# A MATHEMATICAL MODEL OF THE HUMAN THERMAL SYSTEM

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by
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# ABSTRACT

Mathematical model of the human thermal system, which has been greatly developed in recent years, has applications in many areas. It is used to evaluate the environmental conditions in buildings, in car industry, in textile industries, in the aerospace industry, in meteorology, in medicine, and in military applications. In these disciplines, the model can serve for research into human performance, thermal acceptability and temperature sensation, safety limits.

Present study investigates the mathematical modeling of the passive part of the human thermal system. The Bio-Heat Equation is derived in order to solve the heat transfer phenomena in the tissue and with environment. It is assumed that the body is exposed to combination of the convection, evaporation and radiation which are taken into account as boundary conditions when solving the Bio-Heat Equation. Finite difference technique is used in order to find out the temperature distribution of human body. The derived equation by numerical method is solved by written software called Bio-Thermal. Bio-Thermal, is used to determine temperature distribution at succeeding time step of the viscera, lung, brain all tissue type of the torso, neck, head, leg, foot, arm, hand, and mean temperature of torso, neck, head, leg, foot, arm, hand. Additionally, for overall body, mean temperature of the bone tissue, muscle tissue, fat tissue, and skin tissue and mean temperature of the total body can be obtained by Bio-Thermal Software. Also, the software is to be capable of demonstrating the sectional view of the various body limbs and full human body.

In order to verify the present study, predictions of the present system model are compared with the available experimental data and analytical solution and show good agreement is achieved.

# ÖZ

Günümüzde, insan termal sisteminin matematik modellemesi birçok alanda kullanılmakta ve bu alanlar gitgide artmaktadır. Başlıca alanlar ise; teksil sektörü, binaların ısıtma ve havalandırma sistemleri, uzay çalışmaları, araba sanayi, ilaç sanayi ve ordu uygulamalarıdır.

Bu çalışmada insan termal sisteminin matematiksel modellemesi incelenmektedir. Hücre içinde ve çevresinde olan ısı transfer bağıntılarını çözebilmek için Bio-Isı Denklemi çıkartılmıştır. Çalışmada insan vücudunun konveksiyon, ışınım ve buharlaşma etkilerinin toplamına maruz kaldığı kabul edilmiştir. Sıcaklık dağılımını bulmak için sonlu farklar tekniği kullanılmıştır. Elde edilen denklemler Bio-Thermal adında bir yazılım kullanarak çözülmüştür. Yazılım, daha önce hazırlanmış denklem takımlarını çözerek ortam koşulları belirtilmiş bir insanın beyin, kas, yağ, deri, akciğer, iç organ ve kemik dokusunun sıcaklık dağılımını verebilmektedir. Aynı zamanda kafa, boyun, gövde, bacak, ayak, el, kolunda sıcaklık dağılımını hesaplayabilmekte ve bütün bu uzuvların iç katmanlarindaki yağ, kas, kemik gibi dokularındaki sıcaklık dağılımınıda bulabilmektedir. Aynı zamanda Bio-Thermal yazılımını kullanarak incelemek istediginiz uzvun kesit olarak sıcaklık dağılımınıda görebilirsiniz.

Çalışmanın doğrulugunu ispatlamak için, çalışmanın sonunda elde edilen değerler, uygun olan deneysel datalar ve analitik çözümlerle karşılaştırılmıştır. Bu çalışmadaki model sonuçlarının gerek deneysel sonuçlar ile gerekse analitik çözüm ile uyumlu olduğu görülmüştür.

# TABLE OF CONTENTS

# LIST OF FIGURES

# LIST OF TABLES

# NOMENCLATURE

$c_p$: Specific heat (J/kgK)

D: External diameter of the limb (m)

g: Acceleration of gravity (=9.8 $m/s^2$)

Gr: Grashof Number

h: Convective heat transfer coefficient ($W/m^2K$)

k: Thermal conductivity (W/mK)

K: Coefficient for Forced Convection Evaporation Heat Loss

Nu: Nusselt number

Pr: Prandtl number

$P_v$: Vapour Pressure of Water at Air Temperature

q: Heat flux ($W/m^2$)

Re: Reynolds number

RH: Relative Humidity of air

t: Time (s)

T: Temperature (°C)

v: Velocity of air (m/s)

V: Control volume

$\alpha$ : Thermal diffusivity $\left[\alpha = \frac{k}{\rho\, Cp}\right]$

$\beta$ : Thermal expansion coefficients

$\varepsilon$ : Emissivity of the Human Body

$\sigma$ : Stefan-Boltzmann Constant ($W.m^{-2}.K^{-4}$) (= $5.67051\times10^{-8}$ $W.m^{-2}.K^{-4}$)

$\nu$ : Kinematic viscosity ($m^2/s$)

$\rho$ : Density ($kg/m^3$)

$\phi$ : Relative Humidity

Subscripts

a: ambient air

art: artery

bl: blood

c: convection

d: diffusional

e: evaporation

e: Control volume face between P and E

f: film

free: free convection

forced: forced convection

i: Designation of the r location of discrete nodal points.

j: Designation of the $\phi$ location of discrete nodal points

k: Designation of the z location of discrete nodal points.

m: metabolic

r: radiation

s: surface

s: sweat

v: vapour

Superscript

n: Time level

# CHAPTER 1

# INTRODUCTION

It is very important to know how the human body will behave under different environmental conditions of air temperature, humidity and wind velocity. With the rapid development of computer technology, the study of simulating human thermal behavior is moving towards more and more precise methods. In the areas of aerospace, automotive industry and heating, ventilating and air-conditioning engineering, predictions of human thermal response are greatly in demand. Simulating the human thermal behavior is done by two major ways: simulating the passive system of human thermal system and simulating the active system of human thermal system.

The active system is simulated by active modeling, which predicts regulatory responses such as shivering, vasomotion, and sweating. The main purpose of the active model is that it regulates the passive heat transfer model and it is responsible for the maintenance of the human body's temperature. The passive system is modeled by passive modeling, which simulates the physical human body and the heat transfer phenomena occurring in human body and at its surface. Description of the passive model is done by equations resultant from the application of the heat and mass balances to a tissue control volume.By applying the theories of heat transfer and thermodynamics processes, we can predict the thermal behaviour of the entire human body or a part of it.

The purpose of present study is to develop an unclothed human passive system model which can numerically calculate the temperature distribution of human body at different environmental conditions. Software called Bio-Thermal is developed to solve the mathematical equations and calculate the temperature distribution of viscera, lung, brain, and bone, muscle, fat, skin temperature of torso, neck, head, leg, foot, arm, hand. In addition, mean temperature of torso, neck, head, leg, foot, arm and hand can be calculated. For overall body, mean temperature of the bone tissue, the muscle tissue, fat tissue, skin tissue and total body are calculated. Also, the software is to be capable of demonstrating the sectional view of the various body limbs and full human body.

Present study gives the mathematical modeling of human thermal system from general information to specific information.

In Chapter 2, the previous important mathematical models about the human thermal system done by scientists are given. In addition to this, the experimental and theoretical studies used in verification of our model are introduced in this chapter.

In Chapter 3, firstly, the general information about the mathematical modeling and its types are given briefly. Then, human thermal system is explained. The mechanisms called heat-loss mechanisms and heat producing mechanisms which are activated when human body is exposed to hot and cold environmental conditions, are explained. Lastly, mathematical modeling of human thermal system is introduced by giving the heat transfers inside the tissue and heat exchange with environment.

In Chapter 4, present study is introduced. The assumptions, correlations and thermophysical properties, which are used in our model, are given. Apart from that, in this chapter, Bio-Heat Equation is derivated in differential form

In Chapter 5, solution technique which is used for solving the Bio-Heat Equation is explained. Also, the finite difference forms of Bio-Heat Equation for every nodes of the human body are given in this chapter.

In Chapter 6, software which is developed to solve the equation derived in Chapter 5 is introduced. The process of the program is explained step by step. In addition, flow chart of the program is given in order to follow the steps easily.

In Chapter 7, the verification of the program is introduced by using the experimental and theoretical studies done by earlier scientist. The graphics of the temperature of various limbs versus time are obtained. In addition, by using the visual part of our software some sectional views of the human body are given in this chapter.

In Chapter 8, the conclusion of the study and the future work are given.

# CHAPTER 2

# LITERATURE SURVEY

Since the beginning of the medical research, the scientists have been interesting the physiological system and its applications. They have tried to find the most accurate model of the human thermal system for simulating the reactions to the different environmental conditions. Like the evaluation of the every modeling process, the mathematical modeling of the human thermal system has been developing from easy to more complex ones.

Firstly, Pennes modeled the single element of the human body. Pennes, who was one of the earliest workers in this area, developed the so-called BIO-HEAT EQUATION in order to calculate the steady state temperature distribution in human arm in 1948. The human forearm was resembled as a cylinder. The model includes conduction of heat in the radial direction of the cylinder, metabolic heat generation in the tissue, convection of heat by the circulating blood, heat loss from the surface of the skin by convection, radiation and evaporation. The importance of the modeling is that it is essentially applicable to any cylinder element of the body (Wissler, 1998).

From 1960's, the researchers have been starting to model the entire human body. Due to the development in computer technologies, more complex modeling of the human body could be done. Multi layered modeling of the human body is one of them. The best known are Wyndham and Atkin's Model, Crosbie Model, and Gagge Model.

Wyndham and Atkin's Model was probably the first model, which introduced a transient model for a human body in 1960. They approximated the human body by a single cylinder and divided the cylinder into a number of thin concentric layers. The finite difference technique was used to approximate the equation. A set of resulting first-order differential equations was solved by using an analog computer. At the surface of the cylinder the rate of heat loss to the surroundings is the sum of the heat loss due to the convection, radiation and evaporation (Fan et.al.1999).

The weakness of this model was that Wyndham and Atkins compared their models' predications with a wide variety of published data which were limited accuracy. In addition, these works did not include the effect of blood flow in the transfer of heat inside the tissue.

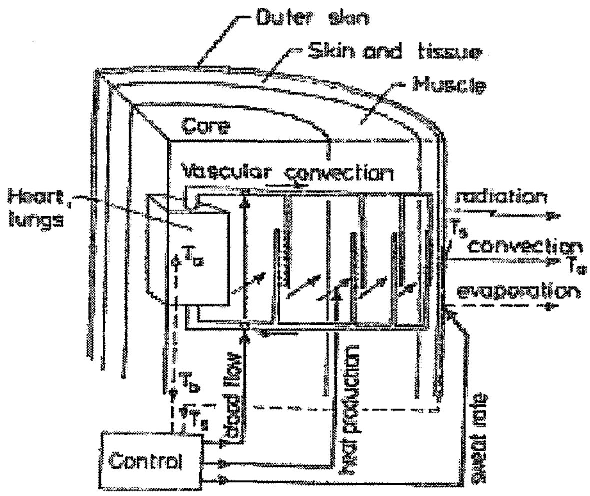



Figure 2.1. Schematic view of Wyndham and Atkins Model.
(Cena and Clark, 1981))

Another multi layered model is Crosbie Model, which is also transient. However, different from other models, Crosbie is the first scientist to simulate the physiological temperature regulation on an analog computer. The physiological thermal regulation mechanism had not been taken into account in modeling until Crosbie. He approximated the values of the thermal conductivity of tissue, metabolic rate, and the rate of heat loss by evaporation as a function of the mean body temperature (Fan et.al.1999). In addition, Crosbie assumed the heat flow from the core to the skin to be unidirectional. This one-dimensional model is divided into three layers: the core layer, the muscle layer and the skin layer. The core layer is the source of basal metabolism and the muscle layer is the source of increased metabolism caused by exercising or shivering. At skin layer, the heat is lost by radiation, convection and evaporation. In the inner layer, only the basal metabolic rate and heat conduction are considered. A simulator was designed to predict steady state values of skin and deep body temperature, metabolic rate, and evaporative heat loss. If the time constants for the various thermal changes are introduced, the simulator can also predict the dynamic responses to sudden shifts in environmental temperature (Cena and Clark, 1981)

Gagge Model is another famous transient model depicting the human body as multi layer. The body is considered to be a single cylinder which is divided into two concentric shells, an outer skin layer and a central core representing internal organs, bone, muscle, and subcutaneous tissue. Therefore, it is called “CORE and SHELL” model. Gagge assumed that the temperature of each layer is uniform. Heat is generated inside the core and transferred to the skin both by the blood and tissue conduction.

Energy balances equations written for the core and skin include heat storage effects, conductive heat transfer between adjacent tissue layers, and convective heat transfer due to the blood flow. In the core energy balance sensible and latent heat loss caused by respiration and the effects of metabolic heat generated during exercise and shivering are considered. In skin layers, heat exchange occurs between the skin surface and surroundings due to the convection, radiation and the evaporation of the moisture. All of these are included in the skin energy balance.

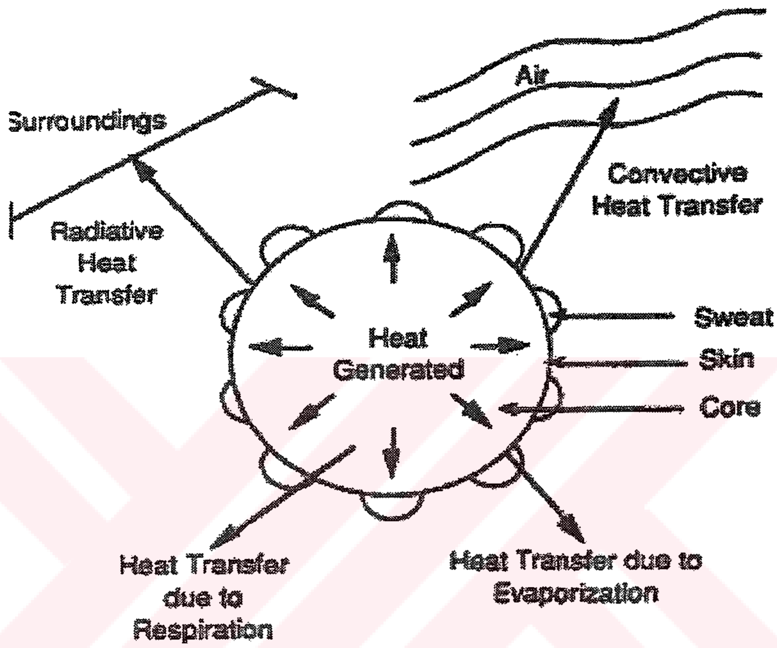



Figure 2.2. Schematic view of Gagge Model.(Smith, 1991)

Gagge developed a software program to solve the energy balance equations for core and skin temperature as a function of time using the accompanying control equations, which are derived from active model. Environmental conditions and activity levels are assigned by user. This program can predict man's thermal response for simulations between 15-60 minutes long. It is generally applicable during moderate levels of activity and uniform environmental conditions described by dry bulb temperatures in the range 5°C-45°C and humidity down to 10%. The strength of the Gagge model is that, it is less complex and easier to use than Stolwijk model and or Wissler model. However, this simplicity limits the amount of information it can provide with any given simulation and restricts the model applications. Another weakness of the Gagge model is found in the control equations. There is considerable variation of the constants in these equations from one publication to the next. This raises questions as to why such variations appear, how these constants are calculated, and one value should be used as opposed to another (Smith, 1991).

During the development of these models given above, some scientists interested in multi element model rather than multi layer model. In multi element model, the human body is depicted as several parts, such as arm, leg; instead of depicted as single multi layered element.

First multi element steady state model called First Version of Wissler Model was developed by Wissler in 1961. Wissler extended the Pennes' model of the forearm to obtain the temperature distribution of the entire body. In addition to the Pennes' model, Wissler took into account the heat loss through the respiration and the countercurrent heat exchange between the large arteries and large veins. Model was composed of interconnected cylinders. The human body is subdivided into six cylinders: the head, torso, two arms and two legs. Each element is assumed to have a uniformly distributed metabolic heat generation, to have a uniformly blood supply, to be homogenous and an isotropic cylinders. The elements consist of four concentric layers: bone, muscle, fat and skin. The ***steady state*** bio-heat equation was solved analytically for each cylinder. At skin layers, heat loss occurs by convection, radiation and evaporation (Fiala, 1999)

In 1963, Wissler developed the Second Version of the Wissler Model, which is transient version. This version is more complex than the previous model. Second version of the Wissler's model considered many important factors such as the variability of the local blood flow rate, the thermal conductivity, the rate of local heat generation by metabolic reactions, the geometry of the human body, heat loss through the respiratory system, sweating. These factors had not been considered by previous researchers. This unsteady state model is an extension of Wyndham and Atkins' model given by single element only. It was developed in 1963 (Shitzer and Chato, 1991). Different from first version, the second version included additional terms describing heat exchange between blood and tissue, both in capillary bed and in the large blood vessels. Finite difference solution scheme used in second version enabled Wissler to expand his model considerably. The body was divided into15 elements to represent the head, thorax, abdomen and proximal, medial and distal segments of the arms and legs. Each element is divided into four layers of core, muscle, fat and skin. Also, each element has three part vascular system, which are arteries, veins, and capillaries. The large arteries and veins are modeled using an arterial blood pool and a venous blood pool, respectively. Metabolic reactions were considered the main source of the heat. . (Smith,1991).

In Second Version of Wissler Model, energy balance equations are written for each tissue in each element and include heat storage, metabolic heat generation, convective heat transfer due to the blood flow in the capillaries, arterial and venous pools, conductive heat transfer in the radial direction, or the direction normal to the body centerline. Conduction through longitudinal and tangential axis is neglected. Perfect heat transfer is assumed between the capillaries and surrounding tissue. Sensible and latent heat losses due to respiration are divided equally between the head and thorax. Heat exchange processes with the environment through convection, radiation and the evaporation of the moisture are included as boundary conditions at the skin surface (Smith, 1991).

Wissler solved this model by using Crank-Nicholson's implicit finite difference method. The radial distance of each element was divided into 15 points and simultaneous equations of each element were solved by repeated use of the Gaussian elimination method. Computed results from a Fortran program were compared to experimental data for varying environmental conditions and activity levels. Rectal and mean skin temperatures, volumetric oxygen consumption rates, and weight losses were recorded using subjects exercising in air at temperatures 10°C to 30°C. Results from Wissler model closely approximated the experimental data.

The second version of Wissler Model has many advantages over previous models. Firstly, It improves the Gagge and Stolwijk models in a number of ways. The passive system used is the detailed representation of the temperature profiles in the human body. The circulatory system, which consists of division of blood vessels into arteries, capillaries and veins, is more realistic than any other models. Additionally, each element communicates with adjacent element through blood flow unlike the Stolwijk model where each tissue layer communicates solely with the central blood pool. Further, the Wissler model is designed for a wide range of applications, for example deep sea diving. Finally, the derivation of the control system equations for blood flow rates and metabolic rates is based on the body's oxygen requirements. This approach is a more accurate than any previous method. However, it is also limited to applications involving uniform environmental conditions surrounding each individual body part. Also the model does not allow for alternative blood flow pathways that occur during vasodilation and vasoconstriction. Another weakness is that it is unclear what methods were used to determine values for variables and constants used in Wissler model (Smith, 1991).

In 1970, Stolwijk developed another model. The Stolwijk model used 5 cylindrical elements to represent the body trunk, arms, legs, hands, and the feet and a spherical element for the head. Each element is divided into four concentric shells representing core, muscle, fat and skin tissue layers. Metabolic heat generated during exercise and shivering is divided among the muscle tissue layers. A network blood vessel is used to transport blood from a central blood pool, or the heart, to each tissue layer. Energy balance was given a set of 24 equations which are written for each tissue layer and blood pool. In all tissue energy balance heat storage effects, conductive heat transfer with adjacent tissue layers, convective heat transfer due to the blood flow and basal metabolic heat generation are included. Equation coupling the various body regions is the one of the central blood pool which assumes that venous blood returning to heart is mixed perfectly. Additionally, latent heat loss due to respiration is divided equally between the head core and trunk core, which seems inappropriate because these losses occur predominantly in the head. All skin energy balance equations include heat losses due to convection, radiation and the evaporation of the moisture at the body surface. The results of this derivation were presented in the form of a Fortran program. Experimental data, specifically rectal and skin temperatures, weight losses, and volumetric oxygen consumption rates, were used to validate the accuracy of the model for sedentary subjects in heat and cold stress conditions and for subjects exercising in environments described by air temperatures in the range 20°C-30°C and a relative humidity of 30%.

The control system of the Stolwijk's model is divided into three parts: the thermoreceptors, the integrative systems, and the effectors. The effectors mechanisms are sweating, vasodilatation, vasoconstriction and shivering. Each of these mechanisms can be represented by an equation (Cena and Clark, 1981).

The Stolwijk model is strength in many aspects. Firstly, it is capable of calculating the spatial temperature distribution in the individual body elements. Secondly, Stolwijk model connects these individual elements through blood flow in the arteries, veins, and thus, offers an improved representation of the circulatory system and its effect on the distribution of heat within the body. With these improvements, the Stolwijk model can provide information describing the overall thermal response of the body as well as local responses to varying ambient conditions and physical activity.

[R8] Lastly, the control system of Stolwijk's model is quite detailed. It is divided into three subsystems: the sensing-comparing mechanism, the error signal summation mechanism, and the regulatory mechanism (Arkin and Shitzer, 1984).

However, besides these advantages there are number of drawbacks in the description of the Stolwijk model. This model disregards the effects of the rate of change of skin temperature on the regulatory systems. Also, the local influences are described rather generally and without any theoretical and experimental verification. Finally, conductive heat transfer is considered only in the radial direction or the direction normal to the body centerline. By neglecting the spatial tissue temperature gradients in the angular and axial directions, he restricted the use of his model to simulations involving uniform environmental conditions (Smith, 1991).

Gordon (Fan et al. 1971) has developed a model especially to simulate and predict the physiological responses of the human to cold exposure in 1977. It uses much new physiological data. The body is idealized as 14 cylindrical and spherical angular segments. Each segment consists of several concentric tissue layers, for instance five for the abdomen (core, bone, muscle, fat, skin). Each tissue layer is subdivided into angular shells and a finite-difference formulation is written for the central modal point of each such shell. The control system of this model is characterized by the input signals. These signals are based on the variation in the head core temperature, the skin temperature and also of the heat flux through the skin over the whole body.

Arkin and Shitzer,1984, has developed model of thermoregulation in the human body. They modeled both the passive and active system of human thermal system. In this model, the body is divided into 14 cylinders which are further subdivided into four radial layers. This model is two-dimensional model in radial and tangential directions. The model takes into consideration air velocities, radiation heat loss, heat transfer by convection and evaporation of sweat.

Fiela et al, 1999, has developed passive model for the human thermal system. The body is divided into 15 cylindrical and spherical body elements. The model takes into account the heat transfer with blood circulation, conduction, clothes, convection, radiation, respiration, evaporation. The equations are solved by using the finite difference technique.

# CHAPTER 3

# HUMAN THERMAL SYSTEM

## 3.1 Mathematical Modeling

Mathematical modeling means to find out the mathematical relations that characterise the internal structure of the delimited system and to describe the interdependencies between the input and output variables.

In constructing a mathematical model, first job is to decide which characteristics of the object or system of intereset are going to be represented in the model. In order to make such definitions, it is necessary that the purpose of making the model be defined as clearly as possible. After the recognition of the problem and its initial study, the next step is making certain idealizations and approximations to eliminate unnecessary information and to simplify as much as possible. The third step is the expression of the entire situation in symbolic terms in order to change the real model to a mathematical model in which the real quantities and processes are replaced by mathematical symbols and relations. After the problem has been transformed into symbolic terms, the resulting mathematical system is studied using appropriate mathematical techniques, which are used to make theorems and predictions from the empirical point of view. The final step in the model-building process is the comparison of the results predicted on the basis of the mathematical work with the real world. The most desirable situation is that the phenomena actually observed are accounted for in the conclusions of the mathematical study and that other predictions are subsequently verified by experiment.

Figure 3.1 represents the steps of mathematical modeling. The solid lines in the figure indicate the process of building, developing, and testing a mathematical model as we have outlined it above. The dashed line is used to indicate an abbreviated version of this process which is often used in practice (WEB_1 (1995)).

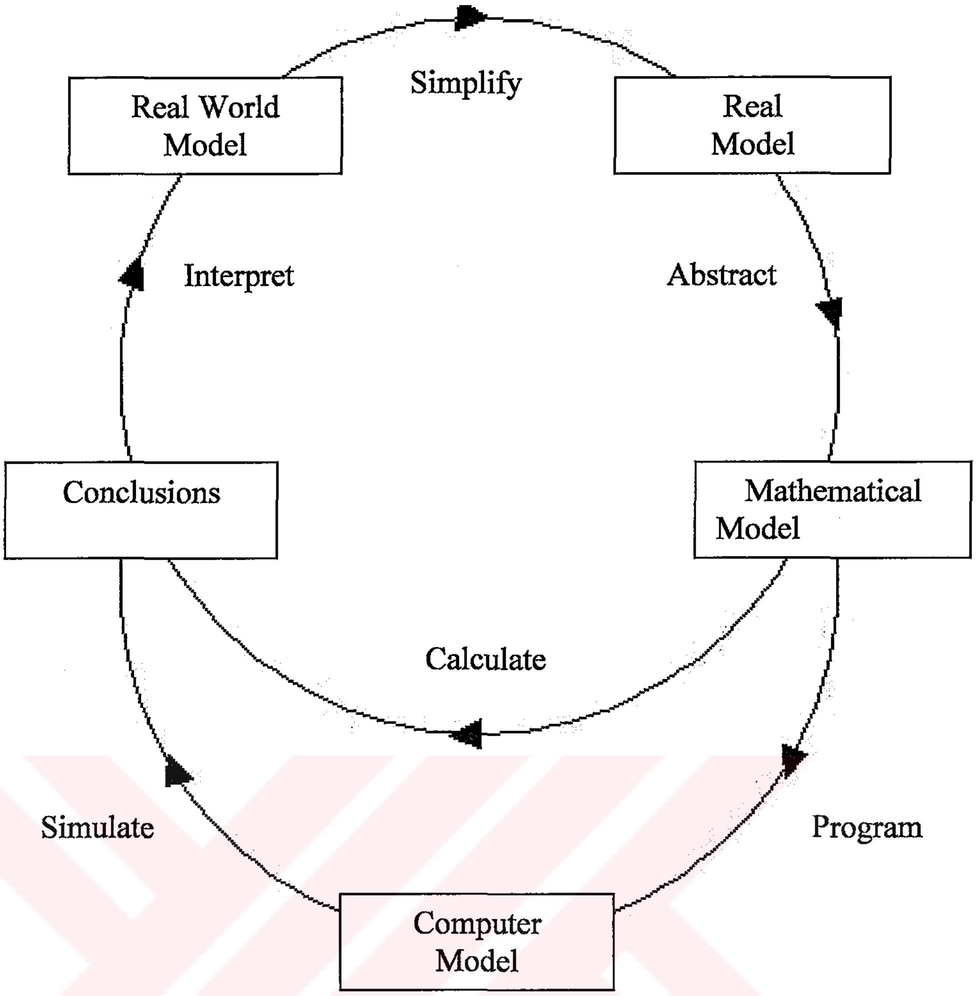



Figure 3.1. Scheme of the steps of mathematical modeling (WEB_1 (1995)).

Mathemetical modeling can be classified into two group as quantitative modeling and qualitative modelling.

### 3.1.1 Quantitative Modeling

Quantitative models of systems can be obtained in two basic ways: by theoretical modeling and by experimental modeling of that system. Many times the two methods are combined in engineering practice.

### a)Theoretical Modeling

Theoretical modeling presupposes the knowledge of essential physical laws of that process. This has to be quite profound in order to obtain a satisfying process model. The model is found out by determining the independent and dependent variables and by stating between them some balance equations, phenomenological laws, state equations that, in general, lead to a system of ordinary and/or partial differential equations or system of difference equations. Its advantage lies in availability of modeling during the planning period of the system since the method does not need the existence of a real process. In this phase, it is also important to find out the influence of different design parameters on the process performance.

In order to obtain a theoretical model that can be handled easily and computed explicitly, some proper assumptions and approximations, whenever necessary, are always made.

### b)Experimental modeling

Experimental modeling is known in the literature as process identification. For identification the process has to be ready for operation and suitable signal generating and recording devices must be available. The mathematical model is determined from the measured signals by using adequate identification methods. The identification methods work with discrete-time signals, therefore they always lead to discrete-time models, that make further computer simulations easy. Applying process identification, often higher model performance can be reached, especially for linearizable processes with less or poor knowledge about their internal behaviour .

## 3.1.2 Qualitative Modeling

Qualitative modeling represents the most recently developed and still under further research domain in finding out mathematical models for systems. The qualitative modeling approach is based on human experience, on knowledge, on pattern recognition. Expert systems, fuzzy sets and neural networks are some keywords that require at least powerful computer tools for successful modelling.

## 3.2 Human Thermal System

Living creatures may be classified into two categories based on their blood temperature: cold blooded and warm blooded animals. "Cold blooded" animals, are those whose body temperature fluctuate with the temperature of the environment. "Warm blooded" animals, constitute a group which exhibit a tendency to stability in the normal body states.

The survival of mammals depends on their ability to control and maintain body temperatures within the range essential to life. This regulation has to be achieved under widely environment, exercise, disease or conditions.

It is well known that the temperature of each of their organs and entire organism should remain within the vital range of 0-42°C, with most of the internal temperatures controlled within the range of 35-39°C.Variability in the endurance time to the extreme of this wider range among the different organs is also great. Some organs such as brain, may suffer irreversible damage should their temperature be allowed to be exceed a much closer limit. The functioning of the other organs such a heart may be slowed down or even impaired should their temperature be kept too low within that range (Gutfinger,1975).

The body temperature of human beings remains relatively constant, despite considerable change in the external conditions. In order to maintain a constant core temperature, the body must balance the amount of heat it produces and absorbs with the amount it loses; this is **thermoregulation**. Thermoregulation maintains the core temperature at a constant set point, averages 36.2°C, despite fluctuations in heat absorption, production and loss. The core temperature fluctuates by about 0.6°C and is lowest around 3 a.m, and highest around 6 p.m. (Despopoulos and Silbernagl, 2003).

The body's thermoregulatory response is determined by feedback from the thermoreceptors. The control center for body temperature and central thermosensors are located in the hypothalamus. The hypothalamus receives afferent input from peripheral thermoreceptors, which are located in the skin, and central thermoreceptors, which is sensitive to the temperature of blood, located in the body core. The hypothalamus maintains the core temperature in vital range by initiating appropriate heat-producing or heat-loss reflex mechanisms by comparing the actual core temperature with the set-point value. There are two centers in the hypothalamus which are concerned with temperature control; the posterior one concerned with protection against cold and the anterior one concerned with protection against heat. These two centers are depicted schematically in Figure 3.2.

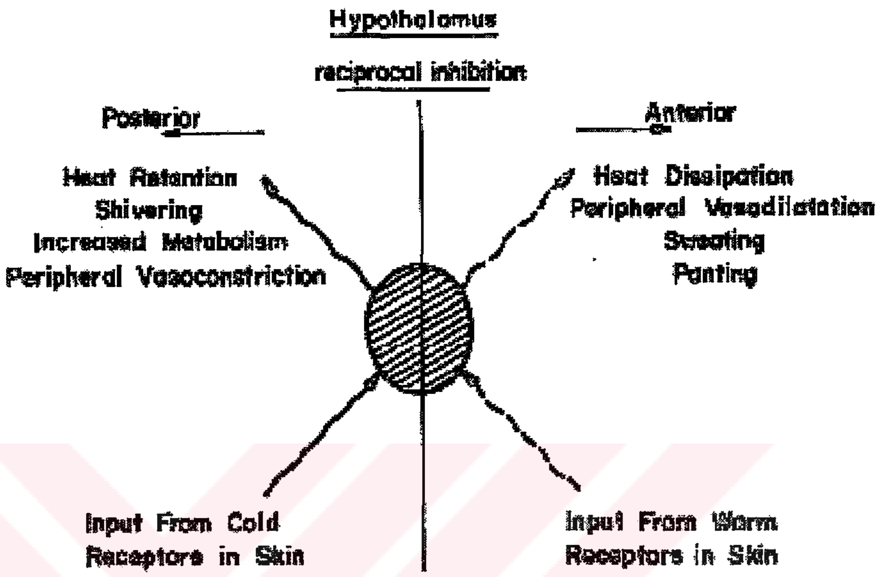



Figure 3.2. Schematic View of Mammalian Temperature Control Center and its Functions (Gutfinger,1975).

### 3.2.1 Heat-Loss Mechanisms

Heat-loss mechanisms protect the body from excessively high temperatures, which can be damaging to the body. Most heat loss occurs through the skin via the physical mechanisms of the heat exchange, such as radiation, conduction, convection, and evaporation. When the core temperature arises above normal, the hypothalamic heat-producing center is inhibited. At the same time, the heat-loss center is activated and so triggers one or both of the following mechanisms.

a) **Vasodilation of cutaneous blood vessels:** Inhibiting the vasomotor fibers serving blood vessels of the skin allows the vessels to dilate. As the skin vasculature swells with warm blood, heat is lost from the shell by radiation, conduction and convection.

b) **Sweating:** If the body is overheated or if the environment is so hot that heat condition not be lost by other, heat loss by evaporation becomes necessary. The sweat glands are strongly activated by sympathetic fibers and spew out large amounts of perspiration. When the relative humidity is high, evaporation occurs much more slowly. In such cases, the heat-liberating mechanisms can not work well, and we feel miserable and irritable.

### 3.2.2 Heat Producing Mechanisms

When the external temperature is cold, the hypothalamic heat-producing mechanisms center is activated. It triggers one or more of the following mechanisms to maintain or increase core body temperature.

a) **Vasoconstriction of cutaneous blood vessels:** Activation of the sympathetic vasoconstrictor fibers serving the blood vessels of the skin causes them to be strongly constricted. As a result, blood is restricted to deep body areas and largely bypasses the skin. Because the skin is separated form deeper organs by a layer of insulating fatty tissue, heat loss from the shell is dramatically reduced and shell temperature drops toward that of the external environment.

b) **Increase in Metabolic Rate**

c) **Shivering**: If the mechanisms described above are not enough to handle the situation, shivering is triggered. Shivering is very effective in increasing body temperature, because muscle activity produces large amount of heat. [3]

## 3.3 Mathematical Modeling of Human Thermal System

A major problem in thermal physiology modeling is the mathematical description of the thermal state of the organism. From the mathematical point of view, the human organism can be separated into two interacting systems of thermoregulation: the *controlling active system* and the *controlled passive system*. Mathematical modelings of these systems are called active model and passive model, respectively.

The active system is simulated by active modeling, which predicts regulatory responses such as shivering, vasomotion, and sweating. The main purpose of the active model is that it regulates the passive heat transfer model and it is responsible for the maintenance of the human body's temperature.

The passive system is modeled by passive modeling, which simulates the physical human body and the heat transfer phenomena occurring in human body and at its surface. For example, the metabolic heat produced within the body, is distributed over body regions by blood circulation and is carried by conduction to the body surface, where, insulated by clothing. In order to loose the heat to the surroundings, the body uses four mechanisms of heat transfer; radiation, conduction, convection, evaporation and respiration. All of these heat transfer phenomena given above are modeled by the passive modeling.

Description of the *passive model* is done by equations resultant from the application of the heat and mass balances to a tissue control volume.

By applying the theories of heat transfer and thermodynamics processes, namely by using the passive modeling of the human thermal system, we can predict the thermal behaviour of the entire human body or a part of it. Information in the change of body is essential for the study of man's tolerance and reaction upon exposure to thermally hostile environment.

## 3.4 Fundamental Equations

All thermoregulation models use the same general equations to describe the heat transfer. Heat is transferred from core to shell and hence to skin surface by conduction and blood convection.

### 3.4.1 Heat Transfer Within the Tıssue

Consider a representative control volume of tissue. This basic element is in continuous thermal communication with surrounding tissues. To maintain equilibrium, thermal energy which is generated inside the element ($q_m'''$) is conducted to adjacent elements, transported (convected) by the blood stream or stored inside the element.

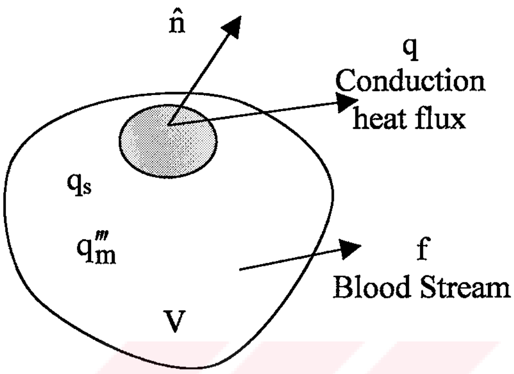



Figure 3.3 Control Volume of Tissue Element

As illustrated in Figure. 3.3, energy-balance equation for this small control volume of tissue is summarized as:

$$\begin{bmatrix} \text{Rate of heat} \\ \text{entering} \\ \text{through the bounding} \\ \text{surface of V} \end{bmatrix} + \begin{bmatrix} \text{Rate of energy} \\ \text{generation} \\ \text{in V} \end{bmatrix} + \begin{bmatrix} \text{Rate of energy} \\ \text{trasported by} \\ \text{blood stream} \\ \text{in V} \end{bmatrix} = \begin{bmatrix} \text{Rate of storage} \\ \text{of energy} \\ \text{in V} \end{bmatrix} \qquad (3.1)$$

#### 3.4.1.1 Conduction

The fundamentals law of heat conduction was developed by Fourier and the conductive heat flux vector is assumed to obey Fourier's law of conduction. Accordingly,

$$q(r,\phi,z,t) = -k(r)\,\nabla T(r,\phi,z,t) \qquad \text{W/m}^2 \qquad (3.2)$$

where the temperature gradient is a vector normal to the isothermal surface, the heat flux vector $q(r,\phi,z,t)$ represents heat flow per unit time, per unit area of the isothermal surface in the direction of the decreasing temperature and k (r) is referred as the thermal conductivity of mammalian tissue which is dependent upon tissue temperature and location. Since the heat flux vector $q(r,\phi,z,t)$ points in the direction of decreasing temperature, the minus sign is included in Equation (3.2) to make heat flow a positive quantity.

The divergence of the temperature T, in the cylindrical coordinate system $(r,\phi,z)$ is given by

$$\nabla T = \hat{u}_r \frac{\partial T}{\partial r} + \hat{u}_\phi \frac{\partial T}{\partial \phi} + \hat{u}_z \frac{\partial T}{\partial z}$$

where $\hat{u}_r, \hat{u}_\phi, \hat{u}_z$ are the unit direction vectors along the $r,\phi,z$ directions respectively.

The total conductive heat through the control surface of tissue is given by

$$-\int_A q \cdot \hat{n}\, dA \qquad (3.3)$$

where A is the surface area of the volume element V, $\hat{n}$ is the outward-drawn normal unit vector to the surface element dA, q is the heat flux vector at dA. The Divergence Theorem is used to convert the surface integral to volume integral.

$$-\int_A q \cdot \hat{n}\, dA = -\int_V \nabla \cdot q(r,\phi,z,t)\, dV \qquad (3.4)$$

After integrating the equation (3.4) on over control volume of the tissue element, the rate of heat, $\nabla q\,(r,\phi,z,t)$, entering through the bounding surface of V is obtained.. The derivation of $\nabla q\,(r,\phi,z,t)$ is given below.

The divergence of the heat flux vector q, in the cylindrical coordinate system $(r,\phi,z)$ is given by

$$\nabla q\,(r,\phi,z) = \frac{1}{r}\frac{\partial}{\partial r}(r\, q_r) + \frac{1}{r}\frac{\partial}{\partial \phi}(q_\phi) + \frac{\partial}{\partial z}(q_z) \qquad (3.5)$$

Applying the equation (3.2) to the three components of the heat flux vector in the $(r, \phi, z)$ directions, the below equations can be taken;

$$q_r = -k\frac{\partial T}{\partial r}, \qquad q_\phi = -k\frac{1}{r}\frac{\partial T}{\partial \phi}, \quad \text{and} \quad q_z = -k\frac{\partial T}{\partial z}$$

To obtain the divergence of the heat flux vector form, the three components of the heat flux vector given above are substituted to equation (3.5)

$$\nabla q(r, \phi, z) = \frac{1}{r}\frac{\partial}{\partial r}(-k \cdot r\frac{\partial T}{\partial r}) + \frac{1}{r}\frac{\partial}{\partial \phi}(-k\frac{1}{r}\frac{\partial T}{\partial \phi}) + \frac{\partial}{\partial z}(-k\frac{\partial T}{\partial z}) \quad \text{then} \tag{3.6a}$$

$$\nabla q(r, \phi, z) = -k\frac{1}{r}(\frac{\partial T}{\partial r} + r\frac{\partial^2 T}{\partial r^2}) - k\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} - k\frac{\partial^2 T}{\partial z^2} \tag{3.6b}$$

$$\nabla q(r, \phi, z) = -k\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right] \tag{3.7}$$

When we substitute $\nabla q(r, \phi, z, t)$ into the equation (3.4), we can get the rate of heat (q) entering through the bounding surfaces of V.

### 3.4.1.2 Metabolic Heat Generation

The rate of energy generation within an organism is defined as the rate of transformation of chemical energy into heat and mechanical work by aerobic and anaerobic metabolic activities. These activities are the sum of the biochemical processes by which food is broken down into simpler compounds with the exchange of energy.

The factors which influence the metabolic heat generation include surface area, age, gender, stress, and hormones. Although metabolic heat generation is related to overall body weight and size, the critical factor is surface area rather than weight itself. This reflects the fact that as the ratio of body surface area to body volume increases, heat loss to the environment increases and the metabolic heat generation must be higher to replace the lost heat. Hence, if two people weight the same, the taller or thinner person will have a higher metabolic heat generation than the shorter or fatter person.

According to above information, the total amount of heat generated in the control volume is given by:

$$q_m = \int_V q_m''' \, dV \tag{3.8}$$

where $q_m'''$ is the specific rate of heat production which may generally be a function of tissue temperature and location. [6] The heat generation rate in the medium, generally specified as heat generation per unit time, per unit volume, is denoted by the symbol $q_m'''$ (r), and given in the units $W/m^3$.

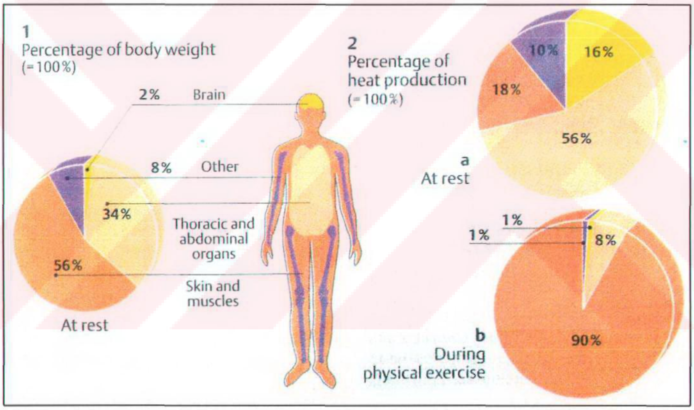



Figure 3.4 Relative Contributions of Organs to Heat Production (Despopoulos and Silbernagl,2003).

The amount of heat produced is determined by energy metabolism. At rest, approximately 56% of total heat production occurs in the internal organs and about 18% in the muscle and skin. During physical exercise, heat production increases several-fold and the percentage of heat produced by muscular work can raise to as much as 90%.

### 3.4.1.3 Convection by the Circulatory System

Heat produced in the body should be absorbed by the bloodstream and conveyed to the body surface. Because all body tissues are poor conductors of heat. If the heat transfer in the body depended on conduction, very large temperature gradients would be needed, and the ability to adapt to varying environmental conditions would be poor. Therefore, the convective flow of blood throughout the body is very important in internal heat transfer (Cooney, 1976).

When there is a significant difference between the temperature of the blood and the tissue through which it flows, convective heat transfer will occur, altering the temperature of both the blood and the tissue (Valvono, 2001).

The effects of the blood circulation to the internal heat distribution within the body can be summarized in three major ways (Cooney, 1976).

1. It minimizes temperature differences within the body. Tissues having high metabolic rates are more highly perfused, and thus are kept at nearly same temperature as less active tissues. Cooler tissues are warmed by blood coming from active organs.

2. It controls effective body insulation in the body skin region. When the body wishes to reject heat, how warm blood flow to the neighborhood of the skin is increased by "vasodilation", and how the blood is bypassed from arteries to veins via deeper channels through "vasoconstriction", when conservation of body heat is vital. These automatic mechanisms either raise or lower the temperature gradient for heat transfer by conduction in the subskin layers.

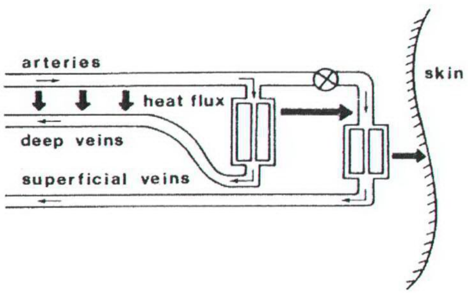



Figure 3.5 Counter Current Heat Exchange in Extremities (Cooney, 1976).

Figure 3.5 indicates that when valve is open, blood flow is routed through superficial capillary bed, allowing efficient transfer of heat to body surface. Blood returning through superficial veins does not exchange significant amounts of heat with deep arterial blood. When valve is closed superficial blood flow is reduced, and most blood returns via deep veins.

3. Countercurrent heat exchange between major arteries and veins often occurs to a significant extent. If heat conservation is necessary, arterial blood flowing along the body's extremities is precooled by loss of heat to adjacent venous streams. This reduces the temperature of the limbs and lowers heat losses. Since most arteries lie deep, while veins occur both in superficial and deep regions, the extent of the arteriovenous heat exchange depends on the route taken back to the body trunk by the venous blood. This is automatically regulated by the vasodilation- vasoconstriction mechanisms.

The capillary bed forms the major site for the live exchange of mass and energy between the blood stream and surrounding tissue. The arterial blood, known temperature $T_{art}$, flows through the capillary bed where complete thermal equilibrium between the blood and tissue is attained. This exchange is a function of several parameters including the rate of perfusion and the vascular anatomy, which vary widely among the different tissues, organs of the body.

The term which represents the rate of energy transported by blood stream is based on Fick's principle. Assuming that, heat is exchanged with the tissue in the capillary bed and that no heat storage occurs in the bloodstream. Hence, total energy exchange between blood and tissue is given

$$q = \int_V \rho_{bl} \times \dot{w}_{bl} \times Cp_{bl} \times (T_{art} - T) \tag{3.9}$$

where $\rho_{bl}$ and $Cp_{bl}$ are the density and specific heat of the blood respectively. Both properties can be taken as constant. $W_{bl}$ is the blood perfusion rate which may generally be a function of location.

### 3.4.1.4 Storage of Thermal Energy

When the body temperature is constant the rate of heat storage in the tissue is zero, in practice it is negligible over long time periods. However, over short periods and severe environments, heat storage in the tissue, which can be an important component of the heat balance, determines the tolerance time for work. Therefore, heat storage is likely to be important in man for periods of up to few hours. In small animals it is important for periods of just a few minutes (Cena and Clark, 1981).

In sum, under transient conditions part of the thermal energy generated or transferred to the control volume may go to alter the amount stored inside it. Thus, the rate of change in storage of thermal energy is given by:

$$q_s = \frac{\partial}{\partial t} \int_V \rho C_p T dV \qquad (3.10)$$

where $C_p$ is the specific heat of the various tissues comprising the organism and is also generally a function of tissue temperature and location.

## 3.4.2 Heat Exchange With Environment

Heat flow inside the human body occurs when the temperature of the body surface is lower than that of the body interior. The body supply to the skin is the chief determinant of heat transport to the skin. Heat loss occurs by the physical processes of radiation, conduction, convection, and evaporation (Despopoulos and Silbernagl, 2003).

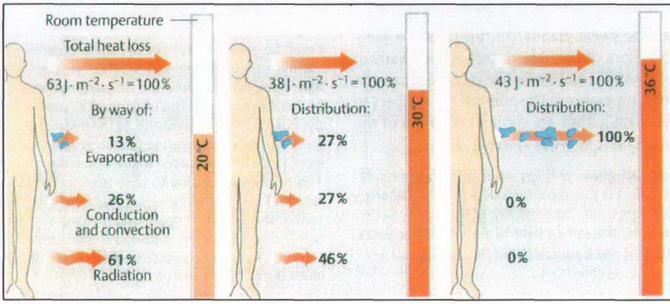



Figure 3.6 Percentage of Heat Loss at Different Environmental Temperatures (Despopoulos and Silbernagl, 2003).

### 3.4.2.1 Radiation

Radiation is the loss of heat in the form of infrared waves. All objects continually radiate energy in accordance with the Stefan-Boltzmann law, i.e., proportionately with the surface area, emissivity, and the fourth power of the absolute temperature. When the surrounding is cooler than the body, net radiative heat loss occurs. Under normal conditions, close to half of body heat loss occurs by radiation. In contrast, when one's surrounding is hotter than the one's body, a net heat gain via radiation occurs (Cooney, 1976).

The amount of incident radiation that is captured by a body depends on its area, the incident flux, and the body's absorptivity. It is common to estimate the absorptivity of a body as equal to its emissivity at the temperature of the surroundings, although this is strictly true only when the body is in radiative equilibrium with the surroundings.

The net rate of heat exchange by radiation between an organism and its environment, usually expressed in terms of unit area of the total body surface.

$$q_r = \varepsilon * \sigma * (T^4{}_s - T^4{}_a) \quad (W/m^2) \tag{3.14}$$

where $\varepsilon$ is the emissivity which is approximately equal to the absorptivity. For incident infrared radiation, the absorptivity of human skin is very high, about 0.97, and is dependent of color. For visible light, the skin has an absorptivity of about 0.65- 0.82, depending on whether it is white or dark, respectively (Cooney, 1976). Stefan-Boltzmann Constant, $\sigma$ , is $5.67051\times10^{-8}$ ($W/m^2K^4$). $T_s$ is the surface temperature of the body or clothes and $T_a$ is the temperature of surroundings.

### 3.4.2.2 Convection

When the body shell transfers heat to the surrounding air, convection also comes into play. Because warm air tends to extend and rise and cool air falls, the warmed air enveloping the body is continually replaced by cooler air molecules. This process, called convection, substantially enhances heat exchange from the body surface to the air, because the cooler air absorbs heat by conduction more rapidly than the already-warmed air.

The movements of fluid which carry heat away from the body surface may be driven by two mechanisms: "free convection" due to density differences in the fluid associated with temperature gradients; or "forced convection", due to external forces such as wind (Cena and Clark, 1981). Pure free convection occurs under stagnant conditions when the velocity of the ambient toward the person is zero. Forced convection heat transfer occurs when the ambient is approaching the body with a definite (and usually steady) velocity (Cooney, 1976).

Convective heat losses from the body are strongly dependent on air velocity. The simplest equation for characterizing convective losses is

$$q_c = h_c * (T_s - T_a) \qquad (W/m^2) \qquad (3.15)$$

$T_s$ is the surface temperature of the body or clothes and $T_a$ is the temperature of surroundings. And $h_c$ is the convective heat transfer coefficient.

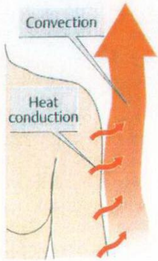



Figure 3.7 Schematic View of Heat Loss by Convection (Despopoulos and Silbernagl, 2003).

