GEOMETRIC ANALYSIS OF HIGH RESOLUTION SPACE IMAGES USING PARAMETRIC APPROACHES CONSIDERING SATELLITE ORBITAL PARAMETERS

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# YÜKSEK ÇÖZÜNÜRLÜKLÜ UYDU GÖRÜNTÜLERİNİN UYDU YÖRÜNGE PARAMETRELERINİ DİKKATE ALAN PARAMETRIK MODELLERLE GEOMETRİK ANALIZİ 

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| ABBREVIATIONS |  |
| :--- | :--- |
|  |  |
| CCD | : Charge-Coupled Device |
| CCRS | : Canada Centre for Remote Sensing |
| CCS | : Camera Coordinate System |
| CICS | : Conventional Inertial Coordinate System |
| CMOS | : Complementary Metal-Oxide Semiconductor |
| CNES | : Centre National d'Ėtudes Spatiales |
| CTCS | : Conventional Terrestrial Coordinate System |
| DEM | : Digital Elevation Model |
| DIMAP | : Digital Image MAP |
| DLT | : Direct Linear Transformation |
| DORIS | : Doppler Orbitography Radiopositioning Integrated by Satellite |
| EOP | : Exterior Orientation Parameter |
| GCP | : Ground Control Point |
| GCS | : Ground Coordinate System |
| GIS | : Geographic Information System |
| GSD | : Ground Sampling Distance |
| GMST | : Greenwich Mean Sidereal Time |
| GNSS | : Global Navigation Satellite System |
| GPS | : Global Positioning System |
| HRG | : High Resolution Geometry |
| HRS | : High Resolution Stereoscopic |
| HRSI | : High Resolution Space Image |
| ICP | : Independent Check Point |
| ICS | : Image Coordinate System |
| IFOV | : Instantaneous Field of View |
| IMU | : Inertial Measurement Unit |
| INS | : Inertial Navigation System |
| IPI | : Institute of Photogrammetry and Geoinformation |
| IRS | : Indian Remote Sensing Satellite |
| LSA | : Least Square Adjustment |
| LOS | : Line-of-Sight |
| MTF | : Modulation Transfer Function |
| NRCS | : Navigation Reference Coordinate System |
| OCS | : Orbital Coordinate System |
| PCS | : Payload Coordinate System |
| PPM | : Piecewise Polynomial Functions |
| PSG | : Pixel Size on Ground |
| RFM | : Rational Function Model |
| RMSE | : Root Mean Square Error |
| SCS | : Scanline Coordinate System |
| SDLT | : Self-Calibratin DLT |
| SPOT | Satellite Pour l'Observation de la Terre |
| SWIR | : Short Wave Infrared |
|  |  |


| VNIR | : Visible and Near Infrared |
| :--- | :--- |
| TDI | : Time Delay and Integration |
| THR | : Very High-Resolution |
| TLS | : Tree-Line Sensor |
| TÜBİTAK | : Türkiye Bilimsel ve Teknik Araştırma Kurumu |
| 2D | : Two Dimensional |
| 3D | : Three Dimensional |

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## GEOMETRIC ANALYSIS OF HIGH RESOLUTION SPACE IMAGES USING PARAMETRIC APPROACHES CONSIDERING SATELLITE ORBITAL PARAMETERS

## SUMMARY

In the last two decades the imaging technology in aerial and space based missions grew up thanks to especially the imaging technology based on linear array sensors and reducing ground sampling distance. Now high resolution space images with about 40 cm resolution are available. This improvement supports developments on geospatial applications of these images. The geometric correction process becomes more important now than in the past for this purpose. This thesis is focused on the geometric analysis of high resolution space images using parametric (rigorous) approaches ignoring non-parametric (deterministic) ones. Parametric approaches consider imaging geometry, orbital and attitude parameters of satellite, and define the real geometry between image and ground on contrary to the non-parametric ones.

The analysed images are only one set of stereo SPOT-5 HRG level 1A images acquired by linear array sensors. So this technology is subjected following brief information about the sensors for data acquisition. Than the existing parametric approaches for the geometric correction of distortions and influences are summarized following definition of the distortions and influences on linear array images.
The generic model which establishes the geometric relationship between image and ground coordinate systems is defined at first (Weser et al., 2008). Then the modification and simplification to generic model are explained taking into account the image characteristics (SPOT Image, 2002; Fotev et al., 2005). The ground coordinates and their accuracies are attained by adjustment process requiring both pre- and bundle adjustment stages.
The test field covering Zonguldak (Turkey), specifications of SPOT-5 HRG level 1A image, brief information about SPOT-5 satellite, and auxiliary data used are presented before the section of MATLAB programming which includes the workflow of the programme GeoSpot-1.0 developed by the author.

The most important issue for the purpose of estimating the true ground coordinate via the stereo images is adjusting the interior orientation components, i.e. look angles to be corrected in the pre-adjustment process, in this thesis. However the effects of exterior orientation parameters on the accuracy evaluation have to be investigated establishing various sets of them. The adjustment requires selection of suitable set of these parameters. The results of geometric analysis are presented with the help of graphical figures and the tables at the end of this thesis. The section Conclusion contains the general overview and comments on the thesis and its results.
Some auxiliary explanations are given in the Appendices.

## YÜKSEK ÇÖZÜNÜRLÜKLÜ UYDU GÖRÜNTÜLERİNİN UYDU YÖRÜNGE PARAMETRELERİNİ DİKKATE ALAN PARAMETRİK MODELLERLE GEOMETRİK ANALİZİ

## ÖZET

Son yirmi yılda doğrusal dizi algılama teknolojisi sayesinde hava ve uzay bazlı görüntüleme teknolojileri önemli ölçüde gelişmiş ve yer örnekleme aralığı küçültmüştür. Günümüzde yaklaşık 40 cm çözünürlüğe sahip uydu görüntüleri elde edilebilmektedir. Bu gelişme bu görüntüler yardımıyla konumsal uygulamaların gelişimini de desteklemiştir. Bu nedenle geometrik düzeltme işlemi geçmişe nazaran daha önemli hale gelmiştir. Bu tez çalışması parametrik olmayan yaklaşımları göz ardı ederek yüksek çözünürlüklü uydu görüntülerinin parametrik modellerle analizini amaçlamaktadır. Parametrik modeller, parametrik olmayanların aksine görüntüleme geometrisini ve uydunun yörünge ve durum parametrelerini dikkate alır ve görüntü ve yeryüzü arasındaki gerçek geometrik ilişkiyi tanımlar.

Analizi yapılan görüntüler doğrusal dizi algılayıcılar tarafindan algılanan bir çift stereo SPOT-5 HRG düzey 1A görüntüsüdür. Bu nedenle veri elde etmek için kullanılan algılayıcılardan bahsedildikten sonra doğrusal dizi algılama teknolojisi hakkında bilgi verilmiştir. Doğrusal dizi görüntülerinin taşıdıkları geometrik bozulma ve etkiler açıklandıktan sonra bunların düzeltilmesi için kullanılan mevcut parametrik modeller tanitılmıştır.
Öncelikle, görüntü ve yer koordinat sistemleri arasındaki geometrik ilişkiyi tanımlayan genel bir model tanımlanmıştır (Weser et al., 2008). Sonrasında kullanılan görüntünün özelliklerini göz önüne alarak genel model değiştirilmiş ve basitleştirilmiştir (SPOT Image, 2002; Fotev et al., 2005). Yer koordinatları ve bunların doğruluğu, öncül ve demet dengeleme adımlarının her ikisine gerek duyan dengeleme işlemiyle elde edilmektedir ve her biri ayrı bölümlerde açıklanmıştır.
Zonguldak'ı kapsayan test alanı, SPOT-5 HRG düzey 1A görüntüsünün özellikleri, SPOT-5 uydusu hakkında özet bilgi, kullanılan ek veriler ve sonrasında yazar tarafından MATLAB ortamında geliştirilen GeoSpot-1.0 yazılımının akış şeması anlatılmıştır.

Bu tezde stereo görüntüler yardımıyla doğru görüntü koordinatlarının elde edilmesi sürecinde en önemli unsur ön dengeleme ile düzeltilen iç yöneltme bileşenleri yani bakış açılarıdır. Bununla birlikte dış yöneltme elemanlarının doğruluk üzerindeki etkileri de farklı parametre setleri oluşturularak incelenmiştir. Dengelemede bu parametrelerin uygun şekilde seçimi gereklidir. Geometrik analiz sonuçları grafik şekiller ve tablolar yardımıyla tezin sonunda sunulmuştur. Sonuç bölümünde ise tez ve sonuçlar hakkında genel bir değerlendirme yapılmaktadır.

Gerekli bazı açıklamalar Ekler bölümünde verilmiştir.

## 1. INTRODUCTION

The high resolution space images (HRSIs) usually contain geometric distortions and influences and are not directly used in geospatial applications. For this reason, correction of geometric distortions and influences are necessary for the production of corrected image related products to allow these products to be registered, compared, combined etc. pixel by pixel and used in a GIS environment (Toutin, 2003a).

Since the first-decades of remote sensing the geometric correction has become more important for some of the following reasons:

- The geometric resolution is sub-meter level while previously it was coarse (for instance Landsat-1 has around $80-100 \mathrm{~m}$ ground sampling distance (GSD)).
- The images are off-nadir viewing whereas previously were nadir viewing.
- The products are digital while previously were hard-copy resulting from image processing.
- The interpretation of final products is performed on the computer whereas previously it was performed as visually.
- The fusion of multi-source images (from different platforms and sensors) is in general use while previously the fusion and integration of multi-source and multi-format data did not exist in the past.

The process of generating a corrected image is called by various terms, i.e. orthoimage generation, georectification, geolocation, georeferencing, geopositioning, geometric correction or direct or indirect sensor orientation. Nevertheless, the main aim in this process is to establish the geometric relationship between image and ground coordinate systems, and remove the geometric distortions and influences of image, assuming image coordinate as observation. The geometric relationship between these two coordinate systems is generally subject of scale, shift and rotation which are generally assumed as parameters. However, the observations and parameters can be set various depending on the adjustment model.

The geometric distortions and influences of the linear array HRSIs are caused by different sources. The characteristics of linear array sensors and their having geometric distortions and influences are briefly explained following the overview of sensors for data acquisition.

### 1.1 Thesis Objectives

The main objective of this thesis is to perform geometrical analysis of stereo SPOT-5 HRG level 1A images based on a linear array imaging technology. The preferred parametric approach is dedicated for these images considering their imaging geometry, and orbital and attitude parameters of satellite. The analysis consists of three main issues. These are:

1. The effects of interior orientation defined by look angles subjected with and without its pre-adjustment.
2. The effects of exterior orientation parameters (EOPs) with and without preadjustment.
3. The correlation among EOPs.

The following steps are required for the issues mentioned above:

- Define the required auxiliary coordinate systems between image and ground coordinate systems.
- Establish generic parametric model; the geometric relationship between image- and ground-coordinate systems will be determined considering imaging geometry and orbital and attitude parameters of satellite.
- Modify and simplify the parametric model for the specifications of the images used in this thesis.

All computations including pre-processing the auxiliary data, pre-adjustment and bundle adjustment are performed in the programme called GeoSpot-1.0 developed by the author in MATLAB environment. Graphical presentations and tables of results helps to discuss the results.

Many images were available for the purpose of this thesis. Nevertheless, of all images acquired by the various sensors, only the auxiliary information required for
the parametric model to be discussed was provided for SPOT-5 HRG images. Thus, these images with 5 m GSD were preferred in this thesis. However various images with higher geometric resolution than SPOT-5 HRG existed. Here, the term "high resolution" has to be discussed. There is no precise definition on this term in the literature. In spite of the fact that SPOT-5 is three years older than IKONOS (panchromatic images with 1 m GSD), SPOT Image names its camera as High Resolution Geometric (HRG). Moreover, the aimed research is independent than the GSD in this thesis. If the geometric relationship of the sensor is known and the auxiliary data is available, the parametric model can be used for all spaceborne, airborne or terrestrial images.

### 1.2 Thesis Outline

The images investigated in this thesis are based on the linear array sensor technology. The background of this technology is described in the second section. The geospatial applications of HRSIs requires geometric correction of these images. So the source of geometric distortions and influences carried by the images are summarized, and the geometric correction methods are explained including the existing approaches in the third section.

The fourth section consists of the auxiliary coordinate systems required to establish the geometric relationship between image and the ground, the generic and the modified and also simplified model, and the geometric analysis. Finally the fifth and the last section concludes the thesis and its results recalled.

Some of the auxiliary information needed are given in the Appendices.

## 2. LINEAR ARRAY SENSORS AND THEIR GEOMETRY

Since the distortions and influences are partly related to the imaging systems, they have to be characterized by understanding their acquisition geometry. This section explains the characteristics of linear array sensors and their geometry.

### 2.1 Linear Array Sensors

Various sensors for data acquisition are available as shown in Figure 2.1. Firstly, the sensors can be divided as passive and active considering the energy source. The passive sensors obtain the energy which comes from an external source while the active sensors are the source of observed energy themselves. Both passive and active sensors have tasks of imaging or non-imaging (Poli, 2005).


Figure 2.1: Sensors for data acquisition (Poli, 2005).

Linear array sensors, generally used in remote sensing applications, are classified as passive, optical and digital sensors aimed imaging. The Radar sensors, which are active and non-optical sensors and aimed imaging, become mostly preferred systems for the purposes such as observation of cloudy areas, generation digital elevation model (DEM), determination of surface-deformation or monitoring the marine-traffic at midnight etc.

The technologic background of the linear array sensors is presented following brief information related to the data acquisition.

### 2.1.1 Technologic background

Linear array sensors used for imaging purposes depends mainly on charge-coupled device (CCD) technology which was invented by George Smith and Willard Boyle at AT\&T Bell Labs in 1969 (Wikipedia, 2007). The CCD is an imaging sensor, consisting of an integrated circuit containing an array of linked, or coupled, lightsensitive capacitors. Complementary metal-oxide semiconductor (CMOS), as an alternative technology, is invented by Frank Wanlass at Fairchild Semiconductor in 1963 (Wikipedia, 2007). CMOS refers to both a particular style of digital circuitry design, and the family of processes used to implement that circuitry on integrated circuits (chips). The CCD technology is used mostly on helicopters, aircrafts and satellites for Earth observation and also for motion capture, human body modelling and object modelling for medical, industrial, archaeological and architecture applications; on the contrary the CMOS technology in linear arrays is used only for close range applications. Only some airborne frame cameras use CMOS in recent years. The advantages of CCD versus CMOS are its superior image performance (as measured in quantum efficiency and noise) and flexibility at the expense of system size (Poli, 2005). Further technical information of CCD and CMOS technologies are available in various references.

### 2.1.2 Geometry of linear array sensors

Sensor elements or detectors in linear array sensors (i.e. pixels) are arranged along a line in the focal plane (Figure 2.2). The observed object, i.e. Earth surface, is projected on the sensor element, and the image is generated by the charge of the related detector. The construction of a linear array is easier than area arrays, and mechanical scanning is not necessary. For covering more area, the arrays are built as


Figure 2.2: Linear array imaging.
longer or designed as a combination of segments. The existing configurations of line design are (Poli, 2005):

- The pixels are placed in a single line (Figure 2.3 a). SPOT-5 HRG has 12000 elements in a line.
- A line consists of two or more segments (Figure 2.3 b). QuickBird with 27000 elements is a combination of six segments, each of 3000 elements (Liedtke, 2002).


Figure 2.3: Various designs of linear arrays.

- Two segments placed parallel on the longer side are staggered as half-element in both directions (Figure 2.3 c ). SPOT-5 Supermode ( 2.5 m GSD) and OrbView-3 panchromatic ( 1 m GSD) images are generated by staggered linear arrays, each pixel has 5 m and 2 m size on the ground, respectively.
- And the segments are placed with overlaps (Figure 2.3 d ). IRS-1C and ALOS PRISM have three and four segments overlapping on each other, respectively. Various acquisition techniques of linear array sensors are available as explained in the following section.


### 2.2 Acquisition Techniques of Linear Array Sensors

A linear array is instantaneously projected on the object. The integration time depends on velocity of platform, for instance 0.88364 ms for IRS-1C panchromatic band ( 5.8 m GSD) or 0.14285 ms for IKONOS panchromatic band. This short time based on the pushbroom technique is not enough to observe sufficient energy from the object. For this reason, the integration time can be extended by other techniques, such as time delay and integration (TDI) or slow-down techniques; or the projection of pixel on the ground can be enlarged producing the image with halved size by staggered line arrays. The acquisition techniques used for linear array sensors are explained below.

### 2.2.1 Pushbroom technique

The basic of pushbroom technique is, as shown by Figure 2.4, a linear array sensor mounted on a moving platform sweeping out a line on the Earth surface during the motion of platform. The instantaneous view plane is perpendicular to the direction of motion. Each elements of linear array generates the charge as a response to the energy from the projected-pixel on the ground during the integration time. The generated charge as a response to the incoming energy is discharged fast enough to independently collect the energy of neighboured pixels projected on the ground (Gupta and Hartley, 1997).

The pixel size on ground (PSG) in the direction of motion $\left(p s g_{x}\right)$ is defined by velocity of platform $(V)$ and line interval $(\Delta t)$ :

$$
\begin{equation*}
p s g_{x}=V . \Delta t \tag{2.1}
\end{equation*}
$$

and the $p s g_{y}$, perpendicular to the direction of motion, becomes:

$$
\begin{equation*}
p s g_{y}=\frac{p \cdot H}{c} \tag{2.2}
\end{equation*}
$$

where $p s g_{x}$ and $p s g_{y}$ are pixel size on ground in the direction of motion $(x)$ and perpendicular to the direction of motion (y), respectively, $V$ is velocity of platform, $\Delta t$ is line interval, $p$ is pixel size $\left(p=p_{x}=p_{y}\right), H$ is flying height, and $c$ is focal length.


Corresponding image


Figure 2.4: Pushbroom technique (left) and corresponding image (right).

Since the integration time is not sufficient for acquisition of enough energy, for instance $\Delta t$ is 0.88364 ms for IRS-1C panchromatic band, 0.75210 ms for SPOT-5 HRG or 0.14285 ms for IKONOS panchromatic band, an extension of $\Delta t$ is necessary by TDI or slow-down techniques with or without the staggered line arrays.

### 2.2.2 TDI technique

The TDI, in other words drift-scan, is a technique based on the principle of multiple generation of the charge as a response to the energy from the projected-pixel on the ground in N stages. Consequently, the object is observed by not only one, but also N
pixels in a line along the motion (Figure 2.5). The final charge is a sum of charges in the previous stages. IKONOS and QuickBird are equipped by the TDI with 13 stages. Nevertheless, QuickBird is launched at altitude of 450 km instead of its planned altitude of 680 km , in order to reduce the GSD for higher geometric resolution. Consequently QuickBird uses also slow-down technique for increasing the imaging time (Jacobsen, 2005). OrbView-3 is other satellite which uses slowdown technique, but this satellite is equipped by staggered line arrays instead of TDI.


Figure 2.5: TDI technique with three stages (Schöder et al., 2001).

### 2.2.3 Slow-down technique

The slow-down technique is based on the principle that permanently changing the view-direction against direction of motion during imaging. So, the imaging time is increased by the reducing the speed of sensor-track on the ground (Jacobsen, 2005). In Figure 2.6, (a) means distance of unchanged view direction against the orbit, (b) means distance of slow down technique. The b/a is 1.4 for OrbView-3 and 1.6 for QuickBird (Topan et al., 2009).


Figure 2.6: Slow-down technique (Jacobsen, 2005).

### 2.2.4 Staggered line array technique

Staggered line arrays depends on the concept that two line arrays are shifted halfpixel along both row and column directions (Figure 2.7). The process of generation an image by this technique consists in quincunx interpolation, deconvolution and denoising. Quincunx interpolation computes radiometric information over a halved pixel, deconvolution compensates for low modulation transfer function (MTF) values for high spatial frequencies and denoising reduces the noise level enhancement due to deconvolution (Latry and Rouge, 2003). The SPOT-5 Supermode ( 2.5 m GSD) and OrbView-3 panchromatic ( 1 m GSD) images are generated by 5 m and 2 m pixel size on the ground, respectively The relation between pixel and GSD is depicted in the Figure 2.7. As comparison of information content of TDI and staggered line arrays, IKONOS panchromatic image with TDI technique has sharper edges than OrbView-3 with staggered line arrays (Topan et al., 2006).


Figure 2.7: Staggered CCD lines (left) and relation of pixel and GSD (right).

### 2.3 Geometric Distortions and Influences of Linear Array HRSIs

The linear array HRSIs have significant geometric distortions denying the use as map base products. The distortions are sourced by acquisition system (i.e. imaging sensor, platform, incidence angle etc.), curvature and rotation of Earth and topographic relief (Toutin, 2003a). These various sources of distortions are explained in the following sections.

### 2.3.1 Geometric distortions of linear array sensors

The images used in this thesis are acquired by linear array sensors with various designs as explained by Figure 2.3 in Section 2.1.2. The distortions of linear array sensor are related to these designs. As general, the distortions corrected by suitable functions are:

- Change of pixel dimension,
- Shift or rotation of the segments in focal plane,
- Line bending or curvature.

In the case of one segment linear array sensor, the geometric distortions are explained in the following:

- The change of pixel dimension affects the image scale (Figure 2.8 a). The error in $y$ direction is highly correlated to the focal length variation, the radial distortion and the scale factor in $y$-direction (Poli, 2005).
- Shift in $x$ - and $y$-directions are possible as depicted in Figure 2.8 b .
- A horizontal rotation in the focal plane is available (Figure 2.8 c ).
- Line bending or curvature distortion is exhibited in focal plane (Figure 2.8 d ).

a) Effects of pixel size change in $x$ - and $y$-directions.

b) Shift in $x$-(above) and in $y$-(below) directions.

c) Horizontal rotation of line array sensor in the focal plane.

c) Line bending or curvature distortion in focal plane.

Figure 2.8: Geometric distortions of one segment linear array sensor.

The unique geometric distortions in two segments linear array sensors are:

- Shift in $x$ - and $y$-directions (Figure 2.9 a). One segment is shifted against its nominal position.
- One segment can be horizontal rotated in focal plane (Figure 2.9 b).

a) Shift of a segment in $y$-(above) and $x$ - (below) directions.

b) Rotation of a segment.

Figure 2.9: Unique geometric distortions of two segments linear array sensor.

The three overlapped linear array segments have following unique distortions:

- The shift and rotation of linear arrays in focal plane causes different locations of each segment at the same integration time (Figure 2.10 a).
- The vertical rotation and different focal length of the overlapped segments change scale in $y$-direction as shown by Figure 2.10 b (Jacobsen, 1998).


### 2.3.2 Distortions of optical system

The existing distortions of optical system are:

- The shift of principal point in $x$ - and $y$-directions,
- The change of focal length (c),
- The symmetric lens distortions,
- The decentering lens distortions,
- The scale variation in $x$ - and $y$-directions.

projection on ground
a) Different location of overlapped segments.
b) Possible vertical rotation and different focal length of overlapped segments.

Figure 2.10: Geometric distortions of three overlapped linear array segments.

The lens distortions can be negligible since the field of view of linear array sensors is generally very narrow (Orun, 1990; Yamakawa, 2004).

### 2.3.3 Sensor geometry

The platforms, artificial satellites in this scope, have a constant sun-synchronous orbit. However the sensor geometry is mainly related to orbit and Earth (elliptic movement, variable Earth gravity etc.). Depending on the acquisition time and the size of image, the influences of sensor geometry are:

- Platform altitude variation in combination with sensor focal length, Earth's flatness and topographic relief changes the pixel size on the ground (Figure $2.11 \mathrm{a})$.
- Platform velocity variations change the line spacing or create line gaps/overlaps (Figure 2.11 b).


Figure 2.11: Influences of sensor geometry.

- Small changes of platform position (in $X, Y$ and $Z$ directions) and rotation angles ( $\omega, \varphi$ and $\kappa$ ) changes the orientation and the shape of images (Michalis, 2005) (Figure 2.11 c).


### 2.3.4 Ground pixel size

The sensors have capability of off-nadir viewing in across-track, in along-track or in flexible directions with the help of mirror or reaction wheels (Jacobsen, 2005). Since their angular instantaneous field of view (IFOV) is constant, this off-nadir viewing allows difference of ground pixel size (Figure 2.12). This difference is seen in $y$ direction of across-track images, in $x$-direction of along-track images and in both $x$ and $y$-directions of flexible images. The images have to be corrected as if observed in nadir view.


Figure 2.12: Difference in ground pixel size in along track view.

### 2.3.5 Influence of earth curvature

The Earth curvature causes the influence of pixel size on the ground (Figure 2.13). This influence is seen in $y$-direction of across-track images, in $x$-direction of alongtrack images and in both $x$ - and $y$-directions of flexible images. This influence is more realized in the images covering longer-distance rather than shorter-distance.


Figure 2.13: Influence of Earth curvature.

### 2.3.6 Influence of Earth rotation

During the image acquisition, the Earth rotates from west to east around itself causing an influence on the image. The sensor acquires the object with longitude $\lambda_{1}$ at time $t_{1}$ whereas it takes the object longitude with $\lambda_{2}$ at time $t_{2}$ (Figure 2.14). Magnitude of this influence is based on the relative velocities of satellite and Earth,
and length of the image even (Richards and Jia, 1999). The corrected image is a leftoriented image. Some satellites such as SPOT-5 have yaw steering mode to recover this influence during the acquisition of image (SPOT Image, 2002).


Figure 2.14: Rotation of Earth during nadir acquisition (above) and corresponding corrected image (below).

### 2.3.7 Influence of topographic relief

Since the Earth surface is not flat, the topographic relief causes shift of pixel position (Figure 2.15). This influence is seen in $y$-direction of an across-track image, in $x$ direction of an along-track image and in both $x$ - and $y$-directions of a flexible image.


Figure 2.15: Influence of topographic relief.

Brief information with respect to geometric correction of these distortions and influences are summarized in the following section.

## 3. GEOMETRIC CORRECTION OF LINEAR ARRAY HRSIs USING PARAMETRIC MODELS

### 3.1 Overview of Geometric Correction of Linear Array HRSIs

The aforementioned distortions and influences related to imaging sensor, platform, Earth curvature, Earth rotation and topographic relief are corrected by the suitable functions during pre- or post-launch calibration. The pre-launch calibration is generally performed in laboratory environment for the correction of following distortions:

- Change of pixel dimension,
- Shift or rotation of segments in focal plane,
- Line bending,
- Lens distortions.

