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ESTIMATING BANKRUPTCY PROBABILITY USING FUZZY LOGIC: AN APPLICATION TO A PANEL OF US AND TURKISH INDUSTRIES

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MİKRO PANEL VERİ ANALİZİ VE BULANIK MANTIK METODOLOJİSİ İLE FİRMALARIN PİYASADAN ÇEKİLME OLASILIKLARININ TAHMİNİ: TÜRK VE AMERİKAN FİRMALARI ÜZERİNE BİR UYGULAMA

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MPD	Merton Default Probability
DD	Distance to Default
EDF	Expected Default Frequency
SP	Stock Price
BVL	Beta Value Line
TD	Total Debt
CF	Cash as Firm Value
NSh	Number of Shares Outstanding
TV	Trading Volume
TS	Trailing Sales
EG	Expected Growth in EPS
PTOM	Pre-tax Operating Margin
St. Dev.	Standard Deviation
IPI	Industrial Production Index
CPI	Consumer Price Index
IR	Interest Rate Spread
SPI	Share Price Index
UR	Unemployment Rate
FOI	Financial Openness Index
GDP	Gross Domestic Product
PS	Price to Sales Ratio
PBV	Price Book Value Ratio
FCFF	Free Cash Flow for the Firm
PE	Price Earning

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ABSTRACT

The main purpose of this study is to show how a Merton Model approach can be used to develop a new measure of company failures' probability independent from their sectors. In this study, a new index, Fuzzy-bankruptcy index, is created which explains the default probability of any firm X, independent from the sector it belongs. In the construction process in order to reduce the relativity of financial ratios due to the fact that their interpretation change with time and according to different sectors, fuzzy logic is used. For the fuzzy process, we used five input variables, four of them are chosen from both factor analysis and clustering and the last input variable calculated from Merton Model. Looking back to the default history of firms, one can find different reasons such as managerial arrogance, fraud and managerial mistakes which are responsible for the very sad endings of well-known companies like Enron, K-Mart and even the country Argentina. Thus, we hope with the help of our Fuzzy-bankruptcy index one could be able to get a better insight into the financial situation a company is in, and it could also prevent credit loan companies from investing in the wrong firm and possibly from losing the entire investment.

This study is organized as seven chapters. Chapter one explains the factor analysis. Chapter two gives the definition of probability of default and outlines the methods for estimating default probability. It reviews the literature on estimating the default probabilities, the Merton Model and its extensions. Chapter three explains the cluster analysis and fuzzy logic. It reviews the literature on clustering and methods of clustering, especially explains the method of how to cluster variables in detail. Second part of chapter three explains fuzzy logic and its applications. It reviews the literature on applications of fuzzy logic and how and why we use fuzzy logic in our model. Chapter four gives the information of our study and describes the model we studied. Chapter five investigates the relationship between macro-economic factors and probability of default and Chapter six concludes. Chapter seven is appendix of our study.

Keywords: Merton Model, Clustering, Fuzzy Logic, Default Probability

ÖZET

Merton Model opsiyon fiyatlama yöntemini kullanarak finansal şirketlerin batma olasılıklarının hesaplanmasında kullanılan bir modeldir. Biz bu çalışmada, Merton Model kullanarak şirketlerin batma olasılıklarını hesapladık. Öncelikle, IMKB100 içinde yer alan finansal şirketler için Merton Model kullanarak batma olasılıklarını hesapladık, değerlerin sıfır ve bire çok yakın olduğunu ya da seneden seneye olan geçişlerde değişkenliğin çok yoğun olduğunu gözlemledik. Bu sonuçlarla çalışmamızdaki ana amacımızı, Merton Modeli de kullanarak, sektörden bağımsız, yani tüm şirketler için yorumlayabileceğimiz ve karşılaştırabileceğimiz bir endeks oluşturmak olarak değiştirdik. Literatürde, şirketlerin finansal durumlarını yorumlayabilmek için çok sayıda finansal rasyolar olduğunu ve bu finansal rasyoların yorumlarının firmaların ait olduğu sektörlerle de ilişkilendiği bilinmektedir. Bu ilişki yüzünden, oluşturmak istediğimiz endeksi bulanık mantık kullanarak oluşturmamız gerektiğine karar verdik. Literatürde çok sayıda yeterli ve anlamlı finansal rasyolarla karşılaştığımızdan, şirketlerin batma olasılıklarını yorumlayabilecek için her bir finansal rasyoya tek tek bakmak yerine; çok sayıda olan finansal rasyolara faktör analizi uygulayarak, aynı açıklamayı yapacak daha az sayıda finansal rasyolar belirledik. Ulaştığımız sayı bulanık mantık uygulamamızı kişiselleştireceğinden, faktör analizi ile elimine ettiğimiz finansal rasyoları kümeleme yöntemi kullanarak bir kez daha elimine ettik. Böylelikle, bulanık mantık kullanarak oluşturacağımız değişkenimizin girdi değişkenlerini belirlemiş olduk. Bu değişkenleri ve arasındaki ilişkiyi inceleyerek, değişkenler arasında kuralları belirleyerek modelimiz oluşturduk. Böylelikle Merton modeli de dâhil ederek, bulanık mantık kullanarak şirketlerin batma olasılıklarını daha hassas ve sektörden bağımsız olarak hesapladık. Son olarak bulduğumuz endeksi makroekonomik göstergelerle açıkladık.

Anahtar Kelimeler: Merton Model, Kümeleme, Bulanık Mantık, Batma Olasılığı

INTRODUCTION

The main purpose of this study is to show how a Merton Model approach can be used to develop a new measure of company failures' probability independent from their sectors. The original Merton model is based on some simplifying assumptions about the structure of the typical firm's finances. The event of default is determined by the market value of the firm's assets in conjunction with the liability structure of the firm.[1]

Merton approach is the most popular approach for estimating default probability by using market information. The model assumes that a firm has equity and certain amount of zero coupon debt that will become due at a future time. Much of the literature follows Merton (1974) by explicitly linking the risk of a firm's bankruptcy to the variability in the firm's asset value and by viewing the market value of firm's equity as the standard call option2 on the market value of firm's asset with strike price equal to the promised payment of corporate debts.3 Merton proposes that the position of the shareholders can be considered as similar to purchasing a call option on the assets of the company, and the price at which they will exercise this option to purchase is equal to the book value of company's debt due for payment in the defined time horizon. In this way, Merton was the first to demonstrate that a firm's option of defaulting can be modeled in accordance with the assumptions of Black and Scholes (1973).

In this study, a new index, Fuzzy-bankruptcy index, is created which explains the default probability of any firm X, independent from the sector it belongs. In the construction process in order to reduce the relativity of financial ratios due to the fact that their interpretation change with time and according to different sectors, fuzzy logic is used. For the fuzzy process, minimum number of input variables is used to construct

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[1] A Merton Model Approach to Assessing the Default Risk of UK Companies, Merxe Tudela and Garry Young; Bank of England; 2003.

[2] Options are known from the financial world where they represent the right to buy or sell a financial value, mostly a stock, for a predetermined price, without having the obligation to do so. For more detail See: Carlsson, C.,Fuller,R.; Fuzzy Sets and Systems and J. Hull, Options, Futures and Derivatives

[3] The Predictive Performance of a Barrier Option Credit Risk Model in an Emerging Market, David Wang, Heng Chih Chou, David Wang, Rim Zaabar**, Journal of Economic Literature**, 2009

the output variable which denotes our bankruptcy probability. There are many different reasons why companies have defaulted over the time. Hot items in these are managerial arrogance, fraud and managerial mistakes. Well-known examples are companies like Enron, K-Mart and even the country Argentina. These causes are related with many other companies that defaulted over time. Thus, with the help of our Fuzzy-bankruptcy index one could be able to get a better insight into the financial situation a company is in, and it could also prevent credit loan companies from investing in the wrong firm and possibly from losing the entire investment.

This study is organized as five chapters. Chapter one starts explaining factor analysis, which is a data reduction method to resume a number of original variables into a smaller set of composite dimensions, or factors. This method is commonly used when developing a questionnaire to see the relationship between the items in the questionnaire and underlying dimensions. It is also used to reduce larger data set of variables to a smaller set of variables that explain important dimensions of variability.

Chapter two gives the definition of probability of default and outlines the methods for estimating default probability. It reviews the literature on estimating the default probabilities, the Merton Model and its extensions. In this chapter, Merton default probabilities of ISE100 financial companies from year 2004 to 2008 are calculated. Distance to Default (DD) and Expected Default Frequency (EDF) are also calculated as functions of Merton default probabilities.

Chapter three explains the cluster analysis and fuzzy logic. In the first part, the literature on clustering and methods of clustering are reviewed by showing special attention how to cluster variables in detail. Cluster analysis groups individuals or objects into clusters so that objects in the same cluster are homogeneous and there is heterogeneity across clusters. Homogeneous means data belongs the same clusters should be as similar as possible and heterogeneity means data belongs the different clusters should be as different as possible. In other words, we can group or cluster the observed data that the similarity of cases within each cluster is maximum and the dissimilarity of cases between clusters is maximum. Second part of chapter three explains fuzzy logic and its applications. It reviews the literature on applications of fuzzy logic. Fuzzy logic is used due to the fact that fuzzy set theory is developed for solving problems in which descriptions of observations are imprecise, vague and uncertain. The term “fuzzy” refers to the situation in which there are no well- defined boundaries of the sets of activities or observations to which the descriptions apply. For example, one can easily assign a firm with Merton default probability is 0.6 to the “class of default firms, because the term default probability does not constitute a well-defined boundary. Chapter four gives the information of the study and describes the model we studied. Chapter five investigates the relationship between macro-economic factors and probability of default.

CHAPTER 1 FACTOR ANALYSIS

Factor Analysis is a branch of statistical science, however because of its extensions on psychology, the technique itself is often mistakenly considered as psychological theory. The origin of the factor analysis is generally ascribed to Charles Spearman[4]. Since his monumental work in developing psychology theory involving a single factor analysis.

Application of factor analysis in fields other than psychology has become very popular since 1950. These fields include such varied disciplines as medical, political science, economics, sociology, taxonomy. Also, there are many individual studies that are difficult to assign to a particular discipline. In economics, evaluating the performance of the systems[5], investment decisions[6], structure of security price changes were the subjects which are examples of factor analysis applications.

When we try to specify the economic relationships by mathematical formulation we encounter problems such as which variables should be included in the equation, what form they should be assume; linear, nonlinear, logarithmic; and whether a single equation may be analyzed independently or a complete model of simultaneous equations should be considered all at once. If the explained variable in one relationship became the explanatory variable in other equation the single relationship can not be analyzed independently. In specifying relationships, it is necessary to know first what variables should be included in the relationship as explanatory variables. One method which will tell us the significance of each explanatory variable in explaining the variation of the explained variable is factor analysis. The basic idea behind the factor analysis is to represent a set of variables by a smaller number of variables. In this case, they are called factors. The variables used in factor analysis should be linearly related to each other. In other words, the variables must be at least moderately correlated to each

[4] English psychologist known for work in statistics as a pioneer of factor analysis and for Spearman's rank correlation coefficient.
[5] Burch, 1972.
[6] Farrar 1962.

other; otherwise the number of factors will be almost the same as the number of original variables. This means that factor analysis is pointless.

Factor analysis could be used for any of the following purposes:

- To reduce a large number of variables to a smaller number of factors for modeling purposes
- To select a subset of variables from a larger set based on which original variables have the highest correlations with the principal component factors.
- To identify clusters of cases
- To create a set of factors to be treated as uncorrelated variables as one approach to handling multicollinearity in such procedures as multiple regression
- To establish that multiple tests measure the same factor, thereby giving justification for administering fewer tests. Factor analysis originated a century ago with Charles Spearman's attempts to show that a wide variety of mental tests could be explained by a single underlying intelligence factor (a notion now rejected, by the way).

In the following, we state the basic procedure of finding the factors by the principal axis method. First, we list all the variables that we think are in some way related. Suppose we have N such variables. We then collect sample data on these variables and compute a symmetric matrix M in such a way that on the diagonal of the matrix, we enter the square of the multiple correlation coefficient[7] of one of the N variables in relation to the other variables. And off the diagonal of the matrix we enter the simple correlation

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[7] To measure the linear correlation between X_2, X_3 and Y we use the correlation of multiple correlation; $R_{1.23}$. The square of $R_{1.23}$ is called the coefficient of determination. It is the ratio of explained variables and total variation of Y. Using simple correlation coefficients,

$$R_{1.23}^2 = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{13}r_{23}}{1 - r_{23}^2}$$

coefficients between each pair of the N variables. The matrix, which we called M, will be like the following:

$$M = \begin{bmatrix} R_{1.234..N}^{2} & r_{12} & \cdots & \cdots \cdots r_{1N} \\ r_{12} & R_{2.134..N}^{2} & \cdots & \cdots r_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ r_{1N} & \cdots & \cdots & \cdots R_{N.123..(N-1)}^{2} \end{bmatrix}_{(N+1)\times(N+1)}$$

Then, we perform row and column operations on matrix M to transform it into a diagonal matrix.

$B^{T}MB = D$, where B is orthogonal matrix[8] and D is diagonal matrix, and $BB^{T}MB = BD$. Since B is orthogonal matrix the transpose of B is the same as inverse of B, which means $BB^{T} = I$. Thus, $MB = BD$, where

$$BD = \begin{bmatrix} m_{11} & m_{12} & \cdots & \cdots & m_{1N} \\ m_{21} & m_{22} & \cdots & \cdots & m_{2N} \\ \cdots & & & & \\ \cdots & & & & \\ m_{N1} & \cdots & \cdots & \cdots & m_{NN} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 & .. & 0 \\ 0 & \lambda_2 & 0 & \cdots & 0 \\ \cdots & \cdots & & & \cdots \\ \cdots & \cdots & & & \cdots \\ 0 & 0 & \cdots & .. & \lambda_N \end{bmatrix}$$

Matrix D is diagonal matrix, on the diagonal are the characteristics roots λ_i and off the diagonal are all zeros. B matrix is called the matrix of characteristic vectors. Choose only positive λ and arrange them in descending order. Multiplying $\sqrt{\lambda_i}$ by its respective column $m_{ji}\,(j = 1,2,\ldots,N)$, we get vectors of coefficients. These coefficients represent the contributions of each variable to each other.

[8] A nonsingular matrix is orthogonal if its inverse and transpose are the same.

Figure 1.1 Matrix Representation of Factor Analysis

	Factor 1	Factor 2	…	Factor K
Variable 1	**Coefficient**	**Coefficient**		**Coefficient**
Variable 2	**…**	**…**	**…**	**…**
….	**…**	**…**	**…**	**…**
…	**…**	**…**	**…**	**…**
Variable N	**…**	**…**	**…**	**…**

The coefficients in each vector whose values are very close may be grouped together and **factors** as defined by the grouped variables.

Now, we can summarize the steps in conducting a factor analysis. There are four basic factor analysis steps:

- data collection and generation of the correlation matrix
- extraction of initial factor solution
- rotation and interpretation
- construction of scales or factor scores to use in further analyses

CHAPTER 2 PROBABILITY OF DEFAULT & METHODS FOR ESTIMATING DEFAULT PROBABILITIES

There are several measures of country risk, one of the simplest and most easily accessible is the rating assigned to a country's debt by ratings agency. These rating agencies measures the default risk rather than the equity risk. In addition, default risk is also affected by many of the factors that affect the equity risk. Note that the most obvious determinant of a company's risk exposure to country risk is how much of the revenues it derives from the country.

2.1. PROBABILITY OF DEFAULT

A technical default is a delay in timely payment of an obligation, or a non-payment all together. If an obligor misses a payment, by even one day, it is said to be in technical default. This delay may be for operational reasons (and so not really a great worry) or it may reflect a short-term cash flow crisis, such as the Argentina debt[9] default for three months. But if the obligor states it intends to pay the obligation as soon as it can, and specifies a time-span that is within (say) one to three months, then while it is in technical default it is not in actual default. If an obligor is in actual default, it is in default and declared as being in default. This does not mean a mere delay of payment. If an obligor does not pay, and does not declare an intention to pay an obligation, it may then be classified by the ratings agencies as being in 'default' and rated 'D'.

[9] J.F. Hornbeck, "Argentina's Sovereign Debt Restructuring", **CRS Report for Congress**, 2004.

Shortly, the default probability is the probability that the value of the assets of the firm will be less than the book value of its liabilities at the maturity. (Default probability is similar to expected default frequency used by Moody's KMV[10])

The probability of default is a function of two factors: [11]

1. Ratio of debt (and other fixed costs) to cash flows: Default is the result of a firm being unable to service its fixed claims. The larger is the size of cash flows on debt obligations and other fixed claims relative to the size of operating cash flows, the higher is the probability of default.

2. Volatility of Cash Flows: Default is triggered when the firm's cash flow is too low to pay its fixed claims. The more volatile the firm's (operating) cash flow, the more likely it is that the firm will face default.

2.2. METHODS FOR ESTIMATING DEFAULT PROBABILITIES

Prior empirical models of corporate default probabilities, reviewed by Jones (1987) and Hillegeist, Keating, Cram and Lundstedt (2004), have relied on many types of covariates, both fixed and time varying. Academics in the fields of accounting and finance have actively studied bankruptcy prediction since the work of Beaver (1966, 1968) and Altman (1968). Altman's z-score is a measure of leverage, defined as the market value of equity divided by the book value of total debt. And a second generation of empirical work is based on qualitative response models, such as logit and probit. Among these, Ohlson (1980) used an o-score method; effectively summarize publicly-available information about the probability of bankruptcy.

[10] KMV is a trademark of KMV Corporation. Stephen Kealhofer, John Mc Quown and Oldrich Vasicek founded KMV Corporation in 1989. On February 11, 2002, Moody's announced that it was acquiring KMV for more than $200 million in cash.
11 Smithson et al. (1990), pp.368-369, and Damodaran, pp.451, 1997.

Credit risk models are gaining popularity in the light of the Basel II[12] accord. The pioneers of estimation of default probabilities are rating agencies like Moody's, Fitch and S&P. They started to publish not only the ratings of companies but also their estimated default probabilities. These estimations are produced from historical data. [13]

2.2.1. Estimating Default Probabilities from Historical Data

In this method, rating agencies produce data for showing that default experience over pre-determined time period of bonds that had a particular rating at the beginning of the period. So, probability of a bond defaulting during a particular year can be calculated from the historical defaulting experience data which was prepared from the rating agencies.

To estimate the default probability from historical data, Stefan Huschens, Konstantin Vogl, and Robert Wania, first consider the homogeneous group. To simplify the model, first they assume all obligors have the same default probability. (i.e., pick all obligors with the same rating from a portfolio). In their study, they characterized obligors by a Bernoulli distribution[14] and each obligor is independent form each other. For estimation of default probabilities one has to take into account that dependence affects the estimation. If one can relax the independence condition the next step is to consider the case of equicorrelation.[15] They found closely connection between the default probability and the default correlation. Also, the estimation procedure has to take into account this relation. In their article, they present estimation methods for both, the general case of a Bernoulli mixture model and the special case of the single-factor model which is used in the Internal Ratings- Based Approach of Basel II.

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[12] Basel II is the second of the Basel accords.

[13] For further reading on estimation methods of rating agencies see Kavvathas (2000) and Carty (1997).

[14] Take the value 1 if the obligor defaults, otherwise take the value 0.

[15] Let $X_1, X_2, \ldots, X_n$ identically distributed random variables, they are said to be equicorrelated if correlation of X_i and X_j is constant, for all i and j distinct.

2.2.2. Estimating Default Probabilities from Bond Prices

Prices of bond's which issued by company can be used for estimate the probability of default for a company. The familiar assumption in this method is the only reason a corporate bond sells for less than a similar risk-free bond is the possibility of default. This assumption is not completely true because in practice the price of a corporate bond is affected by its liquidity. If liquidity will be low than it makes price will be low, too.

2.2.3. Estimating Default Probabilities from Credit Default Swap Spreads or Asset Swap

A Credit Default Swap (CDS)[16] is an instrument where one company buys insurance against another company defaults its obligation. The payoff from the instrument is usually the difference between the face value of a bond issued by the second company and its value immediately after a default. The Credit Default Swap spreads, which can be directly related to the probability of default, is the amount paid per year for protection. Another source of information is the asset swap market. Asset swap spreads provide an estimate of the excess of a bond's yield spread over the LIBOR/swap rate.

2.3. LITERATURE

Merton (1974) used continuous time model and the Black and Scholes's option pricing model, to provide the first comprehensive model on credit spread in order to estimate default probabilities. In his model, the value of the firm's assets is assumed to follow a lognormal diffusion process with a constant volatility.

For estimating default probabilities from CDS, John Hull and Alan White, in the paper of Valuing CDS, test the sensitivity of CDS valuations to assumptions about the

[16] CDS is a swap contract in which the buyer of the CDS makes a series of payments to the seller and, in exchange, receives a payoff if a credit instrument (typically a bond or loan) goes into default (fails to pay). See, Credit Default Swap, Frederic P. Miller, Agnes F. Vandome, John Mc Brewster, VDM Publishing House Ltd., 2009, Page: 92.

expected recovery date. In this study, they also test that whether approximate no arbitrage arguments give accurate valuations. In the second part of the study, they model the default correlations. Evaluating default correlations for more than one firm is an important task in risk management, credit analysis and derivative pricing.[17]

According to Black and Cox (1976), the market value of firm's equity can be viewed as a European down-and-out call option[18] on the market value of firm's asset. If the firm's asset value falls below a certain barrier level, the firm's equity can be knocked out by bankruptcy.

Longstaff and Schwars (1995), Briys and de Varence (1997), Dar-Hsin Chen, Heng-Chih Chou and David Wang (2009), all extend Merton model using barrier option[19]concept. Most empirical research of the barrier option credit risk model focus on developed markets[20].

Dar-Hsin Chen, Heng-Chih Chou and David Wang (2009), and Duan (1994, 2000) transformed-data maximum likelihood estimation method to directly estimate the unobserved model parameters, and compare the predictive ability of the barrier option model to Merton model. They found that the barrier option credit risk model is still more powerful than Merton model in predicting bankruptcy in emerging market[21]. Moreover, the barrier option model predicts bankruptcy much better for electronics firms and for highly-leveraged firms. Also, in 1973, Merton provided the first analytical formula for a down and out call option, which was followed by the more detailed paper by Reiner & Rubinstein (1991), which provides the formulas for all eight types of

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17 See Chusheng Zhou, **An Analysis of Default Correlations and Multiple Defaults**.

18 A call option is deactivated if the price of the underlying falls below a certain price level. If the underlying asset does reach the barrier price level, the down-and-in option becomes a vanilla European call option. If the underlying asset price does not reach the barrier level, the option expires worthless.

19 A barrier option has a prespecified payoff function at its expiration date, T. This payoff function depends on the value of the underlying stock price at the expiration date as well as the time series of stock price leading up to that date. Specifically, the option depends on whether or not the stock price breached a certain price level, called a barrier, during the life of the option. **See, Financial Derivatives: Pricing, Applications and Mathematics,** Cambridge University Press, Jamil Baz, George Chacko.

20 For example: Taiwan Market.

21 Emerging markets are nations with social or business activity in the process of rapid growth and industrialization.

barriers.[22] (Barrier Options can be classified according to the payoff type (Call or Put), knocking type (Knock out or Knock in), barrier type (up and down), relative level of barrier and strike price). Haug (1998) gives a generalization of the set of formulas provided by Reiner & Rubinstein. In the paper of "The Predictive Performance of a Barrier-Option Credit Risk Model in an Emerging Market, he defined a term H, which is barrier level and proportional to the corporate debt. Darrell Duffie, Leondro Saita and Ke Wang (2005), provides maximum likelihood estimators of term structures of conditional probabilities of corporate default, incorporating the dynamics of firm-specific and macroeconomic covariates. They use industrial firms with monthly data spanning 1980 to 2004 and the term structure of conditional future default probabilities depends on a firm's distance to default.[23]

2.4 MERTON MODEL AND ITS EXTENSIONS

2.4.1 Merton Model

In 1974, Robert Merton[24] proposed a model for assessing the credit risk of a company by characterizing the company's equity as European call option, which is written on its assets. Merton Model assumes that a company has a certain amount of zero-coupon debt that will become due at a future time (T)[25]. In other words, the model can be used to

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22 See Appendix I..

23 Darrell Duffie, Multi-period Corporate Default Prediction with Stochastic Covariates, Journal of Financial Economics, 2007.

24 Robert C. Merton is the John and Natty McArthur University Professor at Harvard Business School. In 1997, he and Myron Scholes were awarded the Nobel Prize in Economic Sciences for contributions in the area of option pricing.

25 Credit risk models routinely assume one-year time horizon for debt maturity and subsequent estimation of default probability. One year is perceived as being of sufficient length for a bank to raise additional capital on account of increase in portfolio credit risk (if any). Furthermore, implicit in the regulatory approach to capital requirements is an assumption that if large losses (short of insolvency) are experienced during the analysis period, a bank will take actions such that its probability of remaining solvent during the following period will remain high. Such actions include raising new equity to replace that which has been lost or rebalancing to a safer portfolio such that the remaining equity is adequate to preserve solvency with the specified probability. For bank loan portfolios, substantial rebalancing is usually difficult to accomplish quickly, especially during the periods of general economic distress that are typically associated with large losses. Thus, unless a bank is able to raise new equity by the end of the analysis period, it will begin the next period with a larger-than-desired probability of insolvency. The one-year convention may have arisen largely because, until recently, default rates and rating transition

estimate either the risk neutral probability that the company will default or the credit spread on the debt.

According to Jones, (1984) the default risk implied by the Merton Model is so low that its pricing ability for investment grade bonds is no better than a naïve model that does not consider default risk at all.

The Merton type models are explored from 2001 to 2004 year horizon. As inputs, Merton's model requires the current value of the company's assets, the volatility of the company's assets, the outstanding debt, and the debt maturity. One popular way of implementing Merton's model estimates the current value of the company's assets and the volatility of the assets from the market value of the company's equity and the equity's instantaneous volatility using an approach suggested by Jones et al (1984). A debt maturity date is chosen and debt payments are mapped into a single payment on the debt maturity date in some way. Shortly, Merton model generates the probability of default for each firm in the sample at any given point in time.

We use following notation which is defined for all variables that we use to construct Merton Model.

A_0 : Value of company's assets today

A_t : Value of company's assets at time t

E_0 : Value of company's equity today

E_T : Value of company's equity at time T

D^T : Debt repayment due at time T (T is the maturity of the debt)

σ_A : Volatility of assets (Assumed constant)

σ_E : Instantaneous volatility of equity

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matrices were most easily available at a one-year horizon, and such data are key inputs to conventional portfolio credit risk models. However, Carey (2000) contends that this time horizon is too short.

The company defaults if the value of its assets is less than the promised debt repayment at time T. In other words, model assumes the firm promises to pay D^T to the bondholders at maturity T. If this payment is not met, that is, if the value of the firm's assets at maturity is less than D^T, the bondholders take over the company and shareholders receive nothing. Table 3.1 represents the cases for debt holders and equity holders.

Table 2.1 Default and No Default Cases

Event (At Time T)	**Assets**	**Debtholders**	**Equityholders**
No Default	$A_T > D^T$	D^T	$A_T - D^T$
Default	$A_T < D^T$	A_T	0

The equity of the company is a European call option on the assets of the company with maturity T and exercise price equal to the face value of the debt. Note that, default can be triggered only at maturity and this happens only when $A_T < D^T$.

In other words, the payoff of equityholders is equivalent to a European call option on the assets of the firm with exercise price D^T and maturity T.

$E_T = \max(A_T - D^T, 0)$.

The payoff of debtholders is equivalent to a portfolio which consists of a European put option and debt. The exercise price of the European put option is D^T with maturity T and written on the assets of the firm. So, its value at time T is $\min(A_T, D^T)$, which is equal to $D^T - \max(D^T - A_T, 0)$.

Table 2.2 Value of Portfolio at Time T

	At Time 0	At Time T	
		Exercise Price(D^T) **> Stock Price** (A_T)	**Exercise Price** (D^T) **< Stock Price** (A_T)
Equityholders	European Call	**Not Exercised**	$A_T - D^T$
Debtholders	European Put +Debt	$D^T - A_T$	**Not Exercised** + D^T

2.4.1.1 Assumptions of Merton Model

The model assumes that the underlying value of each firm follows a geometric Brownian motion and that each firm has issued just one zero-coupon bond. As Black and Scholes pricing formula assumes Merton model also assumes that there are no transactions costs and taxes. In addition, there are no dividends during the life of the derivative and there are no riskless arbitrage opportunities. The risk free interest rate is constant and same for all maturities. (r can be known as a function of t) . In addition of all these assumptions; security trading is also continuous. At the end, the price follows a Geometric Brownian Motion with constant drift and volatility. It follows from this that the return has a normal distribution. Following assumptions will undermine Merton model efficiency:

- The assumption that the firm can default only at time T. If the firm's value falls down to minimal level before the maturity of the debt but it is able to recover and meet the debt's payment at maturity, the default would be avoided in Merton's approach. However, you can construct the model on a barrier option and can handle this problem.

- The model does not distinguish among different types of debt according to their seniority, collaterals, covenants or convertibility.

- Default probability for private firms (not listed on the stock exchange) can be estimated only by performing some comparability analysis based on accounting data.

- It is "static" in that the model assumes that once management puts a debt structure in place, it leaves it unchanged even if the firm's assets have increased. As a result, the model cannot capture the behavior of those firms that seek to maintain a constant or target leverage ratio across time.

- Another potential shortcoming of the option based approach is that the stock market may not efficiently incorporate all publicly-available information about default probability into equity prices. In particular, prior studies suggest that the market does not accurately reflect all of the information in the financial statements (Sloan, 1996).

2.4.2 Forecasting Default Probabilities with KMV Merton Model

The KMV-Merton model applies the framework of Merton (1974), in which the equity of the company is European call option on the underlying value of the company with an exercise price equal to the face value of the company's debt. The model recognizes that neither the underlying value of the company nor its volatility is directly observable. Under the model's assumptions both can be inferred from the value of equity, the volatility of equity and several other observable variables by solving two nonlinear simultaneous equations. After inferring these values, the model specifies that the probability of default is the normal cumulative density function of a z-score depending on the firm's underlying value, the firm's volatility and the face value of the firm's debt.
The Merton model makes two important assumptions. The first is that the total value of a company is assumed to follow Geometric Brownian Motion[26],

[26] A stochastic process often assumed for asset prices where the logarithm of the underlying variable follows generalized Wiener Process. See, **"Options Futures and other Derivatives"**, Hull, J.

$$dV = \mu V dt + \sigma_V V dW$$

where,

V : the total value of the company

μ : the expected return on V

σ_V : the volatility of the company value.

dW : standard Weiner Process[27].

The second assumption of the Merton model is that the firm has issued just one discount bond maturing in T periods. Under these assumptions, the equity of the firm is a call option on the underlying value of the company with an exercise price equal to the face value of the firm's debt and a time-to-maturity of T. And, the value of equity as a function of the total value of the company can be defined by the Black-Scholes and Merton Formula. By put-call parity[28], the value of the company's debt is equal to the value of a risk-free discount bond minus the value of a put option written on the company, again with an exercise price equal to the face value of debt and a time-to-maturity of T.

The Merton model stipulates that the equity value of a company today, which is denoted by E_0, satisfies,

$$E_0 = A_0 N(d_1) - e^{-rT} D^T N(d_2) \qquad (1)$$

$$d_1 = \frac{\ln\left(\frac{Ao}{D^T}\right) + \left(r + 0.5\sigma_{A^2}\right)T}{\left(\sigma_A \sqrt{T}\right)} \text{ and } d_2 = d_1 - \sigma_A \sqrt{T}$$

where, N is the cumulative density function of the standard normal distribution, r is the risk-free rate of interest in continuous terms.

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[27] A stochastic process where the change in variable during each short period of time of length x has a normal distribution with expectation equal to 0 and variance equal to x. See **"Options Futures and other Derivatives**, Hull, J.

[28] **Put-Call Parity:** The relationship between price of the European call option and the price of European put option written on same stock when they have the same exercise price and maturity date. $P + S = C + Ke^{-rT}$.

The equation (1), express the value of a company's equity as a function of the value of the company and time.

Now, consider d_1 :

$$d_1 = \frac{\ln\left(\frac{Ao}{D^T}\right)}{\left(\sigma_A\sqrt{T}\right)} + \frac{rT}{\left(\sigma_A\sqrt{T}\right)} + \frac{0.5\sigma_{A^2}T}{\left(\sigma_A\sqrt{T}\right)} = \frac{\ln\left(\frac{Ao}{D^T}\right)}{\left(\sigma_A\sqrt{T}\right)} + \frac{\ln e^{rT}}{\left(\sigma_A\sqrt{T}\right)} + 0.5\sigma_A\sqrt{T}$$

$$d_1 = \frac{\ln\left(\frac{Aoe^{rT}}{D^T}\right)}{\left(\sigma_A\sqrt{T}\right)} + 0.5\sigma_A\sqrt{T}.$$

Let $L = \frac{D^T e^{-rT}}{A_0}$, which denotes **Leverage.** Since $E_0 = A_0 N(d_1) - e^{-rT} D^T N(d_2)$, with replacement of the value of leverage, $E_0 = A_0 N(d_1) - LA_0 N(d_2)$.

Consider the inverse of Leverage, which is denoted by L^{-1}, $L^{-1} = \left[\frac{D^T e^{-rT}}{A_0}\right]^{-1} = \frac{A_0 e^{rT}}{D^T}$.

Since $d_1 = \frac{\ln\left(\frac{Aoe^{rT}}{D^T}\right)}{\left(\sigma_A\sqrt{T}\right)} + 0.5\sigma_A\sqrt{T}$, $d_1 = \frac{-\ln(L)}{\left(\sigma_A\sqrt{T}\right)} + 0.5\sigma_A\sqrt{T}$ and $d_2 = d_1 - \sigma_A\sqrt{T}$.

Shown by Jones, Masan, Rosenfeld (1984), equity value is a function of asset value. By Ito's Lemma, $E_0\sigma_E = \frac{\partial E}{\partial A} A_0\sigma_A$.

Remember that σ_E is the instantaneous volatility of the company's equity at time 0.

$$\sigma_E = \frac{\frac{\partial E}{\partial A} A_0\sigma_A}{E_0} = \frac{\frac{\partial E}{\partial A} A_0\sigma_A}{A_0 N(d_1) - LA_0 N(d_2)} = \frac{N(d_1)\sigma_A}{N(d_1) - LN(d_2)}.$$

(If we know the variables E_0, σ_E, L, T, we can estimate A_0, σ_A.)

Consider the volatility of the equity $\sigma_E = \frac{\frac{\partial E}{\partial A} A_0\sigma_A}{E_0}$, $\frac{\partial E}{\partial A}$ is the **delta of the call option** and equal to $N(d_1)$ and $\frac{\partial E}{\partial A}\frac{E_0}{A_0}$ **is elasticity of the firm's equity** against the value of the assets.

In the KMV-Merton model, the value of the call option is observed as the total value of the company's equity, while the value of the underlying asset (the value of the company) is not directly observable. Thus, while V must be inferred, E is easy to observe in the marketplace by multiplying the company's shares outstanding by its current stock price. Similarly, in the KMV-Merton model, the volatility of equity, σ_E , can be estimated but the volatility of the underlying company, σ_A must be inferred.

Probability of Default, which will be denoted by PD, is equal to $N(-d_2)$. Remember that probability of not defaulting occurs when $A_T \geq D^T$, this happens with probability of $N(d_2)$. This means, PD= $1 - N(d_2) = N(-d_2)$.

Now, we will consider d_2 , which is equal to $d_1 - \sigma_A \sqrt{T}$ and express d_2 in terms of Leverage.

$$d_2 = \frac{-\ln(L)}{\left(\sigma_A \sqrt{T}\right)} +_A \sqrt{T_A} \sqrt{T} \frac{-\ln(L)}{\left(\sigma_A \sqrt{T}\right)_A} \sqrt{T} .$$

To find probability of default, we need to calculate the Leverage (L), volatility of the assets (σ_A) and to know the maturity (T).

Example 2.4.2.1: Suppose a company has $10 million in equity value (based on their current market price and the number of shares outstanding) with an equity volatility of sixty percentage. It also has $10 million in zero-coupon debt outstanding with two years to maturity. The risk free rate is 5%. Now, we will calculate the current value of their assets and the volatility of their assets. With this information, we can calculate the Merton bankruptcy probability of this company.

When solving this problem we will have two equations with two unknowns. One equation will be the Black-Scholes Option formula, which is used to value the firm's equity as a function of its asset value and its asset volatility, as well as other known variables. The equation (1) shows the Black-Scholes formula.

$$E_0 = A_0 N(d_1) - e^{-rT} D^T N(d_2) \qquad (1)$$

where,

$$d_1 = \frac{\ln\left(\frac{Ao}{D^T}\right) + \left(r + 0.5\sigma_{A^2}\right)T}{\left(\sigma_A \sqrt{T}\right)} \text{ and } d_2 = d_1 - \sigma_A \sqrt{T}$$

and

A_t : Value of company's assets at time t

E_T : Value of company's equity at time T

D : Debt repayment due at maturity of the debt

σ_A : Volatility of assets (assumed constant)

σ_E : Instantaneous volatility of equity

The other equation is the relationship, based on Ito's Lemma, between the asset volatility and the equity volatility, which will be shown in equation (2).

$$E_0 \sigma_E = N(d_1) A_0 \sigma_A \qquad (2)$$

Now, to solve the equations; we will use the Newton-Raphson method. The Newton-Raphson method is an iterative procedure to solve equations of the form f(x) = 0. To find the root of f(x), start with an initial guess, x_1, and then adjust each subsequent guess as follows:

$$x_2 = x_1 - \frac{f(x_1)}{f^{'}(x_1)}.$$

When we have a multidimensional problem like Merton Model, the goal is to solve for the unknown parameters A_0 and σ_A such that: $g(A_0, \sigma_A)$ and $h(A_0, \sigma_A)$. Then, if x is a vector containing A_0 and σ_A, F is a vector containing the values $g(A_0, \sigma_A)$ and $h(A_0, \sigma_A)$, then the iterative procedure is rewritten as follows:

$$x_{i+1} = x_i - J^{-1}F(x_i)$$

and

$$\begin{bmatrix} A_{o_{i+1}} \\ \sigma_{A_{i+1}} \end{bmatrix} = \begin{bmatrix} A_{o_i} \\ \sigma_{A_i} \end{bmatrix} - J^{-1} \begin{bmatrix} g(A_0, \sigma_A) \\ h(A_0, \sigma_A) \end{bmatrix}.$$

where J is the Jacobian matrix for the system of equations and $J = \begin{bmatrix} \frac{\partial g}{\partial A_0} & \frac{\partial g}{\partial \sigma_A} \\ \frac{\partial h}{\partial A_0} & \frac{\partial h}{\partial \sigma_A} \end{bmatrix}$.

