How Does Behavior of Banks Affect Financial Instability? An Agent Based Modeling Approach

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Abstract

Structural causes of economic and financial crises has been discussed intensively among academicians, regulating institutions, political and social environments etc especially after the most recent global crisis. Some academicians argued that conventional ways of research in economics and finance were incapable of capturing the mechanisms that lead to the crises. It has been acceleratingly stated that, orthodox economic view that use terms, "representative agent", "rational behavior", "optimization", and "equilibrium" could not explain the aggregate economic behavior in a realistic manner.

We need some other methods to explain the mechanisms of complex interactions in economic and financial organisms. Recently, "heterogeneous interactive agents" and agent based models which take these concepts into account are more widely used to explain microeconomic foundations of macroeconomic behavior.

Another subject that was highly discussed after the crisis was banks' role on the formation of the crises. Most academicians argued that, by leveraging and supplying credits to firms irrationally and shortsightedly, banks triggered the formation of crises. Hyman Minsky's "Financial Instability Hypothesis" became popular again, after the crisis.

In our study we try to explain the effects of banks on financial stability, using a network theoretical agent based approach. We set a theoretical framework of banks and firms where agents interact on game theoretic rules. We later on simulate our model with the help of a C++ program.

Özet

Küresel ekonomik ve finansal kriz sonrasında, krize sebebiyet veren etmenler; akademik, iş, düzenleyici kurumlar, hükümet vb. gibi değişik çevrelerde yoğun olarak tartışılmaya başlandı. Bazı akademisyenler, geleneksel araştırma yöntemlerinin ekonomik ve finansal krizlere yol açan mekanizmaları kavramada yetersiz kaldığını dile getirdiler. "Rasyonel davranış", "optimizasyon", "temsili etmen", "denge" gibi kavramları kullanan ortodoks görüşün toplam ekonomik davranışı gerçekçi bir şekilde açıklayamayacağı savı giderek artan bir şekilde dile getirilmeye başlandı.

Ekonomik ve sosyal yapılardaki kompleks etkileşimlerin mekanizmalarını açıklamak için farklı yöntemlere ihtiyaç duymaktayız. Makroekonomik davranışların mikro ölçekli temellerini açıklamak amacıyla son zamanlarda, "heterojen etkileşimli etmenler" ve bunları dikkate alan "etmen temelli modeller" daha sıklıkla kullanılmaya başlanmıştır.

Krizden sonra yoğun olarak tartışılan bir başka konu da krizlerin oluşumunda bankaların rolüdür. Çok sayıda akademisyen, bankaların irrasyonel ve miyopik kredi arzı ve çok yüksek kaldıraç oranları ile krizlerin oluşumunu tetiklediğini öne sürmektedir. Bu bağlamda, Hyman Minsky'nin "Finansal İstikrar Hipotezi", krizden sonra bir kez daha popüler olmuştur.

Çalışmamızda, ağ teoretik, etmen temelli bir yaklaşım kullanarak bankaların, krizlerin oluşumundaki etkisini açıklamaya çalıştık. Bankalar ve firmaların oyun teorisi kurallarına göre etkileşim gösterdiği bir çerçeve oluşturduk. Daha sonra modelimizi bir C++ programı yardımıyla simüle ettik.

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This humble thesis may have a modest place in economics literature but it has a very important place in my life. The part of my life that I spent during the PhD programme and especially writing this thesis turned out to be a spiritual journey for me. Above all, this experience taught me that real wealth in life is family and real friends.

There are a number of people without whom this dissertation might not have been written, and to whom I am greatly indebted.

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My dear friend Kunter, you were with me and always supported me from the very beginning. You were not a teammate but a real friend. Your support was invaluable. Thank you.

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Finally, and most importantly, I would like to thank my beloved wife who has never stopped supporting and encouraging me even at most desperate times. The time I spent during the PhD program and while writing the thesis actually belonged to my son and wife. Their sacrifice in supporting me during my studies is unforgettable. Similarly, my mother, father, sister, mother-in-law, father-in-law, sister-in-law always encouraged me at times I was about to give up. I feel infinite gratitude for being a member of this family.

To my son, Hüseyin Eren

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Chapter 1

Introduction

"Why had nobody noticed that the credit crunch was on its way?"

This was the question of Queen Elizabeth II to the scholars, when she visited London School of Economics in November 2008. On 17 June 2009, a forum was convened by the British Academy to debate the question. In their letter to the Queen, in which they summarize the views of participants, they stated [1]:

Everyone seemed to be doing their own job properly on its own merit. And according to standard measures of success, they were often doing it well. The failure was to see how collectively this added up to a series of interconnected imbalances over which no single authority has jurisdiction. This, combined with the psychology of herding and the mantra of financial policy gurus, led to a dangerous recipe. Individual risks may rightly have been viewed as small, but the risk to the system as a whole was vast.

There is an important point in the answer of the academicians. They say that

[1] The Letter was signed by Prof. Tim Besley and Prof. Peter Hennessy and reflected opinions of 33 participants of the forum held by British Academy on 17 June 2009: "The Global Financial Crisis - Why Didn't Anybody Noticed?"

everything was seemingly going well on micro level but there where huge risks at macro level. Thus, they claim that the economic system as a whole is something different than the sum of its components.

Banks make a perfect example for the above statement. The economic crisis of 2008 was actually a financial -more specifically credit- crisis. At the micro level, banks behave rationally. For example in corporate finance, they assess -at least theoretically- credit risk of firms in detail with perfectly designed analytic methodologies. On the other hand, a macro assessment of banking behavior is rarely done and bank behavior becomes a good example for coordination failure in the long run. When one or more banks focus on some specific sectors -like construction or energy- and shift credits to those sectors, other banks follow. This in turn increase sectoral leverage in the long run and cause instability in those sectors. It is no surprise why textile is no more popular in Turkey among bankers, and how construction sector grew so rapidly during last decade and ended up with a bubble.

During the years following 2008 -year of global financial crisis- one of the most important concern in the world economy was European debt crisis where Greece was at the center of the debates. A graphical analysis [2] which shows countries' debts to banks in other countries (Figure 1.1), shows the seriousness of the problem.

The report illustrates the connected structure of banks and countries. It states that Greece owes 53.9 billion USD to French banks and 19.3 billion USD to German banks, whereas Italian and Spanish borrowers owe 366, and 118 billion USD to French banks respectively. Taking the debts of countries into account, it is claimed that in the case of default of Greece, investors will be more concerned about their exposure to other risks in the region. Next, it is predicted that borrowing costs of Italy, Ireland, Portugal and Spain will rise and their debt loans will increase. On the other hand, because of the risk aversion of investors, money will migrate to safer economies -like

[2] Holly Epstein Ojalvo, New York Times, 4 November 2011

Germany- in the Eurozone. Therefore, if the Spanish and Italian economies begin to falter, French banks will suffer, given their exposure. And, this will lead the losses beyond the continent.

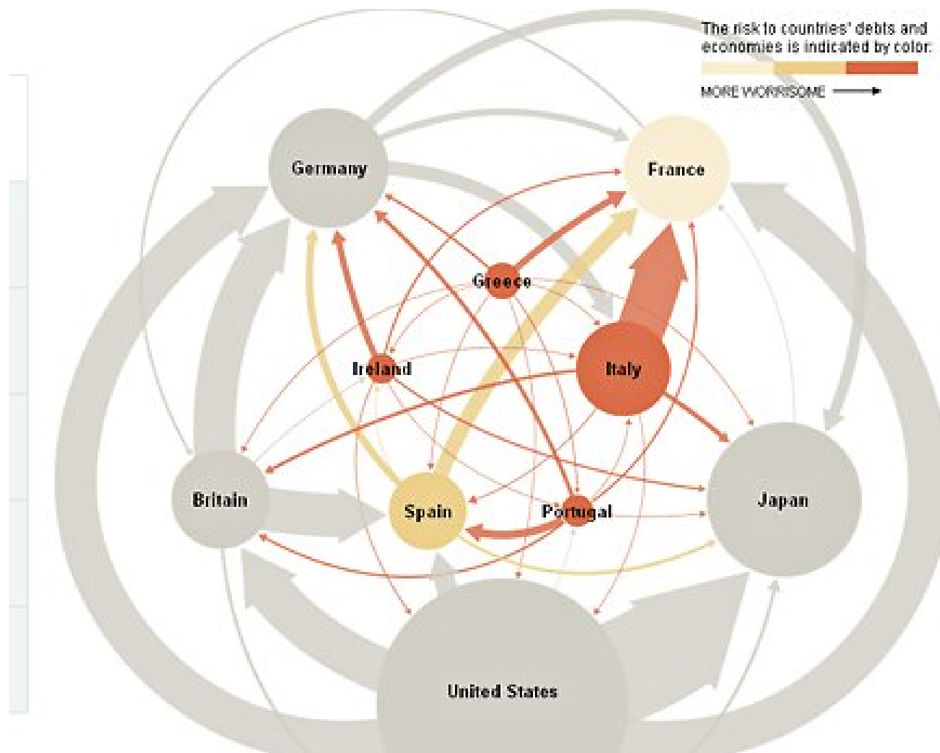


Figure 1.1: The European Debt Crisis, Visualized. By Holly Epstein Ojalvo, 4 November 2011, The New York Times

The graphical analysis of debt structure of countries shows us the importance of default risk of Greece because of the interconnected debt network. It can be observed on the graph that when an individual country defaults, this has 1^{st} and 2^{nd} degree interactions that will affect the world economic system. We can claim that in today's world, banks have the most important role in international economics and that's why they are at the center of debates about global economic crises.

An equally important source of risk exposure that is not mentioned in the analysis is credit derivatives, especially credit default swaps. If we take credit default swaps

-that has a total size of 60 trillion USD at the end of 2007- into account, 1^{st} and 2^{nd} degree interactions mentioned in the previous paragraph gets much more complex.

These two seemingly unrelated cases; the letter to the Queen and the analysis in New York Times, have in fact common points. If we return to the first case, the interesting point in the letter of scholars is the claim about the interconnected structure of economies. They claim that, although there was no problem in risk perception of individual agents, imbalances in economic networks led to the crises. Could this really be a reason that the crisis was unforeseen?

In mainstream economics, economic models are accepted to have either isolated or totally connected network structure and interactions between economic actors are either neglected or considered at minimal level [3]. But, explicit in the previous examples, this is often not the case in real life. The degree and structure of connectedness effect economics. For example in the economics literature, it is generally accepted that employed and unemployed people are part of an homogeneous structure. However, this area has not been researched enough. For example in Turkey, unemployment rate increased like in Europe. However, it has been discussed that, because of strong inter-family relationships, unemployed people suffered much less than those in Europe.

Furthermore, in mainstream economics it is accepted that agents make rational decisions. However, in heterodox economics, rationality is accepted as bounded or procedural. It is accepted that individuals and institutions face severe limitations in their ability to acquire and process information (Lavoie, 2009). Whatever the definition of rationality, what is rational and optimal for an individual may not be rational and optimal for the economic system. For example, in theory, it can be said that banks have different liability structures so they must have different optimal credit strategies. In practice, however, selling out credits (like sub-prime mortgages) to firms

[3]We will mention about this in detail in Chapter 4

or households who do not have suitable ratings may seem rational for bankers who rely on yearly profit or market share for their bonuses. In real life, therefore, after a certain time, all banks behave similarly, and banking behaviors show a diffusing structure. You can see a detailed discussion of critiques to mainstream economics in Chapter 4.

To sum up, we believe that, because of either their interlinkage structure or their links with other economic actors, banks are the most important actors in the economy. Therefore banking behavior needs to be examined in detail in order to understand the formation of economic crises. In addition, we believe that traditional models are inadequate to explain real life economic behavior. As discussed above, links and structure of connectedness are important in analyzing real life economics. As classical methods of economics have shortcomings in understanding these phenomena, a network theoretical approach can provide new insight into economics.

1.1 Problem Statement

Why does the behavior of banks matter so much? Of course behavior of all sectors, classes or institutions are important. As we have seen more dramatically since 2008, the behavior of banks is directly connected to other agents in the economy. If we use a common metaphor and think of banking sector as blood circulation system of a body, then we can see the importance of understanding behavior of banks in changing economic conditions.

However, we cannot be certain about the appropriateness of this metaphor. Perhaps in real life it is much more like endocrine system, and sometimes damage the health of the body by excreting hormones at an excessive level. After the start of global financial crisis in September 2008, and especially after the collapse of Lehman Brothers, the role of banks in the formation of the crisis was one of the most discussed issues. Many claimed that the banks' enormous desire for risk, and aggressive selling strategies were leading reasons for the crisis.

Although debates about banking increased over for the last few years, the subject is not a new one. Hyman Minsky, in his pioneering and illuminating book "Stabilizing an Unstable Economy" (1988) stated the problems within bank behavior clearly and methodically. He claims banking to be a disruptive force that tends to induce and amplify instability even as it is an essential factor if investment and economic growth are to be financed.

Minsky claims that the risks bankers carry are not objective probability phenomena, instead they are uncertainty relations that are subjectively valued (1988). He adds that, in 1970s and 1980s, the increase in the number and complexity of bank problems were largely the result of increased risk exposure of banks in an increasingly cyclical environment. Considering the boom in banking and finance after 80's, in today's world, the problem of increased risk exposure is greatly magnified compared to 70's.

About leverage, Minsky states that high leverage ratio of banks was part of the process that moved the economy toward financial fragility because it facilitated an increase in short-term borrowing (and in leverage) by bank customers: *"the leverage ratio of banks and the import of speculative and Ponzi financing in the economy are two sides of the same coin."*

Minsky also blames bank managers for inducing economic instability. Bank managers always try to maximize their personal fortune as holders of stock options. This leads them emphasize upon growth, which in turn leads to efforts to increase leverage. Increased leverage by banks and ordinary firms, however decreases the margins of safety and thus increases the potential for instability of the economy (Minsky, 1988).

Debate about banks role in the economy and society has not been settled yet. Banks are generally accused of reducing the wealth of societies for their own benefit. Many claim that banks allow an excessive amount of credit to firms and individuals, even when these do not have the ability to repay. On the other hand, banks contribute to economic stability by effective screening. Minsky, himself says that a bank's lending function has three facets: soliciting borrowers, structuring loans, and supervising borrowers.

Banks' effect on financial stability still needs to be investigated. As a matter of fact, banking behavior after initiation of crises is widely studied. On the other hand, we do not know banks' contributory role in formation of economic crises.

1.2 Hypothesis

We believe that calls for deregulation, liberalization, and financial flexibility in the banking system, leads to a speculative and fragile economy. In this study we intend to show the influence of bank behavior on economic trends with the help of credit networks. We believe that banks do not behave reactively but pro-actively i.e. banks' behavior affect economic and financial stability. We propose that banks increase financial and economic instability by excessive risk taking, leveraging, and myopic valuations.

1.3 Background

We can observe that bank behavior directly affects peoples' lives. In times of economic growth, consumption, and so living standards, increase with the help of bank credits. In contrast, some researchers claim that by cutting credit lines in times of recession, banks exacerbate the situation (Bernanke and Gertler, 1996). Indeed, before the most recent crisis banks encouraged firms to increase their leverages. After the crisis, most of the banks cut the credit lines which in turn caused many firms to bankrupt.

According to financial instability hypothesis of Hyman Minsky, households, firms and banks are willing to adopt more risky behavior and strategies in periods of economic boom, or after a long period of high growth (Lavoie,2009, Minsky 1988). Minsky states that, in such situations, banks ease their risk premia as well as their lending criteria, accepting higher debt loads. Agents will hold smaller proportions of liquid assets. This is called *the paradox of tranquility*, which means stability breeds instability. Furthermore, more speculative behavior will lead greater financial fragility. Firms and banks will compete against one another using debt as a lever; households will follow and this will cause stock market prices and real estate values to rise. Finally, the central bank will weigh in and impose credit constraints or raise the benchmark rate.

Higher debt loads and higher rates of interest will erode the fragility of the system, making it difficult to meet the interest payments on existing debt. At this point, banks will change their behavior by tightening their risk premia and lending criteria. This may further lead to a stock market crash unless government stands ready to support aggregate demand and the economy by engaging in large deficit spending (Lavoie, 2009).

As mentioned before, something that is rational and/or optimal for an agent or even for all of the agents may be neither optimal nor rational for the society as a whole. If we think of the credit markets, banks rely on financial tables of firms to access credit worthiness. They may look, as a secondary check, at the supplies or customers of those firms, but greater degrees of connections are never investigated. So, if for example, suppliers of main supplier of a firm are facing difficulties it is almost impossible for a bank to find this out in time with conventional crediting processes.

In economics, generally, aggregate trends in credits are observed but differences in the network structure between financial institutions, firms and households among economies, and the effects of this structure on the progress of the economy is rarely mentioned.

We believe that, a search for modeling the effects of the interaction of agents, especially banks, with other agents, on aggregate outcome is mandatory. Network theory will give us the chance of understanding the dynamics behind this. For example, in real life practice, all bank managers, regardless of the size of the bank, behave similarly. After some time, bank managers' behavior show diffusion for some reason, and they all behave in the same way. Besides the reasons mentioned in the previous section, given by Minsky, salaries and bonuses of bank managers are linked to their market share and sales success. It may however be the case that the size of banks affects their attitude to risk taking, and network theory is likely to be an appropriate way to examine this theory.

To sum up, analyzing the effects of banks' behavior on the economy may provide the opportunity to:

- understand banks' role in the economic trends and in formation of crises,
- gain insight into whether banking regulations or other political instruments may be used to stabilize economies.

1.4 Significance & Purpose

Share of financial sector in world economy increased acceleratingly in the last 30 years. According to Yeldan (2009), one of the most distinguishing features of the 20^{th} century wave of globalization is the ascendancy of finance over industry, and he defines financialization as a phenomenon which can be described by increasing financial motives, volume and impact of financial activities within and among countries. Yeldan states that financialization gave rise to an immense speculation activity, driven by massive capital flows led by myopic expectations. Indeed, Harvey says that *"Something significant has changed in the way capitalism has been working since about 1970"* (Harvey, 1989).

Similarly, Crotty claims: *while the sub-prime mortgage market triggered the crisis, its deep cause is to be found in the flawed institutions and practices of what is often referred to as the New Financial Architecture.* He defines New Financial Architecture as the integration of modern day financial firms and markets with its associated regime of light government regulations (2008).

Similarly, Minsky (1988) claims that there are no effective market barriers to bank expansion and thus to the destabilizing impact of banks upon demand. He also suggests that, to control the disruptive influence that emanates from banking, it is necessary to set limits upon permissible leverage ratios and to constrain the growth of bank equity to a rate that is compatible with noninflationary economic growth.

Consider the possibility that banks have some undiscovered effect on the formation of economic crises. To solve a problem we must first diagnose causes of the problem. So, the problem of determining the function of banking system is very important. If we can find that banks trigger or inhibit economic crises then we can propose methods to warn of economic crises and help to stabilize economies. If it is found, for example, that banks' behavior affect economic stability negatively, then we could propose regulations to offset that behavior. In the opposite case, if banks are actually inhibiting economic crises then we can focus on influencing those behaviors to further stabilize economies.

Our contribution to existing literature on banking behavior will be conceptualizing banking behavior and measuring the impact of it on the economic system, with the help of network theoretic approach that cannot be done with an actor based approach.

Our study will hopefully also enlighten the nature and magnitude of interdependencies and coordination issues in financial system. We aim to do this with the help of several disciplines like economics, mathematical modeling, network theory, complex systems, and graph theory. Even if the focus seems to be more generally on current financial crisis, we believe that banking behavior is not an issue of today but is a part of problems in capitalist economic system. Minsky and others proposed economic policy advices on banking and we believe that our study will contribute them by providing more targeted and specific solutions.

The purpose of this study in general is to investigate the effect of banks' behavior on economic crises. Most studies model banks' behavior after the beginning of the crisis. In contrast we aim to contribute to the literature by examining the behavior of banks before the crisis. In other words, we will try to find out whether banks have a role in the initiation of the crises. If we conclude that banks' contribute to the formation of the crises, we will attempt to propose economic policies to regulate banks that will help to stabilize economies. Our study has also some particular objective like:

- Generating the structure and investigate the properties of the credit-network structure of banking sector in a single economy and in the international financial system,

- Identifying the relationships between banks and other agents by analyzing the credit linkages,

- Simulate the results of different banking policies and analyze how economic behavior responds to different policies,

- To give an alternative explanation of current economic and financial crisis from a financial network perspective.

1.5 Analytical Diagram

In our study we will use an interdisciplinary approach and besides economics we will take advantage of disciplines, complex systems, computer programming, and network theory. Figure 1.2 gives the interdisciplinary structure of our study and the areas we have used in our study in a bottom-up design.

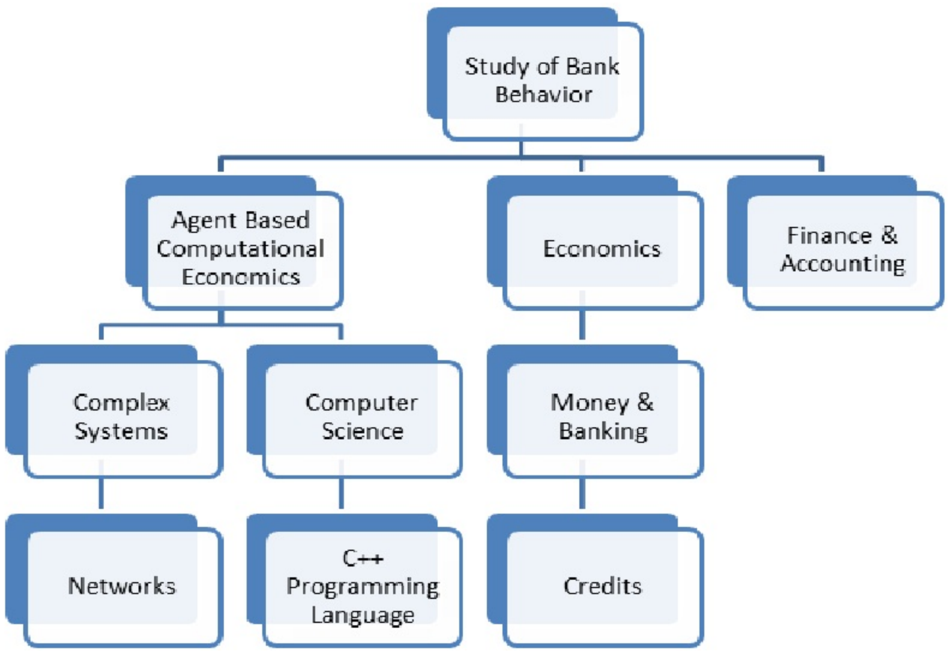


Figure 1.2: Analytical Diagram

1.6 Research Methodology

In academic studies, it is generally accepted that economic models include homogeneous populations. The banking sector is also treated like any other sector, and investigated with this methodology. But real life economics differs from literature. Do banks behave reactively or pro-actively against other economic agents? How does bank behavior differentiate in boom periods and recession times? Is there nonlinear growth of credit lines in boom periods? Is there diffusion among banks' behavior in the long term, and if so, how does this affect economic trends. These are important issues to address.

Banking is a sector where imperfect information has greatest impact. For example, as mentioned earlier, banks assess financial tables of firms that are at most second degree in their credit networks. At most they look at financial tables of customers and customers of customers. Because of this limited view, risk is not priced in a realistic way. This example shows how network links are important in banking behavior. We aim to study whether banks really increase systemic risk by following aggressive and myopic strategies. We believe that using a network approach best suits our purpose of the study.

Credit mechanism are generally ignored in economic studies until recently. As stated by Arestis and Mihailov (2009), the difficulty of modeling credit channels is that they require record keeping while money does not. Another difficulty is gathering data about linkages between banks and other agents.

One way to model credit networks is using balance sheet data. A bank's asset is the liability of a firm so we can set the network with the help of balance sheet data. One difficulty with this method is gathering the data. None of the firms or banks disclose details of their credit liabilities or assets. Another difficulty is that even if we can model a static network with the data available, a dynamic model does not seem to be possible with this method, as banks and firms only disclose financial tables every three months.

In our study we will model a theoretical network of banks, and firms. We will again use a balance sheet approach and make a Monte Carlo simulation of the interaction of the agents. We will start with a simple model of banks and firms in a closed economy. Our model will be composed of the steps in Figure 1.3 [4].

[4]Note that, our model will contain only banks and firms and thus will be simpler than described in the second step of Figure 1.3 However a complete model should contain all kinds of agents. Setting such a model is beyond the capacity of our study and planned as future work.

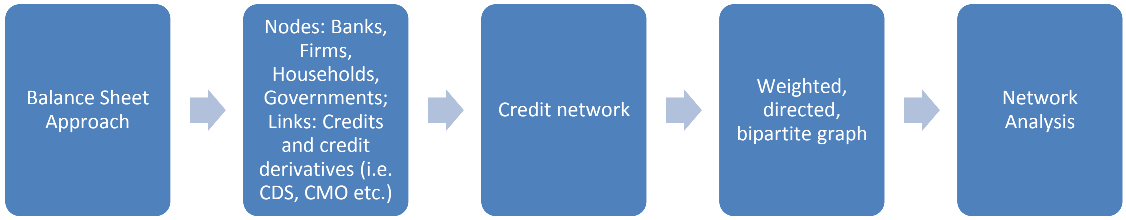


Figure 1.3: Research Steps

In our network model we will define the economic agents as nodes and credit lines between them, as links. So, the link between a bank and a firm will show the risk exposure of the bank on that firm. Credit lines will also be classified according to their maturities.

We can very simply visualize a credit network among firms and banks like the one in Figure 1.4. Blue nodes represent banks and red nodes represent firms. The direction of links shows the credit lines while difference in size of nodes and thickness of links gives variations in size of agents and size of credit lines respectively.

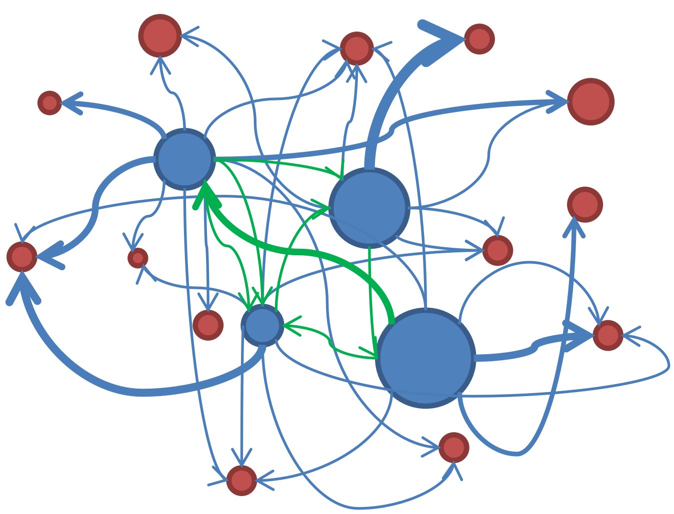


Figure 1.4: A Sample Network

Using agent based modeling methodology, we will be able to impose different scenarios like liquidity, interest rate or output shocks or credit cap regulations to observe the changes in behavior of banks and effects of these changes on the economy. This part is visualized in Figure 1.5.

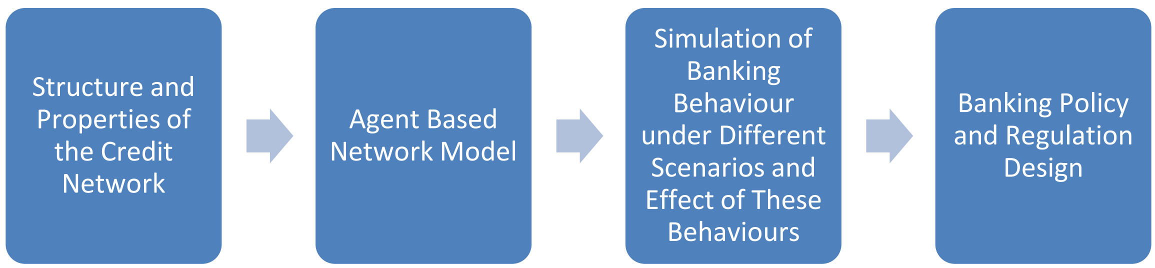


Figure 1.5: Scenario Testing

Chapter 2 will be about economic stability and theories of crises. In Chapter 3 we will try to explain role of banking in (capitalist) economies. In Chapter 4 we will introduce a relatively new approach in economics: Agent Based Modeling. In Chapter 5 we will go through theoretical background of Agent Based Modeling and give a brief discussion of graph theory and networks. Chapter 6 will discuss our base theoretical model. In Chapter 7 we will discuss the results of base scenario simulations and step further by introducing new scenarios. Chapter 8 will conclude our study.

Chapter 2

Economic Stability and Crises

"The essence of this-time-is-different syndrome is simple. It is rooted in the firmly held belief that financial crises are things that happen to other people in other countries at other times; crises do not happen to us, here and now. We are doing things better, we are smarter, we have learned from past mistakes. The old rules of valuation no longer apply" (Reinhart and Rogoff, This Time is Different, 2009).

In his book "Crisis Economics: A Crash Course in the Future of Finance", Nouriel Roubini claims that, mainstream economics aims to prove why and how markets –capitalist markets– work well. On the other hand, a great number of economists have been studying to prove just the opposite. This field –that can be called crisis economics– tries to explain how and why markets fail (2010). In this chapter we want to give some explanatory details on literature of economic crises.

2.1 Crisis Theories

The first economist who wrote on crisis, was John Stuart Mill. Mill believed that crisis were formed by some exogenous changes or shocks. Initially, an exogenous shock causes a bubble in some markets. After prices start to rise, increasing number of players start to speculate on prices, with profit expectation. This mostly increases prices in other goods and markets, which in term expands the bubble further.

With the formation of the bubble in some markets, players seem to make high profits which in turn cause credit conditions become flexible. Another bubble starts to grow in credit markets. When some firms collapse unexpectedly, confidence in the markets start to shrink which at the end abolish the optimistic atmosphere. A psychology that is just the opposite of the first one starts to dominate the market. Credits shrink, prices fall and bankruptcies increase.

Karl Marx was the first who claimed that capitalism itself, is prone to crises and furthermore crises were inseparable from capitalism. According to Marx, only source of capital accumulation is exploiting labor which provides the capitalist a surplus value. He claims that, the historical mission of the bourgeoisie is accumulation for accumulation's sake and production for production's sake (Harvey, 2006).

Capitalists always try to increase the surplus value they exploit. One of the ways to do this, is increasing efficiency of labor. And, one way to increase efficiency of labor is increasing use of machines in production. Capitalists want to increase machine usage both for cost minimization and increase labor efficiency. As capitalists replace labor with machines, this ironically decrease profits, because the share that goes to labor continuously decrease which in turn weaken the buying power of households. In a competitive economy prices decrease and this results a decrease in profits. Decrement in profits motivates the capitalist to cut costs further that finally leads to an economy with overproduction and underemployment. Theory of Marx on economic crises is very much broad to place here, and it is not at the core of our study. And there is a huge literature on Marx's views about crises and capitalism. Interested reader can refer to Marx's Capital as well as Harvey (2006, 2010), Foley (1986), Brenner (2008), Fine (2010), and Mandel (1974).

Irving Fisher, one of the earliest neoclassical economists, developed a model that is named as debt-deflation theory. According to Fisher, as a result of speculative transactions for future profits and/or low interest rates, level of indebtedness increase. This in turn leads a debt liquidation. Then money supply contracts as bank loans are paid off.

Later on the velocity of money decreases, which in turn leads to distress selling. As a result of distress selling, deflation occurs, because money supply is decreased due to paying off loans. The more people try to pay their debts, the more debt grows because real value of money increases. Simultaneously, bankruptcies increase. At the same time, profits also shrink. Total production decreases and this leads to a reduction in employment (Fisher, 1933).

Keynes, one of the most effective economists of 20^{th} century, put forward a new explanation in his book *"The General Theory of Employment, Interest, and Money"* (Keynes (1936)). Until Keynes, it was widely believed that markets were self regularizing. Economists commonly believed that markets will eventually reach equilibrium at full employment level.

Keynes, on the other hand, claimed that when unemployment increase, people will spend less and aggregate demand will decrease, leading a reduction in investment. Finally, wages will be cut and layoffs will increase, and demand will decrease further. Eventually, firms will be forced to cut prices in order to reduce stocks. Price deflation will cause decrease in profits again. According to Keynes, to end this paradoxical cycle government should intervene and increase demand. For more on Keynes, interested reader can refer to Minsky (1975), Skidelsky (2010), and Davidson (2009) besides his book.

We believe all the theories above are invaluable and have righteous points. On the other hand, we believe they miss the active role of banks and other financial institutions in formation of crises. Remember that Marx thought, infinite profit desire of capitalists was leading economies to crisis cycles. We believe that –with the increase of finance in the last few decades of the last century– bankers also have infinite profit desires that lead economies to instability and crisis. Actually, Minsky was the first who mention about bankers role in financial instability. Minsky was an influential academician who drew attention on systemic risk of financial systems in his *Financial Instability Hypothesis.* The theoretical background of our study is based on Minsky's views so we will mention about his views in a separate section (Section 2.3).

2.2 Types of Crises

Reinhart and Rogoff, classify crises as, *"crises defined by quantitative thresholds"*, and *"crises defined by events"*(2009). They sub-classify the first type of crises as inflation, currency crashes, and currency debasement crises. Event triggered crises are banking crises, external debt crises and domestic debt crises.

They define inflation crisis as chronic inflation periods with 40% or higher inflation rate; currency crashes as annual depreciation in excess of 15 percent. And finally they set two types for currency debasement crisis. First type is a 5% or higher reduction in metallic content of coins in circulation, where the second type is defined as a currency reform where the currency is very much depreciated and replaced by a new currency.

As example to event type crises, banking crises are classified in two kinds of events. The first type, systemic crises, is valid if bank runs that lead to closure, merge or takeover by the government are observed. The second type, financial distress, is defined by closure, merge, takeover or large-scale financial assistance by government even if no bank runs are observed.

External or domestic debt crises are defined as failure of government to meet a principal or interest payment on the due date or within the specified grace period.

Besides general crisis theories, there is also a broad literature on banking or financial crises. Studies of banking crises in the past included emerging markets or histories of advanced countries. That was so because it was believed that financial crises had stayed in the past for advanced economies (Reinhart and Rogoff, 2009). This idea was proven to be wrong in the financial turmoil of 2008. Reinhart and Rogoff claim that banking crises affect rich and poor economies similarly and lead to sharp declines in tax revenues almost every time.

Allen and Gale (2007), classify financial crisis theories in two. The first view, which is expanded by Kindleberger (1978), claims that crises occur spontaneously as a result of panics. The second explanation, puts a business cycle theory forward. According to this view, which goes back to Mitchell (1941), when an economy goes into a recession, returns on assets will diminish. As a result, banks will become insolvent because their liabilities are deposits or bonds. That insolvency eventually will trigger bank runs.

One type of banking crises is bank runs which occurs when confidence in banking sector depreciates largely. As confidence to banks disappears deposit holders would run to withdraw their savings (Reinhart and Rogoff, 2009). But banks collect deposits of short term and fund them into long term loans. This time inconsistency makes banks vulnerable to bank runs.

Reinhart and Rogoff –unlike our hypothesis– propose that banking crises do not trigger recessions but they amplify them. A decrease in output growth leads to default of credits which in turn effects banks' lending, that causes further decrease in output. That is called the financial accelerator mechanism (Bernanke and Gertler (1990), Delli Gatti et al. (2010b), Riccetti et al. (2011)) which we will mention in the following chapters.

One factor that is found to trigger banking crises is financial liberalization. Demirguc-Kunt and Detragiache (1998) and Kaminsky and Reinhart (1999) found that financial liberalization affected stability of banking negatively. Kaminsky and Reinhart, studied 26 banking crises in different countries and found that in 18 of those, finance sector had been liberalized in the preceding five years. We believe, products of financial engineering and innovation that surged in the last decade can also be seen as a financial liberalization.

Another factor that is claimed to affect banking crises is the increase in welfare with capital inflows. Reinhart and Reinhart (2008), analyzed welfare boom periods in countries, for 1960-2006, and found that probability of a banking crisis is higher in periods after welfare boom with capital inflows. They also found that countries with the most severe banking crises in 2008 global crisis were the ones that had chronic high current account deficits before the crisis. Similarly, Mendoza and Terrones (2008) found that most of the credit booms in countries end up with financial crises and credit booms were preceded by surges in capital inflows.

2.3 Financial Stability According to Minsky

Discussion in this section and in most part of Chapter 3 are from Hyman Minsky's studies (Minsky, 1982, 1988). We give a brief discussion here that is related to our study. Interested reader can refer to Minsky's studies for a much detailed and illuminating discussion.

According to Minsky, financial instability is strongly related with time inconsistency (1988). Financial and economic practices always include payment commitments, that is payments in financial and economic transaction are rarely done up front. A credit itself is actually a payment commitment.

When a commercial or financial contract is composed, the conditions are set according to the current economic environment and expectations. And expectations are always short sighted. But the economy dynamically changes through time and when due date comes there are totally different economic conditions than the contract date. In Minsky's words: *"Financing of economic activities result in a residue of financial commitments."*

In a capitalist economy, firms always make investment to grow, increase output and profits. Investment is rarely done with just internal sources. Every firm uses external funds, like credits, to finance investment. And those are obtained with some commitments and contracts. Thus while increasing output, and production frontier, investment also changes income distribution and financial structure. When a firm decides to invest it gets credit for the investment and buys the capital. Next we will see a firm with a balance sheet that has capital on assets side and credits (or other finance instruments) on the liability side. So, investment changes the liability structure that finances the assets.

Investment also changes financial relations and payment commitments. In other words, market conditions continuously change from the time of financial contracts, to the cash payment date of the contract, and this causes fluctuations in the economy. That's why cash flows are very important on financial stability.

Assume that the ratio of capital used in production, relative cost of capital, and payback period of investment, increases in time. It becomes more difficult for firms to repay financial debts with cash earned from production. Instead, they obtain new debt to pay the old debts. This is one of the reasons of fluctuations in economies.

We said that cash flows are very important for the financial stability of the economy. Minsky stresses that, to analyze how commitments affect the economy, it is necessary to look at economic units in terms of their cash flows. To have a stable economy, banks and firms must be liquid and solvent. A bank's liquidity and solvency can be obtained by its health of commercial loans. On the other side, a firm's liquidity and solvency means its debt obligations will be done by realized and expected cash flows from sales. A firm must repay its credit debt with the cash obtained with business sales. Specifically, a credit used for an investment project must be repaid with cash created by sales realized from that investment.

Minsky classifies cash flows as income, balance sheet and portfolio type cash flows. Income cash flows are wages, salaries, and payments from production and trade. The second type, balance sheet cash flows, are the interest and principal payments of credit or generally debt obligations. And finally, portfolio cash flows occur when capital or financial assets are sold.

One of the reasons of financial instability, according to Minsky, is financial innovation. Banks, firms, and even households always seek new ways of financial activities. When a new innovation is successful, its developer makes profits and other players follow him.

Although Minsky says that every party seeks for financial innovation, our view is that banks are the impulsive force in this progress. The process is usually like that: one bank offers a new product to its customers which at first can face with some resistance. After a while, if buyers or users of that product or service get -which are almost always speculative- profits, other banks also offer the same product, or other firms demand this product from their banks. There is generally no patent constraint in financial markets. All kinds of financial derivatives is a good example. Other examples are in a very wide range from direct debits systems, to salary payment contracts, and back-office operation services. Actually most of the products or services that seem to be financial innovation are kind of price discrimination which can be researched in another study.

We have seen that there are three kinds of cash flows. And the distribution of these determines the economy's endurance because looking at the weight of kinds of cash payments we can guess how the debt payment obligations are fulfilled. An economy's financial health increases with the ratio of income cash flows. On the other hand, if the portfolio transactions are wide, then this economy is fragile because firms use capital or financial asset incomes to pay their debt commitments.

Now comes the question: how debt obligations are fulfilled? Or in other words, how firms and banks in an economy finance their assets? Minsky, classifies these into three: hedge, speculative, and Ponzi finance[1]. As we noted in the previous paragraph, financial stability is strictly attached to distribution of kinds of cash flows. Hedge financing occurs when a firm fulfills all of its debt payments with realized income cash flows.

In most cases, a firm cannot meet its debt payments just with income cash flows because of time inconsistency. If the firm is rolling over credits to pay debt, this is called speculative finance. As mentioned before, if a firm is issuing new debt or selling assets to pay current debts, this is called Ponzi finance. The difference between hedge finance and speculative-Ponzi finance is that hedge financing units do not engage in portfolio transactions to meet their debt obligations.

Bankers and firms, always try to guess whether a project's cash inflows will meet the debt obligations of the project. In an ideal business world every project pays itself. So, a hedge financing firm do not need large volume of debt. And, this is not actually what bankers want because, hedge financing limits the amount of credit, bankers can sell. If a firm is only hedge financing, only changes in prices and market conditions affects the firm. It will not be affected by the changes in financial markets.

The second type of financing occurs when short term debt is used to fund long term positions. When a firm's cash inflows within a period are not adequate to fulfill the debt payments firm rolls over credits or issues new debt. Actually this is the same with Ponzi finance. Difference between speculative and Ponzi finance is that in speculative finance, cash inflows are greater than principal repayments. On the other

[1]Although the term Ponzi finance is named after a notorious early 20^{th} century fraud, Charles Ponzi, it is not necessarily used for fraudulent events as we will see shortly. Generally, Ponzi finance is used for fulfilling payments on a financial contract by refinancing or selling assets.

hand, in Ponzi finance, operating cash flows to the firm are not even adequate to meet principal part of the credit payments. Speculative financing may be either rolling over or issuing new credit to pay the old debt. Besides, changes in market conditions as prices, speculative units are also vulnerable to changes in financial conditions, like interest rate increases.

Why firms engage in speculative finance? If we think of the payback period of an investment or a project, cash payback is slow at the beginning. While the investment matures they get greater. Ideally, firms use credits that is compatible with the cash flow structure of the project. But, not all projects have definite payback structures. For example, construction firms invest generally with low ratio of internal funds and use credit to start the project, planning to pay back credits with cash from sales. If, market conditions change and sales do not realize as fast as the firm plans, then firm issues new debt to pay maturing credits. The motivation of both the firm and the bank is the belief that, firm's cash inflow will increase in the future. Another reason that banks use speculative -and Ponzi- finance is that they believe, firms' debts will be refinanced or will be met by sale of assets at worst.

Ponzi finance is very much similar to speculative finance, in the manner that both of them use new credit or portfolio sales to payback old credit. As, we told above one difference between them is amount of debt relative to principal part of credit payments. Another difference is that in speculative finance short term cash flows are high enough not to increase financing costs. On the other side, finance costs of Ponzi firms, are greater than income. Thus, whenever a firm engages in Ponzi finance, its total amount of debt increases with cost of finance.

Ponzi financing has not necessarily be related with fraud. If a firm's cash inflow is not adequate to repay its credit principal payments and it is borrowing to pay for credit repayments the firm is Ponzi financing. If interest rates increase or income of a speculative firm decrease, the firm can turn to be a Ponzi financing firm. Opposite progress can cause a Ponzi firm to become a speculative one. Actually, restructuring existing debts can be interpreted as transforming Ponzi firms to speculative and speculative firms to hedge firms, by decreasing debt burden of the firm in the short term.

In addition to principal repayments, a Ponzi firm also uses new credit to pay its interest payments. So, its equity/debt ratio is smaller relative to a speculative firm, other things equal. Bankers care about firms' ability to pay interests and principal with cash inflow. If a firm's cash inflow is not enough to pay for interest payments, bankers make the credit conditions more strict. This, further increases interest costs which increase the probability of bankruptcy.

According to Minsky, the mixture of hedge, speculative, and Ponzi finance in an economy is a major determinant of its stability. An increase in speculative or Ponzi finance also increase the instability of the economy. Firms make financial contracts in which they commit to fulfill payments in the future. Between the time of settlement of contract and deadline of payment, financial conditions change. This is the main reason, that causes a firm's realized financing mode to be different that desired mode.

All three kinds of financing are vulnerable to economic changes. This means, hedge, speculative, and Ponzi finance units are all affected by economic changes like price or sales declines, or aggregate demand shocks. Besides those, speculative and Ponzi kind firms are vulnerable to changes in interest rates. When interest rates increase, cash outflow of speculative and Ponzi finance firms also increase. This happens if either the interest rate of revolving or overnight credits increase or firms finance long term positions with short term debts. Thus they have to obtain new debt on the deadline of the credit. When interest rates increase, speculative and Ponzi financing firms are negatively affected. On the aggregate, if the weight of speculative and Ponzi finance increases in the economy, the financial systems becomes more fragile.

If the situation is such simple, why economic agents do not act in favor of hedge financing economies? Let's think of an economy dominated by hedge financing. As short term interest rates will be lower than long term interest rates, this will allow for speculative profits. Actually, bankers are the leaders for intrusion of speculative arrangements. This triggers an increase in asset prices and firms start to think that they will gain profits by using credit and investing in assets. Mortgage market is a good example for this. Most people use mortgage credit and buy houses with the belief that house prices will continue to increase and their capital gain will be higher than the interest cost. This mechanism in turn triggers an asset bubble. This environment, where capital gains are earned by financial arrangements leads firms to engage in speculative and Ponzi financing because firms believe that their long term capital gains will be higher than short term finance costs.

This part that is explained by Minsky is especially important because it is one of the core reasons why bankers and firms engage in speculative and Ponzi financing arrangements. We want to make some additional notes on it. When the weight of the hedge finance dominates the economy, bankers are optimistic. They make credit conditions easier and firms find it profitable to finance their capital assets or investments with low-rate short term credits. The important point is that, rate of short term credit rates are so low that firms find it profitable to obtain short term debt. And another psychological motivation for this is firms' and bankers' short sighted vision. The length of vision of bankers and firms are limited by the balance sheet term. And, if we use the term introduced by Reinhart and Rogoff (2009), agents always think that *this time is different* and there will not be a crisis again. And this makes introduction of speculative-Ponzi financing arrangements easier. In such an environment, firms widely use short term -mostly revolving liquid- credits and finance their capital assets.

