AN INVESTIGATION INTO MIDDLE SCHOOL MATHEMATICS TEACHERS' SUBJECT MATTER KNOWLEDGE AND PEDAGOGICAL CONTENT KNOWLEDGE REGARDING THE VOLUME OF 3D SOLIDS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF SOCIAL SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

REYHAN TEKİN-SİTRAVA

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY
IN
THE DEPARTMENT OF ELEMENTARY EDUCATION

FEBRUARY 2014

Approval of the Graduate School of Social Sciences

____

Prof. Dr. Meliha ALTUNIŞIK
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Doctor of Philosophy.

____

Prof. Dr. Ceren ÖZTEKİN
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Doctor of Philosophy.

____

Assoc. Prof. Dr. Mine IŞIKSAL-BOSTAN
Supervisor

Examining Committee Members

Assoc. Prof. Dr. Erdinç ÇAKIROĞLU	(METU, ELE)	_________________
Assoc. Prof. Dr. Mine IŞIKSAL-BOSTAN	(METU, ELE)	_________________
Assoc. Prof. Dr. Bülent ÇETİNKAYA	(METU, SSME)	_________________
Assoc. Prof. Dr. Yusuf KOÇ	(Gaziantep U., ELE)	_________________
Assist. Prof. Dr. Didem AKYÜZ	(METU, ELE)	_________________

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : Reyhan TEKİN-SİTRAVA

Signature :

ABSTRACT

AN INVESTIGATION INTO MIDDLE SCHOOL MATHEMATICS TEACHERS' SUBJECT MATTER KNOWLEDGE AND PEDAGOGICAL CONTENT KNOWLEDGE REGARDING THE VOLUME OF 3D SOLIDS

Tekin-Sitrava, Reyhan

Ph.D, Department of Elementary Education

Supervisor: Assoc. Prof. Dr. Mine Işıksal-Bostan

February 2014, 267 pages

The purpose of this study was to examine middle school mathematics teachers' subject matter knowledge and pedagogical content knowledge of the volume of 3D solids. In order to achieve the purpose of the study, four middle school teachers working in public schools in Ankara participated in the study. To get deep and rich answers to research questions asked, qualitative methodology was used. Participants were selected through purposeful sampling. Data was gathered via questionnaire, semi-structured interviews, classroom observations, and field notes. The data was analyzed using constant comparative method.

The findings revealed that although middle school teachers could generate alternative solution methods, they solved questions just using volume formula while teaching the topic to their students. Moreover, they were unable to generate story problems related to the volume of 3D solids using given numbers and terms. The teachers applied teacher-centered instructional strategy to teach the volume of 3D solids to their students. Furthermore, middle school teachers' knowledge of learners, such as interpretation of students' alternative solution methods, identifying their errors and the sources of them, was restricted by their experiences. Moreover, to make the topic more understandable, they altered the order of the sub-topics of 3D solids but their knowledge of the connection the volume of 3D solids with other topics was inadequate. The study showed that middle school teachers used both formative and summative assessment strategies such as informal questioning, homework, paper-pencil test, performance homework and project work. As a result of findings of the study, important implications and recommendations for further studies were suggested.

Keywords: Mathematics education, middle school mathematics teachers, subject matter knowledge, pedagogical content knowledge, the volume of 3D solids,

ÖZ

ORTAOKUL MATEMATİK ÖĞRETMENLERİNİN 3 BOYUTLU CİSİMLERİN HACMİNE İLİŞKİN ALAN VE PEDAGOJİK ALAN BİLGİLERİ ÜZERİNE BİR ÇALIŞMA

Tekin Sitrava, Reyhan
Doktora, İlköğretim Bölümü
Tez Yöneticisi: Doç. Dr. Mine Işıksal Bostan
Şubat 2014, 267 sayfa

Bu çalışmanın amacı, ortaokul matematik ögretmenlerinin 3 boyutlu cisimlerin hacmine iliskin alan bilgilerini ve pedagojik alan bilgilerini incelemektir. Bu amaç doğrultusunda, Ankara'daki devlet okullarında görev yapan dört tane ortaokul matematik öğretmeni çalışmaya katılmıştır. Belirlenen araştırma sorularına derinlemesine ve zengin cevaplar bulabilmek için, nitel araştırma yöntemi kullanılmıştır. Katılımcılar amaçlı örneklem yöntemi ile seçilmiştir. Veri, anket, yarı-yapılandırılmış görüşmeler, sınıf gözlemleri ve gözlem notları ile toplanmıştır. Veriyi analiz etmek için, sürekli karşılaştırmalı analiz yöntemi kullanılmıştır.

Bulgular matematik öğretmenlerinin, 3 boyutlu cisimlerin hacmini hesaplamak için alternatif çözüm yolları geliştirebilmelerine rağmen, öğrencilerine konuyu anlatırken, soruları sadece formül kullanarak çözdüklerini göstermektedir. Ayrıca, öğretmenler, verilen sayılar ve terimleri kullanarak 3 boyutlu cisimlerin hacmi ile ilgili problem kuramamaktadırlar. Öğretmenler, 3 boyutlu cisimlerin hacmini öğretmek için öğretmen merkezli öğretim yöntemini uygulamaktadırlar. Buna ek olarak, öğretmenlerin öğrenci bilgileri, yani öğrencilerin alternatif çözüm yöntemlerini yorumlama, onların hatalarını ve bu hataların kaynaklarını belirleme bilgileri, öğretmenlerin deneyimleri ile sınırlıdır. Ayrıca, konunun daha anlaşılır olması için, 3 boyutlu cisimler ile ilgili alt konuların sırasını değiştirmişler fakat diğer konularla yapılan bağlantılar yetersiz kalmıştır. Ayrıca, çalışma öğretmenlerin, resmi olmayan sorgu sorma ve yazılı sınav gibi değerlendirme yöntemleri kullandığını göstermektedir. Çalışmanın bulguları sonucunda, önemli çıkarımlar yapılmış ve gelecek çalışmalar için önerilerde bulunulmuştur.

Anahtar Kelimeler: Matematik egitimi, ortaokul matematik öğretmenleri, konu alan bilgisi, pedagojik alan bilgisi, 3 boyutlu cisimlerin hacimleri

To my beloved husband,

Kubilay SİTRAVA

ACKNOWLEDGMENTS

I would like to thank many people who have contributed to the accomplishment of this study. First, my deepest gratitude to my supervisor, Assoc. Prof. Dr. Mine IŞIKSAL BOSTAN, who kindly and patiently shared her valuable ideas with me throughout the study. Without her scientific experience and vision, this study could not have been carried out. I will never forget her invaluable support, guidance, and encouragement .

I also would like to thank my ex-advisor and committee member Assoc. Prof. Dr. Yusuf KOÇ. Without his contribution and encouragement, I could not have begun this long journey. I truly thank him for his guidance, wisdom and his belief in me.

I would also thank Assoc. Prof. Dr. Erdinç ÇAKIROĞLU and Assoc. Prof. Dr. Bülent ÇETİNKAYA for being part of this study and guiding me through this journey. When I need feedback, they always agreed to meet and provided the support that I needed. I am grateful to them for their support, ideas, and suggestions which made my research much better and richer. I also thank Assist. Prof. Dr. Didem AKYÜZ for being in my examining committee. Thank you very much for her valuable feedback and encouragement.

Heartfelt thanks also go to the administrators of Evrim College, Jacky DOYEN, Tahsin TORUN and Safiye DEMİR. Without their permission to carry out the study and their subsequent full support, I could not have continued my doctorate. I thank my cousins, Murat DEMİRÖRS and Erdinç SİTRAVA for their support during the development of the instrument. Also, I thank Chris TAYLOR for her patience in proof reading my dissertation.

I would like to express my gratitude to all the teachers and students who participated in my study. Without them, there would have been no study.

I am also grateful to Esra BALGALMIŞ and Bilge YÜREKLİ for their encouragement and support. It was wonderful to share the PhD journey with them.

I also indebted to my dear friends Aslı CAN YALÇIN, Özer GÜLEN, Zehra GÜLSAY, Nesrin PEKER ÇAPIN, Zehra OCAK, Nagehan ORHAN, Rukiye AYAN and Aslıhan OSMANOĞLU. I am very lucky that to have friends that show me such great patience even I did not contact them for a long time due to my work and being under much stress.

In addition, my heartfelt thanks and appreciation for her endless support, great understanding and patience go to my best friend Gülbir GÜR. I am fortunate to have such a friend who has incredible patience, has offered me endless support and has always believed in me. Another special thank you goes to Ramazan SAK whose close friendship, inspiration, help, suggestions and support helped me greatly in keeping me on this path that I have just completed.

Much gratitude and appreciation are due my sister Selda TEKİN ÖZAN, brother-in-law Celal ÖZAN and my nephew Doğukan ÖZAN for your understanding, and support.

I am forever grateful to my father Feyzullah TEKİN and, my mother Ayşe TEKİN. During each phone call, I felt your support, encouragement and belief in me. Without this, I could not possibly complete this study.

Special thanks to my husband KUBİLAY for his incredible patience and everlasting support. This dissertation would not be possible without his considerable assistance and love. He has always been by my side whenever I have needed him. I'm so lucky to have him as my partner in life.

Last, but by no means least, many thanks to my son DEMİR who when I was depressed, motivated me with his smile. Without him, this endeavor would not have been possible.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

LIST OF ABBREVIATIONS

PCK	Pedagogical Content Knowledge
SMK	Subject Matter Knowledge
CK	Curriculum Knowledge
SCK	Specialized Content Knowledge
CCK	Common Content Knowledge
KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Teaching
VDSQ	Volume of 3D Solids Questionnaire
METU	Middle East Technical University
NCTM	National Council of Teachers of Mathematics
MoNE	Ministry of National Education
Pr	Presenter
Std	Student
Stds	Students

CHAPTER I

INTRODUCTION

"If you want to give the students one cup of water, you (the teacher) should have one bucket of water of your own"
(An, Kulm & Wu, 2004, p. 146)

This Chinese saying stated by An et al. (2004) explains that teachers should have extensive and well-organized knowledge for effective teaching. Shulman (1986) commented that there were central questions in the literature, which still have not been answered regarding the knowledge needed for effective teaching. Some of these questions were about the planning of the lessons, the explanations of the subjects, organizing and applying activities, selecting appropriate representations, dealing with students' misconceptions/difficulties, and assessing students' understanding. Briefly, the main focus of these questions is knowing about the subject to be taught and knowing how it will be taught (Grossman, 1990; Ma, 1999). Accordingly, teachers' knowledge became the utmost important issue for effective teaching. Therefore, throughout many years, researchers examined the knowledge that teachers need to know in order to teach effectively (Ball, 1990a; Ball, & Bass, 2002; Ball, Thames, & Phelps, 2008; Carpenter, Fennema, Peterson & Carey, 1988; Gess- Newsome, 1999; Grossman, 1990; Rowland, Huckstep & Thwaites, 2005; Shulman, 1986; 1987).

In many of these studies, researchers explained that teachers should have broad content knowledge for effective teaching which consists of knowledge of the subject and its structures (Ball, 1991; Ball, Thames, & Phelps, 2008; Borko, 2004). In this regard, Ball and her colleagues (2008) pointed out that:

> *Teachers must know the subject they teach. Indeed, there may be nothing more foundational to teacher competency. The reason is simple. Teachers who do not themselves know a subject well are not likely to have the knowledge they need to help students learn this content. At the same time, however, just knowing a subject well may not be sufficient for teaching. One need only sit in a classroom for a few minutes to notice that the mathematics that teachers work with in instruction is not the same mathematics taught and learned in college classes. In addition, teachers need to know mathematics in ways useful for, among other things, making mathematical sense of student work and choosing powerful ways of representing the subject so that it is understandable to students. It seems unlikely that just knowing more advanced mathematics will satisfy all of the content demands of teaching. What seems most important is knowing the mathematics actually used in teaching (p.45).*

It can be understood from statement made by Ball et al. that knowing mathematics for teaching is more than knowing the facts and concepts, applying them to the problems and following procedures to solve the problems correctly. It also means knowing how to make the topics meaningful for students. Actually, teachers' knowledge is amalgam of the subject matter knowledge and the pedagogical content knowledge. The former deals with "what" is to be taught and the latter deals with "how" to teach it (Ma, 1999). In other words, teachers' knowledge includes the subject which is taught and the ways of teaching it (Grossman, 1990).

It is surely beyond doubt that the teacher's knowledge has been described as complex and consisting of various facets. Primarily, Shulman (1986) expressed the complexities of the major categories of teachers' knowledge: subject matter content knowledge (SMCK), pedagogical content knowledge (PCK) and curricular knowledge (CK). Shulman's (1986) SMCK involves knowing the facts, truths and concepts, explaining the reasons for learning these concepts, and relating the concepts within and without the discipline. The second category of teachers' knowledge is PCK which consists of both the knowledge of content and knowledge of pedagogy. Shulman (1986) defined PCK as knowing "the ways of presenting and formulating the subject that make it comprehensible to others" and "an understanding of what makes the learning of topics easy or difficult" (p.9). The last category of teachers’ knowledge is CK which means the knowledge of a program developed for the teaching of particular subjects at a particular level (Shulman, 1986). After Shulman’s introduction of the categories of teachers’ knowledge, many researchers expanded these categories, which are presented and discussed in the literature review of this dissertation (Ball, Thames & Phelps, 2008; Gess-Newsome, 1999; Grossman 1990; Rowland, Huckstep & Thwaites, 2005).

Although the researchers (Ball et al., 2008; Gess-Newsome, 1999; Grossman 1990; Rowland et al., 2005) used different terminology regarding the categories of teachers’ knowledge, all agreed that having sufficient knowledge and being able to use it efficiently is at the heart of teaching mathematics. In this respect, one group of researchers had carried out several studies to investigate either pre-service teachers’ or in-service teachers’ had mathematical content knowledge for teaching. These studies have shown that both pre-service and in-service teachers have limited content knowledge for teaching mathematics (Basturk & Donmez, 2011; Baki, 2013; Contreras, Batanero, Diaz & Fernandes, 2011; Even, 1993; Even & Tirosh, 1995; Hill, Rowan & Ball, 2005; Isiksal, 2006). Various researchers have also conducted studies to investigate the influence of teachers’ knowledge on student learning (Ball, 1990a; Ball, 1991; Borko et al., 1992; Kahan, Cooper & Bethea, 2003; Leinhardt & Smith, 1985; Lenhart, 2010; Ma, 1999). The researchers concluded that teachers’ knowledge influences students learning and teachers’ competence in conveying their knowledge to the students provides more competent students in terms of mathematics. When the importance of teachers’ knowledge is considered, middle school teachers’ knowledge of mathematics is primary source for supplying mathematical teaching to the middle schol students.

In addition to having broad and deep knowledge of mathematics for effective teaching, the knowledge of geometry has a crucial role in teaching and learning mathematics. The US National Council of Teachers of Mathematics [NCTM] (2000) emphasized its prominence by stating “geometry offers an aspect of mathematical thinking that is different from, but connected to, the world of numbers” (p.97). This can be interpreted as when students are engaging in shapes, structures and transformations; they understand the geometrical concepts and also the mathematics behind those concepts. Therefore, it is necessary to have powerful geometry knowledge in addition to mathematics knowledge to be an effective teacher (Maxedon, 2003). Because of the importance of teachers' geometry knowledge, researchers conducted studies to investigate teachers' knowledge related to geometry (Baturo & Nason, 1996; Esen & Cakiroglu, 2012; Fujita & Jones, 2006, Gomes, 2011; Kellogg, 2010; Maxedon, 2003; Ng, 2011; Swafford, Jones and Thornton, 1997). These researchers concluded that teachers' content knowledge on geometry topics were inadequate to teach those topics effectively, which were consistent with the results of the studies related to teachers' knowledge on specific mathematics topics such as fractions, decimals, functions. Since teachers lack the knowledge of geometry topics, their students had difficulties in some of geometry topics. Due to the fact that the teacher who has lack of knowledge about specific topics could not transfer the appropriate knowledge to the students, it is important to reveal teachers' knowledge on specific geometry topics. Based on the previous research studies, the topic with which most of the students had difficulties was the volume of 3D solids (Battista & Clements, 1996; 1998; Ben- Chaim, Lappan & Houang, 1985; Ng, 1998; Olkun, 1999; 2003). Since the students had difficulties with the volume of 3D solids, it is significant to question teachers' knowledge about the volume of 3D solids.

Besides, many reserachers have focused on investigation of mathematics teachers' knowledge in terms of several dimensions. For instance, teachers' understanding of key facts, concepts, and principles related to the mathematics topics, representing mathematical ideas, and providing mathematical explanations and procedures with their justifications were the dimensions of teacher's SMK that had been investigated by several researchers (Ball, 1990a; 1990b; Even, 1993; Haciomeroglu, Haciomeroglu, & Aspinwall, 2007; Isiksal, 2006). In the present study, teachers' alternative solution methods to calculate the volume of 3D solids and generating a story problem regarding the volume of 3D solids were taken as the dimensions to investigate middle school mathematics teachers' SMK. In relation to developing alternative solution methods, the teachers' knowledge was investigated from the point of representing their mathematical ideas (Ball et al.,2008) and comprehending the concepts and principles regarding the topic of the volume of 3D solids (Shulman, 1986). In addition, generating a story problem was another dimension to understand the teachers' SMK. In relation to that, Chapman (2002) emphasized that mathematics teachers thinking about story problems was an important component of understanding their SMK. Consistent with Chapman, Ball (1990a) emphasized that generating story problems or selecting the story problem that represent a given statement were the ways of understanding teacher's knowledge. For this reason, generating a story problem was one of the focuses of the present study.

Similar to teachers' SMK, researchers have investigated teachers' PCK in terms of several dimensions such as knowledge of students' conceptions/ preconceptions/ misconceptions, the ways of eliminating students' misconceptions, the most powerful examples, illustrations and demonstrations (Even, 1993; Even & Tirosh, 1995; Isiksal, 2006; Karahasan, 2010; Kilic, 2011; Leavitt, 1998). In the present study, the instructional strategies that the teachers applied to teach the volume of 3D solids effectively, the assessment strategies that were used to assess student learning regarding the volume of 3D solids were the dimensions that will be considered under the dimension of middle school mathematics teachers' PCK. Moreover, teachers' PCK was analyzed in terms of their knowledge of the solution methods that the students prefer, the errors that the students held, the sources of these errors, the ways of eliminating the errors, and interpreting students' alternative solution methods. Furthermore, relating the topic to the other topics of mathematics or other disciplines and the order of the sub-topics of 3D solids were the other dimensions that were considered under PCK.

As a result, among the different mathematics topics and different dimensions of teachers' SMK and PCK that could be researched, middle school teachers' knowledge on the volume of 3D solids was investigated in terms of developing alternative solution methods, generating a story problems, instructional strategies that the teachers applied, assessment strategies that they used to assess students learning, knowledge about learners, and knowledge about curriculum.

1.1 Statement of the Problem

It is obvious that teachers' content knowledge has crucial role in students' learning thus, teachers should have the knowledge of the facts, truths and concepts, and be able to explain why they are worth learning and how they are related within mathematics and other disciplines. Furthermore, teachers should know how to make the subject more meaningful and understandable for students, determine the topics which the students have difficulty to learn or easy to learn, assess students' preconceptions and misconceptions, present strategies for overcoming the misconceptions, and be able to use the best representation and examples to teach the subject effectively (Shulman, 1986). In this respect, researchers also have important role in students' learning and studies should be conducted to examine teachers' content knowledge in specific topics, and then present the results of the study to teacher educators, curriculum developers and teachers.

All over the world several researchers investigated teachers' knowledge and they concluded that having rich knowledge of mathematics was one of the pillars of effective teaching and students' achievement (Hill & Ball, 2004; Hill, Rowan, & Ball, 2005; Hill, Schilling, & Ball, 2004; Rowan, Schilling, Ball & Miller, 2001). However, Maxedon (2003) claimed that teachers' mathematics knowledge alone was not enough to make the subjects more comprehensible for students. It is necessary to have deep geometry knowledge due to the fact that geometry has crucial role in teaching and learning in other subject areas of the mathematics curriculum and other disciplines (NCTM, 1989; 2000). Contrary to noteworthiness of geometry for mathematics teaching, many researchers did not focus on investigating teachers' knowledge on geometry subjects. In other words, the number of studies regarding teachers' geometry knowledge with respect to the accessible literature was limited (Baturo & Nason, 1996; Fujita & Jones, 2006, Gomes, 2011; Kellogg, 2010; Maxedon, 2003; Ng, 2011; Swafford, Jones and Thornton, 1997). For this reason, there is a need to conduct research studies to investigate teachers' geometry content knowledge. In order to partly satisfy this need, the aim of this study to obtain information related to teachers' content knowledge about specific geometry subject. As with all studies, the focus of this study has been narrowed down with respect to the area of interest of the content that is investigated. The volume of 3D solids was chosen since this is a mathematical topic that is known to be difficult for students (Battista & Clements, 1996; 1998; Ben-Chaim, Lappan & Houang, 1985). Within this context, there needs to be a deeper understanding of the teachers' knowledge concerning the volume of 3D solids for teachers, mathematics educators and curriculum developers to make the topic easier for the students.

Thus, this study aims to answer the following research questions:

1. What is the nature of the four middle school mathematics teachers' subject matter knowledge of the volume of 3D solids?
    1.1. What are the alternative solution methods these four teachers propose to calculate the volume of 3D solids?
    1.2. To what extent are these teachers successful at generating a story problem regarding the volume of 3D solids?
2. What is the nature of the four middle school mathematics teachers' pedagogical content knowledge on the volume of 3D solids?
    2.1. What kind of instructional strategies do these teachers use to teach the volume of 3D solids?
    2.2. To what extent do the teachers recognize their students' knowledge related to the volume of 3D solids?
    2.3. To what extent do the teachers have knowledge of curriculum related to the volume of 3D solids?
    2.4. What kind of assessment types do the teachers apply to assess students' understanding of the volume of 3D solids?

1.2 Definitions of Important Terms

The research questions of the study contain several terms that need to be defined:

Subject Matter Knowledge

Subject matter knowledge was related to teachers' knowledge concerning what they will teach and it includes substantive and syntactic components. The substantive aspect encompasses the knowledge of facts, rules, principles, concepts, and theories in a specific field of mathematics whereas the syntactic component covers knowledge of the process through which knowledge is generated in the field (Schwab, 1978; as cited in Shulman, 1986).

Pedagogical Content Knowledge

The concept of PCK was developed by Shulman and his colleagues in the Knowledge Growth in Teaching Project (Shulman, 1986). In light of the definition proposed by Shulman (1986), PCK is “the particular form of content knowledge that embodies the aspects of content most germane to its teachability” (Shulman, 1986, p.9).

The PCK in this study refers to knowledge of instructional strategies, knowledge of learners, knowledge of curriculum and knowledge of assessment.

Knowledge of Instructional Strategies

One of the PCK dimension is the knowledge of instructional strategies including the teachers’ knowledge of subject specific strategies which represent general approaches for teaching, and topic-specific strategies which are the most effective way for teaching a specific topic (Magnusson et al., 1999).

Knowledge of Learners

The knowledge of the learners is PCK dimension that is related to possessing information about the learners which helps and encourages them to learn specific topic. It refers to the teachers’ knowledge about students’ abilities, prior knowledge, and also involves the gaps in students’ knowledge that should be filled before the subject is presented. Additionally, it consists of the knowledge of students’ misconceptions/difficulties/errors in learning a specific topic (Grossman, 1990; Magnusson, et al., 1999).

Knowledge of Curriculum

Another dimension of PCK is the knowledge of the curriculum, which includes the teachers' knowledge of curriculum goals and objectives, and the teachers’ knowledge of specific curricular programs including their activities and learning goals (Magnusson et al., 1999)

Knowledge of Assessment

Knowledge of assessment is one of the dimensions of teachers' PCK which comprises the teachers' knowledge of what to assess and how to assess students' learning (Magnusson et al., 1999).

1.3 Significance of the Study

In recent years, the most crucial question among researchers is: "What do mathematics teachers need to know to teach effectively?" (Ball, 1990a, 1990b, 2000; Ball, Thames & Phelps, 2008; Borko, 2004; Goulding, Rowland & Barber, 2002; Ma, 1999; Shulman, 1986; 1987). The researchers stated that teachers need to have both subject matter knowledge and pedagogical content knowledge for effective teaching (Ball et al., 2008). That is, teachers should possess the knowledge of both what they will teach and how they will teach. For this reason, the present study has several significant aspects. To begin with, in light of the accessible mathematics education literature, the knowledge of assessment was ignored in many research studies although it is important for understanding students' learning. In order to fill the gap to a certain extent, the study examined mathematics teachers' knowledge of assessment in light of the work of Magnusson et al. (1999). Additionally, the focus was on the knowledge of instructional strategies, knowledge of learners, and knowledge of curriculum in order to present mathematics teachers' PCK. In this way, the mathematics teachers' PCK was examined on several counts and this examination allows us to display the complete picture of mathematics teachers' PCK.

Hill, Rowan and Ball (2005) stated that teachers play an important role in students' effective understanding of mathematics. Since teachers' content knowledge has an important impact on students' achievement, several studies were conducted to investigate teachers' content knowledge related to variety of mathematics subjects all over the world (Ball, 1990a; 1990b; Contreras, Batanero, Diaz & Fernandes, 2011; Even, 1993; Even& Tirosh, 1995; Huang & Kulm, 2012; Livy & Vale, 2011; Nilsson & Lindstrom, 2012; Pino-Fan, Godino, Font & Castro, 2013). When the accessible literature was reviewed, it was noticed that researchers focused on investigating teachers' content knowledge on fractions (Ball, 1990a; Hutchison, 1997), division (Ball, 1990b), probability (Contreras, Batanero, Diaz & Fernandes, 2011), ratio (Livy

& Vale, 2011), polygons (Carreño, Ribeiro & Climent, 2013), derivatives (Pino-Fan, Godino, Font & Castro, 2013) and functions (Even, 1993; Even & Tirosh, 1995; Huang & Kulm, 2012). The literature review showed us that there was a need for more research to investigate teachers' content knowledge on different mathematics topics. Based on the review of the accessible literature, teachers' content knowledge on the volume of 3D solids has not yet been investigated. For this reason, this study is expected to make important contributions to the literature by providing information about teachers' content knowledge on the volume of 3D solids.

Similar to the research studies in the international context, there has been an increase in research about teachers' content knowledge in Turkey. The studies conducted in Turkey dealt with teachers' content knowledge about different topics such as fractions (Isik, Ocal & Kar, 2013, Isiksal, 2006; Isiksal & Cakiroglu, 2008); division (Baki, 2013), variables (Gokturk, Sahin & Soylu, 2013), equal signs (Aygun, Baran-Bulut & İpek, 2013), functions (Hacıomeroglu, Hacıomeroglu & Aspinwall, 2007; Karahasan, 2010), and limit and continuity (Basturk & Donmez, 2011). It was found that the variety of topics concerning teachers' content knowledge investigated by Turkish researchers was limited. In order to obtain the complete picture of teachers' content knowledge, it will be significant to focus on different topics.

Moreover, some researchers have investigated teachers' content knowledge on some geometry topics such as quadrilaterals (Aslan-Tutak, 2009; Fujita & Jones, 2006), area measurement (Baturo & Nason, 1996; Kellogg, 2010), geometric transformations (Gomes, 2011), and solid objects (Bukova-Guzel, 2010). Apart from these studies, only one study was found which aimed to investigate teachers' content knowledge on the volume of 3D solids (Esen & Cakiroglu, 2012). This study was designed to investigate pre-service teachers' knowledge of analyzing students' solution methods and determining the correctness of the solutions. Esen and Cakiroglu (2012) carried out a very specific study which focused on a part of the pre-service teachers' PCK in relation to the volume of 3D solids. However, the researchers (Grossman, 1990; Magnusson et al., 1999) stated that there are many dimensions regarding teachers' PCK such as the knowledge of students' misconceptions on the volume of 3D solids and the reasons for these misconceptions, knowledge of the most useful strategies to teach the topic, knowledge of assessment strategies to understand students' learning on the volume of 3D solids. Therefore, by investigating the teachers' content knowledge of the volume of 3D solids, it is hoped to complete the missing part of the picture of teachers' content knowledge literature.

Contrary to the research studies related to teachers' knowledge of the volume of 3D solids, many studies have been conducted to investigate students' understanding of the volume of 3D solids. The results of these studies revealed that students' achievement related to the volume of 3D solids was low and students had difficulties in the volume of rectangular prism (Battista & Clements, 1996; Ben-Chaim, Lappan & Houang, 1985; Ng, 1998; Olkun, 1999). When the importance of teachers' knowledge on students' achievement is considered; it is noteworthy to investigate teachers' content knowledge relevant to volume of 3D solids.

The significance of the current study is also rooted in the data collected from a real classroom environment. To enrich literature about the teachers' content knowledge, the practical knowledge that teachers actually use in their teaching is very important. Hence, the study contributes to literature through how teachers transfer their SMK to their students and how teachers use their PCK to support students' learning.

Moreover, this study is important for the literature because the participants are experienced teachers. In the related literature, research studies have generally focused on knowledge of pre-service teachers (Ball, 1990a; 1990b; Basturk & Donmez, 2011; Contreras, Batanero, Diaz & Fernandes, 2011; Even, 1993; Even& Tirosh, 1995; Huang & Kulm, 2012; Isiksal, 2006; Livy & Vale, 2011). However, pre-service and novice teachers generally do not have a robust PCK (Magnusson et al., 1999; Shulman, 1987). In this sense, the practice of experienced teachers would provide a valuable example of how teachers transform their SMK and use PCK in their teaching. In addition, the results of the study may provide practical information for other mathematics teachers who teach the same topic in their classes. It is hoped that experienced teachers' rich repertoire of teaching practices may also enrich the other teachers' teaching.

This study has three main powerful aspects. Firstly, by examining teachers' content knowledge in a different topic which has not yet been studied it is expected that this will enrich the literature about teachers' content knowledge. Secondly, since this study investigates experienced teachers, it is hoped that valuable information will be gained concerning how teachers use their content knowledge for effective teaching. Lastly, due to the fact that data was collected via classroom observation, the study provides concrete examples of teachers' content knowledge in relation to a specific topic. As a result, the aim is to contribute to the research literature on teachers' content knowledge by providing a full array of both theoretical and practical data.

CHAPTER II

REVIEW OF THE LITERATURE

The purpose of this study was to provide a picture of the middle school mathematics teachers' knowledge concerning the volume of 3D solids. In this chapter, the theoretical frameworks and related research studies are given. To provide more clarity, this chapter is divided into the following subsections: a) Conceptual frameworks for the types of teachers' knowledge; b) Research studies on mathematics teachers' knowledge on mathematics topics; c) Research studies on mathematics teachers' knowledge on mathematics topics in Turkey; d) Research studies on the way of enhancing mathematics teachers' knowledge on mathematics topics; e) Research studies on mathematics teachers' knowledge on geometry topics; f) Research studies on the way of enhancing mathematics teachers' knowledge on geometry topics; g) Research studies on the volume of three dimensional solids. At the end of the chapter, a summary of the literature review is provided.

2.1 Conceptual Frameworks for the Types of Teachers' Knowledge

High-quality teaching requires everything that teachers must do to increase students' learning (Ball, Thames, & Phelps, 2008). Everything refers to planning lessons, selecting appropriate examples and definitions which make the subjects understandable for students, evaluating students' work, devising and managing homework. Moreover, knowing about students' prior knowledge, identifying their errors/misconceptions/difficulties and the reasons for these errors/misconceptions/ difficulties, generating the ways of overcoming them, and making connections among mathematical topics are the other necessities for high-quality teaching (Ball et al., 2008; NCTM, 2000; Shulman, 1986). In broad terms, high-quality teaching requires both subject matter knowledge and pedagogical content knowledge (Ball et al., 2008). Put differently, high-quality teaching is connected to teachers' content knowledge. Therefore, in recent years, this has led to researchers emphasizing the important role of teachers' content knowledge in teaching (Ball, 1990a, 1990b; Ball, 2000; Ball, Thames & Phelps, 2008; Borko, 2004; Goulding, Rowland & Barber, 2002; Ma, 1999; Shulman, 1987). Moreover, various researchers undertook studies to answer the question of what mathematics teachers need to know to teach effectively (Ball, 1990; Ball, & Bass, 2002; Baki, 2013; Carpenter, Fennema, Peterson & Carey, 1988; Contreras, Batanero, Diaz & Fernandes, 2011; Gess-Newsome, 1999; Livy & Vale, 2011; Masters, 2012; Pino-Fan, Godino, Font & Castro, 2013; Rowland, Huckstep & Thwaites, 2005; Shulman, 1986; 1987). Although some research studies investigated the effect of teachers' knowledge on students' achievement or the level of teachers' knowledge on a particular subject, other research studies examined the basis of teachers' knowledge which was then used as a framework by several researchers. Those frameworks are presented and discussed below. The frameworks by Ball et al. (2008); Gess-Newsome (1999); Grossman (1990); Rowland et al. (2005); and Shulman (1986) were related to teachers' knowledge whereas the framework created by Magnusson et al. (1999) was based on teachers' PCK. For the following sections of this chapter, the frameworks for teachers' knowledge are presented, and then the framework on teachers' PCK (Magnusson et al., 1999) is outlined.

2.1.1 Shulman's Categorization of Teacher's Knowledge

Shulman (1986) commented that the emphasis of previous research studies related to teaching was on how teachers plan lesson and activities, prepare assignments, arrange lesson time, decide questions' level and determine general students' understanding (Shulman, 1986). It can be understood that the focus was not on subject matter knowledge, and ways that teachers' transfer the subject matter to the instruction was not discussed in these studies. In this manner, the content of the lesson taught, the questions asked and the explanations presented were missing from these studies which proved to create serious problems both for policy planning and for research. Shulman considered this to be a "missing paradigm" in teaching studies. As a consequence of "missing paradigm", Shulman noted the following questions: "Where does teacher explanation come from? How do teachers decide what to teach, how to present it, how to question student to about it, and how to deal with problems of misunderstanding?" (p. 8). According to Shulman, responses to these questions should be gained from the teacher's perspective. In order to do this, Shulman and colleagues realized that a more comprehensive theoretical framework was needed. Within this scope, in 1986, Shulman and his colleagues started the "Knowledge Growth in Teaching" project. In this way, they tried to bring to the fore previously unasked questions such as: "What are the sources of teacher knowledge? What does a teacher know and when did he or she come to know it? How is new knowledge acquired, old knowledge retrieved, and both combined to form a new knowledge base?" (Shulman, 1986, p. 8). In order to answer these questions, Shulman and colleagues (1986) aimed to investigate the development of secondary teachers' knowledge of English, biology, mathematics and social studies. Shulman et al. (1986) asked the teachers to read and comment on materials related to the subjects they teach. In order to collect the data, they conducted regular interviews, and observed the process of the secondary teachers' instruction. This study produced large amount of information from which a model concerning categories of teachers' knowledge was created.

According to Shulman and colleagues, teacher's knowledge can be categorized as follows; 1) subject matter knowledge 2) pedagogical content knowledge 3) curricular knowledge.

The first component, content knowledge, involves the structures of subject matter which are defined by Schwab (1978) (as cited in Shulman, 1986) as substantive and syntactic. The former is the way in which the basic concepts and principles are organized in order to associate the facts; the latter is the way in which truth or falsehood, validity or invalidity is established. Shulman (1986) stated that content knowledge requires the teacher to be proficient in the knowledge of the facts and the concepts of their field. Moreover, he explained that a teacher who has adequate content knowledge is able to explain why the accepted truth or facts is worth knowing, how it relates to other conditions within the discipline and without, why the subject is central to the discipline or peripheral. The statement “the teacher need not only understand that something is so; the teacher must further understand why it is so” (p. 9) clearly summarizes the content knowledge.

Pedagogical content knowledge is generally defined as subject matter knowledge for teaching. Namely, it is related to being able to teach the content. Pedagogical content knowledge involves a) knowledge of the most effective representation, b) knowledge of most powerful examples, illustrations, demonstrations and explanations, c) knowledge of the ways of making the subject matter understandable to others, d) knowledge about why the topic is difficult for students and how it will become easy, e) knowledge of students’ conceptions, preconceptions and misconceptions at different grade level, and f) knowledge of how to overcome students’ misconceptions (Shulman, 1986). For Shulman, all these characteristics of pedagogical content knowledge imply that pedagogical content knowledge “represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented and adapted to the diverse interests and abilities of interests and abilities of learners, and presented for instruction” (Shulman, 1987, p. 8).

The last category is curricular knowledge which has two dimensions; lateral curriculum knowledge and vertical curriculum knowledge. The former consists of the knowledge of topics or issues that are being studied at the same time in other subject areas. This knowledge allows teachers to relate mathematics to other subject areas. The latter category is the knowledge of topics or issues that were taught in the preceding year have been taught at the same year and will be taught in later years. This knowledge helps teachers make connections within the topics of a subject.

In 1987, Shulman and colleagues enhanced their model developed in 1986 in the Knowledge Growth in Teaching Project. This model consisted of seven categories; 1) subject matter knowledge, 2) general pedagogical knowledge, 3) curriculum knowledge, 4) pedagogical content knowledge, 5) an understanding of the learners and their characteristics, 6) knowledge of educational ends, purposes, and values, and 7) teachers’ philosophical and historical grounds. Among those categories, PCK is the most important category of teachers’ knowledge for high-quality teaching and defined as;

> *"that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding." (Shulman, 1987, p.8)*

Furthermore, Shulman explained why PCK should attract special interest among seven categories since;

> *"it identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction. Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue." (Shulman, 1987, p.8)*

Following the Shulman model, several research studies were conducted to explore teachers' knowledge and many researchers used Shulman's categorization of teachers' knowledge as a theoretical framework.

2.1.2 Grossman's Categorization of Teacher's Knowledge

Apart from Shulman's categorization, Grossman (1990) categorized teacher's knowledge into four general areas; a) general pedagogical knowledge, b) subject matter knowledge, c) pedagogical content knowledge, and d) knowledge of context as shown in Figure 2.1 below.

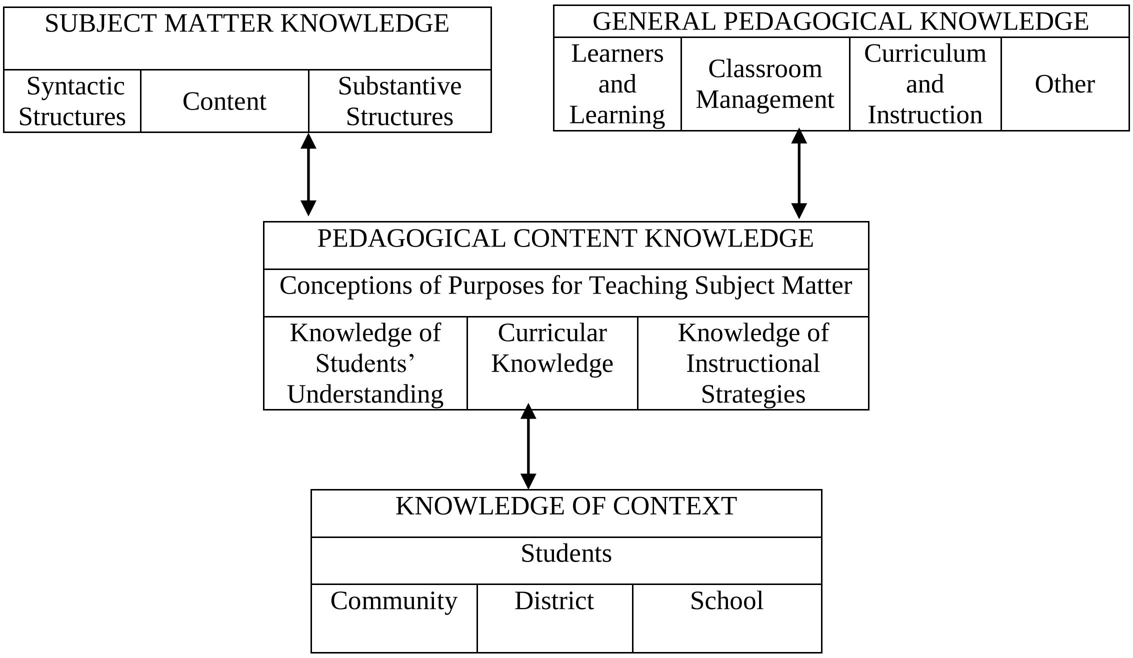


Figure 2.1 Grossman's model of teacher's knowledge (1988, p.9)

The first category is general pedagogical knowledge which was also one of the categories of Shulman's model. Both Grossman and Shulman defined general pedagogical knowledge as knowledge and beliefs related to learners and learning, knowledge of general principles of instruction, knowledge and beliefs concerning classroom management, and knowledge and beliefs about the aims and purposes of education.

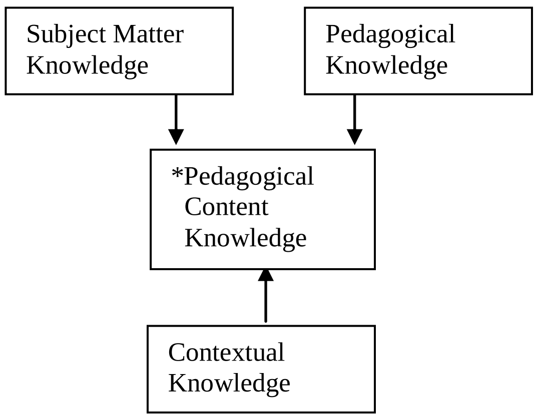
The second category, subject matter knowledge, is another important component of the teacher's knowledge. It includes the knowledge of content and knowledge of the substantive and syntactic structures of a discipline defined by Schwab (1964, as cited in Grossman, 1990). On the one hand, knowledge of content implies knowledge of the major concepts such as facts and truths. On the other hand, the substantive structures of a discipline refer the paradigms within a field which affects the organization of the other fields, and the syntactic structures of a discipline refer to understanding the principles of evidence and proof within a discipline. Knowledge of syntactic and substantive structures are important phenomena for effective teaching since the level of teachers' syntactic and substantive structures of their fields influences their ability to present their disciplines to the students.

The third category of the teacher's knowledge is pedagogical content knowledge. According to Grossman, PCK consists of four components. The first component concerns the knowledge and beliefs about the purposes for teaching a particular subject at a particular grade level. In other words, it is teachers' overarching conception of what it means to teach a particular subject. For example, one mathematics teacher, sees the purpose of teaching problems as teaching students the skills of carrying out a mathematical operation correctly while another defines the purpose of teaching problems as helping students understand the problem first, specifying what is given and what is asked, lastly developing an appropriate strategy. These two perspectives show that teachers' knowledge about the purpose of teaching a subject influences the teachers' teaching style. The second component of PCK is the knowledge of students' understanding, conceptions, and misconceptions of a particular subject. This component of PCK was also specified in Shulman's categorization of teacher's knowledge. Both Grossman and Shulman considered that the teacher who has this knowledge generates appropriate explanations and representations based on their prior knowledge and aims to overcome students' misconceptions. The third component mentioned by Grossman is knowledge of curriculum and curricular materials. This component includes knowing which books and instructional materials are appropriate in order to teach a particular subject effectively. Moreover, it is related to knowledge of the organization of topics in a specific grade level. Furthermore, what students have learnt in the past and will learn in the future are the concerns of this component. Similar to Grossman, Shulman (1986) also explained that the knowledge of the sequence of other topics in the curriculum of the subject area (in this case; mathematics) in the same year, preceding and succeeding years in order to make connections within the topics of the subject. He called this; vertical curriculum knowledge. A final component of PCK is the knowledge of instructional strategies and representations which refers to teaching a particular subject using variety of instructional strategies, examples, models, illustrations, metaphors and simulations to increase students' understanding. Shulman (1986) commented on this component of PCK stating that "the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations-in a word, the ways of representing and formulating the subject that make it comprehensible to others" (p.8).

The last category of the teacher's knowledge is the knowledge of context. It refers to the knowledge of; the district in which teacher works, the school setting, students' academic level and their family background. Although Shulman (1986) did not initially include the knowledge of context in the model developed in 1986, he and colleagues integrated knowledge of educational context in their model in 1987.

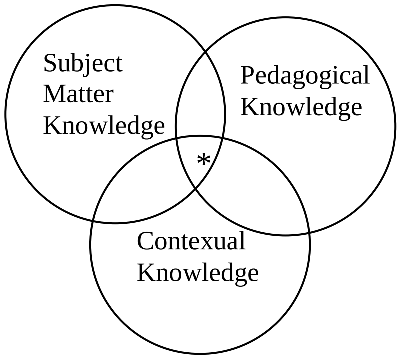
2.1.3 Gess-Newsome's Categorization of Teacher's Knowledge

Gess-Newsome (1999) reviewed the studies of teacher's knowledge which had different dimensions. As a result of her review of research results, she created two distict models of teacher's knowledge: the transformative model, and the integrative model. A schematic of these two models of teacher's knowledge is presented in Figure 2.2 and Figure 2.3, respectively (Gess-Newsome, 1999, p.12).



*= knowledge needed for classroom teaching

Figure 2.2 Gess-Newsome's Transformative Model (1999, p.12)



*= knowledge needed for classroom teaching

Figure 2.3 Gess-Newsome's Integrative Model (1999, p.12)

In the transformative model, the knowledge domains are defined as subject matter knowledge, pedagogical knowledge and knowledge of context which are also the categories of Grossman's model (1990). Similar to Shulman's model (1987), PCK is a distinct category which is a synthesis of the knowledge of subject matter and pedagogy. These domains are useful when they are transformed into PCK. In fact, according to Gess-Newsome, PCK is the only knowledge that is required for effective teaching in the transformative model.

In the integrative model, as with the transformative model, subject matter knowledge, pedagogical knowledge and knowledge of context are defined as the knowledge domains. Contrary to the transformative model, these domains were developed separately and integrated as the part of teaching and PCK is not a distinct category. However, it is the intersection of subject matter knowledge, pedagogical knowledge and contextual knowledge. For effective teaching, a teacher should select the independent knowledge bases of the subject matter, pedagogy, and context, and integrate them to create learning environment. In this model, an effective teacher has well-organized knowledge bases which are easily accessed, integrated, and used flexibly during teaching.

Gess-Newsome (1999) explained the difference between the two models in terms of the knowledge domains by taking an analogy from chemistry.

> *"When two materials are mixed together, they can form a mixture or a compound. In a mixture, the original elements remain chemically distinct, though their visual impact may imply a total integration. Regardless of the level of apparent combination, the parent ingredients in a mixture can be separated through relatively unsophisticated, physical means. In contrast, compounds are created by the addition or*

release of energy. Parent ingredients can no longer be easily separated and their initial properties can no longer be detected. A compound is a new substance distinct from its original ingredients, with chemical and physical properties that distinguish it from all other materials" (p. 11).

Moreover, the two models differ in terms of teaching expertise. According to Gess-Newsome (1999), teachers possess PCK for all topics taught in the transformative model. On the other hand, teachers are flexible in integrating knowledge domains for each topic taught.

The frameworks, mentioned up to this point, are related to the teacher's knowledge, not specifically to that of mathematics teachers'. The frameworks regarding mathematics teacher's knowledge are given and discussed below.

2.1.4 Ball, Thames and Phelps' Categorization of Teacher's Knowledge

In the mid-1980s, a critical development began in the teacher's content knowledge in terms of what teachers know and how they teach (Shulman, 1987). As explained above, Shulman and his colleagues (1986) suggest a special categorization of teacher's knowledge which is the corner stone of the literature. Following Shulman's categorization of teacher's knowledge, other researchers (Gess-Newsome, 1999; Grossman, 1990) developed different frameworks. When these frameworks are examined, it can be seen that they are general and not specific to mathematics education except for that of Rowland et al. (2008). In this context Ball, Thames and Phelps (2008) asserted that a framework related to teacher's content knowledge for mathematics education was needed. Accordingly, they enhanced Shulman's theoretical framework, and developed an approach for mathematics teaching. The focus of their approach was "the work of teaching" which means all the things that teachers do in teaching mathematics. All the things refers to having a deep knowledge of the subject matter, using algorithms in calculations correctly, selecting various and appropriate examples and representations, identifying students' errors, and examining the sources of these errors, being aware of students' preconceptions and misconceptions, and using mathematical language correctly (Ball & Bass, 2002; Ball, Bass & Hill, 2004; Hill, Rowan & Ball, 2005). In other words, the work of teaching refers to what teachers need to do in teaching mathematics which was entitled "mathematical knowledge for teaching" (Ball, Thames & Phelps, 2008). Ball et al. (2008) subdivided Shulman's subject matter knowledge into three types of content knowledge: common content knowledge (CCK), specialized content knowledge (SCK), and horizon knowledge. The specialized content knowledge and common content knowledge are a synthesis of Shulman's subject matter knowledge (Ball et al., 2008). Ball and colleagues (2008) asserted that every person has common content knowledge whether s/he is a mathematics teacher or not. For instance, giving the correct answer to the question "Is 0 an even number" is not special for mathematics teachers. Every person who knows mathematics can answer the question correctly. So, it can be said that s/he has common content knowledge. On the contrary, Ball and colleagues (2008) characterized specialized content knowledge as the knowledge that is unique to the teacher who engages in teaching mathematics. The characteristics of the specialized content knowledge are representing mathematical ideas, providing mathematical explanations and procedures with their justifications, and deciding whether the student's methods are generalizable to other problems. For instance, knowing the representation of $\frac{3}{5}$ or 0.40 using diagrams, explaining why "invert and multiply" rules that work for the division of rational numbers are included in specialized content knowledge.

In addition to SCK and CCK, there is another category of mathematics knowledge for teaching that is; horizon knowledge. In fact, Ball and colleagues were not sure whether horizon knowledge should be included as a third category within subject matter knowledge, so they included it temporarily. Horizon knowledge is defined as "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (Ball et al., 2008, p.403). For example, a mathematics teacher should know that sixth grade students should learn about integers and they will learn four integer operations when they are seventh graders. Namely, it requires knowing what the students will learn ensuing years.

On the other hand, Ball et al. (2008) stated that Shulman's pedagogical content knowledge was specialized into knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). While knowledge of content and students was described as the combination of knowledge about students and knowledge about mathematics; knowledge of content and teaching were described as the combination of knowledge about teaching and knowledge about mathematics (Ball et al., 2008). State differently, the knowledge of content and students requires knowing the topics which the students find easy, difficult or confusing, identifying the students' preconceptions and errors/difficulties/misconceptions, knowing the reasons for these errors/ difficulties/misconceptions, and the ways of responding students' errors/difficulties/ misconceptions. As a second category, the knowledge of content and teaching involves determining the best teaching method and most useful representations, choosing examples that are appropriate for students to begin the topics, and deciding which examples will take students deeper into the content. Additionally, the last category is knowledge of content and curriculum. It represents teachers' knowledge of the programs designed for teaching of particular subjects at a given level, knowledge of the characteristics of the program and knowledge of the variety of instructional materials available for the teaching of a subject.

Figure 2.4 presents the correspondence between Ball et al.'s (2008) map of the domain of content knowledge for teaching and two of Shulman's (1986) initial categories: subject matter knowledge and pedagogical content knowledge.

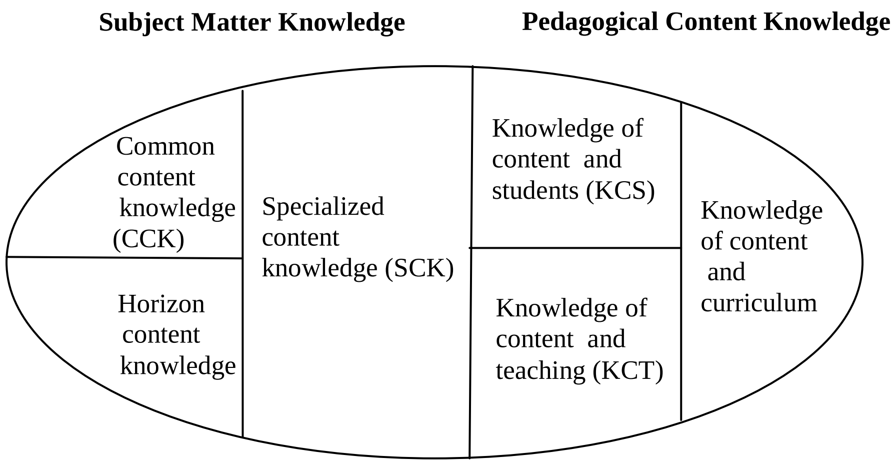


Figure 2.4 Ball et al.'s (2008) map of the domain of content knowledge for teaching and two of Shulman's (1986) initial categories: SMK and PCK (Ball et al., 2008, p. 403)

Figure 2.4 displays how Ball and colleagues (2008) divided Shulman's subject matter knowledge into three categories; common content, specialized content and horizon knowledge. Moreover, the pedagogical content knowledge is defined as the knowledge of content and teaching, knowledge of content and students, and knowledge of content and curriculum. This can be interpreted as Ball and colleagues' categorization of content knowledge being more specific than Shulman's. In this sense, Ball et al. provide a more detailed categorization of content knowledge for mathematics teaching.

Although Ball and colleagues' framework is detailed, it does have some deficiencies and needs refinement and revision (Ball et al., 2008). Firstly, teachers may use a different kind of knowledge for the same situation thus, the categories of knowledge in teaching mathematics may overlap each other and the boundary between the categories is not necessary clear-cut. For example, two teachers (Teacher A and Teacher B) analyzing students' error related to solving equation may respond differently. Teacher A may judge students errors in terms of the stages of the students' solutions, the assumptions that students made and mathematical operations that they used. S/he may think that students may not use the distributive rule while eliminating the parenthesis or they may add the terms which have different variables. Teacher A is using specialized content knowledge in this situation. On the other hand, Teacher B may observe students while they are solving the same kind of problem. In this case, Teacher B may analyze students' error based on their past experiences thus s/he is using knowledge of content and students (Ball et al., 2008). The second deficiency is that there is no specific distinction among the categories of knowledge in teaching mathematics which makes it difficult to measure each one. For example, selecting examples, illustrations and representations that deepen student understanding of the topic of adding rational numbers can be seen as requiring KCT. However, this topic requires the algorithm of adding rational numbers (CCK), explaining the algorithm of adding rational numbers mathematically (SCK) and determining the most effective ways to overcome students' difficulties (KCS) (Ball et al., 2008). The final deficiency emerges from not being able to easily distinguished common content knowledge from the specialized content knowledge for some tasks. Ball and her colleagues referring Figure 2.5 asked "what fraction represents the shaded portion of the two circles shaded".

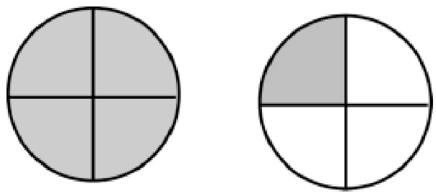


Figure 2.5 Representations of $\frac{5}{8}$ of 2 (Ball et al., 2008, p.404)

When receiving the answer $\frac{5}{8}$ of 2 the next question is posed "Is the knowledge that this is $\frac{5}{8}$ of 2 common? Or is it specialized?" (Ball et al., 2008, p. 403). On one hand, this knowledge can be common content knowledge since some people may use this knowledge in their work. On the other hand, this particular representation can be assessed as specialized content knowledge since a person may not use this knowledge in his daily work.

2.1.5 Rowland, Huckstep, and Thwaites' Categorization of Teacher's Knowledge

Rowland, Huckstep, and Thwaites (2005) developed the "knowledge quartet" model for the mathematical knowledge for teaching. The data was collected from 149 pre-service teachers in the United Kingdom. For the analysis, Rowland et al. (2005) used a grounded approach in order to generate theory. The analysis of the data allowed the researchers to identify four categories of teacher's knowledge. Foundation, the first category, involves teacher's knowledge and understanding of mathematics, knowledge about literature, beliefs about mathematics, and how and why it is learnt. The researchers claimed that foundation is the most essential category of the model, and the remaining three originate from this underpinning. The key components of foundation are the theoretical underpinning of pedagogy, awareness of purpose, identifying pupil errors, overt display of subject knowledge, use of mathematical terminology, adherence to the textbook, and concentration on procedures. The second category is transformation which refers to knowledge used in planning to teach and in the act of teaching. It includes teacher demonstration, use of instructional materials, choice of representations, and choice of examples. This category is similar to the PCK of Shulman and Grossman, and Ball's KCT. Connection is third category of the knowledge quartet and concerns the coherence of the lessons, integration of mathematical content, the sequencing of topics of instruction within and between lessons, and the ordering of tasks and exercises. To put it differently, it means making connections between procedures, making connections between concepts, anticipation of complexity, decisions about sequencing, and recognition of conceptual appropriateness. This category coincides with Shulman's curriculum knowledge, Grossman's pedagogical content knowledge and the horizon knowledge of Ball et al. Contingency is the final category representing unexpected classroom events such as dealing with students' unexpected responses and questions quickly and appropriately. In other words, it concerns classroom events which are impossible to plan for. This category includes responding to student ideas, deviation from the lesson agenda, teacher insight, and responding to the (un)availability of materials and resources.

Up till now, the frameworks related to teacher's knowledge were presented and discussed. In the following, the framework developed by Magnusson, Krajcik and Borko (1999) was given. Different from other frameworks, it was only related to teacher's PCK.

2.1.6 Magnusson, Krajcik and Borko's Categorization of PCK

Magnusson, Krajcik and Borko (1999) developed a theoretical framework for PCK for science teaching which expanded upon the earlier models of teacher's knowledge proposed by Shulman (1987) and Grossman (1990). Initially, they presented a model of teacher's knowledge which identified the relationships between the domains of teacher's knowledge. These domains were (1) subject matter knowledge (both substantive and syntactic structures), (2) general pedagogical knowledge, and (3) knowledge of context, and the centerpiece of teacher's knowledge (4) pedagogical content knowledge (PCK). Magnusson et al. (1999) argue that subject matter knowledge, pedagogical knowledge, and knowledge of context strongly influence pedagogical content knowledge. The model presented in Figure 2.6 represents the knowledge domains of the framework of Magnusson et al. and the relationships among the domains.

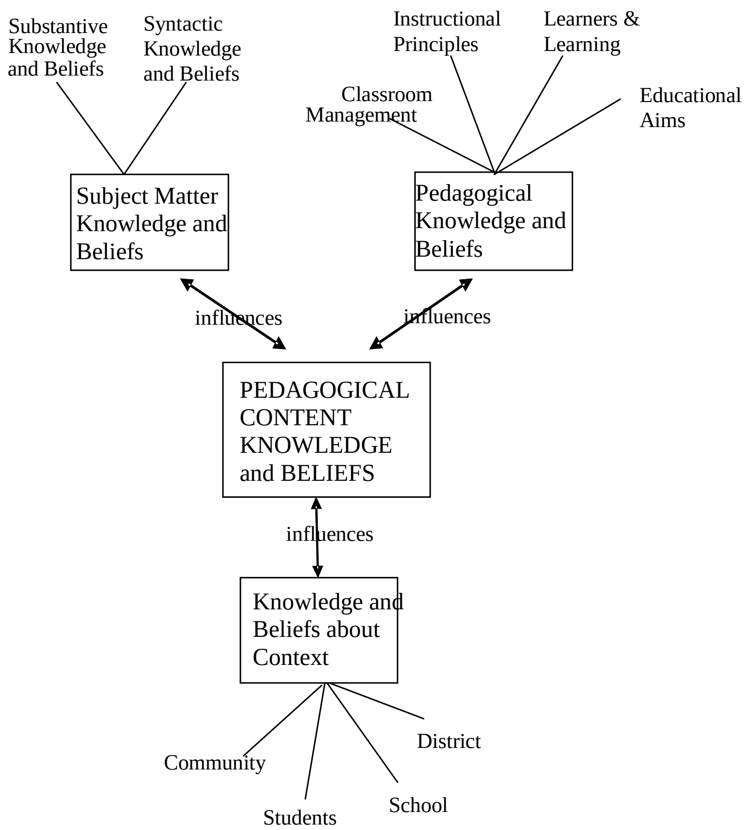


Figure 2.6 A model of the relationship among the domains of teacher's knowledge (Magnusson et al., 1999, p. 98).

The model shown in Figure 2.6 indicated that PCK is influenced by the knowledge and beliefs about subject matter, general pedagogy, and context. Within this context, Magnusson et al. (1999) elaborated on the central components and further delineated categories within PCK as shown in Figure 2.7.

