

A FUZZY LOGIC MODEL FOR THE ISE100 INDEX

ÇİĞDEM ÖZARI
103624006

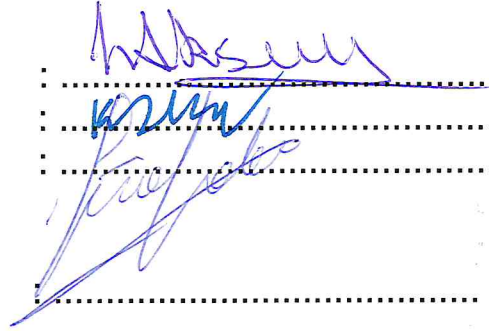
İSTANBUL BİLGİ ÜNİVERSİTESİ
SOSYAL BİLİMLER ENSTİTÜSÜ
FİNANSAL EKONOMİ YÜKSEK LİSANS PROGRAMI

HAKAN AKSOY
2004-2005

A FUZZY LOGIC MODEL FOR THE ISE100 INDEX

CİGDEM ÖZARI
103624006

Dr.HAKAN AKSOY
Assist.Prof. KORAY AKAY
SINA ERDAL



28.06.2005

Toplam Sayfa Sayısı:

Anahtar Kelimeler	Key Words
1) Bulanik Mantik	1) Fuzzy Logic
2) Anomali	2) Anomaly
3) RSI, RVI, MA, BBAND	3) RSI, RVI, MA, BBAND
4) Teknik Analiz	4) Technical Analysis

ACKNOWLEDGEMENT

I am greatly indebted to Dr.Hakan Aksoy for his invaluable contribution to my understanding during the development of this thesis. I would also express my gratitude to Associate Professor Ege Yazgan for his comments and suggestions during the development of this thesis. And, to all my friends for their support.

ABSTRACT

The most important aim of this study is to show the existence of fuzzy logic applications in finance. Fuzzy logic is a system which models complex relationships mathematically. The fuzzy models handle the vague environment in a manner of mathematical language.

In Chapter 2, in order to show the existence of anomalies in stock market, especially in the ISE, efficient market hypothesis is revisited by referring past studies. Among all the anomalies observed in stock markets, the most significant ones are considered here in this study. These are January, Monday, turn of the month- and turn of the year effects.

In Chapter 3, the indicators RSI, RVI, BBAND, MA, which have the most explanatory power within the indicators, used in technical analyses are examined in details with supporting examples and the mathematics behind.

In Chapter 4, general concepts about membership functions, linguistic variables, fuzzy sets, rule generation, fuzzification, defuzzification and optimisation methods are presented in order to construct the fuzzy inference system with application of the ISE 100 index return predictions. We construct a fuzzy model to predict the daily returns of ISE100 with the indicators that we mentioned in chapter 3 and then we made optimisation by using the steepest descent algorithm to make the difference smaller between returns that we found with the real returns.

Consequently, the power of fuzzy logic methodology is scrutinized in return prediction in the ISE 100 index compared to other mathematical methods. By using the four indicators with the rules that we construct by experience; we try to predict the daily returns of the ISE100 index by the period between 1996 to 2004. We find similar correlation in both out_ of_ sample and in_ sample which supports our the results of our model.

