ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE ENGINEERING AND TECHNOLOGY

RESIDUAL STRENGTH ESTIMATION AND IMPERFECTION MODELLING FOR PLASTICALLY DEFORMED STIFFENERS

M.Sc. THESIS

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Department of Naval Architecture and Marine Engineering

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ISTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

PLASTİK ŞEKİL DEĞİŞTİRMEYE MARUZ KALMIŞ STİFNERLERİN MUKAVEMET KAPASİTESİNİN VE KALICI ŞEKİL DEĞİŞTİRMELERİNİN TAYİNİ

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HAZİRAN 2017

İsmail Kahraman, a M.Sc./a Ph.D. student of İTU Graduate School of Science Engineering and Technology student ID 508141008, successfully defended the thesis/dissertation entitled "RESIDUAL STRENGTH ESTIMATION AND IMPERFECTION MODELLING FOR PLASTICALLY DEFORMED STIFFENERS", which he/she prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

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Date of Defense : 9 June 2017

To my family,

FOREWORD

I owe people a debt of gratitude, whom encouraged me; my advisor Asst. Prof. Dr. Tansel Tayyar and my lecturer Assc. Prof. Dr. Bahadır Uğurlu, whom has supported me with their best effort; my family and fiancee, whom were with me during my school life, all my graduate friends, whom have influenced me; my seniors and teachers.

May 2017 İsmail KAHRAMAN

TABLE OF CONTENTS

Page

ABBREVIATIONS

FEM	**:** Finite Element Method
SEA	**:** Sonlu Eleman Analizi
iFEM	**:** Inverse Finite Element Method
KDT	**:** Kinematic Displacement Theory
CPU	**:** Central Processing Unit
DNV	**:** Det Norske Veritas (Norwegian Classification Society)
ABS	**:** American Bureue of Shipping

SYMBOLS

a	: Ratio of the Moment component to Critical Moment
s	: Initial position of the Segment
Δs	: Position through length in Segment
Δc	: Chord length
Θ	: Rotation at the Segment's node
M, M_p, M_o	: Moment, Plastic Moment, Critical Moment
Φ, Φ_{int}, Φ_r	: Curvature Component, Initial ,Residual
r	: Radius of Curvature
σ, σ_y	: Stress Component, Yield Stress
y	: Vertical spatial deflection of Beam
E	: Elasticity Modulus

LIST OF TABLES

LIST OF FIGURES

RESIDUAL STRENGTH ESTIMATION AND IMPERFECTION MODELLING FOR PLASTICALLY DEFORMED STIFFENERS

SUMMARY

Accurate modelling of the deflection curve of a member is essential to assess strength calculations, furthermore a well predicted residual stress distribution presents a better result for residual strength assessment. In a real-time simulation to obtain these results in a desired accuracy demands computer utilization with finite element analysis and modelling. Additionally, measurement of residual stresses is not affordable for every case considering the need of equipment and crew to conduct measurements so they need to be predicted. In this study a new method is proposed for curve fitting of the deflection curve discretization by means of fixed curvature segments employing G^1 tangential continuity. The method uses nodal coordinates and inclination at the 1st node to fit segments through nodes with a fixed curvature within using only either measured or predicted imperfection curve.

Additionally, the method presents the methodology to use curvatures for residual strength assessment employing moment-curvature relationship explained within Euler-Bernoulli beam theory. Since its known that any curvature of a beam leads to strains in fibers, and strains lead to stresses; area integration of the stresses through section leads to the subjected or resultant moment. Using the mentioned relationship for any metarial model or composition of materials for any type of section the moment-curvature relationship can be obtained directly. Thus, it will be possible to predict the reduction in moment carrying capacity due to the damages of the structure considering the residual curvatures and possibly loaded moment could be predicted using moment-curvature diagram.

In the thesis, mesh independency study for the proposed imperfection modelling method is conducted by evaluation of the modelling for 3 different cases and these results are compared to analytical curvature value. For validation of model in prediction of residual curvatures the results are compared with a commercial FEA program, ANSYS using a loading-unloading simulation of a beam for the given moments. It has been seen that proposed method can be used for modelling of any kind of imperfection and assessment of residual strength of beams.

PLASTİK ŞEKİL DEĞİŞTİRMEYE MARUZ KALMIŞ STİFNERLERİN MUKAVEMET KAPASİTESİNİN VE KALICI ŞEKİL DEĞİŞTİRMELERİNİN TAYİNİ

ÖZET

Gerçek uygulamalardaki bir kiriş elemanın doğrultusu hiçbir zaman mükemmel değildir ve bu başlangıç sehimlerinin nihai mukavemet, burkulma ve moment taşıma kapasitesi gibi mukavemet hesaplarında bu sehimlerin yer alması hesaplamaların geçerliliği açısından önem arz etmektedir. Yapılardaki sehimler genel olarak iki kategoriye ayrılabilmektedir, bunlardan biri üretim safhasındaki yüksek sıcaklık farkları ve zorlanmış şekil verme sebebi ile oluşurken, diğer bir sebebi ise kullanım anında meydana gelen ani ve elastik sınır dışındaki yüklemeler ve türevleri sebebiyle olabilmektedir. Türü fark etmeksizin her iki sehim sebebiyle yapıda artık gerilmelerin oluştuğu hali hazırda bilinmektedir. Plastik şekil değiştirmeler kalıcı olsa dahi, artık gerilmeler zaman geçtikçe veya ısıl tavlama ile giderilebilmektedir. Artık gerilmelerin giderilmesi süreci içerisinde yapının artık gerilmelerin mertebesine bağlı olarak mukavemet özelliklerinin farklı olacağı, yapısal sehimlerin göz önüne alınması gerektiği görülebilmektedir. Başlangıç sehimleri temelli bu mukavemet problem, hem artık gerilmeleri gözeterek hem de gözetmeksizin çözülebilecek bir durumdadır.

Başlangıç sehimlerinin tayininde gerçek ölçümler ile belirlenen veriler yer alabilseydi bu veriler mukavemet hesaplarında etkin olarak kullanılabilirdi. Ancak ölçüm yönteminin kolay uygulanamayışı, gerektirdiği maliyet ve çaba sebebiyle bunun yerine literatürde yaygın olarak karşılaşılan başlangıç sehimlerinin burkulma modlarının superpozisyonu şeklinde tayin edilmesi daha kullanışlıdır . Başlangıç sehimlerinin ölçümü veya belirlenmesinin yansıra bu sehimlerin mukavemet hesaplarında nasıl yer edineceği veya kullanılacağı bu çalışmada önerilen yöntem ile gösterilmiştir.