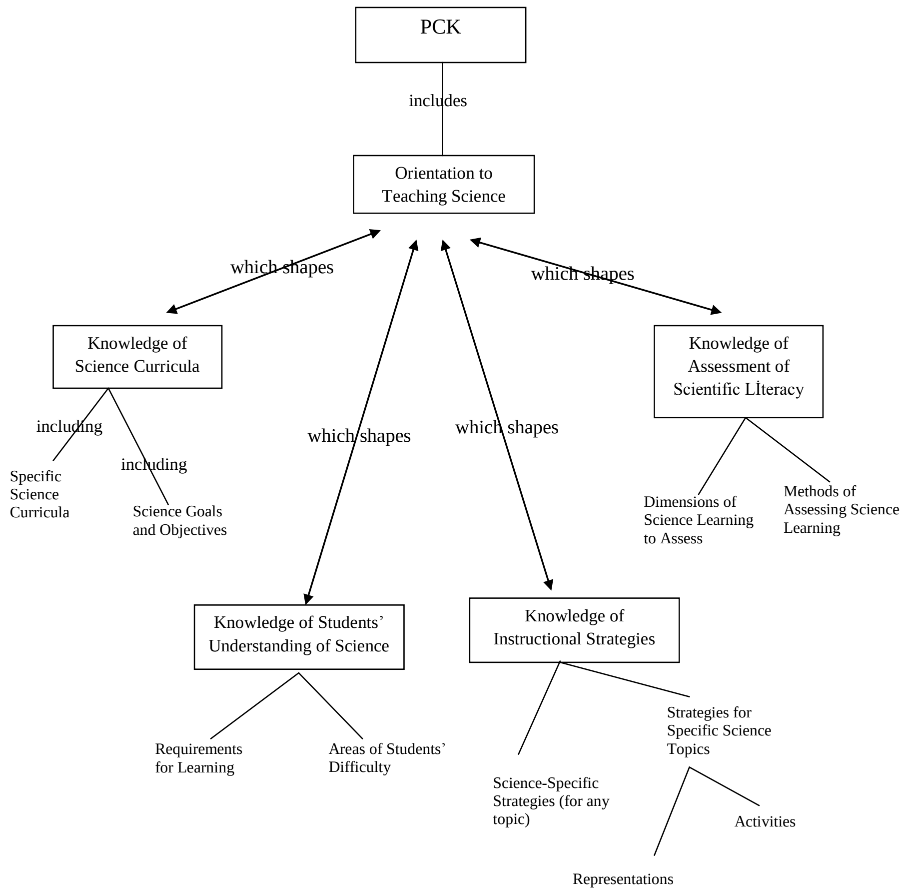


Figure 2.7 Magnusson et al.'s model of PCK showing the components for science teaching (1999, p.99)

Magnusson et al. (1999) defined PCK as "the transformation of several types of knowledge for teaching (including subject matter knowledge)" (p. 95). They stated that pedagogical content knowledge for science teaching consists of five components: a) orientation towards science teaching, b) knowledge and beliefs about science curriculum, c) knowledge and beliefs about students' understanding of specific science topics, d) knowledge and beliefs about assessments in science, and e) knowledge and beliefs about instructional strategies for teaching science (Figure 2.7).

Orientation towards science teaching is the first component, referring to the teachers' knowledge and beliefs about the purposes for teaching a subject at a specific grade level. This is similar to Grossman's category among the four categories of PCK. Although Grossman described this category as "overarching conceptions", Magnusson et al. (1999) stated that this knowledge guides teachers about instructional decisions such as the content of students' homework, the textbooks and other instructional materials and the assessment of students learning.

The second component of PCK is knowledge of instructional strategies consists of two dimensions; knowledge of subject-specific strategies, and knowledge of topic-specific strategies. The former dimension represents general approaches to science teaching and the latter indicates the teacher's knowledge of strategies which are effective tools to teach a particular science topic. Magnusson et al. (1999) examined the knowledge of topic-specific strategies under two sub-categories; topic-specific representations and topic-specific activities (Figure 2.7). Topic-specific representations, which are examples, illustrations, models or analogies, refer to the teacher's knowledge about the ways of presenting topic to help students learn the topic. In addition, topic-specific representations include the knowledge of the advantages and disadvantages of using particular representations. The latter sub-category, topic-specific activities, indicates the demonstrations, simulations, investigations, and experiments that are useful for students to understand a particular concept.

Magnusson et al. (1999) presented knowledge of students' understanding of science as the third component of PCK. It comprises two dimensions: a knowledge of requirements for learning, and knowledge of areas of student difficulty. The first dimension refers to teacher's knowledge concerning the students' prior knowledge about a particular subject. This knowledge informs teachers about the abilities and skills that students need together with the information they lack and therefore should be covered before the subject taught. The second dimension, defined as knowledge of content and students by Ball et al. (2008), is related to teachers' knowledge about the topics that are difficult or easy for students. Teachers should have knowledge about the topics that are difficult for students and the reasons for these difficulties. Besides, teachers should be knowledgeable about students' misconceptions related to a particular subject.

The fourth component of PCK is knowledge of the science curriculum which consists of two dimensions: knowledge of goals and objectives, and knowledge of specific curricular program. Shulman (1988) defined this knowledge as a discrete domain of the knowledge base for teaching called curricular knowledge however, Grossman (1990) and Magnusson et al. (1999) included this component as part of PCK. The first dimension represents the teachers' knowledge of goals and objectives related to the subjects that they are teaching. Moreover, it includes Shulman's vertical curriculum knowledge that is the knowledge of students which was acquired in previous years and that which will be learned in later years. The second dimension involves knowledge of the programs and materials which are relevant to teaching a particular subject.

The final component of PCK is the knowledge of assessment in science again consisting of two dimensions: knowledge of dimensions of science learning to assess and knowledge of the methods of assessment. First dimension denotes that the teacher's knowledge should contain the important concepts that should be assessed for students' learning within a particular subject. The second dimension covers the teacher's knowledge about the methods and the methods of assessing important concepts within a particular subject. In particular, it consists of the knowledge of instruments, procedures or activities which are used to assess students' learning.

Although Magnusson et al. defined PCK as a separate domain of knowledge for teaching, they did not assert that there were clear boundaries between PCK and other knowledge domains (e.g. subject matter knowledge, general pedagogical knowledge) as Ball et al. (2008) expressed.

2.1.7 Conclusions Drawn from Categorization of Teacher's Knowledge

To sum up, different categorizations of teacher's knowledge with different components, sub-components and relations between them have made important contributions to the literature concerning teacher's knowledge. (Ball et al., 2008; Gess-Newsome, 1999; Grossman, 1990; Magnusson et al.,1999; Rowland et al., 2005; Shulman, 1986). Among these researchers, Ball et al., and Rowland et al. developed a framework for the required mathematical knowledge for teaching. Ball et al. developed their model in light of Shulman's model and they divided Shulman's SMK and PCK into different sub-components (SCK, CCK, KCS, KCT). Differing from all the models mentioned above, Rowland et al. (2005) named the categories of teacher's knowledge differently using the terms; foundation, transformation, connection and contingency. Similar to Shulman (1986), the categorizations used by

Grossman (1990) and Gess-Newsome (1999) involve both SMK and PCK. However, they added knowledge of context and pedagogical knowledge in their frameworks. Although Shulman's PCK and SMK are the common knowledge types in different frameworks, Magnusson et al. (1999) focused on only the teacher's PCK. On the other hand, Shulman's curriculum knowledge is not a distinct category in some frameworks. Grossman (1990) and Magnusson et al. (1999) integrated curriculum knowledge in PCK. Moreover, Rowland et al. (2005) defined curriculum knowledge as a category, called connection.

In addition to the theoretical frameworks investigating the nature of teacher' knowledge, many research studies which used those frameworks focused on the teacher's knowledge that was used in classroom. These studies will be reviewed in following sections.

2.2 Research Studies on Mathematics Teachers' Knowledge on Mathematics Topics

Over many years, researchers have given special attention to the investigation of teachers' mathematical content knowledge (Baki, 2013; Ball, 1990a; 1990b; Even, 1993; Even & Tirosh, 1995; Hill, Rowan & Ball, 2005; Leinhardt & Smith, 1985) and this was undertaken from different points of view. Some aimed to investigate mathematics teachers' knowledge on a specific topic in mathematics (Ball, 1990a; 1990b; Contreras, Batanero, Diaz & Fernandes, 2011; Even, 1993; Even& Tirosh, 1995; Huang & Kulm, 2012; Livy & Vale, 2011; Nilsson & Lindstrom, 2012; Pino-Fan, Godino, Font & Castro, 2013). However, others looked at the ways of enhancing teachers' knowledge (An & Wu, 2011; Hill, 2007; Hill & Ball, 2004; Kwong, Joseph, Eric, Khoh, Gek & Eng, 2007) and some explored the relationship between knowledge types (Even, 1993; Hutchison, 1997). Although the aims of these studies were different, there is a common agreement that mathematics teachers' content knowledge has significant role in effective teaching and students' achievement. In order to clarify the important role of the content knowledge that mathematics teachers possess, these research studies are presented and discussed in this part of the literature review. It was found that the studies differed in terms of their participants, namely, some explored pre-service mathematics teachers' knowledge and the remainder investigated in-service mathematics teachers' knowledge. Moreover, some studies explored the ways of enhancing teachers' knowledge. Thus, this section of the literature review is divided into the following three categories; pre-service mathematics teachers' knowledge, in-service mathematics teachers' knowledge and the ways of enhancing mathematics teachers' knowledge.

2.2.1 Research Studies on Pre-Service Mathematics Teachers' Knowledge on Mathematics Topics

To begin, Even (1993), Even and Tirosh (1995), and Huang and Kulm (2012) explored pre-service teachers' knowledge on functions. Even (1993) investigated pre-service secondary teachers' subject matter knowledge and the relationship between subject matter knowledge and pedagogical content knowledge in the context of teaching the concept of function. The data was collected from 152 pre-service teachers in two phases; first, an open-ended questionnaire concerning their knowledge about functions was applied to all the teachers and second 10 of the pre-service teachers were interviewed. The result of the study showed that most of the pre-service teachers did not have broad knowledge related to functions. In fact, only a few could justify the importance and origin of the univalence requirement. Consequently, they did not use modern terms and concept images effectively while describing functions to the students. Moreover, many of them provided a rule to be followed without understanding the concept. Even (1993) implied that pre-service teachers needed better subject matter knowledge to improve their teaching. This can be achieved by developing mathematics courses in line with constructivist approach to teaching and learning (Even, 1993).

As in the study of Even (1993), Even and Tirosh (1995) explored pre-service teachers' knowledge about functions. However, the aim of this study was to investigate pre-service teachers' pedagogical content knowledge in terms of knowledge about the subject matter, and knowledge about students. One hundred and sixty two pre-service secondary mathematics teachers participated in the study. The data was gathered in two phases; an open-ended questionnaire was completed by 162 pre-service teachers, then 10 of the pre-service teachers were interviewed. The result of this study showed that pre-service secondary mathematics teachers did not know the definitions about functions and incorrectly solved problems presented to them. Moreover, they did not explain the logic behind the concepts which is consistent with the outcome of Even's study (1993). Even and Tirosh (1995) inferred that teachers' knowledge of subject matter and knowledge about students needed further investigation in order to improve teacher preparation programs.

Huang and Kulm (2012) conducted a study to explore the knowledge of function of pre-service middle grade mathematics teachers. A survey consisting of 17 multiple-choice items and 8 open-ended items was applied to 115 pre-service teachers and follow-up interviews were undertaken with five of the participants. The survey included the knowledge of; school algebra(SM), advanced algebra (AM), and teaching algebra (TM). The authors found that pre-service teachers' knowledge was limited in all three areas. They performed poorly in selecting appropriate perspectives and using representations of the concept of function. Moreover, they failed to solve quadratic/irrational equations, undertake algebraic manipulation and reasoning, and judging the number of roots of quadratic functions. Furthermore, the participants made mistakes in solving problems using the integration of algebraic and graphic representations of functions. As a result, the authors concluded that the participants' knowledge to teach functions was poor. Therefore, the outcome of all these studies (Even, 1993; Even & Tirosh, 1995; Huang & Kulm, 2012) can be interpreted as pre-service teachers' knowledge not having the mathematical knowledge which a mathematics teacher should possess in order to teach functions. They only had a level of knowledge that anyone who deals with daily mathematics (Even, 1993; Even & Tirosh, 1995; Hutchison, 1997). Regarding the framework that Ball and colleagues (2008) created, it was found that the pre-service teachers had enough common content knowledge, but they had limited specialized content knowledge for mathematics teaching.

In order to develop and improve their conceptual framework related to teachers' content knowledge, Ball and colleagues carried out many studies (Ball, 1990a, 1990b; Hill & Ball, 2004; Hill, Rowan & Ball, 2005). Ball (1990a) conducted a study to investigate pre-service elementary and secondary mathematics teachers' subject matter knowledge on one specific mathematics topic; division with fractions.

The aim of the study was to examine teachers' understanding about mathematics when they entered formal teacher education. Two hundred and fifty two pre-service teachers participated in this study and the data was collected through questionnaires and interviews. Ball (1990a) closely analyzed the pre-service teachers' understanding of division with fractions and the results showed that the teachers had a narrow understanding about the topic. They only knew and could apply the rule "invert and multiply". However, it is not enough to know the rule for effective teaching since discussing the meanings, the relationships, and the procedures of the division of fractions has important role in teaching the subject effectively (Ball, 1990a). Ball (1990a) generally concluded that pre-service teachers' understanding about mathematics was inadequate for teaching mathematics and tended to be rule-based. Similar to Ball (1990), Hutchison (1997) also aimed to make a connection between subject matter knowledge and pedagogical content knowledge related to fractions. Firstly, one pre-service teacher was interviewed prior to the mathematics course to discover her subject matter background and prior pedagogical content knowledge. Secondly, a mathematics educational biography and a structured task interview based on fractions were employed. The result of this study demonstrated that the pre-service teacher faced many problems although she wanted to be a good mathematics teacher. The reason for those problems was mainly due to the lack of her subject matter knowledge which is consistent with result found by Ball (1990a).

Moreover, another study by Ball (1990b) was focused on one aspect of pre-service teachers' subject matter knowledge; understanding of division. Ball (1990b) collected data from 19 pre-service elementary and secondary mathematics teachers. As in her previous study (Ball, 1990a), the results revealed that many of the pre-service teachers could not explain the mathematical reasoning even though they could solve the problems. Moreover, Ball (1990b) claimed that precollege mathematics classes did not provide pre-service teachers with adequate subject matter knowledge for teaching mathematics.

Contreras, Batanero, Diaz and Fernandes (2011) conducted a study based on the framework of Ball et al. (2008) in which they aimed to assess the pre-service teachers' common and specialized content knowledge on probability. The data was collected from 183 pre-service primary school teachers. A task which included two-way table served as the data source. In the first question, teachers were expected to compute a simple probability, a compound probability and a conditional probability in order to explore their CCK. The second question aimed to assess the teachers' SCK regarding probability. To this end, the teachers was asked to identify the mathematical content such as the types of problem, concepts, procedures, properties, and show the mathematical language that they used to solve the given probability problem. The result of this study reported that pre-service teachers' CCK related to probability was limited since they had difficulties in calculating simple, compound and conditional probabilities from a two-way table. This means that pre-service teachers did not have any more knowledge about probability than the person who deals with mathematics on an everyday basis. Furthermore, the researchers reported that most of the pre-service teachers made errors and could not arrive at the correct solution. Contreras et al. (2011) concluded that identifying and classifying mathematical content was not easy for pre-service teachers meaning that their SCK concerning probability was weak. In another study, pre-service teachers' content knowledge on ratio was investigated (Livy & Vale, 2011). Data was collected through the Mathematical Competency, Skills and Knowledge Test but the descriptive statistics and content analysis were only undertaken for the two most difficult items. The results revealed that most of the pre-service teachers were unable to solve whole-whole ratio items. They had difficulty in connecting their mathematical knowledge on ratio with measurement. However, a few pre-service teachers did demonstrate a knowledge of mathematical structure and connection which requires a connection between their knowledge on ratio and measurement. Furthermore, the vast majority of pre-service teachers could not deconstruct the multi-step ratio problem into its component parts thus, these teachers were unable to "identify critical mathematical components within a concept that are fundamental for understanding and applying that concept" (Chick, Baker, Pham, & Cheng, 2006, p.299). In addition, most pre-service teachers had limited knowledge related to standard procedures and solution methods concerning ratio problems.

Apart from the studies on functions, fractions, ratio, probability, and division, Pino-Fan, Godino, Font and Castro (2013) investigated pre-service teachers' knowledge of derivatives in light of the framework of Ball et al., (2008). A 7-task questionnaire was administered to 53 pre-service teachers in Mexico. The results obtained from the data showed that the pre-service teachers had several difficulties in solving tasks related to derivatives. The researchers concluded that these difficulties were related not only to a lack of specialized content knowledge but also to a dearth of common content knowledge. In other words, pre-service teachers had inadequate SCK and CCK on derivatives, as ratio and probability (Contreras et al., 2011; Livy & Vale, 2011).

Different from the studies described in this literature review, Carreño, Ribeiro and Climent (2013) investigated the nature and content of a pre-service teacher's SCK about the concept of polygon. In fact, the researchers did not describe the pre-service teacher's SCK; they focused on the difficulties related to presenting and discussing the borders of SCK, especially in relation to horizon knowledge. As a result of the analysis of the data gathered from the pre-service teacher, questions emerged in relation to both the nature and content of teachers' SCK and horizon knowledge. The teacher was posed questions such as: "Is the identification of the sub-concepts categorized as SCK or a specific knowledge related with connections and thus in the space of horizon knowledge?" (Carreño et al., 2013, p.8). The result of this study manifested that the borders of SCK were not clear. For this reason, some studies did not utilize Ball and colleagues' (2008) categorization of content knowledge as a conceptual framework.

In this section, the studies related to pre-service mathematics teachers' content knowledge for teaching mathematics were briefly summarized. The general trend in the studies was that pre-service teachers' content knowledge was not adequate for teaching mathematics. The literature also includes studies related to in-service mathematics teacher's knowledge and these are presented in the next section.

2.2.2 Research Studies on In-Service Mathematics Teachers' Knowledge on Mathematics Topics

In the literature, many studies investigated pre-service mathematics teachers' content knowledge. Consistent with the results of the studies mentioned above, Shulman (1987) stated that pre-service teachers' content knowledge is not robust because they have had little experience in real classroom context. Although teaching experience does not guarantee adequate content knowledge to teach mathematics, it is one of the important sources of content knowledge (Shulman, 1987). Therefore, studies conducted with in-service teachers provide an insight into whether the teacher's knowledge is sufficiently adequate to teach mathematics effectively. For this reason, studies conducted with in-service teachers are summarized in this part.

To begin, Carpenter, Fennema, Peterson and Carey (1988) investigated 40 first-grade teachers' PCK through the examination of the children's solutions to addition and subtraction word problems. The researchers aimed to explore the teachers' knowledge about the distinctions between different addition and subtraction problem types and the strategies that children use to solve different problems. Moreover, teachers' ability to predict their students' success in solving different types of problems and identifying the strategies used by children to solve problems of different types were the other aims of the study (Carpenter et al., 1988). The result of this study revealed that teachers in this study could distinguish between some of the basic differences of the types of addition and subtraction problems. Moreover, most of the teachers could identify the most frequently used strategies for solving addition and subtraction problems.

Another study conducted with in-service teachers was carried out by Leavitt (1998). The study aimed to explore German mathematics teachers' content knowledge and pedagogical content knowledge. The aims of this study were to investigate German mathematics teachers' knowledge and skills of solving basic mathematics problems correctly, and identify the representations that they prefer to use while solving basic mathematics problems. Multi-digit subtraction, multi-digit multiplication, and division with fraction, perimeter and area were the selected topics. Data was collected through a questionnaire and interview from 20 in-service teachers. The preliminary analysis of the data revealed that the German mathematics teachers' knowledge of multi-digit subtraction, multi-digit multiplication and dealing with perimeter/area were strong. However, their knowledge related to division with fractions was weak which was consistent with the study by Ball (1990b). Even though the teachers' knowledge of computational and solving word problems were adequate, their knowledge of representations was not adequate implying a limited pedagogical content knowledge. The weakest area was the topic of division with fractions in which they had difficulty in connecting the topic with real life.

As discussed above, Ball and colleagues carried out studies to improve the frameworks they had created in relation to the categorization of content knowledge (Hill, 2007; Hill & Ball, 2004). One of the studies was carried out by Hill (2007) which dealt with in-service teachers' specialized and common content knowledge for mathematics teaching. Hill concentrated on two content areas; namely, number and operations and algebra. She used Content Knowledge for Teaching Mathematics (CKTM) measures. Based on the analysis of the data, Hill (2007) asserted that in-service teachers were not adequate in terms of explaining and representing the mathematical ideas in alternative solution methods. Hill (2007) concluded that most of the in-service teachers had either no or a limited amount of SCK in terms of evaluating alternative solution methods however, they knew rules, procedures and algorithms well.

Similar to Leavitt (1998), Masters (2012) explored in-service mathematics teachers' content and pedagogical content knowledge of proportional reasoning and functions. Moreover, the relationship between teachers' knowledge and students' achievement were explored in both topics. To collect data, pre- and post-tests were administered to 137 eighth grade teachers and students. The proportional reasoning items in the tests aimed to explore content knowledge and pedagogical content knowledge, the function items served were the data source for the investigation into content knowledge. As in the studies in which the participants were the pre-service teachers (Even, 1993; Even & Tirosh, 1995), the findings of this study led Masters (2012) to the conclusion that eighth grade teachers' knowledge of proportional reasoning and functions was weak. The researcher reported that as a consequence of teachers' weak knowledge, the level of students' knowledge related to both topics was low which was consistent with previous studies. This means that teachers' content knowledge influences students' knowledge and their academic achievement.

In another study, Nilsson and Lindstrom (2012) investigated in-service teachers' content knowledge regarding probability. Moreover, they aimed to discover the relationship between the teachers' knowledge on probability, their educational level, their experience on teaching and their beliefs about their own understanding of probability concepts. The data was collected through a questionnaire from 24 teachers. The results of the study supported the 2013 study by Contreras et al., which investigated pre-service teachers' knowledge of probability. In light of the analysis of data, the conclusion that the teachers' had limited understanding of probability and in particular their knowledge of probability was procedurally-oriented. Contrary to findings of Shulman (1987), there was no relationship between teachers' knowledge and their experience. In other words, teaching experience does not guarantee a rich content knowledge (Friedrichsen, Lankford, Brown, Pareja, Volkmann, & Abell, 2007). The findings also revealed that teachers did not develop their understanding of probability during their teaching experience. Furthermore, teachers had low confidence in teaching probability due to having difficulties in applying probability tasks.

Apart from those studies mentioned above some researchers designed studies to compare the knowledge of expert and novice teachers. Leinhardt and Smith (1985) compared 4 expert and 4 novice fourth grade mathematics teachers' subject matter knowledge related to fractions. Data was collected via semantic nets, planning nets and flow charts, interviews, card-sorting task and transcription of videotapes of the teachers' classes. The lesson structure knowledge and subject matter knowledge formed the basis of this study. The knowledge about planning and performing lesson in a coherent and fluent way was defined as the knowledge of lesson structure. Moreover, as Shulman (1986) stated, the knowledge of concepts and procedures, and their connections within each and between them was defined as subject matter knowledge. The result of the study indicated that expert teachers had more knowledge of fractions than novice teachers. The result coincided with those of Shulman (1987) who also noted that experienced teachers had more knowledge about the topic. Similar to Leinhardt and Smith (1985), Lucus (2006) investigated 8 in-service and 10 pre-service teachers' subject matter knowledge related to the composition of functions. The particular aim of this study was to explore the influence of teaching experience on SMK. The participants were asked to explain the prerequisites that the students should know before the topic of composition of functions was to be taught and, the participants were asked to describe the main ideas related to the same topic. The analysis of the data showed that the in-service and pre-service teachers' SMK on composition of functions did not differ significantly. This suggests that SMK is not influenced by teaching experience. The findings of this study also support the results of previous studies (Friedrichsen, Lankford, Brown, Pareja, Volkmann, & Abell, 2007; Nilsson & Lindstrom, 2012). However, Shulman (1987) asserted that teaching experience has important role in teacher's having rich SMK.

There are also cross-cultural studies that investigated the content knowledge of American and Chinese teachers (An, Kulm & Wu, 2004; Ma, 1999; Zhou, Peverly & Xin, 2006). Guided by Shulman's (1986) categorization of teacher's knowledge, Zhou and colleagues focused on subject matter knowledge, pedagogical content knowledge and general pedagogical content knowledge of North American and Chinese teachers. Their study focused on 162 American and Chinese 3rd grade mathematics teachers' knowledge about fractions. Results showed that American teachers' knowledge fell behind that of Chinese teachers in terms of concepts, operations and word problems, namely subject matter knowledge and the same result was found for their pedagogical content knowledge. In other words, the Chinese teachers were better than American teachers while identifying important points of teaching the fraction concepts and ensuring students' understanding. However, the performance of Chinese teachers in subject matter knowledge, pedagogical content knowledge and their general pedagogical knowledge specifically the psychological and educational theories were not as sufficient as the American teachers. Ma (1999) compared North American and Chinese elementary school teachers' subject matter knowledge. Consistent with the result of the study by Zhou and colleagues, the Chinese teachers had a better understanding of multi-digit multiplication, division by fractions and the relationship between perimeter and area than the American teachers. Similar to previous research studies (Ma, 1999; Zhou et al., 2006), An, Kulm and Wu (2004) compared PCK of North American and Chinese teachers. Twenty eight mathematics teachers in U.S.A and 33 mathematics teachers in China participated in the study. The results of this study indicated that the mathematics teachers' PCK in the two countries differed significantly. The Chinese teachers focused on gaining the correct conceptual knowledge, and a more rigid development of procedures. On the other hand, the American teachers focused on promoting students' creativity by designing a variety of activities. The results of this study illustrated that each country makes different demands on teachers' pedagogical content knowledge. According to the results of these studies, it can be said that teachers' mathematical content knowledge can vary in different countries. The reason for this variety might be the difference between the type of teacher education program and the education system in different countries.

In this section above, the studies related to in-service mathematics teachers' and pre-service mathematics teachers' content knowledge for teaching various mathematics topics in the international arena have been discussed. In the following section, studies on teachers' knowledge conducted in Turkey context will be reviewed. Due to context-depended nature of teachers' knowledge, examination of these studies is important because they provide information about teachers' knowledge in Turkey.

2.3 Research Studies on Mathematics Teachers' Knowledge on Mathematics Topics in Turkey

Researchers have claimed that both the quality of the mathematics teaching and student achievement depends on teachers' content knowledge (Ball, Hill & Bass, 2005). In order to gain insight about Turkish teachers' content knowledge, researchers focused on investigating teachers' knowledge in a national context (Aslan-Tutak, 2009; Baki, 2013; Basturk & Donmez, 2011; Boz, 2004; Bukova-Guzel, 2010; Butun, 2005; Gokturk, Sahin & Soylu, 2013; Haciomeroglu, Haciomeroglu & Aspinwall, 2007; Isik, Ocal & Kar, 2013; Isiksal, 2006; Isiksal & Cakiroglu, 2008; Karahasan, 2010; Kilic, 2011; Sevis, 2008; Turnuklu, 2005; Yesildere-Imre & Akkoc, 2012). Similar to international studies, Turkish researchers were mostly carried out the research studies with pre-service teachers; these studies are described first followed by work undertaken with in-service teachers.

2.3.1 Research Studies on Pre-Service Mathematics Teachers' Knowledge on Mathematics Topics in Turkey

As in the international arena, many research studies have been conducted to explore pre-service teachers' knowledge on mathematics topics in Turkey (Aygun, Baran-Bulut & Ipek, 2013; Baki, 2013; Basturk & Dönmez, 2011; Boz, 2004; Gokturk, Sahin & Soylu, 2013; Haciomeroglu, Haciomeroglu & Aspinwall, 2007; Isiksal, 2006; Isiksal & Cakiroglu, 2008; Isik, Ocal & Kar, 2013; Karahasan, 2010; Kılıc, 2011; Turnuklu, 2005).

To begin, Haciomeroglu, Haciomeroglu and Aspinwall (2007) investigated 33 pre-service teachers' subject matter knowledge and pedagogical content knowledge on functions similar to the study of Even (1993), and Even and Tirosh (1995). In this study, a function questionnaire, card sorting activity, preparation and analysis of lesson plans on exponential functions, and video teaching episodes were the data collection tools. Consistent with the result of previous studies, the result of this study revealed that the pre-service teachers' SMK was inadequate and too weak to teach functions effectively (Even, 1993; Even & Tirosh, 1995). Lacking neither an adequate nor rich SMK, pre-service teachers were unable to organize lesson plans, select appropriate questions and activities to make the concept easier for students. Furthermore, they were not able to ask critical questions to enhance students' learning. Also, because of lack of their PCK, they could not decide on the important concepts in the exponential functions which are needed by students to draw and understand graph of exponential functions. In addition, the pre-service teachers had difficulty in explaining and discussing the definition and properties of exponential functions since their PCK was not sufficiently comprehensive for teaching. Similarly, Karahasan (2010) conducted a study to understand the extent of pre-service secondary mathematics teachers' pedagogical content knowledge of composite and inverse functions. The data was collected from three pre-service secondary mathematics teachers through observations, interviews, documents, and the use of audiovisual materials. The result of the study is similar to previous studies in that it revealed the low level of the pre-service secondary mathematics teachers' pedagogical content knowledge. On completion of the study, the researcher proposed that in order to ensure both deep and broad subject matter knowledge of pre-service teachers, teacher education programs should provide method courses that cover the topics of secondary mathematics curriculum.

Furthermore, Isiksal and Cakiroglu (2008) conducted a study to investigate pre-service mathematics teachers' knowledge concerning elementary students' misconceptions/difficulties in the division of fractions. In particular, pre-service teachers' knowledge about the sources of students' misconceptions and difficulties and, their strategies to overcome those misconceptions/difficulties were examined through a written question and semi-structured interview. Seventeen pre-service teachers completed a questionnaire about the division of fractions. After analyzing the data, the researchers were grouped the pre-service teachers' knowledge on common conceptions and misconceptions/difficulties under four headings namely; algorithmically based mistakes, intuitively based mistakes, mistakes based on formal knowledge, and misunderstanding on problem. Moreover, the pre-service teachers were asked to suggest strategies to overcome students' misconceptions and difficulties on division of fractions. These were as follows; using multiple representations of the concepts, using different teaching methods, highlighting the importance of practice of computational skills and giving full attention to understanding the problem. Based on these findings, Isiksal and Cakiroglu mentioned the importance of course on method and teaching practice on pre-service teachers' subject matter knowledge and pedagogical content knowledge on various topics in mathematics.

Basturk and Donmez (2011) investigated pre-service teachers' curriculum knowledge concerning limit and continuity. The participants were selected based on their level of subject matter knowledge. In order to determine pre-service teachers' levels of subject matter knowledge related to limit and continuity, a Content Knowledge Questionnaire was administered to 37 pre-service teachers. As a result of the analysis of the questionnaire, four pre-service teachers with different levels of subject matter knowledge on the same topic were interviewed then, they prepared lesson plan concerning the limit and continuity concept and implemented their plan in a microteaching session. The results showed that some pre-service teachers had limited knowledge about the new secondary mathematics curriculum in Turkey. In particular, some were not aware of the concepts which were not included in the new curriculum or had been removed. Also, some teachers did not have any idea about the place of the limit and continuity concepts in the program. The results led the researchers to conclude that pre-service teachers' curriculum knowledge was not adequate. When the results of the Content Knowledge Questionnaire and the analysis of the pre-service teachers' responses of semi-structured interviews were compared, it was realized that the more adequate content knowledge the pre-service teachers had, the more knowledge of the curriculum they had. The pre-service teachers who had adequate content knowledge recognized the benefit of the new curriculum, had a greater desire to implement the goals of the new curriculum and expended more energy in doing so. Furthermore, they paid attention to the order of presentation of concepts and did not include the concepts that had been removed from the new curriculum. Moreover, the researchers specified that the pre-service teachers' curriculum knowledge was rhetorical. That is, their knowledge came from the internet, their friends who were teachers and their students to whom they gave private lessons. In order to increase their curriculum knowledge, researchers recommended that the vision of the new mathematics curriculum might be discussed in the method course, and pre-service and in-service teachers might exchange opinions during teaching practice.

In another study, Kilic (2011) aimed to explore the nature of pre-service secondary mathematics teachers' knowledge of students of algebra. The knowledge of the students refers to students' misconceptions/difficulties/errors, the possible sources of these misconceptions/ difficulties/errors, and the way of eliminating them. Data was collected through interviews, observations, a questionnaire, and written documents from six pre-service secondary mathematics teachers. The results of this study showed that pre-service secondary mathematics teachers' knowledge of students was very limited since they could not identify the students' errors/misconceptions. However, they tried to overcome students' errors and misconceptions by applying a rule or procedures to solve the problem but they did not explain the logic behind the rule or procedures. For this reason, it can be concluded that their basis of their subject matter knowledge was "procedural without reasoning" (p. 23). Moreover, explaining the rules and procedures to eliminate students' errors/misconceptions was an indicator of the weakness of their repertoire of appropriate examples, representations, and teaching strategies. That is, it was the indicator of the weakness in their knowledge of pedagogy. As a result of this study, it can be seen that the teachers' knowledge of students was intertwined with their knowledge of subject matter and knowledge of pedagogy.

Another study related to the pre-service teachers' knowledge of fractions was conducted by Isik, Ocal and Kar (2013). They investigated the level of 36 pre-service teachers' PCK in terms of determining 5th grade students' errors about addition of fractions. The data was collected through asking the participants to explain addition operations through six problem statements and to clarify errors if there were any. The results of the study indicated that the pre-service teachers had difficulty in determining the errors moreover; they made errors while they were explaining students' errors. This indicates that the pre-service teachers' PCK concerning the addition of fractions was weak because they were not aware of students' errors.

With a similar aim to the research of Isik et al. (2013), Gokturk, Sahin and Soylu (2013) investigated 63 pre-service teachers' knowledge on determining students' errors and stating the ways of overcoming those errors. Gokturk et al., focused on the pre-service teachers' knowledge about variables and two questions involving students' incorrect solution methods was used to collect the data. The pre-service teachers were asked to analyze the students' solution methods and determine students' errors. Furthermore, the pre-service teachers were required to specify the ways of overcoming those errors. Contrary to the results the study undertaken by Isik et al. (2013), the pre-service teachers' knowledge on identifying students' errors related to the variables was adequate. They did not have difficulty in determining those errors however, their PCK was not sufficient in terms of overcoming students' errors.

Another study with pre-service teacher participants was conducted by Aygun, Baran-Bulut and Ipek (2013) however; they investigated the teachers' content knowledge and pedagogical content knowledge on the equal sign. As a result, the authors stated that the pre-service teachers focused on the operational meaning of the equal sign. Furthermore, pre-service teachers were able to identify students' errors regarding the equal sign which coincided with the results of the study of Gokturk et al. (2013). Additionally, the pre-service teachers had adequate PCK related to overcoming students' errors.

Baki (2013) conducted a study to evaluate pre-service teachers' knowledge of the algorithm of division associated with place value. In order to collect data, 228 pre-service teachers divided 4057 by 15. The findings of the study showed that most of the teachers were able to do this correctly. However, the majority of the pre-service teachers gave inadequate explanations regarding the division. From these findings, Baki (2013) concluded that the pre-service teachers' knowledge of the algorithm of division associated with place value was inadequate for explanations. Thus it can be seen that most of the pre-service teachers' explanations of the procedures of division were based on the rules.

Furthermore, other researchers (Boz, 2004; Isiksal, 2006; Turnuklu, 2005) investigated the relationship between the pre-service teachers' pedagogical content knowledge and subject matter knowledge on different topics of mathematics. Boz (2004) explored the relationship between PCK and SMK in relation to the simplification of an algebraic statement. A questionnaire consisting of 16 questions was applied to 184 pre-service teachers, and afterwards interviews were conducted with 10 of the participants. The analysis of data showed that most of the pre-service teachers confused the concepts of simplification and solving of equations. This confusion was an indication of the teachers' lack of subject matter knowledge. Due to this lack of knowledge the teachers were unable to determine the mistakes made by the students thus, the pre-service teachers' pedagogical content knowledge was not sufficient. On the other hand, even if some pre-service teachers were able to uncover students' mistakes, they were not able to explain the reason for students' mistakes. Not being able to explain the reasons for the mistakes is an indication of inadequate PCK. In conclusion, Boz (2004) asserted that there was a relationship between teachers' subject matter knowledge and their pedagogical content knowledge.

In another study, Turnuklu (2005) examined the relationship between the pedagogical and mathematical content knowledge of pre-service teachers. To reveal the competency of pre-service teachers' pedagogical content knowledge in mathematics, 45 pre-service teachers were asked to solve four problems. Two of the problems were related to fractions, one concerned decimals and the other was about operations. To determine their existing mathematical content knowledge, the mean of the teachers' grades in their mathematical content courses taken throughout their university education was used. Turnuklu (2005) claimed that there was a relationship between the mathematical content knowledge and pedagogical content knowledge of pre-service teachers. Similarly, Isiksal (2006) aimed to determine the relationships between subject matter knowledge and pedagogical content knowledge regarding the multiplication and division of fractions. The subject matter and pedagogical content knowledge were explored through the teachers' understanding of facts, concepts and principles, their knowledge concerning students' misconceptions and difficulties, their strategies and representations they use to teach multiplication and division of fractions. The results revealed that the pre-service teachers' subject matter knowledge was not conceptually deep. In other words, they did not have enough knowledge to explain the multiplication and division of fractions. Moreover, the pre-service teachers' limited subject matter knowledge affected their pedagogical content knowledge; especially in terms of their knowledge about students' common (mis)conceptions.

In the next section, the studies aiming to explore Turkish in-service mathematics teachers' knowledge about mathematics topics were presented and discussed.

2.3.2 Research Studies on In-Service Mathematics Teachers' Knowledge on Mathematics Topics in Turkey

The studies mentioned above investigated the knowledge of Turkish pre-service mathematics teachers' on different topics. However, it is important to explore the extent of the knowledge of Turkish in-service mathematics teachers because these teachers have experience in the real classrooms and their knowledge directly impacts on student achievement. In this regard, Butun (2005) conducted a qualitative study to investigate in-service elementary mathematics teachers' content knowledge in basic school mathematics concepts. The data was collected from three in-service mathematics teachers via semi-structured interviews and classroom observations additionally, teaching scenarios were discussed in the interview to determine their content knowledge. Butun (2005) concluded that teachers' content knowledge about basic mathematics concepts was disconnected, and they depended on the rules which were shown by the teachers directing students to memorize rules or procedures. Moreover, this study revealed that teachers' aim was to obtain the answer and they did not explain the reasons behind the rules or procedures. Furthermore, the teachers were not successful at forming appropriate representations for the problem. Their inadequacies related to the topics affected their teaching strategies. In light of the result of this study, it can be concluded that the in-service teachers' knowledge was too weak to teach basic school mathematics concepts effectively.

The research studies conducted both internationally and nationally revealed the fact that both pre-service mathematics teachers' and in-service mathematics teachers' knowledge was limited in terms of several mathematics topics such as fractions, functions, divisions, variables and algebra. As emphasized by several researchers (Hill & Ball, 2004; Hill, Rowan, & Ball, 2005; Hill, Schilling, & Ball, 2004; Rowan, Schilling, Ball & Miller, 2001), teachers' knowledge has important role in increasing students' achievement when learning mathematics. Therefore, it is important to improve teachers' knowledge and several research studies aiming to investigate the ways of enhancing teachers' knowledge are presented in the next section.

2.4 Research Studies on the Way of Enhancing Mathematics Teachers' Knowledge on Mathematics Topics

When all the studies presented in this thesis are reviewed, it was understood that teachers' content knowledge is not sufficient to adequate to teach mathematics. Thus, some researchers (An & Wu, 2011; Hill, 2007; Hill & Ball, 2004; Kwong, Joseph, Eric, Khoh, Gek & Eng, 2007) undertook an investigation into the ways of enhancing teachers' knowledge. In order to achieve this aim, Hill and Ball (2004) conducted a study which considered whether a summer workshop component of a professional development institute could help elementary school teachers improve their knowledge of mathematics for teaching. Content Knowledge for Teaching Mathematics (CKTM) was used as a measurement tool. The teachers participated in summer schools for 40 to 120 hours and elementary mathematics topics such as long division and the order of operations were covered. The significant result of the study was that it showed that a professional development program can be effective to improve teachers' mathematical content knowledge for teaching.

In another study, Kwong, Joseph, Eric, Khoh, Gek and Eng (2007) investigated whether teachers' PCK improved significantly after their teacher training program. The researchers concluded that although the teachers' PCK was quite weak at the beginning of their programs, there was significant improvement in teachers' PCK on completion of the training program.

Additionally, An and Wu (2011) focused on in-service teachers' PCK of students' thinking through grading homework, assessing and analyzing misconceptions. The researchers explored the effect of assessing and analyzing misconceptions in student homework in relation to the improvement of in-service teachers' knowledge. The participants were ten 5th and 8th grade teachers and they were divided into two groups for the experimental study. Both groups were assessed using a pre- and post-questionnaire in PCK. The data was collected through both a qualitative and quantitative approach. The assessments of the pre- and post-test questionnaires for teachers and students, classroom observations, interviews, teachers' daily grading logs were served as the data sources. Analysis of the data revealed that grading homework and analyzing misconceptions improved teachers' knowledge of students' thinking. In this way, the teachers' PCK was strengthened since knowledge of students' thinking is a important component of PCK. It gives the teacher clues as to how well students understand mathematical concepts, helps the teacher to decide in which area the students have misconceptions, and then assist the teacher to develop strategies to overcome their misconceptions (Shulman, 1987; An, Kulm & Wu, 2004).

Due to the fact that the Turkish mathematics teachers' content knowledge was not sufficient for effectively teaching mathematics, it is important to explore the ways of developing teachers' knowledge. To end researchers in Turkey have conducted studies to explore how teachers' knowledge can be improved (Sevis, 2008; Yesildere-Imre & Akkoc, 2012).

Sevis (2008) aimed to deduce the effects of a mathematics teaching method course on pre-service elementary mathematics teachers' content knowledge for teaching mathematics. Forty three pre-service mathematics teachers completed an 83-item test at the beginning and after a method course. After analyzing the data, the amount of change in the participants' knowledge for mathematics teaching was measured. According to this analysis, as Hill (2007) and Kwong et al. (2007) stated there was a significant effect of the mathematics teaching method course on pre-service teachers' content knowledge for teaching mathematics.

Similarly, Yesildere-Imre and Akkoc (2012) conducted a case study to examine the development of pre-service teachers' pedagogical content knowledge related to generalizing number patterns during a school practicum (SP). The pre-service teachers' PCK was analyzed in two components: 1) knowledge of students' understanding and difficulties, and 2) knowledge of topic-specific strategies and representations. Three pre-service teachers participated in this study and their lesson plans, micro-teaching lesson plans, interviews and videos served as the data sources. The findings of this study suggested that the pre-service teachers' PCK changed significantly throughout the SP. In detail, the pre-service teachers took the students' understanding of, and difficulties with patterns into account. Moreover, through the SP the pre-service teachers' PCK improved in terms of the way they used pattern-specific strategies. According to the analysis of the data, it can be concluded that observing real classroom practice helped pre-service teachers to improve their PCK. The results imply that selecting mentors with adequate PCK has important role to improve pre-service teachers' PCK via observation. Moreover, the pre-service teachers' PCK improved during the university part of the SP course. As part of the course, pre-service teachers shared the videos of their mentor's lessons with their peers. In this way, they had the opportunity to discuss other mentors' lessons with regard to the PCK's components. As a result of these discussions, pre-service teachers were able to identify a variety of pedagogical approaches.

These research studies (An & Wu, 2011; Hill & Ball, 2004, Hill, 2007; Kwong et al., 2007; Sevis, 2008; Yesildere-Imre & Akkoc, 2012) are very important in the way that they examined how teachers' content knowledge could be developed. Based on the findings of the studies, it can be deduced that teachers' content knowledge might be improved by taking mathematics courses and methods courses during the teacher training period and participating in workshops related to mathematics teaching. These results are important since they point out that teacher educators and policy makers can develop new programs and revise old programs for teacher education. Also, teacher educators and policy makers can organize workshops in which teachers can participate.

2.5 Research Studies on Mathematics Teachers' Knowledge on Geometry Topics

Geometry is one of pillars of mathematics (Atiyah, 2001) and it has a crucial role in teaching and learning mathematics. NCTM (2000) emphasized its prominence by stating that "geometry offers an aspect of mathematical thinking that is different from, but connected to, the world of numbers" (p.97). Thus, while students are engaging in shapes, structures and transformations; they can understand geometrical concepts and also mathematics behind those concepts. In addition, Clements and Battista (1992) commented that geometry can be considered as a tool to provide for interpretation and reflection in our physical environment. In other words it presents a way of describing, analyzing and understanding the world to us.

Moreover, NCTM (1989, 2000) asserted that geometry benefits both teachers and students in other areas of mathematics curriculum and other disciplines. For instance, the circle graph is one of the areas of statistics in mathematics; however, it is also related to geometry. Besides, the topic of geometric probability is related to both geometry and probability. In addition, although symmetry is a area of geometry, it is related to functions and also plays important role in the arts, in design and in science. Similarly, geometry is interrelated with measurement. Steele (2006) stated that there is significant overlap between geometry and measurement, and the overlapping contents are noteworthy in mathematics education. For instance, the area of a square is measured in square units. Finding the area of a geometric figure is related to geometry concepts, and measuring it in square units is relevant to measurement concepts. In brief, geometry is regarded as making an important contribution to learning and teaching other mathematics topics (NCTM, 2000).

In order to use geometry as a tool for teaching mathematics, teachers should also have a broad and deep content knowledge concerning geometry. Vais and Reyhani (2009) stated that exploration, naming, recognition, classification, reasoning, drawing, making relationships between objects in the plane or space, usage of coordinate geometry and geometry transforms were the main issues used to teach geometry at primary and middle schools. This means that teachers who teach in primary and middle schools need to have knowledge of the different and main parts of the geometry, geometric figures and their characteristics and relation between them, the appropriate use of the different types of proof, coordinate geometry and geometric transforms. For this reason, the teachers' knowledge of geometry should be more comprehensive, multi-dimensional and complex. Similar to effective mathematics teaching, successful geometry teaching depends on teachers' content knowledge. On account of its importance, some researchers undertook research to investigate teachers' knowledge on several geometry topics. In this part of the literature review, these studies are presented and discussed (Aslan-Tutak, 2009; Baturo & Nason, 1996; Bukova-Guzel, 2010; Fujita & Jones, 2006, Gomes, 2011; Kellogg, 2010; Maxedon, 2003; Ng, 2011; Swafford, Jones and Thornton, 1997).

2.5.1 Research Studies on Pre-Service Mathematics Teachers' Knowledge on Geometry Topics

Contrary to research studies regarding pre-service teachers' knowledge on mathematics topics, the number of studies conducted to investigate pre-service teachers' knowledge on geometry topics were limited (Aslan-Tutak, 2009; Baturo & Nason, 1996; Bukova-Guzel, 2010; Fujita & Jones, 2006; Gomes, 2011)

Baturo and Nason (1996) investigated pre-service teachers' subject matter knowledge in terms of their understanding about area measurement. In fact, the focus of this study was not only subject matter knowledge, but also the student teachers' knowledge about the nature and discourse of mathematics, their knowledge about mathematics in culture and society, and their dispositions towards mathematics. To this end, sixteen pre-service teachers were interviewed and eight area measurement tasks comprised the data collection tool during the interview. The findings indicated that pre-service teachers' subject matter knowledge regarding area measurement was limited. In other words, their knowledge was incorrect, missing and unconnected. Also, the teachers' could not easily transfer one form of representation to another. Similarly, their lack of knowledge about the nature and discourse of mathematics, and about mathematics in culture and society was disconcerting. Pre-service teachers tended to think that mathematics is a collection of facts, rules and procedures, and stated that knowing mathematics means following set of procedures step-by-step to find correct answer. Moreover, the teachers commented that mathematics can be represented symbolically, and its relationship with real life is little or none. As a result, the teachers had negative feelings towards mathematics and particularly the area of measurement. On completion of the study, Baturo and Nason concluded that the pre-service teachers' limited subject matter knowledge regarding area measurement restricted them in terms of teaching their students in such a way that they would acquire a meaningful understanding of mathematics concepts and processes. Therefore, they did not teach the subject area measurement deeply, their teaching was superficial and their reactions to the students' questions and comments were low-level. Also, the researchers reported that the teachers had difficulties in using multiple representations while teaching area measurement and relating area measurement to other important topics within the mathematics curriculum. The results were in agreement with those of the Fujita and Jones (2006). In other words, the pre-service teachers' PCK and curriculum knowledge were too limited to teach area measurement effectively (Baturo & Nason, 1996).

The study undertaken by Fujita and Jones (2006) investigated pre-service teachers' geometry content knowledge related to defining and classifying quadrilaterals. Two sets of data were collected. First, in order to discover their understanding of relationship between quadrilaterals, a survey was administered to 158 pre-service teachers in their first year of university. Second, a task was applied to 124 pre-service teachers in their third year of university with the purpose of examining their understanding of hierarchical relationships in the classification of quadrilaterals. The results indicated that although the pre-service teachers could draw the figure of quadrilaterals, they could not provide their definitions. Thus, the pre-service teachers' subject matter knowledge on quadrilaterals was inadequate and they lacked sufficient knowledge concerning hierarchical relationship between quadrilaterals.

Similar to Fujita and Jones (2006), Aslan-Tutak (2009) carried out a study to understand pre-service teachers' geometry learning and their geometry content knowledge in the case of quadrilaterals. The study had two investigations, qualitative and quantitative. The former investigation was designed to understand pre-service teachers' geometry learning and their use of effective instructional strategies for students' learning. Three pre-service teachers' participated in this part of the study and the data was collected through individual interviews, observations, field notes taken during the observations and materials used during the geometry instructions. Based on the qualitative investigation, the pre-service teachers' geometry content knowledge was limited and they had problems of classification the quadrilaterals. Although they thought that geometry was an important topic of mathematics in elementary school, they were anxious about teaching geometry because of their limited knowledge. However, they considered that they could increase their geometry knowledge with the help of the experienced teachers working in their schools after they begin teaching in the classroom.

The latter quantitative investigation was performed to compare mathematical knowledge of groups (control and treatment) of the pre-service teachers and to specify the increase of geometry knowledge of pre-service teachers in the experimental group. One hundred and two pre-service teachers participated in this part of the study, and the Content Knowledge for Teaching Mathematics Measures (CKT-M Measures) was administered to the participants as a pre- and post-test. A protocol related to quadrilaterals was applied to the treatment group participants (n= 54) as an intervention and traditional instruction was implemented for the control group participants (n= 48). While analyzing the data, repeated measures ANOVA and mixed ANOVA were used respectively. The analysis of the test results showed that the treatment group participants' geometry knowledge significantly increased following the intervention. However, the control group participants' geometry knowledge also increased but with traditional instruction. Although the knowledge increase of the participants in treatment group was greater than the increase in the control group participants, the difference was not statistically significance. Aslan-Tutak explained this result as the protocol applied to the treatment group not being as effective as she expected.

Another researcher who investigated pre-service teachers' knowledge about geometry was Gomes (2011). She conducted an exploratory study to evaluate pre-service elementary teachers' content knowledge on geometric transformations. She identified 66 pre-service teachers' difficulties/mistakes regarding geometric transformations. A questionnaire concerning three geometric transformations, namely, translation, reflection and quarter turn rotation served as the data source. The findings revealed that the pre-service teachers had knowledge of geometric transformations. However, their knowledge was not adequate to teach this topic and they had some difficulties regarding geometric translations.

Although most of the studies related to pre-service teachers' knowledge on geometry topics were international studies, Bukova-Guzel (2010) conducted a study in Turkey. She aimed to investigate Turkish pre-service mathematics teachers' knowledge about instructional strategies and multiple representations, their knowledge about learners, and their curricular knowledge relevant to solid objects. Semi-structured interviews, lesson plans prepared by the participants and video recordings of instructional applications were the data sources. The findings revealed that the pre-service teachers developed several activities and used real-life materials to enable students to better understand solid objects. It can be concluded that pre-service mathematics teachers' knowledge on instructional strategies and multiple representations was adequate. On the other hand, although pre-service mathematics teachers took the students' prior knowledge into consideration, they did not pay attention to possible students' misconceptions about the topic because of their lack of knowledge about learners. Moreover, they had difficulty in preparing alternative assessment materials to determine students' learning. Regarding the pre-service mathematics teachers' curricular knowledge, the results showed that they were able to relate solid objects to other objects and associate solid objects with plane geometry and functions which are taught at different grade levels. This shows that their horizontal curriculum knowledge and vertical curriculum knowledge were almost sufficient.

2.5.2 Research Studies on In-Service Mathematics Teachers' Knowledge on Geometry Topics

Similar to studies with the aim of investigating in-service mathematics teachers' knowledge on mathematics topics, the number of studies concerning in-service teachers' knowledge on geometry topics was limited. With respect to the available literature, Maxedon (2003) conducted a study to investigate in-service teachers' knowledge under four components: goals of geometry, child development and geometry, geometry curriculum and curriculum content, and geometric concepts. The participants were eight experienced early childhood teachers and the data was collected through interviews. The result of the study revealed that the teachers had sufficient knowledge about the importance of geometry for their students and they could present their own goals when teaching geometry. Moreover, they were familiar with their grade level curricula in terms of the pedagogical aspects such as materials, resources and expectations. However, they were less familiar with subject matter issues such as the content of the geometry curriculum. In other words, they did not know the topics in the grades preceding and following years. Therefore, it could be concluded that the in-service teachers' curriculum knowledge was limited as pre-service teachers (Baturo & Nason, 1996).

These research studies concluded that both pre-service and in-service teachers' knowledge of geometry topics such as geometric transformations, defining and classifying quadrilaterals, area measurement and solid objects was not adequate to teach these topics effectively. For this reason, some researchers proposed to investigate the ways of enhancing teachers' knowledge on geometry topics and these studies are presented in the next part of the literature review.

2.6 Research Studies on the Way of Enhancing Mathematics Teachers' Knowledge on Geometry Topics

As mentioned above, some researchers (An & Wu, 2011; Hill, 2007; Hill & Ball, 2004) aimed to investigate the ways of enhancing teachers' knowledge of mathematics topics since it was considered that teachers' ability to teach mathematics effectively was limited due to their lack of knowledge. These researchers concluded that teachers' mathematical knowledge could be improved through teacher education programs. In the same vein, Swafford, Jones and Thornton (1997) and Kellogg (2010) explored how teachers' knowledge on geometry can be improved. Swafford et al. (1997) conducted a study to examine the effects of an intervention program on teachers' geometry knowledge. Forty-nine middle-grade teachers participated in a 4-week geometry program consisting of content that related to two- and three-dimensional shapes. At the beginning and the end of the intervention program, a pre- and post-test were administered to the teachers to assess their geometry content knowledge. The analysis of the pre- and post-tests showed that teachers' geometry content knowledge increased significantly after the intervention program, especially among 4th and 5th grade teachers. Moreover, Swafford et al. (1997) stated that teachers were more willing to try new instructional approaches, they spent more time and more quality time on geometry instruction, and they were more confident to respond higher levels of student thinking after participating the intervention program.

In another study, Kellogg (2010) claimed that students and pre-service teachers contended with errors/misconceptions/difficulties regarding area and perimeter. For this reason, an alternative instructional method should be used to enhance pre-service teachers' understanding and to overcome students' errors/misconceptions/difficulties. To address this need, Kellogg (2010) investigated how pre-service teachers' knowledge changed and in what way when they engaged in anchored instruction involving web-based microworlds designed for exploring area and perimeter. The study aimed to investigate 12 elementary pre-service teachers' content knowledge and knowledge of students' thinking with respect to principles, relationships, and misconceptions related to area and perimeter. Quantitative (e.g., pre-study questionnaire, and area and perimeter tests) and qualitative research (e.g., interviews, teaching episodes packets) methods were used. The results of this study showed that pre-service teachers' knowledge related to perimeter and area changed in a positive way with intervention. Many pre-service teachers possessed procedural knowledge related to area and perimeter, but they were not aware of students' errors/difficulties/misconceptions prior to intervention. That is, their subject matter knowledge and pedagogical content knowledge were limited before the intervention. After the intervention, most of them considered that the web-based microworlds were an effective tool for them to use and they were better able to address students' difficulties. This result led Kellogg to conclude that pre-service teachers' SMK and PCK improved with the intervention. Improving teachers' knowledge is important since teachers' knowledge is a predictor of student achievement in mathematics, and also in geometry (Hill & Ball, 2004; Hill, Rowan, & Ball, 2005; Hill, Schilling, & Ball, 2004; Rowan, Schilling, Ball & Miller, 2001).

Differing from the studies mentioned above, Ng (2011) aimed to investigate in-service teachers MKT for teaching geometry and the factors that contribute to this knowledge. Ng focused on the number of years of teaching experience, educational level attained, school type (public or private), range of grade levels taught, number of professional development hours completed, and number of college-level geometry courses taken as the factors. One hundred and sixty seven in-service teachers participated in this quantitative study, and the Learning Mathematics for Teaching measures and the Indonesian Educational Survey were served as the data source. Data was statistically analyzed (ANOVA and multiple regression). The findings showed that there was an inverse relationship between the teachers' years of experience and their MKT for teaching geometry. That is, teachers who had taught longer had a lower MKT for teaching geometry. State differently, teaching experience did not guarantee the possession of rich content knowledge (Friedrichsen, Lankford, Brown, Pareja, Volkmann, & Abell, 2007). This result was in contradiction with Shulman (1987) since he asserted that teaching experience was important source of content knowledge. However, the teachers' educational background, such as number of completed professional development hours, and the number of college-level geometry courses taken, and their MKT for teaching geometry were directly related to each other as stated by Hill (2007). On the other hand, whether the teacher taught in a private school or a public school was a factor that affected teachers' MKT for teaching geometry. Teachers who taught in private schools had higher mathematical knowledge for teaching geometry compared to those who taught in public schools. Besides, being a teacher at lower primary grades (grades one to four) and at upper primary grades (grades five to six) had no significance in terms of MKT for teaching geometry. However, teachers who had taught a wider range of grades (grades one to six) had more MKT for teaching than teachers who had taught narrow range of grades. Since experience of teaching at several grades may improve teachers' MKT for teaching, it can be concluded that having rich educational background, teaching in private school, teaching wide range of grades affect teachers' MKT in a positive way.

In the sections given above, the studies related with both pre-service and in-service mathematics teachers' knowledge on geometry in international and national studies were discussed. All these studies showed that mathematics teachers do not have sufficient knowledge to teach geometry topics. Moreover, those studies provided an overview of what has been suggested to develop pre-service and in-service teachers' knowledge for teaching geometry. However, in light of the available literature, the number of studies related to the concept of volume is limited. In the next section, studies on the volume of 3D solids will be presented and discussed.

2.7 Research Studies on the Volume of Three Dimensional Solids

As mentioned above, there are not many research studies concerning the volume of 3D solids in the available literature, only one study was found which investigated teachers' content knowledge on the volume of 3D solids (Esen & Cakiroglu, 2012). However, there are some studies related to students' understanding about the volume of 3D solids (Ben- Chaim, 1985; Battista & Clements, 1996; Ng, 1998; Olkun 1999; 2003). These studies are important since students' understanding related to the volume of 3D solids may help researchers to understand teachers' content knowledge about this topic. For this reason, in this part of literature review, studies regarding the volume of 3D solids are presented.

Esen and Cakiroglu (2012) conducted a qualitative study to explore pre-service teachers' knowledge on using unit cubes to calculate volume. The data was collected from 24 pre-service teachers using a question involving student's method which resulted in an incorrect solution. The pre-service teachers were asked to think about the question, analyze student's solution method and determine the correctness of student's solution. As a result of the data analysis, most of the pre-service teachers did not have any difficulty in determining the volume of prism with non-standard concrete materials. Moreover, all pre-service teachers used the volume formula correctly and their knowledge of volume was based on the formula but this led to pre-service teachers having difficulty in realizing students' error related to the question and the reasons for the error. In fact, some of the pre-service teachers made the same error when calculating the volume of a prism.

Based on the available literature, other studies related to the volume of 3D solids investigated students' understanding of volume. The researchers explained that elementary students have difficulties in measuring the volume of a rectangular prism (Battista & Clements, 1996; Ben-Chaim, Lappan & Houang, 1985; Ng, 1998; Olkun, 1999). The most cited studies were conducted by Ben-Chaim et al. (1985), and Battista and Clements (1996). Ben- Chaim et al. (1985) investigated students' errors in 3D geometry. Approximately 1,000 students in grades five to eight participated in the study and were given a Spatial Visualization Test. On completion of the study, Ben-Chaim et al. (1985) reported four types of errors that the students held in relation to the calculation of the volume of 3D solids. They categorized these errors as two major types which were defined as "dealing with two dimensions rather than three and not counting hidden cubes" (p. 406). In further detail, Ben-Chaim et al. (1985) identified the students' difficulties as "counting the actual number of faces showing, counting the actual number of faces showing and doubling that number, counting the actual number of cubes showing, counting the actual number of cubes showing and doubling that number" (p. 397). Thus, students who count the faces consider three-dimensional solids as two-dimensional, and students who count the visible unit cubes are not aware of three-dimensionality of the solid. Furthermore, some students count the number of unit cubes on the three visible faces and then they do not multiply this number by two to obtain the total. Ben-Chaim et al. (1985) asserted that these students do not recognize the hidden part of the solid.

Battista and Clements (1996) classified students' solution strategies and examined students' difficulties when enumerating the number of unit cubes in a prism. Forty five third grade students and 78 fifth grade students participated in the study and the students were interviewed twice. They classified students' solution strategies into 5 categories with three depending on the students' conceptualization of rectangular prism, one was the use of the volume formula and the last one was a strategy that students used in addition to the four mentioned strategies. According to this study, students conceptualized rectangular prism as faces, unit cubes, and layers. This conceptualization was the reason for students' difficulties in enumerating the number of unit cubes in a prism (Battista & Clements, 1996). The authors explained that students cannot relate to the structure of the rectangular prisms such as the unit cubes, layers and stripes. If students realized that the stripes are formed by unit cubes, and layers are formed by stripes or unit cubes, then they may overcome their difficulties since rectangular prisms will become simpler for students.

Moreover, Battista and Clements (1996) examined students' difficulties in three aspects which were; use of formula, spatial structuring and coordination. The result of this research concluded that 75% of the students used the formula without knowing the reason for the multiplication of the dimensions of rectangular prisms. This means that they only memorized and applied the formula. Besides, Ng (1998) conducted a study related to students' understanding in area and volume. She collected data from 7 participants in grades 4 and 5 using semi-structured interviews. She used geometric tasks involving base ten blocks, tangram activities and questions regarding 2D and 3D. Ng obtained information regarding whether the students knew the structure of a rectangular prism, and whether students comprehend relationship between unit cubes, stripes and layers. She concluded that students viewed 3D solids as a box with six separate edges. Although some recognized the interior of 3D solid, they could not recognize its connecting or shared edges. Moreover, some students conceptualized 3D solids in terms of layers. This means that some students could not realize the structure of 3D solids.