The following distortions are corrected by various correction methods explained together in the post-launch calibration:

- Shift and rotation of three segments, can be determined and corrected by the images with different inclination angles of the same area, in addition to the pre-launch calibration. Since IRS-1C panchromatic camera has three overlapped segments, the study by Jacobsen (1999) is performed to shift images of each overlapped segments using tie points. The maximum shift values are 7 pixels in $x$ - and 30 pixels in $y$-direction.
- Change of focal length and vertical rotation of segments in focal plane can be determined thanks to ground control points (GCPs) located in different heights.
- Variations of sensor geometry can be determined by the orbital parameters of platform and GCPs.
- Differences of ground pixel size are corrected by the inclination angle of
sensor.
- Influence of Earth curvature is corrected by the information of sensor's inclination angle and position of platform.
- Influence of Earth rotation is removed by the information of platformposition and period of acquisition time. This effect can be removed by a control system during the acquisition time, such as in SPOT-5 satellite.
- Influence of topographic relief is corrected by imaging geometry, sensor orientation, orbital parameters of platform, GCPs and DEM.

Some of these distortions are corrected with the pre- or post-launch calibration parameters by the firms before distribution of the images. However, GCPs and DEM are required as additional data for the correction of some distortions and influences such as sensor geometry and topographic relief. The end-user has to correct the images using this additional data by the various mathematical models.

The mathematical models used for this purpose can be classified as parametric and non-parametric. The non-parametric models do not reflect the real geometry of HRSIs while parametric models consider imaging geometry and position and attitude parameters of satellite etc. Since this thesis is focused on the parametric models, the non-parametric models are not summarized. However, the principles and potential of these models such as polynomial transformation, affine projection, direct linear transformation (DLT), self-DLT, sensor and terrain dependent Rational Function Model (RFM) etc. are investigated by the researchers such as Zoej (1997), Wang (1999), Toutin (2003a), Topan (2004), Jacobsen et al. (2005), and Topan and Kutoglu (2009).

### 3.2 Parametric Models

The parametric models usually depend on the colinearity equations explained in the following section.

### 3.2.1 Colinearity equations

The geometric relationship between 2 dimensional (2D) image- and 3 dimensional (3D) object-coordinate systems, independently the optical imaging sensor type, can be established by colinearity equations:

$$
\begin{align*}
& x=x_{0}-c \frac{R_{11} \cdot\left(X-X_{0}\right)+R_{12} \cdot\left(Y-Y_{0}\right)+R_{13} \cdot\left(Z-Z_{0}\right)}{R_{31} \cdot\left(X-X_{0}\right)+R_{32} \cdot\left(Y-Y_{0}\right)+R_{33} \cdot\left(Z-Z_{0}\right)} \\
& y=y_{0}-c \frac{R_{21} \cdot\left(X-X_{0}\right)+R_{22} \cdot\left(Y-Y_{0}\right)+R_{23} \cdot\left(Z-Z_{0}\right)}{R_{31} \cdot\left(X-X_{0}\right)+R_{32} \cdot\left(Y-Y_{0}\right)+R_{33} \cdot\left(Z-Z_{0}\right)} \tag{3.1}
\end{align*}
$$

where $x$ and $y$ are image coordinates of GCP, $x_{0}$ and $y_{0}$ are image coordinates of principal point, $X, Y$ and $Z$ are object coordinates of GCP, $X_{0}, Y_{0}$, and $Z_{0}$ are object coordinates of perspective centre, $R$ is rotation matrix, and $c$ is focal length (Kraus, 1993). $x_{0}, y_{0}$ and $c$ are elements of interior orientation where $X_{0}, Y_{0}, Z_{0}$ and elements of $R$ compose exterior orientation.

This equation allows establishing real geometric relationship between 2D image space and 3D object space whereas other models such as affine projection, DLT etc. do not. Rearrangement of this equation for estimate 3D object coordinates from 2D image coordinates is available when the object exists in stereo-images. The $R$ can be formed by rotation angles, i.e. $\omega, \varphi, \kappa$ in classic photogrammetry or by the combination of roll, pitch, yaw ( $a_{r}, a_{p}$ and $a_{y}$ ) and Keplerian elements (inc, $\Omega, f$ and $w_{p}$ ), or unit vectors estimated by position and velocity vectors of the imaging system in satellite photogrammetry.

For the dynamic images, such as acquired by the linear array sensors, the imagecoordinate component along the flight direction $(x)$ is considered as zero (0) (Weser et al., 2008). This coordinate component associates with the imaging time of the related line of image. Contrary, the exterior orientation parameters (EOPs) are equal for one image in film based or digital frame images and are changed with time for each line of image in the case of linear array imaging. The EOPs in the second case are varied considering the real geometric relationship between image- and object coordinate systems. Figure 3.1 illustrates the relationship between image- and object coordinates in the case of aerial photogrammetry with film-based or digital frame image. In this case only one $R$ matrix consisting of $\omega, \varphi$ and $\kappa$ angles and only one set of $X_{0}, Y_{0}, Z_{0}$ is enough for all image. However if the satellite or shuttle is used as the platform, many other auxiliary coordinate systems, time dependent rotation and shifts between them have to be considered. This issue will be detailed in the section 3.3.


Figure 3.1: Relationship between image and ground coordinate systems in aerial photogrammetry with film based or digital frame images.

The image-coordinates ( $x$ and $y$ ) are mostly considered as observations whereas EOPs are mostly the adjustment parameters in the adjustment procedure. A linearization is required since the colinearity equations are non-linear. The initial values of the unknowns can be estimated by GCPs, or directly measured by Global Navigation Satellite Systems (GNSS), Doppler Orbitography Radiopositioning Integrated by Satellite (DORIS) etc. for positioning and Inertial Measurement Unit (IMU) for determination of rotation angles. The elements of interior orientation can be considered as unknowns if the camera calibration is not confidential, and additional parameters can be applied to overcome systematic errors (Jacobsen, 2008).

### 3.2.2 Existing parametric models

The parametric models developed by the researchers such as Guichard and Toutin in 1993 (Toutin, 2003b), Salamonowicz in 1986 (Salamonowicz, 1986), Gugan in 1987 (Gugan, 1987), Konency et al. in 1987 (Konency et al.,1987), Jacobsen in 1988 (Jacobsen, 2005), Kratky in 1989 (Fristsch and Stalman, 2000), Westin in 1990
(Westin, 1990), Orun and Natarajan in 1994 (Orun and Natarajan, 1994), Radhadevi and Ramachandran in 1994 (Zoej, 1997), El-Manadili and Novak in 1996 (ElManadili and Novak, 1996 ), Poli in 2005 (Poli, 2005) and Jung et al. in 2007 (Jung et al., 2007) are summarized in this section. These models are generated for the motivation of rigorous evaluating the images using their special characteristics with respect to their imaging geometry, sensor orientation and satellite orbital parameters. The following approaches are some of the existing models based on the colinearity equations in photogrammetry, but the investigators take differing parameters into account.

### 3.2.2.1 3D CCRS parametric model by Toutin

The 3 dimensional (3D) Canadian Center for Remote Senssing (CCRS) Parametric Model, developed by Toutin since 1983, benefits from theoretical work in celestial mechanics for better determination of the satellite's osculatory orbit and parameters. The model takes into account the following distortions relative to the global geometry of viewing:

- distortions relative to the platform (position, velocity, orientation),
- the distortions relative to the sensor (orientation angles, IFOV, detection signal integration time),
- the distortions relative to the earth (geoid-ellipsoid, including elevation), and
- the deformations relative to the cartographic projection.

The model integrates the following transformations:

- rotation from the sensor reference to the platform reference,
- translation to the Earth's centre,
- rotation which takes into account the platform time variation,
- rotation to align the $z$-axis with the image centre $\left(M_{0}\right)$ on the ellipsoid,
- translation to the image centre $\left(M_{0}\right)$,
- rotation to align the $y$-axis in the meridian plane,
- rotation to have $x M_{0} y$ tangent to the ellipsoid,
- rotation to align the $x$-axis in the image scan direction, and
- rotation-translation into the cartographic projection.

The final equation which link cartographic coordinates to the image coordinates is given as:

$$
\begin{align*}
& P p+y(1+\delta \gamma X)-\tau H-H_{0} \Delta T^{*}=0  \tag{3.2}\\
& X+\theta H / \cos \chi+\alpha q(Q+\theta X-H / \cos \chi)-Q \Delta R=0 \tag{3.3}
\end{align*}
$$

where
$X=(x-a y)\left(1+h / N_{0}\right)+b y^{2}+c x y$
$H=h-X^{2} / 2 N_{0}$
and $H$ is the altitude of the point corrected for Earth curvature, $H_{0}$ is the satellite elevation at the image centre line, $N_{o}$ is the normal to the ellipsoid, $A$ is mainly a function of the rotation of the Earth, $\alpha$ is the instantaneous field-of-view, $p$ and $q$ are the image coordinates, $P$ and $Q$ are the scale factors in $Y$ and $X$, respectively, $\tau$ and $\theta$ are function of the leveling angles in $Y$ and $X$, respectively, $\Delta T^{*}$ and $\Delta R$ are the nonlinear variations in attitude if they exist $\left(\Delta T^{*}\right.$ : combination of pitch and yaw, $\Delta R$ : roll), $x, y$ and $h$ are the ground coordinates, $b, c, \kappa$ and $\delta \gamma$ are $2^{\text {nd }}$-order parameters, which are a function of the total geometry, e.g., satellite, image and Earth.

In the equations above, $p, q$ are the observations, $x, y, h$ are the known parameters. $b$, $c, \kappa, \delta \gamma, N_{o}, H_{0}$ are determined from latitude of the scene centre and the orbital oscillatory parameters. Therefore the basis of this approach amounts to the determination by least squares solution of the five unknowns $P, Q, \tau, \theta$ and $a$ using collinearity equations, and three unknowns of translation and rotation between the local terrain system and the cartographic system. Thus, eight parameters have to be determined (Zoej, 1997).

Each parameter above represents the physical realities of the full viewing geometry (satellite, sensor, Earth, map projection) and each of these parameters is the combination of several correlated variables of the viewing geometry. The combination of several variables is:

- the orientation of the image is a combination of the platform heading due to orbital inclination, the yaw of the platform, the convergence of the meridian,
- the scale factor in along-track direction is a combination of the velocity, the altitude and the pitch of the platform, the detection signal time of the sensor, the component of the Earth rotation in the along-track direction,
- the levelling angle in the across-track direction is a combination of platform roll, the incidence angle, the orientation of the sensor, the Earth curvature etc.

This model applies to visible and infrared (VIR) images (Landsat 5 and 7, SPOT, IRS, ASTER and KOMPSAT), as well as radar images (ERS, JERS, SIR-C and RADARSAT) with three to six GCPs. This model applied to different image types is robust and not sensitive to GCP distribution, as long as there is no extrapolation in planimetry and elevation (Toutin 2003b).

### 3.2.2.2 Model by Salamonowicz

The model developed by Salamonowicz (1986) is not related to linear array HRSIs. However, this model is referenced for the improving of further parametric models. This model aims to reduce the number of required GCPs using the satellite orientation and position parameters. The steps of processing are:

- The sample positions are corrected for the errors in exist because of periodic variations in the scan rate. The corrected sample $\left(\mathrm{SN}_{\text {corr }}\right)$ is calculated.
- The directions angles $\psi$ and $\theta$, in the along- and across-track directions respectively, are computed from line numbers (LN) and sample numbers ( $\mathrm{SN}_{\text {corr }}$ ).
- The component of a rectangular image coordinate $x_{p}, y_{p}$ and $z_{p}$ are computed by $\psi$ and $\theta$.
- The effect caused by the Earth rotation is removed in the longtitude of GCPs.
- The position vector of the satellite (s) is computed.
- The tangential velocity $\left(v_{s}\right)$ is computed.
- The instantaneous satellite geocentric latitude $\left(\Phi_{\mathrm{s}}\right)$, longitude $\left(\lambda_{\mathrm{s}}\right)$ and azimuth $\left(\mathrm{Az}_{\mathrm{s}}\right)$ are computed.
- The rotation matrix (R) is computed by $\Phi_{\mathrm{s}}, \lambda_{\mathrm{s}}$ and $\mathrm{Az}_{\mathrm{s}}$.
- The roll $\left(\omega_{\mathrm{s}}\right)$, picth $\left(\varphi_{\mathrm{s}}\right)$ and yaw $\left(\kappa_{\mathrm{s}}\right)$ for the $i^{\text {th }}$ point is computed by satellite's roll $\left(\omega_{0}\right)$, picth $\left(\varphi_{\mathrm{o}}\right)$ and yaw $\left(\kappa_{\mathrm{o}}\right)$ values and their rates $(\dot{\omega}, \dot{\varphi}$ and $\dot{\chi}$ respectively) considering time delay $\left(\mathrm{t}_{\mathrm{i}}-\mathrm{t}_{\mathrm{l}}\right)$.
- A rotation matrix (M) is computed as a function of $\omega_{\mathrm{s}}, \varphi_{\mathrm{s}}$ and $\kappa_{\mathrm{s}}$.
- The relation between position of GCP and of satellite is determined using R and M .
- The equations $F_{x}$ and $F_{y}$ are defined.
- The corrections to the estimated parameters are computed.


### 3.2.2.3 Model by Gugan

Gugan's model is an approach developed for the orientation of SPOT images using its dynamic orbital parameters (Gugan, 1987). This model is similar to the model developed by Salamonowicz in 1986 (Zoej, 1997). The model is established between an earth centered coordinate system (e.g. geocentric coordinate system) and image coordinate systems to avoid distortions caused by earth curvature and map projection characteristics.

In the model, the inner orientation is different and simpler than in an aerial photogrammetry. The marks are selected as corner pixels of the image. However, in the case of exterior orientation, the elements of exterior orientation are changing during this period since a SPOT panchromatic image is recorded 9 sec . So the image geometry becomes a dynamic and has a cylindrial perspective. This condition does not allow determining six elements of exterior orientation ( $X_{0}, Y_{0}, Z_{0}, \omega, \varphi, \kappa$ ) and small changes on $x$ and $y$ parallax becomes:
$d X_{0}: p x=d x \quad p y=0$
$d \phi: p x=-z d \phi \quad p y=0$
These two elements cannot be distinguished.
The satellite is moving along a well defined orbit and the EOPs of image can be modelled by consideration of the Keplerian orbital parameters. Of six Keplerian parameters, the semi-minor axis of orbital ellipse (b), and the argument of perigee
$(\omega)$ has very little effect on the image geometry considering very low orbit eccentricity (e).

The true anomaly $(F)$ and the ascending node $(\Omega)$ are modelled by linear angular changes with time because these two parameters are affected by two major components of dynamic motion, i.e. the movement of the satellite along the orbit path and the Earth's rotation:
$F=F_{0}+F_{1} x$
$\Omega=\Omega_{0}+\Omega_{1} x$
The sensor's position $\left(X_{s}\right)$ can be found as following:
$X_{s}=R_{0} \cdot D$
$R_{0}=R_{\Omega^{\prime}}, R_{i} \cdot R_{F^{\prime}}$
$\Omega^{\prime}=180^{\circ}-\Omega$
$i^{\prime}=i-90^{\circ}$
$F^{\prime}=90^{\circ}-(F+\omega)$
$D=(0,0, r)^{T}$
$r=a\left(1-e^{2}\right) /(1+e \cos F)$
where $R_{0}$ is rotation between sensor and geocentric coordinate systems, $i$ is orbit inclination, and $a$ is orbit semi-major axis.

The collinearity equation for one line becomes:
$(0, y,-f)^{T}=s R_{0}\left(X_{A}-X_{s}\right)$
where $s$ is scale, $f$ is focal length, $X_{A}$ is $X, Y, Z$ coordinates of GCP $A$.
The additional attitude rotation defined by $R_{A}$ has to be considered due to the orbit perturbations. So the last equation becomes as following:

$$
\begin{equation*}
(0, y,-f)^{T}=s R_{A} R_{0}\left(X_{A}-X_{s}\right) \tag{3.18}
\end{equation*}
$$

This method of image orientation where the attitude variations are described by drift rates can be used for the handling of long image strips and is particularly flexible to be used with two GCPs. This model is applied, by modification considering the
view-angle, to SPOT level 1A and 1B, MOMS-02, IRS-1C and IKONOS (Zoej, 1997; Zoej and Petrie, 1998; Zoej and Fooami, 1999; Zoej and Sadeghian, 2003) and also to SPOT-5 HRS the along-track pushbroom images as a general sensor model (Dowman and Michalis, 2003; Michalis, 2005).

### 3.2.2.4 Model by Konecny et al.

Konecny et al. (1987) from Institute of Photogrammetry and Geoinformation (IPI), in the Leibniz University Hannover, evaluate the stereo SPOT Level 1A images by a bundle adjustment program BINGO on analytical photogrammetric instrument. A new approach is developed to avoid high correlation among the parameters. The parameters of orientation are estimated by thanks to orbit data and the additional parameters. The exterior orientation of each single CCD line is represented by six parameters as in the case of aerial photography. Nevertheless the parameters are highly correlated.

The flight path from A to E is considered as straight. The projection centre moves linearly from A to E. So the position of projection centre can be calculated as:

$$
\begin{align*}
& X_{0, i}=X_{0, A}+\frac{S_{i}}{S_{A E}}\left(X_{0, E}-X_{0, A}\right)  \tag{3.19}\\
& \frac{i}{n}=\frac{S_{i}}{S_{A E}} \tag{3.20}
\end{align*}
$$

where $X_{0, I}$ is position-vector of the projection centre at time $i, X_{0, A}$ is position-vector of the first line, $X_{0, E}$ is position-vector of the last line, $S_{i}$ is distance from $X_{0, A}$ to $X_{0, i}$, $S_{A E}$ is distance from $X_{0, A}$ to $X_{0, E}$.

The orientation angles ( $\omega, \varphi$ and $\kappa$ ) are regarded as constant. A position-vector of discrete point on a line in the image is:
$X-X_{0}=\lambda R x^{\prime}$
$x^{\prime}=\left(0, y^{\prime},-f\right)$
where $X$ is position-vector of a discrete point, $X_{0}$ is position-vector of projection centre, $\lambda$ is scale factor, $R$ is rotation matrix.

Here, $y^{\prime}$ corresponds to the pixel number $j$ and is related to the centre of the line. In reality, the orientation angles ( $\omega, \varphi$ and $\kappa$ ) are not constant and all six orientation parameters are functions of time. Of all EOPs, $\varphi$ with $X_{0}$ and $\omega$ with $Y_{0}$ are highly correlated assuming the flight in $X$-direction. However the change of $\varphi$ or $X_{0}$ is insignificant in term of ground coordinates.

The angular changes are a function of time $i$ and these are expressed as 8 sets of additional parameters. The parameters are coordinates of the centre point $M$ and the orientation angles ( $\omega, \varphi$ and $\kappa$ ) and additional parameters. The Earth rotation is considered in the model and $t_{A E}$ is defined as real track heading. In order to stabilize the block, a weight is assigned to the parameters $\omega, \varphi$ and $\kappa$. The $t_{A E}, S_{A E}$ and $H_{A E}$ (height difference between A and E) are used for the interpolation of centers of projection between A and E. The GCPs and independent check points (ICPs) are included in the adjustment, even.

In the restitution process the header file of SPOT image is used by CSPOT to compute approximate orientations and sensor specific parameters. In the model, the differences between central perspective and SPOT geometry have to be considered. The modified BINGO results in the following:

- 6 parameters of exterior orientation, i.e. $X_{M}, \omega, \varphi$ and $\kappa$,
- the values and correlations of additional parameters,
- 3D coordinates of object points,
- variances and covariances of the parameters,
- the variance components of the observations.


### 3.2.2.5 BLASPO and CORIKON by Jacobsen

Both programs for the scene orientation by geometric reconstruction BLASPO and CORIKON developed by Jacobsen since 1988, in the IPI of Leibniz University Hannover, are for the handling of satellite linear array images. BLASPO is a program for handling original images linear array images, while CORIKON handles images projected to a plane with constant height. The principle of BLASPO is to reconstruct the image geometry based on the given view-direction, the general orbit information (inclination, eccentricity and semi-major axis) and few GCPs (BLASPO, 2004; Jacobsen, 1998). Because of the high correlation between traditional EOPs, only $Z_{0}$ and $\omega, \varphi$ and $\kappa$ are used as unknowns. Two additional parameters are required for
image- and angular-affinity, and more additional parameters can be used if the geometric problems of image exist. The attitude and the satellite height are improved using GCPs (Jacobsen et al., 2005).

CORIKON is the other program to evaluate the images projected to a plane with constant height such as SPOT-5 level 1B, IRS-1C level 1B, IKONOS Geo, QuickBird Standard and OrthoReady image products. The principle of CORIKON is the computation of the satellite-position using the information of "nominal collection elevation" and "nominal collection azimuth" or the "satellite elevation" and "satellite azimuth". Together with the general orbit information the individual satellite position corresponding to any ground point can be reconstructed (CORIKON, 2003). With ground control points a bias correction is possible after a terrain relief correction. This can be done by 2D-affinity transformation (6 parameters) or just a simple shift (2 parameters). For the case of poor or not available imaging information also the view direction ( 2 parameters) can be adjusted (Büyüksalih et al., 2004).

### 3.2.2.6 Model by Kratky

Kratky developed a model to geometric process of SPOT images considering its dynamic characteristics (Kratky, 1987 and 1989). The transformation between image and ground coordinates is dependent on time and given as:

$$
\left[\begin{array}{l}
X  \tag{3.23}\\
Y \\
Z
\end{array}\right]=\left[\begin{array}{c}
X_{c} \\
Y_{c} \\
Z_{c}
\end{array}\right]+r F_{M}(\kappa, \varphi, \omega)\left[\begin{array}{c}
x^{\prime} \\
y^{\prime} \\
-f
\end{array}\right]
$$

where $X, Y, Z$ are ground coordinates, $X_{c}, Y_{c}, Z_{c}$ is ground coordinates of projection centre, $r$ is scale, $F_{M}(\kappa, \varphi, \omega)$ is rotation matrix, $x^{\prime}$ and $y^{\prime}$ are image coordinates ( $y^{\prime}=0$ ), and $f$ is focal length.

The projection centre is computed as a functions of $y^{\prime}$, position of the centre of image $\left(X_{0}, Y_{0}, Z_{0}\right)$ and linear $(\dot{X}, \dot{Y}, \dot{Z})$ and quadratic $(\ddot{X}, \ddot{Y}, \ddot{Z})$ rates of change, respectively, i.e. the coordinates of projection centre is:

$$
\left[\begin{array}{c}
X_{c}  \tag{3.24}\\
Y_{c} \\
Z_{c}
\end{array}\right]=\left[\begin{array}{c}
X_{0}+y^{\prime} \dot{X}+y^{\prime 2} \ddot{X}+\ldots \\
Y_{0}+y^{\prime} \dot{Y}+y^{\prime 2} \ddot{Y}+\ldots \\
Z_{0}+y^{\prime} \dot{Z}+y^{\prime 2} \ddot{Z}+\ldots
\end{array}\right]
$$

The unknowns are:

- the position of centre of image $\left(X_{0}, Y_{0}, Z_{0}\right)$ and reference attitude elements ( $\kappa$, $\varphi, \omega)_{0}$,
- the linear $(\dot{X}, \dot{Y}, \dot{Z})$ and quadratic $(\ddot{X}, \ddot{Y}, \ddot{Z})$ rates of change, and
- the change of image scale in the direction of scanline.

This model is applied for the orientation of SPOT (Baltsavias and Stallmann, 1992), MOMS-02/D2 (Baltsavias and Stallmann, 2000) and MOMS-02/Priroda (Poli et al., 2000). The model is also investigated and extended for SPOT images by Fritsch and Stallmann (2000).

### 3.2.2.7 Model by Westin

The model developed by Westin is applied on the SPOT and EROS-A1 images (Westin, 1990; Westin and Forsgren, 2001). This model is simplified assuming satellite's orbit circular during the timespan of one scene. Thus four Keplerian elements ( $i$ inclination, $\Omega$ right ascension of the ascending node, $t_{0}$ time at the ascending node and $r_{0}$ orbital radius at $t=t_{0}$ ) are estimated. The radial shape of the orbit is determined by fitting a third-order polynomial in time to the orbital radius derived from the ephemeris. The relative attitude angles can be calculated by integration since the attitude angular velocities are measured on board with sufficient accuracy. The attitude of the satellite is determined by first-order polynomial in time for the rotation angles $\omega, \varphi$ and $\kappa$ as follows:
$\omega=\omega_{0}+\Delta \omega(t)$
$\varphi=\varphi_{0}+\Delta \varphi(t)$
$\kappa=\kappa_{0}+\Delta \kappa(t)$
In totally, seven parameters $\left(i, \Omega, t_{0}, r_{0}, \omega_{0}, \varphi_{0}\right.$ and $\left.\kappa_{0}\right)$ have to be adjusted in this approach.

The author defines seven coordinate systems as follows:

- The Earth centered inertial coordinate system (ECI),
- The local orbital reference system,
- The attitude measurement reference system,
- The sensor coordinate system,
- The SPOT coordinate system,
- The SPOT ephemeris reference system, and
- The ground control point reference system.

The relation between image and ground coordinate system is established as:

$$
\left[\begin{array}{c}
0  \tag{3.26}\\
y \\
-f
\end{array}\right]=\frac{1}{d}{R_{I F}}^{T} R_{F B}{ }^{T} R_{B S}{ }^{T}\left[\begin{array}{c}
X-X_{p} \\
Y-Y_{p} \\
Z-Z_{p}
\end{array}\right]
$$

where $d$ is scaling factor, $X_{p}, Y_{p}, Z_{p}$ are coordinates of satellite position, $R_{I F}$ is flightinertial transformation relating the orbital reference system to the ECI systems, $R_{F B}$ is body-flight transformation relating the attitude reference system to the orbital reference system, $R_{B S}$ is sensor-body transformation relating the sensor coordinate system to the attitude reference system, $f$ is focal length.

The most important difference from the ordinary collinearity equations in aerial photogrammetry is that the sensor $x$-coordinate is always zero, and that parameters of right side apart from $d$ in Equation (3.26) are functions of time (Westin, 1990).

### 3.2.2.8 Trade-off by Orun and Natarajan

The model named Orun \& Natarajan Satellite Sensor Model is developed by Orun and Natarajan (1994) for evaluation of SPOT images. This model is applied to SPOT (Kim and Kang, 2001) and KOMPSAT-1 EOC images (Kim and Im, 2001).

