A GENERAL APPROACH FOR THE SYNTHESIS OF PETRI NET BASED LIVENESS ENFORCING SUPERVISORS IN FLEXIBLE MANUFACTURING SYSTEMS

by

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APPROVAL PAGE

This is to certify that I have read the thesis entitled "A General Approach for the Synthesis of Petri Net Based Liveness Enforcing Supervisors in Flexible Manufacturing Systems" by Umar Suleiman Abubakar and that in my opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science in Electrical and Computer Engineering, the Graduate Institute of Science and Engineering, Melikşah University.

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A General Approach for the Synthesis of Petri Net Based Liveness Enforcing Supervisors in Flexible Manufacturing Systems

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ABSTRACT

In this thesis, a general approach is proposed for the computation of a liveness enforcing supervisor for the Petri net model of a flexible manufacturing system (FMS) prone to deadlocks. The proposed deadlock control policy in this thesis requires a modification to be made to the original Petri net model prone to deadlocks. The modification is simply the addition of a global sink/source place (GP), which is employed temporarily in the design process and then removed when the system becomes live. The proposed method is easy to use, straightforward and has computational simplicity. The applicability of the proposed approach is illustrated through examples from the related literature.

Keywords: Flexible Manufacturing Systems (FMS), Deadlock, Petri nets, liveness enforcing supervisor.

Esnek Üretim Sistemlerinde Petri Ağı Temelli Canlılık Uygulayıcı Gözeticilerin Sentezlenmesi için Genel Bir Yöntem

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ÖZ

Bu tezde, kördüğüm problemi olan bir esnek üretim sisteminin Petri ağı modeli için bir canlılık uygulayıcı gözetici hesaplanması konusunda genel bir yöntem önerilmektedir. Bu tezde önerilen kördüğüm kontrolu yaklaşımında, kördüğüm problemi olan asıl Petri ağı modelinde değişiklik yapılması söz konusudur. Buna göre, tasarım sürecinde geçici olarak kullanılan ve daha sonra asıl Petri ağı modeli canlı olduğunda kaldırılan küresel bir kaynak / yutak mevkisi (a global sink/source place GP) eklenmektedir. Önerilen yöntem basit, kullanımı ve hesaplaması kolaydır. Önerilen yaklaşımın uygulanabilirliği örneklerle gösterilmektedir.

Anahtar Kelimeler: Esnek Üretim Sistemleri (FMS), kördüğüm, Petri ağları, canlılık uygulayıcı gözetici.

DEDICATION

I dedicate this thesis to my late father Alhaji Suleiman Abubakar, who left this world while I was at the beginning of my undergraduate. May his soul rest in the perfect peace of Paradise.

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CHAPTER 1

INTRODUCTION

The advancement in science and technology has brought about complex man-made systems that pose real challenges to both their developers and users. Discrete event systems (DES) are examples of such systems. DESs are dynamic system models with state changes driven by asynchronous occurrences of individual events [1]. DESs are characterized by properties such as process synchronization, concurrent operations and conflicts or resource sharing [2]. Resource allocation systems (RAS) are common in DESs such as flexible manufacturing systems (FMS), workflow management systems and computer operating systems [3].

FMSs as the modern industrial systems are readily adaptable to changes whether predicted or unpredicted, in which machines are able to manufacture parts and have the ability to handle varying levels of production. FMSs allow equipment to be used for more than one purpose. Most FMSs are controlled by computer programs and therefore are automated systems. An FMS consists of a finite number of shared resources such as machines, automated guided vehicles, robots, and buffers. The main aim of FMSs is to offer the speed required to change with market conditions quickly [4].

In an FMS different jobs are carried out by different parts of the system concurrently while sharing limited resources of the system. A situation arises in which one part of the system is holding on some of the limited resources leaving other parts waiting indefinitely for the resources to be released. This situation leads to deadlocks in parts of the system or in the system as a whole. Deadlock in an RAS is a highly undesirable situation and must be dealt with in order to avoid catastrophic results since it affects the overall system's throughput and may lead to complete system failure or stoppage. There are four necessary conditions for the occurrence of deadlocks in RASs [5]. These conditions are given:

1) *Mutual Exclusion*: This means that two or more parts can acquire a resource at the same time. The resource is only exclusively occupied by one part of a system.
2) *No Preemption*: This implies that once a resource is acquired by a part in process it cannot be forcibly removed by any external agent.
3) *Hold and Wait*: This indicates that a process acquires some resources and awaits additional resources.
4) *Circular Wait*: This is a situation whereby there is a set of linearly ordering processes such that each process requests the resources currently held by the next process while the last process requests the resources held by the first.

Deadlock occurs if all of these conditions hold, and cannot happen if any of them does not hold [5]. In manufacturing systems, the first three conditions are already present. Thus, deadlock occurs when two or more jobs enter a circular wait state, and to avoid deadlock in manufacturing systems is to guarantee that there is no circular wait [6]. The first three conditions depend on the physical property of a system and its resources. However, the last is decided by the request, allocation and release of system resources. It is controllable and can be broken by properly assigning the resource of a system, aiming to avoid the occurrence of a circular wait [5].

As the scope of control theory is being extended into the fields of manufacturing systems, robotics, computer systems, communication networks and so on, there is need for different models capable of describing events that characterize the behaviors of these systems. The formal tools that are used to model such systems are finite state machines, graph theory, automata and Petri nets.

Finite state machines are conceptual models for discrete event systems (DESs). They consist of a finite number of states, transitions between these states and actions. States present certain behaviors. A transition indicates a state change and is guided by a condition. Graph theory can be used to describe interactions between activities or operations and resources from which a deadlock control policy can be derived. In graph theory frame work, deadlocks are always related with the circuits of the graph and their occurrences can be detected by simply computing all the circuits [5]. Automata are a powerful modeling tool for systems with an infinite number of states. They provide a comprehensive and structural treatment of the modeling and control of discrete event systems. A number of deadlock control policies that are computationally efficient are developed based on automata [5]. The modeling tool that we use in this research is called Petri nets. Petri nets have been used widely by researchers to model, analyze, design and control FMSs because of their properties which are suitable to detect deadlocks. There are software packages available which make Petri nets easy to be used as systematic tool to model and handle the control of a system in the real world [5].

Several studies have been carried out in the last two decades to deal with the deadlock problems in RAS. Three approaches have been identified for this purpose. The first one is deadlock detection and recovery. This approach employs a mechanism that detects the occurrence of a deadlock in a system and then puts the system back to its deadlock free state. This approach does not eliminate the occurrence of deadlocks, but relies on its ability to handle them when they occur. The efficiency of this method depends on the response time of its implemented algorithms, [3] [7], [8]. The second approach is deadlock avoidance. This method keeps the system away from deadlocks by using a control policy that determines the correct system evaluations among the feasible ones. Even though this method improves system throughput and better utilization of system resources, it does not completely eliminate deadlocks. The third one is deadlock prevention [3], [8], [9]. This approach has received more attention and is a well-defined problem in DES. It is used at the stage of system design and planning, and therefore does not require run-time costs. A control policy is added to the system in such a way that deadlocks never happen in the system. The computation of this method is done off-line in a static way. Control places and related arcs are used for this purpose [3], [8]. Our research is based on the third approach.

Dealing with deadlocks is not an easy task, because there are three important criteria that are considered in evaluating the performance of a liveness-enforcing supervisor: behavioral permissiveness, structural complexity and computational complexity. A maximally permissive optimal supervisor can lead to high utilization of system resources, and a supervisor with a simple structure can reduce the hardware and software costs in the stage of verification, validation and implementation [9]. Researchers try their best to come up with deadlock prevention policies that meet these three criteria. Petri nets are used as mathematical tool to model, analyze and control the FMSs. In Petri net analysis, two techniques are used for deadlock prevention: structural analysis [10], [11], [12] and reachability graph analysis [13], [14]. In structural analysis technique, structural objects of Petri nets, such as siphons and resource transition circuits are used in developing deadlock prevention policies. The control laws are simple and the computational complexity is reduced. However, the technique suffers from structural complexity and the controlled systems obtained are often suboptimal. Details of research works and developments on this techniques, especially by using siphons, can be found in [3], [8], [10], [11], [12], [15], [16], [17], [18], [19], [20], [21].

The reachability graph (RG) analysis technique employs the behavior of a system from its generated RG. Though this analysis technique almost always gives a highly or even maximally permissive liveness-enforcing supervisor, it suffers from the state explosion problem. This is due to the fact that it requires generating all or a part of reachable markings. The theory of region was proposed which is an effective approach [22], [23]. The approach can definitely find an optimal supervisor if there is such a supervisor. The method has structural and computational problems. An important method was proposed in [13], where the RG of a Petri net model suffering from deadlocks is split into two parts: a live-zone (LZ) and a deadlock-zone (DZ). At each iteration, a first-met bad marking (FBM) derived from the reachability graph is selected and control place is designed to prevent the FBM from being reached. The design of the control place is done by using a place invariant (PI) based method proposed in [22]. An FBM is a marking in the DZ, presenting the very first entry from the LZ to the DZ. The drawback of the method is that it cannot guarantee the behavioral optimality of the supervisor, and it is easier to use for systems with a small reachable space.

Another method that combines markings and siphons was proposed in [25]. The method is a selective siphon control policy in which highly permissive behavior can be obtained by a small-sized supervisor. The policy was improved in [26] by avoiding a complete siphon enumeration. However, there is lack of formal proof to show that the policy is definitely maximally permissive in theory [5]. Other related works can be found in [2], [13], [14], [23], [25], [26], [27].

A divide-and-conquer strategy is another approach of liveness-enforcing supervisors which was claimed in [28] to be computationally superior compared with the well-established global-conquer approaches as in [13], [14]. In [29] a computationally efficient divide-and-conquer strategy for the computation of liveness-enforcing supervisors (LES) was presented. The method improves the conventional RG based methods. However, it is necessary to deal with too many submodels when the number of shared resources is big. Therefore the objective of this thesis is to propose a general approach for the computation of a liveness enforcing supervisors for the Petri net model of an FMS without dividing a given PN model into its submodels, and also without transformation or reduction of the given PN model as in [13], [14]. The proposed method is easy to apply and straight forward. The applicability of the proposed method is shown by examples.

The remainder of this thesis is organized as follows. Chapter 2 comprises of basic definitions of Petri net, equations for the computations of the monitors, redundancy check algorithm and simple FMS system and its Petri net model with its reachability graph. Chapter 3 explains the proposed method with an illustrative example to show how the method is applied on a simple Petri net model. Chapter 4 contains application examples of the proposed method on two different Petri net models. Finally Chapter 5 provides the conclusion of the thesis.

CHAPTER 2

PRELIMINARIES

2.1 INTRODUCTION

In this chapter some basic concepts related to this thesis are considered. These include Petri nets, computations of control places (monitors), a method used to identify and eliminate redundant monitors. Finally a simple FMS system and its Petri net model are explained.

2.2 PETRI NETS

Petri net were proposed by C. A. Petri in 1962 in his PhD thesis as net-like mathematical tool for the study of communication with automata [30]. Today, Petri nets are a powerful modeling formalism in many disciplines such as computer science, system engineering, communication and transport systems [31], [32]. Their further development was facilitated by the fact that they combine a well-defined mathematical theory with a graphical representation. Using Petri nets, it is possible to set up algebraic equations, state equations and other mathematical models describing the behavior of systems.

The following definitions are from [5], [9].

2.2.1 Definition 1 Petri net is a four-tuple defined as $N = (P, T, F, W)$, where P and T are finite non-empty and disjoint sets. P is a set of places and T is set of transitions with $P \cup T \neq \emptyset$ and $P \cap T = \emptyset$. $F \subseteq (P \times T) \cup (T \times P)$ is called a flow relation of the net, it is represented by arrows from transitions to places or from places to transitions. $W: (P \times T) \cup (T \times P) \rightarrow$

$\mathbb{N}$ is a mapping which assigns a weight to an arc: $W(x, y) > 0$ iff $(x, y) \in F$, and $W(x, y) = 0$, otherwise, where $x, y \in P \cup T$ and $\mathbb{N}$ is the set of non-negative integers.

2.2.2 Definition 2 A Petri net $N = (P, T, F, W)$ is called an ordinary net, denoted as $N = (P, T, F)$, if $\forall f \in F$, $W(f) = 1$, and is called generalized if $\forall f \in F$, $W(f) > 1$.

2.2.3 Definition 3 A node $x \in P \cup T$, $\bullet x = \{y \in P \cup T | (x, y) \in F\}$ is called the preset of x, while $x\bullet = \{y \in P \cup T | (x, y) \in F\}$ is called the post set of x.

2.2.4 Definition 4 A marking is a mapping $M: P \rightarrow \mathbb{N}$. $M(p)$ represents the number of tokens in a place p. Markings and vectors are usually described using a multiset or formal sum for space economy. $\sum_{p \in P} M(p)p$, is used to denote vector M. A marked Petri net is represented by a pair (N, M_0).

2.2.5 Definition 5 A net is said to be pure (self-loop free) iff $\nexists(x, y) \in (P \times T) \cup (T \times P)$: $(x \,.\, y) \in F \wedge (y \,.\, x) \in F$.

2.2.6 Definition 6 An incidence matrix of net N is a $|P| \times |T|$ integer matrix with $[N](p, t) = W(t, p) - W(p, t)$.

2.2.7 Definition 7 A transition is fired or enabled at marking M if $\forall\, p \in \bullet t$, $M(p) \geq W(p, t)$. This fact is denoted as $M[t\rangle$. Firing a transition yields a new marking M' such that $\forall\, p \in P$, $M'(p) = M(p) - W(p, t) + W(t, p)$, denoted by $M[t\rangle M'$, and M' is called an immediately reachable marking from M. M' is reachable form M if there exists a sequence of transitions $\sigma = t_1, t_2 \ldots, t_n$ and markings $M_1, M_2 \ldots$, and M_{n-1} such that $M[t_1\rangle M_1 [t_2\rangle M_2 [t_3\rangle \ldots M_{n-1} [t_n\rangle M'$ holds and satisfies the state equation $M' = M + [N]\vec{\sigma}$, where $\vec{\sigma}: T \rightarrow \mathbb{N}$ is a vector of non-negative integers called a counting vector, and $\vec{\sigma}(t)$ indicates the algebraic sum of all occurrences of t in σ. $M[\rangle$ is the set of all markings reachable from M by enabling any possible sequence of transitions. $M_0[\rangle$ is called the set of reachable markings of a Petri net N from initial marking M_0, often denoted by $R(N, M_0)$. $R(N, M_0)$ can be graphically expressed by a reachability graph of a net (N, M_0) which is denoted as $G(N, M_0)$. $G(N, M_0)$ is a directed graph which has its nodes in $R(N, M_0)$ as markings, and its arcs are labeled by the transitions of N. An arc from M_1 to M_2 is labeled by the t if $M_1[t\rangle M_2$.

2.2.8 Definition 8 A place is called k-bounded ($k \in \mathbb{N} \setminus \{0\}$) if $\forall M \in R(N, M_0)$: $M(p) \leq k$. A net is k-bounded if every place is k-bounded. A net is safe if it is 1-bounded.

2.2.9 Definition 9 Given a net system (N, M_0) with $N = (P, T, F, W)$, a transition $t \in T$ is live at M_0 if $\forall M \in R(N, M_0)$, $\exists M' \in R(N, M)$, $M'[t\rangle$ holds. A transition is said to be live if it is potentially firable in any marking $R(N, M)$, and the net system (N, M_0) is said to be live. A transition $t \in T$ is dead at M if $\nexists M' \in R(N, M)$ such that $M'[t\rangle$ holds, and the transition cannot fire any more. A net system (N, M_0) with a dead transition is said to be prone to deadlock since it has a transition which is not potentially firable. (N, M_0) is deadlock-free if $\forall M \in R(N, M_0)$, $\exists t \in T$, $M[t\rangle$ holds.

2.2.10 Definition 10 A Petri net is said to be conservative if the total number of tokens of all its reachable markings is constant.

2.3 COMPUTATION OF MONITORS

In the place invariant method proposed in [24] for the computation of monitors, the controlled Petri net of a system has an incidence matrix D made up of both the original Petri net and the added control places with their related arcs. The idea is to force the system to obey constraints which can be grouped in matrix form as follows:

$$L\mu_p \leq b \tag{1}$$

Where: μ_p is the marking vector of the Petri net model (PNM), L is an n_c x n integer matrix representing the place invariant, b is an n_c x 1 integer vector and n_c is the number of constraints of (1). If a non-negative slack variable μ_c is introduced, the inequality constraint becomes equality as follows:

$$L\mu_p + \mu_c = b \tag{2}$$

μ_c is an n_c x 1 integer vector, representing the markings of the control places.

If the incidence matrix of the PNM is given as D_P, the Petri net controller D_c, which is a row vector representing the connection between the control places and the transition can be defined as follows:

$$D_c = -LD_P \qquad (3)$$

The initial marking of the controlled PNM μ_{c0}, which is computed in such a way that the place invariant (PI) of equation (2) is initially satisfied, is given as follows:

$$\mu_{c0} = b - L\mu_{P0} \qquad (4)$$

When dealing with large Petri nets, the incidence matrices tend to be very big, and this is a major drawback of this method. However, a simplified method was proposed in [14] in order to reduce the size of the incidence matrix D_P. Equations 2 and 3 are modified in [14], and there is no need to use the incidence matrix D_P, the computation is done by using the incidence matrix D_{PI} of PI related Petri net.

$$D_c = -L_{PI}\, D_{PI} \qquad (5)$$

Where D_{PI} is the incidence matrix of the PI related net with j places and k transitions, L_{PI} is a $j \times 1$ integer row vector representing the invariant related places, D_c is a $k \times 1$ integer row vector representing the incidence matrix of the monitor.

$$\mu_{c0} = b - L_{PI}\, \mu_{PI0} \qquad (6)$$

Where L_{PI} is place invariant related integer vector, μ_{PI0} is initial marking of place invariant related net.

By definition, initially there is no tokens within the activity places, which means that $L_{PI}\, \mu_{PI0} = 0$. Therefore equation (6) becomes:

$$\mu_{c0} = b \qquad (7)$$

2.4 IDENTIFICATION AND ELIMINATION OF REDUNDANT MONITORS

There are may exist some redundant monitors among the monitors or control places (CPs) computed for liveness enforcing supervisor. If redundant CPs are removed from a live Petri net model (LPN), the Petri net model still maintains its liveness. Removing redundant CPs from a LPN model and leaving only the necessary ones may reduce the structural complexity of the LPN model. Below are redundancy test algorithms proposed in [33] for removing redundant monitors from a LPN model.

Algorithm Redundancy Test: Redundancy test for LES of FMS

```
Input: A live Petri net (LPN) model, denoted by a net system ($(N_0, M_0)$), of an FMS, controlled
        by n CPs; $CP = \{C_1, C_2, ..., C_n\}$;
  (1) [Define] $\beta_0$: the number of reachable markings or states of reachability graph ($R_0$) of ($N_0$, $M_0$).
      [Define for Algorithm A] $\beta_A$: the number of reachable markings or states of $R_A$ of ($N_A$, $M_A$); $n = j + k$, where n: the number of CPs of LPN; j: the number of redundant CPs; k: the number necessary CPs:
      [Define for Algorithm B] $\beta_B$: the number of reachable markings or states of $R_B$ of ($N_B$, $M_B$); $n = l + m$, where n: the number CPs of LPN; l: the number of redundant CPs; m: the number of necessary CPs;
  (2) Apply Algorithm A to ($N_0$, $M_0$) and the resultant net system is denoted as ($N_A$, $M_A$).
  (3) Apply Algorithm B to ($N_0$, $M_0$) and the resultant net system is denoted as ($N_B$, $M_B$).

  Output: If ($j > 0$) [for Algorithm A]
          then Output A = an LPN, denoted a net system ($N_A$, $M_A$), controlled by k
          necessary CPs; there are j redundant CPs;
             if $\beta_A = \beta_0$ then the controlled behaviour of ($N_A$, $M_A$) is the same as ($N_0$, $M_0$)
             if $\beta_A > \beta_0$ then the controlled behaviour of ($N_A$, $M_A$) is more permissive than
              ($N_0$, $M_0$)
          else there is no redundant CPs obtained due to Algorithm A and therefore for
          Algorithm A: Output = Input;

          If ($l > 0$) [for Algorithm B]
```

```
    Then Output B = an LPN, denoted by a net system $(N_B, M_B)$, controlled by m necessary CPs; there are l redundant CPs;
        if $\beta_B = \beta_0$ then the controlled behaviour of $(N_B, M_B)$ is the same as $(N_0, M_0)$
        if $\beta_B > \beta_0$ then the controlled behaviour of $(N_B, M_B)$ is more permissive than $(N_0, M_0)$
    else there is no redundant CPs obtained due to Algorithm B and therefore for Algorithm B: Output = Input;
```

End of Algorithm Redundancy Test

Algorithm A: Front-to-Back (*FTB*) redundancy test for *LES* of FMS.

```
Input: A live Petri net (LPN) model, denoted by a net system $(N_0, M_0)$, of an FMS, controlled by
n CPs; $CP = \{C_1, C_2, \ldots, C_n\}$;
    (1) [Initialize] $N_A := N_0$ ; $M_A := M_0$; $i = 1$; $j = 0$; $k = 0$;
    (2) Remove $C_i$ from $(N_A, M_A)$. Denoted the resultant net system by $(N_i, M_i)$.
    (3) Check the liveness property of $(N_i, M_i)$, compute the reachability graph $(R_i)$ of $C_i$ and
        define $\beta_{Ai}$, i.e., the number of reachable markings of $R_i$;
         If $(N_i, M_i)$ is NOT LIVE
          then put $C_i$ back into $(N_i, M_i)$; $k = k + 1$; which means that $C_i$ is necessary to keep
               the PN model live.
          else [i.e. If $(N_i, M_i)$ is live], $j = j + 1$; which means that $C_i$ is redundant,
          if $\beta_{Ai} = \beta_0$ then the controlled behaviour of $(N_i, M_i)$ is the same as $(N_0, M_0)$
          if $\beta_{Ai} > \beta_0$ then the controlled behaviour of $(N_i, M_i)$ is more permissive than $(N_0, M_0)$
          endif
    (4) $N_A := N_i$ ; $M_A := M_i$
    (5) $i = i + 1$
    (6) if $i \le n$ then go to step 2.
Output: If $(j > 0)$
```

```
    then Output = an LPN, denoted by a net system ($N_A$, $M_A$), controlled by k necessary CPs;
    there are j redundant CPs;
    if $\beta_A = \beta_0$ then the controlled behaviour of ($N_A$, $M_A$)is the same as ($N_0$, $M_0$)
    if $\beta_A > \beta_0$ then the controlled behaviour of ($N_A$, $M_A$) is more permissive than ($N_0$, $M_0$)
      else there is no redundant CPs and therefore Output = Input;
```

End of Algorithm A

Algorithm B: Back-to-Front (*BTF*) redundancy test for *LES* of FMS.

```
Input: A live Petri net (LPN) model, denoted by a net system ($N_0$, $M_0$), of an FMS, controlled by
n CPs; CP = {$C_1$, $C_2$, …, $C_n$};
    (1) [Initialize] $N_B := N_0$ ; $M_B := M_0$; $i = 1$; $l = 0$; $m = 0$;
    (2) Remove $C_i$ from ($N_B$, $M_B$). Denoted the resultant net system by ($N_i$, $M_i$).
    (3) Check the liveness property of ($N_i$, $M_i$), compute the reachability graph ($R_i$) of $C_i$ and
        define $\beta_{Bi}$, i.e., the number of reachable markings of $R_i$;
          If ($N_i$, $M_i$) is NOT LIVE
              then put $C_i$ back into ($N_i$, $M_i$); $m = m + 1$; which means that $C_i$ is necessary to keep
                  the PN model live.
            else [i.e. If ($N_i$, $M_i$) is live], $l = l + 1$; which means that $C_i$ is redundant,
            if $\beta_{Bi} = \beta_0$ then the controlled behaviour of ($N_i$, $M_i$) is the same as ($N_0$, $M_0$)
            if $\beta_{Bi} > \beta_0$ then the controlled behaviour of ($N_i$, $M_i$) is more permissive than ($N_0$, $M_0$)
          endif
    (4) $N_B := N_i$ ; $M_B := M_i$
    (5) $i = i - 1$
    (6) if $i \neq 0$ then go to step 2.
Output: If ($l > 0$)
        then Output = an LPN, denoted by a net system ($N_B$, $M_B$), controlled by m necessary
        CPs;
```

```
there are j redundant CPs;
if $\beta_B = \beta_0$ then the controlled behaviour of $(N_B, M_B)$ is the same as $(N_0, M_0)$
if $\beta_B > \beta_0$ then the controlled behaviour of $(N_B, M_B)$ is more permissive than $(N_0, M_0)$
else there is no redundant CPs and therefore Output = Input;
```

End of Algorithm B

Redundancy test algorithm makes use of both Algorithm A and B. Algorithm A test each CP from number 1 to end, while B tests each CP starting from end to number 1. Both tests may produce the same result or it may be possible to obtain different results [33].

