FLAVOR VIOLATION IN SUPERSYMMETRY

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ABSTRACT

FLAVOR VIOLATION IN SUPERSYMMETRY

This thesis work is meant as an introduction to supersymmetry and its phenomenological implications for flavor-changing phenomena. After a survey of the basic features of the standard model of electroweak interactions, it continues with a through definition and basic derivation of the fundamental concepts of supersymmetric field theories, including superspace, superfield and superpotential.

In a supersymmetric theory, all interactions are to be symmetric under the exchange of bosons and fermions – the superpartners. However, supersymmetry must be an explicitly yet softly broken symmetry of nature, and supersymmetry breaking parameters, the so-called soft terms, give rise to various phenomena observable at present and future experiments. The mixing among different flavors of matter – the flavor violation – is one such phenomenon which exhibits a strong dependence on the structure of the soft terms. In particular, decoupling of superpartners from the particle spectrum at a threshold energy near the ultraviolet scale of the standard model induces sizeable corrections to flavor violating interactions. These corrections are strong enough to disqualify an otherwise viable high-scale flavor model by a confrontation with experiments at low energy. This thesis work focusses a class of flavor models, following from strings or supergravity, and provides a through analysis of their sensitivities to supersymmetric threshold corrections.

ÖZET

SÜPERSİMETRİDE ÇEŞNİ KIRINIMI

Bu tez çalışması süpersimetriye ve süpersimetrinin çeşni degişimi olayındaki implikasyonlarına giriş olarak hazırlanmıştır. Standard Model elektro-zayıf etkileşimlerin temel özelliklerinin incelenmesinden sonra süpersimetrik alan teorilerinin süperuzay, süperalan ve süperpotansiyel gibi temel kavramlarının tanımı ve basit türetimleri çalışılmıştır.

Süpersimetrik bir teoride tüm etkileşimler bosonlar ve fermionların (süpereşlerin) degişimi altında simetrik kalmak durumundadırlar. Buna ragmen süpersimetri doganın açıkça ve yumuşakça kırılmış bir simetrisi olmalıdır ve yumuşak (soft) terimler olarak adlandırılan süpersimetri kırınım parametreleri günümüz ve gelecek deneylerde çeşitli gözlemlenebilir fenomenlerin ortaya çıkmasına sebep olacaktır. Maddenin farklı çeşnilerinin birbirine karışması -çeşni kırınımı- yumuşak (soft) terimlere güçlü baglılık gösteren olaylardan biridir. Özel olarak Standard Model'in morötesi skalasına yakın eşik enerjisinde süpereşlerin parçacık spektrumundan ayrışması çeşni ihlal eden etkileşimlere önemli düzeltmeler getirmektedir. Bu düzeltmeler, aksi taktirde geçerli olan yüksek skala çeşni modelini düşük enerjilerdeki deneylerle geçersiz kılacak kadar güçlüdür.

Bu tez çalışmasında çeşni modellerinin bir sınıfına odaklanarak süpersimetrik eşik düzeltmelerine duyarlılıkları analiz edilmiştir.

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CHAPTER 1

INTRODUCTION

The matter forming up the universe we live in is made up of tiny building blocks held together by appropriate forces. Therefore, a complete picture of nature will arise only after we discover what types of matter (the flavor) and what kinds of forces exist at short distances. As dictated by the quantum theory, the theory of subatomic systems, for probing physical systems of smaller and smaller size one needs to make characteristic energy of scattering processes higher and higher. Hence, the physics of fundamental particles is a high-energy physics.

The so-called standard model of particle physics (SM) is an inside story of the atom, or better, the nucleus. One one hand, it provides a consistent model of how known hadrons (*e.g.* neutron, proton, pion and many more mesons and baryons) are formed from a few fundamental particles – the quarks. On the other hand, it explains how two seemingly unrelated phenomena, electromagnetism and radioactivity, can be tied up to a common origin. It is these virtues of the model and it is its success against numerous experiments that have been performed so far that make it 'the standard model' (Weinberg 1967).

According to SM, there exist two main classes of matter: six leptons (electron, muon, tau lepton and their associated neutrinos) and six quarks (up, down, charm, strange, top and bottom). The quarks form the known kinds of hadrons (*i.e.* mesons and baryons) by a special force that binds them together: the strong force. This strong force is a confining force in that quarks are never liberated as free particles; they are always imprisoned in hadrons.

On the other hand, as we know well from radioactivity, neutron in an unstable nucleus gets converted into proton accompanied by electron and its neutrino. This phenomenon, the radioactivity, requires a distinct force which operates on both leptons and hadrons: the weak force.

Finally, electromagnetic force mediates interactions among the charged matter: electron, muon, tau lepton and all six quarks. The neutrinos are electrically neutral. The quarks posses fractional electric charge. For instance, up quark has got $-2/3$ of the electron's electric charge whereas electric charge of the down quark equals $1/3$ of electron's electric charge.

One of the most important aspects of the SM is that it is a gauge theory *i.e.* quark and lepton fields exhibit exact invariance under a set of symmetry groups. In fact, each of the aforementioned force fields stem from the requirement of local gauge invariance which cannot be implemented unless a mediator – a gauge field – is introduced. In this sense, strong force which binds quarks together follows from invariance of entire SM langangian under the rephasings $\exp\left\{i\sum_{a=1}^{8} f_a(x)\lambda_a\right\}$ where $f_a(x)$ are local functions and λ_a are 3×3 hermitian unit-determinant matrices forming special unitary SU(3) group. This group, the color gauge group $\mathrm{SU}(3)_c$, gives rise to colorless objects, the hadrons, thanks to its confining nature (Wilson 1974). Under this gauge group, each quark is assigned three distinct colors (blue, green, red) not related to electromagnetic spectrum.

The weak force responsible for radioactivity also follows from a gauge principle. Knowing that weak interactions violate parity (Wu et al. 1957), this gauge principle is expected to differentiate between left-handed (the massless particles whose momenta are parallel to their spins) and right-handed (the massless matter whose momenta are anti-parallel to their spins) matter. In fact, weak force is based on invariance of left-handed quarks and leptons under the rephasings $\exp\left\{i\sum_{i=1}^{3} g_i(x)\sigma_i\right\}$ where $g_i(x)$ are functions of coordinates and σ_i are 2×2 hermitian unit-determinant matrices forming special unitary SU(2) group. This group, the isospin group $\mathrm{SU}(2)_L$ correctly generates the nuclear reactions which lead to radioactivity. Under this gauge group, left-handed matter is assigned into doublets. For instance, left-handed up and down quarks and left-handed electron-neutrino and electron form doublets.

Finally, electromagnetism follows form a gauge principle, too. However, the invariance implemented in the SM is based not on the electric charge directly but on hypercharge *i.e.* the difference between particle's electric charge and isospin. For instance, left-handed quark doublets possess $1/3$ hypercharge and left-handed leptons doublets -1. On the other hand, right-handed top quark obtains $4/3$, right-handed tau lepton -2 and right-handed strange quark $-2/3$ hypercharge. Hypercharge is a local invariance of the SM *i.e.* its lagrangian is invariant under rephasing $\exp\{ih(x)Y\}$ of a matter field with hypercharge Y. This invariance forms a unitary one-parameter gauge group, $U(1)_Y$.

In summary, gauge principle is a fundamental notion for explaining fundamental forces in nature, and SM is a gauge theory based on $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge invariance. This invariance comprises all forces in nature, except gravity.

The SM has shown excellent agreement with all the experiments conducted so far. However, it has got a number of problems whose solutions might require a further yet-to-be found extension. These problems can be summarized as follows:

Problem 1 (Gauge Hierarchy Problem): One of the most important features of the SM is the presence of a mass generation mechanism. Indeed, when $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ is an exact invariance of the theory none of the quarks and leptons possesses mass. Their masses are generated via the Higgs field (an $SU(2)_L$ doublet introduced with the same philosophy as Ginzburg-Landau order parameter needed for explaining superconductivity) whose most likely value is zero in the symmetric vacuum (no massive matter) and is non-zero in the broken vacuum. This mismatch between the symmetries of the lagrangian and vacuum state gives rise to spontaneous breakdown of $SU(2)_L \otimes U(1)_Y$ down to electromagnetism (represented by $U(1)_Q$ invariance), and generates masses of quark and leptons while giving rise to a massive neutral vector particle Z and a charged vector particle W. This mechanism (Higgs 1966, Kibble 1967) works consistently and admits a clear physical interpretation only at the classical level, however. Indeed, once quantum mechanical corrections are included one finds that the order parameter sector, the Higgs sector, is destabilized completely. This destabilization is so strong that the 'weak force' becomes as weak as gravity which is in obvious contradiction with experiments *e.g.* the atomic bomb. This quantum anomaly of the Higgs sector can disqualify SM to be an ultimate description of nature, and hence the lesson: *it is necessary to invent a mechanism to stabilize the SM Higgs sector against wild quantum fluctuations.*

Problem 2 (Flavor Problem): The second issue concerns masses of leptons and quarks. Indeed, in the SM these particles receive their masses from the condensation of the Higgs field *i.e.* via its non-vanishing vacuum expectation value. However, the experimentally well-established masses and mixings among quarks (as well as those of the leptons) are neither predicted nor constrained by the model. This problem, the flavor problem, must be understood within the extension of the

SM which solves Problem 1 above. Saying differently, any extension of the SM must be analyzed from the point of view of flavor problem *i.e.* its flavor violation potential must be determined.

In addition to these two problems, the SM may be criticized by its lack of explaining the following phenomena:

- Though electromagnetism and weak force are tied up to a common origin the strong force is left aside. Is there a way of unifying strong force with others? Moreover, gravity is left aside completely. Is there a way of unifying all these four forces of nature into one single force?

- Observations show that approximately 20% of matter in the universe is a non-shining one. Standard model does not have candidate for this. Can it be extended to cover this important component of matter?

- Though fundamental equations are symmetric between matter and anti-matter, the universe we live in seems not so. We are made up of matter but anti-matter is missing. Can SM explain how this asymmetry has arisen?

In the next chapter we will give a detailed discussion of the two main problems above. Then, in Chapter III, we will use observations made in SM as motivations for introducing a new symmetry, the supersymmetry. In Chapter IV we will specialize to minimal supersmmetric standard model – a common prototype model to discuss phenomenological implications of supersymmetry. In Chapter V we will discuss flavor problem in supersymmetric framework. In particular, we will discuss sensitivity of high-scale flavor structures to radiative corrections.

CHAPTER 2

HIERARCHY PROBLEMS IN THE SM

2.1. Flavor Problem in the SM

In the SM there exist three families (generations) of fermions. Flavor physics describes interactions that distinguish between the fermion generations. The fermions experience two types of interactions which are called gauge and Yukawa interactions. Gauge interactions are responsible for where two fermions couple to a gauge boson, and Yukawa interactions responsible for where two fermions couple to a scalar. Within the Standard Model framework (Glashow 1961, Weinberg 1967), there are twelve gauge bosons, related to gauge symmetry which are based on group properties. Now, we can divide behavior of interactions into two categories: interaction and mass bases. In the interaction basis, gauge interactions, each factor group factor in $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ has a single coupling, are diagonal. According to this, the interaction eigenstates have no gauge couplings between fermions of different generations, as well. On the other hand, Yukawa interactions are quite complicated in the interaction basis, the interaction eigenstates do not have well-defined masses since there are Yukawa couplings that involve fermions of different generations. Flavor physics is related to part of the SM that depends on the Yukawa couplings. In the mass basis, Yukawa interactions are diagonal. The mass eigenstates have well defined mass. However, the gauge interactions related to spontaneously broken symmetries (appendix B) can be quite complicated in the mass basis. In particular, the $SU(2)_L$ gauge couplings are not diagonal, that is they *mix quarks of different generations*. Therefore, flavor problem in the SM concerns size and structure of mixings among different quark flavors *i.e.* flavor violation.

There exist 6 different quark flavors u, d, s, c, b, t, 3 different charged leptons e, μ, τ and their corresponding neutrinos ν_e, ν_μ, ν_τ. We can nicely include all these particles into the SM framework, by organizing them into 3 families of quarks and leptons. Thus, we have 3 nearly identical copies of the same $SU(2)_L \otimes U(1)_Y$ structure, with masses as the only difference (further details (Novaes 1999)).

Let us consider the general case of N generations of fermions, and denote ν'_j, l'_j, u'_j, d'_j the members of the weak family j $(j = 1, \ldots, N)$, with definite transformation properties under the gauge group. Owing to the fermion replication, a large variety of fermion–scalar couplings are allowed by the gauge symmetry. The most general Yukawa Lagrangian has the form

$$\mathcal{L}_Y = \overline{Q^I_{Lj}} Y^d_{jk} H d^I_{kR} + \overline{Q^I_{Lj}} Y^u_{jk} \tilde{H} u^I_{kR} + \overline{L^I_{Lj}} Y^\ell_{jk} H \ell^I_{kR}, \tag{2.1}$$

$$Q^I_{Li}(3,2)_{+1/6}, \quad u^I_{Ri}(3,1)_{+2/3}, \quad d^I_{Ri}(3,1)_{-1/3}, \quad L^I_{Li}(1,2)_{-1/2}, \quad \ell^I_{Ri}(1,1)_{-1}. \tag{2.2}$$

In these notations basically mean that, for example, the left-handed quarks, Q^I_L,

are in a triplet (3) of the $SU(3)$ group, a doublet (2) of $SU(2)$ matrix properties and carry hypercharge $Y = Q_{\rm EM} - T_3 = +1/6$,where $H(1,2)_{+1/2}$ is the Standard Model Higgs doublet, and $\tilde{H} = i\sigma_2 H^*$. The index I denotes *interaction eigenstates*. The index $i = 1, 2, 3$ is the *flavor* (or generation) index, explicit form eq(1.16) lagrangian,

$$\begin{aligned} \mathcal{L}_Y \;\; &= \;\; \sum_{jk} \left\{ \left(\bar{u}'_j, \bar{d}'_j\right)_L \left[Y^{(d)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} \; + \; Y^{(u)}_{jk} \begin{pmatrix} \phi^{(0)*} \\ -\phi^{(-)} \end{pmatrix} u'_{kR} \right] \right. \\ & \qquad \left. + \;\; \left(\bar{\nu}'_j, \bar{l}'_j\right)_L \, Y^{(l)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l'_{kR} \right\} \; + \; \text{h.c.}, \end{aligned} \tag{2.3}$$

where encodes Yukawa matrices $Y^{(d)}_{jk}$, $Y^{(u)}_{jk}$ and $Y^{(l)}_{jk}$ (up quarks, down quarks and and leptons respectively) each being 3×3 non-hermitian matrix in the space of fermion flavors.

The Standard Model gauge interactions do not distinguish between the different generations. Another way to state this is to say that the gauge interactions are flavor-blind. The strength of the gauge interactions depends on the gauge quantum numbers given in and not on the flavor index i. Most important for our purposes, the interaction of the $SU(2)_L$ gauge bosons (W^a_μ, $a = 1, 2, 3$) with quarks is given by

$$-\mathcal{L}_W = \frac{g}{2} \overline{Q^I_{Li}} \gamma^\mu \tau^a Q^I_{Li} W^a_\mu. \tag{2.4}$$

The 4×4 matrix γ^μ operates in Lorentz space (it describes the combination of two spin-1/2 quark fields and one spin-1 gauge boson field into a Lorentz scalar) and the

2×2 matrix τ^a operates in the $SU(2)_{\rm L}$ space (it describes the combination of the two quark doublets and the W^a-triplet into an $SU(2)_{\rm L}$ singlet). The coupling $\overline{Q^I_{Li}} Q^I_{Li}$ can be equivalently written as $\overline{Q^I_{Li}} \mathbf{1}_{ij} Q^I_{Lj}$ where the 3×3 unit matrix $\mathbf{1}$ operates in flavor space and makes the universality of the gauge interactions manifest.

The spontaneous symmetry breaking (SSB) mechanism generates the masses of the weak gauge bosons, and gives rise to the appearance of a physical scalar particle in the model, the so-called "Higgs". The fermion masses and mixings are generated through the SSB (the details can be found in (Appendix B) and (Pich 2005)).

To transform to the mass basis, one has to take into account spontaneous symmetry breaking . Within the Standard Model this breaking is the result of a vacuum expectation value assumed by the neutral component of the Higgs doublet, $\phi^0 = \frac{v}{\sqrt{2}}$ with the electroweak breaking scale of order $v \approx 246$ GeV. Upon the replacement $\Re(\phi^0) \rightarrow (v + H^0)/\sqrt{2}$, the Yukawa interactions give rise to mass terms:

$$\mathcal{L}_M = (M_d)_{ij} \overline{d^I_{Li}} d^I_{Rj} + (M_u)_{ij} \overline{u^I_{Li}} u^I_{Rj} + (M_\ell)_{ij} \overline{\ell^I_{Li}} \ell^I_{Rj}, \tag{2.5}$$

where

$$M_f = \frac{v}{\sqrt{2}} Y^f, \tag{2.6}$$

The mass basis corresponds, by definition, to diagonal mass matrices. We can always find unitary matrices V_{fL} and V_{fR} such that

$$V_{fL} M_f V^\dagger_{fR} = M_f^{\rm diag} \tag{2.7}$$

with $M_f^{\rm diag}$ diagonal and real. The mass eigenstates are then identified as

$$\begin{aligned} d_{Li} &= (V_{dL})_{ij} d^I_{Lj} \quad & d_{Ri} &= (V_{dR})_{ij} d^I_{Rj} \\ u_{Li} &= (V_{uL})_{ij} u^I_{Lj} \quad & u_{Ri} &= (V_{uR})_{ij} u^I_{Rj} \\ l_{Li} &= (V_{lL})_{ij} l^I_{Lj} \quad & l_{Ri} &= (V_{lR})_{ij} l^I_{Rj} \\ \nu_{Li} &= (V_{\nu L})_{ij} \nu^I_{Lj} \end{aligned} \tag{2.8}$$

Note that, since the neutrinos are massless, $V_{\nu L}$ is arbitrary.

The charged current interactions (that is the interactions of the charged $SU(2)_{\rm L}$ gauge bosons $W^{\pm}_{\mu} = \frac{1}{\sqrt{2}}(W^1_{\mu} \mp iW^2_{\mu})$), which in the interaction basis are described by ;

$$-\mathcal{L}_{W^{\pm}} = \frac{g}{\sqrt{2}}\overline{u^I_{Li}}\gamma^{\mu}d^I_{Lj}W^+_{\mu} + \text{h.c.}. \tag{2.9}$$

The charged current interaction for quarks in the mass basis is:

$$-\mathcal{L}_{W^{\pm}} = \frac{g}{\sqrt{2}}\overline{u_{Li}}V_{uL}\gamma^{\mu}V^{\dagger}_{dL}d_{Lj}W^+_{\mu} + \text{h.c.}. \tag{2.10}$$

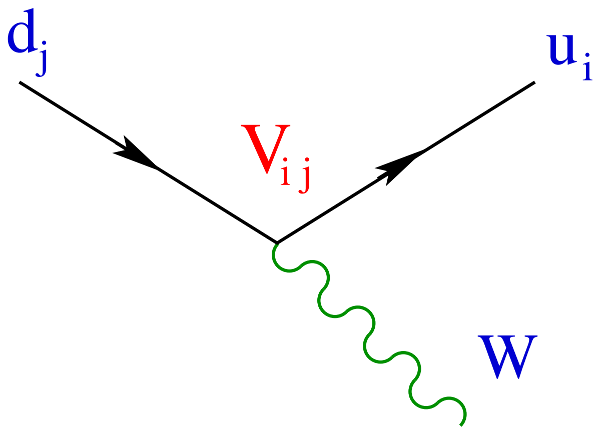

Figure 2.1: Feynman graphs illustrating flavor-violating $W^{\pm}$ couplings.

The 3×3 unitary matrix,

$$V_{\rm CKM} = V_{uL}V^{\dagger}_{dL}, \tag{2.11}$$

is called the Cabibbo-Kobayashi-Maskawa matrix (CKM) *mixing L matrix* for quarks (Cabibbo 1963, Kobayashi and Maskawa 1973). It depends on four parameters: three real angles and one phases. The CKM matrix is a unitary matrix which contains information on the strength of flavor changing decays. Technically, it specifies the mismatch of quantum states of quarks when they propagate freely and when they take part in the weak interactions.

As a result of the fact that V_{CKM} is not diagonal, the $W^{\pm}$ gauge bosons can couple to left-handed quark (mass eigenstates) of different generations. Within the Standard Model, this is the only source of *flavor changing* interactions (Pich 1996, Nir 1998). In general, if massive neutrinos are included in the model then lepton sector also exhibits non-trivial flavor mixings. Experiments are meson factories have already measured all entries of V_{CKM} to a fairly good precision (Eidelman et al. 2004).

The problem is that the SM does not provide an explanation for hierarchy of quark and lepton masses as well as mixing among different quark flavors. As will be seen in Chapter IV, this is also a problem in supersymmetric models, and it is necessary to determine if the model passes tests provided by the existing experimental results.

2.2. The Gauge Hierarchy Problem in the SM

Although the Standard Model provides a very well description to known phenomena, it seems that the Standard Model is still insufficient. It is not the complete story. There are some problems that are not solved with this model, such as the quadratic divergences. Particles receive some quantum corrections from loops. Let us look at these quantum corrections.

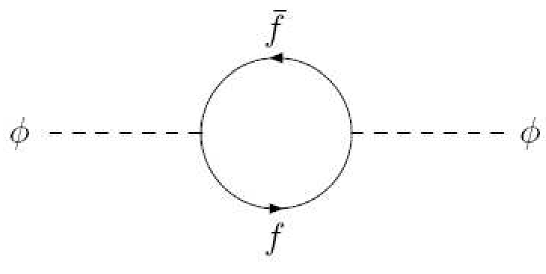

Figure 2.2: A fermion loop contribution to the Higgs boson in the Standard Model.

While fermion masses receive radiative corrections from diagrams, these corrections are logarithmically divergent.

$$\delta m_f \simeq \frac{3\alpha}{4\pi} m_f \ln(\Lambda^2/m_f) \tag{2.12}$$

where Λ is an ultraviolet cutoff. It is the highest energy scale in the calculation. The SM, being an effective theory, is valid below this cutoff scale. Above the cutoff scale some unknown new physics takes place. We do not know what this physics is like? If the SM is a true description of Nature all the way up to Planck scale then $\Lambda \sim M_P$, these corrections are still small,

$$\delta m_f \leq m_f \tag{2.13}$$

However, scalar masses receive quantum corrections from couplings, these corrections are quadratically divergent. When we ignore the gravitational interactions, scalar masses accept the largest quantum corrections.

$$\delta m_H^2 \simeq g_f^2 \int d^4k \frac{1}{k^2} \sim O(\frac{\alpha}{4\pi}\Lambda^2) \tag{2.14}$$

$$\delta m_H^2 \simeq g^2 \int d^4k \frac{1}{k^2} \sim O(\frac{\alpha}{4\pi}\Lambda^2) \tag{2.15}$$

$$\delta m_H^2 \simeq \lambda \int d^4k \frac{1}{k^2} \sim O(\frac{\alpha}{4\pi}\Lambda^2) \tag{2.16}$$

where g_f is from fermion coupling, g is from gauge boson coupling, and λ is from quartic scalar couplings. We expect $M_W \sim m_H$,however $\Lambda \gg M_W$. That is

$$\delta m_H^2 \gg m_H^2 \tag{2.17}$$

The fact that the ratio $\frac{M_P}{M_W}$ is very large poses the hierarchy problem. There are a few technics to control the hierarchy problem and cancelling divergences (Drees 1996, Martin 1997). But they are not simple solutions. An alternative and simpler solution to this problem exist if we introduce new particles with similar masses and appropriate couplings but with a half unit spin difference.Then the δm_H^2 is

$$m_H^2 \simeq O(\frac{\alpha}{4\pi})(\Lambda^2 + m_B^2) - O(\frac{\alpha}{4\pi})(\Lambda^2 + m_F^2) \quad = \quad O(\frac{\alpha}{4\pi})(m_B^2 - m_F^2) \tag{2.18}$$

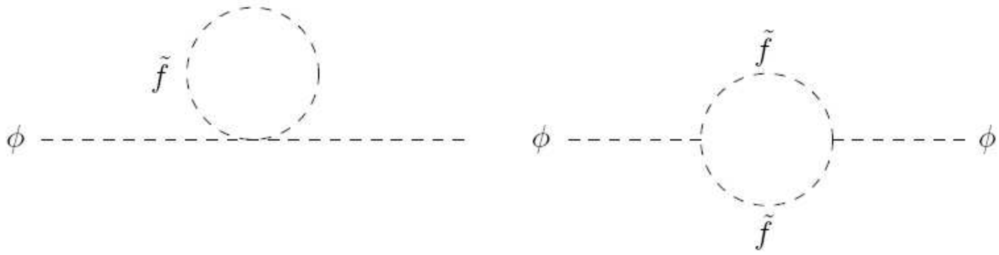

Figure 2.3: Scalar fermion loop contributions to the Higgs self energy.

If the bosons and fermions all have the same masses, then the radiative corrections vanish identically. The only requirement for the hierarchy is preserving the weak scale, so we need only this requirement;

$$| m_B^2 - m_F^2 | \leq 1TeV^2 \tag{2.19}$$

The lesson one learns from these observations is that, scalar masses can be protected against wild radiative corrections if the scalar field under concern couples to fermions and bosons in a correlated fashion *i.e.* couplings to fermions and bosons must be related in a highly tuned way, and moreover, fermion and scalar masses must be equal. Enforcement of such relations on fermionic and bosonic fields is a fine-tuning and thus an unwanted property. However, this fine-tuning impasse would be avoided if these aforementioned relations derive from a symmetry principle. The symmetry principle with these properties is nothing but supersymmetry – a symmetry that exchanges fermions and bosons. In the next chapter we will discuss implications and relevance of this symmetry with motivation obtained by observations made of scalar masses.

CHAPTER 3

SUPERSYMMETRY BASICS

It is easily seen from the examples which are previous chapter that, a new symmetry is needed for stabilizing scalar (the Higgs) mass against violent quantum fluctuations. That must be such a symmetry theory that can protect the Higgs mass from quadratically divergent corrections.

This symmetry model must connect fermions and bosons. There must be a generator of this symmetry that turn a bosonic state into a fermionic one, vice versa. If this were possible, it would imply that bosons and fermions are merely different manifestations of the same state, and in some sense would correspond to an ultimate form of unification. For a long time, it was believed that such a symmetry transformation was not possible to implement physical theories. At present, however, we know that such transformations can be defined, and, in fact, there exist theories that are invariant under such transformations. These transformations are known as Supersymmetry (SUSY) transformations. This new symmetry, which mixes bosons and fermions, is called *Supersymmetry* (*Peskin* 1996, $Martin$ 1997, *Aitchison* 2005).

Let the operator Q be generator of such transformations:

$$Q \mid Boson\rangle \ = \ \mid Fermion\rangle \tag{3.1}$$

$$Q \mid Fermion\rangle \ = \ \mid Boson\rangle \tag{3.2}$$

An exciting feature of the Supersymmetry algebra is that there exist quantum field theories in which the supersymmetry generators Q may be represented in terms of conserved currents ${J_\alpha}^m$:

$$Q_\alpha \ = \ \int d^3 {J_\alpha}^0 \tag{3.3}$$

The currents ${J_\alpha}^m$ are local expressions of the field operators. The algebra is satisfied because of the canonical equal-time commutation relations, and Hilbert space spans a representation of the supersymetry algebra (Wess and Bagger 1992). In parallel to the idea, it is natural to ask if our current quantum field theories exploit all the kinds of symmetries which could exist, consistent with Lorentz invariance. Consider the symmetry "charges" that we are familiar with in the SM, for example an electromagnetic charge of the form

$$Q_\alpha = e\int d^3\psi^\dagger\psi \tag{3.4}$$

or an $SU(2)$ charge (isospin operator) of the form

$$T = g\int d^3\psi^\dagger(\tau/2)\psi \tag{3.5}$$

All such symmetry operators are themselves Lorentz scalars. This implies that when they act on a state of definite spin J, they cannot alter that spin:

$$Q\mid J\rangle = \mid \mathrm{same}J, \mathrm{possibly\ different\ member\ of\ symmetry\ multiplet}\rangle \tag{3.6}$$

It is known that one vector "charge", the 4 momentum operator P_μ generates space-time displacements, and its eigenvalues are conserved 4-momenta. There is also the angular momentum operator represented by an antisymmetric tensor $M_{\mu\nu}$. At this point one can ask if there is a conserved charge $Q_{\mu\nu}$ corresponding to angular momentum operator. To see this, we can consider letting such a charge act on a single particle state with 4-momentum (Ellis 2002) p:

$$Q_{\mu\nu}\mid p\rangle = \mid(\alpha p_\mu p_\nu + \beta g_{\mu\nu})\mid p\rangle \tag{3.7}$$

whose right-hand side follows from the covariance arguments. Now consider a two particle state $\mid p^{(1)}, p^{(2)}\rangle$, and assume that $Q_{\mu\nu}$'s are additive, conserved, and act only one particle at a time, like other known charges. Then

$$Q_{\mu\nu}\mid p^{(1)}, p^{(2)}\rangle = \mid(\alpha(p^{(1)}_\mu p^{(1)}_\nu + p^{(2)}_\mu p^{(2)}_\nu + 2\beta g_{\mu\nu})\mid p^{(1)}, p^{(2)}\rangle \tag{3.8}$$

In an elastic process of the form $1+2\rightarrow 3+4$ we will then need (from conservation of the eigenvalues)

$$p^{(1)}_\mu p^{(1)}_\upsilon + p^{(2)}_\mu p^{(2)}_\upsilon = p^{(3)}_\mu p^{(3)}_\upsilon + p^{(4)}_\mu p^{(4)}_\upsilon \tag{3.9}$$

But we have also 4-momentum conservation:

$$p^{(1)}_\mu + p^{(2)}_\mu = p^{(3)}_\mu + p^{(4)}_\mu \tag{3.10}$$

Hence a common solution of the last two equations give

$$p_\mu^{(1)} = p_\mu^{(3)}, p_\mu^{(2)} = p_\mu^{(4)} \quad ; \quad p_\mu^{(1)} = p_\mu^{(4)}, p_\mu^{(2)} = p_\mu^{(3)} \tag{3.11}$$

which means that only forward or backward scatterings can occur. This is of course unacceptable. The general message, important for us, is that there seems to be no room for further conserved operators with non-trivial Lorentz transformation property. The existing such operators P_μ and $M_{\mu\nu}$ do allow proper scattering process to occur, but imposing any more conservation laws over-restricts the possible configurations. Such was the conclusion of the Coleman-Mandula theorem. Supersymmetries avoid the restrictions of the Coleman-Mandula theorem by relaxing one condition. According to the Coleman-Mandula theorem:

- The S-Matrix is based on a local, relativistic quantum field theory in $4D$ space-time
- There are only a finite number of different particles associated with one particle states with a given mass, and there is an energy band gap between the vacuum and the one particle states.

The theorem states that most general Lie algebra of symmetries of S-Matrix contains energy-momentum operator P_μ , the Lorentz generator $M_{\mu\nu}$, and a finite number of Lorentz-scalar operators. They generalize the notion of Lie algebra to include algebraic systems whose defining relations involve anticommutators as well as commutators. The generators turn out to be "charges" which transform under Lorentz Transformations as spinors; that is to say, objects transforming like a fermionic field. We may denote such a charge by Q_a, the subscript a indicating the spinor component. For such a charge, equation (3.7) will clearly not hold; rather

$$Q_a \mid J\rangle \quad = \quad \mid J \pm 1/2\rangle \tag{3.12}$$

As a result of this, the algebra, Superalgebra, involves commutation as well as anticommutation relations. What is the framework of this algebra? What is it look like? Because our spinorial charge Q_a is a symmetry operator, it must commute with the hamiltonian of the system

$$[Q_a, H] \quad = \quad 0 \tag{3.13}$$

and so must the anticommutator of two different components

$$[\{Q_a, Q_b\}, H] \;=\; 0 \tag{3.14}$$

The spinorial Q's have two components, so as a and b vary the symmetric object $\{Q_a, Q_b\}$ obtains three independent components, and we suspect that it must transform as a spin-1 object . However, as usual in a relativistic theory, this spin-1 object should be described by a 4-vector, not a 3 vector . Further, this 4-vector is conserved. There is only one such conserved 4-vector operator (from Coleman-Mandula theorem) P_μ. So the Q_a's must satisfy an algebra of the form,

$$\{Q_a, Q_b\} \;\sim\; P_\mu \tag{3.15}$$

It is this simple-looking expression that leads to supersymmetry algebra.

3.1. Supersymmetry Algebra

The operators Q and $Q^\dagger$ are fermionic operators, so they carry half-integer spin. Q and $Q^\dagger$ basically satisfy the algebra of commutation and anticommutation relations. Basically,

$$\{Q, Q^\dagger\} \;\propto\; P^\mu \tag{3.16}$$

$$\{Q, P^\mu\} = \{Q^\dagger, P^\mu\} \;=\; 0 \tag{3.17}$$

$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} \;=\; 0 \tag{3.18}$$

where P^μ is momentum i.e translation generator.

In this chapter, we shall follow this philosophy in the rest of the thesis, and develop the idea of supersymmetry in simple terms. We aim at studying a Lagrangian for particles of spins 0 and $\frac{1}{2}$ which exhibits a supersymmetry invariance. We then develop some elegant notions of superspace and superfields, eventually returning to show that our Minimal Supersymmetric extension of the SM Lagrangian may be obtained simply from the superfield formalism. To begin, however, we must warm up by refreshing our knowledge of Lorentz transformations with Poincaré algebra.

