KADIR HAS UNIVERSITY
GRADUATE SCHOOL OF SCIENCE AND ENGINEERING

MULTI-OBJECTIVE NURSE SCHEDULING WITH SHIFT PREFERENCES IN A SURGICAL SUITE

ELİF TEKİN

JUNE, 2017

Elif Tekin M.S. Thesis 2017

MULTI-OBJECTIVE NURSE SCHEDULING WITH SHIFT PREFERENCES IN A SURGICAL SUITE

ELİF TEKİN

Submitted to the Graduate School of Science and Engineering
in partial fulfillment of the requirements for the degree of
Master of Science
in
Industrial Engineering

KADIR HAS UNIVERSITY
2017

KADIR HAS UNIVERSITY
GRADUATE SCHOOL OF SCIENCE AND ENGINEERING

MULTI-OBJECTIVE NURSE SCHEDULING
WITH SHIFT PREFERENCES IN A SURGICAL SUITE

ELİF TEKİN

APPROVED BY:

Asst. Prof. Dr. Esra Ağca Aktunç (Kadir Has University) ____________________

Assoc. Prof. Dr. Funda Samanlıoğlu (Kadir Has University) ____________________

Asst. Prof. Dr. Nezir Aydın (Yıldız Technical University) ____________________

APPROVAL DATE: 01/06/2017

MULTI-OBJECTIVE NURSE SCHEDULING
WITH SHIFT PREFERENCES IN A SURGICAL SUITE

Abstract

Nurse scheduling is a crucial part of hospital personnel scheduling. For effective nurse scheduling, fair workload and job satisfaction should be taken into account, as these factors directly affect the quality of service in healthcare systems. In this thesis project, nurse scheduling constraints for a surgical suite are determined according to government regulations, hospital management requirements, and nurse preferences. Firstly, the nurses can determine their preference for off days based on the priority of importance; for example, for their own planned surgeries, wedding day, or birthday. Then, shifts are assigned subject to the constraints that ensure fair workload in addition to the regulatory constraints. To generate an optimal nurse schedule which provides fair workload and satisfies nurse preferences for off days, a multi-objective integer program with hard and soft constraints is formulated in GAMS and solved using CPLEX as a goal programming model. The main goal is increasing nurse motivation in order to help reduce the mistakes caused by fatigue as a result of working too many shifts. Therefore, service quality can improve in terms of patient care while optimizing the work schedule of nurses. The optimal schedules under various scenarios are compared to show the improvement in the schedule quality in terms of performance measures such as the proportion of nurse preferences met and distribution of unmet preferences.

Keywords: nurse scheduling; integer program; shift preferences; multi-objective optimization; goal programming

HEDEF PROGRAMLAMA KULLANARAK AMELİYATHANE HEMŞİRE ÇİZELGELEME PROBLEMİ ÇÖZÜMÜ

Özet

Hemşire çizelgelemesi, hastane personel çizelgelemesinin önemli bir parçasıdır. Optimum hemşire çizelgelemesi oluşturulurken göz önüne alınan adil iş yükü dağılımı ve iş memnuniyeti sağlık hizmetlerindeki kaliteyi doğrudan etkilemektedir. Bu tez projesinde, ameliyathane hemşire çizelgelemesi oluşturulurken önemli kısıtlar yasaların belirlediği çalışma saatleri, hastane yönetiminin gereksinimleri ve hemşire tercihleri dikkate alınmıştır Öncelikle, hemşireler, önem arz eden öncelikli günler için örneğin, kendi planlı ameliyatları, düğün günü veya doğum günleri gibi günleri belirleyebilirler. Ardından, vardiyalar diğer kısıtlamalara uygun olarak adil iş yükü sağlayacak şekilde tahsis edilir. Adil bir iş yükü dağılımını sağlayan ve izin günleri için hemşire tercihlerini karşılayan en uygun hemşire çizelgelerini oluşturmak için, GAMS programında tüm kısıtlamalara sahip çok amaçlı bir tamsayı programı oluşturuldu ve hedef programlama modeli ile CPLEX kullanılarak çözüldü. Projenin amacı hemşirelerin iş memnuniyetini arttırmak ve orantısız vardiya atamasının sonucunda ortaya çıkan yorgunluğun yol açtığı tıbbi hataları azaltmaktır. Böylece hemşire memnuniyeti arttırılarak hizmet kalitesinin de arttırılması beklenmektedir. Hemşire tercihleri değiştirilerek birbirinden farklı senaryolar için çizelgeler oluşturulmuştur ve hemşire tercihlerinin karşılanma oranları ile fazla mesailerin adil dağılımı gibi performans ölçütlerine bakılarak çizelgeler arasında kıyaslamalar yapılmıştır.

Anahtar Kelimeler: hemşire çizelgeleme; tam sayılı programlama; çok amaçlı en iyileme; 0-1 hedef programlama

Acknowledgements

There are many people who helped to make my years at the graduate school most valuable. First, I thank Asst. Prof. Dr. Esra Ağca Aktunç, my major professor and dissertation supervisor. Having the opportunity to work with her over the years was intellectually rewarding and fulfilling. She has always encouraged me and recommended to trust my selves. She always shows me right ways to reach success in my research.

Many thanks to Department computer staff, who patiently answered my questions and problems on word processing. I would also like to thank to my graduate student colleagues who helped me all through the years full of class work and exams. My special thanks go to Sirun and Şeyma whose friendship I deeply value.

The last words of thanks go to my family. I thank my parents for their patience and encouragement.

Table of Contents

List of Tables

List of Figures

List of Abbreviations

LP: Linear Programming

IP: Integer Programming

GP: Goal Programming

ICN: International Council of Nurses

WHO: World Health Organization

ANA: American Nurses Association

RN: Registered Nurse

NP: Nurse Practitioner

CNM: Certified Nurse-Midwife

CNS: Clinical Nurse Specialist

CRNA: Certified Registered Nurse Anesthetists

TNA: Turkish Nurses Association

BLS: Bureau of Labor Statistics

NCLEX-RN: National Council Licensure Examination

AND: Associates Degree in Nursing

Chapter 1
Introduction

Healthcare systems have experienced drastic changes in the last few decades as a result of increasing population and technological developments that both increased life expectancy and incurred additional costs. There have also been budget cuts that force hospitals to use their resources more efficiently. Healthcare organizations have to work twenty-four hours a day for every day of the year and shift work is used in healthcare organizations in order to provide continuous service for patients. An important resource of a hospital is its surgical suite of operating rooms where nurse scheduling is one of the most challenging workforce planning problems since the quality of care provided by nurses depends greatly on their shift assignments and, thus, job satisfaction. Compared to the globally increasing demand for healthcare, the supply and retention of qualified nursing staff has been insufficient (Wright and Mahar, 2013). This makes nurse scheduling one of the most significant issues in terms of hospital managements' expectations to use their budget wisely, patient expectations of continuous high quality care, and nurses' expectations of fair workload distribution.

Nurse scheduling is the task of specifying the work pattern for individual nurses and it can be generated manually by a head nurse or using software systems for scheduling. In most hospital units, nurse schedules are still manually developed by experienced head nurses who have to gather information from nurses about their time conflicts and have to adjust schedules as needed. However, developing a fair schedule manually is time consuming and difficult because of the vast number of possible schedules and all the constraints to consider. In practice, nurse schedules are typically developed for a period of four weeks. These schedules can be flexible, changing every period, or fixed. Fixed (or cyclical) schedules generally provide good solutions, but they cannot easily address staff preferences or fluctuating demand (Zurn, 2004)**.**While generating nurse schedules, significant constraints should be considered such as observing work regulations, distinguishing between permanent and temporary staff, ensuring that night and weekend shifts are distributed fairly, allowing for leave and days off, and accommodating a range of employee preferences (Clark and Walker, 2011). This project addresses the flexible scheduling of nurses in a surgical suite where the nurse preferences for shifts are used as input to generate fair shift assignments.

This thesis project is organized as follows. In Chapter 2, extensive literature review about relevant work from nurse scheduling is presented. In Chapter 3, model development for nurse scheduling is presented. In Chapters 4 and 5, solution method and computational results are mentioned, respectively. Finally, conclusion of the study results are explained in Chapter 6.

Chapter 2
Literature Review

In this chapter, a comprehensive overview of existing literature on nurse scheduling is provided, which is a special case of personnel scheduling for healthcare systems.

2.1 History of Nursing

Nursing is a crucial field of the health care system. Nurses have various duties such as taking care of ill or injured people, providing health care to people, prevention of illness, disabled, and dying people. In addition to their duties, promoting of a safe environment, research, and participation in shaping health policy and in patient and health systems management (ICN, 2017). Nursing abilities can be categorized as professional responsibility and accountability, knowledge-based practice, client-focused provision of service, and ethical practice (ICN, 2017).

In principle, the nursing profession has been around since the beginning of time, though has substantially developed the course of history. The military has always been playing a significant role in improving nursing education all over the world. First of all, the English nurse Florence Nightingale is known as the founder of modern nursing in the 19th and 20th century. She not only is the founder of nursing as it existed at that time but also lay the foundation for nursing as a profession. Her renowned nursing career began within the Crimean War that took place in the mid 1850's, tending to injured soldiers on the battlefield. Bellevue Hospital in New York City, the first school in the United States, established in 1873 according to Florence Nightingale's nursing principles was the Training School for Nurses

(Egenes, 2009). Moreover, Florence Nightingale had also contribution for Turkish modern nursing. She came to Istanbul in 1854 to provide care to wounded the Turkish and allied soldiers of the Crimean War. Despite of the fact that nursing in Turkey has a very long history, written documentation about nursing did not exist in the history. The nursing profession emerged in Turkey with the demands for caregivers during the War of Tripoli in 1911 and the Balkan War in 1912. At the time of the Ottoman Empire, women from wealthy families were joined up voluntarily in a 6-month nursing course in Istanbul. Safiye Huseyin Elbi was the first volunteer nurse to treat wounded soldiers on the battlefield and in hospitals (Alpar and Bahcecik, 2009).

Many accomplished and effective nurses have developed high quality health care and provided the modern nursing that still exists today. Some of the illustrious nurses can be mentioned as; Clarissa Harlowe Barton, was known as "angel in the battlefield" during the Civil War, was a nurse who founded the American Red Cross soon after the Civil War. Linda Richards and Agnes Elizabeth Jones helped to create a number of nursing schools throughout the U.S. and Japan during the mid to late 1800's. Mary Mahoney became the first African-American woman to fulfill nursing training and become a registered nurse in 1879. Today, 19.3 million nurses and midwives exist according to the World Health Organization's (WHO) Report, (2011). In the United States, the total number of nurses is 4.011.911 in the 2016 according to Redi-Data Inc. including number of registered nurse 3.184.283, and the number of licensed practical nurse is 827.628. There are an estimated 7.3 million nurses and midwives in the WHO European Region. This number is not sufficient to meet demand. In Turkey, number of nurses is 125.191 according to Ministry of Health report in 2012. Health education and health career report in Turkey (2010), the number of nurses per 100.000 people in Turkey is very low compared to the average of the countries in the European Region compared to the average of EU member countries (Health education and health career report in Turkey, 2010).

2.2 Types of Nurses

Some of the important branch of nursing examples; ambulatory care nursing, cardiac nursing, dental nursing, emergency nursing, genetics nursing, home health nursing, medical-surgical nursing, neurosurgical nursing, nurse midwifery, public health nursing, oncology nursing, radiology nursing, surgical nursing, urology nursing etc. In this thesis project, cardiovascular nursing, general surgery nursing and neurosurgical nursing are mentioned. Cardiovascular nurses, also known as cardiac nurses, provide comprehensive cardiovascular care for patients with acute and chronic heart disease (Graduate Nursing, 2017). General surgery nurses help in general surgery procedures handle several tasks. They care for patients before, during and after surgery. Neurosurgical nurses consider patients with dysfunctions of the nervous system.

There are various types of nurses and types of nursing jobs. Types of nurses each category of nurse brings specialized knowledge and skills to health care teams and workplaces. Nurse types can be classified as: Registered Nurses (RNs), Nurse Practitioner (NPs), Certified Nurse-Midwife (CNM), Clinical Nurse Specialist (CNS), and Certified Registered Nurse Anesthetists (CRNA) according to American Nurses Association.

2.2.1 Registered Nurses (RNs)

In general, registered nurses help in patients with illnesses, injuries and medical conditions in hospitals and medical settings. They complete at least an associate’s degree in nursing (ADN) or a Bachelor of Science degree in nursing (BSN) and has successfully passed the National Council Licensure Examination (NCLEX-RN) certification exam in the United States (AND, 2017). They specialize in a field of practice, and assist physicians according to their departments. Registered nurses have to meet a minimum required of practice hours that outlined by a country, state, province or similar licensing body in order to obtain a nursing license and undertake continuing education. Registered nurses (RNs) have significant responsibilities which can be listed as providing health promotion, counseling and education, coordinating care with other healthcare professionals, direct and supervise care delivered by other healthcare personnel like LPNs and nurse aides, making decisions about necessaries based on interpret patient information, conducting research, managing medications, wound care, and other personalized interventions (American Nurses Association, 2017).

RNs provide health care in crucial locations for public such as hospitals, nursing homes, medical offices, ambulatory care centers, community health centers, schools, and retail clinics. They also practice in different areas such as camps, prisons, sporting events and tourist destinations (American Nurses Association, 2017).

2.2.2 Advanced Practice Registered Nurses

Advanced practice registered nurse (APRN) is a general term given to a registered nurse with post-graduate education in nursing. Advanced practice registered nurses consist of four types of advanced practice nurse including nurse practitioner (NP), certified nurse mid-wife (CNM), clinical nurse specialist (CNS), and certified registered nurse anesthetists (CRNA) (ANA, 2017).

2.2.3 Nurse Practitioner (NP)

Nurse Practitioners (NPs) are usually educated in a master's program. They provide a broad range of primary and preventive health care services, prescribe medication, and diagnose and treat common minor illnesses and injuries. They can work under the supervision of a physician (ANA, 2017). Their responsibilities can be listed as diagnose diseases, prescribe meds and initiate treatment plans. Their work places are clinics, nursing homes, hospitals, and private offices.

2.2.4 Certified Nurse-Midwife (CNM)

Midwives are primary health care providers to women during lifelong. They provide woman gynecological and low-risk obstetrical care in hospitals, birth centers, and homes. According to the Bureau of Labor Statistics (BLS, 2014), there are currently over 6.000 nurse-midwives working in the United States, midwifery is a career on the rise.

2.2.5 Clinical Nurse Specialist (CNS)

Clinical nurse specialists (CNSs) are advanced practice nurse who can provide expert advice related to specific conditions. CNSs work in such as hospitals, clinics, nursing homes, private offices, and community-based settings, they care a wide range of physical and mental health problems. A clinical nurse specialist work in a particular area of medicine. These areas include setting, population, disease, health problem, care type. Clinical nurse specialists working area where are indicated such as in a clinic, critical care or emergency room. Besides, nurse specialists serve for a specific population, such as children, or adults. They can focus on a specific disease such as cancer, heart disease or surgical.

2.2.6 Certified Registered Nurse Anesthetists (CRNA)

CRNAs provide anesthesia for surgeons, anesthesiologists, dentists, podiatrists, and other qualified healthcare professionals (ANA, 2017). They are certified to provide a full spectrum of anesthesia care, including for surgical procedures, and may work with healthy to very sick individuals of all ages and all levels of acuity (Nurse Journal, 2017). CRNAs practice in every health care settings such as traditional hospital surgical suites and obstetrical delivery rooms; critical access hospitals; ambulatory surgical centers; the offices of dentists, podiatrists, ophthalmologists, plastic surgeons, and pain management specialists (ANA, 2017).

2.2.7 Operating Room Nursing (OR)

Operating rooms are the areas where enclosed and set apart for occupancy the performance of certain procedures. Operating rooms are vital importance for patients' health. Operating room units should meet comfort conditions and design parameters such as humidity, temperature, air flow, air flow types. For example, temperature of operating rooms should be 21 ˚C ± 3 ˚C, humidity should be between 30 % - 65 %. In operating rooms, decisions on nurses and operating rooms are usually made strategic, tactical and operational levels. Operating rooms are the most profitable and critical hospital resource as well as the most expensive resource. Operating room units form approximately 33 % of the total cost of the health institution in order to provide all of the indoor conditions (Balaras, Dascalaki and Gaglia, 2007). In order to optimization for operating rooms, minimizing the waiting time, maximizing utilization of resources in other departments, minimizing staffing cost, the number and type of operating rooms available, and the number of surgeons or surgical groups should be considered (Martinelly, Baptiste, and Maknoon, 2014).

In terms of scheduling, all surgeries in operating rooms can be categorized into two broad categories: elective and non-elective surgeries. While non-elective surgeries may include urgent cases, elective surgeries have been scheduled ahead of time before the schedule has been closed (Narges, Hosseini and Taaffe, 2013). Nurses play a significant role in operating rooms. Thus, nurse scheduling is very important in terms of meeting patient demand. The objective is to use nurses efficiently and to produce a balanced workload. The quality of the schedule may be measured as penalty costs or scheduling costs (Cheang, Li, Lim, and Rodrigues, 2003).

In this study, operating room nurses are considered for elective surgeries. Therefore, nurses who demand of work for elective nurses can be determined and planned accurately before operations. Cardiovascular surgery, general surgery, and neurosurgery nurse scheduling is studied in this project.

2.3 Nursing in Turkey

The Ministry of Health administers the health care system in Turkey. Privatization has been growing rapidly in all sectors however, private health care systems have become particularly common in recent decades (Tatar, 2011). Most private health care centers have total or partial agreements with the General Health Insurance Institute to provide some or all curative health care services. Private insurance companies, mostly working internationally, provide some employees with private insurance but employees are still required to pay a national insurance premium. Nurses worked for a long time without any legislative definition. The first law to regulate nursing was enacted in 1954 and revised in 2007 with some intermittent regulation between these major enactments. Nursing now includes specialized fields including occupational health nursing.

Three types of basic educational programs have been offered since 1920 in Turkey which can be listed as diploma programs, associate-degree programs, and baccalaureate programs. Additionally, graduate education in nursing has been offered since 1968. The first legislation regulating professional nursing in Turkey was passed in 1954. Nurses who do hold baccalaureate degrees work primarily in big cities and at university and private hospitals. The law outlined educational requirements for those desiring to become professional nurses. In April 2007, a revision of the 1954 nursing law was passed. With the introduction of the 2007 law, the specific responsibility and authority of nurses will be determined according to their educational level. This revised law consist of a description of nursing, criteria for entrance to a nursing school, criteria for nursing educational program content, criteria for graduate specialist education, clarification of nursing roles and responsibilities.

When graduate nursing school, nurses have to pass the exam for Public Personnel Selection (KPSS) in order to serve in public services in Turkish health care system. Candidates who pass this examination can start working in public institutions. Those working in the public sector are called nursing candidates for one year. The diplomas of those candidates who are successful at the end of the one-year period are approved by the Ministry of Health. Although no central and periodic examinations are provided to evaluate the proficiency of nurses, other than the initial employment exams, institutions offer continuous education programs for their employees. The Ministry of Health, the largest employer, carries out continuous education for its employees in line with its own regulation for in-service training (The Ministry of Health, 2007). The first Turkish Nurses Association was established in 1933 and it has been a member of the International Council of Nurses since 1949. Currently there are about 6000 members of the Association (Turkish Association of Nurses, 2007). It has been through this Turkish Nurses Association that the nurses in Turkey have been able to have their voice heard by the Ministry of Health and other official organizations within the Turkish government (Dal and Kitiş, 2008). In addition to the Turkish Nurses Association, 25 special branch associations add to the Association of Research and Development in Nursing, the aim of which is to support and develop research in nursing. Some of the examples of special branch associations include the following: the association of occupational health nursing, the association of children's surgery nursing, the society of Turkish surgery nurses, the association of intensive care nurses, the association of nephrology, dialysis, and transplantation nurses, the association of psychiatry nurses, the association of neurosurgical nurses, the association of diabetes nurses, the association of oncology nurses, and the association of child nursing. The Journal of Turkish Nurses was founded and it has been published continuously since 1953 (Bayık, 2002).

2.4 Nurses' Job Problems

Job dissatisfaction among nurses bring about to costly labor disputes, turnover, and risk to patients. Nursing problems can be listed as hospital shortage staffing level, long working hours, and patient concerns. Inadequate staffing of nurses can not only endanger patient health and safety, and lead to greater complexity of care, but also have an influence on nurses' mental and psychical health and safety by increasing occupational nurse fatigue, injury rate, and job dissatisfaction. When healthcare costs increase, decreasing the number of staff nurses is often seen as the sensible and effortless way to solve financial problems. Inappropriate staffing or increased workload give rise to critical problems and stress for nurses.

National Nurses United (NNU) is working on implementing federal legislation to establish nationwide nurse-to-patient ratio guidelines. Nurse-to-patient ratios indicate that the number of nurses on a shift plays a significant role in patient safety and quality of care. Table 2.1 shows the minimum registered nurse-to-patient ratios for each unit according to the U.S. federal regulations (NNU, 2017). We make use of the provided nurse-to-patient ratio for operating rooms in our case study.

Table 2.1: Proposed federal registered nurse-to-patient ratios in the U.S.

Intensive/Critical Care	1:2	Medical/Surgical	1:4
Neonatal Intensive Care	1:2	Psychiatric	1:4
Operating Room	1:1	Rehabilitation	1:5
Post-anesthesia	1:2	Burn Unit	1:2
Pediatrics	1:3	Intermediate Care	1:4
Emergency Room	1:3	Coronary Care	1:2
Well Baby Nursery	1:6	Acute Respiratory Care	1:2

2.5 Personnel Scheduling

Personnel scheduling can be defined as generating and arranging duty timetables for employees while considering certain work regulations, coverage constraints, or personnel preferences in order to satisfy the demand for the goods and services of organizations. Personnel scheduling has been studied widely for several decades in various application areas such as transportation systems, healthcare systems, call centers, emergency services, restaurants, hotels, and retail stores. First of all, the required number of staff should be determined based on their skills and staff members are assigned to shifts in order to meet the service needs. Then, staff members are assigned to their tasks so as to satisfy work requirements at various times. Shift assignment should take into account work regulations determined by governments or other authorities, distinguish between full-time and part-time staff, distribute shifts fairly among staff, and consider staff preferences. These constraints can be classified as hard constraints, that cannot be violated, and soft constraints, that can be violated at a cost. Finding an optimal solution that minimizes the costs can be complicated regarding all these constraints at the same time.

2.6 Nurse Scheduling

Nurse rostering problem (NRP) or nurse scheduling problem (NSP), which is a type of personnel scheduling, have been studied over the years. In the literature, Warner considered on nurse scheduling problem and solutions in the 1976. Tien and Kamiyama Warner (1976), Tien and Kamiyama (1982), Bradley and Martin (1990), Siferd and Benton (1992), Hung (1995), Cheang et al. (2003), Ernst et al. (2004) are comprehensive research about personnel scheduling and rostering. Manual and automated methods are used for scheduling. Nurse scheduling can be arranged cyclical or non-cyclical scheduling. This project focuses on the nurse scheduling problem in a surgical suite that has been regarded as one of the most challenging healthcare personnel scheduling problems. Several studies have been published on various aspects of computerized healthcare personnel scheduling since the 1960's (Burke, 2004). Nurse scheduling is specifying the work pattern for individual nurses and it can be generated manually by a head nurse or using software systems for scheduling. In most hospital units, nurse schedules are still manually developed by experienced head nurses who have to gather information from nurses about their time conflicts and have to adjust schedules as needed. However, developing a fair schedule manually is time consuming and difficult because of all the constraints to consider. Nurse schedules are typically developed for a period of 4 weeks. These schedules can be flexible, changing every month, or be fixed. Fixed or cyclical schedules generally provide good solutions, but they cannot easily address staff preferences and fluctuating demand. In Figure 2.1 illustrate that flowchart for nurse scheduling (Creately, 2017)

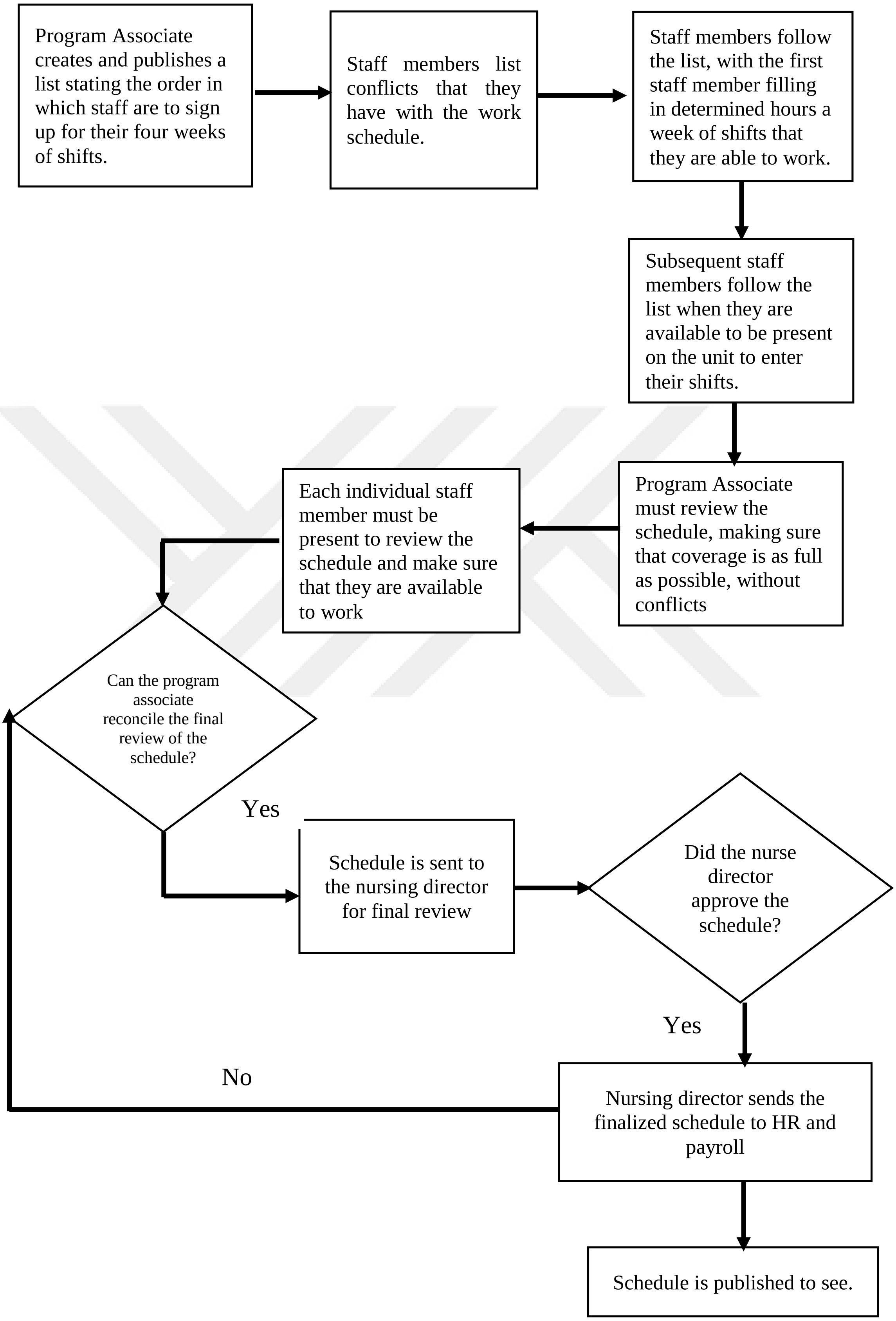


Figure 2.1: Nurse scheduling flowchart

2.6.1 Types of Nurse Scheduling Policies

There are different types of hospitals that have various administrative procedures all around the world. This situation leads to different sort of nurse scheduling problems.

