ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE ENGINEERING AND TECHNOLOGY

NUMERICAL MODELLING OF UNIFORM FLOW OVER A POROUS PLANE WITH SUCTION PERPENDICULAR TO THE SURFACE BY USING SEMI ANALYTICAL NUMERICAL METHODS

M.Sc. THESIS

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Department of Aeronautics Astronautics Engineering

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ISTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

YÜZEYE DİK YÖNDE EMME OLAN GÖZENEKLİ ORTAMA GELEN ÜNİFORM AKIŞIN YARI ANALİTİK YÖNTEMLERLE SAYISAL MODELLENMESİ

YÜKSEK LİSANS TEZİ

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ARALIK 2017

Cemre Melike Pehlivan, a M.Sc. student of ITU Graduate School of Science Engineering and Technology student ID **511151107**, successfully defended the thesis entitled "**NUMERICAL MODELLING OF UNIFORM FLOW OVER A POROUS PLANE WITH SUCTION PERPENDICULAR TO THE SURFACE BY USING SEMI ANALYTICAL NUMERICAL METHODS**", which she prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

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Date of Submission : 17 November 2017
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This work is dedicated to my family…

FOREWORD

I would like to give my special thanks to my thesis advisor Assoc. Prof. Aytac Arikoglu for his support and my committee member Prof. İbrahim Ozkol for his comprehensive recommendations during my undergraduate and graduate education. Also I would like to express my gratitude to all of whom supported me to complete this master thesis especially Res. Assist. Hayriye Pehlivan Solak.

December 2017

Cemre Melike PEHLİVAN
Astronautical Engineer

TABLE OF CONTENTS

ABBREVIATIONS

CGL	**:** Chebyshev-Gauss-Lobatto
DQM	**:** Differential Quadrature Method
FDM	**:** Finite Difference Method
FDQ	**:** Fourier Expansion-Based Differential Quadrature
FEM	**:** Finite Element Method
FVM	**:** Finite Volume Method
MoM	**:** Method of Moments
ODE	**:** Ordinary Differential Equation
PDE	**:** Partial Differential Equation
PDQ	**:** Polynomial-Based Differential Quadrature
Q&C	**:** Quan and Chang's Approach
WRM	**:** Weighted Residual Method

LIST OF TABLES

Page

LIST OF FIGURES

Page

NUMERICAL MODELLING OF UNIFOM FLOW OVER A POROUS PLANE WITH SUCTION PERPENDICULAR TO THE SURFACE BY USING SEMI ANALYTICAL NUMERICAL METHODS

SUMMARY

In this thesis numerical modelling of uniform flow over a porous plane with suction in vertical direction to the plane by using semi analytical numerical methods were carried out. Semi analytical numerical methods are differential quadrature method and method of moments.

In the first section purpose of the thesis and literature review were given. Then in the theory section, required theoretical information was given. Continuity equation, momentum equation and Biot’s theory of poroelasticity were used as governing equations. Reynold and Darcy are dimensionless numbers. In the mathematical modelling section, firstly problem was defined. In this study the flow is 2 dimensional, incompressible, viscous and Newtonian. So density and viscosity are constant. By using equations in the theory section, dimensionless governing equations of the problem were found. After that employing the suggestion by Deng and Martinez, stream function was selected. Final form of the fluid layer equation and porous layer equation were found. Defining boundary conditions at the interface is the most important part of the mathematical modelling. In this study continuity of the velocity vector and shear stress were used as boundary conditions.

In other sections, semi analytical numerical methods were explained and velocity distribution in the fluid layer and porous layer are found. In the result section, graphs were plotted and were interpreted in the section. Also approximate estimate of the error calculation was done. According to this, it is seen that error was reduced with increasing number of grid points.

It is easily seen from the graphics that vertical velocity increases from top to the bottom because of the suction perpendicular to the surface. And at the bottom no horizontal velocity component exists. Thanks to this study alternative semi analytical solution to a two dimensional flow is derived. Without empirical constants meaningful solutions are obtained.

YÜZEYE DİK YÖNDE EMME OLAN GÖZENEKLİ ORTAMA GELEN ÜNİFORM AKIŞIN YARI ANALİTİK YÖNTEMLERLE SAYISAL MODELLENMESİ

ÖZET

Bugüne kadar gözenekli ortam hakkında çok fazla çalışma yapılmıştır. Çünkü gözenekli ortam günlük hayatımızda her alanda karşımıza çıkmaktadır. Küçük ölçekte düşündüğümüzde kanın mikroskobik seviyedeki akışından büyük ölçekte jeotermal bilimlere kadar geniş bir alanda gözenekli ortam problemini görebilmek mümkündür. Gelişen teknoloji ile beraber ortaya çıkan çalışmalar daha kapsamlı sonuçlar vermeye başladıkça gözenekli ortam günden güne daha büyük bir önem kazanmıştır. Biyomekanik sistemlerdeki taşıma işlemlerinden havacılıkta kullanılan kabin içi filtre tasarımına kadar geniş bir uygulama alanında yapılan çalışmalar günümüzde de devam etmektedir. Nükleer mühendislik, biyomedikal sistemler, havacılık, jeotermal bilimler sadece birkaç örnektir. Bu kadar sık karşılaştığımız bir ortama gözenekli ortam diyebilmemiz için bazı şartlara ihtiyacımız vardır. İlk şartımız malzeme kendi boyutları ile karşılaştırıldığında çok küçük boşluklara sahip olmalıdır ve bu boşluklar hava ya da su gibi akışkanlar ile dolu olmalıdır. İkinci şart ise akışkan katı malzemenin bir ucundan girip diğer ucundan çıkabilmelidir. Bahsedilen bu iki şart sağlandığında bulunan ortam gözeneklidir kabulü yapılabilir. Bu çalışmada yüzeye dik yönde emme olan gözenekli bir ortama gelen üniform akışın yarı analitik yöntemlerle sayısal modellenmesi üzerine çalışılmıştır.

Bu çalışmada yüzeye dik yönde emme olan gözenekli ortama gelen üniform akışın iki farklı yarı analitik yöntem kullanarak sayısal modellemesi yapılmıştır. Kullanılan yarı analitik yöntemler diferansiyel kuadratur yöntemi (DQM) ve moment yöntemidir (MoM). Birinci bölümde öncelikle çalışmanın önem ve içeriğinden bahsedilmiş ve sonra konu ile ilgili yapılan diğer çalışmalar hakkında bilgi verilmiştir.

İkinci bölümde matematiksel modelleme kısmında gerekli olan teorik bilgilerden bahsedildi. Öncelikle bölümün amacı ve kapsamından bahsedildi. Devamında akışkan hareketini yöneten denklemler verildi. Süreklilik ve momentumun korunumu denklemleri akışkanı tanımlamak için kullanılan denklemlerdir. Ayrıca akışkanı tanımlayan denklemleri düzenlerken ihtiyaç duyulan boyutsuz sayılar hakkında da bilgi verildi. Reynold sayısı ve Darcy sayısı matematiksel modelleme bölümünde kullanılan boyutsuz sayılardır. Teori kısmının bir diğer önemli kısmı ise gözenekli ortam akışını tanımlayan modeli belirlemektir. Akış modellerini tanımlayan denklemler ilk olarak deneysel çalışma sonucu elde edilen ve deneysel katsayılar içeren denklemlerdir. Gözenekli ortamda akışı tanımlayan ilk yasa Henry Darcy tarafından 1856 yılında geliştirilmiştir. Darcy yasası Reynold sayısının 1'den küçük olduğu, yani düşük hızlı, sıkıştırılamaz ve Newtonyen akışkanlar için geçerlidir. Darcy denklemi deneysel bir bağıntıdır ve Reynold sayısının 1den büyük olduğu, yani yüksek hızlı akışlarda Darcy denklemi geçersiz olmaya başlar. Çünkü Darcy denklemi akışın doğrusal olmayan etkisini modelleyemez. Ayrıca denklem viskoz

etkileri de içermemektedir. 1947 yılında Brinkman tarafından Darcy denklemi tekrar düzenlenmiştir ve Darcy denkleminin içermediği viskoz etkiler Brinkman denkleminde sağlanmıştır. Diğer önemli denklem ise 1931 yılında Richard tarafından geliştirilmiştir.

Üçüncü bölüm matematiksel modelleme kısmına ayrılmıştır. Giriş kısmında öncelikle bölümün içeriğinden, problemin detaylarından ve akışı tanımlamak için yapılan kabullerden bahsedilmiştir. Bu çalışmada iki boyutlu akış için sıkıştırılamaz, viskoz ve Newtonyen olduğu kabulü yapılmıştır. Problem gözenekli ortam ve akışkan ortam olmak üzere iki farklı ortamdan meydana gelmektedir. Girişin devamında gözenekli ortamı tanımlayan model seçimi yapılmıştır. Gözenekli ortamı tanımlayan denklem için Biot'un poroelastisite teorisinden faydalanılmıştır. Referans olarak kullanılan Deng ve Martinez'in çalışmasında ise Brinkman denklemi kullanılmıştır. Kabuller yapılıp akışkan ve gözenekli ortamları tanımlayan modeller belirlendikten sonra yüzeye dik yönde emme olan gözenekli akış probleminin matematiksel modellemesine geçilmiştir. Bu kısmı tamamlarken teori bölümünde verilmiş olan akışkan hareketini yöneten süreklilik ve momentum denklemleri ile boyutsuz sayılardan yararlanılmıştır. Problemi matematiksel olarak tanımladıktan sonra iki farklı bölüm için de denklemlerin boyutsuz halleri bulunmuştur. Gözenekli ortamın matematiksel ifadesi dördüncü dereceden lineer bir diferansiyel denklem iken akışkan ortamın matematiksel ifadesi dördüncü dereceden lineer bir diferansiyel denklem olarak bulunmuştur. Problemin matematiksel ifadesini bulduktan sonra diğer bir husus bulunan denklemleri çözebilmek için gereken yeter sayıdaki başlangıç ve sınır şartlarını belirlemektir. Bu çalışmada iki ortam için de bulunan denklemleri çözebilmek için toplamda 8 tane şart gerekmektedir. Problemin üst yüzeyinde iki hız bileşeninin de sıfıra eşit olması, alt yüzeyde sadece emme kaynaklı y yönünde hız bileşeninin oluşu, iki yüzeyin kesişim noktasında da hızların sürekliliği ve kayma gerilmesinin sürekliliği başlangıç ve sınır şartları olarak belirlenmiştir. Özellikle iki ortamın kesişiminde kullanılan sınır şartları farklı problemlere ve çalışmalara göre değişiklik göstermektedir ve sadece bu sınır şartları üzerine yapılan farklı çalışmalar mevcuttur.

Dördüncü bölümde çalışmada kullanılan yarı analitik yöntemlerden biri olan diferansiyel kuadratur yöntemi anlatılmıştır. Öncelikle genel hatlarıyla yöntemin tarihçesi ve yapısı verilmiştir. DQM ilk defa Bellman tarafından 1971 yılında ortaya konmuştur. Devamında yöntemin içinde bulunan ağırlıklı katsayıları hesaplamak için geliştirilen farklı yaklaşımlar ve bu yaklaşımların birbirlerine göre avantaj ve dezavantajlarından bahsedilmiştir. Diferansiyel kuadratur yönteminde çözümün hassasiyeti hem düğüm noktası sayısına hem de düğüm noktalarının dağılımına bağlıdır. Lineer türden denklemlerin çözümünde eşit aralıklı düğüm noktası seçimi yeterliyken lineer olmayan denklemlerde durum değişmektedir. Çalışmada sınır şartlarına yaklaşıldığında sonuçlar kötüleştiği için sınırlara doğru daha sık adım aralıklarının kullanıldığı Chebyshev-Gauss-Lobatto nokta dağılımı tercih edilmiştir. Diferansiyel kuadratur yönteminin detayları verildikten sonra akışkan ortam ve gözenekli ortam denklemleri DQM ile çözülmüştür.

Beşinci bölüm ise ağırlıklı artıklar yöntemlerinden moment yöntemine ayrılmıştır. Moment yöntemi 1947 yılında Yamada tarafından geliştirilmiştir ve 1951 yılında Fujita yöntemin gelişmesine katkıda bulunmuştur. 4 madde takip edilerek bütün ağırlıklı artıklar yöntemiyle sonuca ulaşmak mümkündür. Öncelikle bilinmeyen katsayılar ile problemi tanımlayacak olan bir polinom oluşturulur. İkinci adımda bu polinom başlangıç ve sınır şartları tarafından sağlanır ve dolayısı ile bu şartlara göre bilinmeyen sayısı düşürülür. Üçüncü adımda problemi tanımlayan artık fonksiyon belirlenir. Ve son adımda ağırlıklı artık fonksiyonları sıfıra eşitlenerek bilinmeyen tüm katsayılar bulunarak problem çözülmüş olur.

Altıncı bölüm ise sonuç bölümüdür. Bu bölümde gözenekli ortam ve akışkan ortam için DQM ve MoM yöntemleri kullanılarak bulunan x ve y yönündeki hız bileşenlerinin grafikleri çizilmiş ve hata hesabı yapılmıştır.

Son bölüm olan yedinci bölümde de sonuçlar değerlendirilip çalışma tamamlanmıştır.

1. INTRODUCTION

In the introduction section, firstly purpose of the thesis was given for creating general point of view and then important developments of flow in the porous media were shared.

1.1 Purpose of Thesis

Fluid flow behavior in porous media is conundrum. This study deals with study on numerical modelling of uniform flow over a porous plane with suction perpendicular to the plane by using semi analytical numerical methods. In recent years, the study of flow over a porous plane with downward suction has gained a lot of importance because porous plane with downward suction has wide range of application areas. Several sciences that have carried out many researchers in this field are geothermal science, biological transport process in biological sciences, flow filtration and energy engineering disciplines. A porous medium is defined by the total volume fraction, which are fluid layer and porous layer. In contrast to the dynamics of large fluid compartments, the dynamics of fluids in porous media is more complicated. There are many reasons for this. First and the most important reason is efficient dissipation of the fluid's kinetic energy in porous media. The other reason is complexity of the internal dynamic for multiple fluids. Be able to make correct assumptions and identify negativities, many scientists focus on this issue. Most research issue through porous media problems are focus on the defining interface.

In both theoretical perspective and experimental perspective, the analysis of interface conditions is very important and updated. Many different types of boundary conditions at the water porous interface have been discussed. In this study continuity of velocity and continuity of shear stress are used as a boundary conditions as seen in the mathematical modelling section. Another important thing is defining porous flow model. As will mentioned in the theory section, there are 4 different porous flow models which are Brinkman equation, Darcy's equation, Richard's equation and Biot's theory of poroelasticity.

The equation of uniform flow over a porous plane with downward suction is solved by using different numerical techniques. One of them is differential quadrature method and the other one is the method of moments. In the next subtitle of the study, historical development of the studies about porous media is given. Then required background to understand the mathematical modelling of the problem was explained. After this, mathematical modelling of the problem was given. And numerical methods are defined and solution procedures were mentioned. In conclusion, results of each numerical method is found and compared according to each other.

1.2 Literature Review

In recent years rather a lot of interest has been taken in the development of flow in the porous media by reason of its importance in a variety of natural phenomena, biological sciences, energy stocking systems, geothermal sciences and industrial processes. The developments started with experimental studies in 1856 and have been continuing to this day by using different boundary conditions and variable geometrical structures.

First empirical studies are done by Darcy in 1856 [1]. It was an important starting point for theoretical studies because it is impossible to solve empirically three dimensional more general cases. Darcy’s study is the extension to more general cases that are difficult to perform experimentally. This kind of cases deserving theoretical studies. The first studies which are especially difficult to perform experimentally are groundwater flows and oil recovery processes. After Darcy’s study, theoretical studies also started.

A large number fluid particles flowing in pores are seen in the porous media. It is very difficult to determine initial and final position of the fluid particles in the flow. Flow through channel in porous media problems are known as Berman flow. Because he was the first one to interest with this issue in 1953. Flow in a channel with a permeable bottom and uniform outward suction is studied by Berman. He investigated effect of wall porosity on the two dimensional laminar flow of an incompressible fluid in a channel. According to Berman’s work, the effect of the channel dimensions, position coordinates and fluid properties on the velocity components and the pressure are defined. He reduced the Navier Stokes equation to the third order, nonlinear, ordinary differential equation [2]. Further contribution has been made by Sellars in 1955 [3]. He extended the problem that is studied by Berman by using very high Reynold numbers. In 1956 Seo Young studied on numerical modelling of heat transfer in a channel with fluid saturated porous media. By using finite element techniques, the two dimensional equation were solved [4]. In 1979 the flow in a channel with one porous wall is studied by Green [5].

More particularly flow through porous media problems are focused on the boundary conditions at the interface separating the pore fluid from the porous medium flow. This has been an important research issue since the Beavers and Joseph in 1967 [6]. These authors postulated a discontinuity in the interfacial tangential velocity. In 1995, Ochoa-Tapia and Whitaker postulated continuity of tangential velocity and discontinuity of shear stress [7]. In 1999, Cieszko and Kubik adopted discontinuity of both tangential velocity and shear stress [8]. Deng and Martinez postulated continuity of tangential and discontinuity of shear stress in 2005 [9]. They used Brinkman-extended Darcy law relationship to define the porous medium. Continuity of the velocity vector at the viscous zone interface is studied by Bars and Worster in 2006 [10]. Multiple domain models which include more complex formulations such as the Brinkman equation are retained in the upscaling process by Auriault in 2009 [11].

2. THEORY

2.1 Introduction

In this section, the problem described in different point of views. In order to understand mathematical modelling of the uniform flow over a porous plane with suction perpendicular to the plane easily requires to theoretical knowledge. Before mathematical modelling, some required informations were given for understanding easily such as governing equations, dimensionless numbers, different pore flow models and so on. Understanding the equations that are mentioned above are necessary to understand the core of the problem. After the preliminary informations, mathematical modeling was given.

2.2 Governing Equations

Fundamental governing equations of the fluid dynamics which are the basic equations of the physical systems are conservation of mass (the equation of continuity), conservation of momentum (the Navier Stokes equations) and conservation of energy equations (first law of thermodynamics). Complete understanding of these equations make it possible to apply other specific cases and tasks. Governing equations are based upon conservation of mass, conservations of energy and Newton's second law.

2.2.1 Continuity equation

The starting point of the all conservation law is

$$\frac{D\alpha}{Dt} = \frac{\partial \alpha}{\partial t} + (\bar{V}.\nabla)\alpha \tag{2.1}$$

in which α is any property of the fluid, D represent the material derivative, ∇ is the divergence operator, t is time and $\bar{V}$ is the velocity.

Basically the law of conservation of mass is

$$m = \rho V = const. \tag{2.2}$$

in which ρ is density and V is volume. According to principle of mass conservation, observed fluid size and shape will change, its mass will remain same. Mathematical representation of the mass conservation is

$$\frac{D}{Dt}m = \frac{D}{Dt}(\rho V) = \rho\frac{DV}{Dt} + V\frac{D\rho}{Dt} \tag{2.3}$$

Be able to arrange equation, alternative expressions were given. The rate of volume increase of a particle per unit volume is equal to total strain rate. This relation was given in equation (2.4)

$$\frac{1}{V}\frac{DV}{Dt} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} \tag{2.4}$$

in which ε represents the strain rate. Another expression for strain rate that was written in equation (2.5) is relate with velocity

$$\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \nabla.\bar{V} = div\bar{V} \tag{2.5}$$

The improved expression for rate of volume increase of a particle is arranged in equation (2.6).

$$\frac{DV}{Dt} = Vdiv\bar{V} \tag{2.6}$$

Equation (2.6) is substituted to the equation (2.3). Then all sides of the equation are divided by volume and equation of continuity is obtained in its general form.

$$\frac{D\rho}{Dt} + \rho div\bar{V} = 0 \tag{2.7}$$

Equation (2.7) is equal to the equation (2.8)

$$\frac{\partial\rho}{\partial t} + div(\rho\bar{V}) = 0 \tag{2.8}$$

Equation (2.8) is called as continuity equation in conservation form. It refers to unsteady flow in a compressible flow.