2.5 FMS EXAMPLE

In this section an example of modeling of an FMS and the computation of its reachability graph is considered. Fig. 2.1 shows an FMS from [13] consisting of two machines M1 and M2 each of which can process one part at a time and one robot which can hold one part at a time. The FMS has two input/output buffers, I/O1 and I/O2 through which parts enter the FMS. We consider only two parts: P1 and P2. It is assumed that there are no parts initially in the system [14]. The production sequences are as follows:

PART 1 (P1): M1 → Robot → M2; PART 2 (P2): M2 → Robot → M1

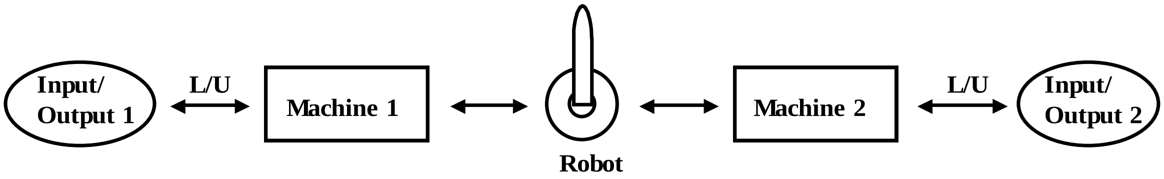

Figure 2.1. An example FMS.

Fig. 2.2 is the PNM of the FMS depicted in Fig. 2.1. There are eleven places in the PNM, P = {p1-p6, p11-p13, p21-p22} and eight transitions, T = {t1-t8}. There are six activity places, P_A = {p1-p6}, which represent the operations of activities of M1, R and M2, and M2, R and M1 for the part type P1and P2 respectively, three resource places P_R = {p11-p13} and two sink/source places P_A = {p21-p22}. The number of tokens in the sink/source places, p21 and p22 represent the number of concurrent activities that can take place for part types P1 and P2 respectively. The initial marking of p12 is one, as robot R can hold one part at a time. Similarly the initial markings of p11 and p13 are all one as machines can process one part at a time.

From the reachability graph (RG) of the PNM, it can be verified that the uncontrolled PNM is prone to deadlocks. There are 20 states within RG, 5 of which are bad states, in the dead zone (DZ) and 15 of which are good states within the live zone (LZ). Therefore a control policy is required to prevent these 5 states within the DZ from being reached in order to get the live PNM.

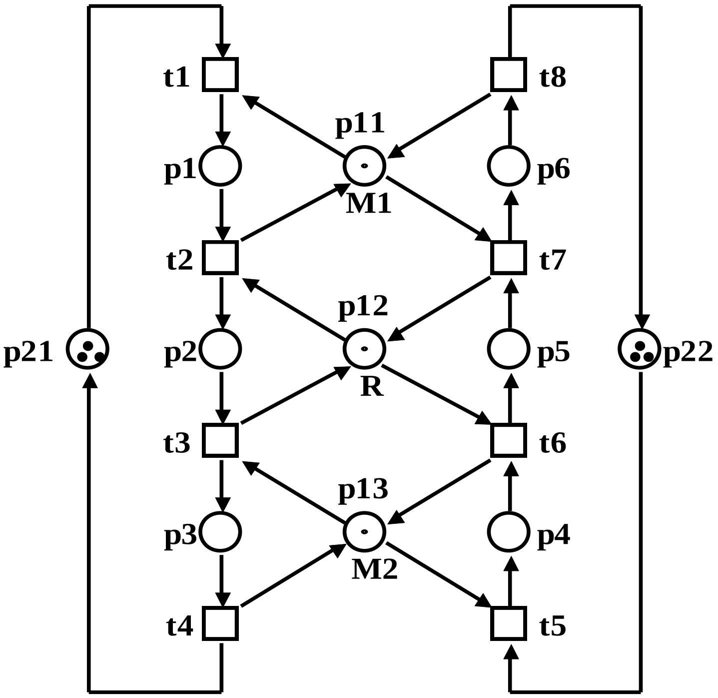

Figure 2.2. Petri net Model (PNM) of the FMS for the two production sequences.

The reachability graph of the FMS is shown in Fig. 2.3 indicating the 20 states within the RG. States 8 and 13 are deadlock states while states 10, 11 and 12 are bad states leading to the deadlock states. These five states form the DZ while the remaining 15 states are good sates; within the LZ.

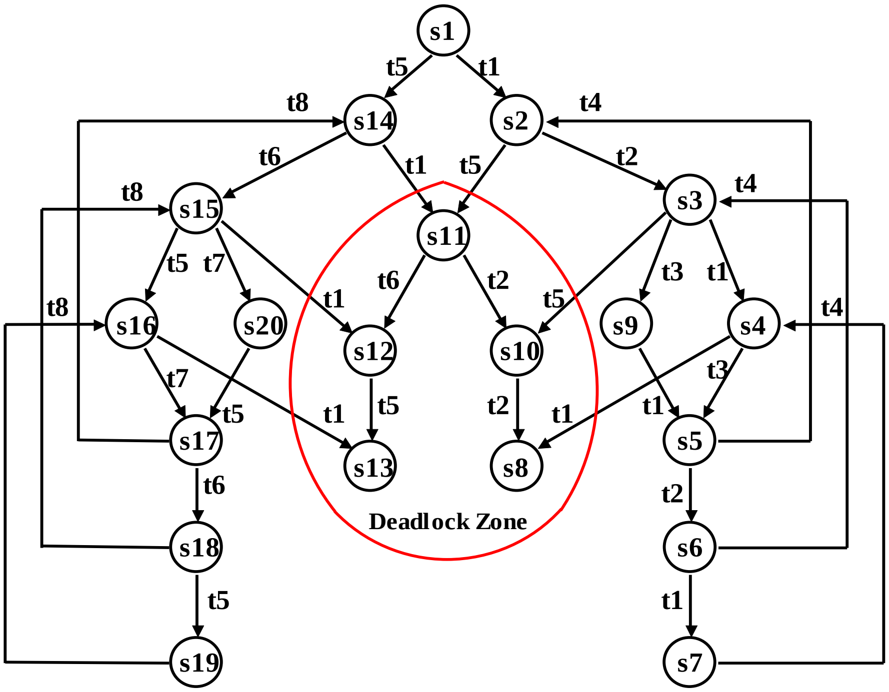

Figure 2.3. The RG of the PNM.

The markings of the states of the RG are shown in Table 2.1.

Table 2.1. States of the computed RG of the PNM.

$s_1 = p_{11} + p_{12} + p_{13} + 3p_{21} + 3p_{22}$	$s_{11} = p_1 + p_4 + p_{12} + 2p_{21} + 2p_{22}$
$s_2 = p_1 + p_{12} + p_{13} + 2p_{21} + 3p_{22}$	$s_{12} = p_1 + p_5 + p_{13} + 2p_{21} + 2p_{22}$
$s_3 = p_2 + p_{11} + p_{13} + 2p_{21} + 3p_{22}$	$s_{13} = p_1 + p_4 + p_5 + 2p_{21} + p_{22}$
$s_4 = p_1 + p_2 + p_{13} + p_{21} + 3p_{22}$	$s_{14} = p_4 + p_{11} + p_{12} + 3p_{21} + 2p_{22}$
$s_5 = p_1 + p_3 + p_{12} + p_{21} + 3p_{22}$	$s_{15} = p_5 + p_{11} + p_{13} + 3p_{21} + 2p_{22}$
$s_6 = p_2 + p_3 + p_{11} + p_{21} + 3p_{22}$	$s_{16} = p_4 + p_5 + p_{11} + 3p_{21} + p_{22}$
$s_7 = p_1 + p_2 + p_3 + 3p_{22}$	$s_{17} = p_4 + p_6 + p_{12} + 3p_{21} + p_{22}$
$s_8 = p_1 + p_2 + p_4 + p_{21} + 2p_{22}$	$s_{18} = p_5 + p_6 + p_{13} + 3p_{21} + p_{22}$
$s_9 = p_3 + p_{11} + p_{12} + 2p_{21} + 3p_{22}$	$s_{19} = p_4 + p_5 + p_6 + 3p_{21}$
$s_{10} = p_2 + p_4 + p_{11} + 2p_{21} + 2p_{22}$	$s_{20} = p_6 + p_{12} + p_{13} + 3p_{21} + 2p_{22}$.

CHAPTER 3

THE PROPOSED METHOD AND ITS ALGORITHM

3.1 INTRODUCTION

In this chapter, a new approach for synthesis of Petri net based liveness enforcing supervisors in FMS is proposed. In the proposed method, reachability graph (RG) is used for tackling deadlock problems of a given Petri net model (PNM). The RG of a PNM suffering from deadlocks has two partitions: dead zone (DZ) and live zone (LZ). The DZ comprises of deadlock states together with bad states leading deadlock states while the LZ has the good states. The aim of this method is to prevent a PNM of an FMS from reaching all states within DZ while allowing every state within LZ to be reached.

There are three categories of places in a PNM of an FMS: *resource places* P_R, *activity (operational) places* P_A, and *sink/source places* $P_{S/S}$. Resource places represent the shared/non-shared resources. Activity (operational) places represent an action to process a part in a production sequence. The number of tokens initially deposited into sink/source places represent the number of the concurrent activities which can take place in a production sequence [27]. In this proposed method, we only focus on the markings of activity places when the RG of a PNM is computed. We consider only the activity places which have tokens.

3.2 THE PROPOSED METHOD ALGORITHM

The method proposed employs a global sink/source place (GP) in computing the liveness enforcing supervisors in an iterative way. In this control policy the reachability graph (RG) of the given PNM is generated by a Petri net analysis tool called INA [34], which gives both the LZ as the first strongly connected components, and the DZ, as the strongly connected components other than the first one of a given PNM. At each iteration, starting from one token, the number of tokens in the GP is increased by one and the RG of the net is computed. If the net is live, the number of tokens in the GP is increased by one and the RG is computed again. When the net is not live, the RG of the related net is divided into a DZ and a LZ. The latter constitutes the good states of the RG which represents optimal solution. The objective here is to prevent all states within the DZ from being reached, because they are considered as bad markings (BM). From a BM we consider only the markings of the activity places. A monitor (control place) with its related arcs and initial marking is computed to prevent the BM from being reached [9], [14], by means of a place invariant (PI). Computed PI is implemented in such a way that the sum of tokens within the subset of the activity places must be at most one token less than their current number.

After the PNM becomes live, a redundancy test as proposed in [33] is carried out to remove any redundant monitor from the computed monitors. Finally, a live controlled Petri net model with all necessary control places as liveness enforcing supervisor is obtained. The proposed method provides optimal permissiveness on some Petri net models and near optimal on others. The method is straight forward and easy to use. The algorithm of the proposed deadlock prevention policy is as follows.

Algorithm: Synthesis of a liveness enforcing supervisor by means of a global sink/source place (GP)

Input: A Petri net model (PNM) of an FMS prone to deadlocks.

Output: A live controlled Petri net model.

```
Step 1: Identify the input and output transitions of all sink/source places $P_{S/S}$ and use them
        for adding a global sink/source place (GP) to the PNM. The addition of the GP will
        be made in such a way that its input transitions are input transitions of all $P_{S/S}$ and
        its output transitions are output transitions of all $P_{S/S}$. The resultant net system is
        $PNM_B = PNM + GP$.

Step 2: for ($B = 1; B \leq k$ ; B++)
```

```
/* B is the number of tokens in the GP, k is the sum of initial tokens in all sink/source Places */
{
2.B.1: Compute the $RG_B$ of the $PNM_B$,
        if $PNM_B$ is live,
            then consider a new net, i.e. go to Step 2.B.1
            else compute the $LZ_B$ and $DZ_B$ of the $RG_B$
        endif

2.B.2: From each BM of $DZ_B$, define a place-invariant PI.

2.B.3: Compute a monitor C, for each PI using the simplified invariant-based method [13].

2.B.4: If there are more than one monitor computed for $PNM_B$ then carry out the
        redundancy test to eliminate any redundant monitors by using the method
        [33].

2.B.5: Add necessary monitors computed in the previous step within $PNM_B$ ($PNM_B$
:= $PNM_B$ + computed monitors)
}

Step 3:  Obtain the live controlled PNM by adding all the necessary monitors computed in
         Step 2 within the PNM.

Step 4:  Exit
```

End of Algorithm

3.3 ILLUSTRATIVE EXAMPLE

The success of any algorithm for solving a problem is measured by its ability to work on an example and also its ability to generalize on other examples that it has yet to see. If the method or algorithm works on every example, then we can claim its success since it is generalized. In this section, an example of synthesizing liveness enforcing supervisor based on the method proposed in the previous section on a simple uncontrolled PNM of an FMS from [10] is considered. The PNM shown in Fig. 3.1 is an S^3PR (a System of Simple Sequential Process with Resources) model. It is verified that there are 95 states in the RG, of which 11 states are within DZ, representing the bad marking to be dealt with, and 84 states are in the LZ, representing good states or legal markings. The objective here, by applying the proposed method, is to obtain a live PNM while preventing the 11 bad states from being reached.

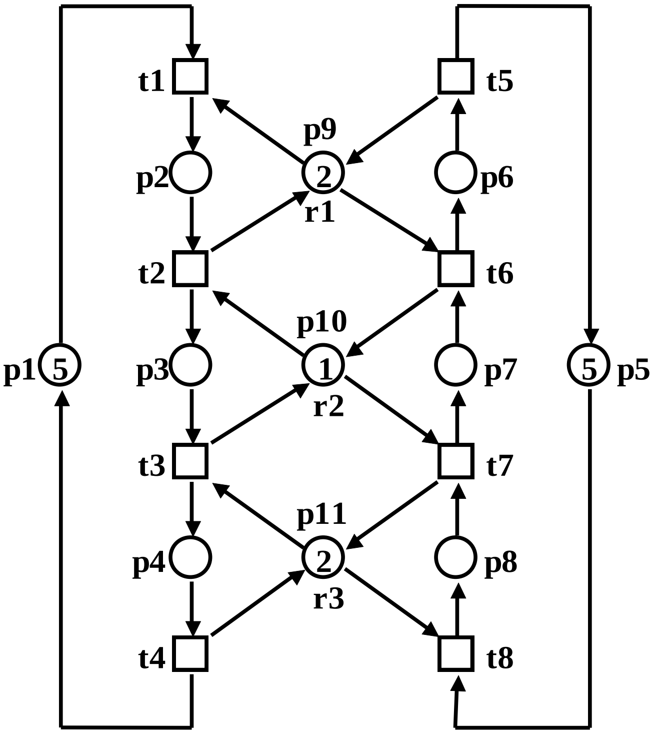

Figure 3.1. S^3PR Petri net model of an FMS from [10].

In the PNM there are six activity places P_A = {p2-p4, p6-p8}, three shared resource places P_R = {p9-p11}, and two sink/source places $P_{S/S}$ = {p1, p5}.

Step 1: The input and output transitions of GP are ${}^{\bullet}GP$ = {t4, t5} and $GP^{\bullet}$ = {t1, t8} respectively. The new Petri net model obtained with the addition of the GP, PNM_B = PNM_B + GP is shown in Fig. 3.2.

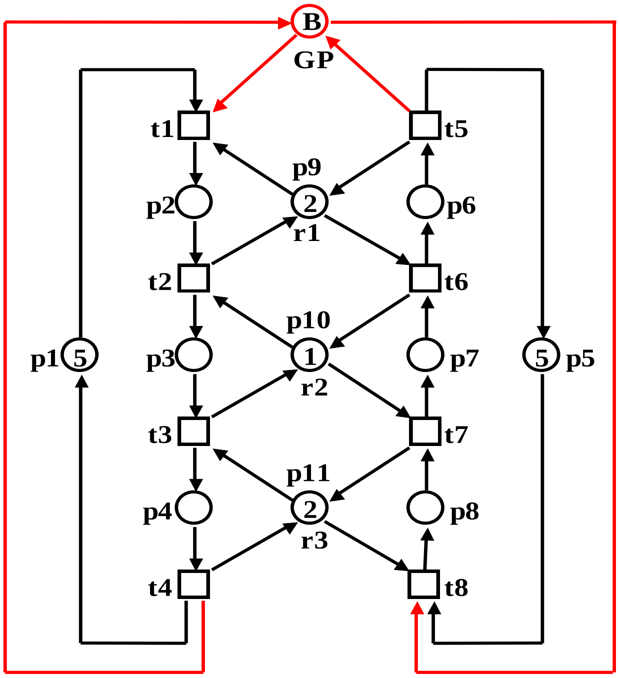

Figure 3.2. Net; PNM_B = PNM + GP.

Step 2: for (B = 1; B ≤ 5; B++)

Step 2.1.1: (B = 1), when one token is deposited in the GP, as shown in Fig. 3.3, the net PNM_1 is live with 7 good states. B: = B++ (B = 2).

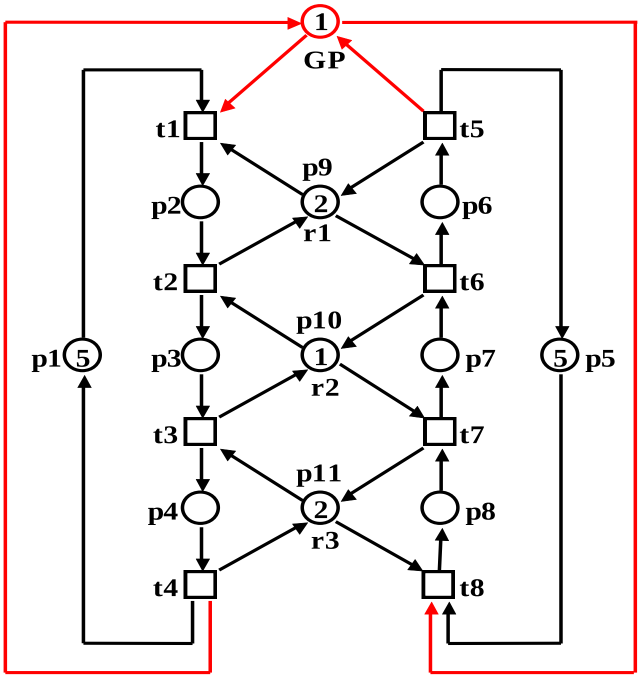

Figure 3.3. Live PNM$_1$ with 7 good states.

Step 2.2.1: (B = 2), When two tokens are deposited in the GP, the net PNM$_2$ is obtained as shown in Fig. 3.4. The PNM$_2$ is live with 25 good states. B: = B++ (B = 3).

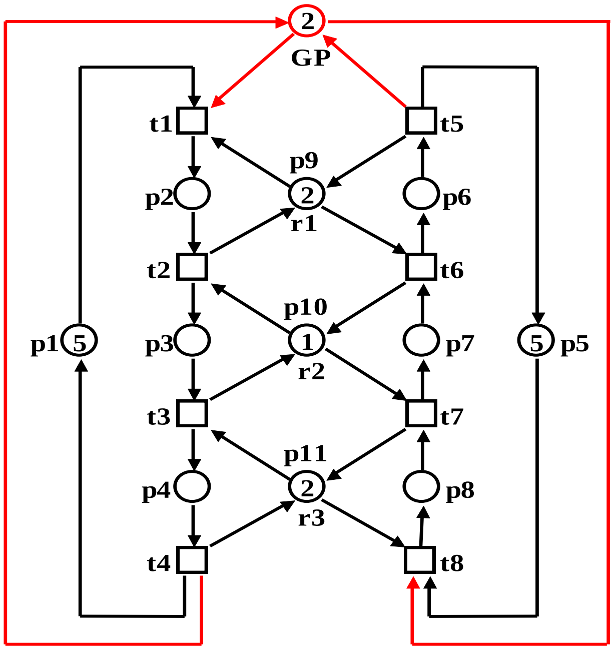

Figure 3.4. Live PNM$_2$ with 25 good states.

Step 2.3.1: (B = 3), the net PNM_3, shown in Fig. 3.5, is not live. The reachability graph RG_3 computed for the PNM_3 has 53 good states in the LZ_3 and 2 bad states BM_1 and BM_2 within the DZ_3.

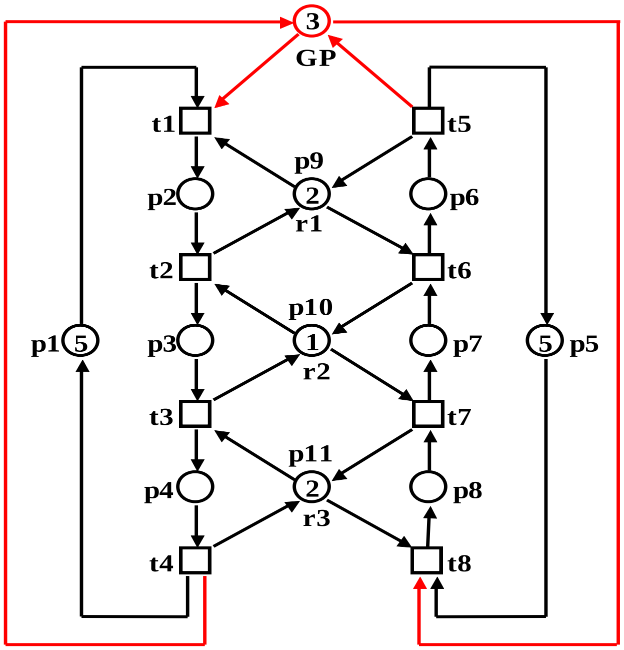

Figure 3.5. The PNM_3.

Step 2.3.2: The markings of activity places of BM_1 and BM_2 are shown in Table 3.1

Table 3.1. The markings of activity places of BM_1 and BM_2.

State nr.	p2	p3	p4	p6	p7	p8
22	2	0	0	0	1	0
46	0	1	0	0	0	2

The place invariants PI_1 and PI_2 for the BM_1 and BM_2 respectively are:

$$PI_1 = \mu_2 + \mu_7 \leq 2$$

$$PI_2 = \mu_3 + \mu_8 \leq 2$$

Step 2.3.3: The computation of the monitors C_1 and C_2 are carried out as follows:

$$L_{PI1} = \begin{array}{cc} p2 & p7 \\ [1 & 1] \end{array}$$

$$D_{PI1} = \begin{array}{c} \\ \left[\begin{array}{cccc} t1 & t2 & t6 & t7 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{array}\right. \end{array} \begin{array}{c} \\ \left.\begin{array}{c} \\ \end{array}\right] \end{array} \begin{array}{c} \\ p2 \\ p7 \end{array}$$

$$D_{PI1} = \begin{array}{ccccc} t1 & t2 & t6 & t7 & \\ 1 & -1 & 0 & 0 & p2 \\ 0 & 0 & -1 & 1 & p7 \end{array}$$

$$D_{C1} = -\, L_{PI1} \,.\, D_{PI1} = -\,[1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$D_{C1} = [1 \; -1 \; -1 \; 1]$$

$$\text{Therefore, } D_{C1} = \begin{array}{cccc} t1 & t2 & t6 & t7 \\ [-1 & 1 & 1 & -1] \end{array}$$

$$\mu_{0(c1)} = 2$$

$$L_{PI2} = \begin{array}{cc} p3 & p8 \\ [1 & 1] \end{array}$$

$$D_{PI2} = \begin{array}{ccccc} t2 & t3 & t7 & t8 & \\ 1 & -1 & 0 & 0 & p3 \\ 0 & 0 & -1 & 1 & p8 \end{array}$$

$$D_{C2} = -\, L_{PI2} \,.\, D_{PI2} = -\,[1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$D_{C2} = -\,[1 \; -1 \; -1 \; 1]$$

$$\text{Therefore, } D_{C2} = \begin{array}{cccc} t2 & t3 & t7 & t8 \\ [-1 & 1 & 1 & -1] \end{array}$$

$$\mu_{0(c\,2)} = 2$$

The computed monitors are shown in Table 3.2.

Table 3.2. Monitors C_1 and C_2.

C_i	$^\bullet C_i$	$C_i^\bullet$	$\mu_{0(c\,i)}$
C_1	t2, t6	t1, t7	2
C_2	t3, t7	t2, t8	2

Step 2.3.4: Redundancy test carried out shows that both C_1 and C_2 are necessary.

Step 2.3.5: The controlled $PNM_3 := PNM_3 + C_1 + C_2$ is shown in Fig. 3.6. It is live with 53 good states. This is the optimal live behavior for the controlled PNM_3.

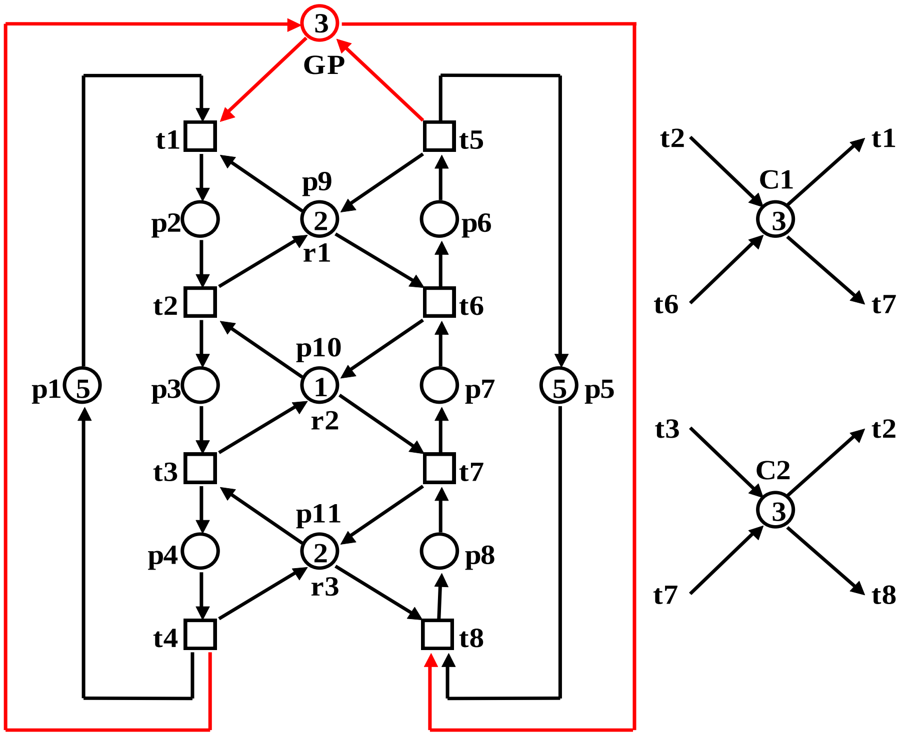

Figure 3.6. The controlled $PNM_3 := PNM_3 + C_1 + C_2$.

