha\tau_\alpha{}^2 + 4A\kappa_\alpha{}'' + 26A\kappa_\alpha + 20\kappa_\alpha{}^3 - 34A\kappa^3 + 4\kappa_\alpha{}''\right)$$

$$+ 21A\kappa_\alpha{}' f^7 (A+1) + Af^6\left(15A\kappa_\alpha + 25\kappa_\alpha + 2\kappa_\alpha{}''\right) + A\kappa_\alpha{}' f^5 (7-A) - 8A\kappa_\alpha f^4$$

$$d_{11} = A^2 f^{12}\left(210\tau_\alpha{}'\kappa_\alpha{}^4 - 756\tau_\alpha{}^3\kappa_\alpha{}'\kappa_\alpha + 756\tau_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2 - 210\kappa_\alpha{}'\tau_\alpha\kappa_\alpha{}^3\right)$$

$$+ A^2 f^{13}\begin{pmatrix} -378\tau_\alpha\kappa_\alpha{}^4 - 210\tau_\alpha{}'^2\tau_\alpha - 126\tau_\alpha{}^5 + 112\kappa_\alpha{}''\tau_\alpha\kappa_\alpha \\ -504\tau_\alpha{}^3\kappa_\alpha{}^2 - 336\kappa_\alpha{}'\tau_\alpha{}'\kappa_\alpha - 168\kappa_\alpha{}'^2\tau_\alpha + 70\tau_\alpha{}''\tau_\alpha{}^2 + 56\tau_\alpha{}''\kappa_\alpha{}^2 \end{pmatrix}$$

$$+ Af^9\begin{pmatrix} -60\kappa_\alpha{}'^2\tau_\alpha + 20\tau_\alpha{}''\kappa_\alpha{}^2 + 15A\tau_\alpha{}'' + 15\tau_\alpha{}''\tau_\alpha{}^2 - 45\tau_\alpha{}'^2\tau_\alpha \\ -465A\tau_\alpha{}^3 + 40\kappa_\alpha{}''\kappa_\alpha\tau_\alpha - 830A\tau_\alpha\kappa_\alpha{}^2 - 120\kappa_\alpha{}'\tau_\alpha{}'\kappa_\alpha \end{pmatrix}$$

$$+ Af^8\begin{pmatrix} 40\tau_\alpha{}'\kappa_\alpha{}^2 + 95A\tau_\alpha{}' - 10A\kappa_\alpha{}'\kappa_\alpha\tau_\alpha + 20A\tau_\alpha{}'\kappa_\alpha{}^2 \\ -80\tau_\alpha{}'\tau_\alpha{}^2 + 30A\tau_\alpha{}'\tau_\alpha{}^2 - 220\kappa_\alpha{}'\tau_\alpha\kappa_\alpha \end{pmatrix} + Af^6\left(39\tau_\alpha{}' + 33A\tau_\alpha{}'\right)$$

$$- A^2 f^{10}\left(917\kappa_\alpha{}'\tau_\alpha\kappa_\alpha + 126\tau_\alpha{}'\kappa_\alpha{}^2 + 455\tau_\alpha{}'\tau_\alpha{}^2\right) + A\tau_\alpha{}' f^4 - A\tau_\alpha f^3$$

$$+ Af^7\left(6A\tau_\alpha{}'' + 38A\tau_\alpha{}^3 + 6\tau_\alpha{}'' + 112A\tau_\alpha + 158\kappa_\alpha{}^2\tau_\alpha - 38\tau_\alpha{}^3\right) + Af^5\left(29A\tau_\alpha + 54\tau_\alpha + \tau_\alpha{}''\right)$$

$$
\begin{aligned}
d_{12} = {} & A^2 f^{11}\left(756\tau_\alpha'\tau_\alpha\kappa_\alpha^3 - 756\kappa_\alpha'\tau_\alpha^2\kappa_\alpha^2 + 210\tau_\alpha'\tau_\alpha^3\kappa_\alpha - 210\tau_\alpha^4\kappa_\alpha'\right) \\
& + A^2 f^{10}\begin{pmatrix} 140\tau_\alpha''\tau_\alpha\kappa_\alpha - 126\kappa_\alpha^5 + 70\kappa_\alpha''\tau_\alpha^2 - 168\kappa_\alpha'^2\kappa_\alpha - 210\tau_\alpha'^2\kappa_\alpha - 504\tau_\alpha^2\kappa_\alpha^3 \\ -378\tau_\alpha^4\kappa_\alpha - 420\kappa_\alpha'\tau_\alpha'\tau_\alpha + 56\kappa_\alpha''\kappa_\alpha^2 \end{pmatrix} \\
& + Af^8\begin{pmatrix} -90\kappa_\alpha'\tau_\alpha'\tau_\alpha - 60\kappa_\alpha'^2\kappa_\alpha + 30\tau_\alpha''\kappa_\alpha\tau_\alpha - 930A\tau_\alpha^2\kappa_\alpha \\ -45\tau_\alpha'^2\kappa_\alpha + 20\kappa_\alpha''\kappa_\alpha^2 + 15\kappa_\alpha''\tau_\alpha^2 + 15A\kappa_\alpha'' - 245A\kappa_\alpha^3 \end{pmatrix} \\
& + Af^7\begin{pmatrix} -20\kappa_\alpha'\kappa_\alpha^2 - 130\kappa_\alpha'\tau_\alpha^2 + 60A\kappa_\alpha' - 10\tau_\alpha'\tau_\alpha\kappa_\alpha \\ +45A\tau_\alpha'\kappa_\alpha\tau_\alpha - 10A\kappa_\alpha'\kappa_\alpha^2 + 10A\kappa_\alpha'\tau_\alpha^2 \end{pmatrix} - A^2 f^9\left(525\tau_\alpha'\tau_\alpha\kappa_\alpha + 315\kappa_\alpha'\kappa_\alpha^2 + 700\kappa_\alpha'\tau_\alpha^2\right) \\
& + Af^6\left(52A\tau_\alpha^2\kappa_\alpha + 84\kappa_\alpha\tau_\alpha^2 + 6A\kappa_\alpha'' + 44A\kappa_\alpha + 70\kappa_\alpha^3 - 26A\kappa^3 + 6\kappa_\alpha''\right) \\
& + 21A\kappa_\alpha' f^5\left(15A + 21\right) + Af^4\left(-3A\kappa_\alpha + 21\kappa_\alpha + \kappa_\alpha''\right) - 3A\kappa_\alpha' f^3 + 4A\kappa_\alpha f^2
\end{aligned}
$$

$$
\begin{aligned}
d_{13} = {} & A^2 f^{10}\left(252\tau_\alpha'\kappa_\alpha^4 - 630\tau_\alpha^3\kappa_\alpha'\kappa_\alpha + 630\tau_\alpha'\tau_\alpha^2\kappa_\alpha^2 - 252\kappa_\alpha'\tau_\alpha\kappa_\alpha^3\right) \\
& + A^2 f^9\begin{pmatrix} -378\tau_\alpha\kappa_\alpha^4 - 168\tau_\alpha'^2\tau_\alpha - 84\tau_\alpha^5 + 140\kappa_\alpha''\tau_\alpha\kappa_\alpha \\ -462\tau_\alpha^3\kappa_\alpha^2 - 420\kappa_\alpha'\tau_\alpha'\kappa_\alpha - 210\kappa_\alpha'^2\tau_\alpha + 56\tau_\alpha''\tau_\alpha^2 + 70\tau_\alpha''\kappa_\alpha^2 \end{pmatrix} \\
& - A^2 f^8\left(1015\kappa_\alpha'\tau_\alpha\kappa_\alpha + 70\tau_\alpha'\kappa_\alpha^2 + 315\tau_\alpha'\tau_\alpha^2\right) \\
& + Af^7\begin{pmatrix} -45\kappa_\alpha'^2\tau_\alpha + 15\tau_\alpha''\kappa_\alpha^2 + 20A\tau_\alpha'' + 6\tau_\alpha''\tau_\alpha^2 - 18\tau_\alpha'^2\tau_\alpha \\ -305A\tau_\alpha^3 + 30\kappa_\alpha''\kappa_\alpha\tau_\alpha - 640A\tau_\alpha\kappa_\alpha^2 - 90\kappa_\alpha'\tau_\alpha'\kappa_\alpha \end{pmatrix} \\
& + Af^6\begin{pmatrix} 70\tau_\alpha'\kappa_\alpha^2 + 110A\tau_\alpha' + 5A\kappa_\alpha'\kappa_\alpha\tau_\alpha + 15A\tau_\alpha'\kappa_\alpha^2 \\ -22\tau_\alpha'\tau_\alpha^2 + 15A\tau_\alpha'\tau_\alpha^2 - 110\kappa_\alpha'\tau_\alpha\kappa_\alpha \end{pmatrix} \\
& + Af^5\left(4A\tau_\alpha'' + 7A\tau_\alpha^3 + 4\tau_\alpha'' + 108A\tau_\alpha + 5A\kappa_\alpha^2\tau_\alpha + 177\kappa_\alpha^2\tau_\alpha - 7\tau_\alpha^3\right) \\
& + Af^4\left(21\tau_\alpha' + 11A\tau_\alpha'\right) + Af^3\left(-5A\tau_\alpha + 22\tau_\alpha\right)
\end{aligned}
$$

$$
\begin{aligned}
d_{14} = {} & A^2 f^9 \left(630\tau_\alpha'\tau_\alpha\kappa_\alpha^3 - 630\kappa_\alpha'\tau_\alpha^2\kappa_\alpha^2 + 120\tau_\alpha'\tau_\alpha^3\kappa_\alpha - 120\tau_\alpha^4\kappa_\alpha'\right) \\
& + A^2 f^8 \begin{pmatrix} 112\tau_\alpha''\tau_\alpha\kappa_\alpha - 126\kappa_\alpha^5 + 56\kappa_\alpha''\tau_\alpha^2 - 210\kappa_\alpha'^2\kappa_\alpha - 168\tau_\alpha'^2\kappa_\alpha \\ -378\tau_\alpha^2\kappa_\alpha^3 - 252\tau_\alpha^4\kappa_\alpha - 336\kappa_\alpha'\tau_\alpha'\tau_\alpha + 70\kappa_\alpha''\kappa_\alpha^2 \end{pmatrix} \\
& - A^2 f^7 \left(301\tau_\alpha'\tau_\alpha\kappa_\alpha + 315\kappa_\alpha'\kappa_\alpha^2 + 518\kappa_\alpha'\tau_\alpha^2\right) + A\kappa_\alpha' f^3 \left(7 - A21\right) + Af^2\left(-7A\kappa_\alpha + 3\kappa_\alpha\right) \\
& + Af^6 \begin{pmatrix} -36\kappa_\alpha'\tau_\alpha'\tau_\alpha - 45\kappa_\alpha'^2\kappa_\alpha + 12\tau_\alpha''\kappa_\alpha\tau_\alpha - 512A\tau_\alpha^2\kappa_\alpha \\ -18\tau_\alpha'^2\kappa_\alpha + 15\kappa_\alpha''\kappa_\alpha^2 + 6\kappa_\alpha''\tau_\alpha^2 + 20A\kappa_\alpha'' - 175A\kappa_\alpha^3 \end{pmatrix} \\
& + Af^5 \left(20\kappa_\alpha'\kappa_\alpha^2 - 44\kappa_\alpha'\tau_\alpha^2 + 60A\kappa_\alpha' + 22\tau_\alpha'\tau_\alpha\kappa_\alpha + 21A\tau_\alpha'\kappa_\alpha\tau_\alpha - 20A\kappa_\alpha'\kappa_\alpha^2 + 8A\kappa_\alpha'\tau_\alpha^2\right) \\
& + Af^4 \left(4A\tau_\alpha^2\kappa_\alpha + 45\kappa_\alpha\tau_\alpha^2 + 4A\kappa_\alpha'' + 36A\kappa_\alpha + 55\kappa_\alpha^3 - 9A\kappa^3 + 4\kappa_\alpha''\right)
\end{aligned}
$$