The challenge in our case is to determine the elements of the Jacobian matrix, which involves relatively messy partial derivatives.

Let $g(A_0, \sigma_A) = A_0 N(d_1) - e^{-rT} D^T N(d_2) - E_0$, $h(A_0, \sigma_A) = N(d_1) A_0 \sigma_A - E_0 \sigma_E$.

To find the Jacobian matrix, we need to calculate the partial derivatives of the function g, which we derive from Black-Scholes formula (i.e. we need to calculate $\frac{\partial g(A_0, \sigma_A)}{\partial A_0}$, and $\frac{\partial g(A_0, \sigma_A)}{\partial \sigma_A}$.) [29]and partial derivatives of the function h, which we derive from Ito's Lemma i.e. we need to calculate $\frac{\partial h(A_0, \sigma_A)}{\partial A_0}$, and $\frac{\partial h(A_0, \sigma_A)}{\partial \sigma_A}$.) .

For the first step we calculate the partial derivative of the function g with respect to σ_A (i.e. we calculate $\frac{\partial g(A_0, \sigma_A)}{\partial \sigma_A}$.)

$$\frac{\partial g(A_0, \sigma_A)}{\partial \sigma_A} = A_0 \frac{\partial N(d_1)}{\partial \sigma_A} - De^{-rT} \frac{\partial N(d_2)}{\partial \sigma_A} \qquad (1)$$

$$\frac{\partial g(A_0, \sigma_A)}{\partial \sigma_A} = A_0 \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial \sigma_A} - De^{-rT} \frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2}{\partial \sigma_A} \qquad (2)$$

[29] Since the function is Black-Scholes, the partial derivative is the Greek Letter vega, which meqasues the sensitivityty ofoption price to the underlying parameter volatility.

To find the partial derivatives (to find the value of equation (2)), first we need to calculate $\frac{\partial N(d_1)}{\partial d_1}, \frac{\partial d_1}{\partial \sigma_A}, \frac{\partial N(d_2)}{\partial d_2}, \frac{\partial d_2}{\partial \sigma_A}$.

$$\frac{\partial N(d_1)}{\partial d_1} = \frac{1}{\sqrt{2\Pi}} e^{\frac{-d_1^{\,2}}{2}}, \quad \frac{\partial N(d_2)}{\partial d_2} = \frac{1}{\sqrt{2\Pi}} e^{\frac{-d_1^{\,2}}{2}} \frac{A_0}{D} e^{rT}$$

For the next step, we calculate the derivative of d_1 with respect to σ_A,

$$\frac{\partial d_1}{\partial \sigma_A} = \frac{0.5 * 2\sigma_A T \sigma_A \sqrt{T} - \sqrt{T}(\ln\left(\frac{A_0}{D}\right) + rT + 0.5\sigma_A^2 + T)}{\sigma_A^2 T} = \frac{\sigma_A^2 T^{\frac{3}{2}} - T^{\frac{1}{2}}(\ln\left(\frac{A_0}{D}\right) + T(r + 0.5\sigma_A^2))}{\sigma_A^2 T}$$

Thus,

$$\frac{\partial d_1}{\partial \sigma_A} = \frac{\sigma_A^2 T^{\frac{3}{2}} - T^{\frac{1}{2}}(\ln\left(\frac{A_0}{D}\right) + T(r + 0.5\sigma_A^2))}{\sigma_A^2 T}.$$

$$\frac{\partial d_2}{\partial \sigma_A} = \frac{0.5 * 2\sigma_A T \sigma_A \sqrt{T} - \sqrt{T}(\ln\left(\frac{A_0}{D}\right) + rT - 0.5\sigma_A^2 + T)}{\sigma^2{}_A T} = \frac{-\sigma_A^2 T^{\frac{3}{2}} - T^{\frac{1}{2}} \ln\left(\frac{A_0}{D}\right) - T^{\frac{3}{2}} r + T^{\frac{3}{2}} 0.5\sigma_A^2)}{\sigma_A^2 T}$$

$$\frac{\partial d_2}{\partial \sigma_A} = \frac{-\left[\ln\left(\frac{A_0}{D}\right) + (r + 0.5\sigma_A^2)T\right] T^{\frac{1}{2}}}{\sigma_A^2 T}$$

Now, we can use partial equations to find the value in equation (2).

$$\frac{\partial g(A_0, \sigma_A)}{\partial \sigma_A} = A_0 \frac{1}{\sqrt{2\Pi}} e^{\frac{-d_1^{\,2}}{2}} \frac{\sigma_A^2 T^{\frac{3}{2}} - T^{\frac{1}{2}}(\ln\left(\frac{A_0}{D}\right) + T(r + 0.5\sigma_A^2))}{\sigma_A^2 T} -$$

$$De^{-rT} \frac{1}{\sqrt{2\Pi}} e^{\frac{-d_1^{\,2}}{2}} \frac{A_0}{D} e^{rT} \frac{-\left[\ln\left(\frac{A_0}{D}\right) + (r + 0.5\sigma_A^2)T\right] T^{\frac{1}{2}}}{\sigma_A^2 T} \qquad (3)$$

Then we make simple mathematical calculations to equation (3) and find the following equation.

$$\frac{\partial g(A_0,\sigma_A)}{\partial \sigma_A} = A_0 N^{'}(d_1)\sqrt{T}$$

Now, we need to calculate the derivative of the function g with respect to A_0. The following equation can be found from simple mathematical calculation.

$$\frac{\partial g(A_0,\sigma_A)}{\partial A_0} = N(d_1)$$

After these mathematical calculations, we find the first row of the Jacobian matrix, to calculate the second row of the matrix; we need to calculate the partial derivatives of the function h, which is derived from Ito's Lemma. Since, $h(A_0,\sigma_A) = N(d_1)A_0\sigma_A - E_0\sigma_E$.

$$\frac{\partial h(A_0,\sigma_A)}{\partial A_0} = \frac{\partial N(d_1)}{\partial A_0} A_0\sigma_A + \sigma_A N(d_1) \qquad (4)$$

$\frac{\partial N(d_1)}{\partial A_0} = \frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial A_0}$, since $\frac{\partial N(d_1)}{\partial d_1} = \frac{1}{\sqrt{2\Pi}} e^{\frac{-d_1^2}{2}}$, we need to calculate derivative of d_1 with respect to A_0,

$$\frac{\partial d_1}{\partial A_0} = \frac{\frac{1/D}{A_0/D}\sigma_A\sqrt{T}}{\left(\sigma_A\sqrt{T}\right)^2} = \frac{1}{A_0\sigma_A\sqrt{T}}$$

From equation (4), $\frac{\partial h(A_0,\sigma_A)}{\partial A_0} = \frac{1}{\sqrt{2\Pi}} e^{\frac{-d_1^2}{2}} \frac{1}{A_0\sigma_A\sqrt{T}} A_0\sigma_A + \sigma_A N(d_1)$

$$\frac{\partial h(A_0,\sigma_A)}{\partial A_0} = N(d_1)\sigma_A + \frac{N^{'}(d_1)}{\sqrt{T}}$$

Now, we will calculate the partial derivative of the function h with respect to σ_A.

$$\frac{\partial h(A_0,\sigma_A)}{\partial \sigma_A} = \frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial \sigma_A}\sigma_A A_0 + A_0 N(d_1)$$

Since we calculate the partial derivatives of $\frac{\partial N(d_1)}{\partial d_1}$ and $\frac{\partial d_1}{\partial \sigma_A}$.

$$\frac{\partial h(A_0,\sigma_A)}{\partial \sigma_A} = A_0 N(d_1) + A_0 \sigma_A N'(d_1)\left[\frac{-\ln\left(\frac{A_0}{De^{-rT}}\right)}{\sigma_A^2\sqrt{T}} + \frac{\sqrt{T}}{2}\right].$$

With all these information, we can construct the Jacobian matrix.

$$J = \begin{bmatrix} N(d_1) & A_0\sqrt{T}N'(d_1) \\ N(d_1)\sigma_A + \frac{N'(d_1)}{\sqrt{T}} & A_0 N(d_1) + A_0\sigma_A N'(d_1)\left(\frac{-\ln\left(\frac{A_0}{De^{-rT}}\right)}{\sigma_A^2\sqrt{T}} + \frac{\sqrt{T}}{2}\right) \end{bmatrix}$$

We found $A_0 = 18.905$ million and $\sigma_A = 32.87\%$ with four iterations when starting with initial guess when $A_0 = 25$ and $\sigma_A = 20\%$. The following tables, Table 2.3.A and Table 2.3.B, show the first four iterations of this

Table 2.3.A Results of Iterations

	Iteration 1	Iteration 2	Iteration 3	Iteration 4
$\mathbf{A_0}$	25	19.05	18.92	18.91
$\boldsymbol{\sigma_A}$	20%	28.78%	32.72%	32.87%
$\mathbf{d_1}$	3.74	2.03	1.83	1.82
$\mathbf{d_2}$	3.45	1.63	1.36	1.35
$\mathbf{N(d_1)}$	0.9999	0.9789	0.9660	0.9654
$\mathbf{N(d_2)}$	0.9997	0.9479	0.9135	0.9119
$\mathbf{N'(d_1)}$	0.0004	0.0506	0.0754	0.0765
Black-Scholes	15.9518	10.0680	10.0317	10.0000
g	5.95	0.068	0.0137	0
h	-1.0005	-0.6334	-0.0184	0

Table 2.3.B Jacobian and Inverse Jacobian Matrix

	Iteration 1	Iteration 2	Iteration 3	Iteration 4
J	$\begin{bmatrix} 1 & 0.013 \\ 0.2 & 24.965 \end{bmatrix}$	$\begin{bmatrix} 0.9791 & 1.363 \\ 0.318 & 17.079 \end{bmatrix}$	$\begin{bmatrix} 0.966 & 2.017 \\ 0.369 & 16.366 \end{bmatrix}$	$\begin{bmatrix} 0.965 & 2.017 \\ 0.371 & 16.336 \end{bmatrix}$
$\mathbf{J}^{-1}$	$\begin{bmatrix} 1 & -0.01 \\ -0.008 & 0.04 \end{bmatrix}$	$\begin{bmatrix} 1.049 & -0.084 \\ -0.019 & 0.06 \end{bmatrix}$	$\begin{bmatrix} 1.086 & -0.134 \\ -0.025 & 0.064 \end{bmatrix}$	$\begin{bmatrix} 1.088 & -0.137 \\ -0.025 & 0.064 \end{bmatrix}$

We can calculate the Merton bankruptcy of this company with the results from the fourth iteration. Probability of Default s equal to $N(-d_2)$. This means, PD= 1 - $N(d_2) = N(-d_2)$. PD of this company is equal to $N(-d_2) = N(-1.35) = 0.089$.

2.4.2.1 Simplification of Distance to Default

The simplified expression contains only observable parameters and distance to default can be computed without solving nonlinear equations. The simplification is based on three assumptions.

1) Assume $N(d_1)$ is close to 1.
2) The limit of the drift term is equal to 0.
3) When calculating distance to default, use the book (face) value of debt instead of the leverage ratio.

$$DD = d_1 - \sigma_A\sqrt{T} = \frac{\ln(A_0 / D^T) + (r + \frac{1}{2}\sigma_{A^2})T}{\sigma_A\sqrt{T}} - \sigma_A\sqrt{T}$$

$$DD = \frac{\ln(A_0 / D^T) + (r - \frac{1}{2}\sigma_{A^2})T}{\sigma_A\sqrt{T}}$$

Now, since the drift term is very small, close to the zero, than $(r - \frac{1}{2}\sigma_{A^2})T = 0$.

Rewrite the explanation of the volatility of the assets by using Ito's Lemma, we found the following expression:

$$DD = \frac{\ln(A_0 / D^T)}{E_0\sigma_E\sqrt{T}A_0}$$

Now assume time to maturity is equal to 1 year. So, $DD = \frac{\ln(A_0 / D^T)}{E_0\sigma_E A_0}$. Since the leverage ratio, which is denoted by L, $L = \frac{D^T}{A_0}$.

$$DD = \frac{\ln(A_0 / D^T)}{E_0\sigma_E A_0} = -\frac{\ln(L)}{E_0\sigma_E A_0}.$$

The DD is simply the number of standard deviations of a firm that is away from the default point within a specified time horizon. It is an ordinal measure of the firm's default risk. Chan-Lau and Say (2006) proposed the distance-to-capital as an alternative tool to forecasting bank default risk. The distance-to-capital is constructed the same way as the distance-to-default except that the default point is proposed as the capital thresholds, which is defined by the prompt-correction-action (PCA) frameworks. PCAs are typically rules-based frameworks, where rules are based on specific levels of a bank's risk adjusted capital. The most commonly used capital threshold is the minimum capital adequacy ratio defined by the Basel II.

To determine the probability of default for a company, there are essentially three steps to be taken, as defined by Crosbie and Bohn (2003):

- Make an estimation of the asset value and the asset volatility[30]: In this step, estimation is made of the asset value and asset volatility of the firm based on the market value and volatility of equity and book value of liabilities.

- Calculate the distance to default[31]: The distance-to-default is calculated from the asset value and asset volatility and the book value of liabilities.

- Calculate the default probability: The default probability is a direct function of the distance-to-default.

The smaller DD, the larger the probability that the firm will default on its debt. It can be used to rank different firms according to their creditworthiness[32]. You can find more information about DD in appendix II.

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[30] The Black-Scholes model is used to calculate these values. Based upon the data collected, it is just matted of applying the formula like Löffler and Posch (2007) did to Enron (2001).

[31] The distance-to-default is a function combined of several variables: value of assets, asset volatility and value of liabilities. But there is also another variable that is part of the distance-to-default: the asset drift rate. To calculate its value, the Capital Asset Pricing Model (CAPM) comes into the as a standard procedure for estimating expected returns.

[32] In the literature one can sometimes see the risk free interest rate replaces with the asset value growth rate. The distance to default measure is then linked to real world default probabilities instead of risk neutral ones. Risk neutral probabilities are typically used for pricing purposes and real world probabilities for risk management purposes.

2.5 DEFAULT & SURVIVAL PROBABILITIES

According to the study of Paul Dunne and Alan Hughes, they shown smaller companies grew faster than larger companies, and age is negatively related to growth. In addition, their study shown that smaller companies had higher death rates but the largest and smallest companies was least vulnerable to takeover.[33]

What logically follows is that when the probability of default for company X is higher than for company Y, company X pays a higher interest on its debt than does company Y.

2.5.1 Bivariate Normal Distribution

Assume X and Y are two random variables with the following joint probability density function.

$$\psi(x,y) = \frac{1}{2\Pi\Pi_X \sigma_Y \sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho_{xy}{}^2)}\theta(x,y)\right),$$

where

$$\theta(x,y) = \left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_Y}{\sigma_Y}\right) + \left(\frac{y-\mu_Y}{\sigma_Y}\right)^2 .$$

Then $\psi(x,y)$ is known as the bivariate normal probability density function and is specified with five parameters; μ_X , μ_Y , σ_X , σ_Y and $\rho_{X,Y}$, μ_X and μ_Y is expectation of the random variables X and Y, respectively. σ_X , σ_Y is standard deviation of the random variables X and Y, respectively. $\rho_{X,Y}$ is the correlation coefficient of X and Y. Knowing the default probability of each single party and joint default probability of both parties, we can obtain the default correlation. Evaluating default correlations or the probabilities of default by more than one firm is an important task in credit analysis, derivatives pricing, and risk management. However, default correlations can not be

33 Paul Dunne and Alan Hughes, "Age, Size, Growth and Survival", UK Companies in the 1980s", **The Journal of Industrial Economics,** 1994.

measured directly, multiple default modeling is technically difficult, and most existing credit models can not be applied to analyze multiple defaults.[34] One way to estimate default correlation uses historical data.[35]The problem with this information is well known and has many applications because defaults for bonds are rare, there are not enough time series data available to accurately estimate default correlations. In addition, these applications do not use firm specific information. Therefore, we cannot recognize the default correlation between Exxon and Chevron could be very different from that between Exxon and Wal-Mart. Default correlation analysis has many applications in asset pricing and risk management. Due to the rapid growth in the credit derivatives market and the increasing importance of measuring and controlling default risks in portfolios of loans, derivatives and other securities, the importance default correlation analysis has been widely recognized by the financial industry in recent years.[36]

Let $D_X(t)$ and $D_Y(t)$ be two random variables that describes the default status of the firms X and Y; respectively.

$$D_X(t) = \begin{cases} 1, & \text{if firm X defaults at time t} \\ 0 & \text{otherwise} \end{cases}$$

Notice that, a random variable will be called Bernoulli variable if it takes values α when a success occurs and a value β when a failure occurs. Here we shall assume $\alpha = 1$ and $\beta = 1$.

If we assume that firm X and Y's default status **are independent** then

$$P(D_X(t) = 1 \quad \text{and} \quad D_Y(t) = 1) = P(D_X(t)).P(D_Y(t)).$$

However, the independency assumption is not accurate in general. It is reasonable to assume that when one firm X defaults, the other firm Y may have higher probability of

[34] Chusheng Zhou, An Analysis of Default Correlations and Multiple Defaults, **Review of Financial Studies, 2001**, pp. 555-576**.**
[35] Lucas, 1995.
[36] J.P Morgan.

default. Perhaps firms work together or both firms X and Y affected from the general economy. In other words, two firms (X and Y) may have a positive correlation. Also, we should consider the opposite direction.

We define the **default correlation** of the firms X and Y, $Cor(D_X(t), D_Y(t))$ as

$$Cor(D_X(t), D_Y(t)) = \frac{Cov(D_X(t), D_Y(t))}{\sqrt{Var(D_X(t))}\sqrt{Var(D_Y(t))}},$$

where, $Cov(D_X(t), D_Y(t))$ denotes the covariance of the default random variables $D_X(t)$ and $D_Y(t)$; for any i, $Var(D_i(t))$ is the variance of the default random variable $D_i(t)$.

Remember that $Cov(D_X(t), D_Y(t)) = E(D_X(t)D_Y(t)) - E(D_X(t))E(D_Y(t))$. Since $D_X(t)$ and $D_Y(t)$ are Bernoulli random variables; we can calculate the expected value, variance as follows:

- **The expected value of the default random variable of firm X :**

$E(D_X(t)) = 1P(D_X(t) = 1) + 0P(D_X(t) = 0) = P(D_X(t) = 1)$.

- **The variance of the default random variable of firm X:**

  Since $Var(D_X(t)) = E(D_X(t)^2) - E(D_X(t))^2$ (from the definition of variance)[37]; and $E(D_X(t)) = P(D_X(t) = 1)$ and $E(D_X(t)^2) = 1^2 P(D_X(t) = 1) + 0^2 P(D_X(t) = 0)$; $Var(D_X(t)) = P(D_X(t) = 1) - P(D_X(t) = 1)^2$. Thus the variance of default random variable is equal to multiplication of default probability with survival probability. (i.e. $Var(D_X(t)) = P(D_X(t) = 1)(1 - P(D_X(t)))$)

Now, consider $P(D_X \cap D_Y)$ which represents the probability of joint default at time T, where T is the maturity. This probability is simply $P(D_X \cap D_Y) = \psi(x, y)$, where $\psi(x, y)$ is bivariate normal distribution. In addition to the joint probability of default, we

[37] See Appendix II.

can derive other useful probabilities. [38]

Consider the conditional probability; $P(D_X I D_Y)$. It represents the probability of default of firm X at time T conditional on the default of firm Y at time T. We can calculate the conditional default probability with the help of joint default probability of default at time T.

$$P(D_X I D_Y) = \frac{P(D_X \cap D_Y)}{P(D_Y)}.$$

Now, consider **the probability of at least one of the firm defaults** at time T which will be denoted $P(D_X \cup D_Y)$ and $P(D_X \cup D_Y) = P(D_X) + P(D_Y) - P(D_X \cap D_Y)$. In addition, $P(D_X) + P(D_Y) - P(D_X \cap D_Y) = E(D_X) + E(D_Y) - E(D_X D_y)$. Since, $E(D_X(t)D_Y(t))$ can be different from $E(D_X(t))E(D_Y(t))$, it implies that estimates of the probability can also be very sensitive to the default correlation.

We can easily deduce the **survival probabilities** from the default probabilities. Let $P(S_X)$ denotes the probability of survival of firm X at time T. Then,
$P(S_X) = 1 - P(D_X)$.

- **The probability of at least one of the firms X and Y survives;** $P(S_X \cup S_Y)$**:**

$$P(S_X \cup S_Y) = 1 - P(D_X \cap D_Y).$$

- **The probability of default of firm X and survival of firm Y at time T;** $P(D_X \cap S_Y)$**:**

$$P(D_X \cap S_Y) = P(D_X) - P(D_X \cap D_Y).$$

- **The probability of survival of firm X and survival of firm Y at time T;** $P(S_X \cap D_Y)$**:**

$$P(S_X \cap D_Y) = P(S_X) - P(S_X \cap S_Y).$$

[38] L. Cathcart, L.El Jahel; Multiple Default's and Metron Model, **Journal** of **Fixed Income**, 2004, Vol: 14, Pages 60-69, ISSN: 1059-8596.

- **The probability of survival of firm X at time T conditional on the survival of firm Y at time T;** $P(S_X I S_Y)$:

$$P(S_X I S_Y) = \frac{P(S_X \cap S_Y)}{P(S_Y)}.$$

$P(D_X = 1) = P(A_T < D^T) = P(A_0 < D^T e^{-rT})$, remember that we denote leverage with L and defined as $L = \frac{D^T e^{-rT}}{A_0}$. The probability of X firm's asset value at time zero (now) smaller than the present value of financial debt is equal the probability of firm X's asset value at time T smaller than the financial debt at time T.

2.6 An APPLICATION of PUBLIC COMPANIES in TURKEY

We have examined the data of fifty ISE[39] companies over the period 2004 and 2008. The inputs to the Merton model include the volatility of stock returns, total debt of the firm, the risk free interest rate and the time period. For the risk free rate, we use the One Year Treasury Constant Maturity Rate obtained from the TUIK[40]. Risk free rates can be found in appendix part 7.3. In addition, we assume the maturity, T is equal to 1 (one year).

As a first application, the volatility of ISE stocks have been calculated by using the return data retrieved from ISE website. In this calculation logarithmic returns have been used. Then the unobservable parameters A_0 and σ_A have been computed simultaneously by using equations derived from the Ito's Lemma and Put-Call Parity. Having all these parameters computed $N(-d_2)$ of Black-Scholes equation is retrieved. All these computations are repeated for individual companies and for each year, between 2004 and 2008. Table 2.3 illustrates MPD values of ISE100 compan

[39] İstanbul Stock Exchange.
[40] Turkish Statistical Institute.

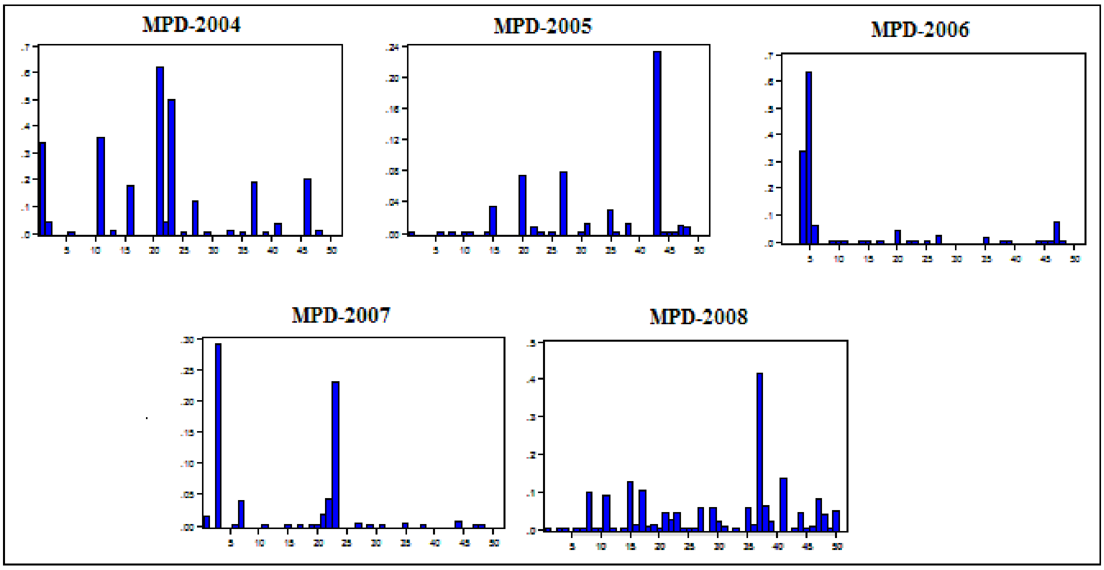
Table 2.4 MPD of Companies Selected from ISE100 (2004-2008)

	MDP-2004	MDP-2005	MDP-2006	MDP-2007	MDP-2008
ACIBD	**0.3401**	0.0008	0.0000	0.0164	0.0001
ADEL	0.0414	0.0000	0.0000	0.0000	0.0000
AEFES	0.0000	0.0000	0.0000	0.2906	0.0001
AKSA	0.0000	0.0000	0.3371	0.0000	0.0007
ALARK	0.0000	0.0000	0.6349	0.0000	0.0000
ALTIN	0.0005	0.0005	0.0582	0.0023	0.0017
ANACM	0.0000	0.0000	0.0000	0.0394	0.0039
ASELS	0.0000	0.0000	0.0000	0.0000	0.0966
AYGAZ	0.0000	0.0000	0.0000	0.0000	0.0015
BAGFS	0.0000	0.0011	0.0000	0.0000	0.0019
BANVT	0.3573	0.0003	0.0003	0.0012	0.0875
BROVA	0.0000	0.0000	0.0000	0.0000	0.0001
BUCIM	0.0114	0.0000	0.0000	0.0000	0.0000
CIMSA	0.0000	0.0004	0.0002	0.0000	0.0033
DYHOL	0.0000	0.0352	0.0000	0.0000	0.1251
ECILC	0.1775	0.0000	0.0000	0.0000	0.0110
EDIP	0.0000	0.0000	0.0003	0.0020	0.1014
EGEEN	0.0000	0.0000	0.0000	0.0000	0.0063
EGGUB	0.0000	0.0000	0.0000	0.0008	0.0135
ENKAI	0.0000	0.0727	0.0388	0.0001	0.0020
EREGL	0.6178	0.0000	0.0000	0.0180	0.0451
FENIS	0.0414	0.0084	0.0018	0.0417	0.0252
FRIGO	0.4991	0.0001	0.0000	0.2285	0.0427
FROTO	0.0000	0.0000	0.0000	0.0000	0.0000
GOLDS	0.0031	0.0000	0.0024	0.0000	0.0014
GOODY	0.0000	0.0000	0.0000	0.0000	0.0005
GSDHO	0.1160	0.0778	0.0240	0.0041	0.0571
HEKTS	0.0000	0.0000	0.0000	0.0000	0.0000
HURGZ	0.0000	0.0000	0.0000	0.0000	0.0561
IHLAS	0.0000	0.0000	0.0000	0.0000	0.0222
IPMAT	0.0000	0.0127	0.0000	0.0001	0.0081
IZOCM	0.0000	0.0000	0.0000	0.0000	0.0000
KAPLM	0.0109	0.0000	0.0000	0.0000	0.0000
KARTN	0.0000	0.0000	0.0000	0.0000	0.0000
KCHOL	0.0001	0.0302	0.0138	0.0031	0.0577
KENT	0.0000	0.0000	0.0000	0.0000	0.0116
KUTPO	0.1886	0.0000	0.0000	0.0000	0.4152
METUR	0.0000	0.0122	0.0000	0.0000	0.0616
MİPAZ	0.0000	0.0000	0.0019	0.0000	0.0210
MRDIN	0.0000	0.0000	0.0000	0.0000	0.0000
OKANT	0.0323	0.0000	0.0000	0.0000	0.1366
OLMKS	0.0000	0.0000	0.0000	0.0000	0.0000
PARSN	0.0000	**0.2314**	0.0000	0.0000	0.0007
PENGD	0.0000	0.0018	0.0003	0.0059	0.0450
PNSUT	0.0000	0.0002	0.0004	0.0000	0.0027
PTOFS	0.2023	0.0002	0.0007	0.0000	0.0072
SAHOL	0.0000	0.0101	0.0733	0.0000	0.0809
TATKS	0.0080	0.0069	0.0004	0.0000	0.0410
TCELL	0.0000	0.0000	0.0000	0.0000	0.0000
VESTL	0.0000	0.0000	0.0000	0.0000	0.0465

From Table 2.3; we can see that for the year 2004, MPD of Acıbadem is equal to 0.34 and one year later the default probability decreases to 0.0008. At 2006 it remains nearly the same or so close to zero. For instance, from the model if follows that PARSN's Merton default probability was 23.14% based on our data for the year 2005. However, MPD of PARSN for 2004 and 2006 take values so close to one. The MPD of TCELL is almost surely zero for all years, we examined. In addition, the MPD of VESTL is different from zero only at the year 2008.

The following graphs illustrate MPD for fifty companies which belong to the ISE100 for each year between 2004 and 2008. In addition, we calculate these probabilities by solving two equations with two unknowns. Two unknown parameters are the value of assets of the firm and its volatility. To solve these equations, we use excel goal seek function[41] in excel. We can not use goal seek function many times as other functions such as correlation or multiplication in excel. To run goal seek function many times we create a macro which will be explained in appendix IV.

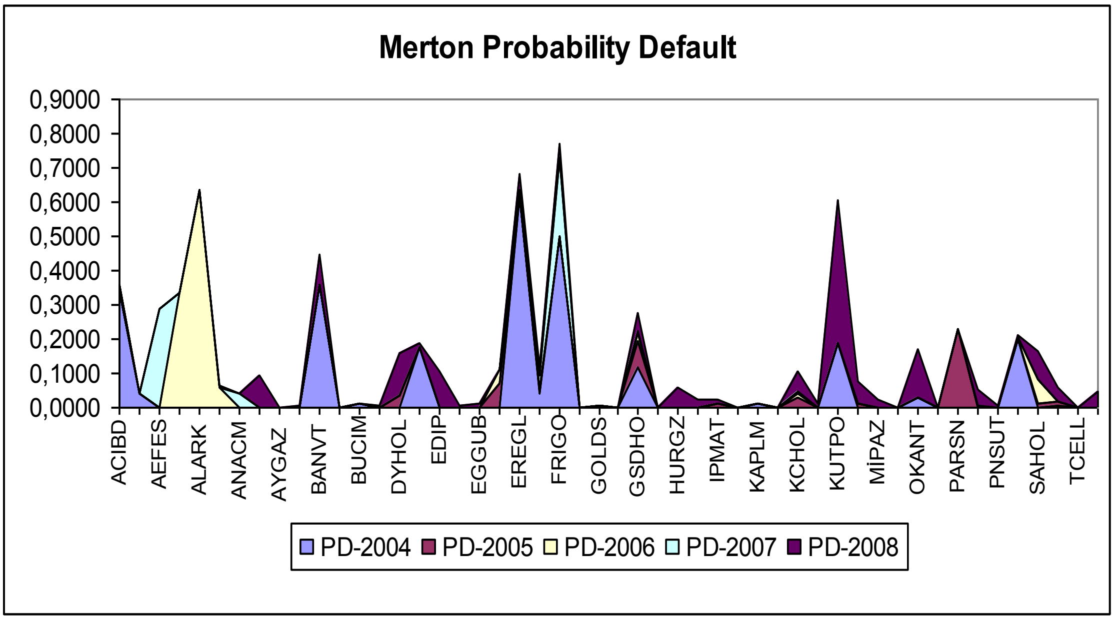
Graph 2.1 MPD of Selected IMKB Companies



[41] Goal seek function can be used when you know the result of a formula, but not the input values required by the formula to decide the result, reverse calculation

From Graph 2.1, one can easily mentioned that Merton probability of default take values close to zero or one. Also, it changes rapidly in one year. We think that it is not accurate to say that any firm X can be default with nearly hundred percentage, after one year later it can be default with nearly zero percentage.

Graph 2.2 MPD of Selected Companies from ISE100



Graph 2.2 illustrates the MPD for all fifty companies for all years between 2004 and 2008**.** (Different colors illustrate different years). From the above graph, we can see that MPD values are not increasing or decreasing function by the time. It changes rapidly and nonsense. One year it takes value which is close to one, after that year its value close to zero.

Since DD is a function of MPD, we can easily calculate DD of each company. Table 2.4 illustrates DD values of each company for all years.

Table 2.5 DD of Selected ISE100 Companies (2004-2008)

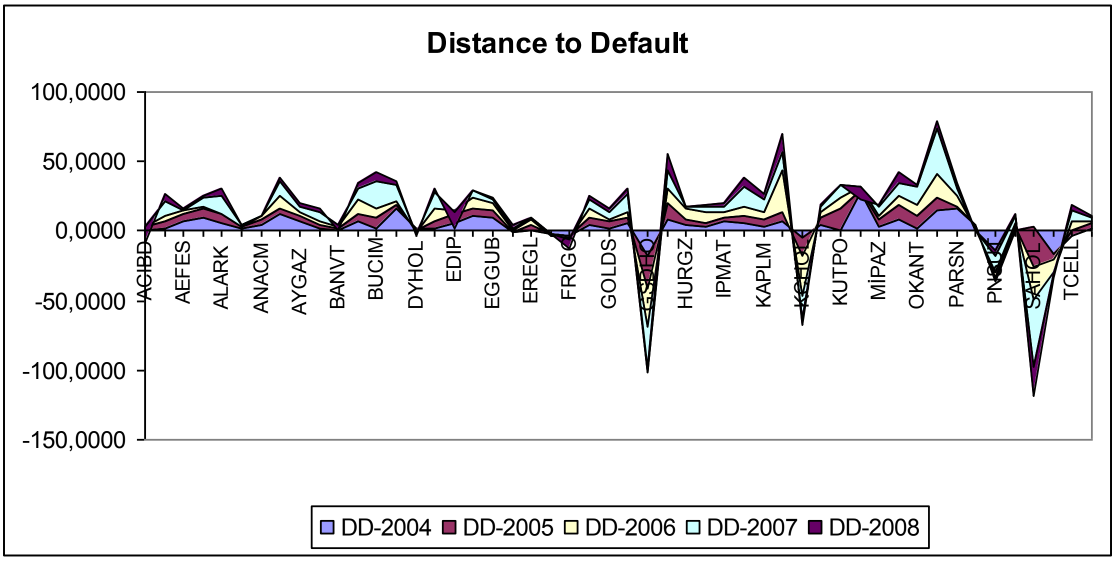
	DD-2004	DD-2005	DD-2006	DD-2007	DD-2008
ACIBD	0.7820	1.8645	0.1882	0.3106	-12.1122
ADEL	1.9130	4.6354	4.4418	10.7553	5.0830
AEFES	6.6760	5.8626	1.7813	0.5655	1.2225
AKSA	10.2374	6.0936	0.7135	6.5240	2.4472
ALARK	5.9498	5.7318	0.7842	13.2675	4.4321
ALTIN	1.6276	1.6276	-0.0449	0.7090	1.0082
ANACM	4.4208	3.5366	2.6609	0.8972	0.0501
ASELS	12.1741	3.7320	9.3941	10.9506	1.8649
AYGAZ	7.0194	4.0880	2.0732	4.8166	2.3852
BAGFS	2.0282	2.4154	2.8915	6.4978	2.7986
BANVT	0.4325	1.0680	1.3232	1.5483	-4.1883
BROVA	7.3573	5.2600	9.8937	8.2861	3.8453
BUCIM	1.9443	7.2438	6.4341	20.4941	5.7919
CIMSA	16.0748	2.3664	2.9366	11.6846	2.2905
DYHOL	1.6466	-1.3672	-1.5107	-0.4800	-1.1720
ECILC	1.5329	5.9413	8.3713	11.9882	2.6463
EDIP	5.4600	7.3758	1.6731	-1.2834	-11.7706
EGEEN	10.6772	5.3307	7.6411	5.0637	1.0389
EGGUB	9.2172	6.0727	5.2884	2.2871	1.0085
ENKAI	-0.8463	0.2344	0.8702	1.9913	2.2245
EREGL	0.7579	3.7025	3.5941	1.6194	0.6073
FENIS	-1.7581	-0.6139	-0.5966	0.3143	0.2167
FRIGO	-3.0367	-0.2330	-1.5207	-1.0700	-6.5729
FROTO	4.2042	6.0133	5.9330	6.1574	3.4119
GOLDS	2.4662	4.0494	2.2961	4.2601	2.8169
GOODY	6.1855	4.1083	3.9172	13.0768	3.0025
GSDHO	-17.6451	-23.2083	-28.0075	-32.2577	1.6626
HEKTS	8.3193	11.4588	10.2467	14.2190	11.1770
HURGZ	3.8540	4.9754	7.1989	1.1723	-0.0985
IHLAS	3.0299	3.3185	7.5200	3.9785	1.6552
IPMAT	7.1974	2.0842	4.8636	3.4374	2.2491
IZOCM	5.1457	5.9615	6.6150	14.6239	6.1476
KAPLM	2.5383	5.4359	5.0033	10.0143	4.2288
KARTN	7.2201	6.5421	30.5370	12.2752	13.6705
KCHOL	-4.8132	-13.3920	-28.4253	-16.7535	-3.7979
KENT	4.9431	4.2494	4.4045	4.1476	1.2350
KUTPO	1.0594	14.5642	8.8672	8.2181	1.1309
METUR	**28.4669**	1.3660	2.1887	0.4972	-9.4179
MİPAZ	2.7205	5.9720	2.8964	6.1214	0.5468
MRDIN	8.8708	10.3643	6.0016	9.4457	7.2592
OKANT	2.0924	9.5007	7.8673	12.6525	1.7492
OLMKS	15.2403	8.5849	16.9193	33.6296	5.3103
PARSN	16.6018	1.3153	8.1293	5.2110	3.1641
PENGD	4.0674	0.6160	-3.2534	-0.8353	-0.2276
PNSUT	-36.3628	2.9622	3.2335	13.0020	2.6676
PTOFS	**0.9871**	2.3937	2.2190	5.0002	1.8768
SAHOL	2.9866	-29.1463	-22.4933	-48.8560	-21.6965
TATKS	-16.3521	-3.5726	-9.7183	-1.3283	-2.3254
TCELL	-4.0383	5.1352	5.6189	8.1364	3.8301
VESTL	1.9012	4.0051	1.0609	3.3169	0.4204

For instance, from table 2.4 the DD for PTOFS is 0.9871 which means that PTOFS was, based on this model for the trading year 2004, less than 1 standard deviation away from its default: 0.9871, standard deviation to be precise. This indicates that PTOFS was more close to a default than it might have looked like. But before we will draw our conclusions, let's have a look at expected default frequencies of companies.

