PERIODIC VEHICLE ROUTING PROBLEM WITH TWO TYPES OF VISITS

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AUGUST 2015

PERIODIC VEHICLE ROUTING PROBLEM WITH TWO TYPES OF VISITS

A THESIS SUBMITTED TO

THE GRADUATE SCHOOL OF

Social Sciences

IZMIR UNIVERSITY OF ECONOMICS

BY

OKAN ALTINKÖK

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

IN THE GRADUATE SCHOOL OF Social Sciences

AUGUST 2015

M.S. THESIS EXAMINATION RESULT FORM

Approval of the Graduate School of Social Sciences

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ABSTRACT

PERIODIC VEHICLE ROUTING PROBLEM WITH TWO TYPES OF VISITS

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Vehicle routing problem is a contemporary topic and it has been studied since 1950's. In last decades, the importance of supply chain management and distribution channels has increased for today's competitive environment. Vehicle routing problem (VRP) is the problem in which vehicles departure from one or more depot(s), visit customers and finally return to the depot(s). Generally, VRP aims to minimize the total distance of the vehicles' routes and therefore decrease transportation costs. Periodic vehicle routing problem (PVRP) is a variant of the classical vehicle routing problem. PVRP is based on visiting customers once or more during a planning horizon with multiple days. The problem decides the customers to be visited and the routes for each day. In general, the goal is to minimize the total length of the routes. Organizing the visit days properly provides an important advantage to decrease delivery costs and the number of vehicles. PVRP with two types of visits (PVRP2TV) involves two types of visits that must be made on consecutive days in PVRP setting. The first type visit to the customer is for collecting demand information and increasing the visibility of the products at the store, the second type of visit is for delivering the goods. The first type visits take place with small and fast vehicles with time capacity constraints. On the other hand, the second type visits realized with relatively large scale, slow and physical capacitated vehicles. Hence using the same routes for the consecutive days might not be feasible. In this thesis, We develop a mathematical programming model and a special lower bound algorithm We also propose a heuristic algorithm based on variable neighborhood search (VNS) and conduct computational tests on the modified versions of the widely used test instances. This thesis is funded by The Scientific And Technological Research Council of Turkey (TÜBİTAK) as TÜBİTAK 1001 research and development program with grant number 213M425.

Keywords: Vehicle routing problem, periodic vehicle routing problem, mathematical programming, lower bound algorithm heuristic methods, VNS.

ÖZET

PERİYODİK ARAÇ ROTALAMA PROBLEMİ İKİ ZİYARET TİPİ OLAN

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Araç rotalama problemi 1950'li yıllardan beri çalışılan modern bir konudur. Son yıllarda, günümüzün rekabetçi ortamında, tedarik zinciri yönetimi ve dağıtım kanallarının önemi ciddi bir biçimde artmıştır. Araç rotalama problemi, bir aracın bir veya daha fazla depodan ayrılıp tüm müşterileri ziyaret ederek tekrar depo veya depolara dönüş yapmasını amaçlayan problemdir. Genellikle, araç rotalama probleminin amacı toplam kat edilen mesafeyi en azlayarak taşıma maliyetini düşürmektir. Periyodik araç rotalama problemi, klasik araç rotalama probleminin bir varyantıdır. Periyodik araç rotalama problemi, planlama periyodu içerisinde bulunan birden fazla günde müşterileri ziyaret sıklıklarına göre bir veya birden fazla ziyaret edilmesi üzerine kurulu bir problemdir. Problem her bir gün için ziyaret edilecek müşterilere ve bu müşterilerin ziyaret edilmesi için en uygun rotaya karar verir. Genellikle, ana amaç müşteri ziyaretleri için belirlenen rotaları en küçüklemektir. Müşterilerin ziyaret günlerini organize etmek, taşıma maliyetini ve araç sayısını düşürmek için önemli bir avantaj sağlar. Periyodik araç rotalama problemi 2 farklı ziyaret tipi, birbirini takip eden günlerde ziyaret edilmesi zorunlu olan iki farklı amaçlı ziyaret içermektedir. İlk ziyaret, talep bilgisini edinmek ve ürünlerin mağazadaki görünürlüğünü arttırmak üzere yapılan çalışmaları yapabilmek amaçlarıyla yapılmaktadır. Bu ziyaretler, görece küçük, hızlı araçlarla yapılmaktadır ve zaman kapasitesine sahiptir. Diğer taraftan, ikinci tip araçlar görece büyük, yavaş ve fiziksel kapasiteye sahiptir. Bu araçların ziyaret amacı ise ürünlerin müşteriye teslim edilmesidir. Daha önce de belirtilmiş olduğu gibi eğer bir müşteri herhangi bir küçük araç ile ziyaret edilmiş ise takip eden günde büyük araç ile ziyaret edilmeli ve ürünler müşteriye teslim edilmelidir. Bu sebeple, kapasite ve özellikleri farklı bu araçlarla birbirini takip eden günlerde aynı rotaları kullanarak ziyaretlerin gerçekleştirilmesi mümkün olmayabilir. Bu çalışmada, yeni bir alt sınır algoritması ve sezgisel yöntem algoritması geliştirilmiştir. Literatürde sıkça kullanılan örnek problemler üzerinde geliştirilen yöntemler test edilmiş ve sonuçları gösterilmiştir. Bu araştırma, Türkiye Bilimsel ve Teknolojik Araştırma Kurumu (TÜBİTAK) tarafından 1001 araştırma geliştirme programı kapsamında, 213M425 numaralı proje olarak desteklenmektedir.

Anahtar Kelimeler: Araç rotalama problemi, periyodik araç rotalama problemi, matematiksel programlama, alt sınır algoritması, sezgisel yöntem, DKA.

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Periodic Vehicle Routing Problem with Two Types of Visits

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August,31 2015

Chapter 1

Introduction

Vehicle routing problem (VRP) is a contemporary topic and it has been studied since 1950's. In early times, VRP has not been studied widely because it's a complex problem and the level of technology was not sufficient enough. In 1990's, with developing technology the number of studies on vehicle routing problem began to increase.

In VRP, the vehicles depart from one or multiple depots, visit customers and finally return to the depot(s). VRP was firstly introduced by Dantzig and Ramser (1959). Generally, VRP aims to minimize the total distance of the vehicles' routes so as to decrease transportation costs. One may also aim to minimize the number of vehicles in VRP. Moreover, VRP protects the environment by decreasing the carbon emission. In last decades, the importance of supply chain management and distribution channels has increased for today's competitive environment. In consequence, firms started to give more importance to VRP

There are many variants of VRP derived in order to cope with the requirements of the real life applications. The reason behind this is the operational differences of the companies. The necessity of some customers to be visited in certain time windows, the joint pickup and delivery operations of cargo firms, the requirement of visiting some customer with the same vehicle and with predefined order precedence relations between customers are some of the reasons for defining and studying different variants of the problem. Some of these real life appications in the literature are; vehicle routing problem with time windows (Solomon, 1987), heterogeneous fleet vehicle routing problem (Gendreau et al., 1999), vehicle routing problem with bachauls (Toth and Vigo, 1999), vehicle routing problem with pickup and delivery (Dumas et al., 1991), Multi-depot vehicle routing problems (Vidal et al., 2012), periodic vehicle routing problem (Francis and Smilowitz, 2006), vehicle routing problem with dynamic demand (Wen et al., 2010).

Periodic vehicle routing problem (PVRP) is a derivative of vehicle routing problem which is well studied in the literature. This problem is based on visiting customers once or more in a given period. The problem decides the customers to be visited and the routes for each period. In general, the goal is to minimize to total length of the routes. Organizing the visit days properly provides an important advantage to decreasing costs and the number of vehicles. Beltrami and Bodin (1974) introduced PVRP under the context of municipal waste collection.