ÖZET

Bu çalışmada, etkin piyasalar hipotezinin formlarından zayıf etkinlik formunu test edebilmek için ya da başka bir deyişle teknik analiz kullanılabilişliğini ölçebilmek için anomalilerin varolup olmadığını incelemekle başladık. En sık gözlenen Pazartesi, yıl dönümü, ay dönümü, ocak ayı anomalilerinin ISE100 endeksi için 1987'den günümüze kadar olan günlük getirileri kullanarak gözlemlenebilirliğini inceledik. Böylece teknik analiz kullanılabilişğine karar verdik ve çalışmamızın devamını teknik indikatörleri inceleyerek sürdürdük. Matematiksel incelemeler sonucunda hareketli ortalama, hareketli ortalamayla hesaplanan bollinger band, göreceli güç endeksi ve hacimle hesaplanan göreceli güç endeksinin diğer indikatörlere göre piyasayı daha iyi ve farklı açılardan da incelediğini gözlemledik ve bu indikatörleri bulanık mantık modelimizde kullanmaya karar verdik. Bulanık mantığı kullanmamızda ki en önemli sebep ya da bulanık mantığı klasik mantıktan ayıran temel fark bilinen anlamda matematiğin sadece aşırı uç değerlerine izin vermesidir. Bu yüzden de klasik matematiksel yöntemlerle karmaşık sistemleri modellemek ve kontrol etmek zordur. Biz de günlük getirileri tahmin etmeye çalıştığımız için bulanık mantığın da finasta bir uygulaması olacağını düşünerek bir model oluşturduk. Çalışmamızın en son bölümünde bulanık mantığın temel kavramlarından olan ve modelimizi kurmamız için gerekli ve yeterli temel kavramlardan bahsettik. Modelimizde daha önce incelediğimiz teknik indikatörleri ve deneyimler sonucu karar verdiğimiz mantık kurallarını kullanarak günlük getirileri hesapladık. Bulduğumuz getirilerle gerçek getirileri birbirine eşitleyerek modelimizdeki değişiklikleri belirledik ve modelimizi oluştururken kullanmadığımız datalarla modelimizi test ettik.

LIST OF TABLES

TABLE

2.1 Statistical for the daily returns.....	9
2.2 Statistical for the turn of the returns.....	11
2.3 Statistical for the turn of the year returns.....	12
2.4.1 Statistical for the monthly returns.....	14
2.4.2 Statistical for the monthly returns.....	14
4.1 Rules between RSI and RVI.....	32
4.2 Rules between MA and BBAND.....	32
4.3 Operations on Fuzzy Set (and, or, not).....	34
4.4 Truth table for p implies q	34
4.5 Range of variables.....	43
4.5.A Range of variables.....	43

LIST OF FIGURES

FIGURE

3.1 Relationship between moving average and return.....	18
3.2 Relative Strength Index (RSI).....	20
3.3 Relative Volume Index (RVI).....	21
3.4 The relations between upper band, lower band and return (BBAND).....	21
4.1 Membership function of RSI.....	29
4.2 Summary of fuzzy if then rules.....	37
4.3 Defuzzification methods.....	38
4.4 Summary of fuzzy system.....	40
4.5 Surface that shows relation between RSI and RVI.....	41
4.6 Surface that shows relation between MA and BBAND.....	42
4.7 Fuzzy Logic Model.....	44
4.8 R square before optimization.....	45
4.9 Convergence of steepest descent method.....	46
4.10 R square after optimization.....	49
4.11 Correlation of actual and estimated output.....	50
4.12 Membership function of the middle layer 1.....	51
4.13 Membership function of the middle layer 2.....	51
4.14 Membership function of the output	52
4.15 Output of the testing model.....	53

CHAPTER 1.INTRODUCTION	3
CHAPTER 2.EXISTENCE OF ANOMALIES	4
2.1.Efficient Market Hypothesis	4
2.1.1. Weak form Efficiency	5
2.1.2 Semistrong Form Efficiency	5
2.1.3.Strong Form Efficiency.....	5
2.2. Calendar Effect.....	7
2.2.1. The Day of the Week Effect (Monday Effect).....	7
2.2.2. The Turn Of the Month Effect	10
2.2.3. The Turn of the Year Effect	11
2.2.4. January Effect.....	13
CHAPTER 3.TECHNICAL ANALYSIS & INDICATORS.....	15
3.1. Technical Analysis	15
3.1.1. Moving Average.....	16
3.1.2. Relative Strength Index (RSI)	18
3.1.3. Relative Volatility Index (RVI).....	20
3.1.4. Bollinger Bands.....	21
CHAPTER 4. FUZZY LOGIC MODELLING OF THE ISE.....	24
4.1. Fuzzy Sets	28
4.1.1. Linguistic Variables and Fuzzy If-Then Rules	29
4.2. Fuzzification & Rule Generation	30
4.3. Operations on Fuzzy Sets.....	33
4.4 Defuzzification.....	37
4.5. Graphs of the Relations between Inputs.....	41

4.6. MODELLING	43
4.6.1. Steepest Descent Method	46
4.6.2. Results of the Optimisation	48
4.6.3. Testing The Model	53
4.7 Summary of the algorithm for the fuzzy & optimisation processes.....	54
CHAPTER 5.CONCLUSION.....	56
REFERENCES.....	58

CHAPTER 1. INTRODUCTION

Questioning the validity of technical analysis or chartering became a popular controversial issue among academicians in last decades with the development of computer techniques and return prediction simulations. Even though bombarded continuously from “fundamentalists” and “efficient market supporters”, technical analysis found its route and application to stock return predictions in almost every intermediary institutions and investment banks.