$$
\begin{aligned}
d_{15} = {} & A^2 f^8 \left(210\tau_\alpha'\kappa_\alpha^4 - 360\tau_\alpha^3\kappa_\alpha'\kappa_\alpha + 360\tau_\alpha'\tau_\alpha^2\kappa_\alpha^2 - 210\kappa_\alpha'\tau_\alpha\kappa_\alpha^3\right) \\
& + A^2 f^7 \begin{pmatrix} -252\tau_\alpha\kappa_\alpha^4 - 84\tau_\alpha'^2\tau_\alpha - 36\tau_\alpha^5 + 112\kappa_\alpha''\tau_\alpha\kappa_\alpha - 288\tau_\alpha^3\kappa_\alpha^2 \\ -336\kappa_\alpha'\tau_\alpha'\kappa_\alpha - 168\kappa_\alpha'^2\tau_\alpha + 28\tau_\alpha''\tau_\alpha^2 + 56\tau_\alpha''\kappa_\alpha^2 \end{pmatrix} \\
& - A^2 f^6 \left(707\kappa_\alpha'\tau_\alpha\kappa_\alpha - 14\tau_\alpha'\kappa_\alpha^2 + 133\tau_\alpha'\tau_\alpha^2\right) + 4Af^2\tau_\alpha' + Af(2\tau_\alpha - 6A\tau_\alpha) \\
& + Af^5 \begin{pmatrix} -18\kappa_\alpha'^2\tau_\alpha + 6\tau_\alpha''\kappa_\alpha^2 + 15A\tau_\alpha'' + \tau_\alpha''\tau_\alpha^2 - 3\tau_\alpha'^2\tau_\alpha \\ -129A\tau_\alpha^3 + 12\kappa_\alpha''\kappa_\alpha\tau_\alpha - 270A\tau_\alpha\kappa_\alpha^2 - 36\kappa_\alpha'\tau_\alpha'\kappa_\alpha \end{pmatrix} \\
& + Af^4 \left(44\tau_\alpha'\kappa_\alpha^2 + 70A\tau_\alpha' + 7A\kappa_\alpha'\kappa_\alpha\tau_\alpha + 6A\tau_\alpha'\kappa_\alpha^2 - 2\tau_\alpha'\tau_\alpha^2 + 3A\tau_\alpha'\tau_\alpha^2 - 22\kappa_\alpha'\tau_\alpha\kappa_\alpha\right) \\
& + Af^3 \left(A\tau_\alpha'' - A\tau_\alpha^3 + \tau_\alpha'' + 52A\tau_\alpha - 4A\kappa_\alpha^2\tau_\alpha + 63\kappa_\alpha^2\tau_\alpha - \tau_\alpha^3\right)
\end{aligned}
$$

$$
\begin{aligned}
d_{16} = {} & A^2 f^7 \left(360\tau_\alpha'\tau_\alpha\kappa_\alpha^3 - 360\kappa_\alpha'\tau_\alpha^2\kappa_\alpha^2 + 45\tau_\alpha'\tau_\alpha^3\kappa_\alpha - 45\tau_\alpha^4\kappa_\alpha'\right) \\
& + A^2 f^6 \begin{pmatrix} 56\tau_\alpha''\tau_\alpha\kappa_\alpha - 84\kappa_\alpha^5 + 28\kappa_\alpha''\tau_\alpha^2 - 168\kappa_\alpha'^2\kappa_\alpha - 84\tau_\alpha'^2\kappa_\alpha - 192\tau_\alpha^2\kappa_\alpha^3 \\ -108\tau_\alpha^4\kappa_\alpha - 168\kappa_\alpha'\tau_\alpha'\tau_\alpha + 56\kappa_\alpha''\kappa_\alpha^2 \end{pmatrix} \\
& - A^2 f^5 \left(91\tau_\alpha'\tau_\alpha\kappa_\alpha + 189\kappa_\alpha'\kappa_\alpha^2 + 238\kappa_\alpha'\tau_\alpha^2\right) - 3A^2 f\kappa_\alpha' - 2A\kappa_\alpha \\
& + Af^4 \begin{pmatrix} -6\kappa_\alpha'\tau_\alpha'\tau_\alpha - 18\kappa_\alpha'^2\kappa_\alpha + 2\tau_\alpha''\kappa_\alpha\tau_\alpha - 174A\tau_\alpha^2\kappa_\alpha \\ -3\tau_\alpha'^2\kappa_\alpha + 6\kappa_\alpha''\kappa_\alpha^2 + \kappa_\alpha''\tau_\alpha^2 + 15A\kappa_\alpha'' - 63A\kappa_\alpha^3 \end{pmatrix} \\
& + Af^3 \begin{pmatrix} 22\kappa_\alpha'\kappa_\alpha^2 - 6\kappa_\alpha'\tau_\alpha^2 + 30A\kappa_\alpha' + 8\tau_\alpha'\tau_\alpha\kappa_\alpha \\ +4A\tau_\alpha'\kappa_\alpha\tau_\alpha - A\kappa_\alpha'\kappa_\alpha^2 + 2A\kappa_\alpha'\tau_\alpha^2 \end{pmatrix} \\
& + Af^2 \left(-4A\tau_\alpha^2\kappa_\alpha + 5\kappa_\alpha\tau_\alpha^2 + A\kappa_\alpha'' + 14A\kappa_\alpha + 11\kappa_\alpha^3 - A\kappa^3 + \kappa_\alpha''\right)
\end{aligned}
$$

$$d_{17}=A^2f^6\left(120\tau_\alpha'\kappa_\alpha{}^4-135\tau_\alpha{}^3\kappa_\alpha'\kappa_\alpha+135\tau_\alpha'\tau_\alpha{}^2\kappa_\alpha{}^2-120\kappa_\alpha'\tau_\alpha\kappa_\alpha{}^3\right)$$
$$+A^2f^5\begin{pmatrix}-108\tau_\alpha\kappa_\alpha{}^4-24\tau_\alpha'^2\tau_\alpha-9\tau_\alpha{}^5+56\kappa_\alpha''\tau_\alpha\kappa_\alpha-117\tau_\alpha{}^3\kappa_\alpha{}^2\\-168\kappa_\alpha'\tau_\alpha'\kappa_\alpha-84\kappa_\alpha'^2\tau_\alpha+8\tau_\alpha''\tau_\alpha{}^2+28\tau_\alpha''\kappa_\alpha{}^2\end{pmatrix}$$
$$-A^2f^4\left(301\kappa_\alpha'\tau_\alpha\kappa_\alpha-42\tau_\alpha'\kappa_\alpha{}^2+31\tau_\alpha'\tau_\alpha{}^2\right)+Af\left(10A\tau_\alpha-3A\kappa_\alpha{}^2\tau_\alpha+4\kappa_\alpha{}^2\tau_\alpha\right)$$
$$+Af^3\begin{pmatrix}-3\kappa_\alpha'^2\tau_\alpha+\tau_\alpha''\kappa_\alpha{}^2+6A\tau_\alpha''+\tau_\alpha''\tau_\alpha{}^2-3\tau_\alpha'^2\tau_\alpha\\-33A\tau_\alpha{}^3+2\kappa_\alpha''\kappa_\alpha\tau_\alpha-52A\tau_\alpha\kappa_\alpha{}^2-6\kappa_\alpha'\tau_\alpha'\kappa_\alpha\end{pmatrix}$$
$$+Af^2\left(10\tau_\alpha'\kappa_\alpha{}^2+23A\tau_\alpha'+2A\kappa_\alpha'\kappa_\alpha\tau_\alpha+A\tau_\alpha'\kappa_\alpha{}^2\right)$$

$$d_{18}=A^2f^5\left(135\tau_\alpha'\tau_\alpha\kappa_\alpha{}^3-135\kappa_\alpha'\tau_\alpha{}^2\kappa_\alpha{}^2+10\tau_\alpha'\tau_\alpha{}^3\kappa_\alpha-10\tau_\alpha{}^4\kappa_\alpha'\right)$$
$$+A^2f^4\begin{pmatrix}16\tau_\alpha''\tau_\alpha\kappa_\alpha-36\kappa_\alpha{}^5+8\kappa_\alpha''\tau_\alpha{}^2-84\kappa_\alpha'^2\kappa_\alpha-24\tau_\alpha'^2\kappa_\alpha-63\tau_\alpha{}^2\kappa_\alpha{}^3\\-27\tau_\alpha{}^4\kappa_\alpha-48\kappa_\alpha'\tau_\alpha'\tau_\alpha+28\kappa_\alpha''\kappa_\alpha{}^2\end{pmatrix}$$
$$-A^2f^3\left(9\tau_\alpha'\tau_\alpha\kappa_\alpha+63\kappa_\alpha'\kappa_\alpha{}^2+62\kappa_\alpha'\tau_\alpha{}^2\right)$$
$$+Af^2\left(-3\kappa_\alpha'^2\kappa_\alpha-36A\tau_\alpha{}^2\kappa_\alpha-3\tau_\alpha'^2\kappa_\alpha+\kappa_\alpha''\kappa_\alpha{}^2+6A\kappa_\alpha''-7A\kappa_\alpha{}^3\right)$$
$$+Af\left(6A\kappa_\alpha'+6\kappa_\alpha'\kappa_\alpha{}^2\right)+A\left(2A\kappa_\alpha-2\kappa_\alpha{}^3\right)$$

$$d_{19}=A^2f^4\left(45\tau_\alpha'\kappa_\alpha{}^4-30\tau_\alpha{}^3\kappa_\alpha'\kappa_\alpha+30\tau_\alpha'\tau_\alpha{}^2\kappa_\alpha{}^2-45\kappa_\alpha'\tau_\alpha\kappa_\alpha{}^3\right)$$
$$+A^2f^3\begin{pmatrix}-27\tau_\alpha\kappa_\alpha{}^4-3\tau_\alpha'^2\tau_\alpha-\tau_\alpha{}^5+16\kappa_\alpha''\tau_\alpha\kappa_\alpha-28\tau_\alpha{}^3\kappa_\alpha{}^2\\-48\kappa_\alpha'\tau_\alpha'\kappa_\alpha-24\kappa_\alpha'^2\tau_\alpha+\tau_\alpha''\tau_\alpha{}^2+8\tau_\alpha''\kappa_\alpha{}^2\end{pmatrix}$$
$$-A^2f^2\left(71\kappa_\alpha'\tau_\alpha\kappa_\alpha-22\tau_\alpha'\kappa_\alpha{}^2+3\tau_\alpha'\tau_\alpha{}^2\right)$$
$$+Af\left(-2A\tau_\alpha\kappa_\alpha{}^2+A\tau_\alpha''-4A\tau_\alpha{}^3\right)+3A^2\tau_\alpha'$$

$$d_{20}=A^2f^3\left(30\tau_\alpha'\tau_\alpha\kappa_\alpha{}^3-30\kappa_\alpha'\tau_\alpha{}^2\kappa_\alpha{}^2+\tau_\alpha'\tau_\alpha{}^3\kappa_\alpha-\tau_\alpha{}^4\kappa_\alpha'\right)$$
$$+A^2f^2\begin{pmatrix}2\tau_\alpha''\tau_\alpha\kappa_\alpha-9\kappa_\alpha{}^5+\kappa_\alpha''\tau_\alpha{}^2-24\kappa_\alpha'^2\kappa_\alpha-3\tau_\alpha'^2\kappa_\alpha-12\tau_\alpha{}^2\kappa_\alpha{}^3\\-3\tau_\alpha{}^4\kappa_\alpha-6\kappa_\alpha'\tau_\alpha'\tau_\alpha+8\kappa_\alpha''\kappa_\alpha{}^2\end{pmatrix}$$
$$-A^2f\left(9\kappa_\alpha'\kappa_\alpha{}^2-\tau_\alpha'\tau_\alpha\kappa_\alpha+7\kappa_\alpha'\tau_\alpha{}^2\right)+A^2\left(\kappa_\alpha{}^3-4\tau_\alpha{}^2\kappa_\alpha+\kappa_\alpha''\right)$$

$$d_{21}=A^2f^2\left(10\tau_\alpha'\kappa_\alpha^{\ 4}-3\tau_\alpha^{\ 3}\kappa_\alpha'\kappa_\alpha+3\tau_\alpha'\tau_\alpha^{\ 2}\kappa_\alpha^{\ 2}-10\kappa_\alpha'\tau_\alpha\kappa_\alpha^{\ 3}\right)$$
$$+A^2f\left(-3\tau_\alpha\kappa_\alpha^{\ 4}+2\kappa_\alpha''\tau_\alpha\kappa_\alpha-3\tau_\alpha^{\ 3}\kappa_\alpha^{\ 2}-6\kappa_\alpha'\tau_\alpha'\kappa_\alpha-3\kappa_\alpha'^2\tau_\alpha+\tau_\alpha''\kappa_\alpha^{\ 2}\right)$$
$$+A^2\left(4\tau_\alpha'\kappa_\alpha^{\ 2}-7\kappa_\alpha'\tau_\alpha\kappa_\alpha\right)$$

şeklinde olup burada

$$A=\sqrt{\frac{h^2}{f^2+h^2}}.$$

E ise,

$$I=f^4h^4+2f^2h^6+\kappa_\alpha^{\ 2}f^8h^2+4\kappa_\alpha^{\ 2}f^6h^4+6\kappa_\alpha^{\ 2}f^4h^6+4\kappa_\alpha^{\ 2}f^2h^8+f^{10}\tau_\alpha^{\ 2}$$
$$+4f^8h^2\tau_\alpha^{\ 2}+6f^6h^4\tau_\alpha^{\ 2}+4f^4h^6\tau_\alpha^{\ 2}+f^2h^8\tau_\alpha^{\ 2}+2f^9h\tau_\alpha\kappa_\alpha+8f^7h^3\tau_\alpha\kappa_\alpha$$
$$+12f^5h^5\tau_\alpha\kappa_\alpha+8f^3h^7\tau_\alpha\kappa_\alpha+2fh^9\tau_\alpha\kappa_\alpha+h^8+h^{10}\kappa_\alpha^{\ 2}$$

olmak üzere

$$E=\frac{I^{5/2}h}{\left(f^2+h^2\right)^{3/2}}.$$

(2.9) eşitliğinde D=0 yapan sıfırdan farklı sabit κ_α ve τ_α değerleri yoktur. Böylece şu sonucu verebiliriz:

Sonuç 2.3: Sabit olmayan eğrilik ve burulmaya sahip bir rektifiyan eğrinin küresel izdüşüm eğrisi, küre üzerinde bir helis değildir.

