ON THE TRANSIENT ANALYSIS OF TRANSFER LINES

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ON THE TRANSIENT ANALYSIS OF TRANSFER LINES

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M.S. THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"ON THE TRANSIENT ANALYSIS OF TRANSFER LINES"** completed by **Görkem Sariyer** under supervision of **Prof. Dr. Cemal Dinçer** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

ON THE TRANSIENT ANALYSIS OF TRANSFER LINES

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May 2009

A manufacturing flow line, transfer line, is defined in literature as a serial production system in which parts are worked sequentially by machines : pieces flow from the first machine, in which they are still raw parts, to the last machine where the process cycle is completed and the finished parts leave the system. In another words, a transfer line corresponds to a manufacturing system consisting of a number of work stations in series integrated into one system by a common transfer mechanism and a control system. In literature many studies have been done, and a lot of paper are published about transfer lines. However, not so many has been done on the stochastic behavior and the transient analysis of these systems. This thesis studies the transient and stochastic behaviour of short transfer lines ,derives the distribution of these systems, finds the number of outputs of these systems, and calculates the performance measures, such as mean and variance. Also extensions are discussed ,conclusions and directions for future work are given in the last section of the thesis.

Keywords: Transfer Lines, Distribution of Throughput, Stochastic Behaviour, Mean and Variance of Throughput, Transient Analysis.

ÖZ

TRANSFER HATLARININ GEÇİCİ DURUM ANALİZİ

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Transfer hatları,literatürde seri üretim sistemleri olarak tanımlanmaktadır. Bu sistemlerde işlenmemiş parçalar, hammaddeler,sisteme dışarıdan girerler, ve sırayla seri olarak bağlanmış makinelerde işlenerek, işlenmiş çıktı olarak sistemi terkederler.Seri üretim hatları parça akışını ve üretim aşamalarının etkileşimini en basit biçimde temsil etmektedir. Bu sistemlerle ilgili geniş bir literatür bulunmaktadır. Fakat, söz konusu rassal süreç evriminin belirlenmesindeki zorluk sebebiyle başarım ölçütlerinin yüksek sıra momentleri kullanılarak seri üretim hatlarının geçici durum çözümlemeleri üzerinde çok çalışma yapılmamıştır. Bu tezde, seri üretim hatlarının geçici durumu incelenmektedir. Bu analizin kullanılmasıyla,kalıcı durum analizi kolaylıkla yapılabilecektir. Transfer hatlarının performansları olasılık ve istatistik teoremleri kullanılarak incelenmektedir. İşlenmiş madde,çıktısayısı, bu çıktıların dağılımları, beklenen değerleri, ve beklenen değerden sapmaları istatistik teoremleri kullanılarak yapılmaktadır.Bu tezde önerilen yöntem, üssel dağılan aravarış ve işgörü sürelerine sahip çok parçalı sistemlere de uygulanmıştır. Bu çalışmadan çıkarılabilecek sonuçlar açıkça dile getirilmiş ve yeni araştırma noktalarına da işaret edilmiştir.

Anahtar Kelimeler: Seri Üretim Hatları, Çıktı Sayısının Dağılımı, Stokastik Davranışlar ve İstatistiksel Özellikler, Geçici Durum Çözümlemesi, Çıktının Beklenen Değeri ve Beklenen Değerden Sapması.

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To my mother and father

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Chapter 1

INTRODUCTION

Manufacturing is the transformation of material into something useful and portable. A manufacturing system is a set of machines, transportation elements, computers, storage buffers, and other items that are used together for manufacturing. Subsets of manufacturing systems, are sometimes called cells, work centers or work stations. Manufacturing systems are very important in Industrial Engineering. Engineers must decide carefully how, where and when to manufacture.

In this thesis, transfer lines which can be defined as a manufacturing system with a very special structure,are studied. There are two main reasons for studying transfer lines. The first one is that they are of economic importance as they are generally used in high volume production. Secondly the transfer lines represent the simplest form of the interactions of manufacturing stages and their decoupling by means of buffers.

Service stations or machines and the buffer storages are the main parts of the transfer line. $M_1,\ M_2,\ M_3..,M_n$ and $B_0,\ B_1,\ B_2...,B_n$ represent the linear network of machines and the buffers of the transfer line. General proceeding of the system can be explained as : Material flows from outside the system to B_0, in order to be controlled and to be lost if the first machine is busy , then to M_1, in order to be processed in the first machine, then to B_1, in order to be controlled and be lost if the second machine is not ready for taking and processing another part, and so forth until it reaches B_n after which it leaves the system as an output. In graphical representation squares and triangles can be used to represent machines and buffers respectively.

There have been many studies and a vast literature about the transfer lines. Most of the studies focused on the steady-state analysis. In steady state analysis initial conditions have no effect on the the system performance.Because in such analysis the behaviour of the system is taken into consideration as the time from initialization becomes very large. Steady-state analysis are easier to perform because the equations can be easily simplified in the limit, and other techniques such as balance equations can be used. So, steady-state operations can be modeled as Markov Chains.One of the disadvantage of using steady-state analysis is in real systems there are many situations in which time horizon of the operation can be terminate and in such situations using steady-state analysis can not be suitable and efficient. So, in manufacturing systems and in queuing theory, there are many situations in which transient analysis can be used. But, transient results are more difficult to obtain,and they are more complicated.

In this thesis, we first try to derive the distribution of the throughput or output, which can be defined as number of parts produced by a transfer line. Then by using this distribution we can calculate the mean and variance of these throughput rate. The crucial point in these transfer lines is both inter-arrival and processing times are exponentially distributed. Inter-arrival time is the time that passes between the sequential inputs. Processing time is the time which is sent to process the input in each machine of the transfer line. We use exponential distribution for the inter-arrival and processing times, because this distribution is mathematically simple and it can be used to model wide range of situations. If the probability, that an event will occur in a small time interval , is very small and if the occurrence of this event is statistically independent of the occurrence of other events, and the probability that more than one event will occur during a time interval is negligible (approaches to zero) then time interval between the occurrence of events of this type is exponentially distributed. The most important property of the exponential service distribution is the memoryless property.

Since, transfer lines with high efficiencies and low variances are generally preferred, the results that are found in this thesis can be used to help to design economically feasible transfer lines, which is the main aim of an Industrial Engineer.

In the next chapter of this thesis, we show the review of the literature survey by giving the some information about the proposals of the both steady-state and transient analysis. In Chapter 3, we give and explain the proposed model. And in the last chapter , we give the conclusions and some future research.

Chapter 2

LITERATURE SURVEY

In literature there has been many papers on the calculation of expectation, by using first order moment, of the steady-state performance measures. However, there are not enough papers which analyze the transient behavior of transfer lines. Also deriving the distributions , and calculating the variance , by using second order moments, are generally disregarded.

In this chapter , in the first section we give the characteristics and the literature review of transient behaviour of the transfer lines, and in the following section we give the models for the performance measures of steady-state analysis of transfer lines.

2.1 Transient Analysis Of Transfer Lines

In the early 1960s, many works are done on the transient behaviour of the transfer lines. There are some papers written by Kendall[72], Keilson and Kooharian and Prabhu[71] on *M/G/1* queues. Bailey[15] [16], Heathcote and Winer [57] published some proposals on the *M/M/1* queues. Karlin and McGregor [69] studied the Markov chains, and proved the previously found results for finite state and infinite state birth-and-death processes. These are used to develop *M/G/1* and

GI/M/1 queuing systems by Kendall [72] and to *GI/G/1* systems by the help of Cohen [28]. Instead of using birth-death processes , using the numerical integration method of Kolmogorov differential equations and randomization technique are used to analyze such systems by Gross and Harris[54] .

Kleinrock[73] can be a good reference when analyzing these systems. The transient behaviours of the *M/G/1* and *GI/M/1* queues are studied in detail by Prabhu [97], Stanford,Pagurek and Woodside [112], Asmussen[9], Ott[91], Middleton[85],Heyman [61], Neuts[89].

Here is a quotation which is taken from Odani and Roth[103] : *Much important work is done on approximate or exact numerical solution techniques that can be used to investigate the transient behaviour (e.g. Neuts[89],Rider[100],Collings and Stoneman[29], Chang [26],Grassman[52],,and Rothkopf and Oren[105].*

2.1.1 A Classification of the Transient Analysis of Transfer Lines

In order to classify the literature on the transient analysis of the serial line production systems, due to main differences in the publications we can use three dimensions : applications objective, system assumptions, and methodology used in resolving problems.

2.1.1.1 Applications Objective

Most convenient way of classification is made due to the objective of the study. These objectives can be performance evaluation, transient period determination, determination of bounds on the system performance measures.

a. Performance Evaluation

Performance evaluation is the impact of a policy decision or design change on the behaviour of a system. This change can be measured by observation or can be estimated by using methodology. The main goal of performance evaluation is to decide on the performance criteria, which can be single measure or set of measures. Some of the most important performance measures can be listed as:

- Number of parts produced per unit time : throughput or output rate
- Number of inputs in the queues :queue length
- Percentage of time that a machine is busy : utilization
- Number of inputs in the system : congestion level
- Unfinished work : workload
- Percentage of time the machine is blocked : blocking probability
- Percentage of time a machine is idle : idle period
- Average time that a part spends in the system : throughput time
- Average time that a part spends in the queue : queuing time

If we want to classify the publications from the performance evaluation criteria viewpoint , we can list the literature as follows :

- Queue Length Publications : Sohraby and Zhang [110],Bertsimas and Nakazato[18], Jean-Marie and Robert[65], Lee and Li[77][78],Abate and Whitt[3][4][5], Odani and Roth[90], Klutke and Seiford[74], Morris and Perros[86], Towsley[118].
- Congestion Level Criteria Publications : Tan and Knessl[117], Abate and Whitt[5], Lin and Cochran[81], Baccelli and Makowski[13]
- Workload Criteria Publications : Tan and Knessl[117]
- Utilization Criteria Publications : Gopalan and Dinesh Kumar[50][51], Browne and Steele[21]

- Blocking Probability Criteria Publications : Yunus[127], Morris and Perros[86], Saito and Machihara[107], Gopalan and Dinesh Kumar[51]
- Idle Period Criteria Publications : Sohraby and Zhang[110] ,Gopalan and Dinesh Kumar[50]
- Queuing Time Criteria Publications : Lavenberg[75], Karpelevitch and Kreinin[70] ,Serfozo eta al. [109], Lin and Cochran[81]
- Throughput Time Criteria Publications : Csenki[31] and Gall[40]

The working paper by B. Deler and C. Dincer[130] also gives appropriate method in order to calculate the performance measures, such as mean and variance of the throughput rate, of relatively short transfer lines. These calculations corresponds the transient state, so this method gives any performance measure of interest in any arbitrary time.

b. Bounds On The Performance Measures

Numerical techniques, simulation and approximation techniques can be used when tractable techniques become insufficient in analyzing the model of the system. When we do not have exact results, bounds can be very useful to test approximations, or they are simply the only information that can be obtained for certain measures. Also, a bound may exhibit similar behaviour to the measure itself.It would be interest to prove bounds on the performance criteria as well as their distance from optimality for a variety of serial line production systems.

If we look at the literature, Jackson[63] derives the joint queue length distribution for the steady-state. And Massey obtains a stochastic upper bound for transient joint queue length distribution.

Xie[128] addresses the performance evaluation and optimization of continuos flow transfer lines composed of two machines separated by a buffer or finite capacity. Machines are subject to time dependent failures. Times to repair and times to failure of the machines are random variables with general distribution. For the purpose of performance evaluation, a set of evaluation equations that determines continuos state variables at epochs of discrete event is established. Based on the evaluation equations, they prove the concavity of the throughput rates of the machines and derive gradient estimators. Unbiasedness and strong consistency of the gradient estimators are proved. Finally a design optimization problem that maximizes a concave function of a throughput rate and design parameter is addressed.

J.Li and M.Meerkov[80] derives bounds for variability measures. Since all variability measures of for a single machine line can be calculated easily, these bounds provides analytical tools for analysis and design of serial production line.

2.1.1.2 System Assumptions

Number of simplifying assumptions on the features such as line length, arrival type, queue discipline, multiple servers and multiple part types, make a transfer line system to represent the simplest form of the interactions of manufacturing stages and their decoupling by means of buffers.

The main assumptions that most of the results in the literature are based on them can be explained as follows :

The first machine is never starved and the last machine is never blocked. Processing time, uptime, downtime, arrival time and all other random variables are independent. The setup time and transfer time through the buffer can be negligible. The failures are single-machine failures and either time dependent and operation dependent failures. When a failure occurs it can be reworked when the machine is up again, the work resumes exactly at the point stops. Interarrival times, processing times, time to failure, time to repair of machines are generally assumed to be exponentially distributed when time is continuos and geometrically distributed when time is discrete. (Papadopoulus and Heavey[95]).

Due to these basic assumptions we can classify the literature according to the line length, arrival type, non-exponential service time distribution, parallel servers and multiple part times.

- Line length
  - Single Machine : Towsley[118], Yunus[127], Brownee and Steele[21], Daigle and Magalhaes[32], Karpelevitch and Kreinin[70], Lee and Li[77][78], Conolly and Langaris[30], Saito and Machihara[107], Xie and Knessl[126], Jean-Marie and Robert[65], Wall and Worthington[121], Sohraby and Zhang[110], Tan and Knessl[117]
  - Longer Lines With More Than One Machine : Klutke and Seiford[74], Csenki[31], Deler and Sabuncuoğlu[36], Gopalan and Dinesh Kumar[51], Lee and Roth[79], Lin and Cochran[81], Morris and Perros[86], Gall[40]
- Arrival type
  - Switched Poisson Arrival : Lee and Li[77]

- Markov Modulated Poisson Arrival : Lee and Li[78]
- Batch Arrivals : Sohraby and Zhang[110], Bohm and Mohanty[20]
- Non Poisson Arrival : Lee and Roth[79], Lin and Cochran[81]

- Non-exponential service time distribution
  - Non-Markovian : Bohm and Mohanty[20], Abate and Whitt[5] , Sohraby and Zhang[110] , Wall and Worthington [121], Jean Marie and Robert[65], Gall[40], Browne and Steele[21], Lin and Cochran[81], Morris and Perros[86], Bertsimas and Nakazato[18]
  - Parallel Servers : Saito and Machihara[107], Yunus[127], Browne and Steele[21], Lee and Li[77], Gopalan and Kumar[50], Xie and Knessl[125], Sohraby and Zhang[110]
  - Multiple Part Types : Daigle and Magalhaes[32]
  - Setup Time : Narahari and Viswanadham[88], Bai and Elhafsi[14]

As it can be seen on the classification of the literature, there has been studies on the parallel servers. Existence of the parallel servers makes the real-life serial line manufacturing systems difficult to deal with. There are two main reasons of building the production systems with parallel servers. One of these reasons can be given as to achieve a greater reliability.This is important property for the system, when some of the machines are much less reliable then the others. By using these machines with parallel connection, one may build the production system more reliable. The other reason can also be given as to achieve greater production rate. This property is very useful when some of the operations are much slower than the others. Again, by using parallel connection among machines, one may produce more items in a given time.

