Model selection criteria for Multivariate GARCH models

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ABSTRACT

In this study, theory of copula is used to establish criteria to select between multivariate GARCH models. It is common to use Multivariate GARCH models to study effects of shocks between markets. There are several MGARCH models which scholars have applied to different markets data. The problem of preferring one model to other model is not discussed by scholars. To do so, first buttermilk problem is identified for MGARCH models, and then a hypothesis testing approached is introduced to check which model collects better the relationships of two markets. In next step several dependency modeling approaches are checked for standardized residuals of MGARCH models. Among them, Alternating Conditional Expectations regression method is selected as model selection criterion for MGARCH models. To check the consistency of criterion, real world data, OPEC oil prices and Chinese stock Indices, has been used for empirical study to check which model captures relationship between volatility spillover effects of Oil prices on Chinese stock market.

Dedication

To my father who taught me mathematics

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1. Introduction

Forecasting the future loss of portfolio is important for hedge fund managers. Portfolio managers want to decrease their exposure to risk (so called volatility) in the markets and investors want to take more benefit from their positions in risky assets. That's why estimating tomorrow's risk and finding the impact of shocks to the portfolio is major concern in asset management. Loads of researchers have been concerned with finding relationships between shocks and volatilities of markets. After introducing ARCH[1] models by Engle (1982), scholars agreed that tomorrow volatilities can be predictable. The study of volatility forecast has been in a variety of approaches, some of which focused on how to model volatilities and some are simply empirical suggestions. Meantime, studying the effect of markets on each other became a common interest of scholars. This analysis emerged after introducing multivariate GARCH[2] models. The first MGARCH model, VEC model, was introduced by Bollerslev et al. (1988) in their study of CAPM model with time varying variances; they found that conditional covariance matrix of asset returns to vary over time and risk premium of asset returns to be influenced by variance of returns. After introducing VEC model, many scholars tried to understand the co-movements of financial returns using MGARCH models. Some scholars indicate that risk management and asset allocation relate to finding and updating optimal hedging positions. Silvennoinen et al. (2007) and Bauwens et al. (2006) have provided a precious literature review on MGARCH models.

To date, none of the scholars have presented how to distinguish among different MGARCH models. In fact, nobody has studied the effectiveness of MGARCH models to check whether the MGARCH model has captured all co-movements of two markets or there is still some dependency left between two markets after taking out the covariance matrix of MGARCH model. This problem has been identified in this study

[1] Autoregressive Conditional Heteroscedasticity
[2] General Autoregressive Conditional Heteroscedasticity

and named as buttermilk problem. After identifying buttermilk in MGARCH models, I tried to find how much fat is left in buttermilk of MGARCH models.

Statistically speaking, the question is what relationship between standardized residuals of MGARCH model is. To investigate this relationship, several dependency models have been studied. These dependency models include linear dependency models, such as Pearson correlation, Kendal's tau and Spearman rho, non linear dependency models, such as copulas, Randomized Dependence Coefficient, Distance or Brownian correlation measure and Alternating Conditional Expectations. Among these models, ACE provides better measure to distinguish the fats of buttermilk. Once ACE is collected as dependency measure, then this criterion applied to several famous and well known MGARCH models to check consistency of criterion. The empirical results confirms the consistency of ACE criterion for selecting proper MGARCH model that captures better the relationship of two markets

This study is organized as follows. Section 2 reviews literature on volatility modeling, section 3 discusses the problem of MGARCH models, section 4 reviews dependency literature and section 5 investigates the criterion on real data, and shows how to select among MGARCH models. The data used in this research is bivariate sets of OPEC oil and Chinese stock indices. To estimate the MGARCH models RATS software, and to estimate the copula functions and dependency measures R Software are used. All codes are presented in appendix C.

2. Literature Review

In this section, volatility modeling for markets discussed and mainly focused on Multivariate risk models introduced by scholars. The very famous models for volatility of markets are ARCH and GARCH models which is the main area of this study.

2.1. Univariate volatility

High frequency observations of financial time series of financial assets are in fact independent or uncorrelated, while the series contain higher order dependence (Teräsvirta (2009)). One way to model this dependency is using ARCH models or in general form of them, namely the GARCH model. The GARCH model was developed independently by Bollerslev (1986) and Taylor (1986) to capture volatilities of financial assets. It is clearly known that the asset returns do not possess constant variance over a period of time (see Bollerslev et al. (1988)). Teräsvirta (2009) prepared an overview of univariate models of conditional heteroscedasticity. Till now, several types of the GARCH model have been introduced to model volatilities. Some try to study asymmetric behavior such as, the GJR-GARCH model of Glosten et al. (1993), the NAGARCH model of Engle and Ng (1993) which model non-linear asymmetric response to news, and the quadratic GARCH (QGARCH) of Sentana (1995). Other extensions of GARCH family are: Integrated GARCH (IGARCH), Exponential GARCH (EGARCH), Markov-switching GARCH and Threshold GARCH (TGARCH) which modeled by Engle and Bollerslev (1986), Nelson (1991), Hamilton and Susmel (1994), and Zakoian (1994) respectively.

The key formula of all GARCH models is that the shocks can be decomposed in the following:

(1) $$\varepsilon_t = \sigma_t \eta_t, \quad \eta_t \sim i.i.d(0,1)$$

With forecasted conditional variances we are able to find the measure of risk whilst investing in an asset. GARCH model tries to model variance of the current error term as a function of the actual sizes of the lag error terms and lag variances. So GARCH

(1,1) model will make use of both error term square and variance of the first lag as the independent variables of the current variance to make regression coefficient estimations for the model. The GARCH (1, 1) is given by:

(2) $y_t = \mu_t + \varepsilon_t,$

(3) $\varepsilon_t = \sigma_t \eta_t, \quad \eta_t \sim \text{i.i.d}(0,1)$

(4) $\sigma_t{}^2 = \alpha_0 + \alpha_1 \varepsilon^2{}_{t-1} + \beta_1 \sigma^2{}_{t-1}$

$\sigma_t{}^2$, is the time period conditional variance of the error terms (ε_t) in the regression equations (2), η_t in equation (3), is normally distributed standardized residual that is used as test statistics. Equation (4) gives the values of the conditional variance $\sigma_t{}^2$ at time period, t.

2.2. Multivariate GARCH Models

Modeling financial volatility of markets and studying the effects of these volatilities on each other is a trending concern of scholars. For modeling dynamic relationship of market volatilities, MGARCH models are applied to asset returns of financial time series. Multivariate volatility is applied to estimating dynamic and optimal hedge ratio or calculating the risk minimizing portfolio. The very first MGARCH model, VEC-GARCH model of Bollerslev, Engle, and Wooldridge (1988), is based on conditional variance-covariance and its lags. Later on, a restricted form of VEC-GARCH model called BEKK model after Baba et al. (1987), developed and became more popular between as it was easier to estimate due to less complexity in model estimation. Another class of MGARCH models is on the decomposition of the conditional covariance matrix into conditional standard deviations and correlations (Silvennoinen et al. (2007)). Bollerslev (1990) proposed Constant Conditional Correlation (CCC–) GARCH model but time varying conditional variances and covariances. Tse and Tsui (2002) investigated volatility and correlation transmission and spillover effects by adopting VECH representation based on the conditional variances and the conditional correlations. They applied their proposed model to exchange rate and stock market indices. Engle

(2002) introduced a Dynamic Conditional Correlation (DCC)–GARCH model which its dynamic conditional correlation structure is similar to Tse and Tsui (2002) model.

Applications of MGARCH models are not just restricted to stock markets. Chan et al. (2005) used MGARCH models to investigate tourist arrival rate between four tour leading countries. Moreover, MGARCH models are widely applied to commodity markets to define hedging strategies especially on oil shocks contagion effect. There is a number of contributions to the literature on the relationship between oil prices and stock index market or other financial and macroeconomic variables. Some researchers provided the literature on the impact of oil prices on the exchange rates (see Zhang et al., (2008); Narayan et al. (2008)). Several studies have also provided new insights into the effect of oil price changes on stock market (see Narayan and Narayan, (2010); Cong et al. (2008); Park and Ratti, (2008)). MGARCH models also have been used to analyze and estimate the volatilities of crude oil prices, exchange rates and stock markets. Asymmetric BEKK model is used by Agren (2006) as strong evidence to show the impact of volatilities of oil prices onto the stock markets in Norway, Japan, UK and US but rather a weak evidence for the oil price effect on Swedish stock market. A research on mutual effects between the S&P oil sector stock index and the oil spot and futures prices is reported by Hammoudeh et al. (2004). Kim et al. (2015) examine spillover effects of the recent U.S. financial crisis on five emerging Asian countries by estimating conditional correlations of financial asset returns across countries using MGARCH models. Lien et al. (2002), evaluate hedging performance of MGARCH models in future markets. Jondeau (2015) investigated the properties of a portfolio composed of a large number of assets driven by a strong MGARCH process with heterogeneous parameter.

The common aspect of all MGARCH models is their decomposition of shocks into the variance matrix and standardized errors. This is generalization of what we have seen in univariate modeling:

$$\textbf{(5)} \quad \boldsymbol{\varepsilon_t = H_t^{1/2}\eta_t}\,, \quad \boldsymbol{\eta_t \sim i.i.d\ (0, I_n)}$$

Where $\mathbf{H_t}$ is the conditional variance-covariance matrix, ($\boldsymbol{\varepsilon_t}$) is time-varying shocks vector and $\boldsymbol{\eta_t}$ is standardized errors vector, which is I.I.D with mean zero and variance of vector$\boldsymbol{I}$. As in univariate all multivariate GARCH models differ just in the way they are modeling variance matrix. In this study just two of MGARCH models are discussed but the methodology can be applied to other models as well.

2.3. BEKK Model

One of important and widely used MGARCH models is the BEKK model, named after Baba, Engle, Kraft and Kroner (1987). The final version of BEKK is discussed in Engle and Kroner (1995). The BEKK model is simple form of VEC-GARCH model on the condition of positive definite covariance process, which makes it is easier to verify stationary conditions of covariance process. `Diagonal BEKK' and `scalar BEKK' are introduced to decrease the number of parameters to be estimated. Kroner and Ng(1998) illustrate the asymmetric behavior of time-varying covariance of asset returns.