Success breeds a disregard of the possibility of failure; the absence of serious financial difficulties over a substantial period leads to the development of a euphoric economy in which increasing short-term financing of long positions becomes a normal way of life (Minsky (1988)). As, we mentioned above, while the time span since the last crisis passes by, economic agents, policy makers and even academicians start to become optimistic. The logical reason for that may be anything. This time is different because internet made everything fast and everyone can reach to information; this time is different because derivatives allowed firms to hedge their risks and anything else. Even if some warning signals are observed, those are not seen or heard. Speculative-Ponzi financing increases, asset bubble enlarges and economy becomes more fragile.

According to Minsky, it is conventionally believed that, demand for money is related to the level of income and money is valuable because it obviates the need for a double coincidence of wants for transactions to take place. On the other hand, capital assets can be financed by different combinations of debt and equity. Besides income related reasons, there is also a demand for money to make payments for financial commitments. As cash flows from an investment or project are not certain, speculative financing increases banks' and firms' need to hold liquid assets for precautionary purposes.

When liquidity increases, a profit opportunity is created. In Minsky's words: *"Any set of financial instruments or market organizations that can finance positions in capital assets and can offer a good measure of the protection offered by money holdings can borrow low and lend high: that is, can make on the carry"*. This creates a hierarchy of liquid assets and leads firms, to use low-rate credits and finance liquid assets. For example, a dealer may borrow from a finance company to hold its inventory of automobiles, the finance company may use commercial paper to borrow from an insurance company, which has outstanding commitments on take out mortgages, and so forth (Minsky, 1988).

While the asset bubble continues to enlarge, firms demand more funds as due date of projects get closer. At some point, some event causes the asset bubble to crash. The reason may be anything that make people believe that asset bubble will not go long. At that point, the elasticity of supply of funds also decrease because the volume of financing available from commercial banks slow, and interest rates increase sharply. The best example is Asian financial crisis in 1997. The increase in interest rates reduce net worth of assets and profits on the carry trade. Then people start to sell assets in a panic to make their lost at minimum that in turn leads to a crisis. As average human brain can remember only short term past, after a crisis, bankers and firms seem to get their lessons from the crisis. They start to take hedge finance positions.

Chapter 3

Role of Banking in Economy

"What is breaking into a bank compared with founding a bank?" Bertolt Brecht

In modern economics there are lenders who have excess money and borrowers who need money for their expenses or investments. Borrowers need secure long-term lending, and lenders want to get their money back whenever they want. We can define a bank as an intermediary between the desires of borrowers and lenders (Krugman, 2009).

Krugman (2009)starts the history of modern banks with goldsmiths that accept peoples' coins to keep safely as their shops had secure vaults [1]. At some point, goldsmiths discovered that they could lend these coins in return for interest. In case, deposit holders could withdraw their money, goldsmiths started to keep a fraction of the money in their vaults and lent the rest.

In earlier banking practices commercial bank loans were supposed to be self-liquidating: proceeds of a loan would be used to finance the acquisition of a specific stock of goods, and the sale of these goods would repay the debt. This meant that the cash flow to fulfill the contractual commitment was clearly visible when the loan was made, in the sense that the completion of well-defined transactions would furnish

[1] We have evidence that modern banking started much before. For example Shull (2010) mentions about banks in 10th century in Abbasid Caliphate

the means for payment. As explained in Chapter 2, Minsky calls this kind of lending as hedge financing.

Before going through function of banks we will explain capital financing theories shortly to understand the theories of capital financing motives of firms.

3.1 Capital Financing Theories

We will use dynamic trade-off financing theory in our model, but it will be useful to explain all theories of capital financing. There are three main theories of capital financing. The first is Modigliani-Miller Theorem (Modigliani and Miller: 1958, 1961, 1963), which is accepted to be the beginning of the theory of business finance (Frank and Goyal, 2005).

There are four propositions in Modigliani-Miller Theorem, first of which says that under certain conditions, firms' debt-equity ratio does not affect their market value. The second proposition, which is more related with our study, claims that a firm's leverage has no effect on its weighted average cost of capital. Next proposition is about market value and says that it is independent from the firm's dividend policy. And the final proposition claims that equity holders are indifferent about firm's dividend policy.

The second theory of corporate finance is the pecking order theory. Myers (1984), claims that a firm prefers internal financing to external, and if it uses external finance, prefers debt to equity. In other words, retained earnings are preferred to debt and debt is preferred to equity.

One motivation for pecking order theory is adverse selection. The idea behind adverse selection is that only owners and managers of a firm can know the firm's real value. Outside investor –bankers in our research– can only guess these values (Myers, 1984; Myers and Majluf, 1984). Agency theory implies that an outside funding requires managers to explain firm, project to investors and to be open to outside due diligence. Managers, generally prefer internal financing instead of this.

The last theory of corporate finance is trade-off theory. According to trade-off theory, firms evaluate the trade-off between costs and benefits of various leverage plans. Benefit of increasing leverage is paying less corporate tax, while cost is increasing probability of bankruptcy. According to Kraus and Litzenberger (1973), optimal leverage level is determined according to the trade-off between tax benefits of debt and deadweight cost of bankruptcy. If this trade–off is evaluated for a single period, the firm is said to follow a static trade-off. In this model, investors are risk neutral and face a tax on wealth from bonds at the end of periods. As the investor is risk neutral, he prefers the security with higher after–tax profit. Static trade–off theories assume that there are no transaction costs of issuing or purchasing securities.

Dynamic trade-off theory that we will use in our model, states that firms have a target leverage but a firm's realized leverage may differ than target leverage. If deviations from target leverage are removed gradually over periods, the firm is said to show a target adjustment or dynamic trade-off behavior (1973). In this model, financing decision of the firm depends on financing margin that the firm anticipates in the next period. In other words dynamic trade-off theory states that firms let their leverage fluctuate within an optimal range.

A recent study by Danis et al. (2012) claims that profitable firms want higher leverage to shield substantial cash flows from taxes. They claim that dynamic trade-off is relevant to a substantial fraction of the large sample of firms. Dudley (2007), claims that profitability and interest rates imply a narrower debt ratio range, while higher volatility implies a wider debt ratio range. He also states that, more profitable firms will increase their leverage ratios by more when their debt ratio reaches the lower extremity of their ratio range.

3.2 Banks' Functions

Banks' role as a component in the financial system, and their role in the economy is indispensable. Even strongest opponents of banks accept that a financial and economic system without banks is inconceivable. Banks' main function is intermediation between borrowers and lenders. Thus, Banks' primary functions are accepting deposits and granting credits. They also have other secondary functions as agencies. This means function like, transfer of funds, collection of cheques, periodic payments on behalf of customers. Banks also involve in brokerage and speculative investment actions.

3.2.1 Banks as Lenders

Firms almost always make use of external financing for their investments. Banks are the institutions that maintain this external financing for the firms. They are also business making entities that aim to maximize their profits. So, they have to believe that their lending will be totally paid back with accrued interest on time in the future. Stiglitz and Greenwald (2003) define banks' function of lending as in the following paragraph:

"Given its equity, the bank must decide how much to lend, how thoroughly to screen loan applicants, how much to retain in government T-bills, how many funds to acquire through deposits, what interest rate to charge on loans, and what interest to pay on deposits. There are other decisions: how much to spend on monitoring loans, how much of the portfolio to devote to real estate, and within real estate, to, say, commercial properties. For each borrower, the bank must decide on the size of the loan that it is willing to provide, the non-price terms (such as collateral), and what restrictions to impose on other borrowing by the firms to which it lends."

As we will explain shortly, firms will pay back their debt in one of three ways: cash flows created with operational activities, revolving debt, and selling some assets or increasing debt.

In the neoclassical economic theory, it is believed that banking has no effect on economic progress because banks' effects are totally repressed by money supply, and interest rate changes that is controlled by Central Bank. In this view, banking is mechanic, static and has a passive structure (Minsky, 1988).

Composition of bank portfolios is not important in neoclassical view. But in reality banking is a dynamic innovative process. Bankers are like other businessman who look for more profits. According to Minsky, bank activities affect cyclical behavior of prices, incomes and employment besides volume and distribution of finance. In addition, money supply is not controlled by Central Banks but it is an endogenously determined variable, where supply is responsive to demand.

Banking is not a business of lending, because banks' do not owe the money they lend (Minsky, 1988). Banks' business is to supply funds to economic agents who spend more than their income: households who want to spend their future income, or firms those want to invest. Banks get those funds from other agents who spend less than their income and save. Bank business is intermediating effectively between these two groups.

Two points are important for bankers and the society: banks lend others' money, and banks are profit maximizing entities. So, bankers must be sure about the credibility of a customer they lend. In other words, banks must be sure that the credit will be paid on time. This point gives us the core business of banking [2].

Core business of banking is borrowing (accepting deposit) from some parties and granting credit to others. Bankers actively look for increasing their profits and equity by charging fee for their services and putting a spread between the interest rates they

[2] If we think of non-cash credits for example, a bank guarantees to make specified payments if its customer fails to pay. So, bank must examine and believe in credit worthiness of its customer.

pay and get. In other words, banks make profit by earning more money on assets than they pay for liabilities.

Balance sheets and income tables are very important for a research on microeconomic foundations of macroeconomics. First, they keep the data of financing types. Second, balance sheet approach is very much appropriate for a network model like ours. Although at first sight it seems very complicated, balance sheet of a bank is actually simple. On liabilities side, there are deposits (demand deposits or time deposits), and funds from other financial institutions. Those may be syndication / securitization credits, or interbank debts[3]. And needless to say, there is equity on liabilities side.

On the assets side, there are various kinds of money and loans. Loans are any kind of credits to households, firms, and governments. Banks can also invest in financial markets by purchasing financial assets to use their excess funds.

When a bank invests in financial markets, like buying treasury bonds, acceptances, or securities, this does not include a customer relationship. On the other hand, when a bank lends to a firm, there is an already started customer relationship process.

In Minsky's words, a banks lending function has three dimensions: *soliciting borrowers, structuring loans, and supervising borrowers.* So banker's first duty is to analyze the credibility of the borrower diligently and to be sure that, the money will be paid back.

Specifically the money will be paid back in one of three ways: cash from sales, refinancing debt, and selling assets or increasing debt. Actually, structuring debt repayments is very important for both the banker and the firm to make repayments. Because, time inconsistencies in credit structure may cause a hedge firm to transform a speculative firm and a speculative one to a Ponzi.

[3] There are also some non-cash liabilities like acceptances, letters of credits etc.

Minsky states that, physical productivity of capital assets is not important for finance sector. What finance sector evaluates is profit yielding capacity. So, the market price of a capital asset depends upon current expectations of future profits and the way expected profits are transformed into a present value. In his words:

"To Wall Street the technical capacity of a Boeing 747 to deliver seat-miles is of secondary importance; what is important is the ability of an organization in a particular market and economic situation to operate 747s profitability. Similarly, whether nuclear power plants produce electricity, damage the environment, or are safe is not important from a Wall Street perspective; what is vital is the calculation of expected costs and revenues."

There are three ways for the borrowers to pay their credit back: cash flow from operations, refinancing or rolling over debts, and selling assets or increasing debt. As mentioned in the previous chapter, those represent hedge, speculative and Ponzi financing respectively. The cash flow structure of the credit must be designed so that, proceeds from the investment that is funded must be enough to fulfill the credit paybacks, which corresponds to hedge financing. Assume a very simple commercial loan where, a stock of goods is funded, and cash from sales of the stock are used for credit payments. This is an example to hedge financing model where debt is paid with cash from operations.

In some cases, cash flow from operations is not enough to fulfill the credit repayments. Or it may be operationally difficult to determine which cash flow belongs to which part of production. For example, assume a production company that produces and sells goods continuously. There are always batches of goods that are being dispatched. In that kind of production, firm needs external funding, most of the time. As a solution, a general limit for credit is determined and firm uses credits up to that limit. Risk of the firm varies within that limit throughout the period.

In this kind of financing, bank cares about profitability of the firm instead of a single project, while determining credibility. And, this leads to speculative financing. Speculative financing, urge the firm to refinance its debt to fulfill existing obligations. Bankers grant this kind of credit because they believe that long term profits will be enough to cover credit repayments. So, while participating in such financing, bankers always take the probability of refinancing into consideration. As, a matter of fact refinancing is an important part of banking process which secures higher profits in the short run.

When a bank and a firm get into a credit relationship, it is not always profitability of the project or the firm that maintains the credit to be granted. Actually, banks mostly demand collateral for granting credits when they see risk. Although in banking practices it is widely accepted that it's not the collateral that gets the credit in many cases collateral structure is analyzed for the credit decision. When a firm's cash inflows are not enough to fulfill debt obligations and firm cannot revolve credits, last choice to repay credits is sale of assets. And this is the last kind of paying credits back.

Assume a firm that goes through a construction project. Cash flows from sales of that project may be very uncertain and may span a long period. In this case it may be difficult for bankers to fully introduce credit paybacks at the right time with cash inflows. And sales may go slower than planned. In that case, cash flow from sales may be lower than enough to pay even interest payment. Just revolving the credit is not enough so, to fulfill its obligations, firm must get new credit or sell some fixed assets. As we explained in the previous chapter this process leads the firms to Ponzi kind of financing.

Using information up to now, we can say that repayment structure of credits that are set by bankers are very important for the stability of the economy. If the majority of credits are planned so that credit payments are done with cash flows from operations the economy converges to a more robust structure. On the other side, if bankers mainly consider expected value of collaterals while granting credit lines, and engage in speculative and Ponzi finance, financial structure becomes fragile against shocks.

In practice, banks' liabilities are short termed and assets are long termed. Banks want to protect themselves from that time inconsistency which left many banks unprotected against bank runs in the past. Thus, it is in bankers favor to decrease average term of credits. Banks make short term credits more attractive and firms engage in those credits. As a result, weight of speculative finance in the economy increase.

According to Minsky, if a bank increases its leverage with a nondecreasing ratio of "profits / total assets", banks' profit goes up. This causes all the banks increase their credit supplies so fast that, prices of capital assets, investment output, and consumption goods also rise. As a result, households and firms increase their short term borrowing which in turn, increases financial fragility. In Minsky's words: *the leverage ratio of banks and the import of speculative and Ponzi financing in the economy are two sides of a coin.*

Minsky also attribute importance to behavior of bank managers. A bank's profitability leads an increase in its share prices. And this increase is also very important for the professional managers of the bank because they are holders of stock options and bank shares. All the top managers of banks are trying to increase their personal wealth as everybody. And fastest way of this is with the help of increasing share prices. So, bank managers will attempt to increase leverage, which will increase earnings per share that in turn increase share price. This motivation for managers to increase leverage, on the other hand, will make the economy more fragile.

Although we think that Minsky is right to a point we do not think that bank share prices is not the only reason for the bank managers to increase leverage. In the high-level competitive structure of capitalist economies, all the banks are trying to increase their profitability and market shares. The question that which one is more important is the subject of another study but all bank managers from top to down are evaluated in a yearly even not quarterly basis for their performance. Performance criteria of bankers are market share and profitability in varying weights. This leads bank managers to be short sighted in their analyses and decisions. Best example to this phenomenon is sub-prime mortgage crisis in 2008. Another example may be credit card sales in Turkey in recent years.

All of the banks have risk management departments that claim to measure risks of the bank and value at risk significantly by analyzing the composition of the financial tables of the bank. On the other hand, as the last crisis showed, risk that bankers carry cannot be objectively measured, and they are subjectively valued. Bankers generally decide the amount of risk on any unit, branch, department, or firm with the help of their experience. And this experience reflects the near past.

According to Minsky, there are no effective market barriers to bank expansion and to the destabilizing impact of banks upon demand. Minsky, claims that to control the disruptive influence that emanates from banking, it is necessary to set limits upon permissible leverage ratios and to constrain the growth of bank equity to a rate that is compatible with noninflationary economic growth (Minsky, 1988). Once more, we experienced how this is true in the last financial crisis. Most of the risk in the crisis emanated from derivatives those are mostly produced by banks to beware leverage limits.

Chapter 4

Agent Based Modeling

At the beginning of the Introduction Chapter we have mentioned about the question of the Queen of England to the academicians and their respective answer. We are giving three sentences from the answer again:

- *Everyone seemed to be doing their own job properly on its own merit. And according to standard measures of success, they were often doing it well.*
- *The failure was to see how collectively this added up to a series of interconnected imbalances over which no single authority has juridiction...*
- *Individual risks may rightly have been viewed as small, but the risk to the system as a whole was vast.*

The important point in these sentences is that, they admit that, they were inadequate to model aggregate behavior with the help of individual behavior because the whole system is something different than the sum of its components. Therefore, is there a way to reach aggregate behavior of economics with the help of individual behavior and if so, how?

4.1 Critiques to Mainstream Approach

The dominating framework throughout 20^{th} century was Walrasian equilibrium model. Walrasian equilibrium is a precisely formulated set of conditions under which feasible allocations of goods and services can be price-supported in an economic system organized on the basis of decentralized markets with private ownership of productive resources (Tesfatsion and Judd, 2006).

In this framework, there is a finite number of price-taking profit-maximizing firms who produce goods and services of known type and quality, a finite number of consumers with exogenously determined preferences who maximizes their utility of consumption taking prices and dividend payments as given, and a Walrasian Auctioneer (or equivalent clearinghouse construct) that determines prices to ensure each market clears (Tesfatsion and Judd, 2006).

As Tesfatsion and Judd note, Walrasian equilibrium answers the question whether efficient allocations can be supported through decentralized market prices. On the other hand, it does not address, and was not meant to address, how production, pricing, and trade actually take place in real-world economies (Tesfatsion and Judd, 2006).

After the most recent financial turmoil, it is acceleratingly discussed that mainstream economic models were unsuccessful in setting an effective paradigm for macroeconomic behavior. Many academicians have been arguing that simplifying and generalistic terms in mainstream methodology like *representative agent, equilibrium, rational behavior* were unrealistic to explain real life economics.

Actually, the matter of relation and dynamics from micro to macro is the subject of other sciences like physics, biology, sociology etc. And similar to other sciences, a "reductionist approach" has been used in economics, especially in mainstream economics.

The theoretical background of the reductionist approach in mainstream economics goes back to Lucas (1976). Lucas claimed that, when generating government policy variables, traditional models, did not correctly take into account the dependence of private agent behavior on government policy rules. So, they could misguide the policy makers about the effectiveness of their policy rule decisions. In other words, while aggregate macroeconomic models up to that time capture correlations between macroeconomic aggregates, they could not capture the causal structure that generated them. And this deficiency made macroeconomic models useless for policy analysis because they did not allow conditional prediction.

Therefore, Lucas proposed achieving aggregate behavior by first defining microeconomic units and attributing identical parameters to them. This way of thinking lead the economists model the individuals (household, or firm) as homogeneous agents that have similar preferences and rationality. The logic behind this, is assuming that individual reflects the general expectations and beliefs of the society. Because of this, homogeneous individual was called representative agent.

It was believed that, analyzing the behavior of representative agent and then summing up all the individuals would give the aggregate behavior of the economy. This proposition lead to a reductionist approach in mainstream economics that is just summing up the market outcomes of individuals. But, the problem with this approach is that, there are no assumptions on isolated individuals to reach the aggregate behavior (Kirman and Hildenbrand, 1988). And this problem is usually avoided by assuming that economy behaves like an individual.

Delli Gatti et al. (2010a)), claims that Lucas critique is wrong in three ways. First, they find the Lucas critique theoretically empty, because the subject of assessing a given model is structural or not is an empirical question. Second, in representative agent approach individual preferences are not affected by policy changes, and this does not reflect the real practice. Finally, in representative agent models, aggregation is done with the assumption that individuals have homothetic and identical preferences which is again, unrealistic.

As noted by Delli Gatti et al. (2010a), to be able to use reductionist approach, we must know that there is linear interaction between individuals. In their words: *"In terms of dynamical system theory, this means, that the eigenvalues of the whole (high level system) are linear combinations of the eigenvalues of the parts (low-level systems)."* It is obvious that in real world cases where there is information asymmetry, this is not true.

In mainstream economics, representative approach framework, where the macroeconomic system can be visualized as the behavior of the average individual became standard. On the other hand, this approach ignores communication and interaction among individuals and coordination problems. Interaction among agents are only through price mechanism which can be true if only there is complete information (Delli Gatti et al., 2007).

Another point that interaction between agents are ignored in mainstream economics is perception of equilibrium. In mainstream economics a system is in equilibrium if all the parts composing the system are in equilibrium. But as noted by biologist Stuart Kaufman: *"An organism in equilibrium is already dead"*.

As claimed by Delli Gatti et al. (2007), developments in quantum physics in the 20^{th} century caused changes in our understanding of equilibrium also. With the help of these developments, a new holistic approach other than reductionist approach is accepted. According to this approach, aggregate is different from the sum of its components because of the interactions between them.

If we think a physical system, the particles (atoms, sub-atomic particles etc.) belonging the system are always dynamic and interacting with each other but the whole system is stable. Similarly, instead of equilibrium it is better to mention about stability in economics. There are continuous interactions between individuals of a system

–in opposite directions– and the whole system is still stable. So, the equilibrium of a system does not require all of its particles are in equilibrium but rather, stability or statistical equilibrium. Statistical equilibrium is defined as the statistical distribution describing the aggregate phenomena being stable or in other words: *"state of the macroscopic equilibrium maintained by a large number of transitions in opposite directions"* (Feller, 1957).

Another critique directed to mainstream economics is rational behavior. In mainstream framework, agents are accepted to behave rationally, with having full information set about the market. Rational expectations theory assume that individuals, having all the information set, optimize their behavior. For example they maximize their lifelong utilization, or income. Second, it is assumed that a rational agent does not make systematic forecasting errors. In reality it is neither possible nor feasible to spend time to get optimal quantity of information. In addition, even if information set is full, decisions are made according to some expectations in the future, and results in the future in turn depends on actions taken today. In reality no one can exactly know the optimal solution. So, individuals follow conventions, customs, rules of thumb, or imitate actions taken by neighbors, or the ones whose behavior they imagine to be better informed (Lavoie, 2009).

Stiglitz and Gallegati (2011), claim that economic theory based on representative agent methodology cannot explain financial crises, bankruptcies, domino's effect, systemic risk, and any pathology in general. According to them, representative agent paradigm does not allow to understand the interplay between the micro and the macro levels and the coordination failure. They add that:

"We might argue that the RA model is partly to blame for the crisis, for in those models, there is no such thing as systemic risk; policy makers, comforted by the notion that they were following 'best practices' of the most advanced monetary theories in taming inflation, assuring the stability of the economy, paid no attention to the far

more important issues of financial structure. In the straightjacket of this methodology, it is hardly surprising that the standard macro framework is without any help in understanding the current events."

Finally, we will cite from Gaffeo et al. (2008), for a summary of critiques to general equilibrium models:

- the conventional general equilibrium theory has difficulties in finding a role for monetary exchange,
- the equilibrium is neither unique nor locally stable under general conditions,
- the introduction of a representative agent is done without paying any attention to composition and aggregation fallacies,
- any tâtonnement process occurs in a meta-time, and implies that the formation of prices precedes the process of exchange, instead of being the result of it.

4.2 From Representative Agents to Heterogeneous Interacting Agents

To summarize the previous section, we can say that the problems with representative agent framework are:

- Agents are homogeneous and all have the same preferences those do not vary with time,
- All agents have access to full set of information in the market and there is no information asymmetry,
- Agents are isolated and they do not interact with each other.

But the case for reality is just the opposite of the above. In real markets, players do not have full access to all set of information. There are information asymmetries. Players have different preferences those evolve in time. And most importantly, agents observe each other and learn from each other. For example, if an individual wants to buy a computer or a mobile phone, he will most probably observe his friends, or ask someone who, he believes, knows technological products well. As another example, Zweig (2008) claims that, when an individual increases volume of a stock in his portfolio by 10%, people in their neighbourhood increase volume of the same stock in their portfolio by 2%, on the average.

In reality, markets are believed to show much complexities than assumed by mainstream models. A complex system is defined as Flake (1998), a system that is composed of interacting units and shows emergent properties. In a complex system, properties arising from the interactions of the units are not the properties of individual units themselves.

Economic and financial markets are not only complex but also adaptive that evolve in time. Tesfatsion and Judd (2006), gives three definition of a complex adaptive system which we will quote correspondingly:

- A complex adaptive system is a complex system that includes reactive units, i.e., units capable of exhibiting systematically different attributes in reaction to changed environmental conditions.

- A complex adaptive system is a complex system that includes goal-directed units, i.e., units that are reactive and that direct at least some of their reactions towards the achievement of built-in (or evolved) goals.

- A complex adaptive system is a complex system that includes planner units, i.e., units that are goal-directed and that attempt to exert some degree of control over their environment to facilitate achievement of these goals.

Representative agent model is also inadequate for the purpose of our study. In this framework it is impossible to model lenders, borrower, and credit links between them. Therefore we need another framework that allows interaction at an agent based level, to study credit markets. In the last decade a new term, which we think is more appropriate for our model, *Heterogeneous Interacting Agents*, is proposed instead of representative agents.

We believe, accepting that agents have differences in their preferences, ability to reach information, and linkages with others will give us better grounded microeconomic foundations for macroeconomic behavior. Indeed, a group of academicians –Mauro Gallegati, Domenico Delli Gatti, Joseph Stiglitz, Bruce Greenwald, Alberto Russo, Edoardo Gaffeo, and Stefano Battiston– showed in a series of papers that a framework which models the interconnected structure between banks, households, and firm allow to better understand systemic risk, bankruptcies, and domino effects in the financial sector, and their effects on output (Delli Gatti et al., 2005, 2007, 2010a, 2010b; Gallegati and Palestrini, 2010; Battiston et al., 2011).

Another important attribute of heterogeneous interacting agents is, as noted in the term itself, the agents' interacting with each other. An individual's position within the society, the nature of games and the level of information shape individual behavior. Furthermore, a player's well-being is related with both its own and neighbors' decision (Galeotti et al., 2010). So, a model of heterogeneous interacting agents must include relations and links between the agents.

4.3 A New Paradigm: Agent Based Models

We have seen that using heterogeneous interacting agents is more advantageous than representative agents. But what will be the methodology of modeling heterogeneous interacting agents? We have noted that heterogeneous interacting agents have different and evolving preferences, and they observe and learn from each other. Whatsoever, relations between players are also important as we will see. It is impossible to model and solve this kind of a framework with the help of some optimization techniques. We believe that actually there is not a state of pure equilibrium in real economics. Thus solving some minimization and maximization problems is not enough for our purpose.

To develop solid micro-founded models, we have to develop a methodology that take into account the interactions of economic agents and their links in a networked economy (Stiglitz and Gallegati, 2011). Agent Based Modeling is the methodology of analyzing heterogeneous interacting agent framework, based on simple rules of behavior and interaction. An agent based model (ABM) is a computer simulation of interaction between many heterogeneous interacting agents. In this methodology, we define a dynamic interaction rule for the agents. Heterogeneity, behavior and interaction rules generate a real world like complexity.

Agent based models are open, dynamic, non-linear systems far from equilibrium. According to Gallegati, they have an evolutionary process of differentiation, selection, and amplification which provides the system with novelty and is responsible for its growth in order and complexity. In agent based modeling, we do not know the resulting aggregate dynamics and empirical regularities a priori, but instead observe the statistical distribution. Actually, in agent based modeling, the modeler does not try to find an equilibrium state but instead observes and watches whether an equilibrium states emerges over time. In an agent based model, at any time, each agent behaves according to its current situation, behavior rules, conditions of its neighbors, and according to rules of interaction.

In the agent-based methodology, computational simulations may imitate working of either an isolated market, or an entire multi-market economy. Units in the simulation are microeconomic agents those are typically firms, workers, consumers, financial intermediaries and so on. In agent based modeling, equilibrium is not a matter of consideration. We let the market behave according to the natural rules of local actions of interacting participants (Delli Gatti et al., 2010a).

An agent in an agent based model refers broadly to bundled data and behavioral methods representing an entity constituting part of a computationally constructed world (Tesfatsion and Judd, 2006). Thus, agents may be, individuals like consumers and workers, social groups like firms and classes, or institutions like markets. Agents may even be biological entities like crops, livestock and forest or physical entities like regions, or infrastructure (Tesfatsion and Judd, 2006). An agent may also be composed of other agents as a firm includes workers and managers, or a credit market includes banks and firms.

The history of developments in agent based modeling does not go very old and is parallel to the improvements in subareas of computational sciences, like computer-based modeling and machine learning. To our knowledge the oldest study of agent based modeling belongs to Holland and Miller (1991), in which they call their model *artificial adaptive agent model*. They capture attention in this relatively early study to the inadequacy of standard economic tools to answer questions about the way in which economic agents make choices when confronted by a perpetually novel and evolving world. They conclude that even limited agent based models can give huge understanding of decentralized, adaptive and emergent systems. In another study, Arthur et al. (1997), asserted that any economy that is composed of millions of individuals, may and should be described as a complex, adaptive, dynamic system.

Studies in agent based modeling accelerated in 2000's and this sped up especially after the global financial crisis at 2008. Although it is already an infant area, agent based modeling promises to maintain rich insight about dynamics of interaction among agents and microeconomic foundations of macroeconomic behavior. Gaffeo et al. (2008), claim that even a very simple agent-based computational laboratory can challenge more structured Dynamic Stochastic General Equilibrium models in mimicking comovements over the business cycle. Delli Gatti et al. (2008) claims that agent-based models can easily outperform traditional ones in explaining a wide range of disparate aggregate phenomena such as fluctuating growth, financial contagion, bankruptcy chains, firms' sizes and growth rates distributions, and much more by means of a unifying framework.

Interest to agent based models is also growing recently, especially after the financial turmoil of 2008. Farmer and Foley (2009), propose:

Agent-based models potentially present a way to model the financial economy as a complex system, as Keynes attempted to do, while taking human adaptation and learning into account, as Lucas advocated. Such models allow for the creation of a kind of virtual universe, in which many players can act in complex –and realistic– ways. In some other areas of science, such as epidemiology or traffic control, agent based models already help policy-making.

Agent based models are simulation programs that model artificial economies. As noted before, agents (or objects in computer programming language) may be any kind of economic player like household, firm, bank, central bank, or government. We first define attributes that characterize these entities: behavior, learning, and interaction rules for the agents, and an environment where the player interact (Gaffeo et al., 2008). We attribute behavior rules so that they are parallel to what we observe in real life economics. This can be obtained by real life observation, controlled social and economic experiments or survey data. Note that in mainstream economics, behavioral rules are described axiomatically and then market outcomes are derived.

After defining behavior and interaction rules, and writing proper simulation program, the progress of the artificial economy is observed which maintains an empirical and normative conceptualization. According to Gaffeo et al. (2008), compatibility of artificial and real historical data demonstrates how aggregate structures of interest for macroeconomists -such as business cycles, price inflation, or underemployment of resources- are effectively attainable starting from a given microstructure. We think that, trying to achieve a one to one correspondence between artificial and real time data is neither feasible nor necessary. Instead, we have to look for a correspondence between statistical distribution and reaction of the system to a change in some parameters or variables, or idiosyncrasies.

Despite its advantages, ABM is said to have some disadvantages relative to previous models. Tesfatsion and Judd (2006), claims that in agent based modeling the modeler must construct a dynamically complete design, which means that the model must permit and fully support the playing out of agent interactions over time without intervention of the modeler. And this requires intensive experimentation over a wide range of plausible initial specifications.

Second disadvantage is said to be the difficulty of validating results of an agent based model with empirical data. Some researchers claim that a meaningful empirical validation for agent based models is not possible. According to Tesfatsion and Judd (2006), agent based models generate outcome distributions for theoretical economic systems with explicitly articulated microfoundations. These outcome distributions generally have multiple equilibria. On the other hand, real world is a single time-series realization arising from a poorly understood data generating process. In their words:

Even if an agent-based computational economic model were to accurately embody this real-world data generating process, it might be impossible to verify this accuracy using standard statistical procedures. For example, an empirically observed outcome might be a low-probability event lying in a relatively small peak of the outcome distribution for this true data-generating process, or in a thin tail of this distribution.

On the other hand there are various agent based models that try to empirically validate their results. Fagiolo et al. (2007) is a very good and detailed discussion on problems and different methodologies on empirical validation of agent based modelling [1]. According to them, as there is no consensus among agent-based modelers on the techniques to construct and analyze models, empirical validation methodologies are so different. Agent based models, they claim, show differences in four key dimensions: nature of the objects under study, the goal of the analysis, the modelling assumptions, and the method of sensitivity analysis that is used.

For more information on agent based modeling, interested reader can refer to Axelrod (1997a; 1997b), Bonabeau (2002), Bratley et al. (1987), Epstein and Axtell (1996), Gilbert and Troitzsch (2005), and Tesfatsion and Judd (2006) for far more detailed discussions of agent based modeling.

As we have mentioned before, agent based modeling fits quite well for understanding financial markets. Studies in this area sped up especially in the last decade. Gallegati et al. (2003), studied one of the earliest agent based model in financial markets. They modeled a network of banks and firms and found that, small idiosyncratic shocks can generate large scale aggregate fluctuations. In a similar study, Delli Gatti et al. (2005), conclude that their model results with a skewed distribution of firms' size and a Laplace distribution in the rate of change of output.

Agliari et al. (2006), model production and investment behavior of financially constrained firms in an uncertain environment with capital market imperfections, building upon Greenwald and Stiglitz (1993). They keep track of the evolution of the first two moments of the distribution of agents according to the degree of financial fragility. They found multiple steady states that show different dynamical properties depending upon the chosen configuration of parameters.

Battiston et al. (2011) model a network of only production firms those are linked by -inside- trade credits. They claim that with the help of an agent based model with realistic interaction rules, main factors of financial fragility may be investigated. In

[1] Interested reader can refer to Fagiolo et al. (2007) for detailed discussion on empirical validation of agent based models.

addition, agent based modeling opens the way to a new class of models for endogenous business fluctuations based on interactions and credit relationships. Those factors are claimed to be:

- Role of credit relationships in generation of bankruptcies,
- Role of interest rate and policies to prevent large number of avalanches,
- Role of network structure and interactions,
- Policies to make more robust financial structure against avalanches.

Delli Gatti et al. (2009; 2010b), model a credit network which includes, banks, downstream firms that produce consumption goods and upstream firms that produce investment goods. In their model, there are inside credit links between downstream and upstream firms and outside credit links between two kind of firms and banks. They found that, a business cycle at the macroeconomic level can develop as a consequence of the agents involved. In addition they conclude that, bankruptcy of one agent can cause bankruptcy of one or more other agents in a snowball effect. The size of this effect depends on the network structure and the incidence of nonperforming loans on balance sheets of agents involved.

Stiglitz and Gallegati (2011), develop a credit network including households, firms, and banks. Agents are connected by inside or outside credit. Inside credit is credit linkages between firms that belong to different layers of the same industry, while outside credit means linkage between banks and firm. They claim that, heterogeneous agent approach provides an alternative, one which has already proven its metal in helping to understand the interlinkages which helped give rise to the crisis.

To sum up, we can say that including credit linkages in economic models via agent based modeling may provide us invaluable insight about banks' role in economic trends and specifically in financial stability that is the core of our study.

Chapter 5

Network Theory

As it is noted in the previous chapter, relations between individuals in an interconnected system is also important for aggregate results. An agent's well-being is also related with its behavior and decisions, besides its place in the society and behavior of neighbors. Thus, an agent's links with others are important for aggregate outcomes. Agent based modeling is closely related with and uses theoretical framework of network theory which itself is derived from graph theory. In this chapter we will give a brief introduction of graph and network theories and their applications on economics. Actually there are very good books about network theory which we have also referred for this chapter. Interested reader can refer to Newman (2010) for a general introduction to and applications of network theory in various disciplines, Easley and Kleinberg (2010)) for an introduction of network theory in economics, and Jackson (2010) and Goyal (2009) for more advanced study of network theory in economics.

5.1 Graph Theory

Roots of network theory lies at graph theory from mathematics. We will give a very brief introduction to graph theory here. Interested readers can refer to the books by

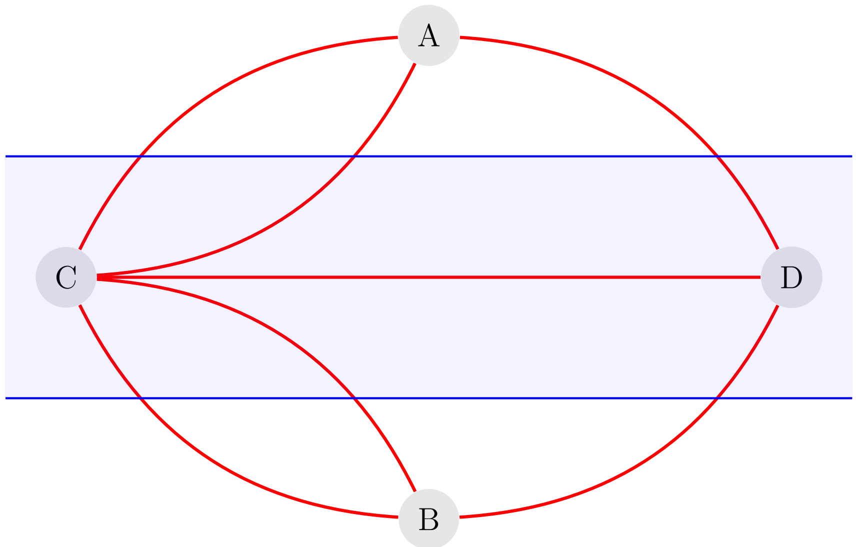
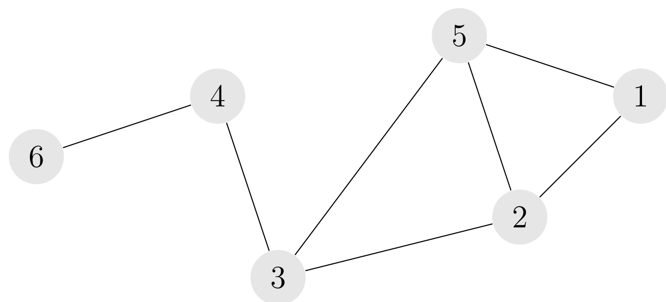


Figure 5.1: Köninsberg Seven Bridge Problem

Bondy and Murty (2008), Diestel (2010), Diestel (2010) for excellent discussions of the theory.

Graph theory goes back to Mathematician Leonard Euler. Euler was the one who succeeded to solve the seven bridges problem of Köninsberg (Kaliningrad/Russia today). Köninsberg is divided by a large river and two islands on it. The islands are connected with a bridge. One of them is connected with two bridges to each side of the river, and the other is connected with one bridge to each side. It makes total of seven bridges. The problem was whether it was possible to walk the city by passing all of the bridges once and only once (Caldarelli, 2007). Euler simplified the map of the city to a graph as in Figure 5.1. He later proved that there is no path that passes each bridge once and only once. Note that, A and B represent two sides of the river, while C and D represent two islands, and the links between them are the bridges.

A graph G is an ordered pair (V, E) where V is a set of vertices, and E of edges such that $E \subseteq [V]^2$, i.e. the elements of E are 2-element subsets of V. An incidence function ψ_G associates each edge of G and an unordered pair of vertices. If e is an edge and u and v are vertices such that $\psi_G(e) = \{u, v\}$, then e is said to join u and v, and the vertices are called the ends of e (Bondy and Murty, 2008). In network theory, *node* is also commonly used instead of vertex. Below is a simple representation of a graph in which, $V = \{1, 2, 3, 4, 5, 6\}$ and $E = \{(1, 2), (1, 5), (2, 3), (2, 5), (3, 4), (3, 5), (4, 6)\}$.



For two graphs $A = \{V_A, E_A\}$ and $B = \{V_B, E_B\}$, $A \cup B := (V_A \cup V_B, E_A \cup E_B)$ and $A \cap B := (V_A \cap V_B, E_A \cap E_B)$. If $A \cap B = \emptyset$ then A and B are disjoint. If $V_A \subseteq V_B$ and $E_A \subseteq E_B$, then A is a subgraph of B, i.e. $A \subseteq B$. And B is called a super-graph of A. A *spanning subgraph* is the one that has all the vertices of the super-graph.

There are various types of graphs that are important in graph theory. In a *complete graph*, all pair of vertices are linked by an edge. The opposite of *complete graph* is *empty graph* which has no edges. A *bipartite graph* has two subsets of vertices A and B, such that all edges have one end from A and one end from B. Bipartite graphs are especially important for our study because our model consists of a bipartite network composed of banks and firms. If every vertex in A is connected with every vertex in B then this is a complete bipartite graph. A complete bipartite graph with one vertex in A or B is called a star.

If there is an edge between vertices a, and b then they are called adjacent, or neighbors i.e. ab is an edge of G. A graph is a *connected graph* if for every partition of the vertices into two subsets, say A and B, there is at least one edge that has one end in A and one in B. In other words if we can connect any two vertices of a graph with edges then it is a connected graph. The *neighborhood* of a vertex is the set of vertices that are linked to it. So, $N_i(d) = \{j : d_{ij} = 1\}$.

In mathematical computations and computer programs a network is denoted with incidence or adjacency matrices. For the graph $G = \{V, E\}$, where $N_V = n$ and $N_E = m$, the incidence matrix $I_G := (a_{ve})$, gives the number of times, vertex v and edge e are incident. For the same graph, adjacency matrix $A_G = (b_{uv})$, gives how many edges are there between vertices u and v. In practice, adjacency matrix is more convenient and easy to use than incidence matrix, because generally a graph includes more edges than vertices.

Another important term for a graph is vertex degrees. The degree of a vertex v, $d(v)$, is the number of edges those are incident with v. A vertex with zero degree is called an isolated vertex. Average degree of a graph is twice the number of edges in the graph:

$$\sum_{v \in V} d(v) = 2m$$

A *walk* in a graph is a sequence $W := v_0 e_1 v_1 ... v_{l-1} e_l v_l$, whose terms are alternatively vertices and edges (not necessarily distinct), such that v_{i-1} and v_i are the ends of e_i. A *path* is a walk where all the vertices are distinct. The *length* of a path is the number of edges on it. It is generally more convenient to denote a path with just its vertices. So we can denote path P as $v_0 v_1 ... v_k$. Two paths that have no common vertices except the beginning and ending ones are called vertex independent paths.

The distance between two vertices, $d(x, y)$, is the length of the shortest path between them. Maximum distance between two vertices in a graph is called its *diameter*. A vertex is central if its greatest distance from any other vertex is as small as possible. This distance is the radius of the graph.

For $P = v_0 v_1 ... v_l$, $C := P + v_l v_0$ is a *cycle*. Minimum length of a cycle in a graph is called *girth* and the maximum length of a cycle is called the graph's *circumference*. A graph is with no cycle is an acyclic graph. A connected acyclic graph is called a *tree*. In a tree there is only one path between any two vertices. By definition, each component of an acyclic graph is a tree, so an acyclic graph can also be named as a forest.

Type of graphs we discussed above are not always adequate to model real life problems where there is a direction of relationship. One of two web sites may have a link to the other, while the other has not. Or, a researcher may cite another while the opposite is untrue. With such networks we have to use directed networks. A directed graph (digraph) $D = \{V, A\}$, consists of the vertex set V and the arc set A with an incidence function ψ_D that associates with each arc of D an ordered pair of vertices of G. If a is an arc and $\psi_D(a) = (u, v)$ then a is said to join u to v. Notice that in a directed graph, order of the vertex pair is important. So, $\{u, v\}$ tells that the direction of the relationship is from u to v, while $\{v, u\}$ is just the opposite.

In a directed graph, if arc a is from vertex u to v then it is said that u dominates v and u is called the tail of a, and v is the head. The vertices that dominate a vertex are its in-neighbors; and the vertices that are dominated by the vertex are its out-neighbors. The in-degree of a vertex is the number of vertices those dominate the vertex; and similarly out-degree of a vertex is the number of vertices those are dominated by the vertex.

5.2 Networks

A network is simply a collection of objects in which some pairs of these objects are connected by links. The objects may be of any kind: people, computers, firms, banks, societies, countries, or websites. And links may be any kind of relationship: friendship, internet connection, trade, credit, joint research, or hyperlinks. The importance of networks is that we can model many real world cases from a network perspective.

A network is actually nothing else but a graph. As we have mentioned before, many real life problems can be modeled as networks. Any kind of society in real world can be thought as a collection of nodes, and links between them. To distinguish differences between networks we attribute some characteristics to them.

Degree Distribution

One of the basic characteristics of a network is its degree distribution. *"The degree distribution of a network is a description of the relative frequencies of nodes that have different degrees. That is, P(d) is the fraction of nodes that have degree d under a degree distribution P."* (Jackson, 2010). For example a network where all the nodes (vertices) have degree k, is a k-regular network, and $P(k) = 1$, and $P(d) = 0$ for $k \neq d$.

A very common degree distribution of networks is *scale-free*, or power distribution, which is $P(d) = cd^{-\gamma}$, where $c > 0$ is a scalar. This distribution is called scale free because, if all degrees are multiplied by the same factor k, relative probabilities of degrees remain the same. Scale free distribution can be observed in large graphs that are very common in nature. For more information on scale free networks the reader can refer to Caldarelli (2007).

We know that the number of edges between two nodes is the distance between them, and diameter of a network is the largest distance in the network. Average of distances in a network is the average path length of the network and one of the characteristics that define a network.

Centrality

Another characteristic that defines a network is the measure of centrality that defines how central (important, influent etc.) a node is. Centrality has different definitions and formulations according to what is desired to be measured. Centrality can be calculated in terms of which has various definitions. Jackson (2010) classifies centrality measures into four main groups:

1. degree centrality: measure of how connected a node is,
2. closeness centrality: how easily a node can reach other nodes,
3. betweenness centrality: how important a node is in connecting other nodes,
4. neighbour's characteristics: how important, central, or influential a node's neighbors are.

