AN INTERACTIVE APPROACH FOR MULTICRITERIA DECISION MAKING;

A LOGISTICS APPLICATION

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JUNE 2014

AN INTERACTIVE APPROACH FOR MULTICRITERIA DECISION MAKING;
A LOGISTICS APPLICATION

A THESIS SUBMITTED TO

THE GRADUATE SCHOOL OF SOCIAL SCIENCES

OF

IZMIR UNIVERSITY OF ECONOMICS

BY

ANIL KAYA

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

OF

MASTER OF ARTS

IN

THE GRADUATE SCHOOL OF SOCIAL SCIENCES

JUNE 2014

Approval of the Graduate School of Social Sciences

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I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Arts.

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This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Arts.

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ABSTRACT

AN INTERACTIVE APPROACH FOR MULTICRITERIA DECISION MAKING; A LOGISTICS APPLICATION

Kaya, Anıl

M.A, Department of Logistics Management

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June 2014, 57 pages

In this study, we work on multiple criteria supplier selection problem. We assume a quasi-concave utility function that represents the preferences of the decision maker (DM). We generate convex cones based on the pairwise comparisons of DM. Then, we build a mathematical model to determine the minimum number of pairwise comparisons required to eliminate all alternatives but the best one. Using the properties of the optimal cones and the pairwise comparisons, we develop two interactive algorithms. We select the pairs of alternatives to be asked to the DM based on the probability that an alternative is preferred to another one. After each pairwise comparison, we calculate new probabilities for unselected pairwise questions. We implement our algorithms to find the best supplier. We conduct computational experiments on generated instances. We evaluate our algorithms and compare with mathematical models according to the minimum number of required questions

Keywords: multi-criteria decision making, supplier selection, interactive approach

ÖZET

ÇOK AMAÇLI KARAR VERME İÇİN BİR ETKİLEŞİMLİ YAKLAŞIM; BİR LOJİSTİK UYGULAMASI

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Ortak Tez Yöneticisi: Yrd. Doç. Dr. Özgür ÖZPEYNİRCİ

Haziran 2014, 57 sayfa

Bu çalışmada çok amaçlı tedarikçi seçim problemi üstünde çalışmaktayız. Karar vericinin tercihlerini temsil eden bir içbükeyimsi fayda fonksiyonu olduğunu kabul etmekteyiz. Karar vericinin ikili karşılarştırmalarına dayanarak konveks konlar üretmekteyiz. Daha sonra, en iyi alternatif dışındaki bütün alternatifleri elemede gerekli olan en az soru sayısını belirlemek için matematisel model kuruyoruz. Optimal konilerin özelliklerini ve ikili karşılaştırmaları kullanarak iki etkileşimli algoritma geliştiriyoruz. Karar vericiye soracağımız alternatif ikilisini seçmek için bir alternatifin bir diğerinden daha iyi olma olasılığını hesaplıyoruz. Her ikili karşılaştırma sonrasında seçilmemiş ikili soruların olasılıklarını hesaplıyoruz. En iyi tedarikçiyi bulmak için algoritmalarımızı uyguluyoruz. Oluşturulmuş örnekler üzerinde hesaplamalı deneylerimizi gerçekleştiriyoruz. Gerekli olan en az soru sayısına göre algoritmalarımızı değerlendirip matematiksel model ile karşılaştırıyoruz.

Anahtar Kelimeler : çok amaçlı karar verme, tedarikçi seçimi, etkileşimli yaklaşım

To my parents

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my supervisors Assist. Prof. Dr. Özgür ÖZPEYNİRCİ and Assist. Prof. Dr. Selin ÖZPEYNİRCİ for their priceless support and guidance. I am deeply indebted to them for their advises, encouragements and providing me with a perfect atmosphere for research.

I would like to thank the members of my thesis committee: Prof. Dr. Mehmet Cemali DİNÇER, Assoc. Prof. Dr. Yiğit KAZANÇOĞLU, Assist. Prof. Dr. Muhittin Hakan DEMİR, for their comments and suggestions.

I would like to thank all the faculty members of Department of Logistics Management for their support. I thank all of my friends who supported me in the writing of this thesis.

I am grateful to my parents for all their support since my birth. They believed in me and encouraged me with their best wishes. I also thank my brother Umut for the continuous support.

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CHAPTER 1

INTRODUCTION

There are many definitions of supply chain management (SCM) that imply not only the flow of materials but also the network of organizations. Simchi-Levi et al. (1999) defines SCM as,

"a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system wide costs while satisfying service level requirements."

Supply chain management, which is a crucial statement for a business process, has an extensive content from supplier of materials to demand in a store. The business operations in the supply chain provide an opportunity to reduce costs for firms. Davis (1993) states that supply chain success depends on the uncertainty cycle, which includes four main items such as manufacturing, supplier performance, customer deliveries and customer demand. For this reason, supplier performance, that is a main component of the uncertainty cycle, has a great effect on supply chain and business success.

The selection and evaluation of suppliers is a complex problem. Choosing the right suppliers requires the evaluation of quantitative and qualitative factors. Many firms need to evaluate supplier performance using criteria in different areas such as quality, lead time, price, cost, etc. (Köksalan and Wallenius,2012). It is clear that, the selection of suppliers and evaluation of supplier performance are multiple criteria decision making (MCDM) problems.

MCDM methods are suitable for solving decision and planning problems involving multiple criteria. Generally, there is not a unique optimal solution for many MCDM problems. MCDM methods assume a decision maker (DM) or a group of DMs own the problem. DM’s preferences directly affect the best solution.

In this study, we focus on the supplier selection process which is a fundamental multiple criteria decision making application on supply chain management. We assume there is a single DM. The best supplier is selected in the group of suppliers. In many MCDM approaches, finding the best solution with the minimum number of questions is a basic matter. We aim to develop an interactive algorithm to select the best alternative by asking few questions to the DM.

We ask DM to compare pairs of alternatives and assume DM has a quasi-concave utility function. We discuss the quasi-concave utility function in detail in Chapter 2. We get the preferences of DM to eliminate one of the alternatives. After each question, we get more information about DM preferences. According to DM’s response, we determine an inferior alternative that is eliminated in future analysis. We want to ask minimum number of pairwise comparison questions in order to find the best alternative.

The outputs of this thesis are:

1) Mathematical models that compute the least possible number of pairwise questions necessary to find the best alternative
2) Strategies to reduce the number of pairwise questions and two interactive algorithms for finding the best solution using these strategies

We develop mathematical models to determine the minimum possible number of questions. We generate all cones that can be generated by one or two pairwise comparisons. We find the set of cones that eliminates all alternatives with minimum number of required questions. Eliminated alternatives that are not used in further questions improve this method. The mathematical program aims to identify the best solution with the minimum number of required questions and find strategies for selecting the pairwise questions.

We analyze the results of the computational test and propose two interactive algorithms to select the best alternative by asking questions to the DM. We test our interactive algorithms for each instance, with different utility function and weight combinations. Moreover we compare the performance of our interactive algorithms with the results of mathematical models. We evaluate our interactive algorithms by the minimum number of required questions. On the other hand, our interactive algorithms are compared with each other. We implement our algorithms to a supplier selection problem.

The mathematical model finds minimum number of questions required the best alternative. However, this model assumes explicit knowledge of the DMutility function and the weights of the objectives. On the other hand, interactive algorithms do not know the DM utility function, but assume DM has a quasi-concave utility function and try to identify the best solution by asking pairwise questions. The number of questions asked by the mathematical model is a lower bound for the interactive algorithms.

In the next chapter, we present a detailed review of the literature. In Chapter 3, we provide a background for multiple criteria decision making and MCDM approaches. In Chapter 4, we introduce mathematical models used for algorithms and we present the findings and the analysis of computational test. We develop two interactive algorithms and we present the implementation of our interactive algorithms in Chapter 5. The last chapter concludes the thesis and provides future research directions.

CHAPTER 2

BACKGROUND

2.1 Definitions

MCDM is about solving problems with multiple and conflicting criteria. In general, we assume that there are m decision alternatives, p criteria and *one decision maker* (DM). We use the preferences of DM to find the best alternative which satisfies all objectives.

Alternatives; $k = \{1, 2, 3,\ldots, m\}$

Criteria ; $q = \{1,\ldots,p\}$

We use the notation of Özpeynirci and Köksalan (2010) to give necessary definitions. Consider $x, x^* \in X$. Point x^* is *dominate* point If $x_q^* \leq x_q$ for all q and $x_q^* < x_q$ at least one q. If $x_q^* < x$ for all q, x^* is *strictly dominate* point. If there is no point x^* that dominates x, then x is said to be *nondominated*. A point x is said to be *weakly nondominated* if there is no point x^* such that $x_q^* < x$ for all q. Figure 2.1 provides a representation of dominated and nondominated points, ideal point, nadir point.

Alternative x_i dominates alternative x_j if x_i is no worse than x_j in all objectives and better in at least one. If there exists no alternative that dominates x_i, x_i is said to be non-dominated.

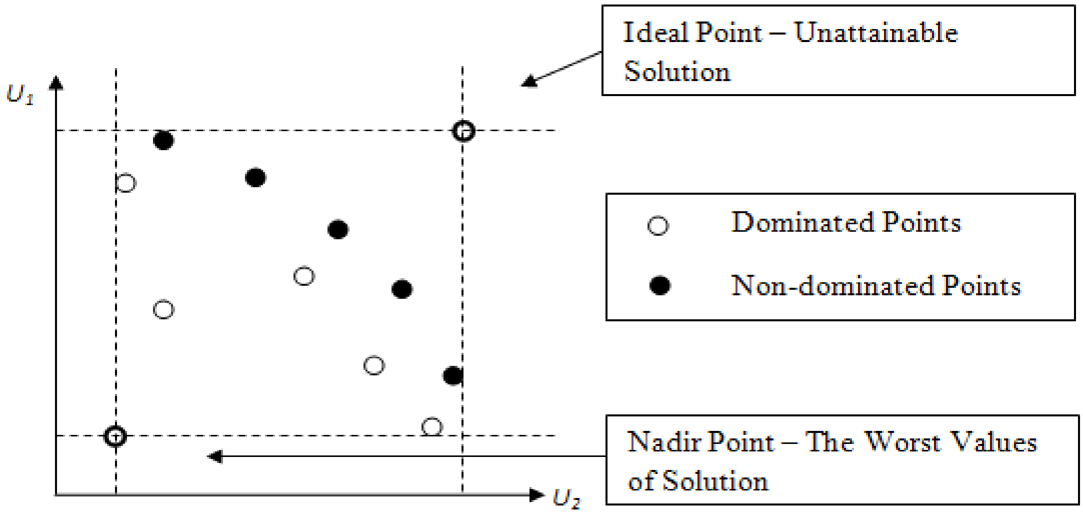


Figure 2.1: Dominated and Non-dominated Points, Köksalan and Wallenius (2012)

The ideal and the nadir points are two important concepts in MCDM. The ideal point has the best values for each objective. This point is unattainable or utopian solution. The nadir point represents the worst values for each objective (Figure 2.1).

