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THE REDUNDANCY ALLOCATION PROBLEM: A TAXONOMIC REVIEW

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YEDEKLİĞİN KULLANILDIĞI SİSTEM GÜVENİLİRLİĞİ OPTİMİZASYONU ÜZERİNE TAKSONOMİK BİR LİTERATÜR ARAŞTIRMASI

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ABSTRACT

THE REDUNDANCY ALLOCATION PROBLEM: A TAXONOMIC REVIEW

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Reliability, which can be described as the probability that a system operates on a continuous basis without failure for a pretedetermined mission time, is an important measure of system performance. Being parallel to the increasing complexity of systems, the results of the system’s unreliability have become severe in terms of cost, effort, lives, etc., therefore the need for developing more reliable systems have become very important. In this content, reliability optimization problem is an important type of optimization problems because of its wide practical applications in real-world such as manufacturing systems, telecommunication systems, transportation systems and electrical power systems.

In this study, a special type of reliability optimization problems which is called as the redundancy allocation problem is discussed, and a comprehensive literature survey in this field is presented based on a novel classification methodology. To analyze the latest trends in this area, the main focus is especially on papers which are presented in the last decade.

KEY WORDS: redundacy allocation, reliability optimization, literature review.

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ÖZ

YEDEKLİĞİN KULLANILDIĞI SİSTEM GÜVENİLİRLİĞİ OPTİMİZASYONU ÜZERİNE TAKSONOMİK BİR LİTERATÜR ARAŞTIRMASI

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Güvenilirlik, yaygın olarak sistem performans ölçütlerinden biri olarak ele alınmaktadır. Sistemlerin her geçen gün artan karmaşıklık düzeyi nedeniyle, güvenilirlik düzeyi düşük sistemlere ilişkin maliyet, performans, ömür vb. sistem parametreleri açısından ciddi sıkıntılarla karşılaşılmakta olup, sistem güvenilirliğinin artırılması çok önemli bir ihtiyaç halini almıştır. Bu kapsamda, güvenilirlik optimizasyonu problemi; üretim, telekomünikasyon, ulaşım, elektrik güç sistemlerinin tasarımı gibi pek çok gerçek hayat probleminde uygulama alanı bulan yapısıyla önemli bir optimizayon problemi türü halini almıştır.

Bu çalışmada, güvenilirlik optimizasyonu probleminin özel bir türü olan, yedekliğin kullanıldığı sistem güvenilirliği optimizasyonu problemi üzerine odaklanılmış olup, özellikle 2000'li yıllardan sonra yayımlanan çalışmalar üzerinden literatürde yer alan mevcut model ve yöntemler özetlenmekte ve bu kapsamda literatürün sınıflandırılmasına ilişkin geliştirilen yeni bir yaklaşım çerçevesinde detaylı bir literatür araştırması sunulmaktadır.

ANAHTAR SÖZCÜKLER: yedekli atama, güvenilirlik optimizasyonu, literatür araştırması.

Danışman: Prof.Dr. Berna DENGİZ, Başkent Üniversitesi, Endüstri Mühendisliği Bölümü.

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LIST OF SYMBOLS AND ABBREVATIONS

ACO	Ant Colony Optimization
AUGMECON	Augmented Epsilon Constraint
BBMOPSO	Bare- Bones Multi-Objective Particle Swarm Optimization
CE-NRGA	Controlled Elitism Non-dominated Ranked Genetic Algorithm
DC	Degrade Ceiling
DE	Differential Evolution
DSAMOPSO	Dynamic Self-Adaptive Multi-Objective Particle Swarm
FLC	Fuzzy Logic Controller
GA	Genetic Algorithm
GDA	Great Deluge Algorithm
HBMO	Honey Bee Mating Optimization
ICA	Imperialist Competitive Algorithm
IA	Immune Algorithm
MC	Monte Carlo
MS	Multi State
MCS	Monte Carlo Simulation
MSS	Multistate System
NN	Neural Network
NSGA II	Non-dominated Sorting Genetic Algorithm II
PDMOSA	Pareto Domination based Multi-Objective Simulated Annealing
PSA	Pareto Simulated Annealing
PSO	Particle Swarm Optimization
RAP	Redundancy Allocation Problem
RSM	Response Surface Methodology
OSSO	Orthogonal Simplified Swarm Optimization
SA	Simulated Annealing Algorithm
SMOSA	Suppapitnarm Multi-Objective Simulated Annealing
TS	Tabu Search
UGF	Universal Genearating Function
UMOSA	Ulungu Multi-Objective Simulated Annealing
VND	Variable Neighborhood Descent
VNS	Variable Neighborhood Search
WMOSA	Weight Based Multi-Objective Simulated Annealing

1. INTRODUCTION

An industrial system is can be described as a collection of components which is arranged in a specific design to achieve desired functions with acceptable performance. Reliability is a fundamental performance measure for the safe operation of any modern technological system. Reliability is defined as a system's ability to perform its intended function, without fail, for a time interval, under predetermined conditions. This attribute has far reaching consequences on the durability, availability, and life cycle cost of a product or system [1], and is of great importance to the end user/engineer. As being parallel to the increasing complexity of the systems today, reliability optimization plays a key role in engineering design and has been effectively applied to enhance system performance.

In realibility theory, the ways for providing improved reliability in a system design, can be listed as follows: (a) increasing component reliability; (b) using redundant components in a parallel manner; (c) a combination of (a) and (b); and (d) reassignment of interchangeable components [2].

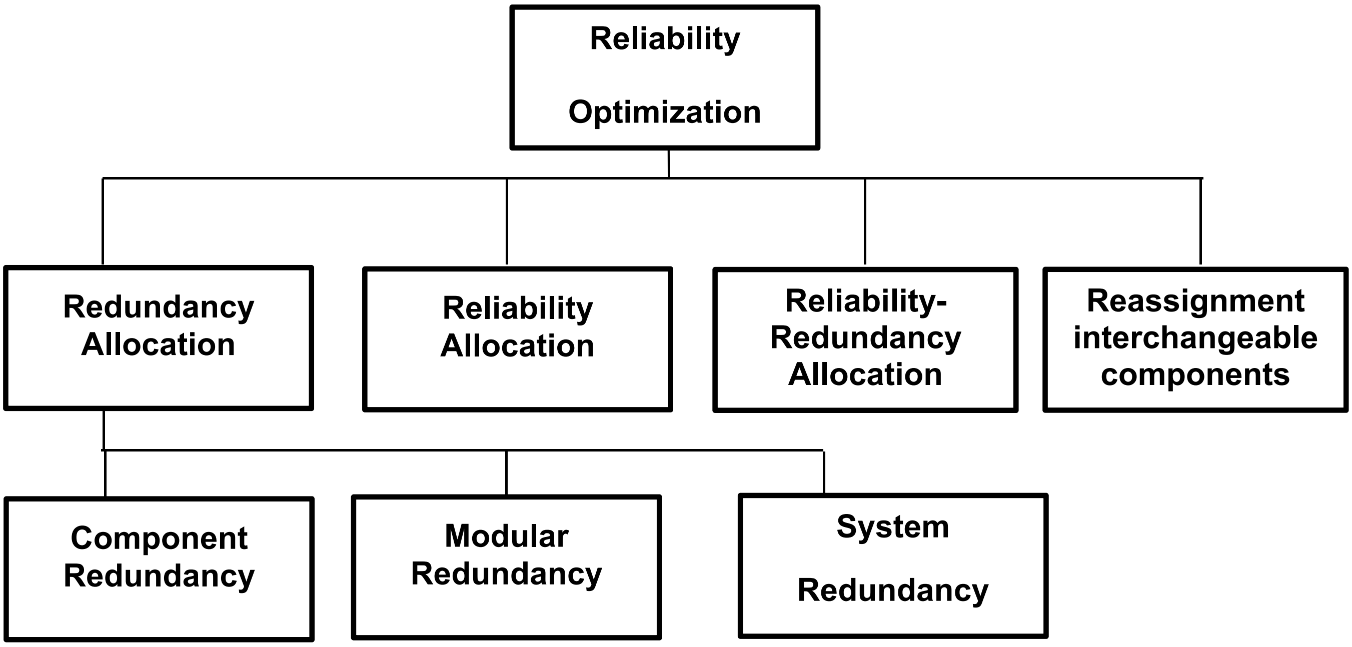

Figure 1.1 Reliability Optimization Problems

The redundancy allocation problem (RAP) is a well-known and complex design problem in reliability optimization field. The RAP is useful for system designs which are largely assembled and manufactured using off-the-shelf components and also, have relatively high reliability requirements such as most of the electronic systems today. In this study, by regarding its wide scope, the focus is mainly on RAP.

RAPs can be categorized under three headings: i) component redundancy, ii) modular redundancy, iii) system redundancy as depicted in Figure 1.1. The detailed information related to these will be presented in the following sections.

Component redundancy, which is in the scope of this study, has very important role in engineering design to increase the system performance in terms of the realiability. Often two different component redundancy techniques are taken into consideration. One of them is parallel redundancy where all redundant units are in parallel and working simultaneously. This method is useful when the system is required to operate for a long period of time without interruption. The other method is standby redundancy where one of redundant units begins to work only when the active one failed. This method is usually employed when the replacement takes a negligible amount of time and does not cause system failure. The detailed information related to these will be discussed in following sections.

In literature, there are few surveys which review the literature of the reliability optimization problems. This study aims to contribute to the previous literature surveys mentioned above. To analyze the latest trends and give an idea to researchers for future research direction, the main focus is especially on papers which are presented in the last decade, but also a summary is presented on the previous works. This research reviews the related studies in the RAP field, based on a novel classification methodolgy for the RAP literature. This developed taxonomy will be a usefull new resource considering all the aspects of RAP areas for researchers studying in this field.

The organization of the study is given as follows. A brief history of the RAP literature, RAP definition, a novel RAP taxonomy and epistemology of the RAP literature are presented in Section 2. In Section 3, the related studies in the RAP field is presented, based on this novel RAP taxonomy, especially focusing mainly on papers presented in the last decade. Section 4 includes conclusions and a discussion of future research directions.

2. REDUNDANCY ALLOCATION PROBLEM

2.1. A Brief History of the RAP Literature

The RAP is one of the most important reliability optimization problems in the designing phase of the parallel-series systems, network systems and other systems with various structures. RAP is a complex combinatorial optimization problem, which has a broad application in the real-world, such as in computer network design [3], consumer electronics [4], software systems design [5], network design [6]).

An overview and summary of work in the RAP field, in terms of different approaches used, is presented in [2;7;8]. Yearout [9] discusses the literature related to standby redundancy. Also, in their study Kuo and Prasad [10] present system reliability optimization methods. Then, more recently new advancements in optimal reliability allocation problems are presented in [11].

2.2. RAP Definition

In RAPs, the main goal is to increase the possibility that a sufficient number of components will survive when a failure occurs and the system will still continue to its intended function by adding some additional functionally identical components to the system.