2.2.2 Momentum equation

The principle of conservation of momentum is an application of Newton’s second law of motion.

$$F = ma \tag{2.9}$$

in which m is mass and a is acceleration. Each element of the equation (2.9) is divided by the volume. We worked with velocity and density instead of acceleration and mass. Hence we arranged equation (2.9)

$$\rho \frac{D\bar{V}}{Dt} = \frac{F}{V} = f_{body} + f_{surface} \tag{2.10}$$

in which f is applied force per unit volume on the fluid particle.

It is divided into two different types which are body forces and surface forces. The body forces apply to the entire mass and formulation is

$$f_{body} = \rho g \tag{2.11}$$

in which g is acceleration of gravity and ρ is density.

The surface forces that are shown in Figure 2.1 are applied by external stresses. All stresses that are seen in the Figure 2.1 are positive.

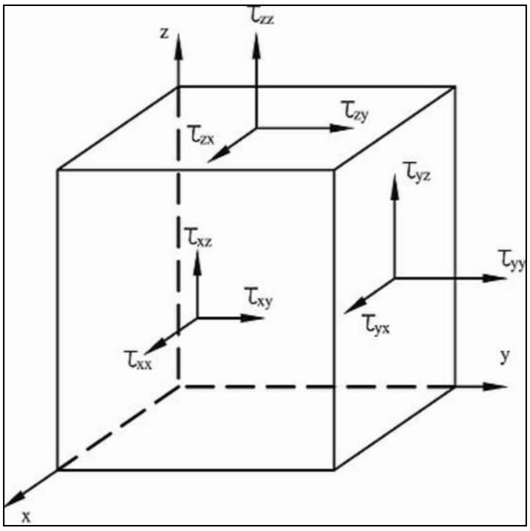

Figure 2.1: Notation for stresses.

The stress tensor can be written as

$$\tau_{ij} = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix} \tag{2.12}$$

Net force on the element in the x direction due to stress is written in the equation (2.13).

$$dF_{x,net} = \left(\frac{\partial \tau_{xx}}{\partial x} d_x \right) d_y d_z + \left(\frac{\partial \tau_{yx}}{\partial y} d_y \right) d_x d_z + \left(\frac{\partial \tau_{zx}}{\partial z} d_z \right) d_x d_y \tag{2.13}$$

$dF_{x,net}$ is divided by volume $d_x d_y d_z$. Hence force per unit volume is found and written in the equation (2.14)

$$f_{surface} = \frac{\partial \tau_{ij}}{\partial x_j} = \nabla.\tau_{ij} \tag{2.14}$$

Equation (2.14) is substituted to the equation (2.10).

$$\rho \frac{DV}{Dt} = \rho g + \nabla.\tau_{ij} \tag{2.15}$$

It remains only to express $\nabla.\tau_{ij}$ in terms of the velocity V. This is done by relating τ_{ij} to ε_{ij} by using some viscous deformation rate law for a Newtonian Fluid. The simpler form of the viscous deformation law is given in the equation (2.16).

$$\tau_{xx} = -p + K\varepsilon_{xx} + C_2 divV \tag{2.16}$$

Indicial notation form of the simpler form of the viscous deformation law is written in the equation (2.17).

$$\tau_{ij} = -p\delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \delta_{ij} \lambda divV \tag{2.17}$$

in which λ is volume viscosity or dilatational viscosity. Hence representation of $\nabla.\tau_{ij}$ in terms of the velocity component is calculated.

Representations of surface forces and body forces in desired form are substituted to the equation (2.10). Found equation is the famous equation of motion that name is Navier Stokes equation that named after Claude Louis Navier and George Gabriel Stokes. Indicial notation of the Navier Stokes equation as a single vector equation is

$$\rho \frac{Du_i}{Dt} = \rho g_i + \frac{\partial}{\partial x_j}\left[\left(-p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \delta_{ij}\lambda \frac{\partial u_k}{\partial x_k}\right)\right] \tag{2.18}$$

If the flow is incompressible, $\delta_{ij}\lambda \frac{\partial u_k}{\partial x_k}$ is equal to zero. The flow is incompressible in this study. Vectorial representation of the Navier Stokes equation in three different directions is

x direction

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{2.19}$$

y direction

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + z\frac{\partial v}{\partial z}\right) = -\frac{\partial P}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \tag{2.20}$$

z direction

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}\right) = -\frac{\partial P}{\partial w} + \rho g_w + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \tag{2.21}$$

2.3 Dimensionless Numbers

There are too many reasons for using dimensionless numbers. To begin with, dimensionless numbers are not just numbers. They contain significant informations about characteristic of a given flow. Dimensionless numbers give information about how the system will behave and allow for comparisons between different systems. Also helps us to scale problem. This minimize the numerical errors of the study. In this way complexity of the problem abates. In the subtitle of the dimensionless numbers section, some important dimensionless numbers that was used in the mathematical section were given.

2.3.1 Reynolds number

The history of the Reynolds number that back to in 1908 that is still valid today. Reynold is the ratio of inertial forces to viscous forces. Formulation of the Reynolds number is

$$\mathrm{Re} = \frac{\rho V L}{\mu} \tag{2.22}$$

in which ρ is density, L is length, V is velocity and μ is dynamic viscosity of fluid. Reynolds number is used to analyze different flow regimes such as laminar, turbulent and transitional. High Reynolds number means that inertial forces are higher than viscous forces. When the inertial forces are dominant, flow is named as turbulent. Smaller Reynolds number means viscous forces are bigger than inertial forces. When the viscous forces are dominant, flow is named as laminar

2.3.2 Darcy number

Darcy number gives information about effect of the permeability of the medium versus its cross sectional area. Formulation of the Darcy number is

$$Da = \frac{k_p}{H^2} \tag{2.23}$$

in which k_p is permeability of the porous medium and H is the total height of the channel.

2.4 Porous Flow Models

The importance of research in flow, heat and mass transfer in porous media are increase during the past several decades. The importance of this research comes from in many engineering applications. Viscous flow through porous media has a wide range of application area such as thermal insulation, air filter technology, petroleum industries, electronic cooling, geothermal systems, biological systems and so on [12]. As a consequence of these wide range of application area defining equations that describing viscous flow through porous media attract considerable great theoretical and experimental attention from the scientific community. Be able to reduce complexity of these physical problems, porous media models are improved by scientist and they are used widely in the literature. Fundamentally there are three porous media models used in the theoretical studies. These three models are Brinkman's equation, Darcy's equation and Richard's equation. Each porous media model has own specific advantages.

2.4.1 Brinkman equation

As mentioned before, there has been increasing rate of interest on heat and fluid flows through porous media from varying disciplines such as engineering and science. The Brinkman equations describe flow through porous media where momentum transport by shear stresses in the fluid is important.

The Brinkman equation is applicable for fluid that moves at high velocities and high permeability areas. The common problem type that is solved by using Brinkman equation is combinations of free flow and porous media. These type of problems are seen in the filtration problem and separation problem of the chemical reaction engineering such as modelling of porous catalysts in monolithic reactors. In the Brinkman equation, velocity and pressure are dependent parameters.

$$\frac{\partial(\varepsilon_p \rho)}{\partial t} + div(\rho u) = Q_{br} \tag{2.24}$$

in which Q_{br} is fluid source, ε_p represent porosity. The momentum equation in the Brinkman form is

$$\frac{\rho}{\varepsilon_p}\left(\frac{\partial u}{\partial t} + (u.div)\frac{u}{\varepsilon_p}\right) = -divp + div\left[\frac{1}{\varepsilon_p}\left\{\mu(divu + (divu)^T) - \frac{2}{3}\mu(divu)I\right\}\right]\text{v}$$

$$-\left(k^{-1}\mu + \frac{Q_{br}}{\varepsilon_p^2}\right)u + f \tag{2.25}$$

in which I is identity matrix, T denotes temperature, μ is the dynamic viscosity, k denotes the porosity tensor and f represents volume forces and gravity

2.4.2 Darcy's law equation

Darcy's Law equation has been investigated the flow of fluids through permeable material by a French hydraulic engineer named Henry Darcy in 1856. In the Darcy's equation fluid viscosity, pore structure and pressure gradient are the base parameters in order to define the fluid velocity. It has a major solving effectiveness for low fluid velocity due to friction resistance. Darcy's velocity is given by

$$u = -\frac{k}{\mu}(divp + \rho g divh) \tag{2.26}$$

in which k is the porous media permeability, u is Darcy's velocity

2.4.3 Richard's equation

The Richard's equation has been introduced by Richards in 1931. The Richard's equation is basic theoretical equation for vertical unsaturated flow. It has many variations but both forms of Richard's equation are used in unsaturated-zone modelling. By using Reynolds Transport Theorem, one directional flow in an unsaturated porous media is

$$\frac{\partial \theta}{\partial t} + \frac{\partial q}{\partial z} = 0 \tag{2.27}$$

in which θ and q are unknowns. Experimental expression that is proven by Darcy is

$$q = -Ki \tag{2.28}$$

in the above expression, K is hydraulic conductivity and i is hydraulic gradient. This expression is only valid for saturated flow. Formulation of the hydraulic gradient is

$$i = \frac{\partial}{\partial z}\left(\frac{p}{\rho g} + z\right) \tag{2.29}$$

Mathematical representation of the hydraulic conductivity is

$$\left(\frac{p}{\rho g} + z\right) = H \tag{2.30}$$

in which H is equal to suction head and gravity head

In unsaturated conditions, the expression for Darcy velocity is

$$q = -K\frac{\partial H}{\partial z} \tag{2.31}$$

This expression should be valid for unsaturated conditions. Be able to satisfy this, hydraulic conductivity H should be function of both θ and z

$$q = -K\frac{\partial(\psi + z)}{\partial z} \tag{2.32}$$

$$\frac{\partial \psi}{\partial z} = \frac{\partial \psi}{\partial \theta}\frac{\partial \theta}{\partial z} \tag{2.33}$$

in which θ is the volumetric soil water content, $\frac{\partial \theta}{\partial z}$ is the gradient of water content in vertical direction, $\frac{\partial \psi}{\partial \theta}$ is specific water capacity.

$$q = -K(\frac{\partial \psi}{\partial \theta}\frac{\partial \theta}{\partial z} + 1) \tag{2.34}$$

soil water diffusivity D is equal to

$$D = K\frac{\partial \psi}{\partial \theta} \tag{2.35}$$

D is substituted to the equation

$$q = -\left[D\frac{\partial \theta}{\partial z} + K\right] \tag{2.36}$$

One directional Richard’s Equation in an unsaturated porous media is

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[D\frac{\partial \theta}{\partial z} + K\right] \tag{2.37}$$

Thanks to θ based form, calculation is faster than alternatives.

3. MATHEMATICAL MODELLING

3.1 Introduction

In the present study, numerical modelling of uniform flow over a porous plane with downward suction by using semi analytical numerical methods was studied. Numerical models that mentioned before are differential quadrature method and method of moments. Schematic diagram of the study that a steady uniform flow passes through a porous plane is seen in the Figure 3.1.

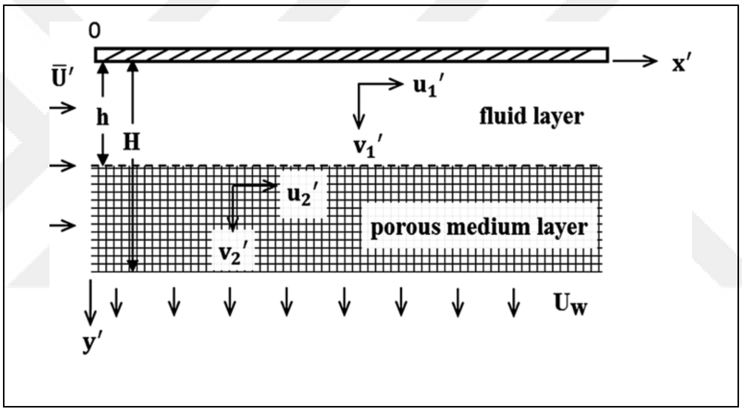

Figure 3.1: Schematic representation of the problem.

In the mathematical modelling section, the problem was divided into two different sections such as fluid layer and porous layer as seen in the schematic representation. The subscript 1 and 2 were used to separate the fluid layer and the porous layer from each other. According to this two different layer, the governing equations and dimensionless governing equations were derived. Remaining variables such as U, H and so on that are seen in the figure were explained in the section of governing equations and dimensionless governing equations. After obtaining final mathematical expressions of the layers, boundary conditions were defined.

Before starting to mathematical modelling, the assumptions that are followed during the modelling are given.

- The first one is defining dimension. The problem is considered two dimensional;
- The fluid is Newtonian;
- The fluid is viscous and incompressible;
- Viscosity of the fluid is constant;
- Gravity effect is neglected;
- Porous medium is saturated, isotropic and homogeneous;
- Solid matrix is unmovable and rigid;

3.2 Deciding Porous Flow Models

Darcy's, Brinkman's and Richard's equations are popularly employed to treat the porous media flow. However, these equations contain empirical constants that lead to cumbersome during the solving stages. In this study, Biot's theory of poroelasticity is used. By using Biot's theory, I overcome the problem without evaluation of the empirical constants. Creation of the discontinuity is the another negativity of the Darcy, Brinkman and Richard's equation. For example Navier Stokes equations is second order and Darcy is first order equation. This situation also leads to discontinuity in the interface of the two different layers. Be able to solve this difficulty, different boundary conditions are used at the interface of the problem. This is the most difficult part of the mathematical modelling section. In this study, Biot's theory of poroelasticity is used and with Brinkman extended Darcy's equation.

3.3 Governing Equations

The continuity equation and the equation of motion were used to describe the fluid. As mentioned before, fluid flow is two dimensional. The Navier Stokes equations in x and y directions were used to express the flow.

3.3.1 Fluid layer

The continuity equation is seen in the equation (3.1)

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \tag{3.1}$$

in which subscript 1 denotes fluid layer, u_1 and v_1 are components of the velocity in x and y direction.

Equations of motion in x and y direction are seen in the equation (3.2) and (3.3)

$$u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} = -\frac{1}{\rho}\frac{\partial P_1}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2}\right) \tag{3.2}$$

$$u_1 \frac{\partial v_1}{\partial x} + v_1 \frac{\partial v_1}{\partial y} = -\frac{1}{\rho}\frac{\partial P_1}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2}\right) \tag{3.3}$$

in which μ is dynamic viscosity of fluid, ρ is density of fluid and P is pressure in the fluid layer.

3.3.2 Porous layer

In the theory section of the study, three different porous flow models that are taken place in literature were given. In the present paper, simplified version of Biot's theory of poroelasticity is used to describe the porous medium flow and Brinkman-extended Darcy equation as the equation of motion was used by Deng and Martinez who are owners of the reference study.

The continuity equation for the porous layer is given by the equation (3.4)

$$\frac{\partial(nu_2)}{\partial x} + \frac{\partial(nv_2)}{\partial y} = 0 \tag{3.4}$$

in which subscript 2 denotes porous layer, n is porosity, u_2 and v_2 are components of the velocity in x and y direction.

Equations of motion in x and y direction are given by in the equation (3.5) and (3.6)

$$\frac{\gamma n u_2}{K} = -\frac{\partial P_2}{\partial x} + \mu\left(\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2}\right) \tag{3.5}$$

$$\frac{\gamma n v_2}{K} = -\frac{\partial P_2}{\partial y} + \mu\left(\frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial y^2}\right) \tag{3.6}$$

in which K is the hydraulic conductivity, P_2 is pressure in porous layer.

3.4 Dimensionless Governing Equations

Nondimensionalization is a technique that can simplify complex mathematical problem and reduces the number of variables in the problem. It is most desirable technique such complex problems that contain so many variables and equations. This method creates relationship between variables in terms of nondimensional parameters such as Reynolds number, Darcy number, Prandtl number, Froude number and so on. In this part of the study, all equations that are found in the governing equations section via continuity and Navier Stokes equations are converted to dimensionless governing equations. Thus the labor of calculation reduced.

3.4.1 Fluid layer

Dimensionless variables that are used in the nondimensionalization process of the continuity equation and Navier Stokes equation were given. Dimensionless variables for coordinates are given in the equation (3.7)

$$x^* = \frac{x}{H} \qquad y^* = \frac{y}{H} \tag{3.7}$$

in which H is the total height of the channel

Dimensionless variables for velocity components are given in the equation (3.8)

$$u_1^* = \frac{u_1}{U_w} \qquad v_1^* = \frac{v_1}{U_w} \tag{3.8}$$

in which U_w is the uniform suction velocity at porous wall

Dimensionless variable for pressure was given in the equation (3.9)

$$P_1^* = \frac{P_1}{\rho U_w^2} \tag{3.9}$$

Dimensionless velocity components in equation (3.8) and dimensionless coordinates in equation (3.7) were substituted to continuity equation in equation (3.10).

$$\frac{U_w}{H}\frac{\partial u_1^*}{\partial x^*} + \frac{U_w}{H}\frac{\partial v_1^*}{\partial y} = 0 \tag{3.10}$$

Thus continuity equation in equation (3.1) was converted to dimensionless form in equation (3.11).

$$\frac{\partial u_1^*}{\partial x^*}+\frac{\partial v_1^*}{\partial y}=0 \tag{3.11}$$

Dimensionless velocity components in equation (3.8), dimensionless coordinates in equation (3.7) and dimensional pressure coordinate in equation (3.9) are substituted to x component of the momentum equation in equation (3.2) and equation (3.12) was found.

$$\frac{u_1^* U_w U_w}{H}\frac{\partial u_1^*}{\partial x^*}+\frac{v_1^* U_w U_w}{H}\frac{\partial u_1^*}{\partial y^*}=-\frac{\rho U_w^2}{\rho H}\frac{\partial P_1^*}{\partial x^*}+\frac{\mu}{\rho}\left(\frac{U_w}{H^2}\frac{\partial^2 u_1^*}{\partial x^{*2}}+\frac{U_w}{H^2}\frac{\partial^2 u_1^*}{\partial y^{*2}}\right) \tag{3.12}$$

All components of the equation (3.12) were multiplied with $\frac{H}{U_w U_w}$ and equation (3.13) was found.

$$u_1^*\frac{\partial u_1^*}{\partial x^*}+v_1^*\frac{\partial u_1^*}{\partial y^*}=-\frac{\partial P_1^*}{\partial x^*}+\frac{\mu}{\rho H U_w}\left(\frac{\partial^2 u_1^*}{\partial x^{*2}}+\frac{\partial^2 u_1^*}{\partial y^{*2}}\right) \tag{3.13}$$

in which $\frac{\mu}{\rho H U_w}$ is inverse of the dimensionless Reynolds number. This dimensionless number is substituted to momentum equation in x direction and equation (3.14) was found.

$$u_1^*\frac{\partial u_1^*}{\partial x^*}+v_1^*\frac{\partial u_1^*}{\partial y^*}=-\frac{\partial P_1^*}{\partial x^*}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u_1^*}{\partial x^{*2}}+\frac{\partial^2 u_1^*}{\partial y^{*2}}\right) \tag{3.14}$$

Be able to calculate dimensionless form of the y component of the momentum equation, dimensionless velocity components in equation (3.8), dimensionless coordinates in equation (3.7) and dimensional pressure coordinate in equation (3.9) are substituted to the equation (3.3) and equation (3.15) was found.

$$\frac{u_1^* U_w U_w}{H}\frac{\partial v_1^*}{\partial x^*}+\frac{v_1^* U_w U_w}{H}\frac{\partial v_1^*}{\partial y^*}=-\frac{\rho U_w^2}{\rho H}\frac{\partial P_1^*}{\partial y^*}+\frac{\mu}{\rho}\left(\frac{U_w}{H^2}\frac{\partial^2 v_1^*}{\partial x^{*2}}+\frac{U_w}{H^2}\frac{\partial^2 v_1^*}{\partial y^{*2}}\right) \tag{3.15}$$

