T.C.
TURGUT ÖZAL UNIVERSITY
GRADUATE SCHOOL OF SCIENCE AND ENGINEERING
ELECTRICAL AND COMPUTER ENGINEERING

# THRESHOLD OPTIMIZATION IN DECENTRALIZED DETECTION PROBLEMS AND OPTIMAL JAMMING IN THE PRESENCE OF UNCERTAINTY

A THESIS
FOR THE DEGREE OF MASTER OF SCIENCE AND ENGINEERING

By
Hakan SOKU

SUPERVISOR
Asst. Prof. Dr. Suat BAYRAM

Ankara, 2015

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## ACADEMICAL ETHICS DECLARATION PAGE

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

25/12/2015

Hakan SOKU

## APPROVAL

"Threshold Optimization In Decentralized Detection Problems And Optimal Jamming In The Presence Of Uncertainty" submitted by Hakan SOKU in partial fulfillment of the requirements as a thesis, 25/12/2015, for the degree of Master of Science in Electrical and Computer Engineering Department, Turgut Ozal University by,

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**Asst. Prof. Dr. Suat BAYRAM (Supervisor)**

**……….**

**Prof. Dr. Etem KÖKLÜKAYA**

**……….**

**Asst. Prof. Dr. Ali ALPAN**

# PREFACE

This thesis consists of two different parts. In the first part of this thesis, in a decentralized detection problem, it is shown that setting thresholds as random variables instead of deterministic ones can improve the performance according to the restricted Bayes criterion. And the second part of the thesis, in the presence of partial information, it is obtained that the optimal policy for an average power constrained jammer is to allocate its power among at most three different power levels.

I would like to, especially, thank Asst. Prof. Dr. Suat Bayram for providing me great research opportunities and environment as my advisor. His patience, generosity and inspirational personality have been a great admiration for me. Also I would like to thanks Prof. Dr. Etem Köklükaya and Asst. Prof. Dr. Ali Alpan for agreeing to serve on my thesis committee.

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# ÖZET

SOKU, Hakan. Merkezi Olmayan Sezim Problemlerinde Eşik Değer Optimizasyonu Ve Belirsizlik Varliğinda En Uygun Sinyal Boğma Stratejisi, Yüksek Lisans tezi, Ankara, 2015

Tezin ilk kısmında, kısıtlı Bayes yaklaşımı ile merkezi olmayan sezim problemleri üzerinde çalışılmaktadır. Hangi Hipotezin doğru olduğu ile ilgili tüm kararlar, şartlı bağımsız gözlemler yoluyla yerel sensörler tarafından verilir. Sonrasında, bu kararlar nihai karar için füzyon merkezine iletilir. Geleneksel yaklaşımda, yerel sensörlerin ve füzyon merkezinin tüm eşik değerleri rasgele olmayan değişkenler olarak düşünülmüş, yerel sensörler ve füzyon merkezinin test istatistikleri için verilen kritere göre optimize edilmektedir. Bu kısımda, kısıtlı Bayes kriterine göre, eşik değerlerin rasgele olmayan değişkenler yerine rasgele değişkenler olarak atanması sonucunda performansın arttığı görülmektedir. Simulasyonlar yoluyla teorik sonuçlar incelenmektedir.

Tezin ikinci kısmında, ortalama bir güçle sınırlandırılmış, hedef sinyal hakkında kısmi bilgiler bulunduran sinyal boğucu için optimal sinyal boğma stratejisi elde edilmektedir. Neyman-Pearson çerçevesinde ele alınmakta ve kısmı bilginin sebep olduğu belirsizliği işlemek amacıyla Hodges-Lehmann Kuralı kullanılmaktadır. Amacımız, Sinyal bozucudaki kısmi bilgi miktarına göre ayarlanmış eşik değer seviyesinin altındaki bir alıcının uygun olan en küçük sezim olasılığını muhafaza ederken, akıllı alıcı sisteminin "beklenen" sezim olasılığına NP kriterine göre olabilecek en küçük değeri vermektir. Kısmi bilginin varlığında, ortalama bir güç ile sınırandırılmış sinyal boğucu için en uygun ilke olarak, en fazla üç farklı güç seviyesi arasından kendi gücünü ayırabildiği sonucuna varılmıştır. Bunlara ilave olarak, güç rasgeleleştirmesinin rasgele olmayan sinyal boğma yaklaşımları üzerinde gelişmeler sağlayıp sağlayamayacağı senaryolarını belirlemek için yeter koşullar elde edilmektedir. Son olarak, teorik bulguları incelemek için simülasyonlar sunulmaktadır.

**Anahtar Sözcükler:**

1. Merkezi olmayan sezim
2. Rasgele eşik değer
3. Kısıtlı Bayes
4. Sinyal boğma
5. Sezim
6. Radar

## ABSTRACT

SOKU, Hakan. Threshold Optimization In Decentralized Detection Problems And Optimal Jamming In The Presence Of Uncertainty, Master Thesis, Ankara, 2015

In the first part of this thesis, restricted Bayes approach is studied in a decentralized detection problem. All decisions on which hypothesis is true are made by local sensors through conditionally independent observations. Then, these decisions are transmitted to the fusion center for the final decision. In the conventional approach, all thresholds of local sensors and the fusion center are considered as deterministic variables and optimized according to the given criterion for given test statistics of local sensors and the fusion center. In this part, it is shown that setting thresholds as random variables instead of deterministic ones can improve the performance according to the restricted Bayes criterion. Theoretical results are investigated through simulations.

In the second part of the thesis, the optimal jamming strategy is obtained for an average power constrained jammer that operates in the presence of partial information about the target signal. A Neyman-Pearson (NP) framework is considered, and the Hodges-Lehmann rule is employed in order to handle the uncertainty caused by partial information. The goal is to make the "expected" detection probability of a smart receiver operating according to the NP criterion as minimum as possible while keeping the least favorable detection probability of the receiver below a threshold level that is set based on the amount of partial information at the jammer. It is obtained that, in the presence of partial information, the optimal policy for an average power constrained jammer is to allocate its power among at most three different power levels. In addition, sufficient conditions are obtained to determine scenarios in which power randomization can or cannot provide improvements over the deterministic (i.e., fixed power) jamming approach. Finally, Simulations are performed in order to investigate the theoretical findings.

**Key Words:**

1. Decentralized detection
2. Random threshold

**3.** Restricted Bayes
**4.** Jamming
**5.** Detection
**6.** Radar

# CONTENTS

## SYMBOLS AND ABBREVIATIONS

| | |
|---|---|
| SNR | : Signal to Noise Ratio |
| PDF | : Probability Density Function |
| NP | : Neyman – Pearson |
| AWGN | : Additive White Gaussian Noise |
| ML | : Maximum Likelihood |
| BPSK | : Binary Phase Shift Keying |
| BASC | : Binary Asymmetric Channel |
| PSO | : Particle Swarm Optimization |

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# INTRODUCTION

Decentralized detection problem was first presented in [1], and have been studied extensively in recent years. A limited capacity of the wireless channel is one of the most important issues raised in decentralized detection problems. In [2], decentralized detection problem is studied in a binary hypothesis-testing framework under a limitation on the capacity of wireless channel, over which the maximum transmission rate is specified by R bits of information per unit time. It is shown that using R identical binary sensors is asymptotically the optimal strategy, where the number of observation at the local sensors converges to infinity and observations are modeled as an identical and independent Gaussian or exponential random variables. As an alternative approach to addressing limited capacity of the wireless channel, data reduction at the local sensors is considered in terms of the optimal quantizer design according to both Bayesian and Neyman-Pearson criteria [3]. In the presence of a full uncertainty about a distribution of the additive noise at the local sensors, a universal decentralized scheme is proposed in [4], which is operating under the bandwidth constrained communication channels between sensors and the fusion center. The error probability is shown to decay exponentially with a rate which is bounded from below by the noise range for bounded noise and by SNR for unbounded noise.

The frequency of occurrence of one hypothesis happens to be much higher than the others under some scenarios in decentralized detection problems [5, 6]. In those circumstances, the censoring scheme is proposed for sensor networks operating under the limited energy resources and limited wireless channel capacity in order to exploit this gap between the frequencies of occurrence of hypotheses [5]. Under the censoring scheme, local sensors do not transmit the observation for which the corresponding value of the local likelihood ratio is in the censoring interval, which means that the related observation is not considered as informative and simply discarded [5]. The censoring scheme is also studied for the sensor networks including randomized decision rules and operating in presence of uncertainty about the observations at the local sensors [6].

In the collaborative human decision making problem under a binary hypothesis testing framework, thresholds used by the individual human agents to decide on which hypothesis is true are modeled as random variables because of mainly unpredictability and cognitive limitations of humans [7]. Decentralized detection schemes find extensive applications areas in defense systems; thereby it is important to develop novel schemes performing better than conventional ones. To that end, in Chapter 1, thresholds of local sensors and the fusion center are set as random variables to improve the performance over the conventional approach, in which thresholds are deterministic, according to restricted Bayes criterion in a decentralized detection problem under a binary hypothesis testing framework. It turns out that optimum random thresholds are dependent on each other and the PDF (probability density function) of each consists of at most two point masses. This case can in fact be considered as dependent randomization of decision rules of local sensors and the fusion center, where each decision rule is selected from a set of deterministic decision rules based on the realization of the associated discrete random variable [8]. In our case, the set of deterministic decision rules consists of the decision rules with the same test statistics but different deterministic thresholds. Dependent randomization of decision rules is studied in [8] under the Neyman-Pearson framework in a general sense. In this part of the thesis, we focus on threshold randomization, which is a special case of randomization of decision rules, under restricted Bayes framework. Focusing on threshold randomization instead of generic randomization of decision rules provides us with doing quantitative and detailed analysis of the proposed scheme. To our best knowledge, there is no study considering effects of replacing deterministic thresholds with random ones on decentralized detection under restricted Bayes framework.