2.6.1.1 Centralized Scheduling

One of the administrative types is centralized scheduling which means one administrative department in a hospital adjusts all the personnel scheduling. The advantage of centralized scheduling is the opportunity for cost reduction through better utilizing of resources. On the hand, personnel can feel unfair the rosters because of the management (Emerging Soft, 2017).

Advantages

- High level control
- Provide for scheduling is easy to locate
- Better utilizing resources

Disadvantages

- Less efficient scheduling practices
- Requires dedicated schedulers
- Personnel may feel that roster are unfair

2.6.1.2 Decentralized Scheduling

Decentralized Scheduling allows all personnel of an organization to reserve their resources through a standardized booking process. This involves a "first come, first serve" (Emerging Soft, 2017).

Advantages

- More efficient resource scheduling
- It does not require dedicated schedulers

Disadvantages

- Less control over scheduling
- Requires a scheduling system to be in place

2.6.1.3 Self-Scheduling and Unit Scheduling

Self-scheduling means that personnel scheduling is generated manually by the staff. It is time consuming, however it has advantage that the nurses cooperate and they can give an advice.

The schedules are organized locally by a head nurse or unit manager, the process is sometimes called unit scheduling.

2.6.1.4 Cyclical Scheduling and Non-Cyclical Scheduling

Cyclical scheduling is known as fixed scheduling, on the other hand non-cyclical scheduling is known as flexible (Silvestro and Silvestro, 2000).

Cyclical Scheduling

Cyclical scheduling known as fixed scheduling and static scheduling.
There are some advantages;

- Personnel know their schedule in advance.
- The work is organized fairly.
- Unhealthy work rotations are avoided.
- Low run-time overhead

There are some drawbacks;

- It is not flexible enough to deal with complex nurse scheduling problems.
- It cannot address flexible work regulations, fluctuating personnel demands and personal preferences.
- It can operate only stable demand and low variability.

- It cannot directly apply to real nurse scheduling problem.
- It is difficult to expand the schedule

Non-Cyclical Scheduling

Non-cyclical known as flexible scheduling.

There are some advantages;

- It doesn't need to a higher level decision to provide a precise number of skilled personnel.
- It can operate uncertain demand and high variability.
- It can apply easily to real nurse scheduling problem.

2.6.2 Solution Approaches for Nurse Scheduling

Many articles and researches have been published on various aspects of computerized healthcare personnel scheduling since the 1960's. The purpose of most approaches is optimizing an objective function with certain constraints. Early examples of optimization of nurse schedules are Miller, Pierskalla, and Rath (1976); Trivedi and Warner (1976); Warner (1976); and Warner and Prawda (1972). There are optimization approaches which are known as mathematical programming, linear and integer programming, goal programming/multi-criteria approaches, and artificial intelligence methods. The relevant literature reviewed is grouped based on the solution approach used in Table 2.2 (Burke et al., 2004). Most nurse scheduling problems are complicated to solve. However, hospital administration's desire optimal personnel scheduling with a wide range of complex constraints.

The quality of nurse scheduling directly affects the service quality and job satisfaction (Marques, I., Captivo, M.E., and Vaz Pato, M., 2015). The schedule must provide suitably qualified nurses to meet the demand based on the number of patients in the wards or a predetermined patient-to-nurse ratio, satisfy work regulations, nurse preferences, and other hospital-specific constraints. The most commonly used objectives of nurse scheduling problems are versions of cost minimization, that may either directly consider recruitment costs, nurse wages, and overtime costs or consider penalty costs for violating a set of soft constraints. Nurse

Table 2.2: Nurse scheduling approaches

NURSE SCHEDULING APPROACHES							
		3. Artificial Intelligence Methods			5. Meta Heuristic Scheduling		
1. Optimizing Approaches Mathematical Approach	**2. Goal Programming**	**3.1. Declarative and constraint programming**	**3.2. Expert systems and knowledge based**	**4. Heuristic**	**5.1. Simulated annealing**	**5.3. Genetic algorithms**	**5.2. Tabu search**
➢ Linear and Integer Programming ➢ Abernathy et al. (1973) ➢ Warner and Prawda (1972) ➢ Trivedi and Warner (1976) ➢ Miller, Pierskalla, and Rath (1976) ➢ Bailey and Field (1985) ➢ Rosenbloom and Goertzen (1987) ➢ Jaumard, Semet, and Vovor (1998) ➢ Millar and Kiragu (1998) ➢ Isken (2004) ➢ Moz and Pato (2004)	➢ Arthur and Ravindran (1981) ➢ Musa and Saxena (1984) ➢ Ozkarahan and Bailey (1988) ➢ Franz et al. (1989) ➢ Chen and Yeung (1993) ➢ Berrada, Ferland, and Michelon (1996) ➢ Jaszkiewicz (1997) ➢ Burke et al. (2002),	➢ Okada and Okada (1988), ➢ Weil et al. (1995) ➢ Darmoni et al. (1995), ➢ Meisels, Gudes, and Solotorevski (1995) ➢ Meyer auf'm Hofe (1997) ➢ Muslija,Gaertner, and Slany (2000) ➢ Li, Lim, and Rodrigues (2003)	➢ Smith, Bird, and Wiggins (1979) ➢ Bell, Hay, and Liang (1986) ➢ Chen and Yeung (1993) ➢ Petrovic, Beddoe and Vanden Berghe (2003)	➢ Smith (1976) ➢ Blau and Sear (1983) ➢ Anzai and Miura (1987), ➢ Kostreva and Jennings (1991) ➢ Schaerf and Meisels (1999)	➢ Isken and Hancock (1991) ➢ Brusco and Jacobs (1995)	➢ Tanomaru (1995) ➢ Aickelin and Dowsland (2003) ➢ Kawanaka et al. (2001)	➢ Dowsland (1998) ➢ Dowsland and Thompson (2000) ➢ Burke, De Causmaecker, and Vanden Berghe (1999) ➢ Valouxis and Housos (2000) ➢ Ikegami and Niwa (2003) ➢ Bellanti et al. (2004),

scheduling in surgical suites is especially a significant issue since the surgical suite has high operating costs and workforce planning directly impacts the efficiency of not only this unit, but also other units of the hospital that are related to the surgical suite (Health Care Financial Management Association Report, 2005).

In this project, we focus on operating room nurses. Briefly, operating theatre (OT), which consists of operating rooms (ORs), and recovery rooms, management is a crucial and complex part for administrative of hospitals. According to Health Care Financial Management Association Report, operating rooms (ORs) have been estimated to account for more than 40% of a hospital's total revenues, and it is also greatest source of revenues as well as the largest cost center for hospitals (Guerriero and Guido R. 2011) . The main objectives are to guarantee the optimal medical resources utilization, the delivery of surgery at the right time, the maximization of profitability, and optimizing patient waiting time. For optimizing operation room management, several researchers utilize the use of mathematical and simulation models, and quantitative techniques plays (Ozkarahan I., 2000). Their goal is to show how operational research solutions can be provided to the surgical planning and scheduling effectively. For example; by I.Ozkarahan (2000); Jeroen Belien and Erik Demeulemeester (2008); B. Cardoen, E. Demeulemeester, and J. Belien (2009).

In this thesis, the solution approach used is based on optimization and goal programming. Next, the related literature in these areas are reviewed.

2.6.2.1 Optimization Methods: Mathematical Programming

Linear and Integer Programming in Nurse Scheduling

Mathematical programming methods such as linear and integer programming are widely used for solving nurse scheduling problems. Several researchers have been studied on nurse scheduling problems using mathematical methods. The nurse scheduling problem has attracted many researchers to study (e.g., Warner DM, Prawda J., 1972; Abernathy, W. J., N. Baloff, Warner DM., 1973; Trivedi, V. M. and M. Warner, 1976; Miller HE, Pierskalla WP, Rath GJ., 1976; Rosenbloom ES, Goertzen NF,1976; Harmeier PE., 1991; Jaumard, B., F.

Semet, and T. Vovor, 1998; Moz, M. and M. Pato, 2004; Wright, D.P., and Mahar, S., 2013).

While most of the mathematical programs have a single objective function subject to a restricted set of constraints as a result of oversimplifying assumptions, many mathematical models are based on goal programming or multi- objective decision making. Many goal programming models to solve nurse scheduling problems have been published by Trivedi VM et al., (1981), Musa AA et al. (1984), Ozkarahan I, and Bailey JE. et al. (1988), Azaiez, M. and S. S. Sharif (2005). There are also some studies in Turkey using goal programming for solving nurse rostering problems. Moreover, numerous researchers have employed multi-objective approach to address the nurse scheduling problem.

Abernathy et al. (1973), presented mathematical (stochastic) programming techniques to solve the general staffing problem. The approach has been applied to a hypothetical example application. The number of people required to fulfill the stochastically varying personnel demands is not pre-determined in this paper. This study is very a successful framework for researchers in realistic problems for the future times.

Warner and Prawda (1972), proposed a mixed-integer quadratic programming formulation to compute the number of nurses from a predetermined skill category to assume a number of shifts per day. The goal function considers minimizing the difference between number of lower limit nurses and the variables. 8 hours shifts are used with three shifts. A disadvantage of the approach is that an accurate forecast of personnel demand cannot be reliable for a period.

Warner (1976), expanded on the previous formulation (Warner and Prawda, 1972) by indicating weights or fairness levels. Before the optimization starts, certain parts of the scheduling are implemented manually, weekends are assigned by hand. The mathematical programming algorithm involves a search for feasibility and an improvement of the objective. The algorithm was implemented in several hospitals in the United States. It allows for a very fair evaluation of the obtained schedules.

Trivedi and Warner (1976), utilized a branch and bound algorithm to arrange the short-term assignment of nurses from different units also known as float nurses, for shortage of personnel. These method overcome small-scale problems only and as such are unconnected to the modern nurse scheduling. On the other hand, in this papers discuss the concept of float personnel and which has formally modelled the idea. The employment of floating staff still plays a significant role in current hospital systems. There does not currently exist a methodology to deal effectively with floating staff and this is a possible area of future research.

Miller, Pierskalla, and Rath (1976), generated formulation of the personnel requirements in terms of minimum number of personnel per day, without specifying shifts. Time constraints with nurse preferences and staffing coverage are weighted against each other. When Warner's study (Warner, 1976) compare Miller, Pierskalla, and Rath the number of unwanted shift patterns is much higher, so decreasing the complexity of the problem. The time related constraints are divided into two groups: the feasibility set and the non-binding constraints. A cyclic coordinate descent algorithm is applied to look for a nearly optimal solution. A comparison with a branch and bound algorithm demonstrates that the algorithm by Miller et al. requires a much lower calculation time. The problems are very basic and the obtained solutions are not always feasible.

Bailey and Field (1985), arrange a general mathematical model for the nurse scheduling problem. Their definition of the cost function is the sum of the cost for utilizing a shift type multiplied by the number of occurrences of that shift type in the schedule. Bailey and Field use to set schedules by manually or by a linear programming. In their study, idle time was reduced in schedules and applied a 12-hour scheduling period. This approach is one of the few published models that allow shifts to begin at any time during the day, despite of the fact that it is not utilized in current hospitals.

Rosenbloom and Goertzen (1987), improved an integer programming algorithm for cyclical scheduling. This algorithm is involved three stages. The first stage is a set of possible schedules is generated and the resulting schedules are evaluated relating to work regulations and work patterns. In the second stage, the minimum daily coverage constraints are analyzed with an integer program. The third stage converts the solution into work patterns for each nurse.

Jaumard, Semet, and Vovor (1998), proposed the linear programming model which applies column generation and branch and bound to generate flexible nurse scheduling. It can be researched fully of the set of feasible solutions. The objectives include coverage constraints, salary costs, and care quality. Assumption of the model is that the demand periods are roughly decomposed into smaller periods so that a given shift covers a given demand period entirely or not at all. Preliminary tests have been carried out based on real data from a large hospital and the results are approved by head nurses.

Isken (2004), improved mathematical programming techniques which implicit tour scheduling model that is part of a scheduling analysis tool for a comprehensive US hospital. It tackles real hospital scheduling problems that, without the tour types, would be too large. The major aim is to reduce labor costs while meeting the fluctuating coverage requirements over a one week planning period. In this study implicit model involves full time and part time tours. It also allows for assigning different shifts throughout the week and for flexible starting days.

Moz and Pato (2004), solved re-rostering nurse problem which is the based on real hospital case, by assigning float nurses, but Moz and Pato search for replacements within the ward the hard constraints satisfied, they aim at minimizing the difference between the original schedule and the newly generated one.

2.6.2.2 Goal Programming

Goal programming (GP) is an extension of linear programming (LP). The major difference is that GP model does not optimize (maximize/minimize) the objective directly, as in the linear programming. Instead, it attempts to minimize the deviations between the desired goals and the realized results. These goals are prioritized in a hierarchy of importance. GP is a significant type of multiple criteria optimization. It was developed to solve problems with multiple objectives (Rifai, 1994). A GP model researches simultaneously considering several objectives or goals that are considerable to a decision maker. Goal programming method requires the decision maker to set goals for each objective that wishing to obtain. A preferred solution is then defined as the one which minimizes the deviations from the set goals. Goal programming can assist decision makers in effective short- and long range strategic planning.

In real life, all problems have several conflicting objectives. For instance, today healthcare system operate in a competitive environment which is heavily influenced by external factors including regulators, healthcare personnel, suppliers, patients, competitors. Moreover, profit maximization, improved relationships with the patients, maximized service quality, minimized employee unrest, and minimized personnel dissatisfaction. These objectives may conflict and so compete for scarce resources.

To determine the prioritization, each goal is explained as an equation and a deviational variable(s) assigned to it. Deviational variables can be positive or negative. A positive deviational variable (d+) represents over achievement of the goal. A negative deviational variable (d–) represents under achievement of the goal. If the desire is not to under achieve the goal, d– should be driven to zero. To the contrary, if d+ is driven to zero, the overachievement of the goal will not be realized.

Arthur and Ravindran (1981), suggested a two phase goal programming heuristic for the nurse scheduling problem. In this study, staff size according to priority sequence, the number of staff with requests or preferences, staff dissatisfaction, and the deviation between scheduled and desired staffing levels are minimized. The shifts are heuristically assigned to the personnel members at the end of the scheduling process.

Ozkarahan and Bailey (1988), proposed three basic objective functions for their goal programming approach. The first goal is to minimize the deviation between the number of nurses scheduled and the demand, for each period of the day (called time-of-day scheduling). The second goal minimizes the deviations between the sum of days on work patterns and the size of the work force (called day-of-week scheduling). With this goal, the system tries to schedule people according to their contract or work agreement. The third goal combines the day-of-week and time-of-day scheduling problems. Since the computational size of the studied problems is very large, they suggest the division of the work into two phases: one to determine schedules for the day-of-week and time-of-day schedules and one to assign people to the proposed schedules. By employing a heuristic assignment of schedules, the algorithm solves the most important shift times and days for individual nurses. Ozkarahan (1991), presents a goal programming approach for a decision support system. The model aims at maximizing the utilization of full time personnel, minimizing over- and understaffing, and minimizing several kinds of personnel costs. It provides support for staffing decisions and for nurses' preferences. In comparison with the other problems discussed in this review (see the tables of comparison in Appendix B), the problem dimensions are very small. Ozkarahan (and Bailey) presented a very comprehensive linear model for nurse rostering (and partly also for staf.ng). The problem had to be decomposed in order to reduce the dimensions but applications are still limited to small size problems.

Franz et al. (1989), developed a multi-objective integer linear program for health care staff working at different locations, called multi-clinic health regions. In this paper, objectives are minimization of travel costs and maximization of the quality of service by regarding personal preferences.

Chen and Yeung (1993), utilized goal programming with expert systems. The assignment of shift types to personnel members is carried out by the expert system part of the approach. The time related constraints on personal schedules and attempts to cover personnel demands are assisted by goal programming.

Burke et al. (2002), introduce a new multi-criteria approach after having implemented mainly cost function driven (Burke et al., 2001b) meta heuristics for nurse rostering in Belgian hospitals (see also Burke et al., 2001a, 2003, Burke, De Causmaecker, and Vanden Berghe, 1999 in Section 3.5). A curial benefit of the multi-criteria method is

Azaiez, M.N., Sharif, S.S. (2005), utilized five goals which can be explained as Goal 1: it minimizes the deviations between the sum of actual days on and the minimum required days on, Goal 2: It attempts to have in the schedule more day shifts than night shifts for all nurses, Goal 3: It avoids assigning a nurse to work a day shift and the night shift of the following day, Goal 4: It avoids off–on–off patterns. This goal attempts to have minimum isolated day or night shifts, for all nurses, and Goal 5: This goal consists on minimizing isolated days off.

Ozkarahan I. and Topaloglu S. (2004), developed tour scheduling problem considering the employee work preferences using goal programming in Turkey. The goal programming model consists of eleven goals in order to minimize the deviations between the desired goals and realized results. These goals can be listed as; required number of employees, assignment of employees to the required shifts types, preferred number of work hours for each employees, minimum deviation to upper bound for the total understaffed work hours, minimum deviation to upper bound for overstaffed work hours, minimum deviation from the preferred shift types of employees, minimum deviation from the consecutive working day requirements, minimum deviation from the employees' weekend-off requests, minimum deviation from the allowed limit for the under-assigned work hours of employees, minimum deviation from the allowed limit for the over assigned work hours of employees, minimum deviation from the allowed over-assigned work hours for each planning period of a work day.

In this chapter, the most relevant literature in nurse scheduling is reviewed and in the next chapter, we provide our problem definition and the model formulation used to solve this problem.

Chapter 3

Problem Definition and Model Development

3.1 Problem Definition

In this study, the operating room nurses are considered in terms of working hours and their job satisfaction. We consider shift preferences of operating room nurses while generating nurse scheduling model inasmuch as operating room nursing is a highly stressful job. They feel physically and psychologically exhausted because of vital surgeries. Our problem is related to nurses who work for elective surgical suites where cardiovascular, general, and neurological surgeries. Shift preferences of nurses are given the highest priority while meeting the demand-related constraints. Different preference scenarios and three types of minimum requirement scenarios are generated.

We develop a multi-objective integer programming model with hard and soft constraints and solve the model using goal programming. The model is formulated to produce the best possible monthly schedule in terms of meeting nurse preferences and fair distribution of workload. We illustrate the performance of the proposed model with an example under various shift preference scenarios and different demand scenarios. The proposed model can easily be used in practice to produce the best possible nurse schedule given a certain shift preference scenario by adjusting the penalty cost, α.

3.2 Mathematical Model Development

The nurse scheduling model proposed in this study is an integer programming model using goal programming method with the objectives of minimizing schedule cost and maximizing job satisfaction for nurses. The model is used to test two main approaches which can be listed as deterministic and stochastic approach, each one considering a different way of planning nurse scheduling in terms of their specialties. Scenarios replicate nurse scheduling; the quality of each scenario is assessed on the basis of performance indicators. The different scenarios are then tested on different demand data-sets in order to make comparisons.

The assumptions of our mathematical model for our multi-objective nurse scheduling problem are as follows:

1. Minimum number of required nurses in each shift is deterministic and known based on experience of demand since mostly elective surgeries are performed in the surgical suite.
2. There are three 8-hour shifts in a day: (1) from 7:00-15:00 day shift, (2) from 15:00-23:00 evening shift, and (3) 23:00- 07:00 night shift for 7 days a week and 24 hours a day.
3. There are two skill classes of nurses: regular nurses and intern nurses.
4. Regular nurses are classified into three groups based on their specialty: (1) cardiovascular surgery, (2) general surgery, and (3) neurosurgery.
5. Minimum staff level requirements must be satisfied.
6. Each nurse has to work at least 24 hours and at most 72 hours per week.
7. Nurses cannot work for more than 6 consecutive working days.
8. A nurse should not work more than 3 consecutive night shifts.
9. Each nurse works at most one shift a day. This is especially a necessary constraint for a surgical suite because of the high service level expectations and more arduous workload than other hospital units.

The notation used in the model formulation are explained below.

Sets

I set of nurses

S set of specialties

I_s set of nurses in specialty $s \in S$

J set of shifts in a day, where $j = 1$ for day shift, $j = 2$ for evening shift, and $j = 3$ for night shift

T set of days in the scheduling period (a month), $T = \{1,2, \dots ,28\}$

W set of weeks in the scheduling period (a month), $W = \{1,2,3,4\}$

Parameters

$$p_{ijt} = \begin{cases} 1, & \text{if nurse } i \in I \text{ prefers to work shift } j \in J \text{ on day } t \in T \\ 0, & \text{otherwise} \end{cases}$$

R number of regular nurses

NR_{sjt} minimum number of regular nurses in specialty $s \in S$ required for shift $j \in J$ on day $t \in T$

NI_{jt} minimum number of intern nurses required for shift $j \in J$ on day $t \in T$

h_L lower limit on the number of hours that a nurse can work per week

h_U upper limit on the number of hours that a nurse can work per week

cr_i cost of a regular nurse $i \in \{1,2, \dots , R\}$ per shift

c_i cost of an intern nurse $i \in \{R + 1, \dots , |I|\}$ per shift

Decision Variables

$$x_{ijt} = \begin{cases} 1, & \text{if nurse } i \in I \text{ is assigned to shift } j \in J \text{ on day } t \in T \\ 0, & \text{otherwise} \end{cases}$$

Model Formulation

$$\text{Min} \quad cr_i\left(\sum_{i=1}^{R}\sum_{j\in J}\sum_{t\in T}x_{ijt}\right)+c_i\left(\sum_{i=R+1}^{|I|}\sum_{j\in J}\sum_{t\in T}x_{ijt}\right) \tag{1}$$

Subject to

$$\sum_{i\in I_s}x_{ijt}\geq NR_{sjt}, \quad \forall s\in S, j\in J, t\in T \tag{2}$$

$$\sum_{i=R+1}^{|I|}x_{ijt}\geq NI_{jt}, \quad \forall j\in J, t\in T \tag{3}$$

$$\sum_{t=7(w-1)+1}^{7w}\sum_{j\in J}x_{ijt}\geq h_L/8, \quad \forall\, i\in I, w\in W \tag{4}$$

$$\sum_{t=7(w-1)+1}^{7w}\sum_{j\in J}x_{ijt}\leq h_U/8, \quad \forall\, i\in I, w\in W \tag{5}$$

$$\sum_{j\in J}x_{ijt}\leq 1, \quad \forall\, i\in I\,, t\in T \tag{6}$$

$$x_{i3t}+x_{i1(t+1)}\leq 1, \quad \forall\, i\in I\,, t\in T \tag{7}$$

$$\sum_{j\in J}\sum_{t=k}^{k+6}x_{ijt}\leq 6, \quad \forall\, i\in I\,, k=1,2,\dots,|T|-6 \tag{8}$$

$$\sum_{t=k}^{k+3}x_{i3t}\leq 3, \quad \forall\, i\in I\,, k=1,2,\dots,|T|-3 \tag{9}$$

$$\sum_{t\in T}x_{i3t}\geq 3, \quad \forall\, i\in I \tag{10}$$

$$x_{ijt}-p_{ijt}\leq 0, \quad \forall\, i\in I, j\in J, t\in T \tag{11}$$

$$\sum_{t \in T} x_{i3t} - \left(\sum_{t \in T} x_{i1t} + \sum_{t \in T} x_{i2t} \right) \leq 0, \quad \forall\, i \in I \tag{12}$$

$$\left(1 - \sum_{j \in J} x_{ijt}\right) + \sum_{j \in J} x_{ij(t+1)} + \left(1 - \sum_{j \in J} x_{ij(t+2)}\right) \leq 2,$$

$$\forall\, i \in I\,, t = 1,2,\ldots,|T| - 2 \tag{13}$$

$$\sum_{j \in J} x_{ijt} + \left(1 - \sum_{j \in J} x_{ij(t+1)}\right) + \sum_{j \in J} x_{ij(t+2)} \leq 2,$$

$$\forall\, i \in I\,, t = 1,2,\ldots,|T| - 2 \tag{14}$$

$$x_{ijt} \in \{0, 1\}, \forall\, i \in I, j \in J, t \in T \tag{15}$$

The initial objective function (1) minimizes the total cost of nurses assigned to shifts. Constraints (2) and (3) ensure that the required numbers of regular nurses and intern nurses are met, respectively, for each shift on each day. Constraints (4) and (5) bound the total weekly hours assigned to a nurse using the minimum and maximum allowed working hours, respectively. Constraint (6) avoids the assignment of more than one shift per day to a nurse. Assigning a day shift followed by a night shift or a night shift right before a day shift is prevented by constraint (7). Constraint (8) and (6) together ensure that a nurse work for at most 6 consecutive days. According to constraint (9), a nurse can be assigned at most 3 consecutive night shifts. Constraint (10) ensures that each nurse is assigned at least 3 night shifts in a month. Constraint (11) avoids shift assignments on days that are not preferred by a nurse, in other words, over-assignment. Constraint (12) requires the total night shifts assigned to be at most as many as the total day and evening shifts assigned to a nurse. Constraints (13) and (14) avoid the “0-1-0” or “1-0-1” types of assignments where a day on would be between two days off or a day off would be between two days on, which are both undesired cases from the nurse’s perspective. Constraint (15) defines the binary decision variables.

In this model, constraints (2)-(10) are hard constraints that cannot be violated and constraints (11)-(14) are soft constraints that can be violated at a cost in order to obtain a feasible schedule. In the following chapter, the goal programming model is formulated by incorporating the penalty of violating these soft constraints in the objective function.

Chapter 4

Solution Methodology

Goal Programming

Goal programming (GP) is used to solve linear programs with multiple objectives. The goal represented in constraint i can be overachieved or underachieved: d_i^+ and d_i^- deviation variables can be used for these goals. The goals are added to the constraint set with d_i^+ and d_i^- acting as the slack and surplus variables. One of the earliest applications of the goal programming method was presented by Charnes et al. (1955). To illustrate how goal programming is implemented in general, consider a linear program as follows.

$$
\begin{aligned}
Min \quad & z = cx \text{ (Objective function)} \\
s.t. \quad & A_1x \leq b_1 \text{ (Functional constraint)} \\
& A_2x \leq b_2 \text{ (Goal inequality)} \\
& \quad x \geq 0 \text{ (Non-negativity constraint)}
\end{aligned}
$$

A set of constraints in the form of inequalities represent the goals to be achieved. Therefore, the non-negative slack and surplus variables are introduced to formulate these constraints as equalities as shown below. Undesired deviations from the target values are minimized in the achievement function.

$$
\begin{aligned}
Min \quad & z' = cx + \textstyle\sum_{i \in I} c_i d_i^- \\
s.t. \quad & A_1x \leq b_1 \\
& A_2x + d^+ - d^- = b_2 \\
& x, d^+, d^- \geq 0
\end{aligned}
$$

In this study, goal programming is utilized as a solution method for nurse scheduling model. We use goal programming method to design a procedure for searching for most preferred solution that minimizes the objective function. In the model presented in Chapter 3, soft constraints are (11)-(14). In order to penalize the violation of these soft constraints, appropriate decision variables should be added to the model as explained below.