Degree centrality, is the ratio $d_i/(n-1)$ where n is the number of nodes in the network. As maximum degree of a node is $n-1$ and minimum is 0, degree centrality is between 0 and 1. Degree centrality is very simple but for some kinds of networks it can be very enlightening. For example, if we are studying a network of citations between researchers, degree centrality of a node represents how many cites it has, and it is a very good representation of its influence (Newman, 2010).

Closeness centrality, measures how close a node is to others, in other words it gives the average distance between a node and all other nodes. Average distance from node i to all other nodes is calculated by:

$$\bar{d}_i = \frac{1}{n-1}\sum_{j\neq i} d_{ij}$$

As, $\bar{d}_i$ gives low values for more central nodes, and high values for low central ones, researchers prefer using multiplicative inverse of $\bar{d}_i$ as closeness centrality. Thus,

$$C_i = \frac{n-1}{\sum_j d_{ij}}$$

In some measures a node itself is also included and $\bar{d}_{ij}$ is calculated as $(\sum_j d_{ij})/n$. Newman (2010), notes that this measure of closeness centrality has two problems. First, range of maximum and minimum closeness centrality values for a network is very small which makes comparison difficult. Second, if two nodes are not connected then $d_{ij} = \infty$. So for an unconnected network $l_i = \infty$ for all nodes and $C_i = 0$. To overcome this problem another measure of closeness centrality is proposed:

$$C_i' = \frac{1}{n-1} \sum_{j \neq i} \frac{1}{d_{ij}}$$

Betweenness centrality measures how often a vertex lies on paths between other nodes. There are various formulations for betweenness centrality (see Newman, 2010). If $P_i(kj)$ is the number of paths between nodes k and j that include node i, and $P(kj)$ be total number of paths between k and j. Then betweenness centrality of a node is:

$$CB_i = \sum_{i \neq k \neq j} \frac{P_i(kj)/P_{kj}}{(n-1)(n-2)/2}$$

There are many other measures of centrality that are used for other kinds of measurements. For example, *Katz prestige* measures the sum of the prestige of a node's neighbors divided by their degrees. So, Katz prestige of a node is:

$$P_i^K = \sum_{j \neq j} \frac{P_j^K}{d_j}$$

As seen the equation is recursive. So to solve the equation, let $\hat{g} = g_{ij}/d_j$ be the normalized adjacency matrix so that the sum across any nonzero column is equal to 1. Then we can write the relationship in the above equation as $P^K = \hat{g}P^K$ which in turn gives $(I - \hat{g})P^K = 0$, where P^K is the nx1 vector and I is the identity matrix.

Another measure of centrality is eigenvector centrality or Bonacich centrality. If C^e denotes the eigenvector centrality of a network then the eigenvector centrality of a node is proportional to the sum of the centrality of its neighbors (Jackson 2010). $\lambda C^e = gC^e$, where C^e is an eigenvector of g and λ is its corresponding eigenvalue.

Groups

In some networks, some group of nodes show interesting properties that are important in social and economic studies. For example, a maximal complete subgraph of a network is called a *clique*. In a clique, each node is linked to all of the other nodes. A *k-clique* is a maximal subset of vertices such that maximum distance between any two nodes is k. (Note that a clique is also a 1-clique.)

Cliques in a network is commonly measured in terms of triples or clustering. Clustering coefficient is the coefficient fraction of paths of length two in the network that are closed. To better understand clustering, we can think of all links that come from node i and look how many of the neighbors know each other. Referring to Jackson (2010) again, we can formulate clustering as:

$$c = \frac{\textit{(number of closed paths of length two)}}{\textit{(number of paths of length two)}}$$

Or in graph terms we can write the clustering measure of a node as:

$$Cl_i = \frac{\sum\limits_{i \neq j \neq k} d_{ij} d_{jk} d_{ik}}{\sum\limits_{i \neq j \neq k} d_{ij} d_{ik}}$$

Above equation gives clustering for a single node, we can get the clustering coefficient of the whole network as:

$$Cl = \frac{\sum\limits_{i; i \neq j \neq k} d_{ij} d_{jk} d_{ik}}{\sum\limits_{i; i \neq j \neq k} d_{ij} d_{ik}}$$

we can also calculate the average clustering coefficient of a network:

$$Cl_{avg} = \sum_i Cl_i / n$$

To measure clustering in directed graphs we can use the definition of transition and look at the percentage of transitive triples. In this approach we simply look at the situations where i has a directed link to j, and j has a directed link to k, and then look whether i has a directed link to k. In social terms, if x knows y and y knows z, then does x know z? The fraction of transitive triples is:

$$Cl_{TT} = \frac{\sum_{i;i \neq j \neq k} d_{ij} d_{jk} d_{ik}}{\sum_{i;i \neq j \neq k} d_{ij} d_{jk}}$$

A relaxation of the term *clique* is *k-plex*. A k-plex of a network is the maximal subset of the network where each node is connected to at least $n - k$ of the other nodes. The k-plex is a useful term to discover groups in networks in real life (Newman, 2010). A different but similar definition that is related to k-plex is *k-core* which is the maximal subset of a network such that each node is linked to at least k other nodes within the group. We can call a k-core as (n-k)-plex.

Remember that a component in a graph is the maximal subset of vertices such that there is a path between every pair of nodes. A k-component is defined as the maximal subset of vertices such that each is reachable from each of the others by at least k vertex-independent paths.

5.3 Social and Economic Networks

Networks were subject of research in disciplines like physics and computer science for decades. In sociology, networks have also been studied since the first half of 20^{th} century. On the other hand, in economics networks has become popular in the last decade.

Study of networks in social sciences goes back to 1933, when psychiatrist Jacob Moreno first presented his series of studies about friendship among school children. Moreno later published his studies in a book: *Who Shall Survive*. He called his diagrams of friendship between students –where he showed boys and girls with triangles and circles and friendship between them as links– as sociograms which are nothing else but networks (Newman, 2010).

Later network theory had been applied in too many various real life cases including but not limited to, collaboration between researchers, friendship and romance among students, corporate governance, musicians, actors, criminals, and business professionals.

A very important and famous study of social networks belongs to Padgett and Ansell (1993) which explains how Medici family got the power in 13^{th} century Florence. Medici family, rose to the power in business and politics although they were not the strongest or richest family. In the study, a network that shows inter family marriages between families is set. If there is a marriage between members of two families then there is link between nodes of those families. At that time marriage was the reason of a strong relationship between families. The network of inter-family marriages showed that, Medici's formed marriage links with other strong families that maintained a key role to them in the network of families.

Another interesting study belongs to Moody (2001), that is about interracial friendship relations between school students. The study shows that friendship between students from same ethnicity is denser. This phenomenon is called *homophily* and is an important part of sociological network studies.

As noted, study of networks in economics is still at infancy. Conventionally, interaction among players was anonymous and centralized, and furthermore unique device of interaction among players in economic models was price. Now increasing number of researches show that, individual behavior is formed by social interaction. Network modeling enables us to understand various subjects in economics like diffusion of innovations, variations in crime, differences in trust and cooperativeness, peer effects in academic performance, extensive use of personal contacts by both employers and workers in labor markets Goyal (2009). Social interaction in a society may explain criminal activity on one side (Glaeser et al., 1996), and choice of pension policy on the other (Duflo and Saez, 2003). Network studies may help us to understand how ethnicity affects participation in welfare programs (Bertrand et al., 2000), or membership to a community influences rate of investment (Banerjee and Munshi, 2004). In summary, social interaction affects individual behavior which in turn affects aggregate outcome.

A difference of network study in economics than other disciplines is the role of social efficiency (Goyal (2009)). With the help of network theory, we can understand why actual outcomes resulting from social interactions are not always socially desirable outcomes.

5.4 Network Formation

Almost all of the networks in real life are dynamic networks so that their structure change over time. So it becomes important how networks form and change; in other words how new nodes born and form links with other nodes over time. We can simply classify networks as static and dynamic networks. And there are two kinds of dynamic networks: random and strategic. For further information about network formation the reader can refer to Jackson (2010).

5.4.1 Static Network Formation

Although not very common in social and economic networks, we want to mention about static network formation for comparison to the following section. There are various ways of forming a static network. Most simply, we can chose M links randomly between n nodes. Note that a network with n nodes has $n(n-1)/2$ potential links.

Or we can list all possible settings with number of links ranging between zero and $n(n-1)/2$, and choose one of them randomly.

Next, we can connect to nodes with probability p where link formation between different nodes is mutually exclusive. In this binomial link formation setting, a network with n nodes has m links with probability $p^m(1-p)^{\frac{n(n-1)}{2}-m}$ and the probability that any given node i has d links is:

$$\binom{n-1}{d} p^d (1-p)^{n-1-d}$$

For large n and small p, we can write $(1-p)^{n-1-d}$ as $(1-p)^{n-1}$. We can write $(1-p)^{n-1} = (1-(n-1)p/(n-1))^{n-1}$ which in turn can be written as $e^{-(n-1)p}$. And, again for large n and small p, we can assume that $\binom{n-1}{d} = (n-1)^d/d!$. Finally we can write the above formula as:

$$\frac{e^{-(n-1)p}((n-1)p)^d}{d!}$$

So degree distribution is approximated by a Poisson distribution. Actually Poisson random networks are most commonly used type of static network formation.

Although static random networks may provide some insight about social networks they lack most characteristics of them because real life networks almost always show dynamic properties. For example, static networks do not show the high clustering properties that are common in many social networks (Jackson, 2010). You can refer Jackson (2010) for more information in static random graph models.

5.4.2 Dynamic Network Formation

Random Network Formation

When a new node is born, there are two ways that node is attached to existing ones. The node can randomly create links with the existing ones or the nodes that will be linked can be selected according to their current degrees. The second way is called preferential attachment. In this second approach, older nodes will have higher chance of growing and will have higher degrees in time. There are also approaches those are in between pure random selection and preferential attachment. Hybrid models may produce high clustering properties that are observed in many real life networks. In random growing networks, older nodes have higher degrees which gives a characteristic of age based homophily that is common in many social networks.

A very basic model of dynamic random link formation that is a variation of Poisson random-network is provided by Jackson (2010). New nodes are born over time and link to existing nodes randomly. Nodes are indexed by the time they are born, $i = \{0, 1, 2...\}$. The number of links of node i at time t is $d_i(t)$. Specifically, $d_i(i)$ is the number of links formed when node i is born and, $d_i(t) - d_i(i)$ is the number of links formed between time i and t.

The model starts with $m + 1$ node each born at $\{0, 1, ...m\}$. After that time, each new node chooses m nodes randomly among existing ones and forms link with them. Thus, degree distribution of node i at time $t > m$ will be

$$m + \frac{m}{i+1} + \frac{m}{i+2} + ... + \frac{m}{t}$$

For large t, degree distribution above becomes

$$m\left(1 + log\left(\frac{t}{i}\right)\right).$$

And nodes that have expected degree less than d are those such that

$$m\left(1 + log\left(\frac{t}{i}\right)\right) < d$$

which can be rewritten as $i > te^{1-d/m}$. The nodes with expected degree less than d are the ones born after $te^{1-d/m}$, and the fraction of nodes with expected degree less than d is:

$$F_t(d) = 1 - e^{-\frac{d-m}{m}}$$

.

Another approach of network formation is preferential attachment. The idea of preferential attachment in networks belongs to Price (1965, 1976). He studied citation networks between scientists and proposed the idea that a paper would have citations in time proportional to the number of citations it already has. Later, Barabási and Albert (1999) modeled an undirected version of Price model.

We will quote a basic preferential attachment model by Jackson (2010). Like in the random growth model above we assume that nodes are indexed according to the time they are born and they form m links at the time of birth. This time, a new node forms links with existing nodes with a probability, proportional to their degrees. The probability that link is formed between a new node t and node i is

$$m\frac{d_i(t)}{\sum_{j=1}^{t} d_j(t)}$$

$\sum_{j=1}^{t} d_j(t) = 2tm$, because there are tm total links in the network at time t. So above formula becomes $d_i(t)/2t$. This is actually the rate of change of degree i with time, assuming a continuous time approximation. With a continuous-time approximation we can think that, node i's degree changes over time with the equation:

$$\frac{d_{i(t)}}{dt} = \frac{d_i(t)}{2t}$$

where initial condition is $d_i(i) = m\left(\frac{t}{i}\right)^{1/2}$

Let $i_{t(d)}$ be the node that has degree d at time t, $d_{i_{t(d)}} = d$. Then,

$$\frac{i_{t(d)}}{t} = \left(\frac{m}{d}\right)^2$$

The fraction of nodes with degree smaller than d at time t is the proportion born after node $i_t(d) = t(m/d)^2$. Thus, the distribution function is given by:

$$F_t(d) = 1 - m^2 d^{-2}$$

,

and the probability density function is,

$$f_t(d) = 2m^2 d^{-3}$$

.

There are also other models those are mixture of random growth and preferential attachment models. In our model we will also use an hybrid model of network formation. For deeper insight about these models the reader can refer to Jackson(2010).

Strategic Network Formation

In most of the real life social and economic networks, deliberate choice plays an important role in network formation. For example in bank-firm networks, price is an important factor of link formation. In some social networks political view determines links while in others just friendship or economic class may be important in. With random growing networks we can understand how network get their form. On the other hand, strategic network formation provides understanding about why networks get this particular structure.

There are two challenges in modeling strategic networks (Jackson, 2010). First challenge is, to explicitly model the payoffs of various networks. This is important because rightly modeling the costs and benefits enables us to correctly measure the outcome to the society. The second is, to model how individual incentives translate into network outcomes. We believe that agent based modeling may solve the second problem.

One aspect of strategic network formation which is important in economic network modeling is that, it provides information about the trade-off between individual incentives and social welfare. We believe that network modeling gives better insight than conventional models on this issue. According to Jackson (2010), even if there exist transfers so that individuals can be subsidized to maintain relationships that are in society's interest but are not in their own interests, it can still be impossible to maintain socially efficient networks under some reasonable restrictions on transfers.

Assume that $u_i : G(N) \to R$ is the utility function where, $u_i(g)$ denotes the utility, player i gets from network g. A network is pairwise stable if no player wants to delete a link and no pair of nodes want to form a link mutually. A network g is network–stable if (Jackson, 2010):

1. $u_i(g) \geq u_i(g - ij)$ and $u_j(g) \geq u_j(g - ij)$, $\forall ij \in g$ and
2. if $u_i(g + ij) > u_i(g)$ then $u_j(g + ij) < u_j(g)$, $\forall ij \notin g$

Pairwise stability has some weaknesses. First, it examines only single link addition or deletion. And, it considers only deviations by at most a pair of players at a time. Although deletion or addition of a single link may not be pairwise stable, adding or deleting a group of links may increase utility of some players (Jackson (2010)).

Another concept in strategic network formation is efficiency. A network is efficient with respect to utility functions $(u_1, ...u_n)$ if $\sum_i u_i(g) \geq \sum_i u_i(g')$ for all $g' \in G(N)$.

Pairwise stability and efficiency may be used in economic network studies and may answer the question of how observed outcomes differ from socially desirable ones.

Chapter 6

The Model

6.1 Introduction

Most of the studies about credit networks (or simply credit markets) put firms at the core of their analyses. The recent literature have mainly focused on aggregate or idiosyncratic shocks to firms and effects of those shocks to aggregate economy (Bernanke and Blinder 1988, 1992; Bernanke and Gertler 1989, 1990, 1996, 1999). As discussed by Delli Gatti et al. (2010b), previous studies of financial networks were using a representative agent methodology. But, these studies cannot model the complex structure of modern credit networks. Delli Gatti et al. (2010b)) claims that there are three reasons why aggregate approach to credit markets cannot model real world.

The first point is that in representative agent approach, the shock is uniform across agents. But in real world an idiosyncratic shock can as well lead to financial distress. Indeed, Gabaix (2011) empirically shows that idiosyncratic firm-level shocks explain an important part of aggregate movements and provide a microfoundation for aggregate shocks. Further, all shocks are not exogenous. Banks' policies like aggressive selling may cause endogenous shocks as well.

Second, Delli Gatti et al. compel the pro-cyclicality, which is stated in Minsky's Financial Instability Hypothesis (1988). Conventional studies cannot model pro-cyclical behavior of agents. In times of growth, both firm and banks are willing to increase their leverage which increases the fragility in the next financial distress. Indeed, a recent article [1] on UK Banks, shows that, mean leverage (measured as total debt over total book value) of top 200 UK banks increased from about 28% in 1960 to 60% in 2005.

Finally, they mention about financial accelerator. If a firm goes bankrupt, the financial situation of banks it owes will also get worse. Those banks will in turn increase interest rates or tighten credit conditions for other firms[2]. So, an idiosyncratic shock may lead to an avalanche of bankruptcies.

6.2 The Environment

Our model is built upon Delli Gatti et al. (2010b) and Riccetti, Russo, and Gallegati (2011). A real economy includes banks, firms, and households, and the markets in the economy are labor, goods, and credit markets. As our study focuses on banks, we exclude households from our model, both to observe bank behavior separately and to keep the model simple. Excluding households from the model we also omit labor market and goods market in the economy[3] and make simplifying assumptions for these markets.

Our model is simply a no-government/no-household debt-closed economy, which is named as basic skeletal capitalist economy (Minsky, 1988). Minsky states that, in this kind of economy, it is strictly true that, money is created as bankers go about their business of arranging for the financing of trade, investment, and positions

[1] www.voxeu.com/article/firm-bank-relationships

[2] However, in our view this may not be the case in real life economics if there is a strong belief that banks will be bailed out or if there are Too Big to Fail Policies.

[3] Actually, it would be subject of another and broader study where households are included and consumer credits are analyzed.

in capital assets. An increase in the quantity of money through bank lending to business transforms a desire for investment or capital assets into an effective demand; the creation of money is part of the mechanism by which a surplus is forced and allocated to the production of investment outputs.

In our model we have firms and banks where firms are indexed by $i = 1, 2, ...I$ and banks by $z = 1, 2, ...Z$. Firms produce all kinds of consumption goods and sell their output to households. Banks, on the other side, grant credit to firms to supply their capital that is necessary for production. Thus, we will model a bipartite directed credit network between banks and firms.

As in Greenwald and Stiglitz (1993) and Riccetti et al. (2011), we assume that prices are exogenous and firm specific. Prices are normally distributed random variables that fluctuate around a common average. We also assume that firms sell all the output they produce. On firms' side, there are no factors that add to growth, like productivity increase, population growth, human capital etc. So, we are purely interested in business cycles. As emphasized in Riccetti et al. (2011), using an exogenous price mechanism we cannot analyze price inflation dynamics. On the other hand, we believe it will be possible to analyze debt-deflation dynamics which is proposed by Fisher (see Chapter 2) and later developed by Hyman Minsky.

The second and more important market for our study is the credit market where firms obtain credits from banks. A firm's production frontier is determined by its total capital which is the sum of its net worth and debt. As there is no lending between firms, all of the debt is bank debt.

Banks finance firms with their equity and deposits. As our model lacks interbank credit market and households, we simply assume that banks can find any amount of deposit they want.

6.3 The Firm

6.3.1 Capital and Production

Previously we mentioned that we will use a balance sheet approach in our model. Balance sheets and cash flows are fundamental in understanding links among agents in the firm-bank linkages in a network theory framework. We are assuming a firm with a simple balance sheet like below.

Assets	**Liabilities**
TA Total Assets	$\hat{B}$ Total Bank Debt A Net Worth

A firm's liabilities are composed of its net worth (or equity) and its total debt. We assume that debt is used only from banks and there is no credit relationship between firms. A firm's production capacity is bounded with its total capital which is its debt plus net worth:

$$K_{i,t} = A_{i,t} + \hat{B}_{i,t} \tag{6.1}$$

The level of production of firm i, at period t, is an increasing concave function of its total capital, $K_{i,t}$:

$$Y_{i,t} = \phi K_{i,t}^{\beta} \tag{6.2}$$

where $\phi > 1$ and $0 < \beta < 1$ are uniform parameters across firms. The equation above is the financially constrained output function.

As explained in Greenwald and Stiglitz (1993), and Delli Gatti et al. (2010b), Eq. 6.2 is the solution of an optimization of the firm. The problem of the firm is maximizing expected profits $E(\pi_i)$, net of bankruptcy costs, C_i, weighted by the probability of bankruptcy, Ω_i. This definition lets profits to be an increasing function of output, Y_i, given total capital K_i: $\pi_i = \pi(Y_i; K_i)$. Bankruptcy costs are assumed to increase with the firms size: $C_i = C(Y_i)$. And the probability of bankruptcy. So maximization problem of the firm:

$$max_{Y_i} V(Y_i; K_i) = E(\pi(Y_i; K_i)) - C(Y_i)\Omega(Y_i; K_i) \tag{6.3}$$

The solution to Eq. 6.3 is:

$$Y_i = argmax V(Y_i; K_i) = f(K_i) \tag{6.4}$$

with $f' > 0$. Eq. 6.2 can be considered an element in the set of functional forms consistent with Eq. 6.4.

As mentioned before, production function is a concave function. This is because there are decreasing returns to financial possibility frontier. A given increase in total capital causes a lower increase in output if total capital is already high. The first order condition for Eq. 6.4 is $V_Y(Y_i; A_i = 0)$ so that

$$f' = -\frac{V_{YA}(Y_i; K_i)}{V_{YY}(Y_i; K_i)}$$

The slope of the output function is positive ($f' > 0$) if the numerator and denominator above have different signs. Greenwald and Stiglitz (1993) implicitly assume that numerator is negative and denominator is positive. We have to sign the second order derivatives of the objective function with respect to its arguments to derive the sign of the first order derivative of the output function. The returns to total capital are captured by the second derivative of the output function:

$$f'' = -\frac{\sigma}{\sigma K_i}\frac{V_{YA}(Y_i; K_i)}{V_{YY}(Y_i; K_i)}$$

If the sign of the second derivative is negative then there are decreasing returns to total capital as we assumed. To determine the sign of the second derivative we have to set the sign of the third order derivatives, V_{YAY} and V_{YYY}, but this is out of the scope of our study (Delli Gatti et al. (2010b)).

For capital accumulation setting of a firm we will use dynamic trade-off theory[4] like Riccetti et al. (2011). Every firm has a target debt level, $B^*_{i,t}$, for each period, which is determined by net worth and its target leverage.

$$B^*_{i,t} = A_{i,t}\ell_{i,t} \tag{6.5}$$

where $A_{i,t}$ is the net worth of firm, and $\ell_{i,t}$ is the leverage level target, set by firm. The firm follows an adapting behavior to set the target leverage. If the expected price level is larger than the interest cost of the firm, target leverage is set by increasing the leverage of previous period by a percentage which is randomly determined. Conversely, if expected price level is less than the interest cost, last period's leverage is decreased by a random percentage to set the current target leverage. In other words, if the firm is profitable, it is motivated to increase leverage level and vice versa. And percentage change in the leverage level differs among firms. So,

$$\ell_{i,t} = f(pe_{i,t}, r^*_{i,t-1}) \tag{6.6}$$

where pe is the expected price, which is a modified exponential smoothing of recent observed firm specific prices. And, $r^*_{i,t-1}$ is the weighted average interest cost of the firm in the previous period. The specific form of the leverage setting adaptive rule is:

[4] For a brief discussion of capital financing theories see Section 3.1.

$$\ell_{i,t} = \ell_{i,t-1}(1 \pm \Delta l_{max} \cdot rand) \tag{6.7}$$

where Δl_{max} is a parameter that sets the maximum leverage change between the two periods and is multiplied by a random number drawn by a uniform distribution between 0 and 1.

We get $pe_{i,t}$ fully from Riccetti et al. (2011) and set it specifically as:

$$pe_{i,t} = \frac{(0.6p_{i,t-1} + 0.36p_{i,t-2} + 0.04p_{i,t-3})}{\sqrt{1 + A_{i,t-1}/A_{t-1}^{max}}}$$

where A_t^{max} is the maximum net worth at period t. The coefficients in the numerator mean that firm's expected price is determined by a weighted average of most recent three years' realized prices. Weights of previous year, two years and three years before are 60%, 36%, and 4% respectively.

We can combine equations 6.6 and 6.7 as below:

$$\ell_{i,t} = \begin{cases} \ell_{i,t-1} \cdot (1 + \Delta l_{max} \cdot rand) & pe_{i,t} \geq r^*_{i,t-1} \\ \ell_{i,t-1} \cdot (1 - \Delta l_{max} \cdot rand) & pe_{i,t} < r^*_{i,t-1} \end{cases} \tag{6.8}$$

Using target leverage mechanism, a firm determines target debt level. The firm demands credit if this target debt level is greater than the difference between total amount of credits at the end of the previous period and credit payback at the current period (i.e. current debt stock).

$$B_{i,t} = max(B^*_{i,t} - (\hat{B}_{i,t-1} - \tilde{B}_{i,t}), 0) \tag{6.9}$$

where $\tilde{B}_{i,t}$ is the total amount of expiring credits at period t[5]. Total amount of debt for the new term, becomes the sum of balance at the end of the previous

[5]The credits are obtained with terms to expiration $d = \{1, 2, ...D\}$. This will be explained later.

term and amount of credit used in the current term minus expiring credits: $\hat{B}_{i,t} = \hat{B}_{i,t-1} + B_{i,t} - \tilde{B}_{i,t}$. New amount of debt stock will be used in Equation 6.1.

6.3.2 Firm Profit

Profit of a firm i, at period t is determined by:

$$\pi_{i,t} = p_{i,t}Y_{i,t} - \sum_{z} r^{*}_{z,i,t-1}\hat{B}_{z,i,t-1} \tag{6.10}$$

where $p_{i,t} = u + \nu_{i,t}$ is the price of the product. u is the expected gross profit and $\nu_{i,t}$ is the random component for each firm in each period which is normally distributed with mean zero and variance σ^2.

Second term on right hand side gives total interest cost of the firm. $r^{*}_{z,i,t}$ is the weighted average interest rate for the credit balance at period t, used by firm i from bank z. Remember that, since a firm can use credits with term $d \leq D$, at any period, a bank and a firm may have multiple credit links with different terms.

We assume that there are no dividends and the firm adds up its profit to its net worth. As we know the profit now, we can set the net worth equation:

$$A_{i,t} = A_{i,t-1} + \pi_{i,t} \tag{6.11}$$

6.3.3 Bankruptcy

Bankruptcy is a very important mechanism in capitalist economy because it is a way of transforming speculative and Ponzi units into hedge and speculative units (Minsky (1988)). In our model we will consider two kinds of bankruptcy. A firm goes bankrupt if it is either insolvent or illiquid.

Equity Bankruptcy

For the first type of bankruptcy, if $A_{i,t} \leq 0$ (or using Eq.6.11, $\pi_{i,t} \leq A_{i,t-1}$) the firm loses all of its net worth and goes bankrupt.

Liquidity Bankruptcy

Another kind of bankruptcy occurs if the firm's profit is not enough to pay its credit and interest payments in the next period. Minsky claims that, in order to analyze how financial commitments affect the economy, it is necessary to look at economic units in terms of their cash flows (1988). Minsky adds that, ordinary business needs to be liquid and solvent. That means the payment commitments on debts must lie within bounds given by realized and expected cash flows. We set our liquidity bankruptcy mechanism on this idea of Minsky.

We first assume that firms hold a ratio (θ) of their total capital on liquid assets. So in case of emergency, the firm can create a cash of θK_i. We also assume that the profit comes in the form of cash. A firm is assumed to be liquid if its cash flow is positive. This means that a firm is liquid if the firm's cash inflow is greater than or equal to cash outflow. In our model cash outflows are interest payments and principal payments. Therefore liquidity of the firm is determined by:

$$liq = \pi_{i,t} + \theta K_{i,t} - \sum_{z} r^*_{z,i,t} \hat{B}_{z,i,t} - \tilde{B}_{i,t} \tag{6.12}$$

If liq is negative we say that firm is illiquid. In this case, we check for the relative size of the negative liquidity in the next step. We assume that, if the absolute value of the ratio of negative liquidity to the production is above a certain level ($lqlim$ in Eq. 6.13) the firm goes bankrupt. On the other hand, if the liquidity need is below that level, firm does not go bankrupt but banks deny to grant credit to the firm[6]. Literally if lig is negative then,

$$liquidity\ bankruptcy = \begin{cases} true & if \quad |lig/Y| > lqlim \\ false & if \quad |lig/Y| \leq lqlim \end{cases} \tag{6.13}$$

We assume that if a firm goes bankrupt, a new player with a relatively small net worth, enters the market.

6.4 Banks

A bank has a balance sheet similar to the one below:

Assets	**Liabilities**
L Credits	D Deposits
R Reserve Accounts	CB Central Bank borrowing
	A Net Worth

For simplicity we will assume for the beginning that there is no interbank or Central Bank funding. So CB will be omitted in our model. Reserve accounts will be accepted as equal to a proportion of deposits. So $R = \varepsilon D$ where $0 \leq \varepsilon < 1$.

The balance sheet equation is $L_{z,t} + R_{z,t} = D_{z,t} + A_{z,t}$ or $A_{z,t} = L_{z,t} + R_{z,t} - D_{z,t}$. We can simplify this as $A_{z,t} = L_{z,t} - (1 - \varepsilon)D_{z,t}$. We assume, a bank can raise any amount of funds in the form of deposits. Thus, $D_{z,t} = \frac{1}{1-\varepsilon}(L_{z,t} - A_{z,t})$.

[6]In reality, banks may increase interest rates and/or demand collaterals for the credits if the firm is illiquid. For simplicity, we will assume that bank denies to grant credit to firm.

6.4.1 Interest Rate Setting

We can represent the equation for interest rate, set by bank z, for firm i, at period t as below:

$$r_{z,i,t} = rmin_t + f_1(\cdot) + f_2(\cdot) \tag{6.14}$$

Where $rmin_t$ is the interest rate floor which is assumed to be set due to Central Bank policies. We say that f_1 is the function related to bank's parameters and f_2 is the function that returns parameters that relate firms.

For f_1 we can set parameters as net worth, market share, leverage, liquidity ratio, capital adequacy ratio. And for f_2 we can also determine similar parameters like leverage, liquidity, net worth.

To keep the model simple, we will first take capital adequacy ratio for bank; leverage and net worth for firm. So our interest rate equation is:

$$r_{z,i,t} = rmin_t + f_1(CAR_{i,t}) + f_2(l_{i,t}, A_{i,t}) \tag{6.15}$$

where we assume

$$f_1(CAR_{i,t}) = \gamma CAR^{-\gamma} \tag{6.16}$$

where $CAR = A/L$ and,

$$f_2(l_{i,t}, A_{i,t}) = \alpha \left(\frac{l_{i,t}}{1 + \frac{A_{i,t}}{A_t^{max}}} \right)^{\alpha} \tag{6.17}$$

We will also consider the other variables above, in alternative scenarios. And in another scenario we will add a term of $f_3(\cdot)$ where banks take into account the GDP growth of the last period x_{t-1}.

6.4.2 Bank Profit

The profit of the bank equals:

$$\pi_{z,t} = \sum_i r^*_{z,i,t}\hat{B}_{z,i,t} - rmin_t D_{z,t} - c(A_{z,t} + D_{z,t}) - (1 - RR)npl_{z,t} \tag{6.18}$$

c: cost, proportional to banks size.

$npl_{z,t}$: non performing loans of bank z, at period t.

RR: Recovery rate

As in firms, bank profit is also fully added to net worth: $A_{z,t} = A_{z,t-1} + \pi_{z,t}$. We assume that a bank goes bankrupt if its net worth is less than or equal to zero (i.e. $A_{z,t} \leq 0$). When a bank goes bankrupt, a new bank with a net worth of A_0, enters the market.

6.5 Partner Selection

In our model, firms may establish credit links with multiple banks. At the beginning of each period, firms with positive credit requirements, $(B_{i,t})$, ask for credit to current banks, plus n other banks. Those n banks are selected randomly among the banks that have no link with the firm. Total number of banks a firm can establish credit links in one period is limited with MB.

For the banks matched, the amount of credit that may be released due to capital adequacy requirements, is calculated. Specifically, the amount of credit a bank can release is:

$$crd_{z,t} = \frac{A_{z,t}}{CAR^*} - L_{z,t} \tag{6.19}$$

where CAR^* is the legally set minimum capital adequacy ratio.

Next, interest rate for each matching is calculated. Firm establishes a credit link with the bank with $\min(r_{z,i,t})$. If the available credit from the bank due to CAR constraint is not enough for the firms credit need, then partial usage is allowed and firm switches to the bank with next best interest rate.

As mentioned before, each credit has an expiration term, which is the number of periods to credit deadline. Credit term is a discrete random variable. Specifically, the credit term is $d = \{1, 2, ...D\}$. To make the model more realistic we assume that shorter term for a credit is more probable (i.e. $p(d = 1) > p(d = 2) > ... > p(d = D)$). We assume that credit term is exponentially distributed. Let p be the probability related to cumulative distribution function of the term. Then $p = 1 - e^{-\lambda x}$ and

$$x = (-1/\lambda) ln(1 - p)$$

.

To set the term variable we first get p via a uniform random generator. Using p we set x and then integer term value.

$$d = \begin{cases} 1 & x < 2 \\ 2 & 2 \leq x < 3 \\ \vdots & \vdots \\ D-1 & D-1 \leq x < D \\ D & D \leq x \end{cases} \tag{6.20}$$

6.6 Summary

In our base model we modeled a bipartite directed credit network of banks and firms. We omit households, government and any other institutions for simplification. Excluding households, we will be able to omit labor and goods market and model just credit market and its effects on the economy.

Firms sell all of their output to households. Prices are determined exogenously, firm specific and normally distributed. There are no growth enhancing factors like population growth, productivity increase etc. Firms produce according to their total capital which is the sum of their net worth plus the credit they obtained from banks.

Each firm decide its target leverage in the upcoming period and according to that target leverage determine the amount of credit to use. If the price expectation of the firm for its goods is higher than the firm's interest cost rate, firm decides to increase its leverage ratio relative to last term leverage, within an upper limit. If interest cost rate is higher, firm decreases its target leverage ratio.

After determining target leverage, firm gets credit from banks if necessary. Thus, sum of net worth (equity) and bank credit is equal to total capital which determines the level of production. Price of goods are determined according to a normal distribution random variable. Profit of the firm is revenue minus interest payments for the period.

We assume that firms add profit to their net worth. So, net worth increases if the firm has positive profit. If net worth becomes negative, the firm goes bankrupt. There is a another kind of bankruptcy in our model: liquidity bankruptcy. In real life practice, liquidity bankruptcy is also as important as equity bankruptcy. If a firm cannot create cash from its operations or cannot pay its debt it will go bankrupt no matter how profitable the business is.

Firms in our model hold a ratio of total capital as liquid assets. In case of emergency a firm can create cash by selling that ratio of assets. If this ratio plus profit of the period is higher than credit interest and principal payments that firm is accepted as liquid in that period. Else the firm is illiquid and now we look at the size of that insolvency. If absolute value of the ratio of liquidity need to total production is above a threshold, the firm is accepted as bankrupt. If the ratio is below the threshold, firm continues its business but this time banks reject to grant credit.

We simplify bank balance sheet so that it includes, credits on assets, and liabilities and net worth on liabilities side. Banks grant credit to firms. Interest rate setting rule has three terms. First one is $rmin$ which is the minimum or benchmark interest rate in the market. We can say that it is the policy rate determined by the central bank. The second term is the term related with the bank. In the base model bank care about its capital adequacy ratio (CAR) when pricing a credit. If CAR of the bank is high, bank sets lower interest rate because it has adequate capital. Final term is related to the firm. It includes, leverage of the firm and its relative net worth among other terms. A firm with high leverage may obtain credit with higher rate.

Banks obtain their income with interest payments from firms. Their expenses are interest paid for deposits, pro rata cost on total assets, and uncollected part of non performing loans.

Firms, may establish credit linkages with multiple banks. At the beginning of each period, firms that need credit, ask banks for funding. Number of banks, firms can have credit linkages are bounded above. On the other hand, firms ask credit to banks they already have credit links, a determined number of random banks. Firm get interest rates from banks. They first go to bank with minimum rate to obtain credit.

When a firm asks credit to a bank, the bank first controls whether it has adequate credit limit due to capital adequacy ratio limitations. In Basel accord and in many countries, minimum capital adequacy ratio is determined legally. In our model we also set this rule. Banks' minimum capital adequacy ratio must be higher than or equal to the minimum CAR requirement.

If the bank's CAR limitation is not enough to grant the credit amount fully, credit is partially used. Then the firm goes to another bank to get rest of the credit.

We allow for long term credits in our model. Credit terms are determined randomly according to exponential distribution. As Minsky also stated, one of the reasons of instability in financial markets is time inconsistency. So it is important to allow variations in credit terms.

This concludes our model. In this section we have modeled a bipartite directed credit network between banks and firms. In the next section we will simulate our model to observe the long term aggregate behavior.

Chapter 7

Simulation

7.1 Baseline Model

As explained in Chapter 4, in agent based models aggregate behavior is not observed by just simply solving optimization equations, because resulting aggregate behavior cannot be foreseen. In agent based models, economic behavior is observed using simulation models. In our model, we set numerous computer simulations in C++ programming language to explore the dynamics of our model.

We assume that our model consists of $I = 500$ firms and $Z = 50$ banks and run the simulation for $T = 1000$ periods. At the beginning, net worth of each firm equals to 10 and bank to 20. The initial leverage of firms is 1.

As mentioned before, we assume that if a firm goes bankrupt, it is replaced by a new firm that has a relatively small net worth, which equals to 2. On the other hand, if a bank goes bankrupt, the new bank will have a net worth of 20.

If a firm goes bankrupt, its credits are then classified as nonperforming loans and those credits are considered banks' loss with a ratio of (1-RR). If a bank goes bankrupt, its credit receivables are taken over by the new bank[1]. The initial para-

[1]This is actually the real practice. In some cases the receivables may be also taken over by government. We chose the first way for practicality

Parameter	Value	Explanation
ϕ	3	production function of the firm, Eq. 6.2
β	0.7	production function of the firm, Eq. 6.2
Δl_{max}	0.1	target leverage setting function, Eq. 6.8
u	0.1	expected gross profit, Eq. 6.10
σ^2	0.01	variance of random variable in profit function, Eq. 6.10
θ	0.5	ratio of total capital that can be liquidized, Eq. 6.12
$lqlim$	0.4	level of relative liquidity for liquidity bankruptcy, Eq. 6.13
ε	0	ratio of reserve deposits, Sec. 6.4
$rmin$	0.02	interest rate floor, Eq. 6.14
γ	0.02	interest rate setting, bank parameter, Eq. 6.16
α	0.02	interest rate setting, firm parameter, Eq. 6.17
RR	0.5	recovery rate in case of default, Eq. 6.18
c	0.005	bank operational costs, Eq. 6.18
CAR^*	0.12	minimum legal capital adequacy ratio, Sec. 6.5
MB	15	maximum number of banks that a firm can work, Sec. 6.5
n	2	number of banks other than current banks, firms ask credit, Sec. 6.5
λ	0.4	exponential distribution parameter for credit term, Sec. 6.5
D	4	maximum number of credit terms
A_i	10	Initial net worth of a firm
A_z	20	Initial net worth of a bank

Table 7.1: Initial parameters of the base model

meters set for the first simulation of the base model in Chapter 6 are given in Table 7.1.

We decided not to confirm our results with empirical data because, as mentioned before, our model lacks some real world characteristics, such as household behavior, technological progress, interbank market etc. Also our model lacks a realistic labor and goods market. Like Delli Gatti et al. (2010b) and Ricetti et al. (2011), we determined the parameter set according to some empirical regularities.

In our baseline model banks consider only capital adequacy ratio requirements for pricing. So, in this model banks, do not behave aggressively, or try to increase their profits or market share. Using this baseline model we will later analyze the changes in the economic dynamics when banks adapt some other behavior rules to increase their market shares, profits, total assets. We give the results of the first run of the base model with parameters above, in Table 7.2.

	Minimum	**Mean**	**Maximum**	**Std. Dev.**
Bad Debt Ratio %	0.00	8.31	55.45	4.49
Bank Defaults %	0.00	0.89	32.00	2.59
Bank Net Worth	1105.48	105128.41	255565.00	66340.62
Total Debt	5253.34	29130.71	67058.10	11754.73
Firm Defaults %	0.00	4.95	8.60	1.07
Firm Net Worth	5885.27	45915.80	143453.00	35626.83
Aggregate Production	12428.20	32597.18	58123.80	9279.12
Interest Rate %	5.88	5.96	6.08	0.04
Leverage	0.52	1.28	2.29	0.39
Growth %	-15.28	0.09	12.29	2.32

Table 7.2: Base Model Results

The results given in the table are valid for periods 0-1000. The time interval between 0-200 is actually a period of initialization and setting up, so some outstanding maximum values are observed which are related to this interval. Excluding the first 200 periods does not change the main results and conclusions. To give a better visualisation of resulting trends we present periods 200 - 1000 in the related graphs.

Figures 7.1 and 7.2 show aggregate production, and growth rates of the economy respectively. As in Delli Gatti et al. (2010b) and Riccetti et al. (2011), we observe irregular fluctuation patterns that show very significant differences among sub-periods. This is both because of exogenous pricing and complex adaptive structure of the system. On the other hand, growth rates fluctuate in a band that fits to the real case. Standard deviation of growth rates is 2.32%. Following this, growth was found to be negatively skewed (-0.37) and has an excessive kurtosis (8.38), similar to Delli Gatti et al. (2010b).

Figure 7.3 gives the debt dynamics of the simulation and Figure 7.4 gives the course of non performing loans. Total debt follows a very similar pattern to aggregate production. Bad debt ratio also follows a realistic pattern, even if it is a bit higher than the real data.

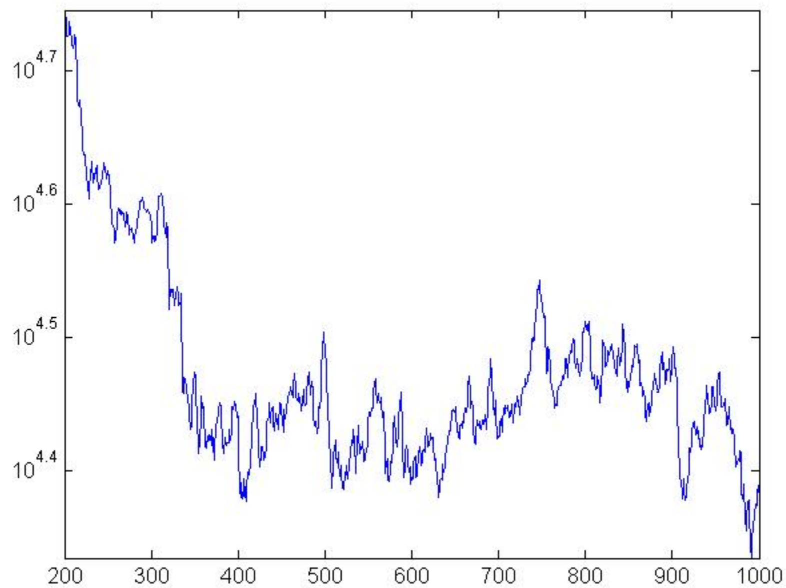


Figure 7.1: Aggregate Production in the Baseline Model

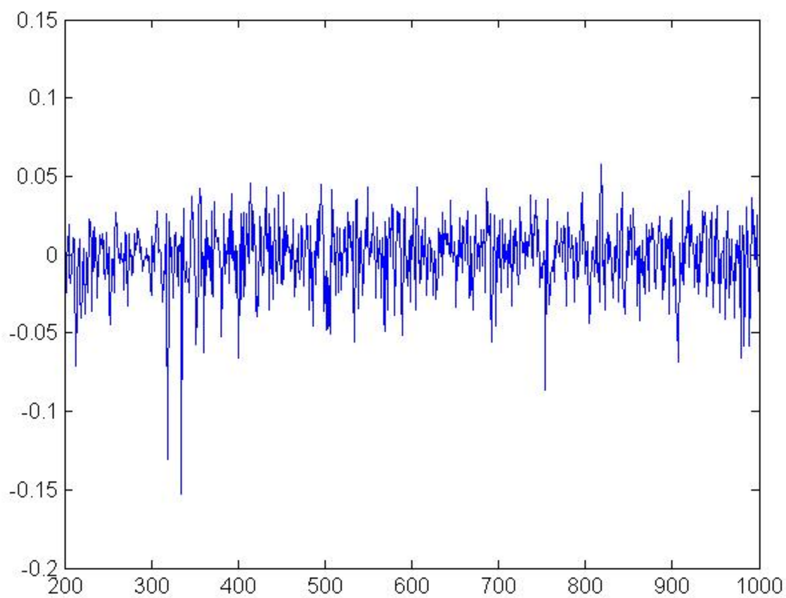


Figure 7.2: Aggregate Growth Rates

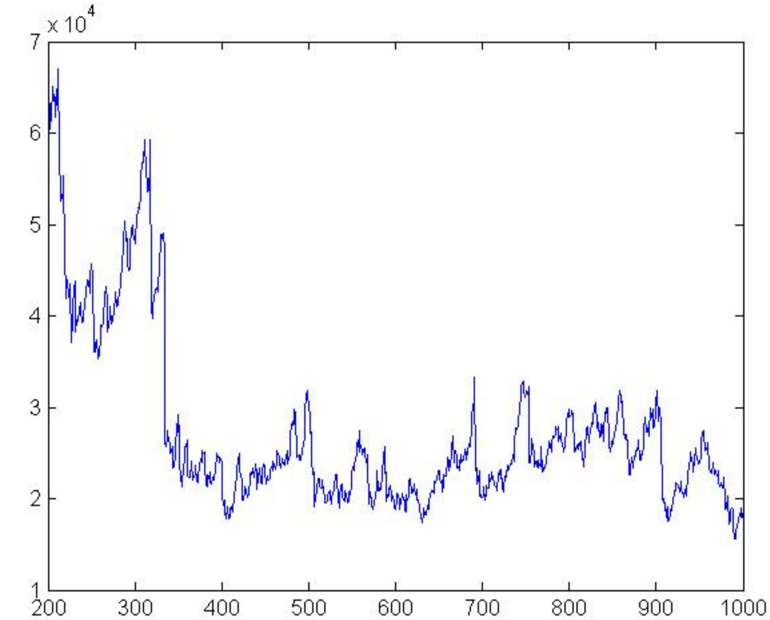


Figure 7.3: Total Amount of Credits

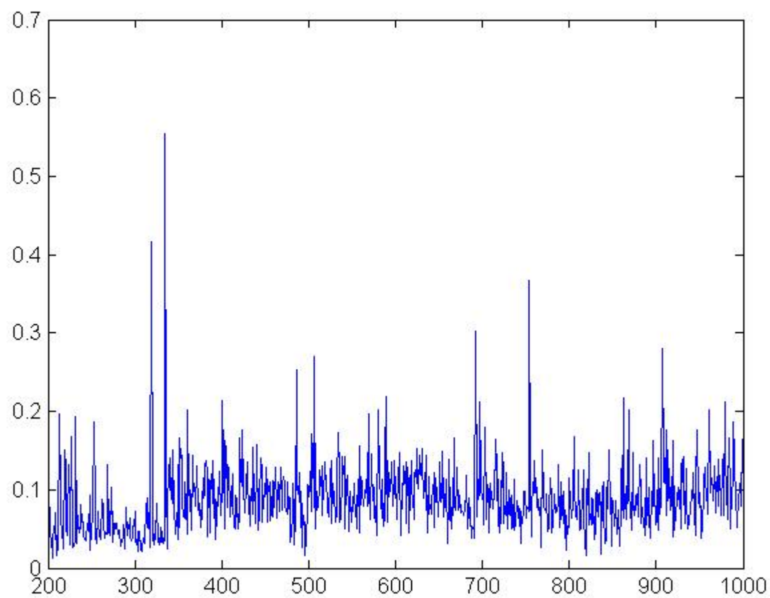


Figure 7.4: Ratio of Non Performing Loans

Figure 7.5 and Figure 7.6 give the default numbers of banks and firms respectively. Firm defaults follow a stable pattern, while bank default stabilize after initializing in the first 100 periods.