The RAP can be applied in different system structures, including series, parallel, network, parallel-series, k-out-of-n and the like. The series-parallel system, as depicted in Figure 2.1 (i.e. $k_i = 1, \forall i$) is a common system structure that is used in most of the system designs. The conventional RAP for a series-parallel system pertains to a system of s subsytems in series, and each subsytem is configured with n_i components in parallel. Redundant components may be either active or in a standby mode. For each subsystem there are m_i functionally equivalent components that can be selected. Each available component has different levels of cost, weight, reliability and other characteristics. There is an unlimited supply of each of the m_i choices. When a component is selected, the same choice of is used for all n_i parallel components. The problem can be described as deciding the component types and levels of redundancy to maximize the reliability under the system level constraints such as cost, weight, volume and etc. [12].

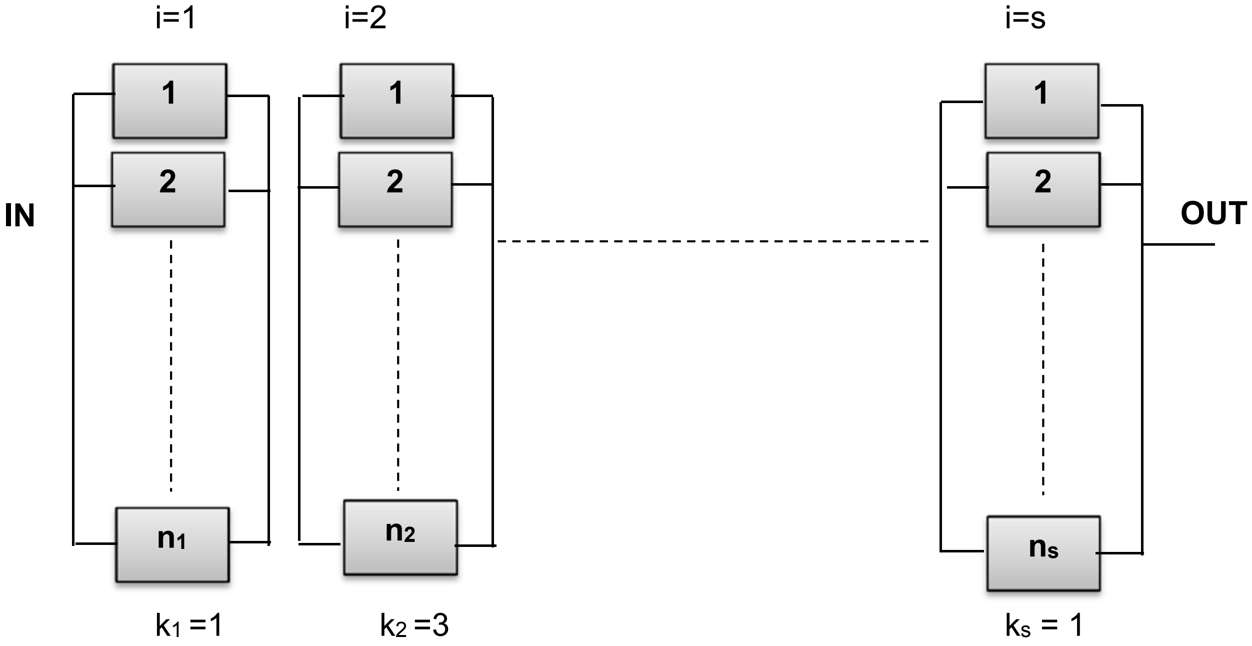

Figure 2.1 Series-parallel System [13]

With the aim of finding the optimum number of redundancies, the RAP can be formulated as maximization of the system reliability under the given cost, weight etc. constraints, or the minimization of the system cost, weight, etc. under the condition that the system reliability is equal or greater than a predetermined level. The basic assumptions and the problem formulations related to RAPs are stated below:

Assumptions:

1) Unlimited supply for each components,
2) Failures of individual components are mutually statistically independent,
3) Failed components do not damage the system,
4) There is no preventive maintenance,
5) System weight ad system cost are linear combinations of component weight and cost

Notations:

x_{ij}: quantity of the j^{th} component of subsytem i
c_{ij}: cost of the j^{th} component of subsytem i
w_{ij}: weight of the j^{th} component of subsytem i
m_i: number of available components for subsystem i
k_i: minimum number of operating components required for subsystem i
s: number of subsystems

Problem 1. (Maximize Reliability)

max $R(t_0; x)$,

s.t.

$$\sum_{i=1}^{s}\sum_{j=1}^{m_i} c_{ij} x_{ij} \le C ,$$

$$\sum_{i=1}^{s}\sum_{j=1}^{m_i} w_{ij} x_{ij} \le W ,$$

$$\sum_{j=1}^{m_i} x_{ij} \ge k_i \text{ for i=1,2,…,s}$$

$$x_{ij} \in \{0,1,2,\ldots\}$$

where R is the system reliability, C and W are the system cost and weight, respectively.

Problem 2. (Minimize Cost)

min C (x) = $\sum_{i=1}^{s}\sum_{j=1}^{m_i} c_{ij} x_{ij}$,

s.t.

$$R(t_0; x) \ge R ,$$

$$\sum_{i=1}^{s}\sum_{j=1}^{m_i} w_{ij} x_{ij} \le W ,$$

$$\sum_{j=1}^{m_i} x_{ij} \ge k_i \text{ for i=1,2,…,s}$$

$$x_{ij} \in \{0,1,2,\ldots\}$$

Chern [14] showed that even a simple redundancy allocation problem in series systems with linear constraints is NP-hard. This implies that it is unlikely an exact algorithm exists with computational requirements that increase less than exponentially with problem size. Also, RAPs are characterised by non-convex and combinatorial search spaces and require a considerable amount of computational effort to find exact optimal solutions [15]. To deal with these problems, a number of algorithms which can be categorised as mathematical programming (approximation or exact), heuristic and meta-heuristics have been used to find optimal solutions to the problems discussed above. The surrogate worth tradeoff, the Lagrange multiplier, and geometric programming methods and their variants can be counted under the approximation algorithms [16;17]. These methods used a kind of trial and error approaches in order to obtain integer solutions [18]. The approximation techniques were popular when exact solution algorithms were under-developed. Hence, their popularity decreased with the advandement of exact algorithms, such as integer programming, branch-and-bound, and dynamic programming [19].

The mathematical programming techniques (approximation and exact algorithms), are not sufficient for complex and large scale problems, such as real life network reliability and redundancy allocation optimisation problems [20;21]. Although the heuristic and meta-heuristic approaches (such as Genetic Algorithms, Simulated Annealing and Tabu-Search) yield solutions which are approximate, they can efficiently handle complexity [22;23], also hybrid optimization techniques are another promising direction in this area. They may combine heuristic methods, neural network, some local search methods, and all kinds of metaheuristics to improve computational efficiency or with exact methods to reduce the search space. Also, two metaheuristic algorithms can also be combined such as Genetic Algorithm and Simulated Annealing or Ant Colony Algorithm.

2.3. Need for a RAP Taxonomy

The size and growth rate of the RAP literature needs a systematic way to classify the various contributions in order to provide a general understading on the existing literature, and also the way ahead in terms of future research direction. Hence, in this study a novel taxonomy for RAP is presented.

According to Reisman [24], a useful taxonomy,

"… will display the similarities and the differences among the various contributions graphically, symbolically or both, thus will demonstrate the relationship of all contributions and the practical applications of each to other. It will provide a framework by which all of the existing knowledge can be systematically filed and therefore recalled efficiently and effectively… “

Beside being a tool for systematic storage, basic motivations and uses for a taxonomy can be summarized as follows [25]:

- It draws the boundaries of the interested subject domain.
- It efficiently and effectively displays all of that domain’s attributes.
- It is an effective and efficient way for the user to identify the sub-fields in the related subject domain and to understand the relationship between these sub-fields and the main frame.
- It is an effective and efficient way for the user to organize his or her knowledge management about the domain in terms of teaching, learning, storing and recalling.
- It is an effective and efficient way for the user to identify the lively topics in the related literature which is very important for researchers, funding agencies and other decision makers.

Any taxonomy is mainly dependent on the definition of the boundaries of the universe it classifies, hence the developed classification in this study has to be expanded being parallel to enlargement in the scope of the RAP.

2.3.1. RAP Taxonomy

RAP deserves to be considered as a seperate and distinct field as the result of the vast literature devoted to this problem type. The increasing interest in RAP makes a systematic eleboration of this field more important in helping researchers as well as attracting potential new researchers to this field.

The new RAP model classification developed in the scope of this study and the new taxonomy are presented in Figure 2.2, Figure 2.3 and in Table 2.1, respectively.

According to the developed classification approach. For classify a RAP model, first of all one has to decide the system configuration such as series, parallel, series-parallel, non-series parallel. At the second step, each of these configurations can be arranged by using homogenous or heterogenous components such as homogenous series parallel, heterogenous parallel etc. Next, the states of these components are taken into consideration (e.g. heterogenous series-parallel multi state system etc.) After deciding the state of the components, characteristics of the design parameters are considered. Design parameters can be deterministic or non-deterministic. Non-deterministic problems can be categorised under six headings: i) stochastic uncertainty, ii) interval uncertainty, iii) fuzzy unceratinty, iv) intiutionistic fuzzy and vague sets, v) fuzzy-random uncertainty, vi) chaos uncertainty (e.g. heterogenous series-parallel fuzzy multi state system) And then, the applied redundancy strategy is taken into consideration There are three different redundancy strategies that can be employed such as active, standby and mixed (combination of active and standby). As presented in Table 2.1, the standby redundancy is categorized under three headings: i) cold, ii) hot and iii) warm (e.g. heterogenous series-parallel multi state system with active redundancy).

After deciding the sytem model, a classification can be made according to the solution methods (i.e. mathematical programming, heuristic and meta heuristics), and optimization objectives (i.e. single objective or multi objective).

And finally after applying all of the steps explained above, the RAP model will have been categorized considering all aspects of it (e.g. A multi objective heterogenous series-parallel multi state system with active redundancy using hybrid particle swarm optimization and local search).