3.1.1. Poincaré Algebra and Spinors

Supersymmetry algebra is a mathematical formalism for describing the relation between bosons and fermions (Ramond 1990, Mohapatra 1996). In a supersymmetric world, every boson would have a partner fermion of equal mass, and vice versa. To explore the consequences of this assertion and to attempt at explain why the present-day world does not appear supersymmetric, physicists and mathematicians have developed an algebraic method for describing the symmetries involved. Traditional symmetries in physics are generated by objects that transform under various representations of the Poincaré group. Supersymmetries, on the other hand, are generated by objects that transform under the spinor representations of Poincare algebra. According to the spin-statistics theorem, bosonic fields commute while fermionic fields anticommute. In order to combine the two kinds of fields into a single object the introduction of a grading under which the bosons are the even elements and the fermions are the odd elements is required. We need to extend our Poincaré algebra to the new formalism (Peskin and Schroeder 1995).

$$\begin{aligned}
\mathbf{P} : x_\rho \to x_\rho^{'} &= \lambda^\rho_\sigma x^\sigma + a^\rho \\
&= x_\rho + \omega^\rho_\sigma x^\sigma + a^\rho \\
&= \exp[-i\frac{\omega^{\mu\nu}}{2}M_{\mu\nu} - ia^\mu P_\mu]x^\rho
\end{aligned} \tag{3.19}$$

so that for infinitesimal rotations and translations one obtains

$$x_\rho^{'} \to x_\rho - i\frac{\omega^{\mu\nu}}{2}M_{\mu\nu}x^\rho - ia^\mu P_\mu x^\rho \tag{3.20}$$

with differential operator equivalents

$$\begin{aligned}
P_\mu &= i\partial_\mu \\
M_{\mu\nu} &= -i(x_\mu\partial_\nu - x_\nu\partial_\mu)
\end{aligned} \tag{3.21}$$

satisfying

$$\begin{aligned}
[P_\mu, P_\nu] &= 0 \\
[M_{\mu\nu}, P_\rho] &= i(g_{\nu\rho}P_\mu - g_{\mu\rho}P_\nu) \\
[M_{\mu\nu}, M_{\rho\sigma}] &= i(g_{\nu\rho}M_{\mu\sigma} + g_{\mu\sigma}M_{\nu\sigma} - g_{\mu\rho}M_{\nu\sigma} - g_{\nu\sigma}M_{\mu\rho})
\end{aligned} \tag{3.22}$$

Let us note that general Lorentz generators include both spin and orbital parts:

$$M_{\mu\nu} \;=\; -i(x_\mu\partial_\nu - x_\nu\partial_\mu) + \frac{1}{2}\sum^{\mu\nu} \tag{3.23}$$

where

$$\sum^{\mu\nu} \;=\; \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu) \tag{3.24}$$

$$\{\gamma^\mu, \gamma^\nu\} \;=\; 2g^{\mu\nu} \tag{3.25}$$

3.1.2. Lorentz Transformation of Ψ_L and Ψ_R

The fermion wavefunctions, or fields, have four components , not two. However, the simplest SUSY theory (Peskin 1996, Csaki 1996) involves a complex scalar field and two-component fermionic field. We first aim to understand the nature of the two-component fields which together constitute a Dirac spinor. This difference has to do with different ways the two parts of the 4-component Dirac field transform under Lorentz transformations. Understanding how this works is important for us to be able to write down SUSY transformations. We write

$$\Psi \;=\; \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} \equiv \begin{pmatrix} \psi \\ \chi \end{pmatrix} \tag{3.26}$$

The Dirac equation gives then

$$(\mathbf{E}\text{-}\sigma.\mathbf{p})\chi \;=\; m\psi$$
$$(\mathbf{E}+\sigma.\mathbf{p})\psi \;=\; m\chi \tag{3.27}$$

Notice that as $m \to 0$, eq.(3.26) becomes $\sigma.p = E\psi_0$, and $E \to |p|$, , and hence the zero mass limit of (3.26):

$$(\sigma.\mathbf{p}/|p|)\psi_0 = \psi_0 \tag{3.28}$$

which means that ψ_0 is an eigenstate of the helicity operator. For $m \neq 0$, ψ and χ have well-defined Lorentz transformation properties, and they are the two-component spinors. Although not helicity eigenstates, ψ and χ are eiegnstates of γ_5, in the sense that in the chiral representation, the projection operators P_L and

P_R, defined via

$$P_L = \frac{1-\gamma^5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$
$$P_R = \frac{1+\gamma^5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (3.29)$$

satisfy

$$P_L\Psi = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} = \begin{pmatrix} \Psi_L \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix} \quad (3.30)$$

and

$$P_R\Psi = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} = \begin{pmatrix} 0 \\ \Psi_R \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ \overline{\chi}^{\dot\alpha} \end{pmatrix} \quad (3.31)$$

Therefore, P_L and P_R decompose Ψ into two different helicity representations. It is easy to check that $P_R P_L = 0$, $P_R^2 = P_L^2 = 1$. The eigenvalue of γ_5 is called chirality, ψ has chirality +1, and χ has chirality -1. We can now start analyzing basic transformations properties in Poincaré algebra:

$$\begin{aligned}
\frac{1}{2}\sum^{\mu\nu} &= \frac{i}{4}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu) \\
&= \frac{i}{4}\left[\begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma^\nu \\ \overline{\sigma}^\nu & 0 \end{pmatrix} - \begin{pmatrix} 0 & \sigma^\nu \\ \overline{\sigma}^\nu & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}\right] \\
&= \frac{i}{4}\begin{pmatrix} \sigma^\mu\overline{\sigma}^\nu - \sigma^\nu\overline{\sigma}^\mu & 0 \\ 0 & \overline{\sigma}^\mu\sigma^\nu - \sigma^\mu\overline{\sigma}^\mu \end{pmatrix} \\
&= i\begin{pmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \overline{\sigma}^{\mu\nu} \end{pmatrix}
\end{aligned} \quad (3.32)$$

Under a Lorentz transformation

$$\Psi^{'}(x') = S(\Lambda)\Psi(x) \quad (3.33)$$

where

$$S(\Lambda)^{-1}\gamma^\mu S(\Lambda) = \Lambda^\mu{}_\nu\gamma^\nu \quad (3.34)$$

The $S(\Lambda)$ consistent with this is given by

$$S(\Lambda) = \exp\{-\frac{i}{2}\omega_{\mu\nu}\frac{1}{2}\sum^{\mu\nu}\} \;\; = \;\; \exp\{\frac{1}{2}\omega_{\mu\nu}\begin{pmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \overline{\sigma}^{\mu\nu} \end{pmatrix}\}$$
$$= \begin{pmatrix} \exp\{(\frac{1}{2}\omega_{\mu\nu}\sigma^{\mu\nu}\} & 0 \\ 0 & \exp\{(\frac{1}{2}\omega_{\mu\nu}\overline{\sigma}^{\mu\nu}\}) \end{pmatrix} \tag{3.36}$$

so that

$$S(\Lambda)\Psi \;\; = \;\; \begin{pmatrix} \exp\{\frac{1}{2}\omega_{\mu\nu}\sigma^{\mu\nu}\}\Psi_L \\ \exp\{\frac{1}{2}\omega_{\mu\nu}\overline{\sigma}^{\mu\nu}\}\Psi_R \end{pmatrix} = \begin{pmatrix} S(\Lambda)_L\Psi_L \\ S(\Lambda)_R\Psi_R \end{pmatrix} \tag{3.37}$$

which read explicitly

$$\begin{pmatrix} \psi_\alpha \\ \overline{\chi}^{\dot{\alpha}} \end{pmatrix} \;\; = \;\; \begin{pmatrix} S(\Lambda)_\alpha{}^\beta\psi_\beta \\ S(\Lambda)^{\dot{\alpha}}{}_{\dot{\beta}}\overline{\chi}^{\dot{\beta}} \end{pmatrix} \tag{3.38}$$

One notes that

$$\begin{aligned} (\sigma^{\mu\nu})^\dagger &= (\sigma^\mu\overline{\sigma}^\nu - \sigma^\nu\overline{\sigma}^\mu)^\dagger \\ &= (\overline{\sigma}^\nu\sigma^\mu - \overline{\sigma}^\mu\sigma^\nu) \\ &= -\overline{\sigma}^{\mu\nu} \end{aligned} \tag{3.39}$$

and hence

$$S_L(\Lambda)^\dagger = S_R(\Lambda)^{-1} \tag{3.40}$$

and

$$\begin{aligned} \overline{\Psi'} &= \Psi'\gamma^0 \\ &= (S\Psi)^\dagger\Psi'\gamma^0 \Rightarrow \Psi^\dagger\gamma^0 S^{-1} = \overline{\Psi}S^{-1} \end{aligned} \tag{3.41}$$

with

$$S^\dagger\gamma^0 \;\; = \;\; \gamma^0 S^{-1} \tag{3.42}$$

These results are important for our purposes.

3.1.3. Charge Conjugation

The charge conjugation operator $\Psi \to \Psi^c$ transforms the Dirac equation by changing the sign of the charge e

$$((i\partial_\mu + eA_\mu) - m)\Psi^c \;=\; 0$$

The charge conjugation operator is interpreted as converting a particle into its antiparticle and vice versa (Ramond 1990, Peskin and Schroeder 1995). The charge-conjugated spinor is given by

$$\Psi^c = C\overline{\Psi}^T \tag{3.43}$$

with the property $C\gamma^{\mu T}C^{-1} = -\gamma^\mu$. Similarly, Lorentz transformation operator can be shown to satisfy $CS(\Lambda)^{-1T} = S(\Lambda)C$. Therefore, one finds from eq.(3.40)

$$\begin{aligned} \Psi^{c'} &= (C\overline{\Psi}^T)' = C\overline{\Psi}'^T \\ &= C(\overline{\Psi}S^{-1})^T \\ &= CS^{-1T}\overline{\Psi}^T = SC\overline{\Psi}^T \equiv S\Psi^c \end{aligned} \tag{3.44}$$

Specializing to chiral representation (appendix A) one finds

$$\begin{aligned} C &= -i\gamma^0\gamma^2 \\ &= -i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma^2 \\ \sigma^{-2} & 0 \end{pmatrix} = \begin{pmatrix} i\sigma^2 & 0 \\ 0 & -i\sigma^2 \end{pmatrix} \end{aligned} \tag{3.45}$$

so that

$$\begin{aligned} \overline{\Psi}^T &= (\Psi^\dagger\gamma^0)^T \\ &= [\begin{pmatrix} \Psi_L^\dagger \,,\; \Psi_R^\dagger \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}]^T \\ &= \begin{pmatrix} \Psi_R^\dagger \,,\; \Psi_L^\dagger \end{pmatrix}^T \equiv \begin{pmatrix} \Psi_R^* \\ \Psi_L^* \end{pmatrix} \end{aligned} \tag{3.46}$$

Thus, the charge conjugation simply flips ψ and χ:

$$\begin{aligned} \Psi^c &= C\overline{\Psi}^T \\ &= \begin{pmatrix} i\sigma^2\Psi_R^* \\ -i\sigma^2\Psi_L^* \end{pmatrix} = \begin{pmatrix} (\Psi_R)^c \\ (\Psi_L)^c \end{pmatrix} \end{aligned} \tag{3.47}$$

More explicitly, for

$$\Psi = \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} = \begin{pmatrix} \psi_\alpha \\ \overline{\chi}^{\dot{\alpha}} \end{pmatrix} \tag{3.48}$$

its charge conjugation reads to be

$$\Psi^c = \begin{pmatrix} (\Psi_R)^c \\ (\Psi_L)^c \end{pmatrix} = \begin{pmatrix} \chi_\alpha \\ \overline{\psi}^{\dot{\alpha}} \end{pmatrix} \tag{3.49}$$

where we introduced the un-dotted and dotted spinor indices via

$$\begin{aligned} \chi_\alpha &= \varepsilon_{\alpha\beta}(\overline{\chi}^{\dot{\beta}})^* \equiv \varepsilon_{\alpha\beta}\chi^\beta \\ \overline{\psi}^{\dot{\alpha}} &= \varepsilon^{\dot{\alpha}\dot{\beta}}(\psi_\beta)^* \equiv \varepsilon^{\dot{\alpha}\dot{\beta}}\overline{\psi}_{\dot{\beta}} \end{aligned} \tag{3.50}$$

As a result of these, one concludes that

$$\overline{\psi}_{\dot{\alpha}} \equiv (\psi_\alpha)^* \quad ; \quad \chi^\alpha \equiv (\overline{\chi}^{\dot{\alpha}})^* \tag{3.51}$$

$$\begin{aligned} \varepsilon_{\alpha\beta} = \varepsilon_{\dot{\alpha}\dot{\beta}} &= i\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ \varepsilon^{\alpha\beta} = \varepsilon^{\dot{\alpha}\dot{\beta}} &= -i\sigma^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \end{aligned} \tag{3.52}$$

At this point it is useful to check how manifest the Lorentz transformation properties. Using $\psi'_\alpha = S_L(\Lambda)_\alpha^{\ \beta}\psi_\beta$ we get

$$\begin{aligned} \psi'^{\alpha} &= \varepsilon^{\alpha\beta}\psi'_\beta \\ &= \varepsilon^{\alpha\beta}S_L(\Lambda)_\alpha^{\ \gamma}\psi_\gamma \\ &= \underbrace{\varepsilon^{\alpha\beta}S_L(\Lambda)_\alpha^{\ \gamma}\varepsilon_{\gamma\delta}}\psi^\delta = (S_L(\Lambda)^{-1})^{T\alpha}{}_\delta\psi^\delta \end{aligned} \tag{3.53}$$

$$(-i\sigma^2)\exp(\frac{1}{2}\omega_{\mu\nu}\sigma^{\mu\nu})(i\sigma^2) \equiv \exp\left(\frac{1}{2}\omega_{\mu\nu}(-\sigma^{\mu\nu})^T\right) \tag{3.54}$$

It is also useful to see that

$$\begin{aligned} \psi\chi \equiv \psi^\alpha\chi_\alpha &= \psi^\alpha\varepsilon_{\alpha\beta}\chi^\beta = -\psi^\beta\varepsilon_{\alpha\beta}\chi^\alpha \to \psi^\beta\varepsilon_{\beta\alpha}\chi^\alpha = \psi^\beta\chi_\beta \equiv \psi\chi \\ &= \varepsilon^{\alpha\beta}\chi_\beta\psi_\alpha = -\varepsilon^{\alpha\beta}\psi_\alpha\chi_\beta = \varepsilon^{\beta\alpha}\psi_\alpha\chi_\beta \to \psi^\beta\chi_\beta \equiv \psi\chi \end{aligned} \tag{3.55}$$

where we can now make the invariance manifest:

$$\begin{aligned}
\chi\psi \to \chi^{'}\psi^{'} &= \chi^{'\alpha}\psi^{'}_{\alpha} \\
&= (S_L(\Lambda)^{-1})^{T\alpha}{}_{\beta}\chi^{\beta}(S_L(\Lambda)_{\alpha}^{\ \gamma})\psi_{\gamma} \\
&= (S_L(\Lambda)^{-1})_{\beta}^{\ \alpha}\chi^{\beta}(S_L(\Lambda))_{\alpha}^{\ \gamma})\psi_{\gamma} \\
&= \chi^{\beta}(S_L(\Lambda)^{-1})_{\beta}^{\ \alpha}(S_L(\Lambda))_{\alpha}^{\ \gamma}\psi_{\gamma} \\
&= \chi^{\beta}\delta_{\beta}^{\ \gamma}\psi_{\gamma} = \chi^{\beta}\psi_{\beta} = \chi\psi
\end{aligned} \tag{3.56}$$

Similarly, one can show Lorentz invariance of

$$\overline{\chi}\overline{\psi} \equiv \overline{\chi}_{\dot\alpha}\overline{\psi}^{\dot\alpha} = \overline{\psi}_{\dot\alpha}\overline{\chi}^{\dot\alpha} \qquad \overline{\psi}\overline{\chi} = (\chi\psi)^{\dagger} = (\psi\chi)^{\dagger} \tag{3.57}$$

as well. In summary,

$$\begin{aligned}
\psi^{'}_{\alpha} &= S_L(\Lambda)_{\alpha}^{\ \beta}\psi_{\beta} \\
\psi^{'\alpha} &= (S_L(\Lambda)^{-1})^{T\alpha}{}_{\beta}\psi^{\beta} \equiv \psi^{\beta}(S_L(\Lambda)^{-1})_{\beta}^{\ \alpha} \\
\overline{\chi}^{'\dot\alpha} &= (S_L(\Lambda)^{-1})^{\dagger\dot\alpha}{}_{\dot\beta}\overline{\chi}^{\dot\beta} \equiv S_R(\Lambda)^{\dot\alpha}{}_{\dot\beta}\overline{\chi}^{\dot\beta} \\
\overline{\chi}^{'}_{\dot\alpha} &= (S_L(\Lambda))^{*}_{\dot\alpha}{}^{\dot\beta}\overline{\chi}_{\dot\beta} = (S_R(\Lambda)^{-1})_{\dot\alpha}{}^{\dot\beta}\overline{\chi}_{\dot\beta} \equiv \overline{\chi}_{\dot\beta}(S_R(\Lambda)^{-1})^{\dot\beta}{}_{\dot\alpha}
\end{aligned} \tag{3.58}$$

where one also recalls that

$$\begin{aligned}
\chi\psi &\equiv \chi^{\alpha}\psi_{\alpha} = -\chi_{\alpha}\psi^{\alpha} \\
\overline{\chi}\overline{\psi} &\equiv \overline{\chi}_{\dot\alpha}\overline{\psi}^{\dot\alpha} = -\overline{\chi}^{\dot\alpha}\overline{\psi}_{\dot\alpha}
\end{aligned} \tag{3.59}$$

Electrically neutral fermions are represented by Majorana spinors. They are given by

$$\Psi^{c}_{M} = \overline{\Psi}_{M} = \begin{pmatrix} \psi_{\alpha} \\ \overline{\chi}^{\dot\alpha} \end{pmatrix} \tag{3.60}$$

As a result of charge conjugation and transformation properties, the Majorana mass term reads as

$$\begin{aligned}
\frac{1}{2}m\overline{\Psi}_m\Psi_m &= \frac{1}{2}m\left(\Psi^{\dagger}_{R}\ ,\ \Psi^{\dagger}_{L}\right)\begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} \\
&= \frac{1}{2}m(\Psi^{\dagger}_{R}\Psi_L + \Psi^{\dagger}_{L}\Psi_R) \\
&= \frac{1}{2}m[(\psi^{\dot\alpha})^{*}\Psi_{\alpha} + (\Psi_{\alpha})^{*}\overline{\psi}^{\dot\alpha}]
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2}m[\psi^\alpha\psi_\alpha + \overline{\psi}_{\dot\alpha}\overline{\psi}^{\dot\alpha}] \\
&= \frac{1}{2}m(\psi\psi + \overline{\psi\psi}) \\
&= \frac{1}{2}m[\psi\psi\text{+h.c}].
\end{aligned}
$$

Here, before closing, we note that leptons and quarks are Dirac spinors; they are not electrically neutral. However, the fermionic partners of gauge bosons and that of the neutral component of the Higgs doublets are all Majorana spinors. In this sense, Majorana spinors turn out to be rather common objects of supersymmetric models.

3.1.4. The Vector Current

From (3.58) we should have the transformation properties

$$
\begin{aligned}
\psi'_\alpha = S_L(\Lambda)_\alpha{}^\beta\psi_\beta \quad &\longrightarrow \quad (\sigma^{\mu\nu})_\alpha{}^\beta = \frac{1}{4}(\sigma^\mu\overline{\sigma}^\nu - \sigma^\nu\overline{\sigma}^\mu)_\alpha{}^\beta \\
\overline{\chi}'^\alpha = S_R(\Lambda)^{\dot\alpha}{}_{\dot\beta}\overline{\chi}^{\dot\beta} \quad &\longrightarrow \quad (\overline{\sigma}^{\mu\nu})^{\dot\alpha}{}_{\dot\beta} = \frac{1}{4}(\sigma^\mu\overline{\sigma}^\nu - \sigma^\nu\overline{\sigma}^\mu)^{\dot\alpha}{}_{\dot\beta}
\end{aligned} \tag{3.62}
$$

so that $(\sigma^\mu)_{\alpha\dot\alpha}$and $(\overline{\sigma}^\mu)^{\alpha\dot\alpha}$ are seen to generate right transformations. We can make two types of vector currents:

$$
\chi\sigma^\mu\overline{\chi} = \chi^\alpha(\sigma^\mu)_{\alpha\dot\alpha}\overline{\chi}^{\dot\alpha} = (\Psi_R)^\dagger\sigma^\mu(\Psi_R) = \overline{\Psi}\gamma^\mu P_R\Psi \tag{3.63}
$$

$$
\psi\overline{\sigma}^\mu\psi = \overline{\psi}_{\dot\alpha}(\overline{\sigma}^\mu)^{\dot\alpha\alpha}\psi_\alpha = (\Psi_L)^\dagger\sigma^\mu(\Psi_L) = \overline{\Psi}\gamma^\mu P_L\Psi \tag{3.64}
$$

Consider first their hermitian conjugates:

$$
(\chi_1\sigma^\mu\overline{\chi}_2)^* = \chi_2\sigma^\mu\overline{\chi}_1 \quad \text{and} \quad (\Psi_1\gamma^\mu P_R\overline{\Psi}_2)^\dagger = \Psi_2\gamma^\mu P_R\overline{\Psi}_1 \tag{3.65}
$$

$$
(\overline{\psi}_1\overline{\sigma}^\mu\overline{\psi}_2)^\dagger = \psi_2\overline{\sigma}^\mu\psi_1 \quad \text{and} \quad (\Psi_1\gamma^\mu P_L\overline{\Psi}_2)^\dagger = \Psi_2\gamma^\mu P_L\overline{\Psi}_1 \tag{3.66}
$$

Their transpositions give

$$
\begin{aligned}
\chi_1\sigma^\mu\overline{\chi}_2 &= \chi_1^\alpha(\sigma^\mu)_{\alpha\dot\alpha}\overline{\chi_2}^{\dot\alpha} = -\overline{\chi_2}^{\dot\alpha}(\sigma^\mu)_{\alpha\dot\alpha}\chi_1^\alpha \\
&= -\overline{\chi}_{2\dot\beta}\varepsilon^{\dot\alpha\dot\beta}(\sigma^\mu)_{\alpha\dot\alpha}\varepsilon^{\alpha\beta}\chi_{1\beta} = \overline{\chi}_{2\dot\beta}\varepsilon^{\dot\alpha\dot\beta}(\sigma^{\mu T})_{\alpha\dot\alpha}\varepsilon^{\alpha\beta}\chi_{1\beta} \\
&= \overline{\chi}_{2\dot\beta}\underbrace{[(i\sigma^2)(\sigma^{\mu T})(-i\sigma^2)]^{\dot\beta\beta}}\chi_{1\beta} = -\overline{\chi}_{2\dot\beta}(\overline{\sigma}^\mu)^{\dot\beta\beta}\chi_{1\beta} \\
&- [\sigma^2(\sigma^{\mu T})\sigma^2]^{\dot\beta\beta} = -\overline{\sigma}^{\dot\beta\beta}
\end{aligned} \tag{3.67}
$$

likewise

$$\begin{aligned}
\chi_1\sigma^\mu\overline{\chi_2} &= -\overline{\chi}_2\overline{\sigma}^\mu\chi_1 \\
\psi_1\sigma^\mu\overline{\psi_2} &= -\overline{\psi}_2\sigma^\mu\overline{\psi}_1
\end{aligned} \tag{3.68}$$

These relations follow from

$$\begin{aligned}
\overline{\Psi_1}\gamma^\mu P_L\Psi_2 &= (\overline{\Psi_1}\gamma^\mu P_L\Psi_2) \\
&= -\Psi_2^\dagger P_L^T\gamma^{\mu T}\overline{\Psi_1}^T \\
&= \overline{\Psi}_2^c C P_L^T\gamma^{\mu T}C^{-1}\Psi_1^c \\
&= \overline{\Psi}_2^c C P_L^T C^{-1}C\gamma^{\mu T}C^{-1}\Psi_1^c \\
&= -\overline{\Psi}_2^c\gamma^\mu P_R\Psi_1^c
\end{aligned} \tag{3.69}$$

and

$$\begin{aligned}
\overline{\Psi_1}\gamma^\mu P_L\Psi_2 &= \Psi_{1L}^\dagger\overline{\sigma}^\mu\Psi_{2L} = \overline{\psi_{1\dot{\alpha}}}(\sigma^\mu)^{\dot{\alpha}\alpha}\psi_{2\alpha} = \overline{\psi}_1(\sigma^\mu)\psi_2 \\
\overline{\Psi_2}^c\gamma^\mu P_R\Psi_1^c &= \Psi_{1L}^c{}^\dagger\sigma^\mu\Psi_{1L}^c = (\overline{\psi}_2^{\dot{\alpha}})^*(\sigma^\mu)_{\alpha\dot{\beta}}(\overline{\psi}_1^{\dot{\beta}}) \\
&= \psi_2^\alpha(\sigma^\mu)_{\alpha\dot{\beta}}\overline{\psi}_1^{\dot{\beta}} = \psi_2(\sigma^\mu)\overline{\psi}_1
\end{aligned} \tag{3.70}$$

According to (3.69) the rules of charge conjugation may be summarized as

$$\Psi^T = -\overline{\Psi}^c C \quad ; \quad \overline{\Psi}^T = C^{-1}\overline{\Psi}^c \tag{3.71}$$

$$C\gamma^{\mu T}C^{-1} = -\gamma^\mu \tag{3.72}$$

$$C\gamma^{5T}C^{-1} = -\gamma^5 \tag{3.73}$$

As a result of these;

$$\Psi_i = \begin{pmatrix} \psi_{i\alpha} \\ \overline{\chi}_i^{\dot{\alpha}} \end{pmatrix}, \Psi_i^c = \begin{pmatrix} \chi_{i\alpha} \\ \overline{\psi}_i^{\dot{\alpha}} \end{pmatrix} \tag{3.74}$$

$$\overline{\psi_1}(\overline{\sigma}^\mu)\psi_2 = -\psi_2(\sigma^\mu)\overline{\psi}_1 = -\overline{\Psi}_2^c\gamma^\mu P_R\Psi_1^c = \overline{\Psi}_1\gamma^\mu P_L\Psi_2 \tag{3.75}$$

$$\overline{\psi_2}(\overline{\sigma}^\mu)\psi_1 = -\psi_1(\sigma^\mu)\overline{\psi}_2 = -\overline{\Psi}_1^c\gamma^\mu P_R\Psi_2^c = \overline{\Psi}_2\gamma^\mu P_L\Psi_1 \tag{3.76}$$

$$\chi_1(\sigma^\mu)\overline{\chi}_2 = -\overline{\chi}_2(\overline{\sigma}^\mu)\chi_1 = -\overline{\Psi}_2^c\gamma^\mu P_L\Psi_1^c = \overline{\Psi}_1\gamma^\mu P_R\Psi_2 \tag{3.77}$$

$$\chi_2(\sigma^\mu)\overline{\chi}_1 = -\overline{\chi}_1(\overline{\sigma}^\mu)\chi_2 = -\overline{\Psi}_1^c\gamma^\mu P_L\Psi_2^c = \overline{\Psi}_2\gamma^\mu P_R\Psi_1 \tag{3.78}$$

$$\overline{\psi_1}(\overline{\sigma}^\mu)\chi_2 = -\chi_2(\sigma^\mu)\overline{\psi}_1 = -\overline{\Psi}_2^c\gamma^\mu P_R\Psi_1^c = \overline{\Psi}_1\gamma^\mu P_L\Psi_2 \tag{3.79}$$

$$\overline{\chi_2}(\overline{\sigma}^\mu)\psi_1 = -\psi_1(\sigma^\mu)\overline{\chi}_2 = -\overline{\Psi}_1^c\gamma^\mu P_R\Psi_2^c = \overline{\Psi}_2\gamma^\mu P_L\Psi_1 \tag{3.80}$$

and also the scalar ones

$$\begin{aligned}
\chi_1\psi_2 = \psi_2\chi_1 &= \overline{\Psi}_2^c P_L \Psi_1^c = \overline{\Psi}_1 P_L \Psi_2 &(3.81)\\
\overline{\psi}_1\overline{\chi}_2 = \overline{\chi}_2\overline{\psi}_1 &= \overline{\Psi}_1^c P_R \Psi_2^c = \overline{\Psi}_2 P_L \Psi_1 &(3.82)\\
\psi_1\psi_2 = \psi_2\psi_1 &= \overline{\Psi}_2^c P_L \Psi_1^c = \overline{\Psi}_1 P_L \Psi_2 &(3.83)\\
\chi_1\chi_2 = \chi_2\chi_1 &= \overline{\Psi}_2^c P_L \Psi_1^c = \overline{\Psi}_2 P_L \Psi_1 &(3.84)\\
\overline{\psi}_1\overline{\psi}_2 = \overline{\psi}_2\overline{\psi}_1 &= \overline{\Psi}_2^c P_R \Psi_1^c = \overline{\Psi}_1 P_R \Psi_2 &(3.85)\\
\overline{\chi}_1\overline{\chi}_2 = \overline{\chi}_2\overline{\chi}_1 &= \overline{\Psi}_2^c P_R \Psi_1^c = \overline{\Psi}_2 P_R \Psi_1 &(3.86)
\end{aligned}$$

This subsection summarizes the transformation properties of vector and scalar bilinears of two-component spinors, together with their four-component counterparts.

3.2. SUSY-Poincaré Algebra

Supersymmetry is of considerable interest among physicists and mathematicians. It follows from a theorem proved by Haag, Sohnious and Lopuszanski. They proved that supersymmetry algebra is the only graded Lie algebra of symmetries of the S-matrix consistent with relativistic quantum field theory (Wess and Bagger 1992). Before we begin, however, we first recall the supersymmetry algebra:

$$[P^\mu, Q_\alpha] = [P^\mu, \overline{Q}^{\dot{\alpha}}] = 0 \tag{3.87}$$

since translation only x not the spinors. Now consider the generator of angular momentum:

$$\begin{aligned}
[M^{\mu\nu}, Q_\alpha] &= -i(\sigma^{\mu\nu})_\alpha^{\ \beta} Q_\beta\\
[M^{\mu\nu}, \overline{Q}^{\dot{\alpha}}] &= -i(\overline{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\ \dot{\beta}} \overline{Q}^{\dot{\beta}}
\end{aligned} \tag{3.88}$$

since

$$\begin{aligned}
Q'_\alpha = (1 + \frac{1}{2}\omega_{\mu\nu}\sigma^{\mu\nu})_\alpha^{\ \beta} Q_\beta &= Q_\alpha + \frac{i}{2}\omega_{\mu\nu}[M^{\mu\nu}, Q_\alpha]\\
\overline{Q}'^{\alpha} = (1 + \frac{1}{2}\omega_{\mu\nu}\overline{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\ \dot{\beta}} \overline{Q}^{\dot{\beta}} &= \overline{Q}^{\dot{\alpha}} + \frac{i}{2}\omega_{\mu\nu}[M^{\mu\nu}, \overline{Q}^{\dot{\alpha}}]
\end{aligned} \tag{3.89}$$

Moreover,

$$\{Q_\alpha, Q_\beta\} = \{\overline{Q}_{\dot{\alpha}}, \overline{Q}_{\dot{\beta}}\} = 0 \tag{3.90}$$

since

$$[P^\mu, \{Q_\alpha, Q_\beta\}] = \{[P^\mu, Q_\alpha], Q_\beta\} + \{Q_\alpha, [P^\mu, Q_\beta]\} = 0 \tag{3.91}$$

The indices $(\alpha, \beta, \dot{\alpha}, \dot{\beta})$ run from one to two and denote two-component Weyl spinors. The indices (μ, ν) run from zero to three and identify Lorentz four vectors. Therefore one finds,

$$\{Q_\alpha, \overline{Q}_{\dot{\beta}}\} = 2(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu \quad , \quad \{\overline{Q}^{\dot{\alpha}}, Q_\beta\} = 2(\overline{\sigma}^\mu)^{\alpha\dot{\beta}} P_\mu \tag{3.92}$$

as the only possibility to close the algebra. These equations give rise to SUSY-Poincaré Algebra.

It is useful to discuss positive-definiteness of energy as well. Using the relations

$$\begin{aligned} 4\sigma^{\mu\nu} &= \sigma^\mu \overline{\sigma}^\nu - \sigma^\nu \overline{\sigma}^\mu \\ 2g^{\mu\nu} &= \sigma^\mu \overline{\sigma}^\nu + \sigma^\nu \overline{\sigma}^\mu \end{aligned} \tag{3.93}$$

one obtains

$$4\sigma^{\mu\nu} + 2g^{\mu\nu} = 2\sigma^\mu \overline{\sigma}^\nu \tag{3.94}$$

so that

$$\begin{aligned} \sigma^\mu \overline{\sigma}^\nu &= g^{\mu\nu} + 2\sigma^{\mu\nu} \\ \mathrm{Tr}[\sigma^\mu \overline{\sigma}^\nu] &= 2g^{\mu\nu} \end{aligned} \tag{3.95}$$

Using these relations, one shows that

$$\begin{aligned} (\overline{\sigma}^\nu)^{\dot{\beta}\alpha} \{Q_\alpha, \overline{Q}_{\dot{\beta}}\} &= 2(\overline{\sigma}^\nu)^{\dot{\beta}\alpha} (\sigma^\mu_{\alpha\dot{\beta}}) P_\mu \\ &= 2Tr[\overline{\sigma}^\nu \sigma^\nu] P_\mu \\ &= 4g^{\mu\nu} P_\mu = 4P_\nu \end{aligned} \tag{3.96}$$

and, for $\nu = 0$, one finds

$$\begin{aligned} 4P^0 &= (\overline{\sigma}^0)^{\dot{\beta}\alpha} \{Q_\alpha, \overline{Q}_{\dot{\beta}}\} \\ &= \delta^{\dot{\beta}\alpha} \{Q_\alpha, \overline{Q}_{\dot{\beta}}\} \\ &= Q_\alpha \overline{Q}_{\dot{\beta}} + \overline{Q}_{\dot{\beta}} Q_\alpha \\ &= Q_\alpha (Q_\alpha)^* + (Q_\alpha)^* Q_\alpha \end{aligned} \tag{3.97}$$

which is manifestly nonnegative. That energy, P_0, either vanishes or takes positive values implies that the vacuum state $\mid 0\rangle$ has to have strictly vanishing energy:

$$\underbrace{<0|P^0|0>=0}\Longleftrightarrow\underbrace{Q_\alpha|0>=0} \tag{3.98}$$

$$\text{vacuum energy is zero} \Longleftrightarrow \text{SUSY is manifest} \tag{3.99}$$

where SUSY is short-hand for supersymmetry. This exact vanishing of the vacuum energy reminds one at once the cosmological constant problem. Indeed, the vacuum energy density arising from even the quark-hadron phase transition turns out to be far beyond the experimental result. Had we lived in a strictly supersymmetric world we would have no such problem; a small breaking of supersymmetry would generate the requisite experimental value. However, the lowest likely scale of supersymmetry breaking lies somewhere thousand times the proton mass, and supersymmetry brings up no possibility of nullifying the vacuum energy. One notes here that, a true solution of the cosmological constant problem should exist in far infrared via, presumably, a modification of the Einstein gravity.

The Coleman-Mandula theorem concludes that the most general Lie algebra of symmetries of the S-matrix contains the energy-momentum operator P_μ, the Lorentz rotation generator $M_{\mu\nu}$. The operators Q act in a Hilbert space with positive definite metric eq.(3.93).

3.2.1. Supersymmetry Multiplets

We proceed drive some physical consequences of the results obtained in previous section. In a theory which is supersymmetric, the operators Q, generators of the symmetry, will commute with the Hamiltonian. The energy-momentum four-vector P_μ commutes with the supersymmetry generators Q_α and $Q_{\dot\alpha}$. The mass operator P^2 is a Casimir operator, so irreducible representations of the supersymmetry algebra must have equal masses. Indeed,

$$[P^\mu, Q_\alpha] = [P^\mu, \overline{Q}_{\dot\alpha}] = 0 \Longrightarrow [P^2, Q_\alpha] = [P^2, \overline{Q}_{\dot\alpha}] = 0 \tag{3.100}$$

where P^2 is the Casimir operator of the SUSY-Poincaré algebra. This implies that mass must common for all members of a multiplet (finite dimension representation). However, this is not true for $W^2 = -m^2 J^2$. This can be seen from

$$W^\mu = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} P^\nu M_{\rho\sigma} \tag{3.101}$$

$$W^2 = -m^2 J^2 \text{ where } J^i = \frac{1}{2}\varepsilon^{ijk} M_{jk} \tag{3.102}$$

The main reason is that spins of members of a multiplet change by actions of Q_α and $\overline{Q}_{\dot{\beta}}$. As aresult we have:

- Q_α and $\overline{Q}_{\dot{\beta}}$ change fermion number by 1 unit (boson $\longleftrightarrow$ fermion)
- $\{Q_\alpha, \overline{Q}_{\dot{\beta}}\} = 2(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu$ does not change the fermion number.

We can prove that every representation of the supersymmetry algebra contains an equal number of bosonic and fermionic states. We begin by introducing a fermion number operator N_F, such that $(-)^{N_F}$ has eiegenvalue $+1$ on bosonic states that -1 on fermionic states. It follows immediately that

$$(-1)^{N_F} Q_\alpha = -Q_\alpha (-1)^{N_B} \tag{3.103}$$

Then, for any finite dimensional representation of the algebra, we find

$$\begin{aligned} \mathrm{Tr}[(-1)^{N_F}\{Q_\alpha, \overline{Q}_{\dot{\beta}}\}] &= \mathrm{Tr}[-Q_\alpha(-1)^{N_F}\overline{Q}_{\dot{\beta}} + (-1)^{N_F}\overline{Q}_{\dot{\beta}} Q_\alpha] \\ &= \mathrm{Tr}[-Q_\alpha(-1)^{N_F}\overline{Q}_{\dot{\beta}} + (-1)^{N_F}\overline{Q}_{\dot{\beta}} Q_\alpha] = 0 \end{aligned} \tag{3.104}$$

so that multiplet is to contain equal numbers of fermionic and bosonic degrees of freedom. More explicitly, the identity

$$\mathrm{Tr}[(-1)^{N_F}\{Q_\alpha, \overline{Q}_{\dot{\beta}}\}] = \mathrm{Tr}[(-1)^{N_F} 2(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu] = 2\mathrm{Tr}[(-1)^{N_F}] \ \ or \ \ (\sigma^\mu)_{\alpha\dot{\beta}} P_\mu = 0 \tag{3.105}$$

proves that $2\mathrm{Tr}[(-1)^{N_F}]$ vanishes for a system with non-vanishing 4-momentum.