In the specification of the BEKK model, each coefficient of past values depends on several parameters. Consider an *n* dimensional vector $\boldsymbol{y_t}$ follows the general BEKK (*p; q; K*) model, given by:

(6) $$\boldsymbol{y_t = \mu_t + \varepsilon_t}$$

(7) $$\boldsymbol{\varepsilon_t = H_t^{1/2}\eta_t\,,\quad \eta_t \sim i.i.d\ (0, I_n)}$$

(8) $$\mathbf{H_t = C'C + \sum_{i=1}^{p}\sum_{k=1}^{K} A'_{ik}\varepsilon_{t-i}\varepsilon'_{t-i}A_{ik} + \sum_{j=1}^{q}\sum_{k=1}^{K} B'_{jk}H_{t-j}B_{jk}}$$

Where $\mathbf{A_{ik}}$ and $\mathbf{B_{ik}}$ are n-dimensional square matrices and ***C*** *is the n-dimensional lower triangular matrix.* As in Propositions 2.2 and 2.3 of Engle and Kroner (1995), it is useful to impose restrictions for the purpose of model identification. Now consider the widely-used BEKK (1,1) specification:

(9) $$\mathbf{H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B}$$

(10) $\mathbf{A} = \begin{bmatrix} \mathbf{a_{11}} & \mathbf{a_{12}} \\ \mathbf{a_{21}} & \mathbf{a_{22}} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{b_{11}} & \mathbf{b_{12}} \\ \mathbf{b_{21}} & \mathbf{b_{22}} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} \mathbf{c_{11}} & \mathbf{0} \\ \mathbf{c_{21}} & \mathbf{c_{22}} \end{bmatrix}, \boldsymbol{\varepsilon_t} = [\boldsymbol{\varepsilon_{1,t}} \;\; \boldsymbol{\varepsilon_{2,t}}], \boldsymbol{\varepsilon_t \varepsilon'_t} = \begin{bmatrix} \boldsymbol{\varepsilon^2{}_{1,t}} & \boldsymbol{\varepsilon_{1,t}\varepsilon_{2,t}} \\ \boldsymbol{\varepsilon_{1,t}\varepsilon_{2,t}} & \boldsymbol{\varepsilon^2{}_{2,t}} \end{bmatrix}$

$\mathbf{H_t}$ is the conditional variance-covariance matrix of shocks ($\boldsymbol{\varepsilon_t}$). Engle and Kroner (1995) proposed that $\mathbf{H_t}$ is required to be positive definite for all values of the disturbances. By expanding matrix form, we can see relationship of volatilities of both markets and effect of shocks of each market on volatilities:

(11)

$$\begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} = \begin{bmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{bmatrix}' \begin{bmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}' \begin{bmatrix} \varepsilon^2{}_{1,t-1} & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon^2{}_{2,t-1} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}' \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

So

(12) $$h_{11,t} = (c_{11}{}^2 + c_{21}{}^2) + (a_{11}{}^2\varepsilon^2{}_{1,t-1} + 2a_{11}a_{21}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}{}^2\varepsilon^2{}_{2,t-1}) + +(b_{11}{}^2 h_{11,t-1} + 2b_{11}b_{21}h_{12,t-1} + b_{22}{}^2 h_{22,t-1})$$

(13) $$h_{12,t} = c_{21}(c_{11} + c_{22}) + a_{11}a_{12}\varepsilon^2{}_{1,t-1} + (a_{11}a_{22} + a_{12}a_{21})\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}a_{22}\varepsilon^2{}_{2,t-1} (b_{11}b_{12}h_{11,t-1} + (b_{11}b_{22} + b_{12}b_{21})h_{12,t-1} + b_{21}b_{22}h_{22,t-1})$$

(14) $$h_{22,t} = (c_{22}{}^2 + c_{21}{}^2) + (a_{12}{}^2\varepsilon^2{}_{1,t-1} + 2a_{12}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{22}{}^2\varepsilon^2{}_{2,t-1}) + +(b_{12}{}^2 h_{11,t-1} + 2b_{12}b_{22}h_{12,t-1} + b_{22}{}^2 h_{22,t-1})$$

The BEKK model captures the effects on the current conditional volatility of own innovations[3] and lagged volatility as well as the cross market shocks and the volatility transmission of other markets. One application of BEKK model is to calculate Value-at-Risk (VaR) and conditional expected shortfall (ES).

To estimate BEKK model, we need to use optimization methods. Similar to univariate GARCH models, we can estimate BEKK model using numerical non liner

[3] In time series analysis (or forecasting) — as conducted in statistics, signal processing, and many other fields — the innovation is the difference between the observed value of a variable at time t and the optimal forecast of that value based on information available prior to time t.

optimization methods. If we assume that the error disturbances follow multivariate normal distribution and variance-covariance matrix as:

(15) $$H_t = E(\varepsilon_t \varepsilon_t' | \varepsilon_{t-1})$$

Then use log-likelihood function is given:

(16) $$L = -\frac{1}{2}\sum_{t=1}^{T}(\log\det H_t + \varepsilon_t' H_t^{-1}\varepsilon_t) = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\ln(|H_t|) - \frac{1}{2}\varepsilon_t' H_t^{-1}\varepsilon_t$$

Subject to:

(17) $$y_t = \mu_t + \varepsilon_t$$

(18) $$H_t = C'C + A'\varepsilon_{t-1}\varepsilon_{t-1}' A + B' H_{t-1} B$$

(19) $H_t \succcurlyeq 0$; Means H is Positive Definite variance covariance condition

To maximize **L**, the nonlinear maximization method introduced by Brendt et al (1974) is used. In case that error disturbances joint distribution is not joint normally distributed, then Quasi ML (QML) estimation has been used. By the way, RATS software can estimate BEKK model for which we can use this code:

```
-> garch(p=1, q=1, mv=bekk, pmethod=simplex, piters=5) Oil Index
```

Moreover, for testing BEKK model, we can find eigne values of **K** matrix which is given in (20). According to Silvennoinen et al. (2009), BEKK model is covariance stationary if and only if the Eigen values of K are less than one in modulus.

(20) $$K = \sum\sum A \otimes A + \sum\sum B \otimes B$$

Where ⊗ denotes the Kronecker product of two matrices. With the following command in R, we can easily calculate **K** as:

```
-> K=Kronecker (A,A)+Kronecker (B,B)
```

2.4. CCC GARCH Models

Bauwens et al. (2006) describe CCC-GARCH Models as nonlinear combinations of univariate GARCH models in which conditional variances and the conditional correlation matrix or another measure of dependence between the individual series is separated from each other. Bollerslev (1990) proposed CCC-GARCH models in which the conditional correlations are constant and the conditional covariances are proportional to the product of the corresponding conditional standard deviations. The CCC model is defined as:

$$\boldsymbol{H_t} = \boldsymbol{D_t R D_t} = \rho_{ij}\sqrt{h_{iit}h_{jjt}} \quad (21)$$

Where:

$$\boldsymbol{D_t} = diag(h_{11t}^{1/2} \dots h_{NNt}^{1/2}) \quad (22)$$

h_{iit} can be defined as any univariate GARCH model and **R** is a symmetric positive definite matrix containing constant conditional correlations: **R** $=\rho_{ij}$ and $\rho_{ii}=1$. The first introduced CCC-GARCH model was:

$$h_{iit} = \omega_i + \alpha_i \varepsilon_{1,t-1}^2 + \beta_i h_{ii,t-1} \quad (23)$$

Which is GARCH (1,1) process. Tae-Hwy, Long, Xiangdong (2006) introduced copula-based MGARCH (C-MGARCH) model with uncorrelated dependent errors, which are generated through a linear combination of dependent random variables. This model is an extension of CCC-GARCH model.

2.4.1. VAR-CCC-GARCH

VAR-CCC-GARCH model was proposed by Ling and McAleer (2003) and later applied by several researchers such as Chan et al. (2005), Hammoudeh et al. (2009) and Arouri et al. (2011).

This model is special case of the multivariate CCC-GARCH of Bollerslev (1990). In this model correlations between market shocks are considered to be constant which makes estimation easier. The bivariate VAR1 -GARCH (1,1) is used to find conditional correlation of OPEC oil and Financials No Bank Industry Index Stock Index. The conditional mean equation of both markets is:

$$Y_t = c + \emptyset Y_{t-1} + \varepsilon_t \quad (24)$$

$$\varepsilon_t = H_t^{1/2}\, \eta_t \quad (25)$$

Where the volatility across both markets is:

$$h_t^{Index} = C_{COM} + a_{11}(\varepsilon_{t-1}^{Index})^2 + b_{11} h_{t-1}^{Index} + a_{12}(\varepsilon_{t-1}^{OPEC})^2 + b_{12} h_{t-1}^{OPEC} \quad (26)$$

$$h_t^{OPEC} = C_{OPEC} + a_{21}(\varepsilon_{t-1}^{OPEC})^2 + b_{21} h_{t-1}^{OPEC} + a_{22}(\varepsilon_{t-1}^{Index})^2 + b_{22} h_{t-1}^{Index} \quad (27)$$

For estimating VAR-CCC-GARCH model, we can use RATS codes as below:

```
system(model=var1)

variables Oil Index

lags 1

end(system)

garch(p=1, q=1, model=var1, mv=CC, variance=varma, pmethod=simplex, piters=5) Oil Index
```

2.5. Tests for MGARCH models

According to Silvennoinen et al. (2007), there are two approaches to test MGARCH models. The general misspecification test approaches to check the adequacy of an estimated model and the other approach tries to test the model against a parametric extension, named specification tests. The later is mostly designed for CCC-GARCH models. Ling and Li (1997) have introduced a test statistics to test the adequacy of MGARCH models which can be used for all models. Later on Tse and Tsui (1999) criticize the power of Ling and Li (1997) approach. Duchesne (2004) provided a generalization of Ling and Li (1997) test. Duchesne (2004), a general class of matricial measures of dependence is proposed, that corresponds to sample autocovariance matrices of the vector time series of squared (standardized) residuals and cross products of (standardized) residuals. He derived the asymptotic distribution of these residual autocovariance matrices, using an approach similar to Li and Mak(1994) approach for univariate GARCH models. Tse (2000) introduced a Lagrange Multiplier test for the constant-correlation hypothesis in a MGARCH model. The test examines the restrictions imposed on a model which encompasses the CCC–GARCH. Bera and Kim (2002) have suggested a test for constancy of correlation in bivariate CCC–GARCH model.

3. Buttermilk! Problem of MGARCH models

In many of papers about commodity markets BEKK, DCC GARCH and VAR-CCC-GARCH models are mostly used by scholars to study co-movements of volatility of oil and stock markets. It is common among these studies that they compare and contrast several MGARCH models results but selecting one model over another is missing in all of these studies. There is no methodology presented in strong literature of MGARCH models that can say which MGARCH model capture better the dependency of markets better than the other models.

Actually the basic assumption of all MGARCH models is that the variance-covariance matrix captures the whole comovement and all relationship of both markets. This can be translated to alternative hypothesis that there is no relationship between the remaining data of markets after driving off the variance-covariance matrix. That’s the main problem of MGARCH models which I call buttermilk. This is similar to the process of producing butter. Let’s consider we have two different processes to produce butter from milk. After processing the milk and getting milk whatever remains is called buttermilk, literally means there is still some fat in the remaining milk. So we need another process to separate more fat from butter milk. In this case, if we can measure how much fat is left in butter milk of each process then we can decide which churning process performs better. MGARCH models are similar to milk process as they are a process of capturing variance-covariance of two markets. So if we consider variance as fats then buttermilk is the same as standardized residuals. Now we want to know if there are fats in buttermilk and how much fat we have left.

Recall equation (5) from MGARCH model:

(28) $$\boldsymbol{\varepsilon_t = H_t^{1/2}\eta_t}\,, \quad \boldsymbol{\eta_t \sim i.i.d\,(0, I_n)}$$

Where $\boldsymbol{\eta}_t$ are time-series random variables that are independent and identically distributed with a mean of **0** and variance of ***I***. This equation is heart of MGARCH models and claims that all relationship between two (or more) markets is modeled via equation (28). Actually this means that there is no relationship left between standardized residuals. Here I will use estimated $(\hat{\eta}_{1,t}, \hat{\eta}_{2,t})$ data to check if the MGARCH model's claim is correct or not. The actual values of $(\hat{\eta}_{1,t}, \hat{\eta}_{2,t})$ random variables are given by matrix multiplication of the inverse of the square root of variance-covariance matrix $(\mathbf{H}_t^{-1/2})$ and the MGARCH error terms as following equation:

$$(29) \qquad \boldsymbol{\eta_t = H_t^{-1/2} \varepsilon_t}, \quad \boldsymbol{\eta_t \sim i.i.d\ (0, I_n)}$$

Now the question is to identify dependency of estimated random variables of two markets $(\hat{\eta}_{1,t}, \hat{\eta}_{2,t})$. To answer this question we need more literature on dependency modeling. Also to avoid complexity of notation, assume that X=$\hat{\eta}_{1,t}$ is estimated standardized residuals of first market and Y =$\hat{\eta}_{2,t}$ is estimated standardized residuals of second market. From now, we are trying to study dependency of random variables of X and Y.