Again all components of the equation (3.15) were multiplied with $\frac{H}{U_w U_w}$ and equation (3.16) was found.

$$u_1^* \frac{\partial v_1^*}{\partial x^*} + v_1^* \frac{\partial v_1^*}{\partial y^*} = -\frac{\partial P_1^*}{\partial y^*} + \frac{\mu}{\rho H U_w}\left(\frac{\partial^2 v_1^*}{\partial x^{*2}} + \frac{\partial^2 v_1^*}{\partial y^{*2}}\right) \tag{3.16}$$

in which $\frac{\mu}{\rho H U_w}$ is inverse of the dimensionless Reynolds number. This dimensionless number is substituted to momentum equation in y direction and equation (3.17) was found.

$$u_1^* \frac{\partial v_1^*}{\partial x^*} + v_1^* \frac{\partial v_1^*}{\partial y^*} = -\frac{\partial P_1^*}{\partial y^*} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_1^*}{\partial x^{*2}} + \frac{\partial^2 v_1^*}{\partial y^{*2}}\right) \tag{3.17}$$

After calculations dimensionless form of the momentum equation in x and y direction were found. Unknown variable P_1^* was seen in dimensionless momentum equation both in x direction and y direction. Same unknown variables were eliminated from both of the equations. Be able to complete this process first derivative of the momentum equation in x direction was taken with respect to y and was written in equation (3.18).

$$\frac{\partial u_1^*}{\partial y^*}\frac{\partial u_1^*}{\partial x^*} + u_1^* \frac{\partial^2 u_1^*}{\partial x^* \partial y^*} + \frac{\partial v_1^*}{\partial y^*}\frac{\partial u_1^*}{\partial y^*} + v_1^* \frac{\partial^2 u_1^*}{\partial y^{*2}} = -\frac{\partial P_1^*}{\partial x^* \partial y^*} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^3 u_1^*}{\partial x^{*2} \partial y^*} + \frac{\partial^3 u_1^*}{\partial y^{*3}}\right) \tag{3.18}$$

Also first derivative of the momentum equation in y direction was taken with respect to x and result was written in equation (3.19).

$$\frac{\partial u_1^*}{\partial x^*}\frac{\partial v_1^*}{\partial x^*} + u_1^* \frac{\partial^2 v_1^*}{\partial x^{*2}} + \frac{\partial v_1^*}{\partial x^*}\frac{\partial v_1^*}{\partial y^*} + v_1^* \frac{\partial^2 v_1^*}{\partial y^* \partial x^*} = -\frac{\partial P_1^*}{\partial y^* \partial x^*} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^3 v_1^*}{\partial x^{*3}} + \frac{\partial^3 v_1^*}{\partial y^{*2} \partial x^*}\right) \tag{3.19}$$

Equation (3.18) is subtracted from equation (3.19), thereby pressure was eliminated from the momentum equation. Final state of the dimensionless equation for fluid layer was written in the equation (3.20)

$$\frac{\partial u_1^*}{\partial y^*}\frac{\partial u_1^*}{\partial x^*} + u_1^* \frac{\partial^2 u_1^*}{\partial x^* \partial y^*} + \frac{\partial v_1^*}{\partial y^*}\frac{\partial u_1^*}{\partial y^*} + v_1^* \frac{\partial^2 u_1^*}{\partial y^{*2}} - \frac{\partial u_1^*}{\partial x^*}\frac{\partial v_1^*}{\partial x^*} - u_1^* \frac{\partial^2 v_1^*}{\partial x^{*2}} - \frac{\partial v_1^*}{\partial x^*}\frac{\partial v_1^*}{\partial y^*} - v_1^* \frac{\partial^2 v_1^*}{\partial y^* \partial x^*} =$$

$$= \frac{1}{\mathrm{Re}}\left(\frac{\partial^3 u_1^*}{\partial x^{*2} \partial y^*} + \frac{\partial^3 u_1^*}{\partial y^{*3}} - \frac{\partial^3 v_1^*}{\partial x^{*3}} - \frac{\partial^3 v_1^*}{\partial y^{*2} \partial x^*}\right) \tag{3.20}$$

And then stream function was defined. Reference author of the stream function is

Deng and Martinez. Stream function that is suggestion by Deng and Martinez is written as follows in equation (3.21).

$$\psi_1 = \left(\bar{U} - x\right) f\left(y\right) \qquad 0 \le y \le \xi \tag{3.21}$$

in which $\bar{U}$ is dimensionless velocity, ξ is the interface position and defines as $\xi = \frac{h}{H}$.

By using stream function, u and v components of the velocity in fluid layer were calculated.

u that is x component of the velocity is written in the equation (3.22)

$$u_1^* = \frac{\partial \psi_1}{\partial y} = \left(U - x\right) f^{'}\left(y\right) \tag{3.22}$$

And v that is y component of the velocity is written in the equation (3.23)

$$v_1^* = -\frac{\partial \psi_1}{\partial x} = f\left(y\right) \tag{3.23}$$

Equation (3.20) is rearranged with new velocity components. Equation (3.22) and equation (3.23) were substituted to the equation (3.20). Be able to make this arrangement, first, second and third order partial derivative of the x and y component of the velocity were taken.

Required derivative calculation of the x component of the velocity with respect to x and y are given in the equation (3.24)

$$\left\{\begin{matrix} \dfrac{\partial u_1^*}{\partial x^*} = -f^{I}(y) & \dfrac{\partial^2 u_1^*}{\partial y^{*2}} = (\bar{U} - x) f^{III}(y) \\ \dfrac{\partial u_1^*}{\partial y^*} = (\bar{U} - x) f^{II}(y) & \dfrac{\partial^3 u_1^*}{\partial y^{*3}} = (\bar{U} - x) f^{IV}(y) \\ \dfrac{\partial^2 u_1^*}{\partial x^* \partial y^*} = -f^{II}(y) & \dfrac{\partial^3 u_1^*}{\partial x^{*2} \partial y^*} = 0 \end{matrix}\right\} \tag{3.24}$$

Required derivative calculation of the y component of the velocity with respect to x and y are given in the equation (3.25)

$$\left\{\begin{array}{ll} \dfrac{\partial v_1^*}{\partial y^*} = f^I(y) & \dfrac{\partial^2 v_1^*}{\partial x^{*2}} = 0 \\ \dfrac{\partial v_1^*}{\partial x^*} = 0 & \dfrac{\partial^3 v_1^*}{\partial x^{*3}} = 0 \\ \dfrac{\partial^2 v_1^*}{\partial y^* \partial x^*} = 0 & \dfrac{\partial^3 v_1^*}{\partial y^{*2} \partial x^*} = 0 \end{array}\right\} \tag{3.25}$$

Derivatives in the equation (3.24) and (3.25) are substituted to the equation (3.20) and equation (3.26) is found.

$$-(U-x) f^{"}(y) f^{'}(y) - (U-x) f^{'}(y) f^{II}(y) + (U-x) f^{I}(y) f^{II}(y) +$$

$$+(U-x) f(y) f^{III}(y) = \frac{1}{\mathrm{Re}}\left(0 + (U-x) f^{IV}(y) - 0 - 0\right) \tag{3.26}$$

Equation (3.26) was rearranged and final form of the fluid layer is found in the equation (3.27).

$$\mathrm{Re}\left(f(y) f^{III}(y) - f^{I}(y) f^{II}(y)\right) = f^{IV}(y) \tag{3.27}$$

3.4.2 Porous layer

Again dimensionless variables that are used in the nondimensionalization process of the continuity equation and Navier Stokes equation for porous layer were given. Dimensionless variables for velocities in the porous layer with subscript 2 are given in the equation (3.28)

$$u_2^* = \frac{u_2}{U_w} \qquad v_2^* = \frac{v_2}{U_w} \tag{3.28}$$

Dimensionless variable for pressure was given in the equation (3.29)

$$P_2^* = \frac{P_2}{\rho U_w^2} \tag{3.29}$$

Dimensionless velocity components in equation (3.28) and dimensionless coordinates in equation (3.7) were substituted to continuity equation in equation (3.4) and equation (3.30) is obtained.

$$\frac{nU_w}{H}\frac{\partial u_2^*}{\partial x^*} + \frac{nU_w}{H}\frac{\partial v_2^*}{\partial y} = 0 \tag{3.30}$$

All sides of the equation (3.30) was multiplied with the $\frac{H}{nU_w}$. Thus continuity equation in equation (3.4) transformed to the dimensionless form and it is written in the equation (3.31)

$$\frac{\partial u_2^*}{\partial x^*}+\frac{\partial v_2^*}{\partial y}=0 \tag{3.31}$$

Then dimensionless velocity components in equation (3.28), dimensionless coordinates in equation (3.7) and dimensional pressure coordinate in equation (3.29) are substituted to x component of the momentum equation in equation (3.5) and equation (3.32) was found.

$$\frac{\gamma n U_w u_2^*}{K}=-\frac{\rho U_w^2}{H}\frac{\partial P_2^*}{\partial x}+\frac{\mu U_w}{H^2}\left(\frac{\partial^2 u_2^*}{\partial x^2}+\frac{\partial^2 u_2}{\partial y^2}\right) \tag{3.32}$$

All elements of the equation (3.32) were multiplied with $\frac{H}{\rho U_w^2}$ and equation (3.33) was found.

$$0=-\frac{\partial P_2^*}{\partial x^*}+\frac{\mu}{\rho H U_w}\left(\frac{\partial^2 u_2^*}{\partial x^{*2}}+\frac{\partial^2 u_2^*}{\partial y^{*2}}\right)-\frac{\gamma n H}{K\rho U_w}u_2^* \tag{3.33}$$

Reynold and Darcy number are substituted to the equation (3.33) and equation (3.34) was obtained.

$$.\,0=-\frac{\partial P_2^*}{\partial x^*}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u_2^*}{\partial x^{*2}}+\frac{\partial^2 u_2^*}{\partial y^{*2}}\right)-\frac{n}{D_a\,\mathrm{Re}}u_2^* \tag{3.34}$$

Then dimensionless velocity components in equation (3.28), dimensionless coordinates in equation (3.7) and dimensional pressure coordinate in equation (3.29) are substituted to y component of the momentum equation in equation (3.6) and equation (3.35) was found.

$$\frac{\gamma n U_w v_2^*}{K}=-\frac{\rho U_w^2}{H}\frac{\partial P_2^*}{\partial y^*}+\frac{\mu U_w}{H^2}\left(\frac{\partial^2 u_2^*}{\partial x^{*2}}+\frac{\partial^2 v_2^*}{\partial y^{*2}}\right) \tag{3.35}$$

All elements of the equation (3.35) were multiplied with $\frac{H}{\rho U_w^2}$ and equation (3.36) was found.

$$0=-\frac{\partial P_2^*}{\partial y^*}+\frac{\mu}{\rho H U_w}\left(\frac{\partial^2 v_2^*}{\partial x^{*2}}+\frac{\partial^2 v_2^*}{\partial y^{*2}}\right)-\frac{\gamma n H}{K\rho U_w}v_2^* \tag{3.36}$$

Reynold and Darcy number are substituted to the equation (3.36) and equation (3.37) was obtained.

$$0 = -\frac{\partial P_2^*}{\partial y^*} + \frac{1}{\text{Re}}\left(\frac{\partial^2 v_2^*}{\partial x^{*2}} + \frac{\partial^2 v_2^*}{\partial y^{*2}}\right) - \frac{n}{D_a \text{Re}} v_2^* \tag{3.37}$$

After calculations dimensionless form of the momentum equation of porous layer in x and y direction were found. Unknown variable P_2^* was seen in dimensionless momentum equation both in x direction and y direction. Same unknown variables were eliminated from both of the equations.

Be able to complete this process first derivative of the momentum equation in x direction was taken with respect to y and was written in equation (3.38).

$$0 = -\frac{\partial^2 P_2^*}{\partial x^* \partial y^*} + \frac{1}{\text{Re}}\left(\frac{\partial^3 u_2^*}{\partial x^{*2} \partial y^*} + \frac{\partial^3 u_2^*}{\partial y^{*3}}\right) - \frac{n}{D_a \text{Re}} \frac{\partial u_2^*}{\partial y^*} \tag{3.38}$$

Also first derivative of the momentum equation in y direction was taken with respect to x and result was written in equation (3.39).

$$0 = -\frac{\partial P_2^*}{\partial y^* \partial x^*} + \frac{1}{\text{Re}}\left(\frac{\partial^3 v_2^*}{\partial x^{*3}} + \frac{\partial^3 v_2^*}{\partial y^{*2} \partial x^*}\right) - \frac{n}{D_a \text{Re}} \frac{\partial v_2^*}{\partial x^*} \tag{3.39}$$

Equation (3.39) is subtracted from equation (3.38), thereby pressure was eliminated from the momentum equation. Final state of the dimensionless equation for fluid layer was written in the equation (3.40)

$$0 = \frac{1}{\text{Re}}\left(\frac{\partial^3 u_2^*}{\partial x^{*2} \partial y^*} + \frac{\partial^3 u_2^*}{\partial y^{*3}} - \frac{\partial^3 v_2^*}{\partial x^{*3}} - \frac{\partial^3 v_2^*}{\partial y^{*2} \partial x^*}\right) - \frac{n}{D_a \text{Re}}\left(\frac{\partial u_2^*}{\partial y^*} - \frac{\partial v_2^*}{\partial x^*}\right) \tag{3.40}$$

Stream function adopted by Deng and Martinez is given in the equation (3.41)

$$\psi_2 = (\overline{U} - x).g(y) \qquad \xi \le y \le 1 \tag{3.41}$$

By using stream function, u and v components of the velocity in fluid layer were calculated.

u that is x component of the velocity is written in the equation (3.42)

$$u_2^* = \frac{\partial \psi_2}{\partial y} = (\bar{U} - x).g^I(y) \tag{3.42}$$

And v that is y component of the velocity is written in the equation (3.43)

$$v_2^* = -\frac{\partial \psi_2}{\partial x} = g\left(y\right) \tag{3.43}$$

Equation (3.40) is rearranged with new velocity components. Equation (3.42) and equation (3.43) were substituted to the equation (3.40). Be able to make this arrangement, first, second and third order partial derivative of the x and y component of the velocity were taken.

Required derivative calculation of the x component of the velocity with respect to x and y are given in the equation (3.44)

$$\left\{ \begin{matrix} \frac{\partial u_2^*}{\partial y^*} = (\bar{U} - x) g^{II}(y) & \frac{\partial u_2^*}{\partial x^*} = -g^I(y) \\ \frac{\partial^2 u_2^*}{\partial y^{*2}} = (\bar{U} - x) g^{III}(y) & \frac{\partial^2 u_2^*}{\partial x^{*2}} = 0 \\ \frac{\partial^3 u_2^*}{\partial y^{*3}} = (\bar{U} - x) g^{IV}(y) & \frac{\partial^3 u_2^*}{\partial x^{*2} \partial y^*} = 0 \end{matrix} \right\} \tag{3.44}$$

Required derivative calculation of the y component of the velocity with respect to x and y are given in the equation (3.45)

$$\left\{ \begin{matrix} \frac{\partial v_2^*}{\partial x^*} = 0 & \frac{\partial^3 v_2^*}{\partial y^{*3}} = g^{III}(y) \\ \frac{\partial^2 v_2^*}{\partial y^{*2}} = g^{II}(y) & \frac{\partial^3 v_2^*}{\partial y^{*2} \partial x^*} = \end{matrix} \right\} \tag{3.45}$$

Derivatives in the equation (3.44) and (3.45) are substituted to the equation (3.20) and equation (3.46) is found.

$$0 = \frac{1}{\mathrm{Re}} \left((\bar{U} - x) g^{IV}(y) \right) - \frac{n}{D_a \mathrm{Re}} \left((\bar{U} - x) g^{II}(y) \right) \tag{3.46}$$

All compents of the equation (3.46) is multiplied with Re and divided by $(\bar{U} - x)$ and finally equation (3.47) was found.

$$0 = g^{IV}(y) - \frac{n}{D_a} g^{II}(y) \tag{3.47}$$

3.5 Boundary Conditions

In this type of physical problems, an interface section in between fluid layer and porous layer is very critical part of the problem. Many different types of boundary conditions at the interface are improved by different researchers. For instance, continuity of tangential velocity, continuity of both shear stress and normal stress, discontinuity of shear stress, discontinuity of both tangential velocity and interfacial velocity are seen as boundary conditions in different studies. Correct specification of boundary conditions is the essential part of the mathematical modelling. In this study, the continuity of the velocity vector and shear stress are used as the boundary conditions. In addition to these equations, an extra equation that is derived is used as the boundary condition. Dimensionless boundary conditions are;

- At the top of the fluid layer (y=0), x and y components of the velocity are zero.

$$\begin{cases} u_1 = (\bar{U} - x).f^{I}(0) \\ v_1 = f(0) \end{cases} \text{at y=0} \Rightarrow \begin{cases} f^{I}(0) = 0 \\ f(0) = 0 \end{cases} \tag{3.48}$$

- At the bottom of the porous layer (y=1), horizontal component of the vecity is equal to zero, only vertical component of the velocity exists.

$$\begin{cases} u_2 = (\bar{U} - x).g^{I}(1) \\ v_1 = g(1) \end{cases} \text{at y=1} \Rightarrow \begin{cases} g^{I}(1) = 0 \\ g(1) = 1 \end{cases} \tag{3.49}$$

- As mentioned before, at the interface section of the fluid and porous layer $y = \xi$ continuity of velocity vector is satisfied

$$g^{I}(\xi) = f^{I}(\xi) \qquad g(\xi) = f(\xi) \tag{3.50}$$

- At the interface fluid layer and porous layer, continuity of the shear stress is satisfied. Formulation of the shear stress is

$$\tau = \mu \frac{\partial u}{dy} \tag{3.51}$$

Shear stress in fluid layer is

$$\tau = \mu \frac{\partial u_1}{dy} = \mu(\bar{U} - x) f^{II}(y) \tag{3.52}$$

Shear stress in porous layer is

$$\tau = \mu \frac{\partial u_2}{dy} = \mu(\bar{U} - x) g^{II}(y) \tag{3.53}$$

Equation (3.52) and equation (3.53) are combined and equation (3.54) is found

$$\mu(\bar{U} - x) g^{II}(y) = \mu(\bar{U} - x) f^{II}(y) \Rightarrow g^{II}(y) = f^{II}(y) \tag{3.54}$$

- In addition to this, equation (3.27) and equation (3.47) are combined by integrating them and equation (3.55) is obtained

$$g^{III}(\xi) = f^{II}(\xi) + \frac{n}{Da} f^{I}(\xi) + R(f^{I^2}(\xi) - f(\xi) f^{II}(\xi)) \tag{3.55}$$

4. DIFFERENTIAL QUADRATURE METHOD

4.1 Introduction

Every process in nature can be expressed by differential equations. For this reason, differential equations take an important place in science and engineering because engineers can examine the change of the critical variables that have a place in system thanks to differential equations. And also, it is easier to understand physical phenomena through differential equations. In addition, a differential equation is a way to describe the physical pheonomenas in the same language through all over the world by constructing mathematical models. Poisson, Navier Stokes, Helmholtz, linear wave and advection convection equations are example of partial differential equation. Poisson's equation is a partial differential equation of elliptic type with a wide range of applications such as electrostatics, mechanical engineering and so on. Navier Stokes equations describe the motion of the fluid. Helmholtz equation simulates the microwaves. All of these equations that we only mention a few examples are partial differential equations. Be able to developing result to these equations is as important as identification of the equation.

To solve these partial differential equations, many methods are used such as finite difference, finite volume and finite element and so on. All the time, there are expected properties from the selected method that has the advantage compared to others. While deciding on methods that will be applied to the equation, some criteria like that higher accuracy, less computation time by using less grid point, the stability of the solution are very important.

