THE HIGGS SECTOR OF THE MSSM WITH GENERAL SOFT BREAKING TERMS

A Thesis Submitted to
the Graduate School of Engineering and Sciences of
İzmir Institute of Technology
in Partial Fulfillment of the Requirements for the Degree of

MASTER OF SCIENCE

in Physics

by
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January 2008
İZMİR

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ACKNOWLEDGEMENTS

During my thesis work, several people deserve to be thanked for their help and support. First of all, I would like to express my gratitude to my supervisor Prof. Dr. Durmuş Ali DEMİR who has believed and supported me since the beginning of this thesis. He is always an excellent role model for me to be a good scientist with his patience and comprehensive knowledge. I am grateful for him to give the chance to me to study with him.

I would like to thank Assoc. Prof. Dr. Levent SOLMAZ for his contribution to various stages of this thesis and helping me improve my knowledge about supersymmetry.

I am thankful to my friends and my colleagues in Izmir Institute of Technology Physics Department for their friendly and encouraging approach to me.

Finally, I would like to express my gratitude and love to my family which has always supported and encouraged me.

This work was partially supported by the Turkish Academy of Sciences (TÜBA through the GEBIP grant (through D. A. Demir).

ABSTRACT

THE HIGGS SECTOR OF THE MSSM WITH GENERAL SOFT BREAKING TERMS

This thesis is basically established upon an analysis of determining the effects of the non-holomorphic soft supersymmetry breaking terms on the mass spectrum of the minimal supersymmetric Standard Model (MSSM). We give an overview concerning the fundamental concepts of supersymmetry (SUSY) after providing a brief discussion about the problems of Standard Model (SM) of particle physics. Then we discuss in detail that SUSY is not the exact symmetry of nature and it must be broken. In general, the breakdown of the global supersymmetry is parameterized by a set of operators. These operators could be both holomorphic or non-holomorphic in structure. Hence, in theories like the (MSSM) that do not contain any gauge singlets, these non-holomorphic supersymmetry breaking terms are soft and do not pose any problem for gauge hierarchy.

In this thesis, in particular we study the impacts of the non-holomorphic soft-breaking terms on the Higgs sector of the MSSM. We analyze Higgs sector in conjunction with the chargino sector so as to single out the effects of non-holomorphic trilinear couplings from the μ parameters. Since the aforementioned sectors are two of the prime concerns of experiments at the LHC, we expect that our results will be testable in near future.

ÖZET

GENEL YUMUŞAK KIRICI TERİMLERE SAHİP MSSM’DE HIGGS SEKTÖRÜ

Bu tez, holomorf olmayan yumuşak kırıcı terimlerin Minimal Süpersimetrik Standard Model’in kütle spektrumlarındaki etkilerini gösteren analize dayanmaktadır. Parçacık Fiziği’nin Standart Modeli’ndeki sorunlardan kısaca bahsettikten sonra, süpersimetrinin (SUSY) temel özeliklerini içeren bir özet verdik. Ardından, neden süpersimetrinin evrenin tam bir simetrisi olamayacağını ve kırılmış olması gerektiğini ayrıntılı bir şekilde tartıştık. Genel olarak, global süpersimetri kırılması bir grup operatörle parametrize edilir. Bu operatörler hem holomorf hem de holomorf olmayan yapılarda olabilirler. Dahası, MSSM gibi ayar singletleri içermeyen teorilerde bahsi geçen holomorf olmayan süpersimetri kırıcı terimler yumuşak olmalıdır ve herhangi bir ayar hiyerarşisine yol açmamaları gereklidir.

Bu tez çalışmasında, yumuşak kırıcı terimlerin MSSM’deki Higgs sektörü üzerindeki etkilerini inceledik. Holomorf olmayan trilinear kuplajların μ parametrelerinden yola çıkarak, Higgs sektörünü chargino sektörü ile birlikte analiz ettik. Bu sekörler LHC’ deki deneylerde üzerinde durulan temel iki sektör olduklarından, sonuçlarımızın gelecek zamanlarda test edilebileceklerini düşünüyoruz.

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CHAPTER 1

INTRODUCTION

Over the second half of the twentieth century, the Standard Model (SM) of particle physics has opened a new era in understanding of the elementary particles and explanation of three of four fundamental interactions of Nature the so–called electromagnetic, weak and strong interactions. The Standard Model incorporates these three fundamental interactions that must obey certain gauge symmetries. Thus, the SM basically depends on a certain gauge principle according to which all the forces of Nature are mediated by the exchange of the gauge fields of the corresponding local gauge symmetry group. Hence the gauge group of the Standard Model is represented as $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ where $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ stand for strong, weak and electromagnetic interactions respectively.

In the Standard Model, this gauge symmetry forbids particles to gain their mass terms. That is why, we need to break the symmetry and allows particles to have their masses. Hence, in the SM, the origin of both gauge and fermion masses is explained with the help of the electroweak symmetry breaking (EWSB). This spontaneous symmetry breaking is implemented by means of the Higgs mechanism. According to Higgs mechanism, all particles have their masses depending on their Yukawa couplings by interacting with Higgs field, whereas the Higgs potential describes the self interactions of H. Thus, the particle responsible for this Higgs mechanism (Higgs 1964), is called the Higgs boson. Although this mechanism is a very elegant theory, the Higgs boson has not been observed yet at any high energy experiments and it remains as the most important motivation for construction of the future colliders. In Chapter 2, we will examine the properties of Higgs particle and we will point out the serious problem about the radiative correction to the Higgs boson mass in detail.

Although, the SM has also been confirmed in numerous high energy experiments with extreme good precision in the past decades, it can not provide any explanation concerning the unification of fundamental forces including gravity , the hierarchy problem between the electroweak and gravity scale, cold dark matter and so on. In Chapter 2, we will give a brief overview about these unanswered problems and we focus on the hierarchy problem that is the main inspiration of the new physics beyond the Standard Model. Then, we will introduce an elegant solution as a further symmetry for stabilizing the dangerously large radiative corrections to Higgs mass. This new symmetry is called as supersymmetry (SUSY) whose basic concepts will be discussed in detail in Chapter 3.

Supersymmetry (SUSY) is a space–time symmetry that relates fermions to bosons by means of the supersymmetric transformation. It provides for fermions and bosons to be represented in a single representation so–called superfield (Wess, et al. 1969). However, if supersymmetry were an exact symmetry of Nature, each fermion and each boson would have a superpartner with the same mass and the same quantum numbers except their spins. However, there is no experimental evidence to prove these kind of degeneracy in masses, it is concluded that the supersymmetry must be broken at low energies. In the Chapter 2, we will give the reasons why supersymmetry must be broken in a safe way not regenerating the hierarchy problem as well as we will therein introduce the minimal supersymmetric extension of the Standard Model (MSSM) by giving its particle spectrum, gauge structure, superpotential.

In general, the breakdown of global supersymmetry is parameterized by a set of operators with dimensionality less than four. Each operator thus comes with an associated mass scale, which must fall in the TeV domain if supersymmetry is the correct description of Nature beyond Fermi energies. In Chapter 4, we will discuss in detail that the operators which break the supersymmetry must be soft $i.e.$ quadratic divergences must not be regenerated. These soft breaking terms include some trilinear contributions which are usually a replica of the superpotential with superfields being replace by their scalar components. Hence, the mass terms for scalars as well as their trilinear couplings are soft operators (Chung, et al. 2003).

However, the most general list of supersymmetry breaking operators involve novel structures beyond these aforementioned holomorphic trilinear symmetry breaking terms which are gauge invariant and do not consist of any conjugated fields (Girardello and Grisaru 1982). Indeed, trilinear couplings, for example, can have both $A\phi\phi\phi$ type holomorphic structure as well as $A'\phi^{\star}\phi\phi$ type non-holomorphic structure. There is nothing wrong in considering the non-holomorphic structures since they are perfectly soft if there are no gauge singlets in the theory like the MSSM (Girardello and Grisaru 1982). In this sense, MSSM provides a perfect arena for analyzing the important consequences of the non-holomorphic soft-breaking terms as will be discussed in Chapter 4.

Furthermore, the last section of Chapter 4 is devoted to analyse the impact of the non–holomorphic soft–breaking terms on the Higgs sector of the MSSM. In this sence, we will analyze Higgs sector in conjunction with the chargino sector so as to single out the effects of non-holomorphic trilinear couplings from the μ parameters. It is clear that an independent knowledge of μ can be obtained from chargino sector via certain observable called as $b \to s\,\gamma$ whose branching ratio is expected to place rather strict on the sparticle contribution. The consequence of this restriction, it provides a unique way of determining the allowed range of non-holomorphic trilinear coupling. Since, Higgs and chargino sectors are also two of the prime concerns of experiments at the LHC, we expect that our results will be testable in near future.

In the last Chapter, we will conclude the thesis with the discussion of the result of our analysis as well as implying the impact of non-holomorphic terms on various observables.

CHAPTER 2

PROBLEMS IN THE STANDARD MODEL (SM)

Throughout the history, some crucial questions asked to understand the structure of universe have been of significance for scientists. *e.g.* : What is the fundamental structure of matter forming the universe? What are the fundamental particles and how do they interact with each other? All these questions point to a theory of particle physics is called as "Standard Model ". The Standard Model **(SM)** (Salam 1967, Glashow 1961, Weinberg 1967) gives an elegant and successful description for explaining the strong, weak and electromagnetic interactions of the fundamental particles.These interactions can be represented in terms of unitary gauge groups, so the gauge group of the SM is

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \tag{2.1}$$

in which, $SU(3)_c$ stands for the strong interactions, $SU(2)_L$ for weak interactions and $U(1)_Y$ for electromagnetic interactions. Each gauge group possesses a number of gauge bosons according to their number of generators which are,

$$\begin{aligned} 8\, gauge\, bosons\, (gluons;\, G^a_\mu) \quad &\longrightarrow \quad SU(3)_c \\ 3\, gauge\, bosons\, (W^\mp, Z^0) \quad &\longrightarrow \quad SU(2)_L \\ 1\, gauge\, boson\, (\gamma) \quad &\longrightarrow \quad U(1)_Y \end{aligned} \tag{2.2}$$

The main properties of the vector gauge bosons are as follows. The gluons G^a_μ are electrically neutral and carry color quantum number. The consequence of being colorful characteristic of the gluons, they interact not only with quarks but also with themselves. The photon is electrically neutral and non-self interacting spin-1 boson. The intermediate vector bosons of the strong and electromagnetic interactions are massless. However, because of the very short range of the weak force, the self-interacting gauge bosons of the weak interactions must be very heavy. That's why, we need an explanation concerning why gluons and photons stay massless while the weak gauge bosons gain their masses. Due to the fact that the symmetry which is responsible for weak interactions must be broken, associated gauge bosons $W^{\mp}, Z^0$ gain their mass terms. Thus, the SM comes up with a successful method which gives masses to both fermions and gauge bosons without violating the gauge invariance. This method is called the *Electroweak Symmetry Breaking* (EWSB) or the *Spontaneous Symmetry Breaking* (SSB) which can be demonstrated with gauge symmetry groups;

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \longrightarrow SU(3)_C \otimes U(1)_{em} \tag{2.3}$$

The spontaneous symmetry breaking is implemented by means of a mechanism, the so-called "Higgs Mechanism " (Higgs 1964). This mechanism establishes on adding a new extra $SU(2)$ doublet

$$\Phi \equiv \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \tag{2.4}$$

is such that the neutral component of the Higgs field H^0 acquires a non-zero vacuum expectation value $v \sim \langle \Phi \rangle$. Every fermion gains its mass which is determined bye the strength of its coupling (Yukawa coupling) to the Higgs field. Due to the different strength couplings, all fermions have different masses. The successful consequences of the Higgs mechanism, while $W^{\mp}$ and Z^0 get their mass terms according to their couplings to the Higgs field both the photon and gluons remain massless because they have no couplings to the Higgs field. ($SU(3)_C \otimes U(1)_{em}$ is protected.) Thus, the outcome of the self interaction of the Higgs field, the Higgs mechanism also provides a new particle so–called the **Higgs boson** about which we have not had any evidence to prove its existence in the high energy experiments up to now.

Even though the Standard Model (SM) has an impressive theoretical framework to explain the basic constituents of matter (leptons and quarks) and their interactions (strong, weak, electromagnetic) being quite precise agreement with experimental data, it is not a full description of nature on account of the some unsolved problems in the theory. These problems can be listed as follows:

- **Hierarchy Problem**: The hierarchy problem in the SM is a significant destabilization issue on the Higgs mass. In other words, when the radiative corrections to Higgs mass are taken into account, it is easily recognized that Higgs mass is quadratically divergent (high dependence on the ultraviolet cutoff scale $\Lambda \sim 10^{19} GeV$). It is implied that tree level Higgs mass is in $10^{2}\,(GeV)$ order while the quantum corrections are in $10^{19}\,(GeV)$.

- **Electroweak Symmetry Breaking**: Although the electroweak symmetry breaking gives an answer how the elementary particles gain their masses by interacting with the Higgs field, the Higgs sector is not constrained by any symmetry. It is just put into theory by hand for satisfying symmetry breaking and it is not clear whether it is fundamental or not. Another issue about symmetry breaking is that the scalar particle, Higgs boson, which is required by the theory has not been observed yet.

- **Matter-Antimatter Asymmetry**: The SM does not give any information concerning the fact that why the universe is made of matter instead of antimatter or both. More seriously, although the fundamental equations demonstrate the equivalence the amounts of matter and antimatter, we can't observe antimatter as much as matter. The SM also can not give any reasonable answer this asymmetry between matter and antimatter.

- **Gauge Coupling Unification**: The idea of the gauge coupling unification is based on that all symmetries have the same gauge coupling at the high energies ($\Lambda_{GUT} \simeq 10^{15} - 10^{16}\,GeV$) and they diversify at the low energies according to the renormalization group evolution. The gauge unification is the basic motivation of the gauge unification theory (GUT) and the string theories which attempt to incorporate all fundamental interactions including gravity. However the experimental results of the values of the low energy gauge couplings show that the SM can not unify the gauge couplings accurately.