The small changes $\left(d \omega_{s}\right)$ in $\omega_{s}$ is not distinguished from a small change $\left(d Y_{s}\right)$ in $Y_{s}$. Similarly, small changes $\left(d \varphi_{0}\right)$ in $\varphi_{s}$ and $\left(d x_{0}\right)$ in $x_{s}$, can not be differentiated. Then, replacing $x_{i}$ in place of $t$ in the model, the EOPs are formulated as follows:

$$
\begin{align*}
& X_{s}=X_{0}+a_{1} x_{i}+b_{1} x_{i}^{2} \\
& Y_{s}=Y_{0}+a_{2} x_{i}+b_{2} x_{i}{ }^{2} \\
& Z_{s}=Z_{0}+a_{3} x_{i}+b_{3} x_{i}{ }^{2}  \tag{3.27}\\
& \kappa_{s}=\kappa_{0}+a_{4} x_{i}+b_{4} x_{i}{ }^{2}
\end{align*}
$$

where $X_{s}, Y_{s}, Z_{s}$ are coordinates of satellite position, $X_{0}, Y_{0}, Z_{0}$ are coordinates of central linear array, $a_{i}, b_{i}=$ coefficients ( $\left.i: 1(1) 4\right), \kappa_{s}$ is yaw angle of satellite position, $\kappa_{0}$ is yaw angle of central linear array.

The rotation matrix $R$ is given as follows:
$R=\left[\begin{array}{ccc}\cos \left(\kappa_{0}+a_{4} x_{i}+b_{4} x_{i}^{2}\right) & \cos \omega \sin \left(\kappa_{0}+a_{4} x_{i}+b_{4} x_{i}^{2}\right) & \sin \omega \sin \left(\kappa_{0}+a_{4} x_{i}+b_{4} x_{i}^{2}\right) \\ -\sin \left(\kappa_{0}+a_{4} x_{i}+b_{4} x_{i}^{2}\right) & \cos \omega \cos \left(\kappa_{0}+a_{4} x_{i}+b_{4} x_{i}^{2}\right) & \sin \omega \cos \left(\kappa_{0}+a_{4} x_{i}+b_{4} x_{i}^{2}\right) \\ 0 & -\sin \omega & \cos \omega\end{array}\right]$
where $w$ is the tilt angle of image.
Some non-negligible points of model are found:

- A pseudo code for the iterative determination of $x$ image coordinate which is set to zero on the left hand side of collinearity equation is established.
- Partial pivoting is invoked during the least-squares solution of the normal equations by Gaussian elimination.
- Highly correlated parameters $a_{3}$ and $b_{3}$ are eliminated.
- In order to check for consistency with the photogrammetric solutions, eight parameters $\left(a_{1}, \ldots, a_{4} ; b_{1}, \ldots, b_{4}\right)$ are set to zero on certain runs.
3.2.2.9 Model by Radhadevi and Ramachandran

In 1994, Radhadevi and Ramanchandran developed a parametric model based on the collinearity equations (Zoej, 1997). The relationship between sensor and ground coordinate system is established as:

$$
\begin{align*}
& {\left[\begin{array}{c}
0 \\
g_{s} \\
-f
\end{array}\right]=\lambda M\left[\begin{array}{c}
X-X_{P} \\
Y-Y_{P} \\
Z-Z_{P}
\end{array}\right]}  \tag{3.29}\\
& M=R_{G F} R_{F B} R_{B S} \tag{3.30}
\end{align*}
$$

$R_{G F}$ is flight-geocentric transformation matrix consisting in three rotations of the osculating parameters $\Omega, i$ and $v$. These parameters and $r$ are functions of time:

$$
\begin{align*}
& \Omega(t)=\Omega_{0}+\Omega_{1} t \\
& i(t)=i_{0}+i_{1} t \\
& v(t)=v_{0}+v_{1} t  \tag{3.31}\\
& r(t)=r_{0}+r_{1} t+r_{2} t^{2}+r_{3} t^{3}
\end{align*}
$$

$R_{F B}$ is body-flight transformation matrix which is a function of roll, pitch and yaw angles. These angles define the angular relationship between the ideal and actual platform coordinates, being a function of time as follows:

$$
\begin{align*}
& \omega(t)=\omega_{0}+\omega_{1} t+\omega_{2} t^{2}+\omega_{3} t^{3} \\
& \phi(t)=\phi_{0}+\phi_{1} t+\phi_{2} t^{2}+\phi_{3} t^{3}  \tag{3.32}\\
& \kappa(t)=\kappa_{0}+\kappa_{1} t+\kappa_{2} t^{2}+\kappa_{3} t^{3}
\end{align*}
$$

$R_{B S}$ is sensor-body transformation matrix. This matrix is a function of angles of rotation which define the angular relationship between the PAN and actual platform coordinate system derived from the looking angles of the detector. These angles are treated as fixed parameters in the model. $R_{B S}$ includes the interior orientation parameters such as looking angle, alignment offset etc. which are treated as constants. A co-factor matrix for observations and a weight matrix for parameter estimates into the system are applied.

### 3.2.2.10 Model by El-Manadili and Novak

El-Manadili and Novak (1996) developed a model derived from DLT. DLT is a modified model by Abdel-Aziz and Karara (1971) based on collinearity equations. In the model, the systematic distortions caused by Earth rotations and GSD variations due to off-nadir viewing are corrected. In order to remove the effects of satellite deviations from nominal positions, perturbations in satellite velocity vectors and rate of changes of the sensor attitude angles. The corrections to the image coordinates are as follows:
$\delta_{x}=c_{1}+c_{2} x_{i}+c_{3} \Delta T$
$\delta_{y}=c_{4}+c_{5} y_{i}+c_{6} \Delta T$
where $c_{j}$ is constant values $(j=1,2, . ., 6), x_{i}, y_{i}$ are the metric image coordinates with respect to the scan-line coordinate system, $\Delta T$ is time with respect to the frame centre.

The geometric corrections are applied to the image coordinates $(u, v)$ for systematic errors caused by the rotation of the Earth and off-nadir viewing. In the case of Earth rotation, the effects are due to the orbit inclination (i), incidence angle of the scene $(I)$ and latitude of the frame centre $(\Phi)$; and in the case of off-nadir viewing, the source of error is incidence angle of the scene.

### 3.2.2.11 Model by Poli

Poli developed a model as flexible as possible and adaptable to a wide class of linear array sensors (Poli, 2005). The model for the purpose of indirect georeferencing estimates the parameters which model the internal and external orientation in a
bundle adjustment with least squares methods, considering the satellite's orbit and the integrations of observations on the external orientation, provided by instruments carried on board, while the internal orientation modelling takes into account the lens distortions and the CCD lines displacement in the focal plane. Namely, the systematic errors caused by the lens and CCD lines are corrected and the integration of any available Global Positioning System (GPS) and Inertial Navigaton System (INS) is included in the model (Poli, 2007).

In the case of multi lens, the relative position $\left(j, d_{x j}, d_{y j}, d_{z j}\right)$ and attitude $\left(\alpha_{j}, \beta_{j}, \gamma_{j}\right)$ of each lens with respect to the nadir one are imported into the collinearity equations. Piecewise Polynomial Functions (PPM) is invoked for the modelling of sensor external orientation depending on time. The platform trajectory is divided into the segments, according to the PPM approach, and in the each segments the sensor external orientation is modelled by second-order polynomials depending on $\bar{t}$ (the time of acquisition of a generic image line). The PPM approach is extended in order to consider the integration and correction of GPS and INS observations. Thus the position and attiude of each image line $l$ belonging the segment $i$, indicated with $\left[\begin{array}{lllll}X_{C} & Y_{C} & Z_{C}, & \omega_{C}, & \varphi_{C}, \kappa_{C},\end{array}\right]$ are modelled as sum of the measured position and attitude data for that line $\left[X_{\text {instr }}, Y_{\text {instr }}, Z_{\text {instr }}, \omega_{\text {instr }}, \varphi_{\text {instr }}, \kappa_{\text {instr }}\right]_{l}^{i}$ plus the second-order polynomial functions depending on $\bar{t}$ :

$$
\begin{align*}
& X_{C}(\bar{t})=X_{\text {instr }}^{i}+X_{0}^{i}+X_{1}^{i} \bar{t}+X_{2}^{i} t^{-2} \\
& Y_{C}(\bar{t})=Y_{\text {instr }}^{i}+Y_{0}^{i}+Y_{1}^{i} \bar{t}+Y_{2}^{i} t^{-} \\
& Z_{C}(\bar{t})=Z_{\text {instr }}^{i}+Z_{0}^{i}+Z_{1}^{i} \bar{t}+Z_{2}^{i} \bar{t}^{-2} \\
& \omega_{C}(\bar{t})=\omega_{\text {instr }}^{i}+\omega_{0}^{i}+\omega_{1}^{i} \bar{t}+\omega_{2}^{i} \bar{t}^{2}  \tag{3.34}\\
& \varphi_{C}(\bar{t})=\varphi_{\text {instr }}^{i}+\varphi_{0}^{i}+\varphi_{1}^{i} \bar{t}+\varphi_{2}^{i} \bar{t}^{-2} \\
& \kappa_{C}(\bar{t})=\kappa_{\text {instr }}^{i}+\kappa_{0}^{i}+\kappa_{1}^{i} \bar{t}+\kappa_{2}^{i} \bar{t}^{2}
\end{align*}
$$

where the constant terms $\left[\begin{array}{llllll}X_{0} & Y_{0}, & Z_{0} & \omega_{0}, & \varphi_{0}, & \kappa_{0}\end{array}\right]_{i}$ compensate for the shifts and angular drifts between the image system and the GPS and INS systems, whereas the linear and quadratic terms $\left[X_{1}, Y_{1}, Z_{1}, \omega_{1}, \varphi_{1}, \kappa_{1}\right]_{i}$ and $\left[\begin{array}{llllll}X_{2} & Y_{2}, & Z_{2} & \omega_{2}, & \varphi_{2}, & \kappa_{2}\end{array}\right]$ model the additional systematic errors contained in the GPS and INS measurements. This model is applied for the optical images such as

ASTER VNIR, EROS-A1, MISR, MOMS-2P, SPOT-5 HRS, TLS (Poli, 2002, 2003, 2004, 2005 and 2007).
3.2.2.12 LOS vector adjustment model by Jung et al.

The Line-of-Sight (LOS) vector adjustment model is developed by Jung et al. (2007), depending on the core idea that only the LOS vector is adjusted during the geometric correction of SPOT-5 high resolution geometry (HRG) imagery. This model assumes:

- The satellite is moving along well-defined close-to-circular elliptical orbit.
- The predicted orbit recorded in the header file is close to the true satellite orbit.

Using all parameters available in the header file, LOS vector adjustment model performs a restitution of satellite imagery using additional corrections terms, which adjusts the LOS vector as a function of image lines and pixels for exterior orientation. Orbital parameters ( $\Omega$, Inc, $f+w_{p}$ and $r$ ) are estimated using position and velocity vectors ( $\vec{S}$ and $\vec{V}$, respectively), with the expressions such as linear function of time for $\Omega$, Inc, and third order polynomial function of time for $f+w_{p}$ and $r$.

The relationship between the satellite postion in an orbit and its ground position on the Earth is formulated as following:
$\vec{p}-\vec{s}=\mu \vec{u}$
where $\vec{p}$ denotes the vector from Earth centre to a point on the Earth's surface, $\vec{s}$ denotes the vector from Earth centre to the satellite, $\vec{u}$ is the LOS vector, and $\mu$ is arbitrary factor. The $\vec{p}$ and $\vec{u}$ can be transformed into local orbital coordinate system using the rotation matrices $M_{A}$ (the rotation matrix attitude measurement through local orbital coordinate system) and $M_{E}$ (the rotation matrix local orbital through Earth-centred inertial coordinate system) as following:
$M_{E}^{T}\left[\begin{array}{l}p_{x} \\ p_{y} \\ p_{z}\end{array}\right]-\left[\begin{array}{l}0 \\ 0 \\ r\end{array}\right]=\mu M_{A}\left[\begin{array}{l}u_{x} \\ u_{y} \\ u_{z}\end{array}\right]$
When $\mu$ is eliminated, the equation becomes:
$\frac{r_{11} p_{x}+r_{12} p_{y}+r_{13} p_{z}-b_{13} r}{r_{31} p_{x}+r_{32} p_{y}+r_{33} p_{z}-b_{33} r}=\frac{u_{x}}{u_{z}}$
$\frac{r_{21} p_{x}+r_{22} p_{y}+r_{23} p_{z}-b_{23} r}{r_{31} p_{x}+r_{32} p_{y}+r_{33} p_{z}-b_{33} r}=\frac{u_{y}}{u_{z}}$
The elements of LOS vector ( $u_{x}, u_{y}$ and $u_{z}$ ) are the functions of the look angles in $X$ $\left(\psi_{o x}\right)$ and in $Y\left(\psi_{o y}\right)$ in the attitude measurement coordinate system:
$u_{x}=\tan \left(\psi_{o x}\right)$
$u_{y}=\tan \left(\psi_{o y}\right)$
$u_{z}=-1$
The residuals $f_{1}$ and $f_{2}$ can be formulated:
$f_{1} \equiv \tan ^{-1}\left[\frac{r_{11} p_{x}+r_{12} p_{y}+r_{13} p_{z}-b_{13} r}{r_{31} p_{x}+r_{32} p_{y}+r_{33} p_{z}-b_{33} r}\right]+\psi_{o x}=-\psi_{S x}+\psi_{o x}=0$
$f_{2} \equiv \tan ^{-1}\left[\frac{r_{21} p_{x}+r_{22} p_{y}+r_{23} p_{z}-b_{23} r}{r_{31} p_{x}+r_{32} p_{y}+r_{33} p_{z}-b_{33} r}\right]+\psi_{o y}=-\psi_{S y}+\psi_{o y}=0$
where $\psi_{S x}$ and $\psi_{S y}$ are the look angles defined by the vector between the ground and satellite positions. The $f_{1}$ and $f_{2}$ have to be zero; however these are not zero because of the uncertainty of orbital parameters. Residuals can be removed by adding error look angles in $X$ and $Y\left(\psi_{E x}\right.$ and $\left.\psi_{E y}\right)$ as follows:

$$
\begin{align*}
& f_{1}(i, j)+\psi_{E x}(i, j)=-\psi_{S x}(i)+\psi_{o x}(j)+\psi_{E x}(i, j)=0  \tag{3.44}\\
& f_{2}(i, j)+\psi_{E y}(i, j)=-\psi_{S y}(i)+\psi_{o y}(j)+\psi_{E y}(i, j)=0 \tag{3.45}
\end{align*}
$$

The additional correction term ( $\psi_{E x}$ and $\psi_{E_{y} y}$ ) for exterior orientation can be line functions given by:
$F_{1}=k f_{1}+k \psi_{E x}=k f_{1}+a_{0}+b_{0} \cdot i+c_{0} \cdot j=0$
$F_{2}=k f_{21}+k \psi_{E y}=k f_{21}+a_{1}+b_{1} \cdot i+c_{1} \cdot j=0$
where $a_{0}, b_{0}, c_{0}, a_{l}, b_{1}$ and $c_{l}$ are the model parameters evaluated by adjusting the LOS vector, and $k$ is a scaling parameter that is defined by the distance between the ground and the satellite position. The basic model for the LOS vector adjustment model using the method of bias-compensated RFM becomes:

$$
\left[\begin{array}{l}
V_{1}  \tag{3.48}\\
V_{2}
\end{array}\right]_{m n}=\left[\begin{array}{lllllllll}
\frac{\partial F_{1}}{\partial p_{x}} & \frac{\partial F_{1}}{\partial p_{y}} & \frac{\partial F_{1}}{\partial p_{z}} & 1 & i & j & 0 & 0 & 0 \\
\frac{\partial F_{2}}{\partial p_{x}} & \frac{\partial F_{2}}{\partial p_{y}} & \frac{\partial F_{2}}{\partial p_{z}} & 0 & 0 & 0 & 1 & i & j
\end{array}\right]\left[\begin{array}{l}
\delta p_{x_{n}} \\
\delta p_{y_{n}} \\
\delta p_{z_{n}} \\
a_{0_{m}} \\
b_{0_{m}} \\
c_{0_{m}} \\
a_{1_{m}} \\
b_{1_{m}} \\
c_{1_{m}}
\end{array}\right]+k_{m}\left[\begin{array}{l}
f_{1} \\
f_{2}
\end{array}\right]_{m n}
$$

where $V_{1}$ and $V_{2}$ are residuals, $\delta p_{x}, \delta p_{y}$ and $\delta p_{z}$ are corrections for the approximations for $\vec{p}$ on a point $n$ on a single image $m$.

In theory this model requires 3 GCPs with triangular distribution. However the suitable results are obtained using 5 GCPs. The model is compared with the bundle adjustment model by ERDAS 8.7 Leica Photogrammetry Suite over a stereopair of SPOT-5, and more rigorous results are reached.

### 3.2.3 Overview of Existing Parametric Models

Many common and discrete points are available among the existing parametric models. However, the common assumptions of the existing parametric models are:

- the image investigated has a dynamic geometry since its each adjacent line is acquired at a different time.
- the EOPs are changing for each adjacent line, and
- the correlation among the parameters is probably available.

Various approaches are generated considering different assumptions. However, the common approach, rather an obligation, is to represent the EOPs by the polynomials. All existing parametric models given in the previous section, except the models by Toutin (1983) and Jacobsen (1988), prefer the polynomial representation. The main motivation here is that if the polynomial representation is not preferred, the EOPs of each line are considered as single parameters. For instance, the total number of parameters for one image becomes 72000 for 12000 lines of an image if the number of EOPs is 6 , while their number reduces 18 if a 2 nd order polynomial is preferred. The required number of observations is more then 72000 for the first- and more then 18 for the second-case respectively. The required number of GCPs is half of the
number of parameters. The representation of the EOPs by a $2^{\text {nd }}$ order polynomial is formulated above.

$$
\begin{equation*}
P_{j, i}=P_{0}+\dot{P} x_{i}+\ddot{P} x_{i}^{2} \tag{3.49}
\end{equation*}
$$

where $P_{j, i}$ is one of six EOPs of the line $i(j=1(1) 6), P_{0}$ is the EOP for the reference line (generally the centre line of an image), $\dot{P}$ is drift, $\ddot{P}$ is drift rates, $x$ is line position of $i^{\text {th }}$ GCP. The coefficients $P_{0}, \dot{P}$ and $\ddot{P}$ are the new parameters.

No correct or optimal parameter selection is proposed (Kim et al., 2007). Orun and Natarajan (1994) ignore errors in $\omega$ and $\varphi$, and chose the position and $\kappa$ as the parameters with the 2 nd order polynomial representation. Konency et al. (1987) ignore all positional parameters and select angular parameters. Salamonowicz (1986) prefers attitude angles with drifts whereas Radhadevi and Ramachandran (1994) take into account the angular parameters as a function of $3^{\text {rd }}$ order polynomial. The order of the polynomial can be determined by a significance test for the each parameter.

### 3.3 Model and Adjustment for Geometric Analysis

At the beginning of this section, the model which establishes geometric relation between image and ground coordinate systems will be presented in its generic form. Following, the required modification and will be presented in accordance with the imaging geometry of SPOT-5 HRG sensor.

### 3.3.1 Generic Model

The generic model in this thesis is based on Weser et al. (2008). This model establishes a well defined relation between image and ground coordinate systems, considering imaging geometry and orbital and attitude parameters of satellite. This model is modified for SPOT-5, ALOS PRISM and QuickBird images by Weser et al. (2008).
3.3.1.1 Coordinate systems and transformations

The relationship between image and ground coordinate systems can be established following the definitions of auxiliary coordinate systems which are defined in Appendix A.2. The required transformations for establishing this relation are illustrated in Figure 3.2 being as following:

- First transformation is from image coordinate system (ICS) to scanline coordinate system (SCS),
- Second transformation is from SCS to camera coordinate system (CCS),
- Third transformation is from CCS to payload coordinate system (PCS),
- Fourth transformation is from PCS to orbital coordinate system (OCS), and
- Fifth, and last, transformation is from OCS to ground coordinate system (GCS).


Figure 3.2: Transformation from ICS to GCS.

One point on the image is pointed by its image coordinates ( $x_{s}$ and $y_{s}$ ). Since each line of an image is generated by the linear array defining the SCS, the coordinates in SCS ( $x$ and $y$ ) has to be defined by the transformation from ICS to SCS as following:
$\vec{p}_{s}=\left[\begin{array}{c}x_{s} \\ y_{s} \\ 0\end{array}\right]=\left[\begin{array}{l}0 \\ y \\ 0\end{array}\right]$
where $\vec{p}_{s}$ is position vector in SCS. The $x$ coordinate is related to the acquisition time $(t)$ in combination with the acquisition time $\left(t_{0}\right)$ for reference line (generally the centre line of image), time interval per line ( $\Delta t$ ) and also the $x$ coordinate of reference line as following:
$t=t_{0}+\Delta t \cdot\left(x-x_{0}\right)$
$t$ is required to estimate time-dependent EOPs changed for each adjacent line of image.

The second transformation, from SCS to CCS, requires taking into account the interior orientation, in other words the position and the rotation of scanline in the camera symbolized by $\vec{c}_{s}=\left[\begin{array}{lll}x_{s}^{c} & y_{s}^{c} & c\end{array}\right]^{T}$. The new position vector $\left(\vec{p}_{c}\right)$ in CCS becomes:
$\vec{p}_{c}=\vec{p}_{s}-\vec{c}_{s}+\delta \vec{x}$
where $\vec{p}_{c}$ is position vector in CCS, $\delta \vec{x}$ is vector of systematic error corrections (Weser et al., 2008).

The third transformation from CCS to PCS defines the position and the rotation of camera in the payload (or satellite body) as following:
$\vec{p}_{p}=C_{M}+m \cdot R_{C P} \cdot \vec{p}_{c}$
where $\vec{p}_{p}$ is position vector in PCS, $C_{M}$ is the camera mounting parameter, i.e. shift in each three axes, $m$ is scale factor, $R_{C P}$ is rotation from CCS to PCS (Weser et al., 2008). Appendix A. 3 explains the rotation between two 3D coordinate systems.

The fourth transformation, from PCS to OCS, is the subject of a time-dependent rotation parameterized by the angles ( $a_{r}, a_{p}, a_{y}$ ) as following:
$\vec{p}_{o}=R_{P O} \cdot \vec{p}_{p}$
where $\vec{p}_{0}$ is position vector in OCS, and $R_{P O}$ is rotation from PCS to OCS as explained in Appendix A.3.

Finally, the fifth and last transformation is from OCS to GCS as following:
$\vec{P}=\left[\begin{array}{l}X \\ Y \\ Z\end{array}\right]=\vec{P}_{S}+R_{O G} \cdot \vec{p}_{o}$
where $\vec{P}$ is position vector in GCS, $\vec{P}_{s}$ is position vector of satellite, $R_{O G}$ is rotation from OCS to GCS as a function of $\vec{P}_{S}$ and velocity-vector $\left(\vec{V}_{S}\right)$ of satellite (Appendix A.3).

The combined equations from (3.50) to (3.55) yield with the required coordinate transformations:
$\vec{P}=\vec{P}_{s}+R_{O G} \cdot R_{P O} \cdot\left[C_{M}+m \cdot R_{C P} \cdot\left(\vec{p}_{s}-\vec{c}_{s}+\delta \vec{x}\right)\right\rfloor$
Since each adjacent line of image is acquired at a different time, the rotation matrixes $R_{O G}$ and $R_{P O}$ are time-dependent. The $R_{O G}$ is estimated by two ways: the first way is performed using Keplerian angular elements depicted in Appendix A. 3 and Greenwich Mean Sidereal Time (GMST), and the second and easier way is performed using $\vec{P}_{S}$ and $\vec{V}_{S}$.

### 3.3.1.2 Modelling parameters

The EOPs consist the position $\left(\vec{P}_{S}\right)$ and velocity $\left(\vec{V}_{S}\right)$ vectors, and attitude angles $\left(a_{r}, a_{p}, a_{y}\right)$, and can be modelled by the polynomial modelling. For this purpose, these parameters are estimated for each line, and then the polynomial coefficients can be calculated. The Lagrange interpolation is suggested for the estimation of position and velocity vectors of satellite while the linear interpolation is sufficient for the estimation of attitude angles for each line (SPOT Image, 2002). It is required to be given some position sets before and after the estimated position. The Lagrange interpolation can be formulated as following:

$$
\begin{equation*}
P\left(t_{i}\right)=\sum_{e=1}^{g} \frac{P\left(t_{i}\right) \prod_{\substack{d=1 \\ d}}^{g}\left(t_{i}-t_{d}\right)}{\prod_{\substack{d=1 \\ d \neq e}}^{g}\left(t_{e}-t_{d}\right)} \tag{3.57}
\end{equation*}
$$

where $P\left(t_{i}\right)$ is position or velocity of satellite $\left(X_{S}, Y_{S}, Z_{S}, V_{X}, V_{Y}, V_{Z}\right)$ for line $i, t$ is time, $d$ and $e$ are running index, integer, $g$ is number of given positions or velocities. This interpolation is preferred since the shape of orbit defined by the position and velocity of satellite is not linear.

The attitude angles are linearly interpolated using the time-dependent attitude values as following (SPOT Image, 2002):
$a_{j}\left(t_{i}\right)=a_{j}\left(t_{k}\right)+\left(a_{j}\left(t_{k+1}\right)-a_{j}\left(t_{k}\right)\right) \cdot \frac{t_{i}-t_{k}}{t_{k+1}-t_{k}}$
where $a_{j}$ is $a_{r}, a_{p}$ and $a_{y}$, and $t_{k}$ is the time given in metadata $\left(t_{k-1}<t_{i}<t_{k}\right)$.
Following the estimation of all parameters for line of each GCP, the EOPs can be represented by $2^{\text {nd }}$ order polynomial modelling as following:

$$
\begin{equation*}
P_{j, i}=P_{0}+\dot{P} \cdot t_{i}+\ddot{P} \cdot t_{i}^{2} \tag{3.59}
\end{equation*}
$$

where $P_{j, i}$ is one of the EOPs of the line $i(j=1(1) 9$, number of EOP $), P_{0}$ is the EOP for the reference line (generally the centre line of image), $\dot{P}$ is drift, $\ddot{P}$ is drift rates. So the number of parameters reduces significantly as mentioned in section 3.2.3.