For instance, the DD for METUR is 28.4669 which means for the trading year 2004, 28.4669, standard deviation to be precise. In addition, the MPD for METUR is so close to zero.

Graph 2.3 illustrates the distance to default values of the same fifty companies. From the graph we can see that some companies have negative values for all years between 2004 and 2008. Positive Distance to default values increases and negative distance default values decreases each year. In addition, we can say that as time changes, absolute value of distance to default increases. This outcome is expected because companies which we calculate distance to default values are selected from ISE100.

Graph 2.3 DD of Selected Companies from ISE100



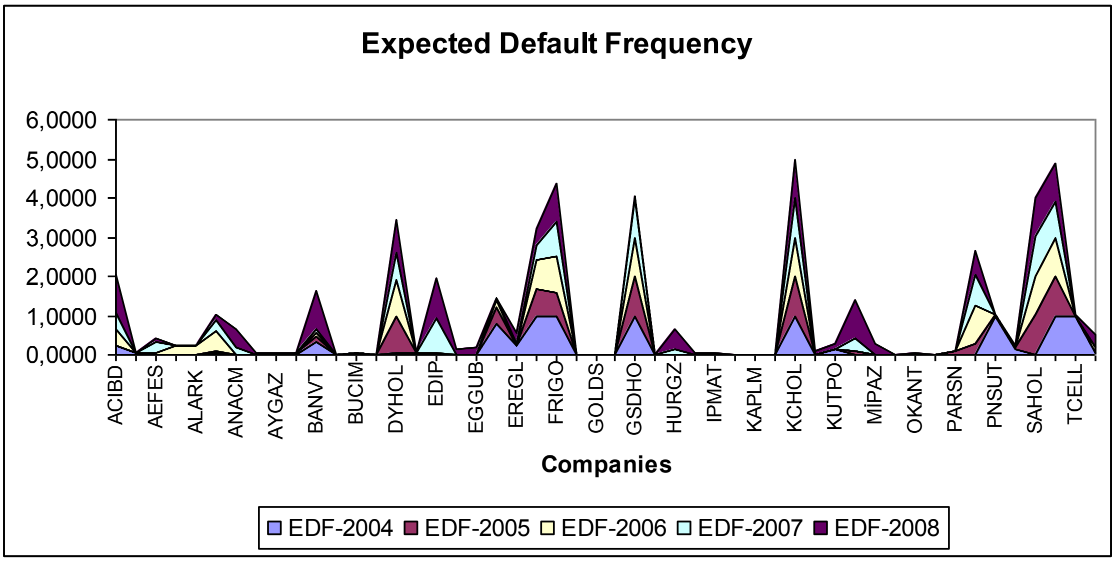
For each year, the EDF's of all companies are shown below. As can be seen, the maximum value for EDF of companies is equal to one.

Table 2.6 EDF of Selected ISE100 Companies (2004-2008)

	EDF-2004	EDF-2005	EDF-2006	EDF-2007	EDF-2008
ACIBD	0.2171	0.0311	0.4253	0.3780	1.0000
ADEL	0.0279	0.0000	0.0000	0.0000	0.0000
AEFES	0.0000	0.0000	0.0374	0.2859	0.1108
AKSA	0.0000	0.0000	0.2378	0.0000	0.0072
ALARK	0.0000	0.0000	0.2165	0.0000	0.0000
ALTIN	0.0518	0.0518	0.5179	0.2392	0.1567
ANACM	0.0000	0.0002	0.0039	0.1848	0.4800
ASELS	0.0000	0.0001	0.0000	0.0000	0.0311
AYGAZ	0.0000	0.0000	0.0191	0.0000	0.0085
BAGFS	0.0213	0.0079	0.0019	0.0000	0.0026
BANVT	0.3327	0.1428	0.0929	0.0608	1.0000
BROVA	0.0000	0.0000	0.0000	0.0000	0.0001
BUCIM	0.0259	0.0000	0.0000	0.0000	0.0000
CIMSA	0.0000	0.0090	0.0017	0.0000	0.0110
DYHOL	0.0498	0.9142	0.9346	0.6844	0.8794
ECILC	0.0627	0.0000	0.0000	0.0000	0.0041
EDIP	0.0000	0.0000	0.0472	0.9003	1.0000
EGEEN	0.0000	0.0000	0.0000	0.0000	0.1494
EGGUB	0.0000	0.0000	0.0000	0.0111	0.1566
ENKAI	0.8013	0.4073	0.1921	0.0232	0.0131
EREGL	0.2243	0.0001	0.0002	0.0527	0.2718
FENIS	0.9606	0.7304	0.7246	0.3767	0.4142
FRIGO	0.9988	0.5921	0.9358	0.8577	1.0000
FROTO	0.0000	0.0000	0.0000	0.0000	0.0003
GOLDS	0.0068	0.0000	0.0108	0.0000	0.0024
GOODY	0.0000	0.0000	0.0000	0.0000	0.0013
GSDHO	1.0000	1.0000	1.0000	1.0000	0.0482
HEKTS	0.0000	0.0000	0.0000	0.0000	0.0000
HURGZ	0.0001	0.0000	0.0000	0.1205	0.5392
IHLAS	0.0012	0.0005	0.0000	0.0000	0.0489
IPMAT	0.0000	0.0186	0.0000	0.0003	0.0123
IZOCM	0.0000	0.0000	0.0000	0.0000	0.0000
KAPLM	0.0056	0.0000	0.0000	0.0000	0.0000
KARTN	0.0000	0.0000	0.0000	0.0000	0.0000
KCHOL	1.0000	1.0000	1.0000	1.0000	0.9999
KENT	0.0000	0.0000	0.0000	0.0000	0.1084
KUTPO	0.1447	0.0000	0.0000	0.0000	0.1290
METUR	0.0000	0.0860	0.0143	0.3095	1.0000
MİPAZ	0.0033	0.0000	0.0019	0.0000	0.2923
MRDIN	0.0000	0.0000	0.0000	0.0000	0.0000
OKANT	0.0182	0.0000	0.0000	0.0000	0.0401
OLMKS	0.0000	0.0000	0.0000	0.0000	0.0000
PARSN	0.0000	0.0942	0.0000	0.0000	0.0008
PENGD	0.0000	0.2689	0.9994	0.7982	0.5900
PNSUT	1.0000	0.0015	0.0006	0.0000	0.0038
PTOFS	0.1618	0.0083	0.0132	0.0000	0.0303
SAHOL	0.0014	1.0000	1.0000	1.0000	1.0000
TATKS	1.0000	0.9998	1.0000	0.9080	0.9900
TCELL	1.0000	0.0000	0.0000	0.0000	0.0001
VESTL	0.0286	0.0000	0.1444	0.0005	0.3371

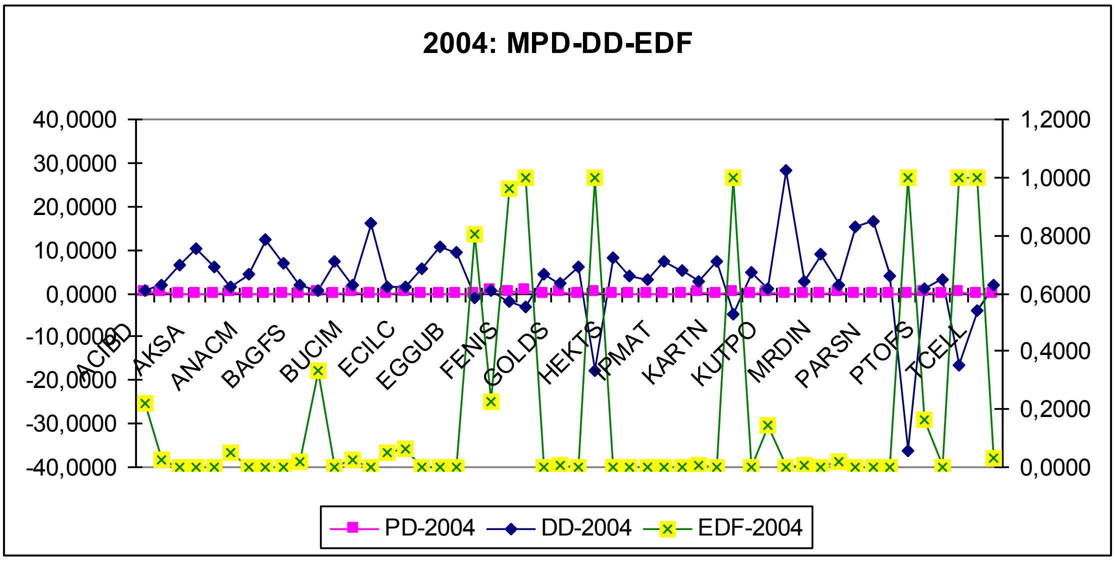
Graph 2.4 illustrates expected default frequencies of the same fifty companies for each year. The companies which have negative distance to default values have greatest expected default frequencies. We can easily see that for almost every company in our study, expected default frequencies increases by time.

Graph 2.4 EDF of Selected Companies from ISE100



Graph 2.5 helps us to see all parameters which we calculate for the companies belong to ISE100 for the year 2004. Each year expected default frequencies increases for nearly all companies in our data and you can analyze this information from graph 2.4.

Graph 2.5 MPD-DD-EDF of Companies



If one wants to see all parameters together, graph 2.5 helps. Graph 2.5 illustrates three parameters (MPD, DD, and EDF) for 2004. From the graph, you can easily see that if DD takes negative values, EDF takes greatest value in a neighborhood range.

After calculating these values, we want to analyze the relationship between these variables. To analyze them, as a first application we calculate the correlation coefficient of these variables. An excerpt of this data is given in Table 2.6. In other words, Table 2.6 illustrates correlation between default parameters.

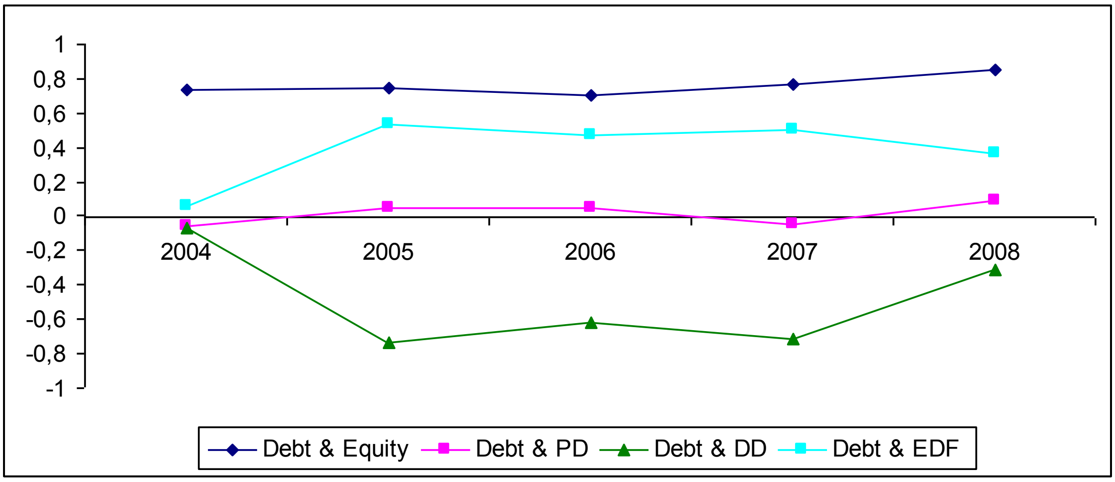
Table 2.7 Correlations between Default Parameters

	Correlation			
Year	**Debt & Equity**	**Debt & MPD**	**Debt & DD**	**Debt & EDF**
2004	0,73	-0,06	-0,07	0,06
2005	0,74	0,05	-0,74	0,54
2006	0,71	0,05	-0,62	0,47
2007	0,76	-0,05	-0,71	0,5
2008	0,85	0,08	-0,31	0,37

From table 2.6 and graph 2.6, we can examine that the correlation between total debt and equity increases by the time, the values are so close to each other and they are highly positively correlated. This means, companies with higher total debt tend to have higher equities and vice versa. In addition, companies with fewer equities tend to have lower total debt. In addition, from the graph 2.6, for years between 2004 and 2008, the correlation between total debt and equity takes nearly same values which are close to 0.8.

The correlation coefficient of total debt and DD take negative values for all years. This means that they are negative correlated and in a negative correlation, as the values of one of the variables increase, the values of the second variable decrease. Likewise, as the value of one of the variable decreases, the value of the other variable increases.

Graph 2.6 Correlations between Default Parameters



Remember that we find MPD with solving two equations. You can find the solutions of the equations for each company for each year in appendix V. In other words, the unobservable parameters A_0 and σ_A for each company, can be found in appendix. Notice that we do not make interpretation with these values. Since these equations can have different values that can be satisfies equations. We checked that different solutions give close values for MPD.

CHAPTER 3 CLUSTER ANALYSIS & FUZZY LOGIC

3.1 CLUSTER ANALYSIS

Cluster analysis divides data into groups (clusters) such that similar data objects belong to the same cluster and dissimilar data objects to different clusters. The resulting data partition improves data understanding and reveals its internal structure. Partition clustering algorithms divide up a data set into clusters or classes, where similar data objects are assigned to the same cluster whereas dissimilar data objects should belong to different clusters. In other words, cluster analysis is an exploratory data analysis tool for organizing observed data such as people, brands, events, companies, countries, etc. into meaningful taxonomies, groups or clusters, which maximizes the similarity of cases within each cluster and maximizes the dissimilarity between clusters or groups that are initially unknown. The term similarity should be understood as mathematical similarity. The term cluster analysis is first used by Robert C. Tryon.[42] In the last few years, the science of cluster analysis has been discovered to be a valuable tool in the physical, economic, finance and biological sciences.

The clustering results depend on the choice of dissimilarity (similarity) so that the natural question is how we should measure the dissimilarity (similarity) between samples. A common choice of dissimilarity function between samples is the Euclidean distance. In metric spaces, similarity is often defined by a distance norm. The distance norm or similarity is usually not known beforehand.

Data can be reveal clusters of different geometrical shapes, sizes and densities. In addition, clusters can be spherical, linear, nonlinear or hollow. The following figures show the examples of cluster types we mentioned, respectively.

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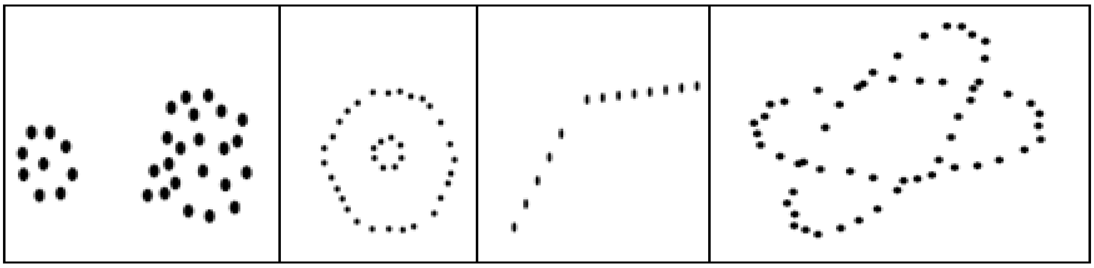
[42] The most thorough treatment of cluster analysis can be found in Robert C. Tryon and Daniel E. Bailey, Cluster Analysis (New York: McGraw-Hill, 1970).

Homogeneity within the clusters means that data that belong to the same cluster should be as similar as possible. Heterogeneity between clusters means data that belong to different clusters should be as different as possible.

FIGURE 3.1 EXAMPLES OF CLUSTER TYPES



The distance between x and y (as data) as considered to be two dimension function satisfying the following properties.

- For every x; $d(x, x) = 0$
- For every x and y; $d(x, y) \geq 0$
- For every x, y; $d(x, y) = d(y, x)$
- For every x, y and z; $d(x, y) + d(y, z) \geq d(x, z)$

In the case of continuous variables, we have a long list of distance functions (which satisfies above properties). Each of distance functions implies different view of data because of their geometry. The following table illustrates the different distance functions with definitions, which are usually measure dissimilarity in cluster analysis.

Table 3.1 Formulas of Distance Functions

Distance Function	Formula (Definition)
Minkowski Distance	$d(x,y) = \sqrt[p]{\sum_{i=1}^{n}(x_i - y_i)^p}$
Hamming Distance	$d(x,y) = \sum_{i=1}^{n} \mid x_i - y_i \mid$
Euclidean Distance	$d(x,y) = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}$
Angular Separation	$d(x,y) = \frac{\sum_{i=1}^{n} x_i y_i}{\left[\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i^2\right]^{1/2}}$
Tchebyschev Distance	$d(x,y) = \max_{i=1,2,..n} \mid x_i - y_i \mid$

The **Minkowski norm** provides a concise, parametric distance function that generalizes many of the distance functions used in the literature. The advantage is that mathematical results can be shown for a whole class of distance functions, and the user can adapt the distance function to suit the needs of the application by modifying the Minkowski parameter. There are several examples of the Minkowski distance, including Hamming distance (usually refereed to as a city-block distance); the Euclidean distance and Tchebyschev distance. They are special case of Minkowski distance when p=1, 2 and infinity.

For example, **Euclidean distance** is the geometric distance between two objects or cases and it is most commonly used one. With Euclidean distances the smaller the distance, the more similar the cases. However, this measure is affected by variables with large size. So, if objects are being compared across variables that have very different variances then the Euclidean distance is not accurate. To handle this problem, you can standardize (normalize) the clustering variables.

For instance, Hamming Distance is a number used to denote the difference between two binary strings. **Hamming distance** was originally conceived for detection and correction of errors in digital communication. In the context of prioritized model checking, the minimum Hamming distance between the state being explored and the set of error states is used as an evaluation function to guide the search. The Tchebyschev distance takes into consideration the maximal distance over the coordinates x and y.

Remember that, distance can be measured in a variety of ways. For example, **Ward's Method** is distinct from other methods because it uses an analysis of variance approach to evaluate the distances between clusters. In general, this method is very efficient. Cluster membership is assessed by calculating the total sum of squared deviations from the mean of a cluster. The criterion for fusion is that it should produce the smallest possible increase in the error sum of squares.[43]

In real applications, there is very often no sharp boundary between clusters so that fuzzy clustering is often better suited for the data. Clustering can also be thought of as a form of data compression, where a large number of samples are converted into a small number of representative prototypes or clusters. Depending on the data and the application, different types of similarity measures may be used to identify classes, where the similarity measure controls how the clusters are formed.

Clustering techniques can be applied to data that is quantitative (numerical), qualitative (Categorical), or a mixture of both. However, having a mixture of different types of data will make the analysis more complicated. In this thesis, the clustering of quantitative data is considered.

Clustering algorithms can be applied to many fields such as marketing, insurance, earthquake studies and city planning. For instance, as application of the insurance; you can identify groups of motor insurance policy holders with a high average claim cost. For the city planning, by clustering you can identify groups of houses according to their

[43] Ward's Method, Cluster Analysis, 2001.

house types, value and geographical location.

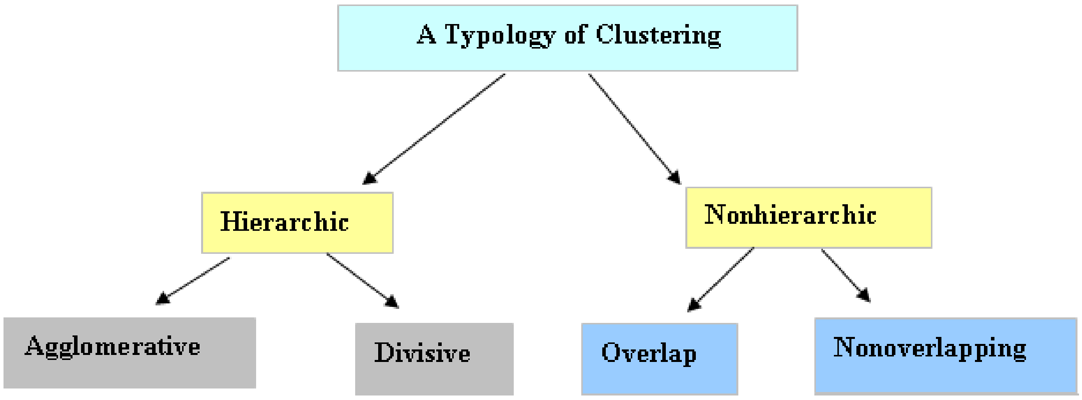
Desirable Properties of a Clustering Algorithm

- Scalability (in terms of both time and space)
- Ability to deal with different data types
- Minimal requirements for domain knowledge to determine input parameters
- Able to deal with noise and outliers
- Insensitive to order of input records
- Incorporation of user-specified constraints
- Interpretability and usability

3.1.1 A TYPOLOGY OF CLUSTERING

To discuss the variety of clustering concepts and algorithms, a classification of them is necessary. Typically, such a classification involves the following three binary oppositions presented by Sneath and Sokal 1973.

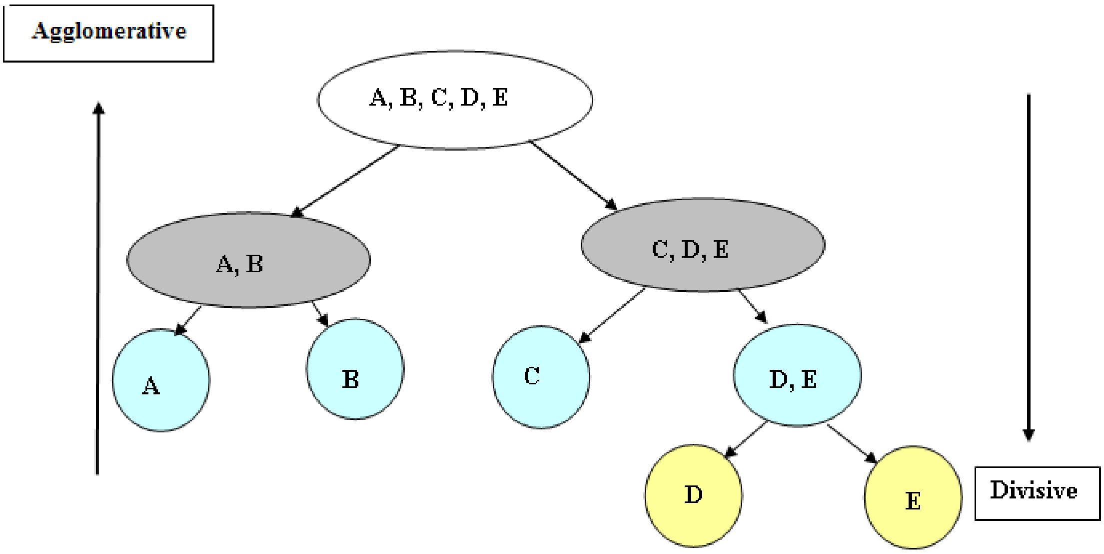
FIGURE 3.2 TYPOLOGY OF CLUSTERING



3.1.1.1 Hierarchic versus Nonhierarchical (Partional) Methods

The clustering techniques in this category produce a graphic representation of data.[44] The construction of graphs is done in two ways, bottom-up[45] and top-down[46]. In hierarchical clustering the data are not partitioned into a particular cluster in a single step. Instead, a series of partitions takes place, which may run from a single cluster containing all objects to n clusters each containing a single object. Hierarchical Clustering is subdivided into agglomerative methods, which proceed by series of fusions of the n objects into groups, and divisive methods, which separate n objects successively into finer groupings. . Hierarchical clustering may be represented by a two dimensional diagram known as dendogram which illustrates the fusions or divisions made at each successive stage of analysis. An example of such a dendogram is defined in the following figure.

FIGURE 3.3 DENDOGRAM



Source: Boris Mirkin, Mathematical Classification and Clustering- Nonconvex Optimization and Its Application, Kluwer Academic Publication.

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[44] Duda et al., 2001.

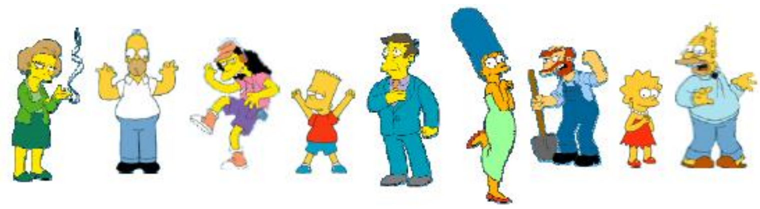
[45] Agglomerative Method: treat each pattern as a single element cluster then merge the closest clusters, repeat until reach to a single data set.

[46] **Divise Method:** Opposite of Agglomerative Method, start with the entire set treated as single cluster and keep splitting it into smaller clusters.

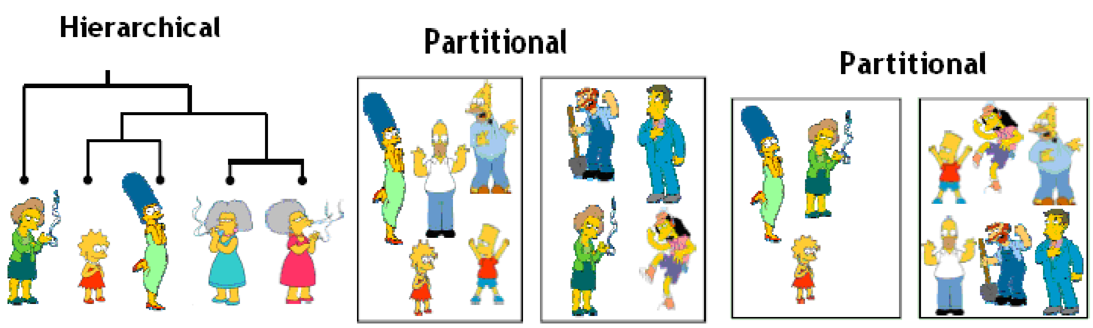
This diagram illustrates which clusters have been joined at each stage of the analysis and the distance between clusters at the time of joining. If there is a large jump in the distance between clusters from one stage to another then this suggests that at one stage clusters that are relatively close together were joined whereas, at the following stage, the clusters that were joined were relatively far apart. This implies that the optimum number of clusters may be the number present just before that large jump in distance.

Hierarchical methods generate clusters as nested structures, in a hierarchical fashion; the clusters of higher levels are aggregations of the clusters of lower level. Nonhierarchical methods result in a set of unnested clusters.

Example 3.1.1.1.1 Assume we have following objects to cluster**.**



We can cluster these objects with partitional or hierarchical clustering. The following graphs are examples of clustering types. Since clustering is subjective; we can separate objects such as females or males, as in last figure or we can separate objects from Simpson’s Family or not.



3.1.1.2 Agglomerative versus Divisive Methods

Agglomerative method starts with each item in its own cluster, find the best pair to merge into a new cluster. Repeat until all clusters are fused together**.** Also, it is called bottom up method. Divisive Method; which is also called top-down method; starts with all the data in a single cluster, consider every possible way to divide the cluster into two. Choose the best division and recursively operate on both sides. This refers to the methods of the hierarchical clustering according to the direction of generating the hierarchy, merging smaller clusters into the larger ones bottom-up (agglomerative) or splitting the larger ones into smaller cluster stop-down (divisive). Agglomerative methods have been developed for processing mostly similarity/dissimilarity data while the divisive method mostly worked with attribute- based information. The above figure shows the direction.

3.1.1.3 Nonoverlapping versus Overlapping Methods

Most clustering methods partition the data into non-overlapping regions, where each point belongs to only one cluster. Overlapping clustering allows items to belong to multiple clusters and it provides o more natural way to discover interesting and useful classes in data.

SPSS[47] has three different procedures that can be used to cluster data: hierarchical cluster analysis, k-means clusters and two step clusters. Hierarchical clustering requires a matrix of distances between all pairs of cases, and k-means cluster requires shuffling cases in and out of clusters and knowing the number of clusters in advances. In a very large data set, if one needs a clustering procedure that can rapidly form clusters on the basis of either categorical or continuous data; neither Hierarchical clustering nor k-means cluster works. In this thesis, we use two step clustering method using SPSS. In two step clustering method in SPSS, we have an option to create a separate cluster for cases that do not well into any other clusters and defined as outlier cluster.

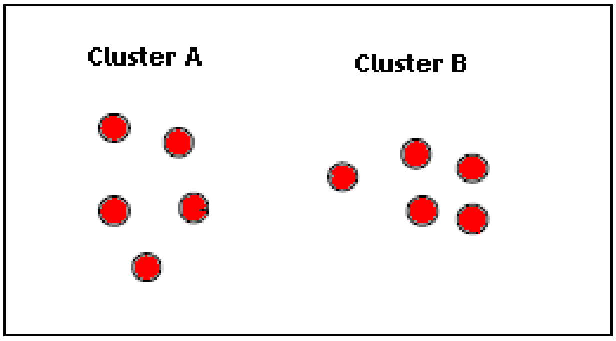
[47] Statistical Package for the Social Sciences.

3.1.2 CLUSTERS

Various definitions of a cluster can be formulated, depending on the objective of clustering. Generally, one may accept the view that a cluster is a group of objects that are more similar to one another than the members of other clusters. As we mentioned before, the term "similarity" should be understood as mathematical similarity, measured in some well-defined sense. In metric spaces, similarity is usually defined by means of a distance norm.

Note that, using cluster analysis, we can also form groups of related variables similar to what we do in factor analysis. On the other hand, an important issue is how to measure the distance between two clusters. Assume we have two clusters; A and B; as in Figure 3.4.

FIGURE 3.4 CLUSTER A & CLUSTER B



There exist three main distance functions to measure the distance between clusters. Single link method, complete link method and group average link method.

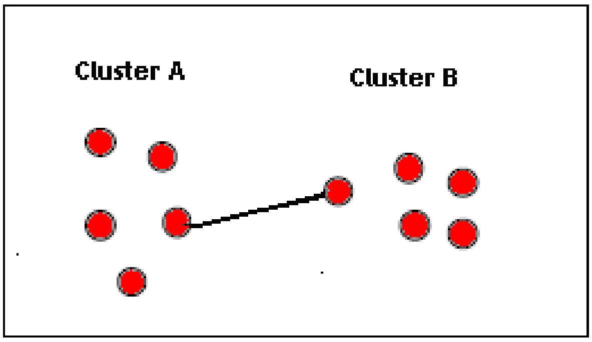
3.1.2.1 Single Link Method

The distance between cluster A and cluster B based on the minimal distance between patterns belonging to A and B. The mathematical definition of the single method, if we denote the distance function of clusters with $d^{'}$,

$$d^{'}(A,B) = \min_{x \in A, y \in B} d(x,y), \text{ where } d(x,y) \text{ is any distance function.}$$

The following figure illustrates the single link method.

FIGURE 3.5 SINGLE LINK METHOD



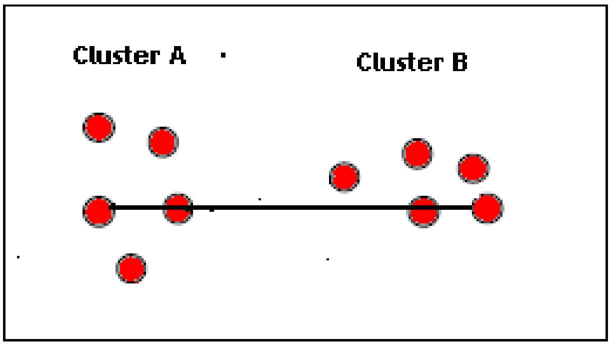
3.1.2.2 Complete Link Method

The distance between cluster A and cluster B based on the maximum distance between patterns belonging to A and B. In other words, this method is at the opposite end of the spectrum, as it based on the distance between two farthest patterns belonging to clusters A and B. The mathematical definition of the complete link method, if we denote the distance function of clusters with $d^{''}$,

$$d^{''}(A,B) = \max_{x \in A, y \in B} d(x,y), \text{ where } d(x,y) \text{ is any distance function.}$$

The following figure illustrates the complete link method.

FIGURE 3.6 COMPLETE LINK METHOD



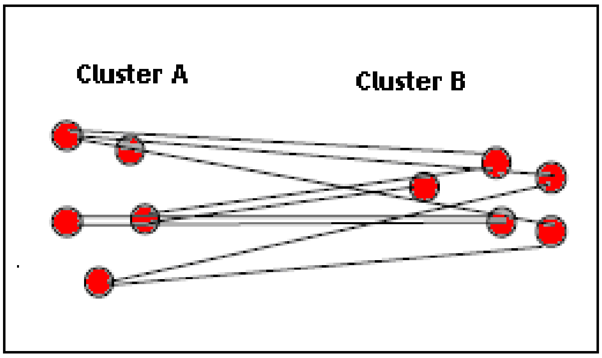
3.1.2.3 Group Average Link

The group average link method considers the average between the distances computed with the all pairs of the pattern, one from each cluster. The mathematical definition of the group average link method, if we denote the distance function of clusters with d''',

$$d'''(A,B) = \frac{1}{card(A)card(B)} \sum_{x \in A, y \in B} d(x,y), \text{ where } d(x,y) \text{ is any distance function.}$$

The following figure illustrates the complete link method.

FIGURE 3.7 GROUP AVERAGE LINK METHOD



In addition, one can use the correlation coefficient as a measure of similarity. To take the absolute value of it before forming clusters because variables with large negative correlation coefficients are just as closely related as variables with large positive coefficients.

3.1.3 FUZZY CLUSTERING

Ruspini (1969) first proposed the idea of fuzzy partitions. After that, fuzzy clustering has grown to be an important tool for data analysis and modeling. Up to that time, it has been applied extensively for diverse tasks such as data analysis, data mining and image processing. The clustering results are largely influenced by how this distance is computed, since it determines the shape of the clusters. The success of fuzzy clustering in various applications may depend very much on the shape of the clusters. In addition, fuzzy clustering can be used as a tool to obtain the partitioning of data. Since clusters can formally be seen as subsets of the data set, one possible classification of clustering methods can be according to whether the subsets are crisp of fuzzy.

3.1.4 LIMITATIONS OF CLUSTERING (ADVANTAGES & DISADVANTAGES OF CLUSTER ANALYSIS)

The different methods of clustering usually give different results. It is important to think carefully to decide which method to use or which clustering variables you will use. The best way to get the correct results you should know your data in detail and know each clustering variables with statistics or properties. The cluster analysis is not stabile when cases are dropped. The cases are dropped because merger of clusters depends on similarity of one case to the cluster. Dropping one case can affect the course in which the analysis progresses. In addition, the choice of variables included in a cluster analysis must be underpinned by conceptual consider because the clustering has no mechanism for differentiating between relevant and irrelevant variables.

Compared to factor analysis, clustering variables identifies the key variables which explain the principal dimensionality in the data, rather than abstract factors; allows much larger correlation or covariance matrices to be analyzed; and greatly simplifies interpretation.

One of the disadvantages of clustering is to find an outlier cluster, which includes the data that does not fit well into any other cluster. We can see this as a disadvantage, because we want to analyze and make predictions from the results of clusters.

3.2 FUZZY LOGIC

In the computational world, there are two broad areas of logic: Crisp Logic and Fuzzy Logic. Crisp Logic arises out of the fundamental concepts of such people Aristotle and Pythagoras who based their work on the idea that everything in the universe can be describe by numerical formulate and relationships. In crisp logic which is also Boolean Logic, problems are simplified by reducing the possible states a variable may have (Black and white, on or off.) The original 0 and 1 or binary set theory was invented by German mathematician Georg Cantor48. The Polish philosopher Jan Lukasiewicz developed the first logic of vagueness in 1920 when he created se with possible membership values 0, 1 and ½.   49 He developed first three valued logic system also in the same period quantum philosopher Black identified the logic with continuous values. But they could not find an application area and these studies did not reach a conclusion.

Fuzzy set theory is developed for solving problems in which descriptions of activities and observations are imprecise, vague, and uncertain. The term “fuzzy” refers to the situation in which there are no well-defined boundaries of the sets of activities or observations to which the descriptions apply. For example, one can easily assign a person seven feet tall to the “class of tall man”. But it would be difficult to justify the inclusion or exclusion of a six-foot tall person to that class, because the term “tall” does not constitute a well-defined boundary. This notion of fuzziness exists almost everywhere in our daily life, such as the “class of red flowers,” the “class of good

[48] German mathematician, who is known as the inventor of the set theory.
[49] Johnson 1972, p. 55

kickers," the "class of expensive cars," or "numbers close to ten," etc. These classes of objects cannot be well represented by classical set theory. In classical set theory, an object is either in a set or not in a set. In optimization, a solution is either feasible or not. Real situations are very often not crisp and deterministic, and they cannot be described precisely.

An object cannot partially belong to a set50. To cope with this difficulty, Lotfi A. Zadeh proposed the fuzzy set theory published in his famous paper "Fuzzy Sets" in Information and Control in 1965. The concept of fuzzy logic, first presented by Zadeh51, who is a well-respected professor in the department of electrical engineering and computer science. Zadeh was a well-respected scholar in control theory before he was working on fuzzy theory. He developed the concept of "state", which forms the basis for modern control theory. In the early 60's, he thought that classical control theory had put to much emphasis on precision therefore could not handle the complex systems. As early as 1962, he wrote to handle biological systems "we need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not describable in terms of probability distribution (Zadeh, 1962). Later he formalized the ideas into his paper Fuzzy Sets. Most of the fundamental concepts in fuzzy process were proposed by Zadeh in late 60's and early in 70's. After the introduction of fuzzy sets in 1965, he proposed the fuzzy algorithm in 1968, fuzzy decision making in 1970 (Bellman &Zadeh), and fuzzy ordering in 1971 (Zadeh). In 1973, he published another seminal paper, "Outline of a new approach to the analysis of complex systems and decision process" (Zadeh, [1973]), which establishes the foundation for fuzzy control. In this paper, he introduced concept of linguistic variables and proposed to use if-then rules to formulate human knowledge. Since then, fuzzy set theory has been rapidly developed by Zadeh himself and numerous researchers, and an increasing number of successful real applications of this theory in a wide variety of unexpected fields have been appearing.

