FAMILY NON-UNIVERSAL U(1)$'$ MODEL

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ABSTRACT

FAMILY NON-UNIVERSAL U(1)$'$ MODEL

This thesis work is devoted to an analysis of dilepton signatures of family non-universal U(1)$'$ model. We first provide a brief overview of Standard Model of particle physics and Supersymmetry then we give an introduction to basic concepts of Minimal Supersymmetric Standard Model (MSSM) and necessities to extend the MSSM with additional symmetry groups. Later we review various existing U(1)$'$ models, then we discuss the effects and results of family non-universality in current and future colliders.

The supersymmetric models extending the MSSM by an additional Abelian gauge factor U(1)$'$ in order to solve the μ problem do generically suffer from anomalies disrupting the gauge coupling unification found in the MSSM. The anomalies are absent if the minimal matter content necessitated by the μ problem is augmented with exotic matter species having appropriate quantum numbers. Recently, it has been shown that anomaly cancellation can also be accomplished by introducing family non-universal U(1)$'$ charges and non-holomorphic soft-breaking terms (Demir, et al. 2005) and keeping the matter content minimal without exotic particles.

We discuss collider signatures of anomaly-free family non-universal U(1)$'$ model by analyzing dilepton production in future colliders. We notice that, both at LHC and NLC, one can establish existance (or absence) of such a Z$'$ boson by simply comparing the number of dilepton production events for electron, muon and tau lepton (Hayreter 2007).

ÖZET

AİLEYE BAĞIMLI U(1)$'$ MODELİ

Bu tez çalışması aileye bağımlı U(1)$'$ modelinin gelecek nesil parçacık çarpıştırıcılarında dilepton sinyallerinin incelenmesi olarak hazırlanmıştır. İlk olarak kısaca Parçacık Fiziğinde Standard Model ve Süpersimetri'yi anlattık ve Minimal Süpersimetrik Standard Model (MSSM)'in temel kavramlarından bahsettik ve ek simetri grupları ile MSSM'i genişletmenin gerekliliğini anlattık. Sonra varolan çeşitli U(1)$'$ modellerini inceledik ve son olarak şimdiki ve gelecek nesil parçacık çarpıştırıcılarında aileye bağımlılığın etkilerini ve sonuçlarını tartıştık.

MSSM'in bir handikapı olan μ sorununu çözmek için MSSM'i ek bir Abelian ayar faktörü ile genişleten süpersimetrik modeller MSSM'in öngördüğü ayar kuplajlarının birleşimini bozan anomaliler gibi başka bir sorunla karşılaşırlar. Bu anomaliler, μ sorununun gerektirdiği minimal madde içeriğinin uygun kuantum numaralarına sahip egzotik madde türleri ile genişletimesiyle ortadan kaldırılabilirler. Son zamanlarda, aileye bağımlı U(1)$'$ yüklerinin ve holomorfik olmayan yumuşak kırıcı termilerin tanımlanması ile ve egzotik parçacıklara gerek olmadanda bu anomalilerin ortadan kalktığı gösterilmiştir (Demir, et al. 2007).

Aileye bağımlı anomalisiz U(1)$'$ modelinin gelecek nesil çarpıştırıcılarda dilepton üretimini analiz ederek çarpıştırıcı sinyallerini inceledik. Ve hem LHC'de hemde NLC'de basitçe elektron, moun ve tau leptonların dilepton üretim sayılarının karşılaştırılmasıyla Z$'$ bozonunun varlığı (yada yokluğu) hakkında bilgi edinebileceğimizi gösterdik (Hayreter 2007).

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CHAPTER 1

INTRODUCTION

Constructed in 1964 by Salam, Glashow and Weinberg The Standard Model (SM) of particle physics was seen to be a perfect structure and an elegant theoretical framework in explaining the particle interactions and the fundamental forces of nature. Passing through several precision experiments in various colliders, the SM became a base structure in phenomenology of particle physics. All the known elementary and force carrier particles with their masses, spins and charges were clearly identified and the further predictions of new elementary and composite particles were testified through upcoming collider experiments. The success of the SM has encouraged physicists to go through deeper investigations and led them ask the ultimate questions; were all the known elementary particles really elementary? and what is the origin of matter?

1.1. The Standard Model

The SM covers three generations of leptons and quarks as elementary particles. Electron (e), muon (μ) and tau (τ) lepton with their associated neutrinos in lepton sector and up (u), down (d), charm (c), strange (s), top (t) and bottom (b) quarks in quark sector, having half-integer spin (s=1/2) all the leptons and quarks obey to Fermi-Dirac statistics and therefore they are called as *fermions*. Besides gravity, which appears to be the first handicap of the SM, all the fundamental forces of nature are described by the exchange of force carrier particles, that is, photon (γ) is responsible for electromagnetic forces, weak forces are transmitted by Z^0, $W^\mp$ and gluons (g) mediate the strong forces. Since all

Table 1.1. Standard Model Fermions

		1st. Generation		**2nd. Generation**		**3rd. Generation**	
Fermions	*Leptons*	*Electron*	e	*Muon*	μ	*Tau*	τ
		Electron − Neutrino	ν_e	*Muon − Neutrino*	ν_μ	*Tau − Neutrino*	ν_τ
	Quarks	*Up*	u	*Charm*	c	*Top*	t
		Down	d	*Strange*	s	*Bottom*	b

these force carrier particles have integer spin (s=1) they obey to Bose-Einstein statistics and thus called as *bosons*.

Table 1.2. Standard Model Bosons

	Electromagnetism		**Weak Interaction**		**Strong Interaction**		**Gravity ???**	
Bosons	$Photon$	γ	$Weak\ bosons$	$W^{\pm}, Z^0$	$Gluons$	g	$Graviton$	G

The SM is based on a gauge principle in which the exchanged bosons are gauge fields of corresponding symmetry groups. The symmetry structure of the SM is,

$$SU(3)_C \times SU(2)_L \times U(1)_Y \tag{1.1}$$

where all the gauge bosons are related with the number of generators of corresponding gauge groups. There are 8 gluons G^a_μ of $SU(3)_C$ color (with $3^2 - 1 = 8$ generators), 3 weak bosons W^i_μ of $SU(2)_L$ isospin (with $2^2 - 1 = 3$ generators) and B_μ boson of $U(1)_Y$ hypercharge (with a single generator). At high energies these 12 gauge bosons were mathematically seen to be virtual massless gauge bosons, however at low energies the spontaneous breakdown of symmetries

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM} \tag{1.2}$$

give rise to physical massive gauge bosons *i.e.* Z^0 (neutral) and $W^{\mp}$ (charged). The mechanism behind this symmetry breakdown is so called the Higgs Mechanism, thus SM predicts the existence of a scalar (spin=0) Higgs boson by which all fermions and vector bosons gain their masses.

The gauge structure of SM is chiral sensitive, that is it exhibits a built-in left-right asymmetry which means that left and right handed fermion fields are treated in a completely different manner. In addition to left handed lepton and quark doublets there are also right handed leptons and quarks in singlet structure, therefore the complete matter content of SM becomes,

$$L_i = \begin{pmatrix} e \\ \nu_e \end{pmatrix}_L , \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L , \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L ; \quad E_i^c = e_R, \ \mu_R, \ \tau_R \tag{1.3}$$

$$Q_i = \begin{pmatrix} u \\ d \end{pmatrix}_L , \begin{pmatrix} c \\ s \end{pmatrix}_L , \begin{pmatrix} t \\ b \end{pmatrix}_L ; \quad U_i^c = u_R, \ c_R, \ t_R \quad ; \quad D_i^c = d_R, \ s_R, \ b_R$$

where $i = 1, 2, 3$ is generation index, notice that there is no room for right handed neutrinos ν_{eR} , $\nu_{\mu R}$, $\nu_{\tau R}$ in SM.

The scalar Higgs sector of the SM consists of a single Higgs doublet whose components are neutral (H^0) and charged (H^-) complex scalar Higgs fields,

$$H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix} \tag{1.4}$$

with the classical potential;

$$V = m_H^2 \mid H \mid^2 + \lambda \mid H \mid^4 \tag{1.5}$$

The non-vanishing vacuum expectation value (VEV) of neutral Higgs component

$$\langle H^0 \rangle = \sqrt{-\frac{m_H^2}{2\lambda}} \tag{1.6}$$

triggers the electroweak symmetry breaking and generates masses to fermions and massless vector bosons. Since it is known experimentally that $\langle H \rangle$ is approximately 174 GeV, from measurements of the properties of weak interactions, it must be that m_H^2 is roughly of order $-(100 \ \ GeV)^2$. However the Higgs mass-squared receives enormous quantum corrections from the virtual effects of every particle that couples, directly or indirectly, to the Higgs field. This problem is called as Gauge Hierarchy Problem of the SM.

Even though the SM succeeds in explaining almost all the known phenomena of particle physics, it is insufficient of being a complete theory of fundamental interactions, primarily because of its lack of inclusion of gravity which is one of the fundamental forces of nature and also of reserving no room for right handed neutrinos and finally because of Gauge Hierarchy problem arises in Higgs mass calculations.

1.2. Supersymmetry

Basically Supersymmetry (SUSY) is a symmetry that relates fermions to bosons. A supersymmetric transformation turns a fermionic state into a bosonic one, and vice versa.

$$Q\,|Fermion\rangle = |Boson\rangle \quad , \quad Q\,|Boson\rangle = |Fermion\rangle \tag{1.7}$$

where Q is a supersymmetric transformation operator. Therefore in SUSY every particle has a supersymmetric partner and they are called as superpartners. Thus, matter content of the SM is doubled in SUSY and each superpartner is represented by a ($\sim$) sign above its counter particle representation in SM, i.e. superpartner of a left-handed electron is demonstrated by $\tilde{e_L}$ and named as *selectron*. Each particle differs from its superpartner only by its spin, in the exact symmetry case every particle must be present with their superpartners in nature, since there is no such particles being candidate for superpartners it is said that SUSY is a broken symmetry in nature, and therefore all the superpartners have to be heavier than ordinary particles.

When physicists were struggling with the Gauge Hierarchy Problem of the SM in the early 1970's supersymmetric field theories were being developed for quite different reasons. After a while they realized that if SUSY exists near to the TeV energy scale, it provides the solution for two major puzzles in particle physics. One is the Hierarchy problem of the SM and the other is the unification of weak, strong and electromagnetic interactions.

1.2.1. Hierarchy Problem

In SM, the Higgs boson mass (m_H) was calculated to be near to the mass of Z boson (m_Z) and even less than it in tree level, however, quantum corrections from every particle that couples to the Higgs field yield enormous contributions to its mass. For example, in Figure 1.1.a there is a correction to m_H^2 from a loop containing a Dirac fermion f with mass m_f.

If this Dirac fermion f couples to the Higgs field with a term in Lagrangian $-\lambda_f H\bar{f}f$, then the Feynman diagram in Figure 1.1.a yields a correction

$$\Delta m_H^2 = -\frac{|\ \lambda_f\ |^2}{8\pi^2}\Lambda_{UV}^2 + \ldots \tag{1.8}$$

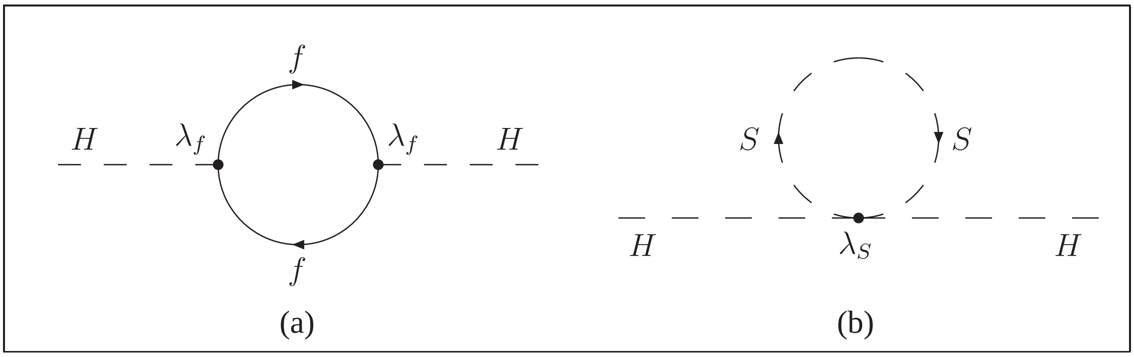

Figure 1.1. Feynman diagrams of fermionic and bosonic one-loop quantum corrections to Higgs mass-squared

where Λ_{UV} is an ultraviolet momentum cutoff used to regulate the loop integral and it is interpreted as the energy scale at which new physics enters to alter the high-energy behavior of the theory. The problem is that if Λ_{UV} is of order Planck Mass (M_P), then this quantum correction to m_H^2 is some 30 orders of magnitude larger than the required value of $m_H^2 \sim -(100GeV)^2$. This is only directly a problem for corrections to the Higgs scalar boson mass-squared, because quantum corrections to fermion and gauge boson masses do not have the direct quadratic sensitivity to Λ_{UV} found in (Equation 1.8). However the quarks and leptons and electroweak gauge bosons of the SM all obtain their masses from $\langle H \rangle$, so that the entire mass spectrum of the SM is directly or indirectly sensitive to the cutoff Λ_{UV}.