Existence of machine breakdowns is also the another complexity which makes the real-life serial line production systems difficult to model. Lin and Cochran[81] make use of simulation based approach in order to study the transient behaviour of assembly line with machine breakdowns. As an another study Jenq[66] develops an approximate algorithm to calculate the mean and variance of the queue length for the discrete-time buffer system with service interruptions.

Generally in the studies of manufacturing systems, the arrival process is assumed to be exponential, because exponential distribution is the only continuos distribution with the lack of memory property. However there are a few papers in the literature which studies the Non-Poisson arrival types. Lee and Li[78] analyze the transient behaviour of Markov modulated Poisson arrival queues with multiple exponential servers. They derive expressions for one step transition probabilities of the queue. Taking advantage of matrix geometric solution for the one step transition probability matrices, they simplify the evaluation of the transient queue length distributions. They also present a methodology to analyze the transient behaviour of a queue length dependent switched Poisson arrival queue. The queue has infinite buffer capacity with an exponential server with its rate driven by a two state Markov Chain. They derive the expressions for one step transition probabilities. The probability distributions of queue length at any arrival epoch can be calculated using the Kolmogorov equations. Taking advantage of the geometric form of the transition probabilities, they further simplify the evaluation of the forward equation. They also explore the queue transient performance as affected by the mean sojourn times of the underlying two-state Markov chain for the arrival process. Finally, Bohm and Mohanty[20] considers a Markovian queuing system in discrete time in which parts arrive from different sources in batches of various size.

First-come-first-serve (FCFS) discipline is the typical queue discipline assumed in the serial line literature. However, Jean-Marie and Robert[65] analyze the transient behaviour of the single server queue under the processor discipline.

The setup time is assumed to be negligible because most of the studies in the literature analyze the production systems with single class of items. Bai and Elhafsi[14]consider a deterministic one-machine two-item production system. They formulate the problem as an optimal control and divide the planning horizon into a transient and steady state period. For the transient period, an algorithm involving a line search procedure is developed and a feedback control policy is found to be the optimal solution. Narahari and Viswanadham[88] discuss two problems for demonstrating the importance of transient analysis : the first problem is concerned with the computation of distribution of time to absorption in

Markov models of production systems with deadlocks or failures, and the second problem shows the relevance of transient analysis to a multi-class production system with significant setup times. Such a facility that involves the production of several classes of items belonging to different priority levels and involving significant setup times is studied by Narahari,Hemochondra and Gaur[87]. If we analyze and compare the numerical experiments of these two studies , we can conclude that transient performance of the system can be significantly different from the performance predicted by the steady-state analysis and also transient analysis has important implications for decision making during design and operation.

2.1.1.3 Methodology Used In Resolving Problems

Analytical approach, heuristics oriented approach and simulation based approach can be the classification of the transient analysis literature based on the methodology used in resolving problems.

a. Analytical Approach

Mathematical methods such as calculus, probability theory or algebraic equations can be possible to use, to obtain exact information on questions of interest, if the relationship that compose the models are simple. But, only special systems have exact solutions due to the complexities introduced by the buffers in the serial line production systems. When we have such complex systems, we can only use approximate methods.

Analytical approach can be classified as exact analysis and approximate analysis.

If a production system models have exact analytical solutions, we can use exact analysis to deal with these models. These models are important for three reasons : One of the main reasons is that exact solutions are better than simulations or approximations when the models fit real system closely. As an another reason, exact solutions provide useful qualitative insight into the behaviour of systems. Finally, the fact that they can be solved rapidly makes them essential parts of decomposition and aggregation methods.

Most of the results of exact analysis of transfer lines are based on the derivation of the joint distributions of the performance criteria of interest via the use of transition probabilities. In order to be able to describe the behaviour of a serial line production system by a Markov process, the distributions have to be of special form : exponential, or continuos time models; geometric, or more generally discrete phase type distribution in case of discrete time models.

The Markov Process is often described by set of difference-differential equations. Abate and Whitt[4][5] use differential equations in order to describe the time-dependent moments of workload process in the *M/G/1* queue. Integral equations are the another way which characterizes the transient behaviour of transfer lines.

Gopalan and Dinesh Kumar[50][51] deal with a merge production system to study the effect of the parameters on utilization and availability, busy period, and blocked period of machines in system. They develop a mathematical model using semi-regenerative phenomena and system of integral equations satisfied by various state probabilities are obtained.

Tan and Knessl[117] derived integral representations for the time dependent distributions functions of the *M/M/1* queue described by the unfinished work and also a finite capacity version of the model where the parts who would cause the unfinished work to exceed a given capacity are rejected and lost. This yields a new set of approximations for describing the transient behaviour of *M/M/1* queue and they also show that their formulas yield some interesting insights into how the process settles to its steady-state distribution.

Bertsimas et al.[18] formulate the problem of finding simultaneously the waiting time distribution and the busy period distribution of the *GI/G/1* queue. Bertsimas further obtain closed form expressions also for the Laplace transforms of the waiting time distribution and the busy period distribution by formulating the problem as two dimensional Lindley process.

Bailey[16] shows that the transient behaviour of the queue length process in the *M/M/1* model can be described using double transforms with respect to time and space. Abate and Whitt[4] discuss how the Laplace transform analysis of Bailey can be persuaded to yield additional insight about time dependent behaviour of the queue length process in the *M/M/1* model. Sharma also used Laplace transforms in developing series of expansion that is known as the methodology of Sharma. Conolly and Langaris discuss series of formula of Sharma for the computation of the time dependent state probabilities in *M/M/1* queuing systems with emphasis on methods that avoid Bessel functions. The last use of Laplace transforms to be mentioned is that they help the development of most of the asymptotic expansions.

The use of moment generating functions is the another way of analyzing the transient behaviour of queuing system. Bacceli and Makowski[13] make use of an exponential martingale in order to calculate the Laplace transform of the length of busy period for the *M/GI/1* queue.

Karpelevitch and Kreinin[70] also treat the generating function of the joint waiting time and queue length distributions at the arrival epochs of Poisson serial lines.

Serial line production systems can also be analyzed by making use of higher order moments. Abate and Whitt[3] shows that factorial moments of the queue length as functions of time when *M/M/1* queue starts empty have interesting structure, which facilitates developing approximations.

Gall[40] makes use of Laplace-Stieltjes transforms to derive the probability of the distribution of the overall sojourn time of a serial line production system in case of a renewal arrival process and arbitrarily distributed identical successive service times. Abate and Whitt[2][3] also characterize time-dependent moments of the workload process in terms of differential equation involving lower moment functions and time-dependent server-occupation probability.

B.Tan and K.Yılmaz[116] use analytical approach to derive Markov chain test for time dependence and homogeneity. In their paper they evaluate the small and large sample properties of Markov chain time-dependence and time-homogeneity tests. They present the Markov chain methodology to investigate various statistical properties of time series. Considering an autoregressive time series and its associated Markov chain representation, they derive analytical measures of statistical power of the Markov chain time dependence and time homogenity tests. They later use Manto Carlo simulations to examine the small-sample properties of these tests. It is found that although Markov chain time dependence test has desirable size and power properties, time homogenity test does not perform well in statistical size and power calculations.

Generally, authors in literature assume empty initial conditions. But, Yunus[127] presents exact expressions for transient blocking probability in $M/M/n$ loss system, assuming initial condition.

Michael Fu and Xiaolan Xie[129] derive estimators of throughput sensitivity to changes in buffer capacity for continuos flow models of a transfer line comprising two machines separated by a buffer of finite capacity, where machines are subject to operation-dependent failures. Both repair and failure times can be general, they need not to be exponentially distributed. The system is hybrid, in the sense that it has both continuos dynamics -as a result of continuos material flow- and discrete events: failures and repairs. The combination of operation-dependent failures and the buffer capacity as the parameter of interest make the derivative estimation problem difficult, in that unlike previous work on continuos flow models, careful use of conditional expectation is required to obtain unbiased estimators.

For relatively short transfer lines it can be easily seen that exact analysis can be appropriate. Unfortunately, it is not appropriate to expect to obtain exact solutions of serial lines with more machines of longer transfer lines. The work on these longer lines involves models that are not tractable, or are subject to numerical problems. For those reasons the use of approximate solutions is the only alternative when dealing with longer transfer lines.

Towsley[118] applies reflection principle for random walks for analyzing

M/M/1 queuing systems. A closed-form solution can be obtained for the probability that exact number of arrivals and departures occur over an integral of length t in an *M/M/1* queuing system that contains a particular number of parts at the beginning of the interval. Application of the reflection principle then allows to determine the latter distribution.

Saito and Machihara[107] analyze the transient behaviour of a Markovian loss system with heterogeneous inputs. They apply the properties of system recovery and covariance functions to the analysis of time congestion measurement whose variance is obtained for a single machine model with multiple servers.

Approximations are proposed using these properties. The transient behaviour of one dimensional reflected Brownian motion associated with *M/M/1* queue model is described by Abate and Whitt[1][2] The results help to describe how a large class of queuing processes approach steady-state. These results provide simple analytical approximations. In another study, Lauchard show that several stochastic variables associated with a finite population queuing system can also be approximated by Brownian motions using weak convergence theorems. The result of this study suggests that queue length, unfinished work, storage occupied and idle time show different limiting behaviour depending on the arrival and service distribution.

A discrete time analysis with its time indexed by packet arrivals, which is a different approach from the conventional transform approach is studied by Lee and Li[78]. They derive expressions for the transition probabilities of the two dimensional Markov chain formed by the queue length and the underlying Markov chain in adjacent interval. Due to the geometric solutions for the one step transition probabilities they are able to devise an efficient algorithm to reduce the complexity of directly evaluating the forward equation.

Klutke and Seiford[74] uses recursive equations. Because the sample path behaviour of any serial line can be described by means of recursive equations mainly for establishing qualitative properties like monotonicity and reversibility. They use these equations to compute the transient expected output times from serial line manufacturing systems with no waiting positions.

The computational requirements like computer memory can be practical in solving discrete time queuing time models by numerical techniques. Also these computational requirements are particularly dependent on the number of discrete time intervals required in the discrete distribution chosen to match the general service time distribution. Wall and Worthington[121] show that the minimum number of points required for matching to the first two moments depends on the size of discrete interval relative to the mean and also on the coefficient of variation.

Classic Erlang loss model is analyzed by Knessl[125]. In constructing the approximations the singular perturbation techniques such as the ray method, boundary layer theory and the method of matched asymptotic expansions are used. Xie and Knessl[126] take similar approach in giving asymptotic expansions for the probability that the system contains a particular number of parts at time t in classic *M/M/1* queue with finite/infinite capacity. Their results are based on both the asymptotic expansion of an exact integral representation for the probability of interest and applying the ray method to a scaled form of the forward Kolmogorov equation describing time evaluation of the probability.

Most appropriate methods are based on decomposition. The common idea is to decompose the analysis of the original model into the analysis of a set of smaller subsystems which are easier to analyze.

Csenki[31]shows that a suitable probabilistic reasoning using Markov chains can be used to obtain the probability mass function and the cumulative distribution function of the joint distribution of the sojourn times. The key method of analysis employed in this study is based on the introduction of equivalent absorbing Markov chains whose state space is partitioned into groups of states which are visited in predetermined order, each one, until absorbtion takes place.

The tandem-network is analyzed by Morris and Perros[86]. They present an approximation algorithm for the analysis of discrete-time tandem network of cut-through trees. The tandem-queue network is analyzed by decomposing into individual queues. Using this approach, the queue length distribution, the packet loss, the blocking probability between queues, and the throughput rate is obtained for different parameters.

In the transient analysis of multi-server discrete time queues, Sohraby and Zhang[110]show that once the spectral decomposition of the probability generating matrix of the arrival process is obtained, the complete solution in the transform domain may be given. Using the complex analysis technique and Cauchy's integral formula, they present an efficient numerical method of the numerical calculation of a few performance measures of interest, namely transient probability of a queue being empty, and the mean of queue length distribution.

Shortly, if we want to give the written papers which deal with analytical approach of transfer lines in transient state, we can give a summary as follows :

- Exact Analysis : Abate and Whitt[1][2][3][4], Bruneel[22], Yunus[127], Gall[40], Baccelli and Makowski[13], Klutke and Seiford[74], Gopalan and Kumar[50], Conolly and Langaris[30], Whitt[123], Gopalan and Kumar[51], Daigle and Magalhaes[32],Tan and Knessl[117], Abate and Whitt[3][4], Narahari and Viswanadham[88], Karpelevitch and Kreinin[70]
- Approximate Analysis : Csenki[31], Bai and Elhafsi[14], Xie and Knessl[126], Jean-Marie and Robert [65], Jacobson[64], Glynn[46], Sohraby and Zhang[110], Lee and Li[78], Whitt[123], Bertsimas and Nakazato[18], Saito and Machihara[107], Morris and Perros[86], Wall and Worthington[121], Bohm and Mohanty[20], Guillemin et al[55], Browne and Steele[21], Asmussen et al[10].

The main problem with analytical method studies is the very narrow applicability of the models: they do not consider the full complexity of general production systems. Another difficulty is the computational effort involved : it is hard to find models that will solve reasonably sized problems efficiently. So, there is a need to develop models that factor in the full complexity of the transfer lines.

b. Heuristics Oriented Approach

In real life, there are many serial line manufacturing systems, which we can not obtain exact solutions due to the complexities. In fact even for simple systems few results are available to characterize the transient response. Unfortunately, there is not much study which uses heuristics approach in literature.