It was Plato who laid the foundation for what would become fuzzy logic, indicating that

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[50] Shu-Jen Chen, Chin-Lai Hwang, in collaboration with Frank P. Hwang, **Fuzzy Multiple Attribute Decision Making,** Germany, 1992, Springer-Verlag, s. 42.
[51] L. Zadeh, 1965.

there was a third region (beyond True and False, beyond Black and White as Grey) where these opposites "tumbled about." Other, more modern philosophers echoed his sentiments, notably Hegel, Marx, and Engels. But it was Lukasiewicz who first proposed a systematic alternative to be valued logic of Aristotle52.

Japanese engineers, with sensitivity to new technology; quickly found that fuzzy controllers were very easy to design and worked very well for many problems. In 1983, Sugeno and Nisnida began the pioneer work on a fuzzy robot, self-parking car that was controlled by calling out commands. Because fuzzy logic provides the tools to classify information into broad, coarse categorizations or groupings, it has infinite possibilities for application which have proven to be much cheaper, simpler and more effective than other systems in handling complex information. Fuzzy logic has extremely broad implications for many fields, not just for electrical engineering and computer technology. Numerous consumer goods especially household products and cameras already incorporate fuzzy logic into their design.

The development of fuzzy set theory to fuzzy technology during the first half of the 1990s has been very fast. More than 16.000 publications have appeared since 1965. Most of them have advanced the theory in many areas. Today; in almost every field; fuzzy logic are able to apply, especially it is widely used in industrial fields. In particular Japanese were implemented the fuzzy logic in washing machines, dishwasher, vacuum cleaners and video cameras. Moreover software and hardware which are intended for fuzzy logic applications are presented to consumer in the market. Even fuzzy microprocessors are also offered to sale. The Panasonic's video camera which automatically eliminate the vibration between shots in the case of hand-held and Matsushita's washing machine that choose washing program according to laundries dirtiness, weight and kind of clothes are interesting examples of fuzzy logic applications.

In economics and finance, fuzzy logic can be applied in modeling complex sales and

[52] C. Lejewski, "Jan Lukasiewicz," **Encyclopedia of Philosophy**, Vol. 5, Mac Millen, NY: 1967, pp. 104-107.

trade systems, cost-benefit analysis, investment evaluations and in portfolio analysis. Also, in banking system fuzzy logic approach is used for the assessment of credit demand or rating credits. Fuzzy systems are used mostly for estimating, decision making and mechanical control systems.

A fuzzy set is a class of objects with a continuum of membership grades. The main idea of fuzzy set theory is quite intuitive and natural: Instead of determining the exact boundaries as in an ordinary set, a fuzzy set allows no sharply defined boundaries because of a generalization of a characteristic function to a membership function53. The membership function, which assigns to each object a grade of membership, is associated with each fuzzy set. Usually, the membership grades are in [0, 1]. When the grade of membership for an object in a set is one, this object is absolutely in that set; when the grade of membership is zero; the object is absolutely not in that set. Borderline cases are assigned numbers between zero and one. Precise membership grades do not convey any absolute significance. They are context-dependent and can be subjectively assessed54.

Aristotle outlined a system of logic that has influenced Western thought for over two thousand years. A syllogism is a conclusion based on two premises. Statements or conclusions are evaluated as either true or false. Truth is regarded as a bivalent variable which assumes a value equal to only true or false (zero or one). This system of logic is powerful and reliable if applied appropriately, and if one has premises that are unquestionably either true or false. Shortly, in conventional logic or in other saying Boolean logic there is only two possible states a variable may have such as we defined before:” Black or White”, “True or False”, and “One or Zero”. However, in fuzzy logic variables can have different values according to their degrees of membership. For example, the concept of “warm room temperature” may be expressed as an interval (e.g. $[21C^0, 25C^0]$) in classical set theory. But, the definition of the warm room temperature does not have well defined boundaries. The definition depends people to people. Suppose, we have a room whose temperature is $25C^0$, we do not say this room is not

53 Masatoshi Sakawa, Fuzzy Sets and Interactive Mul i-objective Optimization, New York, Plenum Press, 1993, s. 36.

54 Shu-Jen Chen, Chin-Lai Hwang, in collaboration with Frank P. Hwang, Fuzzy Multiple Attribute Decision Making, Germany, 1992, Springer Verlag, s. 42.

warm. We may say it is slightly warm. Thus, representation of the concept closer to the human interpretation. As we mentioned before, in fuzzy sets, instead of determining the exact boundaries as in an ordinary set, a fuzzy set allows no sharply defined boundaries because of a generalization of a characteristic function to a membership function.

May be, we can think fuzzy logic as a part of a logic. However, some logicians do not believe the use of fuzzy logic. For example Haack55, a formal logician, has some criticisms about fuzzy logic. She states that there are only two areas where fuzzy logic is "needed". (But, in each case, Haack shows the ultimately classical logic can substitute for fuzzy logic.) The following are Haack's two cases that may require for fuzzy logic:

- **Nature of Truth and Falsity:** Haack argues that True and False are discrete terms. In classical logic, any fuzziness that arises from a statement is due to an imprecise definition of terms. But, Haack says that if it can be shown that fuzzy values are needed indeed fuzzy (meaning not discrete), then a need for fuzzy logic can demonstrated.

- **Utility of Fuzzy Logic:** Haack says that if it can be shown that generalizing classic logic to include fuzzy logic would aid calculations, then fuzzy logic would be needed. However, she argues that data manipulation in a fuzzy system actually becomes more complex. So, fuzzy logic is not necessary.

Haack[56] believes fuzzy logic is not necessary because the calculations are more involved and partial membership values can be eliminated by defining terms more precisely. Fox has responded to Haack's objections. He believes that the following three areas can benefit from fuzzy logic.

- **Requisite Apparatus:** Use fuzzy logic to describe the real world relationships

[55] Haack, "Do we need fuzzy logic?" **Int. Journal. of Man-Mach. Stud.**, Vol. 11, 1979, pp.437-445.
[56] Haack Susan, Deviant Logic and Fuzzy Logic: Beyond the Formalism.

that are inherently fuzzy.

- **Prescriptive Apparatus:** Use fuzzy logic because some data is inherently fuzzy and needs fuzzy calculations.
- **Descriptive Apparatus:** Use fuzzy logic because some inferencing systems are inherently fuzzy.

3.2.1 FUZZY SETS & MEMBERSHIP FUNCTIONS

In this section, we will review the definition of a fuzzy set as well as some of its basic concepts as they apply to later chapters in this thesis. In addition, we give the definition of fuzzy membership functions, their properties their types.

3.2.1.1 Fuzzy Sets

A fuzzy set is represented by a membership function defined on the universe of discourse. The universe of discourse is the space where the fuzzy variables are defined. The membership functions give the grade or degree of membership function within the set of any element of the universe of discourse. The membership function maps the elements of universe onto numerical values in the interval [0, 1]. A membership function value of zero implies that the corresponding element is definitely not an element of the fuzzy set, while a value of unity (1) means that the element fully belongs to the set. A membership function of a fuzzy set is continuous function with range [0, 1]. So, there is nothing fuzzy about a fuzzy set. It is simply a set with a continuous function. For simplicity, we use piecewise linear membership functions such as triangles and trapezoids. Also, membership functions can be continuous curves such as Gaussian membership function and bell membership function. They have an advantage of being smooth and nonzero at all points. However, they are unable to specify asymmetric membership function, which are important in many applications.

Like convential set, a fuzzy set can be used to describe the value of a variable. If a variable can take words or sentences in natural or artificial language as its values, it is called **linguistic variable** where the words are characterized by fuzzy sets. For example, age is a linguistic variable if it values are old, young, very young, quite young, not very old instead of 56, 36, 25,… . Linguistic term is used to express concepts and knowledge in human communication, whereas membership function is useful for processing numeric input data. In our model, the linguistic term is default probability and it values as normal, good, extremely good, bad and extremely bad. Now, we will give mathematical definition of fuzzy sets.

3.2.1.1.1 Definition of a Fuzzy Set

A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X a real number in the interval $[0,1]$. The function value $\mu_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$ .The fuzzy set $\tilde{A}$, is usually denoted by the set of pairs $\tilde{A} = \{x, \mu_{\tilde{A}}(x)), x \in X\}$.

For an ordinary set, $\tilde{A}$

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 & \text{iff} \quad x \in \tilde{A} \\ 0 & \text{iff} \quad x \notin \tilde{A} \end{cases}.$$

When X is a finite set such as; $\{x_1, x_2, ..., x_n\}$ the fuzzy set on X may also be represented as:

$$\tilde{A} = \sum_{i=1}^{n} \frac{x_i}{\mu_{\tilde{A}}(x_i)}.$$

When X is an infinite set, the fuzzy set may be represented as:

$$\tilde{A} = \int \frac{x}{\mu_{\tilde{A}}(x)}.$$

A fuzzy set $\tilde{A}$ of the universe of discourse X is called a **normal fuzzy set** implying that

$$\exists x_i \in X,\ \mu_{\tilde{A}}(x_i) = 1.$$

Example 3.2.1.1 Let $X = \{Tom, John, Mark, Peter\}$ which is a finite set. Evaluated by a girl, the fuzzy set "handsome boys", may be characterized as:

$$\tilde{A} = \{(Tom, 0.7), (John, 0.2), (Mark, 0.8), (Peter, 0.6)\},$$

$$\tilde{A} = \frac{Tom}{0.7} + \frac{John}{0.2} + \frac{Mark}{0.8} + \frac{Peter}{0.6}.$$

Example 3.2.1.2 Consider the set of old people belonging to the universe of people in the age of 0 to 120.

We can define the membership function (μ_{old}) as:

$$\mu_{old}(x) = \begin{cases} 0 & ; \quad 0 \le x \le 60 \\ \dfrac{x-60}{20} & ; \quad 60 < x \le 80 \\ 1 & ; \quad x > 80 \end{cases}$$

Example 3.2.1.3 Suppose we want to model the notion of "high income" with a fuzzy set. Let U denote the set of positive real numbers which represents the possible yearly total income. Assume we believe that no one thought that an income under \$10.000 was high and everyone thought that yearly income over \$100.000 was high, however the proportion p of people that thought that an income x between \$10.000 and \$100.000 was high approximately, $p = \dfrac{x-10}{90}$ for any $x \in [10, 90]$ and the membership function is in the following form.

$$\mu_{high}(x) = \begin{cases} 0 & ; \quad 0 \le x \le 10 \\ \dfrac{x-10}{90} & ; \quad 10 < x \le 100 \\ 1 & ; \quad x > 100 \end{cases}$$

There are various way to get reasonable membership function but we have modeled this membership function with piecewise linear function which is very simple for computations. Most commonly used piecewise linear functions are triangular and trapezoidal functions. Even though, there are situations in which nonlinear membership functions are more suitable, most practitioners have found that triangular and trapezoidal membership functions are sufficient for developing good approximate solutions for the problems they wish to solve.

3.2.1.2 Membership Functions and Fuzzy Numbers

In this section, we will give the definitions and graphs of the membership functions which we use to model the bankruptcy probability**.**

3.2.1.2.1 Triangular Membership Functions

The triangular curve is a function of a vector x; and depends on three scalar parameters a_1, a_2 and a_3. The parameters a_1 and a_3 locate the "feet" of the triangle and the parameter a_2 locates "peak".

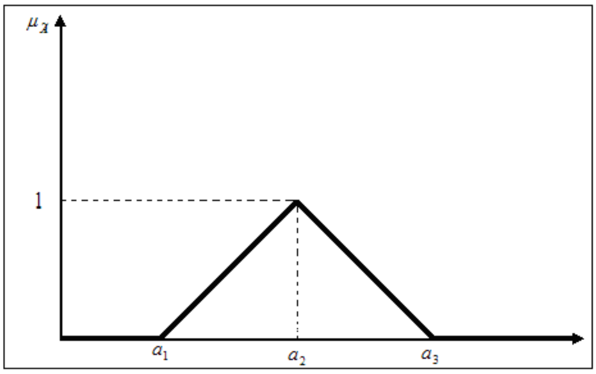
$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , \quad x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & , \quad a_1 \le x < a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & , \quad a_2 \le x < a_3 \\ 0 & , \quad a_3 \le x \end{cases} \quad ; \text{where } a_1 < a_2 < a_3 \qquad (1)$$

In addition, a triangular fuzzy number $\tilde{A}$ also can be defined by a triplet (a_1, a_2, a_3) shown in Figure 4.1. The membership function $\mu_{\tilde{A}}(x)$ is defined as (1)**.** A fuzzy number $\tilde{A}$ is often written as $\tilde{A} = (a_1, a_2 a_3) = (m, \alpha, \beta)$, where $m = a_2$ is the mean of fuzzy number $\tilde{A}$ and $\alpha = a_2 - a_1, \beta = a_3 - a_2$ are the left and right "spreads", respectively. When $\alpha = \beta = 0$, $\tilde{A}$ is considered a crisp number.

Alternatively, defining the interval of confidence at level α, triangular fuzzy number. Is characterized as

$$\tilde{A} = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3],\ \forall \alpha \in [0,1].$$

FIGURE 3.8 A TRIANGULAR FUZZY NUMBER



Now, we will briefly define some important properties which are easy to verify.

i) Addition or subtraction operations on two triangular fuzzy numbers give a triangular trapezoidal fuzzy number.

ii) Multiplication, inverse and division operations on triangular trapezoidal fuzzy number do not necessarily gives a triangular fuzzy number.

iii) Maximum and minimum operations on triangular fuzzy number do not give a triangular fuzzy number. However, in the case of a triangular fuzzy number, we can approximate the results of these operations by a triangular fuzzy number.

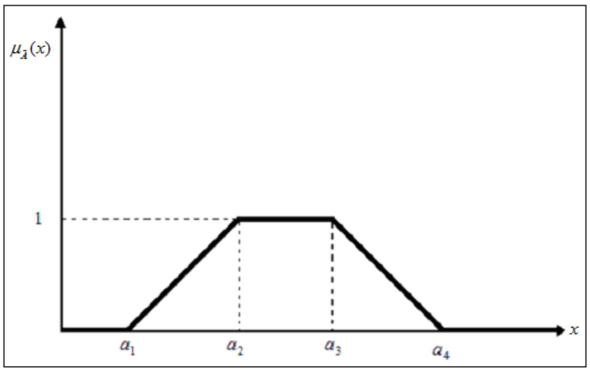
3.2.1.2.2 A Trapezoidal Membership Functions

The trapezoid curve is a function of a vector, x and depends on four scalar parameters a_1, a_2, a_3 and a_4. The parameters a_1 and a_4 locate the "feet" parameters of the trapezoid and the parameters a_2 and a_3 locate the "shoulders".

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , \quad x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & , \quad a_1 \le x \le a_2 \\ 1 & , \quad a_2 < x < a_3 \\ \dfrac{a_4 - x}{a_4 - a_4} & , \quad a_3 \le x \le a_4 \\ 0 & , \quad x > a_4 \end{cases} \quad ; \text{where } a_1 < a_2 < a_3 < a_4 \tag{2}$$

In addition, a Trapezoidal Fuzzy Number can be represented completely by a quadruplet $\tilde{A} = (a_1, a_2, a_3, a_4)$. In this case for $\alpha = 1$ we do not have a point, rather we have a flat line over an interval (a_2, a_3) as shown in following figure.

FIGURE 3.9 A TRAPEZOIDAL FUZZY NUMBER



By the interval of confidence at level α, trapezoidal fuzzy number can be characterized. Thus, $\tilde{A} = [(a_2 - a_1)\alpha + a_1, -(a_4 - a_3)\alpha + a_4], \ \forall \alpha \in [0,1]$.

The membership function of a trapezoidal fuzzy number is characterized as (2). It should be noted that a triangular fuzzy number is a special case of a trapezoidal fuzzy number with $a_2 = a_3$. We can extend all the results of algebraic operations on triangular fuzzy numbers to the trapezoidal fuzzy numbers as well. Some of these are summarized below.

i) Addition or subtraction operations on two trapezoidal fuzzy numbers give a trapezoidal fuzzy number.

ii) Multiplication, inverse and division operations on trapezoidal fuzzy number do not necessarily gives a trapezoidal fuzzy number.

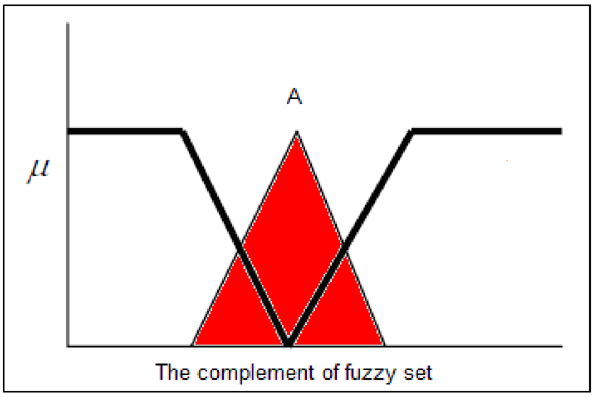
iii) Maximum and minimum operations on trapezoidal fuzzy number do not give a trapezoidal fuzzy number. However, in the case of a trapezoidal fuzzy number, we can approximate the results of these operations by a trapezoidal fuzzy number.

3.2.3 OPERATIONS ON FUZZY SETS

In his very first paper about fuzzy sets[57], L. A. Zadeh suggested the minimum operator for the intersection and the maximum operator for the union of two fuzzy sets.

Applying the fuzzy operators can be thought as the second step of the fuzzy inference system. Let A and B be two fuzzy sets. We say that A and B equal if and only if $\mu_A(x) = \mu_B(x)$ for all x in U (universe). We say that $A \subset B$ if and only if $\mu_A(x) \leq \mu_B(x)$, for all $x \in U$. The complement A^c of the set A is a fuzzy set in U, whose membership function is defined as $\mu_{A^c}(x) = 1 - \mu_A(x)$. The Figure 4.3 illustrates the membership function of complement of any fuzzy set which have triangular membership function.

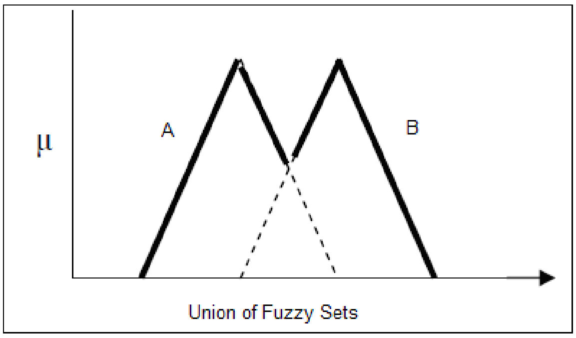
FIGURE 3.10 COMPLEMENT OF A FUZZY SET



[57] L.A. Zadeh, Fuzzy Sets, **Information and Control**, 1965

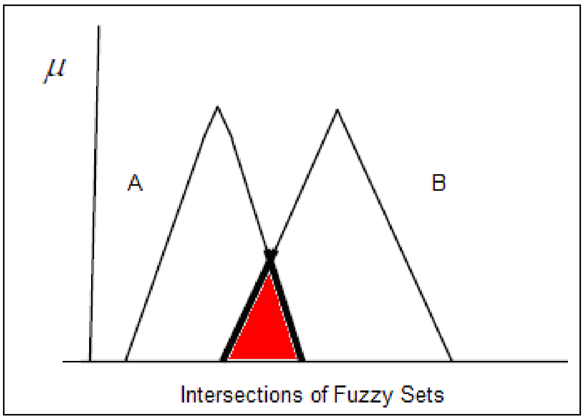
The union of A and B is a fuzzy set $A \cup B \text{ in } U$, whose membership function is defined as $\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$. The Figure 4.4 illustrates the membership function of union of two fuzzy sets which have triangular membership function.

FIGURE 3.11 UNION OF FUZZY SETS



The intersection of A and B is a fuzzy set $A \cap B \text{ in } U$, whose membership function is defined as $\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$. The Figure 4.5 illustrates the membership function of intersection of two fuzzy sets which have triangular membership function.

FIGURE 3.12 INTERSECTION OF FUZZY SETS



In addition to the operations of union and intersection, one can define a number of other ways of forming combinations[58] of fuzzy sets.[59]

[58] Algebraic product, algebraic sum, extension.

There is a wide variety of fuzzy complement, fuzzy union, fuzzy intersection operators, a number of different interpretations of fuzzy "if then" rules were proposed in the literature. Like Zadeh, Godel, Mamdani, Lukasiewicz, Dienes-Rescher implications. For instance in Dienes-Rescher implication they replace the logic operators NOT and OR in NOTP OR Q by the basic fuzzy complement and the basic fuzzy union, respectively. In other words, the fuzzy "if then" rule If A and B is interpreted as a fuzzy relation Q in $\mathrm{U} \times \mathrm{V}$ with the membership function.

In the Mamdani application, which we will explain in the next chapter (Fuzzy Inference System), the fuzzy "if then" rule If A and B is interpreted as a fuzzy relation Q in $U \times V$ with the membership function. Now, we will briefly give definitions of the operations to the membership functions as triangular and trapezoidal fuzzy numbers which we use in our model.

3.2.3.1 Operations on Triangular Fuzzy Number

Equity of Triangular Fuzzy Numbers: Let $\tilde{m} = (m_1, m_2, m_3)$ and $\tilde{n} = (n_1, n_2, n_3)$ be two triangular fuzzy numbers. If $\tilde{m} = \tilde{n}$, then $m_1 = n_1, m_2 = n_2$ and $m_3 = n_3$.

Distance between Triangular Fuzzy Numbers: The distance between $\tilde{m} = (m_1, m_2, m_3)$ and $\tilde{n} = (n_1, n_2, n_3)$ is calculated as

$$d(\tilde{m}, \tilde{n}) = \sqrt{\tfrac{1}{3}\left[(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2\right]},$$

and it is called the vertex method. By definition, $\tilde{m}$ is closer to $\tilde{n}$ as $d(\tilde{m}, \tilde{n})$ approaches to 0.

Algebraic Operations on Triangular Fuzzy Number: We will briefly explain some important properties which are easy to verify.

[59] L.A. Zadeh, Fuzzy Sets, **Information and Control**, 1965

- Addition or subtraction operations upon triangular fuzzy numbers, give a triangular fuzzy number.
- Multiplication, inverse and division operations on triangular fuzzy numbers, do not necessarily give a triangular fuzzy number.
- Maximum and minimum operations on triangular fuzzy number do not necessarily give a triangular fuzzy number.
- However, the results of these operations can be approximated by a triangular fuzzy number.

Now, we define two triangular fuzzy numbers $\tilde{A}$ and $\mathring{B}$ by the triples as $\tilde{A} = (a_1, a_2, a_3)$ and $\mathring{B} = (b_1, b_2, b_3)$.

- **Addition:** $\tilde{A}(+)\mathring{B} = (a_1, a_2, a_3)(+)(b_1, b_2, b_3)$
  $= (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
- **Subtraction:** $\tilde{A}(-)\mathring{B} = (a_1, a_2, a_3)(-)(b_1, b_2, b_3)$
  $= (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

- The **symmetric** (image) of a triangular fuzzy number. is defined as

$$-(\tilde{A}) = (-a_3, -a_2, -a_1).$$

- **Multiplication, Inverse and Division:** For multiplication, inverse and division operations, triplets cannot be used. However the computation can be done using the confidence interval at each level α. For triangular fuzzy number in $\mathbf{R}$ we must decompose the levels in such a way as to examine in the computation the effect of minimum and maximum with positive and negative values when α increases from 0 to ∞. In $\mathbf{R}^+$ the compositions are very simple.

Example 3.2.3.1: Let $\tilde{A}$ and $\mathring{B}$ be two triangular fuzzy numbers defined by the triples as

$\tilde{A} = (3,5,9)$ and $\mathring{B} = (4,7,9)$.

The interval of confidence is then given by

$$\tilde{A}_\alpha = [(5-3)\alpha + 3, -(9-5)\alpha + 9].$$
$$= [2\alpha + 3, -4\alpha + 9].$$

and

$$\overset{\circ}{B}_\alpha = [(7-4)\alpha + 4, -(9-7)\alpha + 9]$$
$$= [3\alpha + 4, -2\alpha + 9].$$

.

The multiplication at each level of $\alpha \in [0,1]$ is given by

$$\tilde{A}_\alpha (.) \overset{\circ}{B}_\alpha = [(2\alpha + 3)(3\alpha + 4), (-4\alpha + 9)(-2\alpha + 9)].$$
$$= [6\alpha^2 + 17\alpha + 12, 8\alpha^2 - 54\alpha + 81].$$

Note that, at $\alpha = 0$ $\tilde{A}_0(\cdot)\tilde{B}_0 = [12, 81]$, and at $\alpha = 1$ $\tilde{A}_1(.)\overset{\circ}{B}_1 = [35,35] = 35.$

3.2.4 FUZZY INFERENCE SYSTEM

There are two type of fuzzy inference system in matlab[60] such as, Mamdani and Sugeno[61] type. These methods operate in the same way when fuzzifying the inputs and applying the fuzzy operators, however their membership functions type of outputs are different. In Sugeno Method, the output membership functions must be constant or linear. Mamdani's fuzzy inference method is the most commonly used fuzzy system. Mamdani's method was among the first control systems built using fuzzy set theory. It was proposed in 1975 by Ebrahim Mamdani as an attempt to control by synthesizing a set of linguistic control rules obtained from experienced human operators.[62]

In our fuzzy model, we use Mamdani method. Because if we use Sugeno method, we can not identify each membership function as we want.

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[60] Matlab is a high-level language and interactive environment that enables you to perform computationally intensive tasks faster than with traditional programming languages such as C, C++, and FORTRAN (www.mathworks.com).

[61] Sugeno, M. and Kang, G.T., "Structure Identification of Fuzzy Model", **Fuzzy Sets and Systems**, Vol: 28, 1988.

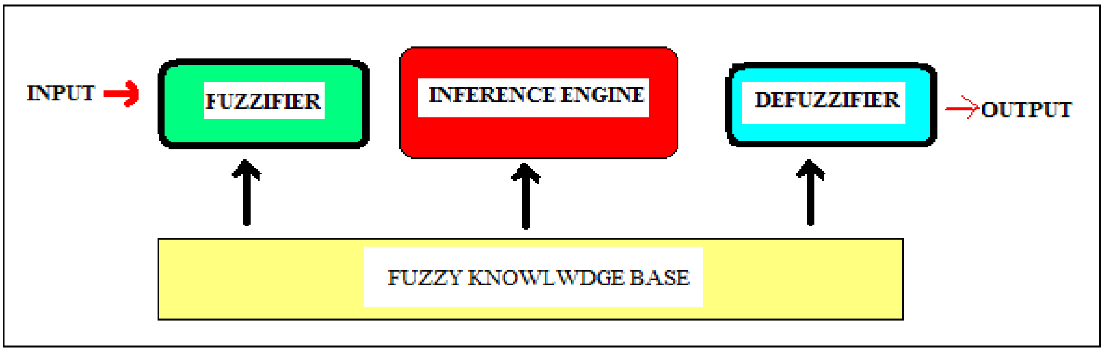
[62] Fuzzy Logic Toolbox, www.mathworks.com/products/fuzzylogic/s.2-20.

Takagi- Sugeno-Kong Method is similar to the Mamdani method in many respects. The first two parts of the fuzzy inference process, fuzzifying the inputs and applying the fuzzy operator are exactly the same. The main difference between Mamdani and Sugeno is that Sugeno output functions are either linear or constant. We can think that they are the types of fuzzy inference systems.

Only Mamdani and Takagi-Sugeno-Kong Method can be used in matlab. Takagi-Sugeno-Kong Method is more compact and computationally efficient representation than Mamdani Method, so Sugeno system lends itself to use the adaptive techniques for constructing fuzzy models. These adaptive techniques can be used to customize the membership function so that fuzzy systems best models the data. However, we did not use the Takagi-Sugeno-Kong method because in Takagi-Sugeno-Kong Method, the output membership function would be linear or constant. It does not let us to define the value of the output. In other words, we cannot say in what percentage the output belongs to each fuzzy set and we cannot identify each membership function as we want.

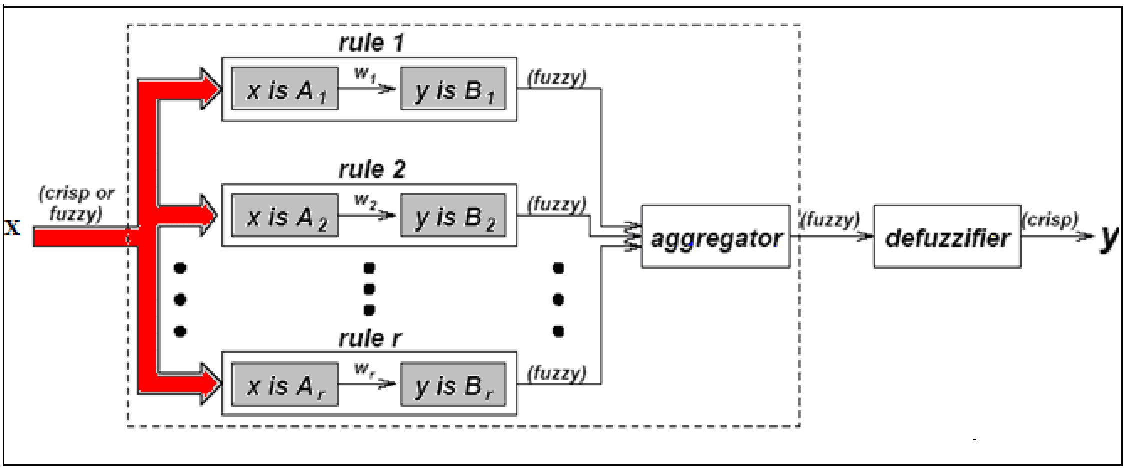
Aggregation is the combination of the consequents of each rule in a Mamdani fuzzy inference system in preparation for defuzzification. Note that, as long as the aggregation method is commutative (which it always should be), then the order in which the rules are executed is unimportant. There exists three aggregation method such as maximum (max), probabilisictor (algebraic sum; probor(a,b)=a + b – ab), and sum (simply the sum of each rule's output set). After the aggregation process, there is a fuzzy set for each output variable that needs defuzzification and defuzzification is the final process of the fuzzy inference system. In other words, after aggregation process, the fuzzy system becomes ready to defuzzification which is final step of fuzzy inference system. Now, we will define the steps of the fuzzy inference system in detail.

FIGURE 3.13 FUZZY INFERENCE SYSTEMS



Fuzzifier converts the crisp input to a linguistic variable using the membership functions stored in the fuzzy knowledge base. Inference engine use "If then" type fuzzy rules converts to fuzzy input to the fuzzy output. The following figure shows, how inference engine handles with the process.

FIGURE 3.14 INFERENCE ENGINE



Defuzzifier converts the fuzzy output of the inference engine to crisp using membership function.

3.2.4.1 Fuzzification

Fuzzification is the first step of the fuzzy inference system. It is the process of decomposing a system input and output into one or more fuzzy sets. Two or more membership functions should be defined for each input and output variables. Fuzzification takes inputs and determines the degree to which they belong to each of the appropriate membership functions.

The process of fuzzification allows the system inputs and outputs to be expressed in linguistic terms so that rules can be applied in a simple manner to express as a complex system. There are five fuzzy sets for our work. They are; extremely bad, bad, normal, good, and extremely good. Fuzzification of the five crisp variables, causes spreading of variables with a distribution profile. For example, as a variable total debt assume take value 20.000 at year 2003, the figure of membership function helps us to identify. In the membership function figure, when x is equal to 20.000 and a vertical line plotted crossing 20.000 at the x-axis, the line intersects with the membership function with "good" and membership function "good" at different points.

All the vertical lines intersect at one or two points. If it intersects at only one point, this means it will be in the range only extremely bad, bad, normal, good, and extremely good and the y-value is equal to 1. In other words, it is belongs to 100% to the fuzzy set. If it intersects at two points; a and b. The sum of the values of y-axis (a+b) is equal to 1. This means that it belongs to one of the fuzzy sets with a% and b% to the other set.

The purpose of fuzzification process is to allow a fuzzy condition in a rule to be interpreted. For instance, the condition, "Total Debt = 20.000" in a rule can be true for all values of "Total Debt", however the confidence factor or membership value of this condition can be derived from membership function graph. An indicator which has a value of 20.000 with a confidence factor of 0.5 (membership value of the club "normal"), it is gradual change of the membership value of the condition "normal" with height that gives fuzzy logic is strength.

3.2.4.2 Rule Generation

A fuzzy rule associates a condition described using linguistic variables and fuzzy sets to come up with a solution. It is a scheme for capturing knowledge that involves imprecision. The main feature of reasoning using these rules is its partial matching capability, which enables an inference to be made from a fuzzy rule even when the rule's condition is only partially satisfied.[63] On the other hand, the conventional rule can not deal with a situation where the condition for rules partially satisfied; it rules are explained in Boolean logic.

In classical logic, a simple proposition P is a linguistic statement within a universe of elements, say U, that can be identified as being a collection of elements in U are strictly true or strictly false. Hence a proposition P is a collection of elements where the truth values for all elements in the set are either all true or false. The truth of the element in the proposition P can be assigned a binary truth value called T. For Boolean classical logic, T is assigned value 1 or 0. Now let P and Q be two simple propositions on the same universe of discourse that can be combined using the logical connectives to form expressions involving the two simple propositions. Such connectives are disjunction, conjunction, negation, implication and equivalence. These connectives can be used to create compound propositions, where a compound proposition is defined as a logical proposition formed by logically connecting two or more simple propositions.

A fuzzy rule has two components like conventional rules: as if-part referred as the antecedent, and a then-part, referred as consequent. The antecedent describes a condition, and the consequent describes a conclusion that can be drawn when the condition holds. Now, we introduce two components.

The structure of fuzzy rule is identical to the conventional rule but there is a main difference which lies in the content of antecedent. The antecedent of a fuzzy rule

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[63] John Yen Langari, **Fuzzy Logic Intelligence, Control and Information, Center for Fuzzy Logic,** Robotics and Intelligent Systems Texas A&M University, Prentice Hall.

describes an elastic condition while the conventional rule describes a grid condition. For example, consider two rules (R1 and R2) below:

R1: If the income of a person is greater than $70.000, then the person is rich.

R2: If the income of a person is high then the person is rich.

The rule R1 is conventional one, because its condition is rigid which means the condition is either satisfied or not. In contrast, R2 is a fuzzy rule because its condition can be satisfied to a degree for those people whose income lies in the boundary of the fuzzy set “high”. In the rule R2, “high” is a fuzzy set defined by a membership function.

Like conventional rules, the antecedent of a fuzzy rule may combine multiple simple conditions into a complex one using three logic connectives: and (conjunction), or (disjunction) and not (negation). For instance, a loan approval system may contain the following fuzzy rule.
“If the income of a person is high and the credit report of a person is fair or the person has a valuable real estate asset then recommends approving the loan”. The rule has three components and a conclusion. Three components have three fuzzy sets which should be defined with membership functions. “High”, ”Fair” and “Valuable” are fuzzy sets.

When applying fuzzy rules we need to use some logical operations of fuzzy sets. The most important thing to realize about fuzzy logical reasoning is the fact that it is a superset of standard Boolean Logic. In other words, if we keep the fuzzy values at their extreme points of 1 for completely true, and 0 for completely false, standard logical operations will hold. However, we know that in fuzzy logic the truth of any statement is a matter of degree so we have to extend the standard logical operations to all real numbers between 0 and 1.

Shortly, in a fuzzy rule-based system, the rules can be represented such as:

If x is in X and y is in Y … then z is in Z

where; x, y and z represents the variables; X, Y, Z are linguistic variables such as extremely good, good, normal, extremely bad and bad.

For two variables with five membership functions, for fuzzy process the total number of rules should be 25 (5x5). For example; if we have two linguistic variables (inputs) such as EBIT and SP and each have five membership functions such as extremely bad, bad, normal, good, and extremely good. We can think as they were numbers such as 1, 2, 3, 4, 5; respectively. Then apply the rule as round $\frac{2EBIT + SP}{3}$ as the membership function of the output below.

Table 3.2 Example for Rule Generation

		EBIT				
		Extremely Bad	**Bad**	**Normal**	**Good**	**Extremely Good**
SP	**Extremely Bad**	Extremely Bad	Bad	Bad	Normal	Good
	Bad	Extremely Bad	Bad	Normal	Normal	Good
	Normal	Bad	Bad	Normal	Normal	Good
	Good	Bad	Normal	Normal	Good	Extremely Good
	Extremely Good	Bad	Normal	Good	Good	Extremely Good

If we have more than two inputs and more than five membership functions, it is perplexing to develop the fuzzy model. For example, if we have three variables and seven membership functions, number of rules increases to 343 (7x7x7). However, we use fuzzy model to simplify the environment and develop the most useful model. If the environment cannot be simplified as we want, it is meaningful to use fuzzy system. Fuzzy logic cannot be used for unsolvable problems. This seems fairly reasonable, but its perception of being a guessing game may lead people to believe that it can be used for anything. An obvious drawback a fuzzy logic is that always accurate. Generally, fuzzy logic, confused with probability theory. In some way they are similar concepts but they do not say the same things. Probability is likelihood that something is true, fuzzy logic is the degree to which something is true.

The consequent of fuzzy rules can be classified into three categories.

1. Crisp Consequent: If … then y = a where is non-fuzzy numeric value or symbolic value.

2. Fuzzy Consequent: If … then y is A, where A is a fuzzy set.

3. Functional Consequent: If x_1 is A_1 and x_2 is A_2 and … x_n is A_n then $y = a_0 + \sum_{i=1}^{n} a_i x_i$, where $a_0, a_1, ...a_n$ are constants.

Generally, fuzzy rules with a crisp consequent can be processed more efficiently. A rule with a fuzzy consequent is easier to understand and more suitable for capturing imprecise human expertise. Finally, rules with a functional consequent can be used to approximate complex nonlinear models using a small number of rules.