Regarding to the very first aim of this study which is to show fuzzy logic applications in finance and in stock return predictions so that modeling and quantifying complex qualitative relationships in order to make human knowledge much more understandable especially to computers, firstly the anomalies of the stock markets are examined. After illustrating that anomalies do exist in stock markets and also in our data set, the Istanbul Stock Exchange, technical analysis and key indicators examined in fine points, which paved the way to construct the basis for fuzzy logic applications in stock return prediction in the ISE. The study of Aksoy “Modelling Stock Market via Fuzzy Rule Based Systems” inspired us expanding the application fuzzy modelling in finance

CHAPTER 2. EXISTENCE OF ANOMALIES

2.1. Efficient Market Hypothesis

The Efficient Market Hypothesis is one of the cornerstones of modern financial economics. It is the idea that information is quickly and efficiently incorporated into asset prices, so that old information can not be used to foretell future price movements. The random walk model of asset prices is an extension of the The Efficient Market Hypothesis, as are the notions that the market can not be consistently beaten, arbitrage is impossible, and "free lunches" are generally unavailable.

The efficient market hypothesis was introduced in the late 1960s and the prevailing view prior to that time was that markets were inefficient. Inefficiency was commonly believed to exist. For example in United States and United Kingdom stock markets. However, earlier work by Kendall (1953) suggested that changes in United Kingdom stock market prices were random. Later work by Brealey and Dryden, and also by Cunningham found that there were no significant dependences in price changes suggesting that the United Kingdom stock market was weak-form efficient. Further to this evidence that the UK stock market is weak form efficient, other studies of capital markets have pointed toward them being semi strong-form efficient.

An excellent review of Efficient Market Hypothesis theory and evidence is provided by Bodie, et al. (1996), and good treatments are provided in most introductory investment textbooks. As Bodie, et al. point out, Fama (1970 and 1991) presents the rigorous underpinnings of the Efficient Market Hypothesis and a review of empirical work. Grossman [1989, articles 1, 2, 3, and 5] and Milgrom (1981) construct compelling models that embody the theoretical argument for the strong-form Efficient Market Hypothesis.

Efficient Market Hypothesis is an attempt to explain why stocks behave the way they do. There are three forms of the Efficient Market Hypothesis.

2.1.1. Weak form Efficiency

The weak-form EMH stipulates that current asset prices already reflect past price and volume information. Consequently, trend analysis is useless for predicting future price changes. It assumes that stock prices fully reflect all historical information, including past returns. Thus, investors would gain little from technical analysis or you can not achieve abnormal profits by using past price changes. In addition, we can say that technical analysis is useless in this form of efficiency. In other words, stock prices follow random walk and past returns are entirely useless for predicting future returns.

2.1.2 Semistrong Form Efficiency

The semistrong-form EMH states that all publicly available information is similarly already incorporated into asset prices, and so a firm's financial statements are of no help in forecasting future price movements and securing high investment returns. In other words, fundamental analysis is of no use.

2.1.3.Strong Form Efficiency

The strong-form EMH stipulates that private information too, is quickly incorporated by market prices and therefore cannot be used to reap abnormal trading profits. It asserts that all information is fully reflected in securities prices. In other words, even insider information is of no use. In the world of the strong form EMH, trying to beat the market becomes a game of chance not skill.

Note that, strong-form efficiency implies semi-strong form efficiency implies weak-form efficiency. However, the reverse is not correct.