DM compares pairs of selected alternatives. We generate cones based on DM preferences. Inferior alternatives are identified and eliminated according to DM's response. We find the best alternative of the nondominated alternatives. In MCDM problems, nondominated points are worth consideration. Dominated points are eliminated directly at the beginning of the decision making process.

We assume DM has a quasi-concave utility function. Quasi-concave utility functions represent human nature properly. We assume that the utility function is not explicitly known. We gather information about the utility function through pairwise comparisons. A representation of the quasi-concave utility function can be seen in Figure 2.2.

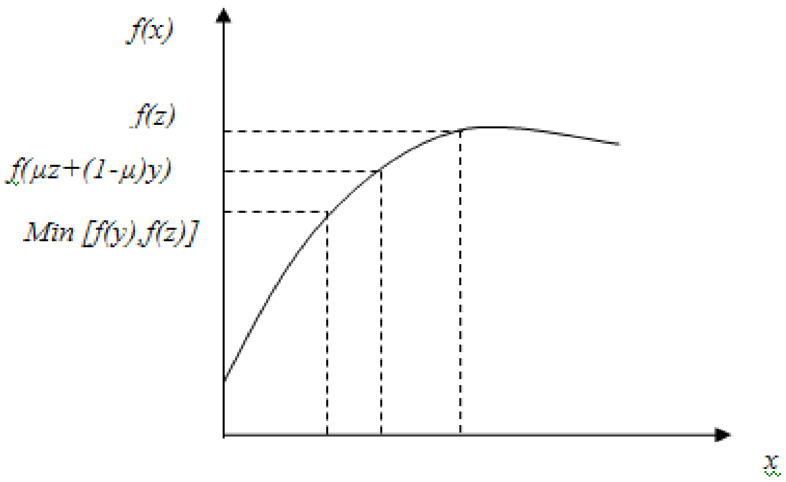


Figure 2.2: Quasi-concave function

We use an underlying utility function for pair-wise comparisons. DM selects preferred alternatives according to this underlying utility function. We estimate linear, quadratic and Tchebycheff utility functions to select alternatives with high preference. For each alternative x_k, we calculate,

Alternatives; $k = \{1, 2, 3, \ldots, m\}$

Criteria ; $q = \{1, \ldots, p\}$

W_q= *The weight set for each q.*

X_{kq}=*Selected alternative values in each q.*

X_q*= *Ideal alternative for each q.*

$$f_k = \sum_{1}^{p} w_q x_{kq}, \quad \forall k$$

For linear utility function,

$$f_k = -\sum_{1}^{p} w_q [x_{kq} - x_q^*]^2, \quad \forall k$$

For quadratic utility function and

$$f_k = -\max_{q=1,\ldots,p} \{w_q (x_{kq} - x_q^*)\}, \quad \forall k, q$$

For Tchebycheff utility function.

2.2 Theory for Eliminating Inferior Alternatives

THEOREM 1 (Korhonen, Wallenius and Zionts, 1984)

Assume a quasi-concave and nondecreasing function f(x) defined in a p-dimensional Euclidean space R^p. Consider distinct points $x_i \in R^p$, $i = 1, \ldots, m$, and any point $x^* \in R^p$ and assume that $f(x_k) < f(x_i)$, $i \neq k$. Then, if $\varepsilon \geq 0$ in the following linear programming problem

$$Max \quad \varepsilon$$

Subject to

$$\sum_{\substack{i=1 \\ i \neq k}}^{m} \mu_i (x_k - x_i) - \varepsilon \geq x^* - x_k, \quad \mu_i \geq 0,$$

It follows that $f(x_k) \geq f(x^*)$.

x_i is preferred alternative than x_k. In theorem we analyze x^* . According to cone, x^* is less preferred alternative than x_k. x^* that is dominated by cone, will be eliminated.

Any alternative which is in the convex cone, is dominated by cone. Alternative in the convex cone will be inferior and this alternative will be eliminated.

Our aim is to find the best solution. We use pairwise questions to eliminate alternatives. In Figure 2.3, we generate cone based on decision maker preferences. If DM prefers x_1 to x_2, then all alternatives in the shaded region are dominated by Cone (x_1, $\underline{x_2}$). x_2 and x_3 cannot be the best alternative. On the other hand, x_1 or x_4 can be optimal solutions. We continue with pairwise comparisons until only one alternative is left, which is the best alternative.

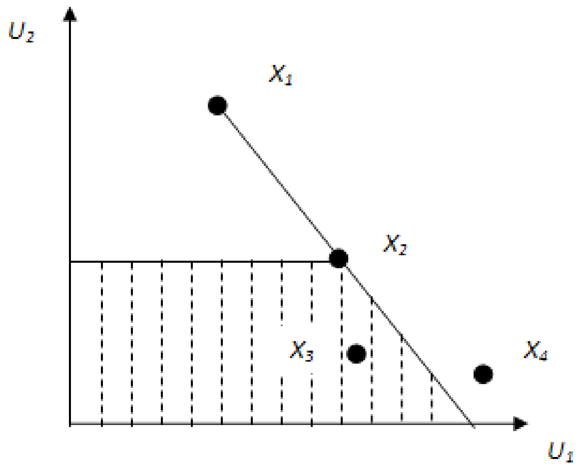


Figure 2.3: The Illustration of Cone

We can see the cone representation in Figure 2.4. Cones are presented by their preferences. The least prefered alternative is underlined. If A is preferred to B, we generate Cone (A,<u>B</u>). If A is preferred to B and E is preferred to B, we generate Cone(A,E,<u>B</u>). It is clear that B is the least preferred alternative in each cone and B is underlined. In this study, we work on single and double cones to find the best alternative with minimum number of questions. We check the necessity of higher degree cones (m>2) in future work.

Single cone

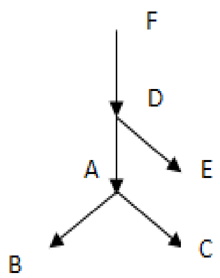
If X_i is preferred to X_j , we generate Cone (X_i, $\underline{X_j}$)

Double cone

If X_i is preferred to X_k, and X_j is preferred to X_k, we generate Cone (X_i, X_j, $\underline{X_k}$)

In general; *m*-cone

If m alternatives are preferred to (m+1) st alternative



Cones: (F,D,A,$\underline{\text{B}}$)

(F,D,A,$\underline{\text{C}}$)

(F,D,$\underline{\text{A}}$)

(F,D,$\underline{\text{E}}$)

(F,$\underline{\text{D}}$)

Figure 2.4: Tree Illustrating the Construction of Cones, Korhonen, Wallenius and Zionts (1984)

CHAPTER 3

LITERATURE REVIEW

We review the literature under two folds: supply chain management and multiple criteria decision making (MCDM). The first section is about the supply chain management. The second section covers an overview of the literature on MCDM, MCDM approaches for supplier selection problem, convex cone methods in MCDM and interactive algorithms.

3.1 Supply Chain Management

In this section, the review of supply chain management (SCM) literature is presented. There are many different definitions of SCM that requires the flow of goods, information and money in the business process. Simchi-Levi et al. (1999) state that supply chain is the combination of suppliers, manufacturers, warehouses and stores. Quantities, locations and time are the important elements of supply chain integration to minimize costs.

It is clear that there is an integration of business operations to minimize the total cost and maximize the profits. Supply chain management covers not only the flow of materials but also the network of organizations. Cooper, Lambert and Pagh (1997) describe SCM that differ from logistics, as a concept for the integration of business operations. They explain three elements of SCM framework that are business processes, management components and supply chain structure. On the other hand, they suggest ten supply chain management components: planning and control, work structure, organization structure, product flow facility structure, information flow facility structure, product structure, management methods, power and leadership structure, risk and reward structure, culture and attitude. There are many definitions of SCM components which are based on successful integration. Stevens (1989) states components of supply chain management such as process structure, planning and control structure, product flow facility structure, information flow, organization structure, management methods, power and leadership structure.

Supply chain management includes all business processes. There is a close relationship between each other. This is to solve problems and design the supply chain for business success. Ganeshan and Harrison (1995) analyze four major decision areas in SCM; location, production, inventory and transportation. These decision areas have both strategic and operational elements. There are three areas in the modeling approaches; network design, simulation and rough cut methods (Ganeshan and Harrison, 1995).

Davis (1993) describes uncertainty cycle in SCM that include supplier performance, manufacturing, customer deliveries and customer demand. Based on Figure 3.1, it is clear that there is a close relationship between the elements of uncertainty cycle. There are three sources such as suppliers, manufacturing, and customers that have impacts on supply chains. SCM success depends on the impact of uncertainty cycle. The author explains some cases for which Hewlett-Packard has developed a decision support system to model supply chain with using tactical tools. Due to supply chain model, the impact of the uncertainties decreases dramatically on business process. Fisher (1997) indicates that a mismatch between the type of product and supply chain is the main cause of the problems for business process. The type of product is an important for an effective supply chain strategy.

Thomas and Griffin (1996) describe the three basic stages in the supply chain. These stages include procurement, production and distribution. They explain operational coordination. Buyer – vendor coordination, production – distribution coordination and inventory – distribution coordination are items in operational coordination.