Some other unanswered questions can be added to this list *e.g.*: cold dark matter problem, cosmological constant problem, neutrino mass problem. Furthermore, the SM gives no information concerning gravitational interaction. All these serious issues need to clarify to construct a fundamental theory of the universe. Especially the hierarchy problem is the main inspiration to believe that the standard model must be a low energy limit of an extended fundamental theory giving solution all these mentioned problems properly. That's why, it is useful to discuss the hierarchy problem in detail to motivate the new physics beyond the standard model.

2.1. Hierarchy Problem

As it has been discussed in the previous section, the SM is not a full story of Nature because it includes several theoretical shortcomings. The ultraviolet sensitivity of the higgs mass, *known as hierarchy problem,* can be given an useful example for these conceptual problems. If one tries to calculate to radiative correction to Higgs mass, resulting from its self couplings, Yukawa couplings to fermions and its coupling to gauge bosons, bring about a quadratic dependency to the ultraviolet cutoff scale (Λ_{UV}). This quadratic divergence problem are present only in the scalar Higgs sector in the SM because the mass of Higgs is not protected by any symmetry while fermions (gauge bosons) are protected by chiral (gauge) symmetry and quantum corrections to their masses are only logarithmically dependent on the cutoff scale (Λ_{UV}) .

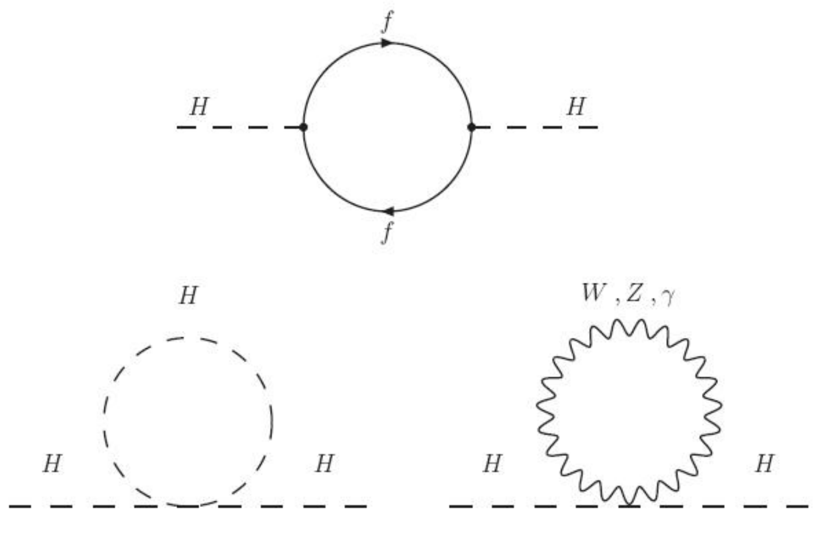

Figure 2.1. The figure in top row corresponds to the Yukawa interaction contribution, the first figure in the second row is the scalar self-interaction contribution, the other corresponds to the gauge interaction contributions to the quadratic divergence.

$$\delta_f \, m_H^2 \sim -\frac{\lambda_f^2}{16\,\pi^2}\Lambda_{UV}^2$$

$$\delta_g \, m_H^2 \sim \frac{g^2}{16\,\pi^2}\Lambda_{UV}^2$$

$$\delta_H \, m_H^2 \sim \frac{\lambda}{16\,\pi^2}\Lambda_{UV}^2 \tag{2.5}$$

All loop contributions to Higgs mass shown in (2.5), where λ_f is the Yukawa couplings λ is the quadratic higgs coupling and g the gauge couplings, have the ultraviolet sensitivity at the high energies. In other words, the mass squared of higgs is expected to be of order $(100\,GeV)^2$ which is the energy scale of the electroweak symmetry breaking (EWSB). However, the radiative corrections to the higgs mass m_H^2 are of order $(\Lambda_{UV})^2$ which can be chosen $(\Lambda_{UV}) \sim M_{Planck} \simeq 10^{19}\,GeV$. The consequences of the contributions of fermion, gauge and higgs loops, the quantum corrections to m_H^2 can be nearly thirty order of magnitude greater than m_H^2 itself. ($\delta\, m_H^2 \gg m_H^2$)

It is obvious that we need new physics theories beyond the standard model in order to tame the higgs mass. These attractive theories come up recent decades which are called " Large Extra dimensions (Arkani, et al. 1998, Antoniadis, et al. 1998), Technicolor (Hill and Simmons 2003) and Supersymmetry (SUSY) (Wess, et al. 1969, Drees 1996, Martin 1997)". We prefer to analyze this stabilization problem under the technic of supersymmetry which introduces a new symmetry to stabilize the Higss mass. In this symmetry, we couple fermion (boson) loop contribution by introducing a new boson (fermion) contribution with same Yukawa (gauge) coupling such that by means of the spin-statistic theorem, the sign of standard fermion loop is opposite to that of boson loop and these contributions cancel exactly each other if they have the same masses.

As it seen in figure (2.2), we design new boson (a partner) for fermion, a new partner for gauge boson called "*gaugino* ". If partners have the same mass as well as same quantum numbers except their spins, the loop contributions then vanish identically. If they do not, in other words, the supersymmetry is broken, then the δm_H^2 is proportional to the mass-squared difference of partners *i.e*: for fermion and its partner contribution is proportional to

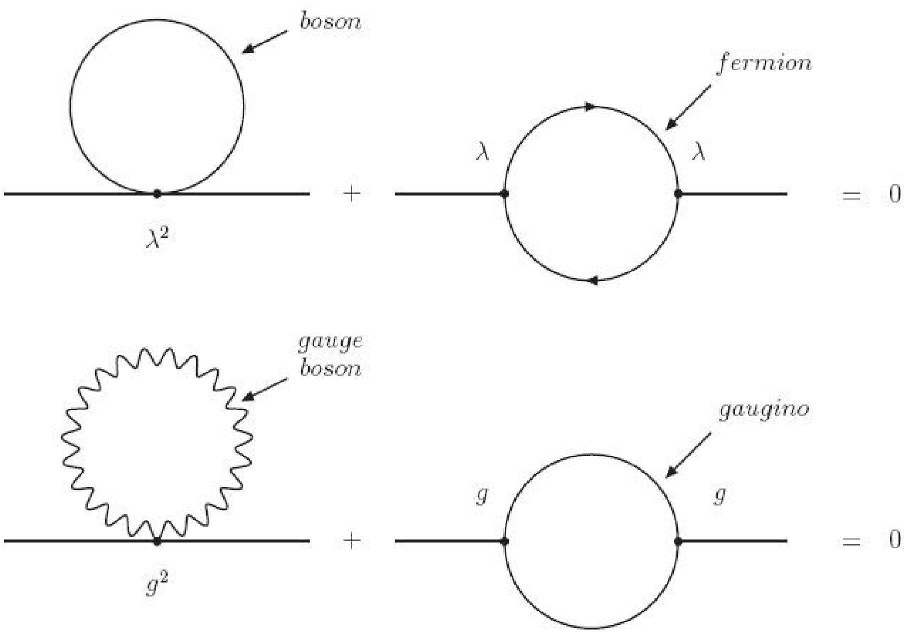

Figure 2.2. Cancellation of the quadratic divergence caused by fermion and boson loops with same couplings to Higgs mass.

$$\delta_f m_h^2 \propto \mid m_f^2 - m_b^2 \mid \tag{2.6}$$

m_f is the mass of the fermion while m_b stand for the related boson mass. It is crucially important to mention that, the cancellation of quadratic divergence remains intact as long as the new particles coming from SUSY or known as *superpartners* appear at an energy scale not too far above TeV scale. Otherwise, huge mass difference between particles and their superpartners would regenerate the hierarchy problem. Thus, if the supersymmetry existed as a exact symmetry of nature then the Standard Model particles and their superpartners would be have the same masses, same quantum numbers except their spins. However, We haven't observed any supersymmetric partners of the particles, that's why SUSY must, of course, be a broken symmetry and the masses of supersymmetric particles differ with those of their SM particles.

Supersymmetry (SUSY) is the one of the most accepted theory beyond the Standard Model and is thought to give an answer not only hierarchy problem (as discussed above) but several problems in SM as well. These important topics explained by SUSY are:

- The gauge coupling unification (de Boer, et al. 1991)
- The explanation of the baryon asymmetry of the universe
- Cold dark matter
- The description how the electroweak symmetry is broken.

Together all these successful predictions, SUSY is believed to be a part of the correct description of the universe. In the next chapter we will discuss the structure and algebra of this elegant symmetry, determine the properties of the theory. Then, we will introduce the minimal extension of supersymmetry and its particle content.

CHAPTER 3

SUPERSYMMETRY

3.1. General Feature of the Supersymmetry

3.1.1. SUSY Algebra

The supersymmetry (SUSY) is a space-time symmetry which relates particles of integer spin with those of half-integer spin. In other words, supersymmetry gives a connection between fermions and bosons. For this aim, SUSY requires a transformation known as *supersymmetric transformation* which turns a bosonic state into a fermionic state and vice versa. An operator *Q* is represented as a generator of the supersymmetric transformation and it must be a form like

$$Q|Boson >= |Fermion >$$

$$Q|Fermion >= |Boson > \tag{3.1}$$

The possible forms of such symmetries are strictly forbidden by the Coleman-Mandula theorem (Coleman and Mandula 1967) with using tensor charges. (it means that there is no other charge with non-trivial transformation properties under $Poinc\acute{a}re$ transformations. More clearly, no non-trivial combination of external (Lorentz transformations) and internal (such as flavor SU(2) or SU(3)) symmetries can be achieved by using just bosonic charges). One can find useful example about the Coleman-Mandula Theorem in Ellis (2002) and Aitchison (2005). However the Coleman-Mandula theorem gives no restriction for using spinor charges which carry spin angular momentum–$1/2$.

For constructing a consistent algebraic scheme, it is necessary to combine the spinorial charge Q_α with the energy momentum operator P^μ and the angular momentum operator $M^{\mu\nu}$ (where P^μ is the momentum generator of space time translation and $M^{\mu\nu}$ is the generator of the Lorentz transformation).

The spinoral charge is a symmetry operator so, it must commute with the Hamiltonian (the temporal component of the P^{μ}) of the system (Golfand and Likthman 1971);

$$[Q_{\alpha}\,, H] = 0 \tag{3.2}$$

It emphasizes that the SM particles and their partners have same masses and the anticommutator of two different components must be

$$[\{Q_{\alpha}\,, Q_{\beta}\}\,, H] = 0 \tag{3.3}$$

The relation (3.3) guarantees that the anticommutation relation of the charges must be proportional to the energy momentum four vector P^{μ} (Aitchoson 2005) because all components of P^{μ} must commute with each other. ($[P^{\mu}\,, P^{\rho}] = 0$)

$$\{Q_{\alpha}\,, Q_{\beta}\} \propto\; P^{\mu} \tag{3.4}$$

The basic commutation and anticommutation relations among P^{μ}, $M^{\mu\nu}$ and Q_{α} can be settled with enlarging Poincaré algebra (Kazakov 2000)

$$\{Q^{i}_{\alpha}\,, \bar{Q}^{j}_{\dot{\beta}}\} = 2\,\delta^{ij}\,(\sigma^{\mu})_{\alpha\dot{\beta}}\,P_{\mu} \tag{3.5}$$

$$\left[P_{\mu}\,, Q^{i}_{\alpha}\right] = [P_{\mu}\,, \bar{Q}^{i}_{\dot{\alpha}}] = 0 \tag{3.6}$$

$$\{Q_{\alpha}\,, Q_{\beta}\} = \{\bar{Q}_{\dot{\alpha}}\,, \bar{Q}_{\dot{\beta}}\} = 0 \tag{3.7}$$

$$\left[Q^{i}_{\alpha}\,, M^{\mu\nu}\right] = \frac{1}{2}\,(\sigma^{\mu\nu})^{\beta}_{\alpha}\,Q^{i}_{\beta} \tag{3.8}$$

$$\left[\bar{Q}^{i}_{\dot{\alpha}}\,, M^{\mu\nu}\right] = -\frac{1}{2}\,\bar{Q}^{i}_{\dot{\beta}}\,(\bar{\sigma}^{\mu\nu})^{\dot{\beta}}_{\dot{\alpha}} \tag{3.9}$$

$$[P^{\mu}\,M^{\sigma\rho}] = i\,(g^{\mu\sigma}\,P^{\rho} - g^{\mu\rho}\,P^{\sigma}) \tag{3.10}$$

$$i\,, j = 1, 2, ..., N \quad ; \alpha\,, \dot{\alpha}\,, \beta\,, \dot{\beta} = 1, 2$$

with using the relations $(4\sigma^{\mu\nu} = i(\sigma^{\mu}\,\bar{\sigma}^{\nu} - \sigma^{\nu}\,\bar{\sigma}^{\mu}))$ and introducing $\sigma^{\mu} = (1, \vec{\sigma})$ and $\bar{\sigma}^{\mu} = (1, -\vec{\sigma})$ where $\vec{\sigma}$ represents the Pauli matrices (see Appendix A), $\alpha\,, \dot{\alpha}\,, \beta\,, \dot{\beta}$ are the spinorial indices, $\mu\,, \nu = 0, .., 3$ are spacetime indices. The algebra (3.5) is called "SuperPoincar$\acute{e}$ algebra ". This SUSY algebra is only possible way to mix integer and half-integer spins and changes statistics. Thus, it is the only non-trivial extension of the set of the spacetime symmetries. This means that it is consistent with the symmetries of the S-Matrix. (Haag, et al. 1975). The simplest case is $i = j = 1$ which is called "$N = 1$ SUSY " corresponds one spinor generator Q_{α} and its conjugate $\bar{Q}_{\dot{\alpha}}$. In this thesis, we consider only unextended $N = 1$ supersymmetry (with minimal particle content). It is useful to mention that with increasing N, the theory also must contain particles with spin greater than 1. This theories are not renormalizable, thus the theories consisting of particles with spin greater than $5/2$ do not have consistent coupling to gravity.

$$
\begin{aligned}
N &\leq 4 \quad \textit{for renormalizable theories} \\
N &\leq 8 \quad \textit{for (super) gravity}
\end{aligned}
$$

In $N = 1$ (global) supersymmetry all particle states fall into irreducible representations of supersymmetry algebra so-called "*supermultiplets*". Each supermultiplet contains both fermionic and bosonic states which are called as "*superpartners*" of each other. As mentioned before, the Equation 3.2 emphasizes that the particles which occupy the same irreducible supermultiplets (particles and their superpartners) must have equal masses and have the same representation of the gauge group, so they must have same color, electric charge, weak isospin degrees of freedom.