Differential quadrature method (DQM) is also a different numerical technique that was presented by Bellman and his associates in the early 1970's is used in the solution of ordinary differential equations (ODE's) or partial differential equations (PDE's) of the initial and boundary value problems. Since that time, this method has been successfully applied to the different kind of problems in engineering and medical science. This method directly aimed to be alternative to the FDM that requires long computation time and it has numerical stability problem. If the DQM compared with the other numerical technique such as the FV, FD and FE methods, this method exhibits higher accuracy and efficiency with little computational effort by using a considerably small number of grid points [13].

4.2 Chronological Development

Differential Quadrature Method that is used for solving ordinary or partial differential equation is developed by Richard Bellman in 1970. In 1971 introductory paper is published by Bellman and Casti. But this paper did not include application or any information about calculations of the weighting coefficients [14]. Subsequent paper that is published by Bellman et al in 1972 include various applications of the DQM and information about the calculation of the weighting coefficients [15]. Results are compared with the exact results.

After these two paper, differential quadrature method is seen in several areas such as hearth model for estimating heart parameters for cardiograms [16]. Thus the method developed quickly. Applications of DQM are found in the different scientific sector such as biosciences, structural mechanics, transport processes and so on. It is claimed that high accurate solutions are found thanks to differential quadrature method by spending minimum computational effort.

DQM is used for the transient analysis of isothermal chemical reactors by Wang in 1982 and it was extended by Naadimuthu for isothermal chemical reactor that involved two initial boundary value equations in terms of partial pressure and temperature.

Civan and Sliepcevich undertook variety application of the quadrature method to engineering problems for different transport phenomena type models. In 1986 an application of the DQM in nuclear engineering problems was undertaken by Passow. Most important improvements are seen after the method is applied to the structural mechanics problems. Subsequent to these works, nonlinear static flexure of thin circular plate is solved with differential quadrature method.

In this thesis mathematical equation of uniform flow over a porous plane with downward suction that is fourth order nonlinear differential equation is solved by using differential quadrature method.

4.3 Structure of the Differential Quadrature Method

The idea of differential quadrature method is first order derivative of a function with respect to a coordinate direction is approximated by a weighted linear sum of all values in the same domain and along same direction. The most critical point of the DQM is computation of the weighting coefficients for the discretization of the first and second order derivatives. Taking into consideration Figure 4.1 that represents the one dimensional problem.

Figure 4.1: One-dimensional problem.

Mathematical representation of the DQM that is expressed before is written in the equation 4.1,

$$f_x(x_i) = \left.\frac{df}{dx}\right|_{x_i} = \sum_{j=1}^{N} a_{ij} f(x_j), \quad \text{for i=1,2,...,N} \tag{4.1}$$

where $f_x(x_i)$ is the first order derivative of the function, N represent the number of grid points in the domain, j represents the grid point (i.e. number of column), i represents the dimension of the problem (i.e. number of row), a_{ij} is the weighting coefficients, x_j is the value of the grid points, $f(x_j)$ is the value of the function at different grid points. Also calculated weighting coefficients a_{ij} are different at different locations according to coordinate axis.

Representation of the DQM formulation through matrices is as seen in equation (4.2).

$$f^{'}(x_1) = \left[a_{11} \ a_{12} \ ...... \ a_{1N}\right]\begin{Bmatrix} f(1) \\ f(2) \\ . \\ . \\ f(N) \end{Bmatrix} \tag{4.2}$$

Two dimensional grid structure can be seen from the Figure 4.2. Number of rows and columns in two dimension is N.

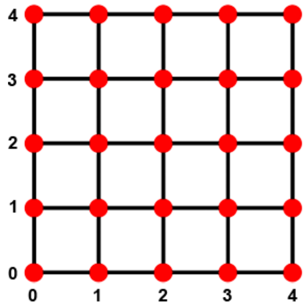

Figure 4.2: Two-dimensional problem.

Two-dimensional matrix representation of the DQM is given in the equation (4.3).

$$\begin{bmatrix} b_{11} & b_{12} & \dots\dots\dots b_{1N} \\ b_{21} & b_{22} & \dots\dots\dots b_{2N} \\ . & . & . \\ . & . & . \\ b_{N1} & b_{N2} & \dots\dots b_{NN} \end{bmatrix} \begin{Bmatrix} f(1) \\ f(2) \\ . \\ . \\ f(N) \end{Bmatrix} = \begin{bmatrix} f'(x_1) \\ f'(x_2) \\ . \\ . \\ f'(x_N) \end{bmatrix} \tag{4.3}$$

Weighting coefficients a_{ij} and b_{ij} are depend on the coordinates, therefore they are different from each other. Weighting coefficients are bridge to link between derivatives and functional values at the mesh points. As can be easily understood from the matrix form, weighting coefficients and functional values are used for computing the derivatives of the function at x_i. Calculations of the derivatives of function at given point is given in equation (4.4).

$$\begin{aligned} & b_{11} f(1) + b_{12} f(2) + \dots + b_{1N} f(N) = f'(x_1) \\ & b_{21} f(1) + b_{22} f(2) + \dots + b_{2N} f(N) = f'(x_2) \\ & \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ & b_{N1} f(1) + b_{N2} f(2) + \dots + b_{NN} f(N) = f'(x_N) \end{aligned} \tag{4.4}$$

Computation of the weighting coefficients that depends on the selected test functions is done by two different fundamental approaches. One of them is polynomial-based differential quadrature (PDQ) and the other one is Fourier expansion-based differential quadrature (FDQ). PDQ is the one that will be used in the thesis.

4.4 Computation of Weighting Coefficients for the First Order Derivative

Calculation of the weighting coefficients which is the most important step of the DQM has three different approaches that belong to Bellman, Quan and Chang, Shu.

4.4.1 Bellman's approaches

Bellman et al. (1972) have two approaches to compute the weighting coefficients a_{ij}. The difference between Bellman's two approaches is the result from the difference in test function.

4.4.1.1 First approach

In Bellman's first approach, test functions are chosen as seen in (4.5),

$$g_k(x) = x^k, \text{ k=0,1,...,N-1} \tag{4.5}$$

where k's interval gives N test functions and rows and columns are taken from 1 up to N. Rows and columns in this amount give the total number of weighing coefficients as NxN. As a consequence the following NxN algebraic equations (4.6) for a_{ij} are obtained.

$$\left\{\begin{array}{l} \sum_{j=1}^{N} a_{ij} = 0 \\ \sum_{j=1}^{N} a_{ij} x_j = 1 \\ \sum_{j=1}^{N} a_{ij} x_j^k = k * x_i^{k-1}, \text{ k=2,3,....,N-1} \end{array}\right\} \text{ for i=1,2,...,N} \tag{4.6}$$

In this approach, if N is large, the matrix is ill-conditioned and for this reason it is difficult to take the inverse of the matrix. In the application of DQM, if Bellman's first approach is used, grid points should be less than 13 for avoiding ill condition situation.

4.4.1.2 Second approach

In the second approach different test function is used by Bellman. The test function was chosen as in the equation (4.7).

$$g_k(x) = \frac{L_N(x)}{(x - x_k) L_N^{(1)}(x_k)}, \text{ k=1,2,....,N} \tag{4.7}$$

where $L_N(x)$ is the Legendre polynomial and first order derivative of the $L_N(x)$ is $L_N^{(1)}(x_k)$. Selected x_k should be the roots of the Legendre polynomial. By using this test function, simple algebraic formulations to compute weighting coefficient a_{ij} is obtained by Bellman et al. are given in equation (4.8) and (4.9).

$$a_{ij} = \frac{L_N^{(1)}(x_i)}{(x_i - x_J)L_N^{(1)}(x_j)}, \text{ for } i \neq j \tag{4.8}$$

$$a_{ii} = \frac{1-2x_i}{2x_i(x_i - 1)} \tag{4.9}$$

When second approach is compared with first approach, it is clear that computation of the weighting coefficients by using second approach is easier. But first approach has greater flexibility in implementation. On the other hand, in the second approach one should use the Legendre polynomials of degree N. Therefore, first approach is easily applied to the practical applications.

4.4.2 Quan and Chang's Approach

To resolve deficiencies that encountered during the calculations of the weighted coefficients in the Bellman's approaches, many studies were done on the issue. One of the most important works is done by Quan and Chang (1989). In the Quan and Chang's approach, Lagrange interpolation polynomials are used as a test functions and there is no restriction for the selection of the grid points. Test function is given in equation (4.10),

$$g_k(x) = \frac{M(x)}{(x - x_k)M^{(1)}(x_k)}, \text{ k=1,2,....,N} \tag{4.10}$$

where calculations of the M(x) given in the equation (4.11)

$$M(x) = (x - x_1)(x - x_2)....(x - x_N) \tag{4.11}$$

Another representation of the equation (4.11) is given in equation (4.12).

$$M^{(1)}(x_i) = \prod_{k=1,k\neq i}^{N} (x_i - x_k) \tag{4.12}$$

After substituting equation (4.11) and (4.12) into (4.10), the formula that is used for calculating the coefficients is obtained and given in equation (4.13) and (4.14).

$$a_{ij} = \frac{1}{(x_j - x_k)} \prod_{k=1,k\neq i}^{N} \frac{(x_i - x_k)}{(x_j - x_k)}, \text{ for } j \neq i \tag{4.13}$$

$$a_{ii} = \sum_{k=1,k\neq i}^{N} \frac{1}{(x_i - x_k)} \tag{4.14}$$

4.4.3 Shu's general approach

Shu's general approach contains all approaches that are Quan and Chang's approach and Bellman's approach. It is based on four basic polynomial type which are given in the equation (4.15), (4.16), (4.17) and (4.18)

$$r_k(x) = x^k - 1 \text{ , k=1,2,...,N} \tag{4.15}$$

$$r_k(x) = \frac{L_N(x)}{(x - x_k)L_N^{(1)}(x_k)} \text{ , k=1,2,...,N} \tag{4.16}$$

$$r_k(x) = \frac{M(x)}{(x - x_k)M^{(1)}(x_k)} \text{ , k=1,2,...,N} \tag{4.17}$$

$$r(x) = 1 \text{ , } r_k(x) = (x - x_{k-1})r_{k-1}(x) \text{ , k=1,2,...,N} \tag{4.18}$$

$M(x)$ is defined in equation (4.11) and (4.12) and $L_N(x)$ is the Legendre polynomial. If $M(x)$ and $L_N(x)$ are compared with each other, it is easily seen that $L_N(x)$ that is only applicable at the Legendre collocation points is a subset of the $M(x)$.

The expressions that are used for the calculating the weighting coefficients defined by the Shu are given in the equation (4.19) and (4.20).

$$a_{ij} = \frac{M^{(1)}(x_i)}{(x_i - x_J)M^{(1)}(x_j)}, \text{ for i} \neq \text{j} \tag{4.19}$$

$$a_{ii} = \frac{M^{(2)}(x_i)}{2M^{(1)}(x_j)} \tag{4.20}$$

Another calculation way for equation (4.20) is given in the equation (4.21). Equation (4.20) and (4.21) are two different formulations to make the same calculations.

$$\sum_{j=1}^{N} a_{ij} = 0 \text{ or } a_{ii} = -\sum_{j=1,\, j\neq i}^{N} a_{ij} \tag{4.21}$$

So far shown formulas are valid for computation of weighting coefficients for the first order derivative. In this thesis work, Shu's general approach is used. Another consideration is computation of weighting coefficients for the second and higher order derivatives.

4.5 Computation of Weighting Coefficients for the Second and Higher Order Derivatives

As seen from the equation (4.22), although calculation of the weighting coefficient is different, expression for computing the second and higher order derivatives is very similar with equation (4.1) for calculating first order derivatives.

$$f_x^{(2)}(x_i) = \left.\frac{d^2 f}{dx^2}\right|_{x_i} = \sum_{j=1}^{N} b_{ij} f(x_j), \quad \text{for i=1,2,...,N} \tag{4.22}$$

Where $f_x^{(2)}(x_i)$ is the second order derivative of the function, N represent the number of grid points in the domain, j represents the grid point (i.e. number of column), i represents the dimension of the problem (i.e. number of row), b_{ij} is the weighting coefficients, x_j is the value of the grid points, $f(x_j)$ is the value of the function at different grid points. Also again calculated weighting coefficients b_{ij} are different at different locations according to coordinate axis.

4.5.1 Weighting coefficients for the second order derivatives

Two approaches will be explained for calculations of the weighting coefficients for the second order derivatives. One of them is Quan and Chang's approach and the other one is Shu's general approach.

4.5.1.1 Quan and Chang's approach

According to Quan and Chang's approach, through Lagrange interpolation polynomials weighting coefficients are calculated as given in the equation (4.23) and (4.24).

$$b_{ij} = \frac{2}{x_j - x_i}\left(\prod_{k=1,k\neq i,j}^{N} \frac{x_i - x_k}{x_j - x_k}\right)\left(\sum_{l=1,l\neq i,j}^{N} \frac{1}{x_i - x_l}\right), \text{ for } i \neq j \tag{4.23}$$

$$b_{ii} = 2\sum_{k=1,k\neq i}^{N-1}\left[\frac{1}{x_i - x_k}\left(\sum_{l=k+1,l\neq i}^{N} \frac{1}{x_i - x_l}\right)\right] \tag{4.24}$$

4.5.1.2 Shu's general approach

According to Shu's general approach calculation of the weighting coefficients for the second and higher order derivatives is similar with coefficients for first order derivatives and polynomial based approximation is used and formulations are given in the equation (4.25) and (4.26).

$$b_{ij} = 2a_{ij}\left(b_{ii} - \frac{1}{x_i - x_l}\right), \text{ for } i \neq j \tag{4.25}$$

$$\sum_{j=1}^{N} b_{ij} = 0 \text{ or } \mathrm{b}_{ii} = -\sum_{j=1,\, j\neq i}^{N} b_{ij} \tag{4.26}$$

4.5.2 Matrix multiplication approach for the second and higher order derivatives

This notation in equation (3.27) is used for second and higher order derivative calculations.

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) \tag{4.27}$$

After reminding equation (4.28) and (4.29) again, a_{ij} for first order derivatives and b_{ij} for second order derivatives, if we substitute DQM approximations in the equation (4.22) to the equation (4.27)

$$\begin{bmatrix} a_{11} & a_{12} & \dots\dots\dots a_{1N} \\ a_{21} & a_{22} & \dots\dots\dots a_{2N} \\ . & . & . \\ . & . & . \\ a_{N1} & a_{N2} & \dots\dots a_{NN} \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_N \end{Bmatrix} = \begin{bmatrix} y_1^{'} \\ y_2^{'} \\ . \\ . \\ y_N^{'} \end{bmatrix} \tag{4.28}$$

$$\begin{bmatrix} b_{11} & b_{12} & \dots\dots\dots b_{1N} \\ b_{21} & b_{22} & \dots\dots\dots b_{2N} \\ . & . & . \\ . & . & . \\ b_{N1} & b_{N2} & \dots\dots b_{NN} \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_N \end{Bmatrix} = \begin{bmatrix} y_1^{"} \\ y_2^{"} \\ . \\ . \\ y_N^{"} \end{bmatrix} \tag{4.29}$$

First derivative is taken and substituted to the equation (4.30).

$$f_x^{(2)}(x_i)=\sum_{k=1}^{N} a_{ik} f_x^{(1)}(x_k) \tag{4.30}$$

Deficit representation of the equation (4.30) is given in the equation (4.31).

$$f_x^{(2)}(x_i)=\sum_{k=1}^{N} a_{ik}\sum_{j=1}^{N} a_{kj} f(x_j)=\sum_{k=1}^{N}\sum_{j=1}^{N} a_{ik}a_{kj} f(x_j) \tag{4.31}$$

If i=1 then, first row elements are found with variable k is given in equation (4.32).

$$f_x^{(2)}(x_1)=\sum_{k=1}^{N}\left(a_{1k}a_{k1} f(x_1)+a_{1k}a_{k2} f(x_2)+\ldots+a_{1k}a_{kN} f(x_{1N})\right)$$

$$=\sum_{k=1}^{N} a_{1k}a_{k1} f(x_1)+\sum_{k=1}^{N} a_{1k}a_{k2} f(x_2)+\ldots+\sum_{k=1}^{N} a_{1k}a_{kN} f(x_{1N}) \tag{4.32}$$

For finding the first row elements without variables, variables of k's are substituted into the equation (4.32) and equation (4.33) is derived.

$$\begin{aligned}
&+\left\{a_{11}a_{11} f(x_1)+a_{12}a_{21} f(x_1)+a_{13}a_{31} f(x_1)+\ldots+a_{1N}a_{N1} f(x_1)\right\}\\
&+\left\{a_{11}a_{12} f(x_2)+a_{12}a_{22} f(x_2)+a_{13}a_{32} f(x_2)+\ldots+a_{1N}a_{N2} f(x_2)\right\}\\
&.\\
&.\\
&+\left\{a_{11}a_{1N} f(x_N)+a_{12}a_{2N} f(x_N)+a_{13}a_{3N} f(x_N)+\ldots+a_{1N}a_{NN} f(x_N)\right\}
\end{aligned} \tag{4.33}$$

If we convert equation (4.33) to the matrix form, equation (4.34) is obtained.

$$\begin{bmatrix}
\left(a_{11}a_{11}+a_{12}a_{21}+\ldots+a_{1N}a_{N1}\right)\ \left(a_{11}a_{12}+a_{12}a_{22}+\ldots+a_{1N}a_{N2}\right)\ \ldots\ldots.\left(a_{11}a_{1N}+a_{12}a_{2N}+\ldots+a_{1N}a_{NN}\right)\\
.\\
.\\
.\\
.\\
\left(a_{N1}a_{11}+a_{N2}a_{21}+\ldots+a_{NN}a_{N1}\right)\ \left(a_{N1}a_{12}+a_{N2}a_{22}+\ldots+a_{NN}a_{N2}\right)\ldots.\left(a_{N1}a_{1N}+a_{N2}a_{2N}+\ldots+a_{NN}a_{NN}\right)
\end{bmatrix} \tag{4.34}$$

It is easily seen matrix multiplication approach from the matrix form and represented in the equation (4.35).

$$\begin{bmatrix} a_{11} & a_{12} & \ldots\ldots\ldots.a_{1N}\\ a_{21} & a_{22} & \ldots\ldots\ldots\ a_{2N}\\ . & . & .\\ . & . & .\\ a_{N1} & a_{N2} & \ldots\ldots.\ a_{NN}\end{bmatrix}\begin{bmatrix} a_{11} & a_{12} & \ldots\ldots\ldots.a_{1N}\\ a_{21} & a_{22} & \ldots\ldots\ldots\ a_{2N}\\ . & . & .\\ . & . & .\\ a_{N1} & a_{N2} & \ldots\ldots.\ a_{NN}\end{bmatrix}\begin{Bmatrix} y_1\\ y_2\\ .\\ .\\ y_N\end{Bmatrix}=\begin{bmatrix} y_1'\\ y_2'\\ .\\ .\\ y_N'\end{bmatrix} \tag{4.35}$$

Matrix multiplication approach for second order derivative is given in the equation (3.36).

$$\begin{bmatrix} y_1^{"} \\ y_2^{"} \\ . \\ . \\ y_N^{"} \end{bmatrix} = A \begin{bmatrix} y_1^{'} \\ y_2^{'} \\ . \\ . \\ y_N^{'} \end{bmatrix} = A \left\{ A \begin{Bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_N \end{Bmatrix} \right\} = A^2 \left[y \right] \tag{4.36}$$

And general form of the this approach is seen from the equation (4.37).

$$\begin{bmatrix} y_1^{m} \\ y_2^{m} \\ . \\ . \\ y_N^{m} \end{bmatrix} = A^m \begin{Bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_N \end{Bmatrix} \tag{4.37}$$

4.6 Grid Point Distribution

In order to solve an engineering problem with differential quadrature method, primarily type of grid distribution should be selected. Two of them that are uniform grid and Chebyshev-Gauss-Lobatto grid will be examined.

4.6.1 Uniform grid

In uniform grid distribution, points are selected at equal intervals in each direction of coordinate axes. As seen from the Figure 4.3, step size is fixed. Selection of the points from uniform grid distribution is very simple. Therefore, it is easily applicable to the problems but accuracy of the results depends on the problem. Grid selection is done by trial and error method to choose the most accurate grid distribution.