### 3.3.2 Modified Model for SPOT-5

Considering specifications and definitions for SPOT-5 HRG and HRS images, one simplification and two modifications are performed by SPOT Image (2002) comparing the generic model given in Section 3.3.1, as following:

- The look angles $\left(\psi_{x}\right.$ and $\left.\psi_{y}\right)$ are defined and used in the model to establish interior orientation (Figure 3.3).
- A navigation reference coordinate system (NRCS) is defined corresponding to PCS (Figure 3.3). Then the rotation from NRCS to OCS is different than in the generic model (Appendix A.3).
- The rotation from OCS to GCS is different than defined in the Appendix A.3.


Figure 3.3: NRCS and look angles ( $\psi_{x}$ and $\psi_{y}$ ) (left) (SPOT Image, 2002), and bundle rays given by the SPOT-5 look angles (Weser et al., 2008).

The look angles ( $\psi_{x}$ and $\psi_{y}$ ) which define the look direction for each element of the linear array are available in the metadata of SPOT-5 image (Figure 3.3). As the simplification, these look angles are equal to the $\vec{p}_{p}$ defining the interior orientation given in equation (3.53) as following:
$\vec{p}_{p}=\left[\begin{array}{c}-\tan \left(\psi_{y}\right) \\ \tan \left(\psi_{x}\right) \\ 1\end{array}\right]=C_{M}+m \cdot R_{C P} \cdot \vec{p}_{C}=s \cdot R_{C P} \cdot\left(\vec{p}_{S}-\vec{c}_{S}\right)$
where $C_{M}=0$ (SPOT Image, 2002). Weser et al. (2008) try to estimate elements of $\vec{c}_{S}$ and $R_{C P}$ while Fotev et al. (2005), Kornus et al. (2006) and Jung et al. (2007) try to use these look angles directly in the adjustment model.

The first modification on the generic model is suggested for the rotation from PCS (or NCRS) to OCS. SPOT Image suggests to multiply $a_{r}$ and $a_{p}$ angles by -1 , so the rotation becomes:
$R_{P O}=R_{X}^{-1}\left(a_{p}\right) \cdot R_{Y}^{-1}\left(a_{r}\right) \cdot R_{Z}^{-1}\left(a_{y}\right)$
However the last equation can be simplified as following, since the attitude angles are very small (Fotev et al., 2005):

$$
R_{P O} \cong\left[\begin{array}{ccc}
1 & -a_{y} & +a_{r}  \tag{3.62}\\
+a_{y} & 1 & -a_{p} \\
-a_{r} & +a_{p} & 1
\end{array}\right]
$$

This last equation can be preferred since the linearization for attitude angles is less complex comparing the trigonometric functions included in the equation 3.61.

The second modification is suggested for the rotation from OCS to GCS as following:

$$
\left[\begin{array}{l}
R_{3,1}^{O G} \\
R_{3,2}^{O G} \\
R_{3,3}^{O G}
\end{array}\right]=\frac{\vec{P}_{S}}{\left\|\vec{P}_{S}\right\|}
$$

$\left[\begin{array}{l}R_{1,1}^{O G} \\ R_{1,2}^{O G} \\ R_{1,3}^{O G}\end{array}\right]=\frac{\vec{V}_{S} \times\left[\begin{array}{l}R_{3,1}^{O G} \\ R_{3,2}^{O G} \\ R_{3,3}^{O G}\end{array}\right]}{\left\|\vec{V}_{S} \times\left[\begin{array}{l}R_{3,1}^{O G} \\ R_{3,2}^{O G} \\ R_{3,3}^{O G}\end{array}\right]\right\|}$
$\left[\begin{array}{l}R_{2,1}^{O G} \\ R_{2,2}^{O G} \\ R_{2,3}^{O G}\end{array}\right]=\frac{\left[\begin{array}{l}R_{3,1}^{O G} \\ R_{3,2}^{O G} \\ R_{3,3}^{O G}\end{array}\right] \times\left[\begin{array}{l}R_{1,1}^{O G} \\ R_{1,2}^{O G} \\ R_{1,3}^{O G}\end{array}\right]}{\left\|\left[\begin{array}{l}R_{3,1}^{O G} \\ R_{3,2}^{O G} \\ R_{3,3}^{O G}\end{array}\right] \times\left[\begin{array}{l}R_{1,1}^{O G} \\ R_{1,2}^{O G} \\ R_{1,3}^{O G}\end{array}\right]\right\|}$

The final equation becomes:
$\vec{P}=\vec{P}_{S}+m \cdot \vec{u}_{3}$
where
$\vec{u}_{3}=R_{O G} \cdot \vec{u}_{2}$
$\vec{u}_{2}=\frac{R_{P O} \cdot \vec{u}_{1}}{\left\|R_{P O} \cdot \vec{u}_{1}\right\|}$

$$
\vec{u}_{1}=\frac{\left[\begin{array}{c}
-\tan \left(\psi_{y}\right)  \tag{3.67}\\
\tan \left(\psi_{x}\right) \\
1
\end{array}\right]}{\|\left[\begin{array}{c}
-\tan \left(\psi_{y}\right) \\
\tan \left(\psi_{x}\right) \\
1
\end{array}\right]} \cong\left[\begin{array}{c}
-\tan \left(\psi_{y}\right) \\
\tan \left(\psi_{x}\right) \\
1
\end{array}\right]
$$

where $\|\|$ denotes norm of vector.
To apply the equation (3.64) into the adjustment model, this equation can be rewritten as:
$\vec{u}_{1}=\frac{1}{m} R_{P O}^{-1} \cdot R_{O G}^{-1}\left(\vec{P}-\vec{P}_{S}\right)$
$\left[\begin{array}{c}-\tan \left(\psi_{y}\right) \\ \tan \left(\psi_{x}\right) \\ 1\end{array}\right]=\frac{1}{m} \cdot R \cdot\left[\begin{array}{c}X-X_{S} \\ Y-Y_{S} \\ Z-Z_{S}\end{array}\right]=\frac{1}{m} \cdot\left[\begin{array}{lll}r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33}\end{array}\right] \cdot\left[\begin{array}{c}X-X_{S} \\ Y-Y_{S} \\ Z-Z_{S}\end{array}\right]$
where $R=R_{P O}^{-1} \cdot R_{O G}^{-1}$. Finally, the first and the second lines are divided by the third line to eliminate the scale factor, similarly within equation 3.1, the equation 3.69 becomes:

$$
\begin{align*}
& F_{\psi_{y}}=\frac{r_{11}\left(X-X_{s}\right)+r_{12}\left(Y-Y_{s}\right)+r_{13}\left(Z-Z_{s}\right)}{r_{31}\left(X-X_{s}\right)+r_{32}\left(Y-Y_{s}\right)+r_{33}\left(Z-Z_{s}\right)}+\tan \left(\psi_{y}\right)=\frac{f_{1}}{f_{3}}+\tan \left(\psi_{y}\right)=0 \\
& F_{\psi_{x}}=\frac{r_{21}\left(X-X_{s}\right)+r_{22}\left(Y-Y_{s}\right)+r_{23}\left(Z-Z_{s}\right)}{r_{31}\left(X-X_{s}\right)+r_{32}\left(Y-Y_{s}\right)+r_{33}\left(Z-Z_{s}\right)}-\tan \left(\psi_{x}\right)=\frac{f_{2}}{f_{3}}-\tan \left(\psi_{x}\right)=0 \tag{3.70}
\end{align*}
$$

This last equation suggested by Kornus et al. (2006) is the mathematical model used in this thesis.

### 3.3.3 Adjustment

The main aim of adjustment is to estimate both residuals $(v)$ of observations $(L)$ and unknowns $(d P)$ of adjustment parameters $(P)$ where observations are consisted by parameters. If the least square adjustment (LSA) is preferred as in this thesis, the condition of $\underline{v}^{T} \underline{v}=\min$ must be expected. The observations are look angles of point $i$ ( $\psi_{x}$ and $\psi_{y}$ ), and the parameters are EOPs and object coordinates of ICPs. These EOPs (position, velocity and attitude angles for line of each GCPs and ICPs) can be represented as following:

$$
\begin{align*}
& \left(X_{S}\right)_{i}=X_{o}+\dot{X} \cdot x_{i}+\ddot{X} \cdot x_{i}^{2} \\
& \left(Y_{S}\right)_{i}=Y_{o}+\dot{Y} \cdot t_{i}+\ddot{Y} \cdot x_{i}^{2} \\
& \left(Z_{S}\right)_{i}=Z_{o}+\dot{Z} \cdot t_{i}+\ddot{Z} \cdot x_{i}^{2} \\
& \left(V_{X}\right)_{i}=V_{X_{o}}+\dot{V}_{X} \cdot x_{i}+\ddot{V}_{X} \cdot x_{i}^{2} \\
& \left(V_{Y}\right)_{i}=V_{Y o}+\dot{V}_{Y} \cdot x_{i}+\ddot{V}_{Y} \cdot x_{i}^{2}  \tag{3.71}\\
& \left(V_{Z}\right)_{i}=V_{Z o}+\ddot{V}_{Z} \cdot x_{i}+\ddot{V}_{Z} \cdot x_{i}^{2} \\
& \left(a_{r}\right)_{i}=a_{r_{o}}+\dot{a}_{r} \cdot x_{i}+\ddot{a}_{r} \cdot x_{i} \\
& \left(a_{p}\right)_{i}=a_{p_{o}}+\dot{a}_{p} \cdot x_{i}+\ddot{a}_{p} \cdot x_{i}^{2} \\
& \left(a_{y}\right)_{i}=a_{y_{o}}+\dot{a}_{y} \cdot x_{i}+\ddot{a}_{y} \cdot x_{i}^{2}
\end{align*}
$$

Additionally, when the tie or ICPs which their adjusted object coordinates must be determined in the adjustment, their ground coordinates $(X, Y$ and $Z)$ are considered as adjustment parameters. The approximate values of their object coordinates can be estimated via stereo images, as depicted in the Appendix A.4. So all parameters in the adjustment are:
$P=\left[\begin{array}{l}X_{o} \dot{X} \ddot{X}^{\prime} Y_{o} \dot{Y} \ddot{Y}^{\prime} Z_{o} \dot{Z} \ddot{Z}_{\ldots} \\ V_{X o} \dot{V}_{X} \ddot{V}_{X} V_{Y o} \dot{V}_{Y} \ddot{V}_{Y} V_{Z o} \dot{V}_{Z} \ddot{V}_{Z} \ldots \\ a_{r o} \dot{a}_{r} \ddot{a}_{r} a_{p o} \dot{a}_{p} \ddot{a}_{p} a_{y o} \dot{a}_{y} \ddot{a}_{y} \ldots \\ X Y Z\end{array}\right]^{T}$
The major important point in the adjustment is the functional model derived from mathematical model. The functional model in this thesis is Gauss-Helmert model based on the establishing linkage between adjusted observations $(\bar{L})$ and the adjusted parameters $(\bar{P})$. Since the LSA requires the linearized form of condition equation, the linearized Gauss-Helmert model can be established as following:

$$
\begin{equation*}
\underline{A} \cdot \underline{d P}+\underline{B} \cdot \underline{v}+\underline{w}=\underline{0} \tag{3.73}
\end{equation*}
$$

where $A$ and $B$ denotes design matrix of parameters and observations, respectively, and $w$ is misclosure vector. $A$ and $B$ result from differentiating the non-linear condition equations using the approximate parameters and observations, respectively. The linearization can be done by a proper method, such as Taylor series ignoring second and higher degree terms. So sub-design matrix of point $i\left(\underline{A}_{i}\right)$ after the simplification using look angles ( $\psi_{x}$ and $\psi_{y}$ ) becomes:
$\underline{A}_{i}=\left[\begin{array}{l}\frac{1}{f_{3}} \frac{\partial f_{1}}{\partial P_{j}}-\frac{f_{1}}{f_{3}^{2}} \frac{\partial f_{3}}{\partial P_{j}} \\ \frac{1}{f_{3}} \frac{\partial f_{2}}{\partial P_{j}}-\frac{f_{2}}{f_{3}^{2}} \frac{\partial f_{3}}{\partial P_{j}}\end{array}\right]_{i}=\left[\begin{array}{l}\frac{1}{f_{3}}\left(\frac{\partial f_{1}}{\partial P_{j}}+\tan \left(\psi_{y}\right) \frac{\partial f_{3}}{\partial P_{j}}\right) \\ \frac{1}{f_{3}}\left(\frac{\partial f_{2}}{\partial P_{j}}+\tan \left(\psi_{x}\right) \frac{\partial f_{3}}{\partial P_{j}}\right)\end{array}\right]_{i}$

Finally the size of $\underline{A}$ reaches $2 \mathrm{n} \times 30$ if all parameters are chosen as adjustment parameters ( n is sum of number of GCPs and ICPs).

The $\underline{d P}$ matrix consists unknown of adjustment parameter as shown in following:
$\underline{d P}=\left[\begin{array}{c}d X_{o} \\ \vdots \\ d \ddot{a}_{y} \\ d X_{I C P} \\ d Y_{I C P} \\ d Y_{I C P}\end{array}\right]$
The size of $\underline{d P}$ matrix is $30 \times 1$ when all parameters is chosen as adjustment parameters.

The $\underline{B}$ matrix related to the observations of point $i$ becomes:
$\underline{B}_{i}=\left[\begin{array}{cc}\frac{1}{\cos \left(\psi_{y}\right)} & 0 \\ 0 & -\frac{1}{\cos \left(\psi_{x}\right)}\end{array}\right]_{i}$
The size of $\underline{B}$ matrix becomes $2 \mathrm{n} \times 2 \mathrm{n}$ when all parameters is chosen as adjustment parameters. And $w$ for point $i$ is:
$\underline{w}_{i}=\left[\begin{array}{l}F_{\psi_{y}}\left(\left.P\right|_{0},\left.\psi_{y}\right|_{0}\right) \\ \left.F_{\psi_{x}}\left(\left.P\right|_{0},\left.\psi_{x}\right|_{0}\right)\right]_{i}, ~\end{array}\right.$
When all parameters are chosen as adjustment parameters, the equation (3.73) can be rewritten with showing size of matrixes as following:
$\underline{A}_{2 n \times 30} \underline{d P}_{30 \times 1}+\underline{B}_{2 n \times 2 n} \underline{v}_{2 n \times 1}+\underline{w}_{2 n \times 1}=\underline{0}_{2 n \times 1}$
The unknowns ( $\underline{d P}$ ) can be estimated ignoring the weight of observations as following:
$\underline{d P}=-\left(\underline{A}^{T}\left(\underline{B} \underline{B}^{T}\right)^{-1} \underline{A}\right)^{-1} \underline{A}^{T}\left(\underline{B} \underline{B}^{T}\right)^{-1} \underline{w}$
So the adjusted parameters become:
$\underline{\bar{P}}=\left.\underline{P}\right|_{0}+\underline{d P}$
The residuals can be estimated as following:
$\underline{v}=B^{T}\left(\underline{B} \underline{B}^{T}\right)^{-1}(\underline{A} \underline{d P}+\underline{w})$
So the adjusted observations become:
$\underline{\bar{L}}=\left.\underline{L}\right|_{0}+\underline{v}$
Finally the validation of adjustment can checked using both adjusted parameters and observations in the mathematical model given by equation (3.70), as following:
$F(\bar{L}, \bar{P}) \stackrel{?}{=} 0$
As depicted in the following section, the pre-adjustment of both observations and parameters is required before bundle adjustment of images in this thesis.

### 3.3.3.1 Pre-adjustment

According to Kornus et al. (2006), the pre-adjustment before the bundle adjustment is a mandatory process. The major motivation of pre-adjustment is the requirement of correction of look angles. Two types of pre-adjustment performed in this thesis. In the first type, the look angles are adjusted assuming the EOPs constant for each mono image (i.e. $\underline{d P}=\underline{0} \rightarrow \underline{A} \cdot \underline{d P}=\underline{0}$ ). So the functional model and the residuals of look angles become:
$\underline{B} \cdot \underline{v}+\underline{w}=0$
$\underline{v}=-\underline{B}^{T}\left(\underline{B} \cdot \underline{B}^{T}\right)^{-1} \underline{w}$
The second type of pre-adjustment depends on an additional step following the first type. Namely the adjusted look angles in the first type are assumed constant (i.e. $\underline{v}=\underline{0} \rightarrow \underline{B} \cdot \underline{v}=\underline{0}$ ), and the parameters are adjusted. The functional model and unknowns become:

$$
\begin{align*}
& \underline{A} \cdot \underline{d P}+\underline{w}=\underline{0}  \tag{3.85}\\
& \underline{d P}=-\left(\underline{A} \cdot \underline{A}^{T}\right)^{-1} \underline{A}^{T} \underline{w} \tag{3.86}
\end{align*}
$$

The adjusted EOPs and observations can be calculated as in equations (3.79) and (3.81) respectively.

The observations and EOPs of each mono image are estimated separately and also using only GCPs. However estimation of the adjusted look angles of ICPs are required in the bundle adjustment. These look angles are estimated using the adjusted look angles of GCPs. Thus, the coefficients of look angles $\left(\psi_{x o}, \dot{\psi}_{x}, \ddot{\psi}_{x} \psi_{y o}, \dot{\psi}_{y}, \ddot{\psi}_{y}\right)$ are estimated using the equation (3.49), and the look angles of ICPs are calculated using their line number ( $x$ ).

The final adjustment following both types of pre-adjustment is bundle adjustment which is mentioned in the following section.

### 3.3.3.2 Bundle adjustment

The bundle adjustment can be performed if at least two stereo images are available. The advantage of bundle adjustment is that all adjustment parameters and observations of at least both two images are adjusted together as following:

$$
\left[\begin{array}{ccc}
\underline{A}_{1} & \cdots & \underline{0}  \tag{3.87}\\
\vdots & \ddots & \vdots \\
\underline{0} & \cdots & \underline{A}_{m}
\end{array}\right] \cdot\left[\begin{array}{c}
\underline{d P}_{1} \\
\vdots \\
\underline{d P}_{m}
\end{array}\right]+\left[\begin{array}{ccc}
\underline{B}_{1} & \cdots & \underline{0} \\
\vdots & \ddots & \vdots \\
\underline{0} & \cdots & \underline{B}_{m}
\end{array}\right] \cdot\left[\begin{array}{c}
\underline{v}_{1} \\
\vdots \\
\underline{v}_{m}
\end{array}\right]+\left[\begin{array}{c}
\underline{w}_{1} \\
\vdots \\
\underline{w}_{m}
\end{array}\right]=\underline{0}
$$

where $m$ is the number of image.
The adjustment parameters are EOPs for GCPs while the correction of ground coordinates are required assuming the ground coordinates as additional parameters for the new point. This new point can be tie point whose true object coordinates are not known, or ICPs whose true coordinates are known. The approximate coordinate of these points can be intersected from at least two images using approximate parameters and uncorrected observations (see Appendix 4).

The root mean-square error (RMSE) of ground coordinates, as the geometric accuracy of images, can be estimated as following:

$$
\begin{align*}
& m_{X}= \pm \sqrt{\frac{\underline{\underline{X^{T}}} \underline{\underline{\Delta X}}}{n}}  \tag{3.88a}\\
& m_{Y}= \pm \sqrt{\frac{\underline{\Delta Y^{T}} \underline{\underline{\Delta Y}}}{n}}  \tag{3.88b}\\
& m_{Z}= \pm \sqrt{\frac{\underline{\underline{Z^{T}}}}{n} \underline{\underline{Z}}}  \tag{3.88c}\\
& \underline{\Delta X}=\underline{\bar{X}}-\underline{X_{e}} \\
& \underline{\Delta Y}=\underline{\bar{Y}}-\underline{Y}_{e}  \tag{3.89b}\\
& \underline{\Delta Z}=\underline{\bar{Z}}-\underline{Z}_{e}
\end{align*}
$$

where $\bar{X}, \bar{Y}$ and $\bar{Y}$ are true ground coordinates, such as observed by GPS and adjusted, and $\underline{X}_{e}, \underline{Y}_{e}$, and $\underline{Z}_{e}$ are estimated coordinates by intersection from at least two images and compensated in the adjustment.

## 4. GEOMETRIC ANALYSIS OF SPOT-5 HRG LEVEL 1A STEREO IMAGES

This section subjects the geometric analysis performed by the suggested modified and simplified mathematical model. At first the test field, images and auxiliary data used in this thesis are mentioned with the description of SPOT-5 satellite and its HRG sensor. Then the workflow of geometric analysis is subjected. Finally the geometric analysis results are presented.

### 4.1 Test Field

Zonguldak test-field is located in Western Karadeniz Region of Turkey. The position of Zonguldak city centre is about $31^{\circ} 47^{\prime} 41^{\prime \prime}$ East and $41^{\circ} 27^{\prime} 25^{\prime \prime}$ North. This testfield has rolling topography with steep and rugged terrain in some parts. The elevation ranges roughly up to 1800 m . Despite urbanised along the sea coast, some agricultural lands and forest areas exist inland. This area is investigated for different aims in the frame of remote sensing researches, such as georeferencing, geometric analysis, DEM generation and validation, deformation monitoring, information content analysis, change detection, urban growth monitoring, environment, forestry, biology etc. The main importance of this test-field is its mountainous characteristic in our scope. Because the real analysis require optimal distributed GCPs in both horizontal and vertical planes whereas many mathematical models reach very precise results on the flat areas.

### 4.2 Description of Satellite, Image, and Auxiliary Data

The investigated images are stereo HRG level 1A images of SPOT-5. The SPOT-5 satellite and its brief specifications are summarized before promoting the image and related metadata information.

### 4.2.1 SPOT-5

SPOT-5, the last satellite of SPOT series, was launched on its sun-synchronous, polar and circular orbit with 832 km altitude by CNES (in French: Centre National d'Ètudes Spatiales; French Space Agency) on $4^{\text {th }}$ May 2002. SPOT-5 carries HRG, HRS (High Resolution Stereoscopic) and Vegetation-2 imaging sensors, and other sensors and instruments such as star tracker, truster, DORIS antenna, payload telemetry antenna and Digital Earth Sensor, which are required for the operation of the satellite, illustrated in Figure 4.1.


Figure 4.1: Imaging sensors and some instruments of SPOT-5 satellite (above) and the CCD array of panchromatic band (below) (SPOT Image, 2002).

Two identical HRG cameras are linear push-broom systems to ensuring multispectral images at four resolution levels, namely the SWIR (short wave infrared) band is at a resolution of 20 m , multispectral mode called HX corresponding to three spectral bands B1 (green), B2 (red) and B3 (near infrared) is at 10 m and panchromatic band
is at 5 m GSD. THR (Very High-Resolution) is a 2.5 m sampled images in the same panchromatic band, namely the thanks to two staggered CCD arrays shifted one from the other 0.5 meter in the cross-track direction and 3.45 m in the along-track direction in focal plane by the process called "Supermode". Both HRG instruments have a pointing mirror, allowing the viewing angle from nadir to vary within a range of $\pm 27^{\circ}$. This tracking capacity allows high revisit frequency and cross-track stereoacquisition capability. HRS is able acquire stereopairs in a single pass by two telescopes with an along-track viewing angle of $20^{\circ}$ forwards and also backwards. Both telescopes have a focal length of 580 mm and a single CCD array consisting of 12000 pixels projected on the ground as 120 km . The $\mathrm{B} / \mathrm{H}$ ratio is close to 0.8 . The ground resolution is 10 m , but the sampling rate is 5 m along the track allowing a higher elevation precision of DEM with $\pm 4 \mathrm{~m}$ RMSE to be obtained (Gleyzes et al. 2003). Vegetation-2 instruments has 1 km geometric resolution and 2250 km wideswath, and its capacity allows covering land mass of the earth in one day (Fontannaz et al. 2002). DORIS antenna supplies the accurate position and velocities and also the absolute dating of the satellite whereas star tracker provides the attitude of the satellite.

### 4.2.2 Images and metadata used

The images used in this thesis are stereo SPOT-5 HRG level 1A panchromatic images, provided by the joint research project, coded 104 Y 050 , between TÜBİTAK (in Turkish: Türkiye Bilimsel ve Teknik Araştırma Kurumu, in English: The Scientific and Technical Research Council of Turkey) and JÜLICH Research Centre (Germany). Level 1A is the definition by SPOT Image meaning the images are only radiometric corrected to compensate the differences of sensitivities among the various elements of a CCD array (SPOT Image, 2002). So the level 1A image carries its original geometry. Specifications of images analysed are summarized in Table 4.1. Both images are in 5 m GSD and 8 bits ( 256 grey values) in Digital Image MAP (DIMAP) format with 12000 by 12000 pixels. The $\mathrm{B} / \mathrm{H}$ ratio of stereo images is 0.54 .

Table 4.1: Specifications for SPOT-5 HRG level 1A images

| Specification | Left image | Right image |
| :---: | :---: | :---: |
| Date | $13^{\text {th }}$ August 2003 | $14^{\text {th }}$ August 2003 |
| Acquisition time for centre line | $09: 03: 30.345268$ | $08: 44: 14.552376$ |
| Time interval per line | 0.00075199509335 sec | 0.00075199660510 sec |

The detailed metadata file provided by the image vendors includes many parameters related to the image. The parameters taken into account in this thesis are manually derived from the metadata file. These parameters are followings:

- Position and velocity vectors (i.e. $\vec{P}_{S}$ and $\vec{V}_{S}$ ) describing the orbit of satellite. These vectors are available in geocentric coordinates in International Terrestrial Reference Frame (ITRF), and produced by DORIS-DIODE system being acquired with 30 second time interval. The accuracy of orbit is reported as $\pm 0.71 \mathrm{~m}, \pm 0.67 \mathrm{~m}$ and $\pm 0.36 \mathrm{~m}$ in $X, Y$, and $Z$ axis (SPOT Image, 2002). The number of samples for positions and velocities and their acquisition time is at 10 and 11 for the images dated $13^{\text {th }}$ and $14^{\text {th }}$ August 2003, respectively.
- Attitude angles ( $a_{r}, a_{p}$ and $a_{y}$ ) in radian. These angles are sampled 8 Hz (i.e. 8 per second) by onboard star tracker of SPOT-5. Totally 150 and 148 time dependent samples are available for the images dated $13^{\text {th }}$ and $14^{\text {th }}$ August 2003, respectively.
- Look angles for each elements of linear array $\left(\psi_{x}\right.$ and $\left.\psi_{y}\right)$ in radian, defining the transformation between scanline and payload coordinate systems. These look angles are given for each 12000 elements of CCD array.

