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A STUDY ON THE STRUCTURAL COMPLEXITY REDUCTION OF PETRI NET BASED LIVENESS-ENFORCING SUPERVISORS IN FLEXIBLE MANUFACTURING SYSTEMS

by

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June 2014
Kayseri, Turkey

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APPROVAL PAGE

This is to certify that I have read the thesis entitled "A Study on the Structural Complexity Reduction of Petri Net Based Liveness Enforcing Supervisors in Flexible Manufacturing Systems" by Muhammad Bashir and that in my opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science in Electrical and Computer Engineering, the Graduate Institute of Science and Engineering, Melikşah University.

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ABSTRACT

Many works have been carried out for the study on deadlock prevention (liveness-enforcing) in flexible manufacturing systems (FMS). Petri nets have been used as a tool to enforce liveness in FMS so as to make deadlocks impossible to occur. Behavioral permissiveness, computational complexity and structural complexity are three criteria to evaluate the performance of a liveness-enforcing Petri net supervisor for FMSs. The reduction of structural complexity involves the reduction of the number of control places (monitors) in liveness-enforcing supervisors. Currently there are some important results in the literature to solve this problem. However, to obtain structurally simple monitors, one has to pay a high computational price. In this study a new method is proposed for obtaining structurally simple monitors with a reasonable computational effort via establishing linear relationships that exist between the place invariants. Structurally simple monitors obtained by the method proposed here provide optimal or near optimal behavioral permissiveness. The applicability of the proposed approach is shown by means of several examples for different classes of Petri nets.

Keywords: Merged place invariants, Deadlock, flexible manufacturing systems (FMS), Petri nets.

ESNEK ÜRETİM SİSTEMLERİNDE CANLILIK SAĞLAYICI GÖZETİCİLERİN YAPISAL KARMAŞIKLIĞININ AZALTILMASI ÜZERİNE BİR ÇALIŞMA

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ÖZ

Esnek üretim sistemlerinde (Flexible Manufacturing Systems – FMS) kördüğümün önlenmesi (canlılık-yürürlüğe koymak) amacıyla şu ana kadar pek çok çalışma yapılmıştır. Petri ağları FMS'te kördüğüm oluşumlarını imkansız hale getirmek için canlılık sağlamak üzere kullanılan bir araçtır. FMS'lerde canlılık-sağlayan bir Petri net denetçisinin performansını değerlendirmek için kullanılan üç kriter davranışsal serbestlik, hesaplama karmaşıklığı ve yapısal karmaşıklıktır. Yapısal karmaşıklığı azaltma, canlılık-uygulayıcı denetçilerdeki kontrol mevkilerinin (monitörlerin) sayısının azaltılmasını içerir. Şu anda bu sorunu çözmek için literatürde bazı önemli sonuçlar vardır. Ancak, yapısal olarak basit monitörler elde etmek için yüksek hesaplama bedeli ödemek zorunludur. Mevki değişmezleri arasında doğrusal ilişkiler kurulması yoluyla makul bir hesaplama çabasıyla yapısal olarak basit monitörler elde etmek için bu çalışmada yeni bir yöntem önerilmiştir. Burada önerilen yöntem ile elde edilen yapısal olarak basit monitörler, optimum veya optimuma yakın davranış serbestliği sağlarlar. Önerilen yaklaşımın farklı Petri ağı sınıflarına uygulanabilirliği çeşitli örneklerle gösterilmiştir.

Anahtar Kelimeler: Birleştirilmiş mevki değişmezleri, Kördüğüm, Esnek üretim sistemleri, Petri ağları.

DEDICATION

I dedicate my thesis work to my parents and my family at large. A special thanks and gratitude goes to my loving parents. Who pray and tough me the best kind of knowledge and tough me that even the largest task can be accomplished if it is done one step at a time.

I also dedicate my thesis to entire people of Kano State for helping us with a lot of prayers day and night.

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TABLE OF CONTENTS

LIST OF TABLES

TABLE

LIST OF FIGURE

CHAPTER 1

INTRODUCTION

In today's world, the economic situation of a country depends on the diversified products contributed to the world market, which is due to revolution of modern industry. For a particular country to involve a transaction with the modern world market, different kinds of diversified products could be contributed to compete with other countries. This can be achieved by replacing an old style fixed hardware sequential system with flexible manufacturing systems (FMSs) which can easily make change to the product design by configuring a supervisory controller [10].

An FMS usually consists of two main parts; a physical system which includes resources (such as machines, robots and a transportation system) shared by several jobs. A management system or decision making system responsible for the control of the physical system to achieve their goal of productivity and works-in-process level [10].

1.1 AIMS AND SCOPE OF FLEXIBLE MANUFACTURING SYSTEMS (FMSS)

Flexible manufacturing has been introduced primarily to achieve some certain objectives. These objectives include: decreasing the lead time (i.e. Speed-up the production), increased machined utilization, improved reliability of the system, increased quality of the product, increased profitable investment and reduced damage of the product by the systems. However, due to the high utilization of resources in the flexible manufacturing system results in the high competition of resources for different jobs. These competition causes the causes the occurrence of deadlock which might upset the advantages of FMSs [10]

1.2 CONDITION FOR DEADLOCK TO OCCUR

Deadlock is an important issue to be considered in design and control of flexible manufacturing systems, since their occurrence causes a lot of damage to the system or might halt the whole system from the operation [5]. In general, Coffman have formulated four necessary conditions for a deadlock occurrence, which are popularly known as Coffman conditions [3], [6], [12]:

1) Mutual exclusion condition: a resource can only be used by one process at a time.
2) Hold and wait condition: processes that use some resources may need another new resource.
3) Non-preemption condition: it is not feasible to remove a resource that is held by a particular process, but a process can only release a resource by an explicit action of that process.
4) Circular-wait-condition: when two or more processes form a circular chain where each process waits for a resources that the next process in the chain holds.

1.3 TOOLS USED TO DEAL WITH DEADLOCK

Many tools have been developed to deal with deadlocks in FMSs [2], [4], [6], [24]. Petri nets, automata and graph theory are the three main tools. In this study we are concerned with Petri net based tools. Petri nets become most essential tools for the study of deadlocks in an FMS. This is due to the fact that they possesses FMS characteristics such as conflicts, concurrency, and casual dependency [8], [28]. Petri nets are widely used in so many areas such as computer and communication networks, manufacturing systems, and automation systems [4], [5]. Generally there are four Petri net based strategies to handle deadlocks in automated manufacturing systems [6], [12], [19], [26], [28]:

1) Deadlock ignoring,
2) Deadlock detection and recovery,
3) Deadlock avoidance,
4) Deadlock prevention.

Deadlock ignoring: in this case, the occurrence of deadlock is ignored due to the negligible amount of probability of their occurrence. Deadlock detection and recovery allows the occurrence of deadlocks, but as soon as the system detects the occurrence of that deadlock the system can be recovered back to its normal position by simply reallocating the resources [6], [11], [14]. Deadlock avoidance determines the possible system evolution at each system state using an online control policy and chooses the correct system evolution [1], [6], [23]. Deadlock prevention is usually achieved by using an off-line computational mechanism to control the request for resources to ensure that deadlocks never occur. Monitors (control places) and related arcs are added to the Petri net model of the system to realize such a control mechanism [2], [3], [5], [6], [11], [28].

Deadlock prevention policies are widely used due to their advantages that the computational mechanism is obtained off-line and once and for all, i.e. deadlocks are totally eliminated. Once deadlocks are eliminated, the system can never enter a deadlock state.

1.4 PERFORMANCE EVALUATION FOR DEADLOCK CONTROL POLICY

The performance for deadlock prevention is evaluated based on the following criteria [5]:

1) Behavioral permissiveness,
2) Structural complexity,
3) Computational complexity.

A maximally permissive supervisor usually leads to sufficient usage of system resources [4], [5], [6]. A supervisor with the minimal number of control places can decrease both hardware and software cost in the stage of validation and implementation [24]. A deadlock control policy with low computational complexity means that it can be applied to complex systems [6], [7].

1.5 PETRI NET BASED DEADLOCK ANALYSIS TECHNIQUES

There are mainly two PN based analysis techniques used for the study of deadlocks: [5], [7], [20], [24], [27], [28]:

1 Structural analysis,

2 Reachability graph analysis.

In the structural analysis, Petri net components, namely siphons and resource transition circuits, are used. Their computation would usually lead to suboptimal behavior of FMS, but the control policy is simple. To prevent siphons from being insufficiently marked, some control places and related arcs are added to their places within the siphons [6], [11], [18]. The number of siphons grows exponentially with the size of the net. In [11] and [12], elementary siphons based approaches were proposed to reduce the number of siphons growing within the complex FMS. However that concept does not provide a maximally permissive behavior. In [14] another concept of avoiding the complete enumeration of siphons was developed, which is an improved method due to the reduced time computation. In [15] another concept was developed for selective siphons control. The relations between uncontrolled siphons and critical markings are identified and a set of siphons is selected by solving a set covering approach for each iteration, the method provide a maximally permissive behavior of Petri net modeled. But it suffer from computational complexity.

The reachability graph (RG) analysis enables one to check certain properties of flexible manufacturing systems (FMS), i.e., liveness, boundedness, synchronization, concurrency and safeness [13], [25]. On the other hand, the RG analysis requires the evaluation of a complete or partial enumeration of reachable states. Therefore, it suffers from the state explosion problem. The theory of regions was developed in [20] as one of the powerful methods of deadlock prevention for deriving a maximally permissive supervisor. However, it is computationally expensive by considering too many inequality constraints [25]. In [21] another policy was developed, which divides the reachability graph into two parts as a live zone (LZ) and a deadlock zone (DZ). The idea is to find the first met bad marking (FBM) from the LZ. However the method is an iterative procedure in which at each iteration an FBM is controlled by adding a control place. The iterations are repeated until the Petri net model is live. The method does not guarantee maximally permissive behavior. In

[24] a RG based method that leads to a maximally permissive liveness supervisor was proposed, where a control place is designed to forbid an FBM, it keeps all legal markings by solving very complex integer linear programming problems (ILPP). Since the method is very complex, a vector covering approach is developed to reduce the sets of legal markings and FBMs by partitioning them into two sets: a minimal covering set of legal markings and a minimal covered set of FBMs. The two sets are the ones considered for designing a supervisor. However, the method suffers from structural complexity problems. The method was later improved by finding the minimal number of control places in [3].

Currently, one of the available methods to reduce the structural complexity of a live Petri net model in the literature was developed in [36]. The method is an iterative procedure aiming to overcome the structural complexity and to ensure that the live Petri net model has a maximally permissive behavior if it exist. The method utilized the used of reachability graph analysis. A vector covering approach is used to compute the minimal covering set of legal markings and the minimal covered set of FBMs. Then, at each iteration a control place is design to reduce as many FBMs as possible. The co-efficients of the PI are computed using integer linear programming problem (ILPP) that ensured the two conditions stated as (i) no marking in the minimal covering set of legal marking are prohibited. (ii) the objectives functions maximizes the number of FBMs that are forbidden by the PI.

In [35], another method was developed for merging two or more siphons for reducing the structural complexity of a live Petri net model. It provide a maximally permissive behavior without using reachability graph or solving integer linear programming problem (ILPP), whereas it relied on the concepts of siphon based control. The method proposed that two or more siphons can be merged if their forbidden sets of makings can be enforced by the same linear invariant constraint. The method is an iterative procedure and it utilized the used of solving first-order equations with the following conditions to be satisfied: (i) a siphon may have a number of FMs. Only one is selected so as to make others forbidden. For this selected marking, the linear constraint is set to a constant k to become a linear-first order equation. (ii) a siphon may have a number of live markings. Only one is selected so as to make others not forbidden either. For this selected marking, the linear constraint is set to a constant $k-1$ to become a linear first-order equation.

In this study, a new method is proposed to reduce the structural complexity in PN based liveness-enforcing supervisors of FMS. The proposed method is structurally and computationally simple and can be applied to complex FMSs modelled with different classes of Petri nets. Some examples are provided to show the significance for the proposed method. The proposed method makes use of the reachability graph analysis. The major contribution of this study is to reduce structural complexity of live Petri net model.

The remainder of this thesis is organized as follows. Some basic concepts of Petri nets are provided in chapter 2. In order to reduce the structural complexity of a given liveness-enforcing supervisor an algorithm is proposed in chapter 3. Applications of the proposed method to three S^3PR Petri net models are provided in chapter 4. Applications of the proposed method to some other Petri net classes such as S^4PR, G-system, S^4R are also considered in chapter 5. Finally conclusions are given in chapter 6.

CHAPTER 2

BASICS OF PETRI NETS

2.1 DEFINITION OF PETRI NETS

A Petri net N is a four-tuple (P,T,F,W) where P and T are finite and non-empty sets. P is a set of places and T is a set of transitions with $P \cup T \neq \phi$ and $P \cap T = \phi$. $F \subseteq (P \times T) \cup (T \times P)$ is called a flow relation of the net, represented by arcs with arrows from places to transitions or from transitions to places. Places are represented by circles while transitions are represented by bars or square boxes. $W : (P \times T) \cup (T \times P) \rightarrow \mathbb{N}$ is a mapping that assigns a weight to an arc: $W(x, y) > 0$ if $(x, y) \in F$, and $W(x, y) = 0$, otherwise, where $x, y \in P \cup T$ and $\mathbb{N}$ is the set of non-negative integers. $N=(P,T,F,W)$ is called an ordinary net, denoted as $N=(P,T,F)$, if $\forall f \in F, W(f) = 1$.

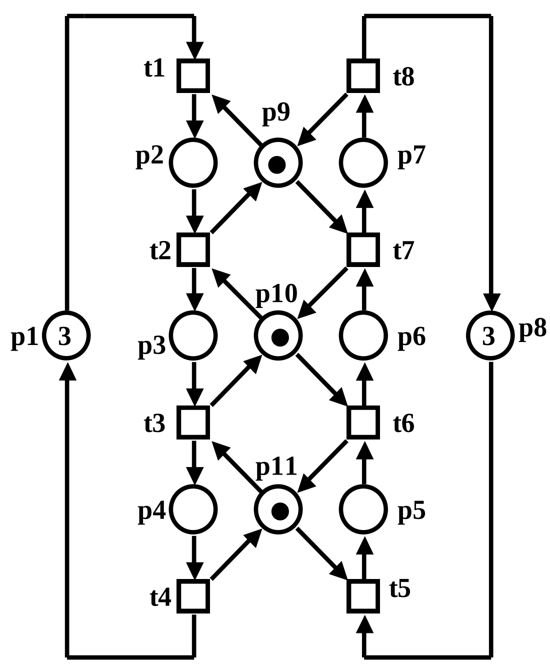


Fig. 2.1. A Petri net example.

Example 2.1

In Fig. 2.1, we can define the net as,

$P = \{p_1, p_2, \cdots, p_{11}\}$
$T = \{t_1, t_2, \cdots, t_8\}$
$F = \{(p_1, t_1), (t_1, p_2), (p_2, t_2), (t_2, p_3), \cdots, (t_5, p_{11})\}$
$\forall p \in P, \forall t \in T,\ \ W(p,t) \leq 1$ or $W(t, p) \leq 1$.

The places in a Petri net model of an FMS, as the one shown in Fig. 2.1, can be partitioned into three parts namely process idle places (P^0), activity places (P_A) and resource places (P_R). In Fig. 2.1, we have $P^0 = \{p_1, p_8\}$, $P_A = \{p_1, p_3, \cdots, p_7\}$ and $P_R = \{p_9, p_{10}, p_{11}\}$.

2.1.1 Definition 1: Preset and Postset

Let $x \in P \cup T$ be a node in $N = (P,T,F,W)$. The preset of x, denoted by ${}^{\bullet}x$, is defined as ${}^{\bullet}x = \{y \in P \cup T / (y, x) \in F\}$ and the postset of x, denoted by $x^{\bullet}$, is defined as $x^{\bullet} = \{y \in P \cup T / (x, y) \in F\}$. Generally, for a set of node X, we have

$$^{\bullet}\mathrm{X} = \bigcup_{x \in \mathrm{X}} {}^{\bullet}x \qquad \text{and} \qquad \mathrm{X}^{\bullet} = \bigcup_{x \in \mathrm{X}} x^{\bullet}$$

That is,

if $\mathrm{X} = \{x_1, x_2, x_3, x_4, x_5, x_6, ..., x_n\}$ then,

$$^{\bullet}\mathrm{X} = {}^{\bullet}x_1 \cup {}^{\bullet}x_2 \cup \cdots \cup {}^{\bullet}x_n$$

$$\mathrm{X}^{\bullet} = \bigcup_{i \in \{1,2,\cdots,n\}} {}^{\bullet}x_i \quad \text{and can also be written as} \quad \mathrm{x} = \bigcup_{x \in \mathrm{x}} {}^{\bullet}x$$

Example 2.2

In Fig. 2.1, we can compute the Preset and Postset of some transitions and places as follows:

${}^{\bullet}t_1 = \{p_1, p_9\}$, ${}^{\bullet}p_2 = \{t_1\}$, ${}^{\bullet}t_2 = \{p_2, p_{10}\}$, ${}^{\bullet}p_3 = \{t_2\}$
$t_1^{\bullet} = \{p_2\}$, $p_2^{\bullet} = \{t_2\}$, $t_2^{\bullet} = \{p_3, p_9\}$, $p_3^{\bullet} = \{t_3\}$

2.1.2 Definition 2: Marking

A marking *(m)* of a Petri net $N = (P, T, F, W)$ is a mapping from P to ℕ where ℕ = $\{0, 1, 2, \cdots\}$, and *m* is a vector of a dimension (1x n) where n is the size of places in the net. Using Fig. 2.1, by redefining Petri net as a marked net or net system with (N, m), where $N = (N, T, F, W)$.

2.1.3 Definition 3: Enabled Transition

Let $t \in T$ be a transition in $N = (P, T, F, W)$ at a marking *m*. Transition t is said to be enabled *if* $\forall p \in {}^{\bullet}t, m(p) \geq W(p, t)$.

Example 2.3

In Fig. 2.2(a), according to Definition 3, t is not enabled as condition three is not satisfied.

$^{\bullet}t_1 = \{p_1, p_2, p_3\}$

$m(p_1) = 1$	$W(p_1, t_1) = 1$	$m(p_1) \geq W(p_1, t_1)$
$m(p_2) = 2$	$W(p_2, t_1) = 2$	$m(p_2) \geq W(p_2, t_1)$
$m(p_3) = 2$	$W(p_3, t_1) = 3$	$m(p_3) \leq W(p_3, t_1)$

It is clear that $m(p_1)$ and $m(p_2)$ satisfy but $m(p_3)$ does not satisfy the enabling condition. Hence t_1 is not enabled in Fig. 2.2(a).

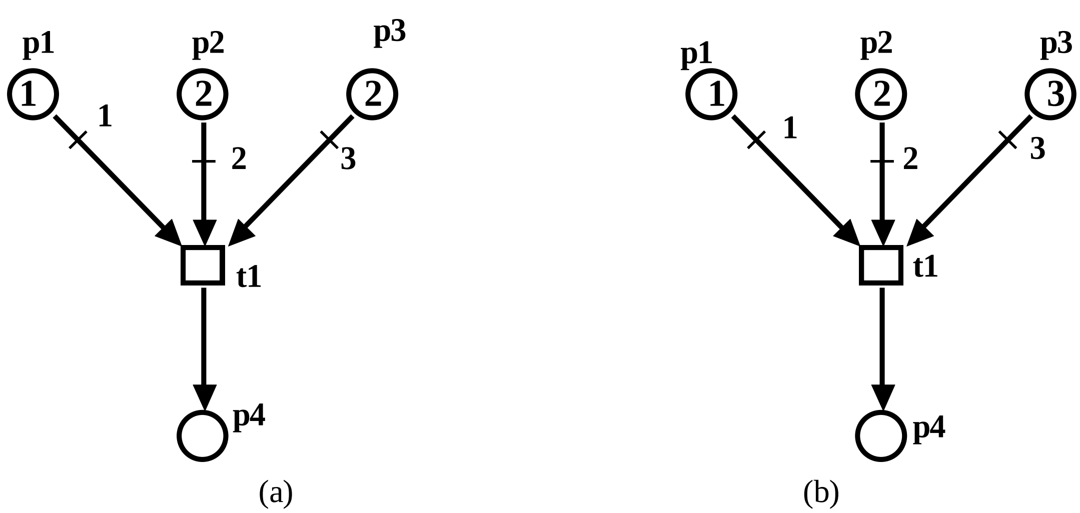


Fig. 2.2. A Petri net model (a) t_1 is not enabled, (b) t_1 is enabled.

By considering Fig. 2.2(b), according to the Definition 3,

$$^{\bullet}t_1 = \{p_1, p_2, p_3\}$$
$$\begin{array}{lll} m(p_1) = 2 & W(p_1, t_1) = 1 & m(p_1) \geq w(p_1, t_1) \\ m(p_2) = 3 & W(p_2, t_1) = 2 & m(p_2) \geq w(p_2, t_1) \\ m(p_3) = 4 & W(p_3, t_1) = 3 & m(p_3) \geq w(p_3, t_1) \end{array}$$

$m(p_1), m(p_2)$ and $m(p_3)$ all satisfy the enabling condition and thus t_1 is enabled.

2.1.4 Definition 4: Firing rule

An enabled t can fire, leading to a new marking m', i.e, $\forall p \in P$, $m'(p) = m(p) + W(t, p) - W(p, t)$. In Fig. 2.3, $m(p_1) = c_1$, $m(p_2) = c_2$ and $m(p_3) = d$. Transition t is enabled when $c_1 \geq a_1$ and $c_2 \geq a_2$. Then according to the Definition 4, $m'(p_1) = c_1 - a_1$, $m'(p_2) = c_2 - a_2$ and $m'(p_3) = d + b_1$. The reachability set of (N, m_0), denoted as $R(N, m_0)$, is the set of reachable markings from the initial marking m_0. Note that in order to represent the number of tokens in a place the symbol 'μ' is also used widely. For example $m(p_1)$ represents the number of tokens in place p_1. Instead, the same is also represented by μ_1.

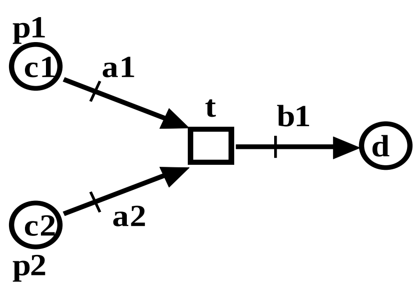


Fig. 2.3. A generalized Petri net.

2.1.5 Definition 5: Boundedness

A place $p \in P$ is said to be bounded *if* $\forall m \in R(N, m_0)$, $\exists k \in \mathbb{N} \, (k \neq 0), m(p) \leq k$.

A Petri net is said to be *k*-bounded if the number of tokens in each place does not exceed a finite number '*k*' at every reachable marking.

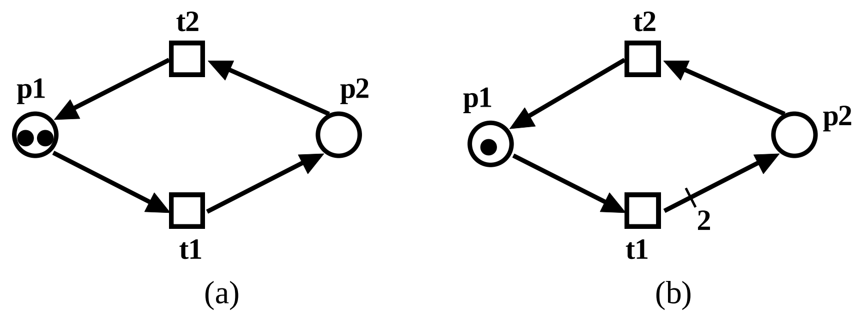


Fig. 2.4 (a) Bounded net example and (b) Unbounded net example.