***ii. f, g* ve *h* sıfırdan farklı sabit olma durumu**

Bu durumda $\alpha(s)$ eğrisinin normu sabit ve dolayısıyla σ ve ρ değerleri sabit ve $\sigma=\rho$ olur**.** β eğrisinin Frenet vektörleri

$$T_\beta=T_\alpha,\quad N_\beta=N_\alpha,\quad B_\beta=B_\alpha$$

ve eğrilik ve burulması da

$$\kappa_\beta=\frac{\kappa_\alpha}{\rho},\quad \tau_\beta=\frac{\tau_\alpha}{\rho}$$

şeklinde bulunur. Buna göre aşağıdaki sonucu verebiliriz.

Sonuç 2.4: *f,g,h* sıfırdan farklı sabitler olmak üzere yer vektörü (2.1) eşitliğinde verilen sabit olmayan eğrilik ve torsiyona sahip $\alpha(s)$ eğrisinin küresel resmi kendisine benzer bir eğridir.

2.2 Sabit Olmayan Eğriliğe ve Sabit Torsiyona Sahip Eğrilerin Küresel Resimleri

Bu durum (2.1) eşitliğindeki f,g,h değerlerine göre iki farklı durumda incelenebilir.

i. $\alpha(s)$ eğrisinin bir rektifiyan eğri olması durumu

$\alpha(s)$ eğrisi sabit olmayan eğriliğe ve sabit torsiyona sahip bir rektifiyan eğri ise kendi yer vektörünün, eğrinin asli normali üzerinde bileşeni yoktur. Bu göre (2.1) eşitliği

$$\alpha(s)=f(s)T_\alpha+h(s)B_\alpha(s)$$

şeklinde yazılır ve $\alpha(s)$ rektifiyan eğrisinin Frenet vektörleri

$$T_\beta=u_1T_\alpha+u_3B_\alpha$$

$$N_\beta=n_1T_\alpha+n_2N_\alpha+n_3B_\alpha$$

$$B_\beta=(u_2n_3-u_3n_2)T_\alpha+(u_3n_1-u_1n_3)N_\alpha+(u_1n_2-u_2n_1)B_\alpha$$

olup burada $u_1,u_2,u_3,n_1,n_2,n_3,v_1,v_2,v_3$ aşağıdaki gibidir.

$$u_1=\frac{\sigma' f+\sigma}{\rho} \qquad u_2=0 \qquad u_3=\frac{\sigma' h}{\rho}$$

$$n_1=\frac{v_1}{\sqrt{v_1^{\,2}+v_2^{\,2}+v_3^{\,2}}} \qquad n_2=\frac{v_2}{\sqrt{v_1^{\,2}+v_2^{\,2}+v_3^{\,2}}} \qquad n_3=\frac{v_3}{\sqrt{v_1^{\,2}+v_2^{\,2}+v_3^{\,2}}}$$

$$v_1=\frac{u_1'}{\rho} \qquad v_2=\frac{u_1\kappa_\alpha-u_3\tau_\alpha}{\rho} \qquad v_3=\frac{u_3'}{\rho}$$

β eğrisinin eğriliği

$$\kappa_\beta=\frac{1}{B^2c^{9/2}}\left(\sum_{i=0}^{10}r_if^i\right)^{1/2} \qquad \textbf{(2.10)}$$

olup burada $r_5=r_7=r_9=0$ ve diğerleri

$r_0=c^6B^2\left(h^2+\kappa_\alpha^{\,2}c^2\right)$

$$r_1 = 2\kappa_\alpha h \tau_\alpha B^2 c^7$$

$$r_2 = Bc^4\left(B\tau_\alpha{}^2 h^2 c^2 - 6Bh^2 c + 4h^2 - 2B\kappa_\alpha{}^2 c^3 + 9B^2 c^2 - 12Bc + 4\right)$$

$$r_3 = -2B^2 \kappa_\alpha \tau_\alpha h c^6$$

$$r_4 = c^2\left(9B^2 h^2 c^2 - 16Bh^2 c + 4h^2 + B^2\kappa_\alpha{}^2 c^4 - 18B^3 c^3 + 36B^2 c^2 - 16Bc\right)$$

$$r_6 = c\left(9B^3 c^3 - 8h^2 - 36c^2 B^2 + 24cB + 12cBh^2\right)$$

$$r_8 = 4h^2 + 12B^2 c^2 - 16Bc$$

$$r_{10} = 4B$$

şeklinde olur. β eğrisinin burulması da

$$\tau_\beta = \frac{\sum_{i=0}^{11} p_i f^i}{B^{3/2} c^{3/2} \sum_{j=0}^{10} r_j f^j} \qquad \textbf{(2.11)}$$

şeklinde olup burada

$$p_0 = c^7 B^{5/2}\left(c^2\kappa_\alpha{}^2\tau_\alpha + 2h^2\tau_\alpha + ch\kappa_\alpha{}'\right)$$

$$p_1 = -hc^6\kappa_\alpha\left(c^2 B^{5/2}\kappa_\alpha{}^2 - 2c^2 B^{5/2}\tau_\alpha{}^2 + 6cB^3 - 4B^2 + 4B^{3/2} - 6cB^{5/2}\right)$$

$$p_2 = c^5\begin{pmatrix} c^2 B^{5/2} h^2 \tau_\alpha{}^3 + 3c^2 B^3 h\kappa_\alpha{}' - 2cB^2 h\kappa_\alpha{}' + 2cB^{3/2} h\kappa_\alpha{}' - 3c^3 B^{5/2}\kappa_\alpha{}^2\tau_\alpha - 2h^2 B^2\tau_\alpha \\ +3cB^3 h^2\tau_\alpha + 2h^2 B^{3/2}\tau_\alpha - 5c^2 B^{5/2} h\kappa_\alpha{}' - 2c^2 B^{5/2} h^2\kappa_\alpha{}^2\tau_\alpha - 5cB^{5/2} h^2\tau_\alpha \end{pmatrix}$$

$$p_3 = c^4 h\kappa_\alpha \begin{pmatrix} +12cB^{5/2} - h^2c^2B^{5/2}\tau_\alpha{}^2 - 4c^3B^{5/2}\tau_\alpha{}^2 - 9c^2B^{7/2} + 2c^3B^{5/2}\kappa_\alpha{}^2 \\ -30cB^2 - 18c^2B^{5/2} + 8B - 8\sqrt{B} + 24cB^{3/2} - 4B^{3/2} + 27c^2B^3 \end{pmatrix}$$

$$p_4 = c^3 \begin{pmatrix} \left(3c^4B^{5/2}\tau_\alpha + 2c^3B^{5/2}h^2\tau_\alpha\right)\kappa_\alpha{}^2 \\ -c^3B^{5/2}h^2\tau_\alpha{}^3 - 6c^3B^3h\kappa_\alpha{}' + 6c^2B^2h\kappa_\alpha{}' - 4h^2\sqrt{B}\tau_\alpha - 6c^2hB^{3/2}\kappa_\alpha{}' \\ -4cB^{3/2}h^2\tau_\alpha + 7c^3hB^{5/2}\kappa_\alpha{}' + 6c^2h^2B^{5/2}\tau_\alpha - 6c^2h^2B^3\tau_\alpha + 10ch^2B^2\tau_\alpha \end{pmatrix}$$

$$p_5 = hc^3 \begin{pmatrix} -B^{5/2}\kappa_\alpha{}^3c^3 \\ -\begin{pmatrix} 36c^2B^3 - 66cB^2 + 36cB^{5/2} - 16B^{3/2} - 18c^2B^{7/2} \\ +32B - 18c^2B^{5/2} - 32\sqrt{B} - 2c^3B^{5/2}\tau_\alpha{}^2 + 48cB^{3/2} \end{pmatrix}\kappa_\alpha \\ -4hcB^{3/2}\tau_\alpha{}' + 3hc^2B^{5/2}\tau_\alpha{}' + 4hcB^2\tau_\alpha{}' - 3hc^2B^3\tau_\alpha{}' \end{pmatrix}$$

$$p_6 = c^2 \begin{pmatrix} 3c^3B^3h\kappa_\alpha{}' - c^4B^{5/2}\kappa_\alpha{}^2\tau_\alpha - 6c^2B^2h\kappa_\alpha{}' - 3c^3B^{5/2}h\kappa_\alpha{}' + 12h^2\sqrt{B}\tau_\alpha \\ +3c^2B^3h^2\tau_\alpha - 14cB^2h^2\tau_\alpha + 2cB^{5/2}h^2\tau_\alpha + 6c^2B^{3/2}h\kappa_\alpha{}' - 3c^2B^{5/2}h^2\tau_\alpha \end{pmatrix}$$

$$p_7 = c^2h\kappa_\alpha \begin{pmatrix} 15c^2B^3 - 6c^2B^{5/2} - 9c^2B^{7/2} - 58cB^2 + 40cB^{3/2} \\ +36cB^{5/2} + 48B - 24B^{3/2} - 48\sqrt{B} \end{pmatrix}$$

$$p_8 = 2ch\left(c^2B^2\kappa_\alpha{}' + 3chB^2\tau_\alpha - 6h\sqrt{B}\tau_\alpha - c^2B^{3/2}\kappa_\alpha{}'\right)$$

$$p_9 = 2ch\kappa_\alpha\left(9cB^2 - 6B^{3/2}c - 16B + 16\sqrt{B} + 8B^{3/2} - 6cB^{5/2}\right)$$

$$p_{10} = 4h^2\tau_\alpha\sqrt{B}$$

$$p_{11} = 4\kappa_\alpha h\left(2B - 2\sqrt{B} - B^{3/2}\right)$$

şeklinde olur. p_{10} terimi göz önüne alınırsa şu sonuç verilebilir.

Sonuç 2.5: Sabit olmayan eğrilik ve sabit burulmaya sahip bir rektifiyan eğrinin küresel izdüşüm eğrisi, küre üzerinde bir düzlemsel eğri olamaz.

(2.10) ve (2.11) eşitliklerinden $\frac{\tau_\beta}{\kappa_\beta}$ oranının türevi hesaplanırsa

$$\left(\frac{\tau_\beta}{\kappa_\beta}\right)' = \frac{D}{E}$$

elde edilir. Burada D aşağıdaki gibidir.

$$D = \sum_{i=0}^{21} d_i h_i \qquad \textbf{(2.12)}$$

$$d_0 = -A^2 \tau_\alpha{}^3 f^{23} \kappa_\alpha{}' \tau_\alpha$$

$$d_1 = -3A^2 \tau_\alpha{}^3 \kappa_\alpha \kappa_\alpha{}' f^{22} - A^2 \tau_\alpha{}^3 f^{21} \left(\kappa_\alpha{}^2 + \tau_\alpha{}^2\right)$$

$$d_2 = A^2 \tau_\alpha f^{21} \left(-3\kappa_\alpha{}' \tau_\alpha \kappa_\alpha{}^2 - 10\kappa_\alpha{}' \tau_\alpha{}^3\right) - 3A^2 \tau_\alpha{}^2 \kappa_\alpha f^{20} \left(\kappa_\alpha{}^2 + \tau_\alpha{}^2\right)$$

$$\begin{aligned} d_3 = {} & A^2 \kappa_\alpha f^{20} \left(-30\kappa_\alpha{}' \tau_\alpha{}^3 - \kappa_\alpha{}' \kappa_\alpha{}^2 \tau_\alpha\right) + A^2 \tau_\alpha f^{19} \left(-9\tau_\alpha{}^4 - 12\kappa_\alpha{}^2 \tau_\alpha{}^2 - 3\kappa_\alpha{}^4\right) \\ & -15A^2 \tau_\alpha{}^3 f^{17} - 20A\tau_\alpha{}^3 f^{15} \end{aligned}$$

$$\begin{aligned} d_4 = {} & A^2 f^{19} \left(-45\tau_\alpha{}^4 \kappa_\alpha{}' - 30\kappa_\alpha{}' \tau_\alpha{}^2 \kappa_\alpha{}^2\right) + A^2 f^{18} \left(\kappa_\alpha{}'' \tau_\alpha{}^2 - 28\tau_\alpha{}^2 \kappa_\alpha{}^3 - \kappa_\alpha{}^5 - 27\tau_\alpha{}^4 \kappa_\alpha\right) \\ & -13A^2 \kappa_\alpha{}' \tau_\alpha{}^2 f^{17} + Af^{16} \left(\kappa_\alpha{}'' \tau_\alpha{}^2 - 42A\tau_\alpha{}^2 \kappa_\alpha\right) + Af^{15} \left(-2A\kappa_\alpha{}' \tau_\alpha{}^2 - 14\kappa_\alpha{}' \tau_\alpha{}^2\right) \\ & - Af^{14} \left(46\kappa_\alpha \tau_\alpha{}^2 + 4A\kappa_\alpha \tau_\alpha{}^2\right) \end{aligned}$$