According to the equation (3.15) the calculation of heat loss due to convection requires the estimation of convection heat transfer coefficient $h_c$, which is calculated from the equation given below

$$Nu = \frac{h_c \times D_{limb}}{k_a} \qquad (3.16)$$

where Nu is the Nusselt Number, $D_{limb}$ is the external diameter of the limb (m), $h_c$ is the convection heat transfer coefficient and $k_a$ is the thermal conductivity of the air (W/mK).

## a) Free Convection

In the situation of density gradients, the body force acts on a fluid. The net effect is the bouncy force, which induces free convection currents. In the most common case, the density gradient is due to a temperature gradient, and body force is due to the gravitational field (Incropera and Dewitt, 1996). For a subject with a mean skin temperature lower than air temperature, the air adjacent to the skin surface will become heated by conduction and will rise due to buoyancy. This is the mechanism by which heat is lost from the body by free convection.

**b) Forced Convection**

When the body is exposed to a wind or is moving through the air, the natural convective boundary-layer flow is displaced and the body losses heat by forced convection. The variables that influenced forced convection are the mean air velocity, the flow direction and the nature of the flow whether it is laminar or turbulent. The degree of turbulence and its scale can have a profound effect upon the heat loss.

### 3.4.2.3 Evaporation

Heat loss by radiation and heat loss by convection alone are unable to maintain adequate temperature homeostasis at high environmental temperatures or during strenuous physical activity. Because water absorbs a great deal of heat before vaporizing, its evaporation from the body surfaces removes large amount of body heat. The water lost by evaporation reaches the skin surface by diffusion and by neuron-activated sweat glands. At temperatures above 36°C or so, heat loss occurs by evaporation only. In addition to this, the surrounding air must be relatively dry in order for heat loss by evaporation to occur. Humid air restricts evaporation. When the air is extremely humid (e.g. in a tropical rain forest), the average person can not tolerate temperatures above 33°C, even under resting conditions. (Despopoulos and Silbernagl,2003).

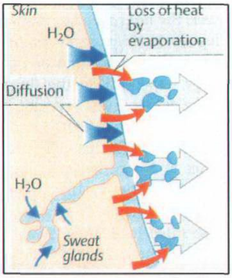



Figure 3.8 Scheme Heat Loss by Evaporation (Despopoulos and Silbernagl,2003).

In human body, evaporative heat losses occur by several mechanisms:

a) Heat losses by diffusion of water through the skin
b) Heat losses by sweat secretion
c) Heat losses by evaporation of water into inspired air.

## a) Heat Losses by Diffusion of Water Through The Skin

Water diffusion through the human skin is part of the “insensible” perspiration. This diffusion totals about 350 ml/day in an average person and is assumed to be proportional to the difference between the vapor pressure of water at the skin temperature and the partial pressure of water vapor in the ambient air.

Inouye (Cooney, 1976) gave the correlation about the diffusional heat loss per unit area.

$$q_d = \frac{4184}{3600} \times (0.35) \times (P_s - P_a) \qquad (\text{W/m}^2) \qquad (3.24)$$

where is the $P_s$ is the vapor pressure of water at skin temperature and $P_a$ is the partial pressure of water vapor in the ambient air. $P_s$ and $P_a$ are in millimeters of mercury and they are calculated from equation (3.25) and (3.26) respectively

The vapor pressure of water at skin temperature, $P_s$, and partial pressure of water vapor in the ambient air, $P_a$, can be calculated by given formulas (Cooney, 1976).

$$P_s = 1.92 T_s - 25.3 \qquad (\text{mmHg}) \qquad (3.25)$$

$$P_a = P_v \times (\%RH) \qquad (\text{mmHg}) \qquad (3.26)$$

In equation (3.25) $T_s$ is the surface temperature of the limb. It is in Celsius (°C). In equation (3.26) $P_v$ is the vapor pressure of water at air temperature in the condition of 1 atm and RH is the relative humidity of the air.

Diffusional heat loss can be calculated after substituting the Ps and Pa value which are calculated from equation (3.25) and (3.26).

## b) Heat Losses By Sweat Secretion

When the heat loss amount is not ample to maintain the core temperature in a suitable range, an automatic mechanism of the body appears. This mechanism for increasing the heat loss is the sweating response, which provides secretion of a dilute electrolyte solution from numerous glands to the skin surface. Then, evaporation from the wetted surface then occurs (Cooney, 1976).

Heat loss by sweat secretion per unit area is given by:

$$q_s = \frac{4184}{3600} \times K_e \times (P_s - P_a) \qquad (W/m^2) \qquad (3.27)$$

where $K_e$ is the coefficient for evaporation heat loss from nude person in air. It is in (kcal/$m^2$hr mmHg). The correlations for $K_e$ which have been experimentally determined by several investigators.

# CHAPTER 4

# DEVELOPMENT OF MODEL

In this chapter, passive modeling of the human thermal system is investigated. Factors that help to build the model are given below. In order to avoid the complexity some assumptions are made. Lastly, the derivation of Bio-Heat Equation for passive system and boundary conditions that act to the human body are given.

As mentioned before, passive modeling of human thermal system deals with the heat transfer phenomena occurring in human body and at its surface. Application of heat balance to a tissue control volume results in equations which simulate the passive system in the mathematical point of view. Solving these equations by chosen method leads to predict the thermal behavior of entire human body or a part of it for different environmental conditions.

The objective of this study is to present the mathematical model of human heat transfer. The model is a multi-segmental, multi-layered representation of the human body with spatial subdivions which simulates the heat transfer phenomena within the body and at its surface. To simulate adequately all these heat-transport phenomena, the present model of the passive system accounts for the geometric and anatomic characteristics of the human body and considers the thermophysical and the basal physiological properties of tissue materials.

In order to avoid the complexity of the passive system in human thermal system the following factors should be considered when attempting such a model.

1) Geometry of organism.
2) Thermophysical and physiological properties of various organs and tissues.
3) Metabolic heat production.
4) Role of blood in heat transfer.
5) Conduction of heat due to the thermal gradients.
6) Thermoregulatory mechanisms in the organism and their functions.
7) The interaction with the environment and air condition.

## 4.1. Geometry of Organism

The human body parts resemble cylinders in appearance; therefore it is convenient to use cylinders to model the human body. The body is divided into 16 concentric cylinders, which depict the head, neck, abdomen (lower torso), thorax (upper torso), upper arms, lower arms, hands, thighs, calves, and feet. A schematic view of the model is given in Figure 4.1.

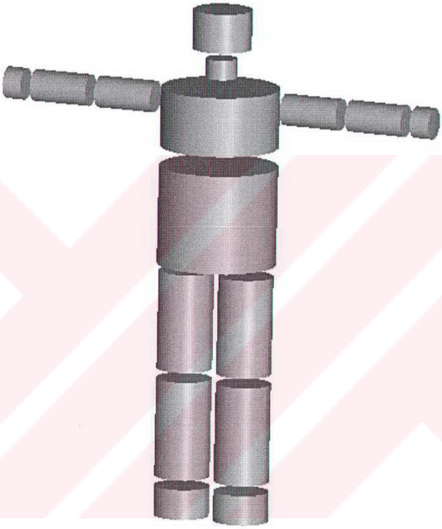


Figure 4.1. Schematic View of the Model

## 4.2. Thermophysical and physiological properties of various organs and tissues.

In our model, each of limbs is subdivided in the radial direction, describing the distributions of the various tissue types throughout the body. According to this division, assuming that the tissue thermal conductivity, specific heat of the tissue, tissue density and the blood perfusion rate of the tissue are segmentally uniform in each layer.

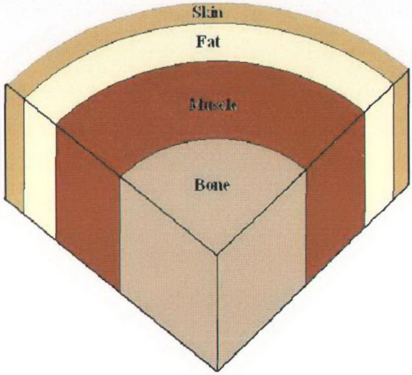



Figure 4.2. Spatial Division of Leg

Each tissue element is assigned specific types, e.g. brain, bone, muscle, fat, skin, lung or viscera. For example, head is divided axially into four concentric shells. In head, the inner shell is the brain tissue which is surrounded by a bone shell. This bone shell is covered with thin layers of fat and skin respectively. The lungs locating in the core of the thorax (upper torso) are surrounded by the rib cage. The other internal organs (viscera), locating in the abdomen (lower torso), partially enclosed in back by pelvic bone. Layers of muscle, fat, and skin encase both thorax and abdomen regions respectively. All remaining body parts are modeled as a single bone core surrounded by muscle, fat and skin. In bone tissue, the investigators suggest that the energy contribution of blood flow and metabolism are negligible. Thus the blood perfusion rate and metabolic heat generation are taken zero (Gutfinger, 1975). The thermal properties of the different body tissues are listed in Table 4.1.

## 4.3 Metabolic Heat Production

The model accounts metabolic heat generation which differs from the layer to layer. Metabolic heat is assumed to be generated uniformly by metabolic and chemical reactions in each section of the cylinder, but the rates are not necessarily equal. Some layers produce much more metabolic heat, such as brain, which has the highest metabolic heat generation amount (13400 W/m$^3$) among the other tissue types. In contrast, bone tissues have no ability to produce heat; therefore, bone's metabolic heat generation value is zero. Tissue type of internal organs (viscera) also has high metabolic heat production ability. For example, in lower torso metabolic heat producing in viscera is 4100 W/m$^3$. In each layer, metabolic heat generation is segmentally uniform can be seen in Table 4.1.

**Table 4.1** Thermal Properties of Body Tissues (Fiala et.al.1999)

| Body Elements | Tissues | r (m) | k (W.m$^{-1}$.K$^{-1}$) | ρ (kg/m$^3$) | $C_p$ (J.kg$^{-1}$.K$^{-1}$) | $w_{bl,0}$x10$^{-3}$ (l.s$^{-1}$m$^{-3}$) | $q_{m,0}$ (W/m$^3$) |
|---|---|---|---|---|---|---|---|
| Head | Brain | 0.0860 | 0.49 | 1,080 | 3,850 | 10.1320 | 13,400 |
| | Bone | 0.1005 | 1.16 | 1,500 | 1,591 | 0 | 0 |
| | Fat | 0.1020 | 0.16 | 850 | 2,300 | 0.0036 | 58 |
| | Skin | 0.1040 | 0.47 | 1,085 | 3,680 | 5.4800 | 368 |
| Neck | Bone | 0.0190 | 1.90 | 1,357 | 1,700 | 0 | 0 |
| | Muscle | 0.0546 | 5.46 | 1,085 | 3,768 | 0.5380 | 684 |
| | Fat | 0.0556 | 5.56 | 850 | 2,300 | 0.0036 | 58 |
| | Skin | 0.0567 | 5.67 | 1,085 | 3,680 | 6.8000 | 368 |
| Throax | Lung | 0.0773 | 0.28 | 550 | 3,718 | 0 | 600 |
| | Bone | 0.0891 | 0.75 | 1,357 | 1,700 | 0 | 0 |
| | Muscle | 0.1234 | 0.42 | 1,085 | 3,768 | 0.5380 | 684 |
| | Fat | 0.1268 | 0.16 | 850 | 2,300 | 0.0036 | 58 |
| | Skin | 0.1290 | 0.47 | 1,085 | 3,680 | 1.5800 | 368 |
| Abdomen | Viscera | 0.0785 | 0.53 | 1,000 | 3,697 | 4.3100 | 4,100 |
| | Bone | 0.0834 | 0.75 | 1,357 | 1,700 | 0 | 0 |
| | Muscle | 0.1090 | 0.42 | 1,085 | 3,768 | 0.5380 | 684 |
| | Fat | 0.1244 | 0.16 | 850 | 2,300 | 0.0036 | 58 |
| | Skin | 0.1260 | 0.47 | 1,085 | 3,680 | 1.4400 | 368 |
| Arms | Bone | 0.0153 | 0.75 | 1,357 | 1,700 | 0 | 0 |
| | Muscle | 0.0343 | 0.42 | 1,085 | 3,768 | 0.5380 | 684 |
| | Fat | 0.0401 | 0.16 | 850 | 2,300 | 0.0036 | 58 |
| | Skin | 0.0418 | 0.47 | 1,085 | 3,680 | 1.1000 | 368 |
| Hands | Bone | 0.0070 | 0.75 | 1,357 | 1,700 | 0 | 0. |
| | Muscle | 0.0174 | 0.42 | 1,085 | 3,768 | 0.5380 | 684 |
| | Fat | 0.0204 | 0.16 | 850 | 2,300 | 0.0036 | 58 |
| | Skin | 0.0226 | 0.47 | 1,085 | 3,680 | 4.5400 | 368 |
| Legs | Bone | 0.0220 | 0.75 | 1,357 | 1,700 | 0 | 0 |
| | Muscle | 0.0480 | 0.42 | 1,085 | 3,768 | 0.5380 | 684 |
| | Fat | 0.0533 | 0.16 | 850 | 2,300 | 0.0036 | 58 |
| | Skin | 0.0553 | 0.47 | 1,085 | 3,680 | 1.0500 | 368 |
| Feet | Bone | 0.0200 | 0.75 | 1,357 | 1,700 | 0 | 0 |
| | Muscle | 0.0250 | 0.42 | 1,085 | 3,768 | 0.5380 | 684 |
| | Fat | 0.0326 | 0.16 | 850 | 2,300 | 0.0036 | 58 |
| | Skin | 0.0350 | 0.47 | 1,085 | 3,680 | 1.5000 | 368 |

## 4.4 Role of Blood in Heat Transfer

The model accounts only heat exchange between the blood capillaries and the tissues. Further exchange of heat between the major venous vessels and the tissue surrounding them is also possible. However, this exchange is not significant compared to the exchange in the capillary bed. Therefore, heat exchanges between large blood vessels themselves and large blood vessels to tissue are not included in the Equation of thermal energy balance for the tissue.

A common assumption is made, based on Fick's principle that blood enters capillaries at the temperature of arterial blood, $T_{art}$, where heat exchanges occurs to bring the temperature to that of the surrounding tissue, T. There is assumed to be no energy transfer either before or after the blood passes through the capillaries, so that the temperature at which it enters the venous circulation is that of the local tissue. Another assumption is that there is no heat storage occurs in the bloodstream when the heat exchange occurs between the capillary bed and the tissue.

Heat exchange with capillary bed and tissue is occurred according to the Equation (3.9). In Equation (3.9), the blood perfusion term, $\dot{w}_{bl}$, is taken homogenous and isotropic and that thermal equilibration occurs in the microcirculatory capillary bed through the tissue layer. For example, in abdomen, the viscera layer is highly perfused than the other layer in abdomen. Blood perfusion rates of different tissues in each cylinder are given in Table 4.1. Also the densities of blood, ρbl, and specific heat of the blood, Cpbl, are taken constant. Their values are 1060 $kg/m^3$ and 3650 kJ/kgK, respectively (Rideout 1991).

## 4.5 The Interaction with the Environment

The model takes into account air velocity around the body, the ambient temperature and the relative humidity of the air in order to calculate the heat losses by convection, evaporation, and radiation. In calculations, the uniform environmental condition is assumed. This means that the ambient air temperature, relative humidity of air, and wind velocity are the same for each limb.

## 4.6 Radiation

In this study, the rate of heat exchange by radiation between an organism and its environment is calculated by using the Stefan-Boltzmann law as given in Equation (4.1). It is assumed that the human body is in radiative equilibrium with the surrounding. Therefore, the absorptivity of the body is taken the same with the body emissivity at the temperature of the surroundings. During the calculation, the value of emissivity of the human body is taken 0.97 which is the standard emissivity value given in literature.

$$q_r = \varepsilon * \sigma * (T^4{}_s - T^4{}_a) \qquad (W/m^2) \qquad (4.1)$$

## 4.7 Evaporation

The amount of heat removed by evaporation is a function of evaporation potential of the environment. It is assumed that, in model, evaporative heat losses occur only by diffusion of water through the skin and by sweat secretion. Heat loss by evaporation of water into inspired air, namely respiration, is neglected in our model.

### 4.7.1 Heat Losses by Diffusion of Water through the Skin

In order to calculate the diffusional heat loss, Inouye's correlation is used (Cooney 1976). This correlation gives the diffusional heat loss per unit area.

$$q_d = \frac{4184}{3600} \times (0.35) \times (P_s - P_a) \qquad (W/m^2) \qquad (4.2)$$

where $P_s$ is the vapor pressure of water at skin temperature and $P_a$ is the partial pressure of water vapor in the ambient air. $P_s$ and $P_a$ are in millimeters of mercury and they are calculated from Equation (4.3) and (4.4), respectively.

The vapor pressure of water at skin temperature, $P_s$, and partial pressure of water vapor in the ambient air, $P_a$, can be calculated by given formulas (Cooney 1976).

$$P_s = 1.92T_s - 25.3 \qquad \text{(mmHg)} \qquad (4.3)$$

$$P_a = P_v \times (\%RH) \qquad \text{(mmHg)} \qquad (4.4)$$

In Equation (4.3) $T_s$ is the surface temperature of the limb. It is in Celsius (°C). In Equation (4.4) $P_v$ is the vapor pressure of water at air temperature in the condition of 1 atm and RH is the relative humidity of the air.

Diffusional heat loss can be calculated after substituting the Ps and Pa values which are calculated from Equation (4.3) and (4.4) into Equation (4.2).

### 4.7.2 Heat Losses by Sweat Secretion

In this study, the formulation of heat loss by sweat secretion per unit area is given by (Cooney 1976):

$$q_s = \frac{4184}{3600} \times K_e \times (P_s - P_a) \qquad (\mathrm{W/m^2}) \qquad (4.5)$$

where $K_e$ is the coefficient for evaporation heat loss from nude person in air. It is in ($\mathrm{kcal/m^2hr\ mmHg}$). Correlations for $K_e$ which have been experimentally determined by several investigators are shown in Table 4.2.

Table 4.2 Evaporation Heat Transfer Coefficient from Nude Person in Air (Cooney 1976)

| $K_e$ (kcal/m$^2$hr mmHg) | Authors | Conditions |
|---|---|---|
| $12.70\ v^{0.634}$ | Clifford | $v > 0.58$ m/s, standing, cross flow |
| $9.660\ v^{0.25}$ | Clifford | $v < 0.51$ m/s |
| $10.17\ v^{0.37}$ | Nelson | $0.15 < v < 3.05$ m/s |
| $18.40\ v^{0.37}$ | Machle and Hatch | - |
| $11.60\ v^{0.40}$ | Wyndham and Atkins | - |
| $19.10\ v^{0.66}$ | Fourt and Powell | - |
| $13.20\ v^{0.60}$ | Fourt and Powell | - |

### 4.1.8 Convection

The heat transfer by convection depends on the air velocities, air properties and size of the limbs. Therefore, in this study, convective heat transfer coefficient, $h_c$, is different for each limb and calculated in the program which is discussed succeeding chapter.

The equation used in our model for characterizing convective losses is given below.

$$q_c = h_c * (T_s - T_a) \qquad (W/m^2) \qquad (4.6)$$

$T_s$ is the surface temperature of the body or clothes, $T_a$ is the temperature of surroundings, $h_c$ is the convective heat transfer coefficient.

Heat transfer coefficient is calculated from the formula given below. As seen, in order to find out the $h_c$, the Nusselt number, which varies according to the convection type, should be known.

$$Nu = \frac{h_c \times D_{limb}}{k_a} \qquad (4.7)$$

where Nu is the Nusselt number, $D_{limb}$ is the external diameter of the limb (m), $h_c$ is the convection heat transfer coefficient and k is the thermal conductivity of the air (W/mK).

According to the outlet temperature ($T_a$), Prandtl number and thermal conductivity (W/mK) and kinematic viscosity of the air ($m^2/s$) are found from WEB_2,2003.

Nusselt number correlations, which are used in our model, and calculation of convection heat transfer coefficient both for free convection and forced convection are given below.

### 4.8.1 Free Convection

In model, Monteith (1973) Nusselt correlation is used to calculate the Nusselt number. According to Monteith, the Nusselt number for free convection from a human body is given by (Cena and Clark 1981).

$$Nu = 0.63\, Gr^{0.25}\, Pr^{0.25} \qquad (4.8)$$

where Pr is the Prandtl Number of the air at given temperature and Gr is the Grashof number which is expressed in Equation (4.9)

$$Gr = \frac{g \times \beta \times (T_s - T_a) \times L^3}{\nu_a^3} \qquad (4.9)$$

$$\beta = \frac{1}{T_f} \qquad (4.10)$$

$$T_f = \frac{(T_s + T_a)}{2} \quad (K) \qquad (4.11)$$

In Equation (4.9), g is the acceleration of gravity (9.8 $m/s^2$) and $\beta$ is the thermal expansion coefficients of the air (1/K) which is given in Equation (4.10), $\nu$ is the kinematic viscosity of the air ($m^2/s$), $T_f$ id the film temperature and $T_s$ is the surface temperature of the body or clothes and $T_a$ is the temperature of surroundings and lastly L is the length of the limb (m).

For free convection, the calculation step of free convection coefficient $(hc)_{free}$ is given below;

1. From Equations (4.11), (4.10), (4.9), film temperature (K), thermal expansion coefficients of the air (1/K), and Grashof number are calculated, respectively.
2. Calculated Grashof number and the Prandtl Number of the air at $T_a$ are substituted to Equation (4.8) in order to find out Nusselt number for free convection.
3. From Equation (4.7), Nusselt Number, which is the function of $h_c$, is obtained.
4. The calculated Nusselt Number from the correlations for free convection labeled with Equation (4.8) is equaled to the Nusselt number, which is the function of of $h_c$, obtained from Equation (4.7) in order to derive $h_c$.
5. After applying these steps we can find the convective heat transfer coefficient for free convection as the function of wind velocity.

### 4.8.2 Forced Convection

The calculation of a rate of heat loss by forced convection requires the estimation of Nusselt number, which depends on the size and shape of the body, the nature of its surface and fluid properties. Some relations are available in literature for common geometric shapes such as cylinder or sphere. Human body is assumed as a smooth cylinder with acceptable accuracy in order to calculate Nusselt number.

In forced convection the Nusselt number for a smooth cylinder is given by (Cena and Clark 1981)

$$Nu = 0.26\, Re^{0.60}\, Pr^{0.33} \qquad \text{for } 10^3 \le Re \le 5\times10^4 \qquad (4.12)$$

$$Nu = 0.026\, Re^{0.81}\, Pr^{0.33} \qquad \text{for } 4\times10^4 \le Re \le 4\times10^5 \qquad (4.13)$$

where Pr is the Prandtl number of the air at given temperature and Re is the Reynolds number which is expressed in Equation (4.14)

$$Re = \frac{D_{\lim b} \times v}{\nu_a} \tag{4.14}$$

In Equation (4.14), $D_{limb}$ is the external diameter of the limb (m), v is the velocity of the wind (m/s) and $\nu$ is the kinematic viscosity of the air ($m^2/s$).

For forced convection, the calculation step of forced convection coefficient $(hc)_{forced}$ is given below;

1. From Equation (4.14) Reynolds Number is calculated.
2. According to the value of Reynolds Number, the Nusselt correlation for forced convection is chosen from Equation (4.12) or Equation (4.13).
3. From Equation (4.7), Nusselt Number, which is the function of $h_c$, is obtained.
4. The calculated Nusselt Number from the correlations for forced convection is equaled to the Nusselt number, which is the function of $h_c$, obtained from Equation (4.7) in order to derive $h_c$.
5. After applying these steps we can find the convective heat transfer coefficient for forced convection as the function of wind velocity.

Nusselt number can be calculated according to the convection type. Nusselt correlations for both free and forced convection can be found in the literature (Cena and Clark 1981) (Cooney 1976) (Incropera and Dewitt1996).

## 4.9 Heat Conduction

Conduction of heat occurs due to the thermal gradients between the tissue and surrounding tissues. In our model, conductive heat transfer between the tissue and surrounding tissue obeys the Fourier's Law of Conduction which is described in Chapter 3. Also, it is assumed that, the tissue thermal conductivity (k), which is given in Table 4.1, is segmentally uniform in each layer.

## 4.10 Thermoregulatory Mechanisms in the Organism and Their Functions

In this model, thermoregulatory mechanism in the organism and their functions are not considered. Therefore; vasomotor activity, sweating, shivering, increased metabolism due to glandular activity, and panting are neglected. These parameters are taken into account in active modeling of the human thermal system, which deals with maintaining the human body's temperature at a constant level.

## 4.11 Passive System Equation

Passive system Equation is the Equation that describes the passive system of human body with the mathematical point of view. Forming the passive system Equation is the final and most important step to finish the mathematical modeling of the human thermal system.

In order to understand the heat transfer phenomena in the tissues The Bio-Heat Equation should be derived for the unit volumetric tissue. According the assumptions given in pervious sections, the Bio-Heat Equation is written. An application of the Fist Law of Thermodynamics, the principle of conservation of energy, requires that the rate of heat gain minus rate of heat loss is equal to the rate of storage in the tissue. The substitution of Equations (3.4), (3.8), (3.9), and (3.10) into Equation (3.1) yields

$$-\int_V \nabla \cdot q(r,\phi,z,t)\,dV + \int_V q'''_m(r)\,dV + \int_V \rho_{bl} \times \dot{w}_{bl}(r) \times Cp_{bl} \times (T_{art} - T)\,dV \qquad (4.15)$$

$$-\frac{\partial}{\partial t}\int_V \rho(r) C_p(r) T\,dV = 0$$

$$\int_V \begin{bmatrix} -\nabla \cdot q(r,\phi,z,t) + q'''_m(r) + \rho_{bl} \times \dot{w}_{bl}(r) \times Cp_{bl} \times (T_{art} - T) \\ \\ -\rho(r) C_p(r) \dfrac{\partial T(r,\phi,z,t)}{\partial t} \end{bmatrix} dV = 0 \qquad (4.16)$$

Equation (4.16) is derived for an arbitrary small volume element V within the vivo tissue, hence the volume V may be chosen so small as to remove the integral; we obtain.

$$\rho(r)C_p(r)\frac{\partial T(r,\phi,z,t)}{\partial t} = -\nabla q(r,\phi,z,t) + q'''_m(r) + \rho_{bl} \times \dot{w}_{bl}(r) \times Cp_{bl} \times (T_{art} - T) \tag{4.17}$$

Substitute the Equation (3.7) into above equation we can get the fundamental equation, which is called Bio-Heat Equation. This equation expresses the fact that, at any time, the sum of the heat transfer through the three directions of the cylinder, the heat produced by it and the heat transported by blood, is equal to the rate of temperature variation $\frac{\partial T}{\partial t}$, at any point.

$$\rho(r)C_p(r)\frac{\partial T(r,\phi,z,t)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + q'''_m(r) + \left[\rho_{bl} \times \dot{w}_{bl}(r) \times Cp_{bl} \times (T_{art} - T)\right] \tag{4.18}$$

Equation (4.18) is referred to as the “Bio-Heat Equation” which was first suggested in this form by Pennes. This differential equation describes the heat dissipation in a homogenous, infinite tissue volume.

According to Equation (4.18), from left to right, the first term is the storage of heat with in the tissue. [$\rho$ ($kg/m^3$) is the density of the tissue, Cp (J/kg K) is the specific heat of the tissue and lastly, t is the time.]. Second term is the heat conduction term, which shows the heat flow from warmer to colder tissue region in radial, tangential, and axial directions. [k is the tissue conductivity ($W.m^{-1}.K^{-1}$), T is the tissue temperature (K)]. Third term, $q'''_m$ ($W/m^3$), is heat produced by metabolism. In addition, last term is the rate of energy transported by blood stream [$T_{art}$ is the arterial temperature, subscript “bl” refers to blood and $\rho_{bl}$, $w_{bl}$, $Cp_{bl}$ describe the density of blood ($kg/m^3$), blood perfusion rate (1/s) and specific heat of the blood (J/kg K) respectively.] This combine effect is balanced by the fourth term which is The Bio-Heat Equation can be applied to all tissue nodes by using the appropriate material constant k, ρ and Cp, the basal heat generation term, $q'''_m$, and basal blood perfusion rate, $w_{bl}$, for each tissue layer.

In order to find out the temperature distribution of the overall body, the Bio-Heat Equation given in Equation (4.20), solved in the radial, tangential and axial directions by using the finite difference technique. The solution method will be discussed in succeeding chapter.

At the outer surface of the human body, heat is removed by a liner combination of convection, radiation, and sweat evaporation. Thus, for each limb the total heat loss due to exposing to the environment is calculated by sum of the radiation heat loss, convection heat loss, and evaporation heat loss.

$$\begin{bmatrix} \text{TOTAL HEAT} \\ \text{LOSS} \end{bmatrix} = h_c \times (T_s - T_a) + \varepsilon \times \sigma \times (T^4{}_s - T^4{}_a) + \frac{4184}{3600} \times (0.35) \times (P_s - P_a) + \frac{4184}{3600} \times K_e \times (P_s - P_a) \qquad (4.19)$$

The boundary conditions of each limb are given in Chapter 5 where the solution method is described.

# CHAPTER 5

# SOLUTION TECHNIQUE

The mathematical modeling of a physiological system results in a description in the terms of equations such as differential equations which can be solved by computers and numerical analysis software.

The main objective of this study is to find out the temperature distribution of the unclothed human body. To achieve this objective, partial derivative form of the Bio-Heat Equation should be solved. The finite-difference technique is used to solve out the Bio-Heat Equation by using explicit method.

In present model, each sector of each tissue shell was divided into nodes. User defines the mesh size of the each limbs and the space of the two nodes in r, $\phi$ and z directions by choosing the sensitivity of the calculations.

Each cylindrical body part was divided into small three dimensional tissue elements as shown in the figure below.

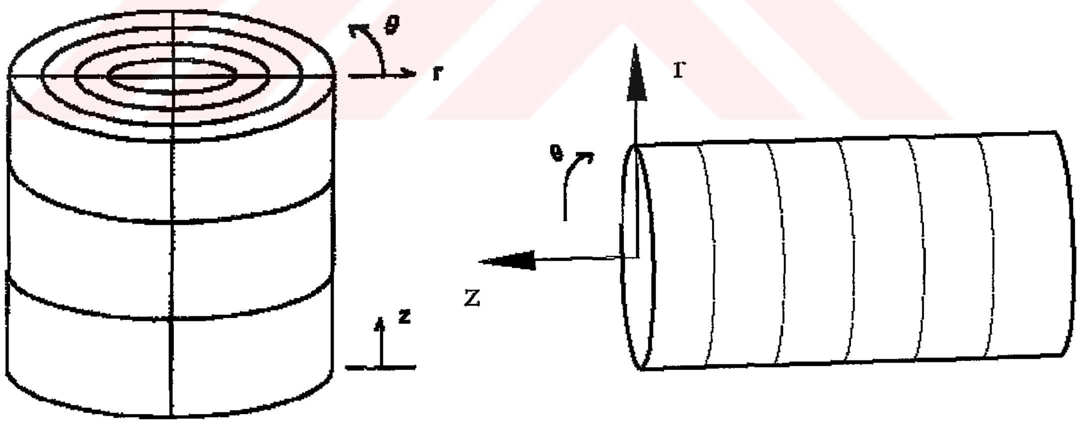



Figure 5.1 Discritization of the domain for vertical and horizontal limbs.

## 5.1 Finite Difference Form of Bio-Heat Equation

The Bio-Heat Equation derived in Chapter 4 should be changed into finite difference form in order to calculate the temperature of the nodes.

$$\rho(r)C_p(r)\frac{\partial T(r,\phi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)$$

$$+q_m'''(r)+\left[\rho_{bl}\times \dot{w}_{bl}(r)\times Cp_{bl}\times(T_{art}-T)\right]$$

Let the coordinate (r, $\phi$ ,z) of a point B at a node by in a cylindrical coordinate system.

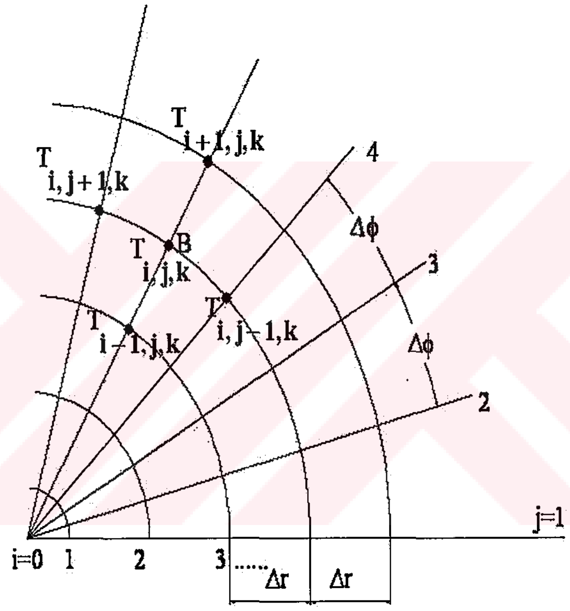



Figure 5.2 An (r, $\phi$ ,z) network in the cylindrical coordinate system

In Figure 5.2, the i, j and k subscripts are used to designate the r, $\phi$ and z locations of discrete nodal points. To obtain the finite-difference form of Bio-Heat Equation, we may use the central difference approximations to the spatial derivatives. The central difference value of the derivatives at the i, j, k nodal point can be written as follows.

$$\left.\frac{\partial T}{\partial r}\right|_{i,j,k}=\frac{T^n_{i+1,j,k}-T^n_{i-1,j,k}}{(\Delta r)^2} \qquad (5.1)$$

$$\left.\frac{\partial T^2}{\partial r^2}\right|_{i,j,k} = \frac{T^n_{i-1,j,k} - 2T^n_{i,j,k} + T^n_{i+1,j,k}}{(\Delta r)^2} \quad (5.2)$$

$$\left.\frac{\partial T^2}{\partial \phi^2}\right|_{i,j,k} = \frac{T^n_{i,j-1,k} - 2T^n_{i,j,k} + T^n_{i,j+1,k}}{(\Delta \phi)^2} \quad (5.3)$$

$$\left.\frac{\partial T^2}{\partial z^2}\right|_{i,j,k} = \frac{T^n_{i,j,k-1} - 2T^n_{i,j,k} + T^n_{i,j,k+1}}{(\Delta z)^2} \quad (5.4)$$

And coordinates of point B is represented at a node by

$$r = i\ \Delta r \quad (5.5)$$

$$\phi = j\ \Delta\phi \quad (5.6)$$

$$z = k\ \Delta z \quad (5.7)$$

At point B, temperature $\partial T(r,\phi,z,t)$, generation rate $q'''_m(r)$ , density of the tissue $\rho(r)$, specific heat of the tissue Cp(r), conduction heat transfer coefficient of the tissue k(r), are blood perfusion rate of the tissue $\dot{w}_{bl}(r)$ , are denoted by

$$T(r,\phi,z,t)\big|_B = T(i\Delta r, j\Delta\phi, k\Delta z) \equiv T_{i,j,k} \quad (5.8)$$

$$q'''_m(r)\big|_B = q'''_m(i\Delta r) \equiv q'''_{m_i} \quad (5.9)$$

$$\rho(r)\big|_B = \rho(i\Delta r) \equiv \rho_i \quad (5.10)$$

$$Cp\,(r)\big|_B = Cp\,(i\Delta r) \equiv Cp_i \tag{5.11}$$

$$k(r)\big|_B = k(i\Delta r) \equiv k_i \tag{5.12}$$

$$\dot{w}_{bl}(r)\big|_B = \dot{w}_{bl}(i\Delta r) \equiv \dot{w}_{bl_i} \tag{5.13}$$

In addition to being discretized in space, the problem must be discretized in time. The integer n is used for this purpose, where

$$t = n\,\Delta t \tag{5.14}$$

And the finite difference approximation to the time derivatives in Bio-Heat Equation is expressed as

$$\left.\frac{\partial T(r,\phi,z,t)}{\partial t}\right|_{i,j,k} = \frac{T^{n+1}_{i,j,k} - T^{n}_{i,j,k}}{\Delta t} \tag{5.15}$$

The equation (5.15) is considered to be a forward-difference approximation to the time derivative.

The subscript n is used to denote the time dependence of T, and the time derivatives is expressed in terms of the difference in temperatures associated with the new (n+1) and previous (n) times. Therefore, calculations must be performed at successive times separated by the interval $\Delta t$.

### 5.1.1 The Finite-Difference Forms of Bio-Heat Equation for Nonzero Values of r

When the equations from (5.1) to (5.15) are substituted into Bio-Heat Equation and by using the explicit method of solution, the finite difference form of the Bio-Heat Equation for non zero values of r can be obtained.

$$k_i\left[\frac{T^n_{i-1,j,k}-2T^n_{i,j,k}+T^n_{i+1,j,k}}{(\Delta r)^2}+\frac{1}{(i\Delta r)}\frac{T^n_{i+1,j,k}-T^n_{i-1,j,k}}{2\Delta r}+\frac{1}{(i\Delta r)^2}\frac{T^n_{i,j-1,k}-2T^n_{i,j,k}+T^n_{i,j+1,k}}{(\Delta\phi)^2}+\frac{T^n_{i,j,k-1}-2T^n_{i,j,k}+T^n_{i,j,k+1}}{(\Delta z)^2}\right]$$

$$+\left[q'''_{m_i}\right]^n+\left[\rho_{bl}\times\dot{w}_{bl_i}\times Cp_{bl}\times(T_{art}-T^n_{i,j,k})\right]=\rho_i C_{p_i}\left[\frac{T^{n+1}_{i,j,k}-T^n_{i,j,k}}{\Delta t}\right] \qquad (5.16)$$

When the node is at the interface of the two different homogeneous tissue types, the thermal properties such as thermal conductivity, density, specific heat, of this interface node can be obtained by considering the neighbor tissue layers.

Obtaining the thermal conductivity of interface node can be given as an example. If it is assumed that the control volume surrounding the grid point P is filled with a material of uniform conductivity $k_P$, and the one around E with a material of uniform conductivity $k_E$, thus the thermal conductivity of interface e in the cylindrical coordinate system can be written by using the harmonic mean of $k_P$ and $k_E$.

Figure 5.3 is used to demonstrate the intersection face between the two homogeneous layers.

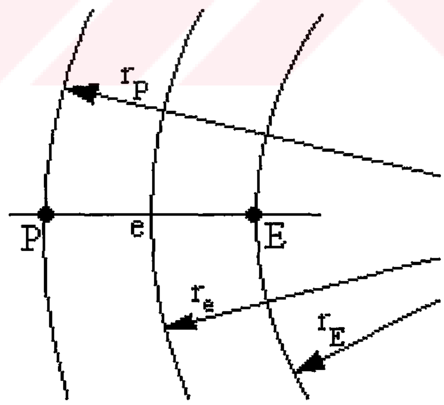



Figure 5.3 Intersection face between two layers

The most straight forward procedure for obtaining the interface thermal properties is to use the harmonic mean of point P and E.

$$k_e = \frac{\ln\left(r_P/r_E\right)}{\dfrac{\ln\left(r_P/r_e\right)}{k_P} + \dfrac{\ln\left(r_e/r_E\right)}{k_E}} = \frac{k_P k_E \ln\left(r_P/r_E\right)}{k_E \ln\left(r_P/r_e\right) + k_P \ln\left(r_e/r_E\right)} \tag{5.16a}$$

Other thermal properties and the physiological properties, such as blood perfusion rate, can be also obtained by using the harmonic mean of the two neighbor tissue layers properties.

### 5.1.2 The Finite-Difference Forms of Bio-Heat Equation For r=0

$$\rho(r)C_p(r)\frac{\partial T(r,\phi,z,t)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)$$
$$+ q_m'''(r) + \left[\rho_{bl} \times \dot{w}_{bl}(r) \times Cp_{bl} \times (T_{art} - T)\right]$$

As seen above, the Bio-Heat Equation appears have a singularities at r=0. To deal with this situation, the Laplacian operator in the cylindrical coordinate system is replaced by the Cartesian equivalent so that Bio-Heat Equation takes form (Özışık 1994b)

as $r \rightarrow 0$

$$\rho(r)\,c_p(r)\frac{\partial T}{\partial t} = k(r)\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + q_m(r) + \rho_{bl} \times \dot{w}_{bl}(r) \times Cp_{bl}(T_{art} - T) \tag{5.17}$$

We construct a circle of radius $\Delta r$, center at r=0. Let $T_0$ be the temperature at r=0 and $T_1$, $T_2$, $T_3$, $T_4$ be the temperatures at the four nodes this circle intersects the x and y axes as seen in the Figure 5.3.

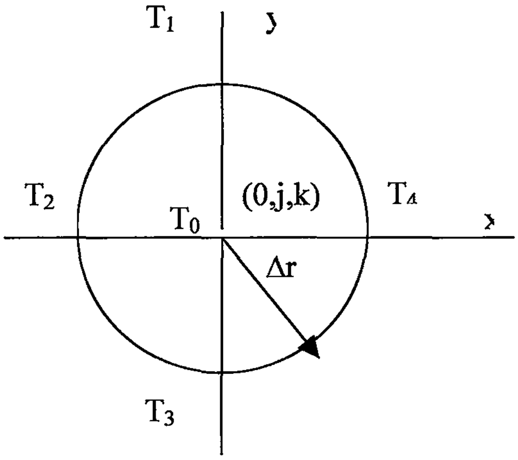



Figure 5.4. Temperature of the four nodes of a circle with center r = 0

Then the finite difference form of this equation about r=0 becomes

$$\rho_0 c_{p0} \frac{\partial T}{\partial t} = k_0 \left[ \frac{T_1 + T_2 + T_3 + T_4 - 4T_0}{(\Delta r)^2} + \frac{\partial^2 T}{\partial z^2} \right] + q_{m0} + \dot{w}_{bl0} \times Cp_{bl} \times \rho_{bl} \times (T_{art} - T) \qquad (5.18)$$

with an error of the order of $(\Delta r)^2$. If we denote $\bar{T}_1$ as the arithmetic mean of the temperatures around the circle of radius $\Delta r_1$. Namely, $\bar{T}_1$ as the arithmetic mean of T1,j,k around the circle of radius Δr with center at r=0.

$$\bar{T}_1 = \frac{T_1 + T_2 + T_3 + T_4}{4} \qquad (5.19)$$

then at r=0 the equation (5.18) becomes

$$\rho_0 c_{p0} \frac{\partial T}{\partial t} = k_0 \left[ 4 \frac{\bar{T}_1 - T_0}{(\Delta r)^2} + \frac{\partial^2 T}{\partial z^2} \right] + q_{m0} + w_{bl0} (c_p \times \rho)_{bl} (T_{art} - T) \qquad (5.20)$$

When equation (5.20) is written in finite difference from, the Bio-Heat Equation can be written as seen in equation (5.21).

$$\rho_0 C_{p_0}\left[\frac{T^{n+1}_{0,j,k}-T^{n}_{0,j,k}}{\Delta t}\right]=k_0\left[4\left(\frac{\bar{T}_1-T^{n}_{0,j,k}}{(\Delta r)^2}\right)+\frac{T^{n}_{0,j,k-1}-2T^{n}_{0,j,k}+T^{n}_{0,j,k+1}}{(\Delta z)^2}\right]+(q'''_{m_0})^n$$

$$+\left[\rho_{bl}\times \dot{w}_{bl_0}\times Cp_{bl}\times(T_{art}-T^{n}_{0,j,k})\right] \qquad (5.21)$$

## 5.2 Boundary Conditions

In our problem, some nodes locate on a boundary surface. To find out the temperature of these nodes, boundary conditions of them should be given. Because of the structure of the finite difference form of the Bio-Heat Equation, on boundary surface there will be a fictitious node which should be given in the term of real node.

Each limb is divided into several planes in z directions. k points out the plane number in z directions and for each limb the last k value is N.

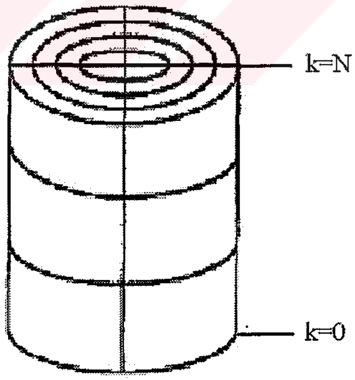



Figure 5.5. Division of z axis of a limb.

Due to the goemetrical and environemntal conditional symetry of each element it is assumed that the human bosy is symetrical according to the j= 1 and j= M. Therefore, the calculation of the temerature distribution of quarter human body is enough to know the whole body temp distribution.

All boundary conditions that we need to solve the problem and the definition of the fictitious node in the term of real node are given in Table 5.1 and Table 5.2.

The boundary conditions given in Table 5.1 are used in solution of the finite difference equation of the **each** limb. Besides to the boundary conditions given in Table 5.1, the solution of Bio-Heat Equation for ***some limb*** needs extra boundary conditions. These extra boundary conditions and the name of the limb are given below in the Table 5.2

The following notation is used both in Table 5.1 and Table 5.2

$$EE = h_c(T - T_a) + \varepsilon\sigma(T^4 - T_a^4) + \frac{4184}{3600}\times(0.35)\times(P_s - P_a) + \frac{4184}{3600}\times K_e \times (P_s - P_a)$$

By using Table 5.1 and Table 5.2, the equations representing the temperature of unclothed human body at (n+1) time step are obtained. These equations are given in AppendixA.

Table 5.1 The Boundary Conditions of Bio-Heat Equation for **each limb.**

| No | Boundary Conditions | Finite Difference Form of BC | Fictitious Temperature |
|---|---|---|---|
| BC1 | $\frac{\partial T}{\partial \phi}=0$ at j=1 | $\left[\frac{T^{n}_{i,j+1,k}-T^{n}_{i,j-1,k}}{2\Delta\phi}\right]=0$ | $T^{n}_{i,j-1,k}=T^{n}_{i,j+1,k}$ |
| BC2 | $\frac{\partial T}{\partial \phi}=0$ at $j=M$ | $\left[\frac{T^{n}_{i,j+1,k}-T^{n}_{i,j-1,k}}{2\Delta\phi}\right]=0$ | $T^{n}_{i,j+1,k}=T^{n}_{i,j-1,k}$ |
| BC3 | $-k\frac{\partial T}{\partial r}=EE$ at $i=P$ | $-k\left[\frac{T^{n}_{i+1,j,k}-T^{n}_{i-1,j,k}}{2\Delta r}\right]=EE$ | $T^{n}_{i+1,j,k}=\frac{2\Delta r\times EE}{-k}+T^{n}_{i-1,j,k}$ |

Table 5.2 The Boundary Conditions of Bio-Heat Equation for **some limb.**

| No | Name of limb | Boundary Conditions | Finite Difference Form of BC | Fictitious Temperature |
|---|---|---|---|---|
| BC4 | Torso<br>Head<br>Hand | $-k\frac{\partial T}{\partial z}=EE$ at k=0 | $-k\left[\frac{T^{n}_{i,j,k+1}-T^{n}_{i,j,k-1}}{2\Delta z}\right]=EE$ | $T^{n}_{i,j,k-1}=\frac{2\Delta z\times EE}{k}+T^{n}_{i,j,k+1}$ |
| BC5 | Torso<br>Head | $-k\frac{\partial T}{\partial z}=EE$ at k = N | $-k\left[\frac{T^{n}_{i,j,k+1}-T^{n}_{i,j,k-1}}{2\Delta z}\right]=EE$ | $T^{n}_{i,j,k+1}=\frac{2\Delta z\times EE}{-k}+T^{n}_{i,j,k-1}$ |
| BC6 | Foot | $\frac{\partial T}{\partial z}=0$ at k=0 | $\left[\frac{T^{n}_{i,j,k+1}-T^{n}_{i,j,k-1}}{2\Delta z}\right]=0$ | $T^{n}_{i,j,k-1}=T^{n}_{i,j,k+1}$ |

# CHAPTER 6

# BIO-THERMAL SOFTWARE

Software called Bio-Thermal is written in Visual Basic language to solve the system of equations of temperature of the human body obtained from Chapter 5. The program consists of two parts called Visual and Module. The Visual part of the program is given in this chapter; however the Module part is given in Appendix A because it is very long to give in this chapter.

In Visual part of the program, first page is "INPUT" page in which user assigns the radius and length of each limb, environmental conditions, initial temperature of the human body and sensitivity of the calculation. The view of the "INPUT" page is given below.

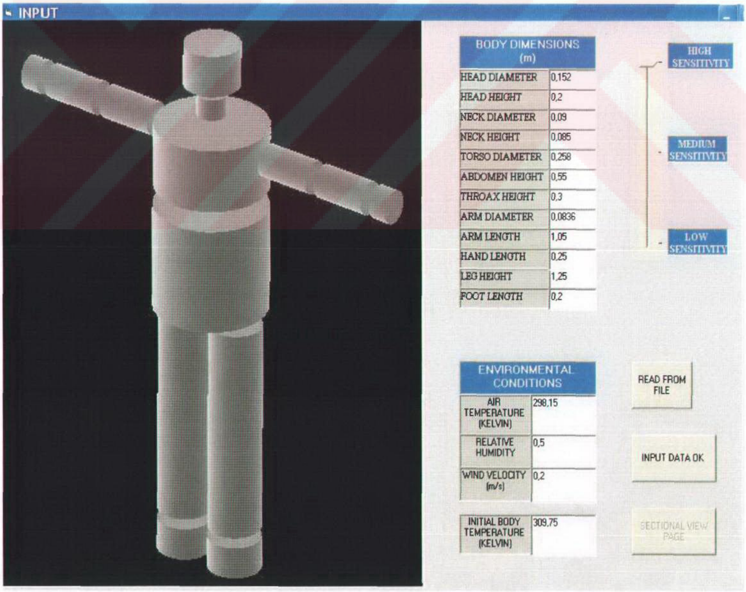



Figure 6.1. Schematic View of the Input Data Form

The default values seen in body dimension part are taken from the literature. The given sizes are the size of the average human body. According to the user preferences, each element's radius and length value can be changed. This leads to our model flexible in body dimensions. Below body dimension part, there is a part called environmental conditions. In this part, the temperature and relative humidity of the air and also wind velocity can be assigned. On the right side of the form there is a cursor which leads to assign the sensitivity of the calculations. User defines the mesh size of the element by selecting the high, medium or low sensitivity button.

After assigning the body dimensions and environmental conditions and defining the calculation sensitivity, "INPUT DATA OK" button is pressed. User should wait few seconds because the assigned values are transformed to the Module part of the program. In a few seconds, "SECTIONAL VIEW" button is activated. Then user presses this button to go to the Sectional View Page.