The position and velocity vectors and attitude angles define the exterior orientation whereas look angles supplies the interior orientation.

### 4.2.3 Points observed by GPS survey

47 points in total are measured with GPS observations collected at 15 seconds intervals in static mode for relative positioning, using TOPCON TURBO-SII GPS receiver and each session took about 15 minutes. The geocentric coordinates of points are produced in WGS-84 by the software package of TOPCON, and their horizontal and vertical accuracies are $\pm 0.06 \mathrm{~m}$ and $\pm 1.00 \mathrm{~m}$, respectively. The points are distributed as homogeneous as on both horizontal and vertical planes (Figure 4.2). However accessibility of the points during field survey was the major obstacle considering mountainous and forestry characteristic of Zonguldak test field. The points were obtained generally on the road intersections, centre of short and narrow bridges etc. which are sharp on the image. The GPS observation of all points was performed by the team from Zonguldak Karaelmas University Department of

Geodesy and Photogrammetry Engineering, and photographed to point them into their correct position on PCI Geomatica OrthoEngine software.


Figure 4.2: Distribution of points on the image dated $13^{\text {th }}$ and $14^{\text {th }}$ August 2003 above and below, respectively.


Figure 4.3: One of the points selected on road intersection (left hand-side) and scene of GPS observation in the field (right hand-side).

### 4.3 Programming

All calculations and graphical presentations in the section 4.4.2 including analysis results of this thesis are coded by the author in the MATLAB 7.0 environment ignoring some usage of functions in MATLAB library such as inversing or transposing the matrix etc. The derived program called GeoSpot-1.0 consist the steps graphically shown in Figure 4.4.


Figure 4.4: Main steps of developed program (GeoSpot-1.0).
The definitions of stages of program GeoSpot-1.0 are explained in the following sections.

### 4.3.1 Loading data

This stage includes the following steps for both images:
i. Reading ID, line $(x)$ and column (y) position, i.e. image coordinates, and GPS surveyed ground ( $X, Y, Z$ ) coordinates of each point,
ii. Reading position $\left(\vec{P}_{S}\right)$ and velocity $\left(\vec{V}_{S}\right)$ of samples on the satellite orbit, and their acquisition time in hour, minute and second mode.
iii. Reading the attitude angles $\left(a_{r}, a_{p}, a_{y}\right)$ and their acquisition time in hour, minute and second mode.
iv. Reading the look angle ( $\psi_{x}$ and $\psi_{y}$ ) of each elements of CCD array.
v. Reading header information including the line and column position for the centre of image, time interval per line and acquisition time for the centre line.

### 4.3.2 Pre-processing

This stage includes the following steps remembering the section 3.3.1.2:
i. The position and velocity for all 47 points considering their line $(x)$ position are estimated using the Lagrange interpolation.
ii. The attitude angles for all 47 points considering their line $(x)$ position are estimated using the linear interpolation.
iii. The coefficients of polynomial modelling ( $P_{0}, \dot{P}$ and $\ddot{P}$ ) for position, velocity and attitudes angles are estimated by $2^{\text {nd }}$ order polynomial. So totally 27 coefficient are estimated with this method.
iv. The look angles for all 47 points considering their column $(y)$ position are estimated using the linear interpolation.
Figure 4.5 illustrates given time dependent positions of satellite both given in metadata and estimated for each points, and also the GPS observed positions of points.

### 4.3.3 Pre-adjustment

This stage varies in two types as mentioned in the Section 3.3.3.1. The first one consists pre-adjustment of look angles assuming the parameters are constant. The second type of pre-adjustment depends on an additional step to the first type, i.e. the adjusted look angles in the first type are assumed constant and the parameters are adjusted. The pre-adjustment is performed using only GCPs.


Figure 4.5: Plotting of positions of both satellite (given in metadata and estimated for each points) and points.

The graphical representation of both types of pre-adjustment is illustrated in Figure 4.6. The second type pre-adjustment runs at most 3 times. However this limit can be changed if it is required. Here $\Delta P=d P_{j+1}-d P_{j}$ where $j$ is iteration number and $\Delta$ is the threshold. The adjusted look angles and parameters of each images are introduced into the bundle adjustment.

### 4.3.4 Bundle adjustment

This stage varies according to the pre-adjustment performed in two types. In the first type, the bundle adjustment compensates adjusted look angles and non-adjusted EOPs derived from the first type pre-adjustment, and also object coordinates of check points. In the second one, both the look angle and parameters introduced into bundle adjustment are adjusted together in the pre-adjustment. The comparative results are presented into the following section.

### 4.4 Geometric Analysis

The results of geometric analysis are presented in this section. At first the organization of the geometric analysis is mentioned to guide the reader. Then the analysis results are presented. Finally the results are concluded.


Figure 4.6: Steps of pre-adjustment.

### 4.4.1 Preface of Geometric Analysis

The geometric analysis is performed on the geometric accuracy on points (GCPs or ICPs) with respect to look angles and parameters. In other words, the ground coordinates of points in meter unit are estimated using look angles and parameters with their approximate and adjusted values, and this estimated coordinates are compared with their GPS observed values. 7 point sets and 19 EOP sets are configured. The distribution of GCPs and ICPs in various configurations on the image dated $13^{\text {th }}$ August 2003 is shown in Figure 4.7. The points are homogeneously distributed $(\mathrm{H})$ locating the ICPs covered by GCPs, i.e. the GCPs are always located at the corners and borders of the images. The grouped distribution (G) depends on the grouping GCPs and ICPs as shown in Figure 4.7. The motivation in the grouped set is to see effects of accumulated distribution of GCPs and ICPs on the image. The $y$ coordinate of ICPs in the set G-A is smaller than 6001, the $x$ coordinate of ICPs in the set G-B is smaller than 6001, and the $x$ and $y$ coordinates of GCPs in the set G-C are smaller and bigger than 6001 , respectively.


Figure 4.7: Various configuration of point distribution. (H: Homogenous distribution, G: Grouped distribution, +: GCP, $\diamond:$ ICP)

The EOP sets are established in various configurations as listed in Table 4.2. Actually, no optimal or correct set of adjustment parameter has been proposed in the previous studies with the choice of adjustment parameters varies (Kim et al., 2007). However the overparameterisation problem occurred when all parameters were chosen as adjustment parameters, caused by high correlation among the parameters occurring diverging of adjustment. The effects of parameter choice on the results can be examined for the various parameter sets listed in Table 4.2. The reader must notice that unknowns of ground coordinates $(X, Y$, and $Z$ ) are added these parameter sets for the ICPs in the bundle adjustment process.

Table 4.2: Configurations of choosing EOP as adjustment parameter

| No | ID | Adjusted EOPs |
| :---: | :--- | :--- |
| 1 | $\mathrm{P}_{\mathrm{S}}$ | $X_{o}, \dot{X}, \ddot{X}, Y_{o}, \dot{Y}, \ddot{Y}, Z_{o}, \dot{Z}, \ddot{Z}$ |
| 2 | $\mathrm{P}_{\mathrm{So}}$ | $X_{o}, Y_{o}, Z_{o}$ |
| 3 | $\mathrm{P}_{\mathrm{S} 1}$ | $\dot{X}, \dot{Y}, \dot{Z}$ |
| 4 | $\mathrm{P}_{\mathrm{S} 2}$ | $\ddot{X}, \ddot{Y}, \ddot{Z}$ |
| 5 | $\mathrm{~V}_{\mathrm{S}}$ | $V_{X o}, \dot{V}_{X}, \ddot{V}_{X}, V_{Y o}, \dot{V}_{Y}, \ddot{V}_{Y}, V_{Z o}, \dot{V}_{Z}, \ddot{V}_{Z}$ |
| 6 | $\mathrm{~V}_{\mathrm{So}}$ | $V_{X o}, V_{Y o}, V_{Z o}$ |
| 7 | $\mathrm{~V}_{\mathrm{S} 1}$ | $\dot{V}_{X}, \dot{V}_{Y}, \dot{V}_{Z}$ |
| 8 | $\mathrm{~V}_{\mathrm{S} 2}$ | $\ddot{V}_{X}, \ddot{V}_{Y}, \ddot{V}_{Z}$ |
| 9 | A | $a_{r o}, \dot{a}_{r}, \ddot{a}_{r}, a_{p o}, \dot{a}_{p}, \ddot{a}_{p}, a_{y o}, \dot{a}_{y}, \ddot{a}_{y}$ |
| 10 | $\mathrm{~A}_{o}$ | $a_{r o}, a_{p o}, a_{y o}$ |
| 11 | $\mathrm{~A}_{1}$ | $\dot{a}_{r}, \dot{a}_{p}, \dot{a}_{y}$ |
| 12 | $\mathrm{~A}_{2}$ | $\ddot{a}_{r}, \ddot{a_{p}}, \ddot{a}_{y}$ |
| 13 | $\mathrm{P}_{\mathrm{S}} \mathrm{V}_{\mathrm{S}}$ | $X_{o}, \dot{X}, \ddot{X}, Y_{o}, \dot{Y}, \ddot{Y}, Z_{o}, \dot{Z}, \ddot{Z}, V_{X o}, \dot{V}_{X}, \ddot{V}_{X}, V_{Y o}, \dot{V}_{Y}, \ddot{V}_{Y}, V_{Z o}, \dot{V}_{Z}, \ddot{V}_{Z}$ |
| 14 | $\left(\mathrm{P}_{\mathrm{S}} \mathrm{V}_{\mathrm{S}}\right)_{o}$ | $X_{o}, Y_{o}, Z_{o}, V_{X o}, V_{Y o}, V_{Z o}$ |
| 15 | $\mathrm{P}_{\mathrm{S}} \mathrm{A}$ | $X_{o}, \dot{X}, \ddot{X}, Y_{o}, \dot{Y}, \ddot{Y}, Z_{o}, \dot{Z}, \ddot{Z}, a_{r o}, \dot{a}_{r}, \ddot{a}_{r}, a_{p o}, \dot{a}_{p}, \ddot{a}_{p}, a_{y o}, \dot{a}_{y}, \ddot{a}_{y}$ |
| 16 | $\left(\mathrm{P}_{\mathrm{S}} \mathrm{A}\right)_{o}$ | $X_{o}, Y_{o}, Z_{o}, a_{r o}, a_{p o}, a_{y o}$ |
| 17 | $\mathrm{~V}_{\mathrm{S}} \mathrm{A}$ | $V_{X o}, \dot{V}_{X}, \ddot{V}_{X}, V_{Y o}, \dot{V}_{Y}, \ddot{V}_{Y}, V_{Z o}, \dot{V}_{Z}, \ddot{V}_{Z}, a_{r o}, \dot{a}_{r}, \ddot{a}_{r}, a_{p o}, \dot{a}_{p}, \ddot{a}_{p}, a_{y o}, \dot{a}_{y}, \ddot{a}_{y}$ |
| 18 | $\left(\mathrm{~V}_{\mathrm{S}} \mathrm{A}\right)_{o}$ | $V_{X o}, V_{Y o}, V_{Z o}, a_{r o}, a_{p o}, a_{y o}$ |
| 19 | $\left(\mathrm{P}_{\mathrm{S}} \mathrm{V}_{\mathrm{S}} \mathrm{A}\right)_{o}$ | $X_{o}, Y_{o}, Z_{o}, V_{X o}, V_{Y o}, V_{Z o}, a_{r o}, a_{p o}, a_{y o}$ |

The results are presented in tables and graphical presentations. The points are located on their image coordinates ( $x$ and $y$ ) whereas the residuals are plotted in meter unit. The scale can be varied for GCPs and ICPs on the same figure.

### 4.4.2 Results Achieved

As mentioned in the previous sections, the ground coordinates of GCPs and ICPs using approximate and adjusted values of look angles and EOPs are estimated and compared with their GPS surveyed coordinates. The results are reported in two sections. The first one includes the results using the first type of pre-adjustment, i.e. adjusting only look angles in the pre-adjustment process. The second one depends on adjustment of both look angles and adjustment parameters in the pre-adjustment process.

### 4.4.2.1 Results based on first type pre-adjustment

The first results of this section listed in Table 4.3 are achieved by comparison between GPS surveyed and estimated ground coordinates of points. The estimation is performed using approximate and adjusted look angles without adjustment of EOPs both in the stages of pre-adjustment and bundle adjustment. In other words the EOPs are assumed as constant in these two stages.

Table 4.3: RMSE of GCPs and ICPs using approximate and adjusted look angles

| Point |  |  | $L$ |  |  | $\bar{L}$ |  |  | B |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| set | type | \# | $m_{X}$ | $m_{Y}$ | $m_{Z}$ | $m_{X}$ | $m_{Y}$ | $m_{Z}$ | $m_{X}$ | $m_{Y}$ | $m_{Z}$ |
| H-0 | GCP | 47 | 1230150.23 | 762556.41 | 1260608.51 | 0.515 | 0.169 | 0.928 | - | - | - |
| H-10 | GCP | 37 | 1230020.35 | 762582.93 | 1260714.43 | 0.515 | 0.169 | 0.928 | 0.603 | 0.219 | 0.975 |
|  | ICP | 10 | 1230630.66 | 762458.25 | 1260216.54 | 6.065 | 5.768 | 6.461 | 5.970 | 5.745 | 6.609 |
| H-20 | GCP | 27 | 1230372.88 | 762579.15 | 1260424.05 | 0.515 | 0.169 | 0.928 | 0.620 | 0.220 | 0.970 |
|  | ICP | 20 | 1229849.58 | 762525.70 | 1260857.50 | 4.782 | 5.335 | 5.943 | 4.827 | 5.310 | 6.174 |
| H-30 | GCP | 17 | 1230851.61 | 762294.90 | 1260277.25 | 0.514 | 0.170 | 0.927 | 0.593 | 0.226 | 0.968 |
|  | ICP | 30 | 1229752.60 | 762704.55 | 1260796.18 | 4.992 | 5.996 | 5.936 | 5.208 | 5.937 | 6.219 |
| H-40 | GCP | 7 | 1229216.73 | 763039.36 | 1261574.80 | 0.516 | 0.168 | 0.929 | 0.569 | 0.236 | 1.045 |
|  | ICP | 40 | 1230313.51 | 762471.86 | 1260439.33 | 6.793 | 6.966 | 6.107 | 7.216 | 6.814 | 6.327 |
| G-A | GCP | 23 | 1226524.57 | 766698.58 | 1260566.64 | 0.530 | 0.157 | 0.937 | 0.678 | 0.194 | 0.925 |
|  | ICP | 24 | 1233614.81 | 758565.60 | 1260648.63 | 24.751 | 33.251 | 7.079 | 25.643 | 32.499 | 6.921 |
| G-B | GCP | 24 | 1234135.02 | 760543.55 | 1258344.29 | 0.509 | 0.174 | 0.923 | 0.500 | 0.203 | 1.010 |
|  | ICP | 23 | 1225978.37 | 764651.13 | 1262966.85 | 5.601 | 5.939 | 9.678 | 5.257 | 5.871 | 10.000 |
| G-C | GCP | 17 | 1224884.38 | 766662.14 | 1262500.62 | 0.528 | 0.157 | 0.937 | 0.720 | 0.214 | 0.892 |
|  | ICP | 30 | 1233124.22 | 760219.98 | 1259535.05 | 29.166 | 54.568 | 6.825 | 30.009 | 53.557 | 6.790 |

$L$ : approximate look angle, $L$ : pre-adjusted look angle, $B$ : bundle adjustment, RMSE $(m)$ is $\pm$ meter

The effect of look angles can be easily realized when the results are examined. The RMSE values are significantly reduced for all point sets and also types. The RMSE of GCPs in all sets are sub-meter level and almost equal with adjusted look angles, meaning possibly the model is fitting on the GCPs very well. However, the RMSEs of ICPs are about $\pm 5 \mathrm{~m}$, i.e. $\pm 1$ pixel, in all sets except G-A and G-C. The RMSEs
larger than $\pm 5 \mathrm{~m}$ in the sets G-A and G-C for ICPs depends on their distribution on the images remembering the Figure 4.7. The reason of this situation is that the adjusted look angles of ICPs are based on the adjusted look angles of GCPs. When the GCPs located on only the right or left half of image, the look angles on outside GCPs, i.e. on the area of ICPs, are extrapolated with the help of adjusted look angles of GCPs. In other words, the half of one line has adjusted look angles by GCPs while other side has extrapolated look angles related to the column number $(y)$ of a line. This problem had not occurred for the ICPs of G-B since the GCPs lay on whole line on this point set.

The residual errors at GCPs and ICPs are plotted in Figures 4.8 for all GCPs (H-0) and in Figure 4.9 and 4.10 for other sets using different scales for GCPs and ICPs. The residual errors are systematic using both approximate and pre-adjusted look angles in the Figure 4.8. If the pre-adjusted look angles are used the errors on GCPs are systematic for all point sets as shown in Figure 4.9 and 4.10 . However this systematic effect is not observed in the bundle adjustment.

ICPs have no systematic errors in all point sets homogenously distributed (H) and in G-B. The ICPs in the sets G-A and G-C have similar systematic errors growing in the bottom side of images. This systematic effect means the GPS surveyed position of the GCPs is on upper-right in planimetry and below in height against their estimated position. Notably, the magnitude and the direction of residual errors of both GCPs


Figure 4.8: Plot of residual errors in planimetry (diagonal) and in height (updown) at all GCPs (H-0). left: using approximate look angles, right: using pre-adjusted look angles.


Figure 4.9: Plot of residual errors in planimetry (diagonal) and in height (up-down) at GCPs and ICPs for homogenously distributed point sets. (above: results of pre-adjusted look angles, below: results of bundle adjusted look angles, H : Homogenous distribution, G: Grouped distribution, $\because:$ GCP, o: ICP).


Figure 4.10: Plot of residual errors in planimetry (diagonal) and in height (up-down) at GCPs and ICPs for grouped point sets (from above to below: G-A, GB, G-C). left: results of pre-adjusted look angles, right: results of bundle adjustment. (H: Homogenous distribution, G: Grouped distribution, $\bullet$ : GCP, o: ICP).
and ICPs are almost similar. For instance the ICP located in the centre of image has equal residual errors with respect to direction and magnitude in all homogenously distributed images in Figure 4.9.

These results mean that the approximate values of EOPs can be assumed correct and used for adjustment of look angles. However the point distribution must have homogenous distribution to reach at least about $\pm 1$ pixel ( $\pm 5 \mathrm{~m}$ ) accuracy for the ICPs. This must be taken into account that the look angles of images are adjusted within running the pre-adjustment for each image individually while they and EOPs are adjusted in bundle adjustment at the same time.

The RMSE of GCPs and ICPs in the bundle adjustment are generally larger than in the results of pre-adjusted look angles. Thus, the bundle adjustment has no improvement on the results according to the Table 4.3. Nevertheless, the systematic effect on the residual errors of GCPs is not observed following the bundle adjustment (Figure 4.9 and 4.10). Here the EOPs are not adjusted and assumed constant, and the ground coordinates of ICPs are assumed as the adjustment parameters in the bundle adjustment.

The second and final results of this section are related to the various EOP sets. The effect of various EOP sets on the accuracy of GCPs and ICPs are analysed presenting the results in Appendix A.5.1. The results of bundle adjustment are almost equal to the first results of this section for the parameter sets from $1^{\text {st }}$ to $11^{\text {th }}$ except the $5^{\text {th }}$ set. In other words the accuracy is generally sub-meter for the GCPs and about $\pm 1$ pixel $( \pm 5 \mathrm{~m})$ for the ICPs on the sets having homogenously distributed points $(\mathrm{H})$. Other EOP sets, i.e. $5^{\text {th }}$ and from $13^{\text {th }}$ to $19^{\text {th }}$, have inaccurate results especially for the GCPs; on the other hand, ICPs have accurate ground coordinates about $\pm 1$ pixel ( $\pm 5$ $\mathrm{m})$. This has to be taken into account that the true ground coordinates of GCPs are estimated using the adjusted both EOPs and look angles intersecting two images where the true ground coordinates of ICPs are some of the adjustment parameters in the bundle adjustment. If the results are assessed following this explanation, the accuracy of GCPs is blurred by the $5^{\text {th }}$ and from $13^{\text {th }}$ to $19^{\text {th }}$ EOP sets. This results means that the unknowns of EOPs are inaccurate as a result of the correlation among the elements of these EOP sets.

The accuracy of point sets is mostly varied by the EOP sets. The adjusted EOP sets excepting the $5^{\text {th }}$ and from $13^{\text {th }}$ to $19^{\text {th }}$, the accuracy of GCPs are generally sub-meter level whereas the accuracy of ICPs are about $\pm 1$ pixel ( $\pm 5 \mathrm{~m}$ ) and between $\pm 1$ pixel ( $\pm 5 \mathrm{~m}$ ) and $\pm 20$ pixels ( $\pm 100 \mathrm{~m}$ ) in the point sets homogenously distributed and grouped, respectively. The results based on these adjusted EOP sets are graphically illustrated in the Figures 4.11 and 4.12. Generally the accuracies of GCPs are smaller than $\pm 1 \mathrm{~m}$ being $m_{Z}>m_{X}>m_{Y}$. The accuracy in height $(Z)$ is generally larger than the accuracy in the planimetry ( $X$ and $Y$ ). Here the accuracy of $Y$ is the highest since this axis is related to the look angles which is important on the estimation of true ground coordinates (see Table 4.3). The effects of the $5^{\text {th }}$ and from $13^{\text {th }}$ to $19^{\text {th }}$ parameter sets can be negligible if the Figures 4.11 and 4.12 are examined. Only the sets V and $\mathrm{V}_{\mathrm{o}}$ including all velocity elements and the constant terms, respectively, are separated from the other parameter sets. The set V has inaccurate accuracy on the GCPs whereas the accuracy of its ICPs is acceptable. The set $\mathrm{V}_{0}$ is blurring the accuracy on the GCPs whereas there is no any problem with the ICPs. Other results based on EOP sets from $13^{\text {th }}$ and $19^{\text {th }}$ are not shown since the accuracy of GCPs is extremely less.

The sets $5^{\text {th }}, 6^{\text {th }}$ and from $13^{\text {th }}$ to $19^{\text {th }}$ cause inaccurate results on the grouped points $(\mathrm{G})$ as similar as in the homogenously distributed points (H). The higher accuracy on the GCPs is reached on the point set G-C whereas others have lower. The cause of these inaccurate results on G-A and G-C is based on the dependency of look angles with column number $(y)$ remembering the similar results achieved in the Table 4.3. The graphical representation of the accuracies of grouped point sets are not shown unlike in Figures 4.12 since there is no any continuity among them.

Finally this can be concluded that the accuracy of GCPs and ICPs are not changing significantly with respect to number of points and the type of EOP sets, excepting $5^{\text {th }}$, $6^{\text {th }}, 13^{\text {th }}-19^{\text {th }}$ set. And the most accurate results are available if the EOPs are used individually. In other words, if position, velocity and attitude angles are adjusted together (EOP sets from $13^{\text {th }}$ to $19^{\text {th }}$ ), the high correlation among them is possibly occurred and the inaccurate results are produced. All residual errors can be plotted for each results depending of each EOP sets. However some of them are chosen and shown in Figure 4.13 instead to show all of them. Here, the general situation is that


Figure 4.11: Graphical representation of accuracy of GCPs and ICPs in point sets homogenously distributed.


Figure 4.12: Graphical representation of accuracy of GCPs and ICPs in point sets homogenously distributed.


All GCPs (from left to right: $\mathrm{P}_{\mathrm{S}}, \mathrm{V}_{1}, \mathrm{~A}$ )



$\mathrm{H}-20$ (from left to right: $\mathrm{P}_{\mathrm{So}}, \mathrm{V}_{1}, \mathrm{~A}_{\mathrm{o}}$ )



$\mathrm{H}-40$ (from left to right: $\mathrm{P}_{\mathrm{S} 1}, \mathrm{~V}_{1}, \mathrm{~A}_{1}$ )




G-B (from left to right: $\mathrm{P}_{\mathrm{S} 2}, \mathrm{~V}_{2}, \mathrm{~A}_{2}$ )
Figure 4.13: Plot of residual errors in planimetry (diagonal) and in height (up-down) at GCPs and ICPs for some point sets with respect to EOP sets.
(H: Homogenous distribution, G: Grouped distribution, $\because$ GCP, o: ICP)
the direction and magnitude of the residuals errors are independent from the adjusted EOP, and they are almost same with the plots in Figure 4.9. This means the EOPs chosen to be adjusted do not affect the direction and magnitude of the residual errors. Here the major domination is based on the look angles adjusted. Only the GCPs located on left-below of the point set G-B have different geometry depending on the $\mathrm{P}_{\mathrm{S} 2}$. Nevertheless this does not change the general condition.

### 4.4.2.2 Results based on second type pre-adjustment

As mentioned several times, the difference between this and the previous sections depends on the pre-adjustment of EOPs. These EOPs are individually pre-adjusted for each image before their use in the bundle adjustment in this second type. Table 4.3 and Figure 4.8 in the previous section are valid for both sections. The results related to this section are summarized in the Appendix A.5.2 and A.5.3. $\overline{L P}$ is the pre-adjusted look angles and EOPs where $B$ is the bundle adjustment. At first, this can be said that both processes ( $\overline{L P}$ and $B$ ) produce equal RMSEs for the point sets including all GCPs (H-0). And secondly the RMSEs based on $\overline{L P}$ are sub-meter whereas they are blurred following the bundle adjustment for the EOPs from $13^{\text {th }}$ to $19^{\text {th }}$. This can not be said that the bundle adjustment improves the RMSEs since some of them becomes equal in both processes ( $\overline{L P}$ and $B$ ) where others of them becomes better or worse. Moreover, bundle adjustment gives inaccurate RMSEs for the GCPs for the EOP sets from $13^{\text {th }}$ to $19^{\text {th }}$.

Figures 4.14 illustrates the plotting of residual errors of $\mathrm{H}-0$ (including only GCPs) for some EOP sets such as $\mathrm{P}_{\mathrm{S}}, \mathrm{V}_{\mathrm{S}}$ and A . The direction and the magnitude of the residuals are almost similar depending on both the second type pre-adjustment (i.e. pre-adjusted look angles and EOPs) and the bundle adjustment. However the same examination has to be performed for the various point sets as shown in Figure 4.15 with various comparison. For instance, the residuals depending on the set $\mathrm{P}_{\mathrm{so}}$ for $\mathrm{H}-$ 10, the pre- and bundle-adjustment have different results growing in the second one. The direction of the residual errors are similar, nevertheless the magnitude on GCPs and ICPs are growing in the bundle adjustment. Similarly the magnitude of residuals on GCPs are growing for the set $\mathrm{V}_{\mathrm{o}}$ for $\mathrm{H}-20$. $\mathrm{A}_{\mathrm{o}}$ has almost equal results with respect to direction and magnitude of accuracies for the point set H-30 and G-A.