In this thesis, we introduce a new extension of PVRP; periodic vehicle routing problem with two types of visits PVRP2TV motivated from a real life application of a soft drink distribution company. Proposed problem requires different aimed visits to customers in consecutive days. PVRP2TV requires a different aimed customer visit in the following day of a regular customer visit day. One can consider the first customer visit for collecting demand information is realized with fast and time capacitated vehicles and the second customer visit for delivering goods is realized with slow and physical capacitated vehicles. Hence using the same routes for the consecutive days might not be possible. To the best of our knowledge, the problem which is studied, has not been studied in the literature before. Hence the problem that is carried out is unique.

A detailed literature review about VRP, PVRP, exact and heuristics solution methods is provided in chapter 2. In chapter 3, basic mathematical models and general notation of VRP and PVRP is shown. In chapter 4, PVRP2TV is explained in detail. Also, proposed mathematical model and computational complexity are provided. Lowerbound algorithm and VNS algorithm are proposed in chapter 5. In chapter 6, test instances and computational results are provided.

Chapter 2

Literature Review

In this chapter, a detailed literature review is provided. In section 2.1 VRP, in next section 2.2 PVRP are examined in detail. In section 2.2.1 exact methods are mentioned. In section 2.2.2 heuristic methods, in section 2.2.3 classical heuristics are referred. Meta heuristics are discussed in section 2.2.4, specifically tabu search, iterative local search and variable neighborhood search.

2.1 Vehicle Routing Problem

Vehicle routing problem was firstly introduced by Dantzig and Ramser (1959). The first solution which contains more than one vehicle is proposed by Clarke and Wright (1964). Traveling salesman problem, which is widely studied in the literature, is a special case of VRP. Laporte (2009) summarize the fifty years history of TSP and recent developments in this field. According to real life requirements, many extensions of VRP are formed. Some of these extensions are VRP with time windows by Solomon (1987), heterogeneous fleet (Gendreau et al., 1999), VRP with backhauls (Toth and Vigo, 1999), pickup and delivery (Dumas et al., 1991), multi-depot VRP (Vidal et al., 2012). In addition, in some articles the main objective of the problem is considered as minimizing the number of vehicles. (Eksioglu et al., 2009) proposed a classification approach for vehicle routing problems.

Cordeau et al. (2002) focuses on VRP extensions and give several examples of its extensions. They state that since VRP is NP-hard problem, only smaller instances can be optimally solved. At the time of that study, we can saw that no exact algorithm can solve instances which has more than 50 customers. They explained the reason that heuristics were started to commonly used for real life practices or for big instances.

2.2 Periodic Vehicle Routing Problem

Periodic vehicle routing problem was defined by Russell and Igo (1979). Several real-life problems such as waste collection and Recycling (Coene et al., 2010) food and drink distribution (Golden and Wasil, 1987) and spare part distribution (Alegre et al., 2007) are modelled as periodic vehicle routing problem. Mourgaya and Vanderbeck (2006) developed a method based on portion customers into regions and generates routes in this regions in order to obtain feasible solutions. While observing major improvements at values of regionalization and workload divisions, extension of total distance is also observed.

Francis et al. (2006) introduced Periodic Vehicle Routing Problem with Service Choice (PVRP-SC). The PVRP-SC is a problem based on PVRP. Like PVRP, PVRP-SC finds routes for every single day over specified period. The main difference is that PVRP-SC is trying to minimize total travel cost minus service benefit. There is a fixed fleet and capacity of vehicles. In this study, visit frequency is a decision variable with a lower bound of pre determined minimum number of visits. Francis and Smilowitz (2006) assume that visiting customers more frequent than customers' requirement, provides a service benefit. Moreover, there are predetermined visiting days. For example, some customers have to be visited daily, some of them have to be visited on Monday, Wednesday, Friday. The last combination is Tuesday and Thursday. Service benefit increases simultaneously with increasing number of visits.

A continuous approximation model is developed by Francis et al. (2006) for PVRP with service choice. They approximate discrete variables and parameters with continuous functions. The main approximation is replacing exact data for node locations and demand volumes. In this way, they divide customers into sub regions. On the other hand, they do not any change for visiting frequencies. Finally, computational results and impacts of their approximations of a problem which has 100 nodes is provided.

2.2.1 Exact Methods

Baldacci et al. (2011) suggested a certain solution method that is based on branch and cut. In 5 of 32 samples that proposed by Cordeau et al. (1997), Baldacci et al. (2011) improved best known results. Also, reached optimal results in 14 samples. Francis et al. (2006) propose a solution algorithm for Periodic Vehicle Routing Problem with Service Choice. The first stage is Lagrangian relaxation. They relax one constraint of the problem and create two different subproblems. Initial lower bound and upper bounds are most eventful for this procedure. If lower and upper bounds reach to ideal values, algorithm reaches to optimal solution. In the contrary case, branch and bound procedure get data from the lagrangian procedure to improve performance. They found that the average gap increases with the number of nodes, the routing sub problems become more difficult to solve and results in weaker lower bounds.

2.2.2 Heuristic Methods

Heuristics methods are developed for solving large problems in a reasonable time. The main trade-off for heuristics methods is obtaining a solution in a reasonable time but, solutions are not optimal. In other words, heuristics methods are saving time and causing lose solution quality.

According to Cordeau et al. (2002) heuristics are mainly divided into two categories. The first one is classical heuristics. Most of them try to quickly obtain a feasible solution and possibly apply to it a post optimization procedure. Well-known classical heuristics are saving heuristics, the sweep algorithm and the Fisher and Jaikumar algorithm. The second one is meta-heuristics. In last decades, there are lots of researches which study on meta-heuristics algorithms which applying mainly local search and population search. Simulated annealing and Tabu Search are two prime example of meta-heuristic.

Four attributes for VRP heuristics which are accepted as performance indicators of heuristics is determined by Cordeau et al. (2002). These attributes are accuracy, speed, simplicity and flexibility. Mostly, heuristics performances are measured based on accuracy and speed. However, they think that simplicity and flexibility are important performance indicators for heuristics. Cordeau et al. (2002) emphasis that *"Accuracy measures the degree of deviation of a heuristic solution value from the optimal value. Since optima and sharp lower bounds are usually unavailable in the case of the VRP, most comparisons have to be made with best known values."* Also, authors explain the most common techniques which are used to report obtained solution in literature. Consistency is an important criterion for a heuristics. A preferable heuristics should perform well for all kinds of problems.

The second attribute is speed. Speed's importance can change case to case. For example, for ambulance relocation problem, speed has a crucial importance. On the other hand, for fleet sizing problem the heuristic can solve the problem in two or three days. From a VRP point of view, it makes sense to spend about half an hour per day on computing the routing problem. The third attribute is simplicity. This attribute might not implemented to VRP heuristics too much. Thus, they are too complex to understand. Simple and short codes have better chance to adapt to other problems. Although, a minimum of complexity is to be expected for good results.