Temel olarak plastik deformasyon sebebiyle oluşan artık gerilmeler plastik gerilme dağılımından, boşaltma gerilme dağılımının çıkartılması ile elde edilebilmektedir. Yükleme geçmişinin bilinmediği bir durumda başlangıç sehimleri veya plastik deformasyonlara bakılarak artık gerilmeleri tayin etmek mümkün olmadığı için, literatürde başlangıç sehimlerinin şiddetine bakılarak yaklaşım yapan yöntemler mevcuttur. Ancak bu yöntem kalıcı deformasyonlar/ başlangıç sehimleri tahmin edilen burkulma mod şekilleri ile örtüşmediğinde bu yaklaşım doğru bir ilişki göstermemektedir. Ayrıca kalıcı şekil değiştirmeye uğramış yapının sonlu elemanlar analizi ile artık gerilmelerini tayin etmek mümkündür . Bunun yansıra, son zamanlarda çalışılmakta olan tersine sonlu elemanlar yöntemi ile mevcut deformasyonlar ile gerilmeleri ilişkilendirebilse de plastik gerilme-şekil değiştirme ilişkisini hakkında henüz bir çözüm sunmamaktadır. Her ne kadar hasarlı veya hasarsız artık gerilme ölçme sistemleri ve yöntemleri mevcut ise de bu yöntemlerin hem zahmetli oluşu hem de ekipman kullanımı dolayısıyla bir ekip ve ödenek gerektirdiğinden akademik veya özel durumlar dışında kullanılması maliyet ve zaman açısında uygulanabilir değildi.

Bu çalışmada, mevcut bilinen bir başlangıç sehimi, eğrilikleri temel alan bir yaklaşımla modellenmiş, ayrıca bu model artık gerilmelerin tayini için moment-eğrilik ilişkisi çerçevesinde kullanılmıştır. Bu modelleme safhasında temel olarak bilinmesi gereken hususlar düğümsel koordinatlar ve ilk düğümdeki dönme değeridir. Ancak ilk düğümdeki dönme değerinin bilinmediği durumlarda ,ya model mutlak olarak dönmenin olmadığı noktadan eşlenik olarak kurulmalı, yada diğer sınır şartı bilinerek bu sınır şartının empoze edilmesi ile ilk düğümdeki iteratif-deneme yöntemleriyle bulunması gerekmektedir. Tez kapsamında yapılan uygulamarda ,hem simetrik özelliklerinden dolayı eşlenik yarı model, hem de asimetriden dolayı iteratif yöntem ile eğriliklerin tayini yapılmıştır. Ayrıca tez kapsamında kullanılan başlangıç sehim modellenme yönteminin ağdan bağımsızlık çalışması yürütülmüştür. Ağdan bağımsızlık çalışması analitik olarak belirlenen bir başlangıç sehiminin farklı düğüm sayılarında ayrıklaştırılarak, hesaplanan eğrilik değerlerinin analitik olarak bulunan eğrilik değeri ile karşılaştırılması ile gerçekleştirilmiştir.

Düğümsel koordinatlardan eğrilik değerleri ile ilişkilendiren formülasyon tez kapsamında eğrisel koordinat ekseni üzerinden tayin edilmiş ve aşağıdaki şekilde elde edilmiştir. İlgili denklem kullanılarak, sadece düğümsel koordinatların ve başlangıç düğümündeki dönme değerinin bilinmesi ile G^1 eğim sürekliliğine uyan yay parçalarının birleşimi ile çökme eğrisini temsil etmek mümkündürBurada

$$\Phi(s) = \frac{2\sin\left(\tan^{-1}\frac{\Delta y(s)}{\Delta x(s)} - \theta(s)\right)}{\Delta c(s)}$$

Aynı yöntem dolu kirişten daha farklı moment-eğrilik ilişkisi sunan I profil kirişi için incelenmiş, fiktif oluşturulmuş bir durum için kalıcı şekil değiştirmelerin tayin edilip artık mukavemet kapasitesinin belirlenmesi ve kirişin hasarlı durumdaki moment-eğrilik ilişkisinin sunulması şeklinde gerçekleştirilmiştir. Farklı malzeme modellerinin uygulanabilirliğinin gösterilmesi amacıyla hem elastik- mükemmel plastik, hem de elastik – plastik malzeme ile uygulanmış , farklı iki modelde de moment-eğrilik ilişkisi elde edilmiş, ayrıca jenerik bir profil enkesiti için hesap yapabilen bir algoritma tez sonunda ek olarak sunulmuştur.

Kirişlerin eğilmesi probleminde sabit eğriliğe sahip bir kiriş parçasında oluşacak gerilme değerleri ,y değeri fiberlerin tarafsız eksenden eğilme doğrultusundaki mesafesi olmak üzere aşağıdaki şekilde elde edilmektedir. Burada Φy değeri ilgili fiberdeki birim şekil değiştirmeyi temsil etmekte, S fonksiyonu ise ilgili birim şekil değiştirmeyi gerilme ile ilişkilendiren fonksiyondur. Lineer malzeme modeli için bu fonksiyon Elastisite modeli ile yazılabilmekte iken lineer olmayan malzeme modelinin dahil edilebilmesi amacıyla fonksiyon olarak seçilmiştir.

$$\sigma(y) = -S[\Phi y']$$

Her fiberde gerilmenin bilindiği durumda, kesite ilgili eğrilik değerinde etkiyen momentin değeri aşağıdaki fonksiyon ile bulunabilmektedir. Burada B(y) fonksiyonu düşey kesit doğrultusunca kalınlık değişimini temsil etmekte, herhangi bir kesitin birim adım fonksiyonları vasıtasıyla tanımlanması sağlayabilmektedir.

$$M_{external}(s) = \int_{y_{top}}^{y_{bottom}} B(y)\sigma(s, y')\, y\, dA$$

Herhangi bir eğriliğe sahip bir kiriş parçası üzerindeki yükleme kaldırıldığında üzerine oluşacak kalıcı deformasyonlar doğrudan eğrilikler kullanılmasıyla, elastik eğrinin yüklenmiş durumdaki eğrilikten çıkartılmasıyla bulunabilmektedir. Tersi bir durumda başlangıç sehimleri üzerinden hesaplanmış eğriliklere sahip kiriş parçasına boşaltma öncesi yüklenmiş olan moment artık eğrilik noktasından lineer yükleme çizgisine paralel çizginin moment-eğrilik eğrisi ile kesiştiği yer olarak aşağıdaki denklemde belirtildiği üzere bulunabilmektedir.

$$M(\Phi) - EI\Phi_{artık} = 0$$

Yapısal elemanların çökme eğrisinin doğru modellenmesi ve temsil edilmesi mukavemet hesaplarının isabetli bir şekilde yapılması ve artık gerilmelerin tayini sebebiyle önem arz etmektedir. Gerçek zamanlı bir sonlu elemanlar simülasyonunda bu işlemlerin gerçekleştirilmesi hem hesaplama zamanı hem de modelleme süreci sebebiyle pratik ve uygulanabilir değildir. Bu çalışmada herhangi bir çökme eğrisinin eğrilik bazında elemanlarla ayrıklaştırarak, ayrıca eğim sürekliliğin sağlanması da gözetilerek modellenmesini sağlayan yeni bir yöntem önerilmiştir. Bunlara ek olarak önerilen yöntem ile Euler-Bernoulli kiriş teorisinde mevcut olan moment-eğrilik ilişkisinin kullanılarak artık gerilmelerin tayini ve artık mukavemet hesaplarının yapılabilirliği sunulmuştur. Sonuçlar dikdörtgen kesitli bir kiriş ve elastik-mükemmel plastik malzeme modeli ayrıca I kesitli bir kiriş ve için sonlu elemanlar analizi ile karşılaştırılmış ve önerilen yöntem doğrulanmıştır.

