FUZZY–SYLLOGISTIC REASONING

A Thesis Submitted to
the Graduate School of Engineering and Sciences of
İzmir Institute of Technology
in Partial Fulfillment of the Requirements for the Degree of
MASTER OF SCIENCE
in Computer Engineering

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July, 2015
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ACKNOWLEDGEMENTS

This thesis is the result of two years continuous work on the syllogistic reasoning during my graduate study in Izmir Institute of Technology. Firstly, I would like to express my gratitude to my thesis advisor Assist. Prof. Dr. Bora İ. KUMOVA for the inspiration and guidance throughout the long process of working on this thesis. I also would like to thank Prof. Dr. Efendi N. NASİBOV, Assist. Prof. Dr. Övgü KINAY, Assist. Prof. Dr. Mustafa ÖZUYSAL and Dr. Engin BÜYÜKAŞIK for their participation as examining committee members.

I am also grateful to all of the Department faculty members for their help and support.

Finally, I would like to thank my family for their support during my study in Turkey.

ABSTRACT

FUZZY – SYLLOGISTIC REASONING

A syllogism is a formal logical scheme used to infer a conclusion from a set of premises. In a categorical syllogism, there are only two premises and each premise and conclusion is given in form a of quantity-quantified relationship between two objects. Different order of objects in premises produce a classification known as syllogistic figures. Ordered combinations of 3 quantifiers with a certain figure, known as moods, provide 256 combinations in total. However, only 25 of them are valid, i.e. conclusion follows from premises. The classical syllogistic system allows to model human thought as reasoning with syllogistic structures. However, a major lack is that there is still no systems that allow to arrive at a decision of syllogisms automatically. This work is an attempt to design a fully algorithmic approach that allows to calculate properties of a whole syllogistic system and provide automated reasoning for given data sets. Since there is a limitation of the classical syllogistic system such as fixed number of crisp quantifiers, advanced fuzzy-quantifiers were introduced to bypass this restriction. Based on the classical syllogistic concept extended by fuzzy-quantifiers, an algorithm for fuzzy-syllogistic reasoning was proposed and integrated into a software system developed for this purpose. Possible applications of syllogistic reasoning, in particular, ontology-based fuzzy-syllogistic reasoning were also discussed.

ÖZET

BULANIK TASIMSAL ÇIKARSAMA

Bir tasım önerme kümelerinden bir sonuç çıkarmak için kullanılan formel bir mantıksal şemadır. Kategorik bir tasımda yalnızca iki adet önerme bulunur ve her bir önerme ve sonuç iki nesne arasındaki nicelik-niceleyici ilişkisinin bir şekli olarak verilidir. Önermelerdeki nesnelerin farklı sıralanışı tasımsal sayılar olarak bilinen bir sınıflandırma üretir. Kip olarak bilinen, 3 niceliğin bir sayı ile birlikte sıralı kombinasyonları 256 adet kombinasyon üretir. Ancak bunların yalnızca 25 tanesi geçerlidir, yani önermelerden doğru sonuç çıkar. Klasik tasım sistemi insan düşüncesinin tasımsal yapılarla çıkarsamasının modellenmesine imkan sağlar. Ancak, çıkarsamaların otomatik olarak sonuca varmasını sağlayan bir sistemin olmaması önemli bir eksikliktir. Bu çalışma bütün bir tasımsal sistemin özelliklerini hesaplamaya izin veren ve verili kümeler için otomatik çıkarsama sağlayan tam algortimik bir yaklaşımın tasarımı için bir girişimdir. Klasik tasımsal sistemde kesin niceleyicilerin belirli bir sayıda olması gibi bir sınırlamayı aşmak için gelişmiş bulanık niceleyiciler önerilmiştir. Klasik tasımsal içeriğin bulanık niceleyicilerle genişletilmesine dayalı bir bulanık tasımsal çıkarsama algoritması ve bu amaçla geliştirilmiş bir yazılım önerilmiştir. Tasımsal çıkarsamaların olası uygulamaları, özellikle ontoloji tabanlı bulanık çıkarsama da ele alınmıştır.

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CHAPTER 1

INTRODUCTION

Reasoning is the ability to make inferences which respectively may be defined as processes of deriving logical conclusions from predefined statements (premises). Reasoning is inseparably linked with the concepts of logic that involves attempts to describe rules or norms by which reasoning operates. The oldest known works associated with the rules of reasoning belongs to the Greek philosopher Aristotle. Despite the fact that Ancient Greeks had no separate logic as an independent science, Aristotle identified logic clearly for the first time as a distinct field of study by introducing the new word, known as "syllogism" (Ward, 2009).

Automated reasoning is an area of computer science and mathematical logic, dedicated to understanding different aspects of reasoning and related to designing algorithms and systems that automate this process. Although the term can be applied to various reasoning tasks, usually automated reasoning is considered with different forms of valid deductive reasoning, like in various applications of automated theorem proving or formal verification.

There are various ways for implementing inference mechanisms in automated reasoning applications, such as inductive, deductive and other types of logical reasoning.

One possible solution may be using approaches based on syllogistic reasoning. A syllogism is a deductive logical scheme used to infer a conclusion from a set of premises. In a categorical syllogism, there are only two premises and each premise and conclusion is given in a form of quantity – quantified relationship between two of 3 possible objects: S, M or P. The S specifies the subject of the conclusion, P is the predicate of the conclusion, and M is the middle term. Furthermore, there are 4 classical syllogistic quantifiers: universal quantifier (All), existential quantifier (Some) and their negations.

Despite the fact that the recently developed many new approaches to old syllogistic problems, there are still many unexplored issues. For example, the classical universal quantifier in the categorical syllogism is too strict because it does not allow for exceptions. However the existential quantifiers is too weak because it quantifies only over at least one object's instance. In daily life reasoning the quantity quantifiers like “most ”, “many ”, or “few ” are used much more often then classical ones, however they are not applicable to classical syllogistic. Such quantifiers that are defined “between” the existential and the universal quantifier are called intermediate quantifiers (Pfeifer et al. 2005). Reasoning with intermediate quantifiers is called fuzzy syllogistic reasoning.

The aim of this thesis is to develop a system, that allows to perform classical and fuzzy – syllogistic reasoning and can be used as a core of various reasoning engines. The current work has been inspired by initial attempts to develop such kind of system, described in (Kumova et al. 2010). However their algorithmic approach for syllogistic reasoning consider just a subset of the full set of possible relationships between 3 objects (P, M and S) in the categorical syllogism. Also any algorithm for generation of those object's relationships is not implemented and no intermediate quantifiers are considered. The current work is an attempt to overcome these limitations and develop a fully algorithmic approach for all steps of the fuzzy – syllogistic reasoning considering the full set of potential object's relationships.

The study also considers the possible applications of a developed system, such as ontology – based syllogistic reasoning and case – based reasoning.

CHAPTER 2

RESEARCH APPROACH

It has been difficult to solve categorical syllogism for people without enough experience in logic. Various external representations of syllogisms, such as Euler and Venn diagrams, linear representation (Englebretsen 1996) and different diagrammatic schemes (in particular, Method of Minimal Representation (Sharma 2013) are traditionally considered as effective tools to support deductive reasoning. Although there is no consensus about the effectiveness of existent representations, experiments provided by (Mineshima et al. 2013), that examined the efficacy of Euler diagram in solving syllogisms, in comparison to sentential reasoning and reasoning with Venn diagrams, show that the performance of the Euler and Venn diagrams was significantly better than the linguistic approaches. (Lemon et al. 1998) claim that linear representation is non-convenient as a representation scheme for general logical inferences. Thus, it seems that representation by using Euler or Venn diagrams is a better option for the modeling of the syllogistic reasoning. Accordingly, Venn diagram representation was chosen as basis in the current work.

In this work, the proposed mathematical model of the syllogistic case is an improved version of the model, described in (Kumova et al. 2010). As it will be shown later, for three symmetrically intersecting sets there are in total 7 possible sub-sets in a Venn diagram, so we have reduced the number of bits needed to encode a syllogistic case from 9 to 7. Also, algorithmic approaches for classical syllogistics will be presented not only for the case of inclusive logic, but for the exclusive logic, too. The exclusive case is particularly interesting as the least studied in modern syllogistics.

After the development of the classical syllogistic system, the fuzzy – syllogistic system as a novel extension of standard system will be proposed and implemented. Our implementation is significantly different from the existing systems, considered in (Peterson 2000, Dubois 1988). Finally, based on the designed system, algorithms for ontology-based fuzzy-syllogistic reasoning will be designed.

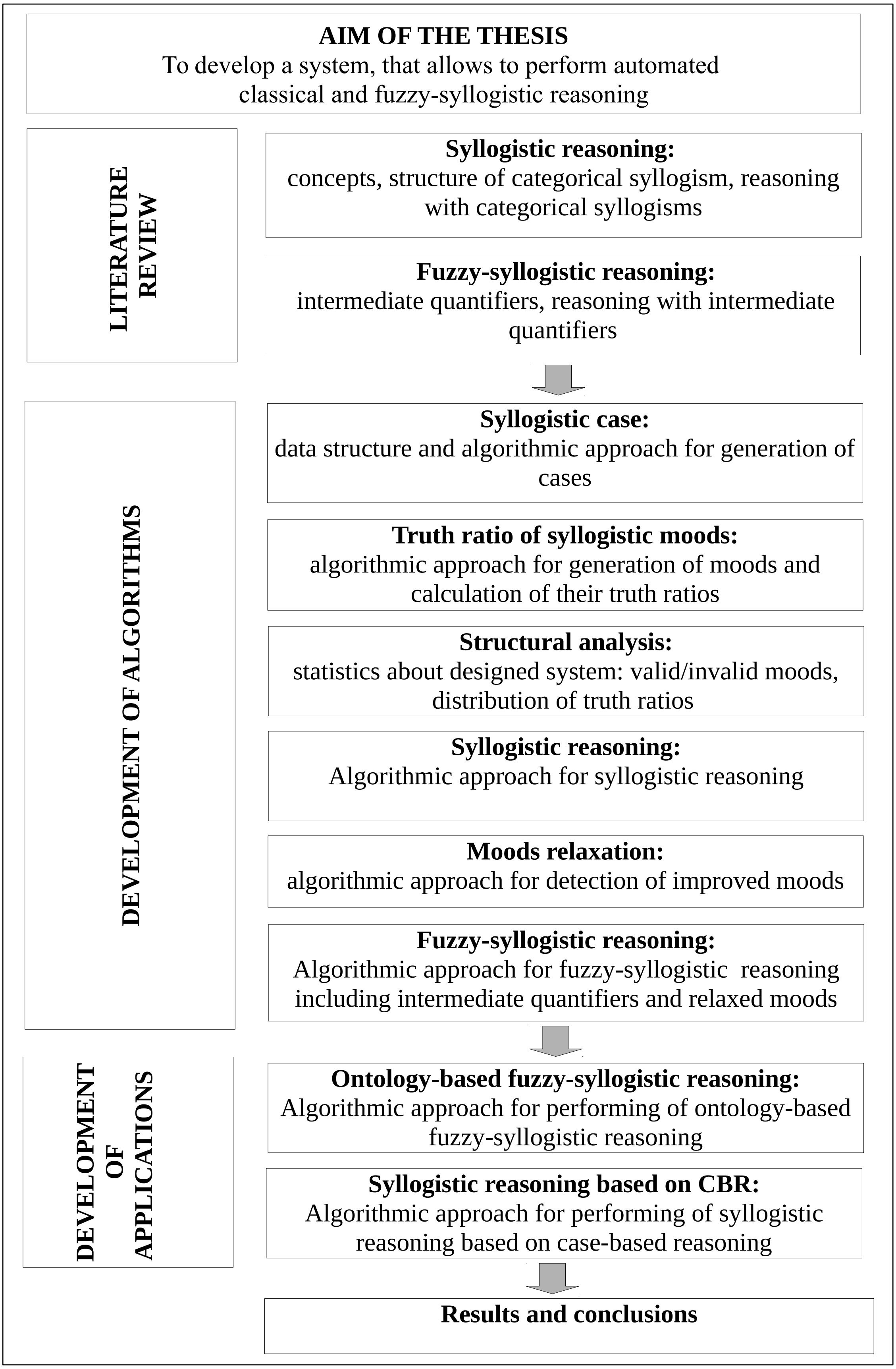


Figure 2.1. Methodological approach

Most of the algorithms mentioned above are implemented within the FSS application. The FSS application includes various statistical information about syllogistic and fuzzy-syllogistic systems, visual representation of syllogistic cases, generator of synthetic data for test purposes and also allows to perform classical and fuzzy-syllogistic reasoning on a given data.

Methodological approach consists of three stages:

- Literature review which includes a literature survey about state of the art of the syllogistic system, such as possible representations of syllogistic reasoning, reasoning in inclusive/exclusive logic and possible applications. Also reasoning with intermediate quantifiers is considered.
- Development of algorithms which includes a description of data structures and ideas of algorithms for all steps of classical and fuzz-syllogistic reasoning with a corresponding pseudo code. During this stage, the FSR_project software system will be developed.
- Development of applications which includes ideas and algorithms for possible applications of a developed system to different problem domains. An implementation of ontology-bases syllogistic reasoning will be proposed . Also a possible application of CBR to current system will be discussed.

A graphical representation of the methodological approach is shown in Fig. 2.1.

The thesis consists of 7 main chapters in addition to appendices. Organization of the chapters follows as:

- Chapters 1-2 consist of the brief introduction includes the motivation of the study and methodological approach that explains the steps on which this thesis was founded;
- Chapter 3 provides a background information about categorical syllogisms, their structure, valid/invalid forms and includes a brief review about current approaches in reasoning with intermediate quantifiers.
- Chapter 4 focuses on algorithmic approaches of implementation of the classical syllogistic system and provide some statistics about it.
- Chapter 5 presents our solution for fuzzy-syllogistic reasoning.
- Chapter 6 considers possible applications of the developed system with proposed solutions for ontology-based fuzzy-syllogistic reasoning and CBR-

based syllogistic reasoning.

- And in the last chapter contributions of this thesis and some limitations of developed system are given.

During development stage of the thesis 2 papers and extended abstract has been accepted in conferences related to artificial intelligence and fuzzy logic:

- Zarechnev, M., Kumova, B.I. (2015). Ontology – based fuzzy – syllogistic reasoning. 28th International Conference on Industrial, Engineering & Other Applications of Applied Intelligent Systems, Seoul, South Korea.
- Zarechnev, M., Kumova, B.İ.(2015). Fuzzy-Syllogistic Reasoning under ontology. Extended abstract. World Congress and School on Universal Logic (UNILOG), June 2015, Istanbul, Turkey.
- Zarechnev, M., Kumova, B.I. (2015). Truth ratios of syllogistic moods**.** Accepted in the 2015 IEEE International Conference on Fuzzy Systems , August 2015 , Istanbul, Turkey, not published yet.

CHAPTER 3

BACKGROUND INFORMATION

The earliest known works related to formal logic belongs to Aristotle. All Aristotle's logic was built on the principle of deduction (sullogismos)(Smith, 2015). In order to understand a deduction and its content it is necessary to investigate Aristotle's whole theory. According to Aristotle, syllogism is “discourse in which, certain things being stated something other than what is stated follows of necessity from their being so” (Aristotle, 1995). But in practice he specified the syllogism as a structure, that contains two premises and a conclusion, each of which is a categorical proposition. The subject and predicate of the conclusion (denoted as S and P respectively) each occur in only one of the premises, together with a third term (the middle, denoted as M) that is found in both premises but not in the conclusion. A syllogism thus argues that because S and P are related in certain ways to M in the premises, they are related in a certain way to one another in the conclusion (see Table 3.3).

In modern syllogistics, the general structure of categorical syllogism can be defined as follow:

$$\begin{array}{c} \psi_1 \, A's \, are \, B's \\ \underline{\psi_2 \, C's \, are \, D's} \\ \psi_3 \, E's \, are \, F's \end{array} \qquad (3.1)$$

where ψ1 , ψ2 and ψ3 are numerical, or more generally, fuzzy quantifiers (e.g. few, many, most (Peterson 1979), and A, B, C, D, E and F are crisp or fuzzy predicates. The predicates A, B, … F are assumed to be related in a specific way, giving rise to different types of syllogisms known as figures which will be explained in advance of this work. (Zadeh 1985).

3.1 Syllogistic Propositions

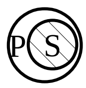
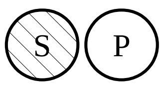
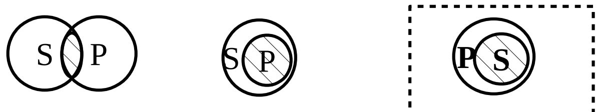
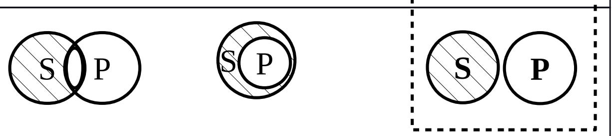
A syllogistic proposition or synonymously, categorical proposition, specifies a quantified relationship between two objects out of S, P or M. We shall denote such relationships with the operator ψ. Four different types are distinguished in the set ψ={A, E, I, O}:

Table 3.1. Syllogistic quantifiers

A	Universal Affirmative	All A are P	E	Universal Negative	All A are not P
I	Particular Affirmative	Some S are P	O	Particular Negative	Some S are not P

The set-theoretical representation of the syllogistic quantifiers by Euler diagrams is presented on Table 3.2. Quantifiers I and O each have three cases respectively. The additional cases, bounded by the dashed line are controversial in the literature. Some do not consider them as valid (Brennan 2007) and some do (Wille 2005). Obviously, the additional case for quantifier I is equal to A, so we can consider A as a special case of I. Analogically, E is a special case of O. We will refer to existential quantifiers I and O that include the additional cases as quantifiers of *inclusive logic*, otherwise as quantifiers of *exclusive logic*. Later on the course of the work, it will be presented that known 24 true moods of the classical categorical syllogism are true only for inclusive existential quantifiers.

Table 3.2. Euler diagrams for syllogistic propositions consist of quantified object relationships (Source: Chakir 2010)

Operator	Proposition	Set – theoretical representation of logical cases
A	All S are P	
E	All S are not P	
I	Some S are P	
O	Some S are not P	

3.2 Square of Opposition

The square of opposition is a diagrammatic representation of the logical relationships holding between certain syllogistic propositions. The concept of the square of opposition formulated by Aristotle in the fourth century BC and considered as first effort to systematize formal symbolic logical inference (Jacquette 2012). The square, traditionally represented as follows (Fig. 3.1):

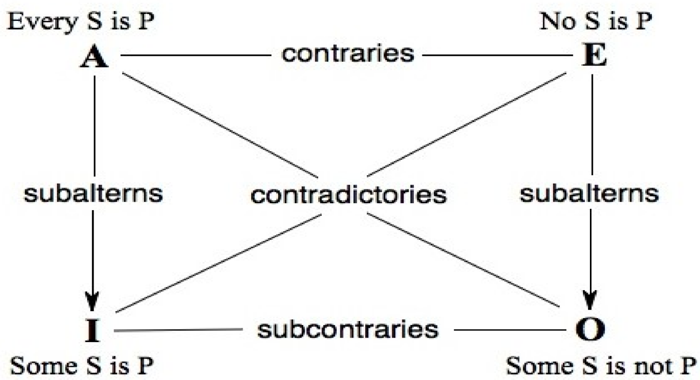


Figure 3.1. Square of opposition
(Source: Parson 2014)

With the assumption that every category contains at least one member, square can be constructed by following statements:

- Pairs of {*"Every S is P"* and *"Some S is not P"*} and {*"No S is P"* and *"Some S is P"*} are contradictories.
- "*Every S is P"* and *"No S is P"* are contraries.
- *"Some S is P"* and *"Some S is not P"* are subcontraries.
- *"Some S is P"* is a subaltern of *"Every S is P"*.
- *"Some S is not P"* is a subaltern of *"No S is P"*.

In terms of logical square of opposition, two propositions are ***contradictory*** if they cannot both be true and they cannot both be false. ***Contrary*** means that two propositions cannot both be true but can both be false. In contrast, two propositions are ***subcontraries*** if they cannot both be false but can both be true. Finally, a proposition is a ***subaltern*** of another if it must be true if its superaltern is true, and the superaltern must be false if the subaltern is false (Parson 2014).

3.3 Syllogistic Figures

Aristotle determined three different figures of syllogisms, according to how the middle term M is related to the other two terms in the premises. In one passage, he says that if one wants to prove S of P syllogistically, one finds a middle M such that either P is predicated of M, and M of S (first figure), M is predicated of both P and S (second figure), or else both P and S are predicated of M (third figure). All syllogisms must fall into one or another of these figures (Spade 2014).

But it is obvious that there is fourth possibility, that P is predicated of M, and S of M. Much later, logicians separated those syllogisms into a different category, now known as fourth figure. Although Aristotle mentioned these syllogisms, he did not group them under a specific figure. Noteworthy that some logicians included these syllogisms under the first figure. However, in modern syllogystics, all four figures are considered.

Table 3.3. Syllogistic figures.

Figure Name	1	2	3	4
Major Premise Minor Premise Conclusion	MψP SψM SψP	PψM SψM SψP	MψP MψS SψP	PψM MψS SψP

3.4 Syllogistic Moods

The mood of a syllogism consists of the three letter names of the propositions that make up that syllogism. For example, mood AAA for Figure 1 denotes the following syllogism:

$$\begin{array}{l} \textit{All M are P} \\ \underline{\textit{All S are M}} \\ \textit{All S are P} \end{array} \qquad (3.2)$$

Since the proposition operator may have four values for ψ, 64 syllogistic moods are possible for every figure and 256 moods for all four figures in total.

3.5 Valid Syllogistic Moods

The vast majority of the 256 possible syllogistic moods are non-valid (the conclusion does not follow logically from the premises). The table below shows the 24 valid forms in case of the inclusive logic.

Table 3.4. Valid syllogistic moods in case of inclusive logic

Figure 1	Figure 2	Figure 3	Figure 4
AAA	AEE	AAI	AEE
AAI	AEO	AII	AEO
AII	AOO	IAI	AAI
EAE	EAE	EAO	EAO
EAO	EAO	OAO	EIO
EIO	EIO	EIO	IAI

Currently, only 24 syllogistic moods are considered as valid. However, using our algorithmic calculation of truth ratios for moods, we have found one further valid mood, which is in the syllogistic figure 4: AAO.

3.6 Examples of Valid/Non-Valid/Invalid Moods

All 24 valid syllogistic moods have mnemonic names, for example, "bArbArA" stands for AAA, "cElArEnt" for EAE, etc.

The most famous example of valid syllogistic mood is BARBARA (AAA-1):

All men are mortal
All Greeks are men (3.3)
All Greeks are mortal

There is only one combination of M, P and S, that satisfies these premises. The corresponding Euler diagram is shown on Fig. 3.2.

One can see that conclusion follows from the premises, so the considered mood is totally valid.

Applying the syllogism above to the mood AAE-1, we get the following syllogism:

All men are mortal — *All M are P*

All Greeks are men — *All S are M* (3.4)

All *Greeks are* ***not*** *mortal* — ***All*** *S* ***are not*** *P*

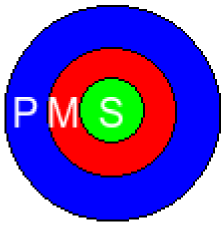


Figure 1:

MψP
SψM
SψP

S (subject): Greeks
P (predicate): Mortal
M (middle): Men

Figure 3.2. Euler diagram for the mood BARBARA-1

There exists only one combination of M, P and S that satisfies premises (see Fig. 3.3).

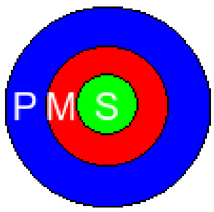


Figure 3.3. Euler diagram for the mood AAE-1

However, conclusion does not follow from the premises (condition in conclusion All S are not P is not satisfied), so we can say that this mood is totally invalid, i.e. it is impossible to find any combination of M, P and S, that satisfies these premises and the conclusion at the same time.

Finally, considering the mood AIA-1, we can construct the following syllogism:

All men are mortal — *All M are P*

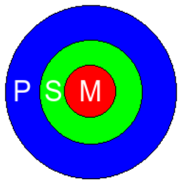
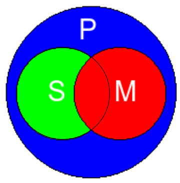
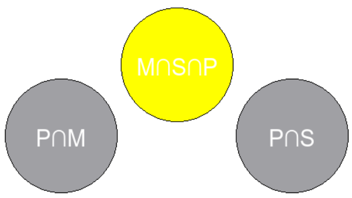
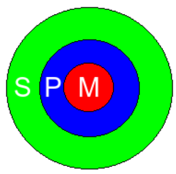
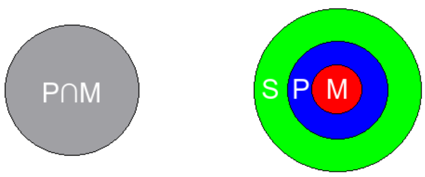
Some *Greeks are men* — ***Some*** *S are M* (3.5)

All Greeks are mortal — *All S are P*

For this mood, there are 9 combinations of M, P and S in total that satisfy the premises (see Table 3.5).

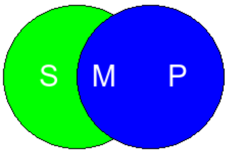
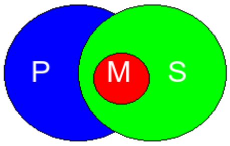
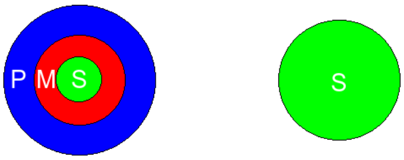
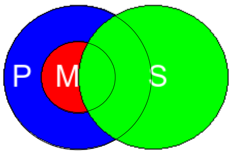
From 9 combinations, 3 of them satisfy the conclusion whereas 6 of them do not, so we can say that this mood is non-valid.

Table 3.5. Euler diagrams for combinations of M, P and S sets that satisfy premises of the syllogistic mood AIA-1

#	Euler diagram	Does conclusion follows from the premises?
1		Yes
2		Yes
3		Yes
4		No
5		No

(Cont. on next page)

Table 3.5. (cont.)

6		No
7		No
8		No
9		No

Manual solving of syllogisms is very time-consuming process, because for certain syllogistic moods there are up to 65 combinations of sets M, P and S, that satisfy premises. So, finding the algorithmic approach for automated decision of syllogisms is the actual problem in the field of natural logic. In the succeeding sections it is going to be proposed a solution to this problem.

3.7 Extensions of the Classical Approach

In recent years various approaches to some essential problems of logic, in particular, to classical syllogistics, were considered in philosophy, linguistics and certain applications of artificial intelligence. However, there are still many unresolved problems in this field, such as issues with weakness and strictness of universal and existensional quantifiers (Pfeifer 2005), reasoning with unlimited numbers of terms, relative quantifiers and so on.

The current approaches to the extension of the syllogistic reasoning can be divided into two independently developing groups:

- approaches, based on introducing new crisp (Peterson 2000) or fuzzy quantifiers (Zadeh 1985, Dubois 1988)(such as much, many, few...), additional to the classical ones.
- approaches, based on the increase of number of terms and premises (N), consisting syllogism (N>2) without introducing new quantifiers (Sommers 1982).

These approaches are briefly discussed in the rest of this chapter.

3.8 Interval Syllogistics: the Dubois's Approach

In a framework, developed by Dubois (Dubois 1993), fuzzy quantifiers, represented by linguistic variables, have interval representation. The example of Dubois's interval syllogism is shown bellow:

[0.05, 0.1] People who have children are single
[0.15, 0.2] People who have children are young (3.6)
[0, 0.1] People who have children are young and single

Given approach is based on calculating of minimum and maximum possible interval values for quantifier in conclusion, considering the intervals, appeared in the premises.

Dubois identified 3 types of quantifiers:

- Imprecise quantifiers: interval quantifiers with the exact boundaries (between 100% and 90% of students get grade AA, less than 10% of students get grade FF , more than 80% of students get grade BB or lower...)
- Precise quantifiers: quantifiers with exact values (48% of students are females)
- Fuzzy quantifiers, interval quantifiers with imprecise boundaries (*most* of students are mails) .