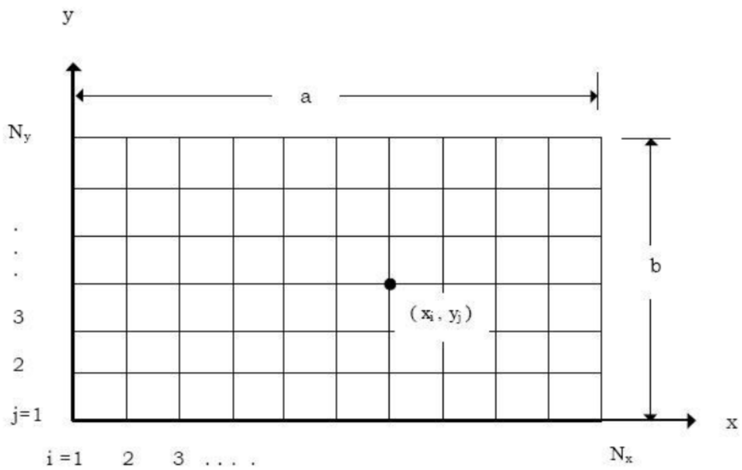

Figure 4.3: Uniform grid distribution.

4.6.2 Chebyshev-Gauss-Lobatto grid

Another option is Chebyshev-Gauss-Lobatto grid. In any problem, the results become worse when close to the boundary conditions. For this reason, increasingly frequent distribution towards boundary conditions point is selected in the Chebyshev-Gauss-Lobatto grid as seen from the Figure 4.4.

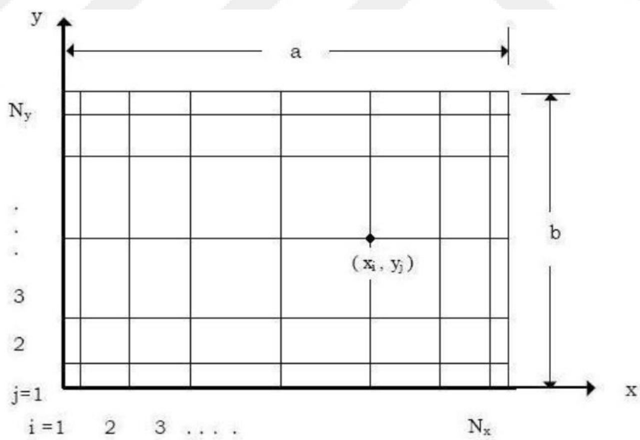

Figure 4.4: Chebyshev-Gauss-Lobatto grid distribution.

Accurate and reliable solutions that are more quickly convergent are obtained through Chebyshev-Gauss-Lobatto grid. Formulation of the Chebyshev-Gauss-Lobatto grid distribution is written in the equation (4.38)

$$x_i = \frac{b-a}{2}\left(1-\cos\frac{i\pi}{N}\right)+a, \text{ i=1,2,...,N} \tag{4.38}$$

4.7 Solution of the Problem with Differential Quadrature Method

As found in the mathematical section of the study, flow is separated into two different layers, the fluid layer and the porous layer. In this problem, equations contain nonlinearities. For this reason, Quan and Chang approach cannot be used. By using lagrange interpolation polynomial, fluid layer formulation is written in the equation (4.39).

$$f(y)=\sum_{i=1}^{m}\frac{\prod_{\substack{j=1\\ j\neq i}}^{m}(y-Y1[j])}{\prod_{\substack{j=1\\ j\neq i}}^{m}(Y1[i]-Y1[j])}F[i] \qquad i=1,2,..,m \tag{4.39}$$

Porous layer equation is given in the equation (4.40)

$$g(y)=\sum_{i=1}^{m}\frac{\prod_{\substack{j=1\\ j\neq i}}^{m}(y-Y2[j])}{\prod_{\substack{j=1\\ j\neq i}}^{m}(Y2[i]-Y2[j])}G[i] \qquad i=1,2,..,m \tag{4.40}$$

in which m represents number of grid points in the domain, Y1[j] and Y2[j] are found according to grid distribution, F[i] and G[i] are unknown values of the functions at different grid points.

In this study Chebyshev-Gauss-Lobatto grid distribution is used. Defined range of the fluid layer starts from 0 and ending at -0.9 and porous layer starts from -0.9 and ending at -1. According to intervals, frequent distribution towards boundary conditions is used.

Firstly, all boundary conditions are substituted to the equation (4.39) and (4.40). Then the equation (4.39) and (4.40) are substituted to the equation (4.41) and (4.42).

$$R2=g^{IV}(y)-\frac{n}{D_a}g^{II}(y) \tag{4.41}$$

$$R1=f^{IV}(y)-\mathrm{Re}\left(f(y)\,f^{III}(y)-f^{I}(y)\,f^{II}(y)\right) \tag{4.42}$$

The equation (4.41) and (4.42) are set to zero and unknown values of the function are found. This stages are followed for 12 different grid points and graphs are shown at the results section.

5. WEIGHTED RESIDUAL METHODS

5.1 Introduction

The method of weighted residuals is a type of approximate solutions of differential equations that are valid only at certain points rather than at each point. All engineering problems could be represented by non-linear differential equations. Mathematical representation of the uniform flow over a porous plane with suction perpendicular to the surface is an example of non-linear differential equations. The method of weighted residual is an approximation technique for solving differential equations. Through the method of weighted residual (WRM) a solution can be approximated numerically. A weighted residual method uses a finite number of functions. The method is a slight extension of that used for boundary value problems.

The basic concept of the WRM is to drive a residual error to zero through a set of conditions. To obtain the approximate solution for the equation given in the differential form, approximation function is selected and is substituted to the differential equation. And the differential function is found in terms of approximation function. Result that is different than the zero is named as residual. This value that was obtained is multiplied by the specific weighted functions and the resulting product is tried to minimize. Five steps of implementing the method of weighted residual can be listed as;

1. The trial function with unknown coefficients is written by expanding unknown solution in a set of basis functions
2. The trial function is satisfied by the boundary conditions and initial conditions. This process reduces the number of unknown coefficients.
3. Residual is defined.
4. Weighted residual is set to zero and equations are solved.

5. The error is examined by setting up successive approximations, and converge is shown the number of basis functions increase.

In mathematical analysis, the Weierstrass approximation theorem shows that every continuous function defined on a closed interval $a \le x \le b$ can be approximated by polynomial function [17]. Calculation of the polynomials with computer facilities is very simple. For the numerical solution of uniform flow over a porous plane with downward suction, weighted residual method is applied by using Weierstrass theorem and trial function is defined as 5th, 6th, 7th, 8th, 9th and 10th order polynomials. There are five types of weighted residual methods which are subdomain method, collocation method, least squares method, Galerkin method and method of moments. Each method is developed in different country and time interval. All methods were unified in 1956 by Crandall [18]. In this thesis uniform flow over a porous plane with downward suction is solved by method of moments.

5.2 Method of Moments

The method of moments is the oldest method of deriving point estimators. Firstly, the method of moments is improved by Yamada in 1947 and Fujita contributed to method in 1951 [18]. At the outset, laminar boundary layer problems and nonlinear transient diffusion problems are solved by using method of moment. In the method of moments, weighted functions are defined as power of independent variations.

$$w_n = \eta^{n-1}; \text{ n=1,2,...,N} \tag{5.1}$$

$$\int_\eta w_n R d\eta = \int_\eta R\eta^{n-1} d\eta = 0; \text{ n=1,2,...,N} \tag{5.2}$$

The method of moments forces the residual to zero.

5.2.1 Solution of the problem by using method of moments

Mathematical representation of the uniform flow over a porous plane with downward suction is found in the mathematical section of the thesis. As mentioned before, the problem is separated two layer which are fluid layer and porous layer. Mathematical representation of each layer are different from each other. Final state of the fluid layer equation is rewritten in the equation (5.3) as found in the equation (3.27).

$$\mathrm{Re}\left(f(y)\, f^{III}(y) - f^{I}(y)\, f^{II}(y) \right) = f^{IV}(y) \tag{5.3}$$

And final state of the porous layer equation is written in the equation (5.4) as found in the equation (3.47).

$$0 = g^{IV}(y) - \frac{n}{D_a} g^{II}(y) \tag{5.4}$$

Be able to calculate f and g functions via method of moments, functions of the form by a polynomial degree 7 with respect to Weierstrass approximation theorem are defined for fluid layer and porous layer. These seventh degree polynomials contain totally 16 unknown coefficients. Trial function of fluid layer f is defined as

$$f = \sum_{i=0}^{7} F[i] y^{i} \tag{5.5}$$

in which F[i] is the leading coefficient.

Explicit representation of the fluid layer formulation is

$$f = F[0] + F[1]y^{1} + F[2]y^{2} + F[3]y^{3} + F[4]y^{4} + F[5]y^{5} + F[6]y^{6} + F[7]y^{7} \tag{5.6}$$

Trial function of porous layer g defined as

$$g = \sum_{i=0}^{7} G[i] y^{i} \tag{5.7}$$

in which G[i] is the leading coefficient. Explicit representation of the porous layer formulation is

$$g = G[0] + G[1]y^{1} + G[2]y^{2} + G[3]y^{3} + G[4]y^{4} + G[5]y^{5} + G[6]y^{6} + G[7]y^{7} \tag{5.8}$$

The trial functions of fluid and porous layer are stated with unknown coefficients. According to theorem every continuous function can be approximated in an interval. Following solution procedure is defining residual function. In this study we have two different residual functions that depend on the layers.

Residual function for fluid layer is calculated as

$$R1 = f^{IV}(y) - \mathrm{Re}\left(f(y)\, f^{III}(y) - f^{I}(y)\, f^{II}(y) \right) \tag{5.9}$$

Residual function of the fluid layer involves the first, second, third and fourth order derivatives of the trial function.

$$f' = F[1] + 2F[2]y + 3F[3]y^2 + 4F[4]y^3 + 5F[5]y^4 + 6F[6]y^5 + 7F[7]y^6 \quad (5.10)$$

$$f'' = 2F[2] + 6F[3]y + 12F[4]y^2 + 20F[5]y^3 + 30F[6]y^4 + 42F[7]y^5 \quad (5.11)$$

$$f''' = 6F[3] + 24F[4]y + 60F[5]y^2 + 120F[6]y^3 + 210F[7]y^4 \quad (5.12)$$

$$f'''' = 24F[4] + 120F[5]y + 360F[6]y^2 + 840F[7]y^3 \quad (5.13)$$

Derivatives of the equation (5.6) as seen in the equation (5.10-5.13) is substituted to the equation (5.9) and R1 is found in terms of unknown coefficients

$$R1 = 24F[4] + 120F[5]y + 360F[6]y^2 + 840F[7]y^3 - \mathrm{Re}(-(2F[2] + 6F[3]y....)) \quad (5.14)$$

The coefficients that the residual function depends on are given in the equation (5.15)

$$R1 = f(F[0], F[1], F[2], F[3], F[4], F[5], F[6], F[7]) \quad (5.15)$$

Residual function for porous layer is calculated as

$$R2 = g^{IV}(y) - \frac{n}{D_a} g^{II}(y) \quad (5.16)$$

Residual function of the fluid layer involves the first, second, third and fourth order derivatives of the trial function.

$$g' = G[1] + 2G[2]y + 3G[3]y^2 + 4G[4]y^3 + 5G[5]y^4 + 6G[6]y^5 + 7G[7]y^6 \quad (5.17)$$

$$g'' = 2G[2] + 6G[3]y + 12G[4]y^2 + 20G[5]y^3 + 30G[6]y^4 + 42G[7]y^5 \quad (5.18)$$

$$g''' = 6G[3] + 24G[4]y + 60G[5]y^2 + 120G[6]y^3 + 210G[7]y^4 \quad (5.19)$$

$$g'''' = 24G[4] + 120G[5]y + 360G[6]y^2 + 840G[7]y^3 \quad (5.20)$$

Derivatives of the equation (5.8) is substituted to the equation (5.16) and R2 is found in terms of unknown coefficients

$$R2 = 24G[4] + 120G[5]x + 360G[6]y^2 + 840G[7]y^3 - \frac{n}{Da}(2G[2] + 6G[3]y + 12G[4]y^2$$

$$+ 20G[5]y^3 + 30G[6]y^4 + 42G[7]y^5) \quad (5.21)$$

The coefficients that the residual function depends on are given in the equation (5.22).

$$R2 = g(G[0], G[1], G[2], G[3], G[4], G[5], G[6], G[7]) \quad (5.22)$$

Before forcing the residual to zero, the number of unknowns are reduced by using boundary conditions. Boundary conditions are written in terms of trial functions. Each boundary condition eliminates one unknown coefficient.

As found before, at the top of the fluid layer x and y components of the velocity are zero. Be able to stagnate fluid at the top, boundary conditions that are $f(0)=0$ and $f^{I}(0)=0$ should be satisfied. When these boundary conditions are satisfied 2 coefficients of the trial function of the fluid layer can be found.

$$f(0)=0 \Rightarrow F[0]=0 \tag{5.23}$$

$$f^{I}(0)=0 \Rightarrow F[1]=0 \tag{5.24}$$

Final stage of the fluid layer equation is

$$f = F[2]y^2 + F[3]y^3 + F[4]y^4 + F[5]y^5 + F[6]y^6 + F[7]y^7 \tag{5.25}$$

The coefficients that the f function depends on are given

$$f = f(F[2], F[3], F[4], F[5], F[6], F[7]) \tag{5.26}$$

Same calculations are done for porous layer equation and intersection point of the porous and fluid layer. As found in the mathematical modelling section, at the bottom of porous layer $ng(1)$ is equal to 1 and $g^{I}(1)$ is equal to 0.

$$ng(1)=1 \Rightarrow G[0]+G[1]+G[2]+G[3]+G[4]+G[5]+G[6]+G[7]=\frac{1}{n} \tag{5.27}$$

Other boundary condition is

$$g'(1)=0 \tag{5.28}$$

$$g'(1)=G[1]+2G[2]+3G[3]+4G[4]+5G[5]+6G[6]+7G[7]=0 \tag{5.29}$$

At the interface of fluid and porous layer continuity of velocity vector is satisfied

$$g(\xi)n = f(\xi) \Rightarrow \tag{5.30}$$

$$\begin{aligned} &n(G[0]+G[1](0.9)+G[2](0.9)^2+G[3](0.9)^3+G[4](0.9)^4+G[5](0.9)^5+G[6](0.9)^6+ \\ &+G[7](0.9)^7)=F[2](0.9)^2+F[3](0.9)^3+F[4](0.9)^4+F[5](0.9)^5+F[6](0.9)^6+F[7](0.9)^7 \end{aligned} \tag{5.31}$$

$$g^{I}(\xi) = f^{I}(\xi) \Rightarrow \tag{5.32}$$

$$G[1]+2G[2](0.9)+3G[3](0.9)^2+4G[4](0.9)^3+5G[5](0.9)^4+6G[6](0.9)^5+7G[7](0.9)^6=$$

$$=2F[2](0.9)+3F[3](0.9)^2+4F[4](0.9)^3+5F[5](0.9)^4+6F[6](0.9)^5+7F[7](0.9)^6 \tag{5.33}$$

At the interface section between porous and fluid layer, continuity of shear stress is satisfied

$$g^{II}(\xi)=f^{II}(\xi)\Rightarrow \tag{5.34}$$

$$2G[2]+6G[3](0.9)+12G[4](0.9)^2+20G[5](0.9)^3+30G[6](0.9)^4+42G[7](0.9)^5=$$

$$2F[2]+6F[3](0.9)+12F[4](0.9)^2+20F[5](0.9)^3+30F[6](0.9)^4+42F[7](0.9)^5 \tag{5.35}$$

The last boundary condition that is obtained by derivation of fluid layer and porous layer equation is

$$g^{III}(\xi)=f^{II}(\xi)+\frac{n}{Da}f^{I}(\xi)+R(f^{I^2}(\xi)-f(\xi)f^{II}(\xi)) \tag{5.36}$$

$$24G[4]+120G[5](0.9)+360G[6](0.9)^2+840G[7](0.9)^3=$$

$$=2F[2]+6F[3](0.9)+12F[4](0.9)^2+20F[5](0.9)^3+30F[6](0.9)^4+42F[7](0.9)^5+$$

$$\frac{n}{Da}(2F[2](0.9)+3F[3](0.9)^2+4F[4](0.9)^3+5F[5](0.9)^4+6F[6](0.9)^5+7F[7](0.9)^6)+$$

$$+R((2F[2](0.9)+3F[3](0.9)^2+4F[4](0.9)^3+5F[5](0.9)^4+6F[6](0.9)^5+7F[7](0.9)^6)^2-$$

$$(F[2]y^2+F[3]y^3+F[4]y^4+F[5]y^5+F[6]y^6+F[7]y^7)*$$

$$*(2F[2]+6F[3](0.9)+12F[4](0.9)^2+20F[5](0.9)^3+30F[6](0.9)^4+42F[7](0.9)^5)) \tag{5.37}$$

Thanks to boundary conditions eight unknown coefficients are found. Remaining informations that are required for calculating eight unknown coefficients are obtained by integration of the residual function. First four equations are valid for fluid layer.

$$\int_0^{0.9} R1dy=0 \tag{5.38}$$

$$\int_0^{0.9} R1ydy = 0 \tag{5.39}$$

$$\int_0^{0.9} R1y^2dy = 0 \tag{5.40}$$

$$\int_0^{0.9} R1y^3dy = 0 \tag{5.41}$$

Last four equations are valid for porous layer.

$$\int_{0.9}^{1} R2dy = 0 \tag{5.42}$$

$$\int_{0.9}^{1} R2ydy = 0 \tag{5.43}$$

$$\int_{0.9}^{1} R2y^2dy = 0 \tag{5.44}$$

$$\int_{0.9}^{1} R2y^3dy = 0 \tag{5.45}$$

The number of equation is enough to solve the problem, 8 equations and 8 unknowns. MATHEMATICA functions are used to solve the problem. The result of the $f(y)$ function that is relate with the y component of the velocity in fluid layer is given in the equation

$$f(y) = -9.433*10^{-14}y + 2.236\,y^2 - 0.541y^3 + 0.528461y^4 - 2.857y^5 + 2.811y^6 - 1.213y^7 \tag{5.42}$$

In the same way, the result of the $f^{'}(y)$ function that is relate with the x component of the velocity in the fluid layer is written in the equation

$$f'(y) = -9.433*10^{-14} + 4.471y - 1.623y^2 + 2.114y^3 - 14.284y^4 + 16.865y^5 - 8.490y^6 \tag{5.43}$$

The result of the $g(y)$ function that gives information about y component of the velocity in porous layer is seen in the equation

$$g(y) = -10373 + 73113y - 221053y^2 + 371638y^3 - 375174y^4 + 227398y^5 - 76613y^6 + 11067y^7 \tag{5.44}$$

In the same way, the result of the $g^{'}(y)$ function that is relate with the x component of the velocity in the porous layer is written in the equation

$$g'(y)=73113-442107y+1.115*10^6y^2-1.501*10^6y^3+1.137*10^6y^4-459680y^5+77470y^6 \quad (5.45)$$

Equation (5.46) represents the shear stress function at the porous wall, when y=1 is substituted to the equation.

$$g'(y)=442107+2.235*10^6y-4.503*10^6y^2+4.548*10^6y^3-2298400y^4+464820y^5 \quad (5.46)$$

Same stages are done for 5th, 6th, 8th, 9th and 10th order polynomials. Obtained polynomials for porous and fluid layers by using 6th order polynomial are given in the equation (5.47) and (5.48).

$$f(y) = -6.6226*10^{-18}y+2.3741y^2-0.6608y^3-1.1823y^4+1.5965y^5-1.1640y^6 \quad (5.47)$$

$$g(y) = -2795.89+17015.02y-43181.14y^2+58510.66y^3-44642.52y^4+18183.86y^5-3088.99y^6 \quad (5.48)$$

Derivatives of the equation (5.47) and (5.48) are given in the equation (5.49) and (5.50)

$$f^{'}(y) = -6.6226*10^{-18}+4.7482y-1.9823y^2-4.7291y^3+7.9825y^4-6.9843y^5 \quad (5.49)$$

$$g^{'}(y) = 17015.02-86362.27y+175531.98y^2-178570.10y^3+90919.32y^4-18533.96y^5 \quad (5.50)$$

Obtained polynomials that define $f(y)$, $g(y)$, $f^{'}(y)$ and $g^{'}(y)$ functions for porous and fluid layers by using 8th order polynomial are given in the equation (5.51)-(5.54)

$$f(y) = 1.9301*10^{-10}y+2.2310y^2-0.5434y^3-0.2104y^4+0.4334y^5-2.5871y^6+2.7140y^7-1.07337y^8 \quad (5.51)$$

$$g(y) = -30536.05+243341.03y-849596.76y^2+1.70*10^6y^3-2.12*10^6y^4+1.70*10^6y^5 - $$
$$-852278.42y^6+244431.82y^7-30705.09y^8 \quad (5.52)$$

$$f^{'}(y) = -6.6226*10^{-18}+4.7482y-1.9823y^2-4.7291y^3+7.9825y^4-6.9843y^5 \quad (5.53)$$

$$g^{'}(y) = 17015.02-86362.27y+175531.98y^2-178570.10y^3+90919.32y^4 -$$
$$-18533.96y^5 \quad (5.54)$$

The graphs of the functions for all polynomials were given in the results section of the study.