The debate about efficient markets has resulted in hundreds and thousands of empirical studies attempting to determine whether specific markets are in fact "efficient" and if so to what degree. Many novice investors are surprised to learn that a tremendous amount of evidence supports the efficient market hypothesis. Early tests of the Efficient Market Hypothesis focused on technical analysis and it is chartists whose very existence seems most challenged by the Efficient Market Hypothesis. And in fact, the vast majority of studies of technical theories have found the strategies to be completely useless in predicting securities prices. However, researchers have documented some technical anomalies that may offer some hope for technicians, although transactions costs may reduce or eliminate any advantage.

In financial literature, in many studies researchers try to test the weak form of efficiency. In our model we also try to test the weak form efficiency with a subtitle, anomalies. An anomaly is an occurrence that can not be explaining by the prevailing theory. In other words, we try to know that if the stock returns follow a seasonable trend. Before 1970s, for the results of the market efficiency tests, the stock prices are independent from each other and follow a normal distribution. Also the expected returns are constant with independency to the time. In addition, when we say efficient market, it means that by using past data you can not predict the future returns. The first studies in this area, which contradicts with this theory, was made by Fama 1965 and Fisher 1966, they observed that one can predict the daily, weekly and monthly returns by using past data. Lo MacKinlay (1988), Conrad and Kaul (1988) observed that the returns for the portfolios consists of stocks of small sized firms is much more predictable than the big sized ones. In addition, French and Roll (1986) found that one can predict the daily and monthly returns.

DeBondt and Thaler (1985, 1987) found that the stocks that losses mostly for the last three- five years make profit much more than the market, in generally in the January and the opposite are also true. They renamed this as a “winner-looser effect”. They reasoned this as the market behaves more than enough to the bad and good news. Banz (1981) founds similar results with DeBondt and Thaler but he reasoned the “winner-looser effect” to the small firm effect. However; Jagadeesh (1990), Lehmann (1990) and Lo MacKinlay (1990) found solutions that contradicts with the above hypothesis.

Since the weak form of efficiency implies that the technical analysis is useless, the studies are intensified around the topic that if the market is weak form efficient.

2.2. Calendar Effect

Calendar Anomalies in stock market returns have been of considerable interest during the last three decades. These anomalies can be listed as the weekend effect, the day of the week effect, January effect... Many researchers examined the foreign exchange and T-bill markets. For example, Cross (1973), French (1980), Gibbon and Hess (1981), Keim and Stambaugh (1984) demonstrate that the distribution of stock returns is presented by different shapes for the different days of the week.

2.2.1. The Day of the Week Effect (Monday Effect)

The weekend effect is examined by comparing implied expected returns on Friday with implied expected returns on Monday, three calendar days later. The null hypothesis is that there is no difference in implied returns between Friday and Monday, whereas the alternative hypothesis is that expected returns increase on Monday following the negative weekend effect.

It is observed that security price changes tend to be negative on Mondays and positive on the other days of the week with Friday being the best of all. In fact, Monday returns are close to zero or negative. In other words; Monday tends to be the worst day to be invested in stocks. Short sellers may be responsible for the weekend effect because they do not want to keep speculative positions open around the weekend. Thus, they close the short positions by buying back on Fridays, and reopen them by short selling on Mondays. Investors should recognize the weekend effect and avoid selling on Mondays. Instead, they should buy stocks on Mondays and sell on Fridays. However, the difference is so small and virtually impossible to take the advantage because of trading costs.

The first study documenting a weekend effect was by M.J. Fields in 1931 in the *Journal of Business* at a time when stocks traded Saturdays. Laurence Harris has studied intraday trading and found that the weekend effect tends to occur in the first 45 minutes of trading as prices fall but on all other days prices rise during the first 45 minutes. Sun Q. Tong W.H.S observed a 'week four effect', not only the returns on Monday but also returns on other days is lower during the fourth week of the month. They suggest that the liquidity selling by individual investors may be this reason. Rogalski (1984) made an observation and found that the negative returns on Mondays caused by the weekend days and by this study, we can rename the day of the week effect or Monday effect as the weekend effect. Smirlock and Starks (1986) made an observation and found the similar results for the period between 1974 and 1983. However, for the period before 1974, they found that the negativity of Monday returns caused by the trading hours on Mondays.