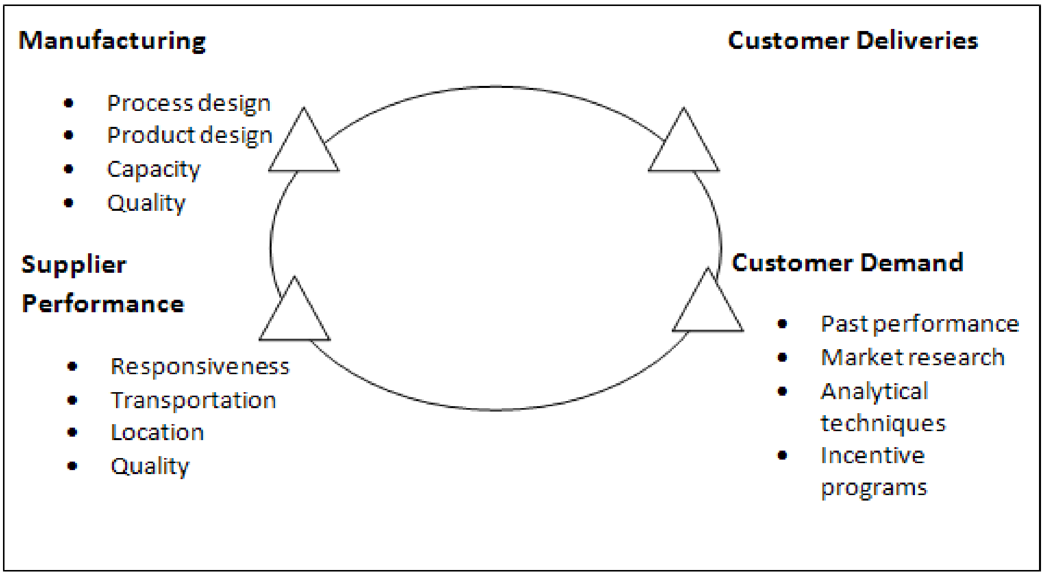


Figure 3.1: The Uncertainty Cycle, Davis (1993)

3.2 Multiple Criteria Decision Making

In this section, we present the review of the multiple criteria decision making (MCDM), MCDM approaches for supplier selection, convex cone method in MCDM and interactive algorithms. MCDM is crucial when a decision maker has to make decisions with multiple and conflicting criteria. MCDM problems are widespread in our life. In personal perspective, we can see many instances. Purchasing a car involves objectives such as price, size, style, safety, comfort that affect our selection. From the business perspective, MCDM problems can be more complicated. Many departments of large companies need to evaluate their performance using criteria in different area such as service, quality, finance etc. In locating a nuclear power plant, many objectives could be considered such as cost, health, environment, safety. These objectives need to be considered during the decision making process.

MCDM methods aim to solve decision and planning problems involving multiple criteria. Generally, there is not a unique optimal solution for many MCDM problems. Decision maker's preferences affect directly optimal solutions. We assume there are *m* decision alternatives, *p* criteria and *one decision maker* who owns the problem. Four main problems considered in MCDM are as follows:

1. The *choice* problem that aims to select the best alternative among the potential alternatives. Optimization problems are examples of choice problem (Köksalan et.al., 2009).

2. The *ranking* problem that is to rank all alternatives with the values on each criteria. This problem can be used to rank academic programs. For choice and ranking problems, we utilize the comparisons in the group of alternatives to find the solution (Köksalan et.al., 2009).

3. The *sorting* problem that is the assignment of alternatives into ordered categories (Köksalan et.al., 2009).

4. The *classification* problem that is the assignment of each alternative into unordered categories. If these categories are just labels, a classification problem can be considered (Zopounidis and Doumpos, 2004).

In this study, we focus on the supplier selection process which is a multiple criteria decision making application on logistics. The best supplier is selected in the group of suppliers so we define supplier selection process as a choice problem.

3.2.1 MCDM Approaches for Supplier Selection Problem

Supply chain management and procurement have been among the most important areas for many companies. Supplier selection problems include multiple and conflicting criteria. The relative performances of suppliers can vary by each criterion. It is clear that the selection of the right suppliers not only depends on their firms' management but also is related to apply right methods for this process.

Ho, Xu and Dey (2009) analyze multiple criteria decision making approaches for the supplier selection problem. They find that the individual approaches are more popular than the integrated approaches. They list the priority of the individual approaches according to their research from the highest to lowest priority, such as data envelopment analysis (DEA), mathematical programming, analytic hierarchy process (AHP), analytic network process (ANP), case-based reasoning, fuzzy set theory, simple multi-attribute rating technique and genetic algorithm. On the other hand, they find the most popular criterion as quality. Delivery, price, manufacturing capability, service, management, technology, research and development, finance, flexibility, reputation, relationship, risk and safety and environment are other popular criteria for the decision makers. Kontis and Vrysagotis (2011) imply that supplier selection has uncontrollable and unpredictable factors that affect decision making process. They analyze the multiple criteria decision making approaches for supplier selection based on DEA.

Soeini et al. (2012) state that supplier selection is a multiple criteria decision making problem. Logistics activities may cover more than 50% of firms' total costs. Many firms want to reduce the number of suppliers to take advantage of good relation with few suppliers. They propose an algorithm that limits the number of suppliers. They use the idea of a knapsack algorithm.

Ng (2007) explains that there is a close relationship between the success of a supply chain and the selection of good suppliers. He proposes a model which is a weighted linear program combine several objectives into a single objective with a weighted linear sum for the multiple criteria supplier selection problem. In this model, the decision maker does not have a subjective role. The proposed model differs from other approaches in analytic hierarchy process due to the decision maker's role. The model can be used in any situation.

Verma and Pullman (1997) examine how managers choose suppliers. They use two methods to test supplier selection process: A liker-type scale is used to identify perceived importance of suppliers and a discrete choice analysis is used to identify actual importance of suppliers. The results show that although managers choose quality in reality they select the low cost supplier.

Katsikeas, Paparoidamis and Katsikea (2004) present IT supplier evaluation criteria in their literature review. In Figure 3.2, the supplier evaluation attributes divide into four categories which are competitive pricing, reliability, service and, technological capability. They report on a supplier performance of distributor firms of information technology products.

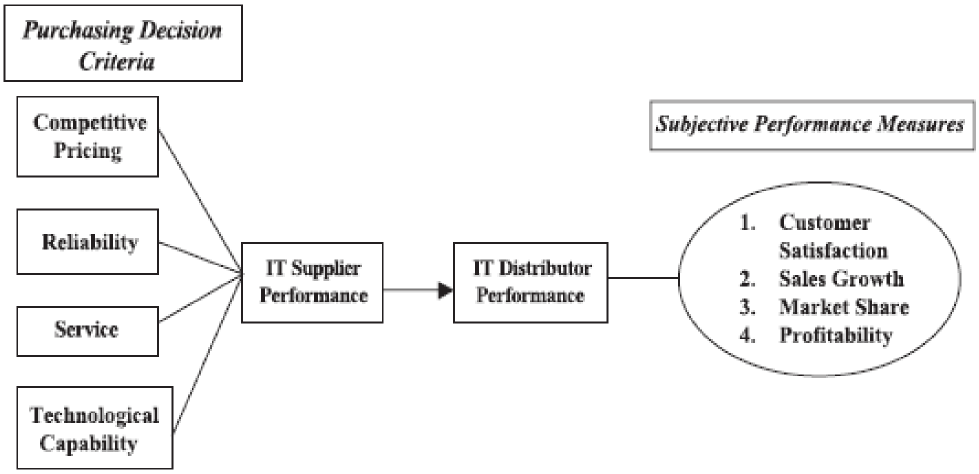


Figure 3.2: The Conceptual framework, Katsikeas, Paparoidamis and Katsikea (1993)

3.2.2 Convex Cone Approach

Korhonen, Wallenius and Zionts (1984) develop an algorithm that generate cones depending on the responses of the decision maker who has a quasi-concave increasing utility function. Quasi-concave utility function represents the human nature well. Inferior alternatives are identifed. They determine the alternative that maximizes decision maker’s utility function. Decision maker compares each adjacent alternative with the reference alternative. If there are not any adjacent efficient solutions, they identify an optimal solution. On the other hand, they represent the construction of cones with the tree representation. Based on tree illustration, an alternative which is an inferior solution, is underlined in each set.

THEOREM 1 (Korhonen, Wallenius and Zionts, 1984) *Assume a quasi-concave and nondecreasing function f(x) defined in a p-dimensional Euclidean space* R^p. *Consider distinct points* $x_i \in R^p$, $i = 1, \ldots, m$, *and any point* $x^* \in R^p$ *and assume that* $f(x_k) < f(x_i)$, $i \neq k$. *Then, if* $\varepsilon \geq 0$ *in the following linear programming problem*

$$Max\ \varepsilon$$

Subject to

$$\sum_{\substack{i=1 \\ i \neq k}}^{m} \mu_i (x_k - x_i) - \varepsilon \geq x^* - x_k, \quad \mu_i \geq 0,$$

It follows that $f(x_k) \geq f(x^*)$.

Consider Figure 3.3. Suppose DM prefers x_1 to x_2. We use this preference information to generate a cone which can be seen in Figure 3.3. Any alternative which is in the convex cone (Region A), is dominated by the cone. Any alternative in the convex cone (Region A) will be inferior and this alternative will be eliminated.