3.1. Criterion and Hypothesis Testing

We can establish a model selection criterion using dependency modeling theory to find out any dependency of standardized residuals of each MGARCH model. The null hypothesis is that **H** matrix captures all relationship and comovement of two markets. This can be translated as there is no relationship between standardized residuals. So we can write:

H_0: $\hat{\eta}_{1,t}, \hat{\eta}_{2,t}$ are stochastically independent

H_1: $\hat{\eta}_{1,t}, \hat{\eta}_{2,t}$ are stochastically dependent

To run this test two MGARCH model have been selected, each from different category of volatility modeling. Then the dependency of standardized residuals of BEKK GARCH model and an extension of CCC-GARCH model is estimated. To avoid complexity, bivariate models are considered for MGARCH models. As the real data is used, after estimating each model, the standardized residuals are estimated for each market data. To do so in RATS 8.0 the following codes are used to estimate each MGARCH model and standardized residuals of MGARCH model:

Standard BEKK RATS codes:

```
garch(p=1, q=1, mv=bekk, pmethod=simplex, piters=5, piters=20, rvectors=rd, hmatrices=hh) Oil Index
```

VAR-CCC-GARCH RATS codes:

```
system(model=var1)
variables Opec CI005022
lags 1
end(system)
garch(p=1, q=1, model=var1, mv=CC, variance=varma, pmethod=simplex, piters=5, piters=20,rvectors=rd, hmatrices=hh) Oil Index
```

This code prepares Standardized residuals

```
dec vect[series] zu(%nvar)
do time=%regstart(),%regend()
  compute %pt(zu,time,%solve(%decomp(hh(time)),rd(time)))
end do time
@mvqstat(lags=1)
#zu
```

4. Dependency Modeling

Pearson correlation coefficient is basic parameter that captures linear dependency of two random variables. Kendall (1938)'s tau rank correlation is another approach to find rank correlation between two random variable. Some scholars study some other non-linear statistics that can be applied to standardized residuals. These are namely, Alternating Conditional Expectations or back fitting algorithm, Kernel Canonical Correlation Analysis, (Copula) Hilbert-Schmidt Independence Criterion, Distance or Brownian Correlation and the Maximal Information Coefficient. (See Breiman et al. (1985), Francis et al. (2002), Gretton et al. (2005), Szekely et al. (2007) and Reshef (2011)). In this study, both linear dependency models and non linear dependency models are studied to check the correlation of fats in buttermilk.

4.1. Pearson Correlation Coefficient

The very famous correlation coefficient between two random variables is Pearson's correlation coefficient. This is calculated by:

(30) $$\rho_{X,Y} = \frac{Cov\ (X,Y)}{\sigma_X \sigma_Y}$$

The claim of buttermilk problem is that if there is significant linear correlation between X and Y. So the parameter used for linear correlation is:

$H_0: \rho_{X,Y} = 0$

$H_1: \rho_{X,Y} \neq 0$

Using t-student distribution for $\rho_{X,Y}$ with *n-2* degrees of freedom, the test statistic is given by Rahman (1968):

(31) $$t = \rho_{X,Y} \sqrt{\frac{n-2}{1-\rho^2{}_{X,Y}}}$$

The R command for preparing such a test is given as:

```
cor(x,y,method="pearson")

cor.test(x,y, method="pearson")$ statistic

cor.test(x,y,method="pearson",alternative = "two.sided")$ p.value
```

4.2. Kendall's Tau and Spearman's rho

Two rank correlation coefficient commonly used as a way of determining association between variables, are Kendall's tau and Spearman's rho. According to Nelsen (1991), Kendall's coefficient introduced by Fechner around 1900, and rediscovered by Kendall (1938). Kendall's tau is a measure of association between two quantities and measure of bivariate concordance (see Joe (2014)). Spearman's rho, named after Charles Spearman, is also another type of nonparametric measure of statistical dependence.

4.2.1. Test for association/correlation between paired samples

Myles Hollander & Douglas A. Wolfe (1973) prepared a test for Kendall's tau rank correlation. The null hypothesis of this test is:

H_0: [$H_{X,Y}(x, y) \equiv F_X(x)F_Y(y)$, for all (x, y) pairs].

They consider alternative hypothesis can be any function of dependence between the X and Y variables. They used Kendall population correlation coefficient as dependence measured by the:

$$\tau = 2\,P\{(Y_2 - Y_1)(X_2 - X_1) > 0\} - 1. \quad (32)$$

The R command cor.test will perform this test:

```
cor.test (x, y, method="kendall")
```

Similarly for Spearman's rho:

```
cor.test (x, y, method="spearman")
```

4.3. Copulas

Recently copulas have had extensive use in financial analysis due to ability to compute the joint distributions for several random variables. Copulas concept were introduced by Sklar (1959), who together with Schweizer (1974, 1983) developed some important aspects of theory of copulas.

Nelson (1999) gives this intuition about copulas. He referred to copulas as "functions that join or couple multivariate distribution functions to their one-dimensional marginal distribution functions" and as "distribution functions whose one-dimensional margins are uniform." He clearly states that these are not definition of copulas. Nelson (1999) explains more precisely, consider X and Y as random variables with distribution functions $F(x)= P[X\leq x]$ and $G(y)=P[Y\leq y]$, respectively, and a joint distribution function $H(x, y) = P[X\leq x , Y \leq y]$. To each pair of real numbers (x, y) we can associate three numbers: F (x), G (y), and H (x, y). Note that each of these numbers lies in the interval [0, 1]. In other words, each pair (x, y) of real numbers leads to a point (F(x), G (y)) in the unit square $[0, 1] \times [0, 1]$, and this ordered pair in turn corresponds to a number H(x, y) in [0, 1]. This correspondence, which assigns the value of the joint distribution function to each ordered pair of values of the individual distribution functions, is indeed a function, named copulas. A copula is a function C that links univariate marginal probabilities of two random variables to their multivariate distribution:

(33) $H_{X,Y}(x, y) = C(F_X(x), F_Y(y))$ where

(34) $C(u, v) = H(F^{-1}{}_X(u), F^{-1}{}_Y(v))$

The joint density function can be obtained from:

(35) $h_{X,Y}(x, y) = c(F_X(x), F_Y(y))\, f_X(x)\, f_Y(y),$

where

$$c(u, v)=\frac{\partial^2 C(u,v)}{\partial u \partial v}$$

In this study, only bivariate copulas are used to determine the joint cumulative distributions of standardized residuals.

4.3.1. Definition

According to Nelson (1999), a two-dimensional Copula is a 2-subcopula C whose domain is $\mathbf{I}^2$. Copula is a real function C: [0, 1] X [0, 1] → [0, 1] with the following properties:

- Groundedness property, for every u,v in $\mathbf{I}$:

$$C(0,v) = C(u,0) = 0$$

$$C(u,1) = u$$

$$C(1, v) = v ;$$

- 2-increasing property:

$$\forall\ u_1, u_2, v_1, v_2 \in \mathbf{I}, \text{ where } u_2 \geq u_1 \text{ and } v_2 \geq v_1 ,$$

$$C(u_2 , v_2)- C(u_1 , v_2)- C(u_2 , v_1)+C(u_1 , v_1) \geq 0 ,$$

Copulas can be considered as joint cumulative distribution functions that act as dependency functions with parameters showing to what degree or measurement level the variables are dependent on one another. This can possibly be done with an application of a well-known, powerful and useful Sklar's theorem (Sklar, 1959). The theorem gives a way of relating directly the joint cumulative distribution function with a special copula and inputs to copulas are the continuous univariate marginal distributions for the variables of interest.

4.3.2. Sklar's theorem

$H_{XY}(x, y) = C(F_X(x), F_Y(y))$ where $F_X(x)$ and $F_Y(y)$ are marginal distributions functions for X and Y respectively. If $F_X(x)$ and $F_Y(y)$ are absolutely continuous, then C is unique.

C: [0,1] X [0,1] → [0,1] and H(x,y) is the joint cumulative distribution function for the two variables.

4.3.3. Rank correlation

As for the case of conventional statistics, using copula as a statistical tool will require one to find means of determining rank correlation between variables. Fortunately we can obtain both coefficients by expressing them as functions of copulas. Schweizer and Wolff (1981) defined Spearman's rho and Kendall's tau rank correlation in terms of copula as:

(36) $$\rho_S = 12 \int_0^1 \int_0^1 \{ C(u,v) \} \, du \, dv - 3$$

(37) $$\rho_\tau = 4 \int_0^1 \int_0^1 C(u,v) \, dC(u,v) - 1$$

Where dC is doubly stochastic measure induced on I2 by C. Schweizer and Wolff (1981) established that for Archimedean copulas, Kendall's tau can be related to the dependence parameter

4.3.4. Student's t-copula

According to Peng et al. (2014) the Student's t-copula distribution is defined by:

(38)

$$C_{\rho,\vartheta} = \int_{-\infty}^{t_\vartheta^{-1}(u)} \int_{-\infty}^{t_\vartheta^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} \left(1 + \frac{x^2 - 2\rho xy + y^2}{\vartheta}\right)^{-\frac{\vartheta+2}{2}} dxdy$$

Where $\vartheta > 0$ is the number of degrees of freedom, $\rho \in [-1, 1]$ is the linear correlation coefficient, t_v is the distribution function of a t-distribution with v degrees of freedom and t_{-v} denotes the generalized inverse function of t_v. Student's t-copula is an elliptical copula that can also be used as an extreme value theorem copula.

The rank correlation coefficient, Kendall's tau (τ) is given by:

(39) $$\tau = \frac{2}{\pi} \arcsin(\rho)$$

For the Student's t-copulas, the relationship between the linear correlation coefficient and Spearman's rho is:

(40) $$\rho s = \frac{6}{\pi}\arcsin(\frac{\rho}{2})$$

To transfer the X and Y, student t distribution is used, for which R codes are as follows:

U <- sapply(1:2, function(y) pt(ETA1[, y], df =2353))

V <- sapply(1:2, function(y) pt(ETA2[, y], df =2353))

To estimate t-copula, “VineCopula” package of R is used:

```
f=1
cop01<-BiCopEst(U1[,1], U1[,2], family=2, method="mle",
se=FALSE, max.df=2352,weights=NA)
CT[f,1]<-cop01$par #so called rho parameter
CT[f,2]<-cop01$par2
tau<-2/pi*asin(cop01$par) #Kendell's tau rank correlation OR
this formula BiCopPar2Tau(family=2, par=cop01$par,cop01$par)
spe.rho<-6/pi*asin(cop01$par/2) #Spearman's rho correlation
CT[f,3]<-tau
CT[f,4]<-spe.rho

cop02<-BiCopEst(U2[,1], U2[,2], family=2, method="mle",
se=FALSE, max.df=2352,weights=NA)
CT[f,5]<-cop02$par #so called rho parameter
CT[f,6]<-cop02$par2
tau<-2/pi*asin(cop02$par) #Kendell's tau rank correlation OR
this formula BiCopPar2Tau(family=2, par=cop01$par,cop01$par)
spe.rho<-6/pi*asin(cop02$par/2) #Spearman's rho correlation
CT[f,7]<-tau
CT[f,8]<-spe.rho
```

4.3.5. Frank copula

Frank copula (1979) can be defined as:

(41) $$C(u,v) = -\theta^{-1} \ln\left\{1 + \frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right\}.$$

The dependency parameter may assume any real value $(-\infty, \infty)$.According to Cherubini et al. (2004), frank Kendall's tau and spearman's rho are as follows respectively:

(42) $$\tau = 1 + 4\frac{[D_1(\theta)-1]}{\theta}$$

(43) $$\rho_s = 1 - 12\frac{[D_2(-\theta)-D_1(-\theta)]}{\theta}$$

Where D_k is Debye function

(44) $$D_k(\theta) = \frac{k}{\theta^k}\int_0^\theta \frac{t^k}{e^t-1}dt \ , k = 1{,}2$$

$$and$$

(45) $$D_k(-\theta) = D_k(\theta) + \frac{k\theta}{k+1}$$

These are the R codes for calculating Debye function:

```
f1 <- function(x) x/(exp(x) - 1)

f2 <- function(x) x^2/(exp(x) - 1)

fu <- function(x) integrate(f1, lower = 0, upper = x)$value

fu2 <- function(x) integrate(f2, lower = 0, upper = x)$value

fl <- function(x) integrate (f1, lower = x, upper = 0)$value

fl2 <- function(x) integrate (f2, lower = x, upper = 0)$value

#Frank Capula
f=2
```

```
cop1<-BiCopEst(U1[,1], U1[,2], family=5, method="mle", se=FALSE,
max.df=2352,weights=NA)
CT[f,1]<-cop1$par
CT[f,2]<-cop1$par2
par=cop1$par; par2=cop1$par2
if (any(par > 0))
{    tau <- 1 - 4/par + 4/par^2 * sapply(par, fu)
     prh<- -1 -24/par^3 * sapply(par, fu2)+12/par^2* sapply(par,
fu)}
if (any(par < 0))
{tau <-  1 - 4/par - 4/par^2 * sapply(par, fl);
 prh <- -1 + 24/(par^3) * sapply (par, fl2)- 12/par^2*
sapply(par, fl)}

CT[f,3]<- tau   #BiCopPar2Tau(family=5, par=cop01$par,
par2=cop01$par2)
CT[f,4]<-prh

cop2<-BiCopEst(U2[,1], U2[,2], family=5, method="mle", se=FALSE,
max.df=2352,weights=NA)
CT[f,5]<-cop2$par
CT[f,6]<-cop2$par2
par=cop2$par; par2=cop2$par2
if (any(par > 0))
{    tau <- 1 - 4/par + 4/par^2 * sapply(par, fu)
     prh<- -1 -24/par^3 * sapply(par, fu2)+12/par^2* sapply(par,
fu)
}
if (any(par < 0))
{tau <-  1 - 4/par - 4/par^2 * sapply(par, fl);
```

```
 prh <- -1 + 24/(par^3) * sapply (par, fl2)- 12/par^2*
sapply(par, fl)}

CT[f,7]<- tau   #BiCopPar2Tau(family=5, par=cop01$par,
par2=cop01$par2)
CT[f,8]<-prh
```

4.4. Regression for dependence modeling and correlation

Another way of finding the linear relationship of two variables is running regression by considering one variable as dependent variable and the other independent variable. In this case, our hypothesis is that there is no correlation between standardized residuals of both markets in each model. In another words, the slope coefficient of regression model is zero.

The R code for running OLS regression is:

```
summary(lm(ETA[,1]~ETA[,2]))
summary(lm(ETA2[,1]~ETA2[,2]))
```

4.5. Distance Correlation and Covariance Statistics

Distance or Brownian correlation and covariance statistics (DCOR) is a measure of dependence between random vectors proposed by Szekely et al. (2007). It is similar to product-moment covariance and correlation, but it is zero only if the random vectors are independent. According to Szekely et al. (2007), the empirical distance dependence measures are based on certain Euclidean distances between sample elements rather than sample moments, yet have a compact representation analogous to the classical covariance and correlation. With following R code from energy package we can estimate DCOR:

```
DCOR(ETA1[,1], ETA1[,2], index = 1.0)
```

4.6. Randomized Dependence Coefficient

Lopez et al. (2013) developed Randomized Dependence Coefficient (RDC) as measure of non-linear dependency. Following R codes estimates RDC:

```
rdc <- function(x,y,k=20,s=1/6,f=sin) {
 x <- cbind(apply(as.matrix(x),2,function(u)rank(u)/length(u)),1)
 y <- cbind(apply(as.matrix(y),2,function(u)rank(u)/length(u)),1)
 x <- s/ncol(x)*x%*%matrix(rnorm(ncol(x)*k),ncol(x))
 y <- s/ncol(y)*y%*%matrix(rnorm(ncol(y)*k),ncol(y))
 cancor(cbind(f(x),1),cbind(f(y),1))$cor[1]}
```

4.7. Alternating Conditional Expectations

Alternating Conditional Expectations (ACE) or back fitting is a very flexible form of additive models, which first optimal transformations from data, and then runs a regression on transferred data. Breiman and Friedman (1985), proposed this non-linear regression technique to ease of finding relationship between variables. Hastie et al. (1986) proposed generalized additive model based on back fitting model of ACE and Generalized Linear Model. ACE tries to transfer both X and Y variables to maximize R^2 between transformed data. (See Faraway (2005) for more detail).

$$Y = \beta_{0+}\beta_1 f(X) + \in$$

"Acepack" library in R allows ACE estimation through following codes:

```
library("acepack")
x<-as.matrix(ETA[,1])
y<-as.matrix(ETA[,2])
a <- ace(y,x)
summary(lm(a$ty~a$tx))$coefficients
```

5. Data and Empirical Results

5.1. Crude oil

Crude oil has become one of the most important commodities over a considerable period of time. The trend and behavior of the crude oil trade has not only affected the Chinese economy but the economy of the whole globe as well. Since it has a huge impact and influences on various human activities, it is considered as one of the physical commodities that set the living standards for the people of China. Over the past few recent years, some macroeconomic variables, such as the exchange rate and the stock indices, have been observed to change their patterns of behavior in conjunction with the change in the value of crude oil, be it increasing or decreasing. This explains why crude oil in China has dynamically and actively been traded on both Chinese Stock exchange and exchange markets, such as those in Shanghai and Shenzhen.

For this study and to collect OPEC Oil prices, I used “QUANDL” package in R. Here are the codes for downloading oil prices in USD:

```
library(Quandl)

## Loading required package: xts
## Loading required package: zoo
##
## Attaching package: 'zoo'
##
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric


Quandl.auth("*****")

# downloading data
 OPECoil2 <- Quandl('OPEC/ORB', start_date = '2004-12-31', end_date =
'2014-10-17')
 write.table(OPECoil2, file = "OPECoil2.txt", sep = "\t")
 str(OPECoil2)

## 'data.frame':    2528 obs. of  2 variables:
##  $ Date : Date, format: "2014-10-17" "2014-10-16" ...
```

```
##  $ Value: num  83.2 81.2 81.9 85.1 85.9 ...
##  - attr(*, "freq")= chr "daily"
```

Then I used USD/RMB exchange rates to convert the oil prices from USD to RMB, Chinese local currency. Figure 1, presents the fluctuations of oil prices during last ten years.

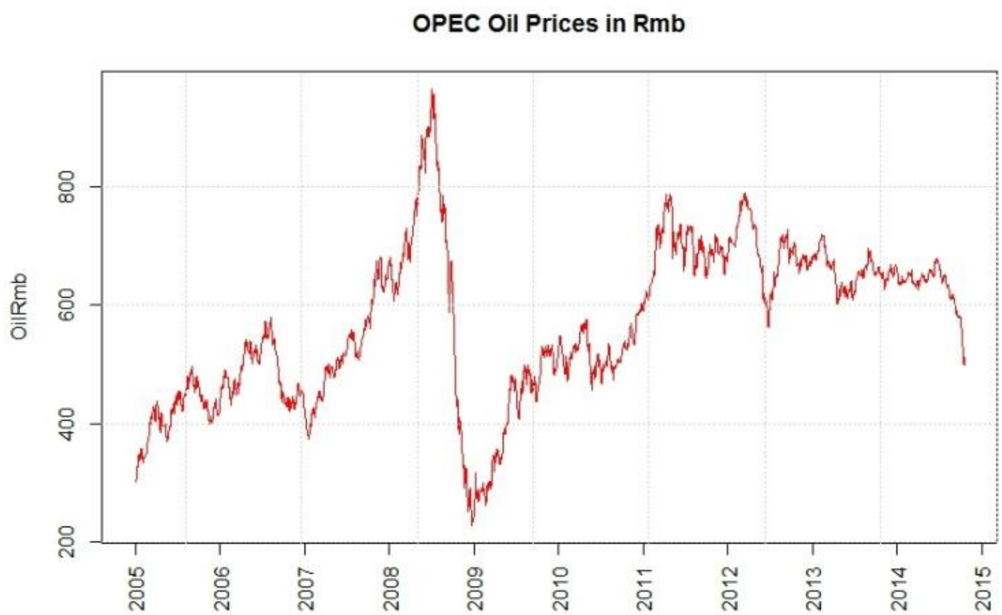

Figure 1-OPEC Oil prices in RMB (2004-2014)

5.2. CHINESE STOCK MARKETS

The Shanghai Stock Exchange is a stock exchange that is based in the city of Shanghai, China. It is one of the two stock exchanges operating independently in the People's Republic of China. Shanghai Stock Exchange is the world's sixth largest stock market. The Shenzhen Stock Exchange is one of China's three stock exchanges, alongside the Shanghai Stock Exchange and Hong Kong Stock Exchange.

The sample data used in this project was obtained from the Wind Data Center. The returns, for all sector indices organized and summarized in table 1, were captured for the time range starting on Dec 31, 2004 to October 17, 2014 on a daily basis. The daily close continuously compounded return data is used for analysis. To prepare descriptive analysis I used R commends as follow:

```
library(moments)
library(FinTS)

## Loading required package: zoo
##
## Attaching package: 'zoo'
##
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

library(rugarch)

## Loading required package: parallel

library(rmgarch)

data <- read.csv("source.csv", header=T)
data2 <- data[,-1]

z<-diff(log(ts(data2)))
heads <- read.csv("~/Industries Indexes/heads.csv", header=F)


dateA=data[,1]

x<-as.Date(dateA, "%m/%d/%Y")

j=1
    plot(x,ts(data2[j]),xlab="",
       ylab=heads[j,2],type="l", main=heads[j,1], col=2,xaxt = "n")

  axis(side=1, at = seq(as.Date("2005/1/1"), as.Date("2015/1/1"),
"years"),
       labels=seq(2005, 2015, by = 1), las=2)

  grid()
```

```
#descriptive table
Result<-matrix(nrow=3, ncol=10)

j=1
for (i in 1:3)
{
  Result[i,1]<-100*mean(z[,j])
```

```
  Result[i,2]<-sd(z[,j])
  Result[i,3]<-unname(skewness(z[,j]))
  Result[i,4]<- unname(kurtosis(z[,j]))
  Result[i,5]<-unname(jarque.test(as.vector(z[,j]))$ statistic)
  Result[i,6]<-unname(jarque.test(as.vector(z[,j]))$ p.value)
  Result[i,7]<-unname(ArchTest(z[,j])$statistic)
  Result[i,8]<-unname(ArchTest(z[,j])$p.value)
  Result[i,9]<-unname(Box.test(z[,j],lag=20,type="Ljung-
Box")$statistic)
  Result[i,10]<-cor(z[,1],z[,j],method="pearson")

  j=j+1

}

write.table(Result, file = "Descriptive-Table1.txt", sep = "\t")
```

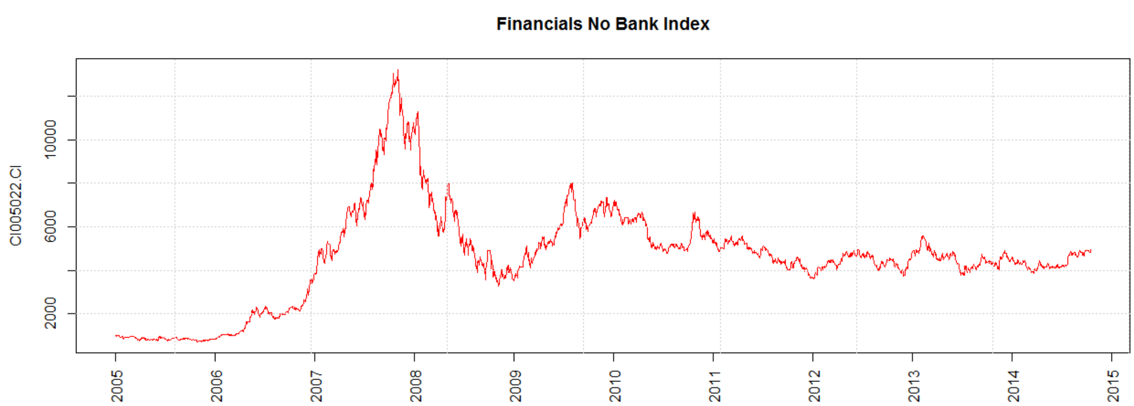

Figure 2- Financials No Bank Industry Stock Index (2004-2014)