The imprecise or precise quantifiers in conclusion are modeled as interval function, fuzzy quantifiers in conclusion are represented by trapezoidal function. The quantifiers in premises are restrictions for the concluding quantifier. So, calculation of the concluding quantifier is an optimization problem and consists in finding the best and worst proportions among the terms of the conclusion according to the corresponding proportions of the terms in the premises. Dubois proposed three syllogistic patterns, two of which (Pattern II and Pattern III) extends the Aristotelian inference scheme. The Pattern I can be used to reproduce the syllogistic figures like in the classical approach but, as shown in (Pereira-Fariña et al. 2014), it is compatible only with Figure 1.

3.9 Fuzzy Syllogism: Zadeh's Approach

As was mentioned before, Zadeh's scheme for general syllogism looks as follows:

$$\frac{\begin{array}{c} Q_1\, A's\, are\, B's \\ Q_2\, C's\, are\, D's \end{array}}{Q\, E's\, are\, F's} \tag{3.7}$$

where Q_1 , Q_2 and Q are fuzzy quantifiers (most, many, some ...) and A, B, C, D, E and F are interrelated fuzzy properties or terms. Zadeh proposed different patterns, based on this general scheme, but with different number of constraints among the terms or fuzzy properties.

Zadeh's model consists of two approaches: the fuzzy-quantification framework and the theory of inferencing.

In the fuzzy-quantification approach, Zadeh defined two types of linguistic quantifiers: absolute (for instance, almost 10, ...) and proportional (for instance, more than half,...). Absolute quantifiers denote absolute quantities (in our case, 10), proportional quantifiers refer to a proportion of cardinalities of elements in the corresponding sets.

Zadeh interpreted absolute and proportional linguistic quantifiers as absolutely or proportional fuzzy-numbers respectively.

The procedure proposed for combining fuzzy numbers with the corresponding fuzzy sets represented in the properties of Σ-count scalar cardinality for fuzzy sets (Zadeh 1983). Σ-count is a simple scalar-valued measure, that helps to replace the intersection and cardinality operations for crisp sets with corresponding fuzzy-operations. Intersection is replaced with a T – norm (usually, min) and cardinality with a scalar-valued cardinality. Thus, Σ-count is the sum of membership values of the elements in the fuzzy sets (Fuzzy Sets and Systems 2003).

In the theory of inference, Zadeh's approach manages the usual quantified statements "Q A's are B's", where Q is a proportional fuzzy number (equivalent to the corresponding linguistic proportional quantifier) and A and B are (fuzzy or crisp) sets.

The inference process is based on quantifier extension principle (QEP), which establishes that:

$$if\ C = f\,(P_1;\ P_2;\ \ldots\ ;\ P_n\,),\ then\ Q = \varphi_f\,(Q_1;\ Q_2;\ \ldots\ ;\ Q_n\,), \qquad (3.8)$$

where C is the conclusion, P_1; P_2; ... ; P_n are the premises, *f* is a function, *Q* is the quantifier of the conclusion, $Q_1;\ Q_2;\ \ldots\ ;\ Q_n$ are the quantifiers of the premises and φ_f is an extension of *f* obtained using the extension principle. The main idea of QEP is to apply the extension principle to *f* to obtain a fuzzy function φ_f that can be directly applied to the corresponding fuzzy numbers. Since fuzzy numbers are used in the calculations, the corresponding arithmetic operations must be performed using fuzzy arithmetic.

3.10 Intermediate Quantifiers: Peterson's Approach and Extensions

The initial detailed conception of intermediate quantifiers was proposed by Peterson (Peterson 1979). Peterson introduced the first version of complete square of opposition with intermediate quantifiers, which was a generalization of the classical square of opposition. In his approach, Peterson considered new quantifiers, such Almost_All, Many and their complements. Thomson (Thomson 1982) extended this approach with the intermediate quantifier Most and introduced a complete square of opposition with contradictions, contraries and subalterns as presented in Fig. 3.4.

Pairs of statements A and E, P and B, T and D compose *contrary pairs* (denoted with the dashed lines), so for each pair, statements in which can not be both true at the same time, however they can be both false (i.e. if the statement A is true (All B are A), statement E (No B are A) cannot be true). Statements K and G, and I and O compose *sub-contrary pairs* (denoted with the dotted lines), so statements in pair can be true simultaneously, so situations where I and O both are true at the same time, are allowed.

Like in classical square of opposition, in Peterson's square pairs of statements A-O, I-E, P-G, and K-B are *contradictory statements*. It means if one statement in pair is true, the other one is false or otherwise.

Peterson noticed the *immediate entailment* of statements. As a consequence, we can show the following valid implications: A→P→T→K→I and E→B→D→G→O. If the statement A is true, it means that statement P is also true, in the same way, due the true P statement, T statement is also true.

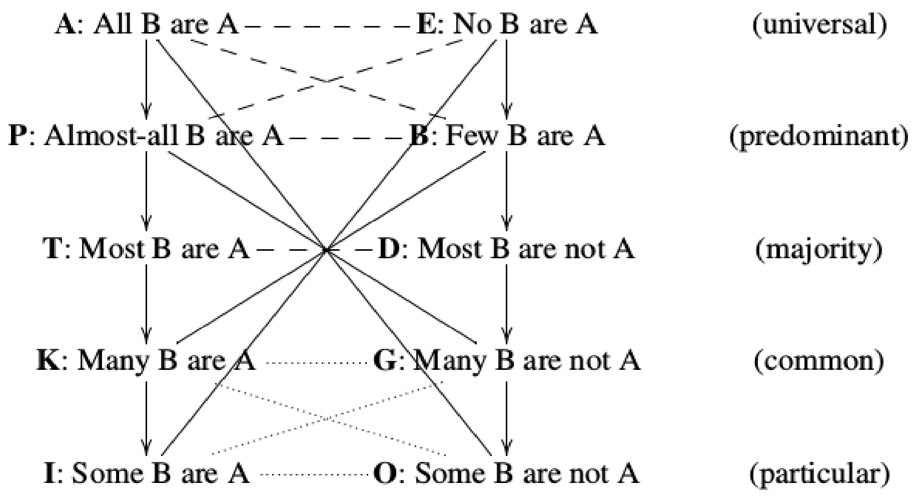


Figure 3.4. Complete square of opposition
(Source: Peterson 2000)

It should be noted that the implications are valid in the given directions only, i.e. from universal term to particular (i.e. in case of affirmative statements, from A to I statements, if statement T is true, it does not mean that P is also true). It is very close to the semantic meanings of the statements: statement A is stronger than P or I, so if A is true, then P, T, K, I must also be true (Turunen 2014).

CHAPTER 4

SYLLOGISTIC SYSTEM

In this chapter algorithmic approaches of implementation of the classical syllogistic system will be considered.

4.1 Algorithmic Decision of Syllogistic Moods

In this section the all steps of algorithmic decision of syllogistic moods with the appropriate data structures will be discussed.

4.1.1 Set – Theoretical Analysis

A Venn diagram for three sets includes 8 subsets which are composed of some combination of inclusions or exclusions of these sets, making the whole system (Venn 1880) (see Fig 4.1 a). We do not consider the complement of [$(M \cup S \cup P)$ in $U] = U \setminus (M \cup S \cup P)$, because only the relations between the three sets, representing syllogistic statements, are important in our model. Thus, for three intersecting sets there are 7 possible subsets in a Venn diagram. If intersections of sets are relaxed and the three sets are labeled as P, M and S, then 109 set relationships are possible (Kumova unp.). At least, one subset of each set is assumed to be non-empty. Excluding sets combinations where at least 2 sets are equivalent finally we end up with 96 set combinations. These 96 relationships are distinct, but re-occur in the 256 moods as basic syllogistic cases. The 7 subsets with the 96 distinct set relationships in case of relaxed symmetry are fundamental to the design of an algorithmic decision of syllogistic moods.

4.1.2 Data Structure for Case Representation

Based on these 7 sub-sets, we have proposed a data - structure for modeling of the syllogistic cases. Each case is presented as a sequence of 7 bits. Each bit is related with a particular subset in a Venn–diagram (see Table 4.1). This structure is efficient in processing and memory consumption: 7 bits are minimal number that allows to recover fully the corresponding Venn – diagram.

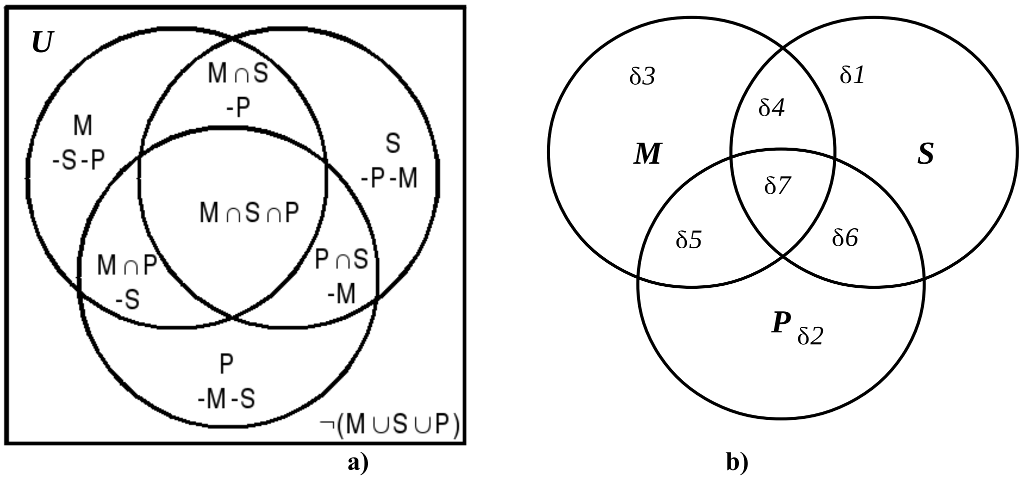


Figure 4.1. Venn – diagram for 3 symmetrically intersecting sets.

Table 4.1. Identification of the seven possible subsets of three sets as distinct spaces

Space ID	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7
Subset	S-(P+M)	P-(M+S)	M-(S+P)	(M∩S)-P	(M∩P)-S	(S∩P)-M	M∩S∩P

4.1.3 Generation of Cases

The efficient way of case generation is using of recursive procedure, with recursion depth is equivalent to number of subsets. On each level of recursion, corresponding bit is added to the described data structure, represented by a dynamical array of bits. On the last level the verification procedure is performed, and if current case passed the verification, it is added to the resulting set of cases.

In our approach the goal is to generate all possible combinations (cases) from 7 elements, where elements are from {0; 1} (the total number of such of permutations is 2^7=128).

4.1.4 Evaluation of Cases

Each case from a resulting set must contain at least one non-zero subset from each of M, P, S sets, and any of the sets (M, P or S) must not be equivalent to each other.

Obviously, for a particular case, if at least one bit from *{δ1, δ4, δ6, δ7}* (subsets, that built-up set M, see Fig.1 b) is set to 1, this case contains elements from M. It gives us a simple criterion for case evaluation:

$$\{cons\_M\}: \forall \Delta_i \text{ from } Cases[128]: \exists \Delta_i: \Delta_{\delta 1} \vee \Delta_{\delta 4} \vee \Delta_{\delta 6} \vee \Delta_{\delta 7} \rightarrow \Delta_i \subset M. \quad (4.1)$$

In the same way:

$$\{cons\_P\}: \forall \Delta_i \text{ from } Cases[128]: \exists \Delta_i: \Delta_{\delta 2} \vee \Delta_{\delta 5} \vee \Delta_{\delta 6} \vee \Delta_{\delta 7} \rightarrow \Delta_i \subset P. \quad (4.2)$$

$$\{cons\_S\}: \forall \Delta_i \text{ from } Cases[128]: \exists \Delta_i: \Delta_{\delta 3} \vee \Delta_{\delta 4} \vee \Delta_{\delta 5} \vee \Delta_{\delta 7} \rightarrow \Delta_i \subset S. \quad (4.3)$$

Applying given criteria at the same time, we obtain a set of 109 cases, which consists of at least one non-empty subset of each S, M and P simultaneously. However, resulting set includes degenerate cases, such as M=P=S (0000001) or M=P etc. To exclude equivalent sets we propose next criterion to evaluate equivalent sets (two sets are equivalent if they have non-empty intersection and empty compliments):

$$\begin{aligned}
&\{S_eq_P\}: \forall \Delta_i \text{ from } Cases[109]: \exists \Delta_i: (\Delta_{\delta 6} \vee \Delta_{\delta 7}) \wedge \neg \Delta_{\delta 1} \wedge \neg \Delta_{\delta 2} \rightarrow \Delta_{i\,[S=P]} \\
&\{P_eq_M\}: \forall \Delta_i \text{ from } Cases[109]: \exists \Delta_i: (\Delta_{\delta 5} \vee \Delta_{\delta 7}) \wedge \neg \Delta_{\delta 2} \wedge \neg \Delta_{\delta 3} \rightarrow \Delta_{i\,[P=M]} \quad (4.4) \\
&\{M_eq_S\}: \forall \Delta_i \text{ from } Cases[109]: \exists \Delta_i: (\Delta_{\delta 4} \vee \Delta_{\delta 7}) \wedge \neg \Delta_{\delta 1} \wedge \neg \Delta_{\delta 3} \rightarrow \Delta_{i\,[M=S]}
\end{aligned}$$

The final set consist of 96 cases Σ=[1,96] (see Appendix A) which are essential data for deciding of syllogistic moods. Euler diagrams for the all cases are listed in Appendix B. Pseudo-code for case-generation is presented in Listing 4.1.

```
generateCases(Δ, rec_depth)
{
      for (int i = 0; i<2; i++)
      {
            Δ_rec_depth= i;           //set correspondent bit to i
      if (rec_depth == 0)   //stop recursion
            {
                  if (cons_S(Δ) ∧ cons_M(Δ) ∧ cons_P(Δ))
                  {
                  //need to exclude equivalent sets
                        //(checking all possible combinations)

                  if not([(S_eq_P(Δ))∧(P_eq_S(Δ))]∨
                        [(S_eq_M(Δ))∧(M_eq_S(Δ))] ∨
                        [(M_eq_P(Δ))∧(P_eq_M(Δ))])

                        append_current_Δ_to_Σ ;
            }
            } else generateCases(Δ, rec_depth - 1)
      }
}
```

Listing 4.1. Procedure of cases-generation

4.1.5 Verification Rules

The basic idea of the algorithm for determining the true and false cases of a given mood is based on selecting the possible set relationships that satisfy premises for considering mood and splitting resulting set into 2 sets of true and false cases according to the conclusion meaning, out of all 96 possible set relationships.

As was discussed above, each mood is presented in form of triple syllogistic quantifiers {A, E, I, O} together with the number of related figure, and total number of moods for all figures is 256.

The validation process includes the validation of each of 3 syllogistic statements (2 premises and conclusion). Since there are 4 syllogistic quantifiers, 4 possible situations for validation functions are possible:

```
All_Are(set1, set2, Δ);
All_AreNot(set1, set2, Δ);                    (4.5)
Some_Are(set1, set2, Δ);
Some_AreNot(set1, set2, Δ);
```

where set1 and set2 represent the sets from {M, P, S}, and Δ is the particular case from Σ=[1,96].

Considering Venn-diagrams for syllogistic propositions (see Table 4.2), we can construct conditions for each quantifier:

- If all elements of first set belong to second set (case All_Are), the all elements of first set must be in intersection of two sets[1]
- If all element of first set is different from second set (case All_AreNot), the intersection of two sets must be empty;
- If some elements of first set belongs to second set (case SomeAre), the intersection of two sets must be non-empty. Additionally, the complement of first set in second set must be non-empty in case of exclusive logic, or may be empty in case of inclusive logic (see Table 4.2, mentioned sets are bounded by dashed line).
- If some elements of first set do not belong to second set (case SomeAreNot), the complement of first set in second set must be non-empty. Additionally, intersection of two sets must be non-empty in case of exclusive logic, or can be empty for inclusive logic.

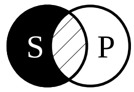
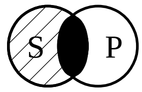
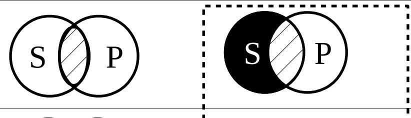
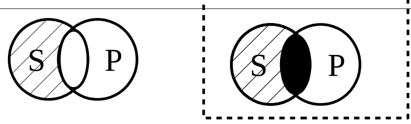
Thereby, the criteria for quantifier A (`All_Are()`) is very similar to the one described in the previous section for equivalent cases. For example, for sets S, P and given case Δ from Σ=[1,96], the pseudo-code of verification function is presented in Listing 4.2.

```
boolean AllAre(S, P, Δ)
{
      return [(Δδ6∨Δδ7)∧¬Δδ1∧ ¬Δδ4]
}
```

Listing 4.2. Verification function for quantifier A and sets S and P

1 Subset filled with black on Venn-diagrams denotes empty subset

Table 4.2. Venn-diagrams for syllogistic propositions

Operator ψ	Venn-diagram for logical cases	Conditions	
		Exclusive logic	Inclusive logic
A		S\P = Ø S∩P ≠ Ø	
E		S\P ≠ Ø S∩P = Ø	
I		S\P ≠ Ø S∩P ≠ Ø	S∩P ≠ Ø
O		S\P ≠ Ø S∩P ≠ Ø	S\P ≠ Ø

The verification function for quantifier E for sets S, P and given case Δ from Σ=[1,96] is presented in Listing 4.3.

```
boolean All_AreNot(S, P, Δ)
{
      return [¬(Δ_δ6 ∨ Δ_δ7)]
}
```

Listing 4.3. Verification procedure for quantifier E and sets S and P

It is clearly from Table 2, that conditions for verification quantifiers I and O is identical in case of exclusive logic. At this point for sets S, P and given case Δ from Σ=[1,96] we can write following verification code (see Listing 4.4):

```
boolean SomeAre_SomeAreNot_Excl(S, P, Δ)
{
      return [(Δ_δ6 ∨ Δ_δ1) ∨ (Δ_δ6 ∨ Δ_δ4) ∨ (Δ_δ7 ∨ Δ_δ1) ∨ (Δ_δ7 ∨ Δ_δ4)]
}
```

Listing 4.4. Verification function for quantifiers I and O and sets S and P
in case of exclusive logic

It should be noted again that the described procedure for quantifiers I and O is valid only in case of exclusive logic. Since our purpose is to develop a universal approach, we can define the type of used logic optionally and implement the quantifiers as below by

```
boolean SomeAre(set1, set2, Δ)
{
      if (SomeAre_SomeAreNot_Excl(set1, set2, Δ)) return true;
      if (use_inclusive_logic)    //need to consider additional
                                  //case for inclusive logic
      {
      return AllAre(set1, set2, Δ);
      }
      return false;
}

boolean SomeAreNot(set1, set2, Δ)
{
      if (SomeAre_SomeAreNot(set1, set2, Δ)) return true;
      if (use_inclusive_logic)
      {
      return AllAreNot(set1,set2, Δ);
      }
      return false;
}
```

Listing 4.5. Generalized verification functions for quantifiers I and O and sets S and P

Conditions for all possible set relationships are summarized in Table 4.3.

4.1.6 Algorithmic Decision of Mood

After implementation of verification functions we can propose a general algorithm for calculating truth/false cases for a given mood Δ (see Fig. 4.2):

INPUT: mood μ: sequence of 3 quantifiers from {A, I, E, O} and number of the figure which corresponds to this mood

OUTPUT: TC_Δ, FC_Δ: 2 lists, containing true and false cases for given mood respectively.

ALGORITHM:

1. **GENERATE 96 possible set combinations** with 7 relationships into a list of cases $\Sigma[96]$
2. **VALIDATE premission_1** of given mood with AllAre(), AllAreNot(), SomeAre(), SomeAreNot() according to the quantifier under all cases from Σ;
3. Construct list of cases Σ_1, that satisfy premise_1
4. **VALIDATE premission_2** of given mood with AllAre(), AllAreNot(), SomeAre(), SomeAreNot() according to the quantifier under cases from Σ_1; construct list of cases Σ_2 , that satisfy premise_1 and premise_2.
5. **VALIDATE conclusion** of given mood with AllAre(), AllAreNot(), SomeAre(), SomeAreNot() according to the quantifier for cases from Σ_2; split Σ_2 to TC_Δ and FC_Δ, according to the meaning of conclusion: if particular case satisfies conclusion, then add this case into TC_Δ, otherwise into FC_Δ.

Table 4.3. Verification rules for quantifiers A, E, I, O

Sets Relationship	A	E	I, O		
			Exclusive logic, (I and O)	Inclusive logic, I	Inclusive logic, O
MP	$(\Delta_{\delta5} \vee \Delta_{\delta7}) \wedge \neg\Delta_{\delta3} \wedge \neg\Delta_{\delta4}$	$\neg\Delta_{\delta5} \wedge \neg\Delta_{\delta7}$	$(\Delta_{\delta5} \wedge \Delta_{\delta3}) \vee (\Delta_{\delta5} \wedge \Delta_{\delta4}) \vee (\Delta_{\delta7} \wedge \Delta_{\delta2}) \vee (\Delta_{\delta7} \wedge \Delta_{\delta4})$	+ for A	+for E
PM	$(\Delta_{\delta5} \vee \Delta_{\delta7}) \wedge \neg\Delta_{\delta2} \wedge \neg\Delta_{\delta6}$		$(\Delta_{\delta5} \wedge \Delta_{\delta2}) \vee (\Delta_{\delta5} \wedge \Delta_{\delta6}) \vee (\Delta_{\delta7} \wedge \Delta_{\delta2}) \vee (\Delta_{\delta7} \wedge \Delta_{\delta6})$	+ for A	+for E
SM	$(\Delta_{\delta4} \vee \Delta_{\delta7}) \wedge \neg\Delta_{\delta1} \wedge \neg\Delta_{\delta6}$	$\neg\Delta_{\delta4} \wedge \neg\Delta_{\delta7}$	$(\Delta_{\delta4} \wedge \Delta_{\delta1}) \vee (\Delta_{\delta4} \wedge \Delta_{\delta6}) \vee (\Delta_{\delta7} \wedge \Delta_{\delta1}) \vee (\Delta_{\delta7} \wedge \Delta_{\delta6})$	+ for A	+for E
MS	$(\Delta_{\delta4} \vee \Delta_{\delta7}) \wedge \neg\Delta_{\delta3} \wedge \neg\Delta_{\delta5}$		$(\Delta_{\delta4} \wedge \Delta_{\delta3}) \vee (\Delta_{\delta4} \wedge \Delta_{\delta5}) \vee (\Delta_{\delta7} \wedge \Delta_{\delta3}) \vee (\Delta_{\delta7} \wedge \Delta_{\delta5})$	+ for A	+for E
SP	$(\Delta_{\delta6} \vee \Delta_{\delta7}) \wedge \neg\Delta_{\delta1} \wedge \neg\Delta_{\delta4}$	$\neg\Delta_{\delta6} \wedge \neg\Delta_{\delta7}$	$(\Delta_{\delta6} \wedge \Delta_{\delta1}) \vee (\Delta_{\delta6} \wedge \Delta_{\delta4}) \vee (\Delta_{\delta7} \wedge \Delta_{\delta1}) \vee (\Delta_{\delta7} \wedge \Delta_{\delta4})$	+ for A	+for E

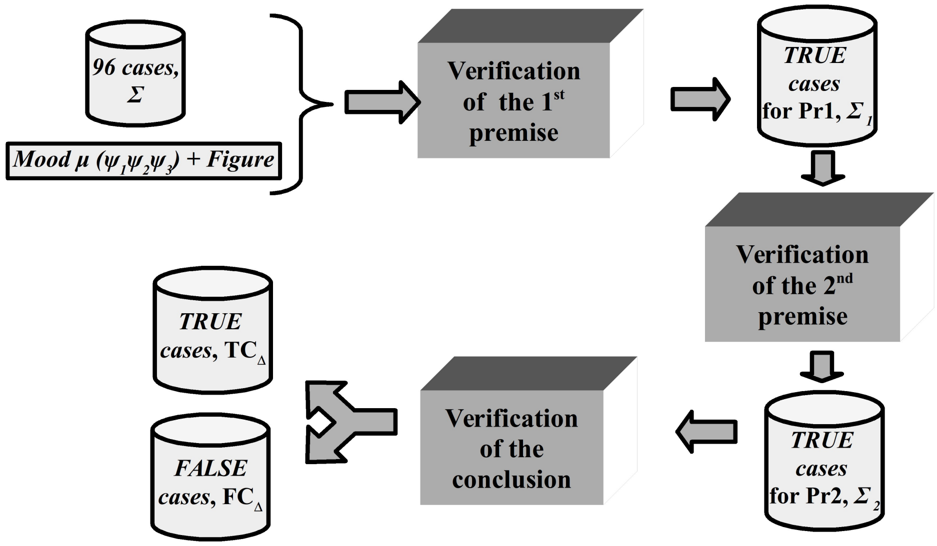


Fig. 4.2. Diagram of algorithm for calculating of truth/false cases for the given mood

4.1.7 Truth Ratio of the Particular Mood

Since we can calculate the number of true and false cases for each mood, we can introduce a measure of truth for a particular mood. We define a truth ratio τ as:

$$\tau = \frac{\|TC\|}{\|TC+FC\|} \quad (4.6)$$

where ||TC|| corresponds to the cardinality of the set of true cases, whereas ||TC+FC|| corresponds to the cardinality of the sum of the sets of true and false cases. Thus, the truth ratio τ becomes a real number, normalized within [0.0, 1.0].

Knowing truth ratio τ we can identify absolutely true (τ = 1.0) and false (τ = 0.0) moods. Absolutely true moods coincide with known valid forms of categorical syllogisms. Absolutely false or particularly non–valid moods are not considered in modern literature.

4.2 Syllogistic reasoning

In this section the algorithm for syllogistic reasoning and sample reasoning are presented.

4.2.1 Algorithm

The main goal of syllogistic reasoning is to find the most suitable moods that match the input data. The input data sets correspond to sets M, P and S. In our model, the criteria of suitability is defined by the mood(s) with the maximum truth ratio.

To verify how the statements of particular mood match the input data, we introduce the measure of similarity of two sets, denoted by φ, that is calculated as a fraction of cardinality of intersection of two sets to cardinality of the first set. For sets S and P, we can calculate φ_{SP} by the following formula:

$$\varphi_{SP} = \frac{\|\delta_6 + \delta_7\|}{\|\delta_1 + \delta_4 + \delta_6 + \delta_7\|} \qquad (4.7)$$

where δ_1, δ_4, δ_6, δ_7 represent the corresponding subsets of sets S and P according to our model (see Figure 4.1 and Table 4.1). The formulas for calculating φ for the rest of possible sets relationships is shown in Table 4.4.

The value φ corresponds to syllogistic quantifiers as below: (see Table 4.5).

Now we can formulate the algorithm for syllogistic reasoning:

INPUT: 3 non-empty sets, labeled as S, P, M

OUTPUT: list of suitable moods, L_ψ.

ALGORITHM:

1. Calculate moods, that fully match with given data (φ for each statement of a particular mood, i.e two premises and a conclusion, must match with corresponding quantifier), put those moods into L'_ψ.
2. Calculate truth ratio τ for all moods from L'_ψ.
3. Put mood(s) with maximum truth ratio τ to the list L_ψ.

Table 4.4. Calculation of φ for all possible sets relationships

Sets Relationship	Formula
MP	$\varphi_{MP}=\frac{\|\delta_5+\delta_7\|}{\|\delta_3+\delta_4+\delta_5+\delta_7\|}$
PM	$\varphi_{PM}=\frac{\|\delta_5+\delta_7\|}{\|\delta_2+\delta_5+\delta_6+\delta_7\|}$
SM	$\varphi_{SM}=\frac{\|\delta_4+\delta_7\|}{\|\delta_1+\delta_4+\delta_6+\delta_7\|}$
MS	$\varphi_{MS}=\frac{\|\delta_4+\delta_7\|}{\|\delta_3+\delta_4+\delta_5+\delta_7\|}$
PS	$\varphi_{PS}=\frac{\|\delta_6+\delta_7\|}{\|\delta_2+\delta_5+\delta_6+\delta_7\|}$

Table 4.5. Relationships between φ and syllogistic quantifiers

Value of φ	Corresponding quantifier	Comments
φ = 1.0	A (All)	Second set contains first set
φ = 0.0	E (All_Not)	Intersection of two sets is empty
φ ∈ (0.0; 1.0)	I (Some) / O (Some_Not)	All intermediate states not including cases for A and E

4.2.2 Sample Reasoning

Let us consider the following example for the syllogistic reasoning. As input data, we have data sets M, P, S that consist the following elements:

Table 4.6. Input data: elements of sets M, P and S

Set	Elements
M	#1, #2, #3, #4, #5, #6, #7, #8, #9, #10, #11, #12, #13, #14, #16, #17
P	#1, #2, #3, #4, #5, #6, #7, #8, #9, #10, #11, #12, #13, #14, #15, #16, #17
S	#1, #2, #3, #4, #5, #6, #7, #8, #9, #10, #13, #14, #17,

According to the given cases, we can calculate the syllogistic case that satisfies input data. The corresponding case (case #20, see Appendix A) is shown in Table 4.7.