$$\begin{aligned} d_5 = {} & A^2 f^{18} \left(-135\tau_\alpha{}^3 \kappa_\alpha{}' \kappa_\alpha - 10\kappa_\alpha{}' \tau_\alpha \kappa_\alpha{}^3\right) \\ & + A^2 f^{17} \left(-36\tau_\alpha{}^5 + 2\kappa_\alpha{}'' \tau_\alpha \kappa_\alpha - 63\tau_\alpha{}^3 \kappa_\alpha{}^2 - 27\tau_\alpha \kappa_\alpha{}^4 - 3\kappa_\alpha{}'^2 \tau_\alpha\right) \\ & - 22A^2 \kappa_\alpha{}' \tau_\alpha \kappa_\alpha f^{16} + Af^{15} \left(2\kappa_\alpha{}'' \kappa_\alpha \tau_\alpha - 40A\tau_\alpha \kappa_\alpha{}^2 - 99A\tau_\alpha{}^3 - 3\kappa_\alpha{}' \tau_\alpha\right) \\ & + Af^{14} \left(-22\kappa_\alpha{}' \tau_\alpha \kappa_\alpha - 3A\kappa_\alpha{}' \kappa_\alpha \tau_\alpha\right) + Af^{13} \left(11A\tau_\alpha{}^3 - 35\kappa_\alpha{}^2 \tau_\alpha - 9A\kappa_\alpha{}^2 \tau_\alpha - 83\tau_\alpha{}^3\right) \end{aligned}$$

$$d_6 = A^2 f^{17}\left(-120\tau_\alpha{}^4\kappa_\alpha{}' - 135\kappa_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2\right)$$
$$+ A^2 f^{16}\left(\kappa_\alpha{}''\kappa_\alpha{}^2 - 9\kappa_\alpha{}^5 - 3\kappa_\alpha{}'^2\kappa_\alpha + 8\kappa_\alpha{}''\tau_\alpha{}^2 - 108\tau_\alpha{}^4\kappa_\alpha - 117\tau_\alpha{}^2\kappa_\alpha{}^3\right)$$
$$- A^2 f^{15}\left(+9\kappa_\alpha{}'\kappa_\alpha{}^2 + 98\kappa_\alpha{}'\tau_\alpha{}^2\right)$$
$$+ Af^{14}\left(-3\kappa_\alpha{}'\kappa_\alpha - 13A\kappa_\alpha{}^3 + \kappa_\alpha{}''\kappa_\alpha{}^2 - 264A\tau_\alpha{}^2\kappa_\alpha + 6\kappa_\alpha{}''\tau_\alpha{}^2\right)$$
$$+ Af^{13}\left(-8\kappa_\alpha{}'\kappa_\alpha{}^2 - A\kappa_\alpha{}'\kappa_\alpha{}^2 - 8A\kappa_\alpha{}'\tau_\alpha{}^2 - 76\kappa_\alpha{}'\tau_\alpha{}^2\right)$$
$$+ Af^{12}\left(4A\tau_\alpha{}^2\kappa_\alpha - 9\kappa_\alpha{}^3 - 5A\kappa_\alpha{}^3 - 159\kappa_\alpha\tau_\alpha{}^2\right)$$

$$d_7 = A^2 f^{16}\left(-360\tau_\alpha{}^3\kappa_\alpha{}'\kappa_\alpha - 45\kappa_\alpha{}'\tau_\alpha\kappa_\alpha{}^3\right)$$
$$+ A^2 f^{15}\left(-84\tau_\alpha{}^5 - 192\tau_\alpha{}^3\kappa_\alpha{}^2 - 108\tau_\alpha\kappa_\alpha{}^4 + 16\kappa_\alpha{}''\tau_\alpha\kappa_\alpha - 24\kappa_\alpha{}'^2\tau_\alpha\right)$$
$$-161\kappa_\alpha{}'\tau_\alpha\kappa_\alpha A^2 f^{14} + Af^{13}\left(-283A\tau_\alpha{}^3 - 242A\tau_\alpha\kappa_\alpha{}^2 - 18\kappa_\alpha{}'^2\tau_\alpha + 12\kappa_\alpha{}''\kappa_\alpha\tau_\alpha\right)$$
$$+ Af^{12}\left(-110\kappa_\alpha{}'\tau_\alpha\kappa_\alpha - 13A\kappa_\alpha{}'\kappa_\alpha\tau_\alpha\right) + Af^9\left(21A\tau_\alpha + 16\tau_\alpha\right) + 28A\tau_\alpha f^7$$
$$+ Af^{11}\left(-28A\kappa_\alpha{}^2\tau_\alpha + 43A\tau_\alpha{}^3 - 93\kappa_\alpha{}^2\tau_\alpha + 12A\tau_\alpha - 133\tau_\alpha{}^3\right)$$

$$d_8 = A^2 f^{15}\left(-360\kappa_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2 - 210\tau_\alpha{}^4\kappa_\alpha{}'\right)$$
$$+ A^2 f^{14}\left(-36\kappa_\alpha{}^5 - 288\tau_\alpha{}^2\kappa_\alpha{}^3 - 24\kappa_\alpha{}'^2\kappa_\alpha + 8\kappa_\alpha{}''\kappa_\alpha{}^2 + 28\kappa_\alpha{}''\tau_\alpha{}^2 - 252\tau_\alpha{}^4\kappa_\alpha\right)$$
$$- A^2 f^{13}\left(+63\kappa_\alpha{}'\kappa_\alpha{}^2 + 322\kappa_\alpha{}'\tau_\alpha{}^2\right) + Af^{12}\begin{pmatrix} 6\kappa_\alpha{}''\kappa_\alpha{}^2 + 15\kappa_\alpha{}''\tau_\alpha{}^2 - 706A\tau_\alpha{}^2\kappa_\alpha + A\kappa_\alpha{}'' \\ -77A\kappa_\alpha{}^3 - 18\kappa_\alpha{}'\kappa_\alpha \end{pmatrix}$$
$$+ Af^{11}\left(-34\kappa_\alpha{}'\kappa_\alpha{}^2 - 170A\kappa_\alpha{}'\tau_\alpha{}^2 + 6A\kappa_\alpha{}' - 10A\kappa_\alpha{}'\tau_\alpha{}^2 - 5A\kappa_\alpha{}'\kappa_\alpha{}^2\right)$$
$$+ Af^{10}\left(-21A\kappa_\alpha{}^3 + \kappa_\alpha{}'' + A\kappa_\alpha{}'' - 17\kappa_\alpha{}^3 + 52A\tau_\alpha{}^2\kappa_\alpha - 171\kappa_\alpha\tau_\alpha{}^2 + 6A\kappa_\alpha\right) + A\kappa_\alpha{}' f^9\left(8A + 7\right)$$
$$+ Af^8\left(11A\kappa_\alpha + 9\kappa_\alpha + \kappa_\alpha{}''\right) + A\kappa_\alpha{}' f^7\left(10 - A\right) + A\kappa_\alpha f^6\left(18 - 4A\right)$$

$$d_9 = A^2 f^{14}\left(-630\tau_\alpha{}^3\kappa_\alpha{}'\kappa_\alpha - 120\kappa_\alpha{}'\tau_\alpha\kappa_\alpha{}^3\right)$$
$$+ A^2 f^{13}\left(-126\tau_\alpha{}^5 - 84\kappa_\alpha{}'^2\tau_\alpha - 252\tau_\alpha\kappa_\alpha{}^4 + 56\kappa_\alpha{}''\tau_\alpha\kappa_\alpha - 378\tau_\alpha{}^3\kappa_\alpha{}^2\right)$$
$$-511A^2\kappa_\alpha{}'\tau_\alpha\kappa_\alpha f^{12} + Af^{11}\left(-45\kappa_\alpha{}'^2\tau_\alpha - 459A\tau_\alpha{}^3 + 30\kappa_\alpha{}''\kappa_\alpha\tau_\alpha - 612A\tau_\alpha\kappa_\alpha{}^2\right)$$
$$+ Af^{10}\left(-20A\kappa_\alpha{}'\kappa_\alpha\tau_\alpha - 220\kappa_\alpha{}'\tau_\alpha\kappa_\alpha\right)$$
$$+ Af^9\left(62A\tau_\alpha{}^3 + 58A\tau_\alpha - 25A\kappa_\alpha{}^2\tau_\alpha - 18\kappa_\alpha{}^2\tau_\alpha - 102\tau_\alpha{}^3\right)$$
$$+ Af^7\left(49A\tau_\alpha + 50\tau_\alpha\right) + 9A\tau_\alpha f^5$$

$$d_{10} = A^2 f^{13}\left(-630\kappa_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2 - 252\tau_\alpha{}^4\kappa_\alpha{}'\right)$$
$$+ A^2 f^{12}\left(-84\kappa_\alpha{}^5 + 56\kappa_\alpha{}''\tau_\alpha{}^2 - 84\kappa_\alpha{}'^2\kappa_\alpha - 462\tau_\alpha{}^2\kappa_\alpha{}^3 - 378\tau_\alpha{}^4\kappa_\alpha + 28\kappa_\alpha{}''\kappa_\alpha{}^2\right)$$
$$- A^2 f^{11}\left(189\kappa_\alpha{}'\kappa_\alpha{}^2 + 602\kappa_\alpha{}'\tau_\alpha{}^2\right)$$
$$+ Af^{10}\left(-45\kappa_\alpha{}'^2\kappa_\alpha - 1044A\tau_\alpha{}^2\kappa_\alpha + 15\kappa_\alpha{}''\kappa_\alpha{}^2 + 20\kappa_\alpha{}''\tau_\alpha{}^2 + 6A\kappa_\alpha{}'' - 189A\kappa_\alpha{}^3\right)$$
$$+ Af^9\left(-50\kappa_\alpha{}'\kappa_\alpha{}^2 - 200\kappa_\alpha{}'\tau_\alpha{}^2 + 30A\kappa_\alpha{}' - 10A\kappa_\alpha{}'\kappa_\alpha{}^2\right)$$
$$+ Af^8\left(88A\tau_\alpha{}^2\kappa_\alpha - 14\kappa_\alpha\tau_\alpha{}^2 + 4A\kappa_\alpha{}'' + 26A\kappa_\alpha + 20\kappa_\alpha{}^3 - 34A\kappa^3 + 4\kappa_\alpha{}''\right)$$
$$+ Af^6\left(15A\kappa_\alpha + 25\kappa_\alpha + 2\kappa_\alpha{}''\right) + A\kappa_\alpha{}' f^5\left(7 - A\right) - 8A\kappa_\alpha f^4 + 21A\kappa_\alpha{}' f^7\left(A + 1\right)$$

$$d_{11} = A^2 f^{12}\left(-756\tau_\alpha{}^3\kappa_\alpha{}'\kappa_\alpha - 210\kappa_\alpha{}'\tau_\alpha\kappa_\alpha{}^3\right)$$
$$+ A^2 f^{13}\left(-378\tau_\alpha\kappa_\alpha{}^4 - 126\tau_\alpha{}^5 + 112\kappa_\alpha{}''\tau_\alpha\kappa_\alpha - 504\tau_\alpha{}^3\kappa_\alpha{}^2 - 168\kappa_\alpha{}'^2\tau_\alpha\right)$$
$$-917\kappa_\alpha{}'\tau_\alpha\kappa_\alpha A^2 f^{10} + Af^9\left(-60\kappa_\alpha{}'^2\tau_\alpha - 465A\tau_\alpha{}^3 + 40\kappa_\alpha{}''\kappa_\alpha\tau_\alpha - 830A\tau_\alpha\kappa_\alpha{}^2\right)$$
$$+ Af^8\left(-10A\kappa_\alpha{}'\kappa_\alpha\tau_\alpha - 220\kappa_\alpha{}'\tau_\alpha\kappa_\alpha\right) + Af^7\left(38A\tau_\alpha{}^3 + 112A\tau_\alpha + 158\kappa_\alpha{}^2\tau_\alpha - 38\tau_\alpha{}^3\right)$$
$$+ Af^5\left(29A\tau_\alpha + 54\tau_\alpha\right) - A\tau_\alpha f^3$$