However, suppose there exists a heavy complex scalar particle S with mass m_S that couples to the Higgs field with a Lagrangian term $-\lambda_S |H|^2 |S|^2$. Then the Feynman diagram in Figure 1.1.b gives a correction

$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2} \left[\Lambda_{UV}^2 - 2m_S^2 \, ln \left(\frac{\Lambda_{UV}}{m_S} \right) + \ldots \right] \tag{1.9}$$

where S is nothing but a bosonic superpartner of Dirac fermion f with a reduced spin ($s = 0$). It is clear that if a SM fermion is accompanied by two complex scalars with $\lambda_S = | \lambda_f |^2$, then the quadratic contributions (Λ_{UV}^2) of Figures 1.1.a and Figure 1.1.b will neatly cancel because of relative minus sign of fermionic and bosonic loops. Consequently, Supersymmetry introduces us two complex scalar fields to enable a cancellation of the quadratically divergent (Λ_{UV}^2) pieces of (Equation 1.8) and (Equation 1.9). And hence, SUSY seems to offer a well cure for the Hierarchy problem of the SM.

CHAPTER 2

MINIMAL SUPERSYMMETRIC STANDARD MODEL

The Minimal Supersymmetric Standard Model (MSSM), containing minimal number of fields and parameters required to construct a realistic model of leptons and quarks, is the minimal extension to the Standard Model (SM) that realizes Supersymmetry. The gauge group of the MSSM is SU(3)$_C\times$SU(2)$_L\times$U(1)$_Y$ which is the same in the SM. But the particle content as is seen from the table (2.1) is enlarged to cover three generations of leptons and quarks, twelve gauge bosons, two Higgs doublets and supersymmetric partners of all these particles. All the chiral and guage fields of the SM now resides in superfields with their associated superpartners in the MSSM.

Table 2.1. Chiral and guage superfields of the MSSM

Superfields		Spin 0	Spin 1/2	Spin 1	$\mathbf{SU(3)_C, SU(2)_L, U(1)_Y}$
$Squarks, Quarks$	Q^i	$(\tilde{u}^i_L, \tilde{d}^i_L)$	(u^i_L, d^i_L)	-	$\mathbf{3}$, $\mathbf{2}$, 1/6
	$\bar{u}^i$	$\tilde{\bar{u}}^i_L(\tilde{u}^i_R)$	$\bar{u}^i_L \sim (u^i_R)^c$	-	$\bar{\mathbf{3}}$, $\mathbf{1}$, -2/3
	$\bar{d}^i$	$\tilde{\bar{d}}^i_L(\tilde{u}^i_R)$	$\bar{d}^i_L \sim (d^i_R)^c$	-	$\bar{\mathbf{3}}$, $\mathbf{1}$, 1/3
$Sleptons, Leptons$	L^i	$(\tilde{e}^i_L, \tilde{\nu}^i_L)$	(e$^i_L, \nu^i_L$)	-	$\mathbf{1}$, $\mathbf{2}$, -1/2
	$\bar{e}^i$	$\tilde{\bar{e}}^i_L(\tilde{e}^i_R)$	$\bar{e}^i_L \sim (e^i_R)^c$	-	$\mathbf{1}$, $\mathbf{1}$, 1
$Higgs, Higgsinos$	H_u	(H^+_u, H^0_u)	$(\tilde{H}^+_u, \tilde{H}^0_u)$	-	$\mathbf{1}$, $\mathbf{2}$, 1/2
	H_d	(H^0_d, H^-_d)	$(\tilde{H}^0_d, \tilde{H}^-_d)$	-	$\mathbf{1}$, $\mathbf{2}$, -1/2
$Gluinos, Gluons$	g	-	$\tilde{g}$	g	$\mathbf{8}$, $\mathbf{1}$, 0
$Wino, W\ boson$	W	-	$\tilde{W}^\pm, \tilde{W}^0$	$W^\pm, W^0$	$\mathbf{1}$, $\mathbf{3}$, 0
$Bino, B\ boson$	B	-	$\tilde{B}$	B	$\mathbf{1}$, $\mathbf{1}$, 0

All the SM fermions with Higgs fields and their superpartners (scalar fermions and fermionic Higgsinos) are members of chiral supermultiplets and vector bosons of the SM with associated superpartners (fermionic gauginos) are placed in gauge supermultiplets in the MSSM.

The MSSM is specified by the choice of superpotential,

$$\widehat{W}_{MSSM} = h_e^{ij}\widehat{E}^c_j\widehat{L}_i\cdot\widehat{H}_d + h_u^{ij}\widehat{U}^c_j\widehat{Q}_i\cdot\widehat{H}_u + h_d^{ij}\widehat{D}^c_j\widehat{Q}_i\cdot\widehat{H}_d + \mu\widehat{H}_u\cdot\widehat{H}_d \tag{2.1}$$

where the first three terms are Yukawa interactions of leptons and quarks, and the last term is self interaction of Higgs fields.

The MSSM was first proposed in 1981 to stabilize the electroweak scale solving the Hierarchy problem of the SM. The Higgs mass of the SM is quadratically divergent (Λ_{UV}^2, where Λ_{UV} is the scale of new physics) and unstable to quantum corrections leading to a weaker electroweak scale than what is observed to be. In the MSSM, the existence of superpartners provide the Higgs boson to inherit stability from its superpartner Higgsino by cancelling the huge contribution coming from quantum corrections. When the Supersymmetry is broken the divergent Λ_{UV}^2 is replaced by $g^2(\tilde{m}^2 - m^2)$, where $\tilde{m}$ is a superpartner mass, m is a typical SM mass and g is the electroweak coupling strength. This also implies that, for not regenerating the quadratic divergency, superpartners must not weigh much larger than the SM particles (roughly below TeV). Therefore the Supersymmetry must rather be softly broken, and consequently soft symmetry breaking operators are introduced in the MSSM.

2.1. Gauge Couplings Unification

In analogy with the unification of electricity and magnetism into electromagnetism in 19th century and especially with the success of electroweak theory which utilizes spontaneous symmetry breaking to unify electromagnetism with weak interaction, people wondered if it might be possible to unify all three groups in a similar manner. This idea became one of the most attractive and strongest predictions of the MSSM. The three extrapolated energy dependent (running) coupling constants of the electroweak and strong forces seem to unify at high energies ($\sim 10^{16}$ GeV.) near Planck scale. This phenomena also gives rise to the idea of a common origin of all fundamental forces of nature. In the SM, such a unification can not be observed at any energy scale.

2.2. Why do we need to extend the MSSM ?

The Minimal Supersymmetric Standard Model (MSSM), devised to solve the gauge hierarchy problem of the standard model of electroweak interactions (SM), suffers from a serious naturalness problem associated with the Dirac mass of Higgsinos in

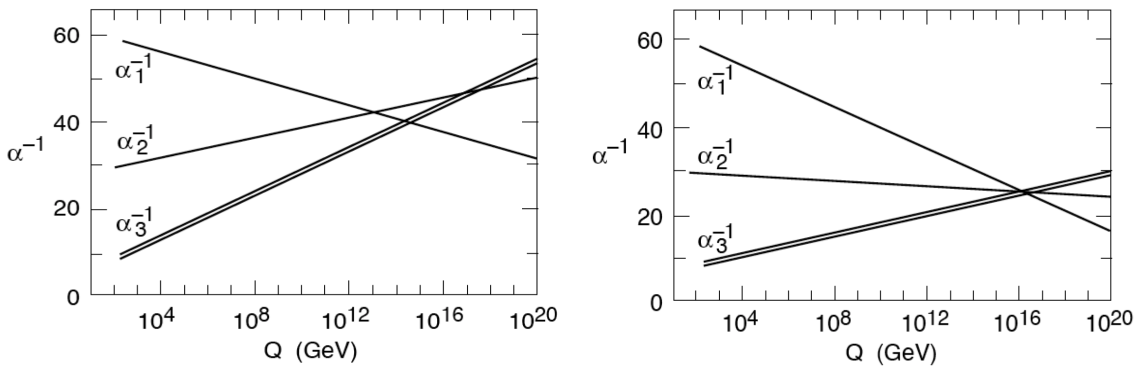

Figure 2.1. Evaluation of $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge couplings to high energy scales in Standard model (left panel) and Supersymmetry (right panel)

the superpotential.

$$\widehat{W} \supset \mu \widehat{H}_u \cdot \widehat{H}_d \tag{2.2}$$

The μ parameter here is nested in the supersymmetric sector of the theory, and its scale is left completely arbitrary as it is not related to the soft supersymmetry-breaking terms (Kim and Nilles 1984, Suematsu and Yamagishi 1995, Jain and Shrock 1995, Nir 1995, Cvetic and Langacker 1996). Having a mass dimension, the μ parameter generates a naturalness problem, since all the natural coefficients have to be dimensionless parameters. A way out of this problem is to generate μ parameter dynamically via the vacuum expectation value (VEV) of some SM-singlet chiral superfield. The extension by a non-SM chiral superfield may or may not involve gauge extension. Concerning the former, the most conservative approach is to extend the gauge structure of the MSSM by an extra Abelian group factor U(1)$'$ along with an additional chiral superfield $\widehat{S}$ whose scalar component generates an effective μ parameter upon spontaneous U(1)$'$ breakdown. The new superpotential then becomes;

$$\widehat{W}_{NEW} = h_e^{ij} \widehat{E}_j^c \widehat{L}_i \cdot \widehat{H}_d + h_u^{ij} \widehat{U}_j^c \widehat{Q}_i \cdot \widehat{H}_u + h_d^{ij} \widehat{D}_j^c \widehat{Q}_i \cdot \widehat{H}_d + h_s \widehat{S} \widehat{H}_u \cdot \widehat{H}_d \tag{2.3}$$

where the μ parameter is replaced by the SM singlet $\widehat{S}$, coupled to SM doublets $\widehat{H}_u$ and $\widehat{H}_d$, whose vacuum expectation value (VEV) along the breakdown of U(1)$'$ generates an effective μ term, that is $\mu_s = h_s \langle \widehat{S} \rangle$. This provides a dynamical solution to the μ problem when $\langle \widehat{S} \rangle \sim \mathcal{O}$ (TeV). What this additional gauge symmetry actually does is to forbid the presence of a bare μ parameter as in (Equation 2.1) (Hewett and Rizzo 1989, Cvetic and Langacker 1996, Hill and Simmons 2002). An important property of U(1)$'$ models is that the lightest Higgs boson weighs significantly heavier than M_Z even at tree level with small $\tan\beta$. Hence the existing LEP bounds (LEP Coll. 2003 , ALEPH Coll. 2005) are satisfied with almost no need for large radiative corrections (Cvetic, et al. 1997, Demir and Pak 1998, Demir and Everett 2004, Han, et al. 2004, Amini 2003). Besides, they offer a rather wide parameter space for facilitating the electroweak baryogenesis (Kang, et al. 2004).

An important problem in U(1)′ models concerns the cancellation of anomalies. Indeed, for making the theory anomaly–free the usual approach to U(1)′ models is to add several exotics to the spectrum (Erler 2000). This naturally happens in U(1)′ models following from SUSY GUTs $e.g.$ E_6 unification. However, this not only causes a significant departure from the minimal structure but also disrupts the gauge couplings unification – one of the fundamental predictions of the MSSM with weak scale soft masses. Therefore, it would be of greatest interest to keep gauge couplings unification with minimal matter content. This has been accomplished in (Demir, et al. 2005) by introducing family non-universal U(1)′ charges in a way solving all anomaly conditions, including the gravitational one.