3.2.2. Massless Supersymmetry Multiplet

Since $W^2 = -m^2 J^2$, the massless particles satisfy $W^2 = 0$. Hence

$$W^\mu = -\lambda p^\mu = \frac{\mathbf{J}.\mathbf{P}}{|P|} p^\mu \tag{3.106}$$

where the constant of proportionality is called the "helicity". We define normalized states

$$< p,\lambda|p,\lambda >= 1 \tag{3.107}$$

with $P^\mu = p^\mu|p,\lambda >$ and $W^\mu = \lambda p^\mu|p,\lambda >$. We can always choose $|p,\lambda >$ such that

$$Q_\alpha|p,\lambda >= 0 \qquad (\alpha = 1,2) \tag{3.108}$$

because if not, $|p,\lambda' >= Q_\alpha|p,\lambda >$ because of $Q_\alpha Q_\alpha = 0$. We go to a particular Lorentz frame with momentum

$$p^\mu = (E,0,0,E) \tag{3.109}$$

where

$$\{Q_\alpha,\overline{Q}_{\dot\beta}\}|p,\lambda >= 2(\sigma^\mu)_{\alpha\dot\beta}P_\mu|p,\lambda >= 4E\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}|p,\lambda > \tag{3.110}$$

Hence

$$\overline{Q}_{\dot 1}|p,\lambda >= 0 \tag{3.111}$$

while

$$< \psi|\psi >= 1 \quad \text{if} \qquad |\psi >= \tfrac{1}{\sqrt{4E}}\overline{Q}_{\dot 2}|p,\lambda >= -\tfrac{1}{\sqrt{4E}}\overline{Q}^{\dot 2}|p,\lambda > \tag{3.112}$$

Now we can show that

$$P_\mu|\psi > \;=\; p^\mu|\psi > \tag{3.113}$$

$$W^\mu|\psi > \;=\; (\lambda-\frac{1}{2})p^\mu|\psi > \tag{3.114}$$

or

$$|\psi > \;=\; |p,\lambda-\frac{1}{2} >= \frac{1}{\sqrt{4E}}\overline{Q}_{\dot 2}|p,\lambda > \tag{3.115}$$

involving a 1/2 unit shift of the angular momentum.

3.2.3. Superspace

Just as Lorentz invariance is inherently manifest in the 4-dimensional Minkowski space, the superspace formalism , originally introduced by Salam and Strathdee (Salam and Strathdee 1974) to extend Minkowski space-time by anti-commuting coordinates, leads one to a higher dimensional spacetime $x_\mu \rightarrow (x_\mu, \theta)$ with Grassmann coordinates θ_α. These coordinates are represented by a Majorana spinor in four-component formalism and by a Weyl spinor in two-component formalism. To formulate a supersymmetric field theory, we must first represent the supersymmetry algebra (3.2) in terms of fields not necessarily living on their mass shells. Anticommuting parameters ξ^α and $\overline{\xi}_{\dot\alpha}$ simplify the task. The superspace is spanned by the coordinates $(x^\mu, \theta_\alpha, \overline{\theta}_{\dot\beta})$ where Grassmann coordinates satisfy: $\{\theta_\alpha, \theta_\beta\} = \{\theta_\alpha, \overline{\theta}_{\dot\beta}\} = \{\overline{\theta}_{\dot\alpha}, \overline{\theta}_{\dot\beta}\} = 0$. Then, under translations

$$x^\mu \longrightarrow x'^\mu = x^\mu + a^\mu \qquad U(a) = 1 - ia^\mu P_\mu \tag{3.116}$$

for Minkowski coordinates, and

$$\begin{aligned} \theta_\alpha &\longrightarrow \theta_\alpha + \xi_\alpha \\ \overline{\theta}^{\dot\alpha} &\longrightarrow \overline{\theta}^{\dot\alpha} + \overline{\xi}^{\dot\alpha} \qquad U(\xi) = 1 - i(\xi Q + \overline{\xi Q}) \end{aligned} \tag{3.117}$$

for Grassmann coordinates.

Then, SUSY-Poincaré algebra can be expressed in terms of the commutators only:

$$[P^\mu, \xi Q] = [P^\mu, \overline{\xi Q}] = 0 \tag{3.118}$$

$$[M^{\mu\nu}, \xi Q] = -i\xi\sigma^{\mu\nu} Q \tag{3.119}$$

$$[M^{\mu\nu}, \overline{\xi Q}] = -i\overline{\xi}\overline{\sigma}^{\mu\nu}\overline{Q} \tag{3.120}$$

$$[\xi Q, \eta Q] = [\overline{\xi Q}, \overline{\eta Q}] = 0 \tag{3.121}$$

$$[\xi Q, \overline{\eta Q}] = 2(\xi\sigma^{\mu\nu}\overline{\eta})P_\mu \tag{3.122}$$

where further details are given in (Wess and Bagger 1992, Mohapatra 1996).

3.2.4. Superspace Translation

It is convenient to express SUSY generators as translation operators in the superspace. Superfields (supermultiplet) provide an elegant and compact description of supersymmetry representations. They simplify the addition and multiplication of representations and prove very useful in the construction of interacting particles. We shall show that superfields may always be constructed from component representations. Component fields may always be recovered from superfields by power series expansion.

We begin with the observation that the supersymmetry algebra may be viewed as a Lie algebra with anticommuting parameters. This motivates us to define a group element via:

$$\begin{aligned} G(x^\mu,\theta,\overline{\theta}) &\equiv \exp\{i(x^\mu P_\mu + \theta Q + \overline{\theta Q})\} \\ G(a^\mu,\xi,\overline{\xi}) &\equiv \exp\{i(a^\mu P_\mu + \xi Q + \overline{\xi Q})\} \end{aligned} \tag{3.123}$$

It is easy to multiply two group elements using Haussdorff's formula because all higher commutators vanish due to SUSY-Poincaré algebra. Indeed, using

$$e^A e^B = e^{(A+B+\frac{1}{2}[A,B]+\ldots)} \tag{3.124}$$

we obtain

$$\begin{aligned} G(x^\mu,\theta,\overline{\theta})G(a^\mu,\xi,\overline{\xi}) &= \exp\{i(x^\mu P_\mu + \theta Q + \overline{\theta Q})\}\exp\{i(a^\mu P_\mu + \xi Q + \overline{\xi Q})\} \\ &= \exp\{i(\xi Q + \overline{\xi Q} + a^\mu P_\mu) + i(Q\theta + \overline{Q\theta} + x^\mu P_\mu) \\ &+ \frac{i^2}{2}[\xi Q + \overline{\xi Q} + a^\mu P_\mu, Q\theta + \overline{Q\theta} + x^\mu P_\mu] + \ldots..\} \end{aligned} \tag{3.125}$$

so that multiplication of two generators gives

$$\begin{aligned} G(x^\mu,\theta,\overline{\theta})G(a^\mu,\xi,\overline{\xi}) &= \exp\{i[(\xi+\theta)Q + (\overline{\xi}+\overline{\theta})\overline{Q} + (x^\mu + a^\mu)P_\mu] \\ &- \frac{1}{2}(\underbrace{[\xi Q,\overline{\theta Q}]}_{2\xi\sigma^\mu\overline{\theta}P_\mu} + \underbrace{[\overline{\xi Q},\theta Q]}_{-2\theta\sigma^\mu\overline{\xi}P_\mu}\} \end{aligned} \tag{3.126}$$

$$= \exp\{i[(\theta+\xi)Q + (\overline{\theta}+\overline{\xi})Q + (x^\mu + a^\mu + i\xi\sigma\theta - i\theta\sigma\overline{\xi})P_\mu]\} \tag{3.127}$$

which is nothing but a translation in the superspace. Hence, the action of $G(a^\mu, \xi, \overline{\xi})$ on the superfield $f(x^\mu, \theta, \overline{\theta})$ is given by

$$
\begin{aligned}
G(a^\mu, \xi, \overline{\xi}) f(x^\mu, \theta, \overline{\theta}) G^{-1}(a^\mu, \xi, \overline{\xi}) &\equiv \exp\{-i(\xi Q + \overline{\xi Q} + a^\mu P_\mu)\} f(x^\mu, \theta, \overline{\theta}) \\
&= f(x^\mu + a^\mu + i\xi\sigma\overline{\theta} - i\theta\sigma\overline{\xi}, \theta + \xi, \overline{\theta} + \overline{\xi}) \\
&= f(x^\mu, \theta, \overline{\theta}) + (a^\mu + i\xi\sigma\overline{\theta} - i\theta\sigma\overline{\xi})\frac{\partial f}{\partial \theta^\alpha} + \overline{\xi}_{\dot{\alpha}} \frac{\partial f}{\partial \overline{\theta}_{\dot{\alpha}}} + ... \\
&= [1 - i\xi^\alpha(Q_\alpha) - i\overline{\xi}_{\dot{\alpha}}(\overline{Q}^{\dot{\alpha}}) - ia^\mu(P_\mu)] f(x^\mu, \theta, \overline{\theta})
\end{aligned}
\tag{3.128}
$$

which is nothing but the linear representations of Q_α and $\overline{Q}_{\dot{\alpha}}$ on superfields. As usual, multiplication of group elements induces a motion in the parameter space. This motion may be generated by the differential operators Q and $\overline{Q}$:

$$P_\mu = i\partial_\mu \tag{3.129}$$

$$iQ_\alpha = -\frac{\partial}{\partial \theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\overline{\theta}^{\dot{\alpha}}\partial_\mu \tag{3.130}$$

$$i\overline{Q}_{\dot{\alpha}} = \frac{\partial}{\partial \overline{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \tag{3.131}$$

Here we use the same letters Q, $\overline{Q}$ for the differential operators as for the group generators because the differential operators do indeed represent infinitesimal group action on the parameter space eq.(3.92). It is useful to recall the important identity

$$\overline{\xi Q} = \xi_{\dot{\alpha}} Q^{\dot{\alpha}} = -\overline{\xi}^{\dot{\alpha}}\overline{Q}_{\dot{\alpha}} \tag{3.132}$$

while analyzing certain quantities. It might be instructive to check this identity by an explicit calculation:

$$
\begin{aligned}
(1 - i\overline{\xi Q})(x^\mu + \overline{\theta}^{\dot{\alpha}}) &= (1 - i\overline{\xi}^{\dot{\alpha}}\overline{Q}_{\dot{\alpha}})(x^\mu + \overline{\theta}^{\dot{\alpha}}) \\
&= x^\mu + \overline{\theta}^{\dot{\alpha}} - \overline{\xi}^{\dot{\alpha}}(-\frac{\partial}{\partial \overline{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial)(x^\mu + \overline{\theta}^{\dot{\alpha}}) \\
&= x^\mu + \overline{\theta}^{\dot{\alpha}} + \overline{\xi}^{\dot{\alpha}} - i\overline{\xi}^{\dot{\alpha}}\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}
\end{aligned}
\tag{3.133}
$$

where the last term at right-hand side equals $i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\xi^{\dot{\alpha}}$. We note, however, the sign change in eq.(3.129). This stems from the fact that the successive product of group elements corresponds to a motion with the order of multiplication reversed.

We now define the covariant derivatives in superspace:

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} - i(\sigma^\mu)_{\alpha\dot{\alpha}}\overline{\theta}^{\dot{\alpha}}\partial_\mu \tag{3.134}$$

$$\overline{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \overline{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \tag{3.135}$$

which necessarily satisfy the anticommutation relations

$$\{D_\alpha, Q_\beta\} = \{D_\alpha, \overline{Q}_{\dot\beta}\} = \{\overline{D}_{\dot\alpha}, Q_\beta\} = \{\overline{D}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} \tag{3.136}$$

$$\{D_\alpha, D_\beta\} = \{\overline{D}_{\dot\alpha}, \overline{D}_{\dot\beta}\} = 0 \tag{3.137}$$

$$\{D_\alpha, \overline{D}_{\dot\beta}\} = 2i(\sigma^\mu)_{\alpha\dot\alpha}\partial_\mu \tag{3.138}$$

It might be instructive to check the last equality explicitly:

$$\begin{aligned}
\{D_\alpha, \overline{D}_{\dot\beta}\} &= (\frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu)_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}\partial_\mu)(-\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu) \\
&+ (-\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu)(\frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu)_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}\partial_\mu) \\
&= \underbrace{-\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} - \frac{\partial}{\partial\overline{\theta}^{\dot\alpha}}\frac{\partial}{\partial\theta^\alpha}} + i\frac{\partial}{\partial\theta^\alpha}\theta^\lambda\sigma^\nu_{\lambda\dot\alpha}\partial_{\dot\alpha} + i\sigma^\mu_{\alpha\dot\beta}\overline{\theta}^{\dot\beta}\partial_\mu\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} \\
&+ i\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}}\sigma^\mu_{\alpha\dot\beta}\overline{\theta}^{\dot\beta}\partial_\mu + i\theta^\lambda\sigma^\nu_{\lambda\dot\alpha}\partial_\nu\frac{\partial}{\partial\theta^\alpha} \\
&+ \sigma^\mu_{\lambda\dot\beta}\overline{\theta}^{\dot\beta}\theta^\lambda\sigma^\nu_{\lambda\dot\alpha}\partial_\mu\partial_\nu + \theta^\lambda\sigma^\nu_{\lambda\dot\alpha}\sigma^\mu_{\alpha\dot\beta}\overline{\theta}^{\dot\beta}\partial_\nu\partial_\mu \\
&= i\sigma^\mu_{\lambda\dot\alpha}\partial_\mu(\frac{\partial}{\partial\theta^\alpha}\theta^\lambda + \theta^\lambda\frac{\partial}{\partial\theta^\alpha}) + i\sigma^\mu_{\alpha\dot\beta}\partial_\mu(\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}}\overline{\theta}^{\dot\beta} + \overline{\theta}^{\dot\beta}\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}}) \\
&+ \sigma^\mu_{\alpha\dot\beta}\sigma^\nu_{\lambda\dot\alpha}(\overline{\theta}^{\dot\beta}\theta^\lambda + \theta^\lambda\overline{\theta}^{\dot\beta})\partial_\mu\partial_\nu \\
&= i\sigma^\mu_{\lambda\dot\alpha}\delta_\alpha{}^\lambda\partial_\mu + i\sigma^\mu_{\alpha\dot\beta}\delta_{\dot\alpha}{}^{\dot\beta}\partial_\mu \\
&= 2i(\sigma^\mu)_{\alpha\dot\alpha}\partial_\mu
\end{aligned} \tag{3.139}$$

Hence the result

$$\begin{aligned}
\{iQ_\alpha, i\overline{Q}_{\dot\alpha}\} &= -\{D_\alpha, \overline{D}_{\dot\alpha}\} \\
\{Q_\alpha, \overline{Q}_{\dot\alpha}\} &= \{D_\alpha, \overline{D}_{\dot\alpha}\}.
\end{aligned} \tag{3.140}$$

3.2.5. General Superfields

We are now ready to introduce superfields and superspace. Elements of superspace are labelled by $\widehat{F}(x,\theta,\overline{\theta})$. Superfields are functions of superspace which should be understood in terms of their power series expansion in θ and $\overline{\theta}$. In general,

$$\begin{aligned}
\widehat{F}(x,\theta,\overline{\theta}) &= f(x) + \theta\phi(x) + \overline{\theta}\overline{\chi}(x) + \theta\theta m(x) \\
&+ \overline{\theta\theta}n(x) + \theta\sigma^\mu\overline{\theta}V(x) + \theta\theta\overline{\theta\lambda}(x) \\
&+ \overline{\theta\theta}\theta\psi(x) + \theta\theta\overline{\theta\theta}d(x)
\end{aligned} \tag{3.141}$$

where we have used $(\theta\theta) = \theta^{\alpha}\theta_{\alpha}$ and $(\overline{\theta\theta}) = \overline{\theta}_{\dot{\alpha}}\theta^{\dot{\alpha}}$. It is easy to see that there are no more terms other than these:

i) any combination having more than two θ's or $\overline{\theta}$ must vanish by their anti-commuting property:

$$\begin{aligned} (\theta\theta)\theta^1 &= \theta^A\theta_A\theta^1 = (\theta^1\theta^2 - \theta^2\theta^1)\theta^1 \\ &= -(\theta^1\theta^2 - \theta^2\theta^1)\theta^1 = -(\theta\theta)\theta^1 \end{aligned} \quad (3.142)$$

where $(\theta\theta)\theta^1 = 0$ and similarly for θ^2 and $(\overline{\theta\theta})\overline{\theta}^{\dot{A}}$.

ii) any higher rank tensorial structures must disappear:

$$(\psi\sigma^{\mu\nu}\chi) = -(\chi\sigma^{\mu\nu}\psi) \quad (3.143)$$

and hence,

$$(\theta\sigma^{\mu\nu}\theta) = 0 \quad (3.144)$$

iii) $(\overline{\theta}\overline{\sigma}^{\mu}\theta) = 0$ does not appear since it can be rewritten using

$$(\theta\sigma^{\mu}\overline{\theta}) = -(\overline{\theta}\overline{\sigma}^{\mu}\theta) \quad (3.145)$$

and finally, we have the condition of result being a Lorentz scalar or pseudoscalar.

The quantatities $f(x), \phi(x), \overline{\chi}(x), m(x), n(x), V(x), \overline{\lambda}(x), \psi(x)$ and d(x) are called component fields. Their geometric character is determined by their transformation properties under the Lorentz group, given that $\widehat{\Phi}(x, \theta, \overline{\theta})$ is a Lorentz scalar or pseudoscalar. We deduce that

- $f(x)$, $m(x)$ and $n(x)$ are complex scalar or pseudoscalar fields
- $\psi(x)$ and $\phi(x)$ are left-handed Weyl spinors
- $\overline{\chi}(x)$ and $\overline{\lambda}$ are right-handed Weyl spinor fields
- $V(x)$ is a four-vector field
- $d(x)$ is a scalar field

which show that a general superfield involves fields of varying transformation properties.

All higher powers of θ and $\overline{\theta}$ vanish. It is easy to verify that linear combinations of superfields are again superfields. Similarly, products of superfields are again superfields because Q and $\overline{Q}$ are linear differential operators eq.(3.128). Thus we see that superfields form linear representations of the supersymmetry algebra. In general, however, the representations are highly reducible. We may eliminate the extra component fields by imposing covariant constraints such as $\overline{D}\widehat{F} = 0$ or $\widehat{F} = \widehat{F}^{\dagger}$. Superfields shift the problem of finding supersymmetry representations to that of finding appropriate constraints. Note that we must reduce superfields without restricting their x-dependence through differential equation in x-space.

Superfields satisfying the condition $\overline{D}\widehat{\Phi} = 0$ are called chiral or scalar superfields. This constraint does not yield a differential equation in x-space. Extra conditions however, often give differential equations. For example, $DD\widehat{\Phi} = \overline{D}\widehat{\Phi} = 0$ yields massless field equations, while $D\widehat{\Phi} = \overline{D}\widehat{\Phi} = 0$ implies $\widehat{\Phi}$ = a constant.

Let us discuss chiral superfields in detail:

$$\overline{D}_{\dot{\alpha}}\widehat{\Phi}(x,\theta,\overline{\theta}) = 0 \tag{3.146}$$

on the superfield $\widehat{\Phi}(x,\theta,\overline{\theta})$ is compatible with SUSY. Because

$$\overline{D}_{\dot{\alpha}}\theta = 0 \tag{3.147}$$

$$\overline{D}_{\dot{\alpha}}y = \overline{D}_{\dot{\alpha}}(x^{\mu} - i\theta\sigma^{\mu}\overline{\theta}) \tag{3.148}$$

$$= +i\theta^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}} - \theta^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}} = 0 \tag{3.149}$$

Superfield $\widehat{\Phi}(x,\theta,\overline{\theta})$ is a function of y and θ only :

$$\begin{aligned}\widehat{\Phi}(x,\theta,\overline{\theta}) &= \widehat{\Phi}(y,\overline{\theta}) \\ &= \varphi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \qquad (3.150)\\ &= \varphi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) \\ &- i\partial_{\mu}\phi\theta\sigma\overline{\theta} + \frac{i}{\sqrt{2}}\theta\theta\partial_{\mu}\psi\sigma^{\mu}\overline{\theta} - \frac{1}{4}\partial_{\mu}\partial^{\mu}\varphi\theta\theta\overline{\theta\theta} \qquad (3.151)\end{aligned}$$

with similar results for $\widehat{\Phi}^{\dagger}$. In general, we have two possibilities of great physical relevance:

$$\begin{aligned} D_{\alpha}\widehat{\Phi}^{\dagger} &= 0 \Longrightarrow \phi^{\dagger} \quad \text{right-handed chiral superfield} \\ \overline{D}_{\dot{\alpha}}\widehat{\Phi} &= 0 \Longrightarrow \phi \quad \text{left-handed chiral superfield} \qquad (3.152)\end{aligned}$$

Vector superfields are defined to satisfy $\widehat{V} = \widehat{V}^\dagger$. It is possible to construct all supersymmetric renormalizable Lagrangians in terms of vector and scalar superfield (Wess and Bagger 1992)).

Fields obeying the chiral conditions (3.152) are called scalar fields or left-handed and right-handed chiral fields, and fields obeying the reality condition $\widehat{\Phi}(x,\theta,\overline{\theta}) = \widehat{\Phi}^\dagger(x,\theta,\overline{\theta})$ are called vector fields. Chiral fields are used to represent matter fields, and vector fields are used to represent gauge fields.

It might be useful to check how component fields transform under SUSY transformations. Hence we consider

$$\begin{aligned}
\delta\widehat{\Phi} &= -i(\xi Q + \overline{\xi Q})\Phi \\
&= -i(\xi^\alpha Q_\alpha + \overline{\xi}^{\dot{\alpha}}\overline{Q}_{\dot{\alpha}})\Phi \\
&= \xi^\alpha(\frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\overline{\theta}^{\dot{\alpha}}\partial_\mu)\Phi + \overline{\xi}^{\dot{\alpha}}(\frac{\partial}{\partial\overline{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu)\Phi \qquad (3.153)
\end{aligned}$$

where after expanding we get

$$\begin{aligned}
\delta\widehat{\Phi} &= \sqrt{2}\xi\psi + 2\xi\theta F - 2i(\theta\sigma^\mu\overline{\xi})(\partial_\mu\phi) + \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi\sigma^\mu\overline{\xi} \\
&+ i\sqrt{2}\overline{\xi}^{\dot{\alpha}}\underbrace{\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\theta^\beta}\partial_\mu\psi_\beta + \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi^\dagger\sigma^\mu_{\alpha\dot{\alpha}}\overline{\xi}^{\dot{\alpha}} + ... \\
&= \sqrt{2}\xi\psi + 2\xi\theta F - 2i(\theta\sigma^\mu\overline{\xi})(\partial_\mu\phi) + i\sqrt{2}\theta\theta\partial_\mu\psi\sigma^\mu\overline{\xi} + ... \\
&= \delta\varphi + \sqrt{2}\theta\delta\psi + \theta\theta\delta F + ... \qquad (3.154)
\end{aligned}$$

Hence scalar, fermionic and F components of a chiral superfield transform as

$$\delta\varphi = \sqrt{2}\xi\psi \qquad (3.155)$$

$$\delta\psi_\alpha = \sqrt{2}\xi_\alpha F - \sqrt{2}i(\partial_\mu\varphi)\sigma^\mu_{\alpha\dot{\alpha}}\xi^{\dot{\alpha}} \qquad (3.156)$$

$$\delta F = \sqrt{2}i\partial_\mu\psi\sigma^\mu\overline{\xi} \qquad (3.157)$$

which are highly suggestive in that under a supersymmetry transformation the scalar component gets converted into the fermionic component and the fermionic does into the scalar component.

3.2.6. Interactions of Superfields

An immediate question that comes to mind concerns the way one can write down interactions among the superfields. For instance, how can we write down

Yukawa interactions in superfield language? To answer this and similar questions, it is useful to consider typical interaction terms among chiral superfields.

It is interesting to observe that appendix A and (Simonsen 1995) product of two chiral superfields gives

$$\begin{aligned}\widehat{\Phi}_i(y,\theta)\widehat{\Phi}_j(y,\theta) &= \varphi_i(y)\varphi_j(y) + \sqrt{2}\theta(\psi_i(y)\varphi_j(y) + \varphi_i(y)\psi_j(y)) \\ &+ \theta\theta[\varphi_i(y)F(y) + \varphi_j(y)F(y) - \psi_i(y)\psi_j(y)] \qquad (3.158)\end{aligned}$$

which contains terms up to θ order, just like a single chiral superfield. However, a similar bilinear with one superfield replaced by its hermitian conjugate gives

$$\begin{aligned}\widehat{\Phi}_i^\dagger(y,\theta)\widehat{\Phi}_j(y,\theta) &= \varphi_i^\dagger(y)\varphi_j(y) + \sqrt{2}\theta\psi_j(y)\varphi_i^\dagger + \sqrt{2}\overline{\theta\psi}_i(y)\varphi_j(y) \\ &+ 2\overline{\theta\psi}_i\theta\psi_j + F_i\varphi_i^\dagger(y)\theta\theta + F_i^\dagger\varphi_i(y)\overline{\theta\theta} + \sqrt{2}\theta\theta\overline{\theta\psi}_iF_j \\ &+ \sqrt{2}\overline{\theta\theta}\theta\psi_iF_j^\dagger + \overline{\theta\theta}\theta_i^\dagger(y)F_j(y) + \overline{\theta\theta}\theta\theta F_i^\dagger(y)F_j(y) \qquad (3.159)\end{aligned}$$

which is quite different than (3.158). In particular, (3.159) is seen to contain higher order terms in θ. In fact, (3.158) behaves as a chiral superfield whereas (3.159) does as a vector superfield.

Consider integration of (3.158) with the measure $d^2\theta$ (which is, of course, identical to derivative operation $\partial^2/\partial\theta^2$). Such an integration (or equivalently, differentiation) gives $\varphi_i(y)F(y) + \varphi_j(y)F(y) - \psi_i(y)\psi_j(y)$ which is nothing but the F component of $\widehat{\Phi}_i(y,\theta)\widehat{\Phi}_j(y,\theta)$. This F term generates holomorphic interactions among the component fields, for instance, their Yukawa interactions. The higher order combination of chiral superfiels, such as $\widehat{\Phi}_i(y,\theta)\widehat{\Phi}_j(y,\theta)\widehat{\Phi}_k(y,\theta)$ also consists of $\theta\theta$ component as the highest order term. One notices that, such holomorphic structures are capable of generating bilinear and trilinear interactions among component fields via their θ component *i.e.* F component. In particular, Yukawa couplings among scalar and fermion fields can be generated via the F component of the trilinear term generated by their associated superfields.

Similarly, consider $\theta\theta\overline{\theta\theta}$ component of (obtained via quartic integration or differentiation) (3.159). It gives, precisely $F_i^\dagger F_j$. It is the highest component of $\widehat{\Phi}_i^\dagger(y,\theta)\widehat{\Phi}_j(y,\theta)$, and its integration over Grassmann numbers yields the D term contribution. The D terms result in quartic interactions among the scalar fields.

In general, in a supersymmetric field theory, the lagrangian of the component fields follows form F and D term contributions. The simplest example is the holomorphic bilinear and trilinear interactions discussed above. Consider the object

$$\widehat{W} = \frac{1}{2}\mu_{ij}\widehat{\Phi}_i\widehat{\Phi}_j + \frac{1}{3}\lambda_{ijk}\widehat{\Phi}_i\widehat{\Phi}_j\widehat{\Phi}_k \tag{3.160}$$

where (i, j, k) run over all fields allowed in a specific model. As follows from discussions above, the F component of this object yields all Yukawa interactions plus a set of quadratic, trilinear and quartic interactions among the scalars. In fact, W is a fundamental object for determining holomorphic interactions among component fields. This quantity, $\widehat{W}$, is called 'superpotential' and it is of fundamental importance for determining interactions among component fields.

CHAPTER 4

The Minimal Supersymmetric Standard Model (MSSM)

We now consider the symmetries of the scattering matrix S in the physical world, that is, those transformations that can be reduced to an interchange of asymptotic states. Before the discovery of supersymmetry, supposedly a symmetry of nature, the only symmetries known were the following: (1) the ones corresponding to the Poincare group; (2) the so called internal global symmetries, both of them ruled by a Lie algebra; and (3) discrete symmetries such as parity (P), charge conjugations (C) and the time reversal (T). In 1967 a theorem due to Coleman and Mandula established rigorously that, under quite general conditions, these are the only symmetries allowed for S matrix if we do not want to induce trivial scattering (fixed angles and speeds) in $2 \to 2$ processes.

The Supersymmetry appears precisely when we assume that the generators of the new symmetry we want to add have a spinorial character instead of a scalar one, therefore transforming under $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations of the Lorentz group (i.e see sec.(3.2.2.)) . Fermionic (spinorial) generators necessarily have an anti-commutative algebra, generically known as a graded Lie algebra. The algebra is not closed with just the SUSY generators, thus it can not be understood as an internal symmetry, but it rather forms an extension of the space-time symmetries of the Poincare group (check previous chapter for algebraic properties).

Following this line of thought, one could relax some other hypotheses of the Coleman-Mandula theorem in order to introduce new theories. SUSY is the only known extension allowed by the S matrix symmetries (3.98). Accepting as the only valid extension of the Coleman-Mandula theorem requires the presence of a graded Lie algebra, and one can show (Haag, Lopuszański and Sohnius theorem) that spinorial generators different from those of SUSY are forbidden.

We have already introduced the basic concepts like "superfields" and "superspace". In general, one extends the usual 4D Minkowski spacetime by adding constant Weyl spinors to obtain the superspace. In one adds just one set of spinorial coordinates $(\theta, \overline{\theta})$ then the supersymmetric theory is called N=1 supersymmetry. The more the spinoral coordinates higher the supersymmetry. In essence, what is done is to add additional Grassmann coordinates $x_\mu \rightarrow \left(x_\mu, \theta, \overline{\theta}\right)$ for obtaining the superspace. The superfields are defined on the superspace, and actions of supersymmetric charges can be represented by appropriate differential operators. All these have been discussed in detail in Chapter III above.

The spinoral character of extra dimensions guarantee that the functions defined in the superspace are necessarily polynomial functions of the $(\theta, \bar{\theta})$. Thus we can decompose the functions (superfields) on this superspace in components of θ^0, θ_α, $\bar{\theta}_{\dot{\alpha}}$, $\theta_\alpha \theta_\beta$, etc. Each of these components will be a function of the space-time coordinates. Similar to usual spacetime, we can define in the superspace scalar superfields as well as vector superfields.

As we have seen in Chapter III, a scalar *chiral* field in superspace has 4 independent component fields (Wess and Bagger 1992, Gates et al. 1983):

$$\Phi_L \quad = \varphi + \sqrt{2}\theta\psi + \theta\theta F \equiv (\varphi, \psi, F) \tag{4.1}$$

$$\Phi_R \quad = \varphi^* + \sqrt{2}\bar{\theta}\bar{\psi} + \bar{\theta}\bar{\theta} F^* \equiv (\varphi^*, \bar{\psi}, F^*) \tag{4.2}$$

where φ is a scalar field, ψ and $\bar{\psi}$ are Weyl spinors (left-handed and right handed Dirac fermions) and F is an auxiliary scalar field. This auxiliary field, in physical world, is not a dynamical field since its equations of motion do not involve time derivatives. To this end we are left with a superfield, whose components represent an ordinary scalar field and an ordinary chiral spinor. So if nature is described by the dynamics of this field we would find a chiral fermion and a scalar with identical quantum numbers. That is *supersymmetry relates particles which differ by spin 1/2.* When a SUSY transformation (Q) acts on a superfield it transforms spin s particles into spin $s \pm 1/2$ particles.

Thus, for a $N = 1$ SUSY, we find that for any chiral fermion there should be a scalar particle with exactly the same quantum numbers. This fact holds on the basis of the absence of quadratic divergences in boson mass renormalization, since for any loop diagram involving a scalar particle there should be a fermionic loop diagram, which will cancel quadratic divergences between each other, though logarithmic divergences remain. In fact, as one recalls from discussions in Chapter II, the quadratic sensitivity of scalar sector to ultraviolet cutoff is the main motivation for introducing supersymmetry.

Supersymmetric interactions can be introduced by means of generalized gauge transformations, and by means of a generalized potential function, the superpotential given at the end of Chapter III. The superpotential encodes Yukawa-type interactions as well as the scalar potential of the model.

As no scalar particles have been found at the electroweak scale we may directly infer that, even if SUSY exists, it must be broken. We can allow SUSY to be broken while maintaining the property that no quadratic divergences arise: its is the so-called Soft-SUSY-Breaking mechanism (Girardello and Grisaru 1981). We can achieve this by introducing only a small set of terms with dimensionful couplings, to with: masses for the components of lowest spin of a supermultiplet and triple scalar interactions. However, other terms like explicit fermion masses for the matter fields would violate the Soft-SUSY-Breaking condition; they have to wait for breakdown of the gauge symmetry.

The MSSM is the minimal supersymmetric extension of the SM. It is introduced by means of an $N = 1$ SUSY, with the minimum number of new particles. Thus, for each fermion f of the SM there are two scalars related to its chiral components called "sfermions" ($\tilde{f}_{L,R}$), for each gauge boson V there is also a chiral fermion: "gaugino" ($\tilde{v}$), and for each Higgs scalar H there is another chiral fermion: "higgsino" ($\tilde{h}$). In the MSSM it turns out that, in order to be able to give masses to up-type and down-type fermions, we must introduce two Higgs doublets with opposite hypercharge, and so the MSSM Higgs sector possesses the structure of the so-called 2HDM (Gunion et al. 1990).

To build the MSSM Lagrangian we must build a Lagrangian invariant under the gauge group $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$, it must also include the superfields with the particle content of the Figure 4.1 and in addition it must contain the terms that break supersymmetry softly. But this Lagrangian violates the baryonic and leptonic number, so we have to introduce an additional symmetry.

In the case of the MSSM this symmetry is the so-called R-symmetry. It is a discrete symmetry which comprises the spin (S), the baryonic number (B) and the leptonic number (L) to generate the so-called R-parity of a field:

$$R = (-1)^{2S+L+3B} \tag{4.3}$$

Clearly, this quantity is 1 for the SM fields and -1 for their supersymmetric partners. In the way the MSSM is implemented R-parity is conserved, this means that R-odd particles (the superpartners of SM particles) can only be created in pairs. This implies that any scattering process must end with the lightest supersymmetric partner, and that particle must be absolutely stable. Though remains outside this thesis work, this lightest supersymmetric partner (LSP) is a viable candidate for dark matter in the universe (Boer 2005). In this sense, SUSY may be envisioned to generate a solution for dark matter problem mentioned in Chapter I, the Introduction of the thesis.

4.1. MSSM field content

Superfield	SM particle	Sparticle	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Matter					
$\hat{L}$	leptons $L = (\nu_l, l)_L$	*sleptons* $\tilde{L} = (\tilde{\nu}_{lL}, \tilde{l}_L)$	1	2	-1
$\hat{R}$	$R = l_L^-$	$\tilde{R} = \tilde{l}_R^+$	1	1	2
$\hat{Q}$	$Q = (u, d)_L$	$\tilde{Q} = (\tilde{u}_L, \tilde{d}_L)$	3	2	1/3
$\hat{U}$	quarks $U = u_L^c$	*squarks* $\tilde{U} = \tilde{u}_R^*$	3^*	1	$-4/3$
$\hat{D}$	$D = d_L^c$	$\tilde{D} = \tilde{d}_R^*$	3^*	1	2/3
$\hat{H}_1$	Higgs $H_1 = (H_1^0, H_1^-)$	*Higgsinos* $\tilde{H}_1 = (\tilde{H}_1^0, \tilde{H}_1^-)$	1	2	-1
$\hat{H}_2$	$H_2 = (H_2^+, H_2^0)$	$\tilde{H}_2 = (\tilde{H}_2^+, \tilde{H}_2^0)$	1	2	1
Gauge					
$\hat{G}$	gluon g_μ	*gluino* $\tilde{g}$	8	0	0
$\hat{V}$	w (W_1, W_2, W_3)	*wino* $(\tilde{W}_1, \tilde{W}_2, \tilde{W}_3)$	1	3	0
$\hat{V}'$	b B^0	*bino* $(\tilde{B}^0$	1	1	0

Figure 4.1: MSSM field content.

The field content of the MSSM consist of the fields of the SM plus all their supersymmetric partners, and an additional Higgs doublet. The figure 4.1 shows all the correspondences and all the fields. All these fields suffer some mixing, so the physical (mass eigenstates) fields look much different from these ones, as shown in Table 4.1. The gauge fields mix up to give the well known gauge bosons of the SM, $W^{\pm}_{\mu}$, Z^0_{μ}, A_{μ}, the gauginos and higgsinos mix up to give the chargino and neutralino fields, and finally the left- and right-chiral sfermions mix among themselves in sfermions of indefinite chirality. Other than this, as we recall from Chapter II, the quarks themselves mix with each other in the way the CKM matrix points.