Figure 4.14: Plot of residual errors in planimetry (diagonal) and in height (up-down) at all GCPs for some EOP sets.


Figure 4.15: Plot of residual errors in planimetry (diagonal) and in height (updown) for some sets of points and EOPs. (left: pre-adj., right: bundle adj., H: Homogenous, G: Grouped, : GCP, o: ICP)

These examples chosen and shown in Figure 4.15 provides the direction and the magnitude of the residual errors can be changed depending on the EOP sets and the processing type (pre- or bundle adjustment). However the direction seems very strong against to be changed in the examples given. Similarly in the section of first type pre-adjustment, the accuracy is mostly varied by the EOP sets. The adjusted EOP sets excepting the $5^{\text {th }}$ and from $13^{\text {th }}$ to $19^{\text {th }}$, the accuracy of GCPs are generally sub-meter level as in the section related to the first type pre-adjustment. The accuracy of ICPs are about $\pm 1$ pixel ( $\pm 5 \mathrm{~m}$ ) and from $\pm 1$ pixel ( $\pm 5 \mathrm{~m}$ ) to $\pm 20$ pixels $( \pm 100 \mathrm{~m})$ in the point sets homogenously distributed and grouped respectively. The results based on these adjusted EOP sets are graphically illustrated in the Figure 4.16. Generally the accuracies of GCPs are smaller than $\pm 1 \mathrm{~m}$ being $m_{Z}>m_{X}>m_{Y}$ like in the first type pre-adjustment. In other words the accuracy in height $(Z)$ is generally larger than the accuracy in the planimetry ( $X$ and $Y$ ). Here the accuracy of $Y$ is the smallest by reason of that this axis is related to the look angles which is important on the estimation of true ground coordinates (see Table 4.3). The effects of the EOP sets $\mathrm{P}_{\mathrm{S}}$ and A both pre- and bundle-adjusted is in the Figure 4.16. The most important results achieved is that bundle adjustment has no advantage on the accuracies of both GCPs and ICPs. The accuracy of ICPs becomes worse with the increasing of their number from 10 to 40 where GCPs have almost equal accuracy for each point sets. Other results based on EOP sets from $13^{\text {th }}$ and $19^{\text {th }}$ are not shown since the accuracy of GCPs is extremely less.

The sets $5^{\text {th }}, 6^{\text {th }}$ and from $13^{\text {th }}$ and $19^{\text {th }}$ cause inaccurate results on the grouped points $(\mathrm{G})$ as similar as in the homogenously distributed points $(\mathrm{H})$. The higher accuracy on the GCPs is reached on the point set G-C whereas others have lower accuracy. The cause of these inaccurate results on G-A and G-C is based on the dependency of look angles with column number $(y)$ remembering the similar results achieved in the Table 4.3. The graphical representation of the accuracies of grouped point sets are not shown in Figure 4.16 since there is no any continuity among them.

### 4.4.3 Overview of Geometric Analysis

The geometric analysis is performed into two sections depending on the preadjustment of EOPs. In the first section only the look angles are pre-adjusted, and


Figure 4.16: Graphical representation of accuracy of GCPs and ICPs in point sets $P_{S}$ and A for the homogenously distributed points. $\overline{L P}$ : pre-adjusted look angles and EOPs, $B$ : bundle adjustment.
various EOPs are compensated in the bundle adjustment process. The effects of preadjusted EOPs are analysed in the second section.

The major domination on the getting accurate ground coordinates is based on the look angles. If the look angles are not pre- or bundle-adjusted, the estimated ground coordinates are extremely different compared to their true values. Usage of the preadjusted look angles assuming the EOPs constant overcomes this problem and the resulting the accuracy is sub-meter level and about $\pm 1$ pixel ( $\pm 5 \mathrm{~m}$ ) for GCPs and ICPs respectively. The pre- or bundle-adjusted EOPs in all analysis do not improve the results. However the assuming of all EOPs as the adjustment parameter is not available since the possible high correlation among them. The EOPs from $1^{\text {st }}$ to $12^{\text {th }}$ excepting $5^{\text {th }}$ and $6^{\text {th }}$ provide accurate results where others cause unstable solutions. The point distribution is the other subject to be examined. The accuracy of the GCPs and ICPs are similar in the point sets having homogenously distributed points. However if the GCPs or ICPs are located on one side of the image, the accuracy of ICPs becomes worse and the residual errors have systematic bias. This should be noted that the systematic effect is seen on the GCPs if the bundle adjustment is not applied.

## 5. CONCLUSION

Many imaging technologies and methods are developed for the wide range of applications. The linear array sensors, one of the imaging technologies, have many widespread usage on the geospatial applications. The geometry and the geometric accuracy provided by the image becomes important in this case. The questions can be "how the image has to be evaluated", "which methods should be used", and "whether auxiliary data is necessary" etc. Many answers for these questions can be found if previous studies and research are reviewed. However the main aim in this thesis is to geometrically analysis HRSIs based on linear array imaging technology using the parametric models and considering the orbital and attitude parameters of satellite.

Many parametric models are developed and have been presented. In this thesis the parametric model establishing the transformation from image to ground and vice versa is first presented in its generic form, and then its modification and simplification for the stereo SPOT-5 HRG level 1A images is explained. The generic model is cited from Weser et al. (2008) where the modified and simplified model is suggested by the SPOT Image (2002) and Fotev et al. (2005). Thus, this thesis does not claim and also not aim to generate a new parametric approach. The main motivation of this thesis is analysing the geometric accuracy of the images mentioned above, and strives to understand which component and parameter has to be considered to obtain accurate ground coordinates.

This section is continued into two parts. At first, both the pre- and bundleadjustments, effects of look angles and EOPs are discussed, and then the further works are suggested.

### 5.1 Discussion of Thesis

The major correction is required for the look angles consisting the inner orientation of the used model in this thesis. If the EOPs, i.e. exterior orientation parameters, are assumed constant and the look angles are compensated in the pre-adjustment process,
the accuracies on GCPs and ICPs can be extremely improved. However, a systematic effect on the GCPs occurs with respect to the direction and the magnitude of their residual errors. The compensation of the EOPs in both pre- and bundle adjustment is not meaningful. Moreover, their various configurations produces inaccurate results on both the homogenously distributed and grouped point sets. On the other hand, the advantage of bundle adjustment has to be discussed. The bundle adjustment recovers the systematic residual errors on the GCPs. Nevertheless, it has no any significant advantage with respect to the accuracy.

The point distribution is the other important issue which has to be considered. The types of point sets are established into two. One of them is based on homogenous distribution decreasing the number of GCPs from 47 to 7 and increasing the number of ICPs from 10 to 40 . The other sets include the located points on one side of the image. The point distribution is very important, since the accuracy of GCPs and ICPs are not significant for the homogenously distributed points. However the similar decision is not valid for the grouped points. If the GCPs lie on a whole line of the image similarly in G-B, its accuracy is higher than the others in comparison G-A and G-C. This reason can be explained by the look angles which are related to the line of image.

Finally the accuracy of GCPs are sub-meter level where ICPs have about $\pm 1$ pixel ( $\pm 5$ meter). If the confidence interval is assumed as $99.7 \%( \pm 3 \sigma)$, the accuracy of ground coordinates of ICPs equal or smaller than $\pm 3$ pixel ( $\sigma= \pm 1$ pixel) are statistically valid. It is interesting that this value (3 pixel) is the minimum required number of adjacent pixels for point determination (Srivastava et al., 1996) on the images having high radiometric resolution. The radiometric resolution of the images investigated in this thesis, stereo SPOT-5 HRG level 1A panchromatic images, have 8 bits (256 grey values) radiometric resolution. This resolution was compared by the other linear array images (Topan et al., 2009), and it is realized that its radiometric quality is sufficient for the point detection within 3 pixels. Thus, the results of this thesis can be scientifically acceptable.

### 5.2 Further Work

Some further works can be suggested as following:

- The generic adjustment model, called collocation, includes the constrains between the adjustment parameters which is not considered in this thesis. So the functional model can be extended by the collocation, and the results can be compared with the results in this thesis.
- Many auxiliary interpolation and extrapolations, such as Lagrange or linear estimation, are preferred both in the pre-processing and the adjustment. The effects of these inevitable methods can be investigated.
- SPOT Image suggests and provides the transformation parameters between level 1A and 1B images. So the level 1B images can be back-projected and their accuracy can be investigated as similar as in this thesis.
Finally, the readers should take note the fact that the further aims can be varied by the many methods and many assumptions to investigate (or analyse) the images with or without high geometric resolution. Thus, much further research based on this scope can be performed. The needed is only the wish.


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## APPENDICES

APPENDIX A. 1 : Symbols
APPENDIX A. 2 : Coordinate Systems
APPENDIX A. 3 : Rotation Around 3D Coordinate Systems
APPENDIX A. 4 : Ground Coordinate Estimation from Stereo Images
APPENDIX A. 5 : Results of Geometric Analysis

## APPENDIX A.1: Symbols* ${ }^{*}$

| $a_{r}, a_{p}, a_{y}$ | : roll, pitch, yaw, respectively. |
| :---: | :---: |
| $\underline{A}$ | : Design matrix of unknowns |
| $\underline{B}$ | : Design matrix of residuals |
| c | : Focal length |
| $\vec{c}$ | : vector consisting exterior orientation parameters |
| $C_{M}$ | : Camera mounting parameter |
| $\underline{d P}$ | : Unknown vector |
| $f$ | : True anomaly |
| $F$ | : Function |
| $g$ | : number of position and velocity of satellite |
| H | : Flying height |
| inc | : Orbital inclination |
| $\underline{l}$ | : Absolute term vector |
| $L$ | : Observation |
| $m$ | : Scale factor |
| $m$ | : Number of images |
| $n$ | : Number of points |
| $N$ | : Ascending node |
| $N^{\prime}$ | : Descending node |
| $N_{c}$ | : Number of column |
| $N_{r}$ | : Number of raw |
| O | : Centre of Earth |
| $p$ | $:$ Pixel size ( $p_{x}, p_{y}$ in $x$ and $y$ direction, respectively) |
| $\boldsymbol{P}$ | : Perigee |
| $\vec{P}$ | : Position vector in GCS |
| $P_{j, i}$ | : One of the parameters of the line $i(j=1(1) 30)$ |
| $\vec{p}_{c}$ | : Position vector in CCS |
| $\vec{p}_{0}$ | : Position vector in OCS |
| $\vec{p}_{p}$ | : Position vector in PCS |

[^0]| $p_{s}$ | : Position vector in SCS |
| :---: | :---: |
| $\vec{P}_{S}$ | Satellite position vector |
| $P_{0}$ | Exterior orientation parameter for the reference line |
| $\dot{P}$ | : Drift |
| $\ddot{P}$ | : Drift rates, |
| $p s g_{x}$ | : Pixel size on ground in the direction of motion (x) |
| $p s g_{y}$ | : Pixel size on ground in the perpendicular direction to the motion (y) |
| $r$ | : Geocentric distance to the satellite |
| $r$ | : Elements of $R$ |
| $R$ | : Rotation matrix around related axis in a right-hand coordinate system |
| $S$ | : Satellite |
| $t$ | : Time |
| $t_{0}$ | : Acquisition time for reference line |
| $v$ | : Residual, |
| V | : Velocity of satellite/platform |
| $\vec{V}_{S}$ | Satellite velocity vector |
| $w_{p}$ | : Argument of perigee |
| w | : Misclosure |
| $x, y$ | : Image coordinates |
| $x_{0}, y_{0}$ | : Image coordinates of principal point |
| $x_{s}, y_{s}$ | : Scanline coordinates |
| $X, Y, Z$ | : Ground/object coordinates |
| $X_{0}, Y_{0}, Z_{0}$ | : Ground/object coordinates of perspective center |
| $\Delta t$ | : Time interval |
| $\Omega$ | : Right ascension of the ascending node |
| $\theta_{G R}$ | : Greenwich Mean Sidereal Time |
| $\omega, \varphi, \kappa$ | : rotation angles around $X, Y$ and $Z$ axis, respectively |
| $\gamma$ | : First point of aries |
| $\delta \vec{x}$ | : vector of systematic error corrections |
| $\pi$ | : Pi constant |
| $\psi_{x}$ | : Look angle in $x$ direction |


| $\psi_{y}$ | $:$ Look angle in $y$ direction |
| :--- | :--- |
| $\left.\right\|_{0}$ | $:$ Initial value of parameters |
| $\\|\\|$ | $:$ Norm of vector |

## APPENDIX A.2: Coordinate Systems

A generic model establishes a geometric relationship between image and ground coordinate systems which the points are measured on both. However, the real relationship is established among the auxiliary coordinate systems. All required coordinate systems are:

1. Image coordinate system (ICS)
2. Scanline coorditane system (SCS)
3. Camera coordinate system (CCS)
4. Payload coordinate system (PCS)
5. Orbit coordinate system (OPS), and
6. Ground coordinate system(GCS)

## Image coordinate system

Image coordinate system is a 2 D coordinate system. Its origin is at the centre of lefttop pixel, and the position is defined by row $(x)$ and column $(y)$ number in pixel unit (Figure A.2.1). The image dimensions are described by total number of rows ( $N_{r}$ ) and of colums $\left(N_{c}\right)$. Both axis are perpendicular to each other.


Figure A.2.1: Image coordinate system.

## Scanline coordinate system

In the case of imaging using linear array sensors, each sensor has its own coordinate system called as scanline coordinate system with its origin is leftmost pixel of the corresponding line (Figure A.2.2). $x_{s}$ is related to the acquisition time of line and always equals to zero. Both axes are perpendicular to each other (Poli, 2005). $N_{c}$ refers number of elements in each line.


Figure A.2.2: Scanline coordinate system.
In an ideal case without lens distortions and geometric errors in the CCD line, the $y_{s}$ in meter can be calculated by the image coordinate $y$ as following:

$$
\begin{equation*}
y_{s}=y \cdot p_{y} \tag{A.2.1}
\end{equation*}
$$

## Camera coordinate system

Camera coordinate system is a 3D right-hand coordinate system. Its origin is at the projection center, with the $y_{c}$ axis being along the $y_{s}$, and the $z_{c}$ axis being orthogonal to the focal plane (Figure B.3).


Figure A.2.3: Camera coordinate system.

## Payload coordinate system

Payload coordinate system, a 3D right-hand coordinate system, is fixed to the payload. Its origin is payload's mass center. The transformation from this coordinate system to orbital coordinate system is a time-dependent rotation parameterised by the three attitude angles ( $a_{r}, a_{p}, a_{y}$ ) around $x, y$ and $z$ axes, respectively.

## Orbital coordinate system

Orbital coordinate system, a 3D right-hand coordinate system, is fixed to the payload. Its origin is payload's mass center, with the $x_{o}$ axis being close to mean flight direction, $x_{o} z_{o}$ plane is on the orbital plane and $y_{o}$ axis completes a right-hand
coordinate system (Figure A.2.4). The transformation from this to ground coordinate system is a time-dependent rotation performed by Keplerian angular elements and GMST, or position and velocity vectors of satellite.

## Ground coordinate system

The ground coordinate system corresponds Conventional Terrestial Coordinate System (CTCS) in this thesis since the GCPs and satellite position vector are defined in this coordinate system. CTCS is a geocentric system that its origin is at the centre of Earth. $Z$ axis points to the conventional origin which is defined as the mean position of the instantaneous pole during the period 1900 to 1905 . The $X Z$ plane contains the mean Greenwhich Observatory and $Y$ axis completes a right-hand coordinate system (Zoej, 1997).


Figure A.2.4: Orbital and ground coordinate systems.

## APPENDIX A.3: Rotation Around 3D Coordinate Systems

Rotation around the $j$ axis in a right-hand coordinate system illustrated in Figure A.3.1. can be written in a matrix form as followings:


Figure A.3.1. Contraclockwise rotations.

$$
\begin{array}{lcc}
\boldsymbol{j} & \text { Contraclockwise } & \text { Clockwise } \\
x & R_{x}=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \omega & -\sin \omega \\
0 & \sin \omega & \cos \omega
\end{array}\right] & R_{x}^{-1}=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \omega & \sin \omega \\
0 & -\sin \omega & \cos \omega
\end{array}\right] \\
y & R_{y}=\left[\begin{array}{ccc}
\cos \varphi & 0 & \sin \varphi \\
0 & 1 & 0 \\
-\sin \varphi & 0 & \cos \varphi
\end{array}\right] & R_{y}^{-1}=\left[\begin{array}{ccc}
\cos \varphi & 0 & -\sin \varphi \\
0 & 1 & 0 \\
\sin \varphi & 0 & \cos \varphi
\end{array}\right] \\
z & R_{z}=\left[\begin{array}{ccc}
\cos \kappa & -\sin \kappa & 0 \\
\sin \kappa & \cos \kappa & 0 \\
0 & 0 & 1
\end{array}\right] & R_{z}^{-1}=\left[\begin{array}{ccc}
\cos \kappa & \sin \kappa & 0 \\
-\sin \kappa & \cos \kappa & 0 \\
0 & 0 & 1
\end{array}\right]
\end{array}
$$

The clockwise rotation is inverse or transpose of contraclockwise rotation matrix since the rotation matrix is orthogonal.

## Elements of Matrix $\boldsymbol{R}_{P O}$

$R$ is the final matrix to transform PCS into GCS. Step by step matrix calculation and elements of both auxiliary and final matrix are summarized. Matrix $R_{P O}$ is calculated as following:

|  | $R_{Z}\left(a_{y}\right)=\left[\begin{array}{ccc}\cos \left(a_{y}\right) & \sin \left(a_{y}\right) & 0 \\ -\sin \left(a_{y}\right) & \cos \left(a_{y}\right) & 0 \\ 0 & 0 & 1\end{array}\right]$ |
| :---: | :---: |
| $R_{Y}\left(a_{p}\right)=\left[\begin{array}{ccc}\cos \left(a_{p}\right) & 0 & -\sin \left(a_{p}\right) \\ 0 & 1 & 0 \\ \sin \left(a_{p}\right) & 0 & \cos \left(a_{p}\right)\end{array}\right]$ | $R_{Y}($ pitch $) R_{Z}($ yaw $)$ |
| $R_{X}\left(a_{r}\right)=\left[\begin{array}{ccc}1 & 0 & 0 \\ 0 & \cos \left(a_{r}\right) & \sin \left(a_{r}\right) \\ 0 & -\sin \left(a_{r}\right) & \cos \left(a_{r}\right)\end{array}\right]$ | $R_{O P}$ |

Elements of matrix $R_{O P}$ become:

$$
\begin{align*}
& R_{1,1}^{O P}=\cos \left(a_{p}\right) \cdot \cos \left(a_{y}\right) \\
& R_{1,2}^{O P}=-\cos \left(a_{p}\right) \cdot \sin \left(a_{y}\right) \\
& R_{1,3}^{O P}=\sin \left(a_{p}\right) \\
& R_{2,1}^{O P}=\cos \left(a_{r}\right) \cdot \sin \left(a_{y}\right)+\sin (w) \cdot \sin \left(a_{p}\right) \cdot \cos \left(a_{y}\right) \\
& R_{2,2}^{O P}=\cos \left(a_{y}\right) \cdot \cos (w)-\sin \left(a_{y}\right) \cdot \sin \left(a_{p}\right) \cdot \sin (w)  \tag{A.3.1}\\
& R_{2,3}^{O P}=-\cos \left(a_{p}\right) \cdot \sin (w) \\
& R_{3,1}^{O P}=\sin \left(a_{r}\right) \cdot \sin \left(a_{y}\right)-\cos \left(a_{r}\right) \cdot \sin \left(a_{p}\right) \cdot \cos \left(a_{y}\right) \\
& R_{3,2}^{O P}=\sin \left(a_{y}\right) \cdot \sin \left(a_{p}\right) \cdot \cos \left(a_{r}\right)+\cos \left(a_{y}\right) \cdot \sin \left(a_{r}\right) \\
& R_{3,3}^{O P}=\cos \left(a_{p}\right) \cdot \cos \left(a_{r}\right)
\end{align*}
$$

## Elements of Matrix $\boldsymbol{R}_{\boldsymbol{O G}}$

Matrix $R_{O G}$ can be established by two ways. The first way is performed by the angular Keplerian elements $\left(f, w_{p}, \Omega\right.$ and $\left.i n c\right)$ shown in Figure A.3.2 and Greenwich Mean Sidereal Time $\left(\theta_{G R}\right)$, the angle between CICS and GCS around $Z$ axis. The second and easier way is performed by the position $\left(\vec{P}_{S}\right)$ and velocity $\left(\vec{V}_{S}\right)$ vectors of satellite.

In the first way, the Keplerian elements can be estimated as following:


Figure A.3.2: Keplerian elements in CICS and GCS.
$\vec{C}=\vec{P}_{S} \times \vec{V}_{S}=\left[\begin{array}{c}C_{X} \\ C_{Y} \\ C_{Z}\end{array}\right]$
$C=\|\vec{C}\|$
$r=\left\|\vec{P}_{s}\right\|$
$i n c=\arccos \left(\frac{C_{Z}}{C}\right)$
$\Omega=\arctan \left(\frac{C_{X}}{C_{Y}}\right)$
$f+w_{p}=\arcsin \left(\frac{Z_{s}}{r \cdot \sin (i n c)}\right)$
where $\times=$ cross product, and $\|\cdot\|=$ norm of the vector.
Matrix $R_{O G}$ is calculated as following:

|  | $\begin{aligned} & R_{Z}\left(\pi-\left(\Omega-\theta_{G R}\right)\right)= \\ & {\left[\begin{array}{ccc} -\cos \left(\Omega-\theta_{G R}\right) & \sin \left(\Omega-\theta_{G R}\right) & 0 \\ -\sin \left(\Omega-\theta_{G R}\right) & -\cos \left(\Omega-\theta_{G R}\right) & 0 \\ 0 & 0 & 1 \end{array}\right]} \end{aligned}$ |
| :---: | :---: |
| $\begin{aligned} & R_{X}\left(\frac{\pi}{2}-i n c\right)= \\ & {\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & \sin (i n c) & -\cos (i n c) \\ 0 & \cos (i n c) & \sin (i n c) \end{array}\right]} \end{aligned}$ | $R_{X}\left(\frac{\pi}{2}-i n c\right) R_{Z}\left(\pi-\left(\Omega-\theta_{G R}\right)\right)$ |
| $\begin{aligned} & R_{Y}\left(\frac{\pi}{2}-\left(f+w_{p}\right)\right)= \\ & {\left[\begin{array}{ccc} \sin \left(f+w_{p}\right) & 0 & -\cos \left(f+w_{p}\right) \\ 0 & 1 & 0 \\ \cos \left(f+w_{p}\right) & 0 & \sin \left(f+w_{p}\right) \end{array}\right]} \end{aligned}$ | $R_{O G}$ |

Elements of matrix $R_{O G}$ becomes:

$$
\begin{align*}
& R_{1,1}^{O G}=-\cos \left(\Omega-\theta_{G R}\right) \sin \left(f+w_{p}\right)-\sin \left(\Omega-\theta_{G R}\right) \cos (\text { inc }) \cos \left(f+w_{p}\right) \\
& R_{1,2}^{O G}=\sin \left(\Omega-\theta_{G R}\right) \sin (\text { inc }) \\
& R_{1,3}^{O G}=\cos \left(\Omega-\theta_{G R}\right) \cos \left(f+w_{p}\right)-\sin \left(\Omega-\theta_{G R}\right) \cos (\text { inc }) \sin \left(f+w_{p}\right) \\
& R_{2,1}^{O G}=-\sin \left(\Omega-\theta_{G R}\right) \sin \left(f+w_{p}\right)+\cos \left(\Omega-\theta_{G R}\right) \cos (\text { inc }) \cos \left(f+w_{p}\right) \\
& R_{2,2}^{O G}=-\cos \left(\Omega-\theta_{G R}\right) \sin (\text { inc })  \tag{A.3.8}\\
& R_{2,3}^{O G}=\sin \left(\Omega-\theta_{G R}\right) \cos \left(f+w_{p}\right)+\cos \left(\Omega-\theta_{G R}\right) \cos (\text { inc }) \sin \left(f+w_{p}\right) \\
& R_{3,1}^{O G}=\sin (\text { inc }) \cos \left(f+w_{p}\right) \\
& R_{3,2}^{O G}=\cos (\text { inc }) \\
& R_{3,3}^{O G}=\sin (\text { inc }) \sin \left(f+w_{p}\right)
\end{align*}
$$

where inc is the orbital inclination, $\Omega$ is right ascension of the ascending node, $f$ is true anomaly, $w_{p}$ argument of perigee, $\theta_{G R}$ is Greenwich Mean Sidereal Time.

In the second way, using $\vec{P}_{S}$ and $\vec{V}_{S}$ elements of $R_{O G}$ becomes:
$\left[\begin{array}{l}R_{3,1}^{O G} \\ R_{3,2}^{O G} \\ R_{3,3}^{O G}\end{array}\right]=\frac{\vec{P}_{S}}{\left\|\vec{P}_{S}\right\|}$
(A.3.9)

$$
\begin{aligned}
& {\left[\begin{array}{l}
R_{2,1}^{O G} \\
R_{2,2}^{O G} \\
R_{2,3}^{O G}
\end{array}\right]=\frac{\left[\begin{array}{l}
R_{3,1}^{O G} \\
R_{3,2}^{O G} \\
R_{3,3}^{O G}
\end{array}\right] \times \vec{V}_{S}}{\left\|\left[\begin{array}{l}
R_{3,1}^{O G} \\
R_{3,2}^{O G} \\
R_{3,3}^{O G}
\end{array}\right] \times \vec{V}_{S}\right\|}} \\
& {\left[\begin{array}{l}
R_{1,1}^{O G} \\
R_{1,2}^{O G} \\
R_{1,3}^{O G}
\end{array}\right]=\frac{\left[\begin{array}{l}
R_{2,1}^{O G} \\
R_{2,2}^{O G} \\
R_{2,3}^{O G}
\end{array}\right] \times\left[\begin{array}{l}
R_{3,1}^{O G} \\
R_{3,2}^{O G} \\
R_{3,3}^{O G}
\end{array}\right]}{\left\|\left[\begin{array}{l}
R_{2,1}^{O G} \\
R_{2,2}^{O G} \\
R_{2,3}^{O G}
\end{array}\right] \times\left[\begin{array}{l}
R_{3,1}^{O G} \\
R_{3,2}^{O G} \\
R_{3,3}^{O G}
\end{array}\right]\right\|}}
\end{aligned}
$$

(A.3.10)
(A.3.11)

Appendix A.4: Ground Coordinate Estimation from Stereo Images
The ground coordinates of a point can be intersected from at least two stereo images (Figure A.4.1).