B := B ++ (B = 4).

Step: 2.4.1: (B = 4), the net PNM_4, shown in Fig. 3.7, is not live. The reachability graph RG_4 computed for PNM_4 has 77 good states in LZ_4 and 1 bad state BM_4 within DZ_4.

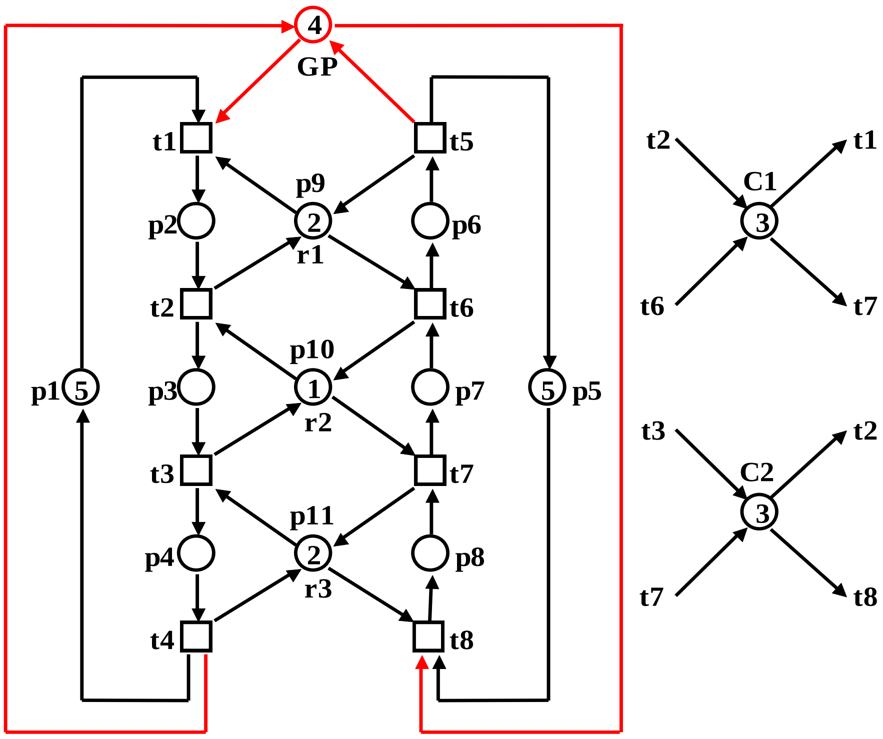

Figure 3.7. The PNM$_4$.

Step 2.4.2: The markings of the activity places BM$_3$ are shown in Table 3.3.

Table 3.3. The markings of the activity places of BM$_3$.

State nr.	p2	p3	p4	p6	p7	p8
22	2	0	0	0	0	2

The place invariant for the BM$_3$ is $PI_3 = \mu_2 + \mu_8 \leq 3$.

Step 2.4.3: The computation of the monitors C$_3$ is carried out as follows:

$$L_{PI3} = \begin{array}{c} \begin{array}{cc} p2 & p8 \end{array} \\ [1 \quad 1] \end{array}$$

$$D_{PI3} = \begin{array}{ccccc} t1 & t2 & t7 & t8 & \\ \left[\begin{array}{c}1\\0\end{array}\right. & \begin{array}{c}-1\\0\end{array} & \begin{array}{c}0\\-1\end{array} & \left.\begin{array}{c}0\\1\end{array}\right] & \begin{array}{c}p2\\p8\end{array} \end{array}$$

$$D_{C3} = - L_{PI3} . D_{PI3} = - [1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$D_{C3} = - [1 \quad -1 \quad -1 \quad 1]$$

$$\text{Therefore, } D_{C3} = \begin{array}{cccc} t1 & t2 & t7 & t8 \\ [-1 & 1 & 1 & -1] \end{array}$$

$\mu_{0(c3)} = 3$

The computed monitor is shown in Table 3.4.

Table 3.4. Monitor C$_3$.

C_i	$^{\bullet}C_i$	$C_i^{\bullet}$	$\mu_{0(ci)}$
C_3	t2, t7	t1, t8	3

Step 2.4.4: Since there is only one monitor computed, the redundancy test is not necessary.

Step 2.4.5: The controlled PNM$_4$ is obtained by adding C$_3$ within the uncontrolled PNM$_4$ (PNM$_4$:= PNM$_4$ + C$_3$) as shown in Fig. 3.8. It is live with 76 good states. This is the optimal live behavior for the controlled PNM$_4$.

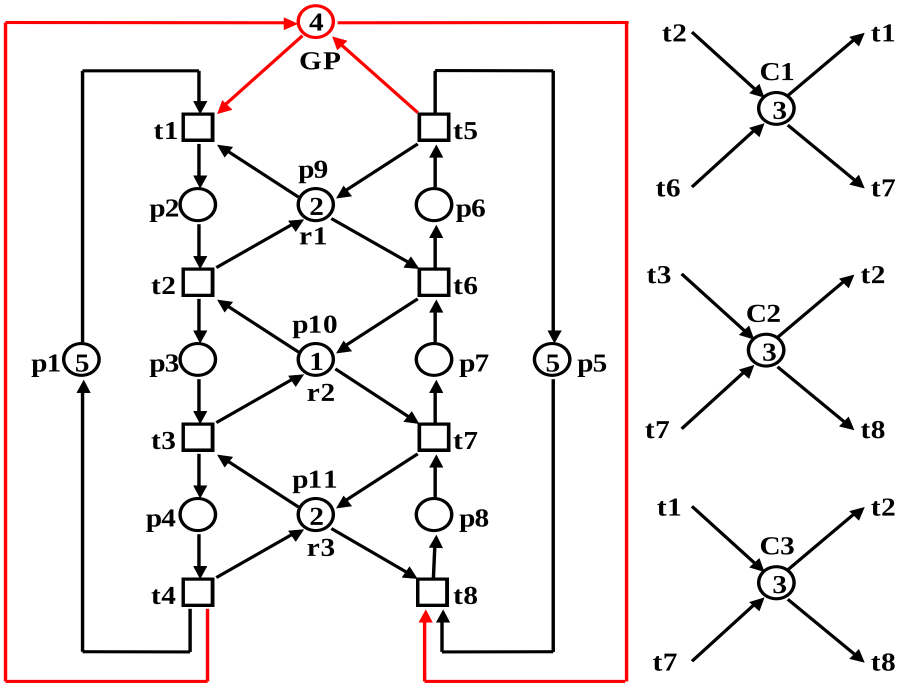

Figure 3.8. The controlled PNM$_4$ (PNM$_4$:= PNM$_4$ + C$_3$).

B := B ++ (B= 5).

Step 2.5.1: (B = 5), the PNM_5 shown in Fig. 3.9 is live with 84 good states. This is the optimal behaviour not only for the PNM_5, but also for the uncontrolled PNM.

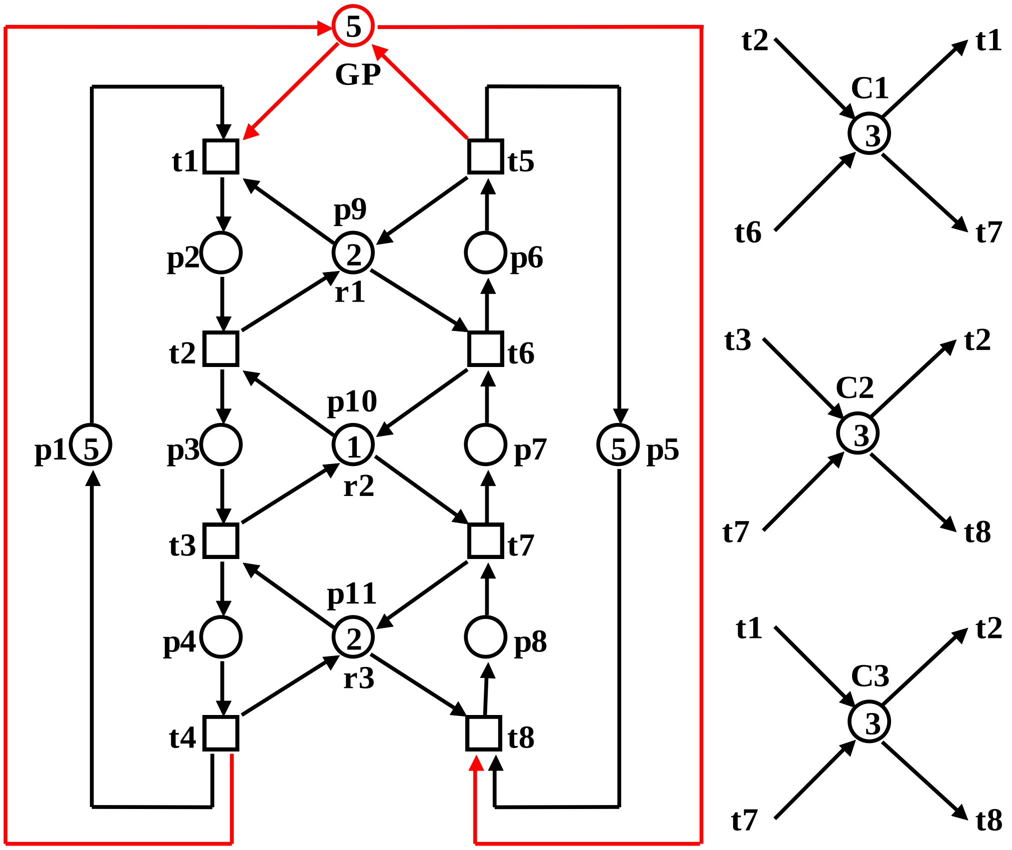

Figure 3.9. Live PNM_5.

Step 3: The design procedure applied in Step 2 is provided in Table 3.6. When the computed necessary monitors are added in the uncontrolled PNM, the controlled PNM is obtained as shown Fig. 3.10. It is verified that this controlled model is live with 84 good states.

Step 4: Exit.

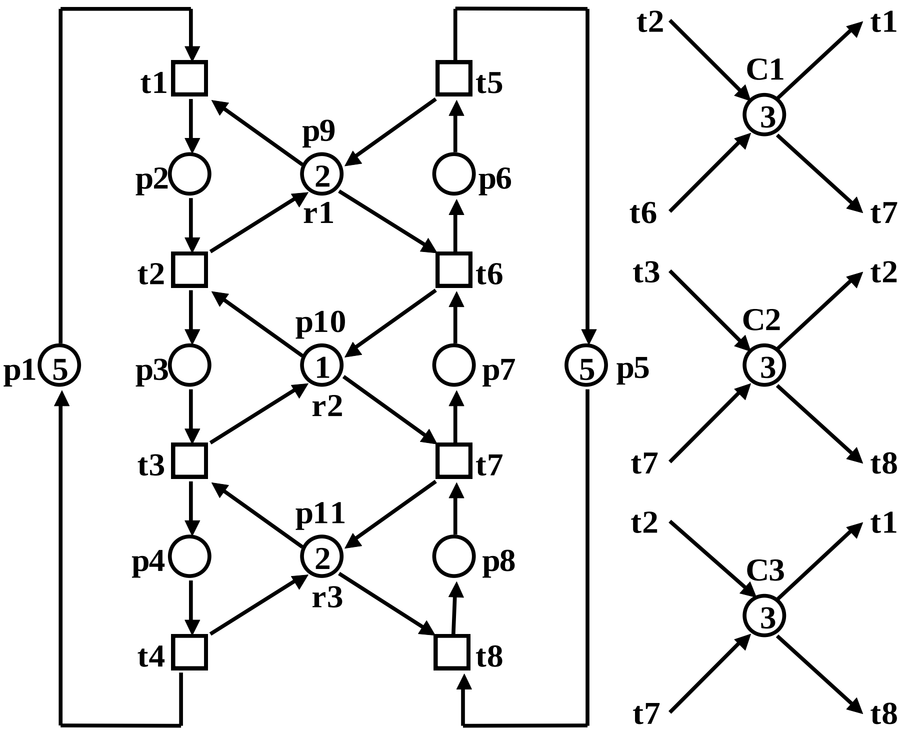

Figure 3.10. The optimally controlled PNM.

Table 3.5. The computed necessary monitors.

C_i	$^{\bullet}C_i$	$C_i^{\bullet}$	$\mu_{0(ci)}$
C_1	t2, t6	t1, t7	2
C_2	t3, t7	t2, t8	2
C_3	t2, t7	t1, t8	3

Table 3.6. The liveness enforcing procedure applied to the S^3PR model.

B	Included C	Is the net live?	# of States in RG	# of states in DZ	# of States in LZ	Computed C	# of states within controlled net	
							RG = LZ	UR
1	–	YES	7	0	7	–		
2	–	YES	25	0	25	–		
3	–	NO	55	2	53	C_1, C_2	53	0
4	C_1, C_2	NO	77	1	76	C_3	76	0
5	C_1,C_2, C_3	YES	84	0	84	–		

CHAPTER 4

APPLICATION EXAMPLES

In this chapter, the proposed method is applied to obtain liveness enforcing supervisor for two Petri net models; an S^3PR PNM from [13] and an AEMG PNM from [34]. Although the method gives optimal solution to the PNM treated in the illustrative example in chapter 3, in the examples considered in this chapter, for both cases liveness with near optimal permissiveness is achieved.

4.1 AN S^3PR PETRI NET EXAMPLE

The PNM of an FMS shown in Fig. 4.1 from [13], suffers from deadlock. It has 26,750 states within the RG, 21,581 of which are in the LZ while remaining 5,169 states are in the DZ.

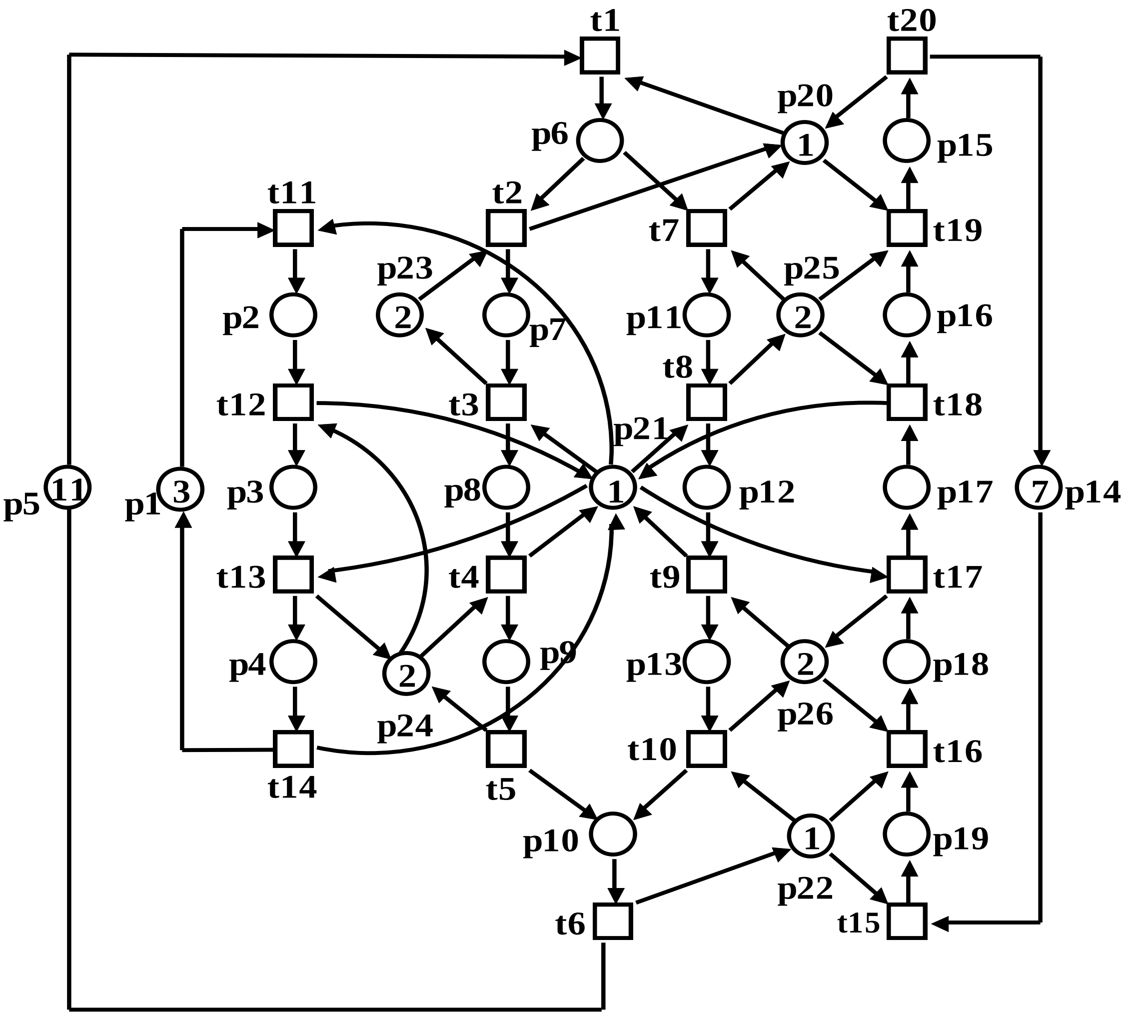

Figure 4.1. S^3PR Petri net model of an FMS from [13].

There are sixteen activity places P_A = {p2-p4, p6-p13, p15-p19}, seven shared resource places P_R = {p20-p26}, and three sink/source places $P_{S/S}$ = {p1, p5, p14}.

Step 1: The input and output transitions of GP are $^\bullet GP$ = {t6, t14, t20} and $GP^\bullet$ = {t1, t11, t15} respectively. The new Petri net model obtained with the addition of the GP, PNM_B = PNM_B + GP is shown in Fig. 4.2.

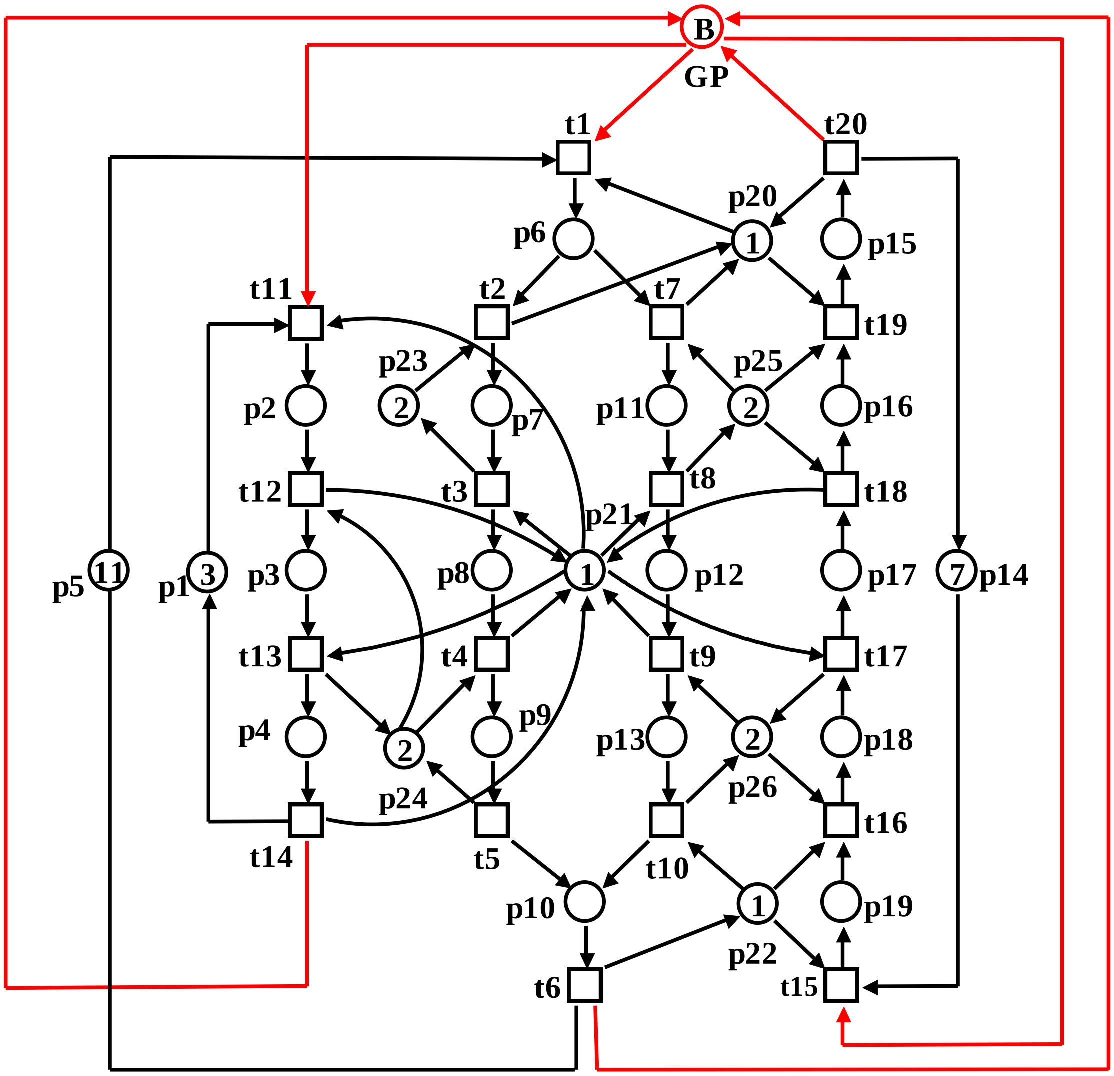

Figure 4.2. Net; $PNM_B = PNM + GP$.

Step 2: for (B = 1; B ≤ 11; B++)

Step 2.1.1: (B = 1), when one token is deposited in the GP, as shown in Fig. 4.3, the net PNM_1 is live with 17 good states. B: = B++ (B = 2).

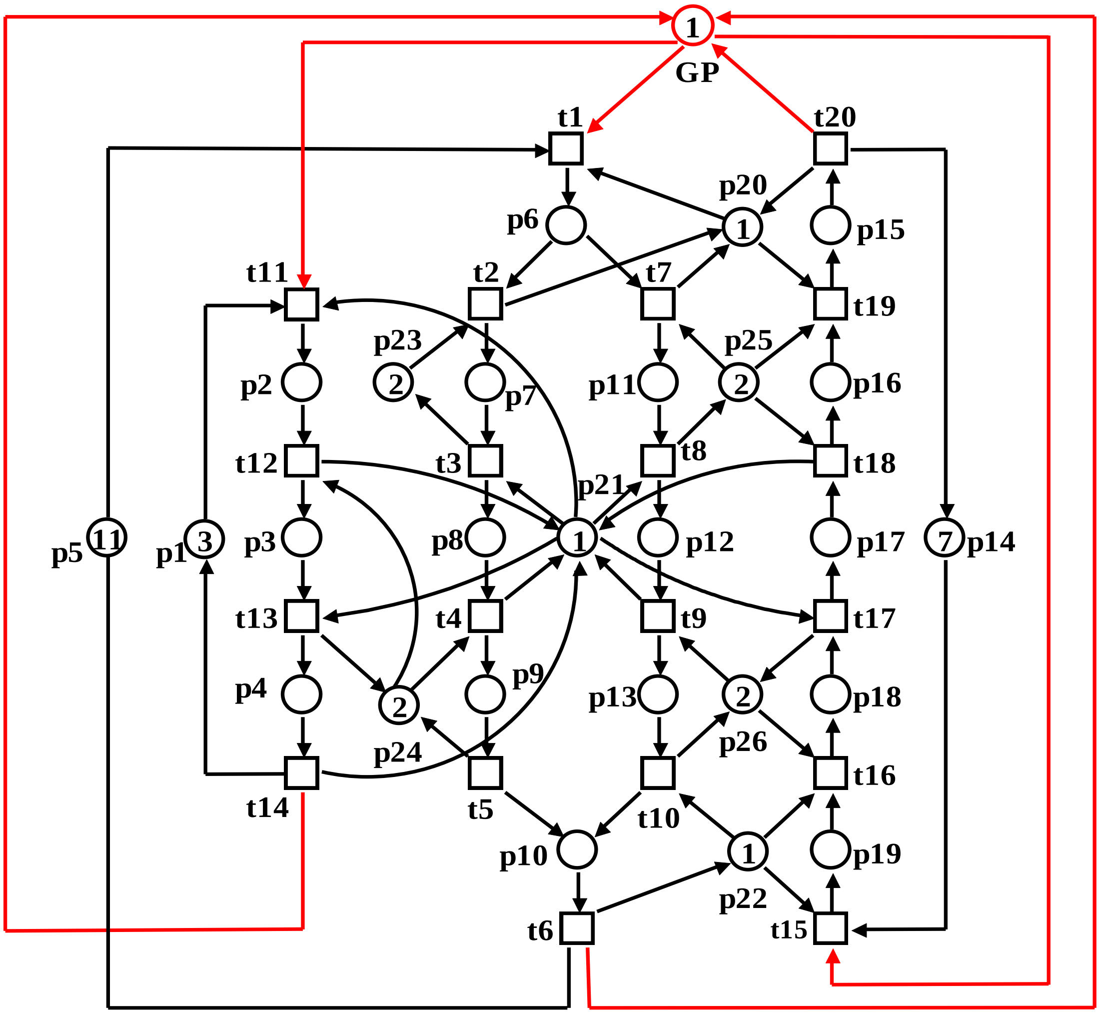

Figure 4.3. Live PNM$_1$ with 17 good states.