Table 4.7. Syllogistic case #20 corresponding to input data

Sub-Set Number	δ1	δ2	δ3	δ4	δ5	δ6	δ7
Syllogistic Case	S-M-P	P-S-M	M-S-P	(M∩S)-P	(M∩P)-S	(S∩P)-M	S∩P∩M
Case	0	1	0	0	1	0	1

Now we need to check all 256 moods for compliance with the input data sets.

For example, for mood AAA-1 corresponding syllogism is:

All M are P $\varphi_{MP} = 1.0$, *statement matches with the input data*
<u>*All S are M*</u> $\varphi_{SM} = 1.0$, *statement matches with the input data* (4.8)
All S are P $\varphi_{SP} = 1.0$, *statement matches with the input data*

For mood AAE-1 corresponding syllogism is:

All M are P $\varphi_{MP} = 1.0$, *statement matches with the input data*
<u>*All S are M*</u> $\varphi_{SM} = 1.0$, *statement matches with the input data* (4.9)
All S are not P $\varphi_{SP} = 1.0 \neq 0.0$, *statement does not match with the input data*

As it is seen from the example, mood AAA-1 fully matches to input data, whereas the conclusion of mood AAE-1 does not match with the input data, so we have to exclude this mood from consideration. In case of inclusive logic, there are 18 moods fully matching with the input data, which constitute the L'_{ψ} (see Table 4.8.)

The moods with maximal truth ratio (AII-1, AII-3, AAI-1, AAA-1) can be considered as the most suitable moods for the given data set.

To check these moods, we can construct Euler diagram for the given data and check all moods visually.

The corresponding Euler diagram is shown on Fig. 4.3.

Table 4.8. Results of the syllogistic reasoning

Mood(s)	τ	Mood(s)	τ
AII-1, AII-3	**1.000**	AIA-1, AIA-3	0.400
AAI-1, AAA-1	**1.000**	IAA-1, IAA-2	0.285
III-1, III-2, III-3, III-4	0.885	IIA-1, IIA-2, IIA-3, IIA-4	0.142
IAI-1, IAI-2	0.714		

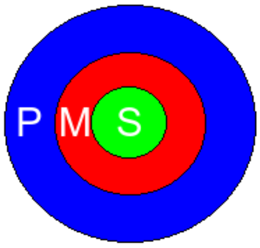


Figure 4.3. Euler diagram for the considering data sets.

Syllogisms, corresponding to resulting moods:

AAA-1:
All M are P
All S are M
All S are P

AAI-1:
All M are P
All S are M
Some S are P

AII-1:
All M are P
Some S are M
Some S are P

AII-3:
All M are P
Some M are S
Some S are P

It is obvious from the Euler diagram that all these syllogisms match with the input data.

In case of exclusive logic, there is only one mood, fully matching with the input data: AAA-1. As shown above, AAA-1 is a valid mood for the given data sets.

4.3 Statistics About Syllogistic Systems

In this section the statistics for the syllogistic systems for the cases inclusive and exclusive logics are presented.

4.3.1 General Statistics

The algorithm introduced allows to calculate various statistical information about the structural properties of syllogistic systems. We have pointed out earlier that, in order to calculate valid moods according to the classical notion (6 true moods for each figure) it is necessary to include the additional cases for the syllogistic propositions I and O (inclusive logic) in the calculation. In the case of inclusive logic, there are 25 valid moods (see Table 3.4). The solutions found are fully consistent with the known valid syllogistic moods, but additionally we have found out that the mood AAO for the Figure 4 is also true. The system obtained is absolutely symmetrical (there are 25 valid and 25 fully invalid moods, which are symmetrical in terms of numbers of true/false cases, see Fig. 4.4). The reason for that is the syllogistic propositions are basically a symmetric sub-set of the 12 distinct set relationships in total between any two sets out of three (Kumova et al. 2010). Therefore, the additional cases for I and O are required to complement the symmetric relationships between the syllogistic propositions. The full list of the moods with their truth ratio for inclusive logic can be found in the Appendix C.

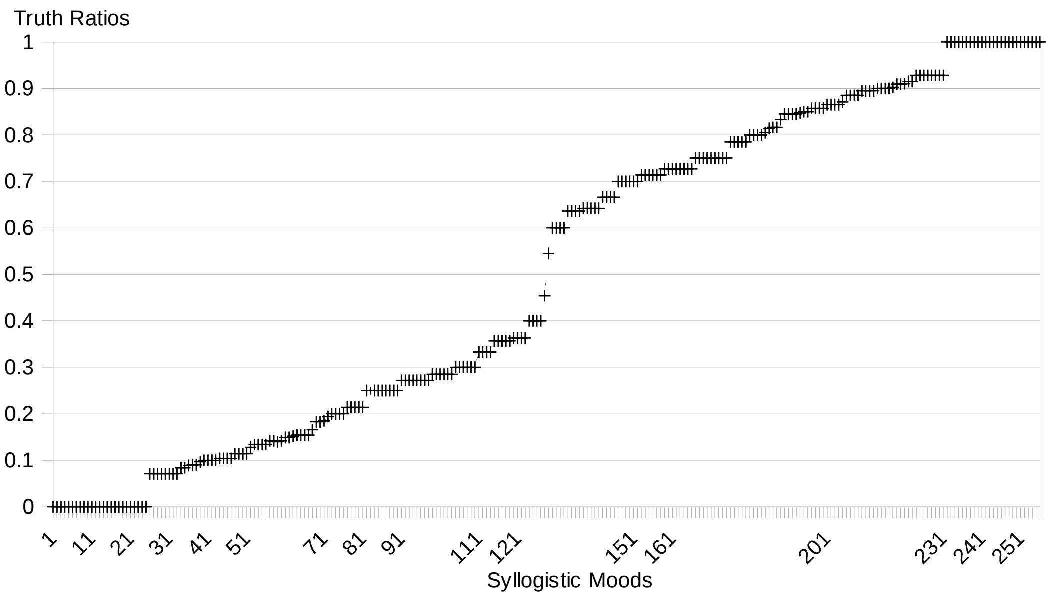


Figure 4.4. 256 moods sorted in ascending order by their truth ratio (inclusive logic).

Every mood has from 0 to 65 true and false cases respectively, which is a real sub-set of the 96 distinct syllogistic cases. The total number of true or false cases varies from one mood to another, from 1 to 72 cases. For instance, mood AAA-1 has only 1 true and 0 false cases, whereas mood AAA-2 has 1 true and 5 false cases. Hence, the truth ratio of AAA-1 is 1.0 and that of AAA-2 is 1/6. The algorithm calculates 6144 syllogistic cases in total, since all cases of the 256 moods map the 96 distinct cases multiple times. It is also interesting that for any given figure the total number of all true cases is equal to all false cases, i.e 768 true and 768 false cases. Thus, we get for all 4 syllogistic figures the total number of 768 x 2 x 4 = 6144 cases.

The system, related to exclusive logic is considered in a separate class. We denote the syllogistic system by ^{2}S, which has an existential and an exclusive version of a universal quantifier and their negations. Valid moods in case of exclusive logic is shown in Table 4.9.: there are 11 valid moods in total. Note that these moods include moods from the list of valid moods for inclusive logic, but there are 2 additional valid moods for Figure 3 (indicated bold).

The distribution of truth ratios of syllogistic moods is asymmetrical. There are only 11 valid and 40 invalid syllogistic moods (see Fig. 4.5). The full list of syllogistic moods with their truth ratios for exclusive logic is given in Appendix D.

Table 4.9. Valid syllogistic moods in case of exclusive logic (System ^{2}S)

Figure 1	Figure 2	Figure 3	Figure 4
$^{2/}AAA$ $^{2/}EAE$	$^{2/}AEE$ $^{2/}EAE$	$^{2/1}IA^{1}I$ $^{2/1}OA^{1}O$ **$^{2/1}OA^{1}I$** **$^{2/1}IA^{1}O$**	$^{2/}AEE$ $^{2/}AA^{1}O$ $^{2/}AA^{1}I$

Each mood of ^{2}S has from 0 to 40 true and from 0 to 48 false cases respectively. The total number of true or false cases in moods varies from 1 to 54.

The algorithm calculates 4432 syllogistic cases in total. The distribution of these cases in syllogistic moods by figures is given in Table 4.10.

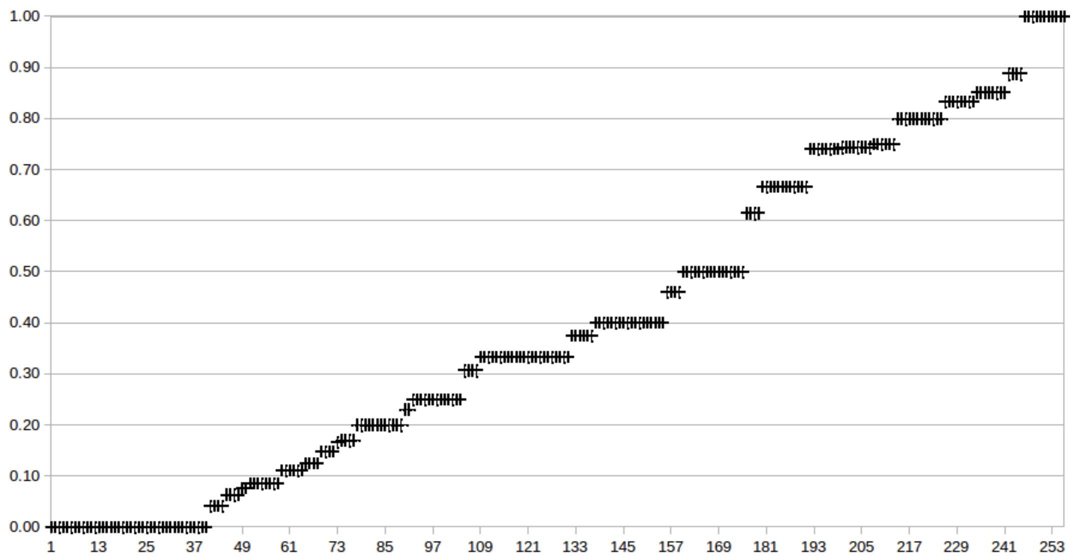


Figure 4.5. 256 moods sorted in ascending order by their truth ratio (exclusive logic).

Table 4.10. The distribution of true/false cases in syllogistic moods by figures for exclusive logic (System ^2S)

	Figure 1	Figure 2	Figure 3	Figure 4
TRUE cases	483	483	483	470
FALSE cases	617	621	645	630
Total:	1100	1104	1128	1100

4.3.2 Structural Properties

In this section the additional structural properties of syllogistic systems, such as distinct mood and point-symmetric moods are considered.

4.3.2.1 Distinct Moods

Out of the 256 moods, there are only 136 distinct mood in S, in terms of identically true and false cases matched per mood and with equal truth ratios. Thus, the syllogistic system consists of 136 inference rules in total for inclusive logic, and 70 in case of exclusive logic. The list of distinct moods for S and ^2S is given in Appendices C-D.

4.3.2.2 Point – Symmetric Moods

As it was noticed before, in case of inclusive logic, the system obtained is fully symmetric in terms of the truth ratio of syllogistic moods. Actually, all these moods are pairwise point-symmetric in terms of the syllogistic cases with which they match and respectively with their truth ratios, too. The list of the first 2 point-symmetric moods is shown in Table 4.11.

Table 4.11. The list of 2 first point-symmetric moods

#	Mood	τ	TC	FC		Mood	τ	TC	FC
1	EIO-1	1.00	11	0		EIA-1	0.00	0	11
2	EIO-2	1.00	11	0		EIA-2	0.00	0	11

Pairs have equal propositional quantifiers, but shifting concluding quantifiers. Almost all moods (250), shift from O to A, in total 63 pairs, or from I to E, in total 62 pairs. The full list of point-symmetric moods is given in Appendix E.

4.4 Fuzzy-Approaches to Systems S/^{2}S

In the following chapters the two fuzzy-approaches to the systems S/^{2}S are described. Actually, non of them is fuzzyfication in terms of classical notion, but just attempts to interpret particular properties of the classical syllogistic system in a fuzzy – manner.

4.4.1 Systems S/^{2}S as Fuzzy-Syllogistic Systems

The system, discussed in (Çakir, 2010) is designed as the fuzzy-syllogistic system of possibilistic argument (in this case, the possibilistic argument stands for truth ratio τ). After utilization of symmetric distributions of truth ratios, a membership function FuzzySyllogisticMood(x) with a symmetrical possibility distribution is defined.

FuzzySyllogisticMood(x) is determined by the linguistic variables such as Certainly/Likely/Uncertainly/Unlikely/Certainly_Not with their corresponding cardinalities. In the considered work (Kumova 2010), out of 256 syllogistic moods, the distribution of truth ratios are as follows: 25 moods have a ratio of 0 (false), 25 have ratio 1 (true), 100 moods have a ratio between 0 and 0.5, 100 have between 0.5 and 1, and 6 moods have a ratio of exactly 0.5. Since the considered system was built up by 41 sets relationships in case of inclusive logic, we can generalize this idea for our systems S/2S.

For the systems $S/^2S$, the distribution of the membership function FuzzySyllogisticMood(x) is shown in Table 4.12.

Table 4.12. The distribution of membership function FuzzySyllogisticMood(x) for cases of inclusive/exclusive logic

Linguistic variable	Inclusive logic	Exclusive logic
Certainly (τ = 1.0)	25	11
Likely	103	70
Uncertainly (τ = 0.5)	0	16
Unlikely	103	119
Certainly_Not (τ = 0.0)	25	40

The graphical representation of the membership function with respect to truth ratio distribution for cases of inclusive/exclusive logic is presented in Fig. 4.6 and Fig. 4.7 respectively.

This approach can be considered as an attempt to fuzzyfication of classical syllogystic system without introducing new quantifiers, but just based on possibilistic distribution of conclusion values under given premises. In fact, it is a simple clustering of 256 syllogistic moods into different groups according to value of τ and it has not been noticed any significant practical value in the considered approach.

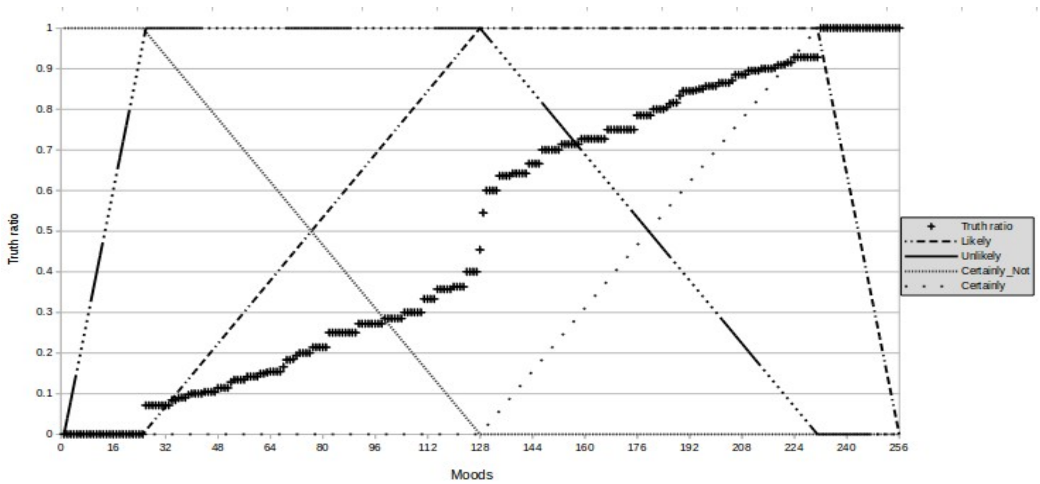


Figure 4.6. Membership function distribution with respect to truth ratio (inclusive logic).

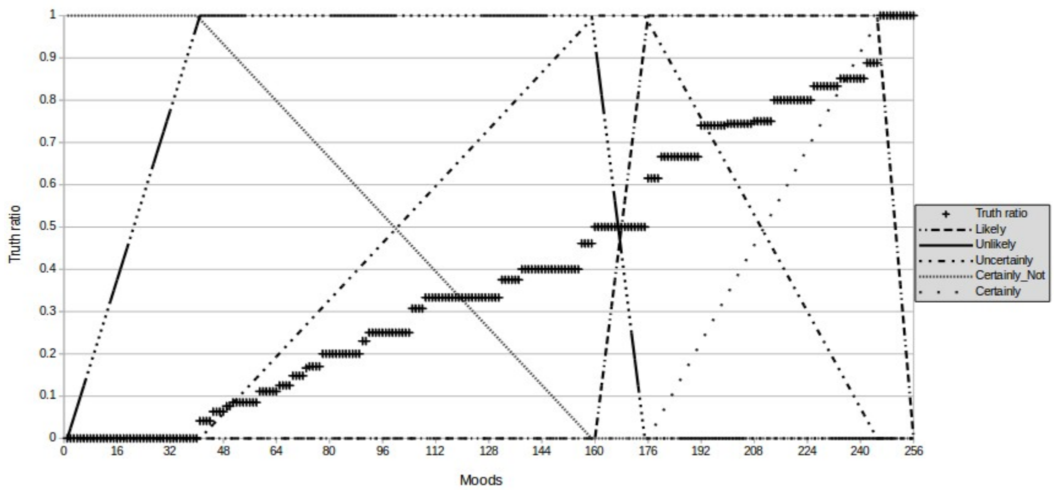


Figure 4.7. Membership function distribution with respect to truth ratio (exclusive logic).

At present, an important application may be the utilization of fuzzy-quantifiers (reasoning with intermediate quantifiers). This problem will be considered in the next chapter.

4.4.2 Improving Truth Ratios of Non-Valid Moods

Another aspect of fuzzyfication can be the improvement of truth ratios of initially non-valid moods.

In the following example, according to the structure of syllogisms, we can apply the fuzzyfication by introducing two new quantifiers such as A' = Almost_All and E' = Almost_None.

Consider the mood AIA-1:

$$\begin{array}{l} \textit{All M are P} \\ \underline{\textit{Some S are M}} \\ \textit{All S are P} \end{array} \qquad (4.10)$$

According to the rules of the classical syllogistics, this syllogism is not valid; there are only 6 valid moods for syllogistic Figure 1, and AIA does not exist in this list. For this mood we have found 10 cases that satisfy the premises, but only 4 of them are additionally satisfy the conclusion, so the truth ratio τ is 4/10.

The Venn-diagrams for true and false cases for AIA-1 is shown in Fig. 4.8. As it is seen, if the complement of S with respect to P and M is not empty ($[S\backslash P \neq \emptyset] \wedge [S\backslash M \neq \emptyset]$) (case b), the conclusion turns out to be wrong. Using fuzzy-quantification we can achieve fully true conclusions under certain conditions. Assuming that the number of elements in the complement of S is much fewer than the cardinality of S and replacing the universal quantifier A by the fuzzy-quantifier A': AlmostAll, we get the following syllogism with true ratio $\tau = 1.0$:

$$\begin{array}{l} \textit{All M are P} \\ \underline{\textit{Some S are M}} \\ \textit{AlmostAll S are P} \end{array} \qquad (4.11)$$

Thus, under certain conditions we can replace one quantifier by another. Obviously, quantifier A' can be considered as a special case of the quantifier I (Almost_All has the "value of similarity of sets" φ close to 1.0, but not 1.0 exactly, it is an extreme value of φ for the quantifier Some). Likewise, the quantifier E' (Almost_None) is a special case of the quantifier O.

Returning to the given example, AlmostAll S are P means that the proportion of elements of S being elements of P is very important. Closer the proportion in S not being P is to 0, higher the probability of satisfaction of the condition AlmostAll S are P.

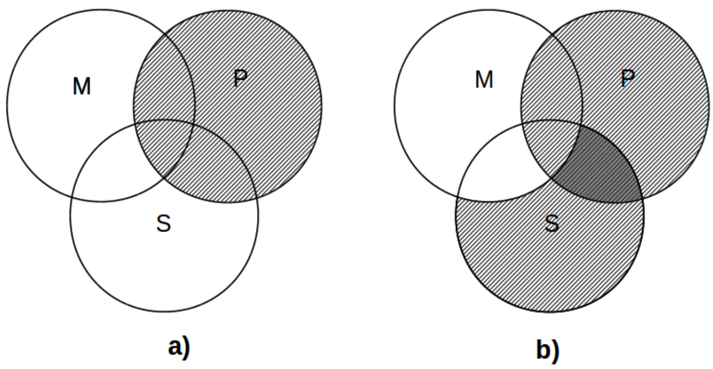


Figure 4.8. Venn - diagrams for true a) and false b) syllogistic cases of the mood AIA-1

Taking this into account, we can replace quantifier A by I and E by O respectively, according to the cardinalities of the given sets.

Applying the fuzzy quantification to mood AIA-1 to the given example, we can obtain 2 modified moods such as AII-1 and III-1. AII is a fully valid mood for the syllogistic Figure 1. So, potentially, for this mood we can increase τ from 0.4 to 1.0.

Moods, of which truth ratios τ can be improved to 1.0, are listed in Table 4.13.

Table 4.13. List of non-valid moods for all figures that can be improved by fuzzyfication (inclusive logic)

Figure 1	
AI**A** ($\tau = 0.071$) →AII	EI**E** ($\tau = 0.272$) →EIO
Figure 2	
AO**E** ($\tau = 0.333$) →AOO	EI**E** ($\tau = 0.272$) →EIO
Figure 3	
AA**A** ($\tau = 0.250$) →AAI A**AA** ($\tau = 0.250$) →AII **AAA** ($\tau = 0.250$) →IAI AI**A** ($\tau = 0.400$) →AII E**AE** ($\tau = 0.200$) →EIO	EA**E** ($\tau = 0.200$) →EAO **EAE** ($\tau = 0.200$) →OAO EI**E** ($\tau = 0.272$) →EIO IA**A** ($\tau = 0.100$) →IAI OA**E** ($\tau = 0.090$) →OAO
Figure 4	
AA**A** ($\tau = 0.0$) →AAI **AAA** ($\tau = 0.0$) →IAI AA**E** ($\tau = 0.0$) →AAO E**AE** ($\tau = 0.200$) →EIO	EA**E** ($\tau = 0.200$) →EAO EI**E** ($\tau = 0.272$) →EIO IA**A** ($\tau = 0.100$) →IAI

Note that the described conversion is not generic, as it depends on the sets relationships and the cardinalities of the real data sets.

Discussed approach can not be applied to the system ^{2}S, because the procedure is based on changing of moods due the transition from inclusive logic to exclusive logic. Thus, in ^{2}S no moods that can be improved from non-valid to valid.

Actually we do not introduce new fuzzy-quantifiers, but attempt to replace one crisp quantifier by another under the certain conditions, so it can be considered as pseudo-fuzzy-quantification and can be applied to only syllogistic system S.

CHAPTER 5

FUZZY-SYLLOGISTIC SYSTEM

The approaches examined by different researchers for the extending of classical syllogistics were briefly discussed in Chapter 3.

By the time of the preparation this work, there has been encountered no general framework dealing with the two approaches discussed in the Section 3.7 at the same time. Particularly, reasoning with the large numbers of premises with crisp and fuzzy quantifiers is considered in (Pereira-Fariña et al. 2014). However, their methods are not well formalized and designed framework looks too abstract and difficult to use. Thus, syllogistic reasoning remains as an incomplete task in terms of general solution for reasoning.

The reasoning with unlimited numbers of terms and premises in the reasoning scheme is beyond the scope of the current work. In this work we concentrate on increasing of the number of classical syllogistic quantifiers and present a generic solution for the reasoning with intermediate quantifiers, and to a certain degree extending the solutions that proposed in the works by (Novák 2008, Turunen 2014).

After the extension of the the classical syllogistic system to the systems based on the distribution of the truth ratios of syllogistic moods (systems S and ^{2}S, discussed in Chapter 4), we introduce the fuzzy-syllogistic system ^{6}S that has 10 new intermediate quantifiers $^{6/5}I$, $^{6/4}I$, $^{6/3}I$, $^{6/2}I$, $^{6/1}I$ and $^{6/5}O$, $^{6/4}O$, $^{6/3}O$, $^{6/2}O$, $^{6/1}O$. Here, in contrast to the approaches, proposed in (Peterson 1979, Peterson 2000, Novák 2008), we explicitly avoid the classical inclusive quantifiers I, O and their linguistic terms Some and SomeNot. Then, in analogy with the Aristotelian square of opposition and its extensions (Peterson 1979, Thompson 1982, Peterson 2000, Novák 2008), we propose our version of the graph of opposition, and list corresponding valid extended fuzzy-syllogistic moods for system ^{6}S.

5.1 Fuzzy-Syllogistic System ^{6}S and Fuzzy-Logical Graph of Opposition $^{6}\Omega$

In this chapter we introduce the new system ^{6}S, that consists of 6 affirmative and 6 negative quantifiers, out of which there are 5 fuzzy quantifiers in each. The proposed system is related to the Peterson's square of opposition, but has significant conceptual differences.

(Peterson 2000, Novák 2008, Murinová et al. 2014, Turunen 2014) consider quantifiers I and O together with new introduced intermediate quantifiers like Many or Most and use them in a complete system together. According to the modern interpretation of the semantic meaning of quantifier I, it can be denoted with the statement “At least one element of... are”. However, we assume that this can be true only in case of inclusive logic. As it has been shown before, there is no difference between quantifiers I and O for inclusive logic, so it is focused on exclusive logic with the of use intermediate quantifiers in ^{6}S.

We modify the list of quantifiers, proposed by Peterson, by including the new quantifiers such as “Half” and “Several” and excluding “Some”. The full list of all quantifiers of ^{6}S is given in Table 5.1:

Table 5.1. Fuzzy-quantifiers, used in ^{6}S

Affirmative	Negative
$^{6/}$**A**: *All* S are P	$^{6/}$**E:** *All* S are *not* P
$^{6/5}$I: *Many* S are P	$^{6/1}$O: *Many* S are *no*t P
$^{6/4}$I: *Most* S are P	$^{6/2}$O: *Most* S are *not* P
$^{6/3}$I: *Half* S are P	$^{6/3}$O: *Half* S are *not* P
$^{6/2}$I: *Several* S are P	$^{6/3}$O: *Several* S are *not* P
$^{6/1}$I: *Few* S are P	$^{6/5}$O: *Few* S are *not* P

Each quantifier is associated with a certain interval (or exact value) of φ (measure of similarity of two sets, introduced in Chapter 4, section 4.2.1): quantifier A corresponds to $\varphi = 1.0$, quantifier $^{6/3}$I (half) to $\varphi = 0.5$, quantifier $^{6/5}$I (Many) to $\varphi \in (0.75 \ldots 1.0)$, as shown in Table 5.2.

Using the quantifiers from Table 5.1. and their ranges of φ, we propose a fuzzy-logical graph of opposition $^{6}\Omega$ (see Figure 5.1).

The dashed lines indicate contrary pairs, straight lines sub-contrary pairs and the arrows denote the relation superaltern–subaltern.