In this work, we will discuss dilepton signatures of U(1)′ models with universal as well as non-universal U(1)′ charges in a comparative fashion. Our discussion will include both lepton (the ILC) and hadron (the LHC) colliders. At the Born level the cross sections are sensitive to Z′exchange only. Therefore, our analysis will have examined Z′properties via dilepton signal. The collider signatures of various U(1)′ models have already been analyzed in the literature (Fiandrino and Taxil 1991, Aguila, et al. 1993, Aguila and Cvetic 1994, Leike 1997, Taxil, et al. 2002, Appelquist, et al. 2003, Carena, et al. 2004, Kang and Langacker 2005,). In addition, the U(1)′ models have also been tested under electroweak precision bounds (Amaldi, et al. 1987, Langacker, et al. 1992, Erler and Langacker 1999, Erler and Langacker 2000).

CHAPTER 3

WHAT IS U(1)$'$ MODEL ?

In U(1)$'$ models the MSSM gauge group is extended to include an extra Abelian group factor at the weak scale: SU(3)$_C$×SU(2)$_L$×U(1)$_Y$×U(1)$'$ with respective gauge couplings g_3, g_2, g_1 and g_1'. The particle spectrum of the model is that of the MSSM plus a SM gauge singlet S charged under only the U(1)$'$ invariance. We employ a rather general U(1)$'$ charge assignment as tabulated in Table 3.1.

Table 3.1. The gauge quantum numbers of chiral superfields of i-th family

	SU(3)$_c$	SU(2)$_L$	U(1)$_Y$	U(1)$'$
Q_i	**3**	**2**	$1/6$	Q'_{Q_i}
U_i^c	$\bar{3}$	**1**	$-2/3$	$Q'_{U_i^c}$
D_i^c	$\bar{3}$	**1**	$1/3$	$Q'_{D_i^c}$
L_i	**1**	**2**	$-1/2$	Q'_{L_i}
E_i^c	**1**	**1**	1	$Q'_{E_i^c}$
H_u	**1**	**2**	$1/2$	Q'_{H_u}
H_d	**1**	**2**	$-1/2$	Q'_{H_d}
S	**1**	**1**	0	Q'_S

There are several sources of U(1)$'$, mainly from Superstrings, Grand Unified Theories (GUT), Extra Dimensions, Dynamical Symmetry Breaking, Little Higgs Models and Stuckelberg Mechanism. Basically U(1)$'$ models are low energy manifestations of these theories. At low energies ,however ,gauge and gravitational triangle anomalies appear in the theory, cancellation of which requires the existence of exotic matter. This not only causes a significant departure from the minimal structure but also disrupts the unification of gauge couplings. Therefore, keeping the theory anomaly-free with minimal matter content and allowing the gauge couplings unification we will focus on family non-universality of charges under U(1)$'$ invariance.

In this work we will study on family non-universal U(1)$'$ model in comparison with models from GUTs . As an example, E(6) ,descending from supersymmetric GUTs, can be broken down to SM gauge structure with an additional U(1),

$$E(6) \to SO(10) \times U(1)_\psi \to SU(5) \times U(1)_\chi \times U(1)_\psi \to G_{SM} \times U(1)'$$

where U(1)′ is a linear combination of $U(1)_\chi$ and $U(1)_\psi$ symmetries,

$$U(1)' = \cos(\theta)U(1)_\psi - \sin(\theta)U(1)_\chi \tag{3.1}$$

with mixing angle θ. Depending on the value of mixing angle there are various models.

Table 3.2. Specific U(1)′ models with $\alpha_1 = (3/8)^{1/2}$ and $\alpha_2 = -(5/8)^{1/2}$

θ	0	-$\pi/2$	arcsin (α_1)	arcsin (α_2)
$U(1)'$	$U(1)_\psi$	$U(1)_\chi$	$U(1)_\eta$	$U(1)_I$

In analyzing the collider signatures of dilepton productions at lepton and hadron colliders we will take $U(1)_\eta$ model to compare with family non-universal U(1)′.

3.1. Anomaly Cancellation and Charge Assignment

One of the most important issues in U(1)′ models is the cancellation of gauge and gravitational triangle anomalies. In fact, it has been shown that (Cheng, et al. 1998, Cheng, et al. 1999, Erler 2000) a number of exotics have to be added to the minimal spectrum for making the theory anomaly-free. However the presence of these additional fields usually destroys the unification of gauge couplings. In this section we will briefly discuss the family dependency of charges under U(1)′ in canceling those triangle anomalies without additional fields preserving the unification of gauge couplings.

As shown in (Demir, et al. 2005), the general U(1)′ charge assignment suffices to solve all anomaly cancellation conditions in a way respecting the gauge invariance of the superpotential. In fact, one finds the solutions (Demir, et al. 2005)

$$\begin{aligned}
Q'_{Q_1} &= Q'_{Q_2} = Q'_{Q_3} = \frac{1}{9}(3Q'_{E_2^c} + 3Q'_{L_2} + Q'_S)\,, \\
Q'_{D_1^c} &= Q'_{D_2^c} = Q'_{D_3^c} = \frac{1}{9}(6Q'_{E_2^c} + 6Q'_{L_2} - Q'_S)\,, \\
Q'_{U_1^c} &= Q'_{U_2^c} = Q'_{U_3^c} = \frac{1}{9}(-12Q'_{E_2^c} - 12Q'_{L_2} - Q'_S)\,, \\
Q'_{L_1} &= -2Q'_{E_2^c} - 3Q'_{L_2}\,,\;\; Q'_{L_3} = -Q'_{E_2^c} - Q'_{L_2}\,, \\
Q'_{E_1^c} &= 3Q'_{E_2^c} + 4Q'_{L_2}\,,\;\; Q'_{E_3^c} = 2Q'_{E_2^c} + 2Q'_{L_2} + Q'_S\,, \\
Q'_{H_d} &= -Q'_{E_2^c} - Q'_{L_2} - Q'_S\,,\;\; Q'_{H_u} = Q'_{E_2^c} + Q'_{L_2}
\end{aligned} \tag{3.2}$$

in terms of the three free charges:

$$Q'_{L_2} = 2 \, , \;\; Q'_{E_2^c} = -3 \, , \;\; Q'_S = 3 \, , \tag{3.3}$$

In this charge assignment it can easily be seen that family non-universality resides only in leptonic sector, that is hadronic part is kept family universal. It is known that different U(1)′ charges for different families lead to a large Z′mediated flavor changing neutral currents (FCNC) (Langacker and Plumacher 2000, Barger, et al. 2004), in hadronic sector FCNCs are suppressed by keeping quark charges family universal under U(1)′;

$$\begin{aligned} Q'_{Q_1} &= Q'_{Q_2} = Q'_{Q_3} \\ Q'_{D_1^c} &= Q'_{D_2^c} = Q'_{D_3^c} \\ Q'_{U_1^c} &= Q'_{U_2^c} = Q'_{U_3^c} \end{aligned}$$

However, in leptonic sector U(1)′ charges are assigned in such a way that they forbid off-diagonal terms in leptons mass matrix. Hence, with identical mass and gauge eigenstates, FCNCs will automatically be absent.

In above charge assignment there is one more aspect needs to be mentioned which is quite important. Family dependence of charges under U(1)′ invariance forbids certain Yukawa couplings in the superpotential, leading to massless fermions in the theory. However, the requisite fermion masses can be induced at loop level by non-holomorphic operators in the soft breaking sector (Hall and Randall 1990, Borzumati, et al. 1999, Demir, et al. 2005). In certain cases, some fermions can not gain their masses neither at tree level nor at any loop level with holomorphic operators, therefore non-holomorphic soft supersymmetry breaking operators necessarily be introduced. Depending on the choice of three free charges the structure of non-holomorphic operators changes, that is which fermions, whose masses are induced by non-holomorphic operators, are decided by the selection of three free charges. A recent work on ”Higgs Boson and Neutrino masses with non-holomorphic operators” is in (Demir, et al. 2007).

3.2. Parametrization

The theory consists of three gauge bosons: the photon, the Z boson and the Z′boson. We parameterize couplings of these vector bosons to fermions via the effective lagrangian (Aguila, et al. 1987):

Table 3.3. The vector boson couplings to fermions with family universal U(1)′. The U(1)′ couplings here are those of $U(1)_\eta$ descending from E(6) supersymmetric GUT (Source: Kang and Langacker 2005)

	γ		Z		Z′	
	v	a	v	a	v	a
ν_e, ν_μ, ν_τ	0	0	1	1	$-\sin\theta_W/3$	$-\sin\theta_W/3$
e^-, μ^-, τ^-	-1	0	$-1 + 4\sin^2\theta_W$	-1	$-\sin\theta_W$	$\sin\theta_W/3$
u, c, t	$2/3$	0	$1 - 8\sin^2\theta_W/3$	1	0	$4\sin\theta_W$
d, s, b	$-1/3$	0	$-1 + 4\sin^2\theta_W/3$	-1	$\sin\theta_W$	$\sin\theta_W/3$

Table 3.4. The vector boson couplings to fermions with family non-universal U(1)′. The U(1)′ charges are determined by using (Equation 3.2) and by the normalization condition that ${g'_1}^2\mathrm{Tr}[{Q'}^2]$ to be equal to the same quantity computed in $U(1)_\eta$ model and the normalization factor $C_{Z'}$ is evaluated as $\sqrt{\frac{5}{52}}$

	γ		Z		Z′	
	v	a	v	a	v	a
ν_e	0	0	1	1	$2\sin\theta_W C_{Z'}$	$-2\sin\theta_W C_{Z'}$
ν_μ	0	0	1	1	$10\sin\theta_W C_{Z'}$	$-2\sin\theta_W C_{Z'}$
ν_τ	0	0	1	1	0	$4\sin\theta_W C_{Z'}$
e^-	-1	0	$-1 + 4\sin^2\theta_W$	-1	$2\sin\theta_W C_{Z'}$	$-2\sin\theta_W C_{Z'}$
μ^-	-1	0	$-1 + 4\sin^2\theta_W$	-1	$10\sin\theta_W C_{Z'}$	$-2\sin\theta_W C_{Z'}$
τ^-	-1	0	$-1 + 4\sin^2\theta_W$	-1	0	$4\sin\theta_W C_{Z'}$
u, c, t	$2/3$	0	$1 - 8\sin^2\theta_W/3$	1	$-2\sin\theta_W C_{Z'}$	$2\sin\theta_W C_{Z'}$
d, s, b	$-1/3$	0	$-1 + 4\sin^2\theta_W/3$	-1	$2\sin\theta_W C_{Z'}$	$-2\sin\theta_W C_{Z'}$

$$\mathcal{L}_{eff} = \frac{g_2}{4\cos\theta_W}\sum_i \bar{f}_i\gamma^\mu\left(v_V^f - a_V^f\gamma^5\right)f_i\mathrm{V}_\mu \tag{3.4}$$

where $V = \gamma, \mathrm{Z}, \mathrm{Z}'$, and f_i stands for any of the quarks or leptons. The U(1)′ gauge coupling g_1' is included in the vector couplings v_V^f and axial-vector couplings a_V^f via the relations

$$v_{Z'}^f = 2\cos\theta_W\left(Q'_{f_L} - Q'_{f_R}\right)\frac{g_1'}{g_2} \quad , \quad a_{Z'}^f = 2\cos\theta_W\left(Q'_{f_L} + Q'_{f_R}\right)\frac{g_1'}{g_2} \tag{3.5}$$

where θ_W is the Weinberg angle, and Q'_{f_L} and Q'_{f_R} are U(1)′ charges of left– and right–handed fermions, respectively.

In writing (Equation 3.2) we have neglected the mixing between Z and Z′bosons. This mixing can stem from kinetic mixing or can be induced after electroweak breaking (Cvetic, et al. 1997, Babu, et al. 1998).In this work we neglect such mixings in accord with the experimental bounds that $\alpha_{\mathrm{Z}-\mathrm{Z}'}$ cannot exceed a few 10^{-3}. This smallness of the mixing puts stringent bounds on the ranges of the soft-breaking masses as it was analyzed in detail in (Cvetic, et al. 1997, Demir, et al. 2005).