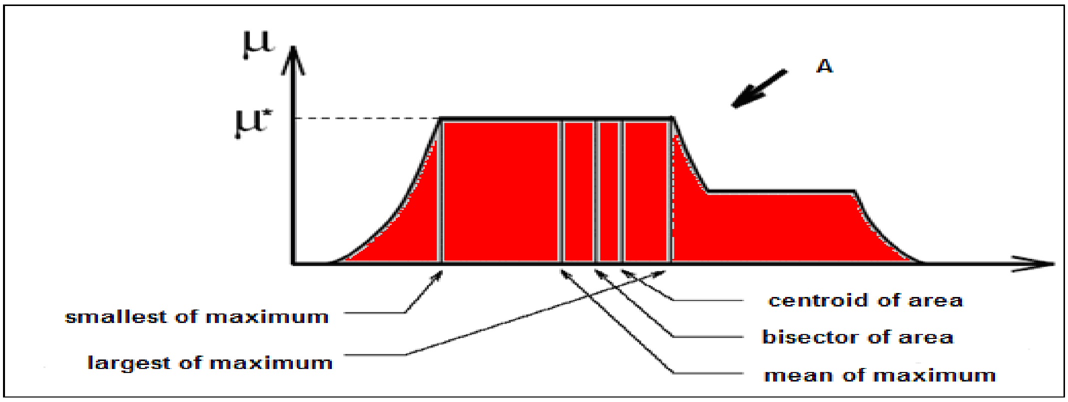
3.2.4.3 Defuzzification

If the conclusion of the fuzzy rule set involves fuzzy concepts, then these concepts will have to be translated back into objective terms before they can be used in practice. The process of converting the fuzzy output to a crisp number is called **defuzzification.** The input for the defuzzification process is a fuzzy set and the output is a single number. Before an output is defuzzified, all the fuzzy outputs of the system are aggregated with a union operator. The union is the maximum of the set of given membership functions and can be expressed as a fuzzy set. Then, by choosing a defuzzification method we need to convert a fuzzy value to an objective term. There are many defuzzification methods but primarily only three of them in common use for the Mamdani Fuzzy Models. Centroid Defuzzification Method (Centroid of Area: COA), Maximum Defuzzification Method, Weighted Average Defuzzification Method. In the Centroid method, the crisp value of the output variable is computed by finding the variable value of center of gravity of the membership function for the fuzzy value. One of the maximum defuzzification methods is mean of maximum method, is used in creating

Fuzzy State Machines for computer gaming development.[64] The largest (smallest) of maximum defuzzification method can be used to yield only two crisp output level for all input values.

In addition, in matlab fuzzy designer only supports five defuzzification methods which are Centroid Defuzzification Method, Bisector Defuzzification Method, Smallest of Maximum (SOM) Defuzzification Method, Largest of Maximum (LOM) Defuzzification Method and Mean of Maximum Defuzzification Method. The following figure shows the defuzzification methods with an example.

FIGURE 3.15 DEFUZZIFICATION METHODS



In the centroid of area defuzzification method, the fuzzy logic controller calculates the area under the scaled membership functions within the range of output variable. After that, fuzzy logic controller uses following integral to calculate the geometric center of this area.

$$COA = \frac{\int_{X_{min}}^{X_{max}} xf(x)dx}{\int_{X_{min}}^{X_{max}} f(x)dx}.$$

where, COA is the center of area, x is the value of linguistic variable, X_{min} is minimum

[64] Perumal, L. and Nagi F.H. **"Fuzzy Control System based on Largest of Maximum Defuzzification"**.

value of the linguistic variable and X_{max} is the maximum value of the linguistic variable. In our model, we use centroid of area method.

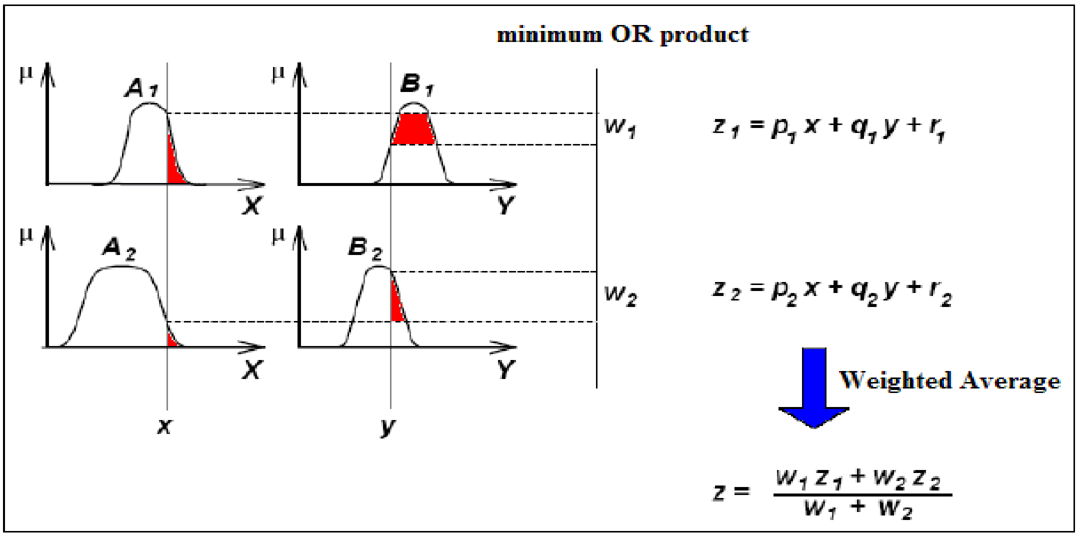
Shortly, to defuzzify the Mamdani style fuzzy inference system, one can choose centroid, bisector (is the vertical line that will divide region into sub-regions of equal area), middle of maximum, smallest of maximum or largest of maximum methods.

Sugeno Fuzzy Models (Sugeno Fuzzy Inference System) was proposed to develop a systematic approach to generate fuzzy rules from a given input- output system.

A typical rule in a Sugeno type fuzzy model has the following form:

If Input1 = x and Input2 = y then Output z = ax + by + c, where a, b and c are reel numbers. For a zero-order Sugeno model, the output level z is constant (i.e. a=b=0).

FIGURE 3.16 SUGENO MODEL



Source: John Yen,Reza Langari; Fuzzy Logic- Intelligence, Control and Information; Prentice Hall, 1999.

For the Sugeno style inference system one can choose weighted average or weighted sum because the output membership function has the form of constant or linear.

"Which defuzzification method is the right one to use?" has no simple answer but the center of area method has mostly used one. We use centroid average method in our model. This method was developed by Sugeno in 1985. It is the most commonly used and very accurate to apply. The only disadvantage of this method is that it is computationally difficult for complex membership functions.

CHAPTER 4 THE MODEL AND DATA

A number of statistical models have been around for decades, and one of the most popular prediction schemes is the Altman model. Edward Altman paired thirty three failed and thirty three non-failed firms in an attempt to control for industry and size differences. He then employed a method called discriminant analysis to a list of twenty two financial ratios. This method builds the best linear model possible so that it can explain the firms as failed or not failed with a little error as possible. The dependent variable in this model denotes the bankruptcy status, takes the value 1 for the company that does not failed and zero otherwise. Altman started with a list of twenty two financial ratios for the independent variables. From the list he chooses five that embrace the best linear model. The best five financial ratios are denoted in the below table.

Table 4.1 Financial Ratios

Working Capital / Total Assets
Retained Earnings / Total Assets
Earnings Before Interest and Taxes / Total Assets
Market Value of Equity / Book Value of Total Debt
Sales / Total Sales

To find the best significant financial ratios for all industries, as a first application we calculate some basic financial ratios which we will define later and then by making factor analysis we eliminate financial ratios that can explain the similar information and gives similar result

4.1 DATA

In order to broaden the usage of Merton Model to different sectors than financial ones, and create a new approach which is more sensitive to bankruptcy probabilities for different sectors, we choose US Market which consists of seventy eight different sectors and 3574 listed companies. The data consist of yearly company observations on financial ratios from the period of January 2005 to December 2008, found in www.usa.gov and www.dataworldbank. In our study, we worked with total 3574 companies (including foreign and not foreign ones) stated in USA, which belong to seventy eight different industries as banking, business equipment, chemicals, internet, retail, energy, healthcare, manufacturing, auto & truck, semiconductor, etc. For each company, we collect and calculate the parameters and/or financial ratios, which are the most significant, mostly used in literature and could be reached easily as shown in Table 4.2. Definitions and usages of financial ratios can be found in part 7.4, in appendix.

Table 4.2 Financial Ratios

Total Debt	EBIT(t-1)	EBIT	EBITDA	Value Line Beta
Revenues Last Year	Enterprise Value / Sales	Enterprise Value/ Trailing Sales	Capital Expenditures	Return on Capital
Number of Shares Outstanding	Non-cash WC as % of Revenues	Current PE (Price Earnings)	Non-Cash Working Capital (WC)	Intangible Assets / Total Assets
Stock Price	Depreciation	Trailing PE	Forward EPS	Net Margin
Trading Volume	Dividends	Book Debt to Capital	Market Debt to Capital	Book Value of Assets
Trailing 12-mounth Revenues	Cash as % Total Assets	Growth in Revenue(Last Year)	Three Year Regression Beta	Free Cash Flow for the Firm
Cash as % Firm Value	Three Year St. Deviation	Expected Growth in EPS	Fixed Assets / Total Assets	Firm Value/Book Value of Capital
PBV Ratio	Market Cap	Price Sales Ratio	SG&A Expenses	Hilo Risk
Firm Value	Forward PE	Dividend Yield	Market Cap	Reinvestment
Enterprise Value	Effective Tax Rate	PEG Ratio	Trailing Net Income	Payout Ratio
EV/EBITDA	Return on Equity	Insider Holdings	Trailing Revenues	EV/EBIT
Net Income	Invested Capital	Institutional Holdings	Cash as % Revenues	Correlation (Market)
Growth in EPS	Change in Non-cash WC	Market (Debt /Equity)		EV/Invested Capital

To construct our default parameter which is going to solve the problem of looking all parameters simultaneously and trying to find the significant ones every time while studying different sectors and gives us the opportunity to embrace all companies' features we need to reduce the number of financial ratios that are given in Table 4.2. With factor analysis we hand down a decision to reduce financial ratios which gives nearly same information for nearly all sectors. As time changes, the explaining default parameter can change or for different sectors different parameters can explain default probabilities.

4.2 FACTOR ANALYSIS & CLUSTERING PART

For the factor analysis, we worked with foreign and non foreign USA companies in USA (3574 companies). As mentioned above factor analysis is a method for investigating whether a number of variables of interest are linearly related to a smaller number of unobservable factors. We factor the variables which are relevant in Table 4.2 and Table 4.3.A, Table 4.3.B and Table 4.3.C illustrates the results.

Table 4.3.A Factor Analysis

Factor	Variance	Cumulative	Difference	Proportion	Cumulative
F1	12.66	12.66	6.96	0.38	0.38
F2	5.7	18.36	1.22	0.17	0.54
F3	4.49	22.85	2.02	0.13	0.68
F4	2.47	25.32	0.88	0.07	0.75
F5	1.59	26.91	0.14	0.05	0.8
F6	1.45	28.36	0.12	0.04	0.84
F7	1.32	29.68	0.24	0.04	0.88
F8	1.08	30.76	0.07	0.03	0.91
F9	1.01	31.77	0.02	0.03	0.94
F10	0.99	32.76	0.01	0.03	0.97
F11	0.97	33.73		0.03	1
Total	33.73	293.16		1	

Table 4.3.B Factor Analysis

	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	C	U
3 Year Regression Beta	-0.14	-0.17	-0.05	0.05	0.22	-0.15	0.07	0.09	0.12	0.27	0.32	0.32	0.67
3Year Standard Deviation	-0.29	-0.17	-0.08	0.15	0.12	-0.10	0.07	0.15	0.06	0.32	0.31	0.39	0.61
NsH	0.71	0.13	0.20	-0.06	0.10	-0.04	-0.02	0.02	0.07	-0.01	0.06	0.59	0.41
Book Debt to Capital	0.05	0.38	-0.54	-0.02	-0.15	0.02	0.06	-0.02	-0.05	-0.11	-0.10	0.50	0.50
Book Value of Assets	0.87	-0.02	0.09	0.03	-0.01	0.19	-0.03	0.00	0.02	-0.18	0.18	0.88	0.12
Capital Expenditures	0.69	-0.06	0.07	0.23	-0.12	0.15	-0.03	0.08	-0.05	0.06	0.23	0.64	0.36
Cash	0.53	-0.08	0.17	-0.25	0.35	-0.09	0.02	-0.09	0.25	-0.15	0.04	0.60	0.40
Firm Value	-0.05	-0.33	-0.08	-0.31	0.46	-0.20	0.05	-0.17	0.40	-0.13	0.10	0.69	0.31
Total Assets (Cash)	-0.16	-0.06	0.12	-0.32	0.48	-0.25	0.12	-0.19	0.42	-0.10	0.05	0.68	0.32
CHG IN NON CASH WC	0.10	0.00	-0.04	0.08	-0.02	0.00	0.05	0.00	-0.03	0.06	0.02	0.03	0.66
Correlation	0.15	0.07	0.15	-0.10	0.11	-0.03	0.00	-0.09	0.09	0.01	0.05	0.09	0.87
CURRENT_PE	-0.30	0.52	0.22	-0.24	-0.17	0.29	-0.16	-0.30	0.06	0.19	0.12	0.75	0.25
EBIT	0.90	-0.01	0.18	0.19	-0.10	0.05	0.16	-0.04	0.18	0.05	-0.05	0.96	0.04
EBIT_1_T_	0.89	0.02	0.17	0.18	-0.06	0.03	0.12	-0.01	0.16	0.04	-0.04	0.90	0.10
EBITDA	0.90	-0.02	0.16	0.18	-0.10	0.08	0.14	-0.06	0.16	0.02	0.00	0.94	0.06
EFF_TAX_RATE	0.14	-0.21	0.05	0.01	-0.19	0.01	0.00	-0.07	-0.11	-0.05	-0.02	0.12	0.88
Enterprise Value	0.81	0.33	0.28	-0.04	-0.02	-0.19	-0.24	0.00	-0.07	0.02	0.00	0.94	0.06
EV Invested Capital	-0.17	0.58	0.49	-0.17	-0.04	-0.23	0.42	-0.02	-0.01	-0.06	0.09	0.87	0.13
EV Trailing Sales	-0.27	0.73	0.22	0.35	0.23	0.18	0.00	-0.10	-0.08	-0.02	-0.06	0.88	0.12
EV_EBIT	-0.27	0.69	0.22	-0.33	-0.06	0.31	-0.12	0.20	0.21	0.02	-0.03	0.90	0.10
EV_EBITDA	-0.27	0.69	0.25	-0.33	-0.03	0.25	-0.06	0.23	0.21	-0.01	-0.06	0.89	0.11
EV_SALES	-0.29	0.72	0.23	0.39	0.25	0.20	-0.01	-0.06	-0.08	0.00	-0.03	0.92	0.08
EG in EPS	-0.31	0.21	0.00	-0.09	-0.02	0.01	0.02	-0.01	0.04	0.26	0.15	0.25	0.73
EG in Revenue	-0.18	0.29	0.15	0.05	0.05	-0.07	0.13	0.18	0.07	0.24	0.15	0.28	0.68
Firm Value	0.83	0.31	0.29	-0.06	0.02	-0.20	-0.25	-0.01	-0.04	0.02	0.00	0.97	0.03
Fixed Assets/ Total Assets	0.07	0.05	0.04	0.47	-0.27	0.13	-0.03	0.14	-0.30	0.14	0.11	0.46	0.54
FORWARD_EPS	0.35	0.00	0.23	0.17	-0.05	-0.06	0.09	0.20	-0.10	-0.05	-0.06	0.27	0.72
FORWARD_PE	-0.25	0.44	0.14	-0.25	-0.14	0.27	-0.19	-0.34	0.06	0.10	0.03	0.59	0.41
Growth in EPS	-0.17	-0.03	-0.02	0.03	0.03	-0.10	0.08	0.02	0.04	0.19	0.18	0.12	0.85
Growth in Revenue Last	-0.03	0.07	0.11	0.03	0.02	-0.04	0.03	0.07	0.03	0.19	0.17	0.09	0.89
HILO Risk	-0.13	-0.21	-0.12	0.20	0.11	-0.07	0.04	0.20	0.02	0.18	0.21	0.25	0.75
Intangible Assets	0.10	0.03	-0.14	0.02	-0.01	0.03	-0.01	-0.03	-0.02	-0.16	-0.10	0.07	0.90
Invested Capital	0.82	0.00	0.03	0.13	-0.07	0.23	-0.11	0.03	-0.02	-0.27	0.23	0.89	0.11
Market Capital	0.74	0.32	0.36	-0.06	0.03	-0.20	-0.19	0.00	-0.03	0.03	-0.01	0.86	0.14
Market D E	0.13	0.10	-0.98	-0.01	0.00	-0.01	0.01	0.00	0.00	0.00	0.00	0.99	0.01
Market Debt to Capital	0.12	0.11	-0.98	-0.01	0.00	-0.01	0.02	0.00	0.00	0.00	0.00	0.99	0.01
Net Income	0.84	0.06	0.23	0.08	0.05	-0.08	0.09	0.12	0.10	0.02	-0.10	0.83	0.17
Net Margin	-0.14	0.39	0.18	0.43	0.28	-0.12	0.14	0.21	0.01	-0.03	0.01	0.54	0.46
NON_CASH_WC	0.33	-0.11	0.10	-0.22	0.13	0.01	-0.05	-0.08	0.12	-0.13	0.01	0.24	0.66
PAYOUT Ratio	0.05	0.00	-0.22	-0.10	0.09	0.01	-0.10	-0.14	0.14	-0.14	-0.10	0.15	0.82
PBV Ratio	-0.11	0.65	0.36	-0.14	-0.15	-0.11	0.35	-0.04	-0.12	-0.04	-0.02	0.76	0.24
PEG Ratio	0.04	0.24	0.22	-0.10	-0.14	0.16	-0.14	-0.22	-0.03	-0.16	-0.09	0.26	0.74
PTOM	-0.09	0.31	0.08	0.59	0.12	0.00	0.09	0.04	-0.07	0.03	0.04	0.49	0.51
PS Ratio	-0.32	0.67	0.32	0.28	0.24	0.10	0.04	-0.03	-0.01	0.02	0.01	0.81	0.19
Reinvestment	0.33	0.01	0.04	0.22	-0.14	0.08	-0.05	0.23	-0.13	0.11	0.21	0.31	0.57
Reinvestment Rate	-0.10	0.00	-0.04	0.02	0.07	0.04	-0.04	-0.05	0.02	-0.03	0.01	0.02	0.86
Last Year Revenues	0.87	-0.16	0.13	-0.23	0.13	0.19	0.16	-0.01	-0.16	0.05	-0.03	0.95	0.05
ROC	0.06	0.10	0.29	0.09	0.16	-0.37	0.39	-0.01	0.03	0.09	-0.05	0.43	0.57
ROE	0.19	0.24	0.24	0.10	0.03	-0.33	0.37	0.21	-0.07	-0.05	-0.16	0.48	0.52
SERIES02	0.49	-0.08	0.08	0.03	0.14	0.12	-0.06	0.10	0.11	-0.06	0.17	0.35	0.61
SG_A_Expenses	0.67	-0.03	0.12	-0.21	0.04	0.03	0.03	-0.07	0.02	-0.17	-0.15	0.57	0.43
Stock Price	0.23	0.25	0.34	0.01	-0.11	0.06	-0.05	0.01	-0.11	-0.05	-0.04	0.27	0.73
Total	0.71	0.08	-0.12	0.08	-0.12	0.18	-0.11	0.03	-0.08	-0.23	0.08	0.65	0.34
Trading Volume	0.09	0.00	0.08	0.15	0.23	-0.01	-0.05	0.24	0.09	0.12	0.28	0.24	0.76
Trailing Revenue	0.87	-0.15	0.14	-0.21	0.14	0.19	0.15	0.02	-0.16	0.05	-0.01	0.95	0.05
TRAILIE	0.82	0.07	0.24	0.08	0.06	-0.09	0.09	0.16	0.09	0.01	-0.08	0.80	0.20
Trailing Net Income	-0.27	0.46	0.19	-0.25	-0.19	0.25	-0.16	-0.39	0.04	0.20	0.11	0.71	0.29
BV of Capital	-0.15	0.62	0.49	-0.15	-0.13	-0.20	0.44	-0.02	-0.09	-0.06	0.09	0.93	0.07
Value Line Beta	-0.05	-0.19	0.00	-0.04	0.33	-0.13	0.01	0.11	0.24	0.25	0.36	0.43	0.52

C: Communality U: Uniqueness

Table 4.3.C Factor Analysis

	Model	Independence	Saturated
Discrepancy	4.27	51.05	0
Chi-square statistic	768.49	9188.46	---
Chi-square Probability	1	0	---
Bartlett chi-square	649.66	8142	---
Bartlett Probability	1	0	---
Parameters	653	59	1770
Degrees of freedom	1117	1711	---

From the results shown in Table 4.3.A, B and C, we have eleven factors and because of the fact that two of them are highly and positively correlated, we eliminate one of them which are not as highly and positively correlated with the remaining factors. So we left out with ten variables to construct our default probability, which is different from Merton default probability but also includes Merton default probability as an input variable. We consider constructing our default probability with fuzzy model aiming to take into account the relativity of different magnitudes of financial factors in different sectors. As mentioned before, the beginning point of fuzzy modeling lies in picking up the input variables and designing fuzzy rules between the input variables. Working out with eleven variables (including Merton Default Probability) in fuzzy modeling and creating fuzzy rules (consisting five linguistic variables; extremely good, good, normal, bad, extremely bad) between them leads to 11^5 rules. Because of the diminishing efficiency and functionality of fuzzy modeling with so many input variables and obviously so many fuzzy rules, we decided to reduce the input variables again with a different process; which is called clustering. The steps of this method applied are as follows:

- Cluster industries with variables we derived from factor analysis
- Find industries between clusters
- Find similar properties of industries which are between clusters
- Reduce financial ratios according to these information

Now, we will give detailed information for the steps which we applied. For the clustering part; we choose to work with only foreign USA companies, where the sensitivity effect of bankruptcy is magnified. Our data in clustering part consists of 992 firms and seventy eight different industries between January 2005 to December 2008. The following table shows the number of companies belong the same industries in our data.

Table 4.4 Number of Companies

Industries	Number of Companies
Bank, Drug, Internet, Computer Software, Thrift, Petroleum, Financial Services, Medical Supplies, Industrial Services, Insurance, Telecom Services, Metals & Mining, Retail Store, Precious Metal, Semiconductor, Electronics.	> 20
Advertising, Chemical, Maritime Entertainment, Medical Services, Telecom Equipments, Biotechnology, Computers, Foreign Telecom, Food Processing, Paper Forest Products, Recreation, Air Transport, Auto Truck, Cable TV, Tobacco, Home Building, Publishing. Educational Services, Wireless Network, Steel, Shoe, Water Utility, Rail Road, Hotel, Restaurant, Power, Oilfield Services & Equipment.	< 20

We cluster the companies with respect to the following variables, which will be defined as follows. In addition; we cluster the companies with variables that we find from the results of factor analysis. Now, we will give some information about our clustering variables.

a. EBIT: Earnings before interest and taxes. EBIT is calculated from the income statement by taking the net income and adding the back taxes and interest expense, plus provision for income taxes. The formula is:

EBIT = Total Revenues – Costs – Depreciation

b. Trailing Sales (TS): A company's revenue from sales over a period of time in the past. Often, a company will use trailing sales over the past twelve months to help forecast its expected sales over the coming twelve months. Trailing sales are useful because they can be known with certainty; however, their predictive value is often limited because of forces outside the company's control.

c. Expected Growth in Earnings per Share (EG): First, we want to give the definition of earnings per share (EPS). It is the dollar amount of the period’s net income that is available to each share of common stock. Basic earnings per share (EPS) are calculated by deducting any preferred dividends due to preferred stock shareholders for the period, and dividing by the weighted average number of common shares of stock outstanding during the period. Diluted earnings per share include an adjustment for common stock equivalents, resulting in 195 earnings per share lower earnings per share than basic EPS.

Expected growth in EPS tells investors how much money per share outstanding a company is expected to make.

d. PRE-TAX Operating Margin (PTOM): Pretax operating margin is calculated by dividing pretax operating earnings, excluding net realized capital gains or losses, interest expense and amortization of other acquired intangible assets by total revenue excluding net realized capital gains or losses.

e. Total Debt (TD): Debt can be explained as owing something of value to another. In business, a debt is usually the obligation to pay for a service or good already received. Debts may be short-term, meaning they must be paid within the next year, or long-term. Total debt includes debt due in less than one year and long- term debt. Long-term debt is debt due in one year or more.

f. Trading Volume (TV): The number of shares, bonds or contracts.

g. Beta- Value Line (BVL): Beta is a measure of a stock’s systematic, or market risk, and offers investors a good indication of an issue’s volatility relative to the overall stock market. The market beta is set at 1, and a stock’s beta is calculated by **Value Line**, based on past stock-price volatility. If equity has a beta of 1, it will probably move in lock step with the broader market. For example, if the market rises (falls) by ten percentage an issue with a beta of 1 will probably increase (decrease) by about the same amount. A beta above 1 indicates that a stock’s volatility is greater than the market. For instance, an issue with a beta of 1.3 has a level of volatility thirty percentages greater then the market average. Hence, given a ten percentage increase (decrease) in the stock market our hypothetical issue will probably climb (fall) about thirteen percentage. The reverse is also true. A stock with a beta of 0.70 has lower volatility than the overall market, and a broad increase (decrease) of 10% would likely result in a 7% gain (loss) for our low-beta issue.

h. Cash as Firm Value (CF): The value of a firm is the value of its business as a going concern. The firm’s business constitutes its assets, and the present assessment of the future returns from the firm’s business constitutes the current value of the firm’s assets. The value of the firm’s assets is different from the bottom line on the firm’s balance sheet. When the firm is bought or sold, the value traded is the ongoing business. The difference between the amount paid for that value and the amount of book assets is usually accounted for as the “good will”.

The value of the firm’s assets can be measured by the price at which the total of the firm’s liabilities can be bought or sold. The various liabilities of the firm are claims on its assets. The claimants may include the debt holders, equity holder, etc.

As we said, we cluster the industries with respect to the variables which we found from the factor analysis. The clustering variables are “Stock Price”, “Trading Volume”, ”Number of Shares Outstanding”, “Total Debt”,” Pre-tax Operating Margin”, “Trailing Sales”, “Expected Growth in EPS”, “Value Line Beta”, “EBIT”, ”Cash as Firm Value”. When we cluster the industries (in other words, we divide up our data set into clusters, where similar data objects are assigned to the same cluster, whereas dissimilar data objects should belong to different clusters), we find that some industries belong to different clusters with different percentages. In addition, we found some sectors that are between clusters. These are industries that might have greater volatilities. For one year process, belonging different clusters can be explained as having greater volatilities than the sectors belonging one cluster (with hundred percentage). Since, we want to find an index which will be explain default probability, the volatile industries are extremely important data for us. If we use this data, we can easily find similar and dissimilar properties of these firms and catch up the variables that can explain the default probability.

If clustering variables are measured on different scales, variables with large values contribute more to the distance measure than variables with small values. In our study, we have clustering variables on different scales; to prevent this issue, we standardized all clustering variables. Standardizing the proximity measure does not change the ratios between different pairs of objects, but can make interpretation clear.

We cluster industries each year between 2005 and 2008 and for each year we obtained two clusters; Cluster 1 and Cluster 2. The following tables (Table 4.5, Table 4.6, Table 4.7, Table 4.8) show the industries which belong to Cluster 2 with hundred percentage and shows the industries between clusters with percentage information for the year 2005. In addition, the following tables do not give the information about Cluster 1 because the industries which do not belong to Cluster 2 belong to Cluster 1. As we said before, for the year 2005, when we cluster seventy eight industries with different number of companies with respect to the clustering variable (which we found from factor analysis), we have two clusters. An expert of this analysis is given in Table 4.5.

Table 4.5 Cluster Analysis of Foreign USA Companies at 2005

	2005		
	Foreign USA Companies		
Cluster 2		**Cluster 1**	**Cluster 2**
Food Processing	**Internet**	0.90	0.10
Foreign Electronics	**Semiconductor**	0.87	0.13
Foreign Telecom Services	**Drug**	0.63	0.37
	Tobacco	0.50	0.50
	Medical Supplies	0.40	0.60
	Auto& Truck	0.25	0.75
	Petroleum	0.17	0.83

Above table gives the information that foreign internet companies in USA belongs 90 % to Cluster 1 and 10 % to Cluster 2. For example, the foreign drug companies in USA belong to 62.5 % to the cluster 1 and 37.5% to the cluster 2. Only the industries Food Processing, Foreign Electronics and Foreign Telecom Services belong to Cluster 2 with 100%. The industries which are not written in Table 4.5 be a member of Cluster 1. With this information, we can say that for the year 2005 the industries Internet, Semiconductor, Drug, Tobacco, Medical Supplies, Auto & Truck and Petroleum were the greatest probability of bankruptcy among the all sectors in our data (seventy eight sectors). Since not belonging one cluster implies that sector has greatest volatility which is also means it has greatest bankruptcy probability. Notice that, they have different probabilities. For instance, we can say that Tobacco sector is more volatile than the internet sector at 2005. Since Tobacco sector is between clusters with 0.5 probabilities; which means Tobacco sector belongs to Cluster 1 and 2 with equal probabilities. However, Internet companies belong to Cluster 1 with higher probability than 0.5, which is 0.9. We can think as Internet companies almost surely belong to Cluster 1.

The following table shows the industries which belong to Cluster 2 with hundred percentage and shows the industries between clusters with percentage information for the year 2006.

Table 4.6 Cluster Analysis of Foreign USA Companies at 2006

2006 Foreign USA Companies			
Cluster 2		**Cluster 1**	**Cluster 2**
Foreign Electronics	**Semiconductor**	0.91	0.09
Tobacco	**Petroleum**	0.86	0.14
Petroleum (Integrated)	**Medical Supplies**	0.86	0.14
Chemical (Diversified)	**Internet**	0.82	0.18
Oilfield Services & Equipment's	**Precious Metals**	0.75	0.25
	Telecom Services	0.56	0.44
	Telecom Equipments	0.50	0.50
	Drug	0.56	0.44
	Food Processing	0.33	0.67
	Auto& Truck	0.25	0.75

For the year 2006, new industries included to belonging both clusters, only Tobacco companies now belong to Cluster 2 with 100%. At year 2005, Tobacco sector were between clusters with equal percentage. We can say that Tobacco companies start to be stable with respect to the clustering variables. The included industries are: Precious Metals, Telecom Services and Equipments, Food Processing. In addition, they are industries become more volatile than the previous year. For year 2006, the most volatile sectors are Telecom Equipments, Telecom Services, and Drug. The Auto & Truck and

Precious Metals sectors have same probabilities for different clusters. In cluster analysis, we cannot say one cluster is better than the other, so we can think that Auto & Truck and Precious Metals sectors are in the same risk group. Since they are between Clusters with same probabilities, we can say the volatility of these sectors are nearly same. Remember that, if one clusters the industries with respect to different clustering variables, the results will be changed. Put differently, if we cluster industries with only five parameters that we found from factor analysis, we can found one cluster or more than two clusters and also we can found the probabilities belonging to each cluster. However, we cluster industries with variables we choose from factor analysis.

Since we measure volatility of companies with respect to being different clusters, it is important to know the probabilities of being each cluster. We should consider the difference between being one cluster with 90% and 50%. What we are trying to mean, a sector belonging to a cluster with 50% is more volatile than a sector belonging a cluster with 90%.

From Table 4.6 we can say that Telecom Services. Telecom Equipment and Drug sectors are more volatile than the sectors Medical Supplies, Internet and Precious Metals.

The Table 4.7 illustrates the industries which belong to Cluster 2 with hundred percentage and show the industries between clusters with percentage information for the year 2007. There are ten sectors which are between clusters and only Petroleum belongs to Cluster 2.

Table 4.7 Cluster Analysis of Foreign USA Companies at 2007

2007 Foreign USA Companies			
Cluster 2		**Cluster 1**	**Cluster 2**
Petroleum	**Drug**	0.92	0.08
	Semiconductor	0.92	0.08
	Internet	0.89	0.11
	Telecom Services	0.85	0.15
	Entertainment	0.83	0.17
	Precious Metals	0.80	0.20
	Telecom Equipments	0.75	0.25
	Food Processing	0.50	0.50
	Metals & Mining	0.50	0.50
	Auto& Truck	0.40	0.60

For the year 2007, the industries Drug, Semiconductor, Internet and Auto & Truck are still belonging to Cluster 1 and 2 with different percentages. Now, there is only one sector (Petroleum) belonging to Cluster 2 with hundred percentages. The other industries which are not written in the above table belong to Cluster 1 with hundred percentage.

If we consider the three year analysis, the first year, Tobacco sector was between clusters. One year later it belongs to Cluster 2 and the third year, it belongs to Cluster1.

The following table shows the industries which belong to Cluster 2 with hundred percentage and show the industries between clusters with percentage information for the year 2008. For the year 2008, Cluster 2 has man members than the other years.

Table 4.8 Cluster Analysis of Foreign USA Companies at 2008

	2008 **Foreign USA Companies**		
Cluster 2		**Cluster 1**	**Cluster 2**
Utility	**Semiconductor**	0.90	0.10
Railroad	**Drug**	0.90	0.10
Precious Metals	**Telecom Services**	0.83	0.17
Power	**Internet**	0.80	0.20
Petroleum	**Medical Supplies**	0.75	0.25
Maritime	**Aerospace / Defense**	0.75	0.25
Hotel & Gaming	**E-Commerce**	0.67	0.33
Entertainment	**Chemical**	0.67	0.33
Diversified Co.	**Telecom Equipment**	0.57	0.43
Chemical	**Metals & Mining**	0.50	0.50
Cable	**Food Processing**	0.50	0.50
Biotechnology	**Auto & Truck**	0.40	0.60

As these clusters are analyzed in more depth, it will be important to look closely at each industries ratio which denotes the percentage of regarding to each cluster. From the above table, we can easily mention that the most volatile industries are Food Processing, Metals & Mining and Telecom Equipment and Auto & Truck. In addition, the Semiconductor industry belongs to Cluster 1 with 90 %. We can say that Semiconductor industry practically belongs to Cluster 1. From above table we can see that E-commerce and Chemical; and Medical Supplies and Aerospace /Defense sectors are between clusters with same probabilities.

Semiconductor, Drug, Internet and Auto & Truck industries are between clusters for all the years between 2005 and 2008. The following table shows the industries between clusters for each year between 2005 and 2008.

Table 4.9 Industries between Clusters

2005	2006	2007	2008
Auto & Truck	**Auto & Truck**	**Auto & Truck**	**Auto & Truck**
Drug	**Drug**	**Drug**	**Drug**
Internet	**Internet**	**Internet**	**Internet**
Semiconductor	**Semiconductor**	**Semiconductor**	**Semiconductor**
Medical Supplies	Medical Supplies		Medical Supplies
	Food Processing	Food Processing	Food Processing
	Telecom Equipment	Telecom Equipment	Telecom Equipment
	Telecom Services	Telecom Services	Telecom Services
		Metals & Mining	**Metals & Mining**
Petroleum	**Petroleum**		
	Precious Metals	**Precious Metals**	
Tobacco			
		Entertainment	
			Chemical
			E-Commerce
			Aerospace/Defense

The cluster analysis shows that the industries Medical Supplies, Food Processing. Telecom Equipment and Telecom Services are also between clusters for different three years. Since we make four year analysis, they are between clusters with 75%. In addition; Metals Mining, Petroleum and Precious Metals are between clusters with 50%. Tobacco, Entertainment, Chemical, E-commerce and Aerospace/ Defense sectors are between clusters for only one year.

The industries between clusters are: **Semiconductor, Drug, Internet and Auto & Truck**. Now, we need to find the similar properties of these industries with respect to the clustering variables. We take the mean and standard deviation of all foreign USA companies clustering variables for each year and all. You can find tables in appendix that illustrates the clustering variables mean and standard deviation values of all seventy eight different industries for each year between 2005 and 2008.

Table 4.10 Descriptive Statistics of All Industries

	ALL									
	SP	**TV**	**NSh**	**TD**	**PTOM**	**EV/TS**	**EG**	**VLB**	**EBIT**	**CF**
2005 Mean	45.61	2099800.68	129.23	866.28	-1.1	18.06	0.18	0.61	819.22	0.14
2005 St. Dev.	534.69	53541582.78	466.02	4524.82	5.54	163.01	0.12	0.56	3307.06	0.34
2006 Mean	50.43	1970577.11	144.23	906.41	-1.47	23.82	0.17	0.88	705.35	0.15
2006 St. Dev.	611.97	49117175.39	468.47	4557.92	7.16	262.36	0.12	0.42	3196.35	0.39
2007 Mean	49.35	2976954.13	154.83	1326.63	-1.79	11.05	0.16	0.9	859.79	0.18
2007 St. Dev.	612.69	64667284.05	485.6	10374.06	8.72	75.41	0.09	0.38	3890.77	0.33
2008 Mean	30.86	2429465.97	175.99	1523.87	-1.28	3.58	0.14	0.99	1052.35	0.43
2008 St. Dev.	405.08	55590958.26	521.34	10558	6.03	29.18	0.12	0.42	4568.08	1.07

As we mentioned before; to find the similar properties of the industries, we use statistical information of the sectors which are between clusters. Table 4.11 shows the mean and standard deviations of the Semiconductor companies for all and each years between 2005 and 2008.