Table 5.2. Fuzzy-quantifiers from ^{6}S associated with φ

Affirmative	φ	Negative	φ
$^{6/}$**A**: *All* S are P	1.00	$^{6/}$**E:** *All* S are *not* P	0.00
$^{6/5}$I: *Many* S are P	(0.75 … 1.00)	$^{6/1}$O: *Many* S are *not* P	(0.00 … 0.25)
$^{6/4}$I: *Most* S are P	(0.50 … 0.75]	$^{6/2}$O: *Most* S are *not* P	[0.25 … 0.50)
$^{6/3}$I: *Half* S are P	0.50	$^{6/3}$O: *Half* S are *not* P	0.50
$^{6/2}$I: *Several* S are P	[0.25 … 0.50)	$^{6/3}$O: *Several* S are *not* P	(0.50 … 0.75]
$^{6/1}$I: *Few* S are P	(0.00 … 0.25)	$^{6/5}$O: *Few* S are *not* P	(0.75 … 1.00)

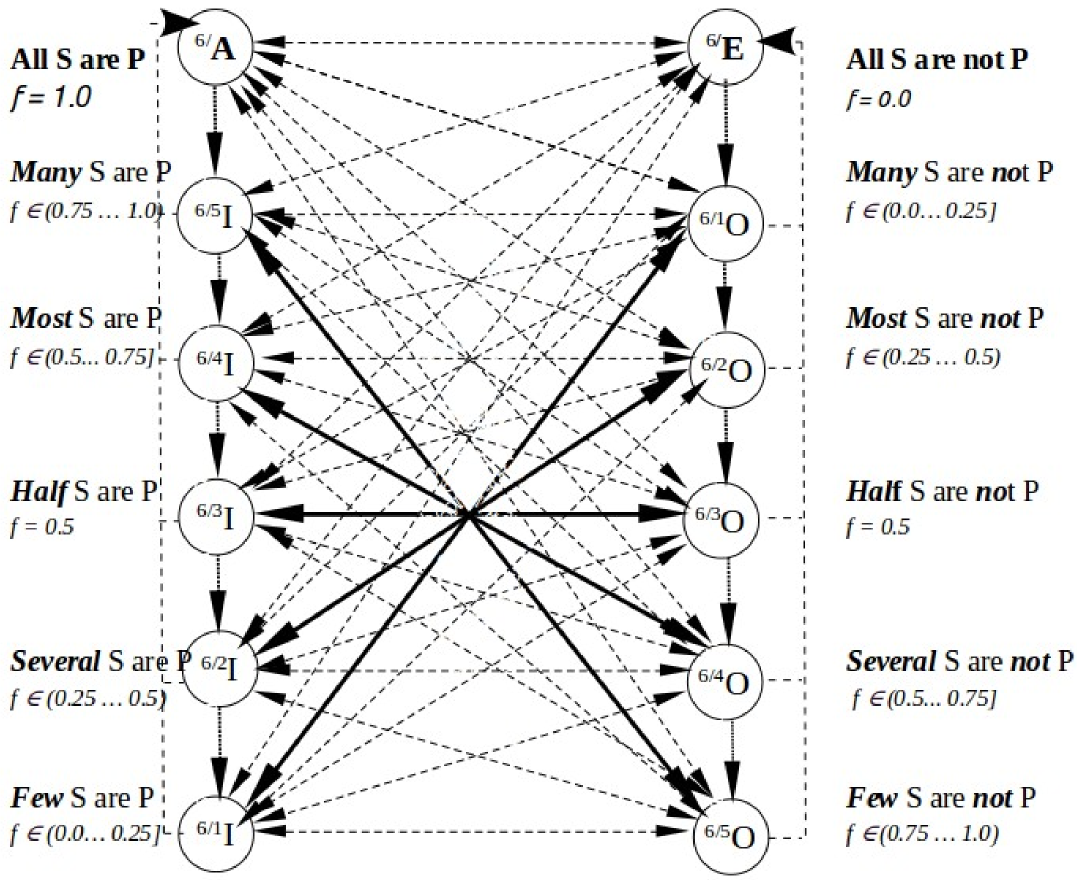


Figure 5.1. Fuzzy-logical graph of opposition $^{6}\Omega$

Note that pairs of quantifiers ($^{6/5}I$; $^{6/1}O$), ($^{6/4}I$; $^{6/2}O$), ($^{6/3}I$; $^{6/3}O$), ($^{6/2}I$; $^{6/4}O$) and ($^{6/1}I$; $^{6/5}O$) share the same ranges of φ, so paired quantifiers are interchangeable in accordance with immediate entailment of statements.

5.2 Valid Moods in ^{6}S

Based on the semantics of the intermediate statements (in particular on rule of immediate entailment) and the fuzzy-logical graph of opposition, we determine the valid intermediate syllogisms.

All valid moods of the fuzzy-syllogistic system ^{6}S are listed below.

Figure 1:

Valid moods in the Aristotelian model:

1) Affirmative: AAA-AAI-AII

2) Negative: EAE-EAO-EIO

In our model:

Table 5.3. Valid moods in ^{6}S for Figure 1.

Pattern A (Affirmative)					
$^{6/}$**AAA**					
$^{6/}AA^{5}I$	$^{6/}A^{5}I^{5}I$				
$^{6/}AA^{4}I$	$^{6/}A^{5}I^{4}I$	$^{6/}A^{4}I^{4}I$			
$^{6/}AA^{3}I$	$^{6/}A^{5}I^{3}I$	$^{6/}A^{4}I^{3}I$	$^{6/}A^{3}I^{3}I$		
$^{6/}AA^{2}I$	$^{6/}A^{5}I^{2}I$	$^{6/}A^{4}I^{2}I$	$^{6/}A^{3}I^{2}I$	$^{6/}A^{2}I^{2}I$	
$^{6/}AA^{1}I$	$^{6/}A^{5}I^{1}I$	$^{6/}A^{4}I^{1}I$	$^{6/}A^{3}I^{1}I$	$^{6/}A^{2}I^{1}I$	$^{6/}A^{1}I^{1}I$
Pattern E (Negative)					
$^{6/}$**EAE**					
$^{6/}EA^{1}O$	$^{6/}E^{5}I^{1}O$				
$^{6/}EA^{2}O$	$^{6/}E^{5}I^{2}O$	$^{6/}E^{4}I^{2}O$			
$^{6/}EA^{3}O$	$^{6/}E^{5}I^{3}O$	$^{6/}E^{4}I^{3}O$	$^{6/}E^{3}I^{3}O$		
$^{6/}EA^{4}O$	$^{6/}E^{5}I^{4}O$	$^{6/}E^{4}I^{4}O$	$^{6/}E^{3}I^{4}O$	$^{6/}E^{2}I^{4}O$	
$^{6/}EA^{5}O$	$^{6/}E^{5}I^{5}O$	$^{6/}E^{4}I^{5}O$	$^{6/}E^{3}I^{5}O$	$^{6/}E^{2}I^{5}O$	$^{6/}E^{1}I^{5}O$

Figure 2:

Valid moods in Aristotelian model

1) AXX: **AEE**-AEO-AOO

2) EXX: **EAE**-EAO-EIO

In our model:

Table 5.4. Valid moods in ^{6}S for Figure 2.

Pattern $^{6/}AXX$					
$^{6/}$**AEE**					
$^{6/}AE^{1}O$	$^{6/}A^{1}O^{1}O$				
$^{6/}AE^{2}O$	$^{6/}A^{1}O^{2}O$	$^{6/}A^{2}O^{2}O$			
$^{6/}AE^{3}O$	$^{6/}A^{1}O^{3}O$	$^{6/}A^{2}O^{3}O$	$^{6/}A^{3}O^{3}O$		
$^{6/}AE^{4}O$	$^{6/}A^{1}O^{4}O$	$^{6/}A^{2}O^{4}O$	$^{6/}A^{3}O^{4}O$	$^{6/}A^{4}O^{4}O$	
$^{6/}AE^{5}O$	$^{6/}A^{1}O^{5}O$	$^{6/}A^{2}O^{5}O$	$^{6/}A^{3}O^{5}O$	$^{6/}A^{4}O^{5}O$	$^{6/}A^{5}O^{5}O$
Pattern $^{6/}EXX$					
EAE					
$^{6/}EA^{1}O$	$^{6/}E^{5}I^{1}O$				
$^{6/}EA^{2}O$	$^{6/}E^{5}I^{2}O$	$^{6/}E^{4}I^{2}O$			
$^{6/}EA^{3}O$	$^{6/}E^{5}I^{3}O$	$^{6/}E^{4}I^{3}O$	$^{6/}E^{3}I^{3}O$		
$^{6/}EA^{4}O$	$^{6/}E^{5}I^{4}O$	$^{6/}E^{4}I^{4}O$	$^{6/}E^{3}I^{4}O$	$^{6/}E^{2}I^{4}O$	
$^{6/}EA^{5}O$	$^{6/}E^{5}I^{4}O$	$^{6/}E^{4}I^{5}O$	$^{6/}E^{3}I^{5}O$	$^{6/}E^{2}I^{5}O$	$^{6/}E^{1}I^{5}O$

Figure 3:

Valid moods in Aristotelian model

1) XXI: AAI-IAI; (AAI)-AII

2) XXO: EAO-OAO; (EAO)-EIO

In our model:

Table 5.5. Valid moods in ^{6}S for Figure 3.

Pattern $^{6/}XXI$					
$^{6/}AA^{5}I$	$^{6/5}IA^{5}I$	$^{6/4}IA^{5}I$	$^{6/3}IA^{5}I$	$^{6/2}IA^{5}I$	$^{6/1}IA^{5}I$
$^{6/}AA^{4}I$	$^{6/5}IA^{4}I$	$^{6/4}IA^{4}I$	$^{6/3}IA^{4}I$	$^{6/2}IA^{4}I$	$^{6/1}IA^{4}I$
$^{6/}AA^{3}I$	$^{6/5}IA^{3}I$	$^{6/4}IA^{3}I$	$^{6/3}IA^{3}I$	$^{6/2}IA^{3}I$	$^{6/1}IA^{3}I$
$^{6/}AA^{2}I$	$^{6/5}IA^{2}I$	$^{6/4}IA^{2}I$	$^{6/3}IA^{2}I$	$^{6/2}IA^{2}I$	$^{6/1}IA^{2}I$
$^{6/}AA^{1}I$	$^{6/5}IA^{1}I$	$^{6/4}IA^{1}I$	$^{6/3}IA^{1}I$	$^{6/2}IA^{1}I$	$^{6/1}IA^{1}I$

(Cont. on next page)

Table 5.5. (cont.)

$^{6/}A^{5}I^{5}I$ $^{6/}A^{5}I^{4}I$ $^{6/}A^{5}I^{3}I$ $^{6/}A^{5}I^{2}I$ $^{6/}A^{5}I^{1}I$	$^{6/5}I^{5}I^{5}I$ $^{6/5}I^{5}I^{4}I$ $^{6/5}I^{5}I^{3}I$ $^{6/5}I^{5}I^{2}I$ $^{6/5}I^{5}I^{1}I$	$^{6/4}I^{5}I^{5}I$ $^{6/4}I^{5}I^{4}I$ $^{6/4}I^{5}I^{3}I$ $^{6/4}I^{5}I^{2}I$ $^{6/4}I^{5}I^{1}I$	$^{6/3}I^{5}I^{5}I$ $^{6/3}I^{5}I^{4}I$ $^{6/3}I^{5}I^{3}I$ $^{6/3}I^{5}I^{2}I$ $^{6/3}I^{5}I^{1}I$	$^{6/2}I^{5}I^{5}I$ $^{6/2}I^{5}I^{4}I$ $^{6/2}I^{5}I^{3}I$ $^{6/2}I^{5}I^{2}I$ $^{6/2}I^{5}I^{1}I$	$^{6/1}I^{5}I^{5}I$ $^{6/1}I^{5}I^{4}I$ $^{6/1}I^{5}I^{3}I$ $^{6/1}I^{5}I^{2}I$ $^{6/1}I^{5}I^{1}I$
$^{6/}A^{4}I^{5}I$ $^{6/}A^{4}I^{4}I$ $^{6/}A^{4}I^{3}I$ $^{6/}A^{4}I^{2}I$ $^{6/}A^{4}I^{1}I$	$^{6/5}I^{4}I^{5}I$ $^{6/5}I^{4}I^{4}I$ $^{6/5}I^{4}I^{3}I$ $^{6/5}I^{4}I^{2}I$ $^{6/5}I^{4}I^{1}I$	$^{6/4}I^{4}I^{5}I$ $^{6/4}I^{4}I^{4}I$ $^{6/4}I^{4}I^{3}I$ $^{6/4}I^{4}I^{2}I$ $^{6/4}I^{4}I^{1}I$	$^{6/3}I^{4}I^{5}I$ $^{6/3}I^{4}I^{5}I$ $^{6/3}I^{4}I^{5}I$ $^{6/3}I^{4}I^{5}I$ $^{6/3}I^{4}I^{5}I$	$^{6/2}I^{4}I^{5}I$ $^{6/2}I^{4}I^{4}I$ $^{6/2}I^{4}I^{3}I$ $^{6/2}I^{4}I^{2}I$ $^{6/2}I^{4}I^{1}I$	
$^{6/}A^{3}I^{5}I$ $^{6/}A^{3}I^{4}I$ $^{6/}A^{3}I^{3}I$ $^{6/}A^{3}I^{2}I$ $^{6/}A^{3}I^{1}I$	$^{6/5}I^{3}I^{5}I$ $^{6/5}I^{3}I^{4}I$ $^{6/5}I^{3}I^{3}I$ $^{6/5}I^{3}I^{2}I$ $^{6/5}I^{3}I^{1}I$	$^{6/4}I^{3}I^{5}I$ $^{6/4}I^{3}I^{4}I$ $^{6/4}I^{3}I^{3}I$ $^{6/4}I^{3}I^{2}I$ $^{6/4}I^{3}I^{1}I$	$^{6/3}I^{3}I^{5}I$ $^{6/3}I^{3}I^{4}I$ $^{6/3}I^{3}I^{3}I$ $^{6/3}I^{3}I^{2}I$ $^{6/3}I^{3}I^{1}I$		
$^{6/}A^{2}I^{5}I$ $^{6/}A^{2}I^{4}I$ $^{6/}A^{2}I^{3}I$ $^{6/}A^{2}I^{2}I$ $^{6/}A^{2}I^{1}I$	$^{6/5}I^{2}I^{5}I$ $^{6/5}I^{2}I^{4}I$ $^{6/5}I^{2}I^{3}I$ $^{6/5}I^{2}I^{2}I$ $^{6/5}I^{2}I^{1}I$	$^{6/4}I^{2}I^{5}I$ $^{6/4}I^{2}I^{4}I$ $^{6/4}I^{2}I^{3}I$ $^{6/4}I^{2}I^{2}I$ $^{6/4}I^{2}I^{1}I$			
$^{6/}A^{1}I^{5}I$ $^{6/}A^{1}I^{4}I$ $^{6/}A^{1}I^{3}I$ $^{6/}A^{1}I^{2}I$ $^{6/}A^{1}I^{1}I$	$^{6/5}I^{1}I^{5}I$ $^{6/5}I^{1}I^{4}I$ $^{6/5}I^{1}I^{3}I$ $^{6/5}I^{1}I^{2}I$ $^{6/5}I^{1}I^{1}I$				
Pattern $^{6/}XXO$					
$^{6/}EA^{1}O$ $^{6/}EA^{2}O$ $^{6/}EA^{3}O$ $^{6/}EA^{4}O$ $^{6/}EA^{5}O$	$^{1}OA^{1}O$ $^{1}OA^{2}O$ $^{1}OA^{3}O$ $^{1}OA^{4}O$ $^{1}OA^{5}O$	$^{2}OA^{1}O$ $^{2}OA^{2}O$ $^{2}OA^{3}O$ $^{2}OA^{4}O$ $^{2}OA^{5}O$	$^{3}OA^{1}O$ $^{3}OA^{2}O$ $^{3}OA^{3}O$ $^{3}OA^{4}O$ $^{3}OA^{5}O$	$^{4}OA^{1}O$ $^{4}OA^{2}O$ $^{4}OA^{3}O$ $^{4}OA^{4}O$ $^{4}OA^{5}O$	$^{5}OA^{1}O$ $^{5}OA^{2}O$ $^{5}OA^{3}O$ $^{5}OA^{4}O$ $^{5}OA^{5}O$
$^{6/}E^{5}I^{1}O$ $^{6/}E^{5}I^{2}O$ $^{6/}E^{5}I^{3}O$ $^{6/}E^{5}I^{4}O$ $^{6/}E^{5}I^{5}O$	$^{1}O^{5}I^{1}O$ $^{1}O^{5}I^{2}O$ $^{1}O^{5}I^{3}O$ $^{1}O^{5}I^{4}O$ $^{1}O^{5}I^{5}O$	$^{2}O^{5}I^{1}O$ $^{2}O^{5}I^{2}O$ $^{2}O^{5}I^{3}O$ $^{2}O^{5}I^{4}O$ $^{2}O^{5}I^{5}O$	$^{3}O^{5}I^{1}O$ $^{3}O^{5}I^{2}O$ $^{3}O^{5}I^{3}O$ $^{3}O^{5}I^{4}O$ $^{3}O^{5}I^{5}O$	$^{4}O^{5}I^{1}O$ $^{4}O^{5}I^{2}O$ $^{4}O^{5}I^{3}O$ $^{4}O^{5}I^{4}O$ $^{4}O^{5}I^{5}O$	$^{5}O^{5}I^{1}O$ $^{5}O^{5}I^{2}O$ $^{5}O^{5}I^{3}O$ $^{5}O^{5}I^{4}O$ $^{5}O^{5}I^{5}O$
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Table 5.5. (cont.)

$^{6/}E^{3}I^{1}O$ $^{6/}E^{3}I^{2}O$ $^{6/}E^{3}I^{3}O$ $^{6/}E^{3}I^{4}O$ $^{6/}E^{3}I^{5}O$	$^{1}O^{3}I^{1}O$ $^{1}O^{3}I^{2}O$ $^{1}O^{3}I^{3}O$ $^{1}O^{3}I^{4}O$ $^{1}O^{3}I^{5}O$	$^{2}O^{3}I^{1}O$ $^{2}O^{3}I^{2}O$ $^{2}O^{3}I^{3}O$ $^{2}O^{3}I^{4}O$ $^{2}O^{3}I^{5}O$	$^{3}O^{3}I^{1}O$ $^{3}O^{3}I^{2}O$ $^{3}O^{3}I^{3}O$ $^{3}O^{3}I^{4}O$ $^{3}O^{3}I^{5}O$		
$^{6/}E^{2}I^{1}O$ $^{6/}E^{2}I^{2}O$ $^{6/}E^{2}I^{3}O$ $^{6/}E^{2}I^{4}O$ $^{6/}E^{2}I^{5}O$	$^{1}O^{2}I^{1}O$ $^{1}O^{2}I^{2}O$ $^{1}O^{2}I^{3}O$ $^{1}O^{2}I^{4}O$ $^{1}O^{2}I^{5}O$	$^{2}O^{2}I^{1}O$ $^{2}O^{2}I^{2}O$ $^{2}O^{2}I^{3}O$ $^{2}O^{2}I^{4}O$ $^{2}O^{2}I^{5}O$			
$^{6/}E^{1}I^{1}O$ $^{6/}E^{1}I^{2}O$ $^{6/}E^{1}I^{3}O$ $^{6/}E^{1}I^{4}O$ $^{6/}E^{1}I^{5}O$	$^{1}O^{1}I^{1}O$ $^{1}O^{1}I^{2}O$ $^{1}O^{1}I^{3}O$ $^{1}O^{1}I^{4}O$ $^{1}O^{1}I^{5}O$				

Figure 4:

Valid moods in Aristotelian model:

1) AAI-IAI
2) **AEE**-AEO
3) EAO-EIO
4) AAO

In our model:

Table 5.6. Valid moods in ^{6}S for Figure 4.

$^{6/}AA^{5}I$ $^{6/}AA^{4}I$ $^{6/}AA^{3}I$ $^{6/}AA^{2}I$ $^{6/}AA^{1}I$	$^{6/}\mathbf{AEE}$	$EA^{1}O$ $EA^{2}O$ $EA^{3}O$ $EA^{4}O$ $EA^{5}O$	$AA^{1}O$ $AA^{2}O$ $AA^{3}O$ $AA^{4}O$ $AA^{5}O$
$^{6/5}IA^{5}I$ $^{6/5}IA^{4}I$ $^{6/5}IA^{3}I$ $^{6/5}IA^{2}I$ $^{6/5}IA^{1}I$	$^{6/}AE^{1}O$	$^{6/}E^{5}I^{1}O$ $^{6/}E^{5}I^{2}O$ $^{6/}E^{5}I^{3}O$ $^{6/}E^{5}I^{4}O$ $^{6/}E^{5}I^{5}O$	
$^{6/4}IA^{5}I$ $^{6/4}IA^{4}I$ $^{6/4}IA^{3}I$ $^{6/4}IA^{2}I$ $^{6/4}IA^{1}I$	$^{6/}AE^{2}O$	$^{6/}E^{4}I^{1}O$ $^{6/}E^{4}I^{2}O$ $^{6/}E^{4}I^{3}O$ $^{6/}E^{4}I^{4}O$ $^{6/}E^{4}I^{5}O$	

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Table 5.6. (cont.)

${}^{6/3}IA^5I$ ${}^{6/3}IA^4I$ ${}^{6/3}IA^3I$ ${}^{6/3}IA^2I$ ${}^{6/3}IA^1I$	${}^{6/}AE^3O$	${}^{6/}E^3I^1O$ ${}^{6/}E^3I^2O$ ${}^{6/}E^3I^3O$ ${}^{6/}E^3I^4O$ ${}^{6/}E^3I^5O$	
${}^{6/2}IA^5I$ ${}^{6/2}IA^4I$ ${}^{6/2}IA^3I$ ${}^{6/2}IA^2I$ ${}^{6/2}IA^1I$	${}^{6/}AE^4O$	${}^{6/}E^2I^1O$ ${}^{6/}E^2I^2O$ ${}^{6/}E^2I^3O$ ${}^{6/}E^2I^4O$ ${}^{6/}E^2I^5O$	
${}^{6/1}IA^5I$ ${}^{6/1}IA^4I$ ${}^{6/1}IA^3I$ ${}^{6/1}IA^2I$ ${}^{6/1}IA^1I$	${}^{6/}AE^5O$	${}^{6/}E^1I^1O$ ${}^{6/}E^1I^2O$ ${}^{6/}E^1I^3O$ ${}^{6/}E^1I^4O$ ${}^{6/}E^1I^5O$	

Totally, we have introduced 285 valid moods for the system ${}^6S$. Note that due the transitivity of quantifiers ${}^{6/}I$ and ${}^{6/}O$ in ${}^6S$, the number of valid moods may be increased in accordance with the fuzzy-logical graph of opposition ${}^6\Omega$.

5.3 Sample Reasoning

Consider the example from Chapter 4, section 4.2. As input data, we have data sets M, P and S that consist of the following elements:

Table 5.7. Input data: elements of sets M, P and S

Set	Elements
M	#1, #2, #3, #4, #5, #6, #7, #8, #9, #10, #11, #12, #13, #14, #16, #17
P	#1, #2, #3, #4, #5, #6, #7, #8, #9, #10, #11, #12, #13, #14, #15, #16, #17
S	#1, #2, #3, #4, #5, #6, #7, #8, #9, #10, #13, #14, #17,

After performing the reasoning under above data, we get the following results (see Table 5.8).

The results is consistent with the results obtained in the section 4.2. However, the number of moods in this case considerably smaller because of using of exclusive logic. Note that there are 4 new moods, which do not exist in section 4.2: A^1OA-3, ${}^5I^1OA$-4, ${}^5I^1OA$-4 and ${}^1O^1OA$-4 resulting from the transitivity of quantifiers ${}^{6/}I$ and ${}^{6/}O$ in ${}^6S$.

Table 5.8. Results of fuzzy-syllogistic reasoning

	Moods	Truth ratio, τ
Valid moods, τ = 1.0	$^{6/}$AAA-1	1.00
Non-valid moods, τ ∈ (1.0 … 0.0)	^{5}IAA-2	0.375
	^{1}OAA-2	0.375
	A^{5}IA-3	0.500
	A^{1}OA-3	0.500
	^{5}I^{5}IA-4	0.170
	^{5}I^{1}OA-4	0.170
	^{5}I^{1}OA-4	0.170
	^{1}O^{1}OA-4	0.170

Considering another example, where input sets defined as follows:

Table 5.9. Input data: elements of sets M, P and S for the 2nd example

Set	**Elements**
M	#4, #5, #6, #7, #8, #9, #10, #11, #12, #13, #14, #15, #16
P	#4, #5, #6, #7, #8, #9, #10, #11, #12, #13
S	#1, #2, #3, #4, #5, #6, #7, #8, #9, #10

We get the following results (see Table 5.10).

According to our model, mood $^{6/}$A^{1}O^{1}O-2 is valid (truth ratio τ = 1.0).

The corresponding syllogism is:

All *P are M*
<u>***Few*** *S are not M*</u> (5.1)
Few *S are not P*

A simple verification shows that the syllogism is correct.

Table 5.10. Results of fuzzy-syllogistic reasoning for the 2nd example

	Moods	Truth ratio, τ
Valid moods, $\tau = 1.0$	$^{6/}A^{1}O^{1}O$-2	1.00
Non-valid moods, $\tau \in (1.0 \ldots 0.0)$	$^{6/}A^{5}I^{1}O$-2; $^{6/}A^{4}I^{1}O$-4;	0.928
	$^{6/}A^{2}O^{1}O$-4	0.928
	$^{1}O^{4}I^{3}O$-3; $^{1}O^{5}I^{1}O$-1; $^{1}O^{1}O^{1}O$-1	0.915
	$^{5}I^{1}O^{1}O$-1	0.895
	$^{5}I^{4}I^{5}I$-3	0.885
	$^{1}O^{2}O^{1}O$-3	0.871
	$^{1}O^{1}O^{5}I$-1	0.865
	$^{5}I^{4}I^{1}O$-3	0.857
	$^{5}O^{1}O^{1}O$-1	0.850
	$^{5}I^{2}O^{5}I$-3	0.845
	$^{1}O^{4}I^{5}I$-3	0.845
	$^{5}I^{4}O^{5}I$-3	0.816
	$^{1}O^{2}O^{5}I$-3	0.814
	$^{6/}A^{5}I^{5}I$-2	0.714
	$^{6/}A^{1}O^{5}I$-2	0.666
	$^{6/}A^{2}O^{5}I$-4	0.642

CHAPTER 6

APPLICATIONS OF SYLLOGISTIC REASONING

Since reasoning is associated with thinking, cognition and intellect, it is an important component in architectures of proactive and deliberative AI agents. Cognitive architecture requires mechanisms that draw inferences using internal knowledge structures as a part of reasoning component. Deductive reasoning is an important and widely studied form of inference that lets one combine general and specific beliefs to conclude others that they entail logically (Langley 2006). At this point of view, the syllogistic system involves different deductive inference schemes and may be the essential solution that implements such mechanisms like deductive reasoning.

According to the (Swarup at al. 2005), one of the central goals of a cognitive architecture should attain an ability to collect and use experience about its environment. The agent should adapt to its environment, and become increasingly better at solving new tasks. Thus, the agent should be able to use information from previously encountered tasks to enhance the learning of new tasks in case the tasks are similar. The syllogistic system can be adapted for learning tasks in case of continuous flows of incoming data. Based on concepts of case-based reasoning we propose an algorithm for iterative syllogistic reasoning. For each portion of new data, we perform only few calculations based on previous experience and it can be really effective in case of big volumes of incoming data.

Another possible application of syllogistic system is inference on the Semantic Web. Inference on the Semantic Web is the basic mechanism for various tasks such as the consistency check of an ontology in order to improve data integration, the construction of a concept taxonomy etc. The main technique for inference implementations in existing solutions is using reasoners, based on deductive inference schemes. Since syllogistic reasoner includes various deductive inference rules, it seems possible to use them for reasoning under ontologies.

6.1 Syllogistic Reasoning in Cognitive Architectures

Syllogistic system is closely linked to the grammatical structure of natural language due to the fact that syllogistic structures are part of the natural logic that we normally use. Medieval logicians tried to extend particular fields of natural logic in such a way to find various patterns of reasoning with very simple inference rules, that allowing quantifiers with shared features and at the same time staying close to linguistic syntax.

Syllogistic reasoning has also captured the attention of cognitive scientists, who try to infer conclusions about what goes on in the human brain when we combine predicates and reason about objects (Logic In Action 2014).

As part of theory of human cognition, syllogistic systems can be used in different cognitive systems representing simple reasoning component. The one attempt of modeling of human syllogistic reasoning in Soar Cognitive architecture belongs to Polk and Newell (Polk at al. 1988). The syllogistic reasoning system in their work modeled many of the answer patterns that people give on a variety of syllogistic problems.

We propose framework architecture for syllogistic agent (see Fig. 6.1) that includes fuzzy-syllogistic reasoner (FSR) as a main reasoning component which works with ontology-bases data structures.

In the sample scenario we are planning to work with ontologies, constructed from text corpora. Sample architecture includes sensors, actuators, memory and reasoning component. Sensor1... SensorN are used to read environment properties (in our case text corpora) and store data in temporary buffers (Information Bases). At this stage we can build text ontologies using various statistical approaches.

Generated ontologies are the input for Synthesizer module that construct new ontology based on certain algorithms.

The ontology is permanently updated by Synthesizer and this ontology consists current Agent's knowledge about Environment. At the same time Executor can retrieve data from knowledge base (KB) (reading cycle is proceeded according to the system events) and performs reasoning process (produce conclusion about input data). Executor controls actuators that affect the Environment. The key elements of architecture (Synthesizer and Executor) use the functionality of FSR for performing appropriate operations.