There are studies which were carried out in Turkey to investigate students' understanding of the volume of 3D solids. In his dissertation, Olkun (1999) examined 4th grade students' understanding of rectangular solids made of unit cubes. The data was collected from four 4th grade students through interviews, and he applied treatment to decide whether students' understanding improved with instruction. He concluded that students used less viable strategies for the problems which were presented pictorially and after instruction; they used different strategies for concrete and pictorial representations.

In another study, Olkun (2003) aimed to designate 4th, 5th, 6th and 7th grade students' strategies while finding the number of unit cubes in rectangular prism. Three hundred and fourteen students participated in the study. The result showed that many students, even 7th graders, had difficulty in finding the number of unit cubes in 3D solids. The reasons for students' difficulty might be that elementary students tend to use a formula to calculate the volume of the prism. When the question is asked in a different way, such as asking the number of unit cubes in the prism, many students become confused. Moreover, the teachers' knowledge of volume is based on using a volume formula which effects students' understanding of volume (Esen & Cakiroglu, 2012).

In conclusion, all the studies reported indirectly or directly that students' achievement related to volume of 3D solids is low. They suggested that students' achievement may improve with the help of effective teaching. (Ben- Chaim, 1985; Battista & Clements, 1996; Ng, 1998; Olkun 1999; 2003). In order to provide effective teaching for students, researchers asserted that teachers' content knowledge has significant role (Chinnapan & Lawson, 2005; Even, 1993; Even & Tirosh, 1995; Hill, Rowan & Ball, 2005). Since teachers' content knowledge is important in relation to students' achievement; it will be noteworthy to investigate teachers' content knowledge relevant to the volume of 3D solids. For that purpose, the present study aimed to investigate middle school teachers' knowledge of the volume of 3D solids.

2.8 Summary of the Literature Review

In light of the studies reviewed in this section, different models explain teachers' content knowledge (Ball et al., 2008; Gess-Newsome, 1999; Grossman, 1990; Rowland et al., 2005; Shulman, 1987). Although some models focus on teachers' knowledge in terms of all aspects, others provide an approach related to the teacher's knowledge for teaching mathematics.

A review of the literature indicated that effective teaching requires knowing the ways of making the subject more understandable for the students (Ball et al., 2008). Thus, teachers have a crucial role in students' understanding of mathematics (Isiksal, 2006). Leinhardt and Smith (1985) stated that the teacher is the only person who determines what to teach, when to teach and how to teach. Therefore, teachers should understand mathematics on a deep level and know how to make mathematics meaningful for the students. In other words, teachers should have adequate content knowledge in order to determine what, when and how the subjects will be taught (Ball 1991; Ball et al., 2008; Borko, 2004; Ma, 1999). However, if the teachers do not have adequate knowledge, then they may transfer their inadequate knowledge to their students. In order to provide an insight in relation to teachers' inadequate knowledge, several studies were conducted (Contreras, Batanero, Diaz & Fernandes, 2011; Even, 1993; Even & Tirosh, 1995; Hill, Rowan & Ball, 2005; Livy & Vale, 2011; Pino-Fan, Godino, Font, Castro, 2013). These studies concluded that not only pre-service but also in-service teachers have limited content knowledge on variety topics and the situation in Turkey is almost the same. More specifically, Turkish pre-service and in-service teachers have insufficient knowledge to explain the meanings of the concepts and procedures, use multiple representations and materials, identify students' errors/difficulties/misconceptions, determine the sources of these errors/difficulties/misconceptions, find strategies to overcome these problems and evaluate students' solutions (Baki, 2013; Basturk & Donmez, 2011; Butun, 2005; Haciomeroglu, 2009; Isiksal, 2006; Karahasan, 2010; Kilic, 2011; Turnuklu, 2005; Yesildere-Imre & Akkoc, 2012). With respect to the topics and the participants of the studies, most of the researchers investigated pre-service teachers' content knowledge on the topics such as functions, limit and continuity, fractions, simplification and solving equations. As stated in the significance of the current study and in this section, there were a few studies focusing on in-service teachers' content knowledge. However, the in-service teacher's knowledge effects students' learning directly. Therefore, it is crucial to explore in-service teachers' knowledge for teaching mathematics. Moreover, in terms of the topic that was examined, there is no investigation on teachers' knowledge related to the volume of 3D solids in the accessible literature. In an attempt to examine in-service teachers' knowledge of the volume of 3D solids is believed to contribute theoretically to teachers' knowledge literature and practically for mathematics teachers. As a result this study, it is expected to make a contribution to the literature in terms of filling the missing part of mathematics teachers' content knowledge literature. In addition, since the participants were experienced mathematics teachers, the results of the current study should provide practical information for mathematics teachers who teach the same topics in their classes. Also, the experienced teachers' teaching practices may enrich other teachers' teaching as well.

CHAPTER III

METHOD

The purpose of this study was to investigate the knowledge of four middle school mathematics teachers concerning the topic of the volume of 3D solids. The study focused on the middle school teachers' subject matter knowledge and pedagogical content knowledge related to the topic.

This chapter gives a full account of the research design and the implementation. Within this perspective, it covers the details of research questions, design of the study, participants of the study, context in which the study took place, data collection and analysis techniques that were used. In addition, the issues of trustworthiness, researcher's role and bias, ethics, assumptions, and time schedule of the study are addressed at the end of this chapter.

3.1 Research Questions

This qualitative case study explores the following research questions.

1. What is the nature of the four middle school mathematics teachers' subject matter knowledge of the volume of 3D solids?
    1.1. What are the alternative solution methods these four teachers propose to calculate the volume of 3D solids?
    1.2. To what extent are these teachers successful at generating a story problem regarding the volume of 3D solids?
2. What is the nature of the four middle school mathematics teachers' pedagogical content knowledge on the volume of 3D solids?
    2.1. What kind of instructional strategies do these teachers use to teach the volume of 3D solids?

2.2. To what extent do the teachers recognize their students' knowledge related to the volume of 3D solids?

2.3. To what extent do the teachers have knowledge of curriculum related to the volume of 3D solids?

2.4. What kind of assessment types do the teachers apply to assess students' understanding of the volume of 3D solids?

3.2 Research Design

In order to investigate the four middle school mathematics teachers' knowledge of the volume of 3D solids, a qualitative research methodology was used to support methodological perspective and to reveal the findings of the study.

Qualitative research has been defined by several researchers in the literature. For instance, Denzin and Lincoln (2005) described qualitative research as follows:

> *Qualitative research is a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible. These practices transform the world. They turn the world into series of representations, including field notes, interviews, conversations, photographs, recordings and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative research study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them (p.3).*

Merriam (1998) defined qualitative research as an umbrella, which covers different aspects of inquiry that helps us understand and explain the phenomena in its particular context. According to Patton (1987), qualitative research is an endeavor to understand situations in a natural setting from the participants' perspective. In other words, it is important to understand "what it means for participants to be in that [natural] setting, what their lives are like, what's going on for them, what their meanings are, what the world looks like in that particular setting" (p.1).

Although different researchers categorized qualitative research in education under different types (Creswell, 2007; Merriam, 1998; Miles & Huberman, 1994; Yin, 2003), there are certain characteristics that apply to all types of qualitative research. In general, these characteristics are; the source of the data is a natural setting, the researcher is the key instrument for data collection and analysis, multiple data sources are used, data is collected in the form of words or pictures instead of numbers, data is analyzed inductively, and the process is as important as the product (Creswell, 2007; Denzin & Lincoln, 2005; Frankel & Wallen, 2006; Merriam, 1998).

As described above, various researchers have presented different types of qualitative research designs. Creswell (2007) defined the different types of qualitative as; narrative, phenomenological, grounded theory, ethnographic, and case studies. Similarly, Merriam (1998) presented the following five different types of qualitative research; basic or generic, ethnography, phenomenology, grounded theory, and case study. Although Creswell (2007) and Merriam (1998) proposed five different types of qualitative research, they are not totally distinct from each other and emphasized that these types work in conjunction.

Researchers sometimes wish to explore and gain insight into a particular phenomenon (Corbin & Strauss, 2008) by asking how questions (Frankel & Wallen, 2006). The important issues in answering those questions are gathering data in natural settings (Bogdan & Biklen, 1998) and entering people's minds (Patton, 2002). In this respect, interviews and observations are the data collecting techniques used in qualitative research (Frankel & Wallen, 2006). For this study in order to understand middle school mathematics teachers' knowledge of the volume of 3D solids, it was necessary to collect data via interviews and observations thus qualitative research was employed. In this qualitative case study, in which I was part of the study as the researcher, I employed a variety of data collection tools, and tried to portray a whole picture of the teachers' knowledge. The following section presents information regarding qualitative case studies is presented and then the design of the current study will be discussed.

3.2.1 Case Study Research

Creswell (2007) stated that one of the types of qualitative research is case study. He denoted that the researcher conducting a case study aims to develop an in-depth description and analysis of a case or multiple cases within a bounded system using multiple sources of data. Moreover, s/he selects an event, a program, an activity or more than one individual as the unit of analysis of the study.

Merriam (1998) referred to a case study design being "employed to gain an in-depth understanding of the situation and meaning for those involved. The interest is in process rather than outcomes, in the context rather than a specific variable, in discovery rather than confirmation." (p. 19). Similar to Creswell (2007), Merriam emphasized that researchers should describe the case which might be a person, a program or a group. Yin (1994) added to the definition of a case study by stating that:

> *A case study is an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident…Case study inquiry copes with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result relies on multiple sources of evidence, with data needing to converge in a triangulating fashion, and as another result benefits from the prior development of theoretical propositions to guide data collection and analysis. (p. 13)*

As can be seen from these definitions, the most important property of the case study is the situation in which the study is located and its context (Creswell, 2007; Merriam, 1998; Yin, 2003). According to Merriam (1998) and Creswell (2007), defining the case within a bounded system has very crucial role in the case study. However, according to Yin (2003), the boundaries between phenomenon and context may not be obvious.

Researchers have also categorized case studies into three groups according to their intent and the size of the bounded case (Creswell, 2008; Merriam, 1998; Stake, 2005). Influenced by the categorization undertaken by Stake (2005), Creswell (2007) categorized case study in terms of the intent of the case analysis. The categories are intrinsic, single instrumental and multiple case studies. In an intrinsic case study, the researcher focuses on the case since it presents an unusual or unique situation. The intrinsic case study takes place because of the case itself is of interest. Creswell defined the single instrumental case study as the study where researcher focuses on an issue or concern, and then selects one bounded case to illustrate this issue. In instrumental case study, the case is the secondary interest but it plays essential role in understanding of something else. Finally, in a multiple case study, the researcher selects multiple cases to display different perspectives of the same issue. Merriam

(1998) categorized the case study into descriptive, interpretive and evaluative. The aim of the descriptive case study is to present basic information about the phenomenon, an interpretive case study aims to obtain a rich and thick description to develop conceptual categories or support theoretical assumptions about the phenomenon. Finally, the evaluative case study, involves description, evaluation and judgment.

The current study was characterized from Creswell and Merriam's definitions of the case study. According to Merriam (1998), the aim of the case study is to "gain in-depth understanding of the situation and meaning for those who are involved" (Merriam, 1998, p. 19). Since the purpose of the current study was to gain a deeper understanding of the nature of middle school mathematics teachers' knowledge related to the volume of 3D solids, a case study approach was appropriate. The cases were four middle school mathematics teachers. The cases were bounded by both the grade level that teachers taught; 8th grade students in elementary school in Ankara and their teaching experience which was more than 10 years.

With respect to Creswell and Merriam's categorizations of the case study, the current study is an interpretive and single case study. The reason for being interpretive case study is to obtain rich and thick description about the middle school mathematics teachers' knowledge of the volume of 3D solids. Due to the fact that the participants of the study were experienced middle school mathematics teachers; study is single case study.

Apart from the categorizations of case studies given above (Creswell, 2008; Merriam, 1998; Stake, 2005), Yin (2003, 2009) further categorized case studies into; single-case holistic and multiple-case holistic designs, and single-case embedded and multiple-case embedded designs. The number of cases in the study indicates whether a study is single-case or multiple-case, and the number of unit of analysis refers to whether it is an embedded or holistic design. In the light of the definitions of designs given by Yin (2003), the single-case embedded design is a common design in case studies where it involves more than one unit of analysis. The model for the multiple-case embedded design is given in Figure 3.1 below.

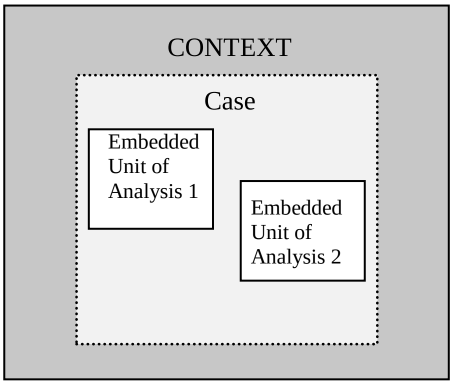


Figure 3.1 Single-case embedded (multiple units of analysis) design (Yin, 2003, p. 40)

The research design of the current study was a single-case embedded design (Yin, 2003). The case was four experienced middle school mathematics teachers, and the subject matter knowledge and pedagogical content knowledge of middle school mathematics teachers were the embedded "unit of analysis". The context of the study is Elementary Schools in Ankara. In Figure 3.2, the model of the study with respect to single-case embedded design is given.

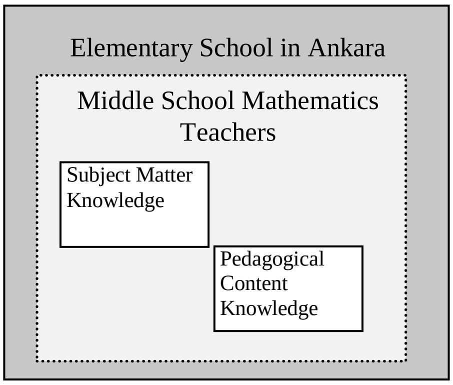


Figure 3.2 Single-case embedded (two units of analysis) design

3.3 Sampling and Selection of the Participants

In this part of the method chapter, the cases of the study, four middle school mathematics teachers, were described.

In the research studies, the aim of the study had crucial role while selecting the participants. If the researcher aims to generalize results of the study from sample to the population, probability sampling method is suitable for that study. However, if generalization as a statistical concern is not the aim of the study, the non-probability sampling method is useful (Merriam, 1998). Since it was not intended to generalize the result, a non-probability sampling method was the most appropriate sampling strategy for this study. In order to obtain a richer and deeper understanding related to the middle school teachers' knowledge of the volume of 3D solids, the participants should be selected from among the people from whom the most knowledge can be gained, can be accessed easily and with whom the most time can be spent (Merriam, 1998). Therefore, purposive sampling method, the most common form of non-probability sampling, was appropriate to achieve the purpose of the study.

Merriam (1998) emphasized that determining the selection criteria is essential in choosing participants. The current study had three criteria for the sampling procedure. The first criterion was related to selecting the elementary schools so that I could easily access the teachers meaning that the schools should be close enough for me to observe the four teachers' classroom sequentially. So that after observing one teacher, I had enough time to travel to the next school and observe the next teacher. The second criterion of the sampling procedure was selecting participants with respect to the grade level they taught. Since the national curriculum determines that the volume of 3D solids is an 8^{th} grade topic, the selected teachers should teach 8^{th} graders. The final selection criterion was being experienced teachers, namely at least 5 years teaching experience as suggested by Berliner (2001). The reason for selecting experienced teachers was that they should have a deeper and rich knowledge regarding the topic, the volume of 3D solids, because of teaching the topic many times. This was supported by Grossman (1990) who stated that teaching experience in real classroom is one of the crucial sources of teachers' knowledge.

There is no rule regarding the number of participants in qualitative research. In fact, it depends on the research questions and the data collection methods (Merriam, 1998). Moreover, it is not possible to conduct the study with everybody whose characteristics are suitable for the researcher. Thus, the researcher decided the number of participants. In the current study, the data was collected via the volume of 3D solids questionnaire, interview and classroom observation. In this sense, collecting the data through classroom observation became important factor to determine the number of participants. All the mathematics teachers taught the topic, at almost the same time. If each teacher had a four-hour mathematics lesson in one class in a week and there were 30-lesson hour in each school, thus, I could work with 7 teachers at most. After the meeting the potential participants, I took their weekly schedule to make a classroom observation schedule for me. In order to not to miss some observations of their teaching, I selected six middle school mathematics teachers (5 female and 1 male) whose schedules did not overlap. After analyzing the data gathered from four middle school teachers, I realized that the data was saturated. In other words, the data collected from two middle school teachers did not give any additional information regarding the teachers' knowledge of the volume of 3D solids. Furthermore, these two teachers taught in the same school as two of the other teachers and this was another reason for not including these two teachers in the study. According to Marshall and Roseman (2006), context has an important role in determining behavior since people's behaviors change from context to context. In this study, context was the elementary school in which teachers taught. I thought that teachers' knowledge may change according to their school. In which case I decided not to choose teachers who worked in the same school. Furthermore, I choose four successful elementary schools on the basis that teachers will tend to develop their knowledge in relation to their students proficiency.

To sum up, using the determined criteria, I selected four experienced middle school mathematics teachers working in the different elementary schools located close to each other in Ankara. The detailed information about the teachers was presented below.

3.3.1 Mrs. Kaya

Mrs. Kaya has been teaching elementary school mathematics in a public school for 31 years. She is one of the two mathematics teachers working at the same public school. She graduated from one of the best universities in Turkey with a bachelor's degree in mathematics from a faculty of arts and sciences. Before starting to teach in elementary school, she tutored elementary school students for the national exams. She is interested in enriching her teaching with different activities and representations whenever possible. Moreover, she encourages her students by preparing games, activities and manipulative related to mathematics. She thinks that mathematics should not be only lesson for students; they also enjoy learning about mathematics.

3.3.2 Mrs. Akay

Mrs.Akay has 32-years experience in teaching mathematics in public schools. She is the only mathematics teacher in her school. She graduated from a training institute, and then she went to a college to complete her education. After graduation, she taught in high school for 25 years. Later she transferred to elementary school and has worked there for 7 years. She genuinely enjoys teaching mathematics and working with elementary school students. She is interested in researching new activities, in acquiring up to date scientific knowledge and in participating in mathematics competitions with her students.

3.3.3 Mr. Esen

Mr. Esen has been teaching in an elementary school for 12 years. He is one of the two mathematics teachers working at this school. He has bachelor’s degree in department of mathematics from a faculty of arts and sciences. He has participated in different in-service trainings (e.g. classroom management, introduction to new elementary mathematics curriculum). He pays attention to use manipulative while teaching mathematics. Furthermore, he tries to link mathematics with daily life since he believes that by this linking mathematics the students’ learning will be more meaningful and permanent.

3.3.4 Mrs. Uzun

Similar to Mrs. Akay, Mrs. Uzun has 32-years elementary school teaching experience. She has been teaching in a public school and is one of the three mathematics teachers working at the same school. She graduated from a training institute. She is interested in applying different activities and representations, and using materials whenever possible for effective teaching.

3.4 Context of the Study

Baxter and Jack (2008) have emphasized that researchers should take into consideration the context when conducting case study. Due to the fact that the main focus of this study is Turkish middle school mathematics teachers, describing basic characteristics of Turkish education system and brief information about the elementary schools that the participants of the study worked would be useful to understand the study.

3.4.1 Turkish Education System

In Turkey, there are about 10 million students at the primary education levels with more than 500 000 teachers (MoNE, 2010/2011). Primary education involves the education and training of children in the age group of 6 to 14. It is eight-year compulsory education for all male and female children and is free at public schools.

In the last ten years, efforts have been attempted to improve and develop the education system. One of the efforts was the new curricula which are being implemented for primary and secondary schools since 2004. The mathematics curriculum highlights the importance of classroom environment where the students are more active and they research, discover, solve problems, and share their solutions. Also, it emphasizes the idea of associating mathematics within itself and other subjects. The primary mathematics curriculum has five learning areas: Numbers, geometry, algebra, probability and statistics, and measurement (MoNE, 2009). There is a spiral approach for each learning areas which was based on constructivist approach. The curriculum is enriched with teaching activities, manipulative usage, technology usage and multiple assessment methods. Moreover, it aims to provide mathematics teachers the flexibility of changing the places of the topics given in the curriculum (Bulut, 2007). Furthermore, teaching with different instructional methods was emphasized in the new mathematics curriculum (MoNE, 2004). In conclusion, the mathematics curriculum highlights the importance of classroom environment where the students solve problems, share their ideas and solutions, do group work and use mathematics in their daily lives and professional practices (Bulut, 2007). Moreover, it encourages teachers to apply activities, manipulative, variety instructional strategies and different assessment strategies.

However, this mathematics curriculum was changed in 2013 and the new mathematics curriculum started to be implemented in 5 graders in 2013-2014 academic year (MoNE, 2013).

3.4.2 Setting of the Study

Besides, due to the fact that people's behaviors change from context to context, the research should be conducted in a real context (Marshall & Roseman, 2006). Therefore, in this study, the real context which teachers' knowledge could be examined is their school and the classroom in which teachers teach. This study was carried out in the context of four elementary schools in Ankara, Turkey. Each teacher worked in different but geographically close schools. There were about 400 students in each school and, the number of students in each class, which was observed, was about 20-25 and the students were generally aged 14. The students in all schools had 4 hour-mathematics lesson every week. Furthermore, the students had opportunities to use materials related to 3D solids during their mathematics lesson.

3.5 Data Collection

The detailed description of the phenomenon studied in qualitative research is obtained in three basic ways; interview, observation and documents (Frankel & Wallen, 2006). Merriam (1998) stated that the interview is the most commonly used data collection tool in qualitative studies in order to obtain specific information from the participants. In other words, researcher aims to ascertain what is on participants' mind, what they think or how they feel about something (Frankel & Wallen, 2006). With the observations, researcher has the opportunity to observe the participants' behavior in the real-life settings (Frankel & Wallen, 2006). Finally, Merriam (1998) specified that documents are the third major source of data in qualitative research including; personal papers, public records, and artifacts.

To obtain deep information related to the four middle school mathematics teachers' knowledge of the volume of 3D solids, the data was collected via a questionnaire concerning the volume of 3D solids, interviews, classroom observations and field notes during the spring semester of 2011-2012 academic year. Table 3.1 presents the time schedule for the data collection.

Table 3.1 Time schedule for data collection

Date	Events
July 2011-October 2011	• Development of data collection tools (questionnaire, observation protocol and interview protocol) • Selecting and meeting the participants
November 2011- January 2012	• Pilot study of the instrument • Obtaining permission from the METU Ethical Committee and Ankara Provincial Directorate for National Education
February 2012- March 2012	• Data analysis of pilot study • Revision on the instruments in light of the pilot study • Preparation of the last version of instruments
March 2012- May 2012	• Data collection

3.6 Data Collection Tools

The purpose of this study was to investigate four middle school mathematics teachers' subject matter knowledge and pedagogical content knowledge concerning the volume of 3D solids. In order to achieve the purpose of the study, following data collection tools were used: 1) Volume of 3D solids questionnaire; 2) Interviews following the questionnaire; 3) Classroom observation of participants' teaching; 4) Field notes. Each data source is explained in detail in the following sections.

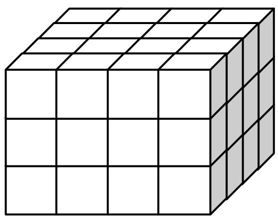
3.6.1 Volume of 3D Solids Questionnaire

In order to examine the four middle school mathematics teachers' knowledge of the volume of 3D solids, the Volume of 3D Solids Questionnaire (VDSQ) was developed by the researcher based on the related literature (Ball, Thames & Phelps, 2008; Battista & Clements, 1996; Ng, 1998). The Turkish version of the questionnaire items is provided in Appendix A.

The questionnaire consisted of 10 open-ended structured questions with sub-dimensions used to assess middle school teachers' subject matter knowledge and pedagogical content knowledge of the volume of 3D solids. The questions were prepared based on the subjects covered in the elementary school mathematics curriculum.

All questions in the questionnaire were prepared by the researcher. A table of specification for the questionnaire items was prepared and is given in Appendix B. The questions and the table of specification were checked by mathematics educator and two experienced mathematics teachers to determine the content validity. The reviewers reached an agreement. A detailed description of questions 1 and 2 in the questionnaire is given below.

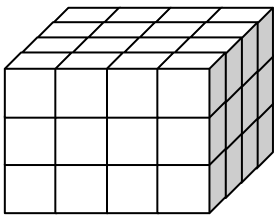
Question 1.



How many unit cubes constitute the square prism?

a) Write down all the methods that you know which could be used to answer the question.
b) What method(s) do your students use to answer this question?
c) Which error(s) do you think your students will make in answering this question?
d) What may be the reasons for these errors? Please explain.
e) Which teaching techniques/materials/strategies do you use to overcome these errors?

Question 2.



How many unit cubes remain when one layer of unit cubes is removed from all faces of square prism?

a) Write down all the methods that you know which could be used to answer the question.

b) What method(s) do your students use to answer this question?

c) Which error(s) do you think your students will make in answering this question?

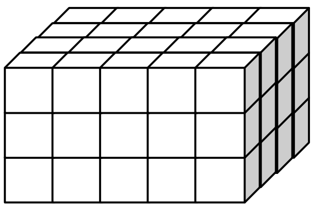
d) What may be the reasons for these errors? Please explain.

e) Which teaching techniques/materials/strategies do you use to overcome these errors?

For both 1st and 2nd questions there were the same five sub-items. The 1st sub-item aims to determine the nature of the middle school mathematics teachers' subject matter knowledge. Particularly, it aimed to assess the teachers' knowledge of alternative solution methods to calculate the volume of prism given in the figure. The remainder of the sub-items of both the 1st and 2nd questions were prepared to evaluate middle school mathematics teachers' pedagogical content knowledge. Thus, the sub-items from b to d were asked to investigate teachers' knowledge of learner.

The 3rd, the 4th, the 5th, 6th and 7th questions were designed to investigate only the middle school mathematics teachers' PCK. Specifically, they were designed to assess the middle school teachers' knowledge of learners. The questions were as follows:

Question 3.



Most of the students in Mr. Aslan's class made the same error in the question "Find the volume of rectangular prism" They gave the answer 94.

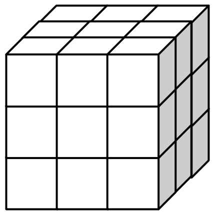
a) What method(s) do Mr. Aslan's students use to answer this question?

b) What are the elementary students' errors which caused them to give the wrong answer?

c) What may be the reasons for these errors? Please explain.

d) Which teaching techniques/materials/strategies would you use to overcome these errors?

Question 4.



Mrs. Aksoy asked the volume of the cube and her students gave the answer 27. Mrs. Aksoy realized that her students solved the question using different solution methods. Although some solution methods were correct, some of them were incorrect.

a) What solution methods were used by the students who solved the problem correctly? Please explain.

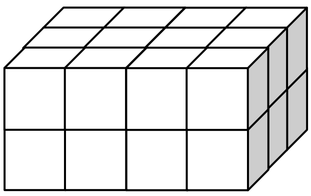
b) What solution methods were used by the students who gave the wrong answer? Please explain.

c) What errors caused the students to make a mistake? Please explain.

d) What are the reasons for the students' errors that gave the wrong answer?

e) Which teaching techniques/materials/strategies would you use to overcome these errors?

Question 5.



Students; Ela, Eren, Kuzey, Yagmur and Berke calculated the volume of prism, presented above, in different ways but they found the same result. Their solutions were given below:

Ela's Solution: 26 x 2= 52 8 x 2= 16 52- 16= 36 36- 12= 24	***Eren's Solution:*** 6 + 6= 12 4 + 4= 8 12 + 8 + 4= 24
Kuzey's Solution: 4 x 3= 12 12 x 2= 24	***Yagmur's solution:*** 6 + 6= 12 4 + 4 + 4 = 12 12 + 12= 24

Berke's Solution:

4x 3 x 2= 24

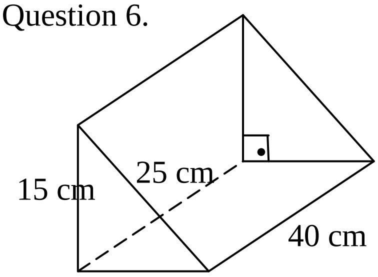
a) Explain students' solution methods in your own words.

b) If any student made errors in answering this question, then what may be the reasons for these errors? Please explain.

c) Which teaching techniques/materials/strategies would you use to overcome these errors?

d) Which method(s) did your students use to obtain the correct answer?

Question 6.



A piece of cheese was cut into a right triangular prism on the left side. The cheese was cut into 20 equal slices, what is the volume of each slice?

Mr. Acar asks the class the question given above and he encounters different solution methods.

Yankı's Solution:

$a^2 = b^2 + c^2$

$25^2 = 15^2 + c^2$

$625 = 225 + c^2$

$400 = c^2 \Rightarrow c = 20$

$V = \frac{15.20}{2}.40$

V= 6000

The volume of one slice :

$\frac{6000}{20} = 300$

Asya's Solution:

$V = \frac{15.25}{2}.40$

V= 7500

The volume of one slice:

$\frac{7500}{20} = 375$

Yaman's Solution:

$a^2 = b^2 + c^2$

$25^2 = 15^2 + c^2$

$625 = 225 + c^2$

$400 = c^2 \Rightarrow c = 20$

$\frac{40}{20} = 2$

$V = \frac{15.20}{2}.2$

V= 300

Ada's Solution:	***Ilgaz's Solution:***
$a^2 = b^2 + c^2$	$a^2 = b^2 + c^2$
$25^2 = 15^2 + c^2$	$25^2 = 15^2 + c^2$
$625 = 225 + c^2$	$625 = 225 + c^2$
$400 = c^2 \Rightarrow c= 20$	$400 = c^2 \Rightarrow c= 20$
$V= \frac{40.20.15}{2}$	$\frac{15.20}{2} = \frac{25.x}{2} \Rightarrow x= 12$
$V= 6000$	$V= \frac{40.25.12}{2} \Rightarrow V= 6000$
The volume of one slice: $\frac{6000}{20} = 300$	The volume of one slice: $\frac{6000}{20} = 300$

a) In your opinion, what process do Mr. Acar's students consider when giving their answer?

b) For those students who gave the wrong answer, describe the errors that they made.

c) Which teaching techniques/materials/strategies would you use to overcome these errors?

Question 7.

The base length of the square prism model is 6 cm and the length of side- face height is 5 cm. Ceren and Cemre who calculated the volume of this model solved the question in different ways.

Ceren's Solution:	***Cemre's Solution:***
$V= \frac{6.\ 6.\ 5}{3}$	$a^2 = b^2 + c^2$
$V= \frac{180}{3} = 60\,cm^3$	$5^2 = 3^2 + c^2$
	$25 = 9 + c^2$
	$16 = c^2$
	$c= 4$
	$V= \frac{6.\ 6.\ 4}{3}$
	$V= \frac{144}{3} = 48\,cm^3$

a) According to you, what were Ceren and Cemre thinking when they developed these methods of solving the question?

b) For those students who gave the wrong answer, describe the errors that they made.

c) For those students who gave the wrong answer, describe the reasons for these errors? Please explain.

d) Which teaching techniques/materials/strategies would you use to overcome these errors?

The sub-items of the 3rd, 4th and 5th questions were designed to assess the middle school teachers' knowledge of learners. Specifically, the 1st sub-item of questions 3, 5, 6 and 7, and 1st and 2nd sub-item of the 4th question examined the middle school teachers' knowledge related to the interpretations of students' alternative solution methods. The 2nd sub-item of questions 3rd, 6th and 7th and 3rd sub-item of the 4th question were designed to assess the middle school teachers' knowledge concerning the students' errors. Additionally, the middle school teachers' knowledge on the possible sources of elementary students' errors was examined through the 3rd sub-item of questions 3 and 7, the 4th sub-item of the 4th question and the 2nd sub-item of the 5th question. The strategies that the middle school teachers use to overcome errors were assessed through the 4th sub-item of questions 3 and 7, the 5th sub-item of the 4th question and the 3rd sub-item of questions 5 and 6. The last sub-item (4th sub-item) of the 5th question was prepared to evaluate the middle school teachers' knowledge of the students' preferences among solution methods related to the volume of 3D solids.

Contrary to the other questions the 8th question, was related to cone and was prepared to investigate middle school teachers' knowledge of SMK. Specifically, the middle school teachers' knowledge on generating a story problem was evaluated as a one of the dimensions of their SMK.

The question was as follows:

Question 8.

Using a cornet, the length of arc, 15, radius and 54, generate a story problem which involves the volume formula.

The 9th question as shown below was developed to evaluate middle school teachers' knowledge of assessment.

Question 9. What methods do you use to assess the students' knowledge related to the volume of 3D solids?

Lastly, as presented below, the 10th question aimed to assess the middle school teachers' knowledge of the curriculum.

Question 10. What other topic or topics within mathematics or other lessons do you use to teach the volume of 3D solids?

3.6.2 Semi-Structured Interview

Interviews can provide a special kind of information which is not observable (Merriam, 1998) and they are the most important sources of information in case study research (Yin, 2003). Feelings, thoughts, and intentions cannot be observed and the researcher has to ask questions to elicit this information. In this way, the researcher enters into the interviewee's mind (Patton, 2002). Thus, in order to obtain a more complete picture of the middle school teachers' knowledge of the volume of 3D solids, interviews were conducted as one of the data sources for this study.

The way in which the interview is structured is important in determining the type of interview to use (Merriam, 1998). Merriam categorized interviews under three headings; highly-structured, semi-structured, and unstructured. In highly-structured interviews, the questions and their order are predetermined. In semi-structured interviews, the questions or issues to be explored are determined but neither the order of the questions nor the exact questions are predetermined. In this situation, the researcher uses more open-ended questions. Unstructured interviews are useful when the researcher wants to ascertain information about an issue in order to formulate questions for subsequent interviews; this last type is rarely used to collect data in qualitative research.

In this study, information gathered from the Volume of 3D Solids Questionnaire (VDSQ) was limited to a general description of the four middle school teachers' knowledge. Thus, interviews were conducted with 4 middle school teachers to clarify and expand on their responses to the VDSQ. Moreover, it was necessary to develop more accurate and detailed picture of the four middle school teachers' knowledge of the volume of 3D solids by asking the teachers questions related to the VDSQ but which required longer and more detailed responses. In order to obtain information related to teachers' knowledge, semi-structured type of interview were held. Example interview questions asked were provided in English in Appendix C and in Turkish in Appendix D. Furthermore, the dimensions of teachers' knowledge which were measured with the interview were presented in Table 3.2 below.

Table 3.2 The dimensions of teachers' knowledge stated in the interview

SMK

- Developing alternative solution methods
- Generating a story problems

PCK

- Knowledge of Learners
    - ✓ Students' preferences among solution methods
    - ✓ Interpretations of students' alternative solution methods
    - ✓ Students' errors and the sources of these errors
    - ✓ The strategies to overcome elementary students' errors
- Knowledge of Curriculum
    - ✓ Connection with other topics
- Knowledge of Assessment

The semi-structured interview was conducted to allow the researcher to ask important further questions to get deeper understanding regarding middle school teachers' responses to the questions on VDSQ. Additionally, this type of interview allows for changing or asking additional questions in relation to the participants' responses. Therefore, semi-structured interviews were important in the collection of data in that the researcher may gain additional detailed insights into the teachers' knowledge on the volume of 3D solids. The sample questions that I asked during the interview were as follows. Mrs. Kaya specified that one of the students' errors might be over-counting the common unit cubes on the adjacent faces in question 2.

Regarding this error, I asked her "What do you mean by saying common unit cubes?" and "Which unit cubes are the common unit cubes?". As another example, Mrs. Uzun explained that she gave homework to assess students' understanding. At this point, I asked her "How did you assess students understanding via homework?" and "Can you explain the method that you use?". Furthermore, all teachers stated that one of the strategies to eliminate students' errors might be using manipulative. In relation to this strategy, the questions, "How do you use the manipulative to overcome the errors?" and "What is the benefits of using the manipulative to students?" were asked during the interviews.

The interviews consisted of two parts; (1) background questions about middle school teachers, and (2) questions based on the responses to the VDSQ. During the interviews, the middle school teachers explained their reasoning behind their responses to the VDSQ. With the permission of the participants all the interviews were video-taping using a digital camera. The duration of all interviews was approximately 40 minutes.

3.6.3 Classroom Observation

Although interviews are the most commonly used data collection tool in qualitative research, Merriam (1998) stated that "observational data represents a firsthand encounter with the phenomenon of interest rather than a secondhand account of the world obtained in the interview" (p.94). In other words, with the observations, researcher has an opportunity to observe the participants' behavior in the real-life. In this regard, to obtain a complete picture of the issue, which was under investigation, observations are another important source of data in qualitative studies.

Although the data collected via interview, and questionnaires provided rich and valuable data, it is not a complete picture of the teacher's SMK and PCK. Therefore, for a full description of teachers' knowledge, their teaching of the volume of 3D solids was observed. The dimensions of teachers' knowledge which were measured with the classroom observation were presented in Table 3.3 below.

Table 3.3 The dimensions of teachers' knowledge measured with the observation

SMK • Developing alternative solution methods
PCK • Knowledge of Learners ✓ Students' preferences among solution methods ✓ Students' errors ✓ The strategies to overcome elementary students' errors • Knowledge of Instructional Strategy ✓ Teacher-centered instruction ✓ Less teacher-centered enriched with class discussion • Knowledge of Curriculum ✓ Connection with other topics ✓ Changing the order of the topics • Knowledge of Assessment

I observed the middle school teachers' teaching with the use of observation protocol including points related to teachers' SMK and PCK such as knowledge of alternative soluton methods, knowledge of learners, and knowledge of curriculum. These points were provided to help me what to look for. For instance, the points related to knowledge of curriculum that I focused on during the observation were "The teacher connects the topic with the other topics in mathematics" and "The teacher alters the order of the sub-topics of the volume of 3D solids". Observation protocol was filled after every observation of each middle school teacher. English version and Turkish version of observation protocol were presented in the Appendix E and the Appendix F, respectively.

All observations were video-taped by the use of a digital camera with the permission of the participants, school administrators and Ankara Provincial Directorate for National Education. I took field notes as much as I could during observations. The classroom observations were scheduled as given in Table 3.4 and each participant was given a pseudonym. In addition, the data collected from observations was used to triangulate the analysis of data gathered via interviews and questionnaire.

Table 3.4 The time schedule of classroom observation of the teachers

Day	Monday	Tuesday		Wednesday	Thursday			Friday
Lesson Hour	10.50-12.20	11.05-12.35	13.00-14.30	9.10-10.40	9.00-10.30	11.05-12.35	13.15-14.45	10.50-12.20
Teachers	Mrs. Kaya	Mrs. Uzun	Mr. Esen	Mrs. Akay	Mrs. Kaya	Mrs. Uzun	Mrs. Akay	Mr. Esen

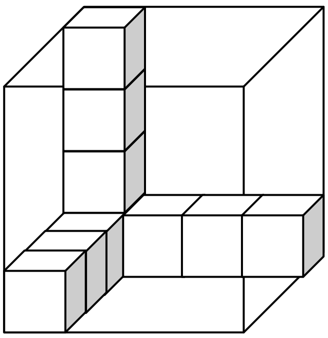
3.7 The Pilot Study

Marshall and Roseman (2006) stated that pilot study of a research allows the researcher to review the instruments and refine them if necessary, to increase self-confidence and self-efficacy in conducting the research, to recognize and resolve any problems regarding the research, before commencing the main study. For these reasons, conducting pilot study is essential for the researcher to conduct main study effectively. Furthermore, the pilot study will determine that is required for the participants to complete the questionnaire.

When selecting the participants for the pilot study, the criteria were; their teaching experience, convenient access to the schools and teacher's for the researcher. Four experienced mathematics teachers and one pre-service mathematics teachers participated in the pilot study. The reason for selecting both experienced and pre-service teachers for the pilot study was to obtain different perspectives in relation to the questionnaire. Two of the middle school teachers had 6-years teaching experience and the other two had 7-years teaching experience. Having taught the topic of the current study in a real classroom, these middle school teachers were able to share their observations and experience of the students' knowledge and attitudes towards the volume of 3D solids. The pre-service teacher was one of the successful students in the elementary mathematics education program at METU and had taken the Teaching Method courses shortly before the pilot study. Therefore, it was assumed that he had a rich knowledge regarding the volume of 3D solids and would potentially have a different point of view in relation to the topic.

In the first phase of the pilot study, the VDSQ was given to the 5 participants with sufficient time allowed for the teachers to complete all the questions before the interviews were conducted. The participants were asked to answer all questions and write their answers in detail. In the second phase, an interview lasting approximately 40 minutes was conducted with each participant. During the interview, the quality of the questions and areas, which were not clear to the participants, were discussed. In light of the pilot study and the suggestions made by the participants, changes were made to the questionnaire. Question 3, presented below, was removed from the questionnaire since its sub-items were the same as question 1. Moreover, the preliminary analysis of the pilot study revealed that the information regarding middle school teachers' knowledge of the volume of 3D solids gathered from questions 1 and 3 was the same and no additional information related to middle school teachers' knowledge on the volume of 3D solids would be obtained.

Question 3 (eliminated following the pilot study).



Calculate the volume of the cube.

a) What other information from other mathematics areas or other subjects in the curriculum would you use to teach the volume of a cube?

b) Write down all the methods that you know which could be used to solve the question.

c) Which method or methods do your students use to solve this question?

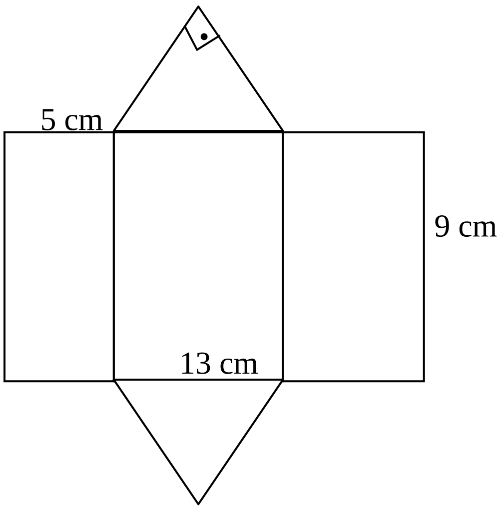
d) Which errors might your students make when solving this question?

e) What could be the sources of these errors? Please explain.

f) Which teaching methods/ materials/ strategies do you use to overcome these errors? Please explain how you use these methods, materials and/or strategies?

Following the pilot study question 5, presented below was also eliminated from the questionnaire since it did not appear to give any valuable information about the middle school teachers' knowledge of the volume of 3D solids. For the sub-item (b) in this question, the participants stated that the volume formula was the only method that could be used in this question. The participants also specified the volume formula as a solution method to calculate the volume of 3D solids in response to other questions. In other words, this information was also acquired from the other questions. Furthermore, in sub-item (d), the participants focused on not being able to draw the closed figure of 3D solids with the help of its net. In other words, the participants did not state that there was any error regarding the volume of 3D solids. Conversely, they determined that students could not decide which edges coincided with which edge when closing the net of a solid. In fact, this error was not related to the volume of 3D solids. That means that this information was not valuable for the study. For this reason, in the pilot study question 5 was eliminated.

Question 5 (eliminated following the pilot study).



What is the volume of right triangular prism whose net was given on the left if its net was fold to make a box.

a) What other information from other mathematics areas or other subjects in the curriculum would you use to teach the volume of a cube?

b) Write down all the methods that you know which could be used to solve the question.

c) Which method or methods do your students use to solve this question?

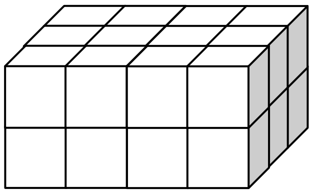
d) Which errors might your students make when solving this question?

e) What could be the sources of these errors? Please explain.

f) Which teaching methods/ materials/ strategies do you use to overcome these errors? Please explain how you use these methods, materials and/or strategies?

Question 4 in the pilot study as given below, was composed of 6 students' solution methods. The participants in the pilot study could not explain Damla's solution, and they stated that the operations "24 – 4= 20; 20 + 4= 24" were meaningless. Thus, Damla's solution was removed from the questionnaire.

Question 4 (eliminated following the pilot study).

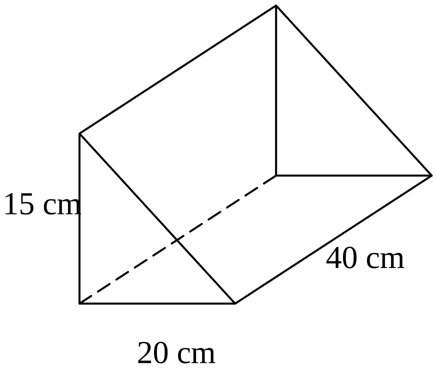


For the prism given above; Ela, Eren, Kuzey, Damla, Yağmur and Berke calculated the volume of prism in different ways but found the same result. Their solutions were given below:

Ela'nın çözümü: 26 x 2= 52 8 x 2= 16 52- 16= 36 36- 12= 24	***Eren'in çözümü:*** 6 + 6= 12 4 + 4= 8 12 + 8 + 4= 24
Kuzey'in çözümü: 4 x 3= 12 12 x 2= 24	***Damla'nın çözümü:*** 12 x 2= 24 24 – 4= 20 20 + 4= 24
Yağmur'un çözümü: 12 x 2= 24	***Berke'nin çözümü:*** 4 x 3 x 2= 24

Also, question 6, as given below, was revised following the pilot study due to the fact that it included two correct solution methods of students. The sub-question (c) was; "if one of the solution methods was incorrect, what could be the source of this error" and sub question (d) asked; "which teaching methods/ materials/ strategies would you use to overcome these errors", respectively. Since both of the solution methods, given in the question 6, were correct, it was meaningless to ask the sub-questions (c) and (d). In order to obtain the information related to the participants' knowledge concerning the sources of students' errors and the strategies that could be used to overcome those errors, three different solution strategies were added in keeping with the suggestions from the participants in pilot study.

Question 6 (revised following the pilot study).



A piece of cheese was cut into a right triangular prism on the left side. The cheese was cut into 20 equal slices, what is the volume of each slice?

Mr. Acar asks the class to solve the problem and he encounters different solution methods.

1. çözüm yolu:

$V=\frac{15.20}{2}.40$

V= 6000

The volume of one slice

$\frac{6000}{20}=300$

2. çözüm yolu:

$\frac{40}{2}=20$

$V=\frac{15.20}{2}.2$

V= 300

c) If one of the solution methods was incorrect, what could be the source of this error?

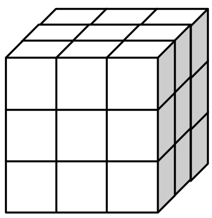
d) Which teaching methods/ materials/ strategies would you use to overcome these errors?

As a result of the feedback from the pilot study, it was also necessary to make question 7 clearer by adding more information. The original first sentence of the question was “The base of the pyramid model is 6 cm and the length of the side-face height is 5 cm” and the words ‘square’ and ‘length’ were added as follows: “The **length** of the base of the **square** pyramid model is 6 cm and the length of the side-face height is 5 cm”.

In addition, other changes were made. Question 1 consisted of two questions with the same figure and the same sub-items and was numbered as 1-i and 1-ii. The participants in the pilot study commented that writing two sub-questions in one question may cause confusion and the participants might forget to answer question 1-ii. For this reason, these questions 1-i and 1-ii were numbered as question 1 and question 2, respectively in the main study.

Furthermore, question 4, presented below, was added to the questionnaire. This question was comprised of a teacher asking her students to calculate the volume of the cube. Although her students’ answers were the same, their solution methods were different. The main issue of the question, was to explain students’ solution methods rather than asking for the calculation of the volume of the cube.

Question 4 (added following the pilot study).



Mrs. Aksoy asked the volume of the cube and her students gave the answer 27. Mrs. Aksoy realized that her students solved the question using different solution methods. Although some solution methods were correct, some of them were incorrect.

a) What solution methods were used by the students who solved the question correctly? Please explain.
b) What solution methods were used by the students who gave the wrong answer? Please explain.
c) What errors caused the students to make a mistake? Please explain.
d) What are the reasons for the students errors that gave the wrong answer
e) Which teaching techniques/materials/strategies would you use to overcome these errors?

The first version of the questionnaire, used in pilot study, was provided in Appendix G.

3.8 Data Analysis

In qualitative research, the process of data analysis begins when the researcher starts to collect data. In other words, data analysis and data collection occurs concurrently (Merriam, 1998). There is no easy set of procedures to apply during the data analysis process. Therefore, the researcher should make sense of the data by working with the data, organizing data, searching for patterns, discovering what is important and what is to be learned, and determining what you will tell people (Bogdan & Biklen, 1998).

In order to make data analysis process easier, various authors have suggested some data analysis strategies (Merriam, 1998; Miles & Huberman, 1994; Yin, 2003). Miles and Huberman (1994) presented three strategies; data reduction, data display, and conclusion drawing/verification. On the other hand, Yin (2003) described five techniques for analysis: pattern matching, linking data to propositions, explanation building, time-series analysis, logic models, and cross-case synthesis. However, Merriam (1998) categorized qualitative data analysis strategies under the following six categories: ethnographic analysis, narrative analysis, phenomenological analysis, constant comparative method, content analysis and analytic induction. According to Glaser and Strauss (1967), the constant comparative method involves identifying a phenomenon, event or set of interest and generating a theory. Due to the fact that the purpose of this study was to identify and to produce an in-depth description of middle school mathematics teachers' knowledge of the volume of 3D solids, I chose this constant comparative method.

In this study, to produce an in-depth description of middle school teachers' subject matter knowledge and pedagogical content knowledge on the volume of 3D solids, questionnaires, semi-structured interviews, classroom observations and field notes of 4 middle school teachers were analyzed. To begin with, I transcribed all the interviews and videos of classroom observation, and then I created and organized the files. I read all the texts, made margin notes and formed initial codes based on the transcripts, the middle school teachers' notes on the questionnaire, the field notes that I took during the data collection period, and the related literature. Then, I compared the codes in the same data set. Based on these comparisons, I tried to generate the categories, which were the components of PCK and SMK. I continued the comparisons until the categories were saturated which means that no new categories emerged. After the comparisons within the same data set were completed, I compared the categories with the data set of the other teachers. Then, I labeled the categories based on the participants' statements and the related literature. Lastly, I integrated the categories to create the themes.

More specifically, I coded the data with respect to the research questions. The aim of the first research question was to examine the nature of middle school mathematics teachers' subject matter knowledge on calculating the volume of 3D solids. In order to identify the teachers' SMK, their knowledge on alternative solution methods that could be used to calculate the volume of 3D solids was investigated. As a result of the data analysis of the VDSQ, interview, and observation, four codes, volume formula, systematic counting, layer counting and column/row iteration, were emerged similar to the categories stated in the study of Battista and Clements (1996). Another dimension of teachers' SMK was generating a story problem. The data gathered from VDSQ and interview was analyzed to identify teachers' knowledge on this dimension. In the finding section, this dimension was discussed with respect to descriptions of teachers' responses, directly.

Furthermore, the aim of the second research question is to investigate middle school teachers' PCK involving four dimensions. To investigate teachers' knowledge of instructional strategy, data collected from classroom observation was analyzed. As a result of the analysis, teachers' knowledge of instructional strategy was coded as teacher-centered instruction and less-teacher centered enriched with class discussion.

The second dimension of teachers' PCK was knowledge of learner. It analyzed based on four issues. The first issue, teachers' knowledge on students' preferences among solution methods, was identified based on the data gathered from VDSQ, interview, and observation. The data was coded as the solution strategies, volume formula, systematic counting, layer counting and column/row iteration. The second issue of teachers' knowledge of learners was teachers' interpretations of students' alternative solution methods. Regarding this issue, the data, came from

VDSQ and interview, was coded based on two aspects: correctness of students' solution methods, and correctness of teachers' interpretations. Moreover, students' errors and the sources of these errors, and the strategies to overcome the errors were identified through the data collected from VDSQ, interview, and observation. In relation to the students' errors, the following codes emerged from the data. The codes were as follows: focusing on the faces of 3D solids, over-counting the common unit cubes on the adjacent faces, conceptual errors and computational errors. Although the error, named as focusing on the faces of 3D solids, was described by Ben-Chaim et al. (1985), the rest of them were emerged from the teachers' explanations. The sources of students' errors were coded as not being able to think solids as three-dimensional, not being able to comprehend the structure of 3D solids, not being able to concretize 3D solids, lack of conceptual knowledge, students' carelessness, and not thinking about the concepts deeply. These codes were taken from the literature (Battista & Clements, 1996). Lastly, the strategies to overcome students' errors were coded as using manipulative and re-explaining the misunderstood part of the topic.

The third dimension of teachers' PCK was their knowledge of curriculum. The data gathered from VDSQ, interview and classroom observation was analyzed to identify teachers' knowledge on connection the topic with other topics and changing the order of the topics. In the finding section, this dimension was discussed with respect to descriptions of teachers' responses, directly. Teachers' knowledge of assessment was the last dimension of teachers' PCK. The data was coded as summative and formative assessment strategies, which were statedin theliterature (Lankford, 2010).

After the codes were determined, categories were formed (Merriam, 2009). The codes were put under categories which were the dimensions of SMK and PCK. The codes and categories painstakingly were discussed with a mathematics educator. Before finalizing the data analysis, similar categories were combined and the names of some cateries were changed. At the end of this process, a final coding scheme was created (Appendix H).

To ensure the dependability during the coding procedure, which is also explained in the trustworthiness section, I discussed the codes with my advisor, firstly and then with my thesis committee members. After reaching agreement with my advisor and the thesis committee members, I asked a Ph.D. student in the mathematics education department at METU to act as a second coder on approximately 25% of the data. The second coder was trained about the dimensions of teachers' SMK and PCK on the volume of 3D solids. I also explained the data analysis framework of the study and gave the coding scheme (Appendix H) to her to make clear the codes and their meanings. Both of us analyzed the data and then, we compared our initial codes to see the commonalities and differences between our codes. The inter-rater reliability was calculated about 95% through the use of formula suggested by Miles and Huberman (1994). The inconsistencies were discussed once more and a consensus was finally reached. All the sub-issues related to the teachers' knowledge of the volume of 3D solids is described in detail in the Chapter IV.

3.9 Trustworthiness

Validity and reliability are important issues that all researchers should take into consideration when designing a study, analyzing the data and judging the quality of the study (Patton, 2002). These issues in qualitative research are different from those in quantitative research (Yildirim & Simsek, 2006). In quantitative research, validity is defined as "referring to the appropriateness, correctness, meaningfulness, and usefulness of the specific inferences researchers make based on the data they collect" (Fraenkel & Wallen, 2006, p. 151) and reliability "refers to the consistency of the scores obtained-how consistent they are for each individual from one administration of an instrument to another and from one set of items to another (Fraenkel & Wallen, 2006, p. 157). However, different views exist about the validity and reliability concepts and different terminology was used instead of using validity and reliability in qualitative research (Creswell, 2007; Lincoln & Guba, 1985; Merriam, 1998; Miles & Huberman, 1994; Stake, 1995; Yin, 2003). Lincoln and Guba (1985) used the terms credibility, dependability, transferability, and confirmability rather than using internal validity, reliability, external validity, and objectivity, respectively. According to Lincoln and Guba, these terms form the trustworthiness of the research which exhibits the quality of qualitative research. In the following part, evidence of the trustworthiness of the study are given.

3.9.1 Credibility

Credibility in qualitative research, which refers to internal validity, is related to the congruence of the research findings and the reality (Merriam, 1998). In order to enhance credibility of qualitative research, Merriam (1998) suggested the following six strategies; triangulation, member checks, long-term observation, peer-examination or peer debriefing, participatory or collaborative modes of research and the researcher's biases. In this study, triangulation, member checks, peer examination and long-term observation were employed to ensure credibility.

Triangulation is defined as using multiple sources of data which confirm the findings of the study (Yin, 2003). There are four types of triangulation; data triangulation, investigator triangulation, theory triangulation, and methodological triangulation (Patton, 2002). In the current study, data triangulation and investigator triangulation were used. Data triangulation is achieved by comparing data from more than one participant or source. In the current study, I worked with 4 middle school teachers more than one individual, using multiple sources of data including questionnaire, semi-structured interviews, classroom observations and field-notes. Moreover, the investigator triangulation method was applied to increase the credibility of the study. The investigator triangulation was achieved by comparing and checking the data analysis and interpretation with more than one researcher. In order to ensure investigator triangulation, another researcher coded the data and the codes were examined by my advisor and thesis committee members.

Additionally, member checking, which refers to the participants checking the data, categories and interpretations were used (Merriam, 1998). During the interview, the participant teachers and I discussed their responses to the questionnaire. In this way, I ensured that whether I interpreted the teachers' responses s correctly.

In addition, peer examination was applied to ensure the credibility of the study. Merriam (1998) defined peer examination as "asking colleagues to comment on the findings as they emerge." (p.204). I asked one of my colleagues with experience in qualitative research, to participate in coding and categorizing process of my study. The second coder was trained in the issues of the middle school teachers' knowledge of the volume of 3D solids and I explained the data analysis process to this second coder. Then we analyzed the data separately following a data analysis process. Then, we discussed if any inconsistencies existed and reached full-consensus. Both coders analyzed the data which contained pseudonyms for the participants in order to eliminate any bias. Also, I regularly discussed the findings of the study with my advisor and thesis committee members throughout the data analysis process.

The process of long-term observation also helped me ensure credibility. I spent about 4 lesson-hours with the teachers every week over a period of two months. During this time, I observed the teachers' classes, spent time, and talked about teaching, learners, context, and curriculum.

3.9.2 Dependability

The second criteria to ensure the trustworthiness of the qualitative research is dependability which refers to reliability in quantitative research. Reliability is defined as " ….the consistency of the scores obtained- how consistent they are for each individual from one administration of an instrument to another and from one set of items to another" (Frankel & Wallen, 2006, p.157). In qualitative research, obtaining the same results is not an issue. However, achieving results which are dependable and consistent with the data is an issue (Merriam, 1998). To ensure whether the results are dependable or not, the investigator's position, triangulation and audit trail are the strategies that can be used. Triangulation is one of the strategies to increase the dependability as well as increase the credibility of the study (Merriam, 1998). Therefore, data triangulation, and investigator triangulation, were employed in the current study as explained above. Moreover, the investigator's position was used to increase the dependability of the study by explaining the theory behind the study, the criteria for selecting the participants, and the context of the study (Merriam, 1998). In addition, I discussed the research design of the study, how I collected and analyzed the data, how I derived the categories and how I interpreted the categories clearly (Merriam, 1998). Thus, an audit trail was employed to ensure the dependability of the study.

3.9.3 Transferability

Transferability, is the third criteria that ensures the trustworthiness of qualitative research referring to external validity. In other words, it is related to the generalizability of the results of the study however, Merriam (1998) pointed out that generalizability is not the concern of qualitative research. Nevertheless, it is possible to achieve generalizability through a thick description of the study and obtaining sufficient data. In this study, the context of the study, the criteria for selecting the participants, the number of the participants, the data collection and analysis methods, and the time schedule of the study were explained in detail in the method section. In this way, a rich and thick description regarding the study was provided to ensure the transferability. Moreover, in order to obtain sufficient data, there was more than one middle school teacher participant which allowed me to increase the transferability of the study. By providing sufficient data and a rich description regarding the study, thus, the findings of the study could be easily shared with other researchers and mathematics teachers to understand the nature of teachers' mathematical knowledge of the volume of 3D solids.

3.9.4 Confirmability

The fourth criteria to establish trustworthiness in qualitative research is confirmability referring to objectivity. Shenton (2004) emphasized that to ensure the confirmability of the study, the results of the study must be based on the experiences and ideas of the participants rather than the researcher. He specified that confirmability is established using triangulation to reduce the effect of investigator bias. In addition, a detailed description on the methodology of the study is another strategy to ensure the confirmability of the study. In this study, triangulation and detailed description of the methodology of the study was used to establish the confirmability. Moreover, I used direct quotations (verbatims) in order to decrease the amount of inferences that I might make.

3.10 Researcher Role and Bias

In qualitative research, the researcher has important role when collecting and analyzing the data (Merriam, 1998). Due to the fact that researcher is the primary instrument throughout the study, s/he may find what s/he wants to find and interprets the data how s/he wants (Johnson, 1997). This researcher bias is a potential threat to validity since "…qualitative research is open ended and less structured than quantitative research" (Johnson, 1997, p. 284). He further stated that reflexivity is a tool to understand researcher bias this involves self-awareness and critical self-reflection about his/her potential bias. In this sense, it is the responsibility of the researcher to monitor and try to control their biases. Thus, it was very important that I undertook to reduce my possible bias throughout the study.

Before the study, as a researcher, I met with the participants a few times to explain the purpose and the data collection procedures of the study in detail. During these meetings, we had the opportunity to get to know each other personally which made the participants and me more comfortable during the data collection process. I also made sure that the participants knew that all the responses to the questionnaire and the content of video-taped, taken during the interview and classroom observations, were confidential. Moreover, I explained that I was the only person who had access to the data and the data was analyzed with pseudonyms given to the participants to eliminate the bias. As a result of this, participants told me that they were willing to participate in my study and share their knowledge and experiences objectively.