Goal 1: We modify the soft constraint (11) as follows:

$$x_{ijt} - p_{ijt} - o_{ijt} + u_{ijt} = 0, \qquad \forall\, i \in I, j \in J, t \in T \tag{16}$$

where the binary decision variables $o_{ijt} \in \{0,1\}, \forall\, i \in I, j \in J, t \in T$, and $u_{ijt} \in \{0,1\}, \forall\, i \in I, j \in J, t \in T$, represent the over- and under-assignment of shifts based on nurse preferences, respectively. It would be undesirable if a nurse is assigned to a shift that the nurse does not prefer as well as if a nurse is not assigned a shift that the nurse prefers to work. Therefore, both of these variables are penalized using the following objective function:

$$\text{minimize} \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} \left(\alpha o_{ijt} + \beta u_{ijt}\right) \tag{17}$$

where α is the penalty cost for assignment of a shift that a nurse does not prefer and β is the penalty cost for not assigning a shift that a nurse prefers, such that $\alpha \geq \beta$ because over-assignment is even more undesirable than under-assignment.

Goal 2: Soft constraint (12) is modified as follows to allow for deviations:

$$\sum_{t \in T} x_{i3t} - \left(\sum_{t \in T} x_{i1t} + \sum_{t \in T} x_{i2t}\right) - d_i^+ + d_i^- = 0 \quad \forall\, i \in I \tag{18}$$

where $d_i^+ \in \{0,1\}, \forall i \in I$, and $d_i^- \in \{0,1\}, \forall i \in I$, are the positive and negative deviations from the goal of assigning less night shifts than day and evening shifts in total. The positive deviations from this goal is penalized by adding the following objective function:

$$\text{minimize} \sum_{i \in I} d_i^+ \tag{19}$$

Goal 3: Soft constraint (13) is modified as follows to allow for deviations:

$$\left(1-\sum_{j\in J} x_{ijt}\right)+\sum_{j\in J} x_{ij(t+1)}+\left(1-\sum_{j\in J} x_{ij(t+2)}\right)-da_{it}^{+}+da_{it}^{-}=2$$

$$\forall\, i \in I\,, t=1,2,\dots,|T|-2 \qquad (20)$$

where $da_{it}^{+} \in \{0,1\}, \forall\, i \in I, t \in T$, and $da_{it}^{-} \in \{0,1\}, \forall\, i \in I, t \in T$, are the positive and negative deviations from the goal of avoiding isolated days on. Only the positive deviations from this goal is penalized by adding the following objective function:

$$\text{minimize} \sum_{i\in I}\sum_{t\in T} da_{it}^{+} \qquad (21)$$

Goal 4: Soft constraint (14) is modified as follows to allow for deviations:

$$\sum_{j\in J} x_{ijt}+\left(1-\sum_{j\in J} x_{ij(t+1)}\right)+\sum_{j\in J} x_{ij(t+2)}-db_{it}^{+}+db_{it}^{-}=2$$

$$\forall\, i \in I\,, t=1,2,\dots,|T|-2 \qquad (22)$$

where $db_{it}^{+} \in \{0,1\}, \forall\, i \in I, t \in T$ and $db_{it}^{-} \in \{0,1\}, \forall\, i \in I, t \in T$ are the positive and negative deviations from the goal of avoiding isolated days off. Only the positive deviations from this goal is penalized by adding the following objective function:

$$\text{minimize} \sum_{i\in I}\sum_{t\in T} db_{it}^{+} \qquad (23)$$

As a result of these goal formulations, the initial objective function is modified as follows.

$$\text{Minimize } z = cr_i\left(\sum_{i=1}^{R}\sum_{j\in J}\sum_{t\in T} x_{ijt}\right)+c_i\left(\sum_{i=R+1}^{|I|}\sum_{j\in J}\sum_{t\in T} x_{ijt}\right)$$

$$+\sum_{i\in I}\sum_{j\in J}\sum_{t\in T}\left(\alpha o_{ijt}+\beta u_{ijt}\right)+\sum_{i\in I} d_i^{+}+\sum_{i\in I}\sum_{t\in T} da_{it}^{+}+\sum_{i\in I}\sum_{t\in T} db_{it}^{+} \qquad (24)$$

The objective function (24) is the weighted sum of five different objectives: (i) minimizing the total cost of nurses assigned to shifts as in (1), (ii-v) minimizing the total violation of soft constraints as shown in (17), (19), (21), and (23), respectively.

Next, we describe our case study and present the computational results of our proposed model in terms of various performance measures.

Chapter 5
Computational Results

In this chapter, we provide a numeric example for illustrating the nurse scheduling model presented above. Consider a surgical suite in which 30 nurses are employed, 5 of whom are intern nurses who are still in training. The other 25 nurses are regular nurses with more experience and they have higher priority in terms of shift preferences and taking time off work. Regular nurses are split into three groups based on their specialties: 10 nurses are in cardiovascular surgery ($s = 1$), 8 nurses are in general surgery ($s = 2$), and 7 nurses are in neurological surgery ($s = 3$). The planning horizon is four weeks (28 days, including the weekends). The minimum required number of regular nurses of each specialty and intern nurses in each shift can be either deterministic or stochastic. When most of the surgeries performed are elective surgeries, deterministic demand scenarios can be used. When there are uncertainties about the demand for nurses due to possible emergency surgeries, stochastic demand scenarios can be used. We present the deterministic demand scenarios and the computational results based on these scenarios followed by the stochastic demand scenarios in this chapter.

5.1 Deterministic Demand Scenario

5.1.1 Demand Scenario 1

The minimum required number of regular nurses of each specialty and intern nurses in each shift are provided in Table 5.1. These numbers are assumed for each day since there are as many elective surgeries on the weekends as on the weekdays. The cost of a regular nurse per shift is $cr_i = 3, i = 1, 2, \ldots, 25$ and the cost of an intern per shift is $c_i = 2, i = 26, 27, \ldots, 30$.

Table 5.1: Minimum nurse requirements in DS 1

Shifts (j)	**1**	**2**	**3**
NR_{1jt}	2	2	1
NR_{2jt}	2	2	1
NR_{3jt}	2	2	1
NI_{jt}	1	1	1

In order to measure the performance of the model solutions, we define the following terms (25)-(27).

Let ω be the number of nurses who are over-assigned, i.e., that are assigned a shift they did not prefer to work, as shown below.

$$\omega = \sum_{i \in I:\, \exists\, o_{ijt}=1} 1 \tag{25}$$

Let ρ_o be the ratio of the total number of over-assigned shifts to the total number of assigned shifts, i.e., the ratio of shifts assigned that nurses do not prefer to work on, as shown below.

$$\rho_o = \frac{\sum_{i,j,t} o_{ijt}}{\sum_{i,j,t} x_{ijt}} \tag{26}$$

Let ρ_u be the ratio of the total number of under-assigned shifts to the maximum possible number of under-assigned shifts, i.e., the total number of shifts that nurses prefer to work on, as shown below.

$$\rho_u = \frac{\sum_{i,j,t} u_{ijt}}{\sum_{i,j,t} p_{ijt}} \tag{27}$$

The optimization model is formulated using GAMS 24.6 and solved using CPLEX 12.6 software on a 2.20 GHz Windows laptop computer with 6 GB RAM.

Base Case Scenario 0 (All 0):

In our Base Case Scenario 0, all shift preferences are set to zero, i.e., $p_{ijt} = 0, \forall i \in I, j \in J, t \in T$. Assuming that $\alpha = \beta = 1$, the optimal value for the objective function (24) in this scenario is $z^* = 1940$, where the total number of shifts assigned is $\sum_{i,j,t} x_{ijt} = \sum_{i,j,t} o_{ijt} = 506$. In this case, the proportion of over-assignment (out of 2520 shifts) is $\rho_o = 100\%$.

Base Case Scenario 1 (All 1):

The other extreme is having all nurses available on all shifts over the planning horizon, i.e., $p_{ijt} = 1, \forall i \in I, j \in J, t \in T$. Assuming that $\alpha = \beta = 1$, the optimal value for the objective function (24) is $z^* = 3451$, where, again, $\sum_{i,j,t} x_{ijt} = 506$, and $\sum_{i,j,t} u_{ijt} = 2014$. In this case, since there cannot be any over-assignment, $\rho_o = 0\%$. These base case scenarios show us the limits on the objective function value and the limits on the quantity $\sum_{i,j,t} o_{ijt}$, as well as ρ_o, when $\alpha = \beta = 1$ at the two extreme nurse preference values.

Four different Preference Scenarios (PS) are developed to test the performance of the scheduling model. These scenarios, PS*n* (*n* = 1, 2, 3, 4), are designed such that *n**20% of all shifts in a month are preferred by nurses. Therefore, PS1 is the most restrictive one out of these scenarios. The problem is solved for these scenarios with different penalty cost, α, values for not meeting the nurse preferences and all the problem instances are solved to optimality within at most 1000.05 seconds. We present the optimal solutions of the proposed model for the four scenarios for various α values in Table 5.2 – Table 5.4 below. In all of the problem instances, it is observed that constraint (12), regarding the "0-1-0" type of assignment, is never violated, i.e., $\sum_{\mathrm{i,t}} \mathrm{da}^{+}_{\mathrm{i,t}} = 0$ in all cases.

Table 5.2: Results for PS1 and PS4 – DS 1

	PS1						PS4					
α	CPU Time (s)	z^*	$\sum_{i,t} db^+_{i,t}$	ρ_o	ρ_u	ω	CPU Time (s)	z^*	$\sum_{i,t} db^+_{i,t}$	ρ_o	ρ_u	ω
1	5.38	2228	2	0.197	0.784	30	4.42	3078	0	0.133	0.782	18
2	5.11	2625	2	0.197	0.784	30	4.11	3146	0	0.133	0.782	18
3	5.22	3024	2	0.197	0.784	30	2.72	3215	0	0.133	0.782	18
4	5.45	3419	2	0.197	0.784	30	4.33	3282	0	0.133	0.782	18
5	15.64	3816	2	0.197	0.784	30	4	3348	0	0.133	0.782	18

When PS1 is used, at any α level the same optimal schedule is obtained. In this solution, 0.197 of assignments (397 of 2016) are over-assignments made for all 30 nurses as can be seen from Table 5.2. When PS4 is assumed, the ratio of over-assigned shifts is reduced to 13.3% (67 of 504) and the number of nurses affected by over-assignment is reduced to 18. In PS4, there is also no "1-0-1" type of assignment in the optimal solution. It is observed that the ratio of over-assigned shifts is significantly reduced to 12.6 % for $\alpha = 1$ when PS2 is assumed and the minimum possible value for this ratio is 11.8 % in this scenario as shown in Table 5.3 The number of nurses affected by over-assignment, ω, is reduced to 27 as α is increased. The ratio, ρ_o, is further reduced to 11.3 % when PS3 is used and the minimum possible value becomes 8.5 % in this scenario as shown Table 5.4. The number of nurses affected by over-assignment is reduced to 25 as α is increased.

Additionally, in PS2, at $\alpha = 4$ and $\alpha = 5$, three nurses are assigned 8 more night shifts than their total day and evening shifts in the optimal schedule, i.e., $\sum_{i \in I} d_i^+ = 24$ at $\alpha = 5$. The sum of these positive slack variables increases in PS3 to 42 and 46 for the same α values, respectively. Also, when PS3 is used, the number of "1-0-1" type assignments made increases as α increases, whereas, in PS2, this number is fixed at 2 for all α values.

Table 5.3: Results for PS2 – DS 1

α	CPU Time(s)	z^*	$\sum_{i\in I} d_i^+$	$\sum_{i,t} db_{i,t}^+$	ρ_o	ρ_u	ω
1	16.17	2324	0	2	0.126	0.686	30
2	19.52	2514	0	2	0.126	0.684	30
3	19.19	2704	0	2	0.126	0.684	30
4	85.81	2882	20	2	0.118	0.660	27
5	72.20	3060	24	2	0.118	0.660	27

Table 5.4: Results for PS3 – DS 1

α	CPU Time (s)	z^*	$\sum_{i\in I} d_i^+$	$\sum_{i,t} db_{i,t}^+$	ρ_o	ρ_u	ω
1	772	2678	0	6	0.113	0.739	29
2	42.31	2785	11	14	0.103	0.731	29
3	41.98	2888	14	16	0.101	0.729	29
4	1000.09	2977	42	17	0.087	0.711	25
5	1000.05	3063	46	19	0.085	0.708	25

In Figure 5.1, it is shown that the impact of increasing the penalty cost for over-assignments, α, diminishes as the number of preferred shifts is increased in the preference matrix. This is mostly due to the reduction in the number of over-assignments required in the optimal schedule.

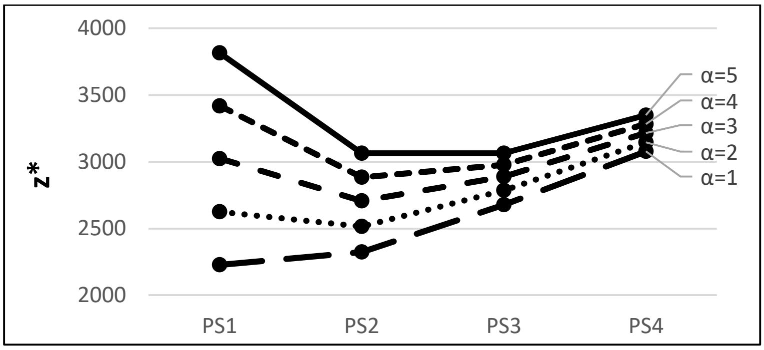


Figure 5.1: The optimal objective function values for the four preference scenarios at different α levels.

The PS1 and PS4 scenarios result in constant ρ_o values at all α levels as shown in Figure 5.2. As the number of preferred shifts is increased by the same amount, in other words as we move from PS1 to PS4, the improvement in the value of ρ_o diminishes at all α levels. Our case study shows that the job satisfaction of nurses measured in terms of ρ_o can be increased as much as 6% in a scenario like PS2.

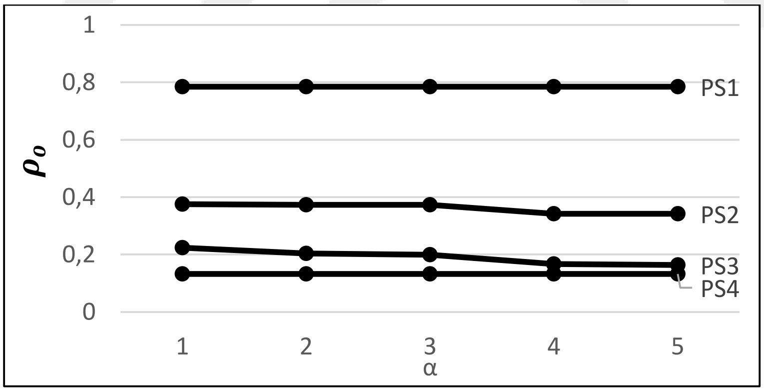


Figure 5. 2: The ratio of over-assignment for the four preference scenarios at different α levels.

We illustrate the performance of the proposed model with an example under various shift preference scenarios. Computational results show that the multi-objective nature of the model leads to higher job satisfaction for nurses in terms of the performance measures evaluated, by avoiding the assignment of shifts that are not preferred, by avoiding isolated days on or off, and by avoiding disproportionate night shift assignments. The proposed model can easily be used in practice to produce the best possible nurse schedule given a certain shift preference scenario by adjusting the penalty cost, α.

5.1.2 Demand Scenario 2

In this section, a different demand scenario is used for solving and illustrating nurse scheduling problem while changing nurse requirements according to weekdays and weekends. We accept that nurse demand is higher on all Fridays and Saturdays than other days during the planning horizon, as shown in Tables 5.5 and 5.6. All problem instances assuming different preference scenarios are solved based on Demand Scenario 2 at different α levels.

Table 5.5: Minimum nurse requirements (except Fridays and Saturdays) in DS 2

Shifts (j)	**1**	**2**	**3**
NR_{1jt}	2	2	1
NR_{2jt}	2	2	1
NR_{3jt}	2	2	1
NI_{jt}	1	1	1

Table 5.6: Minimum nurse requirements (Fridays and Saturdays) in DS 2

Shifts (j)	**1**	**2**	**3**
NR_{1jt}	3	2	1
NR_{2jt}	3	2	1
NR_{3jt}	2	2	2
NI_{jt}	1	1	1

Base Case Scenario 0 (All 0):

In our Base Case Scenario 0, all shift preferences are set to zero, i.e., $p_{ijt} = 0, \forall i \in I, j \in J, t \in T$. Assuming that $\alpha = \beta = 1$, the optimal value for the objective function (24) in this scenario is $z^* = 2095$, where the total number of shifts assigned is $\sum_{i,j,t} x_{ijt} = \sum_{i,j,t} o_{ijt} = 544$. In this case, the proportion of over-assignment (out of 2520 shifts) is $\rho_o = 100\%$.

Base Case Scenario 1 (All 1):

The other extreme is having all nurses available on all shifts over the planning horizon, i.e., $p_{ijt} = 1, \forall i \in I, j \in J, t \in T$. Assuming that $\alpha = \beta = 1$, the optimal value for the objective function (24) is $z^* = 3529$, where, again, $\sum_{i,j,t} x_{ijt} = 544$, and $\sum_{i,j,t} u_{ijt} = 1976$. In this case, since there cannot be any over-assignment, $\rho_o = 0\%$.

Four different Preference Scenarios (PS) are developed to test the performance of the scheduling model. These scenarios, PSn (n = 1, 2, 3, 4), are designed such that n*20% of all shifts in a month are preferred by nurses. Therefore, PS1 is the most restrictive one out of these scenarios. The problem is solved for these scenarios with different penalty cost, α, values for not meeting the nurse preferences and all the problem instances are solved to optimality within at most 1000.05 seconds as shown in Tables 5.7-5.10. In all of the problem instances, it is observed that constraint (12), regarding the "0-1-0" type of assignment, is never violated, i.e., $\sum_{i,t} da^+_{i,t} = 0$ in all cases.

Table 5.7: Results for PS1 – DS 2

α	CPU Time (s)	z^*	$\sum_{i \in I} d^+_i$	$\sum_{i,t} db^+_{i,t}$	ρ_o	ρ_u	ω
1	1000.05	2339	0	5	0.205	0.740	30
2	1000.05	2752	0	5	0.205	0.740	30
3	1000.02	3165	0	5	0.205	0.740	30
4	1000.02	3578	0	5	0.205	0.740	30
5	1000.02	3991	0	5	0.205	0.740	27

When PS1 is used, at any α level the same optimal schedule is obtained. In this solution, 20.5 % of assignments (413 of 2016) are over-assignments made for all 30 nurses as can be seen from Table 9. When PS4 is assumed, the ratio of over-assigned shifts is reduced to 13.3% (67 of 504) and the number of nurses affected by over-assignment is reduced to 18. In PS4, there is also no “1-0-1” type of assignment in the optimal solution.

It is observed that the ratio of over-assigned shifts is significantly reduced to 12.5 % for $\alpha = 1$ when PS2 is assumed and the minimum possible value for this ratio is 11.8 % in this scenario as shown in Table 5.8. The number of nurses affected by over-assignment, ω, is reduced to 24 as α is increased. The ratio, ρ_o, is further reduced to 11 % when PS3 is used and the minimum possible value becomes 9 % in this scenario as shown Table 5.9 . The number of nurses affected by over-assignment is reduced to 24 as α is increased.

Additionally, in PS2, at $\alpha = 4$ and $\alpha = 5$, three nurses are assigned 6 more night shifts than their total day and evening shifts in the optimal schedule, i.e., $\sum_{i \in I} d_i^+ = 18, \sum_{i \in I} d_i^+ = 24,$ respectively. The sum of these positive slack variables increases in PS3 to 46 and 54 for the same α values, respectively. Also, when PS3 is used, the number of “1-0-1” type assignments made increases as α increases, whereas, in PS2, this number is fixed at 2 for all α values.

Table 5.8: Results for PS2 – DS 2

α	CPU Time (s)	z^*	$\sum_{i \in I} d_i^+$	$\sum_{i,t} db_{i,t}^+$	ρ_o	ρ_u	ω
1	1002.84	2403	3	10	0.125	0.648	29
2	1000.03	2585	6	6	0.124	0.646	28
3	1000.02	2774	6	8	0.124	0.646	28
4	1000.01	2955	18	8	0.120	0.634	27
5	1000.03	3134	24	12	0.118	0.658	24

Table 5.9: Results for PS3 – DS 2

α	CPU Time (s)	z^*	$\sum_{i \in I} d_i^+$	$\sum_{i,t} db_{i,t}^+$	ρ_o	ρ_u	ω
1	1000.03	2758	17	9	0.11	0.71	29
2	1000.02	2864	27	21	0.10	0.71	28
3	1000.05	2962	29	19	0.10	0.71	28
4	1000.02	3056	46	24	0.09	0.69	27
5	1000.02	3144	54	30	0.09	0.69	24

Table 5.10: Results for PS4 – DS 2

α	CPU Time (s)	z^*	$\sum_{i \in I} d_i^+$	$\sum_{i,t} db_{i,t}^+$	ρ_o	ρ_u	ω
1	1000.02	3157	0	3	0.133	0.763	18
2	1000.03	3224	0	3	0.133	0.763	18
3	1000.02	3292	0	4	0.133	0.763	18
4	1000.02	3358	0	3	0.133	0.763	18
5	1000.03	3425	0	3	0.133	0.763	18

5.1.3 Demand Scenario 3

In this scenario, we accept that minimum number of required nurses in all Fridays and Saturdays during 4 weeks is changing. Tables 5.11-5.15 demonstrate the minimum nurse requirements depending on days.

Table 5.11: Minimum nurse requirements (except Fridays and Saturdays) in DS 3

Shifts (j)	**1**	**2**	**3**
NR_{1jt}	2	2	1
NR_{2jt}	2	2	1
NR_{3jt}	2	2	1
NI_{jt}	1	1	1

Table 5.12: Minimum nurse requirements for Fridays, $t = 5, 19$, in DS 3

Shifts (j)	**1**	**2**	**3**
NR_{1jt}	3	2	2
NR_{2jt}	3	2	2
NR_{3jt}	3	2	2
NI_{jt}	2	1	1

Table 5.13: Minimum nurse requirements for Saturdays, $t = 6, 20$, in DS 3

Shifts (j)	**1**	**2**	**3**
NR_{1jt}	3	2	2
NR_{2jt}	3	2	2
NR_{3jt}	2	2	2
NI_{jt}	2	1	1

Table 5.14: Minimum nurse requirements for Fridays, $t = 12, 26$, in DS 3

Shifts (j)	**1**	**2**	**3**
NR_{1jt}	3	3	2
NR_{2jt}	2	2	2
NR_{3jt}	2	2	2
NI_{jt}	1	1	1

Table 5.15: Minimum nurse requirements for Saturdays, $t = 13, 27$, in DS 3

Shifts (j)	**1**	**2**	**3**
NR_{1jt}	3	3	2
NR_{2jt}	2	2	2
NR_{3jt}	3	2	2
NI_{jt}	1	1	1

Base Case Scenario 0 (All 0):

In our Base Case Scenario 0, all shift preferences are set to zero, i.e., $p_{ijt} = 0, \forall i \in I, j \in J, t \in T$. Assuming that $\alpha = \beta = 1$, the optimal value for the objective function (24) in this scenario is $z^* = 2129$, where the total number of shifts assigned is $\sum_{i,j,t} x_{ijt} = \sum_{i,j,t} o_{ijt} = 544$. In this case, the proportion of over-assignment (out of 2520 shifts) is $\rho_o = 100\%$.

Base Case Scenario 1 (All 1):

The other extreme is having all nurses available on all shifts over the planning horizon, i.e., $p_{ijt} = 1, \forall i \in I, j \in J, t \in T$. Assuming that $\alpha = \beta = 1$, the optimal value for the objective function (24) is $z^* = 3545$, where, again, $\sum_{i,j,t} x_{ijt} = 544$, and $\sum_{i,j,t} u_{ijt} = 1976$. In this case, since there cannot be any over-assignment, $\rho_o = 0\%$.

The problem is solved for PS1-PS4 with different penalty cost, α, values for not meeting the nurse preferences and all the problem instances are solved to optimality within at most 1000.03 seconds as shown in Tables 5.16-5.19. In all of the problem instances, it is observed that constraint (12), regarding the "0-1-0" type of assignment, is never violated, i.e., $\sum_{i,t} da^+_{i,t} = 0$ in all cases.

Table 5.16: Results for PS 1 – DS 3

α	CPU Time (s)	z^*	$\sum_{i \in I} d^+_i$	$\sum_{i,t} db^+_{i,t}$	ρ_o	ρ_u	ω
1	1000.02	2418	4	12	0.207	0.732	30
2	1000.02	2605	6	14	0.207	0.732	30
3	1000.03	2793	6	15	0.207	0.732	30
4	1000.03	2976	18	17	0.207	0.732	30
5	1000.03	3155	24	21	0.207	0.732	30

When PS1 is used, at any α level the same optimal schedule is obtained. In this solution, 20.7% of assignments (413 of 2016) are over-assignments made for all

30 nurses as can be seen from Table 5.19. When PS4 is assumed, the ratio of over-assigned shifts is reduced to 13.3% (67 of 504) and the number of nurses affected by over-assignment is reduced to 18. In PS4, there is also no "1-0-1" type of assignment in the optimal solution. It is observed that the ratio of over-assigned shifts is significantly reduced to 13 % for $\alpha = 1$ when PS2 is assumed and the minimum possible value for this ratio is 12 % in this scenario as shown in Table 5.17. The number of nurses affected by over-assignment, ω, is reduced to 27 as α is increased. The ratio, ρ_o, is further reduced to 11.3% when PS3 is used and the minimum possible value becomes 9.3% in this scenario as shown Table 5.18. The number of nurses affected by over-assignment is reduced to 26 as α is increased.

Additionally, in PS2, at $\alpha = 4$ and $\alpha = 5$, three nurses are assigned 6 more night shifts than their total day and evening shifts in the optimal schedule, i.e., $\sum_{i \in I} d_i^+ = 18, \sum_{i \in I} d_i^+ = 24,$ respectively. The sum of these positive slack variables increases in PS3 to 46 and 54 for the same α values, respectively. Also, when PS3 is used, the number of "1-0-1" type assignments made increases as α increases, whereas, in PS2, this number is fixed at 2 for all α values. When α increases, z* values also increase, however, the values of ρ_o,ρ_u, ω will be constant after $\alpha = 5$.

Table 5.17: Results for PS 2 – DS 3

α	CPU Time (s)	z*	$\sum_{i \in I} d_i^+$	$\sum_{i,t} db_{i,t}^+$	ρ_o	ρ_u	ω
1	1000.02	2779	11	12	0.13	0.64	30
2	1000.02	2880	21	16	0.12	0.64	30
3	1000.02	2984	22	18	0.12	0.64	30
4	1000.03	3085	34	21	0.12	0.63	29
5	1000.03	3187	42	33	0.12	0.62	27

Table 5.18: Results for PS 3 – DS 3

α	CPU Time (s)	z^*	$\sum_{i\in I} d_i^+$	$\sum_{i,t} db_{i,t}^+$	ρ_o	ρ_u	ω
1	1000.02	2779	11	12	0.113	0.710	29
2	1000.02	2880	21	16	0.104	0.704	29
3	1000.02	2984	22	18	0.103	0.704	29
4	1000.02	3085	34	21	0.097	0.696	27
5	1000.02	3187	42	33	0.093	0.690	26

Table 5.19: PS4 – DS 3

α	CPU Time (s)	z^*	$\sum_{i\in I} d_i^+$	$\sum_{i,t} db_{i,t}^+$	ρ_o	ρ_u	ω
1	1000.02	2418	0	9	0.133	0.759	18
2	1000.02	2605	0	9	0.133	0.759	18
3	1000.02	2793	0	9	0.133	0.759	18
4	1000.02	2976	0	9	0.133	0.759	18
5	1000.02	3155	0	9	0.133	0.759	18

5.1.4 Comparison of Scenario Results

Figure 5.3 illustrates that, for a given preference scenario, z^* values increase when the minimum required number of regular nurses is increased in different demand scenarios. Also, z^* values increase when all preferences are equal to 1, mostly due to the positive under-assignment variable values.