Roth[103] shows that expected queue length approaches to an equilibrium value in an approximately exponential manner. Based on this observation a heuristic is proposed for approximating the transient expected queue length for Markovian systems by scaling the numerical solution of an *M/M/1* system.

Lee and Roth[104] derive an approximation for transient expected queue length. Using these results, they propose a heuristic for estimating transient behaviour. This work is application oriented and their objective is to provide a closed-form expression for non-specialists to use in practice.

A. Dolgui, B. Finel and F. Vernadat[34] studies another heuristics approach for transfer lines balancing. They deal with the optimal balancing transfer lines where the operations in each work-station are grouped into blocks. All operations of the same block are executed simultaneously by one spindle head. Spindle heads of the same workstations are activated simultaneously.

Jaber Abu Qudeiri , Hidehiko Yamahato[120] also studied heuristics approach of transfer lines by using genetic algorithm for buffer size and work-station capacity in production lines. They tried to find an optimal design for Serial-parallel production line (S-PPL).

E. M. Feit and S. David Wu[38] uses heuristics approach in order to find the performance information of transfer lines with uncertain machine. They consider production line design where the information available for equipment under consideration is unreliable, incomplete or erroneous. They also develop an analytic procedure for information gathering, that reduces uncertainty by systematically improving the information critical for overall design performance. They also report test results using real design cases from General Motors.

For more general systems, it is not possible to obtain transient solutions which are sufficiently accurate to verify the accuracy of approximate solutions generated by heuristics.

c. Simulation based approach

Simulation can be defined as a symbolic or numerical abstraction of a manufacturing system on a computer. Simulation is probably the most widely used modelling technique for manufacturing systems design and operation. Because simulation modelling is very flexible in terms of model development. However, the development of a simulation model is expensive. Its disadvantages are that they are costly and time-consuming to build, and it is not an optimization or a generative technique.

Simulation is more efficient than analytical formulations of production system problems. Since there is no need to make any unnecessary simplifying assumptions, there is no doubt about feasibility with simulation.

Lin and Cochran[81] study the transient behaviour of assembly line for the often encountered dynamic event of machine breakdown, in much detailed by computer simulation. Due to the system and job characteristics complexity, they utilize the discrete event simulation that provides a modelling power unavailable from analytical models. To combine the advantages of analytical and simulation methods, the simulation results of assembly line transient behaviour are modeled by dynamic meta models in the form of first order continuos exponential delay functions which are the solutions to continuos first order differential equations.

The integrated research approach of using simulation in modelling system details and analytical method in describing system transient behaviour is a powerful methodology for analyzing complex large scale systems.

In literature there are two other papers written on the simulation based approach : Deler & Sabuncuoğlu[36] and Glynn and Heidelberger[47].

Mahyar Mahinzeim[83] also used simulation approach in analyzing transfer lines. His study employs a simulation-based design methodology to investigate the performance of two models of manufacturing systems. In the first model the dynamic behaviour of a single parallel-machine stage with unreliable work stations is modelled as a Markov process. A similar analytical method for evaluating the performance of a buffered production line is presented in second model. A simple approach towards coding and simulating the model is presented and numerical example based on these simulation models indicate that the approach is viable.

Philippe Lavoie, Jean Pierre Kenne and Ali Garbahi[76] also used simulation based approach in order to study the optimization of production control policies in failure-prone homogenous transfer lines. A method consisting of an analytical form combined discrete/continuos simulation modeling, design of experiments and response surface methodology is used to optimize set of transfer lines, with one parameter per machine, for up to seven machines.

2.2 A Classification Of The Steady-State Analysis of Transfer Lines

Meaning of Steady State : This is often confusing concept. It does not mean that the system is settling down in any sense. Knowing the steady state distribution tells us nothing about the dynamics of the system, or what state it will be in at any given time. The existence of a steady state distribution says nothing about the state of the system at any time in the future. The steady state distribution tells something about the state of our knowledge of the system and its future. If we know that the state at time 0, we might have a good idea of the state at time 1.

There are many papers which analyze the transient behaviour of transfer lines in literature. One of these papers can be given as Dallery and Gershwin[33]. These paper is very important because it includes detailed information of the steady-state behaviour of the transfer lines. It gives information about the major classes of models such as synchronous, asynchronous and continuos classes. It also includes the major features of the steady-state behaviour such as failures and repair probabilities , blocking and processing times. And it also explains the relationships among models. Exact and approximate methods in order to measure the performance is also given. It is well known that exact methods can be used efficiently for small systems and the approximate methods can be quite appropriate for larger systems.

There are two other papers which study the steady-state behaviour of serial production lines and published before 1992. These three papers analyze these systems in a wide perspective and give a lot of knowledge. The other of these paper is written by Papadopoulus and Heavey[95]. In this paper one can find a bibliography of material concerned with modeling and production and transfer lines using queuing network is provided. A systematic categorization of the queuing network models can be done due to their assumptions.

The last paper, which is published before 1992, is written by Buzacott and Shanthikumar[24]. They give detailed information about design issues of flow lines as well as various types of manufacturing systems such as automatic transfer lines, job shops, flexible assembly systems, and multiple cell systems. Approaches to resolving the design issues of these systems using queuing models are also reviewed.

After than, many papers are written on the steady-state behaviour of transfer lines based on the previously written three papers and studies.

Hendricks[59] describes the output process of a serial production line of n machines with exponential processing time distributions and finite buffer capacities. In developing this technique extensive exact results are used to examine the effects of line length, buffer capacity and buffer placement on the inter-departure distribution, correlation structure, and variability of the output process of production line. These results are used in determining the extend to which buffer allocation can be used to control the variability of the output process.

Papadopoulus[92][93] considers the throughput rate of multistation reliable production lines with no intermediate buffers. Processing times at the service stations are independent exponential random variables, possibly with different means. The results provide the distribution function of the holding time at stations. Also the mean performance of multi-station production lines with and without inter-station buffers is determined exactly by generating state-space model automatically and solving the system of equations or approximately by decomposition (Tan and Yeralan[114]).

Hong,Glassey and Seong[62] develop a technique in order to analyze *n-machine* production line with unreliable machines and random processing times. The behaviour of the n-machine line is approximated by behaviours of the *(n-1) machine* lines based on the decomposition technique proposed by Gershwin[44].

The output process of transfer line of n machines with general processing time distributions and finite buffer capacities is examined by Hendricks and Mc Clain[60]. In order to observe the effects of line length, buffer capacity and buffer placement on the interdeparture distribution and correlation structure of the output process of the production line, they used simulation.

Glassey and Hong[45] develop an efficient method to analyze the behaviour of an unreliable *n-stage* transfer line with *(n-1)* finite storage buffers.

Heavey, Papadopoulus and Browne[58] examine unreliable multi station series production lines. The first station is never starved and the last station is never blocked. The processing time at all stations are Erlang distributed with the number of phases allowed to vary for each station. They propose a methodology for generating the associated set of linear equations. These set of linear equations are solved via the use of the Successive Over Relaxation (SOR) method with a dynamically adjusted relaxation factor.Referring to the throughput rate of the production lines, many numerical results are documented. These results are of use for comparison purposes against approximate results which exist in the literature.

Frein, Commault and Dallery[39] propose an analytical method for the performance evaluation of closed loop production lines with unreliable machines and finite buffers. They assume that machines have deterministic processing times but are subject to failures. They approximate the behaviour of this system by a continuos flow model that is analyzed with a decomposition technique. Numerical experiments show that the results provided by this method are fairly accurate.

Dinçer and Donmez[37] propose a Markov model for transfer lines consisting of n-reliable machines with Erlang processing times and finite buffers.the arrivals to the system is Poisson distributed. A program coded in C which is capable of solving the Markov model of a three-machine transfer line is also developed. They calculate the mean throughput rate, machine utilization, expected value of WIP level and the variance of WIP.

Man-Soo Han and Dong-Jo Park[56] also studies the steady-state mean of transfer lines. Using Taylor series expansion and probability generating function technique, they present an approximation method for the analysis of the average steady-state throughput of serial production lines with unreliable machines, finite buffers and quality inspection machines. Employing the approximation method, they propose an analytical method for the optimal buffer allocation to achieve a desired output. The proposed methods are validated via computer simulations.

The performance measures of almost all studies correspond to the steady-state average production rates and steady-state average buffer levels. However, the essence of transfer lines is their variability. The only published papers that deal with the calculation of the variance of the behaviour of a transfer line over a limited period are Lavenberg and Miltenburg[75] .

Lavenberg[75] derives an expression for Laplace-Stieltjes transform of the steady-state distribution of the queuing time for *M/G/1* finite capacity queue. The expression can be differentiated readily in order to obtain higher order moments of the steady-state queuing time. He concludes that the higher order moments about the origin of the steady-state queuing time for the *M/G/1* finite capacity queue does not depend on the service time distribution only through its first $m+1$ moments. Also they show that the mean queuing time does not necessarily increase as the variance of the service time increases.

Miltenburg models the transfer line with an ergodic Markov chain. He presents a procedure for calculating the variance of the number of units produced by the transfer line during a period of length t cycles. He concludes that as the number of cycles t gets large, the influence of the initial starting state diminishes. Finally he uses the mean and variance to construct an interval estimate for the number of units produced during a shift. This interval estimate is an operational guide for the production manager.

Ou and and Gershwin[42] derive a closed form expression for the variance of the lead time distribution of a two machine transfer line with a finite buffer and Gershwin analyzes the variance of tandem production system.

Wu[124], contributes to two related problem categories for transfer lines. First algorithms are developed to calculate the variance of the number of departures at a fixed time interval from transfer lines with finite buffer inventories. Second, the variance calculation is incorporated into planning and design procedures of transfer lines and also basic issues such as due time performance, quota setting and characterization of departure process are considered.

Tan[113] determines analytically the variance of the throughput rate of a n-machine production line with no-intermediate buffers and time-dependent failures. The analytical method shows a closed-form expression for the variance of throughput rate, is based on determining the limiting variance of the sojourn time in a specific state of an irreducible recurrent Markov process from the probability of visiting that state at an arbitrary instant given an initial state. The corresponding procedure can also be applied to determine the variance of the throughput rate of various arrangements of workstations including series, parallel, series-parallel systems provided that the instantaneous availabilities of these systems can be written explicitly.

Philippe Cirput and Max-Olivier Hongler[27] , by using a fluid modeling approach, study the fluctuations around the average throughput delivered by simple production system. A special attention is paid to the buffered production dipole for which an explicit estimation for a stationary variance of the throughput is calculated.

B. Tan[115] also studies the asymptotic variance rate of the output of a transfer line with no buffer storage and cycle-dependent failures. In his study, the variability properties of the output of transfer lines are investigated. The asymptotic variance rate of the output of an n-station synchronous transfer line with no-interstation buffers and cycle-dependent failures is analytically determined. Unlike the other studies, the analytical method presented in this study yields a closed form expression for the asymptotic variance rate of the output. The method is based on general result derived for irreducible recurrent Markov chains. Numerical results show that the asymptotic variance rate of the output does not monotonically increase as the number of stations in the transfer line increases.

J.Li and M.Meerkov[80] also addresses the problem of production variability in serial manufacturing lines with unreliable machines. In their study Bernoulli statistics of reliability assumed. Three problems are considered: the problem of production variance, the problem of constant demand satisfaction and the problem of random demand satisfaction generated by another production line. For all problems, bounds on the respective variability measures are derived. These bounds state that the production variability of a line with many machines is smaller than that of single machine.

Adar A. Kalir, Subhash C. Sarin[67] derive a method for reducing inter-departure time variability in serial production lines. The inter-departure time variability is an important measure in production lines. Higher variability means less predictability in output. In this paper a strategy for the reduction of the effects of line parameters (line length and buffer capacity) on interdeparture time variability is proposed and studied via simulation. Numerical results show that in situations where output predictibility is more of a problem than capacity, this strategy constitutes an effective alternative.

2.3 Summary

Works on the transient behaviour of the output process of transfer lines are scarce in literature, although this analysis gives chance to analyze the system at any arbitrary time. Also, it is also given in this chapter that the derived methods for calculating the variance of transfer lines are not sufficient. It is also dictated that there are little implications about the closed form solution techniques. For these reasons, we tried to develop a new method which calculates the performance measures of both mean and variance of transfer lines in transient-state.

Chapter 3

MODELLING

In this chapter we use distribution functions of serial n-machine transfer lines , based on the total number of throughput of the system up to time t, and then propose an analytical method for estimating the performance measures of these lines with reliable machines and finite buffers. In order to estimate the performance measures and make conclusions, we derive mean and variance functions of these transfer lines.

3.1 The Basic Model Assumptions And The Used Notation

3.1.1 The Basic Model Assumptions

- This transfer line is a serial arrangement of a finite number of n machines and *n-1* in-process-buffers. Each machine can operate on one unit of product at a time and has internal storage capacity for that unit.
- The arrival process is assumed to be Poisson.
- The machines M_j(j=1,..,n) have mutually independent processing time that are also exponentially distributed with density function $f_j(t)$ =

$\mu_j e^{-\mu_j t}, \mu_j > 0$.

- The first buffer of the line is assumed to have zero capacity. So, if the new part arrives the system when the machine is busy, this part is assumed to return into its source. So the system is thought as memoryless. Also, the last buffer is considered to have infinite capacity, so that finished parts are ready to be delivered into customers.

- All machines are reliable. So all outputs are assumed to be produced as unsurprisingly.

- No batching and setup times are considered.

- The output process is not neccessarily stationary. As mentioned earlier, the system is considered as in a transient state. So, steady-state distribution for the output of the system under consideration may not exist.

- The production line assumes idle and empty initial conditions.

- The arrival rate and service rate are assumed to be different.

3.1.2 The Used Notation

$N_j(t)$	:	random variable of the number of parts that have left machine j up to time t in an n-machines transfer line, j=$1,\ldots,n$
l	:	number of parts leaving the system at an instance in time
n	:	number of machines in the system
λ	:	arrival rate to the system
μ_j	:	service rate of machine $j, j=1,\ldots,n$
T^i_λ	:	arrival time for part i
$T^i_{\mu_j}$	:	service time for part i on machine j, $j=1,\ldots,n$
T^i_d	:	departure time for part i
$f_{\lambda,\mu_j}(t)$	:	probability density function with parameters λ and μ_j
$F_{\lambda,\mu_j}(t)$	:	cumulative distribution function with parameters λ and μ_j
$E[N_n(t)]$	:	mean of the throughput, the number of parts leaving the *n-machine* system at time t.
$Var[N_n(t)$	:	variance of the throughput, the number of parts leaving the *n-machine* system at time t.