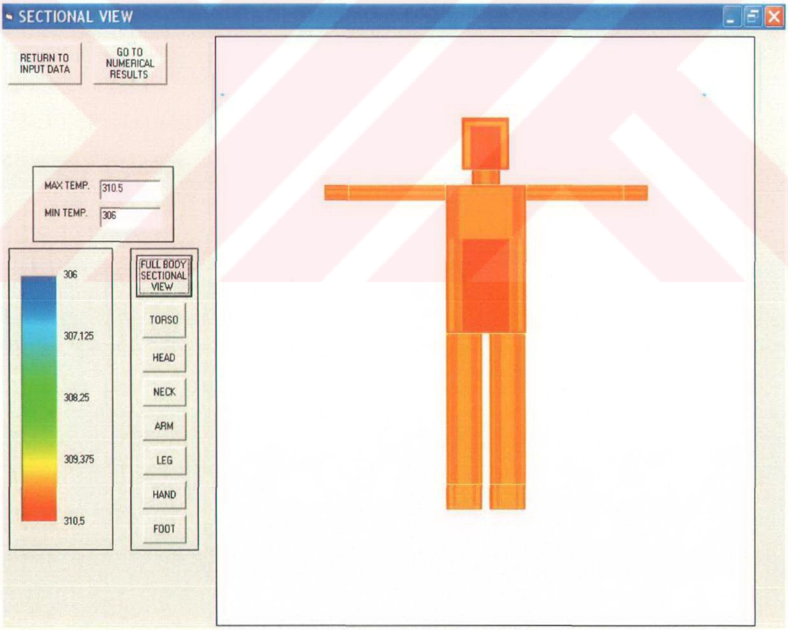
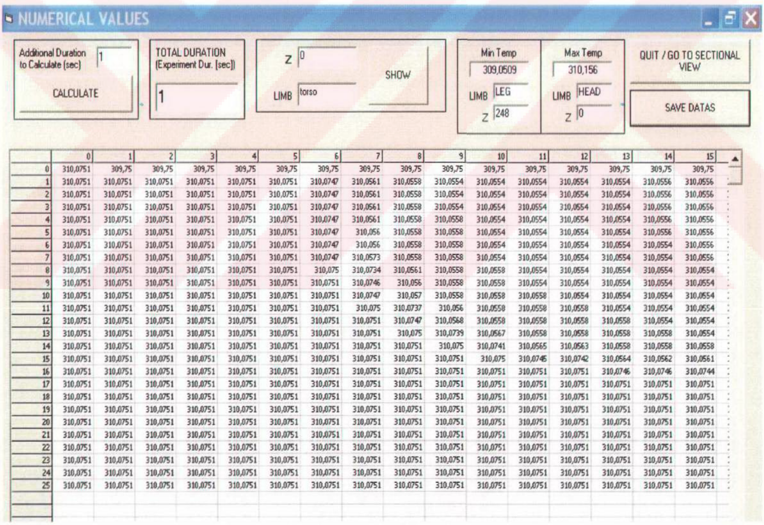



Figure 6.2. Schematic View of the Sectional View Form

"SECTIONAL VIEW" page is used to see sectional view of the temperature distribution of the full body or the chosen limb. In this page, minimum temperature and maximum temperature text boxes are used to assign the temperature range which you would like to use by drawing the sectional view. With the help of these text boxes, user can select the color range of the sectional view. Below the minimum and maximum temperature text boxes, there is a temperature scale which define the temperature value corresponding to the color used in sectional view.

As seen in Figure 6.2, buttons on the right hand side of the temperature scale lead to us to select the which part of the human body's sectional view is wanted to see the temperature distribution. In Sectional View page, there is a button labeled "GO TO NUMERICAL RESULTS". Pressing this button leads to show the form called Numerical Calculations in which user assigns the total time for simulation.

In Figure 6.3 the schematic view of the Numerical Values Form can be seen.




| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 310,0751 | 309,75 | 309,75 | 309,75 | 309,75 | 309,75 | 309,75 | 309,75 | 309,75 | 309,75 | 309,75 | 309,75 | 309,75 | 309,75 | 309,75 | 309,75 |
| 1 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0747 | 310,0561 | 310,0558 | 310,0554 | 310,0554 | 310,0554 | 310,0554 | 310,0554 | 310,0556 | 310,0556 |
| 2 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0747 | 310,0561 | 310,0558 | 310,0554 | 310,0554 | 310,0554 | 310,0554 | 310,0554 | 310,0556 | 310,0556 |
| 3 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0747 | 310,0561 | 310,0558 | 310,0554 | 310,0554 | 310,0554 | 310,0554 | 310,0554 | 310,0556 | 310,0556 |
| 4 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0747 | 310,0561 | 310,0558 | 310,0558 | 310,0554 | 310,0554 | 310,0554 | 310,0554 | 310,0556 | 310,0556 |
| 5 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0747 | 310,056 | 310,0558 | 310,0558 | 310,0554 | 310,0554 | 310,0554 | 310,0554 | 310,0556 | 310,0556 |
| 6 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0747 | 310,056 | 310,0558 | 310,0558 | 310,0554 | 310,0554 | 310,0554 | 310,0554 | 310,0554 | 310,0556 |
| 7 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0747 | 310,0573 | 310,0558 | 310,0558 | 310,0554 | 310,0554 | 310,0554 | 310,0554 | 310,0554 | 310,0556 |
| 8 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,075 | 310,0734 | 310,0561 | 310,0558 | 310,0558 | 310,0554 | 310,0554 | 310,0554 | 310,0554 | 310,0554 |
| 9 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0746 | 310,056 | 310,0558 | 310,0558 | 310,0554 | 310,0554 | 310,0554 | 310,0554 | 310,0554 |
| 10 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0747 | 310,057 | 310,0558 | 310,0558 | 310,0558 | 310,0554 | 310,0554 | 310,0554 | 310,0554 |
| 11 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,075 | 310,0737 | 310,056 | 310,0558 | 310,0558 | 310,0558 | 310,0554 | 310,0554 | 310,0554 |
| 12 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0747 | 310,0568 | 310,0558 | 310,0558 | 310,0558 | 310,0558 | 310,0554 | 310,0554 |
| 13 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,075 | 310,0739 | 310,0567 | 310,0558 | 310,0558 | 310,0558 | 310,0558 | 310,0554 |
| 14 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,075 | 310,0741 | 310,0565 | 310,0563 | 310,0558 | 310,0558 | 310,0558 |
| 15 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,075 | 310,0745 | 310,0742 | 310,0564 | 310,0562 | 310,0561 |
| 16 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0746 | 310,0746 | 310,0744 |
| 17 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 |
| 18 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 |
| 19 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 |
| 20 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 |
| 21 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 |
| 22 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 |
| 23 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 |
| 24 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 |
| 25 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 | 310,0751 |

Figure 6.3. Schematic View of the Numerical Values Form

User firstly, assign the time duration wanted to see the temperature distribution into the "ADDITIONAL TIME DURATION" text box, then press the "CALCULATE" button in order to start the calculation for finding the temperature distribution of the human body. In addition, the duration declaring the calculated time can be seen by "TOTAL DURATION" text box. During the calculation, user can see the maximum and minimum temperature value in the whole human body and name of the limb that has minimum or maximum temperature. In "NUMERICAL VALUES" Form, "SHOW" button is used to see the temperature value of given plane of limb that assigned by user. User assigns the name of the limb and which plane he would like to see the temperature.

After completing the calculation duration, by pressing the "save data" button all these temperature value given below are written in a folder called "thesis data". In this folder, the temperature values of the viscera, lung, torso bone, torso muscle, torso fat, torso skin, neck bone, neck muscle, neck fat, neck skin, brain, head bone, head fat, head skin, leg bone, leg muscle, leg fat, leg skin, foot bone, foot muscle, foot fat, arm bone, arm muscle, arm fat, arm skin, hand bone, hand muscle, hand fat, hand skin, mean temperature of torso, mean temperature of neck, mean temperature of head, mean temperature of leg, mean temperature of foot, mean temperature of arm, mean temperature of hand , mean temperature of the bone tissue for overall body, mean temperature of the muscle tissue for overall body, mean temperature of the fat tissue for overall body, mean temperature of the skin tissue for overall body, mean temperature of the total body are stored.

The outline of the program is given below.

Step 1: Assign the body dimensions, environmental conditions, initial temperature of the body and sensitivity of the calculation in Input Page of the Program.
Step 2: Press "Input Data Ok" Button
Step 3: Generate element mesh and calculate mesh size of each limb.
Step 4: Module part of the program assigns the thermophysical properties of the tissues.
Step 5: Module part of the program calculates the properties of air.
Step 6: Press "Go to Numerical Results" button.
Step 7: Assign total time for simulation.
Step 8: Press "Calculate" button.
Step 9: Check the type of convection whether it is forced or free.
Step 10: Calculate the heat transfer coefficient according to the convection type.
Step 11: Solve temperature distribution of the body
Step 12: Increase the time step.

Step 13: Solve new temperature distribution of the body.

Step 14: Check the simulation period if it is expired or not

Step 15: If the simulation time is not expired go to Step 11.

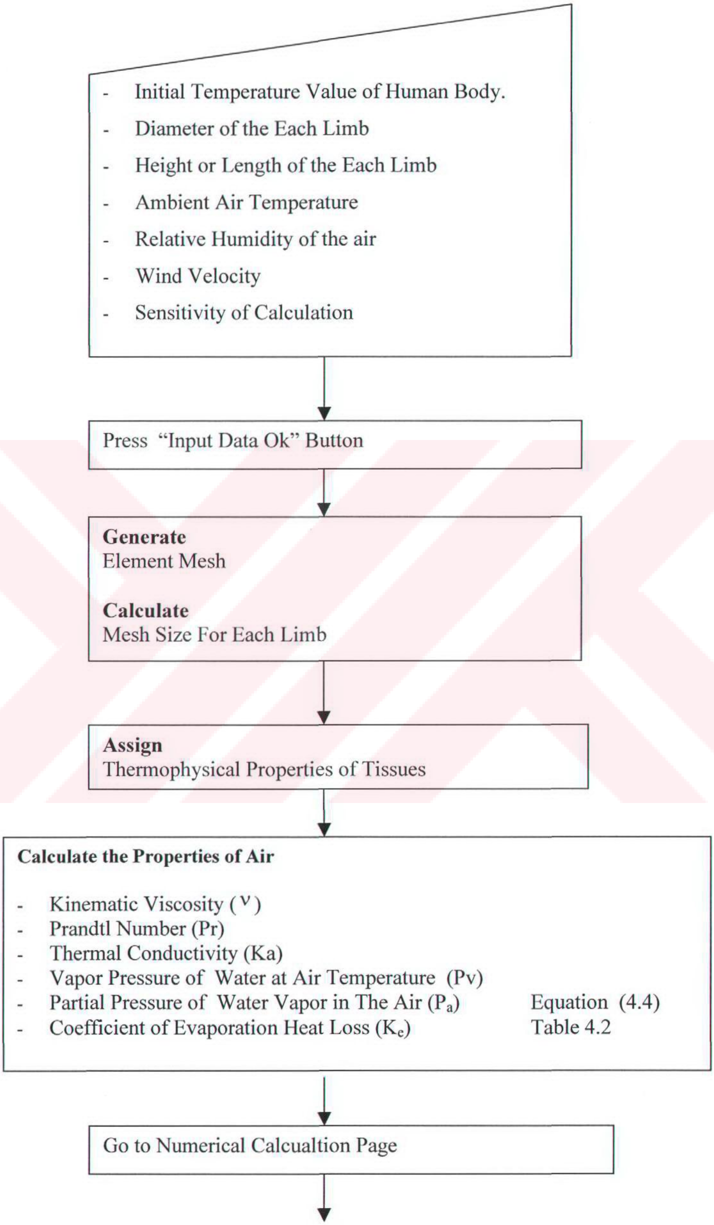
Step 16: If the simulation time is expired, save the temperature value of the human body and its limbs in a folder by using the "Save Data" button in "Numerical Value" page.

Step 17: Press "Go to Sectional View "button to go to the "Sectional View" page in order to see the temperature distribution section of the chosen limb or full human body.

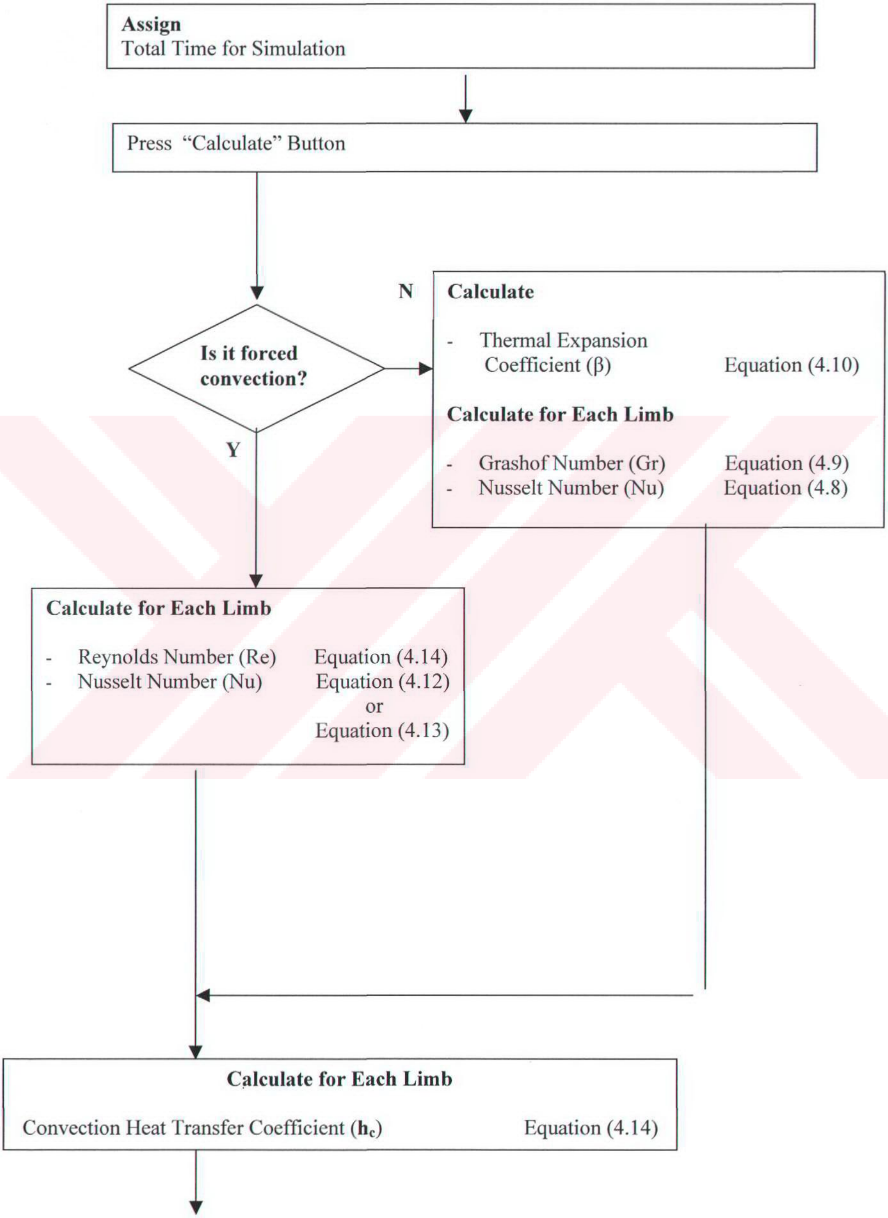
Step 18: Evaluate the saved data and draw conclusion.

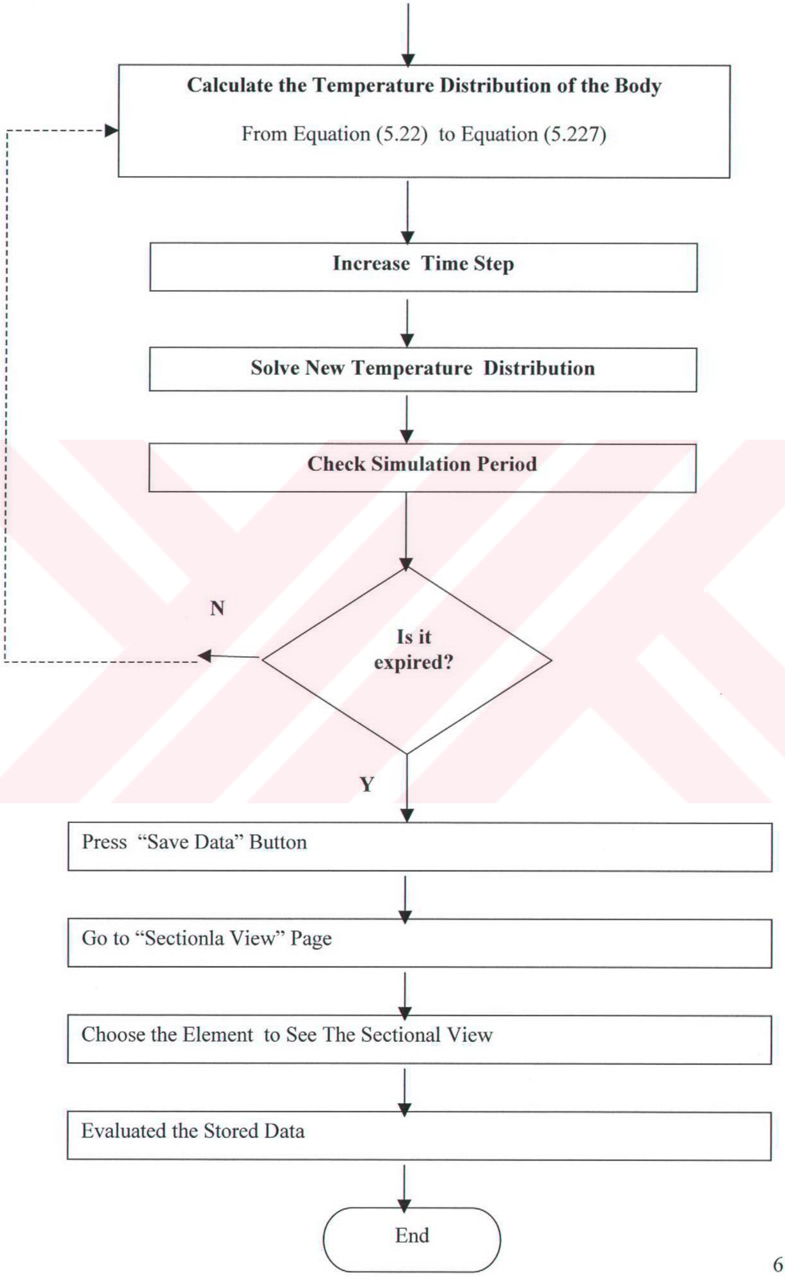
Step 19: End Program.

In order to follow the steps easily, the flow chart of the program is given below.



- Initial Temperature Value of Human Body.
- Diameter of the Each Limb
- Height or Length of the Each Limb
- Ambient Air Temperature
- Relative Humidity of the air
- Wind Velocity
- Sensitivity of Calculation
Press "Input Data Ok" Button
Generate
Element Mesh
Calculate
Mesh Size For Each Limb
Assign
Thermophysical Properties of Tissues
Calculate the Properties of Air
- Kinematic Viscosity ($\nu$)
- Prandtl Number (Pr)
- Thermal Conductivity (Ka)
- Vapor Pressure of Water at Air Temperature (Pv)
- Partial Pressure of Water Vapor in The Air ($P_a$) Equation (4.4)
- Coefficient of Evaporation Heat Loss ($K_e$) Table 4.2
Go to Numerical Calcualtion Page


Assign
Total Time for Simulation
Press "Calculate" Button
Is it forced convection?
N
Y
Calculate
- Thermal Expansion Coefficient (β) Equation (4.10)
Calculate for Each Limb
- Grashof Number (Gr) Equation (4.9)
- Nusselt Number (Nu) Equation (4.8)
Calculate for Each Limb
- Reynolds Number (Re) Equation (4.14)
- Nusselt Number (Nu) Equation (4.12) or Equation (4.13)
Calculate for Each Limb
Convection Heat Transfer Coefficient (hc) Equation (4.14)


Calculate the Temperature Distribution of the Body
From Equation (5.22) to Equation (5.227)
Increase Time Step
Solve New Temperature Distribution
Check Simulation Period
N
Is it expired?
Y
Press "Save Data" Button
Go to "Sectionla View" Page
Choose the Element to See The Sectional View
Evaluated the Stored Data
End

# CHAPTER 7

# RESULTS and DISCUSSION

A model of unclothed human passive thermal model has been developed. As in every computer model, present model should be verified in order to establish the accuarcy of the model. The verification process is done by comparison of model predictions with the experimental data and the analytical solution. Not only does this process establish the accuracy of the model, but it is also helps to define the range of conditions for which the model is applicable.

## 7.1 Verification of The Bio-Thermal Program

The temperature distribution of the human whole body or part of it are obtained at given Cases in Table 7.1 from the Bio-Thermal Program and the results are compared with the experimental data or analytical solution or predicted model done by earlier scientist.

Table 7.1. Conditions of Experimental Results and Analytical Solution.

| Case | Air Temperature (°C) | Relative Humidity | Wind Velocity (m/s) | Initial Temperature (°C) |
|---|---|---|---|---|
| 1 | 26 | %50 | 0.05 | 36 |
| 2 | 35 | %90 | 0.05 | 32 |
| 3 | 0 | %45 | 0.05 | 0 |
| 4 | 24.6 | 50% | 0.1 | 34 |

The computer program is used to find out the temperature distribution of human body to each cases given in Table 7.1. For each computer run, the steps given in Chapter 6 are followed.

**CASE 1**

Case 1 given in Table 7.1 is the experiment condition of the Pennes (Wissler, 1998). He tried the found out the steady state temperature distribution in human arm. As seen in Table 7.1, CASE 1 corresponds to 26°C and 50% relative humidity in natural convection conditions. The initial temperature of the arm is 36°C.

The same conditions are used in order to compare calculated data. The study done by Pennes tired to solve steady state temperature distribution in human arm; therefore, in present model, the data is taken when our system reached the steady state condition.

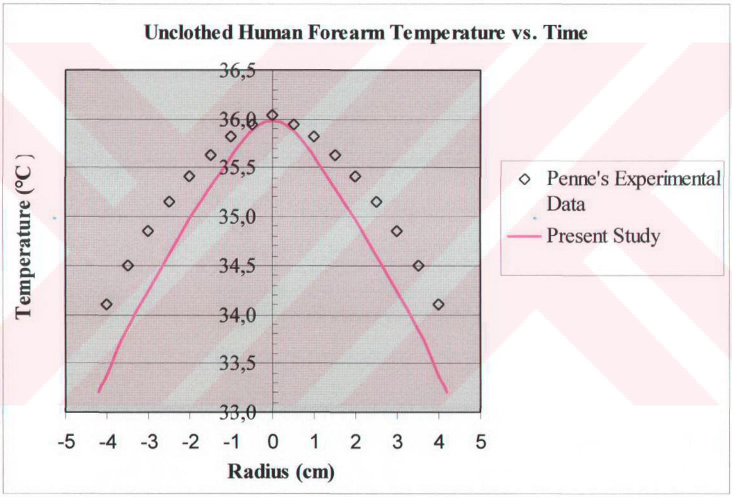



Figure 7.1. Comparison of the Human Forearm Temperature with Pennes Data

As seen in Figure 7.1 present study results follow the same trend and show a good agreement with Penne's Experimental Data.

## CASE 2

The experiment in condition of Case 2 is done by Takata et al, 2001. During the experiment, unclothed skin temperatures at the arm are measured in a room at 35°C and relative humidity 90% for half an hour. Initially, the subject temperature is measured 32°C. In addition to this, the experimental results are analyzed by using the Gagge Model, which is two-node model of human thermophysical responses.

The conditions given in Case 2 is used in present study and as seen in Figure 7.2 our model shows better agreement with the experimental data than calculated data by Takada et al. The reason for this that, to simulate the skin temperature of arm Takada et al used Gagge Model which depicts the human body as a cylinder which consists of only core and skin. However, present model's approximation has more accuracy than Gagge Model because in present model the human body is divided into multi-layered 16 cylinders which lead to show good agreement with experimental data.

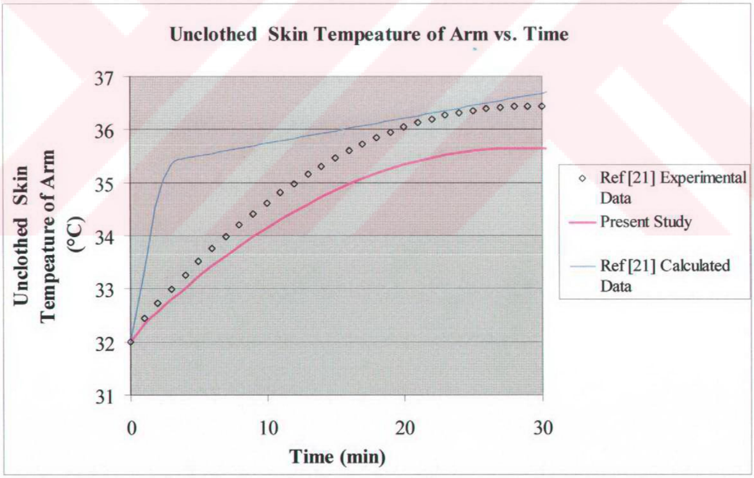



Figure 7.2. Comparison of the Unclothed Skin Temperature of Arm with Takata et al Data.

## CASE 3

Eberhart (Fiala et al., 1999) solves the temperature within a homogenous cylinder of muscle tissue with an analytical method. The condition as described in Case 3 is used. The cylinder is initially in thermal equilibrium with the environment at 0°C. At t=0 the cylinder suddenly supplied with a constant rate of blood flow therefore the temperature of cylinder rises quickly before converging asymptotically to the final steady state.

The calculated temperature of the muscle tissue in leg is compared with an analytical solution of Eberhart. As seen in the Figure 7.3, calculated muscle tissue temperature in leg by present model shows good agreement with the analytical solution.

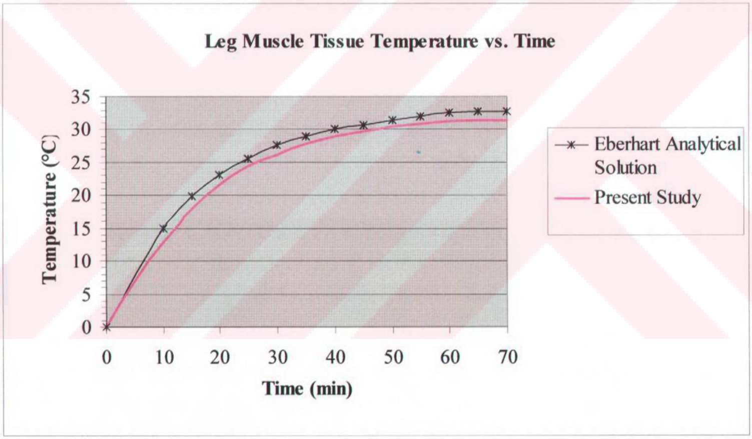
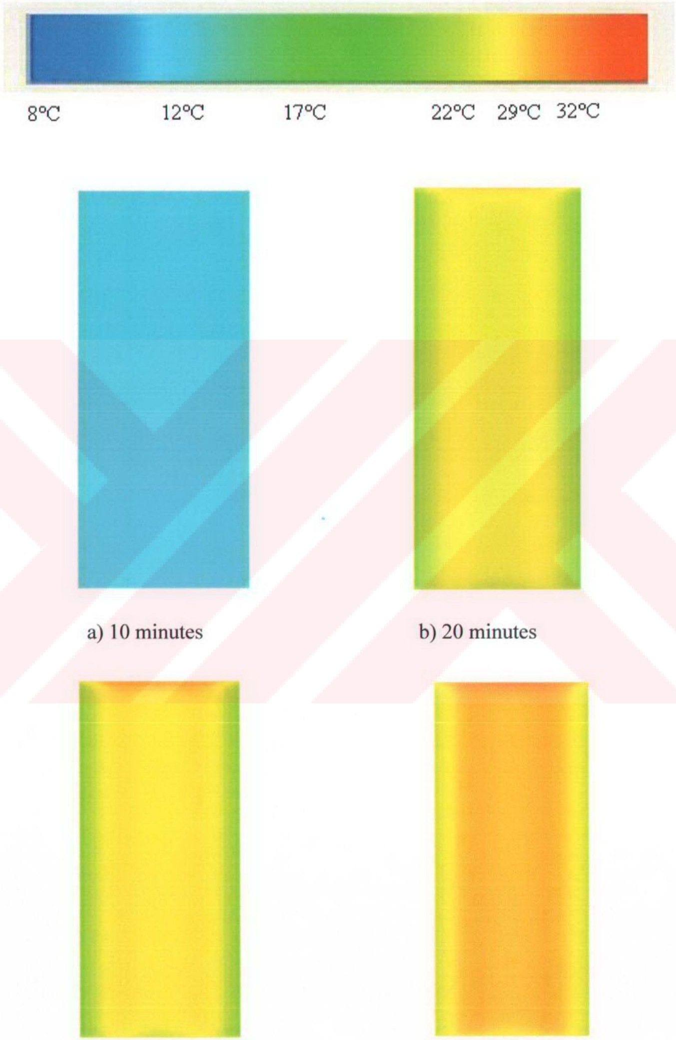



Figure 7.3. Comparison of the Muscle Tissue Temperature of Leg with Eberhart Analytical Solution

Some sectional view of the leg is taken at 10 minutes, 20 minutes, order to simulate the changes in temperature with time.




a) 10 minutes    b) 20 minutes

c) 40 minutes    d) 60 minutes

Figure 7.4. Simulation of Temperature Changes in Leg with Time

## CASE 4

The numerical values in Case 4 are the condition of a model done by Arkin and Shitzer. This model is combined model of the human thermal system, namely it includes both active and passive system simulations.

Although we only modeled the passive system of the human thermal system it can be seen that it is good agreement with the calculated data by Arkin and Shitzer. The deviation between two graphs is caused by the human thermal responses such as vasoconstriction, vasodilation, sweating or shivering.

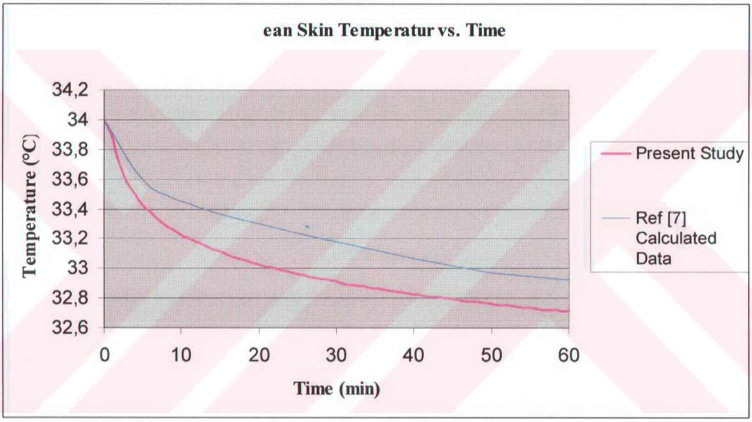



Figure 7.5. Comparison of the Mean Skin Temperature of Human Body with Arkin and Shitzer's Data.

According to present model's calculated data, the temperature changes in mean skin temperature of the human body with time are given below.

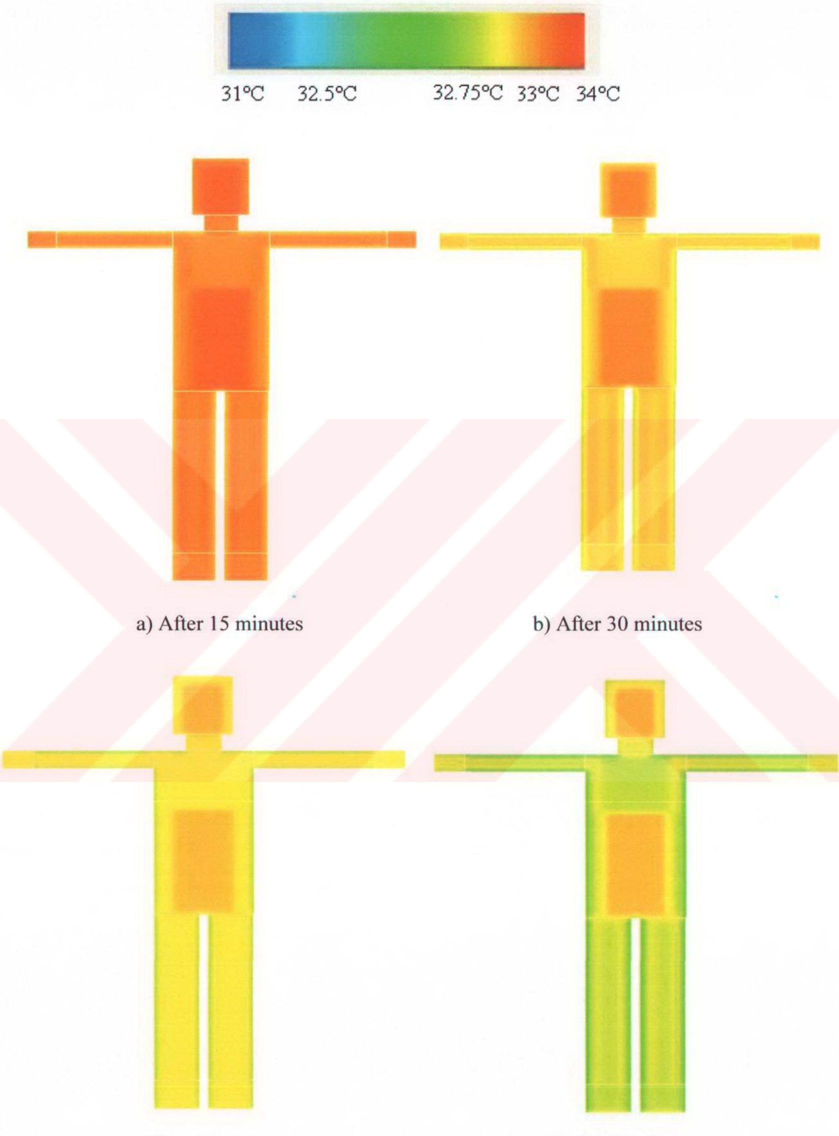



Figure 7.6. Simulation of Temperature Changes in Human Body with Time

## SCENARIO

Apart from comparison with other studies, a scenario is written in order to show the temperature changes in mean body and skin with respect to time. In this scenario, at t=0 a man enters the Room 1 with condition of 28.1°C and 43% relative humidity. This room condition is in the range of thermal comfort zone defined by ASHRAE. After 10 minutes, he enters a room which is hotter than previous room with 47.8°C room temperature and 27% relative humidity. This room's condition is similar to the sauna condition descried in WEB_4, 2003. After 10 minutes he enters a different room with same temperature value with Room 2 but higher relative humidity which is 99%. After 20 minutes, he enters the fourth room which is at 47.8°C and 0% relative humidity. After 10 minutes, he enters again Room1.

Table 7.2 Conditions of Room in the Scenario

| Room | Air Temperature (°C) | Relative Humidity | Time Duration (min) |
|---|---|---|---|
| 1 | 28.1 | 50% | 10 |
| 2 | 47.8 | 27% | 10 |
| 3 | 47.8 | 99% | 20 |
| 4 | 47.8 | 0% | 10 |

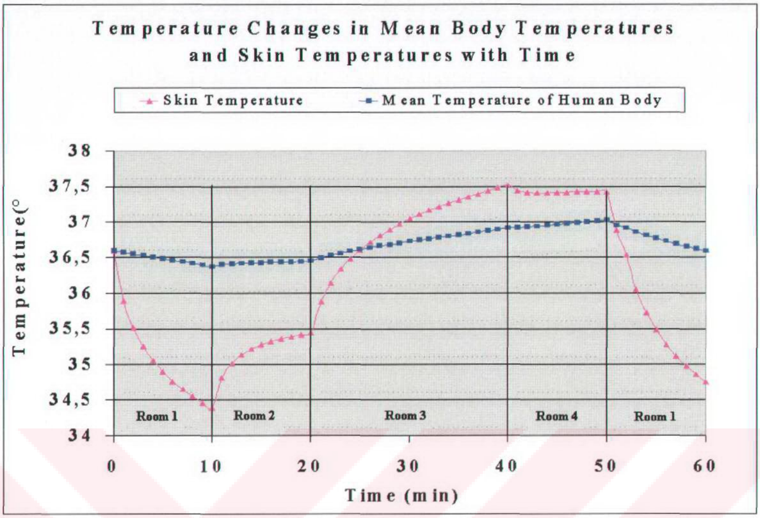



Figure 7.7. Simulation of Temperature Changes in Human Body with Time

As seen in the Figure above, the temperature of skin is affected dramatically when the environmental condition changes. Skin temperature reached its highest value when the man is in Room 3. The reason for this is that, when the temperature and the relative humidity of the environment rises heat loss by diffusion of water through skin and heat loss by evaporation by sweat decreases. This causes high temperature value in the body which is very harmful even fatal for human.

# CHAPTER 8

# CONCLUSIONS

In present model, a transient three dimensional mathematical modeling of the human passive system has been developed using finite difference technique. In order to increase the accuracy of the model, the human body is divided into 16 cylinders and each cylinder is divided into four radial layers which represent the tissue type such as bone, fat, muscle, skin, viscera, lung, and brain. According to assumptions, the Bio-Heat Equation is derived and written for each tissue element. As a result of this, from Equation 5.22 to 5.234, which are given in Appendix A, are obtained.

In present study, is is assumed that the human body dissipates heat by the combination of convection, radiation and evaporation.

A program, which is written in Visual Basic called Bio-Thermal, is used to determine temperature distribution at succeeding time step of the viscera, lung, brain all tissue type of the torso, neck, head, leg, foot, arm, hand, and mean temperature of torso, neck, head, leg, foot, arm, hand. Additionally, for overall body, mean temperature of the bone tissue, muscle tissue, fat tissue, and skin tissue and mean temperature of the total body can be obtained by Bio-Thermal Software. Also, the software is to be capable of demonstrating the sectional view of the various body limbs and full human body. The software is given in Appendix B.

Unlike previous models, in this model, software user defines the dimensions of the body limbs and environmental conditions. Additionally, user may choose the sensitivity of the calculation according to desired sensitivity. All these properties of the software make our model flexible. Also, the model is capable of showing the sectional view of the chosen part of the human body or itself. The visual form of the calculation, sectional view of the human body, gives idea to the user about which part of the body is getting cold or hot faster.

To verify the accuracy of the model, computer simulation results are compared with available experimental data, analytical solution and calculated data of previous studies for different environmental conditions. Based on these comparisons, some conclusions are drawn regarding model accuracy and applicability.

In addition a scenario is written for simulating the temperature at abnormal conditions. In this scenario, it could be seen that the mean skin temperature of the human body remained almost constant although, the skin temperature of the body fluctuated with varying environmental conditions.

The present model can be enlarged by taking into account the heat loss by respiration, clothing insulating factor and metabolic heat generation during physical activities. These could be the future work of other studies.

# APPENDIX A

The equations given below representing the temperature of human body at (n+1) time step is given in this Appendix A.

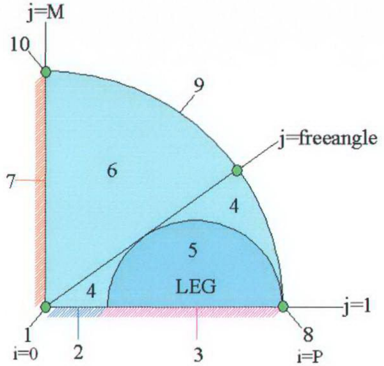
Sectional view of the each limb is given before explaning the equations. First sectional view is at k=0 . Then sectional view of other planes are given. The view is numberized in region. Each region has its own temperature equation. The boundary conditions which help us to solve the unknown temperature are given in Table 5.1 and Table 5.2.

The descriptions of the regions and their nodes are explained below. In order to simplify the equations the following abbreviations are used. It is important to say that for each limb the value of P, N, and M are changed.

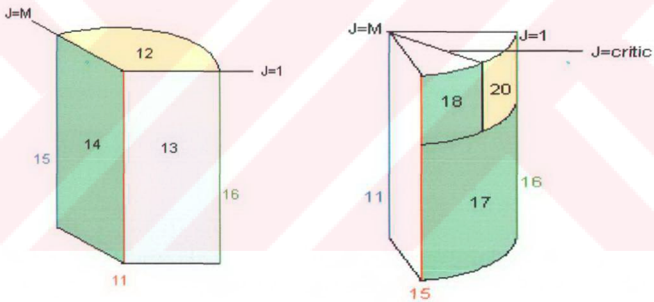
| | | |
|---|---|---|
| Torsointervalr = P | Torsointervalz = N | $\left(\frac{\text{Torsointerval}\phi}{4}\right)+1=M$ |
| Neckintervalr = P | (Neckintervalz -1)= N | $\left(\frac{\text{Neckinterval}\phi}{4}\right)+1=M$ |
| Headintervalr = P | Headintervalz = N | $\left(\frac{\text{Headinterval}\phi}{4}\right)+1=M$ |
| Armintervalr = P | (Armintervalz -1)= N | $\left(\frac{\text{Arminterval}\phi}{4}\right)+1=M$ |
| Legintervalr = P | (Legintervalz -1)= N | $\left(\frac{\text{Leginterval}\phi}{4}\right)+1=M$ |
| Handintervalr = P | Handintervalz = N | $\left(\frac{\text{Handinterval}\phi}{4}\right)+1=M$ |
| Footintervalr = P | Footintervalz = N | $\left(\frac{\text{Footinterval}\phi}{4}\right)+1=M$ |

# TORSO

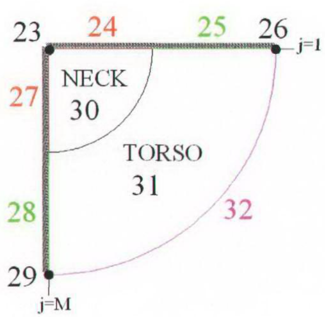
**At k=0**




**At k=1 to (N-1)**




**At k=N**



**Region 1**

Substituting the node (0,0,0) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.21).

$$\rho_0^n\, Cp_0^n\left[\frac{T_{0,0,0}^{n+1}-T_{0,0,0}^{n}}{\Delta t}\right]=k_0^n\left[4\left(\frac{\overline{T}_1-T_{0,0,0}^{n}}{(\Delta r)^2}\right)+\frac{T_{0,0,-1}^{n}-2T_{0,0,0}^{n}+T_{0,0,1}^{n}}{(\Delta z)^2}\right]+(q'''_{m_0})^n \tag{5.22}$$
$$+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,0}^{\,n}\times(T_{art}-T_{0,0,0}^{n})\right]$$

In Equation (5.22) $T_{0,0,-1}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used. The temperature of the node (0,0,0) at next time step (n+1) is given below.

$$T_{0,0,0}^{n+1}=\alpha_0^n\times\Delta t\times\left[4\left(\frac{\overline{T}_1-T_{0,0,0}^{n}}{(\Delta r)^2}\right)+\frac{1}{(\Delta z)^2}\left(T_{0,0,1}^{n}-2T_{0,0,0}^{n}+\left(\frac{2\Delta z\times EE}{k_0^n}\right)+T_{0,0,1}^{n}\right)\right] \tag{5.23}$$
$$+\left[(q'''_{m_0})^n\times\frac{\Delta t}{\rho_0^n\, Cp_0^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl0}^{\,n}\times(T_{art}-T_{0,0,0}^{n})\times\frac{\Delta t}{\rho_0^n\, Cp_0^n}\right]+T_{0,0,0}^{n}$$

**Region 2 and 3**

Substituting the node (i,1,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n\, Cp_i^n\left[\frac{T_{i,1,0}^{n+1}-T_{i,1,0}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{i-1,1,0}^{n}-2T_{i,1,0}^{n}+T_{i+1,1,0}^{n}}{(\Delta r)^2}+\dfrac{T_{i+1,1,0}^{n}-T_{i-1,1,0}^{n}}{2i(\Delta r)^2}+\\ \dfrac{T_{i,0,0}^{n}-2T_{i,1,0}^{n}+T_{i,2,0}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{i,1,-1}^{n}-2T_{i,1,0}^{n}+T_{i,1,1}^{n}}{(\Delta z)^2}\end{array}\right] \tag{5.24}$$
$$+(q'''_{m_i})^n+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,i}^{\,n}\times(T_{art}-T_{i,1,0}^{n})\right]$$

In Equation (5.24), $T^{n}_{i,0,0}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used.

For Region 2, in Equation (5.24) $T^{n}_{i,1,-1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used. At Region 2, the temperature of the node (i,1,0) at next time step (n+1) is given below.

$$T^{n+1}_{i,1,0} = \alpha^{n}_{i} \times \Delta t \times \left[ \frac{T^{n}_{i-1,1,0} - 2T^{n}_{i,1,0} + T^{n}_{i+1,1,0}}{(\Delta r)^2} + \frac{T^{n}_{i+1,1,0} - T^{n}_{i-1,1,0}}{2i(\Delta r)^2} + \frac{T^{n}_{i,2,0} - 2T^{n}_{i,1,0} + T^{n}_{i,2,0}}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{1}{(\Delta z)^2}\left( T^{n}_{i,1,1} - 2T^{n}_{i,1,0} + \left( \frac{2\Delta z \times EE}{k^{n}_{i}} \right) + T^{n}_{i,1,1} \right) \right]$$
$$+ \left[ (q'''_{m_i})^{n} \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl_i} \times (T_{art} - T^{n}_{i,1,0}) \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + T^{n}_{i,1,0} \qquad (5.25)$$

For Region 3, in Equation (5.24) $T^{n}_{i,1,-1}$ is the temperature value of leg. Therefore, instead of node Torso($T^{n}_{i,1,-1}$), Leg($T^{n}_{i,1,\,legintervaz-1}$) should be substituted. At Region 3, the temperature of the node (i,1,0) at next time step (n+1) is given below.

$$T^{n+1}_{i,1,0} = \alpha^{n}_{i} \times \Delta t \times \left[ \frac{T^{n}_{i-1,1,0} - 2T^{n}_{i,1,0} + T^{n}_{i+1,1,0}}{(\Delta r)^2} + \frac{T^{n}_{i+1,1,0} - T^{n}_{i-1,1,0}}{2i(\Delta r)^2} + \frac{T^{n}_{i,2,0} - 2T^{n}_{i,1,0} + T^{n}_{i,2,0}}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{Leg(T^{n}_{i,1,\,leg\,int\,ervalz-1}) - 2T^{n}_{i,1,0} + T^{n}_{i,1,1}}{(\Delta z)^2} \right]$$
$$+ \left[ (q'''_{m_i})^{n} \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,1,0}) \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + T^{n}_{i,1,0} \qquad (5.26)$$

**Region 4, 5 and 6**

Substituting the node (i,j,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n \, Cp_i^n \left[\frac{T_{i,j,0}^{n+1} - T_{i,j,0}^n}{\Delta t}\right] = k_i^n \left[\begin{array}{l} \dfrac{T_{i-1,j,0}^n - 2T_{i,j,0}^n + T_{i+1,j,0}^n}{(\Delta r)^2} + \dfrac{T_{i+1,j,0}^n - T_{i-1,j,0}^n}{2i(\Delta r)^2} + \\ \dfrac{T_{i,j-1,0}^n - 2T_{i,j,0}^n + T_{i,j+1,0}^n}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T_{i,j,-1}^n - 2T_{i,j,0}^n + T_{i,j,1}^n}{(\Delta z)^2} \end{array}\right] \tag{5.27}$$

$$+ (q_{m_i}''')^n + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,i}^n \times (T_{art} - T_{i,j,0}^n)\right]$$

For Region 4 and Region 6, in Equation (5.27) $T_{i,j,-1}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used. At Region 4, the temperature of the node (i,j,0) at next time step (n+1) is given below.

$$T_{i,j,0}^{n+1} = \alpha_i^n \times \Delta t \times \left[\begin{array}{l} \dfrac{T_{i-1,j,0}^n - 2T_{i,j,0}^n + T_{i+1,j,0}^n}{(\Delta r)^2} + \dfrac{T_{i+1,j,0}^n - T_{i-1,j,0}^n}{2i(\Delta r)^2} + \\ \dfrac{T_{i,j-1,0}^n - 2T_{i,j,0}^n + T_{i,j+1,0}^n}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{1}{(\Delta z)^2}\left(T_{i,j,1}^n - 2T_{i,j,0}^n + \left(\dfrac{2\Delta z \times EE}{k_i^n}\right) + T_{i,j,1}^n\right) \end{array}\right] \tag{5.28}$$

$$+ \left[(q_{m_i}''')^n \times \frac{\Delta t}{\rho_i^n \, Cp_i^n}\right] + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,i}^n \times (T_{art} - T_{i,j,0}^n) \times \frac{\Delta t}{\rho_i^n \, Cp_i^n}\right] + T_{i,j,0}^n$$

For Region 5, in Equation (5.27) $T_{i,j,-1}^n$ is the temperature value of leg. Therefore, instead of node Torso($T_{i,j,-1}^n$), Leg($T_{i,j,\text{legintervaz}-1}^n$) should be substituted. At Region 3, the temperature of the node (i,j,0) at next time step (n+1) is given below.

$$T_{i,j,0}^{n+1} = \alpha_i^n \times \Delta t \times \left[ \begin{array}{l} \dfrac{T_{i-1,j,0}^n - 2T_{i,j,0}^n + T_{i+1,j,0}^n}{(\Delta r)^2} + \dfrac{T_{i+1,j,0}^n - T_{i-1,j,0}^n}{2i(\Delta r)^2} + \\ \dfrac{T_{i,j-1,0}^n - 2T_{i,j,0}^n + T_{i,j+1,0}^n}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{Leg(T_{i,1,legintervalz-1}^n) - 2T_{i,j,0}^n + T_{i,j,1}^n}{(\Delta z)^2} \end{array} \right] \quad (5.29)$$

$$+ \left[ (q_{m_i}''')^n \times \frac{\Delta t}{\rho_i^n \, Cp_i^n} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}_{bl_i}^n \times (T_{art} - T_{i,j,0}^n) \times \frac{\Delta t}{\rho_i^n \, Cp_i^n} \right] + T_{i,j,0}^n$$

**Region 7**

Substituting the node (i, M,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n \, Cp_i^n \left[ \frac{T_{i,M,0}^{n+1} - T_{i,M,0}^n}{\Delta t} \right] = k_i^n \left[ \begin{array}{l} \dfrac{T_{i-1,M,0}^n - 2T_{i,M,0}^n + T_{i+1,M,0}^n}{(\Delta r)^2} + \dfrac{T_{i+1,M,0}^n - T_{i-1,M,0}^n}{2i(\Delta r)^2} + \\ \dfrac{T_{i,M-1,0}^n - 2T_{i,M,0}^n + T_{i,M+1,0}^n}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T_{i,M,-1}^n - 2T_{i,M,0}^n + T_{i,M,1}^n}{(\Delta z)^2} \end{array} \right] \quad (5.30)$$

$$+ (q_{m_i}''')^n + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}_{bl_i}^n \times (T_{art} - T_{i,M,0}^n) \right]$$

In Equation (5.30), $T_{i,M+1,0}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T_{i,M,-1}^n$ is also the fictitious temperature. Boundary Condition 4 given in Table 5.2 is used to eliminate this fictitious temperature. At Region 7, the temperature of the node (i,M,0) at next time step (n+1) is given below.

$$T_{i,M,0}^{n+1} = \alpha_i^n \times \Delta t \times \left[ \begin{array}{l} \dfrac{T_{i-1,1,0}^n - 2T_{i,1,0}^n + T_{i+1,1,0}^n}{(\Delta r)^2} + \dfrac{T_{i+1,1,0}^n - T_{i-1,1,0}^n}{2i(\Delta r)^2} + \\ \dfrac{T_{i,M-1,0}^n - 2T_{i,M,0}^n + T_{i,M-1,0}^n}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{1}{(\Delta z)^2} \left( T_{i,M,1}^n - 2T_{i,M,0}^n + \left( \dfrac{2\Delta z \times EE}{k_i^n} \right) + T_{i,M,1}^n \right) \end{array} \right] \quad (5.31)$$

$$+ \left[ (q_{m_i}''')^n \times \frac{\Delta t}{\rho_i^n \, Cp_i^n} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}_{bl_i}^n \times (T_{art} - T_{i,M,0}^n) \times \frac{\Delta t}{\rho_i^n \, Cp_i^n} \right] + T_{i,M,0}^n$$