The final attribute is flexibility. Cordeau et al. (2002) stated that *"A good VRP heuristic should be flexible enough to accommodate the various side constraints encountered in a majority of real-life applications."* Then, they analyze the most popular classical heuristics according to these attributes. Saving algorithm is simple, quick and flexible but accuracy of the algorithm is not good enough. One of saving algorithm extensions, two-matching algorithm has really good accuracy. On the other hand, this extension removes two of saving heuristics best features which are speed and simplicity. In addition this extension has low level flexibility. The sweep algorithm is simple but does not seem to superior to saving algorithm both in terms of accuracy and speed. There are two of its extensions are 1-petal and 2-petals. These extensions provide accuracy and speed to sweep algorithm. On the other hand, levels of complexity of algorithms are very high. The Fisher and Jaikumar algorithm is not simple and its speed is unsettled. Moreover, its accuracy is average level. Meta-heuristics is also analyzed by Cordeau et al. (2002) Generally, metaheuristics generates more accurate solutions but, they are more complex and slow than the classical heuristics. The best metaheuristic for VRP is Tabu search. Taburoute heuristic of Gendreau et al. (1999) is accurate but complexity of heuristic is high. Moreover, speed of the heuristic and flexibility is relatively high. The Taillard tabu search algorithm is one of the best available in terms of accuracy. Also, its flexibility and simplicity are reasonable. Computation times are not reported. The adaptive memory procedure of Rochat and Taillard is one of the most powerful ideas. Its accuracy is perfect. Furthermore, it's simple and flexible. The granular tabu search algorithm of Toth and Vigo (1999) can quickly perform and produces high quality solutions. Also, it's relatively easy to implement. The unified tabu search algorithm of Cordeau et al. (1997) obtains excellent marks on flexibility. It is the simplest of all implementations. Furthermore, it is highly accurate and well on speed.

2.2.3 Classical Heuristics

Classical heuristics are mostly developed between 1960s and 1990s. These heuristics generally, based on a basic idea that provides a feasible solution. However, generally, the solution quality is below the acceptable level. These methods are mostly used before technological development.

The most known classical heuristics that applied to vehicle routing problems are; saving algorithm proposed by Clarke and Wright (1964), sequential improvement method proposed by Mole and Jameson (1976) and the sweep algorithm proposed by Wren (1971).

The first stage of model, which is proposed by Mourgaya and Vanderbeck (2006), is tactical planning model. This tactical planning process based on minimizing distances and balancing workload. While assigning customers to vehicles they consider reginalization; concentrate routes in limited area due to in real life problems, practitioners appreciate routes organized by area code and driver specialization by assigning drivers to routes that they are familiar with. Second stage defines a VRP. By doing this they improved reginalization and workload balance compared to the literature. They solved instances (01-02-03-17-25-26) proposed by Cordeau et al. (1997) and the average results Show 16% improvement in reginalization criteria, 70% improvement in workload balance while distance criteria is worse with a 20% increase. They form clusters of customers minimizing total cost which leads to seperation and homogenity. Then, they try to optimize second and third stage while minimizing the deviation from clustering decisions. Second stage is an operational model optimizing sequencing decisions. At the initial step they set the maximum workload bound to its initial solution value and optimize reginalization by using LP-based rounding heuristic as an improvement heuristic. During the rounding heuristic they include artificial columns that are useful to guarantee the existence of a feasible solution which may help to stabilize the column generation procedure.

2.2.4 Meta-Heuristics

Meta-heuristics are more complex and accurate than classical heuristics. The main objective of the meta-heuristics is searching quickly the solution space and finding a high quality solution. In recent years, meta-heuristics are very popular in the literature. Tabu search, variable neighborhood search methods are mostly used meta-heuristics for vehicle routing problems.

2.2.4.1 Tabu Search

Tabu search is a meta-heuristic algorithm which proposed by Glover (1977). Tabu search basically, includes neighborhood search and local search procedures. So, Tabu search has a memory. This memory keeps last moves and prevent algorithm from repeating this moves in a certain iteration. This provide algorithm to avoid to stuck a local optima.

Cordeau et al. (1997) developed a tabu search method for three different derivative of vehicle routing problem. (PVRP, Periodic Traveling Salesman Problem and Multi Depot Vehicle Routing Problem). They generated an initial solution by organizing customers in an increasing coordinates order to make a randomized circle around the depot. Then, assigned each customer a visit combination. For each day, a customer is randomly chosen and added to route by using GENI heuristic, if capacity not exceeded, up to available vehicle number. GENI constructs a tour by insertion a vertex between two other; considering 4-opt modification and selecting the least cost insertion. The last route which is equal to vehicle number may be infeasible. Tabu search algorithm also takes these infeasible solutions into consideration. Also, by considering routing, over capacity and over duration costs, a cost function is generated for the purpose of measuring a solution. Neighborhood of a solution is obtained by all solutions by removing a customer from its assigned route on that day or replacing the visit combination by another combination. They declare the removed customers attribute as tabu and forbid reinserting customers into the route again until a better feasible solution is obtained. An attribute's aspiration level is set to routing cost if feasible at the initial solution and infinity if infeasible. They set a penalized cost function of solution s and identified the solution minimizing that. After performing removals and insertions using the GENI heuristic, aspiration level updated at every feasible solution found. Also, if the solution is feasible they divide the overcapacity and duration cost penalty factors to a parameter, otherwise multiply with it. When their results are compared to best known results for the PVRP instances, proposed tabu search method's computational results obtain better results for 24 of 32 samples.

2.2.4.2 Iterated Local Search

A hybrid optimization algorithm is proposed by Cacchiani et al. (2014) which includes both heuristics and exact fixtures. They proposed a heuristic algorithm (SC-heur) based on the LP-relaxation of the model. A column generation approach is applied to solve LP-relaxation. In addition, ESPPRC and CYPLEX are used for the subproblems as solvers. Also for solbing subproblems, iterated local search (ILS) approach is used which is generating a sequence of local optima. ESPPRC aims to find the shortest path between 2 nodes; start and finish are 2 copies of the depot. They considered the following possible neighborhoods; insertion, removal, moving, replacing and swapping. ESPPRC impose to have a path without cycles going from and back to the depot. The problem is

NP-hard and may have negative cost cycles. To deal with it they proposed ILS. One iteration of ILS includes following 3 steps: Perturbation includes all steps while Local Search includes only insertion and removal steps because it only performs the move bringing most significant improvement. Acceptance Decision is based on when a new better local optimum is found. Current incumbent and the best solution is compared at each iteration and if it is better, incumbent solution is updated. Their stopping criteria was exiting the procedure if a negative cost path found after a certain number of iterations else increasing the number of iterations to allocate more time with a limit. ILS algorithm for the ESPPRC works as follows; the set of columns initialized by applying VNS with a limited number of iterations which obtains feasible solutions in a second. New columns generated at most 1000 times at every iteration and the procedure iterated until an integer solution is found. They also used a list of forbidden customers for each day because fixing a route may lead some days for a customer become infeasible. In order to avoid direct rush of this method to local optima, second mechanism leads escaping that local optima and allows previously fixed variables to release. They used a tabu list to avoid cycling between fixing and releasing by not allowing last 40 variables released. Stopping criterion herecx., it is seen that they obtain on average an improvement overall algorithms. For instances p01- p32 they improved 2 best known solutions and found best known solutions for 10 instances.