In order to implement optimal random thresholds which are dependent on each other, one way is to allow the fusion center to control all thresholds of local sensors. However, this requires extensive communication capacity and increase in the level of centralization opposed to nature of decentralized detection as stated in [8] for dependent randomization of decision rules. There are two alternative ways proposed in [8] for generic implementation of dependent randomization of decision rules under Neyman-Pearson framework. However, they can be easily adapted to our case of dependent randomization of thresholds as follows. One of alternative ways is to make all local sensors and the fusion center implement a predefined sequence of the different sets of thresholds without any communication among themselves. The other alternative is to use the common clock in all local sensors and the fusion center to choose which threshold to use.

In Chapter 1, we consider the restricted Bayes criterion to handle the uncertainty in the observations of the local sensors. More specifically, the aim of this chapter is to investigate effects of setting and accordingly optimizing thresholds as random variables instead of deterministic variables under the restricted Bayes framework, whose aim is to minimize the global probability of error while keeping the worst-case global probability of error below the predefined level [9,10]. We also consider the likelihood ratio as a test statistic for the fusion center, which is a common use in practice. Uncertainty in the transmission of decisions of local sensors to the fusion center is modeled by a binary asymmetric channel (BASC).

To the best of our knowledge, restricted Bayes approach has never been studied before in decentralized detection problems. Therefore, this thesis also aims to illustrate the use of this approach in the presence of uncertainty about observations at the local sensors. Along with the illustration of restricted Bayes approach in sensor networks, it is also shown how to optimize restricted Bayes criterion by employing dependent random thresholds at the local sensors and the fusion center.

The effects of power randomization (time sharing) have been investigated for average power constrained jammers in some studies such as [11-13]. The power randomization for symmetric unimodal jammer noise is investigated in the presence of binary signaling and the maximum likelihood (ML) receivers in [11]. Allocating the power of the jammer between two different power levels is shown to maximize the average error probability under some conditions. It is proved that the optimal allocation policy is to randomize the jammer power between at most two different levels. The same problem is studied in [12] for the $M$-ary case and the ML receiver operating over an additive white Gaussian noise (AWGN) channel. Achieving an increase in the symbol error rate is shown to be possible through randomizing the jammer power. Various optimal power allocation policies corresponding to different signal-to-noise ratio (SNR) conditions are developed.

The optimal probability density function (PDF) for a jammer is investigated in [14] for the case of a smart jammer subject to a power constraint and a smart receiver operating under the Neyman-Pearson (NP) criterion, both of which are instantaneously kept fully informed of each other. It is proved that the optimal PDF converges to Gaussian under certain conditions. In [15], the designs of a filter function at a smart receiver aiming to maximize the SNR and the spectral density of a noise of a smart jammer aiming to minimize the SNR are studied in a game theory framework.

In [13], unlike [11] and [12], generic PDFs for a jammer running over an arbitrary additive noise channel and generic detectors at the receiver running under the NP framework are considered in developing the optimal power allocation strategy for a jammer, where the goal is to make the detection probability at the receiver as minimum as possible. Both the jammer and the receiver are smart in a way that they have full information about each other. It is shown that randomizing the jammer power between at most two different levels is the optimal allocation strategy for an average power constrained jammer. In practice, however, a smart jammer needs to learn about a receiver by certain means such as previous measurements (experience). For this reason, in most cases, a jammer has partial instead of full information about the receiver. To the best of our knowledge, the optimal power distribution of an average power constrained jammer has not been characterized in the presence of partial information about the receiver in the previous studies.

In Chapter 2, it is shown that an average power constrained jammer can handle partial information by allocating its power among at most three different levels instead of two different levels, which is the optimal power allocation policy when the jammer has full information on the receiver as discussed in [13]. The Hodges-Lehmann rule (restricted Bayes approach) [9,10] is adopted to obtain the optimal policy of allocating the power of a jammer running over an arbitrary noise channel in the presence of partial information. The goal is reducing the "expected" detection probability of a smart receiver running under the NP framework and under the constraint of keeping the least favorable detection probability of the receiver below a predefined level, which is determined according to the uncertainty level at the jammer. In addition, sufficient conditions are derived to check if power randomization can provide benefits with respect to a fixed power jamming strategy. Finally, simulations are performed to investigate theoretical findings.

# CHAPTER 1

# THRESHOLD OPTIMIZATION ACCORDING TO RESTRICTED BAYES CRITERION IN DECENTRALIZED DETECTION PROBLEMS

In Section 1.1, the problem formulation is given, and derivations for optimum random thresholds are provided. Section 1.2 studies the statistical characterization of the optimum random thresholds along with the calculation of the optimal PDFs of thresholds. A sufficient and necessary condition for the improvability of the conventional approach through replacing deterministic thresholds by random ones is provided in Section 1.3. Finally, Section 1.4 studies a numerical example to investigate theoretical results.

## 1.1 PROBLEM FORMULATION

Consider the decentralized detection problem, in which each of local sensors decides on which hypothesis is true then all decisions of the local sensors are transmitted to the fusion center where the final decision is made. We have $N$ local sensors and the observation vector $\mathbf{x}_i \in \mathbb{R}^K$ at the local sensor $i$ can be expressed under binary hypotheses as follows:

$$\mathcal{H}_0 \ : \ \mathbf{x}_i = \mathbf{s}_{i0} + \mathbf{n}_i \qquad , \qquad \mathcal{H}_1 \ : \ \mathbf{x}_i = \mathbf{s}_{i1} + \mathbf{n}_i \tag{1}$$

Where $\mathbf{n}_i$ is the background noise with PDF $p_{\mathbf{n}_i}(\cdot)$. The signals are modeled as random vectors $\mathbf{s}_0 = [\mathbf{s}_{10}^T \mathbf{s}_{20}^T \ldots \mathbf{s}_{N0}^T]^T$ with the estimated PDF $p_{\mathbf{s}_0}(\cdot)$ under the hypothesis $\mathcal{H}_0$ and $\mathbf{s}_1 = [\mathbf{s}_{11}^T \mathbf{s}_{21}^T \ldots \mathbf{s}_{N1}^T]^T$ with the estimated PDF $p_{\mathbf{s}_1}(\cdot)$ under the hypothesis $\mathcal{H}_1$ . In practice, true PDFs of the signals can be very different from the estimated ones due to estimation errors in obtaining $p_{\mathbf{s}_0}(\cdot)$ and $p_{\mathbf{s}_1}(\cdot)$ [9, 10]. In other words, there exists uncertainty in the PDFs of the signals. In the restricted Bayes criterion, the worst-case scenario is also involved to take this uncertainty into account [9, 10]. Therefore, in the restricted Bayes framework, the least-favorable PDFs of the signals corresponding to the worst-case scenario are also considered along with the estimated ones [9, 10]. Let us denote the least favorable PDFs of the signals under hypotheses $\mathcal{H}_1$ and $\mathcal{H}_0$ with $p_{\mathbf{s}_1}^{ls}(\cdot)$ and $p_{\mathbf{s}_0}^{ls}(\cdot)$, respectively. Observations of the local sensors are also assumed to be conditionally independent. It should be noted that the least-favorable PDFs of the signals may be dependent on thresholds, that is, when the thresholds of the local sensors and the fusion center change the least-favorable PDFs may accordingly change in some cases.

The fusion center and all local sensors employ fixed test statistics. Let us denote the test statistic at the local sensor *i* with $\mathcal{T}_i(\cdot)$ and at the fusion center with $\mathcal{L}(\cdot)$. In the conventional approach, thresholds of the fusion center and local sensors are considered as deterministic variables, and optimized according to the given criterion [8, 16], which is the restricted Bayes criterion in this chapter.

Let us denote the threshold of the local sensor *i* with $\boldsymbol{\eta}_i$ and the threshold of the fusion center with $\boldsymbol{\tau}$. In this part of the thesis, while a random variable is denoted in bold font, its realization is depicted without bold font. We define the random vector $\boldsymbol{\eta}$ with the PDF $p_{\boldsymbol{\eta}}(\cdot)$ consisting of all thresholds of the local sensors: $\boldsymbol{\eta} = [\boldsymbol{\eta}_1 \boldsymbol{\eta}_2 \cdots \boldsymbol{\eta}_N]^T$.

The decision rule at the local sensor *i* is denoted with $\phi_i$, where $\phi_i(x_i) = 1$ if $\mathcal{T}_i(x_i) \geq \eta_i$, otherwise $\phi_i(x_i) = 0$. The observation received at the fusion center from the local sensor *i* is denoted with $\mathbf{u}_i$, where $u_i \in \{0,1\}$. Define the random vector **u** consisting of all observations received at the fusion center from the local sensors: $\mathbf{u} = [\mathbf{u}_1 \mathbf{u}_2 \cdots \mathbf{u}_N]^T$. The fusion center makes a final decision based on the observation **u**, that is, $\phi(u) = 1$ if $\mathcal{L}(u) \geq \tau$, otherwise $\phi(u) = 0$, where $\phi(\cdot)$ is the decision rule employed at the fusion center.