Step 2.2.1: (B = 2), when two tokens are deposited in the GP, as shown in Figure 4.4, the net PNM$_2$ is live with 132 good states. B: = B++ (B = 3).

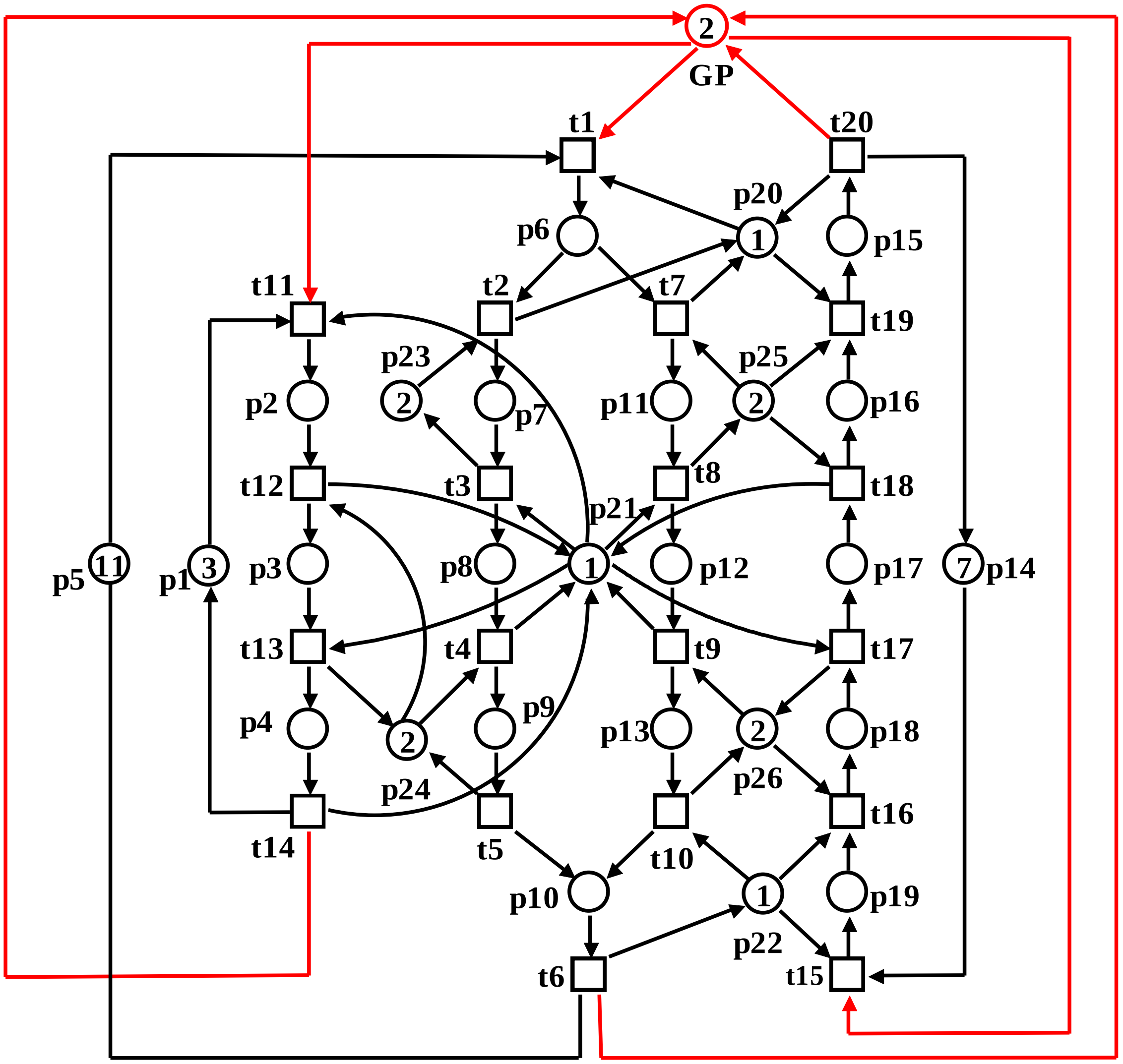

Figure 4.4. Live PNM$_2$ with 132 good states.

Step 2.3.1: (B = 3), when three tokens are deposited in the GP, as shown in Fig. 4.5, the net PNM$_3$ is not live. The reachability graph RG$_3$ computed for PNM$_3$ has 632 good states in the LZ$_3$ and 5 bad markings, BM$_2$, BM$_3$, BM$_4$ and BM$_5$ within the DZ$_3$.

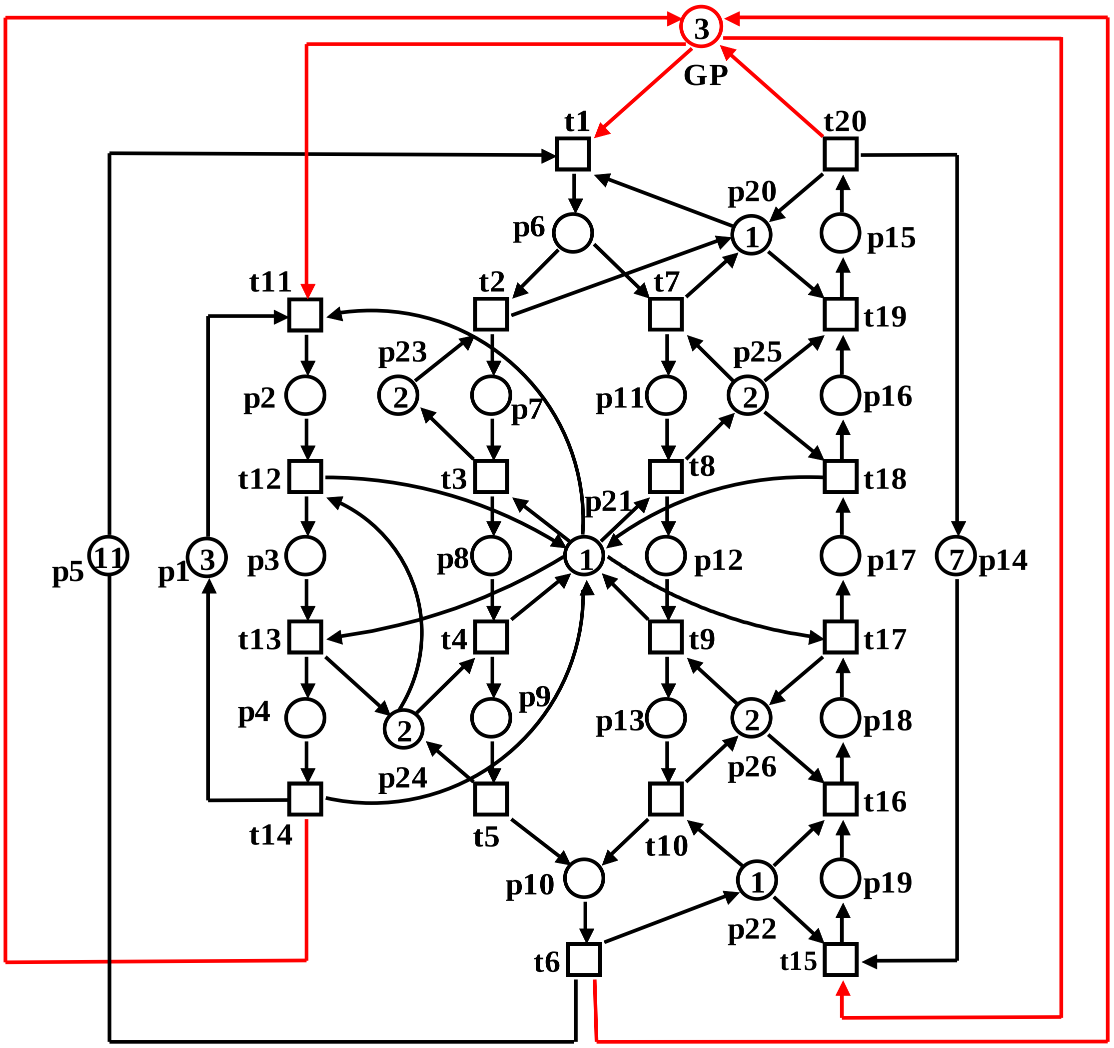

Figure 4.5.The PNM$_3$.

Step 2.3.2: The markings of activity places of BM$_1$, …, BM5 are shown in Table 4.1.

Table 4.1. The markings of the activity places of BM$_1$, …, BM5.

State nr.	p 2	p 3	p 4	p 6	p 7	p 8	p 9	p 10	p 11	p 12	p 13	p 15	p 16	p 17	p 18	p 19
279	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	1
292	0	0	0	0	0	0	0	0	2	0	0	0	0	1	0	0
360	0	0	0	0	0	0	0	0	0	1	0	0	0	0	2	0
425	0	2	0	0	0	1	0	0	0	0	0	0	0	0	0	0
433	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The place invariants PI$_1$, PI$_2$, PI$_3$, PI$_4$ and PI$_5$ for BM$_1$, BM$_2$, BM$_3$, BM$_4$ and BM$_5$ respectively are:

$PI_1 = \mu_{13} + \mu_{19} \leq 2$

$PI_2 = \mu_{11} + \mu_{17} \leq 2$

$PI_3 = \mu_{12} + \mu_{18} \leq 2$

$PI_4 = \mu_3 + \mu_8 \leq 2$

$PI_5 = \mu_2 + \mu_3 \leq 2$

Step 2.3.3: The computation of the monitors C_1, C_2, C_3, C_4 and C_5 are carried out as follows:

$$L_{PI1} = \begin{array}{cc} p13 & p19 \\ [1 & 1] \end{array}$$

$$D_{PI1} = \begin{array}{ccccc} t9 & t10 & t15 & t16 & \\ 1 & -1 & 0 & 0 & p13 \\ 0 & 0 & 1 & -1 & p19 \end{array}$$

$$D_{C1} = -L_{PI1} \,.\, D_{PI1} = -[1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$D_{C1} = -[1 \quad -1 \quad 1 \quad -1]$$

$$\text{Therefore, } D_{C1} = \begin{array}{cccc} t9 & t10 & t15 & t16 \\ [-1 & 1 & -1 & 1] \end{array}$$

$\mu_{0(c1)} = 2$

$$L_{PI2} = \begin{array}{cc} p11 & p17 \\ [1 & 1] \end{array}$$

$$D_{PI2} = \begin{array}{ccccc} t7 & t8 & t17 & t18 & \\ 1 & -1 & 0 & 0 & p11 \\ 0 & 0 & 1 & -1 & p17 \end{array}$$

$$D_{C2} = -L_{PI2} \,.\, D_{PI2} = -[1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$D_{C2} = -[1 \quad -1 \quad 1 \quad -1]$$

Therefore, $D_{C2} = \begin{array}{cccc} t7 & t8 & t17 & t18 \\ [-1 & 1 & -1 & 1] \end{array}$

$\mu_{0(c2)} = 2$

$L_{PI3} = \begin{array}{cc} p12 & p18 \\ [1 & 1] \end{array}$

$$D_{PI3} \begin{array}{cc} \begin{array}{cccc} t8 & t9 & t16 & t17 \end{array} & \\ \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} & \begin{array}{c} p12 \\ p18 \end{array} \end{array}$$

$$D_{C3} = - L_{PI3} \,.\, D_{PI3} = - [1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$D_{C3} = - [1 \quad -1 \quad 1 \quad -1]$

Therefore, $D_{C3} = \begin{array}{cccc} t8 & t9 & t16 & t17 \\ [-1 & 1 & -1 & 1] \end{array}$

$\mu_{0(c3)} = 2$

$L_{PI4} = \begin{array}{cc} p3 & p8 \\ [1 & 1] \end{array}$

$$D_{PI4} = \begin{array}{cc} \begin{array}{cccc} t12 & t13 & t3 & t4 \end{array} & \\ \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} & \begin{array}{c} p3 \\ p8 \end{array} \end{array}$$

$$D_{C4} = -L_{PI4} \,.\, D_{PI4} = - [1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$D_{C4} = - [1 \quad -1 \quad 1 \quad -1]$

Therefore, $D_{C4} = \begin{array}{cccc} t12 & t13 & t3 & t4 \\ [-1 & 1 & -1 & 1] \end{array}$

$\mu_{0(c4)} = 2$

$L_{PI5} = \begin{array}{cc} p2 & p3 \\ [1 & 1] \end{array}$

$$D_{PI5} = \begin{array}{cccc} t11 & t12 & t13 & \\ \left[\begin{matrix} 1 \\ 0 \end{matrix}\right. & \begin{matrix} -1 \\ 1 \end{matrix} & \left.\begin{matrix} 0 \\ -1 \end{matrix}\right] & \begin{matrix} p2 \\ p3 \end{matrix} \end{array}$$

$$D_{C5} = -L_{PI5} \,.\, D_{PI5} = -\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

$$D_{C3} = -\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

$$\text{Therefore, } D_{C5} = \begin{array}{ccc} t11 & t12 & t13 \\ [-1 & 0 & 1] \end{array}$$

$$\mu_{0(c5)} = 2$$

The computed monitors are shown in Table 4.2.

Table 4.2. Computed monitors C_1, C_2, C_3, C_4 and C_5.

C_i	$^\bullet C_i$	$C_i{}^\bullet$	$\mu_{0(ci)}$
C_1	t10, t16	t9, t15	2
C_2	t8, t18	t7, t17	2
C_3	t9, t17	t8, t16	2
C_4	t4, t13	t3, t12	2
C_5	t13	t11	2

Step 2.3.4: Redundancy test carried out shows that all five computed monitors are necessary.

Step 2.3.5: The controlled $PNM_3 := PNM_3 + C_1 + C_2 + C_3 + C_4 + C_5$ is shown in Fig. 4.6. It is live with 632 good states. This is the optimal live behaviour for the controlled PNM_3.

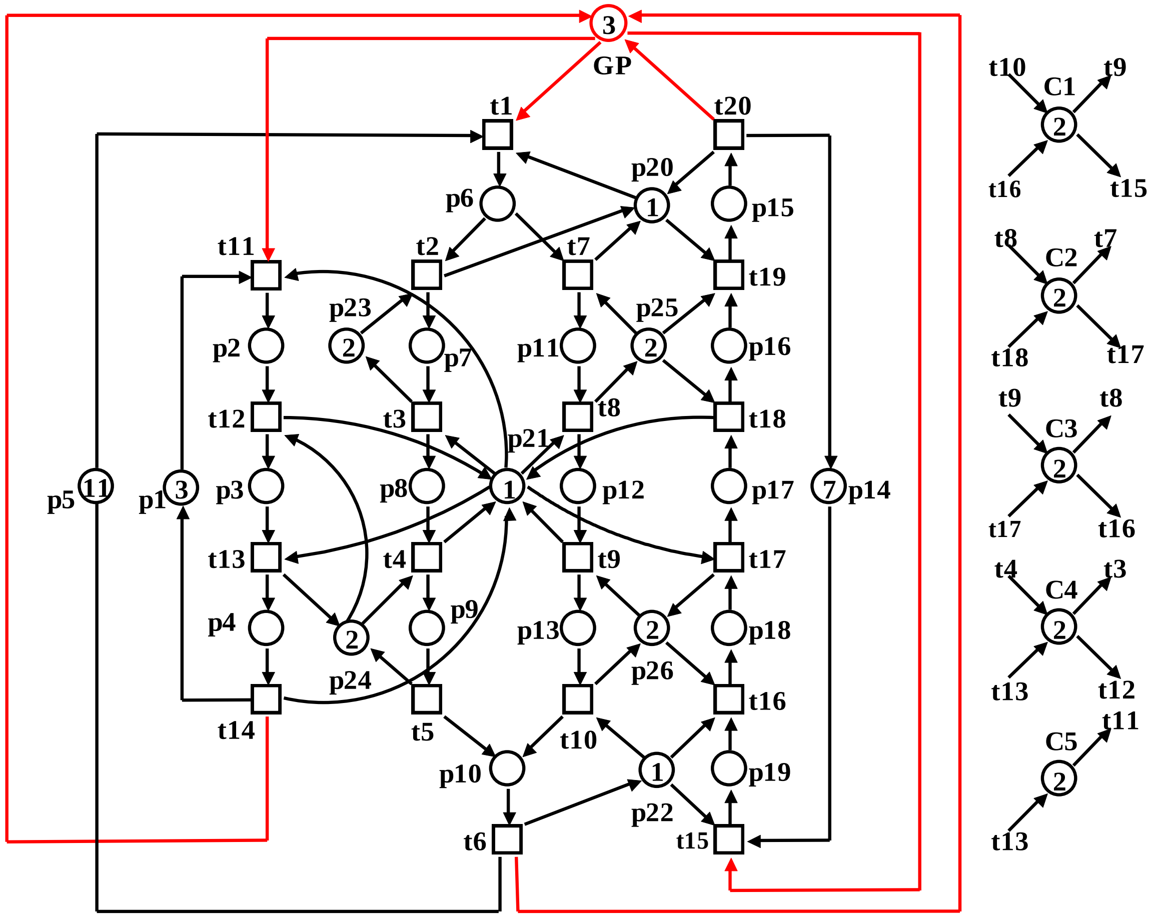

Figure 4.6. The controlled PNM$_3$:= PNM$_3$ + C$_1$ + C$_2$ +C$_3$ + C$_4$ + C$_5$.

B:= B ++ (B = 4).

Step 2.4.1: (B = 4), the PNM$_4$, shown in Fig. 4.7, is not live. The reachability graph RG$_4$ computed for the PNM$_4$ has 2,104 good states in the LZ$_4$ and 2 bad states BM$_6$ and BM$_7$ within the DZ$_4$.

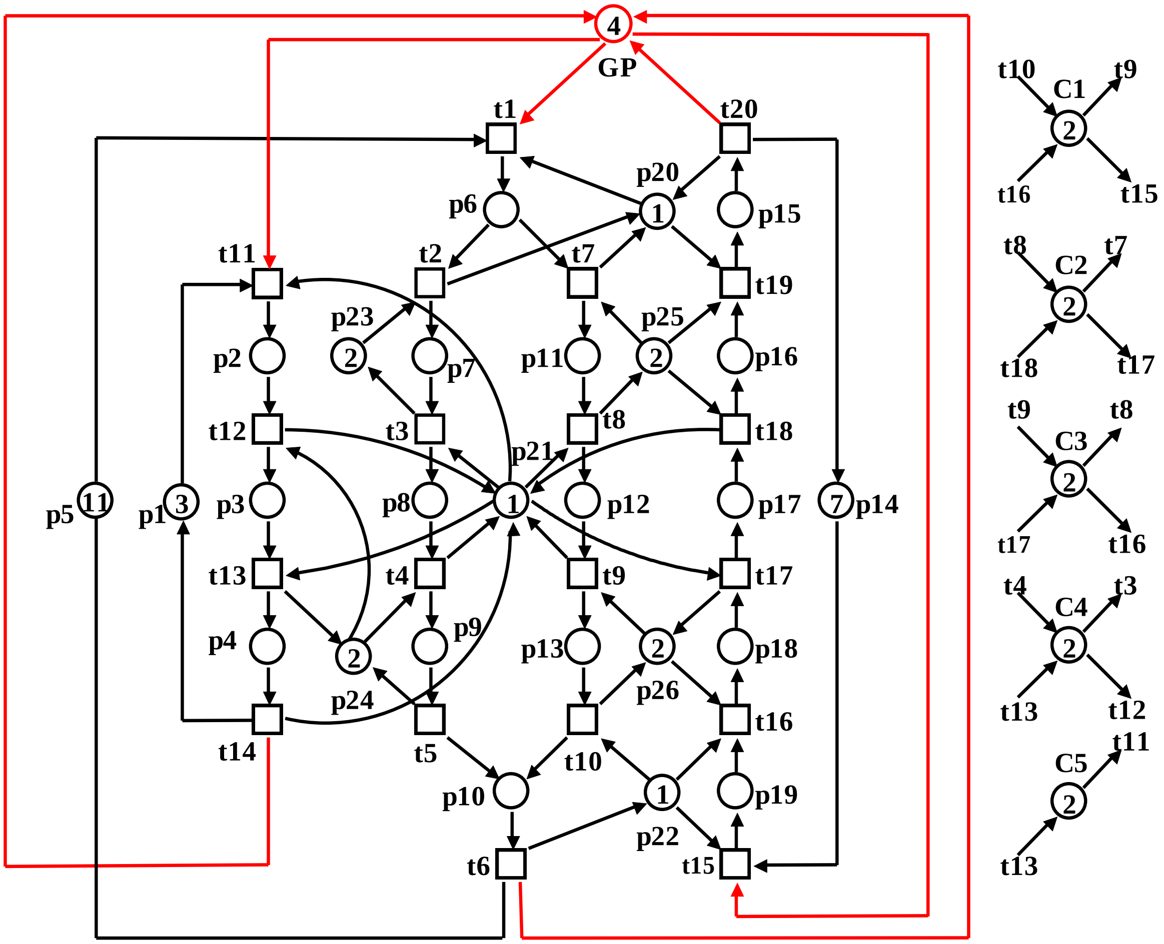

Figure 4.7. The uncontrolled PNM4.

Step 2.4.2: The markings of the activity places of BM$_6$ and BM$_7$ are shown in Table 4.3.

Table 4.3. The markings of the activity of BM$_6$ and BM$_7$.

State nr.	P 2	P 3	P 4	P 6	P 7	P 8	P 9	P 10	P 11	P 12	P 13	P 15	P 16	P 17	P 18	P 19
729	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1
1389	0	0	0	0	0	0	0	0	2	0	0	0	0	0	2	0

The place invariants PI$_6$ and PI$_7$ for BM$_6$ and BM$_7$ are:

$$PI_6 = \mu_{12} + \mu_{13} + \mu_{18} + \mu_{19} \leq 3$$

$$PI_7 = \mu_{11} + \mu_{18} \leq 3$$

Step 2.4.3: The computation of monitors C$_6$ and C$_7$ are carried out as follows:

$$L_{PI6} = \begin{array}{cccc} p12 & p13 & p18 & p19 \\ [1 & 1 & 1 & 1] \end{array}$$

$$D_{PI6} = \begin{array}{cc} & \begin{array}{cccccc} t8 & t9 & t10 & t15 & t16 & t17 \end{array} \\ & \left[\begin{array}{cccccc} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array}\right] \begin{array}{c} p12 \\ p13 \\ p18 \\ p19 \end{array} \end{array}$$

$$D_{C6} = -L_{PI6} \,.\, D_{PI6} = -[1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C6} = -[1 \quad 0 \quad -1 \quad 1 \quad 0 \quad -1]$$

$$\text{Therefore, } D_{C6} = \begin{array}{cccccc} t8 & t9 & t10 & t15 & t16 & t17 \\ [-1 & 0 & 1 & -1 & 0 & 1] \end{array}$$

$\mu_{0(c6)} = 3$

$$L_{PI7} = \begin{array}{cc} p11 & p18 \\ [1 & 1] \end{array}$$

$$D_{PI7} = \begin{array}{c} \begin{array}{cccc} t7 & t8 & t16 & t17 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{array}{c} p11 \\ p18 \end{array} \end{array}$$

$$D_{C7} = -L_{PI7} \,.\, D_{PI7} = -[1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$D_{C7} = -[1 \quad -1 \quad 1 \quad -1]$$

$$\text{Therefore, } D_{C7} = \begin{array}{cccc} t7 & t8 & t16 & t17 \\ [-1 & 1 & -1 & 1] \end{array}$$

$\mu_{0(c7)} = 3$

The computed monitors are shown in Table 4.4.

Table 4.4. Computed monitors C_6 and C_7

C_i	${}^{\bullet}C_i$	$C_i{}^{\bullet}$	$\mu_{0(ci)}$
C_6	t10, t17	t8, t15	3
C_7	t8, t17	t7, t16	3

Step 2.4.4: Redundancy test carried out shows that the two computed monitors are necessary.

Step 2.4.5: The controlled $PNM_4 = PNM_4 + C_6 + C_7$ is shown in Fig. 4.8. It is live with 2,104 good states. This is the optimal live behaviour for the controlled PNM_4.