Table 4.11 Descriptive Statistics of the Industry: Semiconductor (Between Clusters)

SEMICONDUCTOR	SP	TV	NSh	TD	PTOM	EV/ TS	EG	VLB	EBIT	CF
All Mean	9.36	725932.01	325.87	250.86	-0.5	10.22	0.22	0.98	491.44	0.72
All St. Dev.	9.17	2396277.35	1081.66	568.5	4.54	67.3	0.11	0.52	1352.24	1.3
2005 Mean	10.29	294371.05	298.83	244.51	0.15	3.12	0.2	0.74	803.22	0.33
2005 St. Dev.	6.78	672097.1	1068.54	565.11	0.32	2.02	0.06	0.76	1674.71	0.36
2006 Mean	10.99	533954.95	316.23	252.07	0.1	30.24	0.23	1.17	376.14	0.27
2006 St. Dev.	9.56	998316.42	1097.25	568.75	0.4	124.24	0.11	0.43	1173.86	0.26
2007 Mean	10.71	1620146.62	331.56	220.7	0.09	4.67	0.22	1.02	455.97	0.38
2007 St. Dev.	11.54	4504342.85	1123.29	551.14	0.38	9.99	0.11	0.38	1424.32	0.37
2008 Mean	5.44	455255.43	356.86	286.16	-1.82	-1.29	0.24	1.04	493.75	1.62
2008 St. Dev.	7.53	970393.1	1116.24	626.37	8.08	7.88	0.15	0.29	1360.2	2.06

Table 4.12 Descriptive Statistics of the Industry: Internet (Between Clusters)

INTERNET	SP	TV	NSh	TD	PTOM	EV/TS	EG	VLB	EBIT	CF
All Mean	10.29	294371.05	298.83	244.51	0.15	3.12	0.2	0.74	803.22	0.33
All St. Dev.	6.78	672097.1	1068.54	565.11	0.32	2.02	0.06	0.76	1674.71	0.36
2005 Mean	10.99	533954.95	316.23	252.07	0.1	30.24	0.23	1.17	376.14	0.27
2005 St. Dev.	9.56	998316.42	1097.25	568.75	0.4	124.24	0.11	0.43	1173.86	0.26
2006 Mean	10.71	1620146.62	331.56	220.7	0.09	4.67	0.22	1.02	455.97	0.38
2006 St. Dev.	11.54	4504342.85	1123.29	551.14	0.38	9.99	0.11	0.38	1424.32	0.37
2007 Mean	5.44	455255.43	356.86	286.16	-1.82	-1.29	0.24	1.04	493.75	1.62
2007 St. Dev.	7.53	970393.1	1116.24	626.37	8.08	7.88	0.15	0.29	1360.2	2.06
2008 Mean	15.68	302994.6	52.38	9.79	-0.47	17	0.21	0.9	163.23	0.38
2008 St. Dev.	46.21	549909.1	54.12	20.55	2.7	37.84	0.12	0.4	827.34	0.71

Table 4.12 shows the mean and standard deviations of the Internet companies for all and each years between 2005 and 2008.

Table 4.13 Descriptive Statistics of the Industry: Drug (Between Clusters)

DRUG

	SP	TV	NSh	TD	PTOM	EV/ TS	EG	VLB	EBIT	CF
All Mean	10.17	377235.58	188.37	660.38	-7.28	54.84	0.13	0.89	691.78	0.5
All St. Dev.	15.21	899302.64	519.27	2675.76	14.32	500.21	0.08	0.53	2712.44	1.17
2005 Mean	11.89	318453.09	176.86	411.44	-7.27	19.45	0.13	0.52	695.41	0.22
2005 St. Dev.	14.3	957752.63	539.53	1527.34	13.81	30.65	0.05	0.55	2431.04	0.16
2006 Mean	11.79	402909.59	186.86	656.19	-8.09	175.37	0.13	0.98	573.58	0.3
2006 St. Dev.	16.73	877208.66	518.56	2885.72	14.76	967.87	0.05	0.41	2361.65	0.36
2007 Mean	9.89	457125.18	194.22	581.87	-7.82	12.36	0.15	1.05	730	0.47
2007 St. Dev.	16.3	1039490.17	523.18	2071.17	16.7	31.62	0.09	0.48	2916.35	0.52
2008 Mean	7.13	330454.45	195.56	992.02	-6.02	4.08	0.09	1.04	769.05	0.98
2008 St. Dev.	13.15	706295.52	509.24	3728	12	18.9	0.1	0.49	3073.75	2.12

Table 4.14 shows the mean and standard deviations of the Auto & Truck companies for all and each years between 2005 and 2008.

Table 4.13 shows the mean and standard deviations of the Drug companies for all and each years between 2005 and 2008.

Table 4.14 Descriptive Statistics of the Industry: Auto & Truck (Between Clusters)

AUTO-TRUCK

	SP	TV	NSh	TD	PTOM	EV/TS	EG	VLB	EBIT	CF
All Mean	11.89	318453.09	176.86	411.44	-7.27	19.45	0.13	0.52	695.41	0.22
All St. Dev.	14.3	957752.63	539.53	1527.34	13.81	30.65	0.05	0.55	2431.04	0.16
2005 Mean	11.79	402909.59	186.86	656.19	-8.09	175.37	0.13	0.98	573.58	0.3
2005 St. Dev.	16.73	877208.66	518.56	2885.72	14.76	967.87	0.05	0.41	2361.65	0.36
2006 Mean	9.89	457125.18	194.22	581.87	-7.82	12.36	0.15	1.05	730	0.47
2006 St. Dev.	16.3	1039490.17	523.18	2071.17	16.7	31.62	0.09	0.48	2916.35	0.52
2007 Mean	7.13	330454.45	195.56	992.02	-6.02	4.08	0.09	1.04	769.05	0.98
2007 St. Dev.	13.15	706295.52	509.24	3728	12	18.9	0.1	0.49	3073.75	2.12
2008 Mean	14.3	957752.63	539.53	1527.34	13.81	30.65	0.05	0.55	2431.04	0.16
2008 St. Dev.	11.79	402909.59	186.86	656.19	-8.09	175.37	0.13	0.98	573.58	0.3

We analyze that the mean of stock price and trading volume of these industries are smaller than the all sector means for all years between 2005 and 2008. In addition the mean of **Total Debt** and **EBIT** are also smaller for the industries semiconductor, internet and drug. Only for the Auto & Truck, the mean of total debt and EBIT are bigger than the sector means. So we can not generalize that the industries between clusters have smaller total debt and EBIT values.

After that, we look at the correlation matrix of the clustering variables for the industries between clusters. The following table shows the correlation matrix of the clustering variables. Remember that the correlation matrix is symmetric so we did not write the values of correlation below the diagonal.

Table 4.15 Correlation Matrix

	NSh	CF	TS	EG	PTOM	SP	TD	TV	VLB	EBIT
NSh	1	-0.28	-0.03	-0.36	-0.31	0.04	0.41	0.45	-0.08	0.65
CF		1	-0.33	0.15	-0.32	-0.22	-0.20	-0.18	0.06	-0.27
EV/TS			1	0.39	0.03	0.48	-0.16	0.35	0.05	-0.13
EG				1	-0.12	0.24	-0.34	0.02	0.76	-0.40
PTOM					1	0.19	0.03	0.21	-0.01	0.16
SP						1	0.11	0.29	-0.21	0.20
TD							1	-0.56	-0.13	0.89
TV								1	0.01	0.07
VLB									1	-0.20
EBIT										1

We analyze the correlation coefficient matrix and consider each clustering variable. After that we find the clustering variable which have the greatest correlation coefficient with the other clustering variables. For example, we consider total debt, look at the correlation coefficient with all other clustering variables and than take the maximum and find that EBIT have the greatest correlation coefficient with total debt. After considering the clustering variable expected growth in EPS and we find that it has greatest correlation coefficient again with EBIT. The following two figures show the maximum correlation coefficients between clustering variables.

FIGURE 4.1 MAXIMUM CORRELATION COEFFICIENT - I

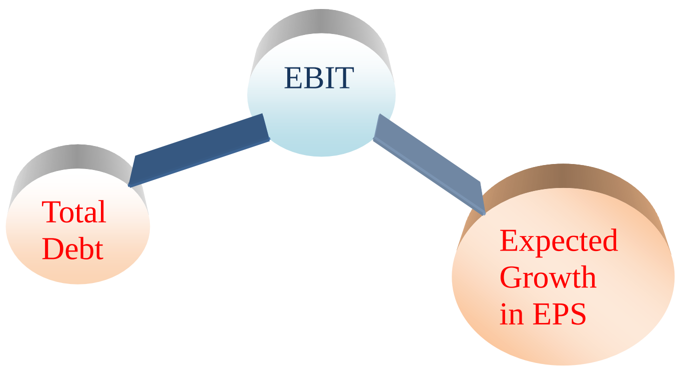
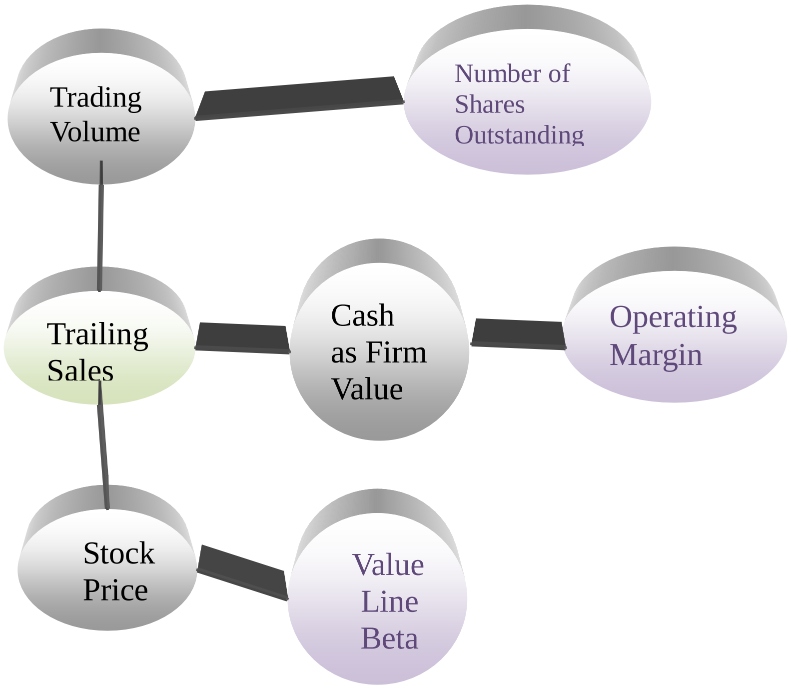


FIGURE 4.2 MAXIMUM CORRELATION COEFFICIENT – II



The above figure shows that the variable "number of shares outstanding" has the greatest correlation coefficient with the variable "trading volume". The variable "trailing sales" has the greatest correlation coefficient with the variable "trading volume". The other connections are similar that we do not need to explain.

We need to cluster the industries that they belong to each cluster distinctly. The main idea behind that, if they are between clusters, their structure changes more than the other industries which mean that they have greater volatilities. We cluster the industries with the variables that we find from factor analysis. We eliminate the variables which are in similar categories before clustering. Now, we require finding variables which can cluster the industries distinctly, which cannot measure the volatility of the industries.

As we find out before the industries between clusters have similar properties. They have both smaller total debt and EBIT values and expected growth in EPS has greatest correlation with EBIT. To make distinct clusters, we will choose the variable "expected growth in EPS" among these three variables. To choose the other clustering variables, we necessitate to find the variables which has no relationship or nearly no relationship. We look at the smallest correlation coefficient between clustering variables and choose the variable "trading volume", "value line beta", "cash as firm value" and "pre-tax operating margin". We again cluster the industries for each year between 2005 and 2008 with respect to variables "Expected Growth in EPS", "Trading Volume", "Value Line Beta", "Cash as Firm Value" and "Pre-tax Operating Margin". The expected results show that now we have distinct clusters. In other words, the clustering variables EBIT, total debt, stock price, trailing sales and number of shares outstanding of the industries constitute intersecting clusters.

The situation of not belonging one cluster that means being between clusters can be identified with the variables EBIT, total debt, stock price, trailing sales and number of shares outstanding. In addition to these, they are variables that can determine the volatility of the companies and if volatility increases, we can say that probability of bankruptcy increases.

To apply fuzzy process, we have five variables that we derive with clustering and we need to calculate Merton default probabilities of all industries in our data. The following part represents how we calculate MPD values of industries.

4.3 THE MODEL (PROBABILITY OF DEFAULT)

In our study, to calculate the default probabilities of the firms, we begin by examining foreign USA companies between 2005 and 2008. We have 992 firms to estimate the probability of default. Notice that, we have firms from different industries.

As we said before; the probability of default can be measured with a model that uses company's financial information to obtain an indication of how likely the firm is to enter distress in the near future[65]. This financial information, in the Merton Model, consists of several variables. A short description of each one will be given below.

The inputs to the Merton Model include the volatility of stock returns, debt of the firm, the risk free rate and the time period. The volatility of assets is estimated from the past three year stock return data for each month. For the risk free rate, we use the 1-Year Treasury Constant Maturity Rate. As an option based model and uses balance sheet of the firms, we use time period as one year.

As we said before, with above known parameters, according to Merton Model we have two equations with two unknowns. If we find the unknown parameters, we can calculate the MPD of companies. As in section 2.6, we have two equations which can be calculated from Ito's Lemma and Black and Scholes Formula. Ito's Lemma defines a relation between the asset volatility and equity volatility. Black-Scholes is used to value the firm's equity as a function of its asset value and its assets volatility. In these equations, we have two unknown parameters (current value of the assets and volatility of the assets). To find these unknown parameters, we need to find values for them that would satisfy both equations. To solve these equations, we use excel goal seek function[66]. However, we should find unknown parameters for each company. In excel we cannot use goal seek function many times as other functions such as correlation or multiplication. To run goal seek function many times we create a macro which will be explained in appendix II. With the help of this macro, we calculate Merton default probability, distance to default and expected default frequencies of each company for each year between 2005 and 2008. In addition we can solve these equations with the help of Newton-Raphson numerical procedure.

As mentioned before, industries which are between clusters are more volatile than industries which belong to one cluster (with hundred percentage). However, when we calculate the MPD of these industries, for each year none of them has greater

65 Saretto, 2004.

66 Goal seek function can be used when you know the result of a formula. but not the input values required by the formula to decide the result. reverse calculation

probability than 0.5. Only Tobacco sector has probability 0.5 at 2005. Following tables illustrates MPD of the industries which are between clusters. This also indicates the necessity of building a default probability.

Table 4.16 MPD of the Industries which are Between Clusters in 2005

2005 - Industries Between Clusters	
Internet	**0.2741**
Semiconductor	**0.0001**
Drug	**0.0736**
Tobacco	**0.5000**
Medical Supplies	**0.1562**
Auto & Truck	**0.2500**
Petroleum	**0.1288**

The table above illustrates the average MPD values of different sectors in 2005. The MPD values of sectors are calculated by finding the average of MPD values of all companies in each sector. In other words, this table illustrates MPD values of each industry which are between clusters with respect to the variables that we found from factor analysis.

Table 4.17 MPD of the Industries which are Between Clusters in 2006

2006 - Industries Between Clusters	
Semiconductor	**0.000451**
Petroleum	**0.00023**
Medical Supplies	**0.04**
Internet	**0.0203**
Precious Metals	**0.0003**
Telecom Services	**0.00024**
Telecom Equipments	**0.00033**
Drug	**0.0003**
Food Processing	**0.0004**
Auto & Truck	**0.0000**

The industry of Medical Supplies has the greatest Merton default probability than the other industries which are between clusters in 2006. The second risky sector is Medical Supplies. To find the probability; first we calculate MPD of each company, which belongs Medical Supplies sector. Then, by calculating average value, we can easily find the MPD value of Medical Supplies.

Table 4.18 MPD of the Industries which are Between Clusters in 2007

2007 - Industries Between Clusters	
Semiconductor	**0.078**
Internet	**0.2441**
Telecom Services	**0.2873**
Drug	**0.1151**
Precious Metals	**0.0024**
Telecom Equipments	**0.3025**
Food Processing	**0.0548**
Auto & Truck	**0.02135**
Metals & Mining	**0.3702**
Entertainment	**0.1684**

The value of 0.078, in the Table 4.18, shows MPD of the Semiconductor sector. Metals & Mining has the greatest Merton default probability than the other industries which are between clusters in 2007. The second risky sector is Telecom Equipment sector. In 2007, all industries have greater Merton default probabilities which can be explained with the global crisis. From cluster analysis, we analyze that the sectors Food Processing, Metals & Mining and Auto Truck are most volatile sectors for the year 2007. From Merton Model, we found that Auto & Truck and Food processing sectors have smaller MPD values. To consider both models, to construct a new bankruptcy probability, we should use MPD and the clustering variables in fuzzy process.

Table 4.19 MPD of the Industries which are Between Clusters in 2008

2008 - Industries Between Clusters	
Semiconductor	**0.0127**
Drug	**0.0626**
Telecom Services	**0.0599**
Internet	**0.1117**
Aerospace / Defense	**0.000**
E – Commerce	**0.016**
Chemical	**0.0024**
Telecom Equipments	**0.0506**
Metals & Mining	**0.052**
Food Processing	**0.0079**
Auto & Truck	**0.0077**

Internet sector has the greatest default probability than the other industries which are between clusters in 2008. The second risky sector is Drug. If we compare the results with the previous year, we can say that it becomes much more stable.

Now, we define the default correlations of sector X and Y. $Cor(D_X(t), D_Y(t))$ as

$$Cor(D_X(t), D_Y(t)) = \frac{Cov(D_X(t), D_Y(t))}{\sqrt{Var(D_X(t))}\sqrt{Var(D_Y(t))}}.$$

For good measure, we anxious about default correlations of industries. These calculations are non- mandatory for our probability default index. However, we want to calculate the probabilities or probability ranges for default correlations. Regarding this, we can analyze cross-sectoral relationship. Table 5.20 illustrates default correlations of industries: Internet and Semiconductor, Internet and Drug, Drug and Medical Supplies and Auto & Truck and Petroleum.

Table 4.20 Upper and Lower Bounds (2005)

Correlation	Internet & Semiconductor	Internet & Drug	Medical Supplies & Drug	Auto Truck & Petroleum
0.1	0.32	0.34	0.31	0.35
0.2	0.33	0.35	0.33	0.36
0.3	0.35	0.37	0.35	0.38
0.4	0.37	0.38	0.36	0.39
0.5	0.38	0.4	0.38	0.41
0.6	0.4	0.42	0.4	0.43
0.7	0.42	0.44	0.42	0.45
0.8	0.43	0.44	0.44	0.47
0.9	0.44	0.47	0.47	0.5
-0.1	0.26	0.28	0.25	0.29
-0.2	0.24	0.26	0.24	0.27
-0.3	0.23	0.25	0.22	0.26
-0.4	0.21	0.23	0.21	0.24
-0.5	0.2	0.22	0.19	0.23
-0.6	0.18	0.2	0.17	0.21
-0.7	0.17	0.19	0.16	0.2
-0.8	0.15	0.17	0.14	0.18
-0.9	0.13	0.15	0.12	0.16
Maximum	0.44	0.47	0.47	0.5
Minimum	0.13	0.15	0.12	0.16

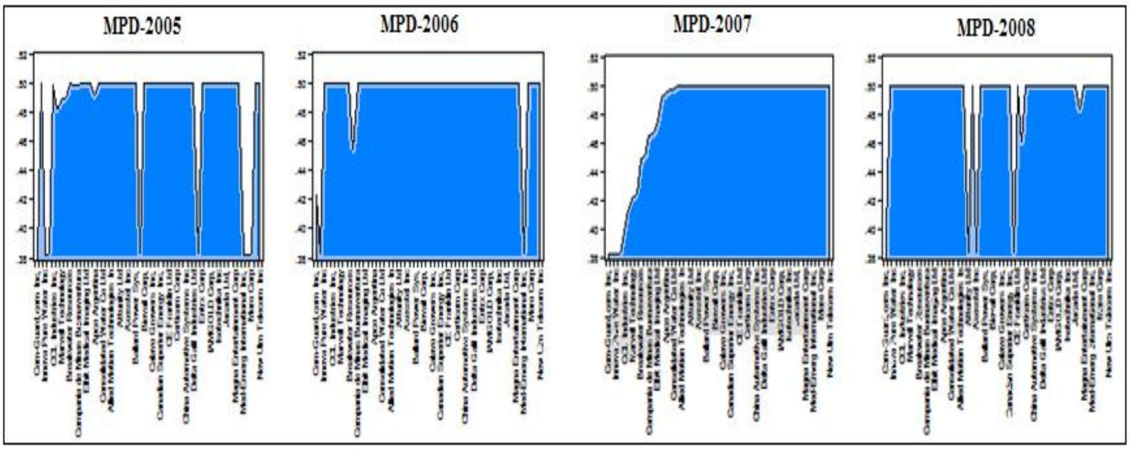
Table 4.21 illustrates the boundaries of default correlation of the following sectors. For instance, the bold values 0.08 and 0.43 represent the minimum and maximum default correlation of the sectors Auto & Truck and Semiconductor. A correlation greater than 0.8 generally describes as strong, whereas a correlation less than 0.5 generally describes as weak. For sectors Auto & Truck and Semiconductor, the minimum value of the correlation is 0.08 and the maximum value of the correlation is 0.43 (which is also smaller than 0.5), which means they are positively correlated, and the correlation is weak. For the following sectors, there is no negative correlation, which describes if any of these sector defaults, we expect any other sector can be default.

Table 4.21 Default Correlation Boundaries of Industries

	2005		2006		2007		2008	
	Min	Max	Min	Max	Min	Max	Min	Max
Auto Truck & Internet	**0.21**	**0.53**	**0.06**	**0.37**	**0.13**	**0.43**	**0.09**	**0.41**
Auto Truck & Drug	**0.14**	**0.47**	**0.06**	**0.37**	**0.09**	**0.43**	**0.07**	**0.41**
Auto Truck & Semiconductor	**0.12**	**0.4**	**0.06**	**0.37**	**0.08**	**0.43**	**0.06**	**0.43**
Internet & Drug	**0.15**	**0.47**	**0.06**	**0.39**	**0.16**	**0.5**	**0.1**	**0.46**
Internet & Semiconductor	**0.13**	**0.4**	**0.06**	**0.39**	**0.14**	**0.48**	**0.09**	**0.42**
Drug & Semiconductor	**0.07**	**0.39**	**0.06**	**0.43**	**0.11**	**0.47**	**0.07**	**0.42**

For each year between 2005 and 2008, the following graph shows the Merton default probabilities of the same foreign USA companies.

Graph 4.1 MPD of Foreign USA Companies



4.4 Fuzzy Model for Bankruptcy Probability

In our fuzzy model, we have five fuzzy variables as inputs and for each input we have five membership functions that two of them are trapezoids and three of them are triangles. We need to characterize each membership function. In other words, we need to find the border points of the each membership function. First we find the maximum, minimum, standard deviation and the mean of the each input. By using these values and expert opinion, we decide the borders of each membership function as seen in the following tables.

Table 4.22.A Range of the Variables

	Max	Min	Mean	St. Dev
SP	20	-1	-0.0321	0.7641
EBIT	15	-1	-0.0158	0.9372
NSh	15	-1	0.0344	1.1141
EV/TS	15	-10	-0.0227	0.8423
MPD	1	0	0.022	0.2883
Output	1	0	0.4853	0.0366

Table 4.22.B Range of the Variables

Extremely Bad	A	B	C	D	Bad	E	F	G
SP	-1	-1	0.25	0.47	**SP**	0.027	2.14	4.08
EBIT	-1	-1	0.70	4.72	**EBIT**	1.79	3.82	6.34
NSh	-1	-1	0.6	4.68	**NSh**	2.89	4.73	7.13
EV/TS	-10	-10	-7.5	-4.93	**EV/TS**	-7.78	-4.24	-1.99
MPD	0.52	0.82	1	1	**MPD**	0.39	0.54	0.86
Output	0	0	0.1	0.3	**Output**	0.1	0.3	0.5

Table 4.22.B Range of the Variables

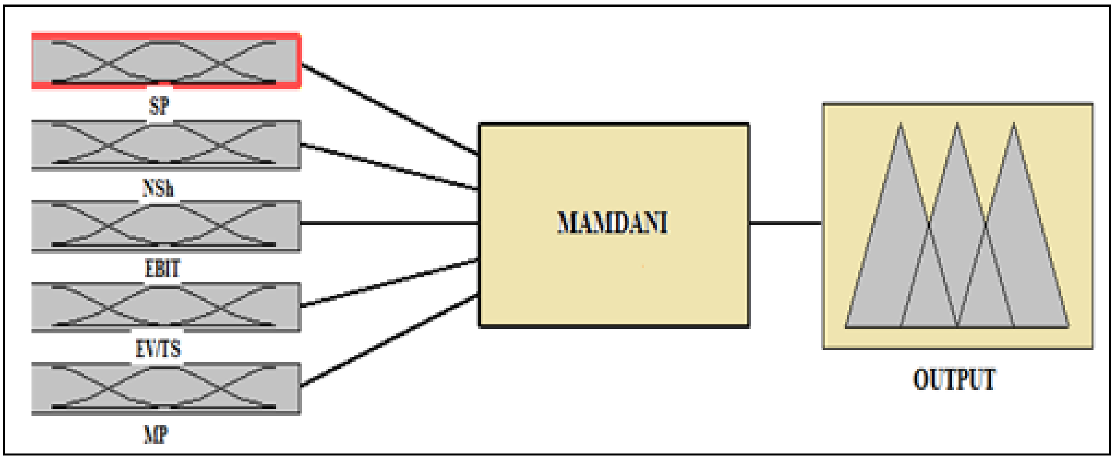
Good	H	I	J	Extremely Good	K	L	M	N
SP	5.36	10.36	16.61	**SP**	14.25	17.9	22.1	38.9
EBIT	6.38	9.16	13	**EBIT**	8.68	10.65	15	15
NSh	8.17	9.65	10.99	**NSh**	9.23	11.42	15	15
EV/TS	-1.17	3.67	7.29	**EV/TS**	2.70	8.62	15	15
MPD	0,009	0,036	0,059	**MPD**	0	0	0.0025	0.033
Output	0.5	0.7	0.9	**Output**	0.7	0.9	1	1

Table 4.22.C Range of the Variables and Model Properties

Normal	O	P	Q	Model Properties	
SP	1.08	4.47	6.08	**And Method**	Minimum
EBIT	4.7	6.61	8.68	**Or Method**	Maximum
NSh	4.47	6.53	9.09	**Implication**	Minimum
EV/TS	-3.16	-1.11	0.68	**Aggregation**	Maximum
MPD	0.054	0.2579	0.521	**Defuzzification**	Centroid
Output	0.41	0.5	0.59	**Type**	Mamdani

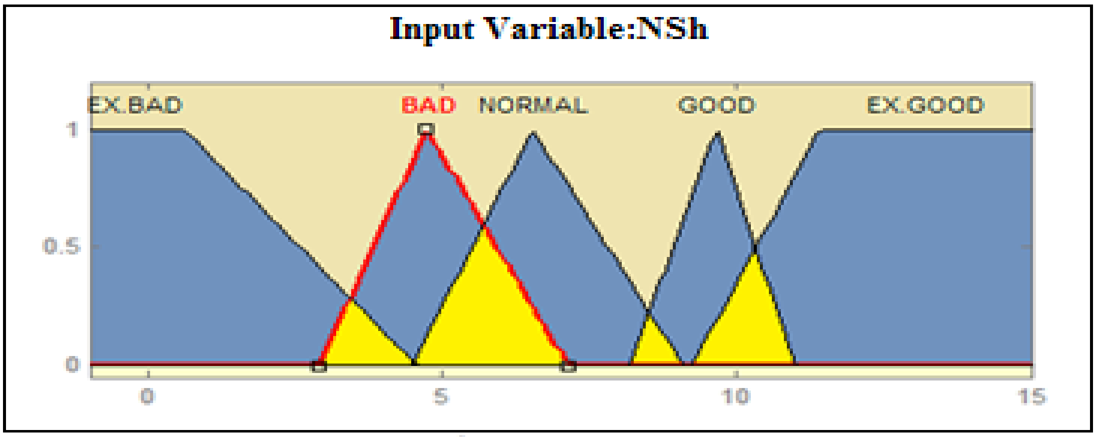
In the above tables; A, B, C and D are borders of the first membership function, which is trapezoid; E, F and G are the borders of the second membership function, which is triangular; O, P and Q are the borders of the second triangular and H, I and J are borders of the last triangular. F, P and I are the head values of the triangular and others are the bottom values. To define a trapezoid, we need five borders. For the first trapezoid the borders are A, B, C and D and for the other trapezoid the borders are K, L, M and N. In our fuzzy logic process, we use and method with taking minimum values of variables. We use and method with taking maximum value of variables. For the implication process, we use minimum and for the aggregation process, we use maximum method. As we explained before, for the defuzzification, we use centroid method. The following figure summaries the fuzzy system which we used.

FIGURE 4.3 FUZZY SYSTEM OF OUR MODEL



The membership functions are defined as “Extremely Bad”, “Bad”, “Normal”, and “Good”, “Extremely Good”. Membership functions can be continuous but we use piecewise linear functions such as triangular and trapezoidal membership functions. The triangular membership functions are “Normal”, “Bad”, “Good”, “Extremely Good” and “Extremely Bad” has the shape of trapezoidal because of their over situation. Graph 4.2 illustrates the membership functions of the input variable; NSh. You can find the graphs of membership functions for the other input variables in appendix, 7.10.

Graph 4.2 Membership Functions of Input Variable; NSh



In previous graph, extremely bad, bad, normal, good, extremely good are functions mapping a bankruptcy scale. A point on that scale has five “truth values”- one for each of the five functions. For the particular bankruptcy shown, the five truth values could be interpreted as describing the bankruptcy probability as, say not good, slightly normal etc. We determine the ranges of the membership functions with doing some statistics.

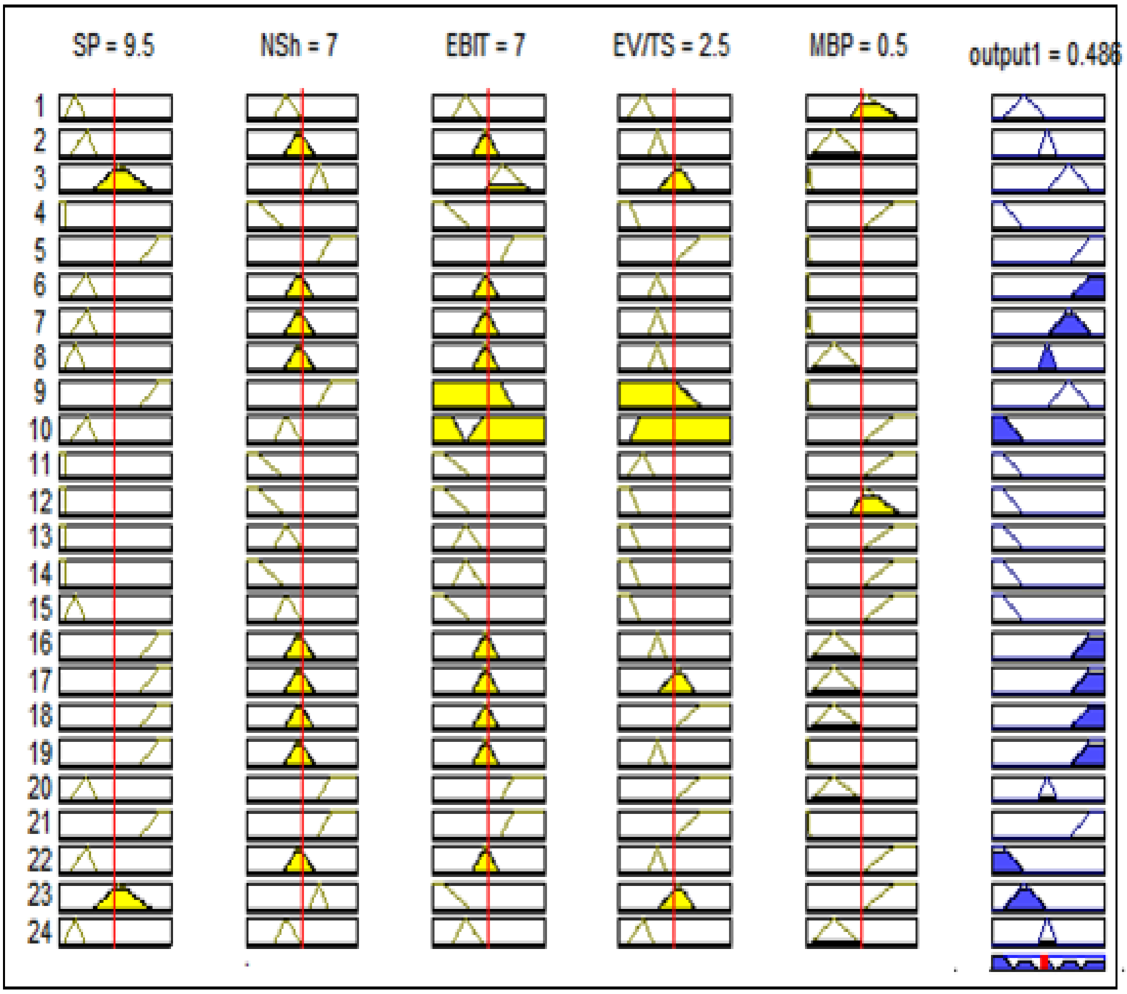
To sum up, we have five input variables and one output, which we said fuzzy bankruptcy index and denoted FBI, which was produced from these five variables. One of the input variables is the default probability that we calculated from Merton Model which is denoted by MPD. The other input variables are SP, NSh, EBIT and EV/TS.

For the next step of modeling, we need to define the rules. Notice that, if we have only two variables with five membership functions for each, the total number of rules should be 25. We have five input variables with five membership functions, so the rules can be represented such as:

If x_1 is A and x_2 is in B and x_3 is in C and x_4 is in D and x_5 is in E then z is in Z; where $x_1, x_2, ..., x_5$ represent variables and A, B, C, D, E and Z are linguistic variables such as “extremely good”, “good”, “normal”, “bad” and “extremely bad”. We decide rules for indicators according to mean and standard deviation applications.

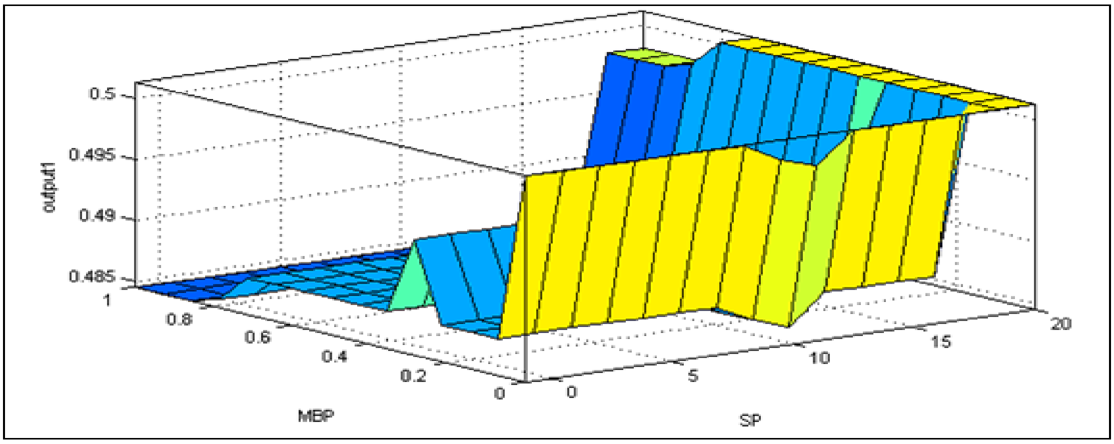
Figure 4.4 illustrates the rules that we use to construct FBI. The following figure is the summary of the topics that we mentioned above. It shows that the effect of one fuzzy if then rule when calculating the output. If the rule is with and application or something different, it is not important, we should know what this means in mathematical sense. After finding the output as a fuzzy set we should defuzzy it to understand what this really means.

FIGURE 4.4 SUMMARY OF "FUZZY IF THEN" RULES



After determining the membership functions and rules, we use fuzzy inference system in matlab to create output for our model. As we mentioned before, we use Mamdani type fuzzy inference system and the system consist of five parts.

Graph 4.3 Surface that Shows Relationship between SP, MPD and Output



By looking above graph, we can easily examine the relations SP and MPD with the output space. So, after investigate the graph, you can change up your mind and decide to use another indicator the best fits the output. You can find the graph of MPD, output with other input variables in appendix. The graph 7.7.A-B-C illustrates the relationship between input variables, MPD and output, which we called FBI.

Steps of the Fuzzy Process,

- Define the data
- Define the borders of the membership functions
- Define the model that develop the output, FBI
- Define the rules
- Fuzzification
- Defuzzification
- Constructing FBI

CHAPTER 5 MACRO ECONOMIC FACTORS & PROBABILITY OF DEFAULT

Researches were carried out to study the probability of default on the individual firm's level. However, only since recent years have the researches about the probability of default taken into account the influence of macro economic conditions. Same empirical results have been found. Standard credit risk models by Vasicek (1987, 1991, 2002), following the option-based approach of Merton (1974), allow for business cycle effects generally via one or more unobserved systemic risk factors. Helwege and Kleiman (1996) studied the relationship between recession and actual default rate. They
model growth in GDP (Gross Domestic Product) as a dummy variable with a value of 1 when growth exceeds 1.5% per year and 0 when it falls below that level. Friedson et al (1997) refined the model by taking into consideration interest rate. Studying the corporate bond of 1971-1995, the authors found a relation between macroeconomic conditions and the probability of default. Later, Wilson (1997) found out that the single factor model explains only 23.9% of the US systematic risk, the substantial correlation remaining is explained by the second or thirdfactors, a single-factor systematic risk model is not enough to capture all. The author constructed a model which allows for the macroeconomic variables (GDP, rate of government spending, regional housing price index) to influence a firm's probability of default using a pooled logit regression, and further confirmed this relationship between macro factors and the probability of default. [67]

In our study, to analyze the relationship between EDF / MPD and macro economy, the least square dummy variable model is chosen. This model makes possible the analysis of relationship between probability of default and macroeconomy. The model is displayed as:

$$\ln(FBI_{i,t}) = \alpha_0 + \sum \alpha_i D_i + \beta \ln(IPI) + \chi \ln(CPI) + \gamma \ln(UR) + \varphi \ln(SPI) + u_{i,t}$$

where

[67] Yiping Qu; Macro Factors and Probability of Default; Journal of Economics, Finance and Business Administration.

$FBI_{i,t}$: Fuzzy Bankruptcy Index of the i^{th} industry over the time t.

IPI : Industrial Production Index

CPI :Consumer Price Index

UR :Unemployment Rate

SPI :Share Price Index

D_i : Dummy variable for certain industry

$u_{i,t}$: Random variable assumed to be independent and identically normally distributed

i : Certain industry

t : Time

A log-linear model is used in order to capture the percentage changes instead of the normal unitchanges. First difference is taken for the variables that are not stationary.The variables considered are Industrial Production Index (IPI), Consumer Price Index (CPI), Unemployment Rate (UR), Interest Rate Spread (IRS), Share prices (SP) and Financial Openness Index (FOI), during the period of January 2005 to December 2008.