2.2.4.3 Variable Neighborhood Search

VNS is a local search based metaheuristic first proposed by Mladenović and Hansen (1997). The VNS approach successfully applied to other variants of the VRPs. To the best of our knowledge variable neighborhood search method firstly proposed by Hemmelmayr et al. (2009) for PVRP. The main aim of Hemmelmayr et al. (2009) is applying VNS method to PVRP without time windows. Hemmelmayr et al. (2009) obtain a initial solution by assign all customer a visit day combination randomly. After all customers assigned to visit day combination, savings algorithm which proposed by Clarke and Wright (1964) is used for constructing the routes. After applying savings algorithm, move and cross exchange operators apply to obtaining better results. The move operator, deletes a node or a segment from one route and add this node or segment into another route at the same day. For example, if the problem consists of 6 customers and 2 vehicles departure from the depot; the customer visit frequency for the first vehicle is 0-1- 2-3-4-0, for the second vehicle 0-5-6-0, move operator subtracts customers 2 and 3 from the first vehicle's route and imply into second vehicle's route. after move operator is run; the new routes of vehicles become 0-1-4-0 for the first vehicle and 0-5-2-3-6-0 for the second vehicle. The cross exchange operator exchanges two segments of different routes. For example if we consider an 8 customer and 2 vehicle instance, the visit sequence of customers are 0-1-2-3-4-0 for the first vehicle and 0-5-6-7-8-0 for the second vehicle. After cross-exchange operator is run, the new routes of vehicles 1 and 2 becomes 0-1-6-7-4-0 and 0-5-2-3-8-0 respectively. For PVRP there should also be neighborhoods that change visit combinations. They use neighborhoods in which visit combinations of only limited number of customers are changed. A visit combination is selected randomly for each of these customers. The metric here is the maximum number of customers for which combination is changed. In their algorithm, shaking uses the set of neighborhoods based on move operator up to 3 customers, cross exchange operator up to 6 customers and change combination up to 6 customers. They made analysis of neighborhood structures and examined that applying respectively change combination-move-cross exchange neighborhood structures is provided better solution. Thus, move and cross exchange perform better when customers assign on the right day at the beginning. At later stages move and cross operators perform well. Use of different neighborhoods is lower at early iterations. By these operators, a jump realized in the solution space. Then, for finding a local minimum, one of the most popular iterative improvement procedure 3-opt is applied which is introduced by Lin (1965). This method basically deletes three arcs and build new three arcs between nodes in the same routes. The local search ends when a move was found which provide better solution and repeat this procedure until termination condition is met. In addition, Hemmelmayr et al. (2009) combined VNS with simulated annealing method because VNS only accept improving solution. However, in some cases accepting non-improving solution can be provide better solution at the end. Acceptance decision is based on comparing solutions with incumbent solution and accepting all improved solutions and with linear annealing scheme (SA) the inferior solutions with a temperature determined. They group instances with larger average distances between customers set the initial temperature to 122 and for the other instances temperature is set to 7 and decreased it at every 1000 iteration lasting at 0. If infeasibility occurs, they used weighted linear penalty function to violate these constraints. These weights are adjusted dynamically; increased if capacity exceeded and decreased if it is feasible between the predetermined lower and upper bounds. For PTSP shaking is applied intra routes due to there is one route for every day. And 2-opt is used for local search in order to be faster due to each tour includes more customers than PVRP. They also added a penalty to the objective function for an empty day to lead visit at least one customer every day. They tested the algorithm on both old and new data sets. At the old data sets the examined their solution outperformed at larger instances and higher visit frequencies. If the instances with visit frequency is 1 would be excluded they could obtain better results. The results obtained by this method are compared with other heuristics methods on widely used instances in the literature. Most of the best known results are obtained or improved by VNS method. Moreover, VNS method proposed by Hemmelmayr et al. (2009) is faster than tabu search method Cordeau et al. (1997) for large problems.

Vidal et al. (2013) proposed insert and swap operators. Insert moves a visit from one rote to another. For example, if a problem with 5 customers and two vehicles is considered the route of first vehicle is 0-1-2-3-0 and second vehicle is 0-4-5-0. Insert subtracts customer 2 from the first vehicles route and interts it into second vehicles route. Swap operator changes two customers from different routes. For example if we consider an 6 customer and 2 vehicle instance, the visit sequence of customers are 0-1-2-3-0 for the first vehicle and 0-4-5-6-0 for the second vehicle. After swap operator is run customers 2 and 5 changes positions and, the new routes of vehicles 1 and 2 become 0-1-5-3-0 and 0-4-2-6-0 respectively.

Chapter 3

Mathematical Models and Notation

The purpose of this section is to provide basic mathematical models and common notation that is used in this thesis. The mathematical model and notation of vehicle routing problem and periodic vehicle routing problem is provided at the first and second parts respectively.

3.1 Vehicle Routing Problem

Vehicle Routing Problems (VRP) are generally established on one or multiple vehicles that leave warehouse to visit pre-determined nodes minimizing distance. The basic mathematical model and notation of vehicle routing problems are as follows;

Sets

N: set of customers, $(i, j \in N)$

N_0: set of customers including depot, $N_0 = N \cup \{0\}$

Parameters

q: physical capacity of vehicle

m: number of vehicles

c_{ij}: cost of traveling from customer i to customer j

d_i: demand of customer i

Decision Variables

x_{ij}: binary variable, 1 if arc ij is used, 0 otherwise.

u_i: amount of load on vehicle after visiting customer i

P(1)

$$min \sum_{i \in N_0} \sum_{j \in N_0} c_{ij} x_{ij} \tag{3.1}$$

$$Subjectto;$$

$$\sum_{i \in N_0} x_{ij} = 1 \qquad j \in N \tag{3.2}$$

$$\sum_{j \in N_0} x_{ij} = 1 \qquad i \in N \tag{3.3}$$

$$\sum_{i \in N_0} x_{i0} \leq m \tag{3.4}$$

$$\sum_{j \in N_0} x_{0j} \leq m \tag{3.5}$$

$$u_i - u_j + x_{ij} q \leq q - d_j \qquad i \in N, j \in N \tag{3.6}$$

$$u_i \leq q \qquad i \in N_0 \tag{3.7}$$

$$u_i \geq d_i \qquad i \in N_0 \tag{3.8}$$

$$x_{ij} \in \{0, 1\} \qquad i, j \in N_0 \tag{3.9}$$

Objective function (3.1) aims to minimize the total distance. Equations (3.2) and (3.3)ensures that a vehicle arrive and departure for every customer. Equations (3.4) and (3.5) states that number of outflows from depot cannot exceed the vehicle number m. Equations (3.6), (3.7) and (3.8) provide subtour elimination and prevent cumulative demand of the actualized visits to exceed the vehicle capacity.

3.2 Periodic Vehicle Routing Problem

Periodic vehicle routing problems (PVRP) are one of the frequently studied varients of vehicle routing problems in the literature and there are plenty of derivatives of these problems. The difference between periodic vehicle routing problems and standard vehicle routing problems is planning vehicle routes and customer visits in a determined period. There are frequencies that determine number of visits within planning periods of customers and visit days are determined according to those frequencies. Generally, the aim is to find a solution satisfying customer demands on time as well as making visits with minimum vehicle and minimum cost based on customer demands.