**Region 8**

Substituting the node (P,1,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n \left[\frac{T_{P,1,0}^{n+1} - T_{P,1,0}^{n}}{\Delta t}\right] = k_i^n \left[\begin{array}{l} \frac{T_{P-1,1,0}^{n} - 2T_{P,1,0}^{n} + T_{P+1,1,0}^{n}}{(\Delta r)^2} + \frac{T_{P+1,1,0}^{n} - T_{P-1,1,0}^{n}}{2P(\Delta r)^2} + \\ \frac{T_{P,0,0}^{n} - 2T_{P,1,0}^{n} + T_{P,2,0}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2} + \frac{T_{P,1,-1}^{n} - 2T_{P,1,0}^{n} + T_{P,1,1}^{n}}{(\Delta z)^2} \end{array}\right] + (q'''_{m_P})^n + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,P}^{n} \times (T_{art} - T_{P,1,0}^{n})\right] \qquad (5.32)$$

In Equation (5.32), $T_{P,0,0}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T_{P,1,-1}^{n}$ is the temperature value of leg. Therefore, instead of node Torso($T_{P,1,-1}^{n}$), Leg($T_{P,1,\,legintervaz-1}^{n}$) should be substituted. Also $T_{P+1,1,0}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. At Region 8, the temperature of the node (P,1,0) at next time step (n+1) is given below.

$$T_{P,1,0}^{n+1} = \alpha_P^n \times \Delta t \times \left[\begin{array}{l} \frac{1}{(\Delta r)^2}\left(T_{P-1,1,0}^{n} - 2T_{P,1,0}^{n} + \left(\frac{2\Delta r \times EE}{-k_P^n}\right) + T_{P-1,1,0}^{n}\right) \\ + \frac{1}{2P(\Delta r)^2}\left[\left(\frac{2\Delta r \times EE}{-k_P^n}\right) + T_{P-1,1,0}^{n} - T_{P-1,1,0}^{n}\right] \\ + \frac{T_{P,2,0}^{n} - 2T_{P,1,0}^{n} + T_{P,2,0}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2} + \frac{Leg(TP,1,legintervaz-1) - 2T_{P,1,0}^{n} + T_{P,1,1}^{n}}{(\Delta z)^2} \end{array}\right] + \left[(q'''_{m_P})^n \times \frac{\Delta t}{\rho_P^n Cp_P^n}\right] + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,P}^{n} \times (T_{art} - T_{P,1,0}^{n}) \times \frac{\Delta t}{\rho_P^n Cp_P^n}\right] + T_{P,1,0}^{n} \qquad (5.33)$$

**Region 9**

Substituting the node (P,j,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n\left[\frac{T_{P,j,0}^{n+1}-T_{P,j,0}^n}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\frac{T_{P-1,j,0}^n-2T_{P,j,0}^n+T_{P+1,j,0}^n}{(\Delta r)^2}+\frac{T_{P+1,j,0}^n-T_{P-1,j,0}^n}{2P(\Delta r)^2}+\\ \frac{T_{P,j,0}^n-2T_{P,j,0}^n+T_{P,j,0}^n}{P^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{P,j,-1}^n-2T_{P,j,0}^n+T_{P,j,1}^n}{(\Delta z)^2}\end{array}\right] \quad (5.34)$$

$$+(q'''_{m_P})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,P}^n\times(T_{art}-T_{P,j,0}^n)\right]$$

In Equation (5.34), $T_{P,j,-1}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used. Also $T_{P+1,j,0}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 9, the temperature of the node (P,j,0) at next time step (n+1) is given below.

$$T_{P,j,0}^{n+1}=\alpha_P\times\Delta t\times\left[\begin{array}{l}\frac{1}{(\Delta r)^2}\left(T_{P-1,j,0}^n-2T_{P,j,0}^n+\left(\frac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,j,0}^n\right)\\ +\frac{1}{2P(\Delta r)^2}\left[\left(\frac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,j,0}^n-T_{P-1,j,0}^n\right]\\ +\frac{T_{P,j-1,0}^n-2T_{P,j,0}^n+T_{P,j+1,0}^n}{P^2(\Delta r)^2(\Delta\phi)^2}+\frac{1}{(\Delta z)^2}\left(T_{P,j,1}^n-2T_{P,j,0}^n+\left(\frac{2\Delta z\times EE}{k_i^n}\right)+T_{P,j,1}^n\right)\end{array}\right] \quad (5.35)$$

$$+\left[(q'''_{m_P})^n\times\frac{\Delta t}{\rho_P^n Cp_P^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,P}^n\times(T_{art}-T_{P,j,0}^n)\times\frac{\Delta t}{\rho_P^n Cp_P^n}\right]+T_{P,j,0}^n$$

**Region 10**

Substituting the node (P,M,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n\left[\frac{T_{P,M,0}^{n+1}-T_{P,M,0}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{P-1,M,0}^{n}-2T_{P,M,0}^{n}+T_{P+1,M,0}^{n}}{(\Delta r)^2}+\dfrac{T_{P+1,M,0}^{n}-T_{P-1,M,0}^{n}}{2P(\Delta r)^2}+\\ \dfrac{T_{P,M-1,0}^{n}-2T_{P,M,0}^{n}+T_{P,M+1,0}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{P,M,-1}^{n}-2T_{P,M,0}^{n}+T_{P,M,1}^{n}}{(\Delta z)^2}\end{array}\right]+(q'''_{m_P})^n \quad (5.36)$$

$$+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{blP}^{n}\times(T_{art}-T_{P,M,0}^{n})\right]$$

In Equation (5.36), $T_{P,M+1,0}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T_{P,M,-1}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used. Also $T_{P+1,M,0}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 10, the temperature of the node (P,M,0) at next time step (n+1) is given below.

$$T_{P,M,0}^{n+1}=\alpha_P^n\times\Delta t\times\left[\begin{array}{l}\dfrac{1}{(\Delta r)^2}\left(T_{P-1,M,0}^{n}-2T_{P,M,0}^{n}+\left(\dfrac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,M,0}^{n}\right)\\ +\dfrac{1}{2P(\Delta r)^2}\left[\left(\dfrac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,M,0}^{n}-T_{P-1,M,0}^{n}\right]\\ +\dfrac{T_{P,M-1,0}^{n}-2T_{P,M,0}^{n}+T_{P,M-1,0}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{1}{(\Delta z)^2}\left(T_{P,M,1}^{n}-2T_{P,M,0}^{n}+\left(\dfrac{2\Delta z\times EE}{k_i^n}\right)+T_{P,M,1}^{n}\right)\end{array}\right] \quad (5.37)$$

$$+\left[(q'''_{m_P})^n\times\frac{\Delta t}{\rho_P^n Cp_P^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{blP}^{n}\times(T_{art}-T_{P,M,0}^{n})\times\frac{\Delta t}{\rho_P^n Cp_P^n}\right]+T_{P,M,0}^{n}$$

**Region 11**

Substituting the node (0,0,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.21).

$$\rho_0^n Cp_0^n\left[\frac{T_{0,0,k}^{n+1}-T_{0,0,k}^{n}}{\Delta t}\right]=k_0^n\left[4\left(\frac{\overline{T}_1-T_{0,0,k}^{n}}{(\Delta r)^2}\right)+\frac{T_{0,0,k-1}^{n}-2T_{0,0,k}^{n}+T_{0,0,k+1}^{n}}{(\Delta z)^2}\right]+(q'''_{m_0})^n \tag{5.38}$$

$$+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,0}^{n}\times(T_{art}-T_{0,0,k}^{n})\right]$$

In Equation (5.38) there is no fictitious temperature. The temperature of the node (0,0,k) at next time step (n+1) is given below.

$$T_{0,0,0}^{n+1}=\alpha_0^n\times\Delta t\times\left[4\left(\frac{\overline{T}_1-T_{0,0,0}^{n}}{(\Delta r)^2}\right)+\frac{1}{(\Delta z)^2}\left(T_{0,0,1}^{n}-2T_{0,0,0}^{n}+\left(\frac{2\Delta z\times EE}{k_0^n}\right)+T_{0,0,1}^{n}\right)\right] \tag{5.39}$$

$$+\left[(q'''_{m_0})^n\times\frac{\Delta t}{\rho_0^n\, Cp_0^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl0}^{n}\times(T_{art}-T_{0,0,0}^{n})\times\frac{\Delta t}{\rho_0^n\, Cp_0^n}\right]+T_{0,0,0}^{n}$$

**Region 12**

Substituting the node (i,j,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.16).

$$\rho C_p\left[\frac{T_{i,j,k}^{n+1}-T_{i,j,k}^{n}}{\Delta t}\right]=k\left[\frac{T_{i-1,j,k}^{n}-2T_{i,j,k}^{n}+T_{i+1,j,k}^{n}}{(\Delta r)^2}+\frac{1}{(i\Delta r)}\frac{T_{i+1,j,k}^{n}-T_{i-1,j,k}^{n}}{2\Delta r}+\frac{1}{(i\Delta r)^2}\frac{T_{i,j-1,k}^{n}-2T_{i,j,k}^{n}+T_{i,j+1,k}^{n}}{(\Delta\phi)^2}+\frac{T_{i,j,k-1}^{n}-2T_{i,j,k}^{n}+T_{i,j,k+1}^{n}}{(\Delta z)^2}\right] \tag{5.40}$$

$$+(q'''_{m_{i,t}})^n+\left[\rho_{bl}\times\dot{w}_{bl}\times Cp_{bl}\times(T_{art}-T_{i,j,k}^{n})\right]$$

In Equation (5.40) there is no fictitious temperature. The temperature of the node (i,j,k) at next time step (n+1) is given below.

$$T_{i,j,k}^{n+1}=\alpha_i^n\times\Delta t\times\left[\begin{array}{l}\frac{T_{i-1,j,k}^n-2T_{i,j,k}^n+T_{i+1,j,k}^n}{(\Delta r)^2}+\frac{1}{(i\Delta r)}\frac{T_{i+1,j,k}^n-T_{i-1,j,k}^n}{2\Delta r}+\\ \frac{1}{(i\Delta r)^2}\frac{T_{i,j-1,k}^n-2T_{i,j,k}^n+T_{i,j+1,k}^n}{(\Delta\phi)^2}+\frac{T_{i,j,k-1}^n-2T_{i,j,k}^n+T_{i,j,k+1}^n}{(\Delta z)^2}\end{array}\right] \tag{5.41}$$

$$+\left[(q'''_{m_i})^n\times\frac{\Delta t}{\rho_i^n\ Cp_i^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl_i}^n\times(T_{art}-T_{i,j,k}^n)\times\frac{\Delta t}{\rho_i^n\ Cp_i^n}\right]+T_{i,j,k}^n$$

**Region 13**

Substituting the node (i,1,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.16).

$$\rho_i^n\ Cp_i^n\left[\frac{T_{i,1,k}^{n+1}-T_{i,1,k}^n}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\frac{T_{i-1,1,k}^n-2T_{i,1,k}^n+T_{i+1,1,k}^n}{(\Delta r)^2}+\frac{T_{i+1,1,k}^n-T_{i-1,1,k}^n}{2i(\Delta r)^2}+\\ \frac{T_{i,0,k}^n-2T_{i,1,k}^n+T_{i,2,k}^n}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{i,1,k-1}^n-2T_{i,1,k}^n+T_{i,1,k+1}^n}{(\Delta z)^2}\end{array}\right]+(q'''_{m_i})^n \tag{5.42}$$

$$+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl_i}^n\times(T_{art}-T_{i,1,k}^n)\right]$$

In Equation (5.42), $T_{i,0,k}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. The temperature of the node (i,1,k) at next time step (n+1) is given below.

$$T_{i,1,k}^{n+1}=\alpha_i^n\times\Delta t\times\left[\begin{array}{l}\frac{T_{i-1,1,k}^n-2T_{i,1,k}^n+T_{i+1,1,k}^n}{(\Delta r)^2}+\frac{T_{i+1,1,k}^n-T_{i-1,1,k}^n}{2i(\Delta r)^2}+\\ \frac{T_{i,2,k}^n-2T_{i,1,k}^n+T_{i,2,k}^n}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{i,1,k-1}^n-2T_{i,1,k}^n+T_{i,1,k+1}^n}{(\Delta z)^2}\end{array}\right] \tag{5.43}$$

$$+\left[(q'''_{m_i})^n\times\frac{\Delta t}{\rho_i^n\ Cp_i^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl_i}^n\times(T_{art}-T_{i,1,k}^n)\times\frac{\Delta t}{\rho_i^n\ Cp_i^n}\right]+T_{i,1,k}^n$$

**Region 14**

Substituting the node (i,M,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.16).

$$\rho_i^n Cp_i^n\left[\frac{T_{i,M,k}^{n+1}-T_{i,M,k}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\frac{T_{i-1,M,k}^{n}-2T_{i,M,k}^{n}+T_{i+1,M,k}^{n}}{(\Delta r)^2}+\frac{T_{i+1M1,k}^{n}-T_{i-1,M,k}^{n}}{2i(\Delta r)^2}+\\ \frac{T_{i,M-1,k}^{n}-2T_{i,M,k}^{n}+T_{i,M+1,k}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{i,M,k-1}^{n}-2T_{i,M,k}^{n}+T_{i,M,k+1}^{n}}{(\Delta z)^2}\end{array}\right]$$

$$+(q'''_{m_i})^n+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl_i}^{n}\times(T_{art}-T_{i,M,k}^{n})\right] \tag{5.44}$$

In Equation (5.44), $T_{i,M+1,k}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. The temperature of the node (i,1,k) at next time step (n+1) is given below.

$$T_{i,M,k}^{n+1}=\alpha_i^n\times\Delta t\times\left[\begin{array}{l}\frac{T_{i-1,M,k}^{n}-2T_{i,M,k}^{n}+T_{i+1,M,k}^{n}}{(\Delta r)^2}+\frac{T_{i+1,M,k}^{n}-T_{i-1,M,k}^{n}}{2i(\Delta r)^2}+\\ \frac{T_{i,M-1,k}^{n}-2T_{i,M,k}^{n}+T_{i,M+1,k}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{i,M,k-1}^{n}-2T_{i,M,k}^{n}+T_{i,M,k+1}^{n}}{(\Delta z)^2}\end{array}\right]$$

$$+\left[(q'''_{m_i})^n\times\frac{\Delta t}{\rho_i^n\, Cp_i^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl_i}^{n}\times(T_{art}-T_{i,M,k}^{n})\times\frac{\Delta t}{\rho_i^n\, Cp_i^n}\right]+T_{i,M,k}^{n} \tag{5.45}$$

**Region 15**

Substituting the node (P,M,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.16).

$$\rho_P^n Cp_P^n\left[\frac{T_{P,M,k}^{n+1}-T_{P,M,k}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\frac{T_{P-1,M,k}^{n}-2T_{P,M,k}^{n}+T_{P+1,M,k}^{n}}{(\Delta r)^2}+\frac{T_{P+1,M,k}^{n}-T_{P-1,M,k}^{n}}{2P(\Delta r)^2}+\\ \frac{T_{P,M-1,k}^{n}-2T_{P,M,k}^{n}+T_{P,M+1,k}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{P,M,k-1}^{n}-2T_{P,M,k}^{n}+T_{P,M,k+1}^{n}}{(\Delta z)^2}\end{array}\right] \tag{5.46}$$

$$+(q'''_{m_P})^n+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl_P}^{n}\times(T_{art}-T_{P,M,k}^{n})\right]$$

In Equation (5.46), $T^{n}_{P,M+1,k}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. Also $T^{n}_{P+1,M,k}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 15, the temperature of the node (P,M,k) at next time step (n+1) is given below.

$$T^{n+1}_{P,M,k} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{aligned} & \frac{1}{(\Delta r)^2}\left( T^{n}_{P-1,M,k} - 2T^{n}_{P,M,k} + \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,M,k} \right) \\ & + \frac{1}{2P(\Delta r)^2}\left[ \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,M,k} - T^{n}_{P-1,M,k} \right] \\ & + \frac{T^{n}_{P,M-1,k} - 2T^{n}_{P,M,k} + T^{n}_{P,M-1,k}}{P^2(\Delta r)^2(\Delta\phi)^2} + \left[ \frac{T^{n}_{P,M,k-1} - 2T^{n}_{P,M,k} + T^{n}_{P,M,k+1}}{(\Delta z)^2} \right] \end{aligned} \right] \tag{5.47}$$

$$+\left[ (q'''_{m_P})^{n} \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,M,k}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,M,k}$$

**Region 16 and 21**

Substituting the node (P,1,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.16).

$$\rho^{n}_{P}\, Cp^{n}_{P} \left[ \frac{T^{n+1}_{P,1,k} - T^{n}_{P,1,k}}{\Delta t} \right] = k^{n}_{i} \left[ \begin{aligned} & \frac{T^{n}_{P-1,1,k} - 2T^{n}_{P,1,k} + T^{n}_{P+1,1,k}}{(\Delta r)^2} + \frac{T^{n}_{P+1,1,k} - T^{n}_{P-1,1,k}}{2P(\Delta r)^2} + \\ & \frac{T^{n}_{P,0,k} - 2T^{n}_{P,1,k} + T^{n}_{P,2,k}}{P^2(\Delta r)^2(\Delta\phi)^2} + \frac{T^{n}_{P,1,k-1} - 2T^{n}_{P,1,k} + T^{n}_{P,1,k+1}}{(\Delta z)^2} \end{aligned} \right] \tag{5.48}$$

$$+(q'''_{m_P})^{n} + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,1,k}) \right]$$

In Equation (5.48), $T^{n}_{P,0,k}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used.

For Region 16, $T^{n}_{P+1,1,k}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. In conclusion, at Region 16, the temperature of the node (P,1,k) at next time step (n+1) is given below.

$$T^{n+1}_{P,1,k} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{array}{l} \dfrac{1}{(\Delta r)^2}\left( T^{n}_{P-1,1,k} - 2T^{n}_{P,1,k} + \left( \dfrac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,1,k} \right) \\ + \dfrac{1}{2P(\Delta r)^2}\left[ \left( \dfrac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,1,k} - T^{n}_{P-1,1,k} \right] \\ + \dfrac{T^{n}_{P,2,k} - 2T^{n}_{P,1,k} + T^{n}_{P,2,k}}{P^2(\Delta r)^2(\Delta\phi)^2} + \left[ \dfrac{T^{n}_{P,1,k-1} - 2T^{n}_{P,1,k} + T^{n}_{P,1,k+1}}{(\Delta z)^2} \right] \end{array} \right] \tag{5.49}$$
$$+ \left[ (q'''_{m_P})^{n} \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,1,k}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,1,k}$$

For Region 21, in Equation (5.49) $T^{n}_{P+1,1,k}$ is the temperature value of arm. Therefore, instead of node Torso($T^{n}_{P+1,1,k}$), Arm($T^{n}_{P+1,1,armintervaz-1}$) should be substituted.

At Region 21, the temperature of the node (P,1,k) at next time step (n+1) is given below.

$$T^{n+1}_{P,1,k} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{array}{l} \dfrac{1}{(\Delta r)^2}\left( T^{n}_{P-1,1,k} - 2T^{n}_{P,1,k} + \left( \dfrac{2\Delta r \times EE}{-k^{n}_{P}} \right) + \text{Arm}\,(T^{n}_{i,j,armintervaz-1}) \right) \\ + \dfrac{1}{2P(\Delta r)^2}\left[ \text{Arm}\,(T^{n}_{i,j,armintervaz-1}) - T^{n}_{P-1,1,k} \right] \\ + \dfrac{T^{n}_{P,2,k} - 2T^{n}_{P,1,k} + T^{n}_{P,2,k}}{P^2(\Delta r)^2(\Delta\phi)^2} + \left[ \dfrac{T^{n}_{P,1,k-1} - 2T^{n}_{P,1,k} + T^{n}_{P,1,k+1}}{(\Delta z)^2} \right] \end{array} \right] \tag{5.50}$$
$$+ \left[ (q'''_{m_P})^{n} \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,j,k}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,j,k}$$

**Region 17, 18, 19 and 20**

Substituting the node (P,j,k) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n\left[\frac{T_{P,j,k}^{n+1}-T_{P,j,k}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\frac{T_{P-1,j,k}^{n}-2T_{P,j,k}^{n}+T_{P+1,j,k}^{n}}{(\Delta r)^2}+\frac{T_{P+1,j,k}^{n}-T_{P-1,j,k}^{n}}{2P(\Delta r)^2}+\\ \frac{T_{P,j-1,k}^{n}-2T_{P,j,k}^{n}+T_{P,j+1,k}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{P,j,k-1}^{n}-2T_{P,j,k}^{n}+T_{P,j,k+1}^{n}}{(\Delta z)^2}\end{array}\right] \quad (5.51)$$

$$+(q'''_{m_P})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,P}^{n}\times(T_{art}-T_{P,j,k}^{n})\right]$$

For Region 17, 18 and 19, in Equation (5.51) $T_{P+1,j,k}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. For these region, the temperature of the node (P,j,k) at next time step (n+1) is given below.

$$T_{P,j,k}^{n+1}=\alpha_P^n\times\Delta t\times\left[\begin{array}{l}\frac{1}{(\Delta r)^2}\left(T_{P-1,j,k}^{n}-2T_{P,j,k}^{n}+\left(\frac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,j,k}^{n}\right)\\ +\frac{1}{2P(\Delta r)^2}\left[\left(\frac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,j,k}^{n}-T_{P-1,j,k}^{n}\right]\\ +\frac{T_{P,j-1,k}^{n}-2T_{P,j,k}^{n}+T_{P,j+1,k}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\left[\frac{T_{P,j,k-1}^{n}-2T_{P,j,k}^{n}+T_{P,j,k+1}^{n}}{(\Delta z)^2}\right]\end{array}\right] \quad (5.52)$$

$$+\left[(q'''_{m_P})^n\times\frac{\Delta t}{\rho_P^n Cp_P^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,P}^{n}\times(T_{art}-T_{P,j,k}^{n})\times\frac{\Delta t}{\rho_P^n Cp_P^n}\right]+T_{P,j,k}^{n}$$

For Region 20, in Equation (5.51) $T_{P+1,j,k}^{n}$ is the temperature value of arm. Therefore, instead of node Torso($T_{P+1,j,k}^{n}$), Arm($T_{P+1,j,\text{legintervaz}-1}^{n}$) should be substituted.

At Region 20, the temperature of the node (P,j,k) at next time step (n+1) is given below.

$$T_{P,j,k}^{n+1} = \alpha_P^n \times \Delta t \times \left[ \begin{array}{l} \frac{1}{(\Delta r)^2}\left( T_{P-1,j,k}^n - 2T_{P,j,k}^n + \left( \frac{2\Delta r \times EE}{-k_P^n} \right) + Arm\,(T_{i,j,arminterva\,z-1}^n) \right) \\ + \frac{1}{2P(\Delta r)^2}\left[ Arm\,(T_{i,j,arminterva\,z-1}^n) - T_{P-1,j,k}^n \right] \\ + \frac{T_{P,j-1,k}^n - 2T_{P,j,k}^n + T_{P,j+1,k}^n}{P^2(\Delta r)^2(\Delta\phi)^2} + \left[ \frac{T_{P,j,k-1}^n - 2T_{P,j,k}^n + T_{P,j,k+1}^n}{(\Delta z)^2} \right] \end{array} \right] \tag{5.53}$$

$$+ \left[ (q_{m_P}''')^n \times \frac{\Delta t}{\rho_P^n\, Cp_P^n} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,P}^n \times (T_{art} - T_{P,j,k}^n) \times \frac{\Delta t}{\rho_P^n\, Cp_P^n} \right] + T_{P,j,k}^n$$

**Region 23**

Substituting the node (0,0,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.21).

$$\rho_0^n\, Cp_0^n \left[ \frac{T_{0,0,N}^{n+1} - T_{0,0,N}^n}{\Delta t} \right] = k_0^n \left[ 4\left( \frac{\overline{T}_1 - T_{0,0,N}^n}{(\Delta r)^2} \right) + \frac{T_{0,0,N-1}^n - 2T_{0,0,N}^n + T_{0,0,N+1}^n}{(\Delta z)^2} \right] + (q_{m_0}''')^n \tag{5.54}$$

$$+ \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,0}^n \times (T_{art} - T_{0,0,N}^n) \right]$$

For Region 23, in Equation (5.54) $T_{0,0,N+1}^n$ is the temperature value of neck. Therefore, instead of node Torso ($T_{0,0,N+1}^n$), Neck ($T_{0,0,1}^n$) should be substituted.

At Region 23, the temperature of the node (0,0,N) at next time step (n+1) is given below.

$$T_{0,0,N}^{n+1}=\alpha_0^n\times\Delta t\times\left[4\left(\frac{\overline{T}_1-T_{0,0,N}^n}{(\Delta r)^2}\right)+\frac{1}{(\Delta z)^2}\left(NECK(T_{0,0,1}^n)-2T_{0,0,N}^n+T_{0,0,N-1}^n\right)\right] \tag{5.55}$$

$$+\left[(q_{m_0}''')^n\times\frac{\Delta t}{\rho_0^n\,Cp_0^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl0}^n\times(T_{art}-T_{0,0,N}^n)\times\frac{\Delta t}{\rho_0^n\,Cp_0^n}\right]+T_{0,0,N}^n$$

**Region 24 and 25**

Substituting the node (i,1,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n\,Cp_i^n\left[\frac{T_{i,1,N}^{n+1}-T_{i,1,N}^n}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{i-1,1,N}^n-2T_{i,1,N}^n+T_{i+1,1,N}^n}{(\Delta r)^2}+\dfrac{T_{i+1,1,N}^n-T_{i-1,1,N}^n}{2i(\Delta r)^2}+\\ \dfrac{T_{i,0,N}^n-2T_{i,1,N}^n+T_{i,2,N}^n}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{i,1,N-1}^n-2T_{i,1,N}^n+T_{i,1,N+1}^n}{(\Delta z)^2}\end{array}\right] \tag{5.56}$$

$$+(q_{m_i}''')^n+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,i}^n\times(T_{art}-T_{i,1,N}^n)\right]$$

In Equation (5.56), $T_{i,0,N}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used.

For Region 24, in Equation (5.56) $T_{i,1,N+1}^n$ is the temperature value of neck. Therefore, instead of node Torso($T_{i,1,N+1}^n$), Neck($T_{i,1,1}^n$) should be substituted. At Region 24, the temperature of the node (i,1,N) at next time step (n+1) is given below.

$$T_{i,1,N}^{n+1}=\alpha_i^n\times\Delta t\times\left[\begin{array}{l}\dfrac{T_{i-1,1,N}^n-2T_{i,1,N}^n+T_{i+1,1,N}^n}{(\Delta r)^2}+\dfrac{T_{i+1,1,N}^n-T_{i-1,1,N}^n}{2i(\Delta r)^2}+\\ \dfrac{T_{i,2,N}^n-2T_{i,1,N}^n+T_{i,2,N}^n}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{i,1,N-1}^n-2T_{i,1,N}^n+NECK(T_{i,1,1}^n)}{(\Delta z)^2}\end{array}\right] \tag{6.57}$$

$$+\left[(q_{m_i}''')^n\times\frac{\Delta t}{\rho_i^n\,Cp_i^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,i}^n\times(T_{art}-T_{i,1,N}^n)\times\frac{\Delta t}{\rho_i^n\,Cp_i^n}\right]+T_{i,1,N}^n$$

For Region 25, in Equation (5.56) $T^{n}_{i,1,N+1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 5 given in Table 5.2 is used.

At Region 25, the temperature of the node (i,1,N) at next time step (n+1) is given below.

$$T^{n+1}_{i,1,N}=\alpha^{n}_{i}\times\Delta t\times\left[\begin{array}{l}\dfrac{T^{n}_{i-1,1,N}-2T^{n}_{i,1,N}+T^{n}_{i+1,1,N}}{(\Delta r)^2}+\dfrac{T^{n}_{i+1,1,N}-T^{n}_{i-1,1,N}}{2i(\Delta r)^2}+\\ \dfrac{T^{n}_{i,2,N}-2T^{n}_{i,1,N}+T^{n}_{i,2,N}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{1}{(\Delta z)^2}\left[T^{n}_{i,1,N-1}-2T^{n}_{i,1,N}+\left(\dfrac{2\Delta z\times EE}{-k^{n}_{i}}\right)+T^{n}_{i,1,N-1}\right]\end{array}\right] \quad (6.58)$$

$$+\left[(q'''_{m_i})^{n}\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl_i}\times(T_{art}-T^{n}_{i,1,N})\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+T^{n}_{i,1,N}$$

**Region 26**

Substituting the node (P,1,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P}\,Cp^{n}_{P}\left[\frac{T^{n+1}_{P,1,N}-T^{n}_{P,1,N}}{\Delta t}\right]=k^{n}_{i}\left[\begin{array}{l}\dfrac{T^{n}_{P-1,1,N}-2T^{n}_{P,1,N}+T^{n}_{P+1,1,N}}{(\Delta r)^2}+\dfrac{T^{n}_{P+1,1,N}-T^{n}_{P-1,1,N}}{2P(\Delta r)^2}+\\ \dfrac{T^{n}_{P,0,N}-2T^{n}_{P,1,N}+T^{n}_{P,2,N}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T^{n}_{P,1,N-1}-2T^{n}_{P,1,N}+T^{n}_{P,1,N+1}}{(\Delta z)^2}\end{array}\right] \quad (5.59)$$

$$+(q'''_{m_P})^{n}+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,P}\times(T_{art}-T^{n}_{P,j,N})\right]$$

In Equation (5.59), $T^{n}_{P,0,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used.

In addition, $T^{n}_{P,1,N+1}$ is the temperature value of leg. Therefore, instead of node Torso($T^{n}_{P+1,1,N}$), Arm($T^{n}_{P,1,\text{armintervaz}-1}$) should be substituted. Also $T^{n}_{P,1,N+1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 5 given in Table 5.2 is used.

At Region 26, the temperature of the node (P,1,N) at next time step (n+1) is given below.

$$T^{n+1}_{P,1,N}=\alpha^{n}_{P}\times\Delta t\times\left[\begin{array}{l}\frac{1}{(\Delta r)^2}\left(T^{n}_{P-1,1,N}-2T^{n}_{P,1,N}+\left(\frac{2\Delta r\times EE}{-k^{n}_{P}}\right)+\text{Arm}(T^{n}_{i,j,\text{armintervaz}-1})\right)\\+\frac{1}{2P(\Delta r)^2}\left[\text{Arm}(T^{n}_{i,j,\text{armintervaz}-1})-T^{n}_{P-1,1,N}\right]\\+\frac{T^{n}_{P,2,N}-2T^{n}_{P,1,N}+T^{n}_{P,2,N}}{P^2(\Delta r)^2(\Delta\phi)^2}+\left[T^{n}_{P,1,N-1}-2T^{n}_{P,1,N}+\left(\frac{2\Delta z\times EE}{-k^{n}_{P}}\right)+T^{n}_{P,1,N-1}\right]\end{array}\right] \tag{5.60}$$
$$+\left[(q'''_{m_P})^{n}\times\frac{\Delta t}{\rho^{n}_{P}\,Cp^{n}_{P}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,P}\times(T_{art}-T^{n}_{P,1,N})\times\frac{\Delta t}{\rho^{n}_{P}\,Cp^{n}_{P}}\right]+T^{n}_{P,1,N}$$

**Region 27 and 28**

Substituting the node (i,M,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{i}\,Cp^{n}_{i}\left[\frac{T^{n+1}_{i,M,N}-T^{n}_{i,1,N}}{\Delta t}\right]=k^{n}_{i}\left[\begin{array}{l}\frac{T^{n}_{i-1,M,N}-2T^{n}_{i,M,N}+T^{n}_{i+1,M,N}}{(\Delta r)^2}+\frac{T^{n}_{i+1,M,N}-T^{n}_{i-1,M,N}}{2i(\Delta r)^2}+\\\frac{T^{n}_{i,M-1,N}-2T^{n}_{i,M,N}+T^{n}_{i,M+1,N}}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{T^{n}_{i,M,N-1}-2T^{n}_{i,M,N}+T^{n}_{i,M,N+1}}{(\Delta z)^2}\end{array}\right] \tag{5.61}$$
$$+(q'''_{m_i})^{n}+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,i}\times(T_{art}-T^{n}_{i,1,M})\right]$$

In Equation (5.61), $T^{n}_{i,M+1,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used.

For Region 27, in Equation (5.61) $T^{n}_{i,M,N+1}$ is the temperature value of neck. Therefore, instead of node Torso($T^{n}_{i,M,N+1}$), Neck($T^{n}_{i,M,1}$) should be substituted.

At Region 27, the temperature of the node (i,M,N) at next time step (n+1) is given below.

$$T^{n+1}_{i,M,N}=\alpha^{n}_{i}\times\Delta t\times\left[\begin{array}{l}\dfrac{T^{n}_{i-1,M,N}-2T^{n}_{i,M,N}+T^{n}_{i+1,M,N}}{(\Delta r)^2}+\dfrac{T^{n}_{i+1,M,N}-T^{n}_{i-1,M,N}}{2i(\Delta r)^2}+\\ \dfrac{T^{n}_{i,M-1,N}-2T^{n}_{i,M,N}+T^{n}_{i,M-1,N}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T^{n}_{i,M,N-1}-2T^{n}_{i,M,N}+NECK(T^{n}_{i,M,1})}{(\Delta z)^2}\end{array}\right]$$
$$+\left[(q'''_{m_i})^{n}\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl_i}\times(T_{art}-T^{n}_{i,M,N})\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+T^{n}_{i,M,N} \quad (5.62)$$

For Region 28, in Equation (5.61) $T^{n}_{i,M,N+1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 5 given in Table 5.2 is used.

At Region 28, the temperature of the node (i,M,N) at next time step (n+1) is given below.

$$T^{n+1}_{i,M,N}=\alpha^{n}_{i}\times\Delta t\times\left[\begin{array}{l}\dfrac{T^{n}_{i-1,M,N}-2T^{n}_{i,M,N}+T^{n}_{i+1,M,N}}{(\Delta r)^2}+\dfrac{T^{n}_{i+1,M,N}-T^{n}_{i-1,M,N}}{2i(\Delta r)^2}+\\ \dfrac{T^{n}_{i,M-1,N}-2T^{n}_{i,M,N}+T^{n}_{i,M-1,N}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{1}{(\Delta z)^2}\left[T^{n}_{i,M,N-1}-2T^{n}_{i,M,N}+\left(\dfrac{2\Delta z\times EE}{-k^{n}_{p}}\right)+T^{n}_{i,M,N-1}\right]\end{array}\right]$$
$$+\left[(q'''_{m_i})^{n}\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl_i}\times(T_{art}-T^{n}_{i,M,N})\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+T^{n}_{i,M,N} \quad (5.63)$$

**Region 29**

Substituting the node (P,M,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n \left[\frac{T_{P,M,N}^{n+1}-T_{P,M,N}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{P-1,M,N}^{n}-2T_{P,M,N}^{n}+T_{P+1,M,N}^{n}}{(\Delta r)^2}+\dfrac{T_{P+1,M,N}^{n}-T_{P-1,M,N}^{n}}{2P(\Delta r)^2}+\\ \dfrac{T_{P,M-1,N}^{n}-2T_{P,M,N}^{n}+T_{P,M+1,N}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{P,M,N-1}^{n}-2T_{P,M,N}^{n}+T_{P,M,N+1}^{n}}{(\Delta z)^2}\end{array}\right] \quad (5.64)$$

$$+(q'''_{m_P})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{blP}^{n}\times(T_{art}-T_{P,M,N}^{n})\right]$$

In Equation (5.64), $T_{i,M+1,N}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T_{P,M,N+1}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 5 given in Table 5.2 is used. Also $T_{P+1,M,N}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 29, the temperature of the node (P,M,N) at next time step (n+1) is given below.

$$T_{P,M,N}^{n+1}=\alpha_P^n\times\Delta t\times\left[\begin{array}{l}\dfrac{1}{(\Delta r)^2}\left(T_{P-1,M,N}^{n}-2T_{P,M,N}^{n}+\left(\dfrac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,M,N}^{n}\right)\\ +\dfrac{1}{2P(\Delta r)^2}\left[\left(\dfrac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,M,N}^{n}-T_{P-1,M,N}^{n}\right]\\ +\dfrac{T_{P,M-1,N}^{n}-2T_{P,M,N}^{n}+T_{P,M-1,N}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}\\ +\dfrac{1}{(\Delta z)^2}\left[T_{P,M,N-1}^{n}-2T_{P,M,N}^{n}+\left(\dfrac{2\Delta z\times EE}{-k_P^n}\right)+T_{P,M,N-1}^{n}\right]\end{array}\right] \quad (5.65)$$

$$+\left[(q'''_{m_P})^n\times\frac{\Delta t}{\rho_P^n\, Cp_P^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{blP}^{n}\times(T_{art}-T_{P,M,N}^{n})\times\frac{\Delta t}{\rho_P^n\, Cp_P^n}\right]+T_{P,M,N}^{n}$$

**Region 30 and 31**

Substituting the node (i,j,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n Cp_i^n\left[\frac{T_{i,j,N}^{n+1}-T_{i,j,N}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{i-1,j,N}^{n}-2T_{i,j,N}^{n}+T_{i+1,j,N}^{n}}{(\Delta r)^2}+\dfrac{T_{i+1,j,N}^{n}-T_{i-1,j,N}^{n}}{2i(\Delta r)^2}+\\ \dfrac{T_{i,j-1,N}^{n}-2T_{i,j,N}^{n}+T_{i,j+1,N}^{n}}{i^2(\Delta r)^2(\Delta \phi)^2}+\dfrac{T_{i,j,N-1}^{n}-2T_{i,j,N}^{n}+T_{i,j,N+1}^{n}}{(\Delta z)^2}\end{array}\right] \tag{5.66}$$

$$+(q'''_{m_i})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,i}^{\,n}\times(T_{art}-T_{i,j,N}^{n})\right]$$

For Region 30, in Equation (5.66) $T_{i,j,N+1}^{n}$ is the temperature value of neck.

Therefore, instead of node Torso($T_{i,j,N+1}^{n}$), Neck($T_{i,j,1}^{n}$) should be substituted.

At Region 30, the temperature of the node (i,j,N) at next time step (n+1) is given below.

$$T_{i,j,N}^{n+1}=\alpha_i^n\times\Delta t\times\left[\begin{array}{l}\dfrac{T_{i-1,j,N}^{n}-2T_{i,j,N}^{n}+T_{i+1,j,N}^{n}}{(\Delta r)^2}+\dfrac{T_{i+1,j,N}^{n}-T_{i-1,j,N}^{n}}{2i(\Delta r)^2}+\\ \dfrac{T_{i,j-1,N}^{n}-2T_{i,j,N}^{n}+T_{i,j+1,N}^{n}}{i^2(\Delta r)^2(\Delta \phi)^2}+\dfrac{T_{i,j,N-1}^{n}-2T_{i,j,N}^{n}+Neck(T_{i,j,1})}{(\Delta z)^2}\end{array}\right] \tag{5.67}$$

$$+\left[(q'''_{m_i})^n\times\frac{\Delta t}{\rho_i^n\,Cp_i^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,i}^{\,n}\times(T_{art}-T_{i,j,N}^{n})\times\frac{\Delta t}{\rho_i^n\,Cp_i^n}\right]+T_{i,j,N}^{n}$$

For Region 31, in Equation (5.66) $T_{i,j,N+1}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 5 given in Table 5.2 is used.

At Region 25, the temperature of the node (i,j,N) at next time step (n+1) is given below.

$$T_{i,j,N}^{n+1} = \alpha_i^n \times \Delta t \times \left[ \begin{array}{l} \dfrac{T_{i-1,j,N}^n - 2T_{i,j,N}^n + T_{i+1,j,N}^n}{(\Delta r)^2} + \dfrac{T_{i+1,j,N}^n - T_{i-1,j,N}^n}{2i(\Delta r)^2} + \\ \dfrac{T_{i,j-1,N}^n - 2T_{i,j,N}^n + T_{i,j+1,N}^n}{i^2(\Delta r)^2(\Delta\phi)^2} \\ + \dfrac{1}{(\Delta z)^2}\left[ T_{i,j,N-1}^n - 2T_{i,j,N}^n + \left( \dfrac{2\Delta z \times EE}{-k_P^n} \right) + T_{i,j,,N-1}^n \right] \end{array} \right] \tag{5.68}$$

$$+\left[ (q'''_{m_i})^n \times \frac{\Delta t}{\rho_i^n \, Cp_i^n} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,i}^{\,n} \times (T_{art} - T_{i,j,N}^n) \times \frac{\Delta t}{\rho_i^n \, Cp_i^n} \right] + T_{i,j,N}^n$$

**Region 32**

Substituting the node (P,j,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n \left[ \frac{T_{P,j,N}^{n+1} - T_{P,j,N}^n}{\Delta t} \right] = k_i^n \left[ \begin{array}{l} \dfrac{T_{P-1,j,N}^n - 2T_{P,j,N}^n + T_{P+1,j,N}^n}{(\Delta r)^2} + \dfrac{T_{P+1,j,N}^n - T_{P-1,j,N}^n}{2P(\Delta r)^2} + \\ \dfrac{T_{P,j-1,N}^n - 2T_{P,j,N}^n + T_{P,j+1,N}^n}{P^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T_{P,j,N-1}^n - 2T_{P,j,N}^n + T_{P,j,N+1}^n}{(\Delta z)^2} \end{array} \right] \tag{5.69}$$

$$+(q'''_{m_P})^n + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}_{blP}^{\,n} \times (T_{art} - T_{P,j,N}^n) \right]$$

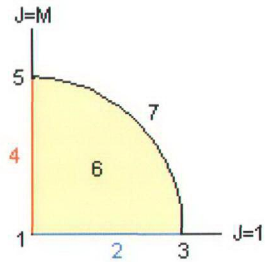
In Equation (5.69), $T_{P,j,N+1}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 5 given in Table 5.2 is used. In addition, $T_{P+1,j,N}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 32, the temperature of the node (P,j,N) at next time step (n+1) is given below.

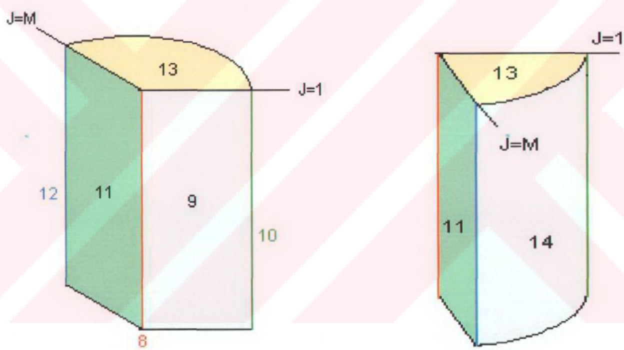
$$T^{n+1}_{P,j,N} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{aligned} & \frac{1}{(\Delta r)^2}\left( T^{n}_{P-1,j,N} - 2T^{n}_{P,j,N} + \left(\frac{2\Delta r\times EE}{-k^{n}_{P}}\right) + T^{n}_{P-1,j,N}\right) \\ & + \frac{1}{2P(\Delta r)^2}\left[\left(\frac{2\Delta r \times EE}{-k^{n}_{P}}\right) + T^{n}_{P-1,j,N} - T^{n}_{P-1,j,N}\right] \\ & + \frac{T^{n}_{P,j-1,N} - 2T^{n}_{P,j,N} + T^{n}_{P,j+1,N}}{i^2(\Delta r)^2(\Delta\phi)^2} \\ & + \frac{1}{(\Delta z)^2}\left[T^{n}_{P,j,N-1} - 2T^{n}_{P,j,N} + \left(\frac{2\Delta z\times EE}{-k^{n}_{P}}\right) + T^{n}_{P,j,N-1}\right] \end{aligned} \right] + \left[(q'''_{m_P})^{n} \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}}\right] + \left[\rho_{bl}\times Cp_{bl}\times \dot{W}^{n}_{bl\,P}\times(T_{art} - T^{n}_{P,j,N})\times\frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}}\right] + T^{n}_{P,j,N} \qquad (5.70)$$

## NECK

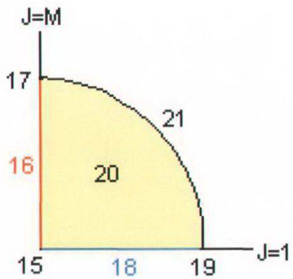
**At k=1**




**At k=2 to (N-1)**




**At k=N**



**Region 1**

Substituting the node (0,0,1) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.21).

$$\rho_0^n Cp_0^n\left[\frac{T_{0,0,1}^{n+1}-T_{0,0,1}^{n}}{\Delta t}\right]=k_0^n\left[4\left(\frac{\overline{T}_1-T_{0,0,1}^{n}}{(\Delta r)^2}\right)+\frac{T_{0,0,0}^{n}-2T_{0,0,1}^{n}+T_{0,0,2}^{n}}{(\Delta z)^2}\right]+(q'''_{m_0})^n$$
$$+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,0}^{n}\times(T_{art}-T_{0,0,1}^{n})\right] \tag{5.71}$$

In Equation (5.71) $T_{0,0,0}^{n}$ is the temperature value of torso. Therefore, instead of node Neck($T_{0,0,0}^{n}$), Torso($T_{0,0,\text{torsointervalz}}^{n}$) should be substituted. At Region 1, the temperature of the node (0,0,1) at next time step (n+1) is given below.

$$T_{0,0,1}^{n+1}=\alpha_0^n\times\Delta t\times\left[4\left(\frac{\overline{T}_1-T_{0,0,1}^{n}}{(\Delta r)^2}\right)+\frac{1}{(\Delta z)^2}\left(\text{Torso}(T_{0,0,\text{torsointervalz}})-2T_{0,0,1}^{n}+T_{0,0,2}^{n}\right)\right]$$
$$+\left[(q'''_{m_0})^n\times\frac{\Delta t}{\rho_0^n\,Cp_0^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,0}^{n}\times(T_{art}-T_{0,0,1}^{n})\times\frac{\Delta t}{\rho_0^n\,Cp_0^n}\right]+T_{0,0,1}^{n} \tag{5.72}$$

**Region 2**

Substituting the node (i,1,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n\,Cp_i^n\left[\frac{T_{i,1,1}^{n+1}-T_{i,1,1}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{i-1,1,1}^{n}-2T_{i,1,1}^{n}+T_{i+1,1,1}^{n}}{(\Delta r)^2}+\dfrac{T_{i+1,1,1}^{n}-T_{i-1,1,1}^{n}}{2i(\Delta r)^2}+\\[2ex]\dfrac{T_{i,0,1}^{n}-2T_{i,1,1}^{n}+T_{i,2,1}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{i,1,0}^{n}-2T_{i,1,1}^{n}+T_{i,1,2}^{n}}{(\Delta z)^2}\end{array}\right]+(q'''_{m_i})^n \tag{5.73}$$
$$+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,i}^{n}\times(T_{art}-T_{i,1,1}^{n})\right]$$

In Equation (5.73), $T^{n}_{i,0,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T^{n}_{i,1,0}$ is the temperature value of torso. Therefore, instead of node Neck($T^{n}_{i,1,0}$), Torso($T^{n}_{i,1,torsointervalz}$) should be substituted. At Region 2, the temperature of the node (i,1,1) at next time step (n+1) is given below.

$$T^{n+1}_{i,1,1} = \alpha^{n}_{i} \times \Delta t \times \left[ \begin{array}{l} \dfrac{T^{n}_{i-1,1,1} - 2T^{n}_{i,1,1} + T^{n}_{i+1,1,1}}{(\Delta r)^2} + \dfrac{T^{n}_{i+1,1,1} - T^{n}_{i-1,1,1}}{2i(\Delta r)^2} \\ + \dfrac{T^{n}_{i,2,1} - 2T^{n}_{i,1,1} + T^{n}_{i,2,1}}{i^2(\Delta r)^2(\Delta\phi)^2} \\ + \dfrac{\text{Torso}(T^{n}_{i,j,\text{torso interval}z}) - 2T^{n}_{i,1,1} + T^{n}_{i,1,2}}{(\Delta z)^2} \end{array} \right] + \left[ (q'''_{m_i})^{n} \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] \tag{5.74}$$

$$+ \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,1,1}) \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + T^{n}_{i,1,1}$$

**Region 3**

Substituting the node (P,1,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P}\, Cp^{n}_{P} \left[ \frac{T^{n+1}_{P,1,1} - T^{n}_{P,1,1}}{\Delta t} \right] = k^{n}_{i} \left[ \begin{array}{l} \dfrac{T^{n}_{P-1,1,1} - 2T^{n}_{P,1,1} + T^{n}_{P+1,1,1}}{(\Delta r)^2} + \dfrac{T^{n}_{P+1,1,1} - T^{n}_{P-1,1,1}}{2P(\Delta r)^2} + \\ \dfrac{T^{n}_{P,0,1} - 2T^{n}_{P,1,1} + T^{n}_{P,2,1}}{P^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T^{n}_{P,1,0} - 2T^{n}_{P,1,1} + T^{n}_{P,1,2}}{(\Delta z)^2} \end{array} \right] + (q'''_{m_P})^{n} \tag{5.75}$$

$$+ \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,1,1}) \right]$$

In Equation (5.75), $T^{n}_{P,0,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T^{n}_{P,1,0}$ is the temperature value of torso. Therefore, instead of node Neck($T^{n}_{P,1,0}$), Torso($T^{n}_{P,1,torsointervaz}$) should be substituted. Also $T^{n}_{P+1,1,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. At Region 3, the temperature of the node (P,1,1) at next time step (n+1) is given below.

$$T^{n+1}_{P,1,1} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{array}{l} \dfrac{1}{(\Delta r)^2}\left( T^{n}_{P-1,1,1} - 2T^{n}_{P,1,1} + \left( \dfrac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,1,1} \right) \\ + \dfrac{1}{2P(\Delta r)^2}\left[ \left( \dfrac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,1,1} - T^{n}_{P-1,1,1} \right] \\ + \dfrac{T^{n}_{P,2,1} - 2T^{n}_{P,1,1} + T^{n}_{P,2,1}}{P^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{Torso(T_{P,1,torsointervaz-1}) - 2T^{n}_{P,1,1} + T^{n}_{P,1,2}}{(\Delta z)^2} \end{array} \right] \tag{5.76}$$

$$+ \left[ (q'''_{m_P})^{n} \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,1,1}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,1,1}$$

**Region 4**

Substituting the node (i, M,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{i}\, Cp^{n}_{i} \left[ \frac{T^{n+1}_{i,M,1} - T^{n}_{i,M,1}}{\Delta t} \right] = k^{n}_{i} \left[ \begin{array}{l} \dfrac{T^{n}_{i-1,M,1} - 2T^{n}_{i,M,1} + T^{n}_{i+1,M,1}}{(\Delta r)^2} + \dfrac{T^{n}_{i+1,M,1} - T^{n}_{i-1,M,1}}{2i(\Delta r)^2} + \\ \dfrac{T^{n}_{i,M-1,1} - 2T^{n}_{i,M,1} + T^{n}_{i,M+1,1}}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T^{n}_{i,M,0} - 2T^{n}_{i,M,1} + T^{n}_{i,M,2}}{(\Delta z)^2} \end{array} \right] + (q'''_{m_i})^{n}$$

$$+ \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,M,1}) \right] \tag{5.77}$$

In Equation (5.77), $T^{n}_{i,M+1,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T^{n}_{i,M,0}$ is the temperature value of torso. Therefore, instead of node Neck($T^{n}_{i,M,0}$), Torso($T^{n}_{i,M,torsointervaz}$) should be substituted. At Region 4, the temperature of the node (i,M,0) at next time step (n+1) is given below.

$$T^{n+1}_{i,M,1} = \alpha^{n}_{i} \times \Delta t \times \left[ \begin{array}{l} \dfrac{T^{n}_{i-1,1,1} - 2T^{n}_{i,1,1} + T^{n}_{i+1,1,1}}{(\Delta r)^2} + \dfrac{T^{n}_{i+1,1,1} - T^{n}_{i-1,1,1}}{2i(\Delta r)^2} \\ + \dfrac{T^{n}_{i,M-1,1} - 2T^{n}_{i,M,1} + T^{n}_{i,M-1,1}}{i^2(\Delta r)^2(\Delta\phi)^2} \\ + \dfrac{1}{(\Delta z)^2}\left( \text{Torso } (T_{i,j,\text{torsointer valz}}) - 2T^{n}_{i,M,1} + T^{n}_{i,M,2} \right) \end{array} \right] \tag{5.78}$$

$$+ \left[ (q'''_{m_i})^{n} \times \frac{\Delta t}{\rho^{n}_{i}\ Cp^{n}_{i}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,i} \times (T_{art} - T^{n}_{i,M,1}) \times \frac{\Delta t}{\rho^{n}_{i}\ Cp^{n}_{i}} \right] + T^{n}_{i,M,1}$$

**Region 5**

Substituting the node (P,M,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P}\, Cp^{n}_{P} \left[ \frac{T^{n+1}_{P,M,1} - T^{n}_{P,M,1}}{\Delta t} \right] = k^{n}_{i} \left[ \begin{array}{l} \dfrac{T^{n}_{P-1,M,1} - 2T^{n}_{P,M,1} + T^{n}_{P+1,M,1}}{(\Delta r)^2} + \dfrac{T^{n}_{P+1,M,1} - T^{n}_{P-1,M,1}}{2P(\Delta r)^2} + \\ \dfrac{T^{n}_{P,M-1,1} - 2T^{n}_{P,M,1} + T^{n}_{P,M+1,1}}{P^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T^{n}_{P,M,0} - 2T^{n}_{P,M,1} + T^{n}_{P,M,k+2}}{(\Delta z)^2} \end{array} \right] \tag{5.79}$$

$$+ (q'''_{m_P})^{n} + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,M,1}) \right]$$

In Equation (5.79), $T^{n}_{P,M+1,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T^{n}_{P,M,0}$ is the temperature value of torso. Therefore, instead of node Neck ($T^{n}_{P,M,0}$), Torso ($T^{n}_{P,M,torsointervaz}$) should be substituted. Also $T^{n}_{P+1,M,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 5, the temperature of the node (P,M,1) at next time step (n+1) is given below.

$$T^{n+1}_{P,M,1} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{aligned} & \frac{1}{(\Delta r)^2}\left( T^{n}_{P-1,M,1} - 2T^{n}_{P,M,1} + \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,M,1} \right) \\ & + \frac{1}{2P(\Delta r)^2}\left[ \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,M,1} - T^{n}_{P-1,M,1} \right] \\ & + \frac{T^{n}_{P,M-1,1} - 2T^{n}_{P,M,1} + T^{n}_{P,M-1,1}}{P^2(\Delta r)^2(\Delta\phi)^2} + \left[ \frac{Torso(T_{P,M,torsointervalz}) - 2T^{n}_{P,M,1} + T^{n}_{P,M,2}}{(\Delta z)^2} \right] \end{aligned} \right] \tag{5.80}$$

$$+ \left[ (q'''_{m_P})^n \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,M,1}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,M,1}$$

**Region 6**

Substituting the node (i,j,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{i}\, Cp^{n}_{i} \left[ \frac{T^{n+1}_{i,j,1} - T^{n}_{i,j,1}}{\Delta t} \right] = k^{n}_{i} \left[ \begin{aligned} & \frac{T^{n}_{i-1,j,1} - 2T^{n}_{i,j,1} + T^{n}_{i+1,j,1}}{(\Delta r)^2} + \frac{T^{n}_{i+1,j,1} - T^{n}_{i-1,j,1}}{2i(\Delta r)^2} + \\ & \frac{T^{n}_{i,j-1,1} - 2T^{n}_{i,j,1} + T^{n}_{i,j+1,1}}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{T^{n}_{i,j,0} - 2T^{n}_{i,j,1} + T^{n}_{i,j,2}}{(\Delta z)^2} \end{aligned} \right] + (q'''_{m_i})^n \tag{5.81}$$

$$+ \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,j,1}) \right]$$

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In Equation (5.81) $T^{n}_{i,j,0}$ is the temperature value of torso. Therefore, instead of node Neck($T^{n}_{i,j,0}$), Torso($T^{n}_{i,j,torsointervaz}$) should be substituted. At Region 6, the temperature of the node (i,j,1) at next time step (n+1) is given below.

$$T^{n+1}_{i,j,1} = \alpha^{n}_{i} \times \Delta t \times \left[ \frac{T^{n}_{i-1,j,1} - 2T^{n}_{i,j,1} + T^{n}_{i+1,j,1}}{(\Delta r)^2} + \frac{T^{n}_{i+1,j,1} - T^{n}_{i-1,j,1}}{2i(\Delta r)^2} + \frac{T^{n}_{i,j-1,1} - 2T^{n}_{i,j,1} + T^{n}_{i,j+1,1}}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{\text{Torso}\,(T^{n}_{i,j,torsointer\ valz}) - 2T^{n}_{i,j,1} + T^{n}_{i,j,2}}{(\Delta z)^2} \right] \tag{5.82}$$

$$+\left[ (q'''_{m_i})^{n} \times \frac{\Delta t}{\rho^{n}_{i}\ Cp^{n}_{i}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,j,1}) \times \frac{\Delta t}{\rho^{n}_{i}\ Cp^{n}_{i}} \right] + T^{n}_{i,j,1}$$

**Region 7**

Substituting the node (P,j,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P}\, Cp^{n}_{P} \left[ \frac{T^{n+1}_{P,j,1} - T^{n}_{P,j,1}}{\Delta t} \right] = k^{n}_{i} \left[ \frac{T^{n}_{P-1,j,1} - 2T^{n}_{P,j,1} + T^{n}_{P+1,j,1}}{(\Delta r)^2} + \frac{T^{n}_{P+1,j,1} - T^{n}_{P-1,j,1}}{2P(\Delta r)^2} + \frac{T^{n}_{P,j,1} - 2T^{n}_{P,j,1} + T^{n}_{P,j,1}}{P^2(\Delta r)^2(\Delta\phi)^2} + \frac{T^{n}_{P,j,0} - 2T^{n}_{P,j,1} + T^{n}_{P,j,2}}{(\Delta z)^2} \right] + (q'''_{m_P})^{n} \tag{5.83}$$

$$+\left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,j,1}) \right]$$

In Equation (5.83), $T^{n}_{P+1,j,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. In addition, $T^{n}_{P,j,0}$

is the temperature value of torso. Therefore, instead of node Neck($T^n_{P,j,0}$), Torso($T^n_{P,j,torsointervaz}$) should be substituted.