3.2 Organization

We first tried to evaluate the derivations of longer transfer lines, i.e. four-machines-three-in-process-buffers , five-machines-four-in-pocess-buffers systems. And we aimed to make conclusions and generalizations about long transfer lines. However, we see that the longer the line is, the more complex the system and its distribution derivation is. Because when the number of machines and buffers between machines increase, the different cases relating to random variable of total number of arrival and processing time occurs as a power function. We mean that if we have an atomic model, we have a unique case. But if the system contains two-machines and one-in-process-buffer between machines, we have 2 cases, buffer between the machines can be full or empty. When the transfer line gets a little longer as three-machines-two-in-pocess-buffers system, number of cases jumps to 4; first buffer between first second machine can be empty or full and the second buffer between second and third machines can be full or empty. Moreover we derive 8 and 16 cases of different derivations of distribution functions for four-machine-three-in-process-buffer and five-machine-four-in-process-buffer transfer lines respectively. Generally if we have n-machines-*(n-1)* in-process-buffers transfer line we have $2^{(n-1)}$ cases of distribution functions. So, we conclude that in order to analyze longer lines, we need to decompose them to smaller lines, or group them related to their parameters, i.e. the machines with same parameters forms the first group, the machines with other same parameters forms the second group and so on. After grouping these machines of the long transfer line we can use equations which are given as the closed-form expressions for distribution of sum of exponential random variables by S. V. Amari and R. B. Misra [7] to calculate the performance measures of interest. (see Chapter – IV Conclusions and Future Work). After than, we tried to develop more efficient method in order to calculate the mean and variance of the throughput rate of short transfer lines in an arbitrary time. In developing these methods we use many axioms and theorems of Probability, Statistics and Stochastic Processes. Total probability formula, conditional probability, independent random events, probability density and cumulative density functions, expected value and variance of the random variable, sum of different random variables, convolutions are the main subjects which our method based on. In consequetive sections of this chapter, we give some information and explanations about atomic model, two-machines-one-in-process-buffer, three machines-two-in-process-buffers models respectively. All equations corresponding to calculation of mean and variance of throughput rate of the developed method and the computation of these methods with different parameters are given in Appendix-section.

3.3 The Single Machine System

The system under consideration can be shown graphically as follows :

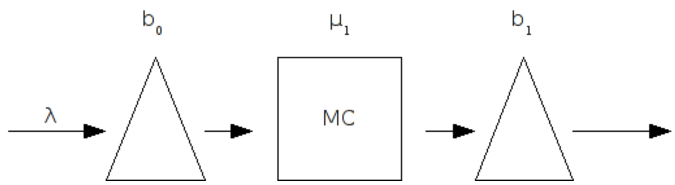

Figure-I: $T_\lambda^i \sim \exp(\lambda), T_{\mu_1}^i \sim \exp(\mu_1) \forall i; b_0 = 0 \& b_1 = \infty$

This system is also named as atomic model. In this system we have only one machine. The buffer before the machine has a zero capacity, and the finished product buffer has infinite capacity. Raw materials and parts, which can be termed as inputs, arrive the system with rate λ . If the machine is empty, these parts enter the machine, and are being processed with rate μ . However, if the machine is busy with processing the material that is entered to the system previously, these parts are considered as to return into their source. In the proposed method the arrival and processing times are thought as the random variable. So, for a single machine model the random variables are given as T_λ^i and $T_{\mu_j}^i$. The throughput leaving the system at time t , namely the distribution function of the atomic model, can be derived as follows[130] :

$$N(t) = N_1(t) = \begin{cases} 0 & \text{if} \quad 0 \le t < T_\lambda^1 + T_{\mu_1}^1 \\ 1 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 \le t < \sum_{i=1}^{2}(T_\lambda^i + T_{\mu_1}^i) \\ 2 & \text{if} \quad \sum_{i=1}^{2}(T_\lambda^i + T_{\mu_1}^i) \le t < \sum_{i=1}^{3}(T_\lambda^i + T_{\mu_1}^i) \\ \dots & \dots \quad \dots \\ l-1 & \text{if} \quad \sum_{i=1}^{l-1}(T_\lambda^i + T_{\mu_1}^i) \le t < \sum_{i=1}^{l}(T_\lambda^i + T_{\mu_1}^i) \\ l & \text{if} \quad \sum_{i=1}^{l}(T_\lambda^i + T_{\mu_1}^i) \le t < \sum_{i=1}^{l+1}(T_\lambda^i + T_{\mu_1}^i) \\ l+1 & \text{if} \quad \sum_{i=1}^{l+1}(T_\lambda^i + T_{\mu_1}^i) \le t < \sum_{i=1}^{l+2}(T_\lambda^i + T_{\mu_1}^i) \\ \dots & \dots \quad \dots \end{cases} \tag{3.1}$$

Due to our assumptions the time-line which shows arrival, processing and departure times can be depicted as:

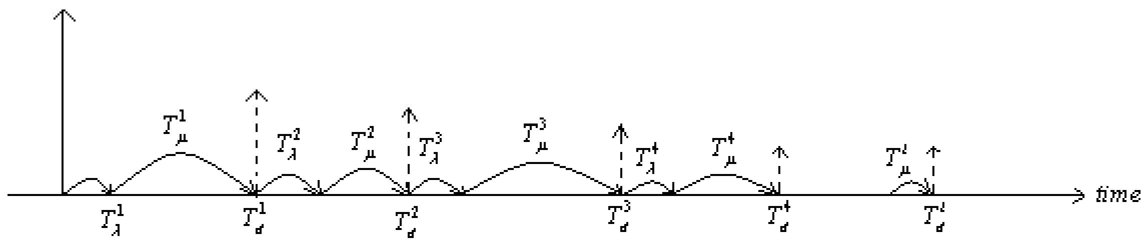

Figure-II: Graphical representation of arrival, processing and departure times of atomic model

This distribution is used with a different approach in this thesis. We analyse the distribution by seperating it into two parts:

i) $P(N_1(t)=0)$ if $0 \leq t < T_\lambda^1 + T_{\mu_1}^1$ so $P(N_1(t)=0) = P(T_\lambda^1 + T_{\mu_1}^1 > t)$. Here we have a new random variable, $T_\lambda^1 + T_{\mu_1}^1$, which is the sum of two independent exponential random variables of T_λ^1(time passes until the first part arrives) and $T_{\mu_1}^1$(time passes until the first part is processed in the machine).The sum of these random variables have hypoexponential distribution with parameters λ and μ_1, where $\lambda \neq \mu_1$. In this case we say that no part is processed until time t, which means that the total time passes until the first part arrives and being processed $(T_\lambda^1 + T_{\mu_1}^1)$ is greater than t. So in order to find $P(N_1(t)=0)$ we need to find $P(T_\lambda^1 + T_{\mu_1}^1 > t)$, which is the complementary cumulative distribution function of hypoexponentially distributed random variable, $T_\lambda^1 + T_{\mu_1}^1$.

$$P(N_1(t)=0) = P(T_\lambda^1 + T_{\mu_1}^1 > t) = 1 - P(T_\lambda^1 + T_{\mu_1}^1 \leq t) = 1 - F_{T_\lambda^1 + T_{\mu_1}^1}(t) = 1 - (1 - \frac{\mu}{\mu-\lambda}e^{-\lambda t} + \frac{\lambda}{\mu-\lambda}e^{-\mu t}) = \frac{\mu}{\mu-\lambda}e^{-\lambda t} - \frac{\lambda}{\mu-\lambda}e^{-\mu t} \tag{3.2}$$

It can be seen from the equation (3.2) if t goes to 0, result of the complementary cumulative distribution function of the hypoexponentially distributed random variable gives "$\frac{\mu}{\mu-\lambda} - \frac{\lambda}{\mu-\lambda}$", which is "1"(the maximum value that a cumulative distribution function can take), for all values of λ and μ,where $\lambda \neq \mu_1$.

It is also observed that,the equation (3.2), has the value of 0 when t approaches infinity(the minimum value that a cumulative distribution function can take). So one can write that : $0 \leq P(N_1(t) = 0) \leq 1$.

ii) A general formula can be written for any number of outputs, except 0, by the following formula:

$$P(N_1(t) = l) = P(\sum_{i=1}^{l}(T_\lambda^i + T_{\mu_1}^i) \leq t < \sum_{i=1}^{l+1}(T_\lambda^i + T_{\mu_1}^i)),\ l = 1, 2, 3... \tag{3.3}$$

In equation (3.3), all T_λ^i and $T_{\mu_1}^i$ are independent random variables. Here, we again use the sum of two independent random variables. The left-side of the equation (3.3) contains two independent random variables having Erlang or Gamma distributions with parameters (λ, l) and (μ, l).Since l is an integer valued Erlang distributions can be used. So, the random variable $\sum_{i=1}^{l}(T_\lambda^i + T_{\mu_1}^i)$ has Erlang distribution with parameters $(\lambda + \mu_1, l)$. Similarly, for the right-side of the equation (3.3), the random variable $\sum_{i=1}^{l+1}(T_\lambda^i + T_{\mu_1}^i)$ has Erlang distribution with parameters $(\lambda + \mu_1, l + 1)$.

If we define the random variable of the left-side of the equation (3.3), as S_l, which is the sum of l sequential phases of exponential distributions with parameters $(\lambda + \mu_1)$, then $S_l = \sum_{i=1}^{l}(T_\lambda^i + T_{\mu_1}^i)$. Similarly, the random variable of the right-side of the equation (3.3) can be defined as S_{l+1} , which is the sum of $l + 1$ sequential phases of exponential distributions with parameters $(\lambda + \mu_1)$, then $S_{l+1} = \sum_{i=1}^{l+1}(T_\lambda^i + T_{\mu_1}^i)$.Besides, there are two events in the equation (3.3): $S_l \leq t$ and $S_{l+1} > t$. If these two events were independent, the probability equation could be written as the product of probabilities of these events, i.e. $P(N_1(t) = l) = P(S_l \leq t) * P(S_{l+1} > t) = F_{S_l}(t) * (1 - F_{S_{l+1}}(t))$. However these two events are not independent. It is clear that the total time which passes until *(l+1)*th part leaves the system as an output,depends on the time which passes until the lth part leaves the system.As a result the probability equation can not be written as the product of the event's probabilities.

On the other hand the equation(3.3) can be written as the conditional probabilities by two different ways as follows :

$$P(N_1(t) = l) = P(S_{l+1} > t \mid S_l \leq t) * P(S_l \leq t) \quad = P(S_{l+1} > t \mid S_l \leq t) * F_{S_L}(t) \tag{3.4}$$

or

$$P(N_1(t) = l) = P(S_l \leq t \mid S_{l+1} > t) * P(S_{l+1} > t) = P(S_l \leq t \mid S_{l+1} > t) * (1 - F_{S_{l+1}}(t)) \tag{3.5}$$

In the equation (3.4) we see that the only difference between the random variables of S_l and S_{l+1} is the arrival and processing time of *(l+1)*th part. In words, the conditional probability says, the time that passes until lth part leaves the system,i.e. z,is smaller than t,where the time that passes until *(l+1)*th part leaves the system is greater than t. As a result under the given condition of $S_l < t$, the time that *(l+1)*th part spends in the system ($T_\lambda^{l+1} + T_{\mu_1}^{l+1}$) is greater than $t - z$. Also, we can say that $T_\lambda^{l+1} + T_{\mu_1}^{l+1}$ is the random variable of two exponentially distributed random variables with parameters λ and μ_1 respectively. As it is given earlier, this sum has hypoexponential distribution with parameters λ and μ_1.

Equation (3.4) can therefore be rewritten as

$$P(N_1(t) = l) = P(S_{l+1} > t \mid S_l \le t) * P(S_l \le t) = \int_0^t \int_{t-z}^{\infty} f_{T_\lambda^{l+1} + T_{\mu_1}^{l+1}}(u) * f_{S_l}(z) du dz = \int_0^t (1 - F_{T_\lambda^{l+1} + T_{\mu_1}^{l+1}}(t-z)) * f_{S_l}(z) dz \tag{3.6}$$

Now, we can substitute the density and cumulative functions of random variables into equation (3.6). Substituting the cumulative distribution function of hypoexponential distribution into $F_{T_\lambda^{l+1} + T_{\mu_1}^{l+1}}(t-z)$ and density function of Erlang distribution into $f_{S_l}(z)$ yields

$$P(N_1(t) = l) = \int_0^t (1 - (1 - \frac{\mu_1}{\mu_1 - \lambda} e^{-\lambda(t-z)} + \frac{\lambda}{\mu_1 - \lambda} e^{-\mu(t-z)})) * \frac{(\lambda + \mu_1)^l z^{l-1} e^{-(\lambda+\mu_1)z}}{(l-1)!} dz. \tag{3.7}$$

The proposed method for calculating the mean and variance of this model, which based on fundamental calculus, probability and statistics theorems, is given in Appendix A.1.

3.4 Two-Machines-One-In-Process-Buffer System

The transfer line corresponding to two-machines-one-in-process-buffer system is depicted in Figure-III:

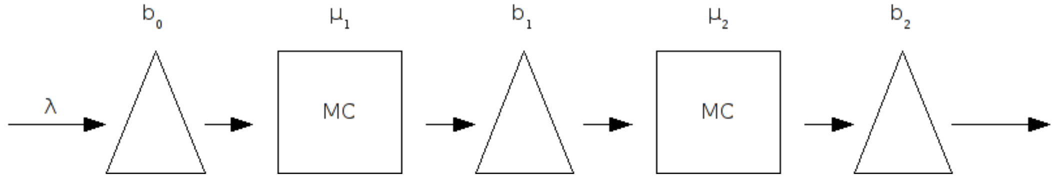

Figure-III: $T_\lambda^i \sim \exp(\lambda), T_{\mu_j}^i \sim \exp(\mu_j) \forall i, j = 1, 2; b_0 = 0, b_1 \ge 0, \& b_2 = \infty$

In this system, there is a buffer between the first and second machines. This buffer can be empty or full when the new part comes. There occurs an increase in the number of the sources of variability due to the more number of machines and buffers in the system of interest and this leads to existence of two mutually exclusive and collectively exhaustive events that describe the behaviour of the system. First event is taken into consideration if the buffer between machines is empty, and the second event is taken into consideration if there are parts in the buffer. In this system, the random variables that are used in modelling are $T^i_\lambda, T^i_{\mu_1}$ and $T^i_{\mu_2}$. This system can be analyzed in two different cases, where the first case is given as $T^i_\lambda + T^i_{\mu_1} \geq T^{i-1}_{\mu_2}$ and the second case is given as $T^i_\lambda + T^i_{\mu_1} < T^{i-1}_{\mu_2}$, based on the two different events which are mentioned previously.