1. INTRODUCTION

Axis of a real beam member is not perfectly straight, and it is essential to take into account the effect of imperfection when compression forces are considered (Schafer & Pekoz 1998). These imperfections can be classified into two categories: imperfections generated from manufacturing processes, and permanent deformations generated from inelastic bending of the structure, caused by accidents, extreme waves or any type of impact loads. It is clear that both types of imperfection make residual stresses on the structure. Even if the plastic strains are permanent, the residual stresses after a plastic deformation will disappear by time or by heat treatment. During this normalizing period, residual strength behaviour of the structure will be different from the stress free condition. Therefore, it is very necessary to consider the effect of residual stresses for a reliable strength calculation after rebound of an inelastic loading. This imperfection based residual strength problem can be studied with or without residual stresses.

The predictions would be much accurate if the actual measurements were used for the imperfection, but common approach is to predict the imperfection geometry according to the buckling modes of the structure (Pastor 2013). Additionally it is possible to obtain magnitude of imperfections through measurement procedures and adapting the corresponding measurement into strength calculations.Furthermore a method to fit curves for a beam deflection is presented in the study.

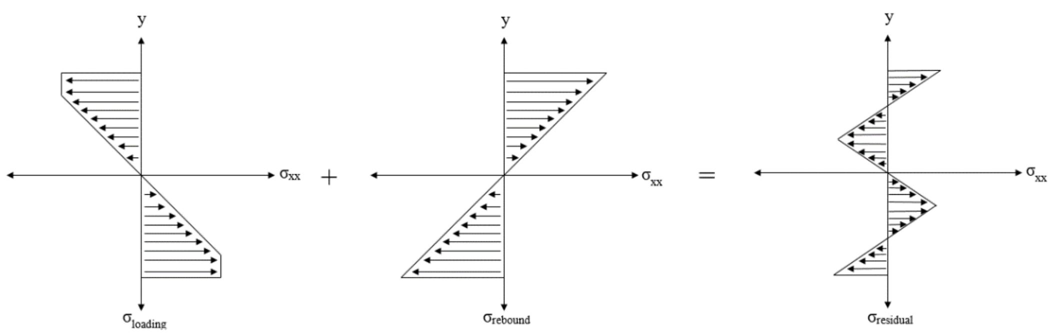

Figure 1.1 . Residual stress distribution in plastically deformed section subjected to bending moment.

Fundamentally, residual stresses are obtained by subtracting the rebound stresses from the inelastic stress distribution, as shown in Figure 1.1, which requires the plastic stress distribution to be known. If the loading history is unknown, there are some prediction methods for residual stress distribution, depending on the magnitudes of the imperfections (Schafer & Pekoz 1998), but these methods does not show a good correlation if the deflection pattern is not similar with the predicted buckling mode (Pastor, 2013 and Schafer & Pekoz, 1998). Additionally, permanent deformed configuration can be simulated by using a finite element analysis (FEA) with solid elements, which is generally a time-consuming process (Pastor, 2013). Furthermore, there are several destructive or non-destructive measurement methods for detecting imperfection and residual stresses, to be later used as input data for FEA ; these methods, however, are not practical considering the required effort and duration for measurement.

A recently developed method, inverse FEM (Tessler & Spangler, 2003, 2005), determines the stress distribution using displacements and is adopted for ship structures (Kefal & Oterkus, 2016). However, this method is currently available for the elastic region and cannot offer representation of the plastic deformation.

1.1 Purpose of Thesis

The motivation of this study is to find a simple and robust method for residual strength calculation of beam type structures that have permanent deflection and when the loading history is unknown; only the material properties and initial imperfection are required as input for the proposed method.

The main ideas of the method is quite straightforward: as the deflections can be obtained by using curvatures, curvatures can be obtained from the deflections. Thus, initial curvature can be calculated from the initial geometry, and since curvature represents the strain distribution, stress values can be calculated from the strain using the constitutive relations, consequently giving the internal forces from the equilibrium relations. Briefly, if the curvature is known, bending moment can be calculated, and vice-versa. Bending moment-curvature relationship can be derived either as a function or as a diagram.Finally, The proposed method presents a solution to obtain deflections through a known loading, by the derivation of the curvatures either by graph or substition into moment-curvature function. In reverse, with a known deflection or imperfection the corresponding moments of loading can be obtained by derivation of the curvatures with the proposed curve –fitting method. Validation of the proposed method was achieved with the comparison of an FEA procedure where plastic moment deformation from the proposed method has been introduced prior to a rebounding process. The resultant permanent deflections along a rectangular cross section beam has been compared and it is seen that proposed method performs an appropiate correlation with the results obtained in FEA.

1.2 Literature Review

Residual stress and imperfections exist on any structure, as a result of manufacturing processes or repairing after breakage and collisions. Cold forming has been indicated as the reason of residual stress distribution over thickness. Even though residual stresses introduced intentionally in some structures which is surely known to subject either tension or compression loads; ship structures aren't subjected to stress variation in a single direction, rather than that, subjects to a cyclic load over a large range both in compression and tension. As a result, it is significant to have an idea about residual stress on cold formed members. Looking through past studies to understand this phenomenon was needed.

Pastor et al (2012) has suggested the actual manufacturing process of cold forming to be modelled by FEA, residual stress along with initial imperfections have transferred to the nonlinear buckling analysis (Progressive Collapse Analysis) to initiate a buckling mode by modelling an actual, realistic progress instead of the statistical methods. Schafer and Pekoz (1998) have emphasized that distribution and magnitude of residual stress in cold formed profiles, as well as the longtidunal imperfections, takes a significant place to define buckling mode shapes of the profile. They have shown that profiles, which is cold formed, characterizes a variable longitudinal imperfection wave lengths which better to be seen in transformed frequency spectrum. They have shown in a set of nonlinear finite element analysis that residual stress and imperfection magnitude of cold formed steels affect buckling behavior. Within this study, it has shown that KDT can be used to obtain residual stresses both in elastic and plastic region as well as can present imperfections which can be used as initial state in nonlinear buckling analysis.

In last decades there has been new regulations offered to ship owners, which is hull monitoring in three levels. First level indicates the motion data to be analyzed on board, while the second level indicates stresses on ship's hull to be monitored instantly and post-processed for awareness of crew. It has been noted in ABS regulations (2014) that, monitoring system has threshold levels which indicates warnings, so that actions may be taken and safety at sea has achieved. In order to achieve the prediction, DNV regulations (2011) require ships over 180 meters to be equipped with monitoring systems, which also involves the structural health monitoring; this includes the strain-gauges located at amidships, as near as possible to amidships at bottom and top, and quarterly lengths at the deck, and at also starboard and port in all the cases. Determination of the residual stress distribution remained after the collision or freak wave encounter can be used to predict the remaining load carrying capacity of the structure. Thus, it would help monitoring systems to predict a threshold which is in safe region of structural health state of the hull. Cai, Jiang and Lodejiwks (2015) discussed that load carrying capacity of structures were underestimated due to removal of plastically deformed areas in determination of residual strength in the ship structures and in contrast, overestimated due to ignoring residual stresses at initial stage. The corresponding research considers both residual stresses and initial deformations and investigates the local behavior for a stiffened plate laid between primary girders subjected to an impact load which simulates weigh fall over deck, to estimate residual stresses and imperfections for nonlinear buckling analysis.