$$d_{12} = A^2 f^{11}\left(-756\kappa_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2 - 210\tau_\alpha{}^4\kappa_\alpha{}'\right)$$
$$+ A^2 f^{10}\left(-126\kappa_\alpha{}^5 + 70\kappa_\alpha{}''\tau_\alpha{}^2 - 168\kappa_\alpha{}'^2\kappa_\alpha - 504\tau_\alpha{}^2\kappa_\alpha{}^3 - 378\tau_\alpha{}^4\kappa_\alpha + 56\kappa_\alpha{}''\kappa_\alpha{}^2\right)$$
$$- A^2 f^9\left(315\kappa_\alpha{}'\kappa_\alpha{}^2 + 700\kappa_\alpha{}'\tau_\alpha{}^2\right) + 21A\kappa_\alpha{}' f^5(15A + 21)$$
$$+ Af^8\left(-60\kappa_\alpha{}'^2\kappa_\alpha - 930A\tau_\alpha{}^2\kappa_\alpha + 20\kappa_\alpha{}''\kappa_\alpha{}^2 + 15\kappa_\alpha{}''\tau_\alpha{}^2 + 15A\kappa_\alpha{}'' - 245A\kappa_\alpha{}^3\right)$$
$$+ Af^7\left(-20\kappa_\alpha{}'\kappa_\alpha{}^2 - 130\kappa_\alpha{}'\tau_\alpha{}^2 + 60A\kappa_\alpha{}' - 10A\kappa_\alpha{}'\kappa_\alpha{}^2 + 10A\kappa_\alpha{}'\tau_\alpha{}^2\right)$$
$$+ Af^6\left(52A\tau_\alpha{}^2\kappa_\alpha + 84\kappa_\alpha\tau_\alpha{}^2 + 6A\kappa_\alpha{}'' + 44A\kappa_\alpha + 70\kappa_\alpha{}^3 - 26A\kappa^3 + 6\kappa_\alpha{}''\right)$$
$$+ Af^4\left(-3A\kappa_\alpha + 21\kappa_\alpha + \kappa_\alpha{}''\right) - 3A\kappa_\alpha{}' f^3 + 4A\kappa_\alpha f^2$$

$$d_{13} = A^2 f^{10}\left(-630\tau_\alpha{}^3\kappa_\alpha{}'\kappa_\alpha - 252\kappa_\alpha{}'\tau_\alpha\kappa_\alpha{}^3\right) + Af^3(-5A\tau_\alpha + 22\tau_\alpha)$$
$$+ A^2 f^9\left(-378\tau_\alpha\kappa_\alpha{}^4 - 84\tau_\alpha{}^5 + 140\kappa_\alpha{}''\tau_\alpha\kappa_\alpha - 462\tau_\alpha{}^3\kappa_\alpha{}^2 - 210\kappa_\alpha{}'^2\tau_\alpha\right)$$
$$-1015\kappa_\alpha{}'\tau_\alpha\kappa_\alpha A^2 f^8 + Af^7\left(-45\kappa_\alpha{}'^2\tau_\alpha - 305A\tau_\alpha{}^3 + 30\kappa_\alpha{}''\kappa_\alpha\tau_\alpha - 640A\tau_\alpha\kappa_\alpha{}^2\right)$$
$$+ Af^6\left(5A\kappa_\alpha{}'\kappa_\alpha\tau_\alpha - 110\kappa_\alpha{}'\tau_\alpha\kappa_\alpha\right) + Af^5\left(+7A\tau_\alpha{}^3 + 108A\tau_\alpha + 5A\kappa_\alpha{}^2\tau_\alpha + 177\kappa_\alpha{}^2\tau_\alpha - 7\tau_\alpha{}^3\right)$$

$$d_{14} = A^2 f^9\left(-630\kappa_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2 - 120\tau_\alpha{}^4\kappa_\alpha{}'\right) + Af^2(-7A\kappa_\alpha + 3\kappa_\alpha)$$
$$+ A^2 f^8\left(-126\kappa_\alpha{}^5 + 56\kappa_\alpha{}''\tau_\alpha{}^2 - 210\kappa_\alpha{}'^2\kappa_\alpha - 378\tau_\alpha{}^2\kappa_\alpha{}^3 - 252\tau_\alpha{}^4\kappa_\alpha + 70\kappa_\alpha{}''\kappa_\alpha{}^2\right)$$
$$- A^2 f^7\left(315\kappa_\alpha{}'\kappa_\alpha{}^2 + 518\kappa_\alpha{}'\tau_\alpha{}^2\right) + Af^6\begin{pmatrix}-45\kappa_\alpha{}'^2\kappa_\alpha + 12\tau_\alpha{}''\kappa_\alpha\tau_\alpha - 512A\tau_\alpha{}^2\kappa_\alpha \\ +15\kappa_\alpha{}''\kappa_\alpha{}^2 + 6\kappa_\alpha{}''\tau_\alpha{}^2 + 20A\kappa_\alpha{}'' - 175A\kappa_\alpha{}^3\end{pmatrix}$$
$$+ Af^5\left(20\kappa_\alpha{}'\kappa_\alpha{}^2 - 44\kappa_\alpha{}'\tau_\alpha{}^2 + 60A\kappa_\alpha{}' - 20A\kappa_\alpha{}'\kappa_\alpha{}^2 + 8A\kappa_\alpha{}'\tau_\alpha{}^2\right)$$
$$+ Af^4\left(4A\tau_\alpha{}^2\kappa_\alpha + 45\kappa_\alpha\tau_\alpha{}^2 + 4A\kappa_\alpha{}'' + 36A\kappa_\alpha + 55\kappa_\alpha{}^3 - 9A\kappa^3 + 4\kappa_\alpha{}''\right) + A\kappa_\alpha{}' f^3(7 - A21)$$

$$d_{15} = A^2 f^8\left(-360\tau_\alpha{}^3\kappa_\alpha{}'\kappa_\alpha - 210\kappa_\alpha{}'\tau_\alpha\kappa_\alpha{}^3\right)$$
$$+ A^2 f^7\left(-252\tau_\alpha\kappa_\alpha{}^4 - 36\tau_\alpha{}^5 + 112\kappa_\alpha{}''\tau_\alpha\kappa_\alpha - 288\tau_\alpha{}^3\kappa_\alpha{}^2 - 168\kappa_\alpha{}'^2\tau_\alpha\right)$$
$$-707\kappa_\alpha{}'\tau_\alpha\kappa_\alpha A^2 f^6 + Af^5\left(-18\kappa_\alpha{}'^2\tau_\alpha - 129A\tau_\alpha{}^3 + 12\kappa_\alpha{}''\kappa_\alpha\tau_\alpha - 270A\tau_\alpha\kappa_\alpha{}^2\right)$$
$$+ Af^4\left(7A\kappa_\alpha{}'\kappa_\alpha\tau_\alpha - 22\kappa_\alpha{}'\tau_\alpha\kappa_\alpha\right) + Af^3\left(-A\tau_\alpha{}^3 + 52A\tau_\alpha - 4A\kappa_\alpha{}^2\tau_\alpha + 63\kappa_\alpha{}^2\tau_\alpha - \tau_\alpha{}^3\right)$$
$$+ Af(2\tau_\alpha - 6A\tau_\alpha)$$

$$d_{16} = A^2 f^7 \left(-360\kappa_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2 - 45\tau_\alpha{}^4\kappa_\alpha{}'\right)$$
$$+ A^2 f^6 \left(-84\kappa_\alpha{}^5 + 28\kappa_\alpha{}''\tau_\alpha{}^2 - 168\kappa_\alpha{}'^2\kappa_\alpha - 192\tau_\alpha{}^2\kappa_\alpha{}^3 - 108\tau_\alpha{}^4\kappa_\alpha + 56\kappa_\alpha{}''\kappa_\alpha{}^2\right)$$
$$- A^2 f^5 \left(189\kappa_\alpha{}'\kappa_\alpha{}^2 + 238\kappa_\alpha{}'\tau_\alpha{}^2\right) + Af^4 \begin{pmatrix} -18\kappa_\alpha{}'^2\kappa_\alpha - 174A\tau_\alpha{}^2\kappa_\alpha \\ +6\kappa_\alpha{}''\kappa_\alpha{}^2 + \kappa_\alpha{}''\tau_\alpha{}^2 + 15A\kappa_\alpha{}'' - 63A\kappa_\alpha{}^3 \end{pmatrix}$$
$$+ Af^3 \left(22\kappa_\alpha{}'\kappa_\alpha{}^2 - 6\kappa_\alpha{}'\tau_\alpha{}^2 + 30A\kappa_\alpha{}' - A\kappa_\alpha{}'\kappa_\alpha{}^2 + 2A\kappa_\alpha{}'\tau_\alpha{}^2\right)$$
$$+ Af^2 \left(-4A\tau_\alpha{}^2\kappa_\alpha + 5\kappa_\alpha\tau_\alpha{}^2 + A\kappa_\alpha{}'' + 14A\kappa_\alpha + 11\kappa_\alpha{}^3 - A\kappa^3 + \kappa_\alpha{}''\right) - 3A^2 f\kappa_\alpha{}' - 2A\kappa_\alpha$$

$$d_{17} = A^2 f^6 \left(-135\tau_\alpha{}^3\kappa_\alpha{}'\kappa_\alpha - 120\kappa_\alpha{}'\tau_\alpha\kappa_\alpha{}^3\right)$$
$$+ A^2 f^5 \left(-108\tau_\alpha\kappa_\alpha{}^4 - 9\tau_\alpha{}^5 + 56\kappa_\alpha{}''\tau_\alpha\kappa_\alpha - 117\tau_\alpha{}^3\kappa_\alpha{}^2 - 84\kappa_\alpha{}'^2\tau_\alpha\right)$$
$$- 301\kappa_\alpha{}'\tau_\alpha\kappa_\alpha A^2 f^4 + Af^3 \left(-3\kappa_\alpha{}'^2\tau_\alpha - 33A\tau_\alpha{}^3 + 2\kappa_\alpha{}''\kappa_\alpha\tau_\alpha - 52A\tau_\alpha\kappa_\alpha{}^2\right)$$
$$+ 2A\kappa_\alpha{}'\kappa_\alpha\tau_\alpha Af^2 + Af\left(10A\tau_\alpha - 3A\kappa_\alpha{}^2\tau_\alpha + 4\kappa_\alpha{}^2\tau_\alpha\right)$$

$$d_{18} = A^2 f^5 \left(-135\kappa_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2 - 10\tau_\alpha{}^4\kappa_\alpha{}'\right) + Af\left(6A\kappa_\alpha{}' + 6\kappa_\alpha{}'\kappa_\alpha{}^2\right) + A\left(2A\kappa_\alpha - 2\kappa_\alpha{}^3\right)$$
$$+ A^2 f^4 \left(-36\kappa_\alpha{}^5 + 8\kappa_\alpha{}''\tau_\alpha{}^2 - 84\kappa_\alpha{}'^2\kappa_\alpha - 63\tau_\alpha{}^2\kappa_\alpha{}^3 - 27\tau_\alpha{}^4\kappa_\alpha + 28\kappa_\alpha{}''\kappa_\alpha{}^2\right)$$
$$- A^2 f^3 \left(63\kappa_\alpha{}'\kappa_\alpha{}^2 + 62\kappa_\alpha{}'\tau_\alpha{}^2\right) + Af^2\left(-3\kappa_\alpha{}'^2\kappa_\alpha - 36A\tau_\alpha{}^2\kappa_\alpha + \kappa_\alpha{}''\kappa_\alpha{}^2 + 6A\kappa_\alpha{}'' - 7A\kappa_\alpha{}^3\right)$$

$$d_{19} = A^2 f^4 \left(-30\tau_\alpha{}^3\kappa_\alpha{}'\kappa_\alpha - 45\kappa_\alpha{}'\tau_\alpha\kappa_\alpha{}^3\right)$$
$$+ A^2 f^3 \left(-27\tau_\alpha\kappa_\alpha{}^4 - \tau_\alpha{}^5 + 16\kappa_\alpha{}''\tau_\alpha\kappa_\alpha - 28\tau_\alpha{}^3\kappa_\alpha{}^2 - 24\kappa_\alpha{}'^2\tau_\alpha\right)$$
$$- 71\kappa_\alpha{}'\tau_\alpha\kappa_\alpha A^2 f^2 + Af\left(-2A\tau_\alpha\kappa_\alpha{}^2 - 4A\tau_\alpha{}^3\right)$$

$$d_{20} = A^2 f^3 \left(-30\kappa_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2 - \tau_\alpha{}^4\kappa_\alpha{}'\right) - A^2 f\left(9\kappa_\alpha{}'\kappa_\alpha{}^2 + 7\kappa_\alpha{}'\tau_\alpha{}^2\right)$$
$$+ A^2 f^2 \left(-9\kappa_\alpha{}^5 + \kappa_\alpha{}''\tau_\alpha{}^2 - 24\kappa_\alpha{}'^2\kappa_\alpha - 12\tau_\alpha{}^2\kappa_\alpha{}^3 - 3\tau_\alpha{}^4\kappa_\alpha + 8\kappa_\alpha{}''\kappa_\alpha{}^2\right)$$
$$+ A^2\left(\kappa_\alpha{}^3 - 4\tau_\alpha{}^2\kappa_\alpha + \kappa_\alpha{}''\right)$$

$$d_{21} = A^2 f^2 \left(-3\tau_\alpha{}^3\kappa_\alpha{}'\kappa_\alpha - 10\kappa_\alpha{}'\tau_\alpha\kappa_\alpha{}^3\right) - 7\kappa_\alpha{}'\tau_\alpha\kappa_\alpha A^2$$
$$+ A^2 f\left(-3\tau_\alpha\kappa_\alpha{}^4 + 2\kappa_\alpha{}''\tau_\alpha\kappa_\alpha - 3\tau_\alpha{}^3\kappa_\alpha{}^2 - 3\kappa_\alpha{}'^2\tau_\alpha\right)$$

şeklinde olur. (2.12) eşitliğinde D=0 yapan sıfırdan farklı sabit κ_α ve τ_α değerleri yoktur. Böylece şu sonucu verebiliriz:

Sonuç 2.6: Sabit olmayan eğrilik ve sabit torsiyona sahip bir rektifiyan eğrinin küresel izdüşüm eğrisi, küre üzerinde bir helis olamaz.