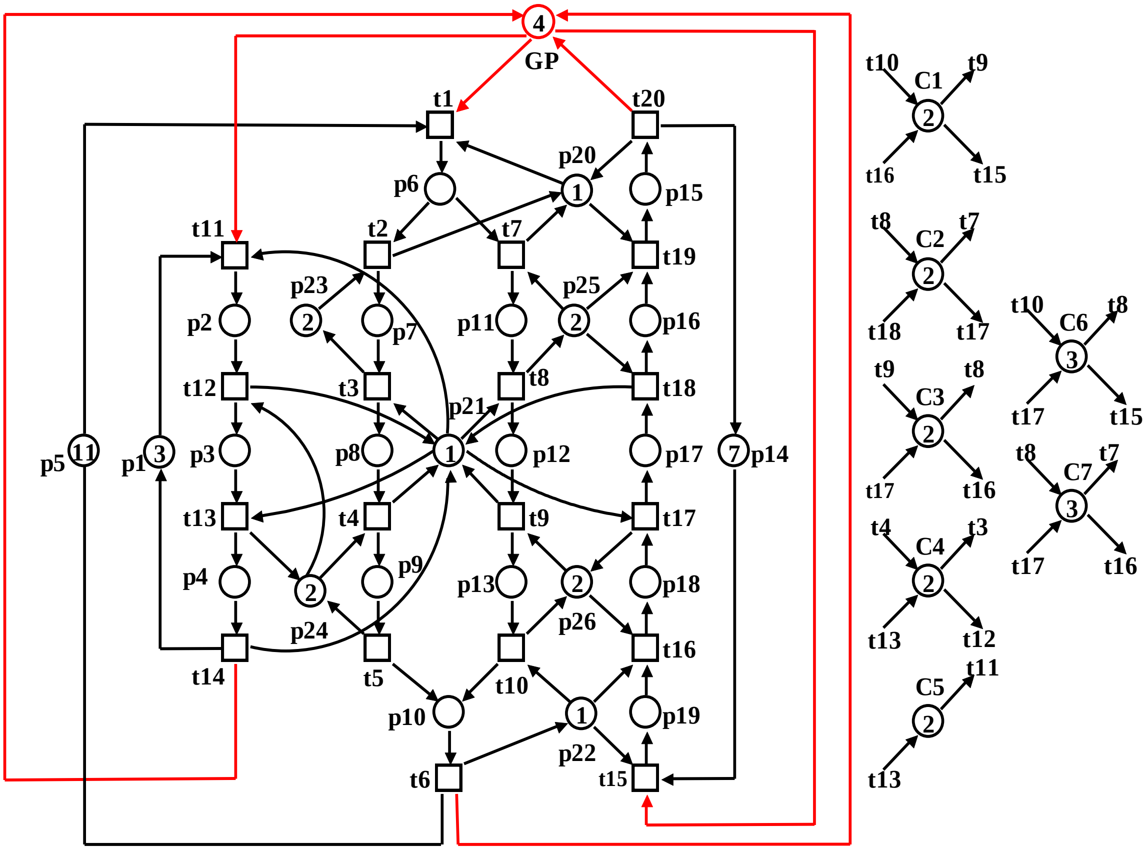

Figure 4.8. The controlled $PNM_4 := PNM_4 + C_6 + C_7$.

B:= B ++ (B = 4).

Step 2.5.1: (B = 5), the PNM_5, shown in Fig. 4.9, is not live. The reachability graph RG_5 computed for the PNM_5 has 5,190 good states in the LZ_5 and 2 bad states BM_8 and BM_9 within the DZ_5.

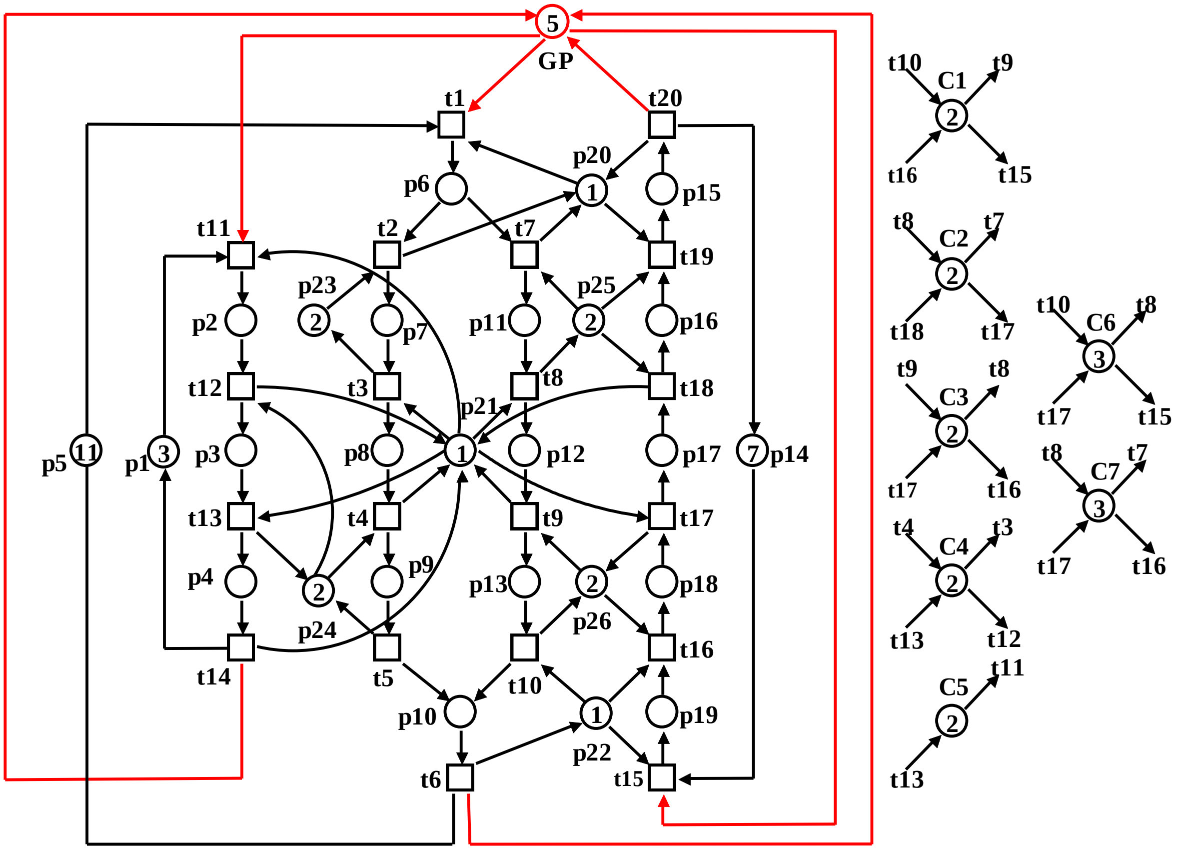

Figure 4.9. The uncontrolled model PNM$_5$.

Step 2.5.2: The markings of the activity places of BM$_8$ and BM$_9$ are shown in Table 4.5.

Table 4.5.The markings of the activity places of BM$_8$ and BM$_9$.

State nr.	p2	p3	p4	p6	p7	p8	p9	p10	p11	p12	p13	p15	p16	p17	p18	p19
1335	0	0	0	0	0	0	0	0	2	1	0	0	0	0	1	1
1334	0	0	0	0	0	0	0	0	2	0	1	0	0	0	1	1

The place invariants PI$_8$ and PI$_9$ for BM$_8$ and BM$_9$ respectively are:

$$PI_8 = \mu_{11} + \mu_{12} + \mu_{18} + \mu_{19} \leq 4$$

$$PI_9 = \mu_{11} + \mu_{13} + \mu_{18} + \mu_{19} \leq 4$$

Step 2.5.3: The computation of the monitors C$_8$ and C$_9$ are carried out as follows:

$$L_{PI8} = \begin{array}{cccc} p11 & p12 & p18 & p19 \\ [1 & 1 & 1 & 1] \end{array}$$

$$D_{PI8} = \begin{array}{c} \begin{array}{cccccc} t7 & t8 & t9 & t15 & t16 & t17 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ p11 \\ p12 \\ p18 \\ p19 \end{array}$$

$$D_{C8} = -L_{PI8} \,.\, D_{PI8} = -\,[1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C8} = -\,[1 \quad 0 \quad -1 \quad 1 \quad 0 \quad -1]$$

$$\text{Therefore, } D_{C8} = \begin{array}{cccccc} t7 & t8 & t9 & t15 & t16 & t17 \\ [-1 & 0 & 1 & -1 & 0 & 1] \end{array}$$

$\mu_{0(c8)} = 4$

$$L_{PI9} = \begin{array}{cccc} p11 & p13 & p18 & p19 \\ [1 & 1 & 1 & 1] \end{array}$$

$$D_{PI9} = \begin{array}{c} \begin{array}{ccccccc} t7 & t8 & t9 & t10 & t15 & t16 & t17 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ p11 \\ p13 \\ p18 \\ p19 \end{array}$$

$$D_{C9} = -\,L_{PI9} \,.\, D_{PI9} = -\,[1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C9} = -\,[1 \quad -1 \quad 1 \quad -1 \quad 1 \quad 0 \quad -1]$$

$$\text{Therefore, } D_{C9} = \begin{array}{ccccccc} t7 & t8 & t9 & t10 & t15 & t16 & t17 \\ [-1 & 1 & -1 & 1 & -1 & 0 & 1] \end{array}$$

$\mu_{0(c9)} = 4$

The computed monitors are shown in Table 4.6.

Table 4.6. Computed monitors C_8 and C_9.

C_i	$^\bullet C_i$	$C_i{}^\bullet$	$\mu_{0(ci)}$
C_8	t9, t17	t7, t15	4
C_9	t8, t10, t17	t7, t9, t15	4

Step 2.4.4: Redundancy test carried out shows that the two computed monitors are necessary.

Step 2.4.5: The controlled $PNM_5 = PNM_5 + C_8 + C_9$ is shown in Fig. 4.10. It is live with 5,190 good states. This is the optimal live behaviour for the controlled PNM_5.

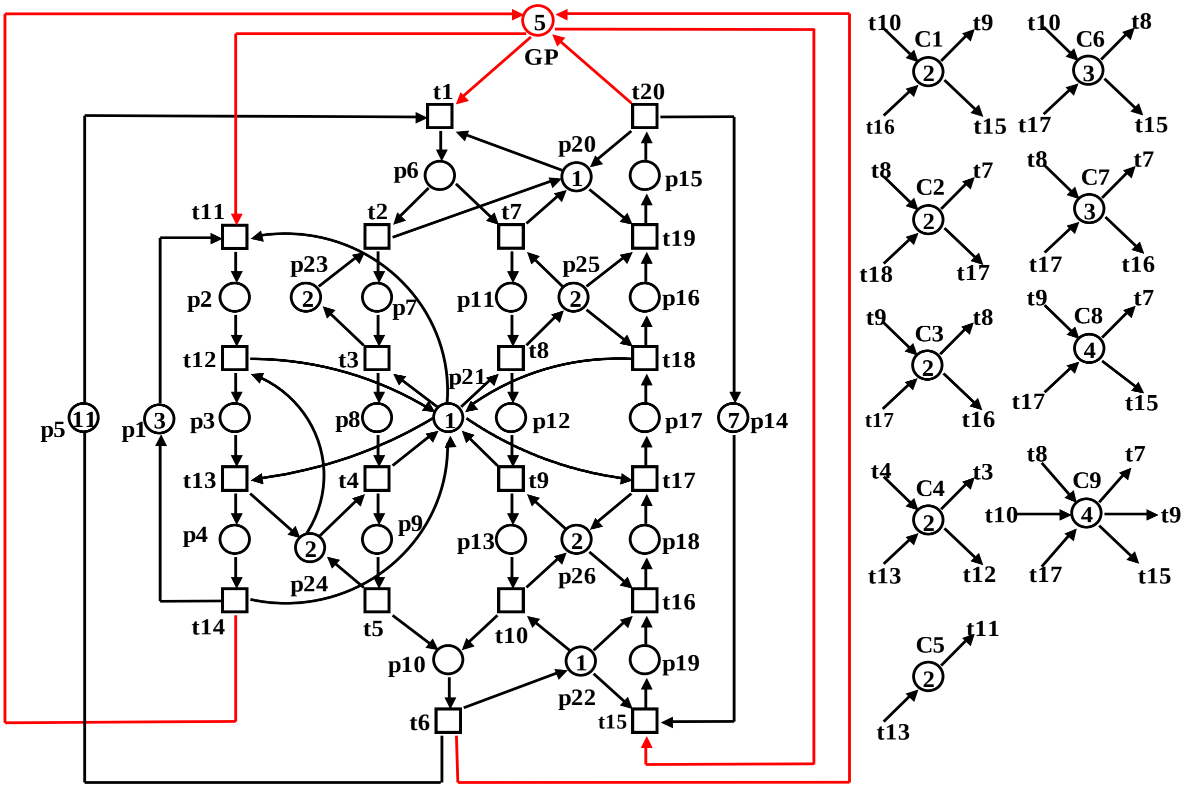

Figure 4.10. The controlled $PNM_5 := PNM_5 + C_8 + C_9$.

B:= B ++ (B = 6).

Step 2.6.1: (B = 6), The PNM_6, shown in Fig. 4.11, is not live. The reachability graph RG_6 computed for the PNM_6 has 9,878 good states in the LZ_6 and 10 bad markings

BM$_{10}$, BM$_{11}$, BM$_{12}$, BM$_{13}$, BM$_{14}$, BM$_{15}$, BM$_{16}$, BM$_{17}$, BM$_{18}$ and BM$_{19}$ within the DZ$_6$.

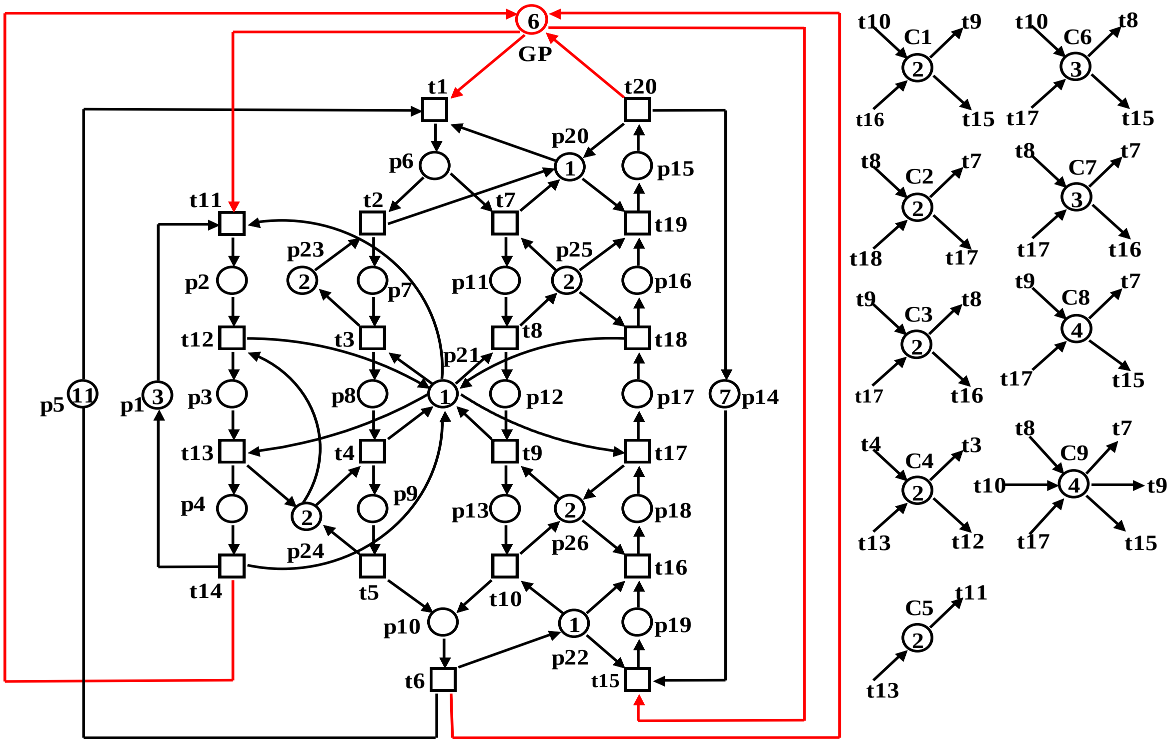

Figure 4.11. The PNM$_6$.

Step 2.6.2: The markings of the activity places of BM$_{10}$, …, BM$_{19}$ of the PNM$_6$ are shown in Table 4.7.

Table 4.7. The markings of the activity places of BM$_{10}$, …, BM$_{19}$.

State nr.	p2	p3	p4	p6	p7	p8	p9	p10	p11	p12	p13	p15	p16	p17	p18	p19
1141	0	1	0	0	0	1	1	0	0	0	0	0	0	0	2	1
1148	0	0	0	0	0	1	2	0	0	0	0	0	0	0	2	1
1661	0	0	0	1	2	0	0	0	1	0	0	0	1	1	0	0
1955	0	0	0	0	0	1	2	0	0	0	1	0	0	0	1	1
2027	0	1	0	0	0	1	1	0	0	0	1	0	0	0	1	1
2570	0	0	0	1	2	0	0	0	0	0	0	0	2	1	0	0
3273	1	1	0	0	0	0	1	0	0	0	1	0	0	0	1	1
3816	1	0	0	0	0	0	2	0	0	0	1	0	0	0	1	1
3974	1	1	0	0	0	0	1	0	0	0	0	0	0	0	2	1
9689	1	0	0	0	0	0	2	0	0	0	0	0	0	0	2	1

The place invariants PIs for the BMs respectively are:

PI$_{10}$ = $\mu_3 + \mu_8 + \mu_9 + \mu_{18} + \mu_{19} \leq 5$

$PI_{11} = \mu_8 + \mu_9 + \mu_{18} + \mu_{19} \leq 5$

$PI_{12} = \mu_6 + \mu_7 + \mu_{11} + \mu_{16} + \mu_{17} \leq 5$

$PI_{13} = \mu_8 + \mu_9 + \mu_{13} + \mu_{18} + \mu_{19} \leq 5$

$PI_{14} = \mu_3 + \mu_8 + \mu_9 + \mu_{13} + \mu_{18} + \mu_{19} \leq 5$

$PI_{15} = \mu_6 + \mu_7 + \mu_{16} + \mu_{17} \leq 5$

$PI_{16} = \mu_2 + \mu_3 + \mu_9 + \mu_{13} + \mu_{18} + \mu_{19} \leq 5$

$PI_{17} = \mu_2 + \mu_9 + \mu_{13} + \mu_{18} + \mu_{19} \leq 5$

$PI_{18} = \mu_2 + \mu_3 + \mu_9 + \mu_{18} + \mu_{19} \leq 5$

$PI_{19} = \mu_2 + \mu_9 + \mu_{18} + \mu_{19} \leq 5$

Step 2.6.3: The computation of the monitors C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, C_{15}, C_{16}, C_{17}, C_{18} and C_{19} are carried out as follows:

$$L_{PI10} = \begin{array}{ccccc} p3 & p8 & p9 & p18 & p19 \\ {[1} & 1 & 1 & 1 & 1] \end{array}$$

$$D_{PI10} = \begin{array}{c} \\ \left[\begin{array}{cccccccc} 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{array}\right] \end{array} \begin{array}{cccccccc|c} t3 & t4 & t5 & t12 & t13 & t15 & t16 & t17 & \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & p3 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & p8 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & p9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & p18 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & p19 \end{array}$$

$$D_{C10} = -L_{PI10} \,.\, D_{PI10} = -\,[1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C10} = -\,[1 \quad 0 \quad -1 \quad 1 \quad -1 \quad 1 \quad 0 \quad -1]$$

Therefore, $$D_{C10} = \begin{array}{cccccccc} t3 & t4 & t5 & t12 & t13 & t15 & t16 & t17 \\ {[-1} & 0 & 1 & -1 & 1 & -1 & 0 & 1] \end{array}$$

$\mu_{0(c10)} = 5.$

$$\begin{array}{cc} & \begin{array}{cccc} p8 & p9 & p18 & p19 \end{array} \\ L_{PI11} = & [\begin{array}{cccc} 1 & 1 & 1 & 1 \end{array}] \end{array}$$

$$D_{PI11} = \begin{array}{c} \begin{array}{cccccc} t3 & t4 & t5 & t15 & t16 & t17 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ p8 \\ p9 \\ p18 \\ p19 \end{array}$$

$$D_{C11} = - L_{PI11} \,.\, D_{PI11} = - [1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C11} = - [1 \quad 0 \quad -1 \quad 1 \quad 0 \quad -1]$$

$$\text{Therefore, } D_{C11} = \begin{array}{c} \begin{array}{cccccc} t3 & t4 & t5 & t15 & t16 & t17 \end{array} \\ [\begin{array}{cccccc} -1 & 0 & 1 & -1 & 0 & 1 \end{array}] \end{array}$$

$\mu_{0(c11)} = 5.$

$$\begin{array}{cc} & \begin{array}{ccccc} p6 & p7 & p11 & p16 & p17 \end{array} \\ L_{PI12} = & [\begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \end{array}] \end{array}$$

$$D_{PI12} = \begin{array}{c} \begin{array}{cccccccc} t1 & t2 & t3 & t7 & t8 & t17 & t18 & t19 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ p6 \\ p7 \\ p11 \\ p16 \\ p17 \end{array}$$

$$D_{C12} = - L_{PI12} \,.\, D_{PI12} = - [1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C12} = - [1 \quad 0 \quad -1 \quad 0 \quad -1 \quad 1 \quad 0 \quad -1]$$

$$\text{Therefore, } D_{C12} = \begin{array}{c} \begin{array}{cccccccc} t3 & t4 & t5 & t12 & t13 & t15 & t16 & t17 \end{array} \\ [\begin{array}{cccccccc} -1 & 0 & 1 & 0 & 1 & -1 & 0 & 1 \end{array}] \end{array}$$

$\mu_{0(c12)} = 5.$

$$L_{PI13} = \begin{array}{c} \begin{array}{ccccc} p8 & p9 & p13 & p18 & p19 \end{array} \\ \left[\begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \end{array}\right] \end{array}$$

$$D_{PI13} = \begin{array}{cc} \begin{array}{cccccccc} t3 & t4 & t5 & t9 & t10 & t15 & t16 & t17 \end{array} & \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} & \begin{array}{c} p8 \\ p9 \\ p13 \\ p18 \\ p19 \end{array} \end{array}$$

$$D_{C13} = -\,L_{PI13} \,.\, D_{PI13} = -\,[1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C13} = -\,[1 \quad 0 \quad -1 \quad 1 \quad -1 \quad 1 \quad 0 \quad -1]$$

$$\text{Therefore, } D_{C13} = \begin{array}{c} \begin{array}{cccccccc} t3 & t4 & t5 & t9 & t10 & t15 & t16 & t17 \end{array} \\ \left[\begin{array}{cccccccc} -1 & 0 & 1 & -1 & 1 & -1 & 0 & 1 \end{array}\right] \end{array}$$

$\mu_{0(c13)} = 5.$

$$L_{PI14} = \begin{array}{c} \begin{array}{cccccc} p3 & p8 & p9 & p13 & p18 & p19 \end{array} \\ \left[\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right] \end{array}$$

$$D_{PI14} = \begin{array}{cc} \begin{array}{cccccccccc} t3 & t4 & t5 & t9 & t10 & t11 & t12 & t15 & t16 & t17 \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} & \begin{array}{c} p2 \\ p8 \\ p9 \\ p13 \\ p18 \\ p19 \end{array} \end{array}$$

$$D_{C14} = -L_{PI14} \,.\, D_{PI14} = -\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C14} = -\begin{bmatrix} 1 & 0 & -1 & 1 & -1 & 1 & -1 & 1 & 0 & -1 \end{bmatrix}$$

$$\text{Therefore, } D_{C14} = \begin{array}{c} \begin{array}{cccccccccc} t3 & t4 & t5 & t9 & t10 & t11 & t12 & t15 & t16 & t17 \end{array} \\ \begin{bmatrix} -1 & 0 & 1 & -1 & 1 & -1 & 1 & -1 & 0 & 1 \end{bmatrix} \end{array}$$

$\mu_{0(c14)} = 5$.

$$L_{PI15} = \begin{array}{c} \begin{array}{cccc} p6 & p7 & p16 & p17 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{PI15} = \begin{array}{cccccccc} t1 & t2 & t3 & t7 & t17 & t18 & t19 & \\ 1 & -1 & 0 & -1 & 0 & 0 & 0 & p6 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & p7 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & p16 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & p17 \end{array}$$

$$D_{C15} = -L_{PI15} \,.\, D_{PI15} = -\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C15} = -\begin{bmatrix} 1 & 0 & -1 & -1 & 1 & 0 & -1 \end{bmatrix}$$

$$\text{Therefore, } D_{C15} = \begin{array}{c} \begin{array}{ccccccc} t1 & t2 & t3 & t7 & t17 & t18 & t19 \end{array} \\ \begin{bmatrix} -1 & 0 & 1 & 1 & -1 & 0 & 1 \end{bmatrix} \end{array}$$

$\mu_{0(c15)} = 5$.

$$L_{PI16} = \begin{array}{c} \begin{array}{cccccc} p2 & p3 & p9 & p13 & p18 & p19 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{PI16} = \begin{array}{cccccccccc|c} t4 & t5 & t9 & t10 & t11 & t12 & t13 & t15 & t16 & t17 & \\ \hline 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & p2 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & p3 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p9 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & p13 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & p18 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & p19 \end{array}$$

$$D_{C16} = -L_{PI16} \,.\, D_{PI16} = -\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C16} = -\begin{bmatrix} 1 & -1 & 1 & -1 & 1 & 0 & -1 & 1 & 0 & -1 \end{bmatrix}$$

Therefore, $D_{C16} = \begin{array}{cccccccccc} t4 & t5 & t9 & t10 & t11 & t12 & t13 & t15 & t16 & t17 \\ [-1 & 1 & -1 & 1 & -1 & 0 & 1 & -1 & 0 & 1] \end{array}$

$\mu_{0(c16)} = 5.$

$$L_{PI17} = \begin{array}{ccccc} p2 & p9 & p13 & p18 & p19 \\ [1 & 1 & 1 & 1 & 1] \end{array}$$

$$D_{PI17} = \begin{array}{ccccccccc|c} t4 & t5 & t9 & t10 & t11 & t12 & t15 & t16 & t17 & \\ \hline 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & p2 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p9 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & p13 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & p18 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & p19 \end{array}$$

$$D_{C17} = -L_{PI17} \,.\, D_{PI17} = -\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C17} = -\begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & 0 & -1 \end{bmatrix}$$

$$\text{Therefore, } D_{C17} = \begin{array}{ccccccccc} t4 & t5 & t9 & t10 & t11 & t12 & t15 & t16 & t17 \\ [-1 & 1 & -1 & 1 & -1 & 1 & -1 & 0 & 1] \end{array}$$

$\mu_{0(c17)} = 5.$

$$L_{PI18} = \begin{array}{ccccc} p2 & p3 & p9 & p18 & p19 \\ [1 & 1 & 1 & 1 & 1] \end{array}$$

$$D_{PI18} = \begin{array}{cccccccc|c} t4 & t5 & t11 & t12 & t13 & t15 & t16 & t17 & \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & p2 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & p3 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & p9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & p18 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & p19 \end{array}$$

$$D_{C18} = -\ L_{PI18} \,.\, D_{PI18} = -\ [1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C18} = -\ [1 \quad -1 \quad 1 \quad 0 \quad -1 \quad 1 \quad 0 \quad -1]$$

$$\text{Therefore, } D_{C18} = \begin{array}{cccccccc} t4 & t5 & t11 & t12 & t13 & t15 & t16 & t17 \\ [-1 & 1 & -1 & 0 & 1 & -1 & 0 & 1] \end{array}$$

$\mu_{0(c18)} = 5.$

$$L_{PI19} = \begin{array}{cccc} p2 & p9 & p18 & p19 \\ [1 & 1 & 1 & 1] \end{array}$$

$$D_{PI19} = \begin{array}{ccccccc|c} t4 & t5 & t11 & t12 & t15 & t16 & t17 & \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & p2 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & p9 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & p18 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & p19 \end{array}$$

$$D_{C19} = - L_{PI19} \,.\, D_{PI19} = - \begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C19} = - \begin{bmatrix}1 & -1 & 1 & -1 & 1 & 0 & -1\end{bmatrix}$$

$$\text{Therefore,}\;\; D_{C19} = \begin{array}{c} \begin{array}{ccccccc} t4 & t5 & t11 & t12 & t15 & t16 & t17 \end{array} \\ \begin{bmatrix} -1 & 1 & -1 & 1 & -1 & 0 & 1 \end{bmatrix} \end{array}$$

$\mu_{0(c19)} = 5$.