Two different distribution functions are used in of finding the probabilities and obtaining the mean and variance values, using the total probability formula, for these two cases.

Case - I : where $T^i_\lambda + T^i_{\mu_1} \geq T^{i-1}_{\mu_2}$

$$N(t) = N_2(t) = \begin{cases} 0 & \text{if} \quad 0 \leq t < T^1_\lambda + T^1_{\mu_1} + T^1_{\mu_2} \\ 1 & \text{if} \quad T^1_\lambda + T^1_{\mu_1} + T^1_{\mu_2} \leq t < T^2_{\mu_2} + \sum_{i=1}^{2}(T^i_\lambda + T^i_{\mu_1}) \\ 2 & \text{if} \quad T^2_{\mu_2} + \sum_{i=1}^{2}(T^i_\lambda + T^i_{\mu_1}) \leq t < T^3_{\mu_2} + \sum_{i=1}^{3}(T^i_\lambda + T^i_{\mu_1}) \\ \cdots & \cdots \quad \cdots \\ l-1 & \text{if} \quad T^{l-1}_{\mu_2} + \sum_{i=1}^{l-1}(T^i_\lambda + T^i_{\mu_1}) \leq t < T^l_{\mu_2} + \sum_{i=1}^{l}(T^i_\lambda + T^i_{\mu_1}) \\ l & \text{if} \quad T^l_{\mu_2} + \sum_{i=1}^{l}(T^i_\lambda + T^i_{\mu_1}) \leq t < T^{l+1}_{\mu_2} + \sum_{i=1}^{l+1}(T^i_\lambda + T^i_{\mu_1}) \\ l+1 & \text{if} \quad T^{l+1}_{\mu_2} + \sum_{i=1}^{l+1}(T^i_\lambda + T^i_{\mu_1}) \leq t < T^{l+2}_{\mu_2} + \sum_{i=1}^{l+2}(T^i_\lambda + T^i_{\mu_1}) \\ \cdots & \text{if} \quad \cdots \end{cases} \tag{3.8}$$

Case - II where $T^i_\lambda + T^i_{\mu_1} < T^{i-1}_{\mu_2}$

$$N(t) = N_2(t) = \begin{cases} 0 & \text{if} \quad 0 \leq t < T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 \\ 1 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 \leq t < T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{2} T_{\mu_2}^i) \\ 2 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{2} T_{\mu_2}^i) \leq t < T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{3} T_{\mu_2}^i) \\ \cdots & \cdots \quad \cdots \\ l-1 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l-1} T_{\mu_2}^i) \leq t < T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l} T_{\mu_2}^i) \\ l & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l} T_{\mu_2}^i) \leq t < T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l+1} T_{\mu_2}^i) \\ l+1 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l+1} T_{\mu_2}^i) \leq t < T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l+2} T_{\mu_2}^i) \\ \cdots & \text{if} \quad \cdots \end{cases} \tag{3.9}$$

For two different cases of two-machines-one-in-process-buffer transfer line we have two different distribution functions for two different cases. So, we first derive all neccessary probability functions for Case-I and then similarly derive these functions for Case-II .

For **Case-I**, where $T_\lambda^i + T_{\mu_1}^i \geq T_{\mu_2}^{i-1}$,the distribution functions can be analysed in two parts.

i) $P(N_2(t) = 0)$ if $0 \leq t < T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1$ so $P(N_2(t) = 0) = P(T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 > t)$. Here, the new random variable, $T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1$, is the sum of three independent exponential random variables of T_λ^1(time passes until the first part arrives) $T_{\mu_1}^1$(time passes until the first part is processed in the first machine), and $T_{\mu_2}^1$(time passes until the second part is processed in the second machine).The distribution of the sum of these random variables having form $\lambda \neq \mu_1 \neq \mu_2$ are obtained from Suprasad V. Amari and Ravindra B. Misra[7]. Since $\lambda \neq \mu_1 \neq \mu_2$,we use the derivations for *"case #b"* from the paper. The reliability and cumulative distribution functions are derived by first obtaining the coefficients of the random variables as in [7].

So, the coefficients of the random variables can be obtained as

$$A_1 = \frac{\mu_1}{\mu_1-\lambda} * \frac{\mu_2}{\mu_2-\lambda} \ , A_2 = \frac{\lambda}{\lambda-\mu_1} * \frac{\mu_2}{\mu_2-\mu_1} \ , A_3 = \frac{\lambda}{\lambda-\mu_2} * \frac{\mu_1}{\mu_1-\mu_2} \tag{3.10}$$

where λ is the rate of inputs and μ_1, mu_2 are service rates of machines 1 and 2 respectively.

The reliability and the cumulative distribution functions can be written as

$$R(t) = \sum_{i=1}^{n} A_i * e^{(-\alpha_i * t)} = \frac{\mu_1}{\mu_1-\lambda} * \frac{\mu_2}{\mu_2-\lambda} * e^{(-\lambda t)} + \frac{\lambda}{\lambda-\mu_1} * \frac{\mu_2}{\mu_2-\mu_1} * e^{(-\mu_1 t)} + \frac{\lambda}{\lambda-\mu_2} * \frac{\mu_1}{\mu_1-\mu_2} * e^{(-\mu_2 t)} \tag{3.11}$$

$$F_{T_\lambda^1+T_{\mu_1}^1+T_{\mu_2}^1}(t) = 1-R(t) = 1-\frac{\mu_1}{\mu_1-\lambda} * \frac{\mu_2}{\mu_2-\lambda} * e^{(-\lambda t)} - \frac{\lambda}{\lambda-\mu_1} * \frac{\mu_2}{\mu_2-\mu_1} * e^{(-\mu_1 t)} - \frac{\lambda}{\lambda-\mu_2} * \frac{\mu_1}{\mu_1-\mu_2} * e^{(-\mu_2 t)}. \tag{3.12}$$

Since the total time passed until the first part arrives and being processed $(T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1)$ is greater than t, no output is left the system until time t. So $P(N_1(t) = 0)$ can be found by using $P(T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 > t)$, which is the complementary cumulative distribution function of the random variable $T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1$.So;

$$P(N_2(t) = 0) = P(T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 > t) = 1 - P(T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 \le t) = 1 - F_{T_\lambda^1+T_{\mu_1}^1+T_{\mu_2}^1}(t) = 1-(1-\frac{\mu_1}{\mu_1-\lambda} * \frac{\mu_2}{\mu_2-\lambda} * e^{(-\lambda t)} - \frac{\lambda}{\lambda-\mu_1} * \frac{\mu_2}{\mu_2-\mu_1} * e^{(-\mu_1 t)} - \frac{\lambda}{\lambda-\mu_2} * \frac{\mu_1}{\mu_1-\mu_2} * e^{(-\mu_2 t)}) = \frac{\mu_1}{\mu_1-\lambda} * \frac{\mu_2}{\mu_2-\lambda} * e^{(-\lambda t)} + \frac{\lambda}{\lambda-\mu_1} * \frac{\mu_2}{\mu_2-\mu_1} * e^{(-\mu_1 t)} + \frac{\lambda}{\lambda-\mu_2} * \frac{\mu_1}{\mu_1-\mu_2} * e^{(-\mu_2 t)} \tag{3.13}$$

As can be seen from the equation (3.13) when t goes to 0, result of the cumulative distribution function of the sum of the independent random variables gives "$\frac{\mu_1}{\mu_1-\lambda} * \frac{\mu_2}{\mu_2-\lambda} + \frac{\lambda}{\lambda-\mu_1} * \frac{\mu_2}{\mu_2-\mu_1} + \frac{\lambda}{\lambda-\mu_2} * \frac{\mu_1}{\mu_1-\mu_2}$", which is "1"(the maximum value that a cumulative distribution function can take), for all values of λ, μ_1 and μ_2 where $\lambda \neq \mu_1 \neq \mu_2$. And it is also observed that when t goes to infinity, the equation(10), has the value of "0"(the maximum value that a cumulative distribution function can take). So one can write that : $0 \leq P(N_1(t) = 0) \leq 1$,which is the one of the main axioms of probability theory.

ii) A general formula can be written for any number of outputs, except 0, using the derivation of the number of outputs for the two-machines-one-in-process-buffer transfer line (see equation (3.9)).

$$P(N_2^{cs-1}(t) = l) = P(\sum_{i=1}^{l}(T_\lambda^i + T_{\mu_1}^i) + T_{\mu_2}^l \leq t < \sum_{i=1}^{l+1}(T_\lambda^i + T_{\mu_1}^i) + T_{\mu_2}^{l+1}) \tag{3.14}$$

where all T_λ^i,$T_{\mu_1}^i$ and $T_{\mu_2}^i$ are independent random variables. The left-side of the equation (3.14) contains the random variables $\sum_{i=1}^{l}(T_\lambda^i+T_{\mu_1}^i)$ and $T_{\mu_2}^l$ which have Erlang distribution with parameters $(\lambda+\mu_1, l)$ and exponential distribution with parameter μ_2 respectively. The right side of the equation contains the *(l+1)*th level of the same random variables with the same distributions.

There are two dependent events in equation (3.14), as explained in Section 3.3, case ii. We can write this equation by conditional probabilities as

$$P(N_2^{cs-1}(t) = l) = P(\sum_{i=1}^{l+1}(T_\lambda^i+T_{\mu_1}^i)+T_{\mu_2}^{l+1} > t \mid \sum_{i=1}^{l}(T_\lambda^i+T_{\mu_1}^i)+T_{\mu_2}^l \leq t)*P(\sum_{i=1}^{l}(T_\lambda^i+ T_{\mu_1}^i) + T_{\mu_2}^l \leq t) \tag{3.15}$$

or

$$P(N_2^{cs-1}(t)=l)=P(\sum_{i=1}^{l}(T_\lambda^i+T_{\mu_1}^i)+T_{\mu_2}^l\leq t\mid\sum_{i=1}^{l+1}(T_\lambda^i+T_{\mu_1}^i)+T_{\mu_2}^{l+1}>t)*P(\sum_{i=1}^{l+1}(T_\lambda^i+T_{\mu_1}^i)+T_{\mu_2}^{l+1}>t) \tag{3.16}$$

In equation (3.15) the difference between the random variables of $\sum_{i=1}^{l+1}(T_\lambda^i+T_{\mu_1}^i)+T_{\mu_2}^{l+1}$ and $\sum_{i=1}^{l}(T_\lambda^i+T_{\mu_1}^i)+T_{\mu_2}^l$ is the arrival and total processing time of *(l+1)*th part and the subtraction of $T_{\mu_2}^l$(time passes until lth part is processed in second machine) from these total time. In words, the conditional probability says, the time that passes until *(l+1)*th part leaves the system,i.e. z,is smaller than t,where the time that passes until *(l+1)*th leaves the system is greater than t. As a result under the given condition of $\sum_{i=1}^{l}(T_\lambda^i+T_{\mu_1}^i)+T_{\mu_2}^l\leq t$, the time that $l+1$. part spends in the system $(T_\lambda^{l+1}+T_{\mu_1}^{l+1})$ is greater than $t-z$. Equation (3.15) can be rewritten as

$$P(N_2^{cs-1}(t)=l)=P(T_\lambda^{l+1}+T_{\mu_1}^{l+1}+T_{\mu_2}^{l+1}-T_{\mu_2}^l>t-z)*F_{\sum_{i=1}^{l}(T_\lambda^i+T_{\mu_1}^i)+T_{\mu_2}^l}(z)=\int_0^t(1-F_{T_\lambda^{l+1}+T_{\mu_1}^{l+1}+T_{\mu_2}^{l+1}-T_{\mu_2}^l}(t-z))*f_{\sum_{i=1}^{l}(T_\lambda^i+T_{\mu_1}^i)+T_{\mu_2}^l}(z)dz\ . \tag{3.17}$$

In order to find the density function of the random variable of $\sum_{i=1}^{l}(T_\lambda^i+T_{\mu_1}^i)+T_{\mu_2}^l$,we can use the convolution formula given in Appendix C.1. If $Z:=\sum_{i=1}^{l}(T_\lambda^i+T_{\mu_1}^i)+T_{\mu_2}^l$, $X:=\sum_{i=1}^{l}(T_\lambda^i+T_{\mu_1}^i)$, $Y:=T_{\mu_2}^l$, then $Z=X+Y$, where $X\sim Erlang(\lambda+\mu_1,l)$ and $Y\sim Exponential(\mu_2)$. So, $f_Z(z)$ is obtained as :

$$f_Z(z) = \int_0^z f_X(x) * f_Y(z-x)dx = \int_0^z \frac{(\lambda+\mu_1)^l * x^{l-1} * e^{-(\lambda+\mu_1)*x}}{(l-1)!} * \mu_2 * e^{-\mu_2*(z-x)}dx \tag{3.18}$$