Binary asymmetric Channel (BASC) is assumed, and the crossover probabilities are defined as $c_{0i} = p(\mathbf{u}_i = 1 \,|\, \phi_i(x_i) = 0)$ and $c_{1i} = p(\mathbf{u}_i = 0 \,|\, \phi_i(x_i) = 1)$ for *i* = 1, 2…, *N*.

When the signal and thresholds of local sensors are given, observations ($\mathbf{u}_i$ for $i$ = 1, 2..., $N$) at the fusion center are independent from each other. Accordingly, we have the following:

$$p(u \mid \eta, s_k, \mathcal{H}_k) = \prod_{i=1}^{N} p(u_i \mid \eta_i, s_{ik}, \mathcal{H}_k) \tag{2}$$

The probability of local sensor $i$ deciding on $\mathcal{H}_1$ when the threshold and the signal are given is denoted with $\mathrm{F}_{ik}(\eta_i, s_{ik}) = p(\phi_i(\mathrm{x}_i) = 1 \mid \eta_i, s_{ik}, \mathcal{H}_k)$. Then, $p(u_i \mid \eta_i, s_{ik}, \mathcal{H}_k)$ can be expressed as follows:

$$p(\mathbf{u}_i = 1 \mid \eta_i, s_{ik}, \mathcal{H}_k) = (1 - c_{1i})\,\mathrm{F}_{ik}(\eta_i, s_{ik}) + c_{0i}(1 - \mathrm{F}_{ik}(\eta_i, s_{ik})) \tag{3}$$

$$p(\mathbf{u}_i = 0 \mid \eta_i, s_{ik}, \mathcal{H}_k) = (1 - c_{0i})(1 - \mathrm{F}_{ik}(\eta_i, s_{ik})) + c_{1i}\,\mathrm{F}_{ik}(\eta_i, s_{ik}) \tag{4}$$

The PDFs of the observations received at the fusion center for the expected and the worst-case scenarios can be calculated as follows:

$$p(u \mid \mathcal{H}_k) = \int_{\mathbb{R}^N} \int_{\mathbb{R}^{KN}} p(u \mid \eta\ , s_k, \mathcal{H}_k) p_{s_k}(s_k) p_\eta(\eta)\, \mathrm{d}s_k \mathrm{d}\eta$$

$$p^{ls}(u \mid \mathcal{H}_k) = \int_{\mathbb{R}^N} \int_{\mathbb{R}^{KN}} p(u \mid \eta\ , s_k, \mathcal{H}_k) p^{ls}_{s_k}(s_k) p_\eta(\eta)\, \mathrm{d}s_k \mathrm{d}\eta$$

In this thesis, the fusion center is assumed to use the likelihood ratio as a test statistic. When the thresholds of the local sensors are given, the likelihood ratio can be calculated as follows:

$$\mathcal{L}(u) = \frac{\int_{\mathbb{R}^K} p(u \mid \eta\ , s_1, \mathcal{H}_1) p_{s_1}(s_1) \mathrm{d}s_1}{\int_{\mathbb{R}^K} p(u \mid \eta\ , s_0, \mathcal{H}_0) p_{s_0}(s_0) \mathrm{d}s_0} = \frac{\mathrm{G}_1(u, \eta)}{\mathrm{G}_0(u, \eta)} \tag{5}$$

Where $\mathrm{G}_k(u, \eta) = \int_{\mathbb{R}^{KN}} p(u \mid \eta\ , s_k, \mathcal{H}_k) p_{s_k}(s_k)\, \mathrm{d}s_k$ for $k$ = 0, 1. For convenience, let us also define $\mathrm{G}_k^{ls}(u,\eta) = \int_{\mathrm{R}^{KN}} p(u \mid \eta, s_k, \mathcal{H}_k) p_{s_k}^{ls}(s_k)\, \mathrm{d}s_k$ for $k$ = 0, 1.

Since $u_i \in \{0,1\}$, the size of the set consisting of the possible realizations of **u** is $2^N$. Therefore, $\mathcal{L}(u)$ can take $2^N$ different values corresponding to a possible realization of **u**. Let us arrange the values of $\mathcal{L}(u)$ in ascending order as $l_1, l_2, \ldots, l_{2^N}$ with corresponding values of $u$ denoted with $u^1$, $u^2, \ldots$, $u^{2^N}$, that is, $l_i = \mathcal{L}(u^i) = \frac{G_1(u^i, \eta)}{G_0(u^i, \eta)}$.

For convenience let us represent all thresholds with the random vector $\boldsymbol{\theta}$ having the PDF $p_{\boldsymbol{\theta}}(\cdot)$, which is defined as $\boldsymbol{\theta} = [\boldsymbol{\eta}^T \boldsymbol{\tau}]^T$. Then, probabilities of the fusion center deciding on $\mathcal{H}_1$ given that the true hypothesis is $\mathcal{H}_1$ (the global detection probability) for the expected and the worst-case scenarios can be calculated as follows:

$$\mathrm{P_D}(\theta) = \sum_{i=0}^{2^N-1} \mathrm{P_{D_i}}(\theta) \tag{6}$$

$$\mathrm{P_D^{ls}}(\theta) = \sum_{i=0}^{2^N-1} \mathrm{P_{D_i}^{ls}}(\theta) \tag{7}$$

Where $\mathrm{P_{D_0}}(\theta) = \mathrm{I}(\tau \le l_1)$,

$\mathrm{P_{D_i}}(\theta) = \mathrm{I}(l_i < \tau \le l_{i+1})\left(\mathrm{G}_1(u^{i+1},\eta) + \mathrm{G}_1(u^{i+2},\eta) + \cdots + \mathrm{G}_1(u^{2^N},\eta)\right)$, similarly

$\mathrm{P_{D_i}^{ls}}(\theta) = \mathrm{I}(\tau \le l_1)$, $\mathrm{P_{D_i}^{ls}}(\theta) = \mathrm{I}(l_i < \tau \le l_{i+1})\left(\mathrm{G}_1^{ls}(u^{i+1},\eta) + \mathrm{G}_1^{ls}(u^{i+2},\eta) + \cdots + \mathrm{G}_1^{ls}(u^{2^N},\eta)\right)$

for $i$ = 1, 2,…,( $2^N$ -1) . Here, $\mathrm{I}(\cdot)$ denotes an indicator function: If the event A is true then I(A) = 1, otherwise I(A) = 0.

The probabilities of the fusion center deciding on $\mathcal{H}_1$ given that the true hypothesis is $\mathcal{H}_0$ (the global false alarm probability) for the expected and the worst-case scenarios can be computed as follows:

$$\mathrm{P_F}(\theta) = \sum_{i=0}^{2^N-1} \mathrm{P_{F_i}}(\theta) \tag{8}$$

$$\mathrm{P_F^{ls}}(\theta) = \sum_{i=0}^{2^N-1} \mathrm{P_{F_i}^{ls}}(\theta) \tag{9}$$

Where $\mathrm{P_{F_0}}(\theta) = \mathrm{I}(\tau \le l_1)$,

$\mathrm{P_{F_i}}(\theta) = \mathrm{I}(l_i < \tau \le l_{i+1})\left(\mathrm{G_0}(u^{i+1},\eta) + \mathrm{G_0}(u^{i+2},\eta) + \cdots + \mathrm{G_0}(u^{2^N},\eta)\right)$, similarly

$\mathrm{P_{F_0}^{ls}}(\theta) = \mathrm{I}(\tau \le l_1)$, $\mathrm{P_{Fi}^{ls}}(\theta) = \mathrm{I}(l_i < \tau \le l_{i+1})\left(\mathrm{G_0^{ls}}(u^{i+1},\eta) + \mathrm{G_0^{ls}}(u^{i+2},\eta) + \cdots + \mathrm{G_0^{ls}}(u^{2^N},\eta)\right)$

for $i$ = 1, 2,…,( $2^{\mathrm{N}}$ -1).

The global error probabilities at the fusion center for the expected and the worst-case scenarios can be calculated as follows:

$$\mathrm{P_E}(\theta) = \pi_1(1-\mathrm{P_D}(\theta)) + (1-\pi_1)\mathrm{P_F}(\theta) \tag{10}$$

$$\mathrm{P_E^{ls}}(\theta) = \pi_1(1-\mathrm{P_D^{ls}}(\theta)) + (1-\pi_1)\mathrm{P_F^{ls}}(\theta) \tag{11}$$

Where $\pi_1$ is the prior probability of hypothesis $\mathcal{H}_1$.

In the restricted Bayes criterion, the goal is to minimize the global probability of error corresponding to the expected scenario under the constraint on the global error probability corresponding to the worst-case scenario.

Therefore, in the conventional approach, the following optimization problem is solved to obtain optimum deterministic thresholds:

$$\begin{aligned} &\min_{\theta} \quad \mathrm{P_E}(\theta) \\ &\text{Subject to} \quad \mathrm{P_E^{ls}}(\theta) \le \beta, \end{aligned} \tag{12}$$

where $\beta$ is a predefined parameter determined based on the level of uncertainty [9,10,17]. Let us denote the optimal deterministic thresholds with $\theta^{opt}$, then in the conventional approach the global error probabilities corresponding to the expected and the worst-case scenarios are $\mathrm{P_E}(\theta^{opt})$ and $\mathrm{P_E^{ls}}(\theta^{opt})$, respectively. The restricted Bayes criterion generalizes the minimax and the Bayes criteria, and includes them as special cases. In (12), as $\beta$ increases the restricted Bayes criterion converges to the Bayes criterion, and after some value of $\beta$ the restricted Bayes becomes equivalent to the Bayes criterion [9,10]. Similarly, as $\beta$ decreases the restricted Bayes criterion converges to the minimax criterion, and at the minimum value of $\beta$ the restricted Bayes becomes equivalent to the minimax criterion [9,10]. In fact, the minimum value of $\beta$ is the probability of error when the minimax criterion is employed.