Another aspect of the supermultiplets is that the fermionic degrees of freedom and the bosonic degrees of freedom in the same supermultiplet must be equal. (Martin 1997)

$$n_F = n_B \tag{3.11}$$

where n_F and n_B represent fermionic and bosonic degrees of freedom respectively.

In the next section, we will state the minimal extension of the Standard Model which is called as the Minimal Supersymmetric Standard Model (MSSM) and its particle content as well as the Lagrangian representing these particle interactions.

3.2. The Minimal Supersymmetric Standard Model (MSSM)

The simplest supersymmetric extension of the Standard Model is the so-called Minimal Supersymmetric Standard Model (MSMM) which contains minimal number of superpartners and interactions. In the supersymmetric extension of the Standard Model, every known fundamental particle falls into either chiral or gauge (vector) supermultiplet representation with the associated superpartner. It is very instructive to discuss the properties of these supermultiplets in order to construct the supersymmetric model properly.

- **Chiral (Matter) Supermultiplets** : It is the supermultiplet which is nothing but the combination of a two component Weyl fermion, a complex scalar field. Chiral supermultipltes classifies fermions whose left-handed parts transform differently under the gauge groups than the right-handed parts, Higgs bosons and their bosonic superpartners.

- **Gauge (Vector) Supermultiplets**: The vector bosons (spin-1) of the Standard Model and their fermionic superpartners (spin-$\frac{1}{2}$) are placed in gauge (vector) supermultiplets which have equal fermionic and bosonic degrees of freedom.

3.2.1. The Particle Content of the MSSM

The supermultiplets whether chiral or gauge, consist of ordinary particles and their superpartners with spin differing by $1/2$ unit. The superpartners (spin-0) of the SM fermions (quarks and leptons) are constructed by adding “s” which stands for “scalar”. Thus they are generically called *squarks* and *sleptons* represented with the same symbols with their SM particles but with a tilde using to denote the superpartners of the Standard Model particles. Due to the chirality (the left-handed or right-handed) of the fermions, their superpartners have different representation (as seen in Table 3.1) *i.e*: left-handed selectron $\tilde{e}_L$ and right-handed selectron $\tilde{e}_R$. It is important to keep in mind that the handedness of the superparticles do not refer to the helicity of them but to that of their SM fermions. Moreover in the SM neutrinos are always left-handed, so the superpartners of the neutrinos (sneutrinos) mus be left-handed denoted as $\tilde{\nu}$.

Table 3.1. Chiral Supermultiplets of the MSSM

Superfield	Bosons (spin 0)	Fermions (spin 1/2)	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\widehat{Q}$	$(\tilde{u}_L \ \tilde{d}_L)$	$(u_L \ d_L)$	$\mathbf{3}$	$\mathbf{2}$	$\mathbf{1/6}$
$\widehat{U}$	$\tilde{u}_R$	u_R^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{-2/3}$
$\widehat{D}$	$\tilde{d}_R$	d_R^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1/3}$
$\widehat{L}$	$(\tilde{\nu} \ \tilde{e}_L)$	$(\nu \ e_L)$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{-1/2}$
$\widehat{E}$	$\tilde{e}_R$	e_R^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\widehat{H}_u$	$(H_u^+ \ H_u^0)$	$(\tilde{H}_u^+ \ \tilde{H}_u^0)$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1/2}$
$\widehat{H}_d$	$(H_d^0 \ H_d^-)$	$(\tilde{H}_d^0 \ \tilde{H}_d^-)$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{-1/2}$

One important feature of Table (3.1) deserves clear explanation. In contrast to the SM, the MSSM requires two Higgs doublets $H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$ and $H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$ with opposite hypercharge ($Y = \frac{1}{2}$ and $Y = -\frac{1}{2}$ respectively). The decisive reasons why we need two Higgs doublets in the SUSY theories can be given as follows:(Dawson 1996 and Kazakov 2000)

- Due to the fact that Higgs is a scalar particle, it can only reside in a chiral supermultiplet with a fermionic superpartner the so-called *Higgsino* which would have hypercharge either $Y = \frac{1}{2}$ or $Y = -\frac{1}{2}$. It is important to mention that all the Standard Model fermions, with the third components of their weak isospins T_3, uphold a delicate balance where the traces $Tr(Y^3)$ and $Tr(T_3T_3Y)$ are both zero when all left-handed fermions are taken consideration. For instance,

$$\begin{array}{cccccccccccc} Tr(Y^3) & = & 3 & \big(\frac{1}{27} & +\frac{1}{27} & -\frac{64}{27} & +\frac{8}{27}\big) & -1 & -1 & +8 & = 0. \\ & & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \\ & & colour & u_L & d_L & u_R & d_R & \nu_L & e_L & e_R & \end{array}$$

This elegant feature protects the theory from anomalies (Martin 1997). However, introducing a new particle with non-zero hypercharge (Higgsino) destroys these relations (these traces have non-zero values) and gives rise to triangle $SU(2)_L$ and $U(1)$ gauge anomalies which would lead to unphysical divergences of the theory. If, however, two Higgs doublets are introduced with opposite hypercharge, the above-mentioned cancelations are reestablished. That's why, we need to add a new Higgs doublet to spoil this anomaly problem and make the SUSY theories sensible.

- Two Higgs doublets are also necessary to provide the mass terms of both down and up-type quarks. H_u whose hypercharge is $Y = \frac{1}{2}$ gives mass to up-type quarks with electric charge $\frac{2}{3}$ when H_d with $Y = -\frac{1}{2}$ gives mass to down-type quarks with electric charge $-\frac{1}{3}$.

The vector bosons of the SM are accomodated in gauge supermultiplets with their fermionic superpartners which are called as *gauginos* (shown in table (3.2)).

Table 3.2. Gauge Supermultiplets of the MSSM

Superfield	Bosons (spin 1)	Fermions (spin 1/2)	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\widehat{G^a}$	g^a	$\tilde{g}^a$	**8**	**1**	**0**
$\widehat{W}$	$W^{\mp}$	$\tilde{W}^{\mp}$	**1**	**3**	**0**
$\widehat{B}$	B^0	$\tilde{B}^0$	**1**	**1**	**0**

The mediator of the $SU(3)$ color gauge interactions is gluon g^a whose color-octet superpartner the *gluino* $\tilde{g}^a$. Thus, the electroweak symmetry $SU(2)_L \otimes U(1)_Y$ has spin-1 gauge bosons W^+, W^-, W^0 and B^0 while their spin-$\frac{1}{2}$ superpartners are $\tilde{W}^+$, $\tilde{W}^-$, $\tilde{W}^0$ and $\tilde{B}^0$ called *winos* and *bino*. After the electroweak symmetry breaking, the eigenstates of W^0 and B^0 mix to give mass to Z^0 and γ whose superpartners $\tilde{Z}^0$ and $\tilde{\gamma}$ the so-called *zino* and *photino* respectively. Furthermore, the spin–$\frac{1}{2}$ superpartners of the Higgs bosons, the higgsinos, will mix with winos and the bino to give mass eigenstates: 2 charginos $\chi^{\pm}_{1,2}$ and 4 neutralinos χ^0_i with $i = 1, 2, 3, 4$.

3.3. The Lagrangian of the MSSM

The MSSM lagrangian can be considered as two different fundamental parts which are given as

$$L = L_{SUSY} + L_{Soft}. \quad (3.12)$$

L_{Soft} is the soft breaking lagrangian term which will be discussed in detail in the next chapter, needs for breaking supersymmetry and giving mass terms to the supersymmetric particles. First term of L is considered as the supersymmetric Lagrangian denoted as L_{SUSY} which consists of the gauge invariant kinetic terms corresponding to the SU$(3)_C$, SU$(2)_L$ and U$(1)_Y$ gauge groups, the same gauge interactions terms as the SM, the Yukawa interaction terms and the scalar potential are derived from the *superpotential* which is an analytic function containing terms with just 2 and 3 chiral superfields. The superpotential must not contain terms with more than 3 chiral superfields because these kind of terms would yield non-renormalizable interaction in the lagrangian. It is important to mention that SUSY does not allow to superpotential to consist of the complex conjugates of the chiral superfields. The most general form of the superpotential is given as explicitly:

$$\widehat{W}_{MSSM} = -\mu \widehat{H}_d \cdot \widehat{H}_u + \widehat{Q} \cdot \widehat{H}_u \mathbf{Y_u} \widehat{U} + \widehat{H}_d \cdot \widehat{Q} \mathbf{Y_d} \widehat{D} + \widehat{H}_d \cdot \widehat{L} \mathbf{Y_e} \widehat{E} \quad (3.13)$$

where the gauge and family indices have been suppressed and $\mathbf{Y_u}$, $\mathbf{Y_d}$ and $\mathbf{Y_e}$ are the Yukawa coupling 3×3 matrices of u-type quarks, down-type quarks and leptons respectively. The first three terms in the superpotential are nothing but a superspace generalization of the Yukawa interaction in the SM. These are necessary for determining the masses and CKM (Cabibbo-Kobayashi-Maskawa) mixing angles of the SM fermions after the neutral component of H_u and H_d get their VEV's (vacuum expectation value). The μ term in the superpotential is just the supersymmetric version of the Higgs mass in the Standard Model. The dot "**.**" notation corresponds to, for instance,

$\widehat{Q} \cdot \widehat{H}_u \equiv \widehat{Q}^T\,(i\sigma_2)\,\widehat{H}_u = \epsilon_{ij}\widehat{Q}^i\widehat{H}_u^j$ with $\epsilon_{12} = -\epsilon_{21} = 1$. Since the superpotential must be holomorphic, the $(\widehat{Q} \cdot \widehat{H}_u\mathbf{Y_u}\widehat{U})$ Yukawa terms can not be replaced by something like $(\widehat{Q} \cdot \widehat{H}_d^*\mathbf{Y_u}\widehat{U})$ and $(\widehat{H}_d \cdot \widehat{Q}\mathbf{Y_d}\widehat{D})$ terms can not be replaced by $(\widehat{H}_u^* \cdot \widehat{Q}\mathbf{Y_d}\widehat{D})$, because these terms with the complex conjugate of the superfields are forbidden by the structure of the supersymmetry.

In principle the superpotential can consist of other terms like

$$\widehat{W}' = \mu^{'}\widehat{L} \cdot \widehat{H}_d + \widehat{L} \cdot \widehat{Q}\mathbf{Y_L}\widehat{E} + \widehat{L} \cdot \widehat{Q}\mathbf{Y}'_{\mathbf{L}}\widehat{D} + \widehat{U} \cdot \widehat{D}\mathbf{Y_B}\widehat{D} \tag{3.14}$$

which are forbidden in the SM by Lorentz invariance. The first three terms imply the lepton-number (L) violating interactions. The latter is the baryon-number (B) violating interaction in the lagrangian. Since both effects are not observed in nature, these terms must be suppressed or be excluded. Therefore, in the MSSM, one imposes a new discrete and multiplicative symmetry so–called **R-Parity** which enforces the baryon and lepton number conservation in the superpotential. For each particle R-Parity is defined as

$$P_R = (-1)^{3(B-L)+2S} \tag{3.15}$$

where B and L stand for baryon number and lepton number respectively while S represents the spin of the particle. The R-parity assignment requires that all Standard Model particles and the Higgs bosons have even R-parity ($P_R = +1$) while all supersymmetric particles have odd R-parity ($P_R = -1$). If R-parity is exactly conserved, then the interactions of superpartners are essentially same as in the SM, two of three particles at any vertex are replaced by superpartners. It is also affirmative to mention the extremely important consequences of R-parity conservation:

- There is no mixing between the Standard Model particles and supersymmetric particles.

- Sparticles are created in pairs in particle collisions. In other words, every interaction vertex in the theory contains even number of sparticles. These particles are heavy and highly unstable and decay quickly into lighter states.

- The lightest sparticle called as "lightest supersymmetric particle or **LSP**" must absolutely be stable and must eventually be produced at the end of a decay chain initiated by the decay of a heavy unstable supersymmetric particle. The LSP should be electrically and color neutral and weakly interacting in order to be consistent with cosmological constraints and therefore it can be an attractive candidate for "cold dark matter" (Jungman, et al. 1996), an important component of the non-baryonic dark matter required in several models of cosmology.

We will take into account only the R-parity conserved case for superpotential. However, one can consider R-parity broken case in which either L or B are not conserved. (In R-parity violating case, the LSP is not stable and will decay into the SM particles. So the collider signatures of R-violating case can be very different from the R-parity conserved case.) Moreover, both L and B is broken, then proton would decay very rapidly. In order to avoid this kind of inconsistencies all couplings in (Equation 3.14) are almost zero. In other words, one or the other (or both) of the interactions is assumed absent (see Haber, et al. 1995 for further discussions of the theory where the R-parity broken).

In the next subsection, we will discuss in detail that what kind of interactions the supersymmetric lagrangian of the MSSM contains and how the Yukawa interaction terms and the so-called *F-terms* are derived from the superpotential.