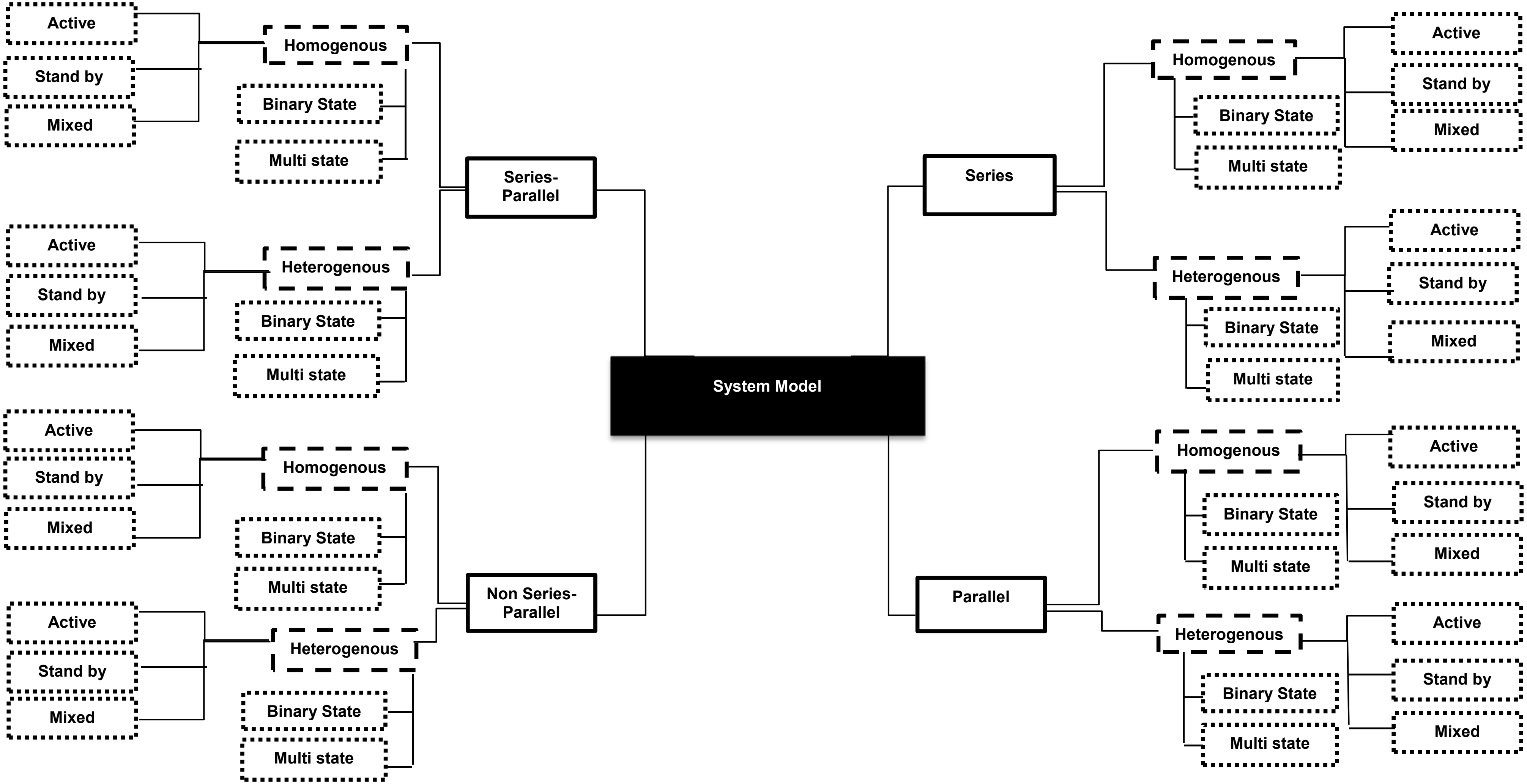

Figure 2.2 RAP System Model

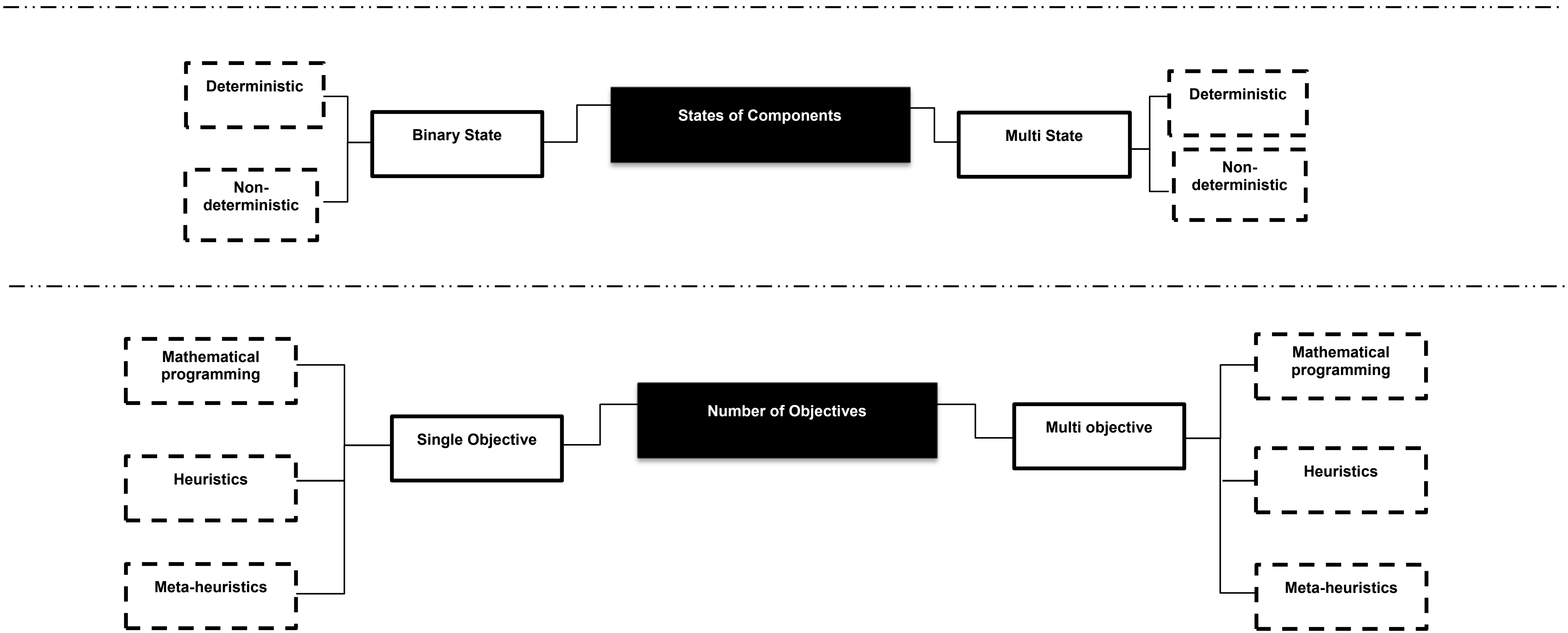

Figure 2.3 RAP Solution Approaches

Table 2.1 Taxonomy of the RAP literature

1. System Configuration	**2. States of Components**	**5. Type of Parameters**
1.1. Series	2.1. Binary State	5.1. Deterministic
1.1.1. Homogenous	2.2. Multi State	5.2.Non-deterministic
1.1.2. Heterogenous	**3. Number of objectives**	5.2.1.Stochastic
1.2. Parallel	3.1. Single Objective	5.2.2.Interval
1.2.1. Homogenous	3.2. Multi Objective	5.2.3.Fuzzy
1.2.2. Heterogenous	**4. Redundancy Strategy**	5.2.4.Intuitionistic fuzzy and vague sets
1.3. Series-Parallel	4.1. Active	5.2.5.Stochastic-fuzzy
1.3.1. Homogenous	4.2. Standby	5.2.6.Chaos
1.3.2. Heterogenous	4.2.1. Cold	**6. Solution Methods**
1.4.Non Series-Parallel	4.2.2. Hot	6.1. Mathematical Programming
1.4.1. Homogenous	4.2.3. Warm	6.2. Heuristics
1.4.2. Heterogenous	4.2.4. Mixed	6.3. Meta-heuristics

2.4. Analysis on the RAP literature

2.4.1. Literature Search Process

During the literature search process, a wide set of academic databases such as EBSCO Inspec, Scopus, Ei Compendex, and ISI Web of Science were utilized to compile information on the RAP. The databases were searched by using “redundancy allocation problem” and “redundancy-optimization” key words. This exact phrases were searched in “Subject/Title/Abstract” field options. By doing this, the irrelevant items beyond the scope of the study were eliminated. Also, bibliographical entries that refer to studies in languages other than English were eliminated.

2.4.2. Statistical findings

The 1394 bibliographical entities between 1969-2015, which included academic journals, book chapters, technical reports, and articles from various conference proceedings were examined. In Table 2.2 the details of the compiled bibliography are presented.

Table 2.2 List of different types of studies in the RAP literature

Entity Type	#
Academic journal	1121
Proceeding	229
Technical Report	41
Book chapters	3
TOTAL	**1394**

In Table 2.3, the total 1121 RAP articles are listed in descending order with respect to in which academic journals they have been published. It can be seen that “IEEE transactions on Reliability” and “Reliability Engineering and System Safety” are the most preferred journals for the RAP researchers. They account together for approximately 54% of all RAP articles published in refereed journals. “Journal of Heuristics” is in the third order as it has been depicted in Table 2.3. This situation shows parallelism with the increasing number of RAP articles in which heuristics and meta heuristic solution approaches used especially in recent years.

Table 2.3 RAP articles with respect to academic journals

Journal Title	#
IEEE Transactions on Reliability	457
Reliability Engineering and System Safety	144
Journal of Heuristics	116
International Journal of Quality&Reliability Engineering	113
International Journal of Engineering	78
Computers and Industrial Engineering	52
International journal of Applied Science and Engineering	37
Indian Journal of Industrial and Applied Mathematics	24
International Journal of Industrial and Systems Engineering	14
Expert Systems with Applications	12
Fuzzy Sets and Systems	11
Jounal of Computational Science	9
Computers in Industry	7
International Journal of Modern Mathematical Sciences	6
International Journal of Applied Operational Research	5
Applied Mathematics and Computation	4
Simulation Modeling Practice and Theory	4
Operations Research Letters	3
Computers and Operations Research	3
Engineering Optimization	3
Others	19
TOTAL	**1121**

The bar chart in Figure 2.4 shows that the RAP literature continues to grow steadily without losing its attraction since 1969-1973 period. In fact, this steady upward trend is an interesting result when the lenght of the time horizon of interest is considered (i.e. nearly a half century). Also, according to the Figure 2.4, it can be argued that the saturation point for the RAP literature has not been arrived at yet. Espacially during the last decade, 344 papers were reported in literature, with a maximum of 132 papers in 2013. This number was only 28 during the 1969-1973 period.

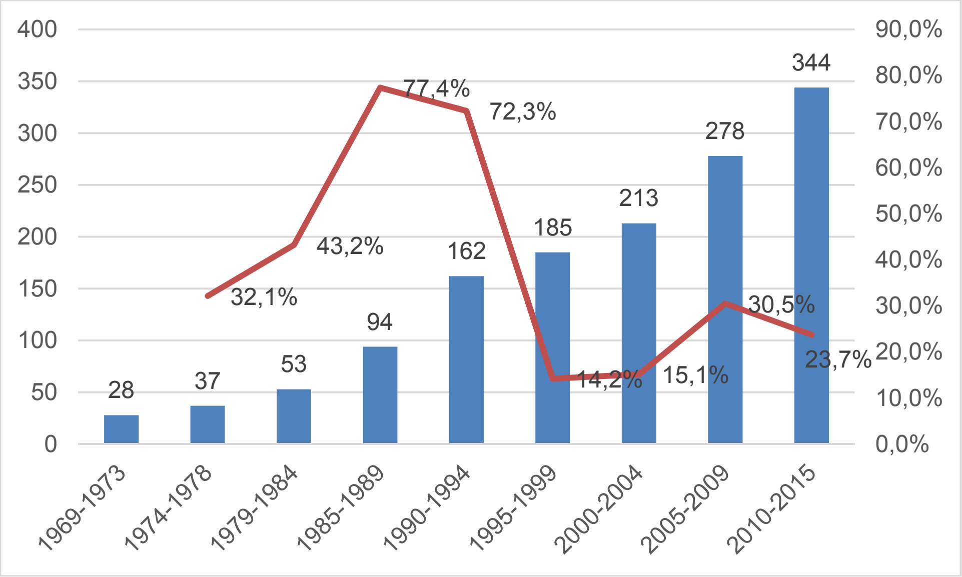

Figure 2.4 Distribution of RAP papers published from 1969-2015

Also in Figure 2.4, the fluctuations in growth rates according the former periods are presented. As it can be seen, this growth rate gets its highest value (77,4%) between 1985-1989 and 1990-1994 periods.