The **Industrial Production Index** is an economic indicator which measures real production output, which includes manufacturing, mining and utilities. IPI is an important indicator for economic forecasting and is often used to measure inflation pressures as high levels of industrial production can lead to sudden changes in prices.

The **Consumer Price Index** is a monthly measurement of inflation. It is one of the red-hot economic indicators that are carefully dissected by the financial markets. It is compiled by the Bureau of Labor Statistics and is based upon a 1982 Base of 100. CPI is an indicator of changes in consumer prices. It is obtained by comparing, over time the cost of a fixed basket of goods and services purchased by consumers. Since the basket contains goods and services of unchanging or equivalent quantity and quality, the index reflects only pure price change. CPI does not include sales price of houses. Instead, it calculates the monthly equivalent of owning a house.

Unemployment rates give the numbers of unemployed persons as a percentage of the civilian labor force. In the least-square dummy variable model, unemployment rate is denoted with UR.

Share Price Index represents the performance of the whole stock market, as a proxy. It therefore reflects investors' sentiment on the state of the economy, which is denoted SPI.

We calculate the coefficient of determination R^2 with the provided information from the regression and find 0.5782 which means around 58% of the total variation in FBI is explained by this regression model.

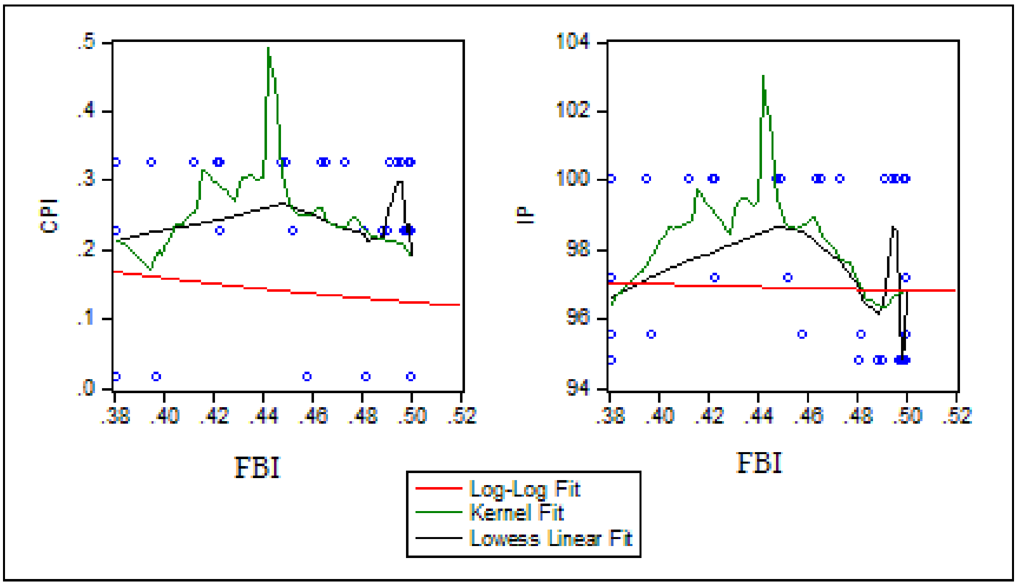
Table 5.1 Estimates of Fixed Effects for Industry

Parameter	Estimate	Standard Error	T	Sig.
ln (IPI)	-3.17	1.80	-2.294	0.022
ln (CPI)	-3.07	0.564	-5.455	0.000
ln (SPI)	-4.91	0.324	-8.066	0.000
ln (UR)	5.12	1.76	2.564	0.023

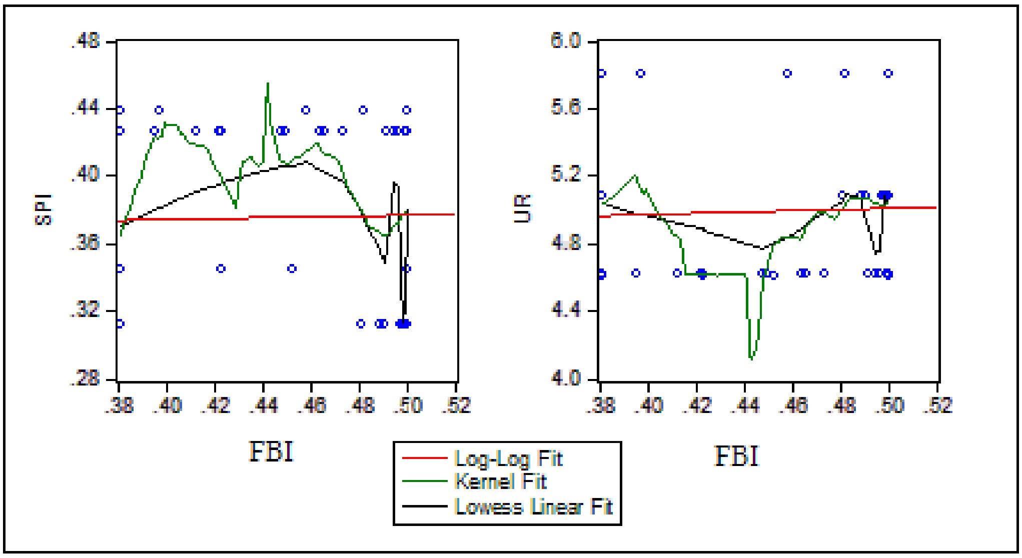
From Table 5.1 we can easily see that if Industrial Production increases by 1%, the Fuzzy bankrupcy probability, which is represented by FBI as dependent variable in the model will on average decrease by 3.17% holding other variables constant. The coefficients of other variables are interpreted in the same way.

Graph 5.1.A and 5.2.B illustrates the relationship between fuzzy index with macro economic factors that we use in our least-square.

Graph 5.1.A FBI & CPI and MPD & IP



Graph 5.1.B MPD & SPI and MPD & UR

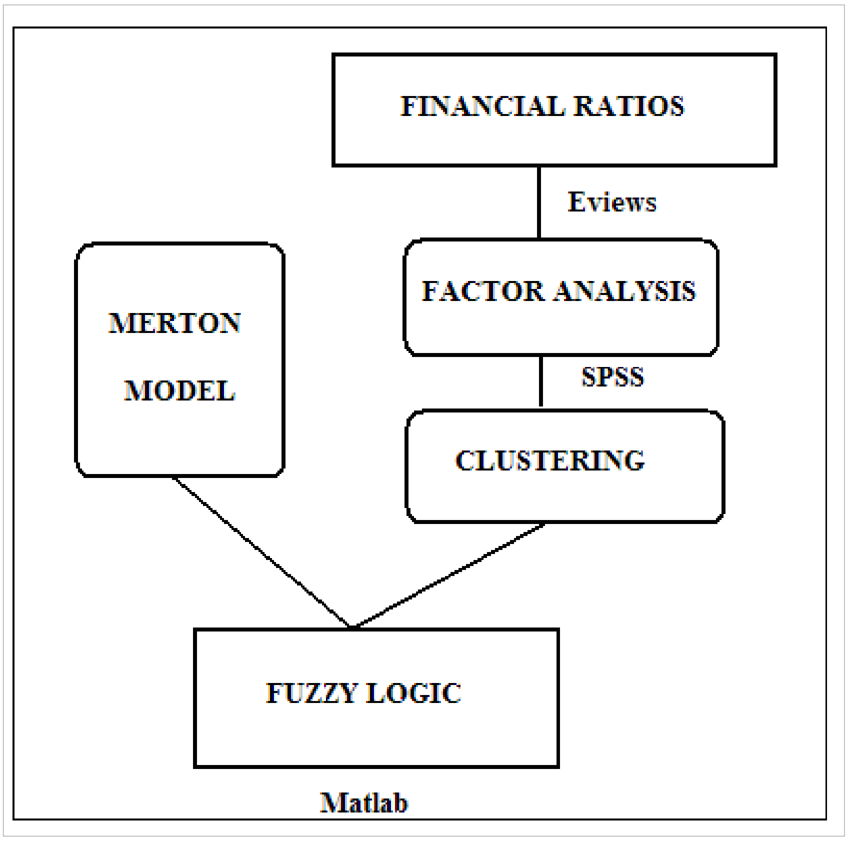


Different industries react to the changes of macro factors with different degree and direction.

CONCLUSION

In this study we construct a default parameter, which we named FBI, by using fuzzy process. For the fuzzy process, we have five input variables, four of them are chosen from both factor analysis and clustering and the last input variable calculated from Merton Model. We use an algorithm which was written in Matlab to apply the fuzzy process. For clustering part, we use SPSS and for the factor analysis, we use E-views. The following figure illustrates the main topics of our study.

Figure I.1 Summary of Topics with Computer Programs



At first, we have introduced an approximation to the Merton model for the ISE100 companies and we analyze that Merton model for bankruptcy probability is not sufficient. To explicate the values which found from Merton Model is perplexing. The Merton bankruptcy probabilities are so close to zero or one, or the values change so extremely which cannot be possible or possibly to explain. Because of this, we try to create a new variable (FBI), which also includes Merton Model as an input variable or in other words, we create a new approach which is more sensitive to bankruptcy probabilities.

In addition, we want to create an index, which is nearly accurate to all securities. To do this, our data consists of seventy eight different sectors. We calculate MPD values of these companies and with the help of these values we calculate the MPD values of sectors. To determine other input variables of our fuzzy process, we use factor analysis and clustering. First, we use factor analysis and find ten financial ratios[68]. For the fuzzy process, ten input variables with five membership functions[69] are too many to be significant. We have two options, one is to reduce the number of membership functions to three (bad, normal, good) or reduce the number of input variables. We decided to reduce the number of input variables and use clustering (If we decided to reduce the number of membership functions, we cannot cope with the difficulty of explaining all sectors). We cluster the companies with variables we found from factor analysis. In other words, we cluster the industries each year and from clustering results, we reduce the financial ratios which imply bankruptcy probability. Since cluster analysis is an exploratory data analysis tool for organizing observed data into meaningful group or clusters; we can easily examine the volatile sectors. Our main point is to find the sectors that are between clusters. Being between clusters illustrates that firms are not stable. After finding the sectors between clusters, we decide to find similar properties of these sectors. In addition, we seek the similarities of the financial ratios of firms which are between clusters. We illustrate that they are the properties that makes companies between clusters. To show these, we eliminate the properties and cluster industries again with these variables and find again two clusters. However, now the sectors belong to each cluster with hundred percentage. Thus, we find our financial ratios that can measure the sensitivity of bankruptcy without considering the sectors. In other words, we found our input variables for the fuzzy process.

---

[68] SP, TV, Nsh, TD,PTOM, EV/TS, EG, VLB, EBIT, CF.

[69] Normal, Bad, Extremely Bad, Good and Extremely Good.

Now we have five input variables to construct a new parameter which indicates bankruptcy probabilities. It includes Merton Model and some significant financial ratios (SP, Nsh, EBIT, EV/TS). As we said before, we use fuzzy logic to construct our output variable, named FBI. To use fuzzy logic, first we analyze the input variables and their meanings. By the help of these information we can construct the membership functions and fuzzy rules between our input variables.

We compare the values of FBI and MPD, their covariance is positive. This means their values increase and decrease at the same time. Notice that we construct FBI because MPD takes values so close to 0, which we cannot easily make estimation with this knowledge. Our results show that FBI is much more sensitive than the MPD.

At the end, we explain FBI index with macro-economic factors. FBI is calculated using both market information as well as firm's individual profile. One of the input that creates FBI is based on well-known Black_Scholes option pricing equation, it combines asset's value, asset risk and the firm's leverage ratio into a unique measure of default risk. The exogenous variables in the model represent the general macro economic state. We analyzed the relationship of macroeconomic factors with the probability of default parameter which we construct with fuzzy logic. Using the multifactor fixed effect model, the study verified the effect of the macro factors on probability of default. Different industries react to the changes of macro factors with different degree.

APPENDICES

I. BARRIER OPTIONS

Barrier options are options that are either activated or deactivated when the price of the underlying security passes through some predefined value which is called the barrier. Barrier options have eight different varieties:

Down and out Call Option: A call option that is deactivated if the price of the underlying falls below a certain price level. If the underlying asset does reach the barrier price level, the down-and-in option becomes a vanilla European call option. If the underlying asset price does not reach the barrier level, the option expires worthless.

Up and in Call Option: A call option that is activated if the price of the underlying rises above a certain price level.

Up and out Call Option: A call option that is deactivated if the price of the underlying rises above a certain price level.

Down and in Call Option: A call option that is activated if the price of the underlying falls below a certain price level.

Up and in Put Option: A put option that is activated if the price of the underlying rises above a certain price level.

Up and out Put Option: A put option that is deactivated if the price of the underlying rises above a certain price level.

Down and in Put Option: A put option that is activated if the price of the underlying falls below a certain price level.

Down and out Put Option: A put option that is deactivated if the price of the underlying falls below a certain price level.

The value of these options is dependent not only upon the value of the underlying security at option expiration, but also the path that the underlying takes prior to expiration. They're therefore known as **path-dependent options**. Investors may choose type of barrier options rather than directly purchasing the vanilla option, as option premiums tend to be significantly lower for knock-in options.

II. VARIANCE FORMULATION

To show equation (1), follow the following steps, we use only definition and simple mathematical calculations.

$$\mathrm{Var}(D_X(t)) = E(D_X(t)^2) - E(D_X(t))^2 \qquad (1)$$

$$\begin{aligned}
&= E[(D_X(t) - E(D_X(t)))^2] = E[D_X(t)^2 - 2D_X(t)E(D_X(t)) + [E(D_X(t))]^2] \\
&= E[D_X(t)^2] - E[2D_X(t)E(D_X(t))] + E[E(D_X(t))^2] \\
&= E[D_X(t)^2] - 2E(D_X(t))E[D_X(t)] + [E(D_X(t))]^2 \\
&= E[D_X(t)^2] - 2E[D_X(t)]^2 + [E(D_X(t))]^2 \\
&= E(D_X(t)^2) - E(D_X(t))^2 .
\end{aligned}$$

III. INTEREST RATES FOR ISE100 APPLICATION

Following table shows the risk-free interest rate which we use in the application of ISE100 index. To calculate the MPD values of each company, one parameter is the risk free interest rate. As said, we take following information from TUİK.

Table III.1 Risk Free Interest Rates between 2004-2008 Risk-free Interest Rates

Year	Treasury Bill Rate
2004	24.7%
2005	16.3%
2006	18.1%
2007	18.4%
2008	19.2%

IV. MACRO FOR GOAL SEEK FUNCTION

As said before, to solve two equations with two unknown parameters, we use goal seek function in the Excel. However, to do this process for many times we need to write a macro, which is as follows.

```
Sub solve All()
Dim cell Change As Range
Dim cell Goal As Range
Dim cell Constraint As Range
Set cell Change = Active Sheet. Range("K3:L3")
Set cell Goal = Active Sheet. Range("M3")
Set cell Constraint = Active Sheet. Range("K3")

Do
Solver Reset
Solver Add Cell Ref:=cell Constraint. Address. Relation:=2. Formula Text:="0"
Solver Ok Set Cell:=cell Goal. Address. _
Max Min Val:=2. By Change:=cell Change. Address
SOLVER . Solver Solve User Finish:=True
Solver Finish Keep Final:= 1
Set cell Change = cell Change. Offset (1. 0)
' Msg Box  cell Change. Row
Set cell Goal = cell Goal. Offset (1. 0)
```

```
Set cell Constraint = cell Constraint. Offset (1. 0)
' Msg Box cell Goal. Row
Loop While Trim (cell Goal. Text) <> "" 'until goal cell is empty
End Sub
```

V. RESULTS OF ISE100 APPLICATION

Table V.1 and Table V.2 illustrate the unknown parameters of the Merton model. Notice that, we have two equations with two unknowns. We found the unknown parameters, by the help of this information we calculate the default probabilities. Because of the properties of our variables, we have some constraints. For instance, they should be greater than zero. Thus, we take a negligible region that the equations almost surely satisfied.

Table V.1 Volatility of Firm's Asset Returns

	2004	2005	2006	2007	2008
ACIBD	1.34	0.36	0.16	0.37	0.09
ADEL	1.03	0.37	0.47	0.24	0.52
AEFES	0.26	0.17	0.18	1.02	0.26
AKSA	0.17	0.23	1.24	0.28	0.45
ALARK	0.29	0.30	2.43	0.16	0.43
ALTIN	0.34	0.32	0.43	0.31	0.32
ANACM	0.33	0.25	0.18	0.53	0.29
ASELS	0.39	0.53	0.47	0.37	1.45
AYGAZ	0.21	0.35	0.27	0.30	0.50
BAGFS	0.24	0.46	0.31	0.30	0.69
BANVT	1.13	0.26	0.28	0.35	0.25
BROVA	0.20	0.51	0.46	0.31	0.77
BUCIM	0.66	0.39	0.51	0.10	0.39
CIMSA	0.32	0.38	0.46	0.19	0.56
DYHOL	0.16	0.30	0.16	0.18	0.41
ECILC	1.59	0.46	0.43	0.27	1.13
EDIP	0.28	0.14	0.31	0.22	0.15
EGEEN	0.34	0.32	0.33	0.45	0.38
EGGUB	0.28	0.39	0.37	0.42	0.43
ENKAI	0.15	0.49	0.52	0.31	0.49
EREGL	2.35	0.32	0.27	0.59	0.50
FENIS	0.30	0.27	0.24	0.44	0.39
FRIGO	0.58	0.20	0.16	0.52	0.18
FROTO	0.23	0.27	0.22	0.23	0.44
GOLDS	0.65	0.45	0.52	0.26	0.65
GOODY	0.28	0.31	0.39	0.17	0.57
GSDHO	0.12	0.09	0.07	0.06	0.92
HEKTS	0.52	0.47	0.53	0.34	0.45
HURGZ	0.51	0.43	0.34	0.21	0.43
IHLAS	0.24	0.34	0.19	0.35	0.64
IPMAT	0.27	0.69	0.72	0.56	0.70
IZOCM	0.38	0.43	0.34	0.25	0.64
KAPLM	1.06	0.65	0.60	0.36	0.60
KARTN	0.87	0.55	0.21	0.52	0.47
KCHOL	0.14	0.12	0.07	0.09	0.24
KENT	0.31	0.52	0.44	0.32	0.46
KUTPO	1.12	0.53	0.32	0.25	2.05
METUR	0.32	0.47	0.27	0.18	0.16
MİPAZ	0.36	0.37	0.74	0.28	0.41
MRDIN	0.24	0.37	0.44	0.26	0.49
OKANT	1.08	0.29	0.66	0.41	1.59
OLMKS	0.18	0.44	0.27	0.17	0.61
PARSN	0.20	1.49	0.33	0.37	0.72
PENGD	0.38	0.29	0.16	0.26	0.39
PNSUT	0.04	0.46	0.66	0.21	0.69
PTOFS	1.10	0.34	0.40	0.27	0.54
SAHOL	0.21	0.07	0.10	0.04	0.10
TATKS	0.10	0.19	0.11	0.15	0.27
TCELL	0.10	0.25	0.26	0.21	0.50
VESTL	0.23	0.10	0.20	0.17	0.47

Table V.2 Market Value of Firm's Asset Return

	2004	2005	2006	2007	2008
ACIBD	131.48	172.62	218.51	260.07	410.16
ADEL	28.46	34.10	40.02	42.32	52.27
AEFES	1290.27	1826.63	2.69	2.60	4.03
AKSA	754.89	744.27	875.85	709.95	910.92
ALARK	358.64	577.05	590.77	740.18	926.41
ALTIN	134.07	220.78	266.54	259.97	313.72
ANACM	629.24	815.30	960.53	991.24	1256.62
ASELS	291.52	416.22	426.21	507.29	510.27
AYGAZ	940.36	1115.27	1923.98	1699.85	1843.17
BAGFS	102.08	111.49	118.81	129.47	204.53
BANVT	126.09	137.23	152.48	226.25	273.53
BROVA	10.88	12.20	9.72	8.18	6.98
BUCIM	179.77	172.11	200.31	263.52	331.33
CIMSA	631.88	693.77	856.03	1007.75	932.64
DYHOL	1.01	1.45	1.63	2.62	3.46
ECILC	563.58	980.98	1228.64	1711.86	1803.80
EDIP	86.42	91.15	83.72	111.54	180.22
EGEEN	45.99	33.31	40.62	39.61	61.35
EGGUB	99.78	105.98	111.20	141.69	165.66
ENKAI	3927.37	4028.27	4612.44	5.01	6.82
EREGL	4681.18	6027.50	7120.79	7922.33	9816.78
FENIS	44.31	59.53	65.69	62.75	76.26
FRIGO	14.40	20.45	21.02	20.39	16.16
FROTO	2080.14	1877.31	2008.24	2061.91	2025.69
GOLDS	153.98	156.51	182.64	194.31	190.83
GOODY	277.43	263.77	284.02	279.13	303.03
GSDHO	1192.72	1776.10	2434.54	2598.72	682.30
HEKTS	55.62	66.38	71.72	76.96	79.67
HURGZ	647.28	724.36	751.75	1228.84	1641.56
IHLAS	551.58	592.24	527.63	522.23	570.98
IPMAT	28.88	50.17	60.90	32.07	430.26
IZOCM	96.22	136.82	173.00	169.54	155.53
KAPLM	33.76	30.60	33.40	33.24	33.20
KARTN	187.91	178.93	189.91	153.77	152.92
KCHOL	10388.26	23767.61	35160.69	36205.88	47932.77
KENT	202.30	201.19	210.67	246.10	323.67
KUTPO	4848.34	84.27	89.29	94.01	91.99
METUR	77.44	19.79	19.56	21.89	20.68
MİPAZ	15.42	23.54	25.91	23.58	53.35
MRDIN	25.52	187.67	77.49	78.48	225.90
OKANT	137.91	10.75	8.94	8.46	13.70
OLMKS	13.75	139.35	152.88	171.69	171.63
PARSN	134.28	118.31	151.21	196.52	253.99
PENGD	84.91	60.71	49.62	57.00	70.15
PNSUT	45.25	266.61	263.67	303.35	318.95
PTOFS	257.66	2991.37	3146.19	3219.98	3584.33
SAHOL	2857.66	46601.75	50156.63	56913.10	13033.54
TATKS	29341.85	248.20	241.07	286.19	398.12
TCELL	241.43	5866.90	6611.36	7716.56	9158.45
VESTL	1475.17	1647.95	1900.86	1699.00	1451.92

From Table V.2, we can say that for the year 2004, market value of TCELL's assets return is nearly 241.

Table V.4.-V.8 illustrates correlation between our parameters from Merton model for each year, respectively.

Table V.4 Correlation between Merton Model Parameters at 2004

	D^T	E_0	σ_E	$N(d_1)$	$N(d_2)$	σ_A	A_0	E_0/D^0	A_0/D^0	MPD
D^T	1	0.7348	-0.0541	0.022	0.0482	-0.1467	0.9809	-0.0359	-0.0359	-0.0482
E_0		1	0.1975	0.0536	-0.2285	0.1586	0.8527	-0.0721	-0.0721	0.2285
σ_E			1	-0.4071	-0.8554	0.9497	0.0117	-0.0813	-0.0814	0.8554
$N(d_1)$				1	0.6566	-0.1516	0.0331	0.0452	0.0453	-0.6566
$N(d_2)$					1	-0.7349	-0.025	0.0643	0.0644	-1
σ_A						1	-0.0708	-0.046	-0.0461	0.7349
A_0							1	-0.0485	-0.0485	0.025
E_0/D^0								1	1	-0.0643
A_0/D^0									1	-0.0644
MPD										1

From the above table, we can say that the correlation between volatility of asset returns and total debt is negative. In addition, the correlation between volatility of asset returns and volatility of equity is equal to 0.9497, which is nearly 1. In other words, they are highly positively correlated. In a positive correlation, as the values of one of the variables increase, the values of the second variable also increase. Likewise, as the value of one of the variables decreases, the value of the other variable also decreases. We want to mean that volatility of asset return decreases implies decrease in volatility of equity.

Table V.5 Correlation between Merton Model Parameters at 2005

	D^T	E_0	σ_E	$N(d_1)$	$N(d_2)$	σ_A	A_0	E_0/D^0	A_0/D^0	MPD
D^T	1	0.7428	-0.0017	-0.2163	-0.05	-0.2808	0.9909	-0.0397	-0.0397	0.05
E_0		1	-0.1088	-0.154	-0.0162	-0.3043	0.826	-0.08	-0.08	0.0162
σ_E			1	-0.3675	-0.8402	0.8256	-0.0234	0.0259	0.0259	0.8402
$N(d_1)$				1	0.5318	0.1	-0.2127	0.0564	0.0564	-0.5318
$N(d_2)$					1	-0.6268	-0.0451	0.0478	0.0478	-1
σ_A						1	-0.2975	0.1213	0.1213	0.6268
A_0							1	-0.0495	-0.0495	0.0451
E_0/D^0								1	1	-0.0478
A_0/D^0									1	-0.0478
MPD										1

Table V.6 Correlation between Merton Model Parameters at 2006

	D^T	E_0	σ_E	$N(d_1)$	$N(d_2)$	σ_A	A_0	E_0/D^0	A_0/D^0	MPD
D^T	1	0.7085	0.0354	-0.6416	-0.0505	-0.1884	0.9913	-0.0732	-0.0732	0.0505
E_0		1	-0.0018	-0.4374	-0.04	-0.1545	0.795	-0.1244	-0.1244	0.04
σ_E			1	-0.4948	-0.9283	0.9406	0.0293	-0.1095	-0.11	0.9283
$N(d_1)$				1	0.52	-0.243	-0.6321	0.1028	0.103	-0.52
$N(d_2)$					1	-0.865	-0.05	0.0667	0.0672	-1
σ_A						1	-0.1914	-0.0142	-0.0147	0.865
A_0							1	-0.086	-0.086	0.05
E_0/D^0								1	1	-0.0667
A_0/D^0									1	-0.0672
MPD										1

Table V.7 Correlation between Merton Model Parameters at 2007

	D^T	E_0	σ_E	$N(d_1)$	$N(d_2)$	σ_A	A_0	E_0/D^0	A_0/D^0	MPD
D^T	1	0.7625	-0.0824	0.0419	0.0486	-0.3036	0.9907	-0.0721	-0.0721	-0.0486
E_0		1	-0.0776	0.0857	0.0855	-0.2235	0.8435	-0.116	-0.116	-0.0855
σ_E			1	-0.7274	-0.8083	0.8551	-0.0848	-0.0511	-0.0513	0.8083
$N(d_1)$				1	0.9202	-0.5098	0.0529	0.0806	0.0807	-0.9202
$N(d_2)$					1	-0.6635	0.0584	0.0808	0.081	-1
σ_A						1	-0.2991	0.1367	0.1366	0.6635
A_0							1	-0.0843	-0.0843	-0.0584
E_0/D^0								1	1	-0.0808
A_0/D^0									1	-0.081
MPD										1

Table V.8 2008 Correlation between Merton Model Parameters at 2008

	D^T	E_0	σ_E	$N(d_1)$	$N(d_2)$	σ_A	A_0	E_0/D^0	A_0/D^0	MPD
D^T	1	0.847	-0.033	-0.3216	-0.085	-0.1851	0.9805	-0.0552	-0.0552	0.085
E_0		1	-0.0436	-0.1277	-0.0006	-0.1153	0.935	-0.0708	-0.0707	0.0006
σ_E			1	-0.0731	-0.8092	0.8941	-0.0382	-0.1313	-0.1316	0.8092
$N(d_1)$				1	0.4127	0.335	-0.2618	0.1323	0.1323	-0.4127
$N(d_2)$					1	-0.5775	-0.0568	0.095	0.0954	-1
σ_A						1	-0.1662	0.0039	0.0036	0.5775
A_0							1	-0.063	-0.063	0.0568
E_0/D^0								1	1	-0.095
A_0/D^0									1	-0.0954
MPD										1

VI. DEFINITIONS OF RATIOS

In this section, we give the descriptions of the variables that we use in factor analysis.[70]

Book Debt Ratio: This is the book estimate of the debt ratio, obtained by dividing the cumulated value of debt by the cumulated value of debt plus the cumulated book value of equity for the entire sector.

Book Value: The dollar amount recorded on the balance sheet for an asset, liability, or equity. Subtract the accumulated depreciation from the historical cost to get the book value of an asset. Adjust the maturity value of the liability for any discount or premium to calculate the book value of a liability. The book value of a liability is usually the amount of cash required to pay off the obligation at that point in time, and the amount at which the asset or liability is "carried on the books." The book value of an asset represents the cost of the asset that has not been put on the income statement as an expense.

Capital (Book Value): This is the book value of debt plus the book value of common equity, as reported on the balance sheet

Capital Expenditure: Outlay charged to a long-term asset account. A capital expenditure either adds a fixed asset unit or increases the value of an existing fixed asset. An example is a new motor for a truck.

Cash to Current Liabilities Ratio: Computation that measures a company's ability to satisfy short-term financial obligations immediately and is therefore a good liquidity measure. The ratio equals cash plus near-cash and marketable securities divided by current liabilities.

[70] Dictionary of Accounting Terms, Dictionary of Financial and Business Terms,

Dividend Yield (Funds): Indicated yield represents return on a share of a mutual fund held over the past twelve months. Assumes fund was purchased 1 year ago. Reflects effect of sales charges (at current rates), but not redemption charges.

Dividend Yield (Stocks): Indicated yield represents annual dividends divided by current stock price.

Dividends per Share: Dividends paid for the past 12 months divided by the number of common shares outstanding, as reported by a company. The number of shares often is determined by a weighted average of shares outstanding over the reporting term.

EBITDA: Estimated by adding depreciation and amortization back to operating income (EBIT).

Effective Rate: A measure of the time value of money that fully reflects the effects of compounding.

Enterprise Accounting: Accounting for the entire business rather than its subdivisions (e.g., department).

Enterprise Value / Sales (EV/S): The EV/S is just a quick way to understand how investors feel about a company. If the EV/S goes up by a 10th or down by a 10th of a point, there is no definitive point that suddenly turns a healthy company into a sick company or vice versa.

Earnings per Share (EPS): The dollar amount of the period's net income that is available to each share of common stock. Basic EPS are calculated by deducting any preferred dividends due to preferred stock shareholders for the period, and dividing by the weighted average number of common shares of stock outstanding during the period. Diluted earnings per share include an adjustment for common stock equivalents, resulting in 195 earnings per share lower earnings per share than basic EPS.

Free Cash Flow for the Firm (FCFF): A measure of financial performance that expresses the net amount of cash that is generated for the firm, consisting of expenses, taxes and changes in net working capital and investments.

Growth in Earnings per Share: EPS Growth Rate ratio is expressed as a percentage and it shows the relative growth of EPS over the last two reporting periods. A minus sign indicates negative growth from last year. If the previous year's EPS-basic is zero earnings per share growth rate is not defined.

Net Income: Divided by average total assets; also called rate earned on total assets. Other versions of return on investment exist, such as net income before interest and taxes divided by average total assets. Return on investment is a commonly used measure to evaluate divisional performance.

Non-Cash Working Capital / Sales:
Non-cash Working Capital = Inventory + Other Current Assets + Accounts Receivable - Accounts Payable - Other Current Liabilities
(Current assets excluding cash - Current liabilities excluding interest bearing debt). For the sector, we use cumulated values for each of the variables.

Payout Ratio: Ratio of cash dividends declared to earnings for the period. It equals dividends per share divided by earnings per share. Stockholders investing for income favor a higher ratio. Stockholders looking for capital gains tolerate low ratios when earnings are being reinvested to finance corporate growth. Assume cash dividends of $100,000, net income of $400,000, and outstanding shares of 200,000. The payout ratio equals 25% ($.50/$2.00).

Price Earning (PE) Ratio: Assume XYZ Corporation sells for $25.50 per share and has earned $2.55 per share this year; $25. 50 = 10 times $2.55.

XYZ stock sells for 10 times earnings.

P/E = Current stock price divided by trailing annual earnings per share or expected annual earnings per share.

A steady decrease in the P/E ratio reflects decreasing investor confidence in the growth potential of the entity. Some companies have high P/E multiples reflecting high earnings growth expectations. Young, fast-growing companies often have high P/E stocks with multiples over 20. A company's P/E ratio depends on many factors such as risk, earnings trend, quality of management, industry conditions, and economic factors.

Price Book Value (PBV) Ratio: Price Book value is the ratio of the market value of equity to the book value of equity. (i.e. It is the measure of shareholders' equity in the balance sheet. If we consider price book value ratio and returns on equity, it is not surprising to see firms which have high returns on equity selling for well above book value and firms which have low returns on equity selling at or below book value.

Reinvestment Rate (Risk): The danger that when a debt security matures, possible investments will be at a lower rate of interest. Short-term debt is subject to reinvestment-rate risk. Contrast with interest-rate risk, which is the chance that while an investor holds a debt security, the interest rate will rise, making the investor's security less valuable. Interest-rate risk is associated with long-term debt.

Reinvestment Rate = (Net Capital Expenditures + Change in WC) / EBIT (1-t).

Return on Equity (ROE): A financial analysis tool that measures how well a company generates earnings compared to the amount of capital shareholders have invested in the firm. The formula is net income divided by average common shareholders' equity. If the company has preferred stock, the preferred stock dividends are subtracted from net income before dividing by equity.

Return on stockholders' investments. It is a variant of the DuPont formula, which equals total assets (investment) turnover, times the profit margin, times the equity multiplier:

$$\frac{\text{Income}}{\text{Equity}} = \frac{\text{Revenues}}{\text{Total Assets}} \times \frac{\text{Income}}{\text{Revenues}} \times \frac{\text{Total Assets}}{\text{Equity}}.$$

Return on Investment (ROI): A measure of the earning power of assets. The ratio reveals the firm's profitability on its business operations and thus serves to measure management's effectiveness. It equals net income divided by average total assets; also called rate earned on total assets. Other versions of ROI exist, such as net income before interest and taxes divided by average total assets. Return on investment is a commonly used measure to evaluate divisional performance.

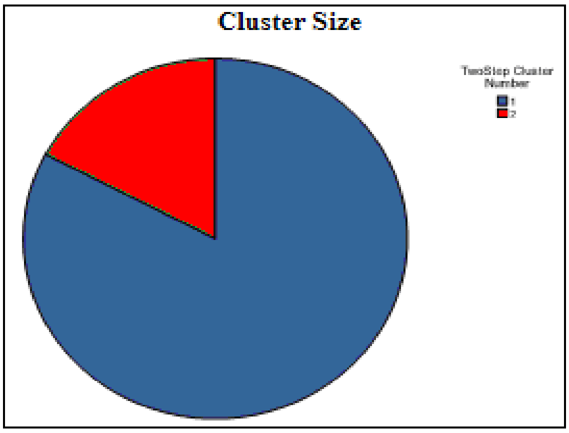
$$ROI = \frac{Net \quad Income}{Invested \quad Capital} = \frac{Net \quad Income}{Sales} \times \frac{Sales}{Invested \quad Capital}$$

Turnover Ratio: Measure of a particular asset's activity (e.g., sales, cost of sales). The average asset balance for the period is used equal to the beginning balance plus the ending balance divided by 2. A turnover ratio is an activity ratio. By looking at the turnover of an asset in terms of generating revenue, the accountant can properly appraise a company's ability to manage assets efficiently. Examples are the turnover in fixed assets (sales/fixed assets), accounts receivable (sales/accounts receivable), and inventory (cost of sales/inventory)

VII. CLUSTERING RESULTS

In this section, we put the results of cluster analysis. Graph VII.1 and VII.2 gives information of cluster percentages and percent rate of industries within clusters for year 2005.

Graph VII.1 Clusters Percentage at 2005



From the following graph, we can easily see that cluster 2 seems to be outlier.

Graph VII.2 Within Cluster Percentage of Industry (2005)

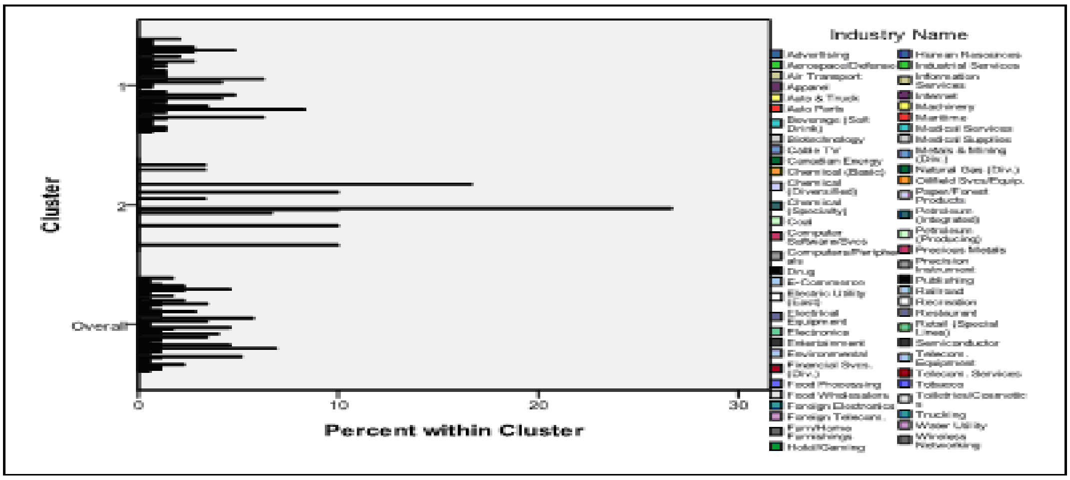


Table VII.1 and VII.2 gives statistical information for Cluster 1 and 2 and union of them at 2005. If one analyze parameters with this information in detail and consider this information together with logical sense, one can classify clusters. We would like to remind clustering does not order clusters from best to worse or vice versa. However, you can order them with analyzing statistical information. The difficulty is to consider all parameters together.