3.3.1. Supersymmetric Part of the MSSM Lagrangian

In order to constitute a gauge invariant SUSY lagrangian, we have to collect all terms not only consisting of the Yukawa and gauge interactions but also interaction terms which are invariant under the supersymmetric transformations. Hence we expect that all scalar and fermion fields must be in the same representation of the gauge group. In renormalization limit, the mass dimension of any term in the lagrangian must be less than or equal to 4. Then the SUSY lagrangian takes form as

$$L_{SUSY} = L_{Kinetic} + L_{Gauge} - L_{Yukawa} - V_F. \tag{3.16}$$

where the lagrangian of the kinetic terms and gauge interactions terms are given respectively,

$$
\begin{aligned}
L_{Kinetic} &= \sum_i (D_\mu \phi_i)^\dagger (D^\mu \phi_i) - \frac{1}{4}\sum_a (F_{\mu\nu})_a \, F_a^{\mu\nu} \\
&+ \frac{i}{2}\sum_i \bar{\psi}_i \, \not{D} \, \psi_i + \frac{i}{2}\sum_a \bar{\lambda}_a \, \not{D} \, \lambda_a
\end{aligned}
\tag{3.17}
$$

$$
\begin{aligned}
L_{Gauge} = & - \sqrt{2}\sum_a g_a \, \phi_i^\dagger \, (T^a)_{ij} \, \bar{\psi}_j \, P_L \bar{\lambda}^a \\
& - \frac{1}{2}\sum_a D^a D_a
\end{aligned}
\tag{3.18}
$$

where D terms are determined as $D^a = \phi_i^\dagger g_a (T^a)_{ij} \phi_j$. "$L_{Gauge}$ " and "$L_{Kinetic}$" represent all interactions of all MSSM particles with gauge bosons and fermions. ϕ_i (ψ_i) is the scalar (Majorana fermion) component of the chiral superfield $\hat{\Psi}$ while λ^a is the Majorana gauge superpartner of the corresponding gauge boson and $F_{\mu\nu}$ is the gauge boson field strength. The derivative D_μ and D^μ are gauge invariant derivatives appropriate to the particle representation which the fields belong and the relation between $\not{D}$ and D_μ is determined as $\not{D} = \gamma^\mu \, D_\mu$ where γ^μ represent the Dirac matrices (Appendix A).

The terms in $L_{Kinetic}$ determine how particles interact with gauge bosons. The latter lagrangian part describes the interactions of gauginos with matter particles and Higgs multiplets where T^a represent the appropriate dimensional matrix representation of the gauge symmetry generators and g_a are the SM gauge couplings. P_L is the one of the helicity operators and it is defined as

$$
P_L = \frac{1-\gamma^5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.
\tag{3.19}
$$

Last two terms of the supersymmetric lagrangian the so-called L_{Yukawa} and V_F are obtained by the superpotential. In order to obtain the interactions come from the superpotential, we take the derivatives of the W with respect to the scalar components of the superfields and, for Yukawa interaction term L_{Yukawa}, we multiply with fermionic part of the two superfield ψ_i and ψ_j for the aim of giving mass terms of the quarks and leptons of the SM.

$$
\begin{aligned}
L_{Yukawa} &= \epsilon_{ij}\left[E\,Y_e\,L^i\,H_d^j + D\,Y_d\,Q^i\,H_d^j + U\,Y_u\,Q^i\,H_u^j + \mu\,\tilde{H}_u^i\,\tilde{H}_d^j\right] \\
&+ \epsilon_{ij}\left[\tilde{E}\,Y_e\,L^i\,\tilde{H}_d^j + \tilde{D}\,Y_d\,Q^i\,\tilde{H}_d^j + \tilde{U}\,Y_u\,Q^i\,\tilde{H}_u^j\right] \\
&+ \epsilon_{ij}\left[E\,Y_e\,\tilde{L}^i\,\tilde{H}_d^j + D\,Y_d\,\tilde{Q}^i\,\tilde{H}_d^j + U\,Y_u\,\tilde{Q}^i\,\tilde{H}_u^j\right] + h.c \\
&= \sum_{i,j}\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\psi_i\psi_j + h.c
\end{aligned}
\tag{3.20}
$$

Finally, the last term in the supersymmetric lagrangian, the V_F term, gives the Higgs masses and describes scalar mass terms and scalar interactions. V_F is introduced by the square of the so-called *F-terms* given as $F_i \equiv \partial W(\phi)/\partial\phi_i$.

$$
V_F = \sum_i \left|\frac{\partial W(\phi)}{\partial\phi_i}\right|^2 = |F_i|^2 \tag{3.21}
$$

Then the total supersymmetric lagrangian is;

$$
\begin{aligned}
L_{SUSY} &= \sum_i (D_\mu\phi_i)^\dagger(D^\mu\phi_i) - \frac{1}{4}\sum_a (F_{\mu\nu})_a\,F_a^{\mu\nu} \\
&+ \frac{i}{2}\sum_i \bar{\psi}_i\,\not{D}\,\psi_i + \frac{i}{2}\sum_a \bar{\lambda}_a\,\not{D}\,\lambda_a \\
&- \sqrt{2}\sum_a g_a\,\phi_i^\dagger\,(T^a)_{ij}\,\bar{\psi}_j\,P_L\bar{\lambda}^a - \frac{1}{2}\sum_a\left[\phi_i^\dagger g_a(T^a)_{ij}\phi_j\right]^2 \\
&- \left[\sum_{i,j}\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\psi_i\psi_j + h.c\right] - |F_i|^2
\end{aligned}
\tag{3.22}
$$

In the next chapter, we will discuss the soft supersymmetry breaking in the MSSM in detail. We will focus on both the holomorphic and non-holomorphic terms in the soft breaking lagrangian so we will have a chance to obtain the whole picture regarding the supersymetry breaking. Thus, we will analyse the importance of the non-holomorphic terms in the Higgs sector in comparison with other sectors are not affected by the general breaking terms in the MSSM.

CHAPTER 4

SOFT SUPERSYMMETRY BREAKING IN THE MSSM

So far, we have considered just the supersymmetry conserving part of the lagrangian. As mentioned in Chapter- 2, in the unbroken supersymmetry, every SM particle is degenerate in mass with its corresponding superpartner comes from SUSY. However, if the particles and their superpartners had had the same masses, they would have already been discovered. Hence, we haven't have any evidence about superpartners, it is concluded that the supersymmetry must be broken at low energies. From the theoretical perspective, it is excepted that SUSY is broken spontaneously analogous to the electroweak symmetry in the SM. However, none of the field in the MSSM can have non-zero expectation value (v.e.v) needed for SUSY breaking without destroying the gauge invariance.

The most common thought is that the supersymmetry breaking occurs in a sector which is called as *hidden sector* and the with the help of the messenger fields (differ depending on the scenario we consider. The most popular ones are gauge-mediated, gravity-mediated, anomaly mediated and gaugino mediated), supersymmetry breaking is mediated to the *visible sector* by flavor blind interactions and the lagrangian terms which are belong to the particles of the MSSM are generated. These effective breaking terms are incorporated with the lagrangian in such a way that they must not spoil the excellent cancelation of the quadratic divergence in the Higgs mass in order not to regenerate the dangerous UV divergences, the so-called hierarchy problem (as discussed in Chapter-2). That's why, the masses of the superpartners differ with those of their SM particles at the scale not too far from the TeV scale. It means that we need to break supersymmetry *softly* to prevent the theory from this kind of divergences.

The part of the lagrangian contains all scale dependent soft breaking terms are generically denoted as L_{Soft} in which all terms are consistent with gauge symmetries of the SM and do not cause any quadratic divergences. The supersymmetry breaking terms are often assumed to be flavor-independent and/or gauge independent at the high energy scale and split as they evolve to low energy scale under the renormalization group equations-RGEs (Falck 1985).

We will firstly deal with the soft breaking terms which do not contain any hermitian conjugates of any scalar fields. Then, we will discuss the additional non-holomorphic terms which respect gauge invariance and R-parity but violate the holomorphicity of the soft breaking lagrangian part.

4.1. The Holomorphic Soft Breaking Terms

Even though we do not know the origin and dynamical mechanism of the supersymmetry breaking, fortunately, it is possible to write the effective soft breaking terms that have mass dimension two or three by means of the restriction of both gauge and Lorentz invariance. The terms of soft SUSY breaking are categorized by Grisaru and Girardello (1982) and listed as follows:

- Soft scalar mass square terms: $m^2_{ij}\, \phi^\dagger_i\, \phi_j$
- Soft gaugino mass terms: $\frac{1}{2} M_a\, \bar{\lambda}^a\, \lambda^a, +h.c$
- Soft bilinear scalar interactions: $b_{ij}\, \phi_i\, \phi_j + h.c$
- Soft *trilinear* scalar interactions terms: $A_{ijk} \phi_i\, \phi_j\, \phi_k + h.c$

Besides, the terms having mass dimension four and more or the terms like ϕ^3, $\bar{\psi}\, \psi$ can not added to the soft breaking lagrangian because they lead the quadratic divergences in the theory (Chung, et al. 2005). Finally, the supersymmetric breaking lagrangian L$_{soft}$ takes the form explicitly,

$$
\begin{aligned}
-\mathcal{L}_{soft} \;=\;& \widetilde{Q}^\dagger \mathbf{m}^2_{\widetilde{Q}} \widetilde{Q} + \widetilde{U}^\dagger \mathbf{m}^2_{\widetilde{U}} \widetilde{U} + \widetilde{D}^\dagger \mathbf{m}^2_{\widetilde{D}} \widetilde{D} + \widetilde{L}^\dagger \mathbf{m}^2_{\widetilde{L}} \widetilde{L} + \widetilde{E}^\dagger \mathbf{m}^2_{\widetilde{E}} \widetilde{E} \\
&+ m^2_{H_u} H^\dagger_u H_u + m^2_{H_d} H^\dagger_d H_d + \left[-\mu B H_d \cdot H_u + \text{h.c.}\right] \\
&+ \left[\widetilde{Q} \cdot H_u \mathbf{Y}^{\mathbf{A}}_{\mathbf{u}} \widetilde{U} + \widetilde{Q} \cdot H_d \mathbf{Y}^{\mathbf{A}}_{\mathbf{d}} \widetilde{D} + \widetilde{L} \cdot H_d \mathbf{Y}^{\mathbf{A}}_{\mathbf{e}} \widetilde{E} + \text{h.c.}\right] \\
&- \frac{1}{2}\left[m_{\widetilde{g}} \lambda^a_{\widetilde{g}} \lambda^a_{\widetilde{g}} + M_2 \lambda^i_{\widetilde{W}} \lambda^i_{\widetilde{W}} + M_1 \lambda_{\widetilde{B}} \lambda_{\widetilde{B}} + \text{h.c.}\right]. \qquad (4.1)
\end{aligned}
$$

Here $\mathbf{m}^2_{\widetilde{Q},\cdots,\widetilde{E}}$ are the soft mass-squares of the scalar fermions, $\mathbf{Y}^{\mathbf{A}}_{\mathbf{u},\mathbf{d},\mathbf{e}}$ are their associated holomorphic trilinear couplings, and finally, $m_{\widetilde{g}}$, M_2, M_1 are, respectively, the masses of color, isospin and hypercharge gauginos which are called as the gluino, wino and bino.

Thus, *a* term in gluino labels $SU(3)_C$ the gauge group and runs from 1 to 8 while i term in the wino terms stands for $SU(2)_L$ gauge group and runs from 1 to 3. The Higgs sector is described by soft masses $m^2_{H_u}$, $m^2_{H_d}$ and B term that mixes the scalar component of two Higgs doublets. In the space of fermion flavors, $\mathbf{m}^2_{\widetilde{Q},\cdots,\widetilde{E}}$ are mass squared hermitian matrices in family space whereas $\mathbf{Y^A_{u,d,e}}$, like Yukawa matrices themselves, are non-hermitian matrices. Thus, the bilinear B term and trilinear soft breaking terms $\mathbf{Y^A_{u,d,e}}$ have forms like those of the superpotential in (Equation 3.14).

It is crucial to mention that these soft breaking terms have holomorphicity which means that any trilinear interaction term in the soft lagrangian does not include the hermitian conjugate of any fields. Indeed, as we will discuss the next section, some other terms can be devised with respecting the gauge invariance but violating the holomorphicity of the lagrangian. However, they are necessary to obtain both the more general feature of the supersymmetry breaking and the complete understanding of the MSSM phenomenology as well as its astrophysical and cosmological implications.

4.2. The Non-Holomorphic Soft Breaking Terms

As discussed in the previous section, the Equation 4.1 has been believed to include all possible soft supersymmetry breaking terms without violating R-parity as well as gauge invariance and not regenerating the quadratic divergences in many supersymmetric theories which do not have pure gauge singlets in their particle spectrum.

However, as has been shown explicitly in (Bagger and Poppitz 1993), in supersymmetric theories without pure gauge singlets (like the MSSM), the holomorphic supersymmetry breaking terms do not necessarily represent the general set of soft-breaking operators. Indeed, one may consider some additional non-holomorphic terms which respect the gauge symmetries. These terms are also "soft", which do not cause any quadratic divergences and do not violate the R-parity (Frere, et al. 2000). Thus, such terms have shown to occur among flux-induced soft terms within intersecting brane models (Camara, et al. 2004).