***ii. f, g* ve *h* sıfırdan farklı sabit olma durumu**

Sabit olmayan eğriliğe ve sabit torsiyona sahip eğrilerin eğriliği ve burulması bölüm (2.1) *(ii)'* deki değerlerle aynı olur. Bu durumda şu sonucu verebiliriz.

Sonuç 2.7: *f,g,h* sıfırdan farklı sabitler olmak üzere yer vektörü (2.1) eşitliğinde verilen sabit olmayan eğrilik ve sabit torsiyona sahip $\alpha(s)$ eğrisinin küresel resmi kendisine benzer bir eğridir.

2.3 Sabit Eğriliğe ve Sabit Olmayan Torsiyona Sahip Eğrilerin Küresel Resimleri

Bu durum (2.1) eşitliğindeki f, g, h değerlerine göre iki farklı durumda incelenebilir.

***i.* α eğrisinin rektifiyan eğri olması durumu**

$\alpha(s)$ eğrisi sabit eğriliğe ve sabit olmayan torsiyona sahip bir rektifiyan eğri ise kendi yer vektörünün, eğrinin asli normali üzerinde bileşeni yoktur. Bu göre (2.1) eşitliği

$$\alpha(s) = f(s)T_\alpha + h(s)B_\alpha(s)$$

şeklinde yazılır ve $\alpha(s)$ rektifiyan eğrisinin Frenet vektörleri

$$T_\beta = u_1 T_\alpha + u_3 B_\alpha$$

$$N_\beta = n_1 T_\alpha + n_2 N_\alpha + n_3 B_\alpha$$

$$B_\beta = (u_2 n_3 - u_3 n_2) T_\alpha + (u_3 n_1 - u_1 n_3) N_\alpha + (u_1 n_2 - u_2 n_1) B_\alpha$$

olup burada $u_1, u_2, u_3, n_1, n_2, n_3, v_1, v_2, v_3$ aşağıdaki gibidir.

$$u_1=\frac{\sigma' f+\sigma}{\rho} \qquad u_2=0 \qquad u_3=\frac{\sigma' h}{\rho}$$

$$n_1=\frac{v_1}{\sqrt{v_1^{\,2}+v_2^{\,2}+v_3^{\,2}}} \qquad n_2=\frac{v_2}{\sqrt{v_1^{\,2}+v_2^{\,2}+v_3^{\,2}}} \qquad n_3=\frac{v_3}{\sqrt{v_1^{\,2}+v_2^{\,2}+v_3^{\,2}}}$$

$$v_1=\frac{u_1'}{\rho} \quad v_2=\frac{u_1\kappa_\alpha-u_3\tau_\alpha}{\rho} \quad v_3=\frac{u_3'}{\rho}$$

β eğrisinin eğriliği

$$\kappa_\beta=\frac{1}{B^2c^{9/2}}\left(\sum_{i=0}^{10} r_i f^i\right)^{1/2} \tag{2.13}$$

olup burada $r_5=r_7=r_9=0$ ve diğerleri

$r_0=c^6B^2\left(h^2+\kappa_\alpha{}^2c^2\right)$

$r_1=2\kappa_\alpha h\tau_\alpha B^2c^7$

$r_2=Bc^4\left(B\tau_\alpha{}^2h^2c^2-6Bh^2c+4h^2-2B\kappa_\alpha{}^2c^3+9B^2c^2-12Bc+4\right)$

$r_3=-2B^2\kappa_\alpha\tau_\alpha hc^6$

$r_4=c^2\left(9B^2h^2c^2-16Bh^2c+4h^2+B^2\kappa_\alpha{}^2c^4-18B^3c^3+36B^2c^2-16Bc\right)$

$r_6=c\left(9B^3c^3-8h^2-36c^2B^2+24cB+12cBh^2\right)$

$r_8=4h^2+12B^2c^2-16Bc$

$r_{10}=4B$

şeklinde olur. β eğrisinin burulması da

$$\tau_\beta = \frac{\sum_{i=0}^{11} p_i f^i}{B^{3/2} c^{3/2} \sum_{j=0}^{10} r_j f^j} \qquad \textbf{(2.14)}$$

şeklinde olup burada

$$p_0 = c^7 B^{5/2} \tau_\alpha \left(c^2 \kappa_\alpha{}^2 + 2h^2 \right)$$

$$p_1 = -hc^6 \left(c^2 B^{5/2} \kappa_\alpha{}^3 + 6cB^3 \kappa_\alpha - 4B^2 \kappa_\alpha - 6cB^{5/2} \kappa_\alpha + 4B^{3/2} \kappa_\alpha - 2c^2 B^{5/2} \tau_\alpha{}^2 \kappa_\alpha - hcB^{5/2} \tau_\alpha{}' \right)$$

$$p_2 = c^5 \tau_\alpha \left(c^2 B^{5/2} h^2 \tau_\alpha{}^2 - 2h^2 B^2 - 2c^2 B^{5/2} h^2 \kappa_\alpha{}^2 - 3c^3 B^{5/2} \kappa_\alpha{}^2 + 2h^2 B^{3/2} - 5cB^{5/2} h^2 + 3cB^3 h^2 \right)$$

$$p_3 = c^4 h \begin{pmatrix} -h^2 c^2 B^{5/2} \tau_\alpha{}^2 \kappa_\alpha + 24cB^{3/2} \kappa_\alpha - 9c^2 B^{7/2} \kappa_\alpha - 4c^3 B^{5/2} \tau_\alpha{}^2 \kappa_\alpha - 18c^2 B^{5/2} \kappa_\alpha \\ +12cB^{5/2} \kappa_\alpha + 2c^3 B^{5/2} \kappa_\alpha{}^3 - 4B^{3/2} \kappa_\alpha - 8\kappa_\alpha \sqrt{B} + 8B\kappa_\alpha - 30cB^2 \kappa_\alpha + 27c^2 B^3 \kappa_\alpha \\ +2chB^{3/2} \tau_\alpha{}' + 3c^2 hB^3 \tau_\alpha{}' - 2chB^2 \tau_\alpha{}' - 4c^2 hB^{5/2} \tau_\alpha{}' \end{pmatrix}$$

$$p_4 = c^3 \tau_\alpha \begin{pmatrix} \left(3c^4 B^{5/2} + 2c^3 B^{5/2} h^2 \right) \kappa_\alpha{}^2 \\ -c^3 B^{5/2} h^2 \tau_\alpha{}^2 - 4h^2 \sqrt{B} - 4cB^{3/2} h^2 + 10ch^2 B^2 - 6c^2 h^2 B^3 + 6c^2 h^2 B^{5/2} \end{pmatrix}$$

$$p_5 = hc^3 \begin{pmatrix} -B^{5/2} \kappa_\alpha{}^3 c^3 \\ -\begin{pmatrix} 36c^2 B^3 - 66cB^2 + 36cB^{5/2} - 16B^{3/2} - 18c^2 B^{7/2} \\ +32B - 18c^2 B^{5/2} - 32\sqrt{B} - 2c^3 B^{5/2} \tau_\alpha{}^2 + 48cB^{3/2} \end{pmatrix} \kappa_\alpha \\ -4hcB^{3/2} \tau_\alpha{}' + 3hc^2 B^{5/2} \tau_\alpha{}' + 4hcB^2 \tau_\alpha{}' - 3hc^2 B^3 \tau_\alpha{}' \end{pmatrix}$$

$$p_6 = c^2 \tau_\alpha \left(-c^4 B^{5/2} \kappa_\alpha{}^2 + 3c^2 B^3 h^2 - 3c^2 B^{5/2} h^2 + 2cB^{3/2} h^2 - 14cB^2 h^2 + 12h^2 \sqrt{B} \right)$$

$$p_7 = c^2 h \kappa_\alpha \begin{pmatrix} 15c^2 B^3 - 6c^2 B^{5/2} - 9c^2 B^{7/2} - 58cB^2 + 40cB^{3/2} \\ +36cB^{5/2} + 48B - 24B^{3/2} - 48\sqrt{B} \end{pmatrix}$$
$$-2c^2 h \tau_\alpha{}' \left(chB^2 - chB^{3/2} \right)$$

$$p_8 = 6ch^2 \tau_\alpha \left(cB^2 - 2\sqrt{B} \right)$$

$$p_9 = 2ch\kappa_\alpha \left(9cB^2 - 6B^{3/2}c - 16B + 16\sqrt{B} + 8B^{3/2} - 6cB^{5/2}\right)$$

$$p_{10} = 4h^2\tau_\alpha \sqrt{B}$$

$$p_{11} = 4\kappa_\alpha h\left(2B - 2\sqrt{B} - B^{3/2}\right)$$

şeklinde olur. p_{10} dan dolayı şu sonucu verebiliriz.

Sonuç 2.8: *f,g,h* sıfırdan farklı sabitler olmak üzere yer vektörü (2.1) eşitliğinde verilen sabit eğrilik ve sabit olmayan torsiyona sahip rektifiyan eğrilerin küresel resmi düzlemsel bir eğri değildir.

(2.13) ve (2.14) eşitliklerinden $\dfrac{\tau_\beta}{\kappa_\beta}$ oranının türevi hesaplanırsa

$$\left(\frac{\tau_\beta}{\kappa_\beta}\right)' = \frac{D}{E}$$

elde edilir. Burada *D* ve *E* şunlardır.

$$D = \sum_{i=0}^{21} d_i h_i \tag{2.15}$$

$$d_0 = A^2\tau_\alpha{}^3\tau_\alpha{}'\kappa_\alpha f^{23}$$

$$d_1 = 3A^2\tau_\alpha{}^2\kappa_\alpha{}^2\tau_\alpha{}' f^{22} - A^2\tau_\alpha{}^3 f^{21}\left(\kappa_\alpha{}^2 + \tau_\alpha{}^2\right)$$

$$d_2 = A^2\tau_\alpha f^{21}\left(10\tau_\alpha{}'\tau_\alpha{}^2\kappa_\alpha + 3\kappa_\alpha{}^3\tau_\alpha{}'\right) - 3A^2\tau_\alpha{}^2\kappa_\alpha f^{20}\left(\kappa_\alpha{}^2 + \tau_\alpha{}^2\right)$$

$$d_3 = A^2\kappa_\alpha f^{20}\left(30\tau_\alpha{}'\tau_\alpha{}^2\kappa_\alpha + \kappa_\alpha{}^3\tau_\alpha{}'\right)$$
$$+ A^2\tau_\alpha f^{19}\left(\tau_\alpha{}''\tau_\alpha - 9\tau_\alpha{}^4 - 12\kappa_\alpha{}^2\tau_\alpha{}^2 - 3\tau_\alpha{}'^2 - 3\kappa_\alpha{}^4\right)$$
$$-10A^2\tau_\alpha{}'\tau_\alpha{}^2 f^{18} + A\tau_\alpha f^{17}\left(\tau_\alpha{}''\tau_\alpha - 15A\tau_\alpha{}^2 - 3\tau_\alpha{}'^2\right)$$
$$-12A\tau_\alpha{}'\tau_\alpha{}^2 f^{16} - 20A\tau_\alpha{}^3 f^{15}$$

$$d_4 = A^2 f^{19}\left(45\tau_\alpha{}'\tau_\alpha{}^3\kappa_\alpha + 30\tau_\alpha{}'\tau_\alpha\kappa_\alpha{}^3\right)$$
$$+ A^2 f^{18}\left(-28\tau_\alpha{}^2\kappa_\alpha{}^3 - \kappa_\alpha{}^5 + 2\tau_\alpha{}''\tau_\alpha\kappa_\alpha - 3\tau_\alpha{}'^2\kappa_\alpha - 27\tau_\alpha{}^4\kappa_\alpha\right)$$
$$-16\tau_\alpha{}'\tau_\alpha\kappa_\alpha A^2 f^{17} + Af^{16}\left(-42A\tau_\alpha{}^2\kappa_\alpha + 2\tau_\alpha{}''\tau_\alpha\kappa_\alpha - 3\tau_\alpha{}'^2\kappa_\alpha\right)$$
$$+ Af^{15}\left(A\tau_\alpha{}'\kappa_\alpha\tau_\alpha - 18\tau_\alpha{}'\kappa_\alpha\tau_\alpha\right) - Af^{14}\left(46\kappa_\alpha\tau_\alpha{}^2 + 4A\kappa_\alpha\tau_\alpha{}^2\right)$$