Step 2.6.4: Redundancy test carried out shows that only three of the computed monitors are necessary, C_{12}, C_{14} and C_{16}. The remaining seven are redundant and are therefore removed. The necessary monitors are also renumbered in order to follow the regular sequence of numbering for convenience. Thus C_{11} becomes C_{10}, C_{13} becomes C_{14} and C_{16} becomes C_{12}.

The necessary monitors C_{10}, C_{11}, and C_{12} are shown in Table 4.8.

Table 4.8. Necessary monitors C_{10}, C_{11} and C_{12}.

C_i	$^{\bullet}C_i$	$C_i{}^{\bullet}$	$\mu_{0(ci)}$
C_{10}	t3, t8, t19	t1, t17	5
C_{11}	t5, t10, t13, t17	t3, t9, t12, t15	5
C_{12}	t5, t10, t13, t17	t4, t9, t11, t15	5

Step 2.6.5: The controlled $PNM_6 := PNM_6 + C_{10} + C_{11} + C_{12}$ is shown in Fig. 4.12. It is live with 9,878 good states. This is the optimal live behaviour for the controlled PNM_6.

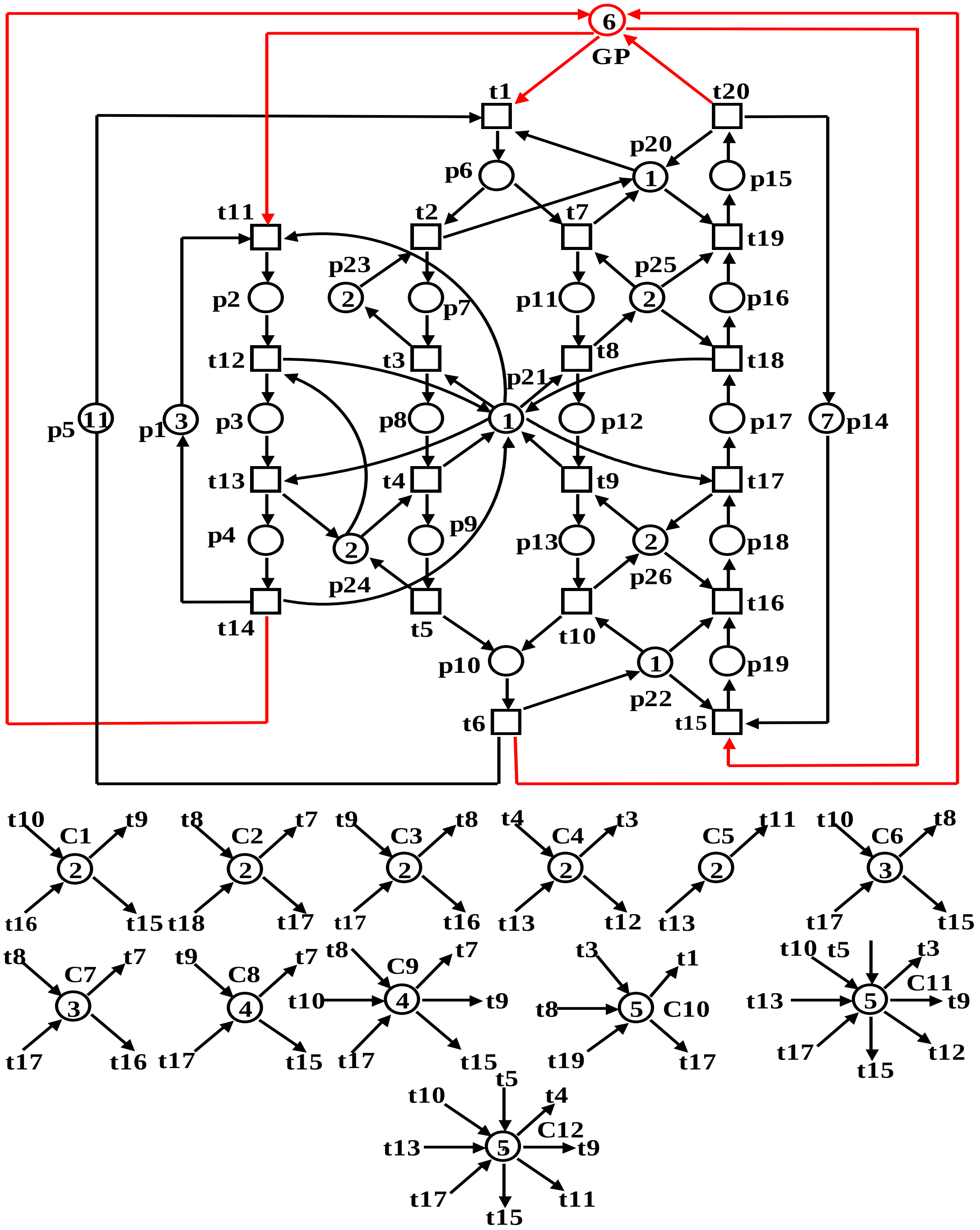

Figure 4.12. The controlled PNM$_6$:= PNM$_6$ + C_{10} + C_{11} + C_{12}.

B:= B ++ (B = 7).

Step 2.7.1: (B = 7), The PNM$_7$, shown in Figure 4.13, is not live. The reachability graph RG$_7$ computed for the PNM$_7$ has 15,013 good states in the LZ$_7$ and 4 bad states BM$_{13}$, BM$_{14}$, BM$_{15}$ and BM$_{16}$ within the DZ$_7$.

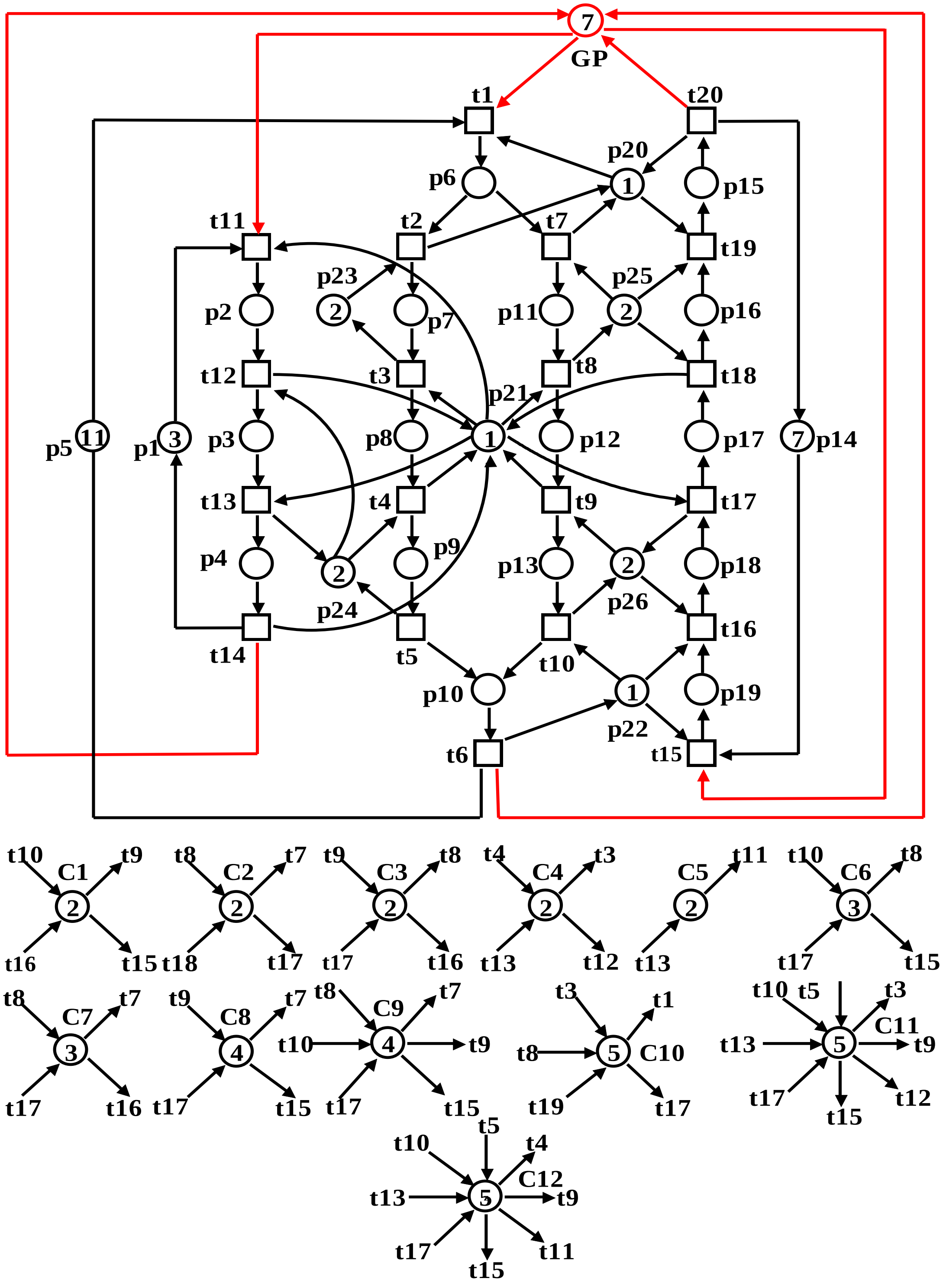

Figure 4.13. The PNM$_7$.

Step 2.7.2: The markings of the activity places of BM$_{13}$, …, BM$_{16}$ are shown in Table 4.9.

Table 4.9. The markings of the activity places of BM_{13} ,…, BM_{16}.

State nr.	p 2	p 3	p 4	p 6	p 7	p 8	p 9	p 10	p 11	p 12	p 13	p 15	p 16	p 17	p 18	p 19
168	0	0	0	0	0	1	2	0	2	0	0	0	0	0	1	1
896	0	1	0	0	0	1	1	0	2	0	0	0	0	0	1	1
3116	1	0	0	0	0	0	2	0	2	0	0	0	0	0	1	1
4337	1	1	0	0	0	0	1	0	2	0	0	0	0	0	1	1

The place invariants $PI_{13,}$ PI_{14}, PI_{15} and PI_{16} for BM_{13}, BM_{14}, BM_{15} and BM_{16} respectively are:

$$PI_{13} = \mu_8 + \mu_9 + \mu_{11} + \mu_{18} + \mu_{19} \leq 6$$

$$PI_{14} = \mu_3 + \mu_8 + \mu_9 + \mu_{11} + \mu_{18} + \mu_{19} \leq 6$$

$$PI_{15} = \mu_2 + \mu_9 + \mu_{11} + \mu_{18} + \mu_{19} \leq 6$$

$$PI_{16} = \mu_2 + \mu_3 + \mu_9 + \mu_{11} + \mu_{18} + \mu_{19} \leq 6$$

Step 2.7.3: The computation of the monitors C_{13}, C_{14}, C_{15} and C_{16} are carried out as follows:

$$L_{PI13} = \begin{array}{ccccc} p8 & p9 & p11 & p18 & p19 \\ [1 & 1 & 1 & 1 & 1] \end{array}$$

$$D_{PI13} = \begin{array}{cccccccccc} & t3 & t4 & t5 & t7 & t8 & t15 & t16 & t17 & \\ \left[\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\right. & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \left.\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\right] \end{array}$$

$$D_{PI13} = \begin{array}{c|cccccccc|c} & t3 & t4 & t5 & t7 & t8 & t15 & t16 & t17 & \\ & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & p8 \\ & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & p9 \\ & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & p11 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & p18 \\ & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & p19 \end{array}$$

$$D_{C13} = -L_{PI13} \, . \, D_{PI13} = -[1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C13} = -[1 \quad 0 \quad -1 \quad 1 \quad -1 \quad 1 \quad 0 \quad -1 \,]$$

Therefore, $D_{C13} = \begin{array}{cccccccc} t3 & t4 & t5 & t7 & t8 & t15 & t16 & t17 \\ [-1 & 0 & 1 & -1 & 1 & -1 & 0 & 1] \end{array}$

$\mu_{0(c13)} = 6$

$$L_{PI14} = \begin{array}{cccccc} p3 & p8 & p9 & p11 & p18 & p19 \\ \left[1\right. & 1 & 1 & 1 & 1 & \left.1\right] \end{array}$$

$$D_{PI14} = \begin{array}{c} \\ \end{array}\begin{array}{cccccccccc|c} t3 & t4 & t5 & t7 & t8 & t12 & t13 & t15 & t16 & t17 & \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & p3 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p8 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p9 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & p11 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & p18 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & p19 \end{array}$$

$$D_{C14} = -L_{PI14} \,.\, D_{PI14} = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C14} = -[1 \quad 0 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad 0 \quad -1\,]$$

Therefore, $$D_{C14} = \begin{array}{cccccccccc} t3 & t4 & t5 & t7 & t8 & t12 & t13 & t15 & t16 & t17 \\ \left[-1\right. & 0 & 1 & -1 & 1 & -1 & 1 & -1 & 0 & \left.1\right] \end{array}$$

$\mu_{0(c14)} = 6$

$$L_{PI15} = \begin{array}{ccccc} p2 & p9 & p11 & p18 & p19 \\ \left[1\right. & 1 & 1 & 1 & \left.1\right] \end{array}$$

$$D_{PI15} = \begin{array}{ccccccccc|c} t4 & t5 & t7 & t8 & t11 & t12 & t15 & t16 & t17 & \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & p2 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p9 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & p11 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & p18 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & p19 \end{array}$$

$$D_{C15} = -\,L_{PI15} \,.\, D_{PI15} = -\,[1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C15} = -[1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad 0 \quad -1]$$

Therefore, $$D_{C15} = \begin{array}{cccccccccc} & t4 & t5 & t7 & t8 & t11 & t12 & t15 & t16 & t17 \\ [& -1 & 1 & -1 & 1 & -1 & 1 & -1 & 0 & 1] \end{array}$$

$\mu_{0(c15)} = 6$

$$L_{PI16} = \begin{array}{ccccccc} & p2 & p3 & p9 & p11 & p18 & p19 \\ [& 1 & 1 & 1 & 1 & 1 & 1] \end{array}$$

$$D_{C16} = \begin{array}{ccccccccccc} t4 & t5 & t7 & t8 & t11 & t12 & t13 & t15 & t16 & t17 & \\ \left[\begin{matrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{matrix}\right. & \begin{matrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{matrix} & \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{matrix} & \begin{matrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{matrix} & \begin{matrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} & \begin{matrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} & \begin{matrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{matrix} & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{matrix} & \left.\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{matrix}\right] & \begin{matrix} p2 \\ p3 \\ p9 \\ p11 \\ p18 \\ p19 \end{matrix} \end{array}$$

$$D_{C16} = -L_{PI16} \, . \, D_{PI16} = -[1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$D_{C16} = -[1 \quad -1 \quad 1 \quad -1 \quad 1 \quad 0 \quad -1 \quad 1 \quad 0 \quad -1]$$

Therefore, $$D_{C16} = \begin{array}{ccccccccccc} & t4 & t5 & t7 & t8 & t11 & t12 & t13 & t15 & t16 & t17 \\ [& -1 & 1 & -1 & 1 & -1 & 0 & 1 & -1 & 0 & 1] \end{array}$$

$\mu_{0(c16)} = 6$

Step 2.7.4: Redundancy test carried out shows that only three of the computed monitors are necessary, C_{14}, C_{15} and C_{16} while C_{13} is redundant and is therefore removed. The necessary monitors are also renumbered in order to follow the regular sequence of numbering for convenience. Thus C_{14} becomes C_{13}, C_{15} becomes C_{14} and C_{16} becomes C_{15}.

The computed necessary monitors C_{13}, C_{14} and C_{15} are shown in Table 4.10.

Table 4.10. Necessary monitors C_{13}, C_{14} and C_{15}.

C_i	$^\bullet C_i$	$C_i{}^\bullet$	$\mu_{0(ci)}$
C_{13}	t5, t8, t13, t17	t3, t7, t12, t15	6
C_{14}	t5, t8, t12, t17	t4, t7, t11, t15	6
C_{15}	t5, t8, t13, t17	t4, t7, t11, t15	6

Step 2.7.5: The controlled PNM_7 := PNM_7 + C_{13} + C_{14} + C_{15} is shown in Fig. 4.14. It is live with 15,013 good states. This is the optimal live behaviour for the controlled PNM_7.

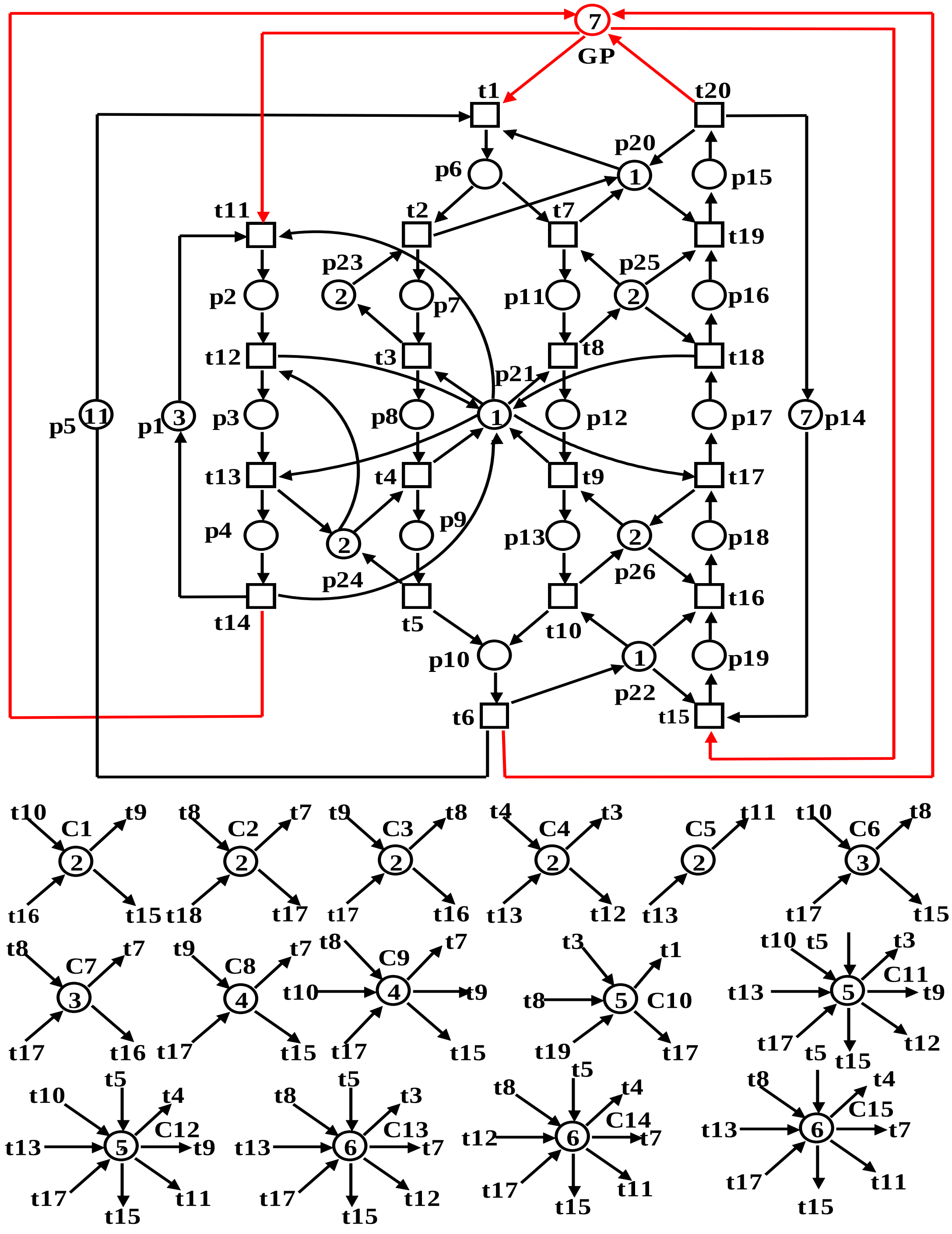

Figure 4.14. The controlled PNM7:= PNM7 + C13 + C14 + C15.

B:= B ++ (B = 8).

Step 2.8.1: (B = 8), when eight tokens are deposited in the GP, as shown in Fig. 4.15, the net PNM8 is live with 18,972 good states. B: = B++ (B = 9).

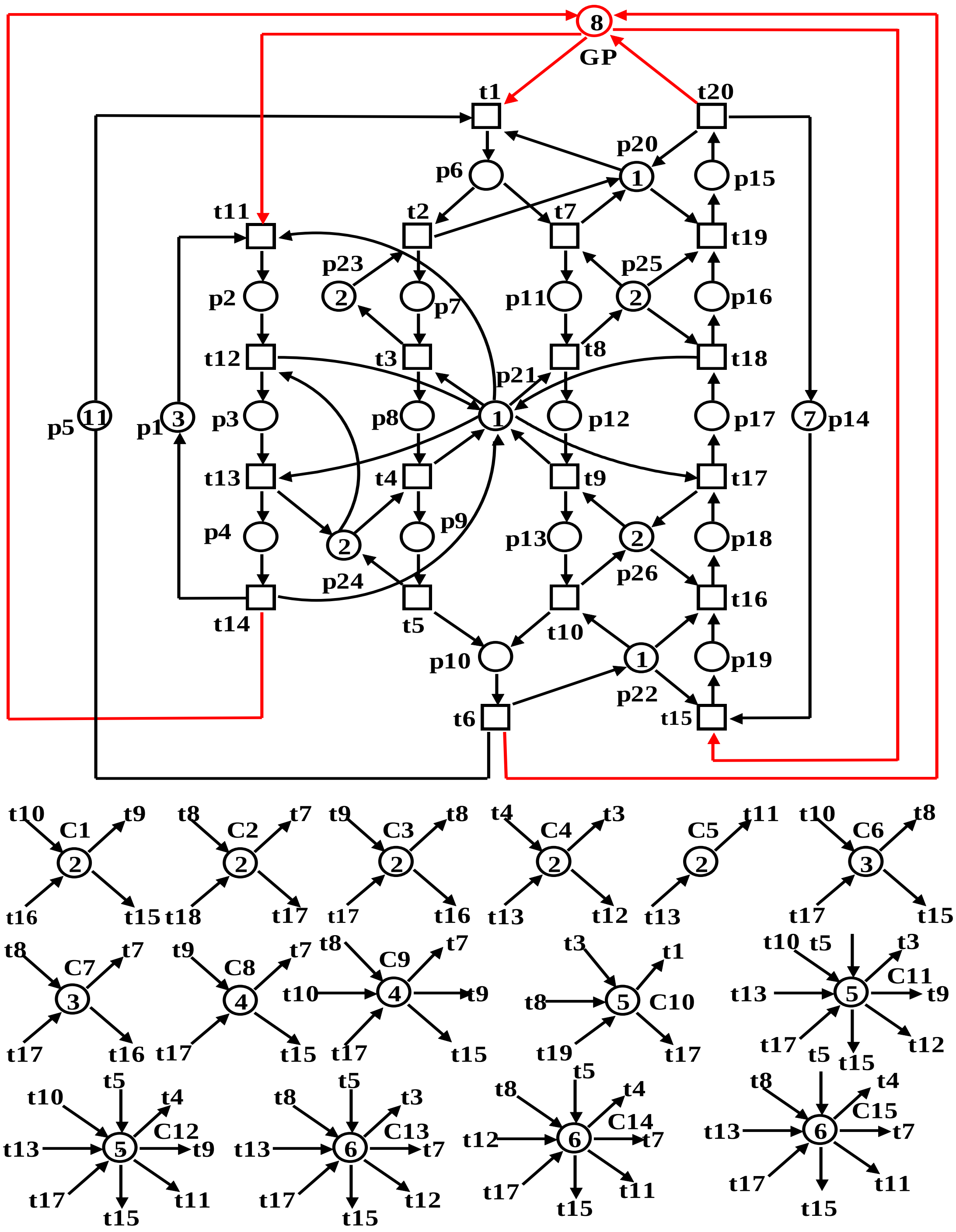

Figure 4.15. The controlled PNM8.