In the sense, for constructing a complete picture of the MSSM phenomenology, we must add these non-analytic (non-holomorphic) terms include the hermitian conjugate of at least one MSSM scalar field. Indeed, one may consider, for instance, triscalar couplings with the hermitian conjugates of the Higgs fields, so the soft breaking sector must necessary include

$$-\mathcal{L}'_{soft} = \widetilde{Q} \cdot H_d^C \mathbf{Y'}_u^A \widetilde{U} + \widetilde{Q} \cdot H_u^C \mathbf{Y'}_{\mathbf{d}}^{\mathbf{A}} \widetilde{D} + \widetilde{L} \cdot H_u^C \mathbf{Y'}_{\mathbf{e}}^{\mathbf{A}} \widetilde{E} + \text{h.c.} \tag{4.2}$$

in addition to those in (Equation 4.1). Here $\mathbf{Y'}^{\mathbf{A}}_{\mathbf{u,d,e}}$ are non-holomorphic trilinear couplings which do not need to bear any relationship to the holomorphic ones in $\mathbf{Y}^{\mathbf{A}}_{\mathbf{u,d,e}}$ in (Equation 4.1). These two classes of trilinear couplings are perfectly soft and must be taken into account when confronting the MSSM predictions with experimental data. Being 3×3 complex non-hermitian flavor matrices, the non-holomorphic trilinears $\mathbf{Y'}^{\mathbf{A}}_{\mathbf{u,d,e}}$, like $\mathbf{Y'}^{\mathbf{A}}_{\mathbf{u,d,e}}$, contribute to various phenomena ranging from flavor-changing LR (s)currents to electric dipole moments. The general analysis of these of $\mathbf{Y'}^{\mathbf{A}}_{\mathbf{u,d,e}}$ in regard to MSSM phenomenology have also been studied in Çakır et al. (2005).

It is very important to analyse the effects of these non-holomorphic terms to be added to the lagrangian in order to understand whole MSSM phenomenology. As a result of the soft supersymmetry breaking, there are 32 mass eigenstates: 2 charginos, 4 neutralinos, 4 Higgs bosons, 6 charged sleptons, 3 sneutrinos, 12 squarks and the gluino in addition to the new phases and mixing angles in comparison with the SM. The aforementioned non-holomorphic soft breaking terms result in inserting new parameters to the mass eigenstates of this MSSM mass spectrum. However, some mass eigenstates are not affected by the existence of new non-holomorphic parameters because these mass eigenstates do not have both holomorphic $\mathbf{Y}^{\mathbf{A}}_{\mathbf{u,d,e}}$ and non-holomorphic $\mathbf{Y'}^{\mathbf{A}}_{\mathbf{u,d,e}}$ trilinear couplings. In other words, they do not contain any trilinear interaction terms (for instance, the terms represent the Higgs-sfermion-sfermion interactions). However, as will discussed in the following sections, it is crucial to mention that, the signals of these mass eigenstates without non-holomorphic soft terms must be compared with the others which include the effects of non-holomorphic trilinear couplings. Hence, this comparison has an affirmative role to reveal the distinctive features of these non-analytic soft breaking terms in collider experiments. For this purpose, in the following sections, we will take into consideration the chargino, sfermion and higgs sector in the MSSM with general soft breaking terms.

4.3. The Neutralino Sector

As a result of the electroweak symmetry breaking, fields which have different $SU(2)_L \otimes U(1)_Y$ quantum numbers can mix if they have the same representation in the gauge group $SU(3)_C \otimes U(1)_{EM}$ with the same baryon, lepton and color quantum numbers. In this sense, the neutral higgsinos ($\tilde{H}_u^0$ and $\tilde{H}_d^0$) and the neutral gauginos ($\tilde{B}$ and $\tilde{W}^0$) combine to form mass eigenstates called *neutralinos*.

Before the electroweak symmetry breaking, the $\tilde{B}^0$ and $\tilde{W}^0$ have their mass terms given by just the soft SUSY breaking which are given in (Ellis, et al.1984)

$$-\frac{1}{2}\left(M_1\,\tilde{B}^0\cdot\,\tilde{B}^0+M_2\,\tilde{W}^0\cdot\,\tilde{W}^0\right). \tag{4.3}$$

where M_1 represents the mass of bino while M_2 stands for the mass of wino. In addition of these terms, the mixing terms must occur between one of higgsinos ($\tilde{H}_u^0$, $\tilde{H}_d^0$) with one of the gauginos ($\tilde{B}^0$, $\tilde{W}^0$). These terms coming from interactions are given explicitly (Gunion and Haber 1984)

$$L_{int}=-\sqrt{2}\,g\left[(\,\phi_i^\dagger\,T^a\,\psi_i)\cdot\lambda^a+h.c\right]-\frac{1}{2}\left(\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\psi_i\psi_j+h.c\right) \tag{4.4}$$

where $T^a = \sigma^a/2$ and σ^a ($a = 1,2,3$) are usual Pauli matrices (given in Appendix A) and λ^a stands for chiral superfields for gauginos. The first term presents the couplings of a Higgs boson to a gaugino and a higgsino when the neutral Higgs fields H_u^0 and H_d^0 acquire their vacuum expectation values denoted as $\langle H_u^0\rangle \equiv (\frac{v_u}{\sqrt{2}})$ and $\langle H_d^0\rangle \equiv (\frac{v_d}{\sqrt{2}})$ respectively. Besides, the second term which is the mixing terms between $\tilde{H}_u^0$ and $\tilde{H}_d^0$ must be added to construct the mass terms of neutralinos. Finally, in the gauge basis $\psi^0 = (\tilde{B}^0, \tilde{W}^0, \tilde{H}_u^0, \tilde{H}_d^0)$, the part of the lagrangian represents the neutralino mass terms is

$$L_{neutralino}=-\frac{1}{2}\,(\psi^0)^T\mathbf{M}_{\tilde{N}}\,\psi^0 \tag{4.5}$$

where the neutralino mass matrix $\mathbf{M}_{\tilde{N}}$ is given as,

$$\begin{pmatrix} M_1 & 0 & -M_Z\, c_\beta\, s_W & M_Z s_\beta\, s_W \\ 0 & M_2 & M_Z\, c_\beta\, c_W & -M_Z\, s_\beta\, c_W \\ -M_Z\, c_\beta\, s_W & M_Z\, c_\beta\, c_W & 0 & -\mu \\ M_Z\, s_\beta\, s_W & -M_Z\, s_\beta\, c_W & -\mu & 0 \end{pmatrix} \tag{4.6}$$

The entries $s_W = \sin\theta_W$ and $c_W = \cos\theta_W$ (where θ_W is defined as the electroweak mixing angle as well as $s_\beta = \sin\beta$ and $c_\beta = \cos\beta$ (where $\tan\beta = v_u/v_d$) are introduced as

$$\begin{aligned} \sin\theta_W &\equiv \frac{g_Y}{\sqrt{g_Y^2+g_2^2}} \;;\cos\theta_W \equiv \frac{g_2}{\sqrt{g_Y^2+g_2^2}} \\ \cos\beta &\equiv \frac{v_d}{\sqrt{v_d^2+v_u^2}} \;;\sin\beta \equiv \frac{v_u}{\sqrt{v_d^2+v_u^2}} \end{aligned} \tag{4.7}$$

where g_2 and g_Y are the gauge couplings of the two gauge groups $SU(2)_L$ and $U(1)_Y$ respectively. The mass eigenstates (a linear combination of the four neutralino states) and mass eigenvalues are found by diagonalizing the mass matrix (4.6). The corresponding neutralino eigenstates are usually denoted by $\tilde{\chi}_i^0$ ($i = 1, ..., 4$) and by convention, these are labeled in ascending order, so that $m_{\tilde{\chi}_1} < m_{\tilde{\chi}_2} < m_{\tilde{\chi}_3} < m_{\tilde{\chi}_4}$. In the special limit, if M_1 and M_2 are small compared to M_Z and $|\mu|$, then the lightest neutralino $\tilde{\chi}_1^0$ would be nearly a pure photino $\tilde{\gamma}$. Thus, if M_1 and M_1 are small comparison with M_2 and $|\mu|$, then the lightest neutralino would be nearly a pure bino $\tilde{B}^0$. (for detail analysis see Giudice, et al. 1996)

4.4. Chargino Sector

The charged analogues of neutralinos are called *charginos* which are nothing but the mixtures of the charged higgsinos ($\tilde{H}_u^+$ and $\tilde{H}_d^-$) and the charged $SU(2)_L$ gauginos ($\tilde{W}^-$ and $\tilde{W}^+$). In order to construct the mass matrix of the charginos, all interactions terms coming from the interaction lagrangian (4.4), the mixing between two charged higgsinos and interactions among the higgs bosons and a charged gaugino and a charged higgsino must be considered. Then, the lagrangian that determines the chargino mass terms is given as

$$L_{chargino} - \frac{1}{2}(\psi^{\pm})^T \, \mathbf{M}_{\tilde{\chi}^{\pm}_{1,2}} \, \psi^{\pm} + h.c \tag{4.8}$$

where $\psi^+ = \left(\tilde{W}^+, \tilde{H}_u^+\right)$, $\psi^- = \left(\tilde{W}^-, \tilde{H}_d^-\right)$, and

$$\mathbf{M}_{\tilde{\chi}^{\pm}_{1,2}} = \begin{pmatrix} 0 & \mathbf{X^T} \\ \mathbf{X} & 0 \end{pmatrix} \; ; \mathbf{X} = \begin{pmatrix} M_2 & \sqrt{2}\, M_W s_\beta \\ \sqrt{2}\, M_W c_\beta & \mu \end{pmatrix} \tag{4.9}$$

Since $\mathbf{X} \neq \mathbf{X^T}$, two distinct 2×2 matrices are needed for the diagonalization. Thus, the charginos $\tilde{\chi}^+_{1,2}$ are the linear combination comes from the diagonalization of $\mathbf{X}^\dagger\mathbf{X}$ and the charginos $\tilde{\chi}^-_{1,2}$ are the combination that diagonalize the matrix $\mathbf{X}\mathbf{X}^\dagger$. After the diagonalization, the two chargino masses $M_{\tilde{\chi}^{\pm}_{1,2}}$ are found to be

$$M^2_{\tilde{\chi}^{\pm}_{1,2}} = \frac{1}{2}\left[M_2^2 + \mu^2 + 2M_W^2 \mp \sqrt{(M_2^2 - \mu^2)^2 + 4M_W^4 \cos^2 2\beta + 4M_W^2\, \Delta_{\tilde{\chi}^{\pm}_{1,2}}}\right] \tag{4.10}$$

where $\Delta_{\tilde{\chi}^{\pm}_{1,2}} = (M_2^2 + \mu^2 + 2M_2\, \mu\, \sin 2\beta)$ and the states are ordered such that $M_{\tilde{\chi}_1} \leq M_{\tilde{\chi}_2}$. In the special limit in which M_2 and μ are taken real, the eigenvalues of charginos are then given approximately by $M_{\tilde{\chi}^{\pm}_1} \approx M_2$ and $M_{\tilde{\chi}^{\pm}_2} \approx |\mu|$. In this limit, we have the approximate degeneracies $M_{\tilde{\chi}^{\pm}_1} \approx M_{\tilde{\chi}^0_2}$. (for detail analysis, see Peskin 1997)

It is extremely important to emphasize that both the neutralino and the chargino sector in the MSSM do not influenced by the non-holomorphic supersymmetry breaking terms. By means of this independency, we can analyze the chargino or/and neutralino sector in conjunction with different sector with general trilinear interaction terms to determine the impacts of these additional symmetry breaking parameters successfully.

4.5. The Sfermion Sector

As discussed in the previous sections, any scalar fields with same electric charge, R-parity, lepton and quantum numbers can mix each other. This means that, the mass eigenstates of the squarks and sleptons of the MSSM should be obtained by the diagonalization of three $6\times\ 6$ squared-mass matrices for up-type squarks, down-type squarks and charged sleptons and one $3\times\ 3$ matrix for sneutrinos. Fortunately, this mixing problem is overcome with the phenomenological constraints implying very small mixing angles. It is useful to keep in mind that the Yukawa couplings are proportional to the associated fermion masses. Hence, only terms involving the Yukawas of the third generation particles and their soft breaking couplings can contribute significantly to the sfermion masses. The first and the second family squarks and sleptons have negligible Yukawa couplings when the scale expected for the mass of the sfermion is considered.

Because of this reason, we take into account just the particles of the third family. For instance, the top squark mass terms are determined by the presence of V_F and the D-terms in the supersymmetric lagrangian as well as the soft-breaking terms are given both in (Equation 4.1) and (Equation 4.2). Moreover, the trilinear interactions (including both holomorphic and non-holomorphic terms) allow the scalar partners of the left and right handed fermions with notationally simplifying definitions $\left(\mathbf{m}^2_{\tilde{Q}}\right)_{33}\equiv m^2_{\tilde{t}_L}$, $\left(\mathbf{m}^2_{\tilde{U}}\right)_{33}\equiv m^2_{\tilde{t}_R}$ to mix in order to form the mass eigenstates. Here, the Yukawa coupling is given for top squark $(\mathbf{Y_u})_{33}\equiv h_t$ and the squared-mass of top quark is $m_t^2(H)=h_t^2\mid H_u^0\mid^2$ while the holomorphic trilinear coupling is $\left(\mathbf{Y^A_u}\right)_{33}\equiv h_tA_t$, and the non-holomorphic ones is $\left(\mathbf{Y'^A_u}\right)_{33}\equiv h_tA'_t$. Here proportionality of $\left(\mathbf{Y^A_u}\right)_{33}$ and $\left(\mathbf{Y'^A_u}\right)_{33}$ to the top Yukawa coupling is no more than an assumption; in full generality of the soft-breaking sector there is no reason to expect such relations to hold. Putting all terms together, we have a squared-mass matrix for the top squarks which is;

$$M^2_{\tilde{t}}=\begin{pmatrix} m^2_{LL} & m^2_{LR} \\ m^2_{RL} & m^2_{RR}\end{pmatrix} \tag{4.11}$$

with m^2_{LL}, m^2_{LR}, m^2_{RL} and m^2_{RR} terms given explicitly,

$$\begin{aligned} m^2_{LL} &= m^2_{\tilde{t}_L}+m_t^2-\frac{1}{4}\left(g_2^2-\frac{1}{3}g_Y^2\right)\left(\mid H_u^0\mid^2-\mid H_d^0\mid^2\right) && (4.12)\\ m^2_{LR} &= h_tA_t^\star H_u^{0\star}-h_t\mu H_d^0-h_tA'^\star_t H_d^0 && (4.13)\\ m^2_{RL} &= h_tA_tH_u^0-h_t\mu^\star H_d^{0\star}-h_tA'_tH_d^{0\star} && (4.14)\\ m^2_{RR} &= m^2_{\tilde{t}_R}+m_t^2-\frac{1}{3}g_Y^2\left(\mid H_u^0\mid^2-\mid H_d^0\mid^2\right). && (4.15)\end{aligned}$$

where top mass is determined as $m_t^2(H) = h_t^2 \mid H_u^0 \mid^2$ and note that the off-diagonal terms are proportional to top mass m_t. The hermitian top squark mass matrix can be diagonalized by a unitary matrix to give two different eigenvalues due to the m_{LR}^2 and m_{RL}^2 terms. The mass terms of two top squarks are;

$$\begin{aligned} m_{\tilde{t}_{1,2}}^2 &= \frac{1}{2}\Bigg[m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + 2m_t^2 + \frac{1}{2}\cos 2\beta M_Z^2 \\ &\pm \sqrt{\left(m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2 + \left(\frac{1}{2} - \frac{4}{3}s_W^2\right)\cos 2\beta M_Z^2\right)^2 + 4m_t^2\left(A_t - \cot\beta(\mu + A_t')\right)^2}\Bigg]. \end{aligned}$$

so that $m_{\tilde{t}_1}^2 < m_{\tilde{t}_2}^2$ with given s_W and $\cot\beta$ in (Equation 4.7).