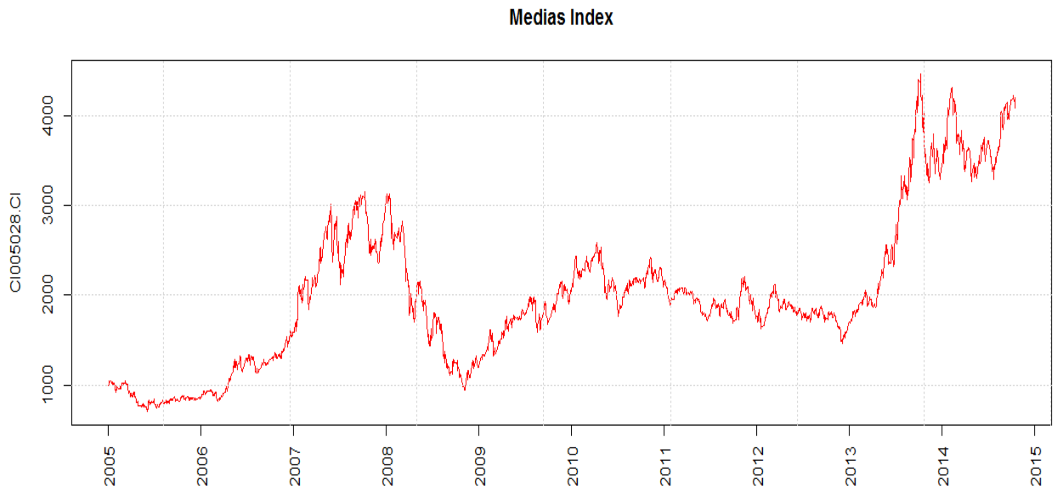

Figure 3- Media Industry Index (2004-2014)

Table 1 presents the descriptive statistics for market returns. This also gives the range of average returns for these sector indices which shows Financials No Bank index has highest return (0.068%). The unconditional standard deviation provides the measure for the volatilities log-return series ranging from 0.0165 (OPEC oil prices in Rmb) to 0.0260 (for Financials No Bank).

Skewness coefficients are seen to be negative for OPEC Oil and Media Industry Index the indices resulting that the return series are skewed to the right and tailed in left. But for Financials No Bank Index, skewness is positive indicating market return series of this index are skewed to the left and tailed to right. Kurtosis statistic estimate values are significantly greater than 3 showing leptokurtic property. These two estimates indicate asymmetric probability distributions for market returns which deviate from normal distribution. This is strongly supported by the Jarque-Bera test statistic results for the series. JB test statistic results show high values and in doing so, they clearly reject the null hypothesis of normality for all the log-returns.

Table 1–Descriptive Analysis

Index Name	OPEC Oil prices in Rmb	Financials No Bank Index	Medias Index
Index Code	Oil Rmb	CI005022.CI	CI005028.CI
Mean*100	0.0224	0.0679	0.0598
Standard Deviation	0.017	0.026	0.024
Skewness	-0.127	0.034	-0.514
Kurtosis	23.117	4.969	4.854
Jarque-Bera Test	39698.68***	380.592***	440.704***
JB (p-value)	0	0	0
Arch Test	130.82***	241.727***	172.619***
p-value	0	0	0
Ljung–Box test (Lag=20)	178.725***	31.878**	34.446**
p-value	0	0.044	0.0233
Pearson Correlation	1	0.134	0.077

The Ljung-box Q statistics are greater than the critical chi-square value indicating serial correlation for the time-series log-returns (in 5% level). The test for conditional heteroscedasticity with statistical significance suggests volatilities are conditional on previous returns; hence the need for the use of GARCH models for further analysis of data.

5.3. Standard BEKK Results

The estimated BEKK model coefficients for spillover effect of Oil prices on each index are given by table 2 and table 3. The mean coefficient of OPEC Oil returns in table2 is significant (in 5%level) indicating drift level in OPEC Oil return series. The results of BEKK model for Financials No Bank Industry Stock Index can be written in the matrix form:

$$\hat{C} = \begin{bmatrix} 0.0011 & 0 \\ 0.00016 & 0.001 \end{bmatrix}, \hat{A} = \begin{bmatrix} 0.245 & 0.069 \\ -0.026 & 0.142 \end{bmatrix}, \hat{B} = \begin{bmatrix} 0.968 & -0.013 \\ 0.003 & 0.988 \end{bmatrix}$$

To test covariance stationary condition for this index, recall formula (20) which can be estimated through following R codes, the K matrix is:

```
A<- matrix(   c(0.245,-0.026,0.069, 0.142),    nrow=2 ,   ncol=2)

B<- matrix(   c(0.968,0.003, -0.013, 0.988),    nrow=2 ,   ncol=2)

A%*%t(A)+B%*%t(B)

n<-kronecker (A,A)+kronecker (B,B)
```

	[,1]	[,2]	[,3]	[,4]
[1,]	0.997049	0.004321	0.004321	0.00493
[2,]	-0.00347	0.991174	-0.00183	-0.00305
[3,]	-0.00347	-0.00183	0.991174	-0.00305
[4,]	0.000685	-0.00073	-0.00073	0.996308

And by setting det (K – λI) = 0 the following Eigen values are obtained:

```
eign(n)

is.positive.definite(n)

eigen(n)$values

[0.9973, 0.9930, 0.9926+0.0033995i, 0.9926516-0.00339i]
```

The absolute value of Eigen values is less than one which fulfills the condition of covariance stationarity. Moreover, if the matrices are expressed as AA ′ and BB ′ then sum of parameter pairs is less than one.

All A matrix coefficients for Financials No Bank Industry Stock Index are significant indicating the effect of previous shocks on current variances and transmitting effect of previous shocks between oil market and Financials No Bank Industry Stock Index. B matrix coefficients, except $\hat{b}_{21}$, indicate the information transmission of previous volatilities between two those markets.

Table 2- BEKK Results for OPEC Oil and Financials No Bank Industry Index

	Variable	Coefficients	Std Error	T-Stat	p-value
1	Mean(OPEC)	0.00053	0.0003	2.43	0.014
2	Mean(CI005022)	0.00030	0.0005	0.62	0.532
3	C(1,1)	0.0011	0.0002	6.75	0
4	C(2,1)	0.00016	0.0005	0.35	0.728
5	C(2,2)	0.001	0.0004	3.42	0.001
6	A(1,1)	0.245	0.0191	12.82	0
7	A(1,2)	0.069	0.0249	2.78	0.005
8	A(2,1)	-0.026	0.0096	-2.69	0.007
9	A(2,2)	0.142	0.0155	9.15	0
10	B(1,1)	0.968	0.0047	205.36	0
11	B(1,2)	-0.013	0.0059	-2.27	0.023
12	B(2,1)	0.003	0.0023	1.43	0.154
13	B(2,2)	0.988	0.0027	363.41	0
Log Likelihood=12188.27, AIC=-10.349 m, HQ=-10.337(log) , FPE= -10.349					

Table 3- BEKK Results for OPEC Oil and Media Industry Index

	Variable	Coefficients	Std Error	T-Stat	p-value
1	Mean(OPEC)	0	0.0003	1.45	0.148
2	Mean(CI005028)	0	0.0004	0.52	0.605
3	C(1,1)	0.001	0.0002	6.64	0
4	C(2,1)	0.002	0.0004	6.36	0
5	C(2,2)	0	0.0051	-0.01	0.991
6	A(1,1)	0.21	0.0146	14.36	0
7	A(1,2)	0.028	0.0202	1.4	0.162
8	A(2,1)	-0.023	0.0065	-3.52	0
9	A(2,2)	0.209	0.0146	14.34	0
10	B(1,1)	0.962	0.0073	132.04	0
11	B(1,2)	0.416	0.0645	6.45	0
12	B(2,1)	0.066	0.0253	2.61	0.009
13	B(2,2)	-0.959	0.0081	-118.9	0

Log Likelihood=12386.84, AIC=-10.518 HQ=-10.506(log) FPE= -10.518

Then standardized residuals $\hat{\eta}_t$ of each market are estimated and histogram graph of them are shown in figure 4 and 5. Now the dependency of both standardized residuals is estimated through several dependency parameters to check if there is still some dependency left. The following R codes are used to graph the histograms:

```
par(mfrow=c(2,1))

hist (ETA[, 1], breaks=400, col="red", main = "Histogram of standardized residuals of Oil market- BEKK Model  ", xlab="" )

hist (ETA[, 2], breaks=400, col="Green", main = "Histogram of standardized residuals of Stock market Index- BEKK Model  ", xlab="")
```

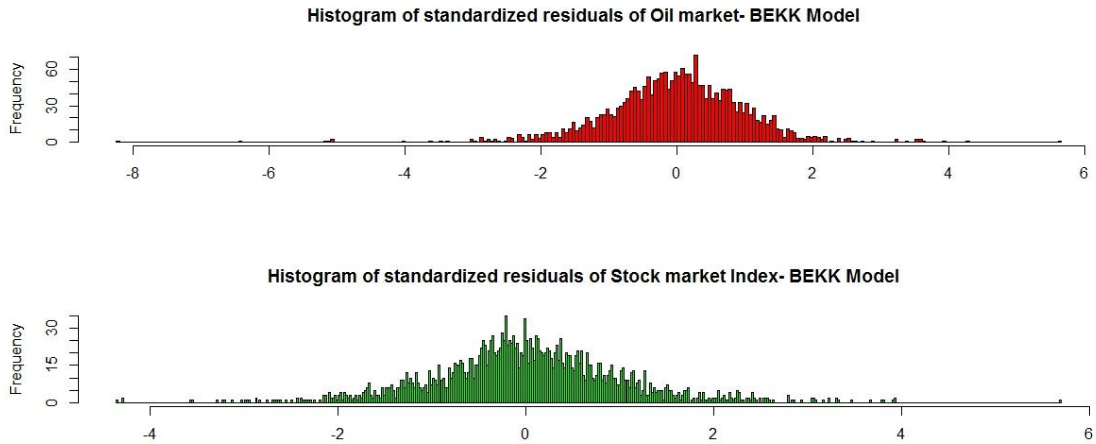

Figure 4 – Histogram of standardized residuals BEKK Model:

(OPEC Oil and Financials No Bank Industry Index)

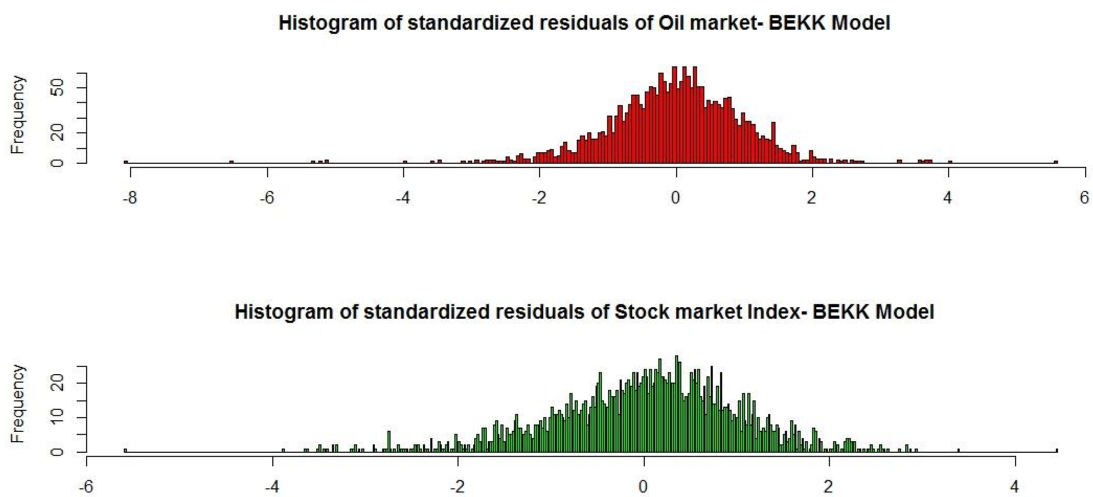

Figure 5- Histogram of standardized residuals of BEKK Model

(OPEC Oil and Media Industry Index)

5.4. VAR-CCC-GARCH Results

The results of applying VAR-CCC-GARCH Model is presented in table 4 and 5. The coefficients of mean model in Financials No Bank Industry Index equation are not significant as well as constant vector in variance estimation of both markets. It is interesting to find 13.17% significant constant correlation between OPEC Oil and Financials No Bank Industry Index markets as well as 8.62% significant constant correlation between OPEC Oil and Media Industry Index.