Additional notation and PVRP mathematical model which are required for periodic vehicle routing problems are as follows :

Sets

T: number of days in planning period

S: possible visit schedules

S_i: possible visit schedules of customer i(minimum visit frequency is f_i)

Parameters

a_{st}: 1 if schedule s requires a visit on day t, 0 otherwise

f_i: minimum visit frequency of customer i in planning period

d_{it}: delivery amount to customer i on day t

Decision Variables

x_{ijt}: binary variable, 1 if arc ij is used on day t, 0 otherwise.

y_{is}: binary variable, 1 is customer i is assigned to schedule s, 0 otherwise

u_{it}: total amount of load on vehicle after visiting customer i on day t

P(2)

$$min \sum_{i \in N_0} \sum_{j \in N_0} \sum_{t=1}^{T} c_{ij} x_{ijt} \tag{3.10}$$

$$Subjectto;$$

$$\sum_{j \in N_0} x_{ijt} = \sum_{s \in S_i} a_{st} y_{is} \qquad i \in N, t = 1, \dots, T \tag{3.11}$$

$$\sum_{i \in N_0} x_{ijt} = \sum_{s \in S_i} a_{st} y_{is} \qquad j \in N, t = 1, \dots, T \tag{3.12}$$

$$\sum_{i \in N} x_{0it} \leq m \qquad t = 1, \dots, T \tag{3.13}$$

$$\sum_{s \in S_i} y_{is} = 1 \qquad i \in N \tag{3.14}$$

$$u_{it} - u_{jt} + q x_{ij} < q - d_j \qquad i \in N, j \in N, t = 1, \dots, T \tag{3.15}$$

$$d_i \leq u_i \leq q \qquad i \in N, t = 1, \dots, T \tag{3.16}$$

$$x_{ijt} \in \{0, 1\} \qquad i, j \in N_0, t = 1, \dots, T \tag{3.17}$$

$$y_{is} \in \{0, 1\} \qquad i \in N, s \in S_i \tag{3.18}$$

Feasible schedule sets S_i includes visit day combinations that can be realized visits within a period, considering visit frequency f_i of customer i. For example when 4 days planning period is examined, for a customer with visit frequency 2, these combinations are 1st and 3rd days or 2nd and 4th days.

Objection function (3.10) aims to minimize the total distance as correspond with vehicle routing problems. Constraints (3.11) and (3.12) guarantee arrival and departure of a vehicle if customer will be visited on day t. Equation (3.13) states that number of outflows from depot cannot exceed the vehicle number m. Constraint (3.14) forces model to assign a visit schedule to every customer i. Constraints (3.15) and (3.16) prevent subtours, in other words to enter in a loop between specific customers and prevent cumulative demand of the actualized visits to exceed the vehicle capacity.

Chapter 4

Periodic Vehicle Routing Problem with Two Types of Visits

In this chapter, PVRP2TV is studied. Mathematical model of problem and computational complexity are submitted. Lower bound is developed and tests are performed, heuristic studies are reported.

4.1 Problem Definition

Periodic vehicle routing problem with two types of visits (PVRP2TV) is an extension of PVRP. The proposed problem requires a different aimed customer visit in the following day of a regular customer visit day. One can consider the first customer visit for collecting demand information and the second customer visit for delivering goods. The first type visit is realized with small scale, fast and time capacitated vehicles, whereas the second type visit is realized with relatively large scale, slow and physical capacitated vehicles. Hence using the same routes for the consecutive days is not possible. Pre-sellers visit customers for getting information about any change in demand. Also, meeting customers needs, arranging products in shelves, inform customers about promotions and new products are other benefits gained through pre-sellers visits. This task can not be assigned to trucks because, these activities take a substantial amount of time. In addition, pre-sellers are more literate people than truck drivers. Our main motivation is considering both type of vehicles might provide a better result than solving a PVRP for a single type of vehicle and arranging other type of vehicle according to obtained solution from PVRP.

4.2 Mathematical Model

Periodical vehicle routing problem, that is known in first section of this thesis, is considered from a different angle. By taking Coca Cola Beverage working system, which is the source of inspiration of this thesis, into consideration; a periodic vehicle routing problem, that includes two kinds of vehicles, is studied. Capacities of two kinds of vehicles are different from each other in terms of quantity and type on this problem. While one vehicle has physical capacity, the other has capacity time wisely. Furthermore the other constraint is the customer which is visited by one vehicle, should be visited by the other type of vehicle one day after. In real life, first vehicle represents people who receive orders and organize shelves in store and second vehicle represents transport vehicle that delivers order, which is received one day before, to store. Best of our knowledge, that type of derivative of periodic vehicle problem is not studied in literature. In addition to PVRP notation, the mathematical problem and typical notation of PVRP2TV as following;

Sets

V: Set of vehicle types $V = \{1, 2\}$ (1: demand collecting, 2: delivery)

Parameters

a_{stv}: 1 if schedule s requires a visit on day t for vehicle type v, 0 otherwise

d_{iv}: demand of customer i for vehicle type v (time for vehicle type 1, physical for vehicle type 2)

q_v: capacity of vehicle type v (time for vehicle type 1, physical for vehicle type 2)

m_v: available number of vehicles for vehicle type v

r_{ij}: total time for traveling from customer i to customer j and serving to customer j

Decision Variables

x_{ijtv}: binary variable, 1 if arc ij is used on day t with vehicle type v, 0 otherwise.

u_{itv}:total amount of load on vehicle after visiting customer i on day t

with vehicle type $v\backslash$total time used by vehicle

P(3)

$$min \sum_{i\in N_0}\sum_{j\in N_0}\sum_{v=1}^{2}\sum_{t=1}^{T} c_{ij}x_{ijtv} \tag{4.1}$$

$$Subjectto;$$

$$\sum_{j\in N_0\backslash\{i\}} x_{ijtv} = \sum_{s\in S_i} y_{is}a_{stv} \quad i\in N, t=1,\ldots,T, v\in\{1,2\} \tag{4.2}$$

$$\sum_{j\in N_0\backslash\{i\}} x_{jitv} = \sum_{s\in S_i} y_{is}a_{stv} \quad i\in N, t=1,\ldots,T, v\in\{1,2\} \tag{4.3}$$

$$\sum_{i\in N} x_{0itv} \le m_v \quad t=1,\ldots,T, v\in\{1,2\} \tag{4.4}$$

$$\sum_{s\in S_i} y_{is} = 1 \quad i\in N \tag{4.5}$$

$$u_{itv} - u_{jtv} + q_v x_{ijtv} \le q_v - d_{jv} \quad i,j\in N, t=1,\ldots,T, v\in\{2\} \tag{4.6}$$

$$d_{iv} \le u_{itv} \le q_v \quad i\in N, t=1,\ldots,T, v\in\{2\} \tag{4.7}$$

$$u_{itv} - u_{jtv} + (q_v + r_{ij})x_{ijtv} \le q_v - r_{ij} \quad i,j\in N, t=1,\ldots,T, v\in\{1\} \tag{4.8}$$

$$d_{iv} \le u_{itv} \le q_v \quad i\in N, t=1,\ldots,T, v\in\{1\} \tag{4.9}$$

$$x_{ijtv} \in \{0,1\} \quad i,j\in N_0,$$

$$t=1,\ldots,T, v\in\{1,2\} \tag{4.10}$$

$$y_{is} \in \{0,1\} \quad i,j\in N, s\in S_i \tag{4.11}$$

The objective function 4.1 minimizes the total cost of delivering goods to all customers in a given period. Equation 4.2 and 4.3 ensures that if each customer $i \in I$ and customer $j \in J$ will be visited on day $t \in T$ by vehicle v, can be visited only one time by vehicle v. Equation 4.4 ensures maximum number of vehicle can not be exceeded on day d by vehicle v. Equation 4.5 ensures that each customer $i \in I$ should be assigned to a schedule. Equation 4.6 and 4.7 are subtour elimination constraints for trucks. Equation 4.8 and 4.9 are subtour elimination constraints for pre-sellers.

Within some approaches in literature, there isn't any constraints related to customer visit days. For example; for 4-days-period problem, a customer that needs 2-days visits can be visited on first and second days. However because of lack of method that is suitable with real life implementation and as a result of literature review; problem is modelled with feasible schedule approach. Regarding to visit frequency and period length, customers are visited according to pre specified lines. For example, considering that a customer's period length is 4 days and visit frequency is 2 days; a schedule that includes customer visit on first and third day or second and fourth day can be selected. As it is specified in PVRP2TV problem, there are visits by 2 different kind of vehicle. These visits are realized on sequential days. If S1 schedule includes 1st and 3rd days for vehicle type 1, then it includes 2nd and 4th days for vehicle type 2. Thus a visit method that is more convenient with real life implementation is used.