In the case of thresholds being random variables, the aim is to obtain optimum PDFs of the thresholds which minimize the average global probability of error corresponding to the expected scenario while keeping the average global error probability corresponding to the worst-case scenario below the predefined level:

$$\begin{aligned} &\min_{p_{\boldsymbol{\theta}}(\cdot)} \quad \mathrm{E}_{\boldsymbol{\theta}}\{\mathrm{P_E}(\boldsymbol{\theta})\} \\ &\text{Subject to} \quad \mathrm{E}_{\boldsymbol{\theta}}\{\mathrm{P_E^{ls}}(\boldsymbol{\theta})\} \leq \beta \,. \end{aligned} \tag{13}$$

It should be noted that when the value of $\beta$ is high enough so that the constraint on the average global error probability corresponding to the worst-case scenario becomes ineffective, then the optimization problem in (13) reduces to the minimization of average global probability of error corresponding to the expected scenario, which is the optimization problem of the Bayes criterion. Under this case, replacing deterministic thresholds by random ones is useless since optimal PDFs of random thresholds which minimize the average global error probability corresponding to the expected scenario consist of only one point mass, which means that random thresholds are indeed deterministic ones.

Since instantaneous error probabilities are equivalent to average error probabilities for deterministic thresholds, all error probabilities mentioned throughout the rest of this chapter are averaged ones.

## 1.2 CHARACTERIZATION AND CALCULATION OF OPTIMAL SOLUTION

The following proposition shows that optimal PDF of each of the thresholds consists of at most two point masses, and the optimal thresholds depend on each other.

**Proposition 1**: *Assume that* $\mathrm{P_E}(\theta)$ *and* $\mathrm{P_E^{ls}}(\theta)$ *are continuous functions, and* $\theta$ *belongs to a finite closed set. Then, the optimum PDF for* $\boldsymbol{\theta}$ *is in the form of* $p_\theta(\theta) = \lambda\delta(\theta-\theta_1)+(1-\lambda)\delta(\theta-\theta_2)$*, where* $0 \le \lambda \le 1$.

**Proof:** The proof is similar with the proof of Proposition 1 in [13].

We can reformulate (13) by employing the results in Proposition 1:

$$\begin{aligned} &\min_{\{\lambda,\theta_1,\theta_2\}} \quad \lambda\mathrm{P_E}(\theta_1)+(1-\lambda)\mathrm{P_E}(\theta_2) \\ &\text{subject to } \lambda\mathrm{P_E^{ls}}(\theta_1)+(1-\lambda)\mathrm{P_E^{ls}}(\theta_2) \le \beta\,. \end{aligned} \tag{14}$$

Techniques for obtaining solution of (14) are extensively studied in [18]. In Section 1.4, the particle swarm optimization (PSO) algorithm is used to solve the problem in (14).

Proposition 1 characterizes the optimal PDFs of random thresholds together with the optimum way of implementing them. Specifically, the optimal PDFs consist of at most two point masses, and the optimum way of employing thresholds is to sync them together. To exemplify, for a given time interval T, all local sensors and the fusion center synchronously employ the corresponding thresholds specified by $\theta_1$ during the time interval $\lambda\mathrm{T}$, and they use the thresholds specified by $\theta_2$ in a synchronous manner in the rest part of the time interval (1- $\lambda$)T. According to Proposition 1, the only restriction on the thresholds is them being finite, which is emphasized in there by the statement that $\theta$ must be confined in finite set, which is already the case in practice. Thanks to results in Proposition 1, the optimization problem in (13) is reformulated in a manageable form, which has been already well studied.

## 1.3 A NECESSARY AND SUFFICIENT CONDITION FOR THE NON-IMPROVABILITY

Next, a necessary and sufficient condition is presented for the non-improvability of the conventional approach through replacing deterministic thresholds by random ones. To that end, define the auxiliary function $J(\mathrm{t}) = \inf(\mathrm{P_E}(\theta) \mid \mathrm{P_E^{ls}}(\theta) = \mathrm{t})$. Then, we have the following proposition:

**Proposition 2:** *Conventional approach cannot be improved through replacing deterministic thresholds by random ones if and only if there exist* $\xi \le 0$ *such that*

$$J(\mathrm{t}) \ge (\mathrm{t} - \beta)\xi + J(\beta) \qquad \forall t \tag{15}$$

**Proof:** The proof is based on the approach in the proof of Proposition 3 in [13]. We only present the sufficiency of the condition due to space limitation. Consider a generic PDF for $\boldsymbol{\theta}$ as $p_{\boldsymbol{\theta}}(\theta) = \lambda\delta(\theta - \theta_1) + (1-\lambda)\delta(\theta - \theta_2)$, then we have $t_1 = \mathrm{P_E^{ls}}(\theta_1)$ and $t_2 = \mathrm{P_E^{ls}}(\theta_2)$. Based on the condition in the proposition, we have the followings $J(\mathrm{t_1}) \ge (\mathrm{t_1} - \beta)\xi + J(\beta)$ and $J(\mathrm{t_2}) \ge (\mathrm{t_2} - \beta)\xi + J(\beta)$. Therefore, employing $p_{\boldsymbol{\theta}}(\theta)$ gives the following relation:

$$\mathrm{P_E}(\theta_1) + (1-\lambda)\mathrm{P_E}(\theta_2) \ge \lambda J(t_1) + (1-\lambda)J(t_2) \ge J(\beta) - \xi(\beta - (\lambda t_1 + (1-\lambda)t_2)) \ge \mathrm{P_E}(\theta^{opt})$$

, because of $\xi \ge 0$ and $\lambda t_1 + (1-\lambda)t_2 \le \beta$.

In Proposition 2, we assume that $\mathrm{P_E^{ls}}(\theta^{opt}) = \beta$, which is the case in practice since $\beta$ is set by the designer based on the uncertainty level. If $J(t)$ is first-order continuously differentiable, we have the relation $\xi = J'(\beta)$.

In some circumstances, deterministic thresholds turn out to be optimal. In those cases, there is no need to engage in the optimization problem in (13). Proposition 2 specifies these circumstances completely, and beforehand gives us certain information about the form of the solution.

## 1.4 NUMERICAL RESULTS

Consider the decentralized detection problem with two local sensors, and scalar observations at the local sensor $i$ are given as follows:

$$\mathcal{H}_0 \quad : \mathbf{x}_i = \mathbf{n} \qquad , \qquad \mathcal{H}_1 \quad : \mathbf{x}_i = s + \mathbf{n} \tag{16}$$

where $\boldsymbol{s}$ is a random variable with the PDF in the form of $p_s(s) = 0.5\delta(s-A) + 0.5\delta(s+A)$ where $\delta(\cdot)$ is the Dirac delta function and the value of A is estimated based on previous experience. In this model, the signal under $\mathcal{H}_1$ employs binary modulation, namely, binary phase shift keying (BPSK). The background noise $\mathbf{n}$ is symmetric Gaussian mixture with the PDF:

$$p_{\mathbf{n}}(n) = \sum_{i=1}^{\mathrm{M}} \omega_i \psi_i (n - \mu_i) \tag{17}$$

where M is the number of Gaussian components in the mixture noise PDF, $\mu_i$ is the mean values of the Gaussian components, $\sum_{i=1}^{\mathrm{M}} \omega_i = 1$, $\omega_i \geq 0$, $\psi_i(y) = (1/\sqrt{2\pi}\,\sigma_i)\exp\{-y^2/(2\sigma_i^2)\}$ for $i$ = 1, . . . , M , with $\sigma_i$ being the standard deviations of the Gaussian components. All parameters are adjusted to make the PDF symmetric around the origin.

The local sensors employ the following test statistics $\mathcal{T}_1(x_1) = x_1^2$ and $\mathcal{T}_2(x_2) = x_2^2$. For this example, we can present $\mathrm{F}_{ik}(\eta_i, s_{ik})$ in the closed form expression:

$$\mathrm{F}_{ik}(\eta_i, s_{ik}) = p\left((\mathrm{n}+\mathrm{s}_{ik})^2 \geq \eta_i \mid \eta_i, \mathrm{s}_{ik}, H_k\right) =$$

$$\sum_{m=1}^{\mathrm{M}} \omega_m \left( Q\left(\frac{\sqrt{\eta_i} - s_{ik} - \mu_m}{\sigma_m}\right) + Q\left(\frac{\sqrt{\eta_i} + s_{ik} - \mu_m}{\sigma_m}\right)\right),$$

where $s_{i0} = 0$ and $s_{i1} \in \{-A, A\}$, and Q-function is given as $\mathrm{Q(x)} = (1/\sqrt{2\pi})\int_x^{\infty} e^{-t^2/2} dt$. Based on previous experience, A is assumed to be estimated as 5, but it is also assumed to be known for sure that $A \geq 3$.

In this case, the estimated PDF for the signal is obtained by inserting 5 for A in $p_s(s)$ since A is estimated as 5, and the least-favorable PDF is obtained by inserting 3 for A in $p_s(s)$ since in the scenario studied in the remaining part of the section the maximum value of means of the gaussian components in the background noise is 2, to which 3 is the closest value A can take. In this example, the least-favorable PDF of the signal is independent from the thresholds for the scenario studied in the remaining part of the section.