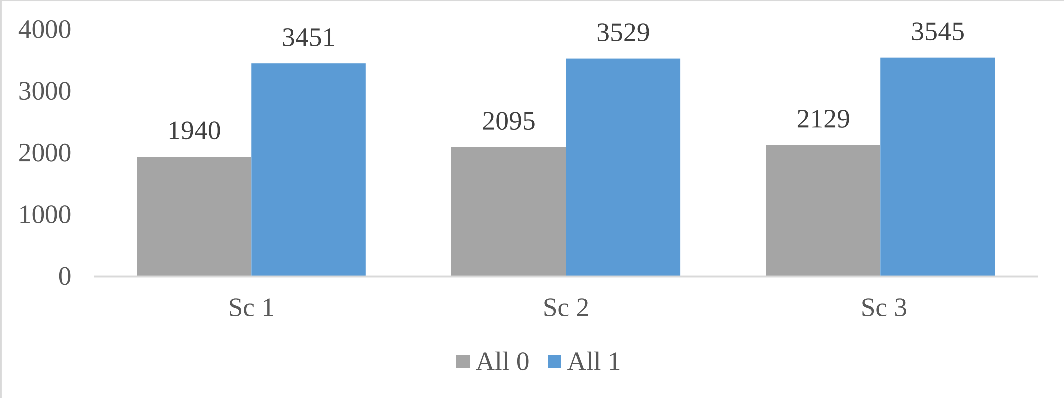


Figure 5.3: z^* values for three different demand scenarios

Figure 5.4 and Figure 5.5 indicate that the number of nurses assigned to shifts on each day changes between 17 and 19, during 28 days based on All 0 and All 1 preference scenarios, respectively. Number of nurses assigned is higher on Fridays and Saturdays as forced by the minimum requirements.

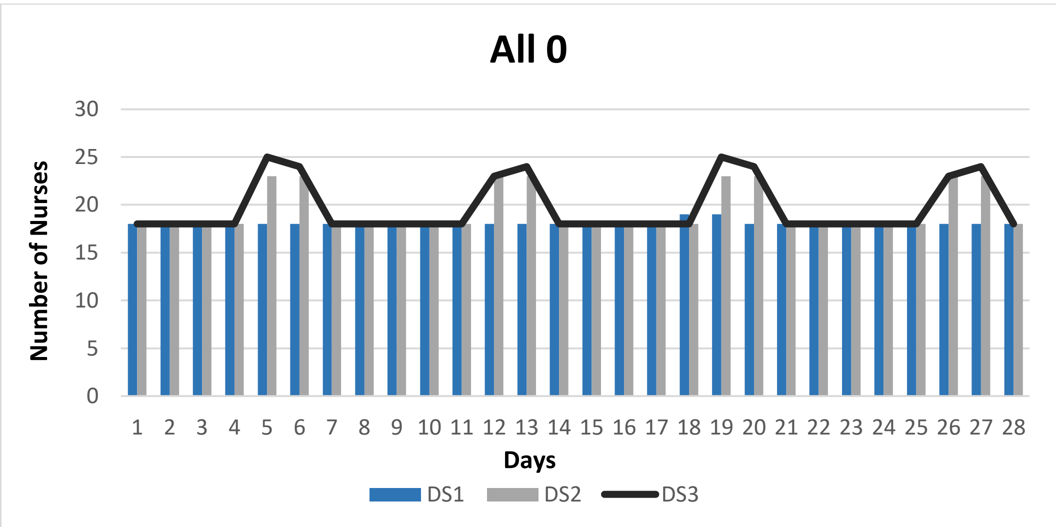


Figure 5.4: Number of nurses at all preference 0

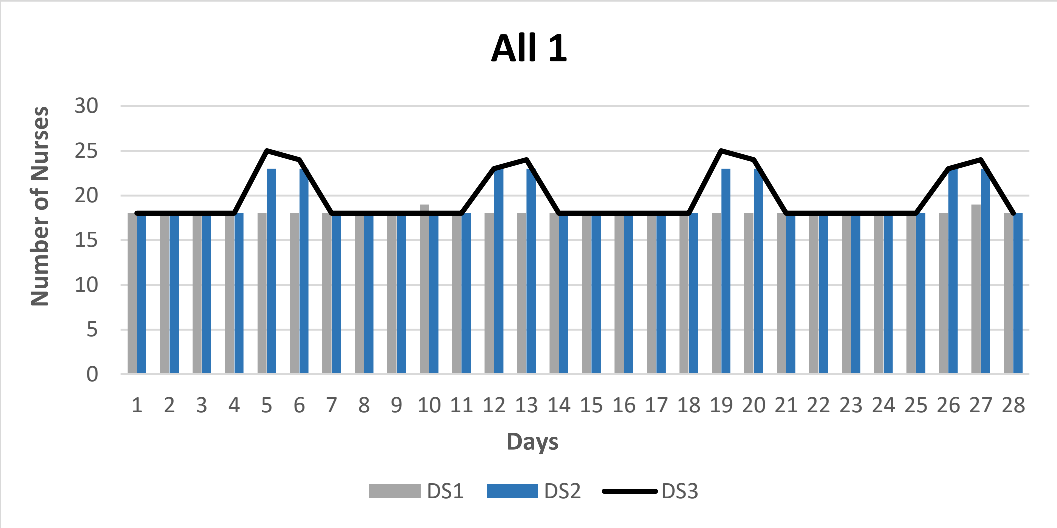
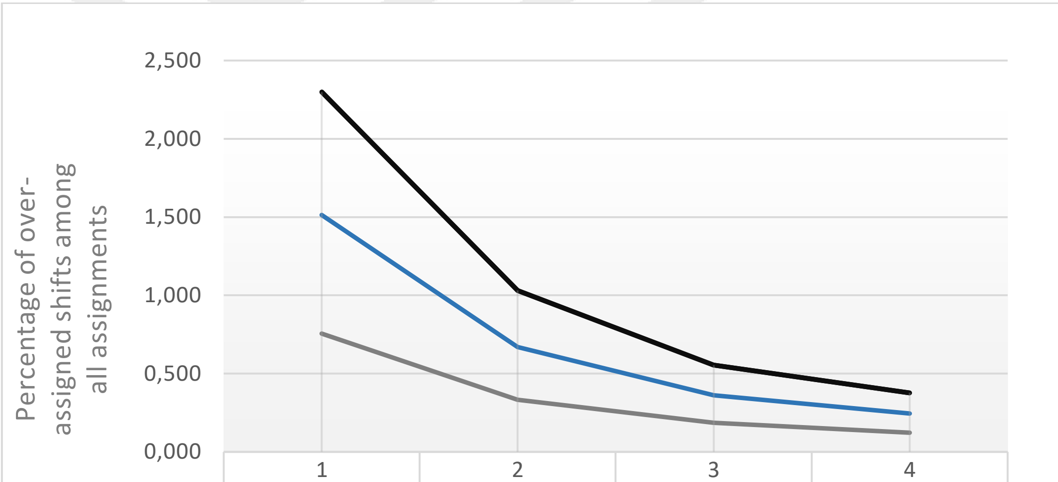


Figure 5.5: Number of nurses at all preference 1



	1	2	3	4
DS1	0,785	0,361	0,192	0,132
DS2	0,759	0,337	0,176	0,123
DS3	0,755	0,332	0,186	0,121

Figure 5.6: Percentage of over assigned among all x assignments for different preference scenarios.

Figure 5.6 demonstrates the percentage of over-assigned shifts among all assignments for preference scenarios 1-4 and demand scenarios which consist of different minimum number of required nurses. We can see that the percent of over-assigned shifts decreases as the demand increases, in general, and as the preference matrix is less restricted.

5.2 Stochastic Demand Scenario

In this stochastic demand scenario, the nurse scheduling model is arranged using three demand scenarios with shift preferences (PS1, PS2, PS3, PS4, Base Case Scenario 0, and Base Case Scenario 1). In the objective function, the demand scenarios have different probabilities as shown in Table 5.20.

Table 5.20: Probability distribution for demand scenarios

Demand Scenarios ($\boldsymbol{\theta}$)	**1**	**2**	**3**
P_θ	0.25	0.50	0.25

Stochastic Model Development

The model formulation is modified using the following notation.

Set

Θ set of demand scenarios, $\boldsymbol{\theta = 1}$ for the first demand scenario, $\boldsymbol{\theta = 2}$ for the second demand scenario, $\boldsymbol{\theta = 3}$ for the third demand scenario.

Parameters

$$p_{ijt}^{\theta} = \begin{cases} 1, & \text{if nurse } i \in I \text{ prefers to work shift } j \in J \text{ on day } t \in T \\ & \text{in demand scenario } \theta \in \Theta \\ 0, & \text{otherwise} \end{cases}$$

NR_{sjt}^{θ} minimum number of regular nurses in specialty $s \in S$ required for shift $j \in J$ on day, $t \in T$, demand scenario $\theta \in \Theta$

NI_{jt}^{θ} minimum number of intern nurses required for shift $j \in J$ on day $t \in T$ demand scenario $\theta \in \Theta$

P_θ probability of demand scenario $\theta \in \Theta$

Decision Variables

$$x_{ijt}^{\theta} = \begin{cases} 1, & \text{if nurse } i \in I \text{ is assigned to shift } j \in J \text{ on day } t \in T \\ & \text{in demand scenario } \theta \in \Theta \\ 0, & \text{otherwise} \end{cases}$$

Model Formulation

$$\text{Min} \quad \sum_{\theta \in \Theta} \left\{ \left[cr_i * \left(\sum_{i=1}^{R} \sum_{j \in J} \sum_{t \in T} x_{ijt}^{\theta} \right) + c_i * \left(\sum_{i=R+1}^{|I|} \sum_{j \in J} \sum_{t \in T} x_{ijt}^{\theta} \right) \right] * P_{\theta} \right\} \tag{28}$$

Subject to

$$\sum_{i \in I_s} x_{ijt}^{\theta} \geq NR_{sjt}^{\theta}, \quad \forall \; \theta \in \Theta, s \in S, j \in J, t \in T \tag{29}$$

$$\sum_{i=R+1}^{|I|} x_{ijt}^{\theta} \geq NI_{jt}^{\theta}, \quad \forall \; \theta \in \Theta, j \in J, t \in T \tag{30}$$

$$\sum_{t=7(w-1)+1}^{7w} \sum_{j \in J} x_{ijt}^{\theta} \geq h_L/8, \quad \forall \; \theta \in \Theta, i \in I, w \in W \tag{31}$$

$$\sum_{t=7(w-1)+1}^{7w} \sum_{j \in J} x_{ijt}^{\theta} \leq h_U/8, \quad \forall \; \theta \in \Theta, i \in I, w \in W \tag{32}$$

$$\sum_{j \in J} x_{ijt}^{\theta} \leq 1, \quad \forall \, \theta \in \Theta, i \in I \,, t \in T \tag{33}$$

$$x_{i3t}^{\theta} + x_{i1(t+1)}^{\theta} \leq 1, \quad \forall \, \theta \in \Theta, i \in I \,, t \in T \tag{34}$$

$$\sum_{j \in J} \sum_{t=k}^{k+6} x_{ijt}^{\theta} \leq 6, \quad \forall \, \theta \in \Theta, \qquad i \in I \,, k = 1,2, \dots, |T| - 6 \tag{35}$$

$$\sum_{t=k}^{k+3} x_{i3t}^{\theta} \leq 3, \quad \forall \, \theta \in \Theta, i \in I \,, k = 1,2, \dots, |T| - 3 \tag{36}$$

$$\sum_{t \in T} x_{i3t}^{\theta} \geq 3, \quad \forall \, \theta \in \Theta, i \in I \tag{37}$$

$$x_{ijt}^{\theta} - p_{ijt}^{\theta} \leq 0, \quad \forall\, \theta \in \Theta, i \in I, j \in J, t \in T \tag{38}$$

$$\sum_{t\in T} x_{i3t}^{\theta} - \left(\sum_{t\in T} x_{i1t}^{\theta} + \sum_{t\in T} x_{i2t}^{\theta}\right) \leq 0, \quad \forall\, \theta \in \Theta, i \in I \tag{39}$$

$$\left(1 - \sum_{j\in J} x_{ijt}^{\theta}\right) + \sum_{j\in J} x_{ij(t+1)}^{\theta} + \left(1 - \sum_{j\in J} x_{ij(t+2)}^{\theta}\right) \leq 2,$$

$$\forall\, \theta \in \Theta, i \in I, t = 1,2,\ldots,|T| - 2 \tag{40}$$

$$\sum_{j\in J} x_{ijt}^{\theta} + \left(1 - \sum_{j\in J} x_{ij(t+1)}^{\theta}\right) + \sum_{j\in J} x_{ij(t+2)}^{\theta} \leq 2,$$

$$\forall\, \theta \in \Theta, i \in I, t = 1,2,\ldots,|T| - 2 \tag{41}$$

$$x_{ijt}^{\theta} \in \{0, 1\}, \forall\, \theta \in \Theta, i \in I, j \in J, t \in T \tag{42}$$

In this model, the index θ is added in order to solve the scheduling problem for different demand scenarios 1, 2, and 3.

Next, we present the reformulation of soft constraints and the objective function using goal programming.

Goal 1: We modify the soft constraint (11) as follows:

$$x_{ijt}^{\theta} - p_{ijt}^{\theta} - o_{ijt}^{\theta} + u_{ijt}^{\theta} = 0, \quad \forall\, \theta \in \Theta, i \in I, j \in J, t \in T \tag{43}$$

where the binary decision variables $o_{ijt}^{\theta} \in \{0, 1\}, \forall\, \theta \in \Theta, i \in I, j \in J, t \in T$, and $u_{ijt}^{\theta} \in \{0, 1\}, \forall\, \theta \in \Theta, i \in I, j \in J, t \in T$, represent the over- and under-assignment of shifts based on nurse preferences, respectively. It would be undesirable if a nurse is assigned to a shift that the nurse does not prefer as well as if a nurse is not assigned a shift that the nurse prefers to work. Therefore, both of these variables are penalized using the following objective function:

$$\text{minimize} \sum_{\theta \in \Theta} \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} \left(\alpha o_{ijt}^{\theta} + \beta u_{ijt}^{\theta} \right) \tag{44}$$

where α is the penalty cost for assignment of a shift that a nurse does not prefer and β is the penalty cost for not assigning a shift that a nurse prefers, such that $\alpha \geq \beta$ because over-assignment is even more undesirable than under-assignment.

Goal 2: Soft constraint (12) is modified as follows to allow for deviations:

$$\sum_{t \in T} x_{i3t}^{\theta} - \left(\sum_{t \in T} x_{i1t}^{\theta} + \sum_{t \in T} x_{i2t}^{\theta} \right) - {d^{\theta}}_{i}^{+} + {d^{\theta}}_{i}^{-} = 0 \quad \forall\, \theta \in \Theta, i \in I \tag{45}$$

where ${d^{\theta}}_{i}^{+} \in \{0,1\}, \forall\, \theta \in \Theta, i \in I,$ and ${d^{\theta}}_{i}^{-} \in \{0,1\}, \forall\, \theta \in \Theta, i \in I,$ are the positive and negative deviations from the goal of assigning less night shifts than day and evening shifts in total. The positive deviations from this goal is penalized by adding the following objective function:

$$\text{minimize} \sum_{i \in I} {d^{\theta}}_{i}^{+} \tag{46}$$

Goal 3: Soft constraint (13) is modified as follows to allow for deviations:

$$\left(1 - \sum_{j \in J} x_{ijt}^{\theta} \right) + \sum_{j \in J} x_{ij(t+1)}^{\theta} + \left(1 - \sum_{j \in J} x_{ij(t+2)}^{\theta} \right) - {da^{\theta}}_{it}^{+} + {da^{\theta}}_{it}^{-} = 2$$

$$\forall\, \theta \in \Theta, i \in I\,, t = 1,2,\ldots,|T| - 2 \tag{47}$$

where ${da^{\theta}}_{it}^{+} \in \{0,1\}, \forall\, \theta \in \Theta, i \in I, t \in T,$ and ${da^{\theta}}_{it}^{-} \in \{0,1\}, \forall\, \theta \in \Theta, i \in I, t \in T$, are the positive and negative deviations from the goal of avoiding isolated days on. Only the positive deviations from this goal is penalized by adding the following objective function:

$$\text{minimize} \sum_{i \in I} \sum_{t \in T} {da^{\theta}}_{it}^{+} \tag{48}$$

Goal 4: Soft constraint (14) is modified as follows to allow for deviations:

$$\sum_{j\in J} x^{\theta}_{ijt} + \left(1 - \sum_{j\in J} x^{\theta}_{ij(t+1)}\right) + \sum_{j\in J} x^{\theta}_{ij(t+2)} - {db^{\theta}}^{+}_{it} + {db^{\theta}}^{-}_{it} = 2$$

$$\forall\, \theta \in \Theta, i \in I\,, t = 1,2, \dots, |T| - 2 \tag{49}$$

where ${db^{\theta}}^{+}_{it} \in \{0,1\}, \forall\, \theta \in \Theta, i \in I, t \in T$ and ${db^{\theta}}^{-}_{it} \in \{0,1\}, \forall\, \theta \in \Theta, i \in I, t \in T$ are the positive and negative deviations from the goal of avoiding isolated days off. Only the positive deviations from this goal is penalized by adding the following objective function:

$$\text{minimize} \sum_{i\in I} \sum_{t\in T} {db^{\theta}}^{+}_{it} \tag{50}$$

As a result of these goal formulations, the initial objective function is modified as follows.

$$\begin{aligned}
\text{Minimize } z = \sum_{\theta\in\Theta} \Bigg\{ \Bigg[& cr_i * \left(\sum_{i=1}^{R} \sum_{j\in J} \sum_{t\in T} x^{\theta}_{ijt} \right) + c_i * \left(\sum_{i=R+1}^{|I|} \sum_{j\in J} \sum_{t\in T} x^{\theta}_{ijt} \right) \\
& + \sum_{i\in I} \sum_{j\in J} \sum_{t\in T} \left(\alpha o^{\theta}_{ijt} + \beta u^{\theta}_{ijt} \right) + \sum_{i\in I} {d^{\theta}}^{+}_{i} + \sum_{i\in I} \sum_{t\in T} {da^{\theta}}^{+}_{it} \\
& + \sum_{i\in I} \sum_{t\in T} {db^{\theta}}^{+}_{it} \Bigg] * P_{\theta} \Bigg\}
\end{aligned} \tag{51}$$

The objective function (51) is the weighted sum of five different objectives: (i) minimizing the total cost of nurses assigned to shifts as in (1), (ii-v) minimizing the total violation of soft constraints as shown in (44), (46), (48), and (50), respectively. The demand scenario probabilities are used as coefficients in this objective function.

5.2.1 Computational Results

In this case, the problem is solved for these demand scenarios with penalty cost $\alpha = 3$ and all the problem instances are solved to optimality within at most 1000.03 seconds. We present the optimal solutions of the proposed model for each demand scenario separately for the six preference scenarios for various α values in Tables 5.21 – 5.26 below. Table 5.27 presents the weighted sum of the results the demand scenario probabilities are the weights.

Table 5.21: Results of all preference 0

θ	P_θ	Probability	$\sum_{i,j,t} x_{ijt}^{\theta}$	$\sum_{i,j,t} o_{ijt}^{\theta}$	$\sum_{ij,t} u_{ijt}^{\theta}$	$\sum_{i} \mathrm{d}^{\theta}{}_{i}^{+}$	$\sum_{i,t} da^{\theta}{}_{it}^{+}$	$\sum_{i,t} \mathrm{db}^{\theta}{}_{i,t}^{+}$
1	0.25	0.25	506	834	0	0	0	0
2	0.50	0.50	533	0	0	0	0	7
3	0.25	0.25	552	757	0	0	0	9

Table 5.22: Results of all preference 1

θ	P_θ	Probability	$\sum_{i,j,t} x_{ijt}^{\theta}$	$\sum_{i,j,t} o_{ijt}^{\theta}$	$\sum_{i,j,t} u_{ijt}^{\theta}$	$\sum_{i} \mathrm{d}^{\theta}{}_{i}^{+}$	$\sum_{i,t} da^{\theta}{}_{it}^{+}$	$\sum_{i,t} \mathrm{db}^{\theta}{}_{i,t}^{+}$
1	0.25	0.25	506	3	508	0	0	10
2	0.50	0.50	533	0	0	0	0	12
3	0.25	0.25	552	3	427	0	0	16

Table 5.23: Results of PS1

θ	P_θ	Probability	$\sum_{i,j,t} x_{ijt}^{\theta}$	$\sum_{i,j,t} o_{ijt}^{\theta}$	$\sum_{i,j,t} u_{ijt}^{\theta}$	$\sum_{i} \mathrm{d}^{\theta}{}_{i}^{+}$	$\sum_{i,t} da^{\theta}{}_{it}^{+}$	$\sum_{i,t} \mathrm{db}^{\theta}{}_{i,t}^{+}$
1	0.25	0.25	506	631	0	0	0	2
2	0.50	0.50	533	0	0	0	0	13
3	0.25	0.25	552	633	90	0	0	17

Table 5.24: Results of PS2

θ	P_θ	Probability	$\sum_{i,j,t} x_{ijt}^{\theta}$	$\sum_{i,j,t} o_{ijt}^{\theta}$	$\sum_{i,j,t} u_{ijt}^{\theta}$	$\sum_{i} d{\theta}_{i}^{+}$	$\sum_{i,t} da_{it}^{\theta+}$	$\sum_{i,t} db{\theta}_{i,t}^{+}$
1	0.25	0.25	506	384	0	0	1	17
2	0.50	0.50	533	0	0	0	0	11
3	0.25	0.25	552	385	89	0	0	19

Table 5.25: Results of PS3

θ	P_θ	Probability	$\sum_{i,j,t} x_{ijt}^{\theta}$	$\sum_{i,j,t} o_{ijt}^{\theta}$	$\sum_{i,j,t} u_{ijt}^{\theta}$	$\sum_{i} d{\theta}_{i}^{+}$	$\sum_{i,t} da_{it}^{\theta+}$	$\sum_{i,t} db{\theta}_{i,t}^{+}$
1	0.25	0.25	515	389	358	1	30	86
2	0.50	0.50	536	12	0	4	26	91
3	0.25	0.25	554	375	325	0	24	85

Table 5.26: Results of PS4

θ	P_θ	$\sum_{i,j,t} x_{ijt}^{\theta}$	$\sum_{i,j,t} o_{ijt}^{\theta}$	$\sum_{i,j,t} u_{ijt}^{\theta}$	$\sum_{i} d{\theta}_{i}^{+}$	$\sum_{i,t} da_{it}^{\theta+}$	$\sum_{i,t} db{\theta}_{i,t}^{+}$
1	0.25	513	232	488	1	20	74
2	0.50	535	4	0	4	23	85
3	0.25	554	215	377	0	18	67

Table 5.27: Results for stochastic demand

Preference Scenarios	z^*	$\sum_{i,j,t,\theta} P_\theta x_{ijt}^{\theta}$	$\sum_{i,j,t,\theta} P_\theta o_{ijt}^{\theta}$	$\sum_{i,j,t,\theta} P_\theta u_{ijt}^{\theta}$	$\sum_{i,\theta} P_\theta d{\theta}_{i}^{+}$	$\sum_{i,t,\theta} P_\theta da_{it}^{\theta+}$	$\sum_{i,t,\theta} P_\theta db{\theta}_{i,t}^{+}$
All 0	2707	531	398	0	0	0	6
1	2512	531	316	23	0	0	11
2	2146	531	192	22	0	0	15
3	2398	535	197	171	2	27	88
4	2174	534	114	216	2	21	78
All 1	1757	531	2	234	0	0	13

Aggregate Schedule

The weighted sum of the three separate schedules generated for different demand scenarios includes fractional values for the shift assignment, i.e., $0 \leq \sum_{\theta} P_{\theta} x_{ijt}^{\theta} \leq 1$. In order to produce a single schedule that can be used in practice based on stochastic demand scenarios, we propose an aggregate schedule in which the weighted sum of $|\Theta|$ schedules, x_{ijt}^{θ}, is calculated with the demand scenario probabilities as weights and a shift assignment is made if the corresponding weighted sum for a nurse-shift-day assignment takes a value that is greater than 0.25. The transformation rule for the aggregate schedule can be represented as follows.

$$x_{ijt} = \begin{cases} 0, & \text{if } \sum_{\theta} P_{\theta} x_{ijt}^{\theta} \leq 0.25 \\ 1, & \text{if } \sum_{\theta} P_{\theta} x_{ijt}^{\theta} > 0.25 \end{cases} \quad (52)$$

Then, the performance measures are recalculated according to the assumed preference scenario in the solution. We compute staffing cost that is the total cost of regular nurses and intern nurses, which is the initial part of the objective function. Table 5.28 shows the resulting measures for the aggregate schedule.

Base Case Scenario 0 (All 0):

In our Base Case Scenario 0, all shift preferences are set to zero, i.e., $p_{ijt} = 0, \forall i \in I, j \in J, t \in T,\ \theta \in \Theta$. Assuming that $\alpha = \beta = 3$, the optimal value for the objective function (51) in this scenario is $z^* = 2707$, where the total number of shifts assigned is $\sum_{i,j,t} x_{ijt} = \sum_{i,j,t} o_{ijt} = 650$.

Base Case Scenario 1 (All 1):

The other extreme is having all nurses available on all shifts over the planning horizon, i.e., $p_{ijt} = 1, \forall i \in I, j \in J, t \in T, \theta \in \Theta$. Assuming that $\alpha = \beta = 3$, the optimal value for the objective function (24) is $z^* = 1757$, where, again, $\sum_{i,j,t} x_{ijt} = 535$. In this case, since there cannot be any over-assignment, $\rho_o = 0\%$. These base case scenarios show us the limits on the objective function value and the limits on the quantity $\sum_{i,j,t} o_{ijt}$, as well as ρ_o, when $\alpha = \beta = 3$ at the two extreme nurse preference values.

Table 5.28: Results of aggregate schedule

Preference Scenarios	Staffing cost	$\sum_{i,j,t} x_{ijt}$	$\sum_{i,j,t} o_{ijt}$
All 0	1849	650	650
1	1712	597	2492
2	1643	577	655
3	1724	606	195
4	1700	598	82
All 1	1521	535	0

The aggregate schedule results in a higher number of shift assignments, as shown in Figure 5.7, since the stochastic demand requires the surgical unit to be over-prepared for emergency surgeries.