CHAPTER 4

DILEPTON SIGNATURES OF U(1)′

In this section we will analyze the family non-universal U(1)′ model by considering its signatures for dilepton production at lepton and hadron colliders, separately. We will investigate distinctive signatures of the U(1)′ model under concern with respect to a typical family universal U(1)′ model which we choose to be the $\mathrm{U}(1)_\eta$ model following from E(6) GUT. The requisite vector and axial-vector couplings of photon, Z and Z′bosons are tabulated in Table 3.3 and Table 3.4 for family universal and non-universal models, respectively.

4.1. Transition Amplitude of the Scattering Process

In general, the $2 \to 2$ scattering process $f\,\bar{f} \to \ell^+\ell^-$ is

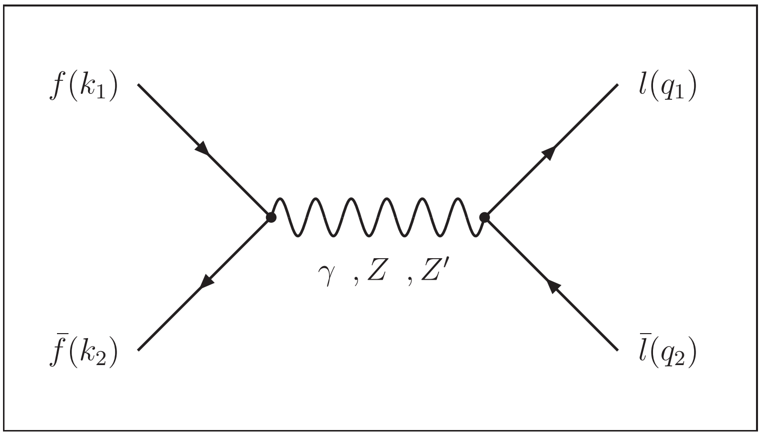

Figure 4.1. A generic scattering process

where f stands for quarks (hadron colliders) or leptons (lepton colliders) carrying momentums k_1, k_2 and ℓ for any of the charged leptons with momentums q_1, q_2. This process proceeds with γ, Z and Z′exchanges in the s-channel when ℓ is not identical to f, and in both s and t channels when $f \equiv \ell$. If center of mass energy of the collider is high enough then Z′effects can be disentangled from those of γ and Z.

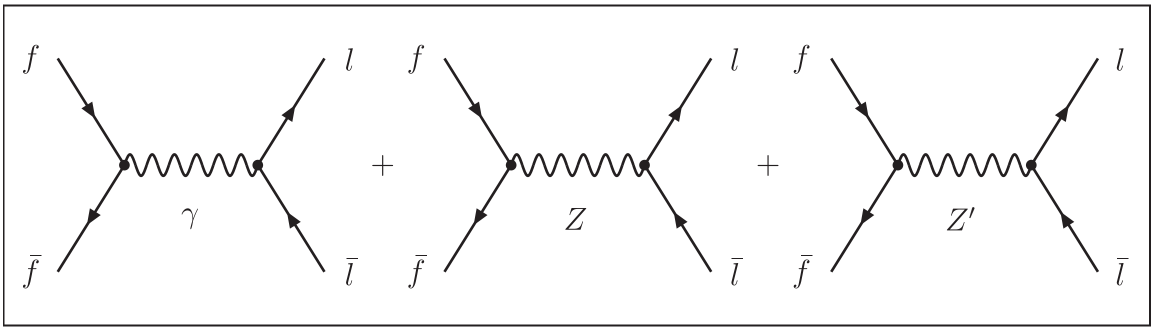

Figure 4.2. Contributions of three vector bosons

The transition amplitude of each processes is labeled as $\mathcal{A}_\gamma, \mathcal{A}_\mathcal{Z}$ and $\mathcal{A}_{\mathcal{Z}'}$ respectively.

$$\mathcal{A}_\gamma = \bar{f}\gamma^\mu\,(i\,Q_f\,e)\;f\cdot\left(\frac{-i\,g_{\mu\nu}}{s-m_\gamma^2+im_\gamma\Gamma_\gamma}\right)\cdot l\,\gamma^\nu\,(i\,Q_l\,e)\,\bar{l}$$

$$\mathcal{A}_Z = G^2\bar{f}\gamma^\mu[v_Z^f-a_Z^f\gamma^5]f\cdot\left(\frac{i\,g_{\mu\nu}}{s-m_Z^2+im_Z\Gamma_Z}\right)\cdot l\gamma^\nu[v_Z^l-a_Z^l\gamma^5]\,\bar{l} \tag{4.1}$$

$$\mathcal{A}_{Z'} = G^2\bar{f}\gamma^\mu[v_{Z'}^f-a_{Z'}^f\gamma^5]f\cdot\left(\frac{i\,g_{\mu\nu}}{s-m_{Z'}^2+im_{Z'}\Gamma_{Z'}}\right)\cdot l\gamma^\nu[v_{Z'}^l-a_{Z'}^l\gamma^5]\,\bar{l}$$

where G = $g_2/4\cos\theta_W$, s is invariant mass, $m_\gamma = 0$ and $\Gamma_\gamma = 0$ since photon is massless and totally stable and $g_{\mu\nu}$ is the metric tensor. Vector and axial-vector couplings of Z and Z′ bosons are parameterized as in Table 3.3 and Table 3.4.

Switching to spinor representation;

$$\mathcal{A}_\gamma = \bar{v}(k_2)\gamma^\mu\,(i\,Q_f\,e)\;u(k_1)\cdot\left(\frac{-i\,g_{\mu\nu}}{s-m_\gamma^2+im_\gamma\Gamma_\gamma}\right)\cdot\bar{u}(q_1)\,\gamma^\nu\,(i\,Q_l\,e)\,v(q_2)$$

$$\mathcal{A}_Z = G^2\bar{v}(k_2)\gamma^\mu[v_Z^f-a_Z^f\gamma^5]u(k_1)\cdot\left(\frac{i\,g_{\mu\nu}}{s-m_Z^2+im_Z\Gamma_Z}\right)\cdot\bar{u}(q_1)\gamma^\nu[v_Z^l-a_Z^l\gamma^5]\,v(q_2)$$

$$\mathcal{A}_{Z'} = G^2\bar{v}(k_2)\gamma^\mu[v_{Z'}^f-a_{Z'}^f\gamma^5]u(k_1)\cdot\left(\frac{i\,g_{\mu\nu}}{s-m_{Z'}^2+im_{Z'}\Gamma_{Z'}}\right)\cdot\bar{u}(q_1)\gamma^\nu[v_{Z'}^l-a_{Z'}^l\gamma^5]\,v(q_2) \tag{4.2}$$

The total transition amplitude including the contributions of all these three vector bosons and its squared are;

$$\mathcal{A}\left(f\,\bar{f} \rightarrow \ell^{+}\ell^{-}\right) = \mathcal{A}_{\gamma} + \mathcal{A}_{\mathcal{Z}} + \mathcal{A}_{\mathcal{Z}'} \tag{4.3}$$

$$|\mathcal{A}\left(f\,\bar{f} \rightarrow \ell^{+}\ell^{-}\right)|^{2} = \mathcal{A}\left(f\,\bar{f} \rightarrow \ell^{+}\ell^{-}\right) \times \mathcal{A}\left(f\,\bar{f} \rightarrow \ell^{+}\ell^{-}\right)^{\star} \tag{4.4}$$

with conjugate transposed of each amplitude being;

$$\mathcal{A}_{\gamma}^{\dagger} = \bar{v}(q_2)\gamma^{\alpha}\,(-i\;Q_l\;e)\;u(q_1)\cdot\left(\frac{i\;g_{\alpha\beta}}{s-m_{\gamma}^{2}-im_{\gamma}\Gamma_{\gamma}}\right)\cdot\bar{u}(k_1)\,\gamma^{\beta}\,(-i\;Q_f\;e)\,v(k_2)$$

$$\mathcal{A}_{Z}^{\dagger} = G^{2}\bar{v}(q_2)\gamma^{\alpha}[v_{Z}^{l}+a_{Z}^{l}\gamma^{5}]u(q_1)\cdot\left(\frac{-i\;g_{\alpha\beta}}{s-m_{Z}^{2}-im_{Z}\Gamma_{Z}}\right)\cdot\bar{u}(k_1)\gamma^{\beta}[v_{Z}^{f}+a_{Z}^{f}\gamma^{5}]\,v(k_2)$$

$$\mathcal{A}_{Z'}^{\dagger} = G^{2}\bar{v}(q_2)\gamma^{\alpha}[v_{Z'}^{l}+a_{Z'}^{l}\gamma^{5}]u(q_1)\cdot\left(\frac{-i\;g_{\alpha\beta}}{s-m_{Z'}^{2}-im_{Z'}\Gamma_{Z'}}\right)\cdot\bar{u}(k_1)\gamma^{\beta}[v_{Z'}^{f}+a_{Z'}^{f}\gamma^{5}]\,v(k_2) \tag{4.5}$$

Then, using the projection operators the amplitude-squared becomes;

$$
\begin{aligned}
|\mathcal{A}|^2 &= \left(\frac{Q_l^2 Q_f^2 e^4}{s^2}\right) Tr[(\not{k}_2 - m_f)\gamma^\mu(\not{k}_1 + m_f)\gamma_\alpha] Tr[(\not{q}_1 + m_l)\gamma_\mu(\not{q}_2 - m_l)\gamma^\alpha] \\
&+ G^2\left(\frac{Q_l Q_f e^2}{s(s - m_Z^2 - i m_Z \Gamma_Z)}\right) \\
&\times Tr[(\not{k}_2 - m_f)\gamma^\mu(\not{k}_1 + m_f)[v_Z^f + a_Z^f\gamma^5]\gamma_\alpha] Tr[(\not{q}_1 + m_l)\gamma_\mu](\not{q}_2 - m_l)[v_Z^l + a_Z^l\gamma^5] \\
&+ G^2\left(\frac{Q_l Q_f e^2}{s(s - m_{Z'}^2 - i m_{Z'} \Gamma_{Z'})}\right) \\
&\times Tr[(\not{k}_2 - m_f)\gamma^\mu(\not{k}_1 + m_f)[v_{Z'}^f + a_{Z'}^f\gamma^5]\gamma_\alpha] Tr[(\not{q}_1 + m_l)\gamma_\mu](\not{q}_2 - m_l)[v_{Z'}^l + a_{Z'}^l\gamma^5] \\
&+ G^2\left(\frac{Q_l Q_f e^2}{s(s - m_Z^2 + i m_Z \Gamma_Z)}\right) \\
&\times Tr[(\not{k}_2 - m_f)\gamma^\mu[v_Z^f - a_Z^f\gamma^5](\not{k}_1 + m_f)\gamma_\alpha] Tr[(\not{q}_1 + m_l)\gamma_\mu[v_Z^l - a_Z^l\gamma^5](\not{q}_2 - m_l)\gamma^\alpha] \\
&+ G^4\left(\frac{1}{(s - m_Z^2 + i m_Z \Gamma_Z)(s - m_Z^2 - i m_Z \Gamma_Z)}\right) \\
&\times Tr[(\not{k}_2 - m_f)\gamma^\mu[v_Z^f - a_Z^f\gamma^5](\not{k}_1 + m_f)[v_Z^f + a_Z^f\gamma^5]\gamma_\alpha] \\
&\times Tr[(\not{q}_1 + m_l)\gamma_\mu[v_Z^l - a_Z^l\gamma^5](\not{q}_2 - m_l)[v_Z^l + a_Z^l\gamma^5]\gamma^\alpha] \\
&+ G^4\left(\frac{1}{(s - m_Z^2 + i m_Z \Gamma_Z)(s - m_{Z'}^2 - i m_{Z'} \Gamma_{Z'})}\right) \\
&\times Tr[(\not{k}_2 - m_f)\gamma^\mu[v_Z^f - a_Z^f\gamma^5](\not{k}_1 + m_f)[v_{Z'}^f + a_{Z'}^f\gamma^5]\gamma_\alpha] \\
&\times Tr[(\not{q}_1 + m_l)\gamma_\mu[v_Z^l - a_Z^l\gamma^5](\not{q}_2 - m_l)[v_{Z'}^l + a_{Z'}^l\gamma^5]\gamma^\alpha] \\
&+ G^2\left(\frac{Q_l Q_f e^2}{s(s - m_{Z'}^2 + i m_{Z'} \Gamma_{Z'})}\right) \\
&\times Tr[(\not{k}_2 - m_f)\gamma^\mu[v_{Z'}^f - a_{Z'}^f\gamma^5](\not{k}_1 + m_f)\gamma_\alpha] Tr[(\not{q}_1 + m_l)\gamma_\mu[v_{Z'}^l - a_{Z'}^l\gamma^5](\not{q}_2 - m_l)\gamma^\alpha] \\
&+ G^4\left(\frac{1}{(s - m_{Z'}^2 + i m_{Z'} \Gamma_{Z'})(s - m_Z^2 - i m_Z \Gamma_Z)}\right) \\
&\times Tr[(\not{k}_2 - m_f)\gamma^\mu[v_{Z'}^f - a_{Z'}^f\gamma^5](\not{k}_1 + m_f)[v_Z^f + a_Z^f\gamma^5]\gamma_\alpha] \\
&\times Tr[(\not{q}_1 + m_l)\gamma_\mu[v_{Z'}^l - a_{Z'}^l\gamma^5](\not{q}_2 - m_l)[v_Z^l + a_Z^l\gamma^5]\gamma^\alpha] \\
&+ G^4\left(\frac{1}{(s - m_{Z'}^2 + i m_{Z'} \Gamma_{Z'})(s - m_{Z'}^2 - i m_{Z'} \Gamma_{Z'})}\right) \\
&\times Tr[(\not{k}_2 - m_f)\gamma^\mu[v_{Z'}^f - a_{Z'}^f\gamma^5](\not{k}_1 + m_f)[v_{Z'}^f + a_{Z'}^f\gamma^5]\gamma_\alpha] \\
&\times Tr[(\not{q}_1 + m_l)\gamma_\mu[v_{Z'}^l - a_{Z'}^l\gamma^5](\not{q}_2 - m_l)[v_{Z'}^l + a_{Z'}^l\gamma^5]\gamma^\alpha]
\end{aligned}
\tag{4.6}
$$