Figure A.4.1: Point intersection from sreteo images.
The ground coordinates can be estimated by the mathematical model derived from the colinearity equations as following:
$\left[\begin{array}{c}X \\ Y \\ Z\end{array}\right]=\left[\begin{array}{c}X_{0} \\ Y_{0} \\ Z_{0}\end{array}\right]+m \underline{R}\left[\begin{array}{c}x-x_{o} \\ y-y_{o} \\ -c\end{array}\right]$
Since the ground coordinates of a point has to be equal from the derivation of both images, the following equation can be written:
$\left[\begin{array}{c}X \\ Y \\ Z\end{array}\right]=\left[\begin{array}{c}X_{0} \\ Y_{0} \\ Z_{0}\end{array}\right]_{1}+m_{1} \underline{R}_{1}\left[\begin{array}{c}x-x_{o} \\ y-y_{o} \\ -c\end{array}\right]_{1}=\left[\begin{array}{c}X_{0} \\ Y_{0} \\ Z_{0}\end{array}\right]_{2}+m_{2} \underline{R}_{2}\left[\begin{array}{c}x-x_{o} \\ y-y_{o} \\ -c\end{array}\right]_{2}$
It yields the system in matrix notation that:

$$
\begin{aligned}
& {\left[\underline{R}_{1}\left[\begin{array}{c}
x-x_{o} \\
y-y_{o} \\
-c
\end{array}\right]_{1}-\underline{R}_{2}\left[\begin{array}{c}
x-x_{o} \\
y-y_{o} \\
-c
\end{array}\right]_{2}\right] \cdot\left[\begin{array}{c}
m_{1} \\
m_{2}
\end{array}\right]=\left[\left[\begin{array}{c}
X_{0} \\
Y_{0} \\
Z_{0}
\end{array}\right]_{2}-\left[\begin{array}{c}
X_{0} \\
Y_{0} \\
Z_{0}
\end{array}\right]_{1}\right]} \\
& \underline{A} \cdot \underline{m}=\underline{b} \\
& \underline{m}=\left(\underline{A}^{T} \underline{A}\right)^{-1} \underline{A}^{T} \underline{b}
\end{aligned}
$$

(A.4.3)

Replacing these scales ( $m_{1}$ and $m_{2}$ ) the ground coordinates of a point can be estimated in equation A.4.1.

Appendix A.5.1. Results based on first type pre-adjustment

| Point | set | H-0 | H-10 |  | H-20 |  | H-30 |  | H-40 |  | G-A |  | G-B |  | G-C |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | type | GCP | GCP | ICP | GCP | ICP | GCP | ICP | GCP | ICP | GCP | ICP | GCP | ICP | GCP | ICP |
| EOP | \# | 47 | 37 | 10 | 27 | 20 | 17 | 30 | 7 | 40 | 23 | 24 | 24 | 23 | 17 | 30 |
| 1. $\mathrm{P}_{\mathrm{S}}$ | $m_{X}$ | 0.597 | 0.596 | 5.979 | 0.592 | 4.838 | 0.515 | 5.284 | 0.427 | 7.293 | 11.454 | 20.291 | 0.686 | 15.140 | 39.595 | 109.251 |
|  | $m_{Y}$ | 0.215 | 0.221 | 5.743 | 0.265 | 5.325 | 0.343 | 5.982 | 0.696 | 6.838 | 8.953 | 16.239 | 0.829 | 15.683 | 24.061 | 88.602 |
|  | $m_{Z}$ | 0.943 | 0.978 | 6.622 | 0.965 | 6.199 | 0.947 | 6.295 | 0.919 | 6.397 | 12.699 | 14.102 | 1.012 | 9.242 | 42.480 | 96.538 |
| 2. $\mathrm{P}_{\mathrm{s} 0}$ | $m_{X}$ | 0.597 | 0.595 | 5.976 | 0.587 | 4.846 | 0.532 | 5.284 | 0.487 | 7.128 | 5.685 | 12.244 | 0.602 | 4.989 | 8.962 | 10.880 |
|  | $m_{Y}$ | 0.215 | 0.208 | 5.739 | 0.191 | 5.323 | 0.200 | 5.981 | 0.699 | 6.672 | 9.426 | 19.381 | 0.342 | 5.727 | 15.136 | 32.999 |
|  | $m_{Z}$ | 0.943 | 0.972 | 6.610 | 0.947 | 6.176 | 0.924 | 6.246 | 0.455 | 6.412 | 7.261 | 21.801 | 1.028 | 10.720 | 11.264 | 21.426 |
| 3. $\mathrm{P}_{\mathrm{S} 1}$ | $m_{X}$ | 0.606 | 0.601 | 5.971 | 0.616 | 4.828 | 0.595 | 5.210 | 1.211 | 7.309 | 4.659 | 18.626 | 0.875 | 4.940 | 6.621 | 21.072 |
|  | $m_{Y}$ | 0.213 | 0.225 | 5.749 | 0.227 | 5.313 | 0.236 | 5.940 | 0.920 | 6.704 | 10.236 | 19.621 | 0.648 | 5.620 | 12.590 | 40.075 |
|  | $m_{\text {Z }}$ | 0.950 | 0.971 | 6.612 | 0.960 | 6.181 | 0.950 | 6.231 | 1.937 | 6.048 | 6.100 | 28.842 | 1.292 | 10.926 | 7.578 | 22.468 |
| 4. $\mathrm{P}_{\mathrm{s} 2}$ | $m_{X}$ | 0.599 | 0.599 | 5.972 | 0.613 | 4.835 | 0.591 | 5.234 | 0.589 | 7.160 | 5.637 | 21.670 | 0.634 | 4.990 | 5.808 | 20.886 |
|  | $m_{Y}$ | 0.215 | 0.223 | 5.742 | 0.219 | 5.310 | 0.232 | 5.945 | 0.630 | 6.786 | 11.725 | 21.277 | 0.542 | 5.580 | 10.105 | 43.183 |
|  | $m_{Z}$ | 0.944 | 0.965 | 6.608 | 0.949 | 6.173 | 0.932 | 6.219 | 0.686 | 6.382 | 7.377 | 36.448 | 1.052 | 11.047 | 6.976 | 16.473 |
| 5. $\mathrm{V}_{\mathrm{s}}$ | $m_{X}$ | 0.735 | 939489.984 | 5.971 | 857488.941 | 4.819 | 1426273.558 | 5.167 | 925835.426 | 6.855 | 2788377.306 | 24.144 | 0.394 | 5.296 | 1223649.079 | 69.468 |
|  | $m_{Y}$ | 0.181 | 1193598.058 | 5.743 | 601294.331 | 5.305 | 1748692.408 | 5.907 | 578400.232 | 6.804 | 1392198.459 | 32.283 | 0.348 | 5.298 | 941124.562 | 50.211 |
|  | $m_{Z}$ | 1.091 | 1058066.953 | 6.601 | 881630.195 | 6.175 | 1617679.447 | 6.217 | 955731.588 | 6.777 | 2736008.555 | 9.318 | 1.229 | 12.913 | 1284129.062 | 88.309 |
| 6. $\mathrm{V}_{0}$ | $m_{X}$ | 0.637 | 0.371 | 5.972 | 1.873 | 4.819 | 6.360 | 5.170 | 14.250 | 7.218 | 1489.062 | 29.475 | 77.553 | 5.235 | 7173.940 | 42.218 |
|  | $m_{Y}$ | 0.205 | 0.415 | 5.738 | 0.625 | 5.337 | 3.025 | 5.999 | 3.631 | 6.856 | 526.387 | 31.650 | 46.626 | 5.756 | 1597.374 | 55.948 |
|  | $m_{Z}$ | 0.986 | 0.641 | 6.603 | 2.388 | 6.169 | 7.097 | 6.237 | 15.892 | 6.483 | 1460.937 | 6.132 | 79.687 | 9.686 | 6894.834 | 11.020 |
| 7. $\mathrm{V}_{1}$ | $m_{X}$ | 0.602 | 0.603 | 5.972 | 0.614 | 4.828 | 0.579 | 5.207 | 0.614 | 7.169 | 0.825 | 26.239 | 0.518 | 5.290 | 1.230 | 34.041 |
|  | $m_{Y}$ | 0.214 | 0.221 | 5.742 | 0.229 | 5.310 | 0.252 | 5.937 | 0.218 | 6.825 | 0.393 | 32.862 | 0.167 | 5.930 | 0.371 | 54.108 |
|  | $m_{Z}$ | 0.948 | 0.980 | 6.609 | 0.991 | 6.180 | 1.034 | 6.232 | 1.404 | 6.367 | 0.833 | 7.491 | 1.051 | 10.061 | 2.793 | 10.958 |
| 8. $\mathrm{V}_{2}$ | $m_{X}$ | 0.613 | 0.614 | 5.971 | 0.630 | 4.826 | 0.594 | 5.207 | 0.574 | 7.188 | 0.714 | 26.023 | 0.513 | 5.474 | 1.762 | 33.091 |
|  | $m_{Y}$ | 0.211 | 0.223 | 5.744 | 0.229 | 5.311 | 0.240 | 5.938 | 0.116 | 6.811 | 0.228 | 32.900 | 0.221 | 5.931 | 0.315 | 54.086 |
|  | $m_{\text {Z }}$ | 0.960 | 0.984 | 6.609 | 0.986 | 6.174 | 1.022 | 6.223 | 1.402 | 6.375 | 0.868 | 7.147 | 1.045 | 10.378 | 1.408 | 9.203 |
| 9. A | $m_{X}$ | 0.601 | 0.600 | 5.971 | 0.621 | 4.827 | 0.587 | 5.200 | 0.564 | 7.183 | 0.645 | 23.024 | 0.584 | 5.246 | 2.011 | 26.994 |
|  | $m_{Y}$ | 0.214 | 0.215 | 5.743 | 0.208 | 5.309 | 0.203 | 5.935 | 0.164 | 6.825 | 0.399 | 32.445 | 0.221 | 5.678 | 0.509 | 48.639 |
|  | $m_{Z}$ | 0.946 | 0.974 | 6.607 | 0.980 | 6.172 | 0.992 | 6.216 | 1.134 | 6.327 | 0.975 | 5.146 | 1.076 | 7.012 | 2.723 | 63.261 |
| 10. $\mathrm{A}_{0}$ | $m_{X}$ | 0.601 | 0.600 | 5.970 | 0.617 | 4.824 | 0.589 | 5.199 | 0.605 | 7.185 | 0.451 | 25.632 | 0.473 | 5.271 | 0.441 | 30.178 |
|  | $m_{Y}$ | 0.214 | 0.218 | 5.744 | 0.215 | 5.309 | 0.211 | 5.935 | 0.121 | 6.824 | 0.432 | 32.788 | 0.268 | 5.882 | 0.558 | 53.995 |
|  | $m_{Z}$ | 0.946 | 0.972 | 6.605 | 0.968 | 6.169 | 0.969 | 6.213 | 1.092 | 6.319 | 0.623 | 6.907 | 0.974 | 10.040 | 0.478 | 6.756 |
| 11. $\mathrm{A}_{1}$ | $m_{X}$ | 0.601 | 0.604 | 5.971 | 0.623 | 4.828 | 0.597 | 5.209 | 0.654 | 7.166 | 1.076 | 24.397 | 0.494 | 5.196 | 1.384 | 29.259 |
|  | $m_{Y}$ | 0.214 | 0.222 | 5.743 | 0.222 | 5.310 | 0.228 | 5.937 | 0.235 | 6.811 | 0.307 | 32.520 | 0.164 | 5.869 | 0.337 | 53.713 |
|  | $m_{Z}$ | 0.947 | 0.975 | 6.610 | 0.971 | 6.178 | 0.983 | 6.227 | 1.270 | 6.333 | 1.331 | 5.738 | 1.077 | 9.881 | 1.252 | 7.381 |
| 12. $\mathrm{A}_{2}$ | $m_{X}$ | 0.601 | 0.606 | 5.971 | 0.623 | 4.828 | 0.593 | 5.208 | 0.637 | 7.210 | 0.611 | 24.741 | 0.510 | 5.208 | 0.846 | 29.259 |
|  | $m_{Y}$ | 0.214 | 0.220 | 5.743 | 0.223 | 5.310 | 0.230 | 5.936 | 0.115 | 6.817 | 0.412 | 32.565 | 0.238 | 5.871 | 0.543 | 53.713 |
|  | $m_{Z}$ | 0.946 | 0.970 | 6.609 | 0.963 | 6.171 | 0.966 | 6.216 | 1.044 | 6.329 | 1.110 | 6.010 | 0.970 | 9.857 | 0.849 | 7.381 |
| 13. $\mathrm{P}_{\mathrm{s}} \mathrm{V}_{\mathrm{s}}$ | $m_{X}$ | 94.687 | 3363754.206 | 5.993 | 2480619.438 | 4.850 | 3299827.426 | 5.415 | 1194847.286 | 46.426 | 2214311.619 | 19.880 | 35111.533 | 14.855 | 94018.431 | 405.849 |
|  | $m_{Y}$ | 25.882 | 2038666.201 | 5.690 | 1358110.296 | 5.314 | 2058852.485 | 5.954 | 868950.819 | 39.492 | 667503.266 | 28.373 | 25961.300 | 24.132 | 53648.759 | 351.632 |
|  | $m_{Z}$ | 91.547 | 3371220.737 | 6.579 | 2457473.719 | 6.147 | 3315454.877 | 6.513 | 1239793.430 | 46.702 | 2135298.602 | 7.112 | 36660.399 | 20.767 | 95661.648 | 432.227 |
| 14. $\left(\mathrm{P}_{\mathrm{s}} \mathrm{V}\right)_{0}$ | $m_{X}$ | 0.331 | 614.359 | 5.976 | 1540.174 | 4.835 | 4310.204 | 5.231 | 82961.642 | 7.521 | 15111.947 | 25.055 | 1033.349 | 5.257 | 156780.579 | 28.222 |
|  | $m_{Y}$ | 0.332 | 189.215 | 5.750 | 418.697 | 5.319 | 1157.833 | 5.948 | 24295.106 | 6.924 | 7559.826 | 32.487 | 288.964 | 5.907 | 90957.713 | 52.399 |
|  | $m_{Z}$ | 0.454 | 597.950 | 6.604 | 1490.247 | 6.166 | 4165.599 | 6.222 | 80604.260 | 6.493 | 15147.503 | 7.293 | 995.090 | 9.823 | 160271.802 | 7.026 |
| 15. $\mathrm{P}_{\mathrm{S}} \mathrm{A}$ | $m_{X}$ | 6.614 | 93.776 | 5.977 | 86.367 | 4.817 | 949.367 | 5.014 | 461575.423 | 220.047 | 6967.113 | 16.205 | 1476.484 | 51.142 | 24930.954 | 77.463 |
|  | $m_{Y}$ | 2.311 | 81.840 | 5.743 | 423.180 | 5.298 | 647.874 | 5.942 | 330422.466 | 89.154 | 10035.801 | 12.922 | 802.246 | 30.264 | 18424.392 | 52.487 |
|  | $m_{Z}$ | 6.196 | 65.991 | 6.595 | 178.262 | 6.132 | 598.511 | 5.963 | 423234.552 | 241.121 | 6705.377 | 19.952 | 916.408 | 62.691 | 7794.052 | 109.205 |
| 16. $\left(\mathrm{P}_{\mathrm{S}} \mathrm{A}\right)_{0}$ | $m_{X}$ | 1.762 | 16.701 | 5.971 | 42.951 | 4.814 | 85.024 | 5.178 | 157.044 | 7.171 | 1253.439 | 27.873 | 93.470 | 5.195 | 1882.853 | 28.279 |
|  | $m_{Y}$ | 0.852 | 37.442 | 5.739 | 169.735 | 5.319 | 304.137 | 5.982 | 457.906 | 6.885 | 250.475 | 31.354 | 551.640 | 5.837 | 673.271 | 52.796 |
|  | $m_{Z}$ | 1.814 | 14.821 | 6.601 | 63.885 | 6.150 | 124.554 | 6.186 | 326.735 | 6.329 | 343.224 | 6.602 | 245.020 | 10.208 | 1359.740 | 7.005 |
| 17. $\mathrm{V}_{\mathrm{s}} \mathrm{A}$ | $m_{X}$ | 437352.820 | 2677225.250 | 5.986 | 1710947.167 | 4.855 | 1855043.749 | 5.274 | 573282.242 | 29.712 | 378437.359 | 24.165 | 939513.405 | 25.168 | 502784.361 | 241.554 |
|  | $m_{Y}$ | 547928.028 | 1152129.924 | 5.736 | 1269998.173 | 5.307 | 497372.024 | 5.881 | 404705.516 | 79.708 | 363825.969 | 50.175 | 598629.738 | 20.127 | 380398.629 | 303.424 |
|  | $m_{Z}$ | 492436.992 | 2622689.290 | 6.613 | 1818760.336 | 6.176 | 1912697.458 | 6.220 | 542825.278 | 39.172 | 397867.069 | 46.751 | 967996.618 | 17.352 | 715962.609 | 227.476 |
| 18. $\left(\mathrm{V}_{\mathrm{S}} \mathrm{A}\right)_{0}$ | $m_{X}$ | 0.324 | 13.270 | 5.972 | 31.407 | 4.827 | 81.115 | 5.200 | 529.367 | 7.189 | 2329.453 | 25.978 | 253.662 | 5.176 | 8761.155 | 32.633 |
|  | $m_{Y}$ | 0.327 | 5.025 | 5.743 | 19.449 | 5.311 | 27.017 | 5.936 | 109.334 | 6.809 | 1439.783 | 31.855 | 278.665 | 5.845 | 11109.504 | 53.114 |
|  | $m_{Z}$ | 0.532 | 11.531 | 6.604 | 34.412 | 6.166 | 96.601 | 6.211 | 505.031 | 6.319 | 2568.341 | 6.660 | 365.584 | 9.971 | 11187.116 | 7.550 |
| $\begin{array}{r} 19 \\ \left(\mathrm{P}_{\mathrm{S}} \mathrm{~V}_{\mathrm{S}} \mathrm{~A}\right)_{0} \end{array}$ | $m_{X}$ | 6.472 | 40264.446 | 5.969 | 55840.525 | 4.809 | 51106.442 | 5.194 | 150584.540 | 7.934 | 352592.286 | 7.633 | 116492.271 | 5.182 | 410555.029 | 13.172 |
|  | $m_{Y}$ | 2.387 | 21629.191 | 5.759 | 23043.296 | 5.320 | 18432.817 | 5.942 | 107670.593 | 6.939 | 268285.143 | 14.417 | 30460.778 | 5.753 | 330881.852 | 38.387 |
|  | $m_{Z}$ | 6.596 | 38231.576 | 6.569 | 52804.785 | 6.128 | 47947.779 | 6.176 | 143467.394 | 6.757 | 350676.991 | 24.954 | 109597.512 | 9.907 | 415125.033 | 11.705 |

Appendix A.5.2. Results based on second type pre-adjustment for homogenously distributed points