Furthermore, we can also find the cumulative distribution of the random variable $T_\lambda^{l+1} + T_{\mu_1}^{l+1} + T_{\mu_2}^{l+1} - T_{\mu_2}^l$, by using the convolution formula for difference of the random variables given in Appendix C.2. If $W := T_\lambda^{l+1} + T_{\mu_1}^{l+1} + T_{\mu_2}^{l+1} - T_{\mu_2}^l$, $U := T_\lambda^{l+1} + T_{\mu_1}^{l+1} + T_{\mu_2}^{l+1}$, $V := T_{\mu_2}^l$, then $W = U - V$, where $U \sim Hypo(\lambda, \mu_1, \mu_2)$, and its cumulative density function is derived in part **(i)**,$V \sim Exponential(\mu_2)$. Then

$$F_U(t) = 1 - \frac{\mu_1}{\mu_1-\lambda} * \frac{\mu_2}{\mu_2-\lambda} * e^{(-\lambda t)} - \frac{\lambda}{\lambda-\mu_1} * \frac{\mu_2}{\mu_2-\mu_1} * e^{(-\mu_1 t)} - \frac{\lambda}{\lambda-\mu_2} * \frac{\mu_1}{\mu_1-\mu_2} * e^{(-\mu_2 t)}$$

and

$$F_W(t-z) = \int_0^\infty F_U(t-z+v) * f_V(v)dv = \int_0^\infty (1 - \frac{\mu_1}{\mu_1-\lambda} * \frac{\mu_2}{\mu_2-\lambda} * e^{(-\lambda(t-z+v))} - \frac{\lambda}{\lambda-\mu_1} * \frac{\mu_2}{\mu_2-\mu_1} * e^{(-\mu_1(t-z+v))} - \frac{\lambda}{\lambda-\mu_2} * \frac{\mu_1}{\mu_1-\mu_2} * e^{(-\mu_2(t-z+v))}) * \mu_2 * e^{-\mu_2 v}dv. \tag{3.19}$$

Substituting the equations (3.18) and (3.19) in equation (3.17) yields

$$P(N_2^{cs-1}(t) = l) = \int_0^t (1 - F_W(t-z)) * f_Z(z)dz = \int_0^t ((1 - \int_0^\infty (1 - \frac{\mu_1}{\mu_1-\lambda} * \frac{\mu_2}{\mu_2-\lambda} * e^{(-\lambda(t-z+v))} - \frac{\lambda}{\lambda-\mu_1} * \frac{\mu_2}{\mu_2-\mu_1} * e^{(-\mu_1(t-z+v))} - \frac{\lambda}{\lambda-\mu_2} * \frac{\mu_1}{\mu_1-\mu_2} * e^{(-\mu_2(t-z+v))}) * \mu_2 * e^{-\mu_2 v}dv)) * \int_0^z \frac{(\lambda+\mu_1)^l * x^{l-1} * e^{-(\lambda+\mu_1)*x}}{(l-1)!} * \mu_2 * e^{-\mu_2*(z-x)}dx)dz \tag{3.20}$$

For Case-II, where $T_{\lambda}^{i}+T_{\mu_1}^{i} < T_{\mu_2}^{i-1}$, then we have a different distribution function (see equation 3.9). So, these density and cumulative distribution functions can be derived as :

i) $P(N_2^{cs-2}(t)=0)$ if $0 \leq t < T_{\lambda}^{1}+T_{\mu_1}^{1}+T_{\mu_2}^{1}$. The derivations and equations are the same with Case-I,part i. So, equations of (3.10), (3.11), (3.12), (3.13) can be used in this part.

ii) The derivations of the density and cumulative distributions for Case-II will be obtained from equation (3.9).

$$P(N_2^{cs-2}(t)=l) = P(T_{\lambda}^{1}+T_{\mu_1}^{1}+(\sum_{i=1}^{l} T_{\mu_2}^{i}) \leq t < T_{\lambda}^{1}+T_{\mu_1}^{1}+(\sum_{i=1}^{l+1} T_{\mu_2}^{i})) \tag{3.21}$$

where all T_{λ}^{i},$T_{\mu_1}^{i}$ and $T_{\mu_2}^{i}$ are independent random variables. In the left-side of the equation (3.18)we have the random variable $\sum_{i=1}^{l} T_{\mu_2}^{i}$ and $T_{\lambda}^{1}+T_{\mu_1}^{1}$(total time that passes until the first part arrives and being processed in the first machine), which have Erlang distribution with parameters (μ_2, l) and hypoexponential distribution with parameters (λ, μ_1) respectively. The right side of this equation contains the *(l+1)*th level of the same random variables with the same distributions.

As mentioned earlier,equation (3.21) can be written as the conditional probabilities by two different ways as follows

$$P(N_2^{cs-2}(t)=l) = P(T_{\lambda}^{1}+T_{\mu_1}^{1}+(\sum_{i=1}^{l+1} T_{\mu_2}^{i}) > t \mid T_{\lambda}^{1}+T_{\mu_1}^{1}+(\sum_{i=1}^{l} T_{\mu_2}^{i}) \leq t) * P(T_{\lambda}^{1}+ T_{\mu_1}^{1}+(\sum_{i=1}^{l} T_{\mu_2}^{i}) \leq t) \tag{3.22}$$

or

$$P(N_2^{cs-2}(t)=l) = P(T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l} T_{\mu_2}^i) \leq t \mid T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l+1} T_{\mu_2}^i) > t) * P(T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l+1} T_{\mu_2}^i) > t) \tag{3.23}$$

The difference between the random variables of $T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l+1} T_{\mu_2}^i)$ and $T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l} T_{\mu_2}^i)$, in equation (3.22) is the processing time of *(l+1)*th part part in the second machine. The time that passes until lth part leaves the system,i.e. z,is smaller than t,while the time that passes until *(l+1)*th part leaves the system is greater than t. As a result under the given condition of $T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l} T_{\mu_2}^i) \leq t$, the time that *(l+1)*th part spends in the system,$T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l+1} T_{\mu_2}^i)$,is greater than $t-z$. Equation (3.22) can now be written as

$$P(N_2^{cs-2}(t)=l) = P(T_{\mu_2}^{l+1} > t-z) * F_{T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l} T_{\mu_2}^i)}(z) = \int_0^t (1 - F_{T_{\mu_2}^{l+1}}(t-z)) * f_{T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l} T_{\mu_2}^i)}(z) dz \tag{3.24}$$

The density function of the random variable of $T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l} T_{\mu_2}^i)$can be obtained by using convolution formula given in Appendix C.1. If $Z := T_\lambda^1 + T_{\mu_1}^1 + (\sum_{i=1}^{l} T_{\mu_2}^i)$, $X := T_\lambda^1 + T_{\mu_1}^1$, $Y := \sum_{i=1}^{l} T_{\mu_2}^i$, then $Z = X + Y$, where $X \sim Hypo(\lambda, \mu_1)$ and $Y \sim Erlang(\mu_2, l)$. So, $f_Z(z)$ can be derived as

$$f_Z(z) = \int_0^z f_X(x) * f_Y(z-x)dx = \int_0^z \frac{\lambda*\mu_1}{\mu_1-\lambda} * (e^{-\lambda*x} - e^{-\mu_1*x}) * \frac{\mu_2^l*(z-x)^{l-1}*e^{-\mu_2*(z-x)}}{(l-1)!}dx \tag{3.25}$$

The random variable $T_{\mu_2}^{l+1}$ in equation (3.24) has exponential distribution with cumulative distribution function $F_{T_{\mu_2}^{l+1}}(t) = 1 - e^{-\mu_2*t}$. Substituting this cumulative distribution function and equation (3.25) in equation (3.24), we get

$$P(N_2^{cs-2}(t) = l) = \int_0^t ((1 - (1 - e^{-\mu_2*(t-z)}) * (\int_0^z \frac{\lambda*\mu_1}{\mu_1-\lambda} * (e^{-\lambda*x} - e^{-\mu_1*x}) * \frac{\mu_2^l*(z-x)^{l-1}*e^{-\mu_2*(z-x)}}{(l-1)!}dx))dz. \tag{3.26}$$

All probabilities for $P(N_2(t) = l)$ where $l = 0, 1, 2, ...$ are calculated. Finally, we will investigate the occurrence probabilities of Case-I and Case-II. If the condition of ,$T_\lambda^i + T_{\mu_1}^i \geq T_{\mu_2}^{i-1}$, is satisfied then Case-I occurs, so we use the distribution function which is derived for Case-I. Here, the random variables are T_λ^i,$T_{\mu_1}^i$ and $T_{\mu_2}^{i-1}$,all have exponential distribution. The sum of these two exponential distribution ($T_\lambda^i + T_{\mu_1}^i$), has hypoexponential distribution with parametersλ and μ_1.By using this distributions and convolution formula given in Appendix C.2. The probability of satisfying this condition can be calculated as

$$P(T_\lambda^i + T_{\mu_1}^i \geq T_{\mu_2}^{i-1}) = P(T_\lambda^i + T_{\mu_1}^i - T_{\mu_2}^{i-1} \geq 0) = 1 - \int_0^\infty (1 - \frac{\mu_1}{\mu_1-\lambda}e^{-\lambda(x+y)} + \frac{\lambda}{\mu_1-\lambda}e^{-\mu_1(x+y)}) * \mu_2 * e^{-\mu_2*y}dy \tag{3.27}$$

Since there are only two conditions, the probability of occurrence of Case-II is easily calculated as $1 - P(T_\lambda^i + T_{\mu_1}^i \geq T_{\mu_2}^{i-1})$. Case-II occurs when the condition $T_\lambda^i + T_{\mu_1}^i < T_{\mu_2}^{i-1}$ is satisfied, so that we can use the distribution function which is derived for this case. Here, the random variables are T_λ^i,$T_{\mu_1}^i$ and $T_{\mu_2}^{i-1}$,all have exponential distribution.Also,the sum of two exponential distribution (T^i_λ + $T^i_{\mu_1}$) has hypoexponential distribution with parameters λ and μ_1. By using this distributions and convolution formula given in Appendix C.2. The probability of satisfying this condition can be calculated as

$$P(T^i_\lambda + T^i_{\mu_1} < T^{i-1}_{\mu_2}) = P(T^i_\lambda + T^i_{\mu_1} - T^{i-1}_{\mu_2} < 0) = \int_0^\infty (1 - \frac{\mu_1}{\mu_1-\lambda}e^{-\lambda(x+y)} + \frac{\lambda}{\mu_1-\lambda}e^{-\mu_1(x+y)}) * \mu_2 * e^{-\mu_2 * y} dy. \tag{3.28}$$

The proposed method, for calculating the mean and variance of this model, which based on fundamental calculus, probability and statistics theorems , is given in Appendix A.2.

3.5 Three-Machines-Two-In-Process-Buffers System

The transfer line corresponding to three-machines-two-in-process-buffers system is depicted in Figure-IV:

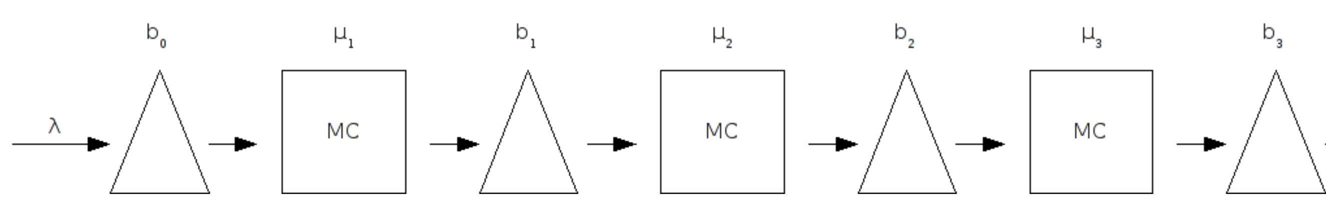

Figure-IV:$T^i_\lambda \sim \exp(\lambda), T^i_{\mu_j} \sim \exp(\mu_j) \forall i, j = 1, 2, 3; b_0 = 0, b_1 \geq 0, b_2 \geq 0 \& b_3 = \infty$

In this system, there are two in-process-buffers : between the first and second machines and second and the third machines. These two buffers can be empty or full when the new part comes. So, there occurs an increase in the number of the sources of variability due to the more number of machines and buffers in the system of interest and this leads to existence of four mutually exclusive and collectively exhaustive events that describe the behaviour of the system, where first event is taken into consideration if both of the first and second buffers between machines is empty. The second event is taken into consideration if the first buffer between the first and second machine is empty, however the second buffer between the second and third machine is consisting of some parts. The third event is taken into consideration if both of the buffers between machines are consisting of inputs. Finally, the fourth event is taken into consideration if the first buffer between machines is containing some inputs, and the second buffer is empty and contains no parts.

In this system the random variables that are used in modelling are $T_\lambda^i, T_{\mu_1}^i$ and $T_{\mu_2}^i$ and $T_{\mu_3}^i$. This system can be analyzed in four different cases, based on the four different events, where the first case is given as $T_\lambda^i + T_{\mu_1}^i \geq T_{\mu_2}^{i-1}$ and $T_\lambda^i + T_{\mu_1}^i + T_{\mu_2}^i \geq T_{\mu_2}^{i-1} + T_{\mu 3}^{i-1}$ .The second case can be given as $T_\lambda^i + T_{\mu_1}^i \geq T_{\mu_2}^{i-1}$,..and $T_\lambda^i + T_{\mu_1}^i + T_{\mu_2}^i < T_{\mu_2}^{i-1} + T_{\mu 3}^{i-1}$. The third case can be represented as $T_\lambda^i + T_{\mu_1}^i < T_{\mu_2}^{i-1}$ and $T_{\mu_3}^{i-2} \geq T_{\mu_2}^{i-1}$. And the final case can be given as $T_\lambda^i + T_{\mu_1}^i < T_{\mu_2}^{i-1}$ and $T_{\mu_3}^{i-2} < T_{\mu_2}^{i-1}$ holds for every part i.