Fig 2.4(a) shows a typical example of bounded net in which the number of tokens in circulation are constantly maintained. In Fig. 2.4(b), the reachability graph the of net keeps expanding and it can never be terminated. Hence we can conclude that it is unbounded.

2.1.6 Definition 6: Safeness

A place $p \in P$ is said to be safe *if* $\forall m \in R(N, m_0), \quad m(p) \leq 1$. A Petri net is said to be safe if all of its places are safe. A place '*p*' is safe if it contains no more than one token.

2.1.7 Definition 7: Liveness

Let $t \in T$ be a transition in (N, m_0). Transition t is said to be live *if* $\forall m \in R(N, m_0), \exists m' \in R(N, m_0)$, such that $m'[t>$. (N, m_0) is live *if* $\forall t \in T, t$ is live. A transition is said to be live if for all markings of the Petri net there is a firing sequence, which takes the net to a marking, in which the transition is enabled.

2.1.8 Definition 8: Deadlock

N is dead under M_0 iff $\nexists\, t \in T$, $M_0[t>$ holds. *t* is not enabled. A deadlock is usually an undesirable condition that when occurring, it blocks the whole or a part of the running processes. It might also cause a catastrophic result such as long downtime and low utilization of resources. Another idea is deadlocks-freeness, in which some transitions are firable while some are totally dead. Fig. 2.5 shows all these three cases.

2.2 MATHEMATICAL TREATMENT OF PETRI NET USING LINEAR ALGEBRA

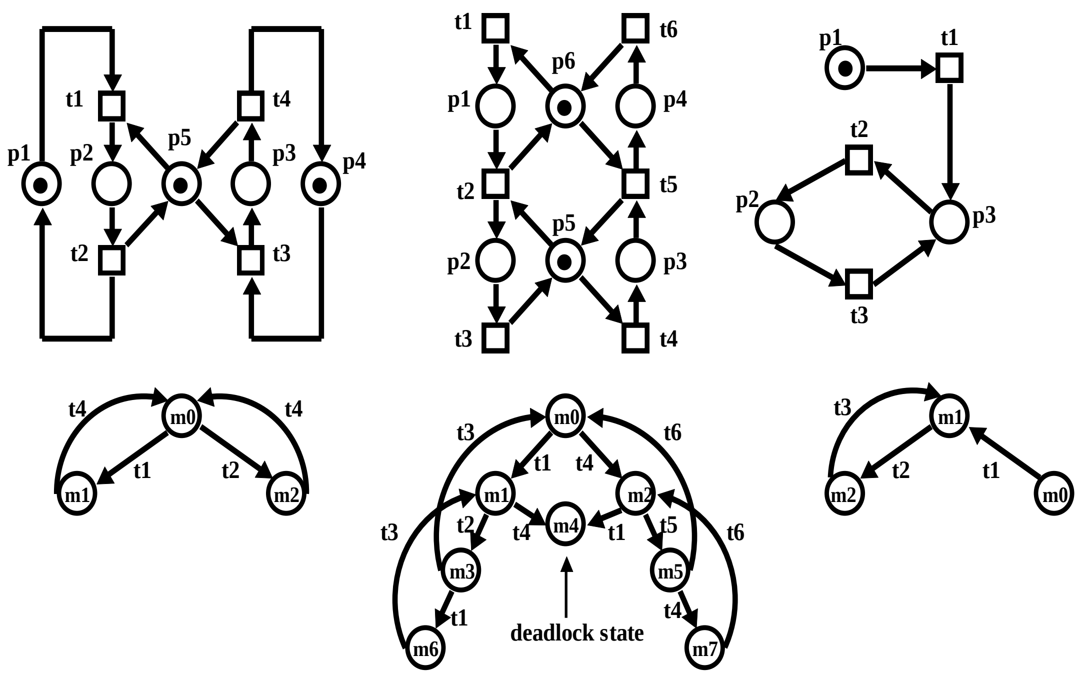
2.2.1 Definition 9: Incidence matrix

Let $N=(P,T,F,W)$ be a net system. Its incidence matrix, denoted by $[N]$, is the matrix of size $|P|\times|T|$ with $[N](p,t)=W(t,p)-W(p,t)$, where $|P|$ is the cardinality of P and $|T|$ is the cardinality of T. However $[N]$ can be divided into two matrices as

$$[N]=[N]^{+}-[N]^{-},$$

where $[N]^{+}$ is the integer matrix of directed inputs arcs.

$[N]^{-}$ is the integer matrix of directed outputs arcs.



(a) (b) (c)

(a)
$m_0=p_1+p_4+p5$
$m_1=p_2+p_4$
$m_2=p_1+p_3$

(b)
$m_0=p_5+p_6$, $m_5=p_4+p_5$
$m_1=p_1+p_5$, $m_6=p_1+p_2$
$m_2=p_3+p_6$, $m_7=p_3+p_4$
$m_4=p_1+p_3$

(c)
$m_0=p_1$
$m_1=p_3$
$m_2=p_2$

Fig. 2.5. (a) a live Petri net, (b) Deadlock in a Petri net system and (c) A deadlock free Perti net (live locked Petri net).

The incidence matrix of the Petri net shown in Fig. 2.5(b) is as follows.

$$[N]^+ = \begin{array}{c} \begin{array}{cccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{array}, \qquad [N]^- = \begin{array}{c} \begin{array}{cccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{array}$$

$$\text{Hence, } [N] = [N]^+ - [N]^- = \begin{array}{c} \begin{array}{cccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{array}$$

Due to the flexibility of Petri nets as a mathematical tool, it is possible to develop a software package to simulate the behavior of an FMS.

2.2.2 Definition 10: Firing sequence

Let σ be a transition sequence. Then the parikh vector of σ, denoted by $\vec{\sigma}$, is given as $\vec{\sigma} = [\#\sigma(t_1), \#\sigma(t_2), \cdots, \#\sigma(t_n)]$, and $|T| = n,$ where $\#\sigma(t_i)$ denotes the number of appearances in the sequence σ.

By considering the net system shown in Fig. 5(b), $m_7 = \begin{array}{c} \begin{array}{cccccc} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 \end{array} \\ [\begin{array}{cccccc} 0 & 0 & 1 & 1 & 0 & 0 \end{array}]^T \end{array}$. Hence, the firing sequence and parikh vector corresponding to that marking are given as $\sigma = t_4 t_5 t_4$ and $\vec{\sigma} = \begin{array}{c} \begin{array}{cccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \end{array} \\ [\begin{array}{cccccc} 0 & 0 & 0 & 2 & 1 & 0 \end{array}] \end{array}$. The advantage of parikh vector is that any marking can be found given an incidence matrix and the initial marking as

$$m = m_0 + [N].\vec{\sigma} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.1)$$

By considering Fig. 5(b), we can prove Eq. (2.1) by using the incidence matrix and the initial marking are given as

$$[N]=\begin{array}{c c} \begin{array}{cccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \end{array} & \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 & -1 & 1 \end{bmatrix} & \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{array} \end{array} \quad \text{and} \quad m_0 = \begin{array}{cccccc} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 \\ [0 & 0 & 0 & 0 & 1 & 1]^T \end{array}$$

To find the new marking m_7 using $\vec{\sigma}=\begin{bmatrix}0 & 0 & 0 & 2 & 1 & 0\end{bmatrix}^T$, we have

$$m_7=\begin{bmatrix}0\\0\\0\\0\\1\\1\end{bmatrix}+\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix}0\\0\\0\\2\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\1\\1\end{bmatrix}+\begin{bmatrix}0\\0\\1\\1\\-1\\-1\end{bmatrix}=\begin{bmatrix}0\\0\\1\\1\\0\\0\end{bmatrix}$$

$m_7=\begin{bmatrix}0 & 0 & 1 & 1 & 0 & 0\end{bmatrix}^T$. Hence, we prove it as given above.

2.2.3 Definition 11: Place invariant

A P-vector is a column vector L: $P \rightarrow \mathbb{Z}$, denoted by P. $\forall p \in P$, $L(p) \in \mathbb{Z}$ and a P-vector L is a place invariant if $\mathrm{L} \neq 0$ and $\mathrm{L}^{\mathrm{T}}[N]=0^T$, where $\mathbb{Z}$={…, -2, -1, 0, 1, 2, … }. The major advantage of a place invariant (PI) is to keep the capacity of tokens to be in circulation within it. In Fig. 2.1, the PIs and their supports are,

$$\|L_1\|=\{p_4, p_5, p_{11}\} \qquad \mathrm{L}_1=\begin{pmatrix}0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}$$

$$\|L_2\|=\{p_3, p_6, p_{10}\} \qquad \mathrm{L}_2=\begin{pmatrix}0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0\end{pmatrix}$$

$$\|L_3\|=\{p_2, p_7, p_9\} \qquad \mathrm{L}_3=\begin{pmatrix}0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0\end{pmatrix}$$

$$\|L_4\|=\{p_1, p_2, p_3, p_4\} \qquad \mathrm{L}_4=\begin{pmatrix}1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{pmatrix}$$

$$\|L_5\|=\{p_5, p_6, p_7, p_8\} \qquad \mathrm{L}_5=\begin{pmatrix}0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0\end{pmatrix}$$

2.2.4 Definition 12: T-invariant

A T-vector is a column vector $H:T \to \mathbb{Z}$, denoted by T. $\forall t \in T$, $H(t) \in \mathbb{Z}$ and a T-vector H is the transition invariant *if* $[N]H = 0$ and $H \neq 0$. By considering a T-vector, its advantage is to return to the initial marking from a certain marking. In Fig. 2.1, the T-vectors are:

$$H_1 = \begin{array}{c} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{array}\begin{array}{c} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \\ t_8 \end{array} \quad \text{and} \quad H_2 = \begin{array}{c} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \end{array}\begin{array}{c} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \\ t_8 \end{array}$$

However, since $[N]H = 0, H = \overrightarrow{\sigma}$. By considering the equation above, we have

$$m = m_0 + [N]H \quad \text{where } H = \overrightarrow{\sigma}$$

Hence, $m = m_0$

2.2.5 Definition 13: Siphons

Let $S \subseteq P$ be a non-empty set of places. S is called a siphon if ${}^{\bullet}S \subseteq S^{\bullet}$, where

$${}^{\bullet}\mathrm{S} = \bigcup_{p \in s} {}^{\bullet}p \qquad \text{and} \qquad \mathrm{S}^{\bullet} = \bigcup_{p \in s} p^{\bullet}$$

A siphons is a set of places, which remains permanently unmarked once the all tokens are lost. When this occurs, the transitions associated with the siphon are permanently disabled. For this reason siphons are extensively studied in the literature [15]. Siphons play a very important role in deadlock prevention due to their feature related to a deadlock. All existing PI satisfy the definition of siphons but there are marked siphons. By considering the net in Fig. 2.1, let us check whether S_1, S_2 and S_3 are siphons or not.

$$S_1 = \{p_4, p_7, p_9, p_{10}, p_{11}\}$$
$$S_2 = \{p_4, p_6, p_{10}, p_{11}\}$$
$$S_3 = \{p_4, p_7, p_{10}, p_{11}\}$$

We have

$$^{\bullet}S_1 = {}^{\bullet}p_4 \cup {}^{\bullet}p_7 \cup {}^{\bullet}p_9 \cup {}^{\bullet}p_{10} \cup {}^{\bullet}p_{11} = \{t_2, t_3, t_4, t_6, t_7, t_8\}$$
$$S_1^{\bullet} = p_4^{\bullet} \cup p_7^{\bullet} \cup p_9^{\bullet} \cup p_{10}^{\bullet} \cup p_{11}^{\bullet} = \{t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8\}$$

Therefore $^{\bullet}S_1 \subset S_1^{\bullet}$ and S_1 is a siphon.

$$^{\bullet}S_2 = {}^{\bullet}p_4 \cup {}^{\bullet}p_6 \cup {}^{\bullet}p_{10} \cup {}^{\bullet}p_{11} = \{t_3, t_4, t_6, t_7\}$$
$$S_2^{\bullet} = p_4^{\bullet} \cup p_6^{\bullet} \cup p_{10}^{\bullet} \cup p_{11}^{\bullet} = \{t_2, t_3, t_4, t_5, t_6, t_7\}$$

Therefore $^{\bullet}S_2 \subset S_2^{\bullet}$ and S_2 is a siphon.

$$^{\bullet}S_3 = {}^{\bullet}p_4 \cup {}^{\bullet}p_7 \cup {}^{\bullet}p_{10} \cup {}^{\bullet}p_{11} = \{t_3, t_4, t_6, t_7\}$$
$$S_3^{\bullet} = p_4^{\bullet} \cup p_7^{\bullet} \cup p_{10}^{\bullet} \cup p_{11}^{\bullet} = \{t_2, t_3, t_4, t_5, t_6, t_8\}$$

Therefore $^{\bullet}S_3 \not\subset S_3^{\bullet}$ and S_3 is not a siphon.

However, an emptied siphon is the one that cause deadlocks (dead transitions). For that reason siphons are classified into minimal siphons and strict minimal siphons. A siphon is said to be minimal if there does not exist a siphon contained in it as a proper subset. While a siphon is said to be strict minimal if it is minimal and does not contain a marked trap. A strict minimal siphon is denoted as SMS for short [7]. However, among the strict minimal siphons they are still divided into essential siphons and dominated siphons. Hence only essential siphons get emptied and need to be controlled to avoid deadlock occurrence. To control siphons, a complementary set of a siphon is evaluated, which stands as place invariant for that siphon.

A complementary sets of emptiable siphon is defined as a set of places (set of places that steal tokens from the siphon) that when added to the structure of emptiable siphon, it completes the number of place invariants formed by that emptiable siphon. The complementary sets of emptiable siphons are used as place invariants to control the emptiable siphons from losing all its tokens. Hence it prevents the siphon from being unmarked.

Example:

To illustrate the structure of emptied siphons and to show the computation of their complementary sets, the Petri net model shown in Fig. 2.6 is considered. This model has six siphons three of which can be unmarked. As shown in Fig. 2.7, the emptiable siphons are

$S_1 = \{p_3, p_6, p_7, p_8, p_9\}$, $S_2 = \{p_3, p_5, p_8, p_9\}$ and $S_3 = \{p_2, p_6, p_7, p_8\}$ with their corresponding complementary sets $[S_1] = \{p_1, p_2, p_4, p_5\}$, $[S_2] = \{p_2, p_4\}$ and $[S_3] = \{p_1, p_5\}$, respectively.

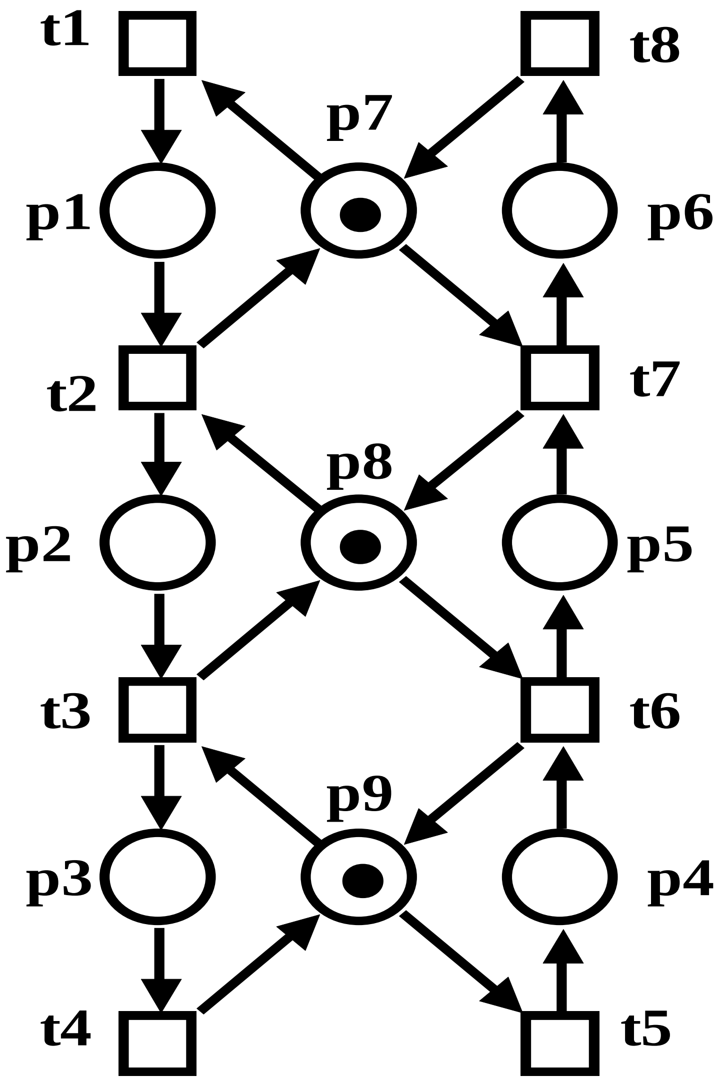


Fig. 2.6. A Petri net example.

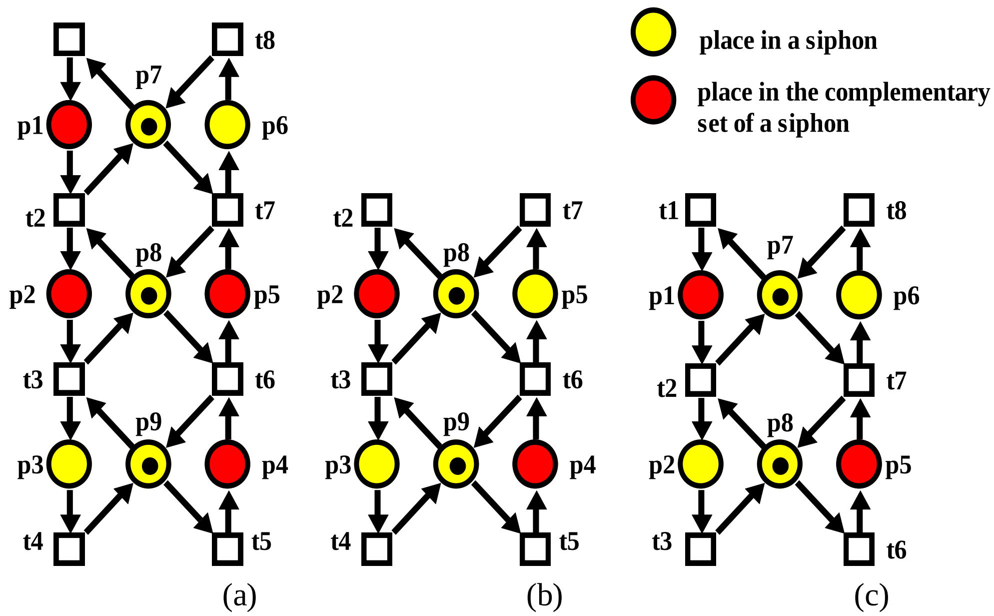


Fig. 2.7 (a) Siphon S_1 and its complementary set $[S_1]$. (b) Siphon S_2 and its complementary set $[S_2]$. (c) Siphon S_3 and its complementary set $[S_3]$.

2.2.6 Definition 14: Control Depth Variable

The controlled depth variable is defined as the least number of tokens that place invariants can hold obviously is equal to or greater than 1 to achieve a control purposed [12]. It is represented by the symbol η.

CHAPTER 3

THE PROPOSED ALGORITHM FOR STRUCTURAL COMPLEXITY REDUCTION OF LIVENESS-ENFORCING SUPERVISOR

In this study, the aim is to develop a method that would generate a maximally permissive behavior or near-optimal behavior with a minimal supervisory structure, i.e., a supervisory structure that has fewer number of control places and directed arcs. The major advantages of generating fewer number of control places and fewer number of arcs are stated as follows [36]:

i) It reduces the cost of implementation and validation drastically in a flexible manufacturing system due to the use of a very compact supervisor.

ii) It reduces the running costs of daily maintenance for the plant of flexible manufacturing industries because of the less number of monitors to be controlled.

iii) It also reduces the time taken for a particular product to be produced as a result of a small number of a controller to be used.

In our study, the idea is to compute the set of control places for a particular net using a first-met bad marking (FBM) based method. The control places computed are the objective constraint that would be reduced to a number as minimum as possible. The main idea is to identify the place invariants that should be merged to form a resultant place invariant. A linear relationship is built up among the sets of place invariants that have been identified to be merged. A systematic approach would follow to reduce these sets of control places to be possibly minimum which has the same behavior with the original sets of control places.

Two or more place invariants can be merged together if they have a common intersecting elements between them. This stands as a core condition. However, there are some supporting conditions that play very vital role for merging two or more place invariants named as subsidiary conditions. These subsidiary conditions are: (i) structural orientation of place invariants and (ii) the initial number of tokens of place invariants.

Let us explain these two subsidiary conditions. Consider a Petri net model with two processes A and B running in an opposite manner with shared resources between them. Process A has an elements of $p_2, p_3, p_4,$ and p_5 and are connected consecutively, while process B has an elements of $p_7, p_8, p_9,$ and p_{10} which are connected in a consecutive manner. Assume that the place invariants are given as follows: $PI_1 = \mu_2 + \mu_7 \leq 2$, $PI_2 = \mu_2 + \mu_8 \leq 2$ and $PI_3 = \mu_2 + \mu_9 \leq 2$. First of all, the core condition is satisfied because of the common intersecting element i.e. p_2. For structural orientation of place invariants PI_1, PI_2 and PI_3, the elements that are not part of the common intersecting element (i.e. p_7, p_8, p_9) are belong to one process and is connected consecutively. Such kind of structural place invariants can be easily merged together. Place invariants with the same initial number of tokens could be originating from one strict minimal siphon. For example if PI_1, PI_2 and PI_3 have the same number of initials tokens. Then it may be possible to merge these *PIs*. In general, for two or more place invariants to be merged, the elements that are not part of the common intersecting elements (i.e. non-intersecting elements among the place invariants to be merged) should be connected consecutively.

For each possible set of place invariants, linear equations between the possible merged place invariants are established. In forming the linear equations, the number of tokens of a first-met bad marking (a_n) play a vital role for finding the relationship between the unknown co-efficients and the initial number of tokens of a merged place invariant. After obtaining the relationship between the unknown co-efficients of merged place invariants, possible values would be assigned to the co-efficients for obtaining a final merged place invariant. The assigned values should be as minimum as possible and most of the time it takes the values of the cardinalities in the set of place invariants.

Example 3.1

Let us consider the Petri net model in Fig. 3.1(a) together with place invariants computed using an FBM based method. The Petri net model 1 has five control places shown in Table 3.1 together with their place invariants. To analyze the importance of the subsidiary conditions in identifying the place invariants that could be merged, let us consider place invariants $PI_1, PI_3,$ and PI_5. Place invariants $PI_1, PI_3,$ and PI_5 have a common intersecting element (i.e. p_2), that satisfied the core-condition for merging two or more place invariants. For subsidiary condition (i.e. S.C. 1), let us consider $PI_1, PI_3,$ and PI_5. The intersecting element belongs to one particular process while the rest of the elements (i.e. $p_{11}, p_{12},$ and p_{13}) belong to another particular process. From the structural orientation, $p_{11}, p_{12},$ and p_{13} are connected consecutively. This kind of structural orientation of the place invariants can easily be merged together.

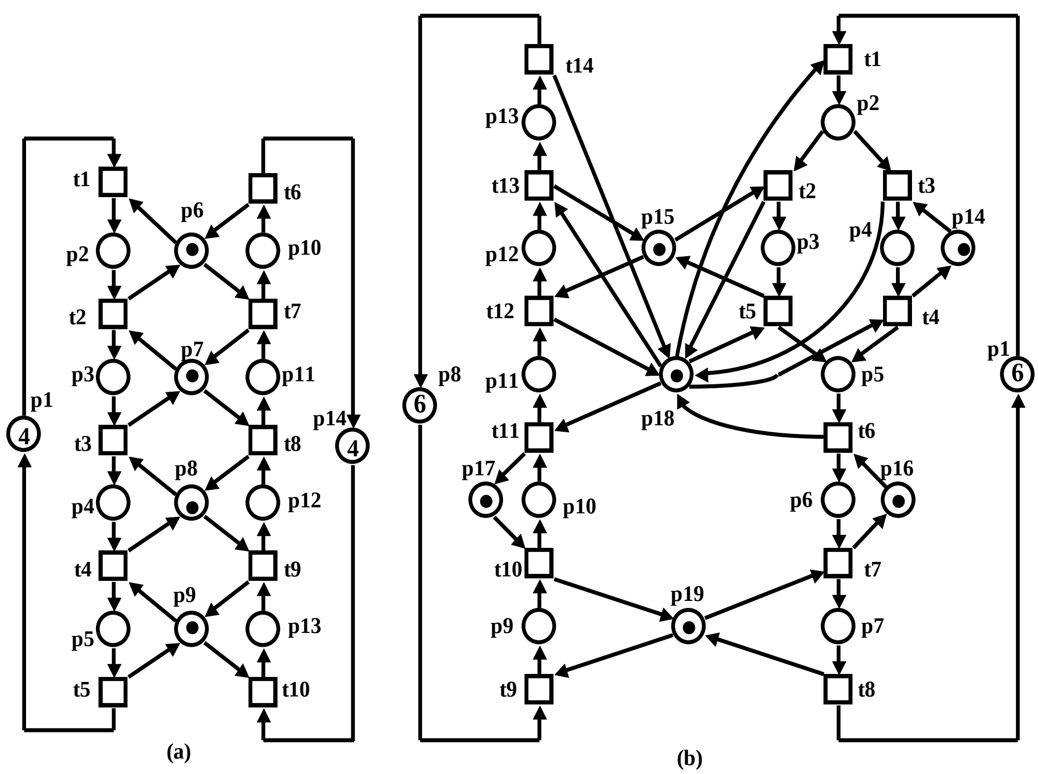


Fig. 3.1 (a) Petri net model 1 used to illustrate the possible merged place invariants. (b) Petri net model 2 used it to illustrate some place invariants that cannot be merged.

Table 3.1. Place invariants and control places computed for the PNM shown in Fig. 3.1(a).

FBMi	*PIi*	C_i	$^{\bullet}C$	$C^{\bullet}$	*M0*
$\mu_2 = 1,\ \mu_{13} = 1$	$PI_1 = \mu_2 + \mu_{13} \le 1$	C_1	t_2, t_9	t_1, t_{10}	1
$\mu_3 = 1,\ \mu_{13} = 1$	$PI_2 = \mu_3 + \mu_{13} \le 1$	C_2	t_3, t_9	t_2, t_{10}	1
$\mu_2 = 1,\ \mu_{12} = 1$	$PI_3 = \mu_2 + \mu_{12} \le 1$	C_3	t_2, t_8	t_1, t_9	1
$\mu_4 = 1,\ \mu_{13} = 1$	$PI_4 = \mu_4 + \mu_{13} \le 1$	C_4	t_4, t_9	t_3, t_{10}	1
$\mu_2 = 1,\ \mu_{11} = 1$	$PI_5 = \mu_2 + \mu_{11} \le 1$	C_5	t_2, t_7	t_1, t_8	1

Table 3.2 Place invariants and Control places computed for the S[3]PR shown in Fig. 3.1(b).

FBMi	*PI*	$^{\bullet}C$	$C^{\bullet}$	*M0*
$\mu_3 = 1, \mu_{11} = 1$	$PI_1 = \mu_3 + \mu_{11} \le 1$	t_5, t_{12}	t_2, t_{11}	1
$\mu_{11} = 1, \mu_{12} = 1$	$PI_2 = \mu_{11} + \mu_{12} \le 1$	t_{13}	t_{11}	1
$\mu_2 = 1, \mu_3 = 1, \mu_4 = 1$	$PI_3 = \mu_2 + \mu_3 + \mu_4 \le 2$	t_4, t_5	t_1	2
$\mu_2 = 1, \mu_4 = 1, \mu_{12} = 1$	$PI_4 = \mu_2 + \mu_4 + \mu_{12} \le 2$	t_2, t_4, t_{13}	t_1, t_{12}	2
$\mu_5 = 1, \mu_6 = 1, \mu_9 = 1, \mu_{10} = 1$	$PI_5 = \mu_5 + \mu_6 + \mu_9 + \mu_{10} \le 3$	t_7, t_{11}	t_4, t_5, t_9	3
$\mu_3 = 1, \mu_6 = 1, \mu_9 = 1, \mu_{10} = 1$	$PI_6 = \mu_3 + \mu_6 + \mu_9 + \mu_{10} \le 3$	t_5, t_7, t_{11}	t_2, t_6, t_9	3
$\mu_3 = 1, \mu_5 = 1, \mu_9 = 1, \mu_{10} = 1$	$PI_7 = \mu_3 + \mu_5 + \mu_9 + \mu_{10} \le 3$	t_6, t_{11}	t_2, t_4, t_9	3
$\mu_2 = 1, \mu_4 = 1, \mu_6 = 1, \mu_9 = 1, \mu_{10} = 1$	$PI_8 = \mu_2 + \mu_4 + \mu_6 + \mu_9 + \mu_{10} \le 4$	t_2, t_4, t_7, t_{11}	t_1, t_6, t_9	4

Secondly, let us consider Petri net model 2 in Fig 3.1.(b) together with control places and place invariants computed using an FBM based method. The Petri net model 2 has eight control places shown in Table 3.2 together with their place invariants. To show the importance of the subsidiary conditions in identifying the place invariants that could be merged together, let us consider place invariants PI_3 and PI_8. Both place invariants PI_3 and PI_8 have common intersecting elements (i.e. p_2, p_4) that satisfy the core condition for merging two or more place invariants. For subsidiary conditions, let us analyze the structural orientation of the place invariants PI_3 and PI_8. Both intersecting elements (i.e.

p_2, p_4) belong to one particular process while the rest of the elements i.e. p_3, p_9 and p_{10} are not connected consecutively. Hence, structurally it is not possible to merge the place invariants PI_3 and PI_8, Even though, they satisfy the core condition.

If some place invariants cannot be merged, then they are left as they are. It is to say that, all the place invariants computed using FBM based method should all be covered in the final simplified set of merged place invariants.

In the following algorithm it is assumed that, an uncontrolled Petri net model (PNM) with a set of control places obtained by an FBM based method together with their related place invariants (PI) are given. Our objective is to reduce the set of control places such that the supervisory structure is reduced.

3.1 ALGORITHM: STRUCTURAL COMPLEXITY REDUCTION OF LIVENESS ENFORCING SUPERVISORS

Input: The PN model of an FMS prone to deadlocks, a set of monitors $(C_1, C_2, \cdots, C_n)$ to enforce liveness on this PNM obtained by an FBM based method together with their related place invariants (PIs) i.e. $PI_1, PI_2, ..., PI_n$.

Output: reduced monitors, i.e. $[C_1, C_2, ..., C_m]$, *m*<*n*, to enforce liveness on the PNM with similar behavioral permissiveness.

1. Identify $Z_i = \{Z_1, Z_2, Z_3, \cdots\}$

   Where Z_i is the set of possible place invariants that can be merged together.

2. For each Z_i

   2.i.1. If $Zi = PI1 \cap PI2 \cap PI3 \cap \cdots = \phi$

   Exit

Else 2.i.2. A tentative draft resulting merged place invariant (mPI_i) is computed as:

$$mPI_i = \alpha_1\mu_1 + \alpha_2\mu_2 + \alpha_3\mu_3 + \cdots + \alpha_n\mu_n \le k \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3.1)$$

Where $\alpha_1, \alpha_2, \alpha_3, \cdots$ are the co-efficients of merged place invariants.

k is the initial number of tokens for merged placed invariants.

$\mu_1, \mu_2, \mu_3, \cdots$ are the possible elements of merged place invariants.

2.i.3. Establish a linear equation between the possible merged place invariants as follows:

$$a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 + \cdots + a_n\alpha_n = k+1 \quad \text{..........................} \quad (3.2)$$

$$a_2\alpha_2 + a_3\alpha_3 + a_4\alpha_4 + \cdots + a_{n+1}\alpha_{n+1} = k+1 \quad \text{..........................} \quad (3.3)$$

$$a_3\alpha_3 + a_4\alpha_4 + a_5\alpha_5 + \cdots + a_{n+2}\alpha_{n+2} = k+1 \quad \text{..........................} \quad (3.4)$$

$$\vdots$$

Where $a_1, a_2, a_3, \cdots, a_n, a_{n+1}, a_{n+2}, a_{n+3}, \cdots$ are FBM co-efficients of each element in the set of possible place invariants that would be merged.

2.i.4. Compute a linear relationship that exists among the co-efficients of resultant merged place invariants.

2.i.5. assign values to the co-efficients of the linear relationship that exists among them.

2.i.6. Evaluate the value of k in any one of the equations.

2.i.7. Substitute the corresponding value for each co-efficient in the resultant draft possible merged place invariant has formulated.

2.i.8. Compute the monitor C_i by using the mPI_i.
End of algorithm.

3.2 RULES TO BE CONSIDERED IN THE IMPLEMENTATION OF THE PROPOSED ALGORITHM

Rule 1.

To evaluate the co-efficients for each element of the resultant merged place invariant, it is assumed that all possible co-efficients for each place invariant to be merged would be equated to a one more than a certain constant (i.e. $k+1$) to form a linear equation.

Example 3.2.1.

For better understanding, let us illustrate rule 1 by an example. Assume that the following place invariants are identified to be merged together. $\mu_5+\mu_6+\mu_9+\mu_{10}\le 3$, $\mu_3+\mu_6+\mu_9+\mu_{10}\le 3$, $\mu_3+\mu_5+\mu_9+\mu_{10}\le 3$. Then according to the rule 1, the right hand side of all place invariants should be equated to $k+1$ as follows:

$$\mu_5+\mu_6+\mu_9+\mu_{10}\le k+1$$

$$\mu_3+\mu_6+\mu_9+\mu_{10}\le k+1$$

$$\mu_3+\mu_5+\mu_9+\mu_{10}\le k+1$$

Rule 2.

There are some exceptions to the Rule 1. This is due to the control depth variable, the value used for control depth variable is greater than one for some few place invariants. In such case, all possible co-efficients for each place invariant to be merged would be equated to a value used for a control depth more than a certain constant (i.e. $k+\eta$).

Example 3.2.2.

For better understanding, let us illustrate the rule 2 by an example. Assume that the following place invariants are identified to be merged together. $\mu_3+\mu_{11}+\mu_{12}\le 1$, and $\mu_2+\mu_3+\mu_4+\mu_{12}\le 2$. It is clear to see that the control depth variable (i.e. η) used is all more than one. Then according to the rule 2, the right hand side of all the place invariants should be equated to $k+\eta$ as follows:

$$\mu_3+\mu_{11}+\mu_{12}\le k+2$$

$$\mu_2+\mu_3+\mu_4+\mu_{12}\le k+2$$

Rule 3.

Despite the fact that, the number of variables of co-efficients to be solved is greater than the number of equations generated from the co-efficients of possible place invariants to be merged, an alternative approach would follow to simplify the computation. It is assumed that all their common elements have equal value for the co-efficient in the resulting merged place invariant.

Example 3.2.3.

For better understanding, let us illustrate rule 3 by an example. Assume that the following equations are given to evaluate the unknown variables.

$$\mu_5 + \mu_6 + \mu_9 + \mu_{10} \leq k + 1$$

$$\mu_3 + \mu_6 + \mu_9 + \mu_{10} \leq k + 1$$

It is well known that to evaluate the unknown variables in mathematics, the number of variables should be equal to the number of equations to be generated. In this example the unknown variables are five while the number of equations generated are only two. Hence to solve that problem, rule 3 take care of that case by assuming all the common elements within the equations to be evaluated must have equal values.

Rule 4.

To have a maximally permissive behavior or near optimal behavior, the value of the co-efficient for their common elements among the possible place invariants to be merged together should be greater than or equal to their cardinalities.

Example 3.2.4.

For clear understanding of the rule 4, let us illustrate it by an example. Assume that the following equations are given to evaluate the unknown variables as follows:

$$\mu_5 + \mu_6 + \mu_9 + \mu_{10} \leq k + 1$$

$$\mu_3 + \mu_6 + \mu_9 + \mu_{10} \leq k + 1$$

There are some common elements between the two equations i.e. $\mu_6, \mu_9,$ and μ_{10}. Hence their values are the cardinality of the elements (i.e. $\mu_6, \mu_9,$ and μ_{10}) in the given sets of the equations. In this case the cardinality of the elements in the given equations is two. Hence their values for $\mu_6, \mu_9,$ and μ_{10} are all two.

Rule 5.

There are some exceptions to the rule 4. Some place invariants (i.e. computed using first-met bad marking based method) belong to one particular process. In such case, to have maximally permissive behavior with reduced structure, the value of the co-efficients of their common elements among the possible place invariants to be merged should be one less than the total cardinalities or greater than that value.

Example 3.2.5.

For clear understanding of rule 5. Let us illustrate it by an example. Assume that the following equations are given to evaluate the unknown variables as follows:

$$\mu_3 + \mu_{11} \leq 1$$

$$\mu_{11} + \mu_{12} \leq 1$$

If μ_{11} and μ_{12} are connected consecutively and belong to a particular process, in that case the values for their common element (i.e. μ_{11}) should be one less than the cardinality of the element in the given equations. In this example the cardinality of the common intersecting element is two, while the value of the common intersecting element (i.e. μ_{11}) is one.

3.3 ILLUSTRATIVE EXAMPLE

To demonstrate the proposed structural complexity reduction method, let us consider the PNM of an FMS [29] shown in Fig. 3.2, monitors are due to FBM variant Liveness-enforcing supervisor [34]. The net has 14 places and 10 transitions. Their places can be considered to be the collection of $P^0 = \{p_1, p_{14}\}$, $P_A = \{p_2, p_3, p_4, p_5, p_{10}, p_{11}, p_{12}, p_{13}\}$ and $P_R = \{p_6, p_7, p_8, p_9\}$. The net has 48 reachable states in which there are 17 bad states and 31 good states.

Control places (monitors) computed for this PNM shown in Fig. 3.2 are provided in Table 3.3, together with their *PIs.* The controlled PNM obtained by including the five control places shown in Table 3.3 into the uncontrolled PNM shown in Fig. 3.2 is live and can reach 31 good state.

Table 3.3. Place invariants and control places computed for the PNM shown in Fig. 3.2.

FBM_i	PI_i	C_i	${}^{\bullet}C$	$C^{\bullet}$	M_0
$\mu_2 = 1,\ \mu_{13} = 1$	$PI_1 = \mu_2 + \mu_{13} \le 1$	C_1	t_2, t_9	t_1, t_{10}	1
$\mu_3 = 1,\ \mu_{13} = 1$	$PI_2 = \mu_3 + \mu_{13} \le 1$	C_2	t_3, t_9	t_2, t_{10}	1
$\mu_2 = 1,\ \mu_{12} = 1$	$PI_3 = \mu_2 + \mu_{12} \le 1$	C_3	t_2, t_8	t_1, t_9	1
$\mu_4 = 1,\ \mu_{13} = 1$	$PI_4 = \mu_4 + \mu_{13} \le 1$	C_4	t_4, t_9	t_3, t_{10}	1
$\mu_2 = 1,\ \mu_{11} = 1$	$PI_5 = \mu_2 + \mu_{11} \le 1$	C_5	t_2, t_7	t_1, t_8	1

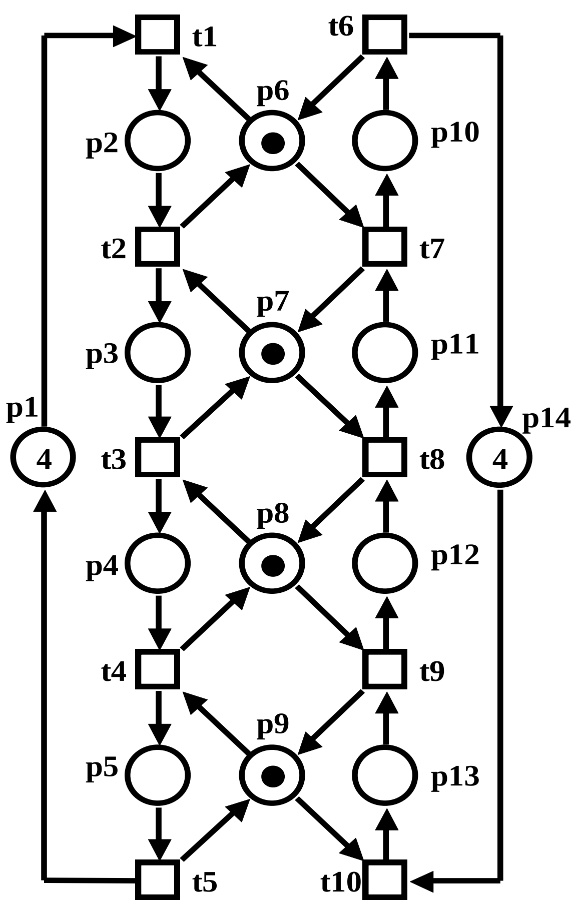


Fig 3.2. A Petri net model (PNM) of an FMS.

Step 1:

The possible set for place invariants to be merged together are evaluated as follows:

Remarks:

By looking the place invariants computed in Table 3.3, the place invariants $PI_1, PI_3,$ and PI_5 have common intersecting element i.e. (p_2) that satisfies the core-condition. The non-intersecting elements i.e. $p_{11}, p_{12},$ and p_{13} are connected consecutively that satisfies the structural orientation for merged place invariants. These would enable the place invariants $PI_1, PI_3,$ and PI_5 to be merged easily. Also it is the same thing when considering $PI_2,$ and PI_4 it has common intersecting element of (p_{13}) with a satisfied structural orientation of a place invariants.

$$Z_1 = \{PI_1, PI_3, PI_5\}$$

$$Z_2 = \{PI_2, PI_4\}$$

Step 2: ($i = 1,$ first iteration)

Let us consider Z_1 first to apply the procedure step by step from 2.1.1 to 2.1.8

Step 2.1.1:

Z_1 has an elements of PI_1, PI_3 and PI_5. The intersecting element among these place invariants are as:

$$PI_1 \cap PI_3 \cap PI_5 = \{p_2\}$$

Step 2.1.2:

A draft merged place invariant mPI_1 should have a form as in Eq. 3.1.1.

$$mPI_1 = \alpha_2\mu_2 + \alpha_{11}\mu_{11} + \alpha_{12}\mu_{12} + \alpha_{13}\mu_{13} \le k \tag{3.1.1}$$

Step 2.1.3:

In this step, linear equations are established by using the co-efficients of the possible place invariants to be merged.

$$a_2 = 1,\ a_{11} = 1,\ a_{12} = 1, \text{ and } a_{13} = 1$$

$$\alpha_2 + \alpha_{13} = k + 1 \tag{3.1.2}$$

$$\alpha_2 + \alpha_{12} = k + 1 \tag{3.1.3}$$

$$\alpha_2 + \alpha_{11} = k + 1 \tag{3.1.4}$$

Step 2.1.4:

This step provide a linear relationship that exists among the co-efficients to be solved in the equations generated in the step 2.1.3.

By equating Eqs. (3.1.2) and (3.1.3), the relationship exists between the unknown co-efficients can be solved:

$$\alpha_2 + \alpha_{13} = \alpha_2 + \alpha_{12}$$

$$\alpha_{13} = \alpha_{12}$$

Also, by equating Eqs. (3.1.2) and (3.1.3), to get the relationship exists between the co-efficients to be solved.

$$\alpha_2 + \alpha_{12} = \alpha_2 + \alpha_{11}$$

$$\alpha_{13} = \alpha_{12}$$

Hence, this shows that $$\alpha_{11} = \alpha_{12} = \alpha_{13} \tag{3.1.5}$$

Step 2.1.5:

To have a minimum structure the values for α_{11}, α_{12} and α_{13} should be as minimum as possible. Let us assume that $\alpha_{11} = \alpha_{12} = \alpha_{13}$=1. Also the values of the co-efficients for the common element should be greater than or equal to the value of its cardinality according to the rule 3 in order to obtain a maximally permissive or near optimal behavior i.e. $\alpha_2 \geq 3$.

Step 2.1.6:

By considering Eq. (3.1.1), the value of k can be obtained as:

$$\alpha_2 + \alpha_{13} = k + 1$$

$$\alpha_2 + 1 = k + 1$$

$$\alpha_2 = k$$

Since, $|p_2| = 3$, if $\alpha_2 = 3$, then the value of $k = 3$.

Step 2.1.7:

Finally after computing all the co-efficients for the resultant merged place invariants, the resultant merged place invariant can be written as:

$$mPI_1 = 3\mu_2 + \mu_{11} + \mu_{12} + \mu_{13} \le 3 \tag{3.1.7}$$

Step 2.1.8:

The monitor C_1 is computed for mPI_1 as follows:

$$\mu_0(C_1) = 3$$

$$Dc_1 = -L_{mPI_1} \,.\, D_{mPI_1}$$

$$Dc_1 = -\begin{array}{c} p_2\ p_{11}\ p_{12}\ p_{13} \\ \begin{bmatrix} 3 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{cc} \begin{array}{cccccc} t_1 & t_2 & t_7 & t_8 & t_9 & t_{10} \end{array} & \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} & \begin{array}{c} p_2 \\ p_{11} \\ p_{12} \\ p_{13} \end{array} \end{array}$$

$$Dc_1 = \begin{array}{c} \begin{array}{cccccc} t_1 & t_2 & t_7 & t_8 & t_9 & t_{10} \end{array} \\ \begin{bmatrix} -3 & 3 & 1 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

Step 2: ($i = 2$, second iteration).

Z_2 is considered in the second iteration.

Step 2.2.1:

Z_2 has elements of PI_2 and PI_4. The intersection between these place invariant is:

$$PI_2 \cap PI_4 = \{p_{13}\}$$

Step 2.2.2:

This step presents the resulting draft possible merge place invariant.

$$mPI_2 = \alpha_3\mu_3 + \alpha_4\mu_4 + \alpha_{13}\mu_{13} \le k \tag{3.1.8}$$

Step 2.2.3:

In this step, linear equations are formed between the co-efficients of ossible place invariants to be merged as follows:

$$a_3 = 1, a_4 = 1, \text{and } a_{13} = 1$$

$$\alpha_3 + \alpha_{13} = k + 1 \tag{3.1.9}$$

$$\alpha_4 + \alpha_{13} = k + 1 \tag{3.1.10}$$

Step 2.2.4:

This step aims to find the linear relationships that exist among the unknown co-efficients in Eqs. (3.1.9) and (3.1.10). This is done by equating Eq. (3.1.9) and (3.1.10):

$$\alpha_3 + \alpha_{13} = \alpha_4 + \alpha_{13}$$

$$\alpha_3 = \alpha_4$$

Step 2.2.5:

To have a minimum structure, the values for α_3 and α_4 should be as minimum as possible. Let us assume that $\alpha_3 = \alpha_4 = 1$. Also the values of the co-efficients for the common element should be greater than or equal to the cardinality according to the rule 3 in order to obtain a maximally permissive behavior i.e. $\alpha_2 \geq 2$.

Step 2.2.6:

By considering Eq. (3.1.9), the value of *k* can be obtained as:

$$\alpha_3 + \alpha_{13} = k + 1$$

$$\alpha_{13} + 1 = k + 1$$

$$\alpha_{13} = k$$

Since, $|p_{13}| = 2$, if $\alpha_{13} = 2$, then the value of $k = 2$.

Step 2.2.7:

Finally after computing all co-efficients, the resultant merged place invariant can be written as:

$$mPI_2 = \mu_3 + \mu_4 + 2\mu_{13} \leq 2 \tag{3.1.11}$$

Step 2.2.8:

The monitor C_2 is computed for mPI_2 :

$$\mu_0(C_2) = 2$$

$$Dc_2 = -L_{mPI_2} \, . \, D_{mPI_2}$$

$$Dc_2 = -\begin{array}{c} \begin{array}{ccc} p_3 & p_4 & p_{13} \end{array} \\ \begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \end{array} \begin{array}{c} \begin{array}{ccccc} t_2 & t_3 & t_4 & t_9 & t_{10} \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} p_3 \\ p_4 \\ p_{13} \end{array}$$

$$Dc_2 = \begin{array}{c} \begin{array}{ccccc} t_2 & t_3 & t_4 & t_9 & t_{10} \end{array} \\ \begin{bmatrix} -1 & 0 & 1 & 2 & -2 \end{bmatrix} \end{array}$$

Finally after applying the above algorithm, the net has only two merged place invariants mPI_1 and mPI_2 as shown in Eqs. (3.1.7) and (3.1.11). These merged *PIs* and monitors computed are shown in Table 3.4. When this first set of reduced monitors C_1 and C_2 are added to the PNM given in Fig. 3.2 the controlled model is obtained. It is verified that the controlled model is live with maximally permissive behavior.

Table 3.4. The merged *PIs* and computed monitors.

C_i	mPI_i	${}^\bullet C$	$C^\bullet$	M_0
C_1	$mPI_1 = 3\mu_2 + \mu_{11} + \mu_{12} + \mu_{13} \le 3$	$3t_2, t_7$	$3t_1, t_{10}$	3
C_2	$mPI_2 = \mu_3 + \mu_4 + 2\mu_{13} \le 2$	$t_4, 2t_9$	$t_2, 2t_{10}$	2

CHAPTER 4

APPLICATION TO S^{3}PR PETRI NET MODEL EXAMPLES

In order to show the applicability of the proposed method, this chapter includes some application examples related to S^{3}PR Petri net models for different manufacturing systems. The examples considered have been studied in several research papers and research works.

4.1 EXAMPLE 4.1

The S^{3}PR Petri net model (PNM) of an FMS [29] is shown in Fig. 4.3, with the set of control places computed using FBM variant liveness-enforcing supervisor [34]. The net has 20 places and 15 transitions. Their places can be considered to be the collection of $P^0 = \{p_1, p_6, p_{11}\}$, $P_A = \{p_2, p_3, p_4, p_5, p_7, p_8, p_9, p_{10}, p_{12}, p_{13}, p_{14}, p_{15}\}$ and $P_R = \{p_{16}, p_{17}, p_{18}, p_{19}, p_{20}\}$ The net has 354 reachable states in which there are 65 bad states and 289 good states.

Control places (monitors) computed for this PNM shown in Fig. 4.1 are provided in Table 4.1, together with their *PIs*. The controlled PNM obtained by including eight control places shown in Table. 4.1 into the uncontrolled PNM shown in Fig. 4.1 is live and can reach 289 good state.

Table 4.1. Place invariants and control places computed for the PNM shown in Fig. 4.1.

FBM_i	PI_i	C_i	$^\bullet C_i$	$C_i^\bullet$	M_0
$\mu_2 = 1, \mu_3 = 1, \mu_4 = 1$	$\mu_2 + \mu_3 + \mu_4 \leq 2$	C1	t4	t1	2
$\mu_2 = 1, \mu_4 = 1, \mu_8 = 1$	$\mu_2 + \mu_4 + \mu_8 \leq 2$	C2	t2, t4, t8	t1, t3, t7	2
$\mu_3 = 1, \mu_7 = 1, \mu_9 = 1$	$\mu_3 + \mu_7 + \mu_9 \leq 2$	C3	t3, t7, t9	t2, t6, t8	2
$\mu_3 = 1, \mu_9 = 1, \mu_{13} = 1$	$\mu_3 + \mu_9 + \mu_{13} \leq 2$	C4	t3, t9, t13	t2, t8, t12	2

$\mu_7 = 1, \mu_8 = 1, \mu_9 = 1$	$\mu_7 + \mu_8 + \mu_9 \leq 2$	C5	t9	t6	2
$\mu_8 = 1, \mu_9 = 1, \mu_{13} = 1$	$\mu_8 + \mu_9 + \mu_{13} \leq 2$	C6	t9, t13	t7, t12	2
$\mu_7 = 1, \mu_{12} = 1, \mu_{14} = 1$	$\mu_7 + \mu_{12} + \mu_{14} \leq 2$	C7	t7, t12, t14	t6, t11, t13	2
$\mu_{12} = 1, \mu_{13} = 1, \mu_{14} = 1$	$\mu_{12} + \mu_{13} + \mu_{14} \leq 2$	C8	t14	t11	2

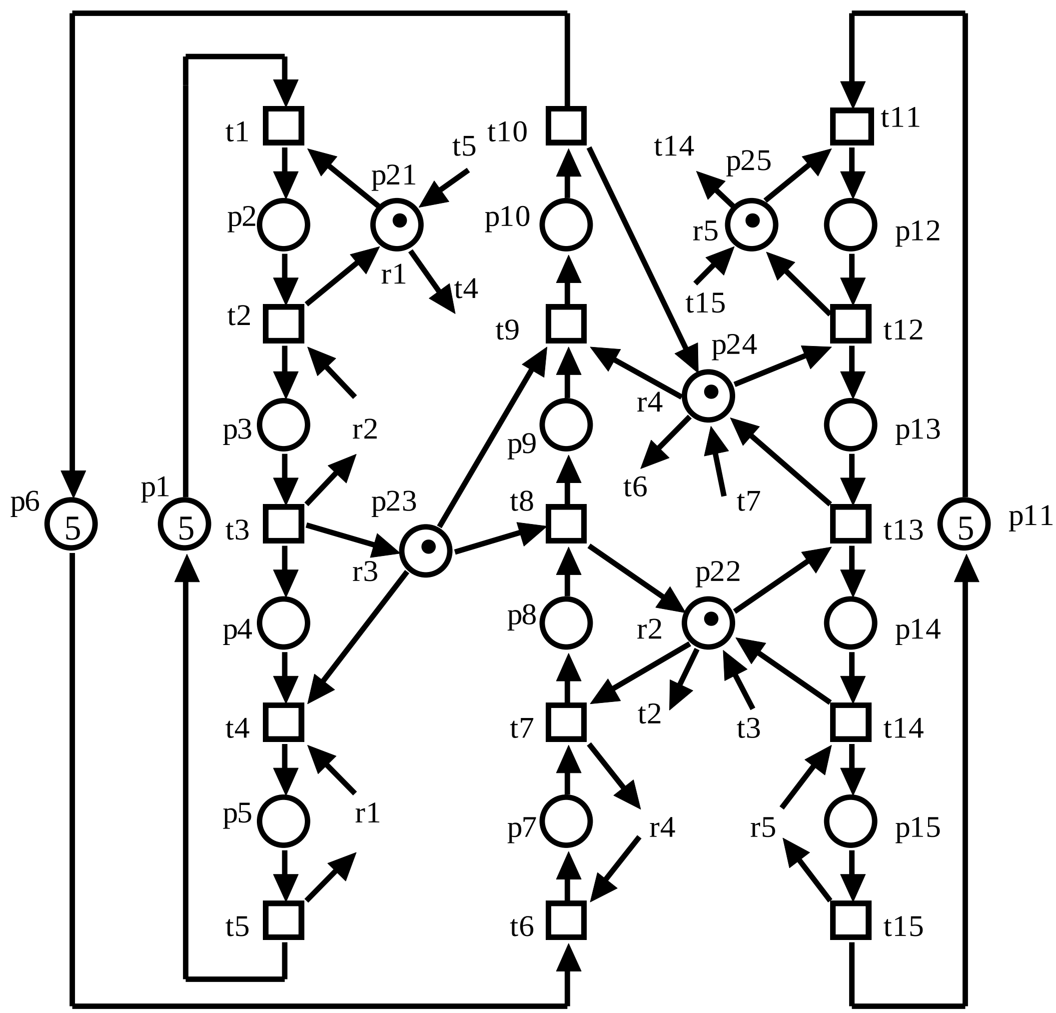


Fig. 4.1 An S^3PR Petri net model (PNM).

Step 1:

In this step, it is necessary to identify the possible place invariants that could be merged together, such that we have a minimum structure.

$$Z_1 = \{PI_1, PI_2\}$$

$$Z_2 = \{PI_3, PI_4, PI_5, PI_6\}$$

$$Z_3 = \{PI_7, PI_8\}$$

Step 2: ($i = 1$, first iteration).

In the first iteration, let us consider *Z1.*

Step 2.1.1:

The common intersecting elements among the place invariants PI_1 and PI_2 are as follows:

$$PI_1 \cap PI_2 = \{p_2, p_4\}$$

Step 2.1.2:

A tentative equation for a merged place invariant can be written as:

$$mPI_1 = \alpha_2\mu_2 + \alpha_3\mu_3 + \alpha_4\mu_4 + \alpha_8\mu_8 \le k \tag{4.1.1}$$

Step 2.1.3:

Linear equations are established by using the co-efficients of possible place invariants to be merged:

$$\alpha_2 + \alpha_3 + \alpha_4 = k + 1 \tag{4.1.2}$$

$$\alpha_2 + \alpha_4 + \alpha_8 = k + 1 \tag{4.1.3}$$

Step 2.1.4:

In this step, a linear relationship is established that exists between the co-efficients of possible place invariants to be merged by using Eqs. (4.1.2) and (4.1.3).

$$\alpha_2 + \alpha_3 + \alpha_4 = \alpha_2 + \alpha_4 + \alpha_8$$

$$\alpha_3 = \alpha_8 \tag{4.1.4}$$

Also, according to rule 3. $\alpha_2 = \alpha_4$ (4.1.5)

Remarks.

By default, for maximally permissive or near optimal behavior the common elements have a value more than or equal to their cardinalities in their place invariants to be merged. However according to the rule 5, if any one of the place invariants involved only one particular process, their values for common intersecting elements should be one less than the cardinalities or greater than that value.

Step 2.1.5:

Having obtaining the linear relationships among the unknown co-efficients, a suitable value is selected for the co-efficients in Eqs. (4.1.4) and (4.1.5), the possible values to choose should be the values of the cardinality of the elements in the place invariants under consideration. Although, it is not necessary to choose the values of the cardinalities of the elements in the set of place invariants under considerations. However choosing a small value is more desirable because it yields a minimal structure.

$$|p_2| = 2, \text{ and } \quad |p_4| = 2$$

Hence, let $\alpha_3 = \alpha_8 = 1$

By the rule 5, $\alpha_2 = \alpha_4 = 1$

Step 2.1.6:

By using Eq. (4.1.2), the value of k can be computed as:

$$\alpha_2 + \alpha_3 + \alpha_4 = k + 1$$

$$k = 2$$

Step 2.1.7:

The resulting merged place invariant mPI_1 is:

$$mPI_1 = \mu_2 + \mu_3 + \mu_4 + \mu_8 \le 2 \tag{4.1.6}$$

Step 2.1.8:

The monitor C_1 is computed for mPI_1 :

$$\mu_0(C_1) = 2$$

$$Dc_1 = -L_{mPI_1} \cdot D_{mPI_1}$$

$$Dc_1 = -\begin{array}{cccc} p_2 & p_3 & p_4 & p_8 \\ [1 & 1 & 1 & 1] \end{array} \begin{array}{c} \begin{array}{cccccc} t_1 & t_2 & t_3 & t_4 & t_7 & t_8 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \end{array} \begin{array}{c} p_2 \\ p_3 \\ p_4 \\ p_8 \end{array}$$

$$Dc_1 = \begin{array}{cccccc} t_1 & t_2 & t_3 & t_4 & t_7 & t_8 \\ [-1 & 0 & 0 & 1 & -1 & 1] \end{array}$$

Step 2: $(i = 2, \text{second iteration})$.

In the second iteration, let us consider Z2.

Step 2.2.1:

The common intersecting element among the place invariants PI_3, PI_4, PI_5 and PI_6 as follows:

$$PI_3 \cap PI_4 \cap PI_5 \cap PI_6 = \{p_9\}$$

Step 2.2.2:

A tentative equation for a merged place invariant can be written as:

$$mPI_2 = \alpha_3\mu_3 + \alpha_7\mu_7 + \alpha_8\mu_8 + \alpha_9\mu_9 + \alpha_{13}\mu_{13} \leq k \tag{4.1.7}$$

Step 2.2.3:

Linear equations are established by using the co-efficients of possible place invariants to be merged as:

$$\alpha_3 + \alpha_7 + \alpha_9 = k + 1 \tag{4.1.8}$$

$$\alpha_3 + \alpha_9 + \alpha_{13} = k + 1 \tag{4.1.9}$$

$$\alpha_7 + \alpha_8 + \alpha_9 = k + 1 \tag{4.1.10}$$

$$\alpha_8 + \alpha_9 + \alpha_{13} = k + 1 \tag{4.1.11}$$

Step 2.2.4:

Relationships are established that exist between the co-efficients of the place invariants to be merged.

By equating Eqs. (4.1.8) and (4.1.9):

$$\alpha_3 + \alpha_7 + \alpha_9 = \alpha_3 + \alpha_9 + \alpha_{13}$$

$$\alpha_7 = \alpha_{13} \tag{4.1.12}$$

By equating Eqs. (4.1.9) and (4.1.11):

$$\alpha_3 + \alpha_9 + \alpha_{13} = \alpha_8 + \alpha_9 + \alpha_{13}$$

$$\alpha_3 = \alpha_8 \tag{4.1.13}$$

Step 2.2.5:

It is clear to see that one of the place invariants to be merged belongs to a particular process, $\alpha_9 \geq 3$.

$$|p_9| = 4$$

$$\text{Let} \qquad \alpha_7 = \alpha_{13} = 2$$

$$\alpha_3 = \alpha_8 = 1$$

$$\text{According to the rule 3.} \qquad \alpha_9 = 3$$

Step 2.2.6:

Eq. (4.1.8) is used to find the value of *k*.

$$\alpha_3 + \alpha_7 + \alpha_9 = k + 1$$

$$1 + 2 + 3 = k + 1$$

$$k = 5$$

Step 2.2.7:

Finally the resultant merged place invariant i.e. mPI_2 is:

$$mPI_2 = \mu_3 + 2\mu_7 + \mu_8 + 3\mu_9 + 2\mu_{13} \leq 5 \tag{4.1.14}$$

Step 2.2.8:

The monitor C_2 is computed for mPI_2 as:

$$\mu_0(C_2) = 5$$

$$Dc_2 = -L_{mPI_2} \cdot D_{mPI_2}$$

$$Dc_2 = -\begin{array}{ccccc} p_3 & p_7 & p_8 & p_9 & p_{13} \\ [1 & 2 & 1 & 3 & 2] \end{array} \begin{array}{c} \begin{array}{cccccccc} t_2 & t_3 & t_6 & t_7 & t_8 & t_9 & t_{12} & t_{13} \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \end{array} \begin{array}{c} p_3 \\ p_7 \\ p_8 \\ p_9 \\ p_{13} \end{array}$$

$$Dc_2 = \begin{array}{cccccccc} t_2 & t_3 & t_6 & t_7 & t_8 & t_9 & t_{12} & t_{13} \\ [-1 & 1 & -2 & 1 & -2 & 3 & -2 & 2] \end{array}$$

Step 2: $(i = 3, \text{third iteration})$.

In the third iteration, let us consider Z_3.

Step 2.3.1:

The common intersecting elements between the place invariants PI_7 and PI_8 are as follows:

$$PI_7 \cap PI_8 = \{p_{12}, p_{14}\}$$

Step 2.3.2:

A tentative equation for a merged place invariant can be written as:

$$mPI_3 = \alpha_7\mu_7 + \alpha_{12}\mu_{12} + \alpha_{13}\mu_{13} + \alpha_{14}\mu_{14} \le k \tag{4.1.15}$$

Step 2.3.3:

Linear equations are established by using the co-efficients of possible place invariants to be merged as:

$$\alpha_7 + \alpha_{12} + \alpha_{14} = k + 1 \tag{4.1.16}$$

$$\alpha_{12} + \alpha_{13} + \alpha_{14} = k + 1 \tag{4.1.17}$$

Step 2.3.4:

A relationship is established that exist between the unknown co-efficients of place invariants to be merged. By equating Eqs. (4.1.16) and (4.1.17), we have:

$$\alpha_7 + \alpha_{12} + \alpha_{14} = \alpha_{12} + \alpha_{13} + \alpha_{14}$$

$$\alpha_7 = \alpha_{13} \tag{4.1.18}$$

Also, according to the rule 3. $\quad \alpha_{12} = \alpha_{14} \quad (4.1.19)$

Remarks:

In choosing the values for the unknown co-efficients of a merged place invariant, a suitable value must be selected. The suitable value should be the cardinalities of the elements among the set of place invariants under consideration. Is does not mean that, it is the only value that can satisfy the liveness of the Petri net model, whereas it is the minimum value that satisfied both the liveness and provide a minimal structure as well as maximally permissive behavior.

Step 2.3.5:

Since one of the place invariants to be merged together is belong to a one particular process, $\alpha_{12} \geq 1$.

$$|p_{12}| = 2 \text{ and } |p_{14}| = 2$$

$$\text{Let} \qquad \alpha_7 = \alpha_{13} = 1$$

$$\text{According to the rule 3.} \qquad \alpha_{12} = \alpha_{14} = 1$$

Step 2.3.6:

Eq. (4.1.16) is used to find the value of k as follows:

$$\alpha_7 + \alpha_{12} + \alpha_{14} = k + 1$$

$$1 + 1 + 1 = k + 1$$

$$k = 2$$

Step 2.3.7:

Finally, the resulting merged place invariant i.e. mPI_3 is:

$$mPI_3 = \mu_7 + \mu_{12} + \mu_{13} + \mu_{14} \leq 2 \tag{4.1.20}$$

Step 2.3.8.

The monitor C_2 is computed for mPI_3 as:

$$\mu_0(C_3) = 2$$

$$Dc_3 = -L_{mPI_3} \cdot D_{mPI_3}$$

$$Dc_3 = -\begin{array}{cccc} p_7 & p_{12} & p_{13} & p_{14} \end{array}\left[\begin{array}{cccc} 1 & 1 & 1 & 1 \end{array}\right] \begin{array}{c} \begin{array}{cccccc} t_6 & t_7 & t_{11} & t_{12} & t_{13} & t_{14} \end{array} \\ \left[\begin{array}{cccccc} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{array}\right] \end{array} \begin{array}{c} \\ p_7 \\ p_{12} \\ p_{13} \\ p_{14} \end{array}$$

$$Dc_3 = \begin{array}{c} \begin{array}{cccccc} t_6 & t_7 & t_{11} & t_{12} & t_{13} & t_{14} \end{array} \\ \left[\begin{array}{cccccc} -1 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \end{array}$$

Finally after applying the proposed algorithm, three merged place invariants namely mPI_1, mPI_2 and mPI_3 as shown in Eqs. (4.1.6), (4.1.14) and (4.1.20) are computed. For these merged *PIs* and monitors computed by using these *PIs* shown in Table 4.2. When monitors C_1, C_2 and C_3 are added to the PNM given in Fig. 4.1 the controlled model is obtained. It is verified that this controlled model is live, with maximally permissive behavior, which has 289 good states.

Table 4.2 The merged PIs and computed monitors.

mPIi	*Ci*	${}^{\bullet}C$	$C^{\bullet}$	M_0
$mPI_1 = \mu_2 + \mu_3 + \mu_4 + \mu_4 + \mu_8 \leq 2$	*C1*	*t4, t8*	*t1, t7*	*2*
$mPI_2 = \mu_3 + 2\mu_7 + \mu_8 + 3\mu_9 + 2\mu_{13} \leq 5$	*C2*	*t3, t7, 3t9, 2t13*	*t2, 2t6, 2t8, 2t12*	*5*
$mPI_3 = \mu_7 + \mu_{12} + \mu_{13} + \mu_{14} \leq 2$	*C3*	*t7, t14*	*t6, t11*	*2*

Table 4.3 compares the original monitors computed by using an FBM variant method with that of obtained by the proposed method. From the Table 4.3, it is clear to observe that the number of control places obtained with the proposed method is as minimum as possible when compared with the original monitors computed by using FBM variant method.

Table 4.3 Performance comparison between the original monitors and the reduced reduced monitors obtained with the proposed method.

LES	# monitors	# arcs	# tokens	% permissiveness
Original monitors	8	34	16	100
Reduced monitors	3	22	9	100

4.2 EXAMPLE 4.2

The S^3PR Petri net model (PNM) of an FMS [2] is shown in Fig. 4.2, with the set of control places computed by using an FBM variant method [34]. The net has 19 places and 14 transitions. Places can be considered to be the collection of $P^0 = \{p_1, p_8\}$, $P_R = \{p_{14}, p_{15}, p_{16}, \cdots, p_{19}\}$ and $p_A = \{p_2, p_3, \cdots, p_7, p_9, \cdots, p_{13}\}$. The net has 282 reachable states in which there are 77 bad states and 205 good states.

Control places (monitors) computed for this PNM shown in Fig. 4.2 are provided in Table 4.4, together with their *PIs*. The controlled PNM obtained by including the eight control places shown in Table 4.4 into the uncontrolled PNM shown in Fig. 4.2 is live and can reach 205 good states.

Table 4.4. Place invariants and control places computed for the S^3PR shown in Fig. 4.2.

FBM_i	PI_i	C_i	$^\bullet C_i$	$C_i^\bullet$	M_0
$\mu_3 = 1, \mu_{11} = 1$	$\mu_3 + \mu_{11} \leq 1$	C1	t5, t12	t2, t11	1
$\mu_{11} = 1, \mu_{12} = 1$	$\mu_{11} + \mu_{12} \leq 1$	C2	t13	t11	1
$\mu_2 = 1, \mu_3 = 1, \mu_4 = 1$	$\mu_2 + \mu_3 + \mu_4 \leq 2$	C3	t4, t5	t1	2
$\mu_2 = 1, \mu_4 = 1, \mu_{12} = 1$	$\mu_2 + \mu_4 + \mu_{12} \leq 2$	C4	t2, t4, t13	t1, t12	2
$\mu_5 = 1, \mu_6 = 1, \mu_9 = 1, \mu_{10} = 1$	$\mu_5 + \mu_6 + \mu_9 + \mu_{10} \leq 3$	C5	t7, t11	t4, t5, t9	3
$\mu_3 = 1, \mu_6 = 1, \mu_9 = 1, \mu_{10} = 1$	$\mu_3 + \mu_6 + \mu_9 + \mu_{10} \leq 3$	C6	t5, t7, t11	t2, t6, t9	3
$\mu_3 = 1, \mu_5 = 1, \mu_9 = 1, \mu_{10} = 1$	$\mu_3 + \mu_5 + \mu_9 + \mu_{10} \leq 3$	C7	t6, t11	t2, t4, t9	3
$\mu_2 = 1, \mu_4 = 1, \mu_6 = 1, \mu_9 = 1, \mu_{10} = 1$	$\mu_2 + \mu_4 + \mu_6 + \mu_9 + \mu_{10} \leq 4$	C8	t2, t4, t7, t11	t1, t6, t9	4

Step 1:

It is necessary to identify the possible place invariants that could be merged:

$$Z_1 = \{PI_5, PI_6, PI_7, PI_8\}$$
$$Z_2 = \{PI_1, PI_2\}$$
$$Z_3 = \{PI_3, PI_4\}$$

Step 2: $(i = 1,$ first iteration$)$.

In the first iteration, let us consider Z_1.

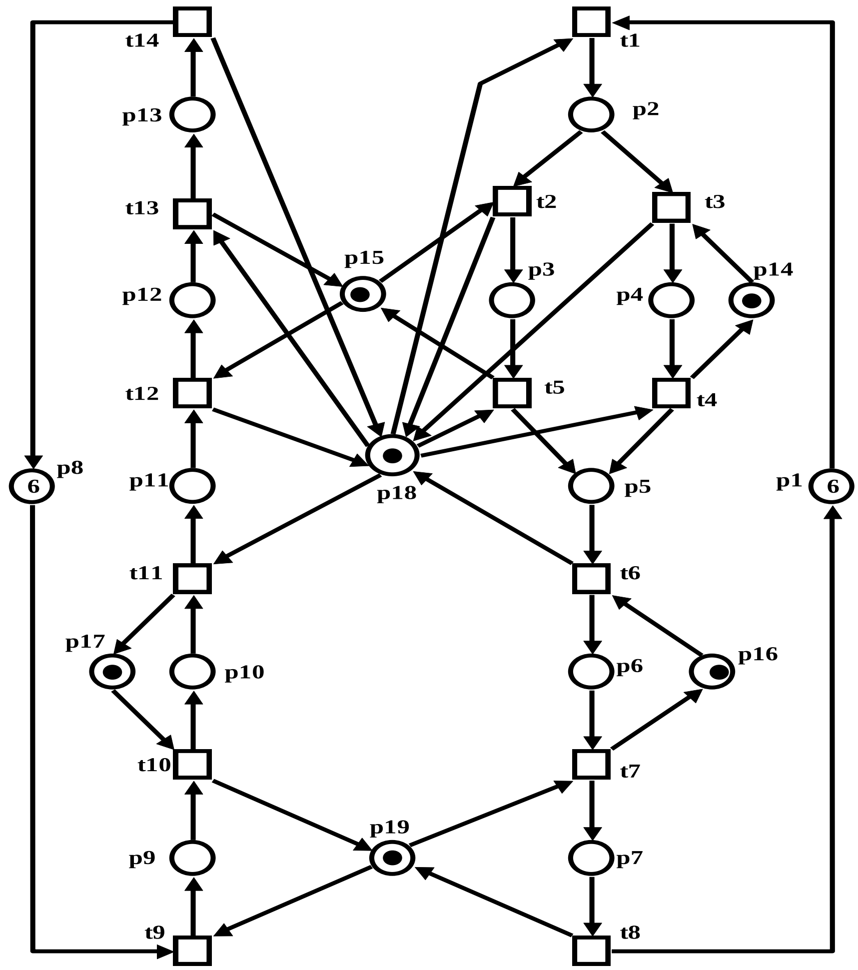


Fig. 4.2. A S³PR Petri net model of an FMS [2].

Step 2.1.1:

The common intersecting elements among the place invariants PI_5, PI_6, PI_7 and PI_8 are as follows:

$$PI_5 \cap PI_6 \cap PI_7 \cap PI_8 = \{p_9, p_{10}\}$$

Step 2.1.2:

A tentative equation for a merged place invariant can be written as:

$$mPI_1 = \alpha_2\mu_2 + \alpha_3\mu_3 + \alpha_4\mu_4 + \alpha_5\mu_5 + \alpha_6\mu_6 + \alpha_9\mu_9 + \alpha_{10}\mu_{10} \le k \tag{4.2.1}$$

Step 2.1.3:

Linear equations are established by using the co-efficients of possible place invariants to be merged.

$$\alpha_5 + \alpha_6 + \alpha_9 + \alpha_{10} = k + 1 \tag{4.2.2}$$

$$\alpha_3 + \alpha_6 + \alpha_9 + \alpha_{10} = k + 1 \tag{4.2.3}$$

$$\alpha_3 + \alpha_5 + \alpha_9 + \alpha_{10} = k + 1 \tag{4.2.4}$$

$$\alpha_2 + \alpha_4 + \alpha_6 + \alpha_9 + \alpha_{10} = k + 1 \tag{4.2.5}$$

Step 2.1.4:

Linear relationships are established that exist among the unknown co-efficients of a place invariants to be merged.

By equating Eqs. (4.2.2) and (4.2.3).

$$\alpha_5 + \alpha_6 + \alpha_9 + \alpha_{10} = \alpha_3 + \alpha_6 + \alpha_9 + \alpha_{10}$$

$$\alpha_5 = \alpha_3$$

Also, by equating Eqs. (4.2.3) and (4.2.4):

$$\alpha_3 + \alpha_6 + \alpha_9 + \alpha_{10} = \alpha_3 + \alpha_5 + \alpha_9 + \alpha_{10}$$

$$\alpha_5 = \alpha_6$$

Hence, it shows that $\alpha_3 = \alpha_5 = \alpha_6$ (4.2.6)

Also, by equating Eqs. (4.2.3) and (4.2.5)

$$\alpha_3 + \alpha_6 + \alpha_9 + \alpha_{10} = \alpha_2 + \alpha_4 + \alpha_6 + \alpha_9 + \alpha_{10}$$

$$\alpha_3 = \alpha_2 + \alpha_4 \tag{4.2.7}$$

Also according to the rule 3. $\alpha_9 = \alpha_{10}$ (4.2.8)

Step 2.1.5:

Having obtaining the linear relationships among the unknown co-efficients, a suitable value is selected for the co-efficients in Eqs. (4.2.6) and (4.2.7). The possible values to choose should be the values of the cardinality of the elements in the place invariants under consideration.

$$|p_9| = 4 \text{ and } |p_{10}| = 4$$

Let $\alpha_2 = \alpha_4 = 1$

$$\therefore \qquad \alpha_3 = \alpha_5 = \alpha_6 = 2$$

According to rule 4. $\alpha_9 = \alpha_{10} = 4$

Step 2.1.6:

By using Eq. (4.2.2), the value of k can be computed as:

$$\alpha_5 + \alpha_6 + \alpha_9 + \alpha_{10} = k + 1$$

$$2 + 2 + 4 + 4 = k + 1$$

$$k = 11$$

Step 2.1.7:

Finally, the resulting merged place invariant mPI_1 is:

$$mPI_1 = \mu_2 + 2\mu_3 + \mu_4 + 2\mu_5 + 2\mu_6 + 4\mu_9 + 4\mu_{10} \le 11 \tag{4.2.9}$$

Step 2.1.8:

The monitor C_1 is computed for mPI_1 as follows:

$$\mu_0(C_1) = 11$$

$$Dc_1 = -L_{mPI_1} \, . D_{mPI_1}$$

$$Dc_1 = -\begin{array}{c} \begin{array}{ccccccc} p_2 & p_3 & p_4 & p_5 & p_6 & p_9 & p_{10} \end{array} \\ [\begin{array}{ccccccc} 1 & 2 & 1 & 2 & 2 & 4 & 4 \end{array}] \end{array} \begin{array}{cc} \begin{array}{cccccccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 & t_9 & t_{10} & t_{11} \end{array} & \\ \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} & \begin{array}{c} p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_9 \\ p_{10} \end{array} \end{array}$$

$$Dc_1 = \begin{array}{c} \begin{array}{cccccccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 & t_9 & t_{10} & t_{11} \end{array} \\ [\begin{array}{cccccccccc} -1 & -1 & 0 & -1 & 0 & 0 & 2 & -4 & 0 & 4 \end{array}] \end{array}$$

Step 2: ($i = 2$, second iteration).

In the second iteration, let us consider Z_2.

Step 2.2.1:

The common intersecting element between the place invariants PI_1 and PI_2 is as follows:

$$PI_1 \cap PI_2 = \{p_{11}\}$$

Step 2.2.2:

A tentative equation for a merged place invariant can be written as follows:

$$mPI_2 = \alpha_3\mu_3 + \alpha_{11}\mu_{11} + \alpha_{12}\mu_{12} \le k \tag{4.2.10}$$

Step 2.2.3:

Linear equations are established by using the co-efficients of possible place invariants to be merged.

$$\alpha_3 + \alpha_{11} = k + 1 \tag{4.2.11}$$

$$\alpha_{11} + \alpha_{12} = k + 1 \tag{4.2.12}$$

Step 2.2.4:

Relationships are established that exists among the unknown co-efficients of place invariants to be merged.

By equating Eqs. (4.2.11) and (4.2.12).

$$\alpha_3 + \alpha_{11} = \alpha_{11} + \alpha_{12}$$

$$\alpha_3 = \alpha_{12} \tag{4.2.13}$$

Also according to the rule 5. $\alpha_{11} \ge 1$

Step 2.2.5:

By selecting a value that is suitable for the co-efficient in Eq. (4.2.13), the possible values to be choose should be the values of the cardinality of the elements in the place invariants under consideration.

$$|p_{11}| = 2$$

Let $\alpha_3 = \alpha_{12} = 1$

According to the rule 5. $\alpha_{11} = 1$

Step 2.2.6:

Eq. (4.2.11) is used to find the value of k as:

$$\alpha_3 + \alpha_{11} = k + 1$$
$$1 + 1 = k + 1$$
$$k = 1$$

Step 2.2.7:

Finally, the resulting merged place invariant mPI_2 is:

$$mPI_2 = \mu_3 + \mu_{11} + \mu_{12} \le 1 \qquad (4.2.14)$$

Step 2.2.8:

The monitor C_2 is computed for mPI_2 as follows:

$$\mu_0(C_2) = 1$$

$$Dc_2 = -L_{mPI_2} \,.\, D_{mPI_2}$$

$$Dc_2 = -\begin{array}{c} \begin{array}{ccc} p_3 & p_{11} & p_{12} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} \begin{array}{ccccc} t_2 & t_5 & t_{11} & t_{12} & t_{13} \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \end{array} \begin{array}{c} p_3 \\ p_{11} \\ p_{12} \end{array}$$

$$Dc_2 = \begin{array}{c} \begin{array}{ccccc} t_2 & t_5 & t_{11} & t_{12} & t_{13} \end{array} \\ \begin{bmatrix} -1 & 1 & -1 & 0 & 1 \end{bmatrix} \end{array}$$

Step 2: ($i = 3$, third iteration).

Let us now consider Z_3, which has elements of PI_3 and PI_4.

Step 2.3.1:

The common intersecting elements between the place invariants PI_3 and PI_4 that could be merged are computed as:

$$PI_3 \cap PI_4 = \{p_2, p_4\}$$

Step 2.3.2:

A tentative equation of a merged place invariant can be written as:

$$mPI_3 = \alpha_2\mu_2 + \alpha_3\mu_3 + \alpha_4\mu_4 + \alpha_{12}\mu_{12} \le k \tag{4.2.15}$$

Step 2.3.3:

Linear equations are established by using the co-efficients of place invariants to be merged as:

$$\alpha_2 + \alpha_3 + \alpha_4 = k + 1 \tag{4.2.16}$$

$$\alpha_2 + \alpha_4 + \alpha_{12} = k + 1 \tag{4.2.17}$$

Step 2.3.4:

Relationships are established that exists among the unknown co-efficients of a place invariant to be merged.

By equating Eqs. (4.2.16) and (4.2.17), we have

$$\alpha_2 + \alpha_3 + \alpha_4 = \alpha_2 + \alpha 4 + \alpha_{12}$$

$$\alpha_3 = \alpha_{12} \tag{4.2.18}$$

Also according to the rule 3. $\alpha_2 = \alpha_4$

Also according to the rule 5. $\alpha_2 \ge 1$

Step 2.3.5:

Having obtaining the linear relationships among the unknown co-efficients, a suitable value is selected for the co-efficients in Eq. (4.2.18). The possible values to choose should be the values of the cardinality of the elements in the place invariants under consideration.

$$|p_2| = 2 \text{ and } |p_4| = 2$$

Let $\alpha_3 = \alpha_{12} = 1$

According to the rule 5. $\alpha_2 = \alpha_4 = 1$

Step 2.3.6.

Eq. (4.2.16) is used to find the value of k .

$$\alpha_2 + \alpha_3 + \alpha 4 = k + 1$$

$$1+1+1 = k + 1$$

$$k = 2$$

Step 2.3.7:

Finally, the resulting merged place invariant mPI_3 is:

$$mPI_3 = \mu_2 + \mu_3 + \mu_4 + \mu_{12} \leq 2 \tag{4.2.19}$$

Step 2.3.8:

The monitor C_3 is computed for mPI_3 as follows:

$$\mu_0(C_3) = 2$$

$$Dc_3 = -L_{mPI_3} \,.\, D_{mPI_3}$$

$$Dc_3 = -\begin{array}{c} \begin{array}{cccc} p_2 & p_3 & p_4 & p_{12} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{cc} \begin{array}{ccccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_{12} & t_{13} \end{array} & \\ \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} & \begin{array}{c} p_2 \\ p_3 \\ p_4 \\ p_{12} \end{array} \end{array}$$

$$Dc_3 = \begin{array}{c} \begin{array}{ccccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_{12} & t_{13} \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & 1 & 1 & -1 & 1 \end{bmatrix} \end{array}$$

Finally after applying the proposed algorithm, three merged place invariants namely mPI_1, mPI_2 and mPI_3 as shown in Eqs. (4.2.6), (4.2.14) and (4.2.20) are computed. These merged *PIs* and monitors computed by using these *PIs* are shown in Table 4.5. When monitors C_1, C_2 and C_3 are added to the PNM given in Fig. 4.2 the controlled model is obtained. It is verified that this controlled model is live, with maximally permissive behavior, which has 205 good states.

Table 4.5. The merged PIs and computed monitors.

mPIi	*Ci*	$^{\bullet}C$	$C^{\bullet}$	M_0
$mPI_1 = \mu_{12} + 2\mu_3 + \mu_4 + 2\mu_5 + 2\mu_6 + 4\mu_9 + 4\mu_{10} \leq 11$	*C1*	*2t7, 4t11*	*t1, t2, t4, 4t9*	*11*
$mPI_2 = \mu_3 + \mu_{11} + \mu_{12} \leq 1$	*C2*	*t5, t13*	*t2, t11*	*1*
$mPI_3 = \mu_2 + \mu_3 + \mu_4 + \mu_{12} \leq 2$	*C3*	*t4, t5, t13*	*t1, t12*	*2*

Table 4.6 compares original monitors computed by using an FBM variant method with that of obtained by the proposed method. From the Table 4.6, it is clear to observe that the number of control places obtained with the proposed method is as minimum as possible when compared with the original monitors computed by using an FBM variant method.

Table 4.6 Performance comparison between the original monitors and the reduced monitors obtained with the proposed method.

LES	# monitors	# arcs	# tokens	% permissiveness
Original monitors	8	37	19	100
Reduced monitors	3	22	14	100

4.3 EXAMPLE 3

The S^3PR Petri net model (PNM) of an FMS [30] is shown in Fig. 4.3, with the set of control places computed by using an FBM variant method [34]. The net has 26 places and 20 transitions. Places can be considered to be the collection of $P^0 = \{p_1, p_5, p_{14}\}$, $P_R = \{p_{20}, p_{21}, p_{22}, p_{23}, p_{24}, p_{25}, p_{26}\}$ and $p_A = \{p_2, p_3, p_4, p_6, \cdots, p_{13}, p_{15}, \cdots, p_{19}\}$. The net has 9572 reachable states in which there are 4177 bad states and 5395 good states.

Control places (monitors) computed for this PNM shown in Fig. 4.3 are provided in Table 4.7, together with their *PIs*. The controlled PNM obtained by including the thirteen control places shown in Table 4.7 into the uncontrolled PNM shown in Fig. 4.3 is live and can reach 3475 good states.

Table 4.7. Place invariants and control places computed for the S^3PR shown in Fig. 4.3.

FBM_i	PI_i	C_i	${}^\bullet C_i$	$C_i^\bullet$	$\mu_0(C_i)$
$\mu_{12} = 1, \mu_{15} = 1$	$\mu_{12} + \mu_{15} \leq 1$	*C1*	*t9, t16*	*t8, t15*	*1*
$\mu_{13} = 1, \mu_{15} = 1$	$\mu_{13} + \mu_{15} \leq 1$	*C2*	*t10, t16*	*t9, t15*	*1*
$\mu_{12} = 1, \mu_{16} = 1$	$\mu_{12} + \mu_{16} \leq 1$	*C3*	*t9, t17*	*t8, t16*	*1*
$\mu_{11} = 1, \mu_{17} = 1$	$\mu_{11} + \mu_{17} \leq 1$	*C4*	*t8, t18*	*t7, t17*	*1*
$\mu_{11} = 1, \mu_{16} = 1$	$\mu_{11} + \mu_{16} \leq 1$	*C5*	*t8, t17*	*t7, t16*	*1*
$\mu_{11} = 1, \mu_{15} = 1$	$\mu_{11} + \mu_{15} \leq 1$	*C6*	*t8, t16*	*t7, t15*	*1*

$\mu_3 = 2$, $\mu_8 = 1$	$\mu_3 + \mu_8 \leq 2$	*C7*	*t4, t13*	*t3, t12*	*2*
$\mu_2 = 1$, $\mu_3 = 2$	$\mu_2 + \mu_3 \leq 2$	*C8*	*t13*	*t11*	*2*
$\mu_3 = 1$, $\mu_8 = 1$, $\mu_9 = 1$, $\mu_{15} = 1$, $\mu_{16} = 1$	$\mu_3 + \mu_8 + \mu_9 + \mu_{15} + \mu_{16} \leq 4$	*C9*	*t5, t13, t17*	*t3, t12, t15*	*4*
$\mu_2 = 1$, $\mu_3 = 1$, $\mu_9 = 1$, $\mu_{15} = 1$, $\mu_{16} = 1$	$\mu_2 + \mu_3 + \mu_9 + \mu_{15} + \mu_{16} \leq 4$	*C10*	*t5, t13, t17*	*t4, t11, t15*	*4*
$\mu_6 = 1$, $\mu_7 = 2$, $\mu_{17} = 1$, $\mu_{18} = 1$	$\mu_6 + \mu_7 + \mu_{17} + \mu_{18} \leq 4$	*C11*	*t3, t7, t19*	*t1, t17*	*4*
$\mu_6 = 1$, $\mu_7 = 2$, $\mu_8 = 1$, $\mu_9 = 1$, $\mu_{15} = 1$, $\mu_{16} = 1$, $\mu_{18} = 1$	$\mu_6 + \mu_7 + \mu_8 + \mu_9 + \mu_{15} + \mu_{16} + \mu_{18} \leq 7$	*C12*	*t5, t7, t17, t19*	*t1, t15, t18*	*7*
$\mu_6 = 1$, $\mu_7 = 2$, $\mu_9 = 2$, $\mu_{15} = 1$, $\mu_{16} = 1$, $\mu_{17} = 1$	$\mu_6 + \mu_7 + \mu_9 + \mu_{15} + \mu_{16} + \mu_{17} \leq 7$	*C13*	*t3, t5, t7, t18*	*t1, t4, t15*	*7*

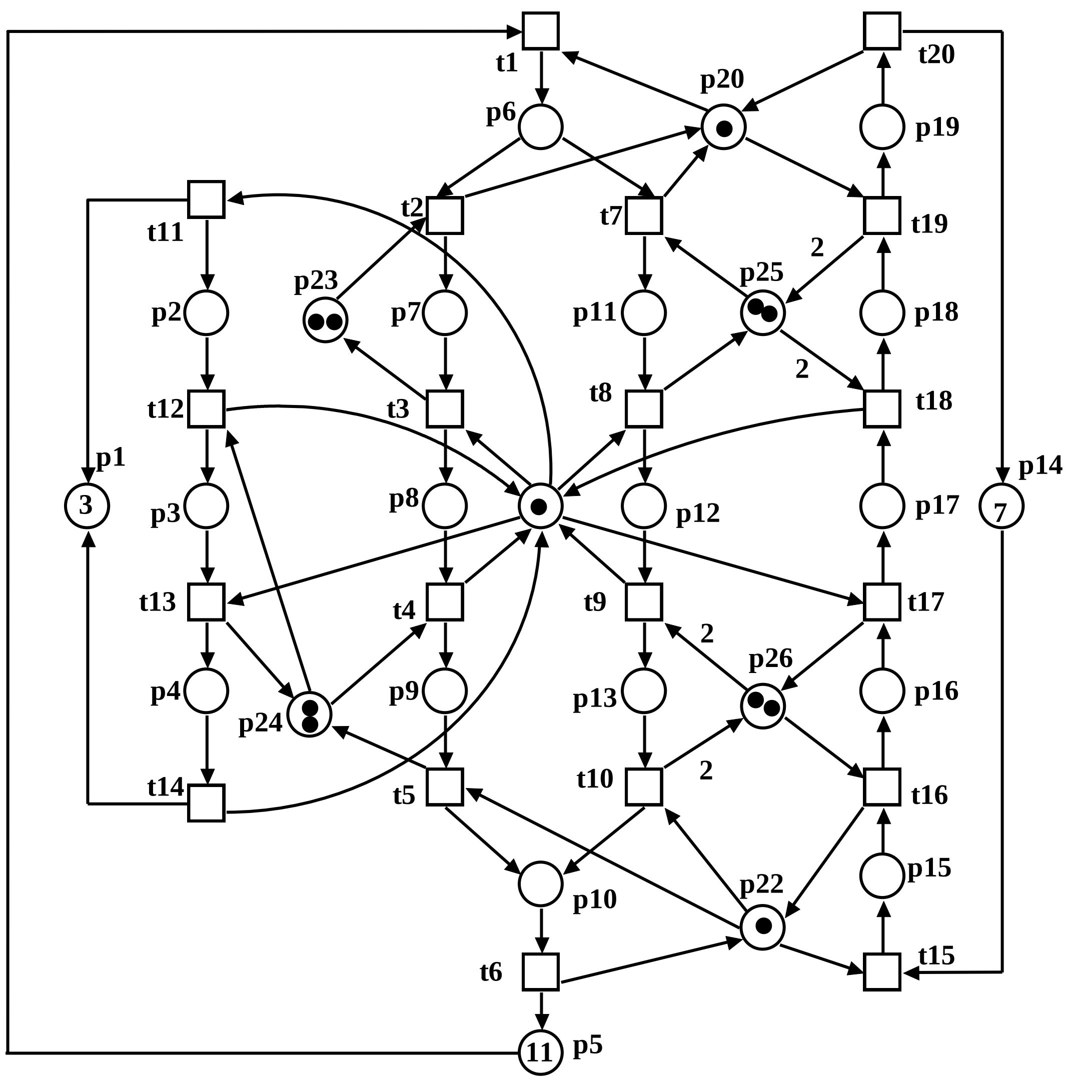


Fig. 4.3 An S^{3}PR Petri net model from [30].

Step 1:

Possible place invariants that could be merged are identified as follows:

$$Z_1 = \{PI_1, PI_2, PI_6\}$$
$$Z_2 = \{PI_3, PI_5\}$$
$$Z_3 = \{PI_7, PI_8\}$$
$$Z_4 = \{PI_4, PI_{11}\}$$
$$Z_5 = \{PI_9, PI_{10}\}$$
$$Z_6 = \{PI_{12}, PI_{13}\}$$

Step 2: ($i = 1$, first iteration).

Firstly, let us consider Z_1, which has elements of PI_1, PI_2 and PI_6.

Step 2.1.1:

The common intersecting elements among the place invariants PI_1, PI_2 and PI_6 are:

$$PI_1 \cap PI_2 \cap PI_6 = \{p_{15}\}$$

Step 2.1.2:

A tentative equation for a merged place invariant can be written as:

$$mPI_1 = \alpha_{11}\mu_{11} + \alpha_{12}\mu_{12} + \alpha_{13}\mu_{13} + \alpha_{15}\mu_{15} \leq k \tag{4.3.1}$$

Step 2.1.3:

Linear equations are established by using the co-efficients of possible place invariants to be merged.

$$\alpha_{12} + \alpha_{15} = k + 1 \tag{4.3.2}$$

$$\alpha_{13} + \alpha_{15} = k + 1 \tag{4.3.3}$$

$$\alpha_{11} + \alpha_{15} = k + 1 \tag{4.3.4}$$

Step 2.1.4:

Linear relationships are established that exist among the unknown co-efficients of place invariant to be merged.

By equating Eqs. (4.3.2) and (4.3.3), we have

$$\alpha_{12} + \alpha_{15} = \alpha_{13} + \alpha_{15}$$

$$\alpha_{12} = \alpha_{13}$$

Also, by equating Eqs. (4.3.3) and (4.3.4), we have

$$\alpha_{13} + \alpha_{15} = \alpha_{11} + \alpha_{15}$$
$$\alpha_{13} = \alpha_{11}$$

Hence, it shows that $\alpha_{11} = \alpha_{12} = \alpha_{13}$ (4.3.5)

Also according to the rule 4. $\alpha_{15} \geq 3$

Step 2.1.5:

Having obtained the linear relationships among the unknown co-efficients, a suitable value is selected for the co-efficients in Eq. (4.3.5). The possible values to choose should be the values of the cardinality of the elements in the place invariants under consideration.

$$|p_{15}| = 3$$

Let $\alpha_{11} = \alpha_{12} = \alpha_{13} = 1$

According to the rule 4. $\alpha_{15} = 3$

Step 2.1.6:

By using Eq. (4.3.2), the value of *k* can be computed as:

$$\alpha_{12} + \alpha_{15} = k + 1$$
$$1 + 3 = k + 1$$
$$k = 3$$

Step 2.1.7:

The resulting merged place invariant mPI_1 is:

$$mPI_1 = \mu_{11} + \mu_{12} + \mu_{13} + 3\mu_{15} \leq 3 \quad (4.3.6)$$

Step 2.1.8:

The monitor C_1 is computed for mPI_1 as follows:

$$\mu_0(C_1) = 3$$

$$Dc_1 = -L_{mPI_1} . D_{mPI_1}$$

$$Dc_1 = -\begin{array}{cccc} p_{11} & p_{12} & p_{13} & p_{15} \\ [1 & 1 & 1 & 3] \end{array} \begin{array}{c} \begin{array}{cccccc} t_7 & t_8 & t_9 & t_{10} & t_{15} & t_{16} \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \end{array} \begin{array}{c} p_{11} \\ p_{12} \\ p_{13} \\ p_{15} \end{array}$$

$$Dc_1 = \begin{array}{cccccc} t_7 & t_8 & t_9 & t_{10} & t_{15} & t_{16} \\ [-1 & 0 & 0 & 1 & -3 & 3] \end{array}$$

Step 2: $(i = 2,$ second iteration).

Let us consider Z2, which has elements of PI_3 and PI_5.

Step 2.2.1:

The common intersecting element between the places invariants PI_3 and PI_5 that could be merged together is defined as:

$$PI_3 \cap PI_5 = \{p_{16}\}$$

Step 2.2.2:

A tentative equation for a merged place invariant can be written as:

$$mPI_2 = \alpha_{11}\mu_{11} + \alpha_{12}\mu_{12} + \alpha_{16}\mu_{16} \le k \quad (4.3.7)$$

Step 2.2.3:

Linear equations are established by using the co-efficients of possible place invariants to be merged as:

$$\alpha_{12} + \alpha_{16} = k + 1 \quad (4.3.8)$$

$$\alpha_{11} + \alpha_{16} = k + 1 \quad (4.3.9)$$

Step 2.2.4:

A relationship is established that exist among the unknown co-efficients of place invariants to be merged.

By equating Eqs. (4.3.8) and (4.3.9), we have

$$\alpha_{12} + \alpha_{16} = \alpha_{11} + \alpha_{16}$$

$$\alpha_{12} = \alpha_{11} \quad (4.3.10)$$

Also according to the rule 4. $\alpha_{16} \geq 2$

Step 2.2.5:

Having obtained the linear relationships among the unknown co-efficients, a suitable value is selected for the co-efficients in Eq. (4.3.10). The possible values to choose should be the values of the cardinality of the elements in the place invariants under consideration.

$$|p_{16}| = 2$$

Let $\alpha_{11} = \alpha_{12} = 1$

According to the rule 4. $\alpha_{16} = 2$

Step 2.2.6:

By using Eq. (4.3.8), the value of k can be computed as:

$$\alpha_{12} + \alpha_{16} = k + 1$$

$$\alpha_{16} = k$$

$$k = 2$$

Step 2.2.7:

Finally, the resulting merged place invariants mPI_2 is:

$$mPI_2 = \mu_{11} + \mu_{12} + 2\mu_{16} \leq 2 \tag{4.3.11}$$

Step 2.2.8:

The monitor C_2 is computed for mPI_2 as follows:

$$\mu_0(C_2) = 2$$

$$Dc_2 = -L_{mPI_2} \,.\, D_{mPI_2}$$

$$Dc2 = -\begin{array}{c} \begin{array}{ccc} p_{11} & p_{12} & p_{16} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{cc} \begin{array}{ccccc} t_7 & t_8 & t_9 & t_{16} & t_{17} \end{array} & \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} & \begin{array}{c} p_{11} \\ p_{12} \\ p_{16} \end{array} \end{array}$$

$$Dc_2 = \begin{array}{c} \begin{array}{ccccc} t_7 & t_8 & t_9 & t_{16} & t_{17} \end{array} \\ \begin{bmatrix} -1 & 0 & 1 & -2 & 2 \end{bmatrix} \end{array}$$

Step 2: ($i = 3$, third iteration).

Let us consider Z3, which has elements of PI_7 and PI_8.

Step 2.3.1:

The common intersecting elements between the place invariants PI_7 and PI_8 that could be merged is defined as:

$$PI_7 \cap PI_8 = \{p_3\}$$

Step 2.3.2:

A tentative equation for a merged place invariant can be written as:

$$mPI_3 = \alpha_2\mu_2 + \alpha_3\mu_3 + \alpha_8\mu_8 \leq k \tag{4.3.12}$$

Step 2.3.3:

Linear equations are established by using the co-efficients of possible place invariants to be merged.

Remarks:

In all previous examples considered, the number of tokens of FBMs were all 1 for the elements. This shows that the value of "a" in establishing linear equations is "one". Therefore no value was attached to the unknown co-efficients in the previous examples. However, in this example, some values of tokens in the FBMs are greater than one.

$$a_3 = 2$$

$$2\alpha_3 + \alpha_8 = k + 1 \tag{4.3.13}$$

$$\alpha_2 + 2\alpha_3 = k + 1 \tag{4.3.14}$$

Step 2.3.4:

A relationship is established that exists between the unknown co-efficients of place invariant to be merged.

By equating Eqs. (4.3.13) and (4.3.14), we have

$$2\alpha_3 + \alpha_8 = \alpha_2 + 2\alpha_3$$

$$\alpha_8 = \alpha_2 \tag{4.3.15}$$

Also according to the rule 4. $\alpha_{16} \geq 2$

Step 2.3.5:

Having obtained the linear relationships among the unknown co-efficients, a suitable value is selected for the co-efficients in Eq. (4.3.15). The possible values to choose should be the values of the cardinality of the elements in the place invariants under consideration.

$$|p_3| = 2$$

Let $\alpha_2 = \alpha_8 = 1$

According to the rule 4. $\alpha_3 = 2$

Step 2.3.6:

By using Eq. (4.3.13), the value of k can be computed as:

$$2\alpha_3 + \alpha_8 = k + 1$$

$$2\alpha_3 = k$$

$$k = 4$$

Step 2.3.7:

Finally, the resulting merged place invariants mPI_3 is:

$$mPI_3 = \mu_2 + 2\mu_3 + \mu_8 \leq 4 \tag{4.3.16}$$

Step 2.3.8:

The monitor C_3 is computed for mPI_3 as follows:

$$\mu_0(C_3) = 4$$

$$Dc_3 = -L_{mPI_3} . D_{mPI_3}$$

$$Dc_3 = -\begin{array}{c} \begin{array}{ccc} p_2 & p_3 & p_8 \end{array} \\ [1 \quad 2 \quad 1] \end{array} \begin{array}{c} \begin{array}{ccccc} t_3 & t_4 & t_{11} & t_{12} & t_{13} \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ p_2 \\ p_3 \\ p_8 \end{array}$$

$$
\begin{array}{cccccc}
 & t_3 & t_4 & t_{11} & t_{12} & t_{13} \\
Dc_3 = & [-1 & 1 & -1 & -1 & 2]
\end{array}
$$

Step 2: $(i = 4,\ \text{fourth iteration})$.

In the fourth iteration Z4 is considered, which has an elements of PI_4 and PI_{11}.

Step 2.4.1:

The common intersecting element between the place invariants PI_4 and PI_{11} is defined as:

$$PI_4 \cap PI_{11} = \{p_{17}\}$$

Step 2.4.2:

A tentative equation for a merged place invariant can be written as:

$$mPI_4 = \alpha_6\mu_6 + \alpha_7\mu_7 + \alpha_{11}\mu_{11} + \alpha_{17}\mu_{17} + \alpha_{18}\mu_{18} \le k \tag{4.3.17}$$

Step 2.4.3:

Linear equations are established by using the co-efficients of possible place invariants to be merged as:

$$a_6 = a_{11} = a_{17} = a_{18} = 1,\ a_7 = 2$$

$$\alpha_{11} + \alpha_{17} = k + 1 \tag{4.3.18}$$

$$\alpha_6 + 2\alpha_7 + \alpha_{17} + \alpha_{18} = k + 1 \tag{4.3.19}$$

Step 2.4.4:

A relationship is established that exists between the unknown co-efficients of place invariants to be merged.

By equating Eqs. (4.3.18) and (4.3.19).

$$\alpha_{11} + \alpha_{17} = \alpha_6 + 2\alpha_7 + \alpha_{17} + \alpha_{18}$$

$$\alpha_{11} = \alpha_6 + 2\alpha_7 + \alpha_{18} \tag{4.3.20}$$

Also according to the rule 4. $\alpha_{17} \ge 2$

Step 2.4.5:

Having obtained the linear relationships among the unknown co-efficients, a suitable value is selected for the co-efficients in Eq. (4.3.20). The possible values to choose should be the values of the cardinality of the elements in the place invariants under consideration.

$$|p_{17}| = 2$$

Let $\alpha_6 = \alpha_7 = \alpha_{18} = 1$

Hence, $\alpha_{11} = 4$

According to the rule 4. $\alpha_{17} = 4$

Step 2.4.6:

Eq. (4.3.18) is used to find the value of k as:

$$\alpha_{11} + \alpha_{17} = k + 1$$

$$4 + 4 = k + 1$$

$$k = 7$$

Step 2.4.7:

Finally, the resulting merged place invariants mPI_4 is:

$$mPI_4 = \mu_6 + \mu_7 + 4\mu_{11} + 4\mu_{17} + \mu_{18} \le 7 \tag{4.3.21}$$

Step 2.4.8:

The monitor C_4 is computed for mPI_4 as follows:

$$\mu_0(C_4) = 7$$

$$Dc_4 = -L_{mPI_4} . D_{mPI_4}$$

$$Dc_4 = -\begin{array}{c} \begin{array}{ccccc} p_6 & p_7 & p_{11} & p_{17} & p_{18} \end{array} \\ \begin{bmatrix} 1 & 1 & 4 & 4 & 1 \end{bmatrix} \end{array} \begin{array}{cc} \begin{array}{cccccccc} t_1 & t_2 & t_3 & t_7 & t_8 & t_{17} & t_{18} & t_{19} \end{array} & \\ \begin{bmatrix} 1 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} & \begin{array}{c} p_6 \\ p_7 \\ p_{11} \\ p_{17} \\ p_{18} \end{array} \end{array}$$

$$Dc_4 = \begin{array}{c} \begin{array}{cccccccc} t_1 & t_2 & t_3 & t_7 & t_8 & t_{17} & t_{18} & t_{19} \end{array} \\ \begin{bmatrix} -1 & 0 & 1 & -3 & 4 & -4 & 3 & 1 \end{bmatrix} \end{array}$$

Step 2: ($i = 5$, fifth iteration).

In the fifth iteration, let us consider Z_5 which has elements of PI_9 and PI_{10}.

Step 2.5.1:

The common intersecting elements between the place invariants PI_9 and PI_{10} that could be merged are as follows:

$$PI_9 \cap PI_{10} = \{p_3, p_9, p_{15}, p_{16}\}$$

Step 2.5.2:

A tentative equation for a merged place invariant can be written as:

$$mPI_5 = \alpha_2\mu_2 + \alpha_3\mu_3 + \alpha_8\mu_8 + \alpha_9\mu_9 + \alpha_{15}\mu_{15} + \alpha_{16}\mu_{16} \le k \quad (4.3.22)$$

Step 2.5.3:

Linear equations are established by using the co-efficienst of possible place invariants to be merged.

$$\alpha_3 + \alpha_8 + \alpha_9 + \alpha_{15} + \alpha_{16} = k + 1 \quad (4.3.23)$$

$$\alpha_2 + \alpha_3 + \alpha_9 + \alpha_{15} + \alpha_{16} = k + 1 \quad (4.3.24)$$

Step 2.5.4:

A relationship is established that exists between the unknown co-efficients of place invariants to be merged.

By equating Eqs. (4.3.23) and (4.3.24).

$$\alpha_3 + \alpha_8 + \alpha_9 + \alpha_{15} + \alpha_{16} = \alpha_2 + \alpha_3 + \alpha_9 + \alpha_{15} + \alpha_{16}$$
$$\alpha_2 = \alpha_8 \quad (4.3.25)$$

But according to the rule 3. $\alpha_3 = \alpha_9 = \alpha_{15} = \alpha_{16}$

Also according to the rule 4. $\alpha_3 \ge 2$

Step 2.5.5:

Having obtained the linear relationships among the unknown co-efficients, a suitable value is selected for the co-efficients in Eq. (4.3.25). The possible values to choose should be the values of the cardinality of the elements in the place invariants under consideration.

$$|p_3| = 2,\ |p_9| = 2,\ |p_{15}| = 2 \text{ and } |p_{16}| = 2$$

$$\text{Let} \quad \alpha_2 = \alpha_8 = 1$$

According to the rule 4. $\alpha_3 = \alpha_9 = \alpha_{15} = \alpha_{16} = 2$

Step 2.5.6:

Eq. (4.3.23) is used to find the value of k as:

$$\alpha_3 + \alpha_8 + \alpha_9 + \alpha_{15} + \alpha_{16} = k + 1$$

$$2 + 1 + 2 + 2 + 2 = k + 1$$

$$k = 8$$

Step 2.5.7:

Finally, the resulting merged place invariants mPI_5 is:

$$mPI_5 = \mu_2 + 2\mu_3 + \mu_8 + 2\mu_9 + 2\mu_{15} + 2\mu_{16} \leq 8 \tag{4.3.26}$$

Step 2.5.8:

The monitor C_5 is computed for mPI_5 as follows:

$$\mu_0(C_5) = 8$$

$$Dc_5 = -L_{mPI_5} \,.\, D_{mPI_5}$$

$$Dc_5 = -\begin{array}{c} \begin{array}{cccccc} p_2 & p_3 & p_8 & p_9 & p_{15} & p_{16} \end{array} \\ \begin{bmatrix} 1 & 2 & 1 & 2 & 2 & 2 \end{bmatrix} \end{array} \begin{array}{cc} \begin{array}{ccccccccc} t_3 & t_4 & t_5 & t_{11} & t_{12} & t_{13} & t_{15} & t_{16} & t_{17} \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} & \begin{array}{c} p_2 \\ p_3 \\ p_8 \\ p_9 \\ p_{15} \\ p_{16} \end{array} \end{array}$$

$$Dc_5 = \begin{array}{c} \begin{array}{ccccccccc} t_3 & t_4 & t_5 & t_{11} & t_{12} & t_{13} & t_{15} & t_{16} & t_{17} \end{array} \\ [-1 \quad -1 \quad 2 \quad -1 \quad -1 \quad 2 \quad -2 \quad 0 \quad 2] \end{array}$$

Step 2: $(i = 6,$ sixth iteration).

In the sixth iteration, let us consider Z_6 which has an element of PI_{12} and PI_{13}.

Step 2.6.1:

The common intersecting elements between the place invariants PI_{12} and PI_{13} that could be merged as:

$$PI_{12} \cap PI_{13} = \{p_6, p_7, p_9, p_{15}, p_{16}\}$$

Step 2.6.2:

A tentative equation for a merged place invariant can be written as:

$$mPI_6 = \alpha_6\mu_6 + \alpha_7\mu_7 + \alpha_8\mu_8 + \alpha_9\mu_9 + \alpha_{15}\mu_{15} + \alpha_{16}\mu_{16} + \alpha_{17}\mu_{17} + \alpha_{18}\mu_{18} \le k \qquad (4.3.27)$$

Step 2.6.3:

Linear equations are established by using the co-efficients of possible place invariants to be merged.

$$a_7 = 2,\ a_9 = 2$$

$$\alpha_6 + 2\alpha_7 + \alpha_8 + \alpha_9 + \alpha_{15} + \alpha_{16} + \alpha_{18} = k + 1 \qquad (4.3.28)$$

$$\alpha_6 + 2\alpha_7 + 2\alpha_9 + \alpha_{15} + \alpha_{16} + \alpha_{17} = k + 1 \qquad (4.3.29)$$

Step 2.6.4:

A relationship is established that exists between the unknown co-efficients of place invariants to be merged.

By equating Eqs. (4.3.28) and (4.3.29).

$$\alpha_6 + 2\alpha_7 + \alpha_8 + \alpha_9 + \alpha_{15} + \alpha_{16} + \alpha_{18} = \alpha_6 + 2\alpha_7 + 2\alpha_9 + \alpha_{15} + \alpha_{16} + \alpha_{17}$$

$$\alpha_8 + \alpha_{18} = \alpha_9 + \alpha_{17} \qquad (4.3.30)$$

But according to the rule 3. $\alpha_6 = \alpha_7 = \alpha_9 = \alpha_{15} = \alpha_{16}$

Also according to the rule 4. $\alpha_6 \ge 2$

Step 2.6.5:

Having obtained the linear relationships among the unknown co-efficients, a suitable value is selected for the co-efficients in Eq. (4.3.30). The possible values to be choose should be the values of the cardinality of the elements in the place invariants under consideration.

$|p_6| = 2$, $|p_7| = 2$, $|p_9| = 2$, $|p_{15}| = 2$ and $|p_{16}| = 2$

Let $\alpha_{17} = \alpha_{18} = 1$

According to the rule 4. $\alpha_3 = \alpha_9 = \alpha_{15} = \alpha_{16} = 2$

$$\text{Hence,} \quad \alpha_8 = 2$$

Step 2.6.6:

Eq. (4.3.23) is used to find the value of k as:

$$\alpha_6 + 2\alpha_7 + \alpha_8 + \alpha_9 + \alpha_{15} + \alpha_{16} + \alpha_{18} = k + 1$$

$$2 + 4 + 2 + 2 + 2 + 2 + 1 = k + 1$$

$$k = 14$$

Step 2.6.7:

Finally, the resulting merged place invariants mPI_6 is:

$$mPI_6 = 2\mu_6 + 2\mu_7 + 2\mu_8 + 2\mu_9 + 2\mu_{15} + 2\mu_{16} + \mu_{17} + \mu_{18} \le 14 \tag{4.3.31}$$

Step 2.6.8:

The monitor C_6 is computed for mPI_6 as follows:

$$\mu_0(C_6) = 14$$

$$Dc_6 = -L_{mPI_6} \,.\, D_{mPI_6}$$

$$Dc_6 = -\begin{array}{c} \begin{array}{cccccccc} p_6 & p_7 & p_8 & p_9 & p_{15} & p_{16} & p_{17} & p_{18} \end{array} \\ \begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{cc} \begin{array}{ccccccccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_7 & t_{15} & t_{16} & t_{17} & t_{18} & t_{19} \end{array} & \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} & \begin{array}{c} p_6 \\ p_7 \\ p_8 \\ p_9 \\ p_{15} \\ p_{16} \\ p_{17} \\ p_{18} \end{array} \end{array}$$

$$Dc_6 = \begin{array}{c} \begin{array}{ccccccccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_7 & t_{15} & t_{16} & t_{17} & t_{18} & t_{19} \end{array} \\ \begin{bmatrix} -2 & 0 & 0 & 0 & 2 & 2 & -2 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

Finally after applying the above algorithm, the net has only six merged place invariants mPI_1, mPI_2, mPI_3, mPI_4, mPI_5 and mPI_6 as shown in Eqs. (4.3.6), (4.3.11), (4.3.16), (4.3.21), (4.3.26) and (4.3.31).. These merged *PIs* and monitors computed are shown in Table

4.8. When this set of reduced monitors C_1, C_2, C_3, C_4, C_5 and C_6 are added to the PNM given in Fig. 4.3 the controlled model is obtained. It is verified that this controlled model is live, with the same sub-optimal behavior, which has 3475 good states.

Table 4.8 The merged *PIs* and computed monitors.

PIi	*Ci*	*C*		M_0
$mPI_1 = \mu_{11} + \mu_{12} + \mu_{13} + 3\mu_{15} \leq 3$	*C1*	*t10, 3t16*	*t7, 3t15*	*3*
$mPI_2 = \mu_{11} + \mu_{12} + 2\mu_{16} \leq 2$	*C2*	*t9, 2t13*	*t7, 2t16*	*2*
$mPI_3 = \mu_2 + 2\mu_3 + \mu_8 \leq 4$	*C3*	*t4, 2t13*	*t3, t11,t12*	*4*
$mPI_4 = \mu_6 + \mu_7 + 4\mu_{11} + 4\mu_{17} + \mu_{18} \leq 7$	*C4*	*t3, 4t8, 3t18, t19*	*t1, 3t7, 4t17*	*7*
$mPI_5 = \mu_2 + 2\mu_3 + \mu_8 + 2\mu_9 + 2\mu_{15} + 2\mu_{16} \leq 8$	*C5*	*2t5, 2t13, 2t17*	*t3, t4, t11, t12, 2t15*	*8*
$mPI_6 = 2\mu_6 + 2\mu_7 + 2\mu_8 + 2\mu_9 + 2\mu_{15} + 2\mu_{16} + 2\mu_{17} + 2\mu_{18} \leq 14$	*C6*	*2t5, 2t7, t17, t19*	*2t1, 2t15*	*14*

Table 4.9 compares original monitors computed by using an FBM variant method with that of obtained by the proposed method. From the Table 4.9, it is clear to observe that the number of control places obtained with the proposed method is as minimum as possible when compared with the original monitors computed by using an FBM variant method.

Table 4.9 Performance comparison between the original monitors and the reduced monitors obtained with the proposed method.

LES	# monitors	# arcs	# tokens	% permissiveness
Original monitors	13	61	36	64.41
Reduced monitors	6	59	38	64.41

CHAPTER 5

APPLICATION OF THE PROPOSED METHOD TO DIFFERENT CLASSES OF PETRI NET MODELS FOR FMSS

5.1 INTRODUCTION

Petri nets have become one of the most powerful tools to handle deadlock problems in flexible manufacturing systems (FMS). Different types of Petri net sub-classes proposed such as S^3PR, S^4PR, S^4R, WS^3PR, ES^3PR, LS^3PR etc [3]. All these types of Petri net sub-classes are used to study deadlock problems in flexible manufacturing systems.
Three S^3PR Petri net models are considered in the previous chapter to show the application of the proposed method. This chapter shows the applicability of the method to other sub-classes of Petri nets apart from the S^3PR Petri nets considered in the previous chapter.

5.2 AN S^4PR PETRI NET EXAMPLE

The S^4PR Petri net model (PNM) of an FMS [31] is shown in Fig. 5.2, with a set of control places computed by using the an FBM variant method [34]. The net has 25 places and 19 transitions, there places can be considered to be the collection of $P^0 = \{p_8, p_{12}, p_{20}\}$, $P_R = \{p_{18}, p_{19}, \cdots, p_{25}\}$ and $P_A = \{p_1, p_2, \cdots, p_7, p_9, \cdots, p_{11}, p_{13}, p_{14}, \cdots, p_{17}\}$. The net has 9378 reachable states in which there are 546 bad states and 8832 good states.

Control places (monitors) computed for this PNM shown in Fig. 5.1 are provided in Table 5.1, together with their *PIs*. The controlled PNM obtained by including the eight control places shown in Table 5.1 into the uncontrolled PNM shown in Fig. 5.1 is live and can reach 8576 good states.

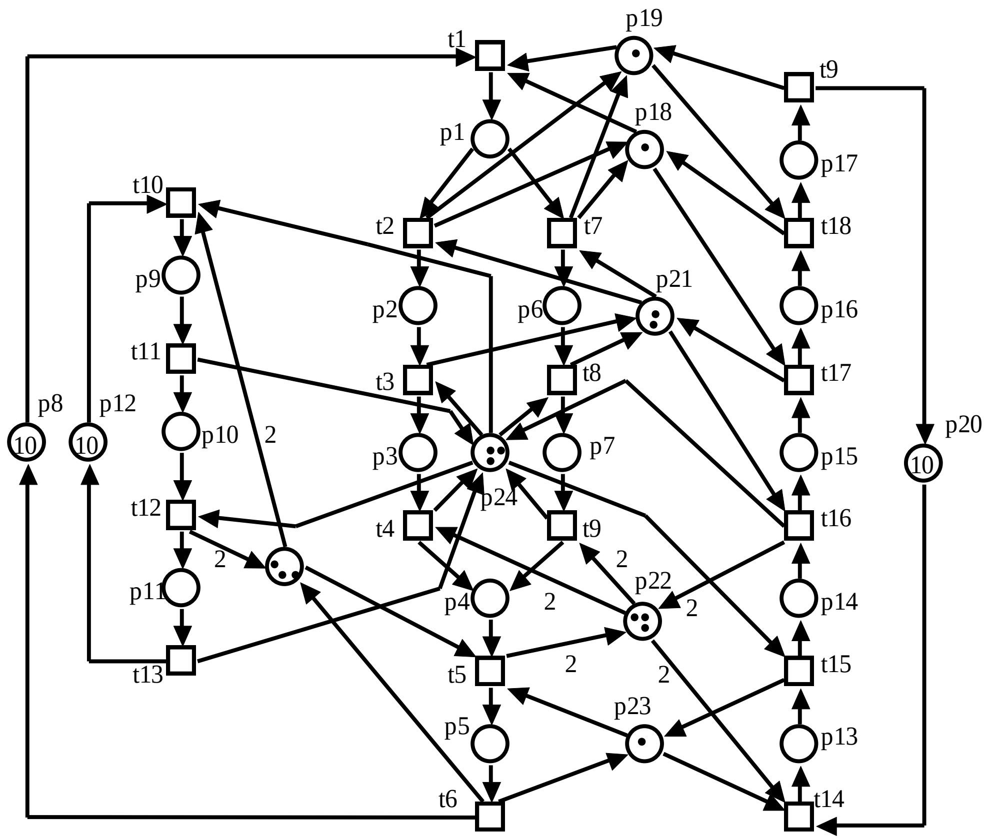


Fig. 5.1. A Petri net model of an S^{4}PR net [31].

Table 5.1. Place invariants and control places for the S^{3}PR shown in Fig. 5.1.

FBM_i	PI_i	C_i	$^\bullet C_i$	$C_i^\bullet$	M_0
$\mu_1 = 1, \mu_{15} = 2$	$\mu_1 + \mu_{15} \le 2$	*C1*	t2, t7, t17	t1, t16	2
$\mu_3 = 1, \mu_7 = 2, \mu_{13} = 1$	$\mu_3 + \mu_7 + \mu_{13} \le 3$	*C2*	t4, t9, t15	t3, t8, t14	3
$\mu_2 = 1, \mu_3 = 1, \mu_6 = 1, \mu_7 = 1, \mu_{13} = 1$	$\mu_2 + \mu_3 + \mu_6 + \mu_7 + \mu_{13} \le 4$	*C3*	t4, t9, t15	t2, t7, t14	4
$\mu_2 = 1, \mu_3 = 1, \mu_6 = 1, \mu_7 = 1, \mu_{14} = 1$	$\mu_2 + \mu_3 + \mu_6 + \mu_7 + \mu_{14} \le 4$	*C4*	t4, t9, t16	t2, t7, t15	4
$\mu_1 = 1, \mu_2 = 1, \mu_3 = 1, \mu_7 = 1, \mu_{13} = 1, \mu_{15} = 1$	$\mu_1 + \mu_2 + \mu_3 + \mu_7 + \mu_{13} + \mu_{15} \le 5$	*C5*	t4, t7, t9, t15, t17	t1, t8, t14, t16	5
$\mu_1 = 1, \mu_2 = 1, \mu_3 = 1, \mu_7 = 1, \mu_{14} = 1, \mu_{15} = 1$	$\mu_1 + \mu_2 + \mu_3 + \mu_7 + \mu_{14} + \mu_{15} \le 5$	*C6*	t4, t7, t9, t17	t1, t8, t15	5
$\mu_1 = 1, \mu_3 = 1, \mu_6 = 1, \mu_7 = 1, \mu_{13} = 1, \mu_{15} = 1$	$\mu_1 + \mu_3 + \mu_6 + \mu_7 + \mu_{13} + \mu_{15} \le 5$	*C7*	t2, t4, t9, t15, t17	t1, t3, t14, t16	5
$\mu_1 = 1, \mu_3 = 1, \mu_6 = 1, \mu_7 = 1, \mu_{14} = 1, \mu_{15} = 1$	$\mu_1 + \mu_3 + \mu_6 + \mu_7 + \mu_{14} + \mu_{15} \le 5$	*C8*	t2, t4, t9, t17	t1, t3, t15	5

Step 1:

Possible place invariants that could be merged are identified as:

$$Z_1 = \{PI_5, PI_6, PI_7, PI_8\}$$

$$Z_2 = \{PI_2, PI_3, PI_4\}$$

Step 2: $(i = 1,\ \text{first iteration})$.

In the first iteration, let us consider Z_1.

Step 2.1.1:

The common intersecting elements among the place invariants PI_5, PI_6, PI_7 and PI_8 are:

$$PI_5 \cap PI_6 \cap PI_7 \cap PI_8 = \{p_1, p_3, p_7, p_{15}\}$$

Step 2.1.2:

A tentative equation for a merged place invariant can be written as:

$$mPI_1 = \alpha_1\mu_1 + \alpha_2\mu_2 + \alpha_3\mu_3 + \alpha_6\mu_6 + \alpha_7\mu_7 + \alpha_{13}\mu_{13} + \alpha_{14}\mu_{14} + \alpha_{15}\mu_{15} \le k \qquad (5.1.1)$$

Step 2.1.3:

Linear equations are established by using the co-efficients of possible place invariants to be merged.

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_7 + \alpha_{13} + \alpha_{15} = k + 1 \qquad (5.1.2)$$

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_7 + \alpha_{14} + \alpha_{15} = k + 1 \qquad (5.1.3)$$

$$\alpha_1 + \alpha_3 + \alpha_6 + \alpha_7 + \alpha_{13} + \alpha_{15} = k + 1 \qquad (5.1.4)$$

$$\alpha_1 + \alpha_3 + \alpha_6 + \alpha_7 + \alpha_{14} + \alpha_{15} = k + 1 \qquad (5.1.5)$$

Step 2.1.4:

Linear relationships are established that exists among the unknown co-efficients of place invariants to be merged.

By equating Eqs. (5.1.2) and (5.1.3):

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_7 + \alpha_{13} + \alpha_{15} = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_7 + \alpha_{14} + \alpha_{15}$$

$$\alpha_{13} = \alpha_{14} \qquad (5.1.6)$$

Also, by equating Eqs. (5.1.3) and (5.1.5).

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_7 + \alpha_{14} + \alpha_{15} = \alpha_1 + \alpha_3 + \alpha_6 + \alpha_7 + \alpha_{14} + \alpha_{15}$$

$$\alpha_2 = \alpha_6 \qquad (5.1.7)$$

Also according to the rule 3. $\alpha_1 = \alpha_3 = \alpha_7 = \alpha_{15}$ (5.1.8)

Step 2.1.5:

Having obtained the linear relationships among the unknown co-efficients, a suitable value is selected for the co-efficients in Eqs. (5.1.6), (5.1.7) and (5.1.8). The possible values to be selected should be the values of the cardinality of the elements in the place invariants under consideration.

$$|p_9| = 4 \text{ and } |p_{10}| = 4$$

Let $\alpha_{13} = \alpha_{14} = 3$

$$\alpha_2 = \alpha_6 = 3$$

According to the rule 4. $\alpha_1 = \alpha_3 = \alpha_7 = \alpha_{15} = 4$

Step 2.1.6:

Eq. (5.1.2) is used to find the value of k as:

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_7 + \alpha_{13} + \alpha_{15} = k + 1$$

$$4 + 3 + 4 + 4 + 3 + 4 = k + 1$$

$$k = 21$$

Step 2.1.7:

Finally, the resulting merged place invariant mPI_1 is:

$$mPI_1 = 4\mu_1 + 3\mu_2 + 4\mu_3 + 3\mu_6 + 4\mu_7 + 3\mu_{13} + 3\mu_{14} + 4\mu_{15} \leq 21 \qquad (5.1.9)$$

Step 2.1.8:

The monitor C_1 is computed for mPI_1 as follows:

$$\mu_0(C_1) = 21$$

$$Dc_1 = -L_{mPI_1} . D_{mPI_1}$$

$$Dc_1 = -\begin{array}{c} \begin{array}{cccccccc} p_1 & p_2 & p_3 & p_6 & p_7 & p_{13} & p_{14} & p_{15} \end{array} \\ \begin{bmatrix} 4 & 3 & 4 & 3 & 4 & 3 & 3 & 4 \end{bmatrix} \end{array} \begin{array}{cc} \begin{array}{ccccccccccc} t_1 & t_2 & t_3 & t_4 & t_7 & t_8 & t_9 & t_{14} & t_{15} & t_{16} & t_{17} \end{array} & \\ \begin{bmatrix} 1 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} & \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_6 \\ p_7 \\ p_{13} \\ p_{14} \\ p_{15} \end{array} \end{array}$$

$$Dc_1 = \begin{array}{c} \begin{array}{ccccccccccc} t_1 & t_2 & t_3 & t_4 & t_7 & t_8 & t_9 & t_{14} & t_{15} & t_{16} & t_{17} \end{array} \\ \begin{bmatrix} -4 & 1 & -1 & 4 & 1 & -1 & 4 & -3 & 0 & -1 & 4 \end{bmatrix} \end{array}$$

Step 2: $(i = 2,\ \text{second iteration})$.

In the second iteration, let us consider Z_2.

Step 2.2.1:

The common intersecting elements between the place invariants PI_2, PI_3 and PI_4 is as follows:

$$PI_2 \cap PI_3 \cap PI_4 = \{p_3, p_7\}$$

Step 2.2.2:

A tentative equation for a merged place invariant can be written as:

$$mPI_2 = \alpha_2\mu_2 + \alpha_3\mu_3 + \alpha_6\mu_6 + \alpha_7\mu_7 + \alpha_{13}\mu_{13} + \alpha_{14}\mu_{14} \le k \tag{5.1.10}$$

Step 2.2.3:

Linear equations are established by using the co-efficients of possible place invariants to be merged.

$$\alpha_2 + 2\alpha_7 + \alpha_{13} = k + 1 \tag{5.1.11}$$

$$\alpha_2 + \alpha_3 + \alpha_6 + \alpha_7 + \alpha_{13} = k + 1 \tag{5.1.12}$$

$$\alpha_2 + \alpha_3 + \alpha_6 + \alpha_7 + \alpha_{14} = k + 1 \tag{5.1.13}$$

Step 2.2.4:

Linear relationships are established that exist among the unknown co-efficients of place invariants to be merged.

By equating Eqs. (5.1.11) and (5.1.12).

$$\alpha_3 + 2\alpha_7 + \alpha_{13} = \alpha_2 + \alpha_3 + \alpha_6 + \alpha_7 + \alpha_{13}$$

$$\alpha_7 = \alpha_2 + \alpha_6 \tag{5.1.14}$$

Also, by equating Eqs. (5.1.12) and (5.1.13).

$$\alpha_2 + \alpha_3 + \alpha_6 + \alpha_7 + \alpha_{14} = \alpha_2 + \alpha_3 + \alpha_6 + \alpha_7 + \alpha_{13}$$

$$\alpha_{13} = \alpha_{14} \tag{5.1.15}$$

According to the Rule 3: $\alpha_3 = \alpha_7$ (5.1.16)

Also according to the rule 5. $\alpha_3 \geq 3$

Step 2.2.5:

Having obtained the linear relationships among the unknown co-efficients, a suitable value is selected for the co-efficients in Eqs. (5.1.14), (5.1.15) and (5.1.16). The possible values to be selected should be the values of the cardinality of the elements in the place invariants under consideration.

$$|p_3| = 3 \text{ and } |p_7| = 3$$

$$\text{Let} \quad \alpha_2 = \alpha_6 = 2$$

$$\therefore \quad \alpha_7 = 4$$

According to the rule 4. $\alpha_3 = \alpha_7 = 4$

Step 2.2.6:

By using Eq. (4.1.11), the value of k can be computed as:

$$\alpha_3 + 2\alpha_7 + \alpha_{13} = k + 1$$

$$4 + 8 + 3 = k + 1$$

$$k = 14$$

Step 2.2.7:

Finally, the resulting merged place invariant mPI_2 is:

$$mPI_2 = 2\mu_2 + 4\mu_3 + 2\mu_6 + 4\mu_7 + 3\mu_{13} + 3\mu_{14} \leq 14 \tag{5.1.17}$$

Step 2.2.8:

The monitor C_2 is computed for mPI_2 as follows:

$$\mu_0(C_2) = 14$$

$$Dc_2 = -L_{mPI_2} . D_{mPI_2}$$

$$Dc_2 = -\begin{array}{c} \begin{array}{cccccc} p_2 & p_3 & p_6 & p_7 & p_{13} & p_{14} \end{array} \\ \begin{bmatrix} 2 & 4 & 2 & 4 & 3 & 3 \end{bmatrix} \end{array} \begin{array}{cc} \begin{array}{ccccccccc} t_2 & t_3 & t_4 & t_7 & t_8 & t_9 & t_{14} & t_{15} & t_{16} \end{array} & \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} & \begin{array}{c} p_2 \\ p_3 \\ p_6 \\ p_7 \\ p_{13} \\ p_{14} \end{array} \end{array}$$

$$Dc_2 = \begin{array}{c} \begin{array}{ccccccccc} t_2 & t_3 & t_4 & t_7 & t_8 & t_9 & t_{14} & t_{15} & t_{16} \end{array} \\ \begin{bmatrix} -2 & -2 & 4 & -2 & -2 & 4 & -3 & 0 & 3 \end{bmatrix} \end{array}$$

Finally after applying the above algorithm, the net has only two merged place invariants mPI_1 and mPI_2 one other place invariant as shown in Eqs. (5.1.9) and (5.1.17). These merged *PIs* and monitors computed are shown in Table 5.2. When this set of reduced monitors C_1, C_2 and C_3 are added to the PNM given in Fig. 5.1 the controlled model is obtained. It is verified that this controlled model is live, with a near optimal behavior, which has 8576 good states.

Table 5.2. The merged PIs and computed monitors.

PI_1 and $mPIi$	Ci	$^{\bullet}C$	$C^{\bullet}$	M_0
$mPI_1 = 4\mu_1 + 3\mu_2 + 4\mu_3 + 3\mu_6 + 4\mu_7 + 3\mu_{13} + 3\mu_{14} + 4\mu_{15} \leq 21$	*C1*	*t2, 4t4, t7,4t9, 4t17*	*4t1, t3, t8, 3t14, t16*	*21*
$mPI_2 = 2\mu_2 + 4\mu_3 + 2\mu_6 + 4\mu_7 + 3\mu_{13} + 3\mu_{14} \leq 14$	*C2*	*4t4, 4t9, 3t16*	*2t2, 2t3, 2t7, 2t8, 2t14*	*14*
$PI_1 = \mu_1 + \mu_{15} \leq 2$	*C3*	*t2, t7, t17*	*t1, t16*	*2*

Table 5.3 compares the original monitors computed by using an FBM variant method with that of the proposed method. From the Table 5.3, it is clear to observe that the number of control places obtained with the proposed method is as minimum as possible when compared with the original monitors computed by using an FBM variant method.

Table 5.3 Performance comparison between the original monitors and the reduced monitors obtained with the proposed method.

LES	# monitors	# arcs	# tokens	% permissiveness
Original monitors	8	55	33	97.10
Reduced monitors	3	50	37	97.10

5.3 A G-SYSTEM PETRI NET EXAMPLE

The G-system Petri net of an FMS [32] is shown in Fig. 5.2, with the set of control places computed by using anFBM variant method [34]. The net has 23 places and 18 transitions. There places can be considered to be the collection of $P^0 = \{p_{11}, p_{17}, p_{18}\}$, $P_R = \{p_{22}, p_{23}, \cdots, p_{27}\}$ and $P_A = \{p_1, p_2, \cdots, p_4, p_6, \cdots, p_4, p_6, \cdots, p_{10}, p_{12}, \cdots, p_{16}, p_{10},$ $p_{12}, \cdots, p_{16}, \quad p_{13}, p_{14}, \cdots, p_{17}\}$ The net has 68531 reachable states in which there are 2131 bad states and 66400 good states.

Control places (monitors) computed for this PNM shown in Fig. 5.2 are provided in Table 5.4, together with their *PIs*. The controlled PNM obtained by including the eleven control places shown in Table 5.4 into the uncontrolled PNM shown in Fig. 5.2 is live and can reach 62682 good states.

Table 5.4. Place invariants and monitors for the *G-System* net shown in Fig. 5.2.

FBM_i	PI_i	C_i	$^\bullet C_i$	$C_i^\bullet$	M_0
$\mu_1 = 2, \mu_{14} = 2$	$\mu_1 + \mu_{14} \le 3$	*C1*	t2, t6, t16	t1, t15	3
$\mu_2 = 1, \mu_6 = 1, \mu_{13} = 3$	$\mu_2 + \mu_6 + \mu_{13} \le 4$	*C2*	t3, t7, t15	t2, t6, t14	4
$\mu_2 = 1, \mu_3 = 1, \mu_6 = 1, \mu_{12} = 1, \mu_{13} = 2$	$\mu_2 + \mu_3 + \mu_6 + \mu_{12} + \mu_{13} \le 5$	*C3*	t4, t7, t15	t2, t6, t13	5
$\mu_2 = 1, \mu_6 = 1, \mu_7 = 1, \mu_{12} = 1, \mu_{13} = 2$	$\mu_2 + \mu_6 + \mu_7 + \mu_{12} + \mu_{13} \le 5$	*C4*	t3, t8, t15	t2, t6, t13	5
$\mu_3 = 2, \mu_7 = 1, \mu_{12} = 3$	$\mu_3 + \mu_7 + \mu_{12} \le 5$	*C5*	t4, t8, t14	t3, t7, t13	5
$\mu_1 = 2, \mu_2 = 1, \mu_{13} = 3, \mu_{14} = 1$	$\mu_1 + \mu_2 + \mu_{13} + \mu_{14} \le 6$	*C6*	t3, t6, t16	t1, t14	6
$\mu_1 = 2, \mu_6 = 1, \mu_{13} = 3, \mu_{14} = 1$	$\mu_1 + \mu_6 + \mu_{13} + \mu_{14} \le 6$	*C7*	t2, t7, t16	t1, t14	6
$\mu_1 = 2, \mu_2 = 1, \mu_3 = 1, \mu_{12} = 1, \mu_{13} = 2, \mu_{14} = 1$	$\mu_1 + \mu_2 + \mu_3 + \mu_{12} + \mu_{13} + \mu_{14} \le 7$	*C8*	t4, t6, t16	t1, t13	7
$\mu_1 = 2, \mu_6 = 1, \mu_7 = 1, \mu_{12} = 1, \mu_{13} = 2, \mu_{14} = 1$	$\mu_1 + \mu_6 + \mu_7 + \mu_{12} + \mu_{13} + \mu_{14} \le 7$	*C9*	t2, t8, t16	t1, t13	7
$\mu_1 = 2, \mu_3 = 1, \mu_6 = 1, \mu_{12} = 1, \mu_{13} = 2, \mu_{14} = 1$	$\mu_1 + \mu_3 + \mu_6 + \mu_{12} + \mu_{13} + \mu_{14} \le 7$	*C10*	t2, t4, t7, t16	t1, t3, t13	7
$\mu_1 = 2, \mu_2 = 1, \mu_7 = 1, \mu_{12} = 1, \mu_{13} = 2, \mu_{14} = 1$	$\mu_1 + \mu_2 + \mu_7 + \mu_{12} + \mu_{13} + \mu_{14} \le 7$	*C11*	t3, t6, t8, t16	t1, t7, t13	7

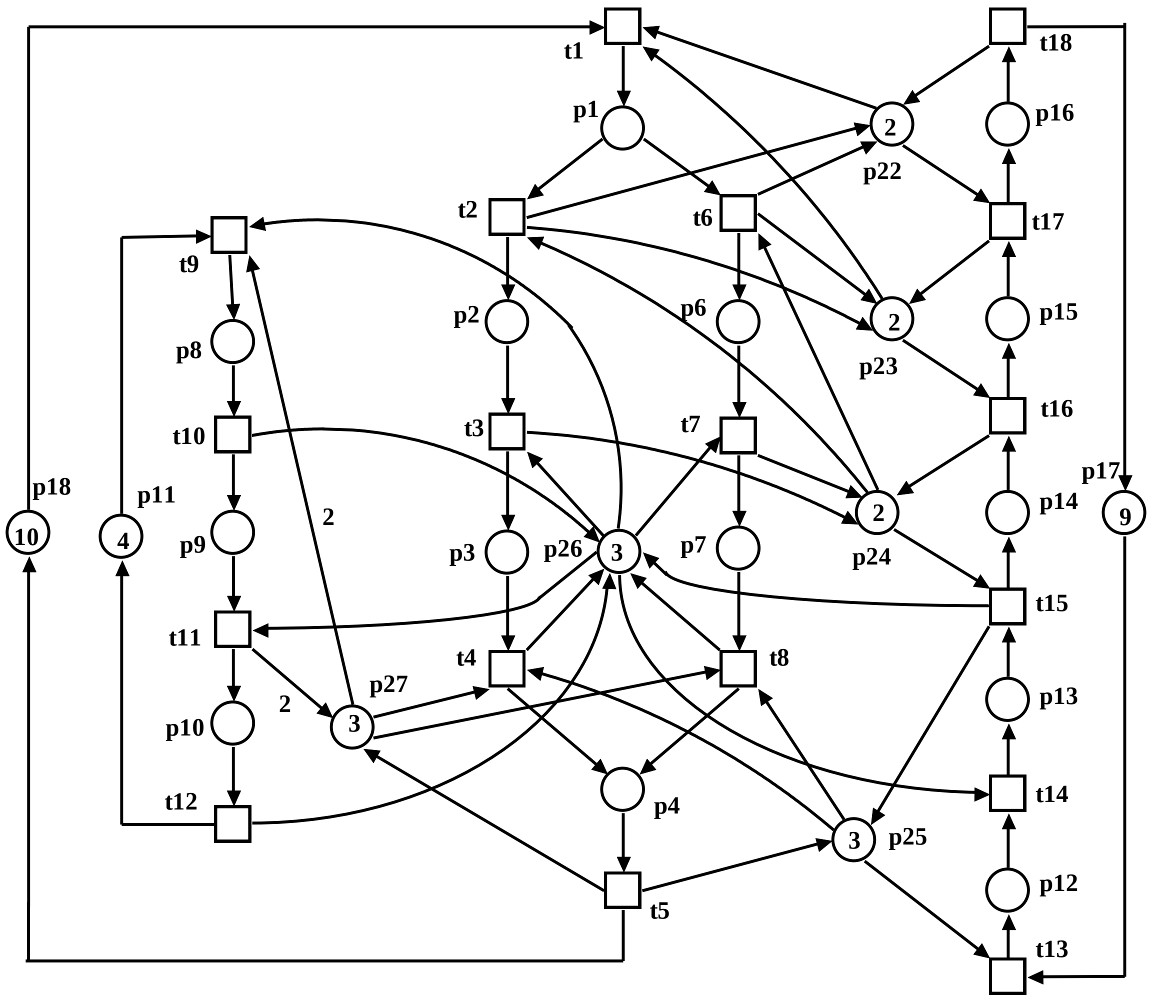


Fig. 5.2. The simplified G-System net (PNM) from [32].

Step 1:

Possible place invariants that could be merged are identified as:

$$Z_1 = \{PI_8, PI_9, PI_{10}, PI_{11}\}$$

$$Z_2 = \{PI_3, PI_4, PI_5\}$$

$$Z_3 = \{PI_6, PI_7\}$$

The same procedure of the previous examples are applied to the set Z_1, Z_2 and Z_3. The net has only three merged place invariants namely, mPI_1, mPI_2 and mPI_3 together with the remaining two place invariants (i.e. place invariants that could not be merged with any given place invariant in Table 5.4).

These merged *PIs* with two other place invariants (PI_1 and PI_2) and monitors computed are all shown in Table 5.5. When this set of monitors C_1, C_2, C_3, C_4 and C_5 are added to the PNM given in Fig. 5.2 the controlled model is obtained. It is verified that this controlled model is live, with a near optimal behavior, which has 63859 good states.

Table 5.5. The merged PIs and computed monitors.

PI_i and $mPIi$	Ci	$^\bullet C_i$	$C_i^\bullet$	M_0
$mPI_1=2\mu_2 + 2\mu_3 + 2\mu_6 + 2\mu_7 + 3\mu_{12} + 3\mu_{14} \leq 14$	C1	2t4, 2t8, 3t15	2t2, 2t6, 3t13	14
$mPI_2=2\mu_1 + \mu_2 + \mu_6 + 2\mu_{13} + 2\mu_{14} \leq 12$	C2	t2, t3, t6, t7, 2t16	2t1, 2t14	12
$mPI_3=4\mu_1 + 2\mu_2 + 2\mu_3 + 2\mu_6 + 2\mu_7 + 4\mu_{12} + 4\mu_{13} + 4\mu_{14} \leq 27$	C3	2t2, 2t4, 2t6, 2t8, 4t16	4t1, 4t13	27
$PI_1=\mu_1 + \mu_{14} \leq 3$	C4	t2, t6, t16	t1, t15	3
$PI_2=\mu_2 + \mu_6 + \mu_{13} \leq 4$	C5	t3, t7, t15	t2, t6, t14	4

Table 5.6 compares the original monitors computed by using an FBM variant method with that of the proposed method. From the Table 5.6, it is clear to observe that the number of control places obtained with the proposed method is as minimum as possible when compared with the original monitors computed by using an FBM variant method.

Table 5.6 Performance comparison between the original monitors and the reduced monitors obtained with the proposed method.

LES	# monitors	# arcs	# tokens	% permissiveness
Original monitors	11	63	62	94.40
Reduced monitors	5	55	60	96.17

5.4 AN S^{4}R PETRI NET MODEL EXAMPLE

The S^{4}R Petri net model of an FMS [33] is shown in Fig. 5.3, control places (monitors) are due to an FBM variant method [34]. The net has 23 places and 18 transitions. There places can be considered to be the collection of $P_0 = \{p_1, p_5, p_{13}\}$, $P_R = \{p_{18}, p_{19}, \cdots, p_{23}\}$ and $P_A = \{p_2, p_3, p_4, p_6, \cdots, p_{12}, p_{14}, \cdots, p_{17}\}$. The net has 19300 reachable states in which there are 935 bad states and 18365 good states.

Control places (monitors) computed for the PNM shown in Fig. 5.3 are provided in Table 5.7, together with their *PIs*. The controlled PNM obtained by including the eleven control places shown in Table 5.7 into the uncontrolled PNM shown in Fig. 5.3 is live and can reach 17101 good states.

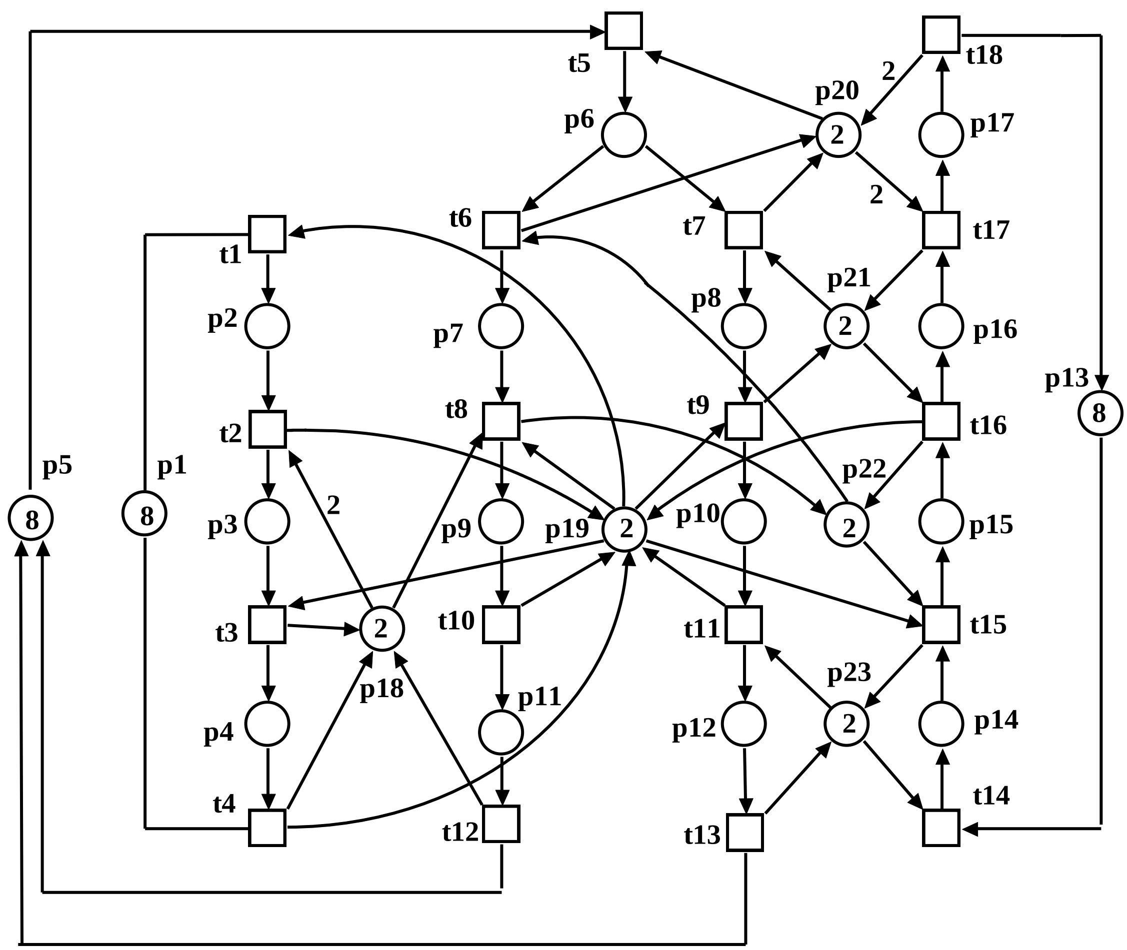


Fig. 5.3. An S^4R Petri net model (PNM) from [33].

Table 5.7. Place invariants and monitors computed for the S^4R net shown in Fig. 5.3.

FBM_i	PI_i	C_i	$^\bullet C_i$	$C_i^\bullet$	M_0
$\mu_2 = 2, \mu_3 = 1$	$\mu_2 + \mu_3 \leq 2$	$C1$	$t3$	$t1$	2
$\mu_8 = 2, \mu_{15} = 2$	$\mu_8 + \mu_{15} \leq 3$	$C2$	$t9, t16$	$t7, t15$	3
$\mu_{10} = 2, \mu_{14} = 2$	$\mu_{10} + \mu_{14} \leq 3$	$C3$	$t11, t15$	$t9, t14$	3
$\mu_6 = 1, \mu_8 = 1, \mu_{15} = 2, \mu_{16} = 1$	$\mu_6 + \mu_8 + \mu_{15} + \mu_{16} \leq 4$	$C4$	$t6, t9, t17$	$t5, t15$	4
$\mu_2 = 1, \mu_3 = 1, \mu_8 = 2, \mu_{15} = 1$	$\mu_2 + \mu_3 + \mu_8 + \mu_{15} \leq 4$	$C5$	$t3, t9, t16$	$t1, t7, t15$	4
$\mu_2 = 1, \mu_3 = 1, \mu_{10} = 1, \mu_{14} = 2$	$\mu_2 + \mu_3 + \mu_{10} + \mu_{14} \leq 4$	$C6$	$t3, t11, t15$	$t1, t9, t14$	4
$\mu_8 = 2, \mu_{10} = 1, \mu_{14} = 2, \mu_{15} = 1$	$\mu_8 + \mu_{10} + \mu_{14} + \mu_{15} \leq 5$	$C7$	$t11, t16$	$t7, t14$	5
$\mu_2 = 1, \mu_3 = 1, \mu_6 = 1, \mu_7 = 1, \mu_8 = 1, \mu_{15} = 1, \mu_{16} = 1$	$\mu_2 + \mu_3 + \mu_6 + \mu_7 + \mu_8 + \mu_{15} + \mu_{16} \leq 6$	$C8$	$t3, t8, t9, t17$	$t1, t5, t15$	6
$\mu_6 = 1, \mu_7 = 1, \mu_8 = 1, \mu_{10} = 1, \mu_{14} = 2, \mu_{15} = 1, \mu_{16} = 1$	$\mu_6 + \mu_7 + \mu_8 + \mu_{10} + \mu_{14} + \mu_{15} + \mu_{16} \leq 7$	$C9$	$t8, t11, t17$	$t5, t14$	7

Step 1:

Possible place invariants that could be merged are identified as:

$$Z_1 = \{PI_3, PI_7\}$$

$$Z_2 = \{PI_1, PI_6\}$$

$$Z_3 = \{PI_2, PI_5\}$$

$$Z_4 = \{PI_4, PI_8\}$$

The same procedure of the previous examples are applied to the set Z_1, Z_2 Z_3 and Z_4. The net has only four merged place invariants namely mPI_1, mPI_2, mPI_3 and mPI_4 together with the remaining one place invariant (i.e. place invariant (PI_9) that could not be merged with any given place invariant in the set).

These four merged *PIs* with one place invariant and monitors computed are all shown in Table 5.8. When this set of monitors C_1, C_2, C_3, C_4 and C_5 are added to the PNM given in Fig. 5.3 the controlled model is obtained. It is verified that this controlled model is live, with 17244 good states.

Table 5.8. The merged PIs and computed monitors.

$mPIi+PI_9$	Ci	$^{\bullet}C$	$C^{\bullet}$	M_0
$mPI_1=\mu_8 + 3\mu_{10} + 3\mu_{14} + \mu_{15} \leq 11$	*C1*	*3t11, 2t15, t16*	*t7, 2t9, 3t14*	*11*
$mPI_2=3\mu_2 + 3\mu_3 + \mu_{10} + \mu_{14} \leq 8$	*C2*	*3t3, t11, t15*	*3t1, t9, t14*	*8*
$mPI_3=\mu_2 + \mu_3 + 2\mu_8 + 2\mu_{15} \leq 7$	*C3*	*t3, 2t9, 2t16*	*t1, 2t7, 2t15*	*7*
$mPI_4=\mu_2 + \mu_3 + 3\mu_6 + \mu_7 + 3\mu_8 + 3\mu_{15} + 3\mu_{16} \leq 14$	*C4*	*t3, 2t6, t8, 3t9, 3t17*	*t1, 3t5, 3t15*	*14*
$PI_9=\mu_6 + \mu_7 + \mu_8 + \mu_{10} + \mu_{14} + \mu_{15} \leq 7$	*C5*	*t8, t11, t17*	*t5, t14*	*7*

Table 5.9 compares the original monitors computed by using an FBM variant method with that of the proposed method. From the Table 5.9, it is clear to observe that the number of control places obtained with the proposed method is as minimum as possible when compared with the original monitors computed by using an FBM variant method.

Table 5.9 Performance comparison between the original monitors and reduced monitors obtained with the proposed method.

LES	# monitors	# arcs	# tokens	% permissiveness
Original net	9	43	38	93.11
Reduced net	5	54	47	93.89

CHAPTER 6

CONCLUSIONS

In this thesis, a new method has been proposed for reducing the structural complexity of a given Petri net based Liveness-enforcing supervisor of an FMS suffering from deadlocks. It is assumed that an uncontrolled PNM of an FMS suffering from deadlocks is given together with a liveness-enforcing supervisor consisting of a set of monitors and their place invariants (*PIs*). Then the proposed structural complexity reduction algorithm considers the PIs from which some of the PIs are merged based on some criteria. From the merged PIs new set of monitors are computed. Experimental studies show that the number of monitors are greatly reduced while maintaining the same or better behavioral permissiveness compared with the ones obtained with the original liveness-enforcing supervisor.

The proposed method requires solving some simple linear equalities. Therefore it is computationally simpler than the methods currently available in the literature. It is shown that the proposed method is not confined to a sub-class of Petri nets. Therefore it is applicable to reduce the structural complexity of Petri net based supervisors of all Petri net classes currently available in the literature.

The main assumption of the proposed method is that the given liveness-enforcing supervisor must be computed by using an FBM variant method. Therefore further studies are necessary to extend the proposed method in order to reduce liveness enforcing supervisors computed by using other synthesis approaches.

REFERENCES

[1] C. Hyuebo, T.k. Kumaran and R.A. Wysk, “Graph theoretic deadlock detection and resolution for flexible manufacturing systems,” *IEEE Trans. Robot Automat*, Vol. 11, No. 2, pp. 413-421, 1995.

[2] Y.F. Chen, Z.W. Li, M. Khalgui and O. Mosbahi, “Design of a maximally permissive Liveness-enforcing Petri net supervisor for flexible manufacturing systems.” *IEEE Transactions on Automation Science and Engineering*, Vol. 8, No. 2, pp. 374-393, 2011.

[3] Y.F. Chen, Z.W. Li, “Optimal Supervisory Control of Automated manufacturing Systems”. *CRC Press, Talor & Francis Group.* (2013).

[4] Y.F. Chen, and G. Liu, “Computation of minimal siphons in Petri nets by using binary decision diagram,” *ACM Trans. Embed. Computer Syst.*Vol.12, No. 1, article 3. (2013).

[5] Y.F. Chen, Z.W. Li, “Design of maximally permissive Liveness enforcing supervisor with a compressed supervisory structure for flexible manufacturing systems”, *Automatica*, 47, pp. 1028-1038, 2011.

[6] D.Y. Chao, “fewer monitors and more efficient controllability for deadlock control in S^3PR”, *The Computer Journal* 53, pp. 1783-1798, 2010.

[7] D.Y. Chao, “Direct minimal empty siphon computation using MIP”, *Int. J Manufac. Technol.*, 45, pp. 397-405, 2009.

[8] J. Ezepeleta, J.M. Colom, and J. Martinez, “A Petri nets based deadlock prevention policy for flexible manufacturing systems,” *IEEE Trans. Robot Autom.*, Vol. 11, No. 2, pp. 173-184, 1995.

[9] A. Ghaffari, N. Reng, and X.L. Xie, “ Design of a live and maximally permissive Petri nets controller using the theory of regions.” *IEEE Transactions on Robotics and Automation*, Vol. 19, No.1, pp. 137-142, 2003.

[10] H. Zandang, G. Lee, “Application of Petri nets for deadlock analysis and avoidance in flexible manufacturing systems”, *Int. J. Adv Manufac. Technol*. 25, pp. 735-742, 2005.

[11] Z.W. Li, and M.C. Zhou, "A maximally permissive deadlock prevention policy for FMS based on Petri net siphon control and theory of regions", *IEEE Transactions on Automations Science and Engineering*, Vol. 5, No. 1, pp. 182-188, 2008.

[12] Z.W. Li and M.C. Zhou, "Control of elementary and dependent siphons in Petri nets and their application", *IEEE Transactions on Systems, Man, and Cybernetics, Part A: Syst. Hum*, Vol. 38, No. 1, 133-148, 2008.

[13] L. Hong, D.Y. Chao, "Improved controllability Test for dependent siphons in S^3PR based on elementary siphons", *Asian Journal of control*. 12, pp. 377-391, 2010.

[14] L. Pirrodi, R. Cordone and I. fumagalli, "Combined siphon and marking generation for deadlock prevention in Petri nets", *IEEE Transactions on Systems, Man, and Cybernetics, Part A: Syst. Hum,* Vol. 39, pp. 650-661, 2009.

[15] L. Pirrodi, R. Cordone and I. Fumagalli, "Selective siphon control for deadlock prevention in Petri nets", *IEEE Transactions on Systems, Man, and Cybernetics,* Part A, Vol. 38, No. 6, pp. 1337-1348, 2008.

[16] A. Wang, M.C. Zhou, "An effective algorithm to find elementary siphons in a class of Petri nets," *IEEE Transactions on Systems, Man, Cybernetics Part A*, Vol. 39, No. 4, pp. 912-922, 2009.

[17] S.A. Reviolitis, "Implicit siphon control and its role in the Liveness enforcing supervision of sequential resource allocation systems", *IEEE Transactions on Systems, Man, and Cybernetics,* Part A, Vol.37, No. 3, pp. 319-324, 2007.

[18] M. Uzam, Z.W. Li and M.C. Zhou, "Identification and elimination of redundant control places in Petri nets based liveness-enforcing supervisors of FMS", *Int. J. Adv. Manufac. Technol,* Vol. 35, pp. 150-168, 2007.

[19] M. Uzam, "The use of Petri nets reduction approach for an optimal deadlock prevention policy for flexible manufacturing systems", *Int. J. Adv. Manuf. Technol.*, Vol. 23, No. 3, pp. 204-219, 2004.

[20] M. Uzam "An optimal deadlock prevention policy for flexible manufacturing systems using Petri nets models with resource and the theory of regions", *Int. J. Adv. Manuf. Technol.*, Vol. 19, No. 3, pp. 193-208, 2002.

[21] M. Uzam and M.C. Zhou, "An improved iterative synthesis method for liveness-enforcing supervisor of flexible manufacturing system", *International Journal of Production Research*. Vol. 44, No. 10, pp. 1987-2030, 2006.

[22] M. Uzam and M.C. Zhou, An iterative synthesis approach to Petri nets based deadlock prevention policy for flexible manufacturing systems", *IEEE Transactions on Systems, Man, and Cybernetics, Part B*, Vol. 35, No. 6, 1193-1202, 2007.

[23] R.A. Wysk, S.Y. Neng, and J. Sanjay, "Resolution of deadlock in flexible manufacturing systems: Avoidance and Recovery approaches", *J. Manuf. Syst.*, Vol. 13, pp.128-136, 1994.

[24] Y.F. Chen, Z.W. Li, "On structural minimality of optimal supervisors for flexible systems" *Automatica*. Vol. 48, pp. 2647-2656, 2012.

[25] Y. S. Huang, Y. L. Pan, "An improved maximally permissive deadlock prevention policy based on the theory of regions and reduction approach", *IET Control Theory Appl.* Vol. 5, pp. 1069-1078, 2011.

[26] Z.W. Li, W. NaiQi and M.C. Zhou, "Deadlock control of Automated Manufacturing System Based on Petri nets-A literature review," *IEEE Transactions on Systems, Man, and Cybernetics, Part C*, Vol. 42, No. 4, pp. 437-460, 2012.

[27] Z.W. Li and N. Wei, "A deadlock control policy for FMS using elementary siphons of Petri nets and an MIP approach", *IEEE Int. Conference. Ind. Technol*., pp. 938-943, 2005.

[28] Z.W. Li and M.C. Zhou, "Elementary siphons of Petri nets and their application to deadlock prevention in flexible manufacturing system", *IEEE Transactions on Systems, Man, and Cybernetics*", *Part A*, Vol. 34, No.1, pp. 38-51, 2004.

[29] S.G. Wang, C.Y. Wang, M.C. Zhou, "Controllability Conditions of resultant siphons in a class of Petri nets", *IEEE Transactions on Systems, Man, and Cybernetics, Part A*, Vol. 42, No. 5, pp. 1206-1215, 2012.

[30] Y.F. Chen, Z.W. Li, and M.C. Zhou, "Most permissive liveness-enforcing Petri net Supervisors for flexible manufacturing systems", *International Journal of Production Research*. Vol. 50, No. 22, pp. 6357-6371, 2012.

[31] Z. Chunfu, Z.W Li, K. Barkaoui, "Monitor Design for Siphon Control in S4R Nets," *International Journal of Innovative Computing Information and Control*. Vol. 7, No. 12, pp. 6677-6690, 2011.

[32] M. Zhao, H. Yifan, "An Iterative Method for Synthesizing Non-blocking Supervisors for a Class of generalized Petri net using Mathematical Programming", *Discrete Event Dynamic Systems-Theory and Applications*. Vol. 23, No. 1, pp. 3-26, 2013.

[33] H. Hesuan, M.C. Zhou, Z.W. Li "Supervisor Optimization for Deadlock Resolution in Automated Manufacturing Systems with Petri Nets", *IEEE Transactions on Automation Science and Engineering.* Vol. 8, No. 4. pp. 794-804, 2011.

[34] M. Uzam, Z.W. Li, and U.S. Abubakar, "A General Approach for the Computation of a Liveness Enforcing Supervisor for the Petri Net Model of an FMS", *Inter. Workshops Petri Nets for Adaptive Discrete-Event Systems ADECS*, within the 35th

international Conference on Application and Theory of Petri nets and Concurrency, Tunis, Tunisia, 15 pages, June 23-27, 2014.

[35] G.Y. Liu, D. Chao, M. Uzam, "A merging method for the siphon-based FMS maximally permissive controller with simpler structures", *IMA J. of Mathematical Control and Infom. Adv. Access*, doi:10.1093/imamci/dtn029, published online August 21, 2013.

[36] Y.F. Chen, Z.W. Li, M.C. Zhou, "Behaviorally Optimal and Structurally Simple Liveness-Enforcing Supervisors of Flexible Manufacturing Systems", *IEEE Transactions on Systems, Man, and Cybernetics*, Part A, Syst. And Humans,Vol. 42, No. 3, pp. 615-628, 2012.