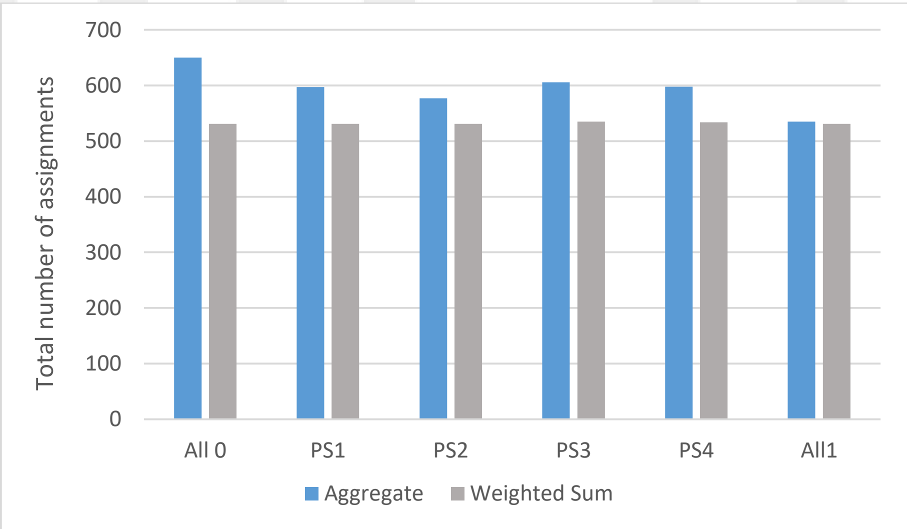


Figure 5.7: Comparison of the total number of shift assignments for the weighted sum solution and the aggregate schedule

Chapter 6

Conclusions

In this study, we focus on the nurse scheduling problem in a surgical suite where cardiovascular, general, and neurological surgeries are performed. Shift preferences of nurses are given the highest priority while meeting the demand-related constraints. We develop a multi-objective integer programming model with hard and soft constraints and solve the model using goal programming. The model is formulated to produce the best possible schedule in terms of meeting nurse preferences and fair distribution of workload. We illustrate the performance of the proposed model with an example under various shift preference scenarios. Computational results show that the multi-objective nature of the model leads to higher job satisfaction for nurses in terms of the performance measures evaluated, by avoiding the assignment of shifts that are not preferred, by avoiding isolated days on or off, and by avoiding disproportionate night shift assignments. The proposed model can easily be used in practice to produce the best possible nurse schedule given a certain shift preference scenario by adjusting the penalty cost, α, and given a demand scenario. Moreover, we considered stochastic demand scenarios based on the same preference scenarios. Our stochastic demand model provides a more applicable schedule for nurses that is the aggregate schedule developed, allowing the surgical unit to be more prepared for unexpected increases in demand. This model can be used for emergency surgeries which cause fluctuating demand.

This study can be extended in various directions. The proposed goal programming model can be modified to include emergency surgeries as a separate specialty. Our model considers a wide set of constraints already; however, there are other scheduling constraints in the literature or in practice that can easily be added to this model such as constraints dealing with annual vacations or minimum number of weekend days off.

References

[1] Abernathy, W. J., N. Baloff, J. C. Hershey, and S. Wandel (1973) "*A Three-Stage Manpower Planning and Scheduling Model A Service-Sector Example,"* Operations Research, 22, 693- 711.

[2] Associates Degree in Nursing (AND), http://www.associates-degree-in-nursing.org/, accessed on May 22, 2017.

[3] American Nurses Association http://www.nursingworld.org/EspeciallyForYou/AdvancedPracticeNurses, Accessed on May 22, 2017.

[4] Alpar, S. E., Bahcecik, N. (2009) *"Nursing Education in Turkey: From Past to Present,"* Nurse Education Today, 29, 698-703.

[5] Arthur JL, Ravindran A. (1981) "A *Multiple Objective Nurse Scheduling Model,"*IIE Transactions, 13(1), 55–60.

[6] Azaiez, M. and S. S. Sharif (2005) *"A 0–1 Goal Programming Model for Nurse Scheduling,"* Computers & Operations Research, 32, 491–507.

[7] Bag N., Ozdemir, M.N., Eren T. (2012) *"0-1 Hedef Programlama ve ANP Yöntemi ile Hemşire Çizelgeleme Problemi Çözümü,"* International Journal of Engineering Research and Development, Vol.4, No.1.

[8] Balaras, C.A., Dascalaki, E., Gaglia, A. (2007) *"HVAC and Indoor Thermal Conditions in Hospital Operating Rooms,"* Energy and Buildings, 39, 454–470.

[9] Baker, K.R., (1976) *"Workforce Allocation in Cyclical Scheduling Problems: A Survey*," Operational Research Quarterly (1970-1977), 27(1), 155-167.

[10] Bayık, A., Erefe, I., and Özsoy, S. (2002*) "At Last Century Development of Nursing Profession in Turkey as a Female Occupation,"* Nursing Forum (5)6, 16-25.

[11] Bechtold, S., Brusco, M., and Showalter, M. (1991) *"A Comparative Evaluation of Labor Tour Scheduling Method,"* Decision Sciences, 22(4), 683–699.

[12] Belien J., Demeulemeester E. (2008) "*A Branch-And-Price Approach for Integrating Nurse and Surgery Scheduling,"* European Journal of Operational Research, (189):652–668.

[13] Berrada I, Ferland JA, Michelon P. (1996) *"A Multi-Objective Approach to Nurse Scheduling with Both Hard and Soft Constraints,"* Socio-Economic Planning Sciences 30(3):183–93.

[14] Bureau of Labor Statistics, https://www.bls.gov/, Accessed on May 22, 2017.

[15] Burke, E.K., De Causmaecker, P., Berghe, G.V., and Van Landeghem, H. (2004) "*The State of the Art of Nurse Rostering,"* Journal of Scheduling, 7(6), 441-499.

[16] Bradley, D., and Martin, J. (1991) *"Continuous Personnel Scheduling Algorithms: A Literature Review,"* Journal of the Society for Health Systems, 2(2), 8-23.

[17] Cardoen B., Demeulemeester E., and Belien J. (2009) *"Operating Room Planning and Scheduling: A Literature Review,"* European Journal of Operational Research.

[18] C. Di Martinelly, P. Baptiste, and M.Y. Maknoon (2014*) "An Assessment of the Integration of Nurse Timetable Changes with Operating Room Planning and Scheduling,"* Vol. 52, No. 24, 7239–7250.

[19] Charnes, A., Cooper, W.W., and Ferguson, R.O. (1955) *"Optimal Estimation of Executive Compensation by Linear Programming,"* Management Science, Vol. 1, No. 2 (Jan., 1955), pp. 138-151.

[20] Cheang, B., H. Li,A. Lim, and B. Rodrigues (2003*) "Nurse Rostering Problems–A Bibliographic Survey,"* European Journal of Operational Research, 151, 447–460.

[21] Clark, A. R., Walker, H. (2011) *"Nurse Rescheduling with Shift Preferences and Minimal Disruption,"* Journal of Applied Operational Research, 3(3), 148–162.

[22] Creately, *Current Nursing Staff Scheduling Process* https://creately.com/diagram/examplev2/izg30ep42/Current%20Nursing%20Staff%20Scheduling%20Process, accessed on May 24,2017.

[23] Dal, U., Kitiş, Y. (2008) *"The Historical Development and Current Status of Nursing in Turkey,"* OJIN: The Online Journal of Issues in Nursing, 13(2).

[24] Denton B., Viapiano J., and Vogl A. (2007) *"Optimization of Surgery Sequencing and Scheduling Decisions Under Uncertainty,"* Health Care Manage Sci, 10, 13–24.

[25] Egenes, K. J. (2009). *"Issues and Trends in Nursing: Essential Knowledge for Today and Tomorrow"*, "History of Nursing, Chapter 1," Jones and Bartlett Publishers, *200*, 1-26.

[26] EmergingSoft https://emergingsoft.wordpress.com/2014/12/08/centralized-vs-decentralized-scheduling/, accessed on May 20, 2017.

[27] Ernst, A.T., Jiang, H., Krishamoorty, M., Owens, B., Sier, D. (2004) *"Staff Scheduling and Rostering: A Review Of Applications, Methods And Models,"* European Journal of Operational Research, 153, 3–27.

[28] Guerriero F., Guido R. (2011) *"Operational Research in the Management of the Operating Theatre: A Survey,"* Health Care Manag Sci, 14:89–114

[29] Harmeier PE. (1991) *"Linear Programming for Optimization of Nurse Scheduling,"* Computers in Nursing, 9(4), 149–51.

[30] HCFM Association. (2005). "*Achieving Operating Room Efficiency through Process Integration,"* Technical report.

[31] World Health Organization, *"Health Education and Health Career Report in Turkey 2010,"*.

[32] Hosseini N., Taaffe K. (2013) *"Operating Room Configurations and Policies for Accommodating Elective and Non-elective Surgeries"* Proceedings of the 2013 Industrial and Systems Engineering Research Conference.

[33] International Council of Nurses (ICN), http://www.icn.ch/who-we-are/icn-definition-of-nursing/, accessed on May 10, 2017.

[34] Jaumard, B., F. Semet, and T. Vovor (1998) *"A Generalized Linear Programming Model for Nurse Scheduling,"* European Journal of Operational Research, 107, 1-18.

[35] Lourenco H., Paixao J., R. Portugal (2001) *"Multi Objective Metaheuristics for the Bus–Driver Scheduling Problem,"* Transportation Science, 35(3), 331–341.

[36] Marques, I., Captivo, M.E., and Vaz Pato, M. (2015) *"A Bicriteria Heuristic for an Elective Surgery Scheduling Problem,"* Health Care Management Science, 18(3), 251-266.

[37] Miller HE, Pierskalla WP, Rath GJ. (1976) "*Nurse Scheduling Using Mathematical Programming,"* Operations Research, 24(5), 857–870.

[38] Moz, M. and M. Pato (2004) *"Solving the Problem of Rerostering Nurse Schedules with Hard Constraints: New Multi Commodity Flow Models,"* Annals of Operations Research, 128, 179-197.

[39] Musa AA, Saxena U. (1984) *"Scheduling Nurses Using Goal-Programming Techniques,"* IIE Transactions, 16(3), 217–221.

[40] National Nurses United, *"National Campaign for Safe RN-to-Patient Staffing Ratios*," http://www.nationalnursesunited.org/issues/entry/ratios, accessed on May 16, 2017.

[41] Nurse Journal, http://nursejournal.org/advanced-practice/advanced-practice-nursing-fact-sheet/, accessed on May 20, 2017.

[42] Parr, D., Thompson, J.M., (2007) *"Solving the Multi-Objective Nurse Scheduling Problem with a Weighted Cost Function,"* Annals of Operations Research, 155, 279–288.

[43] Rosenbloom ES, Goertzen NF. (1976) *"Nurse Scheduling Using Mathematical Programming"* Operations Research 31:857–70.

[44] Silvestro, R. and C. Silvestro, (2000) *"An Evaluation Of Nurse Rostering Practices in The National Health Service*", Journal Advanced Nursing, 32(3), 525-535.

[45] Tatar, M., Mollahaliloğlu S, Sahin B, Aydin S, Maresso A, Hernández-Quevedo C. (2011) "*Health System in Transition"* Turkey Health System review, 13(6), 15-53.

[46] Tien, J. and A. Kamiyama, (1982) *"On Manpower Scheduling Algorithms,*" Society for Industrial and Applied Mathematics, 24(3), 275-287.

[47] Tsaia, C. C., LeebA, C. J. (2009) "*Optimization of Nurse Scheduling Problem with a Two-Stage Mathematical Programming Model,"* Pacific Management Review,*"* 15(4) 503-516.

[48] Trivedi, V. M. and Warner M. (1976) *"A Branch and Bound Algorithm for Optimum Allocation of Float Nurses,"* Management Science, 22(9), 972-981.

[49] Trivedi VM. (1981) *"A Mixed-Integer Goal Programming Model for Nurse Service Budgeting,"* Operations Research, 29(5), 1019–34.

[50] Topaloglu S., Ozkarahan I. (2004) *"An Implicit Goal Programming Model For The Tour Scheduling Problem Considering The Employee Work Preferences,"* Annals of Operations Research 128, 135-158.

[51] Topaloglu, S. (2006) *"A Multi-Objective Programming Model For Scheduling Emergency Medicine Residents,"* Computers & Industrial Engineering, 51, 375–388.

[52] O'Grady, E. T. (2008). "*Advanced practice registered nurses: The impact on patient safety and quality,*"

[53] Ozkarahan I, Bailey JE. (1988) *"Goal Programming Model Subsystem of a Flexible Nurse Scheduling Support System*," IIE Transactions 20(3):306–16.

[54] Ozkarahan I. (2000) "*Allocation of Surgeries to Operating Rooms by Goal Programing*," Journal of Medical Systems, 24 (6):339–378.

[55] Warner, M. and J. Prawda, (1972) "*A Mathematical Programming Model For Scheduling Nursing Personnel In A Hospital,"* Management Science, 19, 411.422.

[56] Warner M. (1976) "*Scheduling Nursing Personnel According To Nursing Preferences: A Mathematical Programming Approach*" Operations Research, 24(5): 842–56.

[57] Warner, M. (1976) *"Nurse Staffing, Scheduling, and Reallocation in The Hospital*", Hospital & Health Services Administration, 77.90.

[58] Winston, W. L., & Goldberg, J. B. (2004). "*Operations Research: Applications and Algorithms*," Boston: Duxbury press., vol3.

[59] World Health Organization (WHO), *"Nursing"*, http://www.who.int/topics/nursing/en/, accessed on May 16, 2017.

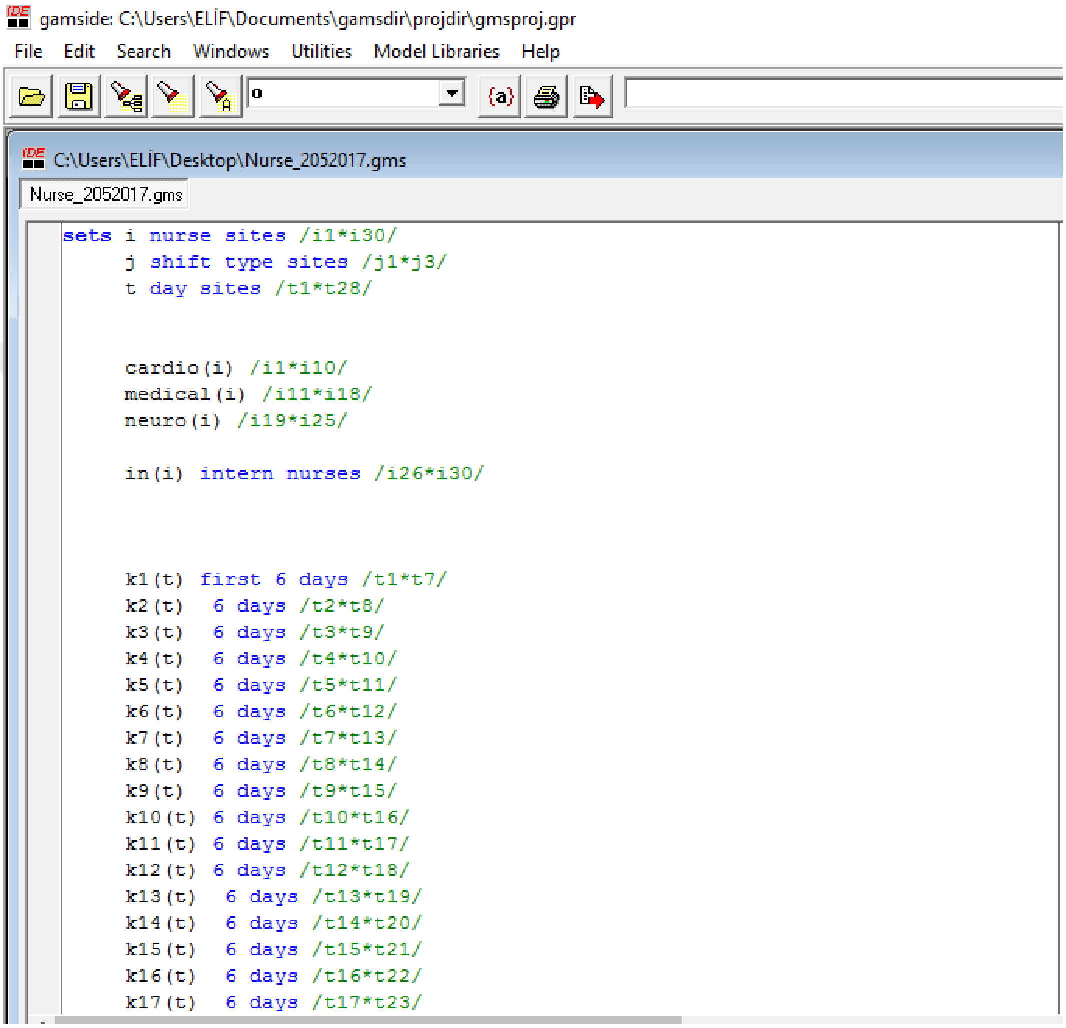
[60] Wright, D.P., and Mahar, S. (2013) *"Centralized Nurse Scheduling to Simultaneously Improve Schedule Cost and Nurse Satisfaction*," Omega, 41(6), 1042-1052.

[61] Van den Bergh, J., Beliën, J., De Bruecker, P., and Demeulemeester, E. (2013) "*Personnel Scheduling: A Literature Review,*" European Journal of Operational Research, 226(3), 367-385.

[62] Z. Sinuany-Stern, Y. Teomi, (1986) *"Multi-Objective Scheduling Plans For Security Guards"* Journal of the Operational Research Society 37 (1) 67–77.

[63] Zurn P, Dal Poz M, Stilwell B and Adams O (2004). "*Imbalance in the Health Workforce. Health Human Resources*," 2, 13, WHO, Geneva, Switzerland. http://www.human resources-health.com/content/pdf/1478-4491-2-13.