4.3 Computational Complexity

It is proven that PVRP is a variant of VRP and both VRP and PVRP are NP-hard. To the best of out knowledge, if a special case of a problem is NP-hard, than this problem admitted as NP-hard. In this section, PVRP2TV P(3) is transformed into PVRP (P2) with some assumptions and its shown that PVRP is a special case of PVRP2TV.

Theorem PVRP2TV is NP-hard in strong sense.

Proof If it is assumed that;

- The capacity of pre-sellers and capacity of trucks are exactly equal. (q_1=q_2=q)
- Service time should be grater than zero and less than minimum of demand of customer i ∈ I and demand of customer j ∈ J divided by 2. ($0 < \ell_j < \min_i\{d_i + d_j\}/2$)
- Travel time of pre-sellers is equal to sum of demands of customer $i \in I$ and customer $j \in J$ divided by 2 minus service time of customer $j \in J$. (r_{ij} = ((d_i+d_j)/2)-ℓ_j)
- Travel time should be grater than 0. ($r_{ij} > 0$)

Regarding above assumptions all route decisions made for trucks, will be valid for pre-sellers. In other words, no need to make route decisions for pre-sellers, optimal routes optimal for both trucks and pre-sellers. Therefore, binary variable x_{ijtv} will be donated as x_{ijt}. The binary variable y_{itv} will be donated as y_{it}. In this context, PVRP2TV is transformed into standard PVRP which is a NP-hard problem in strong sense. Therefore PVRP2TV is NP-hard in strong sense.

Chapter 5

Lower bound and Heuristics Algorithm

5.1 Lower bound Algorithm

A branch and cut method is developed in order to find the lower bound. While lower bound is calculated in the method, customers set, coordinates of customers and visit frequencies of customers are out inputs. On first step, a classical PVRP math model without subtour constraints is solved for obtaining an initial solution. After obtaining an initial solution with subtours, a branch and cut algorithm runs. While branch and cut algorithm's iteration number less than 100 and total number of constraints added less than 1000 constraints, the algorithm detect all subtours. Following, for all detected subtours, a subtour constraint added as a new constraint to the model. Until to reach the limit of iteration or subtour constraints, the lowerbound algorithm terminated.

```
Input:
N nodes customers set
Coordinates of customers
Visit frequencies of customers
Output:
Total travel cost
begin
    Solve PVRP without subtour elemination constraints
    while iteration number less than 100 and total subtour
    elemination constraints added less than 1000 do
        Search for subtours
        Detect subtours
        Add subtour elemination constraints
```

Algorithm 1: Lower bound Algorithm

5.2 VNS Algorithm

In this chapter, developed VNS algorithm is proposed in detail. Proposed VNS algorithm, consists 3 main steps. The first step is generating an initial solution. For generating an initial solution, a simple heuristic methods is used to ensure obtaining a feasible solution. To the best of our knowledge, for VNS algorithm, beginning with a high quality initial solution is not provided an advantage to reach a better final solution. Even, beginning with a low quality solution can be preferable. Applying neighborhood structures is the second step. Swap operator, move operator and change visit combination operators are used for getting a random feasible solution. In third step, an iterative local search method is applied to improved the solution. If a new incumbent solution which is obtained by applying neighborhood operators and iterative local search is better than, the new incumbent solution is accepted. Otherwise, next neighborhood is performed to get another random solution. This VNS procedure is proposed by Mladenović and Hansen (1997). Steps of VNS are shown in figure 2.

```
begin
    Generate an initial solution (f(x))
    k=1
    while k<=12 do
        if k<=3 then
            Apply swap operator (f(x'))
            else if k>3 and k<=6 then
                Apply move operator (f(x'))
                else
                    Apply change visit combination operator(f(x'))
                end
            end
        end
    end
    Apply iterative local search(f(x"))
    if New solution is better (f(x")<f(x)) then
        Accept the new solution
    end
    else
        k=k+1
    end
end
```

Algorithm 2: Variable Neighborhood Search

5.2.1 Initial Solution

To generate an initial solution, a simple heuristic method is developed. In this heuristic method, customer sets, possible schedule sets that each customer can be assigned, visit frequencies of customers, demand of customers, vehicle capacities information are used as inputs.

Input:
N nodes customers set
Customers possible schedules
Visit frequencies of customers
Demand information of customers
Vehicle capacities
Output:
Initial solution
begin
    Assign customers to schedules
    Assign customers to vehicles
    Create routes according to ascending index order
    Calculate the total cost
    Obtain initial solution

Algorithm 3: Initial Solution

The first step of generating initial solution, customers are assigned to schedule by taking descending demand information of customers into consideration. More precisely, customers organized in descending order according to their demands. After that, customers respectively assigned to schedules that have minumum load. While minimum loaded schedule is determining by considering the highest loaded day in schedule. For example, schedule 1 consist of day 1 and day 3, schedule 2 includes day 2 and 4. Total demand assigned to day 1 is 100, day 2 is 120, day 3 is 110 and day 4 is 90. Than, maximum loaded day in schedule 1 is determined as 110. Maximum loaded day in schedule 2 is determined as 120. After this step, minumum loaded schedule is stated as schedule 1. So, the next customers which has highest demand is assigned to schedule

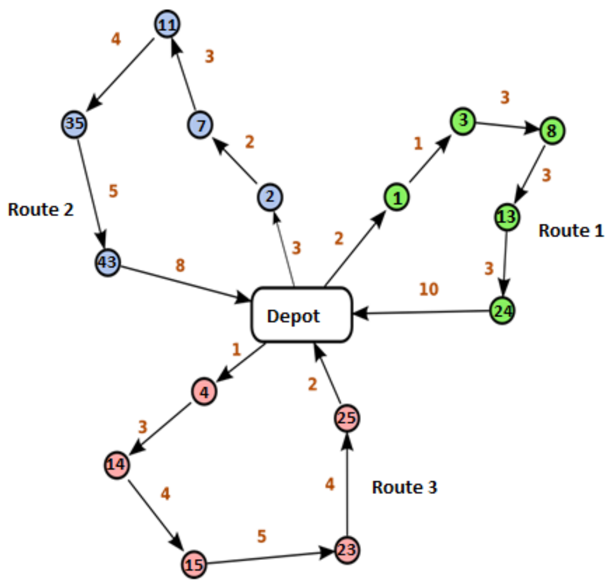

Figure 5.1: Initial Solution

1. In the next step, customers are assigned to vehicles with the same logic of assigning customers to schedule. These two steps are applied for increasing the chance of obtaining a feasible solution at the beginning. In the last step of generating initial solution, customers in the vehicle organized respectively ascending order according to customer number indexes.

5.2.2 Neighborhood Structures

In this section, neighborhood structures swap operator, move operator and change visit combination operator that used in algorithm are listed and logic of the structures are explained.

5.2.2.1 Swap operator

Swap operator changes two customers from different routes while keeping routes feasible.

```
begin
    Choose a day in period randomly
    Choose a random vehicle type
    Choose a random customer on chosen day
    Determine a customer from a different vehicle on the same day
    Make a feasibility check
    Change customers routes each other
```

Algorithm 4: Swap operator

For example if we consider an 6 customer and 2 vehicle instance, the visit sequence of customers are 0-1-2-3-0 for the first vehicle and 0-4-5-6-0 for the second vehicle. After swap operator is run customers 2 and 5 changes positions and, the new routes of vehicles 1 and 2 become 0-1-5-3-0 and 0-4-2-6-0 respectively. Application of swap operator to VNS algorithm is as follows;

5.2.2.2 Move operator

Move operator is a neighborhood structure that widely applied to VNS algorithm in the literature. Move operator moves a visit from one route to another.