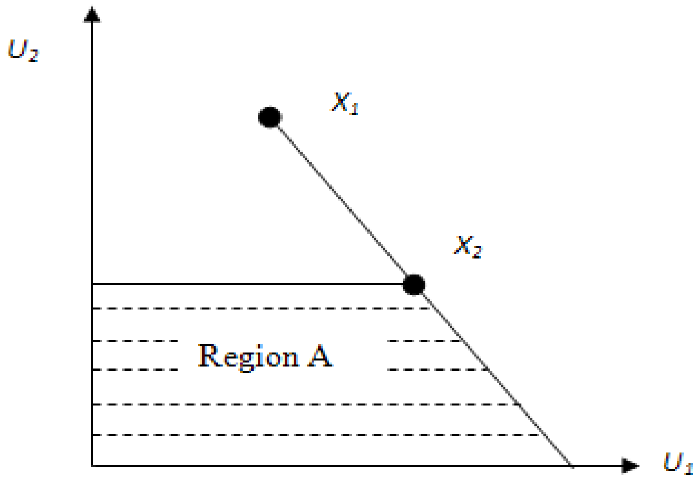


Figure 3.3: The Illustration of Cone, Korhonen, Wallenius and Zionts (1984)

Köksalan, Karwan and Zionts (1984) construct and use dummy alternatives in order to reduce number of questions. They combine the approach of Korhonen, Wallenius and Zionts(1984) with the idea of using dummy alternatives in cone generators. The dummy alternatives they propose are convex combinations of the existing alternatives. The cone generated with a dummy alternative is better than the cone with existing alternative. Instead of comparing x_1 and x_2, x_d and x_2 are compared. They get extra region and the alternatives in this region can be eliminated from further consideration. They demonstrate a cone of inferior solutions for using dummy alternatives in Figure 3.4. If DM prefers x_1 to x_2, alternatives in Region A will be eliminated. If DM prefers x_d to x_2, alternatives in Region A and alternatives in Region B will be eliminated.

Köksalan and Taner (1989) make improvements to reduce required number of pairwise questions. They develop variations of the dummy alternatives. They use dummy alternatives that are dominated alternatives. Dummy alternatives are used as cone generator. Instead of comparing x_1 and x_2, x_1 and x_d are compared in Figure 3.5.

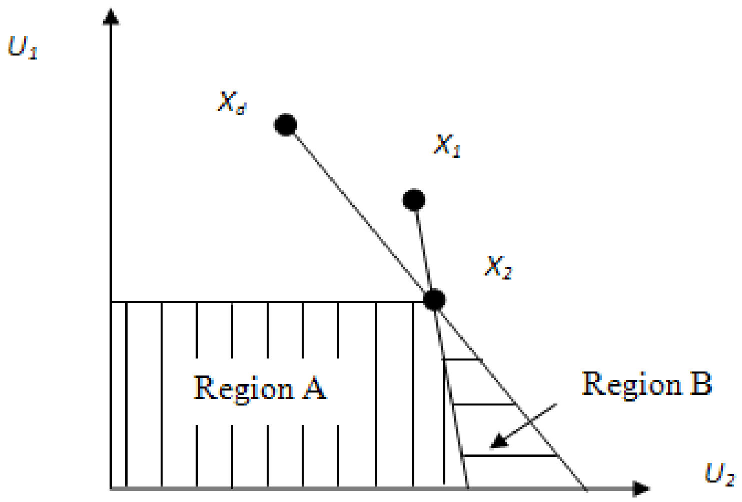


Figure 3.4: Cones with dummy alternatives, Köksalan, Karwan and Zionts (1984)

If DM prefers x_1 to x_2, alternatives in Region A will be eliminated. If DM prefers x_1 to x_d, alternatives in Region B will be eliminated. They get extra region and the alternatives in this region can be eliminated from further consideration.

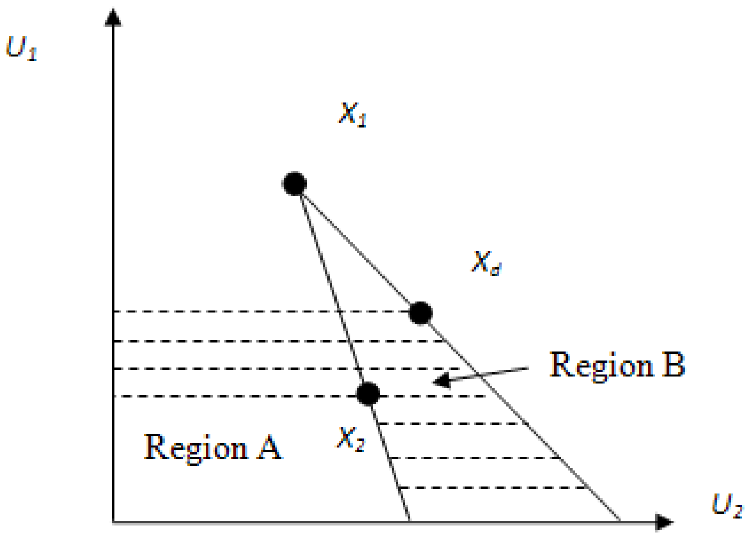


Figure 3.5: Cones with dummy alternatives, Köksalan and Taner (1989)

Köksalan (1989) develop an approach to reduce the total number of required questions. The decision maker has a quasiconcave utility function. He uses two different utility functions, one is quadratic and the other is Tchebyshev utility function. He uses the ideal point as an evaluation criteria. He selects alternatives as cone generator. The selected alternatives are closest to an ideal point in euclidean distance. He identifies and ranks alternatives. Highly ordered alternatives are used as cone generators. In each iteration, he uses alternatives that maximize utility functions, change with the least preferred cone generator.

Taner and Köksalan (1991) experiment to see the effect of cones. They use two different utility functions: quadratic and linear utility function. They estimate utility functions using the decision maker preferences. They select alternatives that have high rankings. Their approach has two variations: finding the best alternative and finding the worst alternative.

Lahdelma, Salminen and Kuula (2002) evaluate two methods: Salminen's piecewise linear prospect (PLP) theory (Salminen,1994) and the convex cone method by Korhonen, Wallenius and Zionts (KWZ) (Korhonen et al.,1984). They use randomly generated non-dominated alternatives in the test. The piece linear prospect method is more efficient than the convex cone method when there are 3 criteria or more. For two criteria problems, the convex cone method is slightly better than the piecewise linear prospect method.

Dehnokhalaji et al. (2011) generate convex cone to get more preference information. They develop an approach to find a strict partial order for a set of multiple criteria alternatives. Dehnokhalaji et al. (2014) extend their previous research and propose an algorithm to find a strict total order for a set of multiple criteria alternatives. They utilize the idea of convex cone based partial order.

3.2.3 Interactive Algorithms

Interactive algorithms gather information from the DM when needed throughout the algorithm. In the following steps, they use this information to make a decision. The preferences of DM can give some information about the utility function. In this study, we use an iterative algorithm, where, in each iteration, we calculate the possibility that an alternative is preferred to another one for each pair. According to some function of these possibilities, we select the pair to be demonstrated to the DM for comparison. According to DM's answer, we update the possibilities in the next iteration.

MCDM is considered to solve decision with multiple and conflicting criteria. There is not a unique optimal solution for many problems. Luque et al. (2011) state that there are many methods to solve multiple criteria decision making problems. Their aim is to find the best solution of many optimal solutions. MCDM methods must help the decision maker in decision making process. For this reason, interactive methods are useful for MCDM problems. We get information about feasible solutions.

Köksalan and Wallenius (2012) state that interactive approaches are used for multiple criteria decision making problems. They describe the main structure of the interactive algorithms which are developed by other authors. Köksalan and Wallenius (2012) define the main structure of an interactive algorithm as follows:

- *It keeps a best known alternative as the incumbent throughout.*
- *It asks the DM to compare pairs of alternatives.*
- *Using the preference information provided by the DM, it generates all possible preference cones. Every subset of alternatives whose least preferred member is known can be used to generate a cone.*
- *It eliminates all alternatives that are dominated by any of the cones.*
- *It continues asking the DM preference information and eliminating alternatives inferior to cones until a single alternative is left.*

Köksalan and Ulu (2003) propose an interactive approach, assuming an underlying additive linear utility function for the sorting problem. They use the preferences of the DM to assign alternatives to different categories. Köksalan and Özpeynirci (2009) propose an interactive approach that combines UTADIS and Köksalan and Ulu (2003) approaches, assuming an underlying additive utility function. They find the priority of categories to classify all the alternatives. DM assigns alternatives to their categories, if it is feasible and they place all alternatives based on DM past preferences. Buğdacı et al. (2012) propose an interactive probabilistic sorting method. They calculate the probability for each unassigned alternative. They find the critical probability level. Unassigned alternative probability is compared with the critical probability level to assign alternatives to class.

Luque, Ruiz and Miettinen (2011) define a global formulation which includes several interactive methods. DM prefers to provide preference information. This preference can change according to the interactive method used in the program.

CHAPTER 4

FINDING THE MINIMUM NUMBER OF PAIRWISE QUESTIONS

In this chapter, we develop two mathematical programming models to find the minimum number of questions. With the first model, we find the alternatives eliminated by each cone and then this information is used as an input in the second model that finds set of cones that eliminate all alternatives but the best alternative with minimum number of questions. The inputs and outputs of the two models are shown in Figure 4.1.

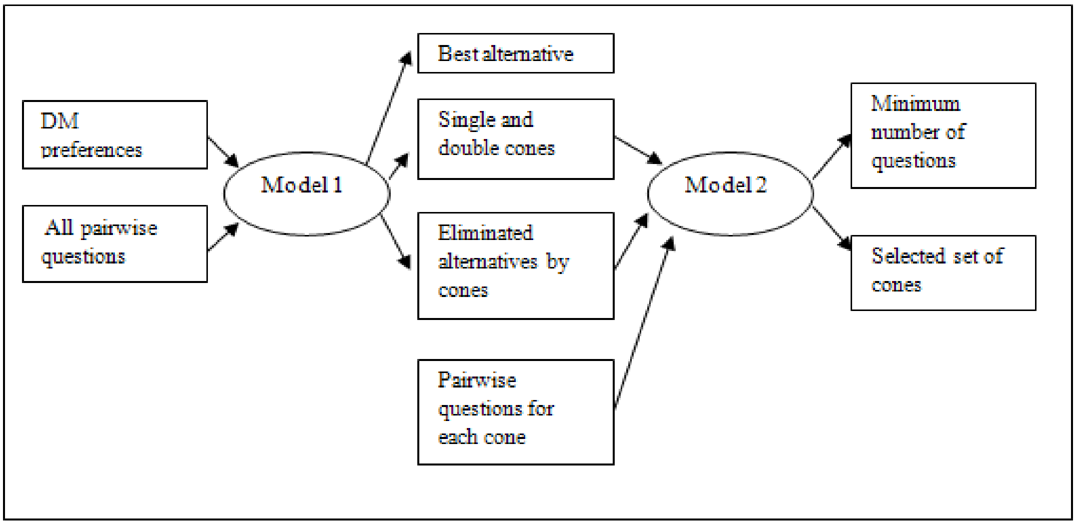


Figure 4.1: Mathematical Models - Finding the Minimum Number of Questions

We generate utility functions to represent the decision maker's preferences. We aim to develop an interactive algorithm to select the best alternative by asking few questions to the decision maker. We use the results of mathematical programming as a guide while developing our interactive algorithm.