The results suggest that past shocks of the stock Index have significant effect on both markets' volatilities. This follows by the significance effect of previous volatilities of Industry Index on both markets.

Table 4- VAR-CCC-GARCH for OPEC Oil and Financials No Bank Industry Index

Variable	Coefficients	Std Error	T-Stat	p-value
Mean Model (OPEC)				
1. OPEC{1}	0.2356	0.0215	10.95412	0
2. CI005022{1}	-0.0302	0.01	-3.00989	0.0026134
Mean Model(CI005022)				
3. OPEC{1}	0.0352	0.032	1.09966	0.271482
4. CI005022{1}	-2.52E-04	0.0212	-0.01191	0.990494
5. C(1)	8.12E-07	4.69E-07	1.73021	0.083592
6. C(2)	2.96E-06	1.42E-06	2.08535	0.037038
7. A(1,1)	0.0554	8.54E-03	6.4833	0
8. A(1,2)	-0.00562	4.90E-03	-1.14854	0.250747
9. A(2,1)	-0.00431	0.0119	-0.36249	0.716982
10. A(2,2)	0.0316	5.99E-03	5.27395	1.3E-07
11. B(1,1)	0.9286	0.011	84.28923	0
12. B(1,2)	0.0704	0.0361	1.95238	0.050893
13. B(2,1)	0.00216	0.0632	0.03424	0.972682
14. B(2,2)	0.9639	9.09E-03	106.0655	0
15. R(2,1)	0.1317	0.02	6.57741	0
Log Likelihood= 12239.1089, AIC=-10.39, HQ=-10.35, (log) FPE= -10.39				

Table 5- VAR-CCC-GARCH for OPEC Oil and Media Industry Index

Variable	Coefficients	Std Error	T-Stat	p-value
Mean Model(OPEC)				
1. OPEC{1}	0.2264	0.0236	9.59149	0
2. CI005028{1}	-0.0121	9.61E-03	-1.25888	0.208073
Mean Model(CI005028)				
3. OPEC{1}	-0.0168	0.0257	-0.65422	0.512967
4. CI005028{1}	0.0274	0.0246	1.11712	0.263942
5. C(1)	1.31E-06	4.83E-07	2.71221	0.006684
6. C(2)	4.14E-06	1.63E-06	2.53472	0.011254
7. A(1,1)	0.0546	7.63E-03	7.16248	0
8. A(1,2)	-0.0197	4.80E-03	-4.10752	4E-05
9. A(2,1)	9.01E-03	0.0143	0.62844	0.529717
10. A(2,2)	0.051	7.39E-03	6.89753	0
11. B(1,1)	0.9452	8.50E-03	111.1612	0
12. B(1,2)	-8.11E-03	0.0341	-0.23816	0.811756
13. B(2,1)	0.0156	0.0816	0.19115	0.84841
14. B(2,2)	0.9405	9.25E-03	101.6583	0
15. R(2,1)	0.0862	0.0204	4.23057	2.33E-05
Log Likelihood=12457.76, AIC=-10.57, HQ=-10.56, (log) FPE= -10.57				

Now standardized residuals $\hat{\eta}_t$ of each market are estimated and graphed in figure 6 and 7 using following R codes:

```
par(mfrow=c(2,1))
hist (ETA2[, 1], breaks=400, col="red", main = "Histogram of standardized residuals of Oil market- VAR-CCC-GARCH Model  ", xlab="")
hist (ETA2[, 2], breaks=400, col="green", main = "Histogram of standardized residuals of Stock market Index- VAR-CCC-GARCH Model  ", xlab="")
```

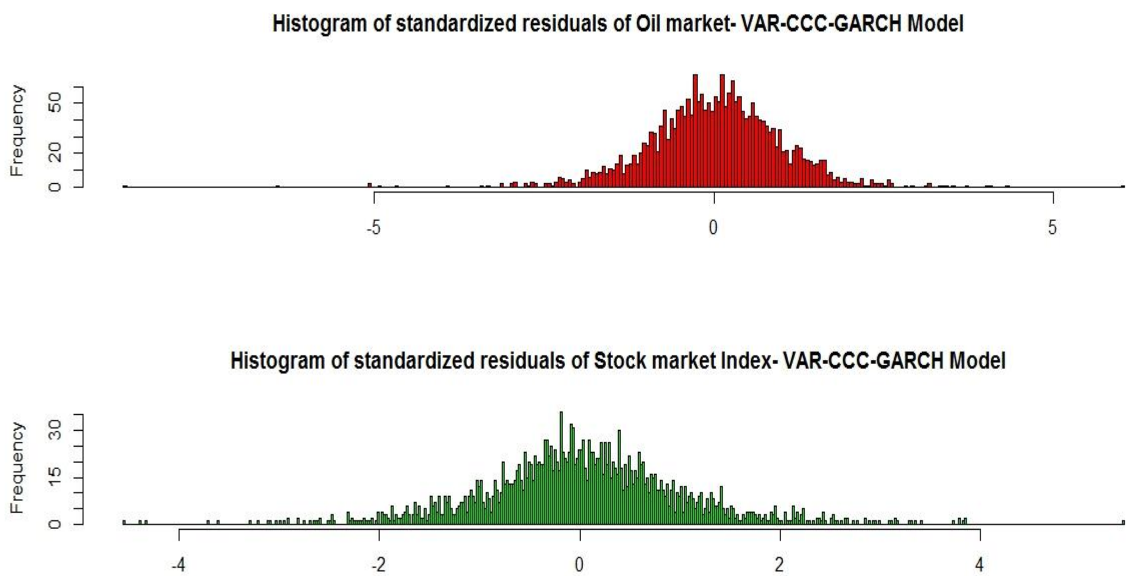

Figure 6- Histogram of standardized residuals VAR-CCC-GARCH Model

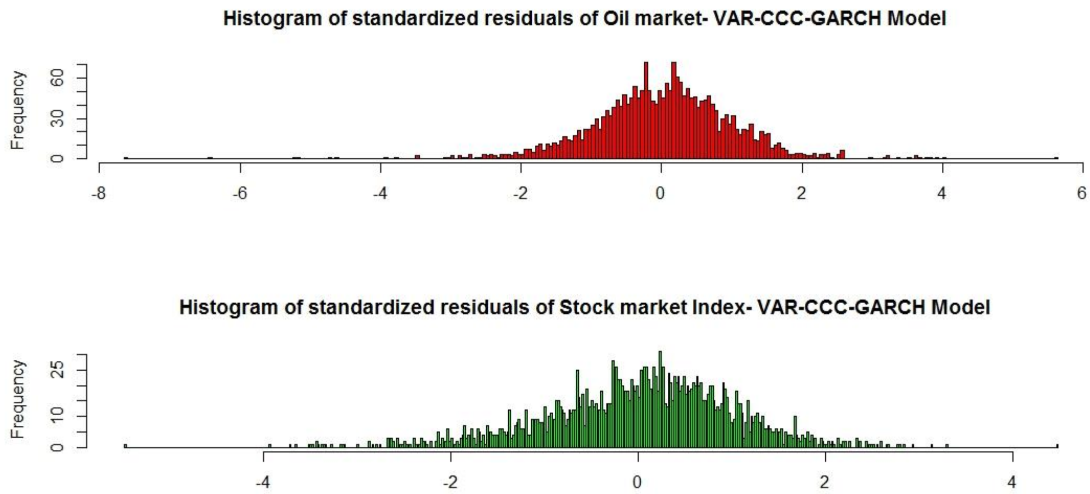

Figure 7- Histogram of standardized residuals of VAR-CCC-GARCH

5.5. Linear Dependency modeling estimation

The results of linear dependency estimation between standardized residuals of BEKK model in each market data series are presented in table 6 and 7 which indicate small (close to zero) linear dependency between standardized residuals of both MGARCH models. This means linear dependency modeling doest help to select between models.

Table 6- Linear Dependency estimation of standardized residuals

(OPEC Oil and Financials No Bank Industry Index)

GARCH Model	Dependency parameter	Correlation	Test statistics	P-value
BEKK Model	Pearson	0.0037	0.1817	0.8558
	kendall's tau	0.0028	0.2007	0.8409
	Spearman rho	0.0035	2163777410	0.8671
VAR GARCH Model	Pearson	-0.0002	-0.0082	0.9934
	kendall's tau	-0.0013	-0.0916	0.927
	spearman	-0.0018	2175241670	0.9294

Table 7- Linear Dependency Estimation of standardized residuals BEKK

(OPEC Oil and Financials No Bank Industry Index)

GARCH Model	Dependency parameter	Correlation	Test statistics	P-value
BEKK Model	Pearson	-0.0032	-0.1552	0.8767
	kendall's tau	-0.0087	-0.634	0.5261
	Spearman rho	-0.013	2.2E+09	0.5279
VAR GARCH Model	Pearson	-4.00E-04	-0.0204	0.9837
	kendall's tau	-0.0038	-0.2742	0.784
	spearman	-0.0057	2.18E+09	0.7809

5.6. Copula Estimation for dependency

As the linear dependency modeling was not helpful to find fats in buttermilk, the copula method is used to find any other dependency between two markets' standardized residuals. As mentioned before, Student t density distribution is used to estimate joint distribution of data. Afterwards, the parameters of two introduced copula, t-copula and frank copula are estimated. The results are provided in table (8) and table (9).

The Kendall's tau and Spearman's rho parameters derived from copula estimation of all markets in both MGARCH models are close to zero. This also suggests that the copula methodology is not indicating any dependency between standardized residuals.

Table 8- Copula estimation of standardized residuals of BEKK Model

(OPEC Oil and Financials No Bank Industry Index)

GARCH Model		Student-T Copula	Frank Copula
BEKK Model	Par1	0.006	0.041
	Par2	31.544	0
	Kedall's Tau	0.004	0.005
	Spearman's rho	0.006	-0.007
VAR GARCH Model	Par1	0.003	0.009
	Par2	38.914	0
	Kedall's Tau	0.002	0.001
	Spearman's rho	0.003	-0.002

Table 9- Copula estimation of standardized residuals of BEKK Model

(OPEC Oil and Financials No Bank Industry Index)

GARCH Model		Student-T Copula	Frank Copula
BEKK Model	Par1	-0.0029	-0.0591
	Par2	57.5247	0
	Kedall's Tau	-0.0018	-0.0066
	Spearman's rho	-0.0028	0.0098
VAR GARCH Model	Par1	0.001	-0.0065
	Par2	91.7742	0
	Kedall's Tau	6.00E-04	-7.00E-04
	Spearman's rho	9.00E-04	0.0011

5.7. Regression on standardized residuals

The results of running simple linear regression on standardized residuals of each model are presented in table (10) and table (11)-see the appendix A and B for more detail. The results indicating that there is no significant relationship between standardized residuals of both models. The linear regression results are not promising to select between models.