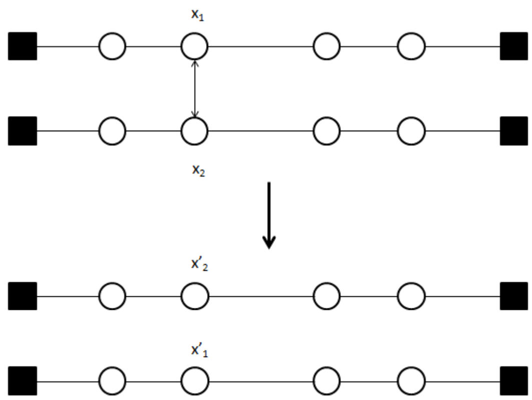

Figure 5.2: Swap Operator

begin
- Choose a random day in period randomly
- Choose a random vehicle type
- Choose a customer on chosen day randomly
- Determine a different route on the same day
- Make a feasibility check
- Move customer from its own route to new route

Algorithm 5: Move operator

For example, if a problem with 5 customers and two vehicles is considered the route of first vehicle is 0-1-2-3-0 and second vehicle is 0-4-5-0. Insert subtracts customer 2 from the first vehicles route and inserts it into second vehicle's route. In this operator, move action is realized between vehicles which are on the same day in the period. Also, in order to keep vehicles' routes feasible, considering vehicles' capacities plays a crucial role.

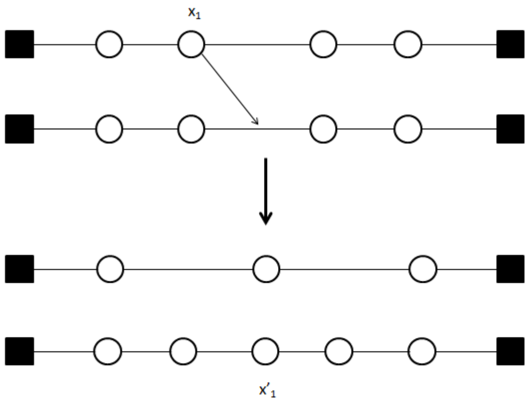

Figure 5.3: Move Operator

5.2.2.3 Change visit combination

Change visit combination operator is last neighborhood structure that is applied in algorithm. This operator change visit combination of a chosen customer with another possible schedule. At that point, the most important difference of change visit combination is a change affects both vehicle types. Thus, customers should be visited consecutive days by different type of vehicles. If one type of vehicle's visit day combination is changed, the other type of vehicle's visit day combination must be changed.

begin
- Choose a random vehicle type
- Choose a customer randomly
- Determine possible schedules of the customer
- Determine visit day combination of the customer by other vehicle type
- Feasibility check
- Move customer from its own visit day combination to new one for both vehicle type

Algorithm 6: Change visit day operator

For instance, a customer is visited on day 3 by a truck. It is known that it was visited on day 2 by a pre-seller. Changing visit day of the customer by truck from on day 3 to day 2, than visit realized by pre-seller should be changed from day 2 to day 1. All of these procedures are performed, possible schedules of customers must be considered.

5.2.3 Local Search

In this study, 2-opt algorithm is used for local search process. 2-opt algorithm is proposed by Croes (1958) to solving traveling salesman problem. 2-opt is a iterative local search algorithm. The main idea behind this local search method is that finding local minimum by selecting a pair of arcs and cross them. After cross the chosen arcs, reversing the necessary part of the route. Hence our instances are symmetric, the part

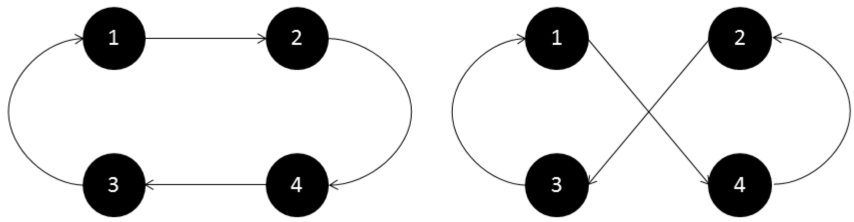

Figure 5.4: 2-opt

of the route which will be reversed is not an important decision. In addition, 2-opt procedure cross all possible arcs and choose the one which provide best improvement.

Chapter 6

Computational Results

6.1 Test Instances

In order to examine mathematical model and lower bound algorithm, we use instance problems that are compiled in Cordeau et al. (1997) article and also have used in many VRP and PVRP articles. Instances that between 1-10 are proposed for VRP by Eilon et al. (1971) and compiled by Christofides and Beasley (1984) for PVRP. 11th instance is proposed by Russell and Igo (1979), 12th instance is proposed by Cook and Russell (1978), 13th instance is introduced by Russell and Gribbin (1991). Instances between 14-31 are proposed by Chao et al. (1995), Cordeau et al. (2001), Angelelli and Speranza (2002), Baptista et al. (2002), Cacchiani et al. (2014), Baldacci et al. (2011) are can be exemplified for articles that include other instances. There are 42 instance problems in our instance sets. Minimum number of customer is 20 and maximum number of customer is 417. Coordinate data of customer, demand data, visit frequency, number of possible visit combination and these combinations are included within data set. The given information about these instances are listed in 6.1.

Table 6.1: Test Instances

Instance Number	Number of Customers(N)	Number of Vehicles(M)	Number of Days in Period(T)
p01	51	3	2
p02	50	3	5
p03	50	1	5
p04	75	5	2
p05	75	6	5
p06	75	1	10
p07	100	4	2
p08	100	5	5
p09	100	1	8
p10	100	4	5
p11	139	4	5
p12	163	3	5
p13	417	9	7
p14	20	2	4
p15	38	2	4
p16	56	2	4
p17	40	4	4
p18	76	4	4
p19	112	4	4
p20	184	4	4
p21	60	6	4
p22	114	6	4
p23	168	6	4
p24	51	3	6
p25	51	3	6
p26	51	3	6
p27	102	6	6
p28	102	6	6
p29	102	6	6
p30	153	9	6
p31	153	9	6
p32	153	9	6
pr01	48	2	4
pr02	96	4	4
pr03	144	6	4
pr04	192	8	4
pr05	240	10	4
pr06	288	12	4
pr07	72	3	6
pr08	144	6	6
pr09	216	9	6
pr10	288	12	6

6.2 Modifications on instances

As mentioned earlier, our problem has not been studied before in the literature. So, instances used in literature are not fit our problem. As a consequence, we modify the instances which are proposed by Cordeau et al. (1997). In this section, the modifications are explained. Parameters used for trucks which is second type of vehicle are directly taken from Cordeau et al. (1997) instances. The required additions are made for pre-sellers. The first required parameter for pre-sellers is demand information. Demand for pre-sellers corresponds to spending time to each customer visits. The demand information is generated from demand information of trucks. All customer demands for pre-sellers are equalized to trucks demands. The second required parameter is the time spending while pre-sellers travel between customers. This parameter is generated as a function of distance matrix. The following function shows the generation procedure. Let c_{ij} be the distance between customer i and customer j. Let r_{ij} be the travel time between customer i and customer j. C is average distance between all customers. D is average demand information. $r_{ij} = cij * D/C$ The last required parameter is capacity of pre-sellers. We demean travel times to demands. We assumed that the capacity of pre-sellers should double the pre-sellers capacity based on trucks.