Since we are studying at very high energies we can count all fermions as massless ($k_1^2 = k_2^2 = q_1^2 = q_2^2 = 0$). Then, by the help of trace theorems and identities of gamma matrices the amplitude-squared becomes;

$$
\begin{aligned}
|\mathcal{A}|^2 &= \left(\frac{8Q_l^2Q_f^2e^4}{s^2}\right)(u^2+t^2) \\
&+ G^2\left(\frac{Q_lQ_fe^2}{s(s-m_Z^2-im_Z\Gamma_Z)}\right)8\left[v_Z^fv_Z^l(u^2+t^2)+a_Z^fa_Z^l(u^2-t^2)\right] \\
&+ G^2\left(\frac{Q_lQ_fe^2}{s(s-m_{Z'}^2-im_{Z'}\Gamma_{Z'})}\right)8\left[v_{Z'}^fv_{Z'}^l(u^2+t^2)+a_{Z'}^fa_{Z'}^l(u^2-t^2)\right] \\
&+ G^2\left(\frac{Q_lQ_fe^2}{s(s-m_Z^2+im_Z\Gamma_Z)}\right)8\left[v_Z^fv_Z^l(u^2+t^2)+a_Z^fa_Z^l(u^2-t^2)\right] \\
&+ G^4\left(\frac{1}{(s-m_Z^2+im_Z\Gamma_Z)(s-m_Z^2-im_Z\Gamma_Z)}\right) \\
&\times 8\left[({v_Z^f}^2{v_Z^l}^2+{v_Z^f}^2{a_Z^l}^2+{a_Z^f}^2{v_Z^l}^2+{a_Z^f}^2{a_Z^l}^2)(u^2+t^2)+4(v_Z^fa_Z^fv_Z^la_Z^l)(u^2-t^2)\right] \\
&+ G^4\left(\frac{1}{(s-m_Z^2+im_Z\Gamma_Z)(s-m_{Z'}^2-im_{Z'}\Gamma_{Z'})}\right) \\
&\times \{8[(v_Z^fv_{Z'}^fv_Z^lv_{Z'}^l+v_Z^fv_{Z'}^fa_Z^la_{Z'}^l+a_Z^fa_{Z'}^fv_Z^lv_{Z'}^l+a_Z^fa_{Z'}^fa_Z^la_{Z'}^l)(u^2+t^2)] \\
&+ \quad 8[(v_Z^fa_{Z'}^fv_Z^la_{Z'}^l+v_Z^fa_{Z'}^fa_Z^lv_{Z'}^l+a_Z^fv_{Z'}^fv_Z^la_{Z'}^l+a_Z^fv_{Z'}^fa_Z^lv_{Z'}^l)(u^2-t^2)]\} \\
&+ G^2\left(\frac{1}{s(s-m_{Z'}^2+im_{Z'}\Gamma_{Z'})}\right)8\left[v_{Z'}^fv_{Z'}^l(u^2+t^2)+a_{Z'}^fa_{Z'}^l(u^2-t^2)\right] \\
&+ G^4\left(\frac{1}{(s-m_{Z'}^2+im_{Z'}\Gamma_{Z'})(s-m_Z^2-im_Z\Gamma_Z)}\right) \\
&\times \{8[(v_{Z'}^fv_Z^fv_{Z'}^lv_Z^l+v_{Z'}^fv_Z^fa_{Z'}^la_Z^l+a_{Z'}^fa_Z^fv_{Z'}^lv_Z^l+a_{Z'}^fa_Z^fa_{Z'}^la_Z^l)(u^2+t^2)] \\
&+ \quad 8[(v_{Z'}^fa_Z^fv_{Z'}^la_Z^l+v_{Z'}^fa_Z^fa_{Z'}^lv_Z^l+a_{Z'}^fv_Z^fv_{Z'}^la_Z^l+a_{Z'}^fv_Z^fa_{Z'}^lv_Z^l)(u^2-t^2)]\} \\
&+ G^4\left(\frac{1}{(s-m_{Z'}^2+im_{Z'}\Gamma_{Z'})(s-m_{Z'}^2-im_{Z'}\Gamma_{Z'})}\right) \\
&\times 8\left[({v_{Z'}^f}^2{v_{Z'}^l}^2+{v_{Z'}^f}^2{a_{Z'}^l}^2+{a_{Z'}^f}^2{v_{Z'}^l}^2+{a_{Z'}^f}^2{a_{Z'}^l}^2)(u^2+t^2)+4(v_{Z'}^fa_{Z'}^fv_{Z'}^la_{Z'}^l)(u^2-t^2)\right]
\end{aligned}
$$

We need to take average over initial-state polarizations because of unpolarized incoming beams and sum over final-state ones since our detectors can probe final-state polarizations, then the amplitude-squared of the generic proceess (Equation 4.4) takes the form

$$
\begin{aligned}
\langle\ |\mathcal{A}\left(f\,\bar{f}\rightarrow\ell^+\ell^-\right)|^2\rangle_{polar.} &= F(s;v,a)\,[(s+t)^2+t^2] \\
&+ G(s;v,a)\,[(s+t)^2-t^2] \qquad (4.7)
\end{aligned}
$$

where $F(s;v,a)$ and $G(s;v,a)$ are given by (Aguila, et al. 1987)

$$F(s;v,a)=2\sum_{\alpha,\beta}\frac{(v_\alpha^f\,v_\beta^f+a_\alpha^f\,a_\beta^f)\,(v_\alpha^l\,v_\beta^l+a_\alpha^l\,a_\beta^l)}{(s-M_\alpha^2+iM_\alpha\Gamma_\alpha)(s-M_\beta^2-iM_\beta\Gamma_\beta)}$$

and

$$G(s;v,a)=2\sum_{\alpha,\beta}\frac{(v_\alpha^f\,a_\beta^f+v_\beta^f\,a_\alpha^f)\,(v_\alpha^l\,a_\beta^l+v_\beta^l\,a_\alpha^l)}{(s-M_\alpha^2+iM_\alpha\Gamma_\alpha)(s-M_\beta^2-iM_\beta\Gamma_\beta)}\,. \qquad (4.8)$$

With the invariant kinematical variables;

$$s = (k_1 + k_2)^2 = (q_1 + q_2)^2$$
$$u = (k_1 - q_2)^2 = (k_2 - q_1)^2$$
$$t = (k_1 - q_1)^2 = (k_2 - q_2)^2 \tag{4.9}$$

In these expressions α and β label intermediate vector bosons $i.e.$ γ, Z and Z′. The Γ_α designates widths of the vector bosons: $\Gamma_\gamma = 0$ (absolutely stable) and $\Gamma_{\rm Z} = 2.4952\,{\rm GeV}$. The Z′width $\Gamma_{\rm Z'}$ is a model-dependent quantity, and while making numerical estimates in what follows we will take $\Gamma_{\rm Z'} = \Gamma_{\rm Z}$. Moreover, in accord with the U(1)$_\eta$ model parameter space, we take $g_1' = g_1$.

4.2. The Linear Collider Signatures

We first examine U(1)′ model at a high-energy linear collider (such as the International Linear Collider (ILC) project under preparation) running at $\sqrt{s} = 500$ GeV. The basic processes we consider are $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$ where we discard e^+e^- final states simply for avoiding the t-channel contributions. Since leptons do not interact strongly there is only QED contributions. The differential cross section of lepton-antilepton pair production is simply given by

$$\begin{aligned} d\sigma(e^+e^- \to \ell^+\ell^-) &= \frac{1}{2E_A 2E_B |v_A - v_B|} \frac{d^3\vec{q_1}}{(2\pi)^3 2E_1} \frac{d^3\vec{q_2}}{(2\pi)^3 2E_2} \\ &\times |\mathcal{A}(e^+e^- \to \ell^+\ell^-)|^2 (2\pi)^4 \delta^{(4)}(k_1 + k_2 - q_1 - q_2) \end{aligned} \tag{4.10}$$

where k_1, k_2, q_1 and q_2 are four momenta, and with some basic assumptions;

$$E_A = k_1 \quad , \quad E_B = k_2 \quad , \quad E_1 = q_1 \quad , \quad E_2 = q_2 \quad ,$$
$$E_A = E_B \quad and \quad \vec{Q} = \vec{k_1} + \vec{k_2}$$

the differential cross section is;

$$\begin{aligned} d\sigma(e^+e^- \to \ell^+\ell^-) &= \frac{1}{32\pi^2 s} \frac{d^3\vec{q_1}}{q_1^0} \frac{d^3\vec{q_2}}{q_2^0} \\ &\times |\mathcal{A}(e^+e^- \to \ell^+\ell^-)|^2 \delta^3(\vec{Q} - \vec{q_1} - \vec{q_2}) \delta(Q^0 - q_1^0 - q_2^0) \end{aligned}$$

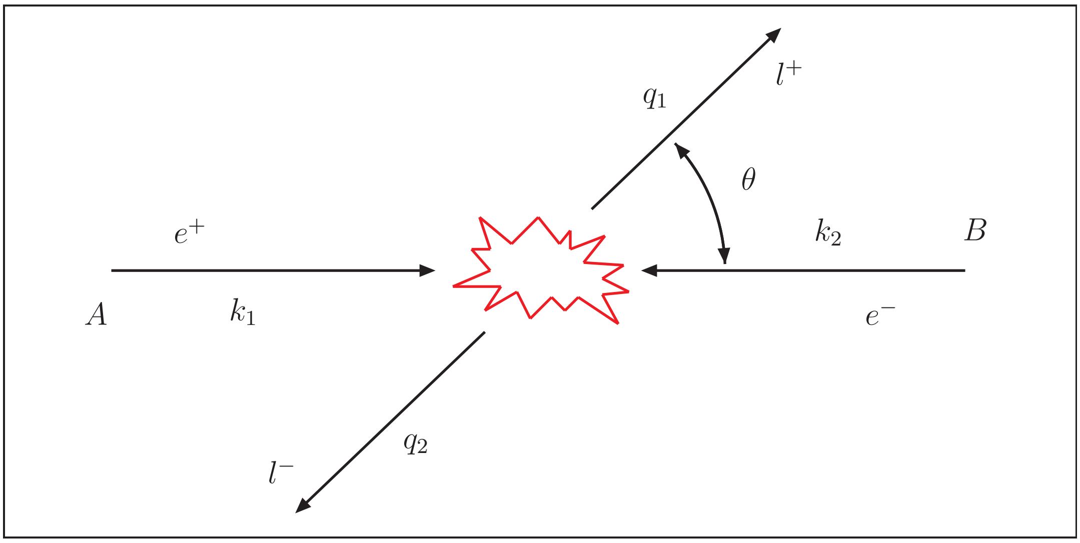

Figure 4.3. Electron-positron annihilation into lepton-antilepton pair in linear collider