Table 10- Standardized residuals regression

(OPEC Oil and Financials No Bank Industry Index)

BEKK	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.02109	0.0205	-1.029	0.304
slope	0.0037	0.02036	0.182	0.856

VAR-CCC-GARCH	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.015685	0.020629	0.76	0.447
slope	-0.00059	0.020646	-0.029	0.977

Table 11- Standardized residuals regression

(OPEC Oil and Media Industry Index)

BEKK	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.01141	0.020503	-0.557	0.578
slope	-0.00316	0.020382	-0.155	0.877

VAR-CCC-GARCH	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.016353	0.020632	0.793	0.428
slope	-0.00042	0.020639	-0.02	0.984

5.8. Distance Correlation and Covariance Statistics

The estimation results of Distance Correlation and Covariance is presented in Table (12) and Table (13). Table (12) Indicates 3.75% distance correlation for BEKK model and 3.82% for that of VAR-CCC-GARCH model. Similarly, the difference of DCOR estimation of both models in table (13) is close to zero (4.53% against 4.32%).

Table 12- DCOR (OPEC Oil and Financials No Banks Industry Index)

	DCOR
BEKK	0.03750254
VAR-CCC-GARCH	0.03820621

Table 13- DCOR (OPEC Oil and Media Industry Index)

	DCOR
BEKK	0.04539
VAR-CCC-GARCH	0.043289

5.9. Randomized Dependence Coefficient

The results of running RDC is presented in table (14) and table (15). The RDC shows there is 8.31 % dependency in contrast to that of VAR-CCC-GARCH 9.51% between standardized residuals of OPEC Oil and Financials No Banks Industry Index MGARCH models. Similarly, for standardized residuals of OPEC Oil and Media Industry index: RDC estimation of BEKK, 8.31%, is slightly different than that of VAR-CCC-GARCH, 8.49%. This indicates deference between models.

Table 14- RDC estimation (OPEC Oil and Financials No Banks Industry Index)

	RDC
BEKK	0.08317179
VAR-CCC-GARCH	0.09512199

Table 15- RDC estimation (OPEC Oil and Media Industry Index)

	RDC
BEKK	0.086999
VAR-CCC-GARCH	0.089436

5.10. Alternating Conditional Expectations

The results of ACE algorithm are presented in table (16) and table (17). All results are providing significant slope coefficient for regression of transformed X and Y. this confirms there is still dependency between standardized residuals. In addition, the slope coefficient of BEKK model in table (16) is greater than that of VAR-CCC-GARCH model and in table (17) the slope coefficient of BEKK model is smaller than that of VAR-CCC-GARCH model. Now the question arises if the difference between these coefficients is statistically different than zero? If the answer is yes then we can say the resulting slopes are statistically different from each other, leading us to selection between models. Hopefully, the answering to this question is provided by t-test based on this formula:

$$t = \frac{\widehat{slope}_1 - \widehat{slope}_2}{\sqrt{\frac{Var(\widehat{slope}_1)}{n} + \frac{Var(\widehat{slope}_1)}{n}}}, df = 2n - 4$$

This in R can be provided by:

```
#estimating ACE
x<-as.matrix(ETA1[,1]) ; y<-as.matrix(ETA1[,2]); a <- ace(y,x)
fit<-lm(a$ty~a$tx); s1 <- summary(fit)$coefficients
x<-as.matrix(ETA2[,1]); y<-as.matrix(ETA2[,2]); a <- ace(y,x)
fit<-lm(a$ty~a$tx)fit<-lm(a$ty~a$tx) ;s2 <- summary(fit)$coefficients
#difference test
db <- (s2[2,1]-s1[2,1]); sd <- sqrt(s2[2,2]^2/2351+s1[2,2]^2/2351);df <- 2*2351-4
```

td <- db/sd; td; 2*pt(-abs(td), df)

Table 16- Standardized residuals ACE regression

(OPEC Oil and Financials No Banks Industry Index)

BEKK	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-1.78E-17	2.06E-02	0	1
slope	1.216	0.3158	3.851	0.000121***
VAR-CCC-GARCH				
(Intercept)	1.68E-18	2.06E-02	0	1
slope	1.155	0.272	4.246	2.26E-05***
Student t Test for difference of slopes			-7.10791	1.3e-12***

Table 17- Standardized residuals ACE regression

(OPEC Oil and Media Industry Index)

BEKK	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-4.71E-18	2.06E-02	0	1
slope	1.1930	0.3208	3.719	0.000205***
VAR-CCC-GARCH				
(Intercept)	3.06E-19	2.06E-02	0	1
slope	1.363	0.3291	4.141	3.58E-05***
Student t Test for difference of slopes			17.90849	1.967408e-69

It is obvious by student t test statistics of both data sets; the slope coefficient's difference is statistically different than zero. Now we can choose between MGARCH models. In table (16), the VAR-CCC-GARCH has smaller slope coefficient than BEKK, so here we choose the former model. In table (17), the BEKK has smaller slope coefficient than VAR-CCC-GARCH, so here we choose the former model which is BEKK.

5.11. Model Selection

Comparing the results shown in table (6) and table (7), provides that Kendal's tau estimated based on BEKK standardized residuals and the ones derived from VAR-CCC-GARCH model are almost close to zero. Also Pearson correlation coefficient confirms the same result. Although dependency parameters are close to zero, non significant coefficients indicates that these criteria are not able to model the dependency. Referring to table (8) and table (9), the dependency parameters derived from copula estimation are not promising, leading us to apply other models for dependency modeling of variables. The simple regression applied to data is not providing any promising result (see table 10 and table 11). Although applying Distance Correlation and Covariance (DCOR) and RDC methods are providing some indications, their results are slightly different from between models.

Finally, Alternating Conditional Expectations method is also tested on the models which provide significant dependency between transformed standardized residuals of models. Moreover, a student t statistics is checked for testing statistical difference of slope confidents of provided regression.

5.12. Consistency of criterion

After finding the proper criteria to choose between models, ACE, now the questions arise for consistency of the criterion. In other words, can we apply this criterion to other MGARCH models?

To answer this question, ACE regression has been applied to several MGARCH models and similar results are obtained (see table (18) and table (19)). These MGARCH models are mainly in context of BEKK and CCC-GARCH categories of MGARCH models just differ how to model mean equation (considering VAR or not) or variance (VARMA variance, E-GARCH, DCC and EWMA). (See appendix C for more detail of RATS codes for OPEC oil and Financials no Banks industry index). DCC model for OPEC oil and Media industry index didn't converge, so the results are not reported in table (19).

Interestingly, Table (18) indicates VAR1-BEKK model has smallest slope coefficient and table (19) indicates Standard BEKK model has smallest slope coefficient among other MGARCH models. This happens while all slope coefficients are significant, which confirms the consistency of criterion. To check that the coefficients are statistically different from each other or not, the student t test provided based on difference of slope of each MGARCH model and smallest slope in each table (Var1-BEKK for table (18) and BEKK for table (19)). The results of student t test are presented in table (20) and table (21). It is obvious from table (20) and table (19) that slope of selected MGARCH model is statistically different from that of other MGARCH models. This provides that we can distinguish which MGARCH model is preferable regarding to less relationship between standardized residuals.

Table 18- ACE (OPEC Oil and Financials No Banks Industry Index)

BEKK	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-1.78E-17	2.06E-02	0	1
slope	1.216	0.3158	3.851	0.000121***
VAR-CCC-GARCH				
(Intercept)	1.68E-18	2.06E-02	0	1
slope	1.155	0.272	4.246	2.26E-05***
CCC-GARCH				
(Intercept)	2.70E-18	2.05E-02	0	1
slope	1.1520	0.2525	4.564	5.27E-06***
DCC				
(Intercept)	-1.78E-18	2.05E-02	0	1
slope	1.1350	0.2607	4.354	1.40E-05***
CC with VARMA variances				
(Intercept)	1.27E-17	2.05E-02	0	1
slope	1.1510	0.2565	4.488	7.52E-06***
EWMA with t-errors				
(Intercept)	3.29E-17	2.05E-02	0	1
slope	1.1610	0.2487	4.667	3.23E-06***
CC-EGARCH with asymmetry				
(Intercept)	-4.18E-18	2.05E-02	0	1
slope	1.1590	0.2607	4.447	9.13E-06***
VAR(1)- BEKK				
(Intercept)	-2.11E-17	2.06E-02	0	1
slope	1.0820	0.3426	3.157	0.00161**

Table 19- ACE (OPEC Oil and Media Industry Index)

BEKK	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-4.71E-18	2.06E-02	0	1
slope	1.1930	0.3208	3.719	0.000205***
VAR-CCC-GARCH				
(Intercept)	3.06E-19	2.06E-02	0	1
slope	1.3630	0.3291	4.141	3.58E-05***
CCC-GARCH				
(Intercept)	-2.75E-18	2.06E-02	0	1
slope	1.2560	0.3364	3.735	0.000192***
DCC				
(Intercept)				
slope				
CC with VARMA variances				
(Intercept)	-1.63E-17	2.06E-02	0	1
slope	1.2760	0.3434	3.717	0.000206
EWMA with t-errors				
(Intercept)	-1.07E-17	2.05E-02	0	1
slope	1.2260	0.2543	4.819	1.53E-06
CC-EGARCH with asymmetry				
(Intercept)	1.21E-17	2.06E-02	0	1
slope	1.2870	0.3583	3.592	0.000335
VAR(1)- BEKK				
(Intercept)	1.71E-17	2.06E-02	0	1
Slope	1.2370	0.3287	3.763	0.000172

Table 20- ACE regression slopes Student t-test, comparing with VAR-BEKK

(OPEC Oil and Financials No Banks Industry Index)

	slopes	t value	Pr(>|t|)
VAR1-BEKK model	1.08171		
BEKK	1.216109	13.98548	1.41E-43***
VAR-CCC-GARCH	1.155011	8.124412	5.70E-16***
CCC-GARCH	1.152378	8.050975	1.03E-15***
DCC	1.13504	6.005923	2.05E-09***
CC with VARMA variances	1.15148	7.903542	3.35E-15***
EWMA with t-errors	1.160718	9.048349	2.08E-19***
CC-EGARCH with asymmetry	1.159457	8.755466	2.79E-18***

Table 21- ACE regression slopes Student t-test, comparing with BEKK

(OPEC Oil and Media Industry Index)

	slopes	t value	Pr(>|t|)
BEKK model	1.193117		
VAR-CCC-GARCH	1.362883	17.90849	1.97E-69***
CCC-GARCH	1.256347	6.594954	4.72E-11***
DCC			
CC with VARMA variances	1.276393	8.592268	1.15E-17***
EWMA with t-errors	1.225764	3.866147	0.000112***
CC-EGARCH with asymmetry	1.286707	9.435698	5.95E-21***
VAR(1)- BEKK	1.237171	4.649944	3.41E-06***

6. Conclusion

In this research a new model selection criterion is introduced to be able select among MGARCH models and find out which model is preferable due to better modeling dependency of markets. To do so, first the buttermilk problem is identified in literature of MGARCH models. Then several dependency models are introduced to identify the dependency of standardized residuals of each model.

Among several MGARCH models, first two famous ones in commodity markets named Standard BEKK and VAR –CCC-GARCH models are chose and applied to OPEC oil and Financials No Bank Industry Index daily return prices of Chinese stock markets. Afterwards, standardized residuals of each market of each model are used to estimate several dependency models such as Pearson correlation coefficient, copulas, Kendal's tau, spearman rho, simple regression and kernel density transfer regression.