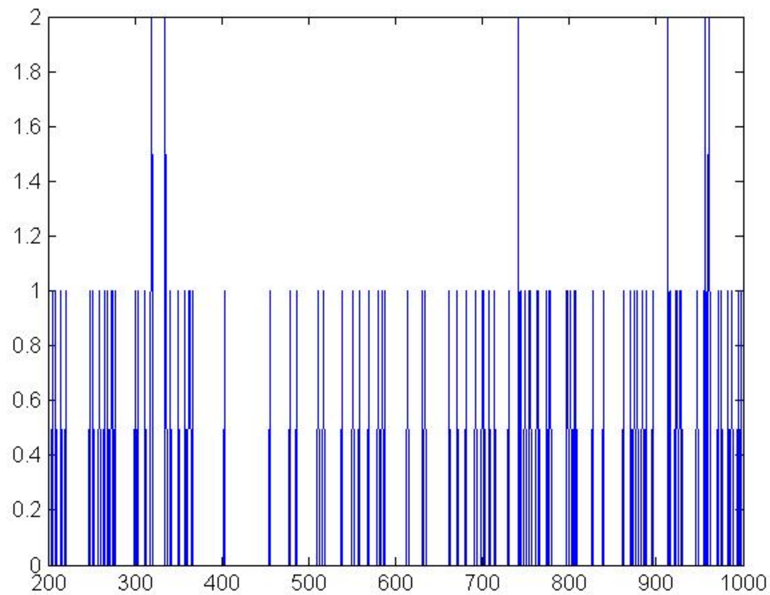


Figure 7.5: Bank Defaults

Figure 7.7 gives the leverage ratios of firms throughout the simulation. As in the case of aggregate production, leverage rates show a realistic pattern after the first 300 periods. However, the pattern is still too volatile. One reason may be exogenous pricing, but leverage rates stay in a realistic band after all.

Figure 7.8 gives the progress of interest rates, which in this case are weighted average values. Interest rates show an increasing pattern because both leverage of banks and firms increase which in turn increase interest rates (Eq. 6.14).

Figures 7.9 and 7.10 show the size distribution of banks and firms at the end of the simulation. Similar to Delli Gatti et al. (2010b) and Riccetti et al. (2011), agents become rapidly heterogeneous, starting from the beginning. Both firm size distribution and bank size distribution at the end of the simulation are right skewed.

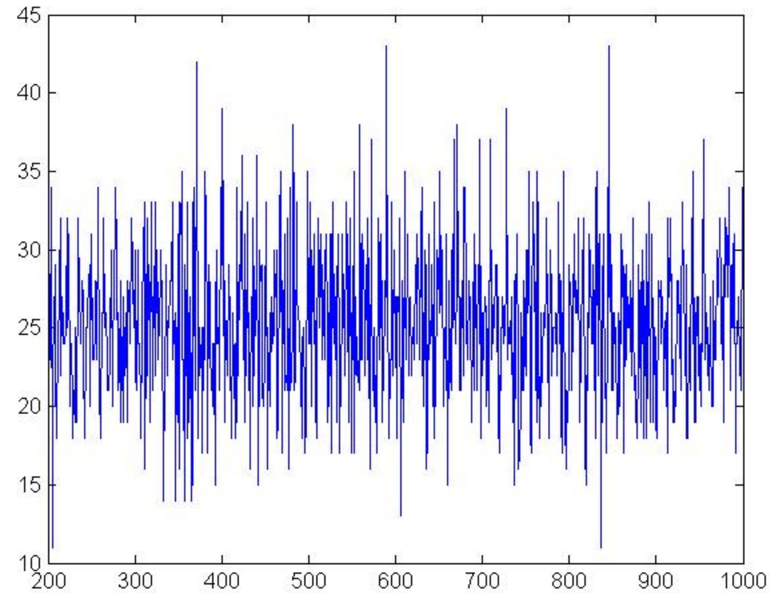


Figure 7.6: Firm Defaults

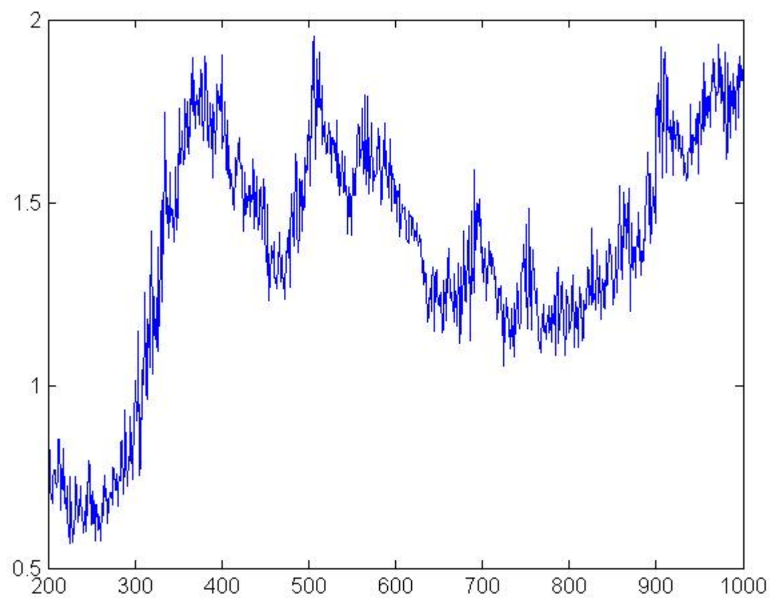


Figure 7.7: Leverage Progress

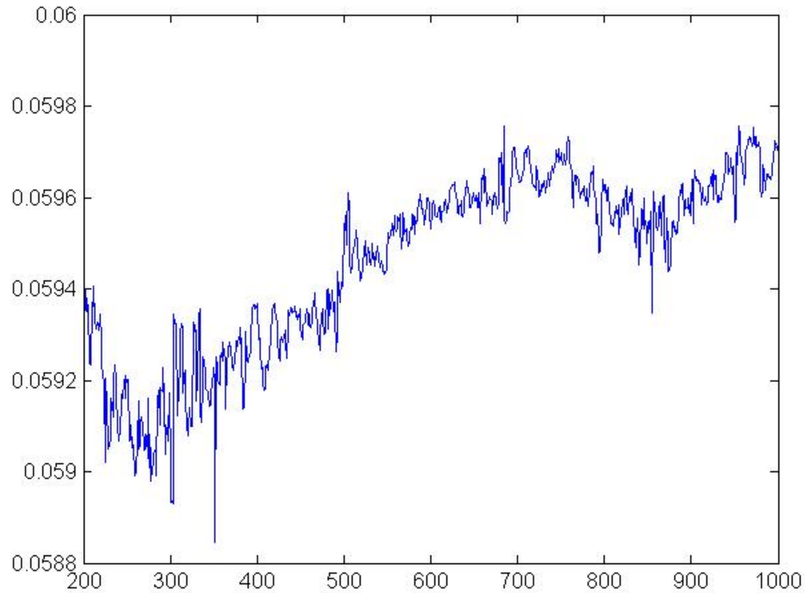


Figure 7.8: Weighted Average Interest Rates

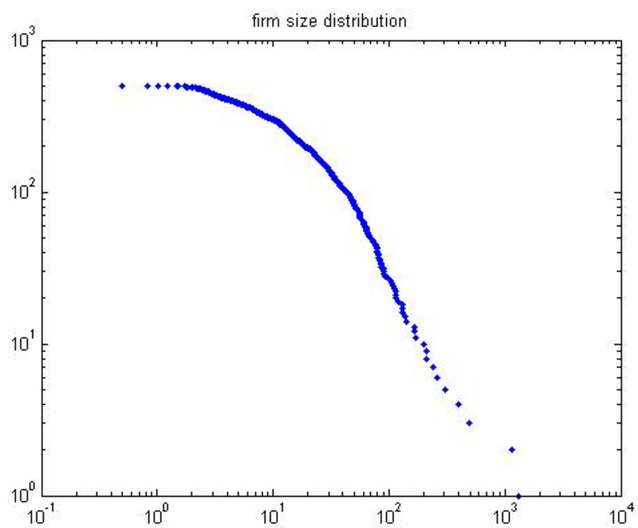


Figure 7.9: Firm Size Distribution

Figure 7.11 shows the degree distribution of banks at the end of the simulation. Similarly degree distribution of the network is also asymmetric as in reference studies because of the endogenous partner selection mechanism. The mechanism is similar to

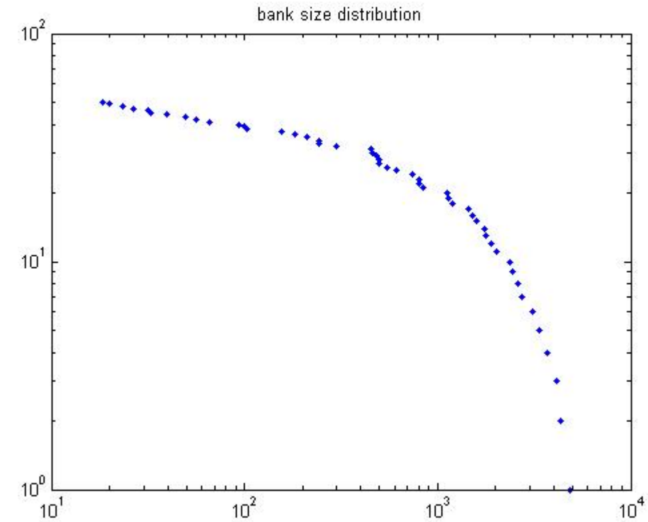


Figure 7.10: Bank Size Distribution

ones in network based financial accelerator models (Delli Gatti et al. 2010b, Riccetti et al. 2011). Banks with higher net worth, which have higher capital adequacy ratio, can provide lower interest rates and thus increase market share. On the firm side, there is a similar mechanism. More robust firms can get lower interest rates, that in turn creates further growth. On the other hand, when a firm goes bankrupt, the banks it owes writes off bad loans. If the debt of the firm to the bank is too high, the bank itself may even go bankrupt. If this is not the case, the bank's net worth decreases, which leads to a decrease in capital adequacy ratio. In turn, the bank increases interest rate for other firms, and this increases interest burden of the firms. This mechanism, which is called "financial accelerator" in the literature, causes avalanches of bankruptcies.

Our results, which are parallel to previous agent base financial accelerator studies, show that idiosyncratic shocks lead to macroeconomic fluctuations, and this in turn increases fragility of agents.

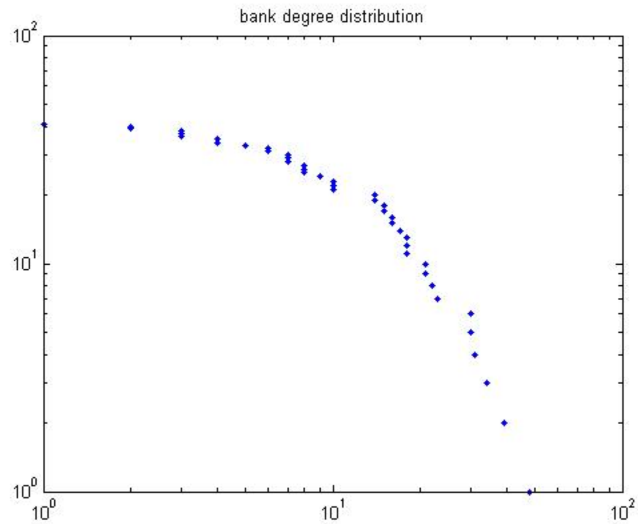


Figure 7.11: Bank Degree Distribution

7.1.1 Robustness Check

To test the robustness of our results, we ran 100 simulations[2]. The results are given in Table 7.3. Our results show that base model is robust, and do not show high volatility among simulations with different seeds. Mean values of parameters do not have high standard deviations among different simulations. We will use the results presented in Table 7.3 to compare with results of various scenarios in the following section.

The results given in the table are valid for periods 0-1000. The time interval between 0-200 is actually a period of initialization and setting up, so some outstanding maximum values are observed which are related to this interval. Excluding the first 200 periods does not change the main results and conclusions.

In neither base simulation nor robustness tests were credit asymmetries observed. This means there is no situation where banks offer credit while firms do not demand it.

[2] In C++ programming language, we used time(NULL) as a seed so each time we run the simulation, different random number are generated

	min	mean	max	std dev
Bad debt ratio mean %	7.69	8.34	8.86	0.23
Bad debt ratio max %	29.84	44.41	72.35	9.37
Bad debt ratio std. dev. %	3.67	4.06	4.51	0.18
Bank defaults mean %	0.79	1.03	1.45	0.14
Bank defaults max %	16	25.18	36.00	4.58
Bank defaults std. dev. %	2.11	2.52	2.90	0.17
Bank net worth mean	49954.74	86255.04	144326.33	19025.01
Bank net worth max	79587.80	176827.15	378882.00	56649.14
Bank net worth std. dev.	20588.03	44475.33	103964.61	15638.91
Total debt min	5233.99	5250.44	5263.53	6.28
Total debt mean	24882.44	27326.41	31300.94	1387.93
Total debt max	37865.50	55817.71	97673.60	10862.90
Total debt std. dev.	4624.98	8188.75	15920.61	2273.61
Firm defaults mean %	4.93	4.99	5.07	0.02
Firm defaults max %	7.80	8.58	9.60	0.38
Firm defaults std. dev. %	0.99	1.04	1.08	0.02
Firm net worth min	5799.13	5930.19	6051.88	56.06
Firm net worth mean	30898.55	39736.76	57704.58	5380.13
Firm net worth max	55070.20	99447.87	211498.00	28572.48
Firm net worth std. dev.	8614.76	21586.64	53546.45	8266.49
Aggregate production min	12411.80	12425.75	12436.90	5.33
Aggregate production mean	28868.08	31044.20	34266.52	1133.60
Aggregate production max	38204.30	48283.84	65440.70	5682.50
Aggregate production std. dev.	3209.67	6058.20	11165.52	1725.48
Interest rate min %	5.86	5.91	5.95	0.02
Interest rate mean %	5.95	5.97	5.98	0.01
Interest rate max %	6.08	6.09	6.09	0.002
Interest rate std. dev. %	0.03	0.03	0.04	0.003
Leverage min	0.00	0.61	0.96	0.13
Leverage mean	0.08	1.25	1.43	0.14
Leverage max	0.33	2.18	2.35	0.19
Leverage std. dev.	0.19	0.32	0.44	0.05
Growth min %	-22.38	-12.35	-7.84	2.94
Growth mean %	0.08	0.10	0.12	0.01
Growth max %	11.21	13.09	14.78	0.72
Growth std. dev. %	2.02	2.18	2.32	0.06

Table 7.3: Robustness Check Results

7.1.2 Sensitivity Analysis

According to Fagiolo et al. (2007), an agent-based modeler must perform a detailed sensitivity analysis which, at the very least, should explore how the results depend on (i) micro-macro parameters, (ii) initial conditions, and (iii) across-run variability induced by stochastic elements. They add that, apart from sampling the space of parameters and initial conditions, researchers need to check the robustness of the results against changes in (i) the distribution of random variables generating noise in the system, (ii) timing and updating mechanisms, and (iii) level of aggregation of microeconomic variables.

In our study, we have made sensitivity analysis tests for the parameters in Table 7.1. We have gradually changed one parameter each time, while keeping others constant. The detailed results of the tests are given in tables from B.1 to B.17. We interpret the results for each parameter, below.

$\underline{\phi}$**:** Remember that ϕ is the multiplier parameter in the production function: $Y_{i,t} = \phi K_{i,t}^{\beta}$. We increased ϕ from 2 to 4 with steps of 0.2. The results are given in Table B.1.

With increasing ϕ, growth standard deviation, skewness, kurtosis, as well as minimum, average, and maximum growth rates increase (Figure 7.12). Bank default rates are not significantly affected by change in ϕ, but firm defaults slightly decrease with increasing ϕ. And finally, interest rates are not affected by changes in ϕ, whereas mean leverage decrease and leverage standard deviation increase. Another significant, although expected, result is that non performing loan percentage increases with ϕ.

$\underline{\beta}$**:** The parameter β is the exponential parameter of production function: $Y_{i,t} = \phi K_{i,t}^{\beta}$. In the initial simulation β has the value 0.7. We gradually increased β from 0.6, to 0.8 in interval of 0.02. A summary of the results are given on Table B.2. The results are parallel with changes in ϕ, but this time, changes are exponential. Effects of changes in β on descriptive statistics of growth are plotted in Figure 7.13. In addition to the results in growth features we observe in the figure, mean values of bank and firm defaults, interest rate and leverage, all decrease with increasing

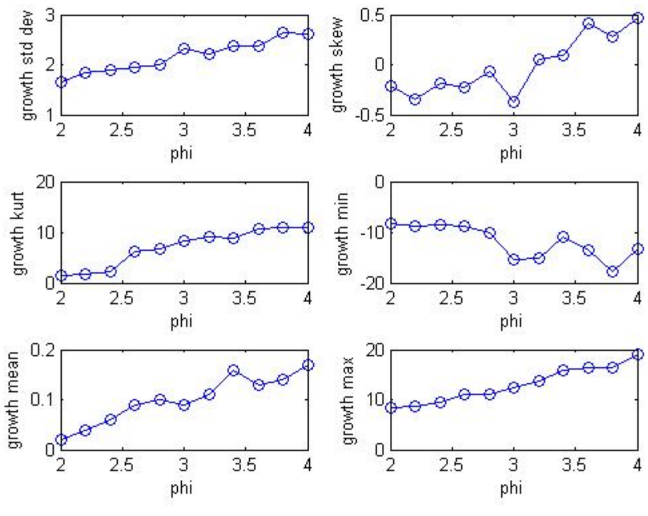


Figure 7.12: Sensitivity analysis of ϕ on growth

β. On the other hand the standard deviation of interest rates and leverage increase significantly.

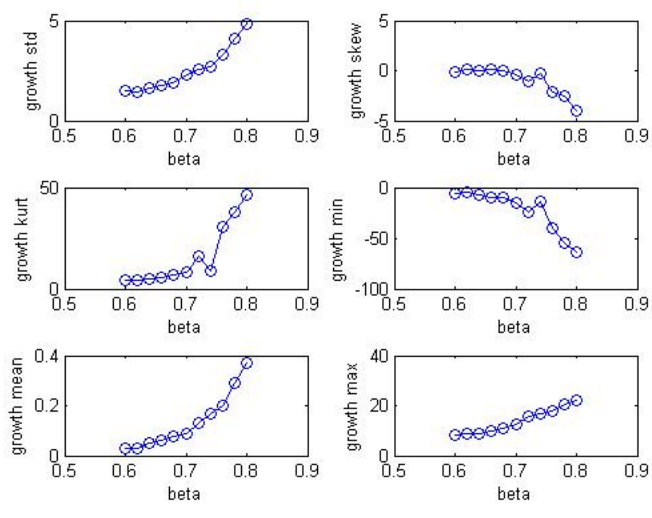


Figure 7.13: Sensitivity analysis of β on growth

<u>Δl_{max}:</u> We have introduced parameter Δl_{max} in Equation 6.7: $\ell_{i,t} = \ell_{i,t-1}(1 \pm \Delta l_{max} \cdot rand)$. It gives the maximum percentage of change in a firm's leverage, relative to previous period. We change value of Δl_{max} from 0.02 to 0.22 in intervals of 0.02.

Results on the parameters are presented in Table B.3. Results on growth variables are presented in Figure 7.14. We observe on the figure that effect of change in Δl_{max} on kurtosis, minimum, mean, and maximum levels of growth is limited and stays at almost the same level after a certain point. On the other hand, standard deviation of growth continues increasing parallel to Δl_{max}.

In addition to changes in growth features, non performing loans, bank defaults, and firm default rate all increase significantly with increasing Δl_{max}. Average interest rate rises from 5.86%, to 6.01%, while standard deviation of interest rate declines gradually. Finally, mean leverage and standard deviation of leverage increase significantly (Table B.3).

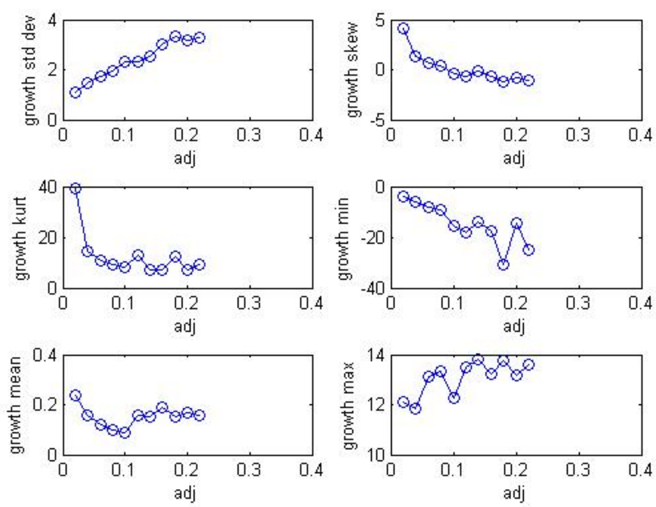


Figure 7.14: Sensitivity analysis of Δl_{max} on growth

$\underline{u}$**:** u is the expected gross profit which determines the price of the firm in profit function, 6.10. In each period, sale price of the firm is determined according to the equation: $p_{i,t} = u + v_{i,t}$. We run simulations with u, varying from 0.02 to 0.22 in intervals of 0.02. The results are presented in Table B.4. Growth variables change as seen in Figure 7.15. Standard deviation first decrease sharply and then increase gradually; average growth stays constant with u greater than 0.08, but there is no clear trend for other variables. Bank defaults show a similar pattern to growth standard deviation, while firm defaults gradually decrease. Interest rates also increase gradually while interest rate standard deviation first decrease sharply and then remain horizontal.

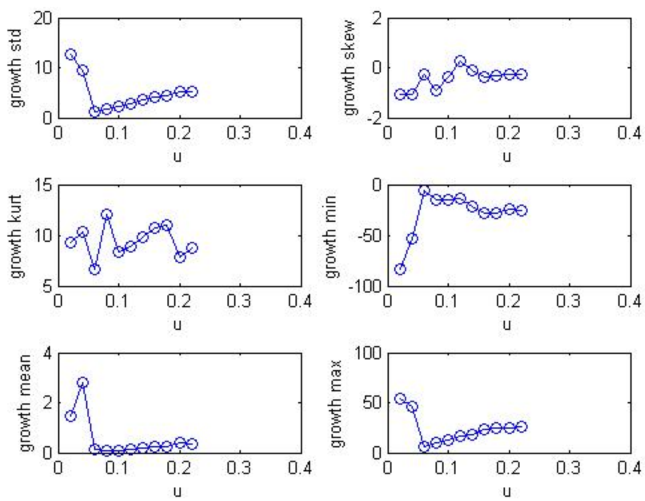


Figure 7.15: Sensitivity analysis of u on growth

$\underline{\sigma^2\textbf{:}}$ σ^2 is also part of random price function, and is the variance of the term $v_{i,t}$. The results of sensitivity analysis of σ^2 are given in Table B.5. Standard deviation of growth initially increases and then decreases after $\sigma^2 = 0.2$. Skewness decreases with variance and mean growth rate first increases and then stays at constant level. As expected, bank defaults, firm defaults, and non performing loan percentage all increase sharply with variance. Interest rates, mean leverage and standard deviation of leverage decrease with increasing variance. On the other hand, standard deviation of interest rates first increases and then stays constant.

$\underline{\theta\textbf{:}}$ In our model, liquidity bankruptcy plays an important role. Bankers are concerned about cash flows while assessing the credibility of a firm. We assume that a determined portion of firms' total capital is liquid (Equation 6.12). A bank's net cash flow consists of profits plus liquid assets, less interest and principal payments.

Resulting growth trends with changing θ are given in Figure 7.16. Growth standard deviation, kurtosis, and mean growth stay steady for approximately $\theta \geq 0.5$. Skewness increases gradually with increasing θ, and is positive for $\theta \geq 0.70$.

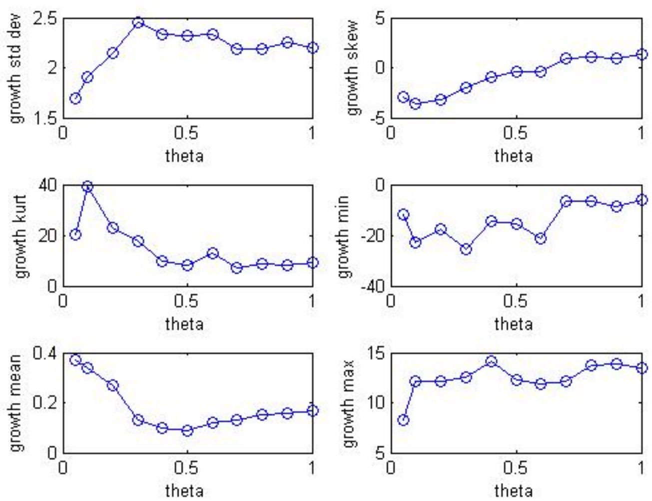


Figure 7.16: Sensitivity analysis of θ on growth

An interesting case occurs with sensitivity analysis of θ (see Figure 7.17). With increasing θ, both median and mean bad debt increase and then stay steady, and median bad debt converges to mean bad debt. Bank and firm defaults, on the other hand, decrease sharply with increasing θ. Standard deviation of interest rates decrease sharply with increasing θ from 0.09 to 0.02. Finally, both leverage, and standard deviation of leverage increase significantly.

lqlim: The parameter *lqlim*, determines whether a firm is accepted as liquidity bankrupt. If $liq < 0$, the firm cannot create cash from its operations. Then if $|liq/Y| > lqlim$, (ratio of liquidity need to production level is above a certain level) the firm is accepted as liquidity bankrupt.

Detailed results of the sensitivity analysis are presented on Table B.7. The results of growth are presented in Figure 7.18. With increasing *lqlim*, standard deviation of growth gradually decreases, while mean growth shows no significant change. In

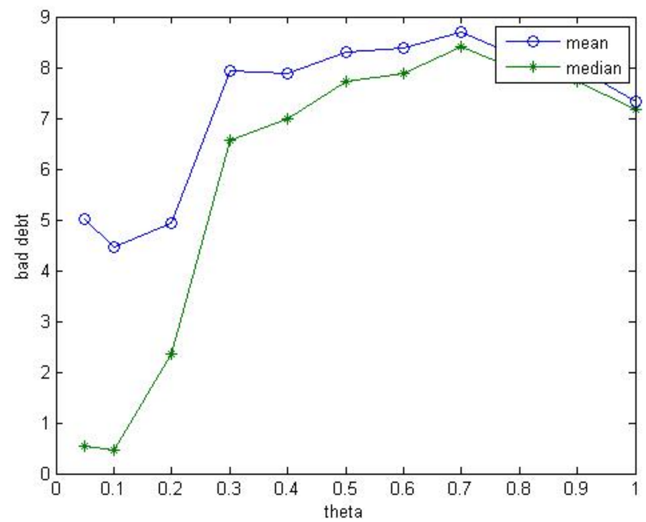


Figure 7.17: Sensitivity analysis of θ on bad debt

addition, there are significant decreases in bad debt ratio, bank defaults, firm defaults, standard deviation of interest rates, and standard deviation of leverage. The results support the idea that, if banks support firms in need of liquidity, this will decrease the financial fragility. We believe however that for this to happen, all banks must cooperate.

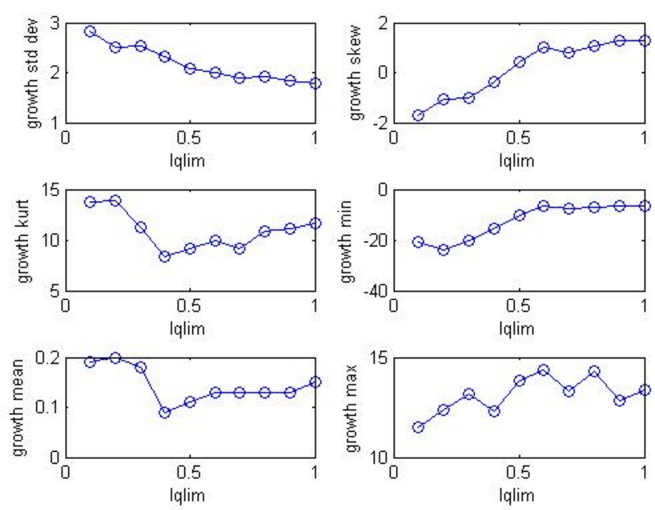


Figure 7.18: Sensitivity analysis of $lqlim$ on growth

rmin: Next parameter for sensitivity analysis is $rmin$, which is the minimum or policy interest rate. In our model, this rate is also interest rate paid for deposits (see Section 6.4) [3]. Detailed results are on Table B.8.

Effects of changing $rmin$ on growth dynamics are presented in Figure 7.19. Standard deviation of growth decrease and then follows a steady trend for $rmin \geq 0.05$, and mean growth rate becomes horizontal after a certain level of $rmin$. On the other hand, non performing loans, bank defaults, and firm defaults all decrease steadily.

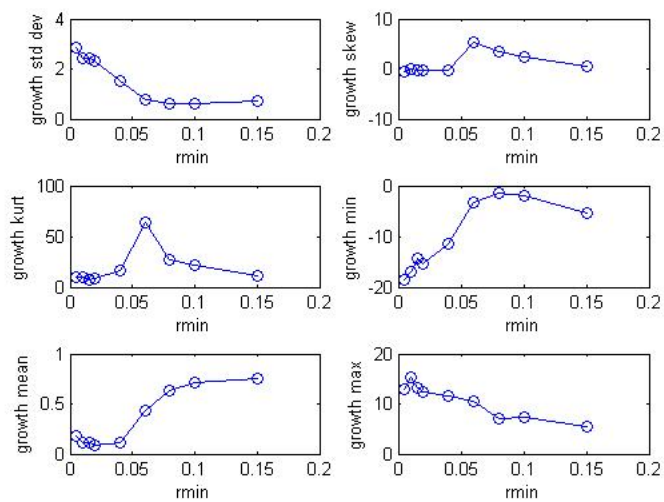


Figure 7.19: Sensitivity analysis of $rmin$ on growth

$\underline{\gamma}$**:** γ is used in interest rate setting formula: $r_{z,i} = rmin + f_1(\cdot) + f_2(\cdot)$ where $f_1(CAR_{i,t}) = \gamma CAR^{-\gamma}$. We observed the changes in results by changing γ from 0.005, to 0.05 in intervals of 0.005 (see Table B.9). The changes in growth figures are seen on Table 7.20. Standard deviation of growth steadily decreases with increasing γ while mean growth stays at almost the same level at $\gamma \geq 0.01$. With increasing γ, bad debt, bank defaults, and total debt increase, while firm default rate shows no significant difference.

[3] In our study, $rmin$ is exogenous, but for studies about central bank intervention, an endogenous mechanism for $rmin$ would be enlightening

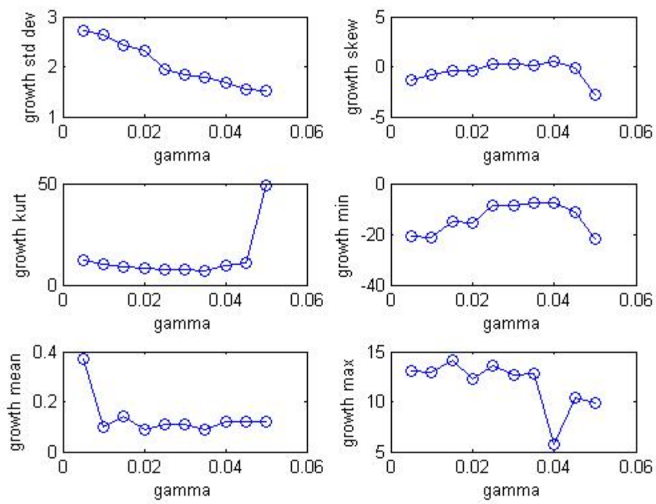


Figure 7.20: Sensitivity analysis of γ on growth

$\underline{\alpha}$**:** Next parameter is used in turn firm–specific part of interest rate pricing, given in Equation 6.17. Results of sensitivity analysis for α are shown in Table B.10. We have gradually increased α from 0.005, to 0.05 in intervals of 0.05. The results shown in Table B.10. With increasing α, the standard deviation of growth decreases steadily. On the other hand, other statistics do not show any regularity; mean growth stays almost at the same level, total debt and bank defaults decrease sharply with increasing α, while firm defaults are not affected. In contrast, both interest rates and standard deviation of interest rates increase.

RR: When a firm goes bankrupt, banks granted credit are able to collect their receivables at a ratio of RR. We have simulated our model with RR values between 0.1 and 1 at intervals of 0.1. The results are given in Table B.11. In Figure 7.21 results on growth statistics with increasing RR values can be seen. Standard deviation, and kurtosis of growth decrease, while skewness and minimum growth increase. On the other hand, mean and maximum growth rate stay steady.

As expected, bank defaults decrease and firm defaults are unaffected by increasing recovery rate. Mean interest rate and standard deviation of interest rate gradually decrease.

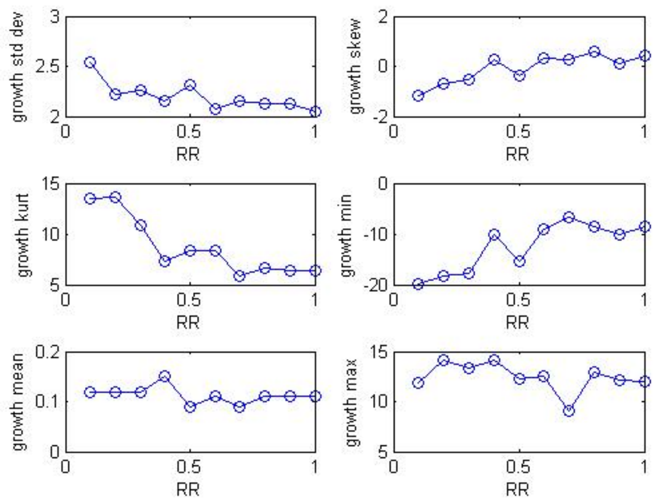


Figure 7.21: Sensitivity analysis of RR on growth

<u>CAR^*</u>**:** The amount of total credit that can be granted by a bank is limited by minimum capital adequacy ratio (CAR). CAR requirement is set by Basel requirements, in order to limit the amount of risk a bank can take and thus protect depositors. According to Basel requirements, credits are weighted according to their risk. In our model, we let $CAR = A/L$ for simplicity as there are no risk differentiation among assets. Minimum CAR requirements differ among countries. We run simulations with minimum capital adequacy ratio, from 0.04, to 0.22, increasing with 0.02 steps. Table B.13, gives detailed results and results on growth figures are given in Figure 7.22. It is noteworthy that as CAR^* increases, average growth rate decreases and standard deviation of growth first increases and then decreases after a certain level.

Changes in other variables are shown on Figure 7.23. With increasing CAR^*, bank defaults and leverage decrease significantly; and interest rates and firm defaults

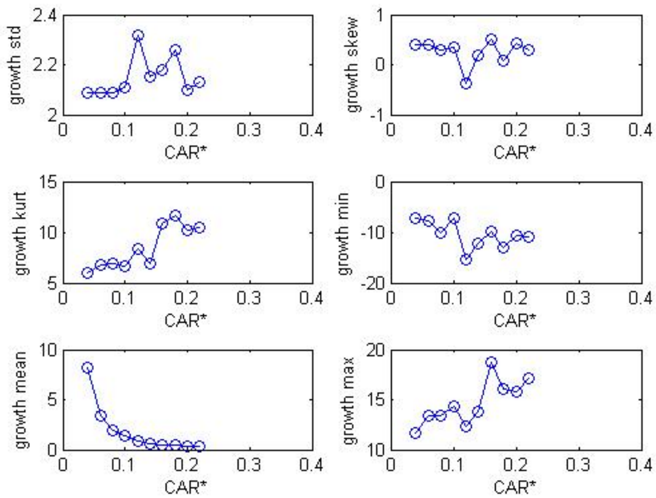


Figure 7.22: Sensitivity analysis of CAR^* on growth

decrease slightly. Standard deviation of interest rates move within a band. We can say that increasing CAR^* stabilizes the economy while decreasing mean growth rate.

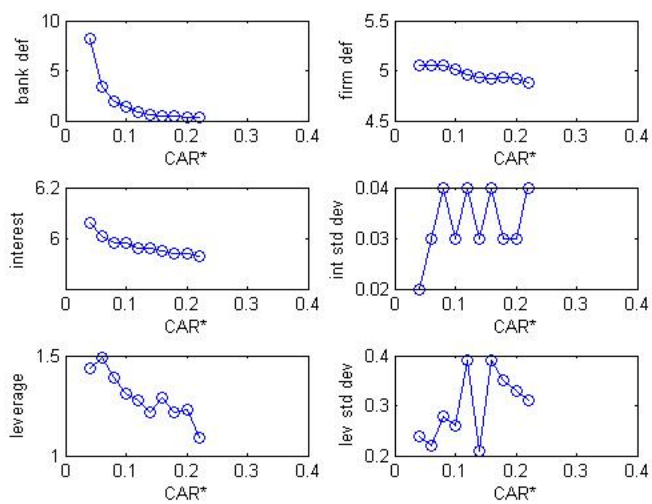


Figure 7.23: Sensitivity analysis of CAR^* on defaults, interest, and leverage

<u>MB :</u> Although not set by written rules, number of banks a firm does business with varies among countries. Firms prefer working with at most 3 banks in some countries, while the number is much higher in others. For example, in countries where financial

crises are frequent, firms prefer to work with as many banks as possible. In our original simulation above, the maximum number of banks a firm can work with is 15. We run sensitivity analyses with various MB between 7 and 25. Our simulations brought no significant variations in any result with increasing MB (see Table B.14). The same is true for the number of banks, n, that a firm asks for interest rates every term, other than current banks.

$\underline{\lambda :}$ In our model credit term is determined randomly with the help of exponential distribution. Cumulative distribution function of exponential function is given by $1-e^{-\lambda x}$. A sample plot of cumulative distribution function of exponential distribution is given in Figure 7.24. To get the term of a credit we first get a real number, uniformly distributed between 0 and 1 [4]. This number gives the cumulative distribution value of the exponential distribution. Using that value and given λ, we get x from $1-e^{-\lambda x}$. Then we find the term using Equation 6.20.

λ is important because higher values increase the probability of long–term credits. Note that we limit maximum term with D (which is 4 in our base model) in our model. Increasing λ therefore actually causes average credit term, converge to allowed maximum term.

We have simulated our model with λ values between 0.1 and 1.0 and incrementing by 0.1. Table B.16 shows detailed results. The results of growth statistics are shown in Figure 7.25. With increasing λ, standard deviation of growth also increases gradually, while average growth rate follows an opposite pattern. Minimum and maximum growth rates also decrease with increasing λ. On the other hand, bank defaults decrease, and firm defaults increase.

Figure 7.26 gives bank and firm defaults rates, interest rate, interest rate standard deviation, leverage and leverage standard deviation. With increasing λ, (and increasing credit terms), bank defaults decrease and firm defaults increase gradu-

[4] We used 'random' library of C++ to get a random number in a specific distribution

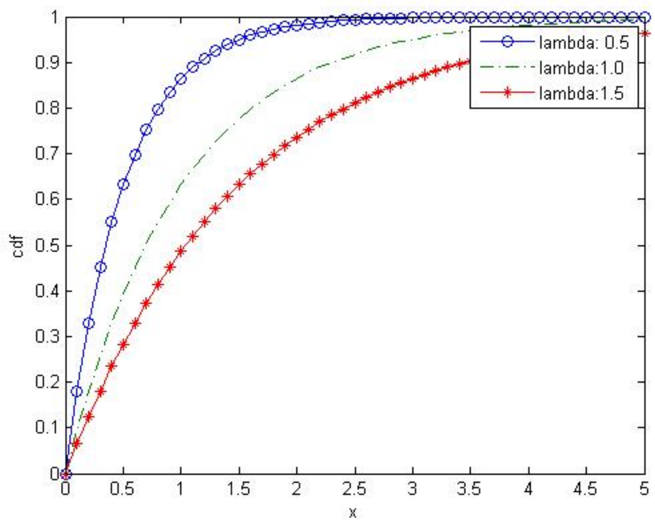


Figure 7.24: Cumulative distribution function

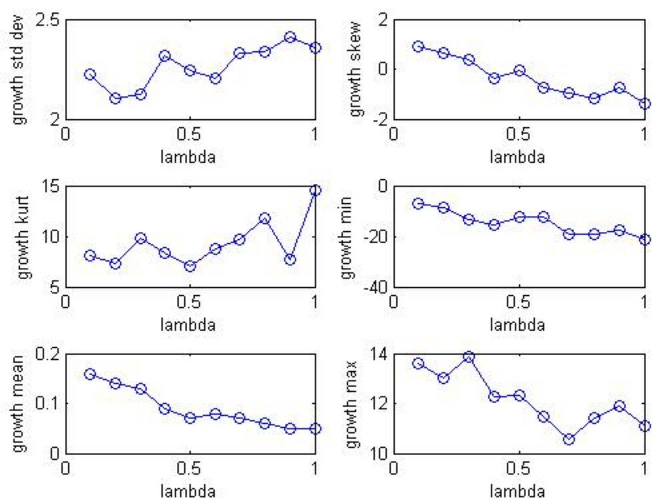


Figure 7.25: Sensitivity analysis of λ on growth

ally. Weighted average interest rates decrease with an increasing standard deviation. Finally, leverage decreases but standard deviation of leverage shows no significant change.

<u>D :</u> Maximum term of credits is limited by D, which we believe is an important factor of financial structure. Sensitivity analysis was performed by increasing D from

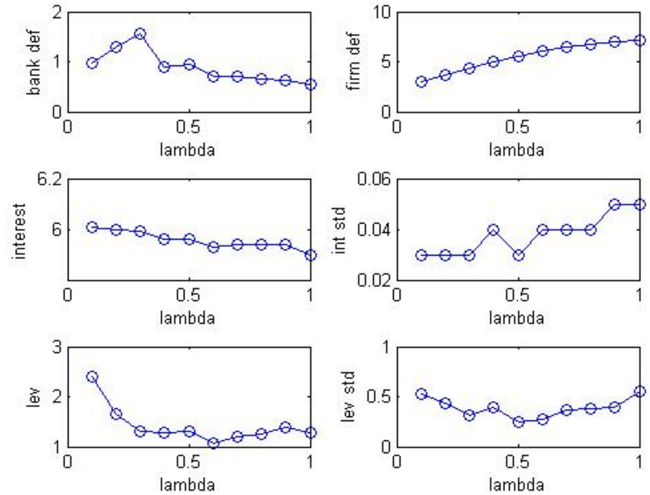


Figure 7.26: Sensitivity analysis of λ on default, interest rate, and leverage

2 to 10. The results are given in Table B.17 and the results of growth statistics with increasing D are given in Figure 7.27. Average growth rate increases with increasing D, while standard deviation, skewness and kurtosis stays highly stable after $D = 3$. Similarly, other variables also show a steady trend after $D = 3$.