This anomaly presents the interesting question: Could the effect be caused by the moods of market participants? Because people are generally better in better moods on Fridays and before holidays, but are generally grumpy on Mondays. Rystrom and Benson (1989) attribute the negative Monday returns to the people feeling less optimistic on Mondays.

In addition, Miller (1988) suggests that the negative returns on Mondays are due to individuals selling rather than the institutions. He argues that individuals are like to sell on Mondays because they have time to decide on weekend.

The study for the Istanbul Stock Exchange that Muradoglu and Oktay made in 1993 for the five years period between the dates 1988 to 1992; they observed that the highest average return is on Friday and the lowest average return is on Tuesday. Also, Balaban observed the similar results for the period 1988 to 1994 for the Istanbul Stock Exchange. He observed that the highest average return is on Fridays and it is approximately two times of the other days and the highest volatility is on Mondays. We looked the data between 1987 and the beginning of 2005 and find the lowest average return on Tuesdays and the highest volatility on Mondays and the maximum positive return on Fridays as easily seen below table.

Table 2.1 Statistical Results For The Daily Returns

	Monday	Tuesday	Wednesday	Thursday	Friday
Mean	0,176	-0,0917	0,2229	0,3648	0,6056
Variance	13,89	11,98	10,418	11,625	10,323
St.Deviation	3,727	3,4619	3,2277	3,4095	3,2129
Mean/St.Deviation	0,047	-0,0265	0,0690	0,1069	0,1885

The above figure shows that the negative returns are only on Tuesdays with the value -0.0917 and Fridays have the maximum positive return with the value 0.6056. By the way, Mondays have the maximum variance; in addition the standard deviation of the each day is so near to each other.

Shortly, the negative returns on Monday are generated by the fact that stocks tend to open lower on Mondays than from what happens during the day.

The returns from intraday returns on Monday (the price changes from open to close on Monday) are not the culprits in creating the negative returns. The Monday effect is worse for small stocks than for larger stocks.

- The Monday effect is no worse following three-day weekends than two-day weekends.
- Monday returns are more likely to be negative if the returns on the previous Friday were negative. In fact, Monday returns are, on average, positive following positive Friday returns, and are negative 80% of the time following negative Friday returns.

There are some who have argued that the weekend effect is the result of bad news being revealed after the close of trading on Friday and during the weekend. They point to the fact that more negative earnings reports are revealed after close of trading on Friday. Even if this were a widespread phenomenon, the return behavior would be inconsistent with a rational market, since rational investors would build in the expectation of the bad news over the weekend into the price before the weekend, leading to an elimination of the weekend effect.

As a final note, the negative returns on Mondays cannot be just attributed to the absence of trading over the weekend. The returns on days following trading holidays, in general, are characterized by abnormally positive, not negative, returns.

2.2.2. The Turn Of the Month Effect

It is studied by comparing implied returns three (may be N days such that N is smaller and equal to 5) trading days prior to the turn of the month with implied returns four trading days after the turn of the month. The first study documented by Lakonishok and Smith in 1988 and they found that the first the last and the first two days of the month have the largest average return. When the ratio of the returns to be positive on the turn of the months was %56, the other days the ratio was %52.

Also, they noted that the average returns on the turn of the month exceed the total average returns of the months. Lauterbach and Ungar found the similar results for the Israel Stock Market in 1991. Also, a turn-of-the-month effect is documented by Ariel (1987) where higher mean stock returns occur during the initial days of a trading month than during days later in the month.

For the Istanbul Stock Exchange, Ozmen (1992) observed the period between January 1988 and February 1992. But, he compared returns three trading days prior to the turn of the month with returns for three trading days after the turn of the month. He found that the turn of the month effect is not seen for the Istanbul Stock Exchange. We looked the period from 1987 to the beginning of the month returns three trading days prior to the turn of the month with implied returns three trading days after the turn of the month and found that it exists.

Table 2.2 Statistical Results For The Turn Of The Month Returns

	<u>First&Last</u> three days of the months	The other Days of the Month
Mean	0,0043	0,0020
Variance	0,001363	0,001149
St.Deviation	0,0369	0,0339
Mean/St.Deviation	0,1172	0,0593

The mean of the turn of the month days are nearly twice times of the other days but both of them have small values for the standard deviations. They seem to converge to their means on these days. In addition, the ratio of mean over standard deviation of the turn of the month is twice of the other days because of their means ratio.