Furthermore, the duration of completion of the questionnaire was determined according to their needs and to avoid the teachers feeling under too much pressure. Additionally, the participants' interview times were arranged in terms of timing so they could take their time to respond. During the interviews, I explained that there were no correct answers to the questions. Moreover, I emphasized that the only think that I expected them was to explain their ideas in as detailed manner as possible. In addition, the participants' explanations during the interview were summarized after every question and I asked participants whether I had understood their point of view correctly.

The classroom observation was the second data source of the study to triangulate the data gathered from the interview. During the classroom observation, I video-taped the lecture. However, I might have had an effect on the flow of the lessons as well as students' behaviors because of process of video-taping. In order to reduce potential disturbance due to the video-taping, I spent time in the classrooms before the data collection period in order for the participants and the students to become accustomed to the camera. I also assured the participants that the videos would only be used for research purposes. In this way, I tried to encourage them to act naturally during data collection process.

Through explaining my purpose and data collection process clearly to the participants, undertaking the research with voluntary participants, trying to make the participants comfortable during the data collection process, and checking my understanding with the participants regarding their explanations; I aimed to reduce researcher bias. I hoped that clarifying my own biases will help readers understand my position, and thus validate the study.

3.11 Ethical Considerations

In order to be able to conduct the study, first, I took permission from the Ethical Committee at METU (Appendix I) and the Ankara Provincial Directorate for National Education (Appendix J). They confirmed that the study had no potential to harm the participants or the students in the classes. For the video-taping during the interview and classroom observation, the permission was taken from the Ankara Provincial Directorate for National Education and METU Ethical Committee. Additionally, I talked with the school administrators about conducting the research in their schools and obtained their approval. Then I identified the mathematics teachers who were willing to participate in the study and they signed the consent form.

Frankel and Wallen (2006) pointed out that there are three important issues related to the ethics in research; avoiding the deception of the participants, protecting of the participants from harm and ensuring the confidentiality of the data. In this regard, I ensured all participants that there would be no harm or deception during the research process that would violate the participants' rights. In order to ensure confidentially, I made sure that no one else knew the names of the participant teachers and the school in which they worked. Also, only myself, my advisor, and the second coder had access to the data collected for the study. For this study I gave all the participants pseudonyms in this study. Furthermore, the participants were informed that they could leave the study at any point in time.

3.12 Assumptions of the Study

There were several assumptions attached to this study. First, as explained above, all the participants were experienced teachers and they had taught the topic, the volume of 3D solids, for many years. Due to the fact that teaching experience is one of the major sources of teachers' knowledge (Grossman, 1990), I assumed that they had rich repertoire of SMK and PCK which were the focus of this study.

Another assumption of this study is that I would spend a long time in participants' classroom to observe their teaching and this would mean that I would be able to obtain data pertaining to their knowledge on the volume of 3D solids in a real sense. After spending time with the participants in their classes, I conducted semi-structured interviews with them and for this reason, it was assumed that the participants expressed and shared their knowledge on the volume of 3D solids clearly and honestly during the interview.

3.13 Time Schedule

The phases of the research were as follows:

Table 3.5 Time schedule for the research

Date	Events
January 2011- June 2011	• Planning the design of the study
July 2011-October 2011	• Development of data collection tools (questionnaire, observation protocol and interview protocol) • Selecting and meeting the participants
November 2011- January 2012	• Pilot study of the instrument • Obtaining permission from the METU Ethical Committee and Ankara Provincial Directorate for National Education
February 2012- March 2012	• Data analysis of the pilot study • Revision of the instruments in light of the pilot study • Preparation of the last version of instruments
March 2012- May 2012	• Data collection

June 2012- August 2012	• Transcription of the video-taped of interviews and classroom observations
September 2012- November 2012	• Data analysis
December 2012- …..	• Writing up the dissertation

In the next chapter, the findings of the study are presented.

CHAPTER IV

FINDINGS

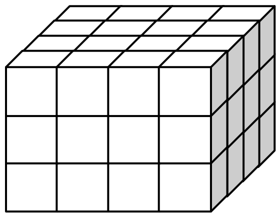
The general aim of this study is to examine the knowledge of four middle school mathematics teachers of the volume of 3D solids. This chapter presents the findings of the research study under two main sections and related sub-sections. In the first section, the four middle school mathematics teachers' subject matter knowledge about the volume of 3D solids was analyzed under two headings: knowledge of alternative solution methods, and knowledge of generating a story problem. The second section summarizes the four middle school mathematics teachers' pedagogical content knowledge of the volume of 3D solids. This section was subdivided into fourheadings: knowledge of instructional strategy, knowledge of learners, knowledge of the curriculum and knowledge of assessment. Under the each heading of middle school teachers' SMK and PCK, the detailed explanation was summarized with related vignettes taken from the interviews and the observations.

4.1 Middle School Mathematics Teachers' Subject Matter Knowledge

One of the aims of this study was to investigate the middle school mathematics teachers' subject matter knowledge of the volume of 3D solids. The analysis revealed that the middle school teachers' subject matter knowledge showed variety based on the data gathered from the questionnaire, interview, classroom observation and field notes. In this manner, the analysis of SMK of the middle school teachers referred to the investigation of teachers' knowledge of alternative solution methods, and their knowledge of how to generate a story problem. The analysis was based on available literature, participants' explanations and my own experiences with the data.

4.1.1 Knowledge of Alternative Solution Methods

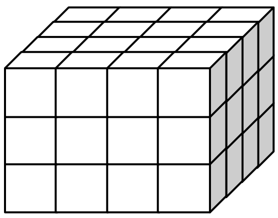
The middle school teachers' knowledge of alternative solution methods emerged from the data as one of the dimensions of the teachers' SMK. In the volume of 3D solids questionnaire (VSDQ) (Appendix A), the middle school teachers asked to propose alternative solution methods for the questions related to the calculation of the volume of 3D solids. The middle school teachers were specifically asked the following questions:



How many unit cubes constitute the square prism?

a) Write down all the methods that you know which could be used to answer the question.

Figure 4.1 Question 1



How many unit cubes remain when one layer of unit cubes is removed from all faces of square prism?

a) Write down all the methods that you know which could be used to answer the question.

Figure 4.2 Question 2

The analysis of the questionnaire, and interview transcripts revealed that the four middle school teachers proposed the following four alternative solution methods; *volume formula*, *systematic counting*, *layer counting* and *column/row iteration.* According to the analysis of the data, *volume formula* was emphasized by all participant teachers. However, *systematic counting* was proposed to calculate the volume of 3D solids by Mrs. Akay and Mr. Esen. On the other hand, Mrs. Kaya and

Mrs. Uzun proposed *layer counting* and *column/row iteration* methods to answer the questions. The summary of the solution methods for the each question is given in Table 4.1 and discussed in the following section.

Table 4.1 The solution methods proposed by the middle school teachers

	Mrs. Kaya	**Mrs. Akay**	**Mr. Esen**	**Mrs. Uzun**
Question 1	• Volume formula • Layer Counting • Column/Row Iteration	• Volume formula • Systematic counting	• Volume formula • Systematic counting	• Volume formula • Layer Counting • Column/Row Iteration
Question 2	• Volume formula	• Volume Formula	• Systematic counting	**Could not develop any correct method**

4.1.1.1 Volume Formula

One of the alternative solution methods that the middle school teachers proposed to calculate the volume of 3D figures was *volume formula*. Battista and Clements (1996) defined *volume formula* as multiplying the depth, the width and the height of the prism. Based on the analysis of the data, all the middle school teachers defined *volume formula* as multiplying the lengths of three edges or multiplying the area of the base of the prism by its height.

As shown in Table 4.1, four teachers proposed *volume formula* to calculate the volume of 3D solids. As an example, Mrs. Kaya's explanation for the question 1 was presented below.

> The bases and the height, namely the volume could be calculated with the volume formula. How is that done? It can be achieved by counting the number of unit cubes in the height and in the edges of bases by finding the volume from a x b x c or by saying the base is a rectangle and [you] multiply the base with the height [of the prism] to answer the question.
>
> *Taban ve yükseklik yani hacim formülü ile hacim hesap edilebilir. O da nasıl? Yükseklik ve tabanın kenarlarındaki birim küpleri sayarak a.b.c. den hacmini buldurarak yapılabilir. Ya da tabanı bir dikdörtgendir ve*

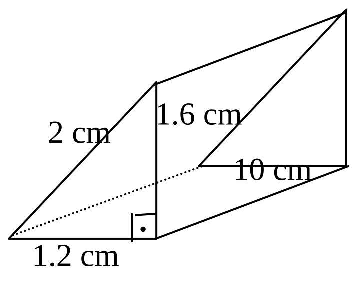
soruyu çözmek için tabanın alanını, [prizmanın] yükseklikle çarparsınız denebilir.

In addition, Mrs. Akay proposed *volume formula* method to calculate the volume of 3D solids and stated:

> In fact, when the students are in 6^{th} grade, we explain that multiplying the width, depth and height gives the volume with the help of associative rule. They can find it [the volume] with this method. When they are in the 8^{th} grade, we formulate this, multiplying the area of the base by the height. They see all phases from the primary school. The most advanced form is using the formula of multiplying the area of the base with the height.
>
> *Aslında öğrencilere 6. Sınıfa geldiğinde birleşme özelliğinden yararlanarak en, boy,ve yüksekliğin çarpımının hacim olduğunu anlatıyoruz. [Öğrenciler] bu methodla bulabilirler. Öğrenciler 8. sınıfa geldiği zaman bunu formülize ediyoruz, Taban Alanı x yükseklik. İlkokuldan itibaren bütün aşamaları görüyorlar. En gelişmiş halide formülle yapılan, taban alanı x yükseklik.*

Based on the data gathered from classroom observation, all the teachers frequently used the *volume formula* method to calculate the volume of 3D solids in their lessons. The examples from each teacher's lesson are presented below.

The first example related to using the *volume formula* method was observed in Mrs. Kaya's lesson.



What is the volume of chock whose shape is right triangle prism in the figure?

(*Şekildeki dik üçgen prizma şeklindeki takozun hacmi kaç cm^3 tür?)*

Figure 4.3 An example from Mrs. Kaya's lesson

Mrs. Kaya explained that to calculate the volume of a prism, the base area of the prism and its height are multiplied. Then she clarified that the base of the prism is a right triangle with right edges of 1.2 cm and 1.6 cm and the height of the prism is 10 cm. Then she solved the problem as follows:

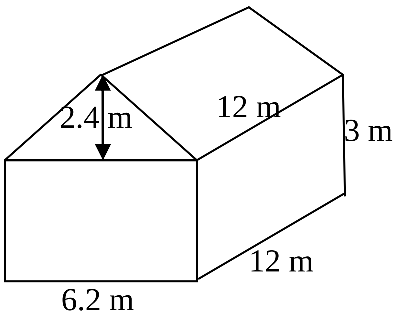
V= the base area x the height

$$V = \frac{1.2 \times 1.6}{2} . 4$$

V= 0.96 x 4

V= 3. 84 cm^3

Mrs. Akay asked her students to solve the following problem taken from the textbook.



Calculate the volume of the barn.

(Yandaki ahırın hacmini hesaplayınız.)

(MoNE Textbook 8th grade, 2010, p. 130)

Figure 4.4 An example from Mrs. Akay's lesson

After the teacher asked the problem, students tried to solve it. After a short time, Mrs. Akay explained the solution using the *volume formula* method. She wrote the formula on the board and put the numbers on the formula. Her solution is presented below:

$$\text{Volume of the barn} = \frac{\text{volume of the}}{\text{triangular prism}} + \frac{\text{volume of the}}{\text{rectangular prism}}$$

Volume of the triangular prism = area of the base x height

$$= \frac{6.2 \times 2.4}{2} . 12$$

$$= 89.28 \text{ m}^3$$

Volume of the rectangular prism = area of the base x height

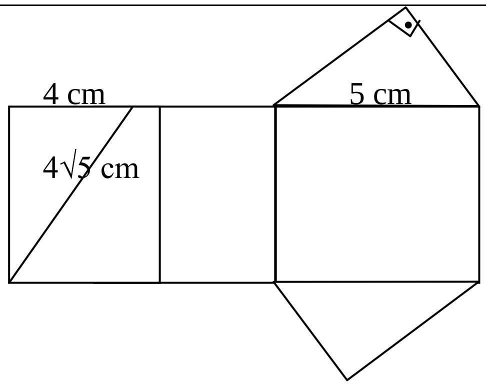
$$= 6.2 \times 3 \times 12$$

$$= 223.\ 2 \text{ m}^3$$

Total volume = 89.23 + 223.2

$$= 312.48 \text{ m}^3$$

Similar to Mrs. Kaya, Mr. Esen asked his students a problem pertaining to a triangular prism as given below:



What is the volume of the triangular prism ?

(Yanda verilen üçgen prizmanın hacmi kaçtır?)

Figure 4.5 An example from Mr. Esen's lesson

As the first step to solve the problem, Mr. Esen aimed to find the unknown length of the side-face [the height] of the prism by applying the Pythagorean Theorem. He defined the height as h and his solution was as follows:

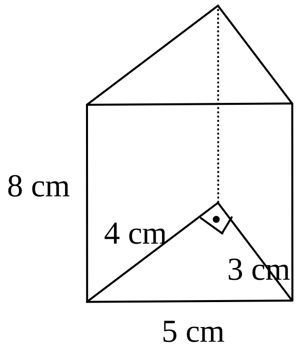
$(4\sqrt{5})^2 = h^2 + 4^2$

$80 = h^2 + 16$

$h^2 = 64$

$h = 8$ cm

Then he stated that the length of unknown edges of the triangle should be found to calculate the base area of the triangular prism. Accordingly, he explained that one of the right edges of the triangle was 4 when the net of the triangular prism was closed. After that, he reminded his students of the 3,4,5 triangle which he previously taught. Then, he drew the closed figure of the triangular prism and calculated its volume using the volume formula.



$$V = \frac{3 \times 4}{2} \times 8$$
$$V = 6 \times 8$$
$$V = 48 \text{ cm}^3$$

Figure 4.6 Mr. Esen's solution of the problem presented in Figure 4.5

Lastly, Mrs. Uzun asked a problem related to the calculation of the volume of a regular hexagonal prism and she solved the problem using the volume formula method.

Problem: Find the volume of regular hexagonal prism in which the edge of the base is 4 cm and the height is 10 cm.

Tabanının bir kenarı 4 cm ve yüksekliği 10 cm olan düzgün altıgen prizmanın hacmini bulunuz.

Before calculating the volume of the prism, Mrs. Uzun reminded her students how to calculate the area of the hexagon. Then she calculated the volume of regular hexagonal prism, as follows:

Area of the hexagon= $\frac{6a^2\sqrt{3}}{4}$

Volume of the prism= $\frac{6a^2\sqrt{3}}{4}$ x h

$$V = \frac{6 \text{x} 4^2\sqrt{3}}{4} \text{x} 10$$
$$= 24\sqrt{3} \text{x} 10$$
$$= 240\sqrt{3} \text{ cm}^3$$

All four middle schoolteachers proposed the *volume formula* method as one of the ways to calculate the volume of 3D figures and all the teachers mostly used this method in their lessons (Table 4.1).

4.1.1.2 Systematic Counting

The analysis of the data revealed that 2 of middle school teachers proposed *systematic counting* as an alternative method to calculate the volume of 3D solids. Battista and Clements (1996) defined the method as "students counts cubes systematically, attempting to count both inside and outside cubes. He or she might, for instance, count the cubes on all the outside faces, and then attempt to determine how many are in the center." (p. 263).

Mrs. Akay and Mr. Esen emphasized the *systematic counting* method to calculate the volume of 3D figures. Both teachers explained that if the prism was presented as in the question 1, the unit cubes might be counted on both inside and outside of the prism. Similarly, Mr. Esen specified this method for the solution of question 2 as presented below:

> Now, do we count the cubes on the outer faces? If we count them, here [the length of the width, depth and height] lessened. The reminder of the cubes constitute a rectangular prism. The rest of the unit cubes could be counted one by one.
>
> *Şimdi dış yüzeydeki küpleri alıyoruz demi? Onları alınca burası küçülür. Geri kalan da dikdörtgenler prizması olur. Kalan küpler birer birer sayılabilir.*

Although this method was specified in the literature (Battista & Clements, 1996) as an elementary students' method to calculate the volume of prism, only 2 of the 4 middle school teachers (Mrs. Akay and Mr. Esen) emphasized this as a solution to questions 1 and 2 on the questionnaire. However, Mrs. Kaya and Mr. Uzun did not use this method in their lessons (Table 4.1).

4.1.1.3 Layer Counting

The analysis revealed that middle school teachers gave *layer counting* as another alternative solution method. Similar to the *volume formula* and *systematic counting* methods, Battista and Clements (1996) explained layer counting as "the student conceptualizes the set of cubes as forming a rectangular array organized into layers." (p. 263). In other words, *layer counting* means counting the number of unit cubes in one layer, and then multiplying this number by the number of layers or using addition to obtain the total.

As shown in Table 4.1, only two participants (Mrs. Kaya and Mrs. Uzun) proposed layer counting to solve question 1. Mrs. Kaya's explanation is presented below.

> I thought that there are 16 unit cubes at the bottom layer and there are 3 layers. For this reason, the total could be calculated as 16+16+16.
>
> *Alt katmanda 16 birim küp var ve 3 katman var diye düşündüm. Bu yüzden, toplam16+16+16 şeklinde hesaplanabilir.*

Mrs. Kaya used this method to solve a problem in her lesson. She asked the students how they would calculate the number of unit cubes in the rectangular prism given below.

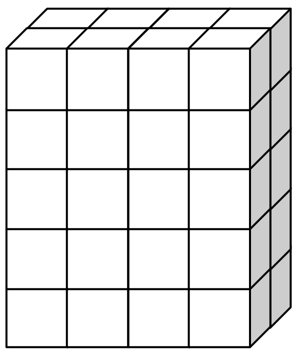


Figure 4.7 An example from Mrs. Kaya's lesson

Students said that they could find its volume by multiplying three edges (width, depth and height) of the rectangular prism. In other words, students proposed to use the *volume formula* method. Then Mrs. Kaya showed her students different methods to calculate the number of unit cubes. One of her method is given in Figure 4.8:

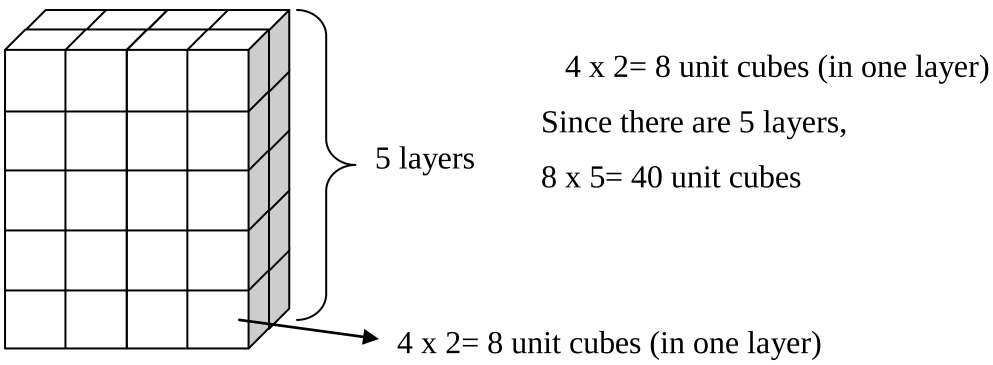


Figure 4.8 An example of using *layer counting* from Mrs. Kaya's lesson

On the other hand, although Mrs. Uzun referred to *layer counting* method during interview, she did not explain this method in her lesson. However, Mrs. Akay and Mr. Esen did not explain the *layer counting* method during their lessons or the interview.

4.1.1.4 Column/Row Iteration

The analysis of the data showed that middle school teachers proposed *column/row iteration* as an alternative solution method to calculate the volume of 3D solids. Similar to other methods, this was also defined by Battista and Clements (1996) as "students count the number of cubes in one row or column and use skip-counting (pointing to successive rows or columns) to get total" (p.263).

Similar to the *layer counting* method, Mrs. Kaya and Mrs. Uzun emphasized the use of the *column/row iteration* method only for question 1 (Table 4.1). Below is the related vignette from Mrs. Uzun's interview.

> The number of unit cubes in each row might be counted. Namely, I thought that there are 4 unit cubes in each row. How many rows are there? 12 rows; 4 x 12= 48.
>
> *Her sıradaki birim küpler sayılabilir. Yani her sırada 4 tane birim küp var diye düşündüm. Kaç tane sıra var? 12 sıra. 4 x 12= 48.*

Another example of using *column/row iteration* was explained by Mrs. Kaya as shown in Figure 4.9:

> There were 3 unit cubes here, 3 here, 3 here…[Presented in the figure]. Namely, it occurs by counting. It was not 1,2,3; by counting 3 by 3.
>
> *Burada 3 tane küp vardır, burada da 3, burada da 3[şekilde gösterildi]. Yani sayarak olmuş oluyor 1,2,3 diye değil de 3 er 3 er sayarak.*

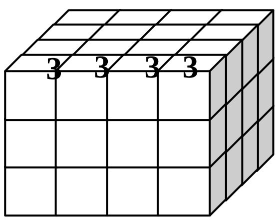


Figure 4.9 Mrs. Kaya's explanation of example of using *column/row iteration*

Additionally, Mrs. Kaya focused on this method to solve the question that she presented in her lessons. As mentioned before, Mrs. Kaya asked her students the number of unit cubes in the rectangular prism given in Figure 4.7.

As stated above, the students solved the problem using a *volume formula*, then Mrs. Kaya solved it using the *layer counting* method. Moreover, Mrs. Kaya solved the problem using the *column/row iteration* method as follows:

Mrs. Kaya wrote the number of unit cubes in each column by adding the previous one to obtain the total.

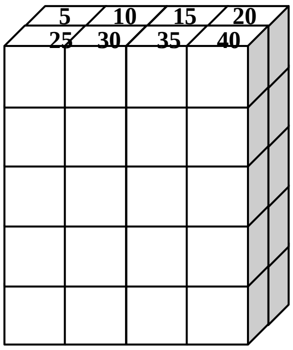


Figure 4.10 An example of using *column/row iteration* from Mrs. Kaya's lesson

As with the *layer counting* method, Mrs. Akay and Mr. Esen did not refer to this method to calculate the volume of 3D solids (Table 4.1).

To summarize the analysis of the data it showed that the four middle school teachers proposed the four following methods; *volume formula, systematic counting, layer counting and column/row iteration*. Although the first method was indicated as a way of solving the problem in the question 1 by all the teachers, *systematic counting* was denoted by Mrs. Akay and Mr. Esen to solve questions 1 and 2. *Layer counting* and *column/row iteration* methods were only specified by Mrs. Kaya and Mrs. Uzun for solving question 1 (Table 4.1).

4.1.2 Knowledge of Generating a Story Problem

As presented above, based on the analysis of the data, one of the dimensions included in the middle school teachers' SMK was their knowledge of generating story problems. In this study, this knowledge type was investigated under middle school teachers' SMK since Ball (1990a) defined as teacher's being able to "understand the subject in sufficient depth to be able to represent it appropriately and in multiple ways-with story problems, pictures, situations, and concrete materials" (p. 458). In the VDSQ (Appendix A), the middle school teachers asked to generate a story problem regarding the volume of 3D solids using the numbers and terms as given in Figure 4.11.

Using a cornet, the length of arc, 15, radius and 54, generate a story problem which involves the volume formula.

Figure 4.11 Question 8

Based on the analysis of the questionnaire and interview transcripts, Mrs. Akay, Mr. Esen and Mrs. Uzun did not want to generate a problem. After a while, Mrs. Uzun outlined the problem given below:

> 54 can be the length of the arc and 15 can be the radius. Because the arc is longer than the radius. Like this.
>
> *54 yay uzunluğu olabilir, 15 de yarıçap olabilir. Çünkü yay, yarıçaptan uzundur. O şekilde.*

However, Mrs. Uzun did not generate a story problem which includes the volume of the cone. She only guessed the meaning of the numbers in the question. Similar to Mrs. Uzun, Mrs. Akay tried to generate a problem but she was not able to use the terms, radius and the length of the arc as shown below. For this reason, her question did not meet the expectations.

> The problem could be; find the volume of the cone whose base area is 54 and height is 15. But the problem should include the length of the arc and the radius.
>
> *Soru taban alanı 54, yüksekliği 15 olan koninin hacmini bulunuz şeklinde olabilir. Ama sorunun yay uzunluğu ve yarıçapı içermesi gerekiyor.*

Furthermore, Mr. Esen could not make any interpretation regarding the problem which could be created using the cornet, the length of arc, 15, radius and 54. Only Mrs. Kaya was able to generate a problem and she solve it correctly. Her problem was as follows:

> Find the volume of the cornet in which the length of the arc is 54 and the length of the generatrix, namely the radius of sector, is 15.
>
> *Yay uzunluğu 54 cm ve ana doğrusunun uzunluğu yani daire diliminin yarıçapı 15 cm olan külahın hacmini bulunuz.*

Generating story problems, which "give us a view of [teachers'] understanding" (Ball, 1990a, p.453) is one of the dimensions of SMK. However, three teachers (Mrs. Akay, Mr. Esen and Mrs. Uzun) were not successful at generating a problem. On the other hand, Mrs. Kaya was able to generate an appropriate problem using the terms and the numbers.

4.2 Middle School Mathematics Teachers' Pedagogical Content Knowledge

As it was presented, one of the aims of this study was to examine middle school mathematics teachers' pedagogical content knowledge on the volume of 3D solids. The analysis of the data collected from the questionnaire, interview and classroom observation revealed that the middle school teachers' pedagogical content knowledge (PCK) could be categorized under the following four dimensions; knowledge of instructional strategy, knowledge of learners, knowledge of curriculum and knowledge of assessment. These dimensions of teachers' PCK are explained in detail in the following sub-sections.

4.2.1 Middle School Mathematics Teachers' Knowledge of Instructional Strategy

Another dimension of middle school teachers' PCK was knowledge of instructional strategy. Teachers' knowledge of topic-specific instructional strategies was emerged from the data. In the current study, topic-specific instructional strategies involve appropriate strategies to teach particular mathematics topics. Based on the analysis of the data gathered from observations, topic-specific strategy implemented by teachers were teacher-centered and less teacher-centered enriched with class discussion.

4.2.1.1 Teacher-Centered Instruction

In the current study, teacher-centered instruction refers to providing clear explanations and examples, checking students' understanding by asking them questions and using manipulative to help students envisage the 3D solids.

The data revealed that all the teachers mostly applied teacher-centered approach to teach the volume of 3D solids. Initially, they introduced the topic, for instance; the volume of prism. Then they asked questions to identify students' prior knowledge regarding the topic. Below is a transcript of part of Mr. Esen's lesson:

> ***Mr. Esen:*** *What is the volume?*
> ***Std:*** *The multiplication of the area of the base and the height.*
> ***Mr. Esen:*** *Okay, how can we calculate the volume of all prisms?*
> ***Stds:*** *By multiplying the area of the base and the height.*

After eliciting students' prior knowledge about the volume of the prism, all teachers provide clear explanation of calculating the volume of 3D solids. As an example, a further except from Mr. Esen's lesson is given.

> In its simplest form, it is the multiplication of the width, depth and height. The multiplication of the three edges [of the prism]. When I said multiplication of the width and depth, you understood that to be the area of the base. In that case, what is the volume of all prisms? The multiplication of the area of the base and the height. If the base is a triangle, then it is the multiplication of the area of the triangle and the height. If the base is rectangle, then it is the multiplication of the area of the rectangle and the height. Briefly, V= area of the base x the height.
>
> *En basit haliyle en, boy [derinlik], yüksekliğin çarpımıdır. [Prizmanın] Üç tane kenarının çarpımı. En ve boy [derinlik] dediğim zaman, taban alanı olduğunu anlıyorsunuz. O halde, bütün prizmaların hacmi nedir? Taban alanı ile yüksekliğin çarpımı. Eğer taban üçgense üçgenin alanı ile yüksekliğin çarpımı. Dikdörtgense, dikdörtgenin alanı ile yüksekliğin çarpımı. Kısaca, V= Taban alanı x yükseklik.*

The example provided above from Mr. Esen's explanation was a highly representative example of Mrs. Kaya, Mrs. Akay and Mrs. Uzun's explanations in calculating the volume of 3D solids.

Later, all teachers provided an exemplary problem, which was first solved by the teacher. At this point, they emphasized the important points of the problem that students should be aware of. After that, the students worked on the other problems on the blackboard. One of the problems from Mrs. Uzun's lesson was:

Question: Find the volume of square prism with one edge of length 3 cm and a height of 7 cm.

Soru:Tabanının bir kenar uzunluğu 3 cm ve yüksekliği 7 cm olan kare prizmanın hacmini bulunuz.

While students were copying the question into their notebooks, Mrs. Uzun drew the figure of the square prism (Figure 4.12) on the board. Then she wrote the given lengths on the figure so the students could visualize the prism. Mrs. Uzun encouraged the students to solve the question on the board to check students' learning.

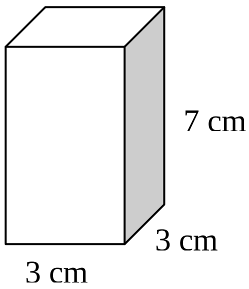


Figure 4.12 Figure of the question Mrs. Uzun asked

Like Mrs. Uzun, all participating teachers used similar figures for all questions they asked and solved when teaching the volume of 3D solids. Apart from explaining the question using representations, the four teachers also used manipulative to help the students envisage and visualize 3D solids. For instance, Mrs. Kaya constructed a 2x3x4 rectangular prism with unit cubes and then asked the number of unit cubes in the rectangular prism. The figure of 2x3x4 rectangular prism was given below.

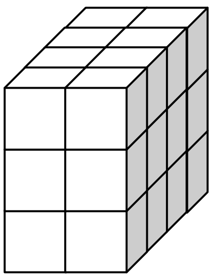


Figure 4.13 Figure used by Mrs. Kaya to indicate 2x3x4 rectangular prism

Using the figure, Mrs. Kaya said:

> As you can see, there are 8 unit cubes in the 1st layer. Because there are 3 layers, 8 is multiplied by 3. What did we do? By calculating the number of unit cubes on the 1st layer, actually we calculated the base area of the prism. Then since the height of the prism is 3, by multiplying by 3, we multiplied the base by the height.
>
> *Gördüğünüz gibi, birinci katmanda 8 birim küp var. 3 katman olduğu için, 8 ile 3'ü çarptık. Ne yaptık? Birinci katmandaki birim küp sayısını bularak aslında prizmanın taban alanını bulduk. Sonra 3 ile çarparak, taban alanını yükseklikle çarptık. Çünkü prizmanın yüksekliği 3.*

Using the prism formed by unit cubes, she aimed to help the students comprehend the logic behind the multiplication of the lengths of three edges.

In this type of instruction, teachers implemented activities to attract students' attention and the students were the passive listeners during the activity. For instance, Mrs. Akay conducted a small activity using manipulative to make the students realize the relationship between the volume of prism and pyramid. She utilized one hollow square prism and one hollow square pyramid, which had the same length of the edges of the base and the same height. She filled the pyramid with water and afterwards poured this water from the pyramid to the prism. She repeated this until the prism was filled with water. At the end of the activity, Mrs. Akay explained that to fill the prism with water, they should pour water three times. In other words, she explained that the volume of prism equals to three times of the volume of pyramid. Similar activities were performed in other teachers' lessons. As it could be seen, the teachers used the manipulative and performed the activity on their own. In other words, these activities were teacher demonstrations and not student investigations. To sum up, all teachers' instruction was mostly dependent on the transfer of knowledge from teacher to learners.

4.2.1.2 Less Teacher-Centered Enriched with Class Discussion

The analysis of the data showed that less teacher-centered enriched with class discussion was another topic-specific instructional strategy implemented by two teachers (Mrs. Kaya and Mrs. Akay) to teach the volume of 3D solids. The basis of this strategy is that teacher is not the only source of the knowledge. Teachers shared the responsibility of explaining the topic with their students; thus, there was a good amount of dialog between the students and the teacher. Thus, questioning and discussions were integrated into teaching process. Also, teachers had less control on the learning process than they had in the teacher-centered instruction.

Contrary to the use of teacher-centered instruction, only two teachers (Mrs. Kaya and Mrs. Akay) applied less teacher-centered enriched with class discussion to teach the volume of 3D solids. Both teachers required students make presentations related to the topic before they taught it. In this manner, during our conversations, Mrs. Kaya explained:

> At the beginning of the semester, I divided the students into groups. Each group determined the topic that they wanted to present. Then, I explained my criteria for them to get high points for their presentations.
>
> *Dönemin başında öğrencileri gruplara ayırdım. Her grup anlatmak istediği konuya karar verdi. Daha sonra, yüksek puan almaları için sunumlarından neler beklediğimi anlattım.*

When I asked what her requirements were, she replied:

> I wanted them to explain topic using representations, daily-life examples, and manipulative. The important issue for me was to discuss the topic with the class.
> G*österimler, günlük hayat örnekleri, ve materyal kullanarak konuyu anlatmaları istedim. Benim için en önemli noktanın konuyu sınıfla tartışmaları olduğunu vurguladım.*

While collecting the data, I observed Mrs. Kaya's students' presentation regarding the volume of 3D solids. As Mrs. Kaya's requirements, students tried to create discussion environment by questioning their friends. For instance,

> ***Presenter-1 (Pr-1):*** *How can we know how much water that this box [showing the rectangular prism] can contain?*
> ***Student-1 (Std-1):*** *We could calculate the volume of the box [showing the rectangular prism].*
> ***Pr-1:*** *Yes, that is correct. But how?*
> ***Std-2:*** *We can calculate the area of the base of the box. Then we multiply it by the height of the box.*
> ***Pr-1:****Yes, you are right. To calculate the amount of water in the box [the rectangular prism], we can multiply the base area of the box by its height.*

Afterwards, the students wrote the formula on the board. As Mrs. Kaya requested, the presenters emphasized daily-use of the volume by asking the question as how much water this box takes.

Moreover, Mrs. Akay applied a less teacher-centered enriched with class discussion as a topic-specific strategy to teach the calculation of the volume of 3D solids. Part of her instructional strategy is presented below.

> ***Mrs. Akay:*** *I want you to think about the relationship between the volume of cone and cylinder. In the next lesson, we will discuss this question. Each student should think about the question.*
> **In the next lesson**
> ***Mrs. Akay:*** *I asked a question yesterday. What do you think is the relationship between the volume of a cone and a cylinder.*
> ***Std-1:*** *The volume of the cone can be calculated by dividing the volume of cylinder by 3.*
> ***Mrs. Akay:*** *Why?*
> ***Std-2:*** *We learnt this in the private course.*
> ***Mrs. Akay:*** *Okay, so you know how to calculate the volume of cone. But how can you explain the reason for dividing the volume of cylinder by 3?*
> ***Std-3:*** *Teacher, we can do the same thing as you did when teaching the volume of the pyramid. We can fill water in the cone and pour it into a cylinder then repeat until the cylinder fills with water. We can count how many times we pour water from the cone into cylinder.*
> ***Mrs. Akay:*** *Good, as your classmate said, we did the same thing when learning about the volume of the pyramid. The relationship between the volume of a prism and a pyramid is the same as the relationship between the volume of a cone and a cylinder. In other words, we can say that while calculating the volume of cone, we divide the volume of cylinder by 3. Is that correct? Can we explain like this?*
> ***Stds:*** *Yes.*
> ***Mrs. Akay:*** *Is this explanation sufficient?*
> ***Stds:*** *Yes.*
> ***Std-2:*** *Teacher, we explained the volume of pyramid in the same way.*
> ***Mrs. Akay:*** *Now I am asking whether this explanation is sufficient. Or does it need some additional information?*
> ***Std-4:*** *Hımm…The base and height of them should be the same.*
> ***Mrs. Akay:*** *Good. This is important. Now, what can you say related to calculating the volume of a cone?*
> ***Std-5:*** *The division of "the multiplication of the area of the base with height" by 3.*
> ***Mrs. Akay:*** *Yes, you are correct. Now, I will write the formula on the board. At the same time, you can copy it into your notebooks. Then we will solve some problems.*

Regarding the less teacher-centered enriched with class discussion instruction, this can be seen in the excerpts from the transcripts of Mrs. Kaya and Mrs. Akay's lessons. Both teachers shared the responsibility of explaining the topic with the students. The students were more active during the learning process. By questioning the whole class and encouraging them to make a presentation related to the volume of 3D solids, Mrs. Kaya and Mrs. Akay applied less teacher-centered enriched with class discussion instruction. The next section presents the teachers' knowledge of learners.

4.2.2 Middle School Teachers' Knowledge of Learners

The third dimension of middle school teachers' PCK was the knowledge of learners. As a result of the analysis of the data, the teachers' knowledge of learners in relation to the calculation of the volume of 3D solids were identified under four areas, namely; the students' preferences among solution methods, the interpretations of students' alternative solution methods, students' errors and the sources of these errors, and the strategies to overcome the errors in the volume of 3D solids.

4.2.2.1 Middle School Teachers' Knowledge of the Students' Preferences among Solution Methods

In order to understand the teachers' knowledge of the solution methods that the students prefer in order to solve the questions regarding the volume of 3D solids, the data gathered from questionnaire, interview and classroom observation were analyzed. According to this analysis, the teachers gave different solution methods that students might prefer to solve the questions related to the volume of 3D solids. Table 4.2 gives a summary of the teachers' knowledge of students' preferences for different solution methods that they would use to calculate the volume of 3D solids. Table 4.2 was given below.

Table 4.2 Students' preferences on different solution methods

	Mrs. Kaya	**Mrs. Akay**	**Mr. Esen**	**Mrs. Uzun**
Strategies	• Volume formula	• Systematic counting • Volume formula	• Systematic counting • Volume formula	• Layer counting • Volume formula

As it was seen in the Table 4.2, four teachers considered that their students would use the *volume formula*. However, one teacher (Mrs. Kaya) only gave a single suggestion. Additionally, 2 teachers also chose *systematic counting* and another teacher suggested that the students would use *layer counting*. Mrs. Kaya said that her students always used volume formula to calculate the volume of 3D solids and she explained this as follows:

> Students focus on reaching the correct solution via the shortest way. They find using formula very practical. They prefer to do calculations by memorizing the formula.
>
> *Öğrenciler kısa yoldan doğru sonuca ulaşmaya odaklanıyorlar. Formül kullanmayı çok pratik buluyorlar. Formülü ezberleyip işlem yapmayı tercih ediyorlar.*

This is supported by Mrs. Kaya's response to question 2 in the VDSQ:

> Students could eliminate the outer faces of the prism one by one. To find the remainder of the unit cubes, they could subtract the length of the edges and then use volume formula. They could reach the answer by counting the unit cubes one by one but they prefer to solve with the formula.
>
> *Öğrenciler dış yüzeyleri çıkarabilirler. Kalan küpleri bulmak için, kenar sayılarını eksiltip hacim formülünü kullanabilirler. Küpleri tek tek sayarak da sonuca ulaşırlar fakat formülle çözmeyi tercih ederler.*

Mrs. Akay and Mr. Esen responded that their students could use systematic counting method. That is, they can count the number of unit cubes one by one to calculate the volume of prism. Mrs. Akay commented as follows:

> Students do not like using the formula. They prefer to do by counting if the solid is comprised of the unit cubes. It is difficult even for 8^{th} grade students to use the formula, so they count the unit cubes.
>
> *Formül kullanmayı sevmiyorlar. Eğer cisim birim küplerden oluşmuşsa sayarak yapmayı tercih ederler. Yani 8. Sınıf öğrencisine bile formülü kullanmak zor geldiği için birim küpleri sayarlar.*

Mrs. Uzun considered that her students might use the layer counting method to calculate the volume of the prism given in Figure 4.14 and said:

> Students mostly find one layer by multiplying 4 by 4. Then they find the 2^{nd} layer and 3^{rd} layer. Thus the students mostly use the layer counting method if the figure is given visually.

Öğrenciler daha çok 4 ile 4 ü çarparak bir katmanı bulurlar. Sonra, 2. katmanı, 3. katmanı bulurlar. Yani görsel olarak şekil verildiyse daha çok katman hesabını kullanırlar.

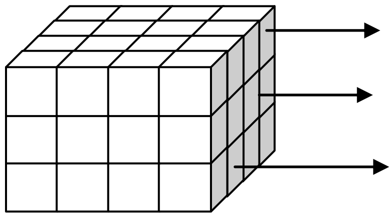


Figure 4.14 Mrs. Uzun's presentation of using the solution method: Layer counting

In conclusion, according to participating teachers, most of the elementary students prefer to use volume formula method to calculate the volume of 3D solids. The data gathered from classroom observations supported this finding since the participating teachers and their students mostly used the volume formula while calculating the volume of 3D solids in their mathematics lessons.

4.2.2.2 Middle School Teachers' Interpretations of Students' Alternative Solution Methods

In order to examine middle school teachers' knowledge on students' alternative solution methods, the data gathered from the VDSQ and interviews were analyzed.

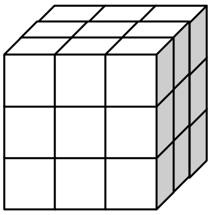
In VDSQ, the teachers were presented with students' alternative solution methods, asked to interpret these solutions and asked to give the reasons for their interpretations. The four middle school teachers' interpretations of students' solution methods displayed diversity in terms of the correct or incorrect interpretations as presented in the next sub section.

4.2.2.2.1. Middle School Teachers' Correct Interpretations of Students' Correct Solution Methods

According to the middle schoolteachers, the elementary students solved the given questions correctly using three different solution methods: volume formula, layer counting and systematic counting. The teachers stated that the students used these methods for solving different questions. In the following subsection, the teachers' interpretations were presented in terms of the method that students used.

4.2.2.2.1.1 Volume Formula

The analysis of the data revealed that the middle school teachers were able to interpret students' solution methods easily if students solved the question with volume formula. Question 4 (Figure 4.15) is given as an example.



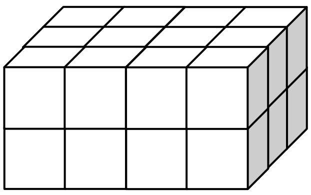
Mrs. Aksoy asked the volume of the cube and her students gave the answer 27. Mrs. Aksoy realized that her students solved the question using different solution methods. Although some solution methods were correct, some of them were incorrect.

a) What solution methods were used by the students who solved the problem correctly? Please explain.

Figure 4.15 Question 4

All the middle school teachers stated that students found the volume of prism presented in question 4 as 27 using the volume formula method. For instance, Mrs. Uzun explained that "the base area of the prism is 9 (3x3) and the height of the prism is 3. So, the volume is 27 (9x3)."

Additionally, Mrs. Kaya, Mrs. Akay and Mr. Esen agreed that Kuzey used the volume formula method to calculate the volume of prism in Question 5 (Figure 4.16).



Students; Ela, Eren, Kuzey, Yagmur and Berke calculated the volume of prism, presented above, in different ways but they found the same result. Their solutions were given below:

Ela's Solution: 26 x 2= 52 8 x 2= 16 52- 16= 36 36- 12= 24	***Eren's Solution:*** 6 + 6= 12 4 + 4= 8 12 + 8 + 4= 24
Kuzey's Solution: 4 x 3= 12 12 x 2= 24	***Yağmur's solution:*** 6 + 6= 12 4 + 4 + 4 = 12 12 + 12= 24
Berke's Solution: 4x 3 x 2= 24	

b)Explain students' solution methods in your own words.

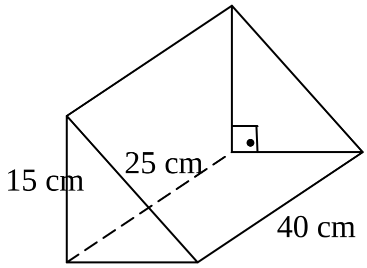
Figure 4.16 Question 5

Regarding Kuzey's solution method, Mrs. Kaya explained her response as follows.

> Kuzey used the formula which was the multiplication of the base area of the prism and the height of the prism. He found the base area and then he multiplied it with the height. In other words, he found the volume.
>
> *Kuzey, taban alanı x yükseklik formülünü kullanmış. Taban alanını bulmuş ve yükseklikle çarpmış.Yani hacmi bulmuş.*

Apart from Kuzey, four teachers interpreted Berke's solution in question 5 as multiplying the length of three dimensions [width, height and depth] of the prism. In other words, according to the teachers, Berke used volume formula.

Furthermore, the middle school teachers were able to explain students' solution method easily if students had solved the question 6 using the volume formula (Figure 4.17)



A piece of cheese was cut into a right triangular prism on the left side. The cheese was cut into 20 equal slices, what is the volume of each slice?

Mr. Acar asks the class the question given above and he encounters different solution methods.

Yankı's Solution:

$a^2 = b^2 + c^2$

$25^2 = 15^2 + c^2$

$625 = 225 + c^2$

$400 = c^2 \Rightarrow c = 20$

$V = \frac{15.20}{2}.40$

$V = 6000$

The volume of one slice

$\frac{6000}{20} = 300$

Asya's Solution:

$V = \frac{15.25}{2}.40$

$V = 7500$

The volume of one slice

$\frac{7500}{20} = 375$

Yaman's Solution:

$a^2 = b^2 + c^2$

$25^2 = 15^2 + c^2$

$625 = 225 + c^2$

$400 = c^2 \Rightarrow c = 20$

$\frac{40}{20} = 2$

$V = \frac{15.20}{2}.2$

$V = 300$

Ada's Solution:

$a^2 = b^2 + c^2$

$25^2 = 15^2 + c^2$

$625 = 225 + c^2$

$400 = c^2 \Rightarrow c = 20$

$V = \frac{40.20.15}{2}$

$V = 6000$

The volume of one slice

$\frac{6000}{20} = 300$

Ilgaz's Solution:

$a^2 = b^2 + c^2$

$25^2 = 15^2 + c^2$

$625 = 225 + c^2$

$400 = c^2 \Rightarrow c = 20$

$\frac{15.20}{2} = \frac{25.x}{2} \Rightarrow x = 12$

$V = \frac{40.25.12}{2} \Rightarrow V = 6000$

The volume of one slice

$\frac{6000}{20} = 300$

a) In your opinion what process do Mr. Acar's students consider when giving their answer.

Figure 4.17 Question 6

In this question, the middle school teachers explained that Yankı, Yaman, and Ada calculated the volume of prism using volume formula. Mrs. Kaya explained Yankı's solution as follows:

> He found the right edge (c) from the Pythagorean Theorem. Then he found the volume of the figure using the volume formula. So, he multiplied the area of the base by the height and since there will be 20 slices, he divided the result by 20 and found the volume of one slice.
>
> *Önce Pisagordan dik kenarı (c) bulmuş. Sonra hacim formülünü kullanarak cismin hacmini bulmuş. Yani, taban alanı ile yüksekliği çarpmış. 20 tane dilim olacağı için 20 dilime bölmüş ve bir tanesinin hacmini bulmuş.*

Examples of teachers' interpretations of Yaman and Ada's solution were given in below. For the former student, Mr. Esen's explanation is:

> Yaman calculated the right edge (c). Then, because of it was necessary to divide the cheese into 20 equal slices, he determined the height of the figure to be 2. Afterwards, he did by considering the volume of figure. The base area x the height.
>
> *Yaman dik kenarı (c) hesaplamış. Sonra 20 eşit dilime bölündüğü için cismin yüksekliğini 2 bulmuş. Daha sonra bir cismin hacmini düşünerek yapmış. Taban alanı x yükseklik.*

For the second student, Mrs. Akay explained:

> Ada's solution is correct, the volume. Taking the base area x the height, she made the calculations and found the result. Then she divided the result by 20 and found the volume of one slice. Correct.
>
> *Ada'nın çözümü doğru, hacim. Taban alanı x yükseklik olduğu için işlemleri yapmış ve sonucu bulmuş. Ondan sonra 20 ye bölmüş, 1 dilimin hacmini bulmuş. Doğru.*

Although Ilgaz solved the question using volume formula, two of the teachers had difficulty in understanding her slightly complicated solution. This student did not multiply the right edges of the triangle, which was the base of the prism, to find its area. Instead, she calculated the area of the triangle multiplying the length of the hypothenuse by the length of the height of it. Only Mrs. Kaya and Mrs. Akay were able explain Ilgaz's solution. Mrs. Kaya stated that:

> Ilgaz calculated the right edge (c) as 20. Then she determined the height of a triangle, using the equivalence of areas of the triangle, as 12. Afterwards, she calculated the volume of the figure from the multiplication of the base area by 40. Lastly, she divided the result by 20.

Ilgaz önce dik kenarı (c) bulmuş, 20. Daha sonra üçgenin yüksekliğini belirlemiş. O yüksekliği üçgenin alanın birbirine eşitliğinden 12 olarak bulmuş. Ondan sonra cismin hacmini taban alanı çarpı 40 dan bulmuş. En son 20 dilime bölmüş.

Initially, Mrs. Akay had difficulty in explaining Ilgaz's solution since this teacher could not comprehend the meaning of "x" in the solution easily. After understanding what "x" was, she clarified the other operations giving a similar explanation to that presented by Mrs Kaya.

As in the previous examples, all middle school teachers were able to interpret students' solutions if they solved the question using the volume formula. Additional example was related to question 7 (see Figure 4.18)

The base length of the square prism model is 6 cm and the length of side-face height is 5 cm. Ceren and Cemre who calculated the volume of this model solved the question in different ways.

Ceren's Solution:

$$V= \frac{6.\ 6.\ 5}{3}$$

$$V= \frac{180}{3} = 60\,cm^3$$

Cemre's Solution:

$a^2 = b^2 + c^2$

$5^2 = 3^2 + c^2$

$25 = 9 + c^2$

$16 = c^2$

$c= 4$

$$V= \frac{6.\ 6.\ 4}{3}$$

$$V= \frac{144}{3} = 48\,cm^3$$

a) According to you, what were Ceren and Cemre thinking when they developed these methods of solving the problem?

Figure 4.18 Question 7

All four teachers interpreted Cemre's solution in a similar way. For example, Mrs. Akay explained the process as follows:

Cemre found c [the height of the pyramid] to be 4 using the Pythagorean Theorem. Then she used the volume formula; multiplying of the base area by the height, and dividing it by 3. It was correct.

Cemre, Pisagor'dan c'nin [piramitin cisim yüksekliği] 4 olduğunu bulmuş. Ondan sonra hacim formülünü, (TA x yükseklik) / 3'ü kullanmış. Bununki doğru.

As shown in the examples, all the teachers interpreted students' solution method as volume formula since they thought that most students prefer to use this method to calculate the volume of 3D solids. Moreover, with respect to the data gathered from classroom observation, it was seen that the middle school teachers generally used volume formula when demonstrating the calculation of the volume of 3D solids to their students. However, some of the teachers were able to correctly interpret students' solutions by the different methods, given in the following sub-sections.

4.2.2.2.1.2 Layer Counting

Only one of the teachers explained that students might use the layer counting method to calculate the volume of 3D solids correctly. In question 4 (Figure 4.15), Mrs. Uzun proposed that:

> *Students might count the number of unit cubes on the bottom layer. There are 9 cubes. Then they might think that there are 27 cubes on the three layers.*
>
> *Öğrenciler alt sıradaki birim küpleri sayabilirler. 9 küp var. Sonra, 3 sırada 27 küp olacağını düşünmüş olabilirler.*

Furthermore, Mrs. Uzun stated that Kuzey's solution in question 5 (Figure 4.16) method was layer counting as shown in her explanation given below.

> I thought that Kuzey's solution was correct. Namely, he found the top layer here. There was one more on the below. He multiplied this by 2. This is first layer; this is second layer (Figure 4.19).
>
> *Kuzey'in çözümünü doğru buluyorum. Yani şurada üst yüzünü bulmuş. Bundan bir tane daha altta var. 2 ile çarpmış. Bu birinci sıra, bu da ikinci sıra.*

Kuzey's Solution:
4 x 3= 12
12 x 2= 24
The figure presents the operation 4 x 3= 12

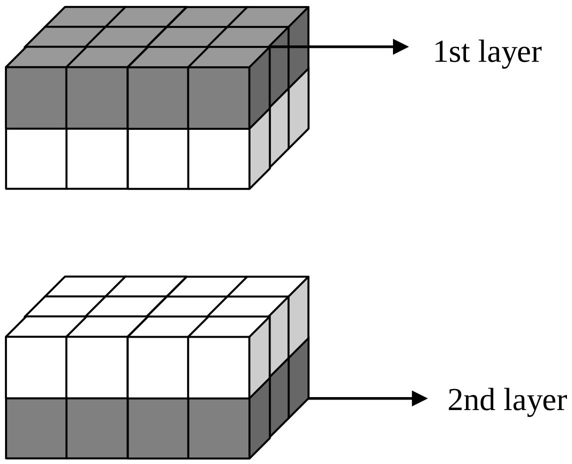


Figure 4.19 Mrs. Uzun's interpretation of Kuzey's solution

Although Mrs. Kaya used layer counting method while she was calculating the volume of 3D solids (Table 4.1), she did not interpret students' solution methods in terms of using layer counting method. Furthermore, neither Mrs. Akay nor Mr. Esen explained that students might use layer counting method and they did not use this method themselves when they were calculating the volume of 3D solids (Table 4.1).

4.2.2.2.1.3 Systematic Counting

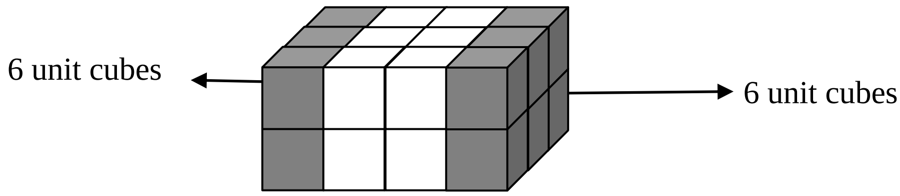
Apart from volume formula and layer counting methods, the teachers suggested that students might use the systematic counting method to calculate the volume of 3D solids. Three teachers (Mrs. Kaya, Mr. Esen and Mrs. Uzun) interpreted the students' solution method in question 4 (Figure 4.15) as a systematic counting method. As an example, the vignette of Mr. Esen is given below:

> The students who were able to solve the question correctly can see everywhere of the cube. S/he was able to concretize [the cube]. S/he can count the unit cubes here.
>
> *Doğru çözen öğrenciler bu verilen küpün her tarafını görebilir. [Küpü] Somutlaştırabilmiştir. Buradaki birim küpleri saymış olabilir.*

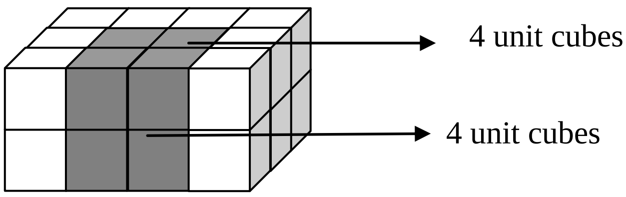
Similar to the responses to question 4, the middle school teachers explained that students might solve question 5 (Figure 4.16) using a systematic counting method. For example, both Mrs. Kaya and Mr. Esen interpreted Eren's solution as systematic counting method and their interpretations are given below.

> Mrs. Kaya: Eren counted cubes in his own way. However, the method was correct. He counted the unit cubes correctly. In his opinion, that 6 cubes, he might have taken 6 + 6 (the side-faces) (Figure 4.20). Then, he might have taken 4 + 4 and he might have added 4 at the end [of the operation].
>
> *Küpleri kendine göre saymış. Ama yöntem doğru. Küpleri doğru saymış. Kendince şu 6 tanesini, 6+6 almış alabilir. Sonra 4+4 almış olabilir, [işlemin] sonuna bir 4 eklemiş olabilir.*

Eren's Solution:
6 + 6= 12
4 + 4= 8
12 + 8 + 4= 24
The figure presents the operation 6+6 = 12



It presents the operation 4+4 = 8



The last 4 unit cubes in the operation 12 + 8 + 4 = 24

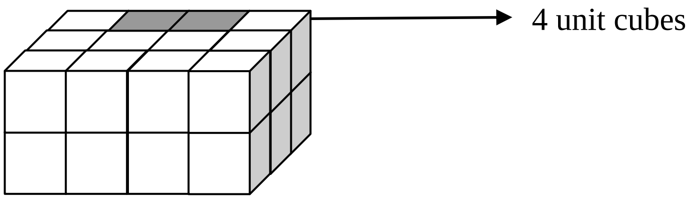


Figure 4.20 Mrs. Kaya's interpretation of Eren's solution

Mr. Esen interpreted Eren's solution in the following different way.

> 6 + 6= 12, he did that correctly. 4. He found there and there (indicating the two side-faces) (Figure 4.21), he gave the answer as 12. After that, he said 4, 4 (indicating the the front and back faces) giving 8. After that, he said that 4 remained between them. He added 4 and he found the result to be 24.
>
> *6+6= 12, doğru yapmış. Şurayla şurayı bulmuş (yan yüzler), 12 demiş. Ondan sonra şurasını 4,4 demiş (ön ve arka yüzler), 8. Ondan sonra arada 4 kalıyor demiş. 4 ü de toplamış. 24 ü bulmuş.*

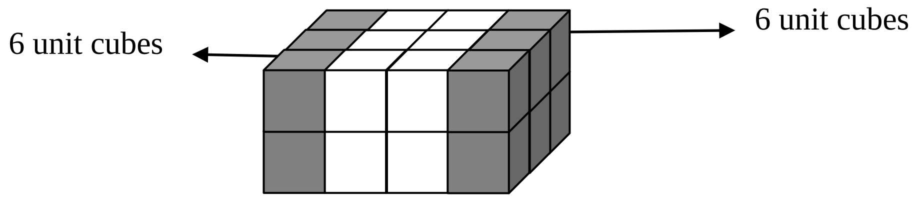
Eren's Solution:

6 + 6= 12

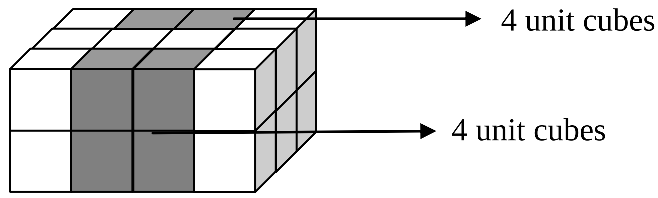
4 + 4= 8

12 + 8 + 4= 24

The figure presents the operation 6+6 = 12



It presents the operation 4+4 = 8



The last 4 unit cubes in the operation 12 + 8 + 4 = 24

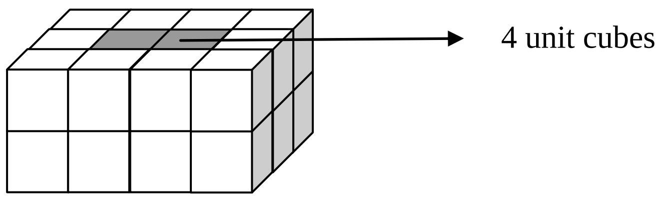


Figure 4.21 Mr. Esen's interpretation of Eren's solution

In addition, Mrs. Kaya and Mr. Esen interpreted that Yağmur's solution, presented in question 5 (Figure 4.16), was systematic counting method. Mrs. Kaya's interpretation of Yağmur's solution is as follows.

> Yağmur counted the cubes, she counted 6, 6 here (indicating the cubes). Then she counted 4, 4, 4 (Figure 4.22). She added all of them together . This method could be used. It is more regular method.
>
> *Yağmur küpleri saymış. Burada 6, 6 saymış. Sonra 4, 4, 4 saymış. Hepsini toplamış. Bu da tercih edilebilir. Daha düzenli bir yöntem.*

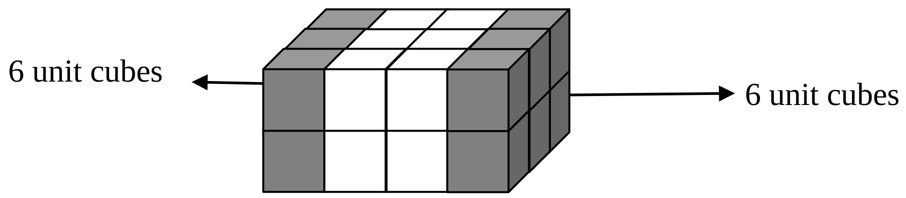
Yağmur's solution:

6 + 6= 12

4 + 4 + 4 = 12

12 + 12= 24

The figure presents the operation 6 + 6= 12



The figure presents the operation 4 + 4 + 4= 12

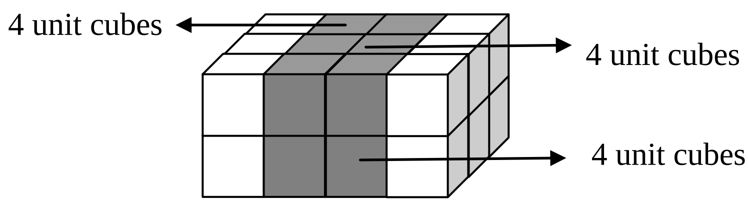


Figure 4.22 Mr. Kaya's interpretation of Yağmur's solution

The data analysis showed that three teachers (Mrs. Kaya, Mr. Esen and Mrs. Uzun) explained that students might calculate the volume of 3D solids using the systematic counting method.