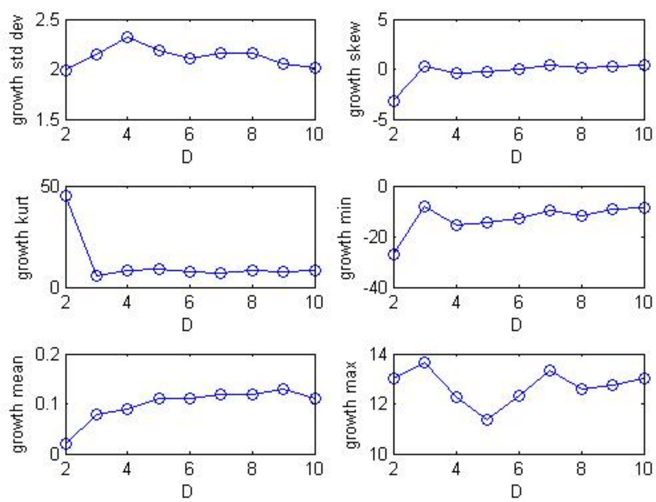


Figure 7.27: Sensitivity analysis of D on growth

7.2 Testing Different Scenarios

In our base model, we have constructed a mechanism where we believe banks care about financial robustness while pricing. From that point, we will simulate different scenarios to test whether differences in bank behavior leads a change in financial fragility and instability. We will then compare the results of new scenarios with average values of robustness tests.

7.2.1 Bank Leverage in Pricing

In this part, we will slightly adjust the pricing mechanism in order to see what happens when banks take their own leverage into account when pricing. This approach to pricing again seems to be similar to the previous one because banks try to preserve financial robustness. We modify Equation 6.11 and set $f_1(\cdot)$ as $\gamma * \ell_{z,t}^{\gamma}$ where $\ell_{z,t}$ is the leverage of bank z at time t, which is simply D/A. The results of this setting are summarized in Table 7.4. Interest rate and leverage course are given in Figures in 7.28 and 7.29.

The results given in the table are valid for periods 0-1000. As, the interval between 0-200 is actually a period of initialization and setting up, some outstanding maximum values are observed in this interval. Excluding the first 200 periods does not change the main results and conclusions. To give a better visualisation of resulting trends we present periods 200 - 1000 in the related graphs.

In this scenario mean values for bad debt ratio, bank defaults, and firm defaults are even higher than the highest respective means in the base model. Total bank net worth decreases due to increased bank defaults. On the other hand, output, growth, interest rate and leverage ratios are similar to the base model on both average, minimum and maximum values (except interest rate has a higher standard deviation than the base model).

	Minimum	Mean	Maximum	Std. Dev.
Bad Debt Ratio %	0.00	9.45	51.78	3.75
Bank Defaults %	0.00	4.87	26.00	4.04
Bank Net Worth	2617.77	12943.91	69579.00	12643.82
Total Debt	5236.77	18728.14	36519.10	4519.38
Firm Defaults %	0.00	5.44	8.80	1.06
Firm Net Worth	5936.38	23602.14	57738.60	11770.63
Aggregate Production	12414.20	23792.44	32830.00	2785.97
Interest Rate %	5.83	5.99	6.05	0.07
Leverage	0.66	1.52	2.34	0.41
Growth %	-11.96	0.10	13.10	2.10

Table 7.4: Results with Bank Leverage Pricing Mechanism

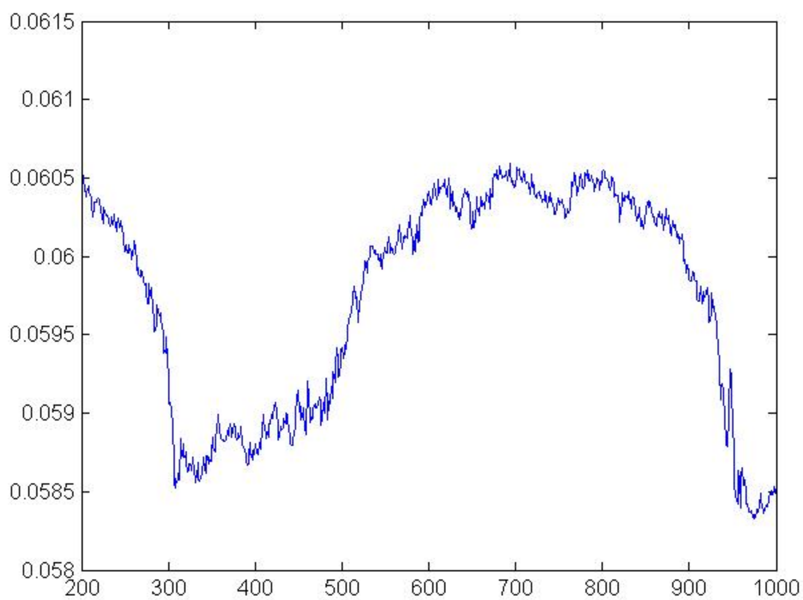


Figure 7.28: Interest Rates in Leverage Pricing

Minsky claims that to control the disruptive influence of banking, it is necessary to limit leverage ratios and constrain the growth of bank equity to a rate that is compatible with noninflationary economic growth (Minsky, 1988). Our results in this scenario weakly support Minsky's claim. The results of leverage scenario point to a slightly less stable economy. Thus, when there is no limit on leverage of banks, the economy is more volatile, even if banks are concerned about their leverage.

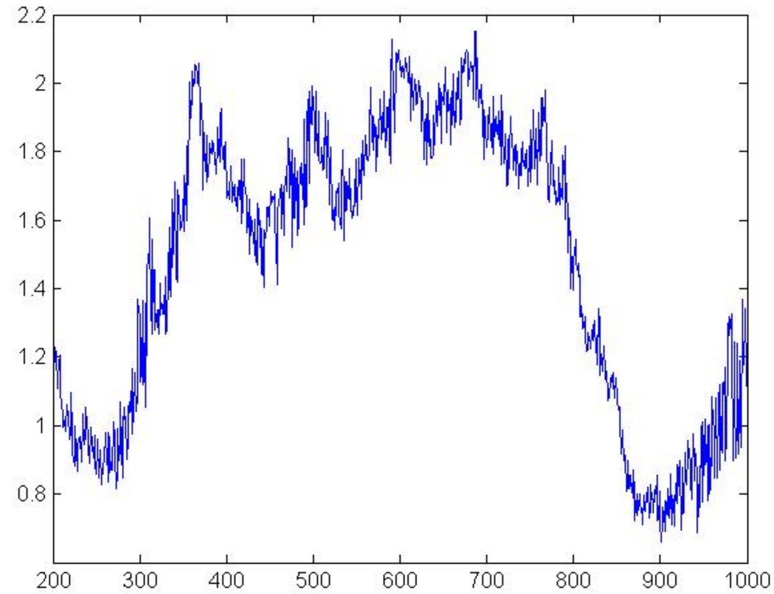


Figure 7.29: Leverage in Leverage Pricing

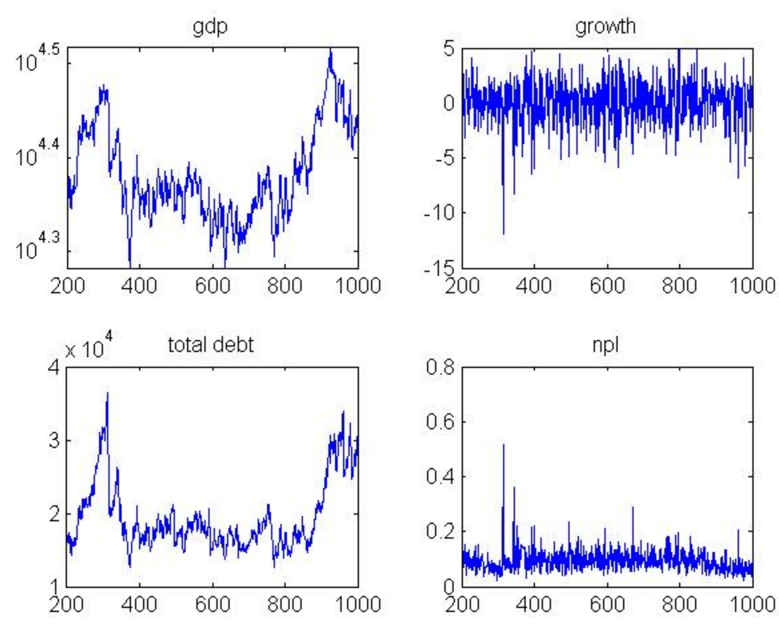


Figure 7.30: Output, growth, debt and bad debt in Leverage Pricing

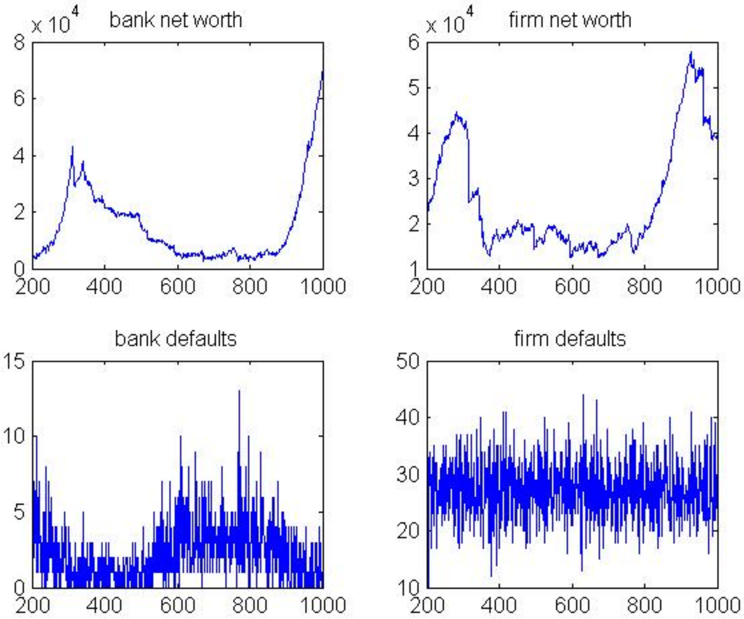


Figure 7.31: Size and default figures of banks and firms in Leverage Pricing

7.2.2 Cost of Equity

In the base model, we assumed that firms can fully liquidize equity in the case of bankruptcy, that is $A_{i,t} = A_{i,t-1} + \pi_{i,t}$. In real life however, if a firm is in a difficult financial condition, there is a cost of equity liquidation. Moreover, most firms pay dividends and tax, so $A_{i,t}$ is not fully transferred to the next term. In this section we will test a more realistic version of Eq. 6.11 that is:

$$A_{i,t+1} = \upsilon A_{i,t} + \pi_{i,t} \tag{7.1}$$

where υ is the ratio of equity that can be liquidized in case of need.

In this part we will analyze the results of the simulation with this setting. In our first test, we have assumed υ to be equal to 0.5. Later we will conduct sensitivity analysis by changing this value. The results with $\upsilon = 0.5$ are given in Table 7.5.

	Minimum	Mean	Maximum	Std. Dev.
Bad Debt Ratio %	0.40	22.65	62.65	8.89
Bank Defaults %	0.00	11.08	42.00	6.78
Bank Net Worth	20.00	423.86	1192.91	147.73
Total Debt	865.48	1296.55	5226.03	287.63
Firm Defaults %	0.40	10.66	20.00	1.48
Firm Net Worth	520.72	629.91	3433.35	119.08
Aggregate Production	3146.06	3596.03	12423.00	420.61
Interest Rate %	6.01	6.07	6.13	0.01
Leverage	1.04	3.14	5.56	0.46
Growth %	-23.40	-0.02	12.40	4.64

Table 7.5: Results of Costly Liquidation Scenario

The results given in the table are valid for periods 0-1000. As, the interval between 0-200 is actually a period of initialization and setting up, some outstanding maximum values are observed in this interval. Excluding the first 200 periods does not change the main results and conclusions. To give a better visualisation of resulting trends we present periods 200 - 1000 in the related graphs.

The course of interest rates in this mechanism is given in Fig. 7.32. Compared to the base model, level of interest rates increase but standard deviation decreases.

Aggregate production, growth rate, total debt, and non-performing loan rate figures are given in Table 7.33. In this mechanism, mean growth is below the minimum mean level that is seen in robustness check results (Table 7.3) while its standard deviation is higher. On the other hand, compared to the base model, total debt is significantly lower, while mean bad debt ratio is much higher.

Total bank and firm net worth and amount of bank and firm defaults are given in Table 7.34. Size of bank and firms are significantly lower in the current scenario, while the ratio of defaults greatly increases.

Finally, aggregate leverage ratio of production sector is given in Figure 7.35. In this scenario, mean leverage ratio and standard deviation of leverage increase significantly.

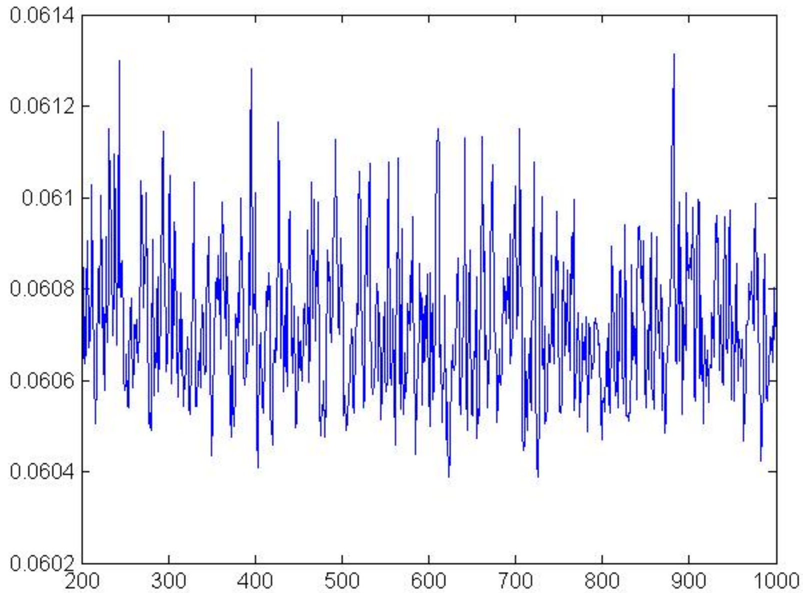


Figure 7.32: Interest rates in costly equity liquidation mechanism

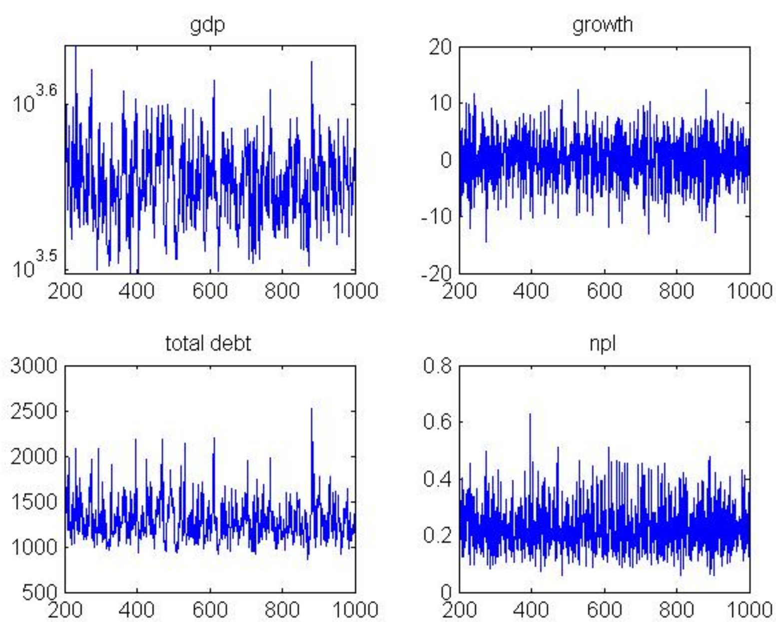


Figure 7.33: Output, growth, debt, and bad debt in costly equity liquidation mechanism

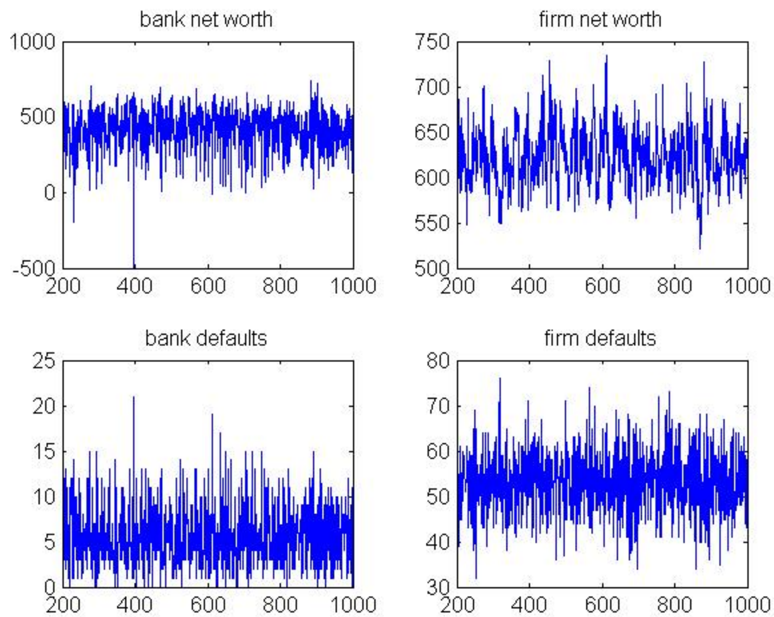


Figure 7.34: Size and default figures of banks and firms in costly equity liquidation mechanism

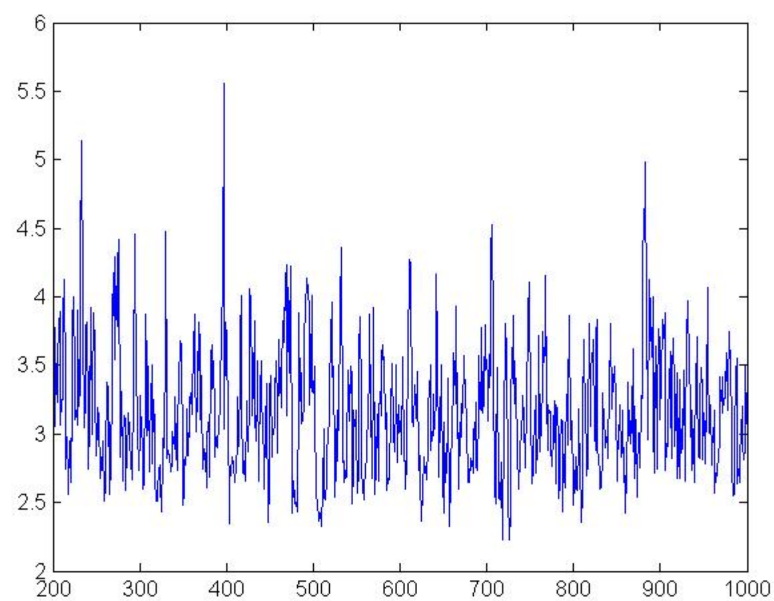


Figure 7.35: Leverage in costly equity liquidation mechanism

Sensitivity analysis of this scenario was performed by increasing υ from 0.1 to 0.9 by 0.1 increments. The results are in Appendix, Table B.20. As expected, financial stability increases with the ratio of the equity that can be liquidated. While interest rates do not differ significantly, standard deviation of interest rates decrease with increasing υ. Growth rate throughout the simulation do not show a regularity with different υ values, but as υ increases, standard deviation of growth, mean bad debt ratio, bank and firm defaults decrease; on the other hand total net worth of banks and firms, and aggregate production increase.

We believe this variation in liquidation of equity, which can also be interpreted as cost of equity, is more realistic according to the base model. As cost of equity increases, financial instability also increases, which in turn affects banks' lending conditions. This, in turn, causes an increase in fluctuations of growth.

When banks increase dividends, or when retained earnings fall for any reason, financial fragility increases. When instability increases, asset prices decrease and cost of liquidizing equity increases further.

7.2.3 Market Share Pricing

In real practice, banks attribute importance to market share and sometimes give up current profit to increase market share. We will now test what happens when banks prioritize increasing market share when pricing credits. In this setting, a bank with lower market share sets lower interest rates to increase its market share. So, the interest rate equation in $f_1(\cdot)$ in Eq. 6.14 becomes:

$$f_1(MS_{i,t}) = \gamma MS^{\gamma} \tag{7.2}$$

The results of this pricing are given in the Table 7.6. The results given in the table are valid for periods 0-1000. As, the interval between 0-200 is actually a period

	Minimum	Mean	Maximum	Std. Dev.
Bad Debt Ratio %	0.00	8.60	43.72	3.74
Bank Defaults %	0.00	3.22	22.00	2.94
Bank Net Worth	3742.90	41225.72	77623.50	19509.42
Total Debt	5250.66	21152.52	38959.30	4028.85
Firm Defaults %	0.00	5.35	9.20	1.05
Firm Net Worth	5822.35	26959.19	50379.40	8598.55
Aggregate Production	12425.90	25651.27	31484.20	2395.69
Interest Rate %	5.60	5.82	5.86	0.03
Leverage	0.73	1.41	2.22	0.30
Growth %	-10.69	0.08	11.46	2.19

Table 7.6: Results with Market Share Pricing Mechanism

of initialization and setting up, some outstanding maximum values are observed in this interval. Excluding the first 200 periods does not change the main results and conclusions. To give a better visualisation of resulting trends we present periods 200 - 1000 in the related graphs.

The most significant result of this scenario is observed in interest rates, as seen in Figure 7.36. There are sharp and sudden decreases in interest rates. Also the minimum, mean, and maximum values for interest rate is below the figures for interest rate in base model, as expected. This is because banks with lower market shares decrease interest rates to gain market share, which in turn, pulls market interest rates down. We can say that, when banks take into account market share for pricing, the effect is increasing financial instability. Comparing Figures 7.8 and 7.36, we observe that short term fluctuations are higher in market share pricing mechanism.

Aggregate output, growth, total debt and non-performing loan figures are given in Figure 7.37. The average growth rate throughout the simulation is slightly lower than average growth of base model. Similarly, average output is also lower than that of base model.

Total net worth and defaults of banks and firms are given in Figure 7.38. A large percentage of bank defaults in this scenario is noteworthy: 1.03% in base model

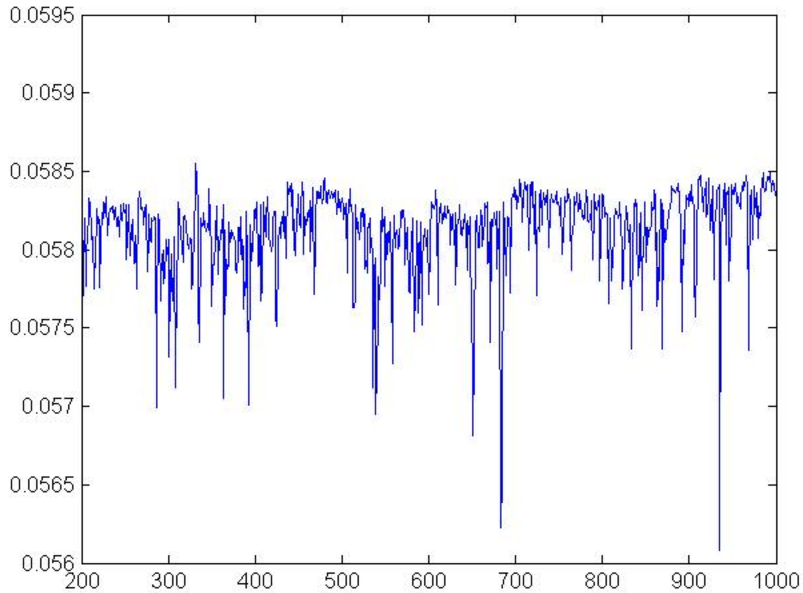


Figure 7.36: Interest Rates in Market Share Pricing Mechanism

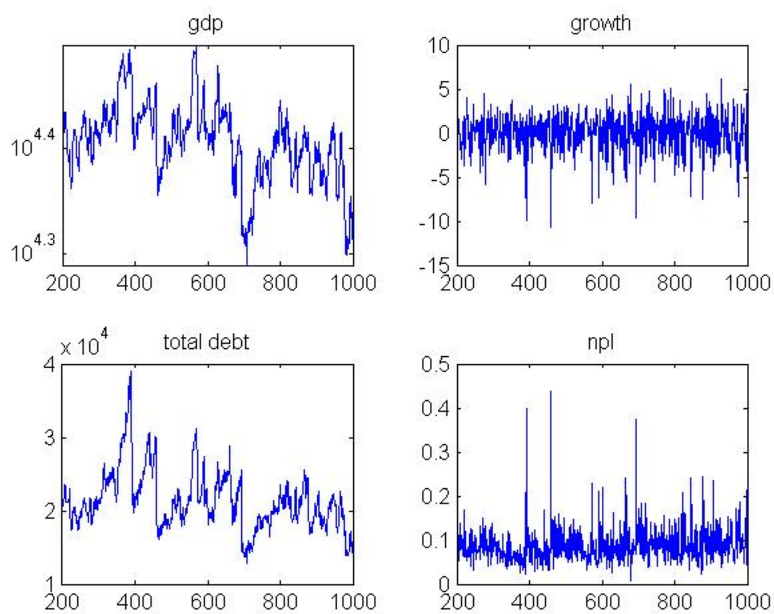


Figure 7.37: Output, growth, debt, and bad debt in market share pricing

and 3.22% in market share pricing mechanism. One cause of this may be that when small banks with low market share set low interest rates, their financial situation is weakened. As expected, as a result of the this, mean and maximum values for bank net worth and total debt are less than (or close to the minimum levels) the minimum values of the base model (compare Tables 7.3 and 7.6).

Another significant result is that, average ratio of firm defaults is above even the maximum of average firm defaults in the robustness check tests of the base model. Parallel to this, average and maximum values of firm net worth in the market share pricing scenario, are below the minimum values of the base model. The same is also true for the aggregate production.

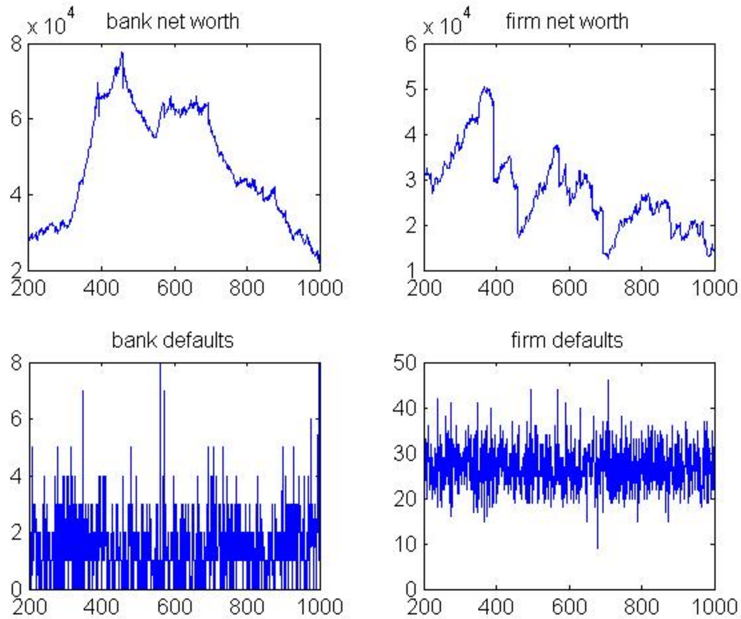


Figure 7.38: Size and default figures of banks and firms in market share pricing

We can claim that when banks prioritize their market shares in pricing, financial instability increases while there is no significant effect on growth.

7.2.4 Recent Economic Growth

In this section, we assume that banks become optimistic in growth times and pessimistic in times of recession. We will change our interest rate equation such that banks lower interest rate when economy grows and vice versa. Thus, the interest rate setting equation of Equity 6.14 becomes:

$$r_{z,i,t} = rmin_t + f_1(CAR_{i,t}) + f_2(l_{i,t}, A_{i,t}) - \rho k \tag{7.3}$$

where k is the rate of growth in output in the previous period and ρ is a smoothing coefficient. In our first testing we let ρ to be 0.10.

The results of this mechanism are given in Table 7.7. The results given in the table are valid for periods 0-1000. As, the interval between 0-200 is actually a period of initialization and setting up, some outstanding maximum values are observed in this interval. Excluding the first 200 periods does not change the main results and conclusions. To give a better visualisation of resulting trends we present periods 200 - 1000 in the related graphs.

In this scenario, bad debt ratio increases to 9.62% from 8.34% in base model. Mean growth rate fall from 0.10%, to 0.07%; while standard deviation of growth increases from 2.18% to 2.23%. Average interest rate rise to 6.03% from 5.97%, while standard deviation of interest rate increases significantly from 0.04% to 0.17%.

Finally, firm and bank defaults also increase significantly. Firm defaults rise from 4.99% to 5.48%, with standard deviation increasing from 0.02% to 1.09%. Similarly, bank defaults increase to 4.96% from 1.03% while standard deviation of bank defaults increase to 4.32% from 0.14%. These results show that when banks calibrate their pricing according to past growth of output, which is a common real life practice, both economic and financial instability increase.

	Minimum	Mean	Maximum	Std. Dev.
Bad Debt Ratio %	0.00	9.62	44.11	3.88
Bank Defaults %	0.00	4.96	24.00	4.32
Bank Net Worth	1705.70	11528.51	40106.90	11081.87
Total Debt	5262.46	17846.98	32345.10	3290.88
Firm Defaults %	0.00	5.48	10.00	1.09
Firm Net Worth	68.18	20273.42	41433.80	6646.01
Aggregate Production	12435.90	23093.32	30243.10	2179.18
Interest Rate %	4.87	6.03	6.73	0.17
Leverage	0.86	1.59	2.34	0.31
Growth %	-10.72	0.07	15.13	2.23

Table 7.7: Results with Growth Pricing Mechanism

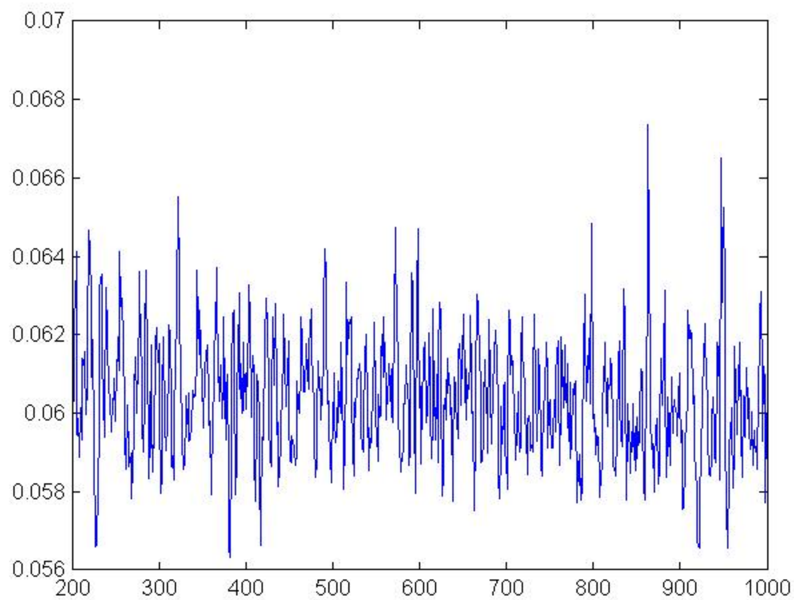


Figure 7.39: Interest Rates in Growth Pricing

We performed a sensitivity analysis of this scenario by varying values of ρ from 0.04 to 0.2 in intervals of 0.02. The results are given in Table B.21. The aim was to understand the changes in results when banks attribute various weights to past economic growth. The effects of changes on ρ on growth figures are given in Figure 7.43. The most noteworthy result is that the more banks attack importance to past growth, the more volatile the economy becomes. Other results show irregularities.

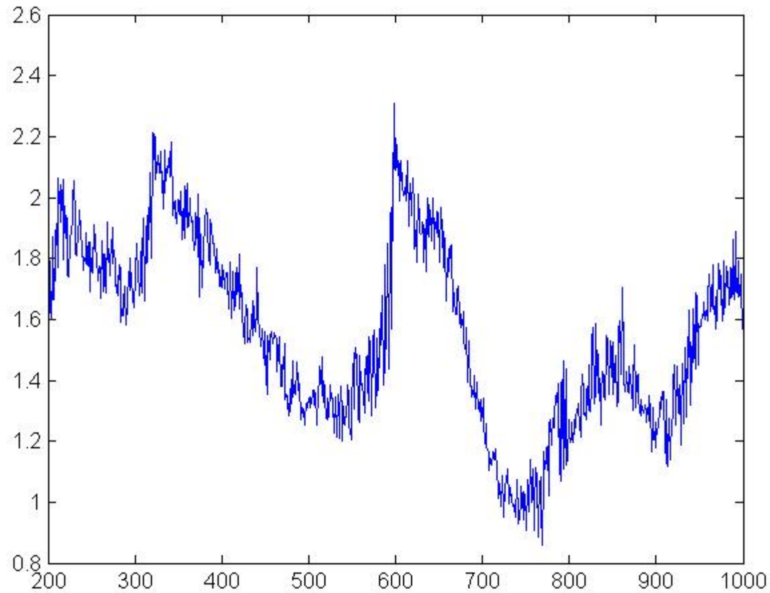


Figure 7.40: Leverage in Growth Pricing

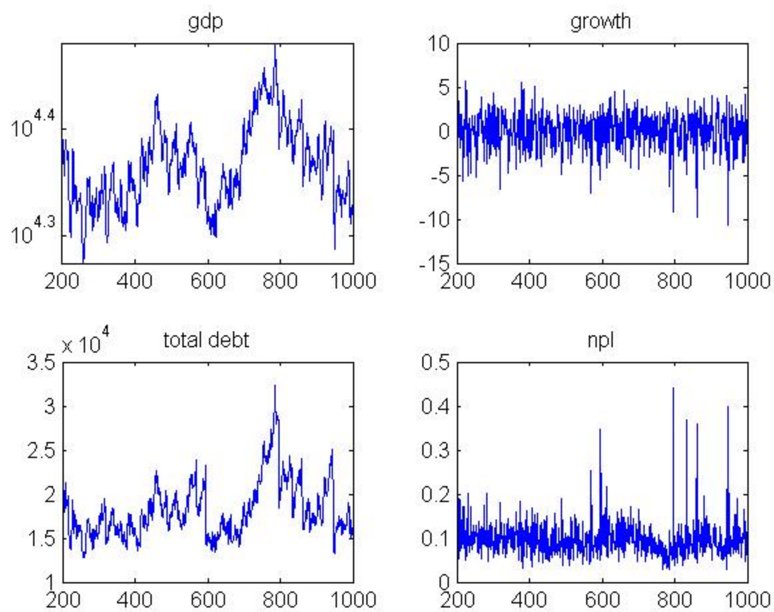


Figure 7.41: Output, growth, debt and bad debt in Growth Pricing

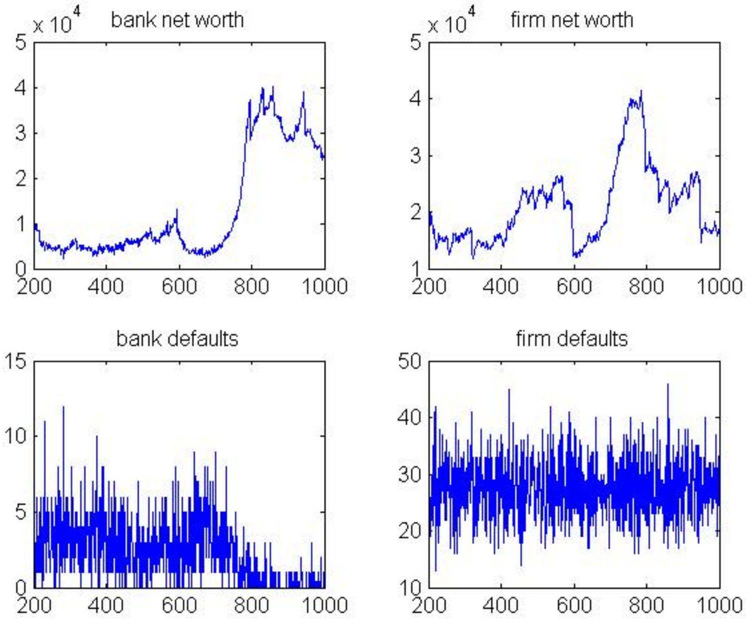


Figure 7.42: Size and default figures of banks and firms in Growth Pricing

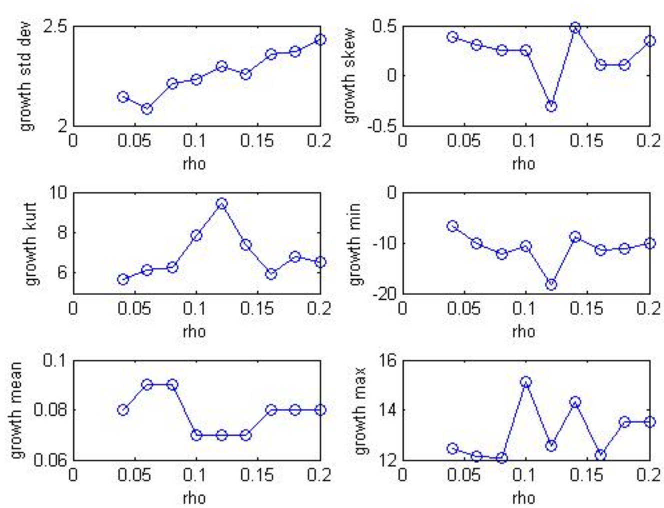


Figure 7.43: Sensitivity analysis of ρ on growth

The effects of sensitivity analysis on bank and firm defaults, interest rates, and leverage are given in Figure 7.44. There is a slight decrease in bank defaults with increasing ρ. On the other hand, mean market interest rates decrease but standard deviation of interest rates increase when we increase ρ.

Looking at our results in growth pricing scenario, we can say that when banks consider past performance of the economy and set their future predictions according to this, economic and financial instability increase. Bankers and other financial agents set their future forecasts taking into consideration past performance of the economy. In addition, as Minsky stated (1988), they almost always look at the most recent past, which often causes inaccurate predictions. In our test we can say that, this behavior not only causes the players make inaccurate predictions but also increases the extent of instability[5].

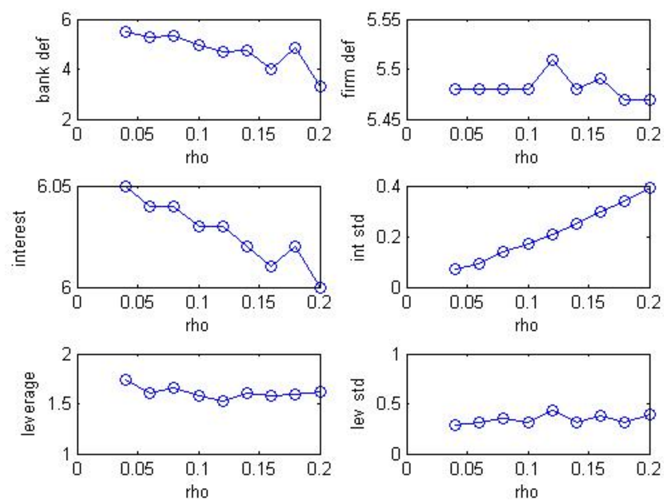


Figure 7.44: Sensitivity analysis of ρ on defaults, interest, and leverage

7.2.5 Including Revolving Credits

Up to now, credits in our model were all investment (or project) credits that have constant interest rates till the end of the credit term. In practice, firms very often

[5]We believe that this test may give more realistic results with endogenous pricing. And a mechanism which not only takes last year performance but a weighted average of past few years may be better. This may be a good research subject for future research.

use revolving credits, which are used for operating needs. An upper limit to the line of credit is determined and firms use and pay freely within that limit.

Revolving credits are important for the stability of the economy. Banks prefer revolving credits in their portfolios because they can easily adjust interest rate. Thus, they have relatively greater flexibility in controlling their credit portfolio.

When it increases the weight of revolving credits, a bank decreases the risk of time inconsistency of its balance sheet. On the other hand, because of speculative motivations, firms engage in revolving credits and they may even finance their long term investments with revolving credits. But doing so they increase the risk of time inconsistency.

To sum up, revolving credits are also part of financial markets, however banks have speculative motivations to increase weight of revolving credits to excessive levels. They start to fund long term investments with revolving credits. According to Central Bank of Turkey, ratio of revolving credits to total credits for Turkish Banks was 70% at the end of 2011.

We now change our model and define two kinds of credits: investment and revolving credits. Each term, when a firm gets credit, a determined ratio (μ) of that credit is set as revolving credit, while the rest is investment type. Interest rate of investment credits are determined at the beginning and stays constant till the end of the credit term. On the other hand, interest rate of revolving credits are updated every term with the same formula used at the beginning.

Results of the model with revolving credits are given in Table 7.8. There is a very slight increase in bad debt ratio and its standard deviation relative to base model: bad debt ratio from 8.34%, to 8.62%, and standard deviation from 4.06%, to 4.25%. An increase in bank default rate (with a high increase in standard deviation) is also noteworthy. On the firm side, firms' default rate increases from 4.99%, to 5.32%, and standard deviation increases sharply: from 0.02%, to 1.14%. Standard deviation of interest rates rise to 1.12% from 0.04%. Mean growth rate also increases slightly and standard deviation of growth increases from 2.18% in base model to 2.34% in revolving credits. Note that maximum standard deviation among 100 runs in robustness check was 2.32%. Thus, we can say that increase in growth standard deviation is noteworthy.

	Minimum	**Mean**	**Maximum**	**Std. Dev.**
Bad Debt Ratio %	0.00	8.62	42.33	4.25
Bank Defaults %	0.00	0.96	30.00	2.47
Bank Net Worth	765.72	121005.06	356494.00	111138.88
Total Debt	5240.32	24975.76	81845.30	16056.23
Firm Defaults %	0.00	5.32	9.00	1.14
Firm Net Worth	5829.09	41928.88	175695.00	48026.73
Aggregate Production	12417.20	29223.39	67195.80	13730.26
Interest Rate %	4.46	5.91	6.06	0.12
Leverage	0.50	1.53	2.29	0.54
Growth %	-11.02	0.09	11.41	2.34

Table 7.8: Results with Revolving Credits

The results given in the table are valid for periods 0-1000. As, the interval between 0-200 is actually a period of initialization and setting up, some outstanding maximum values are observed in this interval. Excluding the first 200 periods does not change the main results and conclusions. To give a better visualisation of resulting trends we present periods 200 - 1000 in the related graphs.

Interest rate and leverage motions in revolving credits pricing mechanism are given in Figures 7.45 and 7.46. There are clear short term fluctuations in interest rates caused by the periodic adjustment of interest rates of revolving credits.

Total production, growth, total debt, and non-performing loans are given in Figure 7.47. Total worth of banks and firms and number of defaults are given in Figure 7.48.

We performed a sensitivity analysis by gradually increasing μ from 0.1 to 0.9. The results are given in Table B.22. With increasing rate of μ, standard deviation of growth also increases, while mean production stays at the same level. Similarly,

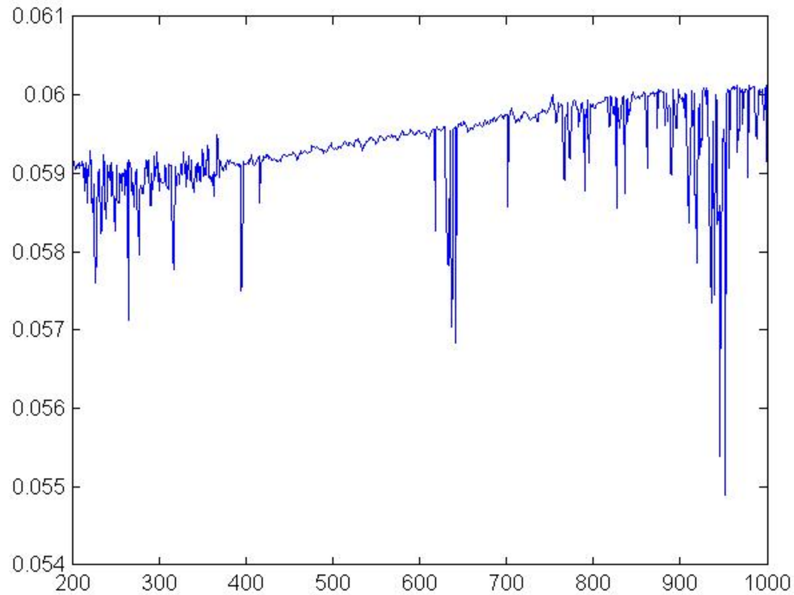


Figure 7.45: Interest rates in model with revolving credits

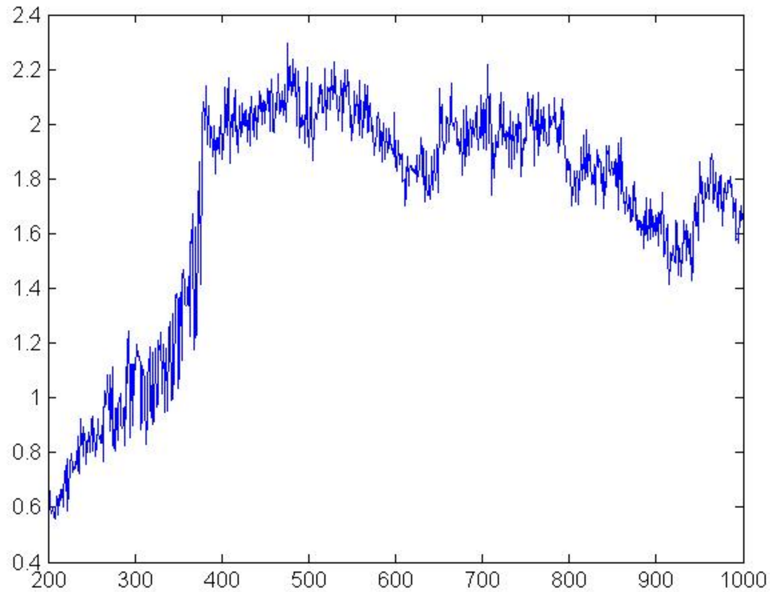


Figure 7.46: Leverage in the model with revolving credits

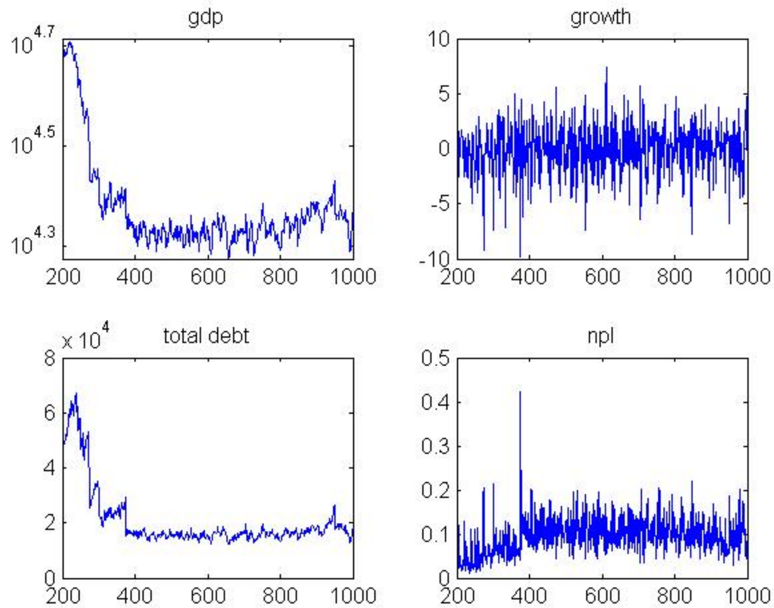


Figure 7.47: Output, growth, debt and bad debt in model with revolving credits

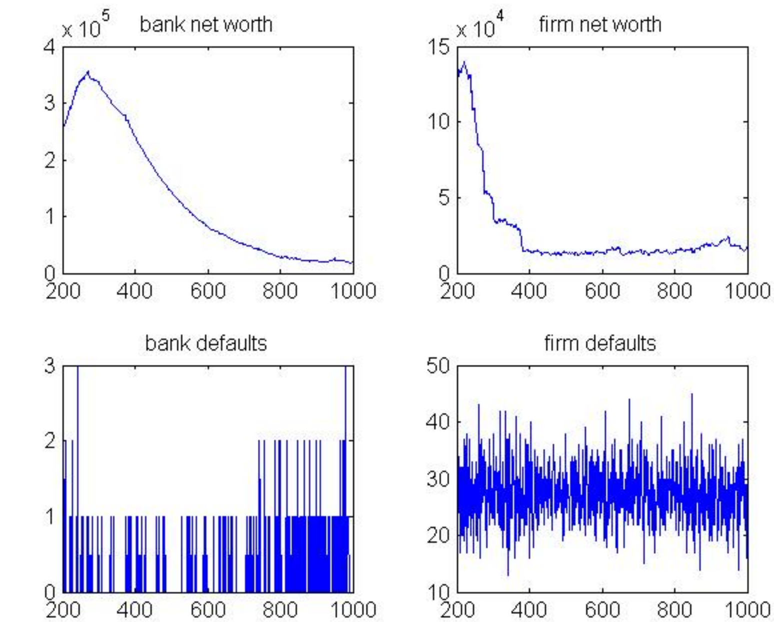


Figure 7.48: Size and default figures of banks and firms in model with revolving credits

standard deviations of interest rates also increase although mean interest rate do not change significantly.