Requirements in structural health monitoring lead to emergence of different methods to be applied. Kefal, Oterkus (2016) has suggested Inverse Finite Element Method to be applied for the case of stress and displacement monitoring of Panamax containership. Four node quadrilateral inverse elements (iQS4) were used to model parallel midbody of the ship and iFEA has been conducted to obtain corresponding stress-displacement distribution along the section. They have shown that stress and displacements over parallel midbody, equipped with adequate number of strain gauge rosettes, could be obtained by iFEM within a range of accuracy.

Residual stresses may not appear only with cold forming, naturally they exist in a structural member subjected to load which leads to yielding at any point, followed by an elastic spring back explained by Timoshenko (1930). Shakedown theorems explain that a structure subjected to cyclic large range loads have a limit where structure stays in a state where deformations stay in elastic range, or form a hysteresis curve within plastic range. Jones (1975) suggested the concept of shakedown limit that can be used instead of ultimate strength as failure criteria for hull girder, since the major loads acting on hull girder consist of wave loads acting not monotonically but cyclic. He, furthermore, modelled ship structure as beam ignoring the buckling phenomena occurs at the plates . Residual stresses could indicate where material yields first, either in tension or compression, therefore a well predicted residual stress in hull girder inevitably leads to a well predicted shakedown limit. Beam-column model doesn’t account for local buckling, hence a model which accounts for local plate buckling in prediction of should have been investigated. Zhang, Paik and Jones (2016) investigated shakedown limit for Suezmax-Class double-hull tanker considering buckling effect on compressive elements. They have shown that a structure, consists of subcomponents, such as hull girder could have shakedown limit. Further, they assessed that a hull girder might fail before reaching to assumed ultimate strength subjected cyclic loads leading to shakedown.

2. CONCEPT OF THE MODEL

2.1 Main Concept of Deflection Modelling

Elastica is the one of the best nonlinear theory for the finite strain calculations of beam deflection, where large deflection approach becomes ineffective. Elastica method describes a formulation for the deflection curve using slopes of the curve in curvilinear coordinate system as follows as shown in Figure 2.1

$$\frac{dx}{ds} = \cos\theta(s) \tag{2.1}$$

$$\frac{dy}{ds} = \sin\theta(s) \tag{2.2}$$

Where $\theta(s)$ starting slope of the segment, $\Delta y(s)$ is vertical displacement at the ends of the segment, $\Delta x(s)$ is vertical displacement at the ends of the segment.

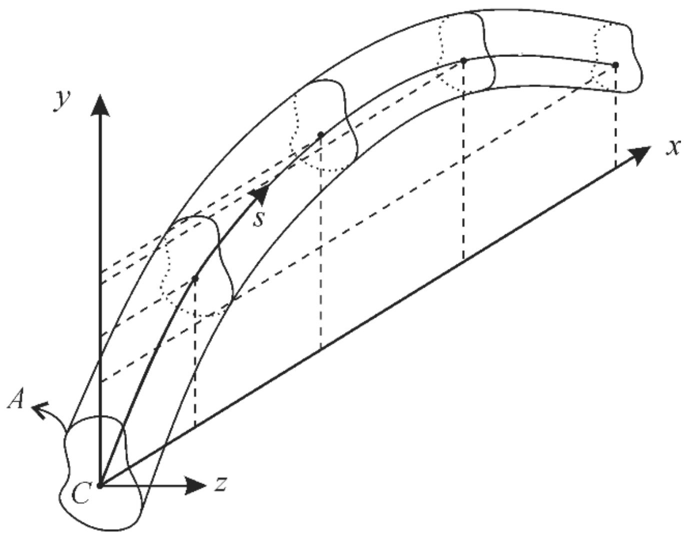

Figure 2.1 Deflection curve of a 1-D structure.

If we take the integration of eq. 2.1 and eq. 2.2, following expression for the locations of the deflection curve can be defined as follows, where Δs presents length of the segment.

$$y(s+\Delta s)-y(s)=\int_{s}^{s+\Delta s}\sin\theta(\bar{s})\,d\bar{s} \tag{2.3}$$

$$x(s+\Delta s)-x(s)=\int_{s}^{s+\Delta s}\cos\theta(\bar{s})\,d\bar{s} \tag{2.4}$$

Moreover, curvature ($\Phi(s)$) expression gives us the following relation:

$$\Phi(s)=\frac{d\theta}{ds} \tag{2.5}$$

By taking the integration of the Eq. 2.5, change in the slope values can be calculated in terms of curvature values in curvilinear system as given in Eq. 2.6. with the assumption that curvature is constant during Δs,along a segment.

$$\theta(s+\Delta s)-\theta(s)=\Phi(s)\Delta s \tag{2.6}$$

The displacement formulations Equation 2.7 and Equation 2.8 can be obtained by submitting Equation 2.6 into Equation 2.3 and Equation 2.4. as follows:

$$\Delta y(s)=\frac{1}{\Phi(s)}\left[\cos\theta(s)-\cos\theta(s+\Delta s)\right] \tag{2.7}$$

$$\Delta x(s)=\frac{1}{\Phi(s)}\left[\sin\theta(s+\Delta s)-\sin\theta(s)\right] \tag{2.8}$$

It is essential to know the slope at the first segment for the starting point of the numerical calculation so that the rest of slopes can be obtained, it could be either obtained by using the other boundary condition or using a conjugate model which the beam is modelled as starting from extremum point (ie. symmetry) of deflection where where slope angle is known to be zero at any condition . Slope angles at the following segments can be calculated from Equation 2.6 using current the curvature value of the segment. Finally, displacements of the segment can be calculated using Equation 2.7 and Equation 2.8.

With an available deflection curve, curvature distribution becomes available to be obtained through explained procedure, in reverse the deflection can be obtained by using set of equations relating the curvature and spatial coordinates of the deflection.

2.2 Imperfection Modelling

If a beam element with initial deflection is discretized into finite number of segments and nodes, it is assumed that each segment may have a different curvature but curvature does not change within the segment as seen in Figure 2.2. Displacements at each node are known or can be measured (Tayyar 2012).

From the geometric considerations slope angle of the chord ($\varphi(s)$) can be expressed as follows:

$$\varphi(s)=\theta(s)+\frac{\Delta\theta(s)}{2}=\tan^{-1}\frac{\Delta y(s)}{\Delta x(s)} \qquad (2.9)$$

From Equation 2.9, $\Delta\theta$ can be obtained as follows:

$$\Delta\theta(s)=2\left(\tan^{-1}\frac{\Delta y(s)}{\Delta x(s)}-\theta(s)\right) \qquad (2.10)$$

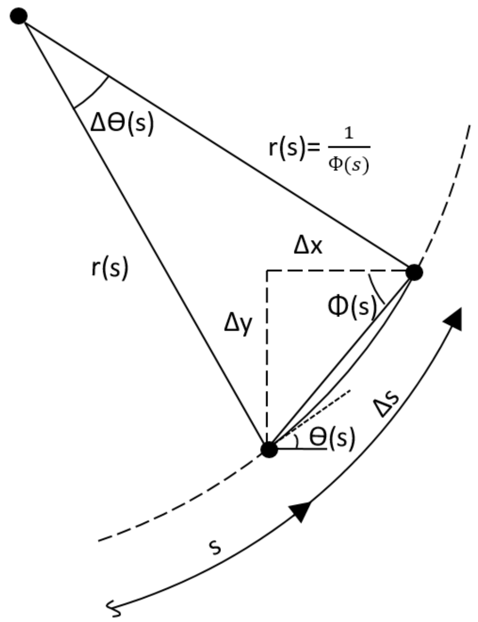

Figure 2.2 Geometry of a segment displacement.