4.1 Model for Finding the Alternatives Eliminated by Each Cone

In the model that finds the alternatives eliminated by each cone, we use decision maker preferences and all pairwise questions as input data. We generate all single and double cones.

Below, we present two models (i) single cone model and (ii) double cone model. For the single cone model, we assume Cone(x_i, $\underline{x_k}$) is generated when DM prefers alternative i to alternative k. The model checks if Cone(x_i, $\underline{x_k}$) dominates alternative t for all possible (i,k,t) triples. For the double cone model, we assume Cone(x_i, x_s, $\underline{x_k}$) is genereated when DM prefers alternatives i and s to alternative k. The model checks if this cone dominates alternative t for all possible (i,s,k,t) quadruplets.

The model that finds alternatives eliminated by one single cone, is given below:

Sets

Alternatives; $k, i, t = \{1, 2, 3, \dots, m\}$

Criteria numbers; $q = \{1, 2, \dots, p\}$

Positive Variable

Positive variable; μ

Variable ε

Model Single Cone

Objective Function;

$$Max \quad Z = \varepsilon$$

The objective is maximizing ε to check whether Cone(x_i, $\underline{x_k}$) dominates alternative x_t

Subject to;

$$\mu \ (x_{kq} - x_{iq}) - \varepsilon \ \geq x_{tq} - x_{kq}, \quad \forall q$$

$$\mu \ \geq 0,$$

If $\varepsilon \geq 0$ in the solution of the model, x_t which is dominated by Cone (x_i, $\underline{x_k}$), is eliminated. If $\varepsilon < 0$ in the solution of the model, x_t which is non-dominated, is not eliminated by Cone (x_i, $\underline{x_k}$).

The model that finds alternatives eliminated by one double cone, is represented as following.

Sets

Alternatives; $i, s, k, t = \{1, 2, 3, \ldots, m\}$

Criteria numbers; $q = \{1, 2, \ldots, p\}$

Positive Variable

Positive variable; μ_i

Positive variable; μ_s

Variable ε

Model Double Cone

Objective Function;

$$Max \quad Z = \varepsilon$$

The objective is maximizing ε to check whether Cone(x_i, x_s, $\underline{x_k}$) dominates alternative x_t

Subject to;

$$\mu_i (x_{kq} - x_{iq}) + \mu_s (x_{kq} - x_{sq}) - \varepsilon \geq x_{tq} - x_{kq}, \quad \forall q$$

$$\mu_i \geq 0,$$

$$\mu_s \geq 0,$$

If $\varepsilon \geq 0$ in the solution of the model, x_t which is dominated by Cone (x_i, x_s, $\underline{x_k}$), is eliminated. If $\varepsilon < 0$ in the solution of the model, x_t which is non-dominated, is not eliminated by Cone (x_i, x_s, $\underline{x_k}$).

We have to solve Model (i,k,t) for all possible alternative triples (i,k,t). Instead, we can solve a single (but large) model which is the combination of all possible models The model that finds alternatives eliminated by all single cones, is represented as follows.

Sets

Alternatives; $k, i, t = \{1, 2, 3, \ldots, m\}$

Criteria numbers; $q = \{1, 2, \ldots, p\}$

Positive Variable

Positive variable; μ_{ikt}

Variable ε_{ikt}

Model Combined Single Cones

Objective Function;

$$Max \quad Z = \sum_{i=1}^{m} \sum_{\substack{k=1 \\ f(x_i) > f(x_k)}}^{m} \sum_{\substack{t=1 \\ t \neq i \\ t \neq k}}^{m} \varepsilon_{ikt}$$

The objective function combines the objective functions of the individual models written for all possible (i, k, t) triplets.

Subject to;

$$\mu_{ikt}(x_{kq} - x_{iq}) - \varepsilon_{ikt} \geq x_{tq} - x_{kq} \qquad \forall q, i, k, t$$

$$\mu_{ikt} \geq 0 \qquad \forall i, k, t$$

The constraint set includes the constraints of the individual models for all (i, k, t) triplets. If $\varepsilon_{ikt} \geq 0$ in the solution of the model, x_t which is dominated by Cone (x_i, $\underline{x_k}$), is eliminated. If $\varepsilon_{ikt} < 0$ in the solution of the model, x_t which is non-dominated, is not eliminated by Cone (x_i, $\underline{x_k}$). The above model considers all single cones. We develop a similar model for double cones.

Sets

Alternatives; $k, i, s, t = \{1, 2, 3, \ldots, m\}$

Criteria numbers; $q = \{1, 2, \ldots, p\}$

Positive Variable

Positive variable; μ_{ikt}

Variable

Variable; ε_{iskt}

Model Combined Double Cones

Objective Function;

$$Max \quad Z = \sum_{i=1}^{m} \sum_{\substack{s=1 \\ s \neq i}}^{m} \sum_{\substack{k=1 \\ f(x_i) > f(x_k) \\ f(x_s) > f(x_k)}}^{m} \sum_{\substack{t=1 \\ t \neq i \\ t \neq s \\ t \neq k}}^{m} \varepsilon_{iskt}$$

The objective function combines the objective functions of the individual models written for all possible (i, s, k, t).

Subject to;

$$\mu_{ikt}(x_{kq} - x_{iq}) + \mu_{skt}(x_{kq} - x_{sq}) - \varepsilon_{iskt} \geq x_{tq} - x_{kq} \qquad \forall i, s, t, k, q$$

$$\mu_{ikt} \geq 0 \qquad \forall i, k, t$$

The constraint set includes the constraints of the individual models for all (i, s, k, t). If $\varepsilon_{iskt} \geq 0$ in the solution of the model, x_t which is dominated by Cone (x_i, x_s, $\underline{x_k}$), is eliminated. If $\varepsilon_{iskt} < 0$ in the solution of the model, x_t which is non-dominated, is not eliminated by Cone (x_i, x_s, $\underline{x_k}$).

We generate all single and double cones. We find the alternatives eliminated by each cone. On the other hand, the best alternative is found by this model.

4.2 Model for Finding the Minimum Number of Questions

In the model that finds the minimum number of questions required to reach the best alternative, we use the set of alternatives eliminated by each cone, pairwise questions required for each cone and all single and double cones generated as input data which are the output data from the first model. We find the best alternative and the selected set of cones with minimum number of questions.

Sets

Alternatives; $i, j = \{1, 2, 3, \ldots, m\}$

Cones; $c = \{1, 2, 3, \ldots, n\}$

Decision Variables

$$x_{ij} = \begin{cases} 1 & \text{if we ask DM to compare alternatives } i \text{ and } j \\ 0 & \text{otherwise} \end{cases}$$

$$y_c = \begin{cases} 1, & \text{if cone } c \text{ is selected} \\ 0, & \text{otherwise} \end{cases}$$

Parameters

$$a_{ij}^c = \begin{cases} 1, & \text{if cone } c \text{ requires the comparison of alternatives } i \text{ and } j \\ 0, & \text{otherwise} \end{cases}$$

$$b_i^c = \begin{cases} 1, & \text{if cone } c \text{ dominates alternative } i \\ 0, & \text{otherwise} \end{cases}$$

Model for Finding Minimum Number of Questions

Objective Function;

$$Min\ Z = \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j \\ f(x_i) > f(x_j)}}^{m} x_{ij}$$

The objective function is to minimize the number of pairwise questions asked to the DM.

Subject to;

$$\sum_{c=1}^{n} b_i^c \cdot y_c \geq 1 \qquad \forall i (i \neq i^*)$$

The constraint pick cones to dominate all alternatives except the best one.

$$a_{ij}^c \cdot y_c \leq x_{ij} \qquad \forall i, j, c, \ f(x_i) > f(x_j)$$

The constraint provides that a cone cannot be used if a required question is not asked.

$$x_{ij} \in \{0,1\}$$
$$y_c \in \{0,1\}$$

We mention that *x* and *y* are binary variables. We find the set of cones that eliminates all alternatives except the best one with minimum number of questions.

4.3 Computational Test and Analysis of the Results

In the computational tests, we consider five different levels for the number of alternatives as 10, 30, 50, 80, 100. We use three different utility functions (linear, quadratic, Tchebyshev). In each utility function, three different set of weights are used ((1/3,1/3,1/3),(0.7,0.2,0.1),(0.1,0.6,0.3)). We take five instances for each combination, which are shown in Table 4.1. Total number of problems is 225.

Table 4.1: The Characteristic of Computational Test

	Level				
Characteristic	**1**	**2**	**3**	**4**	**5**
Problem Size	10	30	50	80	100
DM Value Function Type	Linear	Quadratic	Tchebycheff		
DM Weight	0.33, 0.33, 0.33	0.7, 0.2, 0.1	0.1, 0.6, 0.3		

We use a 674 non-dominated altenatives with three criteria instance used by Özpeynirci, Köksalan and Lokman (2013). We randomly select the required number of nondomianted alternatives among this data set. We use GAMS and C++ programming language to implement our models. Combined double cone model cannot be implemented for 80 and 100 alternatives, because of the incapabilites of C++ programming language.