The results on growth statistics are presented in Figure 7.49. The changes on bank and firm defaults, interest rates and leverage are shown in Figure 7.50. There is a very slight decrease in bank and firm defaults, and mean interest rate when we increase μ. On the other hand, standard deviation of interest rates increase with μ. There is no significant pattern on leverage.

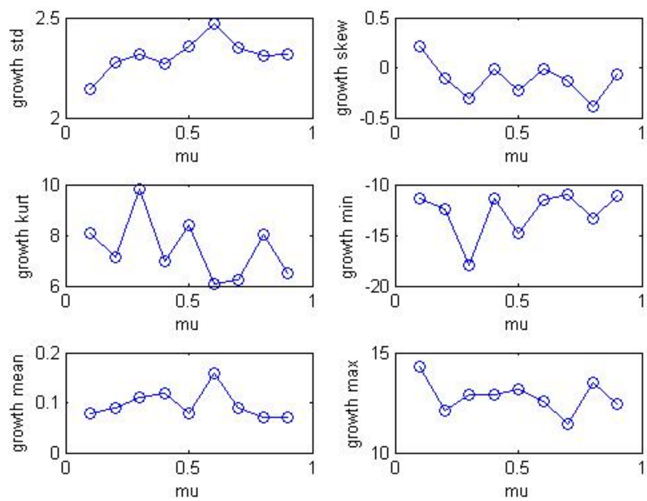


Figure 7.49: Sensitivity analysis of μ on growth

In times of growth, financial institutions make short term credits more attractive, and firms include more short term debt in their portfolios. We have also confirmed this at sectoral financial tables of Turkish Central Bank data. Our results confirm that an increase in ratio of revolving credits lead to an increase in instability of economy and finance. Results of sensitivity analysis also confirm this finding.

7.2.6 Market Share Pricing When Revolving Credits Exist

Finally we combined market share pricing and revolving credits models to get a more realistic frame. The results of this scenario are given on Table 7.9.

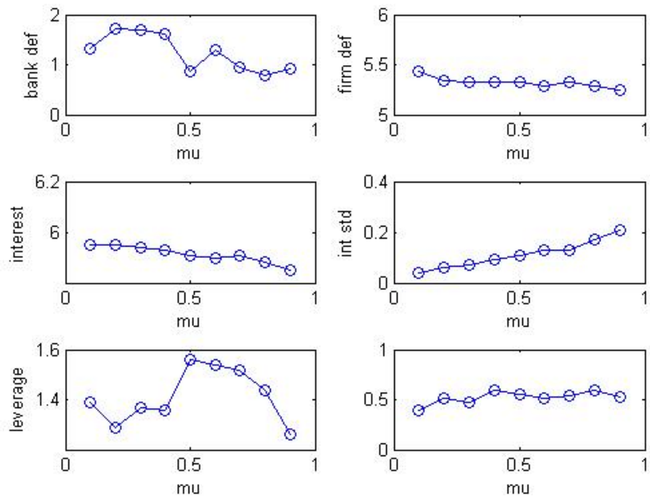


Figure 7.50: Sensitivity analysis of μ on default, interest rate, and leverage

	Minimum	**Mean**	**Maximum**	**Std. Dev.**
Bad Debt Ratio %	0.00	6.81	49.00	4.40
Bank Defaults %	0.00	3.47	30.00	3.49
Bank Net Worth	341.18	146995.02	386298.00	101175.74
Total Debt	5240.59	32778.88	115188.00	17952.02
Firm Defaults %	0.00	5.01	8.40	1.15
Firm Net Worth	5936.31	71362.29	224095.00	56709.35
Aggregate Production	12417.40	35359.10	59846.30	10248.20
Interest Rate %	4.40	5.57	5.84	0.20
Leverage	0.27	0.96	1.97	0.36
Growth %	-22.34	0.12	12.14	2.37

Table 7.9: Results with Revolving Credits

The results given in the table are valid for periods 0-1000. As, the interval between 0-200 is actually a period of initialization and setting up, some outstanding maximum values are observed in this interval. Excluding the first 200 periods does not change the main results and conclusions. To give a better visualisation of resulting trends we present periods 200 - 1000 in the related graphs.

The most important of these interesting results is that this mechanism further increases instability. Remember that mean bad debt ratio was 8.60% and 8.62% in market share pricing, and revolving credits scenarios respectively. While mean bad debt ratio decreases in the current scenario, standard deviation of bad debt increases to 4.4% (it was 3.74% and 4.25% in previous scenarios).

Mean bank defaults are almost at the same level with market share mechanism, and below revolving credit mechanism; on the other hand, standard deviation of bank default is now 3.49%, while it was 2.94%, and 2.47% previously.

Total debt, and standard deviation of total debt also increase relative to previous mechanisms. Firm defaults show no significant difference, remaining at the same level with market share pricing, and revolving credit scenario. Output growth rate rises to 0.12% from 0.08%, and 0.09%. On the other hand, standard deviation of growth increases 2.37%. Note that, minimum growth rate is -22.34%, while it was -10.69%, and -11.02% previously.

Finally and most importantly, standard deviation of interest rates increase to 0.20%, while it was 0.03 and 0.12 in the previous scenarios. Short term fluctuations and sharp movements in the current model are clearly seen(Compare Figure 7.51 with Figures 7.45 and 7.36).

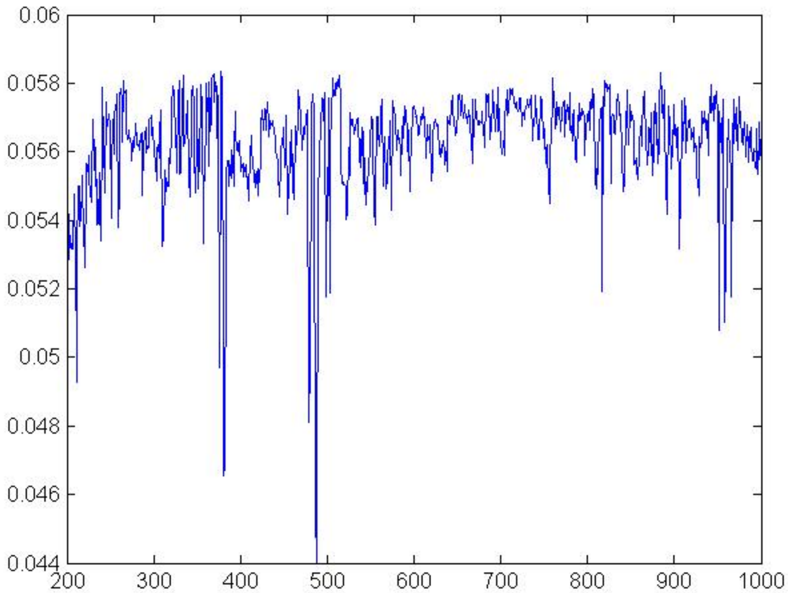


Figure 7.51: Interest rates with market share pricing and revolving credits

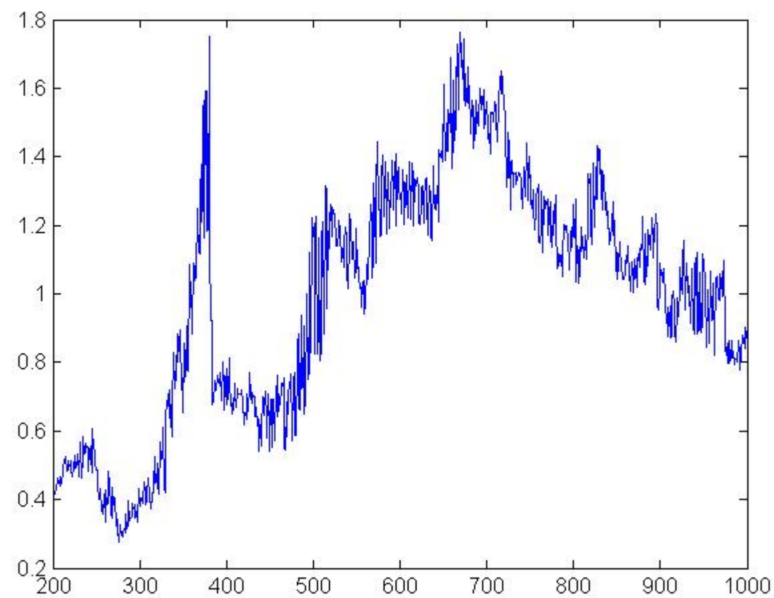


Figure 7.52: Leverage with market share pricing and revolving credits

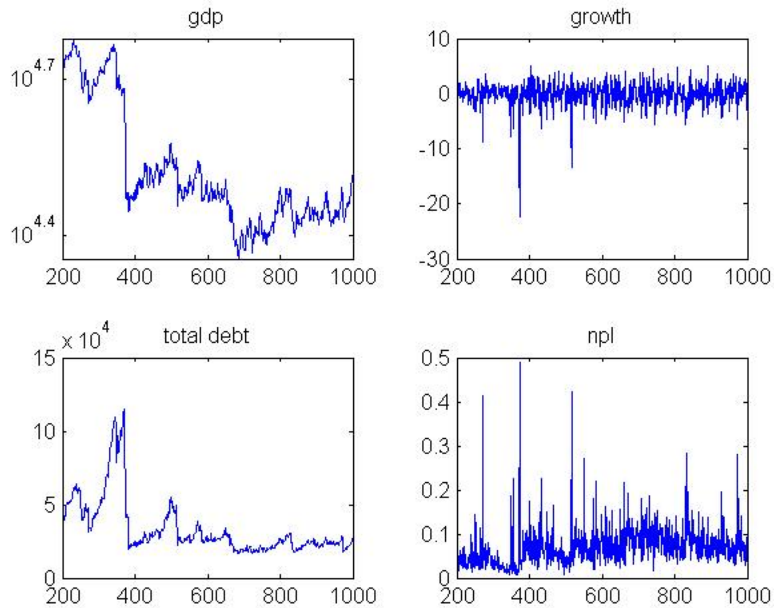


Figure 7.53: Output, growth, debt and bad debt in model with market share pricing and revolving credits

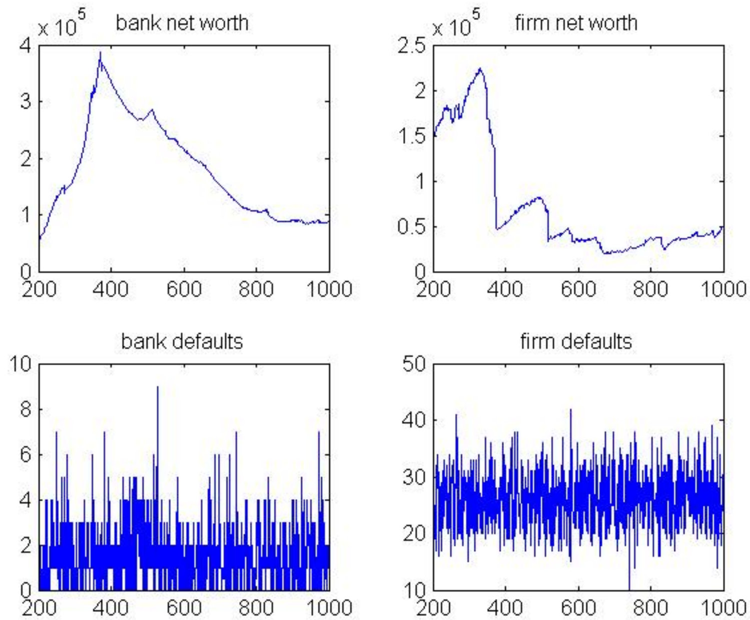


Figure 7.54: Size and default figures of banks and firms in model with market share pricing and revolving credits

Chapter 8

Conclusion

8.1 Introduction and Motives

The credit crunch after financial turmoil in 2008 was so sudden and huge that except few, no one expected such a huge crisis. After the crisis, debates, researches, and publications about the reasons of crisis increased in a great manner. The most important research question after the crisis was: how the crisis evolved? Crisis economics became popular once again.

Some researches explained the crisis with historical explanations from Marxian, Keynesian or other points of view. Some claimed that the economic world was much different than it was in the history, so new explanations, and methodologies must be used.

The size and diversity of crisis economics is huge and may actually be subject of another study. The history of economic literature about economic crises goes back John Stuart Mill. He explained crises with exogenous shocks, which create bubbles in the markets. Those bubbles in turn was causing speculation, expanding the bubble further. Marx, on the other hand, related crises with capitalism itself. According to him, capitalists always look for increasing the rate of exploitation of labor which is the single source of value. Thus they increase productivity which decrease share of labor force, causing a decrease in buying power. In a world with competitive markets, this leads capitalists to decrease prices causing falling profits. Reduction in profits leads the capitalists cutting the costs, leaving the economy with overproduction and underemployment.

Fisher, later explained crises with debt-deflation mechanism where cycles of debt liquidation, distress selling, deflation, and later decrease in profits and increase in bankruptcies and unemployment follow each other.

Minsky later developed debt-deflation theory and was the first who mentioned about banks' role in formation of crises. He claimed that cash flows were very important to understand the stability of an economy, and classified cash flows into three: income, balance sheet, and portfolio type. Income cash flows are originated from ordinary business and sales. Balance sheet cash flows occur because of some debt obligations and portfolio cash flows emanate due to sale of capital or financial assets.

Minsky makes another classification about how firms fulfill their financial obligations: hedge, speculative, and Ponzi finance. In hedge financing, debt obligations are done with income cash flows. In the second type, income cash flows are not enough to fulfill debt obligations (even they are enough for interest payment), so firms rollover credits. And finally, if firms sell assets or increase credits this is called Ponzi finance. Stability of an economy is determined by the weights of three kind of financing. Bankers are always in innovative search of increasing their profits and this is one of the reasons to increase ratio of speculative and Ponzi financing in the economy.

There are studies which show that banks' behavior after the initialization of crises amplify the size of the crisis. On the other hand, we believe banks also play role in increasing instability even not in formation of crises. Although all the crisis theories have righteous points, banks' contribution to formation of crises must also be questioned.

Marx, in the past, pointed the profit desire of capitalists as the reason of crises. We think that profit desire of bankers is another reason for crises. Minsky, three decades ago, claimed that banks have instabilizing effects on economies. We believe that, with methodologies that became available in the last decade we will be able to conceptualize effects of banking and measure the impact of it.

Although at one side we think like Reinhart and Rogoff (2009) –this crisis is not much different from other crises in the history– on the other side it has different characteristics. First, the world financialized in a hyper rate in the second half of the 20^{th} century which accelerated especially in the last 20 years. Second, the world economy became much more complicated and interconnected in the last few decades.

With the high interconnectedness in the markets, it was very difficult to see the effects of a cause. At the micro level, everything was seemed to be well, and risk level was moderate. On the other hand, cumulative risk of the macro system was huge, as understood after the crisis. Thus, it is understood that, similar to some other sciences like physics and chemistry, the dynamics of the sum is different than the sum of its components. So, an important research question arose: How can we reach to macroeconomic behavior with the help of microeconomic behaviors? In other words, if the agents are shaping the economy, are there microeconomic foundations of macroeconomic outcomes?

The higher ratio of finance in the world economy made banks and other financial institutions very important. Remember that during the last crisis, banks were at the core of concern, and the crisis accelerated after the collapse of Lehman Brothers. After the crisis, it was also understood that systemic risk emanating from financial institutions was great.

The second difference in this crisis, interconnected structure of markets, is another point of concern. This point gained attraction in the past decade. It is understood that, players in an economy are neither isolated nor independent; they observe and affect each other, and learn from others. This interconnected behavior is an explanation of why the whole is different than the sum of its components in economics.

In our research we tried to,

- understand banks' role in economic trends and stability (instability),
- gain insight whether policy regulations on banking may contribute to stabilize economies.

After defining our research question and hypothesis, we had to decide on the methodology of modeling the economy. Mainstream economics conventionally modeled aggregate economic behavior with a reductionist approach. In that approach, it was thought that players, or agents in an economy, are similar, isolated, and all behave rationally. Every agent believed to have access to full information set in the market and maximize their utility, or profit using this information. This approach is known as representative agent approach. This is because it is believed that, an agent has the average properties of the society.

There are critiques turned to the reductionist approach. First of all, there are no assumptions about how to reach aggregate behavior from the behavior of individuals. This problem is solved by just assuming that the whole economy is just like an individual (and that individual is the representative agent). But as we have been pointing from the very beginning, the whole is different from the sum of its components in an economy.

Another critique is that players lack full information in real practice which makes them to fully optimize their behavior. Actually, we believe that fully optimizing one's behavior is impossible even if there is access to full information set. First, outcomes in the future are bound to the decisions given today and they are not only related to decisions of a single player but others' behavior also. And, here comes another problem? How can we define a state of equilibrium? Actually, equilibrium is a term used by mainstream economists which is again a very divisive issue. In reality markets are dynamic and there are continuous interactions in opposite sides. We can think of an economic system just like a physical system. There are continuous dynamic interactions which make the system stable. In mainstream economics, a system is accepted as in state of equilibrium if all the components are in equilibrium. But this is actually impossible in a dynamic, interacting system. Thus, instead of searching for an equilibrium point, which we think do not exists, it is better to study on stability. A term that is proposed instead of equilibrium is statistical equilibrium, which is defined as state of the macroscopic equilibrium maintained by a large number of transitions in opposite directions.

We believe that conventional methods are not enough to explain aggregate behavior, so we need another method to model microeconomic foundations of macroeconomic results. Remember that in real life, agents observe each other, learn from each other and affect each other. In addition, players have different preferences that evolve in time. Instead of representative agent, we can call real life players as heterogeneous interactive agents, a term that is proposed in the last decade.

The methodology to model heterogeneous interactive agents is agent base modeling. This kind of a model cannot be solved with just some optimization equations because there are heterogeneities among players and there is not a state of equation. Agent based models are observed with the help of computer simulations. The idea behind agent based modeling is that small heterogeneities on micro level cause huge complexities on aggregate level. This complexities can be grasped with the help of computer simulations. In an agent based model, players and behavior rules of players are defined. Moreover, interaction rules between players are also defined and the evolution of the system is observed.

Studies in agent based modeling accelerated in the last decade. We believe that although it has a long way to go, agent based modeling promises too much to gain insight about microeconomic foundations of macroeconomic behavior. After the crisis in 2008, many academicians also proposed to use agent based modeling for policy making researches.

An agent based model makes use of network theory. A network is simply a collection of nodes that are connected by links. Networks are studied in a very wide range of applications from physics, to sociology, traffic control, or economics. A node in a network may be any kind of agent: a household, a bank, firm of government. The links between the nodes may be any kind of relationship: credit linkage, partnership, resemblance, or trade relationship.

Lack of credit relationship was one of the shortages of the conventional models. We believe that, studying credit network via an agent based modeling perspective may provide insight about role of banking and debt relationships in financial instability.

We mentioned that agent based modeling uses network theory which itself is derived from graph theory. We gave some information about graph theory and networks in Chapter 5. You can refer to that chapter for a basic introduction or publications we refer in that chapter for more advanced study.

Our base model consists of banks and firms. We modeled a bipartite directed credit network. To keep it simple, we omitted households, government and other institutions. Excluding households, we will be able to omit labor and goods markets, and model just credit market and its effects on the economy. The base model is explained in Chapter 6 in detail. The reader can refer to that chapter or Section 6.6 for a brief discussion.

8.2 Simulation, Results and Findings

We first simulated the base model. Then we ran the base model for 100 times to control the robustness of the model. We observed irregular fluctuations in the model, similar to previous studies. Except high fluctuations at the first 200 periods, which we believe are because of setting up and initialization of the program, we believe the patterns observed are realistic. In the next step we changed parameters and made sensitivity analyses. The detailed results can be seen in Section 7.1. The important point here is that, even small changes in small parameters may cause increase in fluctuations and financial instability.

One point we have to note here is that, our aim is not observing real data with our simulation. First, our model is a simplification of real world and lacks some markets in real economy. Second, our aim is to observe results of interactions and changes in results by changing basic parameters.

Next we tested different scenarios of our model. First, we changed the setting such that bank cared about its own leverage ratio in pricing. This scenario is actually not very much different than base model. Banks again care about their financial robustness when pricing.

In the next step, we make the model more realistic. In the base model, equity has no cost and transferred to the next term without loss. In reality, a firm cannot fully liquidize its net worth in case of bankruptcy. To conceptualize this reality, we changed the net worth mechanism such that a firm transfers its equity to the next term with a determined ratio, i.e. $A_{i,t+1} = \upsilon A_{i,t} + \pi_{i,t}$.

In reality banks most often set prices to increase their market share. This may actually seem as an investment. Doing this, they give up today's profit for higher future profits. Thus, a bank with low market share will set lower interest rates to increase its market share.

In financial and economic markets, players generally care about past economic data to forecast future. And, as Minsky also stated, they generally care about near future. In financial markets, people become optimistic when recent economic data is good and vice versa. Thus, we changed pricing mechanism in the base model a little, by adding a term to interest rate formula: ρk. Here, k is the growth rate of previous period and ρ is a smoothing coefficient.

In the next scenario we adapted a major change in the model. Up to now, the credits were assumed to be investment credits. Interest rate of a credit stays constant throughout the period. On the other hand, most of the credits are revolving credits in real life. Revolving credits are set with a limit and firm can use and payback credit anytime required. At the end of every three month, firm pays interest calculated on the average usage of credits. We adapted a simplified version of revolving credit mechanism. Term and payment structure is the same as investment credits. On the other hand, interest rate of the credit changes every period according to interest setting rule.

Finally, we set a scenario when there are revolving credits and banks set interest rates according to their market shares.

The summary of results of the scenarios are given in the Table 8.1.

Mean bad debt ratio in all scenarios, except the last one, are higher than base model. In the last model, standard deviation of bad debt ratio is higher than the base model. Average bank and firm defaults in all of the scenarios are much higher than the base model. Standard deviations are also higher, except for bank defaults in revolving credits scenario.

	Base Model		**Lev**		**Cost of Eq**		**Mrk Shr**		**Grwth**		**Rev Crd**		**Rev & Mrk Sh**	
	Avg	**StD**	**Avg**	**StD**	**Avg**	**StD**	**Avg**	**StD**	**Avg**	**StD**	**Avg**	**StD**	**Avg**	**StD**
Bad Debt Ratio %	8.34	4.06	9.45	3.75	22.65	8.89	8.6	3.74	9.61	3.88	8.62	4.25	6.81	4.40
Bank Defaults %	1.03	2.52	4.87	4.04	11.08	6.78	3.22	2.94	4.96	4.32	0.96	2.47	3.47	3.49
Bnk Net Wrth(000)	86.26	44.48	12.94	12.64	0.42	0.15	41.23	19.51	11.53	11.08	121.01	111.14	147.00	101.18
Total Debt (000)	27.33	8.19	18.73	4.52	1.30	0.29	21.15	4.03	17.85	3.29	24.98	16.06	32.78	17.95
Firm Defaults %	4.99	1.04	5.44	1.06	10.66	1.48	5.35	1.05	5.48	1.09	5.32	1.14	5.01	1.15
Frm Net Wrth (000)	39.74	21.59	23.60	11.77	0.63	0.12	26.96	8.60	20.27	6.65	41.93	48.03	71.36	56.71
Aggr. Prod. (000)	31.04	6.06	23.79	2.79	3.60	0.42	25.65	2.40	23.09	2.18	29.22	13.73	35.36	10.25
Interest Rate %	5.97	0.03	5.99	0.07	6.07	0.01	5.82	0.03	6.03	0.17	5.91	0.12	5.57	0.20
Leverage	1.25	0.32	1.52	0.41	3.14	0.46	1.41	0.3	1.59	0.31	1.53	0.54	0.96	0.36
Growth %	0.10	2.18	0.10	2.10	-0.02	4.64	0.08	2.19	0.07	2.23	0.09	2.34	0.12	2.37

Table 8.1: Summary of Results with Different Scenarios

Average interest rates increase in leverage, cost of equity, and growth scenarios. Standard deviation of interest rates increase also in leverage and growth scenarios. In market share scenario, mean interest rate decrease and its standard deviation stay the same. On the other hand, sharp and sudden decreases in interest rates are common in this scenario (see Figure 7.36), which is not actually a surprise. When we implement revolving credits scenario, mean interest rates fall but standard deviation of interest rates increase to 0.12 from 0.03. Finally, when banks set interest rates according to market share, in existence of revolving credits, mean interest rates fall further but standard deviation goes up significantly, to 0.20.

When we look at growth figures, mean growth rate is only higher in the last scenario but note that it is equal to the maximum of mean growth rates in our robustness tests of base model. In addition, standard deviation of growth in this scenario is 2.37 while maximum observed value was 2.32 in base model.

To sum up, we first set a base model where banks behave in favor of financial robustness. Then we applied various scenarios to observe how differences in behavior of banks may instabilize finance and economy. We observed that, changes in banks' credit or pricing attributes may have noteworthy effects and increase instability. We can say that as banks behave in a way to increase their profits or market share without caring financial robustness, financial and economic instability increase.

8.3 Future Work

Our model aimed to model bank behavior and observe its effects on aggregate economic behavior. We believe that we succeeded to show effects of banking behavior on financial stability but our work is just a small beginning and there is a long way to go. Including all of the players, and interactions in the economy is far above the aim of this study. We are giving a list of potential future work on the subject.

Actually, relations, and financial structure on micro level are hidden in balance sheets of firms and banks. Thus, we believe that the more detailed a model grasps structure of balance sheets the more realistic it will be.

Below is a list of further work that would be useful in achieving macroeconomic behavior of financial markets.

- Our model lacks a rating system. In our model banks set interest rate considering their financial robustness. They deny to grant credit to a firm if the firm is insolvent or set higher interest rates to firms with low rating. But, implementation of a more complicated and realistic rating system used by banks will improve the model.

- Lack of households is also a very important point. Ratio and importance of retail credits, and credit cards increased very much in the last two decades. So, without including households and retail credits, we cannot fully model banking behavior.

- Size distribution of firms and banks in an economy is also very important. In reality few banks or firms may have a great share on the economy. In addition, in some countries banks belong to some business groups. Thus, decomposing between large firms and small firms may be enlightening for the aim of our study.

- Our model do not have a strong customer loyalty mechanism. Actually, in out model, a firm goes to current banks plus a number of new random banks. This is a kind of weak customer loyalty. But a stronger model, that differentiates interest rates according to age of the customer may also be implemented to make the model more realistic.

- A mechanism where, banks bargain for current credits and may have the opportunity to carry them to other banks may also be useful.

- In our model, if a firm goes bankrupt, a new firm with a relatively small net worth enters the market. New firm's net worth is fully composed of equity. We can differentiate the mechanism where banks supply credits to firms to establish business.

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Appendix A

Simulation Program

Here we give a full listing of the simulation we have coded in C++. Our program has three classes: firmType, bankType, and economy. Main program listing is below.

Main Program

```
#include <iostream>
#include <vector>
#include <cmath>
#include <random>
#include <cstdlib>
#include <ctime>
#include <fstream>
#include "economy.h"

using namespace std;
using namespace std::tr1;

int main()
{
  std::tr1::mt19937 eng;            //seed random variables
  eng.seed((unsigned int)time(NULL));

  economy econ;              //define an economy object called econ.
  econ.set_initials();             //initialize parameters

  for(short t = 1; t <= 1000; t++)
  {
    cout << t << "  ";             //period number
    econ.update_term();           //credit terms is decreased by 1

    for (short z = 0; z < Z; z++)    //initialize npl matrix
      for (short i = 0; i < I; i++)
        econ.npl[z][i] = 0;
    econ.change_bankrupt_firm();     //renewal of bankrupt firm
    econ.change_bankrupt_bank();     //renewal bankrupt bank
    econ.set_mrktshr();               //set market share of banks
    for (short i = 0; i < I; i++)    //target leverage for firms
      econ.fset_lev(eng, i, t);
```

```
    econ.fset_Bstar();           //calculate target debt level firms
    econ.use_credit(eng);        //firms use credit
                       //new credit and intrate written to Bnew and rnew
    econ.fset_li(t);            //set the realised leverage for each firm
    econ.write_leverage();        //write the leverage to the file
    for (short i = 0; i < I; i++)   //update price matrix of each firm
      econ.firms[i].set_p(eng);
    econ.set_production();        //calculate new K and Y
    econ.write_gdp();           //write the results to the file
    econ.set_Aipri();           //calculate profit and net worth
    econ.set_liquidity();        //set bankruptcy
    econ.set_eqbankrupt();        //determine the bankrupt firms
    econ.set_npl();             //set npl matrix using bankrupt firms
    econ.write_baddebt();        //find bad debt ratio and write file
    econ.set_Azprz();           //set profit, and net worth for banks
    econ.payback_credit();        //payback expiring credits
    econ.set_bankrupt_bank();      //determine bankrupt banks
    econ.write_default();        //write # of bankrupt banks&firms
    econ.consolidate_credits();     //consolidate Bnew & B, rnew & r
    econ.write_total_debt();      //write total credits to file
    econ.write_net_worth();       //write total net worth to file
    econ.write_avg_interest();      //write interest rate to file
  }

  econ.firm_size_dist();
  econ.bank_size_dist();
  econ.firm_degree_dist();
  econ.bank_degree_dist();
  return 0;
}     //end of main
```

Class: economy

```cpp
#ifndef economy_H
#define economy_H

#include <iostream>
#include <vector>
#include <cmath>
#include <algorithm>
#include <fstream>
#include <random>
#include <ctime>
#include "firmType.h"
#include "bankType.h"

using namespace std;
using namespace tr1;

//declaration of global constants
const short I = 500;        //# of firms
const short Z = 50;         //# of banks
const short D = 4;          //max term of credit

const float phi = 3.0;        //phi in production function
const float beta = 0.7;       //beta in prod function
const float adj = 0.1;        //lev = lev(t-1)(1 +- adj.rand)
const float u = 0.10;       //in price function
const float sigmasq = 0.01;     //variance of r.v. v in price function
const float theta = 0.5;      //ratio of total capital that can be liquidized
const float lqlimit = 0.4;      //if negative cash flow / Y > lqlimit -> lqbankruptcy = true
const float eps = 0;        //ratio of reserve accounts for banks
const float rmin = 0.02;      //interest rate floor set by
const float gamma = 0.02;     //f1(.) in interest rate setting
const float alpha = 0.02;     //f2(.) in interest rate setting
const float RR = 0.5;       //recovery rate
const float c = 0.005;        //cost proportional to bank size
const float CARmin = 0.12;      //legally set minimum capital adequacy ratio
const short MB = 15;        //max number of banks, a firm is allowed to work with
const short n = 2;          //# of banks that are asked for credit every term
const float beg_Ai = 10;      //beginning net worth for firms
const float beg_Az = 70;      //beginning net worth for banks
const float beg_li = 1.0;     //beginning leverage for firms
const float lambda = 0.4;     //for exponential distribution in get_d() function

class economy
{
public:
  vector<firmType> firms;             //vector for firms
```

```
  vector<bankType> banks;           //vector for banks
  vector< vector< vector<float> > > B;    //vector for total amount of
      existing credits bank-firm-term
  vector< vector< vector<float> > > r;    //vector for weighted average
      cost of interest. if a new credit is released
                          // with an already existing term, existing rate
                              is updated by calculating the weighted avg
                          //with the new released part
  vector< vector< vector<float> > > Bnew;
  vector< vector< vector<float> > > rnew;
  vector< vector<float> > npl;

  //economy functions
  void set_initials();

  //functions for firms
  float fget_Amax();
  float fget_pe(short i);
  float fget_rstar(short i);
  void fset_lev(std::tr1::mt19937 & eng, short i, short t);
  void fset_Bstar();
  float fget_crd_need(short i);
  void use_credit(std::tr1::mt19937 & eng);
  void set_production();
  float get_int_rate(short z, short i);
  void set_Aipri();
  void set_Azprz();
  void set_liquidity();
  void set_eqbankrupt();
  void set_bankrupt_bank();
  void set_npl();
  float get_avgAi();
  float get_avgAz();
  float getGDP();
  void update_term();
  void change_bankrupt_firm();
  void change_bankrupt_bank();
  void write_gdp();
  void consolidate_credits();
  void payback_credit();
  void fset_li(short time);
  void write_leverage();
  void write_baddebt();
  void write_default();
  void set_mrktshr();
  void write_total_debt();
  void write_net_worth();
  void write_avg_interest();
  void firm_size_dist();
  void bank_size_dist();
  void firm_degree_dist();
  void bank_degree_dist();
};
void economy::set_initials()
```

```
{
  firms.resize(I);             //set the number of firms to I
  banks.resize(Z);             //set the number of banks to Z

  npl.resize(Z);               //set the matrix npl[Z][I]
  for (short z = 0; z < Z; z++)
    npl[z].resize(I);

 B.resize(Z);                //set the matrix B[Z][I][D]
  for(short z = 0; z < Z; z++)
  {
    B[z].resize(I);
    for (short i = 0; i < I; i++)
      B[z][i].resize(D);
  }

 Bnew.resize(Z);               //set the matrix Bnew[Z][I][D]
  for(short z = 0; z < Z; z++)
  {
    Bnew[z].resize(I);
    for (short i = 0; i < I; i++)
      Bnew[z][i].resize(D);
  }

  r.resize(Z);                //set the matrix r[Z][I][D]
  for(short z = 0; z < Z; z++)
  {
    r[z].resize(I);
    for (short i = 0; i < I; i++)
      r[z][i].resize(D);
  }

  rnew.resize(Z);              //set the matrix rnew[Z][I][D]
  for(short z = 0; z < Z; z++)
  {
    rnew[z].resize(I);
    for (short i = 0; i < I; i++)
      rnew[z][i].resize(D);
  }

  for (short i = 0; i < I; i++)
  {
    firms[i].Ai = beg_Ai;        //set beginning net worth for firms
    firms[i].Bi = 0;           //debt is zero
    firms[i].li = beg_li;        //beginning leverage is set equal to beg_li
    firms[i].pri = 0;          //profit of i is equal to zero
    firms[i].K = 0;            //capital  = 0
    firms[i].Y = 0;            //output = 0
    firms[i].eqbankrupt = false;    //equity bankrupt Ai <= 0
    firms[i].lqbankrupt = false;    //liquidity bankrupt Luca's idea
    firms[i].lqbad = false;       //firms liquidity is negative
    for (short x = 0; x < 3; x++)
      firms[i].p[x] = u;         //set beginning price array = u
```

```
  }
  for(short z = 0; z < Z; z++)
  {
    banks[z].Az = beg_Az;          //set beginning net worth for banks
    banks[z].L = 0;               //initialize the credits to 0
    banks[z].D = 0;               //initialize deposits to 0
    banks[z].Res = eps * banks[z].D;     //reserve deposit is equal to
        eps * D (0 for now)
    banks[z].prz = 0;             //profit of z is equal to 0
    banks[z].set_CAR();             //capital adequacy ratio is equal to 0
    banks[z].bankrupt = false;         //bankrupt is false
  }
}
void economy::update_term()          //decrease the term of the credits
    by one as one period passed
{
  for (short z = 0; z < Z; z++)
  {
    for (short i = 0; i <I; i++)
    {
      for (short d = 1; d < D; d++)
      {
        B[z][i][d-1] = B[z][i][d];
        r[z][i][d-1] = r[z][i][d];
      }
        B[z][i][D-1] = 0;
        r[z][i][D-1] = 0;
    }
  }
}
void economy::change_bankrupt_firm()         //change the bankrup firms
    with a new one. for now net worth of the new firm is set to 2.
{
  for (short i = 0; i <I; i++)
  {
    if ((firms[i].eqbankrupt == true) || (firms[i].lqbankrupt == true))
    {
      firms[i].Ai = 2;
      firms[i].Bi = 0;
      firms[i].li = beg_li;
      firms[i].K = 0;
      firms[i].Y = 0;
      firms[i].pri = 0;
      for (int x = 0; x <3; x++)
        firms[i].p[x] = u;
      firms[i].eqbankrupt = false;
      firms[i].lqbankrupt = false;

      for (short z = 0; z < Z; z++)      //credits are set to zero for
          the new firm
        for (short d = 0; d < D; d++)
        {
          B[z][i][d] = 0;
          Bnew[z][i][d] = 0;
```

```
            r[z][i][d] = 0;
            rnew[z][i][d] = 0;
          }
      }
   }
}
void economy::change_bankrupt_bank()      //bankrupt bank is replaced
    with a new bank. the new bank has net worth equal to beg_Az (100)
{                                          //credit lines of the
    old bank are preserved for the new bank
   for (short z = 0; z < Z; z++)
   {
      if (banks[z].bankrupt == true)
      {
         banks[z].Az = beg_Az;
         banks[z].prz = 0;
         banks[z].set_CAR();
         banks[z].bankrupt = false;
         banks[z].D = 0;
         banks[z].Res = eps * banks[z].D;
      }
   }
}
void economy::set_mrktshr()
{
   float totalcrdt = 0;
   for (short z = 0; z < Z; z++)
      totalcrdt += banks[z].L;
   if (totalcrdt == 0)
      for (short z = 0; z < Z; z++)
      banks[z].mrktshr = 1.0 / Z;
   else
      for (short z = 0; z < Z; z++)
      banks[z].mrktshr = banks[z].L / totalcrdt;
}
void economy::fset_lev(std::tr1::mt19937 & eng, short i, short t)
{
   uniform_real_distribution<double> dis(0,1);
   float rand = dis(eng);

   float targetlev;
   if (fget_pe(i) >= fget_rstar(i))
      targetlev = firms[i].li * (1 + adj * rand);
   else
      targetlev = firms[i].li * (1 - adj * rand);

   if (targetlev < 0.01)
      targetlev = 0.01;

   firms[i].tlev = targetlev;
}
float economy::fget_pe(short i)
{
```

```
  float pe = (0.6 * firms[i].p[0] + 0.36 * firms[i].p[1] + 0.04 * firms[i].p[2]) / (sqrt(1 + firms[i].Ai / fget_Amax()));
  return pe;
}
float economy::fget_rstar(short i)
{
  float interest = 0;
  float principal = 0;
  float r_star;

  for (short z = 0; z < Z; z++)
    for (short d = 0; d < D; d++)
    {
      interest += r[z][i][d] * B[z][i][d];
      principal += B[z][i][d];
    }

  if (principal == 0)
    r_star = 0;
  else
    r_star = interest / principal;
  return r_star;
}
void economy::fset_Bstar()
{
  for (short i = 0; i < I; i++)
    firms[i].BStar = firms[i].Ai * firms[i].tlev;
}
void economy::use_credit(std::tr1::mt19937 & eng)    //core function of the program. setting of new credit line!!!!!
{
  for(short i = 0; i < I; i++)
  {
    if (firms[i].lqbad == false)
    {
      float remCrdt = fget_crd_need(i);

        vector<short> curBnks;       //banks currently being worked together
        vector<short> otherBnks;     //other banks
        vector<short> askBnks;       //the banks that the firm will go and ask interest rate

        for (short z = 0; z < Z; z++)      //set the vector curBnks. banks that firm has credit line currently
        {
          bool found = false;
            for (short d = 0; d < D; d++)
            {
              if (B[z][i][d] > 0)
              {
                found = true;
                break;
              }
```

```
        }
    if (found == true)
      curBnks.push_back(z);
    }

    for (short z = 0; z < Z; z++)      //set the vector otherBnks. banks that has no credit line
    {
      bool fnd = false;
      for (short x = 0; x < curBnks.size(); x++)
      {
        if (curBnks[x] == z)
        {
          fnd = true;
          break;
        }
      }

      if (fnd == false)
        otherBnks.push_back(z);
    }

    random_shuffle(otherBnks.begin(), otherBnks.end());      // permutate other banks so the firm will choose randomly from other banks
    random_shuffle(curBnks.begin(), curBnks.end());          // permutate current banks
    for (short x = 0; x < curBnks.size(); x++)               //first add current banks to the vector askBnks
      askBnks.push_back(curBnks[x]);

    short k = n;                          //now add other banks to the list. every period the firm will go n other banks.
                                          //if the number of other banks <= n then ask all of them
    if (n > otherBnks.size())
      k = otherBnks.size();

    if (askBnks.size() < MB)
      for (short x = 0; x < k; x++)                //add other banks to askBnks
        askBnks.push_back(otherBnks[x]);

    random_shuffle(askBnks.begin(), askBnks.end());      //change the order of askBnks
    if (askBnks.size() > MB)                   //if the number of banks in the list is >= maximum number of banks allowed (MB) erase the
      while (askBnks.size() > MB)              //banks until it is equal to MB
        askBnks.erase(askBnks.end());

    vector<float> intRates;                    //calculate interest rate for each bank and add the int rate to vector intRates
```

```
if (askBnks.size() > 0)
{
  for(short x = 0; x < askBnks.size(); x++)
    intRates.push_back(get_int_rate(askBnks[x], i));
}

short pos = -1;
short minBankNo = -1;
float minRate = -1.0;

while (askBnks.size() > 0 && remCrdt > 0)      //while there is
    need for credit && any bank remains
{
  minBankNo = askBnks[0];              //get the min int rate and #
      of banks
  minRate = intRates[0];
  pos = 0;
  for (short x = 1; x < askBnks.size(); x++)
  {
    if (intRates[x] < minRate)
    {
      minRate = intRates[x];
      minBankNo = askBnks[x];
      pos = x;
    }
  }

  uniform_real_distribution<double> dis(0,1);      //set the term
      of the credit. credit term is exponentially distributed
     with parameter lambda
  float p = dis(eng);
  float x;
  short term  = 2;

  x = -1 * log(1 - p) * (1 / lambda);

  if (x < 1) term = 1;
  else if (x > D-1) term = (D-1);
  else
  {
    for (short y = 1; y < (D-1); y++)
    if (x > y)
      term = y;
  }

  if (pos == -1)
    remCrdt = 0;
  else
  {
    askBnks.erase(askBnks.begin() + pos);             //delete the
        min int rate
    intRates.erase(intRates.begin() + pos);            //delete
        the bank giving that rate
```

```
                float crdt_avail = banks[minBankNo].get_available_crdt(); // check whether the bank has enough capital to obtain the credit

                if (crdt_avail > remCrdt)
                {
                  Bnew[minBankNo][i][term] += remCrdt;
                  rnew[minBankNo][i][term] = minRate;
                  firms[i].Bi += remCrdt;
                  banks[minBankNo].L += remCrdt;
                  banks[minBankNo].set_CAR();
                  banks[minBankNo].D = (1 / (1 - eps)) * (banks[minBankNo].L - banks[minBankNo].Az);
                  if (banks[minBankNo].D < 0) banks[minBankNo].D = 0;
                  banks[minBankNo].Res += eps * banks[minBankNo].D;
                  remCrdt = 0;
                }
                else if (crdt_avail > 0)          //if the banks available crdt limit is not enough use partially available crdt and go
                                                  //to the bank next in askBnks
                {
                  Bnew[minBankNo][i][term] += crdt_avail;
                  rnew[minBankNo][i][term] = minRate;
                  firms[i].Bi += crdt_avail;
                  banks[minBankNo].L += crdt_avail;
                  banks[minBankNo].set_CAR();
                  banks[minBankNo].D = (1 / (1 - eps)) * (banks[minBankNo].L - banks[minBankNo].Az);
                  if (banks[minBankNo].D < 0) banks[minBankNo].D = 0;
                  banks[minBankNo].Res += eps * banks[minBankNo].D;
                  remCrdt = remCrdt - crdt_avail;       //go to other banks for the remaining part
                }
              }
            }
        }
    }
}
void economy::fset_li(short time)       //after using the credit set the leverage for the firm. it will be used to set the target leverage of the
{                                                //next term
    for(short i = 0; i < I; i++)
    {
        float lvg;
        float debt = 0;
        for (short z = 0; z < Z; z++)
            for (short d = 1; d < D; d++)       //expiring credits are not counted
                debt += B[z][i][d] + Bnew[z][i][d];

        lvg = debt / firms[i].Ai;