Physically reciprocal of the curvature is equal to the radius of the curvature. The equation of the curvature is defined as a function of chord length (Δc) , and slope angle ($\Delta\theta$) from geometric considerations as follows:

$$\Phi(s)=\frac{1}{r}=\frac{2\sin\left(\frac{\Delta\theta(s)}{2}\right)}{\Delta c(s)} \tag{2.11}$$

By substituting Eq. 2.10 into Eq. 2.11, curvature of the segment can be defined in terms of the displacements and the starting slope ($\theta(s)$) angle of the segment as follows.

$$\Phi(s)=\frac{2\sin\left(\tan^{-1}\frac{\Delta y(s)}{\Delta x(s)}-\theta(s)\right)}{\Delta c(s)} \tag{2.12}$$

Using the Eq.12, it is possible to model curve from curvatures, which obeys G^1 type continuity. It essential to know the first segment slope angle to start the calculation. Therefore, slope angle of the deflection curve should be measured or iteratively obtained with consideration of the boundary conditions or the numerical calculation should be started from extremum point of the deflection curve.

2.3 Curvature-External Load Relationship

Following expressions are limited with elastic perfectly plastic material model and rectangular cross section structure to achieve closed form equations for a plain presentation. Complex material models and different cross sections can be easily adapted numerically (Tayyar et al., 2014, Tayyar, 2016).

Elastic stress behaviour at fibres is defined as follows according to the curvature value as follows: where *y* and *E* represents the location of the fibre from centroid of the cross section and elasticity modulus of the material, respectively.

$$\sigma(y)=-E\Phi y \tag{2.13}$$

Critical value of the y where outer fibre starts yielding due to curvature value can be submitted from Eq. 13, where σ_0 represents yielding stress of the material as follows:

$$y_{cr}=-\sigma_0/(E\Phi) \tag{2.14}$$

Equilibrium of external moments with stress distribution or resultant of the internal forces can be calculated as follows:

$$M_{external}(s) = \int \sigma(s,y)\, y\, dA \tag{2.15}$$

Moment-curvature can be obtained by submitting Eq. 2.13 and Eq. 2.14 into Eq. 2.15

$$M(\Phi) = \begin{cases} \displaystyle\int_{-\frac{h}{2}}^{\frac{h}{2}} -E\,\Phi\, y^2\, dA & y_{cr} > \dfrac{h}{2} \\ \displaystyle\int_{-\frac{y_{cr}}{2}}^{\frac{y_{cr}}{2}} -E\,\Phi\, y^2\, dA + 2\int_{\frac{y_{cr}}{2}}^{\frac{h}{2}} -\sigma_0\, y\, dA & y_{cr} \le \dfrac{h}{2} \end{cases} . \tag{2.16}$$

Fully plastic moment: M_p and critical moment: M_o value where yielding starts for a rectangular bar is given as follows:

$$M_0 = \sigma_0 b h^2 / 6 \tag{2.17}$$

$$M_p = 1.5 M_0 \tag{2.18}$$

Therefore, moment value can be defined in terms of M_o value as follows:

$$M(a) = a M_0 \tag{2.19}$$

itical value of a is unity and curvature values can be obtained in terms of M_o value from Eq. 2.16 and Eq. 2.14 as follows: where Φ_e and Φ_p represents elastic and inelastic curvature values, respectively.

$$\Phi(a) = \begin{cases} \Phi_e = -\dfrac{2\sigma_0}{Eh} a & |a| \le a_{cr} \\ \Phi_p = -\dfrac{2\sigma_0}{Eh}\sqrt{\dfrac{2}{6-4a}} & a_{cr} < |a| < 1.5 \end{cases}$$

(2.20)

2.4 Spring-back Mechanism

Curvatures after the unloading can be calculated just by subtracting the elastic curvature values for the loads from the inelastic curvature values as seen in Figure 2.3.

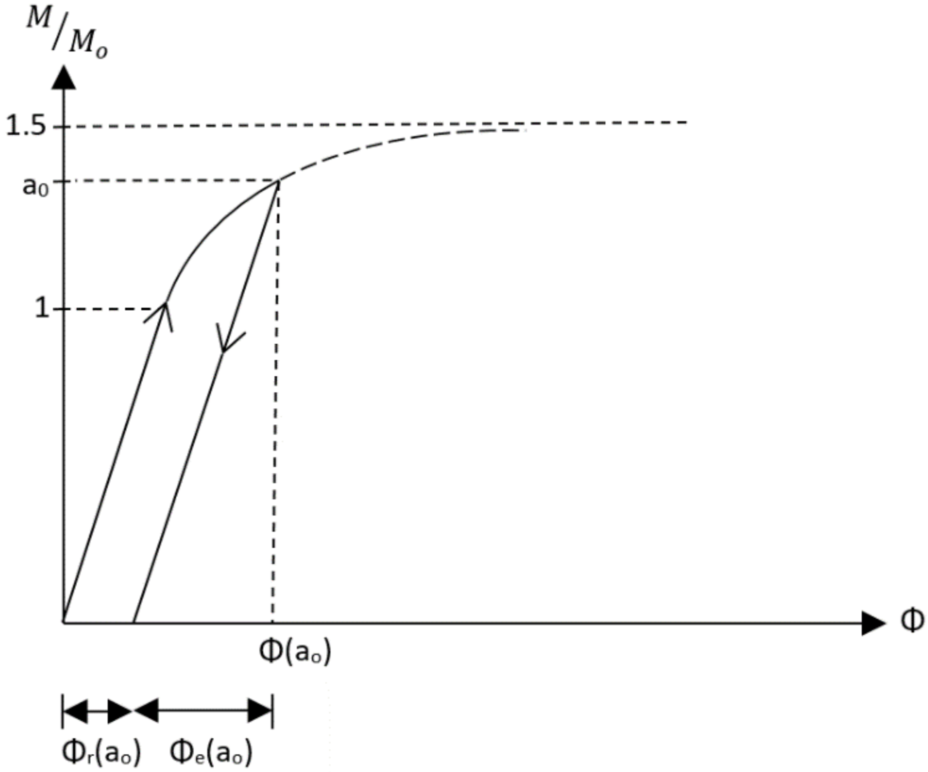

Figure 2.3 Spring-back mechanism of a member in terms of curvature- moment relationship.