Appendix A: GAMS Model for Nurse Scheduling




```
sets i nurse sites /i1*i30/
     j shift type sites /j1*j3/
     t day sites /t1*t28/


     cardio(i) /i1*i10/
     medical(i) /i11*i18/
     neuro(i) /i19*i25/

     in(i) intern nurses /i26*i30/



     k1(t) first 6 days /t1*t7/
     k2(t)  6 days /t2*t8/
     k3(t)  6 days /t3*t9/
     k4(t)  6 days /t4*t10/
     k5(t)  6 days /t5*t11/
     k6(t)  6 days /t6*t12/
     k7(t)  6 days /t7*t13/
     k8(t)  6 days /t8*t14/
     k9(t)  6 days /t9*t15/
     k10(t) 6 days /t10*t16/
     k11(t) 6 days /t11*t17/
     k12(t) 6 days /t12*t18/
     k13(t)  6 days /t13*t19/
     k14(t)  6 days /t14*t20/
     k15(t)  6 days /t15*t21/
     k16(t)  6 days /t16*t22/
     k17(t)  6 days /t17*t23/
```

```
        k18(t)  6 days /t18*t24/
        k19(t)  6 days /t19*t25/
        k20(t)  6 days /t20*t26/
        k21(t)  6 days /t21*t27/
        k22(t)  6 days /t22*t28/

        n1(t)   night /t1*t4/
        n2(t)   night /t2*t5/
        n3(t)   night /t3*t6/
        n4(t)   night /t4*t7/
        n5(t)   night /t5*t8/
        n6(t)   night /t6*t9/
        n7(t)   night /t7*t10/
        n8(t)   night /t8*t11/
        n9(t)   night /t9*t12/
        n10(t)   night /t10*t13/
        n11(t)   night /t11*t14/
        n12(t)   night /t12*t15/
        n13(t)   night /t13*t16/
        n14(t)   night /t14*t17/
        n15(t)   night /t15*t18/
        n16(t)   night /t16*t19/
        n17(t)   night /t17*t20/
        n18(t)   night /t18*t21/
        n19(t)   night /t19*t22/
        n20(t)   night /t20*t23/
        n21(t)   night /t21*t24/
        n22(t)   night /t22*t25/
        n23(t)   night /t23*t26/
        n24(t)   night /t24*t27/
```

```
     n25(t)    night /t25*t28/

     at(t)     avoid 010 or 101 /t1*t26/

;

Table p(i,j,t)

$call xls2gms I=PreferenceNurses.xls O=NursePreference R=Preference!A1:AE91
$include "NursePreference"

;

table NR11(j,t) requirement for regular nurses on shift j of day t

      t1  t2  t3 t4 t5 t6 t7 t8  t9 t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21 t22 t23 t24 t25 t26 t27 t28
j1     2   2   2  2  2  2  2  2   2  2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
j2     2   2   2  2  2  2  2  2   2  2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
j3     1   1   1  1  1  1  1  1   1  1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1

table NR21(j,t) requirement for regular nurses on shift j of day t

      t1  t2  t3 t4 t5 t6 t7 t8  t9 t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21 t22 t23 t24 t25 t26 t27 t28
j1     2   2   2  2  2  2  2  2   2  2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
j2     2   2   2  2  2  2  2  2   2  2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
j3     1   1   1  1  1  1  1  1   1  1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
```

```
table NR31(j,t) requirement for regular nurses on shift j of day t
*$call xls2gms I=PreferenceNurses.xls O="RegularsRequired" R="NursesRequired!B7:AC8"
*$include "RegularsRequired"
      t1  t2  t3 t4 t5 t6 t7 t8  t9 t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21 t22 t23 t24 t25 t26 t27 t28
j1     2   2   2  2  2  2  2  2   2  2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
j2     2   2   2  2  2  2  2  2   2  2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
j3     1   1   1  1  1  1  1  1   1  1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1

table NI1(j,t) requirement for intern nurses on shift j of day t
*$call xls2gms I=PreferenceNurses.xls O="InternsRequired" R="NursesRequired!B2:AC3"
*$include "InternsRequired"
        t1  t2  t3 t4 t5 t6 t7 t8  t9 t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21 t22 t23 t24 t25 t26 t27 t28
j1       1   1   1  1  1  1  1  1   1   1   1   1   1   1   1   1   1   1   1  1   1   1   1   1   1   1   1   1
j2       1   1   1  1  1  1  1  1   1   1   1   1   1   1   1   1   1   1   1  1   1   1   1   1   1   1   1   1
j3       1   1   1  1  1  1  1  1   1   1   1   1   1   1   1   1   1   1   1  1   1   1   1   1   1   1   1   1


table NR12(j,t) requirement for regular nurses on shift j of day t

      t1  t2  t3 t4 t5 t6 t7 t8  t9 t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21 t22 t23 t24 t25 t26 t27 t28
j1     2   2   2  2  3  3  2  2   2  2   2   3   3   2   2   2   2   2   3   3   2   2   2   2   2   3   3   2
j2     2   2   2  2  2  2  2  2   2  2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   3   2
j3     1   1   1  1  1  1  1  1   1  1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
```

```
table NR22(j,t) requirement for regular nurses on shift j of day t

      t1  t2  t3 t4 t5 t6 t7 t8  t9 t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21 t22 t23 t24 t25 t26 t27 t28
j1     2   2   2  2  3  3  2  2   2  2   2   3   3   2   2   2   2   2   3   3   2   2   2   2   2   3   3   2
j2     2   2   2  2  2  2  2  2   2  2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
j3     1   1   1  1  1  1  1  1   1  1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1

table NR32(j,t) requirement for regular nurses on shift j of day t
*$call xls2gms I=PreferenceNurses.xls O="RegularsRequired" R="NursesRequired!B7:AC8"
*$include "RegularsRequired"
      t1  t2  t3 t4 t5 t6 t7 t8  t9 t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21 t22 t23 t24 t25 t26 t27 t28
j1     2   2   2  2  2  2  2  2   2  2   2   2   3   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
j2     2   2   2  2  2  2  2  2   2  2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
j3     1   1   1  1  2  2  1  1   1  1   2   2   2   1   1   1   1   1   2   2   1   1   1   1   1   2   2   1

table NI2(j,t) requirement for intern nurses on shift j of day t
*$call xls2gms I=PreferenceNurses.xls O="InternsRequired" R="NursesRequired!B2:AC3"
*$include "InternsRequired"
      t1  t2  t3 t4 t5 t6 t7 t8  t9 t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21 t22 t23 t24 t25 t26 t27 t28
j1     1   1   1  1  1  1  1  1   1   1   1   1   1   1   1   1   1   1   1  1   1   1   1   1   1   1   1   1
j2     1   1   1  1  1  1  1  1   1   1   1   1   1   1   1   1   1   1   1  1   1   1   1   1   1   1   1   1
j3     1   1   1  1  1  1  1  1   1   1   1   1   1   1   1   1   1   1   1  1   1   1   1   1   1   1   1   1
```

```
table NR13(j,t) requirement for regular nurses on shift j of day t

     t1  t2  t3 t4 t5 t6 t7 t8  t9 t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21 t22 t23 t24 t25 t26 t27 t28
j1    2   2   2  2  3  3  2  2   2  2   2   3   3   2   2   2   2   2   3   3  2   2   2   2   2   3   3   2
j2    2   2   2  2  2  2  2  2   2  2   2   3   3   2   2   2   2   2   2   2  2   2   2   2   2   3   3   2
j3    1   1   1  1  2  2  1  1   1  1   1   2   2   1   1   1   1   1   2   2  1   1   1   1   1   2   2   1


table NR23(j,t) requirement for regular nurses on shift j of day t

      t1  t2  t3 t4 t5 t6 t7 t8  t9 t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21 t22 t23 t24 t25 t26 t27 t28
j1     2   2   2  2  3  3  2  2   2  2   2   2   2   2   2   2   2   2   3   3   2   2   2   2   2   2   2   2
j2     2   2   2  2  2  2  2  2   2  2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
j3     1   1   1  1  2  2  1  1   1  1   1   2   2   1   1   1   1   1   2   2   1   1   1   1   1   2   2   1

table NR33(j,t) requirement for regular nurses on shift j of day t
*$call xls2gms I=PreferenceNurses.xls O="RegularsRequired" R="NursesRequired!B7:AC8"
*$include "RegularsRequired"
      t1  t2  t3 t4 t5 t6 t7 t8  t9 t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21 t22 t23 t24 t25 t26 t27 t28
j1     2   2   2  2  3  2  2  2   2  2   2   2   3   2   2   2   2   2   3   2   2   2   2   2   2   2   3   2
j2     2   2   2  2  2  2  2  2   2  2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
j3     1   1   1  1  2  2  1  1   1  1   1   2   2   1   1   1   1   1   2   2   1   1   1   1   1   2   2   1

table NI3(j,t) requirement for intern nurses on shift j of day t
*$call xls2gms I=PreferenceNurses.xls O="InternsRequired" R="NursesRequired!B2:AC3"
*$include "InternsRequired"
      t1  t2  t3 t4 t5 t6 t7 t8  t9 t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21 t22 t23 t24 t25 t26 t27 t28
j1     1   1   1  1  2  2  1  1   1   1   1   1   1   1   1   1   1   1   2  2   1   1   1   1   1   1   1   1
j2     1   1   1  1  1  1  1  1   1   1   1   1   1   1   1   1   1   1   1  1   1   1   1   1   1   1   1   1
j3     1   1   1  1  1  1  1  1   1   1   1   1   1   1   1   1   1   1   1  1   1   1   1   1   1   1   1   1
```

```
BINARY VARIABLE x1(i,j,t) 1 if nurse i is the shift j of day t
                x2(i,j,t) 1 if nurse i is the shift j of day t
                x3(i,j,t) 1 if nurse i is the shift j of day t

                y1(i,j,t) 1 if nurse i did not prefer shift j on day t but was assigned
                y2(i,j,t) 1 if nurse i did not prefer shift j on day t but was assigned
                y3(i,j,t) 1 if nurse i did not prefer shift j on day t but was assigned

                u1(i,j,t)   1 if nurse i prefered shift j on day t but was not assigned
                u2(i,j,t)   1 if nurse i prefered shift j on day t but was not assigned
                u3(i,j,t)   1 if nurse i prefered shift j on day t but was not assigned
;
VARIABLE Z1 first objective function value
;

INTEGER VARIABLE dp1(i)   positive difference between total day shifts and total night shifts
                 dp2(i)   positive difference between total day shifts and total night shifts
                 dp3(i)   positive difference between total day shifts and total night shifts
                 dn1(i)   negative difference between total day shifts and total night shifts
                 dn2(i)   negative difference between total day shifts and total night shifts
                 dn3(i)   negative difference between total day shifts and total night shifts

        dap1(i,t)         avoid 010 assignment positive difference
        dap2(i,t)         avoid 010 assignment positive difference
        dap3(i,t)         avoid 010 assignment positive difference

        dan1(i,t)         avoid 010 assignment negative difference
        dan2(i,t)         avoid 010 assignment negative difference
        dan3(i,t)         avoid 010 assignment negative difference
```

```
         dbp1(i,at)        avoid 101 assignment positive difference
         dbp2(i,at)        avoid 101 assignment positive difference
         dbp3(i,at)        avoid 101 assignment positive difference

         dbn1(i,at)        avoid 101 assignment negative difference
         dbn2(i,at)        avoid 101 assignment negative difference
         dbn3(i,at)        avoid 101 assignment negative difference
;

Equations Eq21(j,t)  Staff requrement
          Eq22(j,t)  Staff requrement
          Eq23(j,t)  Staff requrement

           Eq311(j,t)  cardio regular nurse
           Eq312(j,t)  cardio regular nurse
           Eq313(j,t)  cardio regular nurse

           Eq321(j,t)  medical
           Eq322(j,t)  medical
           Eq323(j,t)  medical

           Eq331(j,t)  brain
           Eq332(j,t)  brain
           Eq333(j,t)  brain


        Eq411(i)   Minimum working hours requirement week 1
        Eq421(i)   Minimum working hours requirement week 2
        Eq431(i)   Minimum working hours requirement week 3
        Eq441(i)   Minimum working hours requirement week 4

        Eq412(i)   Minimum working hours requirement week 1
        Eq422(i)   Minimum working hours requirement week 2
        Eq432(i)   Minimum working hours requirement week 3
        Eq442(i)   Minimum working hours requirement week 4

        Eq413(i)   Minimum working hours requirement week 1
        Eq423(i)   Minimum working hours requirement week 2
        Eq433(i)   Minimum working hours requirement week 3
        Eq443(i)   Minimum working hours requirement week 4

          Eq511(i)   Maximum working hours requirement week 1
          Eq521(i)   Maximum working hours requirement week 2
          Eq531(i)   Maximum working hours requirement week 3
          Eq541(i)   Maximum working hours requirement week 4

          Eq512(i)   Maximum working hours requirement week 1
          Eq522(i)   Maximum working hours requirement week 2
          Eq532(i)   Maximum working hours requirement week 3
          Eq542(i)   Maximum working hours requirement week 4

          Eq513(i)   Maximum working hours requirement week 1
          Eq523(i)   Maximum working hours requirement week 2
          Eq533(i)   Maximum working hours requirement week 3
          Eq543(i)   Maximum working hours requirement week 4

          Eq61(i,t) Working requirement(one shift per day)
          Eq62(i,t) Working requirement(one shift per day)
          Eq63(i,t) Working requirement(one shift per day)
```

```
Eq71(i,t) Working requirement (no day shift after night shift)
Eq72(i,t) Working requirement (no day shift after night shift)
Eq73(i,t) Working requirement (no day shift after night shift)

Eq811(i) max consecutive working days
Eq821(i) max consecutive working days
Eq831(i) max consecutive working days
Eq841(i) max consecutive working days
Eq851(i) max consecutive working days
Eq861(i) max consecutive working days
Eq871(i) max consecutive working days
Eq881(i) max consecutive working days
Eq891(i) max consecutive working days
Eq8101(i) max consecutive working days
Eq8111(i) max consecutive working days
Eq8121(i) max consecutive working days
Eq8131(i) max consecutive working days
Eq8141(i) max consecutive working days
Eq8151(i) max consecutive working days
Eq8161(i) max consecutive working days
Eq8171(i) max consecutive working days
Eq8181(i) max consecutive working days
Eq8191(i) max consecutive working days
Eq8201(i) max consecutive working days
Eq8211(i) max consecutive working days
Eq8221(i) max consecutive working days
```

```
Eq812(i) max consecutive working days
Eq822(i) max consecutive working days
Eq832(i) max consecutive working days
Eq842(i) max consecutive working days
Eq852(i) max consecutive working days
Eq862(i) max consecutive working days
Eq872(i) max consecutive working days
Eq882(i) max consecutive working days
Eq892(i) max consecutive working days
Eq8102(i) max consecutive working days
Eq8112(i) max consecutive working days
Eq8122(i) max consecutive working days
Eq8132(i) max consecutive working days
Eq8142(i) max consecutive working days
Eq8152(i) max consecutive working days
Eq8162(i) max consecutive working days
Eq8172(i) max consecutive working days
Eq8182(i) max consecutive working days
Eq8192(i) max consecutive working days
Eq8202(i) max consecutive working days
Eq8212(i) max consecutive working days
Eq8222(i) max consecutive working days
```

```
Eq813(i) max consecutive working days
Eq823(i) max consecutive working days
Eq833(i) max consecutive working days
Eq843(i) max consecutive working days
Eq853(i) max consecutive working days
Eq863(i) max consecutive working days
Eq873(i) max consecutive working days
Eq883(i) max consecutive working days
Eq893(i) max consecutive working days
Eq8103(i) max consecutive working days
Eq8113(i) max consecutive working days
Eq8123(i) max consecutive working days
Eq8133(i) max consecutive working days
Eq8143(i) max consecutive working days
Eq8153(i) max consecutive working days
Eq8163(i) max consecutive working days
Eq8173(i) max consecutive working days
Eq8183(i) max consecutive working days
Eq8193(i) max consecutive working days
Eq8203(i) max consecutive working days
Eq8213(i) max consecutive working days
Eq8223(i) max consecutive working days
```

```
Eq911(i) max consecutive night shifts
Eq921(i) max consecutive night shifts
Eq931(i) max consecutive night shifts
Eq941(i) max consecutive night shifts
Eq951(i) max consecutive night shifts
Eq961(i) max consecutive night shifts
Eq971(i) max consecutive night shifts
Eq981(i) max consecutive night shifts
Eq991(i) max consecutive night shifts
Eq9101(i)  max consecutive night shifts
Eq9111(i)  max consecutive night shifts
Eq9121(i)  max consecutive night shifts
Eq9131(i)  max consecutive night shifts
Eq9141(i)  max consecutive night shifts
Eq9151(i)  max consecutive night shifts
Eq9161(i)  max consecutive night shifts
Eq9171(i)  max consecutive night shifts
Eq9181(i)  max consecutive night shifts
Eq9191(i)  max consecutive night shifts
Eq9201(i)  max consecutive night shifts
Eq9211(i)  max consecutive night shifts
Eq9221(i)  max consecutive night shifts
Eq9231(i)  max consecutive night shifts
Eq9241(i)  max consecutive night shifts
Eq9251(i)  max consecutive night shifts
```

```
Eq912(i) max consecutive night shifts
Eq922(i) max consecutive night shifts
Eq932(i) max consecutive night shifts
Eq942(i) max consecutive night shifts
Eq952(i) max consecutive night shifts
Eq962(i) max consecutive night shifts
Eq972(i) max consecutive night shifts
Eq982(i) max consecutive night shifts
Eq992(i) max consecutive night shifts
Eq9102(i)  max consecutive night shifts
Eq9112(i)  max consecutive night shifts
Eq9122(i)  max consecutive night shifts
Eq9132(i)  max consecutive night shifts
Eq9142(i)  max consecutive night shifts
Eq9152(i)  max consecutive night shifts
Eq9162(i)  max consecutive night shifts
Eq9172(i)  max consecutive night shifts
Eq9182(i)  max consecutive night shifts
Eq9192(i)  max consecutive night shifts
Eq9202(i)  max consecutive night shifts
Eq9212(i)  max consecutive night shifts
Eq9222(i)  max consecutive night shifts
Eq9232(i)  max consecutive night shifts
Eq9242(i)  max consecutive night shifts
Eq9252(i)  max consecutive night shifts
```

```
Eq913(i) max consecutive night shifts
Eq923(i) max consecutive night shifts
Eq933(i) max consecutive night shifts
Eq943(i) max consecutive night shifts
Eq953(i) max consecutive night shifts
Eq963(i) max consecutive night shifts
Eq973(i) max consecutive night shifts
Eq983(i) max consecutive night shifts
Eq993(i) max consecutive night shifts
Eq9103(i)  max consecutive night shifts
Eq9113(i)  max consecutive night shifts
Eq9123(i)  max consecutive night shifts
Eq9133(i)  max consecutive night shifts
Eq9143(i)  max consecutive night shifts
Eq9153(i)  max consecutive night shifts
Eq9163(i)  max consecutive night shifts
Eq9173(i)  max consecutive night shifts
Eq9183(i)  max consecutive night shifts
Eq9193(i)  max consecutive night shifts
Eq9203(i)  max consecutive night shifts
Eq9213(i)  max consecutive night shifts
Eq9223(i)  max consecutive night shifts
Eq9233(i)  max consecutive night shifts
Eq9243(i)  max consecutive night shifts
Eq9253(i)  max consecutive night shifts
```

```
Eq101(i)
Eq102(i)
Eq103(i)
Eq1111(i)
Eq1121(i)
Eq1131(i)
Eq1141(i)
Eq1151(i)
Eq1161(i)
Eq1171(i)
Eq1181(i)
Eq1191(i)
Eq11101(i)
Eq11111(i)
Eq11121(i)
Eq11131(i)
Eq11141(i)
Eq11151(i)
Eq11161(i)
Eq11171(i)
Eq11181(i)
Eq11191(i)
Eq11201(i)
Eq11211(i)
Eq11221(i)
Eq11231(i)
Eq11241(i)
Eq11251(i)
Eq11261(i)
```

```
Eq1112(i)
Eq1122(i)
Eq1132(i)
Eq1142(i)
Eq1152(i)
Eq1162(i)
Eq1172(i)
Eq1182(i)
Eq1192(i)
Eq11102(i)
Eq11112(i)
Eq11122(i)
Eq11132(i)
Eq11142(i)
Eq11152(i)
Eq11162(i)
Eq11172(i)
Eq11182(i)
Eq11192(i)
Eq11202(i)
Eq11212(i)
Eq11222(i)
Eq11232(i)
Eq11242(i)
Eq11252(i)
Eq11262(i)
```

```
Eq1113(i)
Eq1123(i)
Eq1133(i)
Eq1143(i)
Eq1153(i)
Eq1163(i)
Eq1173(i)
Eq1183(i)
Eq1193(i)
Eq11103(i)
Eq11113(i)
Eq11123(i)
Eq11133(i)
Eq11143(i)
Eq11153(i)
Eq11163(i)
Eq11173(i)
Eq11183(i)
Eq11193(i)
Eq11203(i)
Eq11213(i)
Eq11223(i)
Eq11233(i)
Eq11243(i)
Eq11253(i)
Eq11263(i)
```

```
Eq1211(i)
Eq1221(i)
Eq1231(i)
Eq1241(i)
Eq1251(i)
Eq1261(i)
Eq1271(i)
Eq1281(i)
Eq1291(i)
Eq12101(i)
Eq12111(i)
Eq12121(i)
Eq12131(i)
Eq12141(i)
Eq12151(i)
Eq12161(i)
Eq12171(i)
Eq12181(i)
Eq12191(i)
Eq12201(i)
Eq12211(i)
Eq12221(i)
Eq12231(i)
Eq12241(i)
Eq12251(i)
Eq12261(i)
```

```
Eq1212(i)
Eq1222(i)
Eq1232(i)
Eq1242(i)
Eq1252(i)
Eq1262(i)
Eq1272(i)
Eq1282(i)
Eq1292(i)
Eq12102(i)
Eq12112(i)
Eq12122(i)
Eq12132(i)
Eq12142(i)
Eq12152(i)
Eq12162(i)
Eq12172(i)
Eq12182(i)
Eq12192(i)
Eq12202(i)
Eq12212(i)
Eq12222(i)
Eq12232(i)
Eq12242(i)
Eq12252(i)
Eq12262(i)
```

```
Eq1213(i)
Eq1223(i)
Eq1233(i)
Eq1243(i)
Eq1253(i)