At Region 7, the temperature of the node (P,j,1) at next time step (n+1) is given below.

$$T^{n+1}_{P,j,1} = \alpha^n_P \times \Delta t \times \left[ \begin{array}{l} \dfrac{1}{(\Delta r)^2}\left( T^n_{P-1,j,1} - 2T^n_{P,j,1} + \left(\dfrac{2\Delta r \times EE}{-k^n_P}\right) + T^n_{P-1,j,1} \right) \\ + \dfrac{1}{2P(\Delta r)^2}\left[ \left(\dfrac{2\Delta r \times EE}{-k^n_P}\right) + T^n_{P-1,j,1} - T^n_{P-1,j,1} \right] \\ + \dfrac{T^n_{P,j-1,1} - 2T^n_{P,j,1} + T^n_{P,j+1,1}}{P^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{\text{Torso}\,(T^n_{i,j,torsointervalz}) - 2T^n_{P,j,1} + T^n_{P,j,2}}{(\Delta z)^2} \end{array} \right] \tag{5.84}$$

$$+\left[ (q'''_{m_P})^n \times \frac{\Delta t}{\rho^n_P\, Cp^n_P} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^n_{bl\,P} \times (T_{art} - T^n_{P,j,1}) \times \frac{\Delta t}{\rho^n_P\, Cp^n_P} \right] + T^n_{P,j,1}$$

**Region 8**

Substituting the node (0,0,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.21).

$$\rho^n_0\, Cp^n_0 \left[ \frac{T^{n+1}_{0,0,k} - T^n_{0,0,k}}{\Delta t} \right] = k^n_0 \left[ 4\left( \frac{\overline{T}_1 - T^n_{0,0,k}}{(\Delta r)^2} \right) + \frac{T^n_{0,0,k-1} - 2T^n_{0,0,k} + T^n_{0,0,k+1}}{(\Delta z)^2} \right] + (q'''_{m_0})^n \tag{5.85}$$

$$+\left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^n_{bl\,0} \times (T_{art} - T^n_{0,0,k}) \right]$$

In Equation (5.85) there is no fictitious temperature. The temperature of the node (0,0,k) at next time step (n+1) is given below.

$$T^{n+1}_{0,0,k}=\alpha^{n}_{0}\times\Delta t\times\left[4\left(\frac{\overline{T}^{n}_{1}-T_{0,0,0}}{(\Delta r)^{2}}\right)+\left[\frac{T^{n}_{0,0,k-1}-2T^{n}_{0,0,k}+T^{n}_{0,0,k+1}}{(\Delta z)^{2}}\right]\right] \tag{5.86}$$

$$+\left[(q'''_{m_0})^{n}\times\frac{\Delta t}{\rho^{n}_{0}\,Cp^{n}_{0}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,0}\times(T_{art}-T^{n}_{0,0,k})\times\frac{\Delta t}{\rho^{n}_{0}\,Cp^{n}_{0}}\right]+T^{n}_{0,0,k}$$

**Region 9**

Substituting the node (i,1,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.16).

$$\rho^{n}_{i}\,Cp^{n}_{i}\left[\frac{T^{n+1}_{i,1,k}-T^{n}_{i,1,k}}{\Delta t}\right]=k^{n}_{i}\left[\begin{array}{c}\dfrac{T^{n}_{i-1,1,k}-2T^{n}_{i,1,k}+T^{n}_{i+1,1,k}}{(\Delta r)^{2}}+\dfrac{T^{n}_{i+1,1,k}-T^{n}_{i-1,1,k}}{2i(\Delta r)^{2}}+\\ \dfrac{T^{n}_{i,0,k}-2T^{n}_{i,1,k}+T^{n}_{i,2,k}}{i^{2}(\Delta r)^{2}(\Delta\phi)^{2}}+\dfrac{T^{n}_{i,1,k-1}-2T^{n}_{i,1,k}+T^{n}_{i,1,k+1}}{(\Delta z)^{2}}\end{array}\right]+(q'''_{m_i})^{n} \tag{5.87}$$

$$+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,i}\times(T_{art}-T^{n}_{i,1,k})\right]$$

In Equation (5.87), $T^{n}_{i,0,k}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. At Region 9, the temperature of the node (i,1,k) at next time step (n+1) is given below.

$$T^{n+1}_{i,1,k}=\alpha^{n}_{i}\times\Delta t\times\left[\begin{array}{c}\dfrac{T^{n}_{i-1,1,k}-2T^{n}_{i,1,k}+T^{n}_{i+1,1,k}}{(\Delta r)^{2}}+\dfrac{T^{n}_{i+1,1,k}-T^{n}_{i-1,1,k}}{2i(\Delta r)^{2}}+\\ \dfrac{T^{n}_{i,2,k}-2T^{n}_{i,1,k}+T^{n}_{i,2,k}}{i^{2}(\Delta r)^{2}(\Delta\phi)^{2}}+\dfrac{T^{n}_{i,1,k-1}-2T^{n}_{i,1,k}+T^{n}_{i,1,k+1}}{(\Delta z)^{2}}\end{array}\right]+ \tag{5.88}$$

$$+\left[(q'''_{m_i})^{n}\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,i}\times(T_{art}-T^{n}_{i,1,k})\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+T^{n}_{i,1,k}$$

**Region 10**

Substituting the node (P,1,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.16).

$$\rho_P^n Cp_P^n\left[\frac{T_{P,1,k}^{n+1}-T_{P,1,k}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\frac{T_{P-1,1,k}^{n}-2T_{P,1,k}^{n}+T_{P+1,1,k}^{n}}{(\Delta r)^2}+\frac{T_{P+1,1,k}^{n}-T_{P-1,1,k}^{n}}{2P(\Delta r)^2}+\\ \frac{T_{P,0,k}^{n}-2T_{P,1,k}^{n}+T_{P,2,k}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{P,1,k-1}^{n}-2T_{P,1,k}^{n}+T_{P,1,k+1}^{n}}{(\Delta z)^2}\end{array}\right]$$

$$+(q'''_{m_P})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,P}^{n}\times(T_{art}-T_{P,1,k}^{n})\right] \qquad (5.89)$$

In Equation (5.89), $T_{P,0,k}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T_{P+1,1,k}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. In conclusion, at Region 16, the temperature of the node (P,1,k) at next time step (n+1) is given below.

$$T_{P,1,k}^{n+1}=\alpha_P^n\times\Delta t\times\left[\begin{array}{l}\frac{1}{(\Delta r)^2}\left(T_{P-1,1,k}^{n}-2T_{P,1,k}^{n}+\left(\frac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,1,k}^{n}\right)\\ +\frac{1}{2P(\Delta r)^2}\left[\left(\frac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,1,k}^{n}-T_{P-1,1,k}^{n}\right]\\ +\frac{T_{P,2,k}^{n}-2T_{P,1,k}^{n}+T_{P,2,k}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\left[\frac{T_{P,1,k-1}^{n}-2T_{P,1,k}^{n}+T_{P,1,k+1}^{n}}{(\Delta z)^2}\right]\end{array}\right] \qquad (5.90)$$

$$+\left[(q'''_{m_P})^n\times\frac{\Delta t}{\rho_P^n Cp_P^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,P}^{n}\times(T_{art}-T_{P,1,k}^{n})\times\frac{\Delta t}{\rho_P^n Cp_P^n}\right]+T_{P,1,k}^{n}$$

**Region 11**

Substituting the node (i,M,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.16).

$$\rho_i^n\, Cp_i^n \left[\frac{T_{i,M,k}^{n+1}-T_{i,M,k}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{i-1,M,k}^{n}-2T_{i,M,k}^{n}+T_{i+1,M,k}^{n}}{(\Delta r)^2}+\dfrac{T_{i+1Ml,k}^{n}-T_{i-1,M,k}^{n}}{2i(\Delta r)^2}+\\ \dfrac{T_{i,M-1,k}^{n}-2T_{i,M,k}^{n}+T_{i,M+1,k}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{i,M,k-1}^{n}-2T_{i,M,k}^{n}+T_{i,M,k+1}^{n}}{(\Delta z)^2}\end{array}\right]$$
$$+(q'''_{m_i})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,i}^{n}\times(T_{art}-T_{i,M,k}^{n})\right] \qquad (5.91)$$

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n Equation (5.91), $T_{i,M+1,k}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. The temperature of the node (i,M,k) at next time step (n+1) is given below.

$$T_{i,M,k}^{n+1}=\alpha_i^n\times\Delta t\times\left[\begin{array}{l}\dfrac{T_{i-1,M,k}^{n}-2T_{i,M,k}^{n}+T_{i+1,M,k}^{n}}{(\Delta r)^2}+\dfrac{T_{i+1,M,k}^{n}-T_{i-1,M,k}^{n}}{2i(\Delta r)^2}+\\ \dfrac{T_{i,M-1,k}^{n}-2T_{i,M,k}^{n}+T_{i,M-1,k}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{i,M,k-1}^{n}-2T_{i,M,k}^{n}+T_{i,M,k+1}^{n}}{(\Delta z)^2}\end{array}\right] \qquad (5.92)$$
$$+\left[(q'''_{m_i})^n\times\frac{\Delta t}{\rho_i^n\; Cp_i^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,i}^{n}\times(T_{art}-T_{i,M,k}^{n})\times\frac{\Delta t}{\rho_i^n\; Cp_i^n}\right]+T_{i,M,k}^{n}$$

**Region 12**

Substituting the node (P,M,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.16).

$$\rho_P^n\, Cp_P^n\left[\frac{T_{P,M,k}^{n+1}-T_{P,M,k}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{P-1,M,k}^{n}-2T_{P,M,k}^{n}+T_{P+1,M,k}^{n}}{(\Delta r)^2}+\dfrac{T_{P+1,M,k}^{n}-T_{P-1,M,k}^{n}}{2P(\Delta r)^2}+\\ \dfrac{T_{P,M-1,k}^{n}-2T_{P,M,k}^{n}+T_{P,M+1,k}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{P,M,k-1}^{n}-2T_{P,M,k}^{n}+T_{P,M,k+1}^{n}}{(\Delta z)^2}\end{array}\right] \qquad (5.93)$$
$$+(q'''_{m_P})^n+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,P}^{n}\times(T_{art}-T_{P,M,k}^{n})\right]$$

In Equation (5.93), $T^{n}_{P,M+1,k}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. Also $T^{n}_{P+1,M,k}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 12, the temperature of the node (P,M,k) at next time step (n+1) is given below.

$$T^{n+1}_{P,M,k} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{array}{l} \frac{1}{(\Delta r)^2}\left( T^{n}_{P-1,M,k} - 2T^{n}_{P,M,k} + \left(\frac{2\Delta r \times EE}{-k^{n}_{P}}\right) + T^{n}_{P-1,M,k} \right) \\ + \frac{1}{2P(\Delta r)^2}\left[ \left(\frac{2\Delta r \times EE}{-k^{n}_{P}}\right) + T^{n}_{P-1,M,k} - T^{n}_{P-1,M,k} \right] \\ + \left[ \frac{T^{n}_{P,M-1,k} - 2T^{n}_{P,M,k} + T^{n}_{P,M-1,k}}{P^2(\Delta r)^2(\Delta\phi)^2} \right] \\ + \left[ \frac{T^{n}_{P,M,k-1} - 2T^{n}_{P,M,k} + T^{n}_{P,M,k+1}}{(\Delta z)^2} \right] \end{array} \right] \tag{5.94}$$

$$+ \left[ (q'''_{m_P})^{n} \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,M,k}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,M,k}$$

**Region 13**

Substituting the node (i,j,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.16).

$$\rho^{n}_{P}\, Cp^{n}_{P} \left[ \frac{T^{n+1}_{i,j,k} - T^{n}_{i,j,k}}{\Delta t} \right] = k^{n}_{i} \left[ \begin{array}{l} \frac{T^{n}_{i-1,j,k} - 2T^{n}_{i,j,k} + T^{n}_{i+1,j,k}}{(\Delta r)^2} + \frac{T^{n}_{i+1,j,k} - T^{n}_{i-1,j,k}}{2P(\Delta r)^2} + \\ \frac{T^{n}_{i,j-1,k} - 2T^{n}_{i,j,k} + T^{n}_{i,j+1,k}}{P^2(\Delta r)^2(\Delta\phi)^2} + \frac{T^{n}_{i,j,k-1} - 2T^{n}_{i,j,k} + T^{n}_{i,j,k+1}}{(\Delta z)^2} \end{array} \right] \tag{5.95}$$

$$+ (q'''_{m_P})^{n} + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{i,j,k}) \right]$$

In Equation (5.95) there is no fictitious temperature. The temperature of the node (i,j,k) at next time step (n+1) is given below.

$$T_{i,j,k}^{n+1}=\alpha_i^n\times\Delta t\times\left[\begin{array}{l}\dfrac{T_{i-1,j,k}^n-2T_{i,j,k}^n+T_{i+1,j,k}^n}{(\Delta r)^2}+\dfrac{1}{(i\Delta r)}\dfrac{T_{i+1,j,k}^n-T_{i-1,j,k}^n}{2\Delta r}+\\ \dfrac{1}{(i\Delta r)^2}\dfrac{T_{i,j-1,k}^n-2T_{i,j,k}^n+T_{i,j+1,k}^n}{(\Delta\phi)^2}+\dfrac{T_{i,j,k-1}^n-2T_{i,j,k}^n+T_{i,j,k+1}^n}{(\Delta z)^2}\end{array}\right] \tag{5.96}$$

$$+\left[(q'''_{m_i})^n\times\frac{\Delta t}{\rho_i^n\ Cp_i^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl_i}^n\times(T_{art}-T_{i,j,k}^n)\times\frac{\Delta t}{\rho_i^n\ Cp_i^n}\right]+T_{i,j,k}^n$$

**Region 14**

Substituting the node (P,j,k) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n\,Cp_P^n\left[\frac{T_{P,j,k}^{n+1}-T_{P,j,k}^n}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{P-1,j,k}^n-2T_{P,j,k}^n+T_{P+1,j,k}^n}{(\Delta r)^2}+\dfrac{T_{P+1,j,k}^n-T_{P-1,j,k}^n}{2P(\Delta r)^2}+\\ \dfrac{T_{P,j-1,k}^n-2T_{P,j,k}^n+T_{P,j+1,k}^n}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{P,j,k-1}^n-2T_{P,j,k}^n+T_{P,j,k+1}^n}{(\Delta z)^2}\end{array}\right] \tag{5.97}$$

$$+(q'''_{m_P})^n+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{blP}^n\times(T_{art}-T_{P,j,k}^n)\right]$$

In Equation (5.97) $T_{P+1,j,k}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. For these region, the temperature of the node (P,j,k) at next time step (n+1) is given below.

$$T^{n+1}_{P,j,k} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{array}{l} \frac{1}{(\Delta r)^2}\left( T^{n}_{P-1,j,k} - 2T^{n}_{P,j,k} + \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,j,k} \right) \\ + \frac{1}{2P(\Delta r)^2}\left[ \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,j,k} - T^{n}_{P-1,j,k} \right] \\ + \frac{T^{n}_{P,j-1,k} - 2T^{n}_{P,j,k} + T^{n}_{P,j+1,k}}{P^2(\Delta r)^2(\Delta\phi)^2} + \left[ \frac{T^{n}_{P,j,k-1} - 2T^{n}_{P,j,k} + T^{n}_{P,j,k+1}}{(\Delta z)^2} \right] \end{array} \right] \tag{5.98}$$

$$+ \left[ (q'''_{m_P})^{n} \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,j,k}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,j,k}$$

**Region 15**

Substituting the node (0,0,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.21).

$$\rho^{n}_{0}\, Cp^{n}_{0} \left[ \frac{T^{n+1}_{0,0,N} - T^{n}_{0,0,N}}{\Delta t} \right] = k^{n}_{0} \left[ 4\left( \frac{\overline{T}_1 - T^{n}_{0,0,N}}{(\Delta r)^2} \right) + \frac{T^{n}_{0,0,N-1} - 2T^{n}_{0,0,N} + T^{n}_{0,0,N+1}}{(\Delta z)^2} \right] + (q'''_{m_0})^{n} \tag{5.99}$$

$$+ \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,0} \times (T_{art} - T^{n}_{0,0,N}) \right]$$

In Equation (5.99) $T^{n}_{0,0,N+1}$ is the temperature value of head. Therefore, instead of node Neck ($T^{n}_{0,0,N+1}$), Head ($T^{n}_{0,0,0}$) should be substituted.

At Region 15, the temperature of the node (0,0,N) at next time step (n+1) is given below.

$$T^{n+1}_{0,0,N} = \alpha^{n}_{0} \times \Delta t \times \left[ 4\left( \frac{\overline{T}_1 - T^{n}_{0,0,N}}{(\Delta r)^2} \right) + \frac{1}{(\Delta z)^2}\left( T^{n}_{0,0,N-1} - 2T^{n}_{0,0,N} + \mathrm{Head}(T^{n}_{0,0,0}) \right) \right] \tag{5.100}$$

$$+ \left[ (q'''_{m_0})^{n} \times \frac{\Delta t}{\rho^{n}_{0}\, Cp^{n}_{0}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl0} \times (T_{art} - T^{n}_{0,0,N}) \times \frac{\Delta t}{\rho^{n}_{0}\, Cp^{n}_{0}} \right] + T^{n}_{0,0,N}$$

**Region 16**

Substituting the node (i,1,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n Cp_i^n \left[\frac{T_{i,1,N}^{n+1} - T_{i,1,N}^n}{\Delta t}\right] = k_i^n \left[\begin{array}{l} \dfrac{T_{i-1,1,N}^n - 2T_{i,1,N}^n + T_{i+1,1,N}^n}{(\Delta r)^2} + \dfrac{T_{i+1,1,N}^n - T_{i-1,1,N}^n}{2i(\Delta r)^2} + \\ \dfrac{T_{i,0,N}^n - 2T_{i,1,N}^n + T_{i,2,N}^n}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T_{i,1,N-1}^n - 2T_{i,1,N}^n + T_{i,1,N+1}^n}{(\Delta z)^2} \end{array}\right] \quad (5.101)$$

$$+ (q'''_{m_i})^n + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,i}^n \times (T_{art} - T_{i,1,N}^n)\right]$$

In Equation (5.101), $T_{i,0,N}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T_{i,1,N+1}^n$ is the temperature value of head. Therefore, instead of node Neck ($T_{i,1,N+1}^n$), Head ($T_{i,1,0}^n$) should be substituted. At Region 16, the temperature of the node (i,1,N) at next time step (n+1) is given below.

$$T_{i,1,N}^{n+1} = \alpha_i^n \times \Delta t \times \left[\begin{array}{l} \dfrac{T_{i-1,1,N}^n - 2T_{i,1,N}^n + T_{i+1,1,N}^n}{(\Delta r)^2} + \dfrac{T_{i+1,1,N}^n - T_{i-1,1,N}^n}{2i(\Delta r)^2} + \\ \dfrac{T_{i,2,N}^n - 2T_{i,1,N}^n + T_{i,2,N}^n}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T_{i,1,N-1}^n - 2T_{i,1,N}^n + \text{Head}\ (T_{i,1,0}^n)}{(\Delta z)^2} \end{array}\right] \quad (5.102)$$

$$+ \left[(q'''_{m_i})^n \times \frac{\Delta t}{\rho_i^n\ Cp_i^n}\right] + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,i}^n \times (T_{art} - T_{i,1,N}^n) \times \frac{\Delta t}{\rho_i^n\ Cp_i^n}\right] + T_{i,1,N}^n$$

**Region 17**

Substituting the node (P,1,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n\left[\frac{T_{P,1,N}^{n+1}-T_{P,1,N}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{P-1,1,N}^{n}-2T_{P,1,N}^{n}+T_{P+1,1,N}^{n}}{(\Delta r)^2}+\dfrac{T_{P+1,1,N}^{n}-T_{P-1,1,N}^{n}}{2P(\Delta r)^2}+\\ \dfrac{T_{P,0,N}^{n}-2T_{P,1,N}^{n}+T_{P,2,N}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{P,1,N-1}^{n}-2T_{P,1,N}^{n}+T_{P,1,N+1}^{n}}{(\Delta z)^2}\end{array}\right] \quad (5.103)$$

$$+(q'''_{m_P})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{blP}^{n}\times(T_{art}-T_{P,j,N}^{n})\right]$$

In Equation (5.103), $T_{P,0,N}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T_{P,1,N+1}^{n}$ is the temperature value of head. Therefore, instead of node Neck($T_{P,1,N+1}^{n}$), Head($T_{i,1,0}^{n}$) should be substituted. Also $T_{P+1,1,N}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 17, the temperature of the node (P,1,N) at next time step (n+1) is given below.

$$T_{P,1,N}^{n+1}=\alpha_P^n\times\Delta t\times\left[\begin{array}{l}\dfrac{1}{(\Delta r)^2}\left(T_{P-1,1,N}^{n}-2T_{P,1,N}^{n}+\left(\dfrac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,1,N}^{n}\right)\\ +\dfrac{1}{2P(\Delta r)^2}\left[\left(\dfrac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,1,N}^{n}-T_{P-1,1,N}^{n}\right]\\ +\dfrac{T_{P,2,N}^{n}-2T_{P,1,N}^{n}+T_{P,2,N}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{P,1,N-1}^{n}-2T_{P,1,N}^{n}+\text{Head }(T_{i,1,0}^{n})}{(\Delta z)^2}\end{array}\right] \quad (5.104)$$

$$+\left[(q'''_{m_P})^n\times\frac{\Delta t}{\rho_P^n Cp_P^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{blP}^{n}\times(T_{art}-T_{P,1,N}^{n})\times\frac{\Delta t}{\rho_P^n Cp_P^n}\right]+T_{P,1,N}^{n}$$

**Region 18**

Substituting the node (i,M,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n \, Cp_i^n \left[\frac{T_{i,M,N}^{n+1} - T_{i,1,N}^{n}}{\Delta t}\right] = k_i^n \left[\begin{array}{l} \dfrac{T_{i-1,M,N}^{n} - 2T_{i,M,N}^{n} + T_{i+1,M,N}^{n}}{(\Delta r)^2} + \dfrac{T_{i+1,M,N}^{n} - T_{i-1,M,N}^{n}}{2i(\Delta r)^2} + \\ \dfrac{T_{i,M-1,N}^{n} - 2T_{i,M,N}^{n} + T_{i,M+1,N}^{n}}{i^2(\Delta r)^2(\Delta \phi)^2} + \dfrac{T_{i,M,N-1}^{n} - 2T_{i,M,N}^{n} + T_{i,M,N+1}^{n}}{(\Delta z)^2} \end{array}\right] \quad (5.105)$$

$$+ (q_{m_i}^{'''})^n + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl_i}^{n} \times (T_{art} - T_{i,1,M}^{n})\right]$$

In Equation (5.105), $T_{i,M+1,N}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T_{i,M,N+1}^{n}$ is the temperature value of head. Therefore, instead of node Neck($T_{i,M,N+1}^{n}$), Head($T_{i,M,0}^{n}$) should be substituted.

At Region 18, the temperature of the node (i,M,N) at next time step (n+1) is given below.

$$T_{i,M,N}^{n+1} = \alpha_i^n \times \Delta t \times \left[\begin{array}{l} \dfrac{T_{i-1,M,N}^{n} - 2T_{i,M,N}^{n} + T_{i+1,M,N}^{n}}{(\Delta r)^2} + \dfrac{T_{i+1,M,N}^{n} - T_{i-1,M,N}^{n}}{2i(\Delta r)^2} + \\ \dfrac{T_{i,M-1,N}^{n} - 2T_{i,M,N}^{n} + T_{i,M-1,N}^{n}}{i^2(\Delta r)^2(\Delta \phi)^2} + \dfrac{T_{i,M,N-1}^{n} - 2T_{i,M,N}^{n} + \text{Head}\,(T_{i,M,0}^{n})}{(\Delta z)^2} \end{array}\right]$$

$$+ \left[(q_{m_i}^{'''})^n \times \frac{\Delta t}{\rho_i^n \, Cp_i^n}\right] + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl_i}^{n} \times (T_{art} - T_{i,M,N}^{n}) \times \frac{\Delta t}{\rho_i^n \, Cp_i^n}\right] + T_{i,M,N}^{n} \quad (5.106)$$

**Region 19**

Substituting the node (P,M,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n\left[\frac{T_{P,M,N}^{n+1}-T_{P,M,N}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\frac{T_{P-1,M,N}^{n}-2T_{P,M,N}^{n}+T_{P+1,M,N}^{n}}{(\Delta r)^2}+\frac{T_{P+1,M,N}^{n}-T_{P-1,M,N}^{n}}{2P(\Delta r)^2}+\\ \frac{T_{P,M-1,N}^{n}-2T_{P,M,N}^{n}+T_{P,M+1,N}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{P,M,N-1}^{n}-2T_{P,M,N}^{n}+T_{P,M,N+1}^{n}}{(\Delta z)^2}\end{array}\right] \quad (5.107)$$

$$+(q'''_{m_P})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{blP}^{n}\times(T_{art}-T_{P,M,N}^{n})\right]$$

In Equation (5.107), $T_{P,M+1,N}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T_{P,M,N+1}^{n}$ is the temperature value of head. Therefore, instead of node Neck ($T_{P,M,N+1}^{n}$), Head ($T_{P,M,0}^{n}$) should be substituted. Also $T_{P+1,M,N}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 19, the temperature of the node (P,M,N) at next time step (n+1) is given below.

$$T_{P,M,N}^{n+1}=\alpha_P^n\times\Delta t\times\left[\begin{array}{l}\frac{1}{(\Delta r)^2}\left(T_{P-1,M,N}^{n}-2T_{P,M,N}^{n}+\left(\frac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,M,N}^{n}\right)\\ +\frac{1}{2P(\Delta r)^2}\left[\left(\frac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,M,N}^{n}-T_{P-1,M,N}^{n}\right]\\ +\frac{T_{P,M-1,N}^{n}-2T_{P,M,N}^{n}+T_{P,M-1,N}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}\\ +\frac{T_{P,M,N-1}^{n}-2T_{P,M,N}^{n}+\text{Head }(T_{P,M,0}^{n})}{(\Delta z)^2}\end{array}\right] \quad (5.108)$$

$$+\left[(q'''_{m_P})^n\times\frac{\Delta t}{\rho_P^n Cp_P^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{blP}^{n}\times(T_{art}-T_{P,M,N}^{n})\times\frac{\Delta t}{\rho_P^n Cp_P^n}\right]+T_{P,M,N}^{n}$$

**Region 20**

Substituting the node (i,j,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n Cp_i^n \left[\frac{T_{i,j,N}^{n+1}-T_{i,j,N}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{i-1,j,N}^{n}-2T_{i,j,N}^{n}+T_{i+1,j,N}^{n}}{(\Delta r)^2}+\dfrac{T_{i+1,j,N}^{n}-T_{i-1,j,N}^{n}}{2i(\Delta r)^2}+\\ \dfrac{T_{i,j-1,N}^{n}-2T_{i,j,N}^{n}+T_{i,j+1,N}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{i,j,N-1}^{n}-2T_{i,j,N}^{n}+T_{i,j,N+1}^{n}}{(\Delta z)^2}\end{array}\right] \quad (5.109)$$

$$+(q'''_{m_i})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,i}^{\,n}\times(T_{art}-T_{i,j,N}^{n})\right]$$

In Equation (5.109) $T_{i,j,N+1}^{n}$ is the temperature value of head. Therefore, instead of node Neck($T_{i,j,N+1}^{n}$), Head ($T_{i,j,0}^{n}$) should be substituted.

At Region 20, the temperature of the node (i,j,N) at next time step (n+1) is given below.

$$T_{i,j,N}^{n+1}=\alpha_i^n\times\Delta t\times\left[\begin{array}{l}\dfrac{T_{i-1,j,N}^{n}-2T_{i,j,N}^{n}+T_{i+1,j,N}^{n}}{(\Delta r)^2}+\dfrac{T_{i+1,j,N}^{n}-T_{i-1,j,N}^{n}}{2i(\Delta r)^2}+\\ \dfrac{T_{i,j-1,N}^{n}-2T_{i,j,N}^{n}+T_{i,j+1,N}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{i,j,N-1}^{n}-2T_{i,j,N}^{n}+\text{Head }(T_{i,j,0}^{n})}{(\Delta z)^2}\end{array}\right] \quad (5.110)$$

$$+\left[(q'''_{m_i})^n\times\frac{\Delta t}{\rho_i^n\ Cp_i^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,i}^{\,n}\times(T_{art}-T_{i,j,N}^{n})\times\frac{\Delta t}{\rho_i^n\ Cp_i^n}\right]+T_{i,j,N}^{n}$$

**Region 21**

Substituting the node (P,j,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n \left[\frac{T_{P,j,N}^{n+1}-T_{P,j,N}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{P-1,j,N}^{n}-2T_{P,j,N}^{n}+T_{P+1,j,N}^{n}}{(\Delta r)^2}+\dfrac{T_{P+1,j,N}^{n}-T_{P-1,j,N}^{n}}{2P(\Delta r)^2}+\\ \dfrac{T_{P,j-1,N}^{n}-2T_{P,j,N}^{n}+T_{P,j+1,N}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{P,j,N-1}^{n}-2T_{P,j,N}^{n}+T_{P,j,N+1}^{n}}{(\Delta z)^2}\end{array}\right] \quad (5.111)$$

$$+(q'''_{m_P})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,P}^{n}\times(T_{art}-T_{P,j,N}^{n})\right]$$

In Equation (5.111), $T_{P+1,j,N}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. In addition, $T_{P,j,N+1}^{n}$ is the temperature value of head. Therefore, instead of node Neck($T_{P,j,N+1}^{n}$), Head ($T_{P,j,0}^{n}$) should be substituted.

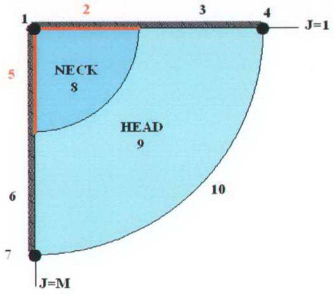
At Region 21, the temperature of the node (P,j,1) at next time step (n+1) is given below.

$$T_{P,j,N}^{n+1}=\alpha_P^n\times\Delta t\times\left[\begin{array}{l}\dfrac{1}{(\Delta r)^2}\left(T_{P-1,j,N}^{n}-2T_{P,j,N}^{n}+\left(\dfrac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,j,N}^{n}\right)\\ +\dfrac{1}{2P(\Delta r)^2}\left[\left(\dfrac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,j,N}^{n}-T_{P-1,j,N}^{n}\right]\\ +\dfrac{T_{P,j-1,N}^{n}-2T_{P,j,N}^{n}+T_{P,j+1,N}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}\\ +\dfrac{1}{(\Delta z)^2}\left[\text{Head }(T_{P,j,0}^{n})-2T_{P,j,N}^{n}+T_{P,j,N-1}^{n}\right]\end{array}\right] \quad (5.112)$$

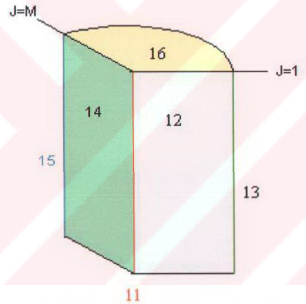
$$+\left[(q'''_{m_P})^n\times\frac{\Delta t}{\rho_P^n\,Cp_P^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,P}^{n}\times(T_{art}-T_{P,j,N}^{n})\times\frac{\Delta t}{\rho_P^n\,Cp_P^n}\right]+T_{P,j,N}^{n}$$

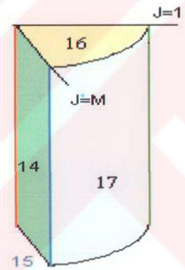
# HEAD

**At k=0**

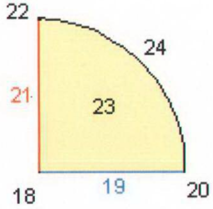



**At k=1 to (N-1)**




**At k=N**



**Region 1**

Substituting the node (0,0,0) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.21).

$$\rho_0^n Cp_0^n \left[\frac{T_{0,0,0}^{n+1} - T_{0,0,0}^n}{\Delta t}\right] = k_0^n \left[4\left(\frac{\overline{T}_1 - T_{0,0,0}^n}{(\Delta r)^2}\right) + \frac{T_{0,0,-1}^n - 2T_{0,0,0}^n + T_{0,0,1}^n}{(\Delta z)^2}\right] + (q'''_{m_0})^n \quad (5.113)$$

$$+\left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,0}^n \times (T_{art} - T_{0,0,0}^n)\right]$$

In Equation (5.113) $T_{0,0,-1}^n$ is the temperature value of neck. Therefore, instead of node Head ($T_{0,0,-1}^n$), Neck ($T_{0,0,\text{neckintervalz}-1}^n$) should be substituted.

At Region 1, the temperature of the node (0,0,0) at next time step (n+1) is given below.

$$T_{0,0,0}^{n+1} = \alpha_0^n \times \Delta t \times \left[4\left(\frac{\overline{T}_1 - T_{0,0,0}^n}{(\Delta r)^2}\right) + \frac{1}{(\Delta z)^2}\left(\text{Neck}\,(T_{0,0,\text{neckintervalz}-1}) - 2T_{0,0,0}^n + T_{0,0,1}^n\right)\right]$$

$$+\left[(q'''_{m_0})^n \times \frac{\Delta t}{\rho_0^n Cp_0^n}\right] + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,0}^n \times (T_{art} - T_{0,0,0}^n) \times \frac{\Delta t}{\rho_0^n Cp_0^n}\right] + T_{0,0,0}^n \quad (5.114)$$

**Region 2-3**

Substituting the node (i,1,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n Cp_i^n \left[\frac{T_{i,1,0}^{n+1} - T_{i,1,0}^n}{\Delta t}\right] = k_i^n \left[\begin{array}{l}\dfrac{T_{i-1,1,0}^n - 2T_{i,1,0}^n + T_{i+1,1,0}^n}{(\Delta r)^2} + \dfrac{T_{i+1,1,0}^n - T_{i-1,1,0}^n}{2i(\Delta r)^2} + \\ \dfrac{T_{i,0,0}^n - 2T_{i,1,0}^n + T_{i,2,0}^n}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T_{i,1,-1}^n - 2T_{i,1,0}^n + T_{i,1,1}^n}{(\Delta z)^2}\end{array}\right] \quad (5.115)$$

$$+ (q'''_{m_i})^n + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,i}^n \times (T_{art} - T_{i,1,0}^n)\right]$$

In Equation (5.115), $T^{n}_{i,0,0}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used.

For Region 2, in Equation (5.115) $T^{n}_{i,1,-1}$ is the temperature value of neck. Therefore, instead of node Head($T^{n}_{i,1,-1}$), Neck($T^{n}_{i,1,neckintervalz-1}$) should be substituted. At Region 2, the temperature of the node (i,1,0) at next time step (n+1) is given below.

$$T^{n+1}_{i,1,0} = \alpha^{n}_{i} \times \Delta t \times \left[ \begin{array}{l} \dfrac{T^{n}_{i-1,1,0} - 2T^{n}_{i,1,0} + T^{n}_{i+1,1,0}}{(\Delta r)^2} + \dfrac{T^{n}_{i+1,1,0} - T^{n}_{i-1,1,0}}{2i(\Delta r)^2} + \\ \dfrac{T^{n}_{i,2,0} - 2T^{n}_{i,1,0} + T^{n}_{i,2,0}}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{Neck(T_{i,1,neckintervalz-1}) - 2T^{n}_{i,1,0} + T^{n}_{i,1,1}}{(\Delta z)^2} \end{array} \right] \quad (5.116)$$
$$+ \left[ (q'''_{m_i})^{n} \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,1,0}) \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + T^{n}_{i,1,0}$$

For Region 3, in Equation (5.115) $T^{n}_{i,1,-1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used. At Region 3, the temperature of the node (i,1,0) at next time step (n+1) is given below.

$$T^{n+1}_{i,1,0} = \alpha^{n}_{i} \times \Delta t \times \left[ \begin{array}{l} \dfrac{T^{n}_{i-1,1,0} - 2T^{n}_{i,1,0} + T^{n}_{i+1,1,0}}{(\Delta r)^2} + \dfrac{T^{n}_{i+1,1,0} - T^{n}_{i-1,1,0}}{2i(\Delta r)^2} + \\ \dfrac{T^{n}_{i,2,0} - 2T^{n}_{i,1,0} + T^{n}_{i,2,0}}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{1}{(\Delta z)^2}\left( T^{n}_{i,1,1} - 2T^{n}_{i,1,0} + \left( \dfrac{2\Delta z \times EE}{k^{n}_{i}} \right) + T^{n}_{i,1,1} \right) \end{array} \right] \quad (5.117)$$
$$+ \left[ (q'''_{m_i})^{n} \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,1,0}) \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + T^{n}_{i,1,0}$$

**Region 4**

Substituting the node (P,1,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n \left[\frac{T_{P,1,0}^{n+1} - T_{P,1,0}^n}{\Delta t}\right] = k_i^n \left[\begin{array}{l} \frac{T_{P-1,1,0}^n - 2T_{P,1,0}^n + T_{P+1,1,0}^n}{(\Delta r)^2} + \frac{T_{P+1,1,0}^n - T_{P-1,1,0}^n}{2P(\Delta r)^2} + \\ \frac{T_{P,0,0}^n - 2T_{P,1,0}^n + T_{P,2,0}^n}{P^2(\Delta r)^2(\Delta\phi)^2} + \frac{T_{P,1,-1}^n - 2T_{P,1,0}^n + T_{P,1,1}^n}{(\Delta z)^2} \end{array}\right] \quad (5.118)$$

$$+(q'''_{mP})^n + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,P}^n \times (T_{art} - T_{P,1,0}^n)\right]$$

In Equation (5.118), $T_{P,0,0}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T_{P,1,-1}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used. Also, $T_{P+1,1,0}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 4, the temperature of the node (P,1,0) at next time step (n+1) is given below.

$$T_{P,1,0}^{n+1} = \alpha_P^n \times \Delta t \times \left[\begin{array}{l} \frac{1}{(\Delta r)^2}\left(T_{P-1,1,0}^n - 2T_{P,1,0}^n + \left(\frac{2\Delta r \times EE}{-k_P^n}\right) + T_{P-1,1,0}^n\right) \\ + \frac{1}{2P(\Delta r)^2}\left[\left(\frac{2\Delta r \times EE}{-k_P^n}\right) + T_{P-1,1,0}^n - T_{P-1,1,0}^n\right] \\ + \frac{T_{P,2,0}^n - 2T_{P,1,0}^n + T_{P,2,0}^n}{P^2(\Delta r)^2(\Delta\phi)^2} + \frac{1}{(\Delta z)^2}\left(T_{P,1,1}^n - 2T_{P,1,0}^n + \left(\frac{2\Delta z \times EE}{k_i^n}\right) + T_{P,1,1}^n\right) \end{array}\right] \quad (5.119)$$

$$+\left[(q'''_{mP})^n \times \frac{\Delta t}{\rho_P^n\, Cp_P^n}\right] + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,P}^n \times (T_{art} - T_{P,1,0}^n) \times \frac{\Delta t}{\rho_P^n\, Cp_P^n}\right] + T_{P,1,0}^n$$

**Region 5-6**

Substituting the node (i, M,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^{n}\,Cp_i^{n}\left[\frac{T_{i,M,0}^{n+1}-T_{i,M,0}^{n}}{\Delta t}\right]=k_i^{n}\left[\begin{array}{l}\dfrac{T_{i-1,M,0}^{n}-2T_{i,M,0}^{n}+T_{i+1,M,0}^{n}}{(\Delta r)^2}+\dfrac{T_{i+1,M,0}^{n}-T_{i-1,M,0}^{n}}{2i(\Delta r)^2}+\\ \dfrac{T_{i,M,0}^{n}-2T_{i,M,0}^{n}+T_{i,M,0}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{i,M,-1}^{n}-2T_{i,M,0}^{n}+T_{i,M,1}^{n}}{(\Delta z)^2}\end{array}\right] \quad (5.120)$$

$$+(q'''_{m_i})^{n}+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl_i}^{n}\times(T_{art}-T_{i,M,0}^{n})\right]$$

In Equation (5.120), $T_{i,M+1,0}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used.

For Region 5, in Equation (5.120) $T_{i,M,-1}^{n}$ is the temperature value of neck. Therefore, instead of node Head($T_{i,M,-1}^{n}$), Neck($T_{i,M,neckintervalz-1}^{n}$) should be substituted.

At Region 5, the temperature of the node (i, M,0) at next time step (n+1) is given below.

$$T_{i,M,0}^{n+1}=\alpha_i^{n}\times\Delta t\times\left[\begin{array}{l}\dfrac{T_{i-1,1,0}^{n}-2T_{i,1,0}^{n}+T_{i+1,1,0}^{n}}{(\Delta r)^2}+\dfrac{T_{i+1,1,0}^{n}-T_{i-1,1,0}^{n}}{2i(\Delta r)^2}+\\ \dfrac{T_{i,M-1,0}^{n}-2T_{i,M,0}^{n}+T_{i,M-1,0}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{1}{(\Delta z)^2}\left(\mathrm{Neck}(T_{i,M,neckinteralz-1})-2T_{i,M,0}^{n}+T_{i,M,1}^{n}\right)\end{array}\right] \quad (5.121)$$

$$+\left[(q'''_{m_i})^{n}\times\frac{\Delta t}{\rho_i^{n}\,Cp_i^{n}}\right]+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl_i}^{n}\times(T_{art}-T_{i,M,0}^{n})\times\frac{\Delta t}{\rho_i^{n}\,Cp_i^{n}}\right]+T_{i,M,0}^{n}$$

For Region 6, in Equation (5.120) $T_{i,M,-1}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used.

At Region 6, the temperature of the node (i,M,0) at next time step (n+1) is given below.

$$T_{i,M,0}^{n+1}=\alpha_i^n\times\Delta t\times\left[\begin{array}{l}\frac{T_{i-1,1,0}^n-2T_{i,1,0}^n+T_{i+1,1,0}^n}{(\Delta r)^2}+\frac{T_{i+1,1,0}^n-T_{i-1,1,0}^n}{2i(\Delta r)^2}+\\ \frac{T_{i,M-1,0}^n-2T_{i,M,0}^n+T_{i,M-1,0}^n}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{1}{(\Delta z)^2}\left(T_{i,M,1}^n-2T_{i,M,0}^n+\left(\frac{2\Delta z\times EE}{k_i^n}\right)+T_{i,M,1}^n\right)\end{array}\right] \tag{5.122}$$

$$+\left[(q_{m_i}''')^n\times\frac{\Delta t}{\rho_i^n\,Cp_i^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl_i}^n\times(T_{art}-T_{i,M,0}^n)\times\frac{\Delta t}{\rho_i^n\,Cp_i^n}\right]+T_{i,M,0}^n$$

**Region 7**

Substituting the node (P,M,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n\,Cp_P^n\left[\frac{T_{P,M,0}^{n+1}-T_{P,M,0}^n}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\frac{T_{P-1,M,0}^n-2T_{P,M,0}^n+T_{P+1,M,0}^n}{(\Delta r)^2}+\frac{T_{P+1,M,0}^n-T_{P-1,M,0}^n}{2P(\Delta r)^2}+\\ \frac{T_{P,M-1,0}^n-2T_{P,M,0}^n+T_{P,M+1,0}^n}{P^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{P,M,-1}^n-2T_{P,M,0}^n+T_{P,M,1}^n}{(\Delta z)^2}\end{array}\right] \tag{5.123}$$

$$+(q_{m_P}''')^n+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl_P}^n\times(T_{art}-T_{P,M,0}^n)\right]$$

In Equation (5.123), $T_{P,M+1,0}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T_{P,M,-1}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used. Also $T_{P+1,M,0}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 7, the temperature of the node (P,M,0) at next time step (n+1) is given below.

$$T^{n+1}_{P,M,0} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{array}{l} \dfrac{1}{(\Delta r)^2}\left( T^{n}_{P-1,M,0} - 2T^{n}_{P,M,0} + \left(\dfrac{2\Delta r \times EE}{-k^{n}_{P}}\right) + T^{n}_{P-1,M,0} \right) \\ + \dfrac{1}{2P(\Delta r)^2}\left[ \left(\dfrac{2\Delta r \times EE}{-k^{n}_{P}}\right) + T^{n}_{P-1,M,0} - T^{n}_{P-1,M,0} \right] \\ + \dfrac{T^{n}_{P,M-1,0} - 2T^{n}_{P,M,0} + T^{n}_{P,M-1,0}}{P^2(\Delta r)^2(\Delta\phi)^2} + \\ \dfrac{1}{(\Delta z)^2}\left( T^{n}_{P,M,1} - 2T^{n}_{P,M,0} + \left(\dfrac{2\Delta z \times EE}{k^{n}_{i}}\right) + T^{n}_{P,M,1} \right) \end{array} \right] \tag{5.124}$$

$$+ \left[ (q'''_{m_P})^{n} \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,M,0}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,M,0}$$

**Region 8-9**

Substituting the node (i,j,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{i}\, Cp^{n}_{i} \left[ \frac{T^{n+1}_{i,j,0} - T^{n}_{i,j,0}}{\Delta t} \right] = k^{n}_{i} \left[ \begin{array}{l} \dfrac{T^{n}_{i-1,j,0} - 2T^{n}_{i,j,0} + T^{n}_{i+1,j,0}}{(\Delta r)^2} + \dfrac{T^{n}_{i+1,j,0} - T^{n}_{i-1,j,0}}{2i(\Delta r)^2} + \\ \dfrac{T^{n}_{i,j-1,0} - 2T^{n}_{i,j,0} + T^{n}_{i,j+1,0}}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T^{n}_{i,j,-1} - 2T^{n}_{i,j,0} + T^{n}_{i,j,1}}{(\Delta z)^2} \end{array} \right]$$

$$+ (q'''_{m_i})^{n} + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,j,0}) \right] \tag{5.125}$$

For Region 8, in Equation (5.125) $T^{n}_{i,j,-1}$ is the temperature value of neck.