The detailed results on 30 alternatives for each instance can be seen in Table 4.2. We have 435 pairwise questions. Besides the minimum number of required questions, Table 4.2 indicates the minimum number of required questions based on the order of distance. For example, for instance no 1, the pair of alternatives with the largest distance among the selected ones is in the rank 174 when we sort the pairs in non-decreasing order of distances. We understand that there is a close relation between the order of distance and required questions.

The detailed results on 5 different alternative sizes can be seen in Tables 4.3, 4.4, 4.5, 4.6 and 4.7. Tables include five instances for each alternative. There are 8 columns in these tables. The first column indicates DM utility functions. The second column indicates the weight set of utility function. We implement a data set for two situations. In the first situation, we generate and use only single cone. In the other situation, we generate and use both single and double cone. We state minimum, maximum and average number of required questions in the tables for all instances. We separate these findings into two parts according to cone types.

Table 4.2: The Detailed Results for 30 Alternatives

No	Instances	N.of Alt.	Utility Function	Weights	N.of Questions (Single Cones)	N.of Questions (Single and Double Cones)	Distance
1	1	30	Quadratic	1	12	11	174
2	1	30	Quadratic	2	9	9	122
3	1	30	Quadratic	3	9	8	158
4	1	30	Linear	1	12	11	174
5	1	30	Linear	2	5	5	113
6	1	30	Linear	3	2	2	160
7	1	30	Tchebycheff	1	10	10	126
8	1	30	Tchebycheff	2	7	7	263
9	1	30	Tchebycheff	3	9	8	158
10	2	30	Quadratic	1	15	14	88
11	2	30	Quadratic	2	6	6	88
12	2	30	Quadratic	3	9	8	153
13	2	30	Linear	1	12	11	91
14	2	30	Linear	2	5	5	357
15	2	30	Linear	3	7	7	153
16	2	30	Tchebycheff	1	12	12	135
17	2	30	Tchebycheff	2	7	7	357
18	2	30	Tchebycheff	3	8	7	153
19	3	30	Quadratic	1	12	12	75
20	3	30	Quadratic	2	8	7	252
21	3	30	Quadratic	3	10	9	99
22	3	30	Linear	1	13	13	78
23	3	30	Linear	2	6	6	176
24	3	30	Linear	3	6	6	88
25	3	30	Tchebycheff	1	11	11	80
26	3	30	Tchebycheff	2	8	7	252
27	3	30	Tchebycheff	3	8	7	94
28	4	30	Quadratic	1	12	12	134
29	4	30	Quadratic	2	10	10	179
30	4	30	Quadratic	3	9	9	200
31	4	30	Linear	1	13	12	134
32	4	30	Linear	2	7	7	234
33	4	30	Linear	3	7	7	200
34	4	30	Tchebycheff	1	11	11	134
35	4	30	Tchebycheff	2	9	9	164
36	4	30	Tchebycheff	3	7	7	119
37	5	30	Quadratic	1	13	13	199
38	5	30	Quadratic	2	6	6	111
39	5	30	Quadratic	3	7	7	237
40	5	30	Linear	1	12	11	133
41	5	30	Linear	2	4	4	146
42	5	30	Linear	3	5	5	90
43	5	30	Tchebycheff	1	13	13	62
44	5	30	Tchebycheff	2	6	6	146
45	5	30	Tchebycheff	3	7	7	238

Table 4.3: The Minimum Number of Questions for 10 Alternatives

10 Alternatives		**Single Cone**			**Single and Double Cone**		
Utility F.	**Weight**	**Min**	**Avr**	**Max**	**Min**	**Avr**	**Max**
Linear	**1**	4	5.6	7	4	5.6	7
	2	4	4.4	5	4	4.4	5
	3	4	5.4	7	4	5.4	7
Tcheb.	**1**	5	5.8	7	5	5.8	7
	2	3	4.4	6	3	4.4	6
	3	4	5.2	7	4	5.2	7
Quadratic	**1**	4	6	8	4	6	8
	2	4	4.6	6	4	4.6	6
	3	5	5.6	7	5	5.6	7

Table 4.4: The Minimum Number of Questions for 30 Alternatives

30 Alternatives		**Single Cone**			**Single and Double Cone**		
Utility F.	**Weight**	**Min**	**Avr**	**Max**	**Min**	**Avr**	**Max**
Linear	**1**	12	12.4	13	11	11.6	13
	2	4	5.4	7	4	5.4	7
	3	2	5.4	7	2	5.4	7
Tcheb.	**1**	10	11.4	13	10	11.4	13
	2	6	7.4	9	6	7.2	9
	3	7	7.8	9	7	7.2	8
Quadratic	**1**	12	12.8	15	11	12.4	14
	2	6	7.8	10	6	7.6	10
	3	7	8.8	10	7	8.2	9

Table 4.5: The Minimum Number of Questions for 50 Alternatives

50 Alternatives		Single Cone			Single and Double Cone		
Utility F.	**Weight**	**Min**	**Avr**	**Max**	**Min**	**Avr**	**Max**
Linear	**1**	12	14.6	19	11	14	18
	2	6	7.6	9	6	7.6	9
	3	7	8.4	9	6	8	9
Tcheb.	**1**	11	12.2	13	11	11.8	13
	2	5	7.2	9	5	7.2	9
	3	6	7.6	9	6	7.2	9
Quadratic	**1**	12	14.6	19	11	14	18
	2	6	7.6	9	6	7.6	9
	3	7	8.4	9	6	8	9

Table 4.6: The Minimum Number of Questions for 80 Alternatives

80 Alternatives		Single Cone		
Utility F.	**Weight**	**Min**	**Avr**	**Max**
Linear	**1**	8	16	19
	2	8	8.4	10
	3	6	7.2	9
Tcheb	**1**	12	13.6	15
	2	6	7.4	10
	3	7	9	12
Quadratic	**1**	16	18.8	21
	2	6	9.8	12
	3	8	8.6	10

Table 4.7: The Minimum Number of Questions for 100 Alternatives

100 Alternatives		**Single Cone**		
Utility F.	**Weight**	**Min**	**Avr**	**Max**
Linear	**1**	17	19.4	21
	2	7	9	11
	3	8	9	10
Tcheb	**1**	12	13.6	15
	2	6	7.6	10
	3	7	9	11
Quadratic	**1**	18	20	23
	2	9	10.2	12
	3	8	9.8	14

Analysis of the Results

We find the best alternative and the set of cones that eliminates all alternatives with minimum number of questions. We calculate euclidean distances between alternatives in each pairwise alternatives. Distance between alternative *a* and *b* is the lenght of the line. For each pairwise alternatives, we calculate,

$$d(a,b)=\sqrt{\sum_{q=1}^{p}(a_q-b_q)^2}$$

We sort the pairs in non-decreasing order of distances. We calculate an average of the distances between alternatives in required questions. We consider first 35% of the pairs because of the average of the distances. Generally, required questions are in the first 35% of the pairs. In Table 4.2, it is clear that there is a close relation between required questions and the order of distances in each pairwise question. We use the distances between alternatives in each pairwise question to develop our interactive algorithms.

When we analyze the results of the computational test, we also observe that the minimum number of required questions depends on the number of eliminated alternatives by each cone.

Combined double cone model cannot be implemented for 80 and 100 alternatives, because of the incapabilites of C++ programming language. We compare single and double cones for 10, 30 and 50 alternatives. When we consider the single and double cones together, there is a slight decrease on the number of questions that is required to find the best alternative compared to considering only the single cones. It is clear that solving the model for double cones creates disadvantage in terms of solution time.

CHAPTER 5

INTERACTIVE ALGORITHM

In this chapter, we develop interactive algorithms. We then compare our interactive algorithms. Two interactive algorithms are compared with the results of mathematical models based on the minimum number of required questions. We implement our interactive algorithms to supplier selection problem.

5.1 Overview of the Algorithm

We aim to develop an interactive algorithm to select the best alternative by asking few questions to the DM. We select the best one in m alternatives with using p criteria. Through the mathematical models defined in Chapter 4, we find minimum number of required questions to reach the best solution and strategiesfor selecting the alternative pairs to be asked to the DM. When we analyze the results of the computational test, we see that there is a close relationship between the distances between the alternatives in each pairwise question and the minimum number of required questions. We use the distances between alternatives in each pairwise question to develop our interactive algorithms. On the other hand, the number of eliminated alternatives by each cone, which affect our selection process, is another question strategy. We calculate the distances between alternatives for each pair. We find the alternatives eliminated by each cone. For each pairwise question, we find the minimum and the maximum values of utility functions for each alternative. We calculate the possibility that an alternative is preferred to another one for each pair and we use this information to determine expected number of eliminated alternatives. We ask the DM to compared the selected alternatives. With new preferences, we calculate the possibility that alternative *a* is preferred to alternative *b* for each (*a*, *b*) pair. This procedure continues until all alternatives are eliminated other than the best alternative. We propose two different approaches. With the first algorithm, we use the sum of eliminated alternatives for each (*a*, *b*) and (*b*, *a*) pair. In the other algorithm, we use the minimum of eliminated alternatives for each (*a*, *b*) and (*b*, *a*) pair while picking the maximum.

5.2 Finding Utility Ranges

In order to calculate the possibility that an alternative is preferred to another one (for each pair of alternatives), we need to find the minimum and the maximum values of utility functions for each alternative.

<u>Sets</u>

Alternatives; $i = \{1, 2, 3, \ldots, m\}$

Criteria numbers; $p = \{1, 2, 3\}$

<u>Positive Variable</u>

Positive variable; μ_L

Positive variable; w_L

<u>Model Maximum Utility Function</u>

Objective Function;

$$Max\ Z = \mu_L(i)$$

The objective is maximizing the utility function for each alternative *i*.

Subject to;

$$\mu_L(i) = \sum_{q=1}^{p} x(i,q) w_L(q), \quad \forall i$$

$$\sum_{q=1}^{p} w_L(q) = 1, \quad \forall i$$

$$\mu_L \geq 0,$$

$$w_L \geq 0,$$

The model provides the maximum value of utility function which is used for possibility calculation.