6.3 Computational Results

In this section computational results are provided for both lower bound algorithm and VNS algorithm. The instances are used mentioned previous section. The algorithm implemented in C programming. The algorithms are run on PC equipped AMD A6-3400M APU with Radeon(tm) HD graphics 1.40 GHz and 4,00 GB installed memory(RAM).

6.3.1 Lower bound Algorithm

The lower bound algorithm is restricted with 3600 seconds for all instances. The following table includes informations about instances and results of lower bound algorithm.CPU times are provided in seconds.

6.3.2 VNS Algorithm

In this section, computational results of VNS algorithm are shown in table 6.3.VNS iteration limit set as 5000 and 2-opt iteration set as 100. Table 6.3 includes instances information, VNS algorithm results and CPU time. Compared results are shown in table 6.4. Table 6.4 includes results of lower bound algorithm, results of VNS algorithm and

Table 6.2: Lower bound algorithm results

Instance Number	Number of Customers	Number of Vehicles	Number of Days in Period	Lowerbound Algorithm
p01	51	3	2	1097
p02	50	3	5	2648
p03	50	1	5	958
p04	75	5	2	1406
p05	75	6	5	3040
p06	75	1	10	1255
p07	100	4	2	1453
p08	100	5	5	3375
p09	100	1	8	1391
p10	100	4	5	2655
p11	139	4	5	1036
p12	163	3	5	1601
p13	417	9	7	2947
p14	20	2	4	3452
p15	38	2	4	3574
p16	56	2	4	5843
p17	40	4	4	2628
p18	76	4	4	4944
p19	112	4	4	7982
p20	184	4	4	14904
p21	60	6	4	3659
p22	114	6	4	6393
p23	168	6	4	9689
p24	51	3	6	7591
p25	51	3	6	8891
p26	51	3	6	8052
p27	102	6	6	36249
p28	102	6	6	36460
p29	102	6	6	36751
p30	153	9	6	116364
p31	153	9	6	116831
p32	153	9	6	117394
pr01	48	2	4	4655
pr02	96	4	4	7525
pr03	144	6	4	8099
pr04	192	8	4	9294
pr05	240	10	4	9306
pr06	288	12	4	11548
pr07	72	3	6	8437
pr08	144	6	6	10773
pr09	216	9	6	14921
pr10	288	12	6	16409

Table 6.3: VNS algorithm results

Instance Number	Number of Customers	Number of Vehicles	Number of Days in Period	Lower bound	VNS	CPU (sec)
p01	51	3	2	1097	1409	1080
p02	50	3	5	2648	3412	713
p03	50	1	5	958	1725	602
p04	75	5	2	1406	2176	2390
p05	75	6	5	3040	5293	817
p06	75	1	10	1255	3335	1738
p07	100	4	2	1453	2396	798
p08	100	5	5	3375	6075	2034
p09	100	1	8	1391	3695	1352
p10	100	4	5	2655	5034	1037
p11	139	4	5	1036	2264	1127
p12	163	3	5	1601	-	-
p13	417	9	7	2947	-	-
p14	20	2	4	3452	-	-
p15	38	2	4	3574	-	-
p16	56	2	4	5843	6800	522
p17	40	4	4	2628	3762	553
p18	76	4	4	4944	8061	694
p19	112	4	4	7982	13169	981
p20	184	4	4	14904	-	-
p21	60	6	4	3659	-	-
p22	114	6	4	6393	11235	1108
p23	168	6	4	9689	18315	1509
p24	51	3	6	7591	9956	578
p25	51	3	6	8891	10036	595
p26	51	3	6	8052	9847	614
p27	102	6	6	36249	62289	1180
p28	102	6	6	36460	62267	786
p29	102	6	6	36751	62937	603
p30	153	9	6	116364	-	-
p31	153	9	6	116831	-	-
p32	153	9	6	117394	-	-
pr01	48	2	4	4655	5107	470
pr02	96	4	4	7525	9881	503
pr03	144	6	4	8099	15989	1196
pr04	192	8	4	9294	-	-
pr05	240	10	4	9306	-	-
pr06	288	12	4	11548	-	-
pr07	72	3	6	8437	12428	1085
pr08	144	6	6	10773	23344	1317
pr09	216	9	6	14921	-	-
pr10	288	12	6	16409	-	-

Table 6.4: VNS-Lowerbound compared results

Instance Number	Number of Customers	Number of Vehicles	Number of Days in Period	Lower bound	VNS	GAP(%)
p01	51	3	2	1097	1409	0.284
p02	50	3	5	2648	3412	0.289
p03	50	1	5	958	1725	0.801
p04	75	5	2	1406	2176	0.548
p05	75	6	5	3040	5293	0.741
p06	75	1	10	1255	3335	1.657
p07	100	4	2	1453	2396	0.649
p08	100	5	5	3375	6075	0.800
p09	100	1	8	1391	3695	1.656
p10	100	4	5	2655	5034	0.896
p11	139	4	5	1036	2264	1.185
p12	163	3	5	1601	-	-
p13	417	9	7	2947	-	-
p14	20	2	4	3452	-	-
p15	38	2	4	3574	-	-
p16	56	2	4	5843	6800	0.164
p17	40	4	4	2628	3762	0.432
p18	76	4	4	4944	8061	0.630
p19	112	4	4	7982	13169	0.650
p20	184	4	4	14904	-	-
p21	60	6	4	3659	-	-
p22	114	6	4	6393	11235	0.757
p23	168	6	4	9689	18315	0.890
p24	51	3	6	7591	9956	0.312
p25	51	3	6	8891	10036	0.129
p26	51	3	6	8052	9847	0.223
p27	102	6	6	36249	62289	0.718
p28	102	6	6	36460	62267	0.708
p29	102	6	6	36751	62937	0.713
p30	153	9	6	116364	-	-
p31	153	9	6	116831	-	-
p32	153	9	6	117394	-	-
pr01	48	2	4	4655	5107	0.097
pr02	96	4	4	7525	9881	0.313
pr03	144	6	4	8099	15989	0.974
pr04	192	8	4	9294	-	-
pr05	240	10	4	9306	-	-
pr06	288	12	4	11548	-	-
pr07	72	3	6	8437	12428	0.473
pr08	144	6	6	10773	23344	1.167
pr09	216	9	6	14921	-	-
pr10	288	12	6	16409	-	-

*GAP = 100 x $\frac{\text{VNS - Lower bound}}{\text{Lower bound}}$

gaps between lower bound algorithm and VNS algortihm.

6.4 Conclusion

In conclusion, We propose a new extension of PVRP which is PVRP2TV. In addition, a lower bound algorithm and VNS algorithm for PVRP2TV are proposed. According to computational results, proposed VNS algorithm performs well for small size instances. The average gap between lower bound and VNS algorithm results are around 20% for small size instances. Instances that have only one vehicle on each day, there are huge gaps between results of lower bound and VNS algorithm. Therefore, swap and move operators can not be applied for these problems. Developing a method for solving these instances can be an essential addition to our heuristic algorithm from flexibility point of view. The average gaps between lower bound and VNS algorithm is about 70% for large scale instances.

Our algorithm should be improved for relatively large size instances. These can be reduce 35-40% with some additions to algorithm. Improving algorithm for only one vehicle on each day and relatively large size instances is most essential future research for this study. Then, using real life data which will be provided by Coca Cola İçecek A.Ş. and comparing results with real costs of Coca Cola İçecek A.Ş. will be our future researches.

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