Therefore, instead of Head($T^{n}_{i,j,-1}$), Neck($T^{n}_{i,j,neckintervalz-1}$) should be substituted.

At Region 8, the temperature of the node (i,j,0) at next time step (n+1) is given below.

$$T^{n+1}_{i,j,0} = \alpha^{n}_{i} \times \Delta t \times \left[ \begin{array}{l} \dfrac{T^{n}_{i-1,j,0} - 2T^{n}_{i,j,0} + T^{n}_{i+1,j,0}}{(\Delta r)^2} + \dfrac{T^{n}_{i+1,j,0} - T^{n}_{i-1,j,0}}{2i(\Delta r)^2} + \\ \dfrac{T^{n}_{i,j-1,0} - 2T^{n}_{i,j,0} + T^{n}_{i,j+1,0}}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{Neck(T_{i,j,neckintervalz-1}) - 2T^{n}_{i,j,0} + T^{n}_{i,j,1}}{(\Delta z)^2} \end{array} \right]$$

$$+ \left[ (q'''_{m_i})^{n} \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,j,0}) \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + T^{n}_{i,j,0} \tag{5.126}$$

For Region 9, $T^{n}_{i,j,-1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used. At Region 9, the temperature of the node (i,j,0) at next time step (n+1) is given below.

$$T^{n+1}_{i,j,0} = \alpha^{n}_{i} \times \Delta t \times \left[ \frac{T^{n}_{i-1,j,0} - 2T^{n}_{i,j,0} + T^{n}_{i+1,j,0}}{(\Delta r)^2} + \frac{T^{n}_{i+1,j,0} - T^{n}_{i-1,j,0}}{2i(\Delta r)^2} + \frac{T^{n}_{i,j-1,0} - 2T^{n}_{i,j,0} + T^{n}_{i,j+1,0}}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{1}{(\Delta z)^2}\left( T^{n}_{i,j,1} - 2T^{n}_{i,j,0} + \left( \frac{2\Delta z \times EE}{k^{n}_{i}} \right) + T^{n}_{i,j,1} \right) \right] \quad (5.127)$$

$$+ \left[ (q'''_{m_i})^{n} \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl_i} \times (T_{art} - T^{n}_{i,j,0}) \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + T^{n}_{i,j,0}$$

**Region 10**

Substituting the node (P,j,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P}\, Cp^{n}_{P} \left[ \frac{T^{n+1}_{P,j,0} - T^{n}_{P,j,0}}{\Delta t} \right] = k^{n}_{i} \left[ \frac{T^{n}_{P-1,j,0} - 2T^{n}_{P,j,0} + T^{n}_{P+1,j,0}}{(\Delta r)^2} + \frac{T^{n}_{P+1,j,0} - T^{n}_{P-1,j,0}}{2P(\Delta r)^2} + \frac{T^{n}_{P,j,0} - 2T^{n}_{P,j,0} + T^{n}_{P,j,0}}{P^2(\Delta r)^2(\Delta\phi)^2} + \frac{T^{n}_{P,j,-1} - 2T^{n}_{P,j,0} + T^{n}_{P,j,1}}{(\Delta z)^2} \right] \quad (5.128)$$

$$+ (q'''_{m_P})^{n} + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,j,0}) \right]$$

In Equation (5.128), $T^{n}_{P,j,-1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used. Also $T^{n}_{P+1,j,0}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 10, the temperature of the node (P,j,0) at next time step (n+1) is given below.

$$T_{P,j,0}^{n+1}=\alpha_P^{n}\times\Delta t\times\left[\begin{array}{l}\frac{1}{(\Delta r)^2}\left(T_{P-1,j,0}^{n}-2T_{P,j,0}^{n}+\left(\frac{2\Delta r\times EE}{-k_P^{n}}\right)+T_{P-1,j,0}^{n}\right)\\+\frac{1}{2P(\Delta r)^2}\left[\left(\frac{2\Delta r\times EE}{-k_P^{n}}\right)+T_{P-1,j,0}^{n}-T_{P-1,j,0}^{n}\right]\\+\frac{T_{P,j-1,0}^{n}-2T_{P,j,0}^{n}+T_{P,j+1,0}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\frac{1}{(\Delta z)^2}\left(T_{P,j,1}^{n}-2T_{P,j,0}^{n}+\left(\frac{2\Delta z\times EE}{k_i^{n}}\right)+T_{P,j,1}^{n}\right)\end{array}\right] \tag{5.129}$$

$$+\left[(q_{m_P}''')^{n}\times\frac{\Delta t}{\rho_P^{n}\,Cp_P^{n}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,P}^{n}\times(T_{art}-T_{P,j,0}^{n})\times\frac{\Delta t}{\rho_P^{n}\,Cp_P^{n}}\right]+T_{P,j,0}^{n}$$

**Region 11**

Substituting the node (0,0,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.21).

$$\rho_0^{n}\,Cp_0^{n}\left[\frac{T_{0,0,k}^{n+1}-T_{0,0,k}^{n}}{\Delta t}\right]=k_0^{n}\left[4\left(\frac{\overline{T}_1-T_{0,0,k}^{n}}{(\Delta r)^2}\right)+\frac{T_{0,0,k-1}^{n}-2T_{0,0,k}^{n}+T_{0,0,k+1}^{n}}{(\Delta z)^2}\right]+(q_{m_0}''')^{n} \tag{5.130}$$

$$+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,0}^{n}\times(T_{art}-T_{0,0,k}^{n})\right]$$

In Equation (5.130) there is no fictitious temperature. The temperature of the node (0,0,k) at next time step (n+1) is given below.

$$T_{0,0,k}^{n+1}=\alpha_0^{n}\times\Delta t\times\left[4\left(\frac{\overline{T}_1-T_{0,0,0}^{n}}{(\Delta r)^2}\right)+\left[\frac{T_{0,0,k-1}^{n}-2T_{0,0,k}^{n}+T_{0,0,k+1}^{n}}{(\Delta z)^2}\right]\right] \tag{5.131}$$

$$+\left[(q_{m_0}''')^{n}\times\frac{\Delta t}{\rho_0^{n}\,Cp_0^{n}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,0}^{n}\times(T_{art}-T_{0,0,k}^{n})\times\frac{\Delta t}{\rho_0^{n}\,Cp_0^{n}}\right]+T_{0,0,k}^{n}$$

**Region 12**

Substituting the node (i,1,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.16).

$$\rho_i^n \, Cp_i^n \left[ \frac{T_{i,1,k}^{n+1} - T_{i,1,k}^{n}}{\Delta t} \right] = k_i^n \left[ \begin{array}{l} \dfrac{T_{i-1,1,k}^{n} - 2T_{i,1,k}^{n} + T_{i+1,1,k}^{n}}{(\Delta r)^2} + \dfrac{T_{i+1,1,k}^{n} - T_{i-1,1,k}^{n}}{2i(\Delta r)^2} + \\ \dfrac{T_{i,0,k}^{n} - 2T_{i,1,k}^{n} + T_{i,2,k}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T_{i,1,k-1}^{n} - 2T_{i,1,k}^{n} + T_{i,1,k+1}^{n}}{(\Delta z)^2} \end{array} \right] + (q'''_{m_i})^n$$

$$+ \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}_{bl_i}^{n} \times (T_{art} - T_{i,1,k}^{n}) \right] \qquad (5.132)$$

In Equation (5.132), $T_{i,0,k}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. At Region 12, the temperature of the node (i,1,k) at next time step (n+1) is given below.

$$T_{i,1,k}^{n+1} = \alpha_i^n \times \Delta t \times \left[ \begin{array}{l} \dfrac{T_{i-1,1,k}^{n} - 2T_{i,1,k}^{n} + T_{i+1,1,k}^{n}}{(\Delta r)^2} + \dfrac{T_{i+1,1,k}^{n} - T_{i-1,1,k}^{n}}{2i(\Delta r)^2} + \\ \dfrac{T_{i,2,k}^{n} - 2T_{i,1,k}^{n} + T_{i,2,k}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T_{i,1,k-1}^{n} - 2T_{i,1,k}^{n} + T_{i,1,k+1}^{n}}{(\Delta z)^2} \end{array} \right] +$$

$$+ \left[ (q'''_{m_i})^n \times \frac{\Delta t}{\rho_i^n \, Cp_i^n} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}_{bl_i}^{n} \times (T_{art} - T_{i,1,k}^{n}) \times \frac{\Delta t}{\rho_i^n \, Cp_i^n} \right] + T_{i,1,k}^{n} \qquad (5.133)$$

**Region 13**

Substituting the node (P,1,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.16).

$$\rho_P^n \, Cp_P^n \left[ \frac{T_{P,1,k}^{n+1} - T_{P,1,k}^{n}}{\Delta t} \right] = k_i^n \left[ \begin{array}{l} \dfrac{T_{P-1,1,k}^{n} - 2T_{P,1,k}^{n} + T_{P+1,1,k}^{n}}{(\Delta r)^2} + \dfrac{T_{P+1,1,k}^{n} - T_{P-1,1,k}^{n}}{2P(\Delta r)^2} + \\ \dfrac{T_{P,0,k}^{n} - 2T_{P,1,k}^{n} + T_{P,2,k}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T_{P,1,k-1}^{n} - 2T_{P,1,k}^{n} + T_{P,1,k+1}^{n}}{(\Delta z)^2} \end{array} \right]$$

$$+ (q'''_{m_P})^n + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}_{blP}^{n} \times (T_{art} - T_{P,1,k}^{n}) \right] \qquad (5.134)$$

In Equation (5.134), $T^{n}_{P,0,k}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T^{n}_{P+1,1,k}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. In conclusion, at Region 13, the temperature of the node (P,1,k) at next time step (n+1) is given below.

$$T^{n+1}_{P,1,k} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{array}{l} \frac{1}{(\Delta r)^2}\left( T^{n}_{P-1,1,k} - 2T^{n}_{P,1,k} + \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,1,k} \right) \\ + \frac{1}{2P(\Delta r)^2}\left[ \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,1,k} - T^{n}_{P-1,1,k} \right] \\ + \frac{T^{n}_{P,2,k} - 2T^{n}_{P,1,k} + T^{n}_{P,2,k}}{P^2(\Delta r)^2(\Delta\phi)^2} + \left[ \frac{T^{n}_{P,1,k-1} - 2T^{n}_{P,1,k} + T^{n}_{P,1,k+1}}{(\Delta z)^2} \right] \end{array} \right] \tag{5.135}$$

$$+ \left[ (q'''_{m_P})^{n} \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,1,k}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,1,k}$$

**Region 14**

Substituting the node (i,M,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.16).

$$\rho^{n}_{i}\, Cp^{n}_{i} \left[ \frac{T^{n+1}_{i,M,k} - T^{n}_{i,M,k}}{\Delta t} \right] = k^{n}_{i} \left[ \begin{array}{l} \frac{T^{n}_{i-1,M,k} - 2T^{n}_{i,M,k} + T^{n}_{i+1,M,k}}{(\Delta r)^2} + \frac{T^{n}_{i+1M,k} - T^{n}_{i-1,M,k}}{2i(\Delta r)^2} + \\ \frac{T^{n}_{i,M-1,k} - 2T^{n}_{i,M,k} + T^{n}_{i,M+1,k}}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{T^{n}_{i,M,k-1} - 2T^{n}_{i,M,k} + T^{n}_{i,M,k+1}}{(\Delta z)^2} \end{array} \right] \tag{5.136}$$

$$+ (q'''_{m_i})^{n} + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl_i} \times (T_{art} - T^{n}_{i,M,k}) \right]$$

In Equation (5.136), $T^{n}_{i,M+1,k}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. The temperature of the node (i,M,k) at next time step (n+1) is given below.

$$T^{n+1}_{i,M,k} = \alpha^n_i \times \Delta t \times \left[ \begin{array}{l} \dfrac{T^n_{i-1,M,k} - 2T^n_{i,M,k} + T^n_{i+1,M,k}}{(\Delta r)^2} + \dfrac{T^n_{i+1,M,k} - T^n_{i-1,M,k}}{2i(\Delta r)^2} + \\ \dfrac{T^n_{i,M-1,k} - 2T^n_{i,M,k} + T^n_{i,M-1,k}}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T^n_{i,M,k-1} - 2T^n_{i,M,k} + T^n_{i,M,k+1}}{(\Delta z)^2} \end{array} \right]$$

$$+ \left[ (q'''_{m_i})^n \times \frac{\Delta t}{\rho^n_i \; Cp^n_i} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^n_{bl\,i} \times (T_{art} - T^n_{i,M,k}) \times \frac{\Delta t}{\rho^n_i \; Cp^n_i} \right] + T^n_{i,M,k} \quad (5.137)$$

**Region 15**

Substituting the node (P,M,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.16).

$$\rho^n_P \, Cp^n_P \left[ \frac{T^{n+1}_{P,M,k} - T^n_{P,M,k}}{\Delta t} \right] = k^n_i \left[ \begin{array}{l} \dfrac{T^n_{P-1,M,k} - 2T^n_{P,M,k} + T^n_{P+1,M,k}}{(\Delta r)^2} + \dfrac{T^n_{P+1,M,k} - T^n_{P-1,M,k}}{2P(\Delta r)^2} + \\ \dfrac{T^n_{P,M-1,k} - 2T^n_{P,M,k} + T^n_{P,M+1,k}}{P^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T^n_{P,M,k-1} - 2T^n_{P,M,k} + T^n_{P,M,k+1}}{(\Delta z)^2} \end{array} \right] \quad (5.138)$$

$$+ (q'''_{m_P})^n + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^n_{bl\,P} \times (T_{art} - T^n_{P,M,k}) \right]$$

In Equation (5.138), $T^n_{P,M+1,k}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. Also $T^n_{P+1,M,k}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 15, the temperature of the node (P,M,k) at next time step (n+1) is given below.

$$T^{n+1}_{P,M,k} = \alpha^n_P \times \Delta t \times \left[ \begin{array}{l} \dfrac{1}{(\Delta r)^2} \left( T^n_{P-1,M,k} - 2T^n_{P,M,k} + \left( \dfrac{2\Delta r \times EE}{-k^n_P} \right) + T^n_{P-1,M,k} \right) \\ + \dfrac{1}{2P(\Delta r)^2} \left[ \left( \dfrac{2\Delta r \times EE}{-k^n_P} \right) + T^n_{P-1,M,k} - T^n_{P-1,M,k} \right] \\ + \left[ \dfrac{T^n_{P,M-1,k} - 2T^n_{P,M,k} + T^n_{P,M-1,k}}{P^2(\Delta r)^2(\Delta\phi)^2} \right] \\ + \left[ \dfrac{T^n_{P,M,k-1} - 2T^n_{P,M,k} + T^n_{P,M,k+1}}{(\Delta z)^2} \right] \end{array} \right] \quad (5.139)$$

$$+ \left[ (q'''_{m_P})^n \times \frac{\Delta t}{\rho^n_P \; Cp^n_P} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^n_{bl\,P} \times (T_{art} - T^n_{P,M,k}) \times \frac{\Delta t}{\rho^n_P \; Cp^n_P} \right] + T^n_{P,M,k}$$

**Region 16**

Substituting the node (i,j,k) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.16).

$$\rho_P^n Cp_P^n\left[\frac{T_{i,j,k}^{n+1}-T_{i,j,k}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{i-1,j,k}^{n}-2T_{i,j,k}^{n}+T_{i+1,j,k}^{n}}{(\Delta r)^2}+\dfrac{T_{i+1,j,k}^{n}-T_{i-1,j,k}^{n}}{2P(\Delta r)^2}+\\ \dfrac{T_{i,j-1,k}^{n}-2T_{i,j,k}^{n}+T_{i,j+1,k}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{i,j,k-1}^{n}-2T_{i,j,k}^{n}+T_{i,j,k+1}^{n}}{(\Delta z)^2}\end{array}\right] \quad (5.140)$$

$$+(q'''_{m_P})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{blP}^{n}\times(T_{art}-T_{i,j,k}^{n})\right]$$

In Equation (5.140) there is no fictitious temperature. The temperature of the node (i,j,k) at next time step (n+1) is given below.

$$T_{i,j,k}^{n+1}=\alpha_i^n\times\Delta t\times\left[\begin{array}{l}\dfrac{T_{i-1,j,k}^{n}-2T_{i,j,k}^{n}+T_{i+1,j,k}^{n}}{(\Delta r)^2}+\dfrac{1}{(i\Delta r)}\dfrac{T_{i+1,j,k}^{n}-T_{i-1,j,k}^{n}}{2\Delta r}+\\ \dfrac{1}{(i\Delta r)^2}\dfrac{T_{i,j-1,k}^{n}-2T_{i,j,k}^{n}+T_{i,j+1,k}^{n}}{(\Delta\phi)^2}+\dfrac{T_{i,j,k-1}^{n}-2T_{i,j,k}^{n}+T_{i,j,k+1}^{n}}{(\Delta z)^2}\end{array}\right] \quad (5.141)$$

$$+\left[(q'''_{m_i})^n\times\frac{\Delta t}{\rho_i^n\, Cp_i^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl_i}^{n}\times(T_{art}-T_{i,j,k}^{n})\times\frac{\Delta t}{\rho_i^n\, Cp_i^n}\right]+T_{i,j,k}^{n}$$

**Region 17**

Substituting the node (P,j,k) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n\left[\frac{T_{P,j,k}^{n+1}-T_{P,j,k}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{P-1,j,k}^{n}-2T_{P,j,k}^{n}+T_{P+1,j,k}^{n}}{(\Delta r)^2}+\dfrac{T_{P+1,j,k}^{n}-T_{P-1,j,k}^{n}}{2P(\Delta r)^2}+\\ \dfrac{T_{P,j-1,k}^{n}-2T_{P,j,k}^{n}+T_{P,j+1,k}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{P,j,k-1}^{n}-2T_{P,j,k}^{n}+T_{P,j,k+1}^{n}}{(\Delta z)^2}\end{array}\right] \quad (5.142)$$

$$+(q'''_{m_P})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{blP}^{n}\times(T_{art}-T_{P,j,k}^{n})\right]$$

In Equation (5.142) $\overset{n}{T}_{P+1,j,k}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. For these region, the temperature of the node (P,j,k) at next time step (n+1) is given below.

$$\overset{n+1}{T}_{P,j,k}=\overset{n}{\alpha}_{P}\times\Delta t\times\left[\begin{array}{l}\dfrac{1}{(\Delta r)^2}\left(\overset{n}{T}_{P-1,j,k}-2\overset{n}{T}_{P,j,k}+\left(\dfrac{2\Delta r\times EE}{-\overset{n}{k}_{P}}\right)+\overset{n}{T}_{P-1,j,k}\right)\\ +\dfrac{1}{2P(\Delta r)^2}\left[\left(\dfrac{2\Delta r\times EE}{-\overset{n}{k}_{P}}\right)+\overset{n}{T}_{P-1,j,k}-\overset{n}{T}_{P-1,j,k}\right]\\ +\dfrac{\overset{n}{T}_{P,j-1,k}-2\overset{n}{T}_{P,j,k}+\overset{n}{T}_{P,j+1,k}}{P^2(\Delta r)^2(\Delta\phi)^2}+\left[\dfrac{\overset{n}{T}_{P,j,k-1}-2\overset{n}{T}_{P,j,k}+\overset{n}{T}_{P,j,k+1}}{(\Delta z)^2}\right]\end{array}\right]$$
$$+\left[(q'''_{m_P})^n\times\frac{\Delta t}{\overset{n}{\rho}_{P}\overset{n}{Cp}_{P}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\overset{n}{\dot{W}}_{bl\,P}\times(T_{art}-\overset{n}{T}_{P,j,k})\times\frac{\Delta t}{\overset{n}{\rho}_{P}\overset{n}{Cp}_{P}}\right]+\overset{n}{T}_{P,j,k} \quad (5.143)$$

**Region 18**

Substituting the node (0,0,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.21).

$$\overset{n}{\rho}_0\overset{n}{Cp}_0\left[\frac{\overset{n+1}{T}_{0,0,N}-\overset{n}{T}_{0,0,N}}{\Delta t}\right]=\overset{n}{k}_0\left[4\left(\frac{\overline{T}_1-\overset{n}{T}_{0,0,N}}{(\Delta r)^2}\right)+\frac{\overset{n}{T}_{0,0,N-1}-2\overset{n}{T}_{0,0,N}+\overset{n}{T}_{0,0,N+1}}{(\Delta z)^2}\right]+(q'''_{m_0})^n$$
$$+\left[\rho_{bl}\times Cp_{bl}\times\overset{n}{\dot{W}}_{bl\,0}\times(T_{art}-\overset{n}{T}_{0,0,N})\right] \quad (5.144)$$

In Equation (5.144) $\overset{n}{T}_{0,0,N+1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 5 given in Table 5.2 is used.

At Region 18, the temperature of the node (0,0,N) at next time step (n+1) is given below.

$$\overset{n+1}{T}_{0,0,N}=\overset{n}{\alpha}_0\times\Delta t\times\left[4\left(\frac{\overline{T}_1-\overset{n}{T}_{0,0,N}}{(\Delta r)^2}\right)+\frac{1}{(\Delta z)^2}\left(\overset{n}{T}_{0,0,N-1}+\left(\frac{2\Delta z\times EE}{-\overset{n}{k}_0}\right)-2\overset{n}{T}_{0,0,N}+\overset{n}{T}_{0,0,N-1}\right)\right] \quad (5.145)$$
$$+\left[(q'''_{m_0})^n\times\frac{\Delta t}{\overset{n}{\rho}_0\overset{n}{Cp}_0}\right]+\left[\rho_{bl}\times Cp_{bl}\times\overset{n}{\dot{W}}_{bl\,0}\times(T_{art}-\overset{n}{T}_{0,0,N})\times\frac{\Delta t}{\overset{n}{\rho}_0\overset{n}{Cp}_0}\right]+\overset{n}{T}_{0,0,N}$$

**Region 19**

Substituting the node (i,1,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n Cp_i^n\left[\frac{T_{i,1,N}^{n+1}-T_{i,1,N}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\frac{T_{i-1,1,N}^{n}-2T_{i,1,N}^{n}+T_{i+1,1,N}^{n}}{(\Delta r)^2}+\frac{T_{i+1,1,N}^{n}-T_{i-1,1,N}^{n}}{2i(\Delta r)^2}+\\ \frac{T_{i,0,N}^{n}-2T_{i,1,N}^{n}+T_{i,2,N}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{i,1,N-1}^{n}-2T_{i,1,N}^{n}+T_{i,1,N+1}^{n}}{(\Delta z)^2}\end{array}\right]$$
$$+(q_{m_i}''')^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,i}^{n}\times(T_{art}-T_{i,1,N}^{n})\right] \tag{5.146}$$

In Equation (5.146), $T_{i,0,N}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T_{i,1,N+1}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 5 given in Table 5.2 is used.

$$T_{i,1,N}^{n+1}=\alpha_i^n\times\Delta t\times\left[\begin{array}{l}\frac{T_{i-1,1,N}^{n}-2T_{i,1,N}^{n}+T_{i+1,1,N}^{n}}{(\Delta r)^2}+\frac{T_{i+1,1,N}^{n}-T_{i-1,1,N}^{n}}{2i(\Delta r)^2}+\\ \frac{T_{i,2,N}^{n}-2T_{i,1,N}^{n}+T_{i,2,N}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{1}{(\Delta z)^2}\left[T_{i,1,N-1}^{n}-2T_{i,1,N}^{n}+\left(\frac{2\Delta z\times EE}{-k_i^n}\right)+T_{i,1,N-1}^{n}\right]\end{array}\right] \tag{5.147}$$
$$+\left[(q_{m_i}''')^n\times\frac{\Delta t}{\rho_i^n\,Cp_i^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,i}^{n}\times(T_{art}-T_{i,1,N}^{n})\times\frac{\Delta t}{\rho_i^n\,Cp_i^n}\right]+T_{i,1,N}^{n}$$

**Region 20**

Substituting the node (P,1,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n\left[\frac{T_{P,1,N}^{n+1}-T_{P,1,N}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\frac{T_{P-1,1,N}^{n}-2T_{P,1,N}^{n}+T_{P+1,1,N}^{n}}{(\Delta r)^2}+\frac{T_{P+1,1,N}^{n}-T_{P-1,1,N}^{n}}{2P(\Delta r)^2}+\\ \frac{T_{P,0,N}^{n}-2T_{P,1,N}^{n}+T_{P,2,N}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{P,1,N-1}^{n}-2T_{P,1,N}^{n}+T_{P,1,N+1}^{n}}{(\Delta z)^2}\end{array}\right] \tag{5.148}$$
$$+(q_{m_P}''')^n+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,P}^{n}\times(T_{art}-T_{P,j,N}^{n})\right]$$

In Equation (5.148), $T^{n}_{P,0,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. Also $T^{n}_{P,1,N+1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 5 given in Table 5.2 is used. In addition, $T^{n}_{P+1,1,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 20, the temperature of the node (P,1,N) at next time step (n+1) is given below.

$$T^{n+1}_{P,1,N}=\alpha^{n}_{P}\times\Delta t\times\left[\begin{array}{l}\dfrac{1}{(\Delta r)^2}\left(T^{n}_{P-1,1,N}-2T^{n}_{P,1,N}+\left(\dfrac{2\Delta r\times EE}{-k^{n}_{P}}\right)+T^{n}_{P-1,1,N}\right)\\ +\dfrac{1}{2P(\Delta r)^2}\left[T^{n}_{P-1,1,N}+\left(\dfrac{2\Delta r\times EE}{-k^{n}_{P}}\right)-T^{n}_{P-1,1,N}\right]\\ +\dfrac{T^{n}_{P,2,N}-2T^{n}_{P,1,N}+T^{n}_{P,2,N}}{P^2(\Delta r)^2(\Delta\phi)^2}+\left[T^{n}_{P,1,N-1}-2T^{n}_{P,1,N}+\left(\dfrac{2\Delta z\times EE}{-k^{n}_{P}}\right)+T^{n}_{P,1,N-1}\right]\end{array}\right]$$
$$+\left[(q'''_{m_P})^{n}\times\frac{\Delta t}{\rho^{n}_{P}\,Cp^{n}_{P}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,P}\times(T_{art}-T^{n}_{P,1,N})\times\frac{\Delta t}{\rho^{n}_{P}\,Cp^{n}_{P}}\right]+T^{n}_{P,1,N} \qquad (5.149)$$

**Region 21**

Substituting the node (i,M,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{i}\,Cp^{n}_{i}\left[\frac{T^{n+1}_{i,M,N}-T^{n}_{i,1,N}}{\Delta t}\right]=k^{n}_{i}\left[\begin{array}{l}\dfrac{T^{n}_{i-1,M,N}-2T^{n}_{i,M,N}+T^{n}_{i+1,M,N}}{(\Delta r)^2}+\dfrac{T^{n}_{i+1,M,N}-T^{n}_{i-1,M,N}}{2i(\Delta r)^2}+\\ \dfrac{T^{n}_{i,M-1,N}-2T^{n}_{i,M,N}+T^{n}_{i,M+1,N}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T^{n}_{i,M,N-1}-2T^{n}_{i,M,N}+T^{n}_{i,M,N+1}}{(\Delta z)^2}\end{array}\right] \qquad (5.150)$$
$$+(q'''_{m_i})^{n}+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,i}\times(T_{art}-T^{n}_{i,1,M})\right]$$

In Equation (5.150), $T^{n}_{i,M+1,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. $T^{n}_{i,M,N+1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 5 given in Table 5.2 is used.

At Region 21, the temperature of the node (i,M,N) at next time step (n+1) is given below.

$$T^{n+1}_{i,M,N}=\alpha^{n}_{i}\times\Delta t\times\left[\begin{array}{l}\dfrac{T^{n}_{i-1,M,N}-2T^{n}_{i,M,N}+T^{n}_{i+1,M,N}}{(\Delta r)^2}+\dfrac{T^{n}_{i+1,M,N}-T^{n}_{i-1,M,N}}{2i(\Delta r)^2}+\\ \dfrac{T^{n}_{i,M-1,N}-2T^{n}_{i,M,N}+T^{n}_{i,M-1,N}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{1}{(\Delta z)^2}\left[T^{n}_{i,M,N-1}-2T^{n}_{i,M,N}+\left(\dfrac{2\Delta z\times EE}{-k^{n}_{p}}\right)+T^{n}_{i,M,N-1}\right]\end{array}\right] \quad (5.151)$$

$$+\left[(q'''_{m_i})^{n}\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl_i}\times(T_{art}-T^{n}_{i,M,N})\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+T^{n}_{i,M,N}$$

**Region 22**

Substituting the node (P,M,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P}Cp^{n}_{P}\left[\frac{T^{n+1}_{P,M,N}-T^{n}_{P,M,N}}{\Delta t}\right]=k^{n}_{i}\left[\begin{array}{l}\dfrac{T^{n}_{P-1,M,N}-2T^{n}_{P,M,N}+T^{n}_{P+1,M,N}}{(\Delta r)^2}+\dfrac{T^{n}_{P+1,M,N}-T^{n}_{P-1,M,N}}{2P(\Delta r)^2}+\\ \dfrac{T^{n}_{P,M-1,N}-2T^{n}_{P,M,N}+T^{n}_{P,M+1,N}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T^{n}_{P,M,N-1}-2T^{n}_{P,M,N}+T^{n}_{P,M,N+1}}{(\Delta z)^2}\end{array}\right] \quad (5.152)$$

$$+(q'''_{m_P})^{n}+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{blP}\times(T_{art}-T^{n}_{P,M,N})\right]$$

In Equation (5.152), $T^{n}_{i,M+1,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T^{n}_{P,M,N+1}$ is the fictitious temperature.

In order to eliminate this fictitious node Boundary Condition 5 given in Table 5.2 is used. Also $T^{n}_{P+1,M,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 22, the temperature of the node (P,M,N) at next time step (n+1) is given below.

$$T^{n+1}_{P,M,N}=\alpha^{n}_{P}\times\Delta t\times\left[\begin{array}{l}\frac{1}{(\Delta r)^2}\left(T^{n}_{P-1,M,N}-2T^{n}_{P,M,N}+\left(\frac{2\Delta r\times EE}{-k^{n}_{P}}\right)+T^{n}_{P-1,M,N}\right)\\ +\frac{1}{2P(\Delta r)^2}\left[\left(\frac{2\Delta r\times EE}{-k^{n}_{P}}\right)+T^{n}_{P-1,M,N}-T^{n}_{P-1,M,N}\right]\\ +\frac{T^{n}_{P,M-1,N}-2T^{n}_{P,M,N}+T^{n}_{P,M-1,N}}{i^2(\Delta r)^2(\Delta\phi)^2}\\ +\frac{1}{(\Delta z)^2}\left[T^{n}_{P,M,N-1}-2T^{n}_{P,M,N}+\left(\frac{2\Delta z\times EE}{-k^{n}_{P}}\right)+T^{n}_{P,M,N-1}\right]\end{array}\right] \tag{5.153}$$

$$+\left[(q'''_{m_P})^{n}\times\frac{\Delta t}{\rho^{n}_{P}\,Cp^{n}_{P}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,P}\times(T_{art}-T^{n}_{P,M,N})\times\frac{\Delta t}{\rho^{n}_{P}\,Cp^{n}_{P}}\right]+T^{n}_{P,M,N}$$

**Region 23**

Substituting the node (i,j,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{i}\,Cp^{n}_{i}\left[\frac{T^{n+1}_{i,j,N}-T^{n}_{i,j,N}}{\Delta t}\right]=k^{n}_{i}\left[\begin{array}{l}\frac{T^{n}_{i-1,j,N}-2T^{n}_{i,j,N}+T^{n}_{i+1,j,N}}{(\Delta r)^2}+\frac{T^{n}_{i+1,j,N}-T^{n}_{i-1,j,N}}{2i(\Delta r)^2}+\\ \frac{T^{n}_{i,j-1,N}-2T^{n}_{i,j,N}+T^{n}_{i,j+1,N}}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{T^{n}_{i,j,N-1}-2T^{n}_{i,j,N}+T^{n}_{i,j,N+1}}{(\Delta z)^2}\end{array}\right] \tag{5.154}$$

$$+(q'''_{m_i})^{n}+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,i}\times(T_{art}-T^{n}_{i,j,N})\right]$$

In Equation (5.154) $T^{n}_{i,j,N+1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 5 given in Table 5.2 is used.

At Region 23, the temperature of the node (i,j,N) at next time step (n+1) is given below.

$$T^{n+1}_{i,j,N}=\alpha^{n}_{i}\times\Delta t\times\left[\begin{array}{l}\dfrac{T^{n}_{i-1,j,N}-2T^{n}_{i,j,N}+T^{n}_{i+1,j,N}}{(\Delta r)^2}+\dfrac{T^{n}_{i+1,j,N}-T^{n}_{i-1,j,N}}{2i(\Delta r)^2}+\\[2ex] \dfrac{T^{n}_{i,j-1,N}-2T^{n}_{i,j,N}+T^{n}_{i,j+1,N}}{i^2(\Delta r)^2(\Delta\phi)^2}\\[2ex] +\dfrac{1}{(\Delta z)^2}\left[T^{n}_{i,j,N-1}-2T^{n}_{i,j,N}+\left(\dfrac{2\Delta z\times EE}{-k^{n}_{P}}\right)+T^{n}_{i,j,,N-1}\right]\end{array}\right] \quad (5.155)$$

$$+\left[(q'''_{m_i})^{n}\times\frac{\Delta t}{\rho^{n}_{i}\ Cp^{n}_{i}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,i}\times(T_{art}-T^{n}_{i,j,N})\times\frac{\Delta t}{\rho^{n}_{i}\ Cp^{n}_{i}}\right]+T^{n}_{i,j,N}$$

**Region 24**

Substituting the node (P,j,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P}\,Cp^{n}_{P}\left[\frac{T^{n+1}_{P,j,N}-T^{n}_{P,j,N}}{\Delta t}\right]=k^{n}_{i}\left[\begin{array}{l}\dfrac{T^{n}_{P-1,j,N}-2T^{n}_{P,j,N}+T^{n}_{P+1,j,N}}{(\Delta r)^2}+\dfrac{T^{n}_{P+1,j,N}-T^{n}_{P-1,j,N}}{2P(\Delta r)^2}+\\[2ex] \dfrac{T^{n}_{P,j-1,N}-2T^{n}_{P,j,N}+T^{n}_{P,j+1,N}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T^{n}_{P,j,N-1}-2T^{n}_{P,j,N}+T^{n}_{P,j,N+1}}{(\Delta z)^2}\end{array}\right] \quad (5.156)$$

$$+(q'''_{m_P})^{n}+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,P}\times(T_{art}-T^{n}_{P,j,N})\right]$$

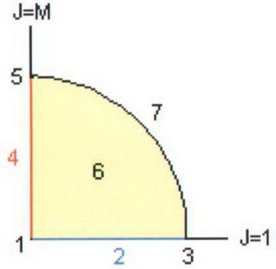
In Equation (5.156), $T^{n}_{P,j,N+1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 5 given in Table 5.2 is used. In addition, $T^{n}_{P+1,j,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 24, the temperature of the node (P,j,N) at next time step (n+1) is given below.

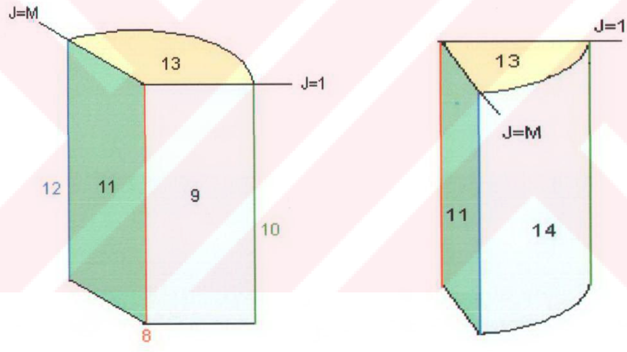
$$
\begin{aligned}
T^{n+1}_{P,j,N} = \alpha^{n}_{P} \times \Delta t \times \Bigg[ & \frac{1}{(\Delta r)^2}\left( T^{n}_{P-1,j,N} - 2T^{n}_{P,j,N} + \left(\frac{2\Delta r \times EE}{-k^{n}_{P}}\right) + T^{n}_{P-1,j,N} \right) \\
& + \frac{1}{2P(\Delta r)^2}\left[ \left(\frac{2\Delta r \times EE}{-k^{n}_{P}}\right) + T^{n}_{P-1,j,N} - T^{n}_{P-1,j,N} \right] \\
& + \frac{T^{n}_{P,j-1,N} - 2T^{n}_{P,j,N} + T^{n}_{P,j+1,N}}{i^2(\Delta r)^2(\Delta \phi)^2} \\
& + \frac{1}{(\Delta z)^2}\left[ T^{n}_{P,j,N-1} - 2T^{n}_{P,j,N} + \left(\frac{2\Delta z \times EE}{-k^{n}_{P}}\right) + T^{n}_{P,j,N-1} \right] \Bigg] \\
& + \left[ (q'''_{m_P})^{n} \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,j,N}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,j,N}
\end{aligned}
\tag{5.157}
$$

**ARM**

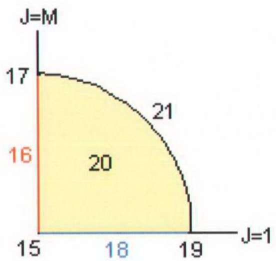
**At k=0**




**At k=2 to (N-1)**




**At k=N**



**Region 1**

Substituting the node (0,0,1) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.21).

$$\rho_0^n Cp_0^n\left[\frac{T_{0,0,1}^{n+1}-T_{0,0,1}^{n}}{\Delta t}\right]=k_0^n\left[4\left(\frac{\overline{T}_1-T_{0,0,1}^{n}}{(\Delta r)^2}\right)+\frac{T_{0,0,0}^{n}-2T_{0,0,1}^{n}+T_{0,0,2}^{n}}{(\Delta z)^2}\right]+(q'''_{m_0})^n + \left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl0}^{n}\times(T_{art}-T_{0,0,1}^{n})\right] \quad (5.158)$$

In Equation (5.158) $T_{0,0,0}^{n}$ is the temperature value of hand. Therefore, instead of node Arm($T_{0,0,0}^{n}$), Hand($T_{0,0,handintervalz}^{n}$) should be substituted. At Region 1, the temperature of the node (0,0,1) at next time step (n+1) is given below.

$$T_{0,0,1}^{n+1}=\alpha_0^n\times\Delta t\times\left[4\left(\frac{\overline{T}_1-T_{0,0,1}^{n}}{(\Delta r)^2}\right)+\frac{1}{(\Delta z)^2}\left(\text{Hand}(T_{0,0,handintervalz})-2T_{0,0,1}^{n}+T_{0,0,2}^{n}\right)\right] + \left[(q'''_{m_0})^n\times\frac{\Delta t}{\rho_0^n Cp_0^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl0}^{n}\times(T_{art}-T_{0,0,1}^{n})\times\frac{\Delta t}{\rho_0^n Cp_0^n}\right]+T_{0,0,1}^{n} \quad (5.159)$$

**Region 2**

Substituting the node (i,1,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n Cp_i^n\left[\frac{T_{i,1,1}^{n+1}-T_{i,1,1}^{n}}{\Delta t}\right]=k_i^n\left[\frac{T_{i-1,1,1}^{n}-2T_{i,1,1}^{n}+T_{i+1,1,1}^{n}}{(\Delta r)^2}+\frac{T_{i+1,1,1}^{n}-T_{i-1,1,1}^{n}}{2i(\Delta r)^2}+\frac{T_{i,0,1}^{n}-2T_{i,1,1}^{n}+T_{i,2,1}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{i,1,0}^{n}-2T_{i,1,1}^{n}+T_{i,1,2}^{n}}{(\Delta z)^2}\right]+(q'''_{m_i})^n + \left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,i}^{n}\times(T_{art}-T_{i,1,1}^{n})\right] \quad (5.160)$$

In Equation (5.160), $\overset{n}{T}_{i,0,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $\overset{n}{T}_{i,1,0}$ is the temperature value of hand. Therefore, instead of node Arm($\overset{n}{T}_{i,1,0}$), Hand($\overset{n}{T}_{i,1,\text{handintervalz}}$) should be substituted. At Region 2, the temperature of the node (i,1,1) at next time step (n+1) is given below.

$$\overset{n+1}{T}_{i,1,1} = \overset{n}{\alpha}_i \times \Delta t \times \left[ \frac{\overset{n}{T}_{i-1,1,1} - 2\overset{n}{T}_{i,1,1} + \overset{n}{T}_{i+1,1,1}}{(\Delta r)^2} + \frac{\overset{n}{T}_{i+1,1,1} - \overset{n}{T}_{i-1,1,1}}{2i(\Delta r)^2} + \frac{\overset{n}{T}_{i,2,1} - 2\overset{n}{T}_{i,1,1} + \overset{n}{T}_{i,2,1}}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{\text{Hand}(\overset{n}{T}_{i,j,\text{handintervalz}}) - 2\overset{n}{T}_{i,1,1} + \overset{n}{T}_{i,1,2}}{(\Delta z)^2} \right] + \left[ (q'''_{m_i})^n \times \frac{\Delta t}{\overset{n}{\rho}_i \, \overset{n}{Cp}_i} \right] \tag{5.161}$$

$$+ \left[ \rho_{bl} \times Cp_{bl} \times \overset{n}{\dot{W}}_{bl\,i} \times (T_{art} - \overset{n}{T}_{i,1,1}) \times \frac{\Delta t}{\overset{n}{\rho}_i \, \overset{n}{Cp}_i} \right] + \overset{n}{T}_{i,1,1}$$

**Region 3**

Substituting the node (P,1,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\overset{n}{\rho}_P \, \overset{n}{Cp}_P \left[ \frac{\overset{n+1}{T}_{P,1,1} - \overset{n}{T}_{P,1,1}}{\Delta t} \right] = \overset{n}{k}_i \left[ \frac{\overset{n}{T}_{P-1,1,1} - 2\overset{n}{T}_{P,1,1} + \overset{n}{T}_{P+1,1,1}}{(\Delta r)^2} + \frac{\overset{n}{T}_{P+1,1,1} - \overset{n}{T}_{P-1,1,1}}{2P(\Delta r)^2} + \frac{\overset{n}{T}_{P,0,1} - 2\overset{n}{T}_{P,1,1} + \overset{n}{T}_{P,2,1}}{P^2(\Delta r)^2(\Delta\phi)^2} + \frac{\overset{n}{T}_{P,1,0} - 2\overset{n}{T}_{P,1,1} + \overset{n}{T}_{P,1,2}}{(\Delta z)^2} \right] + (q'''_{m_P})^n \tag{5.162}$$

$$+ \left[ \rho_{bl} \times Cp_{bl} \times \overset{n}{\dot{W}}_{bl\,P} \times (T_{art} - \overset{n}{T}_{P,1,1}) \right]$$

In Equation (5.162), $T^{n}_{P,0,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T^{n}_{P,1,0}$ is the temperature value of hand. Therefore, instead of node Arm($T^{n}_{P,1,0}$), Hand($T^{n}_{P,1,handintervaz}$) should be substituted. Also $T^{n}_{P+1,1,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. At Region 3, the temperature of the node (P,1,1) at next time step (n+1) is given below.

$$T^{n+1}_{P,1,1} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{array}{l} \frac{1}{(\Delta r)^2}\left( T^{n}_{P-1,1,1} - 2T^{n}_{P,1,1} + \left(\frac{2\Delta r \times EE}{-k^{n}_{P}}\right) + T^{n}_{P-1,1,1} \right) \\ + \frac{1}{2P(\Delta r)^2}\left[ \left(\frac{2\Delta r \times EE}{-k^{n}_{P}}\right) + T^{n}_{P-1,1,1} - T^{n}_{P-1,1,1} \right] \\ + \frac{T^{n}_{P,2,1} - 2T^{n}_{P,1,1} + T^{n}_{P,2,1}}{P^2(\Delta r)^2(\Delta\phi)^2} + \frac{Hand(T^{n}_{P,1,handintervaz}) - 2T^{n}_{P,1,1} + T^{n}_{P,1,2}}{(\Delta z)^2} \end{array} \right]$$
$$+ \left[ (q'''_{m_P})^{n} \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,1,1}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,1,1} \qquad (5.163)$$

**Region 4**

Substituting the node (i, M,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{i}\, Cp^{n}_{i} \left[ \frac{T^{n+1}_{i,M,1} - T^{n}_{i,M,1}}{\Delta t} \right] = k^{n}_{i} \left[ \begin{array}{l} \frac{T^{n}_{i-1,M,1} - 2T^{n}_{i,M,1} + T^{n}_{i+1,M,1}}{(\Delta r)^2} + \frac{T^{n}_{i+1,M,1} - T^{n}_{i-1,M,1}}{2i(\Delta r)^2} + \\ \frac{T^{n}_{i,M-1,1} - 2T^{n}_{i,M,1} + T^{n}_{i,M+1,1}}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{T^{n}_{i,M,0} - 2T^{n}_{i,M,1} + T^{n}_{i,M,2}}{(\Delta z)^2} \end{array} \right] + (q'''_{m_i})^{n}$$
$$+ \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,M,1}) \right] \qquad (5.164)$$

In Equation (5.164), $T^{n}_{i,M+1,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T^{n}_{i,M,0}$ is the temperature value of hand. Therefore, instead of node Arm($T^{n}_{i,M,0}$), Hand($T^{n}_{i,M,handintervaz}$) should be substituted. At Region 4, the temperature of the node (i,M,0) at next time step (n+1) is given below.

$$T^{n+1}_{i,M,1} = \alpha^{n}_{i} \times \Delta t \times \left[ \begin{array}{l} \dfrac{T^{n}_{i-1,1,1} - 2T^{n}_{i,1,1} + T^{n}_{i+1,1,1}}{(\Delta r)^2} + \dfrac{T^{n}_{i+1,1,1} - T^{n}_{i-1,1,1}}{2i(\Delta r)^2} \\ + \dfrac{T^{n}_{i,M-1,1} - 2T^{n}_{i,M,1} + T^{n}_{i,M-1,1}}{i^2 (\Delta r)^2 (\Delta \phi)^2} \\ + \dfrac{1}{(\Delta z)^2}\left( \text{Hand}\,(T^{n}_{i,M,handintervaz}) - 2T^{n}_{i,M,1} + T^{n}_{i,M,2} \right) \end{array} \right] \tag{5.165}$$

$$+ \left[ (q'''_{m_i})^{n} \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,M,1}) \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + T^{n}_{i,M,1}$$

**Region 5**

Substituting the node (P,M,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P} Cp^{n}_{P} \left[ \frac{T^{n+1}_{P,M,1} - T^{n}_{P,M,1}}{\Delta t} \right] = k^{n}_{i} \left[ \begin{array}{l} \dfrac{T^{n}_{P-1,M,1} - 2T^{n}_{P,M,1} + T^{n}_{P+1,M,1}}{(\Delta r)^2} + \dfrac{T^{n}_{P+1,M,1} - T^{n}_{P-1,M,1}}{2P(\Delta r)^2} + \\ \dfrac{T^{n}_{P,M-1,1} - 2T^{n}_{P,M,1} + T^{n}_{P,M+1,1}}{P^2 (\Delta r)^2 (\Delta \phi)^2} + \dfrac{T^{n}_{P,M,0} - 2T^{n}_{P,M,1} + T^{n}_{P,M,k+2}}{(\Delta z)^2} \end{array} \right] \tag{5.166}$$

$$+ (q'''_{m_P})^{n} + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{blP} \times (T_{art} - T^{n}_{P,M,1}) \right]$$

In Equation (5.166) is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T^{n}_{P,M,0}$ is the temperature value of hand. Therefore, instead of node Arm$(T^{n}_{P,M,0})$, Hand $(T^{n}_{P,M,handintervaz})$ should be substituted. Also $T^{n}_{P+1,M,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 5, the temperature of the node (P,M,1) at next time step (n+1) is given below.

$$T^{n+1}_{P,M,1}=\alpha^{n}_{P}\times\Delta t\times\left[\begin{array}{l}\dfrac{1}{(\Delta r)^2}\left(T^{n}_{P-1,M,1}-2T^{n}_{P,M,1}+\left(\dfrac{2\Delta r\times EE}{-k^{n}_{P}}\right)+T^{n}_{P-1,M,1}\right)\\ +\dfrac{1}{2P(\Delta r)^2}\left[\left(\dfrac{2\Delta r\times EE}{-k^{n}_{P}}\right)+T^{n}_{P-1,M,1}-T^{n}_{P-1,M,1}\right]\\ +\dfrac{T^{n}_{P,M-1,1}-2T^{n}_{P,M,1}+T^{n}_{P,M-1,1}}{P^2(\Delta r)^2(\Delta\phi)^2}+\left[\dfrac{Hand(T^{n}_{P,M,handintervaz})-2T^{n}_{P,M,1}+T^{n}_{P,M,2}}{(\Delta z)^2}\right]\end{array}\right] \tag{5.167}$$

$$+\left[(q'''_{m_P})^{n}\times\frac{\Delta t}{\rho^{n}_{P}\,Cp^{n}_{P}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,P}\times(T_{art}-T^{n}_{P,M,1})\times\frac{\Delta t}{\rho^{n}_{P}\,Cp^{n}_{P}}\right]+T^{n}_{P,M,1}$$

**Region 6**

Substituting the node (i,j,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{i}\,Cp^{n}_{i}\left[\frac{T^{n+1}_{i,j,1}-T^{n}_{i,j,1}}{\Delta t}\right]=k^{n}_{i}\left[\begin{array}{l}\dfrac{T^{n}_{i-1,j,1}-2T^{n}_{i,j,1}+T^{n}_{i+1,j,1}}{(\Delta r)^2}+\dfrac{T^{n}_{i+1,j,1}-T^{n}_{i-1,j,1}}{2i(\Delta r)^2}+\\ \dfrac{T^{n}_{i,j-1,1}-2T^{n}_{i,j,1}+T^{n}_{i,j+1,1}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T^{n}_{i,j,0}-2T^{n}_{i,j,1}+T^{n}_{i,j,2}}{(\Delta z)^2}\end{array}\right]+(q'''_{m_i})^{n} \tag{5.168}$$

$$+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,i}\times(T_{art}-T^{n}_{i,j,1})\right]$$

In Equation (5.168) $T^{n}_{i,j,0}$ is the temperature value of hand. Therefore, instead of node Arm($T^{n}_{i,j,0}$), Hand($T^{n}_{i,j,handintervaz}$) should be substituted. At Region 6, the temperature of the node (i,j,1) at next time step (n+1) is given below.

$$T^{n+1}_{i,j,1} = \alpha^{n}_{i} \times \Delta t \times \left[ \frac{T^{n}_{i-1,j,1} - 2T^{n}_{i,j,1} + T^{n}_{i+1,j,1}}{(\Delta r)^2} + \frac{T^{n}_{i+1,j,1} - T^{n}_{i-1,j,1}}{2i(\Delta r)^2} + \frac{T^{n}_{i,j-1,1} - 2T^{n}_{i,j,1} + T^{n}_{i,j+1,1}}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{Hand(T^{n}_{i,j,handintervaz}) - 2T^{n}_{i,j,1} + T^{n}_{i,j,2}}{(\Delta z)^2} \right]$$
$$+ \left[ (q'''_{m_i})^{n} \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,j,1}) \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + T^{n}_{i,j,1} \tag{5.169}$$

**Region 7**

Substituting the node (P,j,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P}\, Cp^{n}_{P} \left[ \frac{T^{n+1}_{P,j,1} - T^{n}_{P,j,1}}{\Delta t} \right] = k^{n}_{i} \left[ \frac{T^{n}_{P-1,j,1} - 2T^{n}_{P,j,1} + T^{n}_{P+1,j,1}}{(\Delta r)^2} + \frac{T^{n}_{P+1,j,1} - T^{n}_{P-1,j,1}}{2P(\Delta r)^2} + \frac{T^{n}_{P,j,1} - 2T^{n}_{P,j,1} + T^{n}_{P,j,1}}{P^2(\Delta r)^2(\Delta\phi)^2} + \frac{T^{n}_{P,j,0} - 2T^{n}_{P,j,1} + T^{n}_{P,j,2}}{(\Delta z)^2} \right] + (q'''_{m_P})^{n}$$
$$+ \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,j,1}) \right] \tag{5.170}$$

In Equation (5.170), $T^{n}_{P+1,j,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. In addition, $T^{n}_{P,j,0}$

is the temperature value of hand. Therefore, instead of node Arm($T^{n}_{P,j,0}$), Hand($T^{n}_{P,j,handintervaz}$) should be substituted.