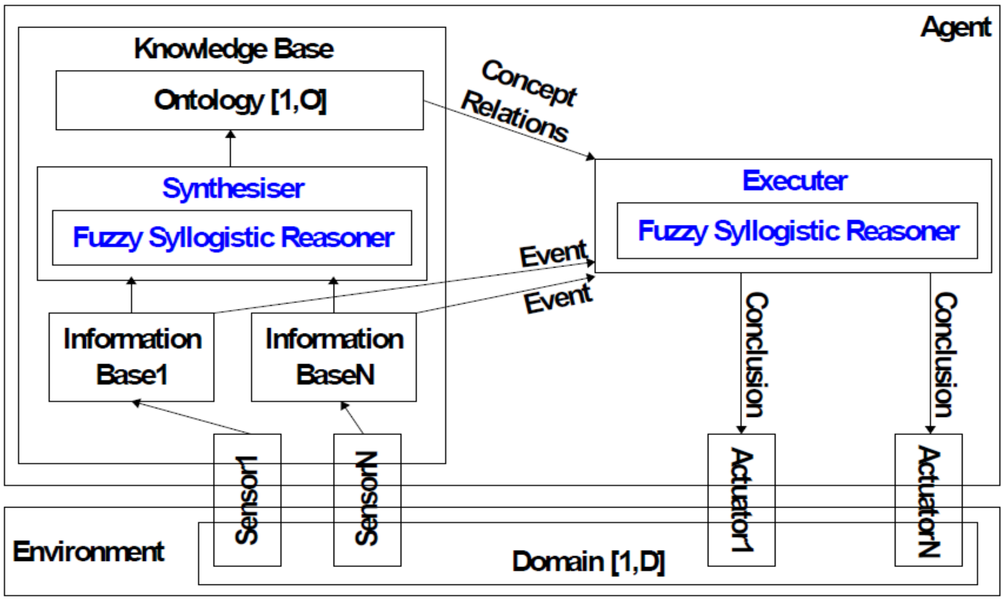

Figure 6.1. Cognitive architecture of a fuzzy syllogistic agent.

The important point that data from sensors is coming continuously in small portions. It is possible to perform reasoning only for new coming data without considering all data in KB for updating global knowledge. The problem is very similar to using of case-based reasoning to learning tasks (Kolodner 1992). Reasoner can become more efficient by remembering old solutions and adapting them to new data rather than having to derive answers from scratch each time.

6.2 Case-Based Reasoning

Case-based reasoning (CBR) is the process of solving new problems based on the previous experience. Considering the syllogistic system as a system with continuous flows of input data (i.e. as a part of learning mechanism of AI agents), there is no need to recalculate suitable case for each iteration of learning process with whole data every time while new data is coming to the system. It is possible to define a simple set of constraints for current syllogistic sets, so if new data satisfy these constraints, the case does not change, otherwise we need to change it. So, for each portion of new-coming data there is a simple procedure for case-defining.

Now we can define an algorithm for syllogistic reasoning based on CBR. The basic steps of CBR cycle (Aamodt 1994) are:

1. RETRIEVE the current case from KB
2. REUSE the knowledge to solve problem
3. REVISE the proposed solution
4. RETAIN current case for next step.

Now we can discuss these steps in detail.

The input data for the first step is the new portion of data (in our case, some elements of sets M, P or S) from sensors of AI agent. We need to retrieve the case from previous step and analyze new data with constraints coming from retrieved case. Actually, constraints are related to zero-bits from case structure (δ1 .. δ7), so we need to check new data only for satisfaction of these constraints. If the case is not changed we can skip the rest of the steps of the current cycle.

If the input data does not satisfy case-constraints, we have to construct an updated case by changing subsets, that are not satisfied, from zero to one. Then, we can apply the second step by searching the moods matching updated case in the KB. We have only 96 predefined cases, so we can use hashing or sorting cases by number of non-empty cases for search optimization. If a matching case is not found we can skip the following steps and clear current state because the current data is not suitable for our model. This situation is possible at the beginning of the agent's work where we have insufficient amount of data (empty S, M, P sets). With high probability, after few working cycles, the input data will be sufficient for performing the reasoning.

On the third step, if the matching case was found in the set of predefined cases, it is necessary to find moods, suitable for the current case.

In the last step we have to save current data, such as the matched case, list of corresponding moods and updated data sets (M, P and S) for the future use.

This procedure provides iterative process of reasoning with the minimum possible numbers of calculation, that can be very effective for the big data sets.

The general algorithm for syllogistic reasoning with CBR is presented bellow:

1. Initialization
   A) Calculate and store the 96 possible syllogistic cases.
   B) Calculate list of corresponding moods for all 96 cases.
2. Define data sets M, P, S
3. Reasoning
   A) From incoming data stream select data related to sets M, P, S

B) For this data provide all steps of CBR cycle

As it is seen, in the step 3A the algorithm involves data-selection for reasoning. This depends on target domain and current task and is open to discussion which data is to retrieve.

6.3 Inference on the Semantic Web. Ontology-Based Fuzzy-Syllogistic Reasoning

Generally, inference on the Semantic Web can be defined as discovering new relationships between resources in existing ontologies, where data is modeled as a set of labeled relationships between resources.

A typical ontology consists of two different types of statements: facts (set of facts is known as assertion component, ABox) and terminological component (conceptualization, associated with the facts, TBox). Statements in TBox can be defined via vocabularies or rule sets. In general, ontologies concentrate on classification methods, defining classes, subclasses and their relationships together with their instances. Rules, on the other hand, concentrate on defining a general mechanism on discovering and generating new relationships based on existing ones, much like logic programs, for example Prolog (Breitman 2007).

Inference mechanisms in the Semantic Web are conceptual tools for performing several tasks such as the finding of relationships between concepts, consistency check of an ontology etc.

Let us consider the simple ontology, that includes a relationship such as “Socrates is a man”. In the Tbox, one of the rules may be defined as "All men are mortal". That means that reasoning engine can understand the notion of "X is Y" and can add the statement “Socrates is mortal” to the set of relationships, although that was not a part of the original data. Thus, by means of inference new relationships which were not existing before can be invented.

Ontologies potentially will be widely used in information systems in the near future, so ontology construction is still an active topic of research. The major problems in building and using ontologies are the knowledge acquisition and the time-consuming construction (Pan et al. 2009). Integration of different ontologies or knowledge exchange between ontology applications is yet another difficulty.

6.3.1 Ontology-Based Fuzzy-Syllogistic Reasoning

Our objective is to implement syllogistic reasoning to ontologies and iteratively quantify ontological relationships among concepts by using FSR. In this process, FSR does not directly produce an ontology. Concepts and relationships of a given ontology are evaluated and altered by the FSR.

Among the various possible ways to construct ontologies for a given domain, the most widely used approaches are the generation of an ontology from the text-based sources (Shamsfard at al. 2003). There are several open-source tools for ontology generation from text corpora available for research purposes, such as Text2Onto (Cimiano at al. 2005), WebKB (Craven et al. 1998) or DLLearner[2].

The most convenient tool for our purposes is Text2Onto, because it allows to generate ontologies automatically and the generated ontology is sufficiently good in terms of numbers of concepts and corresponding relationships extracted from text corpora.

6.3.2 Building a Source Ontology

To generate a source ontology it is necessary to prepare a source data (in our case, text corpus for the particular domain). In case of Text2Onto, the text corpus may be a set of plain text documents, html pages and other unstructured or semi-structured text sources. The integration of this tool with the web search engine seems to be an optimal solution for collecting and preparing a text corpus for a given domain. Furthermore, as a result of stepwise synthesizing concepts and properties, we obtain a domain ontology for a given corpus. The resulting ontology includes a set of nodes, represented by terminal or intermediate nodes, with linked relationships (Fig.6.2. a).

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2 http://dl-learner.org/

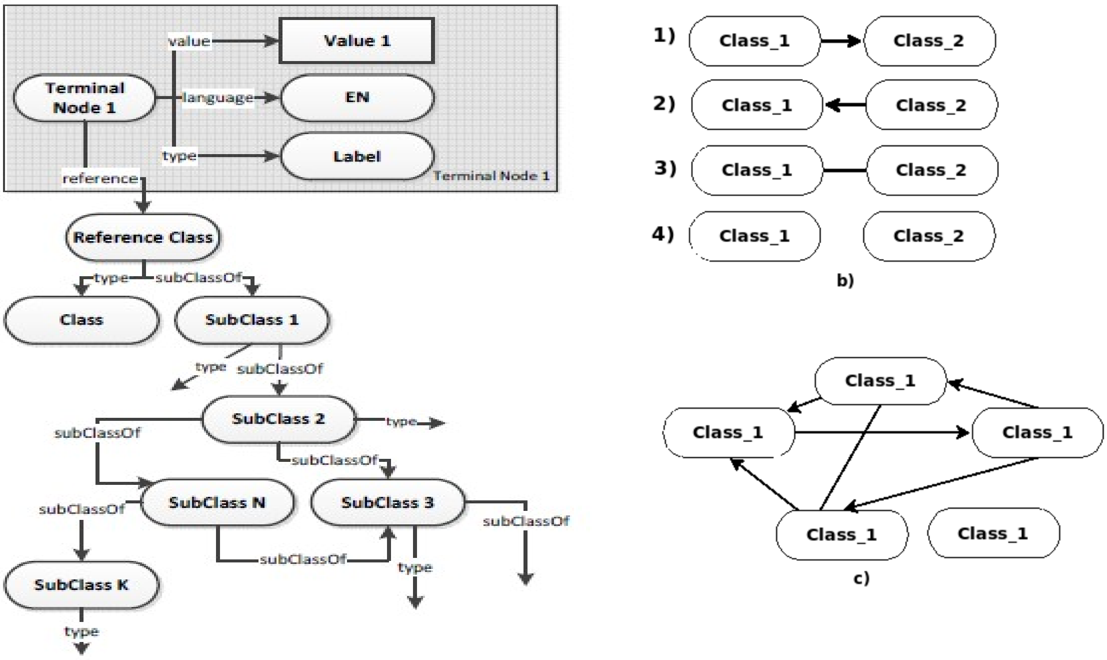

Figure 6.2. Structural schema of a simple ontology (a); 1 – 4 types of possible class relationships (b) and a sample graph of dependencies (c).

6.3.3 Building a Graph of Dependencies

For an existing ontology we can build a graph of dependencies, which reflects quantitative relationships between concepts of the original ontology, so that FSR can be applied.

Such a graph of dependencies contains all concepts and concept attributes of the original ontology and additionally all quantities that contributed to their conceptualizations during the ontology learning process.

Every attribute of a particular concept from a source ontology is further decomposed, e.g. new sub-concepts (subclasses) are created for each attribute value. This helps revealing hidden dependencies. All subclasses, constructed from attributes, must be linked with their parent class by direct link (from attribute-subclass to parent).

Now we can link all concepts (classes) in our graph according to the below procedure.

Let the number of all (sub)classes (terminal and intermediate) be equal to ***N***. For every subclass ***SubCli*** we need to consider ***N-1*** subclasses. Analysis should be performed on pairs. For each pair of subclasses there are following possible conditions:

- ***SubCli*** is subclass for ***SubClj*** (or vice versa) (Fig.6.2. a, Subclass2 and Subclass3): direct link from subclass to superclass on the graph of dependencies;
- ***SubCli*** and ***SubClj*** have no shared subclasses (Fig.6.2. a, Subclass1 and Subclass3): no link between the classes on the graph of dependencies;
- ***SubCli*** and ***SubClj*** have shared subclasses (Fig.6.2. a, Subclass3 and SubclassK): in this case we need to calculate the value of ***F*** as fraction of shared subclasses to number of all subclasses for each of 2 subclasses (nodes), if ***F=1*** for one of the nodes, then this node becomes subclass of the other node and we need to create a direct link from subclass to superclass; if ***F<1***, we need to create a non-direct link (nodes have shared subclasses but each of them is a not superclass of another).

After performing these operations we will get the graph of dependencies, which is a reflection of the input ontology (Fig. 6.2 c).

There are 4 possible types of relationships between the classes (Fig. 6.2 b, 1-4):

- direct link from CLASS_1 to CLASS_2: CLASS_2 includes all elements from CLASS_1, corresponds to the syllogistic quantifier **A**;
- direct link from CLASS_2 to CLASS_1: CLASS_1 includes all elements from CLASS_2, corresponds to the syllogistic quantifier **I** or **O** (some elements of CLASS_1 in CLASS_2);
- non-direct link from CLASS_1 to CLASS_2 (or vice versa): some elements from CLASS_1 in CLASS_2, at the same time, some elements from CLASS_2 in CLASS_1: corresponds to the quantifiers **I, O** (appropriate quantifier can be selected according to cardinality of given sets);
- no link between classes: corresponds to quantifier **E**.

FSR with such a dependency graph enables reasoning with 256 possible fuzzy inferences per triple concept relationships.

In some cases, the transitive concept of a triple can be removed, as that is not included in the conclusion. This helps reducing the complexity of the ontology and increases the level of abstraction over details that are no more required in reasoning.

6.3.4 Reasoning with Ontologies: Procedure

A sample algorithm for ontology-based syllogistic reasoning is presented below :

1. Calculate truth ratios of all 256 syllogistic moods.
2. For given ontology, build dependency graph as was described above.
3. Select the triple of sets for analysis and label them as M, P and S.
4. Construct 4 syllogistic figures and associate quantifiers appropriate for the quantities of the premises from the graph of dependencies; apply all possible quantifiers in conclusions.
5. Calculate truth ratios for all possible moods.
6. Select the moods with the highest truth ratio τ; if $\tau<1.0$, apply the fuzzification if possible.

6.3.5 Sample Application

Let us perform the steps of the algorithm on a sample ontology (Fig. 6.3. a). One can see that there are four classes, HUMANS (with the attribute #gender), PHILOSOPHERS, SCIENTISTS and ARTISTS. Also there are six instances of the class HUMANS.

First of all, we need to distinguish all attributes of each class as separate subclasses. As shown on (Fig. 6.3. b), we have created 2 subclasses of HUMANS, humans_gender_male and humans_gender_female. Subclasses have direct link to their superclass, because this operation can be considered as decomposition of superclass. Thus, all subclasses are part of a superclass and relation between subclass and superclass is “part-of”. In terms of syllogistic quantification this relationship corresponds to quantifier **A** (All elements of subclass are elements of superclass).

We are not interested in the instances of classes but the number of them (the cardinality) for each (sub)class. So, all instances are removed from the graph and cardinalities of each subclass are calculated.

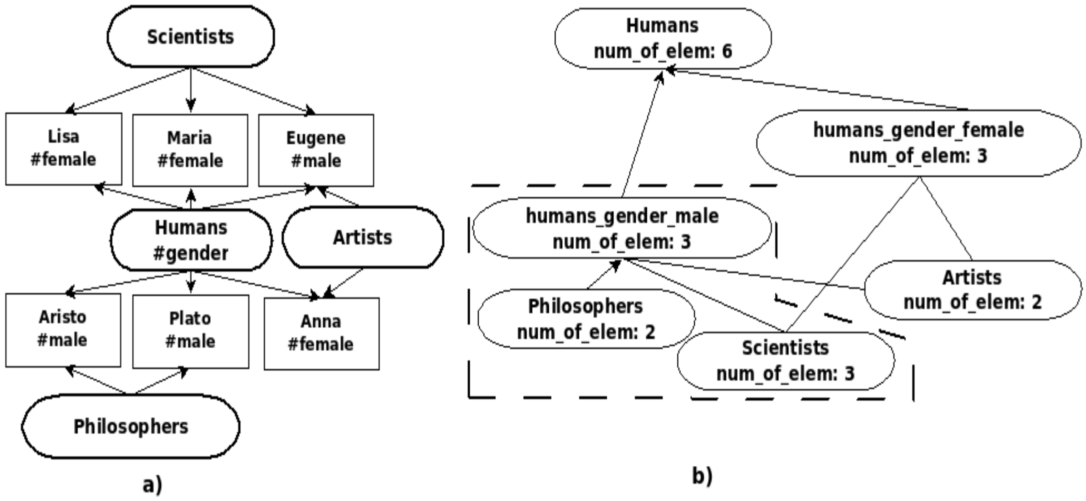

Figure 6.3. Sample ontology (a) and graph of dependencies for sample ontology (b).

Now we can see that all PHILOSOPHERS are male (direct link from subclass to superclass), some of SCIENTISTS and ARTISTS are women, some are man. The constructed graph is suitable for performing by FSR.

To perform reasoning, we need to select 3 classes. For now it looks quite indefinable, but when we embed our system in a real application, like an intelligent agent, the selected classes will be determined by the logic of the agent.

Let us consider the relationships between SCIENTISTS and PHILOSOPHERS classes through people_gender_male class (Fig. 6.3 b, dashed area).

According to the structure of syllogisms, the middle term (M) is humans_gender_male class, predicate (P) is SCIENTISTS class and subject (S) is PHILOSOPHERS class.

So, for four syllogistic figures we have the following combinations:

Figure 1	Figure 2	Figure 3	Figure 4
{I, O}: M P	{I, O}: P M	{I, O}: M P	{I, O}: P M
{A}: S M	{A}: S M	{I, O}: M S	{I, O}: M S
{?}: S P	{?}: S P	{?}: S P	{?}: S P

The problem is to find the most appropriate quantifier in conclusion. The quantifiers for the premises were selected according to the relationships between classes in the graph of dependencies. For example, direct link from PHILOSOPHERS to humans_gender_male (S M) corresponds to the quantifier A, non-direct link from SCIENTISTS to humans_gender_male (P M) can be an I or O quantified relationship.

By calculating truth ratios of all possible moods, we obtain the following results:

Figure 1	Figure 2	Figure 3	Figure 4
IAA=0.285	IAA=0.285	IIA=0.142	IIA=0.142
IAE=0.285	IAE=0.285	IIE=0.144	IIE=0.144
IAI=0.714	IAI=0.714	III=0.885	III=0.885
IAA=0.714	IAA=0.714	IIO=0.857	IIO=0.857
OAA=0.214	OAA=0.333	IOA=0.183	IOA=0.183
OAE=0.357	OAE=0.333	IOE=0.154	IOE=0.154
OAI=0.642	OAI=0.666	IOI=0.845	IOI=0.845
OAA=0.785	OAA=0.666	IOE=0.816	IOE=0.816
	OIA=0.084	OIA=0.134	
	OIE=0.157	OIE=0.104	
	OII=0.845	OII=0.895	
	OIO=0.915	OIO=0.865	
	OOA=0.128	OOA=0.194	
	OOE=0.185	OOE=0.134	
	OOI=0.814	OOI=0.865	
	OOE=0.871	OOE=0.805	

The highest truth ratio is OIO3=0.915. We cannot apply fuzzy-quantification based on moods relaxation to a given mood, because it does not contain A or E quantifiers. Based on the below results we can retrieve the most suitable syllogism for the given data:

O: ***Some*** *Male* ***are not*** *Scientists*
I: ***Some*** *Male* ***are*** *Philosophers*
O: ***Some*** *Philosophers* ***are not*** *Scientist*

with truth ratio τ=0.915. Considering the OIO3 mood, we can see that it has 71 syllogistic cases and only 6 cases are false.

In analogy with previous example, consider following scenario: M= humans_gender_male class, S=PHILOSOPHERS, P=HUMANS. So, we want to investigate the relationship between PHILOSOPHERS and HUMANS.

Possible moods are listed below:

Figure 1	Figure 2	Figure 3	Figure 4
{A}: M P	{I, O}: P M	{A}: M P	{I, O}: P M
{A}: S M	{A}: S M	{I, O}: M S	{I, O}: M S
{?}: S P	{?}: S P	{?}: S P	{?}: S P

After calculating the truth ratio for all moods, only three moods AAA1, AAI1 and AII3 have τ=1.0:

A: ***All*** *Male* ***are*** *Humans*
A: ***All*** *Philosophers* are *Male*
{A, I}: ***All (Some)****Philosophers* ***are*** *Humans*

A: ***All*** *Male* ***are*** *Humans*
I: ***Some*** *Male* ***are*** *Philosophers*
I: ***Some*** *Philosophers* ***are*** *Humans*

Actually, we can remove the link between Philosophers and humans_gender_men and create direct (or non-direct) link between Philosophers and Humans. Performing the same operation for Scientists and Artists classes, it is possible to remove all links to humans_gender_men. Since there is no link, related with the class, we can simply delete this class from the graph. This leads to an increase in the abstraction level and decrease of complexity in the resulting ontology, which is a desired result.

CHAPTER 7

CONCLUSION

Currently, the categorical syllogism is the most studied scheme of deductive reasoning, so mathematical formalization with algorithmic implementation is of great interest to researchers in the fields of AI and cognitive sciences. The main contributions of this thesis are as follows:

- an algorithm for generation of a complete set of syllogistic cases for three sets of input data was proposed;
- an algorithm for calculation of truth ratios of the syllogistic moods was generalized for cases of inclusive/exclusive logics;
- structural properties of syllogistic systems S/^{2}S were revealed and evaluated;
- different aspects of fuzzification in S/^{2}S were considered;
- the extension of classical system with intermediate quantifiers (system ^{6}S) was examined;
- possible applications of the developed system as FSR component in cognitive architectures or ontology-based fuzzy-syllogistic reasoning were suggested.

Most of the algorithms, proposed in this thesis were implemented within the FSR_project which consists of two parts: the fuzzy-syllogistic reasoning engine (FSREngine) and thin client for the FSREngine (FSR_GUI). The FSREngine is implemented as a shared library and can be integrated in various applications related to the syllogistic reasoning. The FRSEngine supports all functions, related to syllogistic/fuzzy-syllogistic reasoning. The FSR_GUI consists of various tools for analyzing structural properties of systems S/^{2}S/^{6}S. Also FSR_GUI allows to perform fuzzy-syllogistic reasoning under a given data, that can be generated by the FSR_GUI or defined by the user (see Appendix F).

The designed systems have some limitations caused by logics applied. Currently, only two premises can be used to infer conclusion, so it is necessary to decompose input data, such as ontologies on triple sets. Overcoming this restriction will allow the system to become a universal mechanism for modeling of decision making.

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APPENDIX A

96 DISTINCT SYLLOGISTIC CASES

96 distinct syllogistic cases calculated in accordance with the algorithm discussed in Section 4.1.4

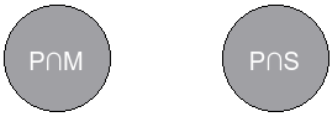
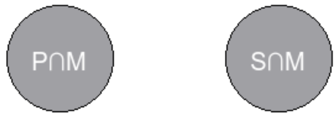
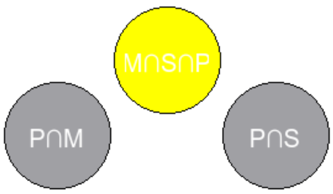
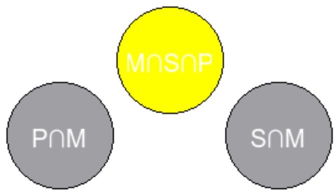
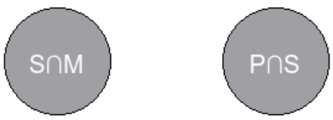
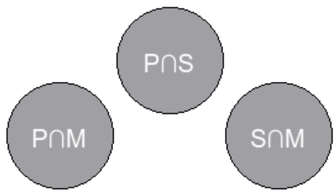
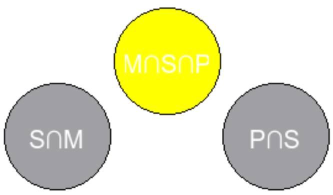
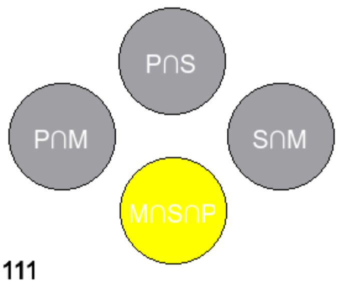
Table A.1. Syllogistic cases

Case	Space combination	Case	Space combination	Case	Space combination	Case	Space combination
1	0000110	25	0101100	49	1001100	73	1101000
2	0000111	26	0101101	50	1001101	74	1101001
3	0001010	27	0101110	51	1001110	75	1101010
4	0001011	28	0101111	52	1001111	76	1101011
5	0001100	29	0110001	53	1010001	77	1101100
6	0001101	30	0110010	54	1010010	78	1101101
7	0001110	31	0110011	55	1010011	79	1101110
8	0001111	32	0110101	56	1010100	80	1101111
9	0010101	33	0110110	57	1010101	81	1110000
10	0010110	34	0110111	58	1010110	82	1110001
11	0010111	35	0111000	59	1010111	83	1110010
12	0011001	36	0111001	60	1011001	84	1110011
13	0011010	37	0111010	61	1011010	85	1110100
14	0011011	38	0111011	62	1011011	86	1110101
15	0011100	39	0111100	63	1011100	87	1110110
16	0011101	40	0111101	64	1011101	88	1110111
17	0011110	41	0111110	65	1011110	89	1111000
18	0011111	42	0111111	66	1011111	90	1111001
19	0100011	43	1000011	67	1100001	91	1111010
20	0100101	44	1000110	68	1100011	92	1111011
21	0100110	45	1000111	69	1100100	93	1111100
22	0100111	46	1001001	70	1100101	94	1111101
23	0101010	47	1001010	71	1100110	95	1111110
24	0101011	48	1001011	72	1100111	96	1111111

APPENDIX B

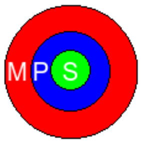
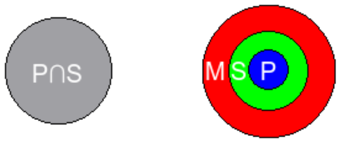
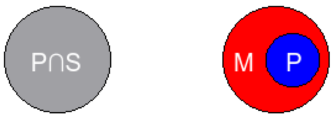
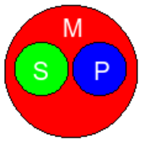
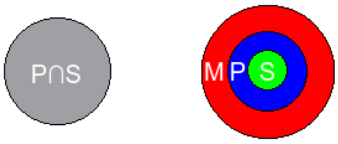
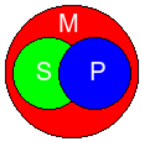
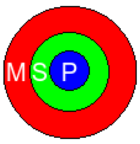
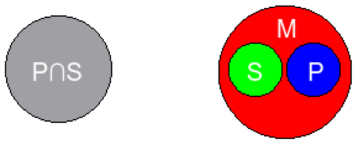
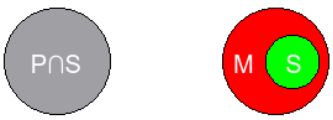
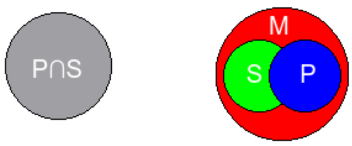
EULER DIAGRAMS FOR THE 96 DISTINCT CASES

Table B.1. Euler diagrams for the 96 syllogistic cases

Case	Euler diagram	Case	Euler diagram
1	 0000110	5	 0001100
2	 0000111	6	 0001101
3	 0001010	7	 0001110
4	 0001011	8	 0001111

(Cont. on next page)

Table B.1. (cont.)