6. RESULTS

After finding equations that define fluid layer and porous layer by using semi analytical numerical methods which are differential quadrature method and method of moments, distribution of each velocity component is found and discussed as follows by employing the relation between stream function and velocity components.

As mentioned before, reference study in this study is Deng and Martinez's study. Be able to obtain close results to the reference study, dimensionless numbers and constants are selected as same with reference study. According to this, Re=5, Da=0.001, n=0.9 and ξ=0.9

Figure 6.1 and Figure 6.2 depict the horizontal component of the velocity in fluid layer in the defined interval ($-0.9 \leq y \leq 0$) by using two different method. At the top of the fluid layer, horizontal component of the velocity is zero as a result of the no slip boundary condition. x component of the velocity increases until reaching the maximum value that is approximately 1.6 at roughly y=0.5. It is not clearly seen from the Figure 6.1 but the best result of the differential quadrature method is obtained by using N=15 grid points and N=4 is the worst result. As seen from the Figure 6.2 in which method of moments approximation is used, the best result of the method of moment approximation is obtained by using 10^{th} degree polynomial. Figure 6.3 and Figure 6.4 demonstrate horizontal velocity distribution in porous layer in the defined interval ($-1 \leq y \leq -0.9$). At the bottom, x component of the velocity is equal to zero as shown in the graphics. Figure 6.5 and Figure 6.6 represents horizontal velocity combination in porous layer and fluid layer. At the end of the graphics, approximate estimate of the error for velocity at different location was added. Vertical velocity component distribution in fluid layer was given in the Figure 6.7 and Figure 6.8. At the top of the fluid layer, no slip boundary condition exists and velocity increases with the fluid depth. Cause of this rate of increase is downward suction at the bottom. Vertical velocity component distribution in porous layer is plotted in the Figure 6.9 and 6.10. Combining fluid layer and porous layer graphics, Figure 6.11 and Figure 6.12 that depict whole velocity profile of the vertical component of the fluid was plotted.

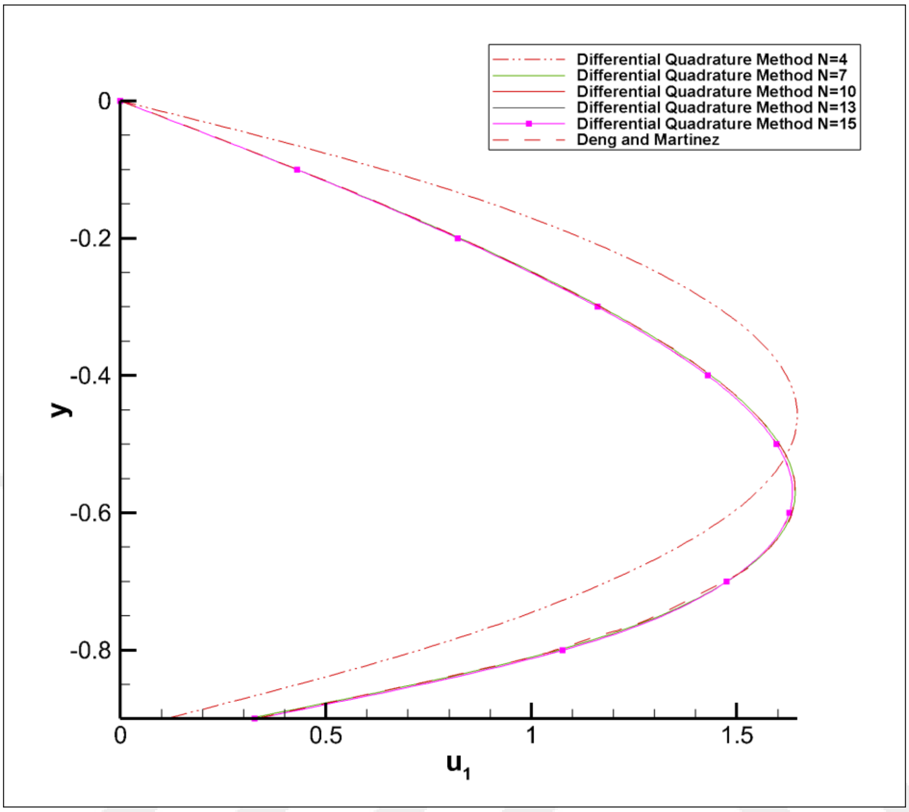

Figure 6.1: Horizontal velocity distribution in fluid layer **(**DQM).

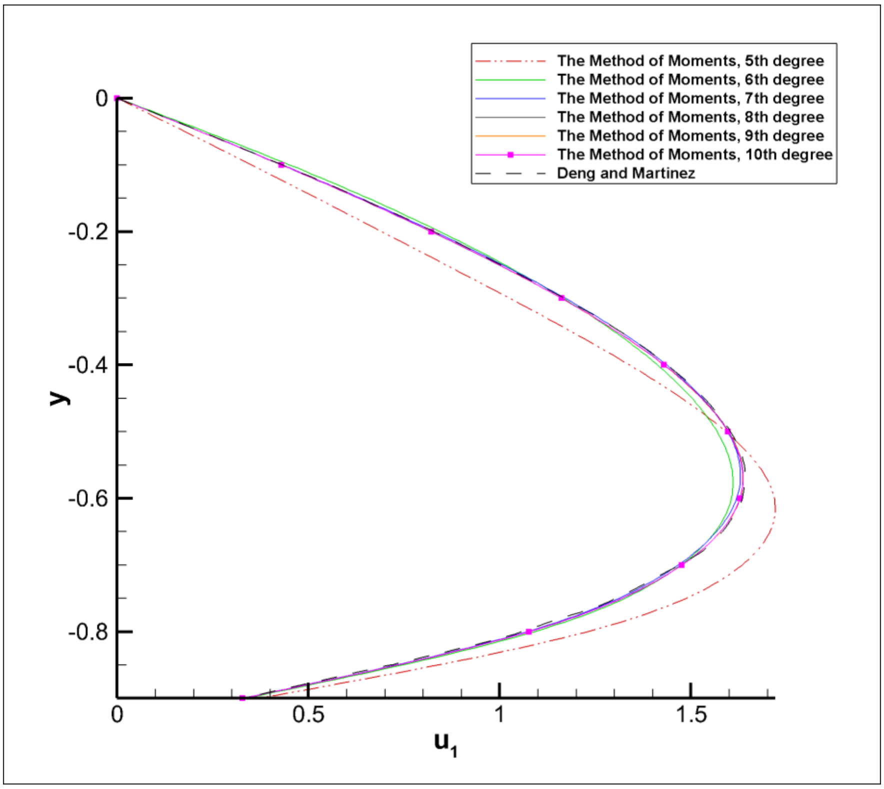

Figure 6.2: Horizontal velocity distribution in fluid layer **(**MoM).

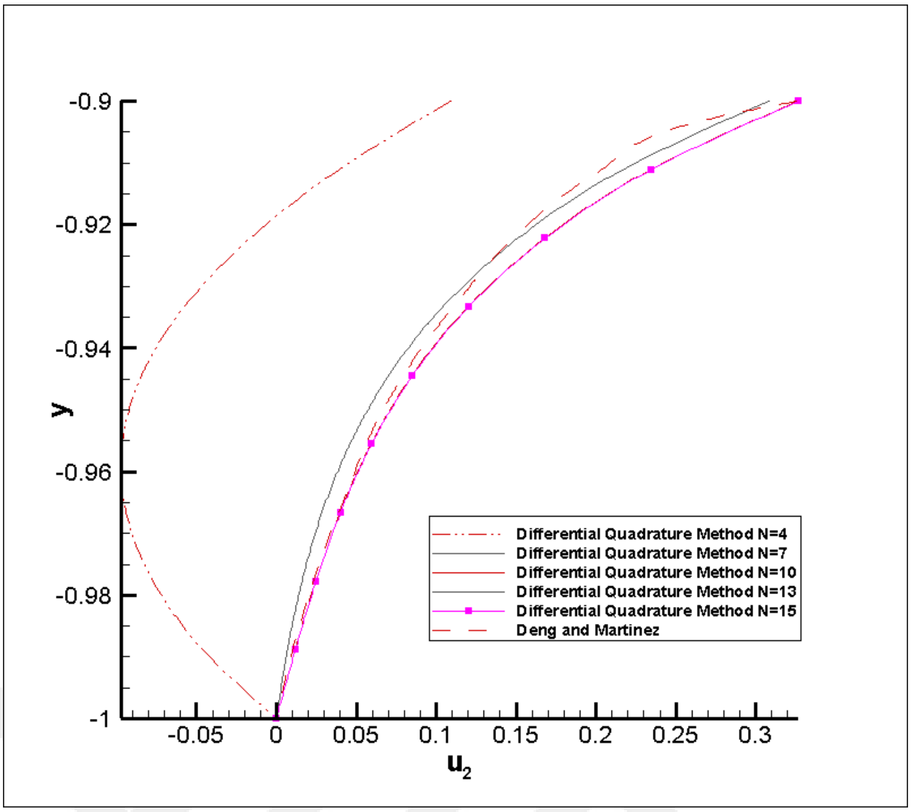

Figure 6.3: Horizontal velocity distribution in porous layer **(**DQM).

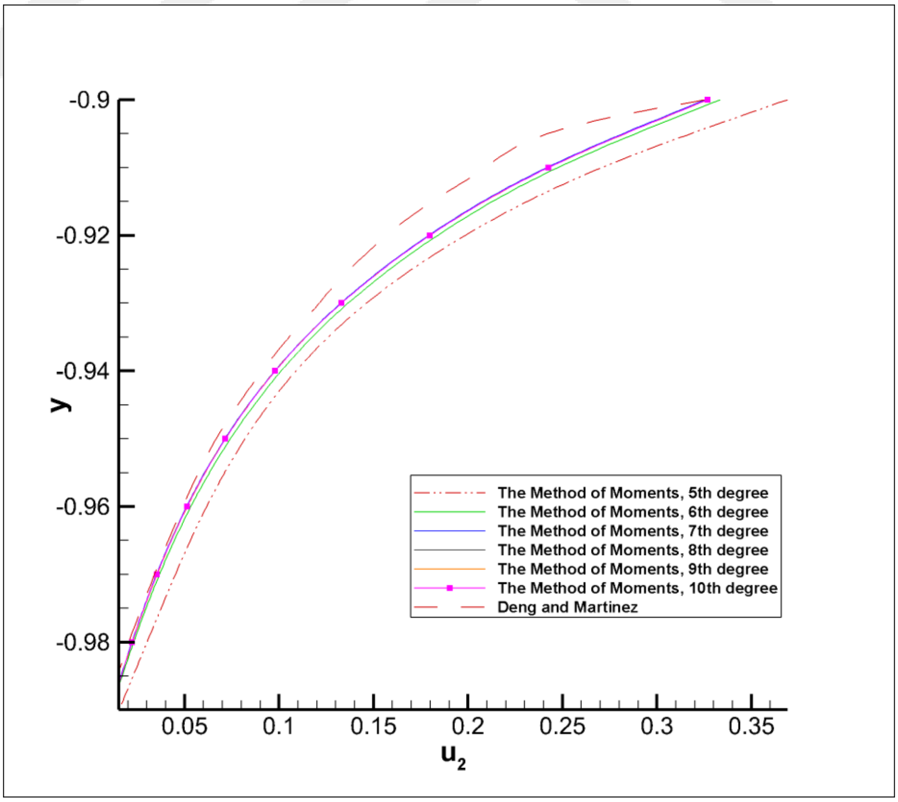

Figure 6.4: Horizontal velocity distribution in porous layer **(**MoM).

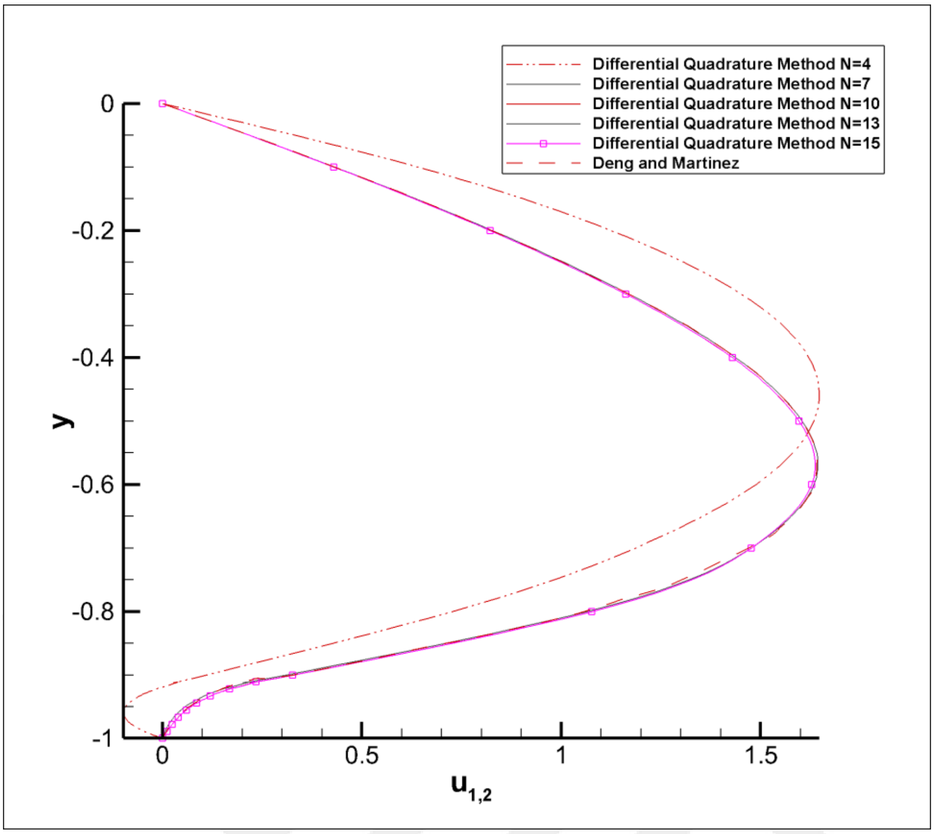

Figure 6.5: Horizontal velocity distribution (DQM).

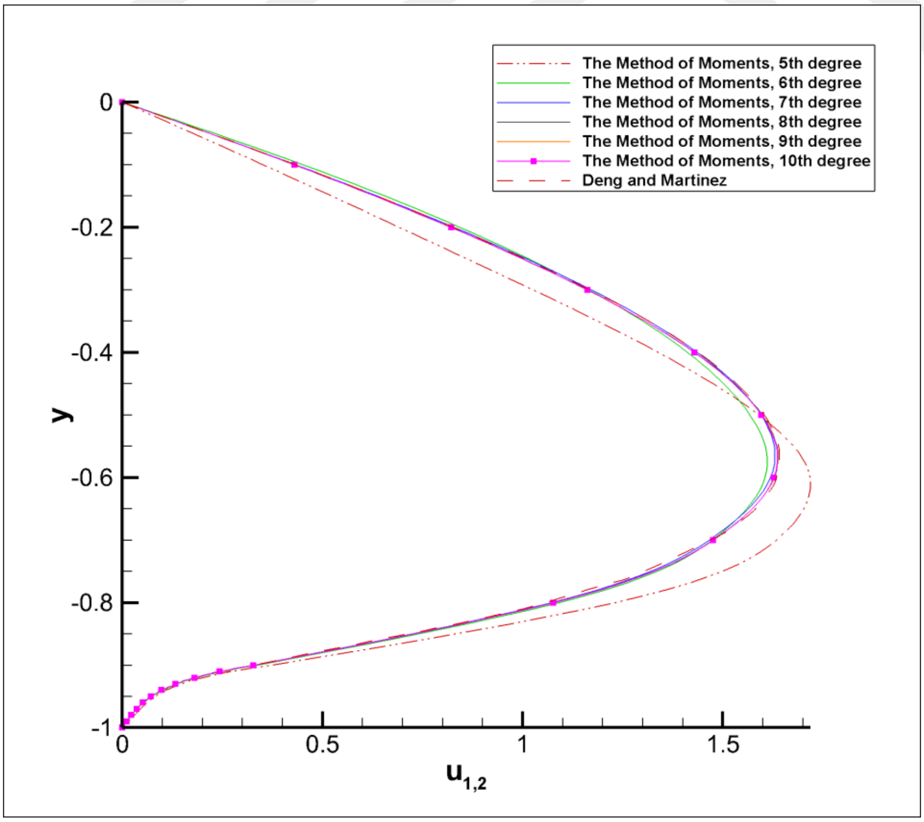

Figure 6.6: Horizontal velocity distribution (MoM).

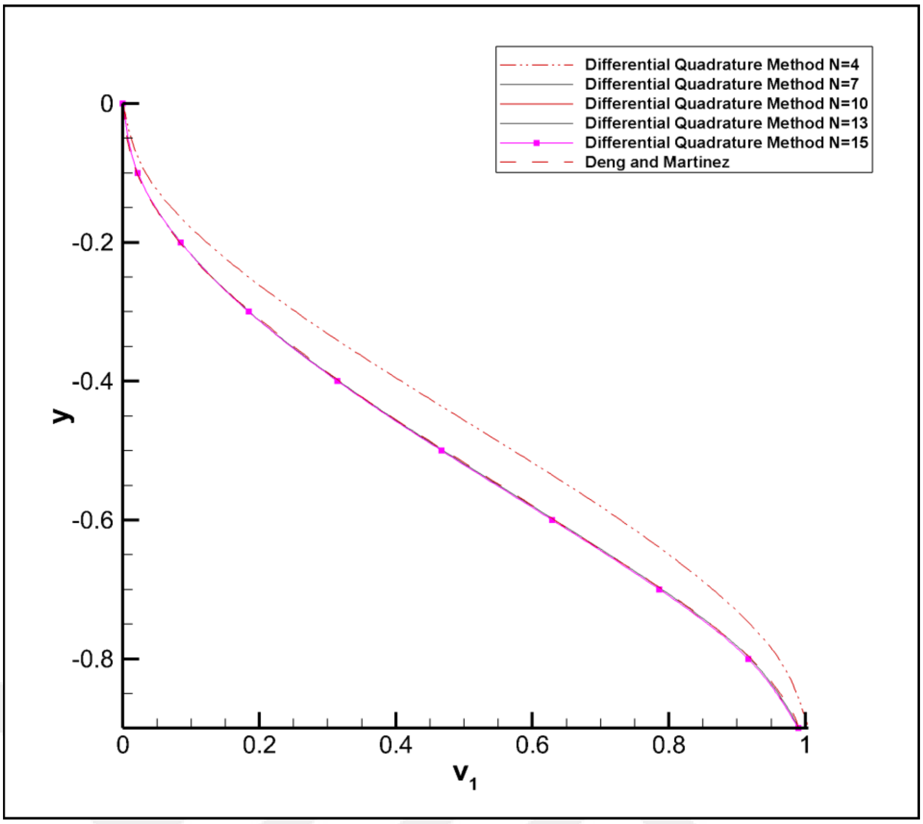

Figure 6.7: Vertical velocity distribution in fluid layer (DQM).