In this system, we first use four different distribution functions for these cases, find their probabilities and use total probability formula to obtain the mean and variance values. The previously derived[130] distributions for Case-I , Case-II, Case-III and Case-IV can be given as follows :

Case - I : where $T_\lambda^i + T_{\mu_1}^i \geq T_{\mu_2}^{i-1}$ and $T_\lambda^i + T_{\mu_1}^i + T_{\mu_2}^i \geq T_{\mu_2}^{i-1} + T_{\mu 3}^{i-1}$.

$$N(t) = N_3(t) = \begin{cases} 0 & \text{if} \quad 0 \le t < T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + T_{\mu_3}^1 \\ 1 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + T_{\mu_3}^1 \le t < \sum_{i=1}^{2}(T_\lambda^i + T_{\mu_1}^i) + T_{\mu_2}^2 + T_{\mu_3}^2 \\ 2 & \text{if} \quad \sum_{i=1}^{2}(T_\lambda^i + T_{\mu_1}^i) + T_{\mu_2}^2 + T_{\mu_3}^2 \le t < \sum_{i=1}^{3}(T_\lambda^i + T_{\mu_1}^i) + T_{\mu_2}^3 + T_{\mu_3}^3 \\ \cdots & \cdots \quad \cdots \\ l-1 & \text{if} \quad \sum_{i=1}^{l-1}(T_\lambda^i + T_{\mu_1}^i) + T_{\mu_2}^{l-1} + T_{\mu_3}^{l-1} \le t < \sum_{i=1}^{l}(T_\lambda^i + T_{\mu_1}^i) + T_{\mu_2}^l + T_{\mu_3}^l \\ l & \text{if} \quad \sum_{i=1}^{l}(T_\lambda^i + T_{\mu_1}^i) + T_{\mu_2}^l + T_{\mu_3}^l \le t < \sum_{i=1}^{l+1}(T_\lambda^i + T_{\mu_1}^i) + T_{\mu_2}^{l+1} + T_{\mu_3}^{l+1} \\ l+1 & \text{if} \quad \sum_{i=1}^{l+1}(T_\lambda^i + T_{\mu_1}^i) + T_{\mu_2}^{l+1} + T_{\mu_3}^{l+1} \le t < \sum_{i=1}^{l+2}(T_\lambda^i + T_{\mu_1}^i) + T_{\mu_2}^{l+2} + T_{\mu_3}^{l+2} \\ \cdots & \text{if} \quad \cdots \end{cases} \tag{3.29}$$

Case -II : where $T_\lambda^i + T_{\mu_1}^i \ge T_{\mu_2}^{i-1}$ and $T_\lambda^i + T_{\mu_1}^i + T_{\mu_2}^i < T_{\mu_2}^{i-1} + T_{\mu 3}^{i-1}$.

$$N(t) = N_3(t) = \begin{cases} 0 & \text{if} \quad 0 \le t < T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + T_{\mu_3}^1 \\ 1 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + T_{\mu_3}^1 \le t < T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{2} T_{\mu_3}^i \\ 2 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{2} T_{\mu_3}^i \le t < T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{3} T_{\mu_3}^i \\ \cdots & \cdots \quad \cdots \\ l-1 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{l-1} T_{\mu_3}^i \le t < T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{l} T_{\mu_3}^i \\ l & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{l} T_{\mu_3}^i \le t < T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{l+1} T_{\mu_3}^i \\ l+1 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{l+1} T_{\mu_3}^i \le t < T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{l+2} T_{\mu_3}^i \\ \cdots & \text{if} \quad \cdots \end{cases} \tag{3.30}$$

Case -III : where $T_\lambda^i + T_{\mu_1}^i < T_{\mu_2}^{i-1}$ and $T_{\mu_3}^{i-2} \ge T_{\mu_2}^{i-1}$

$$N(t) = N_3(t) = \begin{cases} 0 & \text{if} \quad 0 \le t < T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + T_{\mu_3}^1 \\ 1 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + T_{\mu_3}^1 \le t < T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{2} T_{\mu_3}^i \\ 2 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{2} T_{\mu_3}^i \le t < T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{3} T_{\mu_3}^i \\ \cdots & \cdots \quad \cdots \\ l-1 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{l-1} T_{\mu_3}^i \le t < T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{l} T_{\mu_3}^i \\ l & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{l} T_{\mu_3}^i \le t < T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{l+1} T_{\mu_3}^i \\ l+1 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{l+1} T_{\mu_3}^i \le t < T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + \sum_{i=1}^{l+2} T_{\mu_3}^i \\ \cdots & \text{if} \quad \cdots \end{cases} \tag{3.31}$$

Case -IV: where $T_\lambda^i + T_{\mu_1}^i < T_{\mu_2}^{i-1}$ and $T_{\mu_3}^{i-2} < T_{\mu_2}^{i-1}$

$$N(t) = N_3(t) = \begin{cases} 0 & \text{if} \quad 0 \le t < T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + T_{\mu_3}^1 \\ 1 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + T_{\mu_2}^1 + T_{\mu_3}^1 \le t < T_\lambda^1 + T_{\mu_1}^1 + \sum_{i=1}^{2}(T_{\mu_2}^i) + T_{\mu 3}^2 \\ 2 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + \sum_{i=1}^{2}(T_{\mu_2}^i) + T_{\mu 3}^2 \le t < T_\lambda^1 + T_{\mu_1}^1 + \sum_{i=1}^{3}(T_{\mu_2}^i) + T_{\mu 3}^3 \\ \cdots & \cdots \quad \cdots \\ l-1 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + \sum_{i=1}^{l-1}(T_{\mu_2}^i) + T_{\mu 3}^{l-1} \le t < T_\lambda^1 + T_{\mu_1}^1 + \sum_{i=1}^{l}(T_{\mu_2}^i) + T_{\mu 3}^l \\ l & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + \sum_{i=1}^{l}(T_{\mu_2}^i) + T_{\mu 3}^l \le t < T_\lambda^1 + T_{\mu_1}^1 + \sum_{i=1}^{l+1}(T_{\mu_2}^i) + T_{\mu 3}^{l+1} \\ l+1 & \text{if} \quad T_\lambda^1 + T_{\mu_1}^1 + \sum_{i=1}^{l+1}(T_{\mu_2}^i) + T_{\mu 3}^{l+1} \le t < T_\lambda^1 + T_{\mu_1}^1 + \sum_{i=1}^{l+2}(T_{\mu_2}^i) + T_{\mu 3}^{l+2} \\ \cdots & \text{if} \quad \cdots \end{cases} \tag{3.32}$$

Due to the increase in the number of cases and distribution functions, there is a complexity in analyzing the mean and variance of three-machines-two-in-process-buffers system. But, the proposed method can also be used in order to analyze this system. However, this analysis is not given in this thesis. Using this method in analyzing the transfer lines with three and more machines is an extension and can be given as a future work.

3.6 Numerical Results And Source Codes

In the first three sections analytical results corresponding to the evaluation of the stochastic processes for a single machine, two-machines-one-in-process-buffers and three-machines-two-in-process-buffers transfer lines are given. Also, in the Appendix-A, the proposed analytical methods which can be used to compute the mean and variance of these transfer lines are given. We can calculate analytically for small parameters, i.e. arrival and processing rates (λ^i and μ^i_j), small time intervals (t), and a few number of output (l) of the proposed method for calculating the performance measures of these short transfer lines. However, as the parameters gets larger, the calculations get difficult and complex. In order to cope with this difficulty and make the calculations quickly and easily, we use a packet computer programs.

Maple-XII environment is the main packet program which is used in the calculation of performance measures of interest. Matlab is also used for our calculations in order to check the certainty of the results obtained by using Maple-XII. The numerical results corresponding to single-machine and two-machine-one-buffer transfer lines with different input parameters are given in Appendix-B.part by using tables.

In the following sub-section, the basic computer codes, functions and the algorithms are given.

3.6.1 Source Codes

In this part, we give the main functions and theorems that are used in the proposed method, and also in computer codes.

For the evaluation of the corresponding stochastic processes we use $P(N_n(t) = l), n = 1, 2, 3..and\ l = 0, 1, 2..$ formula and in order to obtain the analytical representation of the mean of the number of parts leaving the n-machine transfer line we use $E[N_n(t)] = \sum_{i=1}^{\infty} l * P(N_n(t) = l)$. For obtaining the variance of the number of parts leaving the n-machine transfer line we first calculate $E[N_n(t)^2]$ by using $E[N_n(t)^2] = \sum_{i=1}^{\infty} l^2 * P(N_n(t) = l)$. After that, we can calculate the variance of, the number of parts leaving the n-machine transfer line, $N_n(t)$, with the formula of $Var[N_n(t)] = E[N_n(t)^2] - (E[N_n(t)])^2 = \sum_{i=1}^{\infty} l^2 * P(N_n(t) = l) - (\sum_{i=1}^{\infty} l * P(N_n(t) = l))^2$

By using this formula we can obtain the expected value and the variance of the number of parts leaving the n-machine transfer line, in any arbitrary time, t. So our proposed method gives derivations for transient state. Furthermore, we can also obtain the steady state results by taking the limit of when t goes to infinity.

3.7 Summary

In this section, the transient behaviour of transfer lines with reliable machines and finite buffers is examined and the distribution of the performance measures of interest is derived and given in Appendix-A The proposed method based on the analytical derivation of the distribution of the throughput of the system provides good results for typical transfer lines encountered in real applications. The results are given in Appendix-B.

Chapter 4

EXTENSION, CONCLUSION AND FUTURE RESEARCH

4.1 Extension

In this section, we discuss the certain issues requiring further investigations. These issues can be interpreted as a future research direction on the work accomplished in this thesis.

4.1.1 Longer Tranfer Lines

Since the number of states of the Markov chains grows very fast with the number of machines, the buffer capacities, and the number of phases of the distributions, the tractable models in the transfer line literature are mostly of limited sizes. In our study, a similar approach is applied. As the number of machines and buffers in the system increase, the tractability of the evaluation of the stochastic processes decreases by a considerable amount even if the arrival and service times are exponentially distributed. Therefore it becomes harder to write down the corresponding representations as well as the codes in the Maple-XII environment. So, decomposition and aggregation techniques together with the proposed method should be utilized in order to obtain the distribution of the throughput of the longer lines.

4.1.2 Non-Exponential Distributions

The Markovian class includes systems in which the arrival and service processes can be represented as exponential, Erlangien, hyperexponential, and phase-type random variables. The general assumption in the transfer line literature is that the arrival and service processes are represented via the exponential distribution. It is argued that the distribution type that is the best in reflecting the real system behaviour is the truncated normal distribution.

In this research the machines are considered to have mutually independent processing times that are exponentially distributed. Despite this assumption, the evaluations of the output processes of the corresponding systems also hold for the systems with non-expontial processing times. However, the closed-form expressions for the mean and variance of the throughput rate of the system of interest will differ from the available ones depending on the probability density and distribution functions.

This study can also be extended to the systems with non-Poisson arrival process by relaxing the strict assumption of the exponential distribution.

4.1.3 General Networks

Throughout this thesis, we have been concerned with flow line models. The special structure of these models, ie., a series of machines separated by buffers, allows many results to be obtained. Although flow line structures are often encountered in industry, there also exist manufacturing systems exhibiting more general structures. Among these, assembly systems are of great interest and can be viewed as an extension of the flow line structure.

Actually, these assembly systems can be reduced to a flow line structure in case of satisfying a set of assumptions:

1) The routing is entirely deterministic. That is, the path that each part follows in the system is known in advance and the contents of each buffer are homogenous.

2) The only source of randomness is the variation of processing times of the machines.

3) They should be connected tree-structured networks in which exactly one sequence of machines and buffers connects any two machines in the network. If there are $n_m = n$ machines, then there are n_B=*n-1* in-process-buffers. In fact, this is a generalization of a transfer line.

Then, the proposed method can calculate the mean and variance of the throughput rate of these systems.

4.2 Conclusion and Future Research

In this thesis we propose an analytical method for estimating the performance measures of a serial production line with reliable machines and finite buffers. The transient and steady-state behaviours of this system are determined by using the evaluation of the stochastic processes under consideration. The analysis of the evaluation of the stochastic processes enables us to derive the distribution of the throughput of the system. This distribution is then utilized to find the higher order moments of the throughput in measuring the performance of the system. The method based on the analytical derivation of the distribution provides good results for typical transfer line models encountered in real applications. Moreover, the iterative algorithms coded in Maple XII environment seem to be efficient: in the examples we tested, it always converged and in general rapidly.

This research should be extended to develop analytical methods for the analysis of more complicated systems such as longer transfer lines with non-exponential distributions.and multiple part types. Also, we aim to study the such systems without assuming that arrival time between services are exponential with parameter λ. For that reason we try to study the systems in which inter-arrival times for part i, T^i_λ *i=1,..* is independent identically distributed exponential random variables.,which can be figured as follows:

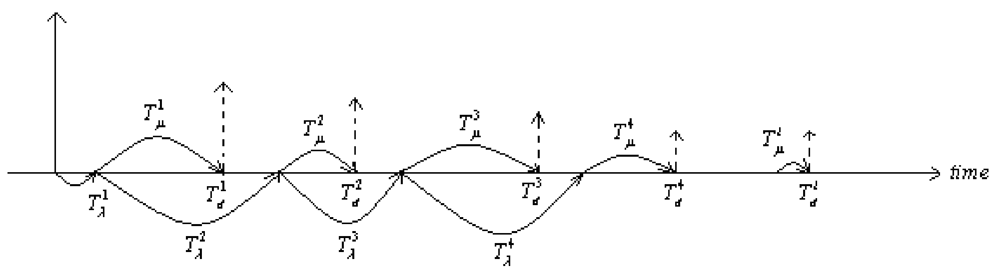

Figure-V: Graphical representation of arrival,processing and departure times of random variables for future research

This research should also be extended to develop analytical methods for the analysis of transfer lines with buffers which have finite and infinite capacities as a future work.

In this thesis, we study the performance measures of short transfer lines in the transient state, at any arbitrarily time. We first tried the derive distribution functions of the number of outputs, for longer transfer lines.Over these derivations we first conclude that for longer transfer lines we have more complex derivations, due to the increase in the number of sources of variability and the number of random variables.As mentioned earlier for *n-machines-(n-1)-in-process-buffers* transfer lines we have 2^{n-1} different cases and distribution functions.For that reason, we decided to study these longer transfer lines, by using decomposition, aggregation and grouping method as a future research, because we thought that in order analyze these longer and more complex systems, we first have to analyze and completely understand the shorter systems.

Then, we started from the simplest systems, made our assumptions and derived our distribution functions for these short systems. We compared and validated our distributions with B.Deler and C. Dinçer [35]. After, we used these distribution functions in order to find the probability, cumulative distribution and probability density functions.We checked these derived probability density and cumulative distribution functions and saw that they satisfy the probability

1) $f(t) \geq 0$ for all $t \geq 0$

2) $\int_0^\infty f(x)dx = 1$

3) $F(t)$ is nondecreasing

4) $F(t)$ is right continuos

5) $\lim_{t\to 0} F(t) = 0$ and $\lim_{t\to\infty} F(t) = 1$

Then, we coded this derivations in Maple-XII environment to see their results for different parameters, and how these derivations are affected by the changes of the time.The results are given in tables in Appendix-B. We analyzed these results and see that the performance measures of expectation and variance are decreasing as t increases. Theoretically we thought that these values would decrease because when we take the *limits* of the derived probability functions as *t goes to infinity these functions goes to 0.*We also compared these results with the results given in B.Deler and C. Dinçer [35], we again see that expected values and variances decreases when t increases in their proposed method.