Eq1263(i)
Eq1273(i)
Eq1283(i)
Eq1293(i)
Eq12103(i)
Eq12113(i)
Eq12123(i)
Eq12133(i)
Eq12143(i)
Eq12153(i)
Eq12163(i)
Eq12173(i)
Eq12183(i)
Eq12193(i)
Eq12203(i)
Eq12213(i)
Eq12223(i)
Eq12233(i)
Eq12243(i)
Eq12253(i)
Eq12263(i)
```

```
Eq131(i,j,t)
Eq132(i,j,t)
Eq133(i,j,t)
```

```
Eq141(i)
Eq142(i)
Eq143(i)
obj1 objective first equation
;

*HARD CONSTRAINTS

*number of intern nurses required per shift on day t
Eq21(j,t)..      x1('i26',j,t)+x1('i27',j,t)+x1('i28',j,t)+x1('i29',j,t)+x1('i30',j,t)=G= NI1(j,t);
Eq22(j,t)..      x2('i26',j,t)+x2('i27',j,t)+x2('i28',j,t)+x2('i29',j,t)+x2('i30',j,t)=G= NI2(j,t);
Eq23(j,t)..      x3('i26',j,t)+x3('i27',j,t)+x3('i28',j,t)+x3('i29',j,t)+x3('i30',j,t)=G= NI3(j,t);
*number of regular nurses required per shift on day t
Eq311(j,t)..    (x1('i1',j,t)+x1('i2',j,t)+x1('i3',j,t)+x1('i4',j,t)+x1('i5',j,t)+x1('i6',j,t)+x1('i7',j,t)+x1('i8',j,t)+x1('i9',j,t)+x1('i10',j,t))=G= NR11(j,t)
Eq312(j,t)..    (x2('i1',j,t)+x2('i2',j,t)+x2('i3',j,t)+x2('i4',j,t)+x2('i5',j,t)+x2('i6',j,t)+x2('i7',j,t)+x2('i8',j,t)+x2('i9',j,t)+x2('i10',j,t))=G= NR12(j,t)
Eq313(j,t)..    (x3('i1',j,t)+x3('i2',j,t)+x3('i3',j,t)+x3('i4',j,t)+x3('i5',j,t)+x3('i6',j,t)+x3('i7',j,t)+x3('i8',j,t)+x3('i9',j,t)+x3('i10',j,t))=G= NR13(j,t)

Eq321(j,t)..        (x1('i11',j,t)+x1('i12',j,t)+x1('i13',j,t)+x1('i14',j,t)+x1('i15',j,t)+x1('i16',j,t)+x1('i17',j,t)+x1('i18',j,t))=G= NR21(j,t);
Eq322(j,t)..        (x2('i11',j,t)+x2('i12',j,t)+x2('i13',j,t)+x2('i14',j,t)+x2('i15',j,t)+x2('i16',j,t)+x2('i17',j,t)+x2('i18',j,t))=G= NR22(j,t);
Eq323(j,t)..        (x3('i11',j,t)+x3('i12',j,t)+x3('i13',j,t)+x3('i14',j,t)+x3('i15',j,t)+x3('i16',j,t)+x3('i17',j,t)+x3('i18',j,t))=G= NR23(j,t);

Eq331(j,t)..        (x1('i19',j,t)+x1('i20',j,t)+x1('i21',j,t)+x1('i22',j,t)+x1('i23',j,t)+x1('i24',j,t)+x1('i25',j,t))=G= NR31(j,t);
Eq332(j,t)..        (x2('i19',j,t)+x2('i20',j,t)+x2('i21',j,t)+x2('i22',j,t)+x2('i23',j,t)+x2('i24',j,t)+x2('i25',j,t))=G= NR32(j,t);
Eq333(j,t)..        (x3('i19',j,t)+x3('i20',j,t)+x3('i21',j,t)+x3('i22',j,t)+x3('i23',j,t)+x3('i24',j,t)+x3('i25',j,t))=G= NR33(j,t);


*minimum weekly hours that a nurse can work
Eq411(i)..          sum(j,  (x1(i,j,'t1')+x1(i,j,'t2')+x1(i,j,'t3')+x1(i,j,'t4')+x1(i,j,'t5')+x1(i,j,'t6')+x1(i,j,'t7')))*12 =G= 24;
Eq421(i)..          sum(j,  (x1(i,j,'t8')+x1(i,j,'t9')+x1(i,j,'t10')+x1(i,j,'t11')+x1(i,j,'t12')+x1(i,j,'t13')+x1(i,j,'t14')))*12 =G= 24;
Eq431(i)..          sum(j,  (x1(i,j,'t15')+x1(i,j,'t16')+x1(i,j,'t17')+x1(i,j,'t18')+x1(i,j,'t19')+x1(i,j,'t20')+x1(i,j,'t21')))*12 =G= 24;
Eq441(i)..          sum(j,  (x1(i,j,'t22')+x1(i,j,'t23')+x1(i,j,'t24')+x1(i,j,'t25')+x1(i,j,'t26')+x1(i,j,'t27')+x1(i,j,'t28')))*12 =G= 24;
```

```
Eq412(i)..          sum(j,  (x2(i,j,'t1')+x2(i,j,'t2')+x2(i,j,'t3')+x2(i,j,'t4')+x2(i,j,'t5')+x2(i,j,'t6')+x2(i,j,'t7')))*12 =G= 24;
Eq422(i)..          sum(j,  (x2(i,j,'t8')+x2(i,j,'t9')+x2(i,j,'t10')+x2(i,j,'t11')+x2(i,j,'t12')+x2(i,j,'t13')+x2(i,j,'t14')))*12 =G= 24;
Eq432(i)..          sum(j,  (x2(i,j,'t15')+x2(i,j,'t16')+x2(i,j,'t17')+x2(i,j,'t18')+x2(i,j,'t19')+x2(i,j,'t20')+x2(i,j,'t21')))*12 =G= 24;
Eq442(i)..          sum(j,  (x2(i,j,'t22')+x2(i,j,'t23')+x2(i,j,'t24')+x2(i,j,'t25')+x2(i,j,'t26')+x2(i,j,'t27')+x2(i,j,'t28')))*12 =G= 24;

Eq413(i)..          sum(j,  (x3(i,j,'t1')+x3(i,j,'t2')+x3(i,j,'t3')+x3(i,j,'t4')+x3(i,j,'t5')+x3(i,j,'t6')+x3(i,j,'t7')))*12 =G= 24;
Eq423(i)..          sum(j,  (x3(i,j,'t8')+x3(i,j,'t9')+x3(i,j,'t10')+x3(i,j,'t11')+x3(i,j,'t12')+x3(i,j,'t13')+x3(i,j,'t14')))*12 =G= 24;
Eq433(i)..          sum(j,  (x3(i,j,'t15')+x3(i,j,'t16')+x3(i,j,'t17')+x3(i,j,'t18')+x3(i,j,'t19')+x3(i,j,'t20')+x3(i,j,'t21')))*12 =G= 24;
Eq443(i)..          sum(j,  (x3(i,j,'t22')+x3(i,j,'t23')+x3(i,j,'t24')+x3(i,j,'t25')+x3(i,j,'t26')+x3(i,j,'t27')+x3(i,j,'t28')))*12 =G= 24;
```

```
*maximum weekly hours that a nurse can work
Eq511(i)..          sum(j,  (x1(i,j,'t1')+x1(i,j,'t2')+x1(i,j,'t3')+x1(i,j,'t4')+x1(i,j,'t5')+x1(i,j,'t6')+x1(i,j,'t7')))*12 =L= 72;
Eq521(i)..          sum(j,  (x1(i,j,'t8')+x1(i,j,'t9')+x1(i,j,'t10')+x1(i,j,'t11')+x1(i,j,'t12')+x1(i,j,'t13')+x1(i,j,'t14')))*12 =L= 72;
Eq531(i)..          sum(j,  (x1(i,j,'t15')+x1(i,j,'t16')+x1(i,j,'t17')+x1(i,j,'t18')+x1(i,j,'t19')+x1(i,j,'t20')+x1(i,j,'t21')))*12 =L= 72;
Eq541(i)..          sum(j,  (x1(i,j,'t22')+x1(i,j,'t23')+x1(i,j,'t24')+x1(i,j,'t25')+x1(i,j,'t26')+x1(i,j,'t27')+x1(i,j,'t28')))*12 =L= 72;

Eq512(i)..          sum(j,  (x2(i,j,'t1')+x2(i,j,'t2')+x2(i,j,'t3')+x2(i,j,'t4')+x2(i,j,'t5')+x2(i,j,'t6')+x2(i,j,'t7')))*12 =L= 72;
Eq522(i)..          sum(j,  (x2(i,j,'t8')+x2(i,j,'t9')+x2(i,j,'t10')+x2(i,j,'t11')+x2(i,j,'t12')+x2(i,j,'t13')+x2(i,j,'t14')))*12 =L= 72;
Eq532(i)..          sum(j,  (x2(i,j,'t15')+x2(i,j,'t16')+x2(i,j,'t17')+x2(i,j,'t18')+x2(i,j,'t19')+x2(i,j,'t20')+x2(i,j,'t21')))*12 =L= 72;
Eq542(i)..          sum(j,  (x2(i,j,'t22')+x2(i,j,'t23')+x2(i,j,'t24')+x2(i,j,'t25')+x2(i,j,'t26')+x2(i,j,'t27')+x2(i,j,'t28')))*12 =L= 72;

Eq513(i)..          sum(j,  (x3(i,j,'t1')+x3(i,j,'t2')+x3(i,j,'t3')+x3(i,j,'t4')+x3(i,j,'t5')+x3(i,j,'t6')+x3(i,j,'t7')))*12 =L= 72;
Eq523(i)..          sum(j,  (x3(i,j,'t8')+x3(i,j,'t9')+x3(i,j,'t10')+x3(i,j,'t11')+x3(i,j,'t12')+x3(i,j,'t13')+x3(i,j,'t14')))*12 =L= 72;
Eq533(i)..          sum(j,  (x3(i,j,'t15')+x3(i,j,'t16')+x3(i,j,'t17')+x3(i,j,'t18')+x3(i,j,'t19')+x3(i,j,'t20')+x3(i,j,'t21')))*12 =L= 72;
Eq543(i)..          sum(j,  (x3(i,j,'t22')+x3(i,j,'t23')+x3(i,j,'t24')+x3(i,j,'t25')+x3(i,j,'t26')+x3(i,j,'t27')+x3(i,j,'t28')))*12 =L= 72;


*only one shift can be worked in a day
Eq61(i,t)..       x1(i,"j1",t)+x1(i,"j2",t)+ x1(i,"j3",t) =L=1;
Eq62(i,t)..       x2(i,"j1",t)+x2(i,"j2",t)+ x2(i,"j3",t) =L=1;
Eq63(i,t)..       x3(i,"j1",t)+x3(i,"j2",t)+ x3(i,"j3",t) =L=1;

*no day shift after a night shift
Eq71(i,t)..       x1(i,"j3",t)+x1(i,"j1",t+1)=L=1;
Eq72(i,t)..       x2(i,"j3",t)+x2(i,"j1",t+1)=L=1;
Eq73(i,t)..       x3(i,"j3",t)+x3(i,"j1",t+1)=L=1;
```

```
*maximum number of consecutive days worked
Eq811(i)..          sum((j,k1),x1(i,j,k1))   =L=6;
Eq821(i)..          sum((j,k2), x1(i,j,k2)) =L=6;
Eq831(i)..          sum((j,k3), x1(i,j,k3)) =L=6;
Eq841(i)..          sum((j,k4), x1(i,j,k4)) =L=6;
Eq851(i)..          sum((j,k5), x1(i,j,k5)) =L=6;
Eq861(i)..          sum((j,k6), x1(i,j,k6)) =L=6;
Eq871(i)..          sum((j,k7), x1(i,j,k7)) =L=6;
Eq881(i)..          sum((j,k8), x1(i,j,k8)) =L=6;
Eq891(i)..          sum((j,k9), x1(i,j,k9)) =L=6;
Eq8101(i)..          sum((j,k10),x1(i,j,k10)) =L=6;
Eq8111(i)..          sum((j,k11),x1(i,j,k11)) =L=6;
Eq8121(i)..          sum((j,k12),x1(i,j,k12)) =L=6;
Eq8131(i)..          sum((j,k13),x1(i,j,k13)) =L=6;
Eq8141(i)..          sum((j,k14),x1(i,j,k14)) =L=6;
Eq8151(i)..          sum((j,k15),x1(i,j,k15)) =L=6;
Eq8161(i)..          sum((j,k16),x1(i,j,k16)) =L=6;
Eq8171(i)..          sum((j,k17),x1(i,j,k17)) =L=6;
Eq8181(i)..          sum((j,k18),x1(i,j,k18)) =L=6;
Eq8191(i)..          sum((j,k19),x1(i,j,k19)) =L=6;
Eq8201(i)..          sum((j,k20),x1(i,j,k20)) =L=6;
Eq8211(i)..          sum((j,k21),x1(i,j,k21)) =L=6;
Eq8221(i)..          sum((j,k22),x1(i,j,k22)) =L=6;
```

```
*maximum number of consecutive days worked
Eq812(i)..          sum((j,k1),x2(i,j,k1))   =L=6;
Eq822(i)..          sum((j,k2),x2(i,j,k2)) =L=6;
Eq832(i)..          sum((j,k3),x2(i,j,k3)) =L=6;
Eq842(i)..          sum((j,k4),x2(i,j,k4)) =L=6;
Eq852(i)..          sum((j,k5),x2(i,j,k5)) =L=6;
Eq862(i)..          sum((j,k6),x2(i,j,k6)) =L=6;
Eq872(i)..          sum((j,k7),x2(i,j,k7)) =L=6;
Eq882(i)..          sum((j,k8),x2(i,j,k8)) =L=6;
Eq892(i)..          sum((j,k9),x2(i,j,k9)) =L=6;
Eq8102(i)..          sum((j,k10),x2(i,j,k10)) =L=6;
Eq8112(i)..          sum((j,k11),x2(i,j,k11)) =L=6;
Eq8122(i)..          sum((j,k12),x2(i,j,k12)) =L=6;
Eq8132(i)..          sum((j,k13),x2(i,j,k13)) =L=6;
Eq8142(i)..          sum((j,k14),x2(i,j,k14)) =L=6;
Eq8152(i)..          sum((j,k15),x2(i,j,k15)) =L=6;
Eq8162(i)..          sum((j,k16),x2(i,j,k16)) =L=6;
Eq8172(i)..          sum((j,k17),x2(i,j,k17)) =L=6;
Eq8182(i)..          sum((j,k18),x2(i,j,k18)) =L=6;
Eq8192(i)..          sum((j,k19),x2(i,j,k19)) =L=6;
Eq8202(i)..          sum((j,k20),x2(i,j,k20)) =L=6;
Eq8212(i)..          sum((j,k21),x2(i,j,k21)) =L=6;
Eq8222(i)..          sum((j,k22),x2(i,j,k22)) =L=6;
```

```
*maximum number of consecutive days worked
Eq813(i)..          sum((j,k1),x3(i,j,k1))  =L=6;
Eq823(i)..          sum((j,k2), x3(i,j,k2)) =L=6;
Eq833(i)..          sum((j,k3), x3(i,j,k3)) =L=6;
Eq843(i)..          sum((j,k4), x3(i,j,k4)) =L=6;
Eq853(i)..          sum((j,k5), x3(i,j,k5)) =L=6;
Eq863(i)..          sum((j,k6), x3(i,j,k6)) =L=6;
Eq873(i)..          sum((j,k7), x3(i,j,k7)) =L=6;
Eq883(i)..          sum((j,k8), x3(i,j,k8)) =L=6;
Eq893(i)..          sum((j,k9), x3(i,j,k9)) =L=6;
Eq8103(i)..          sum((j,k10),x3(i,j,k10)) =L=6;
Eq8113(i)..          sum((j,k11),x3(i,j,k11)) =L=6;
Eq8123(i)..          sum((j,k12),x3(i,j,k12)) =L=6;
Eq8133(i)..          sum((j,k13),x3(i,j,k13)) =L=6;
Eq8143(i)..          sum((j,k14),x3(i,j,k14)) =L=6;
Eq8153(i)..          sum((j,k15),x3(i,j,k15)) =L=6;
Eq8163(i)..          sum((j,k16),x3(i,j,k16)) =L=6;
Eq8173(i)..          sum((j,k17),x3(i,j,k17)) =L=6;
Eq8183(i)..          sum((j,k18),x3(i,j,k18)) =L=6;
Eq8193(i)..          sum((j,k19),x3(i,j,k19)) =L=6;
Eq8203(i)..          sum((j,k20),x3(i,j,k20)) =L=6;
Eq8213(i)..          sum((j,k21),x3(i,j,k21)) =L=6;
Eq8223(i)..          sum((j,k22),x3(i,j,k22)) =L=6;
```

```
*maximum number of consecutive night shifts worked
Eq911(i)..             sum (n1,x1(i,"j3",n1)) =L=3;
Eq921(i)..             sum (n2,x1(i,"j3",n2)) =L=3;
Eq931(i)..             sum (n3,x1(i,"j3",n3)) =L=3;
Eq941(i)..             sum (n4,x1(i,"j3",n4)) =L=3;
Eq951(i)..             sum (n5,x1(i,"j3",n5)) =L=3;
Eq961(i)..             sum (n6,x1(i,"j3",n6)) =L=3;
Eq971(i)..             sum (n7,x1(i,"j3",n7)) =L=3;
Eq981(i)..             sum (n8,x1(i,"j3",n8)) =L=3;
Eq991(i)..             sum (n9,x1(i,"j3",n9)) =L=3;
Eq9101(i)..            sum (n10,x1(i,"j3",n10)) =L=3;
Eq9111(i)..            sum (n11,x1(i,"j3",n11)) =L=3;
Eq9121(i)..            sum (n12,x1(i,"j3",n12)) =L=3;
Eq9131(i)..            sum (n13,x1(i,"j3",n13)) =L=3;
Eq9141(i)..            sum (n14,x1(i,"j3",n14)) =L=3;
Eq9151(i)..            sum (n15,x1(i,"j3",n15)) =L=3;
Eq9161(i)..            sum (n16,x1(i,"j3",n16)) =L=3;
Eq9171(i)..            sum (n17,x1(i,"j3",n17)) =L=3;
Eq9181(i)..            sum (n18,x1(i,"j3",n18)) =L=3;
Eq9191(i)..            sum (n19,x1(i,"j3",n19)) =L=3;
Eq9201(i)..            sum (n20,x1(i,"j3",n20)) =L=3;
Eq9211(i)..            sum (n21,x1(i,"j3",n21)) =L=3;
Eq9221(i)..            sum (n22,x1(i,"j3",n22)) =L=3;
Eq9231(i)..            sum (n23,x1(i,"j3",n23)) =L=3;
Eq9241(i)..            sum (n24,x1(i,"j3",n24)) =L=3;
Eq9251(i)..            sum (n25,x1(i,"j3",n25)) =L=3;
```

```
*maximum number of consecutive night shifts worked
Eq912(i)..          sum (n1,x2(i,"j3",n1)) =L=3;
Eq922(i)..          sum (n2,x2(i,"j3",n2)) =L=3;
Eq932(i)..          sum (n3,x2(i,"j3",n3)) =L=3;
Eq942(i)..          sum (n4,x2(i,"j3",n4)) =L=3;
Eq952(i)..          sum (n5,x2(i,"j3",n5)) =L=3;
Eq962(i)..          sum (n6,x2(i,"j3",n6)) =L=3;
Eq972(i)..          sum (n7,x2(i,"j3",n7)) =L=3;
Eq982(i)..          sum (n8,x2(i,"j3",n8)) =L=3;
Eq992(i)..          sum (n9,x2(i,"j3",n9)) =L=3;
Eq9102(i)..         sum (n10,x2(i,"j3",n10)) =L=3;
Eq9112(i)..         sum (n11,x2(i,"j3",n11)) =L=3;
Eq9122(i)..         sum (n12,x2(i,"j3",n12)) =L=3;
Eq9132(i)..         sum (n13,x2(i,"j3",n13)) =L=3;
Eq9142(i)..         sum (n14,x2(i,"j3",n14)) =L=3;
Eq9152(i)..         sum (n15,x2(i,"j3",n15)) =L=3;
Eq9162(i)..         sum (n16,x2(i,"j3",n16)) =L=3;
Eq9172(i)..         sum (n17,x2(i,"j3",n17)) =L=3;
Eq9182(i)..         sum (n18,x2(i,"j3",n18)) =L=3;
Eq9192(i)..         sum (n19,x2(i,"j3",n19)) =L=3;
Eq9202(i)..         sum (n20,x2(i,"j3",n20)) =L=3;
Eq9212(i)..         sum (n21,x2(i,"j3",n21)) =L=3;
Eq9222(i)..         sum (n22,x2(i,"j3",n22)) =L=3;
Eq9232(i)..         sum (n23,x2(i,"j3",n23)) =L=3;
Eq9242(i)..         sum (n24,x2(i,"j3",n24)) =L=3;
Eq9252(i)..         sum (n25,x2(i,"j3",n25)) =L=3;
```

```
*maximum number of consecutive night shifts worked
Eq913(i)..          sum (n1,x3(i,"j3",n1)) =L=3;
Eq923(i)..          sum (n2,x3(i,"j3",n2)) =L=3;
Eq933(i)..          sum (n3,x3(i,"j3",n3)) =L=3;
Eq943(i)..          sum (n4,x3(i,"j3",n4)) =L=3;
Eq953(i)..          sum (n5,x3(i,"j3",n5)) =L=3;
Eq963(i)..          sum (n6,x3(i,"j3",n6)) =L=3;
Eq973(i)..          sum (n7,x3(i,"j3",n7)) =L=3;
Eq983(i)..          sum (n8,x3(i,"j3",n8)) =L=3;
Eq993(i)..          sum (n9,x3(i,"j3",n9)) =L=3;
Eq9103(i)..         sum (n10,x3(i,"j3",n10)) =L=3;
Eq9113(i)..         sum (n11,x3(i,"j3",n11)) =L=3;
Eq9123(i)..         sum (n12,x3(i,"j3",n12)) =L=3;
Eq9133(i)..         sum (n13,x3(i,"j3",n13)) =L=3;
Eq9143(i)..         sum (n14,x3(i,"j3",n14)) =L=3;
Eq9153(i)..         sum (n15,x3(i,"j3",n15)) =L=3;
Eq9163(i)..         sum (n16,x3(i,"j3",n16)) =L=3;
Eq9173(i)..         sum (n17,x3(i,"j3",n17)) =L=3;
Eq9183(i)..         sum (n18,x3(i,"j3",n18)) =L=3;
Eq9193(i)..         sum (n19,x3(i,"j3",n19)) =L=3;
Eq9203(i)..         sum (n20,x3(i,"j3",n20)) =L=3;
Eq9213(i)..         sum (n21,x3(i,"j3",n21)) =L=3;
Eq9223(i)..         sum (n22,x3(i,"j3",n22)) =L=3;
Eq9233(i)..         sum (n23,x3(i,"j3",n23)) =L=3;
Eq9243(i)..         sum (n24,x3(i,"j3",n24)) =L=3;
Eq9253(i)..         sum (n25,x3(i,"j3",n25)) =L=3;
```

```
*SOFT CONSTRAINTS

*total number of day shifts assigned to each nurse is at least as much as total number of night shifts assigned
Eq101(i)..          sum(t, x1(i,'j3',t))- sum(t, x1(i,'j1',t)+x1(i,'j2',t))-dp1(i)+dn1(i) =E= 0;
*total number of day shifts assigned to each nurse is at least as much as total number of night shifts assigned
Eq102(i)..          sum(t, x2(i,'j3',t))- sum(t, x2(i,'j1',t)+x2(i,'j2',t))-dp2(i)+dn2(i) =E= 0;
*total number of day shifts assigned to each nurse is at least as much as total number of night shifts assigned
Eq103(i)..          sum(t, x3(i,'j3',t))- sum(t, x3(i,'j1',t)+x3(i,'j2',t))-dp3(i)+dn3(i) =E= 0;

*avoid 010 assignment, no day on between two days off
Eq1111(i)..        1-sum(j, x1(i,j,'t1'))+sum(j, x1(i,j,'t2'))+1-sum(j, x1(i,j,'t3'))-dap1(i,'t1')+dan1(i,'t1')=E= 2;
Eq1121(i)..        1-sum(j, x1(i,j,'t2'))+sum(j, x1(i,j,'t3'))+1-sum(j, x1(i,j,'t4'))-dap1(i,'t2')+dan1(i,'t2')=E= 2;
Eq1131(i)..        1-sum(j, x1(i,j,'t3'))+sum(j, x1(i,j,'t4'))+1-sum(j, x1(i,j,'t5'))-dap1(i,'t3')+dan1(i,'t3')=E= 2;
Eq1141(i)..        1-sum(j, x1(i,j,'t4'))+sum(j, x1(i,j,'t5'))+1-sum(j, x1(i,j,'t6'))-dap1(i,'t4')+dan1(i,'t4')=E= 2;
Eq1151(i)..        1-sum(j, x1(i,j,'t5'))+sum(j, x1(i,j,'t6'))+1-sum(j, x1(i,j,'t7'))-dap1(i,'t5')+dan1(i,'t5')=E= 2;
Eq1161(i)..        1-sum(j, x1(i,j,'t6'))+sum(j, x1(i,j,'t7'))+1-sum(j, x1(i,j,'t8'))-dap1(i,'t6')+dan1(i,'t6')=E= 2;
Eq1171(i)..        1-sum(j, x1(i,j,'t7'))+sum(j, x1(i,j,'t8'))+1-sum(j, x1(i,j,'t9'))-dap1(i,'t7')+dan1(i,'t7')=E= 2;
Eq1181(i)..        1-sum(j, x1(i,j,'t8'))+sum(j, x1(i,j,'t9'))+1-sum(j, x1(i,j,'t10'))-dap1(i,'t8')+dan1(i,'t8')=E= 2;
Eq1191(i)..        1-sum(j, x1(i,j,'t9'))+sum(j, x1(i,j,'t10'))+1-sum(j, x1(i,j,'t11'))-dap1(i,'t9')+dan1(i,'t9')=E= 2;
Eq11101(i)..       1-sum(j, x1(i,j,'t10'))+sum(j, x1(i,j,'t11'))+1-sum(j, x1(i,j,'t12'))-dap1(i,'t10')+dan1(i,'t10')=E= 2;
Eq11111(i)..       1-sum(j, x1(i,j,'t11'))+sum(j, x1(i,j,'t12'))+1-sum(j, x1(i,j,'t13'))-dap1(i,'t11')+dan1(i,'t11')=E= 2;
Eq11121(i)..       1-sum(j, x1(i,j,'t12'))+sum(j, x1(i,j,'t13'))+1-sum(j, x1(i,j,'t14'))-dap1(i,'t12')+dan1(i,'t12')=E= 2;
Eq11131(i)..       1-sum(j, x1(i,j,'t13'))+sum(j, x1(i,j,'t14'))+1-sum(j, x1(i,j,'t15'))-dap1(i,'t13')+dan1(i,'t13')=E= 2;
Eq11141(i)..       1-sum(j, x1(i,j,'t14'))+sum(j, x1(i,j,'t15'))+1-sum(j, x1(i,j,'t16'))-dap1(i,'t14')+dan1(i,'t14')=E= 2;
Eq11151(i)..       1-sum(j, x1(i,j,'t15'))+sum(j, x1(i,j,'t16'))+1-sum(j, x1(i,j,'t17'))-dap1(i,'t15')+dan1(i,'t15')=E= 2;
Eq11161(i)..       1-sum(j, x1(i,j,'t16'))+sum(j, x1(i,j,'t17'))+1-sum(j, x1(i,j,'t18'))-dap1(i,'t16')+dan1(i,'t16')=E= 2;
Eq11171(i)..       1-sum(j, x1(i,j,'t17'))+sum(j, x1(i,j,'t18'))+1-sum(j, x1(i,j,'t19'))-dap1(i,'t17')+dan1(i,'t17')=E= 2;
Eq11181(i)..       1-sum(j, x1(i,j,'t18'))+sum(j, x1(i,j,'t19'))+1-sum(j, x1(i,j,'t20'))-dap1(i,'t18')+dan1(i,'t18')=E= 2;
Eq11191(i)..       1-sum(j, x1(i,j,'t19'))+sum(j, x1(i,j,'t20'))+1-sum(j, x1(i,j,'t21'))-dap1(i,'t19')+dan1(i,'t19')=E= 2;

Eq11201(i)..       1-sum(j, x1(i,j,'t20'))+sum(j, x1(i,j,'t21'))+1-sum(j, x1(i,j,'t22'))-dap1(i,'t20')+dan1(i,'t20')=E= 2;
Eq11211(i)..       1-sum(j, x1(i,j,'t21'))+sum(j, x1(i,j,'t22'))+1-sum(j, x1(i,j,'t23'))-dap1(i,'t21')+dan1(i,'t21')=E= 2;
Eq11221(i)..       1-sum(j, x1(i,j,'t22'))+sum(j, x1(i,j,'t23'))+1-sum(j, x1(i,j,'t24'))-dap1(i,'t22')+dan1(i,'t22')=E= 2;
Eq11231(i)..       1-sum(j, x1(i,j,'t23'))+sum(j, x1(i,j,'t24'))+1-sum(j, x1(i,j,'t25'))-dap1(i,'t23')+dan1(i,'t23')=E= 2;
Eq11241(i)..       1-sum(j, x1(i,j,'t24'))+sum(j, x1(i,j,'t25'))+1-sum(j, x1(i,j,'t26'))-dap1(i,'t24')+dan1(i,'t24')=E= 2;
Eq11251(i)..       1-sum(j, x1(i,j,'t25'))+sum(j, x1(i,j,'t26'))+1-sum(j, x1(i,j,'t27'))-dap1(i,'t25')+dan1(i,'t25')=E= 2;
Eq11261(i)..       1-sum(j, x1(i,j,'t26'))+sum(j, x1(i,j,'t27'))+1-sum(j, x1(i,j,'t28'))-dap1(i,'t26')+dan1(i,'t26')=E= 2;

*avoid 010 assignment, no day on between two days off
Eq1112(i)..        1-sum(j, x2(i,j,'t1'))+sum(j, x2(i,j,'t2'))+1-sum(j, x2(i,j,'t3'))-dap2(i,'t1')+dan2(i,'t1')=E= 2;
Eq1122(i)..        1-sum(j, x2(i,j,'t2'))+sum(j, x2(i,j,'t3'))+1-sum(j, x2(i,j,'t4'))-dap2(i,'t2')+dan2(i,'t2')=E= 2;
Eq1132(i)..        1-sum(j, x2(i,j,'t3'))+sum(j, x2(i,j,'t4'))+1-sum(j, x2(i,j,'t5'))-dap2(i,'t3')+dan2(i,'t3')=E= 2;
Eq1142(i)..        1-sum(j, x2(i,j,'t4'))+sum(j, x2(i,j,'t5'))+1-sum(j, x2(i,j,'t6'))-dap2(i,'t4')+dan2(i,'t4')=E= 2;
Eq1152(i)..        1-sum(j, x2(i,j,'t5'))+sum(j, x2(i,j,'t6'))+1-sum(j, x2(i,j,'t7'))-dap2(i,'t5')+dan2(i,'t5')=E= 2;
Eq1162(i)..        1-sum(j, x2(i,j,'t6'))+sum(j, x2(i,j,'t7'))+1-sum(j, x2(i,j,'t8'))-dap2(i,'t6')+dan2(i,'t6')=E= 2;
Eq1172(i)..        1-sum(j, x2(i,j,'t7'))+sum(j, x2(i,j,'t8'))+1-sum(j, x2(i,j,'t9'))-dap2(i,'t7')+dan2(i,'t7')=E= 2;
Eq1182(i)..        1-sum(j, x2(i,j,'t8'))+sum(j, x2(i,j,'t9'))+1-sum(j, x2(i,j,'t10'))-dap2(i,'t8')+dan2(i,'t8')=E= 2;
Eq1192(i)..        1-sum(j, x2(i,j,'t9'))+sum(j, x2(i,j,'t10'))+1-sum(j, x2(i,j,'t11'))-dap2(i,'t9')+dan2(i,'t9')=E= 2;
Eq11102(i)..       1-sum(j, x2(i,j,'t10'))+sum(j, x2(i,j,'t11'))+1-sum(j, x2(i,j,'t12'))-dap2(i,'t10')+dan2(i,'t10')=E= 2;
Eq11112(i)..       1-sum(j, x2(i,j,'t11'))+sum(j, x2(i,j,'t12'))+1-sum(j, x2(i,j,'t13'))-dap2(i,'t11')+dan2(i,'t11')=E= 2;
Eq11122(i)..       1-sum(j, x2(i,j,'t12'))+sum(j, x2(i,j,'t13'))+1-sum(j, x2(i,j,'t14'))-dap2(i,'t12')+dan2(i,'t12')=E= 2;
Eq11132(i)..       1-sum(j, x2(i,j,'t13'))+sum(j, x2(i,j,'t14'))+1-sum(j, x2(i,j,'t15'))-dap2(i,'t13')+dan2(i,'t13')=E= 2;
Eq11142(i)..       1-sum(j, x2(i,j,'t14'))+sum(j, x2(i,j,'t15'))+1-sum(j, x2(i,j,'t16'))-dap2(i,'t14')+dan2(i,'t14')=E= 2;
Eq11152(i)..       1-sum(j, x2(i,j,'t15'))+sum(j, x2(i,j,'t16'))+1-sum(j, x2(i,j,'t17'))-dap2(i,'t15')+dan2(i,'t15')=E= 2;
Eq11162(i)..       1-sum(j, x2(i,j,'t16'))+sum(j, x2(i,j,'t17'))+1-sum(j, x2(i,j,'t18'))-dap2(i,'t16')+dan2(i,'t16')=E= 2;
Eq11172(i)..       1-sum(j, x2(i,j,'t17'))+sum(j, x2(i,j,'t18'))+1-sum(j, x2(i,j,'t19'))-dap2(i,'t17')+dan2(i,'t17')=E= 2;
Eq11182(i)..       1-sum(j, x2(i,j,'t18'))+sum(j, x2(i,j,'t19'))+1-sum(j, x2(i,j,'t20'))-dap2(i,'t18')+dan2(i,'t18')=E= 2;
Eq11192(i)..       1-sum(j, x2(i,j,'t19'))+sum(j, x2(i,j,'t20'))+1-sum(j, x2(i,j,'t21'))-dap2(i,'t19')+dan2(i,'t19')=E= 2;
Eq11202(i)..       1-sum(j, x2(i,j,'t20'))+sum(j, x2(i,j,'t21'))+1-sum(j, x2(i,j,'t22'))-dap2(i,'t20')+dan2(i,'t20')=E= 2;
Eq11212(i)..       1-sum(j, x2(i,j,'t21'))+sum(j, x2(i,j,'t22'))+1-sum(j, x2(i,j,'t23'))-dap2(i,'t21')+dan2(i,'t21')=E= 2;
Eq11222(i)..       1-sum(j, x2(i,j,'t22'))+sum(j, x2(i,j,'t23'))+1-sum(j, x2(i,j,'t24'))-dap2(i,'t22')+dan2(i,'t22')=E= 2;