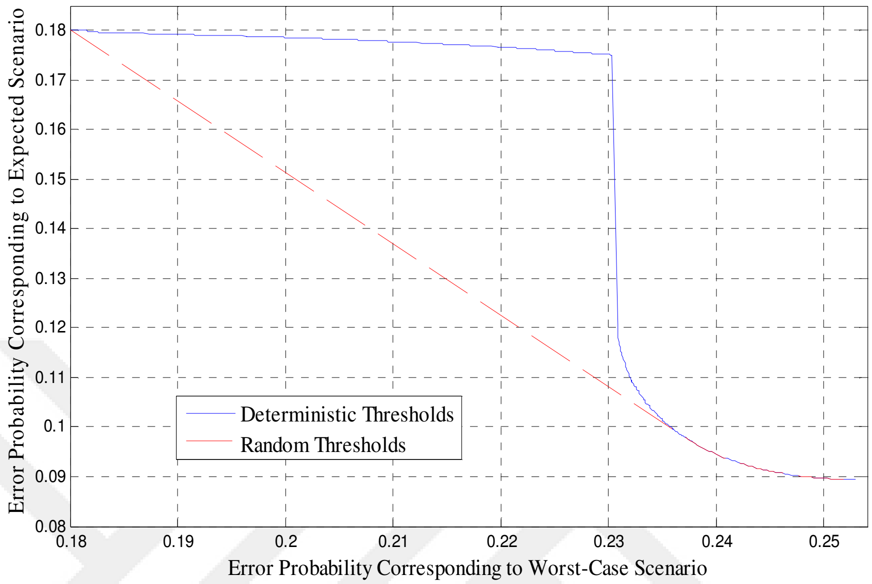


**Figure 1.1:** The curves of error probability corresponding to the expected scenario versus error probability corresponding to the worst-case scenario for the case of using optimal deterministic thresholds and the case of using optimal random thresholds, where $c_{0i} = c_{1i} = 0.01$ for $i$ = 1,2, $M$ = 2, $\mu_1 = -\mu_2 = 2$, $\omega_1 = \omega_2 = 0.5$, $\sigma_1 = \sigma_2 = 0.8$, $\pi_1 = 0.82$.

In the Figure 1.1, the curves of error probability corresponding to the expected scenario versus error probability corresponding to the worst-case scenario are plotted for the case of thresholds being optimal deterministic variables and the case of thresholds being optimal random variables, where $c_{0i} = c_{1i} = 0.01$ for $i$ = 1, 2, $M$ = 2, $\mu_1 = -\mu_2 = 2$, $\omega_1 = \omega_2 = 0.5$, $\sigma_1 = \sigma_2 = 0.8$, $\pi_1 = 0.82$. The optimal random thresholds improve performance over optimal deterministic ones for $\beta \in (0.18, 0.2378)$, which is also confirmed by Proposition 2. In the cases of $\beta = 0.18$ and $\beta = 0.2378$, the restricted Bayes criterion is equivalent to the minimax and Bayes criteria, respectively. And for these cases, using random thresholds does not provide any benefits over deterministic thresholds. It is also interesting to note that the curve corresponding to optimal random thresholds is convex.

| $\beta$ | $\lambda$ | $\eta_{11}$ | $\eta_{12}$ | $\tau_1$ | $\eta_{21}$ | $\eta_{22}$ | $\tau_2$ |
|---|---|---|---|---|---|---|---|
| 0.20 | 0.3621 | 10.4206 | 10.4376 | 3.2090 | 0 | 0.1320 | 0.3597 |
| 0.21 | 0.4550 | 11.3602 | 8.0927 | 0.5258 | 0 | 2.1927 | 0.0916 |
| 0.22 | 0.6974 | 9.9897 | 9.7801 | 1.0889 | 0 | 0.3333 | 0.2290 |
| 0.23 | 0.13 | 0.0866 | 0 | 0.4212 | 9.8691 | 9.7368 | 0.4975 |
| 0.24 | 1 | 9.4285 | 9.4285 | 0.7844 | - | - | - |

**Table 1.1:** Optimal PDFS of random thresholds under the scenario in the Figure 1.1 for various values of $\beta$, $p_{\boldsymbol{\theta}}(\theta)=\lambda\delta(\theta-\theta_1)+(1-\lambda)\delta(\theta-\theta_2)$, Where $\theta_1=[\eta_{11}\eta_{12}\tau_1]$ and $\theta_2=[\eta_{21}\eta_{22}\tau_2]$.

In the Table 1.1, the optimal PDFs of thresholds are presented for various values of $\beta$. From the Table 1.1, it is observed that the optimal PDF of thresholds consists of at most two point masses as stated in Proposition 1.

Because of uncertainty issues, designer sets a upper bound for error performance according to design metrics. Performances higher than the upper bound are not tolerable from the designer's perspective. In our, the upper bound is an error probability corresponding to the worst-case scenario. Therefore, designer first sets a upper bound, which is the worst-case error probability in our case, and then aims to optimize the expected error probability, which is true error probability if all estimations turn out to be perfectly correct. In the Figure 1.1, it is shown that for some values of the upper bound, which is the worst-case error probability, optimal random thresholds outperform optimal deterministic ones in terms of minimizing the expected error probability, which is the approach adopted in restricted Bayes criterion.

# CHAPTER 2

# OPTIMAL JAMMING IN THE PRESENCE OF UNCERTAINTY

In Chapter 2, in section 2.1, the problem formulation is given and derivations for optimal jamming in the presence of uncertainty are provided. Section 2.2 presents an optimization example to illustrate the theoritical results.

## 2.1 PROBLEM FORMULATION AND ANALYSIS

Consider the problem formulation in [13], where a receiver aiming to detect a target under the jamming. Consider hypothesis testing between the presence and absence of the target, and the corresponding observation at the receiver is given as follows:

$$\mathcal{H}_0 : \mathbf{y} = \gamma\mathbf{n} + \boldsymbol{\varepsilon}, \quad \mathcal{H}_1 : \mathbf{y} = \mathbf{s} + \gamma\mathbf{n} + \boldsymbol{\varepsilon} \tag{18}$$

where $\mathcal{H}_1$ and $\mathcal{H}_0$ represent the alternative and null hypotheses, respectively, $\mathbf{s}$ is the target signal with PDF $p_{\mathbf{s}}(\cdot)$, $\mathrm{y} \in \mathbb{R}^{\mathrm{K}}$ is the observation, $\boldsymbol{\varepsilon}$ is the measurement noise with PDF $p_{\boldsymbol{\varepsilon}}(\cdot)$, and $\mathbf{n}$ is the jammer noise with PDF $p_{\mathbf{n}}(\cdot)$, which is learned by the receiver. The jammer allocates the power over time according to the scalar variable $\gamma$, which has a PDF denoted by $p_{\gamma}(\cdot)$. Since the receiver is smart, it is supposed that the receiver learns the value of $\gamma$ instantaneously [14]: A jammer employs the variable $\gamma$ in a pattern, so a receiver can learn that pattern by observing the noise over some time, and accordingly gets the knowledge of $\gamma$ instantaneously. For the sake of simplicity, it is assumed that $\mathrm{trace}\{\mathrm{Cov}(\mathbf{n})\} = 1$; the corresponding constraint on the jammer is given by

$$E_{\gamma}\{\gamma^2\} \leq \beta \tag{19}$$

where $\beta$ represents the constraint level , that is, the jammer can allocate its power under the constraint in (19).

An NP framework is considered, where the receiver aims to maximize detection probability under the constraint that the false alarm can not exceed the constraint level $\alpha$. The decision rule is represented by $\phi_{\gamma}(\mathbf{y})$ at the receiver, which is the probability of selecting $\mathcal{H}_1$ [19]. In practice, the jammer can learn a target signal, which is modeled as $p_{\mathbf{s}}(\cdot)$, through previous experiences [9]. In this context, jammer's knowledge about a target signal includes some degree of uncertainty depending on the circumstances under which the jammer operates. In order to handle the uncertainty issue related to the jammer, we employ the Hodges-Lehmann rule (restricted Bayes approach) [9], [10] in finding the optimal power allocation strategy for the jammer. According to this approach, the jammer aims reducing the "expected" average detection probability at the receiver as much as possible under the constraint that the least favorable (i.e., maximum) average detection probability at the receiver is below a predefined threshold and under the constraint on average power of the jammer. The jammer achieves this aim by allocating its power, which is specified by the PDF of $\gamma$. As the receiver is considered to be smart, the correspondıng detector is accordingly designed as a function of $\gamma$, i.e., $\phi_{\gamma}(\cdot)$, and the receiver can adapt a decision rule for each value of $\gamma$. As in [13], generic decision rules are considered (i.e., suboptimal or optimal), each of the decision rules equates the false alarm rate to $\alpha$.