9	 0010101	14	 0011011
10	 0010110	15	 0011100
11	 0010111	16	 0011101
12	 0011001	17	 0011110
13	 0011010	18	 0011111

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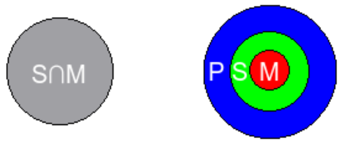
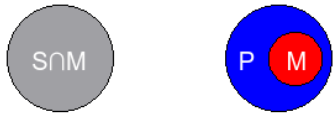
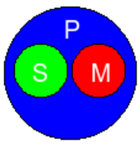
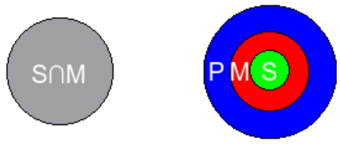
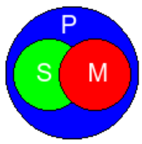
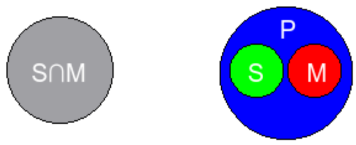
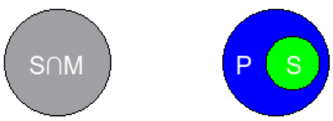
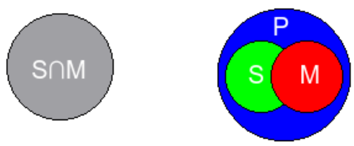
19	 0100011	24	 0101011
20	 0100101	25	 0101100
21	 0100110	26	 0101101
22	 0100111	27	 0101110
23	 0101010	28	 0101111

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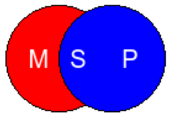
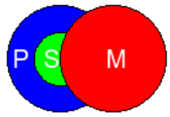
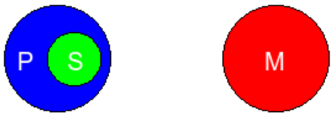
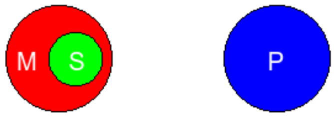
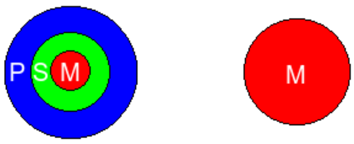
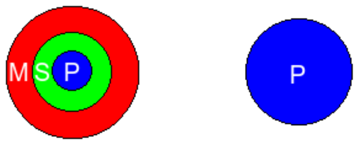
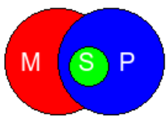
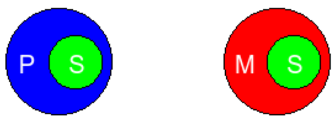
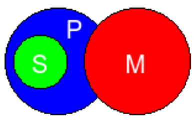
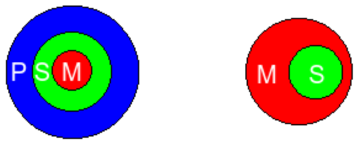
29	 0110001	34	 0110111
30	 0110010	35	 0111000
31	 0110011	36	 0111001
32	 0110101	37	 0111010
33	 0110110	38	 0111011

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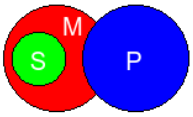
<tr><td>39</td><td>

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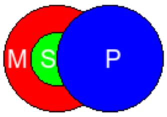
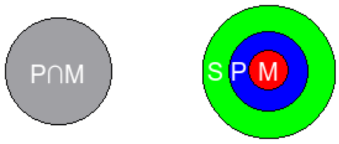
<tr><td>40</td><td>

0111101</td><td>45</td><td>

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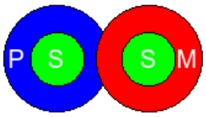
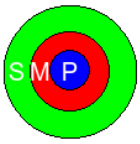
<tr><td>41</td><td>

0111110</td><td>46</td><td>

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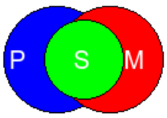
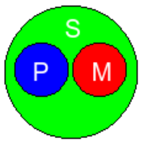
<tr><td>42</td><td>

0111111</td><td>47</td><td>

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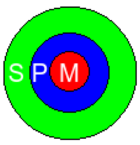
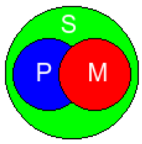
<tr><td>43</td><td>

1000011</td><td>48</td><td>

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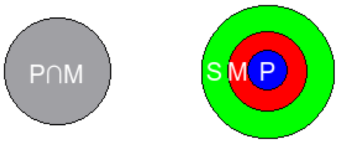
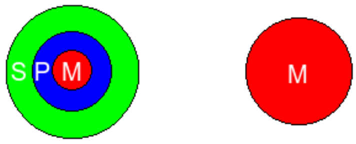
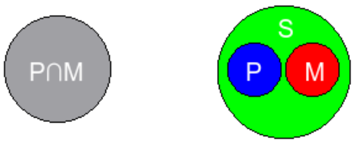
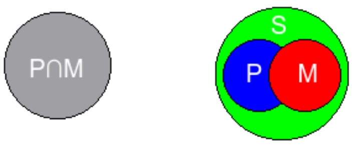
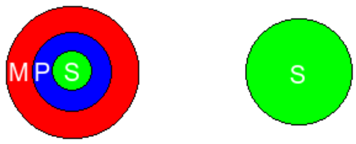
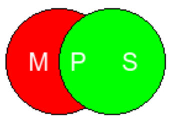
50	 1001101	55	 1010011
51	 1001110	56	 1010100
52	 1001111	57	 1010101
53	 1010001	58	 1010110

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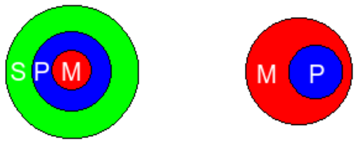
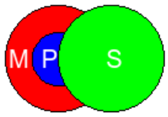
<tr><td>59</td><td>

1010111</td><td>64</td><td>

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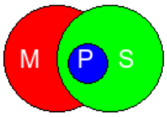
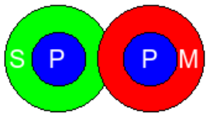
<tr><td>60</td><td>

1011001</td><td>65</td><td>

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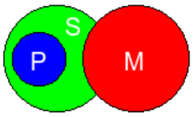
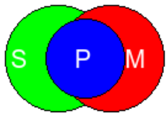
<tr><td>61</td><td>

1011010</td><td>66</td><td>

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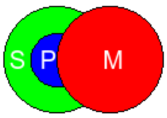
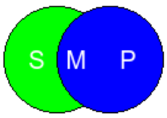
<tr><td>62</td><td>

1011011</td><td>67</td><td>

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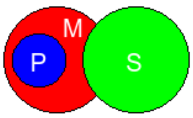
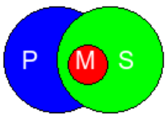
<tr><td>63</td><td>

1011100</td><td>68</td><td>

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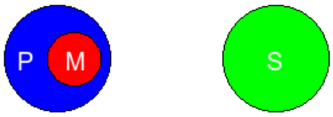
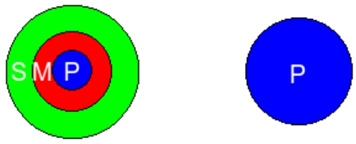
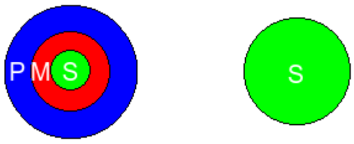
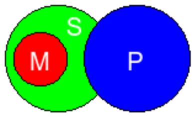
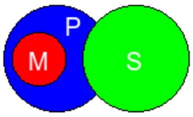
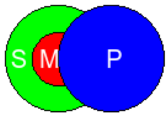
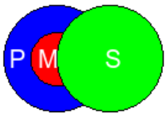
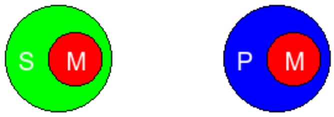
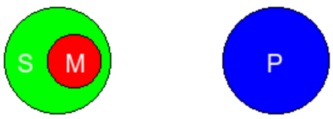
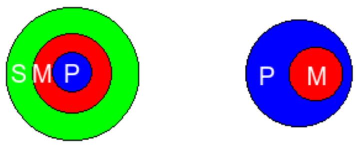
69	 1100100	74	 1101001
70	 1100101	75	 1101010
71	 1100110	76	 1101011
72	 1100111	77	 1101100
73	 1101000	78	 1101101

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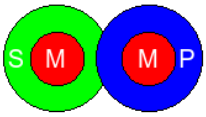
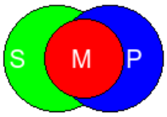
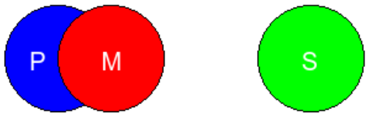
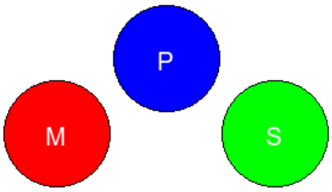
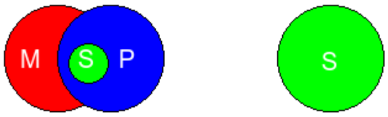
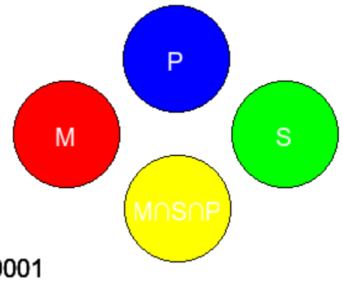
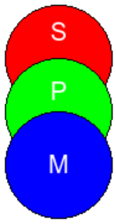
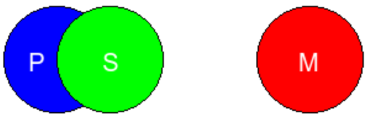
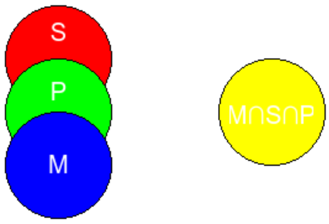
79	 1101110	84	 1110011
80	 1101111	85	 1110100
81	 1110000	86	 1110101
82	 1110001	87	 1110110
83	 1110010	88	 1110111

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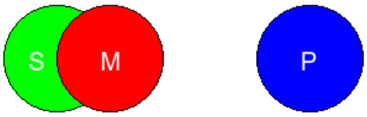
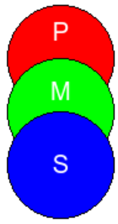
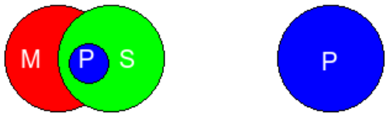
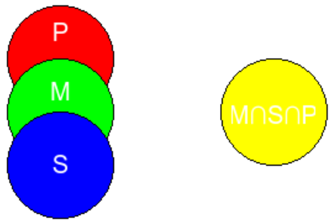
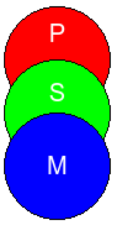
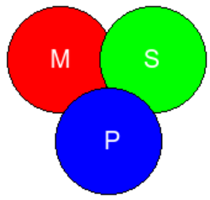
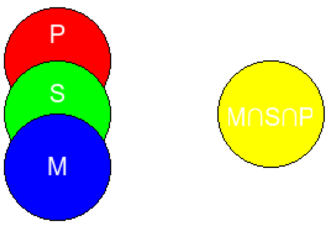
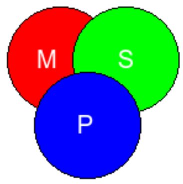
Table B.1. (cont.)

89	 1111000	93	 1111100
90	 1111001	94	 1111101
91	 1111010	95	 1111110
92	 1111011	96	 1111111

APPENDIX C

DISTINCT SYLLOGISTIC MOODS WITH THEIR TRUTH RATIOS FOR SYSTEM S

Table C.1. Distinct syllogistic moods for system S

#	Truth ratio, τ	Moods	Moods in group	t[3]	f	#	Truth ratio, τ	Moods	Moods in group	t	f
1	1.000	EIO-4; EIO-3; EIO-2; EIO-1;	4	11	0	20	0.909	AOI-3;	1	10	1
2	1.000	OAO-3;	1	11	0	21	0.909	OAI-3;	1	10	1
3	1.000	AII-3; AII-1;	2	10	0	22	0.902	OOI-2;	1	65	7
4	1.000	IAI-4; IAI-3;	2	10	0	23	0.900	EOO-4; EOO-3;	2	9	1
5	1.000	AOO-2;	1	9	0	24	0.900	IAO-4; IAO-3;	2	9	1
6	1.000	EAO-4; EAO-3;	2	5	0	25	0.895	IOI-2; IOI-1;	2	60	7
7	1.000	AAI-3;	1	4	0	26	0.895	OII-4; OII-2;	2	60	9
8	1.000	AAI-1; AAA-1;	2	1	0	27	0.885	III-4; III-3; III-2; III-1;	4	62	9
9	1.000	AAO-4; AAI-4;	2	1	0	28	0.871	OOO-3;	1	61	9
10	1.000	AEO-4; AEO-2; AEE-4; AEE-2;	4	1	0	29	0.865	OIO-4; OIO-2;	2	58	9
11	1.000	EAO-2; EAO-1; EAE-2; EAE-1;	4	1	0	30	0.865	OOI-1;	1	58	10
12	0.928	AIO-4; AIO-2;	2	13	1	31	0.865	OOI-4;	1	58	10
13	0.928	AOI-1;	1	13	1	32	0.857	IIO-4; IIO-3; IIO-2; IIO-1;	4	60	11
14	0.928	AOO-4;	1	13	1	33	0.850	IOO-2; IOO-1;	2	57	11
15	0.928	EOO-2; EOO-1;	2	13	1	34	0.847	OOO-2;	1	61	11
16	0.928	OAI-4;	1	13	1	35	0.845	IOI-4; IOI-3;	2	60	1
17	0.928	OAO-4;	1	13	1	36	0.845	OII-3; OII-1;	2	60	13
18	0.915	OIO-3; OIO-1;	2	65	6	37	0.833	AAO-2;	1	5	13
19	0.910	OOO-1;	1	61	6	38	0.816	IOO-4; IOO-3;	2	58	13

(Cont. on next page)

3 t denotes number of true cases, f denotes number of false cases for the corresponding mood

Table C.1. (cont.)

#	Truth ratio, τ	Moods	Moods in group	t	f	#	Truth ratio, τ	Moods	Moods in group	t	f
39	0.814	OOI-3;	1	57	1	64	0.642	OEO-4; OEO-2;	2	9	5
40	0.805	OOO-4;	1	54	1	65	0.636	IEO-4; IEO-3; IEO-2; IEO-1;	4	7	4
41	0.800	AEI-3; AEI-1;	2	4	3	66	0.600	AIO-3; AIO-1;	2	6	4
42	0.800	EAI-4; EAI-3;	2	4	3	67	0.600	AEO-3; AEO-1;	2	3	2
43	0.785	EOI-2; EOI-1;	2	11	3	68	0.545	AOO-3;	1	6	5
44	0.785	OAO-1;	1	11	1	69	0.454	AOA-3;	1	5	6
45	0.785	OEI-4; OEI-2;	2	11	1	70	0.400	AEA-3; AEA-1;	2	2	3
46	0.750	AAO-3;	1	3	1	71	0.400	AIA-3; AIA-1;	2	4	6
47	0.750	EEI-4; EEI-3; EEI-2; EEI-1;	4	3	1	72	0.363	IEA-4; IEA-3; IEA-2; IEA-1;	4	4	7
48	0.750	EEO-4; EEO-3; EEO-2; EEO-1;	4	3	1	73	0.357	AOA-1;	1	5	9
49	0.727	EII-4; EII-3; EII-2; EII-1;	4	8	3	74	0.357	AOE-4;	1	5	9
50	0.727	IEI-4; IEI-3; IEI-2; IEI-1;	4	8	3	75	0.357	OAE-1;	1	5	9
51	0.714	AII-4; AII-2;	2	10	4	76	0.357	OEA-4; OEA-2;	2	5	9
52	0.714	IAI-2; IAI-1;	2	10	4	77	0.333	AAE-2;	1	2	4
53	0.714	IAO-2; IAO-1;	2	10	4	78	0.333	AOE-2;	1	3	6
54	0.700	EOI-4; EOI-3;	2	7	3	79	0.333	OAA-2;	1	3	6
55	0.700	OEI-3; OEI-1;	2	7	3	80	0.333	OAE-2;	1	3	6
56	0.700	OEO-3; OEO-1;	2	7	3	81	0.300	EOE-4; EOE-3;	2	3	7
57	0.666	AOI-2;	1	6	3	82	0.300	OEA-3; OEA-1;	2	3	7
58	0.666	OAI-2;	1	6	3	83	0.300	OEE-3; OEE-1;	2	3	7
59	0.666	OAO-2;	1	6	3	84	0.285	AIE-4; AIE-2;	2	4	10
60	0.666	AAI-2;	1	4	2	85	0.285	IAA-2; IAA-1;	2	4	10
61	0.642	AOI-4;	1	9	5	88	0.272	IEE-4; IEE-3; IEE-2; IEE-1;	4	3	8
62	0.642	AOO-1;	1	9	5	89	0.250	AAA-3;	1	1	3
63	0.642	OAI-1;	1	9	5	90	0.250	EEA-4; EEA-3; EEA-2; EEA-1;	4	1	3

(Cont. on next page)

Table C.1. (cont.)

#	Truth ratio, τ	Moods	Moods in group	t	f	#	Truth ratio, τ	Moods	Moods in group	t	f
93	0.214	OAA-1;	1	3	11	114	0.100	IAA-4; IAA-3;	2	1	
94	0.214	OEE-4; OEE-2;	2	3	11	115	0.097	OOE-2;	1	7	65
93	0.200	AEE-3; AEE-1;	2	1	4	116	0.090	AOE-3;	1	1	10
94	0.200	EAE-4; EAE-3;	2	1	4	117	0.090	OAE-3;	1	1	10
95	0.194	OOA-4;	1	13	54	118	0.089	OOA-1;	1	6	61
96	0.185	OOE-3;	1	13	57	119	0.084	OIA-3; OIA-1;	2	6	65
97	0.183	IOA-4; IOA-3;	2	13	58	120	0.071	AIA-4; AIA-2;	2	1	13
98	0.166	AAA-2;	1	1	1	121	0.071	AOA-4;	1	1	13
99	0.154	IOE-4; IOE-3;	2	11	3	122	0.071	AOE-1;	1	1	13
100	0.154	OIE-3; OIE-1;	2	11	3	123	0.071	EOA-2; EOA-1;	2	1	13
101	0.152	OOA-2;	1	11		124	0.071	OAA-4;	1	1	13
102	0.149	IOA-2; IOA-1;	2	10		125	0.071	OAE-4;	1	1	13
103	0.142	IIA-4; IIA-3; IIA-2; IIA-1;	4	10		126	0.000	EAI-1; EAA-1; EAI-2; EAA-2;	4	0	1
104	0.134	OIA-4; OIA-2;	2	9		127	0.000	AEI-2; AEA-2; AEI-4; AEA-4;	4	0	1
105	0.134	OOE-1;	1	9		128	0.000	AAO-1; AAE-1;	2	0	1
106	0.134	OOE-4;	1	9		129	0.000	AAE-4; AAA-4;	2	0	1
107	0.128	OOA-3;	1	9		130	0.000	AAE-3;	1	0	4
108	0.114	IIE-4; IIE-3; IIE-2; IIE-1;	4	8		131	0.000	EAA-3; EAA-4;	2	0	5
109	0.104	IOE-2; IOE-1;	2	7		132	0.000	AOA-2;	1	0	9
110	0.104	OIE-4; OIE-2;	2	7		133	0.000	IAE-3; IAE-4;	2	0	10
111	0.100	EOA-4; EOA-3;	2	1		134	0.000	AIE-1; AIE-3;	2	0	10
112	0.100	IAA-4; IAA-3;	2	1		135	0.000	OAA-3;	1	0	11
113	0.100	EOA-4; EOA-3;	2	1		136	0.000	EIA-1; EIA-2; EIA-3; EIA-4;	4	0	11

APPENDIX D

DISTINCT SYLLOGISTIC MOODS WITH THEIR TRUTH RATIOS FOR SYSTEM ^{2}S

Table D.1. Distinct syllogistic moods for system ^{2}S

#	Truth ratio, τ	Moods	Moods in group	t[4]	f	#	Truth ratio, τ	Moods	Moods in group	t	f
1	1.000	OAO-3; OAI-3; IAO-3; IAI-3;	4	6	0	8	0.833	OOO-2; OOI-2; OIO-2; OII-2; IOO-2; IOI-2; IIO-2; III-2;	8	40	8
2	1.000	AAA-1;	1	1	0	9	0.800	EOO-2; EOO-1; EOI-2; EOI-1; EIO-2; EIO-1; EII-2; EII-1;	8	8	2
3	1.000	AAO-4; AAI-4;	2	1	0	10	0.800	EAO-4; EAO-3; EAI-4; EAI-3;	4	4	1
4	1.000	AEE-4; AEE-2;	2	1	0	11	0.750	AOO-2; AOI-2; AIO-2; AII-2;	4	6	2
5	1.000	EAE-2; EAE-1;	2	1	0	12	0.750	AAO-3; AAI-3;	2	3	1
6	0.888	OAO-4; OAI-4; IAO-4; IAI-4;	4	8	1	13	0.744	OOO-4; OOI-4; OIO-4; OII-4; IOO-4; IOI-4; IIO-4; III-4;	8	35	12
7	0.851	OOO-1; OOI-1; OIO-1; OII-1; IOO-1; IOI-1; IIO-1; III-1;	8	40	7	14	0.740	OOO-3; OOI-3; OIO-3; OII-3; IOO-3; IOI-3; IIO-3; III-3;	8	40	14

(Cont. on next page)

4 t denotes number of true cases, f denotes number of false cases for the corresponding mood

Table D.1. (cont.)

#	Truth ratio, τ	Moods	Moods in group	t	f	#	Truth ratio, τ	Moods	Moods in group	t	f
15	0.666	AOO-1; AOI-1; AIO-1; AII-1;	4	6	3	28	0.375	OAO-2; OAI-2; IAO-2; IAI-2;	4	3	
16	0.666	EOO-4; EOO-3; EOI-4; EOI-3; EIO-4; EIO-3; EII-4; EII-3;	8	4	2	29	0.333	AAE-2;	1	2	
17	0.615	AOO-4; AOI-4; AIO-4; AII-4;	4	8		30	0.333	EOE-4; EOE-3; EIE-4; EIE-3;	4	2	4
18	0.500	AAO-2; AAI-2;	2	3		31	0.333	OEA-3; OEA-1; IEA-3; IEA-1;	4	2	4
19	0.500	AOA-3; AIA-3;	2	3		32	0.333	OEE-3; OEE-1; IEE-3; IEE-1;	4	2	4
20	0.500	AOO-3; AOI-3; AIO-3; AII-3;	4	3		33	0.333	OEO-3; OEO-1; OEI-3; OEI-1; IEO-3; IEO-1; IEI-3; IEI-1;	8	2	4
21	0.500	EEO-4; EEO-3; EEO-2; EEO-1; EEI-4; EEI-3; EEI-2; EEI-1;	8	2		34	0.333	AOA-1; AIA-1;	2	3	6
22	0.461	OAO-1; OAI-1; IAO-1; IAI-1;	4	6		35	0.307	AOE-4; AIE-4;	2	4	9
23	0.400	AEA-3; AEA-1;	2	2		36	0.307	OAE-1; IAE-1;	2	4	9
24	0.400	AEO-3; AEO-1; AEI-3; AEI-1;	4	2		37	0.250	AAA-3;	1	1	3
25	0.400	OEA-4; OEA-2; IEA-4; IEA-2;	4	4		38	0.250	EEA-4; EEA-3; EEA-2; EEA-1;	4	1	3
26	0.400	OEO-4; OEO-2; OEI-4; OEI-2; IEO-4; IEO-2; IEI-4; IEI-2;	8	4		39	0.250	EEE-4; EEE-3; EEE-2; EEE-1;	4	1	3
27	0.375	OAA-2; IAA-2;	2	3		40	0.250	AOE-2; AIE-2;	2	2	6

(Cont. on next page)

Table D.1. (cont.)

#	Truth ratio, τ	Moods	Moods in group	t	f	#	Truth ratio, τ	Moods	Moods in group	t	f
41	0.250	OAE-2; IAE-2;	2	2	6	56	0.063	OOA-1; OIA-1; IOA-1; IIA-1;	4	3	44
42	0.230	OAA-1; IAA-1;	2	3	10	57	0.041	OOE-2; OIE-2; IOE-2; IIE-2;	4	2	46
43	0.200	AEE-3; AEE-1;	2	1	4	58	0.000	EAO-1; EAI-1; EAA-1; EAO-2; EAI-2; EAA-2;	6	0	1
44	0.200	EAE-4; EAE-3;	2	1	4	59	0.000	AEO-2; AEI-2; AEA-2; AEO-4; AEI-4; AEA-4;	6	0	1
45	0.200	EOE-2; EOE-1; EIE-2; EIE-1;	4	2	8	60	0.000	AAO-1; AAI-1; AAE-1;	3	0	1
46	0.200	OEE-4; OEE-2; IEE-4; IEE-2;	4	2	8	61	0.000	AAE-4; AAA-4;	2	0	1
47	0.170	OOA-4; OIA-4; IOA-4; IIA-4;	4	8	39	62	0.000	AAE-3;	1	0	4
48	0.166	AAA-2;	1	1	5	63	0.000	EAA-3; EAA-4;	2	0	5
49	0.148	OOE-3; OIE-3; IOE-3; IIE-3;	4	8	46	64	0.000	OAE-3; OAA-3; IAE-3; IAA-3;	4	0	6
50	0.125	OOA-2; OIA-2; IOA-2; IIA-2;	4	6	42	65	0.000	EOA-3; EOA-4; EIA-3; EIA-4;	4	0	6
51	0.111	OAA-4; IAA-4;	2	1	8	66	0.000	AOE-3; AIE-3;	2	0	6
52	0.111	OOA-3; OIA-3; IOA-3; IIA-3;	4	6	48	67	0.000	AOA-2; AIA-2;	2	0	8
53	0.085	OOE-1; OIE-1; IOE-1; IIE-1;	4	4	43	68	0.000	OAE-4; IAE-4;	2	0	9
54	0.085	OOE-4; OIE-4; IOE-4; IIE-4;	4	4	43	69	0.000	AOE-1; AIE-1;	2	0	9
55	0.076	AOA-4; AIA-4;	2	1	12	70	0.000	EOA-1; EOA-2; EIA-1; EIA-2;	4	0	10

APPENDIX E

POINT-SYMMETRIC MOODS (FOR SYSTEM S)

Table E.1. Symmetric moods for system S

#	Mood	Truth ratio, τ	t[5]	f	Mood	Truth ratio, τ	t	f
1	EIO-4	1.000	11	0	EIA-4	0.000	0	11
2	EIO-3	1.000	11	0	EIA-3	0.000	0	11
3	EIO-2	1.000	11	0	EIA-2	0.000	0	11
4	EIO-1	1.000	11	0	EIA-1	0.000	0	11
5	OAO-3	1.000	11	0	OAA-3	0.000	0	11
6	AII-3	1.000	10	0	AIE-3	0.000	0	10
7	AII-1	1.000	10	0	AIE-1	0.000	0	10
8	IAI-4	1.000	10	0	IAE-4	0.000	0	10
9	IAI-3	1.000	10	0	IAE-3	0.000	0	10
10	AOO-2	1.000	9	0	AOA-2	0.000	0	9
11	EAO-4	1.000	5	0	EAA-4	0.000	0	5
12	EAO-3	1.000	5	0	EAA-3	0.000	0	5
13	AAI-3	1.000	4	0	AAE-3	0.000	0	4
14	AAI-1	1.000	1	0	AAO-1	0.000	0	1
15	AAA-1	1.000	1	0	AAO-1	0.000	0	1
16	AAO-4	1.000	1	0	AAE-4	0.000	0	1
17	AAI-4	1.000	1	0	AAE-4	0.000	0	1
18	AEO-4	1.000	1	0	AEA-4	0.000	0	1
19	AEO-2	1.000	1	0	AEI-4	0.000	0	1
20	AEE-4	1.000	1	0	AEA-2	0.000	0	1
21	AEE-2	1.000	1	0	AEI-2	0.000	0	1
22	EAO-2	1.000	1	0	EAA-2	0.000	0	1
23	EAO-1	1.000	1	0	EAI-2	0.000	0	1
24	EAE-2	1.000	1	0	EAA-1	0.000	0	1
25	EAE-1	1.000	1	0	EAI-1	0.000	0	1
26	AIO-4	0.928	13	1	AIA-4	0.071	1	13
27	AIO-2	0.928	13	1	AIA-4	0.071	1	13

(Cont. on next page)

5 t denotes number of true cases, f denotes number of false cases for the corresponding mood

Table E.1. (cont.)