If the a value is smaller than 1, curvature should be calculated based on elastic deformation. And, for the greater values of the *a* than unity, plastic formulations should be taken. During a plastic loading process, if the load is unloaded from the system, there should exist a plastic strain and plastic curvature. Because behaviour of the unloading process will be elastic, residual curvature can be calculated with following formula.

$$\Phi_{residual}\left(a_0\right)=\Phi_p\left(a_0\right)-\Phi_e\left(a_0\right)=-\frac{2\sigma_0}{E\,h}\left(\sqrt{\frac{2}{6-4a_0}}-a_0\right) \qquad (2.21)$$

2.5 Residual Strength Calculations

Strength calculations where residual stresses are neglected is very simple and new curvature can be calculated as the summation of initial curvature from Eq. 2.12 with curvature comes from new loading represented with a_1 as follows:

$$\Phi_{new}\left(a_1\right)=\begin{cases}\Phi_{initial}+\Phi_e\left(a_1\right) & a1<1\\ \Phi_{initial}+\Phi_P\left(a_1\right) & 1<a1\leq 1.5\end{cases}. \qquad (2.22)$$

It should be noticed that second order effects are very sensitive when compression forces are considered in Eq. 2.22. Curvature formulation is still Eq.20 where a_{cr}=1.

If residual stress values want to be taken into account, the moment or a_0 value should be determined from roots of Eq. 2.20, where initial curvature values from Eq. 2.12 is submitted.

$$\Phi_{initial} = -\frac{2\sigma_0}{E\,h}\left(\sqrt{\frac{2}{6-4a_0}} - a_0\right) \qquad 1<|a_0|\leq 1.5 \tag{2.23}$$

Now, critical value yields to $a_0$value and new criteria can be expressed as follows:

$$\Phi_{new}(a_1) = \begin{cases} \Phi_{initial}(a_0) + \Phi_e(a_1) & a1 < a_0 \\ \Phi_{initial}(a_0) + \Phi_P(a_1) & a_0 < a1 \leq 1.5 \end{cases} \tag{2.24}$$

3. VALIDATION AND APPLICATION

3.1 Mesh Independency of Imperfection Modelling

Mesh independency of the proposed imperfection modelling has been done using a analytically known curve and comparing the curvatures with the values calculated from the proposed method.

A Half sinus wave with a magnitude of 100 and expressed in following formula;

$$100 * \mathrm{Sin}(0.5 * \pi * x / 500) \qquad 0 \le x \le 500 \tag{3.1}$$

Comparison of results have been proposed in the Table 3.1 and accuracy of 20 division of segments have been decided to be used for the current case .

Table 3.1 Comparison of relative difference between different divisions.

Analytical	20 Division	%	10 Division	%	5 Division	%
0.00093866	0.000940	0.13548	0.000953	1.5502	0.00097	4.03505
0.00078847	0.000785	-0.47941	0.000809	2.6554	0.00085	7.31361
0.00056900	0.000559	-1.70890	0.00059	3.7150	0.00065	12.2152
0.00030499	0.000302	-1.14336	0.000335	9.7252	0.00040	31.3725

3.2 Application of a Bar with I Section

In thesis the analytical relationships have been proposed for a bar with rectangular section where integrals could be evaluated analytically. In a different case where material model changes or shape of section varies than simple and analytical expression the evluation should be done numerically, thus relationship between moment-curvature should be expressed. Since moment should be numerically evaluated based on integration through section, it could be easily obtained by employing simple algorithms but reverse relation from curvature to moment, or residual curvature to moment should be obtained using graphical relationships or by expressing the moment-curvature curve into polynomial form.

In the application the material model has chosen to be elastic-plastic with strain hardening and the strain-stress relationship has drawn in Figure 3.1. With a yield stress of 350 Mpa and elasticity modulus of 200000 Mpa.

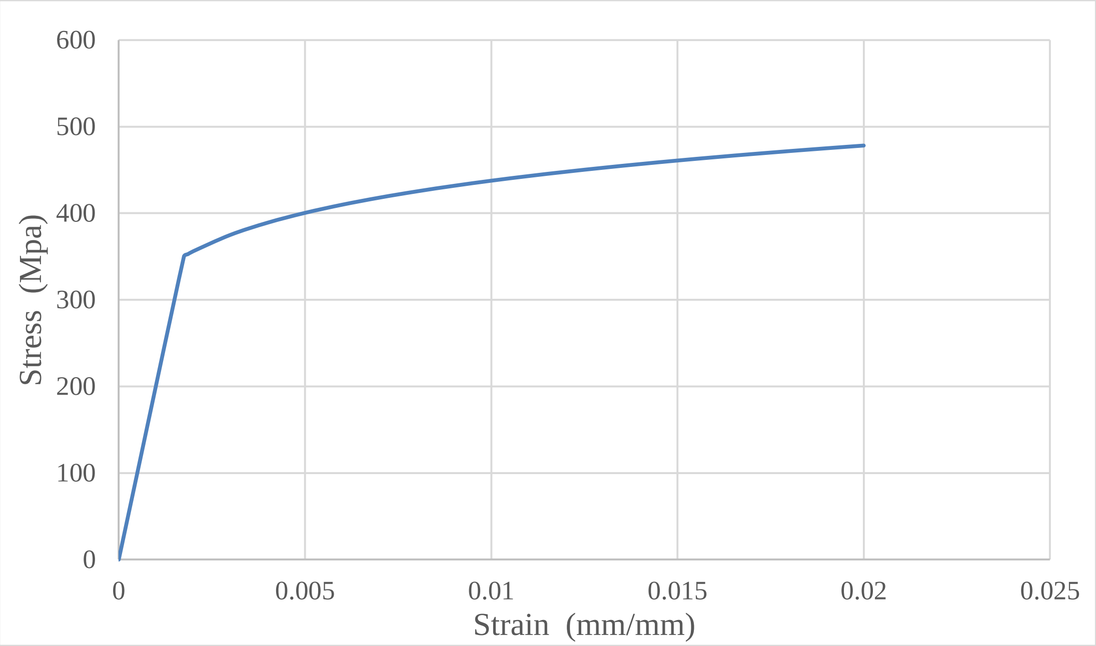

Figure 3.1 Stress-Strain relationship of the material.

Employing Equation 2.15 and 2.13 , moment value can be evaluated at any curvature using numerical integration. Moment- curvature relationship for corresponding I section with dimensions of 100 mm wide flange and 10 mm thickness and 100 mm wide web and 10 mm thickness presented in Figure 3.2.

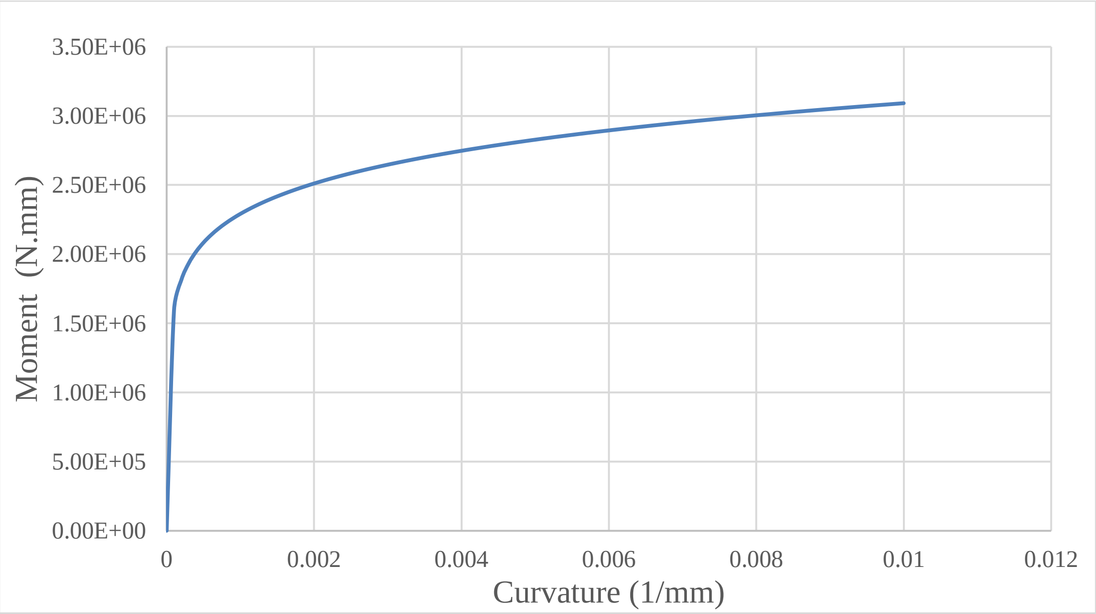

Figure 3.2 Moment-Curvature relationship of the section with the presented material.