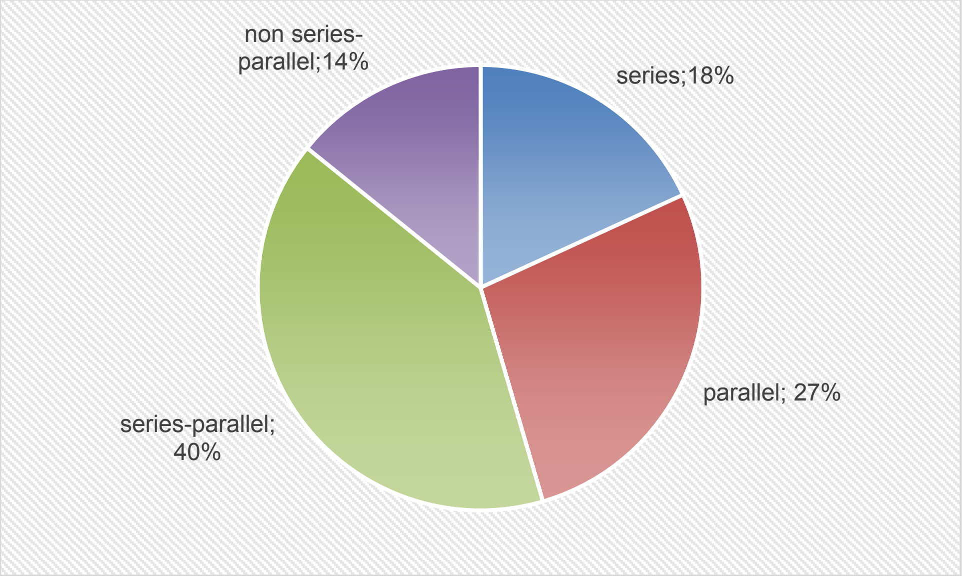

Figure 2.5 Distribution of RAP papers based on the system models

When these total 1391 papers (excluding book chapters) are classified based on the system configuration, it can be seen that 40% of the RAP problems are applied to the series-parallel systems as it is depicted in Figure 2.5. Parallel systems have the second biggest share with 27 percent. While the share of series systems are 18%, this rate is only 14% for non-series parallel systems.

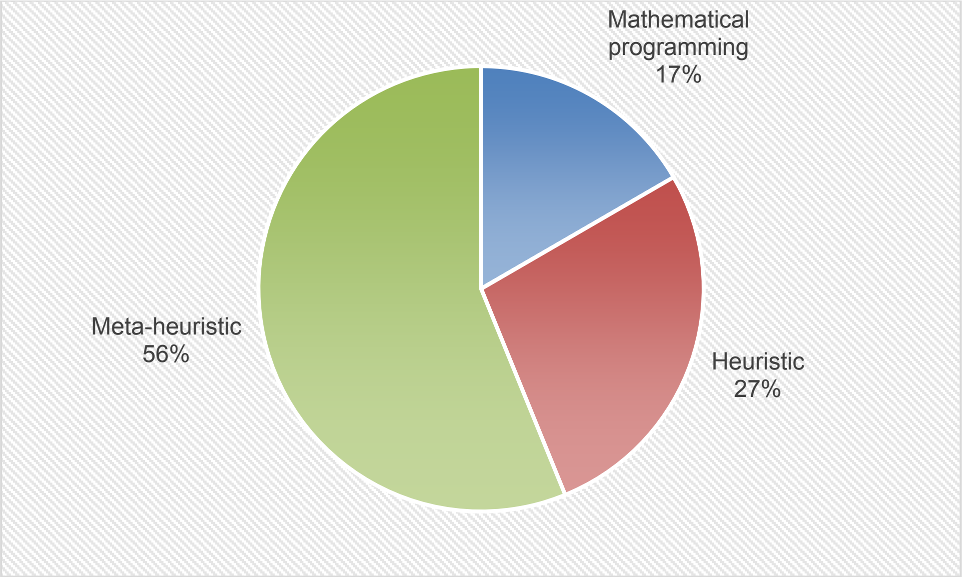

Figure 2.6 Distribution of RAP papers based on the solution approaches

Based on the solution approaches used in RAPs (including 1391 papers), it can be seen that in most of the problems in RAP field meta-heuristic methods (56%) are used as depicted in Figure 2.6. The share of heuristic approaches are 27%, and the share of mathematical programming approachess is 17%.

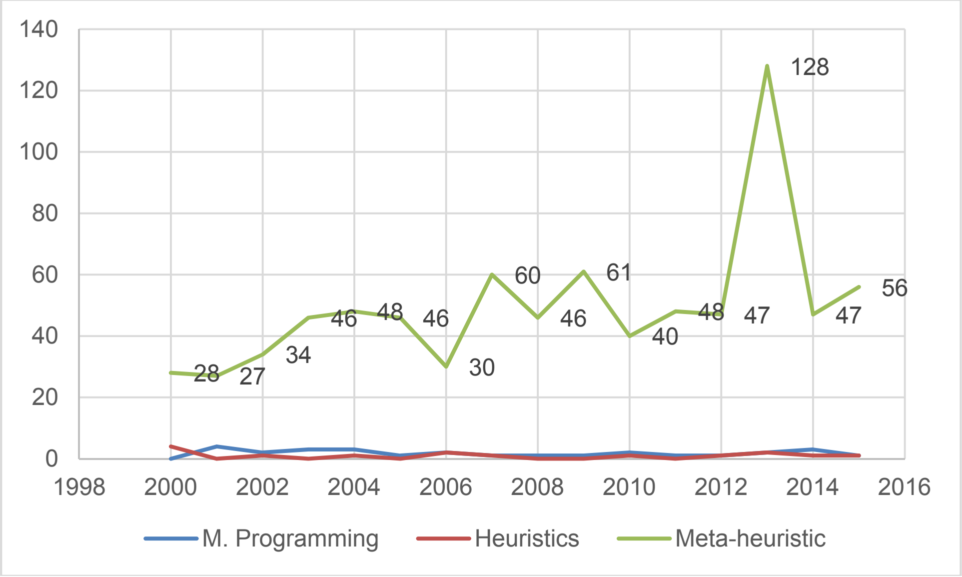

Figure 2.7 Solution methods used in RAP papers in last 15 years

During the last 15 years totally 792 RAP related papers were reported in literature. The graph in Figure 2.7 shows that meta heuristic solution approaches are highly prefered by researchers compared to the other methods, such as mathematical programming and heuristics, during this period of interest. However, mathematical programming and heuristic solution methods have not been completely absent.

Although there are fluctuations in the number of studies between years, there is an upward trend in the usage of meta heuristic approaches. While there are only 28 RAP papers in which meta heuristic techniques used in year 2000, this number goes up to 128 (by nearly quadrupling) and takes its highest value in year 2013. As a result of the large search spaces in RAP field regarding complex engineering systems, meta heuristics play an important role to produce good solutions for decision makers.

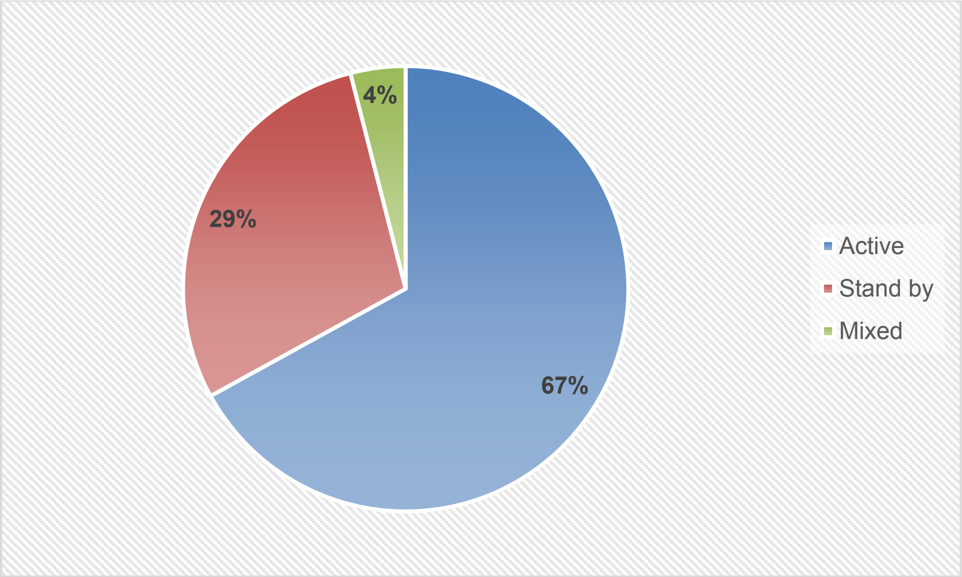

Figure 2.8 Distribution of RAP papers based on the redundancy strategy

Redundancy strategy is another criteria used for classification (including 1391 papers), and the results show that the major redundancy strategy applied in RAP field is active redundancy with a share of 67%. As it can be seen from the chart in Figure 2.8, the share of stand by redundancy is 29% and mixed strategy is employed in only 4% of these total 1391 papers. In fact, this is an expected situation being parallel to the increasing complexity of systems today.

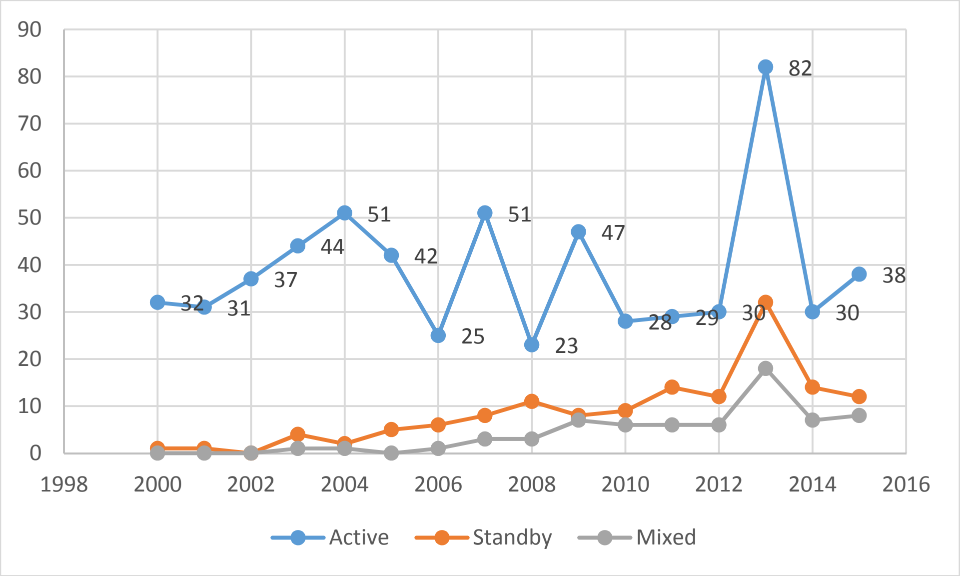

Figure 2.9 Redundancy strategy used in RAP papers in last 15 years

In Figure 2.9, as being parallel to the results depicted in Figure 2.8, active redundancy strategy is the most preferred redundancy strategy type during the last 15 years too. In fact, in many real life situations, standby and mixed redundancy strategies are become more important for system designers as these approaches can provide higher reliability values without increasing the system design parameters such as system cost and weight. Although there is an increase in the number of studies in this area, this field is still somehow under-developed in RAP literature. Hovever, this situation can be regarded as an advantage for researchers studying in this field.