Name	Mass eigenstates	Gauge eigenstates
Higgs bosons	$h^0\ H^0\ A^0\ H^{\pm}$	$H^0_d\ H^0_u\ H^-_d\ H^+_u$
squarks	$\tilde{t}_1\ \tilde{t}_2\ \tilde{b}_1\ \tilde{b}_2$	$\tilde{t}_L\ \tilde{t}_R\ \tilde{b}_L\ \tilde{b}_R$
sleptons	$\tilde{\tau}_1\ \tilde{\tau}_2\ \tilde{\nu}_\tau$	$\tilde{\tau}_L\ \tilde{\tau}_R\ \tilde{\nu}_\tau$
neutralinos	$\tilde{N}_1\ \tilde{N}_2\ \tilde{N}_3\ \tilde{N}_4$	$\tilde{B}^0\ \tilde{W}^0\ \tilde{H}^0_d\ \tilde{H}^0_u$
charginos	$\tilde{C}^{\pm}_1\ \tilde{C}^{\pm}_2$	$\tilde{W}^{\pm}\ \tilde{H}^-_d\ \tilde{H}^+_u$

Table 4.1: The mass and gauge eigenstates of some fields contained in the MSSM spectrum.

4.2. Lagrangian

The MSSM interactions come from three different kinds of sources:

- Superpotential:

$$\widehat{W} = \widehat{U}\mathbf{Y_u}\widehat{Q}\widehat{H}_u + \widehat{D}\mathbf{Y_d}\widehat{Q}\widehat{H}_d + \widehat{E}\mathbf{Y_e}\widehat{L}\widehat{H}_d + \mu\widehat{H}_u\widehat{H}_d \tag{4.4}$$

  The superpotential contributes to the interaction Lagrangian with two different kinds of interactions. The first one is the Yukawa interaction, which is obtained from (4.4) just by replacing two of the superfields by their fermionic components setting the third to its scalar component (these should be clear from analyses presented in Chapter III, for a general supersymmetric theory):

$$\begin{aligned} V_Y &= \epsilon_{ij}\left[EY_eL^iH_d^j + DY_dQ^iH_d^j + UY_uQ^iH_u^j + \mu\tilde{H}_u^i\tilde{H}_d^j\right] \\ &+\epsilon_{ij}\left[\tilde{E}Y_eL^i\tilde{H}_d^j + \tilde{D}Y_dQ^i\tilde{H}_d^j + \tilde{U}Y_uQ^i\tilde{H}_u^j\right] \\ &+\epsilon_{ij}\left[\tilde{E}Y_eL^i\tilde{H}_d^j + \tilde{D}Y_dQ^i\tilde{H}_d^j + \tilde{U}Y_uQ^i\tilde{H}_u^j\right] \\ &+ \text{h.c.}\ . \end{aligned} \tag{4.5}$$

The second kind of interactions are obtained by first computing the F terms, $F = \partial W/\partial\varphi_i$ and squaring:

$$V_W = \sum_i \left|\frac{\partial W(\varphi)}{\partial\varphi_i}\right|^2 \ , \tag{4.6}$$

φ_i being the scalar components of the superfields.

- Interactions related to the gauge symmetry, which contain:
  - the usual gauge interactions
  - the gaugino interactions:

$$V_{\tilde{G}\psi\tilde{\psi}} = i\sqrt{2}g_a\varphi_k\bar{\lambda}^a\left(T^a\right)_{kl}\bar{\psi}_l + \ \text{h.c.} \tag{4.7}$$

    where (φ, ψ) are the spin 0 and spin 1/2 components of a chiral superfield respectively, T^a is a generator of the gauge symmetry, λ_a is the gaugino field and g^a its coupling constant.

  - and the D-terms, related to the gauge structure of the theory, but that do not contain neither gauge bosons nor gauginos:

$$V_D = \frac{1}{2}\sum D^aD^a \ , \tag{4.8}$$

    with

$$D^a = g^a\varphi_i^*\left(T^a\right)_{ij}\varphi_j \ , \tag{4.9}$$

    where again φ_i are the scalar components of the superfields.

- Soft–Breaking interaction terms:

$$V_{\text{soft}}^{\text{I}} = \frac{g}{\sqrt{2}M_W\cos\beta}\epsilon_{ij}\left[\tilde{E}m_eA_e\tilde{L}^iH_d^j + \tilde{D}m_dA_d\tilde{Q}^iH_d^j + \tilde{U}m_uA_u\tilde{Q}^iH_u^j\right] + \ \text{h.c.} \tag{4.10}$$

plus mass terms for the scalar component of each superfield. These trilinear interactions, with dimensionful trilinear couplings A_f, may be viewed as Yukawa interaction in the scalar sector. The supersymmetry breaking effects, though not unique at all, are such that mass-squareds of the scalar fields and triliear couplings are of similar size.

The full MSSM Lagrangian is then:

$$\begin{aligned}\mathcal{L}_{\rm MSSM} &= \mathcal{L}_{\rm Kinetic} + \mathcal{L}_{\rm Gauge} - V_{\tilde{G}\psi\tilde{\psi}} - V_D - V_Y - \sum_i \left|\frac{\partial W(\varphi)}{\partial \varphi_i}\right|^2 \\ & -V^{\rm I}_{\rm soft} - H_d^\dagger m_1^2\, H_d - H_u^\dagger m_2^2\, H_u - m_{d,u}^2 \left(H_d H_u + H_d^\dagger H_u^\dagger\right) \\ & -\frac{1}{2} m\, \psi^a \psi^a - \frac{1}{2} M\, \tilde{w}_i \tilde{w}_i - \frac{1}{2} M'\, \tilde{B}^0 \tilde{B}^0 \\ & -\tilde{L}^\dagger m_{\tilde{L}}^2\, \tilde{L} - \tilde{E}^\dagger m_{\tilde{E}}^2\, \tilde{E} - \tilde{Q}^\dagger m_{\tilde{Q}}^2\, \tilde{Q} - \tilde{U}^\dagger m_{\tilde{U}}^2\, \tilde{U} - \tilde{D}^\dagger m_{\tilde{D}}^2\, \tilde{D}\ , \end{aligned} \quad (4.11)$$

where we have included all of the soft SUSY-breaking terms.

From the Lagrangian (4.11) we can obtain the full MSSM spectrum, as well as their interactions, which contain the usual gauge interactions, the fermion-Higgs interactions that correspond to a 2HDM (Gunion et al. 1990), and the pure SUSY interactions. A very detailed treatment of this Lagrangian, and the process of derivation of the forthcoming results can be found in (Simonsen 1995).

4.2.1. Higgs boson sector

The Higgs sector of the MSSM is that of a 2HDM, with some SUSY restrictions. After expanding (4.11) the Higgs potential reads

$$\begin{aligned} V &= m_1^2\, |H_d|^2 + m_2^2\, |H_u|^2 - m_{d,u}^2\, \left(\epsilon_{ij}\, H_d^i\, H_u^j + \text{h.c.}\right) \\ &+ \frac{1}{8}(g^2 + g'^2)\, \left(|H_d|^2 - |H_u|^2\right)^2 + \frac{1}{2}\, g^2\, |H_d^\dagger\, H_u|^2\, . \end{aligned} \quad (4.12)$$

The neutral Higgs bosons fields acquire a vacuum expectation value (VEV),

$$< H_d >_0 \;=\; \begin{pmatrix} v_d \\ 0 \end{pmatrix}\ ,\ < H_u >_0 = \begin{pmatrix} 0 \\ v_u \end{pmatrix} \quad (4.13)$$

From the physical shell these VEVs must satisfy:

$$M_W^2 \;=\; \frac{1}{2} g^2 (v_u^2 + v_d^2) \equiv g^2 \frac{v^2}{2} \quad (4.14)$$

$$M_Z^2 = \frac{1}{2}(g^2 + g'^2)v^2 \equiv M_W^2 \cos^2\theta_W \quad (4.15)$$

$$\tan\beta = \frac{v_u}{v_d}\ , \quad 0 < \beta < \frac{\pi}{2} \quad (4.16)$$

$$\tan\theta_W = \frac{g'}{g} \quad (4.17)$$

where θ_W and gauge boson masses have already been measured. Here, the additional parameter $\tan\beta$ is an unknown of the model, and it signals the presence of more than one single Higgs doublet.

These VEV's make the Higgs fields to mix up. There are five physical Higgs fields: a couple of charged Higgs bosons ($H^\pm$); a "pseudoscalar" Higgs ($CP = -1$) A^0; and two scalar Higgs bosons ($CP = 1$) H^0 (the heaviest) and h^0 (the lightest). There are also the Goldstone bosons G^0 and $G^\pm$. The relation between the physical Higgs fields and that fields of (4.1) is

$$\begin{pmatrix} -H_d^\pm \\ H_u^\pm \end{pmatrix} = \begin{pmatrix} cos\beta & -sin\beta \\ sin\beta & cos\beta \end{pmatrix} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} \quad (4.18)$$

$$\begin{pmatrix} H_d^0 \\ H_u^0 \end{pmatrix} = \begin{pmatrix} v_d \\ v_u \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{pmatrix} \begin{pmatrix} H^0 \\ h^0 \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} -cos\beta & -sin\beta \\ sin\beta & cos\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \quad (4.19)$$

were α is special to real parts of $H_{u,d}^0$ *i.e.* the neutral Higgs sector. All the masses of the Higgs sector of the MSSM can be obtained with only two parameters, the first one is $tan\beta$, and the second one is a mass; usually this second parameter is taken to be either the charged Higgs mass $m_{H^\pm}$ or the pseudoscalar Higgs mass m_{A^0}. We will take the last option. From (4.12) one can obtain the tree-level mass relations between the different Higgs particles,

$$\begin{aligned} m_{H^\pm}^2 &= m_{A^0}^2 + M_W^2\ , \\ m_{H^0,h^0}^2 &= \frac{1}{2}\left(m_{A^0}^2 + M_Z^2 \pm \sqrt{\left(m_{A^0}^2 + M_Z^2\right)^2 - 4\, m_{A^0}^2\, M_Z^2 \cos^2 2\beta}\right) \end{aligned} \quad (4.20)$$

The immediate consequence of such a constrained Higgs sector, is the existence of absolute bounds (at tree level) for the Higgs masses:

$$0 < m_{h^0} < m_Z < m_{H^0}, \quad m_W < m_{H^\pm} \quad (4.21)$$

where experiments have already bounded m_h, the lightest Higg mass, to be larger than 114 GeV. Therefore, these tree-level relations are far from representing the reality; one needs radiative effects to be incorporated into the Higgs potential. This we do in the analysis given in next chapter.

4.2.2. The SM Interactions

In this part we give some expressions to obtain some MSSM parameters as a function of the SM parametrization.

As stated above, the Higgs VEV's can be obtained by means of (4.17), and the Z mass can be obtained at tree-level via the relation:

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} \ .$$

Fermion masses are obtained from the Yukawa potential (4.5) by letting the neutral Higgs fields acquire their VEV(4.17). The up-type fermions get their masses from the H_u^0 whereas H_d^0 gives masses to down-type fermions, so

$$m_u = h_u v_2 = \frac{h_u\sqrt{2}M_W\sin\beta}{g} \ , \ m_d = h_d v_1 = \frac{h_d\sqrt{2}M_W\cos\beta}{g} \ ,$$

and the Yukawa coupling can be obtained as

$$\lambda_u = \frac{h_u}{g} = \frac{m_u}{\sqrt{2}M_W\sin\beta} \ , \ \lambda_d = \frac{h_d}{g} = \frac{m_d}{\sqrt{2}M_W\cos\beta} \ . \qquad (4.22)$$

4.2.3. Sfermion sector

The sfermion mass terms are determined by the F terms computed from the superpotential (4.6), the D-terms as well as the Soft–Breaking terms (4.11). By letting the neutral Higgs fields get their (4.13), one obtains the following mass matrices:

$$\mathcal{M}_{\tilde{q}}^2 = \begin{pmatrix} M_{\tilde{q}_L}^2 + m_q^2 + \cos 2\beta(-s_W^2)M_Z^2 & m_q M_{LR}^q \\ m_q M_{LR}^q & M_{\tilde{q}_R}^2 + m_q^2 + \cos 2\beta\, Q_q\, s_W^2\, M_Z^2 \end{pmatrix} \qquad (4.23)$$

where Q the electric charge of the corresponding fermion and $s_W = \sin\theta_W$ (Haber and Kane 1985, Ferrera 1985). The mixings among left– and right–chirality squarks,

$M_{LR,RL}$, follow from the F terms (the ones depending on μ) and soft terms (the ones involving A_f):

$$M_{LR}^u = A_u - \mu \cot\beta \ ,$$
$$M_{LR}^d = A_d - \mu \tan\beta \ . \quad (4.24)$$

We define the sfermion mixing matrix as ($\tilde{q'}_a = \{\tilde{q'}_1 \equiv \tilde{q}_L,\ \tilde{q'}_2 \equiv \tilde{q}_R\}$ are the weak-eigenstate squarks, and $\tilde{q}_a = \{\tilde{q}_1, \tilde{q}_2\}$ are the mass-eigenstate squark fields)

$$\tilde{q'}_a = \sum_b R_{ab}^{(q)} \tilde{q}_b \quad (4.25)$$

with the mixing matrix

$$R^{(q)} = \begin{pmatrix} \cos\theta_q & -\sin\theta_q \\ \sin\theta_q & \cos\theta_q \end{pmatrix} \quad (4.26)$$

diagonalizing the mass-sqaured matrix of the sfermion under concern:

$$R^{(q)\dagger} \mathcal{M}_{\tilde{q}}^2 R^{(q)} = \mathrm{diag}\{m_{\tilde{q}_2}^2, m_{\tilde{q}_1}^2\} \qquad (m_{\tilde{q}_2} \geq m_{\tilde{q}_1}) \quad (4.27)$$

This expression is valid for describing the mixing between the left– and right–chiralities of a given sfermion. In other words, it is intra-generational mixing. However, on top of such mixings, there exist mixings among different generations of down and up squarks, separately. These intergenerational mixings are discussed below.

4.2.4. Flavor Changing Neutral Currents

The most general MSSM includes tree-level flavor changing neutral currents (FCNCs) among sfermions. They induce loop-level FCNC interactions among the SM particles. Given the observed smallness of these interactions, tree-level SUSY FCNCs are usually avoided by including one of the two following assumptions: either the SUSY particle masses are very large, and their radiative effects are suppressed by the large SUSY mass scale; or the soft SUSY-breaking squark mass matrices are aligned with the SM quark mass matrix, so that both mass matrices are simultaneously diagonalized. However, if one looks closely, it is easy to realize that the MSSM does not only include the possibility of tree-level FCNCs, but it actually *requires* their existence (Duncan 1983). Indeed, the requirement of $SU(2)_L$ gauge invariance means that the up-left-squark mass matrix can not be simultaneously diagonal with the down-left-squark mass matrix, and therefore these two matrices cannot be simultaneously diagonalized unless both of them are proportional to the identity matrix. However, even then we could not take such a possibility seriously, for the radiative corrections would produce non-zero elements in the non-diagonal part of the mass matrix (i.e. induced by $H^{\pm}$ and $\chi^{\pm}$). All in all, we naturally expect tree-level FCNC interactions mediated by the SUSY partners of the SM particles. As an example, in the MSSM one can not set the FCNC Higgs bosons interactions to zero without inconsistency with UV divergence being absent (Hikasa and Kobayashi 1987). The potentially largest FCNC interactions are those originating from the strong supersymmetric (SUSY-QCD) sector of the model (viz. those interactions involving the squark-quark-gluino couplings and squark-quark-higgsino couplings). In the next chapter we will mainly concentrate on such. These couplings induce FCNC loop effects on more conventional fermion-fermion interactions, like the gauge boson-quark vertices.

In general, sfermions of a given electric charge (say, up squarks) exhibit a rather generic structure of flavor mixings. Typically one has the structure

$$M^2_{\tilde{q}} = \begin{pmatrix} M^2_{LL} & M^2_{LR} \\ M^2_{RL} & M^2_{RR} \end{pmatrix} \tag{4.28}$$

where $(1,1)$ element describes mixing among left-handed sfermions, $(1,2)$ element does mixing among left– and right–handed sfermions, and finally $(2,2)$ element holds for right-handed sfermions. This is a 6×6 mass-squared matrix, and contributions of sfermions to rare processes requires its full diagonalization. The 6 mass-eigenstate squarks exhibit non-negligible flavor-changing vertices with gluinos and quarks. This is the source of SUSY flavor violation and it arises from flavor structures of the squark soft mass-squareds as well as their trilinear couplings.

In computing the contributions of sparticle loops to FCNC processes, sometimes it proves useful to use an approximation scheme instead of full diagonalization of the squark soft mass-squareds (4.28). The idea is to represent SUSY-induced amplitudes in terms of "mass insertions" instead of sparticle mixing angles. In general, we define mass insertion between a sfermion of chirality a in generation i and the one with chirality b and generation j as follows:

$$(\delta_{ab})_{ij} = \frac{(M^2_{ab})_{ij}}{M^2_0} \tag{4.29}$$

where M^2_0 stands for the mean of the diagonal terms. The use of mass insertions provides an easy-to-follow way of sparticle contributions. However, for this method to be applicable the flavor-violating entries of the sfermion mass-squared matrix must be sufficiently small compared to the diagonal ones (Demir 2003).

Of course, low energy meson physics puts stringent constraints on the possible value of the FCNC couplings, especially for the first and second generation squarks which are sensitive to the data on $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ (Gabbiani et al. 1996, Misiak et al. 1997, Buras and Lindner 1998). The third generation system is, in principle, very loosely constrained since present data on $B^0 - \bar{B}^0$ mixing still leaves a wide-enoug room for FCNCs (Barbieri and Giudice 1993).

4.2.5. Renormalization Group Equations (RGE)

Irrespective of if we are discussing SM or MSSM or some other model; the quantities depend on scale at which theory is renormalized. The main reason is that Green functions are truncated at a specific order and thus there is an explicit dependence on the scale of renormalization. For collider processes, for instance, it is necessary to compute all masses and couplings at the scale relevant for the collider. Indeed, even if we are given a set of soft-breaking masses at the Planck scale, for estimating certain physical observables to be measured at the LHC, it is necessary to renormalize all soft masses and coupling down to the scale of $Q = 1$ TeV. The scale-dependence of lagrangian parameters are obtained by solving the Renormalization Group Equations (RGE) that they obey. They are first order coupled differential equations, and running of parameters could be quite substantial. For example, experimental values of gauge couplings at $Q \sim M_Z$ are quite different but their running under RGEs make them unite at a scale $Q \sim 10^{16}$ GeV.

In order to characterize RGE's we need to identify some basic examples for some soft masses at two loop order (Martin and Vaughn 1993). Basically, we can show general constructions of RGE's and then construct an example for our purposes. We consider a general $N = 1$ supersymmetric $\mathrm{SU}(3)_c$ gauge theory. The chiral superfields Φ_i contain a complex scalar ϕ_i and a two-component fermion ψ_i which transform as a (possibly reducible) representation R of the gauge group G. The superpotential is

$$W = \frac{1}{6}Y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu^{ij}\Phi_i\Phi_j + L^i\Phi_i \quad . \tag{1}$$

The RGEs for the gauge coupling and the superpotential parameters Y^{ijk}, μ^{ij} and L^i and the gaugino mass M are known previously. Let $\mathbf{t}^A \equiv (\mathbf{t})_i^{Aj}$ denote the representation matrices for the gauge group G. Then

$$(\mathbf{t}^A\mathbf{t}^A)_i^j \equiv C(R)\delta_i^j \qquad \mathrm{Tr}_R(\mathbf{t}^A\mathbf{t}^B) \equiv S(R)\delta^{AB}$$

define the quadratic Casimir invariant $C(R)$ and the Dynkin index $S(R)$ for the representation R. For the adjoint representation [of dimension denoted by $d(G)$], $C(G)\delta^{AB} = f^{ACD}f^{BCD}$ with f^{ABC} are structure constants of the group. The evolution of the superpotential couplings are given by

$$\frac{d}{dt}Y^{ijk} = Y^{ijp}[\frac{1}{16\pi^2}\gamma_p^{(1)k} + \frac{1}{(16\pi^2)^2}\gamma_p^{(2)k}] + (k \leftrightarrow i) + (k \leftrightarrow j) \tag{4.30}$$

$$\frac{d}{dt}\mu^{ij} = \mu^{ip}\left[\frac{1}{16\pi^2}\gamma_p^{(1)j} + \frac{1}{(16\pi^2)^2}\gamma_p^{(2)j}\right] + (j \leftrightarrow i) \tag{4.31}$$

$$\frac{d}{dt}L^i = L^p\left[\frac{1}{16\pi^2}\gamma_p^{(1)i} + \frac{1}{(16\pi^2)^2}\gamma_p^{(2)i}\right] \tag{4.32}$$

where

$$\gamma_i^{(1)j} = \frac{1}{2}Y_{ipq}Y^{jpq} - 2\delta_i^j g^2 C(i) \tag{4.33}$$

$$\begin{aligned}\gamma_i^{(2)j} &= -\frac{1}{2}Y_{imn}Y^{npq}Y_{pqr}Y^{mrj} + g^2Y_{ipq}Y^{jpq}[2C(p) - C(i)] \\ &+ 2\delta_i^j g^4[C(i)S(R) + 2C(i)^2 - 3C(G)C(i)].\end{aligned} \tag{4.34}$$

In these equations, $C(r)$ always refers to the quadratic Casimir invariant of the representation carried by the indicated chiral superfield, while $S(R)$ refers to the total Dynkin index summed over all of the chiral superfields. The objects $\gamma_i^{(1)j}$ and $\gamma_i^{(2)j}$ arise completely from the wave-function renormalization in the superfield approach.

Given the running of Yukawa couplings Y_{ijk} above, then the trilinear couplings run as follows:

$$\frac{d}{dt}h^{ijk} = \tfrac{1}{16\pi^2}\left[\beta_h^{(1)}\right]^{ijk} + \tfrac{1}{(16\pi^2)^2}\left[\beta_h^{(2)}\right]^{ijk} \tag{4.35}$$

whose structures is similar to that of the Yukawa coupings due to the holomoprhicity of the couplings. Here beta function coefficients are given by

$$\begin{aligned}[\beta_h^{(1)}]^{ijk} &= \frac{1}{2}h^{ijl}Y_{lmn}Y^{mnk} + Y^{ijl}Y_{lmn}h^{mnk} - 2\left(h^{ijk} - 2MY^{ijk}\right) \\ & g^2C(k) + (k \leftrightarrow i) + (k \leftrightarrow j) \end{aligned}\tag{4.36}$$

and

$$\begin{aligned}[\beta_h^{(2)}]^{ijk} &= -\frac{1}{2}h^{ijl}Y_{lmn}Y^{npq}Y_{pqr}Y^{mrk} - Y^{ijl}Y_{lmn}Y^{npq}Y_{pqr}h^{mrk} - Y^{ijl}Y_{lmn}h^{npq}Y_{pqr}Y^{mrk} \\ &+ \left(h^{ijl}Y_{lpq}Y^{pqk} + 2Y^{ijl}Y_{lpq}h^{pqk} - 2MY^{ijl}Y_{lpq}Y^{pqk}\right)g^2\left[2C(p) - C(k)\right] \\ &+ \left(2h^{ijk} - 8MY^{ijk}\right)g^4\left[C(k)S(R) + 2C(k)^2 - 3C(G)C(k)\right] \\ &+ (k \leftrightarrow i) + (k \leftrightarrow j)\end{aligned}\tag{4.37}$$

The running of the soft masses can also be obtained in a similar fashion:

$$\frac{d}{dt}(m^2)_i^j = \frac{1}{16\pi^2}[\beta_{m^2}^{(1)}]_i^j + \frac{1}{(16\pi^2)^2}\left[\beta_{m^2}^{(2)}\right]_i^j \tag{4.38}$$

with beta function coefficients

$$\begin{aligned}[\beta_{m^2}^{(1)}]_i^j &= \frac{1}{2}Y_{ipq}Y^{pqn}(m^2)_n^j + \frac{1}{2}Y^{jpq}Y_{pqn}(m^2)_i^n + 2Y_{ipq}Y^{jpr}(m^2)_r^q \\ &+ h_{ipq}h^{jpq} - 8\delta_i^j MM^\dagger g^2C(i) + 2g^2\mathbf{t}_i^{Aj}\mathrm{Tr}[\mathbf{t}^A m^2]\end{aligned}\tag{4.39}$$

and

$$\begin{aligned}[\beta_{m^2}^{(2)}]_i^j &= -\frac{1}{2}(m^2)_i^l Y_{lmn}Y^{mrj}Y_{pqr}Y^{pqn} - \frac{1}{2}(m^2)_l^j Y^{lmn}Y_{mri}Y^{pqr}Y_{pqn} \\ &- Y_{ilm}Y^{jnm}(m^2)_r^l Y_{npq}Y^{rpq} - Y_{ilm}Y^{jnm}(m^2)_n^r Y_{rpq}Y^{lpq} \\ &- Y_{ilm}Y^{jnr}(m^2)_n^l Y_{pqr}Y^{pqm} - 2Y_{ilm}Y^{jln}Y_{npq}Y^{mpr}(m^2)_r^q \\ &- Y_{ilm}Y^{jln}h_{npq}h^{mpq} - h_{ilm}h^{jln}Y_{npq}Y^{mpq} - h_{ilm}Y^{jln}Y_{npq}h^{mpq} - Y_{ilm}h^{jln}h_{npq}Y^{mpq} \\ &+ \Big[(m^2)_i^l Y_{lpq}Y^{jpq} + Y_{ipq}Y^{lpq}(m^2)_l^j + 4Y_{ipq}Y^{jpl}(m^2)_l^q + 2h_{ipq}h^{jpq}\end{aligned}$$

$$
\begin{aligned}
&- \quad 2h_{ipq}Y^{jpq}M - 2Y_{ipq}h^{jpq}M^{\dagger} + 4Y_{ipq}Y^{jpq}MM^{\dagger}\Big]g^2\left[C(p)+C(q)-C(i)\right] \\
&- \quad 2g^2\mathbf{t}_i^{Aj}(\mathbf{t}^A m^2)_r^l Y_{lpq}Y^{rpq} + 8g^4\mathbf{t}_i^{Aj}\mathrm{Tr}[\mathbf{t}^A C(r)m^2] \\
&+ \quad \delta_i^j g^4 MM^{\dagger}\left[24C(i)S(R)+48C(i)^2-72C(G)C(i)\right] \\
&+ \quad 8\delta_i^j g^4 C(i)(\mathrm{Tr}[S(r)m^2]-C(G)MM^{\dagger}) \qquad (4.40)
\end{aligned}
$$

The full set of RGEs can be found in appendix B taken from (Martin and Vaughn 1993).

CHAPTER 5

FLAVOR VIOLATION

It is essential for the existing and planned colliders and of the meson factories to test the standard model (SM) and determine possible 'new physics' effects on its least understood sectors: breakdown of CP, flavor and gauge symmetries. In the standard picture, both CP and flavor violations are restricted to arise from CKM matrix, and the gauge symmetry breaking is accomplished by introducing the Higgs field. However, the Higgs sector is badly behaved at quantum level; its stabilization against quadratic divergences requires supersymmetry (SUSY) or some other extension of the standard model (SM).

We proceed that Supersymmetric theories are prime candidates to replace the standard electroweak model, among which the minimal extension (MSSM) occupies a special place. It is known from the SUSY perspective due to the null collider searches that yet unobserved supersymmetric spectra implies the existence of a soft symmetry breaking mechanism which might have impact on our perception of the fundamental physics if SUSY is really the way chosen by the mother Nature. The soft breaking sector of the MSSM accommodates novel sources for CP and flavor violations. The Yukawa couplings, which are central to Higgs searches at the LHC, differ from all other couplings in the lagrangian in one aspect: the radiative corrections from sparticle loops depend only on the ratio of the soft masses and hence they do not decouple even if the SUSY breaking scale lies far above the weak scale. In this sense, non-standard hierarchy and texture of Higgs-quark couplings, once confirmed experimentally, might provide direct access to sparticles irrespective of how heavy they might be.

In order to explain the observed flavor mixing patterns and the spectrum of fermion masses, many theoretical and phenomenological models are developed. Radiative mechanisms, textures, family symmetries and the seasaw mechanism can be mentioned among them, which are related with each other to some extend. While the origin of flavor is not known in both of the models, in the minimal supersymmetric theory fermion masses are related with two Higgs doublets contrary to the unique

Higgs doublet of the SM. In the MSSM up(down) type Higgs fields can couple to up (down) quarks at the tree level, however, once radiative corrections are realized the coupling properties of Higgs bosons change, leaving fermion masses and flavor mixing currents disturbed by the loop effects.

Interestingly, SUSY explanation of the flavor mixing observed among fermions could be quite different from what is proposed in the SM. This situation brings opportunities offered by SUSY to have some explanations associated with phenomena like, the hierarchy of charged fermions mass spectra, origin of flavor mixing and CP violation which suffers from an adequate answer within the realm of the SM. Solid examples concerning this issue will be given in the following parts for quark sector only. Here it suffices to stress that instead of the standard electroweak explanation of the observed flavor mixing, it may also be attributed to the soft breaking sector of the SUSY. Naturally, this possibility worsens the *flavor problem* . Nevertheless, the shortcomings of the SM like inadequate explanation of the baryon asymmetry observed in the the universe, no dark matter candidate,...etc. (for MSSM motivations) raise questions on the flavor mixing interpretations of the standard model, even if it faces no serious problem in confrontation with data, for the time being. We expect this situation to change as colliders begin to probe deeper energies where decoupling properties of supersymmetrics particles become more severe.

Related with flavor physics, on the experimental side, high precision determination of the flavor mixing parameters ensured by B meson factories opened up a new era, which will be enriched with the start of the LHC and the ILC . Accumulation of the related data will demand interpretation of quark mixing and CP violation and thereby provide useful hints towards discovering the hidden dynamics behind fermion mass generation and CP violation. On the theoretical side it should be noticed that Yukawa matrices are the sole sources of flavor mixing and fermion masses for the SM. This case is very similar for Minimal Flavour Violation (MFV) SUSY models, in which flavor and CP violation is governed entirely by the CKM matrix .

For general SUSY models the case is more complicated due to additional structures present within those theories. On one hand, flavor mixing observed among quarks is explained within the standard model, further, consistency of SM expectations with experiments is also impressive, on the other hand, there are also important possibilities emerging from the supersymmetric theories that they may alter the whole picture, especially from the viewpoint in which SM is seen as a residue of a higher effective theory. In this respect, flavor physics opens a beautiful door, denoting supersymmetric theories have additional sources of flavor violating terms which could be the hidden reason for the observed quark mixing. This idea may be clarified by the production of the well known quark mixing patterns with the contributions coming from the other sources of flavor mixing terms around the weak scale.

Indeed, it is important to study aspects of supersymmetric theories as general as possible that may give us such hints. As is well known different supersymmetric models predict distinct soft breaking sectors and this can be seen in the superpotential of the Higgs sectors. Interactions of Higgs doublets, especially those related with flavor violation may give us important clues as to which supersymmetric model is to replace the SM and about the mechanism behind the symmetry breaking. Most probably phenomenological approaches will play a crucial role in this direction. Actually, in SUSY models flavor violation may stem from various sources which include not only the Yukawa couplings of the standard theory but also trilinear couplings and soft mass terms of the additional symmetry. This issue is addressed in a recent paper of Chankowski et al.(Chankowski et al 2005) in which a classification of the flavor violating sources is given within the Supergravity (SUGRA) framework.

In this chapter, we will study the MSSM with the consideration of radiative corrections on squark-gluino and squark-higgsino loops . Our calculations for radiative calculations are based on a recent work (Demir 2003) which discusses the radiative corrections to Yukawa couplings from sparticle loops and their impact on FCNC observables and Higgs phenomenology. Notice that, FCNC SUSY contributions do not arise from the mere supersymmetrization of the FCNC in the SM. They originate from the FC couplings of gluinos and neutralinos to fermions and sfermions as stated in (Duncan 1983) previous chapter. When supersymmetry is broken and the heavy degrees of freedoms are integrated out this symmetry of the Higgs sector is also broken, which eventually can change the coupling properties of the Higgs bosons with fermions and/or bosons of the SM.

5.0.6. The Formalism

The superpotential of the MSSM (4.4) encodes the rigid parameters μ and Yukawa couplings $\mathbf{Y_{u,d,e}}$ (of up quarks, down quarks and of leptons) each being a 3×3 non-hermitian matrix in the space of fermion flavors.

The breakdown of supersymmetry is parameterized by a set of soft (*i.e.* operators of dimension ≤ 3) terms (Chung et al. 2005)

$$\begin{aligned}\mathcal{L}_{soft} &= m^2_{H_u} H^\dagger_u H_u + m^2_{H_d} H^\dagger_d H_d + \widetilde{Q}^\dagger \mathbf{m^2_Q} \widetilde{Q} + \widetilde{U} \mathbf{m^2_U} \widetilde{U}^\dagger + \widetilde{D} \mathbf{m^2_D} \widetilde{D}^\dagger + \widetilde{L}^\dagger \mathbf{m^2_L} \widetilde{L} + \widetilde{E} \mathbf{m^2_E} \widetilde{E}^\dagger \\ &+ \left[\widetilde{U} \mathbf{Y^A_u} \widetilde{Q} H_u + \widetilde{D} \mathbf{Y^A_d} \widetilde{Q} H_d + \widetilde{E} \mathbf{Y^A_e} \widetilde{L} H_d + \mu B H_u H_d + \frac{1}{2} \sum_\alpha M_\alpha \lambda_\alpha \lambda_\alpha + \text{h.c.} \right] \end{aligned} \quad (5.1)$$

where trilinear couplings $\mathbf{Y^A_{u,d,e}}$ like Yukawas themselves are non-hermitian flavor matrices whereas the sfermion mass-squareds $\mathbf{m^2_{Q,\dots,E}}$ are all hermitian. In general, all of the parameters in the second line and off-diagonal entries of the sfermion mass-squared matrices are endowed with CP–odd phases; they serve as sources of CP violation beyond the SM. The Yukawa matrices, trilinear couplings and sfermion mass-squareds facilitate flavor violation in processes mediated by sparticle loops. The MSSM possesses 21 mass parameters, 36 mixing angles and 40 CP-odd phases in addition to ones in the SM (Dimopoulos and Sutter 1995). Consequently, there is a 97-dimensional parameter space to be scanned in confronting theory with experiments at M_{weak}. In supergravity or string models the parameters of (4.4) and (5.1) are determined by compactification mechanism and structure of the internal manifold (Bouquet et al. 1984, Hall et al. 1986, Brignole et al. 1997).

The parameters of (4.4) and (5.1) are scale- dependent. They are rescaled to $Q = M_{weak}$ via the MSSM RGEs (Ross and Roberts 1992, Kelley et al. 1991, Castano et al. 1994, Avdeev et al. 1998) and Appendix C with boundary conditions specified at $Q = M_{GUT}$. The RG running of model parameters is crucial. In fact, various phenomena central to supersymmetry phenomenology *e.g.* gauge coupling unification, radiative electroweak breaking, induction of flavor structures even for flavor-blind soft terms are pure renormalization effects. The Yukawa couplings, μ parameter and gauge couplings form a coupled closed set of observables (Demir 2005) in that their scale dependencies are not affected by soft-breaking sector unless some sparticles are decoupled before reaching M_{weak}. Flavor mixings exhibited by

$\mathbf{m^2_Q}$ at $Q = M_{weak}$ can stem from $\mathbf{m^2_{Q,U,D}}$ or $\mathbf{Y_{u,d}}$ or $\mathbf{Y^A_{u,d}}$ or all of them. Therefore, a given pattern of flavor mixings in, for instance, kaon system can be sourced by various flavor matrices in rigid as well as soft sectors of the theory.

The flavor structures at M_{weak} arising from solutions of RGEs are further rehabilitated by taking into account the decoupling of sparticles at the supersymmetric threshold. Indeed, once part of the sparticles are integrated out of the spectrum the effective theory below M_{weak} can exhibit sizeable non-standard effects in certain scattering channels of the SM particles (Demir 2003, Curiel et al. 2003, Atwood et al. 2002). Taking the effective theory below M_{weak} to be two-Higgs-doublet model (2HDM) one finds

$$\begin{aligned} \mathbf{Y_d}^{eff} &= \mathbf{Y_d}(M_{weak}) - \gamma^d + \tan\beta\,\Gamma^d \\ \mathbf{Y_u}^{eff} &= \mathbf{Y_u}(M_{weak}) + \gamma^u - \cot\beta\,\Gamma^u \end{aligned} \tag{5.2}$$

where $\mathbf{Y_{d,u}}(M_{weak})$ are solutions of the corresponding RGEs evaluated at $Q = M_{weak}$, and $\gamma^{d,u}$ and $\Gamma^{d,u}$ are flavor matrices arising from squark-gluino and squark-Higgsino loops. Their explicit expressions can be found in(Demir 2003) and Appendix D.