The same procedure for getting mass terms can be fulfilled for bottom squark $\tilde{b}$ and stau $\tilde{\tau}$ with different mass matrix terms which are determined as;

$$(m_{LL})_{\tilde{b},\tilde{\tau}}^2 = m_{\tilde{b}_L,\tilde{\tau}_L}^2 + m_{b,\tau}^2 + \Delta_{b_L,\tau_L}^2 \tag{4.16}$$

$$(m_{LR})_{\tilde{b},\tilde{\tau}}^2 = h_{b,\tau}A_{b,\tau}^\star H_d^{0\star} - h_{b,\tau}\mu H_u^0 - h_{b,\tau}A'^\star_{b,\tau}H_u^0 \tag{4.17}$$

$$(m_{RL})_{\tilde{b},\tilde{\tau}}^2 = h_{b,\tau}A_{b,\tau}H_d^0 - h_{b,\tau}\mu^\star H_u^{0\star} - h_{b,\tau}A'_{b,\tau}H_u^{0\star} \tag{4.18}$$

$$(m_{RR})_{\tilde{b},\tilde{\tau}}^2 = m_{\tilde{b}_R,\tilde{\tau}_R}^2 + m_{b,\tau}^2 + \Delta_{b_R,\tau_R}^2 \tag{4.19}$$

where $\Delta_{\tau_L}^2$, $\Delta_{b_L}^2$, $\Delta_{\tau_R}^2$ and $\Delta_{b_R}^2$ are given

$$\begin{aligned} \Delta_{\tau_L}^2 &= \frac{1}{4}(g_2^2 - g_Y^2)\left(\mid H_u^0 \mid^2 - \mid H_d^0 \mid^2\right) \\ \Delta_{b_L}^2 &= \frac{1}{4}(g_2^2 + \frac{1}{3}g_Y^2)\left(\mid H_u^0 \mid^2 - \mid H_d^0 \mid^2\right) \\ \Delta_{\tau_R}^2 &= \frac{1}{2}g_Y^2\left(\mid H_u^0 \mid^2 - \mid H_d^0 \mid^2\right) \\ \Delta_{b_L}^2 &= \frac{1}{4}g_Y^2\left(\mid H_u^0 \mid^2 - \mid H_d^0 \mid^2\right) \end{aligned} \tag{4.20}$$

respectively with $A_{b,\tau}$ ($A'_{b,\tau}$) being holomorphic (non-holomorphic) trilinear couplings of the associated sfermion. Here, it is necessary to remark the $\tan\beta$ dependency of the mixing in the sbottom and stau sectors. If $\tan\beta$ is not too large, the sbottoms and staus do not get a very large effect from the mixing so the mass eigenstates are nearly the same as the gauge eigenstates $\tilde{b}_L$, $\tilde{\tau}_L$, $\tilde{b}_R$ and $\tilde{\tau}_R$ (Martin 1997).

It is important to emphasize that, in the limit of flavor-blind soft terms, as in Equations 4.12 and 4.14, the net effect of the non-holomorphic soft terms is seen to replace μ by $\mu + A'^{\star}_f$. This shift alone tells us that the μ parameter seen in the mass terms of charginos (Equation 4.10) and the mass matrix of the neutralinos equation 4.6 is completely different than what is felt by the scalar fermions. Hence, all effects of the non-holomorphic terms reveal in the mass spectrum of particles like sfermions and Higgs bosons that include these general symmetry breaking terms.

In the next section, we will discuss in detail the Higgs sector as a testing ground for examining such general soft breaking terms and show the effects of these terms on expanding the limits of the Higgs boson masses in the collider experiments.

4.6. The Higgs Sector

The Higgs sector in the MSSM is quite complicated due to the fact that there are two complex Higgs doublets which are denoted as

$$\begin{aligned} H_u &= \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} H_u^+ \\ v_u + \phi_u + i\varphi_u \end{pmatrix} \\ H_d &= \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d + \phi_d + i\varphi_d \\ H_d^- \end{pmatrix} \end{aligned} \tag{4.21}$$

where H_u and H_d have the hypercharge $(Y_{H_u} = \frac{1}{2})$ and $(Y_{H_d} = -\frac{1}{2})$ respectively. Therefore, in order to determined the Higgs bosons in the MSSM, we introduce the (clasical) tree-level scalar potential that includes all interaction terms belonging the Higgs bosons come from $|F|^2$ term and $(\frac{1}{2}\sum_a D^a\, D_a)$ term in the lagrangian both equations 3.17 and 3.22 as well as soft breaking terms from L_{Soft}. Then, the scalar potential becomes;

$$
\begin{aligned}
V_{tree} &= V_F + V_D + V_{Soft} \\
&= m_1^2 H_d^\dagger H_d + m_2^2 H_u^\dagger H_u - \mu\, B\, (H_d \cdot H_u + h.c) \\
&+ \frac{1}{8}(g_2^2 + g_Y^2)\left(H_u^\dagger H_u - H_d^\dagger H_d\right)^2 + \frac{1}{2} g_2^2 \mid H_d \cdot H_u \mid^2
\end{aligned} \tag{4.22}
$$

with given $m_1^2 = m_{H_d}^2 + |\mu|^2$, $m_2^2 = m_{H_u}^2 + |\mu|^2$ and B are the soft breaking parameters coming from L_{Soft}. Note that, in the MSSM, because of the presence of a second Higgs doublet, the quartic scalar coupling in V_{tree} are related to the electroweak gauge couplings in contrast, the strength of the Higgs self interaction is an unknown free parameter in the SM.

For preserving the charge conservation in the absolute minimum of the potential, we first must investigate under what condition the minimal of this scalar potential breaks the $SU(2)_L \otimes U(1)_Y$ gauge symmetry while preserving the electroweak symmetry $U(1)_{em}$. More clearly, there must not be any breaking in the charge directions so the charged components of the Higgs doublets can not develop non-vanishing vacuum expectation value (v.e.v). In this sence, by the freedom to make the $SU(2)_L$ gauge transformations, one can always choose vacuum expectation value of one the charged field *i.e* $\langle H_u^+ \rangle = 0$ without loss of the generality. Therefore, at $(\partial V_{tree}/\partial H_u^+ = 0)$, one can obtain automatically $\langle H_d^- \rangle = 0$, so the electromagnetism remains unbroken. After setting $\langle H_u^+ \rangle = 0$ and $\langle H_d^- \rangle = 0$, the scalar potential simply becomes;

$$
\begin{aligned}
V &= m_1^2 \mid H_d^0 \mid^2 + m_2^2 \mid H_u^0 \mid^2 - \mu\, B \left(H_d^0 H_u^0 + h.c\right) \\
&+ \frac{1}{8}(g_2^2 + g_Y^2)\left(\mid H_u^0 \mid^2 - \mid H_d^0 \mid^2\right)^2 .
\end{aligned} \tag{4.23}
$$

where $\langle H_u^0 \rangle = v_u/\sqrt{2}$ and $\langle H_d^0 \rangle = v_d/\sqrt{2}$ so that one can easily write the conditions obtaining from $\partial V_{tree}/\partial H_u^0 = \partial V_{tree}/\partial H_d^0 = 0$

$$
\begin{aligned}
m_1^2 + m_3^2 \tan\beta + \frac{1}{4} M_Z^2 \cos 2\beta &= 0 \\
m_2^2 + m_3^2 \cot\beta - \frac{1}{4} M_Z^2 \cos 2\beta &= 0
\end{aligned}
$$

where our convention is determined as $v_u/v_d \equiv \tan\beta$, $m_3^2 = B\,\mu$ and $M_Z^2 = \frac{1}{4}(v_u^2 + v_d^2)(g_2^2 + g_Y^2)$. These results show that at tree level, supersymmetry imposes strong constraints on the Higgs sector and also indicate some important remarks need to be signified. For a special case $|H_u^0| = |H_d^0|$, the quartic contributions to V are identically zero. Another case requires that one linear combination of H_u^0 and H_d^0 has a negative squared mass near $H_u^0 = H_d^0 = 0$. V is bounded from the relations in order to be stable and independent parameters of V must satisfy the minimization conditions

$$\begin{aligned} m_1^2 + m_2^2 &\geq 2\,m_3^2 \\ m_3^2 &> m_1^2\,m_2^2 \end{aligned} \tag{4.24}$$

As long as these relations must be satisfied the neutral components of the Higgs doublets get their vacuum expectation value (v.e.v) and the electroweak symmetry breaking occur.

It is affirmative to compute the mass terms of the Higgs bosons in the MSSM. After symmetry breaking in the SM, single Higgs doublet leads to one real scalar Higgs boson, as the other three components are eaten by the massive electroweak gauge bosons. In the supersymmetric version, three components of eight degrees of freedom are "eaten" by the longitudinal modes of the $W^{\pm}$ and Z^0 gauge bosons. The five degrees of freedom result in two CP-even neutral real scalar (h^0,H^0), one CP-odd pseudo-scalar (A^0) and two different charged Higgs ($H^{\pm}$). The tree-level mass matrices of the Higgs states can readily be computed from the matrix of second derivatives of the higgs potential (4.23) taken at absolute minimum (Kazakov 2000). Then, the tree-level matrices are;

1. CP-odd components φ_u and φ_d:

$$\mathcal{M}_{odd} = \left.\frac{\partial^2 V}{\partial\varphi_i \partial\varphi_j}\right|_{H_i = v_i} = \begin{pmatrix} m_3^2 \tan\beta & m_3^2 \\ m_3^2 & m_3^2\,\cot\beta \end{pmatrix} \tag{4.25}$$

While computing the eigenvalues of $\mathcal{M}_{odd}$, one can easily find that one eigenvalue is equal to zero and this eigenvalue corresponds to the mass of the Goldstone boson while other non-zero eigenvalue corresponds to the mass of the pseudoscalar Higgs boson denoted as A^0. Mass of the A^0 is then,

$$M_{A^0}^2 \quad = \quad -\frac{2m_3^2}{\sin 2\,\beta} = m_1^2 + m_2^2. \tag{4.26}$$

2. CP-even neutral components ϕ_u and ϕ_d:

$$\begin{aligned} \mathcal{M}_{even} &= \left.\frac{\partial^2 V}{\partial\phi_u \partial\phi_j}\right|_{H_i=v_i} \\ &= \begin{pmatrix} m_3^2 \tan\beta + M_Z^2 \cos^2\beta & -m_3^2 - M_Z^2 \cos\beta\,\sin\beta \\ -m_3^2 - M_Z^2 \cos\beta\,\sin\beta & m_3^2 \cot\beta + M_Z^2 \sin^2\beta \end{pmatrix} \end{aligned} \tag{4.27}$$

The corresponding non-zero mass terms of neutral h^0 and H^0 can be found readily after the diagonalization of the CP-even $\mathcal{M}_{even}$ matrix. The mass terms are then,

$$m_{h^0,H^0}^2 = \frac{1}{2}\{m_{A^0}^2 + M_Z^2 \mp \left[(m_{A^0}^2 + M_Z^2)^2 - 4m_{A^0}^2\,M_Z^2 \cos^2 2\beta\right]^{1/2}\} \tag{4.28}$$

3. Charged components H^- and H^+:

$$\begin{aligned} \mathcal{M}_{charged} &= \left.\frac{\partial^2 V}{\partial H_i^+ \partial H_j^-}\right|_{H_i=v_i} \\ &= \begin{pmatrix} m_3^2 \tan\beta & m_3^2 + M_W \cos\beta \sin\beta \\ m_3^2 + M_W \cos\beta \sin\beta & m_3^2 \cot\beta \end{pmatrix} \end{aligned} \tag{4.29}$$

After completing the diagonalization process one can easily find two Goldstone bosons $G^{\pm}$ and two massive charged Higgs bosons whose mass terms are,

$$m_{H^{\pm}}^2 \quad = \quad M_{A^0}^2 + M_W^2 \tag{4.30}$$

where M_W^2 is defined as $M_W^2 = g_2^2\,(v_u^2 + v_d^2)/2$.