#	Mood	Truth ratio, τ	t	f	Mood	Truth ratio, τ	t	f
28	AOI-1	0.928	13	1	AOE-1	0.071	1	13
29	AOO-4	0.928	13	1	AOA-4	0.071	1	13
30	EOO-2	0.928	13	1	EOA-2	0.071	1	13
31	EOO-1	0.928	13	1	EOA-2	0.071	1	13
32	OAI-4	0.928	13	1	OAE-4	0.071	1	13
33	OAO-4	0.928	13	1	OAA-4	0.071	1	13
34	OIO-3	0.915	65	6	OIA-1	0.084	6	65
35	OIO-1	0.915	65	6	OIA-3	0.084	6	65
36	OOO-1	0.910	61	6	OOA-1	0.089	6	61
37	AOI-3	0.909	10	1	AOE-3	0.090	1	10
38	OAI-3	0.909	10	1	OAE-3	0.090	1	10
39	OOI-2	0.902	65	7	OOE-2	0.097	7	65
40	EOO-4	0.900	9	1	EOA-4	0.100	1	9
41	EOO-3	0.900	9	1	EOA-4	0.100	1	9
42	IAO-4	0.900	9	1	IAA-4	0.100	1	9
43	IAO-3	0.900	9	1	IAA-4	0.100	1	9
44	IOI-2	0.895	60	7	IOE-2	0.104	7	60
45	IOI-1	0.895	60	7	IOE-2	0.104	7	60
46	OII-4	0.895	60	7	OIE-4	0.104	7	60
47	OII-2	0.895	60	7	OIE-4	0.104	7	60
48	III-4	0.885	62	8	IIE-1	0.114	8	62
49	III-3	0.885	62	8	IIE-2	0.114	8	62
50	III-2	0.885	62	8	IIE-3	0.114	8	62
51	III-1	0.885	62	8	IIE-4	0.114	8	62
52	OOO-3	0.871	61	9	OOA-3	0.128	9	61
53	OIO-4	0.865	58	9	OIA-4	0.134	9	58
54	OIO-2	0.865	58	9	OIA-4	0.134	9	58
55	OOI-1	0.865	58	9	OOE-1	0.134	9	58
56	OOI-4	0.865	58	9	OOE-4	0.134	9	58
57	IIO-4	0.857	60	10	IIA-1	0.142	10	60
58	IIO-3	0.857	60	10	IIA-2	0.142	10	60
59	IIO-2	0.857	60	10	IIA-3	0.142	10	60
60	IIO-1	0.857	60	10	IIA-4	0.142	10	60

(Cont. on next page)

Table E.1. (cont.)

#	Mood	Truth ratio, τ	t	f	Mood	Truth ratio, τ	t	f
61	IOO-2	0.850	57	10	IOA-1	0.149	10	57
62	IOO-1	0.850	57	10	IOA-2	0.149	10	57
63	OOO-2	0.847	61	11	OOA-2	0.152	11	61
64	IOI-4	0.845	60	11	IOE-4	0.154	11	60
65	IOI-3	0.845	60	11	IOE-4	0.154	11	60
66	OII-3	0.845	60	11	OIE-3	0.154	11	60
67	OII-1	0.845	60	11	OIE-3	0.154	11	60
68	AAO-2	0.833	5	1	AAA-2	0.166	1	5
69	IOO-4	0.816	58	13	IOA-3	0.183	13	58
70	IOO-3	0.816	58	13	IOA-4	0.183	13	58
71	OOI-3	0.814	57	13	OOE-3	0.185	13	57
72	OOO-4	0.805	54	13	OOA-4	0.194	13	54
73	AEI-3	0.800	4	1	AEE-3	0.200	1	4
74	AEI-1	0.800	4	1	AEE-3	0.200	1	4
75	EAI-4	0.800	4	1	EAE-4	0.200	1	4
76	EAI-3	0.800	4	1	EAE-4	0.200	1	4
77	EOI-2	0.785	11	3	EOE-2	0.214	3	11
78	EOI-1	0.785	11	3	EOE-2	0.214	3	11
79	OAO-1	0.785	11	3	OAA-1	0.214	3	11
80	OEI-4	0.785	11	3	OEE-4	0.214	3	11
81	OEI-2	0.785	11	3	OEE-4	0.214	3	11
82	AAO-3	0.750	3	1	AAA-3	0.250	1	3
83	EEI-4	0.750	3	1	EEE-2	0.250	1	3
84	EEI-3	0.750	3	1	EEE-3	0.250	1	3
85	EEI-2	0.750	3	1	EEE-4	0.250	1	3
86	EEI-1	0.750	3	1	EEE-4	0.250	1	3
87	EEO-4	0.750	3	1	EEA-2	0.250	1	3
88	EEO-3	0.750	3	1	EEA-3	0.250	1	3
89	EEO-2	0.750	3	1	EEA-4	0.250	1	3
90	EEO-1	0.750	3	1	EEA-4	0.250	1	3
91	EII-4	0.727	8	3	EIE-4	0.272	3	8
92	EII-3	0.727	8	3	EIE-4	0.272	3	8
93	EII-2	0.727	8	3	EIE-4	0.272	3	8

(Cont. on next page)

Table E.1. (cont.)

#	Mood	Truth ratio, τ	t	f	Mood	Truth ratio, τ	t	f
94	EII-1	0.727	8	3	EIE-4	0.272	3	8
95	IEI-4	0.727	8	3	IEE-4	0.272	3	8
96	IEI-3	0.727	8	3	IEE-4	0.272	3	8
97	IEI-2	0.727	8	3	IEE-4	0.272	3	8
98	IEI-1	0.727	8	3	IEE-4	0.272	3	8
99	AII-4	0.714	10	4	AIE-4	0.285	4	10
100	AII-2	0.714	10	4	AIE-4	0.285	4	10
101	IAI-2	0.714	10	4	IAE-2	0.285	4	10
102	IAI-1	0.714	10	4	IAE-2	0.285	4	10
103	IAO-2	0.714	10	4	IAA-2	0.285	4	10
104	IAO-1	0.714	10	4	IAA-2	0.285	4	10
105	EOI-4	0.700	7	3	EOE-4	0.300	3	7
106	EOI-3	0.700	7	3	EOE-4	0.300	3	7
107	OEI-3	0.700	7	3	OEE-3	0.300	3	7
108	OEI-1	0.700	7	3	OEE-3	0.300	3	7
109	OEO-3	0.700	7	3	OEA-3	0.300	3	7
110	OEO-1	0.700	7	3	OEA-3	0.300	3	7
111	AOI-2	0.666	6	3	AOE-2	0.333	3	6
112	OAI-2	0.666	6	3	OAE-2	0.333	3	6
113	OAO-2	0.666	6	3	OAA-2	0.333	3	6
114	AAI-2	0.666	4	2	AAE-2	0.333	2	4
115	AOI-4	0.642	9	5	AOE-4	0.357	5	9
116	AOO-1	0.642	9	5	AOA-1	0.357	5	9
117	OAI-1	0.642	9	5	OAE-1	0.357	5	9
118	OEO-4	0.642	9	5	OEA-4	0.357	5	9
119	OEO-2	0.642	9	5	OEA-4	0.357	5	9
120	IEO-4	0.636	7	4	IEA-1	0.363	4	7
121	IEO-3	0.636	7	4	IEA-2	0.363	4	7
122	IEO-2	0.636	7	4	IEA-3	0.363	4	7
123	IEO-1	0.636	7	4	IEA-4	0.363	4	7
124	AIO-3	0.600	6	4	AIA-1	0.400	4	6
125	AIO-1	0.600	6	4	AIA-3	0.400	4	6
126	AEO-3	0.600	3	2	AEA-1	0.400	2	3
127	AEO-1	0.600	3	2	AEA-3	0.400	2	3
128	AOO-3	0.545	6	5	AOA-3	0.454	5	6

APPENDIX F

FSR_PROJECT

Most of the algorithms, discussed in this thesis were implemented within the FSR_project software system. FSR_project consists of two parts: FSREngine and FSR_GUI_Client. The simplified UML diagram (without data types) is shown in Fig. E.1. FSREngine is implemented as a shared library and provides an interface for fuzzy-syllogistic reasoning. FSREngine class supply with the following methods (see Listing E.1).

```
class FSRENGINESHARED_EXPORT FSR_Engine
{
public:
     FSR_Engine(bool A_in_I , bool E_in_O);
     ~FSR_Engine();
     //statistical information about syllogystic system
     void recalculateStatTable(bool A_in_I , bool E_in_O);
     QListStatElem getStatTable();
     QList<QListQChar> getMoods();
     QList<QListBool> getCases();
     QList<groupedMoods> getGroupedMoods();
     QList<distinctMoods> getDistinctMoods();
     QList<symmetricMoods> getSymmetricMoods();
     //get lists of true and false cases for given figure
     bool getTC_FCforFig(int figureNumber, QList<QListInt> &TCList,
                                 QList<QListInt> &FCList);
     //get statistical information by given mood
     elemStatTableCell getStatisticsByMood(QString mood, int figNum);
     QList<fuzzyMoods> calculateRelaxedMoods(bool AlmostA, bool AlmostE);

//Reasoning on real data
  QList<FSRResultElem> SyllReas_ClassicVersion(QList<QString> setM,
           QList<QString> setP, QList<QString> setS, bool A_in_I,
                  bool E_in_O, bool useDistMoods = false);

 QList<FSRRelaxedResultElem> SyllReasWithRelaxedMoods(QList<QString>
           setM, QList<QString> setP, QList<QString> setS, bool
     A_in_I, bool E_in_O, bool useDistMoods = false);

  QList<FSRFuzzyResultElem> SyllReasWithFuzzyMoods(QList<QString>
           setM, QList<QString> setP, Qlist<QString> setS, bool
     A_in_I, bool E_in_O, bool useDistMoods = false);

     //private part of class
}
```

Listing E.1. Open interface of FSREngine class

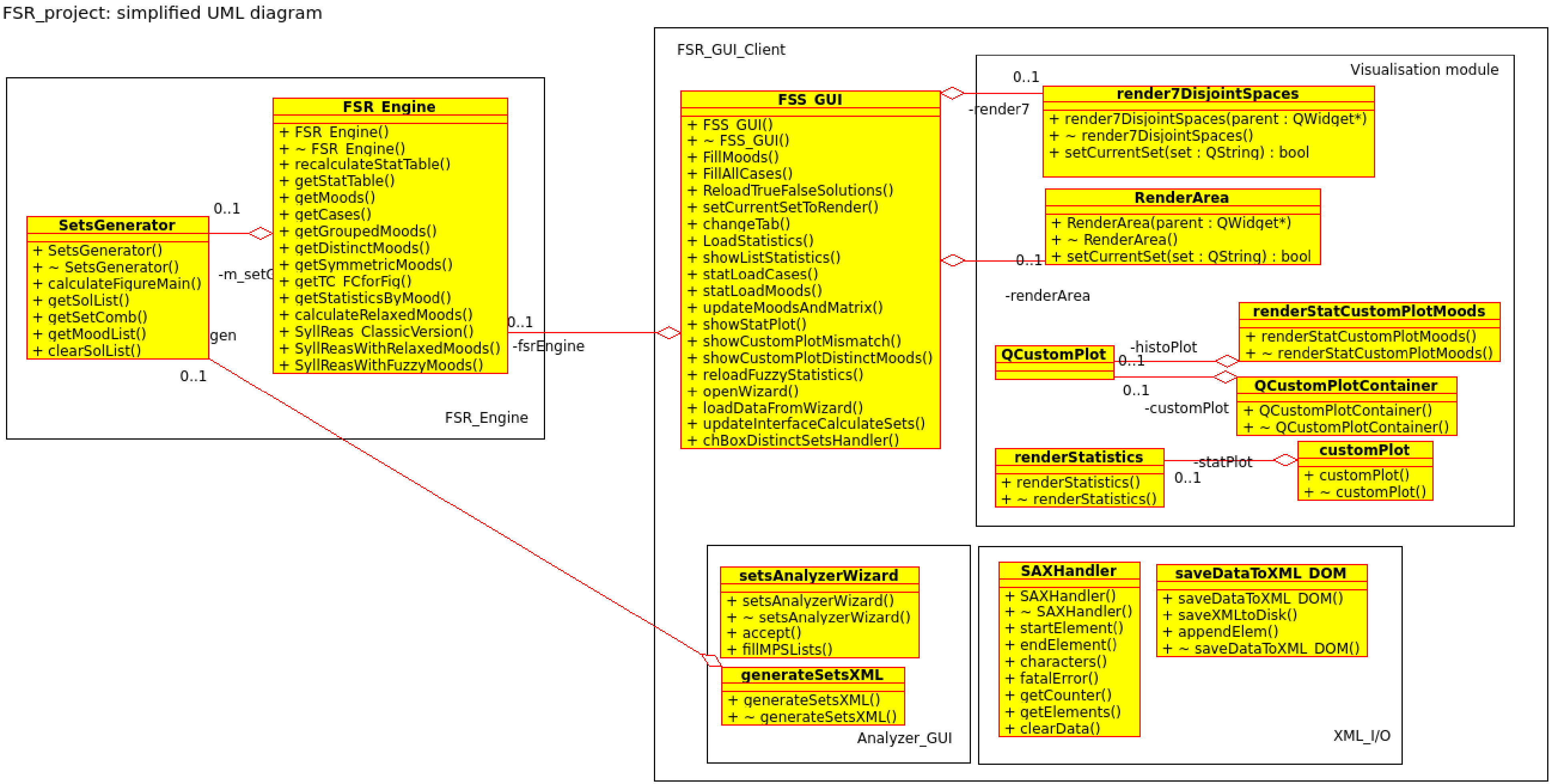


Figure E.1: UML diagram of the FSR_Project

The FSR_GUI_Client is an thin client for FSREngine shared library. It consists of different modules, such as visualization module (includes all classes related to sets visualization, various plots, graphs etc.), XML_module (includes classes, related to XML processing) and Analyzer module (includes classes, related to processing of input data for reasoning).

FSR_GUI_Client: GUI and functionality

GUI of FSR_GUI_Client consists of 5 tabs, such as “Cases”, “Moods”, “Statistics”, “Sets Analysis”, “Mood relaxation” and “Info”. Bellow is a brief description of each tab with elements and functionality.

E.1. Tab “Cases”

The tab “Cases” consists of 96 syllogistic cases with Euler diagram and set-theoretical representation for each case (see Fig. E.2). If the check-box “Sort by the number of spaces” is selected, the list of cases is grouped by the number of space in ascending order.

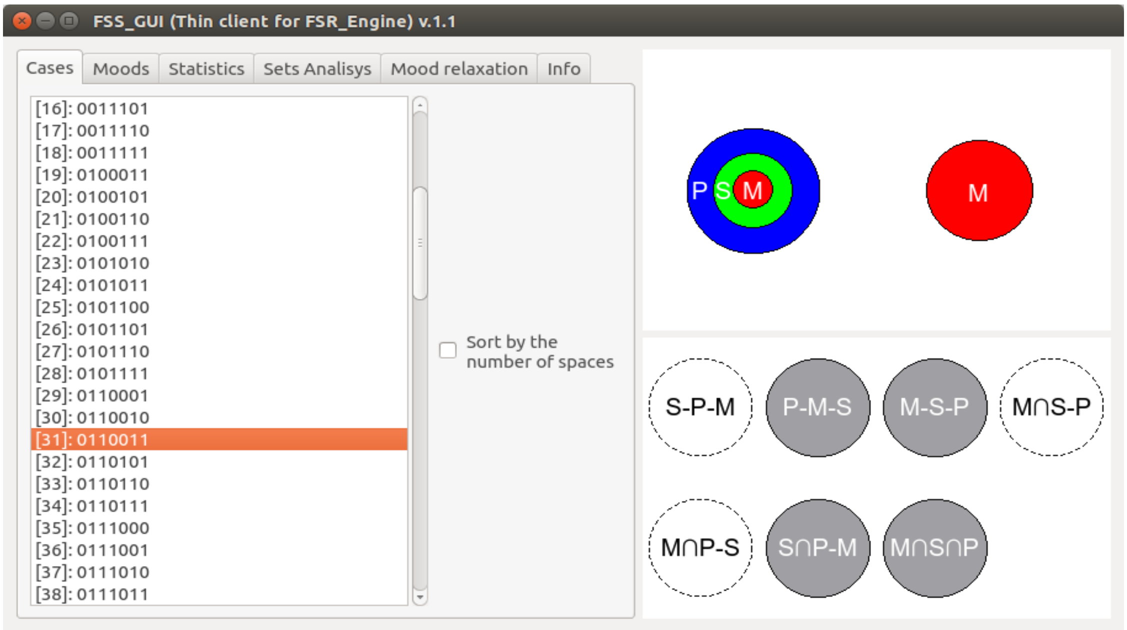


Figure E.2: FSR_GUI Application, tab “Cases”

E.2. Tab "Moods"

The tab "Moods" consist of the list of the moods for each of 4 figures with their truth ratios and detailed information about true/false cases of each mood (see Fig. E.3). For each case the Euler diagram and set-theoretical representation is provided. The check-boxes "A case in I" and "E case in O" enables the ability of using inclusive/exclusive logic in accordance with the algorithms from Section 4.1. The states of these check-boxes affect the whole system, including statistics and analysis modules.

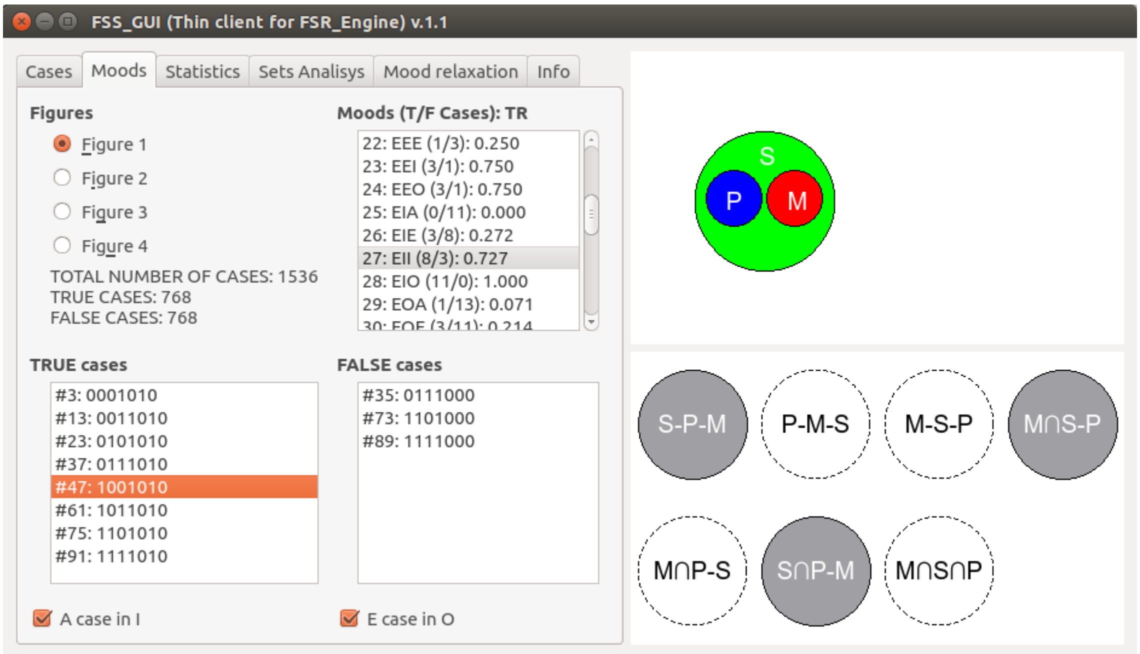


Figure E.3: FSR_GUI Application, tab "Moods"

E.3. Tab "Statistics"

The tab "Statistics" includes various statistical information about systems $S/^2S$ in tabular and graphical form. In group of plots, each plot is produced so that it consists of only true or only false cases for the all of the moods (according to the value of radio-buttons "Only T cases", "Only F cases") .

E.3.1. Statistics: Mood/Figure

If the radio-button "Moods/Figure" is marked, a table is displayed with all moods in alphabetical order grouped by figures and with their truth ratios (see Fig. E.4.).

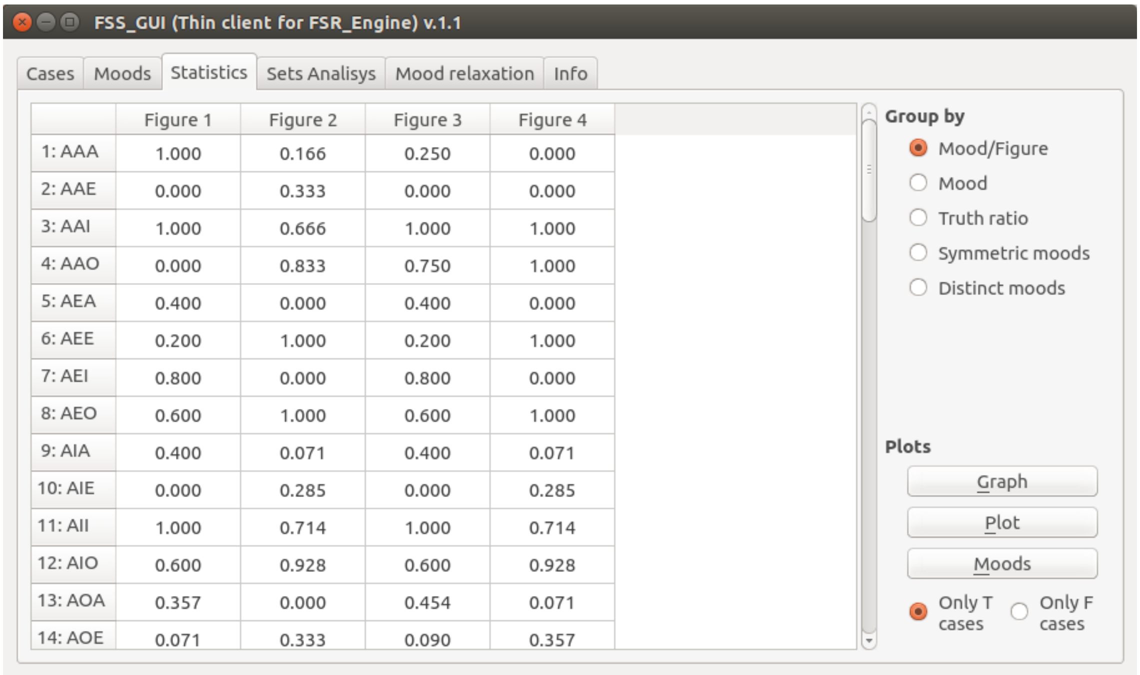


Figure E.4: FSR_GUI Application, tab “Statistics”, moods, grouped by figures with their truth ratios

E.3.2. Statistics: Mood

If the radio-button “Mood” is checked, a table is displayed with all moods with their truth ratios sorted in descending order (see Fig. E.5.). For each mood a detailed information about corresponding of truth/false cases is provided.

E.3.3. Statistics: Truth ratio

If the radio-button “Truth ratio” is checked, a table is displayed with all possible values of truth ratios(see Fig. E.6.). For each group of the truth ratios a detailed information about the corresponding moods is provided.

E.3.4. Statistics: Symmetric moods

If the radio-button “Symmetric moods” is checked, a table is displayed with all 128 symmetric moods (see Fig. E.7.) Note that the function is available only in case of using inclusive logic in accordance with the explanation in Section 4.3.2.2.

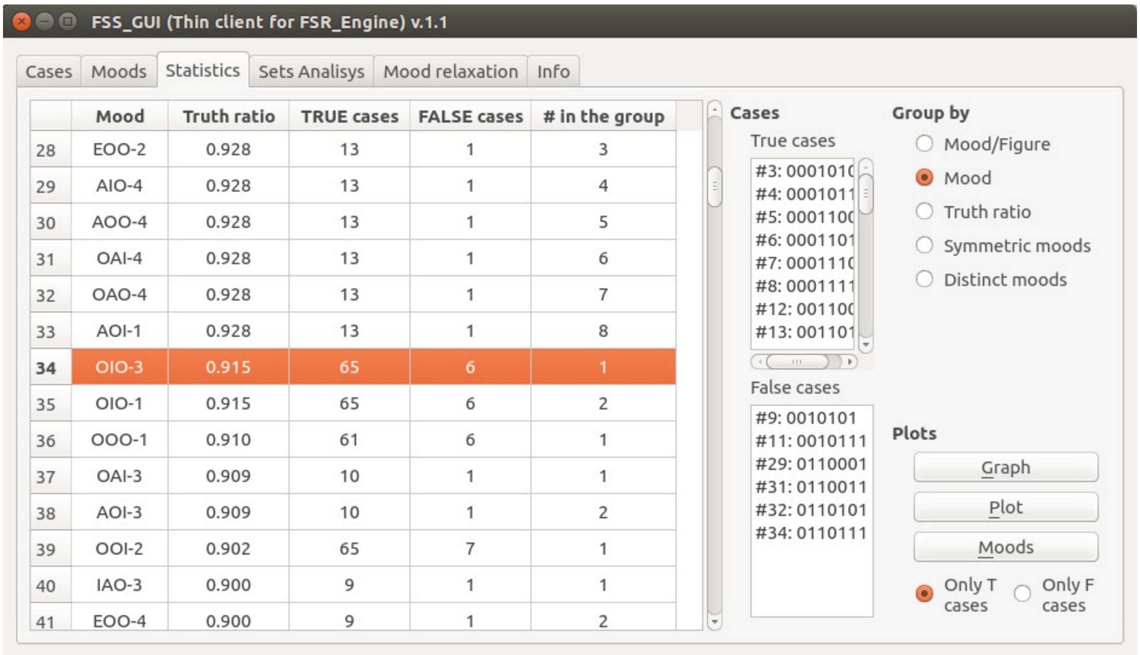


Figure E.5: FSR_GUI Application, tab “Statistics”, moods, grouped by their truth ratios in descending order

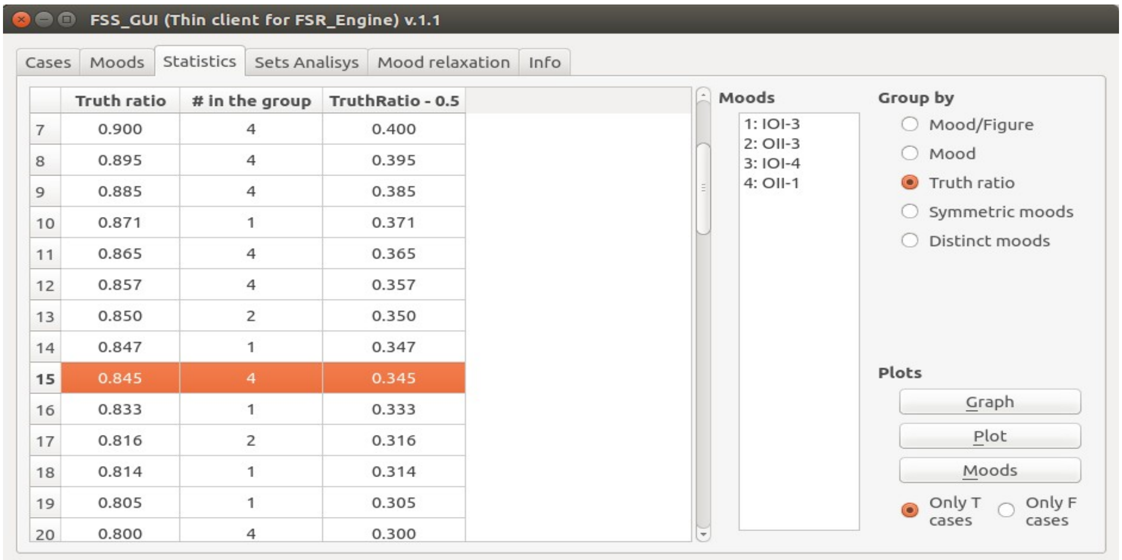
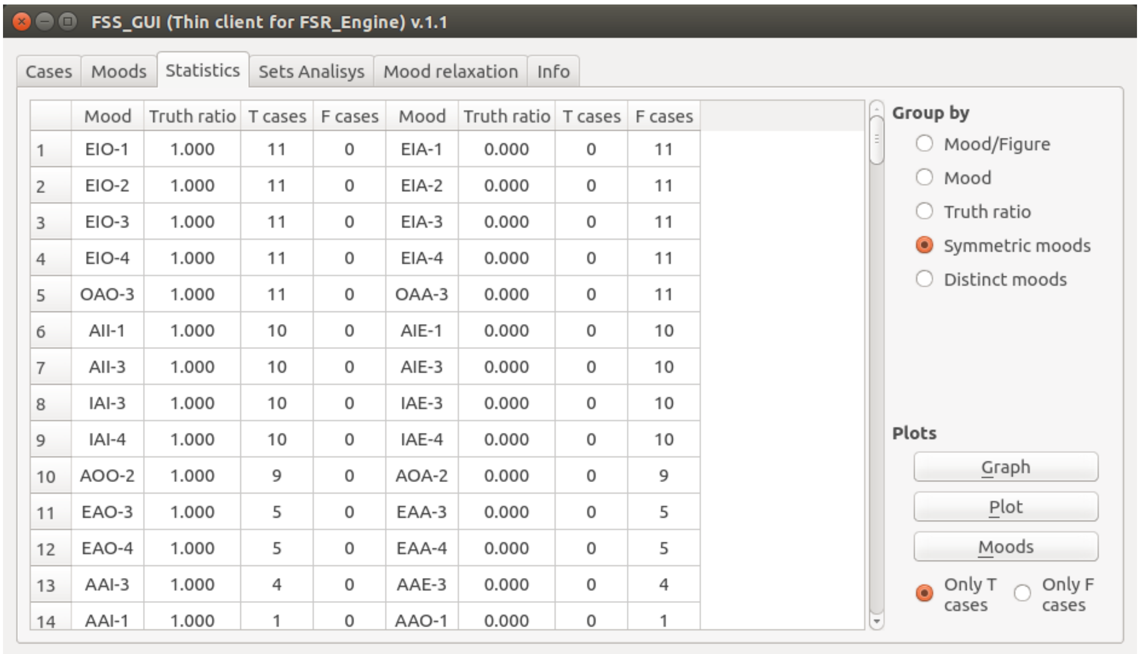