At the center of mass frame;

$$\vec{k_1} = -\vec{k_2} \quad , \quad \vec{q_1} = -\vec{q_2}$$
$$k_1^0 = k_2^0 \quad , \quad q_1^0 = q_2^0$$
$$Q^0 = k_1^0 + k_2^0 = 2k_1^0 = 2k_2^0$$

it becomes

$$d\sigma(e^+e^- \to \ell^+\ell^-) = \frac{1}{64\pi^2 s}\frac{d^3\vec{q_1}}{(q_1^0)^2}|\mathcal{A}(e^+e^- \to \ell^+\ell^-)|^2\delta(k_1^0 - q_1^0) \tag{4.11}$$

the volume element then can be written in the form;

$$\begin{aligned} d^3\vec{q_1} &= |\vec{q_1}|^2 d|\vec{q_1}| \sin\theta d\theta d\varphi \\ &= q_1^{02} dq_1^0 \sin\theta d\theta d\varphi \end{aligned} \tag{4.12}$$

Thus, the differential cross section is;

$$d\sigma(e^+e^- \to \ell^+\ell^-) = \frac{1}{64\pi^2 s} dq_1^0 \sin\theta d\theta d\varphi |\mathcal{A}(e^+e^- \to \ell^+\ell^-)|^2 \delta(k_1^0 - q_1^0) \tag{4.13}$$

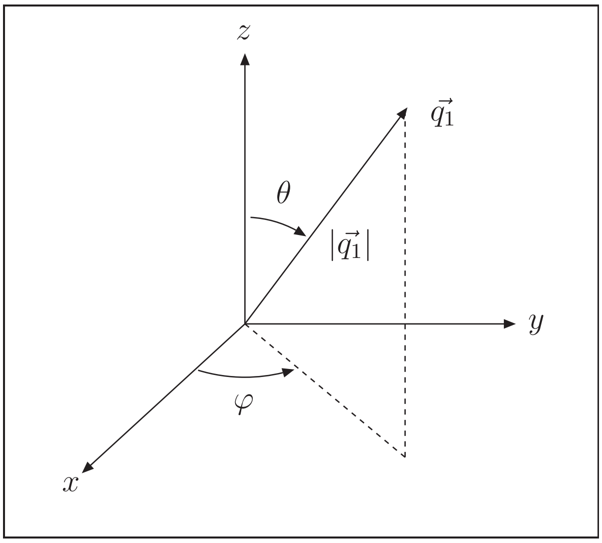

Figure 4.4. Z-axis is the direction of longitudinal momentum

Finally, the cross section of lepton-antilepton production through electron-positron annihilation is;

$$\sigma(e^+e^- \to \ell^+\ell^-) = \frac{1}{32\pi s}\int \sin\theta d\theta |\mathcal{A}(e^+e^- \to \ell^+\ell^-)| \tag{4.14}$$

Rearranging the kinematical variables in (Equation 4.9) we can get the relation;

$$t = -\frac{s}{2}(1-\cos\theta) \quad , \quad and \quad , \quad dt = \frac{s}{2}d(\cos\theta) = -\frac{s}{2}\sin\theta d\theta \tag{4.15}$$

Then substituting into (Equation 4.14), the cross section is;

$$\sigma(e^+e^- \to \ell^+\ell^-) = \frac{1}{16\pi s^2}\int_{-s}^{0} dt |\mathcal{A}(e^+e^- \to \ell^+\ell^-)|^2 \tag{4.16}$$

Depicted in Figure 4.5 are unpolarized $\mu^+\mu^-$ and $\tau^+\tau^-$ production cross sections at a future e^+e^- machine for family universal U(1)′ (in the left panel) and family non-universal U(1)′ (in the right panel) models. For family universal U(1)′ it is seen that $\sigma(e^+e^- \to \mu^+\mu^-)$ and $\sigma(e^+e^- \to \tau^+\tau^-)$ completely overlap. The main reason behind this coincidence is that μ and τ leptons do have identical gauge quantum numbers (including those of under the U(1)′ gauge symmetry) and their mass difference causes only a tiny deviation at such high energies (LEP Coll. 2003, ALEPH Coll. 2005). Consequently, from the left panel of Figure 4.5 one concludes that numbers of muons and tau leptons produced at an e^+e^- collider will be identical (up to systematic and statistical errors in analyzing the experimental data) if the new gauge symmetry, the U(1)′ symmetry under concern, exhibits identical Z′couplings for each fermion (at least lepton) family as happens in the standard electroweak theory.

In clear contrast to the left-panel of Figure 4.5, one observes that $\mu^+\mu^-$ and $\tau^+\tau^-$ differ by an order of magnitude if the U(1)′ symmetry possesses non-universal couplings to fermions (at least leptons). Indeed, $\sigma(e^+e^- \to \mu^+\mu^-)$ is larger than $\sigma(e^+e^- \to \tau^+\tau^-)$ by a factor of 6.5, and this factor is related to U(1)′ charges listed in Table 3.1 and vector and axial-vector couplings in Table 3.4. Therefore, the right-panel of Figure 4.5 alone is sufficient for concluding that the number of $\mu^+\mu^-$ and $\tau^+\tau^-$ events will significantly differ from each other if the new gauge symmetry, the U(1)′ gauge symmetry under concern, exhibits different Z′couplings to different fermion (at least lepton) families.

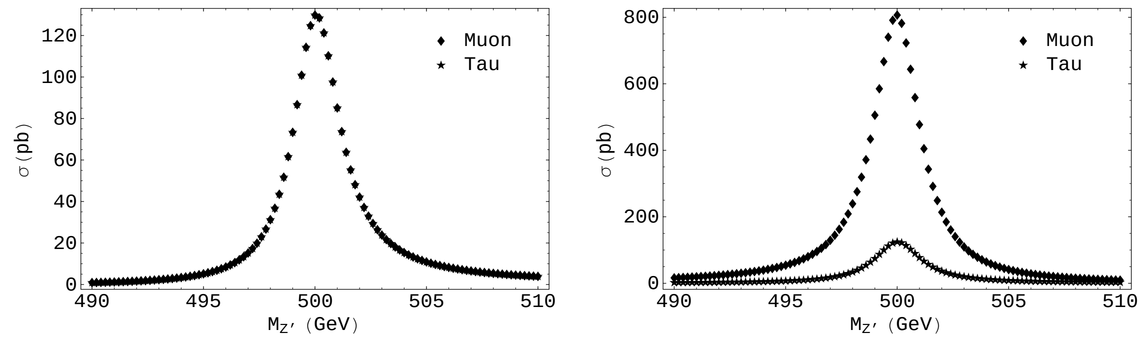

Figure 4.5. The $\mu^+\mu^-$ and $\tau^+\tau^-$ productions at a future e^+e^- collider with $\sqrt{s} = 500\,\mathrm{GeV}$ for family universal U(1)′ (in the left panel) and family non-universal U(1)′ (in the right panel) models. The ratio between family non-universal and family universal cross sections varies with model parameters

Additionally we analyze the U(1)′ model at the Large Electron-Positron (LEP) collider which is closed at 2000 with $\sqrt{s} = 209$ GeV and 140 pb^{-1} luminosity. Figure 4.6 is the production cross sections of muon and tau lepton final states with family non-universal U(1)′. It is clear in Figure 4.6 that family non-universal U(1)′ signal is quite clean and distinguishable as the muon and tau lepton production cross sections are as much as several hundreds of picobarns. However, these productions are observed to be around few picobarns in various analysis (Aguila, et al. 1993, Aguila and Cvetic 1994, Leike 1997, Appelquist, et al. 2003, LEP Coll. 2003, Carena, et al. 2004, ALEPH Coll. 2005) and since such a clear and distinct signal has not been observed in LEP (LEP Coll. 2003, ALEPH Coll. 2005), it can easily be said that family non-universal Z′lies beyond the discovery limit of LEP.

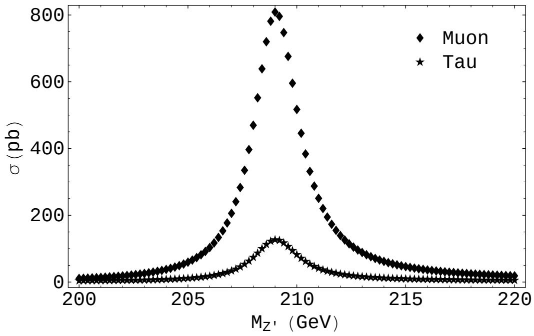

Figure 4.6. Family non-universal Z′at LEP

In conclusion, at linear colliders, which provide a perfect arena for precision measurements, one can determine if the new gauge symmetry, if any, which extends the SM gauge group exhibits family universal or non-universal couplings by simply counting the number of lepton pairs produced. This aspect is quite important since family non-universality might signal anomaly cancellation in Abelian extended models as shown in (Demir, et al. 2005).

4.3. The Hadron Collider Signatures

The most important hadron machine to come up is the Large Hadron Collider (LHC) which is a proton-proton collider running at $\sqrt{s} = 14$ TeV center of mass energy. At the parton level dilepton production processes are started by quark–anti-quark annihilation into lepton pairs via s-channel γ, Z and Z′exchanges. Since hadrons interact strongly, QCD contributions must be included in calculations thus the hadronic cross section is related to the partonic one via

$$\sigma\left(pp \to \ell^+\ell^-\right) = \sum_{q,\bar{q}} C_{q\bar{q}} \int dx_q\, dx_{\bar{q}}\, \mathcal{P}_{q/A}(x_q)\mathcal{P}_{\bar{q}/B}(x_{\bar{q}})\, \sigma\left(q\bar{q} \to \ell^+\ell^-\right) \qquad (4.17)$$

with the partonic cross-section,

$$\sigma(q\bar{q} \to \ell^+\ell^-) = \frac{1}{16\pi\hat{s}^2} \int_{-\hat{s}}^{0} dt |\mathcal{A}(q\bar{q} \to \ell^+\ell^-)|^2 \qquad (4.18)$$

where $\mathcal{P}_{q/A}(x_q)$ is Parton Distribution Function (PDF) standing for probability of finding parton (quark) q within the hadron A with a longitudinal momentum x_q time that of the hadron. Moreover, $C_{q\bar{q}}$ stands for color averaging over initial-state partons and it equals $1/9$ for $q\,\bar{q}$ annihilation. In numerical analysis we used CTEQ5 Mathematica package for PDF's.