4.2.2.2.2 Middle School Teachers' Incorrect or Missing Interpretations of Students' Correct Solution Methods

The middle school teachers were able to explain some of the correct solutions; however, they were unable to interpret the students' correct solution methods for other questions. For example, none of the teachers could understand Ela's solution for question 5 (Figure 4.16). For instance, Mrs. Uzun stated that:

> Now, I did not understand Ela's solution. Where did she get this 26? I did not understand that, namely 26. After that 8 x 2, she found 52 from there, subtracted from here, reached the correct result but I did not understand where she got that 26. Where did 26 come from?
>
> *Ben şimdi bu Ela'nın çözüm yolunu hiç anlamadım. Bu 26 yı nereden bulmuş. Şunu anlayamadım yani 26 yı. Ondan sonrada 8 x 2, şuradan 52 yi bulmuş, buradan da çıkarmış, doğru sonuca ulaşmış ama şu 26 yı nereden bulduğunu anlayamadım. Şu 26 nereden gelmiş.*

In addition, Mrs. Akay's commented:

> The child (Ela) counted the cubes according to her, added, found the difference. The result of 4 x 3 x 2, is 24 isn't it? If this was the correct solution, now, here s 4 x 3= 12 (Figure 4.23). If she found the result to be 24, it was completely wrong for me. I taught the volume, I said that the volume is multiplication of the area of the base and the height. By putting the units in appropriate places, by writing necessary formula and then by indicating operation and result, finally she must explain the operation and the unit.
>
> *Bu çocuk (Ela) buradaki küpleri kendine göre saymış, toplamış, aralarındaki farkını almış. 4 x 3 x 2, sonuç 24 mü çıkıyor? Bu doğru çözüm olsa bile; şimdi burası 4 x 3= 12. Eğer sonucunu 24 bulmuşsa, benim için tamamen yanlıştır. Ben hacmi öğretmişim, hacmin taban alanı x yükseklik olduğunu söylemişimdir. Burada verilen birimleri uygun yerine koyarak, formülüyle birlikte yazıp, ondan sonra işlem ve sonuç deyip, sonuçta işlem ve birimini açıklaması gerekir.*

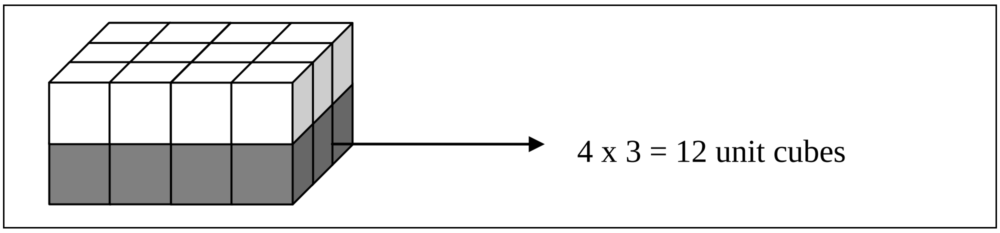


Figure 4.23 Mrs. Akay's interpretation of Ela's solution

Similar to Mrs. Akay and Mrs. Uzun, the other two teachers (Mrs. Kaya and Mr. Esen) could not understand how Ela found 26. Because they did not understand how the number, 26, was achieved, they did not analyze the other operations. In addition, the middle school teachers did not understand some of the correct solution methods that the students' gave for question 6 (Figure 4.17). For instance, Mrs. Uzun could not explain Yaman's solution in question 6, as given below:

> Yaman found the right edge (c). But he divided 40 by 2, here. I could not understand what he wanted to do here. I am confused.
>
> *Yaman dik kenarı (c) bulmuş. Ama burada 40'ı 2'ye bölmüş. Burada ne yapmak istediğini anlayamadım. Burada bir karmaşa yaşadım.*

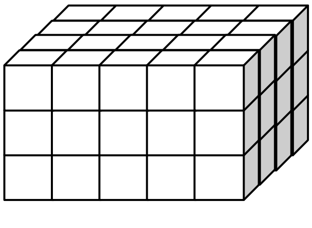
Also Mr. Esen and Mrs. Uzun were not able to explain how Ilgaz calculated the volume of triangular prism in question 6.

It can be seen that the middle school teachers could interpret students' correct solution methods if they were familiar with them. However, middle school teachers were not successful in explaining some solution methods that the students used even if they were correct. That is, in order to comprehend how students think while solving the questions, it was essential that middle school teachers know and use the solution methods.

In the next section, the middle school teachers' interpretations of the students' incorrect solution methods were presented.

4.2.2.2.3 Middle School Teachers' Correct Interpretations of Students' Incorrect Solution Methods

The middle school teachers were able to explain some of the students' incorrect solutions regarding the volume of 3D solids. For instance, regarding question 3 (Figure 4.24) both Mrs. Kaya and Mrs. Uzun stated that students could calculate the surface area of the prism instead of calculating its volume.



Most of the students in Mr. Aslan's class made the same error in the question "Find the volume of rectangular prism" They gave the answer 94.

b) What method(s) do Mr. Aslan's students use to answer this question?

Figure 4.24 Question 3

Mrs. Kaya's vignette related to interpretation of students' incorrect solution in question 3 is presented below.

> Here, the students could calculate the [surface] area [of the prism]. There were 15 unit cubes on the front face, 12 unit cubes on the right face and 20 unit cubes on the top face (Figure 4.25). There are 47 unit cubes on the three faces and 94 unit cubes in total.
>
> *Burada öğrenciler [prizmanın yüzey] alan hesaplamış olabilirler. 15 küp ön yüzde, 12 küp sağ tarafta, 20 küp üst yüzde var. 3 yüzde toplam 47 küp var, toplamda da 94 küp var.*

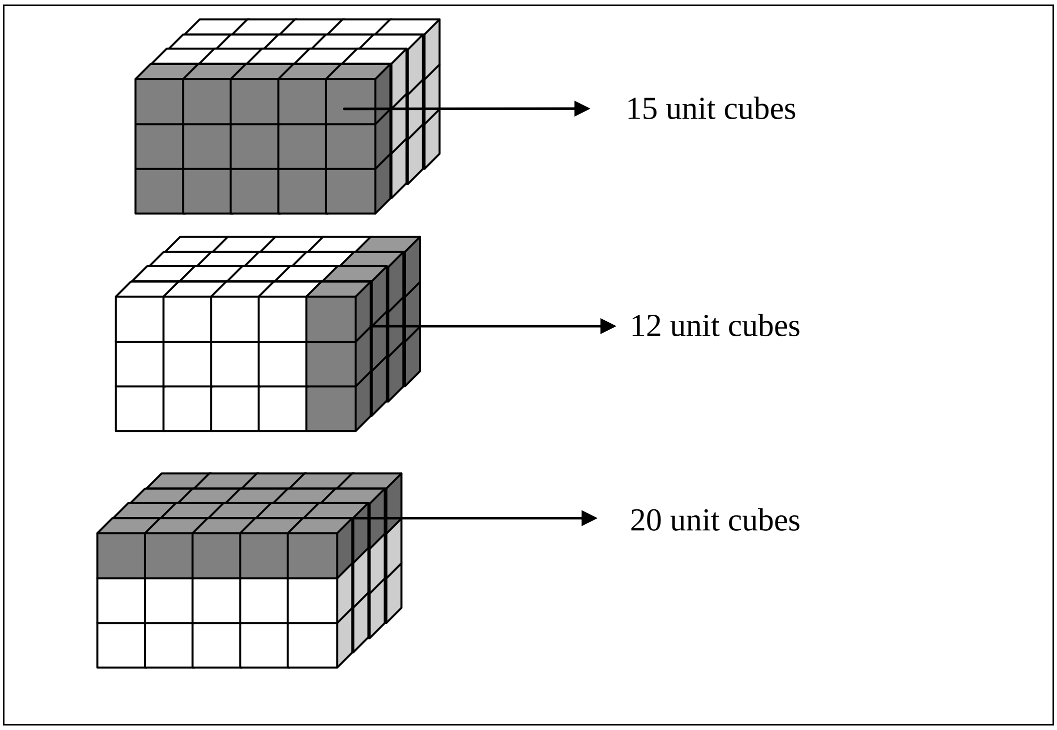


Figure 4.25 Mrs. Kaya's interpretation in question 3

Additional example related to interpretation of the students' incorrect solution in question 3 was stated by Mrs. Uzun.

> They can do it like this. 4 x 3, 4 x 3, 3 x 5, 3 x 5, 4 x 5, 4 x 5 (Figure 4.26) and they can add them up. They may make an error in this way. What is the result of this operation? Wait a minute. 12 + 12 + 15 + 15 + 20 + 20 = 94. That is correct. So students found the surface area.
>
> *Şöyle yapmış olabilirler. 4 x 3, 4 x 3, 3 x 5, 3 x 5, 4 x 5, 4 x 5 ve hepsini toplamış olabilirler. Bu şekilde hata yapmış olabilirler. Yani yüzey alanını bulmuşlardır.*

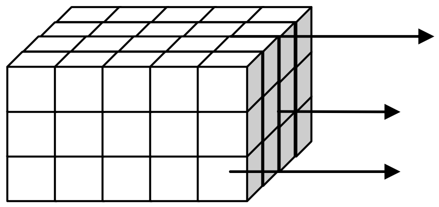


Figure 4.26 Mrs. Uzun's interpretation in question 3

As shown above Mrs. Akay and Mr. Esen could not interpret the students' incorrect solution method in question 3. Furthermore, three middle school teachers (Mrs. Akay, Mr. Esen and Mrs. Uzun) interpreted students' incorrect solution methods in question 4 (Figure 4.15). For instance, Mrs. Akay and Mrs. Uzun stated that students could count the number of unit cubes on the visible faces. According to these two teachers, if students used the incorrect solution method, they could find the answer to be 27. Additionally, Mr. Esen commented that students whose solution method was wrong might have calculated the length of visible edges and he explained it thus:

> Students might think about the length of the edges. The length of visible edges. Nine edges are visible and the length of the edge is 3. The total is 27. Students can solve in this way.
>
> *Kenar uzunluklarını düşünmüş olabilirler. Görünen kenar uzunluklarını. 9 tane kenar görünüyor ve her birinin uzunluğu 3. Toplamda 27. Öğrenciler böyle de çözmüş olabilirler.*

Additional example related to teachers' interpretations of students' incorrect solution methods concerned question 6 (Figure 4.17). According to Mrs. Kaya, Mrs. Akay and Mr. Esen, Asya's solution was incorrect. They all explained it in a similar way. To exemplify, Mr. Esen's interpretation is presented below:

> She said 15 x 25, she multiplied the right edge by the hypotenuse. She multiplied the right edges of the triangle by the hypotenuse to find the area of the triangle. She did not know how the area of triangle can be found. She does not know [the area of the triangle is] half of the multiplication of the right edges. She did it wrong here. I think that Asya's solution was wrong.
>
> *15x25 demiş, dik kenar ile hipotenüsü çarpmış. Üçgenin alanını bulurken hipotenüs ile üçgenin dik kenarını çarpmış. Üçgenin alanının nasıl bulunacağını bilmiyor. [Üçgenin alanının]Dik kenarların*

> *yarısının çarpımı olduğunu bilmiyor. Burada yanlış yapmış. Asya'nın çözümü yanlış gibi görüyorum.*

Additionally, four of the teachers analyzed and interpreted the students' incorrect solution method for question 7 (Figure 4.18). They determined that Ceren's solution was incorrect. Mrs. Uzun's vignette is given as an example.

> Now, the edge is 6, she found the area here. She took the height of the side-face [of the pyramid]. However, she should take the height of the figure instead of the height of the side-face. She knows the volume formula correctly but she confused the height. For this reason, her solution was wrong.
>
> *Şimdi kenarı 6, şurada alanını bulmuş. [Piramitin] Yan yüz yüksekliğini almış. Oysa yan yüz yüksekliği yerine cisim yüksekliğini alması gerekiyordu. Hacim formülünü doğru biliyor fakat yüksekliği karıştırmış. Bu nedenle, yanlış çözmüş.*

In conclusion, middle school teachers were able to explain students' incorrect solution. Nevertheless, for some solution methods, they could not correctly interpret how students solve the question. Middle school teachers' incorrect interpretations are presented in the next sub-section.

4.2.2.2.4 Middle School Teachers' Incorrect or Missing Interpretations of Students' Incorrect Solution Methods

Data gathered from VDSQ and the interviews revealed that the middle school teachers were not able to explain some of students' incorrect solution methods. For instance, Mrs. Akay and Mr. Esen could not correctly interpret students' solution for question 3 (Figure 4.24). Initially, Mrs. Akay stated that students could calculate the number of unit cubes on the visible faces and multiply this number by 2. However, Mrs. Akay did not take the common unit cubes on adjacent faces into consideration while interpreting students' solution in question 3. As can be seen in her explanation as follows:

> What the volume of the figure, 3 x 4 x 5= 60? How did they find 94? They can count the squares. Sometimes, students carry out such operations. I applied all techniques that I know and I could not find the result. I wonder whether they multiplied the visible lengths. (Thinks…) They do like this. There are 5 here, 5,10,15,20,25,30,35 (she counts unit cubes on the front and top face). There are 12 cubes here [on the right

side] (Figure 4.27). So, there are 47 cubes in total [front, top and right side]. They found the number of cubes at the back side and add them up. Namely, they found the surface area.

Cismin hacmi neymiş, 3 x 5 x 4= 60. 94 ü nasıl bulmuşlar? Kareleri saymış olabilirler. Bazen çocuklar öyle bir işlem yapıyorlar. Bildiğim bütün teknikleri uyguluyorum ve ben o sonucu bulamıyorum. Acaba görünen uzunlukları mi çarpıyorlar (düşündü….) Şöyle yaparlar bunlar. Burada 5 tane var, 5,10,15,20,25,30,35 (ön ve üst yüzdekileri saydı). Burada 12 tane var (sağ yan yüz). Toplamda 47 tane küp var. Arka taraftaki yüzleri sayıp toplar. Yani yüzey alanını bulurlar. Bütün yüzleri toplayıp hacim diye söyleyebilir.

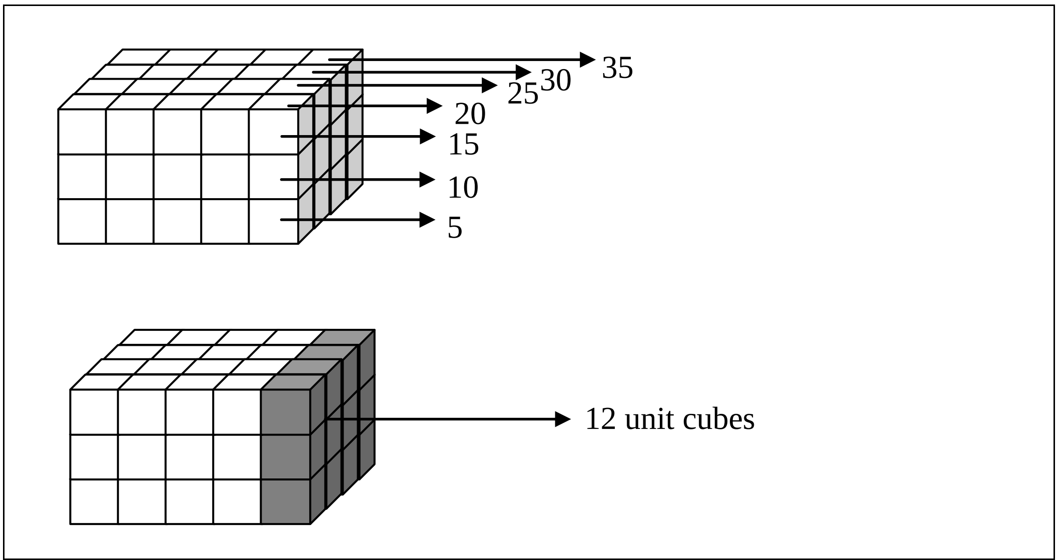


Figure 4.27 Mrs. Akay's interpretation of question 3

While thinking how students can find the volume of the prism in question 3 as 94, Mrs. Akay waited a long time. However, her explanation was not correct since she counted common unit cubes twice and three times. Mrs. Akay did not consider her solution in this respect. Her aim was to find 94 and her wrong solution method provided her to find 94 incidentally. Similar to Mrs. Akay, Mr. Esen had difficulty in explaining students' solution in which the students determined the volume to be 94 and could not justify students' solution in a correct way. Mr. Esen explanation exemplifies this as follows:

How did they find 94? It was difficult for me to know how they solve it. I could not see how they found 94. However, they could have counted the edges. Many students carried out operations from the edges to find the volume. They tried to find the perimeter. They could have done something like that. They may solve it using a visible line segment. They could multiply by 2 again. I do not know whether you counted

> how many there were. If there were 47, then they might find 94 by multiplying by 2.
>
> *Nasıl 94 bulmuşlardır ? Benim onların nasıl çözdüğünü bilmem zor. Nasıl 94 bulduklarını göremiyorum. Ama kenar sayabilirler. Bir sürü öğrenci hacmini bulurken kenarından işlem yapıyorlar. Çevre bulmaya çalışıyorlar. Böyle bir şey yapmış olabilirler. Görünen doğru parçalarının sayılarından gitmiş olabilirler. Tekrar 2 ile çarparak yapmış olabilirler. Bilmiyorum saydınız mı kaç tane var burada. 47 ise daha sonra 2 ile çarparak 94 bulmuş olabilirler.*

Only Mr. Esen stated that the students might add the length of edges but he did not apply his solution method to check whether the result of that solution was 94 and in fact the result calculation did not give 94.

Additionally, Mrs. Uzun was not able to explain Asya's solution in question 6 (Figure 4.17). She stated:

> Why did she multiply 15 and 25? I think that there is a mistake here. But I don't understand what she did.
>
> *Niye 15 ile 25'i çarpmış? Burada bir yanlışlık olduğunu düşünüyorum. Ama ne yaptığını anlayamadım.*

All the teachers attempted to explain the students' solution methods whether they were correct or not. However, the middle school teachers could not make any interpretations on some of students' solution methods. In fact, they did not make any interpretation related to the methods that they had not experienced before. In conclusion, the middle school teachers were able to clarify students' methods if they were familiar with the methods.

4.2.2.3 Middle School Teachers' Knowledge of the Students' Errors and the Sources of These Errors

In order to achieve a deeper understanding of students' errors and the sources of these errors related to calculating the volume of 3D solids, the middle school teachers' knowledge of the students' thinking process and their own experiences as learners were investigated systematically. The middle school teachers emphasized the various errors that the elementary students could make when they were calculating the volume of 3D solids. Here the use of the term 'error', I am referring to the students' (mis)construction of their own knowledge, (mis)understanding of the given terms, concepts, operations and difficulties encountered while solving the given problems. Thus, the analysis of the knowledge of the students' errors refers to the middle school teachers' perception of the mistakes that the students can make when they calculate the volume of 3D solids. In addition to the knowledge of the errors, possible sources of these errors are also discussed below.

Based on the analysis of the data obtained from the questionnaire, interviews, and classroom observations, the four middle school teachers' knowledge of students' errors related to the volume of 3D solids is grouped under four main dimensions namely; focusing on the faces of 3D solids, over-counting the common unit cubes on adjacent faces, conceptual errors, and computational errors. The possible sources of these errors are also discussed under these four dimensions. The dimensions are based on the available literature, participants' statements, and my own experiences with the data.

Although focusing on the faces of 3D solids and conceptual errors were emphasized by all the participating teachers, only 2 of them (Mrs. Kaya and Mrs. Akay) stressed that over-counting the common unit cubes on adjacent faces were among students' errors. Furthermore, computational errors were another error that three participants (Mrs. Kaya, Mrs. Akay and Mr. Esen) interpreted as one of the errors that the students make. Table 4.3 gives a summary the elementary students' errors given by the middle school teachers.

Table 4.3 The elementary students' errors that middle school teachers stated

	Mrs. Kaya	**Mrs. Akay**	**Mr. Esen**	**Mrs. Uzun**
Focusing on the faces of 3D Solids	**x**	**x**	**x**	**x**
Over-counting the common unit cubes on adjacent faces	**x**	**x**		
Conceptual errors	**x**	**x**	**x**	**x**
Computational errors		**x**		

4.2.2.3.1 Focusing on the Faces of 3D Solids

All the middle school teachers stated that focusing on faces of the 3D solid as one of the common students' errors related to calculating the volume of 3D solids. In light of teachers' statements and available literature, this error can be defined as considering all or a subset of the visible faces of 3D solid when calculating its volume. Thus, according to four middle school teachers' knowledge the elementary students who made this error might think that 3D solid is formed by its faces and do not concentrate on inside of the given solid. Based on the analysis, the teachers specified this error in relation to a rectangular prism (Questions 1, 2, 3, and 4). For instance, in relation to question 1, Mrs. Kaya said:

> What the students can do when calculating the volume of this prism? They can count faces that they can see. Namely, they consider this face [front], right face and top face (Figure 4.28).
>
> *Bu prizmanın hacmini hesaplarken öğrenciler ne yapabilir?Kendi gördüğü yüzeyleri sayabilirler. Yani, şu yüzeyi (ön), sağ yüzeyi ve sol yüzeyi düşünürler.*

Additionally, Mr. Esen described the same error for the rectangular prism, as follows:

> Students only counted the visible faces, in other words, they counted the 3 faces which are visible. They may not see the others. This often happens but many students do see all the faces (Figure 4.28).
>
> *Gördükleri dış yüzeyleri sayarlar yani görünen 3 yüzü sayarlar. Diğerini görmeyebilirler. Genelde böyle olur ama gören öğrencilerimiz çok.*

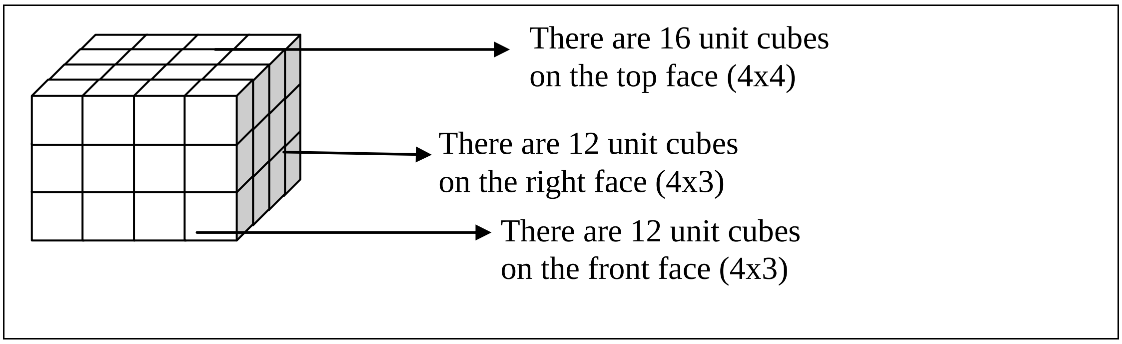


Figure 4.28 Mrs. Kaya and Mr. Esen's interpretations regarding students' error in question 1

Besides, Mrs. Akay and Mrs. Uzun identified focusing on the faces of a 3D solid as the error in question 4. The comments from these two teachers are presented below.

> Mrs. Akay: Students can count these [visible] faces (Figure 4.29). Namely, these 3 faces [by showing visible faces in the figure]. They did

not see backside of the prism. They can say 27 by counting these squares.

Öğrenciler şuradaki[görünen] yüzeyleri sayabilirler. Yani şu 3 yüzü [görünen yüzleri göstererek]. Arka tarafı görmüyor. Şu kareleri sayıp işte 27 tane diyebilirler.

In addition, Mrs. Uzun reflected that:

Students can count the outer unit cubes. In other words, they can count the unit cubes on the visible 3 faces (Figure 4.29).

Dış taraftaki küpleri sayabilirler. Yani görünen 3 yüzdeki küpleri sayabilirler.

In order to clarify the students' error in the question 4, the figure, interpreted by Mrs. Akay and Mrs. Uzun, is presented below:

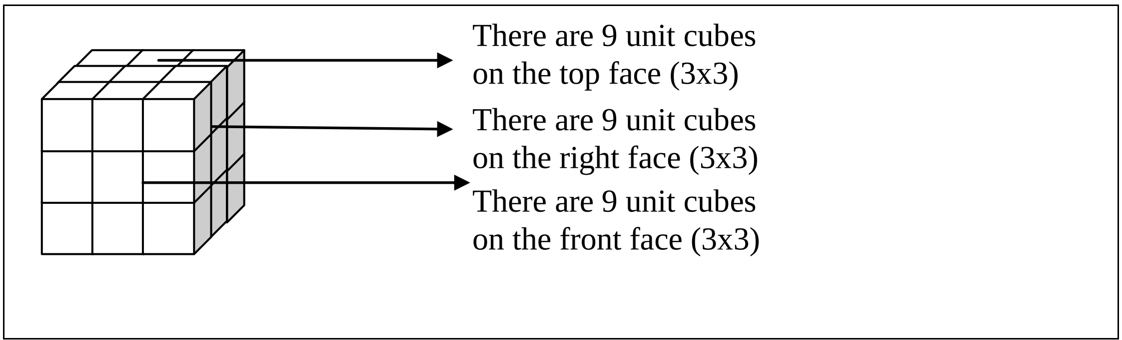


Figure 4.29 Mrs. Akay and Mrs. Uzun's interpretations regarding the students' error in question 4

In the given examples, all the teachers stated that students might focus on three visible faces of 3D solid. In other words, students might count the number of unit cubes on the visible faces. Moreover, Mrs. Uzun specified that students might focus on only one outer face or all outer faces of the 3D solid. Her idea is related to focusing on only one outer face is stated in the below vignette:

They looked at outer face directly. What they did immediately, they multiply 4 by 3 [length and height of the rectangular prism], and said that this was its volume.

Bunlar direk dış yüzüne bakıyorlar. Hemen ne yapıyorlar, 4 ile 3 ü çarpıyorlar [Dikdörtgenler prizmasının uzunluğu ve genişliği] ve hacmi budur diyorlar.

In the next example from Mrs. Uzun is related to focusing on all the outer faces of the 3D solid is presented below.

> Students can sum up the [number of] unit cubes on all the outer faces. They did not see the inside cubes (Figure 4.30).
>
> *Öğrenciler dış yüzlerdeki birim küpleri [sayısını] toplarlar. Içerdekileri görmezler.*

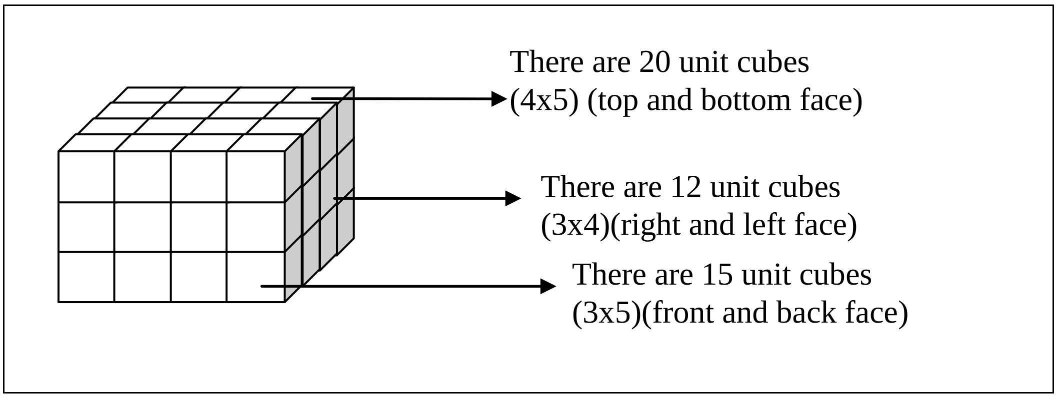


Figure 4.30 Mrs. Uzun's interpretations regarding the students' error in question 3

In terms of the possible sources of the errors related to focusing on the faces of the 3D solids, the participating teachers specified different sources. These sources are summarized in Table 4.4.

Table 4.4 The sources of the error; focusing on the faces of 3D solids

	Mrs. Kaya	**Mrs. Akay**	**Mr. Esen**	**Mrs. Uzun**
Not being able to think of solids as three-dimensional		x		x
Not being able to comprehend the structure of 3D solids				x
Not being able to concretize 3D solids			x	
Students' carelessness	x			

Initially, Mrs. Akay and Mrs. Uzun stated that the reason for focusing on the faces when calculating the volume of 3D solids might be that they are not able to think of solids as three-dimensional. As an example, Mrs. Akay explained:

> Students could not imagine a 3D solid. They focused only on what they see. I think that all their problems emanated from this.

3 boyutlu bir cisim hayal edemiyor. Sadece gördüklerine odaklanıyorlar. Bütün sorun sanıyorum bundan kaynaklanıyor.

Similarly, while clarifying why the students made this error, Mrs. Uzun commented that the students did not think of the rectangular prism as three-dimensional. Moreover, Mrs. Uzun gave another possible source of the error in terms of focusing on the faces of a 3D solid, might be that the students are not being able to comprehend the structure of a 3D solid. The related vignette is given below:

> Students do not see the back side of the rectangular prism. They only calculate [the unit cubes] on the visible faces. In my opinion, they do not know the shape of a prism For example, that, it has 6 faces. For this reason, they only calculate 3 faces.
>
> *Öğrenciler arka tarafı görmüyorlar. Prizmanın sadece ön yüzündekileri [birim küpleri] hesaplıyorlar. Bence onlar prizmayı tanımıyorlar. Mesela 6 yüzü olduğunu. O yüzden sadece 3 yüzü hesaplıyorlar.*

Mr. Esen identified another source of the error. He considered that elementary students may not be able to concretize a 3D solid since they do not see this shape in their daily life. He explained:

> They encounter an object which they have not seen so far in their lives. Namely, students did not concrete prism and they did not envisage. It is normal.
>
> *Bugüne kadar gördükleri objelerle tam karşılaşmadıklarından dolayı. Yani öğrenciler prizmaları somutlaştıramıyor ve gözlerinin önüne getiremiyorlar. Bu da normal.*

In addition to the sources of errors given above, Mrs. Kaya stated that another source of the errors were that students' carelessness in calculating the volume of 3D solids.

In conclusion, focusing on the faces of 3D solids was one of the errors that the four teachers commented on in relation to the students' attempts to calculate the volume of 3D solid. According to the teachers, students who made this error focused on counting the number of unit cubes on the faces and they ignored the unit cubes inside the 3D solid.

4.2.2.3.2 Over-Counting the Common Unit Cubes on the Adjacent Faces

As it was stated, based middle school teachers' knowledge, another error related to calculating the volume of 3D solids was over-counting the common unit cubes on the adjacent faces. With respect to teachers' statements and accessible literature, students might not realize that some unit cubes belong to more than one face of 3D solid, which is formed by unit cubes.

According to the analysis of the data, two teachers (Mrs. Kaya and Mrs. Akay) referred to this error. Both these teachers explained that elementary students might count common unit cubes on the adjacent faces more than once. For instance, Mrs. Akay connected this error to the student response to question 3:

> The students might not realize that the unit cubes here [upper column on the front layer] can have common unit cubes with the unit cubes there [upper column on the right layer] (indicated in the Figure 4.31 with grey color).
>
> *Öğrenciler buradaki küplerle [ön katmandaki üst sıra] şuradaki küplerin [sağ katmandaki üst sıra] ortak küpe sahip olabileceğini farketmiyorlar.*

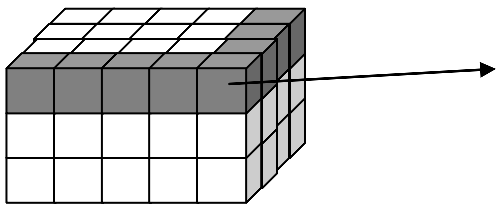


Figure 4.31 Mrs. Akay's interpretations regarding the error in question 3

When the data was analyzed to reveal the possible sources of the error of over-counting the common unit cubes on the adjacent faces, two different sources were identified by Mrs. Kaya and Mrs. Akay. Table 4.5 summarizes the sources of this error.

Table 4.5 The sources of the error; over-counting the common unit cubes on the adjacent faces

	Mrs. Kaya	**Mrs. Akay**	**Mr. Esen**	**Mrs. Uzun**
Students' carelessness	x	x		
Not thinking deeply about the concepts	x	x		

Both teachers stated that the reasons for the over-counting could be students' carelessness and not thinking deeply about the concepts. Pertaining to these sources, Mrs. Akay stated:

> Most of the students focused only on obtaining an answer and did not examine the figure. They counted unit cubes without considering the common unit cubes on the adjacent faces.
>
> *Öğrencilerin çoğu sadece cevabı bulmaya odaklanıyorlar ve şekli incelemezler. Yan yana olan yüzlerdeki ortak küpleri düşünmeden birim küpleri sayarlar.*

As a result, one of the errors made by students when calculating the volume of 3D solid was over-counting the common unit cubes on the adjacent faces. According to the teachers, students were not sufficiently careful and did not think deeply about concepts such as unit cubes and faces when calculating the volume of a 3D solid.

4.2.2.3.3 Conceptual Errors

As presented in Table 4.3, all the participating teachers agreed that one of the main errors was conceptual errors. In this study, this error was defined as students' misunderstanding or confusing the meanings of the concepts. According to the teachers, it emanated from not knowing the meaning of concepts such as the volume of 3D solids, the surface area or the perimeter of 2D figures, the height of the prism, or the height of the side-face of the prism. Table 4.6 presents a summary of the types of conceptual errors.

Table 4.6 The types of conceptual errors

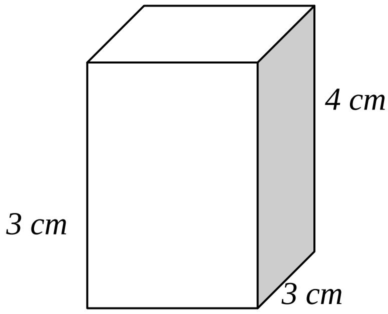
	Mrs. Kaya	**Mrs. Akay**	**Mr. Esen**	**Mrs. Uzun**
Confusing the surface area and volume	**x**			**x**
Confusing the perimeter and volume			**x**	
Confusing the height of 3D solids and the height of side-face	**x**	**x**	**x**	**x**
Not being able to calculate the area of the triangle	**x**	**x**	**x**	
Not being able to apply Pythagorean Theorem		**x**		

Mrs. Kaya and Mrs. Uzun stated that students might not be able to discriminate the meaning of the surface area and volume. For this reason, students can calculate the surface area of the 3D solid instead of calculating its volume. This is illustrated by Mrs. Kaya's explanation below.

> Students might find the surface area. They might find the surface area of the solid. They might confuse the surface area of the solid and the volume of the solids.
>
> *Alan bulabilirler. Cismin alanını bulabilirler. Alanla hacmi karıştırabilirler.*

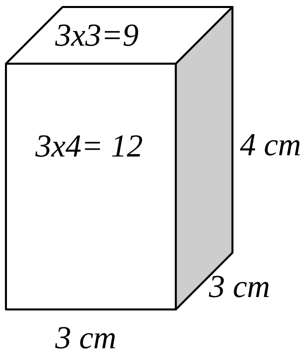
In the interview, Mrs. Uzun provided similar explanations related to this error. However, both teachers seemed not to be aware of it during their lessons since the analysis of the data gathered from the classroom observations showed that their students had made the same error in their lessons. As an example, an extract taken from the observation of Mrs. Uzun's class in which she asked the students the following question.

Question:



What is the volume of square prism presented in the figure?

Solution of one of Mrs. Uzun's students' is given below:



V= 2. 9 + 4. 12
V= 18 + 48
V= 66

As it can be seen in this vignette, student solved the given question by calculating the surface area which reveals that the student confused the surface area and volume of the 3D solid. The teachers' knowledge regarding confusing the surface area and volume of a 3D solid is evidence that the students did have error in calculating the volume of 3D solid.

All teachers identified the possible sources of students' conceptual errors. The sources are presented in Table 4.7.

Table 4.7 The sources of the conceptual errors

	Mrs. Kaya	Mrs. Akay	Mr. Esen	Mrs. Uzun
Not being able to think of solids as three-dimensional				x
Not being able to concretize 3D solids			x	
Lack of conceptual knowledge	x	x	x	x
Students' carelessness	x	x	x	
Not thinking deeply about the concepts	x	x	x	

Mrs. Kaya and Mrs. Uzun stated the possible sources of confusing the surface area and volume of 3D solid in a different way. According to Mrs. Kaya, the source of this error might emanate from a lack of conceptual knowledge as she explained:

> The students do not know the concepts. Namely, they do not know what the volume is or area is. Therefore, they are confused as to when to calculate the volume and when to calculate the area. Students have a lack of conceptual knowledge.
>
> *Öğrenciler kavramları bilmiyorlar. Yani hacim nedir, alan nedir bilmiyorlar. Bu yüzden ne zaman hacim hesaplayacaklarını, ne zaman alan hesaplayacaklarını karıştırıyorlar. Öğrencilerde kavram eksikliği var.*

Mrs. Uzun considered that students might not think of solids as three-dimensional. In fact, according to Mrs. Uzun, students think 3D solids are two-dimensional. For this reason, students confuse the area and volume concepts.

Furthermore, Mr. Esen specified that the elementary students might confuse the perimeter and volume concepts. He explained:

> Students might add the length of the edges of the rectangular prism while calculating its volume. That is, students may calculate the perimeter of 3D solids.

> *Öğrenciler, prizmanın hacmini hesaplarken kenar uzunluklarını toplayabilirler. Yani, prizmanın çevresini hesaplayabilirler.*

As it was stated in Table 4.7, Mr. Esen clarified the reason for making this error as follows:

> I think it is because they are not able to concretize. They could not mentally imagine 3D solids.
>
> *Bana göre somutlaştırılmadığından dolayıdır. Kafalarında 3 boyutlu cisimleri canlandıramıyorlar.*

Additionally, according to all the participating teachers, students might confuse the height of the pyramid and height of the side-face of the pyramid while calculating volume of the prism in question 7 (Figure 4.32).

The base length of the square prism model is 6 cm and the length of side-face height is 5 cm. Ceren and Cemre who calculated the volume of this model solved the question in different ways.

Ceren's Solution:

$V= \frac{6.\ 6.\ 5}{3}$

$V= \frac{180}{3} = 60\,cm^3$

Cemre's Solution:

$a^2 = b^2 + c^2$

$5^2 = 3^2 + c^2$

$25 = 9 + c^2$

$16 = c^2$

$c= 4$

$V= \frac{6.\ 6.\ 4}{3}$

$V= \frac{144}{3} = 48\,cm^3$

b) For those students who gave the wrong answer, describe the errors that they made.

Figure 4.32 Question 7

Mrs. Kaya explained her idea pertaining to question 7 in the following way:

> Ceren's solution was wrong because she did not understand which was the height of the pyramid and which was the height of the side-face of the pyramid. She confused them. She took both of them to be the same height. However, 5 was not the height of the given prism, it was the height of the side-face of the prism.

Ceren'in çözümü yanlış olmuş. Çünkü yan yüz yüksekliği ile cisim yüksekliğini anlamamış. Karıştırmış. Aynı yükseklik olarak almış. Halbuki 5 değil bizim cisim yüksekliğimiz, yan yüz yüksekliğimiz.

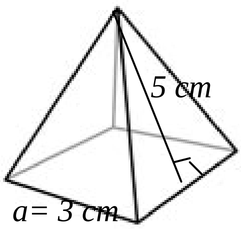
Mrs. Uzun's vignette given below is a further example of this error.

> Ceren took the height of the side-face of the pyramid here. Whereas, she should have taken the height of the pyramid not the height of the side-face of the pyramid.
>
> *Ceren burada yan yüz yüksekliğini almış. Oysa yan yüz yüksekliği değil de cisim yüksekliğini alması gerekiyordu.*

In a similar way, Mrs. Akay and Mr. Esen explained that elementary students might not be able to differentiate between the height of the pyramid and height of the side-face of the pyramid.

This error, confusing the height of pyramid and height of side-face of the pyramid, was observed in Mrs. Akay's lesson. The following excerpt is an example from Mrs. Akay's lesson.

(Mrs. Akay wrote the question below)



Calculate the volume of square pyramid located on the left side?

Before solving the question, the students discussed how they could solve it. One of the students said that the height of the pyramid was 5. Then, Mrs. Akay provided the following explanation:

> It is not 5. It is the height of the side-face [of the pyramid]. You should calculate the height of the pyramid before calculating its volume.
>
> *5 değil. O, [piramidin] yan yüz yüksekliği. Hacmi hesaplamadan önce piramidin yüksekliğini hesaplamalısın.*

According to all the participating teachers, the reason for confusing the height of the pyramid and height of the the side-face of the pyramid was students' lack of conceptual knowledge (Table 4.7). Thus, Mr.Esen explained:

> Ceren did not comprehend the height of the pyramid and height of the side-face [of the pyramid]. She confused these concepts.
>
> *Ceren cisim yüksekliği ile yan yüz yüksekliğini anlamamış. Bu kavramları karıştırmış.*

Similarly, Mrs. Akay interpreted the source of the error in question 7 as:

> Because of saying height, this height misdirected them. It means that the students did not study in detail. They were unaware of the height of the prism.
>
> *Yükseklik dediği için bu yükseklik onları yanıltıyor. Detaylı çalışmıyorlar demek ki. Cismin yüksekliğinden haberleri yok.*

In addition, based on the analysis of the data, three participants (Mrs. Kaya, Mrs. Akay and Mr. Esen) identified another error which was related to the conceptual error. These teachers thought that the students might not know how to calculate the area of triangle. For this reason, they specified that the elementary students might not calculate the base area, which resulted in calculating the volume of triangular prism incorrectly. Mr. Esen explained this by stating that:

> The students who used volume formula might not be able to find the base of the prism. They might ask how to calculate the the base area.
>
> *Hacim formülü kullananlar taban alanını yanlış bulabilirler. Taban alanını nasıl bulayım diyebilir.*

Lastly, Mrs. Akay commented that students might not apply the Pythagorean Theorem, which was denoted as a conceptual error. She identified this error in relation to question 6 (Figure 4.17) as follows:

> Asya could not identify the hypotenuse and the right edges. She did not know how to calculate the area of this triangle. In general, students write $a^2 = b^2 + c^2$ as the Pythagorean Theorem. However, because they do not know what a^2, b^2 and c^2 are, they cannot apply the theorem. For this reason, Asya multiplied 15 by 25 when calculating the area of the triangle and she was wrong.
>
> *Asya hipotenüsün ve dik kenarın ne olduğunu bilmiyor. Bu üçgenin alanını bilmiyor. Genellikle, Pisagor Teoremi deyince öğrenciler $a^2 = b^2 + c^2$ yazarlar. Ama a^2 'nin, b^2 'nin ve c^2 'nin ne olduğunu bilmedikleri için, teoremi uygulayamazlar. Bu yüzden, üçgenin alanını bulurken Asya 15 ile 25'i çarpmış ve yanlış yapmış.*

According to the middle school teachers, the reasons for students' errors, in terms of not being able to calculate the area of the triangle and not being able to apply the Pythagorean Theorem, were students' carelessness and not thinking deeply about the concepts (Table 4.7).

To summarize, the participating teachers stated that students might confuse the concepts when calculating the volume of 3D solid. According to the teachers, this error emanated from not knowing the meaning of concepts such as the volume of 3D solids, the surface area or the perimeter of 2D figures, the height of the prism, the height of side-face of the prism, the area of the triangle and the Pythagorean Theorem. The teachers' considered that the sources of students' conceptual errors might be lack of conceptual knowledge, not being able to concretize 3D solids, not being able to think of solids as three-dimensional, students' carelessness and not thinking deeply about the concepts.

4.2.2.3.4 Computational Errors

In this study, middle school teachers stated that computational errors were another error that elementary students held. The errors in calculating the volume of 3D solid could be defined as a mistake made when undertaking the calculation. Thus, this error can be regarded as multiplication error. Analysis revealed that only one middle school teacher (Mrs. Akay) commented this error in this way:

> If the students consider multiplying the height, the length and the width of the prism [to find the volume of the prism], many of them do not know the multiplication table. They can make a mistake while multiplying the numbers.
>
> *Eğer [prizmanın hacmini bulmak için] prizmanın eni, boyu ve yüksekliğini çarpmayı düşünüyorlarsa bir çoğu çarpım tablosunu bilmiyor, sayıları çarparken hata yapabilirler.*

From the analysis of the data, it was found that one teacher (Mrs. Akay) focused on the students' multiplication error when they were calculating the volume of 3D solid. She considered that, students can make mistakes when doing the calculations and the source of this error was students' carelessness (Table 4.8).

Table 4.8 The sources of the computational errors

	Mrs. Kaya	Mrs. Akay	Mr. Esen	Mrs. Uzun
Students' carelessness		x		

In brief, the middle school teachers categorized the errors under four dimensions; focusing on the faces of a 3D solid, over-counting the common unit cubes on the adjacent faces, conceptual errors, and computational errors. The main sources of the error in focusing on the faces of a 3D solid are; not being able to think of solids as three-dimensional, not being able to comprehend the structure of 3D solids, not being able to concretize 3D solids and carelessness Some middle schoolteachers also stated that students' carelessness and not thinking deeply about the concepts were the sources of over-counting the common unit cubes on the adjacent faces. On the other hand, the middle school teachers believed that a lack of conceptual knowledge, not being able to think about solids as three-dimensional, not being able to concretize 3D solids led the elementary students to make mistakes based on conceptual errors. In addition, students' carelessness was considered to be the main source for their computational errors.

4.2.2.4 Middle School Teachers' Knowledge of the Strategies to Overcome Elementary Students' Errors

In this section, strategies used by middle school teachers to overcome the errors made by the elementary students when calculating the volume of 3D solid were investigated. The middle school teachers suggested various strategies that they could use to eliminate the errors made by the elementary students when calculating the volume of 3D solid. Using the term strategies, I am referring to the approaches, methodologies that the middle school teachers used in their lessons or planned to use when their students make these errors. The four middle school teachers stated two strategies to deal with elementary students' errors in relation to calculation of the volume of 3D solids. The strategies to overcome each error are presented in Table 4.9.

Table 4.9 The strategies to overcome students' errors

Strategy / Students' Errors	**Using manipulative**	**Re-explaining the misunderstood part of the topic**
Focusing on the faces of 3D Solids	**x**	
Over-counting the common unit cubes on adjacent faces	**x**	
Conceptual errors	**x**	**x**
Computational errors		**x**

The first one was using manipulative which was proposed by all the teachers to eliminate the error, focusing on the volume of 3D solids. For example Mrs. Kaya said:

> We can ask them to create 3D solids from the unit cubes. We can count the unit cubes one by one. Moreover, we can calculate the volume [of it]. We see that both are the same. Yes, we can say that the volume refers to the number of unit cubes or the number of unit cubes refers to the volume, namely we can make two-way connection using an if-and-only clause.
>
> *Birim küplerden cisimler yaptırabiliriz. Küpleri tek tek sayarız. Ayrıca hacmini hesaplarız. Ikisinin aynı olduğunu görürüz . Evet, hacmin gerçekten küp sayısı verdiğini söyleyebiliriz veya küp sayısının hacmi verdiğini de yani ikisini de ancak ve ancak şeklinde çift taraflı bağlantı kurabiliriz.*

According to Mrs. Kaya and Mrs. Akay, the students might over-counting the common unit cubes on the adjacent faces of the 3D solid. Mrs. Kaya and Mrs. Akay stated that students might recognize the common unit cubes with the help of manipulative. Pertaining to this, Mrs. Akay stated:

> Many students do not recognize the common unit cubes on two or three adjacent faces when counting the unit cubes. They can count these unit cubes more than once. I might give them a prism formed by unit cubes to examine. By asking questions regarding the common unit cubes, students may notice that a unit cube belongs to more than one face but that unit cube is only one in the prism.
>
> *Birçok öğrenci birim küpleri sayarken 2 veya 3 yüzdeki ortak küpleri farketmiyor. Bu küpleri birden fazla sayabiliyorlar. Öğrencilere birim küplerden oluşmuş prizma verip cismi incelemelerini isterim. Öğrencilere ortak küplerle ilgili sorular sorarak, öğrenciler bir küpün*

birden fazla yüze ait olduğunu fakat o küpten sadece bir tane olduğunu farkedebilirler.

Additionally, Mrs. Kaya, Mr. Esen and Mrs. Uzun stated that teachers might use manipulative to eliminate the students' conceptual errors which were confusing the volume and area, and confusing the volume and perimeter. As Mrs. Kaya explained:

> If we use visual material, for instance we can cover the surface with wrapper, then we can try to find the area of the wrapper because the area is covering, in fact, the surface area. After finding the surface area visually, the student will obtain the answer 94. We can create a figure, fill the inside with the water. We can say that the surface area is the amount of wrapper to be used to cover the visual material and the volume is the amount of water that fill the figure. In this way, we can distinguish between two concepts.
>
> *Görsel materyaller kullanırsak, mesela cismi bir cilt ile kaplayabiliriz. O cildin alanını bulmaya çalışırız. Çünkü alan bir kaplamadır, aslında, yüzey alanı. O yüzey alanını bulduktan sonra görsel olarak 94 e ulaşacak. Öyle bir cisim yaparız ki, içini suyla doldururuz. Alanın kullanılan cildin mikltarı, hacmin de cismi dolduran suyun miktarı olduğunu söyleriz Böylece, ikisinin farklı şeyler olduğu kavramını ayırt edebiliriz.*

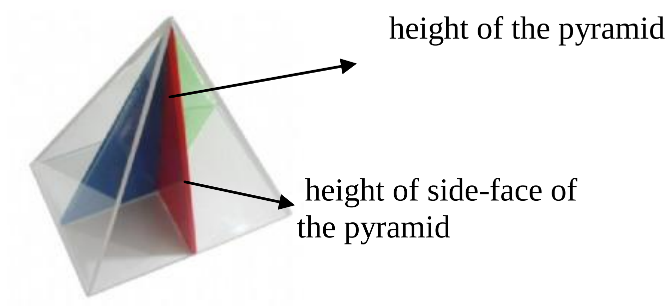
Mrs. Uzun explained that constructing and visualizing the solid using manipulative might be helpful for students to eliminate their conceptual errors and Mrs. Uzun's explanation regarding this error is given below. Specifically, the example is related to confusing the area and volume, which is one of the dimensions of the conceptual error.

> I corrected [the error] using the intelligence cube or by putting their books one on the top of the other. Let's measure the volume of a book. How many books are there? In that case, what is the volume [of whole shape]? Let's find the area [of whole shape]. Are they same? I try to eliminate [the error] by asking these kind of questions.
>
> *Zeka küpünü kullanarak düzeltirim veya kitaplarını üst üste koyarak. Hadi şu kitabımızın hacmini ölçelim. Kaç tane kitap var? Bu durumda, [bütün şeklin] hacmi kaç? Alanını bulalım. Bunlar aynı mı? Sorular sorarak gidermeye çalışırım.*

Furthermore, Mr. Esen, Mrs. Kaya and Mrs. Uzun proposed to use manipulative to eliminate students' conceptual error in question 7. As indicated, the error in this question was confusing the height of the triangular prism and the height of the side-face. The related vignette of Mr. Esen is given below.

> If there is a pyramid that is able to open and close, then the students can physically perceive it, then we can say that this is the height of the triangle, which is at the outside. We can explain that the students can find the area in this way but to find its volume, the height of the pyramid should be measured.
>
> *Eğer açılıp kapanan bir piramit olabilseydi ve ellerine somut olarak verebilsek bu dıştaki üçgenlerin yüksekliği deriz. Bununla alanlarını bulabilirsiniz ama hacmini bulmak için kendi yüksekliğinin olması gerekir diye anlatırız.*

Mrs. Kaya and Mrs. Uzun stated that teachers could use the transparent pyramid model given below, to deal with students' conceptual error in which they confuse the height of the triangular prism with the height of the side-face.



From the knowledge of the participating teachers, the second strategy to eliminate students' errors in calculating the volume of 3D solids was, presented in Table 4.9, re-explaining the misunderstood part of the topic. For instance, Mrs. Akay, Mr. Esen and Mrs. Uzun specified that the students' conceptual error, not being able to distinguish the height of 3D solids from the height of the side-face, might be eliminated by re-explaining the topic. In this manner, the teachers stated that the teacher might explain the difference between the height of 3D solids from height of side-face.

In addition, all teachers specified that students' computational errors could be eliminated by explaining the topic again and presenting many more examples related to the topic. Moreover, they stated that conceptual errors, not being able to apply Pythagorean Theorem and not being able to calculate the area of a triangle, could be eliminated by re-explaining the topic and solving many examples.

This section presented the middle school teachers' knowledge of their learners in terms of elementary students' preferences among solution methods, their interpretations of students' alternative solution methods, and errors held by the elementary students and their possible sources when calculating the volume of 3D solids were given. In addition, middle school teachers' knowledge on the strategies to overcome the errors was explained. In the next section of the result chapter, teachers' knowledge of curriculum will be presented and discussed.

4.2.3 Middle School Teachers' Knowledge of Curriculum

From the analysis of the data, another dimension of the middle school teachers' PCK was determined; this was their knowledge of curriculum. In relation to this dimension two major categories were formed from the data gathered through the questionnaire, classroom observation and interviews. The four middle school teachers' knowledge of curriculum on the volume of 3D solids is discussed in terms of the connections with other topics and changing the order of the topics.

4.2.3.1 Connection with Other Topics

In the current study, the connection with the other topics refers to connecting the topic to the other mathematics areas and topics taught in other courses (e.g. science) taught in previous years, the same year and to be taught in later years. Making the connections between topics is used to help students remember topics learned, relate new topics and previous ones and to encourage them to review the previous topic. The connection with the other topics is undertaken by referring to the previous topic, asking questions that help students to understand the relationship, and reminding them about what they have already learnt it.

From the analysis of the data from the interview, three teachers (Mrs. Kaya, Mrs. Akay and Mrs. Uzun) stated that they connected the volume of 3D solids to the topics taught in previous years (the area of the polygons). Mrs. Kaya stated:

> It is necessary to know how to calculate the area of a polygon to apply the volume of prism. In fact, to find the area of base [of the prism], it is necessary to know how to calculate the area of a polygon.
>
> *Prizmanın hacmi için taban alanını bilmek gerekiyor. Taban alanını bulmak için çokgenlerin alanını bilmek gerekiyor.*

Pertaining to the volume of the pyramid, Mrs. Kaya, Mrs. Akay and Mrs. Uzun said that they made links to the topic that had been taught in previous years and also taught at the same year. Mrs. Akay gave this example:

> When I am teaching the volume of the pyramid, I connect it with the volume of a prism and the area of a polygon. Since the volume of a pyramid is one third of the volume of a prism. For this reason, students should know how the volume of a prism is calculated to find the volume of a pyramid.
>
> *Piramitin hacmini anlatırken prizmanın hacmi ve çokgenlerin alanı ile ilişkilendiriyorum. Çünkü piramitin hacmi, prizmanın hacminin 1/3 'ü. Bu yüzden, piramitin hacmini hesaplamak için, öğrenciler prizmanın hacmini bilmemeliler.*

Thus it can be seen that the topic; the area of a polygon, had been taught in the previous year and the volume of a prism had been taught at the same year. However, Mr. Esen said that he did not connect the volume of 3D solids to the topics taught in previous years, the same year and to be taught in later years. During the interview, Mr. Esen commented:

> I had not connected this topic with the other topics until today. With which topics do I connect? Of course, all the topics is related to each other. Multiplying the numbers is related to the volume, as well. But I do not link [the volume] with any topic in particular.
>
> *Bugüne kadar bu konuyu başka konularla ilişkilendirmedim. Hangi konu ile ilişkilendirebilirim ki? Tabiki mutlaka her konu birbiriyle ilişki içinde. Sayıların çarpılması bile hacimle ilişkilidir. Ama özel olarak [hacim konusunu] bir konuyla ilişkilendirmiyorum.*

Contrary to his explanation in the interview, Mr. Esen did make links to the volume of a pyramid with the volume of a prism like the other teachers in his lessons. In other words, when teaching the volume of the pyramid, in their lessons all the teachers reminded the students about how to calculate the volume of a prism. From the observation of Mrs. Uzun's class this dialogue was recorded:

> ***Mrs. Uzun:*** *How can you calculate the volume of this prism? (showed a triangular prism)*
> ***Student-1 (Std):*** *By multiplying the area of the triangle and the height of the prism.*

> ***Mrs. Uzun:*** *Okay. What about the volume of this? (showed a triangular pyramid)*
> ***Std-2:*** *By multiplying the area of the triangle and the height of the pyramid.*
> ***Mrs. Uzun:*** *You said that the volume of the pyramid and the volume of the prism are the same. But they are not the same. Do you think that if we fill both of them with water, will the amount of water be the same?*
> ***Stds:*** *No*
> ***Mrs. Uzun:*** *In that case, their volumes are not the same. Look at this prism. There are 3 pyramids in it (showing the triangular prism). That is, the volume of prism is equal to the volume of 3 pyramids. In other words, when calculating the volume of pyramid, we divide the volume of prism by 3 if their bases and their height are the same.*

Similar to the way in which Mrs. Uzun taught, the other three teachers taught the volume of the pyramid by relating it to the volume of the prism in their lessons. However, none of the teachers connected the volume of 3D solids with the topics to be taught in later years and topics that had been taught in other courses.

4.2.3.2 Changing the Order of the Topics

In addition to connecting topics with the teachers used, their curriculum knowledge refers to make changes in the order of the sub-topics of 3D solids in the curriculum. In the elementary mathematics curriculum (MoNE, 2006), the topic of 3D solids, started with defining the basic terms of the prism and continued with calculating the surface area then the volume of prism. Up to this point, teachers had not altered the order of the topic. However, in the curriculum, the topic continues to define the terms (e.g. base, lateral faces and height,) related to a pyramid, cone and sphere followed by their surface area. Lastly, the calculation of their volumes was taught. Regarding the order of pyramid, cone and sphere, all the teachers did change the order in the process of teaching. The teachers were asked the reason for this change in the interview. Mr. Esen responded as follows:

> It was not reasonable to teach defining the basic terms related to pyramid, cone and sphere, and calculating the surface area and the volume of pyramid, cone and sphere separately. In my opinion, it is more effective first to teach the definition of the basic terms related to a pyramid, then calculate its surface area and volume. Then these aspects related to a cone should be taught. Lastly, the terms regarding the sphere should be defined and the calculation of the surface area and

> volume of sphere should be taught. In this way, I created coherence for each 3D solid.
>
> *Piramit, koni ve küre ile ilgili temel kavramları tanımlamayı ve onların yüzey alanını ve hacmini hesaplamayı ayrı ayrı öğretmek çok mantıklı değil. Bence önce piramit ile ilgili kavramları, piramitin yüzey alanını ve hacmini hesaplamayı öğretmek daha etkili oluyor. Sonra koni ile ilgili olan konular öğretilmeli. En son da, küre ile ilgili olan terimler tanımlanmalı ve kürenin yüzey alanı ve hacminin hesaplanması öğretilmeli. Bu şekilde her bir geometrik şekil de bütünlük sağlıyorum.*

Moreover, Mrs. Uzun explained:

> If I taught [the topic] of 3D solids according to the sequence given in the curriculum, I would lose time. I reminded the students about the pyramid, cone and sphere in every lesson.
>
> *Eğer 3 boyutlu cisimleri müfredattaki sıraya gore anlatırsam vakit kaybediyorum. Her defasında piramiti, koniyi, küreyi tekrar hatırlatmak gerekiyor.*

All the teachers altered the order of the sub-topics related to 3D solids in order to teach the topics more effectively and to avoid loss of time.

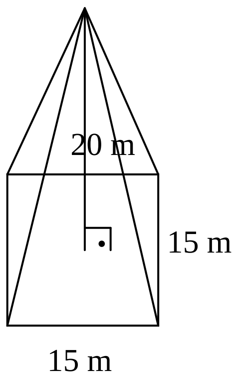
4.2.4 Middle School Teachers' Knowledge of Assessment

The last dimension of the middle school teachers' PCK is presented and discussed in this study was the teachers' knowledge of assessment. The analysis of data collected through the questionnaire, interview and classroom observation revealed that teachers applied formative and summative assessment during their regular instructions. The descriptions of these two types of assessment are presented before discussing how they implemented them.

Formative assessment provides the teacher with information about the students learning during the learning process. Moreover, it provides the students with feedback about their learning. On the other hand, summative assessment evaluates the students' knowledge concerning a specific topic and measures student learning following the completion of a unit in any subject (Lankford, 2010).

The participating teachers applied different formative assessment methods to obtain information about how much students have learned during the lesson. All teachers asked students many questions related to the calculation of the volume of prism after teachers presented a few examples during their lessons. The teachers observed the students while they were solving the questions; they helped the students if they missed important aspects, and gave them feedback about their learning. An example of this process, below is one of the questions that Mrs. Akay presented in her lesson.

Question:



The half of the store, which was a square pyramid-shape, filled with wheat when 300 m^3 wheat was put to the store. How much wheat in the store had previously?

One of the students solved the question on the board. The teacher observed the class, and realized that the students were able to calculate the volume of pyramid. However, students missed an important point, which prevented them from solving the question. Her explanation regarding the missing point was given below.

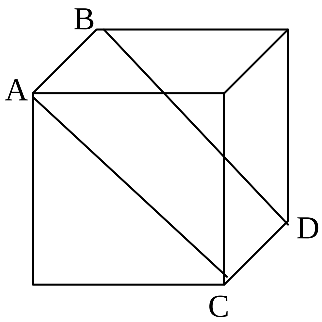
Mrs. Akay: *What was the volume of pyramid?*
Std-1: *1500 m^3*
Mrs. Akay: *Good. How much wheat was there before putting 300 m^3 wheat in the store?*
Std-2: *The half of the store*
Mrs. Akay: *How much?*
Std-1: *Hımmm…750 m^3*
Mrs. Akay: *That is, if 300 m^3 wheat was put in, then there would be 750 m^3 wheat. In this case, how much wheat is there before putting 300 m^3 wheat?*
Stds: *450*
Mrs. Akay: *Yes. Do you understand?*
Stds: *Yes.*

When observing the class Mrs. Akay noticed that some of the students did not understand the question. She helped them by asking questions. As a result of this formative assessment, Mrs. Akay decided to perform more exercises and explained the points in which the students had problems. Moreover, Mrs. Kaya assessed her students’ learning through the group work; she expected a group of students to present the topic to the whole class and to discuss the topic with their classmates.