$$d_5 = A^2 f^{18}\left(10\tau_\alpha{}'\kappa_\alpha{}^4 + 135\tau_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2\right)$$
$$+ A^2 f^{17}\left(-36\tau_\alpha{}^5 + 8\tau_\alpha{}''\tau_\alpha{}^2 - 63\tau_\alpha{}^3\kappa_\alpha{}^2 - 27\tau_\alpha\kappa_\alpha{}^4 - 24\tau_\alpha{}'^2\tau_\alpha + \tau_\alpha{}''\kappa_\alpha{}^2\right)$$
$$- A^2 f^{16}\left(6\tau_\alpha{}'\kappa_\alpha{}^2 + 73\tau_\alpha{}'\tau_\alpha{}^2\right)$$
$$+ Af^{15}\left(-40A\tau_\alpha\kappa_\alpha{}^2 + \tau_\alpha{}''\kappa_\alpha{}^2 - 99A\tau_\alpha{}^3 - 18\tau_\alpha{}'^2\tau_\alpha + 6\tau_\alpha{}''\tau_\alpha{}^2\right)$$
$$+ Af^{14}\left(3A\tau_\alpha{}'\tau_\alpha{}^2 + A\tau_\alpha{}'\kappa_\alpha{}^2 - 62\tau_\alpha{}'\tau_\alpha{}^2 - 6\tau_\alpha{}'\kappa_\alpha{}^2\right)$$
$$+ Af^{13}\left(11A\tau_\alpha{}^3 - 35\kappa_\alpha{}^2\tau_\alpha - 9A\kappa_\alpha{}^2\tau_\alpha - 83\tau_\alpha{}^3\right)$$

$$d_6 = A^2 f^{17}\left(135\tau_\alpha{}'\tau_\alpha\kappa_\alpha{}^3 + 120\tau_\alpha{}'\tau_\alpha{}^3\kappa_\alpha\right)$$
$$+ A^2 f^{16}\left(-24\tau_\alpha{}'^2\kappa_\alpha - 9\kappa_\alpha{}^5 + 16\tau_\alpha{}''\tau_\alpha\kappa_\alpha - 108\tau_\alpha{}^4\kappa_\alpha - 117\tau_\alpha{}^2\kappa_\alpha{}^3\right)$$
$$-111\tau_\alpha{}'\tau_\alpha\kappa_\alpha A^2 f^{15} + Af^{14}\left(12\tau_\alpha{}''\kappa_\alpha\tau_\alpha - 13A\kappa_\alpha{}^3 - 18\tau_\alpha{}'^2\kappa_\alpha - 264A\tau_\alpha{}^2\kappa_\alpha\right)$$
$$+ Af^{13}\left(-82\tau_\alpha{}'\tau_\alpha\kappa_\alpha + 9A\tau_\alpha{}'\kappa_\alpha\tau_\alpha\right)$$
$$+ Af^{12}\left(4A\tau_\alpha{}^2\kappa_\alpha - 9\kappa_\alpha{}^3 - 5A\kappa_\alpha{}^3 - 159\kappa_\alpha\tau_\alpha{}^2\right)$$

$$d_7 = A^2 f^{16}\left(45\tau_\alpha{}'\kappa_\alpha{}^4 + 360\tau_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2\right) - A^2 f^{14}\left(+38\tau_\alpha{}'\kappa_\alpha{}^2 + 231\tau_\alpha{}'\tau_\alpha{}^2\right)$$
$$+ A^2 f^{15}\left(-84\tau_\alpha{}^5 + 28\tau_\alpha{}''\tau_\alpha{}^2 - 192\tau_\alpha{}^3\kappa_\alpha{}^2 - 108\tau_\alpha\kappa_\alpha{}^4 + 8\tau_\alpha{}''\kappa_\alpha{}^2 - 84\tau_\alpha{}'^2\tau_\alpha\right)$$
$$+ Af^{13}\left(-283A\tau_\alpha{}^3 - 242A\tau_\alpha\kappa_\alpha{}^2 + 15\tau_\alpha{}''\tau_\alpha{}^2 + A\tau_\alpha{}'' + 6\tau_\alpha{}''\kappa_\alpha{}^2 - 45\tau_\alpha{}'^2\tau_\alpha\right)$$
$$+ Af^{12}\left(15A\tau_\alpha{}'\tau_\alpha{}^2 + 6A\tau_\alpha{}'\kappa_\alpha{}^2 - 130\tau_\alpha{}'\tau_\alpha{}^2 + 8A\tau_\alpha{}' - 20\tau_\alpha{}'\kappa_\alpha{}^2\right)$$
$$+ Af^{11}\left(-28A\kappa_\alpha{}^2\tau_\alpha + \tau_\alpha{}'' + 43A\tau_\alpha{}^3 + A\tau_\alpha{}'' - 93\kappa_\alpha{}^2\tau_\alpha + 12A\tau_\alpha - 133\tau_\alpha{}^3\right)$$
$$+ Af^{10}\left(9\tau_\alpha{}' + 11A\tau_\alpha{}'\right) + Af^9\left(21A\tau_\alpha + 16\tau_\alpha + \tau_\alpha{}''\right) + 12A\tau_\alpha{}' f^8 + 28A\tau_\alpha f^7$$

$$d_8 = A^2 f^{15}\left(210\tau_\alpha{}'\tau_\alpha{}^3\kappa_\alpha + 360\tau_\alpha{}'\tau_\alpha\kappa_\alpha{}^3\right)$$
$$+ A^2 f^{14}\left(-84\tau_\alpha{}'^2\kappa_\alpha - 36\kappa_\alpha{}^5 + 56\tau_\alpha{}''\tau_\alpha\kappa_\alpha - 288\tau_\alpha{}^2\kappa_\alpha{}^3 - 252\tau_\alpha{}^4\kappa_\alpha\right)$$
$$-329\tau_\alpha{}'\tau_\alpha\kappa_\alpha A^2 f^{13} + Af^{12}\left(-45\tau_\alpha{}'^2\kappa_\alpha - 706A\tau_\alpha{}^2\kappa_\alpha + 30\tau_\alpha{}''\kappa_\alpha\tau_\alpha - 77A\kappa_\alpha{}^3\right)$$
$$+ Af^{11}\left(-140\tau_\alpha{}'\tau_\alpha\kappa_\alpha + 30A\tau_\alpha{}'\kappa_\alpha\tau_\alpha\right)$$
$$+ Af^{10}\left(-21A\kappa_\alpha{}^3 - 17\kappa_\alpha{}^3 + 52A\tau_\alpha{}^2\kappa_\alpha - 171\kappa_\alpha\tau_\alpha{}^2 + 6A\kappa_\alpha\right)$$
$$+ Af^8\left(11A\kappa_\alpha + 9\kappa_\alpha\right) + A\kappa_\alpha f^6\left(18 - 4A\right)$$

$$d_9 = A^2 f^{14}\left(120\tau_\alpha{}'\kappa_\alpha{}^4 + 630\tau_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2\right) - A^2 f^{12}\left(98\tau_\alpha{}'\kappa_\alpha{}^2 + 413\tau_\alpha{}'\tau_\alpha{}^2\right)$$
$$+ A^2 f^{13}\left(-126\tau_\alpha{}^5 + 56\tau_\alpha{}''\tau_\alpha{}^2 - 252\tau_\alpha\kappa_\alpha{}^4 - 378\tau_\alpha{}^3\kappa_\alpha{}^2 + 28\tau_\alpha{}''\kappa_\alpha{}^2 - 168\tau_\alpha{}'^2\tau_\alpha\right)$$
$$+ Af^{11}\left(15\tau_\alpha{}''\kappa_\alpha{}^2 + 6A\tau_\alpha{}'' + 20\tau_\alpha{}''\tau_\alpha{}^2 - 60\tau_\alpha{}'^2\tau_\alpha - 459A\tau_\alpha{}^3 - 612A\tau_\alpha\kappa_\alpha{}^2\right)$$
$$+ Af^{10}\left(-10\tau_\alpha{}'\kappa_\alpha{}^2 + 43A\tau_\alpha{}' + 15A\tau_\alpha{}'\kappa_\alpha{}^2 - 140\tau_\alpha{}'\tau_\alpha{}^2 + 30A\tau_\alpha{}'\tau_\alpha{}^2\right)$$
$$+ Af^9\left(4A\tau_\alpha{}'' + 62A\tau_\alpha{}^3 + 4\tau_\alpha{}'' + 58A\tau_\alpha - 25A\kappa_\alpha{}^2\tau_\alpha - 18\kappa_\alpha{}^2\tau_\alpha - 102\tau_\alpha{}^3\right)$$
$$+ Af^8\left(31\tau_\alpha{}' + 33A\tau_\alpha{}'\right) + Af^7\left(49A\tau_\alpha + 50\tau_\alpha + 2\tau_\alpha{}''\right) + 13A\tau_\alpha{}' f^6 + 9A\tau_\alpha f^5$$

$$d_{10} = A^2 f^{13}\left(630\tau_\alpha'\tau_\alpha\kappa_\alpha{}^3 + 252\tau_\alpha'\tau_\alpha{}^3\kappa_\alpha\right)$$
$$+ A^2 f^{12}\left(112\tau_\alpha''\tau_\alpha\kappa_\alpha - 84\kappa_\alpha{}^5 - 168\tau_\alpha'^2\kappa_\alpha - 462\tau_\alpha{}^2\kappa_\alpha{}^3 - 378\tau_\alpha{}^4\kappa_\alpha\right)$$
$$- 539\tau_\alpha'\tau_\alpha\kappa_\alpha A^2 f^{11} + Af^{10}\left(40\tau_\alpha''\kappa_\alpha\tau_\alpha - 1044A\tau_\alpha{}^2\kappa_\alpha - 60\tau_\alpha'^2\kappa_\alpha - 189A\kappa_\alpha{}^3\right)$$
$$+ Af^9\left(-100\tau_\alpha'\tau_\alpha\kappa_\alpha + 50A\tau_\alpha'\kappa_\alpha\tau_\alpha\right) + Af^6\left(15A\kappa_\alpha + 25\kappa_\alpha\right) - 8A\kappa_\alpha f^4$$
$$+ Af^8\left(88A\tau_\alpha{}^2\kappa_\alpha - 14\kappa_\alpha\tau_\alpha{}^2 + 26A\kappa_\alpha + 20\kappa_\alpha{}^3 - 34A\kappa^3\right)$$

$$d_{11} = A^2 f^{12}\left(210\tau_\alpha'\kappa_\alpha{}^4 + 756\tau_\alpha'\tau_\alpha{}^2\kappa_\alpha{}^2\right)$$
$$+ A^2 f^{13}\left(-378\tau_\alpha\kappa_\alpha{}^4 - 210\tau_\alpha'^2\tau_\alpha - 126\tau_\alpha{}^5 - 504\tau_\alpha{}^3\kappa_\alpha{}^2 + 70\tau_\alpha''\tau_\alpha{}^2 + 56\tau_\alpha''\kappa_\alpha{}^2\right)$$
$$- A^2 f^{10}\left(126\tau_\alpha'\kappa_\alpha{}^2 + 455\tau_\alpha'\tau_\alpha{}^2\right)$$
$$+ Af^9\left(20\tau_\alpha''\kappa_\alpha{}^2 + 15A\tau_\alpha'' + 15\tau_\alpha''\tau_\alpha{}^2 - 45\tau_\alpha'^2\tau_\alpha - 465A\tau_\alpha{}^3 - 830A\tau_\alpha\kappa_\alpha{}^2\right)$$
$$+ Af^8\left(40\tau_\alpha'\kappa_\alpha{}^2 + 95A\tau_\alpha' + 20A\tau_\alpha'\kappa_\alpha{}^2 - 80\tau_\alpha'\tau_\alpha{}^2 + 30A\tau_\alpha'\tau_\alpha{}^2\right)$$
$$+ Af^7\left(6A\tau_\alpha'' + 38A\tau_\alpha{}^3 + 6\tau_\alpha'' + 112A\tau_\alpha + 158\kappa_\alpha{}^2\tau_\alpha - 38\tau_\alpha{}^3\right)$$
$$+ Af^6\left(39\tau_\alpha' + 33A\tau_\alpha'\right) + Af^5\left(29A\tau_\alpha + 54\tau_\alpha + \tau_\alpha''\right) + A\tau_\alpha' f^4 - A\tau_\alpha f^3$$

$$d_{12} = A^2 f^{11}\left(756\tau_\alpha'\tau_\alpha\kappa_\alpha{}^3 + 210\tau_\alpha'\tau_\alpha{}^3\kappa_\alpha\right)$$
$$+ A^2 f^{10}\left(140\tau_\alpha''\tau_\alpha\kappa_\alpha - 126\kappa_\alpha{}^5 - 210\tau_\alpha'^2\kappa_\alpha - 504\tau_\alpha{}^2\kappa_\alpha{}^3 - 378\tau_\alpha{}^4\kappa_\alpha\right)$$
$$- 525\tau_\alpha'\tau_\alpha\kappa_\alpha A^2 f^9 + Af^8\left(30\tau_\alpha''\kappa_\alpha\tau_\alpha - 930A\tau_\alpha{}^2\kappa_\alpha - 45\tau_\alpha'^2\kappa_\alpha - 245A\kappa_\alpha{}^3\right)$$
$$+ Af^7\left(-10\tau_\alpha'\tau_\alpha\kappa_\alpha + 45A\tau_\alpha'\kappa_\alpha\tau_\alpha\right)$$
$$+ Af^6\left(52A\tau_\alpha{}^2\kappa_\alpha + 84\kappa_\alpha\tau_\alpha{}^2 + 44A\kappa_\alpha + 70\kappa_\alpha{}^3 - 26A\kappa^3\right)$$
$$+ Af^4\left(-3A\kappa_\alpha + 21\kappa_\alpha\right) + 4A\kappa_\alpha f^2$$