The physical quark fields are obtained by rotating the original gauge eigenstate fields via the unitary matrices $V^{u,d}_{R,L}$ that diagonalize $\mathbf{Y_{u,d}}^{eff}$:

$$\left(V^d_R\right)^\dagger \mathbf{Y_d}^{eff} V^d_L = \overline{\mathbf{Y_d}}\ , \quad \left(V^u_R\right)^\dagger \mathbf{Y_u}^{eff} V^u_L = \overline{\mathbf{Y_u}} \tag{5.3}$$

where $\overline{\mathbf{Y_d}} = \text{diag.}\left(\overline{h_d}, \overline{h_s}, \overline{h_b}\right)$ and $\overline{\mathbf{Y_u}} = \text{diag.}\left(\overline{h_u}, \overline{h_c}, \overline{h_t}\right)$ are physical Yukawa matrices whose entries are directly related to running quark masses 4.2.2. at $Q = M_{weak}$.

In general, whatever flavor textures are adopted at M_{GUT}, the resulting CKM matrix, $V^{corr}_{CKM} \equiv \left(V^u_L\right)^\dagger V^d_L$, must agree with the existing experimental bounds (Eidelman et al. 2004). Clearly, in the limit of vanishing threshold corrections $\Gamma^{u,d}$ and $\gamma^{u,d}$, physical CKM matrix V^{corr}_{CKM} reduces to V^{tree}_{CKM} computed by diagonalizing $\mathbf{Y_{u,d}}(M_{weak})$. Reiterating, it is with comparison of the predicted CKM matrix, V^{corr}_{CKM}, with experiment that one can tell if a high-scale texture, classified to be viable at tree-level by considering V^{tree}_{CKM} only, is spoiled by the supersymmetric threshold corrections. The experimental bounds on the absolute magnitudes of the CKM entries (at 90% CL) read collectively as:

$$|V_{CKM}^{exp}| = \begin{pmatrix} \boxed{0.9739 \,|\, 0.9751} & \boxed{0.2210 \,|\, 0.2270} & \boxed{0.0029 \,|\, 0.0045} \\ & & \\ \boxed{0.2210 \,|\, 0.2270} & \boxed{0.9730 \,|\, 0.9744} & \boxed{0.0390 \,|\, 0.0440} \\ & & \\ \boxed{0.0048 \,|\, 0.0140} & \boxed{0.0370 \,|\, 0.0430} & \boxed{0.9990 \,|\, 0.9992} \end{pmatrix} \tag{5.4}$$

where left (right) window of $\boxed{|}$ in each entry refers to lower (upper) experimental bound on the associated CKM element. Clearly, the largest uncertainity occurs in $|V_{td}|$. These matrix elements are measured at $Q = M_Z$, and for a comparison with predictions of the effective theory below $Q = M_{weak}$ they have to be scaled from M_Z up to M_{weak}. This can be done without having a detailed knowledge of the particle spectrum of the effective 2HDM at M_{weak} (as emphasized above, the effective theory may consist of some light superpartners in which case beta functions of certain couplings get modified as exemplified by analyses of $b \to s\gamma$ decay in effective supersymmetry (Degrassi et al. 2000, Demir and Olive 2002)) since RG running of the CKM elements is such that $V_{CKM}(1,1)$, $V_{CKM}(1,2)$, $V_{CKM}(2,1)$, $V_{CKM}(2,2)$ and $V_{CKM}(3,3)$ do not evolve with energy scale, to an excellent approximation (Olechowski and Pokorski 1986, Barger et al. 1993). Therefore, it is rather safe to confront the CKM matrix predicted by the effective theory at M_{weak} with the experimental results (5.4) entry by entry excluding, however, $V_{CKM}(1,3)$, $V_{CKM}(3,1)$, $V_{CKM}(2,3)$ and $V_{CKM}(3,2)$ for which renormalization effects can be sizeable.

In the next parts, we will compute supersymmetric threshold corrections to Yukawa couplings of quarks for certain prototype flavor textures defined at $Q = M_{GUT}$. In particular, we will evaluate radiatively corrected CKM matrix as well as couplings of the Higgs bosons to quarks to determine the impact of the decoupling of squarks out of the spectrum at M_{weak} on scattering processes at energies accessible to present and future colliders.

5.1. RGE's, Textures and a Mathematica Package (SUFLA)

Evolution of gauge, Yukawa and soft symmetry breaking terms are described by a set renormalization group equations which are known for the MSSM up to 3–

loops , (see also for 2-loop results which we use in our calculations). Those equations connect the SUSY breaking scale with the GUT scale. Analytical solutions of the RGEs are not known (except in the form of simple renormalization group invariants), but there are a number of softwares that can numerically solve RGEs of the MSSM in certain frameworks. Some of the codes that can be mentioned include Isajet, Softsusy and Suspect , which enable understanding the interesting properties of evolving terms under the RGEs. In order to characterize those terms one can assume them in special forms as hierarchic, diagonal (including universal or non-universal) and democratic structures.

While the exact form of the Yukawa textures or trilinear couplings or that of soft mass terms are not known a priori, string theory or GUT predictions ensure certain candidates. Strongest constraints that can be applied on these textures arise from weak scale observables which are to be supported by additional assumptions like unification of gauge couplings. For instance there are string motivations to imagine Yukawa matrices in certain forms at the high scale and they are to be evolved with the running gauge couplings which are expected to unify at the GUT scale. On the other hand, whatever the form of those structures at the high scale they should respect the existing collider bounds realized at the low scale. In this sense, studying FCNC transitions yields important projections on the allowed forms of string or GUT realizations.

As a matter of the fact, to handle the issue, we use 2-loop Renormalization Group Equations (RGEs) of the Minimal Supersymmetric Standard Model (MSSM) , in a top-down approach. That is we assume strict unification of gauge couplings at the Grand Unified Theory (GUT) scale together with suitable choice of Yukawa matrices which should approximately reproduce correct mass and mixing of quarks, approximate prediction for the mass of MSSM particles in accordance with the SPA point benchmark values, which respects the known constraints for today. Of course we can use an alternative approach in which weak scale parameters are well known from the beginning and used to predict the properties of gauge couplings and Yukawa textures at the GUT scale. Those two approaches are equivalent if threshold corrections are ignored. Since there are well measured quantities like quark mixing matrix, mass of quarks (at least for the third generation) and gauge couplings, the scale of unification is predicted as $\sim 2.5 \times 10^{16}$ GeV in such a typical approach.

Actually, there a number of studies that can successfully reveal the correct form of the CKM matrix under RGEs . It is common in those studies that predicted form of Yukawa matrices bring the CKM with high precision. However when corrections on Higgs couplings are in charge Yukawa matrices are to be deformed, which even has capacity to change the whole picture. Candidates for the form of Yukawa matrices chosen by the mother Nature ranges from simple textures, texture zeros, hierarchic textures to democratic textures. We will actually concentrate on two distinct forms among the mentioneds. Related with, please notice that, the unitary transformations acting on the quark fields also transform $(\text{mass})^2$ matrices, from which indirect relation of existing bounds should be inferred. Obviously, existence of corrections on the entries of Yukawa matrices changes the tree level prediction of the quark mixing matrix and also quark masses with the relaxation of constraints on flavor violating processes.

It is our aim to probe certain forms of Yukawa matrices, trilinear couplings and soft terms under RGEs such that existing bounds on the FCNC processes should be respected for certain forms and for all forms considered they should also (at least approximately) reproduce some of the well known phenomena like quark masses and their mixings when SUSY scale threshold corrections on the Higgs boson couplings are also realized. We use Supersymmetric Parameter Analysis (SPA) top-down data point in order to benchmark our results Fig.5.1.

RGEs of sources of flavor violating terms and their textures are considered, where certain examples are given as subsections. We first discuss in sensitivities of the GUT-scale CKM-ruled hierarhic and democratic Yukawa textures to supersymmetric threshold corrections when trilinear couplings are proportional to Yukawas. We investigate effects of flavor mixings in squark mass-squared matrices on textures analyzed. We determine effects of threshold corrections on Yukawa textures which would not qualify physical tree level.

In general,testing high-scale flavor structures with experimental data involves three basic ingredients:

1. Specification of flavor textures in rigid and soft sectors at the messenger scale (which we take to be the MSSM gauge coupling unification scale $Q = M_{GUT} \sim$

10^{16} GeV).

2. Rescaling of lagrangian parameters to low-scale $Q = M_{weak} \sim$ TeV via renormalization group flow. This stage is particularly important due to (i) largeness of the logs ($\log M_{GUT}/M_{weak}$) involved, and (ii) modifications of flavor structures because of mixings with others.

3. Integration out of the superpartners at M_{weak} to achieve an effective theory which comprises the SM particle spectrum with possible imprints of supersymmety in various couplings. For FCNC phenomenology this step is important as it induces flavor-nonuniversal couplings of gauge and Higgs bosons to fermions.

An analytic treatment of these three steps is simply not possible. Therefore, one needs a dedicated computer code to implement the integration of RGEs from string scale down to the electrowek scale. We prefer to use Mathematica to implement the code, and we name it SUFLA (derived from SUpersymmetric FLAvor violation). SUFLA, after feeding in the flavor structre at the string scale, integrates the RGEs at two loop level (Martin and Vaughn 1993), and after making appropriate conventional changes in the flavor matrices it computes supersymmetric threshold corrections (Demir 2003). The output of the code involves all physical masses and mixings within the MSSM with most general flavor and CP violation properties. The flow diagram of SUFLA is given in Fig. 5.1.

Any high-scale flavor structure specified in step 1 is classified to be phenomenologically viable if it agrees with experimental data after step 3. The first two steps have been widely discussed in literature by identifying flavor violation sources in general supergravity (Bouquet et al. 1984, Chankowski et al. 2005) and confronting them with experimental data on fermion masses and mixings as well as various observables in kaon and beauty systems (Campbell et al. 1987, Hagelin et al. 1994).

So far analysis of the third step above has been restricted to TeV-scale supersymmetry where gauge (Atwood et al. 2002) and Higgs (Demir 2003) bosons have been found to develop flavor-changing couplings to fermions. In particular, emphasis has been put on the couplings of Z (Atwood et al. 2002) and Higgs (Curiel et al. 2003, Arhrib et al. 2005, Foster et al. 2005, Hahn et al. 2005) to $b\overline{s}$

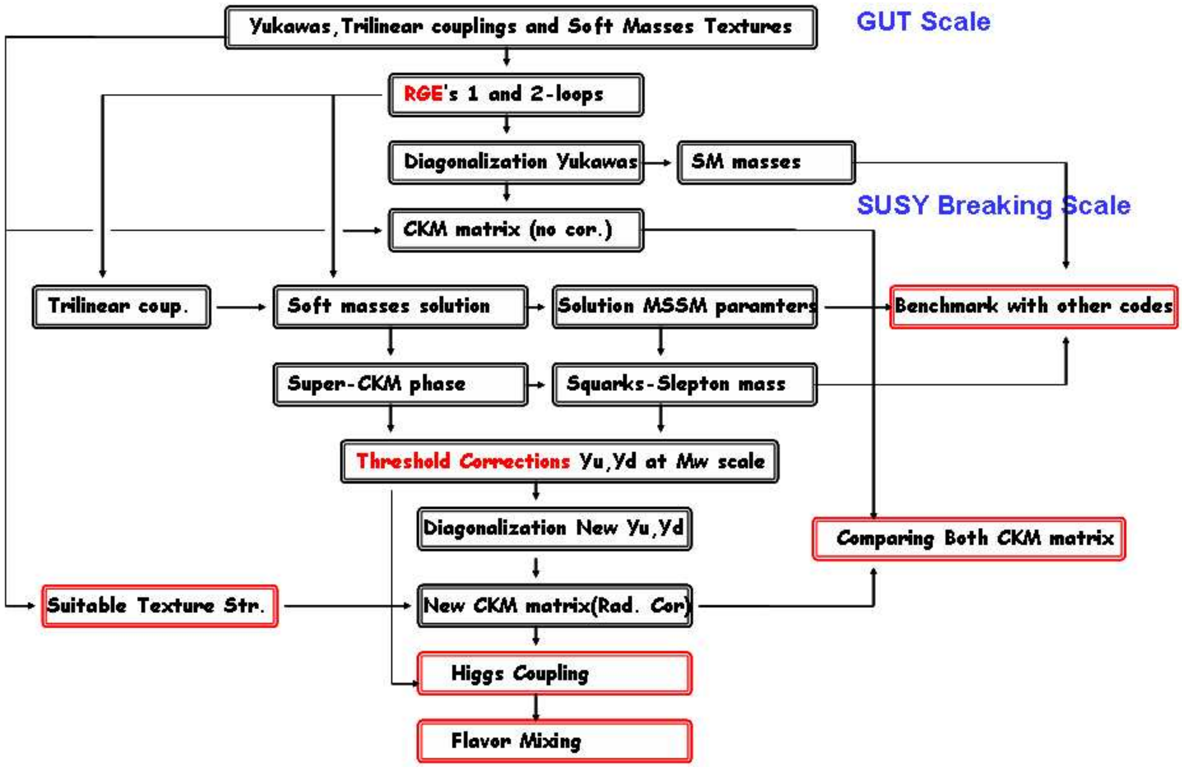

Figure 5.1: Flow diagram for the Mathematica package SUFLA.

since mixing between second and third generation fermions exhibits a theoretically clean and experimentally wide room for new physics. These analyses have led to conclusion that flavor violation sources in sfermion sector can have a big impact on Higgs phenomenology as well as various rare processes in kaon and beauty systems (Demir 2003).

5.2. High-Scale Textures and Threshold Corrections

First of all, for standardization and easy comparison with literature (*e.g.* with the computer codes ISAJET (Paige et al. 2003) and SOFTSUSY (Allanach 2002)) we take SPS1a′ conventions for supersymmetric parameters (Aguilar-Saavedra et al. 2005)

$$\tan\beta = 10\ ,\ m_0 = 70\ \mathrm{GeV}\ ,\ \mathrm{A}_0 = -300\ \mathrm{GeV}\ ,\ \mathrm{m}_{1/2} = 250\ \mathrm{GeV} \qquad (5.5)$$

and completely neglect supersymmetric CP-violating phases, as mentioned before. Instead of scanning a 97-dimensional parameter space for specifying what high-scale parameter ranges are useful for what low-energy observables, which is actually what has to be done, we simplify the analysis by focussing on certain prototype textures at high scale. In general, for any flavor matrix in any sector of the theory there exist, boldly speaking, three extremes: (*i*) completely diagonal, (*ii*) hierarchical, and (*iii*) democratic textures. There are, of course, a continuous infinity of textures among these extremes; however, for definiteness and clarity in our analysis we will focus on these three structures.

5.2.1. Flavor violation from Yukawas and Trilinear couplings

In this subsection we investigate effects of superymmetric threshold corrections on high-scale textures in which Yukawa couplings exhibit non-trivial flavor mixings and so do the trilinear couplings since we take

$$\mathbf{Y^A_{u,d,e}} = A_0 \mathbf{Y_{u,d,e}} \tag{5.6}$$

at the GUT scale. The soft mass-squareds, on the other hand, are taken entirely flavor conserving *i.e.* they are strictly diagonal and universal at the GUT scale. It is with direct proportionality of trilinear couplings with Yukawas and certain ansatze for Yukawa textures that, we will study below sensitivities of certain high-scale Yukawa structures to supersymmetric threshold corrections at the TeV scale.

5.2.1.1. CKM-ruled Texture

We take Yukawa couplings of up and down quarks to be

$$\begin{aligned} \mathbf{Y_u} &= \text{diag}\left(3.5\ 10^{-6}, 1.3\ 10^{-3}, 0.4566\right) \\ \mathbf{Y_d} &= \begin{pmatrix} 6.2368\ 10^{-5} & -1.4272\ 10^{-5} & 5.9315\ 10^{-7}\ e^{0.3146i} \\ 2.4640\ 10^{-4} & 1.07074\ 10^{-3} & -4.0458\ 10^{-5} \\ 1.6495\ 10^{-4}\ e^{1.047i} & 1.81465\ 10^{-3} & 4.8476\ 10^{-2} \end{pmatrix} \end{aligned} \tag{5.7}$$

with no flavor violation in the lepton sector: $\mathbf{Y_e} = \text{diag.}\,(1.9\ 10^{-5}, 4\ 10^{-3}, 0.071)$. The flavor violation effects are entirely encoded in $\mathbf{Y_d}$ which exhibits a CKM-ruled hierarchy in similarity to Yukawa textures analyzed in (Chankowski et al. 2005) *i.e.* this choice of boundary values of the Yukawas leads to correct CKM matrix (Eidelman et al. 2004) at M_{weak} upon integration of the RGEs.

At the weak scale the Yukawa matrices, trilinear couplings and squark soft mass-squareds serve as sources of flavor violation. The trilinear couplings, under two-loop RG running (Ross and Roberts 1992, Kelley et al. 1991, Castano et al. 1994, Avdeev et al. 1998) with boundary conditions (5.6), attain the flavor structures

$$\begin{aligned} \mathbf{Y}_u^A &= \begin{pmatrix} -7.2\ 10^{-3} & 0 & 0 \\ 1.70\ 10^{-6}\ e^{0.5641i} & -2.67 & 2.9\ 10^{-4} \\ 6.24\ e^{1.047i}\ 10^{-3} & 6.8\ 10^{-2} & -532.7 \end{pmatrix} \\ \mathbf{Y}_d^A &= \begin{pmatrix} -0.204 & -0.191 & -0.138\ e^{-1.039i} \\ -0.567 & -3.495 & -1.436 \\ -0.384\ e^{1.046i} & -4.19 & -134.24 \end{pmatrix} \end{aligned} \tag{5.8}$$

both measured in GeV at $M_{weak} = 1$ TeV. Clearly, $\mathbf{Y_u^A}$ is essentially diagonal whereas $(2,3)$, $(3,2)$ and $(2,2)$ entries of $\mathbf{Y_d^A}$ are of the same size.

Though they start with completely diagonal and universal boundary values, the squark soft squared masses develop flavor-changing entries at $M_{weak} = 1$ TeV:

$$\begin{aligned} \mathbf{m_Q^2} &= (533.67\ \text{GeV})^2 \begin{pmatrix} 1.07 & 0.0 & 0.0 \\ 0.0 & 1.07 & -2.2\ 10^{-4} \\ 0.0 & -2.2\ 10^{-4} & 0.86 \end{pmatrix} \\ \mathbf{m_D^2} &= (530.76\ \text{GeV})^2 \begin{pmatrix} 1.01 & 0.0 & 0.0 \\ 0.0 & 1.01 & -1.5\ 10^{-4} \\ 0.0 & -1.5\ 10^{-5} & 0.99 \end{pmatrix} \end{aligned} \tag{5.9}$$

with $\mathbf{m_U^2} = (497.11\ \text{GeV})^2\ \text{diag.}(1.15, 1.15, 0.69)$. The numerical values of the parameters above exhibit good agreement with well-known codes like ISAJET (Paige et al. 2003) and SOFTSUSY (Allanach 2002). The presence of flavor violation in the soft sector of the low-energy theory gives rise to non-trivial corrections to Yukawa couplings and in turn to the CKM matrix. Indeed, use of (5.8) and (5.9) in (Demir 2003) introduces certain corrections to the tree-level Yukawa matrices $\mathbf{Y_{u,d}}(M_{weak})$ to generate $\mathbf{Y_{u,d}^{eff}}$ in (5.2). In fact, V_{CKM}^{tree} (obtained from $\mathbf{Y_{u,d}}(M_{weak})$)

and V^{corr}_{CKM} (obtained from ${\mathbf{Y_{u,d}}}^{eff}$) compare to exhibit spectacular differences:

$$\boxed{|V^{tree}_{CKM}|}\boxed{|V^{corr}_{CKM}|} = \begin{pmatrix} \boxed{0.9746}\boxed{0.9795} & \boxed{0.2241}\boxed{0.2015} & \boxed{0.0037}\boxed{0.0034} \\ \boxed{0.2240}\boxed{0.2014} & \boxed{0.9737}\boxed{0.9788} & \boxed{0.0406}\boxed{0.0375} \\ \boxed{0.0079}\boxed{0.0066} & \boxed{0.0400}\boxed{0.0371} & \boxed{0.99917}\boxed{0.9993} \end{pmatrix} \quad (5.10)$$

where left (right) window of $\boxed{}\boxed{}$ in (i,j)-th entry refers to $|V^{tree}_{CKM}(i,j)|$ ($|V^{corr}_{CKM}(i,j)|$). Clearly, $|V^{tree}_{CKM}|$ agrees very well with $|V^{exp}_{CKM}|$ in (5.4) entry by entry. This qualifies (5.7) to be the correct high-scale texture given experimental FCNC bounds at $Q = M_Z$. However, radiative corrections induced by decoupling of squarks, gluinos and Higgsinos at the supersymmetric threshold $M_{weak} = 1$ TeV is seen to leave a rather strong impact on the CKM entries. Consider for instance $(1,1)$ entries of V^{exp}_{CKM}, V^{tree}_{CKM} and V^{corr}_{CKM}. Present experiments provide a 1.64σ significance to $|V^{exp}_{CKM}(1,1)|$ around a mean value of 0.745 as is seen from (5.4). The tree-level prediction, $|V^{tree}_{CKM}(1,1)|$, takes the value of 0.9746 which is rather close to the center of the experimental interval. However, once supersymmetric threshold corrections are included this tree-level prediction gets modified to $|V^{corr}_{CKM}(1,1)| = 0.9795$. This value is obviously far beyond the existing experimental limits as it is a 13.39σ effect. Similarly, $|V^{corr}_{CKM}(1,2)|$, $|V^{corr}_{CKM}(2,1)|$, $|V^{corr}_{CKM}(2,2)|$ and $|V^{corr}_{CKM}(3,3)|$ are, respectively, 12.36σ, 12.36σ, 11.95σ and 2.30σ effects.

Obviously, deviation of $|V^{corr}_{CKM}(i,j)|$ from $|V^{tree}_{CKM}(i,j)|$ (comparison with experiments at $Q = M_Z$ is meaningful especially for $(i,j) = (1,1), (1,2), (2,1), (3,3)$ entries whose scale dependencies are known to be rather mild (Olechowski and Pokorski 1986, Barger et al. 1993)), when the latter falls well inside the experimentally allowed range, obviously violates existing experimental bounds in (5.4) by several standard deviations. Consequently, supersymmetric threshold corrections entirely disqualify the high-scale texture (5.7) being the correct texture to reproduce the FCNC measurements at the weak scale. This case study, based on numerical values for Yukawa entries in (5.7), manifestly shows the impact of supersymmetric threshold corrections on high-scale textures which qualify viable at tree level. The physical quark fields, which arise after the unitary rotations (5.3), acquire the masses

$$\begin{aligned}\overline{\mathbf{M_u}}(M_{weak}) &= \text{diag.}\left(\simeq 0, 0.545, 149.45\right)\\ \overline{\mathbf{M_d}}(M_{weak}) &= \text{diag.}\left(3.35\ 10^{-3}, 5.76\ 10^{-2}, 2.33\right)\end{aligned} \tag{5.11}$$

all measured in GeV. In this physical basis for quark fields, V_{CKM}^{corr} governs the strength of charged current vertices for each pair of up and down quarks. These mass predictions are to be evolved down to $Q = M_Z$ to make comparisons with experimental results. This evolution depends on the effective theory below M_{weak}. Speaking conversely, the high-scale texture (5.7) has to be folded in such a way that resulting mass and mixing patterns for quarks agree with experiments below the sparticle threshold M_{weak}.

5.2.1.2. Hierarchical Texture

The Yukawa couplings are taken to have the structure (as can be motivated from (?Ko and Kobayashi 2004, Chankowski et al 2005))

$$\begin{aligned}\mathbf{Y_u} &= \begin{pmatrix} 2.6463\ 10^{-4} & 5.8163\ 10^{-4} i & -1.0049\ 10^{-2} \\ -5.8163\ 10^{-4} i & 2.2587\ 10^{-3} & 1.0049\ 10^{-5} i \\ -4.8233\ 10^{-3} & -9.0437\ 10^{-6} i & 0.495 \end{pmatrix}\\ \mathbf{Y_d} &= \begin{pmatrix} 3.9808\ 10^{-4} & 8.1167\ 10^{-4}\ e^{0.734 i} & -1.1431\ 10^{-3} \\ 8.1167\ 10^{-4}\ e^{-0.734 i} & 2.7997\ 10^{-3} & 2.04844\ 10^{-3} i \\ -1.1431\ 10^{-3} & -1.6461\ 10^{-3} i & 4.97\ 10^{-2} \end{pmatrix}\end{aligned} \tag{5.12}$$

with no flavor violation in the lepton sector: $\mathbf{Y_e} = \text{diag.}\left(1.9\ 10^{-5}, 0.004, 0.071\right)$. Here both $\mathbf{Y_u}$ and $\mathbf{Y_d}$ exhibit a hierarchically organized pattern of entries . In a sense, the hierarchic nature of $\mathbf{Y_d}$ in (5.7) is now extended to $\mathbf{Y_u}$ so as to form a complete hierarchic pattern for quark Yukawas at the GUT scale.

At the weak scale, the Yukawa matrices above, trilinear couplings, and squark soft mass-squareds serve as sources of flavor violation. The trilinear couplings, under two-loop RG running (Ross and Roberts 1992, Kelley et al. 1991, Castano et al. 1994, Avdeev et al. 1998) with boundary conditions (5.6), obtain the flavor structures

$$\mathbf{Y}_u^A = \begin{pmatrix} -0.4315 & -1.1442i & 10.637 \\ 1.1466i & -4.4531 & -4.8631\ 10^{-3}i \\ 5.0657 & -0.1046i & -524.07 \end{pmatrix}$$

$$\mathbf{Y}_d^A = \begin{pmatrix} -1.2934 & -2.6494\ e^{0.734i} & 3.1221 \\ 2.6428\ e^{-0.731i} & -9.1395 & 5.2606i \\ 3.4532 & -5.6827i & -135.861 \end{pmatrix} \quad (5.13)$$

both measured in GeV at $M_{weak} = 1$ TeV. Clearly, in contrast to (5.8), now both $\mathbf{Y_u^A}$ and $\mathbf{Y_d^A}$ develop sizeable off-diagonal entries, as expected from (5.12). Though they start with completely diagonal and universal boundary values, the squark soft squared masses develop flavor-changing entries at $M_{weak} = 1$ TeV:

$$\mathbf{m_Q^2} = (533.69\ \text{GeV})^2 \begin{pmatrix} 1.07 & 1.9\ 10^{-5}\ e^{1.144i} & 2.14\ 10^{-3} \\ 1.9\ 10^{-5}\ e^{-1.144i} & 1.07 & 3.17\ 10^{-4}i \\ 2.14\ 10^{-3} & -3.17\ 10^{-4}i & 0.86 \end{pmatrix}$$

$$\mathbf{m_U^2} = (496.76\ \text{GeV})^2 \begin{pmatrix} 1.16 & -6.66\ 10^{-6}i & 9.6\ 10^{-3} \\ 6.66\ 10^{-6}i & 1.16 & -1.4\ 10^{-5}i \\ 9.6\ 10^{-3} & 1.4\ 10^{-5}i & 0.685 \end{pmatrix}$$

$$\mathbf{m_D^2} = (531.07\ \text{GeV})^2 \begin{pmatrix} 1.01 & 3.3\ 10^{-5}\ e^{1.06i} & 3.75\ 10^{-4} \\ 3.3\ 10^{-5}\ e^{-1.06i} & 1.01 & -6.62\ 10^{-4}i \\ 3.75\ 10^{-4} & 6.62\ 10^{-4}i & 0.99 \end{pmatrix} \quad (5.14)$$

whose average values show good agreement with (5.9) but certain off-diagonal entries exhibit significant enhancements when the corresponding entries of Yukawas and trilinear couplings are sizeable.

The flavor-violating entries of Yukawas, trilinear couplings and soft mass-squareds collectively generate radiative contributions $\gamma^{u,d}$, $\Gamma^{u,d}$ to the Yukawa couplings below M_{weak} (Demir 2003). In fact, V_{CKM}^{tree} (obtained from $\mathbf{Y_{u,d}}(M_{weak})$) and V_{CKM}^{corr} (obtained from $\mathbf{Y_{u,d}}^{eff}$) confront as follows:

$$\boxed{|V_{CKM}^{tree}|}\boxed{|V_{CKM}^{corr}|} = \begin{pmatrix} \boxed{0.9745}\boxed{0.9773} & \boxed{0.2243}\boxed{0.2118} & \boxed{0.0049}\boxed{0.0034} \\ \boxed{0.2240}\boxed{0.2116} & \boxed{0.9737}\boxed{0.9766} & \boxed{0.0417}\boxed{0.0379} \\ \boxed{0.0109}\boxed{0.0091} & \boxed{0.0405}\boxed{0.0370} & \boxed{0.99912}\boxed{0.99927} \end{pmatrix} \quad (5.15)$$

where left (right) window of $\boxed{}\boxed{}$ in (i,j)-th entry refers to $|V_{CKM}^{tree}(i,j)|$ ($|V_{CKM}^{corr}(i,j)|$). Clearly, $|V_{CKM}^{tree}|$ falls well inside the 1.64σ experimental interval in (5.4) entry by entry. In this sense, Yukawa matrices in (5.17) qualify to be the correct high-scale textures given present experimental determination of V_{CKM} at $Q = M_Z$. However, this agreement between experiment and theory gets spoiled strongly by the inclusion of supersymmetric threshold corrections. Indeed, as is shown comparatively by (5.20), V_{CKM}^{corr} violates the bounds in (5.4) significantly. More precisely, $|V_{CKM}^{corr}(1,1)|$, $|V_{CKM}^{corr}(1,2)|$, $|V_{CKM}^{corr}(2,1)|$, $|V_{CKM}^{corr}(2,2)|$, $|V_{CKM}^{corr}(3,3)|$ turn out to have 7.65σ, 6.83σ, 6.77σ, 6.79σ, 3.28σsignificance levels, respectively. These significance levels are far beyond the existing experimental 1.64σ intervals depicted in (5.4). As a result, supersymmetric threshold corrections are found to entirely disqualify the high-scale texture (5.12) to be the correct texture to reproduce the FCNC measurements at the weak scale. This case study therefore shows the impact of supersymmetric threshold corrections on high-scale textures which qualify viable at tree level. The physical quark fields, which arise after the unitary rotations (5.3), acquire the masses

$$\begin{aligned} \overline{\mathbf{M_u}}(M_{weak}) &= \mathrm{diag.}\,(0.0065, 0.98, 153.82) \\ \overline{\mathbf{M_d}}(M_{weak}) &= \mathrm{diag.}\,(0.0071, 0.155, 2.37) \end{aligned} \quad (5.16)$$

all measured in GeV. In this physical basis for quark fields, V_{CKM}^{corr} is responsible for charged current interactions in the effective theory below M_{weak}. The morale of the analysis above is that, the high-scale flavor structures (5.12) are to be modified in such a way that V_{CKM}^{corr} agrees with V_{CKM}^{exp} with sufficient precision. Aftermath, the question is to predict quark masses appropriately at $Q = M_{weak}$ so that, depending on the particle spectrum of the effective theory beneath, existing experimental values of quark masses at $Q = M_Z$ are reproduced correctly.

5.2.1.3. Democratic Texture

In this subsection, we take Yukawa couplings to be (as can be motivated from relevant works (Fritzsch and Plankl 1990, Abel et al. 2003, Branco et al. 2004))

$$\mathbf{Y_u} = \begin{pmatrix} 0.1475 & 0.1443 & 0.1458 \\ 0.1443 & 0.1475 & 0.1458 \\ 0.1456 & 0.1458 & 0.1456 \end{pmatrix} \tag{5.17}$$

$$\mathbf{Y_d} = \begin{pmatrix} 0.01583 & 0.01452(1-10^{-2}i) & 0.01553(1-10^{-2}i) \\ 0.01452(1+10^{-2}i) & 0.01944 & 0.01617(1+2\ 10^{-2}i) \\ 0.01551(1+10^{-2}i) & 0.01617(1-2\ 10^{-2}i) & 0.01604 \end{pmatrix}$$

with no flavor violation in the lepton sector: $\mathbf{Y_e} = \text{diag.}\,(1.9\ 10^{-5}, 4\ 10^{-3}, 0.071)$. Here both $\mathbf{Y_u}$ and $\mathbf{Y_d}$ exhibit an approximate democratic structure so that $\mathbf{Y_{u,d}}(M_{weak})$ generate correctly masses and mixings of the quarks at the weak scale. Clearly, in the exact democratic limit two of the quarks from each sector remain massless, and therefore, a realistic flavor structure is likely to come from small perturbations of the exact democratic texture (Fritzsch and Plankl 1990, Abel et al. 2003, Branco et al. 2004). Another important feature of exact democratic texture is that all higher powers of Yukawas reduce to Yukawas themselves up to a multiplicative factor, and this gives rise to linearization of and in turn direct solution of Yukawa RGEs in the form of an RG rescaling of the GUT scale texture (Demir 2005). These properties remain approximately valid for perturbed democratic textures like (5.17).

At the weak scale, the Yukawa matrices above, trilinear couplings, and squark soft mass-squareds serve as sources of flavor violation. The trilinear couplings, under two-loop RG running (Ross and Roberts 1992, Kelley et al. 1991, Castano et al. 1994, Avdeev et al. 1998) with boundary conditions (5.6), obtain the flavor structures

$$\mathbf{Y}_u^A = -\begin{pmatrix} 182.44 & 175.57 & 178.81 \\ 175.69 & 182.32 & 178.81 \\ 178.62 & 178.81 & 178.67 \end{pmatrix}$$

$$
\mathbf{Y}_d^A = -\begin{pmatrix} 44.41 & 40.07\ e^{-0.0117i} & 43.39\ e^{0.0115i} \\ 39.44e^{0.0101i} & 55.46\ e^{-0.0013i} & 44.82\ e^{0.0218i} \\ 43.09\ e^{-0.099i} & 45.17\ e^{-0.0216i} & 44.79\ e^{0.0016i} \end{pmatrix} \quad (5.18)
$$

both measured in GeV at $M_{weak} = 1$ TeV. Though not shown explicitly, each entry of $\mathbf{Y_u^A}$ is complex with a phase around 10^{-7} – 10^{-6} in size.

Though they start with completely diagonal and universal boundary values, the squark soft squared masses develop flavor-changing entries at $M_{weak} = 1$ TeV:

$$
\begin{aligned}
\mathbf{m_Q^2} &= (533.67\ \mathrm{GeV})^2 \begin{pmatrix} 1.0 & 0.0672 & 0.0670 \\ 0.0672 & 1.0 & 0.0673 \\ 0.0670 & 0.0673 & 1.0 \end{pmatrix} \\
\mathbf{m_U^2} &= (497.38\ \mathrm{GeV})^2 \begin{pmatrix} 1.0 & 0.1526 & 0.1524 \\ 0.1526 & 1.0 & 0.1524 \\ 0.1524 & 0.1524 & 1.0 \end{pmatrix} \\
\mathbf{m_D^2} &= (530.59\ \mathrm{GeV})^2 \begin{pmatrix} 1.0 & 5.046\ 10^{-3}\ e^{-0.01i} & 4.826\ 10^{-3}\ e^{0.01i} \\ 5.046\ 10^{-3}\ e^{0.01i} & 1.0 & 5.289\ 10^{-3}\ e^{0.02i} \\ 4.826\ 10^{-3}\ e^{-0.01i} & 5.289\ 10^{-3}\ e^{-0.02i} & 1.0 \end{pmatrix}
\end{aligned} \quad (5.19)
$$

whose average values show good agreement with (5.9) and (5.14). The off-diagonal entries of each squark soft mass-squared are of similar size due to the democratic structure of the Yukawa couplings. The flavor-mixing entries $m^2_{\widetilde{U}}$ are the largest among all three mass squareds.