During the students' presentation, Mrs. Kaya observed the class and noted the points that students did not understand. Similar to Mrs. Kaya, Mrs. Uzun applied formative assessment method through the group work undertaken during her lessons. In her assessment, she grouped the students and gave them different prisms. She asked each group to calculate the volume of the prism that she had given them. Then, she checked whether students' answers were correct. If there were students who had not been able to find the volume of the prisms, then Mrs. Uzun explained the subject again and solved additional examples.

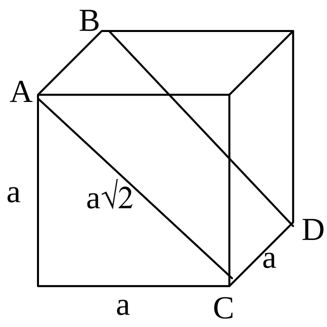
Besides, Mr. Esen and Mrs. Akay asked their students a question and then they said that "for the students who solved the question correctly, I will give them 100 for their class performance".

Mr. Esen asked the following question to the class and he stated that the students who solved this question would get 100 for the class performance grade. He gave students time to solve the question. When students were solving the question, he walked around the classroom and checked the student's work.



If A(ABCD)= 8√2 cm2, then calculate the volume of the cube.

After a while, one student solved the question and the student explained the following solution to his classmates:



I called the length of each edge of the cube as "a". In this case, the length of AC is a√2. I can calculate the area of ABCD by multiplying a and a√2.

A(ABCD)=a x a√2

$8\sqrt{2} = a^2\sqrt{2}$ $\quad$ $a^2 = 8$ *and* $a = 2\sqrt{2}$

$V = a^3 \Longrightarrow (2\sqrt{2})^3 = 16\sqrt{2}$

After checking the solution, Mr. Esen said the students who solve the question correctly that your class performance grade was 100.

On the other hand, one of the formative assessment strategies was that all the teachers gave homework, from the textbook and workbook. During our conversation, Mrs. Kaya explained:

> I give homework after teaching each topic. Thus, I check whether students have understood the topic.
>
> *Her konuyu anlattıktan sonra ödev veririm. Böylece, öğrencilerin konuyu anlayıp anlamadıklarını kontrol ederim.*

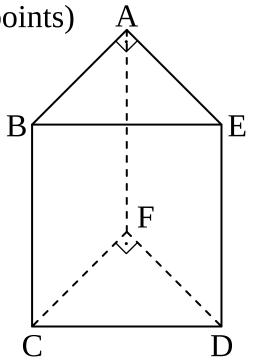
Mrs. Akay and Mr. Esen did not check whether students did homework, however, Mrs. Kaya and Mrs. Uzun checked students' homework and they solved the questions that students could not solve. In this sense, Mrs. Kaya stated:

> Sometimes I prepare a worksheet and give it to the students as homework. They do the worksheet at home and we discuss the questions in the class the next lesson. If they have difficulties in solving some questions, then I solve them and explain the point that they do not understand again.
>
> *Bazen çalışma kağıtları hazırlıyorum ve öğrencilere ödev olarak veriyorum. Onları evde çözüyorlar ve bir ders sonra soruları sınıfta tartışıyoruz. Eğer zorlandıkları sorular varsa, onları çözüyorum ve anlamadıkları noktayı tekrar anlatıyorum.*

As can be seen, the teachers planned to elicit information about their students' learning through homework and to determine what they had missed in relation to that had been taught in the lesson.

On the other hand, the teachers specified that they used paper-pencil test to grade students. Below, a paper-pencil test question posed by Mr. Esen.

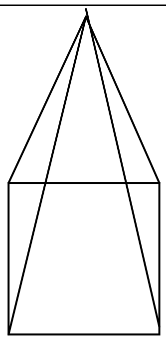
(10 points)



In the triangular prism in the figure; if $|AB| = 6$ cm , $|BC| = 12$ *cm and A(BCDE)= 120 cm*^{2}*,* *then what is the volume of this triangular prism?*

Figure 4.33 An example paper-pencil test question posed by Mr. Esen

Another example was taken from the exam created by Mrs. Uzun.



If the length of the one edge of base of square pyramid, in the figure, is 8 cm and the height of side-face [of the pyramid] is 5 cm, then what is the volume of this triangular prism?

Figure 4.34 An example of paper-pencil test question posed by Mrs. Uzun

In addition, the teachers gave performance homework and projects which are obligatory for the students. Regarding the volume of 3D solids, all the teachers asked the students to construct 3D solids from cartoons and calculate the surface area and volume of them as performance homework. None of the students selected the volume of 3D solidsfor their project.

In conclusion, the teachers used formative assessment by giving homework and asked many questions during the lesson to determine whether their students learnt the topic. Moreover, the teachers implemented summative assessment by giving performance homework and project work, and administering tests to discover how much the students had absorbed concerning what they had taught.

CHAPTER V

CONCLUSION, DISCUSSION, AND IMPLICATIONS

The purpose of this study was to examine four middle school mathematics teachers' subject matter knowledge and pedagogical content knowledge of the volume of 3D solids. In light of this purpose, this chapter presents the conclusion and the discussion of the results, educational implications, recommendations for future research studies, and the limitations of the research study. In other words, the important points mentioned in the analysis part reviewed and discussed with references to previous studies in the literature.

5.1 Conclusions

In this study, in relation to four middle school teachers' subject matter knowledge on the volume of 3D solids, two dimensions, knowledge of alternative solution methods and knowledge of generating a story problem, were examined. Based on the findings of the study, the teachers generated four solution methods to calculate the volume of 3D solids. These methods were named as volume formula, systematic counting, layer counting and column/row iteration by means of the related literature and the participants' statements. Regarding these solution methods, it was seen that although middle school teachers stated these solution methods, they did not solve the questions using the stated methods in their lessons. Besides, middle school teachers had difficulty in generating a story problems related to the volume of 3D solids. Although they tried to use given numbers and terms, they were not successful in generating the story problems. State differently, middle school teachers did not have strong SMK to teach alternative solution methods to their students although they could generate different solution methods. Moreover, teachers' SMK is limited in generating story problems regarding the volume of 3D solids.

Regarding middle school teachers' PCK, their knowledge of instructional strategy, knowledge of learners, knowledge of curriculum and knowledge of assessment were investigated. Based on the findings of the study, it could be concluded that middle school teachers mostly used teacher-centered instructional strategy rather than using less teacher-centered enriched with class discussion strategy. In light of the teachers' knowledge of learners, it could be resulted in that students mostly preferred to use volume formula method while calculating the volume of 3D solids. Besides, the middle school teachers' interpretations of students' alternative solution methods were examined to identify their knowledge of learners. The findings of the study showed that the teachers were not successful in interpreting students' alternative solution methods if they themselves had not used them in their teaching. Moreover, the middle school teachers' knowledge of learners analyzed in terms of their knowledge of students' errors in the volume of 3D solids. They gave the following errors; focusing on the faces of 3D solids, over-counting the common unit cubes on adjacent faces, conceptual errors, and computational errors. Paralel to the errors, six sources of students' errors were identified. The sources were as follows: not being able to think of solids as three-dimensional, not being able to comprehend the structure of 3D solids, not being able to concretize 3D solids, lack of conceptual knowledge, students' carelessness and not thinking deeply about the concepts. Also, with respect to their knowledge of learners, two strategies, using manipulative and re-explaining the misunderstood part of the topic, were proposed by the the middle school teachers. Besides, another dimension of teachers' PCK was the knowledge of curriculum which was discussed in terms of the connection with other topics and changing the order of the topics in the curriculum. Although the middle school teachers made links to the volume of 3D solids with topics taught in previous years or in the same year, they did not connect it with topics that will be taught in the following years. Additionally, they altered the order of the sub-topics of the volume of 3D solids because they realized that there was a problem in the order of the sub-topics. Lastly, the middle school teachers' PCK was investigated through their knowledge of assessment. The teachers used both formative assessment strategies (informal questioning and homework) and summative assessment strategies (paper-pencil test, performance homework and project work) to assess students learning. In the next section, these conclusions are discussed based on the related literature.

5.2 Discussion

The research findings are discussed under two main sections based on the research questions. In the first section, the nature of middle school teachers' subject matter knowledge on the volume of 3D solids is discussed with references to the previous studies. In the second part, the nature of the middle school teachers' pedagogical content knowledge is discussed based on the related literature.

5.2.1 The Nature of Middle School Teachers' Subject Matter Knowledge

Generating alternative solution methods to the given questions is one of the dimensions of teachers' subject matter knowledge that formed the basis of the research study. The data analysis revealed that middle school teachers were able to generate four alternative solution methods to calculate the volume of 3D solids. Of these methods, systematic counting, layer counting and column/row iteration were more complicated than volume formula. To generate these three methods, it is necessary to know more than the multiplication of three edges (Battista & Clements, 1996). For instance, a student using the layer counting method should realize that a prism is composed of the layers. Similarly, to use column/row iteration method, it is necessary to know that a prism formed by columns or rows. To summarize, in order to generate other methods apart from the volume formula, a student needs to have knowledge about the structure of the 3D solids (Battista & Clements, 1996). In this respect, it can be concluded that the middle school teachers did have adequate knowledge of the structure of 3D solids, such as a prism, since they were able to establish the relationship between the units such as layers, columns and unit cubes. This means that generating alternative solution methods to calculate the volume of 3D solids was connected with knowledge of the structure of 3D solids. Since the middle school teachers had that knowledge, they were able to develop alternative solution methods such as layer counting.

Although teachers stated the methods, systematic counting, layer counting and column/row iteration, during the interview, they did not use them when teaching the volume of 3D solids in their lessons. They only used the volume formula to calculate the volume of 3D solids. Singmuang (2002) stated that teachers do not need to use any other alternative solution methods for the given problems since they are able to easily solve the problems using the formula. However, the reason for using volume formula might be that teachers have little understanding of the mathematical concepts (Berenson et al., 1997). Since they had limited understanding of the topic, they would not explain the alternative solution methods to their students and furthermore, they might not encourage students to use these methods even if they could generate them. As a result, it could be concluded that middle school teachers, who focused on using formula, probably had a limited understanding regarding the topic and were reliant on formulas in their teaching. This conclusion was parallel to previous studies (Berenson et al., 1997; Hill 2007) which concluded that teachers did not have sufficiently deep knowledge to explain and present mathematical ideas using alternative solution methods.

On the other hand, the variety of alternative solution methods that middle school teachers generate changed with respect to the way that the researcher asked the questions even if all the questions were related to calculating the volume of 3D solids. Two questions were given to the teachers for them to generate alternative solution methods to calculate the volume of 3D solids. In one of the questions, teachers were expected to directly calculate the number of unit cubes of the prism (Question 1). There was no extra challenge in terms of solving this question and all the middle school teachers were able to use at least two solution methods. This result could be interpreted as for simple questions; the teachers could use more than one strategy. In other words, the middle school teachers did not need broad subject matter knowledge to use alternative solution methods if the volume of 3D solids was asked directly. However, in another question, the teachers were asked to calculate the number of unit cubes of prism when one layer of unit cubes is removed from all faces of prism (Question 2). Since this question has two phases, removing the outer faces and then calculating the number of unit cubes remain, this question is more complicated than the previous one. It is possible that the teachers might not be familiar with this type of question and it was found that the middle school teachers applied, at most, one solution method to solve this question. Two middle school teachers used the volume formula, one applied systematic counting method however, one teacher was unable to solve the question when one layer of unit cubes is removed from all the faces of the prism. This result reveals that if the question was a little complicated and the teachers were not familiar with it, then the teachers focused on using the volume formula. The reason for this might be that teachers had limited knowledge of generating alternative solution methods to solve complicated questions.

To sum up, to use systematic counting, layer counting and column/row iteration methods, the middle school teachers need to have a deep knowledge of the subject matter however, the teachers lacked this knowledge thus they applied volume formula method in their lessons. As Lederman, Gess-Newsome and Latz (1994) explained, teachers' low level of subject matter knowledge influences their instructional decisions. Therefore, the teachers would not be effective in developing children's understanding of a topic (Murphy, 2012). In the current study, when the question was asked indirectly or was complicated, it required rich subject matter knowledge in order to develop a repertoire of different solution methods. Because of having inadequate subject matter knowledge, their repertoire of solution methods was limited as well.

The middle school teachers' subject matter knowledge was also investigated in terms of their knowledge of generating story problems related to the volume of 3D solids. The analysis of the data revealed the fact that the teachers' subject matter knowledge on generating story problems using given numbers and terms was not sufficiently strong. According to analysis of the data, one teacher could not make any interpretation regarding the problem which could be generated by the given numbers and terms. Two other middle school teachers tried to create a problem related to the volume, but their attempts were not appropriate. From the findings, it can be concluded that teachers posed problems without thinking about the mathematical aspects and did not pay attention to how the problems could be solved (Crespo, 2003). In order to present the topics appropriately and in multiple ways as in story problems, teachers should have a deep understanding of the subject (Ball, 1990a), thus, it can be concluded that the teachers had inadequate subject matter knowledge.

Even though these discussions are important part of teachers' knowledge, subject matter knowledge alone does not ensure effective teaching performance (Kahan, Cooper, & Bethea, 2003). Pedagogical content knowledge (PCK) is also necessary and this is discussed in the following sections.

5.2.2 The Nature of Middle School Teachers' Pedagogical Content Knowledge

As a result of the analysis of the data, the middle school teachers' pedagogical content knowledge is present below in the following four dimensions; knowledge of instructional strategy, knowledge of learners, knowledge of curriculum and knowledge of assessment. These dimensions of the middle school teachers' PCK are discussed on the basis of the related literature.

5.2.2.1 Knowledge of Instructional Strategy

The teachers' knowledge of instructional strategy was one of the dimensions of middle school teachers' PCK as used in with several other studies (Grossman, 1990; Magnusson et al., 1999). This dimension has been defined as the teachers' knowledge of subject-specific strategies, presenting broad applications for mathematics teaching, and topic-specific strategies, including ways to represent concepts (pictures, tables, graphs) and instructional strategies (investigations, demonstrations, simulations or problems) facilitating students' learning of specific topics in mathematics (Magnusson et al., 1999).

The research findings revealed that the middle school teachers tended to apply teacher-centered instructional strategy to teach students the volume of 3D solids. The teachers provided clear explanations and examples concerning the topic followed by questioning the students in order to understand how much they had learned. Moreover, they used manipulative to help their students envisage and visualize 3D solids. In the literature, there is support for this finding in that teachers tend to teach using teacher-centered methods (Mellado, 1998). In the current study, the teachers' instructional strategy to teach the volume of 3D solids was parallel to the results of other studies concerning middle school teachers (Friedrichsen &Dana, 2005; Koballa et al., 2005; Magnusson et al., 1999). There might be many reasons for the teachers in the current study to apply teacher-centered instructional strategy when teaching the volume of 3D solids as explained below.

A major influence on the way the middle school teachers' taught was their prior experience (Eick & Reed, 2002). Teachers who lack of experience related to using different instructional strategies in their teaching (Flick, 1996) and have inadequate subject matter knowledge might be reasons for applying teacher-centered strategy to teach the volume of 3D solids.

The complex nature of 3D solids might lead to the middle school teachers implementing teacher-centered instruction. The teachers might think it better to teach the topic directly instead of sharing the responsibility with the students since the teachers might think that the students might not be able to visualize and envisage 3D solids on their own. The middle school teachers may consider that using manipulative would facilitate students' understanding of the structure of 3D solids. In other words, the teachers believe that for the students to learn such a difficult topic that the teacher should orchestrate the instruction. However, Borko and Putnam (1996) claimed the opposite in that the teacher should support students' learning through student-centered instructional strategies rather than engaging in teacher-centered instructional strategies.

Another reason for applying teacher-centered instruction to teach the topic might be the number of concepts that need to be taught. The volume of 3D solids includes main concepts such as the volume of a pyramid, prism, cone and sphere. The middle school teachers might think that students would confuse the calculation of the volume of these shapes if they did not provide clear explanation to discriminate between them. Also, according to the middle school teachers, elements such as; height, base, and height of the side-face, are the other concepts that elementary students might confuse. In order to help students gain a clear idea of these concepts, the teachers might prefer to explain them directly. In addition, since there are many concepts in the topic, by adopting a teacher-centered strategy the teachers might consider that they can help students develop their understanding of all the concepts needed to solving many different problems. This is important in relation for the students to achieve a high score in the national exam. Since calculating the volume of 3D solids is one of the topics in the exam, the middle school teachers are under pressure to solve many questions and this takes time thus they could not apply activities or conduct discussions when teaching the topic.

Lastly, another probable reason for using teacher-centered instruction might be that the topic, the volume of 3D solids, is one of the last topics of the 8^{th} grade curriculum in Turkey (MoNE, 2006). Because of the time constraint, teachers may ignore the topic or teach it superficially.

Understanding teachers' instructional strategies provides an insight into how teachers use their content knowledge, and integrate this knowledge with their PCK with its other dimensions (knowledge of learners, knowledge of curriculum, and knowledge of assessment). A further aspect of PCK is the middle school teachers' knowledge of learners which is discussed in the following section.

5.2.2.2 Knowledge of Learners

The middle school teachers' knowledge of learners is important issue in terms of effective teaching and learning (Ball & Cohen, 1999). The knowledge of learners, one of the dimensions of PCK, was defined as teachers' knowledge of students' prior knowledge, and their misconceptions/difficulties in learning specific topic (Magnusson et al, 1999). In order to facilitate mathematics instruction and learning, this knowledge has a pivotal role in understanding students' thinking process (Fennema, Carpenter, Franke, Levi, Jacobs & Empson, 1996).Thus, the present study aimed to obtain information regarding the teachers' knowledge of students' understanding. To this end, the research findings concerning the middle school teachers' knowledge of the students' preferences among the solution methods that could be used to calculate the volume of 3D solids, the middle school teachers' interpretations of students' alternative solution methods, and the middle school teachers' knowledge concerning to students' errors related to the volume of 3D solids are presented and discussed. Additionally, discussions on the sources of students' errors and the ways of overcoming students' errors are presented.

Middle school teachers stated that most of the students prefer to use volume formula to calculate the volume of 3D solids. According to the teachers, in order to calculate the volume of 3D solids through volume formula, it is sufficient to have knowledge of algorithmic calculations. In other words, students do not need to know the structure of 3D solids when applying the volume formula. However, the teachers specified that students should know about the structure of 3D solids to apply other solution methods such as systematic counting, layer counting and column/row iteration. For instance, the systematic counting strategy requires counting the number of unit cubes exterior and interior of the prism. In that case, students should consider that some unit cubes are common unit cubes on the adjacent faces and these cubes should only be counted once. Moreover, the students should realize that there are unit cubes on the invisible faces and inside the prism. In addition, students should know that the prism is formed by layers in order to apply layer counting strategy which depends on the knowledge of the structure of prism. According to the teachers, because the students' have a limited knowledge of the structure of 3D solids, the students prefer to use a formula to calculate the volume of 3D solids. The middle school teachers' expressions regarding students' preferences among different solution method were consistent with the solution method that the students used when calculating the volume of 3D solids in their lessons. During the classroom observation, it was observed that students had a tendency to calculate the volume of 3D solids through the formula.

On the other hand, similar to their elementary students, the teachers also prefer to use volume formula when teaching the topic. This reveals that middle school teachers possessed the required procedural knowledge regarding the volume of 3D solids. As Fennema and Franke (1992) emphasized that the teacher's conceptual and procedural understanding of a topic influences their teaching. In other words, because the teachers introduced the volume formula in the lesson, students tended to use it. As stated by Zacharos (2006), if instruction involves procedural knowledge and the use of formula, the students will insist on using the formula. This means that the middle schoolteachers were able to identify the most frequently used solution methods by the students to solve the questions related to the volume of 3D solids. The result is consistent with that found by Carpenter et al. (1988) which concluded that teachers had knowledge that allowed them to identify their students' methods to solve addition and subtraction problems.

The data was analyzed to investigate the middle schoolteachers' knowledge concerning the interpretation of the students' alternative solution methods. In a general sense, the teachers were able to interpret students' solution methods if they were familiar with those methods. On the contrary, the middle school teachers were not able to interpret students' alternative solution methods if the teachers themselves had not used them in their teaching. Thus, the teachers' knowledge about their students' methods was limited by their experiences. This result could be interpreted as middle schoolteachers did not having a sufficiently broad knowledge of the students' alternative solution methods. The reason for this might be that the middle schoolteachers did not know how students thought when solving the questions. Based on the accessible literature, the findings of the present study paralleled previous studies that aimed to investigate teachers' knowledge of learners on different mathematic topics (Esen & Cakiroglu, 2012; Hill, 2007; Turnuklu & Yesildere, 2007). For instance, Hill (2007) emphasized that middle schoolteachers did not have sufficient knowledge to explain the mathematical ideas underpinning the students' alternative solution methods regarding number and operations, and algebra. Moreover, Turnuklu and Yesildere (2007) concluded that teachers did not have an adequate knowledge of fractions in order to identify students' incorrect solution methods. In addition, the findings of their study revealed that the middle school teachers had difficulty in interpreting students' incorrect solution methods. Similar to the present study, Esen and Cakiroglu conducted a study to explore teachers' knowledge in relation to their interpretation of students' incorrect solutions regarding the calculation of the volume of different shapes. The researchers concluded that the teachers could not justify the students' incorrect solution methods and could not find the errors in the students' incorrect solutions. On the contrary, Gokturk et al. (2013) claimed that teachers were able to interpret students' incorrect solution methods related to variables. In the present study, the reason for not be able to interpretation of students' alternative solution methods might be the middle school teachers' inadequate subject matter knowledge. As emphasized in the literature, teachers' SMK influences their PCK (Ball, 1991b; Ball & Bass, 2002; Kahan, Cooper & Bethea, 2003; Shulman, 1986). Thus, having a limited SMK may restrict their understanding of students' thinking. As a result, they may have difficulty in interpreting students' alternative solution methods regarding the volume of 3D solids.

The middle school teachers' knowledge of student errors related to the volume of 3D solids was investigated. Moreover, the reasons for students' errors and the strategies to be used to eliminate them were examined. The middle school teachers gave the following four errors; focusing on the faces of 3D solids, over-counting the common unit cubes on adjacent faces, conceptual errors, and computational errors. These errors were consistent with studies in the literature (Battista & Clements, 1996; Ben- Chaim, Lappan & Houang, 1985; Hirstein, 1981). The findings of earlier studies identified similar errors made by elementary students. Battista and Clements (1996) reported that elementary students made the following errors when calculating the volume of prism: a) counting the unit cubes on the visible faces in the picture, b) counting the unit cubes on six faces, c) counting the unit cubes on some visible and hidden faces, d) counting the unit cubes on the front face only, d) counting the unit cubes on faces but not systematically (Battista & Clements, 1996). In a similar vein, Ben-Chaim et al. (1985) identified students' errors made when calculating the number of unit cubes in a prism. The errors were as follows; a) counting the number of visible faces, b) counting the number of visible faces and doubling that number, c) counting the number of visible cubes, and d) counting the number of visible cubes and doubling that number. As it can be seen, the results of both studies stated that students focused on the faces of the prism when calculating its volume. Consistently, the middle school teachers also presented the same error, focusing on the faces of 3D solids that elementary students might make. Moreover, according to Hirstein (1981), elementary students might calculate the surface area of 3D solids when calculating its volume. Similarly, the middle school teachers noted the same error which was considered to be conceptual error. As a result, earlier studies supported the information given here about middle school teachers' knowledge of students' errors related to the calculation of the volume of a geometric shape. This could be interpreted as the middle school teachers' having adequate knowledge of students' errors made when calculating the volume of 3D solids. Although the teachers in the present study had sufficient knowledge on elementary students' errors in the volume, Kilic (2011) and Isik et al. (2013) concluded that the teachers' had limited knowledge to identify student errors in some topics. For instance, the teachers' knowledge regarding algebra and fractions was not sufficient to identify the students' errors. On the other hand, Aygun et al. (2013) and Gokturk et al. (2013) found that the teachers could identify students' errors related to the equal sign and variables. The results of Aygun et al. (2013) and Gokturk et al. (2013) were consistent with the result of current study in terms of the level of the teachers' knowledge in relation to determine the errors of their students to here.

The surprising finding of the study is related to identifying the students' errors in the volume of 3D solids. When the incorrect solution methods of students were presented to the middle school teachers, they could not identify the errors which caused students to give the incorrect answer. However, when the teachers where asked about the possible errors that the students might make when calculating the volume of 3D solids, then they were able to state several errors. The reason for this might be that the middle school teachers did not have adequate knowledge concerning students' thinking (Carpenter, Fennema, & Franke, 1996).

Parallel to the errors, the middle school teachers identified six sources of students' errors in calculating the volume of 3D solids. According to the middle school teachers, the most important source of students' errors was that they were not being able to think of solids as three-dimensional. The middle school teachers thought that students considered three-dimensional solids as two-dimensional. For this reason, the elementary students focused on the faces of 3D solids, and confused volume and area. Similar findings were also given in the literature (Battista et al., 1996; Ben-Chaim et al., 1985; Hirstein, 1981). In the study of Ben-Chaim et al. (1985), the elementary students focused on the faces of the 3D solids when calculating its volume. Consistent with the middle school teachers' knowledge, Ben-Chaim et al. (1985) asserted that students were not aware of the three-dimensionality of the solid. They stated that "dealing with two dimensions rather than three is related to some aspects of spatial visualization ability" (p. 406). The results of Ben-Chaim et al. verified that the teachers' knowledge in terms of the source of students' error was that they were not being able to think of solids as three-dimensional. Moreover, Battista et al. (1996) stated that students considered 3D solids as an uncoordinated set of faces which confirmed the level of middle school teachers' knowledge of the source, as not being able to think solids as three-dimensional. Moreover, the middle school teachers stated that students might confuse volume and area because they were not being able to think of solids as three-dimensional. This source of the students' error was confirmed by Hirstein (1981). He also concluded in his study that elementary students might calculate the surface area of 3D solid instead of calculating its volume because they did not considering solids as three-dimensional.

Another source of the error stated by the middle school teachers was not being able to comprehend the structure of 3D solid. According to the middle school teachers, the students' error of focusing on the faces of 3D solid emanated from not knowing the structure of 3D solid. According to the middle school teachers, students may think that a 3D solid is like an empty box thus, the students may only focus on the faces of 3D solid. For instance, students might not think that the prism is composed of the layers, and that a layer is composed of a column/row. Also, that the column/row is formed by unit cubes. Since they did not know the structure of 3D solids, the students concentrated on the faces of a 3D solid when calculating its volume. In other words, elementary students might not have the appropriate spatial awareness of the structure of 3D solids defined as "the mental act constructing an organization or form for an object or set of objects" (Battista et al., 1996, p.282).

Also, with respect to the middle school teachers' knowledge, another source of students' errors in relation to the faces of 3D solids and confusing the volume and perimeter was not being able to concretize 3D solids. The middle school teachers stated that the elementary students had difficulty in mentally visualizing 3D solids as stated by Olkun (1999; 2001; 2003) . Thus, students were not able to consider the inside of the prism and they concentrated on the outer faces of the 3D solids when calculating its volume. Furthermore, due to not being able to concretize 3D solids, they might confuse the concepts such as perimeter and volume. For this reason, they might calculate the perimeter of the 3D solids instead of its volume.

According to the middle school teachers, the most widespread source of students' errors was the students' carelessness. This consisted of focusing on the faces of 3D solid, over-counting the common unit cubes on the adjacent faces, not being able to calculate the area of the triangle, not being able to apply Pythagorean

Theorem and multiplication error. Due to being careless, students may forget the three-dimensionality of the 3D solids. For this reason, they may focus on the faces of 3D solids. Furthermore, middle school teachers stated that owing to their carelessness, the students did not consider the common unit cubes on the adjacent faces. For the same reason, the students may make errors in the multiplication operation and they may not calculate the area of the triangle when calculating the volume of pyramid. Students may not apply Pythagorean Theorem when finding the length of edges which was necessary for calculating the volume of 3D solids. According to the middle school teachers, this source of error was the least important of students' errors because it is not specific to the volume of 3D solids.

In relation to the sources of students' errors, the middle school teachers specified that students did not think deeply about the concepts. This could be explained as those students did not interpret the concepts such as the volume, relate the concepts with the other concepts that they had previously learnt and their daily life, and give the meaning of the volume formula. According to the teachers, because they did not think deeply about the concepts, the elementary students might make errors such as; over-counting the common unit cubes on the adjacent faces and computational errors. Regarding over-counting the common unit cubes on the adjacent faces, the middle school teachers explained that students might not consider how the unit cubes are placed in a prism. According to the middle school teachers, their students could not establish the relationship between units as Battista and Clements (1996) stated. For this reason, students did not think that some unit cubes belonged to two or three faces. This showed that students were not aware of the structure of 3D solids. Additionally, the middle school teachers specified that one of the sources of computational error was not thinking deeply about the concepts. For instance, students might make an error in multiplication, and the reasons for this kind of error might not consider which numbers should be multiplied to calculate the volume. They may multiply the numbers without understanding the meaning of the numbers. In other words, students might not consider the depth, width and height of the prisms. Additionally, the middle school teachers stated that their elementary students did not think about how to calculate the area of the triangle for the volume of prism and they could not apply Pythagorean Theorem. That is, the students might not know which edges of the triangle should be multiplied to calculate the area of the triangle or they may not know the reasons for dividing the result of multiplication of the base and the height of the triangle by 2. In relation to not being able to apply the Pythagorean Theorem, the students might not discriminate the right edges and the hypotenuse of the triangle or they might not know give the meaning of the Pythagorean Theorem. For these reasons, the source of the errors, not being able to calculate the area of a triangle and not being able to apply Pythagorean Theorem, was not thinking deeply about the concepts.

Finally, the middle school teachers specified a lack of conceptual knowledge as one of the sources of the students' errors in the volume concepts. According to the middle school teachers, confusion of the concepts emanated from the lack of conceptual knowledge. Due to the fact that the elementary students do not comprehend the concepts, it is indispensable in avoiding errors. For instance, middle school teachers stated that the reason for confusing volume and area might be due the students' lack of conceptual knowledge. Put it differently, students did not know what the volume or area was when they were trying to calculate them. If students comprehended the volume and area, then they might not confuse them as stated by Hirstein (1981).

The middle school teachers specified six sources of students' errors in relation to the concept of volume. This result could be interpreted as the middle school teachers having knowledge on the sources of students' errors related to the volume of 3D solids. In order to eliminate these errors, the teachers proposed two strategies, using manipulatives and re-explaining the misunderstood part of the topic and, all the teachers used these strategies during their lessons. Although these strategies were not specific to the volume of 3D solids, the middle school teachers believed that they were the best ways to eliminate student errors. For instance, to eliminate errors, over-counting the common unit cubes on the adjacent faces, the middle school teachers might show the common unit cubes on the adjacent faces by using manipulative. In this way, the students may realize that some unit cubes belong to two or three faces. Moreover, for focusing on the faces of 3D solid, the middle school teachers may use base-ten blocks. In this manner, students may comprehend the structure of prism and concretize the prism in their minds. In addition, middle school teachers may help students to discriminate between the volume and area concepts via using manipulative. Additionally, according to the middle school teachers, re-explaining the misunderstood part of the topic could be used to eliminate all errors that elementary students made. For example, middle school teachers stated that elementary students might confuse the height of 3D solids and the height of side-face of 3D solids. To eliminate this error, middle school teachers may explain these concepts until the students understand.

As a result, middle school teachers had knowledge of learners to identify students' errors, to determine the sources of the errors and the strategies to eliminate these errors. Moreover, they had knowledge of the methods that elementary students used when calculating the volume of 3D solids.

5.2.2.3 Knowledge of Curriculum

In this study, another dimension of PCK was the teachers' knowledge of the curriculum is discussed in terms of the connection with other topics and changing the order of the topics in the curriculum.

Four middle school teachers connected the volume of 3D solids with the area of polygons, which was taught in previous years. By making this connection, it can be said that middle school teachers had vertical curriculum knowledge (Shulman, 1986). However, this relation between topics is not an indicator that teachers' knowledge of curriculum was strong since the area of polygons also uses the volume formula. This means that by relating the volume of 3D solids to the area of polygons may not help students to learn the volume of 3D solids in a meaningful way rather that students are likely only to remember how to calculate the area of polygons. In addition, when connecting these two topics, the teachers commented that students should know how to calculate the area of a polygon to apply the volume formula. This result led me to conclude that the middle school teachers referred to the area of the polygons because it is a prerequisite to calculating the volume of 3D solids.

Additionally, teachers made links to other topics such as the volume of the pyramid with the volume of the prism, taught in the same year. The purpose of connecting the two topics was to use the volume of the prism which they had learned a short time before to make the volume of pyramid more understandable. When connecting these topics, they focused on using manipulative. In this way, the middle school teachers aimed to help students visualize the connection. In other words, they tried to eliminate the error, which arose from the students not being able to concretize 3D solid.

Although teachers made links to the topics taught in previous years or in the same year, they did not connect the volume of 3D solids with topics to be taught in later years. Moreover, they did not make a connection between the volume of 3D solids and topics in other courses. Furthermore, the middle school teachers did not aim to link the topic with the future daily-life of the students. The reason for not connecting the topic with other topics or daily life might be that middle school teachers' knowledge concerning the relationship between the topics might not be well established.

Furthermore, the teachers' knowledge of the curriculum was discussed in terms of changing the order of the topics. It was connected to curriculum saliency and the robust SMK that middle school teachers had (Aydin, 2012). Curriculum saliency refers to teacher's knowledge on the position of the topic in the curriculum (Rollnick, Bennett, Rhemtula, Dharsey, & Ndlovu, 2008). As a result of the middle school teachers' curriculum saliency, they diagnosed a problem in where the sub-topics of 3D solids are located in the curriculum in terms of teaching the volume of 3D solids. To resolve this, they changed the order to make teaching more comprehensible to the students. In addition to their curriculum saliency, it is highly probable that the teachers' robust SMK may have helped them realize the problem. This result was in keeping with that of Basturk and Kilic (2011). They also concluded that teachers who had adequate content knowledge paid attention to the order of presentation of topics if necessary. Furthermore, their teaching experience may have an influence on the middle school teachers in terms of altering the order of the sub-topics of 3D solids in the curriculum. Similarly, Friedrichsen et al. (2007) stated that teachers' prior experience has influences in planning instruction. Throughout their teaching in the previous years, they might notice that the order in the curriculum disrupts the integrity of the sub-topics of 3D solids. According to the middle school teachers, this may prevent students from making relationship between the sub-topics of 3D solids and understanding the topic. Changing the order of the sub-topics of 3D solids revealed the teacher's knowledge and understanding of the topic which might be an indicator of teachers' adequate knowledge of curriculum.

In brief, middle school teachers had limited knowledge in terms of connecting the volume of 3D solids with other topics within mathematics or other lessons. However, they did have adequate knowledge to realize that there was a problem in the order of the sub-topics of the volume of 3D solids and to alter their order.

5.2.2.4 Knowledge of Assessment

Knowledge of assessment is one of the dimensions of teachers' PCK. It comprises teachers' knowledge of what to assess and how to assess students' learning (Magnusson et al., 1999). In the present study, the middle school teachers' knowledge of assessment was discussed in terms of how to assess.

When teaching the topic of the volume of 3D solids, the teachers implemented assessment strategies from the beginning to the end of the topic. Teachers need to be sure that students had a good understanding the previous parts before moving forward. In addition, to be able to learn the volume of one of the 3D solids (e.g. pyramid), students need to know the volume of another 3D solid (e.g. prism). The lack of this knowledge results in potential problems in learning the volume of 3D solids (Sirhan, 2007). For this reason, teachers should assess students' learning from the beginning to the end of the topic.

Magnusson et al. (1999) specified that teachers should know what to assess and how to assess. The middle school teachers used both formative and summative assessment strategies such as informal questioning, homework, paper-pencil test, performance homework and project work. The reasons for using informal questioning, one of the formative assessment strategies, might be to obtain feedback about students' understanding, to determine the points that students were not able to understand, to identify students' errors in the topic. This result was parallel to the result from the study by Lankford (2010). Additionally, teachers gave the students homework, which is another formative assessment strategy. However, they did not check whether the students could solve the questions in the homework, whether students had difficulties regarding the questions, and whether there were points that they could not understand in relation to the topic. The aim of giving homework was to encourage the students to study the topic which was inconsistent with the aim of using formative assessment strategies (Cowie & Bell, 1999; Lankford, 2010). Since the researchers (Cowie & Bell, 1999; Lankford, 2010) emphasized that the aim of using formative assessment was to provide feedback in order to guide their teaching. On the other hand, the middle school teachers applied a paper-pencil test, performance homework and a project as summative assessment strategies which are required by Ministry of Education. MoNE (2006) explained that students present their knowledge and skills by relating the topic to their daily-life during the preparation of performance homework. Moreover, the project work is similar to performance homework but requires creativity and high level skills (MoNE, 2006). Although MoNE (2006) introduced the aims of giving performance homework and project work, the purpose of giving them was not consistent with their aims that were presented by MoNE. Additionally, teachers used a paper-pencil test at the end of the topic. But they did not aim to determine points that students could not understand, the purpose was only to grade students. The interesting finding is that none of the middle school teachers gave quizzes at the end of one sub-topic of the volume of 3D solids and before starting a new one. However, quizzes may help teachers to understand which part of students' learning is lacking.

There might be two reasons for choosing the assessment strategies. One might be that teachers have a tendency to implement assessment strategies with which their understanding was assessed when they were student (Kamen, 1996). Another reason might be that they have to apply certain assessment strategies. In addition, the teachers' teaching style may influence their use of assessment strategies as stated by Lannin et al. (2008). The teachers in this study applied teacher-centered instructional strategy and thus, they may focus on assessing knowledge through traditional assessment strategies.

5.3 Implications

In this study, the four middle school teachers' subject matter knowledge and pedagogical content knowledge of the volume of 3D solids were investigated. In light of the findings, the study has several implications for middle schoolmathematics teachers working with elementary students, mathematics educators, the curriculum developers, textbook writers, and policy makers.

The current study verified the argument in the literature that teachers should have deep and broad subject matter knowledge (Gess-Newsome & Lederman, 1999; Shulman, 1987). Teachers need to recognize that just knowing and applying procedures or formulas does not mean that they have a deep understanding of SMK and PCK for teaching mathematics. Moreover, the middle school teachers' PCK cannot be strong and effective for mathematics teaching without a strong SMK (Maxedon, 2003). In the current study, the middle school teachers had limited SMK and PCK. Generally they did not develop alternative solution methods to calculate the volume of 3D solids and did not generate a story problem involving the volume. Moreover, they were not able to interpret students' solution methods and identify students' errors in relation to the volume of 3D solids. Additionally, they did not have rich repertoire of instructional strategies and they taught the topic through lecturing. Their lack of knowledge of instructional strategies may influence their instructional decisions since they did not know how to apply different instructional strategies. Furthermore, the teachers applied a limited number of assessment strategies (informal questioning, paper-pencil tests, homework). Thus, the middle school teachers should enroll in professional development programs to help them develop their understanding of the mathematics. The middle school teachers in the current study had not participated in any professional development programs. Participating professional development programs has several benefits for the teachers such as; providing opportunities to develop the depth and breadth of their mathematics content knowledge. Furthermore, the middle schoolteachers can identify students' misconceptions/difficulties/errors regarding a particular topic and determine ways to eliminate them. Moreover, the teachers need to increase their repertoire of instructional strategies to teach mathematics. Additionally, professional development programs should be provided to help the teachers to enrich the assessment strategies to determine the level of their students' learning. It is recommended that MoNE organizes middle school teachers

2 professional development programs for teachers and together with the school administration, encourages them to participate in these programs.

In the literature many authors claimed that teaching experience is a crucial source of teachers' PCK (Grossman, 1990; Shulman, 1987; van Driel et al., 2002). However, it does not guarantee a well-developed PCK (Friedrichsen et al., 2007) and this is supported by the findings of the current study since although the four teachers each had at least 12 years experience, their PCK was not robust. For instance, the teachers did not have adequate knowledge of students' errors and the ways of eliminating their errors because they did not focus on the students' errors during their teaching. In particular, they did not take the reasons for students' errors into consideration when they were teaching. They only considered whether students followed the procedure or not when calculating the volume of 3D solids. However, the teachers could have obtained knowledge of students' errors by checking their homework (An et al., 2004) and paper-pencil test but the middle school teachers did not check students' homework. It seems that the purpose of assigning homework was not to understand students' learning but only to make students revise what they have learned in class. Another way of obtaining information about student errors would be through tests but the aim of the paper-pencil test was to grade students. If teachers wish to support their students' learning, then they need to observe the students during lessons to understand their thinking. In addition, homework, performance homework, project, and paper-pencil test could be important indicator of students' learning. In order to do this effectively, the middle school teachers should focus on developing formative and summative assessment strategies to acquire deep and meaningful knowledge of how students learn and comprehend mathematics topics.

Moreover, the study offers implications for curriculum developers and textbook writers. The findings revealed that the middle school teachers applied teacher-centered instructional strategy. Therefore, they had tendency to teach the mathematics by lecturing. During the interview, middle school teachers stated that they want to apply activities but they do not know which activities can help students to facilitate their learning. For this reason, teachers should be supported by useful activities for teaching the topics effectively. In this regard, curriculum developers and textbook writers should include activities which promote students' learning. Additionally, textbook writers may give place to problems which support conceptual understanding rather than directing students to apply the formula. Guidelines for teachers that cover the important issues in mathematics teaching should be clearly presented and explained in the teacher copies of textbooks.

The recommendations for further research and limitations of the current study are presented in the following section.

5.4 Recommendations for the Future Research Studies

This research study was aimed to understand four middle school mathematics teachers' subject matter knowledge and pedagogical content knowledge related to the volume of 3D solids. Based on the findings, related research studies are suggested in this section. Future studies could be conducted to investigate teachers' knowledge about the other important areas of geometry, such as the surface area of 3D solids, triangles and angles. This would serve to present a larger picture of the middle school teachers' knowledge of geometry and since understanding teachers' knowledge of geometry has important role for understanding teachers' knowledge of mathematics (Maxedon, 2003), and this type of research would extend the understanding of the teachers' mathematics knowledge.

In this study, the effect of teachers' knowledge on students' learning was not investigated. However, in the literature it is claimed that the teachers' knowledge has an impact on the students' achievement in mathematics (Hill & Ball, 2004). In order to explore how teachers' SMK and PCK affects students' learning in a certain geometry topic, further studies need to be undertaken.

Researchers have claimed that teachers' orientation has pivotal role among the dimensions of PCK since it guides teachers in making instructional decisions (Brown et al., 2009; Koballa, Glynn, Upson and Coleman, 2005). Although middle school mathematics teachers' PCK was investigated under four dimensions, the teachers' orientation to mathematics teaching was not taken into consideration in the present study. Furthermore, there are many studies regarding teachers' orientation to science teaching but research aiming to investigate teachers' orientation to mathematics teaching is limited. Due to the importance of teachers' orientation on their PCK, research studies should be carried out to identify teachers' orientation to mathematics teaching and its effect on other dimensions of PCK.

The findings also suggest the need for further studies on the possible effects of teaching experience on the development of teachers' knowledge (Grossman, 1990; Shulman, 1987; van Driel et al., 2002). In order to identify the effects of teaching experience on teachers' knowledge, novice teachers and experienced teachers' knowledge about the same topic could be investigated and compared.

As stated by several researchers, PCK is a topic-specific construct (van Driel, et al., 1998; Veal & MaKinster, 1999). Although one of the aims of the present study is to investigate middle school teachers' topic-specific PCK, the literature has identified the need for more topic-specific PCK research. In order to present how topic-specific PCK is, teachers' PCK in different topics could be examined and contrasted.

In addition, pre-service teachers' knowledge of the volume of 3D solids could be investigated in order to provide support for teacher educators to design their methods course to enrich pre-service teachers' SMK and PCK.

In Turkey the elementary and secondary mathematics curriculum was revised in June 2013 (MoNE, 2013). The volume of 3D solids was removed from the 8th grade mathematics curriculum and included in the 10th grade mathematics curriculum. For this reason, further studies need to be done to examine secondary mathematics teachers' knowledge of the volume of 3D solids.

Lastly, quantitative research studies could be performed to investigate pre-service and in-service mathematics teachers' knowledge on several topics in mathematics. In this way, the findings of the study could be generalized to the broader context.

5.5 Limitations of the Study

This study examined four middle school mathematics teachers' knowledge regarding the volume of 3D solids. The findings of this study have made contributions to the literature however, there are also the following limitations to the current study; the selection of the participants, the data collecting instruments and procedures, researcher position, and the topic selected to be studied. These issues are detailed below.

It is indisputable that the selection of the participants is one of the limitations of the study. As stated in the methodology part, four middle school teachers volunteered to participate in the study. Since this was qualitative case study, the findings may change with respect to the participants' backgrounds, experiences, and beliefs. For this reason, different participants could produce different results.

Another limitation was directly related to the data collection instruments. The questionnaire concerning the volume of 3D solids was developed by the researcher and all the items were discussed with a mathematics educator and mathematics teachers. However, the questionnaire may have been influenced by the researcher's beliefs and biases. Moreover, most of the questions in the questionnaire were related to the volume of the prism. Although data gathered from the questionnaire related to teachers' knowledge on the volume of pyramid, cone and sphere were enriched through the classroom observation; the findings of the study were mostly related to the volume of prism. In addition, the findings were restricted to the data gathered from the questionnaire, interview, classroom observation and field notes.

The researcher's position is also a limitation of the study. As explained in the methodology part, one of the data collection tools was classroom observation in which the teachers' lessons were videotaped. The presence of researcher and the camera during the lessons might have had on effect on both the teachers and students. To minimize this, researcher attended a few lessons and videotaped before teachers started teaching the volume of 3D solids. Moreover, researcher's background and beliefs can lead to unintended biases during the data collection and analysis process. To minimize researcher's biases, several data collection tools (such as questionnaire, interviews and classroom observations) were used and the data was analyzed by a second coder. Furthermore, the findings of the study were discussed with my supervisor and thesis committee members throughout the data analysis process.

A final limitation of the study is related to the topic to be studied. In the 8^{th} grade mathematics curriculum, the volume of 3D solids was one of the last topics (MoNE, 2006). Because of the time constraint, teachers may ignore the topic or teach it superficially. The findings of the study might be influenced by this situation.

REFERENCES

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education, 7*, 145-172.

An, S., & Wu, Z. (2011). Enhancing mathematics teachers' knowledge of students' thinking from assessing and analyzing misconceptions in homework. *International Journal of Science and Mathematics in Education, (10*), 717-753.

Aslan-Tutak, F. (2009). *A study of geometry content knowledge of elementary preservice teachers: The case of quadrilaterals.* Unpublished doctoral dissertation, University of South Florida, USA.

Atiyah, M. (2001). Mathematics in the 20th century: Geometry versus algebra. *Mathematics Today, 37*(2), 46- 53.

Aydin, S. (2012). *Examination of chemistry teachers' topic-specific nature of pedagogical content knowledge in electrochemistry and radioactivity.* Unpublished doctoral dissertation, Middle East Technical University, Turkey.

Aygun, B., Baran-Bulut, D., & Ipek, A. S. (2013, Mayıs). *Sınıf öğretmeni adaylarının eşit işaretine yönelik alan bilgileri ve pedagojik alan bilgileri.* 12. Matematik Sempozyumu, Türkiye.

Baki, M. (2013). Sınıf öğretmeni adaylarının bölme işlemi ile ilgili matematiksel Bilgileri ve öğretimsel açıklamaları. *Eğitim ve Bilim, 38* (167), 300-311.

Ball, D. L. (1990a). The mathematical understanding that prospective teachers to teacher education. *Elementary School Journal , 90*, 449-466.

Ball, D. L. (1990b). Prospective elementary and secondary teachers' understanding of division. *Journal of Research in Mathematics Education, 21*, 132-144.

Ball, D. L. (1991). Research on teaching mathematics: Making subject-matter knowledge part of the equation. In J. Brophy (Ed.), *Advances in research on teaching: Vol: 2, Teachers' subject-matter knowledge* (pp.1-48). Greenwich,CT: JAI Press.

Ball, D. L. (2000). Bridging practices: interwining content and pedagogy in teaching and learning to teach. *Journal of Teacher Education, 51* (3), 241-247.

Ball, D. L., & Bass, H. (2002). Toward a practice-based theory of mathematical knowledge for teaching. In B. Davis & E. Simmt (Eds.), *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group*,(pp. 3-14). Edmonton, AB: CMESG/GCEDM.

Ball, D. L., Bass, H., & Hill, H. C. (2004). Knowing and using mathematical knowledge in teaching: Learning what matters, *Proceedings for the 12th Annual Conference of the Southern African Association of Mathematics, Science, and Technology Education, South Africa,* 1-19.

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners:Toward a practice-based theory of professional education. In L. D. Hammond & G.Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice*.San Francisco: Jossey-Bass.

Ball, D. L., Lubienski, S., & Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching*. (4th ed.). New York: Macmillan.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59* (5), 389- 407.

Basturk, S., & Donmez, G. (2011). Examining pre-service teachers' pedagogical content knowledge with regard to curriculum knowledge. *International Online Journal of Educational Sciences, 3*(2), 743-775.

Battista, M. T., & Clements, D. H. (1996). Students' understanding of three-dimensional rectangular arrays of cubes. *Journal for Research in Mathematics Education, 27* (3), 258- 292.

Battista, M. T., & Clements, D. H. (1998). Finding the number of cubes in rectangular cube building. *Teaching Children Mathematics, 4,* 258- 264.

Baturo, A., & Nason R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, *31,* 235-268.

Baxter, P., & Jack, S. (2008). Qualitative case study methodology: Study design and implementation for novice researchers. *The Qualitative Report*, *13*(4), 544-559.

Ben-Chaim, D., Lappan, G. & Houang, R. T. (1985). Visualizing rectangular solids made of small cubes: Analyzing and affecting students' performance. *Educational Studies in Mathematics*, *16,* 389- 409.

Berenson, S., Von Der Valk, T., Oldham, E., Runesson, U., Moreira, Q., & Broekman, H. (1997). An international study to investigate prospective teachers' content knowledge of the area concept. *European Journal of Teacher Education*, *20*(2), 137-150.

Berliner, D. C. (2001). Learning about and learning from expert teachers. *International Journal of Educational Research*, *35,* 463-482.

Bogdan, R. C., & Biklen, S. K. (1998). *Qualitative research for education: An introduction to theory and methods*. Boston: Allyn and Bacon.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher, 33*(8), 3-15.

Borko, H., Eisenhart, M., Brown, C. A., Underhill, R. G., Jones, D. & Agard, P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education*, *23*(3), 194-222.

Borko, H. & Putnam, R. T. (1996). Learning to teach. In D. C. Berliner & R. C. Calfee (eds.) *Handbook of Educations Psychology*. New York, NY: MacMillan.

Boz, N. (2004). *Interactions between knowledge of variables and knowledge about teaching variables.* Unpublished doctoral dissertation, University of Warwick, England.

Brown, P., Friedrichsen, P., & Abell, S. *(*2009*, April). Teachers' knowledge of learners and instructional sequencing in an alternative certification program.* Paper presented at the annual meeting of the American Educational Research Association, *San Diego, CA.*

Bukova-Güzel, E. (2010). An investigation of pre-service mathematics teachers' pedagogical content knowledge, using solid objects. *Scientific Research and Essays,*5(14), 1872-188.

Bulut, M. (2007). Curriculum reform in Turkey: A case of primary school mathematics curriculum. *Eurasia Journal of Mathematics, Science & Technology Education, 3*(3), 203-212.

Butun, M. (2005). *İlköğretim matematik öğretmenlerinin alan eğitimi bilgilerinin nitelikleri üzerine bir çalışma.*Unpublished master thesis, Karadeniz Technical University, Turkey.

Carpenter, T. P., Fennema, E., & Franke, M.L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal, 97* (1), 3-20.

Carpenter, T. P., Fennema, E., Peterson, P. L., & Carey, D. A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary mathematics. *Journal of Reserarch in Mathematics Education, 19* (5), 385-401.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C., & Loef, M. (1989). Using knowledge of children's mathematical thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*, 499–532.

Carreño, E., Ribeiro, C. M., & Climent, N. (2013, February). S*pecialized and horizon content knowledge – Discussing prospective teachers knowledge on polygons.* Paper presented at The Eight Congress of the European Society for Research in Mathematics Education, Turkey.

Carrillo, J., Climent, N., Contreras, L.C., & Muñoz-Catalán, M.C. (2013, February). *Determining specialised knowledge for mathematics teaching*. Paper presented at The Eight Congress of the European Society for Research in Mathematics Education, Turkey.

Chapman, O. (2002). High School Mathematics Teachers' Perspectives of Mathematical Word Problems. In B. Davis & E. Simmt (Eds.), *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group*,(pp. 91-98). Edmonton, AB: CMESG/GCEDM.

Chick, H. L., Baker, M., Pham, T., & Cheng, H. (2006). Aspects of teachers' pedagogical content knowledge for decimals. In J. Novotna, H. Moraova, M. Kraka & N. Stehlikova (Eds.) *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 297-304). Prague: PME.

Chinnapan, M. & Lawson, M. J. (2005). A framework for analysis of teachers' geometric content knowledge and geometric knowledge for teaching. *Journal for Mathematics Teacher Education, 8*, 197-221.

Clements, D. H. & Battista, M. T. (1992). Geometry and spatial understanding. In Dougles A. Grouws (Ed.), *Handbook of Research Mathematics Teaching and Learning* (pp. 420- 464). New York, McMillan Publishing Company.

Contreras, J. M., Batanero, C., Díaz, C., & Fernandes, J. A. (2011, February). *Prospective teachers' common and specialized knowledge in a probability task.* Paper presented at The Seventh Congress of the European Society for Research in Mathematics Education, Poland.

Corbin, J., & Strauss, A. (2008). *Basics of qualitative Research: Techniques and procedures for developing grounded theory* (3rd ed.). Thousand Oaks, CA: Sage.

Cowie, B. & Bell, B. (1999) A model of formative assessment in science education. *Assessment in Education, 6*(1), 101–116.

Crespo, S. (2003). Learning to pose mathematical problems: Exploring changes in pre-service teachers' practices. *Educational Studies in Mathematics, 52*, 243–270.

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five approaches* (2nd ed.). Thousand Oaks, California: Sage Publications, Inc.

Denzin, N.K., & Lincoln, Y.S. (2005). *The SAGE Handbook of Qualitative Research, Third Edition*. Thousand Oaks, CA: Sage Publications.

Eick, C. J., & Reed, C. (2002). What makes an inquiry-oriented science teacher? The influence of learning histories on student teacher role identity and practice. *Science Education, 86*(3), 401-416.

Esen, Y. ,& Cakiroglu, E. (2012). İlköğretim matematik öğretmen adaylarının hacim ölçmede birim kullanmaya yönelik kavrayışları. *MED, 1*, 21-30.

Even, R. (1993). Subject matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education, 24*(2), 94-116.

Even, R., & Tirosh, D. (1995). Subject matter knowledge and knowledge about students as sources of teacher presentations of the subject matter. *Educational Studies in Mathematics, 29*(1), 1-20.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B.(1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education, 27,* 403-434.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). NewYork: Macmillan.

Flick, L. B. (1996). Understanding a generative learning model of instruction: A case study of elementary teacher planning. *Journal of Science Teacher Education, 7(2),* 95-122.

Fraenkel J. R., & Wallen, N. E. (2006). *How to Design and Evaluate Research in Education* (6th ed.). NY: McGraw-Hill.

Friedrichsen, P. M., & Dana, T. M. (2003). Using a card-sorting task to elicit and clarify science-teaching orientations. *Journal of Science Teacher Education, 14*, 291-309.

Friedrichsen, P. M. & Dana, T. M. (2005). Substantive-level theory of highly regarded secondary biology teachers' science teaching orientations. *Journal of Research in Science Teaching, 42,* 218-244.

Friedrichsen, P. M., Lankford, D., Brown, P., Pareja, E., Volkmann, M. J., & Abell, S. K. (2007, April). *The PCK of future science teachers in an alternative certification program.* Paper presented at the National Association for Research in Science Teaching Conference, New Orleans, LA.

Fujita, T, & Jones, K. (2006). P*rimary trainee teachers' understanding of basic geometrical figures in Scotland.* Paper presented at the annual meetings of the International Group For The Psychology Of Mathematics Education, Czech Republic.

Gess-Newsome, J. (1999). Pedagogical content knowledge: An introduction and orientation nature, sources and development of pedagogical content knowledge for science teaching, In J. Gess-Newsome & N. G. Lederman (Eds.). *Examining pedagogical content knowledge: The construct and its implications for science education* (pp.3-17). Boston: Kluwer.

Glaser, B.G., & Strauss, A.L., (1967). *The Discovery of Grounded Theory: Strategies for Qualitative Research.* New York: Aldine.

Gokturk, B., Sahin, O., & Soylu, Y. (2013, Mayıs). *Öğretmen adaylarının degişken kavramına yönelik pedagojik alan bilgilerinin ögrenci hataları baglamında incelenmesi.* 12. Matematik Sempozyumu, Türkiye.

Gomes, A. (2011). Portuguese pre-service elementary teachers' knowledge of geometric transformations: An exploratory study. In Smith, C. (Ed.), *Proceedings of the British Society for Research into Learning Mathematics, 31(3)*, 59-64.

Goulding, M., Rowland, T., & Barber, P. (2002). Does it matter? Primary teacher trainees' subject knowledge in mathematics. *British Educational Research Journal, 28*(5), 689-704.

Grossman, P. L. (1990). *The making of a teacher: Teacher knowledge and teacher education.* New York: Teachers College.

Haciomeroglu, G., Haciomeroglu, E.S., & Aspinwall, L. N. (2007). Prospective secondary teachers' subject matter knowledge and pedagogical content knowledge: The ramifications for teaching. In Lamberg, T., & Wiest, L. R. (Eds.). *Proceedings of the 29th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Stateline (Lake Tahoe), NV: University of Nevada, Reno.

Hill, H. C.(2007). Mathematical knowledge of middle school teachers: Implications for the no child left behind policy initiative. *Educational Evaluation and Policy Analysis, 29* (2), 95-114.

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education, 35*(5), 330-351.

Hill, H. C., Ball, D. L., & Schilling, S. (2004). Developing measures of teachers' mathematics knowledge for teaching. *Elementary School Journal, 105*(1), 11-30.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Hirstein, J. J. (1981). The second national assessment in mathematics: Area and volume. *Mathematics Teacher, 74*, 704-708.

Howald, C. L. (1998). Secondary teachers' knowledge of functions: Subject matter knowledge, pedagogical content knowledge, and classroom practice. *Dissertation Abstracts International 59* (5), 1500.

Huang, R., & Kulm, G. (2012). Prospective middle grade mathematics teachers' knowledge of algebra for teaching. *The Journal of Mathematical Behavior, 31* (4), 417-430.

Hutchison, L. (1997). *Learning for teaching: A case of constructing the bridge between subject matter knowledge and pedagogical content knowledge (Report No. 143).*

Isik, C., Ocal, T., & Kar, T. (2013, February). *Analysis of pre-service elementary teachers' pedagogical content knowledge in the context of problem posing.* Paper presented at The Eight Congress of the European Society for Research in Mathematics Education, Turkey.

Isiksal, M. (2006). *A study on pre-service elementary mathematics teachers' subject matter knowledge and pedagogical content knowledge regarding the multiplication and division of fractions.* Unpublished doctoral dissertation, Middle East Technical University, Turkey.

Isiksal, M., & Cakiroglu, E. (2008). Pre-service teachers' knowledge of students' cognitive processes about the division of fractions. *Hacettepe University Journal of Education, 35,* 175- 185.

Johnson, R. B. (1997). Examining the validity structure of qualitative research. *Education*, *118*, 282-292.

Kagan, D. M. (1992). Professional growth among preservice and beginning teachers. *Review of Educational Research, 62,* 129-169.

Kahan, J.A., Cooper, D.A., & Bethea, K.A. (2003). The role of mathematics teachers' content knowledge in their teaching: A framework for research applied to a study of student teachers. *Journal of Mathematics Teacher Education, 6,* 223-252.

Kamen, M. (1996). A teacher's implementation of authentic assessment in an elementary science classroom. *Journal of Research in Science Teaching, 33,* 859-877.

Karahasan, B. (2010). *Preservice secondary mathematics teachers' pedagogical content knowledge of composite and inverse function.* Unpublished doctoral dissertation, Middle East Technical University, Turkey.

Kellogg, M. S. (2010). *Preservice elementary teachers' pedagogical content knowledge related to area and perimeter: A teacher development experiment investigating anchored instruction with web-based microworlds.* Unpublished doctoral dissertation, University of South Florida, USA.

Kilic, H. (2011). Preservice secondary mathematics teachers' knowledge of students.*Turkish Online Journal of Qualitative Inquiry, 2*(2), 17-36.

Koballa, T. R., Glynn, S. M., Upson, L., & Coleman, D. (2005). Conceptions of teaching science held by novice teachers in an alternative certification program. *Journal of Science Teacher Education, 16*, 287-308.

Kwong, C. W., Joseph, Y. K. K., Eric, C. C. M., Khoh, L.-T., S., Gek, C. K., & Eng, N. L. (2007). Development of mathematics pedagogical content knowledge in student teachers. *The Mathematics Educator*, *10* (2), 27-54.

Lankford, D. (2010). *Examining the pedagogical content knowledge and practice of experienced secondary biology teachers for teaching diffusion and osmosis.* Unpublished doctoral dissertation, University of Missouri, MO, USA.

Lannin, J. K., Abell,, S. K., Arbough, F., Chval, K., Friedrichsen, P. , & Volkmann, M. (2008). Researching Teacher Knowledge: Further Delineating the PCK Construct for Science and Mathematics Education, Paper presented at the annual meeting of the American Educational Research Association, March 24-28, New York, NY.