$$d_{13} = A^2 f^{10}\left(252\tau_\alpha'\kappa_\alpha{}^4 + 630\tau_\alpha'\tau_\alpha{}^2\kappa_\alpha{}^2\right)$$
$$+ A^2 f^9\left(-378\tau_\alpha\kappa_\alpha{}^4 - 168\tau_\alpha'^2\tau_\alpha - 84\tau_\alpha{}^5 - 462\tau_\alpha{}^3\kappa_\alpha{}^2 + 56\tau_\alpha''\tau_\alpha{}^2 + 70\tau_\alpha''\kappa_\alpha{}^2\right)$$
$$- A^2 f^8\left(70\tau_\alpha'\kappa_\alpha{}^2 + 315\tau_\alpha'\tau_\alpha{}^2\right) + Af^4\left(21\tau_\alpha' + 11A\tau_\alpha'\right) + Af^3\left(-5A\tau_\alpha + 22\tau_\alpha\right)$$
$$+ Af^7\left(15\tau_\alpha''\kappa_\alpha{}^2 + 20A\tau_\alpha'' + 6\tau_\alpha''\tau_\alpha{}^2 - 18\tau_\alpha'^2\tau_\alpha - 305A\tau_\alpha{}^3 - 640A\tau_\alpha\kappa_\alpha{}^2\right)$$
$$+ Af^6\left(70\tau_\alpha'\kappa_\alpha{}^2 + 110A\tau_\alpha' + 15A\tau_\alpha'\kappa_\alpha{}^2 - 22\tau_\alpha'\tau_\alpha{}^2 + 15A\tau_\alpha'\tau_\alpha{}^2\right)$$
$$+ Af^5\left(4A\tau_\alpha'' + 7A\tau_\alpha{}^3 + 4\tau_\alpha'' + 108A\tau_\alpha + 5A\kappa_\alpha{}^2\tau_\alpha + 177\kappa_\alpha{}^2\tau_\alpha - 7\tau_\alpha{}^3\right)$$

$$d_{14} = A^2 f^9 \left(630\tau_\alpha{}'\tau_\alpha\kappa_\alpha{}^3 + 120\tau_\alpha{}'\tau_\alpha{}^3\kappa_\alpha\right)$$
$$+ A^2 f^8 \left(112\tau_\alpha{}''\tau_\alpha\kappa_\alpha - 126\kappa_\alpha{}^5 - 168\tau_\alpha{}'^2\kappa_\alpha - 378\tau_\alpha{}^2\kappa_\alpha{}^3 - 252\tau_\alpha{}^4\kappa_\alpha\right)$$
$$- 301\tau_\alpha{}'\tau_\alpha\kappa_\alpha A^2 f^7 + Af^6\left(12\tau_\alpha{}''\kappa_\alpha\tau_\alpha - 512A\tau_\alpha{}^2\kappa_\alpha - 18\tau_\alpha{}'^2\kappa_\alpha - 175A\kappa_\alpha{}^3\right)$$
$$+ Af^5\left(22\tau_\alpha{}'\tau_\alpha\kappa_\alpha + 21A\tau_\alpha{}'\kappa_\alpha\tau_\alpha\right) + Af^2\left(-7A\kappa_\alpha + 3\kappa_\alpha\right)$$
$$+ Af^4\left(4A\tau_\alpha{}^2\kappa_\alpha + 45\kappa_\alpha\tau_\alpha{}^2 + 36A\kappa_\alpha + 55\kappa_\alpha{}^3 - 9A\kappa^3\right)$$

$$d_{15} = A^2 f^8\left(210\tau_\alpha{}'\kappa_\alpha{}^4 + 360\tau_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2\right)$$
$$+ A^2 f^7\left(-252\tau_\alpha\kappa_\alpha{}^4 - 84\tau_\alpha{}'^2\tau_\alpha - 36\tau_\alpha{}^5 - 288\tau_\alpha{}^3\kappa_\alpha{}^2 + 28\tau_\alpha{}''\tau_\alpha{}^2 + 56\tau_\alpha{}''\kappa_\alpha{}^2\right)$$
$$- A^2 f^6\left(-14\tau_\alpha{}'\kappa_\alpha{}^2 + 133\tau_\alpha{}'\tau_\alpha{}^2\right) + 4Af^2\tau_\alpha{}' + Af(2\tau_\alpha - 6A\tau_\alpha)$$
$$+ Af^5\left(6\tau_\alpha{}''\kappa_\alpha{}^2 + 15A\tau_\alpha{}'' + \tau_\alpha{}''\tau_\alpha{}^2 - 3\tau_\alpha{}'^2\tau_\alpha - 129A\tau_\alpha{}^3 - 270A\tau_\alpha\kappa_\alpha{}^2\right)$$
$$+ Af^4\left(44\tau_\alpha{}'\kappa_\alpha{}^2 + 70A\tau_\alpha{}' + 6A\tau_\alpha{}'\kappa_\alpha{}^2 - 2\tau_\alpha{}'\tau_\alpha{}^2 + 3A\tau_\alpha{}'\tau_\alpha{}^2\right)$$
$$+ Af^3\left(A\tau_\alpha{}'' - A\tau_\alpha{}^3 + \tau_\alpha{}'' + 52A\tau_\alpha - 4A\kappa_\alpha{}^2\tau_\alpha + 63\kappa_\alpha{}^2\tau_\alpha - \tau_\alpha{}^3\right)$$

$$d_{16} = A^2 f^7\left(360\tau_\alpha{}'\tau_\alpha\kappa_\alpha{}^3 + 45\tau_\alpha{}'\tau_\alpha{}^3\kappa_\alpha\right)$$
$$+ A^2 f^6\left(56\tau_\alpha{}''\tau_\alpha\kappa_\alpha - 84\kappa_\alpha{}^5 - 84\tau_\alpha{}'^2\kappa_\alpha - 192\tau_\alpha{}^2\kappa_\alpha{}^3 - 108\tau_\alpha{}^4\kappa_\alpha\right)$$
$$- 91\tau_\alpha{}'\tau_\alpha\kappa_\alpha A^2 f^5 + Af^4\left(2\tau_\alpha{}''\kappa_\alpha\tau_\alpha - 174A\tau_\alpha{}^2\kappa_\alpha - 3\tau_\alpha{}'^2\kappa_\alpha - 63A\kappa_\alpha{}^3\right)$$
$$+ Af^3\left(8\tau_\alpha{}'\tau_\alpha\kappa_\alpha + 4A\tau_\alpha{}'\kappa_\alpha\tau_\alpha\right)$$
$$+ Af^2\left(-4A\tau_\alpha{}^2\kappa_\alpha + 5\kappa_\alpha\tau_\alpha{}^2 + 14A\kappa_\alpha + 11\kappa_\alpha{}^3 - A\kappa^3\right) - 2A\kappa_\alpha$$

$$d_{17} = A^2 f^6\left(120\tau_\alpha{}'\kappa_\alpha{}^4 + 135\tau_\alpha{}'\tau_\alpha{}^2\kappa_\alpha{}^2\right) - A^2 f^4\left(-42\tau_\alpha{}'\kappa_\alpha{}^2 + 31\tau_\alpha{}'\tau_\alpha{}^2\right)$$
$$+ A^2 f^5\left(-108\tau_\alpha\kappa_\alpha{}^4 - 24\tau_\alpha{}'^2\tau_\alpha - 9\tau_\alpha{}^5 - 117\tau_\alpha{}^3\kappa_\alpha{}^2 + 8\tau_\alpha{}''\tau_\alpha{}^2 + 28\tau_\alpha{}''\kappa_\alpha{}^2\right)$$
$$+ Af^3\left(\tau_\alpha{}''\kappa_\alpha{}^2 + 6A\tau_\alpha{}'' + \tau_\alpha{}''\tau_\alpha{}^2 - 3\tau_\alpha{}'^2\tau_\alpha - 33A\tau_\alpha{}^3 - 52A\tau_\alpha\kappa_\alpha{}^2\right)$$
$$+ Af^2\left(10\tau_\alpha{}'\kappa_\alpha{}^2 + 23A\tau_\alpha{}' + A\tau_\alpha{}'\kappa_\alpha{}^2\right) + Af\left(10A\tau_\alpha - 3A\kappa_\alpha{}^2\tau_\alpha + 4\kappa_\alpha{}^2\tau_\alpha\right)$$

$$d_{18} = A^2 f^5\left(135\tau_\alpha{}'\tau_\alpha\kappa_\alpha{}^3 + 10\tau_\alpha{}'\tau_\alpha{}^3\kappa_\alpha\right)$$
$$+ A^2 f^4\left(16\tau_\alpha{}''\tau_\alpha\kappa_\alpha - 36\kappa_\alpha{}^5 - 24\tau_\alpha{}'^2\kappa_\alpha - 63\tau_\alpha{}^2\kappa_\alpha{}^3 - 27\tau_\alpha{}^4\kappa_\alpha\right)$$
$$- 9\tau_\alpha{}'\tau_\alpha\kappa_\alpha A^2 f^3 + Af^2\left(-36A\tau_\alpha{}^2\kappa_\alpha - 7A\kappa_\alpha{}^3\right) + A\left(2A\kappa_\alpha - 2\kappa_\alpha{}^3\right)$$

$$d_{19}=A^2f^4\left(45\tau_\alpha'\kappa_\alpha^{\,4}+30\tau_\alpha'\tau_\alpha^{\,2}\kappa_\alpha^{\,2}\right)$$
$$+A^2f^3\left(-27\tau_\alpha\kappa_\alpha^{\,4}-3\tau_\alpha'^2\tau_\alpha-\tau_\alpha^{\,5}-28\tau_\alpha^{\,3}\kappa_\alpha^{\,2}+\tau_\alpha''\tau_\alpha^{\,2}+8\tau_\alpha''\kappa_\alpha^{\,2}\right)$$
$$-A^2f^2\left(-22\tau_\alpha'\kappa_\alpha^{\,2}+3\tau_\alpha'\tau_\alpha^{\,2}\right)+Af\left(-2A\tau_\alpha\kappa_\alpha^{\,2}+A\tau_\alpha''-4A\tau_\alpha^{\,3}\right)+3A^2\tau_\alpha'$$

$$d_{20}=A^2f^3\left(30\tau_\alpha'\tau_\alpha\kappa_\alpha^{\,3}+\tau_\alpha'\tau_\alpha^{\,3}\kappa_\alpha\right)$$
$$+A^2f^2\left(2\tau_\alpha''\tau_\alpha\kappa_\alpha-9\kappa_\alpha^{\,5}-3\tau_\alpha'^2\kappa_\alpha-12\tau_\alpha^{\,2}\kappa_\alpha^{\,3}-3\tau_\alpha^{\,4}\kappa_\alpha\right)$$
$$+\tau_\alpha'\tau_\alpha\kappa_\alpha A^2f+A^2\left(\kappa_\alpha^{\,3}-4\tau_\alpha^{\,2}\kappa_\alpha\right)$$

$$d_{21}=A^2f^2\left(10\tau_\alpha'\kappa_\alpha^{\,4}+3\tau_\alpha'\tau_\alpha^{\,2}\kappa_\alpha^{\,2}\right)+A^2f\left(-3\tau_\alpha\kappa_\alpha^{\,4}-3\tau_\alpha^{\,3}\kappa_\alpha^{\,2}+\tau_\alpha''\kappa_\alpha^{\,2}\right)$$
$$+A^2\left(4\tau_\alpha'\kappa_\alpha^{\,2}-7\kappa_\alpha'\tau_\alpha\kappa_\alpha\right)$$

şeklinde olur. (2.15) eşitliğinde D=0 yapan uydun κ_α ve τ_α değerleri olmadığından şu sonucu verebiliriz.

Sonuç 2.9: Sabit eğriliğe ve sabit olmayan torsiyona sahip rektifiyan eğrilerin küresel resimleri bir helis değildir.

***ii. f, g* ve *h* sıfırdan farklı sabit olma durumu**

Sabit eğriliğe ve sabit olmayan torsiyona sahip eğrilerin eğriliği ve burulması bölüm (2.1) *(ii)'* deki değerlerle aynı olur.Şu sonucu verebiliriz.

Sonuç 2.10: *f,g,h* sıfırdan farklı sabitler olmak üzere yer vektörü 2.1 eşitliğinde verilen sabit eğrilik ve sabit olmayan torsiyona sahip $\alpha(s)$ eğrisinin küresel resmi kendisine benzer bir eğridir.

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