These mass terms fulfill the following relations at tree level;

$$m_{H^\pm} \geq M_W \tag{4.31}$$

$$m_{h^0} \leq M_z|\cos 2\beta| \leq M_Z \tag{4.32}$$

$$m_{h^0}^2 + m_{H^0}^2 = M_{A^0}^2 + M_Z^2 \tag{4.33}$$

If one takes into consideration the inequality (4.32), it is obviously recognized that at tree level, the lightest Higgs boson, h^0, turns out to be lighter than the Z boson. If this inequality were robust the lightest Higgs boson of the MSSM would have been discovered at LEP2. However, fortunately, the radiative corrections to Higgs sector in the MSSM are not negligible and give important contribution to the Higgs boson masses. Furthermore, sizeable radiative corrections are needed to satisfy the LEP bound of $m_{h^0} \gtrsim 114\,\mathrm{GeV}$. The radiative corrections (Haber and Hempfling 1991, Espinosa and Quiros 1991, Drees and Nojiri 1992) are dominated by loops of the top (s)quark, and to a lesser extent, by those of the (s)tau lepton, (s)bottom quark (Choi, et al. 2000; Ibrahim and Nath 2001). Furthermore, at low $\tan\beta$ ($\tan\beta \leq 30$), radiative effects in the Higgs sector drive mainly from the top (s)quarks since other fermions are too light to have significant Yukawa interactions (as discussed in the previous section). A particularly useful framework for computing the radiative corrections in the Higgs sector is effective potential approach (Demir 1999, Pilaftsis and Wagner 1999, Ibrahim and Nath 2002) by considering the top quark and scalar top quark loops. The effective potential including both tree-level and radivative corrections is then given as (Weinberg and Coleman 1973);

$$V_{Higgs} = V_{tree} + \frac{6}{64\pi^2}\left[\sum_{i=1}^{2} m_{\tilde{t}_i}^4(H)\left(\log\frac{m_{\tilde{t}_i}^2(H)}{Q^2} - \frac{3}{2}\right) - 2m_t^4(H)\left(\log\frac{m_t^2(H)}{Q^2} - \frac{3}{2}\right)\right]$$

where V_{tree} is determined as in (Equation 4.22) and Q is the renormalization scale while (s)top masses $m_{\tilde{t}_i}$ are given clearly in sfermion sector. It is necessary to keep in mind that stop masses include the effects of the non-holomorphic trilinear coupling A_t' which causes to shift the μ parameter as $\mu \rightarrow \mu + A_t'$. This shift implies that all effects of scalar top quarks on the Higgs sector, as described in detail in (Haber, et al. 1991, Choi, et al. 2000) for holomorphic soft terms, remain intact except that μ parameter is not the μ parameter in the superpotential.

Leaving aside possibility of CP violation, ($i.e.$ by taking μ, triscalar couplings and gaugino masses all real) via the effective potential, one enables to compute Higgs boson masses. After the including the loop correction terms, the mass-squared term of CP-odd pseudoscalar Higgs boson $M^2_{A^0}$, then becomes

$$M^2_{A^0} = \frac{\mu B}{\sin\beta\cos\beta}\left[1 + \frac{3h_t^2}{32\pi^2}\frac{A_t\left(\mu + A'_t\right)}{\mu B}\mathcal{F}\left(Q^2, m^2_{\widetilde{t}_1}, m^2_{\widetilde{t}_2}\right)\right] \tag{4.34}$$

where $\mathcal{F}\left(Q^2, m^2_{\widetilde{t}_1}, m^2_{\widetilde{t}_2}\right)$ is introduced by

$$\mathcal{F}\left(Q^2, m^2_{\widetilde{t}_1}, m^2_{\widetilde{t}_2}\right) = -2 + \ln\left(\frac{m^2_{\widetilde{t}_1} m^2_{\widetilde{t}_2}}{Q^4}\right) + \frac{m^2_{\widetilde{t}_1} + m^2_{\widetilde{t}_2}}{m^2_{\widetilde{t}_1} - m^2_{\widetilde{t}_2}}\ln\left(\frac{m^2_{\widetilde{t}_1}}{m^2_{\widetilde{t}_2}}\right) \tag{4.35}$$

Second term in (Equation 4.34) comes from the radiative corrections. This additional term includes both holomorphic and non-holomorphic trilinear couplings. Notice that by settting $A_{t'} \longrightarrow 0$ then, the MSSM result is recovered.

Similarly, by adding radiative correction terms to the CP–even Higgs boson masses, the mass-squared matrix of the CP-even components of $H^0_{u,d}$ becomes

$$\mathcal{M}^2 = \begin{pmatrix} M_Z^2\cos^2\beta + M_A^2\sin^2\beta + \Delta^2_{dd} & -\left(M_A^2 + M_Z^2\right)\sin\beta\cos\beta + \Delta^2_{du} \\ -\left(M_A^2 + M_Z^2\right)\sin\beta\cos\beta + \Delta^2_{du} & M_Z^2\sin^2\beta + M_A^2\cos^2\beta + \Delta^2_{uu} \end{pmatrix} \tag{4.36}$$

where Δ^2_{uu}, Δ^2_{du} and Δ^2_{dd} stand for radiative corrections to that particular combination of the Higgs fields. These correction terms are given by;

$$\begin{aligned} \Delta^2_{dd} &= \frac{3\alpha_2}{4\pi}\frac{m_t^4}{M_W^2\sin^2\beta}\frac{\mu'\left(A_t - \mu'\cot\beta\right)}{\left(m^2_{\widetilde{t}_2} - m^2_{\widetilde{t}_1}\right)^2} \\ &\times \left[\left\{\mu'\left(A_t - \mu'\cot\beta\right) - \left(\frac{1}{4} - \frac{2}{3}s_W^2\right)\sin 2\beta\frac{M_Z^2}{m_t^2}\left(m^2_{\widetilde{t}_L} - m^2_{\widetilde{t}_R}\right)\right\}\mathcal{G}\left(m^2_{\widetilde{t}_1}, m^2_{\widetilde{t}_2}\right)\right. \\ &- \left.\sin 2\beta\frac{M_Z^2}{4m_t^2}\left(m^2_{\widetilde{t}_2} - m^2_{\widetilde{t}_1}\right)\log\frac{m^2_{\widetilde{t}_2}}{m^2_{\widetilde{t}_1}}\right] \end{aligned}$$

$$
\begin{aligned}
\Delta_{uu}^2 &= \frac{3\alpha_2}{4\pi}\frac{m_t^4}{M_W^2\sin^2\beta}\frac{A_t\left(A_t-\mu'\cot\beta\right)}{\left(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2\right)^2} \\
&\times \left[\left\{A_t\left(A_t-\mu'\cot\beta\right)-\left(\frac{1}{2}-\frac{4}{3}s_W^2\right)\sin^2\beta\frac{M_Z^2}{m_t^2}\left(m_{\tilde{t}_L}^2-m_{\tilde{t}_R}^2\right)\right\}\mathcal{G}\left(m_{\tilde{t}_1}^2,m_{\tilde{t}_2}^2\right)\right. \\
&+ \left.\left(2-\frac{M_Z^2}{2m_t^2}\sin^2\beta\right)\left(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2\right)\log\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}\right] \\
&+ \frac{3\alpha_2}{4\pi}\frac{m_t^4}{M_W^2\sin^2\beta}\left[\log\left(\frac{m_{\tilde{t}_2}^2 m_{\tilde{t}_1}^2}{m_t^4}\right)-\frac{M_Z^2}{2m_t^2}\sin^2\beta\log\left(\frac{m_{\tilde{t}_2}^2 m_{\tilde{t}_1}^2}{Q^4}\right)\right. \\
&- \left.\left(\frac{1}{2}-\frac{4}{3}s_W^2\right)\sin^2\beta\frac{M_Z^2}{m_t^2}\left(\frac{m_{\tilde{t}_L}^2-m_{\tilde{t}_R}^2}{m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2}\right)\log\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}\right]
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{du}^2 &= \frac{3\alpha_2}{4\pi}\frac{m_t^4}{M_W^2\sin^2\beta}\left[\frac{M_Z^2}{8m_t^2}\sin 2\beta\log\left(\frac{m_{\tilde{t}_2}^2 m_{\tilde{t}_1}^2}{Q^4}\right)\right. \\
&+ \frac{1}{m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2}\left\{-\mu'\left(A_t-\mu'\cot\beta\right)+\left(\frac{1}{8}-\frac{1}{3}s_W^2\right)\sin 2\beta\frac{M_Z^2}{m_t^2}\left(m_{\tilde{t}_L}^2-m_{\tilde{t}_R}^2\right)\right. \\
&+ \left.\frac{M_Z^2}{8m_t^2}\sin 2\beta\left(A_t+\mu'\tan\beta\right)\left(A_t-\mu'\cot\beta\right)\right\}\log\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2} \\
&+ \frac{\mathcal{G}\left(m_{\tilde{t}_1}^2,m_{\tilde{t}_2}^2\right)}{\left(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2\right)^2}\left\{-\mu' A_t\left(A_t-\mu'\cot\beta\right)^2\right. \\
&+ \left.\left.\left(\frac{1}{8}-\frac{1}{3}s_W^2\right)\sin 2\beta\frac{M_Z^2}{m_t^2}\left(m_{\tilde{t}_L}^2-m_{\tilde{t}_R}^2\right)\left(A_t+\mu'\tan\beta\right)\left(A_t-\mu'\cot\beta\right)\right\}\right]
\end{aligned}
$$

where we have introduced a scale–independent loop function $\mathcal{G}\left(m_{\tilde{t}_1}^2,m_{\tilde{t}_2}^2\right)$ is introduced by

$$
\mathcal{G}\left(m_{\tilde{t}_1}^2,m_{\tilde{t}_2}^2\right)=2-\frac{m_{\tilde{t}_1}^2+m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2}\log\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} \tag{4.37}
$$

As mentioned before, the higgs boson masses depend on $\mu + At'$ not At' in isolation. In fact, the lightest Higgs boson mass reads as (Demir 1999, Choi, et al. 2000)

$$m_h^2 \simeq M_Z^2 + \frac{3g_2^2 m_t^4}{8\pi^2 M_W^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t'^2}{M_S^2}\left(1 - \frac{X_t'^2}{12M_S^2}\right)\right] \tag{4.38}$$

where the mean stop mass-squared is given as

$$M_S^2 = \frac{1}{2}(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2) \tag{4.39}$$

is independent of A_t' while the left-right mixing term $X_t' = A_t - (\mu + A'_t)\cot\beta$.

Notice that the MSSM result is recovered by setting $A_t' \to 0$. Thus, in the MSSM limit, $X_t' = X_t$. For a clearer view of the impact of A_t' on the Higgs boson mass, one notes that the upper bound of the lightest Higgs boson mass is shifted by

$$\Delta m_h^2 \sim \frac{3g_2^2 m_t^4}{8\pi^2 M_W^2 M_S^2}\left[X_t'^2 - X_t^2 + \frac{X_t^4 - X_t'^4}{12M_S^2}\right] \tag{4.40}$$

in the presence of the non-holomorphic soft breaking term A_t'. This shift may vary from a few MeVs (for low values of $|A_t'|$) up to tens of GeVs depending on the input parameters. This is an important aspect since it modifies the upper bound of the Higgs boson mass, and in case a Higgs signal below 130 GeV is not observed at the LHC, it provides an explanation for higher values of m_h already in the MSSM (without resorting to NMSSM or U(1)$'$ models which generically yield higher values for m_h).

4.7. Effects of General Soft Breaking Terms on Higgs Sector

As mentioned in section (4.4), the effects of the non-holomorphic trilinear coupling A' parameter can be disentangled from those of the μ parameter if μ is known from an independent source. Clearly, an independent knowledge of μ can be obtained from neutralino or chargino sectors given in section (4.3) and (4.4) either via direct searches or via indirect bounds from certain observables. A readily recalled observable is $b \to s\gamma$ decay (Ciuchini, et al. 1998, Ciuchini 1998, Demir and Olive 2001). In addition one can consider bounds from EDMs or muon $g-2$ and suchlike but for purposes of obtaining a simple yet direct constraint on μ–A_t' relationship $b \to s\gamma$ decay suffices.

The rare radiative decay $b \to s\gamma$ provides an excellent arena for hunting the new physics model because it is accurately measured and its theoretical determination is rather clean (Hewett 1994). Since its characteristic mass scale, the b quark mass m_b, admits direct application of perturbative QCD (Ciuchini, et al. 1998, Kagan and Neubert 1999, Demir and Olive 2001, Misiak, et al. 2007). Moreover, experimental precision has increased over the years at the level of essentially confirming the SM result (Misiak, et al 2007, Barberio, et al. 2005). Therefore, the branching ratio of this decay is expected to place rather stringent limits on the sparticle contributions, and thus, provide an almost unique way of determining the allowed ranges of A'_t. The reason behind this observation is that $b \to s\gamma$ decay is sensitive to both μ (via chargino exchange) and $\mu + A'_t$ (via the stop exchange) as illustrated in Figure 4.1. Therefore, one has both μ and $\mu + A'_t$ at hand simultaneously and thus it becomes possible to disentangle A'_t effects from rest of the soft masses.

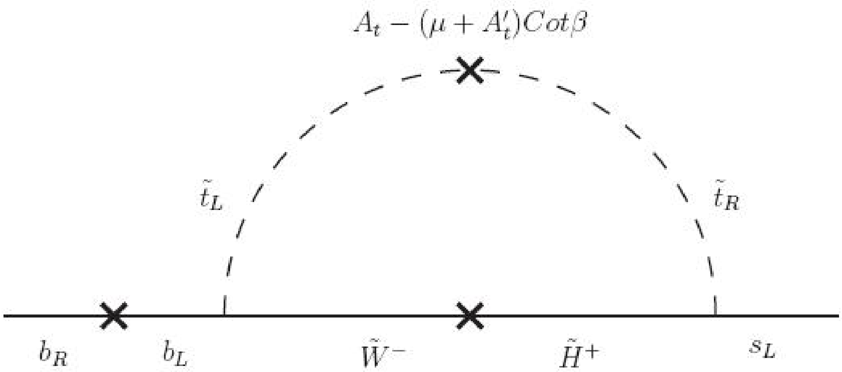

Figure 4.1. The stop–chargino exchange contribution to $b \to s\gamma$ decay (photon can be coupled to any charged line). While the stop mixing is directly sensitive to $\mu + A'_t$ the chargino exchange involves mass of the charged Higgsino, the μ parameter. This process thus involves both μ itself and $\mu + A'_t$ leading thus disentangling of A'_t from rest of the parameters.