```

```
    if (lvg == 0)
      lvg = beg_li;            //if the leverage is zero it is set to beg_li to avoid it will stay at 0

    firms[i].li = lvg;
  }
}
void economy::write_leverage()
{
  float debt = 0;
  float netwrth = 0;

  for (short z = 0; z < Z; z++)
    for (short i = 0; i < I; i++)
      for (short d = 0; d < D; d++)
        debt += B[z][i][d] + Bnew[z][i][d];

  for (short i = 0; i < I; i++)
    netwrth += firms[i].Ai;

  float lev = debt / netwrth;

  ofstream oflev;
  oflev.open("lev.txt", ios_base::app);
  oflev << lev << endl;
  oflev.close();
}
void economy::set_production()          //after using the credit set the capital and output
{
  for (short i = 0; i < I; i++)
  {
    float debt = 0;
    for (short z = 0; z < Z; z++)
      for (short d = 1; d < D; d++)        //expiring credits that have d = 0 are not included
        debt += B[z][i][d] + Bnew[z][i][d];


    if (debt > firms[i].BStar)             //if current debt is greater than target level, use target debt level for production
      debt = firms[i].BStar;              //for simplicity credit payback is not included

    firms[i].K = firms[i].Ai + debt;
    firms[i].Y = phi * pow(firms[i].K, beta);
  }
}
void economy::write_gdp()
{
  float gdp = 0;
  for (short i = 0; i < I; i++)
    gdp += firms[i].Y;

```

```
  ofstream ofgdp;
  ofgdp.open("gdp.txt", ios_base::app);
  ofgdp << gdp << endl;
  ofgdp.close();
}
void economy::set_Aipri()                //after settin
{
  for (short i = 0; i < I; i++)
  {
    float interest = 0;
    for (short z = 0; z < Z; z++)
      for (short d = 1; d < D; d++)
        interest += r[z][i][d] * B[z][i][d] + rnew[z][i][d] * Bnew[z][i
            ][d];

    float pr = firms[i].p[0] * firms[i].Y - interest;
    firms[i].pri = pr;
    firms[i].Ai += pr;
  }
}
void economy::set_liquidity()
{
  for (short i = 0; i < I; i++)
  {
    float prn_pmnt = 0;
    float int_pmnt = 0;

    for (short z = 0; z < Z; z++)
      prn_pmnt += B[z][i][0];

    for (short z = 0; z < Z; z++)
      for (short d = 1; d < D; d++)
        int_pmnt += r[z][i][d] * B[z][i][d] + rnew[z][i][d] * Bnew[z][i
            ][d];

    float liquidity = theta * firms[i].K + firms[i].pri - int_pmnt -
        prn_pmnt;
    if (liquidity < 0)
      firms[i].lqbad = true;
    else
      firms[i].lqbad = false;

    if (firms[i].lqbad == true)
    {
      if (((-1 * liquidity) / (firms[i].Y)) > lqlimit)
        firms[i].lqbankrupt = true;
    }
  }
}
void economy::set_eqbankrupt()          //determine whether the firm is
    bankrupy either by net worth or by liquidity
{
  for (short i = 0; i < I; i++)
  {
```

```
    if (firms[i].Ai <= 0)
      firms[i].eqbankrupt = true;
  }
}
void economy::set_npl()          //set the npl matrix npl[z][i] for the firms that are bankrupt
{
  for(short i = 0; i < I; i++)
  {
    if ((firms[i].eqbankrupt == true) || (firms[i].lqbankrupt == true))
    {
      for (short z = 0; z < Z; z++)
      {
        float debt = 0;

        for (short d = 0; d < D; d++)
        {
          debt += B[z][i][d] + Bnew[z][i][d];
          B[z][i][d] = 0;
          Bnew[z][i][d] = 0;
          r[z][i][d] = 0;
          rnew[z][i][d] = 0;
        }
        npl[z][i] = debt;
        banks[z].L -= debt;
        banks[z].D = (1 / (1 - eps)) * (banks[z].L - banks[z].Az);
        if (banks[z].D < 0) banks[z].D = 0;
        banks[z].Res = eps * banks[z].D;
        banks[z].set_CAR();
      }
    }
  }
}
void economy::write_baddebt()
{
  float baddebt = 0;
  float crdts = 0;
  float nplratio;

  for (short z = 0; z < Z; z++)
    for(short i = 0; i < I; i++)
      baddebt += npl[z][i];

  for (short z = 0; z < Z; z++)
    for (short i = 0; i < I; i++)
      for (short d = 0; d < D; d++)
        crdts += B[z][i][d] + Bnew[z][i][d];


  nplratio = baddebt / (baddebt + crdts);

  ofstream ofbad;
  ofbad.open("bad.txt", ios_base::app);
  ofbad << nplratio << endl;
```

```
  ofbad.close();
}
void economy::set_Azprz()        //set the net worth and profit of the
    bank
{
  for (short z = 0; z < Z; z++)
  {
    float intgain = 0;

    for (short i = 0; i < I; i++)      //calculate interest gain from the
        firms that are not bankrupt
    {
      if ((firms[i].eqbankrupt == false) && (firms[i].lqbankrupt ==
          false))
        for (short d = 0; d < D; d++)
          intgain += r[z][i][d] * B[z][i][d] + rnew[z][i][d] * Bnew[z][i
              ][d];
    }

    float nonpl = 0;           //calculate the npl using npl[z][i]
    for (short i = 0; i < I; i++)
      nonpl += npl[z][i];

    float prz = intgain - rmin * banks[z].D - c * (banks[z].Az + banks[z
        ].D) - (1 - RR) * nonpl;
    banks[z].prz = prz;
    banks[z].Az += prz;
    banks[z].D = (1 / (1 - eps)) * (banks[z].L - banks[z].Az);
    if (banks[z].D < 0) banks[z].D = 0;
    banks[z].Res = eps * banks[z].D;
    banks[z].set_CAR();
  }
}
void economy::payback_credit()        //pay the expiring credits back B[
    z][i][0]
{
  for (short i = 0; i < I; i++)
  {
    if ( (firms[i].eqbankrupt == false) && (firms[i].lqbankrupt == false
        ))
    {
      for (short z = 0; z < Z; z++)
      {
        firms[i].Bi -= B[z][i][0];
        banks[z].L -= B[z][i][0];
        banks[z].set_CAR();
        banks[z].D = (1 / (1 - eps)) * (banks[z].L - banks[z].Az);
        if (banks[z].D < 0) banks[z].D = 0;
        banks[z].Res = eps * banks[z].D;
        B[z][i][0] = 0;
        r[z][i][0] = 0;
      }
    }
  }
```

```
}
void economy::set_bankrupt_bank()
{
  for (short z = 0; z < Z; z++)
    if (banks[z].Az <= 0)
      banks[z].bankrupt = true;
}
void economy::write_default()
{
  short defbank = 0;
  short deffirm = 0;

  for (short z = 0; z < Z; z++)
    if (banks[z].bankrupt == true)
      defbank++;

  for (short i = 0; i < I; i++)
    if (firms[i].eqbankrupt == true || firms[i].lqbankrupt == true)
      deffirm++;

  ofstream ofdefb;
  ofdefb.open("bankdef.txt", ios_base::app);
  ofdefb << defbank << endl;
  ofdefb.close();

  ofstream ofdeff;
  ofdeff.open("firmdef.txt", ios_base::app);
  ofdeff << deffirm << endl;
  ofdeff.close();
}
void economy::consolidate_credits()
{
  for (short z = 0; z < Z; z++)
    for (short i = 0; i < I; i++)
      for (short d = 0; d < D; d++)
      {
        if ((B[z][i][d] + Bnew[z][i][d]) == 0)
          r[z][i][d] = 0;
        else
          r[z][i][d] = ((r[z][i][d] * B[z][i][d]) + (rnew[z][i][d] *
              Bnew[z][i][d])) / (B[z][i][d] + Bnew[z][i][d]); //wghtd
              avg int rt

        B[z][i][d] = B[z][i][d] + Bnew[z][i][d];
        Bnew[z][i][d] = 0;
        rnew[z][i][d] = 0;
      }
}
void economy::write_total_debt()
{
  float debt  = 0;

  for (short z = 0; z < Z; z++)
    for (short i = 0; i < I; i++)
```

```
      for (short d = 0; d < D; d++)
        debt += B[z][i][d];
  ofstream ofdebt;
  ofdebt.open("debt.txt", ios_base::app);
  ofdebt << debt << endl;
  ofdebt.close();
}
void economy::write_net_worth()
{
  float firmNW = 0;
  float bankNW = 0;

  for (short z = 0; z < Z; z++)
    bankNW += banks[z].Az;

  for (short i = 0; i < I; i++)
    firmNW += firms[i].Ai;

  ofstream offnw;
  offnw.open("firmNETW.txt", ios_base::app);
  offnw << firmNW << endl;
  offnw.close();

  ofstream ofbnw;
  ofbnw.open("bankNETW.txt", ios_base::app);
  ofbnw << bankNW << endl;
  ofbnw.close();
}
void economy::write_avg_interest()
{
  float interest = 0;
  float principal = 0;
  float intrate;

  for (short z = 0; z < Z; z++)
    for (short i = 0; i < I; i++)
      for (short d = 0; d < D; d++)
      {
        interest += r[z][i][d] * B[z][i][d];
        principal += B[z][i][d];
      }

  if (principal == 0)
    intrate = 0;
  else
    intrate = interest / principal;

  ofstream ofint;
  ofint.open("intrate.txt", ios_base::app);
  ofint << intrate << endl;
  ofint.close();
}
float economy::fget_Amax()
{
```

```
  float Amax =  firms[0].Ai;
  for (short i = 1; i < I; i++)
    if (firms[i].Ai > Amax)
      Amax = firms[i].Ai;
  return Amax;
}
float economy::fget_crd_need(short i)
{
  float need;
  float crdts = 0;
  float diff;

  for (short z = 0; z < Z; z++)
    for (short d = 1; d < D; d++)
      crdts += B[z][i][d];

  diff = firms[i].BStar - crdts;
  if (diff > 0)
    need = diff;
  else need = 0;
  return need;
}
float economy::get_int_rate(short z, short i)
{
  float y = firms[i].li;
  float car = banks[z].CAR;
  float Az = banks[z].Az;
  float ms = banks[z].mrktshr;
  float blev = banks[z].D / banks[z].Az;
  if (blev < 0.01)
    blev = 0.01;
  if (car < 0.01)
    car = 0.01;
  float interest = rmin + gamma * pow(blev, gamma) + alpha * pow(y,
      alpha);
  return interest;
}
float economy::get_avgAi()
{
  float sum = 0;

  for (short i = 0; i < I; i++)
    sum += firms[i].Ai;

  float avg = sum / I;
  return avg;
}
float economy::get_avgAz()
{
  short num = 0;
  float sum = 0;

  for (short z = 0; z < Z; z++)
  {
```

```
    if (banks[z].bankrupt == false)
    {
      num++;
      sum += banks[z].Az;
    }
  }
  float avg = sum / num;
  return avg;
}
void economy::firm_size_dist()
{
  ofstream ofsize;
  ofsize.open("fsize.txt");
  for (short i = 0; i < I; i++)
    ofsize << firms[i].Ai << endl;
  ofsize.close();
}
void economy::bank_size_dist()
{
  ofstream obsize;
  obsize.open("bsize.txt");
  for (short z = 0; z < Z; z++)
    obsize << banks[z].Az << endl;
  obsize.close();
}
void economy::firm_degree_dist()
{
  short degree[I];

  for (short i = 0; i < I; i++)
  {
    short deg = 0;
    for (short z = 0; z < Z; z++)
    {
      float sum = 0;
      for (short d = 0; d < D; d++)
        sum += B[z][i][d];

      if (sum > 0)
        deg++;
    }

    degree[i] = deg;
  }

  ofstream ofdeg;
  ofdeg.open("fdegdist.txt");
  for (short i = 0; i < I; i++)
    ofdeg << degree[i] << endl;
  ofdeg.close();
}
void economy::bank_degree_dist()
{
  short degree[Z];
```

```
  for (short z = 0; z <Z; z++)
  {
    short deg = 0;
    for (short i = 0; i < I; i++)
    {
      float sum = 0;
      for (short d = 0; d < D; d++)
        sum += B[z][i][d];

      if (sum > 0)
        deg++;
    }

    degree[z] = deg;
  }

  ofstream obdeg;
  obdeg.open("bdegdist.txt");
  for (short z = 0; z < Z; z++)
    obdeg << degree[z] << endl;
  obdeg.close();
}
#endif
```

Class: bankType

```
#ifndef bankType_H
#define bankType_H
#include <iostream>
using namespace std;

extern const float CARmin;

class bankType
{
public:

  float L;          //total amount of credits
  float Az;         //net worth
  float D;          //total amount of deposits
  float Res;          //amount of reserve accounts
  float CAR;          //capital adequacy ratio
  float prz;          //profit of the bank
  bool bankrupt;
  float mrktshr;        //market share

  float get_available_crdt();
  void set_CAR();
};

void bankType::set_CAR()
{
  if (L == 0)
    CAR = 100;
  else
    CAR = Az / L;
}
float bankType::get_available_crdt()
{
  float crdt = Az / CARmin - L;
  return crdt;
}

#endif
```

Class: firmType

```
#ifndef firmType_H
#define firmType_H

#include <iostream>
#include <cmath>
#include <ctime>
#include <random>
#include "economy.h"
using namespace std;
using namespace tr1;
extern const float phi;
extern const float beta;
extern const float u;
extern const float sigmasq;
extern const float theta;
extern const float minprz;
class firmType
{
public:

  float Ai;          //net worth
  float Bi;          //amount of debt
  float li;          //observed leverage
  float K;           //total capital
  float Y;           //total production
  float pri;           //profit of the firm
  float p[3];          //price vector that gives prices of last 3 periods
      . when new price is determined values will slide back
  bool eqbankrupt;       //check for equity bankruptcy
  bool lqbankrupt;       //check for liquidity bankruptcy
  bool lqbad;          //firms liquidity is negative but not as bad as to
      go bankrupt. banks deny to obtain credit
  float tlev;          //target level of leverage for the term
  float BStar;         //target level of debt for the term
  void set_p(std::tr1::mt19937 & eng);
};


void firmType::set_p(std::tr1::mt19937 & eng)
{
  float sd = sqrt(sigmasq);
  normal_distribution<float> normal(0.0, sd);
  float v = normal(eng);
  float prz = u + v;

  p[2] = p[1];
  p[1] = p[0];
  p[0] = prz;
}
#endif
```

Appendix B

Results of Sensitivity Analysis

The results of sensitivity tests are given in the tables below.

Table B.1: ϕ

ϕ	2	2.2	2.4	2.6	2.8	3	3.2	3.4	3.6	3.8	4
Growth std dev (%)	1.66	1.85	1.90	1.96	2.01	2.32	2.20	2.38	2.37	2.64	2.62
Growth skewness	-0.21	-0.34	-0.18	-0.23	-0.06	-0.37	0.06	0.09	0.42	0.28	0.47
Growth kurtosis	1.48	1.64	2.20	6.16	6.61	8.38	9.14	8.90	10.56	11.00	10.86
Growth min (%)	-8.34	-8.70	-8.43	-8.76	-10.23	-15.29	-15.00	-11.05	-13.56	-17.78	-13.27
Growth mean (%)	0.02	0.04	0.06	0.09	0.10	0.09	0.11	0.16	0.13	0.14	0.17
Growth max (%)	8.37	8.60	9.5	10.94	10.95	12.29	13.84	15.96	16.38	16.39	19.06
Bad debt mean (%)	8.91	8.75	8.52	8.17	8.25	8.31	8.29	8.26	8.41	8.44	8.38
Bad debt median (%)	8.51	8.23	8.03	7.68	7.69	7.72	7.49	7.64	7.80	7.87	7.72
Bad debt max (%)	36.95	26.84	34.22	38.08	46.39	55.45	58.38	45.75	50.80	29.53	40.57
Bank default mean (%)	1.47	1.43	1.01	1.02	1.01	0.89	0.99	1.01	1.30	1.07	1.31
Bank default max (%)	28	20	28	26	30	32	26	26	26	52	28
Bank NW mean (000)	13.69	20.83	34.49	61.60	65.19	105.13	102.62	128.69	128.86	179.12	332.46
Bank NW max (000)	25.43	36.25	55.33	120.40	118.99	255.56	191.52	284.04	212.93	380.22	101.04
Total debt min (000)	5.24	5.25	5.24	5.25	5.24	5.25	5.26	5.24	5.25	5.25	5.25
Total debt mean (000)	7.98	10.65	13.72	18.50	22.10	29.13	33.12	40.51	44.33	54.89	72.89
Total debt max (000)	12.65	18.71	22.38	34.70	39.68	67.06	65.73	103.16	71.47	131.8	219.73
Firm default mean (%)	5.73	5.51	5.31	5.16	5.06	4.96	4.92	4.85	4.84	4.78	4.76
Firm default max (%)	8.80	8.80	8.80	9.00	9.00	8.60	7.80	9.00	8.00	8.80	7.80
Aggr. prod. min (000)	7.33	8.66	9.11	10.77	11.59	12.42	13.26	14.07	14.91	15.73	16.56
Aggr. prod. mean (000)	9.41	12.40	15.98	20.93	25.34	32.59	37.60	45.91	50.58	61.92	79.12
Aggr. prod. max (000)	11.58	15.16	22.38	31.71	34.77	58.12	57.86	75.63	70.41	109.83	160.78
Interest min (%)	5.95	5.95	5.94	5.91	5.91	5.88	5.90	5.89	5.90	5.89	5.88
Interest mean (%)	5.99	5.99	5.97	5.96	5.97	5.96	5.96	5.96	5.97	5.96	5.95
Interest max (%)	6.08	6.08	6.08	6.08	6.08	6.08	6.08	6.08	6.08	6.09	6.09
Interest std dev (%)	0.03	0.03	0.03	0.04	0.04	0.04	0.03	0.03	0.03	0.04	0.04
Leverage min	1.00	0.88	0.75	0.69	0.69	0.52	0.69	0.61	0.76	0.58	0.39
Leverage mean	1.45	1.36	1.36	1.24	1.24	1.28	1.20	1.20	1.21	1.26	1.22
Leverage max	2.13	2.15	2.15	2.21	2.18	2.29	2.15	2.23	2.29	2.27	2.32
Leverage std dev	0.20	0.23	0.25	0.28	0.34	0.39	0.29	0.32	0.25	0.31	0.39

Table B.2: β

β	0.6	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80
Growth std dev (%)	1.52	1.46	1.64	1.76	1.88	2.32	2.55	2.70	3.29	4.07	4.82
Growth skewness	-0.15	0.17	-0.01	0.18	0.03	-0.37	-1.07	-0.22	-2.18	-2.57	-3.96
Growth kurtosis	4.34	4.15	5.05	5.80	7.03	8.38	16.28	8.79	30.92	38.10	46.35
Growth min (%)	-5.97	-4.14	-7.04	-9.61	-10.49	-15.29	-24.35	-13.73	-40.53	-54.24	-63.20
Growth mean (%)	0.03	0.03	0.05	0.06	0.08	0.09	0.13	0.17	0.20	0.29	0.37
Growth max (%)	8.29	8.81	8.86	10.03	10.84	12.29	15.89	16.56	17.70	20.36	22.39
Bad debt mean (%)	8.49	8.78	8.70	8.61	8.32	8.31	7.70	7.92	7.08	7.71	6.48
Bad debt median (%)	8.15	8.52	8.35	8.09	7.86	7.72	6.70	7.15	6.12	6.29	4.54
Bad debt max (%)	30.14	25.96	36.09	33.64	38.71	55.45	70.47	43.49	76.54	86.16	89.56
Bank default mean (%)	4.61	1.39	1.36	1.10	1.01	0.89	0.95	0.92	0.75	0.94	0.88
Bank default max (%)	20	16	20	32	30	32	26	18	28	32	22
Bank NW mean (000)	3.55	18.41	27.94	42.12	60.25	105.13	173.30	237.29	669.61	887.87	2709.70
Bank NW max (000)	9.02	32.64	45.32	93.43	85.02	255.56	358.25	554.76	1609.95	2272.77	6470.46
Total debt min (000)	5.24	5.24	5.25	5.25	5.25	5.25	5.25	5.24	5.26	5.24	5.25
Total debt mean (000)	9.77	11.19	13.31	16.41	20.92	29.13	41.65	55.10	100.88	152.20	367.99
Total debt max (000)	13.11	15.16	19.18	29.89	30.32	67.05	101.09	146.06	339.06	612.48	1690.27
Firm default mean (%)	5.34	5.35	5.27	5.20	5.07	4.96	4.91	4.76	4.70	4.65	4.56
Firm default max (%)	9.80	9.20	8.80	8.20	8.80	8.60	9.00	8.20	8.00	8.47	8.00
Aggr. prod. min (000)	9.06	9.75	10.37	11.01	11.70	12.42	13.20	14.01	14.91	15.81	16.80
Aggr. prod. mean (000)	11.47	13.16	15.69	19.02	24.14	32.59	45.06	60.67	111.69	157.48	356.51
Aggr. prod. max (000)	13.77	15.71	19.52	25.13	31.60	58.12	79.42	112.89	262.90	424.10	1047.20
Interest min (%)	6.01	5.96	5.94	5.93	5.93	5.88	5.88	5.86	5.83	5.78	5.67
Interest mean (%)	6.05	5.99	5.98	5.98	5.97	5.96	5.95	5.95	5.92	5.93	5.9
Interest max (%)	6.08	6.08	6.08	6.08	6.08	6.08	6.09	6.08	6.09	6.09	6.09
Interest std dev (%)	0.01	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.05	0.05	0.05
Leverage min	0.9	0.70	0.77	0.54	0.81	0.52	0.48	0.48	0.36	0.37	0.26
Leverage mean	1.24	1.31	1.23	1.29	1.21	1.28	1.08	1.23	1.06	1.26	0.96
Leverage max	2.03	2.00	2.05	2.16	2.17	2.29	2.19	2.15	2.27	2.76	2.35
Leverage std dev	0.19	0.29	0.23	0.37	0.21	0.39	0.35	0.37	0.38	0.51	0.43

Table B.3: Δl_{max}

Δl_{max}	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22
Growth std dev (%)	1.09	1.46	1.75	1.96	2.32	2.33	2.54	2.99	3.35	3.16	3.29
Growth skewness	4.19	1.27	0.61	0.41	-0.37	-0.73	-0.16	-0.71	-1.21	-0.78	-1.13
Growth kurtosis	39.09	14.59	10.75	9.04	8.38	12.77	7.03	7.01	12.21	6.99	9.44
Growth min (%)	-4.12	-6.1	-8.06	-9.08	-15.29	-18.09	-13.74	-17.45	-30.54	-14.48	-25.02
Growth mean (%)	0.24	0.16	0.12	0.1	0.09	0.16	0.15	0.19	0.15	0.17	0.16
Growth max (%)	12.11	11.86	13.12	13.32	12.29	13.5	13.81	13.21	13.74	13.17	13.61
Bad debt mean (%)	1.72	3.47	5.19	7.04	8.31	8.31	9.16	9.74	9.58	9.51	10.11
Bad debt median (%)	0.84	2.68	4.51	6.42	7.72	7.59	8.21	8.71	8.19	7.91	8.76
Bad debt max (%)	13	24.41	26.33	38.38	55.45	58.27	48.18	46.98	72.11	53.02	77.19
Bank default mean (%)	0.07	0.21	0.54	0.77	0.89	2.34	3.8	3.68	5.1	5.5	5.66
Bank default max (%)	6	8	14	22	32	36	34	40	46	42	36
Bank NW mean (000)	2376.50	593.00	280.23	137.21	105.13	63.09	40.83	66.61	41.72	42.29	69.04
Bank NW max (000)	5087.82	1086.72	458.11	216.38	255.56	292.57	126.99	208.32	149.49	135.79	235.22
Total debt min (000)	5.05	5.10	5.15	5.19	5.25	5.29	5.35	5.39	5.43	5.50	5.53
Total debt mean (000)	205.39	65.06	40.58	30.29	29.13	31.15	29.74	32.92	37.53	43.70	48.02
Total debt max (000)	388.93	111.63	70.34	53.15	67.05	90.49	62.84	91.47	101.48	117.38	170.94
Firm default mean (%)	3.57	4.08	4.46	4.75	4.96	5.14	5.24	5.29	5.32	5.41	5.42
Firm default max (%)	7.00	7.00	8.40	7.60	8.60	8.80	9.00	11.00	10.80	10.80	11.80
Aggr. prod. min (000)	12.25	12.30	12.34	12.38	12.42	12.46	12.51	12.54	12.58	12.64	12.67
Aggr. prod. mean (000)	14.78	59.35	41.62	33.65	32.59	34.51	34.08	36.71	41.70	48.61	52.99
Aggr. prod. max (000)	224.99	79.46	52.85	45.84	58.12	58.19	55.22	75.17	88.27	99.81	120.52
Interest min (%)	5.77	5.81	5.82	5.90	5.88	5.90	5.92	5.93	5.93	5.95	5.92
Interest mean (%)	5.86	5.89	5.92	5.95	5.96	5.99	6.01	6.00	6.01	6.01	6.01
Interest max (%)	6.06	6.06	6.07	6.08	6.08	6.08	6.09	6.09	6.09	6.09	6.09
Interest std dev (%)	0.06	0.06	0.06	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
Leverage min	0.23	0.41	0.59	0.65	0.52	0.34	0.4	0.41	0.35	0.32	0.22
Leverage mean	0.48	0.7	0.89	1.11	1.28	1.11	1.24	1.38	1.09	0.95	1.02
Leverage max	1.68	1.81	1.98	2.14	2.29	2.32	2.41	3.35	2.96	3.24	3.99
Leverage std dev	0.29	0.28	0.25	0.23	0.39	0.48	0.58	0.74	0.65	0.72	0.88

Table B.4: u

u	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22
Growth std dev (%)	12.79	9.43	1.07	1.75	2.32	2.72	3.59	4.19	4.48	5.16	5.22
Growth skewness	-1.05	-1.07	-0.28	-0.92	-0.37	0.29	-0.12	-0.36	-0.34	-0.27	-0.25
Growth kurtosis	9.29	10.31	6.59	12.08	8.38	8.92	9.83	10.71	10.97	7.81	8.69
Growth min (%)	-83.32	-52.66	-5.98	-15.34	-15.29	-13.83	-22.02	-27.64	-28.28	-24.36	-25.51
Growth mean (%)	1.49	2.82	0.12	0.09	0.09	0.14	0.20	0.22	0.25	0.42	0.36
Growth max (%)	53.76	46.03	6.28	10.22	12.29	17.01	18.65	22.74	24.01	24.28	26.40
Bad debt mean (%)	7.54	3.72	3.58	5.84	8.31	9.36	9.24	10.22	10.48	10.06	10.18
Bad debt median (%)	7.19	2.30	2.45	4.95	7.72	8.80	8.48	9.70	10.09	8.55	8.56
Bad debt max (%)	24.56	22.39	32.03	51.67	55.45	41.88	44.81	45.31	47.35	57.57	69.24
Bank default mean (%)	1.26	0.56	0.64	0.59	0.89	2.35	3.13	3.29	3.78	4.81	4.53
Bank default max (%)	14.00	16.00	20.00	24.00	32	34.00	40.00	68.00	60.00	46.00	50.00
Bank NW mean (000)	8.62	146.24	275.81	150.60	105.13	115.01	136.67	230.28	386.32	249.84	487.61
Bank NW max (000)	34.58	485.82	635.12	261.84	255.56	415.30	461.75	841.22	1625.90	1260.40	2005.20
Total debt min (000)	1.56	2.91	5.25	5.25	5.25	5.25	5.25	5.24	5.24	5.24	5.26
Total debt mean (000)	2.67	17.27	30.03	24.11	29.13	45.43	79.54	118.55	185.19	243.95	370.52
Total debt max (000)	5.37	45.91	60.61	45.58	67.05	177.53	222.80	387.77	785.65	741.18	1201.70
Firm default mean (%)	6.86	5.37	5.08	5.01	4.96	4.88	4.68	4.60	4.50	4.19	4.19
Firm default max (%)	10.80	9.40	8.00	8.60	8.6	9.40	10.60	12.40	14.40	14.40	17.20
Aggr. prod. min (000)	5.12	7.56	11.70	12.42	12.43	12.42	12.43	12.42	12.41	12.42	12.43
Aggr. prod. mean (000)	18.49	69.40	42.25	29.40	32.58	43.87	66.73	87.88	121.19	151.66	202.64
Aggr. prod. max (000)	51.03	199.58	63.65	40.98	58.12	102.59	149.57	218.52	340.03	360.96	466.67
Interest min (%)	5.78	5.75	5.75	5.86	5.88	5.89	5.91	5.93	5.91	5.94	5.92
Interest mean (%)	5.95	5.88	5.88	5.91	5.96	5.99	6.00	5.99	5.99	6.02	6.01
Interest max (%)	6.07	6.06	6.08	6.07	6.08	6.09	6.09	6.09	6.09	6.10	6.09
Interest std dev (%)	0.08	0.09	0.08	0.05	0.04	0.04	0.04	0.04	0.04	0.04	0.04
Leverage min	0.03	0.04	0.17	0.35	0.52	0.42	0.43	0.24	0.21	0.13	0.14
Leverage mean	0.38	0.31	0.46	0.76	1.28	1.56	1.46	1.66	1.67	1.45	1.43
Leverage max	1.78	1.81	1.92	2.00	2.29	2.45	2.69	2.98	3.09	3.15	3.21
Leverage std dev	0.45	0.43	0.39	0.29	0.39	0.57	0.73	0.88	0.99	0.89	0.95

Table B.5: σ^2

σ^2	0.001	0.005	0.010	0.015	0.020	0.030	0.050	0.070	0.100	0.200	0.400
Growth std dev (%)	2.21	2.00	2.32	2.34	2.58	2.85	2.82	3.69	5.03	6.13	2.99
Growth skewness	0.87	0.26	-0.37	-0.25	-0.81	-0.89	-0.49	-1.36	-0.18	-0.49	-1.42
Growth kurtosis	9.61	7.98	8.38	7.71	9.31	7.83	7.23	10.80	22.72	11.62	15.14
Growth min (%)	-10.88	-9.45	-15.29	-16.65	-18.20	-19.57	-16.93	-26.42	-40.90	-38.16	-24.57
Growth mean (%)	0.08	0.08	0.09	0.12	0.11	0.23	0.19	0.39	0.41	0.36	0.25
Growth max (%)	14.39	12.25	12.29	13.42	12.73	11.12	14.01	13.04	51.35	44.73	16.95
Bad debt mean (%)	8.04	8.32	8.31	8.86	8.83	9.08	8.20	11.65	13.51	15.18	7.58
Bad debt median (%)	7.71	7.93	7.72	8.08	7.91	7.52	6.59	8.34	10.65	11.23	4.75
Bad debt max (%)	41.38	31.19	55.45	53.98	64.36	58.37	67.12	70.54	87.71	87.35	80.27
Bank default mean (%)	3.53	0.91	0.89	2.23	1.92	2.72	3.66	4.34	6.05	7.70	18.80
Bank default max (%)	24.00	20.00	32.00	28.00	32.00	30.00	32.00	46.00	38.00	48.00	54.00
Bank NW mean (000)	18.46	61.71	105.13	80.09	140.21	316.08	351.81	656.46	1220.40	2367.30	2521.40
Bank NW max (000)	100.24	92.24	255.56	245.91	417.36	1022.80	117.25	2382.10	4930.00	8611.30	7971.10
Total debt min (000)	5.24	92.25	5.25	5.25	5.26	5.20	5.15	4.79	4.20	3.69	2.08
Total debt mean (000)	22.16	23.00	29.13	29.98	36.49	62.08	78.36	109.87	137.28	248.57	270.56
Total debt max (000)	48.61	41.83	67.05	67.36	113.64	230.52	246.79	466.34	821.21	1449.40	1464.90
Firm default mean (%)	4.89	4.71	4.96	5.54	6.20	7.25	9.35	10.69	12.73	17.28	18.80
Firm default max (%)	8.00	8.40	8.60	8.80	11.20	11.60	14.60	17.20	20.00	25.80	27.80
Aggr. prod. min (000)	12.42	12.42	12.43	12.42	12.43	12.41	12.42	12.42	12.19	11.98	10.71
Aggr. prod. mean (000)	27.11	27.86	32.58	32.75	37.53	59.42	73.34	112.37	122.69	199.21	251.02
Aggr. prod. max (000)	43.89	38.98	58.12	57.55	86.28	146.16	186.06	351.67	435.88	785.83	716.63
Interest min (%)	5.95	5.92	5.88	5.90	5.87	5.83	5.83	5.79	5.81	5.73	5.79
Interest mean (%)	6.04	5.98	5.96	5.98	5.95	5.95	5.95	5.94	5.94	5.92	5.91
Interest max (%)	6.09	6.08	6.08	6.08	6.09	6.09	6.08	6.10	6.07	6.07	6.06
Interest std dev (%)	0.04	0.03	0.04	0.04	0.04	0.06	0.06	0.07	0.07	0.08	0.07
Leverage min	0.67	0.83	0.52	0.53	0.40	0.25	0.23	0.06	0.07	0.02	0.01
Leverage mean	1.68	1.47	1.28	1.23	1.00	0.98	0.58	1.07	0.97	1.12	0.27
Leverage max	2.61	2.12	2.29	2.26	2.14	3.31	2.83	5.98	15.68	9.73	1.05
Leverage std dev	0.52	0.29	0.39	0.39	0.32	0.60	0.37	1.07	1.04	1.24	0.19

Table B.6: θ

θ	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Growth std dev (%)	1.69	1.91	2.15	2.45	2.34	2.32	2.33	2.19	2.19	2.26	2.20
Growth skewness	-2.87	-3.63	-3.16	-1.99	-0.93	-0.37	-0.36	0.91	1.11	0.96	1.33
Growth kurtosis	20.38	39.01	22.90	17.57	9.52	8.38	12.90	7.28	8.71	8.33	8.96
Growth min (%)	-11.93	-23.02	-17.89	-25.63	-14.53	-15.29	-21.27	-6.69	-6.48	-8.83	-6.13
Growth mean (%)	0.37	0.34	0.27	0.13	0.10	0.09	0.12	0.13	0.15	0.16	0.17
Growth max (%)	8.23	12.09	12.18	12.48	14.17	12.29	11.86	12.14	13.80	13.90	13.43
Bad debt mean (%)	5.01	4.47	4.93	7.93	7.87	8.31	8.38	8.71	8.19	7.98	7.32
Bad debt median (%)	0.54	0.45	2.36	6.56	6.99	7.72	7.87	8.40	7.93	7.73	7.17
Bad debt max (%)	77.54	89.15	75.66	63.06	52.70	55.45	73.12	23.70	21.67	21.45	16.55
Bank default mean (%)	8.98	7.16	5.25	1.18	0.84	0.89	3.47	1.48	1.69	0.43	0.09
Bank default max (%)	38.00	44.00	24.00	24.00	26.00	32	32.00	20.00	24.00	6.00	4.00
Bank NW mean (000)	1404.40	2290.90	466.07	93.89	88.69	105.13	53.82	83.36	115.18	209.23	315.84
Bank NW max (000)	3459.80	5361.30	841.96	226.29	180.25	255.56	223.03	121.59	201.80	283.12	409.61
Total debt min (000)	2.97	3.35	4.56	5.25	5.25	5.25	5.24	5.25	5.24	5.25	5.24
Total debt mean (000)	181.12	272.45	60.72	21.98	22.98	29.13	34.64	41.95	53.35	62.50	71.62
Total debt max (000)	1613.80	1575.60	173.66	72.57	50.48	67.05	73.26	62.95	80.39	94.09	116.89
Firm default mean (%)	8.66	7.40	6.32	6.09	5.68	4.96	4.25	3.66	3.25	2.94	2.71
Firm default max (%)	22.00	21.40	19.00	9.60	9.40	8.6	7.80	6.40	5.60	5.60	5.20
Aggr. prod. min (000)	9.37	11.06	12.42	12.42	12.43	12.43	12.42	12.43	12.42	12.42	12.42
Aggr. prod. mean (000)	406.71	439.73	96.12	34.01	30.28	32.58	34.66	37.68	42.99	46.76	50.65
Aggr. prod. max (000)	764.41	790.99	162.06	74.99	50.73	58.12	45.22	49.75	59.65	63.88	74.00
Interest min (%)	5.69	5.76	5.81	5.86	5.87	5.88	5.96	5.97	5.98	5.97	5.97
Interest mean (%)	5.85	5.86	5.88	5.92	5.94	5.96	6.03	6.01	6.02	6.00	5.99
Interest max (%)	6.07	6.07	6.07	6.08	6.08	6.08	6.09	6.09	6.10	6.07	6.07
Interest std dev (%)	0.09	0.07	0.06	0.04	0.04	0.04	0.04	0.03	0.03	0.02	0.02
Leverage min	0.02	0.03	0.09	0.26	0.40	0.52	0.56	1.05	1.05	1.04	1.05
Leverage mean	0.10	0.11	0.21	0.54	0.83	1.28	1.66	2.66	3.18	4.11	4.52
Leverage max	1.51	1.61	1.57	1.54	1.77	2.29	2.86	3.50	4.18	5.21	5.34
Leverage std dev	0.18	0.15	0.12	0.15	0.21	0.39	0.61	0.44	0.53	0.37	0.42

Table B.7: lqlim

lqlim	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Growth std dev (%)	2.84	2.50	2.53	2.32	2.07	2.01	1.89	1.91	1.85	1.78
Growth skewness	-1.72	-1.07	-0.99	-0.37	0.43	1.04	0.81	1.05	1.29	1.27
Growth kurtosis	13.76	13.94	11.25	8.38	9.20	9.89	9.21	10.87	11.16	11.65
Growth min (%)	-20.75	-24.19	-20.12	-15.29	-10.05	-6.53	-7.54	-7.17	-6.52	-6.36
Growth mean (%)	0.19	0.20	0.18	0.09	0.11	0.13	0.13	0.13	0.13	0.15
Growth max (%)	11.48	12.40	13.18	12.29	13.83	14.37	13.28	14.28	12.86	13.35
Bad debt mean (%)	9.48	9.05	8.79	8.31	7.65	7.03	6.48	5.84	5.47	5.04
Bad debt median (%)	8.06	8.25	8.22	7.72	7.02	6.51	6.00	5.29	5.01	4.51
Bad debt max (%)	68.27	71.52	61.59	55.45	40.11	26.39	27.82	26.54	26.59	35.43
Bank default mean (%)	2.69	2.28	2.04	0.89	0.92	0.52	0.39	0.19	0.10	0.04
Bank default max (%)	44.00	34.00	36.00	32	16.00	16.00	8.00	8.00	6.00	4.00
Bank NW mean (000)	333.92	234.94	158.70	105.13	109.45	131.78	169.71	232.17	287.35	346.89
Bank NW max (000)	920.73	809.87	489.15	255.56	168.71	194.50	258.22	342.78	390.33	475.69
Total debt min (000)	4.84	5.25	5.25	5.25	5.24	5.24	5.25	5.23	5.24	5.24
Total debt mean (000)	60.89	43.41	33.11	29.13	29.93	30.39	32.95	38.39	42.23	47.41
Total debt max (000)	204.60	192.82	106.25	67.05	47.44	42.48	53.21	57.64	68.89	71.59
Firm default mean (%)	6.75	5.93	5.36	4.96	4.60	4.19	3.84	3.56	3.26	3.00
Firm default max (%)	15.40	10.20	9.40	8.6	8.00	7.20	7.00	6.80	6.80	5.20
Aggr. prod. min (000)	11.82	12.42	12.42	12.43	12.42	12.42	12.42	12.41	12.42	12.42
Aggr. prod. mean (000)	62.38	46.83	36.61	32.58	32.16	33.85	36.16	40.24	43.89	48.15
Aggr. prod. max (000)	154.00	116.59	90.82	58.12	40.69	40.21	43.17	49.50	56.86	62.47
Interest min (%)	5.81	5.83	5.86	5.88	5.88	5.91	5.91	5.86	5.89	5.89
Interest mean (%)	5.94	5.95	5.96	5.96	5.96	5.95	5.94	5.94	5.93	5.92
Interest max (%)	6.08	6.08	6.09	6.08	6.09	6.08	6.06	6.07	6.07	6.06
Interest std dev (%)	0.07	0.06	0.06	0.04	0.04	0.04	0.04	0.04	0.03	0.03
Leverage min	0.23	0.29	0.39	0.52	0.77	0.90	0.87	0.79	0.76	0.77
Leverage mean	0.89	1.06	1.15	1.28	1.25	1.29	1.24	1.19	1.17	1.16
Leverage max	2.47	2.23	2.18	2.29	2.29	2.15	2.10	2.14	2.08	1.97
Leverage std dev	0.60	0.60	0.46	0.39	0.29	0.19	0.21	0.25	0.21	0.19

Table B.8: rmin

rmin	0.005	0.010	0.015	0.020	0.040	0.060	0.080	0.100	0.150
Growth std dev (%)	2.85	2.42	2.42	2.32	1.53	0.79	0.59	0.59	0.74
Growth skewness	-0.60	-0.11	-0.19	-0.37	-0.21	5.42	3.38	2.32	0.61
Growth kurtosis	9.52	9.36	7.86	8.38	16.32	63.41	26.85	21.29	11.21
Growth min (%)	-18.66	-17.01	-14.40	-15.29	-11.35	-3.33	-1.33	-2.08	-5.52
Growth mean (%)	0.18	0.12	0.11	0.09	0.12	0.44	0.63	0.72	0.76
Growth max (%)	12.83	15.21	13.18	12.29	11.52	10.65	7.03	7.36	5.45
Bad debt mean (%)	7.64	7.95	8.29	8.31	4.75	1.89	1.05	0.74	0.54
Bad debt median (%)	6.41	6.89	7.54	7.72	3.92	0.82	0.19	0.04	0.00
Bad debt max (%)	48.66	52.54	41.10	55.45	46.39	19.27	13.28	8.89	9.63
Bank default mean (%)	4.40	4.12	2.30	0.89	0.37	0.02	0.00	0.00	0.00
Bank default max (%)	38.00	26.00	34.00	32.00	10.00	4.00	2.00	0.00	0.00
Bank NW mean (000)	42.98	46.08	69.12	105.13	445.68	2919.50	9906.00	1668.40	34421.00
Bank NW max (000)	136.82	167.89	155.27	255.56	815.94	9721.00	3981.50	82299.00	161755.00
Total debt min (000)	5.24	5.25	5.25	5.25	5.24	5.26	5.25	5.25	4.42
Total debt mean (000)	47.80	37.28	32.62	29.13	37.72	172.90	506.44	798.81	1178.40
Total debt max (000)	105.19	90.08	76.71	67.05	83.79	481.94	1796.50	3531.40	4819.60
Firm default mean (%)	4.78	4.92	4.97	4.96	4.85	4.09	2.61	1.36	0.57
Firm default max (%)	9.20	8.20	9.40	8.60	7.60	7.00	6.60	5.40	6.60
Aggr. prod. min (000)	12.42	12.42	12.43	12.43	12.42	12.43	12.42	12.42	11.58
Aggr. prod. mean (000)	45.63	37.89	34.59	32.58	41.85	414.44	1894.30	4463.50	7011.20
Aggr. prod. max (000)	74.25	61.05	56.52	58.12	57.53	1018.90	6212.70	16350.00	22881.00
Interest min (%)	4.46	4.94	5.43	5.88	7.77	9.66	11.72	13.71	18.69
Interest mean (%)	4.52	5.01	5.49	5.96	7.87	9.79	11.79	13.76	18.73
Interest max (%)	4.59	5.09	5.58	6.08	8.06	10.06	12.06	14.05	19.04
Interest std dev (%)	0.03	0.04	0.04	0.04	0.07	0.08	0.07	0.07	0.06
Leverage min	0.48	0.46	0.59	0.52	0.16	0.02	0.02	0.01	0.01
Leverage mean	1.12	1.24	1.32	1.28	0.51	0.17	0.12	0.10	0.07
Leverage max	2.53	2.35	2.33	2.29	1.91	1.73	1.53	1.56	1.59
Leverage std dev	0.50	0.43	0.39	0.39	0.33	0.32	0.26	0.25	0.20