Eq11232(i)..       1-sum(j, x2(i,j,'t23'))+sum(j, x2(i,j,'t24'))+1-sum(j, x2(i,j,'t25'))-dap2(i,'t23')+dan2(i,'t23')=E= 2;
Eq11242(i)..       1-sum(j, x2(i,j,'t24'))+sum(j, x2(i,j,'t25'))+1-sum(j, x2(i,j,'t26'))-dap2(i,'t24')+dan2(i,'t24')=E= 2;
Eq11252(i)..       1-sum(j, x2(i,j,'t25'))+sum(j, x2(i,j,'t26'))+1-sum(j, x2(i,j,'t27'))-dap2(i,'t25')+dan2(i,'t25')=E= 2;
Eq11262(i)..       1-sum(j, x2(i,j,'t26'))+sum(j, x2(i,j,'t27'))+1-sum(j, x2(i,j,'t28'))-dap2(i,'t26')+dan2(i,'t26')=E= 2;

*avoid 010 assignment, no day on between two days off
Eq1113(i)..        1-sum(j, x3(i,j,'t1'))+sum(j, x3(i,j,'t2'))+1-sum(j, x3(i,j,'t3'))-dap3(i,'t1')+dan3(i,'t1')=E= 2;
Eq1123(i)..        1-sum(j, x3(i,j,'t2'))+sum(j, x3(i,j,'t3'))+1-sum(j, x3(i,j,'t4'))-dap3(i,'t2')+dan3(i,'t2')=E= 2;
Eq1133(i)..        1-sum(j, x3(i,j,'t3'))+sum(j, x3(i,j,'t4'))+1-sum(j, x3(i,j,'t5'))-dap3(i,'t3')+dan3(i,'t3')=E= 2;
Eq1143(i)..        1-sum(j, x3(i,j,'t4'))+sum(j, x3(i,j,'t5'))+1-sum(j, x3(i,j,'t6'))-dap3(i,'t4')+dan3(i,'t4')=E= 2;
Eq1153(i)..        1-sum(j, x3(i,j,'t5'))+sum(j, x3(i,j,'t6'))+1-sum(j, x3(i,j,'t7'))-dap3(i,'t5')+dan3(i,'t5')=E= 2;
Eq1163(i)..        1-sum(j, x3(i,j,'t6'))+sum(j, x3(i,j,'t7'))+1-sum(j, x3(i,j,'t8'))-dap3(i,'t6')+dan3(i,'t6')=E= 2;
Eq1173(i)..        1-sum(j, x3(i,j,'t7'))+sum(j, x3(i,j,'t8'))+1-sum(j, x3(i,j,'t9'))-dap3(i,'t7')+dan3(i,'t7')=E= 2;
Eq1183(i)..        1-sum(j, x3(i,j,'t8'))+sum(j, x3(i,j,'t9'))+1-sum(j, x3(i,j,'t10'))-dap3(i,'t8')+dan3(i,'t8')=E= 2;
Eq1193(i)..        1-sum(j, x3(i,j,'t9'))+sum(j, x3(i,j,'t10'))+1-sum(j, x3(i,j,'t11'))-dap3(i,'t9')+dan3(i,'t9')=E= 2;
Eq11103(i)..       1-sum(j, x3(i,j,'t10'))+sum(j, x3(i,j,'t11'))+1-sum(j, x3(i,j,'t12'))-dap3(i,'t10')+dan3(i,'t10')=E= 2;
Eq11113(i)..       1-sum(j, x3(i,j,'t11'))+sum(j, x3(i,j,'t12'))+1-sum(j, x3(i,j,'t13'))-dap3(i,'t11')+dan3(i,'t11')=E= 2;
Eq11123(i)..       1-sum(j, x3(i,j,'t12'))+sum(j, x3(i,j,'t13'))+1-sum(j, x3(i,j,'t14'))-dap3(i,'t12')+dan3(i,'t12')=E= 2;
Eq11133(i)..       1-sum(j, x3(i,j,'t13'))+sum(j, x3(i,j,'t14'))+1-sum(j, x3(i,j,'t15'))-dap3(i,'t13')+dan3(i,'t13')=E= 2;
Eq11143(i)..       1-sum(j, x3(i,j,'t14'))+sum(j, x3(i,j,'t15'))+1-sum(j, x3(i,j,'t16'))-dap3(i,'t14')+dan3(i,'t14')=E= 2;
Eq11153(i)..       1-sum(j, x3(i,j,'t15'))+sum(j, x3(i,j,'t16'))+1-sum(j, x3(i,j,'t17'))-dap3(i,'t15')+dan3(i,'t15')=E= 2;
Eq11163(i)..       1-sum(j, x3(i,j,'t16'))+sum(j, x3(i,j,'t17'))+1-sum(j, x3(i,j,'t18'))-dap3(i,'t16')+dan3(i,'t16')=E= 2;
Eq11173(i)..       1-sum(j, x3(i,j,'t17'))+sum(j, x3(i,j,'t18'))+1-sum(j, x3(i,j,'t19'))-dap3(i,'t17')+dan3(i,'t17')=E= 2;
Eq11183(i)..       1-sum(j, x3(i,j,'t18'))+sum(j, x3(i,j,'t19'))+1-sum(j, x3(i,j,'t20'))-dap3(i,'t18')+dan3(i,'t18')=E= 2;
Eq11193(i)..       1-sum(j, x3(i,j,'t19'))+sum(j, x3(i,j,'t20'))+1-sum(j, x3(i,j,'t21'))-dap3(i,'t19')+dan3(i,'t19')=E= 2;
Eq11203(i)..       1-sum(j, x3(i,j,'t20'))+sum(j, x3(i,j,'t21'))+1-sum(j, x3(i,j,'t22'))-dap3(i,'t20')+dan3(i,'t20')=E= 2;
Eq11213(i)..       1-sum(j, x3(i,j,'t21'))+sum(j, x3(i,j,'t22'))+1-sum(j, x3(i,j,'t23'))-dap3(i,'t21')+dan3(i,'t21')=E= 2;
Eq11223(i)..       1-sum(j, x3(i,j,'t22'))+sum(j, x3(i,j,'t23'))+1-sum(j, x3(i,j,'t24'))-dap3(i,'t22')+dan3(i,'t22')=E= 2;
Eq11233(i)..       1-sum(j, x3(i,j,'t23'))+sum(j, x3(i,j,'t24'))+1-sum(j, x3(i,j,'t25'))-dap3(i,'t23')+dan3(i,'t23')=E= 2;
Eq11243(i)..       1-sum(j, x3(i,j,'t24'))+sum(j, x3(i,j,'t25'))+1-sum(j, x3(i,j,'t26'))-dap3(i,'t24')+dan3(i,'t24')=E= 2;
Eq11253(i)..       1-sum(j, x3(i,j,'t25'))+sum(j, x3(i,j,'t26'))+1-sum(j, x3(i,j,'t27'))-dap3(i,'t25')+dan3(i,'t25')=E= 2;
Eq11263(i)..       1-sum(j, x3(i,j,'t26'))+sum(j, x3(i,j,'t27'))+1-sum(j, x3(i,j,'t28'))-dap3(i,'t26')+dan3(i,'t26')=E= 2;
```

```
*avoid 010 assignment, no day off between two days on
Eq1211(i)..          sum(j, x1(i,j,'t1'))+1-sum(j, x1(i,j,'t2'))+sum(j, x1(i,j,'t3'))-dbp1(i,'t1')+dbn1(i,'t1')=E=2;
Eq1221(i)..          sum(j, x1(i,j,'t2'))+1-sum(j, x1(i,j,'t3'))+sum(j, x1(i,j,'t4'))-dbp1(i,'t2')+dbn1(i,'t2')=E=2;
Eq1231(i)..          sum(j, x1(i,j,'t3'))+1-sum(j, x1(i,j,'t4'))+sum(j, x1(i,j,'t5'))-dbp1(i,'t3')+dbn1(i,'t3')=E=2;
Eq1241(i)..          sum(j, x1(i,j,'t4'))+1-sum(j, x1(i,j,'t5'))+sum(j, x1(i,j,'t6'))-dbp1(i,'t4')+dbn1(i,'t4')=E=2;
Eq1251(i)..          sum(j, x1(i,j,'t5'))+1-sum(j, x1(i,j,'t6'))+sum(j, x1(i,j,'t7'))-dbp1(i,'t5')+dbn1(i,'t5')=E=2;
Eq1261(i)..          sum(j, x1(i,j,'t6'))+1-sum(j, x1(i,j,'t7'))+sum(j, x1(i,j,'t8'))-dbp1(i,'t6')+dbn1(i,'t6')=E=2;
Eq1271(i)..          sum(j, x1(i,j,'t7'))+1-sum(j, x1(i,j,'t8'))+sum(j, x1(i,j,'t9'))-dbp1(i,'t7')+dbn1(i,'t7')=E=2;
Eq1281(i)..          sum(j, x1(i,j,'t8'))+1-sum(j, x1(i,j,'t9'))+sum(j, x1(i,j,'t10'))-dbp1(i,'t8')+dbn1(i,'t8')=E=2;
Eq1291(i)..          sum(j, x1(i,j,'t9'))+1-sum(j, x1(i,j,'t10'))+sum(j, x1(i,j,'t11'))-dbp1(i,'t9')+dbn1(i,'t9')=E=2;
Eq12101(i)..          sum(j, x1(i,j,'t10'))+1-sum(j, x1(i,j,'t11'))+sum(j, x1(i,j,'t12'))-dbp1(i,'t10')+dbn1(i,'t10')=E=2;
Eq12111(i)..          sum(j, x1(i,j,'t11'))+1-sum(j, x1(i,j,'t12'))+sum(j, x1(i,j,'t13'))-dbp1(i,'t11')+dbn1(i,'t11')=E=2;
Eq12121(i)..          sum(j, x1(i,j,'t12'))+1-sum(j, x1(i,j,'t13'))+sum(j, x1(i,j,'t14'))-dbp1(i,'t12')+dbn1(i,'t12')=E=2;
Eq12131(i)..          sum(j, x1(i,j,'t13'))+1-sum(j, x1(i,j,'t14'))+sum(j, x1(i,j,'t15'))-dbp1(i,'t13')+dbn1(i,'t13')=E=2;
Eq12141(i)..          sum(j, x1(i,j,'t14'))+1-sum(j, x1(i,j,'t15'))+sum(j, x1(i,j,'t16'))-dbp1(i,'t14')+dbn1(i,'t14')=E=2;
Eq12151(i)..          sum(j, x1(i,j,'t15'))+1-sum(j, x1(i,j,'t16'))+sum(j, x1(i,j,'t17'))-dbp1(i,'t15')+dbn1(i,'t15')=E=2;
Eq12161(i)..          sum(j, x1(i,j,'t16'))+1-sum(j, x1(i,j,'t17'))+sum(j, x1(i,j,'t18'))-dbp1(i,'t16')+dbn1(i,'t16')=E=2;
Eq12171(i)..          sum(j, x1(i,j,'t17'))+1-sum(j, x1(i,j,'t18'))+sum(j, x1(i,j,'t19'))-dbp1(i,'t17')+dbn1(i,'t17')=E=2;
Eq12181(i)..          sum(j, x1(i,j,'t18'))+1-sum(j, x1(i,j,'t19'))+sum(j, x1(i,j,'t20'))-dbp1(i,'t18')+dbn1(i,'t18')=E=2;
Eq12191(i)..          sum(j, x1(i,j,'t19'))+1-sum(j, x1(i,j,'t20'))+sum(j, x1(i,j,'t21'))-dbp1(i,'t19')+dbn1(i,'t19')=E=2;
Eq12201(i)..          sum(j, x1(i,j,'t20'))+1-sum(j, x1(i,j,'t21'))+sum(j, x1(i,j,'t22'))-dbp1(i,'t20')+dbn1(i,'t20')=E=2;
Eq12211(i)..          sum(j, x1(i,j,'t21'))+1-sum(j, x1(i,j,'t22'))+sum(j, x1(i,j,'t23'))-dbp1(i,'t21')+dbn1(i,'t21')=E=2;
Eq12221(i)..          sum(j, x1(i,j,'t22'))+1-sum(j, x1(i,j,'t23'))+sum(j, x1(i,j,'t24'))-dbp1(i,'t22')+dbn1(i,'t22')=E=2;
Eq12231(i)..          sum(j, x1(i,j,'t23'))+1-sum(j, x1(i,j,'t24'))+sum(j, x1(i,j,'t25'))-dbp1(i,'t23')+dbn1(i,'t23')=E=2;
Eq12241(i)..          sum(j, x1(i,j,'t24'))+1-sum(j, x1(i,j,'t25'))+sum(j, x1(i,j,'t26'))-dbp1(i,'t24')+dbn1(i,'t24')=E=2;
Eq12251(i)..          sum(j, x1(i,j,'t25'))+1-sum(j, x1(i,j,'t26'))+sum(j, x1(i,j,'t27'))-dbp1(i,'t25')+dbn1(i,'t25')=E=2;
Eq12261(i)..          sum(j, x1(i,j,'t26'))+1-sum(j, x1(i,j,'t27'))+sum(j, x1(i,j,'t28'))-dbp1(i,'t26')+dbn1(i,'t26')=E=2;

Eq1212(i)..          sum(j, x2(i,j,'t1'))+1-sum(j, x2(i,j,'t2'))+sum(j, x2(i,j,'t3'))-dbp2(i,'t1')+dbn2(i,'t1')=E=2;
Eq1222(i)..          sum(j, x2(i,j,'t2'))+1-sum(j, x2(i,j,'t3'))+sum(j, x2(i,j,'t4'))-dbp2(i,'t2')+dbn2(i,'t2')=E=2;
Eq1232(i)..          sum(j, x2(i,j,'t3'))+1-sum(j, x2(i,j,'t4'))+sum(j, x2(i,j,'t5'))-dbp2(i,'t3')+dbn2(i,'t3')=E=2;
Eq1242(i)..          sum(j, x2(i,j,'t4'))+1-sum(j, x2(i,j,'t5'))+sum(j, x2(i,j,'t6'))-dbp2(i,'t4')+dbn2(i,'t4')=E=2;
Eq1252(i)..          sum(j, x2(i,j,'t5'))+1-sum(j, x2(i,j,'t6'))+sum(j, x2(i,j,'t7'))-dbp2(i,'t5')+dbn2(i,'t5')=E=2;
Eq1262(i)..          sum(j, x2(i,j,'t6'))+1-sum(j, x2(i,j,'t7'))+sum(j, x2(i,j,'t8'))-dbp2(i,'t6')+dbn2(i,'t6')=E=2;
Eq1272(i)..          sum(j, x2(i,j,'t7'))+1-sum(j, x2(i,j,'t8'))+sum(j, x2(i,j,'t9'))-dbp2(i,'t7')+dbn2(i,'t7')=E=2;
Eq1282(i)..          sum(j, x2(i,j,'t8'))+1-sum(j, x2(i,j,'t9'))+sum(j, x2(i,j,'t10'))-dbp2(i,'t8')+dbn2(i,'t8')=E=2;
Eq1292(i)..          sum(j, x2(i,j,'t9'))+1-sum(j, x2(i,j,'t10'))+sum(j, x2(i,j,'t11'))-dbp2(i,'t9')+dbn2(i,'t9')=E=2;
Eq12102(i)..          sum(j, x2(i,j,'t10'))+1-sum(j, x2(i,j,'t11'))+sum(j, x2(i,j,'t12'))-dbp2(i,'t10')+dbn2(i,'t10')=E=2;
Eq12112(i)..          sum(j, x2(i,j,'t11'))+1-sum(j, x2(i,j,'t12'))+sum(j, x2(i,j,'t13'))-dbp2(i,'t11')+dbn2(i,'t11')=E=2;
Eq12122(i)..          sum(j, x2(i,j,'t12'))+1-sum(j, x2(i,j,'t13'))+sum(j, x2(i,j,'t14'))-dbp2(i,'t12')+dbn2(i,'t12')=E=2;
Eq12132(i)..          sum(j, x2(i,j,'t13'))+1-sum(j, x2(i,j,'t14'))+sum(j, x2(i,j,'t15'))-dbp2(i,'t13')+dbn2(i,'t13')=E=2;
Eq12142(i)..          sum(j, x2(i,j,'t14'))+1-sum(j, x2(i,j,'t15'))+sum(j, x2(i,j,'t16'))-dbp2(i,'t14')+dbn2(i,'t14')=E=2;
Eq12152(i)..          sum(j, x2(i,j,'t15'))+1-sum(j, x2(i,j,'t16'))+sum(j, x2(i,j,'t17'))-dbp2(i,'t15')+dbn2(i,'t15')=E=2;
Eq12162(i)..          sum(j, x2(i,j,'t16'))+1-sum(j, x2(i,j,'t17'))+sum(j, x2(i,j,'t18'))-dbp2(i,'t16')+dbn2(i,'t16')=E=2;
Eq12172(i)..          sum(j, x2(i,j,'t17'))+1-sum(j, x2(i,j,'t18'))+sum(j, x2(i,j,'t19'))-dbp2(i,'t17')+dbn2(i,'t17')=E=2;
Eq12182(i)..          sum(j, x2(i,j,'t18'))+1-sum(j, x2(i,j,'t19'))+sum(j, x2(i,j,'t20'))-dbp2(i,'t18')+dbn2(i,'t18')=E=2;
Eq12192(i)..          sum(j, x2(i,j,'t19'))+1-sum(j, x2(i,j,'t20'))+sum(j, x2(i,j,'t21'))-dbp2(i,'t19')+dbn2(i,'t19')=E=2;
Eq12202(i)..          sum(j, x2(i,j,'t20'))+1-sum(j, x2(i,j,'t21'))+sum(j, x2(i,j,'t22'))-dbp2(i,'t20')+dbn2(i,'t20')=E=2;
Eq12212(i)..          sum(j, x2(i,j,'t21'))+1-sum(j, x2(i,j,'t22'))+sum(j, x2(i,j,'t23'))-dbp2(i,'t21')+dbn2(i,'t21')=E=2;
Eq12222(i)..          sum(j, x2(i,j,'t22'))+1-sum(j, x2(i,j,'t23'))+sum(j, x2(i,j,'t24'))-dbp2(i,'t22')+dbn2(i,'t22')=E=2;
Eq12232(i)..          sum(j, x2(i,j,'t23'))+1-sum(j, x2(i,j,'t24'))+sum(j, x2(i,j,'t25'))-dbp2(i,'t23')+dbn2(i,'t23')=E=2;
Eq12242(i)..          sum(j, x2(i,j,'t24'))+1-sum(j, x2(i,j,'t25'))+sum(j, x2(i,j,'t26'))-dbp2(i,'t24')+dbn2(i,'t24')=E=2;
Eq12252(i)..          sum(j, x2(i,j,'t25'))+1-sum(j, x2(i,j,'t26'))+sum(j, x2(i,j,'t27'))-dbp2(i,'t25')+dbn2(i,'t25')=E=2;
Eq12262(i)..          sum(j, x2(i,j,'t26'))+1-sum(j, x2(i,j,'t27'))+sum(j, x2(i,j,'t28'))-dbp2(i,'t26')+dbn2(i,'t26')=E=2;

Eq1213(i)..          sum(j, x3(i,j,'t1'))+1-sum(j, x3(i,j,'t2'))+sum(j, x3(i,j,'t3'))-dbp3(i,'t1')+dbn3(i,'t1')=E=2;
Eq1223(i)..          sum(j, x3(i,j,'t2'))+1-sum(j, x3(i,j,'t3'))+sum(j, x3(i,j,'t4'))-dbp3(i,'t2')+dbn3(i,'t2')=E=2;
Eq1233(i)..          sum(j, x3(i,j,'t3'))+1-sum(j, x3(i,j,'t4'))+sum(j, x3(i,j,'t5'))-dbp3(i,'t3')+dbn3(i,'t3')=E=2;
Eq1243(i)..          sum(j, x3(i,j,'t4'))+1-sum(j, x3(i,j,'t5'))+sum(j, x3(i,j,'t6'))-dbp3(i,'t4')+dbn3(i,'t4')=E=2;
Eq1253(i)..          sum(j, x3(i,j,'t5'))+1-sum(j, x3(i,j,'t6'))+sum(j, x3(i,j,'t7'))-dbp3(i,'t5')+dbn3(i,'t5')=E=2;
Eq1263(i)..          sum(j, x3(i,j,'t6'))+1-sum(j, x3(i,j,'t7'))+sum(j, x3(i,j,'t8'))-dbp3(i,'t6')+dbn3(i,'t6')=E=2;
Eq1273(i)..          sum(j, x3(i,j,'t7'))+1-sum(j, x3(i,j,'t8'))+sum(j, x3(i,j,'t9'))-dbp3(i,'t7')+dbn3(i,'t7')=E=2;
Eq1283(i)..          sum(j, x3(i,j,'t8'))+1-sum(j, x3(i,j,'t9'))+sum(j, x3(i,j,'t10'))-dbp3(i,'t8')+dbn3(i,'t8')=E=2;
Eq1293(i)..          sum(j, x3(i,j,'t9'))+1-sum(j, x3(i,j,'t10'))+sum(j, x3(i,j,'t11'))-dbp3(i,'t9')+dbn3(i,'t9')=E=2;
Eq12103(i)..          sum(j, x3(i,j,'t10'))+1-sum(j, x3(i,j,'t11'))+sum(j, x3(i,j,'t12'))-dbp3(i,'t10')+dbn3(i,'t10')=E=2;
Eq12113(i)..          sum(j, x3(i,j,'t11'))+1-sum(j, x3(i,j,'t12'))+sum(j, x3(i,j,'t13'))-dbp3(i,'t11')+dbn3(i,'t11')=E=2;
Eq12123(i)..          sum(j, x3(i,j,'t12'))+1-sum(j, x3(i,j,'t13'))+sum(j, x3(i,j,'t14'))-dbp3(i,'t12')+dbn3(i,'t12')=E=2;
Eq12133(i)..          sum(j, x3(i,j,'t13'))+1-sum(j, x3(i,j,'t14'))+sum(j, x3(i,j,'t15'))-dbp3(i,'t13')+dbn3(i,'t13')=E=2;
Eq12143(i)..          sum(j, x3(i,j,'t14'))+1-sum(j, x3(i,j,'t15'))+sum(j, x3(i,j,'t16'))-dbp3(i,'t14')+dbn3(i,'t14')=E=2;
Eq12153(i)..          sum(j, x3(i,j,'t15'))+1-sum(j, x3(i,j,'t16'))+sum(j, x3(i,j,'t17'))-dbp3(i,'t15')+dbn3(i,'t15')=E=2;
Eq12163(i)..          sum(j, x3(i,j,'t16'))+1-sum(j, x3(i,j,'t17'))+sum(j, x3(i,j,'t18'))-dbp3(i,'t16')+dbn3(i,'t16')=E=2;
Eq12173(i)..          sum(j, x3(i,j,'t17'))+1-sum(j, x3(i,j,'t18'))+sum(j, x3(i,j,'t19'))-dbp3(i,'t17')+dbn3(i,'t17')=E=2;
Eq12183(i)..          sum(j, x3(i,j,'t18'))+1-sum(j, x3(i,j,'t19'))+sum(j, x3(i,j,'t20'))-dbp3(i,'t18')+dbn3(i,'t18')=E=2;
Eq12193(i)..          sum(j, x3(i,j,'t19'))+1-sum(j, x3(i,j,'t20'))+sum(j, x3(i,j,'t21'))-dbp3(i,'t19')+dbn3(i,'t19')=E=2;
Eq12203(i)..          sum(j, x3(i,j,'t20'))+1-sum(j, x3(i,j,'t21'))+sum(j, x3(i,j,'t22'))-dbp3(i,'t20')+dbn3(i,'t20')=E=2;
Eq12213(i)..          sum(j, x3(i,j,'t21'))+1-sum(j, x3(i,j,'t22'))+sum(j, x3(i,j,'t23'))-dbp3(i,'t21')+dbn3(i,'t21')=E=2;
Eq12223(i)..          sum(j, x3(i,j,'t22'))+1-sum(j, x3(i,j,'t23'))+sum(j, x3(i,j,'t24'))-dbp3(i,'t22')+dbn3(i,'t22')=E=2;
Eq12233(i)..          sum(j, x3(i,j,'t23'))+1-sum(j, x3(i,j,'t24'))+sum(j, x3(i,j,'t25'))-dbp3(i,'t23')+dbn3(i,'t23')=E=2;
Eq12243(i)..          sum(j, x3(i,j,'t24'))+1-sum(j, x3(i,j,'t25'))+sum(j, x3(i,j,'t26'))-dbp3(i,'t24')+dbn3(i,'t24')=E=2;
Eq12253(i)..          sum(j, x3(i,j,'t25'))+1-sum(j, x3(i,j,'t26'))+sum(j, x3(i,j,'t27'))-dbp3(i,'t25')+dbn3(i,'t25')=E=2;
Eq12263(i)..          sum(j, x3(i,j,'t26'))+1-sum(j, x3(i,j,'t27'))+sum(j, x3(i,j,'t28'))-dbp3(i,'t26')+dbn3(i,'t26')=E=2;
```

```
*preference violations
Eq131(i,j,t)..          x1(i,j,t)-p(i,j,t)-y1(i,j,t)+u1(i,j,t)  =E= 0;
Eq132(i,j,t)..          x2(i,j,t)-p(i,j,t)-y2(i,j,t)+u2(i,j,t)  =E= 0;
Eq133(i,j,t)..          x3(i,j,t)-p(i,j,t)-y3(i,j,t)+u3(i,j,t)  =E= 0;

*assign at least 3 night shifts per month to each nurse
Eq141(i)..            sum(t, x1(i,'j3',t))=G=3;
*assign at least 3 night shifts per month to each nurse
Eq142(i)..            sum(t, x2(i,'j3',t))=G=3;
*assign at least 3 night shifts per month to each nurse
Eq143(i)..            sum(t, x3(i,'j3',t))=G=3;
```

```
Model Nurse_2052017 / Eq21,Eq22,Eq23,Eq311, Eq312,Eq313,Eq321,Eq322,Eq323,Eq331,Eq332,Eq333,Eq411,Eq421,Eq431,Eq441,Eq412,
                      Eq422,Eq432,Eq442,Eq413,Eq423,Eq433,Eq443,
                      Eq511,Eq521,Eq531,Eq541, Eq512,Eq522,Eq532,Eq542, Eq513,Eq523,Eq533,Eq543,
                      Eq61,Eq62,Eq63,
                      Eq71,Eq72,Eq73,

                      Eq811,Eq821,Eq831,
                      Eq841,Eq851,Eq861,Eq871,Eq881,Eq891,Eq8101,Eq8111,Eq8121,Eq8131,Eq8141,Eq8151,Eq8161,Eq8171,
                      Eq8181,Eq8191,Eq8201,Eq8211,Eq8221,

                      Eq812,Eq822,Eq832,
                      Eq842,Eq852,Eq862,Eq872,Eq882,Eq892,Eq8102,Eq8112,Eq8122,Eq8132,Eq8142,Eq8152,Eq8162,Eq8172,
                      Eq8182,Eq8192,Eq8202,Eq8212,Eq8222,

                      Eq813,Eq823,Eq833,
                      Eq843,Eq853,Eq863,Eq873,Eq883,Eq893,Eq8103,Eq8113,Eq8123,Eq8133,Eq8143,Eq8153,Eq8163,Eq8173,
                      Eq8183,Eq8193,Eq8203,Eq8213,Eq8223,

                      Eq911,Eq921,Eq931,Eq941,Eq951,Eq961,Eq971,Eq981,Eq991,Eq9101,Eq9111,
                      Eq9121,Eq9131,Eq9141,Eq9151,Eq9161,Eq9171,Eq9181,Eq9191,Eq9201,Eq9211,Eq9221,Eq9231,Eq9241,
                      Eq9251,

                      Eq912,Eq922,Eq932,Eq942,Eq952,Eq962,Eq972,Eq982,Eq992,Eq9102,Eq9112,
                      Eq9122,Eq9132,Eq9142,Eq9152,Eq9162,Eq9172,Eq9182,Eq9192,Eq9202,Eq9212,Eq9222,Eq9232,Eq9242,
                      Eq9252,

                      Eq913,Eq923,Eq933,Eq943,Eq953,Eq963,Eq973,Eq983,Eq993,Eq9103,Eq9113,
                      Eq9123,Eq9133,Eq9143,Eq9153,Eq9163,Eq9173,Eq9183,Eq9193,Eq9203,Eq9213,Eq9223,Eq9233,Eq9243,
                      Eq9253,
```

```
                      Eq101,Eq102,Eq103,

                      Eq1111,Eq1121,Eq1131,Eq1141,Eq1151,Eq1161,Eq1171,Eq1181,Eq1191,Eq11101,Eq11111,Eq11121,
                      Eq11131,Eq11141,Eq11151,Eq11161,Eq11171,Eq11181,Eq11191,Eq11201,Eq11211,Eq11221,Eq11231,Eq11241,
                      Eq11251,Eq11261,

                      Eq1112,Eq1122,Eq1132,Eq1142,Eq1152,Eq1162,Eq1172,Eq1182,Eq1192,Eq11102,Eq11112,Eq11122,
                      Eq11132,Eq11142,Eq11152,Eq11162,Eq11172,Eq11182,Eq11192,Eq11202,Eq11212,Eq11222,Eq11232,Eq11242,
                      Eq11252,Eq11262,

                      Eq1113,Eq1123,Eq1133,Eq1143,Eq1153,Eq1163,Eq1173,Eq1183,Eq1193,Eq11103,Eq11113,Eq11123,
                      Eq11133,Eq11143,Eq11153,Eq11163,Eq11173,Eq11183,Eq11193,Eq11203,Eq11213,Eq11223,Eq11233,Eq11243,
                      Eq11253,Eq11263,

                      Eq1211,Eq1221,Eq1231,Eq1241,Eq1251,Eq1261,Eq1271,Eq1281,Eq1291,Eq12101,Eq12111,Eq12121,Eq12131,
                      Eq12141,Eq12151,Eq12161,Eq12171,Eq12181,Eq12191,Eq12201,Eq12211,Eq12221,Eq12231,Eq12241,Eq12251,Eq12261,

                      Eq1212,Eq1222,Eq1232,Eq1242,Eq1252,Eq1262,Eq1272,Eq1282,Eq1292,Eq12102,Eq12112,Eq12122,Eq12132,
                      Eq12142,Eq12152,Eq12162,Eq12172,Eq12182,Eq12192,Eq12202,Eq12212,Eq12222,Eq12232,Eq12242,Eq12252,Eq12262,

                      Eq1213,Eq1223,Eq1233,Eq1243,Eq1253,Eq1263,Eq1273,Eq1283,Eq1293,Eq12103,Eq12113,Eq12123,Eq12133,
                      Eq12143,Eq12153,Eq12163,Eq12173,Eq12183,Eq12193,Eq12203,Eq12213,Eq12223,Eq12233,Eq12243,Eq12253,Eq12263,

                      Eq131,Eq132,Eq133,Eq141,Eq142,Eq143,obj1 /
```

```
Option optcr = 0.00;

Solve Nurse_2052017 using MIP minimize Z1;

Parameter xall;
xall=sum((i,j,t), x1.L(i,j,t)*0.25+x2.L(i,j,t)*0.50 +x3.L(i,j,t)*0.25);

Parameter yall;
yall=sum((i,j,t), y1.L(i,j,t)*0.25+y2.L(i,j,t)*0.50 +y3.L(i,j,t)*0.25);

Parameter uall;
uall= sum((i,j,t), u1.L(i,j,t)*0.25+ u2.L(i,j,t)*0.50+ u3.L(i,j,t)*0.25);

Parameter dpall;
dpall= sum((i), dp1.L(i)*0.25+ dp2.L(i)*0.50+ dp3.L(i)*0.25);

Parameter dapall;
dapall= sum((i,t), dap1.L(i,t)*0.25+ dap2.L(i,t)*0.50+ dap3.L(i,t)*0.25);

Parameter dbpall;
dbpall= sum((i,at), dap1.L(i,at)*0.25+ dap2.L(i,at)*0.50+ dap3.L(i,at)*0.25);

display x1.L, y1.L, u1.L, dp1.L, dap1.L, dbp1.L;

execute_unload 'results.gdx',i,j,x1,y1,u1,dp1,dap1,dbp1,x2,y2,u2,dp2,dap2,dbp2,xall,yall,uall,dpall,dapall,dbpall;
```

```
execute 'gdxxrw.exe results.gdx O=results.xls var=x1.L' ;
execute 'gdxxrw.exe results.gdx O=results.xls var=y1.L rng=Sheet2!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls var=u1.L rng=Sheet3!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls var=dp1.L rng=Sheet4!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls var=dap1.L rng=Sheet5!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls var=dbp1.L rng=Sheet6!a1' ;

execute 'gdxxrw.exe results.gdx O=results.xls par=xall rng=Sheetx19!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls par=yall rng=Sheetx20!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls par=uall rng=Sheetx21!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls par=dpall rng=Sheetx22!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls par=dapall rng=Sheetx22!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls par=dbpall rng=Sheetx23!a1' ;


display x2.L, y2.L, u2.L, dp2.L, dap2.L, dbp2.L;

execute_unload 'results.gdx',i,j,x2,y2,u2,dp2,dap2,dbp2;

execute 'gdxxrw.exe results.gdx O=results.xls var=x2.L rng=Sheet7!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls var=y2.L rng=Sheet8!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls var=u2.L rng=Sheet9!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls var=dp2.L rng=Sheet10!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls var=dap2.L rng=Sheet11!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls var=dbp2.L rng=Sheet12!a1' ;
```

```
display x3.L, y3.L, u3.L, dp3.L, dap3.L, dbp3.L;

execute_unload 'results.gdx',i,j,x3,y3,u3,dp3,dap3,dbp3;

execute 'gdxxrw.exe results.gdx O=results.xls var=x3.L rng=Sheet13!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls var=y3.L rng=Sheet14!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls var=u3.L rng=Sheet15!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls var=dp3.L rng=Sheet16!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls var=dap3.L rng=Sheet17!a1' ;
execute 'gdxxrw.exe results.gdx O=results.xls var=dbp3.L rng=Sheet18!a1' ;
*execute 'gdxxrw.exe results.gdx var=x.M rng=NewSheet!f1:i4'
```