2.2.3. The Turn of the Year Effect

A turn-of-the-year effect mostly looks if there exists a trend between the last few days of December together with the first few days of the January.

It has also been studied extensively. Keim (1983) documents a strong seasonal pattern for small firms where returns in January differ significantly from the returns earned during other months of the year. For small firms, January brings higher stock returns with much of this gain occurring in the first five trading days of the month. Large firms, on the other hand, experience negative returns during this period. Therefore, a turn-of-the-year effect appears to be a small firm phenomenon. This characterization, however, does not appear to hold for the weekend or turn-of-the-month seasonal patterns observed in stock return data. We looked the data between 1987 to 2005 beginning and check the null hypothesis that there is no difference in returns between the last days of the December together with first few days of the January and the other days of the year. As seen in the below table, we found that it also exists in our country for the Istanbul Stock Exchange.

Table 2.3 Statistical Results For The Turn of The Year Returns

	Last three days of the December First few days of the January	The other Days of the Year
Mean	0,00979	0,020823
Variance	0,00107	0,044919
St.Deviation	0,0327	0,2119400
Mean/St.Deviation	0,2992	0,09825

From the above table, one can easily examine that the turn of the year days have smaller mean and variance with respect to the other days. In addition, the difference between the mean and the standard deviation is also smaller than the other days. Because of this we found a largest ratio for the mean over standard deviation and it shows the existence of the turn of the year effect.

2.2.4. January Effect

In January, stock returns are inexplicably high and small firms' stocks do better than large firms. In recent years, there is less evidence of a January effect. At the end of the year, investors start to worry about taxes and sell some stocks that are down so the losses can be written off against capital gains then this selling causes stocks to go down near the end of the year and back up in January when investors buy back the stocks they sold. The first study was made by Watchel (1942) and he found that the highest average return is on January and it influenced most the small firms. After a long time, Rozeff and Kinney (1976) looked at the New York Stock Market between the period 1904-1974 and find the highest average return on January and also looked at the average returns, found that the average return on January is nearly seven times greater than the other months. However, Lakonishok and Smidh (1988) looks same things for only the large firms and could not found the same results. So, they thought that only small firms depend on January effect. Gultekin and Gultekin (1983) observed the seventeen countries and they found January effect for the twelve of them. They observed that it is not dependent on small firms; may be depended to taxes.

Note that the January effect is most pronounced for the smallest, riskiest firms in the market and least pronounced for larger, safer firms. A number of explanations have been advanced for the January effect, but few hold up to serious scrutiny. One is that there is tax loss selling by investors at the end of the year on stocks which have gone down to capture the capital gain, driving prices down, presumably below true value, in December, and a buying back of the same stocks (It is to prevent this type of trading that the internal revenue service has a "wash sale rule" that prevent you from selling and buying back the same stock within 45 days. To get around this rule, there has to be some substitution among the stocks. Thus, investor 1 sells stock A and investor 2 sells stock B, but when it comes time to buy back the stock, investor 1 buys stock B and investor 2 buys stock A) in January, resulting in the high

returns. The fact that the January effect is accentuated for stocks that have done worse over the prior year is offered as evidence for this explanation. There are several pieces of evidence that contradict it, though. First, there are countries, like Australia, which have a different tax year, but continue to have a January effect. Second, the January effect is no greater, on average, in years following bad years for the stock market, than in other years.