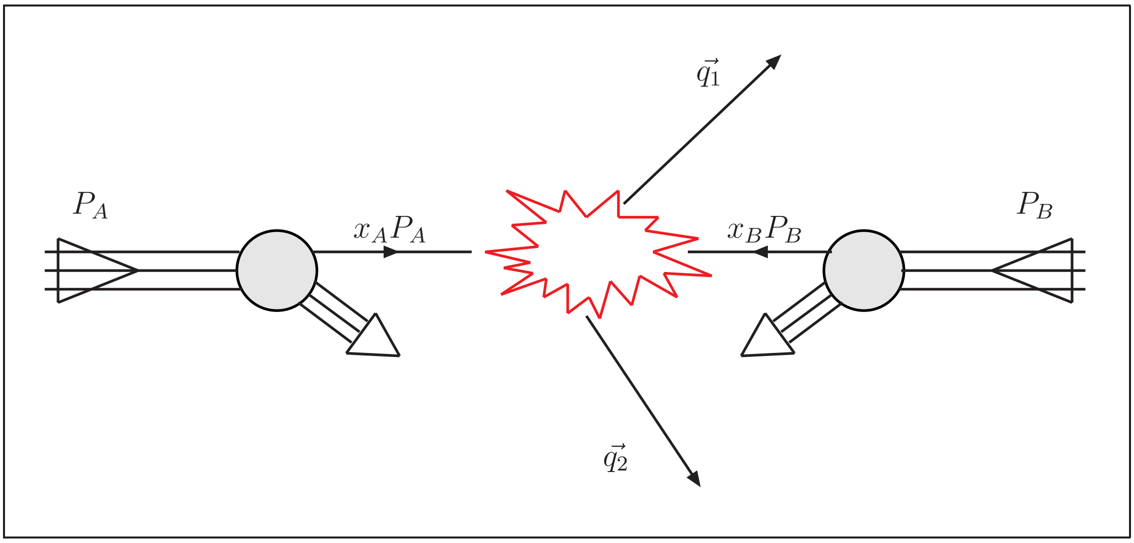

Figure 4.7. A generic two-body parton scattering process

It should be noticed that the partonic cross-sections of up-type and down-type quarks differ by their charges. Therefore the hadronic cross-section can be written in detail

$$\begin{aligned}
\sigma\left(pp \to \ell^+\ell^-\right) &= \frac{1}{9}\int dx_u\ dx_{\bar{u}}\ \mathcal{P}_{u/A}(x_u)\ \mathcal{P}_{\bar{u}/B}(x_{\bar{u}})\ \ \sigma\left(u\bar{u} \to \ell^+\ell^-\right) \\
&+ \frac{1}{9}\int dx_c\ dx_{\bar{c}}\ \mathcal{P}_{c/A}(x_c)\ \mathcal{P}_{\bar{c}/B}(x_{\bar{c}})\ \ \sigma\left(c\bar{c} \to \ell^+\ell^-\right) \\
&+ \frac{1}{9}\int dx_t\ dx_{\bar{t}}\ \mathcal{P}_{t/A}(x_t)\ \mathcal{P}_{\bar{t}/B}(x_{\bar{t}})\ \ \sigma\left(t\bar{t} \to \ell^+\ell^-\right) \\
&+ \frac{1}{9}\int dx_d\ dx_{\bar{d}}\ \mathcal{P}_{d/A}(x_d)\ \mathcal{P}_{\bar{d}/B}(x_{\bar{d}})\ \ \sigma\left(d\bar{d} \to \ell^+\ell^-\right) \\
&+ \frac{1}{9}\int dx_s\ dx_{\bar{s}}\ \mathcal{P}_{s/A}(x_s)\ \mathcal{P}_{\bar{s}/B}(x_{\bar{s}})\ \ \sigma\left(s\bar{s} \to \ell^+\ell^-\right) \\
&+ \frac{1}{9}\int dx_b\ dx_{\bar{b}}\ \mathcal{P}_{b/A}(x_b)\ \mathcal{P}_{\bar{b}/B}(x_{\bar{b}})\ \ \sigma\left(b\bar{b} \to \ell^+\ell^-\right) \qquad (4.19)
\end{aligned}$$

At the parton level the kinematical variables are denoted as $\hat{s}$, $\hat{u}$, $\hat{t}$ and expressed as

$$\begin{aligned}
\hat{s} &= (k_1 + k_2)^2 = (x_A P_A + x_B P_B)^2 = (q_1 + q_2)^2 \\
\hat{u} &= (k_1 - q_2)^2 = (x_A P_A - q_2)^2 = (k_2 - q_1)^2 = (x_B P_B - q_1)^2 \\
\hat{t} &= (k_1 - q_1)^2 = (x_A P_A - q_1)^2 = (k_2 - q_2)^2 = (x_B P_B - q_2)^2 \qquad (4.20)
\end{aligned}$$

for massless initial and final states $\quad P_A^2 = 0,\, P_B^2 = 0,\, q_1^2 = 0,\, q_2^2 = 0\,;$

$$\begin{aligned}
\hat{s} &= 2x_A x_B P_A \cdot P_B = x_A x_B s \\
\hat{u} &= 2x_A P_A q_2 = 2x_B P_B q_1 \\
\hat{t} &= 2x_A P_A q_1 = 2x_B P_B q_2 \qquad (4.21)
\end{aligned}$$

Here, x_A and x_B determine what fraction of the hadron momentum is carried by the parton inside the hadron, thus their values are between 0 and 1,

$$0 < x_A < 1 \quad and \quad 0 < x_B < 1 \tag{4.22}$$

substituting (Equation 4.21) and (Equation 4.22) into (Equation 4.18) and (Equation 4.19)

$$\begin{aligned}
\sigma\left(pp \to \ell^+\ell^-\right) &= \frac{1}{9}\int_0^1 dx_u \int_0^1 dx_{\bar{u}}\ \mathcal{P}_{u/A}(x_u)\ \mathcal{P}_{\bar{u}/B}(x_{\bar{u}})\ \ \sigma\left(u\bar{u} \to \ell^+\ell^-\right) \\
&+ \frac{1}{9}\int_0^1 dx_c \int_0^1 dx_{\bar{c}}\ \mathcal{P}_{c/A}(x_c)\ \mathcal{P}_{\bar{c}/B}(x_{\bar{c}})\ \ \sigma\left(c\bar{c} \to \ell^+\ell^-\right) \\
&+ \frac{1}{9}\int_0^1 dx_t \int_0^1 dx_{\bar{t}}\ \mathcal{P}_{t/A}(x_t)\ \mathcal{P}_{\bar{t}/B}(x_{\bar{t}})\ \ \sigma\left(t\bar{t} \to \ell^+\ell^-\right) \\
&+ \frac{1}{9}\int_0^1 dx_d \int_0^1 dx_{\bar{d}}\ \mathcal{P}_{d/A}(x_d)\ \mathcal{P}_{\bar{d}/B}(x_{\bar{d}})\ \ \sigma\left(d\bar{d} \to \ell^+\ell^-\right) \\
&+ \frac{1}{9}\int_0^1 dx_s \int_0^1 dx_{\bar{s}}\ \mathcal{P}_{s/A}(x_s)\ \mathcal{P}_{\bar{s}/B}(x_{\bar{s}})\ \ \sigma\left(s\bar{s} \to \ell^+\ell^-\right) \\
&+ \frac{1}{9}\int_0^1 dx_b \int_0^1 dx_{\bar{b}}\ \mathcal{P}_{b/A}(x_b)\ \mathcal{P}_{\bar{b}/B}(x_{\bar{b}})\ \ \sigma\left(b\bar{b} \to \ell^+\ell^-\right)
\end{aligned} \tag{4.23}$$

Depicted in Figure 4.8 are $\sigma\left(pp \to e^+e^-\right)$ and $\sigma\left(pp \to \mu^+\mu^-\right)$ for family universal (in the left panel) and non-universal (in the right panel) models. From the left-panel it is clear that the two cross sections coincide, that is, an additional U(1)′ symmetry with universal couplings to fermion (at least lepton) families is expected to lead equal numbers of e^+e^- and $\mu^+\mu^-$ pairs at the LHC. This observation is similar to what we found while analyzing ILC signatures in Section 4.1 above because of the fact that U(1)$_\eta$ model possesses family universal couplings and mass difference between muon and electron cannot induce an observable effect on cross sections at such a high-energy collider (LEP Coll. 2003, ALEPH Coll. 2005).

Similar to the right-panel of Figure 4.5, the right-panel of Figure 4.8 shows e^+e^- and $\mu^+\mu^-$ production cross sections at the LHC with family non-universal U(1)′ model. The panel manifestly shows that $\sigma\left(pp \to e^+e^-\right)$ is approximately 13 times smaller than $\sigma\left(pp \to \mu^+\mu^-\right)$ because of unequal U(1)′ charges of electron and muon tabulated in Table 3.1 as well as their vector and axial-vector couplings given in Table 3.4. Therefore, a family non-universal U(1)′, if any, can have observable signatures at the LHC via dilepton production processes.

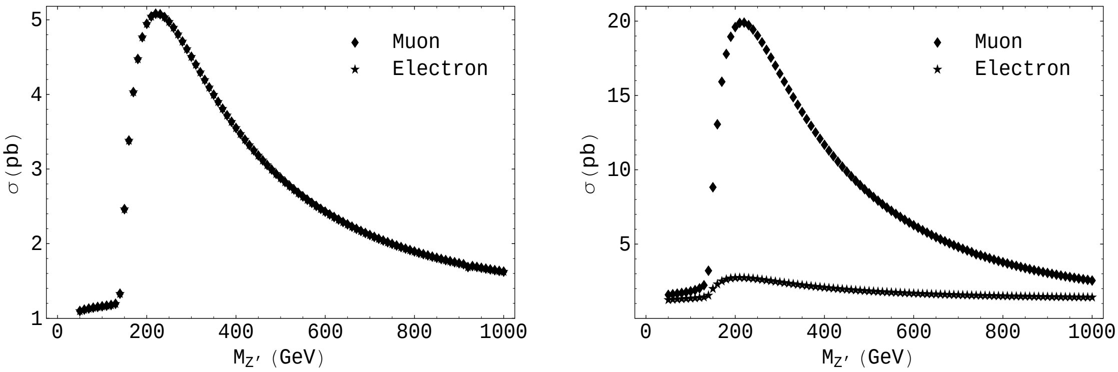

Figure 4.8. The unpolarized e^+e^- and $\mu^+\mu^-$ productions at the LHC for family universal (left panel) and non-universal (in the right panel) U(1)′ models. The ratio between family non-universal and family universal cross sections varies with model parameters

We also examine the family non-universal U(1)′ model at $p-\bar{p}$ collisions with current bounds from Tevatron ($\sqrt{s} = 2$ TeV). Figure 4.9 shows muon and electron production cross sections at Tevatron with family non-universal U(1)′. Nevertheless the CDF (Abe, et al. 1992, Abe, et al. 1995, Abe, et al. 1997) and D0 (Abachi, et al. 1996, Abbott, et al. 1998, Abazov, et al. 2001) experiments are expected to probe Z′roughly in the range of 200-800 GeV masses for various models, thus Tevatron experiments put strong limits on Z′masses in agreement with the limits set by the LEP experiments. As it is understood in Figure 4.9, family non-universal U(1)′ by being out of the limits is excluded at Tevatron with current bounds.

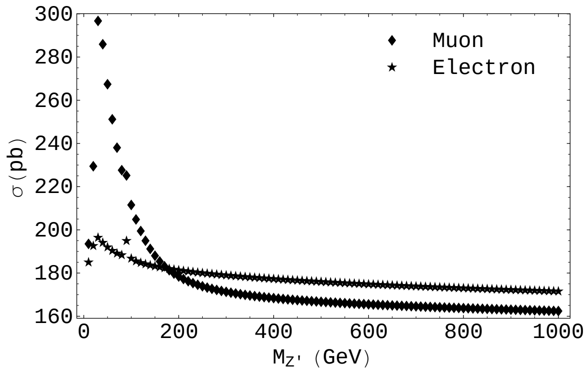

Figure 4.9. Family non-universal Z′at Tevatron

Before closing this section, we put strong emphasis on the fact that family non-universal $U(1)'$ offers observable signatures in dilepton signal in both linear and hadron colliders. In this sense, the LHC, which is expected to start operation in coming years, will be able to establish existence/absence of an additional $U(1)'$ symmetry in general and a family non-universal $U(1)'$ in particular. The latter will have easier observational characteristics because all that matters is the measurement of the ratios of events with different lepton flavors.

CHAPTER 5

CONCLUSION AND OUTLOOK

In this work we have contrasted family universal and non-universal U(1)′ models via their dilepton signatures in future linear (the ILC) and hadron (the LHC) colliders. These production signatures are also observable in current colliders, and there are more stringent bounds on Z′from precision electroweak experiments and from direct searches in LEP (LEP Coll. 2003, ALEPH Coll. 2005) and Tevatron (Abe, et al. 1992, Abe, et al. 1995, Abachi, et al. 1996, Abe, et al. 1997, Abbott, et al. 1998, Abazov, et al. 2001. The limits are model dependent because of the different couplings to fermions but typically the mass of a light Z′is comparable with Z ($\sim\ 200\,\mathrm{GeV}$) and the heavy one is around 500-800 GeV with small mixings (Cvetic, et al. 1997, Langacker 2004, LEP Coll. 2003, ALEPH Coll. 2005).

Figure 4.6 and Figure 4.9 can be used in comparison between current and future colliders. Similar to ILC analysis Figure 4.6 indicates a family non-universal U(1)′ model with current bounds in LEP and Figure 4.9 is family non-universal U(1)′ at Tevatron in a similar fashion with LHC analysis. And again the family non-universality is at the difference in production cross sections of different flavors. As a result, family non-universal Z′is out of limits set by various experiments in LEP and Tevatron.