| Point | set | $\begin{aligned} & \hline \mathrm{H}-0 \\ & \hline \mathrm{GCP} \\ & \hline \end{aligned}$ |  | H-10 |  |  |  | H-20 |  |  |  | H-30 |  |  |  | H-40 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | type |  |  | GCP |  | ICP |  | GCP |  | ICP |  | GCP |  | ICP |  | GCP |  | ICP |  |
|  | \# | 47 |  | 37 |  | 10 |  | 27 |  | 20 |  | 17 |  | 30 |  | 7 |  | 40 |  |
| EOP | Process | $\overline{L P}$ | $B$ | $\overline{L P}$ | B | $\overline{L P}$ | $B$ | $\overline{L P}$ | B | $\overline{L P}$ | B | $\overline{L P}$ | B | $\overline{L P}$ | B | $\overline{L P}$ | B | $\overline{L P}$ | B |
| 1. $\mathrm{P}_{\mathrm{S}}$ | $m_{X}$ | 0.605 | 0.605 | 0.607 | 0.604 | 6.037 | 5.973 | 0.624 | 0.600 | 4.754 | 4.835 | 0.598 | 0.524 | 4.985 | 5.282 | 0.573 | 0.423 | 6.803 | 7.292 |
|  | $m_{Y}$ | 0.212 | 0.212 | 0.217 | 0.218 | 5.800 | 5.747 | 0.218 | 0.263 | 5.374 | 5.326 | 0.224 | 0.341 | 6.042 | 5.983 | 0.234 | 0.693 | 7.013 | 6.840 |
|  | $m_{\text {z }}$ | 0.953 | 0.953 | 0.982 | 0.987 | 6.456 | 6.626 | 0.977 | 0.975 | 5.929 | 6.208 | 0.978 | 0.958 | 5.908 | 6.307 | 1.058 | 0.925 | 6.082 | 6.414 |
| 2. $\mathrm{P}_{\mathrm{s} 0}$ | $m_{X}$ | 0.605 | 0.605 | 0.606 | 0.603 | 6.037 | 5.971 | 0.624 | 0.597 | 4.753 | 4.843 | 0.598 | 0.542 | 4.984 | 5.281 | 0.573 | 0.477 | 6.802 | 7.127 |
|  | $m_{Y}$ | 0.212 | 0.212 | 0.217 | 0.205 | 5.800 | 5.743 | 0.218 | 0.190 | 5.374 | 5.324 | 0.224 | 0.200 | 6.042 | 5.982 | 0.235 | 0.695 | 7.013 | 6.673 |
|  | $m_{z}$ | 0.952 | 0.952 | 0.981 | 0.981 | 6.454 | 6.614 | 0.976 | 0.958 | 5.927 | 6.185 | 0.978 | 0.938 | 5.907 | 6.257 | 1.056 | 0.463 | 6.082 | 6.428 |
| 3. $\mathrm{P}_{\mathrm{S} 1}$ | $m_{X}$ | 0.617 | 0.617 | 0.618 | 0.612 | 6.036 | 5.966 | 0.634 | 0.627 | 4.755 | 4.825 | 0.606 | 0.607 | 4.988 | 5.206 | 0.583 | 1.223 | 6.809 | 7.304 |
|  | $m_{Y}$ | 0.209 | 0.209 | 0.214 | 0.221 | 5.801 | 5.753 | 0.215 | 0.223 | 5.374 | 5.314 | 0.221 | 0.231 | 6.043 | 5.941 | 0.231 | 0.922 | 7.015 | 6.704 |
|  | $m_{z}$ | 0.966 | 0.966 | 0.994 | 0.986 | 6.462 | 6.614 | 0.989 | 0.976 | 5.938 | 6.185 | 0.990 | 0.969 | 5.917 | 6.238 | 1.070 | 1.952 | 6.088 | 6.054 |
| 4. $\mathrm{P}_{\mathrm{S} 2}$ | $m_{X}$ | 0.610 | 0.610 | 0.612 | 0.610 | 6.036 | 5.967 | 0.629 | 0.624 | 4.752 | 4.832 | 0.603 | 0.602 | 4.986 | 5.230 | 0.579 | 0.591 | 6.806 | 7.157 |
|  | $m_{Y}$ | 0.211 | 0.211 | 0.216 | 0.219 | 5.800 | 5.746 | 0.218 | 0.216 | 5.374 | 5.311 | 0.223 | 0.230 | 6.043 | 5.945 | 0.235 | 0.627 | 7.014 | 6.787 |
|  | $m_{Z}$ | 0.958 | 0.958 | 0.986 | 0.979 | 6.459 | 6.610 | 0.980 | 0.963 | 5.932 | 6.178 | 0.981 | 0.949 | 5.913 | 6.226 | 1.059 | 0.705 | 6.087 | 6.392 |
| 5. $\mathrm{V}_{\mathrm{s}}$ | $m_{X}$ | 0.751 | 0.751 | 0.748 | 962043.081 | 6.002 | 5.966 | 0.773 | 2478851.193 | 4.736 | 4.818 | 0.743 | 2581511.487 | 4.994 | 5.175 | 2536096.516 | 808634.245 | 2072611.140 | 15.744 |
|  | $m_{Y}$ | 0.177 | 0.177 | 0.183 | 924231.288 | 5.827 | 5.743 | 0.185 | 1359971.849 | 5.385 | 5.307 | 0.191 | 1473031.690 | 6.057 | 5.989 | 627658.877 | 497754.652 | 450254.949 | 6.961 |
|  | $m_{\text {z }}$ | 1.110 | 1.110 | 1.135 | 1038974.333 | 6.487 | 6.605 | 1.138 | 2455864.367 | 5.997 | 6.189 | 1.136 | 2570855.510 | 5.996 | 6.300 | 2451801.227 | 823513.996 | 1987952.832 | 17.808 |
| 6. $\mathrm{V}_{0}$ | $m_{X}$ | 0.649 | 0.649 | 0.648 | 0.375 | 6.028 | 5.968 | 0.665 | 1.823 | 4.749 | 4.818 | 0.636 | 6.236 | 4.989 | 5.168 | 0.603 | 14.154 | 6.815 | 7.222 |
|  | $m_{Y}$ | 0.201 | 0.201 | 0.206 | 0.411 | 5.807 | 5.741 | 0.208 | 0.614 | 5.377 | 5.337 | 0.214 | 3.007 | 6.046 | 5.999 | 0.226 | 3.595 | 7.018 | 6.857 |
|  | $m_{Z}$ | 1.000 | 1.000 | 1.027 | 0.655 | 6.468 | 6.609 | 1.022 | 2.341 | 5.950 | 6.179 | 1.022 | 6.982 | 5.934 | 6.248 | 1.094 | 15.794 | 6.102 | 6.492 |
| 7. $\mathrm{V}_{1}$ | $m_{X}$ | 0.618 | 0.618 | 0.619 | 0.618 | 6.033 | 5.968 | 0.638 | 0.629 | 4.750 | 4.825 | 0.612 | 0.592 | 4.984 | 5.204 | 0.583 | 0.620 | 6.805 | 7.166 |
|  | $m_{Y}$ | 0.208 | 0.208 | 0.214 | 0.216 | 5.803 | 5.745 | 0.215 | 0.225 | 5.375 | 5.310 | 0.221 | 0.247 | 6.044 | 5.937 | 0.232 | 0.213 | 7.015 | 6.826 |
|  | $m_{\text {Z }}$ | 0.967 | 0.967 | 0.995 | 0.999 | 6.457 | 6.609 | 0.991 | 1.011 | 5.932 | 6.182 | 0.993 | 1.056 | 5.914 | 6.236 | 1.068 | 1.424 | 6.089 | 6.375 |
| 8. $\mathrm{V}_{2}$ | $m_{X}$ | 0.624 | 0.624 | 0.624 | 0.625 | 6.032 | 5.967 | 0.642 | 0.642 | 4.751 | 4.825 | 0.615 | 0.606 | 4.985 | 5.205 | 0.586 | 0.581 | 6.806 | 7.186 |
|  | $m_{Y}$ | 0.207 | 0.207 | 0.212 | 0.219 | 5.804 | 5.747 | 0.213 | 0.225 | 5.376 | 5.312 | 0.219 | 0.237 | 6.044 | 5.938 | 0.230 | 0.114 | 7.015 | 6.811 |
|  | $m_{z}$ | 0.973 | 0.973 | 1.001 | 0.998 | 6.458 | 6.611 | 0.997 | 0.999 | 5.936 | 6.180 | 1.000 | 1.037 | 5.917 | 6.230 | 1.074 | 1.421 | 6.090 | 6.383 |
| 9. A | $m_{X}$ | 0.608 | 0.608 | 0.609 | 0.608 | 6.037 | 5.967 | 0.626 | 0.629 | 4.754 | 4.825 | 0.598 | 0.594 | 4.985 | 5.200 | 0.586 | 0.571 | 6.806 | 7.186 |
|  | $m_{Y}$ | 0.211 | 0.211 | 0.217 | 0.212 | 5.800 | 5.747 | 0.218 | 0.205 | 5.374 | 5.310 | 0.224 | 0.200 | 6.042 | 5.936 | 0.230 | 0.163 | 7.015 | 6.811 |
|  | $m_{z}$ | 0.956 | 0.956 | 0.984 | 0.983 | 6.458 | 6.610 | 0.979 | 0.990 | 5.931 | 6.180 | 0.979 | 1.004 | 5.909 | 6.228 | 1.074 | 1.146 | 6.090 | 6.383 |
| 10. $\mathrm{A}_{0}$ | $m_{X}$ | 0.608 | 0.608 | 0.609 | 0.607 | 6.036 | 5.967 | 0.627 | 0.624 | 4.752 | 4.824 | 0.599 | 0.597 | 4.984 | 5.200 | 0.575 | 0.614 | 6.802 | 7.188 |
|  | $m_{Y}$ | 0.211 | 0.211 | 0.216 | 0.215 | 5.801 | 5.747 | 0.218 | 0.212 | 5.374 | 5.310 | 0.224 | 0.209 | 6.042 | 5.936 | 0.235 | 0.120 | 7.013 | 6.826 |
|  | $m_{z}$ | 0.956 | 0.956 | 0.984 | 0.982 | 6.455 | 6.611 | 0.979 | 0.977 | 5.929 | 6.178 | 0.979 | 0.980 | 5.907 | 6.226 | 1.057 | 1.103 | 6.083 | 6.331 |
| 11. $\mathrm{A}_{1}$ | $m_{X}$ | 0.617 | 0.617 | 0.618 | 0.620 | 6.035 | 5.967 | 0.635 | 0.638 | 4.753 | 4.825 | 0.607 | 0.611 | 4.987 | 5.205 | 0.583 | 0.665 | 6.807 | 7.163 |
|  | $m_{Y}$ | 0.209 | 0.209 | 0.214 | 0.217 | 5.802 | 5.746 | 0.215 | 0.217 | 5.375 | 5.311 | 0.221 | 0.223 | 6.043 | 5.937 | 0.232 | 0.231 | 7.015 | 6.811 |
|  | $m_{z}$ | 0.966 | 0.966 | 0.994 | 0.994 | 6.460 | 6.610 | 0.990 | 0.992 | 5.936 | 6.180 | 0.991 | 1.006 | 5.916 | 6.231 | 1.069 | 1.289 | 6.088 | 6.340 |
| 12. $\mathrm{A}_{2}$ | $m_{X}$ | 0.612 | 0.612 | 0.614 | 0.617 | 6.036 | 5.967 | 0.631 | 0.635 | 4.752 | 4.826 | 0.604 | 0.604 | 4.986 | 5.206 | 0.580 | 0.665 | 6.806 | 7.209 |
|  | $m_{Y}$ | 0.210 | 0.210 | 0.216 | 0.216 | 5.801 | 5.747 | 0.217 | 0.219 | 5.374 | 5.311 | 0.223 | 0.227 | 6.043 | 5.937 | 0.234 | 0.231 | 7.014 | 6.818 |
|  | $m_{z}$ | 0.960 | 0.960 | 0.988 | 0.984 | 6.459 | 6.611 | 0.983 | 0.976 | 5.932 | 6.177 | 0.983 | 0.982 | 5.914 | 6.223 | 1.062 | 1.289 | 6.087 | 6.337 |
| 13. $\mathrm{P}_{\mathrm{S}} \mathrm{V}_{\mathrm{S}}$ | $m_{X}$ | 143.192 | 143.192 | 93.266 | 1436350.889 | 96.580 | 5.969 | 155.675 | 1378011.919 | 128.290 | 4.776 | 54.470 | 2058748.908 | 41.139 | 5.099 | 4.941 | 3829303.264 | 8.163 | 362.688 |
|  | $m_{Y}$ | 39.369 | 39.369 | 25.559 | 1773095.665 | 31.412 | 5.805 | 42.479 | 1686469.231 | 37.916 | 5.325 | 15.636 | 1412877.835 | 14.507 | 5.916 | 1.138 | 2059551.366 | 7.425 | 103.105 |
|  | $m_{\text {z }}$ | 138.181 | 138.181 | 90.182 | 1628973.813 | 94.151 | 6.621 | 150.283 | 1560829.158 | 126.031 | 6.148 | 52.953 | 2075642.521 | 42.380 | 6.073 | 5.241 | 3966074.841 | 8.812 | 359.442 |
| 14. ( $\left.\mathrm{P}_{\mathrm{s}} \mathrm{V}\right)_{0}$ | $m_{X}$ | 0.329 | 0.329 | 0.347 | 42.819 | 6.157 | 5.966 | 0.353 | 64.469 | 4.817 | 4.824 | 0.373 | 792.812 | 4.958 | 5.212 | 0.414 | 7898.025 | 6.675 | 7.260 |
|  | $m_{Y}$ | 0.329 | 0.329 | 0.334 | 45.705 | 5.735 | 5.757 | 0.327 | 44.889 | 5.348 | 5.319 | 0.328 | 253.666 | 6.006 | 5.945 | 0.331 | 5428.240 | 6.968 | 6.885 |
|  | $m_{z}$ | 0.461 | 0.461 | 0.489 | 47.513 | 6.322 | 6.599 | 0.505 | 68.942 | 5.698 | 6.169 | 0.538 | 776.487 | 5.635 | 6.228 | 0.621 | 8214.423 | 5.874 | 6.378 |
| 15. $\mathrm{P}_{\mathrm{S}} \mathrm{A}$ | $m_{X}$ | 6.623 | 6.623 | 6.448 | 94.819 | 10.373 | 5.971 | 7.309 | 86.578 | 9.284 | 4.814 | 8.570 | 952.609 | 9.654 | 5.009 | 7.518 | 482936.244 | 10.240 | 388.881 |
|  | $m_{Y}$ | 2.314 | 2.314 | 2.274 | 82.034 | 4.971 | 5.747 | 2.539 | 423.603 | 5.280 | 5.298 | 2.964 | 648.680 | 5.700 | 5.942 | 1.941 | 344851.187 | 7.007 | 177.722 |
|  | $m_{Z}$ | 6.204 | 6.204 | 6.000 | 65.717 | 9.000 | 6.599 | 6.849 | 178.147 | 7.446 | 6.140 | 8.011 | 599.221 | 7.027 | 5.973 | 8.025 | 444653.596 | 7.880 | 424.213 |
| 16. $\left(\mathrm{P}_{\mathrm{S}} \mathrm{A}\right)_{0}$ | $m_{X}$ | 1.897 | 1.897 | 1.695 | 16.855 | 6.752 | 5.965 | 1.780 | 42.779 | 5.389 | 4.811 | 1.664 | 84.852 | 5.166 | 5.175 | 1.011 | 157.089 | 6.482 | 7.169 |
|  | $m_{Y}$ | 0.891 | 0.891 | 0.838 | 37.434 | 5.513 | 5.743 | 0.853 | 169.686 | 5.260 | 5.319 | 0.818 | 304.033 | 5.861 | 5.982 | 0.647 | 457.713 | 6.837 | 6.886 |
|  | $m_{z}$ | 1.990 | 1.990 | 1.825 | 14.639 | 6.223 | 6.605 | 1.762 | 63.691 | 5.161 | 6.159 | 1.540 | 124.346 | 4.800 | 6.197 | 1.464 | 326.481 | 5.388 | 6.345 |
| 17. $\mathrm{V}_{\mathrm{s}} \mathrm{A}$ | $m_{X}$ | 18.679 | 18.679 | 14.421 | 936297.357 | 17.640 | 5.960 | 21.151 | 1638923.530 | 20.674 | 4.811 | 18.482 | 627999.282 | 19.393 | 5.184 | 94.315 | 606870.208 | 88.631 | 274.730 |
|  | $m_{Y}$ | 5.122 | 5.122 | 4.439 | 1021817.177 | 5.278 | 5.746 | 5.580 | 376023.351 | 6.271 | 5.307 | 5.313 | 449218.938 | 6.477 | 5.791 | 19.858 | 235035.787 | 17.260 | 207.358 |
|  | $m_{z}$ | 1.704 | 1.704 | 9.943 | 984037.447 | 13.746 | 6.609 | 4.853 | 1551614.818 | 5.718 | 6.168 | 8.979 | 720597.936 | 9.936 | 6.080 | 78.560 | 613477.167 | 72.893 | 288.327 |
| 18. $\left(\mathrm{V}_{\mathrm{s}} \mathrm{A}\right)_{0}$ | $m_{X}$ | 0.326 | 0.326 | 0.362 | 12.010 | 6.110 | 5.967 | 0.351 | 22.954 | 4.802 | 4.824 | 0.428 | 10.561 | 4.954 | 5.198 | 0.485 | 184.931 | 6.749 | 7.191 |
|  | $m_{Y}$ | 0.324 | 0.324 | 0.307 | 4.699 | 5.751 | 5.747 | 0.329 | 17.290 | 5.344 | 5.311 | 0.286 | 3.496 | 6.016 | 5.937 | 0.265 | 75.380 | 6.996 | 6.811 |
|  | $m_{z}$ | 0.542 | 0.542 | 0.631 | 9.910 | 6.353 | 6.608 | 0.554 | 28.791 | 5.719 | 6.174 | 0.721 | 15.860 | 5.742 | 6.222 | 0.886 | 181.456 | 6.004 | 6.338 |
| $\text { 19. }\left(\mathrm{P}_{\mathrm{S}} \mathrm{~V}_{\mathrm{S}} \mathrm{~A}\right)_{0}$ | $m_{X}$ | 6.476 | 6.476 | 6.336 | 37849.643 | 9.859 | 5.957 | 7.000 | 47979.542 | 9.133 | 4.810 | 7.285 | 33428.997 | 8.692 | 5.201 | 243746.100 | 244290.321 | 243292.503 | 5.031 |
|  | $m_{Y}$ | 2.391 | 2.391 | 2.370 | 19636.361 | 5.122 | 5.763 | 2.565 | 19318.401 | 5.350 | 5.319 | 2.684 | 12297.413 | 5.694 | 5.943 | 166209.503 | 166702.873 | 167428.486 | 6.559 |
|  | $m_{z}$ | 6.617 | 6.617 | 6.694 | 36143.077 | 8.412 | 6.573 | 7.700 | 45196.468 | 7.704 | 6.141 | 8.134 | 31095.464 | 7.163 | 6.196 | 413010.995 | 413058.838 | 412538.760 | 9.059 |

Appendix A.5.3. Results based on second type pre-adjustment for grouped points

| Point | set | G-A |  |  |  | G-B |  |  |  | G-C |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | type | GCP |  | ICP |  | GCP |  | ICP |  | GCP |  | ICP |  |
|  | \# | 23 |  | 24 |  | 24 |  | 23 |  | 17 |  | 30 |  |
| EOP | Process | $\overline{L P}$ | B | $\overline{L P}$ | $B$ | $\overline{L P}$ | B | $\overline{L P}$ | B | $\overline{L P}$ | B | $\overline{L P}$ | $B$ |
| 1. $\mathrm{P}_{\mathrm{S}}$ | $m_{X}$ | 0.678 | 11.456 | 24.725 | 20.309 | 0.510 | 0.692 | 5.622 | 15.166 | 39.595 | 29.139 | 39.595 | 109.269 |
|  | $m_{Y}$ | 0.193 | 8.954 | 33.253 | 16.244 | 0.199 | 0.827 | 5.985 | 15.690 | 24.061 | 54.563 | 24.061 | 88.595 |
|  | $m_{z}$ | 0.926 | 12.701 | 7.083 | 14.086 | 1.019 | 1.022 | 9.560 | 9.264 | 42.480 | 6.811 | 42.480 | 96.555 |
| 2. $\mathrm{P}_{\mathrm{s} 0}$ | $m_{X}$ | 0.678 | 5.687 | 24.726 | 12.257 | 0.511 | 0.619 | 5.614 | 4.989 | 8.964 | 29.142 | 8.962 | 10.890 |
|  | $m_{Y}$ | 0.193 | 9.427 | 33.253 | 19.385 | 0.199 | 0.346 | 5.984 | 5.725 | 15.137 | 54.563 | 15.136 | 33.003 |
|  | $m_{\text {z }}$ | 0.926 | 7.263 | 7.083 | 21.784 | 1.019 | 1.045 | 9.568 | 10.720 | 11.266 | 6.811 | 11.264 | 21.410 |
| 3. $\mathrm{P}_{\mathrm{S} 1}$ | $m_{X}$ | 0.682 | 4.658 | 24.729 | 18.632 | 0.512 | 0.881 | 5.598 | 4.939 | 6.620 | 29.148 | 6.621 | 21.080 |
|  | $m_{Y}$ | 0.192 | 10.236 | 33.256 | 19.625 | 0.198 | 0.642 | 5.980 | 5.620 | 12.589 | 54.566 | 12.590 | 40.079 |
|  | $m_{z}$ | 0.930 | 6.100 | 7.080 | 28.831 | 1.025 | 1.303 | 9.592 | 10.931 | 7.577 | 6.812 | 7.578 | 22.465 |
| 4. $\mathrm{P}_{\mathrm{S} 2}$ | $m_{X}$ | 0.680 | 5.639 | 24.727 | 21.667 | 0.515 | 0.650 | 5.613 | 4.989 | 5.810 | 29.145 | 5.808 | 20.895 |
|  | $m_{Y}$ | 0.193 | 11.726 | 33.254 | 21.278 | 0.197 | 0.545 | 5.984 | 5.578 | 10.105 | 54.564 | 10.105 | 43.187 |
|  | $m_{z}$ | 0.928 | 7.379 | 7.083 | 36.431 | 1.028 | 1.073 | 9.575 | 11.049 | 6.977 | 6.812 | 6.976 | 16.463 |
| 5. $\mathrm{V}_{\mathrm{s}}$ | $m_{X}$ | 0.717 | 2766973.512 | 24.768 | 23.783 | 0.725 | 2.248 | 5.536 | 5.027 | 1368058.738 | 29.181 | 1223649.079 | 72.216 |
|  | $m_{Y}$ | 0.180 | 2346779.341 | 33.271 | 32.141 | 0.142 | 0.340 | 5.966 | 5.475 | 275362.699 | 54.580 | 941124.562 | 52.107 |
|  | $m_{z}$ | 0.969 | 2873251.756 | 7.054 | 9.662 | 1.235 | 0.903 | 9.703 | 13.305 | 1302242.979 | 6.820 | 1284129.062 | 85.197 |
| 6. $\mathrm{V}_{0}$ | $m_{X}$ | 0.683 | 1490.498 | 24.729 | 26.250 | 0.621 | 70.480 | 5.563 | 5.210 | 7169.615 | 29.153 | 7173.940 | 42.332 |
|  | $m_{\gamma}$ | 0.192 | 526.680 | 33.254 | 32.866 | 0.168 | 48.309 | 5.972 | 5.743 | 1597.907 | 54.568 | 1597.374 | 55.964 |
|  | $m_{z}$ | 0.931 | 1462.314 | 7.081 | 7.483 | 1.133 | 73.466 | 9.645 | 9.625 | 6890.962 | 6.814 | 6894.834 | 11.119 |
| 7. $\mathrm{V}_{1}$ | $m_{X}$ | 0.683 | 0.829 | 24.731 | 26.037 | 0.544 | 0.533 | 5.612 | 5.293 | 1.229 | 29.145 | 1.230 | 34.051 |
|  | $m_{\gamma}$ | 0.191 | 0.392 | 33.256 | 32.905 | 0.191 | 0.162 | 5.983 | 5.931 | 0.369 | 54.564 | 0.371 | 54.111 |
|  | $m_{z}$ | 0.932 | 0.837 | 7.079 | 7.137 | 1.050 | 1.068 | 9.574 | 10.057 | 2.796 | 6.812 | 2.793 | 10.956 |
| 8. $\mathrm{V}_{2}$ | $m_{X}$ | 0.680 | 0.717 | 24.727 | 23.038 | 0.542 | 0.525 | 5.598 | 5.467 | 1.764 | 29.147 | 1.762 | 33.104 |
|  | $m_{\gamma}$ | 0.193 | 0.228 | 33.254 | 32.450 | 0.192 | 0.215 | 5.980 | 5.928 | 0.315 | 54.566 | 0.315 | 54.091 |
|  | $m_{z}$ | 0.928 | 0.871 | 7.082 | 5.144 | 1.048 | 1.061 | 9.589 | 10.389 | 1.408 | 6.812 | 1.408 | 9.199 |
| 9. A | $m_{X}$ | 0.680 | 0.646 | 24.727 | 23.038 | 0.512 | 0.595 | 5.615 | 5.251 | 2.011 | 29.141 | 2.011 | 26.986 |
|  | $m_{\gamma}$ | 0.193 | 0.398 | 33.254 | 32.450 | 0.198 | 0.217 | 5.983 | 5.675 | 0.508 | 54.564 | 0.509 | 48.643 |
|  | $m_{z}$ | 0.928 | 0.977 | 7.082 | 5.144 | 1.022 | 1.089 | 9.569 | 7.024 | 2.724 | 6.812 | 2.723 | 63.276 |
| 10. $\mathrm{A}_{0}$ | $m_{X}$ | 0.680 | 0.453 | 24.727 | 25.647 | 0.513 | 0.488 | 5.613 | 5.265 | 0.442 | 29.144 | 0.441 | 30.191 |
|  | $m_{\gamma}$ | 0.193 | 0.430 | 33.254 | 32.794 | 0.198 | 0.264 | 5.983 | 5.879 | 0.557 | 54.564 | 0.558 | 54.001 |
|  | $m_{z}$ | 0.928 | 0.626 | 7.082 | 6.897 | 1.022 | 0.986 | 9.570 | 10.056 | 0.481 | 6.812 | 0.478 | 6.762 |
| 11. $\mathrm{A}_{1}$ | $m_{X}$ | 0.682 | 1.078 | 24.730 | 24.408 | 0.512 | 0.510 | 5.598 | 5.200 | 1.387 | 29.147 | 1.384 | 29.269 |
|  | $m_{\gamma}$ | 0.192 | 0.307 | 33.255 | 32.524 | 0.198 | 0.159 | 5.980 | 5.870 | 0.338 | 54.566 | 0.337 | 53.716 |
|  | $m_{z}$ | 0.930 | 1.335 | 7.080 | 5.731 | 1.025 | 1.092 | 9.592 | 9.875 | 1.254 | 6.812 | 1.252 | 7.387 |
| 12. $\mathrm{A}_{2}$ | $m_{X}$ | 0.681 | 0.613 | 24.728 | 24.754 | 0.514 | 0.523 | 5.613 | 5.201 | 0.847 | 29.145 | 0.846 | 29.782 |
|  | $m_{\gamma}$ | 0.192 | 0.411 | 33.254 | 32.570 | 0.197 | 0.232 | 5.984 | 5.868 | 0.542 | 54.564 | 0.543 | 53.781 |
|  | $m_{z}$ | 0.929 | 1.113 | 7.082 | 6.002 | 1.027 | 0.989 | 9.575 | 9.869 | 0.853 | 6.812 | 0.849 | 6.974 |
| 13. $\mathrm{P}_{\mathrm{S}} \mathrm{V}_{\mathrm{s}}$ | $m_{X}$ | 418577.182 | 439729.367 | 398066.101 | 73468.889 | 166.470 | 1582595.347 | 83.994 | 21.945 | 2383609.474 | 148.373 | 94018.431 | 148.373 |
|  | $m_{\gamma}$ | 257598.585 | 403629.438 | 300119.123 | 101890.137 | 45.421 | ${ }_{1}^{1914580.592}$ | 18.204 | 13.523 | 2163489.615 | 82.595 | 53648.759 | $\begin{array}{r}82.595 \\ \hline 20453 \\ \hline\end{array}$ |
|  | $m_{z}$ | 311548.658 | 621453.846 | 296393.044 | 90005.618 | 160.697 | 1765500.256 | 87.175 | 35.851 | 2525017.860 | 204.753 | 95661.648 | 204.753 |
| 14. $\left(\mathrm{P}_{\mathrm{s}} \mathrm{V}_{\mathrm{S}}\right)_{0}$ | $m_{X}$ | 0.714 | 20146.873 | ${ }^{24.765}$ | 24.888 | 766360.502 | 636970.239 | 771083.509 | 697.141 | 197465.402 | 0.781 | 156780.579 | 27.647 |
|  | $m_{Y}$ | 0.189 | 8586.594 | 33.262 | 32.432 | 269235.778 | 371120.093 | 253744.220 | 152.490 | 99609.424 | 0.209 | 90957.713 | 52.250 |
|  | $m_{z}$ | 0.964 | 19960.392 | 7.056 | 7.430 | 633107.335 | 584356.114 | 639129.754 | 707.654 | 199323.756 | 0.960 | 160271.802 | 6.988 |
| 15. $\mathrm{P}_{\mathrm{S}} \mathrm{A}$ | $m_{X}$ | 2.471 | 6966.894 | 26.347 | 16.219 | 9.118 | 1447.215 | 10.803 | 49.936 | 24898.881 | 22.344 | 24930.954 | 77.391 |
|  | $m_{Y}$ | 1.221 | 10035.695 | 34.252 | 12.924 | 2.947 | 780.653 | 5.474 | 29.277 | 18406.978 | 50.841 | 18424.392 | 52.365 |
|  | $m_{z}$ | 2.550 | 6705.287 | 6.174 | 19.940 | 8.196 | 897.925 | 12.613 | 60.908 | 7757.400 | 8.552 | 7794.052 | 108.753 |
| 16. $\left(\mathrm{P}_{\mathrm{S}} \mathrm{A}\right)_{0}$ | $m_{X}$ | 0.403 | 1253.452 | 24.412 | 27.891 | 3.298 | 93.044 | 8.024 | 5.194 | 1882.829 | 28.718 | 1882.853 | 28.296 |
|  | $m_{\gamma}$ | 0.320 | 250.472 | 33.112 | 31.359 | 1.309 | 551.117 | ${ }_{6}^{6.443}$ | 5.836 | $\begin{array}{r}673.258 \\ \hline 139567\end{array}$ | 54.383 | $\begin{array}{r}673.271 \\ \hline 1359740\end{array}$ | 52.801 |
|  | $m_{z}$ | 0.587 | 343.222 | 7.289 | 6.592 | 3.416 | 244.505 | 6.793 | 10.212 | 1359.767 | 6.761 | 1359.740 | 7.004 |
| 17. $\mathrm{V}_{\mathrm{s}} \mathrm{A}$ | $m_{X}$ | 5.042 | 380800.668 | 24.990 | 18.639 | 88.572 | 86170.641 | 40.385 | 100.406 | 275365.674 | 24.553 | 502784.361 | 127.657 |
|  | $m_{Y}$ | 3.503 | 351781.019 | 32.712 | 20.229 | 17.423 | 114045.306 | 8.211 | 52.964 | 273554.193 | 53.427 | 380398.629 | 142.244 |
|  | $m_{z}$ | 6.633 | 500046.019 | 20.348 | 9.214 | 123.529 | 179167.514 | 340.239 | 122.982 | 503858.960 | 24.138 | 715962.609 | 147.236 |
| 18. $\left(\mathrm{V}_{\mathrm{S}} \mathrm{A}\right)_{0}$ | $m_{X}$ | 0.664 | 1959.034 | 24.714 | 25.990 | 1.098 | 45.006 | 5.363 | 5.178 | 6529.395 | 29.014 | 8761.155 | 32.800 |
|  | $m_{Y}$ | 0.175 | 1344.455 | 33.274 | 31.858 | 0.080 | 202.876 | 5.911 | 5.843 | 10620.299 | 54.518 | 11109.504 | 53.151 |
|  | $m_{z}$ | 0.890 | 2217.847 | 7.102 | 6.652 | 1.433 | 82.213 | 9.892 | 9.960 | 9134.392 | 6.788 | 11187.116 | 7.627 |
| 19. $\left(\mathrm{P}_{\mathrm{S}} \mathrm{V}_{\mathrm{S}} \mathrm{A}\right)_{0}$ | $m_{X}$ | 0.777 | 416664.708 | 23.738 | 14.204 | 486088.961 | 484249.794 | 500616.408 | 6.322 | 454477.910 | 27.260 | 410555.029 | 21.305 |
|  | $m_{Y}$ | 1.169 | 260970.641 | 32.427 | 20.057 | 134814.403 | 132974.748 | 123105.777 | 6.076 | 285279.246 | 53.168 | 330881.852 | 45.134 |
|  | $m_{z}$ | 0.419 | 408516.337 | 7.825 | 15.700 | 426718.814 | 429948.733 | 442289.121 | 10.934 | 445664.058 | 6.917 | 415125.033 | 6.428 |

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[^0]:    * Symbols used in alphabetic order. The specific symbols used by other researches (e.g. in Section 3.2.2) are ignored.