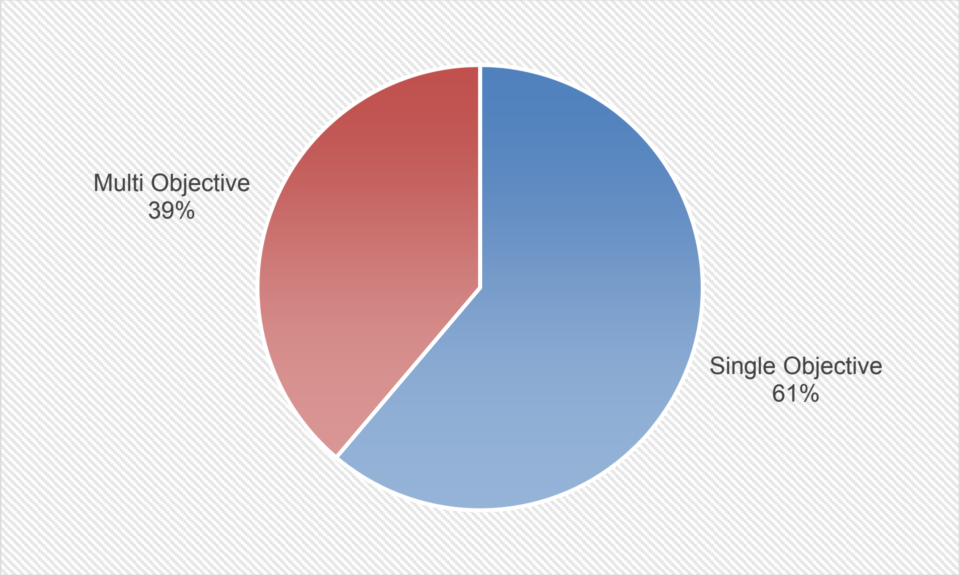

Figure 2.10 Distribution of RAP papers based on the type of optimization

The results in Figure 2.10 show that in 61% of RAP papers only single objective has been taken into consideration. However, in many real life situations involving realibility optimization decision makers are recognized to be multi objective. It means that there exist multiple criteria to be achieved rather than measuring the success of a particular solution via a single criterion. For instance, a decision maker may want to maximize system reliability and minimize the system weight at the same time by adding redundant components into the system of interest.

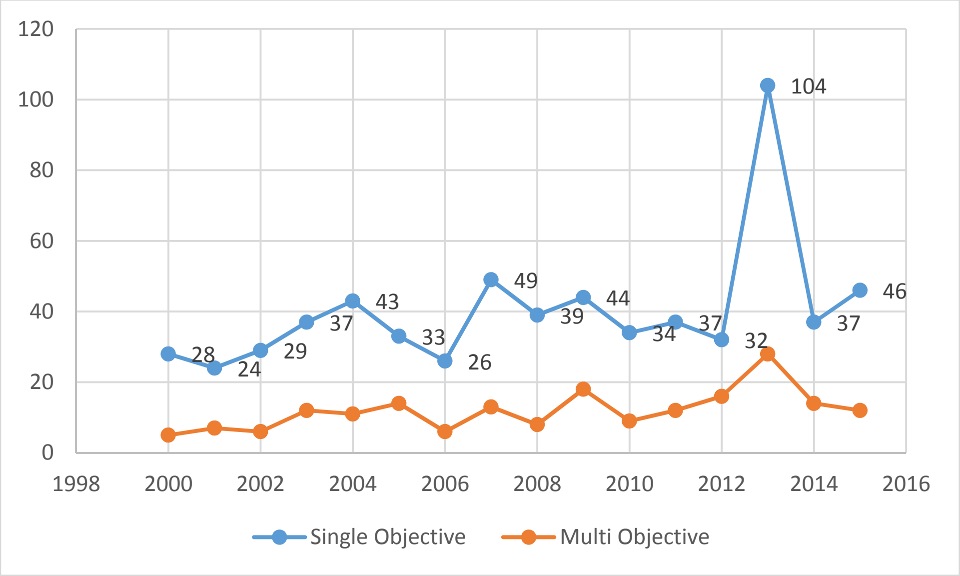

Figure 2.11 Single objective/multi objective RAP papers in last 15 years

The graph in Figure 2.11 depicts the single-objective RAP problem’s dominance in RAP literature during the last 15 years. However, there is a meaningful interest in the number of multi objective RAPs in recent years.

3. TYPES OF REDUNDANCY ALLOCATION PROBLEMS

Based on the classification presented above, in this section mainly redundancy allocation problem types and related literature are presented. Also, recent advancements in RAP field are shared to draw the researchers attention to these promising research areas.

3.1. Binary State Systems (BSS) /Multi State Systems (MMS)

In traditional reliability optimization theory, a system and its components can take only two possible states such as working or failed. These kinds of systems called as binary state systems. But in most of the real World applications such as a power generation plant, plastic recycling systems can perform their intended functions at more than two (but finite) different levels, from perfectly working to completely failed. These systems are called as multi state systems (MSS).

There are abundant publications for binary state models in RAP literature. However, the research for multi state RAP models is somehow under developed. The computational complexity of MSSs may have an important role in this situation. The basic concepts of MSS reliability can be traced back to the 1970s (e.g. Murchland [26]'s study). In RAP literature, the most applied MSS reliability evaluation methods can be described as follows: an extension of binary state models to MSSs, the stochastic search process, the universal generating (UGF) technique, the structure function approach, the monte carlo simulation and recursive algorithm. Levitin et al., [27] are pioneers who use a UGF technique to estimate the availability of a series parallel MSS. For example, Sharma et al, [28] studied a series-paralel multi state RAP problem to decide a system configuration which aims to minimize the system cost under the given reliability and weight constraints. They used a version of ant colony algorithm as a solution procedure. Li et al., [29] studied a MSS series-parallel heterogenous RAP subject to common failures.A summary of related work on MSS reliability is reported by Lisnianski and Levitin, [30]. Also, Yingkui et al., [31] summarizes the latest studies and advancements in MSS reliability area in their work.

General MSS formulation is presented below:

Problem 3:

$\max E(x, T, W^*)$

s.t.

$$g_i(x) \leq b_i, \text{ for } i = 1, 2, .., m$$

$$x \in X.$$

Problem 4:

$\min C_s(x\ )$

s.t.

$$E(x, T, W^*) \geq E_0$$

$$g_i(x) \leq b_i, \text{ for } i = 1, 2, .., m$$

$$x \in X.$$

where E is a measure of the system availability represented by a cumulative demand curve with a known T (MSS operation period) and W* (predetermined MSS performance level).

3.2. Redundancy Strategy

One of the recent advances in reliability optimization studies is modeling the system by considering different redundancy strategies. In most of the studies in RAP literature, as it is discussed in Section 2, generally one redundancy strategy (generally limited to active redundacy) has taken into consideration for modeling the system. However, in practice there are different redundancy strategies: i) active, ii) standby (cold, hot and warm) and iii) mixed.

In active redundancy strategy, all the redundant components operate simultaneously from time zero, but in fact only one of them is in operation in a certain time. However, a standby redundant component is initially unpowered and switched on when it is needed to replace the failed unit.

There are three different standby redundancy strategies such as called cold, warm, and hot standby. A cold-standby redundant component does not fail before it is switched into the power mode, warm-standby redundant components are more prune to operational stresses compared the cold-standby ones. For the hot-standby redundancy case, the failure pattern of component is not affected by the component's situation i.e. in or out operation. Hence, the mathematical formulations for hot-standby and active redundancy strategies are the same. In a system which uses the standby redundancy strategy, the redundant components are put into the operation one by one when a one online component fails.

In this process two alternative ways can be applied. In the first scenario, the system is monitored on a continuous bases to detect the failure and put the redundant component into opeartion via a hardware/software; in the second one, it is assumed that switch failure can occur at any time and there is not a relationship between switch reliability and the number of required switches [32].

For the cold-standby redundancy strategy, the studies reported by Robinson et al., [33], Shankar et al., [34] and Gurov et.al, [35] can be accepted as early examples of in this area. For series–parallel systems, Coit [32] presented an integer programming solution to the RAP in which the system only uses the cold-standby redundancy. Coit and Liu [36] presented a novel mathematical model in redundancy allocation area. In their study, predetermined active or cold-standby redundancy was applied for each subsystem to determine the optimal system design. In 2003, Coit [36] presented an integer programming method for solving a series-parallel RAP. The novelty of this study was including a new decision variable, the selection of active or cold-standby redundancy strategy for each subsystem, to the mathematical model. For the same problem, Tavakkoli-Moghaddam et al., [37] proposed a GA which can be used for large search spaces. Also, again the same mathematical model in Coit [36]'s study was extended in multiobjective assumption by Safari [38] and Chambari et al., [39].

The mathematical model developed by Coit [12] is presented below:

Notations:

s : number of subsystems
t : mission time
C,W : system level cost and weight contraint limits

R(t;z,n): system reliability at time t for the designing vectors z and n

$r_i(t)$: reliability at time t for j^{th} available component for subsytem i

n_i : number of components used in subsystem i (i=1,2,..,s)

$n_{max,i}$: upper bound for n_i

z_i : index of component choice used for a subsystem i

m_i : number of available components for subsystem i

c_{ij}, w_{ij} : cost and weight for the j^{th} component of subsytem i

$\rho_i(t)$: failure detection/switching reliability at time t (Scenario 1)

ρ_i : failure detection/switching reliability at time t (Scenario 2)

Problem 7:

Maximize R(t;z,n)

s.t.

$$\sum_i c_{i,z_i} n_i \le C,\ n_i \in \{1,2,..,n_{max,i}\}$$

$$\sum_i w_{i,z_i} n_i \le W,\ z_i \in \{1,2,..,m_i\}.$$

According to this formulation, with the objective of maximizing system reliability and given weight and cost contraints, this model tries to determine the redundancy strategy, type and amount of components which will be used in each subsystem.