As a conclusion, we can say that, the proposed method can be used in short transfer lines in calculating performance measures of expected value and variance at any arbitrary time. This method can also be applied in real situations by using computer programs effectively. Also we concluded that this method can be developed in order to use for longer transfer lines.Although this thesis analyze the transient behaviour of serial production systems, it can be used in steady state analysis by using limit theorems.

Appendix A

Calculating Performance Measures

A.1 One-Machine-System (Atomic Model)

In section 3.3 we derive distributions and equations for probability calculations of the single-transfer line. By using these equations the expected number of output and the variance of the output for the atomic model can be calculated as follows:

We have two different probability functions as given before : The first was $P(N_1(t) = 0)$ and the second was $P(N_1(t) = l)$ $l = 1, 2, 3...$

However,in calculating the mean and variance of the output,the first case does not make sense,because the expected value and variance of the 0 output is also 0. So, in these calculations we only use the probabilities for $l = 1, 2, 3..$

Expected value of the output of a single-line can be calculated as :

$$E[N_1(t)] = \sum_{l=1}^{\infty} l * P(N_1(t) = l) = \sum_{l=1}^{\infty} l * (\int_0^t (1 - (1 - \frac{\mu_1}{\mu_1 - \lambda} e^{-\lambda(t-z)} + \frac{\lambda}{\mu_1 - \lambda} e^{-\mu(t-z)})) * \frac{(\lambda + \mu_1)^l z^{l-1} e^{-(\lambda + \mu_1) z}}{(l-1)!} dz)$$

For calculating the variance of the output for the atomic model we will first find the $E(N_1(t)^2)$, and,

$$E[N_1(t)^2] = \sum_{l=1}^{\infty} l^2 * P(N_1(t) = l) = \sum_{l=1}^{\infty} l^2 * (\int_0^t (1 - (1 - \frac{\mu_1}{\mu_1-\lambda} e^{-\lambda(t-z)} + \frac{\lambda}{\mu_1-\lambda} e^{-\mu(t-z)})) * \frac{(\lambda+\mu_1)^l z^{l-1} e^{-(\lambda+\mu_1)z}}{(l-1)!} dz)$$

By using these equations we can easily find the variance of the output of the atomic model as :

$$Var[N_1(t)] = E[N_1(t)^2] - (E[N_1(t)])^2 = \sum_{l=1}^{\infty} l^2 * (\int_0^t (1 - (1 - \frac{\mu_1}{\mu_1-\lambda} e^{-\lambda(t-z)} + \frac{\lambda}{\mu_1-\lambda} e^{-\mu(t-z)})) * \frac{(\lambda+\mu_1)^l z^{l-1} e^{-(\lambda+\mu_1)z}}{(l-1)!} dz) - (\sum_{l=1}^{\infty} l * (\int_0^t (1 - (1 - \frac{\mu_1}{\mu_1-\lambda} e^{-\lambda(t-z)} + \frac{\lambda}{\mu_1-\lambda} e^{-\mu(t-z)})) * \frac{(\lambda+\mu_1)^l z^{l-1} e^{-(\lambda+\mu_1)z}}{(l-1)!} dz))^2$$

A.2 Two-Machines-One-In-Process-Buffer Transfer Line

In section 3.4 we derive distributions and equations for probability calculations of the two-machines-one-in-process-buffers transfer line. By using these equations the expected number of output and the variance of the output for the this model can be calculated as follows:

We have two different cases each having different distribution functions as given before. Also each of these functions are analyzed in two parts, where the first parts, $P(N_2(t) = 0)$, of these distributions are the same. However the second part of them,$P(N_2(t) = l) \quad l = 1, 2, 3...$are different.

However,in calculating the mean and variance of the output,the first parts of the cases do not make sense,because the expected value and variance of the 0 output is also 0. So, in these calculations we only use the probabilities for only the outputs of,$l = 1, 2, 3..$

Expected value of the output of a single-line can be calculated as :

$$E[N_2(t)] = \sum_{l=1}^{\infty} l * P(N_2(t) = l) = \sum_{l=1}^{\infty} l * [(P(N_2^{cs-1}(t) * P(T_\lambda^i + T_{\mu_1}^i \geq T_{\mu_2}^{i-1})) + (P(N_2^{cs-2}(t) * (1 - P(T_\lambda^i + T_{\mu_1}^i \geq T_{\mu_2}^{i-1})))]$$

$$= \sum_{l=1}^{\infty} l*[(\int_0^t((1-\int_0^{\infty}(1-\frac{\mu_1}{\mu_1-\lambda}*\frac{\mu_2}{\mu_2-\lambda}*e^{(-\lambda(t-z+v))}-\frac{\lambda}{\lambda-\mu_1}*\frac{\mu_2}{\mu_2-\mu_1}*e^{(-\mu_1(t-z+v))}-\frac{\lambda}{\lambda-\mu_2}*\frac{\mu_1}{\mu_1-\mu_2}*e^{(-\mu_2(t-z+v))})*\mu_2*e^{-\mu_2 v}dv))*\int_0^z\frac{(\lambda+\mu_1)^l*x^{l-1}*e^{-(\lambda+\mu_1)*x}}{(l-1)!}*\mu_2*e^{-\mu_2*(z-x)}dx)dz)*(1-\int_0^{\infty}(1-\frac{\mu_1}{\mu_1-\lambda}e^{-\lambda(x+y)}+\frac{\lambda}{\mu_1-\lambda}e^{-\mu_1(x+y)})*\mu_2*e^{-\mu_2*y}dy))+((\int_0^t((1-(1-e^{-\mu_2*(t-z)})*(\int_0^z\frac{\lambda*\mu_1}{\mu_1-\lambda}*(e^{-\lambda*x}-e^{-\mu_1*x})*\frac{\mu_2^l*(z-x)^{l-1}*e^{-\mu_2*(z-x)}}{(l-1)!}dx))dz)*(\int_0^{\infty}(1-\frac{\mu_1}{\mu_1-\lambda}e^{-\lambda(x+y)}+\frac{\lambda}{\mu_1-\lambda}e^{-\mu_1(x+y)})*\mu_2*e^{-\mu_2*y}dy))]$$

For calculating the variance of the output for the atomic model we will first find the $E(N_2(t)^2)$, and,

$$E[N_2(t)^2] = \sum_{l=1}^{\infty} l^2 * P(N_2(t) = l) = \sum_{l=1}^{\infty} l^2 * [(P(N_2^{cs-1}(t) * P(T_\lambda^i + T_{\mu_1}^i \geq T_{\mu_2}^{i-1})) + (P(N_2^{cs-2}(t) * (1 - P(T_\lambda^i + T_{\mu_1}^i \geq T_{\mu_2}^{i-1})))]$$

$$= \sum_{l=1}^{\infty} l^2*[(\int_0^t((1-\int_0^{\infty}(1-\frac{\mu_1}{\mu_1-\lambda}*\frac{\mu_2}{\mu_2-\lambda}*e^{(-\lambda(t-z+v))}-\frac{\lambda}{\lambda-\mu_1}*\frac{\mu_2}{\mu_2-\mu_1}*e^{(-\mu_1(t-z+v))}-\frac{\lambda}{\lambda-\mu_2}*\frac{\mu_1}{\mu_1-\mu_2}*e^{(-\mu_2(t-z+v))})*\mu_2*e^{-\mu_2 v}dv))*\int_0^z\frac{(\lambda+\mu_1)^l*x^{l-1}*e^{-(\lambda+\mu_1)*x}}{(l-1)!}*\mu_2*e^{-\mu_2*(z-x)}dx)dz)*(1-\int_0^{\infty}(1-\frac{\mu_1}{\mu_1-\lambda}e^{-\lambda(x+y)}+\frac{\lambda}{\mu_1-\lambda}e^{-\mu_1(x+y)})*\mu_2*e^{-\mu_2*y}dy))+((\int_0^t((1-(1-e^{-\mu_2*(t-z)})*(\int_0^z\frac{\lambda*\mu_1}{\mu_1-\lambda}*(e^{-\lambda*x}-e^{-\mu_1*x})*\frac{\mu_2^l*(z-x)^{l-1}*e^{-\mu_2*(z-x)}}{(l-1)!}dx))dz)*(\int_0^{\infty}(1-\frac{\mu_1}{\mu_1-\lambda}e^{-\lambda(x+y)}+\frac{\lambda}{\mu_1-\lambda}e^{-\mu_1(x+y)})*\mu_2*e^{-\mu_2*y}dy))]$$

By using these equations we can easily find the variance of the two-machines-one-in-process-buffer transfer line as :

$$Var[N_2(t)] = E[N_2(t)^2] - (E[N_2(t)])^2$$

Appendix B

Results Obtained by Using Maple-XII

B.1 One-Machine-System (Atomic Model)

As mentioned earlier, in order to make calculations faster and more reliable, a software program, Maple-XII, is used. We first derived the distributions, develop the density and cumulative distribution functions. Then we wrote the equations into the Maple-XII environment and get the results for different parameters. By analyzing these results, we could easily make conclusions. Here we first give the program part and codes written for atomic-model calculations.

Maple-XII program codes for atomic-model

```
restart;

lambda:=a;

mu:=b;

t:=c;
```

```
P(N_1(t)=l):=int((((mu/(mu-lambda))*exp(-lambda*(t-z)))-((lambda/(mu-lambda))*exp(-mu*(t-z))))*(((lambda+mu)^l*z^(l-1)*exp(-(lambda+mu)*z))/(l-1)!),z=0..t);

evalf(P(N_1(t)=l));

EN:=sum(l*int((((mu/(mu-lambda))*exp(-lambda*(t-z)))-((lambda/(mu-lambda))*exp(-mu*(t-z))))*(((lambda+mu)^l*z^(l-1)*exp(-(lambda+mu)*z))/(l-1)!),z=0..t),l=1..infinity);

evalf(EN);

EN2:=sum(l^2*int((((mu/(mu-lambda))*exp(-lambda*(t-z)))-((lambda/(mu-lambda))*exp(-mu*(t-z))))*(((lambda+mu)^l*z^(l-1)*exp(-(lambda+mu)*z))/(l-1)!),z=0..t),l=1..infinity);

evalf(EN2);

Var:=EN2-EN^2;

evalf(Var);
```

By giving different values to the parameters of lambda, mu and t (a,b,c); we obtain different results for expected value and variance. This results are given in Table-I below.

λ	μ	t	$E(N_1(t))$	$Var(N_1(t))$
1	2	2	1.261	0.580
1	2	5	0.938	3.024
1	2	10	0.125	1.068
1	2	15	0.010	0.140

B.2 Two-Machines-One-In-Process-Buffer Transfer Line

Maple-XII program codes for two-machine-one-buffer-transfer line

```
restart;
```

lambda:=a;

mu[1]:=b;

mu[2]:=c;

t := d;

P(N_2^{cs-1}(t)=l):=int((1-int((1-mu[1]*mu[2]*exp(-lambda*(t-z+y))/((mu[1]-lambda) * (mu[2]-lambda))-lambda*mu[2]*exp(-mu[1]*(t-z+y))/((lambda-mu[1]) * (mu[2]-mu[1]))-lambda*mu[1]*exp(-mu[2]*(t-z+y))/((lambda-mu[2]) * (mu[1]-mu[2])))*mu[2]*exp(-mu[2]*y),y=0..infinity)) * int((((lambda+mu[1])ˆl*xˆ(l-1)*exp(-(lambda+mu[1])*x))/(l-1)!)*mu[2]*exp(-mu[2]*(z-x)),x=0..z),z=0..t)

f:=int((lambda*mu[1]*(exp(-lambda*x)-exp(-mu[1]*x))/(mu[1]-lambda)*mu[2]ˆl*(z-x)ˆ(l-1)*exp(-mu[2]*(z-x))/(l-1)!),x=0..z);

P(N_2^{cs-2}(t)=l):=int(exp(-mu[2]*(t-z))*f,z=0..t);

P(case-1):=int((1-(mu[1]/(mu[1]-lambda)*exp(-lambda(x+y)))+(lambda/(mu[1]-lambda)*exp(-mu[1](x+y))))*mu[2]*exp(-mu[2]*y),y=0..infinity);

EN:=sum(l*(P(N_2^{cs-1}(t)=l)*P(case-1)+P(N_2^{cs-2}(t)=l)*(1-P(case-1))),l=1..infinity);

evalf(EN);

EN2:=sum(l^2*(P(N_2^{cs-1}(t)=l)*P(case-1)+P(N_2^{cs-2}(t)=l)*(1-P(case-1))),l=1..infinity);

evalf(EN2);

Var:=EN2-EN^2;

evalf(Var);

By giving different values to the parameters of lambda, mu[1],mu[2]and t (a,b,c,d); we obtain different results for expected value and variance. This results are given in Table-II below.

λ	μ_1	μ_2	t	$E(N_1(t))$	$Var(N_1(t))$
1	2	3	2	0.842	0.715
1	2	3	5	0.829	2.735
1	2	3	10	0.119	1.022
1	2	3	15	0.010	0.136

Appendix C

Used Theorems

C.1 Density Function of Sum of Two Random Variables

Let (X,Y) be a random variable of the continuos type with probability density function *f*. Let Z=X+Y. Then the probability density function of Z is given by

$$f_Z(z) = \int_{-\infty}^{z} f(x, z-x)dx = \int_{-\infty}^{z} f_X(x) * f_Y(z-x)dx$$

C.2 Cumulative Distribution Function of Difference of Two Random Variables

Let (X,Y) be a random variable of the continuos type with probability density function *f*. Let Z=X-Y. Then the probability density function of Z is given by

$$F_Z(z) = \int_{0}^{\infty} P(X < x+y) f_Y(y)dy = \int_{0}^{\infty} F_X(x+y) * f_Y(y)dy$$

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