At Region 7, the temperature of the node (P,j,1) at next time step (n+1) is given below.

$$T^{n+1}_{P,j,1} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{array}{l} \frac{1}{(\Delta r)^2}\left( T^{n}_{P-1,j,1} - 2T^{n}_{P,j,1} + \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,j,1} \right) \\ + \frac{1}{2P(\Delta r)^2}\left[ \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,j,1} - T^{n}_{P-1,j,1} \right] \\ + \frac{T^{n}_{P,j-1,1} - 2T^{n}_{P,j,1} + T^{n}_{P,j+1,1}}{P^2(\Delta r)^2(\Delta \phi)^2} + \frac{Hand(T^{n}_{P,j,handintervaz}) - 2T^{n}_{P,j,1} + T^{n}_{P,j,2}}{(\Delta z)^2} \end{array} \right] \quad (5.171)$$

$$+ \left[ (q'''_{m_P})^{n} \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,j,1}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,j,1}$$

**Region 8-9-10-11-12-13-14**

These region's equations can be obtained by using the same boundary conditions in neck Region (8-9-10-11-12-13-14). Therefore, the temperature equations at t=(n+1) are the same with Neck Region 8-9-10-11-12-13-14.

**Region 15**

Substituting the node (0,0,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.21).

$$\rho^{n}_{0}\, Cp^{n}_{0} \left[ \frac{T^{n+1}_{0,0,N} - T^{n}_{0,0,N}}{\Delta t} \right] = k^{n}_{0} \left[ 4\left( \frac{\overline{T}_1 - T^{n}_{0,0,N}}{(\Delta r)^2} \right) + \frac{T^{n}_{0,0,N-1} - 2T^{n}_{0,0,N} + T^{n}_{0,0,N+1}}{(\Delta z)^2} \right] + (q'''_{m_0})^{n} \quad (5.172)$$

$$+ \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,0} \times (T_{art} - T^{n}_{0,0,N}) \right]$$

In Equation (5.172) $T^{n}_{0,0,N+1}$ is the temperature value of torso. Therefore, instead of node Arm ($T^{n}_{0,0,N+1}$), Torso ($T^{n}_{torsointervalr,armtransphi,armtransz}$) should be substituted.

At Region 15, the temperature of the node (0,0,N) at next time step (n+1) is given below.

$$T^{n+1}_{0,0,N}=\alpha^{n}_{0}\times\Delta t\times\left[4\left(\frac{\overline{T}_1-T^{n}_{0,0,N}}{(\Delta r)^2}\right)+\frac{1}{(\Delta z)^2}\left(T^{n}_{0,0,N-1}-2T^{n}_{0,0,N}+Torso(T^{n}_{torsointervalr,armtransphi,armtransz})\right)\right] \quad (5.173)$$

$$+\left[(q'''_{m_0})^{n}\times\frac{\Delta t}{\rho^{n}_{0}\,Cp^{n}_{0}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl0}\times(T_{art}-T^{n}_{0,0,N})\times\frac{\Delta t}{\rho^{n}_{0}\,Cp^{n}_{0}}\right]+T^{n}_{0,0,N}$$

**Region 16**

Substituting the node (i,1,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{i}\,Cp^{n}_{i}\left[\frac{T^{n+1}_{i,1,N}-T^{n}_{i,1,N}}{\Delta t}\right]=k^{n}_{i}\left[\begin{array}{l}\dfrac{T^{n}_{i-1,1,N}-2T^{n}_{i,1,N}+T^{n}_{i+1,1,N}}{(\Delta r)^2}+\dfrac{T^{n}_{i+1,1,N}-T^{n}_{i-1,1,N}}{2i(\Delta r)^2}+\\ \dfrac{T^{n}_{i,0,N}-2T^{n}_{i,1,N}+T^{n}_{i,2,N}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T^{n}_{i,1,N-1}-2T^{n}_{i,1,N}+T^{n}_{i,1,N+1}}{(\Delta z)^2}\end{array}\right] \quad (5.174)$$

$$+(q'''_{m_i})^{n}+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,i}\times(T_{art}-T^{n}_{i,1,N})\right]$$

In Equation (5.174), $T^{n}_{i,0,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T^{n}_{i,1,N+1}$ is the temperature value of torso. Therefore, instead of node Arm ($T^{n}_{i,1,N+1}$), Torso $(T^{n}_{torso\,intervalr,armtransphi,armtransz})$ should be substituted. At Region 16, the temperature of the node (i,1,N) at next time step (n+1) is given below.

$$T^{n+1}_{i,1,N} = \alpha^{n}_{i} \times \Delta t \times \left[ \begin{array}{l} \dfrac{T^{n}_{i-1,1,N} - 2T^{n}_{i,1,N} + T^{n}_{i+1,1,N}}{(\Delta r)^2} + \dfrac{T^{n}_{i+1,1,N} - T^{n}_{i-1,1,N}}{2i(\Delta r)^2} + \\ \dfrac{T^{n}_{i,2,N} - 2T^{n}_{i,1,N} + T^{n}_{i,2,N}}{i^2(\Delta r)^2(\Delta\phi)^2} + \\ \dfrac{T^{n}_{i,1,N-1} - 2T^{n}_{i,1,N} + Torso(T^{n}_{torso\,intervalr,armtransphi,armtransz})}{(\Delta z)^2} \end{array} \right] \tag{5.175}$$

$$+\left[ (q'''_{m_i})^n \times \frac{\Delta t}{\rho^n_i\, Cp^n_i} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,1,N}) \times \frac{\Delta t}{\rho^n_i\, Cp^n_i} \right] + T^{n}_{i,1,N}$$

**Region 17**

Substituting the node (P,1,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^n_P\, Cp^n_P \left[ \frac{T^{n+1}_{P,1,N} - T^{n}_{P,1,N}}{\Delta t} \right] = k^n_i \left[ \begin{array}{l} \dfrac{T^{n}_{P-1,1,N} - 2T^{n}_{P,1,N} + T^{n}_{P+1,1,N}}{(\Delta r)^2} + \dfrac{T^{n}_{P+1,1,N} - T^{n}_{P-1,1,N}}{2P(\Delta r)^2} + \\ \dfrac{T^{n}_{P,0,N} - 2T^{n}_{P,1,N} + T^{n}_{P,2,N}}{P^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T^{n}_{P,1,N-1} - 2T^{n}_{P,1,N} + T^{n}_{P,1,N+1}}{(\Delta z)^2} \end{array} \right] \tag{5.176}$$

$$+(q'''_{m_P})^n + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{blP} \times (T_{art} - T^{n}_{P,j,N}) \right]$$

In Equation (5.176), $T^{n}_{P,0,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T^{n}_{P,1,N+1}$ is the temperature value of torso. Therefore, instead of node Arm$(T^{n}_{P,1,N+1})$, Torso$(T^{n}_{torso\,intervalr,armtransphi,armtransz})$ should be substituted.

Also $\overset{n}{T}_{P+1,1,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 17, the temperature of the node (P,1,N) at next time step (n+1) is given below.

$$\overset{n+1}{T}_{P,1,N}=\overset{n}{\alpha}_P\times\Delta t\times\left[\begin{array}{l}\frac{1}{(\Delta r)^2}\left(\overset{n}{T}_{P-1,1,N}-2\overset{n}{T}_{P,1,N}+\left(\frac{2\Delta r\times EE}{-\overset{n}{k}_P}\right)+\overset{n}{T}_{P-1,1,N}\right)\\+\frac{1}{2P(\Delta r)^2}\left[\left(\frac{2\Delta r\times EE}{-\overset{n}{k}_P}\right)+\overset{n}{T}_{P-1,1,N}-\overset{n}{T}_{P-1,1,N}\right]\\+\frac{\overset{n}{T}_{P,2,N}-2\overset{n}{T}_{P,1,N}+\overset{n}{T}_{P,2,N}}{P^2(\Delta r)^2(\Delta\phi)^2}\\+\frac{\overset{n}{T}_{P,1,N-1}-2\overset{n}{T}_{P,1,N}+Torso\,(\overset{n}{T}_{torsointervalr,\,armtransphi,\,armtransz})}{(\Delta z)^2}\end{array}\right] \tag{5.177}$$

$$+\left[(q'''_{m_P})^n\times\frac{\Delta t}{\overset{n}{\rho}_P\,\overset{n}{Cp}_P}\right]+\left[\rho_{bl}\times Cp_{bl}\times\overset{n}{\dot{W}}_{bl\,P}\times(T_{art}-\overset{n}{T}_{P,1,N})\times\frac{\Delta t}{\overset{n}{\rho}_P\,\overset{n}{Cp}_P}\right]+\overset{n}{T}_{P,1,N}$$

**Region 18**

Substituting the node (i,M,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\overset{n}{\rho}_i\,\overset{n}{Cp}_i\left[\frac{\overset{n+1}{T}_{i,M,N}-\overset{n}{T}_{i,1,N}}{\Delta t}\right]=\overset{n}{k}_i\left[\begin{array}{l}\frac{\overset{n}{T}_{i-1,M,N}-2\overset{n}{T}_{i,M,N}+\overset{n}{T}_{i+1,M,N}}{(\Delta r)^2}+\frac{\overset{n}{T}_{i+1,M,N}-\overset{n}{T}_{i-1,M,N}}{2i(\Delta r)^2}+\\\frac{\overset{n}{T}_{i,M-1,N}-2\overset{n}{T}_{i,M,N}+\overset{n}{T}_{i,M+1,N}}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{\overset{n}{T}_{i,M,N-1}-2\overset{n}{T}_{i,M,N}+\overset{n}{T}_{i,M,N+1}}{(\Delta z)^2}\end{array}\right] \tag{5.178}$$

$$+(q'''_{m_i})^n+\left[\rho_{bl}\times Cp_{bl}\times\overset{n}{\dot{W}}_{bl\,i}\times(T_{art}-\overset{n}{T}_{i,1,M})\right]$$

In Equation (5.178), $T^{n}_{i,M+1,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T^{n}_{i,M,N+1}$ is the temperature value of torso. Therefore, instead of node Arm($T^{n}_{i,M,N+1}$), Torso($T^{n}_{torso\,intervalr,\,armtransphi,\,armtransz}$) should be substituted.

At Region 18, the temperature of the node (i,M,N) at next time step (n+1) is given below.

$$T^{n+1}_{i,M,N}=\alpha^{n}_{i}\times\Delta t\times\left[\begin{array}{l}\dfrac{T^{n}_{i-1,M,N}-2T^{n}_{i,M,N}+T^{n}_{i+1,M,N}}{(\Delta r)^2}+\dfrac{T^{n}_{i+1,M,N}-T^{n}_{i-1,M,N}}{2i(\Delta r)^2}+\\ \dfrac{T^{n}_{i,M-1,N}-2T^{n}_{i,M,N}+T^{n}_{i,M-1,N}}{i^2(\Delta r)^2(\Delta\phi)^2}\\ +\dfrac{T^{n}_{i,M,N-1}-2T^{n}_{i,M,N}+\text{Head }(T^{n}_{torso\,intervalr,\,armtransphi,\,armtransz})}{(\Delta z)^2}\end{array}\right] \quad (5.179)$$

$$+\left[(q'''_{m_i})^{n}\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,i}\times(T_{art}-T^{n}_{i,M,N})\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+T^{n}_{i,M,N}$$

**Region 19**

Substituting the node (P,M,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P}Cp^{n}_{P}\left[\frac{T^{n+1}_{P,M,N}-T^{n}_{P,M,N}}{\Delta t}\right]=k^{n}_{i}\left[\begin{array}{l}\dfrac{T^{n}_{P-1,M,N}-2T^{n}_{P,M,N}+T^{n}_{P+1,M,N}}{(\Delta r)^2}+\dfrac{T^{n}_{P+1,M,N}-T^{n}_{P-1,M,N}}{2P(\Delta r)^2}+\\ \dfrac{T^{n}_{P,M-1,N}-2T^{n}_{P,M,N}+T^{n}_{P,M+1,N}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T^{n}_{P,M,N-1}-2T^{n}_{P,M,N}+T^{n}_{P,M,N+1}}{(\Delta z)^2}\end{array}\right] \quad (5.180)$$

$$+(q'''_{m_P})^{n}+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{blP}\times(T_{art}-T^{n}_{P,M,N})\right]$$

In Equation (5.180), $T^{n}_{P,M+1,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T^{n}_{P,M,N+1}$ is the temperature value of torso. Therefore, instead of node Arm($T^{n}_{P,M,N+1}$), Torso ($T^{n}_{torsointervalr, armtransphi, armtransz}$) should be substituted.

Also $T^{n}_{P+1,M,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 19, the temperature of the node (P,M,N) at next time step (n+1) is given below.

$$T^{n+1}_{P,M,N} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{array}{l} \frac{1}{(\Delta r)^2}\left( T^{n}_{P-1,M,N} - 2T^{n}_{P,M,N} + \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,M,N} \right) \\ + \frac{1}{2P(\Delta r)^2}\left[ \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,M,N} - T^{n}_{P-1,M,N} \right] \\ + \frac{T^{n}_{P,M-1,N} - 2T^{n}_{P,M,N} + T^{n}_{P,M-1,N}}{i^2 (\Delta r)^2 (\Delta \phi)^2} \\ + \frac{T^{n}_{P,M,N-1} - 2T^{n}_{P,M,N} + \text{Torso}\,(T^{n}_{torsointervalr, armtransphi, armtransz})}{(\Delta z)^2} \end{array} \right] + \left[ (q'''_{m_P})^{n} \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,M,N}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,M,N} \quad (5.181)$$

**Region 20**

Substituting the node (i,j,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n Cp_i^n \left[\frac{T_{i,j,N}^{n+1} - T_{i,j,N}^{n}}{\Delta t}\right] = k_i^n \left[\begin{array}{l} \frac{T_{i-1,j,N}^{n} - 2T_{i,j,N}^{n} + T_{i+1,j,N}^{n}}{(\Delta r)^2} + \frac{T_{i+1,j,N}^{n} - T_{i-1,j,N}^{n}}{2i(\Delta r)^2} + \\ \frac{T_{i,j-1,N}^{n} - 2T_{i,j,N}^{n} + T_{i,j+1,N}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{T_{i,j,N-1}^{n} - 2T_{i,j,N}^{n} + T_{i,j,N+1}^{n}}{(\Delta z)^2} \end{array}\right] \tag{5.182}$$

$$+ (q'''_{m_i})^n + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,i}^{n} \times (T_{art} - T_{i,j,N}^{n})\right]$$

In Equation (5.182) $T_{i,j,N+1}^{n}$ is the temperature value of torso. Therefore, instead of node Arm($T_{i,j,N+1}^{n}$), Torso ($T_{torso\,interval r,\,arm trans phi,\,arm trans z}^{n}$) should be substituted.

At Region 20, the temperature of the node (i,j,N) at next time step (n+1) is given below.

$$T_{i,j,N}^{n+1} = \alpha_i^n \times \Delta t \times \left[\begin{array}{l} \frac{T_{i-1,j,N}^{n} - 2T_{i,j,N}^{n} + T_{i+1,j,N}^{n}}{(\Delta r)^2} + \frac{T_{i+1,j,N}^{n} - T_{i-1,j,N}^{n}}{2i(\Delta r)^2} + \\ \frac{T_{i,j-1,N}^{n} - 2T_{i,j,N}^{n} + T_{i,j+1,N}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2} \\ + \frac{T_{i,j,N-1}^{n} - 2T_{i,j,N}^{n} + \text{Torso}\,(T_{torso\,interval r,\,arm trans phi,\,arm trans z}^{n})}{(\Delta z)^2} \end{array}\right] \tag{5.183}$$

$$+ \left[(q'''_{m_i})^n \times \frac{\Delta t}{\rho_i^n\, Cp_i^n}\right] + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,i}^{n} \times (T_{art} - T_{i,j,N}^{n}) \times \frac{\Delta t}{\rho_i^n\, Cp_i^n}\right] + T_{i,j,N}^{n}$$

**Region 21**

Substituting the node (P,j,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n \left[\frac{T_{P,j,N}^{n+1}-T_{P,j,N}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{P-1,j,N}^{n}-2T_{P,j,N}^{n}+T_{P+1,j,N}^{n}}{(\Delta r)^2}+\dfrac{T_{P+1,j,N}^{n}-T_{P-1,j,N}^{n}}{2P(\Delta r)^2}+\\ \dfrac{T_{P,j-1,N}^{n}-2T_{P,j,N}^{n}+T_{P,j+1,N}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{P,j,N-1}^{n}-2T_{P,j,N}^{n}+T_{P,j,N+1}^{n}}{(\Delta z)^2}\end{array}\right] \quad (5.184)$$

$$+(q'''_{m_P})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,P}^{n}\times(T_{art}-T_{P,j,N}^{n})\right]$$

In Equation (5.184), $T_{P+1,j,N}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. In addition, $T_{P,j,N+1}^{n}$ is the temperature value of torso. Therefore, instead of node Arm($T_{P,j,N+1}^{n}$), Torso ($T_{torso\,intervalr,\,armtransphi,\,armtransz}^{n}$) should be substituted.

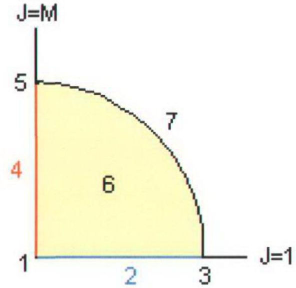
At Region 21, the temperature of the node (P,j,1) at next time step (n+1) is given below.

$$T_{P,j,N}^{n+1}=\alpha_P^n\times\Delta t\times\left[\begin{array}{l}\dfrac{1}{(\Delta r)^2}\left(T_{P-1,j,N}^{n}-2T_{P,j,N}^{n}+\left(\dfrac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,j,N}^{n}\right)\\ +\dfrac{1}{2P(\Delta r)^2}\left[\left(\dfrac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,j,N}^{n}-T_{P-1,j,N}^{n}\right]\\ +\dfrac{T_{P,j-1,N}^{n}-2T_{P,j,N}^{n}+T_{P,j+1,N}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}\\ +\dfrac{1}{(\Delta z)^2}\left[\text{Torso }(T_{torso\,intervalr,\,armtransphi,\,armtransz}^{n})-2T_{P,j,N}^{n}+T_{P,j,N-1}^{n}\right]\end{array}\right] \quad (5.185)$$

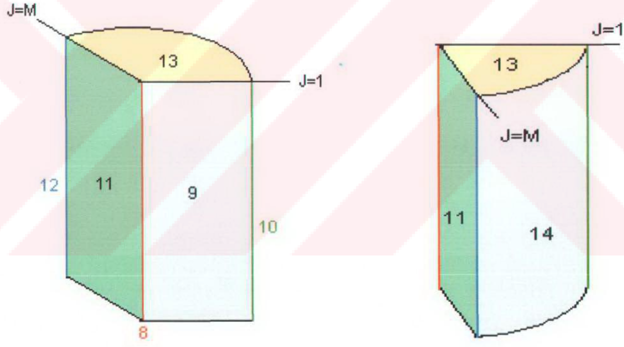
$$+\left[(q'''_{m_P})^n\times\frac{\Delta t}{\rho_P^n\,Cp_P^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,P}^{n}\times(T_{art}-T_{P,j,N}^{n})\times\frac{\Delta t}{\rho_P^n\,Cp_P^n}\right]+T_{P,j,N}^{n}$$

# LEG

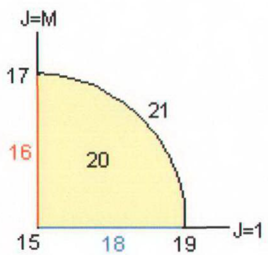
**At k=0**




**At k=2 to (N-2)**




**At k=N-1**



**Region 1**

Substituting the node (0,0,1) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.21).

$$\rho_0^n Cp_0^n\left[\frac{T_{0,0,1}^{n+1}-T_{0,0,1}^n}{\Delta t}\right]=k_0^n\left[4\left(\frac{\bar{T}_1-T_{0,0,1}^n}{(\Delta r)^2}\right)+\frac{T_{0,0,0}^n-2T_{0,0,1}^n+T_{0,0,2}^n}{(\Delta z)^2}\right]+(q_{m_0}''')^n$$

$$+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl0}^n\times(T_{art}-T_{0,0,1}^n)\right] \qquad (5.186)$$

In Equation (5.186) $T_{0,0,0}^n$ is the temperature value of foot. Therefore, instead of node Leg($T_{0,0,0}^n$), Foot($T_{0,0,footintervalz}^n$) should be substituted. At Region 1, the temperature of the node (0,0,1) at next time step (n+1) is given below.

$$T_{0,0,1}^{n+1}=\alpha_0^n\times\Delta t\times\left[4\left(\frac{\bar{T}_1-T_{0,0,1}^n}{(\Delta r)^2}\right)+\frac{1}{(\Delta z)^2}\left(Foot(T_{0,0,footintervalz}^n)-2T_{0,0,1}^n+T_{0,0,2}^n\right)\right] \qquad (5.187)$$

$$+\left[(q_{m_0}''')^n\times\frac{\Delta t}{\rho_0^n Cp_0^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl0}^n\times(T_{art}-T_{0,0,1}^n)\times\frac{\Delta t}{\rho_0^n Cp_0^n}\right]+T_{0,0,1}^n$$

**Region 2**

Substituting the node (i,1,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n Cp_i^n\left[\frac{T_{i,1,1}^{n+1}-T_{i,1,1}^n}{\Delta t}\right]=k_i^n\left[\frac{T_{i-1,1,1}^n-2T_{i,1,1}^n+T_{i+1,1,1}^n}{(\Delta r)^2}+\frac{T_{i+1,1,1}^n-T_{i-1,1,1}^n}{2i(\Delta r)^2}+\frac{T_{i,0,1}^n-2T_{i,1,1}^n+T_{i,2,1}^n}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{i,1,0}^n-2T_{i,1,1}^n+T_{i,1,2}^n}{(\Delta z)^2}\right]+(q_{m_i}''')^n \qquad (5.188)$$

$$+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,i}^n\times(T_{art}-T_{i,1,1}^n)\right]$$

In Equation (5.188), $T^{n}_{i,0,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T^{n}_{i,1,0}$ is the temperature value of foot. Therefore, instead of node Leg($T^{n}_{i,1,0}$), Foot($T^{n}_{i,1,\text{footintervalz}}$) should be substituted. At Region 2, the temperature of the node (i,1,1) at next time step (n+1) is given below.

$$T^{n+1}_{i,1,1} = \alpha^{n}_{i} \times \Delta t \times \left[ \frac{T^{n}_{i-1,1,1} - 2T^{n}_{i,1,1} + T^{n}_{i+1,1,1}}{(\Delta r)^2} + \frac{T^{n}_{i+1,1,1} - T^{n}_{i-1,1,1}}{2i(\Delta r)^2} + \frac{T^{n}_{i,2,1} - 2T^{n}_{i,1,1} + T^{n}_{i,2,1}}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{\text{Foot}(T^{n}_{i,1,\text{footintervalz}}) - 2T^{n}_{i,1,1} + T^{n}_{i,1,2}}{(\Delta z)^2} \right] + \left[ (q'''_{m_i})^{n} \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] \tag{5.189}$$

$$+ \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,1,1}) \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + T^{n}_{i,1,1}$$

**Region 3**

Substituting the node (P,1,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P}\, Cp^{n}_{P} \left[ \frac{T^{n+1}_{P,1,1} - T^{n}_{P,1,1}}{\Delta t} \right] = k^{n}_{i} \left[ \frac{T^{n}_{P-1,1,1} - 2T^{n}_{P,1,1} + T^{n}_{P+1,1,1}}{(\Delta r)^2} + \frac{T^{n}_{P+1,1,1} - T^{n}_{P-1,1,1}}{2P(\Delta r)^2} + \frac{T^{n}_{P,0,1} - 2T^{n}_{P,1,1} + T^{n}_{P,2,1}}{P^2(\Delta r)^2(\Delta\phi)^2} + \frac{T^{n}_{P,1,0} - 2T^{n}_{P,1,1} + T^{n}_{P,1,2}}{(\Delta z)^2} \right] + (q'''_{m_P})^{n} \tag{5.190}$$

$$+ \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,1,1}) \right]$$

In Equation (5.190), $T^{n}_{P,0,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T^{n}_{P,1,0}$ is the temperature value of foot. Therefore, instead of node Leg($T^{n}_{P,1,0}$), Foot($T^{n}_{P,1,footintervaz}$) should be substituted. Also $T^{n}_{P+1,1,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. At Region 3, the temperature of the node (P,1,1) at next time step (n+1) is given below.

$$T^{n+1}_{P,1,1} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{array}{l} \dfrac{1}{(\Delta r)^2}\left( T^{n}_{P-1,1,1} - 2T^{n}_{P,1,1} + \left(\dfrac{2\Delta r \times EE}{-k^{n}_{P}}\right) + T^{n}_{P-1,1,1} \right) \\ + \dfrac{1}{2P(\Delta r)^2}\left[ \left(\dfrac{2\Delta r \times EE}{-k^{n}_{P}}\right) + T^{n}_{P-1,1,1} - T^{n}_{P-1,1,1} \right] \\ + \dfrac{T^{n}_{P,2,1} - 2T^{n}_{P,1,1} + T^{n}_{P,2,1}}{P^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{Foot(T^{n}_{P,1,footintervaz}) - 2T^{n}_{P,1,1} + T^{n}_{P,1,2}}{(\Delta z)^2} \end{array} \right] \tag{5.191}$$

$$+\left[ (q'''_{m_P})^{n} \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,1,1}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,1,1}$$

**Region 4**

Substituting the node (i, M,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{i}\, Cp^{n}_{i} \left[ \frac{T^{n+1}_{i,M,1} - T^{n}_{i,M,1}}{\Delta t} \right] = k^{n}_{i} \left[ \begin{array}{l} \dfrac{T^{n}_{i-1,M,1} - 2T^{n}_{i,M,1} + T^{n}_{i+1,M,1}}{(\Delta r)^2} + \dfrac{T^{n}_{i+1,M,1} - T^{n}_{i-1,M,1}}{2i(\Delta r)^2} + \\ \dfrac{T^{n}_{i,M-1,1} - 2T^{n}_{i,M,1} + T^{n}_{i,M+1,1}}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T^{n}_{i,M,0} - 2T^{n}_{i,M,1} + T^{n}_{i,M,2}}{(\Delta z)^2} \end{array} \right] + (q'''_{m_i})^{n} \tag{5.192}$$

$$+\left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl_i} \times (T_{art} - T^{n}_{i,M,1}) \right]$$

In Equation (5.192), $T^{n}_{i,M+1,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T^{n}_{i,M,0}$ is the temperature value of foot. Therefore, instead of node Leg($T^{n}_{i,M,0}$), Foot($T^{n}_{i,M,footintervaz}$) should be substituted. At Region 4, the temperature of the node (i,M,0) at next time step (n+1) is given below.

$$T^{n+1}_{i,M,1} = \alpha^{n}_{i} \times \Delta t \times \left[ \begin{array}{l} \dfrac{T^{n}_{i-1,1,1} - 2T^{n}_{i,1,1} + T^{n}_{i+1,1,1}}{(\Delta r)^2} + \dfrac{T^{n}_{i+1,1,1} - T^{n}_{i-1,1,1}}{2i(\Delta r)^2} \\ + \dfrac{T^{n}_{i,M-1,1} - 2T^{n}_{i,M,1} + T^{n}_{i,M-1,1}}{i^2 (\Delta r)^2 (\Delta\phi)^2} \\ + \dfrac{1}{(\Delta z)^2}\left( \text{Foot}\,(T^{n}_{i,M,footintervaz}) - 2T^{n}_{i,M,1} + T^{n}_{i,M,2} \right) \end{array} \right] \quad (5.193)$$

$$+ \left[ (q'''_{m_i})^{n} \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,M,1}) \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + T^{n}_{i,M,1}$$

**Region 5**

Substituting the node (P,M,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P} Cp^{n}_{P} \left[ \frac{T^{n+1}_{P,M,1} - T^{n}_{P,M,1}}{\Delta t} \right] = k^{n}_{i} \left[ \begin{array}{l} \dfrac{T^{n}_{P-1,M,1} - 2T^{n}_{P,M,1} + T^{n}_{P+1,M,1}}{(\Delta r)^2} + \dfrac{T^{n}_{P+1,M,1} - T^{n}_{P-1,M,1}}{2P(\Delta r)^2} + \\ \dfrac{T^{n}_{P,M-1,1} - 2T^{n}_{P,M,1} + T^{n}_{P,M+1,1}}{P^2 (\Delta r)^2 (\Delta\phi)^2} + \dfrac{T^{n}_{P,M,0} - 2T^{n}_{P,M,1} + T^{n}_{P,M,k+2}}{(\Delta z)^2} \end{array} \right] \quad (5.194)$$

$$+ (q'''_{m_P})^{n} + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,M,1}) \right]$$

In Equation (5.194), $T^{n}_{P,M+1,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $\overset{n}{T}_{P,M,0}$ is the temperature value of foot. Therefore, instead of node Leg($\overset{n}{T}_{P,M,0}$), Foot ($\overset{n}{T}_{P,M,footintervaz}$) should be substituted. Also $\overset{n}{T}_{P+1,M,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 5, the temperature of the node (P,M,1) at next time step (n+1) is given below.

$$\overset{n+1}{T}_{P,M,1}=\overset{n}{\alpha_P}\times\Delta t\times\left[\begin{array}{l}\dfrac{1}{(\Delta r)^2}\left(\overset{n}{T}_{P-1,M,1}-2\overset{n}{T}_{P,M,1}+\left(\dfrac{2\Delta r\times EE}{-\overset{n}{k_P}}\right)+\overset{n}{T}_{P-1,M,1}\right)\\ +\dfrac{1}{2P(\Delta r)^2}\left[\left(\dfrac{2\Delta r\times EE}{-\overset{n}{k_P}}\right)+\overset{n}{T}_{P-1,M,1}-\overset{n}{T}_{P-1,M,1}\right]\\ +\dfrac{\overset{n}{T}_{P,M-1,1}-2\overset{n}{T}_{P,M,1}+\overset{n}{T}_{P,M-1,1}}{P^2(\Delta r)^2(\Delta\phi)^2}+\left[\dfrac{Foot(\overset{n}{T}_{P,M,footintervaz})-2\overset{n}{T}_{P,M,1}+\overset{n}{T}_{P,M,2}}{(\Delta z)^2}\right]\end{array}\right] \tag{5.195}$$

$$+\left[(q'''_{m_P})^n\times\frac{\Delta t}{\overset{n}{\rho_P}\,\overset{n}{Cp_P}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\overset{n}{\dot{W}}_{blP}\times(T_{art}-\overset{n}{T}_{P,M,1})\times\frac{\Delta t}{\overset{n}{\rho_P}\,\overset{n}{Cp_P}}\right]+\overset{n}{T}_{P,M,1}$$

**Region 6**

Substituting the node (i,j,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\overset{n}{\rho_i}\,\overset{n}{Cp_i}\left[\frac{\overset{n+1}{T}_{i,j,1}-\overset{n}{T}_{i,j,1}}{\Delta t}\right]=\overset{n}{k_i}\left[\begin{array}{l}\dfrac{\overset{n}{T}_{i-1,j,1}-2\overset{n}{T}_{i,j,1}+\overset{n}{T}_{i+1,j,1}}{(\Delta r)^2}+\dfrac{\overset{n}{T}_{i+1,j,1}-\overset{n}{T}_{i-1,j,1}}{2i(\Delta r)^2}+\\ \dfrac{\overset{n}{T}_{i,j-1,1}-2\overset{n}{T}_{i,j,1}+\overset{n}{T}_{i,j+1,1}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{\overset{n}{T}_{i,j,0}-2\overset{n}{T}_{i,j,1}+\overset{n}{T}_{i,j,2}}{(\Delta z)^2}\end{array}\right]+(q'''_{m_i})^n \tag{5.196}$$

$$+\left[\rho_{bl}\times Cp_{bl}\times\overset{n}{\dot{W}}_{bl\,i}\times(T_{art}-\overset{n}{T}_{i,j,1})\right]$$

In Equation (5.196) $T^{n}_{i,j,0}$ is the temperature value of foot. Therefore, instead of node Leg($T^{n}_{i,j,0}$), Foot($T^{n}_{i,j,footintervalz}$) should be substituted. At Region 6, the temperature of the node (i,j,1) at next time step (n+1) is given below.

$$T^{n+1}_{i,j,1} = \alpha^{n}_{i} \times \Delta t \times \left[ \frac{T^{n}_{i-1,j,1} - 2T^{n}_{i,j,1} + T^{n}_{i+1,j,1}}{(\Delta r)^2} + \frac{T^{n}_{i+1,j,1} - T^{n}_{i-1,j,1}}{2i(\Delta r)^2} + \frac{T^{n}_{i,j-1,1} - 2T^{n}_{i,j,1} + T^{n}_{i,j+1,1}}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{\text{Foot}(T^{n}_{i,j,\text{foot intervalz}}) - 2T^{n}_{i,j,1} + T^{n}_{i,j,2}}{(\Delta z)^2} \right] \tag{5.197}$$

$$+ \left[ (q'''_{m_i})^{n} \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,j,1}) \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + T^{n}_{i,j,1}$$

**Region 7**

Substituting the node (P,j,1) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P} Cp^{n}_{P} \left[ \frac{T^{n+1}_{P,j,1} - T^{n}_{P,j,1}}{\Delta t} \right] = k^{n}_{i} \left[ \frac{T^{n}_{P-1,j,1} - 2T^{n}_{P,j,1} + T^{n}_{P+1,j,1}}{(\Delta r)^2} + \frac{T^{n}_{P+1,j,1} - T^{n}_{P-1,j,1}}{2P(\Delta r)^2} + \frac{T^{n}_{P,j,1} - 2T^{n}_{P,j,1} + T^{n}_{P,j,1}}{P^2(\Delta r)^2(\Delta\phi)^2} + \frac{T^{n}_{P,j,0} - 2T^{n}_{P,j,1} + T^{n}_{P,j,2}}{(\Delta z)^2} \right] + (q'''_{m_P})^{n} \tag{5.198}$$

$$+ \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,j,1}) \right]$$

In Equation (5.198), $T^{n}_{P+1,j,1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. In addition, $T^{n}_{P,j,0}$

is the temperature value of foot. Therefore, instead of node Leg($T^{n}_{P,j,0}$), Foot($T^{n}_{P,j,footintervalz}$) should be substituted.

At Region 7, the temperature of the node (P,j,1) at next time step (n+1) is given below.

$$T^{n+1}_{P,j,1} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{array}{l} \frac{1}{(\Delta r)^2}\left( T^{n}_{P-1,j,1} - 2T^{n}_{P,j,1} + \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,j,1} \right) \\ + \frac{1}{2P(\Delta r)^2}\left[ \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,j,1} - T^{n}_{P-1,j,1} \right] \\ + \frac{T^{n}_{P,j-1,1} - 2T^{n}_{P,j,1} + T^{n}_{P,j+1,1}}{P^2(\Delta r)^2(\Delta\phi)^2} + \frac{Foot(T^{n}_{P,j,footintervalz}) - 2T^{n}_{P,j,1} + T^{n}_{P,j,2}}{(\Delta z)^2} \end{array} \right] \tag{5.199}$$

$$+\left[ (q'''_{m_P})^{n} \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,j,1}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,j,1}$$

**Region 8-9-10-11-12-13-14**

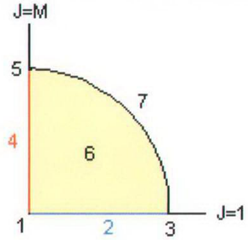
These region's equations can be obtained by using the same boundary conditions in neck Region (8-9-10-11-12-13-14). Therefore, the temperature equations at t=(n+1) are the same with Neck Region 8-9-10-11-12-13-14.

**Region 15-16-17-18-19-20-21**

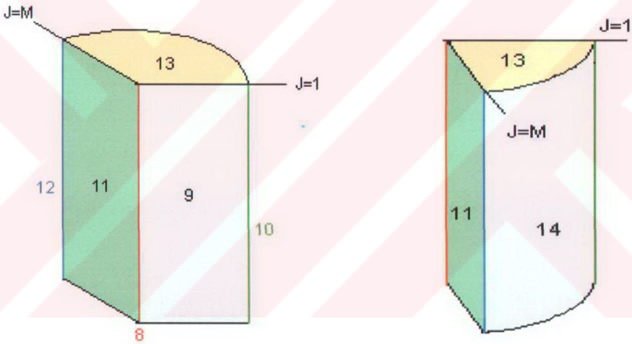
These region's equations can be obtained by using the same boundary conditions in Arm Region (15-16-17-18-19-20-21). Therefore, the temperature equations at t=(n+1) are the same with Arm Region 15-16-17-18-19-20-21.

## HAND

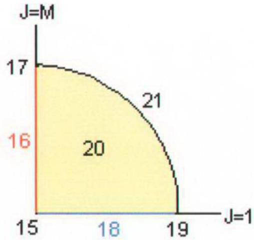
**At k=0**




**At k=1 to (N-1)**




**At k=N**



**Region 1**

Substituting the node (0,0,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.21).

$$\rho_0^n Cp_0^n \left[\frac{T_{0,0,N}^{n+1}-T_{0,0,N}^{n}}{\Delta t}\right]=k_0^n\left[4\left(\frac{\overline{T}_1-T_{0,0,N}^{n}}{(\Delta r)^2}\right)+\frac{T_{0,0,N-1}^{n}-2T_{0,0,N}^{n}+T_{0,0,N+1}^{n}}{(\Delta z)^2}\right]+(q_{m_0}''')^n$$
$$+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,0}^{n}\times(T_{art}-T_{0,0,N}^{n})\right] \quad (5.200)$$

In Equation (5.200) $T_{0,0,N+1}^{n}$ is the temperature value of arm. Therefore, instead of node Hand($T_{0,0,N+1}^{n}$), Arm ($T_{0,0,1}^{n}$) should be substituted.

At Region 1, the temperature of the node (0,0,N) at next time step (n+1) is given below.

$$T_{0,0,N}^{n+1}=\alpha_0^n\times\Delta t\times\left[4\left(\frac{\overline{T}_1-T_{0,0,N}^{n}}{(\Delta r)^2}\right)+\frac{1}{(\Delta z)^2}\left(Arm(T_{0,0,1}^{n})-2T_{0,0,N}^{n}+T_{0,0,N-1}^{n}\right)\right] \quad (5.201)$$
$$+\left[(q_{m_0}''')^n\times\frac{\Delta t}{\rho_0^n Cp_0^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,0}^{n}\times(T_{art}-T_{0,0,N}^{n})\times\frac{\Delta t}{\rho_0^n Cp_0^n}\right]+T_{0,0,N}^{n}$$

**Region 2**

Substituting the node (i,1,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n Cp_i^n\left[\frac{T_{i,1,N}^{n+1}-T_{i,1,N}^{n}}{\Delta t}\right]=k_i^n\left[\frac{T_{i-1,1,N}^{n}-2T_{i,1,N}^{n}+T_{i+1,1,N}^{n}}{(\Delta r)^2}+\frac{T_{i+1,1,N}^{n}-T_{i-1,1,N}^{n}}{2i(\Delta r)^2}+\frac{T_{i,0,N}^{n}-2T_{i,1,N}^{n}+T_{i,2,N}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{i,1,N-1}^{n}-2T_{i,1,N}^{n}+T_{i,1,N+1}^{n}}{(\Delta z)^2}\right] \quad (5.202)$$
$$+(q_{m_i}''')^n+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,i}^{n}\times(T_{art}-T_{i,1,N}^{n})\right]$$

In Equation (5.202), $T^{n}_{i,0,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T^{n}_{i,1,N+1}$ is the temperature value of arm. Therefore, instead of node Hand ($T^{n}_{i,1,N+1}$), Arm ($T^{n}_{i,1,1}$) should be substituted. At Region 2, the temperature of the node (i,1,N) at next time step (n+1) is given below.

$$T^{n+1}_{i,1,N}=\alpha^{n}_{i}\times\Delta t\times\left[\begin{array}{c}\dfrac{T^{n}_{i-1,1,N}-2T^{n}_{i,1,N}+T^{n}_{i+1,1,N}}{(\Delta r)^2}+\dfrac{T^{n}_{i+1,1,N}-T^{n}_{i-1,1,N}}{2i(\Delta r)^2}+\\ \dfrac{T^{n}_{i,2,N}-2T^{n}_{i,1,N}+T^{n}_{i,2,N}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T^{n}_{i,1,N-1}-2T^{n}_{i,1,N}+Arm(T^{n}_{i,1,1})}{(\Delta z)^2}\end{array}\right] \tag{5.203}$$

$$+\left[(q'''_{m_i})^{n}\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,i}\times(T_{art}-T^{n}_{i,1,N})\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+T^{n}_{i,1,N}$$

**Region 3**

Substituting the node (P,1,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P}\,Cp^{n}_{P}\left[\frac{T^{n+1}_{P,1,N}-T^{n}_{P,1,N}}{\Delta t}\right]=k^{n}_{i}\left[\begin{array}{c}\dfrac{T^{n}_{P-1,1,N}-2T^{n}_{P,1,N}+T^{n}_{P+1,1,N}}{(\Delta r)^2}+\dfrac{T^{n}_{P+1,1,N}-T^{n}_{P-1,1,N}}{2P(\Delta r)^2}+\\ \dfrac{T^{n}_{P,0,N}-2T^{n}_{P,1,N}+T^{n}_{P,2,N}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T^{n}_{P,1,N-1}-2T^{n}_{P,1,N}+T^{n}_{P,1,N+1}}{(\Delta z)^2}\end{array}\right] \tag{5.204}$$

$$+(q'''_{m_P})^{n}+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,P}\times(T_{art}-T^{n}_{P,j,N})\right]$$

In Equation (5.204), $T^{n}_{P,0,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T^{n}_{P,1,N+1}$ is the temperature value of arm. Therefore, instead of node Hand

$(T^{n}_{P,1,N+1})$, $Arm(T^{n}_{P,1,1})$ should be substituted. Also $T^{n}_{P+1,1,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 3, the temperature of the node (P,1,N) at next time step (n+1) is given below.

$$T^{n+1}_{P,1,N}=\alpha^{n}_{P}\times\Delta t\times\left[\begin{array}{l}\dfrac{1}{(\Delta r)^2}\left(T^{n}_{P-1,1,N}-2T^{n}_{P,1,N}+\left(\dfrac{2\Delta r\times EE}{-k^{n}_{P}}\right)+T^{n}_{P-1,1,N}\right)\\ +\dfrac{1}{2P(\Delta r)^2}\left[\left(\dfrac{2\Delta r\times EE}{-k^{n}_{P}}\right)+T^{n}_{P-1,1,N}-T^{n}_{P-1,1,N}\right]\\ +\dfrac{T^{n}_{P,2,N}-2T^{n}_{P,1,N}+T^{n}_{P,2,N}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T^{n}_{P,1,N-1}-2T^{n}_{P,1,N}+Arm\ (T^{n}_{P,1,1})}{(\Delta z)^2}\end{array}\right]$$
$$+\left[(q'''_{m_P})^{n}\times\frac{\Delta t}{\rho^{n}_{P}\,Cp^{n}_{P}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,P}\times(T_{art}-T^{n}_{P,1,N})\times\frac{\Delta t}{\rho^{n}_{P}\,Cp^{n}_{P}}\right]+T^{n}_{P,1,N}\qquad(5.205)$$

**Region 4**

Substituting the node (i,M,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{i}\,Cp^{n}_{i}\left[\frac{T^{n+1}_{i,M,N}-T^{n}_{i,1,N}}{\Delta t}\right]=k^{n}_{i}\left[\begin{array}{l}\dfrac{T^{n}_{i-1,M,N}-2T^{n}_{i,M,N}+T^{n}_{i+1,M,N}}{(\Delta r)^2}+\dfrac{T^{n}_{i+1,M,N}-T^{n}_{i-1,M,N}}{2i(\Delta r)^2}+\\ \dfrac{T^{n}_{i,M-1,N}-2T^{n}_{i,M,N}+T^{n}_{i,M+1,N}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T^{n}_{i,M,N-1}-2T^{n}_{i,M,N}+T^{n}_{i,M,N+1}}{(\Delta z)^2}\end{array}\right]\qquad(5.206)$$
$$+(q'''_{m_i})^{n}+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,i}\times(T_{art}-T^{n}_{i,1,M})\right]$$

In Equation (5.206), $T^{n}_{i,M+1,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T^{n}_{i,M,N+1}$ is the temperature value of arm. Therefore, instead of node Hand($T^{n}_{i,M,N+1}$), Arm($T^{n}_{i,M,1}$) should be substituted.

At Region 4, the temperature of the node (i,M,N) at next time step (n+1) is given below.

$$T^{n+1}_{i,M,N}=\alpha^{n}_{i}\times\Delta t\times\left[\begin{array}{l}\dfrac{T^{n}_{i-1,M,N}-2T^{n}_{i,M,N}+T^{n}_{i+1,M,N}}{(\Delta r)^2}+\dfrac{T^{n}_{i+1,M,N}-T^{n}_{i-1,M,N}}{2i(\Delta r)^2}+\\ \dfrac{T^{n}_{i,M-1,N}-2T^{n}_{i,M,N}+T^{n}_{i,M-1,N}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T^{n}_{i,M,N-1}-2T^{n}_{i,M,N}+Arm(T^{n}_{i,M,1})}{(\Delta z)^2}\end{array}\right] \quad (5.207)$$

$$+\left[(q'''_{m_i})^{n}\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl_i}\times(T_{art}-T^{n}_{i,M,N})\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+T^{n}_{i,M,N}$$

**Region 5**

Substituting the node (P,M,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P}Cp^{n}_{P}\left[\frac{T^{n+1}_{P,M,N}-T^{n}_{P,M,N}}{\Delta t}\right]=k^{n}_{i}\left[\begin{array}{l}\dfrac{T^{n}_{P-1,M,N}-2T^{n}_{P,M,N}+T^{n}_{P+1,M,N}}{(\Delta r)^2}+\dfrac{T^{n}_{P+1,M,N}-T^{n}_{P-1,M,N}}{2P(\Delta r)^2}+\\ \dfrac{T^{n}_{P,M-1,N}-2T^{n}_{P,M,N}+T^{n}_{P,M+1,N}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T^{n}_{P,M,N-1}-2T^{n}_{P,M,N}+T^{n}_{P,M,N+1}}{(\Delta z)^2}\end{array}\right] \quad (5.208)$$

$$+(q'''_{m_P})^{n}+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{blP}\times(T_{art}-T^{n}_{P,M,N})\right]$$

In Equation (5.208), $T^{n}_{P,M+1,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T^{n}_{P,M,N+1}$ is the temperature value of arm. Therefore, instead of node Hand($T^{n}_{P,M,N+1}$), Arm ($T^{n}_{P,M,1}$) should be substituted. Also $T^{n}_{P+1,M,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 5, the temperature of the node (P,M,N) at next time step (n+1) is given below.

$$T_{P,M,N}^{n+1}=\alpha_P^n\times\Delta t\times\left[\begin{array}{l}\frac{1}{(\Delta r)^2}\left(T_{P-1,M,N}^n-2T_{P,M,N}^n+\left(\frac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,M,N}^n\right)\\ +\frac{1}{2P(\Delta r)^2}\left[\left(\frac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,M,N}^n-T_{P-1,M,N}^n\right]\\ +\frac{T_{P,M-1,N}^n-2T_{P,M,N}^n+T_{P,M-1,N}^n}{i^2(\Delta r)^2(\Delta\phi)^2}\\ +\frac{T_{P,M,N-1}^n-2T_{P,M,N}^n+Arm\,(T_{P,M,1}^n)}{(\Delta z)^2}\end{array}\right] \tag{5.209}$$

$$+\left[(q_{m_P}''')^n\times\frac{\Delta t}{\rho_P^n\,Cp_P^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,P}^n\times(T_{art}-T_{P,M,N}^n)\times\frac{\Delta t}{\rho_P^n\,Cp_P^n}\right]+T_{P,M,N}^n$$

**Region 6**

Substituting the node (i,j,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n\,Cp_i^n\left[\frac{T_{i,j,N}^{n+1}-T_{i,j,N}^n}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\frac{T_{i-1,j,N}^n-2T_{i,j,N}^n+T_{i+1,j,N}^n}{(\Delta r)^2}+\frac{T_{i+1,j,N}^n-T_{i-1,j,N}^n}{2i(\Delta r)^2}+\\ \frac{T_{i,j-1,N}^n-2T_{i,j,N}^n+T_{i,j+1,N}^n}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{i,j,N-1}^n-2T_{i,j,N}^n+T_{i,j,N+1}^n}{(\Delta z)^2}\end{array}\right] \tag{5.210}$$

$$+(q_{m_i}''')^n+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,i}^n\times(T_{art}-T_{i,j,N}^n)\right]$$

In Equation (5.210) $T_{i,j,N+1}^n$ is the temperature value of arm. Therefore, instead of node Hand($T_{i,j,N+1}^n$), Arm($T_{i,j,1}^n$) should be substituted.