According to the two scenarios, R(t;z,n) is calculated as follows:

Scenario 1:

$$R(t;z,n)=\prod_{i\in A}(1-(1-r_{i,z_i}(t))^{n_i})\prod_{i\in Co}(r_{i,z_i}(t)+\sum_{j=1}^{n_i-1}\int_0^t \rho_i(u)\, f_{i,z_i}^{(j)}(u)\, r_{i,z_i}(t-u)du)$$

Scenario 2:

$$R(t;z,n)=\prod_{i\in A}(1-(1-r_{i,z_i}(t))^{n_i})\prod_{i\in Co}(r_{i,z_i}(t)+\sum_{j=1}^{n_i-1}\rho_i{}^{j}\int_0^t f_{i,z_i}^{(j)}(u)\, r_{i,z_i}(t-u)du)$$

In RAP literature, fewer studies were reported which taking into account active and cold-standby redundancies in a specific system simultaneously (mixed strategy). For instance, Ardakan et al. [40] studied a series-parallel multi objective RAP where mixed redundacy strategy applied. They used NSGA-II algorithm to solve this problem. The results of their study showed that instead of only adding redundant components into a system, which causes an increase in the systemi cost, weight etc., changing the redundancy strategy may be beneficial for improving the realibility of the interested system.

3.3. Single Objective Optimization

In a single objective optimization problem, one criterion is specified as the objective function to be optimised. Mostly the single objective optimization is the dominant type used in reliability optimization problems.

In general, the single objective RAP can be formulated as follow:

Problem 5: (Single Objective RAP general formulation)

$$\text{Minimize } f\,(x_1, x_2, \ldots, x_N)$$

$$\text{s.t.}$$

$$g_i\,(x_1, x_2, \ldots, x_N)\ \ 0 \qquad i=1,2,\ldots,m$$

$$x_l \leq x_j \leq x_u \qquad j=1,2,\ldots,N$$

This is a discrete optimisation problem since the elements of the decision vector $[x_1, x_2,\ldots,x_N]^T$ which specifies the redundancy levels for a set of N components or subsystems are required to be discrete values. The objective function may be either the system's reliability expression (i.e. – f) or the system cost, weight etc. (i.e. f) which is minimized, subject to constraints on the system resources and the redundancy levels given by the functions g_i which are usually separable [2]. The values x_i and x_u are respectively lower and upper limits on the j^{th} component or subsystem redundancy level. The type of parallel redundancy may be total, partial, or standby [2]. There are cases, where the decision variables concern the selection of components or their assignment in a system, without redundancy [20;21;22]. The model stated in Problem 5 assumes that a component or subsystem reliability is known and remains constant throughout the optimisation process. The precise form of f depends on the criterion to be optimised; it is generally a non-linear function however, irrespective of the chosen performance measure. The constraints g_i are also generally non-linear and could be limits imposed on either the reliability of the component, subsystem, or overall system; or on cost, weight, volume or other system attribute. The type of system configuration and problem being analysed also dictate the form of both f and g_i. Among early examples of this type of problem are the cases reported by Bala and Aggarwal [41], Kim and Yum [42], and Deeter and Smith [43] which concerned redundancy allocation in complex systems or networks for their optimal reliability, and that of Coit and Smith [44] which focused on a series-parallel system reliability optimisation. Prasad and Raghavachari [45], considered the problem of the optimal allocation of interchangeable components, to a series-parallel system in order to maximize its reliability, with only one component allowed for each subsystem. Later Prasad and Kuo [2] discussed the optimal allocation of redundant components to both series and complex coherent systems, to maximize their reliability, subject to constraints on the subsystems' reliability and redundancy levels. Munoz and Pierre [17] presented a model that sought to find parallel redundancies at both the component and system levels of a series system that minimized the cost associated with the redundancies, subject to lower bound constraints on both the system reliability and the redundancy levels. You and Chen [46] proposed a model to maximize a series-parallel system reliability, with upper bounds on both the system cost and weight for a given redundancy level. Tavakkoli-Moghaddam et al,, [37] discussed the situation where the decision to be made concerned not just the component type and redundancy levels, but also the type of redundancy strategy to use: whether cold or active standby.

3.4. Multi Objective Optimization

Most of the real world decision making problems in the reliability optimization field require the optimization of more than one objective function simultaneosly, such as the maximization of system reliability, minimization of system cost, weight and 24ort h. Although the single objective optimization models obviously result in improved system reliability as presented in the many reported cases in literature, beside being more appropriate for he real world applications, the multi objective optimization is also very beneficial for providing decision makers with the opportunity in the selection of the most appropriate solution

The models presented by Sakawa [47] and those by Misra and Sharma [48;49] were among the earliest publications found in this category. Sakawa [47] used a surrogate worth trade-off method to solve a multi objective redundancy allocation problem which aims maximizating system reliability and minimizing the system cost of redundancy allocation at the same time. Misra and Sharma [48] considered a multiple component choice redundant series-parallel system in which both the system reliability and cost were optimised subject to a set of constraints on both the system reliability and the number of redundant components. This problem was also presented by Misra and Sharma in [49] as one of two; the other being concerned with maximizing a series-parallel system's reliability and minimizing the system cost and weight subject to a set of expressions related to the redundancy levels of each subsystem. To analyze the research trend in multi objective RAP area, the works reported by Park [50], Dhingra [51], Rao and Dhingra [52], Ravi, Reddy and Zimmerman [53], Coit and Konak [54], Kumar et al., [23], Liang and Lo [55], Safari [56], and Chambari et al. [57], and others can be examined.

The redundancy allocation optimization for multi-objective problems can be modelled as follows:

Problem 6: (Multi Objective RAP general formulation)

$$\text{Minimize } [\, f_1(x), f_2(x), \ldots, f_k(x)]$$

$$\begin{aligned} \text{Subject to } \; & g_i(x) \le 0 & & i=1,2,\ldots,m \\ & x_l \le x_j \le x_u \quad x=(x_1, x_2, \ldots, x_N) & & j=1,2,\ldots,N \end{aligned}$$

The vector of k objective functions $[\, f_1(x), f_2(x), \ldots, f_k(x)]^T$, $(k \ge 2)$ represents the criteria to be optimised, which generally includes the reliability or unreliability of a system, the variance of the reliabilities, the subsystems' reliability, the system's cost, weight, risk, etc. The other parameters and the assumptions of this model are the same as (or similar to) their counterparts given in the single objective formulation.

Unless the situation in the single objective optimization case, in multi objective problems there may not exist a solution which is best respect to all the objectives which are taken into consideration in the formulation phase. In multi objective optimization, there is a solution set, described as Pareto optimal solutions or non-dominated solutions [58], which are superior to the rest of solutions in the search space when all of the objectives taken into consideration, but worse than other solutions in the search space in terms of one or more objectives. As none of the solutions in the-non dominated set can be regarded as absolutely better than one another, decision maker can accept any of them as final solution.

3.5. Deterministic Models

In most of the realibility optimization problems with single objective or multi-objectives, it is assumed that all system design parameters are precisely known.
To deal with these deterministic RAPs, many mathematical programming heuristic and meta-heuristic solution methods were applied in literature. Table 3.1 and Table 3.2 present the examples of different solution appraoches used in non-repairable RAP.

The problem has been studied by using exact approaches, e.g., dynamic programming [58;59;60] branch and bound [61], heuristic and metaheuristic approaches, such as simulated annealing [62], tabu search [63], ant colony optimization [64;65], genetic algorithms [66], variable neighborhood search [67], particle swarm optimization [68], cuckoo search [69], and hybrid algorithms [70;71;72].

It can be easily seen that in the single objective and multi objective deterministic RAPs, the most studied system structure is series-parallel and the main consideration is on active redundancy rather than other redundancy strategies.

Table 3.1 Examples of non-repairable RAP papers using mathematical programming approaches

	Source	System Configuration	Type of parameter	Redundancy Strategy	Solution Method
Single Objective Optimization	Prasad and Kuo (2000)	Series- parallel	Deterministic	Active	Lexicographic order (p&k-ag)
	Prasad, Kuo and Kim (2001)	Series- parallel	Deterministic	Active	Lexicographic search
	Ng and Sancho (2001)	Series- parallel	Deterministic	Active	Hybrid DP/depth first search
	Djerdjour and Rekab (2001)	Series- parallel	Deterministic	Active	Branch and bound
	Coit (2001)	Series-parallel	Deterministic	Cold standby	Integer programming
	Hsieh (2002)	Series- parallel	Deterministic	Active	Two-phase linear programming
	Lee, Kuo and Ha (2003)	Series-parallel	Deterministic	Active	Comparision of max-min approach and NN
	Hsieh (2003)	Series- parallel	Deterministic	Active	Simple linear approximation
	Elegbede, Chu and et al (2003)	Series- parallel	Deterministic	Active	ECAY algorithm
	Coit et. al (2004)	Series- parallel	Deterministic	Active	Weighting method under an IP software package
	Ramirez-Marquez, Coşt and Konak (2004)	Series- parallel	Deterministic	Active	Mixed integer linear programming
	Yalaoui et al. (2005)	Series- parallel	Deterministic	Active	Dp
	Onishi et al. (2007)	Series- parallel	Deterministic	Active	Improved surrogate constraint
	Billionnet (2008)	Series- parallel	Deterministic	Active	Integer linear programming
	Amari (2010)	Series- parallel	Deterministic	Active/warm standby	Linear programming based branch-and-bound

Table 3.1 continuing

	Source	System Configuration	Type of parameter	Redundancy Strategy	Solution Method
Single Objective Optimization	Tannous et al. (2011)	Series-parallel	Deterministic	Warm standby	GA and exact integer programming
	Soltani et al. (2015)	Series-parallel	Deterministic	Active and cold standby	Compromise programming
	Caserta and Voß (2015a)	Series-parallel	Deterministic	Active	A branch and cut algorithm
	Caserta and Voß (2015b)	Series-parallel	Deterministic	Active	Exact dynamic programming approach
	Gago et al (2013)	Series-parallel	Deterministic	Active	Greedy, walkback
Multi Objective Optimization	Coit and Konak (2006)	Series- parallel	Deterministic	Active	The weighting method in conjunction with a heuristic & an IP algorithm
	Onishi et. al (2007)	Series-parallel	Deterministic	Active	Improved surrogate constraint algorithm
	Mahapatra (2009)	Series- parallel	Deterministic	Active	Global criterion method
	Khalili-Damghani and Amiri (2012)	Series-parallel	Deterministic	Active	Epsilon constraint along with dea
	Cao et al. (2013)	Series-parallel	Deterministic	Active	Decomposition approach
	Sadjadi et al. (2014)	Series-parallel	Deterministic	Active and cold standby	Compromise programming

Table 3.2 Examples of non-repairable RAP papers using heuristic and meta-heuristic approaches