In order to calculate subjected moment so that the residual curvature would appear at the measured value , the moment-curvature curve should be either expressed in closed form. $M(\Phi)$ is the function that represents the closed form, the expression to find the moment can be expressed as below.

$$M(\Phi) - EI\Phi_{artık} = 0 \tag{3.2}$$

This formula can be shown in the diagram as below , to find the residual curvature for a moment, or vice versa. Where 1st curve shows procedure to find the sujected moment to lead into residual curvature, 2nd curve shows the unloading where a moment leads to residual curvature.

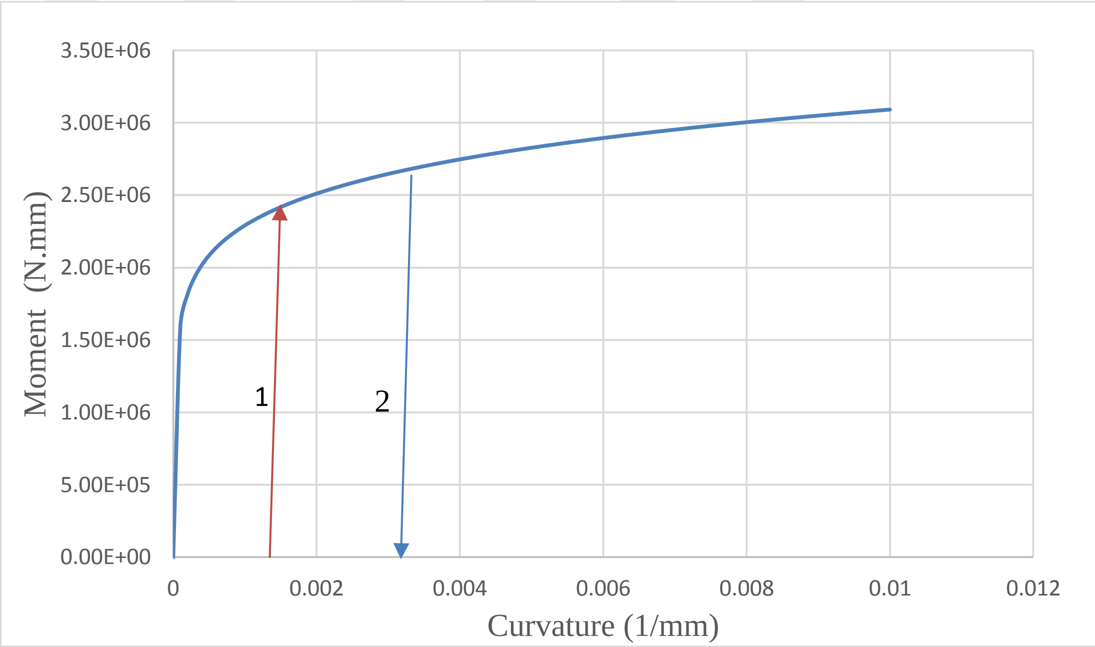

Figure 3.3 Rebounding process and calculation of the subjected moment.

3.3 Validation of Proposed Model

It is challenging to find out corresponding deflected curve or loading history from a permanently deflected curve with FEM. Therefore, moment (a_0) distribution of the rectangular bar for inelastic bending is going to define via proposed method and is going to implement to FEM. Results are going to compared after unloading process with FEM.

A numerical application for a rectangular bar is examined for validation of the method. Main dimensions of the bar are “10 mm X 10 mm” with a 1000 mm length and material is elastic perfectly plastic with 200000 N/mm^2 Elasticity modulus and 300 N/mm^2 yield stress. It is assumed that initial imperfection of the defection curve is in sine form. Result of plastic and residual deflections of the half length of the structure for a 0.1 *L* mid-span deflection are given in Figure 3.1, simulated from the proposed method. Simplified simulation is available in appendix.

Furthermore, comparison of the proposed method with the FEM is been presented in Table 3.1, difference of the proposed method is given in percentage of the error ,varies from the FEM results by 3% in the largest a_{max} achieved. And percentage of

Proposed δplastic/L	FEM $\delta_{plastic}$/L	Proposed $\delta_{residual}$/L	FEM $\delta_{residual}$/L	%Error	a_{max}
0.0453	0.0454	*0.002*	0.0020	0.4800	1.1758
0.0582	0.0583	*0.01*	0.0100	-0.2105	1.3150
0.0709	0.0706	*0.02*	0.0199	-0.2200	1.3796
0.0925	0.0925	*0.04*	0.0398	-0.3822	1.4339
0.1543	0.1508	*0.1*	0.0969	-3.0306	1.4780

error is seen to be varying with the a_{max} value.

Table 3.2 Comparison of FEM results with the proposed method, plastic deformation.

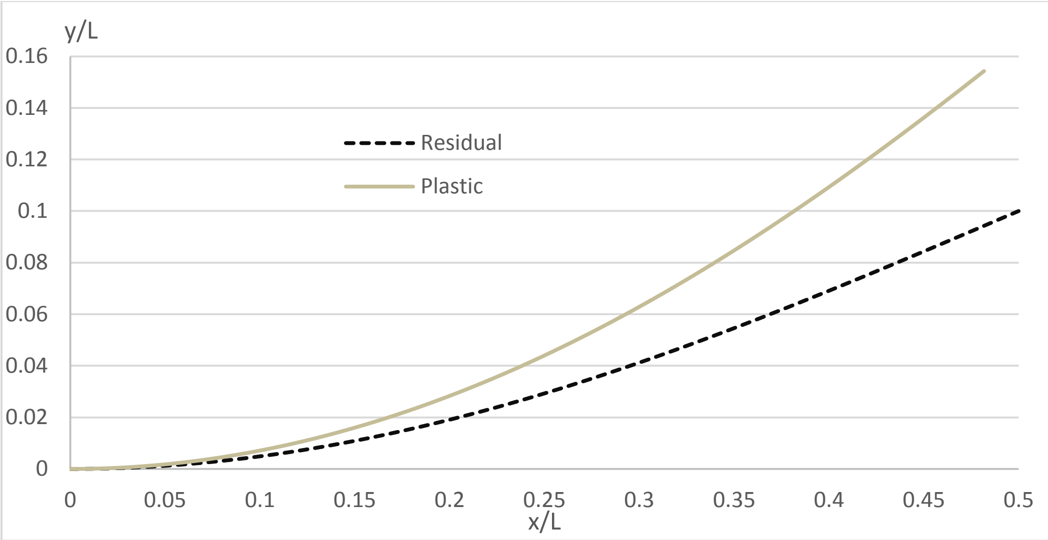

Figure 3.4 Comparison of plastic and residual deformation on the span.