The flavor-violating entries of Yukawas, trilinear couplings and soft mass-squareds collectively generate radiative contributions $\gamma^{u,d}$, $\Gamma^{u,d}$ to the Yukawa couplings below M_{weak} (Demir 2003). In fact, V_{CKM}^{tree} (obtained from $\mathbf{Y_{u,d}}(M_{weak})$) and V_{CKM}^{corr} (obtained from $\mathbf{Y_{u,d}}^{eff}$) confront as follows:

$$
\boxed{|V_{CKM}^{tree}|}\boxed{|V_{CKM}^{corr}|} = \begin{pmatrix} \boxed{0.9748}\boxed{0.9685} & \boxed{0.2229}\boxed{0.2490} & \boxed{0.0083}\boxed{0.0085} \\ \boxed{0.2229}\boxed{0.2489} & \boxed{0.9739}\boxed{0.9674} & \boxed{0.0421}\boxed{0.0463} \\ \boxed{0.0092}\boxed{0.0104} & \boxed{0.0419}\boxed{0.0459} & \boxed{0.99908}\boxed{0.99889} \end{pmatrix} \quad (5.20)
$$

where left (right) window of $\boxed{\ }\boxed{\ }$ in (i,j)-th entry refers to $|V_{CKM}^{tree}(i,j)|$ ($|V_{CKM}^{corr}(i,j)|$). Obviously, $|V_{CKM}^{tree}|$ agrees very well with $|V_{CKM}^{exp}|$ in (5.4) entry by entry. This qualifies (5.17) to be the correct high-scale texture given present experimental determination of V_{CKM} at $Q = M_Z$. The most striking aspect of (5.20) is the fact that supersymmetric threshold corrections push V_{CKM}^{tree} beyond the experimental bounds. More precisely, $|V_{CKM}^{corr}(1,1)|$, $|V_{CKM}^{corr}(1,2)|$, $|V_{CKM}^{corr}(2,1)|$, $|V_{CKM}^{corr}(2,2)|$, $|V_{CKM}^{corr}(3,3)|$ turn out to have 17.22σ, 14.21σ, 14.21σ, 15.22σ, 16.40σ significance levels, respectively. These are obviously far beyond the existing experimental 1.64σ significance intervals depicted in (5.4). As a result, supersymmetric threshold corrections are found to entirely disqualify the high-scale texture (5.17) to be the correct texture to reproduce the FCNC measurements at the weak scale. Here, it is worthy of noting that deviation of $|V_{CKM}^{corr}(i,j)|$ from $|V_{CKM}^{tree}(i,j)|$ (for $i,j = 1,2$) turns out to be similar in size for CKM-ruled (see eq. 5.10) and democratic (see eq. 5.20) textures. It is smallest for the hierarchical texture (see eq. 5.15). Therefore, CKM-ruled texture in (5.7) and democratic one in (5.17) exhibit a pronounced sensitivity to supersymmetric threshold corrections in comparison to hierarchical texture in (5.12).

The physical quark fields, which arise after the unitary rotations (5.3), acquire the masses

$$\begin{aligned} \overline{\mathbf{M_u}}(M_{weak}) &= \text{diag.}\,(0.055, 1.27, 144.78) \\ \overline{\mathbf{M_d}}(M_{weak}) &= \text{diag.}\,(0.099, 0.27, 2.4) \end{aligned} \tag{5.21}$$

all measured in GeV. In this physical basis for quark fields, V_{CKM}^{corr} is responsible for charged current interactions in the effective theory below M_{weak}. The morale of the analysis above is that, the high-scale flavor structures (5.17) are to be modified in such a way that V_{CKM}^{corr} agrees with V_{CKM}^{exp} with sufficient precision. Aftermath, the question is to predict quark masses appropriately at $Q = M_{weak}$ so that, depending on the particle spectrum of the effective theory beneath, existing experimental values of quark masses at $Q = M_Z$ are reproduced correctly.

5.3. Inclusion of Flavor Violation from squark soft masses

In this section we extend GUT-scale flavor structures analyzed in Sec. 3.1 by switching on flavor mixings in certain squark soft mass-squareds. In other words, we maintain Yukawa textures to be one of (5.7), (5.12) or (5.17), and examine what happens to CKM prediction if squared masses of squarks possess non-trivial flavor mixings at the GUT scale.

The effective Yukawa couplings ${\mathbf{Y_{u,d}}}^{eff}$ beneath $Q = M_{weak}$ receive contributions from all entries of $\mathbf{m^2_{Q,U,D}}(M_{weak})$ via respective mass insertions (Demir 2003). Generically, larger the mass insertions larger the flavor violation potential of ${\mathbf{Y_{u,d}}}^{eff}$. Consequently, main problem is to determine the relative strengths of on-diagonal and off-diagonal entries of $\mathbf{m^2_{Q,U,D}}(M_{weak})$ given that they start with a certain pattern of flavor mixings. Take, for instance, $\mathbf{m^2_Q}$ which evolves with energy scale via at single loop level. That this is the case can be seen explicitly by considering, for instance, democratic texture for Yukawas (5.17) together with (5.6) and strict universality and flavor-diagonality of the soft masses, except

$$\mathbf{m^2_Q}(0) = m_0^2 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{5.22}$$

which contributes maximally to each term . Even with such a democratic pattern for Yukawas, trilinear couplings and $\mathbf{m^2_Q}(0)$, however, one obtains at $M_{weak} = 1$ TeV

$$\mathbf{m^2_Q} = (533.37 \text{ GeV})^2 \begin{pmatrix} 1.0 & -0.0512 & -0.0510 \\ -0.0512 & 1.0 & -0.0513 \\ -0.0510 & -0.0513 & 1.0 \end{pmatrix} \tag{5.23}$$

with similar structures for $\mathbf{m^2_U}$ and $\mathbf{m^2_D}$. Alternatively, if one adopts (5.7) or (5.12) setups the off-diagonal entries of squark soft mass-squareds at M_{weak} are found to remain around m_0^2 which are much smaller than the on-diagonal ones. Therefore, Yukawa textures (and hence those of the trilinear couplings) studied in sec.4.2.1 lead one generically to hierarchic textures for squark soft mass-squareds at $Q = M_{weak}$ irrespective of how large the flavor mixings in $\mathbf{m^2_{Q,U,D}}(0)$ might be. In fact, predictions for CKM matrix remain rather close to those in sec.4.2.1. This is actually clear where off-diagonal entries of $\mathbf{m^2_{Q,U,D}}$ are seen to evolve into new mixing patterns via themselves and those of Yukawas and trilinear couplings. In conclusion, evolution of squark soft masses is fundamentally Yukawa-ruled and when Yukawas at the GUT scale are taken to shoot the measured value of CKM matrix, the mass insertions associated with $\mathbf{m^2_{Q,U,D}}(M_{weak})$ are too small to give any significant contribution to ${\mathbf{Y_{u,d}}}^{eff}$.

For generating sizeable off-diagonal entries for $\mathbf{m^2_{Q,U,D}}(M_{weak})$ it is necessary to abandon either Yukawa textures analyzed sec.4.2.1 or proportionality of trilinear couplings with Yukawas. Therefore, we take Yukawa couplings at the GUT scale precisely as (5.17), we maintain (5.6) for both $\mathbf{Y^A_d}$ and $\mathbf{Y^A_e}$, and we take $\mathbf{m^2_U}(0)$ and $\mathbf{m^2_D}(0)$ strictly flavor-diagonal as in all three case studies carried out in previously. However, we take $\mathbf{m^2_Q}(0)$ as in (5.22) above, and $\mathbf{Y^A_u}$ as

$$\mathbf{Y^A_u}(0) = -150 \text{ GeV} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{5.24}$$

which certainly violates (5.6) that enforces trilinears to be proportional to the corresponding Yukawas. Then two-loop RG running from $Q = M_{GUT}$ down to $Q = M_{weak}$ gives

$$\begin{aligned} \mathbf{Y}^A_u &= \begin{pmatrix} -262.087 & -259.342 & -260.709 \\ -259.474 & -261.954 & -260.709 \\ -260.688 & -260.674 & -260.735 \end{pmatrix} \\ \mathbf{Y}^A_d &= \begin{pmatrix} -41.435 & 37.091\ e^{-0.0127i} & -40.408\ e^{0.0124i} \\ -36.171\ e^{0.0102i} & 52.230\ e^{-0.0019i} & -41.558\ e^{0.0228i} \\ 39.983\ e^{-0.0300i} & 42.075\ e^{-0.0224i} & -41.683\ e^{0.0025i} \end{pmatrix} \end{aligned} \tag{5.25}$$

both measured in GeV at $M_{weak} = 1$ TeV. Though not shown explicitly, each entry of $\mathbf{Y^A_u}$ is complex with a phase around 10^{-7} – 10^{-6} in size. On the other hand, squark soft mass-squared at $Q = M_{weak}$ are given by

$$\begin{aligned} \mathbf{m^2_Q} &= (516.58 \text{ GeV})^2 \begin{pmatrix} 1.0 & -0.13 & -0.13 \\ -0.13 & 1.0 & -0.13 \\ -0.13 & -0.13 & 1.0 \end{pmatrix} \\ \mathbf{m^2_U} &= (455.49 \text{ GeV})^2 \begin{pmatrix} 1.0 & -0.3852 & -0.3853 \\ -0.3852 & 1.0 & -0.3853 \\ -0.3853 & -0.3853 & 1.0 \end{pmatrix} \\ \mathbf{m^2_D} &= (532.91 \text{ GeV})^2 \begin{pmatrix} 1.0 & 4.34\ 10^{-3}\ e^{-0.01i} & -4.15\ 10^{-3}\ e^{0.01i} \\ 4.34\ 10^{-3}\ e^{-0.01i} & 1.0 & -4.55\ 10^{-3}\ e^{0.02i} \\ -4.15\ 10^{-3}\ e^{0.01i} & -4.55\ 10^{-3}\ e^{0.02i} & 1.0 \end{pmatrix} \end{aligned} \tag{5.26}$$

where small phases in off-diagonal entries of $\mathbf{m^2_Q}$ and $\mathbf{m^2_U}$ are neglected. A comparison with (5.19) reveals spectacular enhancements in mass insertions pertaining $\mathbf{m^2_Q}$ and $\mathbf{m^2_U}$.

The trilinear couplings (5.25) and squark mass-squareds (5.26) give rise to non-trivial changes in flavor structures of $\mathbf{Y_{u,d}}(M_{weak})$ by generating effective Yukawas $\mathbf{Y_{u,d}}^{eff}$ beneath $Q = M_{weak}$. Then the CKM matrix V^{tree}_{CKM} obtained from $\mathbf{Y_{u,d}}(M_{weak})$ and V^{corr}_{CKM} obtained from $\mathbf{Y_{u,d}}^{eff}$ compare as:

$$\boxed{|V^{tree}_{CKM}| \,\Big|\, |V^{corr}_{CKM}|} = \begin{pmatrix} \boxed{0.9748 \,|\, 0.9637} & \boxed{0.2229 \,|\, 0.2668} & \boxed{0.0083 \,|\, 0.0080} \\ \\ \boxed{0.2229 \,|\, 0.2666} & \boxed{0.9739 \,|\, 0.9626} & \boxed{0.0421 \,|\, 0.0480} \\ \\ \boxed{0.0092 \,|\, 0.0132} & \boxed{0.0419 \,|\, 0.0468} & \boxed{0.99908 \,|\, 0.99888} \end{pmatrix} \tag{5.27}$$

where left (right) window of $\boxed{\;\,|\;\,}$ in (i, j)-th entry refers to $|V^{tree}_{CKM}(i, j)|$ ($|V^{corr}_{CKM}(i, j)|$). Obviously, $|V^{tree}_{CKM}|$ agrees very well with $|V^{exp}_{CKM}|$ as was the case in (5.20). This qualifies (5.17) to be the correct high-scale texture given present experimental determination of V_{CKM} at $Q = M_Z$. However, implementation of supersymmetric threshold corrections is seen to leave a big impact on certain entries of the physical CKM matrix. Indeed, $|V^{corr}_{CKM}(1, 1)|$, $|V^{corr}_{CKM}(1, 2)|$, $|V^{corr}_{CKM}(2, 1)|$, $|V^{corr}_{CKM}(2, 2)|$, $|V^{corr}_{CKM}(3, 3)|$ turn out to have 6.06σ, 23.99σ, 23.89σ, 26.52σ, 4.35σ significance levels, respectively. These are to be contrasted with standard deviations computed for (5.20) in Sec. sec.4.2.1 above. Needless to say, these deviations are far beyond the experimental sensitivities and thus supersymmetric threshold corrections completely disqualify the flavor textures (5.17) in a way different than (5.20) due to new structures (5.22) and (5.24).

Finally, physical quark fields, which arise after the unitary rotations (5.3), acquire the masses

$$\begin{aligned} \overline{\mathbf{M_u}}(M_{weak}) &= \text{diag.}\,(0.138, 1.26, 143.3) \\ \overline{\mathbf{M_d}}(M_{weak}) &= \text{diag.}\,(0.140, 0.304, 2.42) \end{aligned} \tag{5.28}$$

all measured in GeV. These mass predictions are close to those obtained within democratic texture. As in all cases discussed above especially light quark masses fall outside the existing experimental bounds, and choice of the correct high-scale texture must reproduce both V_{CKM}^{corr} and quark masses in sufficient agreement with experiment.

5.4. A Purely Soft CKM?

We have just discussed how prediction for the CKM matrix depends crucially on the inclusion of the supersymmetric threshold corrections. This we did by negation *i.e.* we have taken certain Yukawa textures which are known to generate CKM matrix correctly at tree level, and then included threshold corrections to demonstrate how those the would-be viable flavor structures get disqualified.

In this section we will do the opposite *i.e.* we will take a Yukawa texture which is known not to work at all, and incorporate supersymmetric threshold corrections to show how it can become a viable one, at least approximately. For sure, a highly interesting limit would be to start with exactly diagonal Yukawas at the GUT scale and generate CKM matrix beneath M_{weak} via purely soft flavor violation *i.e.* flavor violation from sfermion soft mass-squareds and trilinear couplings, alone. However, this limit seems difficult to realize, at least for SPS1a$'$ parameter values, since it may require tuning of various parameters, in particular, soft mass-squareds of Higgs and quark sectors (Demir 2003). Even if this is done by a fine-grained scan of the parameter space, it will possibly cost a great deal of fine-tuning. Indeed, threshold corrections depend on ratios of the soft masses (Demir 2003), and generating a specific entry of the CKM matrix can require a judiciously arranged hierarchy among various soft mass parameters – a parameter region certainly away from the SPS1a$'$ point.

Therefore, we relax the constraint of strict diagonality and consider instead GUT-scale Yukawa matrices with five texture zeroes which are known to be completely unphysical as they cannot induce the CKM matrix (Fritzsch and Xing 2000). In fact, this kind of textures has recently been found to arise from heterotic string (Braun et al. 2006) when the low-energy theory is constrained to be minimal supersymmetric model (Braun et al. 2005, Bouchard et al. 2006). Consequently, we take Yukawas at $Q = M_{GUT}$ to be

$$
\begin{aligned}
\mathbf{Y_u} &= \begin{pmatrix} 0 & 9.249\ 10^{-5} & 1.428\ 10^{-3} \\ 1.307\ 10^{-3} & 0 & 0 \\ 0.4675 & 0 & 0 \end{pmatrix} \\
\mathbf{Y_d} &= \begin{pmatrix} 0 & 9.0\ 10^{-5} & 1.3\ 10^{-3} \\ 1.42\ 10^{-3} & 0 & 0 \\ 0.047 & 0 & 0 \end{pmatrix}
\end{aligned} \tag{5.29}
$$

with no flavor violation in the lepton sector: $\mathbf{Y_e} = \mathrm{diag.}\,(1.9\ 10^{-5}, 0.004, 0.071)$. Both $\mathbf{Y_u}$ and $\mathbf{Y_d}$ are endowed with five texture zeroes, and they precisely conform to the structures found in effective theories coming from the heterotic string (Braun et al. 2006). Besides, though left unspecified in (Braun et al. 2006), we take sfermion mass-squareds strictly flavor-diagonal as in Sec. 4.2.1, and let $\mathbf{Y_e^A}$ obey (5.6). For trilinear couplings pertaining to squark sector we take

$$
\begin{aligned}
\mathbf{Y}_u^A(0) &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & -30.469 & -74.029 \\ 0 & -74.029 & -97.406 \end{pmatrix} \\
\mathbf{Y}_d^A(0) &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & -25.241 & -68.185 \\ 0 & -67.545 & -63.990 \end{pmatrix}
\end{aligned} \tag{5.30}
$$

both measured in GeV. These trilinear couplings do not obey (5.6); they are given completely independent flavor structures, in particular, they exhibit $\mathcal{O}(1)$ mixing between second and third generations. The first generation of squarks is decoupled from the rest completely. Two-loop RG running down to $Q = M_{weak}$ modifies GUT-scale textures (5.30) to give

$$
\begin{aligned}
\mathbf{Y}_u^A &= \begin{pmatrix} 0 & -0.157 & -2.426 \\ -1.326 & -75.382 & -183.335 \\ -474.410 & -126.247 & -167.265 \end{pmatrix} \\
\mathbf{Y}_d^A &= \begin{pmatrix} 0 & -0.231 & -3.341 \\ -3.114 & -78.521 & -212.328 \\ -103.062 & -205.742 & -193.530 \end{pmatrix}
\end{aligned} \tag{5.31}
$$

both measured in GeV. The texture zeroes in (5.30) are seen to elevated to small yet nonzero values via RG running. The squark soft mass-squareds, on the other hand, exhibit the following flavor structures at $M_{weak} = 1$ TeV:

$$\begin{aligned}
\mathbf{m^2_Q} &= (560.63\text{ GeV})^2 \begin{pmatrix} 0.936 & -0.029 & -0.036 \\ -0.029 & 1.051 & -0.049 \\ -0.036 & -0.049 & 1.012 \end{pmatrix} \\
\mathbf{m^2_U} &= (523.88\text{ GeV})^2 \begin{pmatrix} 1.155 & -3.1\ 10^{-4} & -2.9\ 10^{-4} \\ -3.1\ 10^{-4} & 1.107 & -5.5\ 10^{-2} \\ -2.9\ 10^{-4} & -5.5\ 10^{-2} & 0.738 \end{pmatrix} \\
\mathbf{m^2_D} &= (548.52\text{ GeV})^2 \begin{pmatrix} 1.043 & -3.72\ 10^{-4} & -3.54\ 10^{-4} \\ -3.72\ 10^{-4} & 0.997 & -5.322\ 10^{-2} \\ -3.54\ 10^{-4} & -5.322\ 10^{-2} & 0.960 \end{pmatrix}
\end{aligned} \tag{5.32}$$

where off-diagonal entries are seen to be hierarchically small so that contributions to ${\mathbf{Y_{u,d}}}^{eff}$ from squark soft mass-squareds are expected to be rather small.

The use of Yukawas, trilinear couplings and squark mass-squareds, all rescaled to $M_{weak} = 1$ TeV via RG running, give rise to modifications in Yukawa couplings after squarks being integrated out. In fact, the CKM matrix V_{CKM}^{tree} obtained from $\mathbf{Y_{u,d}}(M_{weak})$ and V_{CKM}^{corr} obtained from ${\mathbf{Y_{u,d}}}^{eff}$ compare as:

$$\boxed{|V_{CKM}^{tree}|}\boxed{|V_{CKM}^{corr}|} = \begin{pmatrix} \boxed{0.9999}\boxed{0.9751} & \boxed{0.0044}\boxed{0.2216} & \boxed{0.0}\boxed{0.0079} \\ \boxed{0.0044}\boxed{0.2218} & \boxed{0.9999}\boxed{0.9742} & \boxed{0.0}\boxed{0.0412} \\ \boxed{0.0}\boxed{0.0014} & \boxed{0.0}\boxed{0.0419} & \boxed{1.0}\boxed{0.99912} \end{pmatrix} \tag{5.33}$$

where left (right) window of $\boxed{}\boxed{}$ in (i,j)-th entry refers to $|V_{CKM}^{tree}(i,j)|$ ($|V_{CKM}^{corr}(i,j)|$). It is clear that V_{CKM}^{tree} by no means qualifies to be a realistic CKM matrix: $|V_{CKM}^{tree}(i,j)| = 0$ for $(i,j) = (1,3),(3,1),(2,3),(3,2)$; moreover, Cabibbo angle is predicted to be one order of magnitude smaller. In addition, its diagonal elements turn out to be well outside the experimental limits. However, once supersymmetric threshold corrections are included certain entries are found to attain their experimentally preferred ranges. Indeed, $|V_{CKM}^{tree}(1,1)|$ and $|V_{CKM}^{tree}(3,1)|$ fall right at their upper bounds, and $|V_{CKM}^{tree}(1,3)|$ far exceeds the experimental bound. The predictions for these entries are not good enough; they need to be correctly predicted by further arrangements of the GUT-scale textures. Nevertheless, for the main purpose of illustrating how threshold corrections influence flavor structures at the IR end, the results above are good enough for what has to be shown since all other entries turn out to be in rather good agreement with experimental bounds. The case study illustrated here shows that, even unphysical Yukawa textures with five texture zeroes, can lead to acceptable CKM matrix predictions once supersymmetric threshold corrections are incorporated into Yukawa couplings.

The corrected Yukawa couplings lead to the following quark mass spectrum:

$$\begin{aligned} \overline{\mathbf{M_u}}(M_{weak}) &= \text{diag.}\,(0.168, 0.93, 151.6) \\ \overline{\mathbf{M_d}}(M_{weak}) &= \text{diag.}\,(0.0325, 0.0711, 2.31) \end{aligned} \qquad (5.34)$$

all measured in GeV. These predictions are not violatively outside the experimental limits, except for the up quark mass. A rehabilitated choice for the GUT-scale textures (5.29) should lead to a fully consistent prediction for CKM matrix (with much better precision than in, especially the $(1,3)$, $(3,1)$ entries of (5.33) above) together with precise predictions for quark masses (modulo sizeable QCD corrections while running from $Q = M_{weak}$ down to hadronic scale).

CHAPTER 6

CONCLUSION

The theoretical framework called supersymmetry has already had a considerable impact on the development of theoretical physics, in spite of not having been discovered yet. A basic knowledge of supersymmetry is now considered to be an indispensable part of the contemporary high-energy physics. This master of science thesis is meant to be a means of gaining such a basic insight. We have here tried to give a concise and thorough survey of the mathematical and physical foundations of supersymmetry and its phenemonology, as well as giving some familiarity with the most common concepts which appear in the literature. In order to make the work comprehensible for readers not familiar with physics beyond relativistic quantum mechanics and basic quantum field theory, we have also provided, basic notation and other auxiliaries in the body and appendices of the thesis.

The main novelty in this work is the material presented in Chapter V where we discussed effects of supersymmetric threshold corrections on Higgs boson couplings to quarks. The effective theory below the SUSY breaking scale M_{SUSY} consists of a modified Higgs sector; in particular, the tree level Yukawa couplings receive important corrections from sparticle loops. In contrast to the minimal flavor violation scheme, the Yukawa couplings acquire large corrections from those of the heavier ones. Unlike the light quarks, the top and bottom Yukawas remain stuck to their Minimal Flavor violation (MFV) values to a good approximation. Therefore, the SUSY flavor violation sources mainly influence the light sector whereby modifying several processes they participate. These corrections are important even at low $\tan\beta$. The FCNC processes are contributed by both the sparticle loops and Higgs exchange amplitudes. The constraints on various mass insertions can be satisfied by a partial cancellation between these two contributions if M_{SUSY} is close to the weak scale. Therefore, existing bounds on various mass insertions overlook the potentially important contributions coming from Higgs exchange. In this sense, what is done in this thesis work opens up a new avenue for phenomenology of the supersymmetric models.

The material contained in Chapter V implies that high-scale flavor structures (stemming from strings or supergravity) which may be classified viable may be completely disqualified once SUSY threshold corrections are included. This we have shown in Chapter V by analyzing CKM-ruled, Hierarchical and Democratic textures which exhibit good agreement with data in the absence of threshold corrections. However, once such corrections are included we end up with a completely unacceptable correction for various entries of the CKM matrix. Thus, it is important to take into account such corrections while contrasting high-scale textures with experiment.

Apart from thes, we have presented an opposite example of the effects of threshold corrections. Indeed, in general, textures with 5 texture zeroes are known to be completely incapable of producing low-energy data, the CKM matrix. However, we have shown that a recently advocated string model with 5 texture zeroes turn out to show good agreement with experiment once SUSY threshold corrections are included. This shows that, the existing sole flavor matrix, the CKM matrix, may originate at least partially from soft SUSY breaking sector.

The main conclusion of this thesis work is that integration of superpartners near the TeV scale out of the spectrum gives rise to, in the presence of tree-level flavor violation in Yukawa and soft-breaking sectors of the theory, a number of phenomena:

- Down quark Yukawa couplings (to a lesser extent those of the up type quarks) receive large radiative corrections influencing, among other things, the Higgs branching into various quarks. This effect can be directly observed in experiments within LHC.

- The effective, physical CKM matrix turns out to receive rather large corrections from Higgs threshold corrections so that several string or supergravity textures classified viable in the literature turn out to disagree with experiments.

- Several stringy textures which cannot generate a viable CKM matrix under RGE flow turn exhibit good agreement with experiment after the inclusion of SUSY threshold corrections.

We thus conclude that radiative corrections in the presence of SUSY flavor violation can give rise to a number of important phenomena testable at upcoming experiments such as LHC and ILC.

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APPENDIX A

Notation and Conventions.

1.1. Relativistic Notation.

In this report we will adopt standard relativistic units, i.e.

$$\hbar = c = 1. \tag{A.1}$$

A general contravariant and covariant four-vector will be denoted by

$$\left.\begin{array}{lllll} A^{\mu} & = & (A^0; A^1, A^2, A^3) & = & (A^0; \mathbf{A}) \\ A_{\mu} & = & (A_0; -A_1, -A_2, -A_3) & = & (A^0; -\mathbf{A}) \end{array}\right\}. \tag{A.2}$$

The compact "Feynman slash" notation

$$\not{A} = \gamma^{\mu} A_{\mu}, \tag{A.3}$$

will be used. The metric tensor, $g^{\mu\nu}$, which connects A^{μ} and A_{μ}, is defined by

$$g^{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -1). \tag{A.4}$$

Moreover, we will use the (relativistic) summation convention which states that repeated Greek indices, $\mu, \nu, \rho, \sigma, \tau$, are summed from 0 to 3 and latin indices run from 1 to 3 unless specifically indicated to the contrary.

The Minkowski product (the four-product) will be denoted by AB and defined as

$$AB \equiv A^{\mu} B_{\mu} = A^0 B^0 - \mathbf{AB} \tag{A.5}$$

Practical notation for the four-gradients, ∂^{μ} and ∂_{μ}, will be used

$$\partial^{\mu} \equiv \frac{\partial}{\partial x_{\mu}} = (\frac{\partial}{\partial t}; -\nabla), \tag{A.6}$$

$$\partial_{\mu} \equiv \frac{\partial}{\partial x^{\mu}} = (\frac{\partial}{\partial t}; \nabla). \tag{A.7}$$

The totally antisymmetric Levi-Civita tensors in three and four dimensions are respectively defined by

$$\varepsilon_{ijk} = \begin{cases} +1 & \text{, for even permutations of 123} \\ -1 & \text{, for odd permutations} \\ 0 & \text{, otherwise,} \end{cases} \tag{A.8}$$

$$\varepsilon_{\mu\nu\rho\sigma} = \begin{cases} +1 & \text{, for even permutations of 0123} \\ -1 & \text{, for odd permutations} \\ 0 & \text{, otherwise,} \end{cases} \tag{A.9}$$

where

$$\varepsilon_{ijk} = \varepsilon^{ijk}, \tag{A.10}$$

$$\varepsilon_{\mu\nu\rho\sigma} = -\varepsilon^{\mu\nu\rho\sigma}. \tag{A.11}$$

1.2. Pauli Matrices.

The well known Pauli matrices are defined by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.12}$$

and satisfy the commutator relation

$$[\sigma^i, \sigma^j] = 2i\varepsilon^{ijk}\sigma^k, \qquad i, j, k = 1, 2, 3.$$

From this definition it is evident that

$$(\sigma^i)^\dagger = \sigma^i, \qquad i = 1, 2, 3, \tag{A.13}$$

$$(\sigma^i)^2 = 1, \tag{A.14}$$

$$Tr(\sigma^i) = 0. \tag{A.15}$$

For later use, we also introduce[1]

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{A.16}$$

[1]Note that different signs are used in the literature for the definition of this quantity.

and a useful arrangement of these matrices is

$$\sigma^{\mu} = (\sigma^0\,;\,\boldsymbol{\sigma}) = (\sigma^0\,;\,\sigma^1, \sigma^2, \sigma^3).$$

The index structure of the σ-matrices is given by

$$\sigma^{\mu} \;=\; [\sigma^{\mu}_{\alpha\dot{\alpha}}]. \tag{A.17}$$

We now introduce some "Pauli related" matrices defined by

$$\bar{\sigma}^{\mu\,\dot{\alpha}\alpha} \equiv \sigma^{\mu\,\alpha\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon^{\alpha\beta}\sigma^{\mu}_{\beta\dot{\beta}}, \tag{A.18}$$

where the "metrics" ε and $\bar{\varepsilon}$ have been used. By direct computations one can establish the following relations

$$\bar{\sigma}^0 \;=\; \sigma^0 \tag{A.19}$$

$$\bar{\sigma}^i \;=\; -\,\sigma^i, \qquad i = 1, 2, 3. \tag{A.20}$$

Moreover, the following relations are true

$$\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\sigma}^{\dot{\beta}\beta}_{\mu} \;=\; 2\,\delta^{\,\beta}_{\alpha}\delta^{\,\dot{\beta}}_{\dot{\alpha}} \tag{A.21}$$

$$Tr(\sigma^{\mu}\bar{\sigma}^{\nu}) \;=\; 2g^{\mu\nu} \tag{A.22}$$

$$(\sigma^{\mu}\bar{\sigma}^{\nu} + \sigma^{\nu}\bar{\sigma}^{\mu})^{\;\beta}_{\alpha} \;=\; 2\,g^{\mu\nu}\delta^{\,\beta}_{\alpha} \tag{A.23}$$

$$(\bar{\sigma}^{\mu}\sigma^{\nu} + \bar{\sigma}^{\nu}\sigma^{\mu})^{\dot{\alpha}}_{\;\dot{\beta}} \;=\; 2\,g^{\mu\nu}\delta^{\dot{\alpha}}_{\dot{\beta}} \tag{A.24}$$

$$(\sigma^{\mu}\bar{\sigma}^{\nu}\sigma^{\rho} + \sigma^{\rho}\bar{\sigma}^{\nu}\sigma^{\mu}) \;=\; 2\,(g^{\mu\nu}\sigma^{\rho} + g^{\nu\rho}\sigma^{\mu} - g^{\mu\rho}\sigma^{\nu}) \tag{A.25}$$

$$(\bar{\sigma}^{\mu}\sigma^{\nu}\bar{\sigma}^{\rho} + \bar{\sigma}^{\rho}\sigma^{\nu}\bar{\sigma}^{\mu}) \;=\; 2\,(g^{\mu\nu}\bar{\sigma}^{\rho} + g^{\nu\rho}\bar{\sigma}^{\mu} - g^{\mu\rho}\bar{\sigma}^{\nu}) \tag{A.26}$$

$$Tr(\sigma^{\mu}\bar{\sigma}^{\nu}\sigma^{\rho}\bar{\sigma}^{\sigma}) \;=\; 2\,(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma} - i\varepsilon^{\mu\nu\rho\sigma}). \tag{A.27}$$

Most of the above relations are easily proved by direct computations. Besides, Müller-Kirsten and Wiedemann, have proved most of them, and in particular eq. $(A.27)$ which is the most difficult one.

Anti-symmetric matrices $\sigma^{\mu\nu}$ and $\bar{\sigma}^{\mu\nu}$ are defined by

$$\sigma^{\mu\nu} \;=\; \frac{i}{4}\,(\sigma^{\mu}\bar{\sigma}^{\nu} - \sigma^{\nu}\bar{\sigma}^{\mu}), \tag{A.28}$$

$$\bar{\sigma}^{\mu\nu} \;=\; \frac{i}{4}\,(\bar{\sigma}^{\mu}\sigma^{\nu} - \bar{\sigma}^{\nu}\sigma^{\mu}). \tag{A.29}$$

By utilizing the index structure of the σ-matrices, it is easily seen that $\sigma^{\mu\nu}$ and $\bar{\sigma}^{\mu\nu}$ must have the index structure $\sigma^{\mu\nu} = [(\sigma^{\mu\nu})_\alpha^{\ \beta}]$ and $\bar{\sigma}^{\mu\nu} = [(\bar{\sigma}^{\mu\nu})^{\dot\beta}_{\ \dot\alpha}]$. In fact are $\sigma^{\mu\nu}$ and $\bar{\sigma}^{\mu\nu}$ the generators of $SL(2,C)$ in the spinor representations $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$ respectively. The proofs together with the establishment of the below formulae can be found in (3.39),(Ramond 1990):

$$\sigma^{\mu\nu\,\dagger} = -\bar{\sigma}^{\mu\nu}, \tag{A.30}$$

$$\sigma^{\mu\nu} = \frac{1}{2i}\varepsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma}, \tag{A.31}$$

$$\bar{\sigma}^{\mu\nu} = -\frac{1}{2i}\varepsilon^{\mu\nu\rho\sigma}\bar{\sigma}_{\rho\sigma}, \tag{A.32}$$

$$Tr(\sigma^{\mu\nu}) = Tr(\bar{\sigma}^{\mu\nu}) = 0 \tag{A.33}$$

$$Tr(\sigma^{\mu\nu}\sigma^{\rho\sigma}) = \frac{1}{2}\left(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}\right) + \frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}, \tag{A.34}$$

$$Tr(\bar{\sigma}^{\mu\nu}\bar{\sigma}^{\rho\sigma}) = \frac{1}{2}\left(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}\right) - \frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}. \tag{A.35}$$

1.3. Dirac Matrices.

The Dirac γ-matices are defined by the anticommutation (Clifford) relations

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}. \tag{A.36}$$

From the four γ-matrices above, it is possible to define a "fifth γ-matrix" by

$$\gamma_5 \equiv \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 (A.37)$$

It possesses the following properties which follows easily from the definitions $(A.36)$ and $(A.37)$

$$\{\gamma^5, \gamma^\mu\} = 0, \tag{A.38}$$

$$(\gamma^5)^2 = 1. \tag{A.39}$$

We will now state three explicit representations of the γ-matrices, namely the so-called Dirac representation, the Majorana representation, and finally the Chiral representation.

1.3.1. Representations

The lowest non-trivial representation of these matrices is of dimension four. and we will concentrate on this represntation. From now on, we will assume that a four dimensional representation is used.

1.3.1.1. The Dirac Representation or Canonical Basis.

In this particular representation the γ-matrices read

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.40}$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ \bar{\sigma}^i & 0 \end{pmatrix}, \qquad i = 1, 2, 3, \tag{A.41}$$

$$\gamma^5 = \begin{pmatrix} 0 & \sigma^0 \\ \bar{\sigma}^0 & 0 \end{pmatrix}, \tag{A.42}$$

where 1 denotes the 2×2 identity matrix and σ^μ and $\bar{\sigma}^\mu$ are the Pauli matrices defined in the previous section.

1.3.1.2. The Majorana Representation.

In this representation all γ-matricrs are pure imaginary and have the explicit form:

$$\gamma^0 = \begin{pmatrix} 0 & \sigma^2 \\ -\bar{\sigma}^2 & 0 \end{pmatrix}, \tag{A.43}$$

$$\gamma^1 = \begin{pmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix}, \tag{A.44}$$

$$\gamma^2 = \begin{pmatrix} 0 & -\sigma^2 \\ -\bar{\sigma}^2 & 0 \end{pmatrix}, \tag{A.45}$$

$$\gamma^3 = \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & i\sigma^1 \end{pmatrix}, \tag{A.46}$$

and finally

$$\gamma^5 \;=\; \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}. \tag{A.47}$$

1.3.1.3. The Chiral representation or Weyl Basis.

This basis is of particular interest to persons doing SUSY. In this representation the γ-matrices take on the explicite form

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \tag{A.48}$$

$$\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{A.49}$$

APPENDIX B

Spontaneous Symmetry Breaking (SSB)

Let us consider a Lagrangian, which:

1. Is invariant under a group G of transformations.

2. Has a degenerate set of states with minimal energy, which transform under G as the members of a given multiplet.

If one arbitrarily selects one of those states as the ground state of the system, one says that the symmetry becomes spontaneously broken.

A well-known physical example is provided by a ferromagnet: although the Hamiltonian is invariant under rotations, the ground state has the spins aligned into some arbitrary direction. Moreover, any higher-energy state, built from the ground state by a finite number of excitations, would share its anisotropy. In a Quantum Field Theory, the ground state is the vacuum. Thus, the SSB mechanism will appear in those cases where one has a symmetric Lagrangian, but a non-symmetric vacuum.