Before going through the analysis of the proposed optimal power allocation policy for the jammer in the presence of partial information, some remarks are provided about the restricted Bayes approach in the context of the jammer. In practice, the jammer can estimate the probability distribution of the target signal **s**, $p_{\mathbf{s}}(\cdot)$, based on previous observations/experience. Therefore, there exist estimation errors causing uncertainty in the knowledge of $p_{\mathbf{s}}(\cdot)$ [10]. Two conventional approaches, known as "classical" and minimax, are extensively employed in such situations. In the classical approach, the jammer neglects the estimation errors and uses the estimated probability distribution of the signal as if it were the true probability distribution of the signal, $p_{\mathbf{s}}(\cdot)$. In this case, the neglected uncertainty may cause undesired performance for the jammer. The classical approach is the optimum one in the case of perfect estimation of the PDF of a target signal; that is, the jammer has the exact knowledge of the PDF $p_{\mathbf{s}}(\cdot)$. On the other hand, in the minimax approach, the jammer takes the worst-case scenario into account by adopting the least-favorable probability distribution for the target signal. In this case, even if the jammer has the some information on $p_{\mathbf{s}}(\cdot)$ obtained based on previous experience, it is not taken into account at all by the jammer. Therefore, this approach is too conservative in terms of jammer's performance. The minimax approach is the optimum one when the jammer does not have any information on the PDF of the signal; that is, there is full uncertainty in the PDF $p_{\mathbf{s}}(\cdot)$. However, in reality, having full knowledge and full uncertainty for the PDF $p_{\mathbf{s}}(\cdot)$ are the exceptional cases; hence, these two approaches correspond to two extreme cases. In practice, the jammer has partial information on the PDF $p_{\mathbf{s}}(\cdot)$, and there have been various approaches for taking partial information into account such as restricted Bayes, Γ-minimax, mean-max, robust Bayes, and empirical Bayes [9]. In this study, we employ the restricted Bayes approach (Hodges-Lehmann rule) to take the partial information of the jammer into account in developing the optimum power allocation strategy of the jammer.

Denote the estimated and the least-favorable PDFs for the signal **s** as $p_{\mathbf{s}}^{\mathrm{est}}(\cdot)$ and $p_{\mathbf{s}}^{\mathrm{ls}}(\cdot)$, respectively. It is noted that the estimated PDF is the PDF information at the jammer, which is not perfect, and that the least favorable PDF is the one that maximizes the average detection probability at the receiver, which corresponds to the worst-case scenario for the jammer. The "expected" (estimated) and least-favorable detection probabilities of the receiver are calculated, respectively, as

$$\mathrm{P}_{\mathrm{D}_{\mathrm{est}}}(\gamma) = \int_{\mathbb{R}^K} \phi_\gamma(\mathbf{y})\, p_1^{\mathrm{est}}(\mathbf{y})\, d\mathbf{y} \tag{20}$$

$$\mathrm{P}_{\mathrm{D}_{\mathrm{ls}}}(\gamma) = \int_{\mathbb{R}^K} \phi_\gamma(\mathbf{y})\, p_1^{\mathrm{ls}}(\mathbf{y})\, d\mathbf{y} \tag{21}$$

where $p_1^{\mathrm{est}}(\mathbf{y})$ and $p_1^{\mathrm{ls}}(\mathbf{y})$ are, respectively, the estimated and the least-favorable distributions of the observation under the alternative hypothesis. Since **n**, $\boldsymbol{\varepsilon}$ and **s** in (18) are independent, $p_1^{\mathrm{est}}(\mathbf{y})$ and $p_1^{\mathrm{ls}}(\mathbf{y})$ are computed as

$$p_1^{\mathrm{est}}(\mathbf{y}) = \int_{\mathbb{R}^K}\int_{\mathbb{R}^K} \frac{1}{|\gamma|}\, p_{\mathbf{n}}\left(\frac{\mathbf{y}-\mathbf{s}-\mathbf{e}}{\gamma}\right) p_{\boldsymbol{\varepsilon}}(\mathbf{e})\, p_{\mathbf{s}}^{\mathrm{est}}(\mathbf{s})\, d\mathbf{e}\, d\mathbf{s} \tag{22}$$

$$p_1^{\mathrm{ls}}(\mathbf{y}) = \int_{\mathbb{R}^K}\int_{\mathbb{R}^K} \frac{1}{|\gamma|}\, p_{\mathbf{n}}\left(\frac{\mathbf{y}-\mathbf{s}-\mathbf{e}}{\gamma}\right) p_{\boldsymbol{\varepsilon}}(\mathrm{e})\, p_{\mathbf{s}}^{\mathrm{ls}}(\mathbf{s})\, d\mathbf{e}\, d\mathbf{s} \tag{23}$$

Thus, $\mathrm{P}_{\mathrm{D}_{\mathrm{est}}}(\gamma)$ and $\mathrm{P}_{\mathrm{D}_{\mathrm{ls}}}(\gamma)$ in (20)-(21) are given by

$$\mathrm{P}_{\mathrm{D}_{\mathrm{est}}}(\gamma) = \int_{\mathbb{R}^K} \phi_\gamma(\mathbf{y}) \int_{\mathbb{R}^K}\int_{\mathbb{R}^K} \frac{1}{|\gamma|}\, p_{\mathbf{n}}\left(\frac{\mathbf{y}-\mathbf{s}-\mathbf{e}}{\gamma}\right) p_{\boldsymbol{\varepsilon}}(\mathbf{e})\, p_{\mathbf{s}}^{\mathrm{est}}(\mathbf{s})\, d\mathbf{e}\, d\mathbf{s}\, d\mathbf{y} \tag{24}$$

$$\mathrm{P}_{\mathrm{D}_{\mathrm{ls}}}(\gamma) = \int_{\mathbb{R}^K} \phi_\gamma(\mathbf{y}) \int_{\mathbb{R}^K}\int_{\mathbb{R}^K} \frac{1}{|\gamma|}\, p_{\mathbf{n}}\left(\frac{\mathbf{y}-\mathbf{s}-\mathbf{e}}{\gamma}\right) p_{\boldsymbol{\varepsilon}}(\mathbf{e})\, p_{\mathbf{s}}^{\mathrm{ls}}(\mathbf{s})\, d\mathbf{e}\, d\mathbf{s}\, d\mathbf{y}\,. \tag{25}$$

The average detection probabilities of the receiver for the “expected” and the least-favorable cases are given as follows:

$$\mathrm{P}_{\mathrm{D}_{\mathrm{est}}}^{\mathrm{avg}}(\gamma) = \int_{-\infty}^{\infty} p_\gamma(\gamma)\mathrm{P}_{\mathrm{D}_{\mathrm{est}}}(\gamma)\, d\gamma = \mathrm{E}_\gamma\left\{\mathrm{P}_{\mathrm{D}_{\mathrm{est}}}(\gamma)\right\} \tag{26}$$

$$\mathrm{P}_{\mathrm{D}_{\mathrm{ls}}}^{\mathrm{avg}}(\gamma) = \int_{-\infty}^{\infty} p_\gamma(\gamma)\mathrm{P}_{\mathrm{D}_{\mathrm{ls}}}(\gamma)\, d\gamma = \mathrm{E}_\gamma\left\{\mathrm{P}_{\mathrm{D}_{\mathrm{ls}}}(\gamma)\right\} \tag{27}$$

where $\mathrm{P}_{\mathrm{D}_{\mathrm{est}}}(\gamma)$ and $\mathrm{P}_{\mathrm{D}_{\mathrm{ls}}}(\gamma)$ are as in (24) and (25), respectively. In the proposed approach, which is based on the Hodges- Lehmann rule (restricted Bayes approach), the goal is to develop the optimal power allocation strategy of the jammar, i.e., $p_\gamma(\cdot)$, which reduces "expected" average detection probability at the receiver as much as possible under constraints on the least-favorable average detection probability at the receiver and the average power level of the jammer. The proposed problem formulation is stated as follows:

$$\underset{p_\gamma(\cdot)}{\text{minimize}} \;\; \mathrm{P}_{\mathrm{D_{est}}}^{\mathrm{avg}} \tag{28}$$

$$\text{subject to } \; \mathrm{P}_{\mathrm{D_{ls}}}^{\mathrm{avg}} \le \kappa \tag{29}$$

$$\mathrm{E}_\gamma\{\gamma^2\} \le \beta \tag{30}$$

where $\mathrm{P}_{\mathrm{D_{est}}}^{\mathrm{avg}}$ and $\mathrm{P}_{\mathrm{D_{ls}}}^{\mathrm{avg}}$ are as in (26)-(27), and $\kappa$ is the constraint on the least-favorable (i.e., maximum) average detection probability of the receiver. Constraint $\kappa$ is adjusted to compensate the uncertainty due to the errors resulting from the estimation of the signal PDF. In the case of an increase in the uncertainty level, the value of $\kappa$ is reduced to compensate the increased uncertainty. The minimum possible value of $\kappa$ is the one obtained from the solution of the minimax problem, which minimizes $\mathrm{P}_{\mathrm{D_{ls}}}^{\mathrm{avg}}$ under the power constraint $\mathrm{E}_\gamma\{\gamma^2\} \le \beta$. For that value of $\kappa$, the proposed formulation in (28)-(30) reduces to the minimax approach, which ignores the information about the PDF of the signal and considers the worst-case scenario. On the other hand, when the constraint on the least-favorable average detection probability in (29) is not considered, the proposed formulation reduces to the classical approach in which the PDF of the signal is assumed to be known perfectly. In fact, this scenario corresponds to the one studied in [3]. By employing a suitable value for $\kappa$ according to the amount of uncertainty, the proposed formulation in (28)-(30) can handle the amount of uncertainty (partial information) efficiently. The optimization problem in (28)-(30) involves the minimization of the expected value of a function of $\gamma$ (see (26) and (28)) over the PDF of $\gamma$ under the constraints on the expected values of two other functions of $\gamma$ (see (27), (29), and (30)). In such problems, Carathéodory's theorem [20] can be invoked to show that, under certain regularity conditions, the minimizer of the optimization problem in (28)-(30) can be represented by a discrete probability distribution with at most three point masses (see, e.g., [21]). More formally, assuming that $\gamma$ is in a finite closed set and that $\mathrm{P}_{\mathrm{D}_{\mathrm{est}}}(\gamma)$ in (24) and $\mathrm{P}_{\mathrm{D}_{\mathrm{ls}}}(\gamma)$ (25) are continuous functions; the PDF corresponding to an optimal power allocation strategy is given as

$$p_{\gamma}^{\mathrm{opt}}(\gamma)=\sum_{i=1}^{3}\lambda_i\delta(\gamma-\gamma_i) \tag{31}$$