In fact, from the form of the chargino mass matrix given in (4.9), one observes that wino and higgsino components mix as a result of the electroweak symmetry breaking (denoted by a cross on the horizontal line inside the loop), and higgsino mass μ enters the branching ratio in isolation. Unlike chargino sector, as suggested by Figure 4.1, the stop left-right mixing (denoted by a cross on the dashed arc in the loop) depends explicitly on $\mu + A_t'$ as seen also from matrix 4.11. The simultaneous μ and $\mu + A_t'$ dependencies of $b \to s\gamma$ decay, as depicted in Figure 4.1, results thus in a distinction between μ and $\mu + A_t'$, which would not be possible by an analysis of the Higgs sector alone.

Depicted in Figure 4.2 is the dependence of the lightest Higgs boson mass on certain parameters as A_t' takes on a set of values in the negative direction. The numerical results herein correspond to a specific choice of the parameters

$$M_1 = 140,\ M_2 = 280,\ M_3 = 1000,\ M_A = 500, \qquad (4.41)$$
$$A_t = -1600,\ m_{t_L} = 1000,\ m_{t_R} = 200,$$

all in GeV. We fix $\tan\beta = 5$ and do not consider higher $\tan\beta$ values since in this regime A_t' effects are reduced as can be seen from the left-right mixing entry of (4.11). These parameter values are chosen judiciously in that m_h agrees with the LEP II lower bound of $m_h \geq 114$ GeV and $\tan\beta > 2$ when A_t' vanishes (Schael, et al 2006). This choice will help in revealing the effects of A_t' in a transparent way. We will see that typically large negative values of A_t' leads to observable changes where how large it should be depends, of course, on the characteristic scale of soft mass parameters in matrix 4.11.

Figure 4.2a shows how m_h depends on A_t'. It is seen that m_h just agrees with the LEP bound when A_t' is small in magnitude. However, as it grows in negative direction up to $-2.5\ \mathrm{TeV}$ the Higgs boson mass gets gradually shifted towards the $135\ \mathrm{GeV}$ borderline. This clearly shows that a measurement of the Higgs boson mass can imply strikingly different parameter space than one would expect naively from a restricted set of soft-breaking terms given in equation (4.1). In addition, the horizontal behavior of the curves in Figure 4.2.(a) is due to the allowed range of the μ parameter by the $b \to\ s\,\gamma$ bound. That is, μ parameter takes on different values of each selectionpf the A_t' determined via the $b \to\ s\,\gamma$ restriction. This is also reflected in Figure 4.2c.

Shown in Figure 4.2b is the mass splitting between the CP–odd and CP–even Higgs bosons vs. the lightest Higgs boson mass. In the MSSM, due to the radiative corrections A^0 and H^0 degenerate in mass. However, the contributions stemming from

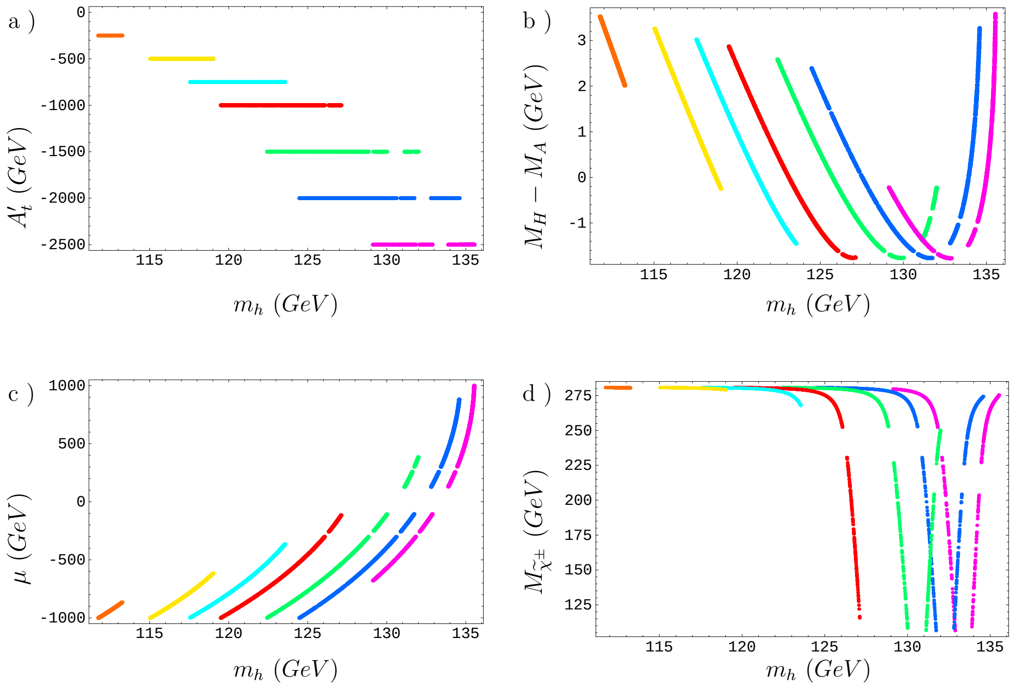

Figure 4.2. The lightest Higgs boson mass vs. certain model parameters after taking into account the $b \to s\gamma$ constraint.

the existence of $A_{t'}$, the mass splitting could be occur between CP–odd and CP–even Higgs bosons. It is clear that, for each value of A'_t a respectable splitting $\sim$ 3.5 GeV can exist. For small values of A'_t, the μ parameter falls in a rather narrow band, that is, bigger the A'_t in the negative direction large the range of μ parameter. This increase in the mass splitting can be measured at the ILC if not at the LHC.

Depicted in Figure 4.2c is the dependence of m_h on μ parameter for different values of A'_t. At low A'_t the μ parameter is preferred to be -1 TeV for m_h to agree with the experiment. However, as A'_t grows to large negative values the μ parameter goes to its mirror symmetric value; $\mu = 1$TeV. This large swing in the allowed range of μ stems solely from the dependence of the stop masses in (Equation 4.11) on μ and A'_t where $b \to s\gamma$ does not allow their sum to exceed a certain threshold due to the rather narrow band of values left to new physics contributions (Misiak, et al. 2006, Barberio, et al. 2006).

Finally, shown in Figure 4.2d is the variation of m_h with the lighter chargino mass $m_{\chi^{\pm}}$ as A'_t varies. One notices how their relationship is modified at large negative

A_t' via especially the region at large m_h. Indeed, as A_t' grows to large negative values the Higgs boson mass is shifted towards $130\ \mathrm{GeV}$ border wherein change of $m_{\chi^\pm}$ with m_h is rather sharp. It is clear that both these masses are measurable at the LHC, and their interdependence can guide one if the model under concern is a minimal one based on (Equation 4.1) or a more general one based on (Equation 4.1) and (Equation 4.2) especially after a fit to model parameters.

In principle, a full experiment on chargino and neutralino masses must determine M_2, M_1, μ and $\tan\beta$ in a way independent of what happens in the sfermion sector. Experimentally, however, realization of this statement can be quite non-trivial; in particular, one might need to determine final states containing only neutralinos or only charginos or neutralinos and charginos (Brhlik and Kane 1998). An extraction of A_t' then follows from constructing relations like the ones illustrated in Figure 4.2.

CHAPTER 5

CONCLUSION

In this thesis work, we examined the impact of the non-holomorphic soft terms on the different sectors in the MSSM in detail. We also gave the main concepts of the supersymmetry as well as the minimal supersymmetric standard model (MSSM) after pointing out the problem in the SM. Then, we focused on the breakdown of the global supersymmetry. We showed explicitly that the soft symmetry breaking sector of the MSSM must in general include the supersymmetry-breaking terms in (4.1) as well as in (4.2). Hence, the presence of these non–holomorphic trilinear couplings given in (4.2) result in several important impacts on various observables. In particular, holomorphic and non–holomorphic soft breaking terms influence radiative corrections to Higgs boson masses, and their size can be examined within the LHC data by forming a cross correlation among Higgs boson mass and other observables.

In this sense, we showed explicitly that the upper bound on the lightest Higgs boson mass is shifted by means of the existence of the non-holomorphic breaking term A_t'. This shift could be as large as 10 GeV depending on the input parameters. This is a vitally important aspect for modifying the upper bound on the Higgs boson mass in the MSSM without introducing extended models like NMSSM or $U(1)'$ that generically yield higher values of m_h.

Furthermore, we showed explicitly that the analysis of the Higgs sector in conjunction with the chargino sector disentangles the effects of non–holomorphic trilinear couplings from μ parameters. As mentioned before, the independent knowledge can be obtained from the chargino sector by means of certain observables like rare radiative B meson decay, $b \to s\,\gamma$. It is important to emphasize that branching ratio of the rare radiative decay $b \to s\,\gamma$ is expected to restrict the sparticle contributions. This restriction plays a crucial role to determine the allowed range of the non–holomorphic trilinear coupling. In particular, as illustrated in Figure 4.2d, a simultaneous knowledge of chargino and Higgs boson masses enables one to search for A_t' effects after a fit to the model parameters. The results advocated here could have important implications for a global analysis of the LHC data.

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APPENDIX A

NOTATIONS AND CONVENTIONS

In this thesis, we will use the standard relativistic units which are

$$\hbar = c = 1. \tag{A.1}$$

A general covariant or contravariant four vector will be symbolized by

$$\begin{aligned} A^{\mu} &= (A^0; A^1, A^2, A^3) = (A^0; \mathbf{A}) \\ A_{\mu} &= (A_0; -A_1, -A_2, -A_3) = (A^0; -\mathbf{A}) \end{aligned} \tag{A.2}$$

and the compact "Feymann slash" given as

$$\not{A} = \gamma^{\mu} A_{\mu}. \tag{A.3}$$

The metric tensor $(g_{\mu\nu})$, which connects convariant four vector with contravariant vector, is defined by

$$g_{\mu\nu} = diag(1, -1, -1, -1) \tag{A.4}$$

A.1. Pauli Matrices

The well known Pauli matrices are defined as

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A.5}$$

and satisfy the commutator relation

$$[\sigma^i, \sigma^j] = 2i\epsilon^{ijk}\sigma_k \quad , \quad \{\sigma^i, \sigma^j\} = 2\delta^{ij} \quad , \quad Tr(\sigma^i \sigma^j) = 2\delta^{ij} \tag{A.6}$$

where ϵ^{ijk} is antisymmetric $\epsilon^{ijk} = \epsilon_{ijk} = 1$ for $i, j, k = 1, 2, 3$.

It is useful to define the anti-symmetric matrices $\sigma_{\mu\nu}$ and $\bar{\sigma}_{\mu\nu}$

$$\begin{aligned}
\sigma^{\mu\nu} &= \frac{i}{4}\left(\sigma^{\mu}\,\bar{\sigma}^{\nu} - \sigma^{\nu}\,\bar{\sigma}^{\mu}\right) \\
\bar{\sigma}^{\mu\nu} &= \frac{i}{4}\left(\bar{\sigma}^{\mu}\,\sigma^{\nu} - \bar{\sigma}^{\nu}\,\sigma^{\mu}\right) \\
Tr(\sigma^{\mu\nu}) &= Tr(\bar{\sigma}^{\mu\nu}) = 0 \\
Tr(\sigma^{\mu\nu}\,\sigma^{\rho\lambda}) &= \frac{1}{2}(g^{\mu\rho}g^{\nu\lambda} - g^{\mu\lambda}g^{\nu\rho}) + \frac{i}{2}\,\epsilon^{\mu\nu\rho\lambda} \\
Tr(\bar{\sigma}^{\mu\nu}\,\bar{\sigma}^{\rho\lambda}) &= \frac{1}{2}(g^{\mu\rho}g^{\nu\lambda} - g^{\mu\lambda}g^{\nu\rho}) - \frac{i}{2}\,\epsilon^{\mu\nu\rho\lambda}
\end{aligned} \tag{A.7}$$

A.2. Dirac Matrices

The Dirac γ matrices are defined by anti–commutation relation

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu} \tag{A.8}$$

where γ^5 given as

$$\gamma_5 \equiv \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\gamma_0\gamma_1\gamma_2\gamma_3. \tag{A.9}$$

which is satisfied the relations

$$\{\gamma^5, \gamma^{\mu}\} = 0 \quad , \quad (\gamma^5)^2 = 1 \tag{A.10}$$

It is useful to state three different representations of the γ-matrices which are Dirac, Majorana, and Chiral representation.

A.2.1. Dirac Representation

The γ-matrices are demonstrated as

$$\begin{aligned}
\gamma^0 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\
\gamma^i &= \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix} \qquad i = 1, 2, 3. \\
\gamma^5 &= \begin{pmatrix} 0 & \sigma^0 \\ \bar{\sigma}^0 & 0 \end{pmatrix}
\end{aligned} \tag{A.11}$$

A.2.2. Majorana Representation

In this representation the γ-matrices are pure imaginary and given as

$$\begin{aligned}
\gamma^0 &= \begin{pmatrix} 0 & \sigma^2 \\ -\bar{\sigma}^2 & 0 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix} \\
\gamma^2 &= \begin{pmatrix} 0 & -\sigma^2 \\ -\bar{\sigma}^2 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & i\sigma^1 \end{pmatrix}
\end{aligned} \tag{A.12}$$

and

$$\gamma^5 = \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix} \tag{A.13}$$

A.2.3. The Chiral Representation

The γ matrices under Chiral representation or Weyl basis, which are very important for SUSY calculations, are

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \quad \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{A.14}$$

and

$$\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{A.15}$$