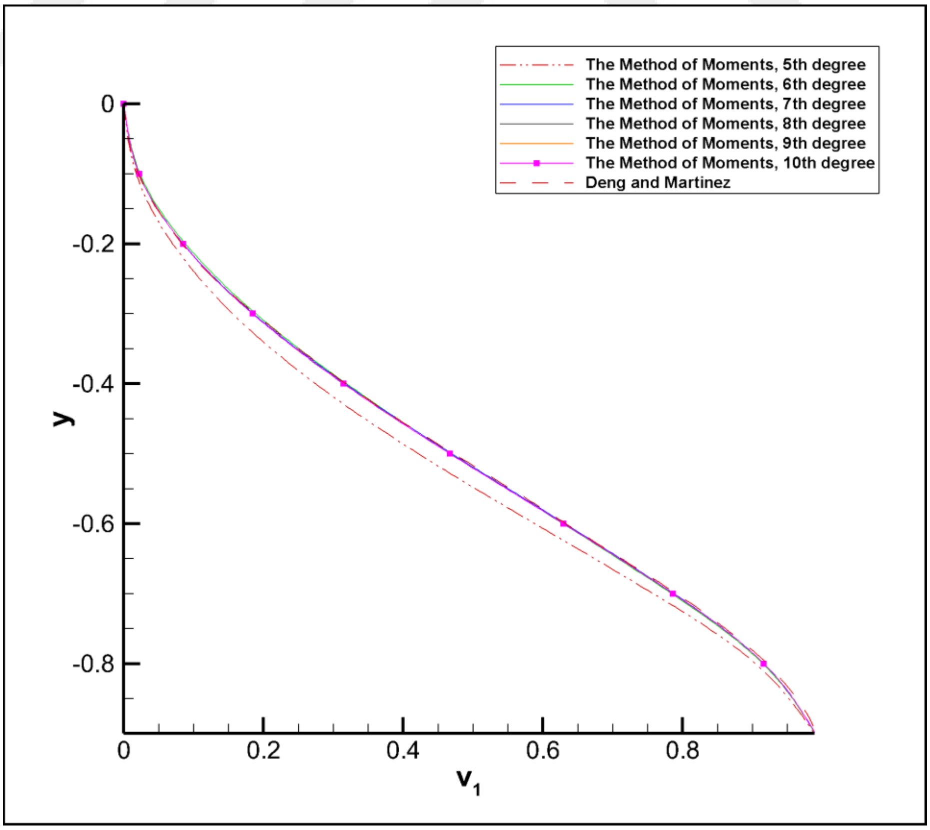

Figure 6.8: Vertical velocity distribution in fluid layer (MoM).

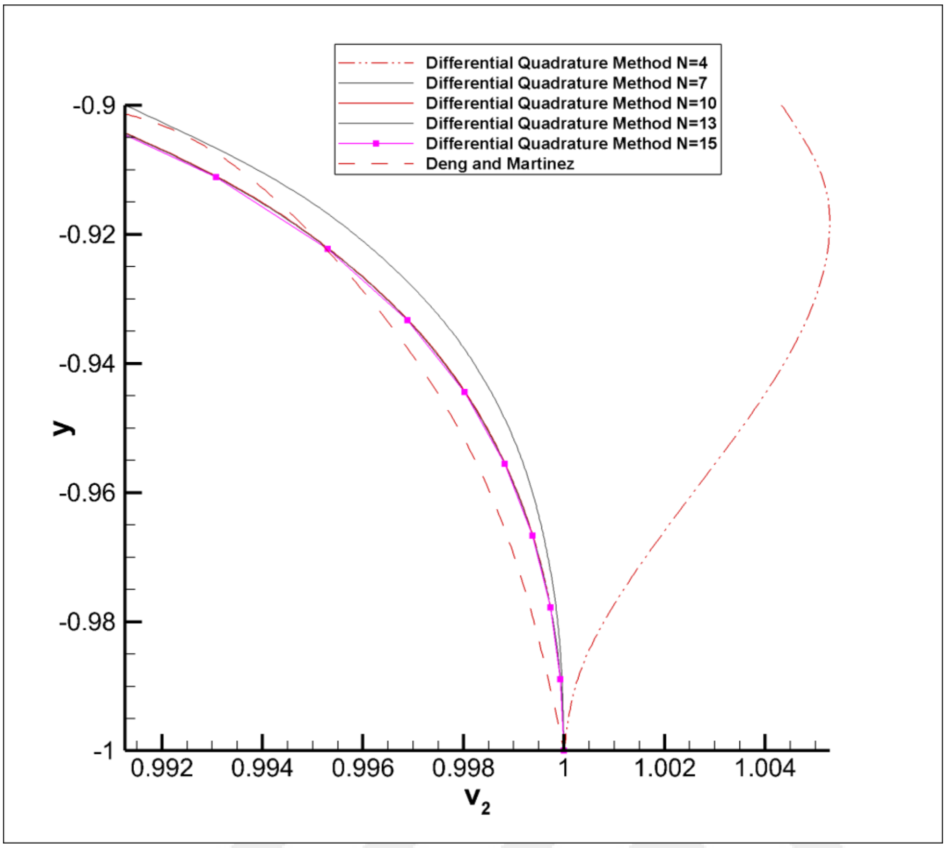

Figure 6.9: Vertical velocity distribution in porous layer (DQM).

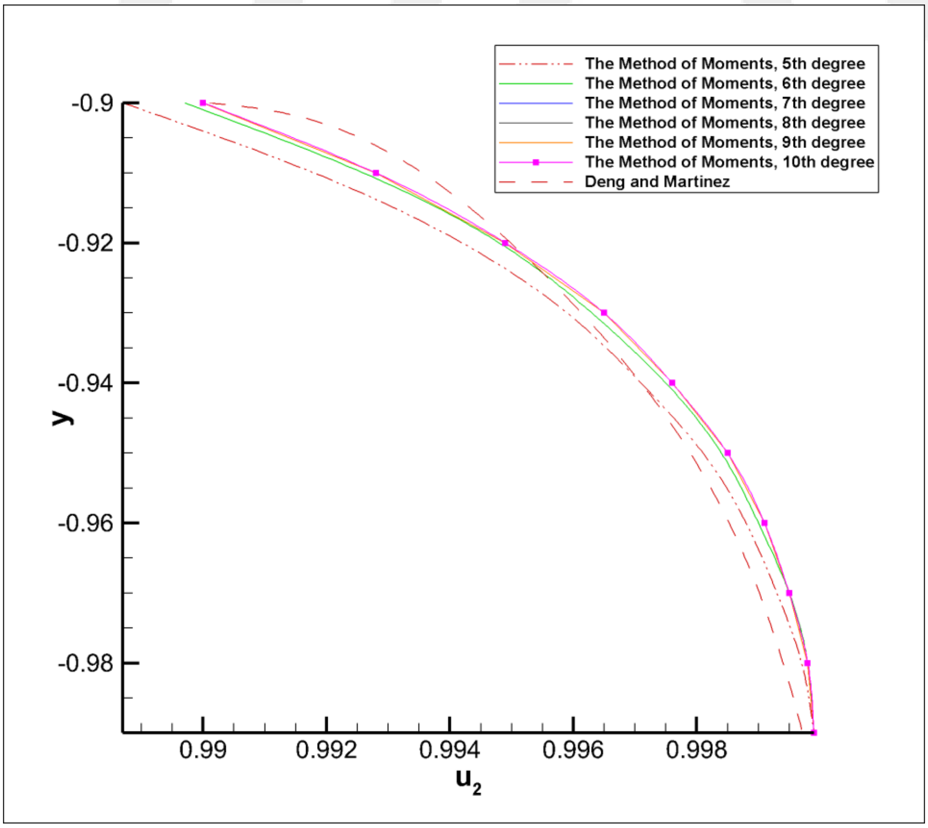

Figure 6.10: Vertical velocity distribution in porous layer (MoM).

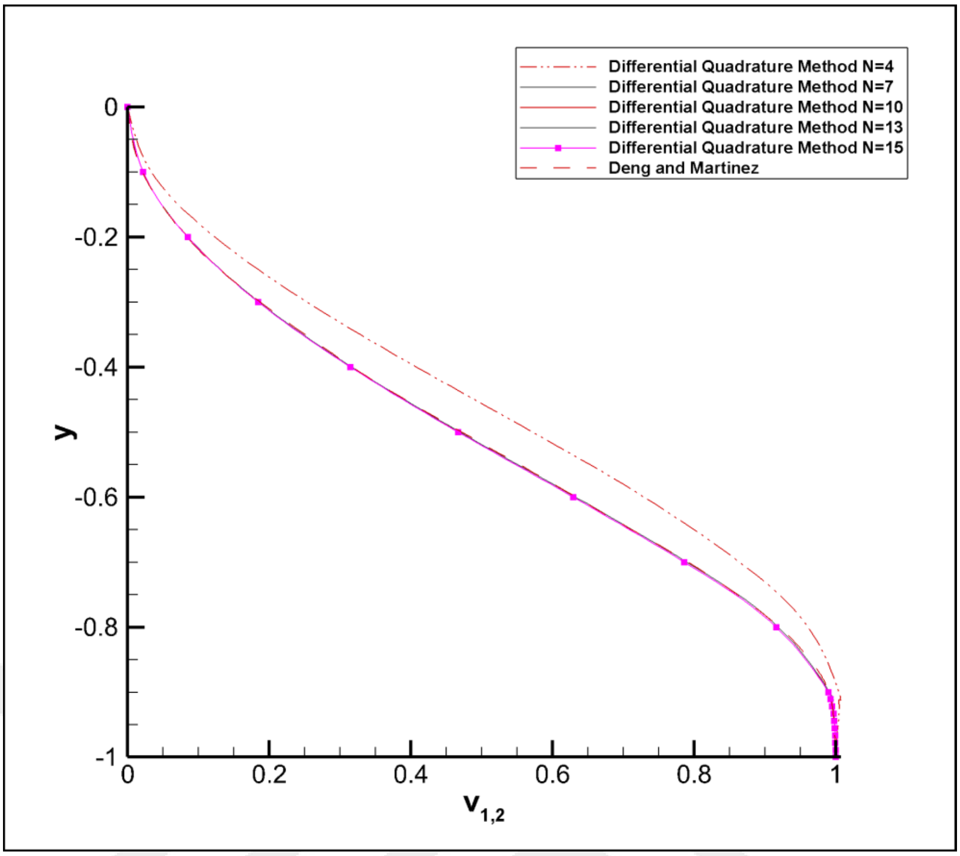

Figure 6.11: Vertical velocity distribution (DQM).

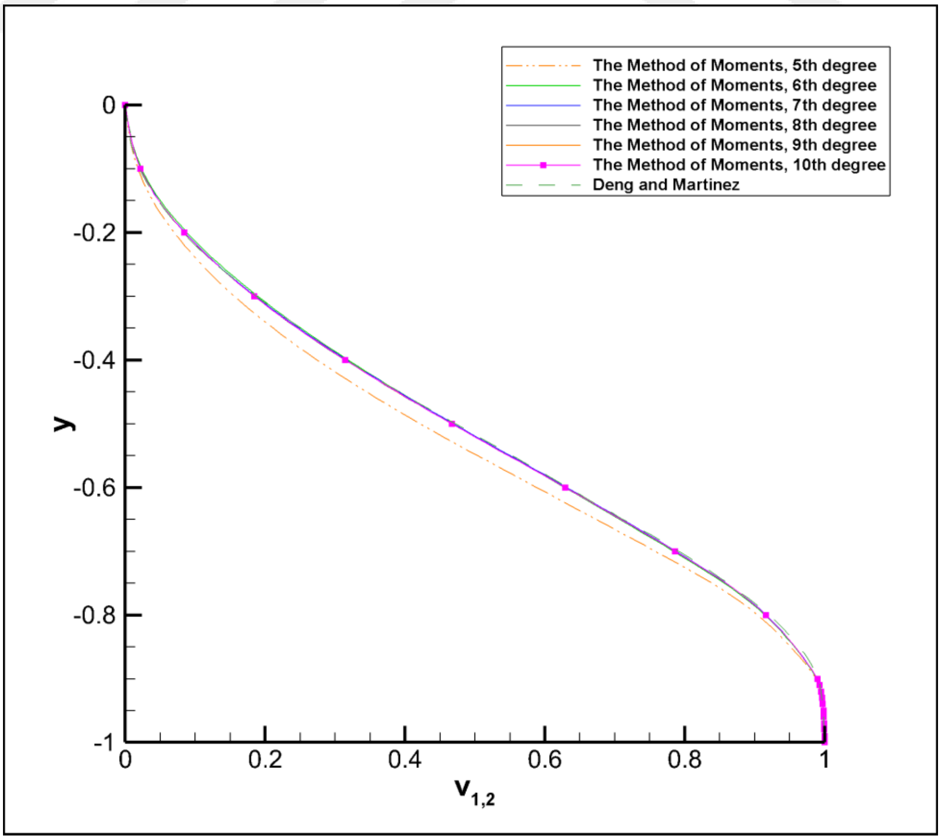

Figure 6.12: Vertical velocity distribution (MoM).

In addition to these figures, the best results are compiled in a graphic. Figure 6.13 and Figure 6.14 are plotted to show that results agree very well.

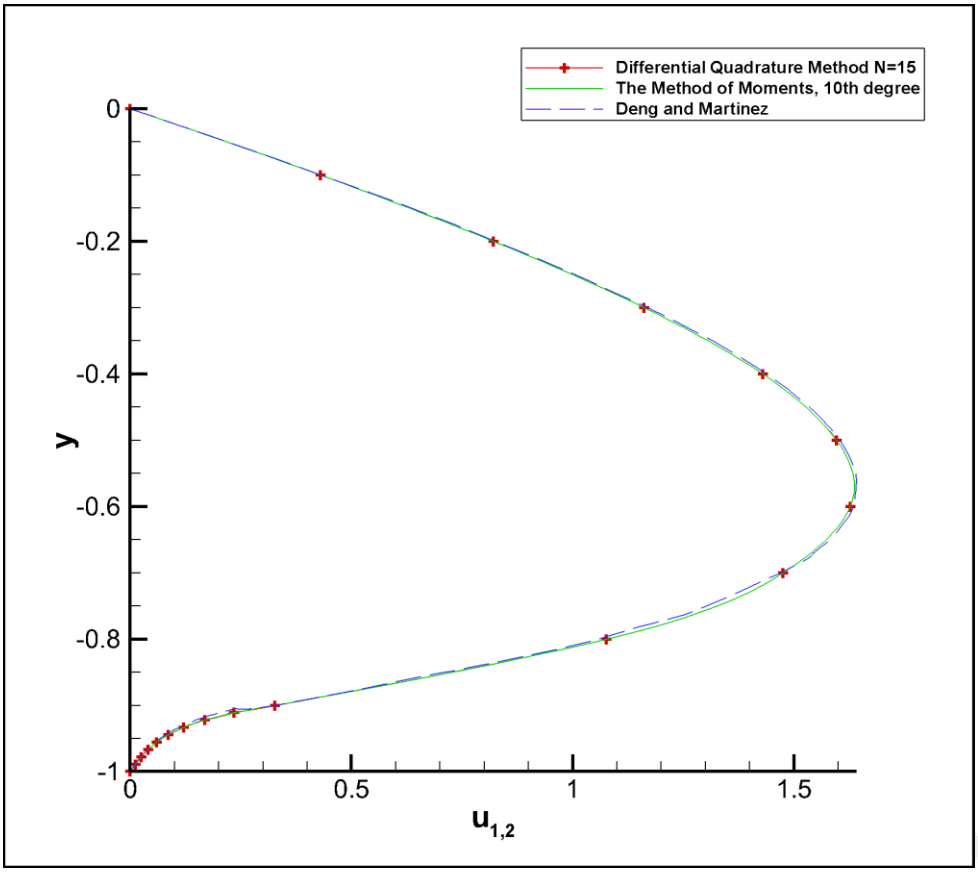

Figure 6.13: Horizontal velocity with high accuracy.

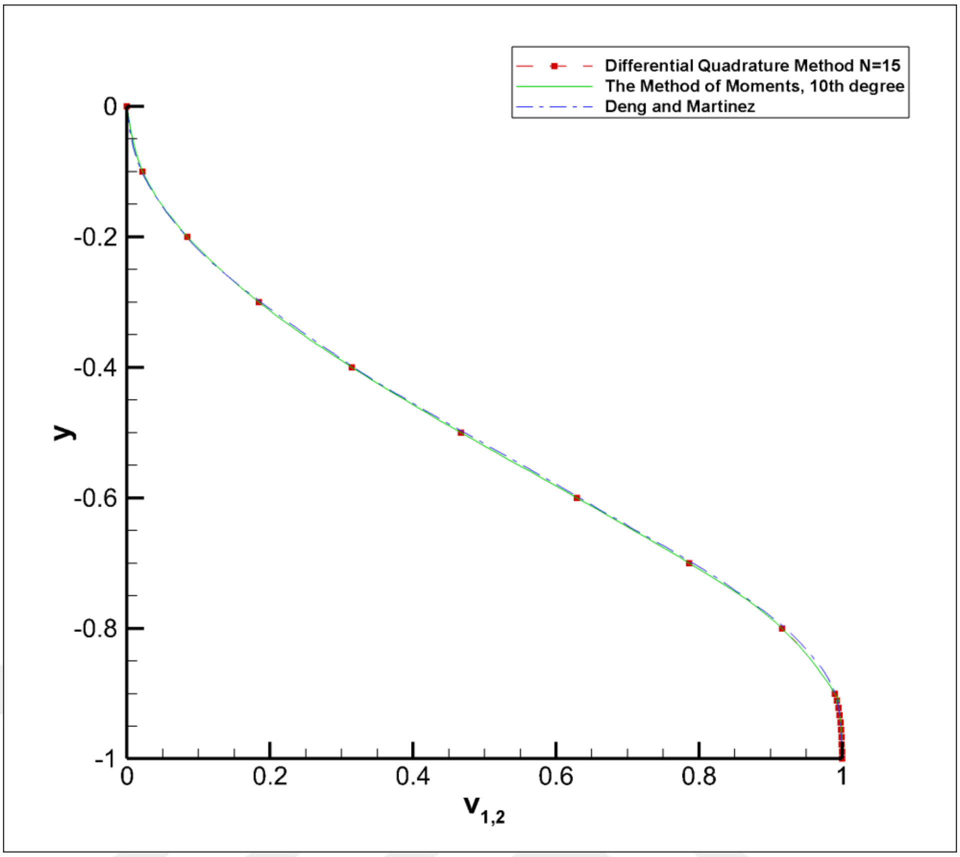

Figure 6.14: Vertical velocity with high accuracy.

At the end of the graphics, the tables that show values of the velocity component in different locations and approximate estimate of the error for velocity are added. Table 6.1 and Table 6.3 that are obtained by using differential quadrature method are values of the vertical and horizontal velocity in different locations. It can be easily seen from the Table 6.2 and the Table 6.4 that error was reduced with increasing number of grid points.