In applying copula method, probability distribution functions from i.i.d standardized residuals, approximated with the t-distribution as long as the dataset is large enough. These estimated PDF's were used as inputs to the bivariate copula functions to determine the estimation of both copula dependence parameters by applying copula maximum likelihood estimation and the joint distribution function for any preferred combination of two return series. Once the estimates of dependence parameters are obtained, the spearman's rho and Kendall's tau directly were estimated as well to show rank correlation between the two standardized residuals series in consideration. In addition, running a regression on the standardized residuals of each MGARCH model is also considered. Besides RDC and Distance Correlation and covariance (DCOR) estimates for dependency are also estimated. None presented very clear difference between variables.

The only criterion which worked very well with MGARCH model's standardized residuals is ACE method. The ACE regression gives significant slope coefficient for transformed standardized residuals. This can help to select among models. To check consistency of ACE, I have tried several other MGARCH models for the data in hand

and then tried ACE regression on remaining of buttermilks. The results were similar and confirming consistency of the ACE regression for selecting among MGARCH models.

It is interesting that for the applied data sets we can prefer BEKK to CCC-GARCH category of GARCH models as the correlation between the standardized residuals of later model is obviously greater than the former model.

For future studies, scholars can apply this methodology to distinguish among several MGARCH models and to study which one under which condition can capture better co-movements of two markets. Moreover, scholars are encouraged to introduce unique dependency criteria for checking the fat density in buttermilk. Even, trying to establish a new MGARCH model which captures the whole dependency of markets would be considered as from now there is a way to check which model is doing well.

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Appendix A- Regression on standardized residuals OPEC Oil- Media Industry Index (CI005028)

BEKK Standardized residuals regression

```
Call:
lm(formula = ETA[, 1] ~ ETA[, 2])

Residuals:
    Min      1Q  Median      3Q     Max 
-7.9247 -0.5651  0.0316  0.6199  5.6527 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.011411   0.020503  -0.557    0.578
ETA[, 2]    -0.003164   0.020382  -0.155    0.877

Residual standard error: 0.9945 on 2351 degrees of freedom
Multiple R-squared:  1.025e-05,     Adjusted R-squared:  -0.0004151 
F-statistic: 0.0241 on 1 and 2351 DF,  p-value: 0.8767
```

VAR-CCC-GARCH Standardized residuals regression

```
Call:
lm(formula = ETA2[, 1] ~ ETA2[, 2])

Residuals:
    Min      1Q  Median      3Q     Max 
-7.6204 -0.5703  0.0316  0.6060  5.6246 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.0163531  0.0206318   0.793    0.428
ETA2[, 2]   -0.0004187  0.0206385  -0.020    0.984

Residual standard error: 1.001 on 2351 degrees of freedom
Multiple R-squared:  1.751e-07, Adjusted R-squared:  -0.0004252 
F-statistic: 0.0004116 on 1 and 2351 DF,  p-value: 0.9838
```

Appendix B- Regression on standardized residuals OPEC Oil- Financials No Bank Industry Index (CI005022)

```
BEKK Standardized residuals regression


Call:
lm(formula = ETA[, 1] ~ ETA[, 2])

Residuals:
    Min      1Q  Median      3Q     Max 
-8.2228 -0.5653  0.0327  0.6147  5.6622 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.02109    0.02050  -1.029    0.304
ETA[, 2]     0.00370    0.02036   0.182    0.856

Residual standard error: 0.9945 on 2351 degrees of freedom
Multiple R-squared:  1.405e-05,       Adjusted R-squared:  -0.0004113 
F-statistic: 0.03303 on 1 and 2351 DF,  p-value: 0.8558


VAR-CCC-GARCH Standardized residuals regression

Call:
lm(formula = ETA2[, 1] ~ ETA2[, 2])

Residuals:
    Min      1Q  Median      3Q     Max 
-8.5873 -0.5725  0.0296  0.5777  5.6326 

Coefficients:
            Estimate Std.Error tvalue Pr(>|t|)
(Intercept)  0.01333    0.02047   0.651    0.515
ETA2[,2]    0.13148    0.02046   6.428 1.57e-10 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.9924 on 2351 degrees of freedom
Multiple R-squared:  0.01727, Adjusted R-squared:  0.01685 
F-statistic: 41.31 on 1 and 2351 DF,  p-value: 1.565e-10
```

Appendix C- MGARCH models RATS codes and results

Standard BEKK - ETA1

```
garch(p=1,q=1,mv=BEKK,pmethod=simplex,piters=20,rvectors=rd,hmatrices=hh      )
gstart gend Opec CI005022
```

McAleer VAR-CCC-GARCH- ETA2

```
open data LogReturn.xls

data(format=xls,org=columns) / Opec CI005022

compute gstart=2,gend=2354

system(model=var1)

variables Opec CI005022

lags 1

end(system)

garch(p=1,q=1,model=var1,mv=CC,variance=varma,pmethod=simplex,piters=20,rve
ctors=rd,hmatrices=hh, MVHSERIES=VarmaHmatrix ) gstart gend Opec CI005022
```

Restricted correlation models- ETA3

garch(p=1,q=1,mv=CC,pmethod=simplex,piters=20,rvectors=rd,hmatrices=hh)
gstart gend Opec CI005022

MV-GARCH, CC - Estimation by BFGS

Convergence in 27 Iterations. Final criterion was 0.0000000 <= 0.0000100

Log Likelihood 12178.1975

Variable	Coefficients	Std	Error	T-Stat	P. Value
1	Mean(1)	4.43E-04	2.52E-04	1.76213	0.078047
2	Mean(2)	6.18E-04	4.50E-04	1.37227	0.169979
3	C(1)	1.42E-06	3.99E-07	3.55564	0.000377
4	C(2)	3.00E-06	1.41E-06	2.11903	0.034088
5	A(1)	0.05348	0.00822	6.50936	0
6	A(2)	0.03251	0.00584	5.56882	3E-08
7	B(1)	0.94254	0.00804	117.2229	0
8	B(2)	0.96277	0.0069	139.503	0
9	R(2,1)	0.12831	0.02093	6.13145	0

DCC –ETA4

garch(p=1,q=1,mv=DCC,pmethod=simplex,piters=20,rvectors=rd,hmatrices=hh)

gstart gend Opec CI005022

MV-GARCH, DCC - Estimation by BFGS

Convergence in 30 Iterations. Final criterion was 0.0000000 <= 0.0000100

Log Likelihood 12185.9191

Variable	Coefficients	Std	Error	T-Stat	P. Value
1	Mean(1)	4.83E-04	2.43E-04	1.98998	0.046593
2	Mean(2)	6.14E-04	4.68E-04	1.313	0.189182
3	C(1)	1.43E-06	4.13E-07	3.46424	0.000532
4	C(2)	3.02E-06	1.62E-06	1.86865	0.061671
5	A(1)	0.054	0.00741	7.29038	0
6	A(2)	0.0334	0.00599	5.58011	2E-08
7	B(1)	0.94206	0.00742	126.8993	0
8	B(2)	0.96201	0.00741	129.8404	0
9	DCC(1)	0.09449	0.03032	3.11597	0.001833
10	DCC(2)	0.38697	0.26628	1.45324	0.146157

CC with VARMA variances-ETA5

garch(p=1,q=1,mv=CC,variance=varma,pmethod=simplex,piters=20,rvectors=rd,hmatrices=hh) gstart gend Opec CI005022

MV-GARCH, CC with VARMA Variances - Estimation by BFGS

Convergence in 52 Iterations. Final criterion was 0.0000000 <= 0.0000100

Log Likelihood 12182.1006

Variable	Coefficients	Std	Error	T-Stat	P. Value
1	Mean(1)	4.52E-04	2.53E-04	1.78367	0.074478
2	Mean(2)	5.54E-04	4.61E-04	1.20206	0.229339
3	C(1)	7.65E-07	4.70E-07	1.6285	0.10342
4	C(2)	3.03E-06	1.37E-06	2.21185	0.026977
5	A(1,1)	0.0533	9.00E-03	5.91853	0
6	A(1,2)	-6.85E-03	5.32E-03	-1.28898	0.197406
7	A(2,1)	-4.50E-03	9.82E-03	-0.45834	0.64671
8	A(2,2)	0.0323	5.89E-03	5.47439	4E-08
9	B(1,1)	0.9333	0.0117	79.76803	0
10	B(1,2)	0.0624	0.0363	1.72068	0.08531
11	B(2,1)	8.98E-03	0.0531	0.1691	0.865717
12	B(2,2)	0.9626	8.38E-03	114.8811	0
13	R(2,1)	0.1323	0.0193	6.84601	0

EWMA with t-errors with an estimated degrees of freedom parameter-ETA6

garch(p=1,q=1,mv=ewma,distrib=t,pmethod=simplex,piters=20,rvectors=rd,hmatrices=hh) gstart gend Opec CI005022

MV-GARCH, EWMA - Estimation by BFGS

Convergence in 2 Iterations. Final criterion was 0.0000018 <= 0.0000100

Log Likelihood 12344.8717

Variable	Coefficients	Std	Error	T-Stat	P. Value
1	Mean(1)	0.000460327	0.000188394	2.44343	0.01454853
2	Mean(2)	-0.000012799	0.000392835	-0.03258	0.97400777
3	Alpha	0.026143736	0.002351885	11.11608	0
4	Shape	6.351573087	0.410603234	15.46888	0

CC-EGARCH with asymmetry- ETA7

garch(p=1,q=1,mv=cc,asymmetric,variances=exp,pmethod=simplex,piters=20,rvectors=rd,hmatrices=hh) gstart gend Opec CI005022

MV-GARCH, CC with E-GARCH Variances - Estimation by BFGS

Convergence in 50 Iterations. Final criterion was 0.0000000 <= 0.0000100

Log Likelihood 12181.6903

Variable	Coefficients	Std	Error	T-Stat	P. Value
1	Mean(1)	0.00046412	0.00023834	1.94732	0.05149659
2	Mean(2)	0.00054376	0.00047395	1.14731	0.25125384
3	C(1)	-0.20894082	0.03035163	-6.88401	0
4	C(2)	-0.10002165	0.03247918	-3.07956	0.00207305
5	A(1)	0.11163861	0.01526422	7.31375	0
6	A(2)	0.08020922	0.01301611	6.1623	0
7	B(1)	0.98584806	0.00295665	333.43394	0
8	B(2)	0.99453264	0.00381372	260.77748	0
9	D(1)	33.04458308	8.89394401	3.7154	0.00020288
10	D(2)	-2.47922141	7.39270213	-0.33536	0.73735305
11	R(2,1)	0.12724105	0.02008214	6.33603	0

VAR(1) model for the mean, BEKK – ETA8

garch(p=1,q=1,model=var1,mv=bekk,pmethod=simplex,piters=20,rvectors=rd,hmatrices=hh) gstart gend Opec CI005022

MV-GARCH, BEKK - Estimation by BFGS

Convergence in 40 Iterations. Final criterion was 0.0000057 <= 0.0000100

Log Likelihood 12245.0620

Variable	Coefficients	Std	Error	T-Stat	P. Value
1	OPEC{1}	0.234143	0.020924	11.19038	0
2	CI005022{1}	-0.03103	0.010641	-2.91639	0.003541
3	OPEC{1}	0.041489	0.029298	1.41611	0.156743
4	CI005022{1}	0.00403	0.021377	0.18853	0.850461
5	C(1,1)	0.001187	0.000167	7.10399	0
6	C(2,1)	0.000213	0.000406	0.52467	0.599814
7	C(2,2)	0.001255	0.000352	3.56122	0.000369
8	A(1,1)	0.243448	0.016301	14.93486	0
9	A(1,2)	0.07225	0.025092	2.87935	0.003985
10	A(2,1)	-0.02081	0.008652	-2.40489	0.016177
11	A(2,2)	0.140079	0.015157	9.24192	0
12	B(1,1)	0.96831	0.003953	244.9857	0
13	B(1,2)	-0.01287	0.005513	-2.33393	0.019599
14	B(2,1)	0.002026	0.002029	0.99839	0.318091
15	B(2,2)	0.988123	0.002558	386.2403	0