Sets

Alternatives; $i = \{1, 2, 3, \ldots, m\}$

Criteria numbers; $p = \{1, 2, 3\}$

Positive Variable

Positive variable; μ_L

Positive variable; w_L

Model Minimum Utility Function

Objective Function;

$$Min\ Z = \mu_L(i)$$

The objective is minimizing the utility function for each alternative i.

Subject to;

$$\mu_L(i) = \sum_{q=1}^{p} x(i,q) w_L(q), \quad \forall i$$

$$\sum_{q=1}^{p} w_L(q) = 1, \quad \forall i$$

$$\mu_L \geq 0,$$

$$w_L \geq 0,$$

The model provides the minimum value of utility function which is used for possibility calculation.

5.3 Possibility Computation

In this section, we show how the possibility that each alternative is preferred to another one is calculated. We utilize a uniform possibility distribution. Let $f_{max}(a)$ and $f_{min}(a)$ be the maximum and minimum utility values for alternative a under the information taken from the DM so far. Let $P(a, b)$ be the possibility that alternative a is preferred to alternative b. Three cases are possible considering $f_{max}(a)$, $f_{min}(a)$, $f_{max}(b)$ and $f_{min}(b)$.

Case 1: If $f_{max}(a) > f_{max}(b)$ and $f_{min}(a) > f_{min}(b)$ (Figure 5.1),

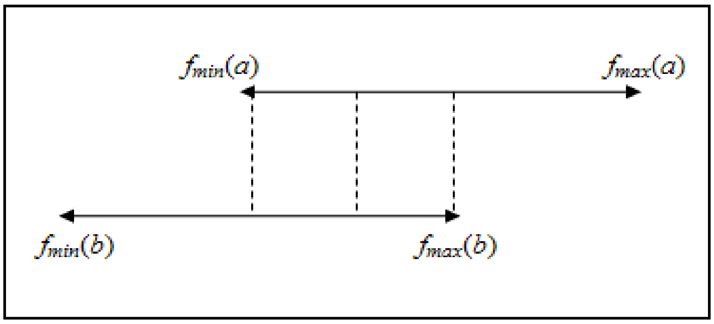


Figure 5.1: Case 1

$$P(a,b) = \frac{\left[f_{\max}(a) - \left(\frac{f_{\max}(b) + f_{\min}(a)}{2} \right) \right] + (f_{\min}(a) - f_{\min}(b))}{f_{\max}(a) - f_{\min}(b)}$$

Case 2: If $f_{max}(a) > f_{max}(b)$ and $f_{min}(a) < f_{min}(b)$ (Figure 5.2),

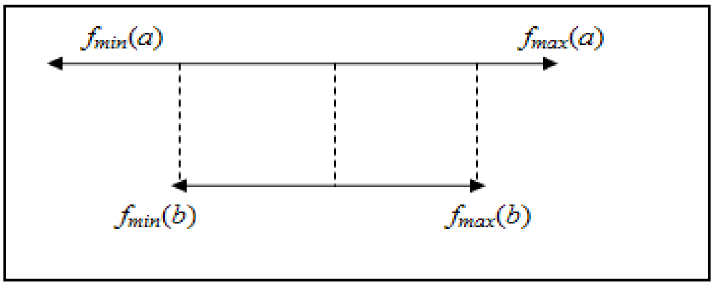


Figure 5.2: Case 2

$$P(a,b) = \frac{f_{\max}(a) - \left(\frac{f_{\max}(b) + f_{\min}(b)}{2} \right)}{f_{\max}(a) - f_{\min}(a)}$$

Case 3: If $max\ f\ (a) > max\ f\ (b)$ and $min\ f\ (a) < min\ f\ (b)$ (Figure 5.3),

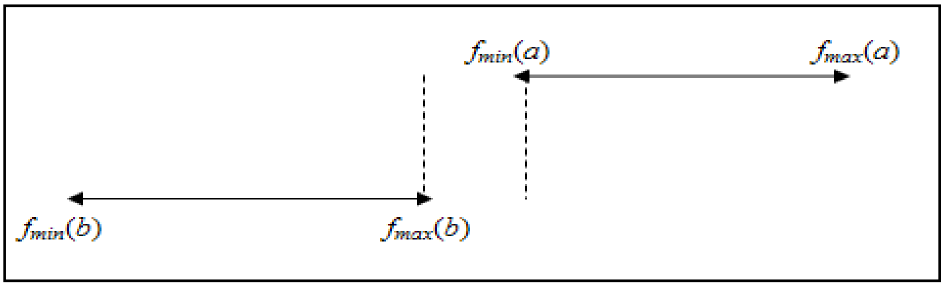


Figure 5.3: Case 3

$$P(a,b) = 1$$

5.4 Selecting the Next Question

In this section, we determine which pairwise question can be used in our selection process. We develop two algorithms using different strategies. Both algorithms use two types of information:

1) $P(a,b)$: the possibility that DM prefers alternative a to altenative b
2) $NE(a,b)$: the number of alternatives that will be eliminated by Cone(x_a,x_b)

For the first algorithm, we find the alternative pair (a,b) that maximizes the following term

$$\max(a,b)\{\min\{P(a,b)\times NE(a,b); P(b,a)\times NE(b,a)\}\}$$

For the second algorithm, we find the alternative pair (a,b) that maximizes the expected number of eliminated alternatives that is computed with the following term

$$E[a,b] = \max(a,b)\{P(a,b) \times NE(a,b) + P(b,a) \times NE(b,a)\}$$

In the first algorithm, we want to maximize the minimum of eliminated alternatives for each pairwise question. In Figure 5.4, it is clear that there are two cones: (x_1, x_2) and (x_2, x_1). We aim to maximize the minimum of eliminated alternatives. Cone (x_2, x_1), which dominates three alternatives, affect our selection process due to having the minimum of eliminated alternatives. This approach, which is a stable method, control both sides in order to find the best alternative with the minimum number of questions.

In the second algorithm, we use the sum of eliminated alternatives for each pairwise question. In Figure 5.1, it is clear that there are two cones: (x_1, x_2) and (x_2, x_1). We use the sum of eliminated alternatives for each pairwise question. Cone (x_2, x_1) dominates three alternatives. Cone (x_1, x_2) dominates four alternatives. There are seven dominated alternatives, that affects our selection process. This approach, which is not a stable method, control only the expected number of eliminated alternatives in order to find the best alternative with the minimum number of questions.

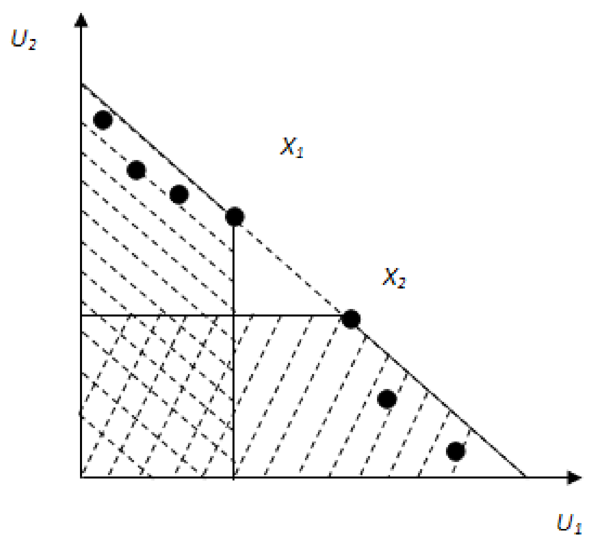


Figure 5.4: Using the Number of Eliminated Alternatives By Each Cone

5.5 The Algorithm

We propose two different approaches to select the best alternative by asking few questions to the DM. With the first algorithm (Algorithm A), we use the minimum of eliminated alternatives for each (*a, b*) and (*b, a*) pair while picking the maximum. In the other algorithm (Algorithm B), we use the sum of eliminated alternatives for each (*a, b*) and (*b, a*) pair.

We present the steps of the Algorithm A,

Step 1: Calculate the distances between each pair of alternatives. Sort the pairs in non-decreasing order of distances and consider first 35% of the pairs.

Step 2: Find the minimum and the maximum values of utility functions for each alternative. Calculate the possibilities.

Step 3: Find expected number of eliminated alternatives for each pair. Ask the DM to compare the pair of alternatives which has the highest minimum number of expected eliminated alternatives.

Step 4: If only one alternative is left, go to step 4. If more than one alternatives are left, go to step 0.

Step 5: The alternative is the most preferred solution. Stop.

We present the steps of the Algorithm B,

Step 1: Calculate the distances between each pair of alternatives. Sort the pairs in non-decreasing order of distances and consider first 35% of the pairs.

Step 2: Find the minimum and the maximum values of utility functions for each alternative. Calculate the possibilities.

Step 3: Find expected number of eliminated alternatives for each pair. Ask the DM to compare the pair of alternatives which has the highest sum of expected eliminated alternatives.

Step 4: If only one alternative is left, go to step 4. If more than one alternatives are left, go to step 0.

Step 5: The alternative is the most preferred solution. Stop.

5.6 Computational Tests

We propose two interactive algorithms to select the best alternative by asking few questions to the DM. One is to use the sum of eliminated alternatives for each sides. The other is to use the minimum of eliminated alternatives for each sides while picking the maximum. We develop mathematical models to determine the minimum possible number of questions. We test to compare two different algorithms under different utility functions. Also two interactive algorithms are compared with the results of mathematical models based on the minimum number of required questions. On the other hand, we compare our algorithms without calculating the distances between alternatives in each pairwise question. The detailed results on 5 different alternative sizes can be seen in Tables 5.1, 5.2, 5.3, 5.4 and 5.5. Tables include five instances for each alternative. There are 8 columns in these tables. The first column indicates DM utility functions. The second column indicates the weight set of utility function. We implement a data set on mathematical models for two situations. In the first situation, we generate and use only single cone which is indicated in the third column. In the other situation, we generate and use both single and double cone which is indicated in fourth column. We implement a data set on algorithms for two situations. A35 and B35 mean that we calculate the distances between alternatives in each pairwise question and we sort the pairs in non-decreasing order of distances and consider first 35% of the pairs. A100 and B100 mean that the order of distances in each pairwise question is not important for us and we consider all pairs. We state the results of our four algorithms in the fifth, sixth, seventh and eighth columns.