Table 2.4.1 Statistical Results For The Monthly Returns

	January	February	March	April	May	June
Mean	0,5976	0,2476	0,0321	0,4384	-0,0038	0,3363
Variance	14,48	16,34	10,74	10,93	13,34	7,7
St.Deviation	3,8057	4,0423	3,2782	3,3068	3,6525	2,7755
Mean/St.Deviation	0,1570	0,0612	0,0098	0,1325	-0,00105	0,1211

Table 2.4.2 Statistical Results For The Monthly Returns

	July	August	September	October	November	December
Mean	0,1615	-0,0574	0,2562	0,1952	0,2618	0,4597
Variance	12,55	9,87	10,61	9,12	13,78	13,4467
St.Deviation	3,5432	3,1431	3,2582	3,0208	3,7125	3,6669
Mean/St.Deviation	0,0456	-0,0182	0,0786	0,0646	0,0705	0,1253

From the above tables, January has the maximum positive return than the other months have and February has the maximum variance like the days of the week. So, we mean that the second input seems to be most changed variable. The month of the maximum negative return changes from country to country, in our country it seems to be on August.

But, it is not the only month that we have negative return. We have negative return on May also with greater variance.

CHAPTER 3. TECHNICAL ANALYSIS & INDICATORS

3.1. Technical Analysis

Technical Analysis has been quite rigorously argued among academicians and practitioners for its validity to let investors to gain extra returns over the market itself. Its popularity is increasing in the last decades and allows the wide usability of technical analysis has been the interest of many academic studies, most of them to support to the validity of it, offending the “fundamentalists” on one side and the “random walk supporters” to the other side.

Technical Analysis is a quantitative analysis to give simple investment decisions. The technique analyst first has to calculate some indicators or draw and interpret graphical charts to make decision. All these processes are required on the assumption that the past prices or returns have some predictive power on the future. The analyst's purpose is to recognize various patterns or signals to decide the position to trade action. The inputs of technical analysis are the stock price and volume. For the stock price we can use not only the closing price. Also, we can use opening, closing, maximum, minimum prices of a stock. From these inputs, we can easily calculate different types of indicators by using simple mathematics. Then; by the help of these indicators, technique analyst makes his/her decision to buy, to sell or do nothing. For the most commonly used indicators are moving average, relative strength index, Bollinger band, exponential moving average, moving average convergence/divergence, momentum, commodity channel index. Due to the power of their explaining we choose to use moving average, bollinger band, relative strength index, relative volume index to use in our model.

3.1.1. Moving Average

The moving average is one of the most useful, objective and oldest indicator that show the average value of a security's price over a period of time. This means that N day moving average is the average of the last N prices. For example; to find the 20 day moving average; add up the closing prices from the past 20 days and divide them by 20. The most commonly used moving averages are the 20, 30, 50, 100, and 200 day averages. (**20 day**: provides a very volatile, choppy line. It isn't the most accurate, but is probably the most useful for short term traders, **30 day**: similar to 20 day but provides a bit more certainty for the trend, **50 day**: moving averages provide a much less volatile, smooth line, **100 day**: similar to the 50 day, it is less volatile, and one of the most widely used for long term trends, **200 day**: even less volatile, more of a rolling chart or smooth line. It doesn't react to quick movements in the stock price therefore it is rarely used.) Each moving average provides a different interpretation on what the stock price will do. Moving averages with different time spans each tell a different story. The shorter the time span, the more sensitive the moving average will be to price changes. The longer the time span, the less sensitive or the more smoothed the moving average will be. Moving averages are used to emphasize the direction of a trend and smooth out price and volume fluctuations or "noise" that can confuse interpretation.

If the price moves above the moving average then buy the stock because when a stock price exceeds its moving average it is believed as a start of a bullish trend (the prices will continue to increase) as a result a BUY signal is given; if the price falls below the moving average, then sell the stock because prices falling below the moving average is considered as a start of bearish trend (the prices will continue to decrease). So, we can easily construct the Buy/Sell decision rule.