	Source	System Configuration	Type of parameter	Redundancy Strategy	Solution Method
Single Objective Optimization	Coit and Liu (2000)	Series- parallel	Deterministic	Active	IP Algorithm
	Kulturel-Konak et al. (2003)	Series- parallel	Deterministic	Active	TS
	Ha (2004)	Non series-parallel, Series-parallel	Deterministic	Active	Tree and scanning (a multi-path heuristic)
	Kim et al. (2004)	Series-parallel	Deterministic	Active	SA
	Liang and Smith (2004)	Series-parallel	Deterministic	Active	ACO
	You and Chen (2005)	Series-parallel	Deterministic	Active	Heuristic (based on greedy method and GA)
	Nahas and Nourelfath (2005)	Series	Deterministic	Active	ACO with local search
	Liang and Wu (2005)	Series-parallel	Deterministic	Active	VND
	Chen andYou (2005)	Series-parallel	Deterministic	Active	IA
	Nahas et al. (2007)	Series-parallel	Deterministic	Active	ACO and DC
	Liang and Chen (2007)	Series-parallel	Deterministic	Active	VNS
	Tavakkoli-Moghaddam et al. (2008)	Series-parallel	Deterministic	Active and Cold standby	GA
	Sadjadi and Soltani (2009)	Series-parallel	Deterministic	Active	Heuristic and GA
	Beji et al. (2010)	Series-parallel	Deterministic	Active	Hybrid PSO with local search
	Safari and Tavakkoli-Moghaddam (2010)	Series-parallel	Deterministic	Active and cold standby	Memetic algorithm
	Ahmadizar and Soltanpanah (2011)	Series	Deterministic	Active	ACO
	Karimi et al. (2011)	Series-parallel	Deterministic	Cold standby	GA and SA
	Safari et al. (2012)	Series-parallel	Deterministic	Active	Annealing-based PSO
	Sadjadi and Soltani (2012)	Series-parallel	Deterministic	Active	Heuristic and HBMO
	Kong, Gao et al (2015)	Series-parallel	Deterministic	Active and cold standby	Simplified particle swarm optimization
	Ouzineb et al (2008)	Series-parallel (multi state)	Deterministic	Active	TS

Table 3.2 continuing

	Source	**System Configuration**	**Type of parameter**	**Redundancy Strategy**	**Solution Method**
Single Objective Optimization	Wattanapongsakorn and Levitan (2001)	Series-parallel	Deterministic	Active	SA
	Lee, Gen & Kuo (2001)	Series-parallel	Deterministic	Active	GA & NN (nonlinear mixed integer programming RAP)
	You and Chen (2005)	Series-parallel	Deterministic	Active	Heuristic Algorithm
	Liang and Chen (2007)	Series-parallel	Deterministic	Active	Variable Neighbourhood Search Algorithm
	Wattanapongsakorn (2004)	Series-parallel	Deterministic	Active	SA
	Nahas et al (2007)	Series-parallel	Deterministic	Active	Ant colony and degraded local search
	Zou, Gao ad Wu (2011)	Series-parallel, bridge	Deterministic	Active	Effective Global Harmony Search (combines HS and PSO)
	Sheikhalishahi et al (2013)	Series, Series-parallel, bridge	Deterministic	Active	A hybrid GA and PSO
	Garg et al (2013)	Series	Deterministic	Active	Artificial Bee Colony
	Ouzineb et al (2010)	Series-parallel	Deterministic	Active	A combination of space partitioning, GA and TS
	Zia and Coit (2010)	Series-parallel	Deterministic	Active	A column generation approach
	Ouzineb et al (2011)	Series-parallel (MS)	Deterministic	Active	GA
	Sharma and Agarwal (2009)	Series-parallel (MS)	Deterministic	Active	ACO
	Chambari et al. (2013)	Series-parallel	Deterministic	Active and cold standby	SA
	Najafi et al. (2013)	Series-parallel	Deterministic	Active	Tuned SA and GA
	Soltani et al. (2013)	Series-parallel	Deterministic	Active	Heuristic and HBMO
	Yeh (2014)	Series-parallel	Deterministic	Active	OSSO
	Levitin, Xing et al (2013)	Series-parallel (MS)	Deterministic	Active	UGF

Table 3.2 continuing

	Source	**System Configuration**	**Type of parameter**	**Redundancy Strategy**	**Solution Method**
Multi objective Optimization	Shelokar et al. (2002)	Non-series parallel, series-parallel	Deterministic	Active	Ant algorithm
	Suman (2003)	Series-parallel	Deterministic	Active	SMOSA, UMOSA, PSA, PDMOSA and WMOSA
	Salazar et al. (2006)	Non-series parallel, series-parallel	Deterministic	Active	NSGA-II
	Coit and Konak (2006)	Series-parallel	Deterministic	Active	Multiple weighted objective heuristic
	Zhao et al. (2007)	Series-parallel	Deterministic	Active	ACO
	Taboada et al. (2007)	Series-parallel	Deterministic	Active	NSGA-II
	Taboada and Coit (2008)	Series-parallel	Deterministic	Active	GA
	Liang and Lo (2010)	Series-parallel	Deterministic	Active	MOVNS
	EbrahimNezhad et al. (2011)	Series-parallel	Deterministic	Active and Cold Standby	NSGA-II
	Safari (2012)	Series-parallel	Deterministic	Active and Cold Standby	NSGA-II
	Chambari et al. (2012)	Series-parallel	Deterministic	Active and Cold Standby	NSGA-II
	EbrahimNezhad et al. (2012)	Series-parallel	Deterministic	Active and Cold Standby	NSGA-II and Memetic algorithm
	Azizmohammadi et al. (2013)	Series-parallel	Deterministic	Active and standby	HMOICA(hybrid ICA and GA)
	Khalili-Damghani et al. (2013)	Series-parallel	Deterministic	Active	DSAMOPSO, AUGMECON, NSGA-II, CTVMOPSO
	Zhang et al. (2014)	Series-parallel	Deterministic	Active	BBMOPSO followed by k-Means and Hierarchical clustering

Table 3.2 continuing

	Source	**System Configuration**	**Type of parameter**	**Redundancy Strategy**	**Solution Method**
Multi Objective Optimization	Garg and Sharma (2012)	Series-parallel	Deterministic	Active	PSO
	Marseguerra et al (2005)	Series-parallel	Deterministic	Active	GA & Monte Carlo simulation
	Coit and Baheranwala (2005)	Series-parallel	Deterministic	Active	Multi-objective GA
	Taboada and Coit (2007)	Series-parallel	Deterministic	Active	Elitist Nondominated Sorting GA 2 (NSGA 2)
	Wattanapongsakorn and Coit (2007)	Series-parallel	Deterministic	Active	GA
	Taboada et al (2007)	Series-parallel	Deterministic	Active	NSGA
	Zhao et al (2007)	Series-parallel	Deterministic	Active	Multi-objective Ant Colony
	Zafiropoulos and Dialynas (2007)	Series-parallel	Deterministic	Active	SA
	Yamachi et al (2006)	Series-parallel	Deterministic	Active	Multi-objective GA
	Zaratelab et al (2015)	Series parallel	Deterministic	Active and cold standby	Knowledge-based archive multi-objective SA
	Ghorabaee, Amiri et al (2015)	Series-parallel(k-out-of n)	Deterministic	Active	NSGA-II
	Khalili-Damghani et al (2014)	Series-parallel	Deterministic	Active	ε-constraint method

3.6. Nondeterministic Models

Non-deterministic models are those in which at least one of the system design parameters are not precisely known. In the classical redundancy optimization theory, it is generally assumed that the design parameters related to system and system performance measures such as system reliability are random variables and evaluated using the probability measure. But, in real world applications, such as space shuttle system, this assumption can not be appropriate in which the estimations of probability distributions of lifetimes of systems and components are very difficult due to uncertainties and imprecision of data. The uncertainty in the reliability estimation is an under-developed area in RAP field.

Based on the 33iteratüre review, the uncertainty can be considered under six categories: i) stochastic uncertainty, ii) interval uncertainty, iii) fuzzy unceratinty, iv) intiutionistic fuzzy and vague sets, v) fuzzy-random uncertainty, vi) chaos uncertainty. Detailed information related to these topics are presented in the following sub-sections. Table 3.3 lists the examples of the related work regarding non deterministic models in non-repairable RAP field.

3.6.1. Stochastic Uncertainty

Rubinstein et. al, [73] presented one of the early 33iter in this area. In their study, they used a GA to maximize the expectation of system reliability for a series parallel RAP with component uncertain properties. However, maximization of the expectation of the reliability estimate may not suffice in many practical cases. Instead, maximizating the system reliability and minimizing the estimation of system reliability uncertainty is the commonly desired situation by system designers. Marseguarre et. al, [74] studied a multi-objective network design problem which aims to balance the dual objectives of high reliability, and low uncertainty in its estimation by using a GA.

3.6.2. Interval Uncertainty

Most of the reliability optimization problems assume that design parameters such as reliabilities of components are a fixed number which lie between zero for the. But, because of the unappropriate storage conditions, the human factor and other environmantal factors, the realibility of a one component can not be spesified to a fixed number. This situation may be valid for other design parameters too. Hence, it will be more appropriate approach to evaluate the design parameters related to a system as a positive imprecise number rather than a fixed real number.

In their study, for the first time Yokota et. al, [75] developed a nonlinear integer programming RAP with with interval coefficients. They used a GA to solve this problem. Gupta et al., [77] studied a constrained single objective RAP for a series system with interval valued component reliabilities. They used a GA for integer variables. Another example is Sahoo et. al, [79]'s study. In this study, they solved a constrained multi-objective RAP for a series-parallel system in which each component has interval valued realibility. They used interval mathematics during the formulation and solved this problem via a GA.

Table 3.3 Examples of non-repairable RAP papers using non-deterministic design parameters

	Source	**System Configuration**	**Type of parameter**	**Redundancy Strategy**	**Solution Method**
Single Objective Optimization	Rubinstein et al. (1997)	Series-parallel	Stochastic component reliability	Active	Simulation and GA
	Coit and Smith (2002)	Series-parallel	Random scale parameter for weibull distribution	Active	GA
	Yeh (2003)	Series-parallel, non series parallel	Stochastic component reliability	Active	MCS-RSM
	Coit and Wattanapongsakorn (2004)	Series-parallel, non series parallel	Stochastic component reliability	Active	Stochastic optimization
	Marseguerra et al. (2005)	Non series parallel	Stochastic component reliability	Active	GA and MC
	Yadavalli et al. (2007)	Series-parallel	Resource chance constraint	Active	Branch and bound
	Li and Hu (2008)	Series-parallel	Random lifetimes	Active and standby	Stochastic comparison
	Reddy et al. (2011)	Non series parallel	Stochastic component reliability	Active	Simulation method
	Tekiner & Coit (2011)	Series-parallel	Stochastic component reliability	Active	Neighborhood search, and linear integer programming
	Gupta et al. (2009)	Series-parallel	Interval component reliability	Active	Advanced GA with interval fitness function
	Sahoo et al. (2010)	Series-parallel, non series parallel	Interval reliability	Active	GA
	Taguchi and Yokota (2011)	Series-parallel	Interval reliability	Active	Hybrid GA, SA and FLC
	Sahoo et al. (2013)	Non series parallel	Interval reliability, cost and amount of resources	Active	GA
	Hou and Wu (2006)	Series-parallel	Fuzzy reliability	Active	Fuzzy simulation-based GA
	Han et al. (2006)	Non series-parallel	Triangular fuzzy numbers	-	Fuzzy fault tree