Where $\sum_{i=1}^{3}\lambda_i=1$ and $\lambda_i\geq 0$ for $i$ = 1,2,3. In other words, the optimal power allocation policy of the jammer involves randomization among *at most three different power levels*. This is different from the conventional on-off jamming or the optimal jamming in the absence of uncertainty, which is performed between at most two different power levels [13]. Based on (26)-(31), the optimization problem in (28)-(30) can be simplified as follows:

$$\begin{aligned}
&\underset{\{\lambda_i,\gamma_i\}_{i=1}^{3}}{\text{minimize}} && \sum_{i=1}^{3}\lambda_i\mathrm{P}_{\mathrm{D}_{\mathrm{est}}}(\gamma_i)\\
&\text{subject to} && \sum_{i=1}^{3}\lambda_i\mathrm{P}_{\mathrm{D}_{\mathrm{ls}}}(\gamma_i)\leq\kappa,\ \sum_{i=1}^{3}\lambda_i{\gamma_i}^2\leq\beta,\\
& && \sum_{i=1}^{3}\lambda_i=1,\ \lambda_i\geq 0,\ \forall i\in\{1,2,3\}.
\end{aligned} \tag{32}$$

The optimization problem in (32), which involves optimization over five variables (utilizing the equality constraint), is computationally much simpler than the problem in (26)-(31), which requires optimization over PDFs. The analytical and numerical techniques in [21] can be employed to obtain the solution of the optimization problem in (32).

It is noted that the optimal jamming strategy in (32) covers the deterministic (no randomization) approach of transmitting at the maximum average power, which is specified by $p_{\gamma}(\gamma)=\delta(\gamma-\sqrt{\beta})$. In other words, in some cases, the solution to (32) can correspond to the deterministic jamming strategy in which the jammer steadily operates at the power limit of $\beta$. In the following propositions, sufficient conditions are presented in order to specify if the solution of the optimal jamming problem in (32) corresponds to the deterministic jamming approach or involves randomization among multiple power levels. These conditions are simple to check and they provide guidelines about the necessity for solving the optimization problem in (32).

Proposition 3 presents sufficient conditions for the superiority of power randomization over the deterministic strategy in the presence of uncertainty, which can be proved based on similar techniques to those in [13].

**Proposition 3:** *Define functions* $\mathrm{F}(a)$ *and* $\mathrm{G}(a)$ *as* $\mathrm{F}(a)=\mathrm{P}_{\mathrm{D}_{\mathrm{est}}}\left(\sqrt{a}\right)$ and $\mathrm{G}(a)=\mathrm{P}_{\mathrm{D}_{\mathrm{ls}}}\left(\sqrt{a}\right)$, *respectively, and assume that* $\mathrm{F}(a)$ *and* $\mathrm{G}(a)$ *are second-order continuously differentiable around* $\beta$. *If* $\mathrm{F}(a)$ *and* $\mathrm{G}(a)$ *are strictly concave and concave at* $a = \beta$, *respectively, that is,* $\mathrm{F}''(\beta)<0$ *and* $\mathrm{G}''(\beta)\leq 0$, *power randomization outperforms the deterministic strategy.*

When the conditions hold, deterministic jamming is not optimal, and the solution of (32) involves a randomization of the jammer power. Next proposition presents a sufficient condition under which the deterministic jamming approach is optimal; that is, power randomization cannot outperform the deterministic strategy in the presence of uncertainty.[1]

**Proposition 4:** *Suppose that* $\gamma\in[0,\gamma_{max}]$, *where* $\gamma_{\max}$ *is the upper limit for* $\gamma$. *Then, the solution of (32) does not involve power randomization if there exists* $\theta\leq 0$ *such that at least one of the following inequalities hold:*

$$\mathrm{P}_{\mathrm{D}_{\mathrm{est}}}\left(\sqrt{a}\right)\geq(a-\beta)\theta+\mathrm{P}_{\mathrm{D}_{\mathrm{est}}}\left(\sqrt{\beta}\right),\ \forall a\in[0,\gamma_{\max}^{2}] \tag{33}$$

$$\mathrm{P}_{\mathrm{D}_{\mathrm{ls}}}\left(\sqrt{a}\right)>(a-\beta)\theta+\kappa,\ \forall a\in[0,\gamma_{\max}^{2}] \tag{34}$$

[1] The proof, omitted since it is simple, is an extension of the proof of Proposition 3 in [13] to the optimal jamming problem in the presence of uncertainty.

When the conditions in Proposition 4 hold, there is no need to solve the problem (32) since the optimal solution is already specified by deterministic jamming.[2]

[2] In Proposition 3 and 4, the goal is to determine if power randomization provides an advantage over deterministic jamming. Therefore, while checking conditions in the propositions, it is convenient to set $\kappa = P_{D_{ls}}(\sqrt{\beta})$ as employed in Section 2.2.

## 2.2 NUMERICAL RESULTS

In this section, numerical results are presented in order to explore the theoretical results in the previous section. Consider the observations modeled as in (35). Signal s is a scalar random variable with a PDF in the form of

$$p_{\mathbf{s}}(\mathrm{s}) = \rho\,\delta(\mathrm{s}-A) + (1-\rho)\,\delta(\mathrm{s}+A) \tag{35}$$

Where $\rho$ is known exactly and $A$ is known with some uncertainty, and the measurement noise $\varepsilon$ and the jammer noise n are taken as symmetric Gaussian mixture noise with PDFs in the form of

$$p_{\boldsymbol{\varepsilon}}(\varepsilon) = \sum_{i=1}^{N} \xi_i \varphi_i(\varepsilon-\mu_i)\,,\; p_{\mathbf{n}}(\mathrm{n}) = \sum_{i=1}^{\tilde{N}} \tilde{\xi}_i \tilde{\varphi}_i(\mathrm{n}-\tilde{\mu}_i) \tag{36}$$

where $N$ and $\tilde{N}$ are Gaussian component numbers in the mixtures, $\mu_i$ and $\tilde{\mu}_i$ are the means of the Gaussian components, $\sum_{i=1}^{N}\xi_i = \sum_{i=1}^{\tilde{N}}\tilde{\xi}_i = 1$, $\xi_i \geq 0$, $\tilde{\xi}_i \geq 0$, $\varphi_i(\mathrm{y}) = (1/\sqrt{2\pi}\,\sigma_i)\exp\{-y^2/(2\sigma_i^{\,2})\}$, and $\tilde{\varphi}_i(\mathrm{y}) = (1/\sqrt{2\pi}\,\tilde{\sigma}_i)\exp\{-y^2/(2\tilde{\sigma}_i^{\,2})\}$, with $\sigma_i$ and $\tilde{\sigma}_i$ depicting the standard deviations of the components. All parameters are assigned in a way to ensure that the mixtures are symmetric around the origin.

The receiver employs the following decision rule, and adjusts its threshold to keep its false alarm probability constant at $\alpha$ as $\gamma$ varies.[3]

$$\phi_\gamma(\mathrm{y}) = \begin{cases} 0, & -\eta_\gamma \leq \mathrm{y} \leq \eta_\gamma \\ 1, & \text{otherwise} \end{cases} \tag{37}$$

where the threshold value $\eta_\gamma > 0$ is adjusted by the receiver to ensure that the false

[3] Due to symmetry, negative values of $\gamma$ are not taken into account in the simulations.

alarm rate is to equal to $\alpha$. From (35), (36), and (37), the false alarm rate is derived as follows:

$$\mathrm{P_F}(\gamma)=\sum_{i=1}^{N}\sum_{j=1}^{\tilde{N}}\xi_i\tilde{\xi}_i\left[Q\left(\frac{\eta_{\gamma}-\mu_i-\gamma\tilde{\mu}_j}{\sqrt{\sigma_i^2+\gamma^2\tilde{\sigma}_j^2}}\right)+Q\left(\frac{\eta_{\gamma}+\mu_i+\gamma\tilde{\mu}_j}{\sqrt{\sigma_i^2+\gamma^2\tilde{\sigma}_j^2}}\right)\right] \tag{38}$$

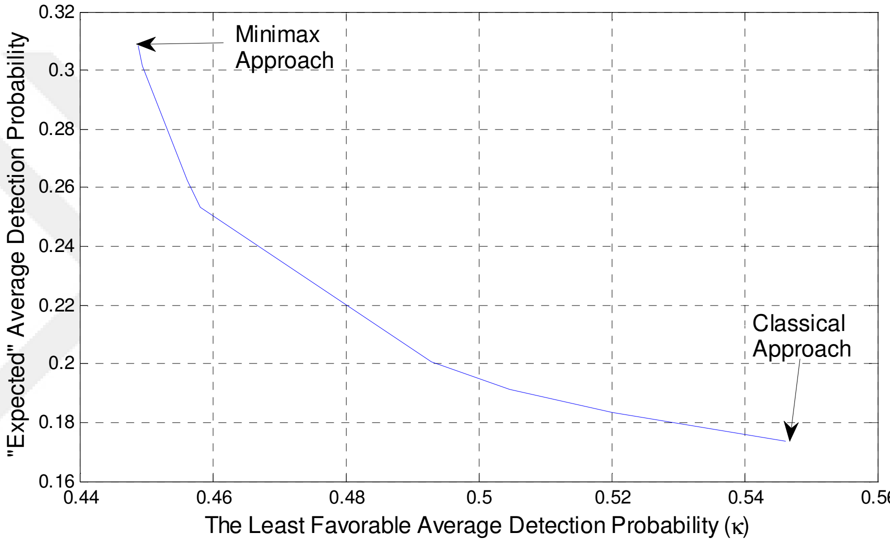


**Figure 2.1:** The "expected" average detection probability versus the least favorable average detection probability $\kappa$, where $\beta=0.5$.