Leavitt, T. A. (2008). *German mathematics teachers' subject content and pedagogical content knowledge.* Unpublished doctoral dissertation, University of Nevada, USA.

Lederman, N., Gess-Newsome, J., & Latz, M. (1994). The nature and development of preservice science teachers' conceptions of subject matter and pedagogy. *Journal of Research in Science Teaching, 31*, 129-146.

Leinhardt, G., & Smith, D. A. (1985). Expertise in mathematics instruction: Subject matter knowledge. *Journal of Educational Psychology*, *77*, 247-271.

Lenhart, S. T. (2010). *The effect of teacher pedagogical content knowledge and the instruction of middle school geometry*. Unpublished doctoral dissertation, Liberty University, Virginia.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry.* Beverly Hills, CA: Sage.

Livy, S., & Vale, C. (2011). First year pre-service teachers' mathematical content knowledge: Methods of solution for a ratio question. *Mathematics Teacher Education and Development, 13*(2), 22-43.

Lucus, C. A. (2006). Is subject matter knowledge affected by experience? The case of composition of functions. In J. Novotna, H. Moraova, M. Kraka & N. Stehlikova (Eds.) *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 97-104). Prague: PME.

Ma, L. (1999). *Knowing and teaching elementary mathematics: teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

Magnusson, S., Krajcik, J., & Borko, H. (1999). Nature, sources and development of pedagogical content knowledge for science teaching. In J. Gess-Newsome & N. G. Lederman (Eds.), *Examining pedagogical content knowledge: The construct and its implications for science education* (pp. 95-132). Boston: Kluwer.

Marshall, C., & Rossman, G. B. (2006). *Designing Qualitative Research* (4th ed.). Thousand Oaks, CA: Sage.

Masters, J. (2012). Eighth grade in-service teachers' knowledge of proportional reasoning and functions: A secondary data analysis. *International Journal for Mathematics Teaching and Learning.* Retrieved April 19, 2012, from http://www.cimt.plymouth.ac.uk/Journal/masters.pdf

Maxedon, S. J. (2003). *Early childhood teachers' content and pedagogical knowledge of geometry.* Unpublished doctoral dissertation, The University of Arizona, USA.

Mellado, V. (1998). The classroom practices of preservice teachers and their conceptions of teaching and learning science. *Science Teacher Education, 82*(2), 197-214.

Merriam, S. B. (1998). *Qualitative research and case study applications in education (Rev. ed.).* San Francisco, CA: Jossey-Bass.

Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation*. San Francisco: Jossey-Bass.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative Data Analysis; An Expanded Source Book*. Sage Publications.

Ministry of National Education [MoNE] (2004). *Talim ve Terbiye Kurulu Baskanlığı, İlkögretim Matematik Dersi (1-5. Sınıflar) Öğretim Programı. (Board of Education, Elemenatry school mathematics curriculum (1-5th grades).* Ankara, Turkey: MoNE.

Ministry of National Education [MoNE] (2006). *İlkögretim matematik dersi ögretim programı 6-8. sınıflar: Ögretim programı ve kılavuzu*. Ankara, Turkey: MoNE.

Ministry of National Education [MoNE] (2009). *İlkögretim matematik dersi ögretim programı 6-8. sınıflar: Ögretim programı ve kılavuzu.* Ankara, Turkey: MoNE.

Ministry of National Education [MoNE] (2010/2011). *Milli eğitim istatistikleri: Örgün eğitim.* Ankara, Turkey: MoNE.

Ministry of National Education [MoNE] (2013). *Ortaokul matematik dersi ögretim programı 5-8. sınıflar.* Ankara, Turkey: MoNE.

Murphy, C. (2012). The role of subject knowledge in primary prospective teachers' approaches to teaching the topic of area. *J Math Teacher Educ, 15*,187–206.

National Council of Teachers of Mathematics. (1989, 2000). *Principles and Standards for School Mathematics.* Reston, VA: Author.

Ng, D. (2011). Indonesian Primary Teachers' Mathematical Knowledge for Teaching Geometry: Implica-tions for Educational Policy and Teacher Preparation Programs. *Asia-Pacific Journal of Teacher Education, 39*(2), 151-164.

Ng, G. L. (1998). *Exploring children's geometrical thinking.* Unpublished doctoral dissertation, University of Oklahoma, Norman.

Nilsson, P., & Lindstrom, T. (2012, July). *Connecting Swedish compolsory school teachers' content knowledge of probability to their level of education,teaching years and self-assessments of probability concepts.* Paper presented at the 12th International Congress on Mathematical Education, Seoul, Korea

Olkun, S. (1999). *Stimulating children's understanding of rectangular solids made of small cubes.* Unpublished doctoral dissertation, Arizona State University, USA.

Olkun, S. (2001). Öğrencilerin hacim formülünü anlamlandırmalarına yardım edelim. *Kuram ve Uygulamada Eğitim Bilimleri Dergisi, 1*, 181–190.

Olkun, S. (2003). Ögrencilere hacim formula ne zaman anlamli gelir?. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 25*, 160-165.

Patton, M.Q. (1987). *How to use qualitative methods in evaluation.* Newbury Park, CA: Sage Publications, Inc.

Patton, M. Q. (2002). *Qualitative evaluation and research methods* (3rd ed.). Thousand Oaks, CA: Sage.

Pino-Fan, L. R., Godino, J. D., Font, V., & Castro, W. F. (2013, February). *Prospective teacher's specialized content knowledge on derivative.* Paper presented at The Eight Congress of the European Society for Research in Mathematics Education, Turkey.

Rollnick, M., Bennett, J., Rhemtula, M., Dharsey, N., & Ndlovu, T. (2008). The place of subject matter knowledge in pedagogical content knowledge: A case study of South African teachers teaching the amount of substance and chemical equilibrium. *International Journal of Science Education, 30 (10),* 1365–1387.

Rowan, B., Schilling, S. G., Ball, D. L., & Miller, R. (2001). *Measuring teachers' pedagogical content knowledge in surveys: An exploratory study.* Ann Arbor: University of Michigan, Consortium for Policy Research in Education.

Rowland, T., Huckstep, P., &Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The Knowledge Quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, *8*(3), 255-281.

Schwab, J. J. (1964). The structure of disciplines: Meanings and significance. In G. W. Ford & L. Pugno (Eds.), *The structure of knowledge and the curriculum.* Chicago: Rand McNalley.

Schwab, J. J. (1978). *Science, curriculum and liberal education*. Chicago: University of Chicago Press.

Sevis, S. (2008). *The effects of a mathematics teaching methods course on pre-service elementary mathematics teachers' content knowledge for teaching mathematics.* Unpublished master thesis, Middle East Technical University, Turkey.

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information, 22,* 63-75.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harward Educational Review, 57*(1), 1-22.

Singmuang, C. (2002). *Thai preservice middle school mathematics teachers' subject matter knowledge and knowledge of students' conceptions of division of rational numbers with respect to their classroom practices.* Unpublished doctoral dissertation, Oregon State University, USA.

Stake, R. E. (1995). *The art of case study research.* Thousand Oaks, CA: Sage Publications, Inc.

Steele, M. D. (2006). M*iddle grades geometry and measurement: Examining change in knowledge needed for teaching through a practice-based teacher education experience.* Unpublished doctoral dissertation, University of Pittsburgh, USA.

Swafford, O. J., Jones, G. A., & Thornton, C. A. (1997). Increased knowledge in geometry and instructional practice. *Journal for Research in Mathematics Education, 28*(4), 467-483.

Turnuklu, E. B. (2005). The relationship between pedagogical and mathematical content knowledge of preservice mathematics teachers. *Eurasian Journal of Educational Research, 21,* 234-247.

Turnuklu, E. B., & Yesildere, S. (2007). The pedagogıcal content knowledge in mathematics: Preservice primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers:The Journal, 1.* Retrieved October 20, 2011 from http://www.k-12prep.math.ttu.edu/journal/contentknowledge/volume.shtml

Vais, A. M., & Reyhani, E. (2009). *The role of geometry in preparation of mathematics teachers.* Retrieved May 20, 2010 from http://www.recsam.edu.my/cosmed/cosmed09/AbstractsFullPapers2009/Abstract/Mathematics%20Parallel%20PDF/Full%20Paper/M04.pdf

Van Driel, J. H., de Jong, O., & Verloop, N. (2002). The development of preservice chemistry teachers' pedagogical content knowledge. *Science Education, 86,* 572-590.

Yesildere-Imre, S. & Akkoc, H. (2012). Investigating the development of prospective mathematics teachers' pedagogical content knowledge of generalising number patterns through school practicum. *Journal of Mathematics Teacher Education, 15,* 207–226.

Yildirim, A. & Simsek, H. (2006). *Sosyal Bilimlerde Nitel Araştırma Yöntemleri.* Seçkin Yayıncılık Ankara.

Yin, R. K. (1994). *Case Study Research: Design and Methods* (2nd ed.). Thousand Oaks, California: Sage Publications.

Yin, R. K. (2003). Case study research: Design and methods (3rd ed.). *Applied Social Research Methods Series, (Volume 5).* Thousand Oaks, CA: Sage Publications, Inc.

Yin, R. K. (2009). Case study research: Design and methods (4th ed.). Thousand Oaks, CA: Sage Publications, Inc.

Zacharos, K. (2006). Prevailing educational practices for area measurement and students' failure in measuring areas. *Journal of Mathematical Behaviour, 25,* 224–239.

Zhou, Z., Peverly, S. T., & Xin, T. (2006). Knowing and teaching fractions: A crosscultural study of American and Chinese mathematics teachers. *Contemporary Educational Psychology*, *31*, 438

APPENDICES

APPENDIX A

THE TURKISH VERSION OF VOLUME OF 3D SOLIDS QUESTIONNAIRE (VDSQ)

Merhaba,

Ben Reyhan TEKİN SİTRAVA. İlköğretim Bölümünde Matematik Eğitiminde doktora yapmaktayım. Prizma, piramit ve koninin hacmi ile ilgili bir araştırma yapıyorum. Bu araştırmada ortaya çıkacak sonuçların matematik eğitime katkısı olacağını düşünüyorum. Bu yüzden sizden düşüncelerinizi ve görüşlerinizi açıkça ifade etmenizi rica ediyorum.

Bana görüşme sürecince söyleyeceklerinizin tümü gizlidir. Araştırma sonuçlarını yazarken, görüştüğüm bireylerin isimlerini kesinlikle rapora yansıtmayacağım.

Başlamadan önce, bu söylediklerimle ilgili sormak istediğiniz bir soru var mı?

Görüşmenin yaklaşık 1 saat süreceğini tahmin ediyorum. Anlamadığınız bir soru veya herhangi bir şey olursa lütfen söyleyin. Şimdi sorulara başlamak istiyorum.

Reyhan TEKİN SİTRAVA
reyhan_tekin@yahoo.com

1. BÖLÜM: Demografik Bilgiler

1. Cinsiyetiniz: ☐ Bay ☐ Bayan

2. Mezun Olduğunuz Lise:

   ☐ Düz Lise ☐ Anadolu Lisesi ☐ Anadolu Öğretmen Lisesi ☐ Meslek Lisesi ☐ Diğer

3. Eğitim Durumu:

   ☐ Lisans ☐ Yüksek Lisans ☐ Doktora

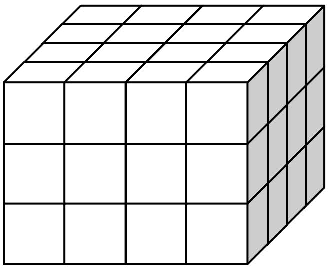
4. Görev Yaptığınız Okul:

5. Hizmet Yılınız:

   ☐ 0-5 yıl ☐ 6-10 yıl ☐ 11-15 yıl ☐ 16-20 yıl ☐ 21-25 yıl ☐ 26-30 yıl ☐ 31-35 yıl ☐ 36 ve fazlası

SORU SETİ

Aşağıda verilen soruların cevaplarını boş bırakılan yerlere veya boş bir kağıda cevaplayabilirsiniz.

1) 

Yanda verilen kare prizmayı oluşturan birim küp kaçtır?

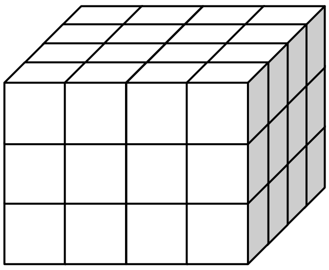
a) Bu problemin çözümünde kullanılabilecek bildiğiniz tüm yöntemleri yazınız.

b) Öğrencileriniz belirttiğiniz yöntemlerden hangisini veya hangilerini kullanarak bu problem çözerler?

c) Öğrencileriniz bu problemi çözerken hangi hataları yapabilirler?

d) Bu hataların kaynağı ne olabilir?

e) Siz bu hataları düzeltmek için hangi öğretim tekniklerini/ materyalleri/stratejileri kullanırsınız?

2) 

Yanda verilen kare prizmanın dış yüzeylerindeki birim küpler çıkartıldığında geriye kaç tane birim küp kalır?

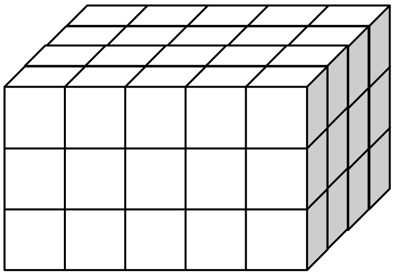
a) Bu problemin çözümünde kullanılabilecek bildiğiniz tüm yöntemleri yazınız.

b) Öğrencileriniz belirttiğiniz yöntemlerden hangisini veya hangilerini kullanarak bu problemi çözerler?

c) Öğrencileriniz bu problemi çözerken hangi hataları yapabilirler?

d) Bu hataların kaynağı ne olabilir?

e) Siz bu hataları düzeltmek için hangi öğretim tekniklerini/ materyalleri/stratejileri kullanırsınız?

3) 

"Yanda verilen dikdörtgenler prizmasının hacmini hesaplayınız" probleminde Mehmet Aslan'ın sınıfındaki öğrencilerin büyük çoğunluğu aynı hatayı yapıyor ve cismin hacmini 94 buluyorlar.

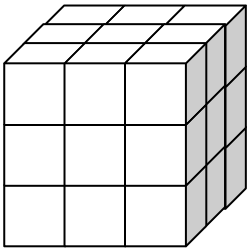
Buna göre;

a) Hata yapan öğrenciler problemi nasıl çözmüş olabilirler?

b) Öğrencilerin bu soruyu yanlış çözmelerine neden olan hataları nelerdir?

c) Bu hataların kaynağı ne olabilir?

d) Siz bu hataları düzeltmek için hangi öğretim tekniklerini/ materyalleri/ stratejileri kullanırsınız?

4) 

Leyla Aksoy yandaki küpün hacmini öğrencilerine sorar ve öğrencileri sonucun 27 olduğunu söyler. Leyla Aksoy öğrencilerinin soruyu farklı yöntemlerle çözdüğünü farkeder. Bu çözüm yollarından bazılarının doğru olmasına rağmen bazıları yanlıştır.

Buna göre;

a) Soruyu doğru çözen öğrenciler ne tür çözüm yolları geliştirmişlerdir? Açıklayınız

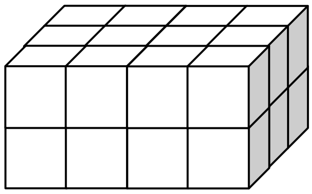
b) Soruyu yanlış çözen öğrenciler ne tür çözüm yolları geliştirmişlerdir? Açıklayınız

c) Soruyu yanlış çözen öğrencilerin hataları nelerdir? Açıklayınız.

d) Soruyu yanlış çözen öğrencilerin hatasının kaynağı nedir? Açıklayınız.

e) Siz bu hataları düzeltmek için hangi öğretim tekniklerini/ materyalleri/ stratejileri kullanırsınız?

5)



Yukarıda verilen dikdörtgen prizmasının hacmini Ela, Eren, Kuzey, Damla, Yağmur ve Berke farklı şekilde çözmüşler fakat aynı cevabı bulmuşlardır. Öğrencilerin çözümleri aşağıda verilmiştir.

Ela'nın çözümü: 26 x 2= 52 8 x 2= 16 52- 16= 36 36- 12= 24	***Eren'in çözümü:*** 6 + 6= 12 4 + 4= 8 12 + 8 + 4= 24
Kuzey'in çözümü: 4 x 3= 12 12 x 2= 24	***Yağmur'un çözümü:*** 6 + 6= 12 4 + 4 + 4 = 12 12 + 12= 24
Berke'nin çözümü: 4x 3 x 2= 24	

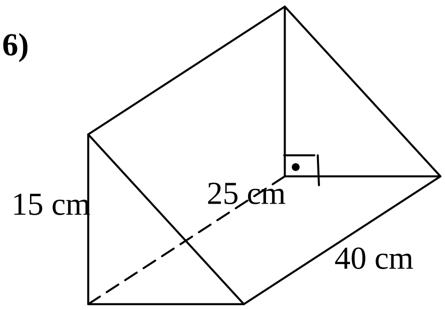
a) Öğrencilerin çözüm yollarını kendi cümlelerinizle açıklayınız.

b) Soruyu yanlış çözen öğrenci veya öğrencilerin hatalarının kaynağı ne olabilir?

c) Siz bu hataları düzeltmek için hangi öğretim tekniklerini/ materyalleri/ stratejileri kullanırsınız?

d) Sizin öğrencileriniz genelde hangi öğrencinin çözüm yolunu tercih ederler?

6)



Yanda dik üçgen prizma şeklinde kesilmiş bir peynir kalıbı bulunmaktadır. Bu peynir kalıbı 20 eşit dilime ayrıldığında elde edilen her bir dilimin hacmi ne kadar olur?

Arda Acar yukarıdaki problemi sınıfta öğrencilerine sorar ve farklı çözüm yolları ile karşılaşır.

Yankı'nın çözümü:

$a^2 = b^2 + c^2$

$25^2 = 15^2 + c^2$

$625 = 225 + c^2$

$400 = c^2 \Rightarrow c = 20$

$V = \frac{15.20}{2}.40$

$V = 6000$

Bir dilimin hacmi:

$\frac{6000}{20} = 300$

Asya'nın çözümü:

$V = \frac{15.25}{2}.40$

$V = 7500$

Bir dilimin hacmi:

$\frac{7500}{20} = 375$

Yaman'ın çözümü:

$a^2 = b^2 + c^2$

$25^2 = 15^2 + c^2$

$625 = 225 + c^2$

$400 = c^2 \Rightarrow c = 20$

$\frac{40}{20} = 2$

$V = \frac{15.20}{2}.2$

$V = 300$

Ada'nın çözümü:

$a^2 = b^2 + c^2$

$25^2 = 15^2 + c^2$

$625 = 225 + c^2$

$400 = c^2 \Rightarrow c = 20$

$V = \frac{40.20.15}{2}$

$V = 6000$

Bir dilimin hacmi:

$\frac{6000}{20} = 300$

Ilgaz'ın çözümü:

$a^2 = b^2 + c^2$

$25^2 = 15^2 + c^2$

$625 = 225 + c^2$

$400 = c^2 \Rightarrow c = 20$

$\frac{15.20}{2} = \frac{25.x}{2} \Rightarrow x = 12$

$V = \frac{40.25.12}{2} \Rightarrow V = 6000$

Bir dilimin hacmi: $\frac{6000}{20} = 300$

a) Sizce Arda Öğretmen'in öğrencileri bu çözüm yollarını geliştirirken ne düşünmüş olabilirler?

b) Eğer bu öğrencilerden soruyu yanlış çözen varsa bu öğrencinin/öğrencilerin hataları nelerdir?

c) Siz bu hataları düzeltmek için hangi öğretim tekniklerini/ materyalleri/ stratejileri kullanırsınırsınız?

7)

Yanda verilen kare piramit şeklindeki maketin tabanının bir kenar uzunluğu 6 cm ve yan yüz yüksekliği 5 cm'dir. Bu maketin hacmini hesaplayan Ceren ile Cemre problemi farklı yollarla çözmüşlerdir.

Ceren'in çözümü:

$V= \frac{6.\ 6.\ 5}{3}$

$V= \frac{180}{3} = 60\,cm^3$

Cemre'nin çözümü:

$a^2 = b^2 + c^2$

$5^2 = 3^2 + c^2$

$25 = 9 + c^2$

$16 = c^2$

$c= 4$

$V= \frac{6.\ 6.\ 4}{3}$

$V= \frac{144}{3} = 48\,cm^3$

a) Sizce Ceren ile Cemre bu çözüm yollarını geliştirirken ne düşünmüş olabilirler?

b) Eğer bu çözüm yollarından biri veya her ikisi yanlış ise yapılan hatalar ne olabilir?

c) Eğer bu çözüm yollarından herhangi biri yanlış ise yapılan hatanın kaynağı ne olabilir?

d) Siz bu hataları düzeltmek için hangi öğretim tekniklerini/ materyalleri/ stratejileri kullanırsınız?

8) Külah, yay uzunluğu, 15, yarıçap ve 54 cm ifadelerini kullanarak hacim bağıntısını içeren bir problem kurunuz.

9) Öğrencilerin üç boyutlu cisimlerin hacmini hesaplamaya ilişkin bilgilerini ölçmek için hangi ölçme araçlarını kullanırsınız?

10) 3 boyutlu cisimlerin hacmini matematik veya diğer derslerdeki hangi konu veya konularla ilişkilendirerek anlatırsınız?

APPENDIX B

A TABLE OF SPECIFICATION FOR THE QUESTIONNAIRE ITEMS

SMK		**PCK**						
		Knowledge of Learners					**Knowledge of Curriculum**	**Knowledge of Assessment**
Alternative Solution Methods	Generation A Story Problem	Students' Preferences among Different Solution Methods	Interpretations of Students' Alternative Solution Methods	Students' Errors	The Sources of These Errors	The Strategies to Overcome Elementary Students' Errors		
1a, 2a	8	1b, 2b, 5d	3a, 4a, 4b, 5a, 6a, 7a	1c, 2c, 3b, 4c, 6b, 7b	1d, 2d, 3c, 4d, 5b, 7c	1e, 2e, 3d, 4e, 5c, 6c,7d,	10	9

APPENDIX C

INTERVIEW PROTOCOL (IN ENGLISH)

SUBJECT MATTER KNOWLEDGE

- What are the solution methods that you use to solve this question?
- Do you know any other methods? If yes, please explain.
- Could you generate a story problem which involves the volume formula using a cornet, the length of arc, 15, radius and 54?
- Could you solve the problem that you generated?

PEDAGOGICAL CONTENT KNOWLEDGE

➢ **Knowledge of Learners**

- What method(s) do your students use to answer this question?
- Which error(s) do you think your students will make in answering this question?
- What may be the reasons for the error?
- Which teaching techniques/materials/strategies do you use to overcome these errors?
- According to you, what was the student thinking when s/he developed the methods of solving the question?

➢ **Knowledge of Curriculum**

- What other topic or topics within mathematics do you use to teach the volume of 3D solids?
- What other topic or topics within other lessons do you use to teach the volume of 3D solids?
- How do you use real life examples to teach the volume of 3D solids?

➢ **Knowledge of Assessment**

- What methods do you use to assess the students' knowledge related to the volume of 3D solids?

APPENDIX D

INTERVIEW PROTOCOL (IN TURKISH)

ALAN BİLGİSİ

- Bu problemin çözümünde hangi yöntemleri kullanırsınız?
- Başka yöntem biliyor musunuz? Eğer biliyorsanız, açıklar mısınız?
- Külah, yay uzunluğu, 15, yarıçap ve 54 cm ifadelerini kullanarak hacim bağıntısını içeren bir problem kurar mısınız?
- Kurduğunuz problemi çözer misiniz?

PEDAGOJİK ALAN BİLGİSİ

➢ **Öğrenci Bilgisi**

- Öğrencileriniz belirttiğiniz yöntemlerden hangisini veya hangilerini kullanarak bu problemi çözerler?
- Öğrencileriniz bu problemi çözerken hangi hataları yapabilirler?
- Bu hataların kaynağı ne olabilir?
- Siz bu hataları düzeltmek için hangi öğretim tekniklerini/materyalleri/stratejileri kullanırsınız?
- Öğrenci soruyu yanlış çözer ve öğretmen, öğrencinin hatasını fark ediyor/etmiyor.
- Öğretmen, öğrencinin hatasını gidermek için ………………….yöntemini kullanıyor.

➢ **Müfredat Bilgisi**

- 3 boyutlu cisimlerin hacmini matematikteki hangi konu veya konularla ilişkilendirerek anlatırsınız?
- 3 boyutlu cisimlerin hacmini diğer derslerdeki hangi konu veya konularla ilişkilendirerek anlatırsınız?
- 3 boyutlu cisimlerin hacmini gerçek hayatla nasıl ilişkilendirirsiniz?

➢ **Ölçme Bilgisi**

- Öğrencilerin üç boyutlu cisimlerin hacmini hesaplamaya ilişkin bilgilerini ölçmek için hangi ölçme araçlarını kullanırsınız?

APPENDIX E

OBSERVATION PROTOCOL (IN ENGLISH)

Teacher: School:

Subject: Date:

	Never	Sometimes	Always	EXPLANATIONS
SUBJECT MATTER KNOWLEDGE				
• Teacher solves the questions using alternative solution methods.				The methods are:
• Teacher takes notice of students' alternative solution methods and explains these methods to the other students.				
PEDAGOGICAL CONTENT KNOWLEDGE				
➢ **Knowledge of Learners**				
• Student solves the questions using different solution methods.				The methods are:
• Student solves the questions incorrectly and teacher does/doesn't recognize student's error.				
• Teacher uses different strategies to overcome student's error.				The strategies are:
➢ **Knowledge of Instructional Strategy**				
• Teacher transfers his/her knowledge to the students. S/he uses teacher-centered instructional method.				
• Teacher creates classroom environment that gives students opportunity to share their knowledge.				

➢ **Knowledge of Curriculum**				
• Teacher connects the topic with **a)** the topics (…………………….) in mathematics **b)** other topics (………………….) in other lessons (………………….) **c)** real life				
• Teacher alters the order of the topic.				The order is:
➢ **Knowledge of Assessment**				
• Teacher uses assessment strategies during the lesson.				The assessment strategies are:
• Teacher uses assessment strategies following the completion of the topic.				The assessment strategies are:

APPENDIX F

OBSERVATION PROTOCOL (IN TURKISH)

Öğretmen: Okul:

Konu: Tarih:

	Hiçbir zaman	Bazen	Her zaman	AÇIKLAMALAR
ALAN BİLGİSİ				
• Öğretmen soruyu farklı çözüm yöntemleri kullanarak çözer.				Çözüm yöntemleri:
• Öğretmen öğrencilerin farklı çözüm yöntemlerini önemser ve bu yöntemleri, diğer öğrencilere açıklar.				
PEDAGOJİK ALAN BİLGİSİ				
➢ **Öğrenci Bilgisi**				
• Öğrenciler soruları farklı çözüm yöntemleri kullanarak çözerler.				Çözüm yöntemleri:
• Öğrenci soruyu yanlış çözer ve öğretmen, öğrencinin hatasını fark eder/etmez.				
• Öğretmen, öğrencinin hatasını gidermek için farklı yöntemler kullanır.				Yöntemler:
➢ **Öğretim Metotları Bilgisi**				
• Öğretmen bilgiyi öğrencilere transfer eder. Öğretmen merkezli öğretim tekniği uygular.				
• Öğretmen, öğrencilerin bilgilerini paylaşacağı bir sınıf ortamı yaratır.				

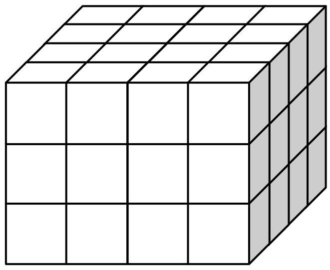
➢ **Müfredat Bilgisi**				
• Öğretmen konuyu a) matematikteki……....…..konuları b) ………….derslerdeki……....konuları c) gerçek hayat ile ilişkilendirir.				
• Öğretmen konunun sırasını değiştirir.				Sıra şu şekildedir.
➢ **Ölçme Bilgisi**				
• Öğretmen, ders esnasında ölçme ve değerlendirme yöntemleri kullanır.				Ölçme ve değerlendirme yöntemleri:
• Öğretmen, konu bittikten sonra ölçme ve değerlendirme yöntemleri kullanır.				Ölçme ve değerlendirme yöntemleri:

APPENDIX G

THE FIRST VERSION OF THE QUESTIONNAIRE, USED IN PILOT STUDY

SORU SETİ

Aşağıda verilen soruların cevaplarını boş bırakılan yerlere veya boş bir kağıda cevaplayabilirsiniz.



i) Yanda verilen kare prizmayı oluşturan birim küp sayı kaçtır?

ii) Yanda verilen kare prizmanın dış yüzeylerindeki birim küpler çıkartıldığında geriye kaç tane birim küp k alır?

1) Yukarıda verilen her bir problem (i-ii) için aşağıdaki soruları cevaplandırınız.

a) Bu konuyu müfredattaki hangi konu veya konular ile ilişkilendirerek anlatırsınız?

b) Bu problemin çözümünde kullanılabilecek bildiğiniz tüm yöntemleri yazınız.

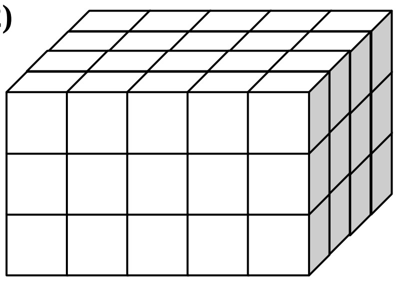
c) Öğrencileriniz belirttiğiniz yöntemlerden hangisini veya hangilerini kullanarak bu problemi çözerler?

d) Öğrencileriniz bu problemi çözerken hangi hataları yapabilirler?

e) Bu hataların kaynağı ne olabilir? Açıklayınız.

f) Siz bu hataları düzeltmek için hangi öğretim tekniklerini/ materyalleri/stratejileri kullanırsınız? Nasıl?

2)



"Yanda verilen dikdörtgenler prizmasının hacmini hesaplayınız" probleminde Hakan Aslan'ın sınıfındaki öğrencilerin büyük çoğunluğu aynı hatayı yapıyor ve cismin hacmini 94 buluyorlar.

Buna göre;

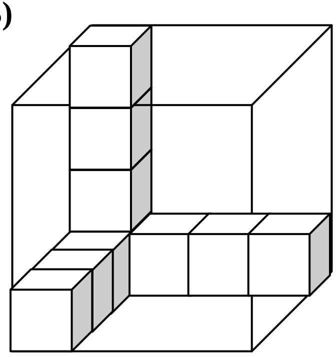
a) Hata yapan öğrenciler problemi nasıl çözmüş olabilirler?

b) Öğrencilerin bu soruda hata yapmalarına neden olan kavram yanılgıları nelerdir?

c) Bu kavram yanılgılarının kaynağı ne olabilir?

d) Siz bu hataları düzeltmek için hangi öğretim tekniklerini/ materyalleri/ stratejileri kullanırsınız? Nasıl?

3)



Yanda verilen büyük küpün hacmi kaçtır?

a) Bu konuyu müfredattaki hangi konu veya konular ile ilişkilendirerek anlatırsınız?

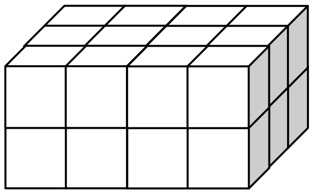
b) Bu problemin çözümünde kullanılabilecek bildiğiniz tüm yöntemleri yazınız.

c) Öğrencileriniz belirttiğiniz yöntemlerden hangisini veya hangilerini kullanarak bu problemi çözerler?

d) Öğrencileriniz bu problemi çözerken hangi hataları yapabilirler?

e) Bu hataların kaynağı ne olabilir? Açıklayınız.

f) Siz bu hataları düzeltmek için hangi öğretim tekniklerini/ materyalleri/ stratejileri kullanırsınız? Nasıl?

4) 

Yukarıda verilen dikdörtgen prizmasının hacmini Ela, Eren, Kuzey, Damla, Yağmur ve Berke farklı şekilde çözmüşler fakat aynı cevabı bulmuşlardır. Öğrencilerin çözümleri aşağıda verilmiştir.

.

Ela'nın çözümü: 26 x 2= 52 8 x 2= 16 52- 16= 36 36- 12= 24	***Eren'in çözümü:*** 6 + 6= 12 4 + 4= 8 12 + 8 + 4= 24
Kuzey'in çözümü: 4 x 3= 12 12 x 2= 24	***Damla'nın çözümü:*** 12 x 2= 24 24 – 4= 20 20 + 4= 24
Yağmur'un çözümü: 12 x 2= 24	***Berke'nin çözümü:*** 4 x 3 x 2= 24

a) Öğrencilerin çözüm yollarını kendi cümlelerinizle açıklayınız.

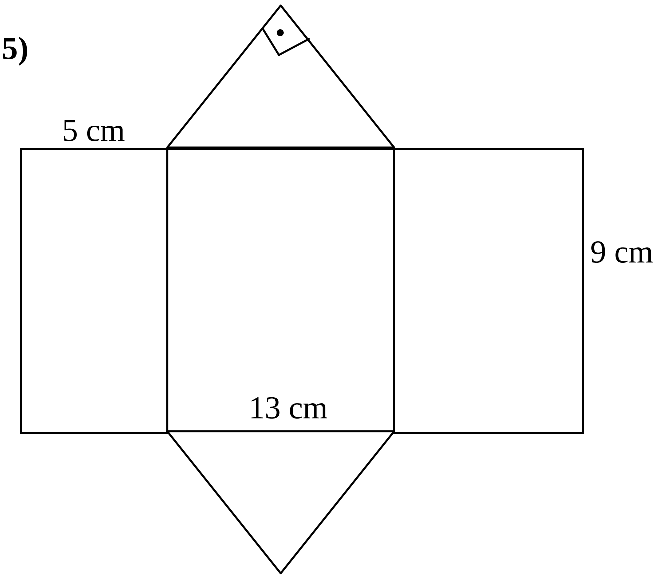
b) Hangi çözüm yolu/yolları doğrudur?

c) Hangi öğrenci veya öğrenciler problemi yanlış çözmüştür? Yanlış çözen öğrenci veya öğrencilerin yanlışlarının kaynağı ne olabilir?

d) Siz bu hataları düzeltmek için hangi öğretim tekniklerini/ materyalleri/ stratejileri kullanırsınız? Nasıl?

e) Sizin öğrencileriniz genelde hangi öğrencinin çözüm yolunu tercih ederler?

5)



Yanda açık hali verilen dik üçgen prizma şeklindeki karton katlanıp kutu yapıldığında kaplayacağı hacmi hesaplayınız.

a) Bu konuyu müfredattaki hangi konu veya konular ile ilişkilendirerek anlatırsınız?

b) Bu problemin çözümünde kullanılabilecek bildiğiniz tüm yöntemleri yazınız.

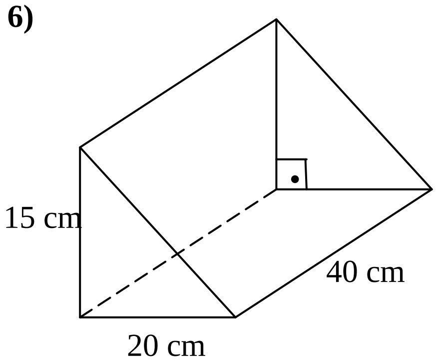
c) Öğrencileriniz belirttiğiniz yöntemlerden hangisini veya hangilerini kullanarak bu problemi çözerler?

d) Öğrencileriniz bu problemi çözerken hangi hataları yapabilirler?

e) Bu hataların kaynağı ne olabilir? Açıklayınız.

f) Siz bu hataları düzeltmek için hangi öğretim tekniklerini/ materyalleri/ stratejileri kullanırsınız? Nasıl?

6)



Yanda dik üçgen prizma şeklinde kesilmiş bir peynir kalıbı bulunmaktadır. Bu peynir kalıbı 20 eşit dilime ayrıldığında elde edilen her bir dilimin hacmi ne kadar olur?

Arda Acar yukarıdaki problemi sınıfta öğrencilerine sorar ve iki farklı çözüm yolu ile karşılaşır.

1. çözüm yolu:

V= $\frac{15.20}{2}.40$

V= 6000

Bir dilimin hacmi: $\frac{6000}{20} = 300$

2. çözüm yolu:

$\frac{40}{2} = 20$

V= $\frac{15.20}{2}.2$

V= 300

a) Sizce Arda Öğretmen'in öğrencileri bu çözüm yollarını geliştirirken ne düşünmüş olabilirler?

b) Bu çözüm yollarından hangisi/hangileri doğrudur?

c) Eğer bu çözüm yollarından biri/ ikisi yanlış ise bu yanlışın kaynağı ne olabilir?

d) Siz bu hataları düzeltmek için hangi öğretim tekniklerini/ materyalleri/ stratejileri kullanırsınız? Nasıl?

e) Bu çözüm yollarından hangi/hangileri sizin öğrencileriniz için daha anlaşılırdır?

7)

Yanda verilen piramit maketinin tabanı 6 cm ve yan yüz yüksekliği 5 cm'dir. Bu maketin hacmini hesaplayan Ceren ile Cemre problemi farklı yollarla çözmüşlerdir.

Ceren'in çözümü:

$$V= \frac{6.\ 6.\ 5}{3}$$

$$V= \frac{180}{3} = 60\,cm^3$$

Cemre'nin çözümü:

$a^2 = b^2 + c^2$

$5^2 = 3^2 + c^2$

$25 = 9 + c^2$

$16 = c^2$

$c= 4$

$$V= \frac{6.\ 6.\ 4}{3}$$

$$V= \frac{144}{3} = 48\,cm^3$$

a) Bu problemin çözümünü müfredattaki hangi konu veya konular ile ilişkilendirerek anlatırsınız?

b) Sizce Ceren ile Cemre bu çözüm yollarını geliştirirken ne düşünmüş olabilirler?

c) Bu çözüm yollarından hangisi/hangileri doğrudur? Neden?

d) Eğer bu çözüm yollarından biri veya her ikisi yanlış ise bu yanlışın/hatanın kaynağı ne olabilir?

e) Siz bu hataları düzeltmek için hangi öğretim tekniklerini/ materyalleri/ stratejileri kullanırsınız? Nasıl?

8) Külah, yay uzunluğu, 15, yarıçap ve 54 cm ifadelerini kullanarak hacim bağıntısını içeren bir problem kurunuz.

APPENDIX H

CODING SCHEME

SUBJECT MATTER KNOWLEDGE

Knowledge of Alternative Solution Methods	
Coding	**Meaning**
Volume Formula	Multiplying the depth, the width and the height of the prism Multiplying the lengths of three edges Multiplying area of the base of 3D solids by its height
Systematic Counting	Counting cubes systematically, attempting to count both inside and outside cubes. He or she might, for instance, count the cubes on all the outside faces, and then attempt to determine how many are in the center
Layer Counting	Counting the number of unit cubes in one layer, and then multiplies this number by the number of layers or uses addition to obtain the total
Column/Row Iteration	Counting the number of cubes in one row or column and uses skip-counting

PEDAGOGICAL CONTENT KNOWLEDGE

Coding	Meaning
Knowledge of Instructional Strategy	
Teacher-centered instruction	Teacher provides clear explanations and examples, checking students' understanding by asking them questions and using manipulative to help students envisage the 3D solids
Less teacher-centered enriched with class discussion	Teacher shares the responsibility of explaining the topic with their students; thus, there was a good amount of dialog between the students and the teacher. Thus, questioning and discussions were integrated into teaching process.
Knowledge of Learners	
Students' Preferences among Solution Methods	
Volume Formula	Multiplying the depth, the width and the height of the prism Multiplying the lengths of three edges Multiplying area of the base of 3D solids by its height
Systematic Counting	Counting cubes systematically, attempting to count both inside and outside cubes. He or she might, for instance, count the cubes on all the outside faces, and then attempt to determine how many are in the center
Layer Counting	Counting the number of unit cubes in one layer, and then multiplies this number by the number of layers or uses addition to obtain the total
Column/Row Iteration	Counting the number of cubes in one row or column and uses skip-counting

Interpretations of Students' Alternative Solution Methods	
Coding	**Meaning**
Teachers' Correct Interpretations of Students' Correct Solution Methods	Students' solution method is correct and teacher explains it correctly
Teachers' Incorrect or Missing Interpretations of Students' Correct Solution Methods	Students' solution method is correct however, teacher could not explain it or teacher's explanation is not true for the solution method
Teachers' Correct Interpretations of Students' Incorrect Solution Methods	Students' solution method is incorrect and teacher explains it correctly
Teachers' Incorrect or Missing Interpretations of Students' Incorrect Solution Methods	Students' solution method is incorrect and teacher could not explain it or teacher's explanation is not true for the solution method
Students' Errors	
Focusing on the faces of 3D solids	Considering all or a subset of the visible faces of 3D solid when calculating its volume
Over-counting the common unit cubes on the adjacent faces	Not realizing some unit cubes belong to more than one face of 3D solid
Conceptual errors	Students' misunderstanding or confusing the meanings of the concepts
Computational errors	A mistake made when undertaking the calculation
Sources of Students' Errors	
Not able to think of solids as three-dimensional	Focusing on the faces of 3D solids and ignoring inside of 3D solids
Not being able to comprehend the structure of a 3D solid	Not realizing that column/row is formed by the unit cubes, the layer is formed by the columns/rows and the prism is formed by the layers

Not being able to concretize a 3D solid	Not envisaging the structure of 3D solids
Lack of conceptual knowledge	Not knowing the concepts or confusing the concepts
Students' carelessness	Not focusing on what is asked in the problem, forgetting the volume formula and doing calculation errors while solving the problem
Not thinking deeply about the concepts	Not interpreting the concepts such as the volume, relate the concepts with the other concepts that they had previously learnt and their daily life, and give the meaning of the volume formula
The Strategies to Overcome Elementary Students' Errors	
Using manipulative	Forming and visualizing 3D solids with unit cubes, showing the concepts which the students confuse such as the height of the pyramid and the height of the side-face of the pyramid
Re-explaining the misunderstood part of the topic	Explaining the topic again and presenting many more examples related to the topic
Knowledge of Curriculum	
Connection with other topics	Connecting the topic to the other mathematics areas and topics taught in other courses (e.g. science) taught in previous years, the same year and to be taught in later years
Changing the order of the topics	Making changes in the order of the sub-topics of 3D solids in the curriculum to teach the topics more effectively

Knowledge of Assessment	
Coding	**Meaning**
Formative assessment	Evaluating the students' knowledge concerning a specific during the learning process
Summative assessment	Evaluating the students' knowledge concerning a specific topic following the completion of a unit in any subject

APPENDIX I

PERMISSION FROM THE ETHICAL COMMITTEE AT METU

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

DUMLUPINAR BULVARI 06800
ÇANKAYA ANKARA/TURKEY
T: +90 312 210 22 91
F: +90 312 210 79 59
ueam@metu.edu.tr
www.ueam.metu.edu.tr

Sayı: 28620816/ 71 – 116

06.02.2014

Gönderilen : Doç. Dr. Mine Işıksal Bostan
İlköğretim Bölümü

Gönderen : Prof. Dr. Canan Özgen
IAK Başkanı

İlgi : Etik Onayı

Danışmanlığını yapmış olduğunuz İlköğretim Bölümü öğrencisi Reyhan Tekin Sitrava'nın "Matematik Öğretmenlerinin Üç Boyutlu Cisimler Hakkındaki Alan Bilgileri/ In-Service Teachers' Mathematical Knowledge For Teaching On Three Dimensional Solids" isimli araştırması "İnsan Araştırmaları Komitesi'' tarafından uygun görülerek gerekli onay verilmiştir.

Bilgilerinize saygılarımla sunarım.

Etik Komite Onayı

Uygundur

06/02/2014

Prof.Dr. Canan Özgen
Uygulamalı Etik Araştırma Merkezi
(UEAM) Başkanı
ODTÜ 06531 ANKARA

APPENDIX J

PERMISSION FROM THE ANKARA PROVINCIAL DIRECTORATE FOR NATIONAL EDUCATION

T.C.
ANKARA VALİLİĞİ
Milli Eğitim Müdürlüğü

BÖLÜM : İstatistik Bölümü
SAYI : B.08.4.MEM.0.06.20.01-60599/13615 17/02/2012
KONU : Araştırma İzni
Reyhan Tekin SİTRAVA

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
(Öğrenci İşleri Daire Başkanlığı)

İlgi : a) MEB Bağlı Okul ve Kurumlarda Yapılacak Araştırma ve Araştırma Desteğine Yönelik İzin ve Uygulama Yönergesi.
b) Üniversiteniz 16/01/2012 tarih ve 764 sayılı yazısı.

Üniversiteniz İlköğretim Anabilim Dalı Doktora Programı öğrencisi Reyhan Tekin SİTRAVA'nın **"İlköğretim Matematik öğretmenlerinin üç boyutlu cisimler hakkındaki alan bilgileri"** konulu tezi ile ilgili çalışma yapma isteği Müdürlüğümüzce uygun görülmüş ve araştırmanın yapılacağı İlçe Milli Eğitim Müdürlüğüne bilgi verilmiştir.

Mühürlü anket örnekleri (10 sayfadan oluşan) ekte gönderilmiş olup, uygulama yapılacak sayıda çoğaltılması ve çalışmanın bitiminde iki örneğinin (CD/disket) Müdürlüğümüz İstatistik Bölümüne gönderilmesini rica ederim.

Gülçin UYSAL
Müdür a.
Müdür Yardımcısı

EKLER : Anket (10 Sayfa)

İl Milli Eğitim Müdürlüğü-Beşevler
İstatistik Bölümü
Bilgi İçin: Nermin ÇELENK

Tel : 223 75 22
Fax: 223 75 22
istatistik06@meb.gov.tr

APPENDIX K

CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Tekin Sitrava, Reyhan
Nationality: Turkish (TC)
Date and Place of Birth: 4 October 1982 , Burdur
Marital Status: Married
email: reyhan_tekin@yahoo.com

EDUCATION

Degree	Institution	Year of Graduation
BS	METU Elementary Mathematics Education	2005
High School	Burdur Anatolian Training High School, Burdur	2000

WORK EXPERIENCE

Year	Place	Enrollment
2012-2013	MEV College ANKARA	Mathematics Teachers
2009- 2011	Evrim College İSTANBUL	Mathematics Teachers
2007- 2009	İTÜ Foundation Schools Dr. Natuk Birkan Elementary School İSTANBUL	Mathematics Teachers
2005- 2006	Gürçağlar College MUĞLA	Mathematics Teachers

FOREIGN LANGUAGES

Advanced English

PUBLICATIONS

A. Papers published in Journals:

Tekin-Sitrava, R. & Işıksal-Bostan, M. (accepted). A study on investigating middle school students' performances, solution strategies and difficulties in calculating the volume of a rectangular prism. *International Journal for Mathematics Teaching and Learning.*

B. Papers Presented in International Conferences:

Tekin-Sitrava, R. & Işıksal-Bostan, M. (2013*). In-Service Mathematics Teacher's Mathematical Knowledge For Teaching: A Case Of Volume Of Prism.* Paper presented at 8th Congress of European Research in Mathematics Education (CERME 8), Antalya, Turkey.

Tekin-Sitrava, R. & Işıksal-Bostan, M. (2012*). Pre-Service Elementary Mathematics Teacher's Mathematical Knowledge For Teaching: A Case Of Volume Of Prism.* Paper presented at 3rd International Conference on New Horizons in Education, Prague, Czech Republic.

Tekin-Sitrava, R., Işıksal-Bostan, M. & Koç, Y. (2011*). A Study on Understanding Relation Between Elementary Students' Problem Solving Performance Related to Volume of 3D Figures and Grade Level.* Paper presented at Emerging Researchers' Conference of European Educational Research Association (ECER), Berlin, Germany.

Tekin-Sitrava, R., Işıksal-Bostan, M. & Koç, Y. (2011*). A Study on Understanding the Volume of Three Dimensional Figures for 7th And 8th Grade Students.* Paper presented at Junior Researchers of European Association for Research on Learning and Instruction (JURE), Exeter, England.

Tekin-Sitrava, R., Işıksal-Bostan, M. & Koç, Y. (2009*). Investigation of Elementary Students'Geometrical Thinking.* Paper presented at 16th International Conference on Learning, Barcelona, Spain.

C.Papers Presented in National Conferences:

Tekin-Sitrava, R., Işıksal-Bostan, M. & Koç, Y. (2010*). İlköğretim Öğrencilerinin Geometrik Düşünme Yapılarının İncelenmesi.* Paper presented at Eğitimde İyi Örnekler, Türkiye.

HOBBIES

Sports, cooking, travelling, reading

APPENDIX L

TURKISH SUMMARY

ORTAOKUL MATEMATİK ÖĞRETMENLERİNİN 3 BOYUTLU CİSİMLERİN HACMİNE İLİŞKİN ALAN VE PEDAGOJİK ALAN BİLGİLERİ ÜZERİNE BİR ÇALIŞMA

1. Giriş

Son yıllarda birçok araştırmacıöğretmen bilgisinin, verimli öğretimin sağlanması için çok önemli bir unsur olduğunu vurgulamıştır (Ball, Thames & Phelps, 2008; Shulman,1986). Öğretmen eğitimiyle ilgili literatür incelendiğinde öğretmen bilgisinin farklı şekillerde tanımlandığı ve bu bilgiyi oluşturan çeşitli değişkenlerden bahsedildiği görülmektedir (Fennema & Franke, 1992; Grossman, 1990; Hill, Ball & Schilling, 2008; Shulman, 1986). Bu konuda yapılan çalışmaların öncülüğünü Shulman'ın (1986, 1987) öğretmen bilgisini tanımladığı ve sınıflandırdığı çalışmaları yapmaktadır. 1986'daki çalışmasında, Shulman öğretmen bilgisini; alan bilgisi, pedagojik alan bilgisi ve öğretim programları bilgisi olarak belirlemiştir. Shulman'a (1986) göre, alan bilgisi, matematik ve matematiğin yapısı hakkındaki bilgidir. Başka bir deyişle, alan bilgisine sahip olan bir öğretmenin matematikteki kavramları, kuralları, teoremleri bilmesi, bunların doğruluklarını ispat edebilmesi ve kavramlar arasındaki ilişkileri kurabilmesi gerekmektedir. Pedagojik alan bilgisi ise konu alan bilgisi ile pedagojik bilginin birleşimidir. Bu bilgi türüne sahip olan bir öğretmenin, konuyu en iyi biçimde anlatabilmesi için kullanması gereken öğretim tekniklerini, örnekleri, gösterimleri ve sunumları bilmesi gerekir. Ayrıca, öğretmenin, öğrencilerin kavram yanılgılarını ve bu kavram yanılgılarının kaynağını bilme, kavram yanılgılarının giderilmesi için kullanılabilecek benzetimler, temsiller, örnekler ve açıklamaları bilmesi de öğretmenin pedagojik alan bilgisine bağlıdır. Shulman'a (1986) göre, öğretmen bilgisinin üçüncü boyutu öğretim programları bilgisidir. Öğretim programı bilgisi, bir öğrenme alanındaki öğretim programı ile ilgili kaynakların (kaynak ders kitapları, somut materyaller, yazılımlar, teknolojik araçlar, vb.) ne zaman ve nasıl kullanacağı bilgisini içermektedir.

Shulman'dan sonra bazı araştırmacılar mevcut kategorileri genişletmiş ve öğretmen bilgisini açıklamak için farklı kategoriler ortaya koymuştur (Ball, Thames & Phelps, 2008; Gess-Newsome, 1999; Grossman 1990; Rowland, Huckstep & Thwaites, 2005). Fakat araştırmacılar, öğretmenlerin yeterli bilgiye sahip olma ve bu bilgiyi etkili bir şekilde kullanılmasının matematik öğretiminin temelini oluşturduğu konusunda hemfikirdir. Bu nedenle, öğretmenlerin çeşitli matematik konularına ilişkin bilgilerinin ve bu bilgilerin, öğrencilerin öğrenmeleri üzerindeki etkisini araştırmak için birçok çalışma yapılmıştır (Baki, 2013; Ball, 1990a; Ball, 1991; Baştürk & Dönmez, 2011; Borko ve diğerleri, 1992; Even & Tirosh, 1995; Hill, Rowan & Ball, 2005; Işıksal, 2006; Kahan, Cooper & Bethea, 2003; Leinhardt & Smith, 1985; Ma, 1999). Bu çalışmalar, öğretmenin matematik bilgisinin, öğrencinin başarısını doğrudan etkilediği sonucuna ulaşmıştır (Kahan ve diğerleri, 2003; Leinhardt & Smith, 1985). Diğer taraftan, verimli matematik öğretimi için, öğretmenin matematik bilgisi kadar geometri bilgisi de önemli bir role sahiptir (Maxedon, 2003). Çünkü geometri matematiğin önemli öğrenme alanlarından biridir (Atiyah, 2001; NCTM, 2000). Ayrıca, öğrenciler şekilleri ve yapıları incelerken, geometri kavramlarını ve bu kavramların matematikle ilişkisini de anlarlar (NCTM, 2000). Başka bir deyişle, geometri bilgisi, öğrencilere ve öğretmenlere matematiğin diğer öğrenme alanlarını öğrenme ve öğretme açısından önemlidir. Bu yüzden, öğretmenlerin geometri konuları ile ilgili bilgilerinin araştırılması literatüre önemli bir katkı sağlayacaktır. Bu bağlamda, bu çalışma, öğrencilerin en çok zorlandıkları konulardan biri olan, 3 boyutlu cisimlerin hacmine ilişkin öğretmen bilgisini araştırarak literatüre katkı sağlamayı hedeflemektedir.

Literatürde, öğretmenlerin farklı matematik konularına ait bilgilerini inceleyen çalışmalar olduğu gibi, öğretmen bilgisini farklı boyutlarda inceleyen çalışmalar da bulunmaktadır (Ball, 1990a; 1990b; Even, 1993; Hacıömeroğlu, Haciömeroğlu, & Aspinwall, 2007; Işıksal, 2006). Bu açıdan incelendiğinde, bu çalışma öğretmenlerin alan bilgisini iki farklı boyutta incelemektedir. İlk olarak, öğretmenlerin alan bilgisini, matematik kavramlarını farklı şekilde ifade etmek (Ball ve diğ., 2008) açısından incelemek için ortaokul matematik öğretmenlerinin 3 boyutlu cisimlerin hacmine ilişkin geliştirdikleri farklı çözüm yöntemleri ele alınmıştır. Bunun yanında, Chapman (2002) ve Ball (1990a) çalışmalarında, matematik problemi oluşturmanın öğretmenlerin alan bilgisini ortaya koymak için bir araç olduğunu belirtmiştir. Bu nedenle, bu çalışmada, ortaokul matematik öğretmenlerinin 3 boyutlu cisimlerin hacmi ile ilgili problem yazma konusundaki bilgileri onların alan bilgisi başlığı altında incelenmiştir.

Diğer taraftan, öğretmenlerin pedagojik alan bilgileri dört farklı boyutta incelenmiştir: öğretim stratejileri bilgisi, öğretim programları bilgisi, öğrenci bilgisi ve ölçme ve değerlendirme bilgisi (Magnusson, Krajcik & Borko, 1999). Ortaokul öğretmenlerinin 3 boyutlu cisimlerin hacmini etkili bir biçimde öğretmek için kullandıkları öğretim stratejileri ile öğrencilerin bu konuda başarı düzeylerini belirlemek için kullandıkları ölçme ve değerlendirme yöntemleri, onların pedagojik alan bilgilerini belirlemek için incelenen alt boyutlardır. Ayrıca, öğretmenlerin pedagojik alan bilgileri, 3 boyutlu cisimlerin hacmi konusunun, matematikteki diğer konularla, diğer derslerdeki konularla ve günlük hayatla ilişkilendirmelerine yönelik bilgileri ile 3 boyutlu cisimlerin hacminin daha anlaşılır olması için öğretmenlerin bu konuya ait alt konuların (prizma, piramit, koni gibi) sırasında yaptıkları değişiklikler açısından da incelenmiştir. Son olarak, öğretmenlerin pedagojik alan bilgileri, öğrencilerin 3 boyutlu cisimlerin hacmi ile ilgili problemleri çözmek için kullandıkları çözüm yollarını bilmesi, öğrencilerin farklı çözüm yollarını açıklayabilmeleri, öğrencilerin hataları, bu hataların nedenlerini ve bu hataları gidermek için kullandıkları yöntemler açısından ele alınmıştır. Sonuç olarak, bu çalışma, 4 tane ortaokul matematik öğretmenlerinin 3 boyutlu cisimlerin hacmine ilişkin, alan bilgisi ve pedagojik alan bilgisini incelemeyi amaçlamaktadır.

1.1. Çalışmanın Önemi ve Alana Sağladığı Katkı

Verimli matematik öğretimi için, öğretmen bilgisinin önemi göz önüne alındığında, öğretmenlerin farklı matematik konularındaki bilgisinin araştırılması literatüre katkı sağlayacaktır.

Mevcut matematik eğitimi literatürü incelendiğinde, birçok çalışmanın matematik öğretmenlerinin ölçme ve değerlendirmeye yönelik bilgilerini araştırmaya odaklanmadığını göstermiştir. Bu yönü ile çalışma matematik literatüründeki bu eksiği gidermeyi hedeflemektedir. Ayrıca çalışma, ortaokul öğretmenlerinin pedagojik alan bilgilerinin öğretim stratejileri bilgisi, öğretim programları bilgisi ve öğrenci bilgisi açısından incelemeyi amaçlamaktadır. Bu açıdan değerlendirildiğinde, çalışma ortaokul matematik öğretmenlerinin pedagojik alan bilgilerini geniş bir çerçevede incelemektedir.

Ayrıca, Hill, Rowan ve Ball (2005), öğretmen bilgisinin öğrenci başarısı üzerindeki etkisinin çok önemli olduğunu belirtmiştir. Bu etki düşünüldüğünde, farklı matematik konularına ilişkin öğretmen bilgisini araştıran çok sayıda çalışma bulunmaktadır. Bu çalışmalarda, genel olarak, kesirler (Ball, 1990a; Hutchison, 1997; Isiksal, 2006; Işıksal & Çakıroğlu, 2008), bölme (Baki, 2013; Ball, 1990b), olasılık (Contreras, Batanero, Diaz & Fernandes, 2011), oran (Livy & Vale, 2011) ve fonsiyonlar (Even, 1993; Even & Tirosh, 1995; Hacıömeroğlu, Hacıömeroğlu & Aspinwall, 2007; Huang & Kulm, 2012; Karahasan, 2010) konularına odaklanılmıştır. Bunun dışında, geometri konularına ilişkin öğretmen bilgisini araştıran çalışmalara bakıldığında, dörtgenler (Aslan-Tutak, 2009; Fujita & Jones, 2006), alan ölçme (Baturo & Nason, 1996; Kellogg, 2010) ve dönüşüm geometrisi (Gomes, 2011) konuları ön plana çıkmaktadır. Yapılan literatür taramasında, öğretmenlerin 3 boyutlu cisimlerin hacmini hesaplamaya yönelik çalışmaya rastlanmamıştır. Öğrencilerle hacim kavramı üzerine yapılan çalışmalarda, öğrencilerin bu konuda zorlandıkları ortaya konulmuştur (Battista & Clements, 1996; Ben-Chaim, Lappan & Houang, 1985; Ng, 1998; Olkun, 1999). Öğrencilerin, öğrenmesinde öğretmen bilgisinin önemi düşünüldüğünde ve literatürde öğretmenlerin bu konuya ilişkin bilgilerini araştıran çalışmaların az sayıda olduğu dikkate alındığında, öğretmenlerin 3 boyutlu cisimlerin hacmine ilişkin bilgilerinin incelenmesinin literatüre katkı sağlayacağı düşünülmektedir.

Çalışma, katılımcıları ve veri toplama yöntemleri ile de alana katkı sağlamaktadır. Öğretmen bilgisi alanında yapılan çalışmaların birçoğu öğretmen adayları ile yapılmıştır (Ball, 1990a; 1990b; Basturk & Donmez, 2011; Contreras, Batanero, Diaz & Fernandes, 2011; Even, 1993; Even& Tirosh, 1995; Huang &

Kulm, 2012; Işıksal, 2006; Livy & Vale, 2011). Fakat Shulman (1987), öğretmen adaylarının alan bilgisinin yeterli olmadığını savunmaktadır. Deneyimli öğretmenlerle yapılan çalışmalar, öğretmenlerin alan bilgilerini öğrencilere nasıl aktardıklarını ve konuların öğretiminde, pedagojik alan bilgilerini nasıl kullandıkları ile ilgili örnekler sunmaktadır. Bu bağlamda, çalışmanın deneyimli (en 10 yıl) ortaokul matematik öğretmenleriyle yapılması önemlidir. Ayrıca, öğretmenlerin hacim konusunu anlattıkları süre boyunca derslerinin gözlemlenmesi, öğretmenlerin bilgisi ile ilgili gerçek örnekler sunma fırsatı verecektir.

Sonuç olarak, çalışmanın teorik ve pratik açıdan, öğretmen bilgisi konusunda alana sağladığı katkı açıkça görülmektedir.

2. Yöntem

Bu çalışmanın amacı, 4 tane deneyimli ortaokul matematik öğretmenin 3 boyutlu cisimlerin hacmine ilişkin alan bilgilerini ve pedagojik alan bilgilerini incelemektir. Bu amaç doğrultusunda, çalışmada aşağıdaki araştırma sorularına cevap aranmıştır.