Table 3.3 continuing

	Source	System Configuration	Type of parameter	Redundancy Strategy	Solution Method
Single Objective Optimization	Yao et al. (2008)	Series-parallel	Triangular fuzzy numbers	Active	Signed distance method to defuzzify
	Mahapatra and Roy (2011)	Series-parallel	Fuzzy reliability, cost and weight	Active	Fuzzy parametric geometric programming
	Lee et al. (2012)	Parallel	Level (λ,ρ) interval-valued fuzzy numbers	Active	Signed distance method to defuzzify
	Mahapatra and Roy (2014)	Non series-parallel	Intuitionistic fuzzy cost	Active	Intuitionistic fuzzy optimization method
	Kumar and Yadav (2012)	Series, parallel	Intuitionistic fuzzy failure rate	Active	Non-linear programming techniques
	Sadjadi and Soltani (2015)	Series-parallel	Interval reliability	Active and cold standby	Min–Max regret criterion and Benders' decomposition method
	Ding and Lisnianski (2008)	Series-parallel (multi state)	Fuzzy availability	Active	UGF
	Ebrahimipour, Asadzadeh et al (2013)	Series-parallel	Fuzzy reliability, cost and weight	Active	Fuzzy inference system
	Pandey et al. (2011)	Series, parallel, non series-parallel	Triangular intuitionistic fuzzy reliability	Active	A method based on the IFS theory
	Jameel and Radhi (2014)	Series-parallel	Fuzzy reliability and flexible constraints	Active	Penalty function mixed with Nelder and Mend's algorithm
	Zhao and Liu (2004)	Non series parallel	Random-fuzzy lifetimes	Standby	Integrated random fuzzy simulation, NN and GA

Table 3.3 continuing

	Source	**System Configuration**	**Type of parameter**	**Redundancy Strategy**	**Solution Method**
Single Objective Optimization	Nematian et al. (2008)	Series-parallel	Random-fuzzy lifetimes	Active /Standby	Integer programming
	Wang and Watada (2009)	Parallel-series	Random-fuzzy lifetimes	Active	Fuzzy random simulation and GA
	Wang et al. (2012)	Series-parallel	Random-fuzzy lifetime	Active	Saddlepoint Approximation
	Feizollahi & Modarres (2012)	Series-parallel	Interval uncertainty	Active	MIP and Benders decomposition
	Soltani et al. (2013)	Series-parallel	Interval uncertainty	Cold standby	Benders decomposition, GA and Enumeration method
	Soltani & Sadjadi (2014)	Series-parallel	Fuzzy uncertainty	Active	Branch and cut
	Feizollahi et al. (2014)	Series-parallel	Budgeted uncertainty	Active	MIP and Benders decomposition
	Chen (2003)	Series, parallel, seriesparallel	Triangular vague set for components reliabilities	Active	A method based on the vague set theor
	Kumar et al. (2006)	Series, parallel	Interval valued trapezoidal vague sets	Active	A method for analyzing the fuzzy system reliability
	Kumar et al. (2007)	Series, parallel	LR type interval valued triangular vague set for component reliability	Active	Tw (the weakest t norm) based arithmetic operation
	Mahapatra & Roy (2009)	Series, parallel, non series-parallel	Triangular intuitionistic fuzzy reliability	Active	(α, β)-cut

Table 3.3 continuing

	Source	**System Configuration**	**Type of parameter**	**Redundancy Strategy**	**Solution Method**
Multi Objective Optimization	Sasaki & Gen (2003)	Series-parallel	Fuzzy objectives	Active	Hybrid GA
	Chen & Liu(2011)	Series-parallel	Type-2 fuzzy lifetime	Standby	Fuzzy Goal programming and Approximation approach based PSO
	Bhunia & Sahoo (2012)	Series-parallel	Interval reliability and cost	Active	GA, Global criterion method, Tchebycheff and weighted Tchebycheff
	Garg, Rani et al (2014)	Series-parallel	Fuzzy design parameters	Active	PSO and GA
	Roy et al. (2014)	Series-parallel	Interval reliability and cost and system entropy	Active	Entropy based region reducing GA
	Zang and Chen (2015)	Series-parallel	Interval reliability and cost	Active	Multi-objective PSO
	Mousavi, Alikar et al (2013)	Series-parallel (multi state)	Fuzzy design parameters	Active	CE-NRGA
	Ebrahimipour and Sheikhalishahi (2011)	Series-parallel	Fuzzy availability	Active	PSO

3.6.3. Fuzzy Uncertainty

The use of fuzzy theory in representing unknown parameters is an alternative to the traditional aproaches used in probabilistic modeling. In many situations fuzzines and randomness of the system design parameters such as component lifetimes are mixed up with eah other. Fuzziness can be used when there is no such a historical data to estimate the design parameters. According to the fuzzy theory, the parameters, constraints and objectives are regarded as fuzzy sets and there is known membership functions and fuzzy numbers related to these fuzzy sets.

In past two decades, fuzzy optimization techniques have been successfully applied to the RAPs. One of the early examples of the fuzzy methodology in reliability engineering can be found in Kaufmann [80]'s study. The main work of fuzzy methodology in reliability engineering can be traced back to the 1980s. Cai, Wen, and Zhang [81] introduced the possibility assumption and the fuzzy state assumption which replaces the probability and binary state assumptions. Dhingra [51], Rao and Dhingra [52] worked on reliability and redundancy apportionment for multi-stage systems using crisp and fuzzy multi-objective optimization problem and used a threshold accepting technique to solve it. Recently, Dengiz et. al, [82] modeled a multiobjective series-parallel RAP in which the component reliabilities are considered as fuzzy parameters and a GA was used as fuzzy optimization technique.

3.6.4. Intuitionistic fuzzy and vague sets

The concept of intuitionistic fuzzy sets (IFS) can be regarded as an alternative approach to define a fuzzy set when available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set [83].

In IFS theory, the degree of membership of an element is measured in the interval form instead of the point valued as in fuzzy set theory. As IFS separates the positive and the negative evidence for the membership of an element in a set, this fact can be regarded as the main advantage of using IFS over the fuzzy sets [84].

Kumar et. al, [85] used a triangular intuitionistic fuzzy set and developed a procedure to generate the membership function and non-membership function of the reliability function by using intuitionistic fuzzy failure rate.

For the first time, Kumar et. al, [86] introduced a new algorithm to generate the membership function and non-membership function of fuzzy reliability of a system in which the components follow different types of intuitionistic fuzzy failure rates contrary to the the classical fuzzy system reliability theory.

3.6.5. Fuzzy-Stochastic Uncertainty

In real-world applications, the system design parameters such as component lifetimes are never precise or completely vague. For instance, the component lifetimes are generally assumed to be exponentially distributed variables with unknown parameters. But, the required historical data to estimate the value of these parameters can not be obtained. In these situations, fuzziness and randomness of the component lifetimes should be considered at the same time and this application results in effectiveness loss in the classical redundancy allocation theory.

Hence, to deal with these challenges, fuzzy stochastic approach is used in which some parameters are evaluated as fuzzy sets and others as random variables. There are limited research in reliability optimization problems which takes into consideration such a hybrid uncertainty. For example, Zhao and Liu [87] modeled three types of system performance based on random fuzzy lifetime parameters for a series-parallel system. They used a hybrid intelligent algorithm to solve this poblem. Recently, Wang et. al, [88] studied a parallel–series system with fuzzy random lifetimes (convex and non-convex lifetimes) which consideres two redundancy allocation models through reliability maximization and cost minimization, respectively. They used a GA to solve this problem. Huang et.al. [89] made two developments with their study in this area. Firstly, they used the saddlepoint approximation to deal with reliability analysis accounting for the time-dependent degradation process and fuzzy random variables. Secondly, two system reliability analysis methods were proposed for different scenarios of reliability modeling processes.

3.6.6. Chaos uncertainty

Chaos theory is a new approach in the analysis of the nonlinear time series. Chaos theory deals with oscillations which are are generated by the deterministic nonlinear model [90].

The usage of chaos theory in reliability optimization can be based on Zou and Liu [91]'s study. In their study, Zou and Li [91] used two real data bases related to software failures, and processed them by using chaos theory methods. With this work, it was reported that the deterministic failure models are more appropriate to the experimental data contrary to the traditional stochastic models. In fact, this results can be considered as a new approach to the classical statistical data processing about the the failure patterns of components.

Based on the literature survey conducted in this study, the only work in RAP field for chaos uncertainty was reported by Rothstein et. al, [92]. In their study, by combining the fuzzy logic and chaos theory, a redundancy optimization problem under chaotic oscillations of parameters was presented and a GA was used as a solution procedure. Because of the lack of related reserch in the usage of chaos uncertainty in RAPs, this area is very promising for researchers who work in RAP field.

4. CONCLUDING REMARKS

In this study, a special type of reliability optimization problems which is called as redundancy allocation have been discussed from different perspectives based on a novel classification methodology, and latest trends in this field, in terms of models, solution methodologies etc., have been presented. The main purpose of this study is to provide researchers working in this field with a framework for future reseach direction.

As it can be seen in the previous sections, a lot of studies have been reported in this vast RAP field. Because of the difficulty in including all problem types in this field into a single review study, within the scope of this study only binary state non-repairable systems which regarding redundancy at component level have been discussed in detail. Hence, it will be beneficial to prepare a similar study for multi-level, multi-state and/or repairable systems in RAP literature.

According to the results presented in Section 2, the most studied type of RAP is the one that seeks to maximize system reliability (those with cost as the objective function are in the minority). But, it is meaningful when a system is in 42iterat a spesific time interval. However, if the interested system is being used beyond a specified time (e.g. artificial satellites, space explorers), other performance measures like average life (i.e. a mean time to failure for a system) and percentile life (i.e. maximum mission time for which system reliability meets at least a spesific value) are relevant in this case.

In the 40% of the RAP papers, the series-parallel system structure is studied, and the types of parallel redundancies applied are those done at the component level with generally active redundancy (67%). The share of stand by redundancy is 29% and mixed strategy is employed in only 4% of the total 1391 papers during 1969-2015 period. Hence, standby strategies are still promising topics in RAP field, especially there is limited work in literature in terms of the cold standby, and mixed redundancy strategies.

The RAPs are generally formulated as single objective (61%) which seeks to maximize an appropriate system performance measure under resource constraints, and more realistic problems involving multiobjective programming are also being considered, but multi objective RAPs for different problem types are still not a saturated area in RAP literature.

Heuristic (27%) and meta-heuristic (56%) algorithms are very popular solution methods in RAP optimisation whether by single or multi-objective. However, the classical methods such as mathematical programming (approximation and exact) have not been completely absent. But, as a result of the large search spaces in RAP field regarding complex engineering systems, the decline in the usage of these classical methods are inevitable. From the point of solution techniques, there are still opportunities to improve the effectiveness and efficiency of available meta-heuristics such as ACO, PSO, IA, TS and GDA, and also some new metaheuristic algorithms such as the harmony search algorithm, artificial bee colony algorithm can be applied to different problem types to achieve improved solutions. Hybrid optimization techniques are also another promising approach in this field such as the combination of heuristic methods, NN, or some local search methods with all kinds of metaheuristics to improve computational efficiency or with exact methods to reduce search space, and also combining two meta-heuristics such as GA and PSO or ACO can be used to provide improved solutions.

Compared to traditional binary-state systems, there are still many unsolved issues in MSS optimal design. And also, there are not enough studies related to multi-level redundancy in RAP literature. Single/multi objective multi-level redundacy problems in which different solution techniques will be applied.

Also, non-deterministic approaches are under-developed areas in RAP field. Therefore, for different problem types in terms of system structure, solution techniques etc., this is still a very promising area to study for researchers who work in RAP field.

To conclude RAP is still a promising field in the scope of reliability optimization with its extended and modified versions. It can be argued that the saturation point for the RAP literature has not been arrived at yet.

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