Step 2.9.1: (B = 9), when nine tokens are deposited in the GP, as shown in Fig. 4.16, the net PNM9 is live with 20,980 good states. B: = B++ (B = 10).

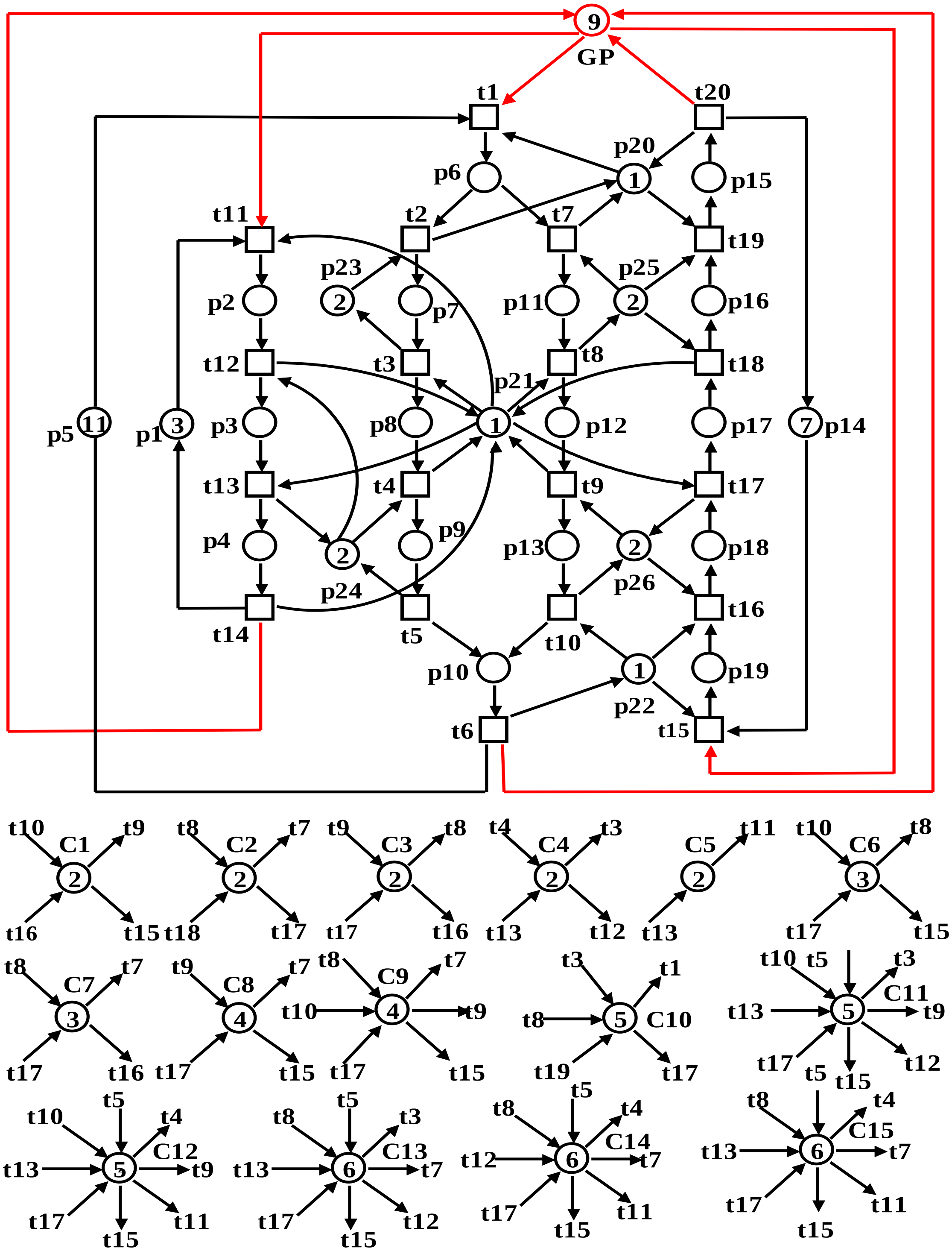

Figure 4.16. The controlled PNM$_9$.

Step 2.10.1: (B = 10) The PNM$_{10}$, shown in Fig. 4.17, is not live. The reachability graph RG$_{10}$ computed for the PNM$_{10}$ has 21,536 good states in the LZ$_{10}$ and 11 bad states

BM_{16}, BM_{17}, BM_{18}, BM_{19}, BM_{20}, BM_{21}, BM_{22}, BM_{23}, BM_{24}, BM_{25} and BM_{26} within the DZ_{10}.

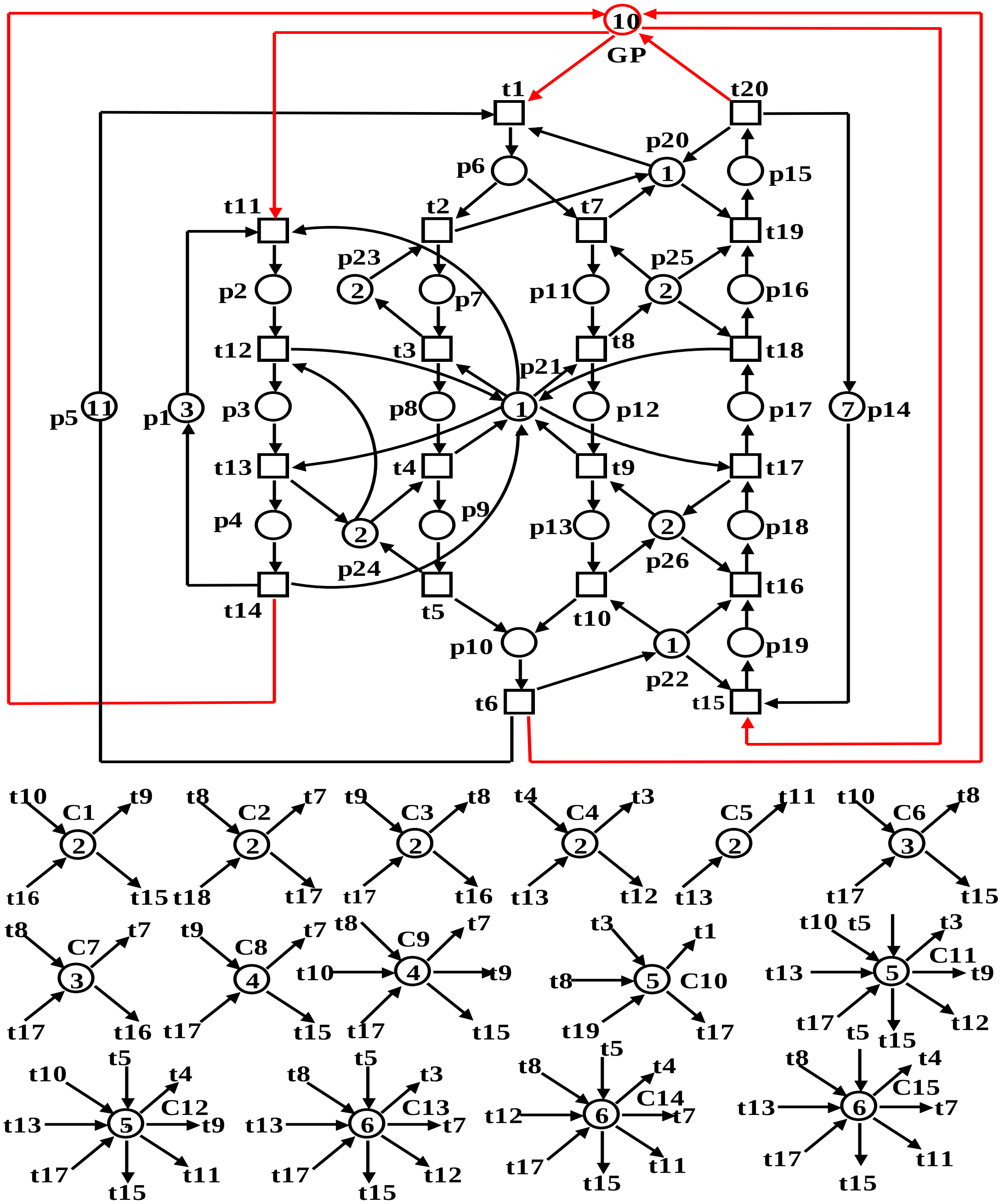

Figure 4.17. The PNM_{10}.

Step 2.7.2: The markings of the activity places of BM_{16}, …, BM_{26} are shown in Table 4.11.

Table 4.11. The markings of the activity places of BM$_{16}$, …, BM$_{26}$.

State nr.	p 2	p 3	p 4	p 6	p 7	p 8	p 9	p 10	p 11	p 12	p 13	p 15	p 16	p 17	p 18	p 19
273	0	0	0	1	2	1	1	0	1	0	0	0	1	0	2	1
274	0	0	0	1	2	0	2	0	1	0	0	0	1	0	2	1
770	0	0	0	1	2	1	1	0	1	0	1	0	1	0	1	1
771	0	0	0	1	2	0	2	0	1	0	1	0	1	1	1	1
893	0	0	0	1	2	1	1	0	0	0	0	0	2	0	2	1
894	0	0	0	1	2	0	2	0	0	0	0	0	2	0	2	1
1431	0	0	0	1	2	0	2	0	1	1	0	0	1	0	1	1
1603	0	0	0	1	2	0	2	0	1	0	0	0	0	1	2	1
4274	0	0	0	1	2	1	1	0	0	0	1	0	2	0	1	1
4275	0	0	0	1	2	0	2	0	0	0	1	0	2	0	1	1
12161	0	0	0	1	2	0	2	0	1	0	1	0	0	1	1	1

The place invariants PIs for the BMs respectively are:

$$PI_{16} = \mu_6 + \mu_7 + \mu_8 + \mu_9 + \mu_{11} + \mu_{16} + \mu_{18} + \mu_{19} \leq 9$$

$$PI_{17} = \mu_6 + \mu_7 + \mu_9 + \mu_{11} + \mu_{16} + \mu_{18} + \mu_{19} \leq 9$$

$$PI_{18} = \mu_6 + \mu_7 + \mu_8 + \mu_9 + \mu_{11} + \mu_{13} + \mu_{16} + \mu_{18} + \mu_{19} \leq 9$$

$$PI_{19} = \mu_6 + \mu_7 + \mu_9 + \mu_{11} + \mu_{13} + \mu_{16} + \mu_{18} + \mu_{19} \leq 9$$

$$PI_{20} = \mu_6 + \mu_7 + \mu_8 + \mu_9 + \mu_{16} + \mu_{18} + \mu_{19} \leq 9$$

$$PI_{21} = \mu_6 + \mu_7 + \mu_9 + \mu_{16} + \mu_{18} + \mu_{19} \leq 9$$

$$PI_{22} = \mu_6 + \mu_7 + \mu_9 + \mu_{11} + \mu_{12} + \mu_{16} + \mu_{18} + \mu_{19} \leq 9$$

$$PI_{23} = \mu_6 + \mu_7 + \mu_9 + \mu_{11} + \mu_{17} + \mu_{18} + \mu_{19} \leq 9$$

$$PI_{24} = \mu_6 + \mu_7 + \mu_8 + \mu_9 + \mu_{13} + \mu_{16} + \mu_{18} + \mu_{19} \leq 9$$

$$PI_{25} = \mu_6 + \mu_7 + \mu_9 + \mu_{13} + \mu_{16} + \mu_{18} + \mu_{19} \leq 9$$

$$PI_{26} = \mu_6 + \mu_7 + \mu_9 + \mu_{11} + \mu_{13} + \mu_{17} + \mu_{18} + \mu_{19} \leq 9$$

Step 2.7.3: The computation of the monitors are carried out as follows:

$$\begin{array}{cccccccccc} & & p6 & p7 & p8 & p9 & p11 & p16 & p18 & p19 \\ L_{PI16} & = & [1 & 1 & 1 & 1 & 1 & 1 & 1 & 1] \end{array}$$

$$D_{PI16} = \begin{array}{cccccccccccc|c} t1 & t2 & t3 & t4 & t5 & t7 & t8 & t15 & t16 & t17 & t18 & t19 & \\ 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & p6 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p7 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p8 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p9 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & p11 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & p16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & p18 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & p19 \end{array}$$

$D_{C16} = - L_{PI16} \,.\, D_{PI16}$

$$= - \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$D_{C16} = - \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & -1 & 1 & 0 & -1 & 1 & -1 \end{bmatrix}$

Therefore, $D_{C16} = \begin{array}{cccccccccccc} t1 & t2 & t3 & t4 & t5 & t7 & t8 & t15 & t16 & t17 & t18 & t19 \\ [-1 & 0 & 0 & 0 & 1 & 0 & 1 & -1 & 0 & 1 & -1 & 1] \end{array}$

$\mu_{0(c16)} = 9$

$L_{PI17} = \begin{array}{ccccccc} p6 & p7 & p9 & p11 & p16 & p18 & p19 \\ [1 & 1 & 1 & 1 & 1 & 1 & 1] \end{array}$

$$D_{PI17} = \begin{array}{cccccccccccc|c} t1 & t2 & t3 & t4 & t5 & t7 & t8 & t15 & t16 & t17 & t18 & t19 & \\ 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & p6 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p7 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p9 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & p11 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & p16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & p18 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & p19 \end{array}$$

$D_{C17} = -L_{PI17} \,.\, D_{PI17}$

$$= - [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{C17} = -[1 \quad 0 \quad -1 \quad 1 \quad -1 \quad 1 \quad 0 \quad 1 \quad 0 \quad -1 \quad 1 \quad -1]$$

$$\text{Therefore, } D_{C17} = \begin{array}{cccccccccccc} t1 & t2 & t3 & t4 & t5 & t7 & t8 & t15 & t16 & t17 & t18 & t19 \\ [-1 & 0 & 1 & -1 & 1 & -1 & 0 & -1 & 0 & 1 & -1 & 1] \end{array}$$

$$\mu_{0(c17)} = 9$$

$$L_{PI18} = \begin{array}{ccccccccc} p6 & p7 & p8 & p9 & p11 & p13 & p16 & p18 & p19 \\ [1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1] \end{array}$$

$$D_{PI18} = \begin{array}{cccccccccccccc|c} t1 & t2 & t3 & t4 & t5 & t7 & t8 & t9 & t10 & t15 & t16 & t17 & t18 & t19 & \\ 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p6 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p7 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p8 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p9 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p11 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & p13 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & p16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & p18 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & p19 \end{array}$$

$$D_{C18} = - L_{PI18} \, . \, D_{PI18}$$

$$= - [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{C18} = - [1 \quad 0 \quad 0 \quad 0 \quad -1 \quad 0 \quad -1 \quad 1 \quad -1 \quad 1 \quad 0 \quad -1 \quad 1 \quad -1]$$

Therefore, $$D_{C18} = \begin{array}{cccccccccccccc} t1 & t2 & t3 & t4 & t5 & t7 & t8 & t9 & t10 & t15 & t16 & t17 & t18 & t19 \\ \end{array}$$
$$\left[\begin{array}{cccccccccccccc} -1 & 0 & 0 & 0 & 1 & 0 & 1 & -1 & 1 & -1 & 0 & 1 & -1 & 1 \end{array}\right]$$

$\mu_{0(c18)} = 9$

$$L_{PI19} = \begin{array}{cccccccc} p6 & p7 & p9 & p11 & p13 & p16 & p18 & p19 \\ \end{array}$$
$$\left[\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

$$D_{PI19} = \begin{array}{ccccccccccccccc} t1 & t2 & t3 & t4 & t5 & t7 & t8 & t9 & t10 & t15 & t16 & t17 & t18 & t19 & \\ 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p6 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p7 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p9 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p11 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & p13 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & p16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & p18 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & p19 \end{array}$$

$$D_{C19} = - L_{PI19} \,.\, D_{PI19}$$

$$= - \left[\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right] \left[\begin{array}{cccccccccccccc} 1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{array}\right]$$

$$D_{C19} = - \left[\begin{array}{cccccccccccccc} 1 & 0 & -1 & 1 & -1 & 1 & 0 & 1 & -1 & 1 & 0 & -1 & 1 & -1 \end{array}\right]$$

Therefore, $$D_{C19} = \begin{array}{cccccccccccccc} t1 & t2 & t3 & t4 & t5 & t7 & t8 & t9 & t10 & t15 & t16 & t17 & t18 & t19 \\ \end{array}$$
$$\left[\begin{array}{cccccccccccccc} -1 & 0 & 1 & -1 & 1 & -1 & 0 & -1 & 1 & -1 & 0 & 1 & -1 & 1 \end{array}\right]$$

$\mu_{0(c19)} = 9$

$$L_{PI20} = \begin{array}{ccccccc} p6 & p7 & p8 & p9 & p16 & p18 & p19 \\ \end{array}$$
$$\left[\begin{array}{ccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

$$D_{PI20} = \begin{array}{c|c} \begin{matrix} t1 & t2 & t3 & t4 & t5 & t7 & t15 & t16 & t17 & t18 & t19 \end{matrix} & \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} p6 \\ p7 \\ p8 \\ p9 \\ p16 \\ p18 \\ p19 \end{matrix} \end{array}$$

$$D_{C20} = - L_{PI20} \, . \, D_{PI20}$$

$$= - \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{C20} = - \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & -1 & 1 & 0 & -1 & 1 & -1 \end{bmatrix}$$

$$\text{Therefore, } D_{C20} = \begin{array}{c} \begin{matrix} t1 & t2 & t3 & t4 & t5 & t7 & t15 & t16 & t17 & t18 & t19 \end{matrix} \\ \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 1 & -1 & 1 \end{bmatrix} \end{array}$$

$\mu_{0(20)} = 9$

$$L_{PI21} = \begin{array}{c} \begin{matrix} p6 & p7 & p9 & p16 & p18 & p19 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{PI21} = \begin{array}{c|c} \begin{matrix} t1 & t2 & t3 & t4 & t5 & t7 & t15 & t16 & t17 & t18 & t19 \end{matrix} & \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} p6 \\ p7 \\ p9 \\ p16 \\ p18 \\ p19 \end{matrix} \end{array}$$

$$D_{C21} = - L_{PI21} \, . \, D_{PI21}$$

$$= - \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{C21} = - \begin{bmatrix} 1 & 0 & -1 & 1 & -1 & -1 & 1 & 0 & -1 & 1 & -1 \end{bmatrix}$$

$$\text{Therefore, } D_{C21} = \begin{array}{ccccccccccc} t1 & t2 & t3 & t4 & t5 & t7 & t15 & t16 & t17 & t18 & t19 \\ \hline -1 & 0 & 1 & -1 & 1 & 1 & -1 & 0 & 1 & -1 & 1 \end{array}$$

$\mu_{0(21)} = 9$

$$L_{PI22} = \begin{array}{cccccccc} p6 & p7 & p9 & p11 & p12 & p16 & p18 & p19 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}$$

$$D_{PI22} = \begin{array}{ccccccccccccc|c} t1 & t2 & t3 & t4 & t5 & t7 & t8 & t9 & t15 & t16 & t17 & t18 & t19 & \\ \hline 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p6 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p7 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p9 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & p11 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & p12 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & p16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & p18 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & p19 \end{array}$$

$$D_{C22} = - L_{PI22} \cdot D_{PI22}$$

$$= - \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{C22} = - \begin{bmatrix} 1 & 0 & -1 & 1 & -1 & 0 & 0 & -1 & 1 & 0 & -1 & 1 & -1 \end{bmatrix}$$

Therefore, $$D_{C22} = \begin{array}{ccccccccccccc} t1 & t2 & t3 & t4 & t5 & t7 & t8 & t9 & t15 & t16 & t17 & t18 & t19 \\ [-1 & 0 & 1 & -1 & 1 & 0 & 0 & 1 & -1 & 0 & 1 & -1 & 1] \end{array}$$

$\mu_{0(c22)} = 9$

$$L_{PI23} = \begin{array}{ccccccc} p6 & p7 & p9 & p11 & p17 & p18 & p19 \\ [1 & 1 & 1 & 1 & 1 & 1 & 1] \end{array}$$

$$D_{PI23} = \begin{array}{c|c} \begin{array}{ccccccccccc} t1 & t2 & t3 & t4 & t5 & t7 & t8 & t15 & t16 & t17 & t18 \end{array} & \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix} & \begin{array}{c} p6 \\ p7 \\ p9 \\ p11 \\ p17 \\ p18 \\ p19 \end{array} \end{array}$$

$D_{C23} = - L_{PI23} \,.\, D_{PI23}$

$$= - \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix}$$

$$D_{C23} = - \begin{bmatrix} 1 & 0 & -1 & 1 & -1 & 0 & -1 & 1 & 0 & 0 & -1 \end{bmatrix}$$

Therefore, $$D_{C23} = \begin{array}{ccccccccccc} t1 & t2 & t3 & t4 & t5 & t7 & t8 & t15 & t16 & t17 & t18 \\ [-1 & 0 & 1 & -1 & 1 & 0 & 1 & -1 & 0 & 0 & 1] \end{array}$$

$\mu_{0(c23)} = 9$

$$L_{PI24} = \begin{array}{cccccccc} p6 & p7 & p8 & p9 & p13 & p16 & p18 & p19 \\ [1 & 1 & 1 & 1 & 1 & 1 & 1 & 1] \end{array}$$

$$D_{PI24} = \begin{array}{ccccccccccccc|c} t1 & t2 & t3 & t4 & t5 & t7 & t9 & t10 & t15 & t16 & t17 & t18 & t19 & \\ \hline 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p6 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p7 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p8 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & p13 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & p16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & p18 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & p19 \end{array}$$

$D_{C24} = - L_{PI24} \,.\, D_{PI24}$

$$= - [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$D_{C24} = - [1 \quad 0 \quad 0 \quad 0 \quad -1 \quad -1 \quad 1 \quad -1 \quad 1 \quad 0 \quad -1 \quad 1 \quad -1]$

$$\text{Therefore, } D_{C24} = \begin{array}{ccccccccccccc} t1 & t2 & t3 & t4 & t5 & t7 & t9 & t10 & t15 & t16 & t17 & t18 & t19 \\ [-1 & 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 & 0 & 1 & -1 & 1] \end{array}$$

$\mu_{0(c24)} = 9$

$$L_{PI25} = \begin{array}{ccccccc} p6 & p7 & p9 & p13 & p16 & p18 & p19 \\ [1 & 1 & 1 & 1 & 1 & 1 & 1] \end{array}$$

$$D_{PI25} = \begin{array}{ccccccccccccc|c} t1 & t2 & t3 & t4 & t5 & t7 & t9 & t10 & t15 & t16 & t17 & t18 & t19 & \\ \hline 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p6 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p7 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & p13 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & p16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & p18 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & p19 \end{array}$$

$D_{C25} = -L_{PI25} \,.\, D_{PI25}$

$$= - [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$D_{C25} = - [1 \quad 0 \quad -1 \quad 1 \quad -1 \quad -1 \quad 1 \quad -1 \quad 1 \quad 0 \quad -1 \quad 1 \quad -1]$

$$\text{Therefore, } D_{C25} = \begin{array}{ccccccccccccc} t1 & t2 & t3 & t4 & t5 & t7 & t9 & t10 & t15 & t16 & t17 & t18 & t19 \\ [-1 & 0 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 0 & 1 & -1 & 1] \end{array}$$

$\mu_{0(c25)} = 9$

$$D_{PI26} = \begin{array}{c} \\ \left[\begin{array}{ccccccccccccc} t1 & t2 & t3 & t4 & t5 & t7 & t8 & t9 & t10 & t15 & t16 & t17 & t18 \\ 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \end{array} \right] \end{array} \begin{array}{l} \\ p6 \\ p7 \\ p9 \\ p11 \\ p13 \\ p17 \\ p18 \\ p19 \end{array}$$

$$L_{PI26} = \begin{array}{cccccccc} p6 & p7 & p9 & p11 & p13 & p17 & p18 & p19 \\ [1 & 1 & 1 & 1 & 1 & 1 & 1 & 1] \end{array}$$

$D_{C26} = -L_{PI26} \,.\, D_{PI26}$

$$= - [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix}$$

$D_{C26} = - [1 \quad 0 \quad -1 \quad 1 \quad -1 \quad 0 \quad -1 \quad 1 \quad -1 \quad 1 \quad 0 \quad 0 \quad -1]$

Therefore, $$D_{C26} = \begin{array}{ccccccccccccc} t1 & t2 & t3 & t4 & t5 & t7 & t8 & t9 & t10 & t15 & t16 & t17 & t18 \\ [-1 & 0 & 1 & -1 & 1 & 0 & 1 & -1 & 1 & -1 & 0 & 0 & 1] \end{array}$$

$\mu_{0(c26)} = 9$

Step 2.10.4: Redundancy test carried out shows that only three of the computed monitors are necessary, C_{18}, C_{22} and C_{26} while the rest of the monitors computed are redundant and are therefore removed. The necessary monitors are also renumbered in order to follow the regular sequence of numbering for convenience. Thus C_{18} becomes C_{16}, C_{22} becomes C_{17} and C_{26} becomes C_{18}.

The necessary monitors C_{16}, C_{17} and C_{18} are shown in Table 4.12.

Table 4.12. Necessary monitors C_{16}, C_{17} and C_{18}.

C_i	$^\bullet C_i$	$C_i{}^\bullet$	$\mu_{0(ci)}$
C_{16}	t5, t8, t10, t17, t19	t1, t9, t15, t18	9
C_{17}	t3, t5, t9, t17, t19	t1, t4, t15, t18	9
C_{18}	t3, t5, t8, t10, t18	t1, t4, t9, t15	9

Step 2.10.5: The controlled $PNM_{10} := PNM_{10} + C_{16} + C_{17} + C_{18}$ is shown in Fig. 4.18. It is live with 21,513 good states. This is the optimal live behaviour for the controlled PNM_{10}.