1. Ortaokul matematik öğretmenlerinin 3 boyutlu cisimlerin hacmine ilişkin alan bilgilerinin doğası nedir?
    - 1.1. Ortaokul matematik öğretmenlerinin 3 boyutlu cisimlerin hacmine ilişkin problemleri çözmek için geliştirdikleri çözüm yöntemleri nelerdir?
    - 1.2. Ortaokul matematik öğretmenleri 3 boyutlu cisimlerin hacmine ilişkin problem kurmadaki başarıları hangi düzeydedir?
2. Ortaokul matematik öğretmenlerinin 3 boyutlu cisimlerin hacmine ilişkin pedagojik alan bilgilerinindoğası nedir?
    - 2.1. Ortaokul matematik öğretmenlerinin 3 boyutlu cisimlerin hacmini anlatırken kullandıkları öğretim stratejileri nelerdir?
    - 2.2. Ortaokul matematik öğretmenleri, 3 boyutlu cisimlerin hacmi ile ilgili öğrenci bilgisine ne ölçüde sahiptirler?
    - 2.3. Ortaokul matematik öğretmenleri, 3 boyutlu cisimlerin hacmi ile ilgili öğretim programları bilgisine ne ölçüde sahiptirler?

2.4. Ortaokul matematik öğretmenleri, 3 boyutlu cisimlerin hacmine ilişkin öğrenci başarısını ölçmek için ne tür ölçme ve değerlendirme yöntemleri kullanırlar?

Araştırma sorularına cevap verebilmek için, nitel araştırma yöntemlerinden biri olan durum çalışması yöntemi kullanılmıştır. Durum çalışması yöntemi bir kişiyi, bir programı veya bir grubu derinliğine ve genişliğine incelemeyi amaçlamaktadır (Merriam, 1998). Ayrıca, durum çalışması, bir veya birkaç durumu sınırlı bir sistem içinde birden fazla veri toplama yöntemi kullanarak derinlemesine incelemek için uygulanır. Bu doğrultuda, 4 tane ortaokul matematik öğretmeninin, 3 boyutlu cisimlerin hacmine ilişkin alan bilgilerini ve pedagojik alan bilgilerini incelemek için en uygun yöntem durum çalışması yöntemidir. Çalışmada incelenen durum 4 tane ortaokul matematik öğretmeni ve bu durumun sınırları ise öğretmenlerin Ankara ilindeki bir ortaokulda 8.sınıfları okutuyor ve 10 yıldan fazla tecrübeye sahip olmalarıdır.

Çalışmanın verileri, 4 tane deneyimli ortaokul matematik öğretmeninden toplanmıştır. Katılımcı öğretmenlere Gül, Esin, Can ve İrem rumuzları verilmiştir. Katılımcı öğretmenler en az 10 yıl öğretmenlik tecrübesi olan ve Ankara ilindeki devlet okullarında 8. sınıf öğrencilerini okutan öğretmenler arasından seçilmiştir. Öğretmenlerin, 3 boyutlu cisimlerin hacmine ilişkin bilgilerini incelemek için araştırmacı tarafından hazırlanan 10 sorudan oluşan soru seti kullanılmıştır (Appendix A). Öğretmenlerin, soru setindeki sorulara verdikleri cevapları detaylandırmaları için onlarla yarı-yapılandırılmış görüşmeler yapılmıştır. Ayrıca, öğretmenlerin, 3 boyutlu cisimlerin hacmi konusunu anlatımları da gözlemlenmiş ve sınıf gözlemi kameraya alınmıştır.

Verilerin analizine başlamadan önce görüşmeler ve ders gözlemleri esnasında çekilen video kayıtları deşifre edilmiştir. Veri kodlama sürecinin ilk aşamasında alan yazınından yararlanarak açık kodlama suretiyle kodlar geliştirilmiştir. Birbirini kapsadığı düşünülen kodlar birleştirilmiştir. Kodlama işlemi iki araştırmacı tarafından gerçekleştirilmiş ve güvenirlik için kodlar tartışmalar sonucunda ortak karar ile belirlenmiştir. Aşağıda kodlama süreci sonucunda elde edilen kodlar sunulmuştur.

ALAN BİLGİSİ	
Farklı Çözüm Yöntemlerine İlişkin Bilgileri	
Hacim Formülü	Yükseklik, derinlik ve genişliğin çarpılması 3 boyutun uzunluğunun çarpılması Taban alanı ile yüksekliğin çarpılması
Sistemli Sayma	Prizmaların içindeki ve dışındaki birim küpleri sistemli bir şekilde sayma. Örneğin, önce dış yüzlerdeki birim küpleri sayıp, ortada kaç tane birim küp olduğunu bularak toplam birim küp sayısına ulaşma
Katman Hesabı	Bir katmandaki birim küpleri hesaplayıp, bu sayıyı toplam katman sayısı ile çarpma veya toplama yaparak sonuca ulaşma
Sütun/Satır Sayma	Bir sütun/satırdaki birim küpleri hesaplayıp, bu sayıyı toplam sütun/satır sayısı ile çarpma veya toplama yaparak sonuca ulaşma
PEDAGOJİK ALAN BİLGİSİ	
Öğretim Stratejileri Bilgisi	
Öğretmen merkezli öğretim	Öğretmen, konuyu açıklar ve örnekleri anlatır, öğrencilerin öğrenmelerini sorular sorarak kontrol eder ve 3 boyutlu cisimleri öğrencilerin zihninde canlandırmalarına yardımcı olmak için materyal kullanır
Tartışma yöntemi ile zenginleştirilmiş daha az öğretmen merkezli öğretim	Öğretmen konunun anlatımını öğrencileri ile paylaşır ve öğretmenler ile öğrenciler arasındaki diyalog fazladır. Yani, soru sorma ve tartışma konunun anlatımı ile bütünleştirilmiştir

Öğrenci Bilgisi	
Öğrencilerin Tercih Edeceği Çözüm Yöntemlerine İlişkin Bilgileri	
Hacim Formülü	Yükseklik, derinlik ve genişliğin çarpılması 3 boyutunun uzunluğunun çarpılması Taban alanı ile yüksekliğin çarpılması
Sistemli Sayma	Prizmaların içindeki ve dışındaki birim küpleri sistemli bir şekilde sayma. Örneğin, önce dış yüzlerdeki birim küpleri sayıp, ortada kaç tane birim küp olduğunu bularak toplam birim küp sayısına ulaşma
Katman Hesabı	Bir katmandaki birim küpleri hesaplayıp, bu sayıyı toplam katman sayısı ile çarpma veya toplama yaparak sonuca ulaşma
Öğrencilerin Farklı Çözüm Yöntemlerini Yorumlama	
Öğretmenlerin, öğrencilerin doğru çözüm yöntemlerini doğru şekilde yorumlamaları	Öğrenciler soruyu doğru çözerler ve öğretmenler, öğrencilerin çözüm yöntemini doğru şekilde yorumlarlar
Öğretmenlerin, öğrencilerin doğru çözüm yöntemlerini yanlış veya eksik şekilde yorumlamaları	Öğrenciler soruyu doğru çözerler fakat öğretmenler, öğrencilerin çözüm yöntemini yorumlayamaz veya eksik şekilde yorumlarlar
Öğretmenlerin, öğrencilerin yanlış çözüm yöntemlerini doğru şekilde yorumlamaları	Öğrenciler soruyu yanlış çözer ve öğretmenler, öğrencilerin çözüm yöntemini doğru bir şekilde yorumlarlar
Öğretmenlerin, öğrencilerin yanlış çözüm yöntemlerini yanlış şekilde yorumlamaları veya yorumlayamamaları	Öğrenciler soruyu yanlış çözerler ve öğretmenler, öğrencilerin çözüm yöntemini yorumlayamaz veya eksik şekilde yorumlarlar

Öğrenci Hataları	
3 boyutlu cisimlerin yüzlerine odaklanma	3 boyutlu cisimlerin hacmini hesaplarken cismin tüm dış yüzlerindeki veya görünen dış yüzlerindeki birim küpleri hesaplama
Yan yana olan yüzlerdeki ortak küpleri birden fazla sayma	3 boyutlu cisimlerin birden fazla yüzüne ait ortak birim küpleri fark etmeme
Kavramsal hatalar	Kavramların anlamını bilmeme, yanlış anlama veya kavramları karıştırma
İşlemsel hatalar	İşlem yaparken hata yapma
Öğrenci Hatalarının Kaynakları	
Cisimleri 3 boyutlu düşünememe	3 boyutlu cisimlerin yüzlerine odaklanma ve içini düşünememe
3 boyutlu cisimlerin yapısını anlamama	Sütun/satırın birim küplerden oluştuğunu, katmanların sütun/satırdan oluştuğunu ve prizmaların katmanlardan oluştuğunu fark edememe
3 boyutlu cisimleri somutlaştıramama	3 boyutlu cisimleri zihninde canlandıramama
Kavramsal bilgi eksikliği	Kavramların anlamını bilmeme, yanlış anlama veya kavramları karıştırma
Dikkatsizlik	Problemlerde sorulana odaklanmama, formülü unutma ve işlem hatası yapma
Kavramları derinlemesine düşünmeme	Hacim, alan gibi kavramları yorumlamama, kavramları daha önce öğrenilmiş kavramlarla veya günlük hayatla ilişkilendirememe ve hacim formülünü anlamlandıramama

Öğrenci Hatalarını Gidermek için Kullanılabilecek Yöntemler	
Materyal kullanma	Birim küplerle 3 boyutlu cisimleri oluşturma ve görselleştirme, piramidin yüksekliğini ve yan yüz yüksekliğini materyal kullanarak öğrencilere gösterme
Konunun anlaşılmayan kısmını tekrar anlatma	Konuyu tekrar anlatma ve konu ile ilgili soru çözme
Müfredat Bilgisi	
Konuları ilişkilendirme	Konuyu, geçmiş yıllarda ve aynı yıl içinde öğretilen veya gelecek yıllarda öğretilecek olan matematikteki diğer konularla veya diğer derslerdeki konularla ilişkilendirme
Konuların sırasını değiştirme	Konunun daha anlaşılır olması için müfredatta 3 boyutlu cisimler ile ilgili konuların sırasını değiştirme
Ölçme ve Değerlendirme Bilgisi	
Biçimlendirici değerlendirme	Öğrenme süreci devam ederken belirli bir konuya ilişkin öğrencinin bilgisini değerlendirme
Düzey belirleyici değerlendirme	Öğrenme süreci bittikten sonra belirli bir konuya ilişkin öğrencinin bilgisini değerlendirme

Bir araştırmada, geçerlilik ve güvenirlik konuları çok önemlidir (Patton, 2002). Araştırmacılar, nitel çalışmalardaki geçerlilik ve güvenilirlik kavramları için farklı terminolojiler kullanmıştır (Creswell, 2007; Lincoln & Guba, 1985; Merriam, 1998; Miles & Huberman, 1994; Stake, 1995; Yin, 2003). Bu çalışmada, Lincoln ve Guba'nın (1985) geçerlilik ve güvenirlik kavramları kullanılmıştır. Çalışmanın geçerliliğini ve güvenirliği sağlamak için farklı yöntemler kullanılmıştır. Çalışmanın verisi üç farklı kaynaktan oluşmaktadır. Yani çalışmada, katılımcılara 3 boyutlu cisimlerin hacmine ilişkin sorular uygulanmış, bu sorulara verdikleri cevapları detaylandırmaları için görüşmeler yapılmış ve 3 boyutlu cisimlerin hacmi konusunu anlatırken sınıf gözlemi yapılmıştır. Sınıfları gözlenen öğrencilerin ve öğretmenlerin, araştırmacıdan ve kameradan etkilenmemeleri için öğretmenler 3 boyutlu cisimlerin hacmi konusunu anlatmaya başlamadan önce de sınıfta gözlemler yapılmış ve kamera kaydı alınmıştır. Araştırmacı görüşme kayıtlarını deşifre ettikten sonra katılımcılarla tekrar görüşerek cevapları hakkında bir daha düşünmesi, eklemek istediği bir şey olup olmadığını sormuştur. Ayrıca, veri analiz sürecinde ikinci bir araştırmacı ile çıkarılan kodlar karşılaştırılarak ne kadar uyumlu oldukları hesaplanmıştır. Tez danışmanı ve tez izleme komitesi ile kodlar tartışılmış ve verilerin analizi tamamlanmıştır.

3. Bulgular

3.1. Ortaokul Matematik Öğretmenlerinin Alan Bilgisi

4 ortaokul öğretmenlerine uygulanan 3 boyutlu cisimlerin hacmine ilişkin soru seti, yapılan görüşmeler ve sınıf gözlemlerinden elde edilen verilerin analizi sonucu, ortaokul öğretmenlerinin alan bilgisi, 3 boyutlu cisimlerin hacmi ile ilgili problemlerin çözümünde kullanılan farklı çözüm yöntemleri ve 3 boyutlu cisimlerin hacmine ilişkin problem kurmaya ilişkin bilgileri incelenmiştir.

3.1.1. Farklı Çözüm Yöntemlerine İlişkin Bilgileri

Soru setinden ve görüşmelerden elde edilen verilerin analizi sonucunda, 4 ortaokul matematik öğretmeni, 3 boyutlu cisimlerin hacmini hesaplamak için 4 farklı çözüm yöntemi belirtmişlerdir. 4 öğretmenin hepsi 3 boyutlu cisimlerin hacmini formül kullanarak hesaplayabileceklerini açıklamışlardır. Ancak öğretmenlerden 2'si

(Esin ve Can) sistemli sayma yönteminin hacim hesaplamada kullanılabileceğini söylemiştir. Diğer taraftan Gül ve İrem Öğretmen katman hesabı ve sütun/satır hesabı yaparak 3 boyutlu cisimlerin hacminin hesaplanabileceğini açıklamışlardır.

Öğretmenlerin sınıf gözlerimden elde edilen veriler, 4 öğretmenin hacim hesaplamaya yönelik soruları çözerken hacim formülünü kullandıklarını göstermiştir. Ayrıca öğretmenler, öğrencilerini formül kullanmaya yönlendirmişlerdir. Esin ve Can Öğretmen, yapılan görüşmede sistemli sayma yöntemini açıklamalarına rağmen bu yöntemi öğrencilerine anlatmamışlardır. Aynı şekilde, İrem Öğretmen de katman hesabı ve sütun/satır hesabını 3 boyutlu cisimlerin hacmini hesaplamak için kullanılabilecek yöntemlerden olduğunu düşünmesine rağmen bu yöntemleri öğrencilerine açıklamamıştır. Gül Öğretmen ise, bildiği tüm yöntemleri (hacim formülü, katman hesabı ve sütun/satır hesabı) öğrencilerine açıklamış ve derste bu yöntemleri kullanarak sorular çözmüştür.

3.1.2. Problem Kurmaya İlişkin Bilgileri

Öğretmenlerin 3 boyutlu cisimlerin hacmi ile ilgili problem kurmaya ilişkin bilgilerini incelemek için soru setindeki 8. soru (Şekil 3.1), 4 ortaokul matematik öğretmenine yöneltilmiştir.

Külah, yay uzunluğu, 15, yarıçap ve 54 cm ifadelerini kullanarak hacim bağıntısını içeren bir problem kurunuz.

Şekil 3.1: 8. soru

Katılımcı öğretmenlerden 3 tanesi (Esin, İrem ve Can) 8.soruda verilen terim ve sayıları kullanarak hacim hesaplamaya yönelik bir problem kuramamışlardır. İrem Öğretmen sayılar ile terimleri eşleştirmiştir. Yapılan görüşmede, İrem öğretmen "*54 yay uzunluğu olabilir, 15 de yarıçap olabilir. Çünkü yay, yarıçaptan uzundur. O şekilde.*" şeklinde açıklama yapmıştır. İrem Öğretmen gibi, Esin Öğretmen de problem kurmayı denemiş fakat yarıçap ve yay uzunluğu terimlerini kullanamamıştır. Bu yüzden de soru kurmayı başaramamıştır. Can Öğretmen ise verilen sayı ve terimleri kullanarak problem kurmaya yönelik hiçbir yorumda bulunamamıştır.

Diğer taraftan, Gül öğretmen sayı ve terimleri kullanarak problem kurmada başarılı olmuş ve kuruduğu problemi doğru bir şekilde çözmüştür. Gül Öğretmen'in kurduğu problem aşağıda verilmiştir.

Yay uzunluğu 54 cm ve ana doğrusunun uzunluğu yani daire diliminin yarıçapı 15 cm olan külahın hacmini bulunuz.

Sonuç olarak, çalışmaya katılan öğretmenlerden 3 tanesinin (Esin, İrem ve Can) hacim ile ilgili problem kurmaya yönelik bilgilerinin zayıf olduğu gözlemlenmiştir..

3.2. Ortaokul Matematik Öğretmenlerinin Pedagojik Alan Bilgisi

Bu çalışmada, 4 ortaokul öğretmeninin pedagojik alan bilgileri dört farklı açıdan incelenmiştir: öğretim stratejileri bilgisi, öğrenci bilgisi, öğretim programları bilgisi ve ölçme ve değerlendirme bilgisi.

3.2.1. Öğretim Stratejileri Bilgisi

Sınıf gözlemlerinden elde edilen veriler, öğretmenlerin, 3 boyutlu cisimlerin hacmini anlatırken çoğunlukla öğretmen merkezli öğretim uyguladıklarını göstermektedir. Öğretmenler konuyu tanıtıp, örneğin prizmanın hacmi, konuyla ilgili öğrencilerin var olan bilgilerini anlamak için sorular sormuşlardır. Mesela, "*Hacim nedir?, Bütün prizmaların hacmini nasıl hesaplarsınız?*" gibi sorular öğretmenlere yöneltilmişitir. Daha sonra, konuyla ilgili açıklamalarda bulunup öğrencilere örnek olması amacıyla kendileri birkaç soru çözmüşlerdir. Bu esnada, sorulardaki önemli noktalarla ilgili açıklamalar yapmışlardır. Dersin geri kalan kısmında, öğrencilere soru sorup onların tahtada çözmesini sağlayarak konuyu pekiştirmeye çalışmışlardır. Konuyu anlatırken öğretmenlerin hepsi materyal kullanmışlar veya şekil çizmişlerdir.

Bunun dışında, bazı derslerde, Gül ve Esin Öğretmen tartışma yöntemi ile zenginleştirilmiş daha az öğretmen merkezli öğretimde uygulamışlardır. Bu öğretim yöntemini uygularken, öğretmenler sadece bilgiyi aktarmamışlar, öğrencilere de konunun anlatımında sorumluluklar vermişlerdir. Öğretmen ve öğrenciler arasındaki etkileşim daha fazladır ve soru sorma ile tartışma, öğrenme sürecinde iç içedir. Gül ve Esin Öğretmen, konuyu kendileri açıklamadan önce öğrencilerden konu ile ilgili sunum yapmalarını istemişlerdir. Sunum esnasında, öğrencilerden tartışma ortamı yaratmalarını, gösterimler, günlük hayat örnekleri ve materyal kullanmalarını istemişlerdir. Böylece, Gül ve Esin Öğretmen öğrenme süreci boyunca öğrencilerin daha aktif olmalarını sağlamışlar ve kendileri daha pasif bir rol üstlenmişlerdir.

3.2.2. Öğrenci Bilgisi

Bu çalışmada, 4 ortaokul matematik öğretmenin 3 boyutlu cisimlerin hacmine ilişkin pedagojik alan bilgileri öğrenci bilgisi açısından da incelenmiştir. Verilerin analizi sonucu, öğretmenlerin öğrenci bilgisi 4 alanda incelenmiştir bunlar: Öğrencilerin tercih edeceği çözüm yöntemlerine ilişkin bilgileri, öğrencilerin farklı çözüm yöntemlerini yorumlama, öğrenci hataları ve öğrenci hatalarının kaynakları ve öğrenci hatalarını gidermek için kullanılabilecek yöntemlere ilişkin bilgileri.

3.2.2.1. Öğretmenlerin, Öğrencilerin Tercih Edeceği Çözüm Yöntemlerine İlişkin Bilgileri

Çalışmaya katılan öğretmenler, öğrencilerinin 3 boyutlu cisimlerin hacmine ilişkin soruları çözmek için hacim formülünü kullanacaklarını belirtmişlerdir. Bununla ilgili, Gül Öğretmen aşağıdaki açıklamayı yapmıştır:

> *Öğrenciler kısa yoldan doğru sonuca ulaşmaya odaklanıyorlar. Formül kullanmayı çok pratik buluyorlar. Formülü ezberleyip işlem yapmayı tercih ediyorlar.*

Bunun dışında, Esin ve Can Öğretmen öğrencilerin sistemli sayma yöntemini tercih edebileceklerini belirtirken, İrem Öğretmen de öğrencilerin katman hesabı yaparak 3 boyutlu cisimlerin hacmi ile ilgili soruları çözebileceklerini belirtmiştir.

3.2.2.2. Öğretmenlerin, Öğrencilerin Farklı Çözüm Yöntemlerini Yorumlama

Öğretmenlerin pedagojik alan bilgilerini, öğrencilerin farklı çözüm yöntemlerini yorumlayabilmeleri açısından incelemek için soru setinde öğrencilerin farklı çözüm yöntemlerini içeren sorular verilmiştir. Yapılan görüşmelerde, öğretmenlerin bu çözüm yöntemlerini yorumlamaları istenmiştir.

Öğretmenler, soruları doğru çözen öğrencilerin hacim formülü, sistemli sayma ve katman hesabı kullanarak soruları doğru bir şekilde çözmüş olabileceklerini belirtmişlerdir. Hacim formülü kullanan öğrencilerin çözüm yöntemlerini, 4 matematik öğretmeni de doğru bir şekilde açıklayabilmişlerdir. Sistemli sayma yöntemi kullanan öğrencilerin çözüm yöntemini Gül, Can ve İrem Öğretmen doğru bir şekilde yorumlarken, katman hesabı yöntemini sadece İrem Öğretmen doğru bir şekilde açıklayabilmiştir.

Öğretmenler, öğrencilerin yanlış olan çözüm yöntemlerini de doğru bir şekilde açıklayabilmişlerdir. Gül ve İrem Öğretmen, öğrencilerin hacim hesaplarken prizmaların yüzey alanını hesaplayabileceklerini belirtmişlerdir. Ayrıca, Esin ve İrem Öğretmen öğrencilerin sadece görünen yüzlerdeki birim küpleri saydıkları için prizmanın hacmini yanlış hesaplamış olabileceklerini açıklamışlardır. Can Öğretmen ise öğrencilerin yanlış çözüm yollarından birinin görünen kenarların uzunluğunu hesaplamak olabileceğini söylemiştir.

Öğretmenler, öğrencilerin çözüm yöntemlerini doğru bir şekilde yorumlayabildikleri gibi bazı çözüm yöntemlerini yanlış yorumlamışlar ve bazılarını yorumlayamamışlardır.

3.2.2.3. Öğretmenlerin, Öğrenci Hataları ve Öğrenci Hatalarının Kaynaklarına İlişkin Bilgileri

Katılımcı öğretmenler, 3 boyutlu cisimlerin hacmini hesaplarken öğrencilerin 4 farklı hata yapabileceklerini belirtmişlerdir. Bu hatalardan 3 boyutlu cisimlerin yüzlerine odaklanmayı 4 matematik öğretmeni de belirtmiştir. Öğretmenlere göre, öğrenciler 3 boyutlu cisimlerin sadece yüzlerine odaklanmakta ve içlerini görmezden gelmektedirler. Daha detaylı açıklamak gerekirse, 4 matematik öğretmeninin hepsi, öğrencilerin prizmaların sadece görünen 3 yüzündeki birim küplerin sayısını hesaplayabileceklerini söylemişlerdir. Bunun dışında, İrem Öğretmen, öğrencilerin sadece bir dış yüzdeki birim küpleri veya tüm dış yüzlerdeki birim küpleri de hesaplayabileceklerini açıklamıştır.

Esin ve İrem Öğretmen, bu hatanın kaynağını, öğrencilerin cisimleri 3 boyutlu düşünememeleri olduğunu belirtmişlerdir. Esin Öğretmen şu şekilde açıklamıştır:

> *3 boyutlu bir cisim hayal edemiyor. Sadece gördüklerine odaklanıyorlar. Bütün sorun sanıyorum bundan kaynaklanıyor.*

Ayrıca, İrem Öğretmen, öğrencilerin 3 boyutlu cisimlerin yapısını anlayamamalarından dolayı bu hatayı yapmış olabileceklerini açıklamıştır. Öğrencilerin prizmaların 6 yüzü olduğunu bilmediklerini, prizmaların gördükleri 3 yüzden oluştuğunu düşündüklerini söylemiştir. Can Öğretmen ise öğrencilerin 3 boyutlu cisimleri somutlaştıramamalarından dolayı sadece cisimlerin dış yüzüne odaklandıklarını açıklamıştır. Bunların dışında, Gül Öğretmen öğrencilerin 3 boyutlu cisimlerin dış yüzlerine odaklanmalarının nedenlerinden birinin dikkatsizlik olduğunu belirtmiştir.

Öğretmenlerin belirttiği hatalardan diğeri ise öğrencilerin, prizmaların yan yana olan yüzlerindeki ortak küpleri birden fazla saymalarıdır. Bu hatayı sadece Gül ve Esin Öğretmen söylemiştir. İki öğretmen de bu hatanın nedenini, öğrencilerin dikkatsizliği ve kavramları derinlemesine düşünmemeleri olduğunu açıklamışlardır. Gül ve Esin Öğretmen, öğrenciler dikkatsiz olduğu için ortak küpleri görmemiş olabileceklerini veya prizmaların yapısını derinlemesine incelemedikleri ve düşünmedikleri için ortak küpleri fark etmemiş olabileceklerini belirtmişlerdir.

Veri analizi sonucunda, öğrencilerin 3 boyutlu cisimlerin hacmine ilişkin hataları arasında kavramsal hatalar olduğu ortaya çıkmıştır. Bu hatalar, hacim ve yüzey alanını karıştırma, hacim ve çevreyi karıştırma, yan yüz yüksekliği ile cisim yüksekliğini karıştırma, üçgenin alanını hesaplayamama ve Pisagor Teoremini uygulayamama olarak belirlenmiştir. Gül Öğretmen'e göre, öğrencilerde kavramsal bilgi eksikliği olduğu için öğrenciler hacim ve yüzey alanı kavramlarını karıştırmışlardır. Diğer taraftan, İrem Öğretmen'e göre, öğrenciler 3 boyutlu cisimleri 2 boyutlu düşündükleri için yüzey alanı hesaplamışlardır. Can Öğretmen ise öğrencilerin hacim ve çevreyi karıştırmalarının nedenini cisimleri zihinlerinde somutlaştıramamaları olarak açıklamıştır. Öğretmenlerin hepsi öğrencilerdeki kavram eksikliği nedeniyle yan yüz yüksekliği ile cisim yüksekliğini karıştırdıklarını belirtmişlerdir. Gül, Esin ve Can Öğretmen, öğrencilerin prizmaların hacmini hesaplarken üçgenin alanını hesaplamada zorlandıklarını açıklamışlardır. Son olarak, Esin Öğretmen, öğrencilerin Pisagor Teoremini uygulayamamalarının öğrencilerin 3 boyutlu cisimlerin hacmini hesaplarken hata yapmalarına neden olabileceğini belirtmiştir. Katılımcı öğretmenler, bu hataların (üçgenin alanını hesaplayamama ve Pisagor Teoremini uygulayamama) nedenini öğrencilerin dikkatsizliği ve konuyu derinlemesine düşünmemeleri olarak tespit etmişlerdir.

Öğretmenlerin bahsettiği son hata işlem hatasıdır. Bu hatayı sadece Esin Öğretmen söylemiş ve hatanın kaynağının öğrencilerin dikkatsizliği olduğunu vurgulamıştır. Esin Öğretmen, öğrencilerin formülü uygularken dikkatsizlikleri nedeniyle çarpma işleminde hata yapabileceklerini belirtmiştir.

Sonuç olarak, öğretmenler, 3 boyutlu cisimlerin hacmini hesaplarken öğrencilerin 4 farklı hata yapabileceklerini tespit etmişler ve bu hataların kaynağı olarak 6 farklı neden ortaya koymuşlardır.

3.2.2.4 Öğretmenlerin, Öğrenci Hatalarını Gidermek için Kullanılabilecek Yöntemlere İlişkin Bilgileri

Veri analizine göre, katılımcı öğretmenlerin hepsi öğrencilerin hatalarını gidermek için 2 farklı yöntem önermişlerdir. Gül ve Esin Öğretmen, öğrencilerin prizmaların yan yana olan yüzlerindeki ortak küpleri birden fazla saydıklarını belirtmişlerdir. Öğretmenler, bu hatayı materyal kullanarak giderebileceklerini açıklamışlardır. Örnek olarak, Esin Öğretmen'in açıklaması aşağıda verilmiştir:

> *Birim küplerden cisimler yaptırabiliriz. Küpleri tek tek sayarız. Ayrıca hacmini hesaplarız. İkisinin aynı olduğunu görürüz . Evet, hacmin gerçekten küp sayısı verdiğini söyleyebiliriz veya küp sayısının hacmi verdiğini de yani ikisini de ancak ve ancak şeklinde çift taraflı bağlantı kurabiliriz.*

Bunun dışında, Gül, Can ve İrem Öğretmen öğrencilerin hacim ile alanı karıştırma, hacim ile çevreyi karıştırma ve cisim yüksekliği ile yan yüz yüksekliğini karıştırma ile ilgili hatalarını gidermek için de materyal kullanımının etkili olacağını belirtmişlerdir. Ayrıca, Esin, Can ve İrem Öğretmen, öğrencilerin cisim yüksekliği ile yan yüz yüksekliğini ayırt etmelerini sağlamak için bu kavramların tekrar anlatılmasının ve bunlara ilişkin çok sayıda soru çözülmesinin yararlı olabileceğini açıklamışlardır. Ayrıca, 4 öğretmen de işlemsel hataları konuyu tekrar anlatarak ve bol bol soru çözerek giderilebileceklerini vurgulamışlardır.

3.2.3. Öğretim Programları Bilgisi

Bu çalışmada, ortaokul öğretmenlerinin pedagojik alan bilgileri, öğretim programları bilgisi açısından da incelenmiştir. Öğretmenlerin öğretim programları bilgisi, konuları ilişkilendirme ve konuların sırasını değiştirme boyutlarında değerlendirilmiştir.

Gül, Esin ve İrem Öğretmen, 3 boyutlu cisimlerden prizma ve piramidin hacmini geçmiş yıllarda anlatılan çokgenlerin alanı ile ilişkilendirmiştir. Ayrıca, piramitlerin hacmini kısa zaman önce öğretilen prizmaların hacmi ile ilişkilendirerek anlatmışlardır. Fakat Can Öğretmen görüşmede 3 boyutlu cisimlerin hacmini anlatırken başka hiçbir konu ile ilişkilendirmediğinden bahsetmesine rağmen konuyu anlatırken piramitlerin hacmini, prizmaların hacmi ile ilişkilendirmiştir. Veri analizi göstermektedir ki öğretmenler 3 boyutlu cisimlerin hacmini, sonraki yıllarda öğretilecek konularla veya diğer derslerde öğretilen konularla ilişkilendirmemişlerdir.

Katılımcı öğretmenlerin hepsi matematik müfredatında 3 boyutlu cisimlerin alt konularının sırasının öğrencilerin konuyu öğrenmesini zorlaştırdığını düşünmektedirleri. Müfredatta, 3 boyutlu cisimlerin anlatım sırası şu şekildedir: Prizma ile ilgili temel kavramlar açıklanır, prizmanın yüzey alanı ve daha sonra hacmi anlatılır (MEB, 2006). Bu noktaya kadar, öğretmenler konunun sırasında değişiklik yapmamışlardır. Daha sonra piramit, koni ve küre ile ilgili temel kavramlar tanıtılır. Bunların yüzey alanları anlatılır. Son olarak da piramit, koni ve kürenin hacmi açıklanır (MEB, 2006). Öğretmenler, piramit, koni ve küreyi müfredatta verilen bu sırayla anlatmanın karışıklığa neden olduğunu, konunun tam olarak anlaşılmadığını ve sürekli geri dönüşler yapıldığı için zaman kaybına yol açtığını düşünmektedirler. Yani öğretmenlere göre, öğrenciler piramidi tam olarak kavramadan koniyi öğrenmeye çalışmaktadırlar. Bu yüzden, öğretmenler konuların pekişmediğini düşünmektedirler. Bu sıralama yerine, öğretmenler, önce piramit ile ilgili kavramları açıklayıp, daha sonra piramidin yüzey alanını ve son olarak da piramidin hacmini anlatmayı tercih etmişlerdir. Aynı sıralamayı koni ve küre içinde yapmışlardır. Böylece, öğretmenlere göre, öğrenciler piramidi anlayıp ve piramit ile ilgili yeterince soru çözdükten sonra, koniyi öğreneceklerdir ve konular kendi içinde bütün sağlayacaktır. Hem öğrenme daha etkili olacaktır hem de zaman kaybı olmayacaktır.

3.2.4. Ölçme ve Değerlendirme Bilgisi

Bu çalışmada, 4 ortaokul öğretmeninin 3 boyutlu cisimlerin hacmi konusunda öğrenci başarısını ölçmek için kullandıkları ölçme ve değerlendirme yöntemleri incelenmiştir.

Dört öğretmenin genel olarak 2 farklı ölçme ve değerlendirme yöntemi uygulamışlardır. Birinci yöntem, biçimlendirici değerlendirme yöntemidir. Bu değerlendirme de öğrenme süreci devam ederken öğrencilerin 3 boyutlu cisimlerin hacmine ilişkin bilgileri değerlendirilmiştir. Öğretmenler, bu değerlendirme yöntemini öğrencilerin konuyu ne kadar anladıklarını öğrenmek amacıyla uygulamışlardır. Ders esnasında öğrencilere konuyla ilgili sorular sormuşlar ve bu soruları öğrencilerin kendi kendilerine çözmesini istemişlerdir. Öğrenciler soruları çözerken onları gözlemlemişler ve eğer öğrencilerin zorlandıkları noktalar var ise o anda öğrencilere yardım etmişlerdir. Böylece, öğrencilerin konuyla ilgili zorlandıkları noktalar hakkında bilgi edinmişler ve bu bilgi ışığında dersin gidişatına karar vermişlerdir.

Öğretmenlerin kullandığı diğer biçimlendirici değerlendirme yöntemi ders sonunda konuyla ilgili verilen ev ödevleridir. Katılımcı öğretmenlerin hepsi ders kitabından veya çalışma kitabından ödev vermelerine rağmen, Esin ve Can öğretmen verdikleri ödevleri kontrol etmemişlerdir. Fakat Gül ve İrem öğretmen, bazen öğrencilerin ödevlerini kontrol etmişler ve öğrencilerin çözemedikleri soruları derste çözmüşlerdir. Böylece, öğrencilerin anlamadıkları kısımları tekrar anlatmışlardır.

Diğer taraftan, öğretmenler öğrenme süreci bittikten sonra öğrencilerin 3 boyutlu cisimlerin hacmine ilişkin bilgilerini ölçmek için düzey belirleyici değerlendirme yöntemlerini uygulamışlardır. Bu ölçme ve değerlendirme yöntemi kapsamında konunun anlatımı bittikten sonra öğretmenler yazılı sınav uygulamışlardır. Ayrıca, öğretmenler, öğrencilere MEB'in zorunlu tuttuğu performans ödevi ve proje görevi vermişlerdir.

Sonuç olarak, bu çalışmada 4 ortaokul matematik öğretmeninin alan bilgileri, 3 boyutlu cisimlerin hacmi ile ilgili problemlerin çözümünde kullanılan farklı çözüm yöntemlerine ve 3 boyutlu cisimlerin hacmi ile ilgili problem kurmaya ilişkin bilgileri incelenmiştir. Çalışma bulguları göstermektedir ki, ortaokul matematik öğretmenleri, 3 boyutlu cisimlerin hacmi ile ilgili problemleri çözmek için 4 farklı çözüm yöntemi geliştirebilmişlerdir ve genel olarak, öğretmenler, 3 boyutlu cisimlerin hacmi ile ilgili problem kurmakta zorlanmışlardır.

Diğer taraftan, 4 ortaokul öğretmeninin pedagojik alan bilgileri dört boyutta incelenmiştir. Öğretmenler, 3 boyutlu cisimlerin hacmini öğretmen merkezli öğretim yöntemi kullanarak anlatmışlardır. Öğretmenlere göre, öğrenciler 3 boyutlu cisimlerin hacmi ile ilgili problemleri çözerken hacim formülü kullanmayı tercih etmektedirler. Öğretmenler, öğrencilerin farklı çözüm yöntemlerinin bir kısmını açıklayabilmişler fakat bir kısmını açıklayamamışlardır. Öğretmenlerin pedagojik alan bilgileri, öğrencilerin 3 boyutlu cisimlerin hacmine ilişkin hatalarını belirleyebilme açısından değerlendirildiğinde, öğretmenlerin 4 farklı hata ve bu hataların kaynağı olarak da 6 farklı kaynak belirttikleri ortaya çıkmıştır. Öğretmenler, öğrencilerin hatalarını yok etmek için 2 farklı yöntem önermişlerdir. Ayrıca, öğretmenler, 3 boyutlu cisimlerin hacmini geçmiş yıllarda ve aynı yılda anlatılan konularla ilişkilendirmişlerdir. İlaveten, öğretmenler, 3 boyutlu cisimlerin alt konuları ile ilgili matematik müfredatındaki sıralamanın verimli öğrenme açısından uygun olmadığını düşünüp sıralamayı değiştirmişlerdir. Son olarak, 4 öğretmen de öğrencilerin konuya ilişkin bilgilerini ölçmek için biçimlendirici ve düzey belirleyici değerlendirme yöntemlerini kullanmışlardır.

Aşağıda çalışmanın bulguları ilgili literatürden yararlanılarak tartışılacaktır.

4. Tartışma, Sonuç ve Öneriler

Çalışmanın bulguları gösteriyor ki, 4 ortaokul matematik öğretmeni 3 boyutlu cisimlerin hacmi ile ilgili problemleri çözmek için 4 farklı çözüm yöntemi belirtmişlerdir. Öğretmenler sistemli sayma, katman hesabı ve sütun/satır hesabı yöntemlerini görüşmelerde açıklamalarına rağmen öğrencilerine bu yöntemleri öğretmemişlerdir. Bununla ilgili olarak, Singmuang (2002) çalışmasında, öğretmenlerin formül kullanarak çözebilecekleri sorular için başka yöntemler geliştirmeye gerek görmediklerini belirtmiştir. Ayrıca öğretmenlerin formül kullanmalarının sebebi matematik kavramları yani hacim kavramı ile ilgili bilgilerinin yetersiz olması olabilir (Berenson ve diğ., 1997). Yani öğretmenlerin hacim kavramına ilişkin bilgileri yetersiz olduğu için öğrencilerine farklı çözüm yöntemlerini anlatmadıkları ve hatta öğrencilerini bu yöntemleri uygulamaları konusunda cesaretlendirmedikleri sonucuna ulaşılabilir. Bu sonuç literatürdeki geçmiş çalışmaların sonuçları ile paralellik göstermiştir (Berenson ve diğ., 1997; Hill, 2007). Ayrıca, çalışmanın bulguları, öğretmenlerin alan bilgilerinin, onların 3 boyutlu cisimlerin hacmine ilişkin problem kurmaları için yeterli olmadığını göstermiştir. Çünkü bir konuyu problemlerle, şekillerle, materyallerle ifade etmek, öğretmenlerin o konuya ait bilgilerinin derinliğini göstermektedir (Ball, 1990a).

Daha öncede belirtildiği gibi, katılımcı öğretmenler 3 boyutlu cisimlerin hacmini öğretmen merkezli öğretim yöntemi ile anlatmışlardır. Öğretmenlerin bu öğretim yöntemini kullanma nedenlerinden bir tanesi, diğer öğretim yöntemleri ile ilgili ders anlatım tecrübelerinin yetersizliği olabilir (Flick, 1996). Ayrıca, yetersiz alan bilgisine sahip olmaları da konuyu öğretmen merkezli öğretim yöntemi ile anlatmalarının bir nedeni olarak düşünülebilinir. Katılımcı öğretmenler, öğrencilerin 3 boyutlu cisimleri zihinlerinde canlandırmaları gerektiğini ve bu konuda öğrenilmesi gereken birçok kavram (prizma, piramit, koni, küre, yükseklik, cisim yüksekliği) olduğunu belirtmişlerdir. Bu nedenlerden dolayı, öğretmenler, öğretmen merkezli öğretim yöntemi ile kavramların net bir şekilde açıklanması ve materyal kullanarak öğrencilerin cisimleri zihinlerinde canlandırmalarına yardım etmeleri gerektiğini düşünmüşlerdir. Halbuki, Borko ve Putnam (1996) öğrenci merkezli öğretim ile öğrencilerin konuyu daha iyi anlayacaklarını belirtmiştir. Son olarak, 3 boyutlu cisimlerin hacmi konusunun 8.sınıf müfredatındaki son konu olması ve öğrencilerin 8.sınıfta lise giriş sınavlarına hazırlanıyor olmaları da öğretmenlerin, öğretmen merkezli öğretim yöntemi uygulamalarının bir nedeni olabilir. Böylece öğretmenler konuyu daha hızlı anlatmış, konuyla ilgili daha fazla soru çözmüş ve zaman sıkıntısı yaşamamış olacaklardır.

Öğretmenler, öğrencilerin hacim sorularını çözerken formül kullanmayı tercih edeceklerini belirtmişlerdir. Öğretmenlerin, öğrencilerin tercih edecekleri çözüm yöntemlerine ilişkin pedagojik alan bilgileri, öğrencilerin ders esnasında kullandıkları çözüm yöntemleri ile tutarlıdır. Öğrencilerin formül kullanmayı tercih etmelerinin nedeni, öğretmenlerin derste soruları formül kullanarak çözmeleri olabilir (Zacharos, 2006). Ayrıca, öğretmenlerin pedagojik alan bilgilerini değerlendirmek için onlardan öğrencilerin farklı çözüm yöntemlerini açıklamaları istenmiştir. Elde edilen bulgular gösteriyor ki eğer öğretmen, öğrencinin çözüm yöntemini daha önceden biliyorsa, bu yöntemi kolaylıkla açıklayabilmiştir. Fakat öğrencinin çözüm yöntemi ile daha önce karşılaşmamışsa veya kendisi o yöntemi hiç kullanmamışsa, öğretmen, öğrencinin çözüm yöntemini açıklamada başarısız olmuştur. Bunun nedeni de öğretmenlerin, öğrencilerin ne düşündüğünü bilmemesi ve yeterli alan bilgisine sahip olmamaları olabilir. Literatürdeki benzer çalışmalarda bu bulguyu desteklemektedir (Carpenter, Fennema, & Franke, 1996; Esen & Çakıroğlu, 2012; Hill, 2007; Türnüklü & Yeşildere, 2007). İlaveten, öğretmenlerin öğrenci bilgisi, öğrencilerin hatalarını ve bu hataların kaynağını belirleme açısından da incelenmiştir. Öğretmenlerin öğrenci hatası olarak belirttiği hatalar, 3 boyutlu cisimlerin hacmi ile ilgili öğrencilerle yapılan çalışmalarda da tespit edilmiştir (Battista & Clements, 1996; Ben-Chaim ve diğ., 1985; Hirstein, 1981). Öğretmenlere, verilen sorularda öğrencilerin muhtemel hataları sorulduğunda, öğretmenler birçok hata belirtebilmişlerdir. Fakat öğrencilerin yanlış çözüm yöntemleri öğretmenlere verilip bu yöntemdeki hataları sorulduğunda, öğretmenler hatayı bulmada zorlanmışlardır. Bunun nedeni ise öğretmenlerin öğrencilerin düşünme yapılarına ilişkin bilgilerinin az olması olabilir (Carpenter, Fennema, & Franke, 1996). Öğretmenler, öğrencilerin hatalarının kaynağı olarak 6 farklı kaynak belirtmişlerdir. Bu kaynaklar, aynı konuda öğrenciler ile yapılan çalışmalarda da tespit edilmiştir (Battista & Clements, 1996; Ben-Chaim ve diğ., 1985; Hirstein, 1981). Son olarak, öğretmenlerin öğrencilerin hatalarını gidermek için belirttiği yöntemler çok genel yöntemlerdir. 3 boyutlu cisimlerin hacmine özel yöntemler değildir. Öğretmenler, 3 boyutlu cisimlerin hacmini, çokgenlerin alanı ile ilişkilendirmişlerdir. Fakat bu ilişkilendirmede öğrencilerin konuyu daha iyi anlamaları amaçlanmamıştır. Çokgenlerin alanı, hacim formülünün bir parçasıdır. Dolayısıyla, öğrenciler hacim formülünü uygulamak için çokgenlerin alanını bilmek zorundadırlar. Bunun dışında, öğretmenler piramidin hacmini prizmanın hacmi ile ilişkilendirerek anlatmışlardır. Böylece, öğretmenler, öğrencilerin piramidin hacmini daha kolay bir şekilde öğrenmesini sağlamaya çalışmışlardır. Veri analizine göre, öğretmenler konuyu başka derslerle veya günlük hayatla ilişkilendirmemişlerdir. Bunun nedeni de öğretmenlerin konular arasındaki bilgileri iyi kurgulanmamış olması olabilir. Öğretmenlerin öğretim programları bilgisi, 3 boyutlu cisimler ile ilgili alt konuların matematik müfredatındaki sırasını değiştirme açısından da incelenmiştir. Çalışmanın bulgularına göre, öğretmenler, 3 boyutlu cisimlere ilişkin konuların müfredattaki sırasının, öğrencilerinin konuyu anlamlı bir şekilde öğrenmesi için uygun olmadığını fark etmişlerdir. Bu problemleri fark etmelerinde öğretmenlerin geçmiş deneyimleri etkili olmuş olabilir (Friedrichsen ve diğerleri, 2007). Ayrıca, Baştürk ve Kılıç (2011) öğretmenlerin alan bilgilerinin de bu problemi fark etmelerinde etkili olabileceğini belirtmiştir. Son olarak, öğretmenlerin pedagojik alan bilgileri, ölçme ve değerlendirme bilgisi açısından da incelenmiştir. Daha önce de belirtildiği gibi, öğretmenler biçimlendirici değerlendirme yöntemi olarak soru sorma ve ev ödevi verme yöntemlerini kullanmışlardır. Öğretmenlerin ders esnasında soru sormalarının amacı, öğrencilerin konuyu anlamaları ile ilgili geri bildirim almak, öğrencilerin konuyla ilgili anlamadıkları noktaları ve öğrencilerin hatalarını tespit etmektir. Lankford'da (2010) çalışmasında, öğretmenlerin aynı amaçlar için ders esnasında soru sorma yöntemini kullandıklarını belirtmiştir. Fakat öğretmenler, verdiklerini ev ödevlerini genellikle kontrol etmemişlerdir. Öğretmenlerin ev ödevi vermelerindeki amaç, öğrencilerin evde ders çalışmasını sağlamaktır. Hâlbuki literatürdeki çalışmalar öğrencilere ev ödevini vermekte ki amacın öğrencilerin konuyu anlamalarına ilişkin geri bildirim almak olduğunu açıklamaktadır (Cowie & Bell, 1999; Lankford, 2010). Diğer taraftan, öğretmenler düzey belirleyici değerlendirme yöntemleri olarak yazılı sınavlar, performans ödevleri ve proje görevleri uygulamışlardır. Bu ölçme ve değerlendirme yöntemlerini tercih etmelerinin nedeni MEB tarafında zorunlu koşulması olabilir (MEB, 2006). Fakat bu değerlendirme yöntemlerini uygulamalarındaki amaç, MEB'in amaçlarıyla örtüşmemektedir. Öğretmenler, öğrencilere not verme amacıyla bu yöntemlere başvurmuşlardır.

Sonuç olarak, öğretmenlerin alan bilgilerini ve pedagojik alan bilgilerini geliştirmek için hizmet içi eğitim seminerlerine katılabilirler. Öğretmenlere bilgilerini geliştirme konusunda yardımcı olmak için müfredatı geliştiren eğitimciler ve kitap yazarları, öğretmenlerin konuyu anlatırken uygulayabilecekleri daha fazla etkinlik örnekleri sunabilirler. Öğretmen kılavuz kitabında ve diğer ders kitaplarında yer alan soruların formül odaklı olmasından ziyade kavramsal anlamaya yönelik sorular olmasına özen gösterebilirler.

Bu konuda yapılacak başka çalışmalar, öğretmenlerin geometri alt öğrenme alanındaki başka konulardaki (3 boyutlu cisimlerin yüzey alanı, üçgenler, açılar gibi) bilgilerini incelemek amacıyla yapılabilinir. Ayrıca, araştırmacılar, öğretmen bilgisinde deneyimin önemini anlamak amacıyla deneyimli ve deneyimsiz öğretmenlerin alan bilgilerini karşılaştırabilirler. Öğretmen eğitimcilere method derslerini düzenlemekte yardımcı olmak için, öğretmen adaylarının 3 boyutlu cisimlerin hacmine ilişkin alan bilgilerini araştıran çalışmalar düzenlenebilir. 2013 yılında düzenlenen matematik müfredatında 3 boyutlu cisimlerin hacmi konusu 10.sınıf müfredatına alınmıştır. Bu nedenle, lise öğretmenlerin bu konudaki bilgileri önem kazanmıştır. Araştırmacılar, lise öğretmenlerinin 3 boyutlu cisimlerin hacmine ilişkin bilgilerini araştırabilirler. Son olarak, öğretmenlerin bu konuyla ilgili bilgilerinin genellenebilmesi için nicel çalışmalar yapılabilir.

Bu çalışmada bazı sınırlılıklar bulunmaktadır. Nitel çalışmalarda katılımcıların özellikleri çok önemli bir role sahiptir. Çalışma başka şartlardaki (çalıştıkları okulun bulunduğu bölge, deneyim, sınıf ortamı gibi) farklı öğretmenlerle yapılmış olsa çalışmanın bulguları farklı olabilirdi. Ayrıca, araştırmacı, öğretmenler konuyu anlatmaya başlamadan önce sınıfta bulunmuş ve video çekimi yapmış olsa bile, verinin sınıf ortamında video kaydı alınarak toplanmış olması, öğretmenleri ve öğrencileri etkilemiş olabilir. Son olarak, 3 boyutlu cisimlerin hacminin 8. sınıf müfredatındaki son konulardan biri olması nedeniyle öğretmenler konuyu daha yüzeysel ve hızlı bir şekilde anlatmış olabilirler. Bu durum çalışmanın bulgularını değiştirmiş olabilir. Yani, bu konu müfredatta daha önce yer almış olsaydı, öğretmenler konuyla ilgili daha fazla etkinlik düzenleyip öğrenci merkezli öğretim yöntemi uygulayabilirlerdi.

Kaynakça

Aslan-Tutak, F. (2009). *A study of geometry content knowledge of elementary pre-service teachers: The case of quadrilaterals.* Unpublished doctoral dissertation, University of South Florida, USA.

Atiyah, M. (2001). Mathematics in the 20th century: Geometry versus algebra. *Mathematics Today, 37*(2), 46- 53.

Baki, M. (2013). Sınıf öğretmeni adaylarının bölme işlemi ile ilgili matematiksel bilgileri ve öğretimsel açıklamaları. *Eğitim ve Bilim, 38* (167), 300-311.

Ball, D. L. (1990a). The mathematical understanding that prospective teachers to teacher education. *Elementary School Journal , 90*, 449-466.

Ball, D. L. (1990b). Prospective elementary and secondary teachers' understanding of division. *Journal of Research in Mathematics Education, 21*, 132-144.

Ball, D. L. (1991). Research on teaching mathematics: Making subject-matter knowledge part of the equation. In J. Brophy (Ed.), *Advances in research on teaching: Vol: 2, Teachers' subject-matter knowledge* (pp.1-48). Greenwich,CT: JAI Press.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59* (5), 389- 407.

Battista, M. T., & Clements, D. H. (1996). Students' understanding of three-dimensional rectangular arrays of cubes. *Journal for Research in Mathematics Education, 27* (3), 258- 292.

Baturo, A., & Nason R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics, 31,* 235-268.

Baştürk, S., & Dönmez, G. (2011). Examining pre-service teachers' pedagogical content knowledge with regard to curriculum knowledge. *International Online Journal of Educational Sciences, 3*(2), 743-775.

Ben-Chaim, D., Lappan, G. & Houang, R. T. (1985). Visualizing rectangular solids made of small cubes: Analyzing and affecting students' performance. *Educational Studies in Mathematics, 16,* 389- 409.

Berenson, S., Von Der Valk, T., Oldham, E., Runesson, U., Moreira, Q., & Broekman, H. (1997). An international study to investigate prospective teachers' content knowledge of the area concept. *European Journal of Teacher Education, 20*(2), 137-150.

Borko, H., Eisenhart, M., Brown, C. A., Underhill, R. G., Jones, D. & Agard, P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education, 23*(3), 194-222.

Borko, H. & Putnam, R. T. (1996). Learning to teach. In D. C. Berliner & R. C. Calfee (eds.) *Handbook of Educations Psychology*. New York, NY: MacMillan.

Carpenter, T. P., Fennema, E., & Franke, M.L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal, 97* (1), 3-20.

Chapman, O. (2002). High School Mathematics Teachers' Perspectives of Mathematical Word Problems. In B. Davis & E. Simmt (Eds.), *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group,*(pp. 91-98). Edmonton, AB: CMESG/GCEDM.

Contreras, J. M., Batanero, C., Díaz, C., & Fernandes, J. A. (2011, February). *Prospective teachers' common and specialized knowledge in a probability task.* Paper presented at The Seventh Congress of the European Society for Research in Mathematics Education, Poland.

Cowie, B. & Bell, B. (1999) A model of formative assessment in science education. *Assessment in Education, 6*(1), 101–116.

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five approaches* (2nd ed.). Thousand Oaks, California: Sage Publications, Inc.

Esen, Y. ,& Çakıroğlu, E. (2012). İlköğretim matematik öğretmen adaylarının hacim ölçmede birim kullanmaya yönelik kavrayışları. *MED, 1*, 21-30.

Even, R. (1993). Subject matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education, 24*(2), 94-116.

Even, R., & Tirosh, D. (1995). Subject matter knowledge and knowledge about students as sources of teacher presentations of the subject matter. *Educational Studies in Mathematics, 29*(1), 1-20.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). NewYork: Macmillan.

Flick, L. B. (1996). Understanding a generative learning model of instruction: A case study of elementary teacher planning. *Journal of Science Teacher Education, 7(2)*, 95-122.

Friedrichsen, P. M., Lankford, D., Brown, P., Pareja, E., Volkmann, M. J., & Abell, S. K. (2007, April). *The PCK of future science teachers in an alternative certification program.* Paper presented at the National Association for Research in Science Teaching Conference, New Orleans, LA.

Fujita, T, & Jones, K. (2006). P*rimary trainee teachers' understanding of basic geometrical figures in Scotland.* Paper presented at the annual meetings of the International Group For The Psychology Of Mathematics Education, Czech Republic.

Gess-Newsome, J. (1999). Pedagogical content knowledge: An introduction and orientation nature, sources and development of pedagogical content knowledge for science teaching, In J. Gess-Newsome & N. G. Lederman (Eds.). *Examining pedagogical content knowledge: The construct and its implications for science education* (pp.3-17). Boston: Kluwer.

Gomes, A. (2011). Portuguese pre-service elementary teachers' knowledge of geometric transformations: An exploratory study. In Smith, C. (Ed.), *Proceedings of the British Society for Research into Learning Mathematics, 31(3)*, 59-64.

Grossman, P. L. (1990). *The making of a teacher: Teacher knowledge and teacher education.* New York: Teachers College.

Hacıömeroğlu, G., Hacıömeroğlu, E.S., & Aspinwall, L. N. (2007). Prospective secondary teachers' subject matter knowledge and pedagogical content knowledge: The ramifications for teaching. In Lamberg, T., & Wiest, L. R. (Eds.). *Proceedings of the 29th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education,* Stateline (Lake Tahoe), NV: University of Nevada, Reno.

Hill, H. C.(2007). Mathematical knowledge of middle school teachers: Implications for the no child left behind policy initiative. *Educational Evaluation and Policy Analysis, 29* (2), 95-114.

Hill, H. C., Ball, D. L., & Schilling, S. (2004). Developing measures of teachers' mathematics knowledge for teaching. *Elementary School Journal, 105*(1), 11-30.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Hirstein, J. J. (1981). The second national assessment in mathematics: Area and volume. *Mathematics Teacher, 74*, 704-708.

Huang, R., & Kulm, G. (2012). Prospective middle grade mathematics teachers' knowledge of algebra for teaching. *The Journal of Mathematical Behavior, 31* (4), 417-430.

Hutchison, L. (1997). *Learning for teaching: A case of constructing the bridge between subject matter knowledge and pedagogical content knowledge (Report No. 143).*

Işıksal, M. (2006). *A study on pre-service elementary mathematics teachers' subject matter knowledge and pedagogical content knowledge regarding the multiplication and division of fractions.* Unpublished doctoral dissertation, Middle East Technical University, Turkey.

Işıksal, M., & Cakıroğlu, E. (2008). Pre-service teachers' knowledge of students' cognitive processes about the division of fractions. *Hacettepe University Journal of Education, 35*, 175- 185.

Kahan, J.A., Cooper, D.A., & Bethea, K.A. (2003). The role of mathematics teachers' content knowledge in their teaching: A framework for research applied to a study of student teachers. *Journal of Mathematics Teacher Education, 6*, 223-252.

Karahasan, B. (2010). *Preservice secondary mathematics teachers' pedagogical content knowledge of composite and inverse function.* Unpublished doctoral dissertation, Middle East Technical University, Turkey.

Kellogg, M. S. (2010). *Preservice elementary teachers' pedagogical content knowledge related to area and perimeter: A teacher development experiment investigating anchored instruction with web-based microworlds.* Unpublished doctoral dissertation, University of South Florida, USA.

Lankford, D. (2010). *Examining the pedagogical content knowledge and practice of experienced secondary biology teachers for teaching diffusion and osmosis.* Unpublished doctoral dissertation, University of Missouri, MO, USA.

Leinhardt, G., & Smith, D. A. (1985). Expertise in mathematics instruction: Subject matter knowledge. *Journal of Educational Psychology*, *77*, 247-271.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry.* Beverly Hills, CA: Sage.

Livy, S., & Vale, C. (2011). First year pre-service teachers' mathematical content knowledge: Methods of solution for a ratio question. *Mathematics Teacher Education and Development, 13*(2), 22-43.

Ma, L. (1999). *Knowing and teaching elementary mathematics: teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

Magnusson, S., Krajcik, J., & Borko, H. (1999). Nature, sources and development of pedagogical content knowledge for science teaching. In J. Gess-Newsome & N. G. Lederman (Eds.), *Examining pedagogical content knowledge: The construct and its implications for science education* (pp. 95-132). Boston: Kluwer.

Maxedon, S. J. (2003). *Early childhood teachers' content and pedagogical knowledge of geometry*. Unpublished doctoral dissertation, The University of Arizona, USA.

Merriam, S. B. (1998). *Qualitative research and case study applications in education (Rev. ed.).* San Francisco, CA: Jossey-Bass.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative Data Analysis; An Expanded Source Book*. Sage Publications.

Milli Eğitim Bakanlığı [MEB] (2006). *İlkögretim matematik dersi ögretim programı 6-8. sınıflar: Ögretim programı ve kılavuzu.* Ankara, Turkey: MEB.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics.* Reston, VA: Author.

Ng, G. L. (1998). *Exploring children's geometrical thinking.* Unpublished doctoral dissertation, University of Oklahoma, Norman.

Olkun, S. (1999). *Stimulating children's understanding of rectangular solids made of small cubes.* Unpublished doctoral dissertation, Arizona State University, USA.

Patton, M. Q. (2002). *Qualitative evaluation and research methods* (3rd ed.). Thousand Oaks, CA: Sage.

Rowland, T., Huckstep, P., &Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The Knowledge Quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*(3), 255-281.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harward Educational Review, 57*(1), 1-22.

Singmuang, C. (2002). *Thai preservice middle school mathematics teachers' subject matter knowledge and knowledge of students' conceptions of division of rational numbers with respect to their classroom practices.* Unpublished doctoral dissertation, Oregon State University, USA.

Stake, R. E. (1995). *The art of case study research.* Thousand Oaks, CA: Sage Publications, Inc.

Türnüklü, E. B., & Yeşildere, S. (2007). The pedagogıcal content knowledge in mathematics: Preservice primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers:The Journal, 1*. Retrieved October 20, 2011 from http://www.k-12prep.math.ttu.edu/journal/contentknowledge/volume.shtml

Yin, R. K. (2003). Case study research: Design and methods (3rd ed.). *Applied Social Research Methods Series*, *(Volume 5).* Thousand Oaks, CA: Sage Publications, Inc.

Zacharos, K. (2006). Prevailing educational practices for area measurement and students' failure in measuring areas. *Journal of Mathematical Behaviour, 25,* 224–239.

APPENDIX M

TEZ FOTOKOPİSİ İZİN FORMU

<u>ENSTİTÜ</u>

Fen Bilimleri Enstitüsü ☐

Sosyal Bilimler Enstitüsü ■

Uygulamalı Matematik Enstitüsü ☐

Enformatik Enstitüsü ☐

Deniz Bilimleri Enstitüsü ☐

<u>YAZARIN</u>

Soyadı : TEKİN-SİTRAVA
Adı : Reyhan
Bölümü : İlköğretim Matematik Öğretmenliği

<u>TEZİN ADI</u>: An Investigation into Middle School Mathematics Teachers' Subject Matter Knowledge and Pedagogical Content Knowledge Regarding the Volume of 3D Solids

<u>TEZİN TÜRÜ</u> : Yüksek Lisans ☐ Doktora ■

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
3. Tezimden bir bir (1) yıl süreyle fotokopi alınamaz. ■

<u>TEZİN KÜTÜPHANEYE TESLİM TARİHİ</u>: