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A NEAR-OPTIMAL APPROACH FOR THE SYNTHESIS OF PETRI NET BASED LIVENESS ENFORCING SUPERVISORS IN FLEXIBLE MANUFACTURING SYSTEMS

by

Tahir Lawan SALEH

June 2014
Kayseri, Turkey

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by

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APPROVAL PAGE

This is to certify that I have read the thesis entitled "A Near-Optimal Approach for the Synthesis of Petri Net Based Liveness Enforcing Supervisors in Flexible Manufacturing Systems" by Tahir Lawan Saleh and that in my opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science in Electrical and Computer Engineering, the Graduate Institute of Science and Engineering, Melikşah University.

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ABSTRACT

In a flexible manufacturing system (FMS), an undesirable situation called deadlocks may occur due to the existence of shared resources. Petri nets (PN) are popular modeling tool used for the analysis, design and control of FMS. In this study, PN models of FMSs are utilized to handle deadlocks that may occur in the system. A new method is proposed for deadlock prevention by using a Global sink/source place (GP). The proposed method is especially effective for a generalized PN classes. All computed control places have weighted arcs due to the approach proposed. The GP is used temporarily in the design steps and is removed when the liveness of the system is obtained. The aim is to obtain an easy to use deadlock prevention policy that will ensure liveness with better behavioral permissiveness while maintaining less computational cost.

Key words: Flexible manufacturing system (FMS), Petri net model (PNM), Global sink/source place (GP), deadlock and liveness.

ESNEK ÜRETİM SİSTEMLERİNDE CANLILIK SAĞLAYICI GÖZETİCİLERİN SENTEZLENMESİ İÇİN OPTİMUMA YAKIN BİR YAKLAŞIM

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ÖZ

Bir esnek üretim sisteminde (Flexible Manufacturing System – FMS), kördüğüm olarak adlandırılan istenmeyen bir durum, paylaşılan kaynakların varlığı sebebiyle oluşabilir. Petri ağları (Petri nets – PN), FMS'in analizi, tasarımı ve kontrolu için kullanılan popüler bir modelleme aracıdır. Bu çalışmada, FMS'lerin Petri ağı modelleri, sistemde oluşabilecek kördüğümlerin üstesinden gelmek için kullanılmaktadır. Küresel bir yutak/kaynak mevkisi (Global sink/source place – GP) kullanarak kördüğüm önlenmesi için yeni bir yöntem önerilmektedir. Önerilen yöntem, özellikle genel Petri ağı sınıflarında etkilidir. Hesaplanan tüm kontrol mevkileri önerilen yaklaşım nedeniyle ağırlıklı oklara sahiptir. GP tasarım adımlarında geçici olarak kullanılır ve sistemin canlılığı elde edildiğinde kaldırılır. Amaç, az hesaplama maliyetiyle daha iyi davranış serbestlikli canlılık sağlayıcı kördüğüm önleme ilkesini kolay bir şekilde elde etmektir.

Anahtar Kelimeler: Esnek üretim sistemleri, Petri ağı modeli, Küresel yutak / kaynak mevkisi, kördüğüm, canlılık.

Dedicated to my parents;

Late father, Mal. Lawan Saleh,

Mother, Safiya Sanusi,

Late aunt Amina Saleh, and

Late grandmother Aisha Umar

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TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURE

LIST OF SYMBOLS AND ABBREVIATIONS

SYMBOL/ABBREVIATION	
AGV	Automated guided vehicle
BM	Bad marking
DES	Discrete-event system
DZ	Dead zone
FMS	Flexible manufacturing system
GP	Global sink/source place
G-System	General system
INA	Integrated Net Analyzer
LPN	Live Petri net
LZ	Live zone
PI	Place invariant
PN	Petri net
PNM	Petri net model
RG	Reachability graph
S^3PR	System of simple sequential processes with resource
S^4PR	Systems of sequential systems with shared resources
S^4R	System of sequential systems with shared resources
SMS	Strict minimal siphons
TPNM	Transformed Petri net model
WAMG	Weighted Automated Marked Graph
$\forall$	for all
$\exists$	there exist
$\nexists$	there not exist
$\in$	Belongs to

$\cup$	Union
$\subseteq$	Includes
iff	if and only if
$\mathbb{N}$	The set of non-negative integers
$\mathbb{N}^{+}$	The set of positive integers
$\emptyset$	Empty set
S	Set of places
W	Weight of an arc

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Rapid change in customer needs on products in time results in continuous modification of products so as to meet the customers' expectations. This is a big challenge to manufacturing processes. This also influences the need for flexible and automated manufacturing systems. Flexible manufacturing systems (FMS) are widely used by manufacturers. An FMS consists of some shared resources such as buffers, fixtures, robots, automated guided vehicles (AGV), and other material-handling devices. It usually exhibits a high degree of resource sharing in order to increase flexibility such that manufacturers can respond to market changes quickly [1]. The use of shared resources in FMS may lead to deadlock since different operations may happen at the same instance. Deadlocks cause some operations to stop from execution and may cause other operations to stop elsewhere in the system. When deadlocks occur, some particular operations will hold on indefinitely waiting for a shared resource that is busy elsewhere in the system. An FMS must be deadlock-free to ensure reliability and efficiency of the manufacturing process.

1.2 DEADLOCK HANDLING TECHNIQUES

A proper model of FMS is done so as to analyze its behavior and make all the necessary control activities to handle the deadlock states. There are three main approaches used for deadlock handling in FMS [2], [3]: deadlock recovery, deadlock avoidance and deadlock prevention. *Deadlock recovery* allows deadlocks to occur, and then detects and puts the system back to a normal state. *Deadlock avoidance* is done online where the system evolution is determined such that a restriction is enforced to the system to ensure the processing of each job is finished [4]. *Deadlock prevention* is done off-line by proper system design with desired properties that will prevent the system from entering deadlock states.

There are various tools used for deadlock detection, avoidance and prevention. These include; graph techniques, finite state machine based models and Petri nets [5]. Petri nets are widely used for the modeling of FMS due to their ability to easily detect the good behavior of a system like deadlock-freeness and boundedness [5].

There are four main Petri net based deadlock prevention techniques in the literature [1]: initial marking configuration, reachability graph analysis, structural analysis and combination technique [1].

The initial marking configuration technique was proposed in [6]. The aim is to prevent deadlocks in a system based on initial markings of source and shared resource places. Initially, the number of tokens in resource places and sink/source places is greater than zero. A relation between initial marking of shared resource and sink/source places is established at which Petri net model (PNM) of an FMS is live, bounded and reversible [1].

A deadlock prevention technique based on structural analysis was proposed in [7]. The technique characterizes deadlock situations in terms of unmarked structural objects called siphons. The aim is to prevent the PNM from entering deadlock by adding some control places (monitors) to the strict minimal siphons (SMS). It ensures that each SMS is not empty or unmarked at any reachable marking [1]. The system is live when there is no empty siphon.

An example to the reachability graph (RG) study of deadlocks using the theory of regions was given in [8]. The technique makes use of the behavior of the system from its RG. The RG of a PNM is categorized into states that are in a dead zone (DZ), including deadlock states and critical states that may lead to deadlocks and a live zone (LZ) representing good states [2]. The aim is to ensure that all states in the DZ are prevented and all states in the LZ are reachable. It is achieved by adding monitors to the uncontrolled model (off-line).

A combined technique, proposed in [9], uses siphons and the theory of regions. The aim is to develop a hybrid approach that combines siphons control and theory of regions to drive a maximally permissible liveness enforcing supervisor for large classes of PNMs. It has two stages: the first is siphons control that adds control places to every strict minimal siphon identified in the original net model so that the siphon is controlled. Second, the theory of regions is used to determine the net supervisor so as to prevent the deadlocks from occurring [9].

1.3 OBJECTIVE OF THE THESIS

There are various approaches for the synthesis of Petri net based liveness enforcing supervisors in FMSs, but some of these approaches could not provide the optimal behavior for some FMSs. However, it is necessary to propose optimal or near optimal approaches that will provide better liveness behavior for FMSs model by generalized PNs. The objective of this study is to propose a computationally efficient PNs based deadlock prevention method with optimal or near optimal permissive behavior for FMSs that are modeled by generalized classes of PNs, such as S^4PR.

The remainder of this thesis is organized as follows. Chapter 2 gives the basics of Petri net, which includes some definitions and computational constrains. It also reviews the computation of monitors and elimination of redundant monitors. A new synthesis approach for liveness enforcing supervisors in generalized PNMs of FMSs is proposed in Chapter 3, which gives an optimal or near optimal liveness behavior of a PNM. The applicability and efficiency of the proposed method to different generalized classes of PNs are shown in Chapter 4. Chapter 5 gives some conclusions.

CHAPTER 2

PETRI NETS BASICS AND COMPUTATION OF MONITORS

2.1 INTRODUCTION

In this chapter, basic PN definitions related to this thesis are considered. In addition, the computation of monitors based on place invariants and redundancy test used for finding redundant monitors for liveness-enforcing supervisors are also recalled.

2.2 PETRI NET DEFINITIONS

Petri nets are graphical and mathematical tool introduced by Carl Adam Petri in 1962 [10]. Since then, they have been used in different fields, such as production systems, computer networks, traffic systems, communication systems, social services, work flow management, etc. [10]. Petri net have been good tool for modeling for modeling due to their ability to provide simple, direct, faithful, and convenient graphical representation of Discrete-event system DESs [11]. They also have the ability to easily detect good behavior of a system like deadlock-freeness and boundedness [5].

A Petri net is a directed bipartite graph which has two nodes representing places (symbolized by circles) and transitions (symbolized by bars or square boxes). A place defines a condition and a transition defines an action that may occur. Transitions and places are connected by directed arcs. Some formal PN definitions are given below [11].

Definition 2.1 A Petri net is a four-tuple $N = (P, T, F, W)$, where P and T are finite and nonempty sets. P is the set of places, and T is the set of transitions with $P \cup T = \emptyset$ and $P \cap T = \emptyset$. $F \subseteq (P \times T) \cup (T \times P)$ is called flow relation of the net, which is represented by arcs with arrows from places to transitions or from transitions to places. $W: (P \times T) \cup (T \times P) \rightarrow \mathbb{N}$ is a mapping that assigns a weight to an arc: $W(f) > 0$ if $f \in F$ and $W(f) = 0$ otherwise, where $N = \{0, 1, 2, ...\}$.

Definition 2.2 A Petri net $N = (P, T, F, W)$ is said to be ordinary if $\forall f \in F$, $W(f) = 1$. N is said to be generalized if $\exists\forall f \in F$, $W(f) > 1$.

An example of a generalized Petri net with $W(t_1,p_2) = W(p_2,t_4) = 2$, and an ordinary Petri net with weighted arcs equal to one ($W(f) = 1$) are shown in Fig. 2.1 and Fig. 2.2 [12].

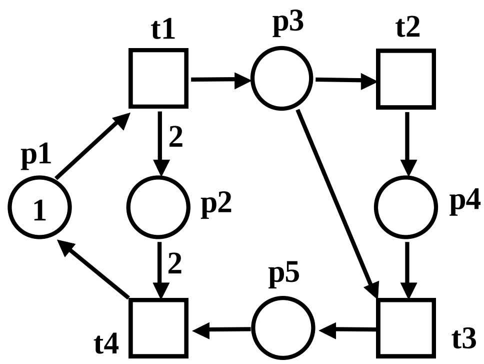

Figure 2.1. A Petri net example.

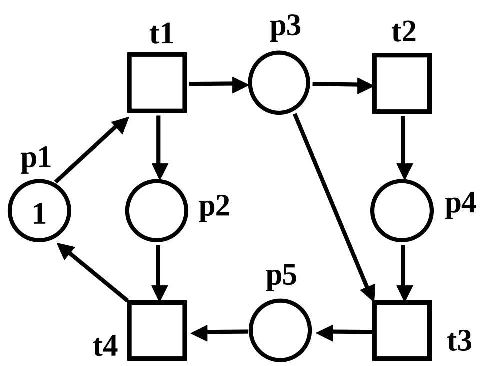

Figure 2.2. An ordinary Petri net.

Definition 2.3 A marking M of a Petri net N is a mapping from P to $\mathbb{N}$. $M(p)$ denotes the number of tokens in place p. A place p is marked by a marking M iff $M(p) > 0$. A subset $S \subseteq P$ is marked by M iff at least one place in S is marked by M. The sum of tokens of all places in S is denoted by $M(S)$, i.e., $M(S) = \sum_{p \in S} M(p)$. S is said to be empty at M iff $M(S) = 0$. (N,M_0) is called a net system or marked net and M_0 is called an initial marking of N.

Definition 2.4 Let $x \in P \cup T$ be a node of $N = (P, T, F, W)$. The preset of x defined as $^{\bullet}x = \{y \in P \cup T | (y, x) \in F$, and the postset of x defined as $x^{\bullet} = \{y \in P \cup T | (x, y) \in F\}$. This notation can be extended to a set of nodes as follows: given $X \subseteq P \cup T$, $^{\bullet}X = \cup_{x \in X} {}^{\bullet}x$, and $X^{\bullet} = \cup_{x \in X} x^{\bullet}$. Note that $^{\bullet\bullet}X$ is the preset of $^{\bullet}X$, and $X^{\bullet\bullet}$ is the postset of $X^{\bullet}$. Given place p, we denote max $\{W(p,t) \mid t \in p^{\bullet}\}$ by max $_p$.

Definition 2.5 A transition $t \in T$ is enabled at a marking M iff $\forall p \in {}^{\bullet}t$, $M(p) \geq W(p,t)$. This fact is denoted as $M[t>$. Firing t yields a new marking M' such that $\forall p \in P$, $M'(p) = M(p) - W(p,t) + W(t,p)$, which is denoted as $M[t>M'$. M' is called an immediately reachable marking from M. Marking M'' is said to be reachable from M if there exist a sequence of transitions $\sigma = t_0, t_1 \ . \ . \ . \ t_n$ and markings M_1, M_2, . . . , and M_n such that $M[t_0>M_1[t_1>M_2 \ . \ . \ . \ M_n[t_n>M''$ holds. The set of markings reachable from M in N is called the reachability set of Petri net (N,M) and denoted as $R(N, M)$.

Definition 2.6 A net $N = (P, T, F, W)$ is pure (self-loop free) iff $\forall x,y \in P \cup T$, $W(x,y) > 0$ implies $W(y,x) = 0$.

Definition 2.7 A pure net $N = (P, T, F, W)$ can be represented by its incidence matrix $[N]$, where $[N]$ is a $|P| \times |T|$ integer matrix with $[N](p,t) = W(t,p) - W(p,t)$. For a place p (transition t), its incidence vector, a row (column) in $[N]$, is denoted by $[N](p, \cdot)([N](\cdot,t))$.

Definition 2.8 A Petri net (N,M_0) is safe if $\forall M \in R(N,M_0)$, $\forall p \in P$, $M(p) \leq 1$ is true. It is bounded if $\exists k \in \mathbb{N}^+$, $\forall M \in R(N,M_0)$, $\forall p \in P$, $M(p) \leq k$. It is said to be unbounded if it is not bounded. A net N is structurally bounded if it is bounded for any initial marking.

Definition 2.9 Given a Petri net (N,M_0), $t \in T$ is live under M_0 iff $\forall M \in R(N,M_0)$, $\exists M' \in R(N,M)$, $M'[t>$. (N,M_0) is live iff $\forall t \in T$, t is live under M_0. (N,M_0) is dead under M_0 iff $\nexists t \in T$, $M_0[t>$. (N,M_0) is deadlock-free (weakly live or live-locked) iff $\forall M \in R(N,M_0)$, $\exists t \in T$, $M[t>$.

Definition 2.10 A P-vector is a column vector $I: P \rightarrow \mathbb{Z}$ indexed by P and a T- vector is a column vector $J: T \rightarrow \mathbb{Z}$ indexed by T, where $\mathbb{Z}$ is the set of integers.

We denote column vectors where every entry equals 0(1) by 0(1). I^T and $[N]^T$ are the transposed versions of vector I and matrix $[N]$, respectively. A $P(T)$-vector is non-negative if no element in it is negative.

Definition 2.11 P-vector I is called a P-invariant (place invariant) iff $I \neq 0$ and $I^T [N]^T = \mathbf{0}^T$. T -vector J is called a T-invariant (transition invariant) iff $J \neq 0$ and $[N]J = 0$.

Definition 2.12 P-invariant I is a P-semiflow if every element of I is non-negative. $||I|| = \{p|I(p) \neq 0\}$ is called the support of I. $||I||^+ = \{p|I(p) > 0\}$ denotes the positive support of P-invariant I and $||I||^- = \{p|I(p) < 0\}$ denotes the negative support of I. I is called a minimal P-invariant if $||I||$ is not a superset of the support of any other one and its components are mutually prime.

Definition 2.13 T-invariant J is a T-semiflow if every element of J is non-negative. $||J|| = \{t|J(t) = 0\}$ is called the support of J. $||J||^+ = \{t|J(t) > 0\}$ denotes the positive support of T-invariant J and $||J||^- = \{t|J(t) < 0\}$ denotes the negative support of J. J is called a minimal T-invariant if $||J||$ is not a superset of the support of any other one and its components are mutually prime. A P-invariant corresponds to a set of places whose weighted token count is a constant for any reachable marking. It follows immediately from the state equation.

2.3 CONTROL PLACE COMPUTATION USING PLACE INVARIANTS

In this thesis, control places (monitors) are computed based on a place invariant (PI) method proposed in [13]. The method uses two equations for computation; Eq. (2.1) for computing the initial markings and Eq. (2.3) for computing the control place arcs connecting control place C_i to the transitions in the uncontrolled Petri net model (PNM).

$$\mu_{CO} + L\mu_{PO} = b \tag{2.1}$$

where: μ_{CO} is the initial marking of the control place,

μ_{PO} is the initial marking of the PNM

L is an integer matrix and b is an integer vector representing some place invariant constraints.

Eq. (2.1) can be written as

$$\mu_{co} = b - L\mu_{PO} \tag{2.2}$$

$$D_C = -LD_P \tag{2.3}$$

where D_c is the control place row matrix representing the connection of control place to the transitions.

D_P is the incidence matrix of the PNM,

L is row matrix representing the place invariants.

A simple method in [2] is provided which reduces the size of the PNM incidence matrix (D_P). Since many places may not be used in the incidence matrix (D_P) for a particular controller computation, the place invariant related incidence matrix (D_{PI}) of the PNM is used. Eqs. (2.2) and (2.3) are now modified based on a place invariant related net.

$$\mu_{co} = b - L_{PI}\mu_{PIO} \tag{2.4}$$

where L_{PI} is place invariant related integer vector,

μ_{PIo} is initial marking of a place invariant related net,

$$D_c = -L_{PI}\, D_{PI} \tag{2.5}$$

where L_{PI} is a j×1 integer row vector representing the invariant related place,

D_{PI} is the incidence matrix (j×k) of a place invariant related net with j places and k transitions.

D_C is a k×1 integer vector representing the incidence matrix of the monitor.

It is known that at initial marking of PNM, the activity places have no tokens, which means that $\mu_{PIO} = 0$. Therefore Eq. (2.4) becomes;

$$\mu_{co} = b \tag{2.6}$$

2.4 REDUNDANCY TEST FOR LIVENESS ENFORCING SUPERVISORS OF FMS

A number of control places CPs are computed in Petri-net-based approaches for deadlock prevention in FMS. It is the fact that some computed control places are redundant in a live Petri net (LPN) model. This increases the structural complexity of LPN, and may reduce the behavioral permissiveness of the LPN. A method was proposed in [5] to identify and eliminate redundant control places in a Petri net based liveness enforcing supervisor. In this section the redundancy test is recalled from [5].

There may exist redundant *CP*s in a live Petri net (*LPN*) model, denoted by a net system (N_0,M_0), controlled by *n CP*s: $CP = \{C_1, C_2, \ldots, C_n\}$. *CP* is called redundant if removing it still keeps the net live. It should be noted that this definition is different from that of a redundant place in literature. Removing the latter does not change the net's reachability graph. Also, redundant *CP*s are not necessarily unique given a set of *CP*s used to make a deadlock-prone net live.

Redundancy Test Algorithm: **Redundancy test for *LES* of FMS.**

Input: A live Petri net (*LPN*) model, denoted by a net system (N_0,M_0), of an FMS, controlled by *n CP*s; $CP = \{C_1, C_2, \ldots, C_n\}$;

1) [Define] β_0: the number of reachable markings or states of reachability graph (R_0) of (N_0,M_0)

   [Defined for Algorithm A] β_A: the number of reachable markings or states of R_A *of* (N_A,M_A); $n = j + k$, where *n*: the number of *CP*s of *LPN; j*: the number of redundant *CP*s; *k*: the number of necessary *CP*s*;*

   [Defined for Algorithm B] β_B: the number of reachable markings or states of R_B of (N_B,M_B); $n = l + m$, where *n*: the number of *CP*s of *LPN; l*: the number of redundant *CP*s; *m*: the number of necessary *CP*s*;*

2) Apply Algorithm A to (N_0,M_0) and the resultant net system is denoted as (N_A,M_A).
3) Apply Algorithm B to (N_0,M_0) and the resultant net system is denoted as (N_B,M_B).

Output:*If* (j>0) [for Algorithm A] *then* Output A = an *LPN*, denoted by a net system (N_A,M_A), controlled by *k* necessary *CP*s; there are *j* redundant *CP*s;

if $\beta_A = \beta_0$ *then* the controlled behaviour of (N_A,M_A) is

the same as (N_0,M_0)

if $\beta_A > \beta_0$ *then* the controlled behaviour of (N_A,M_A) is

more permissive than (N_0,M_0)

else there is no redundant *CP*s obtained due to Algorithm A and therefore

for Algorithm A: Output = Input;

If (*l*>0) [for Algorithm B]

then Output B = an *LPN*, denoted by a net system (N_B,M_B), controlled by *m* necessary *CP*s; there are *l* redundant *CP*s;

if $\beta_B = \beta_0$ *then* the controlled behaviour of (N_B,M_B) is

the same as (N_0,M_0)

if $\beta_B > \beta_0$ *then* the controlled behaviour of (N_B,M_B) is

more permissive than (N_0,M_0)

else there is no redundant *CP*s obtained due to Algorithm B and therefore

for Algorithm B: Output = Input;

---

end Redundancy Test Algorithm

Algorithm A: Front-to-Back (*FTB*) redundancy test for *LES* of FMS.

Input: A live Petri net (*LPN*) model, denoted by a net system (N_0,M_0), of an FMS, controlled by *n CP*s; $CP = \{C_1, C_2, \ldots, C_n\}$;

1) [Initialize] $N_A := N_0$; $M_A := M_0$; $i = 1$; $j = 0$; $k = 0$;
2) Remove C_i from (N_A,M_A). Denote the resultant net system by (N_i,M_i).
3) Check the liveness property of (N_i,M_i), compute the reachability graph (R_i) of (N_i,M_i) and define β_{Ai}, i.e., the number of reachable markings of R_i;

   If (N_i,M_i) is NOT LIVE

   then put C_i back into (N_i,M_i); $k = k + 1$; which means that C_i is necessary

   to keep the *PN* model live,

   else [i.e., *If* (N_i,M_i) is LIVE], $j = j + 1$; which means that C_i is redundant,

   if $\beta_{Ai} = \beta_0$ *then* the controlled behaviour of (N_i,M_i) is

   the same as (N_0,M_0)

   if $\beta_{Ai} > \beta_0$ *then* the controlled behaviour of (N_i,M_i) is

   more permissive than (N_0,M_0)

   endif

4) $N_A := N_i$; $M_A := M_i$
5) $i = i + 1$.
6) *If* $i \le n$ *then* go to step 2.

Output:*If* (j>0)

then Output = an *LPN*, denoted by a net system (N_A,M_A), controlled by *k* necessary

*CP*s; there are *j* redundant *CP*s;

if $\beta_A = \beta_0$ *then* the controlled behaviour of (N_A,M_A) is

the same as (N_0,M_0)

if $\beta_A > \beta_0$ *then* the controlled behaviour of (N_A,M_A) is

more permissive than (N_0,M_0)

else there is no redundant *CP*s and therefore Output = Input;

---

end Algorithm A

Algorithm B: Back-to-Front (*BTF*) redundancy test for *LES* of FMS.

Input: A live Petri net model (*LPN*), denoted by a net system (N_0,M_0), of an FMS, controlled by n *CP*s; $CP = \{C_1, C_2, \ldots, C_n\}$;

1) [Initialize] $N_B := N_0$; $M_B := M_0$; $i = n$; $l = 0$; $m = 0$;
2) Remove C_i from (N_B,M_B). Denote the resultant net system by (N_i,M_i).
3) Check the liveness property of (N_i,M_i), compute the reachability graph (R_i) of (N_i,M_i) and define β_{Bi}, i.e., the number of reachable markings of R_i;

   If (N_i,M_i) is NOT LIVE

   then put C_i back into (N_i,M_i); $m = m + 1$; which means that C_i is necessary

   to keep the *PN* model live,

   else [i.e., *If* (N_i,M_i) is LIVE], $l = l + 1$; which means that C_i is redundant,

   if $\beta_{Bi} = \beta_0$ *then* the controlled behaviour of (N_i,M_i) is

   the same as (N_0,M_0)

   if $\beta_{Bi} > \beta_0$ *then* the controlled behaviour of (N_i,M_i) is

   more permissive than (N_0,M_0)

   endif

4) $N_B := N_i$; $M_B := M_i$
5) $i = i - 1$.
6) *If* $i \neq 0$ *then* go to step 2.

Output:*If* (l>0)

then Output = an *LPN*, denoted by a net system (N_B,M_B), controlled by m

necessary *CP*s; there are l redundant *CP*s;

if $\beta_B = \beta_0$ *then* the controlled behaviour of (N_B,M_B) is

the same as (N_0,M_0)

if $\beta_B > \beta_0$ *then* the controlled behaviour of (N_B,M_B) is

more permissive than (N_0,M_0)

else there is no redundant *CP*s and therefore Output = Input;

---

end Algorithm B

The Redundancy Test Algorithm makes use of both Algorithms A and B. The former tests each *CP* starting from number 1 to the end, i.e., to n, while the latter tests each *CP* starting from number n to 1. Both tests may produce the same result or it may be possible to obtain different outcomes. It depends on the controlled live net system (N_0,M_0) considered. Of course if there is no redundant *CP* in an *LPN*, then the Algorithm Redundancy Test finds no redundant *CP*. In the existence of one or more redundant *CP* in an *LPN*, we may obtain the following results:

1. We may obtain the same set of redundant *CP*s and necessary *CP*s. In this case, the live behaviour of the Petri net model, controlled by the set of necessary *CP*s, may be the same as or more permissive than the original controlled net system, obtained with a smaller number of *CP*s.
2. We may obtain two different sets of redundant *CP*s and necessary *CP*s. The live behaviour of the Petri net model obtained with each set of necessary *CP*s, may be the same as or more permissive than the original controlled net system, obtained with a smaller number of *CP*s.

The Redundancy Test Algorithm is easy to use, very effective and straight forward. Its complexity is, however, exponential with respect to the net size since it requires generating the reachability graph. At the worst cases, Algorithm A and Algorithm B, i.e. BTF and FTB redundancy tests respectively, also exhibit the same exponential complexity. When dealing with a particular case, their performance may vary significantly. The Redundancy Test Algorithm is applicable to any *LPN* consisting of a *PNM*, prone to deadlock, of an FMS, controlled by means of a set of *CP*s. It has been applied to a number of *LPN* currently available within the Petri net based deadlock prevention/liveness enforcing literature with success. The liveness property can be checked and the reachability analysis can be carried out by currently available Petri net analysis tools. In this work, *INA* [14] is used.

CHAPTER 3

SYNTHESIS OF PETRI NET BASED LIVENESS ENFORCING SUPERVISORS IN FLEXIBLE MANUFACTURING SYSTEMS

3.1 INTRODUCTION

In this chapter, a new method is proposed for computing a liveness-enforcing supervisor for a given Petri net model (PNM) of an FMS prone to deadlocks. There may be three groups of places in a PNM of an FMS: *resource places*, *activity places* and *sink/source places*. Resource places represent either shared or non-shared resources and initially there are tokens in these places representing the number of available instances. Activity places represent an action to process a part in a production sequence by a resource (machine, robot, etc.) and initially there are no tokens in these places. Initially, tokens put into sink/source places represent the maximal number of concurrent activities which can take place in a production sequence. In some models it may be possible not to use them. In cyclic models a sink place is also a source place and vice versa.

The proposed method is especially effective for generalized Petri net classes. All computed control places have weighted arcs due to the proposed method. One of the most important features of the proposed method is to transform the given PNM into its conservative form called TPNM. This transformation is obtained by using Algorithm 1. It can be verified that a PNM and its transformed form TPNM obtained by using Algorithm 1 have isomorphic reachability graphs. This means that when we obtain a liveness-enforcing supervisor by using TPNM, the same supervisor is also valid for the original PNM. Then Algorithm 2 is used to compute the control places (monitors) based on the TPNM.

3.2 TRANSFORMATION OF A PETRI NET MODEL OF AN FMS PRONE TO DEADLOCK INTO ITS CONSERVATIVE FORM

The transformation of a PNM of an FMS prone to deadlocks into its conservative form, called TPNM, is necessary within the liveness enforcing method proposed in this study. Therefore this transformation is explained in this section. The basic idea behind this transformation is to obtain a conservative version of the original PNM, to be used in the control place computation. The experimental studies carried out show that for certain problems including generalized Petri net classes, the monitors computed by using TPNM provide more permissive behavior compared with the ones computed by using PNM. Therefore the transformation explained in this section is necessary to obtain a supervisor with more permissive behavior.

It is important to note that the transformation carried out here does not change some important properties of the original PNM. The following shows that both PNM and its conservative form TPNM have isomorphic (the same) reachability graphs. A Petri net (N,M_0) is said to be conservative if the total number of tokens in all its places for all reachable markings is constant.

3.2.1 Isomorphic Petri Nets

Let RG_1 $(S_1,A1)$ and RG_2 (S_2,A_2) represent two reachability graphs of two Petri net models (PNM_1 and PNM_2). Assume that both PNM_1 and PNM_2 suffer from deadlock problems. Let S_1 (respectively S_2) represent the number of states of RG_1 (respectively RG_2) and likewise let A_1 (respectively A_2) represent the number of arcs of RG_1 (respectively RG_2).

RG_1 and RG_2 are said to be isomorphic if there exist a pair of functions $f: S1 \rightarrow S2$ and $g: A1 \rightarrow A2$ such that f associates each element in S_1 with exactly one element in S_2 and vice versa; g associates each element in A_1 with exactly one element in A_2 and vice versa.

If two reachability graphs RG_1 and RG_2 of two Petri nets models PNM_1 and PNM_2 suffering from deadlock are isomorphic, then they must have:

(a) The same number of states.
(b) The same number of arcs.
(c) The same number of dead states.

(d) The same number of bad states in their dead zones (DZs).

(e) The same number of good states in their live zones (LZs).

(f) The same number of connected components.

Now let us consider Algorithm 1

Algorithm 1: Transformation of a PNM of an FMS prone to deadlocks into its conservative form, called TPNM.

Input: A Petri net model (PNM), (N, M_0) of an FMS prone to deadlocks.

Output: Conservative form of the PNM called TPNM, (N',M_0).

Step 1: Identify sink/source (idle) places ($P_{S/S}$), resource places (P_R), and activity places (P_A) of the PNM.

Step 2: Based on the number of product types processed in the FMS, find sub-nets SN_I (I is the number of product types) consisting of P activity places with their input and output arcs and T transitions.

Step 3: Identify the input and output transitions of each sub-net SN_I.

Step 4: for (i = 1; $i \leq I$; i ++)

{

for (n = 1; $n \leq T$; n ++)

{

/* in the first iteration the input transition of SN_I is used ***/**

Identify the weight of input arc for activity place p_n:

$$W(t_n, p_n) = \sum_{j=1}^{J} W(p_j, t_n) - \sum_{k=1}^{K} W(t_n, p_k)$$

/* J is the number of input places of t_n ***/**

/* K is the number of output places of t_n except for p_n ***/**

$$W(p_n, t_{n+1}) = W(t_n, p_n)$$

/* the weight of the output arc from activity place p_n is equal to the weight of the input arc of the same activity place p_n ***/**

}

}

Step 5: Based on the computed input and output arc weights of the activity places, establish TPNM, i.e., a conservative form of the PNM.

---

end of Algorithm 1.

3.3 A NEW SYNTHESIS APPROACH FOR THE LIVENESS ENFORCING IN GENERALIZES PETRI NET MODELS OF FMS

In this section, a new method is proposed for computing a liveness-enforcing supervisor for a given Petri net model (PNM) of an FMS prone to deadlocks. In the monitor computation steps of Algorithm 2, a global sink/source place (GP) is used to make the necessary computation easily in an iterative way. At each iteration, the reachability graph (RG) of the related net is computed. If the net is not live, the RG is divided into dead zone (DZ) and live zone (LZ) as in [8]. The former may contain deadlock states (markings), partial deadlock states, and states which inevitably lead to deadlocks or livelocks. The latter constitutes remaining good states of the RG representing the optimal system behaviour. The control policy is based on the exclusion of the DZ from the RG, while making sure that every state within the LZ can still be reached. All states in the DZ are considered as bad markings (BM) and they are prevented from being reached by means of the simplified invariant-based control method explain in Chapter 2.

From a BM we consider only the markings of activity places. Then, our objective is to prevent the marking of the subset of the activity places of the BM from being reached. Therefore, the marking of the subset of the activity places is characterized as a *PI* of the PNM. In the *PI* relating to a *BM*, the sum of tokens within the subset of the activity places has to be at most one token less than their current value within the BM in order not to reach the BM. A *PI* can be implemented by a control place with its related arcs and initial marking.

The redundancy test recalled in Chapter 2 is used to find out if there are any redundant monitors within computed control places in the computation procedures.

Finally, a live controlled Petri net is obtained by including all necessary control places in the PNM. Although not explained in Algorithm 2, in order to simplify very big PNMs so as to make necessary computation easily as in [2], the Petri net reduction approach can be used. The reachability graph analysis of PNMs can be carried out by currently available Petri net analysis tools. In this work, *INA* [14] is used, in which we are provided with both *LZ*, as the first strongly connected component, and *DZ*, as the strongly connected components other than the first one, of a given PNM. The *DZ* is then considered as the collection of all bad markings (BM_i, i= 1, 2, . . .).

Algorithm 2: Synthesis of liveness-forcing supervisor with weighted arcs

---

Input: A Petri net model (PNM) of an FMS prone to deadlocks

Output: Liveness enforcing supervisor with weighted arcs for the PNM

Step 1: Transform the given PNM into its conservative form TPNM by using Algorithm 1.

Step 2: Define input and output transitions of all sink/source places $P_{S/S}$. Add a global sink/source place (GP) to the TPNM. The input transitions of the GP are input transitions of all sink/source places $P_{S/S}$. Likewise output transitions of the GP are output transitions of all sink/source places $P_{S/S}$. Thus a new net system N_B = TPNM + GP is obtained, where $B \in \mathbb{N}$

Step 3: for (B = 1; B ≤ k; B ++)

/* B is the number of tokens in GP and *k* is the sum of initial tokens in all sink/source places $P_{S/S}$ */

{

3.B.1. Compute the reachability graph RG_B of N_B. If N_B is live, then consider net N_B next net with B + 1, i.e., go to step 3.B.1. Otherwise, define the LZ_B and DZ_B of RG_B.

3.B.2. Define a *PI* for each bad marking (BM) within DZ_B, from the subset of marked activity places of BM.

3.B.3. Compute a monitor *C* for each *PI* using the simplified invariant-based control method.

3.B.4. If the number of monitors computed for N_B is greater than 1, then carry out the redundancy test using the method proposed in [5] to find out the set of necessary monitors $C_{B,i}$; i = 1, 2, 3, . .

3.B.5. Augment all necessary monitors computed in the previous step into N_B (N_B: = N_B + $C_{B,i}$, i = 1, 2, 3, . . .).

}

Step 4: Obtain a live controlled PNM by augmenting all necessary monitors computed in step 3 into the PNM.

Step 5: Exit

---

end of Algorithm 2.

3.4 ILLUSTRATIVE EXAMPLE

In order to show the applicability of the proposed synthesis approach, an example is considered in this section. Fig. 3.1 shows a simple uncontrolled System of Simple Sequential Processes with Resource (S^3PR) PNM of an FMS from [15]. This model suffers from deadlocks. It can be verified that there are 95 states in the RG of this PNM, 11 of which are bad states representing the DZ, while 84 of which are good states representing the LZ. This means that an optimal liveness-enforcing supervisor should provide 84 good states for this PNM. Let us now apply the proposed method to the PNM.

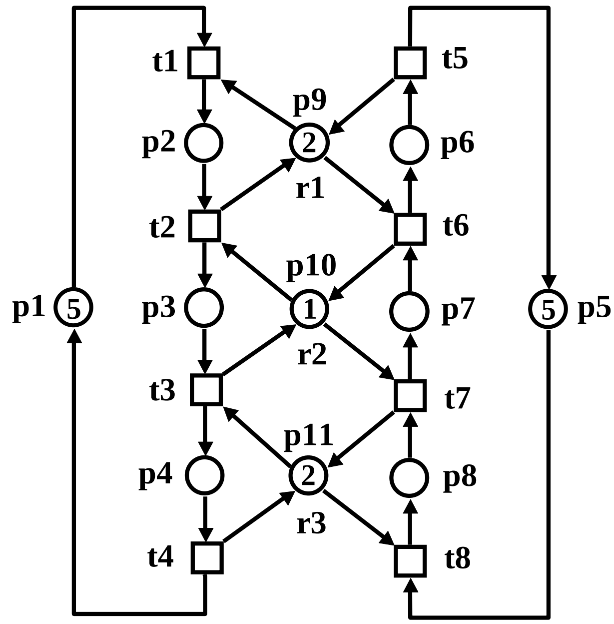

Figure 3.3. S^{3}PR model of FMS from [15].

Step 1: The PNM shown in Fig. 3.1 is considered by Algorithm 1 and then the transformed PNM (TPNM) shown in Fig. 2 is obtained. It is verified that the RG of the TPNM has 95 states, whose DZ includes 11 bad states, and LZ contains 84 good states.

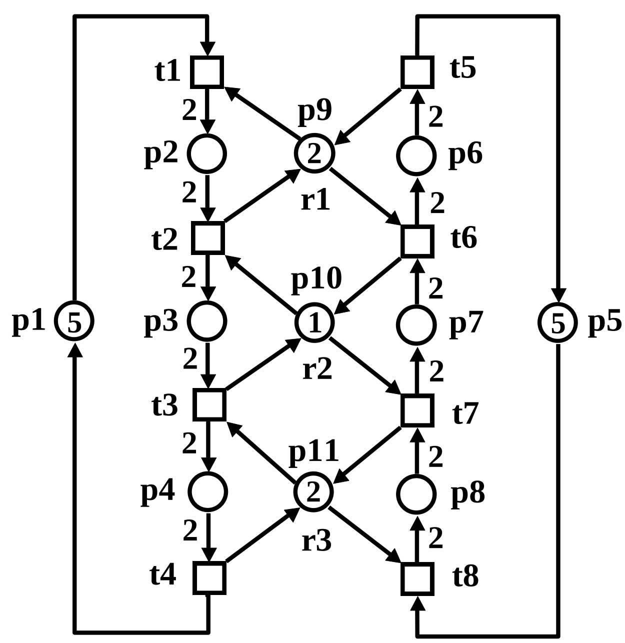

Figure 3.4. Transformed PNM (TPNM).

Step 2: Input transitions of sink/source places p1 and p5 are ${}^{\bullet}p1 = \{t4\}$ and ${}^{\bullet}p5 = \{t5\}$. Likewise output transitions of p1 and p5 are $p1^{\bullet} = \{t1\}$ and $p5^{\bullet} = \{t8\}$. Therefore the input and output transitions of the GP are ${}^{\bullet}GP = \{t4, t5\}$ and $GP^{\bullet} = \{t1, t8\}$. When the GP is added within the TPNM, a new net structure N_B = TPNM + GP is obtained as shown in Fig. 3.3.

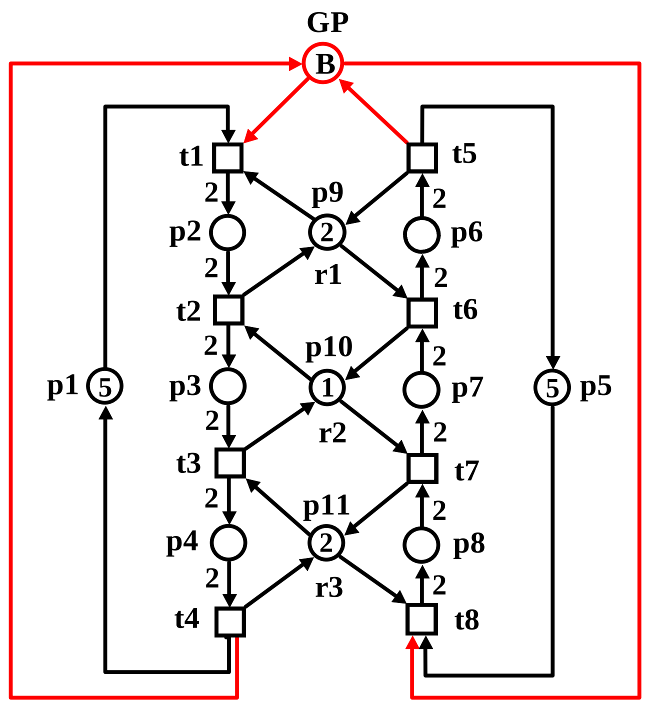

Figure 5.3. The net N_B with N_B = TPNM + GP (S^3PR net).

Step 3:

(B = 1)

Step 3.1.1. When one token is deposited in the GP within the net N_1 shown in Fig. 3.3, the net N_1 is live with 7 good states. Then B: = B + 1 = 2.

(B = 2)

Step 3.2.1. When two tokens are deposited in the GP within the net N_2 shown in Fig. 3.3, the net N_2 is live with 25 good states. Then B: = B + 1 = 3.

(B = 3)

Step 3.3.1. When three tokens are deposited in the GP within the net N_3 shown in Fig. 3.3, the net N_3 is not live. There are 55 states within the RG_3 of N_3. DZ_3 includes 2 bad states BM_1 and BM_2 and LZ_3 contains 53 good states.

Step 3.3.2. The markings of the activity places of BM_1 and BM_2 are shown in Table 3.1.

Table 3.1 The markings of activity places of BM_1 and BM_2.

State number	p2	p3	p4	p6	p7	p8
s_{22}	4	0	0	0	2	0
s_{46}	0	2	0	0	0	4

In order not to reach BM_1 and BM_2 the following place invariants are established respectively:

$PI_1 = \mu_2 + \mu_7 \leq 5$

$PI_2 = \mu_3 + \mu_8 \leq 5$

Step 3.3.3. Monitors C_1 and C_2 are computed in order to enforce PI_1 and PI_2 respectively as follows.

$$D_{PI1} = \begin{array}{c} \begin{array}{cccc} t1 & t2 & t6 & t7 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 & 0 \\ 0 & 0 & -2 & 2 \end{bmatrix} \end{array} \begin{array}{c} p2 \\ p7 \end{array}$$

$$L_{PI1} = \begin{array}{c} \begin{array}{cc} p2 & p7 \end{array} \\ \begin{bmatrix} 1 & 1 \end{bmatrix} \end{array}$$

$$D_{c1} = -L_{PI1}.D_{PI1} = -\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 \\ 0 & 0 & -2 & 2 \end{bmatrix}$$

$$D_{C1} = -\begin{bmatrix} 2 & -2 & -2 & 2 \end{bmatrix}$$

$$D_{C1} = \begin{array}{c} \begin{array}{cccc} t1 & t2 & t6 & t7 \end{array} \\ \begin{bmatrix} -2 & 2 & 2 & -2 \end{bmatrix} \end{array}$$

$$\mu_{0(c1)} = 5$$

$$D_{PI2} = \begin{array}{c} \\ \\ \end{array}\begin{array}{cccc} t2 & t3 & t7 & t8 \\ \end{array}$$

$$D_{PI2} = \begin{bmatrix} 2 & -2 & 0 & 0 \\ 0 & 0 & -2 & 2 \end{bmatrix} \begin{array}{c} p3 \\ p8 \end{array}$$

$$L_{PI2} = \begin{array}{cc} p3 & p8 \\ \end{array}$$

$$L_{PI2} = [1 \quad 1]$$

$$D_{C2} = -L_{PI2}.D_{PI2} = -[1 \quad 1] \begin{bmatrix} 2 & -2 & 0 & 0 \\ 0 & 0 & -2 & 2 \end{bmatrix}$$

$$D_{C2} = -[\,2 \quad -2 \quad -2 \quad 2\,]$$

$$D_{C2} = \begin{array}{cccc} t2 & t3 & t7 & t8 \\ \end{array}$$

$$D_{C2} = [-2 \quad 2 \quad 2 \quad -2]$$

$$\mu_{0(c2)} = 5$$

The computed monitors are shown in Table 3.2.

Table 3.2. Computed monitors C_1 and C_2.

C_i	$^\bullet c_I$	$c_I{}^\bullet$	$\mu_{0(ci)}$
C_1	2t2, 2t6	2t1, 2t7	5
C_2	2t3, 2t7	2t2, 2t8	5

Step 3.3.4. The redundancy test shows that both computed monitors C_1 and C_2 are necessary.

Step 3.3.5. When the computed necessary monitors C_1 and C_2 are augmented in the uncontrolled model N_3, the controlled model of N_3 is obtained as follows: $N_3 := N_3 + C_1 + C_2$ and is shown in Fig. 3.4.

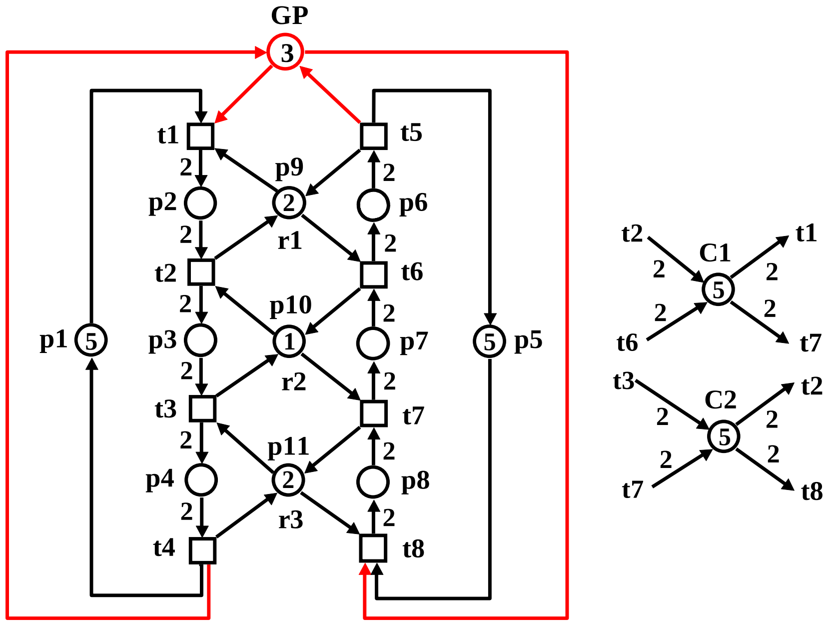

Figure 3.6. The controlled model N_3 ($N_{3:} = N_3 + C_1 + C_2$).

It is verified that the controlled model N_3 shown in Fig. 3.4 is live with 53 good states. This is the optimal live behavior for the controlled model N_3.

Step 3.4.1. The net N_4 considered in this step is shown in Fig. 3.5. It is obtained by increasing the number of tokens in GP as shown in Fig. 3.4.

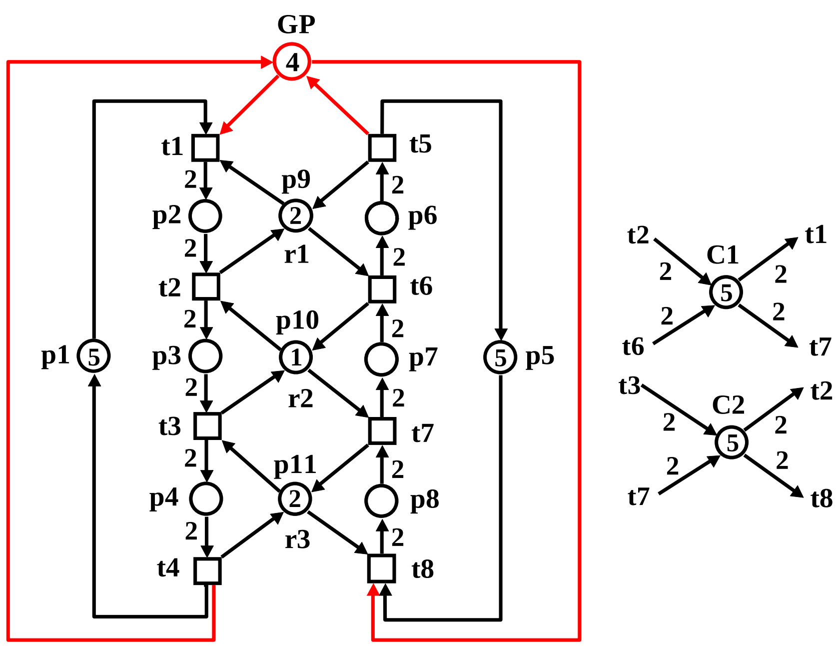

Figure 3.7. The net N_4 (S3PR net).

The net N_4 is not live. There are 77 states in the RG_4 of N_4. The DZ_4 includes 1 bad marking (BM_3) and LZ_4 contains 76 good states.

Step 3.4.2. The markings of the activity places of BM_3 are shown in Table 3.3.

Table 3.3. The markings of the activity places of BM_3.

State number	p2	p3	p4	p6	p7	p8
s_{22}	4	0	0	0	0	4

In order not to reach BM_3, the following place invariant is established: $PI_3 = \mu_2 + \mu_8 \leq 7$.

Step 3.4.3. Monitor C_3 is computed in order to enforce PI_3 as follows.

$$D_{PI3} = \begin{array}{c} \begin{array}{cccc} t1 & t2 & t7 & t8 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 & 0 \\ 0 & 0 & -2 & 2 \end{bmatrix} \end{array} \begin{array}{c} \\ p2 \\ p8 \end{array}$$

$$L_{PI3} = \begin{array}{c} \begin{array}{cc} p2 & p8 \end{array} \\ \begin{bmatrix} 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C3} = -L_{PI3}.D_{PI3} = -\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 \\ 0 & 0 & -2 & 2 \end{bmatrix}$$

$$D_{C3} = -\begin{bmatrix} 2 & -2 & -2 & 2 \end{bmatrix}$$

$$D_{C3} = \begin{array}{c} \begin{array}{cccc} t1 & t2 & t7 & t8 \end{array} \\ \begin{bmatrix} -2 & 2 & 2 & -2 \end{bmatrix} \end{array}$$

$$\mu_{0(c3)} = 7$$

The computed monitor C_3 is shown in Table 3.4.

Table 3.4. Computed monitor C_3.

C_i	${}^{\bullet}c_I$	$c_I{}^{\bullet}$	$\mu_{0(ci)}$
C_3	2t2, 2t7	2t1, 2t8	7

Step 3.4.4. No need to do redundancy test as there is only one monitor computed.

Step 3.4.5 When the computed monitor C_3 is augmented in the uncontrolled model N_4 shown in Fig. 3.5. The controlled model of N_4 is obtained as follows: $N_4 = N_4 + C_3$ and is shown in Fig. 3.6.

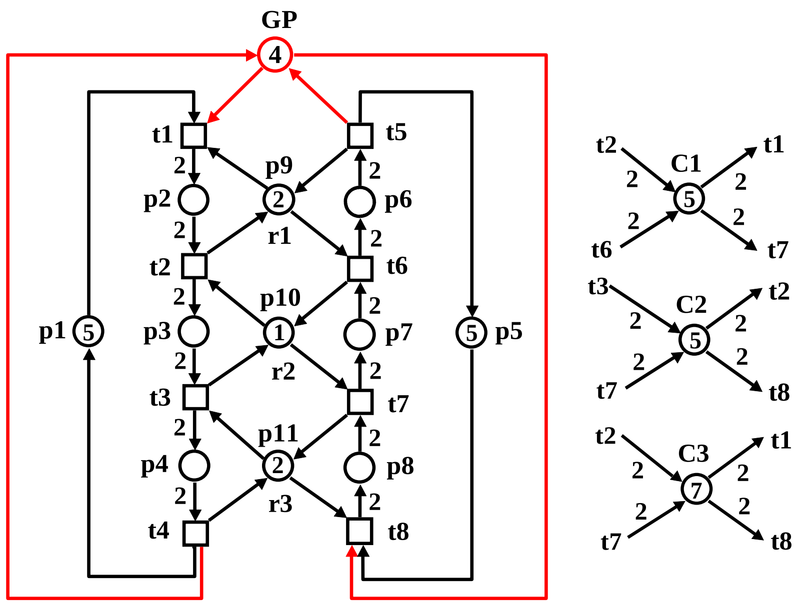

Figure 3.8. The controlled model N_4 (N_4: = $N_4 + C_3$).

It is verified that the controlled model N_4 shown in Fig. 3.6 is live with 76 good states. This is the live optimal behavior for the controlled model N_4

Step 3.5.1. The net N_5 considered in this step is shown in Fig. 3.7. It is obtained by increasing the number of tokens in GP shown in Fig. 3.6.

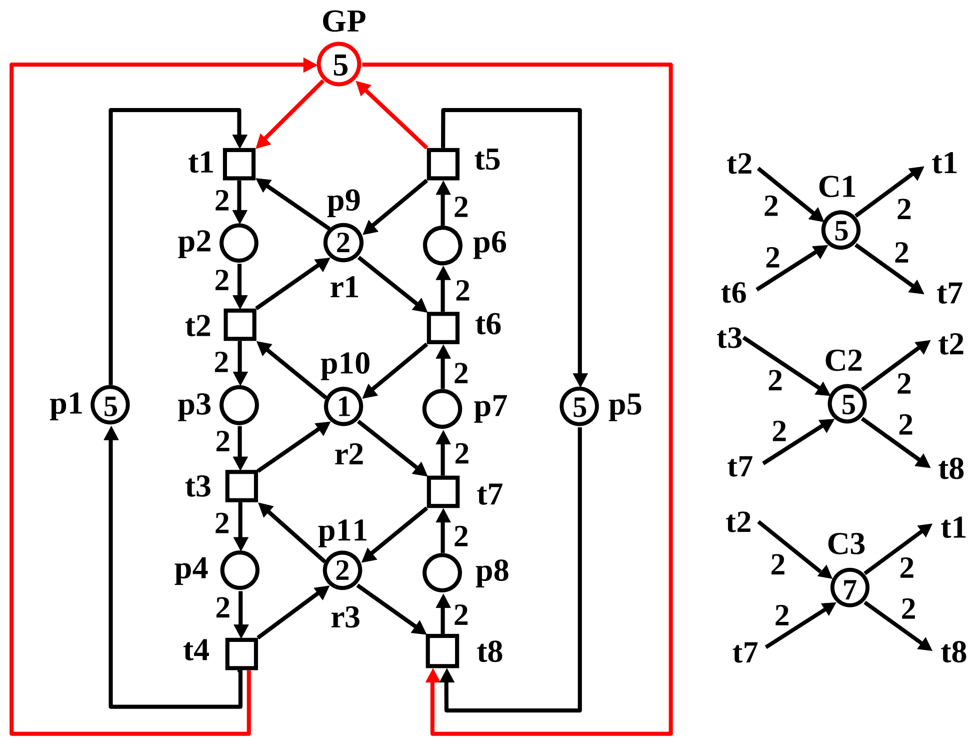

Figure 3.9. The net N_5 (S^3PR net).

The net N_5 is live with 84 good states. Likewise the net N_6, N_7, N_8, N_9, and N_{10} are all live with 84 good states.

Step 4. The live controlled S^3PR PNM shown in Fig. 3.8 is obtained by augmenting 3 necessary monitors provided in Table 3.5 into the uncontrolled model PNM shown in Fig. 3.1. It is live with 84 good states. Permissiveness of the controlled PNM is (84/84) × 100 = 100%. This is the optimal live behavior for PNM. The liveness enforcing procedure applied for the PNM is provided in Table 3.6.

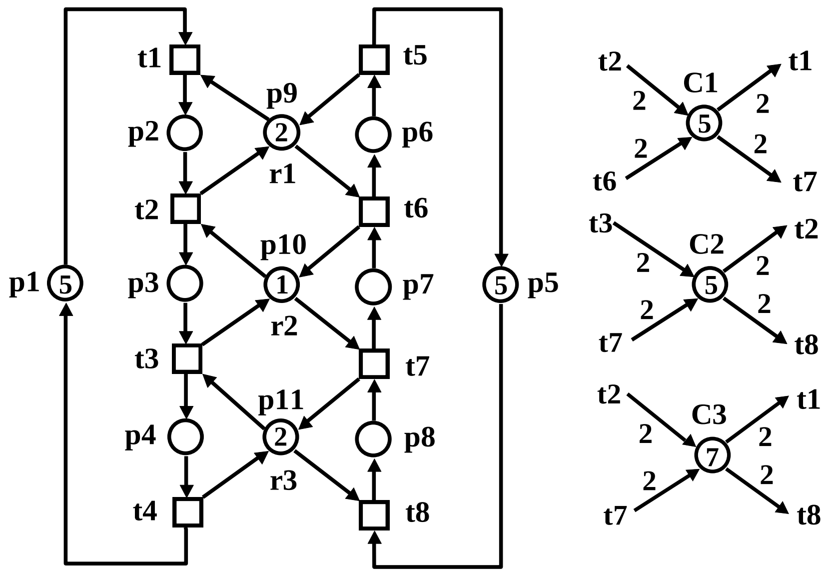

Figure 3.10. The controlled S^3PR net.

Table 3.5. Necessary monitors for the PNM shown in Fig. 3.1.

C_i	$^{\bullet}c_I$	$c_I^{\bullet}$	$\mu_{0(ci)}$
C1	2t2, 2t6	2t1, 2t7	5
C2	2t3, 2t7	2t2, 2t8	5
C3	2t2, 2t7	2t1, 2t8	7

Table 3.6. The liveness enforcing procedure applied for the PNM shown in Fig. 3.1.

B	Included C	Is the net live?	# of states in RG	# of states in DZ	# of states in LZ	Computed C	# of states within controlled net	
							RG = LZ	UR
1	–	YES	7	0	7	–		
2	–	YES	25	0	25	–		
3	–	NO	55	2	53	C_1,C_2	53	0
4	C_1, C_2	NO	77	1	76	C_3	76	0
5	C_1, C_2, C_3	YES	84	0	84	–		
6	C_1, C_2, C_3	YES	84	0	84	–		
7	C_1, C_2, C_3	YES	84	0	84	–		
8	C_1, C_2, C_3	YES	84	0	84	–		
9	C_1, C_2, C_3	YES	84	0	84	–		
10	C_1, C_2, C_3	YES	84	0	84	–		

CHAPTER 4

APPLICATION EXAMPLES

4.1 INTRODUCTION

In this chapter, some example of generalized classes of PNMs such as Weighted Automated Marked Graph (WAMG), Systems of Sequential Systems with Shared Resources (S^4PR) and G-System from the literature are used to show the applicability and effectiveness of the proposed liveness-enforcing approach.

4.2 WAMG MODEL

Fig. 4.1 shows an uncontrolled WAMG PNM of an FMS from [16]. This model is prone to deadlocks. It can be verified that there are 15571 states in the RG of the PNM in which 4159 are bad states representing the DZ, and 11412 are good states representing the LZ. This means that the optimal solution should provide a live net with 11412 good states for this PNM. Now, the proposed method is applied to this model.

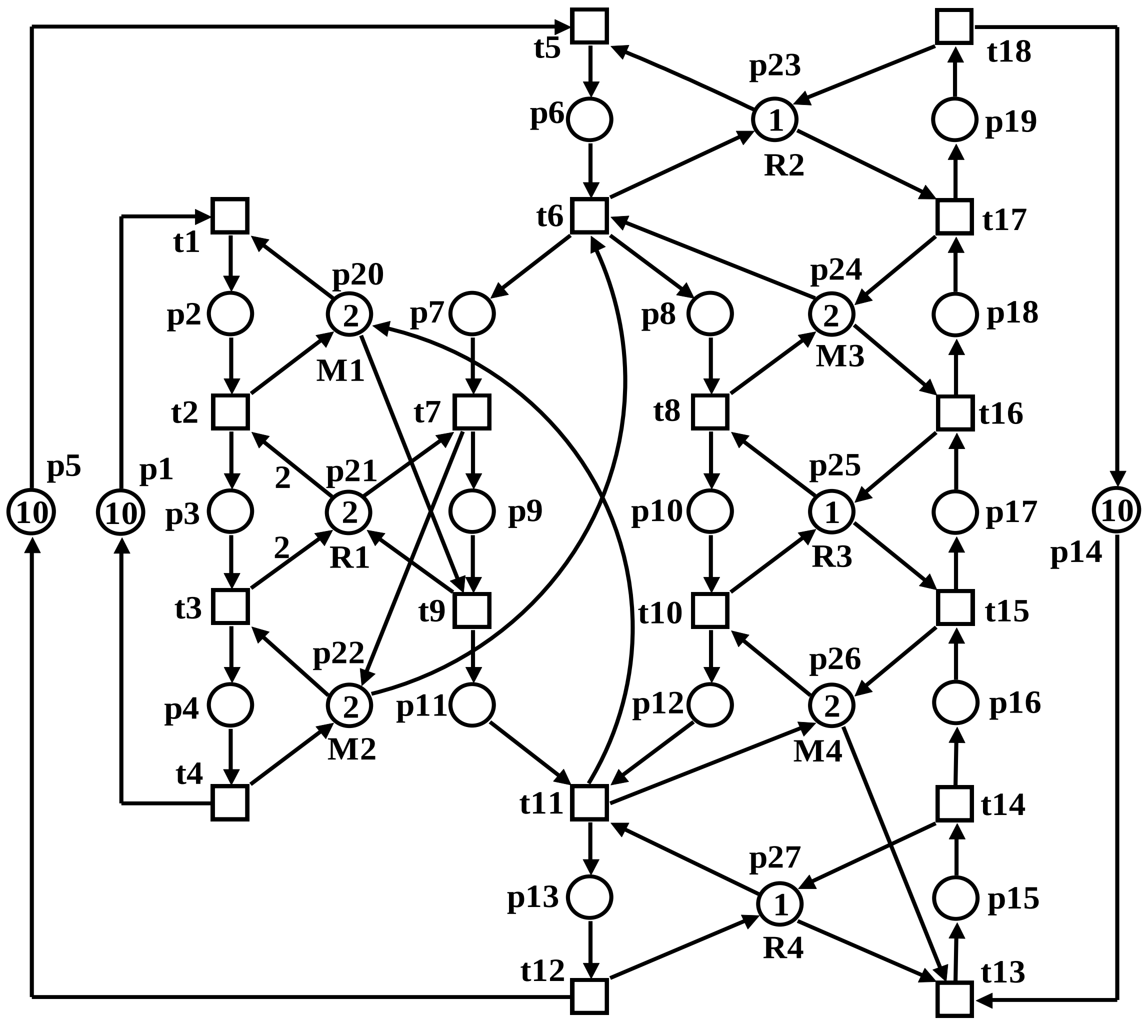

Figure 4.1 WAMG model from [16].

Step 1: The PNM shown in Fig. 4.1 is transformed into its conservative form (TPNM) by using the Algorithm 1. The TPNM is shown in Fig. 4.2. It is verified that the RG of the TPNM has 15571 states, whose DZ includes 4159 bad states, and LZ contains 11412 good states.

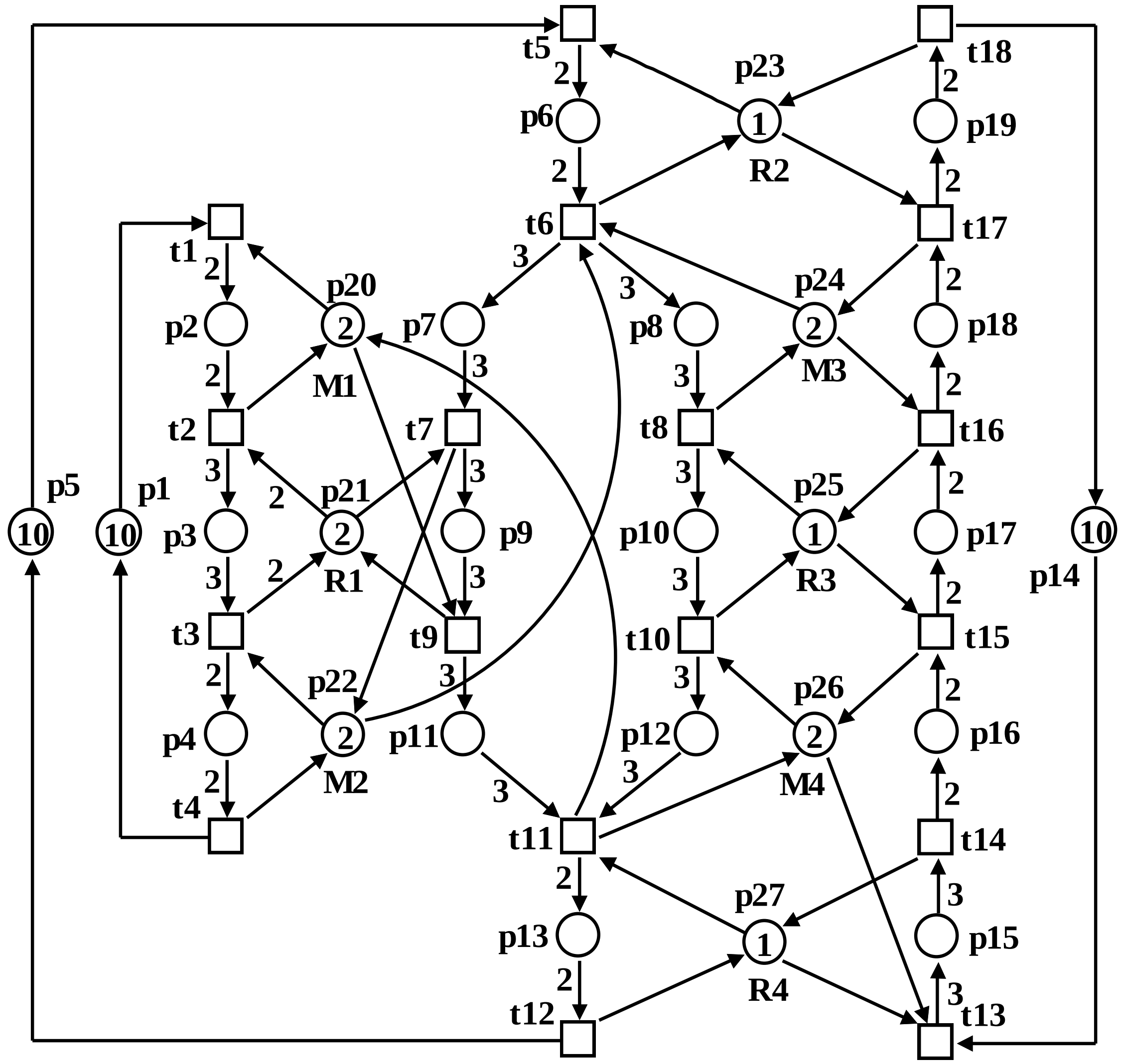

Figure 0. The TPNM.

Step 2: Input transitions of sink/source places p1, p5 and p14 are $^{\bullet}$p1= {t4}, $^{\bullet}$p5 = {t12} and $^{\bullet}$p14 = {t18} respectively. Likewise output transitions of sink/source places p1, p5 and p14 are p1$^{\bullet}$ = {t1}, p5$^{\bullet}$ = {t5} and p14$^{\bullet}$ = {t13} respectively. Therefore the input and output transitions of the GP are $^{\bullet}$GP = {t4, t12, t18} and GP$^{\bullet}$ = {t1, t5, t13}. When the GP is added within the TPNM, a new net structure N_B = TPNM + GP is obtained as shown in Fig. 4.3.

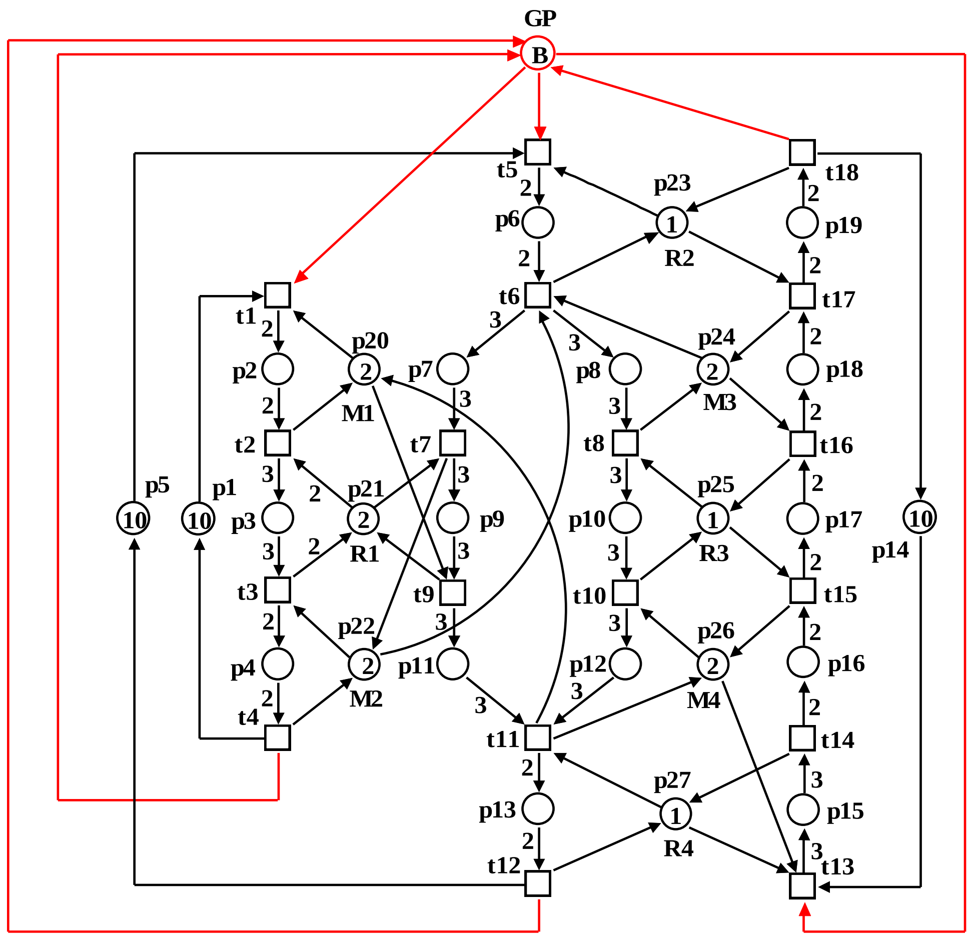

Figure 4.3. The net N_B with N_B = TPNM + GP (WAMG net).

Step 3:

(B = 1)

Step 3.1.1. When one token is deposited in the GP within the net N_1 shown in Fig. 4.3, the net N_1 is live with 20 good states. Then B: = B + 1 = 2.

(B = 2)

Step 3.2.1. When two tokens are deposited in the GP within the net N_2 shown in Fig. 4.3, the net N_2 is live with 181 good states. Then B: = B + 1 = 3.

(B = 3)

Step 3.3.1. When three tokens are deposited in the GP within the net N_3 shown in Fig. 4.3, the net N_3 is not live. There are 931 states within the RG_3 of N_3. DZ_3 includes 21 bad states BM_1, BM_2, BM_3, . . . , BM_{21} and LZ_3 contains 910 good states.

Step 3.3.2. The bad markings of the activity places of BM_1, BM_2, . . . , BM_{21} are shown in Table 4.1.

Table 4.1. Markings of activity places of BM_1, BM_2, . . . , BM_{21}.

State number	p 2	p 3	p 4	p 6	p 7	p 8	p 9	p 10	p 11	p 12	p 13	p 15	p 16	p 17	p 18	p 19
S_{32}	4	0	0	0	0	0	3	0	0	3	0	0	0	0	0	0
S_{50}	0	3	0	0	6	0	0	0	0	6	0	0	0	0	0	0
S_{182}	0	0	0	0	0	6	0	0	6	0	0	0	0	2	0	0
S_{632}	0	0	0	0	0	0	0	3	3	0	0	0	4	0	0	0
S_{869}	0	0	0	2	0	0	0	0	0	0	0	0	0	0	4	0
S_{31}	4	0	0	0	0	0	3	3	0	0	0	0	0	0	0	0
S_{49}	0	3	0	0	6	0	0	3	0	3	0	0	0	0	0	0
S_{181}	0	0	0	0	0	6	3	0	3	0	0	0	0	2	0	0
S_{631}	0	0	0	0	0	0	0	3	3	0	0	3	2	0	0	0
S_{633}	0	0	0	0	0	0	3	3	0	0	0	0	4	0	0	0
S_{30}	4	0	0	0	0	3	3	0	0	0	0	0	0	0	0	0
S_{48}	0	3	0	0	6	3	0	0	0	3	0	0	0	0	0	0
S_{183}	0	0	0	0	3	6	0	0	3	0	0	0	0	2	0	0
S_{630}	0	0	0	0	0	0	3	3	0	0	0	3	2	0	0	0
S_{776}	0	0	0	0	3	0	0	3	0	0	0	0	4	0	0	0
S_{47}	0	3	0	0	6	3	0	3	0	0	0	0	0	0	0	0
S_{179}	0	0	0	0	3	6	3	0	0	0	0	0	0	2	0	0
S_{775}	0	0	0	0	3	0	0	3	0	0	0	3	2	0	0	0
S_{892}	0	3	0	0	6	6	0	0	0	0	0	0	0	0	0	0
S_{178}	0	0	0	0	6	6	0	0	0	0	0	0	0	2	0	0
S_{180}	0	0	0	0	0	6	6	0	0	0	0	0	0	2	0	0

NOTE: Places p7 and p8 are not considered at the same time for determining the *PI* relations, only one of them is taken at a time in order to obtain bounded behavior of the net. Also, in order not to reach bad markings BM_1, BM_2, . . , BM_{21}, the following place invariants are established respectively:

$$PI_1 = \mu_2 + \mu_9 + \mu_{12} \leq 9$$

$$PI_2 = \mu_3 + \mu_7 + \mu_{12} \leq 14$$

$$PI_3 = \mu_8 + \mu_{11} + \mu_{17} \leq 13$$

$PI_4 = \mu_{10} + \mu_{11} + \mu_{16} \leq 9$

$PI_5 = \mu_6 + \mu_{18} \leq 5$

$PI_6 = \mu_2 + \mu_9 + \mu_{10} \leq 9$

$PI_7 = \mu_3 + \mu_7 + \mu_{10} + \mu_{12} \leq 14$

$PI_8 = \mu_8 + \mu_9 + \mu_{11} + \mu_{17} \leq 13$

$PI_9 = \mu_{10} + \mu_{11} + \mu_{15} + \mu_{16} \leq 10$

$PI_{10} = \mu_9 + \mu_{10} + \mu_{16} \leq 9$

$PI_{11} = \mu_2 + \mu_8 + \mu_9 \leq 9$

$PI_{12} = \mu_3 + \mu_7 + \mu_{12} \leq 11$ (μ_8 is ignored)

$PI_{13} = \mu_8 + \mu_{11} + \mu_{17} \leq 10$ (μ_7 is ignored)

$PI_{14} = \mu_9 + \mu_{10} + \mu_{15} + \mu_{16} \leq 10$

$PI_{15} = \mu_7 + \mu_{10} + \mu_{16} \leq 9$

$PI_{16} = \mu_3 + \mu_7 + \mu_{10} \leq 11$ (μ_8 is ignored)

$PI_{17} = \mu_8 + \mu_9 + \mu_{17} \leq 10$ (μ_7 is ignored)

$PI_{18} = \mu_7 + \mu_{10} + \mu_{15} + \mu_{16} \leq 10$

$PI_{19} = \mu_3 + \mu_7 \leq 8$ (μ_8 is ignored)

$PI_{20} = \mu_8 + \mu_{17} \leq 7$ (μ_7 is ignored)

$PI_{21} = \mu_8 + \mu_9 + \mu_{17} \leq 13$

Step 3.3.3. Monitors C_1, C_2, . . . C_{21} are computed in order to enforce PI_1, PI_2, . . . , PI_{21} respectively as follows:

$$D_{PI1} = \begin{array}{cc} \begin{array}{cccccc} t1 & t2 & t7 & t9 & t10 & t11 \end{array} & \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix} & \begin{array}{c} p2 \\ p9 \\ p12 \end{array} \end{array}$$

$$L_{PI1} = \begin{array}{c} \begin{array}{ccc} p2 & p9 & p12 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C1} = -L_{PI1}.D_{PI1} = -\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix}$$

$$D_{C1} = \begin{array}{c} \begin{array}{cccccc} t1 & t2 & t7 & t9 & t10 & t11 \end{array} \\ \begin{bmatrix} -2 & 2 & -3 & 3 & -3 & 3 \end{bmatrix} \end{array}$$

$\mu_{0(C1)} = 9$

$$D_{PI2} = \begin{array}{cc} \begin{array}{cccccc} t2 & t3 & t6 & t7 & t10 & t11 \end{array} & \\ \begin{bmatrix} 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix} & \begin{array}{c} p3 \\ p7 \\ p12 \end{array} \end{array}$$

$$L_{PI2} = \begin{array}{c} \begin{array}{ccc} p3 & p7 & p12 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C2} = -L_{PI2}.D_{PI2} = -\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix}$$

$$D_{C2} = \begin{array}{c} \begin{array}{cccccc} t2 & t3 & t6 & t7 & t10 & t11 \end{array} \\ \begin{bmatrix} -3 & 3 & -3 & 3 & -3 & 3 \end{bmatrix} \end{array}$$

$\mu_{0(C2)} = 14$

$$D_{PI3} = \begin{array}{cc} \begin{array}{cccccc} t6 & t8 & t9 & t11 & t15 & t16 \end{array} & \\ \begin{bmatrix} 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} & \begin{array}{c} p8 \\ p11 \\ p17 \end{array} \end{array}$$

$$L_{PI3} = \begin{array}{c} \begin{array}{ccc} p8 & p11 & p17 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C3} = -L_{PI3}.D_{PI3} = -\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C3} = \begin{array}{c} \begin{array}{cccccc} t6 & t8 & t9 & t11 & t15 & t16 \end{array} \\ \begin{bmatrix} -3 & 3 & -3 & 3 & -2 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C3)} = 13$

$$D_{PI4} = \begin{array}{c|c} \begin{array}{cccccc} t8 & t9 & t10 & t11 & t14 & t15 \end{array} & \\ \begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} & \begin{array}{c} p10 \\ p11 \\ p16 \end{array} \end{array}$$

$$L_{PI4} = \begin{array}{c} \begin{array}{ccc} p10 & p11 & p16 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C4} = -L_{PI4}.D_{PI4} = -\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C4} = \begin{array}{c} \begin{array}{cccccc} t8 & t9 & t10 & t11 & t14 & t15 \end{array} \\ \begin{bmatrix} -3 & -3 & 3 & 3 & -2 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C4)} = 9$

$$D_{PI5} = \begin{array}{cc} \begin{array}{cccc} t5 & t6 & t16 & t17 \end{array} & \\ \begin{bmatrix} 2 & -2 & 0 & 0 \\ 0 & 0 & 2 & -2 \end{bmatrix} & \begin{array}{c} p6 \\ p18 \end{array} \end{array}$$

$$L_{PI5} = \begin{array}{c} \begin{array}{cc} p6 & p18 \end{array} \\ \begin{bmatrix} 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C5} = -L_{PI5}.D_{PI5} = -\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 \\ 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C5} = \begin{array}{c} \begin{array}{cccc} t5 & t6 & t16 & t17 \end{array} \\ \begin{bmatrix} -2 & 2 & -2 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C5)} = 5$

$$D_{PI6} = \begin{array}{cc} \begin{array}{cccccc} t1 & t2 & t7 & t8 & t9 & t10 \end{array} & \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & -3 & 0 \\ 0 & 0 & 0 & 3 & 0 & -3 \end{bmatrix} & \begin{array}{c} p2 \\ p9 \\ p10 \end{array} \end{array}$$

$$L_{PI6} = \begin{array}{c} \begin{array}{ccc} p2 & p9 & p10 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C6} = -L_{PI6}.D_{PI6} = -\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & -3 & 0 \\ 0 & 0 & 0 & 3 & 0 & -3 \end{bmatrix}$$

$$D_{C6} = \begin{array}{c} \begin{array}{cccccc} t1 & t2 & t7 & t8 & t9 & t10 \end{array} \\ \begin{bmatrix} -2 & 2 & -3 & -3 & 3 & 3 \end{bmatrix} \end{array}$$

$\mu_{0(C6)} = 9$

$$D_{PI7} = \begin{array}{c} \begin{array}{ccccccc} t2 & t3 & t6 & t7 & t8 & t10 & t11 \end{array} \\ \begin{bmatrix} 3 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix} \end{array} \begin{array}{c} \\ p3 \\ p7 \\ p10 \\ p12 \end{array}$$

$$L_{PI7} = \begin{array}{c} \begin{array}{cccc} p3 & p7 & p10 & p12 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C7} = -L_{PI7}.D_{PI7} = -\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix}$$

$$D_{\mathbf{C7}} = \begin{array}{c} \begin{array}{ccccccc} t2 & t3 & t6 & t7 & t8 & t10 & t11 \end{array} \\ \begin{bmatrix} -3 & 3 & -3 & 3 & -3 & 0 & 3 \end{bmatrix} \end{array}$$

$\mu_{0(C7)} = 14$

$$D_{PI8} = \begin{array}{c} \begin{array}{ccccccc} t6 & t7 & t8 & t9 & t11 & t15 & t16 \end{array} \\ \begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{c} \\ p8 \\ p9 \\ p11 \\ p17 \end{array}$$

$$L_{PI8} = \begin{array}{c} \begin{array}{cccc} p8 & p9 & p11 & p17 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C8} = -L_{PI8}.D_{PI8} = -\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C8} = \begin{array}{c} \begin{array}{ccccccc} t6 & t7 & t8 & t9 & t11 & t15 & t16 \end{array} \\ \begin{bmatrix} -3 & -3 & 3 & 0 & 3 & -2 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C8)} = 13$

$$D_{PI9} = \begin{array}{c} \begin{array}{ccccccc} t8 & t9 & t10 & t11 & t13 & t14 & t15 \end{array} \\ \begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{c} \\ p10 \\ p11 \\ p15 \\ p16 \end{array}$$

$$L_{PI9} = \begin{array}{c} \begin{array}{cccc} p10 & p11 & p15 & p16 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C9} = -L_{PI9}.D_{PI9} = -[1 \quad 1 \quad 1 \quad 1]\begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C9} = \begin{array}{c} \begin{array}{ccccccc} t8 & t9 & t10 & t11 & t13 & t14 & t15 \end{array} \\ \begin{bmatrix} -3 & -3 & 3 & 3 & -3 & 1 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C9)} = 10$

$$D_{PI10} = \begin{array}{c} \begin{array}{cccccc} t7 & t8 & t9 & t10 & t14 & t15 \end{array} \\ \begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{l} p9 \\ p10 \\ p16 \end{array}$$

$$L_{PI10} = \begin{array}{c} \begin{array}{ccc} p9 & p10 & p16 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C10} = -L_{PI10}.D_{PI10} = -[1 \quad 1 \quad 1]\begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C10} = \begin{array}{c} \begin{array}{cccccc} t7 & t8 & t9 & t10 & t14 & t15 \end{array} \\ \begin{bmatrix} -3 & -3 & 3 & 3 & -2 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C10)} = 9$

$$D_{PI11} = \begin{array}{c} \begin{array}{cccccc} t1 & t2 & t6 & t7 & t8 & t9 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & -3 & 0 \\ 0 & 0 & 0 & 3 & 0 & -3 \end{bmatrix} \end{array} \begin{array}{l} p2 \\ p8 \\ p9 \end{array}$$

$$L_{PI11} = \begin{array}{c} \begin{array}{ccc} p2 & p8 & p9 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C11} = -L_{PI11}.D_{PI11} = -[1 \quad 1 \quad 1]\begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & -3 & 0 \\ 0 & 0 & 0 & 3 & 0 & -3 \end{bmatrix}$$

$$D_{C11} = \begin{array}{c} \begin{array}{cccccc} t1 & t2 & t6 & t7 & t8 & t9 \end{array} \\ \begin{bmatrix} -2 & 2 & -3 & -3 & 3 & 3 \end{bmatrix} \end{array}$$

$\mu_{0(C11)} = 9$

$$D_{PI12} = \begin{array}{c} \begin{array}{cccccc} t2 & t3 & t6 & t7 & t10 & t11 \end{array} \\ \begin{bmatrix} 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix} \end{array} \begin{array}{c} \\ p3 \\ p7 \\ p12 \end{array}$$

$$L_{PI12} = \begin{array}{c} \begin{array}{ccc} p3 & p7 & p12 \end{array} \\ [1 \quad 1 \quad 1] \end{array}$$

$$D_{C12} = -L_{PI12}.D_{PI12} = -[1 \quad 1 \quad 1]\begin{bmatrix} 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix}$$

$$D_{C12} = \begin{array}{c} \begin{array}{cccccc} t2 & t3 & t6 & t7 & t10 & t11 \end{array} \\ [-3 \quad 3 \quad -3 \quad 3 \quad -3 \quad 3] \end{array}$$

$\mu_{0(C12)} = 11$

$$D_{PI13} = \begin{array}{c} \begin{array}{cccccc} t6 & t8 & t9 & t11 & t15 & t16 \end{array} \\ \begin{bmatrix} 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{c} \\ p8 \\ p11 \\ p17 \end{array}$$

$$L_{PI13} = \begin{array}{c} \begin{array}{ccc} p8 & p11 & p17 \end{array} \\ [1 \quad 1 \quad 1] \end{array}$$

$$D_{C13} = -L_{P13}.D_{P13} = -[1 \quad 1 \quad 1]\begin{bmatrix} 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C13} = \begin{array}{c} \begin{array}{cccccc} t6 & t8 & t9 & t11 & t15 & t16 \end{array} \\ [-3 \quad 3 \quad -3 \quad 3 \quad -2 \quad 2] \end{array}$$

$\mu_{0(C13)} = 10$

$$D_{PI14} = \begin{array}{c} \begin{array}{ccccccc} t7 & t8 & t9 & t10 & t13 & t14 & t15 \end{array} \\ \begin{bmatrix} 3 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{c} \\ p9 \\ p10 \\ p15 \\ p16 \end{array}$$

$$L_{PI14} = \begin{array}{c} \begin{array}{cccc} p9 & p10 & p15 & p16 \end{array} \\ [1 \quad 1 \quad 1 \quad 1] \end{array}$$

$$D_{C14} = -L_{PI14}.D_{PI14} = -\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}\begin{bmatrix}3 & 0 & 3 & 0 & 0 & 0 & 0\\ 0 & 3 & 0 & -3 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 3 & -3 & 0\\ 0 & 0 & 0 & 0 & 0 & 2 & -2\end{bmatrix}$$

$$D_{C14} = \begin{array}{c} \begin{array}{ccccccc} t7 & t8 & t9 & t10 & t13 & t14 & t15 \end{array} \\ \begin{bmatrix}-3 & -3 & 3 & 3 & -3 & 1 & 2\end{bmatrix} \end{array}$$

$\mu_{0(C14)} = 10$

$$D_{PI15} = \begin{array}{cc} \begin{array}{cccccc} t6 & t7 & t8 & t10 & t14 & t15 \end{array} & \\ \begin{bmatrix}3 & -3 & 0 & 0 & 0 & 0\\ 0 & 0 & 3 & -3 & 0 & 0\\ 0 & 0 & 0 & 0 & 2 & -2\end{bmatrix} & \begin{array}{c} p7 \\ p10 \\ p16 \end{array} \end{array}$$

$$L_{PI15} = \begin{array}{c} \begin{array}{ccc} p7 & p10 & p16 \end{array} \\ \begin{bmatrix}1 & 1 & 1\end{bmatrix} \end{array}$$

$$D_{C15} = -L_{P15}.D_{P15} = -\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}3 & -3 & 0 & 0 & 0 & 0\\ 0 & 0 & 3 & -3 & 0 & 0\\ 0 & 0 & 0 & 0 & 2 & -2\end{bmatrix}$$

$$D_{C15} = \begin{array}{c} \begin{array}{cccccc} t6 & t7 & t8 & t10 & t14 & t15 \end{array} \\ \begin{bmatrix}-3 & 3 & -3 & 3 & -2 & 2\end{bmatrix} \end{array}$$

$\mu_{0(C15)} = 9$

$$D_{PI16} = \begin{array}{cc} \begin{array}{cccccc} t2 & t3 & t6 & t7 & t8 & t10 \end{array} & \\ \begin{bmatrix}3 & -3 & 0 & 0 & 0 & 0\\ 0 & 0 & 3 & -3 & 0 & 0\\ 0 & 0 & 0 & 0 & 3 & -3\end{bmatrix} & \begin{array}{c} p3 \\ p7 \\ p10 \end{array} \end{array}$$

$$L_{PI16} = \begin{array}{c} \begin{array}{ccc} p3 & p7 & p10 \end{array} \\ \begin{bmatrix}1 & 1 & 1\end{bmatrix} \end{array}$$

$$D_{C16} = -L_{PI16}.D_{PI16} = -\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}3 & -3 & 0 & 0 & 0 & 0\\ 0 & 0 & 3 & -3 & 0 & 0\\ 0 & 0 & 0 & 0 & 3 & -3\end{bmatrix}$$

$$D_{C16} = \begin{array}{c} \begin{array}{cccccc} t2 & t3 & t6 & t7 & t8 & t10 \end{array} \\ \begin{bmatrix}-3 & 3 & -3 & 3 & -3 & 3\end{bmatrix} \end{array}$$

$\mu_{0(C16)} = 11$

$$D_{PI17} = \begin{array}{c} \begin{array}{cccccc} t6 & t7 & t8 & t9 & t15 & t16 \end{array} \\ \begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{l} \\ p8 \\ p9 \\ p17 \end{array}$$

$$L_{PI17} = \begin{array}{c} \begin{array}{ccc} p8 & p9 & p17 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C17} = -L_{PI17}.D_{PI17} = -\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C17} = \begin{array}{c} \begin{array}{cccccc} t6 & t7 & t8 & t9 & t15 & t16 \end{array} \\ \begin{bmatrix} -3 & -3 & 3 & 3 & -2 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C17)} = 10$

$$D_{PI18} = \begin{array}{c} \begin{array}{ccccccc} t6 & t7 & t8 & t10 & t13 & t14 & t15 \end{array} \\ \begin{bmatrix} 3 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{l} \\ p7 \\ p10 \\ p15 \\ p16 \end{array}$$

$$L_{PI18} = \begin{array}{c} \begin{array}{cccc} p7 & p10 & p15 & p16 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C18} = -L_{PI18}.D_{PI18} = -\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C18} = \begin{array}{c} \begin{array}{ccccccc} t6 & t7 & t8 & t10 & t13 & t14 & t15 \end{array} \\ \begin{bmatrix} -3 & 3 & -3 & 3 & -3 & 1 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C18)} = 10$

$$D_{PI19} = \begin{array}{c} \begin{array}{cccc} t2 & t3 & t6 & t7 \end{array} \\ \begin{bmatrix} 3 & -3 & 0 & 0 \\ 0 & 0 & 3 & -3 \end{bmatrix} \end{array} \begin{array}{l} \\ p3 \\ p7 \end{array}$$

$$L_{PI19} = \begin{array}{c} \begin{array}{cc} p3 & p7 \end{array} \\ \begin{bmatrix} 1 & 1 \end{bmatrix} \end{array}$$

$$D_{CI19} = -L_{PI19}.D_{PI19} = -\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -3 & 0 & 0 \\ 0 & 0 & 3 & -3 \end{bmatrix}$$

$$D_{C19} = \begin{array}{cccc} t2 & t3 & t6 & t7 \\ \end{array}$$
$$D_{C19} = \begin{bmatrix} -3 & 3 & -3 & 3 \end{bmatrix}$$

$\mu_{0(C19)} = 8$

$$D_{PI20} = \begin{array}{c} \begin{array}{cccc} t6 & t8 & t15 & t16 \end{array} \\ \begin{bmatrix} 3 & -3 & 0 & 0 \\ 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{c} \\ p8 \\ p17 \end{array}$$

$$L_{PI20} = \begin{array}{c} \begin{array}{cc} p8 & p17 \end{array} \\ \begin{bmatrix} 1 & 1 \end{bmatrix} \end{array}$$

$$D_{CI20} = -L_{PI20}.D_{PI20} = -\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -3 & 0 & 0 \\ 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C20} = \begin{array}{c} \begin{array}{cccc} t6 & t8 & t15 & t16 \end{array} \\ \begin{bmatrix} -3 & 3 & -2 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C20)} = 7$

$$D_{PI21} = \begin{array}{c} \begin{array}{cccccc} t6 & t7 & t8 & t9 & t15 & t16 \end{array} \\ \begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{c} \\ p8 \\ p9 \\ p17 \end{array}$$

$$L_{PI21} = \begin{array}{c} \begin{array}{ccc} p8 & p9 & p17 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C21} = -L_{P21}.D_{P21} = -\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C21} = \begin{array}{c} \begin{array}{cccccc} t6 & t7 & t8 & t9 & t15 & t16 \end{array} \\ \begin{bmatrix} -3 & -3 & 3 & 3 & -2 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C21)} = 10$

The computed monitors are shown in Table 4.2.

Table 4.2 Computed monitors for N_3.

C_i	$^\bullet c_i$	$c_i^\bullet$	$\mu_{0(ci)}$
C_1	2t2, 3t9, 3t11	2t1, 3t7, 3t10	9
C_2	3t3, 3t7, 3t11	3t2, 3t6, 3t10	14
C_3	3t8, 3t11, 2t16	3t6, 3t9, 2t15	13
C_4	3t10, 3t11, 2t15	3t8, 3t9, 2t14	9
C_5	2t6, 2t17	2t5, 2t16	5
C_6	2t2, 3t9, 3t10	2t1, 3t7, 3t8	9
C_7	3t3, 3t7, 3t11	3t2, 3t6, 3t8	14
C_8	3t8, 3t11, 2t16	3t6, 3t7, 2t15	13
C_9	3t10, 3t11, t14, 2t15	3t8, 3t9, 3t13	10
C_{10}	3t9, 3t10, 2t15	3t7, 3t8, 2t14	9
C_{11}	2t2, 3t8, 3t9	2t1, 3t6, 3t7	9
C_{12}	3t3, 3t7, 3t11	3t2, 3t6, 3t10	11
C_{13}	3t8, 3t11, 2t16	3t6, 3t9, 2t15	10
C_{14}	3t9, 3t10, t14, 2t15	3t7, 3t8, 3t13	10
C_{15}	3t7, 3t10, 2t15	3t6, 3t8, 2t14	9
C_{16}	3t3, 3t7, 3t10	3t2, 3t6, 3t8	11
C_{17}	3t8, 3t9, 2t16	3t6, 3t7, 2t15	10
C_{18}	3t7, 3t10, t14, 2t15	3t6, 3t8, 3t13	10
C_{19}	3t3, 3t7	3t2, 3t6	8
C_{20}	3t8, 2t16	3t6, 2t15	7
C_{21}	3t8, 3t9, 2t16	3t6, 3t7, 2t15	13

Step 3.3.4. Redundancy test is carried out on the monitors and found that 14 monitors are necessary. These include: C_1, C_4, C_5, C_6, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, C_{15}, C_{18}, C_{19} and C_{20}.

Step 3.3.5. When the computed necessary monitors are augmented in the uncontrolled model N_3, the controlled model of N_3 is obtained as follows: $N_3 = N_3 + C_1 + C_4 + C_5 + C_6 + C_9 + C_{10} + C_{11} + C_{12} + C_{13} + C_{14} + C_{15} + C_{18} + C_{19} + C_{20}$, and is shown in Fig. 4.4.

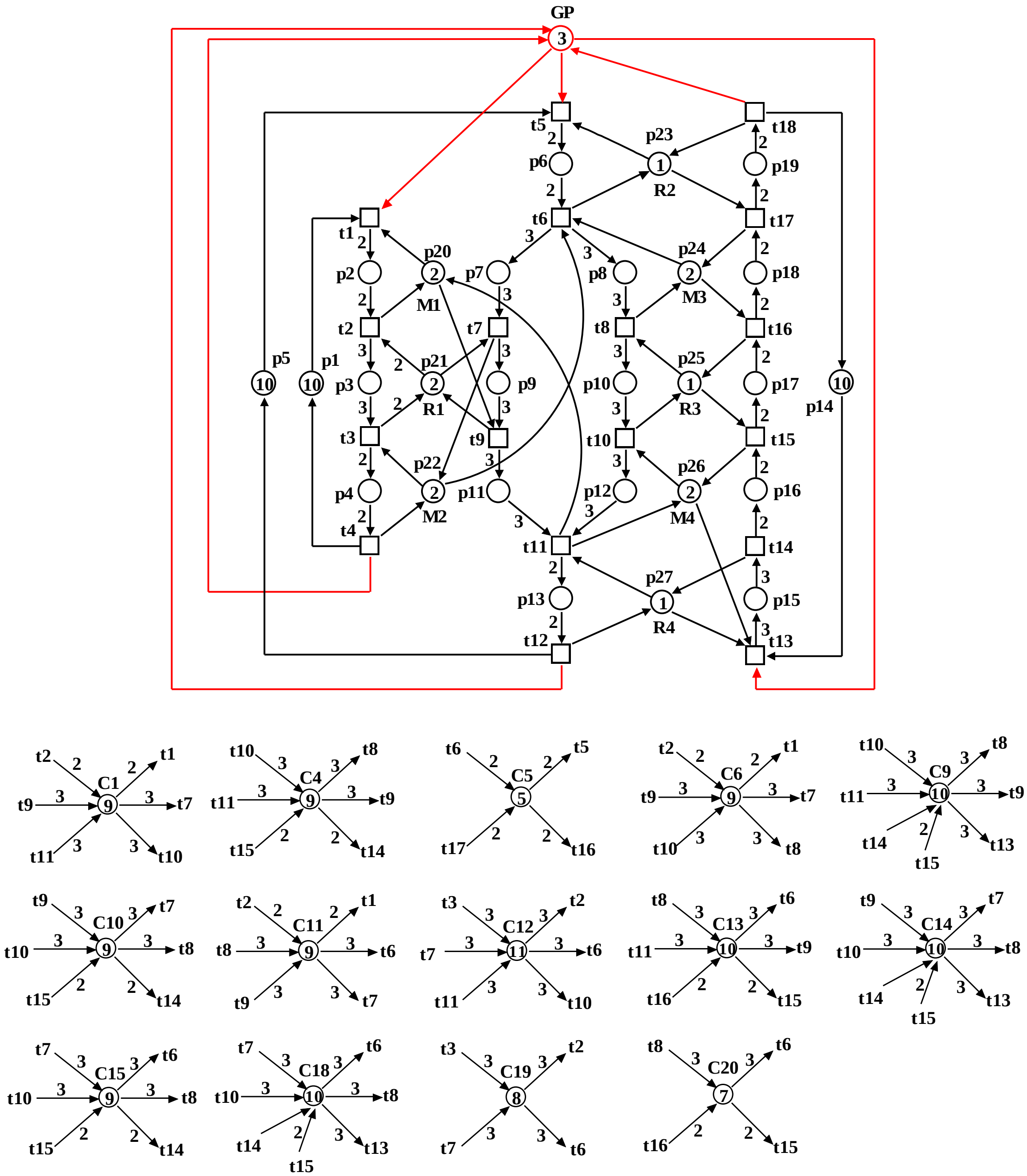

Figure 4.4. The controlled model of N_3 (N_3:= N_3 + 14 necessary computed monitors).

It is verified that the controlled model of N_3 shown in Fig. 4.4 is live with 847 good states.

Step 3.4.1. The net N_4 considered in this step is shown in Fig. 4.5. It is obtained by increasing the number of tokens in GP as shown in Fig. 4.4

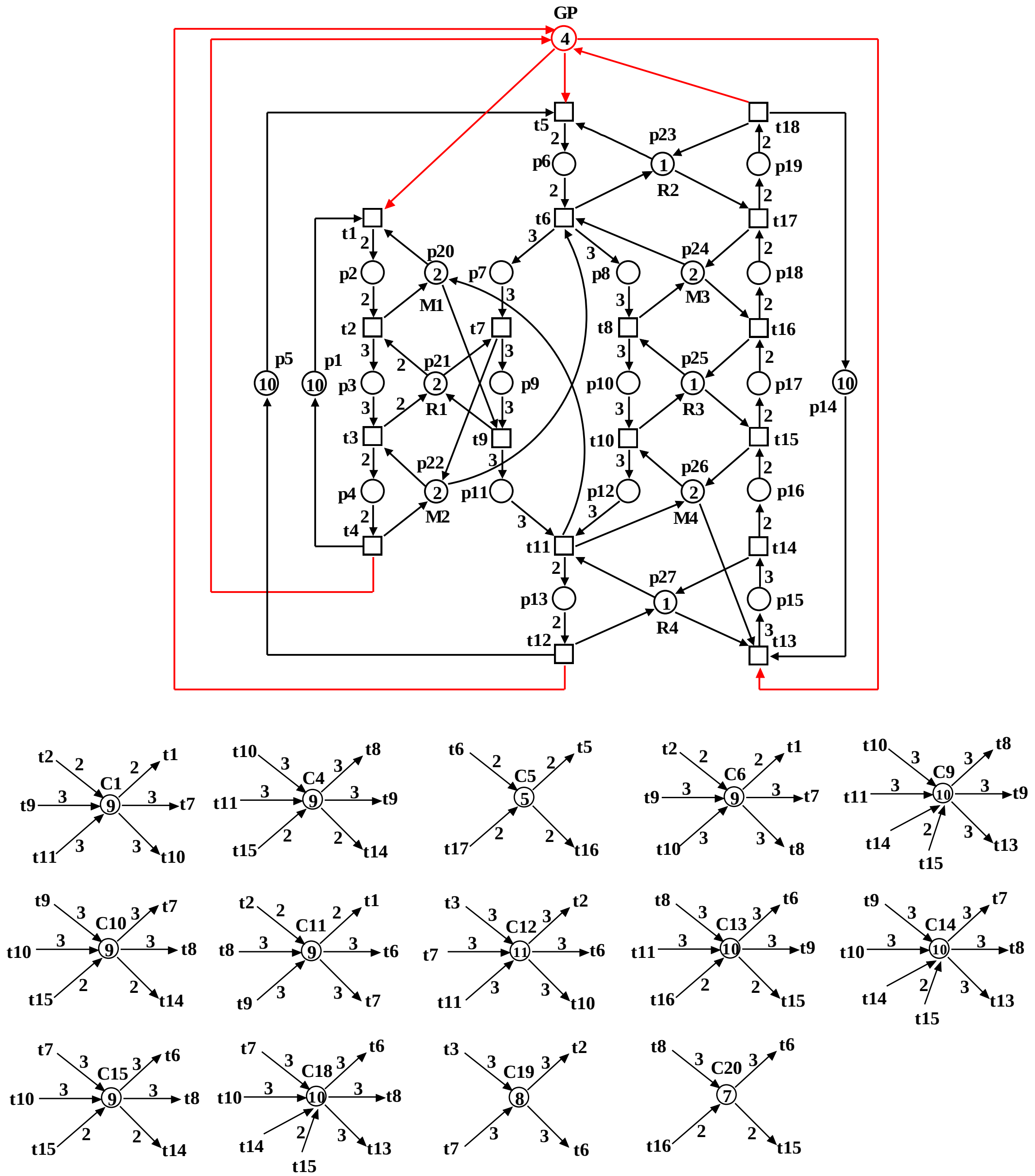

Figure 4.5. The net N_4 (WAMG net).

The net in N_4 is not live. There are 2495 states in the RG_4 of the N_4. The DZ_4 includes 14 bad marking (BM_{22}, BM_{23}, . . . , BM_{35}) and the LZ_4 contains 2481 good states.

Step 3.4.2. The bad markings of the activity places are shown in Table 4.3.

Table 4.3. The markings of the activity places of BM_{22}, BM_{23}, . . . , BM_{35}.

State number	p 2	p 3	p 4	p 6	p 7	p 8	p 9	p 10	p 11	p 12	p 13	p 15	p 16	p 17	p 18	p 19
S_{322}	2	0	0	0	6	6	0	0	0	0	0	0	2	0	0	0
S_{757}	0	0	0	2	0	3	0	0	3	0	0	0	0	2	2	0
S_{1099}	0	0	0	0	0	6	3	0	3	0	0	0	4	0	0	0
S_{1931}	4	0	0	0	6	0	0	3	0	3	0	0	0	0	0	0
S_{1324}	2	0	0	0	6	6	0	0	0	0	0	3	0	0	0	0
S_{1106}	0	0	0	0	0	6	3	0	3	0	0	3	2	0	0	0
S_{1930}	4	0	0	0	6	3	0	0	0	3	0	0	0	0	0	0
S_{1402}	0	0	0	2	0	3	3	0	0	0	0	0	0	2	2	0
S_{1105}	0	0	0	0	3	6	0	0	3	0	0	3	2	0	0	0
S_{2121}	4	0	0	0	6	3	0	3	0	0	0	0	0	0	0	0
S_{2222}	0	0	0	2	3	3	0	0	0	0	0	0	0	2	2	0
S_{1371}	0	0	0	0	3	6	3	0	0	0	0	3	2	0	0	0
S_{1367}	0	0	0	0	3	6	3	0	0	0	0	0	4	0	0	0
S_{1098}	0	0	0	0	3	6	0	0	3	0	0	0	4	0	0	0

In order not to reach bad markings BM_{22}, BM_{23}, . . . , BM_{35}, the following place invariants *PI*s are established respectively:

$PI_{22} = \mu_2 + \mu_8 + \mu_{16} \leq 9$ (μ_7 is ignored)

$PI_{23} = \mu_6 + \mu_8 + \mu_{11} + \mu_{17} + \mu_{18} \leq 11$

$PI_{24} = \mu_8 + \mu_9 + \mu_{11} + \mu_{16} \leq 15$

$PI_{25} = \mu_2 + \mu_7 + \mu_{10} + \mu_{12} \leq 15$

$PI_{26} = \mu_2 + \mu_7 + \mu_{15} \leq 10$ (μ_8 is ignored)

$PI_{27} = \mu_8 + \mu_9 + \mu_{11} + \mu_{15} + \mu_{16} \leq 16$

$PI_{28} = \mu_2 + \mu_7 + \mu_{12} \leq 12$ (μ_8 is ignored)

$PI_{29} = \mu_6 + \mu_8 + \mu_9 + \mu_{17} + \mu_{18} \leq 11$

$PI_{30} = \mu_8 + \mu_{11} + \mu_{15} + \mu_{16} \leq 13$ (μ_7 is ignored)

$PI_{31} = \mu_2 + \mu_7 + \mu_{10} \leq 12$ (μ_8 is ignored)

$PI_{32} = \mu_6 + \mu_8 + \mu_{17} + \mu_{18} \leq 8$ (μ_7 is ignored)

$PI_{33} = \mu_8 + \mu_9 + \mu_{15} + \mu_{16} \leq 13$ (μ_7 is ignored)

$PI_{34} = \mu_8 + \mu_9 + \mu_{16} \leq 12$ (μ_7 is ignored)

$PI_{35} = \mu_8 + \mu_{11} + \mu_{16} \leq 12$ (μ_7 is ignored)

Step 3.4.3. Monitors are computed in order to enforce place invariance *PI*s as follows.

$$D_{PI22} = \begin{array}{c} \begin{matrix} t1 & t2 & t6 & t8 & t14 & t15 \end{matrix} \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{matrix} p2 \\ p8 \\ p16 \end{matrix}$$

$$L_{PI22} = \begin{array}{c} \begin{matrix} p2 & p8 & p16 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C22} = -L_{PI22}.D_{PI22} = -\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$\mathrm{D_{C22}} = \begin{array}{c} \begin{matrix} t1 & t2 & t6 & t8 & t14 & t15 \end{matrix} \\ \begin{bmatrix} -2 & 2 & -3 & 3 & -2 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C22)} = 9$

$$D_{PI23} = \begin{array}{c} \begin{matrix} t5 & t6 & t8 & t9 & t11 & t15 & t16 & t17 \end{matrix} \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{matrix} p6 \\ p8 \\ p11 \\ p17 \\ p18 \end{matrix}$$

$$L_{PI23} = \begin{array}{c} \begin{matrix} p6 & p8 & p11 & p17 & p18 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C23} = -L_{PI23}.D_{PI23} = -\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C23} = \begin{array}{c} \begin{matrix} t5 & t6 & t8 & t9 & t11 & t15 & t16 & t17 \end{matrix} \\ \begin{bmatrix} -2 & -1 & 3 & -3 & 3 & -2 & 0 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C26)} = 11$

$$D_{PI24} = \begin{array}{c} \begin{array}{ccccccc} t6 & t7 & t8 & t9 & t11 & t14 & t15 \end{array} \\ \begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{c} p8 \\ p9 \\ p11 \\ p16 \end{array}$$

$$L_{PI24} = \begin{array}{c} \begin{array}{cccc} p8 & p9 & p11 & p16 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C24} = -L_{PI24}.D_{PI24} = -\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C24} = \begin{array}{c} \begin{array}{ccccccc} t6 & t7 & t8 & t9 & t11 & t14 & t15 \end{array} \\ \begin{bmatrix} -3 & -3 & 3 & 0 & 3 & -2 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C24)} = 15$

$$D_{PI25} = \begin{array}{c} \begin{array}{ccccccc} t1 & t2 & t6 & t7 & t8 & t10 & t11 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix} \end{array} \begin{array}{c} p2 \\ p7 \\ p10 \\ p12 \end{array}$$

$$L_{PI25} = \begin{array}{c} \begin{array}{cccc} p2 & p7 & p10 & p12 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C25} = -L_{PI25}.D_{PI25} = -\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix}$$

$$D_{C25} = \begin{array}{c} \begin{array}{ccccccc} t1 & t2 & t6 & t7 & t8 & t10 & t11 \end{array} \\ \begin{bmatrix} -2 & 2 & -3 & 3 & -3 & 0 & 3 \end{bmatrix} \end{array}$$

$\mu_{0(C25)} = 15$

$$D_{PI26} = \begin{array}{c} \begin{array}{cccccc} t1 & t2 & t6 & t7 & t13 & t14 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix} \end{array} \begin{array}{c} p2 \\ p7 \\ p15 \end{array}$$

$$L_{PI26} = \begin{array}{c} \begin{array}{ccc} p2 & p7 & p15 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C26} = -L_{PI26}.D_{PI26} = -[1 \quad 1 \quad 1]\begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix}$$

$$D_{C26} = \begin{array}{cccccc} t1 & t2 & t6 & t7 & t13 & t14 \\ \end{array}$$
$$D_{C26} = \begin{bmatrix} -2 & 2 & -3 & 3 & -3 & 3 \end{bmatrix}$$

$\mu_{0(C26)} = 10$

$$D_{PI27} = \begin{array}{c} \begin{array}{cccccccc} t6 & t7 & t8 & t9 & t11 & t13 & t14 & t15 \end{array} \\ \begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{c} \\ p8 \\ p9 \\ p11 \\ p15 \\ p16 \end{array}$$

$$L_{PI27} = \begin{array}{c} \begin{array}{ccccc} p8 & p9 & p11 & p15 & p16 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C27} = -L_{PI27}.D_{PI27} = -[1 \quad 1 \quad 1 \quad 1 \quad 1]\begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C27} = \begin{array}{c} \begin{array}{cccccccc} t6 & t7 & t8 & t9 & t11 & t13 & t14 & t15 \end{array} \\ \begin{bmatrix} -3 & -3 & 3 & 0 & 3 & -3 & 1 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C27)} = 16$

$$D_{PI28} = \begin{array}{c} \begin{array}{cccccc} t1 & t2 & t6 & t7 & t10 & t11 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix} \end{array} \begin{array}{c} \\ p2 \\ p7 \\ p12 \end{array}$$

$$L_{PI28} = \begin{array}{c} \begin{array}{ccc} p2 & p7 & p12 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C28} = -L_{PI28}.D_{PI28} = -[1 \quad 1 \quad 1]\begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix}$$

$$D_{C28} = \begin{array}{c} \begin{array}{cccccc} t1 & t2 & t6 & t7 & t10 & t11 \end{array} \\ \begin{bmatrix} -2 & 2 & -3 & 3 & -3 & 3 \end{bmatrix} \end{array}$$

$\mu_{0(C28)} = 12$

$$D_{PI29} = \begin{array}{c} \begin{array}{cccccccc} t5 & t6 & t7 & t8 & t9 & t15 & t16 & t17 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{c} \\ p6 \\ p8 \\ p9 \\ p17 \\ p18 \end{array}$$

$$L_{PI29} = \begin{array}{c} \begin{array}{ccccc} p6 & p8 & p9 & p17 & p18 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C29} = -\mathrm{L_{PI29}}.\mathrm{D_{PI29}} = -\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C29} = \begin{array}{c} \begin{array}{cccccccc} t5 & t6 & t7 & t8 & t9 & t15 & t16 & t17 \end{array} \\ \begin{bmatrix} -2 & -1 & -3 & 3 & 3 & -2 & 0 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C29)} = 11$

$$D_{PI30} = \begin{array}{c} \begin{array}{ccccccc} t6 & t8 & t9 & t11 & t13 & t14 & t15 \end{array} \\ \begin{bmatrix} 3 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{c} \\ p8 \\ p11 \\ p15 \\ p16 \end{array}$$

$$L_{PI30} = \begin{array}{c} \begin{array}{cccc} p8 & p11 & p15 & p16 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C30} = -L_{PI30}.D_{PI30} = -\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C30} = \begin{array}{c} \begin{array}{ccccccc} t6 & t8 & t9 & t11 & t13 & t14 & t15 \end{array} \\ \begin{bmatrix} -3 & 3 & -3 & 3 & -3 & 1 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C30)} = 13$

$$D_{PI31} = \begin{array}{c} \begin{array}{cccccc} t1 & t2 & t6 & t7 & t8 & t10 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix} \end{array} \begin{array}{c} \\ p2 \\ p7 \\ p10 \end{array}$$

$$L_{PI31} = \begin{array}{ccc} p2 & p7 & p10 \\ \end{array}$$
$$L_{PI31} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

$$D_{C31} = -L_{PI31}.D_{PI31} = -\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix}$$

$$D_{C31} = \begin{array}{cccccc} t1 & t2 & t6 & t7 & t8 & t10 \\ \end{array}$$
$$D_{C31} = \begin{bmatrix} -2 & 2 & -3 & 3 & -3 & 3 \end{bmatrix}$$

$\mu_{0(C31)} = 12$

$$D_{PI32} = \begin{array}{c} \begin{array}{cccccc} t5 & t6 & t8 & t15 & t16 & t17 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{c} \\ p6 \\ p8 \\ p17 \\ p18 \end{array}$$

$$L_{PI32} = \begin{array}{c} \begin{array}{cccc} p6 & p8 & p17 & p18 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C32} = -L_{PI32}.D_{PI32} = -\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C32} = \begin{array}{c} \begin{array}{cccccc} t5 & t6 & t8 & t15 & t16 & t17 \end{array} \\ \begin{bmatrix} -2 & -1 & 3 & -2 & 0 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C32)} = 8$

$$D_{PI33} = \begin{array}{c} \begin{array}{ccccccc} t6 & t7 & t8 & t9 & t13 & t14 & t15 \end{array} \\ \begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{c} \\ p8 \\ p9 \\ p15 \\ p16 \end{array}$$

$$L_{PI33} = \begin{array}{c} \begin{array}{cccc} p8 & p9 & p15 & p16 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C33} = -L_{PI33}.D_{PI33} = -\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C33} = \begin{array}{c} \begin{array}{ccccccc} t6 & t7 & t8 & t9 & t13 & t14 & t15 \end{array} \\ \begin{bmatrix} -3 & -3 & 3 & 3 & -3 & 1 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C33)} = 13$

$$D_{PI34} = \begin{array}{c c} \begin{array}{cccccc} t6 & t7 & t8 & t9 & t14 & t15 \end{array} & \\ \begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} & \begin{array}{c} p8 \\ p9 \\ p16 \end{array} \end{array}$$

$$L_{PI34} = \begin{array}{c} \begin{array}{ccc} p8 & p9 & p16 \end{array} \\ [1 \quad 1 \quad 1] \end{array}$$

$$D_{C34} = -L_{PI34}.D_{PI34} = -[1 \quad 1 \quad 1]\begin{bmatrix} 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C34} = \begin{array}{c} \begin{array}{cccccc} t6 & t7 & t8 & t9 & t14 & t15 \end{array} \\ [-3 \quad -3 \quad 3 \quad 3 \quad -2 \quad 2\ ] \end{array}$$

$\mu_{0(C34)} = 12$

$$D_{PI35} = \begin{array}{c c} \begin{array}{cccccc} t6 & t8 & t9 & t11 & t14 & t15 \end{array} & \\ \begin{bmatrix} 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} & \begin{array}{c} p8 \\ p11 \\ p16 \end{array} \end{array}$$

$$L_{PI35} = \begin{array}{c} \begin{array}{ccc} p8 & p11 & p16 \end{array} \\ [1 \quad 1 \quad 1] \end{array}$$

$$D_{C35} = -L_{PI35}.D_{PI35} = -[1 \quad 1 \quad 1]\begin{bmatrix} 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C35} = \begin{array}{c} \begin{array}{cccccc} t6 & t8 & t9 & t11 & t14 & t15 \end{array} \\ [-3 \quad 3 \quad -3 \quad 3 \quad -2 \quad 2\ ] \end{array}$$

$\mu_{0(C35)} = 12$

The computed monitors are shown in Table 4.4.

Table 4.4 Computed monitors for N_4.

C_i	${}^{\bullet}c_I$	$c_I{}^{\bullet}$	$\mu_{0(ci)}$
C_{22}	2t2, 3t8, 2t15	2t1, 3t6, 2t14	9
C_{23}	3t8, 3t11, 2t17	2t5, t6, 3t9, 2t15	11
C_{24}	3t8, 3t11, 2t15	3t6, 3t7, 2t14	15
C_{25}	2t2, 3t7, 3t11	2t1, 3t6, 3t8	15
C_{26}	2t2, 3t7, 3t14	2t1, 3t6, 3t13	10
C_{27}	3t8, 3t11, t14, 2t15	3t6, 3t7, 3t13	16
C_{28}	2t2, 3t7, 3t11	2t1, 3t6, 3t10	12
C_{29}	3t8, 3t9, 2t17	2t5, t6, 3t7, 2t15	11
C_{30}	3t8, 3t11, t14, 2t15	3t6, 3t9, 3t13	13
C_{31}	2t2, 3t7, 3t10	2t1, 3t6, 3t8	12
C_{32}	3t8, 2t17	2t5, t6, 2t15	8
C_{33}	3t8, 3t9, t14, 2t15	3t6, 3t7, 3t13	13
C_{34}	3t8, 3t9, 2t15	3t6, 3t7, 2t14	12
C_{35}	3t8, 3t11, 2t15	3t6, 3t9, 2t14	12

Step 3.4.4. Redundancy test is carried out on the monitors and it is found that 7 out of 14 monitors are necessary. These include: C_{22}, C_{26}, C_{28}, C_{30}, C_{31}, C_{32}, and C_{33}. Monitors C_{12} and C_{13} of previous step are also found redundant in this step. Removal of C_{12} and C_{13} increase the number of live states in N_4 from 2481 to 2554 (addition of 81 live states).

Step 3.4.5. When the computed necessary monitors are augmented in the uncontrolled model N_4, the controlled model of N_4 is obtained as follows: $N_4 := N_4 + C_{22} + C_{26} + C_{28} + C_{30} + C_{31} + C_{32} + C_{33} – C_{12} – C_{13}$, and is shown in Fig. 4.6.

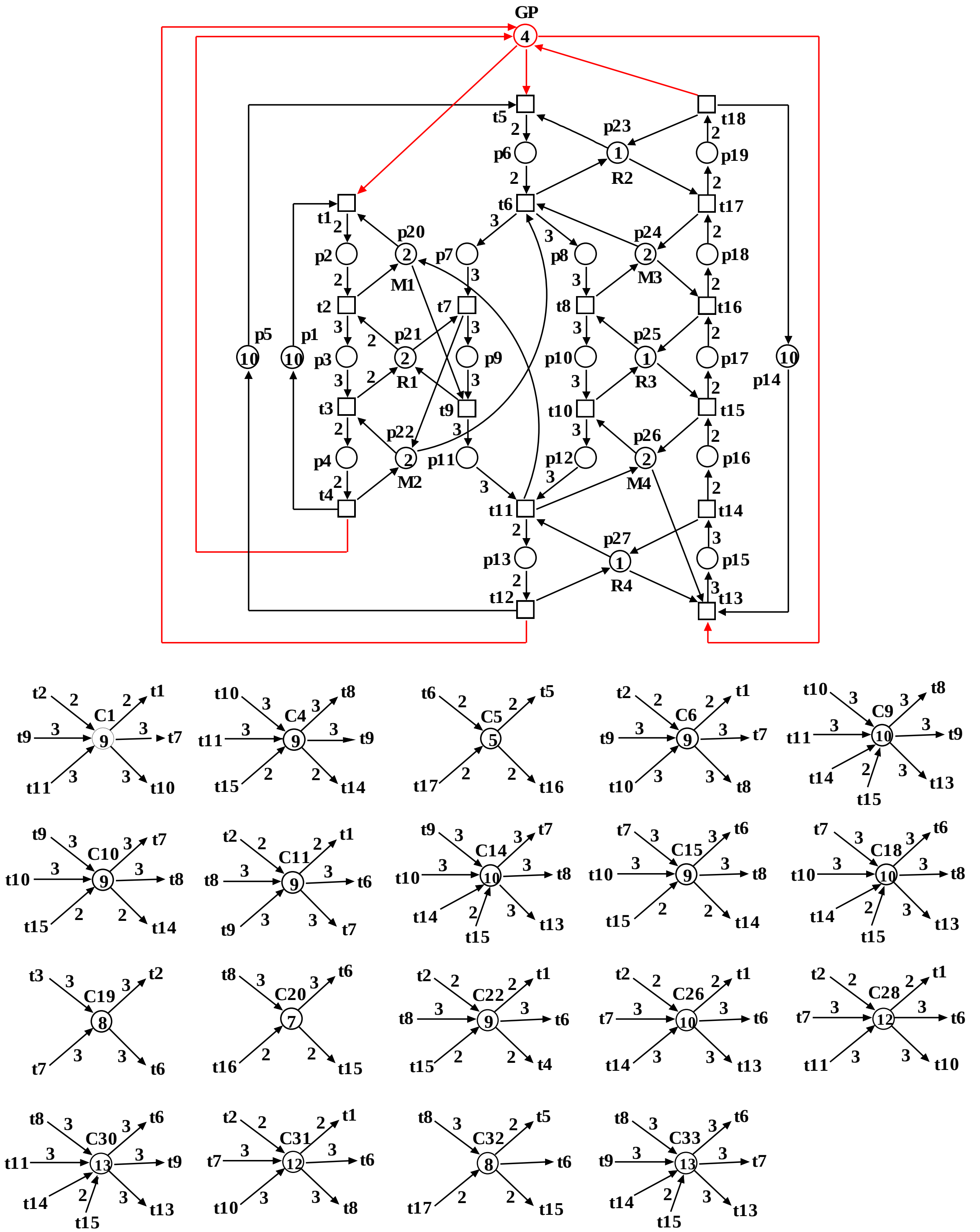

Figure 4.6. The controlled model N4 (N4: = N4 + C22 + C26 + C28 + C30 + C31 + C32 + C33 – C12 – C13).

It is verified that the controlled model of N4 shown in Fig. 4.6 is live with 2554 good states. This is the live optimal behavior for the controlled model N4.

Step 3.5.1. The net N_5 considered in this step is shown in Fig. 4.7. It is obtained by increasing the number of tokens in the GP shown in Fig. 4.6.

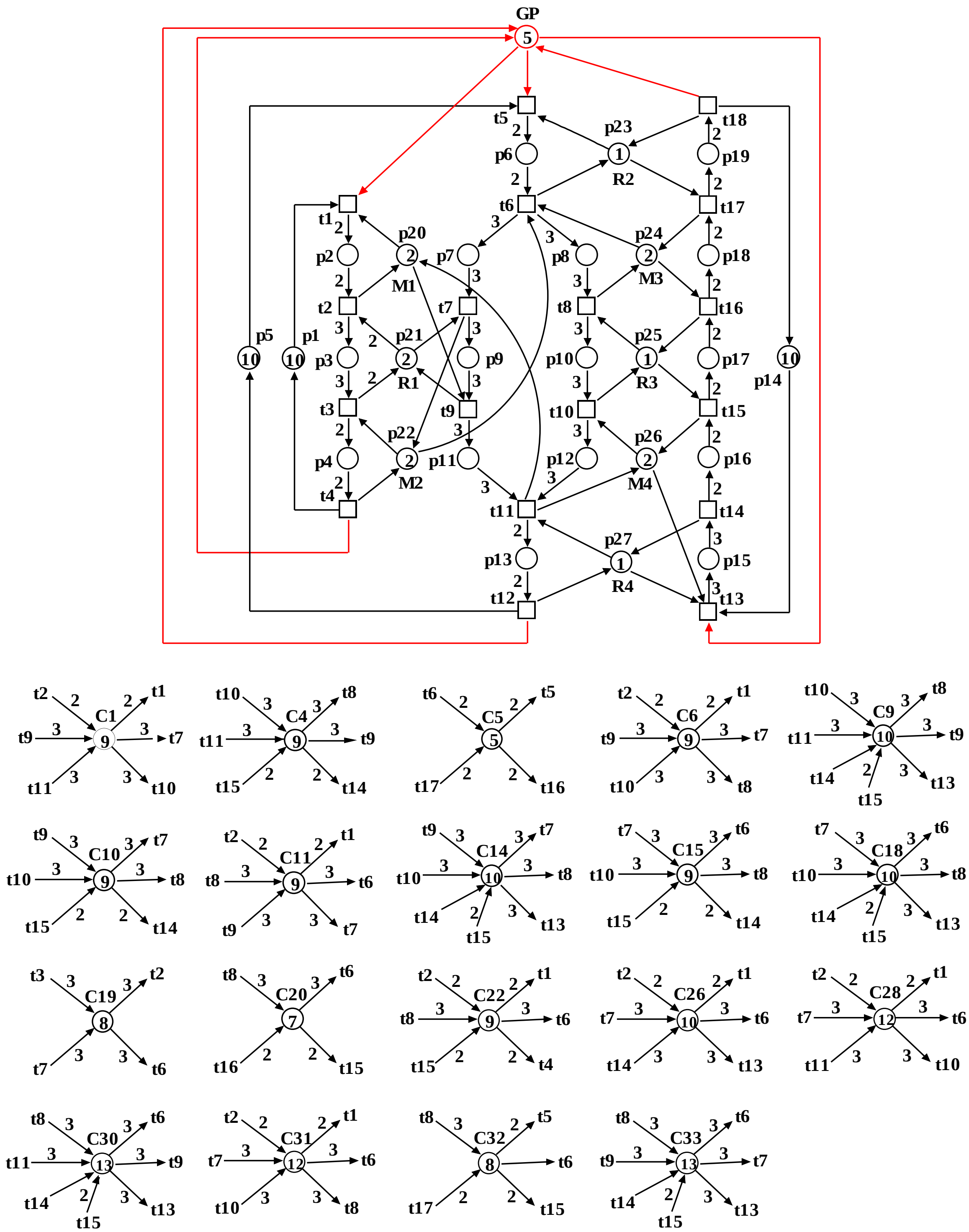

Figure 4.7. The net N_5 (WAMG net).

The net N_5 is not live. There are 5065 states in the RG_5 of N_5. The DZ_5 includes 12 bad marking (BM_{36}, BM_{37}, . . . , BM_{47}) and the LZ_5 contains 5053 good states.

Step 3.5.2. The markings of the activity places are shown in Table 4.5.

Table 4.**5.** The markings of the activity places of BM_{36}, BM_{37}, ... , BM_{47}.

state number	p 2	p 3	p 4	p 6	p 7	p 8	p 9	p 10	p 11	p 12	p 13	p 15	p 16	p 17	p 18	p 19
S_{492}	0	0	0	2	0	3	0	0	3	0	0	0	4	0	2	0
S_{491}	0	0	0	2	0	3	3	0	0	0	0	0	4	0	2	0
S_{882}	0	0	0	2	0	3	0	0	3	0	0	0	4	2	0	0
S_{883}	0	0	0	2	0	3	0	0	3	0	0	3	2	0	2	0
S_{490}	0	0	0	2	3	3	0	0	0	0	0	0	4	0	2	0
S_{881}	0	0	0	2	0	3	0	0	3	0	0	3	2	2	0	0
S_{1630}	0	0	0	2	0	3	3	0	0	0	0	0	4	2	0	0
S_{1631}	0	0	0	2	0	3	3	0	0	0	0	3	2	0	2	0
S_{1629}	0	0	0	2	0	3	3	0	0	0	0	3	2	2	0	0
S_{2953}	0	0	0	2	3	3	0	0	0	0	0	0	4	2	0	0
S_{2954}	0	0	0	2	3	3	0	0	0	0	0	3	2	0	2	0
S_{2952}	0	0	0	2	3	3	0	0	0	0	0	3	2	2	0	0

In order not to reach the bad markings BM_{36}, BM_{37}, . . . , BM_{47}, the following place invariants *PI*s are established respectively:

$$PI_{36} = \mu_6 + \mu_8 + \mu_{11} + \mu_{16} + \mu_{18} \leq 13$$

$$PI_{37} = \mu_6 + \mu_8 + \mu_9 + \mu_{16} + \mu_{18} \leq 13$$

$$PI_{38} = \mu_6 + \mu_8 + \mu_{11} + \mu_{16} + \mu_{17} \leq 13$$

$$PI_{39} = \mu_6 + \mu_8 + \mu_{11} + \mu_{15} + \mu_{16} + \mu_{18} \leq 14$$

$$PI_{40} = \mu_6 + \mu_8 + \mu_{16} + \mu_{18} \leq 10 \text{ } (\mu_7 \text{ is ignored})$$

$$PI_{41} = \mu_6 + \mu_8 + \mu_{11} + \mu_{15} + \mu_{16} + \mu_{17} \leq 14$$

$$PI_{42} = \mu_6 + \mu_8 + \mu_9 + \mu_{16} + \mu_{17} \leq 13$$

$$PI_{43} = \mu_6 + \mu_8 + \mu_9 + \mu_{15} + \mu_{16} + \mu_{18} \leq 14$$

$$PI_{44} = \mu_6 + \mu_8 + \mu_9 + \mu_{15} + \mu_{16} + \mu_{17} \leq 14$$

$$PI_{45} = \mu_6 + \mu_8 + \mu_{16} + \mu_{17} \leq 10 \text{ } (\mu_7 \text{ is ignored})$$

$PI_{46} = \mu_6 + \mu_8 + \mu_{15} + \mu_{16} + \mu_{18} \leq 11$ (μ_7 is ignored)

$PI_{47} = \mu_6 + \mu_8 + \mu_{15} + \mu_{16} + \mu_{17} \leq 11$ (μ_7 is ignored)

Step 3.5.3. Monitors are computed in order to enforce place invariance *PI*s as follows.

$$D_{PI36} = \begin{array}{c} \begin{array}{ccccccccc} t5 & t6 & t8 & t9 & t11 & t14 & t15 & t16 & t17 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{l} \\ p6 \\ p8 \\ p11 \\ p16 \\ p18 \end{array}$$

.

$$L_{PI36} = \begin{array}{c} \begin{array}{ccccc} p6 & p8 & p11 & p16 & p18 \end{array} \\ [1 \quad 1 \quad 1 \quad 1 \quad 1] \end{array}$$

$$D_{C36} = -L_{PI36}.D_{PI36} = -[1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C36} = \begin{array}{c} \begin{array}{ccccccccc} t5 & t6 & t8 & t9 & t11 & t14 & t15 & t16 & t17 \end{array} \\ [-2 \quad -1 \quad 3 \quad -3 \quad 3 \quad -2 \quad 2 \quad -2 \quad 2\ ] \end{array}$$

$\mu_{0(C36)} = 13$

$$D_{PI37} = \begin{array}{c} \begin{array}{ccccccccc} t5 & t6 & t7 & t8 & t9 & t14 & t15 & t16 & t17 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{l} \\ p6 \\ p8 \\ p9 \\ p16 \\ p18 \end{array}$$

.

$$L_{PI37} = \begin{array}{c} \begin{array}{ccccc} p6 & p8 & p9 & p16 & p18 \end{array} \\ [1 \quad 1 \quad 1 \quad 1 \quad 1] \end{array}$$

$$D_{C37} = -\mathrm{L}_{\mathrm{PI37}}.\mathrm{D}_{\mathrm{PI37}} = -[1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C37} = \begin{array}{c} \begin{array}{ccccccccc} t5 & t6 & t7 & t8 & t9 & t14 & t15 & t16 & t17 \end{array} \\ [-2 \quad -1 \quad -3 \quad 3 \quad 3 \quad -2 \quad 2 \quad -2 \quad 2\ ] \end{array}$$

$\mu_{0(C37)} = 13$

$$D_{PI38} = \begin{array}{c} \begin{array}{cccccccc} t5 & t6 & t8 & t9 & t11 & t14 & t15 & t16 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{c} \\ p6 \\ p8 \\ p11 \\ p16 \\ p17 \end{array}$$

$$L_{PI38} = \begin{array}{c} \begin{array}{ccccc} p6 & p8 & p11 & p16 & p17 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C38} = -L_{PI38}.D_{PI38} = -\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C38} = \begin{array}{c} \begin{array}{cccccccc} t5 & t6 & t8 & t9 & t11 & t14 & t15 & t16 \end{array} \\ \begin{bmatrix} -2 & -1 & 3 & -3 & 3 & -2 & 0 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C38)} = 13$

$$D_{PI39} = \begin{array}{c} \begin{array}{cccccccccc} t5 & t6 & t8 & t9 & t11 & t13 & t14 & t15 & t16 & t17 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{c} \\ p6 \\ p8 \\ p11 \\ p15 \\ p16 \\ p18 \end{array}$$

$$L_{PI39} = \begin{array}{c} \begin{array}{cccccc} p6 & p8 & p11 & p15 & p16 & p18 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C39} = -L_{PI39}.D_{PI39}$$

$$= -\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C39} = \begin{array}{c} \begin{array}{cccccccccc} t5 & t6 & t8 & t9 & t11 & t13 & t14 & t15 & t16 & t17 \end{array} \\ \begin{bmatrix} -2 & -1 & 3 & -3 & 3 & -3 & 1 & 2 & -2 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C39)} = 14$

$$D_{PI40} = \begin{array}{c c} & \begin{array}{ccccccc} t5 & t6 & t8 & t14 & t15 & t16 & t17 \end{array} \\ & \left[\begin{array}{ccccccc} 2 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{array}\right] & \begin{array}{c} p6 \\ p8 \\ p16 \\ p18 \end{array} \end{array}$$

$$L_{PI40} = \begin{array}{c} \begin{array}{cccc} p6 & p8 & p16 & p18 \end{array} \\ \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \end{array}\right] \end{array}$$

$$D_{C40} = -\mathrm{L}_{\mathrm{PI40}}.\mathrm{D}_{\mathrm{PI40}} = -\left[\begin{array}{cccc} 1 & 1 & 1 & 1 \end{array}\right] \left[\begin{array}{ccccccc} 2 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{array}\right]$$

$$D_{C40} = \begin{array}{c} \begin{array}{ccccccc} t5 & t6 & t8 & t14 & t15 & t16 & t17 \end{array} \\ \left[\begin{array}{ccccccc} -2 & -1 & 3 & -2 & 2 & -2 & 2 \end{array}\right] \end{array}$$

$\mu_{0(C40)} = 10$

$$D_{PI41} = \begin{array}{c c} & \begin{array}{ccccccccc} t5 & t6 & t8 & t9 & t11 & t13 & t14 & t15 & t16 \end{array} \\ & \left[\begin{array}{ccccccccc} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{array}\right] & \begin{array}{c} p6 \\ p8 \\ p11 \\ p15 \\ p16 \\ p17 \end{array} \end{array}$$

$$\mathrm{L}_{\mathrm{PI41}} = \begin{array}{c} \begin{array}{cccccc} p6 & p8 & p11 & p15 & p16 & p17 \end{array} \\ \left[\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right] \end{array}$$

$$D_{C41} = -L_{PI41}.D_{PI41} = -\left[\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right] \left[\begin{array}{ccccccccc} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{array}\right]$$

$$\mathrm{D}_{\mathrm{C41}} = \begin{array}{c} \begin{array}{ccccccccc} t5 & t6 & t8 & t9 & t11 & t13 & t14 & t15 & t16 \end{array} \\ \left[\begin{array}{ccccccccc} -2 & -1 & 3 & -3 & 3 & -3 & 1 & 0 & 2 \end{array}\right] \end{array}$$

$\mu_{0(C41)} = 14$

$$D_{PI42} = \begin{array}{cc} \begin{array}{cccccccc} t5 & t6 & t7 & t8 & t9 & t14 & t15 & t16 \end{array} & \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} & \begin{array}{c} p6 \\ p8 \\ p9 \\ p16 \\ p17 \end{array} \end{array}$$

$$L_{PI42} = \begin{array}{c} \begin{array}{ccccc} p6 & p8 & p9 & p16 & p17 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C42} = -L_{PI42}.D_{PI42} = -\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C42} = \begin{array}{c} \begin{array}{cccccccc} t5 & t6 & t7 & t8 & t9 & t14 & t15 & t16 \end{array} \\ \begin{bmatrix} -2 & -1 & -3 & 3 & 3 & -2 & 0 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C42)} = 13$

$$D_{PI43} = \begin{array}{cc} \begin{array}{cccccccccc} t5 & t6 & t7 & t8 & t9 & t13 & t14 & t15 & t16 & t17 \end{array} & \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} & \begin{array}{c} p6 \\ p8 \\ p9 \\ p15 \\ p16 \\ p18 \end{array} \end{array}$$

$$L_{PI43} = \begin{array}{c} \begin{array}{cccccc} p6 & p8 & p9 & p15 & p16 & p18 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C43} = -L_{PI43}.D_{PI43} =$$

$$-\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C43} = \begin{array}{c} \begin{array}{cccccccccc} t5 & t6 & t7 & t8 & t9 & t13 & t14 & t15 & t16 & t17 \end{array} \\ \begin{bmatrix} -2 & -1 & -3 & 3 & 3 & -3 & 1 & 2 & -2 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C43)} = 14$

$$D_{PI44} = \begin{array}{cc} \begin{array}{ccccccccc} t5 & t6 & t7 & t8 & t9 & t13 & t14 & t15 & t16 \end{array} & \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} & \begin{array}{c} p6 \\ p8 \\ p9 \\ p15 \\ p16 \\ p17 \end{array} \end{array}$$

.

$$L_{PI44} = \begin{array}{c} \begin{array}{cccccc} p6 & p8 & p9 & p15 & p16 & p17 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C44} = -L_{PI44}.D_{PI44} =$$

$$-\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C44} = \begin{array}{c} \begin{array}{ccccccccc} t5 & t6 & t7 & t8 & t9 & t13 & t14 & t15 & t16 \end{array} \\ \begin{bmatrix} -2 & -1 & -3 & 3 & 3 & -3 & 1 & 0 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C44)} = 14$

$$D_{PI45} = \begin{array}{cc} \begin{array}{cccccc} t5 & t6 & t8 & t14 & t15 & t16 \end{array} & \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} & \begin{array}{c} p6 \\ p8 \\ p16 \\ p17 \end{array} \end{array}$$

$$L_{PI45} = \begin{array}{c} \begin{array}{cccc} p6 & p8 & p16 & p17 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C45} = -L_{PI45}.D_{PI45} = -\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C45} = \begin{array}{c} \begin{array}{cccccc} t5 & t6 & t8 & t14 & t15 & t16 \end{array} \\ \begin{bmatrix} -2 & -1 & 3 & -2 & 0 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C45)} = 10$

$$D_{PI46} = \begin{array}{c} \begin{array}{cccccccc} t5 & t6 & t8 & t13 & t14 & t15 & t16 & t17 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{c} \\ p6 \\ p8 \\ p15 \\ p16 \\ p18 \end{array}$$

$$L_{PI46} = \begin{array}{c} \begin{array}{ccccc} p6 & p8 & p15 & p16 & p18 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C46} = -L_{PI46}.D_{PI46} = -\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C46} = \begin{array}{c} \begin{array}{cccccccc} t5 & t6 & t8 & t13 & t14 & t15 & t16 & t17 \end{array} \\ \begin{bmatrix} -2 & -1 & 3 & -3 & 1 & 2 & -2 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C6)} = 11$

$$D_{PI47} = \begin{array}{c} \begin{array}{ccccccc} t5 & t6 & t8 & t13 & t14 & t15 & t16 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{c} \\ p6 \\ p8 \\ p15 \\ p16 \\ p17 \end{array}$$

$$L_{PI47} = \begin{array}{c} \begin{array}{ccccc} p6 & p8 & p15 & p16 & p17 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_{C47} = -L_{PI47}.D_{PI47} = -\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

$$D_{C47} = \begin{array}{c} \begin{array}{ccccccc} t5 & t6 & t8 & t13 & t14 & t15 & t16 \end{array} \\ \begin{bmatrix} -2 & -1 & 3 & -3 & 1 & 0 & 2 \end{bmatrix} \end{array}$$

$\mu_{0(C47)} = 11$

The computed monitors are shown in Table 4.6.

Table 4.6 Computed monitors for N_5.

C_i	$^{\bullet}c_I$	$c_I^{\bullet}$	$\mu_{0(ci)}$
C_{36}	3t8, 3t11, 2t15, 2t17	2t5, t6, 3t9, 2t14, 2t16	13
C_{37}	3t8, 3t9, 2t15, 2t17	2t5, t6, 3t7, 2t14, 2t16	13
C_{38}	3t8, 3t11, 2t16	2t5, t6, 3t9, 2t14	13
C_{39}	3t8, 3t11, t14, 2t15 2t17	2t5, t6, 3t9, 3t13, 2t16	14
C_{40}	3t8, 2t15, 2t17	2t5, t6, 2t14, 2t16	10
C_{41}	3t8, 3t11, t14, 2t16	2t5, t6, 3t9, 3t13	14
C_{42}	3t8, 3t9, 2t16	2t5, t6, 3t7, 2t14	13
C_{43}	3t8, 3t9, t14, 2t15 2t17	2t5, t6, 3t7, 3t13, 2t16	14
C_{44}	3t8, 3t9, t14, 2t16	2t5, t6, 3t7, 3t13	14
C_{45}	3t8, 2t16	2t5, t6, 2t14	10
C_{46}	3t8, t14, 2t15, 2t17	2t5, t6, 3t13, 2t16	11
C_{47}	3t8, t14, 2t16	2t5, t6, 3t13	11

Step 3.5.4. Redundancy test is carried out on the monitors and it is found that 4 monitors, C_{40}, C_{45}, C_{46}, and C_{47}, are necessary.

Step 3.5.5. When the computed necessary monitors are augmented in the uncontrolled model N_5, the controlled model of N_5 is obtained as follows: $N_5 := N_5 + C_{40} + C_{45} + C_{46} + C_{47}$, and is shown in Fig. 4.8.

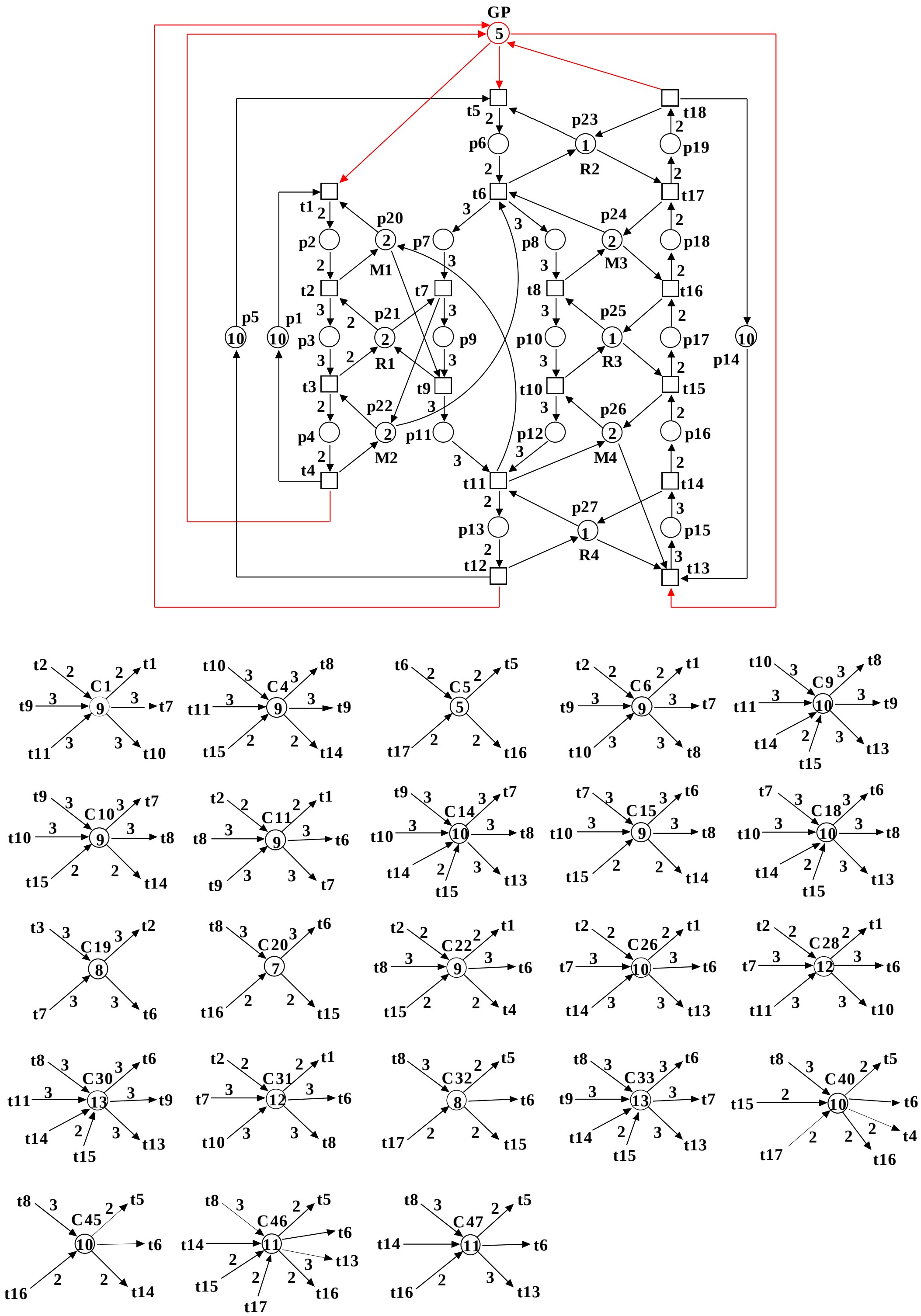

Figure 4.8. The controlled model N_5 (N_5: = N_5 + C_{40} + C_{45} + C_{46} + C_{47}).

It is verified that the controlled model of N_5 shown in Fig. 4.8 is live with 5053 good states. This is the live optimal behavior for the controlled model N_5.

Step 3.6.1. The net N_6 considered in this step is shown in Fig. 4.9. It is obtained by increasing the number of tokens in GP shown in Fig. 4.8.

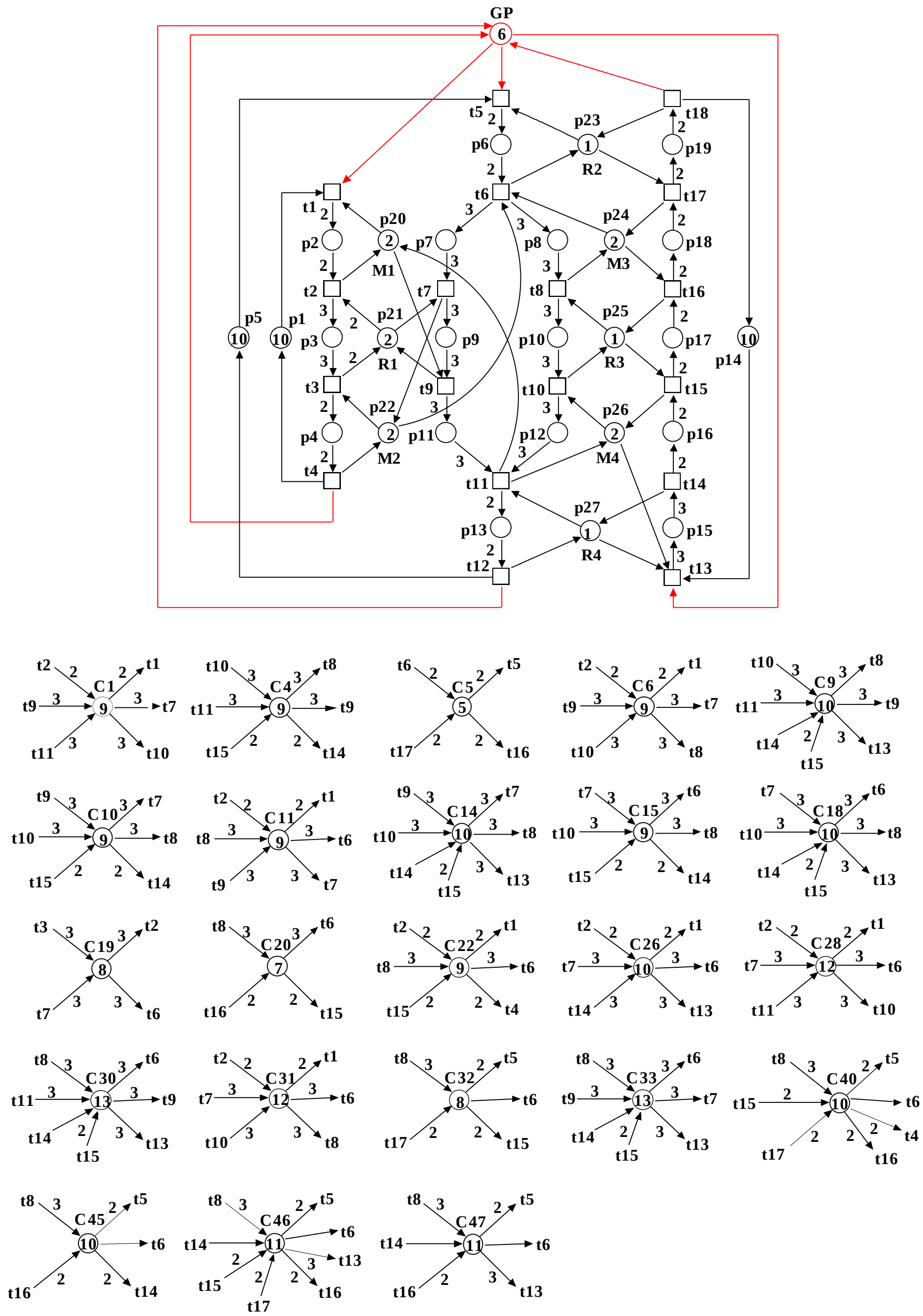

Figure 4.9. The net N_6 (WAMG net).

The net N_6 is live with 7386 good states.

Step 3.7.1. The net N_7 considered in this step is shown in Fig. 4.10. It is obtained by increasing the number of tokens in GP shown in Fig. 4.9.

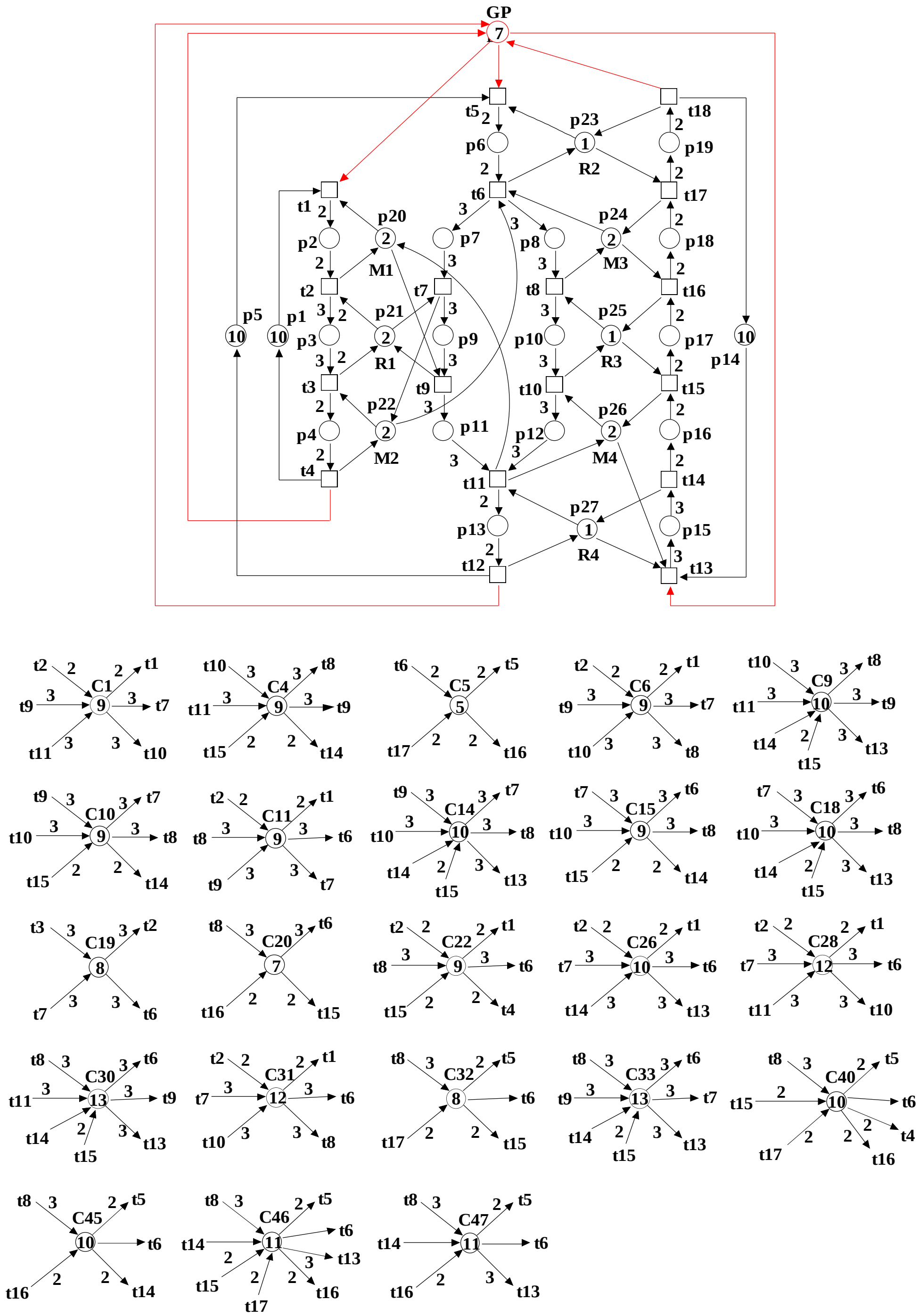

Figure 4.10. The net N_7 (WAMG net).

The net N_7 is live with 8836 good states. The net N_8 (with GP = 8) has 9461 good states. The net N_9 (with GP = 9) has 9643. N_{10} (with GP = 10) has 9676. N_{11} (with GP = 11) has 9679 and N_{12} (with GP = 12) has 9679. The nets N_{13}, N_{14}, . . . , N_{20} with GPs having 13, 14, . . , 20 tokens respectively are all live with 9679 good states. This shows the maximum number of good states that can be reachable for WAMG model using this method.

Step 4: The live controlled WAMG PNM shown in Fig. 4.11 is obtained by augmenting all the 23 necessary monitors provided in Table 4.7 into the uncontrolled WAMG model shown in Fig. 4.1. The net is live with 9679 good states. This is the live behavior for the WAMG PNM using the proposed method.

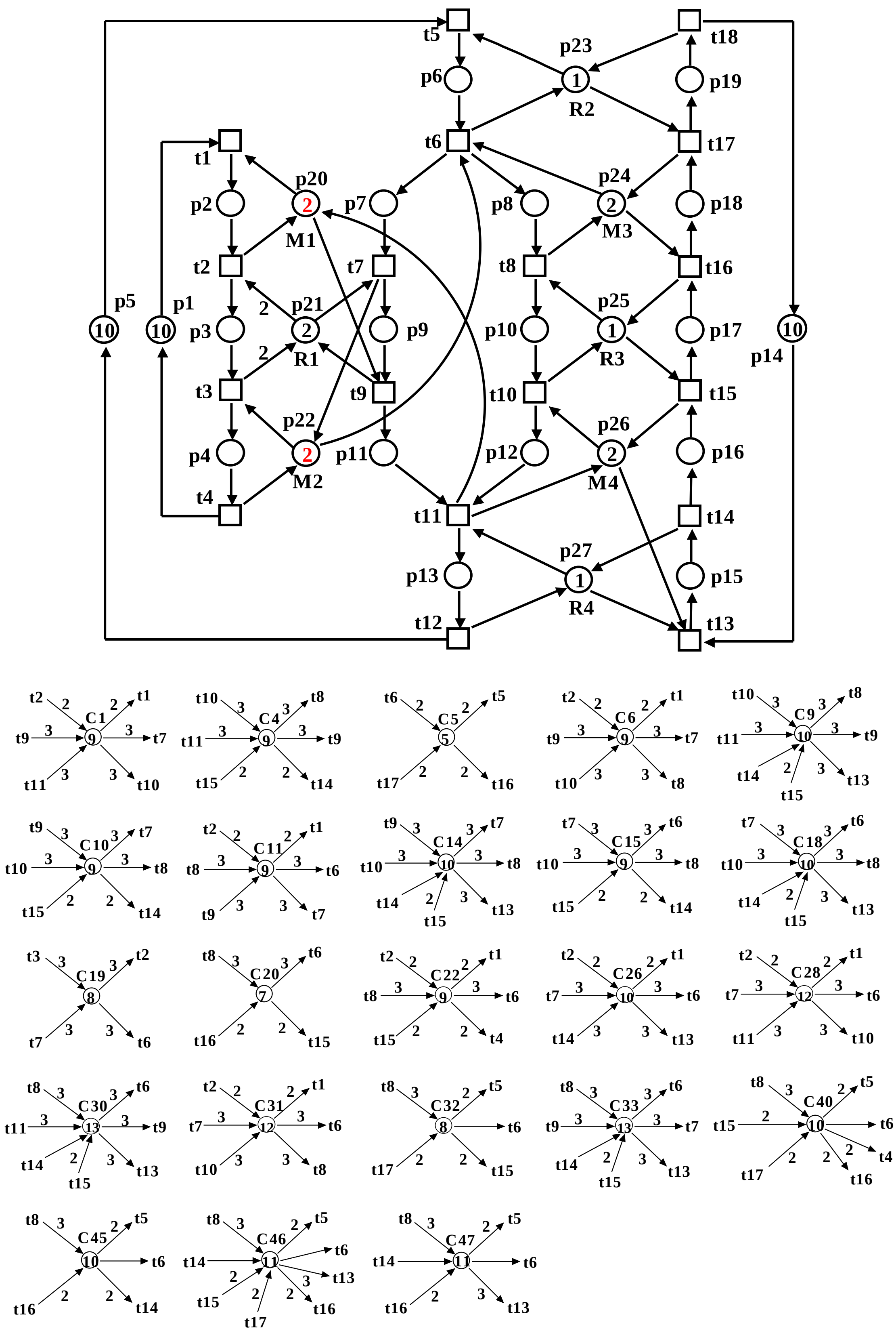

Figure 4.11. The controlled WAMG model.

Table 4.**7** the necessary monitors computed for WAMG model.

C_i	${}^\bullet c_I$	$c_I{}^\bullet$	$\mu_{0(ci)}$
C_1	2t2, 3t9, 3t11	2t1, 3t7, 3t10	9
C_4	3t10, 3t11, 2t15	3t8, 3t9, 2t14	9
C_5	2t6, 2t17	2t5, 2t16	5
C_6	2t2, 3t9, 3t10	2t1, 3t7, 3t8	9
C_9	3t10, 3t11, t14, 2t15	3t8, 3t9, 3t13	10
C_{10}	3t9, 3t10, 2t15	3t7, 3t8, 2t14	9
C_{11}	2t2, 3t8, 3t9	2t1, 3t6, 3t7	9
C_{14}	3t9, 3t10, t14, 2t15	3t7, 3t8, 3t13	10
C_{15}	3t7, 3t10, 2t15	3t6, 3t8, 2t14	9
C_{18}	3t7, 3t10, t14, 2t15	3t6, 3t8, 3t13	10
C_{19}	3t3, 3t7	3t2, 3t6	8
C_{20}	3t8, 2t16	3t6, 2t15	7
C_{22}	2t2, 3t8, 2t15	2t1, 3t6, 2t14	9
C_{26}	2t2, 3t7, 3t14	2t1, 3t6, 3t13	10
C_{28}	2t2, 3t7, 3t11	2t1, 3t6, 3t10	12
C_{30}	3t8, 3t11, t14, 2t15	3t6, 3t9, 3t13	13
C_{31}	2t2, 3t7, 3t10	2t1, 3t6, 3t8	12
C_{32}	3t8, 2t17	2t5, t6, 2t15	8
C_{33}	3t8, 3t9, t14, 2t15	3t6, 3t7, 3t13	13
C_{40}	3t8, 2t15, 2t17	2t5, t6, 2t14, 2t16	10
C_{45}	3t8, 2t16	2t5, t6, 2t14	10
C_{46}	3t8, t14, 2t15, 2t17	2t5, t6, 3t13, 2t16	11
C_{47}	3t8, t14, 2t16	2t5, t6, 3t13	11

The liveness enforcing procedure applied for the PNM is provided in Table 4.8

Table 4.8. The liveness enforcing procedure applied for WAMG model.

B	Included C	Is the net live?	# of states in RG	# of states in DZ	# of states in LZ	Computed C	# of states within controlled net	
							RG = LZ	UR
1	–	YES	20	0	20	–		
2	–	YES	181	0	181	–		
3	–	NO	931	21	910	C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, C_{15}, C_{16}, C_{17}, C_{18}, C_{19}, C_{20}, C_{21},	847	63

Table 4.8 continue.

						Necessary (C_1, C_4, C_5, C_6, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, C_{15}, C_{18}, C_{19}, C_{20})		
4	C_1, C_4, C_5, C_6, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, C_{15}, C_{18}, C_{19}, C_{20}	NO	2495	14	2481	C_{22}, C_{23}, C_{24}, C_{25}, C_{26}, C_{27}, C_{28}, C_{29}, C_{30}, C_{31}, C_{32}, C_{33}, C_{34}, C_{35}. Necessary (C_{22}, C_{26}, C_{28}, C_{30}, C_{31}, C_{32}, C_{33})	2554	0
5	C_1, C_4, C_5, C_6, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, C_{15}, C_{18}, C_{19}, C_{20}, C_{22}, C_{26}, C_{28}, C_{30}, C_{31}, C_{32}, C_{33}	NO	5065	12	5053	C_{36}, C_{37}, C_{38}, C_{39}, C_{40}, C_{41}, C_{42}, C_{43}, C_{44}, C_{45}, C_{46}, C_{47}, Necessary (C_{40}, C_{45}, C_{46}, C_{47})	5053	0
6	C_1, C_4, C_5, C_6, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, C_{15}, C_{18}, C_{19}, C_{20}, C_{22}, C_{26}, C_{28}, C_{30}, C_{31}, C_{32}, C_{33}, C_{40}, C_{45}, C_{46}, C_{47}	YES	7386	0	7386	–		
7	‖	YES	8836	0	8836	–		
8	‖	YES	9461	0	9461	–		
9	‖	YES	9643	0	9643	–		
10	‖	YES	9676	0	9676	–		
11	‖	YES	9679	0	9679	–		
12	‖	YES	9679	0	9679	–		
13	‖	YES	9679	0	9679	–		
14	‖	YES	9679	0	9679	–		
15	‖	YES	9679	0	9679	–		
16	‖	YES	9679	0	9679	–		
17	‖	YES	9679	0	9679	–		
18	‖	YES	9679	0	9679	–		
19	‖	YES	9679	0	9679	–		
20	‖	YES	9679	0	9679	–		

The controlled model of the WAMG net using this method is live with 9679 good states. There are 1733 unreachable states which should have been provided by an optimal live behavior of the WAMG model. The permissiveness of controlled net is (9679/11412) × 100 = 84.81%.

4.3 S^{4}PR NET EXAMPLE

An S^{4}PR model is considered in this section in order to show the applicability of the proposed liveness-enforcing method. Fig. 4.12 shows an S^{4}PR model of an FMS from [17]. This model is prone to deadlocks. There are 9378 states within the RG of these PNM, 546 of these states are in the DZ, while the remaining 8832 states are in the LZ.

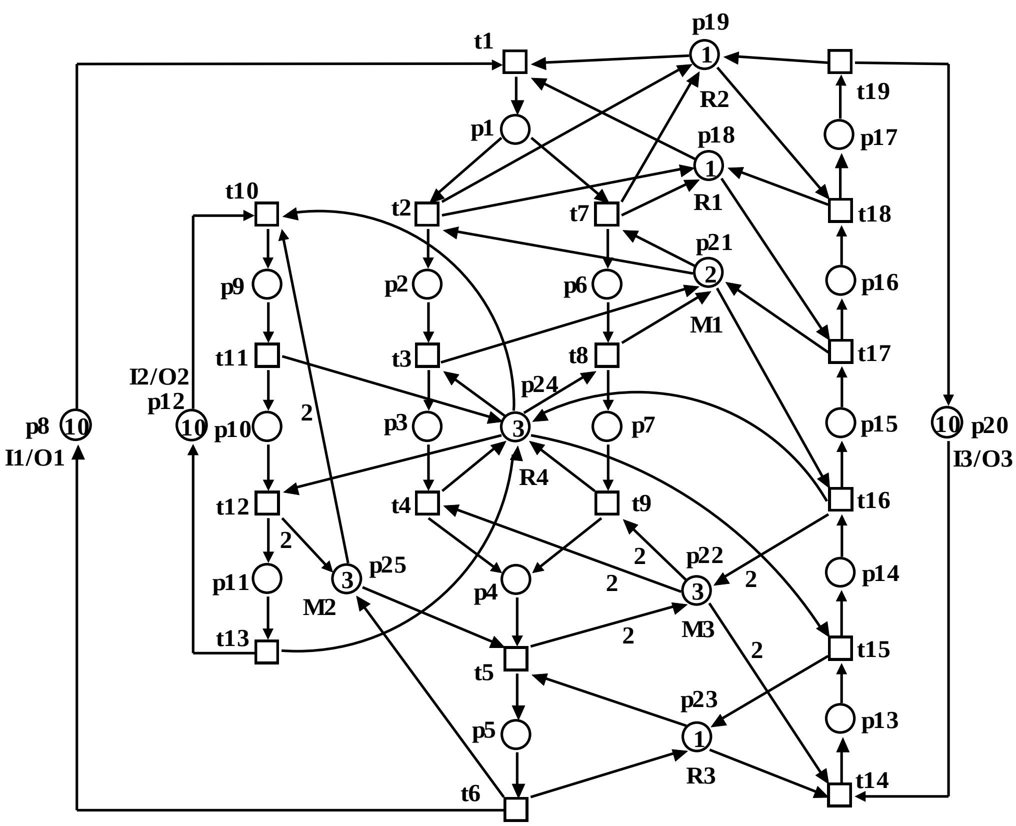

Figure 4.12. A Petri net model of an S^{4}PR net from [17].

The proposed method is applied to the S^{4}PR model shown in Fig. 4.12. The controlled model of the S^{4}PR net is obtained by augmenting 8 necessary monitors that are computed following the steps provided in the proposed method. Table 4.9 shows the liveness enforcing procedure applied to the net and Table 4.10 shows the necessary monitors computed for the S^4PR model respectively.

Table 4.9. The liveness enforcing procedure applied for the S^4PR net.

B	Included C	Is the net live?	# of states in RG	# of states in DZ	# of states in LZ	Computed C	# of states within controlled net	
							RG = LZ	UR
1	–	YES	16	0	16	–		
2	–	YES	119	0	119	–		
3	–	NO	551	1	550	C_1	550	0
4	C_1	NO	1750	4	1746	C_2, C_3, C_4, C_5	1746	0
5	C_2, C_3	NO	4002	18	3984	C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, C_{15}, C_{16}, C_{17}, C_{18}, C_{19}, C_{20}, C_{21}, C_{22}, C_{23} Necessary (C_{11}, C_{20})	3984	0
6	C_1, C_3, C_{11}, C_{20}	NO	6609	12	6597	C_{24}, C_{25}, C_{26}, C_{27}, C_{28}, C_{29}, C_{30}, C_{31}, C_{32}, C_{33}, C_{34}, C_{35} Necessary (C_{27}, C_{28}, C_{33}, C_{34})	6597	0
7	C_1, C_3, C_{11}, C_{20}, C_{27}, C_{28}, C_{33}, C_{34}	YES	8269	0	8269	–		
8	C_1, C_3, C_{11}, C_{20}, C_{27}, C_{28}, C_{33}, C_{34}	YES	8776	0	8776	–		
9	‖	YES	8832	0	8832	–		
10	‖	YES	8832	0	8832	–		
.	.	.	.	.	.	.		
.	.	.	.	.	.	.		
15	‖	YES	8832	0	8832	–		

Table 4.10. Necessary monitors for the S^{4}PR net.

C_i	${}^{\bullet}c_I$	$c_I{}^{\bullet}$	$\mu_{0(ci)}$
C_1	3t2, 3t7, 2t17	3t1, 2t16	6
C_2	2t4, 2t9, 4t15	2t3, 2t8, 4t14	9
C_3	2t4, 2t9, 4t16	2t2, 2t7, 4t15	11
C_4	2t4, 2t9, 4t15	2t2, 2t7, 4t14	11
C_5	t2, 2t4, 3t7, 2t16, 2t17, 2t9	3t1, 2t8, 4t15	14
C_6	3t2, 2t4, t7, 2t9, 2t16, 2t17	3t1, 2t3, 4t15	14
C_7	t2, 2t4, 3t7, 2t9, 4t15, 2t17	3t1, 2t8, 4t14, 2t16	14
C_8	3t2, 2t4, t7, 2t9, 4t15, 2t17	3t1, 2t3, 4t14, 2t16	14

The controlled S^{4}PR PNM in this example is live with 8832 good states. The permissiveness of the controlled net is (8832/8832) × 100 = 100%. This is the optimal live behavior of the S^{4}PR model in this example obtained by using the proposed method.

4.4 G-SYSTEM NET EXAMPLE

Fig. 4.13 shows a G-System net example from [18]. The model is prone to deadlocks.

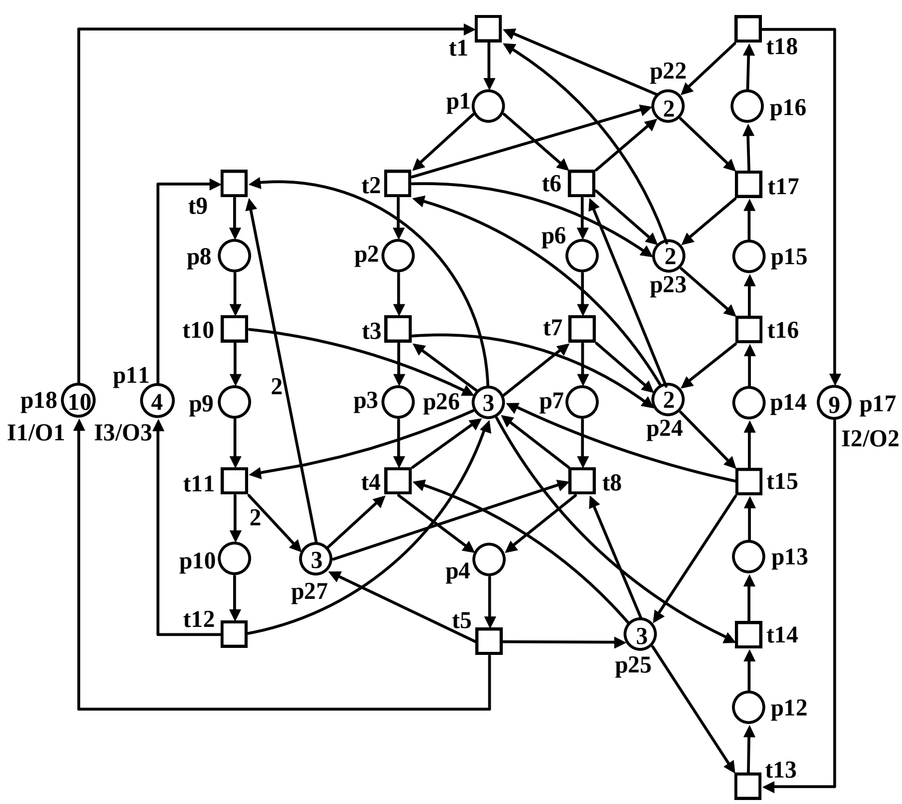

Figure 4.13. A G-System net example from [18].

There are 68531 states within the RG of this model, of which 2131 are bad states. The optimal solution should provide a live behavior with 66400 good states. The controlled model of the G-System net is obtained by augmenting 17 necessary monitors that are computed following the steps provided in the proposed method. Table 4.11 shows the liveness enforcing procedure applied to the G-System net and Table 4.12 shows the necessary monitors computed for the G-system net respectively.

Table 4.11. The liveness enforcing procedure applied for the G-system net.

B	Included C	Is the net live?	# of states in RG	# of states in DZ	# of states in LZ	Computed C	# of states within controlled net	
							RG = LZ	UR
1	–	YES	15	0	15	–		
2	–	YES	117	0	117	–		
3	–	YES	618	0	618	–		
4	–	NO	2398	1	2397	C_1	2397	0
5	C_1	NO	7138	3	7135	C_2, C_3, C_4 Necessary (C_2, C_4)	7135	0
6	C_1, C_3	NO	16645	10	16635	C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14} Necessary (C_6, C_{10}, C_{12})	16635	0
7	C_1, C_3, C_6, C_{10}, C_{12}	NO	30881	8	30873	C_{15}, C_{16}, C_{17}, C_{18}, C_{19}, C_{20}, C_{21}, C_{22} Necessary (C_{16}, C_{18}, C_{19}, C_{21},)	30867	6
8	C_1, C_3, C_6, C_{10}, C_{12}, C_{16}, C_{18}, C_{19}, C_{21}	NO	46399	4	46395	C_{23}, C_{24}, C_{25}, C_{26}	46395	0
9	C_1, C_3, C_6, C_{10}, C_{12}, C_{16}, C_{18}, C_{19}, C_{21}, C_{23}, C_{24}, C_{25}, C_{26}	NO	58258	4	58254	C_{27}, C_{28}, C_{29}, C_{30},	58250	4

Table 4.11 continue.

10	C_1, C_3, C_6, C_{10}, C_{12}, C_{16}, C_{18}, C_{19}, C_{21}, C_{23}, C_{24}, C_{25}, C_{26}, C_{27}, C_{28}, C_{29}, C_{30},	YES	64077	0	64077	–		
11	||	YES	65681	0	65681	–		
12	||	YES	65888	0	65888	–		
13	||	YES	65888	0	65888	–		
14	||	YES	65888	0	65888	–		
15	||	YES	65888	0	65888	–		

Table 4.12 Necessary monitors for the G-system net.

C_i	${}^{\bullet}c_i$	$c_i{}^{\bullet}$	$\mu_{0(ci)}$
C_1	3t2, 3t6, 2t16	3t1, 2t15	9
C_2	2t3, 2t7, 3t15	2t2, 2t6, 3t14	12
C_3	2t3, 2t8, 3t15	2t2, 2t6, 2t13, t14	13
C_4	2t4, 2t8, 2t14	2t3, 2t7, 2t13	11
C_5	2t4, 2t7, 3t15	2t2, 2t6, 2t13, t14	13
C_6	2t4, 2t8, 3t15	2t2, 2t6, 2t13, t14	14
C_7	t2, 2t3, 3t6, t15, 2t16	3t1, 3t14	18
C_8	3t2, t6, 2t7, t15, 2t16	3t1, 3t14	18
C_9	2t4, 2t8, 2t14	2t2, 2t6, 2t13	13
C_{10}	t2, 2t3, 3t6, 2t8, t15, 2t16	3t1, 2t7, 2t13, t14	19
C_{11}	3t2, 2t4, t6, 2t7, t15, 2t16	3t1, 2t3, 2t13, t14	19
C_{12}	t2, 2t4, 3t6, t15, 2t16	3t1, 2t13, t14	19
C_{13}	3t2, t6, 2t8, t15, 2t16	3t1, 2t13, t14	19
C_{14}	t2, 2t4, 3t6, 2t8, t15, 2t16	3t1, 2t13, t14	20
C_{15}	3t2, 2t4, t6, 2t8, t15, 2t16	3t1, 2t3, 2t13, t14	20
C_{16}	t2, 2t4, 3t6, 2t8, 2t14, 2t16	3t1, 2t7, 2t13, 2t15	19
C_{17}	3t2, 2t4, t6, 2t8, 2t14, 2t16	3t1, 2t3, 2t13, 2t15	19

The controlled G-System net in this example is live with 65888 good states. There 512 unreachable states which should have been provided by an optimal live behavior. The permissiveness of the controlled net is (65888/66400) × 100 = 99.23%.

4.5 DISCUSSION

The results obtained for the examples given in this Chapter are summarized in Table 4.13.

Table 4.13 Summary of results.

PNM	# of reachable states	# of unreachable states	Permissiveness (%)	# of necessary monitors	Liveness behavior
S^3PR	84	0	100	3	Optimal
WAMG net	9679	1793	84.81	23	Near optimal
S^4PR	8832	0	100	8	Optimal
G-System	65888	512	99.23	17	Near optimal

The performance comparisons of the deadlock control polices for the examples in the literature and the method proposed in this study are shown in Tables 4.14, 4.15, 4.16 and 4.17. It is clear that the proposed policy can lead to a more permissive behavior for liveness-enforcing Petri net supervisor compared with the other supervisors obtained by using other policies except for the G-System net example.

For WAMG PNM in Fig. 4.1, the comparisons are based on the results from [16].

Table 4.14 Performance comparisons for the WAMG net.

Parameters	Control policy of [18]	Control policy of [19]	Control policy of [16]	The proposed method
# monitors added	7	12	6	23
# of reachable states	6834	7683	8428	9679
Permissiveness (%)	59.88	67.32	73.85	84.81

For S^4PR PNM in Fig. 4.12, the comparisons are based on the results from [20].

Table 4.15 Performance comparisons for the S^4PR model.

Parameters	Control policy of [21]	Control policy of [20] (a)	Control policy of [20] (b)	The proposed method
# monitors added	6	2	2	8
# of reachable states	1952	2570	5198	8832
Permissiveness (%)	22.10	29.10	58.85	100

For G-system in Fig. 4.13, the comparisons are based on G-System net in [18] where the sink and source places are removed.

Table 4.16. Performance comparisons for the G-system net.

Parameters	Control policy of [18]	The proposed method
# monitors added	5	17
# of reachable states	11035	65888
Permissiveness (%)	16.62	99.23

CHAPTER 5

CONCLUSIONS

In this thesis, a new method is proposed to obtain an optimal or near-optimal solution for the synthesis of liveness enforcing supervisor in flexible manufacturing systems (FMS) modeled with generalized classes of Petri nets. The applicability of the proposed approach is shown by means of examples from the literature. The proposed method is not restricted to a particular class of Petri nets. It is tested successfully against different generalized classes of Petri nets including S^3PR, S^4PR, WAMG, G-System and other classes of Petri nets currently available in the literature. The proposed method is generally applicable, easy to use and provides very high behavioral permissiveness. The drawback of the resulting control places is that they are all generalized, i.e., they all have weighted arcs.

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