The existence of flat directions connecting the degenerate states of minimal energy is a general property of the SSB of continuous symmetries. In a Quantum Field Theory it implies the existence of massless degrees of freedom.

2.0.2. Goldstone theorem

Let us consider a complex scalar field $\phi(x)$, with Lagrangian

$$\mathcal{L} \,=\, \partial_\mu\phi^\dagger\partial^\mu\phi - V(\phi)\,, \qquad\qquad V(\phi) \,=\, \mu^2\phi^\dagger\phi + h\left(\phi^\dagger\phi\right)^2 \,. \tag{B.1}$$

$\mathcal{L}$ is invariant under global phase transformations of the scalar field

$$\phi(x) \;\longrightarrow\; \phi'(x) \,\equiv\, \exp\left\{i\theta\right\}\phi(x)\,. \tag{B.2}$$

In order to have a ground state the potential should be bounded from below, i.e. $h > 0$. For the quadratic piece there are two possibilities:

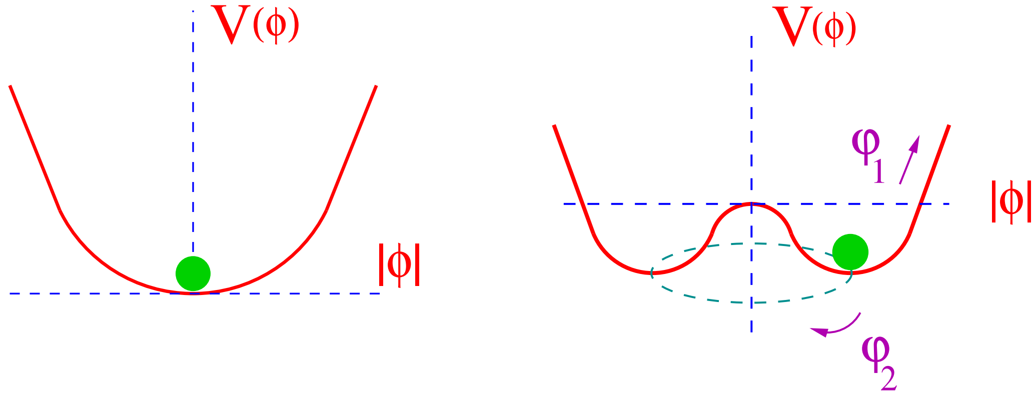

Figure B.1: Shape of the scalar potential for $\mu^2 > 0$ (left) and $\mu^2 < 0$ (right). In the second case there is a continuous set of degenerate vacua, corresponding to different phases θ, connected through a massless field excitation φ_2.

1. $\boldsymbol{\mu^2 > 0}$: The potential has only the trivial minimum $\phi = 0$. It describes a massive scalar particle with mass μ and quartic coupling h.

2. $\boldsymbol{\mu^2 < 0}$: The minimum is obtained for those field configurations satisfying

$$|\phi_0| \;=\; \sqrt{\frac{-\mu^2}{2h}} \;\equiv\; \frac{v}{\sqrt{2}} \;>\; 0\,, \qquad\qquad V(\phi_0) \;=\; -\frac{h}{4}v^4\,. \tag{B.3}$$

Owing to the $U(1)$ phase-invariance of the Lagrangian, there is an infinite number of degenerate states of minimum energy, $\phi_0(x) = \frac{v}{\sqrt{2}}\exp\{i\theta\}$. By choosing a particular solution, $\theta = 0$ for example, as the ground state, the symmetry gets spontaneously broken. If we parametrize the excitations over the ground state as

$$\phi(x) \;\equiv\; \frac{1}{\sqrt{2}}\left[v + \varphi_1(x) + i\,\varphi_2(x)\right]\,, \tag{B.4}$$

where φ_1 and φ_2 are real fields, the potential takes the form

$$V(\phi) \;=\; V(\phi_0) - \mu^2\varphi_1^2 + h\,v\,\varphi_1\left(\varphi_1^2+\varphi_2^2\right) + \frac{h}{4}\left(\varphi_1^2+\varphi_2^2\right)^2\,. \tag{B.5}$$

Thus, φ_1 describes a massive state of mass $m^2_{\varphi_1} = -2\mu^2$, while φ_2 is massless.

The first possibility ($\mu^2 > 0$) is just the usual situation with a single ground state. The other case, with SSB, is more interesting. The appearance of a massless particle when $\mu^2 < 0$ is easy to understand: the field φ_2 describes excitations around a flat direction in the potential, i.e. into states with the same energy as the chosen ground state. Since those excitations do not cost any energy, they obviously correspond to a massless state.

The fact that there are massless excitations associated with the SSB mechanism is a completely general result, known as the Goldstone theorem (Goldstone 1961): if a Lagrangian is invariant under a continuous symmetry group G, but the vacuum is only invariant under a subgroup $H \subset G$, then there must exist as many massless spin–0 particles (Goldstone bosons) as broken generators (i.e. generators of G which do not belong to H).

APPENDIX C

Renormalization Group Equations

The two-loop beta functions for the superpotential parameters are:

$$\frac{d}{dt}\mu = \frac{1}{16\pi^2}\beta_\mu^{(1)} + \frac{1}{(16\pi^2)^2}\beta_\mu^{(2)} \qquad \text{(C.1)}$$

$$\frac{d}{dt}\mathbf{Y}_{u,d,e} = \frac{1}{16\pi^2}\beta_{\mathbf{Y}_{u,d,e}}^{(1)} + \frac{1}{(16\pi^2)^2}\beta_{\mathbf{Y}_{u,d,e}}^{(2)} \qquad \text{(C.2)}$$

with:

$$\beta_\mu^{(1)} = \mu\Big\{ \mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + 3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) - 3g_2^2 - \frac{3}{5}g_1^2\Big\}$$

$$\begin{aligned}\beta_\mu^{(2)} = \mu\Big\{ & -3\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + 3\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + 2\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger) \\ & + \Big[16g_3^2 + \frac{4}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) + \Big[16g_3^2 - \frac{2}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger) \\ & + \frac{15}{2}g_2^4 + \frac{9}{5}g_1^2g_2 + \frac{207}{50}g_1^4\Big\}\end{aligned}$$

$$\beta_{\mathbf{Y}_u}^{(1)} = \mathbf{Y}_u\Big\{ 3\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) + 3\mathbf{Y}_u^\dagger\mathbf{Y}_u + \mathbf{Y}_d^\dagger\mathbf{Y}_d - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\Big\}$$

$$\begin{aligned}\beta_{\mathbf{Y}_u}^{(2)} = \mathbf{Y}_u\Big\{ & -3\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger) - \mathbf{Y}_d^\dagger\mathbf{Y}_d\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) \\ & - 9\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) - 4\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u - 2\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d - 2\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u \\ & + \Big[16g_3^2 + \frac{4}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) + \Big[6g_2^2 + \frac{2}{5}g_1^2\Big]\mathbf{Y}_u^\dagger\mathbf{Y}_u + \frac{2}{5}g_1^2\mathbf{Y}_d^\dagger\mathbf{Y}_d \\ & - \frac{16}{9}g_3^4 + 8g_3^2g_2^2 + \frac{136}{45}g_3^2g_1^2 + \frac{15}{2}g_2^4 + g_2^2g_1^2 + \frac{2743}{450}g_1^4\Big\}\end{aligned}$$

$$\beta_{\mathbf{Y}_d}^{(1)} = \mathbf{Y}_d\Big\{ \mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) + 3\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_u^\dagger\mathbf{Y}_u - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\Big\}$$

$$\beta_{\mathbf{Y}_d}^{(2)} = \mathbf{Y}_d\Big\{ -3\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger)$$

$$
\begin{aligned}
&- 3\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) - 3\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) - 4\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d \\
&- 2\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u - 2\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d + \left[16g_3^2 - \frac{2}{5}g_1^2\right]\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger) \\
&+ \frac{6}{5}g_1^2\mathrm{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger) + \frac{4}{5}g_1^2\mathbf{Y}_u^\dagger\mathbf{Y}_u + \left[6g_2^2 + \frac{4}{5}g_1^2\right]\mathbf{Y}_d^\dagger\mathbf{Y}_d \\
&- \frac{16}{9}g_3^4 + 8g_3^2g_2^2 + \frac{8}{9}g_3^2g_1^2 + \frac{15}{2}g_2^4 + g_2^2g_1^2 + \frac{287}{90}g_1^4\Bigg\}
\end{aligned}
$$

$$
\beta^{(1)}_{\mathbf{Y}_e} = \mathbf{Y}_e\Bigg\{\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) + 3\mathbf{Y}_e^\dagger\mathbf{Y}_e - 3g_2^2 - \frac{9}{5}g_1^2\Bigg\}
$$

$$
\begin{aligned}
\beta^{(2)}_{\mathbf{Y}_e} = \mathbf{Y}_e\Bigg\{ &- 3\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger) \\
&- 3\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) - 4\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e + \left[16g_3^2 - \frac{2}{5}g_1^2\right]\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger) \\
&+ \frac{6}{5}g_1^2\mathrm{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger) + 6g_2^2\mathbf{Y}_e^\dagger\mathbf{Y}_e + \frac{15}{2}g_2^4 + \frac{9}{5}g_2^2g_1^2 + \frac{27}{2}g_1^4\Bigg\}
\end{aligned}
$$

The above results for the MSSM have all appeared before. Now we apply our results of arriving at the two-loop beta functions for the soft-breaking trilinear scalar couplings:

$$
\frac{d}{dt}\mathbf{h}_{u,d,e} = \frac{1}{16\pi^2}\beta^{(1)}_{\mathbf{h}_{u,d,e}} + \frac{1}{(16\pi^2)^2}\beta^{(2)}_{\mathbf{h}_{u,d,e}} \tag{C.3}
$$

$$
\begin{aligned}
\beta^{(1)}_{\mathbf{h}_u} = \;&\mathbf{h}_u\Bigg\{3\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) + 5\mathbf{Y}_u^\dagger\mathbf{Y}_u + \mathbf{Y}_d^\dagger\mathbf{Y}_d - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\Bigg\} \\
&+\mathbf{Y}_u\Bigg\{6\mathrm{Tr}(\mathbf{h}_u\mathbf{Y}_u^\dagger) + 4\mathbf{Y}_u^\dagger\mathbf{h}_u + 2\mathbf{Y}_d^\dagger\mathbf{h}_d + \frac{32}{3}g_3^2M_3 + 6g_2^2M_2 + \frac{26}{15}g_1^2M_1\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(2)}_{\mathbf{h}_u} = \;&\mathbf{h}_u\Bigg\{ - 3\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger) - \mathbf{Y}_d^\dagger\mathbf{Y}_d\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) \\
&- 15\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) - 6\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u - 2\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d - 4\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u \\
&+ \left[16g_3^2 + \frac{4}{5}g_1^2\right]\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) + 12g_2^2\mathbf{Y}_u^\dagger\mathbf{Y}_u + \frac{2}{5}g_1^2\mathbf{Y}_d^\dagger\mathbf{Y}_d \\
&- \frac{16}{9}g_3^4 + 8g_3^2g_2^2 + \frac{136}{45}g_3^2g_1^2 + \frac{15}{2}g_2^4 + g_2^2g_1^2 + \frac{2743}{450}g_1^4\Bigg\} \\
&+\mathbf{Y}_u\Bigg\{ - 6\mathrm{Tr}(6\mathbf{h}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{h}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + \mathbf{h}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger) \\
&- 18\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathrm{Tr}(\mathbf{h}_u\mathbf{Y}_u^\dagger) - \mathbf{Y}_d^\dagger\mathbf{Y}_d\mathrm{Tr}(6\mathbf{h}_d\mathbf{Y}_d^\dagger + 2\mathbf{h}_e\mathbf{Y}_e^\dagger) - 12\mathbf{Y}_u^\dagger\mathbf{h}_u\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) \\
&- \mathbf{Y}_d^\dagger\mathbf{h}_d\mathrm{Tr}(6\mathbf{Y}_d\mathbf{Y}_d^\dagger + 2\mathbf{Y}_e\mathbf{Y}_e^\dagger) - 6\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{h}_u - 8\mathbf{Y}_u^\dagger\mathbf{h}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u
\end{aligned}
$$

$$
\begin{aligned}
&- 4\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{h}_d - 4\mathbf{Y}_d^\dagger \mathbf{h}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d - 2\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{h}_u - 4\mathbf{Y}_d^\dagger \mathbf{h}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u \\
&+ \Big[32g_3^2 + \frac{8}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{h}_u \mathbf{Y}_u^\dagger) + \Big[6g_2^2 + \frac{6}{5}g_1^2\Big]\mathbf{Y}_u^\dagger \mathbf{h}_u + \frac{4}{5}g_1^2 \mathbf{Y}_d^\dagger \mathbf{h}_d \\
&- \Big[32g_3^2 M_3 + \frac{8}{5}g_1^2 M_1\Big]\mathrm{Tr}(\mathbf{Y}_u \mathbf{Y}_u^\dagger) - \Big[12g_2^2 M_2 + \frac{4}{5}g_1^2 M_1\Big]\mathbf{Y}_u^\dagger \mathbf{Y}_u \\
&- \frac{4}{5}g_1^2 M_1 \mathbf{Y}_d^\dagger \mathbf{Y}_d + \frac{64}{9}g_3^4 M_3 - 16g_3^2 g_2^2(M_3 + M_2) - \frac{272}{45}g_3^2 g_1^2(M_3 + M_1) \\
&- 30g_2^4 M_2 - 2g_2^2 g_1^2(M_2 + M_1) - \frac{5486}{225}g_1^4 M_1\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{\mathbf{h}_d}^{(1)} \;=\; & \mathbf{h}_d\Big\{ \mathrm{Tr}(3\mathbf{Y}_d \mathbf{Y}_d^\dagger + \mathbf{Y}_e \mathbf{Y}_e^\dagger) + 5\mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_u^\dagger \mathbf{Y}_u - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\Big\} \\
& + \mathbf{Y}_d\Big\{ \mathrm{Tr}(6\mathbf{h}_d \mathbf{Y}_d^\dagger + 2\mathbf{h}_e \mathbf{Y}_e^\dagger) + 4\mathbf{Y}_d^\dagger \mathbf{h}_d + 2\mathbf{Y}_u^\dagger \mathbf{h}_u + \frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{14}{15}g_1^2 M_1\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{\mathbf{h}_d}^{(2)} \;=\; & \mathbf{h}_d\Big\{ - 3\mathrm{Tr}(3\mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger + \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger + \mathbf{Y}_e \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_e^\dagger) \\
&- 3\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathrm{Tr}(\mathbf{Y}_u \mathbf{Y}_u^\dagger) - 5\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathrm{Tr}(3\mathbf{Y}_d \mathbf{Y}_d^\dagger + \mathbf{Y}_e \mathbf{Y}_e^\dagger) - 6\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d \\
&- 2\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u - 4\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d + \Big[16g_3^2 - \frac{2}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{Y}_d \mathbf{Y}_d^\dagger) \\
&+ \frac{6}{5}g_1^2 \mathrm{Tr}(\mathbf{Y}_e \mathbf{Y}_e^\dagger) + \frac{4}{5}g_1^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u + \Big[12g_2^2 + \frac{6}{5}g_1^2\Big]\mathbf{Y}_d^\dagger \mathbf{Y}_d \\
&- \frac{16}{9}g_3^4 + 8g_3^2 g_2^2 + \frac{8}{9}g_3^2 g_1^2 + \frac{15}{2}g_2^4 + g_2^2 g_1^2 + \frac{287}{90}g_1^4\Big\} \\
& + \mathbf{Y}_d\Big\{ - 6\mathrm{Tr}(6\mathbf{h}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger + \mathbf{h}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger + \mathbf{h}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger + 2\mathbf{h}_e \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_e^\dagger) \\
&- 6\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathrm{Tr}(\mathbf{h}_u \mathbf{Y}_u^\dagger) - 6\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathrm{Tr}(3\mathbf{h}_d \mathbf{Y}_d^\dagger + \mathbf{h}_e \mathbf{Y}_e^\dagger) \\
&- 6\mathbf{Y}_u^\dagger \mathbf{h}_u \mathrm{Tr}(\mathbf{Y}_u \mathbf{Y}_u^\dagger) - 4\mathbf{Y}_d^\dagger \mathbf{h}_d \mathrm{Tr}(3\mathbf{Y}_d \mathbf{Y}_d^\dagger + \mathbf{Y}_e \mathbf{Y}_e^\dagger) - 6\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{h}_d \\
&- 8\mathbf{Y}_d^\dagger \mathbf{h}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d - 4\mathbf{Y}_u^\dagger \mathbf{h}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u - 4\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{h}_u - 4\mathbf{Y}_u^\dagger \mathbf{h}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d \\
&- 2\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{h}_d + \Big[32g_3^2 - \frac{4}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{h}_d \mathbf{Y}_d^\dagger) + \frac{12}{5}g_1^2 \mathrm{Tr}(\mathbf{h}_e \mathbf{Y}_e^\dagger) + \frac{8}{5}g_1^2 \mathbf{Y}_u^\dagger \mathbf{h}_u \\
&+ \Big[6g_2^2 + \frac{6}{5}g_1^2\Big]\mathbf{Y}_d^\dagger \mathbf{h}_d - \Big[32g_3^2 M_3 - \frac{4}{5}g_1^2 M_1\Big]\mathrm{Tr}(\mathbf{Y}_d \mathbf{Y}_d^\dagger) - \frac{12}{5}g_1^2 M_1 \mathrm{Tr}(\mathbf{Y}_e \mathbf{Y}_e^\dagger) \\
&- \Big[12g_2^2 M_2 + \frac{8}{5}g_1^2 M_1\Big]\mathbf{Y}_d^\dagger \mathbf{Y}_d - \frac{8}{5}g_1^2 M_1 \mathbf{Y}_u^\dagger \mathbf{Y}_u + \frac{64}{9}g_3^4 M_3 - 16g_3^2 g_2^2(M_3 + M_2) \\
&- \frac{16}{9}g_3^2 g_1^2(M_3 + M_1) - 30g_2^4 M_2 - 2g_2^2 g_1^2(M_2 + M_1) - \frac{574}{45}g_1^4 M_1\Big\}
\end{aligned}
$$

$$\beta_{\mathbf{h}_e}^{(1)} = \mathbf{h}_e\Big\{ \mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) + 5\mathbf{Y}_e^\dagger\mathbf{Y}_e - 3g_2^2 - \frac{9}{5}g_1^2\Big\}$$
$$+\mathbf{Y}_e\Big\{ \mathrm{Tr}(6\mathbf{h}_d\mathbf{Y}_d^\dagger + 2\mathbf{h}_e\mathbf{Y}_e^\dagger) + 4\mathbf{Y}_e^\dagger\mathbf{h}_e + 6g_2^2M_2 + \frac{18}{5}g_1^2M_1\Big\}$$

$$\beta_{\mathbf{h}_e}^{(2)} = \mathbf{h}_e\Big\{ -3\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger)$$
$$-5\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) - 6\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e + \Big[16g_3^2 - \frac{2}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger)$$
$$+\frac{6}{5}g_1^2\mathrm{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger) + \Big[12g_2^2 - \frac{6}{5}g_1^2\Big]\mathbf{Y}_e^\dagger\mathbf{Y}_e + \frac{15}{2}g_2^4 + \frac{9}{5}g_2^2g_1^2 + \frac{27}{2}g_1^4\Big\}$$
$$+\mathbf{Y}_e\Big\{ -6\mathrm{Tr}(6\mathbf{h}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{h}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + \mathbf{h}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger + 2\mathbf{h}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger)$$
$$-4\mathbf{Y}_e^\dagger\mathbf{h}_e\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) - 6\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathrm{Tr}(3\mathbf{h}_d\mathbf{Y}_d^\dagger + \mathbf{h}_e\mathbf{Y}_e^\dagger)$$
$$-6\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{h}_e - 8\mathbf{Y}_e^\dagger\mathbf{h}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e$$
$$+\Big[32g_3^2 - \frac{4}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{h}_d\mathbf{Y}_d^\dagger) + \frac{12}{5}g_1^2\mathrm{Tr}(\mathbf{h}_e\mathbf{Y}_e^\dagger) + \Big[6g_2^2 + \frac{6}{5}g_1^2\Big]\mathbf{Y}_e^\dagger\mathbf{h}_e$$
$$-\Big[32g_3^2M_3 - \frac{4}{5}g_1^2M_1\Big]\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger) - \frac{12}{5}g_1^2M_1\mathrm{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger) - 12g_2^2M_2\mathbf{Y}_e^\dagger\mathbf{Y}_e$$
$$-30g_2^4M_2 - \frac{18}{5}g_2^2g_1^2(M_1+M_2) - 54g_1^4M_1\Big\}$$

$$\frac{d}{dt}B = \frac{1}{16\pi^2}\beta_B^{(1)} + \frac{1}{(16\pi^2)^2}\beta_B^{(2)} \qquad \text{(C.4)}$$

$$\beta_B^{(1)} = B\Big\{ \mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + 3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) - 3g_2^2 - \frac{3}{5}g_1^2\Big\}$$

$$+\mu\left\{ \mathrm{Tr}(6\mathbf{h}_u\mathbf{Y}_u^\dagger + 6\mathbf{h}_d\mathbf{Y}_d^\dagger + 2\mathbf{h}_e\mathbf{Y}_e^\dagger) + 6g_2^2M_2 + \frac{6}{5}g_1^2M_1\right\}$$

$$\begin{aligned}
\beta_B^{(2)} \;=\; B\Big\{ & -3\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + 3\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + 2\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger) \\
& + \left[16g_3^2 + \frac{4}{5}g_1^2\right]\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) + \left[16g_3^2 - \frac{2}{5}g_1^2\right]\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger) \\
& + \frac{15}{2}g_2^4 + \frac{9}{5}g_1^2g_2^2 + \frac{207}{50}g_1^4\Big\} \\
+\mu\Big\{ & -12\ \mathrm{Tr}(3\mathbf{h}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + 3\mathbf{h}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{h}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + \mathbf{h}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger + \mathbf{h}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger) \\
& + \left[32g_3^2 + \frac{8}{5}g_1^2\right]\mathrm{Tr}(\mathbf{h}_u\mathbf{Y}_u^\dagger) + \left[32g_3^2 - \frac{4}{5}g_1^2\right]\mathrm{Tr}(\mathbf{h}_d\mathbf{Y}_d^\dagger) + \frac{12}{5}g_1^2\mathrm{Tr}(\mathbf{h}_e\mathbf{Y}_e^\dagger) \\
& - \left[32g_3^2M_3 + \frac{8}{5}g_1^2M_1\right]\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) - \left[32g_3^2M_3 - \frac{4}{5}g_1^2M_1\right]\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger) \\
& - \frac{12}{5}g_1^2M_1\mathrm{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger) - 30g_2^4M_2 - \frac{18}{5}g_1^2g_2^2(M_1+M_2) - \frac{414}{25}g_1^4M_1\Big\}\ .
\end{aligned}$$

Finally, we turn to the β-functions for the scalar (mass)^2 terms of the $(m^2)_i^j$ type in the MSSM. It is convenient to define the quantities

$$\mathcal{S} \;=\; m_{H_u}^2 - m_{H_d}^2 + \mathrm{Tr}[\mathbf{m}_Q^2 - \mathbf{m}_L^2 - 2\mathbf{m}_u^2 + \mathbf{m}_d^2 + \mathbf{m}_e^2] \qquad \text{(C.5)}$$

$$\begin{aligned}
\mathcal{S}' \;=\; \mathrm{Tr}\Big[& -(3m_{H_u}^2 + \mathbf{m}_Q^2)\mathbf{Y}_u^\dagger\mathbf{Y}_u + 4\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u + (3m_{H_d}^2 - \mathbf{m}_Q^2)\mathbf{Y}_d^\dagger\mathbf{Y}_d - 2\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d \\
& + (m_{H_d}^2 + \mathbf{m}_L^2)\mathbf{Y}_e^\dagger\mathbf{Y}_e - 2\mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e\Big] \\
& + \left[\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2\right]\left\{m_{H_u}^2 - m_{H_d}^2 - \mathrm{Tr}(\mathbf{m}_L^2)\right\} + \left[\frac{8}{3}g_3^2 + \frac{3}{2}g_2^2 + \frac{1}{30}g_1^2\right]\mathrm{Tr}(\mathbf{m}_Q^2) \\
& - \left[\frac{16}{3}g_3^2 + \frac{16}{15}g_1^2\right]\mathrm{Tr}(\mathbf{m}_u^2) + \left[\frac{8}{3}g_3^2 + \frac{2}{15}g_1^2\right]\mathrm{Tr}(\mathbf{m}_d^2) + \frac{6}{5}g_1^2\mathrm{Tr}(\mathbf{m}_e^2)
\end{aligned}$$

$$\sigma_1 = \frac{1}{5} g_1^2 \Big\{ 3(m_{H_u}^2 + m_{H_d}^2) + \mathrm{Tr}[\mathbf{m}_Q^2 + 3\mathbf{m}_L^2 + 8\mathbf{m}_u^2 + 2\mathbf{m}_d^2 + 6\mathbf{m}_e^2] \Big\}$$

$$\sigma_2 = g_2^2 \left\{ m_{H_u}^2 + m_{H_d}^2 + \mathrm{Tr}[3\mathbf{m}_Q^2 + \mathbf{m}_L^2] \right\}$$

$$\sigma_3 = g_3^2 \mathrm{Tr}[2\mathbf{m}_Q^2 + \mathbf{m}_u^2 + \mathbf{m}_d^2] \ .$$

$$\frac{d}{dt} m^2 = \frac{1}{16\pi^2} \beta_{m^2}^{(1)} + \frac{1}{(16\pi^2)^2} \beta_{m^2}^{(2)} \tag{C.6}$$

$$\begin{aligned}
\beta_{m_{H_u}^2}^{(1)} &= 6\mathrm{Tr}[(m_{H_u}^2 + \mathbf{m}_Q^2)\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u + \mathbf{h}_u^\dagger \mathbf{h}_u] \\
&\quad - 6 g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 + \frac{3}{5} g_1^2 \mathcal{S}
\end{aligned}$$

$$\begin{aligned}
\beta_{m_{H_u}^2}^{(2)} &= -6\mathrm{Tr}\Big[6(m_{H_u}^2 + \mathbf{m}_Q^2)\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u + 6\mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u \\
&\qquad + (m_{H_u}^2 + m_{H_d}^2 + \mathbf{m}_Q^2)\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d \\
&\qquad + \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{m}_Q^2 \mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d + 6\mathbf{h}_u^\dagger \mathbf{h}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u + 6\mathbf{h}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{h}_u \\
&\qquad + \mathbf{h}_d^\dagger \mathbf{h}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{h}_u^\dagger \mathbf{h}_u + \mathbf{h}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{h}_u + \mathbf{Y}_d^\dagger \mathbf{h}_d \mathbf{h}_u^\dagger \mathbf{Y}_u \Big] \\
&\quad + \Big[32 g_3^2 + \frac{8}{5} g_1^2 \Big] \mathrm{Tr}[(m_{H_u}^2 + \mathbf{m}_Q^2)\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u + \mathbf{h}_u^\dagger \mathbf{h}_u] \\
&\quad + 32 g_3^2 \Big\{ 2|M_3|^2 \mathrm{Tr}[\mathbf{Y}_u^\dagger \mathbf{Y}_u] - M_3^* \mathrm{Tr}[\mathbf{Y}_u^\dagger \mathbf{h}_u] - M_3 \mathrm{Tr}[\mathbf{h}_u^\dagger \mathbf{Y}_u] \Big\} \\
&\quad + \frac{8}{5} g_1^2 \Big\{ 2|M_1|^2 \mathrm{Tr}[\mathbf{Y}_u^\dagger \mathbf{Y}_u] - M_1^* \mathrm{Tr}[\mathbf{Y}_u^\dagger \mathbf{h}_u] - M_1 \mathrm{Tr}[\mathbf{h}_u^\dagger \mathbf{Y}_u] \Big\} + \frac{6}{5} g_1^2 \mathcal{S}' \\
&\quad + 33 g_2^4 |M_2|^2 + \frac{18}{5} g_2^2 g_1^2 (|M_2|^2 + |M_1|^2 + \mathrm{Re}[M_1 M_2^*]) + \frac{621}{25} g_1^4 |M_1|^2 \\
&\quad + 3 g_2^2 \sigma_2 + \frac{3}{5} g_1^2 \sigma_1
\end{aligned}$$

$$
\begin{aligned}
\beta^{(1)}_{m^2_{H_d}} \;=\; & \mathrm{Tr}\Big[6(m^2_{H_d}+\mathbf{m}^2_Q)\mathbf{Y}_d^\dagger\mathbf{Y}_d + 6\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d + 2(m^2_{H_d}+\mathbf{m}^2_L)\mathbf{Y}_e^\dagger\mathbf{Y}_e + 2\mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e \\
& + 6\mathbf{h}_d^\dagger\mathbf{h}_d + 2\mathbf{h}_e^\dagger\mathbf{h}_e\Big] - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2 - \frac{3}{5}g_1^2\mathcal{S}
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(2)}_{m^2_{H_d}} \;=\; & -6\mathrm{Tr}\Big[6(m^2_{H_d}+\mathbf{m}^2_Q)\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d + 6\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d \\
& + (m^2_{H_u}+m^2_{H_d}+\mathbf{m}^2_Q)\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d \\
& + \mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{m}_Q^2\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d + 2(m^2_{H_d}+\mathbf{m}^2_L)\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e \\
& + 2\mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e + 6\mathbf{h}_d^\dagger\mathbf{h}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d + 6\mathbf{h}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{h}_d + \mathbf{h}_u^\dagger\mathbf{h}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d \\
& + \mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{h}_d^\dagger\mathbf{h}_d + \mathbf{h}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{h}_d + \mathbf{Y}_u^\dagger\mathbf{h}_u\mathbf{h}_d^\dagger\mathbf{Y}_d + 2\mathbf{h}_e^\dagger\mathbf{h}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e + 2\mathbf{h}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{h}_e\Big] \\
& + \Big[32g_3^2 - \frac{4}{5}g_1^2\Big]\mathrm{Tr}[(m^2_{H_d}+\mathbf{m}^2_Q)\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d + \mathbf{h}_d^\dagger\mathbf{h}_d] \\
& + 32g_3^2\Big\{2|M_3|^2\mathrm{Tr}[\mathbf{Y}_d^\dagger\mathbf{Y}_d] - M_3^*\mathrm{Tr}[\mathbf{Y}_d^\dagger\mathbf{h}_d] - M_3\mathrm{Tr}[\mathbf{h}_d^\dagger\mathbf{Y}_d]\Big\} \\
& - \frac{4}{5}g_1^2\Big\{2|M_1|^2\mathrm{Tr}[\mathbf{Y}_d^\dagger\mathbf{Y}_d] - M_1^*\mathrm{Tr}[\mathbf{Y}_d^\dagger\mathbf{h}_d] - M_1\mathrm{Tr}[\mathbf{h}_d^\dagger\mathbf{Y}_d]\Big\} \\
& + \frac{12}{5}g_1^2\Big\{\mathrm{Tr}[(m^2_{H_d}+\mathbf{m}^2_L)\mathbf{Y}_e^\dagger\mathbf{Y}_e + \mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e + \mathbf{h}_e^\dagger\mathbf{h}_e] + 2|M_1|^2\mathrm{Tr}[\mathbf{Y}_e^\dagger\mathbf{Y}_e] \\
& \qquad - M_1\mathrm{Tr}[\mathbf{h}_e^\dagger\mathbf{Y}_e] - M_1^*\mathrm{Tr}[\mathbf{Y}_e^\dagger\mathbf{h}_e]\Big\} - \frac{6}{5}g_1^2\mathcal{S}' + 33g_2^4|M_2|^2 \\
& + \frac{18}{5}g_2^2g_1^2(|M_2|^2 + |M_1|^2 + \mathrm{Re}[M_1M_2^*]) + \frac{621}{25}g_1^4|M_1|^2 \\
& + 3g_2^2\sigma_2 + \frac{3}{5}g_1^2\sigma_1
\end{aligned}
$$

$$
\beta^{(1)}_{\mathbf{m}^2_Q} \;=\; (\mathbf{m}^2_Q + 2m^2_{H_u})\mathbf{Y}_u^\dagger\mathbf{Y}_u + (\mathbf{m}^2_Q + 2m^2_{H_d})\mathbf{Y}_d^\dagger\mathbf{Y}_d + [\mathbf{Y}_u^\dagger\mathbf{Y}_u + \mathbf{Y}_d^\dagger\mathbf{Y}_d]\mathbf{m}^2_Q + 2\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u
$$

$$+ 2\mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d + 2\mathbf{h}_u^\dagger \mathbf{h}_u + 2\mathbf{h}_d^\dagger \mathbf{h}_d - \frac{32}{3} g_3^2 |M_3|^2 - 6 g_2^2 |M_2|^2 - \frac{2}{15} g_1^2 |M_1|^2 + \frac{1}{5} g_1^2 \mathcal{S}$$

$$\begin{aligned}
\beta^{(2)}_{\mathbf{m}_Q^2} = & - (2\mathbf{m}_Q^2 + 8 m_{H_u}^2)\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u - 4\mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u - 4\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{m}_Q^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u \\
& - 4\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u - 2\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{m}_Q^2 - (2\mathbf{m}_Q^2 + 8 m_{H_d}^2)\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d \\
& - 4\mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d - 4\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{m}_Q^2 \mathbf{Y}_d^\dagger \mathbf{Y}_d - 4\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d - 2\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{m}_Q^2 \\
& - \Big[(\mathbf{m}_Q^2 + 4 m_{H_u}^2)\mathbf{Y}_u^\dagger \mathbf{Y}_u + 2\mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u + \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{m}_Q^2\Big] \mathrm{Tr}(3\mathbf{Y}_u^\dagger \mathbf{Y}_u) \\
& - \Big[(\mathbf{m}_Q^2 + 4 m_{H_d}^2)\mathbf{Y}_d^\dagger \mathbf{Y}_d + 2\mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d + \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{m}_Q^2\Big] \mathrm{Tr}(3\mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_e^\dagger \mathbf{Y}_e) \\
& - 6\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathrm{Tr}(\mathbf{m}_Q^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u) \\
& - \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathrm{Tr}(6\mathbf{m}_Q^2 \mathbf{Y}_d^\dagger \mathbf{Y}_d + 6\mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d + 2\mathbf{m}_L^2 \mathbf{Y}_e^\dagger \mathbf{Y}_e + 2\mathbf{Y}_e^\dagger \mathbf{m}_e^2 \mathbf{Y}_e) \\
& - 4\Big\{\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{h}_u^\dagger \mathbf{h}_u + \mathbf{h}_u^\dagger \mathbf{h}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_u^\dagger \mathbf{h}_u \mathbf{h}_u^\dagger \mathbf{Y}_u + \mathbf{h}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{h}_u\Big\} \\
& - 4\Big\{\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{h}_d^\dagger \mathbf{h}_d + \mathbf{h}_d^\dagger \mathbf{h}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_d^\dagger \mathbf{h}_d \mathbf{h}_d^\dagger \mathbf{Y}_d + \mathbf{h}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{h}_d\Big\} \\
& - \mathbf{h}_u^\dagger \mathbf{h}_u \mathrm{Tr}[6\mathbf{Y}_u^\dagger \mathbf{Y}_u] - \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathrm{Tr}[6\mathbf{h}_u^\dagger \mathbf{h}_u] - \mathbf{h}_u^\dagger \mathbf{Y}_u \mathrm{Tr}[6\mathbf{Y}_u^\dagger \mathbf{h}_u] - \mathbf{Y}_u^\dagger \mathbf{h}_u \mathrm{Tr}[6\mathbf{h}_u^\dagger \mathbf{Y}_u] \\
& - \mathbf{h}_d^\dagger \mathbf{h}_d \mathrm{Tr}[6\mathbf{Y}_d^\dagger \mathbf{Y}_d + 2\mathbf{Y}_e^\dagger \mathbf{Y}_e] - \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathrm{Tr}[6\mathbf{h}_d^\dagger \mathbf{h}_d + 2\mathbf{h}_e^\dagger \mathbf{h}_e] \\
& - \mathbf{h}_d^\dagger \mathbf{Y}_d \mathrm{Tr}[6\mathbf{Y}_d^\dagger \mathbf{h}_d + 2\mathbf{Y}_e^\dagger \mathbf{h}_e] - \mathbf{Y}_d^\dagger \mathbf{h}_d \mathrm{Tr}[6\mathbf{h}_d^\dagger \mathbf{Y}_d + 2\mathbf{h}_e^\dagger \mathbf{Y}_e] \\
& + \frac{2}{5} g_1^2 \Big\{(2\mathbf{m}_Q^2 + 4 m_{H_u}^2)\mathbf{Y}_u^\dagger \mathbf{Y}_u + 4\mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u + 2\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{m}_Q^2 + 4\mathbf{h}_u^\dagger \mathbf{h}_u - 4M_1 \mathbf{h}_u^\dagger \mathbf{Y}_u \\
& \qquad - 4M_1^* \mathbf{Y}_u^\dagger \mathbf{h}_u + 8|M_1|^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u + (\mathbf{m}_Q^2 + 2 m_{H_d}^2)\mathbf{Y}_d^\dagger \mathbf{Y}_d + 2\mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d
\end{aligned}$$