4. CONCLUSION

Curve modelling method from curvatures is expressed for numerical calculation. Curvature modelling from deflection curve locations is introduced using geometric relations. Moment calculation formulas from curvatures of an elastic perfectly plastic rectangular bar is obtained, and formulations of curvatures for the given moment values of an elastic perfectly plastic rectangular bar is submitted from that formulas.

Validity of the proposed method is shown with FEM comparison. Remaining strength capacity of the structure is defined for the plastically deformed structures.

Calculations are separated into two parts; firstly the geometric calculations based on simple usage of curvature values of the structure, second is the moment curvature relationship. Moment curvature relationship is depending on the material model and equilibrium of internal and external forces over cross-section. Therefore, deflection calculation is just a simple numerical calculation of curvatures achieved from moment distribution of the system if moment curvature relationship is obtained initially. By the way proposed method has the advantage of fast response and may become an alternative for hull monitoring. It will be possible to find out loading history of an inelastic deformation if the fracture does not occur and residual stresses are available. Post buckling analysis for residual stress free initial deflected structures can be obtained just by addition of initial curvature to actuated curvature due to loading.

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APPENDICES

APPENDIX A: Simplified simulation of the beam model
APPENDIX B: Moment-Curvature calculator mathematica code

APPENDIX A

Calculation table for a 10 mm midspan residual deflection of rectangular bar described at chapter 3 for 10 segments is given below.

Table A.1. Initial curvature and loading history calculation .

X	Y	Θ	ΔΘ	$\phi_{residual}$	A
0	0.000	0.000			
50	1.231	0.049	0.0492	0.000984	1.4779
100	4.894	0.097	0.0478	0.000953	1.4769
150	10.899	0.142	0.0450	0.000893	1.4748
200	19.098	0.183	0.0410	0.000809	1.4712
250	***29.289***	***0.219***	***0.0361***	***0.000707***	***1.4658***
300	41.221	0.249	0.0303	0.000590	1.4574
350	54.601	0.273	0.0241	0.000465	1.4442
400	69.098	0.291	0.0174	0.000335	1.4224
450	84.357	0.301	0.0105	0.000202	1.3814
500	100.000	0.305	0.0035	0.000067	1.2787

Sub calculations of x = 250 is given below:

Following calculation are for the s where x = 250

$$\Delta y = 29.289 - 19.098 = 10.191 \text{ mm}$$

$$\Delta x = 50 \text{ mm}$$

From Eq. 2.10

$$\Delta\theta(250) = 2\left(\tan^{-1}\frac{10.191}{50} - 0.183\right) = 0.036054 \text{ rad}$$

$$\theta(250) = 0.183 + 0.036054 = 0.2191 \text{ rad}$$

From Eq. 2.11

$$\phi_{residual}(250) = \frac{2\sin\left(\frac{0.0361}{2}\right)}{\sqrt{50^2+10.191^2}} = 0.000707$$

and from Eq. 2.21.

$$0.000707 = -\frac{2\,\sigma_0}{E\,h}\left(\sqrt{\frac{2}{6-4\,a_0}} - a_0\right) \qquad (1<a\leq 1.5)$$

Where the root of the equation is: a =1.4658

Therefore, corresponding external moment subjected to the deflection at x = 250 is

$$M(a) = M_o\,a = \frac{\sigma_0 b h^2}{6}1.4658 = -73287.5 \text{ N mm}$$

It should be noticed that curve length can be submitted from eq.5 as follows:

$$\Delta s = \frac{\Delta\theta}{\phi_{residual}} = 0.036054/0.000707 = 51.0308 \text{ mm}$$

Table for deflection for the moment distribution above.

Table A.2. Deflection calculation for the estimated loadings

X	$\boldsymbol{\phi}$	**ΔΘ**	**Θ**	**ΔY**	**ΔX**
0			0		
50	0.00143	0.071	0.071	1.785	49.978
100	0.00140	0.070	0.141	5.324	49.845
150	0.00134	0.067	0.209	8.770	49.584
200	0.00125	*0.063*	*0.272*	*12.062*	*49.206*
250	**0.00115**	**0.058**	**0.331**	**15.143**	**48.725**
300	0.00103	0.053	0.383	17.961	48.160
350	0.00090	0.046	0.430	20.470	47.536
400	0.00076	0.040	0.470	22.628	46.881
450	0.00062	0.032	0.502	24.399	46.231
500	0.00045	0.02362	0.525	25.736	45.632

Curvature for the corresponding moment is calculated from Eq. 2.20 as follows:

$$\Phi_p = -\frac{2 \times 300}{200000 \times 10}\sqrt{\frac{2}{6 - 4 \times 1.4658}} = 0.001146$$

Δθ can be obtained from Eq. 2.5 as:

$$\Delta\theta = 0.001146 \times 51.0308 = 0.05849\, rad$$

And from the previous iteration, if θ at x=200 is equal to 0.272, θ for the curve where x=250 will be as follows:

$$\theta(250) = \theta(200) + \Delta\theta = 0.272 + 0.05849 = 0.3306\, rad$$

Finally Δx and Δy values can be obtained from Eq. 2.7 and Eq. 2.8, respectively as follows:

$$\Delta x = \frac{1}{0.001146}[\sin(0.3306) - \sin(0.272)] = 48.725\, mm$$

$$\Delta y = \frac{1}{0.001146}[\cos(0.272) - \cos(0.3306)] = 15.143\, mm$$

APPENDIX B

```
ElasticityModulus = 200000;
YieldStress = 350;
FlenchBreadth1[y_] = 100;
FlenchBreadth2[y_] = 100;
WebBreadth[y_] = 10;
Pinitial = 0;
PEnd = 200;
Neck1 = 10;
Neck2 = 190;
B[y_] = FlenchBreadth1 [y] HeavisideTheta[y - Pinitial] -
    (FlenchBreadth1[y] - WebBreadth[y]) HeavisideTheta[y - Neck1] +
    (FlenchBreadth2[y] - WebBreadth[y]) HeavisideTheta[y - Neck2] -
    FlenchBreadth2[y] HeavisideTheta[y - PEnd];
Efunc[y_] = ElasticityModulus y - HeavisideTheta[Abs[y] - YieldStress / ElasticityModulus]
    (ElasticityModulus y - 868.4 Abs[y]^0.128)
StrainFunc[ϕ_, y_] = y ϕ;
StressFunc[ ϕ_, y_] = Efunc[y ϕ] ;
AreaV = ∫_Pinitial^PEnd B[y] ⅆy;
AreaMoment = ∫_Pinitial^PEnd (y B[y]) ⅆy;
Centroid = AreaMoment / AreaV;
MomentV[ϕ_] = HoldForm[∫_Pinitial^PEnd B[y] StressFunc[ϕ , (y - Centroid)] ⅆy];
ReleaseHold[MomentV[0.01]];
Plot[B[y], {y, 0, 200 }]
Plot[Efunc[y], {y, 0, 0.2}]
```

Figure B.1 Mathematica code for calculation of moment.

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