At Region 6, the temperature of the node (i,j,N) at next time step (n+1) is given below.

$$T_{i,j,N}^{n+1}=\alpha_i^n\times\Delta t\times\left[\begin{array}{l}\dfrac{T_{i-1,j,N}^{n}-2T_{i,j,N}^{n}+T_{i+1,j,N}^{n}}{(\Delta r)^2}+\dfrac{T_{i+1,j,N}^{n}-T_{i-1,j,N}^{n}}{2i(\Delta r)^2}+\\ \dfrac{T_{i,j-1,N}^{n}-2T_{i,j,N}^{n}+T_{i,j+1,N}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{i,j,N-1}^{n}-2T_{i,j,N}^{n}+Arm(T_{i,j,1})}{(\Delta z)^2}\end{array}\right] \quad (5.211)$$

$$+\left[(q'''_{m_i})^n\times\frac{\Delta t}{\rho_i^n\ Cp_i^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,i}^{\,n}\times(T_{art}-T_{i,j,N}^{n})\times\frac{\Delta t}{\rho_i^n\ Cp_i^n}\right]+T_{i,j,N}^{n}$$

**Region 7**

Substituting the node (P,j,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n\,Cp_P^n\left[\frac{T_{P,j,N}^{n+1}-T_{P,j,N}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{P-1,j,N}^{n}-2T_{P,j,N}^{n}+T_{P+1,j,N}^{n}}{(\Delta r)^2}+\dfrac{T_{P+1,j,N}^{n}-T_{P-1,j,N}^{n}}{2P(\Delta r)^2}+\\ \dfrac{T_{P,j-1,N}^{n}-2T_{P,j,N}^{n}+T_{P,j+1,N}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{P,j,N-1}^{n}-2T_{P,j,N}^{n}+T_{P,j,N+1}^{n}}{(\Delta z)^2}\end{array}\right]$$

$$+(q'''_{m_P})^n+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,P}^{\,n}\times(T_{art}-T_{P,j,N}^{n})\right]$$

(5.212)

In Equation (5.212), $T_{P+1,j,N}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. In addition, $T_{P,j,N+1}^{n}$ is the temperature value of arm. Therefore, instead of node Hand($T_{P,j,N+1}^{n}$), Arm ($T_{P,j,1}^{n}$) should be substituted.

At Region 7, the temperature of the node (P,j,1) at next time step (n+1) is given below.

$$T^{n+1}_{P,j,N} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{array}{l} \frac{1}{(\Delta r)^2}\left( T^{n}_{P-1,j,N} - 2T^{n}_{P,j,N} + \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,j,N} \right) \\ + \frac{1}{2P(\Delta r)^2}\left[ \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,j,N} - T^{n}_{P-1,j,N} \right] \\ + \frac{T^{n}_{P,j-1,N} - 2T^{n}_{P,j,N} + T^{n}_{P,j+1,N}}{i^2(\Delta r)^2(\Delta\phi)^2} \\ + \frac{1}{(\Delta z)^2}\left[ Arm\ (T^{n}_{P,j,1}) - 2T^{n}_{P,j,N} + T^{n}_{P,j,N-1} \right] \end{array} \right] \tag{5.213}$$

$$+ \left[ (q'''_{m_P})^n \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,j,N}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,j,N}$$

**Region 8-9-10-11-12-13-14**

These region's equations can be obtained by using the same boundary conditions in neck Region (8-9-10-11-12-13-14). Therefore, the temperature equations at t=(n+1) are the same with Neck Region 8-9-10-11-12-13-14.

**Region 15**

Substituting the node (0,0,0) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.21).

$$\rho^{n}_{0}\, Cp^{n}_{0} \left[ \frac{T^{n+1}_{0,0,0} - T^{n}_{0,0,0}}{\Delta t} \right] = k^{n}_{0} \left[ 4\left( \frac{\overline{T}_1 - T^{n}_{0,0,0}}{(\Delta r)^2} \right) + \frac{T^{n}_{0,0,-1} - 2T^{n}_{0,0,0} + T^{n}_{0,0,1}}{(\Delta z)^2} \right] + (q'''_{m_0})^n \tag{5.214}$$

$$+ \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,0} \times (T_{art} - T^{n}_{0,0,0}) \right]$$

In Equation (5.214) $T^{n}_{0,0,-1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used.

At Region 15, the temperature of the node (0,0,0) at next time step (n+1) is given below.

$$T_{0,0,0}^{n+1} = \alpha_0^n \times \Delta t \times \left[ 4\left( \frac{\overline{T}_1 - T_{0,0,0}^n}{(\Delta r)^2} \right) + \frac{1}{(\Delta z)^2}\left( T_{0,0,1}^n + \left( \frac{2\Delta z \times EE}{k_0^n} \right) - 2T_{0,0,0}^n + T_{0,0,1}^n \right) \right]$$

$$+ \left[ (q_{m_0}''')^n \times \frac{\Delta t}{\rho_0^n\, Cp_0^n} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,0}^n \times (T_{art} - T_{0,0,0}^n) \times \frac{\Delta t}{\rho_0^n\, Cp_0^n} \right] + T_{0,0,0}^n \qquad (5.215)$$

**Region 16**

Substituting the node (i,1,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n\, Cp_i^n \left[ \frac{T_{i,1,0}^{n+1} - T_{i,1,0}^n}{\Delta t} \right] = k_i^n \left[ \frac{T_{i-1,1,0}^n - 2T_{i,1,0}^n + T_{i+1,1,0}^n}{(\Delta r)^2} + \frac{T_{i+1,1,0}^n - T_{i-1,1,0}^n}{2i(\Delta r)^2} + \frac{T_{i,0,0}^n - 2T_{i,1,0}^n + T_{i,2,0}^n}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{T_{i,1,-1}^n - 2T_{i,1,0}^n + T_{i,1,1}^n}{(\Delta z)^2} \right]$$

$$+ (q_{m_i}''')^n + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,i}^n \times (T_{art} - T_{i,1,0}^n) \right] \qquad (5.216)$$

In Equation (5.216), $T_{i,0,0}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T_{i,1,-1}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used.

At Region 16, the temperature of the node (i,1,0) at next time step (n+1) is given below.

$$T_{i,1,0}^{n+1} = \alpha_i^n \times \Delta t \times \left[ \frac{T_{i-1,1,0}^n - 2T_{i,1,0}^n + T_{i+1,1,0}^n}{(\Delta r)^2} + \frac{T_{i+1,1,0}^n - T_{i-1,1,0}^n}{2i(\Delta r)^2} + \frac{T_{i,2,0}^n - 2T_{i,1,0}^n + T_{i,2,0}^n}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{1}{(\Delta z)^2}\left( T_{i,1,1}^n - 2T_{i,1,0}^n + \left( \frac{2\Delta z \times EE}{k_i^n} \right) + T_{i,1,1}^n \right) \right] \qquad (5.217)$$

$$+ \left[ (q_{m_i}''')^n \times \frac{\Delta t}{\rho_i^n\, Cp_i^n} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,i}^n \times (T_{art} - T_{i,1,0}^n) \times \frac{\Delta t}{\rho_i^n\, Cp_i^n} \right] + T_{i,1,0}^n$$

**Region 17**

Substituting the node (P,1,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n\left[\frac{T_{P,1,0}^{n+1}-T_{P,1,0}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{P-1,1,0}^{n}-2T_{P,1,0}^{n}+T_{P+1,1,0}^{n}}{(\Delta r)^2}+\dfrac{T_{P+1,1,0}^{n}-T_{P-1,1,0}^{n}}{2P(\Delta r)^2}+\\ \dfrac{T_{P,0,0}^{n}-2T_{P,1,0}^{n}+T_{P,2,0}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{P,1,-1}^{n}-2T_{P,1,0}^{n}+T_{P,1,1}^{n}}{(\Delta z)^2}\end{array}\right] \quad (5.218)$$

$$+(q'''_{mP})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{blP}^{n}\times(T_{art}-T_{P,1,0}^{n})\right]$$

In Equation (5.218), $T_{P,0,0}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T_{P,1,-1}^{n}$ is also the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used. Also, $T_{P+1,1,0}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 17, the temperature of the node (P,1,0) at next time step (n+1) is given below.

$$T_{P,1,0}^{n+1}=\alpha_P^n\times\Delta t\times\left[\begin{array}{l}\dfrac{1}{(\Delta r)^2}\left(T_{P-1,1,0}^{n}-2T_{P,1,0}^{n}+\left(\dfrac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,1,0}^{n}\right)\\ +\dfrac{1}{2P(\Delta r)^2}\left[\left(\dfrac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,1,0}^{n}-T_{P-1,1,0}^{n}\right]\\ +\dfrac{T_{P,2,0}^{n}-2T_{P,1,0}^{n}+T_{P,2,0}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{1}{(\Delta z)^2}\left(T_{P,1,1}^{n}-2T_{P,1,0}^{n}+\left(\dfrac{2\Delta z\times EE}{k_i^n}\right)+T_{P,1,1}^{n}\right)\end{array}\right] \quad (5.219)$$

$$+\left[(q'''_{mP})^n\times\frac{\Delta t}{\rho_P^n Cp_P^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{blP}^{n}\times(T_{art}-T_{P,1,0}^{n})\times\frac{\Delta t}{\rho_P^n Cp_P^n}\right]+T_{P,1,0}^{n}$$

**Region 18**

Substituting the node (i, M,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n Cp_i^n\left[\frac{T_{i,M,0}^{n+1}-T_{i,M,0}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\frac{T_{i-1,M,0}^{n}-2T_{i,M,0}^{n}+T_{i+1,M,0}^{n}}{(\Delta r)^2}+\frac{T_{i+1,M,0}^{n}-T_{i-1,M,0}^{n}}{2i(\Delta r)^2}+\\ \frac{T_{i,M,0}^{n}-2T_{i,M,0}^{n}+T_{i,M,0}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{i,M,-1}^{n}-2T_{i,M,0}^{n}+T_{i,M,1}^{n}}{(\Delta z)^2}\end{array}\right] \quad (5.220)$$

$$+(q'''_{m_i})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl_i}^{n}\times(T_{art}-T_{i,M,0}^{n})\right]$$

In Equation (5.220), $T_{i,M+1,0}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. Also, $T_{i,M,-1}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used.

At Region 18, the temperature of the node (i,M,0) at next time step (n+1) is given below.

$$T_{i,M,0}^{n+1}=\alpha_i^n\times\Delta t\times\left[\begin{array}{l}\frac{T_{i-1,1,0}^{n}-2T_{i,1,0}^{n}+T_{i+1,1,0}^{n}}{(\Delta r)^2}+\frac{T_{i+1,1,0}^{n}-T_{i-1,1,0}^{n}}{2i(\Delta r)^2}+\\ \frac{T_{i,M-1,0}^{n}-2T_{i,M,0}^{n}+T_{i,M-1,0}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{1}{(\Delta z)^2}\left(T_{i,M,1}^{n}-2T_{i,M,0}^{n}+\left(\frac{2\Delta z\times EE}{k_i^n}\right)+T_{i,M,1}^{n}\right)\end{array}\right] \quad (5.221)$$

$$+\left[(q'''_{m_i})^n\times\frac{\Delta t}{\rho_i^n\ Cp_i^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl_i}^{n}\times(T_{art}-T_{i,M,0}^{n})\times\frac{\Delta t}{\rho_i^n\ Cp_i^n}\right]+T_{i,M,0}^{n}$$

**Region 19**

Substituting the node (P,M,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n \left[\frac{T_{P,M,0}^{n+1}-T_{P,M,0}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{c}\frac{T_{P-1,M,0}^{n}-2T_{P,M,0}^{n}+T_{P+1,M,0}^{n}}{(\Delta r)^2}+\frac{T_{P+1,M,0}^{n}-T_{P-1,M,0}^{n}}{2P(\Delta r)^2}+\\ \frac{T_{P,M-1,0}^{n}-2T_{P,M,0}^{n}+T_{P,M+1,0}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{P,M,-1}^{n}-2T_{P,M,0}^{n}+T_{P,M,1}^{n}}{(\Delta z)^2}\end{array}\right] \quad (5.222)$$

$$+(q'''_{m_P})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,P}^{n}\times(T_{art}-T_{P,M,0}^{n})\right]$$

In Equation (5.222), $T_{P,M+1,0}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T_{P,M,-1}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used. Also $T_{P+1,M,0}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 19, the temperature of the node (P,M,0) at next time step (n+1) is given below.

$$T_{P,M,0}^{n+1}=\alpha_P^n\times\Delta t\times\left[\begin{array}{l}\frac{1}{(\Delta r)^2}\left(T_{P-1,M,0}^{n}-2T_{P,M,0}^{n}+\left(\frac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,M,0}^{n}\right)\\ +\frac{1}{2P(\Delta r)^2}\left[\left(\frac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,M,0}^{n}-T_{P-1,M,0}^{n}\right]\\ +\frac{T_{P,M-1,0}^{n}-2T_{P,M,0}^{n}+T_{P,M-1,0}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\\ \frac{1}{(\Delta z)^2}\left(T_{P,M,1}^{n}-2T_{P,M,0}^{n}+\left(\frac{2\Delta z\times EE}{k_i^n}\right)+T_{P,M,1}^{n}\right)\end{array}\right] \quad (5.223)$$

$$+\left[(q'''_{m_P})^n\times\frac{\Delta t}{\rho_P^n\,Cp_P^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,P}^{n}\times(T_{art}-T_{P,M,0}^{n})\times\frac{\Delta t}{\rho_P^n\,Cp_P^n}\right]+T_{P,M,0}^{n}$$

**Region 20**

Substituting the node (i,j,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n Cp_i^n\left[\frac{T_{i,j,0}^{n+1}-T_{i,j,0}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\dfrac{T_{i-1,j,0}^{n}-2T_{i,j,0}^{n}+T_{i+1,j,0}^{n}}{(\Delta r)^2}+\dfrac{T_{i+1,j,0}^{n}-T_{i-1,j,0}^{n}}{2i(\Delta r)^2}+\\ \dfrac{T_{i,j-1,0}^{n}-2T_{i,j,0}^{n}+T_{i,j+1,0}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T_{i,j,-1}^{n}-2T_{i,j,0}^{n}+T_{i,j,1}^{n}}{(\Delta z)^2}\end{array}\right] \tag{5.224}$$

$$+(q'''_{m_i})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,i}^{\,n}\times(T_{art}-T_{i,j,0}^{n})\right]$$

In Equation (5.224), $T_{i,j,-1}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used.

At Region 20, the temperature of the node (i,j,0) at next time step (n+1) is given below.

$$T_{i,j,0}^{n+1}=\alpha_i^n\times\Delta t\times\left[\begin{array}{l}\dfrac{T_{i-1,j,0}^{n}-2T_{i,j,0}^{n}+T_{i+1,j,0}^{n}}{(\Delta r)^2}+\dfrac{T_{i+1,j,0}^{n}-T_{i-1,j,0}^{n}}{2i(\Delta r)^2}+\\ \dfrac{T_{i,j-1,0}^{n}-2T_{i,j,0}^{n}+T_{i,j+1,0}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{1}{(\Delta z)^2}\left(T_{i,j,1}^{n}-2T_{i,j,0}^{n}+\left(\dfrac{2\Delta z\times EE}{k_i^n}\right)+T_{i,j,1}^{n}\right)\end{array}\right] \tag{5.225}$$

$$+\left[(q'''_{m_i})^n\times\frac{\Delta t}{\rho_i^n\, Cp_i^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,i}^{\,n}\times(T_{art}-T_{i,j,0}^{n})\times\frac{\Delta t}{\rho_i^n\, Cp_i^n}\right]+T_{i,j,0}^{n}$$

**Region 21**

Substituting the node (P,j,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n \left[\frac{T_{P,j,0}^{n+1} - T_{P,j,0}^n}{\Delta t}\right] = k_i^n \left[\begin{array}{l} \dfrac{T_{P-1,j,0}^n - 2T_{P,j,0}^n + T_{P+1,j,0}^n}{(\Delta r)^2} + \dfrac{T_{P+1,j,0}^n - T_{P-1,j,0}^n}{2P(\Delta r)^2} + \\ \dfrac{T_{P,j,0}^n - 2T_{P,j,0}^n + T_{P,j,0}^n}{P^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T_{P,j,-1}^n - 2T_{P,j,0}^n + T_{P,j,1}^n}{(\Delta z)^2} \end{array}\right] \tag{5.226}$$

$$+ (q'''_{m_P})^n + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,P}^n \times (T_{art} - T_{P,j,0}^n)\right]$$

In Equation (5.226), $T_{P,j,-1}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 4 given in Table 5.2 is used. Also $T_{P+1,j,0}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

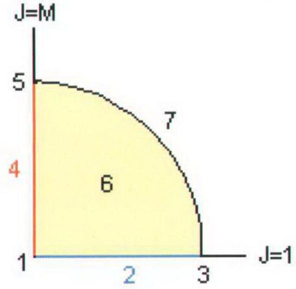
At Region 21, the temperature of the node (P,j,0) at next time step (n+1) is given below.

$$T_{P,j,0}^{n+1} = \alpha_P \times \Delta t \times \left[\begin{array}{l} \dfrac{1}{(\Delta r)^2}\left(T_{P-1,j,0}^n - 2T_{P,j,0}^n + \left(\dfrac{2\Delta r \times EE}{-k_P^n}\right) + T_{P-1,j,0}^n\right) \\ + \dfrac{1}{2P(\Delta r)^2}\left[\left(\dfrac{2\Delta r \times EE}{-k_P^n}\right) + T_{P-1,j,0}^n - T_{P-1,j,0}^n\right] \\ + \dfrac{T_{P,j-1,0}^n - 2T_{P,j,0}^n + T_{P,j+1,0}^n}{P^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{1}{(\Delta z)^2}\left(T_{P,j,1}^n - 2T_{P,j,0}^n + \left(\dfrac{2\Delta z \times EE}{k_i^n}\right) + T_{P,j,1}^n\right) \end{array}\right]$$

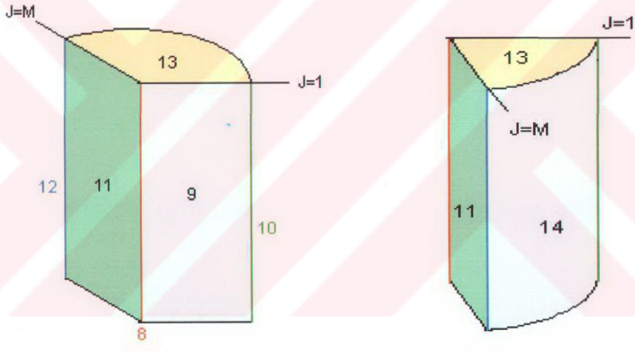
$$+ \left[(q'''_{m_P})^n \times \frac{\Delta t}{\rho_P^n Cp_P^n}\right] + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{blP}^n \times (T_{art} - T_{P,j,0}^n) \times \frac{\Delta t}{\rho_P^n Cp_P^n}\right] + T_{P,j,0}^n \tag{5.227}$$

# FOOT

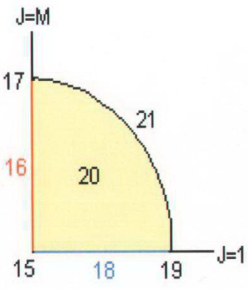
**At k=0**




**At k=1 to (N-1)**




**At k=N**



**Region 1**

Substituting the node (0,0,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.21).

$$\rho_0^n Cp_0^n\left[\frac{T_{0,0,N}^{n+1}-T_{0,0,N}^{n}}{\Delta t}\right]=k_0^n\left[4\left(\frac{\bar{T}_1^n-T_{0,0,N}^n}{(\Delta r)^2}\right)+\frac{T_{0,0,N-1}^n-2T_{0,0,N}^n+T_{0,0,N+1}^n}{(\Delta z)^2}\right]+(q'''_{m_0})^n$$

$$+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,0}^n\times(T_{art}-T_{0,0,N}^n)\right] \quad (5.228)$$

For Region 1, in Equation (5.228) $T_{0,0,N+1}^n$ is the temperature value of leg. Therefore, instead of node Foot($T_{0,0,N+1}^n$), Leg ($T_{0,0,1}^n$) should be substituted.

At Region 1, the temperature of the node (0,0,N) at next time step (n+1) is given below.

$$T_{0,0,N}^{n+1}=\alpha_0^n\times\Delta t\times\left[4\left(\frac{\bar{T}_1^n-T_{0,0,N}^n}{(\Delta r)^2}\right)+\frac{1}{(\Delta z)^2}\left(Leg(T_{0,0,1}^n)-2T_{0,0,N}^n+T_{0,0,N-1}^n\right)\right] \quad (5.229)$$

$$+\left[(q'''_{m_0})^n\times\frac{\Delta t}{\rho_0^n\,Cp_0^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,0}^n\times(T_{art}-T_{0,0,N}^n)\times\frac{\Delta t}{\rho_0^n\,Cp_0^n}\right]+T_{0,0,N}^n$$

**Region 2**

Substituting the node (i,1,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n\,Cp_i^n\left[\frac{T_{i,1,N}^{n+1}-T_{i,1,N}^{n}}{\Delta t}\right]=k_i^n\left[\begin{aligned}&\frac{T_{i-1,1,N}^n-2T_{i,1,N}^n+T_{i+1,1,N}^n}{(\Delta r)^2}+\frac{T_{i+1,1,N}^n-T_{i-1,1,N}^n}{2i(\Delta r)^2}+\\&\frac{T_{i,0,N}^n-2T_{i,1,N}^n+T_{i,2,N}^n}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{i,1,N-1}^n-2T_{i,1,N}^n+T_{i,1,N+1}^n}{(\Delta z)^2}\end{aligned}\right] \quad (5.230)$$

$$+(q'''_{m_i})^n+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,i}^n\times(T_{art}-T_{i,1,N}^n)\right]$$

In Equation (5.230), $T^{n}_{i,0,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T^{n}_{i,1,N+1}$ is the temperature value of leg. Therefore, instead of node Foot ($T^{n}_{i,1,N+1}$), Leg ($T^{n}_{i,1,1}$) should be substituted. At Region 2, the temperature of the node (i,1,N) at next time step (n+1) is given below.

$$T^{n+1}_{i,1,N}=\alpha^{n}_{i}\times\Delta t\times\left[\begin{array}{c}\dfrac{T^{n}_{i-1,1,N}-2T^{n}_{i,1,N}+T^{n}_{i+1,1,N}}{(\Delta r)^2}+\dfrac{T^{n}_{i+1,1,N}-T^{n}_{i-1,1,N}}{2i(\Delta r)^2}+\\ \dfrac{T^{n}_{i,2,N}-2T^{n}_{i,1,N}+T^{n}_{i,2,N}}{i^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T^{n}_{i,1,N-1}-2T^{n}_{i,1,N}+Leg(T^{n}_{i,1,1})}{(\Delta z)^2}\end{array}\right] \quad (5.231)$$

$$+\left[(q'''_{m_i})^{n}\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,i}\times(T_{art}-T^{n}_{i,1,N})\times\frac{\Delta t}{\rho^{n}_{i}\,Cp^{n}_{i}}\right]+T^{n}_{i,1,N}$$

**Region 3**

Substituting the node (P,1,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P}\,Cp^{n}_{P}\left[\frac{T^{n+1}_{P,1,N}-T^{n}_{P,1,N}}{\Delta t}\right]=k^{n}_{i}\left[\begin{array}{c}\dfrac{T^{n}_{P-1,1,N}-2T^{n}_{P,1,N}+T^{n}_{P+1,1,N}}{(\Delta r)^2}+\dfrac{T^{n}_{P+1,1,N}-T^{n}_{P-1,1,N}}{2P(\Delta r)^2}+\\ \dfrac{T^{n}_{P,0,N}-2T^{n}_{P,1,N}+T^{n}_{P,2,N}}{P^2(\Delta r)^2(\Delta\phi)^2}+\dfrac{T^{n}_{P,1,N-1}-2T^{n}_{P,1,N}+T^{n}_{P,1,N+1}}{(\Delta z)^2}\end{array}\right] \quad (5.232)$$

$$+(q'''_{m_P})^{n}+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,P}\times(T_{art}-T^{n}_{P,j,N})\right]$$

In Equation (5.232), $T^{n}_{P,0,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T^{n}_{P,1,N+1}$ is the temperature value of leg. Therefore, instead of node Foot ($T^{n}_{P,1,N+1}$),

$Leg(T^{n}_{P,1,1})$ should be substituted. Also $T^{n}_{P+1,1,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 3, the temperature of the node (P,1,N) at next time step (n+1) is given below.

$$T^{n+1}_{P,1,N}=\alpha^{n}_{P}\times\Delta t\times\left[\begin{array}{l}\frac{1}{(\Delta r)^2}\left(T^{n}_{P-1,1,N}-2T^{n}_{P,1,N}+\left(\frac{2\Delta r\times EE}{-k^{n}_{P}}\right)+T^{n}_{P-1,1,N}\right)\\+\frac{1}{2P(\Delta r)^2}\left[\left(\frac{2\Delta r\times EE}{-k^{n}_{P}}\right)+T^{n}_{P-1,1,N}-T^{n}_{P-1,1,N}\right]\\+\frac{T^{n}_{P,2,N}-2T^{n}_{P,1,N}+T^{n}_{P,2,N}}{P^2(\Delta r)^2(\Delta\phi)^2}+\frac{T^{n}_{P,1,N-1}-2T^{n}_{P,1,N}+Leg(T^{n}_{P,1,1})}{(\Delta z)^2}\end{array}\right]$$
$$+\left[(q'''_{m_P})^{n}\times\frac{\Delta t}{\rho^{n}_{P}Cp^{n}_{P}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,P}\times(T_{art}-T^{n}_{P,1,N})\times\frac{\Delta t}{\rho^{n}_{P}Cp^{n}_{P}}\right]+T^{n}_{P,1,N} \qquad (5.233)$$

**Region 4**

Substituting the node (i,M,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{i}Cp^{n}_{i}\left[\frac{T^{n+1}_{i,M,N}-T^{n}_{i,1,N}}{\Delta t}\right]=k^{n}_{i}\left[\begin{array}{l}\frac{T^{n}_{i-1,M,N}-2T^{n}_{i,M,N}+T^{n}_{i+1,M,N}}{(\Delta r)^2}+\frac{T^{n}_{i+1,M,N}-T^{n}_{i-1,M,N}}{2i(\Delta r)^2}+\\\frac{T^{n}_{i,M-1,N}-2T^{n}_{i,M,N}+T^{n}_{i,M+1,N}}{i^2(\Delta r)^2(\Delta\phi)^2}+\frac{T^{n}_{i,M,N-1}-2T^{n}_{i,M,N}+T^{n}_{i,M,N+1}}{(\Delta z)^2}\end{array}\right] \qquad (5.234)$$
$$+(q'''_{m_i})^{n}+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}^{n}_{bl\,i}\times(T_{art}-T^{n}_{i,1,M})\right]$$

In Equation (5.234), $T^{n}_{i,M+1,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T^{n}_{i,M,N+1}$ is the temperature value of leg. Therefore, instead of node Foot($T^{n}_{i,M,N+1}$), Leg($T^{n}_{i,M,1}$) should be substituted.

At Region 4, the temperature of the node (i,M,N) at next time step (n+1) is given below.

$$T^{n+1}_{i,M,N} = \alpha^{n}_{i} \times \Delta t \times \left[ \begin{array}{l} \dfrac{T^{n}_{i-1,M,N} - 2T^{n}_{i,M,N} + T^{n}_{i+1,M,N}}{(\Delta r)^2} + \dfrac{T^{n}_{i+1,M,N} - T^{n}_{i-1,M,N}}{2i(\Delta r)^2} + \\ \dfrac{T^{n}_{i,M-1,N} - 2T^{n}_{i,M,N} + T^{n}_{i,M-1,N}}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T^{n}_{i,M,N-1} - 2T^{n}_{i,M,N} + Leg(T^{n}_{i,M,1})}{(\Delta z)^2} \end{array} \right] \tag{5.235}$$

$$+ \left[ (q'''_{m_i})^{n} \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl_i} \times (T_{art} - T^{n}_{i,M,N}) \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + T^{n}_{i,M,N}$$

**Region 5**

Substituting the node (P,M,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P} Cp^{n}_{P} \left[ \frac{T^{n+1}_{P,M,N} - T^{n}_{P,M,N}}{\Delta t} \right] = k^{n}_{i} \left[ \begin{array}{l} \dfrac{T^{n}_{P-1,M,N} - 2T^{n}_{P,M,N} + T^{n}_{P+1,M,N}}{(\Delta r)^2} + \dfrac{T^{n}_{P+1,M,N} - T^{n}_{P-1,M,N}}{2P(\Delta r)^2} + \\ \dfrac{T^{n}_{P,M-1,N} - 2T^{n}_{P,M,N} + T^{n}_{P,M+1,N}}{P^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T^{n}_{P,M,N-1} - 2T^{n}_{P,M,N} + T^{n}_{P,M,N+1}}{(\Delta z)^2} \end{array} \right] \tag{5.236}$$

$$+ (q'''_{m_P})^{n} + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl_P} \times (T_{art} - T^{n}_{P,M,N}) \right]$$

In Equation (5.236), $T^{n}_{P,M+1,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T^{n}_{P,M,N+1}$ is the temperature value of leg. Therefore, instead of node Foot($T^{n}_{P,M,N+1}$), Leg ($T^{n}_{P,M,1}$) should be substituted. Also $T^{n}_{P+1,M,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 5, the temperature of the node (P,M,N) at next time step (n+1) is given below.

$$
T_{P,M,N}^{n+1} = \alpha_P^{n} \times \Delta t \times \left[ \begin{array}{l}
\dfrac{1}{(\Delta r)^2} \left( T_{P-1,M,N}^{n} - 2T_{P,M,N}^{n} + \left( \dfrac{2\Delta r \times EE}{-k_P^{n}} \right) + T_{P-1,M,N}^{n} \right) \\
+ \dfrac{1}{2P(\Delta r)^2} \left[ \left( \dfrac{2\Delta r \times EE}{-k_P^{n}} \right) + T_{P-1,M,N}^{n} - T_{P-1,M,N}^{n} \right] \\
+ \dfrac{T_{P,M-1,N}^{n} - 2T_{P,M,N}^{n} + T_{P,M-1,N}^{n}}{i^2 (\Delta r)^2 (\Delta \phi)^2} \\
+ \dfrac{T_{P,M,N-1}^{n} - 2T_{P,M,N}^{n} + Leg\,(T_{P,M,1}^{n})}{(\Delta z)^2}
\end{array} \right]
$$

$$
+ \left[ (q_{m_P}''')^{n} \times \frac{\Delta t}{\rho_P^{n}\, Cp_P^{n}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,P}^{n} \times (T_{art} - T_{P,M,N}^{n}) \times \frac{\Delta t}{\rho_P^{n}\, Cp_P^{n}} \right] + T_{P,M,N}^{n} \tag{5.237}
$$

**Region 6**

Substituting the node (i,j,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$
\rho_i^{n}\, Cp_i^{n} \left[ \frac{T_{i,j,N}^{n+1} - T_{i,j,N}^{n}}{\Delta t} \right] = k_i^{n} \left[ \begin{array}{l}
\dfrac{T_{i-1,j,N}^{n} - 2T_{i,j,N}^{n} + T_{i+1,j,N}^{n}}{(\Delta r)^2} + \dfrac{T_{i+1,j,N}^{n} - T_{i-1,j,N}^{n}}{2i(\Delta r)^2} + \\
\dfrac{T_{i,j-1,N}^{n} - 2T_{i,j,N}^{n} + T_{i,j+1,N}^{n}}{i^2 (\Delta r)^2 (\Delta \phi)^2} + \dfrac{T_{i,j,N-1}^{n} - 2T_{i,j,N}^{n} + T_{i,j,N+1}^{n}}{(\Delta z)^2}
\end{array} \right]
$$

$$
+ (q_{m_i}''')^{n} + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,i}^{n} \times (T_{art} - T_{i,j,N}^{n}) \right] \tag{5.238}
$$

In Equation (5.238) $T^{n}_{i,j,N+1}$ is the temperature value of arm. Therefore, instead of node Hand($T^{n}_{i,j,N+1}$), Arm($T^{n}_{i,j,1}$) should be substituted.

At Region 6, the temperature of the node (i,j,N) at next time step (n+1) is given below.

$$T^{n+1}_{i,j,N} = \alpha^{n}_{i} \times \Delta t \times \left[ \frac{T^{n}_{i-1,j,N} - 2T^{n}_{i,j,N} + T^{n}_{i+1,j,N}}{(\Delta r)^2} + \frac{T^{n}_{i+1,j,N} - T^{n}_{i-1,j,N}}{2i(\Delta r)^2} + \frac{T^{n}_{i,j-1,N} - 2T^{n}_{i,j,N} + T^{n}_{i,j+1,N}}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{T^{n}_{i,j,N-1} - 2T^{n}_{i,j,N} + \mathrm{Arm}(T_{i,j,1})}{(\Delta z)^2} \right] \tag{5.239}$$

$$+\left[ (q'''_{m_i})^n \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,j,N}) \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + T^{n}_{i,j,N}$$

**Region 7**

Substituting the node (P,j,N) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{P} Cp^{n}_{P} \left[ \frac{T^{n+1}_{P,j,N} - T^{n}_{P,j,N}}{\Delta t} \right] = k^{n}_{i} \left[ \frac{T^{n}_{P-1,j,N} - 2T^{n}_{P,j,N} + T^{n}_{P+1,j,N}}{(\Delta r)^2} + \frac{T^{n}_{P+1,j,N} - T^{n}_{P-1,j,N}}{2P(\Delta r)^2} + \frac{T^{n}_{P,j-1,N} - 2T^{n}_{P,j,N} + T^{n}_{P,j+1,N}}{P^2(\Delta r)^2(\Delta\phi)^2} + \frac{T^{n}_{P,j,N-1} - 2T^{n}_{P,j,N} + T^{n}_{P,j,N+1}}{(\Delta z)^2} \right] \tag{5.240}$$

$$+(q'''_{m_P})^n + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,j,N}) \right]$$

In Equation (5.240), $T^{n}_{P+1,j,N}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used. In addition, $T^{n}_{P,j,N+1}$ is the temperature value of leg. Therefore, instead of node Foot($T^{n}_{P,j,N+1}$), Leg ($T^{n}_{P,j,1}$) should be substituted.

At Region 7, the temperature of the node (P,j,1) at next time step (n+1) is given below.

$$T_{P,j,N}^{n+1}=\alpha_P^{n}\times\Delta t\times\left[\begin{array}{l}\dfrac{1}{(\Delta r)^2}\left(T_{P-1,j,N}^{n}-2T_{P,j,N}^{n}+\left(\dfrac{2\Delta r\times EE}{-k_P^{n}}\right)+T_{P-1,j,N}^{n}\right)\\ +\dfrac{1}{2P(\Delta r)^2}\left[\left(\dfrac{2\Delta r\times EE}{-k_P^{n}}\right)+T_{P-1,j,N}^{n}-T_{P-1,j,N}^{n}\right]\\ +\dfrac{T_{P,j-1,N}^{n}-2T_{P,j,N}^{n}+T_{P,j+1,N}^{n}}{i^2(\Delta r)^2(\Delta\phi)^2}\\ +\dfrac{1}{(\Delta z)^2}\left[Arm\,(T_{P,j,1}^{n})-2T_{P,j,N}^{n}+T_{P,j,N-1}^{n}\right]\end{array}\right] \tag{5.241}$$

$$+\left[(q'''_{m_P})^{n}\times\frac{\Delta t}{\rho_P^{n}\,Cp_P^{n}}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,P}^{n}\times(T_{art}-T_{P,j,N}^{n})\times\frac{\Delta t}{\rho_P^{n}\,Cp_P^{n}}\right]+T_{P,j,N}^{n}$$

**Region 8-9-10-11-12-13-14**

These region's equations can be obtained by using the same boundary conditions in neck Region (8-9-10-11-12-13-14). Therefore, the temperature equations at t=(n+1) are the same with Neck Region 8-9-10-11-12-13-14.

**Region 15**

Substituting the node (0,0,0) to the Finite difference form of Bio-Heat Equation at r=0 given in Equation (5.21).

$$\rho_0^{n}\,Cp_0^{n}\left[\frac{T_{0,0,0}^{n+1}-T_{0,0,0}^{n}}{\Delta t}\right]=k_0^{n}\left[4\left(\frac{\overline{T}_1-T_{0,0,0}^{n}}{(\Delta r)^2}\right)+\frac{T_{0,0,-1}^{n}-2T_{0,0,0}^{n}+T_{0,0,1}^{n}}{(\Delta z)^2}\right]+(q'''_{m_0})^{n} \tag{5.242}$$

$$+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,0}^{n}\times(T_{art}-T_{0,0,0}^{n})\right]$$

In Equation (5.242) $T^{n}_{0,0,-1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 6 given in Table 5.2 is used.

At Region 15, the temperature of the node (0,0,0) at next time step (n+1) is given below.

$$T^{n+1}_{0,0,0} = \alpha^{n}_{0} \times \Delta t \times \left[ 4\left( \frac{\bar{T}_{1} - T^{n}_{0,0,0}}{(\Delta r)^{2}} \right) + \frac{1}{(\Delta z)^{2}} \left( T^{n}_{0,0,1} - 2T^{n}_{0,0,0} + T^{n}_{0,0,1} \right) \right]$$
$$+ \left[ (q'''_{m\,0})^{n} \times \frac{\Delta t}{\rho^{n}_{0}\, Cp^{n}_{0}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,0} \times (T_{art} - T^{n}_{0,0,0}) \times \frac{\Delta t}{\rho^{n}_{0}\, Cp^{n}_{0}} \right] + T^{n}_{0,0,0} \qquad (5.243)$$

**Region 16**

Substituting the node (i,1,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho^{n}_{i}\, Cp^{n}_{i} \left[ \frac{T^{n+1}_{i,1,0} - T^{n}_{i,1,0}}{\Delta t} \right] = k^{n}_{i} \left[ \frac{T^{n}_{i-1,1,0} - 2T^{n}_{i,1,0} + T^{n}_{i+1,1,0}}{(\Delta r)^{2}} + \frac{T^{n}_{i+1,1,0} - T^{n}_{i-1,1,0}}{2i(\Delta r)^{2}} + \frac{T^{n}_{i,0,0} - 2T^{n}_{i,1,0} + T^{n}_{i,2,0}}{i^{2}(\Delta r)^{2}(\Delta \phi)^{2}} + \frac{T^{n}_{i,1,-1} - 2T^{n}_{i,1,0} + T^{n}_{i,1,1}}{(\Delta z)^{2}} \right]$$
$$+ (q'''_{m\,i})^{n} + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,1,0}) \right] \qquad (5.244)$$

In Equation (5.244), $T^{n}_{i,0,0}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T^{n}_{i,1,-1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 6 given in Table 5.2 is used.

At Region 16, the temperature of the node (i,1,0) at next time step (n+1) is given below.

$$T^{n+1}_{i,1,0} = \alpha^{n}_{i} \times \Delta t \times \left[ \frac{T^{n}_{i-1,1,0} - 2T^{n}_{i,1,0} + T^{n}_{i+1,1,0}}{(\Delta r)^{2}} + \frac{T^{n}_{i+1,1,0} - T^{n}_{i-1,1,0}}{2i(\Delta r)^{2}} + \frac{T^{n}_{i,2,0} - 2T^{n}_{i,1,0} + T^{n}_{i,2,0}}{i^{2}(\Delta r)^{2}(\Delta \phi)^{2}} + \frac{1}{(\Delta z)^{2}} \left( T^{n}_{i,1,1} - 2T^{n}_{i,1,0} + T^{n}_{i,1,1} \right) \right]$$
$$+ \left[ (q'''_{m\,i})^{n} \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,i} \times (T_{art} - T^{n}_{i,1,0}) \times \frac{\Delta t}{\rho^{n}_{i}\, Cp^{n}_{i}} \right] + T^{n}_{i,1,0} \qquad (5.245)$$

**Region 17**

Substituting the node (P,1,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n\left[\frac{T_{P,1,0}^{n+1}-T_{P,1,0}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\frac{T_{P-1,1,0}^{n}-2T_{P,1,0}^{n}+T_{P+1,1,0}^{n}}{(\Delta r)^2}+\frac{T_{P+1,1,0}^{n}-T_{P-1,1,0}^{n}}{2P(\Delta r)^2}+\\ \frac{T_{P,0,0}^{n}-2T_{P,1,0}^{n}+T_{P,2,0}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{P,1,-1}^{n}-2T_{P,1,0}^{n}+T_{P,1,1}^{n}}{(\Delta z)^2}\end{array}\right] \quad (5.246)$$

$$+(q'''_{mP})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{blP}^n\times(T_{art}-T_{P,1,0}^{n})\right]$$

In Equation (5.246), $T_{P,0,0}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 1 given in Table 5.1 is used. In addition, $T_{P,1,-1}^{n}$ is also the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 6 given in Table 5.2 is used. Also, $T_{P+1,1,0}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 17, the temperature of the node (P,1,0) at next time step (n+1) is given below.

$$T_{P,1,0}^{n+1}=\alpha_P^n\times\Delta t\times\left[\begin{array}{l}\frac{1}{(\Delta r)^2}\left(T_{P-1,1,0}^{n}-2T_{P,1,0}^{n}+\left(\frac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,1,0}^{n}\right)\\ +\frac{1}{2P(\Delta r)^2}\left[\left(\frac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,1,0}^{n}-T_{P-1,1,0}^{n}\right]\\ +\frac{T_{P,2,0}^{n}-2T_{P,1,0}^{n}+T_{P,2,0}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\frac{1}{(\Delta z)^2}\left(T_{P,1,1}^{n}-2T_{P,1,0}^{n}+T_{P,1,1}^{n}\right)\end{array}\right] \quad (5.247)$$

$$+\left[(q'''_{mP})^n\times\frac{\Delta t}{\rho_P^n\, Cp_P^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,P}^n\times(T_{art}-T_{P,1,0}^{n})\times\frac{\Delta t}{\rho_P^n\, Cp_P^n}\right]+T_{P,1,0}^{n}$$

**Region 18**

Substituting the node (i, M,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n Cp_i^n \left[\frac{T_{i,M,0}^{n+1} - T_{i,M,0}^n}{\Delta t}\right] = k_i^n \left[\begin{array}{l} \frac{T_{i-1,M,0}^n - 2T_{i,M,0}^n + T_{i+1,M,0}^n}{(\Delta r)^2} + \frac{T_{i+1,M,0}^n - T_{i-1,M,0}^n}{2i(\Delta r)^2} + \\ \frac{T_{i,M,0}^n - 2T_{i,M,0}^n + T_{i,M,0}^n}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{T_{i,M,-1}^n - 2T_{i,M,0}^n + T_{i,M,1}^n}{(\Delta z)^2} \end{array}\right] \quad (5.248)$$

$$+ (q'''_{m_i})^n + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl_i}^n \times (T_{art} - T_{i,M,0}^n)\right]$$

In Equation (5.248), $T_{i,M+1,0}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. Also, $T_{i,M,-1}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 6 given in Table 5.2 is used.

At Region 18, the temperature of the node (i,M,0) at next time step (n+1) is given below.

$$T_{i,M,0}^{n+1} = \alpha_i^n \times \Delta t \times \left[\begin{array}{l} \frac{T_{i-1,1,0}^n - 2T_{i,1,0}^n + T_{i+1,1,0}^n}{(\Delta r)^2} + \frac{T_{i+1,1,0}^n - T_{i-1,1,0}^n}{2i(\Delta r)^2} + \\ \frac{T_{i,M-1,0}^n - 2T_{i,M,0}^n + T_{i,M-1,0}^n}{i^2(\Delta r)^2(\Delta\phi)^2} + \frac{1}{(\Delta z)^2}\left(T_{i,M,1}^n - 2T_{i,M,0}^n + T_{i,M,1}^n\right) \end{array}\right] \quad (5.249)$$

$$+ \left[(q'''_{m_i})^n \times \frac{\Delta t}{\rho_i^n\, Cp_i^n}\right] + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl_i}^n \times (T_{art} - T_{i,M,0}^n) \times \frac{\Delta t}{\rho_i^n\, Cp_i^n}\right] + T_{i,M,0}^n$$

**Region 19**

Substituting the node (P,M,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n\left[\frac{T_{P,M,0}^{n+1}-T_{P,M,0}^{n}}{\Delta t}\right]=k_i^n\left[\begin{array}{l}\frac{T_{P-1,M,0}^{n}-2T_{P,M,0}^{n}+T_{P+1,M,0}^{n}}{(\Delta r)^2}+\frac{T_{P+1,M,0}^{n}-T_{P-1,M,0}^{n}}{2P(\Delta r)^2}+\\ \frac{T_{P,M-1,0}^{n}-2T_{P,M,0}^{n}+T_{P,M+1,0}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\frac{T_{P,M,-1}^{n}-2T_{P,M,0}^{n}+T_{P,M,1}^{n}}{(\Delta z)^2}\end{array}\right] \quad (5.250)$$

$$+(q'''_{m_P})^n+\left[\rho_{bl}\times Cp_{bl}\times \dot{W}_{bl\,P}^{n}\times(T_{art}-T_{P,M,0}^{n})\right]$$

In Equation (5.250), $T_{P,M+1,0}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 2 given in Table 5.1 is used. In addition, $T_{P,M,-1}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 6 given in Table 5.2 is used. Also $T_{P+1,M,0}^{n}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 19, the temperature of the node (P,M,0) at next time step (n+1) is given below.

$$T_{P,M,0}^{n+1}=\alpha_P^n\times\Delta t\times\left[\begin{array}{l}\frac{1}{(\Delta r)^2}\left(T_{P-1,M,0}^{n}-2T_{P,M,0}^{n}+\left(\frac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,M,0}^{n}\right)\\ +\frac{1}{2P(\Delta r)^2}\left[\left(\frac{2\Delta r\times EE}{-k_P^n}\right)+T_{P-1,M,0}^{n}-T_{P-1,M,0}^{n}\right]\\ +\frac{T_{P,M-1,0}^{n}-2T_{P,M,0}^{n}+T_{P,M-1,0}^{n}}{P^2(\Delta r)^2(\Delta\phi)^2}+\\ \frac{1}{(\Delta z)^2}\left(T_{P,M,1}^{n}-2T_{P,M,0}^{n}+T_{P,M,1}^{n}\right)\end{array}\right] \quad (5.251)$$

$$+\left[(q'''_{m_P})^n\times\frac{\Delta t}{\rho_P^n\, Cp_P^n}\right]+\left[\rho_{bl}\times Cp_{bl}\times\dot{W}_{bl\,P}^{n}\times(T_{art}-T_{P,M,0}^{n})\times\frac{\Delta t}{\rho_P^n\, Cp_P^n}\right]+T_{P,M,0}^{n}$$

**Region 20**

Substituting the node (i,j,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_i^n Cp_i^n \left[\frac{T_{i,j,0}^{n+1} - T_{i,j,0}^{n}}{\Delta t}\right] = k_i^n \left[\begin{array}{l} \dfrac{T_{i-1,j,0}^n - 2T_{i,j,0}^n + T_{i+1,j,0}^n}{(\Delta r)^2} + \dfrac{T_{i+1,j,0}^n - T_{i-1,j,0}^n}{2i(\Delta r)^2} + \\ \dfrac{T_{i,j-1,0}^n - 2T_{i,j,0}^n + T_{i,j+1,0}^n}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T_{i,j,-1}^n - 2T_{i,j,0}^n + T_{i,j,1}^n}{(\Delta z)^2} \end{array}\right] \quad (5.252)$$

$$+ (q'''_{m_i})^n + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,i}^n \times (T_{art} - T_{i,j,0}^n)\right]$$

In Equation (5.252), $T_{i,j,-1}^n$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 6 given in Table 5.2 is used.

At Region 20, the temperature of the node (i,j,0) at next time step (n+1) is given below.

$$T_{i,j,0}^{n+1} = \alpha_i^n \times \Delta t \times \left[\begin{array}{l} \dfrac{T_{i-1,j,0}^n - 2T_{i,j,0}^n + T_{i+1,j,0}^n}{(\Delta r)^2} + \dfrac{T_{i+1,j,0}^n - T_{i-1,j,0}^n}{2i(\Delta r)^2} + \\ \dfrac{T_{i,j-1,0}^n - 2T_{i,j,0}^n + T_{i,j+1,0}^n}{i^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{1}{(\Delta z)^2}\left(T_{i,j,1}^n - 2T_{i,j,0}^n + T_{i,j,1}^n\right) \end{array}\right] \quad (5.253)$$

$$+ \left[(q'''_{m_i})^n \times \frac{\Delta t}{\rho_i^n\, Cp_i^n}\right] + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,i}^n \times (T_{art} - T_{i,j,0}^n) \times \frac{\Delta t}{\rho_i^n\, Cp_i^n}\right] + T_{i,j,0}^n$$

**Region 21**

Substituting the node (P,j,0) to the Finite difference form of Bio-Heat Equation given in Equation (5.16).

$$\rho_P^n Cp_P^n \left[\frac{T_{P,j,0}^{n+1} - T_{P,j,0}^{n}}{\Delta t}\right] = k_i^n \left[\begin{array}{l} \dfrac{T_{P-1,j,0}^n - 2T_{P,j,0}^n + T_{P+1,j,0}^n}{(\Delta r)^2} + \dfrac{T_{P+1,j,0}^n - T_{P-1,j,0}^n}{2P(\Delta r)^2} + \\ \dfrac{T_{P,j,0}^n - 2T_{P,j,0}^n + T_{P,j,0}^n}{P^2(\Delta r)^2(\Delta\phi)^2} + \dfrac{T_{P,j,-1}^n - 2T_{P,j,0}^n + T_{P,j,1}^n}{(\Delta z)^2} \end{array}\right] \quad (5.254)$$

$$+ (q'''_{m_P})^n + \left[\rho_{bl} \times Cp_{bl} \times \dot{W}_{bl\,P}^n \times (T_{art} - T_{P,j,0}^n)\right]$$

In Equation (5.254), $T^{n}_{P,j,-1}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 6 given in Table 5.2 is used. Also $T^{n}_{P+1,j,0}$ is the fictitious temperature. In order to eliminate this fictitious node Boundary Condition 3 given in Table 5.1 is used.

At Region 21, the temperature of the node (P,j,0) at next time step (n+1) is given below.

$$T^{n+1}_{P,j,0} = \alpha^{n}_{P} \times \Delta t \times \left[ \begin{array}{l} \frac{1}{(\Delta r)^2}\left( T^{n}_{P-1,j,0} - 2T^{n}_{P,j,0} + \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,j,0} \right) \\ + \frac{1}{2P(\Delta r)^2}\left[ \left( \frac{2\Delta r \times EE}{-k^{n}_{P}} \right) + T^{n}_{P-1,j,0} - T^{n}_{P-1,j,0} \right] \\ + \frac{T^{n}_{P,j-1,0} - 2T^{n}_{P,j,0} + T^{n}_{P,j+1,0}}{P^2(\Delta r)^2(\Delta\phi)^2} + \frac{1}{(\Delta z)^2}\left( T^{n}_{P,j,1} - 2T^{n}_{P,j,0} + T^{n}_{P,j,1} \right) \end{array} \right] \tag{5.255}$$

$$+ \left[ (q'''_{m_P})^{n} \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + \left[ \rho_{bl} \times Cp_{bl} \times \dot{W}^{n}_{bl\,P} \times (T_{art} - T^{n}_{P,j,0}) \times \frac{\Delta t}{\rho^{n}_{P}\, Cp^{n}_{P}} \right] + T^{n}_{P,j,0}$$