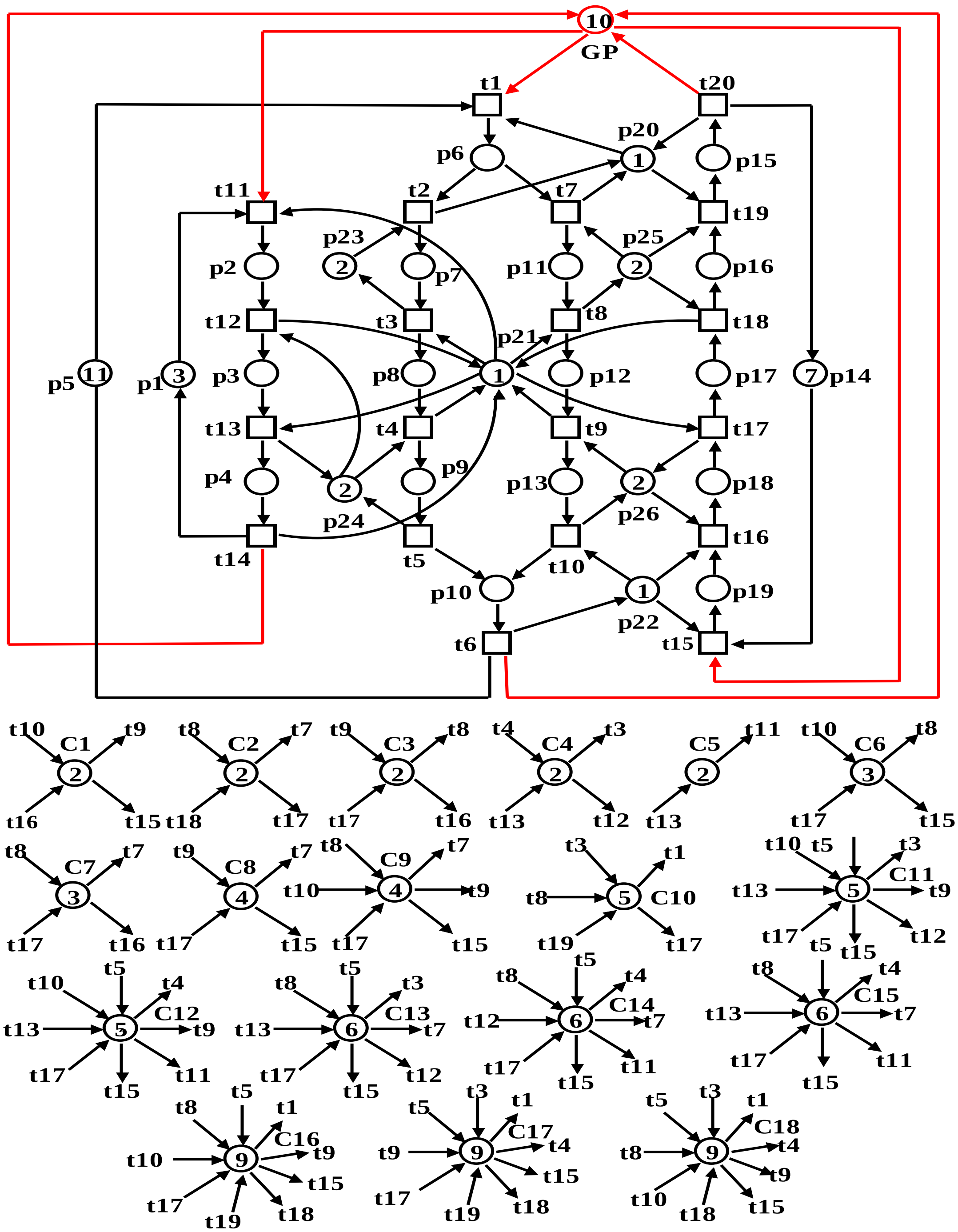

Figure 4.18. The controlled model PNM$_{10}$ (PNM$_{10}$:= PNM$_{10}$ + C$_{16}$ + C$_{17}$ +C$_{18}$).

B := B ++ (B = 11)

Step 2.5.1: (B = 11), the PNM$_{11}$ shown Fig. 4.19 is live with 21,562 good states.

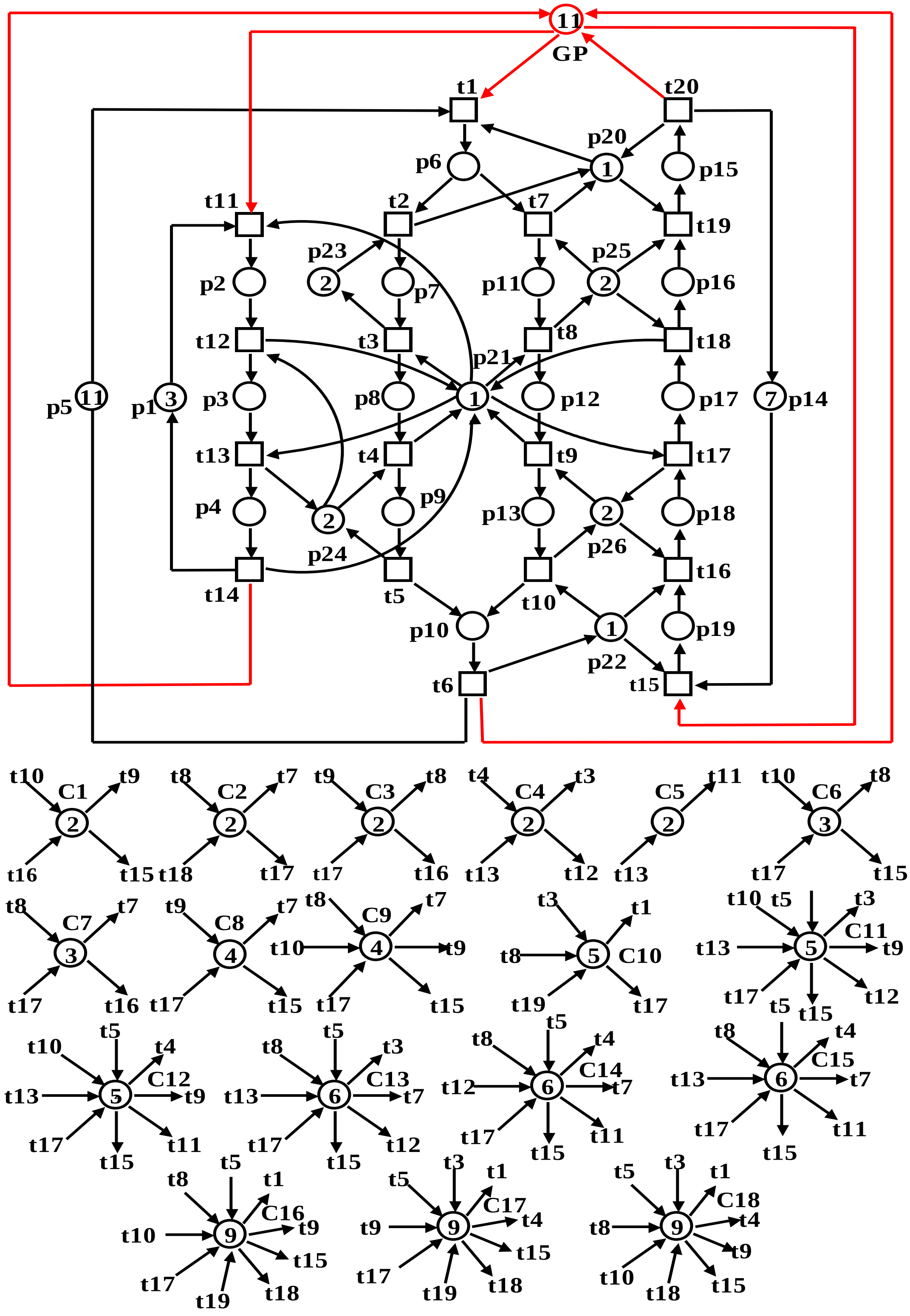

Figure 4.19. The Live PNM$_{11}$.

Step 3: The design procedure applied in Step 2 is provided in Table 4.13. The computed eighteen necessary monitors are added in the uncontrolled PNM, the controlled PNM is obtained as shown in Fig. 4.20. It is verified that this controlled model is live with 21,562 good states.

Step 4: Exit

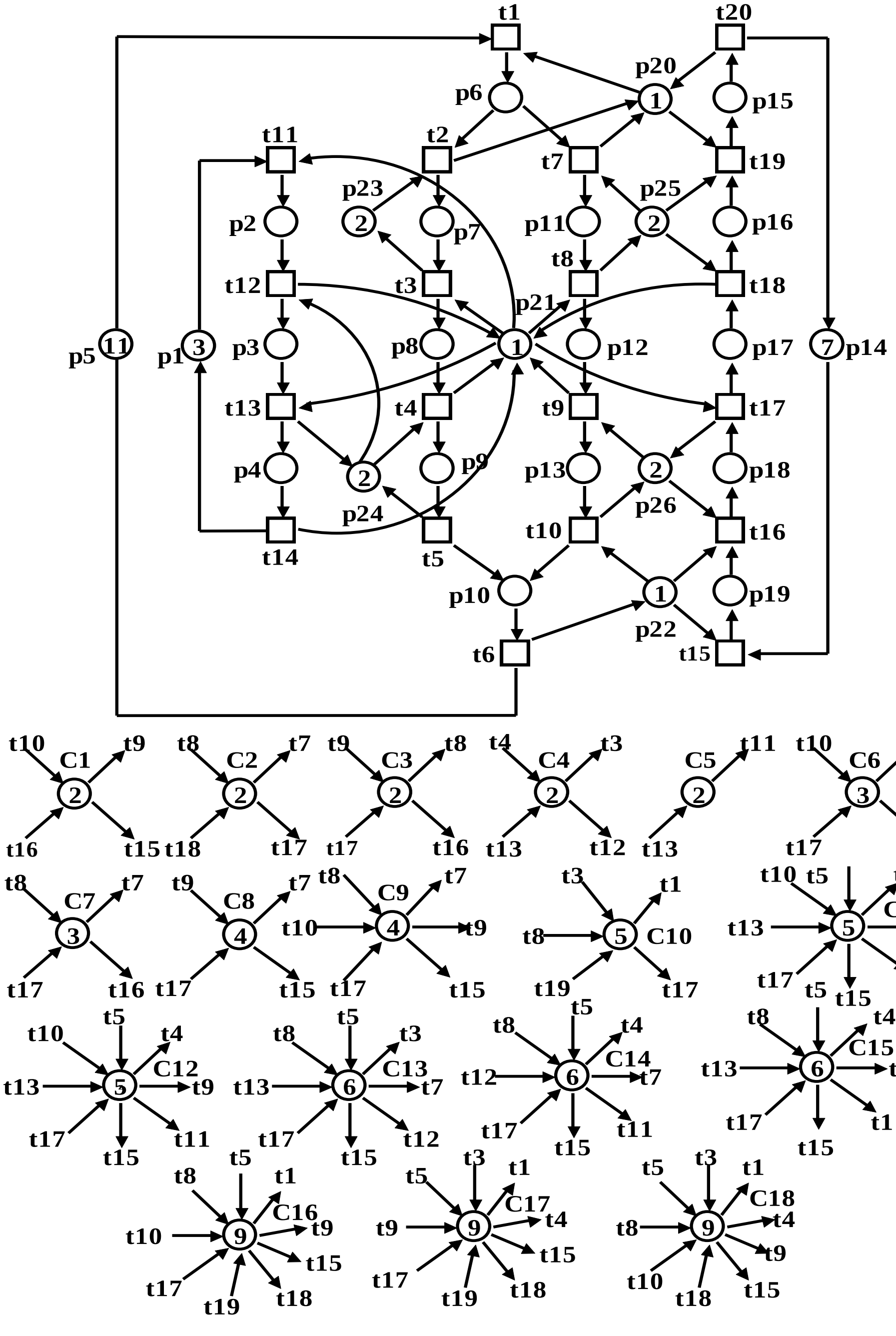

Figure 4.20. The controlled PNM.

Table 4.13. Computed necessary monitors.

C_i	$^{\bullet}C_i$	$C_i^{\bullet}$	$\mu_{0(C_i)}$
C_1	t10, t16	t9, t16	2
C_2	t8, t18	t7, t17	2
C_3	t9, t17	t8, t16	2
C_4	t4, t13	t3, t12	2
C_5	t13	t11	2
C_6	t10, t17	t8, t15	3
C_7	t8, t17	t7, t16	3
C_8	t9, t17	t7, t15	4
C_9	t8, t10, t17	t7, t9, t15	4
C_{10}	t3, t8, t19	t1, t17	5
C_{11}	t5, t10, t13, t17	t3, t9, t12, t15	5
C_{12}	t5, t10, t13, t17	t4, t9, t11, t15	5
C_{13}	t5, t8, t13, t17	t3, t7 ,t12, t15	6
C_{14}	t5, t8, t12, t17	t4, t7, t11, t15	6
C_{15}	t5, t8, t13, t17	t4, t7, t11, t15	6
C_{16}	t5, t8, t10, t17, t19	t1, t9, t15, t18	9
C_{17}	t3, t5, t9, t17, t19	t1, t4, t15, t18	9
C_{18}	t3 t5, t8, t10, t18	t1, t4, t9, t15	9

Table 4.14 shows the liveness enforcing procedure applied to the given PNM of Fig. 4.1. The maximally permissive behaviour of the PNM must provide 21,581 good states. Using our method, we obtained live behaviour with 21,562 good states, 99.91% of the maximally permissive behaviour.

Table 4.14. The liveness enforcing procedure applied to S^3PR PNM.

B	Included C	Is the net live?	# of States in RG	# of States in DZ	# of States in LZ	Computed C	#of states With in Controlled net	
							RG = LZ	UR
1	–	YES	17	0	17	–	17	
2	–	YES	132	0	132	–	132	
3	–	NO	637	5	632	C_1, C_2, C_3, C_4, C_5	632	0
4	C_1, C_2, .., C_5	NO	2,106	2	2,104	C_6, C_7	2,104	0
5	C_1, C_2, .., C_7	NO	5,192	2	5,190	C_8, C_9	5,190	0
6	C_1, C_2, .., C_9	NO	9,888	10	9,878	C_{10}, C_{11}, C_{12} (7 redundant)	9,878	0
7	C_1, C_2, .., C_{12}	NO	15,017	4	15,013	C_{13},C_{14}, C_{15} (1 redundant)	15,013	0
8	C_1, C_2, .., C_{15}	YES	18,972	0	18,972	–	18,972	0
9	C_1, C_2, .., C_{15}	YES	20,980	0	20,980	–	20,980	0
10	C_1, C_2, .., C_{15}	NO	21,536	11	21,525	C_{16},C_{17}, C_{18} (1 redundant)	21,513	12
11	C_1,C_2, .., C_{18}	YES	21,562	0	21,562	–	21,562	19

4.2 AN AEMG PETRI NET EXAMPLE

The PNM of an FMS shown in Fig. 4.21 from [35], suffers from deadlocks. It has 3,136 states within the RG, 1,466 states of which are in the LZ while remaining 1,670 states are in the DZ.

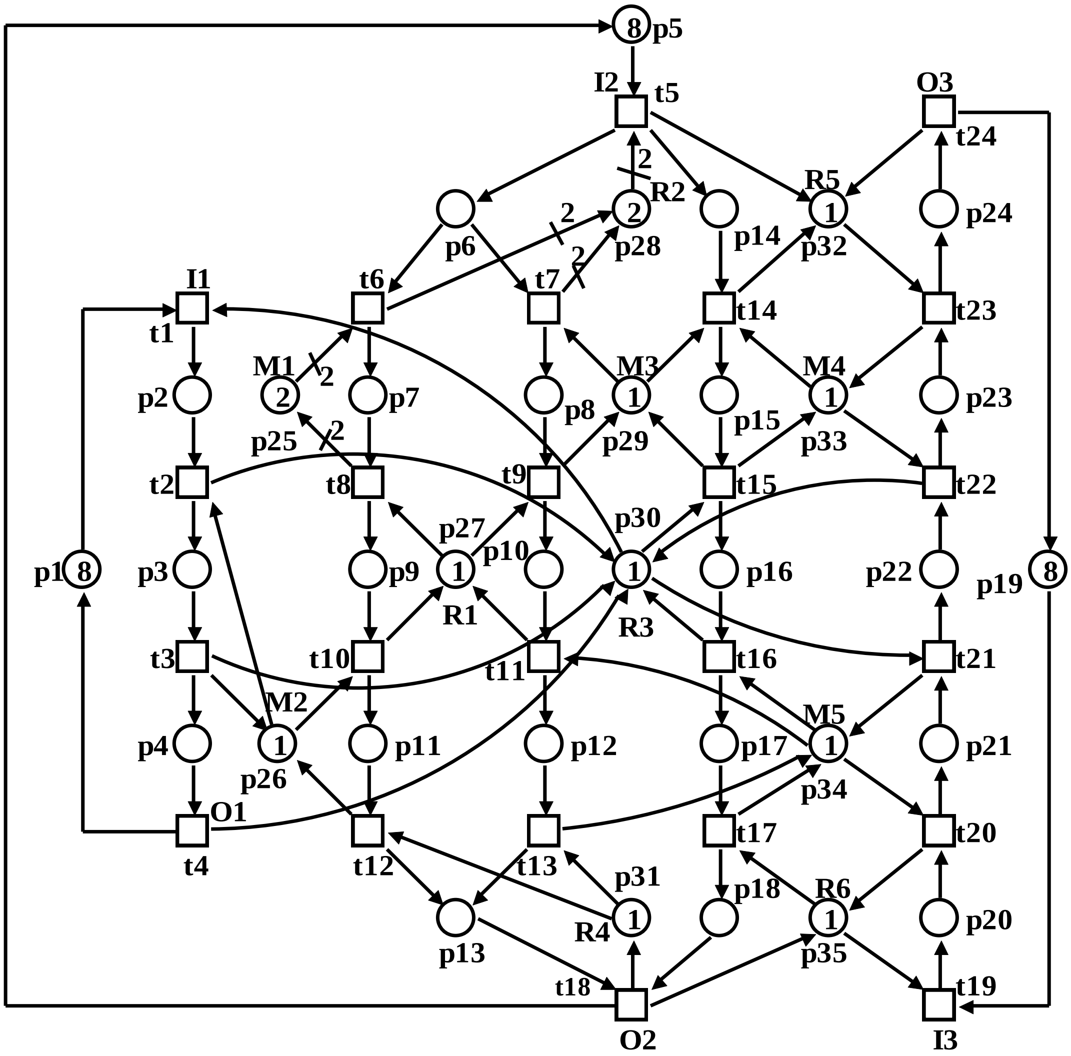

Figure 4.21. An AEMG Petri net model of an FMS from [35].

There are twenty one activity places P_A = {p2-p4, p6-p18, p20-p24}, eleven shared resource places P_R = {p25-p27, p28-p35}, and three sink/source places $P_{S/S}$ = {p1, p5, p19}. Table 4.15 gives the liveness enforcing procedure applied to the AEGM PNM.

Table 4.15 shows the liveness enforcing procedure applied to the AEMG PNM of Fig. 4.21. The optimal solution for this PNM must provide 1,466 reachable good states. By using our method, we obtained permissiveness of 1,330 reachable states which is 90.72% of the optimal solution. Table 4.16 provides the necessary computed monitors and their related arcs.

Table 4.15. The liveness enforcing procedure applied to AEGM PNM.

B	Included C	Is the net live?	# of States in RG	# of states in DZ	# of States in LZ	Computed C	# of states Within controlled net	
							RG = LZ	UR
1	–	YES	47	0	17	–	17	
2	–	NO	446	80	366	C_1,C_2,..,C_{40} (40 redundant)	366	0
3	C_1, C_2,.., C_{40}	NO	991	2	989	C_{41}, C_{42}	989	0
4	C_1, C_2,.., C_{42}	NO	1,305	2	1,303	C_{43} (1 redundant)	1,303	0
5	C_1, C_2,.., C_{43}	YES	1,329	0	1,329	–	1,329	0
6	C_1, C_2,.., C_{43}	YES	1,330	0	1,330	–	1,330	0

Table 4.16. Computed necessary monitors.

C_i	$^\bullet C_i$	$C_i^\bullet$	$\mu_{0(C_i)}$	C_i	$^\bullet C_i$	$C_i^\bullet$	$\mu_{0(C_i)}$
C_1	t3	t1	1	C_{23}	t8, t17, t20	t6, t16, t19	2
C_2	t16, t18	t11, t12, t14	3	C_{24}	t8, t16, t20	t6, t15, t19	2
C_3	t14, t16, t18	t5, t11, t12, t15	3	C_{25}	t9, t17, t20	t7, t16, t19	2
C_4	t15, t18	t5, t11, t12	3	C_{26}	t9, t16, t20	t7, t15, t19	2
C_5	t8, t11, t18	t6, t9, 1t6	3	C_{27}	t6, t7, t17, t20	t5, t16, t19	2
C_6	t11, t18	t7, t16	3	C_{28}	t6, t7, t16, t20	t5, t15, t19	2
C_7	t17, t18, t20	t12, t13, t16, t19	2	C_{29}	t12, t15, t20	t10, t14, t19	2
C_8	t18, t16, t20	t12, t13, t15, t19	2	C_{30}	t10, t15, t20	t8, t14, t19	2
C_9	t18, t15, t20	t12, t13, t14, t19	2	C_{31}	t6, t7, t15, t20	t5, t14, t19	2
C_{10}	t18, t14, t20	t12, t13, t5, t19	2	C_{32}	t8, t15, t20	t6, t14, t19	2
C_{11}	t13, t16, t20	t11, t15, t19	2	C_{33}	t6, t7, t11, t18	t5, t9, t16	3
C_{12}	t13, t15, t20	t11, t14, t19	2	C_{34}	t12, t14, t20	t10, t5, t19	2
C_{13}	t13, t14, t20	t11, t5, t19	2	C_{35}	t10, t14, t20	t5, t8, t19	2
C_{14}	t11, t17, t20	t9, t16, t19	2	C_{36}	t8, t14, t20	t5, t6, t19	2
C_{15}	t11, t16, t20	t9, t15, t19	2	C_{37}	t6, t7, t14, t23	2t5, t22	2
C_{16}	t11, t15, t20	t9, t14, t19	2	C_{38}	t6, t7, t14, t22	2t5, t21	2
C_{17}	t11, t14, t20	t9, t5, t19	2	C_{39}	t6, t7, t14, t21	2t5, t20	2
C_{18}	t9, t14, t20	t7, t5, t19	2	C_{40}	t6, t7, t14, t20	2t5, t19	2
C_{19}	t12, t17, t20	t10, t16, t19	2	C_{41}	t2, t15, t18	t1, t5, t10,t13	4
C_{20}	t12, t16, t20	t10, t15, t19	2	C_{42}	t2, t12, t13, t15	t1, t5, t10, t11	4
C_{21}	t10,t17, t20	t8, t16, t19	2	C_{43}	t3, t7, t10, t18	t2, t5, t15	6
C_{22}	t10, t16, t20	t8, t15, t19	2				

4.3 PERFORMANCE COMPARISON OF DIFFERENT CONTROL POLICIES

4.3.1 Performance Comparison for the S^3PR Example

Table 4.17 shows the performance comparison of our solution on the S^3PR example with other different solutions previously provided on the S^3PR model in the literature.

Table 4.17 Performance comparison of different control policies for the S^3PR model.

Parameters	Ezpeleta *et al.* [16]	Li and Zhou [8]	Huang *et al.* [20]	Li *et al.* [21]	Uzam and Zhou [13]	Our solution
# of monitors added	18	6	12	7	19	18
# of reachable states	6287	6287	12656	16636	21562	21562
Permissiveness (%)	29.13	29.13	58.64	77.09	99.91	99.91

The permissiveness of PNM increases as we move from the left to the right of the Table 4.17. Our solution and the one in [13] give the highest number of reachable states but with different number of monitors. In our solution, 18 monitors are required to obtain the number of reachable states we obtained while in [13] 19 monitors are required. The proposed method is straightforward. The only modification it requires to be made in the original PNM is the addition of the global sink/source place. Also a small number of control places is desirable in the design of a liveness enforcing supervisor as it reduces the structural complexity of a controlled PN model.

4.3.2 Performance Comparison for the AEMG Example

Table 4.18 shows the performance comparison of our solution on the AEGM example with the one provided in [35] for the AEMG example.

Table 4.18 Performance comparison of two control policies for the AEGM model.

Parameters	[35]	Our solution
# of monitors added	15	43
# of reachable states	167	1,330
Permissiveness (%)	11.39	90.72

It can be seen from Table 4.18 that our method provides very high number of reachable states compared with that of [35]. Our solution provides near optimal permissiveness for the AEGM example.

CHAPTER 5

CONCLUSIONS

In this thesis a new method for the synthesis of Petri net based liveness enforcing supervisors in FMS is proposed. The method is simple, straightforward and easy to apply. The applicability of the proposed method to Petri net models suffering from deadlocks is demonstrated through examples. It does not require dividing the given PN model into subnets as in [29]. The only modification it requires in its algorithm is the addition of global sink/source place (GP) which is used temporarily in the computation stage, and it is removed when the net becomes live. The computation is carried out in an iterative way by increasing the number of tokens in the GP at each iteration.

The method provides very high behavioral permissiveness. It is not restricted to certain classes of PN models and it can be applied to many classes of Petri nets currently available in the literature.

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