Table 5.1: The Results of Comparison Between Algorithms and Mathematical Model for 10 Alternatives

10 Alternatives		**Models**		**Algorithms**			
Utility F.	**Weight**	**Single C.**	**Double C.**	**A100**	**B100**	**A35**	**B35**
Linear	**1**	5.6	5.6	6.8	8.4	6.8	7.6
	2	4.4	4.4	6.6	8.4	6.6	7.6
	3	5.4	5.4	7	8	6.8	7.8
Tcheb	**1**	5.8	5.8	7	8.6	6.8	7.8
	2	4.4	4.4	6.6	8.4	6.6	7.6
	3	5.2	5.2	7.2	7.8	7.2	7.2
Quadratic	**1**	6	6	7.4	8.6	7.4	8.2
	2	4.6	4.6	6.6	8.4	6.6	7.6
	3	5.6	5.6	7.2	8.6	7.2	8

Table 5.2: The Results of Comparison Between Algorithms and Mathematical Model for 30 Alternatives

30 Alternatives		Models		Algorithms			
Utility F.	**Weight**	**Single C.**	**Double C.**	**A100**	**B100**	**A35**	**B35**
Linear	**1**	12.4	11.6	17.2	27.8	16.8	26.4
	2	5.4	5.4	12	12	12.2	14.8
	3	5.4	5.4	11	17.4	11.8	19.2
Tcheb	**1**	11.4	11.4	17.8	28.2	17.2	26.8
	2	7.4	7.2	13.4	15	11.6	17.6
	3	7.8	7.2	13.6	21.2	13.6	19.4
Quadratic	**1**	12.8	12.4	17.8	28.2	17.6	26.8
	2	7.8	7.6	13.6	15.2	11.8	17.2
	3	8.8	8.2	13.4	24	13.6	22.6

Table 5.3: The Results of Comparison Between Algorithms and Mathematical Model for 50 Alternatives

50 Alternatives		Models		Algorithms			
Utility F.	**Weight**	**Single C.**	**Double C.**	**A100**	**B100**	**A35**	**B35**
Linear	**1**	15.2	12.2	22.6	46.4	22.4	43
	2	6.8	6.6	15.4	17.6	15.8	16.2
	3	8.8	8.8	18	26	17.4	24.8
Tcheb	**1**	12.2	11.8	23	44.8	22.8	41.4
	2	7.2	7.2	15.8	17.2	15	16.4
	3	7.6	7.2	19.6	36.4	18.6	33.6
Quadratic	**1**	14.6	14	22.6	46.4	22.2	42.6
	2	7.6	7.6	14	16.6	14.4	16
	3	15.2	12.2	22.6	46.4	22.4	43

Table 5.4: The Results of Comparison Between Algorithms and Mathematical Model for 80 Alternatives

80 Alternatives		Models		Algorithms			
Utility F.	**Weight**	**Single C.**	**Double C.**	**A100**	**B100**	**A35**	**B35**
Linear	**1**	16	*	25	61.8	24.8	59.8
	2	8.4	*	18.6	29	18.4	32
	3	7.2	*	18.8	38.8	18.6	39.2
Tcheb	**1**	13.6	*	31.8	66.6	31.6	63.6
	2	7.4	*	27.8	32	21.6	31
	3	9	*	24	43.6	24.4	46.4
Quadratic	**1**	18.8	*	30.8	72.8	30.6	70.6
	2	9.8	*	20.6	31.6	20.2	32.2
	3	8.6	*	19.8	43.2	19	46.6

Table 5.5: The Results of Comparison Between Algorithms and Mathematical Model for 100 Alternatives

100 Alternatives		Models		Algorithms			
Utility F.	**Weight**	**Single C.**	**Double C.**	**A100**	**B100**	**A35**	**B35**
Linear	**1**	19.4	*	33.2	93.8	33	90.8
	2	9	*	19.8	34	20.6	37
	3	9	*	19.4	57.8	18.6	52.2
Tcheb	**1**	13.6	*	32.2	89.2	33	88
	2	7.6	*	23	39	22.6	38.8
	3	9	*	22.8	68	24.4	68.2
Quadratic	**1**	20	*	33	95.6	32.2	92.2
	2	10.2	*	22	39.6	21.8	40
	3	9.8	*	21.8	64.4	21.2	62.4

We test to compare two different algorithms under different utility functions. Also two interactive algorithms are compared with the results of mathematical models based on the minimum number of required questions.

The minimum number of required questions in Algorithm A are less than Algorithm B's in two situations. It is obvious that Algorithm A is more efficient than Algorithm B. There is a great difference between each other according to required questions. However, the required questions in our algorithms match only in few instances with the minimum number of questions in mathematical models.

We calculate the distances between alternatives in each pairwise question. We benefit from non-decreasing order of distances according to the minimum number of required questions. There is a decrease on the number of required questions when we calculate the distances between alternatives in each pairwise question.

The other finding is about the weight set of utility function. The first weight set (0.33, 0.33, 0.33) for all utility functions require more questions than the other weight sets to both mathematical models and algorithms.

5.7 An Application to Supplier Selection Problem

We propose two interactive approaches to multiple criteria selection problems. We implement our algorithms on a supplier selection problem from Benyoucef, Ding and Xie (2003). There are 10 suppliers evaluated on 4 criteria, which are delivery, quality, price and service after sale. After applying Analytic Hierarchy Process (AHP), the corresponding weights of these criteria are found as 0.4, 0.3, 0.2 and 0.1, respectively. We assume that the DM has a linear utility function. The scores of alternative suppliers on 4 criteria are given in Table 5.6.

Table 5.6: Data of supplier selection problem

Supplier	Delivery	Quality	Price	Service after sale
Supplier 1	7.2	6.5	7.2	6.2
Supplier 2	6.5	7.3	6.3	8.2
Supplier 3	6.3	8.2	5.5	4.8
Supplier 4	8.8	7.8	7.8	5.7
Supplier 5	6.3	6.3	5.7	4.7
Supplier 6	5.3	8.2	5.7	5.7
Supplier 7	6.5	5.5	7.8	7.3
Supplier 8	7.3	8.2	4.8	7.3
Supplier 9	9	7.2	7.7	4.8
Supplier 10	6.3	7.3	6.3	6.5

In Table 5.7, we give the results of above supplier selection problem. There are 10 suppliers evaluated on 4 criteria, which are delivery, quality, price and service after sale. We assume that the DM has a linear value function which is indicated in the first column. The second column indicates the weight set of utility function. The results of mathematical models are in the third and fourth columns. The results of our algorithms are indicated in the fifth, sixth, seventh and eighth columns.

Table 5.7: Results of supplier selection problem

10 Suppliers		**Models**		**Algorithms**			
Utility F.	**Weight**	**Single C.**	**Double C.**	**A100**	**B100**	**A35**	**B35**
Linear	0.4, 0.3, 0.2, 0.1	7	7	7	9	8	9

We solve mathematical models and we find the set of cones that eliminates all suppliers except the best one with minimum number of questions. We have to ask minimum 7 pairwise questions to the DM. Algorithm A without calculating the distances between alternatives in each pairwise question (A100) requires 7 pairwise questions that is same result with mathematical models. According to computational tests, the order of distance, which has a good effect on our solution, can reduce the number of questions to the DM. In spite of this generalization, A100 has a better solution than the other algorithms. A100 find the best supplier with 7 pairwise questions that is a optimal solution based on mathematical models.

CHAPTER 6

CONCLUSION AND FUTURE WORK

In this study, we propose two algorithms to discrete alternative multiple criteria selection problems. We use a convex cone method in our study. We use DM preferences in order to generate cones. Inferior alternatives, which are identified by cones, are eliminated directly. We aim to select the best alternative by asking few questions to the DM.

We develop an interactive algorithm to select the best alternative by asking pairwise questions to the DM. One is to maximize the minimum of eliminated alternatives for each pairwise question. The other is to use the sum of eliminated alternatives for each pairwise question. We develop mathematical models to determine the minimum number of questions. In mathematical models, we find two main items such as the minimum number of questions and the question strategies for all problems. These findings are used to develop an interactive algorithm. The distances between alternatives in each pairwise question and the number of eliminated alternatives by each cone have a great effect on the minimum number of required questions. In other words, we solve decision making process easily in spite of our limited time. We find the minimum and the maximum values of utility functions for each alternative in order to calculate the possibility that an alternative is preferred to another one for each pair. We calculate expected eliminated alternatives with the possibility for each pair.

We test to compare two different algorithms under different utility functions. Also two interactive algorithms are compared with the results of mathematical models based on the minimum number of required questions. On the other hand, we compare our algorithms without calculating the distances between alternatives in each pairwise question.

We analyze the results of the computational test. We compare single and double cones. It is obvious that there is a slight decrease on the number of questions that is required to find the best alternative. For 80 and 100 alternatives, we can not find the results of double cones, due to the incapabilites of C++ programming language. It is clear that solving the model for double cones is disadvantageous to our time.

Using the distance in our algorithms, which provides time savings, reduces the number of required questions. In this method, we eliminate nearly the half of all pairwise question.

The required questions in Algorithm A are less than Algorithm B's. There is a great difference between each other according to required questions. However, the required questions in our algorithms match only in few instances with the minimum number of questions in mathematical models.

We implement our algorithms on a supplier selection problem from Benyoucef, Ding and Xie (2003). We find the set of cones that eliminates all suppliers except the best one with minimum number of questions. We have to ask minimum 7 pairwise questions to the DM. Although using the distance in our algorithms reduces the number of required questions according to computational tests, Algorithm A without calculating the distances between alternatives in each pairwise question (A100) has a better solution than the other algorithms. A100 find the best supplier with 7 pairwise questions that is a optimal solution based on mathematical models.

We propose three future research directions; (i) to utilize a triangular possibility distribution using middlemost weights in addition to minimum and maximum values (ii) to check the necessity of higher degree cones (m>2), (iii) to compare our interactive algorithms performances with other algorithms available in the literature, (iv) to implement for group decision making.

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