Table 6.1: Values of the vertical velocity in different locations (DQM).

	y	GRID POINTS											
	0,00	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=11	N=12	N=13	N=14	N=15
VALUES OF THE VERTICAL VELOCITY	-0,10	0,03337	0,02061	0,02248	0,02174	0,02189	0,02177	0,02178	0,02177	0,02177	0,02177	0,02177	0,02177
	-0,20	0,12298	0,08343	0,08700	0,08499	0,08500	0,08472	0,08470	0,08469	0,08468	0,08469	0,08469	0,08469
	-0,30	0,25312	0,18669	0,18870	0,18533	0,18489	0,18441	0,18437	0,18434	0,18433	0,18433	0,18433	0,18433
	-0,40	0,40805	0,32417	0,32127	0,31639	0,31541	0,31473	0,31463	0,31458	0,31457	0,31457	0,31457	0,31457
	-0,50	0,57206	0,48530	0,47590	0,46944	0,46790	0,46707	0,46690	0,46685	0,46683	0,46683	0,46683	0,46683
	-0,60	0,72942	0,65509	0,64024	0,63266	0,63055	0,62964	0,62943	0,62937	0,62935	0,62935	0,62935	0,62935
	-0,70	0,86440	0,81416	0,79740	0,78995	0,78750	0,78659	0,78637	0,78630	0,78628	0,78628	0,78628	0,78628
	-0,80	0,96128	0,93873	0,92489	0,91939	0,91732	0,91658	0,91638	0,91633	0,91631	0,91631	0,91631	0,91631
	-0,90	1,00433	1,00062	0,99361	0,99127	0,99040	0,99011	0,99002	0,99000	0,98999	0,98999	0,98999	0,98999
	-0,91	1,00514	1,00231	0,99617	0,99416	0,99342	0,99318	0,99311	0,99309	0,99309	0,99308	0,99308	0,99308
	-0,92	1,00525	1,00307	0,99786	0,99618	0,99557	0,99538	0,99532	0,99530	0,99530	0,99530	0,99530	0,99530
	-0,93	1,00482	1,00316	0,99893	0,99759	0,99710	0,99695	0,99691	0,99689	0,99689	0,99689	0,99689	0,99689
	-0,94	1,00402	1,00278	0,99956	0,99855	0,99818	0,99807	0,99804	0,99802	0,99802	0,99802	0,99802	0,99802
	-0,95	1,00300	1,00274	0,99977	0,99855	0,99858	0,99843	0,99840	0,99838	0,99836	0,99836	0,99836	0,99836
	-0,96	1,00250	1,00213	0,99990	0,99919	0,99893	0,99885	0,99883	0,99882	0,99882	0,99882	0,99882	0,99882
	-0,97	1,00193	1,00138	1,00004	0,99959	0,99943	0,99939	0,99938	0,99937	0,99937	0,99937	0,99937	0,99937
	-0,98	1,00096	1,00069	1,00006	0,99984	0,99976	0,99974	0,99973	0,99973	0,99973	0,99973	0,99973	0,99973
	-0,99	1,00027	1,00019	1,00003	0,99996	0,99994	0,99994	0,99993	0,99993	0,99993	0,99993	0,99993	0,99993
	-1,00	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000

Table 6.2: Approximate estimate of the error for velocity in vertical direction (DQM).

	y	GRID POINTS										
	0,0000	N=5	N=6	N=7	N=8	N=9	N=10	N=11	N=12	N=13	N=14	N=15
APPROXIMATE ESTIMATE OF THE ERROR	-0,1000	61,9080	8,3233	3,3940	0,6938	0,5696	0,0505	0,0354	0,0106	0,0092	0,0000	0,0000
	-0,2000	47,3979	4,0966	2,3654	0,0109	0,3317	0,0189	0,0128	0,0109	0,0083	0,0000	0,0000
	-0,3000	35,5849	1,0678	1,8200	0,2391	0,2565	0,0233	0,0184	0,0033	0,0005	0,0000	0,0000
	-0,4000	25,8758	0,9033	1,5437	0,3098	0,2173	0,0302	0,0153	0,0038	0,0006	0,0000	0,0000
	-0,5000	17,8777	1,9769	1,3750	0,3285	0,1790	0,0358	0,0114	0,0036	0,0006	0,0000	0,0000
	-0,6000	11,3460	2,3207	1,1972	0,3345	0,1445	0,0338	0,0099	0,0029	0,0000	0,0000	0,0000
	-0,7000	6,1705	2,1026	0,9422	0,3114	0,1156	0,0284	0,0088	0,0027	0,0003	0,0000	0,0000
	-0,8000	2,4019	1,4963	0,5984	0,2262	0,0798	0,0223	0,0055	0,0022	0,0003	0,0001	0,0000
	-0,9000	0,3708	0,7055	0,2358	0,0884	0,0292	0,0089	0,0020	0,0010	0,0003	0,0001	0,0000
	-0,9100	0,2823	0,6169	0,2016	0,0746	0,0243	0,0068	0,0021	0,0005	0,0005	0,0004	0,0000
	-0,9200	0,2173	0,5224	0,1679	0,0617	0,0192	0,0056	0,0023	0,0003	0,0003	0,0000	0,0000
	-0,9300	0,1655	0,4237	0,1344	0,0488	0,0149	0,0044	0,0017	0,0000	0,0000	0,0000	0,0000
	-0,9400	0,1237	0,3218	0,1019	0,0366	0,0113	0,0032	0,0015	0,0003	0,0003	0,0002	0,0000
	-0,9500	0,0259	0,2973	0,1225	0,0035	0,0150	0,0030	0,0020	0,0020	0,0000	0,0000	0,0000
	-0,9600	0,0369	0,2229	0,0717	0,0255	0,0078	0,0022	0,0010	0,0001	0,0001	0,0000	0,0000
	-0,9700	0,0549	0,1340	0,0446	0,0164	0,0040	0,0014	0,0006	0,0001	0,0001	0,0000	0,0000
	-0,9800	0,0270	0,0630	0,0221	0,0079	0,0021	0,0006	0,0003	0,0000	0,0000	0,0000	0,0000
	-0,9900	0,0080	0,0160	0,0067	0,0023	0,0004	0,0002	0,0004	0,0004	0,0004	0,0004	0,0000
	-1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

Table 6.3: Values of the horizontal velocity in different locations (DQM).

VALUES OF THE HORIZONTAL VELOCITY		GRID POINTS											
	y												
	0,00	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=11	N=12	N=13	N=14	N=15
	-0,10	0,64111	0,41649	0,44238	0,43070	0,43147	0,42990	0,42985	0,42976	0,42976	0,42975	0,42975	0,42975
	-0,20	1,12497	0,83704	0,84007	0,82686	0,82345	0,82177	0,82149	0,82135	0,82133	0,82133	0,82133	0,82133
	-0,30	1,45158	1,21767	1,18363	1,16938	1,16419	1,16205	1,16163	1,16149	1,16145	1,16145	1,16144	1,16144
	-0,40	1,62093	1,51439	1,45325	1,43736	1,43193	1,43001	1,42938	1,42925	1,42922	1,42921	1,42921	1,42921
	-0,50	1,63304	1,68325	1,61885	1,60420	1,59841	1,59730	1,59672	1,59659	1,59655	1,59655	1,59655	1,59655
	-0,60	1,48789	1,68027	1,63999	1,63368	1,62841	1,62801	1,62769	1,62764	1,62762	1,62761	1,62761	1,62761
	-0,70	1,18550	1,46148	1,46593	1,47594	1,47526	1,47587	1,47584	1,47590	1,47590	1,47590	1,47590	1,47590
	-0,80	0,72585	0,98290	1,03562	1,06367	1,07234	1,07558	1,07623	1,07644	1,07648	1,07649	1,07649	1,07649
	-0,90	0,10895	0,20057	0,27764	0,30807	0,32058	0,32509	0,32640	0,32679	0,32689	0,32691	0,32691	0,32691
	-0,91	0,03891	0,10679	0,18694	0,21675	0,22875	0,23292	0,23413	0,23449	0,23458	0,23460	0,23460	0,23461
	-0,92	-0,01665	0,03507	0,12114	0,15108	0,16273	0,16664	0,16776	0,16809	0,16818	0,16820	0,16820	0,16820
	-0,93	-0,05772	-0,01636	0,07443	0,10404	0,11516	0,11880	0,11984	0,12015	0,12023	0,12024	0,12025	0,12025
	-0,94	-0,08431	-0,04932	0,04202	0,07032	0,08070	0,08404	0,08500	0,08528	0,08535	0,08537	0,08537	0,08537
	-0,96	-0,09641	-0,06560	0,02019	0,04609	0,05549	0,05848	0,05933	0,05958	0,05965	0,05966	0,05967	0,05967
	-0,97	-0,09403	-0,06701	0,00622	0,02862	0,03669	0,03923	0,03996	0,04017	0,04023	0,04024	0,04024	0,04024
	-0,98	-0,07717	-0,05535	-0,00156	0,01602	0,02221	0,02415	0,02471	0,02487	0,02491	0,02492	0,02492	0,02492
	-0,99	-0,04583	-0,03241	-0,00377	0,00689	0,01042	0,01155	0,01187	0,01196	0,01199	0,01199	0,01199	0,01199
	-1,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 6.4: Approximate estimate of the error for velocity in horizontal direction (DQM)

APPROXIMATE ESTIMATE OF THE ERROR		GRID POINTS										
	y											
	0,00000	N=5	N=6	N=7	N=8	N=9	N=10	N=11	N=12	N=13	N=14	N=15
	-0,10000	53,9319	5,8524	2,7119	0,1785	0,3652	0,0116	0,0209	0,0009	0,0012	0,0000	0,0000
	-0,20000	34,3986	0,3607	1,5976	0,4141	0,2044	0,0341	0,0170	0,0021	0,0009	0,0000	0,0000
	-0,30000	19,2096	2,8759	1,2186	0,4458	0,1842	0,0362	0,0121	0,0034	0,0000	0,0009	0,0000
	-0,40000	7,0352	4,2071	1,1055	0,3792	0,1343	0,0441	0,0091	0,0021	0,0007	0,0000	0,0000
	-0,50000	2,9829	3,9781	0,9132	0,3622	0,0695	0,0363	0,0081	0,0025	0,0000	0,0000	0,0000
	-0,60000	11,4494	2,4561	0,3862	0,3236	0,0246	0,0197	0,0031	0,0012	0,0006	0,0000	0,0000
	-0,70000	18,8836	0,3036	0,6782	0,0461	0,0413	0,0020	0,0041	0,0000	0,0000	0,0000	0,0000
	-0,80000	26,1522	5,0907	2,6371	0,8085	0,3012	0,0604	0,0195	0,0037	0,0009	0,0000	0,0000
	-0,90000	45,6798	27,7590	9,8776	3,9023	1,3873	0,4013	0,1193	0,0297	0,0067	0,0009	0,0000
	-0,91000	63,5625	42,8772	13,7527	5,2447	1,7929	0,5160	0,1535	0,0379	0,0090	0,0013	0,0000
	-0,92000	147,4574	71,0468	19,8182	7,1555	2,3452	0,6694	0,1987	0,0499	0,0113	0,0018	0,0000
	-0,93000	-252,7805	121,9817	28,4585	9,6613	3,0607	0,8687	0,2572	0,0657	0,0150	0,0025	0,0000
	-0,94000	-70,9399	217,3574	40,2371	12,8669	3,9707	1,1271	0,3307	0,0855	0,0199	0,0000	0,0000
	-0,96000	-46,9628	424,9376	56,1952	16,9358	5,1197	1,4343	0,4179	0,1140	0,0201	0,0107	0,0000
	-0,97000	-40,3245	1177,7994	78,2760	21,9993	6,4695	1,8219	0,5278	0,1417	0,0323	0,0000	0,0000
	-0,98000	-39,4345	-3454,6821	109,7162	27,8617	8,0166	2,2544	0,6554	0,1686	0,0321	0,0000	0,0000
	-0,99000	-41,4052	-760,7968	154,6730	33,8800	9,8268	2,6795	0,7858	0,2002	0,0334	0,0000	0,0000
	-1,00000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

Table 6.5 and Table 6.7 that are obtained by using differential quadrature method are values of the vertical and horizontal velocity in different locations. Table 6.6 and Table 6.8 are approximate estimate of the error for velocity in vertical and horizontal direction. Error was reduced with increasing degree of polynomials. If two semi analytical methods compared with each other, DQM is more efficient than the MoM in terms of computational effort. MoM takes more time than DQM.

Table 6.5: Values of the vertical velocity in different locations (MoM).

	y	POLYNOMIAL DEGREE					
	0,00	5th	6th	7th	8th	9th	10th
VALUES OF THE VERTICAL VELOCITY	-0,10	0,0178	0,0230	0,0218	0,0217	0,0218	0,0218
	-0,20	0,0699	0,0882	0,0852	0,0846	0,0847	0,0847
	-0,30	0,1559	0,1893	0,1857	0,1841	0,1843	0,1843
	-0,40	0,2744	0,3188	0,3169	0,3143	0,3145	0,3146
	-0,50	0,4216	0,4687	0,4695	0,4668	0,4668	0,4668
	-0,60	0,5884	0,6286	0,6315	0,6295	0,6293	0,6293
	-0,70	0,7581	0,7842	0,7875	0,7864	0,7863	0,7863
	-0,80	0,9044	0,9149	0,9167	0,9163	0,9163	0,9163
	-0,90	0,9887	0,9897	0,9900	0,9900	0,9900	0,9900
	-0,91	0,9918	0,9926	0,9928	0,9928	0,9928	0,9928
	-0,92	0,9942	0,9948	0,9949	0,9949	0,9949	0,9949
	-0,93	0,9959	0,9963	0,9965	0,9965	0,9965	0,9965
	-0,94	0,9971	0,9975	0,9976	0,9976	0,9976	0,9976
	-0,95	0,9981	0,9984	0,9985	0,9985	0,9985	0,9985
	-0,96	0,9988	0,9990	0,9991	0,9991	0,9991	0,9991
	-0,97	0,9993	0,9995	0,9995	0,9995	0,9995	0,9995
	-0,98	0,9997	0,9998	0,9998	0,9998	0,9998	0,9998
	-0,99	0,9999	0,9999	0,9999	0,9999	0,9999	0,9999
	-1,00	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000

Table 6.6 Approximate estimate of the velocity error in vertical direction(MoM)

	y	POLYNOMIAL DEGREE				
	0,00	6th	7th	8th	9th	10th
APPROXIMATE ESTIMATE OF THE ERROR	-0,10	22,6794	5,1936	0,4710	0,1552	0,0092
	-0,20	20,7193	3,5519	0,7021	0,1117	0,0093
	-0,30	17,6492	1,9152	0,8832	0,1221	0,0043
	-0,40	13,9175	0,6040	0,8228	0,0750	0,0124
	-0,50	10,0471	0,1608	0,5773	0,0028	0,0028
	-0,60	6,3882	0,4697	0,3223	0,0276	0,0022
	-0,70	3,3222	0,4191	0,1373	0,0155	0,0001
	-0,80	1,1482	0,1953	0,0426	0,0010	0,0004
	-0,90	0,1055	0,0336	0,0038	0,0004	0,0001
	-0,91	0,0776	0,0250	0,0027	0,0003	0,0001
	-0,92	0,0597	0,0187	0,0020	0,0002	0,0001
	-0,93	0,0486	0,0134	0,0014	0,0003	0,0002
	-0,94	0,0400	0,0098	0,0026	0,0013	0,0002
	-0,95	0,0324	0,0066	0,0007	0,0004	0,0003
	-0,96	0,0273	0,0039	0,0012	0,0004	0,0003
	-0,97	0,0159	0,0020	0,0020	0,0015	0,0002
	-0,98	0,0080	0,0007	0,0000	0,0001	0,0000
	-0,99	0,0022	0,0001	0,0000	0,0000	0,0000
	-1,00	0,0000	0,0000	0,0000	0,0000	0,0000

Table 6.7: Values of the horizontal velocity in different locations (MoM).

	y	POLYNOMIAL DEGREE					
	0,00	5th	6th	7th	8th	9th	10th
VALUES OF THE VERTICAL VELOCITY	-0,10	0,3515	0,4510	0,4317	0,4291	0,4298	0,4297
	-0,20	0,6913	0,8431	0,8283	0,8201	0,8213	0,8213
	-0,30	1,0259	1,1661	1,1715	1,1610	1,1612	1,1615
	-0,40	1,3390	1,4123	1,4364	1,4304	1,4289	1,4292
	-0,50	1,5905	1,5680	1,5958	1,5984	1,5966	1,5965
	-0,60	1,7175	1,6053	1,6193	1,6283	1,6279	1,6276
	-0,70	1,6333	1,4731	1,4659	1,4753	1,4760	1,4759
	-0,80	1,2282	1,0896	1,0707	1,0759	1,0765	1,0765
	-0,90	0,3691	0,3334	0,3256	0,3268	0,3270	0,3269
	-0,91	0,2707	0,2474	0,2416	0,2425	0,2426	0,2425
	-0,92	0,1984	0,1838	0,1791	0,1798	0,1798	0,1798
	-0,93	0,1462	0,1365	0,1326	0,1330	0,1330	0,1330
	-0,94	0,1089	0,1011	0,0977	0,0980	0,0979	0,0980
	-0,95	0,0819	0,0742	0,0713	0,0715	0,0716	0,0716
	-0,96	0,0616	0,0533	0,0510	0,0512	0,0514	0,0513
	-0,97	0,0451	0,0367	0,0351	0,0353	0,0354	0,0353
	-0,98	0,0301	0,0230	0,0221	0,0222	0,0223	0,0222
	-0,99	0,0153	0,0111	0,0107	0,0108	0,0108	0,0108
	-1,00	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

Table 6.8: Approximate estimate of the error for velocity in horizontal direction (MoM).

	y	POLYNOMIAL DEGREE				
	0,00	6th	7th	8th	9th	10th
APPROXIMATE ESTIMATE OF THE ERROR	-0,10	22,0505	4,4589	0,6087	0,1568	0,0154
	-0,20	18,0044	1,7863	0,9998	0,1546	0,0017
	-0,30	12,0182	0,4652	0,9096	0,0198	0,0250
	-0,40	5,1923	1,6799	0,4237	0,0994	0,0168
	-0,50	1,4343	1,7408	0,1583	0,1077	0,0081
	-0,60	6,9908	0,8664	0,5558	0,0270	0,0160
	-0,70	10,8737	0,4939	0,6372	0,0495	0,0041
	-0,80	12,7184	1,7642	0,4842	0,0511	0,0009
	-0,90	10,6888	2,3986	0,3776	0,0358	0,0125
	-0,91	9,3861	2,4346	0,3761	0,0326	0,0091
	-0,92	7,9525	2,5967	0,3649	0,0056	0,0222
	-0,93	7,1020	2,9607	0,3495	0,0609	0,0669
	-0,94	7,6601	3,5150	0,3503	0,0716	0,0762
	-0,95	10,3548	4,1194	0,3841	0,0214	0,0038
	-0,96	15,5323	4,5360	0,4461	0,2132	0,1459
	-0,97	22,8646	4,5244	0,5007	0,3990	0,2894
	-0,98	31,1156	4,0036	0,4978	0,4149	0,2972
	-0,99	38,0224	3,2444	0,4188	0,1891	0,1132
	-1,00	0,0000	0,0000	0,0000	0,0000	0,0000

7. DISCUSSION AND CONCLUSION

In this study, we consider semi analytical numerical investigation of uniform flow over a porous plane with suction perpendicular to the surface by using semi analytical numerical methods. Semi analytical numerical methods are differential quadrature method and method of moments. Porous media is modeled by using Biot's theory of poroelasticity and in addition to this, Brinkman-extended Darcy equation is used in the reference study. At the interface of the fluid layer and porous layer, continuity of velocity vector and continuity of shear stress are used.

It is easily seen from the graphs that vertical velocity increases from top to the bottom because of the downward suction. The maximum horizontal velocity is 1.64 and at the bottom there is no horizontal velocity. Also Deng and Martinez's study is added to the graphs to be reference. At the end of the graphics, approximate estimate of the error for velocity are added in the table.

This study derived an alternative semi analytical solution to a two dimensional flow field is derived. Flow field of a two dimensional flow is possible to be solved new approaches. Meaningful solutions can be obtained without evaluation of the empirical constants. Present approaches and methods are able to simplify the equations and algorithm process. The technique and method can be applied to the other studies.

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