From discussions in Section 4.1 and Section 4.2 we conclude that in both colliders (depending on systematic and statistical error bars in experimental data) one can establish existence/absence of a family non-universal U(1)′ model. This search is actually easier than direct Z′search since all that matters is the ratio of production cross sections of different lepton flavors.

For having a clearer sense of Z′search at colliders, it would be useful to analyze decay patterns of Z′boson into different flavors of matter. In general, a Z′boson of mass $M_{Z'}$ decays into a fermion f and anti-fermion $\bar{f}$ with a rate

$$\Gamma_{Z'\to f\bar{f}} = M_{Z'}\left(\frac{g_2}{4\cos\theta_W}\right)^2\left(\frac{{v^f_{Z'}}^2+{a^f_{Z'}}^2}{12\pi}\right) \tag{5.1}$$

directly proportional to $M_{Z'}$. Therefore, if a certain number of Z′bosons are produced

(Z′bosons can be copiously produced at the LHC) then their decays into different fermion pairs gives information about the underlying structure of the U(1)′ model.

Indeed, one expects at all grounds

$$\frac{\Gamma_{Z'\to\mu+\mu-}}{\Gamma_{Z'\to\tau+\tau-}} = 1 \quad ; \quad \frac{\Gamma_{Z'\to\mu+\mu-}}{\Gamma_{Z'\to e+e-}} = 1 \tag{5.2}$$

in any U(1)′ model (may it follow from E(6) or from strings) in which Z′couples to each lepton family in a universal fashion.

However, the same ratios of the decay rates become

$$\frac{\Gamma_{Z'\to\mu+\mu-}}{\Gamma_{Z'\to\tau+\tau-}} = 6.5 \quad ; \quad \frac{\Gamma_{Z'\to\mu+\mu-}}{\Gamma_{Z'\to e+e-}} = 13 \tag{5.3}$$

in the U(1)′ model of (Demir, et al. 2005) in which Z′couples to different lepton families differently (as listed in Tables 3.1 and 3.4). That the decay rates can significantly (depending on the model parameters) deviate from unity is a highly interesting signature for collider searches for a family non-universal U(1)′ gauge symmetry.

From the analyzes presented above we conclude that a U(1)′ gauge symmetry with non-universal couplings to lepton families offers unique observational signatures for collider searches via dilepton production.

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APPENDIX A

CONVENTIONS AND FEYNMAN RULES

A.1. Gamma Matrices

Anticommutation relations:

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} \quad , \quad \{\gamma^5, \gamma^\mu\} = 0 \tag{A.1}$$

Definitions of γ^5:

$$\gamma_5 \equiv \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\gamma_0\gamma_1\gamma_2\gamma_3 \tag{A.2}$$

Hermitian conjugates:

$$\gamma^{0\dagger} = \gamma^0 \quad , \quad \gamma^{k\dagger} = -\gamma^k \quad , \quad \gamma^{5\dagger} = \gamma^5 \quad , \quad \gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0 \tag{A.3}$$

Squares:

$$(\gamma^0)^2 = -(\gamma^k)^2 = (\gamma^5)^2 = I \tag{A.4}$$

Dirac representaion:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad , \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \quad , \quad \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \tag{A.5}$$

I is a 2×2 identity matrix, and the 2×2 Pauli matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \quad , \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A.6}$$

which satisfy

$$[\sigma_i, \sigma_j] = 2i\epsilon^{ijk}\sigma_k \quad , \quad \{\sigma_i, \sigma_j\} = 2\delta_{ij} \quad , \quad Tr(\sigma_i\sigma_j) = 2\delta_{ij} \tag{A.7}$$

where ϵ^{ijk} is totally antisymmetric; $\epsilon^{ijk} = \epsilon_{ijk} = 1$ for an even permutation of 1, 2, 3.

A.2. Trace Theorems and Tensor Contractions

Some useful relations involving gamma matrices:

$$\gamma \cdot \mathbf{k} = \gamma_\mu k^\mu \quad , \quad Tr(I) = 4 \quad , \quad Tr(\gamma_\mu) = 0 \quad , \quad Tr(odd\,\#of\,\gamma matrices) = 0 \qquad \text{(A.8)}$$

$$Tr(\gamma_\mu \gamma_\nu) = 4g_{\mu\nu} \quad , \quad Tr(\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma) = 4[g_{\mu\nu} g_{\rho\sigma} - g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho}] \qquad \text{(A.9)}$$

$$Tr(\gamma_5) = 0 \quad , \quad Tr(\gamma_5 \gamma_\mu) = 0 \quad , \quad Tr(\gamma_5 \gamma_\mu \gamma_\nu) = 0 \quad , \quad Tr(\gamma_5 \gamma_\mu \gamma_\nu \gamma_\rho) = 0 \quad ,$$

$$Tr(\gamma_5 \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma) = -4i\epsilon_{\mu\nu\rho\sigma} = 4i\epsilon^{\mu\nu\rho\sigma} \qquad \text{(A.10)}$$

$$\epsilon^{\mu\nu\rho\sigma} = -\epsilon_{\mu\nu\rho\sigma} = \begin{cases} 1 & \text{for even permutations of 0,1,2,3 ;} \\ -1 & \text{for odd permutations ;} \\ 0 & \text{otherwise} \end{cases} \qquad \text{(A.11)}$$

$$\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu\rho\sigma} = -24 \quad , \quad \epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu\rho}{}^{\alpha} = 6g^{\sigma\alpha} \quad ,$$

$$\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu}{}^{\alpha\beta} = -2(g^{\rho\alpha}g^{\sigma\alpha} - g^{\rho\beta}g^{\sigma\beta}) \qquad \text{(A.12)}$$

Summation of polarization states for real vector bosons:

massless, $\sum_\sigma \epsilon^\star_\mu(p,\sigma)\epsilon_\nu(p,\sigma) = -g_{\mu\nu}$

massive, $\sum_\sigma \epsilon^\star_\mu(p,\sigma)\epsilon_\nu(p,\sigma) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{M_V^2}$

A.3. Dirac Spinors

Positive energy spinor $u(p)$:

$$(\not{p} - m)u(p) = 0 \quad , \quad \bar{u}(p)(\not{p} - m) = 0 \qquad \text{(A.13)}$$

with adjoint spinor: $\bar{u}(p) = u^\dagger(p)\gamma^0$,

Negative energy spinor $v(p)$:

$$(\not{p} + m)v(p) = 0 \quad , \quad \bar{v}(p)(\not{p} + m) = 0 \qquad \text{(A.14)}$$

with adjoint spinor : $\bar{v}(p) = v^{\dagger}(p)\gamma^0$.

Projection operators:

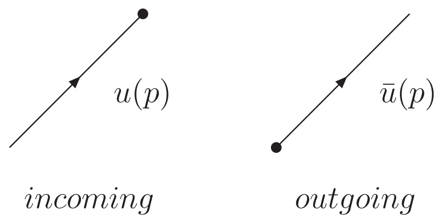
$$\sum_{\lambda} u_{\lambda}(p)\bar{u}_{\lambda}(p) = \not{p} + m \tag{A.15}$$

$$\sum_{\lambda} v_{\lambda}(p)\bar{v}_{\lambda}(p) = \not{p} + m \tag{A.16}$$

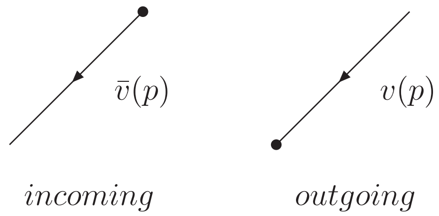
$$\sum_{\lambda,\sigma} \bar{u}_{\lambda}(p)\gamma^{\mu}v_{\sigma}(k)\gamma^{\nu}\bar{v}_{\sigma}(k)u_{\lambda}(p) = Tr[(\not{p} + m)\gamma^{\mu}(\not{k} - m)\gamma^{\nu}] \tag{A.17}$$

A.4. Feynman Rules for Tree Graphs

External Fermion Lines

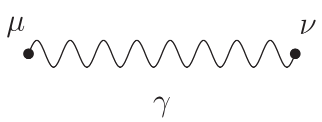


External Antifermion Lines



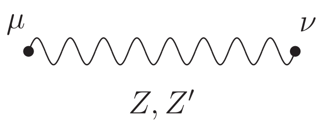
Propagators

Photon:



$$\frac{-ig_{\mu\nu}}{p^2} \tag{A.18}$$

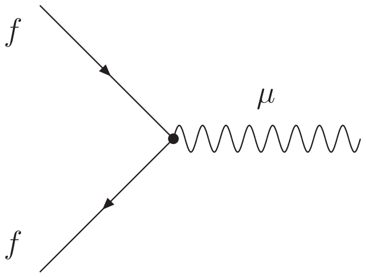
Massive Boson:



$$\frac{-ig_{\mu\nu}}{p^2 - m^2 + i\epsilon} \tag{A.19}$$

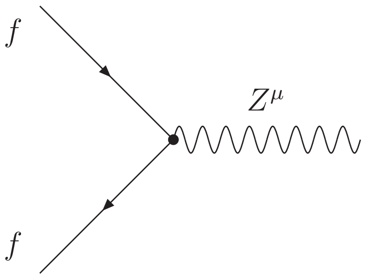
Vertices

Photon - Fermion :



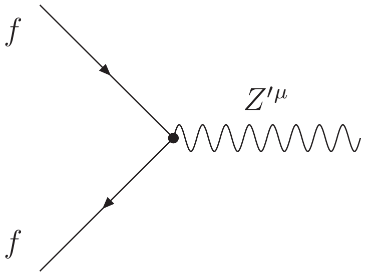
$$-iQ_f e\gamma^\mu \tag{A.20}$$

Z boson - Fermion :



$$-\frac{ig_2\gamma^\mu}{4\cos\theta_W}(v_Z^f - a_Z^f\gamma^5) \tag{A.21}$$

Z′ boson - Fermion :



$$-\frac{ig_2\gamma^\mu}{4\cos\theta_W}(v_{Z'}^f - a_{Z'}^f\gamma^5) \tag{A.22}$$

A.5. Cross Sections and Decay Rates

The differential cross section of a scattering process is given by;

$$\begin{aligned} d\sigma &= \frac{1}{2E_A 2E_B |v_A - v_B|} \left(\prod_f \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} \right) \\ &\times |\mathcal{A}(p_A, p_B \rightarrow \{p_f\})|^2 (2\pi)^4 \delta^{(4)} \left(p_A + p_B - \sum p_f \right) \end{aligned} \quad \text{(A.23)}$$

The differential decay rate of an unstable particle to a given final state is;

$$d\Gamma_V = \frac{1}{2m_V} \left(\prod_f \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} \right) |\mathcal{A}(p_V \rightarrow \{p_f\})|^2 (2\pi)^4 \delta^{(4)} \left(p_V - \sum p_f \right) \quad \text{(A.24)}$$

where E_A and E_B are incident beam energies, v_A and v_B are incoming beam velocities, p_A and p_B are incoming beam momentums, E_f's are final state fermion energies, p_f's are final state fermion momentums and m_V is the mass of intermediate vector boson.

A.6. Physical Constants and Conversion Factors

Physical constants

$$\begin{aligned} c &= 2.998 \times 10^{10} \ \ cm/s \\ \hbar &= 6.582 \times 10^{-22} \ \ MeVs \\ e &= -1.602 \times 10^{-19} \ \ C \\ \alpha &= \frac{e^2}{4\pi\hbar c} = \frac{1}{137} \\ \sin\theta_W{}^2 &= 0.23 \\ g_1 &= \frac{e}{\cos\theta_W} \\ g_2 &= \frac{e}{\sin\theta_W} \\ g_Z &= \frac{g_2}{\cos\theta_W} \\ \Gamma_Z &= 2.4952 GeV \end{aligned}$$

Conversion factors

$$\begin{aligned} \text{1barn} &= 10^{-24} \ \ cm^2 \\ (1GeV)^{-2}(\hbar c)^2 &= 0.3894 \times 10^{-27} \ \ cm^2 = 0.3894 \ \ mbarn \end{aligned}$$