$$+ \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{m}_Q^2 + 2\mathbf{h}_d^\dagger \mathbf{h}_d - 2M_1 \mathbf{h}_d^\dagger \mathbf{Y}_d - 2M_1^* \mathbf{Y}_d^\dagger \mathbf{h}_d + 4|M_1|^2 \mathbf{Y}_d^\dagger \mathbf{Y}_d \Big\}$$

$$+ \frac{2}{5} g_1^2 \mathcal{S}' - \frac{128}{3} g_3^4 |M_3|^2 + 32 g_3^2 g_2^2 (|M_3|^2 + |M_2|^2 + \mathrm{Re}[M_2 M_3^*])$$

$$+ \frac{32}{45} g_3^2 g_1^2 (|M_3|^2 + |M_1|^2 + \mathrm{Re}[M_1 M_3^*]) + 33 g_2^4 |M_2|^2$$

$$+ \frac{2}{5} g_2^2 g_1^2 (|M_2|^2 + |M_1|^2 + \mathrm{Re}[M_1 M_2^*]) + \frac{199}{75} g_1^4 |M_1|^2$$

$$+ \frac{16}{3} g_3^2 \sigma_3 + 3 g_2^2 \sigma_2 + \frac{1}{15} g_1^2 \sigma_1$$

$$\beta_{\mathbf{m}_L^2}^{(1)} = (\mathbf{m}_L^2 + 2m_{H_d}^2) \mathbf{Y}_e^\dagger \mathbf{Y}_e + 2\mathbf{Y}_e^\dagger \mathbf{m}_e^2 \mathbf{Y}_e + \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{m}_L^2 + 2\mathbf{h}_e^\dagger \mathbf{h}_e$$

$$- 6 g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 - \frac{3}{5} g_1^2 \mathcal{S}$$

$$\beta_{\mathbf{m}_L^2}^{(2)} = -(2\mathbf{m}_L^2 + 8m_{H_d}^2) \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_e^\dagger \mathbf{Y}_e - 4\mathbf{Y}_e^\dagger \mathbf{m}_e^2 \mathbf{Y}_e \mathbf{Y}_e^\dagger \mathbf{Y}_e - 4\mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{m}_L^2 \mathbf{Y}_e^\dagger \mathbf{Y}_e$$

$$- 4\mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_e^\dagger \mathbf{m}_e^2 \mathbf{Y}_e - 2\mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{m}_L^2$$

$$- \Big[(\mathbf{m}_L^2 + 4m_{H_d}^2) \mathbf{Y}_e^\dagger \mathbf{Y}_e + 2\mathbf{Y}_e^\dagger \mathbf{m}_e^2 \mathbf{Y}_e + \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{m}_L^2\Big] \mathrm{Tr}(3\mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_e^\dagger \mathbf{Y}_e)$$

$$- \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathrm{Tr}[6\mathbf{m}_Q^2 \mathbf{Y}_d^\dagger \mathbf{Y}_d + 6\mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d + 2\mathbf{m}_L^2 \mathbf{Y}_e^\dagger \mathbf{Y}_e + 2\mathbf{Y}_e^\dagger \mathbf{m}_e^2 \mathbf{Y}_e]$$

$$- 4\Big\{\mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{h}_e^\dagger \mathbf{h}_e + \mathbf{h}_e^\dagger \mathbf{h}_e \mathbf{Y}_e^\dagger \mathbf{Y}_e + \mathbf{Y}_e^\dagger \mathbf{h}_e \mathbf{h}_e^\dagger \mathbf{Y}_e + \mathbf{h}_e^\dagger \mathbf{Y}_e \mathbf{Y}_e^\dagger \mathbf{h}_e\Big\}$$

$$- \mathbf{h}_e^\dagger \mathbf{h}_e \mathrm{Tr}[6\mathbf{Y}_d^\dagger \mathbf{Y}_d + 2\mathbf{Y}_e^\dagger \mathbf{Y}_e] - \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathrm{Tr}[6\mathbf{h}_d^\dagger \mathbf{h}_d + 2\mathbf{h}_e^\dagger \mathbf{h}_e]$$

$$- \mathbf{h}_e^\dagger \mathbf{Y}_e \mathrm{Tr}[6\mathbf{Y}_d^\dagger \mathbf{h}_d + 2\mathbf{Y}_e^\dagger \mathbf{h}_e] - \mathbf{Y}_e^\dagger \mathbf{h}_e \mathrm{Tr}[6\mathbf{h}_d^\dagger \mathbf{Y}_d + 2\mathbf{h}_e^\dagger \mathbf{Y}_e]$$

$$+\frac{6}{5}g_1^2\Big\{(\mathbf{m}_L^2+2m_{H_d}^2)\mathbf{Y}_e^\dagger\mathbf{Y}_e+2\mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e+\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{m}_L^2+2\mathbf{h}_e^\dagger\mathbf{h}_e$$

$$-2M_1\mathbf{h}_e^\dagger\mathbf{Y}_e-2M_1^*\mathbf{Y}_e^\dagger\mathbf{h}_e+4|M_1|^2\mathbf{Y}_e^\dagger\mathbf{Y}_e\Big\}-\frac{6}{5}g_1^2\mathcal{S}'$$

$$+33g_2^4|M_2|^2+\frac{18}{5}g_2^2g_1^2(|M_2|^2+|M_1|^2+\mathrm{Re}[M_1M_2^*])+\frac{621}{25}g_1^4|M_1|^2$$

$$+3g_2^2\sigma_2+\frac{3}{5}g_1^2\sigma_1$$

$$\beta_{\mathbf{m}_u^2}^{(1)} = (2\mathbf{m}_u^2+4m_{H_u}^2)\mathbf{Y}_u\mathbf{Y}_u^\dagger+4\mathbf{Y}_u\mathbf{m}_Q^2\mathbf{Y}_u^\dagger+2\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{m}_u^2+4\mathbf{h}_u\mathbf{h}_u^\dagger$$

$$-\frac{32}{3}g_3^2|M_3|^2-\frac{32}{15}g_1^2|M_1|^2-\frac{4}{5}g_1^2\mathcal{S}$$

$$\beta_{\mathbf{m}_u^2}^{(2)} = -(2\mathbf{m}_u^2+8m_{H_u}^2)\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger-4\mathbf{Y}_u\mathbf{m}_Q^2\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger-4\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u\mathbf{Y}_u^\dagger$$

$$-4\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{m}_Q^2\mathbf{Y}_u^\dagger-2\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{m}_u^2-(2\mathbf{m}_u^2+4m_{H_u}^2+4m_{H_d}^2)\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger$$

$$-4\mathbf{Y}_u\mathbf{m}_Q^2\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger-4\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d\mathbf{Y}_u^\dagger-4\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{m}_Q^2\mathbf{Y}_u^\dagger-2\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{m}_u^2$$

$$-\Big[(\mathbf{m}_u^2+4m_{H_u}^2)\mathbf{Y}_u\mathbf{Y}_u^\dagger+2\mathbf{Y}_u\mathbf{m}_Q^2\mathbf{Y}_u^\dagger+\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{m}_u^2\Big]\mathrm{Tr}[6\mathbf{Y}_u^\dagger\mathbf{Y}_u]$$

$$-12\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathrm{Tr}[\mathbf{m}_Q^2\mathbf{Y}_u^\dagger\mathbf{Y}_u+\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u]$$

$$-4\left\{\mathbf{h}_u\mathbf{h}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger+\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{h}_u\mathbf{h}_u^\dagger+\mathbf{h}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{h}_u^\dagger+\mathbf{Y}_u\mathbf{h}_u^\dagger\mathbf{h}_u\mathbf{Y}_u^\dagger\right\}$$

$$-4\Big\{\mathbf{h}_u\mathbf{h}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger+\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{h}_d\mathbf{h}_u^\dagger+\mathbf{h}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{h}_u^\dagger+\mathbf{Y}_u\mathbf{h}_d^\dagger\mathbf{h}_d\mathbf{Y}_u^\dagger\Big\}$$

$$- 12\left\{\mathbf{h}_u\mathbf{h}_u^\dagger \mathrm{Tr}[\mathbf{Y}_u^\dagger \mathbf{Y}_u] + \mathbf{Y}_u\mathbf{Y}_u^\dagger \mathrm{Tr}[\mathbf{h}_u^\dagger \mathbf{h}_u] + \mathbf{h}_u\mathbf{Y}_u^\dagger \mathrm{Tr}[\mathbf{h}_u^\dagger \mathbf{Y}_u] + \mathbf{Y}_u\mathbf{h}_u^\dagger \mathrm{Tr}[\mathbf{Y}_u^\dagger \mathbf{h}_u]\right\}$$

$$+ \left[6g_2^2 - \frac{2}{5}g_1^2\right]\left\{(\mathbf{m}_u^2 + 2m_{H_u}^2)\mathbf{Y}_u\mathbf{Y}_u^\dagger + 2\mathbf{Y}_u\mathbf{m}_Q^2\mathbf{Y}_u^\dagger + \mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{m}_u^2 + 2\mathbf{h}_u\mathbf{h}_u^\dagger\right\}$$

$$+ 12g_2^2\left\{2|M_2|^2\mathbf{Y}_u\mathbf{Y}_u^\dagger - M_2^*\mathbf{h}_u\mathbf{Y}_u^\dagger - M_2\mathbf{Y}_u\mathbf{h}_u^\dagger\right\}$$

$$- \frac{4}{5}g_1^2\left\{2|M_1|^2\mathbf{Y}_u\mathbf{Y}_u^\dagger - M_1^*\mathbf{h}_u\mathbf{Y}_u^\dagger - M_1\mathbf{Y}_u\mathbf{h}_u^\dagger\right\} - \frac{8}{5}g_1^2\mathcal{S}'$$

$$- \frac{128}{3}g_3^4|M_3|^2 + \frac{512}{45}g_3^2g_1^2(|M_3|^2 + |M_1|^2 + \mathrm{Re}[M_1M_3^*]) + \frac{3424}{75}g_1^4|M_1|^2$$

$$+ \frac{16}{3}g_3^2\sigma_3 + \frac{16}{15}g_1^2\sigma_1$$

$$\beta_{\mathbf{m}_d^2}^{(1)} = (2\mathbf{m}_d^2 + 4m_{H_d}^2)\mathbf{Y}_d\mathbf{Y}_d^\dagger + 4\mathbf{Y}_d\mathbf{m}_Q^2\mathbf{Y}_d^\dagger + 2\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{m}_d^2 + 4\mathbf{h}_d\mathbf{h}_d^\dagger$$

$$- \frac{32}{3}g_3^2|M_3|^2 - \frac{8}{15}g_1^2|M_1|^2 + \frac{2}{5}g_1^2\mathcal{S}$$

$$\beta_{\mathbf{m}_d^2}^{(2)} = -(2\mathbf{m}_d^2 + 8m_{H_d}^2)\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger - 4\mathbf{Y}_d\mathbf{m}_Q^2\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger - 4\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d\mathbf{Y}_d^\dagger$$

$$- 4\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{m}_Q^2\mathbf{Y}_d^\dagger - 2\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{m}_d^2 - (2\mathbf{m}_d^2 + 4m_{H_u}^2 + 4m_{H_d}^2)\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger$$

$$- 4\mathbf{Y}_d\mathbf{m}_Q^2\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger - 4\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u\mathbf{Y}_d^\dagger - 4\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{m}_Q^2\mathbf{Y}_d^\dagger - 2\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{m}_d^2$$

$$- \left[(\mathbf{m}_d^2 + 4m_{H_d}^2)\mathbf{Y}_d\mathbf{Y}_d^\dagger + 2\mathbf{Y}_d\mathbf{m}_Q^2\mathbf{Y}_d^\dagger + \mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{m}_d^2\right]\mathrm{Tr}(6\mathbf{Y}_d^\dagger\mathbf{Y}_d + 2\mathbf{Y}_e^\dagger\mathbf{Y}_e)$$

$$- 4\mathbf{Y}_d\mathbf{Y}_d^\dagger \mathrm{Tr}(3\mathbf{m}_Q^2\mathbf{Y}_d^\dagger\mathbf{Y}_d + 3\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d + \mathbf{m}_L^2\mathbf{Y}_e^\dagger\mathbf{Y}_e + \mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e)$$

$$- 4\Big\{\mathbf{h}_d\mathbf{h}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{h}_d\mathbf{h}_d^\dagger + \mathbf{h}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{h}_d^\dagger + \mathbf{Y}_d\mathbf{h}_d^\dagger\mathbf{h}_d\mathbf{Y}_d^\dagger\Big\}$$

$$- 4\Big\{\mathbf{h}_d\mathbf{h}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger + \mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{h}_u\mathbf{h}_d^\dagger + \mathbf{h}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{h}_d^\dagger + \mathbf{Y}_d\mathbf{h}_u^\dagger\mathbf{h}_u\mathbf{Y}_d^\dagger\Big\}$$

$$- 4\mathbf{h}_d\mathbf{h}_d^\dagger \mathrm{Tr}(3\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_e^\dagger\mathbf{Y}_e) - 4\mathbf{Y}_d\mathbf{Y}_d^\dagger \mathrm{Tr}(3\mathbf{h}_d^\dagger\mathbf{h}_d + \mathbf{h}_e^\dagger\mathbf{h}_e)$$

$$- 4\mathbf{h}_d\mathbf{Y}_d^\dagger \mathrm{Tr}(3\mathbf{h}_d^\dagger\mathbf{Y}_d + \mathbf{h}_e^\dagger\mathbf{Y}_e) - 4\mathbf{Y}_d\mathbf{h}_d^\dagger \mathrm{Tr}(3\mathbf{Y}_d^\dagger\mathbf{h}_d + \mathbf{Y}_e^\dagger\mathbf{h}_e)$$

$$+ \Big[6g_2^2 + \frac{2}{5}g_1^2\Big]\Big\{(\mathbf{m}_d^2 + 2m_{H_d}^2)\mathbf{Y}_d\mathbf{Y}_d^\dagger + 2\mathbf{Y}_d\mathbf{m}_Q^2\mathbf{Y}_d^\dagger + \mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{m}_d^2 + 2\mathbf{h}_d\mathbf{h}_d^\dagger\Big\}$$

$$+ 12g_2^2\Big\{2|M_2|^2\mathbf{Y}_d\mathbf{Y}_d^\dagger - M_2^*\mathbf{h}_d\mathbf{Y}_d^\dagger - M_2\mathbf{Y}_d\mathbf{h}_d^\dagger\Big\}$$

$$+ \frac{4}{5}g_1^2\Big\{2|M_1|^2\mathbf{Y}_d\mathbf{Y}_d^\dagger - M_1^*\mathbf{h}_d\mathbf{Y}_d^\dagger - M_1\mathbf{Y}_d\mathbf{h}_d^\dagger\Big\} + \frac{4}{5}g_1^2\mathcal{S}'$$

$$- \frac{128}{3}g_3^4|M_3|^2 + \frac{128}{45}g_3^2g_1^2(|M_3|^2 + |M_1|^2 + \mathrm{Re}[M_1M_3^*]) + \frac{808}{75}g_1^4|M_1|^2$$

$$+ \frac{16}{3}g_3^2\sigma_3 + \frac{4}{15}g_1^2\sigma_1$$

$$\beta_{\mathbf{m}_e^2}^{(1)} \;=\; (2\mathbf{m}_e^2 + 4m_{H_d}^2)\mathbf{Y}_e\mathbf{Y}_e^\dagger + 4\mathbf{Y}_e\mathbf{m}_L^2\mathbf{Y}_e^\dagger + 2\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{m}_e^2 + 4\mathbf{h}_e\mathbf{h}_e^\dagger$$

$$- \frac{24}{5}g_1^2|M_1|^2 + \frac{6}{5}g_1^2\mathcal{S}$$

$$
\begin{aligned}
\beta^{(2)}_{\mathbf{m}^2_e} \;=\;& -(2\mathbf{m}^2_e + 8m^2_{H_d})\mathbf{Y}_e\mathbf{Y}^\dagger_e\mathbf{Y}_e\mathbf{Y}^\dagger_e - 4\mathbf{Y}_e\mathbf{m}^2_L\mathbf{Y}^\dagger_e\mathbf{Y}_e\mathbf{Y}^\dagger_e - 4\mathbf{Y}_e\mathbf{Y}^\dagger_e\mathbf{m}^2_e\mathbf{Y}_e\mathbf{Y}^\dagger_e \\
& - 4\mathbf{Y}_e\mathbf{Y}^\dagger_e\mathbf{Y}_e\mathbf{m}^2_L\mathbf{Y}^\dagger_e - 2\mathbf{Y}_e\mathbf{Y}^\dagger_e\mathbf{Y}_e\mathbf{Y}^\dagger_e\mathbf{m}^2_e \\
& - \Big[(\mathbf{m}^2_e + 4m^2_{H_d})\mathbf{Y}_e\mathbf{Y}^\dagger_e + 2\mathbf{Y}_e\mathbf{m}^2_L\mathbf{Y}^\dagger_e + \mathbf{Y}_e\mathbf{Y}^\dagger_e\mathbf{m}^2_e\Big]\mathrm{Tr}[6\mathbf{Y}^\dagger_d\mathbf{Y}_d + 2\mathbf{Y}^\dagger_e\mathbf{Y}_e] \\
& - 4\mathbf{Y}_e\mathbf{Y}^\dagger_e\mathrm{Tr}[3\mathbf{m}^2_Q\mathbf{Y}^\dagger_d\mathbf{Y}_d + 3\mathbf{Y}^\dagger_d\mathbf{m}^2_d\mathbf{Y}_d + \mathbf{m}^2_L\mathbf{Y}^\dagger_e\mathbf{Y}_e + \mathbf{Y}^\dagger_e\mathbf{m}^2_e\mathbf{Y}_e] \\
& - 4\Big\{\mathbf{h}_e\mathbf{h}^\dagger_e\mathbf{Y}_e\mathbf{Y}^\dagger_e + \mathbf{Y}_e\mathbf{Y}^\dagger_e\mathbf{h}_e\mathbf{h}^\dagger_e + \mathbf{h}_e\mathbf{Y}^\dagger_e\mathbf{Y}_e\mathbf{h}^\dagger_e + \mathbf{Y}_e\mathbf{h}^\dagger_e\mathbf{h}_e\mathbf{Y}^\dagger_e\Big\} \\
& - 4\mathbf{h}_e\mathbf{h}^\dagger_e\mathrm{Tr}[3\mathbf{Y}^\dagger_d\mathbf{Y}_d + \mathbf{Y}^\dagger_e\mathbf{Y}_e] - 4\mathbf{Y}_e\mathbf{Y}^\dagger_e\mathrm{Tr}[3\mathbf{h}^\dagger_d\mathbf{h}_d + \mathbf{h}^\dagger_e\mathbf{h}_e] \\
& - 4\mathbf{h}_e\mathbf{Y}^\dagger_e\mathrm{Tr}[3\mathbf{h}^\dagger_d\mathbf{Y}_d + \mathbf{h}^\dagger_e\mathbf{Y}_e] - 4\mathbf{Y}_e\mathbf{h}^\dagger_e\mathrm{Tr}[3\mathbf{Y}^\dagger_d\mathbf{h}_d + \mathbf{Y}^\dagger_e\mathbf{h}_e] \\
& + \Big[6g^2_2 - \frac{6}{5}g^2_1\Big]\Big\{(\mathbf{m}^2_e + 2m^2_{H_d})\mathbf{Y}_e\mathbf{Y}^\dagger_e + 2\mathbf{Y}_e\mathbf{m}^2_L\mathbf{Y}^\dagger_e + \mathbf{Y}_e\mathbf{Y}^\dagger_e\mathbf{m}^2_e + 2\mathbf{h}_e\mathbf{h}^\dagger_e\Big\} \\
& + 12g^2_2\Big\{2|M_2|^2\mathbf{Y}_e\mathbf{Y}^\dagger_e - M^*_2\mathbf{h}_e\mathbf{Y}^\dagger_e - M_2\mathbf{Y}_e\mathbf{h}^\dagger_e\Big\} \\
& - \frac{12}{5}g^2_1\Big\{2|M_1|^2\mathbf{Y}_e\mathbf{Y}^\dagger_e - M^*_1\mathbf{h}_e\mathbf{Y}^\dagger_e - M_1\mathbf{Y}_e\mathbf{h}^\dagger_e\Big\} \\
& + \frac{12}{5}g^2_1\mathcal{S}' + \frac{2808}{25}g^4_1|M_1|^2 + \frac{12}{5}g^2_1\sigma_1 \;\; .
\end{aligned}
$$

APPENDIX D

Radiative Corrections

$$
\begin{aligned}
\gamma^d_{ii} &= \frac{2\alpha_s}{3\pi}\ \left(\mathbf{Y}^A_d\right)_{ii} M^\star_g\ I_3\left(M^2_{\tilde{d}^i_L}, M^2_{\tilde{d}^i_R}, |M_g|^2\right) \\
&+ \frac{2\alpha_s}{3\pi}\sum_{j=1}^{3}\left(\mathbf{Y}^A_d\right)_{jj} M^\star_g\ M^4_{\tilde{D}} I_5\left(M^2_{\tilde{d}^i_L}, M^2_{\tilde{d}^j_L}, M^2_{\tilde{d}^j_R}, M^2_{\tilde{d}^i_R}, |M_g|^2\right)\ \left(\delta^d_{ij}\right)_{RR}\left(\delta^d_{ji}\right)_{LL} \\
&+ \frac{(\mathbf{Y}_d)_{ii}}{(4\pi)^2}\ \sum_{j=1}^{3}\ \left(\mathbf{Y}^\dagger_u\right)_{ij}(\mathbf{Y}_u)_{ji}|\mu|^2\ I_3\left(M^2_{\tilde{u}^j_R}, M^2_{\tilde{u}^i_L}, |\mu|^2\right) \\
\Gamma^d_{ii} &= \frac{2\alpha_s}{3\pi}\ (\mathbf{Y}_d)_{ii}\,\mu^\star M^\star_g\ I_3\left(M^2_{\tilde{d}^i_L}, M^2_{\tilde{d}^i_R}, |M_g|^2\right) \\
&+ \frac{2\alpha_s}{3\pi}\sum_{j=1}^{3}(\mathbf{Y}_d)_{jj}\,\mu^\star M^\star_g\ M^4_{\tilde{D}} I_5\left(M^2_{\tilde{d}^i_L}, M^2_{\tilde{d}^j_L}, M^2_{\tilde{d}^j_R}, M^2_{\tilde{d}^i_R}, |M_g|^2\right)\ \left(\delta^d_{ij}\right)_{RR}\left(\delta^d_{ji}\right)_{LL} \\
&+ \frac{(\mathbf{Y}_d)_{ii}}{(4\pi)^2}\ \sum_{j=1}^{3}\ \left(\mathbf{Y}^{A\,\dagger}_u\right)_{ij}(\mathbf{Y}_u)_{ji}\,\mu^\star\ I_3\left(M^2_{\tilde{u}^j_R}, M^2_{\tilde{u}^i_L}, |\mu|^2\right) \qquad \text{(D.1)}
\end{aligned}
$$

$$
\begin{aligned}
\gamma^d_{ij} &= \frac{2\alpha_s}{3\pi}\ \left(\mathbf{Y}^A_d\right)_{ii} M^\star_g M^2_{\tilde{D}}\ I_4\left(M^2_{\tilde{d}^j_L}, M^2_{\tilde{d}^i_L}, M^2_{\tilde{d}^i_R}, |M_g|^2\right)\ \left(\delta^d_{ij}\right)_{LL} \\
&+ \frac{2\alpha_s}{3\pi}\ \left(\mathbf{Y}^A_d\right)_{jj} M^\star_g M^2_{\tilde{D}}\ I_4\left(M^2_{\tilde{d}^j_L}, M^2_{\tilde{d}^j_R}, M^2_{\tilde{d}^i_R}, |M_g|^2\right)\ \left(\delta^d_{ij}\right)_{RR}
\end{aligned}
$$

$$+ \quad \frac{(\mathbf{Y}_d)_{ii}}{(4\pi)^2} \left(\mathbf{Y}_u^\dagger\right)_{ij} (\mathbf{Y}_u)_{jj} |\mu|^2 \; I_3\left(M^2_{\tilde{u}^j_R}, M^2_{\tilde{u}^i_L}, |\mu|^2\right)$$

$$+ \quad \frac{(\mathbf{Y}_d)_{ii}}{(4\pi)^2} \left(\mathbf{Y}_u^\dagger\right)_{ii} (\mathbf{Y}_u)_{ij} |\mu|^2 \; I_3\left(M^2_{\tilde{u}^i_R}, M^2_{\tilde{u}^i_L}, |\mu|^2\right)$$

$$+ \quad \frac{(\mathbf{Y}_d)_{ii}}{(4\pi)^2} \left(\mathbf{Y}_u^\dagger\right)_{jj} (\mathbf{Y}_u)_{jj} |\mu|^2 M^2_{\tilde{U}} \; I_4\left(M^2_{\tilde{u}^j_R}, M^2_{\tilde{u}^j_L}, M^2_{\tilde{u}^i_L}, |\mu|^2\right) \left(\delta^u_{ij}\right)_{LL}$$

$$+ \quad \frac{(\mathbf{Y}_d)_{ii}}{(4\pi)^2} \left(\mathbf{Y}_u^\dagger\right)_{ii} (\mathbf{Y}_u)_{jj} |\mu|^2 M^2_{\tilde{U}} \; I_4\left(M^2_{\tilde{u}^j_R}, M^2_{\tilde{u}^i_R}, M^2_{\tilde{u}^i_L}, |\mu|^2\right) \left(\delta^u_{ij}\right)_{RR}$$

$$\Gamma^d_{ij} \quad = \quad \frac{2\alpha_s}{3\pi} \; (\mathbf{Y}_d)_{ii} \, \mu^\star M_g^\star M^2_{\tilde{D}} \; I_4\left(M^2_{\tilde{d}^j_L}, M^2_{\tilde{d}^i_L}, M^2_{\tilde{d}^i_R}, |M_g|^2\right) \left(\delta^d_{ij}\right)_{LL}$$

$$+ \quad \frac{2\alpha_s}{3\pi} \; (\mathbf{Y}_d)_{jj} \, \mu^\star M_g^\star M^2_{\tilde{D}} \; I_4\left(M^2_{\tilde{d}^j_L}, M^2_{\tilde{d}^j_R}, M^2_{\tilde{d}^i_R}, |M_g|^2\right) \left(\delta^d_{ij}\right)_{RR}$$

$$+ \quad \frac{(\mathbf{Y}_d)_{ii}}{(4\pi)^2} \left(\mathbf{Y}_u^{A\,\dagger}\right)_{ij} (\mathbf{Y}_u)_{jj} \, \mu^\star \; I_3\left(M^2_{\tilde{u}^j_R}, M^2_{\tilde{u}^i_L}, |\mu|^2\right)$$

$$+ \quad \frac{(\mathbf{Y}_d)_{ii}}{(4\pi)^2} \left(\mathbf{Y}_u^{A\,\dagger}\right)_{ii} (\mathbf{Y}_u)_{ij} \, \mu^\star \; I_3\left(M^2_{\tilde{u}^i_R}, M^2_{\tilde{u}^i_L}, |\mu|^2\right)$$

$$+ \quad \frac{(\mathbf{Y}_d)_{ii}}{(4\pi)^2} \left(\mathbf{Y}_u^{A\,\dagger}\right)_{jj} (\mathbf{Y}_u)_{jj} \, \mu^\star M^2_{\tilde{U}} \; I_4\left(M^2_{\tilde{u}^j_R}, M^2_{\tilde{u}^j_L}, M^2_{\tilde{u}^i_L}, |\mu|^2\right) \left(\delta^u_{ij}\right)_{LL}$$

$$+ \quad \frac{(\mathbf{Y}_d)_{ii}}{(4\pi)^2} \left(\mathbf{Y}_u^{A\,\dagger}\right)_{ii} (\mathbf{Y}_u)_{jj} \, \mu^\star M^2_{\tilde{U}} \; I_4\left(M^2_{\tilde{u}^j_R}, M^2_{\tilde{u}^i_R}, M^2_{\tilde{u}^i_L}, |\mu|^2\right) \left(\delta^u_{ij}\right)_{RR} \quad \text{(D.2)}$$

for the off–diagonal elements. These expressions (D.1) and (D.2), with $i, j = 1, 2, 3$, complete the radiative corrections to down quark interactions with

Higgs fields, Repeating a similar analysis for the up quark sector, one finds

$$
\begin{aligned}
\gamma^u_{ii} &= \frac{2\alpha_s}{3\pi}\ \left(\mathbf{Y}^A_u\right)_{ii} M_g^\star\ I_3\left(M^2_{\tilde{u}^i_L}, M^2_{\tilde{u}^i_R}, |M_g|^2\right) \\
&+ \frac{2\alpha_s}{3\pi}\sum_{j=1}^{3}\left(\mathbf{Y}^A_u\right)_{jj} M_g^\star\ M^4_{\tilde{U}} I_5\left(M^2_{\tilde{u}^i_L}, M^2_{\tilde{u}^j_L}, M^2_{\tilde{u}^j_R}, M^2_{\tilde{u}^i_R}, |M_g|^2\right)\ \left(\delta^u_{ij}\right)_{RR}\left(\delta^u_{ji}\right)_{LL} \\
&+ \frac{(\mathbf{Y}_u)_{ii}}{(4\pi)^2}\ \left(\mathbf{Y}^\dagger_d\right)_{ii}(\mathbf{Y}_d)_{ii}\,|\mu|^2\ I_3\left(M^2_{\tilde{d}^i_R}, M^2_{\tilde{u}^i_L}, |\mu|^2\right) \\
\Gamma^u_{ii} &= \frac{2\alpha_s}{3\pi}\ (\mathbf{Y}_u)_{ii}\,\mu^\star M_g^\star\ I_3\left(M^2_{\tilde{u}^i_L}, M^2_{\tilde{u}^i_R}, |M_g|^2\right) \\
&+ \frac{2\alpha_s}{3\pi}\sum_{j=1}^{3}(\mathbf{Y}_u)_{jj}\,\mu^\star M_g^\star\ M^4_{\tilde{U}} I_5\left(M^2_{\tilde{u}^i_L}, M^2_{\tilde{u}^j_L}, M^2_{\tilde{u}^j_R}, M^2_{\tilde{u}^i_R}, |M_g|^2\right)\ \left(\delta^u_{ij}\right)_{RR}\left(\delta^u_{ji}\right)_{LL} \\
&+ \frac{(\mathbf{Y}_u)_{ii}}{(4\pi)^2}\ \left(\mathbf{Y}^{A\,\dagger}_d\right)_{ii}(\mathbf{Y}_d)_{ii}\,\mu^\star\ I_3\left(M^2_{\tilde{d}^j_R}, M^2_{\tilde{d}^i_L}, |\mu|^2\right) \qquad \text{(D.3)}
\end{aligned}
$$

$$
\begin{aligned}
\gamma^u_{ij} &= \frac{2\alpha_s}{3\pi}\ \left(\mathbf{Y}^A_u\right)_{ij} M_g^\star\ I_3\left(M^2_{\tilde{u}^j_L}, M^2_{\tilde{u}^i_R}, |M_g|^2\right) \\
&+ \frac{2\alpha_s}{3\pi}\ \left(\mathbf{Y}^A_u\right)_{ii} M_g^\star M^2_{\tilde{U}}\ I_4\left(M^2_{\tilde{u}^j_L}, M^2_{\tilde{u}^i_L}, M^2_{\tilde{u}^i_R}, |M_g|^2\right)\ \left(\delta^u_{ij}\right)_{LL} \\
&+ \frac{2\alpha_s}{3\pi}\ \left(\mathbf{Y}^A_u\right)_{jj} M_g^\star M^2_{\tilde{U}}\ I_4\left(M^2_{\tilde{u}^j_L}, M^2_{\tilde{u}^j_R}, M^2_{\tilde{u}^i_R}, |M_g|^2\right)\ \left(\delta^u_{ij}\right)_{RR} \\
&+ \frac{(\mathbf{Y}_u)_{ij}}{(4\pi)^2}\ \left(\mathbf{Y}^\dagger_d\right)_{jj}(\mathbf{Y}_d)_{jj}\,|\mu|^2\ I_3\left(M^2_{\tilde{d}^j_R}, M^2_{\tilde{d}^j_L}, |\mu|^2\right)
\end{aligned}
$$

$$
\begin{aligned}
& + \frac{(\mathbf{Y}_u)_{ii}}{(4\pi)^2} \left(\mathbf{Y}_d^{\dagger}\right)_{jj} (\mathbf{Y}_d)_{jj} \, |\mu|^2 M_{\tilde{D}}^2 \; I_4 \left(M^2_{\tilde{d}_R^j}, M^2_{\tilde{d}_L^j}, M^2_{\tilde{d}_L^i}, |\mu|^2\right) \left(\delta^d_{ij}\right)_{LL} \\
& + \frac{(\mathbf{Y}_u)_{ii}}{(4\pi)^2} \left(\mathbf{Y}_d^{\dagger}\right)_{ii} (\mathbf{Y}_d)_{jj} \, |\mu|^2 M_{\tilde{D}}^2 \; I_4 \left(M^2_{\tilde{d}_R^j}, M^2_{\tilde{d}_R^i}, M^2_{\tilde{d}_L^i}, |\mu|^2\right) \left(\delta^d_{ij}\right)_{RR} \\
\Gamma^u_{ij} = & \frac{2\alpha_s}{3\pi} \, (\mathbf{Y}_u)_{ij} \, \mu^\star M_g^\star \; I_3 \left(M^2_{\tilde{u}_L^j}, M^2_{\tilde{u}_R^i}, |M_g|^2\right) \\
& + \frac{2\alpha_s}{3\pi} \, (\mathbf{Y}_u)_{ii} \, \mu^\star M_g^\star M_{\tilde{U}}^2 \; I_4 \left(M^2_{\tilde{u}_L^j}, M^2_{\tilde{u}_L^i}, M^2_{\tilde{u}_R^i}, |M_g|^2\right) \left(\delta^u_{ij}\right)_{LL} \\
& + \frac{2\alpha_s}{3\pi} \, (\mathbf{Y}_u)_{jj} \, \mu^\star M_g^\star M_{\tilde{U}}^2 \; I_4 \left(M^2_{\tilde{u}_L^j}, M^2_{\tilde{u}_R^j}, M^2_{\tilde{u}_R^i}, |M_g|^2\right) \left(\delta^u_{ij}\right)_{RR} \\
& + \frac{(\mathbf{Y}_u)_{ij}}{(4\pi)^2} \left(\mathbf{Y}_d^{A\,\dagger}\right)_{jj} (\mathbf{Y}_d)_{jj} \, \mu^\star \; I_3 \left(M^2_{\tilde{d}_R^j}, M^2_{\tilde{d}_L^i}, |\mu|^2\right) \\
& + \frac{(\mathbf{Y}_u)_{ii}}{(4\pi)^2} \left(\mathbf{Y}_d^{A\,\dagger}\right)_{jj} (\mathbf{Y}_d)_{jj} \, \mu^\star M_{\tilde{D}}^2 \; I_4 \left(M^2_{\tilde{d}_R^j}, M^2_{\tilde{d}_L^j}, M^2_{\tilde{d}_L^i}, |\mu|^2\right) \left(\delta^d_{ij}\right)_{LL} \\
& + \frac{(\mathbf{Y}_u)_{ii}}{(4\pi)^2} \left(\mathbf{Y}_d^{A\,\dagger}\right)_{ii} (\mathbf{Y}_d)_{jj} \, \mu^\star M_{\tilde{D}}^2 \; I_4 \left(M^2_{\tilde{d}_R^j}, M^2_{\tilde{d}_R^i}, M^2_{\tilde{d}_L^i}, |\mu|^2\right) \left(\delta^d_{ij}\right)_{RR} \quad \text{(D.4)}
\end{aligned}
$$