Table B.9: γ

γ	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
Growth std dev (%)	2.71	2.63	2.44	2.32	1.96	1.84	1.80	1.67	1.54	1.53
Growth skewness	-1.32	-0.77	-0.35	-0.37	0.30	0.31	0.15	0.49	-0.18	-2.80
Growth kurtosis	12.09	10.40	9.01	8.38	7.88	7.62	7.10	9.24	11.09	49.32
Growth min (%)	-20.61	-21.11	-14.80	-15.29	-8.94	-8.57	-7.60	-7.62	-11.18	-21.81
Growth mean (%)	0.37	0.10	0.14	0.09	0.11	0.11	0.09	0.12	0.12	0.12
Growth max (%)	13.11	12.98	14.17	12.29	13.56	12.71	12.85	5.80	10.44	9.93
Bad debt mean (%)	7.67	7.34	7.57	8.31	7.91	7.22	7.07	5.02	5.09	4.62
Bad debt median (%)	6.82	6.45	6.73	7.72	7.39	6.56	6.46	36.67	4.33	3.70
Bad debt max (%)	68.34	71.73	47.72	55.45	32.47	35.78	30.39	0.03	44.12	61.73
Bank default mean (%)	2.14	1.77	2.33	0.89	0.81	0.38	0.18	0.03	0.02	0.01
Bank default max (%)	42.00	28.00	42.00	32.00	20.00	10.00	8.00	2.00	2.00	4.00
Bank NW mean (000)	509.76	224.38	85.03	105.13	101.05	161.15	188.48	349.91	436.59	553.88
Bank NW max (000)	2225.50	647.37	266.48	255.56	180.92	270.83	304.06	628.45	768.58	1142.70
Total debt min (000)	5.26	5.25	5.26	5.25	5.25	5.25	5.25	5.25	5.24	5.25
Total debt mean (000)	132.86	67.77	39.03	29.13	25.09	25.58	24.60	30.97	34.46	39.99
Total debt max (000)	834.54	211.90	95.88	67.05	38.29	42.31	51.97	62.22	70.04	117.99
Firm default mean (%)	4.10	4.58	4.85	4.96	4.99	4.95	4.95	4.83	4.76	4.70
Firm default max (%)	10.20	8.60	8.80	8.60	8.40	8.40	8.00	8.00	8.20	8.40
Aggr. prod. min (000)	12.43	12.43	12.43	12.43	12.42	12.42	12.43	12.43	12.42	12.42
Aggr. prod. mean (000)	120.45	62.32	40.51	32.58	29.77	30.50	29.94	36.28	40.14	46.17
Aggr. prod. max (000)	423.70	152.13	79.79	58.12	36.45	39.08	42.05	51.34	60.87	71.86
Interest min (%)	4.45	4.95	5.44	5.88	6.35	6.74	7.07	7.12	7.69	7.64
Interest mean (%)	4.49	4.98	5.49	5.96	6.43	6.85	7.27	7.60	7.96	8.30
Interest max (%)	4.52	5.03	5.56	6.08	6.61	7.14	7.67	8.22	8.76	9.30
Interest std dev (%)	0.02	0.02	0.03	0.04	0.07	0.09	0.11	0.16	0.19	0.25
Leverage min	0.09	0.27	0.48	0.52	0.59	0.63	0.49	0.33	0.21	0.14
Leverage mean	1.21	1.17	1.16	1.28	1.11	1.01	0.99	0.78	0.62	0.51
Leverage max	2.41	2.51	2.18	2.29	2.12	1.98	2.06	2.04	1.82	1.78
Leverage std dev	0.82	0.62	0.48	0.39	0.27	0.23	0.31	0.33	0.33	0.36

Table B.10: α

α	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
Growth std dev (%)	2.60	2.58	2.46	2.32	1.93	1.96	1.81	1.71	1.43	1.36
Growth skewness	-0.52	-0.43	-0.46	-0.37	0.57	0.07	0.18	-1.38	0.68	-0.93
Growth kurtosis	7.57	7.52	10.14	8.38	7.26	6.58	0.02	23.51	9.01	20.91
Growth min (%)	-15.91	-14.23	-18.66	-15.29	-6.22	-8.91	-7.28	-19.03	-5.10	-13.62
Growth mean (%)	0.17	0.11	0.14	0.09	0.09	0.10	0.12	0.12	0.12	0.16
Growth max (%)	13.27	13.18	13.37	12.29	13.06	11.04	11.26	10.49	9.16	10.17
Bad debt mean (%)	7.43	7.67	8.08	8.31	8.28	7.66	6.88	5.81	4.78	3.93
Bad debt median (%)	6.39	6.76	7.63	7.72	7.78	6.97	6.21	5.14	3.97	2.90
Bad debt max (%)	43.94	52.63	57.81	55.45	31.56	25.62	37.17	53.45	21.24	49.22
Bank default mean (%)	1.91	1.93	1.63	0.89	1.06	0.46	0.12	0.04	0.03	0.00
Bank default max (%)	32.00	36.00	28.00	32.00	14.00	14.00	6.00	6.00	4.00	2.00
Bank NW mean (000)	422.94	148.93	144.06	105.13	93.72	145.97	222.99	337.44	418.10	621.11
Bank NW max (000)	1502.70	348.08	459.96	255.56	144.29	212.95	412.46	744.24	687.82	1052.70
Total debt min (000)	5.25	5.25	5.25	5.25	5.25	5.25	5.25	5.25	5.26	5.24
Total debt mean (000)	119.52	52.73	37.37	29.13	23.47	23.66	25.44	30.47	31.86	40.48
Total debt max (000)	550.20	142.60	120.63	67.05	37.75	44.54	56.12	85.11	51.41	80.25
Firm default mean (%)	4.28	4.69	4.88	4.96	5.04	4.95	4.88	4.84	4.73	4.56
Firm default max (%)	9.20	9.20	8.60	8.60	8.20	9.20	8.60	12.43	8.00	8.60
Aggr. prod. min (000)	12.43	12.43	12.43	12.43	12.43	12.42	12.43	12.43	12.43	12.41
Aggr. prod. mean (000)	100.02	50.40	38.65	32.58	28.13	29.07	30.98	36.27	38.18	48.87
Aggr. prod. max (000)	341.95	110.18	93.03	58.12	35.32	38.99	48.45	59.65	47.21	64.59
Interest min (%)	4.36	4.85	5.37	5.88	6.41	6.85	7.30	7.66	8.15	8.52
Interest mean (%)	4.47	4.97	5.46	5.96	6.46	6.93	7.39	7.83	8.28	8.69
Interest max (%)	4.57	5.08	5.98	6.08	6.59	7.08	7.58	8.08	8.58	9.08
Interest std dev (%)	0.07	0.05	0.05	0.04	0.04	0.05	0.06	0.09	0.09	0.12
Leverage min	0.13	0.34	0.44	0.52	0.69	0.62	0.42	0.15	0.28	0.19
Leverage mean	1.25	1.26	1.26	1.28	1.26	1.11	0.97	0.63	0.63	0.50
Leverage max	2.78	2.61	2.19	2.29	2.05	2.08	2.09	1.91	1.81	1.79
Leverage std dev	0.82	0.60	0.39	0.39	0.28	0.28	0.32	0.38	0.32	0.32

Table B.11: RR

RR	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Growth std dev (%)	2.54	2.22	2.26	2.15	2.32	2.07	2.15	2.13	2.13	2.05
Growth skewness	-1.17	-0.71	-0.53	0.27	-0.37	0.34	0.28	0.58	0.09	0.42
Growth kurtosis	13.48	13.68	10.87	7.31	8.38	8.40	5.86	6.70	6.39	6.44
Growth min (%)	-19.92	-18.23	-17.73	-10.03	-15.29	-9.20	-6.63	-8.57	-10.17	-8.52
Growth mean (%)	0.12	0.12	0.12	0.15	0.09	0.11	0.09	0.11	0.11	0.11
Growth max (%)	11.91	14.13	13.31	14.15	12.29	12.49	9.13	12.88	12.16	12.03
Bad debt mean (%)	6.07	5.98	6.47	7.41	8.31	8.97	9.13	9.25	9.07	9.35
Bad debt median (%)	4.95	4.99	5.66	6.97	7.72	8.39	8.37	8.90	8.47	8.76
Bad debt max (%)	63.59	46.22	57.03	36.05	55.45	35.29	36.73	36.42	32.84	36.65
Bank default mean (%)	3.23	3.17	2.65	2.15	0.89	0.70	0.07	0.00	0.00	0.00
Bank default max (%)	48.00	40.00	30.00	30.00	32.00	14.00	4.00	2.00	0.00	0.00
Bank NW mean (000)	224.75	145.29	135.43	112.26	105.13	105.11	144.29	197.51	267.69	298.63
Bank NW max (000)	563.70	389.96	342.23	349.34	255.56	168.04	225.52	280.20	347.33	417.36
Total debt min (000)	5.25	5.25	5.25	5.26	5.25	5.25	5.25	5.23	5.26	5.25
Total debt mean (000)	61.94	50.99	43.81	35.02	29.13	24.47	23.57	23.29	24.05	22.94
Total debt max (000)	173.38	143.05	114.51	79.89	67.05	42.52	35.49	35.59	41.40	34.87
Firm default mean (%)	4.64	4.72	4.78	4.83	4.96	5.04	5.07	5.07	5.08	5.08
Firm default max (%)	8.60	9.20	8.60	9.00	8.60	8.40	8.40	8.00	8.40	8.20
Aggr. prod. min (000)	12.42	12.42	12.43	12.43	12.43	12.43	12.42	12.41	12.43	12.42
Aggr. prod. mean (000)	60.71	51.09	45.28	38.41	32.58	28.81	27.95	27.78	28.29	27.31
Aggr. prod. max (000)	134.93	93.36	82.79	71.74	58.12	39.83	34.30	34.75	34.30	34.78
Interest min (%)	5.86	5.87	5.88	5.89	5.88	5.92	5.89	5.88	5.88	5.86
Interest mean (%)	5.95	5.96	5.94	5.97	5.96	5.96	5.94	5.92	5.91	5.91
Interest max (%)	6.08	6.08	6.08	6.08	6.08	6.08	6.07	6.06	6.07	6.06
Interest std dev (%)	0.06	0.06	0.05	0.05	0.04	0.04	0.03	0.03	0.03	0.03
Leverage min	0.24	0.28	0.41	0.49	0.52	0.72	0.96	1.04	0.98	0.97
Leverage mean	0.88	0.77	0.90	1.11	1.28	1.40	1.43	1.45	1.37	1.45
Leverage max	1.95	2.06	2.06	2.10	2.29	2.27	2.24	2.32	2.25	2.48
Leverage std dev	0.43	0.33	0.42	0.41	0.39	0.32	0.25	0.25	0.27	0.26

Table B.12: c

c	0.001	0.003	0.005	0.010	0.015	0.020	0.030
Growth std dev (%)	2.10	2.26	2.32	2.24	2.11	2.44	2.64
Growth skewness	0.22	0.22	-0.37	0.12	-0.35	-1.24	-1.66
Growth kurtosis	7.05	7.37	8.38	8.41	9.61	12.08	11.45
Growth min (%)	-10.85	-9.98	-15.29	-11.23	-13.92	-18.43	-18.27
Growth mean (%)	0.11	0.12	0.09	0.19	0.10	0.12	0.20
Growth max (%)	13.72	14.89	12.29	13.22	11.66	12.58	10.23
Bad debt mean (%)	8.82	8.69	8.31	7.18	6.38	5.73	3.47
Bad debt median (%)	8.27	8.16	7.72	6.67	5.47	4.43	1.97
Bad debt max (%)	46.68	40.96	55.45	44.43	51.09	71.06	52.99
Bank default mean (%)	0.72	0.94	0.89	1.92	1.88	1.63	1.76
Bank default max (%)	18.00	18.00	32.00	26.00	22.00	28.00	30.00
Bank NW mean (000)	135.81	89.96	105.13	84.61	86.83	244.18	894.82
Bank NW max (000)	207.78	133.78	255.56	295.57	216.69	761.99	4185.00
Total debt min (000)	5.24	5.25	5.25	5.24	5.24	5.25	4.01
Total debt mean (000)	24.76	25.22	29.13	37.89	44.25	99.52	503.57
Total debt max (000)	39.59	44.29	67.05	103.27	98.59	335.06	1947.70
Firm default mean (%)	5.04	4.98	4.96	4.82	4.78	4.35	3.11
Firm default max (%)	8.60	8.40	8.60	9.00	8.00	8.80	9.20
Aggr. prod. min (000)	12.41	12.42	12.43	12.41	12.42	12.43	12.42
Aggr. prod. mean (000)	28.91	29.46	32.58	41.92	45.05	91.76	416.95
Aggr. prod. max (000)	36.96	41.50	58.12	82.85	90.82	233.93	1124.50
Interest min (%)	5.90	5.91	5.88	5.89	5.90	5.86	5.81
Interest mean (%)	5.95	5.96	5.96	5.98	5.97	5.97	5.97
Interest max (%)	6.08	6.08	6.08	6.08	6.08	6.08	6.08
Interest std dev (%)	0.04	0.04	0.04	0.05	0.04	0.05	0.06
Leverage min	0.76	0.85	0.52	0.34	0.50	0.16	0.04
Leverage mean	1.33	1.38	1.28	1.01	0.95	0.08	0.49
Leverage max	2.25	2.26	2.29	2.09	1.87	2.00	2.18
Leverage std dev	0.25	0.26	0.39	0.40	0.32	0.45	0.48

Table B.13: CAR^*

CAR^*	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22
Growth std dev (%)	2.09	2.09	2.09	2.11	2.32	2.15	2.18	2.26	2.10	2.13
Growth skewness	0.40	0.39	0.29	0.36	-0.37	0.19	0.51	0.09	0.44	0.30
Growth kurtosis	6.00	6.72	6.96	6.67	8.38	6.85	10.88	11.67	10.24	10.41
Growth min (%)	-7.20	-7.69	-10.01	-7.19	-15.29	-12.25	-9.78	-13.02	-10.53	-10.93
Growth mean (%)	0.09	0.09	0.09	0.13	0.09	0.11	0.11	0.11	0.09	0.11
Growth max (%)	11.68	13.36	13.37	14.34	12.29	13.81	18.71	16.01	15.84	17.17
Bad debt mean (%)	9.23	9.28	8.78	8.38	8.31	8.16	8.26	8.03	8.00	7.58
Bad debt median (%)	8.67	8.93	8.19	8.24	7.72	7.78	7.84	7.61	7.44	7.18
Bad debt max (%)	34.69	28.69	42.32	32.50	55.45	30.17	36.57	53.21	43.91	42.15
Bank default mean (%)	8.22	3.41	1.96	1.42	0.89	0.65	0.50	0.50	0.39	0.29
Bank default max (%)	28.00	32.00	26.00	24.00	32.00	24.00	18.00	12.00	6.00	8.00
Bank NW mean (000)	6.89	25.58	53.23	47.98	105.13	87.84	136.14	149.68	127.21	199.20
Bank NW max (000)	25.37	45.94	103.24	90.83	255.56	127.14	361.37	372.48	301.98	473.98
Total debt min (000)	5.25	5.25	5.26	5.25	5.25	5.24	4.50	3.80	3.25	2.89
Total debt mean (000)	22.73	22.56	24.78	24.79	29.13	27.32	30.86	31.91	30.41	35.00
Total debt max (000)	34.12	35.53	43.83	44.58	67.05	48.62	78.51	79.43	67.68	91.82
Firm default mean (%)	5.06	5.06	5.05	5.02	4.96	4.93	4.92	4.93	4.92	4.88
Firm default max (%)	8.60	9.00	8.60	8.40	8.60	8.20	9.40	8.40	8.00	9.20
Aggr. prod. min (000)	12.43	12.42	12.43	12.43	12.43	12.42	12.33	11.76	11.26	10.88
Aggr. prod. mean (000)	27.36	27.11	28.58	29.00	32.58	31.38	34.02	34.83	33.76	37.92
Aggr. prod. max (000)	33.63	33.40	39.30	39.42	58.12	48.74	67.67	68.40	65.31	77.78
Interest min (%)	6.00	5.97	5.95	5.94	5.88	5.92	5.89	5.87	5.89	5.85
Interest mean (%)	6.06	6.01	5.98	5.98	5.96	5.96	5.95	5.94	5.94	5.93
Interest max (%)	6.09	6.09	6.10	6.09	6.08	6.08	6.07	6.06	6.06	6.05
Interest std dev (%)	0.02	0.03	0.04	0.03	0.04	0.03	0.04	0.03	0.03	0.04
Leverage min	0.96	0.91	0.71	0.69	0.52	0.83	0.47	0.49	0.56	0.46
Leverage mean	1.44	1.49	1.39	1.31	1.28	1.22	1.29	1.22	1.23	1.09
Leverage max	2.25	2.23	2.26	2.28	2.29	1.99	2.02	1.89	2.09	1.78
Leverage std dev	0.24	0.22	0.28	0.26	0.39	0.21	0.39	0.35	0.33	0.31

Table B.14: MB

MB	7	9	11	13	15	17	19	21	23	25
Growth std dev (%)	2.22	2.04	2.24	2.18	2.32	2.19	2.10	2.08	2.24	2.16
Growth skewness	-0.11	0.39	-0.05	0.13	-0.37	0.28	0.34	0.06	-0.17	-0.02
Growth kurtosis	7.13	8.39	7.81	6.42	8.38	7.13	6.82	8.09	7.83	7.41
Growth min (%)	-11.99	-9.08	-12.27	-9.31	-15.29	-9.36	-9.90	-13.11	-11.75	-12.66
Growth mean (%)	0.09	0.10	0.09	0.10	0.09	0.10	0.10	0.12	0.11	0.09
Growth max (%)	11.61	12.98	12.41	12.69	12.29	13.75	13.95	13.22	13.61	12.90
Bad debt mean (%)	8.09	8.58	8.42	8.40	8.31	8.45	8.29	7.72	8.28	7.89
Bad debt median (%)	7.61	8.19	7.74	7.78	7.72	7.77	7.71	7.06	7.74	7.20
Bad debt max (%)	34.24	38.06	54.76	34.11	55.45	33.73	41.26	53.15	43.19	46.79
Bank default mean (%)	0.82	0.88	0.95	1.17	0.89	0.92	0.75	0.98	0.92	0.97
Bank default max (%)	28.00	24.00	34.00	18.00	32.00	24.00	22.00	20.00	24.00	24.00
Bank NW mean (000)	135.07	81.41	79.41	79.55	105.13	80.30	82.53	106.82	99.17	103.68
Bank NW max (000)	314.61	196.44	150.72	158.21	255.56	193.05	154.86	173.40	243.43	193.19
Total debt min (000)	5.25	5.25	5.25	5.26	5.25	5.25	5.26	5.24	5.25	5.24
Total debt mean (000)	30.59	27.25	26.66	27.28	29.13	27.16	27.09	29.46	28.44	28.87
Total debt max (000)	83.43	57.37	48.67	54.55	67.05	58.87	52.37	63.25	63.51	51.29
Firm default mean (%)	4.98	4.97	5.00	4.99	4.96	5.03	4.97	4.96	4.99	4.99
Firm default max (%)	8.60	8.20	9.20	8.20	8.60	8.20	8.40	8.20	9.20	8.20
Aggr. prod. min (000)	12.42	12.43	12.42	12.43	12.43	12.43	12.43	12.41	12.42	12.42
Aggr. prod. mean (000)	33.57	31.05	30.48	30.98	32.58	31.03	31.03	32.89	31.88	32.09
Aggr. prod. max (000)	66.09	53.40	47.79	48.01	58.12	50.98	47.08	48.12	55.70	48.65
Interest min (%)	5.87	5.91	5.91	5.93	5.88	5.91	5.91	5.90	5.90	5.91
Interest mean (%)	5.95	5.97	5.97	5.96	5.96	5.97	5.96	5.95	5.96	5.96
Interest max (%)	6.08	6.08	6.08	6.08	6.08	6.09	6.08	6.08	6.08	6.08
Interest std dev (%)	0.04	0.04	0.03	0.03	0.04	0.03	0.03	0.04	0.04	0.04
Leverage min	0.49	0.59	0.72	0.65	0.52	0.62	0.62	0.54	0.61	0.59
Leverage mean	1.26	1.30	1.27	1.30	1.28	1.29	1.24	1.05	1.26	1.15
Leverage max	2.23	2.21	2.22	2.16	2.29	2.39	2.20	2.18	2.23	2.21
Leverage std dev	0.45	0.30	0.26	0.30	0.39	0.32	0.28	0.24	0.35	0.31

Table B.15: n

n	1	2	3	4	5	6	7	8
Growth std dev (%)	2.21	2.32	2.21	2.14	2.16	2.25	2.25	2.23
Growth skewness	0.23	-0.37	-0.36	0.41	-0.75	0.10	-0.05	0.25
Growth kurtosis	8.29	8.38	8.76	6.53	15.31	7.66	7.79	8.18
Growth min (%)	-10.32	-15.29	-15.54	-8.15	-20.73	-11.01	-12.27	-10.70
Growth mean (%)	0.11	0.09	0.09	0.09	0.10	0.11	0.11	0.11
Growth max (%)	14.24	12.29	11.76	12.31	14.24	14.54	12.80	8.48
Bad debt mean (%)	8.28	8.31	8.09	8.23	7.61	8.43	8.98	8.48
Bad debt median (%)	7.48	7.72	7.29	7.87	6.99	7.78	8.40	8.07
Bad debt max (%)	43.59	55.45	50.67	32.11	64.21	33.07	37.62	40.05
Bank default mean (%)	0.84	0.89	0.94	0.93	0.69	0.93	0.87	0.96
Bank default max (%)	24.00	32.00	26.00	24.00	22.00	20.00	30.00	20.00
Bank NW mean (000)	89.13	105.13	100.22	101.48	133.00	76.73	48.89	75.55
Bank NW max (000)	165.75	255.56	192.33	195.04	306.53	155.29	92.81	147.76
Total debt min (000)	5.19	5.25	5.25	5.26	5.26	5.25	5.26	5.25
Total debt mean (000)	27.33	29.13	28.31	28.00	30.97	27.03	24.33	26.59
Total debt max (000)	46.86	67.05	64.86	63.30	70.56	54.34	46.65	51.23
Firm default mean (%)	4.99	4.96	5.03	4.99	4.98	4.97	5.03	5.01
Firm default max (%)	8.80	8.60	8.60	9.40	8.60	8.60	8.60	8.60
Aggr. prod. min (000)	12.37	12.43	12.42	12.43	12.44	12.43	12.43	12.42
Aggr. prod. mean (000)	31.07	32.58	31.41	31.65	33.89	30.94	28.67	30.44
Aggr. prod. max (000)	49.53	58.12	46.86	57.87	45.81	49.91	40.83	51.01
Interest min (%)	5.92	5.88	5.91	5.92	5.87	5.93	5.95	5.93
Interest mean (%)	5.96	5.96	5.96	5.96	5.95	5.97	5.98	5.97
Interest max (%)	6.08	6.08	6.08	6.09	6.09	6.09	6.08	6.09
Interest std dev (%)	0.03	0.04	0.04	0.04	0.04	0.03	0.03	0.04
Leverage min	0.67	0.52	0.44	0.60	0.38	0.65	0.75	0.63
Leverage mean	1.23	1.28	1.19	1.29	1.05	1.28	1.44	1.31
Leverage max	2.09	2.29	2.23	2.18	2.28	2.27	2.17	2.29
Leverage std dev	0.30	0.39	0.37	0.34	0.43	0.28	0.34	0.29

Table B.16: λ

λ	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Growth std dev (%)	2.22	2.10	2.12	2.32	2.24	2.20	2.33	2.34	2.41	2.36
Growth skewness	0.92	0.65	0.36	-0.37	-0.03	-0.77	-0.94	-1.16	-0.76	-1.41
Growth kurtosis	8.04	7.25	9.77	8.38	7.10	8.80	9.73	11.73	7.74	14.59
Growth min (%)	-7.28	-8.61	-13.52	-15.29	-12.52	-12.45	-19.13	-19.33	-17.38	-21.36
Growth mean (%)	0.16	0.14	0.13	0.09	0.07	0.08	0.07	0.06	0.05	0.05
Growth max (%)	13.62	13.03	13.89	12.29	12.32	11.46	10.54	11.42	11.89	11.08
Bad debt mean (%)	8.41	8.78	8.68	8.31	8.43	7.38	7.96	8.28	8.52	7.61
Bad debt median (%)	8.16	8.24	8.18	7.72	7.75	6.72	7.59	8.05	8.35	7.74
Bad debt max (%)	24.02	28.39	57.18	55.45	44.68	53.47	66.79	63.28	60.13	66.10
Bank default mean (%)	0.98	1.30	1.57	0.89	0.94	0.70	0.71	0.66	0.62	0.56
Bank default max (%)	24.00	18.00	24.00	32.00	24.00	24.00	26.00	28.00	26.00	22.00
Bank NW mean (000)	133.32	87.42	69.35	105.13	59.42	117.19	85.98	77.79	101.68	194.12
Bank NW max (000)	185.63	191.05	153.45	255.56	108.63	265.81	222.05	208.40	322.31	521.98
Total debt min (000)	5.25	5.25	5.25	5.25	5.26	5.25	5.24	5.25	5.25	5.25
Total debt mean (000)	61.00	46.03	33.94	29.13	20.71	21.90	17.69	15.32	15.37	19.59
Total debt max (000)	89.38	97.63	62.40	67.05	39.93	58.58	51.76	41.34	53.87	70.23
Firm default mean (%)	3.03	3.66	4.36	4.96	5.57	6.03	6.46	6.79	6.95	7.16
Firm default max (%)	5.60	6.60	7.20	8.60	8.80	10.20	10.20	10.60	10.60	11.80
Aggr. prod. min (000)	12.42	12.43	12.43	12.43	12.43	12.43	12.42	12.42	12.42	12.35
Aggr. prod. mean (000)	49.16	43.06	35.80	32.58	25.50	26.57	22.81	20.67	20.53	23.18
Aggr. prod. max (000)	71.32	62.23	48.72	58.12	39.24	46.90	44.07	40.21	48.61	51.70
Interest min (%)	5.97	5.95	5.95	5.88	5.92	5.84	5.87	5.86	5.83	5.81
Interest mean (%)	6.01	6.00	5.99	5.96	5.96	5.93	5.94	5.94	5.94	5.90
Interest max (%)	6.11	6.09	6.09	6.08	6.08	6.08	6.07	6.07	6.07	6.07
Interest std dev (%)	0.03	0.03	0.03	0.04	0.03	0.04	0.04	0.04	0.05	0.05
Leverage min	1.05	0.66	0.63	0.52	0.76	0.49	0.55	0.51	0.53	0.40
Leverage mean	2.41	1.65	1.30	1.28	1.31	1.07	1.19	1.25	1.38	1.27
Leverage max	3.33	2.76	2.45	2.29	2.03	1.96	2.01	1.97	2.09	2.14
Leverage std dev	0.53	0.43	0.32	0.39	0.25	0.28	0.37	0.38	0.39	0.55

Table B.17: D

D	2	3	4	5	6	7	8	9	10
Growth std dev (%)	1.99	2.15	2.32	2.18	2.10	2.16	2.16	2.06	2.01
Growth skewness	-3.17	0.22	-0.37	-0.29	0.05	0.37	0.08	0.25	0.35
Growth kurtosis	45.11	5.37	8.38	8.92	7.61	6.70	8.00	7.24	8.15
Growth min (%)	-27.12	-8.22	-15.29	-14.30	-12.91	-9.57	-12.06	-9.12	-8.73
Growth mean (%)	0.02	0.08	0.09	0.11	0.11	0.12	0.12	0.13	0.11
Growth max (%)	13.00	13.65	12.29	11.38	12.32	13.34	12.61	12.76	13.02
Bad debt mean (%)	3.87	8.87	8.31	8.49	8.37	8.10	8.28	7.97	7.27
Bad debt median (%)	1.60	8.47	7.72	7.82	7.69	7.49	7.86	7.31	6.60
Bad debt max (%)	72.49	30.82	55.45	55.48	48.71	32.96	40.28	31.16	41.12
Bank default mean (%)	0.43	1.88	0.89	1.23	1.06	1.06	1.00	1.02	0.73
Bank default max (%)	18.00	24.00	32.00	22.00	24.00	24.00	26.00	26.00	20.00
Bank NW mean (000)	599.39	29.13	105.13	79.97	81.38	112.16	99.93	119.51	186.85
Bank NW max (000)	1015.10	78.46	255.56	164.03	134.48	200.83	181.13	187.11	336.94
Total debt min (000)	4.77	5.24	5.25	5.24	5.26	5.25	5.24	5.25	5.25
Total debt mean (000)	40.83	14.91	29.13	27.01	29.58	33.74	33.82	36.44	42.67
Total debt max (000)	100.68	27.12	67.05	57.05	49.93	72.58	56.47	69.53	81.39
Firm default mean (%)	7.80	5.88	4.96	4.96	4.74	4.58	4.51	4.45	4.32
Firm default max (%)	11.80	9.60	8.60	8.40	8.60	7.40	7.20	7.80	8.20
Aggr. prod. min (000)	10.81	12.41	12.43	12.41	12.43	12.42	12.42	12.42	12.43
Aggr. prod. mean (000)	36.28	21.37	32.58	30.89	32.76	36.18	35.75	31.92	42.95
Aggr. prod. max (000)	62.53	29.64	58.12	49.92	46.17	55.46	52.33	57.24	71.42
Interest min (%)	5.65	5.93	5.88	5.91	5.91	5.92	5.87	5.93	5.90
Interest mean (%)	5.83	5.99	5.96	5.97	5.97	5.96	5.97	5.97	5.94
Interest max (%)	6.07	6.08	6.08	6.08	6.09	6.09	6.08	6.09	6.09
Interest std dev (%)	0.07	0.04	0.04	0.04	0.03	0.04	0.03	0.04	0.04
Leverage min	0.20	0.51	0.52	0.59	0.75	0.64	0.71	0.64	0.51
Leverage mean	0.75	1.44	1.28	1.32	1.30	1.24	1.35	1.27	1.06
Leverage max	2.12	2.21	2.29	2.22	2.26	2.30	2.28	2.36	2.40
Leverage std dev	0.60	0.48	0.39	0.39	0.27	0.27	0.29	0.30	0.30

Table B.18: Initial Firm Net Worth

$FirmNetWorth$	4	6	8	10	12	15	20	30	40
Growth std dev (%)	2.35	2.36	2.18	2.32	2.19	2.13	2.26	2.00	2.24
Growth skewness	1.03	0.54	0.11	-0.37	-0.63	-0.11	-0.21	-0.35	-0.33
Growth kurtosis	10.32	9.70	7.94	8.38	14.09	8.98	8.52	6.68	5.13
Growth min (%)	-8.11	-10.92	-12.22	-15.29	-20.04	-15.18	-13.85	-13.28	-11.62
Growth mean (%)	0.21	0.15	0.14	0.09	0.06	0.09	0.04	0.07	0.04
Growth max (%)	16.13	17.35	13.95	12.29	14.64	15.56	14.04	10.12	8.85
Bad debt mean (%)	8.69	8.49	8.82	8.31	8.55	8.45	8.66	8.59	8.92
Bad debt median (%)	8.27	8.09	8.27	7.72	7.80	7.89	8.01	7.98	8.55
Bad debt max (%)	34.47	29.15	52.78	55.45	63.46	59.00	50.00	57.49	43.61
Bank default mean (%)	1.93	1.81	1.60	0.89	1.04	1.92	1.76	1.64	0.79
Bank default max (%)	28.00	30.00	28.00	32.00	24.00	36.00	28.00	22.00	32.00
Bank NW mean (000)	49.43	61.54	52.47	105.13	65.79	62.63	52.72	83.59	107.02
Bank NW max (000)	138.64	170.24	126.21	255.56	161.52	158.72	121.27	218.97	333.41
Total debt min (000)	2.10	3.15	4.19	5.25	6.28	6.61	6.50	6.68	6.12
Total debt mean (000)	21.20	23.06	20.50	29.13	21.69	23.44	22.22	22.96	24.27
Total debt max (000)	42.24	54.47	43.07	67.05	42.79	53.79	40.57	52.89	82.45
Firm default mean (%)	5.36	5.38	5.39	4.96	5.46	5.38	5.35	5.38	5.37
Firm default max (%)	8.60	11.40	8.60	8.60	9.60	8.40	9.00	8.60	8.40
Aggr. prod. min (000)	6.54	8.68	10.62	12.43	14.10	16.04	16.45	18.47	17.46
Aggr. prod. mean (000)	26.58	27.96	25.56	32.58	25.96	28.03	26.83	28.06	28.71
Aggr. prod. max (000)	44.23	45.24	36.85	58.12	39.58	50.33	37.81	47.01	65.02
Interest min (%)	5.91	5.89	5.92	5.88	5.89	5.89	5.92	5.85	5.86
Interest mean (%)	5.99	5.98	5.98	5.96	5.96	5.98	5.98	5.97	5.96
Interest max (%)	6.08	6.08	6.08	6.08	6.08	6.08	6.08	6.07	6.08
Interest std dev (%)	0.04	0.05	0.04	0.04	0.04	0.04	0.04	0.05	0.04
Leverage min	0.54	0.46	0.57	0.52	0.47	0.52	0.68	0.41	0.41
Leverage mean	1.45	1.38	1.39	1.28	1.31	1.38	1.46	1.37	1.48
Leverage max	2.23	2.32	2.31	2.29	2.24	2.33	2.34	2.28	2.32
Leverage std dev	0.49	0.59	0.45	0.39	0.50	0.57	0.51	0.59	0.48

Table B.19: Initial Bank Net Worth

$BankNetWorth$	5	8	10	15	20	25	30	50	70
Growth std dev (%)	2.27	2.34	2.37	2.29	2.32	2.09	2.15	2.19	2.11
Growth skewness	0.46	0.95	0.47	-0.11	-0.37	0.25	0.02	-0.05	0.17
Growth kurtosis	12.63	12.83	13.87	10.52	8.38	7.51	6.00	6.53	6.31
Growth min (%)	-14.19	-10.21	-16.19	-15.26	-15.29	-11.72	-10.58	-11.04	-9.02
Growth mean (%)	0.11	0.18	0.09	0.14	0.09	0.13	0.12	0.08	0.08
Growth max (%)	16.98	19.59	20.69	17.79	12.29	12.67	12.58	13.08	13.17
Bad debt mean (%)	8.63	8.29	8.80	8.46	8.31	8.62	8.92	9.28	9.66
Bad debt median (%)	8.13	7.78	8.65	8.06	7.72	8.16	8.22	8.80	9.33
Bad debt max (%)	52.05	37.91	41.85	58.39	55.45	40.33	40.97	38.69	37.26
Bank default mean (%)	1.72	1.27	0.74	1.46	0.89	2.41	2.43	4.62	3.93
Bank default max (%)	22.00	20.00	18.00	26.00	32.00	26.00	24.00	22.00	20.00
Bank NW mean (000)	61.76	82.99	118.89	99.39	105.13	36.16	36.54	15.06	17.07
Bank NW max (000)	196.14	198.60	382.38	266.05	255.56	111.82	107.40	52.05	59.24
Total debt min (000)	1.69	2.51	3.24	4.98	5.25	5.24	5.25	5.25	5.25
Total debt mean (000)	23.13	24.20	25.27	24.33	29.13	21.31	20.08	18.58	17.78
Total debt max (000)	48.31	57.50	80.61	58.93	67.05	48.59	37.13	35.69	30.79
Firm default mean (%)	5.32	5.31	5.39	5.33	4.96	5.41	5.42	5.43	5.51
Firm default max (%)	9.00	11.00	9.00	8.80	8.60	9.40	8.80	8.80	9.00
Aggr. prod. min (000)	9.47	10.54	11.23	12.42	12.43	12.42	12.42	12.42	12.42
Aggr. prod. mean (000)	27.70	29.07	29.33	29.00	32.58	26.44	25.16	23.66	22.93
Aggr. prod. max (000)	50.94	54.49	64.27	52.85	58.12	40.55	38.42	31.19	27.81
Interest min (%)	5.90	5.89	5.87	5.87	5.88	5.88	5.92	5.97	5.96
Interest mean (%)	5.98	5.97	5.95	5.96	5.96	6.00	6.00	6.03	6.03
Interest max (%)	6.07	6.08	6.07	6.08	6.08	6.08	6.08	6.07	6.07
Interest std dev (%)	0.04	0.04	0.04	0.05	0.04	0.04	0.04	0.02	0.02
Leverage min	0.42	0.52	0.48	0.48	0.52	0.48	0.64	0.81	0.68
Leverage mean	1.46	1.38	1.45	1.35	1.28	1.38	1.44	1.54	1.49
Leverage max	2.35	2.42	2.50	2.29	2.29	2.29	2.21	2.27	2.24
Leverage std dev	0.52	0.57	0.52	0.61	0.39	0.49	0.41	0.36	0.37

Table B.20: Ratio of Liquidated Equity, υ

υ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Growth std dev (%)	6.87	6.59	5.88	5.45	4.64	3.74	3.08	2.46	2.03
Growth skewness	-0.76	-0.61	-0.52	-0.49	-0.47	-0.22	-0.28	-0.12	-0.14
Growth kurtosis	8.16	6.58	5.90	4.61	4.03	3.65	3.09	2.83	3.49
Growth min (%)	-55.97	-47.71	-41.14	-31.54	-23.39	-16.42	-12.35	-8.39	-7.45
Growth mean (%)	0.08	0.06	0.03	0.01	-0.01	-0.04	-0.03	-0.02	0.02
Growth max (%)	25.94	20.61	19.78	14.82	12.39	13.51	8.16	6.62	6.83
Bad debt mean (%)	35.99	32.58	29.44	26.06	22.65	18.78	15.68	12.64	10.13
Bad debt median (%)	34.53	31.34	27.82	24.33	20.93	17.49	14.58	12.18	9.80
Bad debt max (%)	75.09	70.30	63.67	60.59	62.65	48.64	34.88	35.78	23.38
Bank default mean (%)	13.17	12.36	11.99	11.38	11.08	10.57	10.45	10.21	9.03
Bank default max (%)	92.00	56.00	54.00	44.00	42.00	36.00	36.00	40.00	26.00
Bank NW mean (000)	0.34	0.36	0.37	0.39	0.42	0.48	0.57	0.74	1.25
Bank NW max (000)	0.86	1.03	1.14	1.18	1.19	1.24	1.48	1.80	2.88
Total debt min (000)	0.37	0.45	0.53	0.68	0.86	1.22	1.63	2.73	4.68
Total debt mean (000)	0.69	0.78	0.91	1.04	1.29	1.66	2.29	3.51	6.40
Total debt max (000)	4.60	4.92	5.14	5.22	5.22	5.24	5.23	5.63	9.55
Firm default mean (%)	21.01	17.59	14.81	12.55	10.66	9.05	7.67	6.64	5.57
Firm default max (%)	50.00	33.60	26.00	22.20	20.00	14.80	12.00	10.40	9.60
Aggr. prod. min (000)	2.06	2.16	2.41	2.64	3.15	3.73	4.70	6.68	10.19
Aggr. prod. mean (000)	2.49	2.62	2.84	3.12	3.59	4.29	5.45	7.51	12.12
Aggr. prod. max (000)	12.43	12.42	12.42	12.43	12.42	12.42	12.42	12.46	16.02
Interest min (%)	6.00	5.97	5.96	6.01	6.01	6.01	6.01	6.01	6.01
Interest mean (%)	6.06	6.06	6.07	6.07	6.07	6.07	6.08	6.08	6.08
Interest max (%)	6.15	6.17	6.13	6.13	6.13	6.12	6.11	6.12	6.09
Interest std dev (%)	0.03	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.004
Leverage min	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05
Leverage mean	3.04	3.13	3.24	3.16	3.14	2.97	2.79	2.50	1.97
Leverage max	5.90	6.23	6.02	5.25	5.56	4.33	3.72	3.29	2.34
Leverage std dev	0.72	0.70	0.60	0.55	0.47	0.36	0.26	0.17	0.12

Table B.21: Growth Pricing Scenario, ρ

ρ	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20
Growth std dev (%)	2.14	2.08	2.21	2.23	2.30	2.26	2.36	2.37	2.43
Growth skewness	0.39	0.31	0.26	0.25	-0.30	0.48	0.11	0.11	0.35
Growth kurtosis	5.67	6.14	6.28	7.84	9.43	7.37	5.94	6.79	6.56
Growth min (%)	-6.80	-10.12	-12.20	-10.71	-18.26	-8.79	-11.33	-11.27	-10.02
Growth mean (%)	0.08	0.09	0.09	0.07	0.07	0.07	0.08	0.08	0.08
Growth max (%)	12.43	12.16	12.09	15.13	12.55	14.31	12.18	13.54	13.53
Bad debt mean (%)	9.81	9.65	9.75	9.62	9.62	9.63	9.67	9.63	9.58
Bad debt median (%)	9.43	9.33	9.28	9.20	9.07	9.17	9.33	9.19	9.16
Bad debt max (%)	29.14	35.01	44.89	44.11	67.07	35.79	45.92	41.12	42.26
Bank default mean (%)	5.48	5.26	5.32	4.96	4.70	4.75	3.99	4.86	3.31
Bank default max (%)	24.00	22.00	22.00	24.00	20.00	22.00	24.00	26.00	22.00
Bank NW mean (000)	7.84	8.91	8.71	11.52	13.68	14.16	15.60	13.64	19.78
Bank NW max (000)	28.48	36.04	29.63	40.10	65.86	36.18	44.79	59.96	74.15
Total debt min (000)	5.24	5.24	5.25	5.26	5.24	5.24	5.25	5.24	5.25
Total debt mean (000)	17.00	17.49	17.61	17.85	17.89	17.54	17.74	17.78	18.01
Total debt max (000)	27.84	28.06	28.71	32.34	38.27	27.36	31.76	30.63	37.74
Firm default mean (%)	5.48	5.48	5.48	5.48	5.51	5.48	5.49	5.47	5.47
Firm default max (%)	9.20	9.00	8.80	10.00	9.20	9.00	8.80	10.00	10.20
Aggr. prod. min (000)	12.42	12.42	12.42	12.43	12.42	12.42	12.42	12.42	12.43
Aggr. prod. mean (000)	22.39	22.81	22.86	23.09	23.13	22.73	22.97	23.02	23.02
Aggr. prod. max (000)	27.96	27.95	27.86	30.24	29.09	28.56	30.91	28.84	29.75
Interest min (%)	5.62	5.43	5.19	4.87	4.65	4.36	4.19	3.76	3.61
Interest mean (%)	6.05	6.04	6.04	6.03	6.03	6.02	6.01	6.02	6.00
Interest max (%)	6.27	6.42	6.62	6.73	7.30	6.94	7.23	7.63	7.68
Interest std dev (%)	0.07	0.09	0.14	0.17	0.21	0.25	0.30	0.34	0.39
Leverage min	0.96	0.85	1.00	0.86	0.68	0.90	0.79	0.86	0.76
Leverage mean	1.74	1.61	1.66	1.58	1.53	1.61	1.58	1.60	1.62
Leverage max	2.35	2.24	2.36	2.34	2.35	2.28	2.32	2.25	2.44
Leverage std dev	0.29	0.31	0.35	0.31	0.43	0.31	0.38	0.32	0.39

Table B.22: Revolving Credits, μ

μ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Growth std dev (%)	2.14	2.28	2.32	2.27	2.36	2.47	2.35	2.31	2.32
Growth skewness	0.21	-0.11	-0.30	-0.01	-0.23	-0.01	-0.13	-0.39	-0.07
Growth kurtosis	8.06	7.12	9.80	6.97	8.38	6.07	6.24	8.03	6.52
Growth min (%)	-11.37	-12.38	-17.97	-11.39	-14.75	-11.57	-11.02	-13.29	-11.08
Growth mean (%)	0.08	0.09	0.11	0.12	0.08	0.16	0.09	0.07	0.07
Growth max (%)	14.28	12.08	12.88	12.87	13.16	12.55	11.41	13.48	12.44
Bad debt mean (%)	8.84	7.88	8.39	8.26	8.88	8.80	8.62	8.30	7.74
Bad debt median (%)	8.52	7.08	7.89	7.72	8.86	8.49	8.55	8.25	7.32
Bad debt max (%)	34.61	42.91	37.06	36.95	49.55	31.94	42.33	42.94	39.72
Bank default mean (%)	1.33	1.72	1.70	1.61	0.88	1.29	0.96	0.79	0.91
Bank default max (%)	18.00	24.00	36.00	32.00	26.00	36.00	30.00	30.00	20.00
Bank NW mean (000)	66.80	70.90	46.94	69.82	150.12	137.95	121.00	223.48	282.41
Bank NW max (000)	166.04	235.76	146.82	203.15	481.31	470.85	356.49	619.83	791.68
Total debt min (000)	5.25	5.25	5.25	5.24	5.23	5.25	5.24	5.25	5.25
Total debt mean (000)	21.25	24.63	22.79	24.82	27.59	26.38	24.97	31.89	36.68
Total debt max (000)	45.59	55.95	46.93	54.92	99.09	119.96	81.84	118.45	128.93
Firm default mean (%)	5.43	5.34	5.33	5.32	5.33	5.29	5.32	5.29	5.24
Firm default max (%)	9.40	9.00	9.20	9.00	9.20	9.20	9.00	9.00	9.00
Aggr. prod. min (000)	12.42	12.42	12.42	12.42	12.41	12.42	12.42	12.42	12.43
Aggr. prod. mean (000)	26.12	28.72	27.46	29.57	31.43	30.65	29.22	34.93	38.73
Aggr. prod. max (000)	45.76	48.48	42.71	49.44	81.92	77.85	67.19	98.15	115.81
Interest min (%)	5.77	5.56	5.50	5.35	5.13	5.04	4.46	4.61	4.19
Interest mean (%)	5.95	5.95	5.94	5.93	5.91	5.90	5.91	5.88	5.85
Interest max (%)	6.07	6.07	6.07	6.07	6.07	6.07	6.07	6.07	6.07
Interest std dev (%)	0.04	0.06	0.07	0.09	0.11	0.13	0.13	0.17	0.21
Leverage min	0.63	0.47	0.60	0.45	0.45	0.46	0.50	0.44	0.36
Leverage mean	1.39	1.29	1.37	1.36	1.56	1.54	1.52	1.44	1.26
Leverage max	2.22	2.27	2.31	2.27	2.37	2.54	2.29	2.32	2.25
Leverage std dev	0.39	0.51	0.47	0.60	0.56	0.52	0.54	0.59	0.53