Figure E.6: FSR_GUI Application, tab “Statistics”, truth ratios grouped by descending order

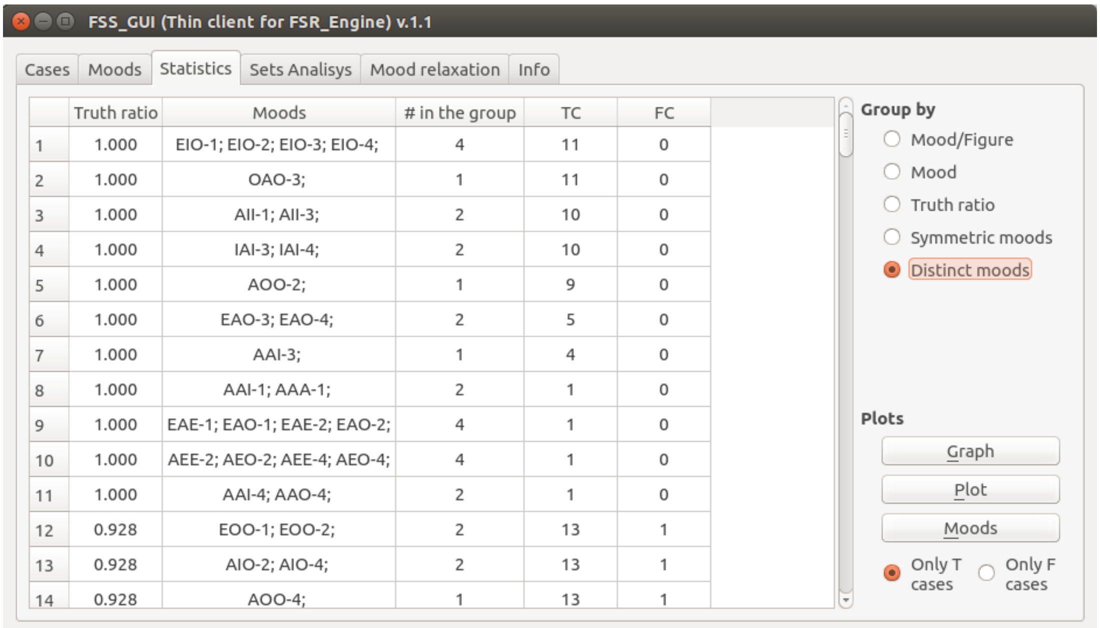


	Mood	Truth ratio	T cases	F cases	Mood	Truth ratio	T cases	F cases
1	EIO-1	1.000	11	0	EIA-1	0.000	0	11
2	EIO-2	1.000	11	0	EIA-2	0.000	0	11
3	EIO-3	1.000	11	0	EIA-3	0.000	0	11
4	EIO-4	1.000	11	0	EIA-4	0.000	0	11
5	OAO-3	1.000	11	0	OAA-3	0.000	0	11
6	AII-1	1.000	10	0	AIE-1	0.000	0	10
7	AII-3	1.000	10	0	AIE-3	0.000	0	10
8	IAI-3	1.000	10	0	IAE-3	0.000	0	10
9	IAI-4	1.000	10	0	IAE-4	0.000	0	10
10	AOO-2	1.000	9	0	AOA-2	0.000	0	9
11	EAO-3	1.000	5	0	EAA-3	0.000	0	5
12	EAO-4	1.000	5	0	EAA-4	0.000	0	5
13	AAI-3	1.000	4	0	AAE-3	0.000	0	4
14	AAI-1	1.000	1	0	AAO-1	0.000	0	1

Figure E.7: FSR_GUI Application, tab “Statistics”, list of symmetric moods

E.3.5. Statistics: Distinct moods

If the radio-button “Distinct moods” is checked, the table is displayed with all of 136 distinct moods in case of inclusive logic and 70 distinct moods in case of exclusive logic (see Fig. E.8.).



	Truth ratio	Moods	# in the group	TC	FC
1	1.000	EIO-1; EIO-2; EIO-3; EIO-4;	4	11	0
2	1.000	OAO-3;	1	11	0
3	1.000	AII-1; AII-3;	2	10	0
4	1.000	IAI-3; IAI-4;	2	10	0
5	1.000	AOO-2;	1	9	0
6	1.000	EAO-3; EAO-4;	2	5	0
7	1.000	AAI-3;	1	4	0
8	1.000	AAI-1; AAA-1;	2	1	0
9	1.000	EAE-1; EAO-1; EAE-2; EAO-2;	4	1	0
10	1.000	AEE-2; AEO-2; AEE-4; AEO-4;	4	1	0
11	1.000	AAI-4; AAO-4;	2	1	0
12	0.928	EOO-1; EOO-2;	2	13	1
13	0.928	AIO-2; AIO-4;	2	13	1
14	0.928	AOO-4;	1	13	1

Figure E.8: FSR_GUI Application, tab “Statistics”, list of distinct moods

E.3.6. Plots: Graph of distinct moods

The plot "Graph of distinct moods" shows relationships between the syllogistic cases for the group of distinct moods (see Fig. E.9.). The plot reveals the moods, containing the minimum or the maximum number of cases and sub-moods (moods, fully contained in other moods). The circles represent the distinct groups of cases whereas the colored links represent group of moods. Thus, the cases inside of a particular mood are connected by the link of the same color.

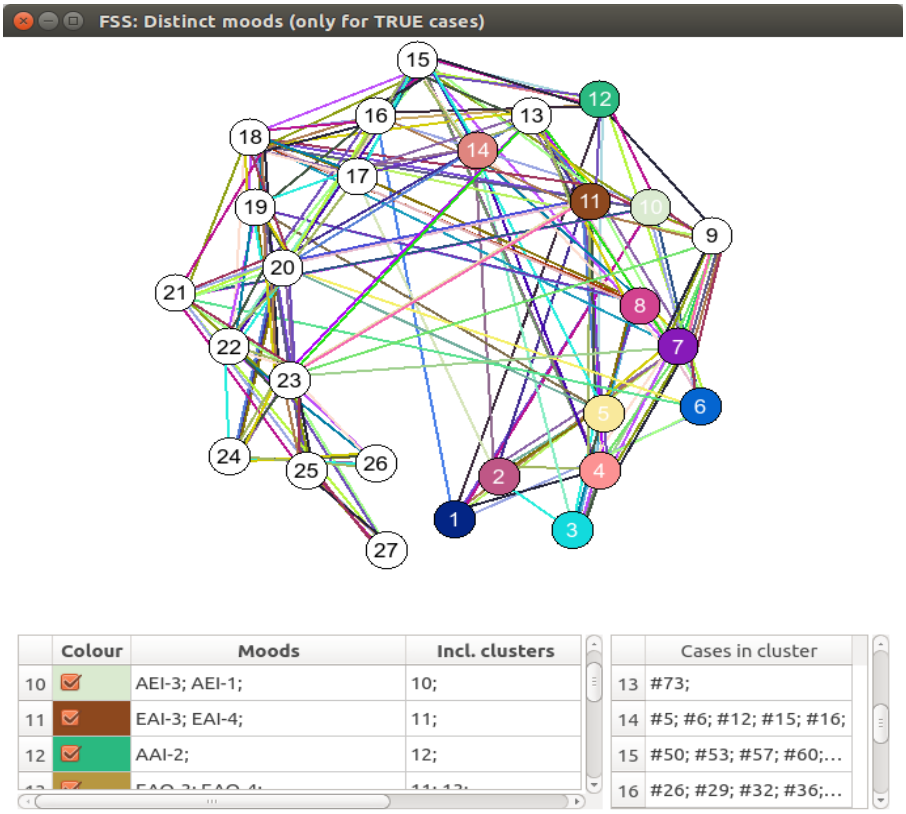


Figure E.9: FSR_GUI Application, tab "Statistics", graph of distinct moods

E.3.7. Plots: Plot of similarity of moods

The plot of similarity of moods shows the moods consisting of cases from a particular mood in increasing order of the value of the mismatching function (see Fig. E.10.). The value of the mismatching function is calculated according to the formula:

$$\textit{Total mismatching} = ||\textit{missing cases}|| + ||\textit{unequal cases}|| \qquad \text{(E.1)}$$

where ||*missing cases*|| denotes the number of missing cases (difference between the number of cases in a chosen mood and any mood which has at least one common case together), ||*unequal cases*|| denotes the number of unequal cases (number of cases in a chosen mood different from the cases in any mood which has at least one common case together). The moods with low value of mismatching is closer to each other in term of shared syllogistic cases.

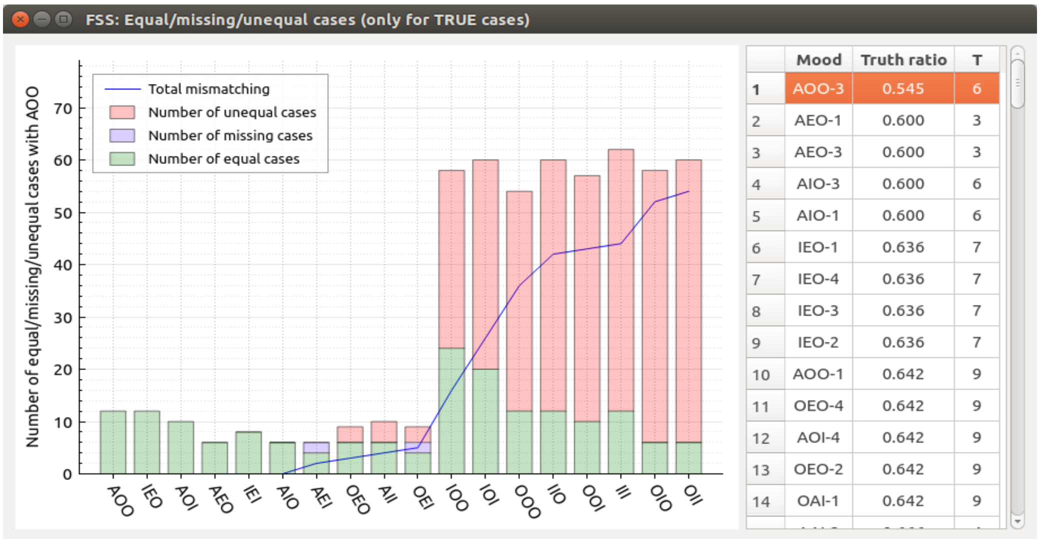


Figure E.10: FSR_GUI Application, tab "Statistics", plot of similarity of moods

E.3.8. Plots: Moods

The plot "Moods" shows the distribution of the syllogistic cases in moods (see Fig. E.11).

If check-box "Distinct moods" is checked, the distribution of cases in distinct moods is shown.

E.4. Tab "Sets Analysis"

The tab "Set Analysis" includes tools for set analysis. Currently, classical syllogistic reasoning (for systems S/^{2}S), reasoning with moods relaxation (discussed in Section 4.4.2, only for system S) and fuzzy-syllogistic reasoning (for system ^{6}S) are available.

All types of reasonings can be performed also with distinct moods. The main tool for performing of reasoning is Sets Analysis Wizard.

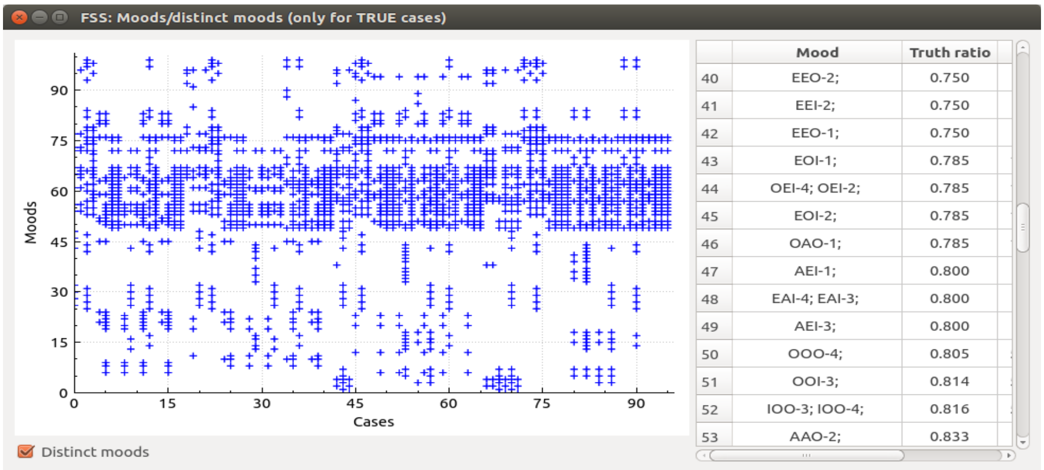


Figure E.11: FSR_GUI Application, tab “Statistics”, plot of case distribution in moods

E.4.1. Sets Analysis Wizard

The Sets Analysis Wizard provides an interface to prepare data for reasoning. It includes two main steps: input data selection and setting of reasoning parameters (see Fig. E.12).

It is possible to use prearranged xml – file with sets description (format of xml – file is defined in Section E.4.2) or generate input data for a particular mood. The sets generator (see Fig. E.13) allows to choose a mood and a number of elements for each set (M, P ans S) (up to 100 elements per set).

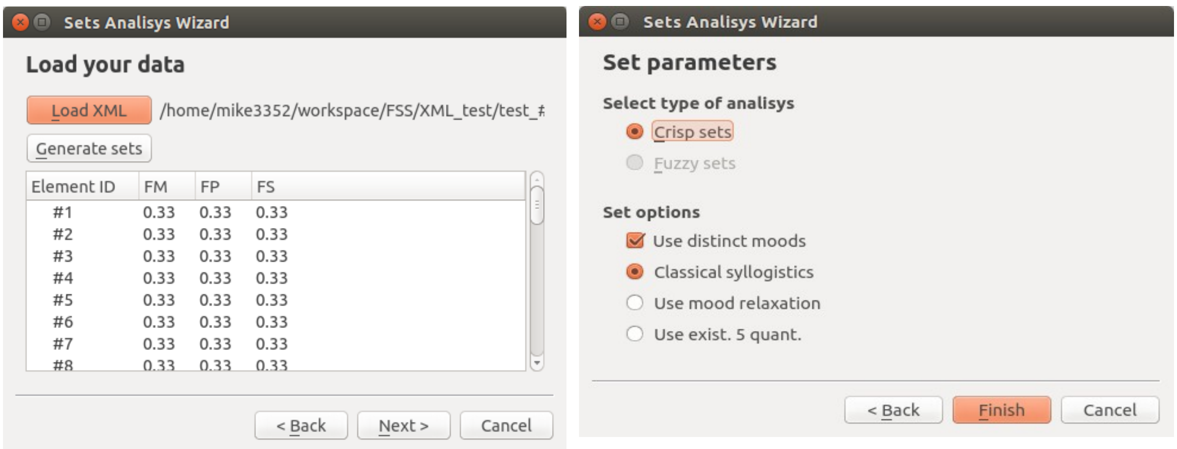


Figure E.12: FSR_GUI Application, Sets Analysis Wizards, data source and parameters selection dialogs

After the input data is loaded, it is necessary to select reasoning options, such as types of reasonings and moods.

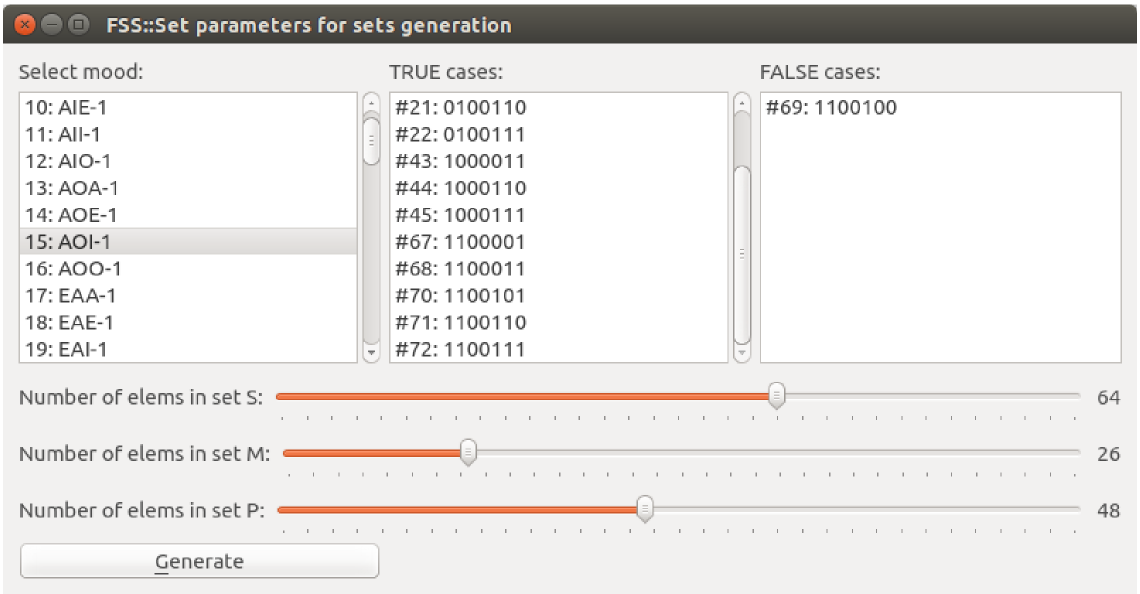


Figure E.13: FSR_GUI Application, Sets Analysis Wizards, generating of input data

E.4.2. Format of the XML file, using for sets definition

Each element of input data takes a string ID and a value of member function for sets M, P and S. If FM, FP or FS is set to 0.0, element is considered as not belonging to the corresponding set. The syntax of XML file is shown in Listing E.2.

```
<?xml version="1.0"?>
<FSS>
<elem ID ="#1">
  <FM>0.5</FM>
  <FP>0.5</FP>
  <FS>0.0</FS>
</elem>
...
</FSS>
```

Listing E.2. Syntax of XML – file for sets definition

E.4.3. Different types of reasoning

It is possible to perform different types of reasoning in the designed system. The output for the classical syllogistic reasoning is shown in Fig. E.14. In the table, the value of φ (for reference, see Section 4.2.1.) is calculated for premises and conclusion.

If the value φ does not satisfy corresponding quantifier, it is marked as “X”. The moods, fully matching with the input data (the input data satisfies premises and conclusion of the corresponding mood) are marked by light green color, whereas the moods with the best truth ratio are marked by green color.

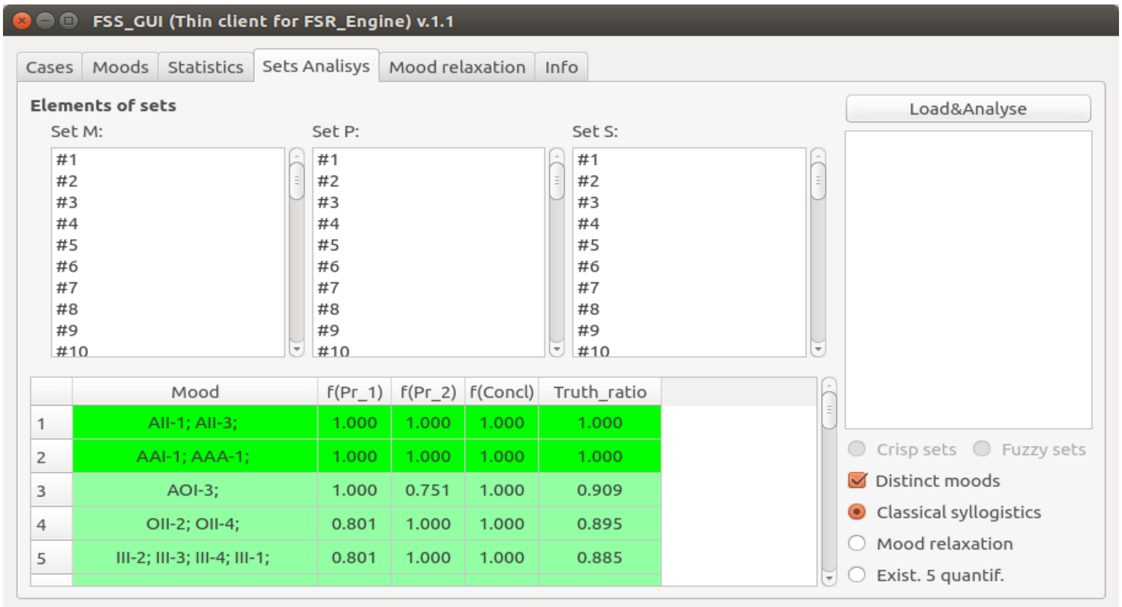


Figure E.14: FSR_GUI Application, tab “Sets Analysis”, results of performing of classical reasoning

The output for reasoning with relaxed moods is shown in Fig. E.15. In the table, in the column “Improved TR” the updated value of truth ratio is shown in case that truth ratio of corresponding mood is improved after applying fuzzification. The moods with the highest truth ratio are marked by green.

The output of fuzzy-syllogistic reasoning is similar to reasoning with relaxed moods. However, the conception of improved mood could not be applied in this case.

E.5. Tab “Mood relaxation”

The relaxed moods are calculated in accordance with the suggestions proposed in the Section 4.4.2. The moods that can be improved are marked by green. The quantifiers (Almost_A or/and Almost_E) can be selected along with a number of syllogistic figure (see Fig. E.16).

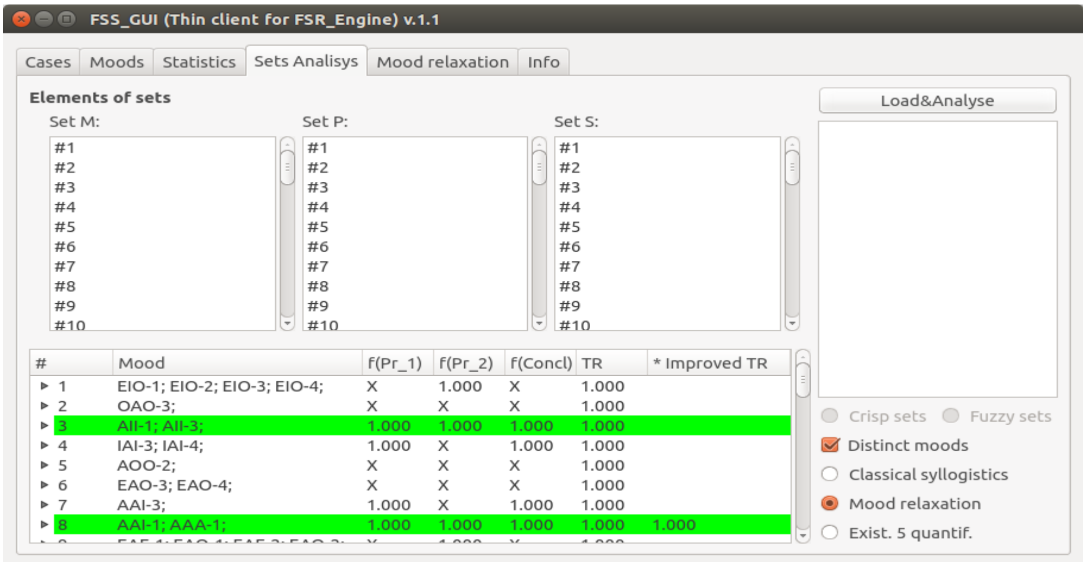


Figure E.15: FSR_GUI Application, tab “Sets Analysis”, results of performing of moods relaxation

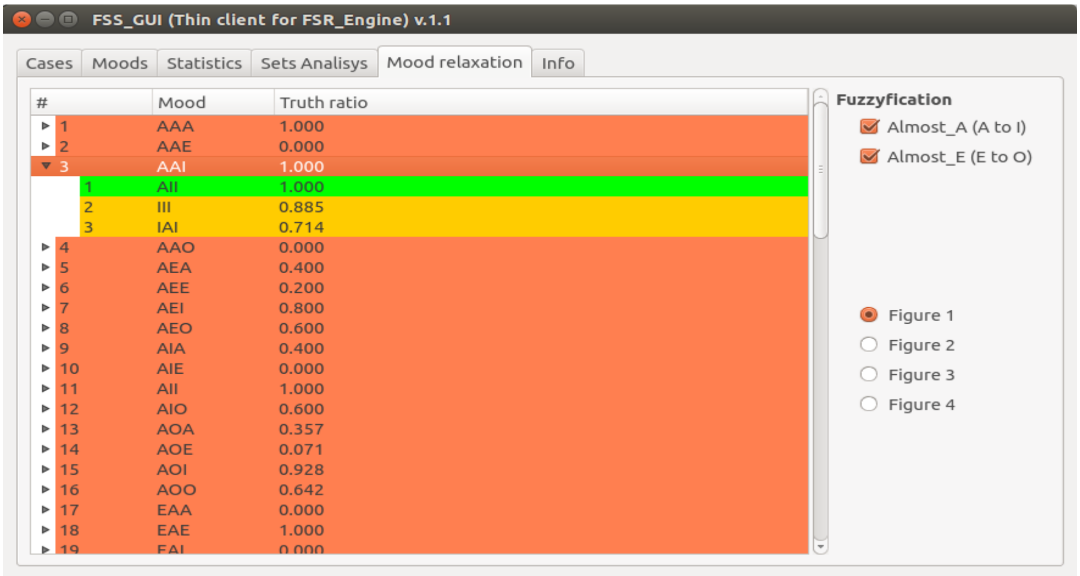


Figure E.16: FSR_GUI Application, tab "Mood relaxation"

6.4 E.6. Tab "Info"

The tab "Info" includes reference information about syllogisms (mainly from http://en.wikipedia.org/wiki/Syllogism) located in a simple HTML-browser with basic navigation tools (see Fig. E.17).

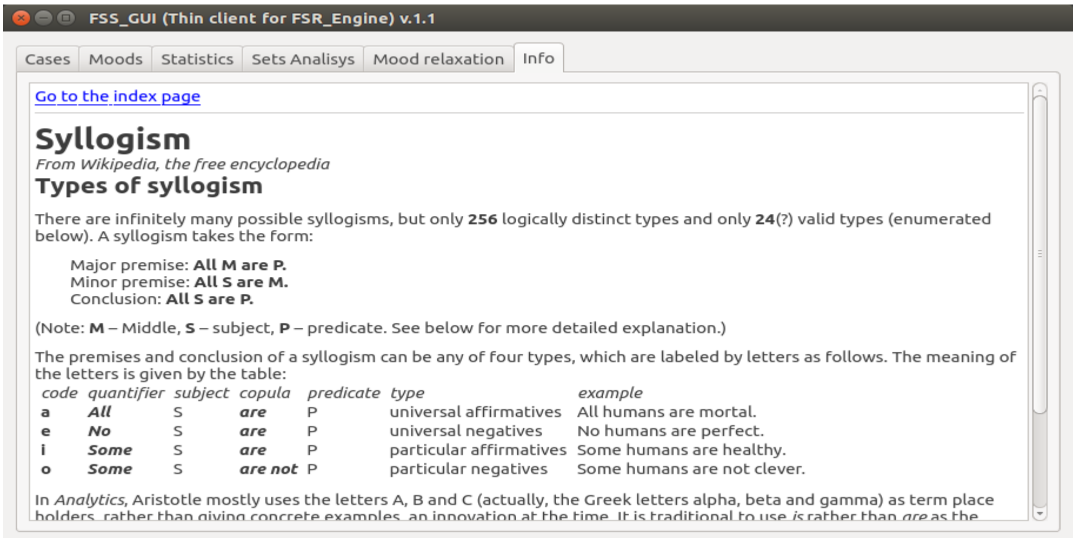


Figure E.17: FSR_GUI Application, tab "Info"