Table VII.1 Cluster Descriptive Statistics (2005)

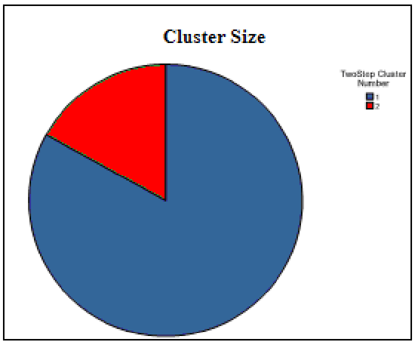
	SP		TV		NSh		TD		PTOM	
2005	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**
Cluster 1	27.72	22.82	341902.94	510713.95	143.69	207.92	861.35	1602.65	0.22	0.21
Cluster 2	59.06	73.81	1377533.50	2334192.59	1458.62	1220.72	14642.38	16347.82	-0.32	3.15
Combined	33.12	38.53	520459.94	1132627.14	370.41	730.53	3237.39	8612.65	0.13	1.32

Table VII.2 Cluster Descriptive Statistics (2005)

	EV/TS		EG		VLB		EBIT		CF	
2005	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**
Cluster 1	2.94	2.56	0.20	0.14	0.80	0.52	603.94	1175.96	0.11	0.14
Cluster 2	3.90	5.25	0.16	0.16	0.91	0.46	10213.53	10198.79	0.09	0.10
Combined	3.10	3.19	0.19	0.14	0.82	0.51	2260.77	5641.96	0.11	0.13

Graphs VII.3 and VII.4 gives information about cluster analysis for year 2006.

Graph VII.3 Clusters Percentage at 2006



Graph VII.4 Within Cluster Percentage (2006)

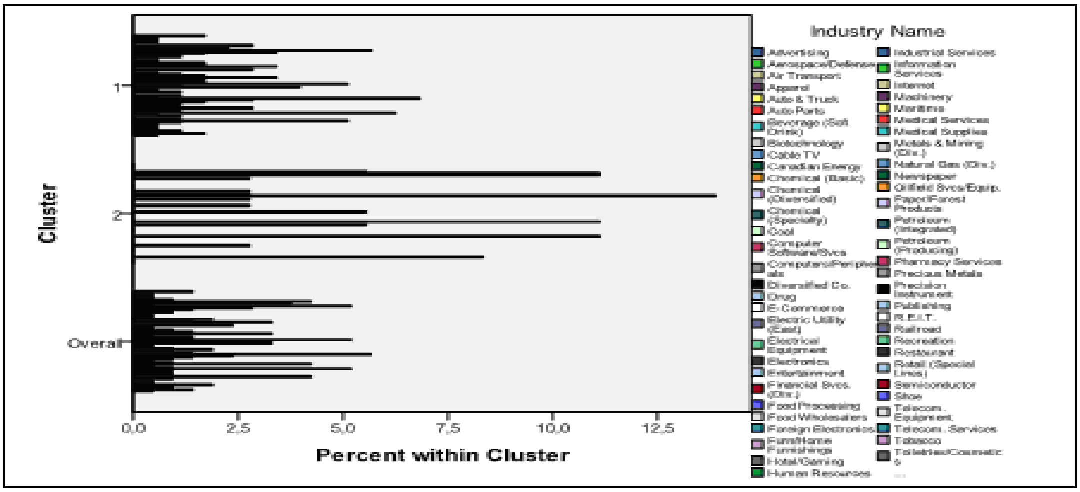


Table VII.3 and VII.4 gives statistical information for Cluster 1 and 2 and union of them at 2006.

Table VII.3 Cluster Descriptive Statistics (2006)

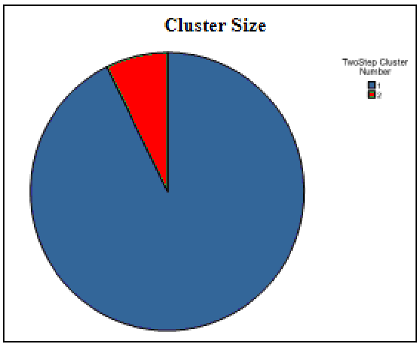
	SP		TV		NSh		TD		PTOM	
2006	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**
Cluster 1	27.34	21.51	298049.91	400533.59	141.56	220.19	923.53	1793.96	0.21	0.21
Cluster 2	56.11	74.44	1400979.83	1672966.15	1561.04	1308.24	14211.20	18135.64	0.09	0.99
Combined	32.22	37.68	485339.90	877278.28	382.60	780.71	3179.93	9068.36	0.19	0.45

Table VII.4 Cluster Descriptive Statistics (2006)

	EV/TS		EG		VLB		EBIT		CF	
2006	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**
Cluster 1	2.77	2.24	0.18	0.12	0.97	0.31	601.72	1190.01	0.12	0.17
Cluster 2	4.39	6.48	0.13	0.11	1.08	0.33	10009.46	11126.69	0.09	0.08
Combined	3.04	3.39	0.17	0.12	0.99	0.31	2199.26	5852.08	0.12	0.16

Graph VII.5 and VII.6 gives information of cluster percentages and percent rate of industries within clusters for year 2007.

Graph VII.5 Cluster Percentage at 2007



Graph VII.6 Within Cluster Percentage (2007)

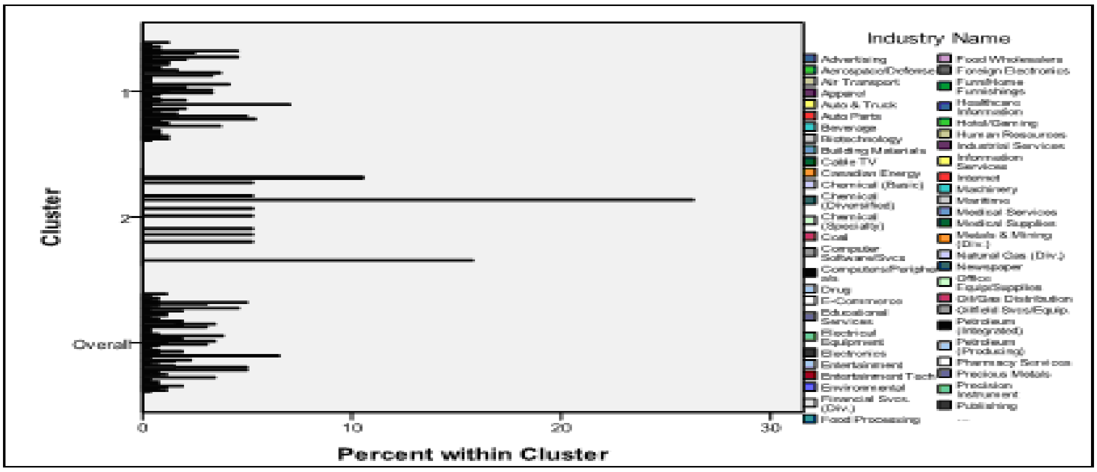


Table VII.5 and VII.6 give statistical information for Cluster 1 and 2 and union of them at 2007.

Table VII.5 Cluster Descriptive Statistics (2007)

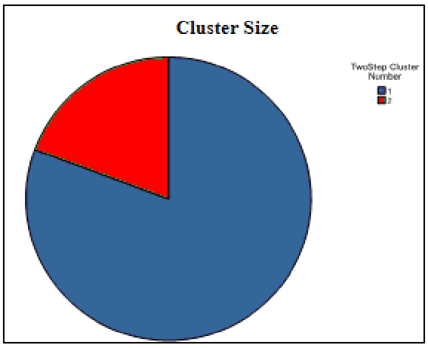
	SP		TV		NSh		TD		PTOM	
2007	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**
Cluster 1	920128.94	1358621.76	198.02	341.08	1294.52	3072.97	0.20	0.24	2.69	2.57
Cluster 2	5895963.79	7201193.80	2390.79	1487.87	24991.33	29176.84	-0.16	1.88	3.21	3.07
Combined	1283747.64	2645369.74	358.26	767.21	3026.21	10300.34	0.17	0.55	2.73	2.61

Table VII.6 Cluster Descriptive Statistics (2007)

	EV/TS		EG		VLB		EBIT		CF	
2007	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**
Cluster 1	0.17	0.10	1.03	0.30	936.87	2025.86	0.13	0.18	26.53	22.34
Cluster 2	0.12	0.09	1.09	0.30	16464.16	13709.88	0.08	0.15	81.25	143.28
Combined	0.16	0.10	1.04	0.30	2071.56	5767.16	0.13	0.18	30.53	45.75

Graph VII.7 and VII.8 give information of cluster percentages and percent rate of industries within clusters for year 2008.

Graph VII.7 Clusters Percentage at 2008



Graph VII.8 Within Cluster Percentage (2008)

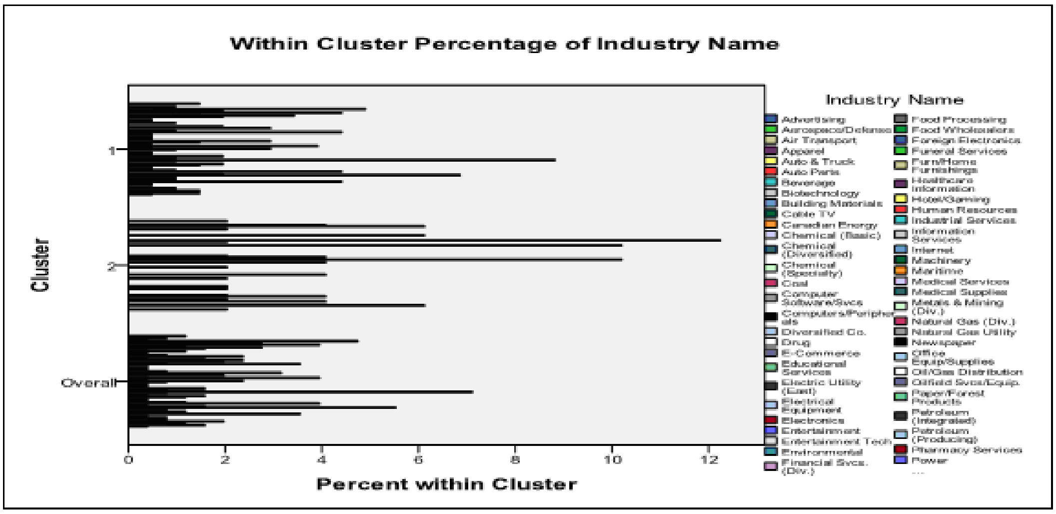


Table VII.7 and VII.8 illustrate statistical information for Cluster 1 and 2 and union of them at 2008.

Table VII.7 Cluster Descriptive Statistics (2008)

	SP		TV		NSh		TD		PTOM	
2008	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**
Cluster 1	14.54	13.79	626904.17	954689.02	187.74	317.72	1331.40	3022.50	0.20	0.18
Cluster 2	31.07	45.28	2727240.92	3495512.96	1266.31	1511.72	11632.65	21675.74	0.17	0.58
Combined	17.74	24.22	1033688.75	1937319.68	396.64	836.06	3326.50	10653.14	0.19	0.30

Table VII.8 Cluster Descriptive Statistics (2008)

	EV/TS		EG		VLB		EBIT		CF	
2008	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**	**Mean**	**St. Dev**
Cluster 1	1.07	1.09	0.15	0.09	1.17	0.37	1000.62	2210.22	0.30	0.45
Cluster 2	2.74	2.97	0.17	0.23	1.19	0.28	8361.19	13239.77	0.11	0.12
Combined	1.39	1.75	0.15	0.13	1.17	0.35	2426.19	6768.93	0.27	0.41

VIII MPD RESULTS OF USA INDUSTRIES

Table VIII.1 illustrates Merton default probabilities of the industries. First, we calculate MPD of each company with solving two equations with two unknowns. Then, taking the average MPD of companies belong to the same industries, we find the MPD of each industry.

In addition, Table VIII.1 illustrates which industries have maximum MPD values. As an extra confirmation, they are nearly same industries which are between clusters.

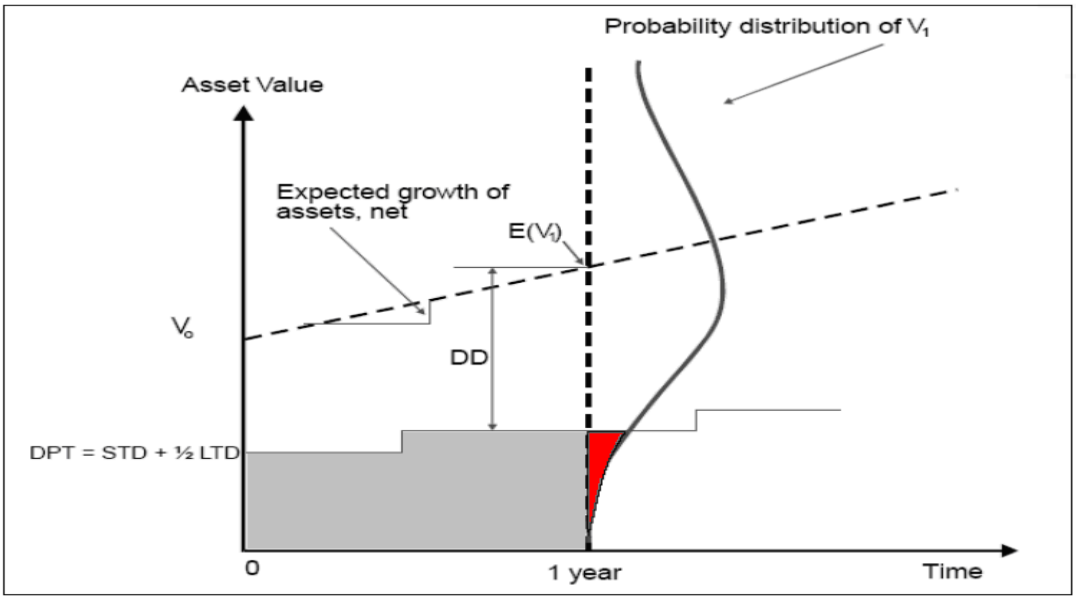
Table VIII.1 Merton Default Probabilities of Industries

Industry Name	MDP-2005	MDP-2006	MDP-2007	MDP-2008
Advertising	0.5000	0.2875	0.7983	0.0082
Aerospace/Defense	0.3333	0.0610	0.3433	0.0000
Air Transport	0.0003	0.0009	0.0256	0.0034
Apparel	0.0092	0.0000	0.0080	0.0029
Auto & Truck	0.2500	0.0000	0.0214	0.0077
Auto Parts	0.1667	0.2189	0.5703	0.3341
Bank	0.0213	0.0002	0.0031	0.0006
Beverage	0.0000	0.0000	0.3371	0.0133
Biotechnology	0.3850	0.1823	0.0475	0.0034
Building Materials	0.3333	0.0000	0.0003	0.0079
Chemical	0.2500	0.0005	0.0000	0.0024
Computer Software/Services	0.1792	0.0178	0.1868	0.0423
Computers/Peripherals	0.0992	0.0000	0.1434	0.0004
Diversified Co.	0.0000	0.0000	0.0067	0.1452
Drug	0.0736	0.0003	0.1151	0.0626
E-Commerce	0.0000	0.0000	0.0107	0.0163
Electrical Equipment	0.1667	0.0001	0.3334	0.0017
Electronics	0.1038	0.2311	0.1278	0.2853
Entertainment Tech	0.3578	0.0002	0.1685	0.1277
Environmental	0.2115	0.0629	0.0914	0.1215
Financial Services. (Div.)	0.0388	0.0000	0.0112	0.0016
Food Processing	0.2000	0.0005	0.0549	0.0079
Industrial Services	0.2280	0.0570	0.1157	0.0882
Insurance	0.0000	0.0000	0.0840	0.0011
Internet	0.2741	0.0203	0.2441	0.1117
Machinery	0.0833	0.0462	0.1821	0.1251
Maritime	0.0000	0.0012	0.0000	0.0136
Medical Services	0.2187	0.4654	0.2394	0.1600
Medical Supplies	0.1562	0.0404	0.2930	0.2972
Metals & Mining (Div.)	0.2409	0.0233	0.3703	0.0526
Oilfield Services/Equip.	0.0000	0.0004	0.1666	0.0013
Petroleum	0.1288	0.0002	0.0326	0.0086
Precious Metals	0.1500	0.0003	0.0024	0.0050
Precision Instrument	0.1831	0.1665	0.3440	0.1250
Recreation	0.1380	0.0001	0.2696	0.0069
Retail Store	0.2000	0.0002	0.2011	0.2192
Semiconductor	0.0001	0.0005	0.0008	0.0127
Telecom. Equipment	0.2063	0.0003	0.3025	0.0507
Telecom. Services	0.2257	0.0002	0.2874	0.0599
Thrift	0.0000	0.0000	0.0000	0.0000
Utility	0.0000	0.0000	0.0000	0.0000
Wireless Networking	0.1429	0.0000	0.0010	0.0000
Maximum	**0.5000**	**0.4654**	**0.7983**	**0.3341**
Second Maximum	**0.3850**	**0.2875**	**0.5703**	**0.2972**
Third Maximum	0.3578	0.2311	0.3703	0.2853
Fourth Maximum	0.3333	0.2189	0.3440	0.2192
Fifth Maximum	0.3333	0.1823	0.3433	0.1600

IX DISTANCE TO DEFAULT

The following figure demonstrates the concept of distance to default. In the figure, STD stands for short-term debt. LTD refers to long-term debt. DPT is default point, which is defined as the sum of STD and 50% of LTD. DD is the abbreviation of distance to default. V_0 refers to the current asset value. V_1 is the expected asset value in one year. The shadow represents probability of default related with the specified DD in the graph.

FIGURE IX.1 DISTANCE TO DEFAULT

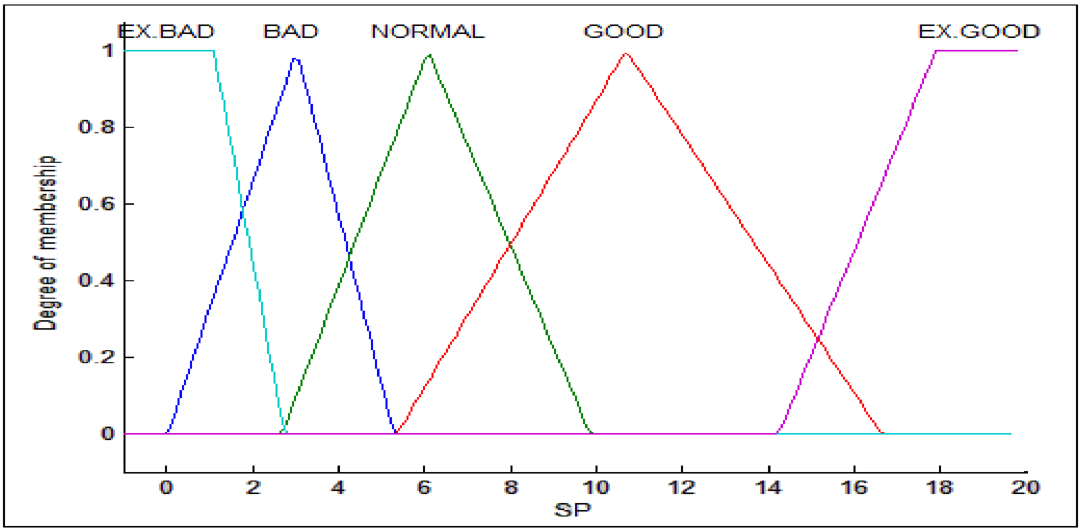


Source: Crouhy. M.. Galai. D., Mark. R., "A comparative analysis of current credit risk models", **Journal of Banking and Finance,** 2000.

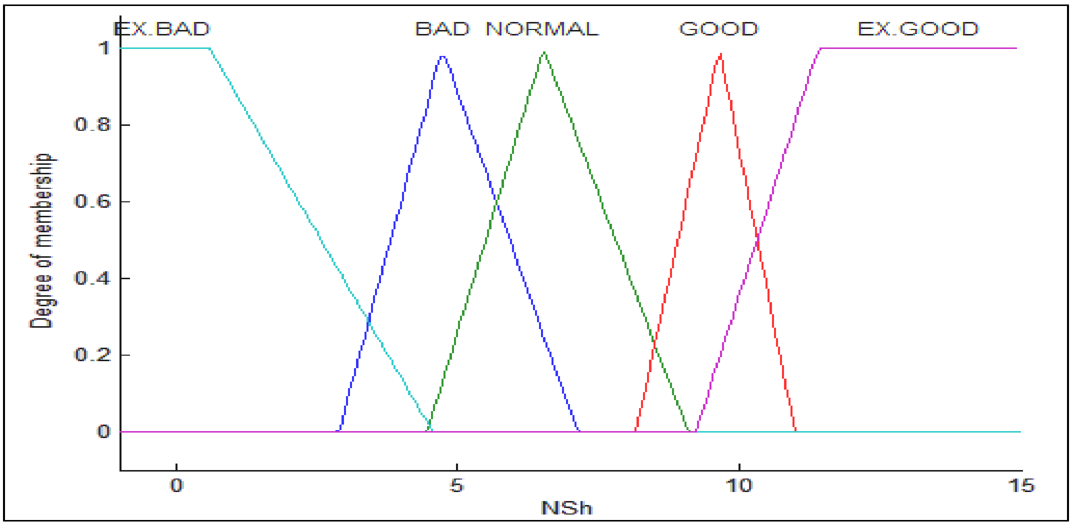
X. MEMBERSHIP FUNCTIONS OF INPUT VARIABLES

As said, in the fuzzy process we have five input variables and following graphs shows membership functions of each input variable. Graph X.1 illustrates membership functions of input variable Stock Price, SP. We have five linguistic variables as extremely bad, bad, normal good and extremely good as seen in the following graph. In addition, following graph also illustrates the boundaries of each membership function. As we defined before, each input variable has five membership functions (extremely bad, bad, normal, good and extremely good).

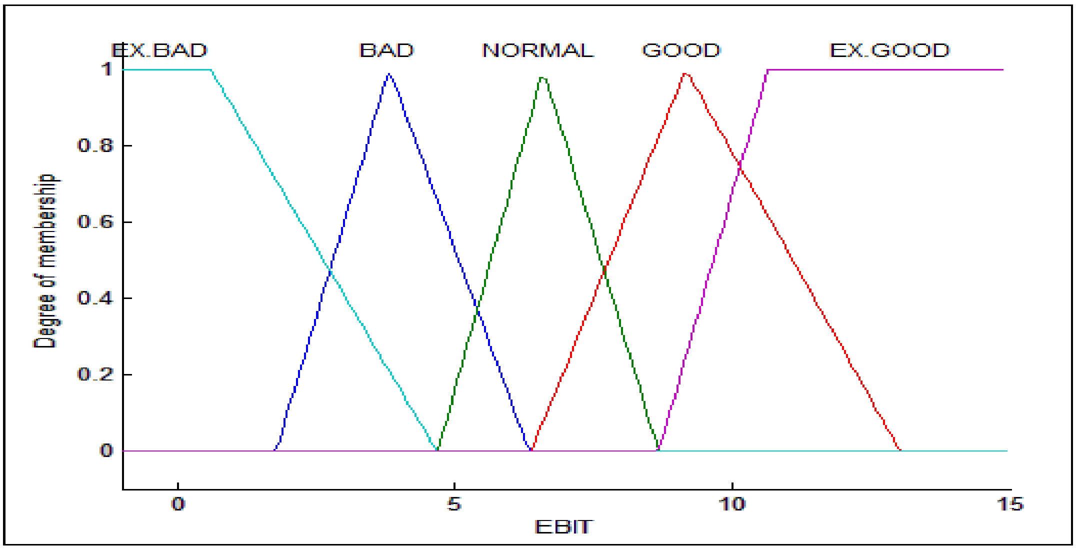
Graph X.1 Membership Function of the Input Variable: SP



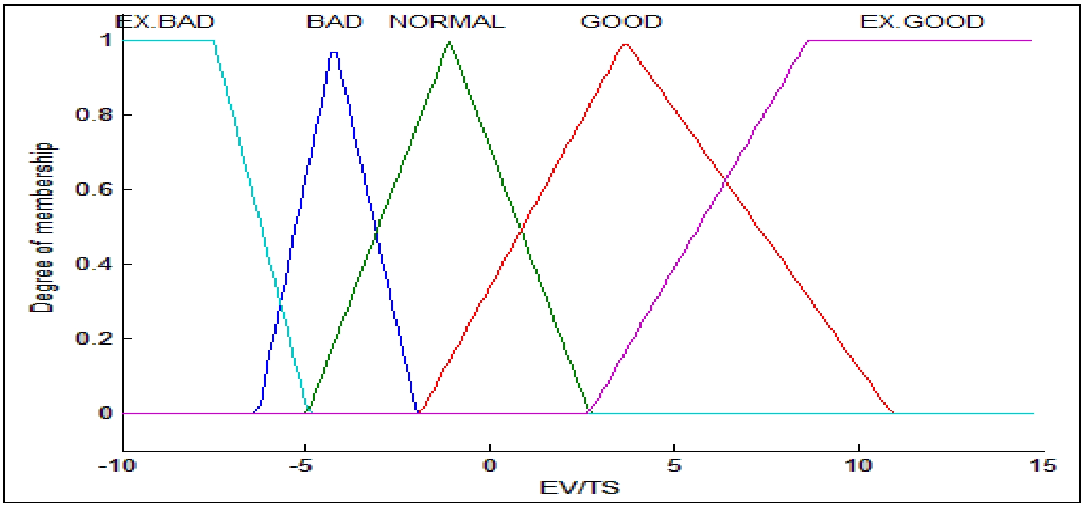
Graph X.2 Membership Function of the Input Variable: NSh



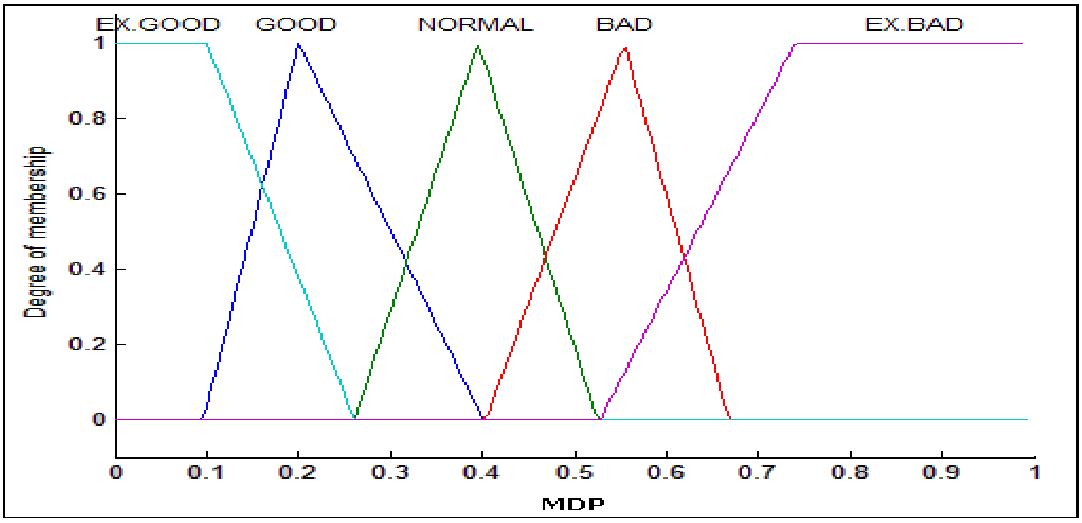
Graph X.3 Membership Function of the Input Variable: EBIT



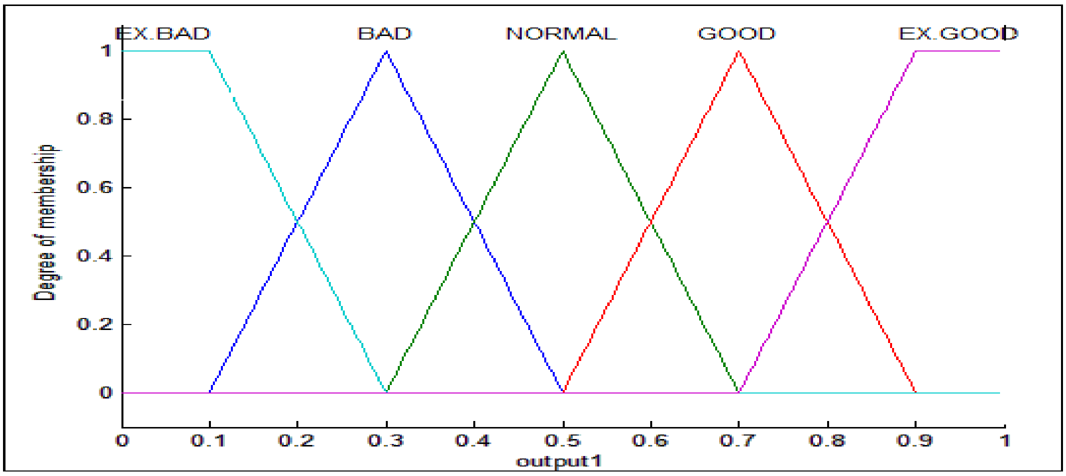
Graph X.4 Membership Function of Input Variable: EV/TS



Graph X.5 Membership Function of the Input Variable: MPD

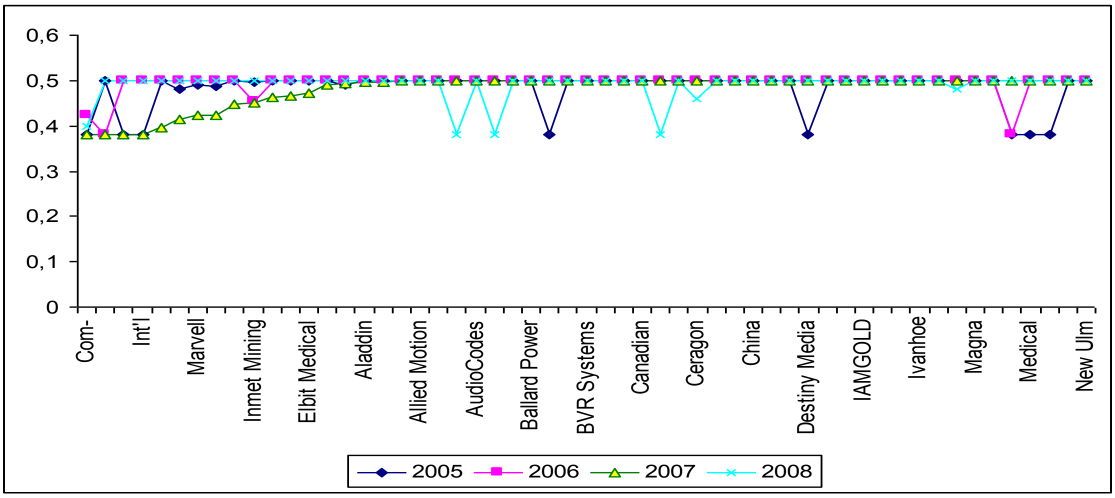


Graph X.6 Membership Function of the Output Variable



In graphs X.1, X.2, X.3, X.4, X.5 and X.6; extremely bad, bad, normal, good, extremely good are functions, which denotes bankruptcy scale. A point on that scale has five "truth values"- one for each of the five functions. We have five membership functions, three of them are triangle and two of them are trapezoids. They all take values between 0 and 1. The five truth values could be interpreted as degree of the bankruptcy.

Graph X.7 Fuzzy Result of Output Variable

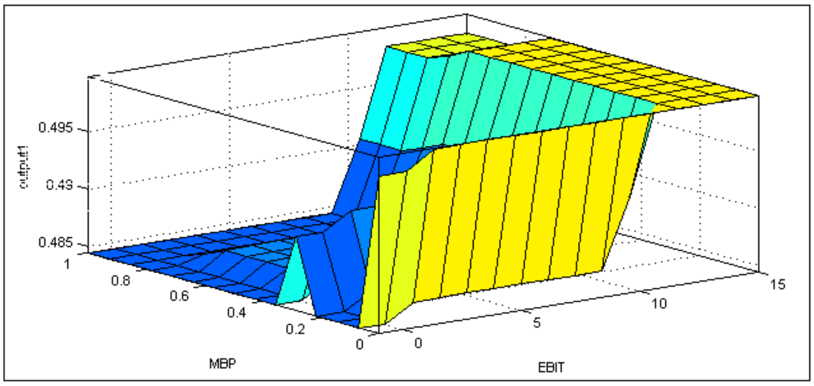


The following table shows the value of fuzzy outputs.

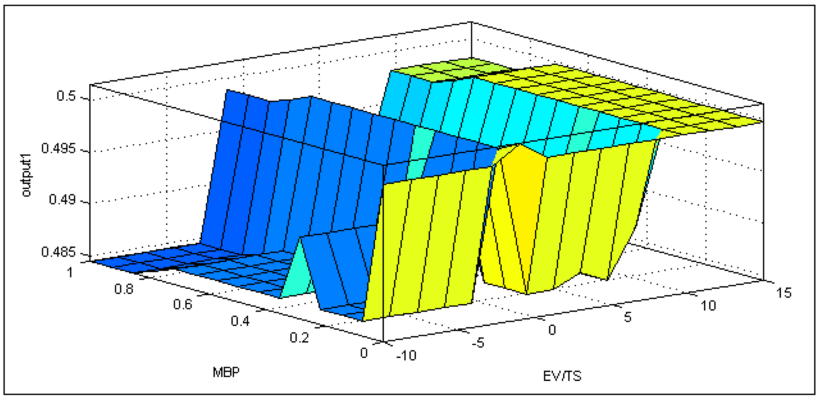
Table X.1 Fuzzy Outputs

Company Name	2005	2006	2007	2008
Com-Guard.com Inc.	0.3814	0.4231	0.3814	0.3979
Environmental Solutions World.	0.5000	0.3814	0.3814	0.5000
Innova Pure Water Inc	0.3814	0.5000	0.3814	0.5000
Int'l Commercial Television	0.3814	0.5000	0.3814	0.5000
CCL Industries Inc.	0.4981	0.5000	0.3961	0.5000
Infosys Technology ADR	0.4810	0.5000	0.4132	0.5000
Marvell Technology	0.4893	0.5000	0.4222	0.5000
Central European Media Enterps.	0.4888	0.5000	0.4238	0.5000
Breakwater Resources	0.5000	0.5000	0.4481	0.5000
Inmet Mining Corp.	0.4979	0.4533	0.4498	0.5000
Compania de Minas Buenaventura	0.4983	0.5000	0.4644	0.5000
Elbit Systems	0.4994	0.5000	0.4660	0.5000
Elbit Medical Imaging Ltd	0.5000	0.5000	0.4736	0.5000
Form Factor Inc.	0.4995	0.5000	0.4917	0.5000
Apco Argentina	0.4908	0.5000	0.4945	0.5000
Aladdin Knowledge	0.5000	0.5000	0.4966	0.5000
Consolidated Water Co Ltd	0.5000	0.5000	0.4966	0.5000
Dorchester Minerals LP	0.4992	0.5000	0.4997	0.5000
Allied Motion Technologies In	0.5000	0.5000	0.5000	0.5000
ATS Automation Tooling Systems	0.5000	0.5000	0.5000	0.5000
Attunity Ltd.	0.5000	0.5000	0.5000	0.3814
Audio Codes Ltd.	0.5000	0.5000	0.5000	0.5000
Axesstel Incorporation	0.5000	0.5000	0.5000	0.3814
B.O.S. Better On Line Solution	0.5000	0.5000	0.5000	0.5000
Ballard Power Sys.	0.5000	0.5000	0.5000	0.5000
Bingo.com Ltd.	0.3814	0.5000	0.5000	0.5000
Biovail Corporation	0.4998	0.5000	0.5000	0.5000
BVR Systems 1998 Ltd	0.5000	0.5000	0.5000	0.5000
Calavo Growers Inc.	0.5000	0.5000	0.5000	0.5000
Camtek Ltd	0.5000	0.5000	0.5000	0.5000
Canadian Superior Energy Inc.	0.5000	0.5000	0.5000	0.5000
Casual Male Retail Group	0.5000	0.5000	0.5000	0.3814
CE Franklin Ltd	0.5000	0.5000	0.5000	0.5000
Ceragon Networks Ltd	0.5000	0.5000	0.5000	0.4590
Certicom Corp	0.5000	0.5000	0.5000	0.5000
CEVA Inc.	0.5000	0.5000	0.5000	0.5000
China Automotive Systems Inc	0.5000	0.5000	0.5000	0.5000
Defense Industries Int'l Inc	0.5000	0.5000	0.5000	0.5000
Delta Galil Industries Ltd	0.5000	0.5000	0.5000	0.5000
Destiny Media Technologies Inc	0.3814	0.5000	0.5000	0.5000
Entrx Corp	0.5000	0.5000	0.5000	0.5000
IAMGOLD Corp.	0.5000	0.5000	0.5000	0.5000
Intertape Polymer Group Inc.	0.5000	0.5000	0.5000	0.5000
Isotechnika Inc	0.5000	0.5000	0.5000	0.5000
Ivanhoe Energy Inc	0.5000	0.5000	0.5000	0.5000
Jacada Ltd.	0.5000	0.5000	0.5000	0.5000
Kirkland Lake Gold Inc	0.5000	0.5000	0.5000	0.4821
Magna Entertainment Corp	0.5000	0.5000	0.5000	0.5000
MDI Inc	0.5000	0.5000	0.5000	0.5000
Med-Emerg International Inc.	0.3814	0.3814	0.5000	0.5000
Medical Nutrition USA Inc	0.3814	0.5000	0.5000	0.5000
Moro Corp	0.3814	0.5000	0.5000	0.5000
Net Sol Technologies Inc	0.5000	0.5000	0.5000	0.5000
New Ulm Telecom Inc	0.5000	0.5000	0.5000	0.5000

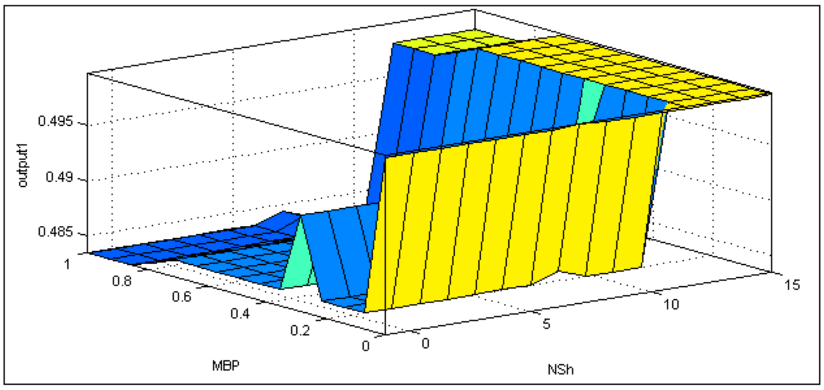
Graph X.8 Surface that Shows Relationship between EBIT, MPD and Output



Graph X.9 Surface that Shows Relationship between EV/TS, MPD and Output



Graph X.10 Surface that Shows Relationship between NSh, MPD and Output



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