where $Q(x)=\left(1/\sqrt{2\pi}\right)\int_x^{\infty}\mathrm{e}^{-u^2/2}du.$. The value of $\eta_\gamma$ is obtained from the relation $\mathrm{P_F}(\gamma)=\alpha$. Then the detection probability can be obtained as follows:

$$\mathrm{P_D}(\gamma)=\sum_{i=1}^{N}\sum_{j=1}^{\tilde{N}}\xi_i\tilde{\xi}_i\left[\rho Q\left(\frac{\eta_{\gamma}-A-\mu_i-\gamma\tilde{\mu}_j}{\sqrt{\sigma_i^2+\gamma^2\tilde{\sigma}_j^2}}\right)+\rho Q\left(\frac{\eta_{\gamma}+A+\mu_i+\gamma\tilde{\mu}_j}{\sqrt{\sigma_i^2+\gamma^2\tilde{\sigma}_j^2}}\right)+\right.$$
$$\left.(1-\rho)Q\left(\frac{\eta_{\gamma}+A-\mu_i-\gamma\tilde{\mu}_j}{\sqrt{\sigma_i^2+\gamma^2\tilde{\sigma}_j^2}}\right)+(1-\rho)Q\left(\frac{\eta_{\gamma}-A+\mu_i+\gamma\tilde{\mu}_j}{\sqrt{\sigma_i^2+\gamma^2\tilde{\sigma}_j^2}}\right)\right] \tag{39}$$

In this example, the uncertainty is originated from $A$, which is estimated based on previous observations (experience). If the estimated value of $A$ is used in (39), then the detection probability in (39) becomes the "expected" detection probability, $\mathrm{P}_{\mathrm{D}_{\mathrm{est}}}(\gamma)$. On the other hand, if the least favorable value of $A$, which means the worst case scenario for the jammer, is used, then the detection probability in (39) becomes the least favorable detection probability, $\mathrm{P}_{\mathrm{D}_{\mathrm{ls}}}(\gamma)$. It should be noted that the least favorable value of $A$ is also dependent on $\gamma$. In the numerical results, the background noise $\varepsilon$ is characterized by parameters $N = 2$, $\xi_1 = \xi_2 = 0.5$, $\mu_1 = -1$, $\mu_2 = 1$, and $\sigma_1 = \sigma_2 = 0.3$, and jammer's standard noise n is specified by $\tilde{N} = 2$, $\tilde{\xi}_1 = 0.1$, $\tilde{\xi}_2 = 0.9$, $\tilde{\mu}_1 = \tilde{\mu}_2 = 0$, $\tilde{\sigma}_1 = 3$, and $\tilde{\sigma}_2 = 1/3$ (see (36)). Jammer standard noise is modeled as a random variable with a zero mean and unit variance, which has heavier tails than Gaussian random variable with the same variance and the zero-mean. In the simulations, $\rho = 0.5$ and $\alpha = 0.001$ are employed. In addition, it is assumed that based on the previous experience, $A$ is estimated as $5$, and it is also observed that $A \leq 6$. In this scenario, the least favorable value of $A$ is $6$ for all values of $\gamma$; thus, the estimated and the least favorable PDFs of the signal are obtained by replacing $A$ with 5 and 6 in (35), respectively.

| $\kappa$ | $\lambda_1$ | $\lambda_2$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ |
|---|---|---|---|---|---|
| 0.4612 | 0.4549 | 0.1465 | 0.6613 | 0.8420 | 0.7034 |
| 0.4882 | 0.5147 | 0.1043 | 0.6754 | 0.7173 | 0.7452 |
| 0.5081 | 0.3853 | 0.1126 | 0.7313 | 0.5622 | 0.7173 |
| 0.5288 | 0.2344 | 0.7481 | 0.5821 | 0.7494 | 0.1664 |

**Table 2.1:** PDFs for the optimum power allocation policy, where $\beta = 0.5$, $p_{\gamma}^{\mathrm{OPT}}(\gamma) = \lambda_1\delta(\gamma - \gamma_1) + \lambda_2\delta(\gamma - \gamma_2) + (1 - \lambda_1 - \lambda_2)\delta(\gamma - \gamma_3)$.

In Figure 2.1, the "expected" average detection probability is plotted against the least favorable average detection probability for $\beta = 0.5$. In the Figure 2.1, the end points correspond to the classical approach (no uncertainty assumption) and the minimax approach (full uncertainty assumption), and the interior part of the curve corresponds to the proposed approach based on the restricted Bayes approach for different values of $\kappa$. In the Figure 2.1, the tradeoff between the “expected” and the least favorable average detection probabilities is clearly observed. In the Table 2.1, the PDFs of the optimal power allocation policy are presented for different values of $\kappa$, where $\beta = 0.5$. As observed from the Table 2.1, the optimal strategy involves randomization among up to three different power levels as stated in the first section of Chapter 2.

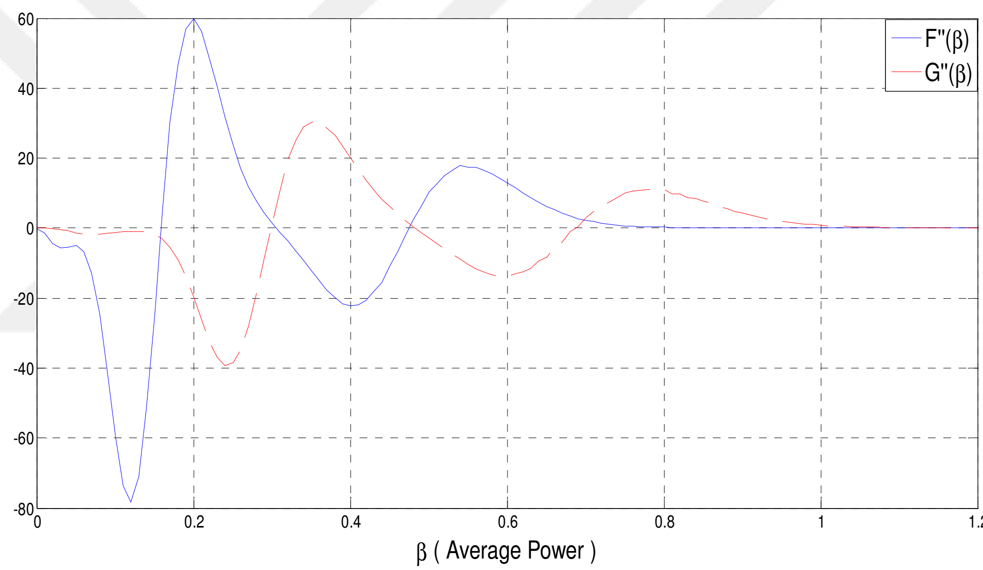


**Figure 2.2:** Plots of $F''(\beta)$ and $G''(\beta)$ in Proposition 3.

Figure 2.2 presents the plots of $F''(\beta)$ and $G''(\beta)$ to investigate Proposition 3. Based on the sufficient conditions in Proposition 3, it is concluded that power randomization is guaranteed to improve the jamming performance over the deterministic approach for $\beta < 0.1586$.

Based on the sufficient conditions in Proposition 4, it is obtained from the numerical calculations that power randomization cannot be the solution of the optimization problem in (32) for $0.228 \le \beta \le 0.248$ and $\beta \ge 0.571$. It should be noted that $\kappa = P_{D_{ls}}(\sqrt{\beta})$ is employed in the implementation of Propositions 3 and 4.

# CHAPTER 3

# CONCLUSIONS

In Section 3.1, concluding remarks and future works are presented.

## 3.1 CONCLUDING REMARKS AND FUTURE WORKS

In this thesis, firstly, the effects of replacing deterministic thresholds of local sensors and the fusion center by random ones have been investigated according to the restricted Bayes criterion. It has been shown that the optimal random thresholds are dependent on each other, and contain at most two point masses. Two methods for the implementation of the optimal random thresholds are proposed. A necessary and sufficient condition has been presented to determine when employing the optimal random thresholds outperforms employing the optimal deterministic ones. Through simulations, the effectiveness of the using the optimal random thresholds in place of the optimal deterministic ones has been observed. Employing the absolute worst-case error probability as a performance metric in place of the average worst-case error probability as a more conservative approach will be investigated as future work. Considering the fact that implementing independent randomized rules is much less costly than implementing dependent randomized ones, optimization of independent random thresholds instead of dependent ones can also be investigated as future study. Effects of adding correlated noises to observations at local sensors can be investigated as another future work.

Secondly, for an average power constrained jammer, the optimal jamming strategy is obtained. A Neyman-Pearson (NP) framework is considered, and the Hodges-Lehmann rule is employed in order to handle the uncertainty caused by partial information. The “expected” detection probability of a smart receiver is made as minimum as possible while keeping the least favorable detection probability of the receiver below a threshold level. It is obtained that the optimal policy for an average power constrained jammer is to allocate its power among at most three different levels in the presence of partial information. Sufficient conditions are obtained to determine scenarios in which power randomization can or cannot provide improvements over the deterministic (i.e., fixed power) jamming approach. Finally, Simulations are performed in order to investigate the theoretical findings.

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