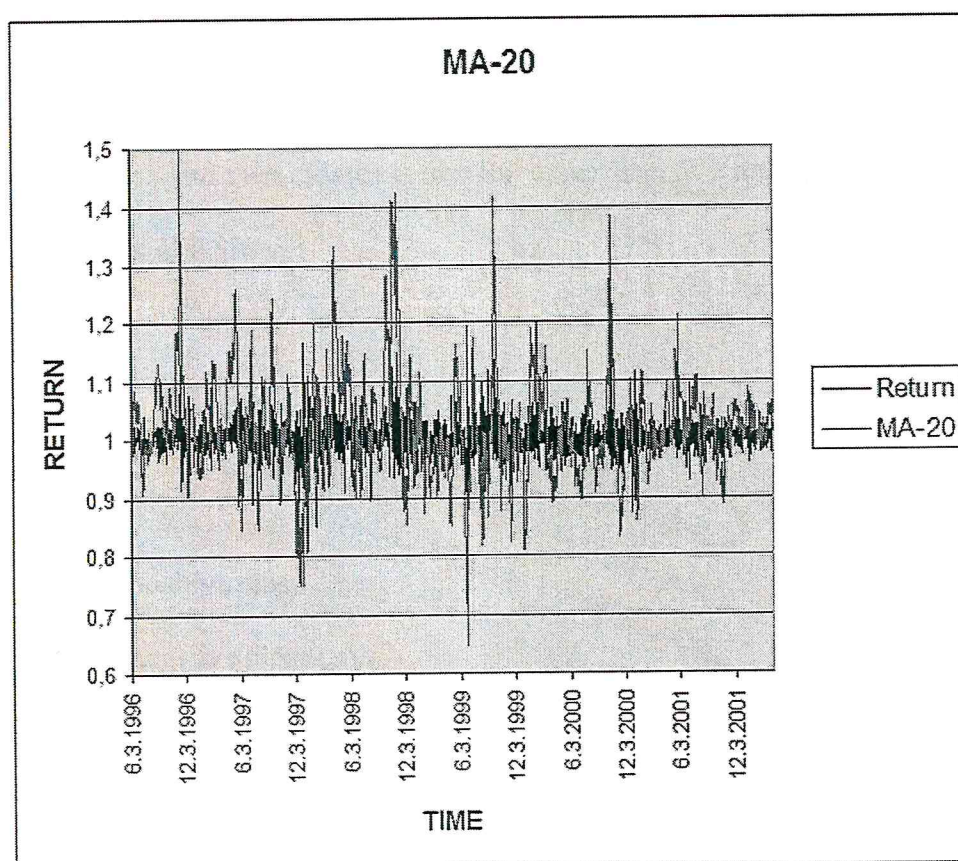
There are different modified versions of the moving average. The most commonly used are Exponential Moving Average and Moving Average Convergence Divergence.

i.Exponential Moving Average (EMA) – It is calculated by applying a percentage of today's closing price to yesterday's moving average value. Use an exponential moving average to place more weight on recent prices.

ii.Moving Average Convergence Divergence (MACD) - quite common, the "MACD" is a trend following momentum indicator that shows the relationship between two moving averages of prices. To Calculate the MACD subtract the 26-day EMA from a 12-day EMA. A 9-day dotted EMA of the MACD called the signal line is then plotted on top of the MACD.

Other less common moving averages include triangular, variable, and weighted moving average. In our model, we calculate the moving average with period 20. We choose period 20 because when period gets smaller, it will be more sensitive and makes more decision to sell and buy. The below table shows the relation between MA-20 and return to make us to decide to buy or sell. If blue line (stock price) exceeds the pink line (moving average), then buy the stock. Otherwise, sell the stock.

Figure 3.1 Relation between Moving Average & Return



3.1.2. Relative Strength Index (RSI)

The Relative Strength Index is one of the most widely used technical indicators among traders. This indicator was first introduced by Welles Wilder in an article in "Futures Magazine" in 1978. The RSI is a so-called *oscillator* because it is an index whose value tends to swing or bounce around between an upper limit value and a lower limit value. In addition the name "Relative Strength Index" is somewhat misleading. In particular, the RSI does not attempt to compare the relative strength of two different securities, but rather the strength of a single security relative to its past performance. The RSI is used primarily to help identify *overbought* or *oversold* conditions in a particular stock. It does this by confirming changes in momentum which, in turn, may signal an imminent change in price direction or trend for a particular stock.

It consists of the use of the separate averages of the asset values that close up and close down. It uses two separate constant trigger line to generate the trading signals. A buy sell is generated when RSI value fall, then breaks above the lower line and a sell signal is generated when RSI value rise and then crosses below the upper line. We use the following formula to calculate the values of RSI:

$$RSI=100-(100/(1+RS)) \text{ where}$$

RS: U/D

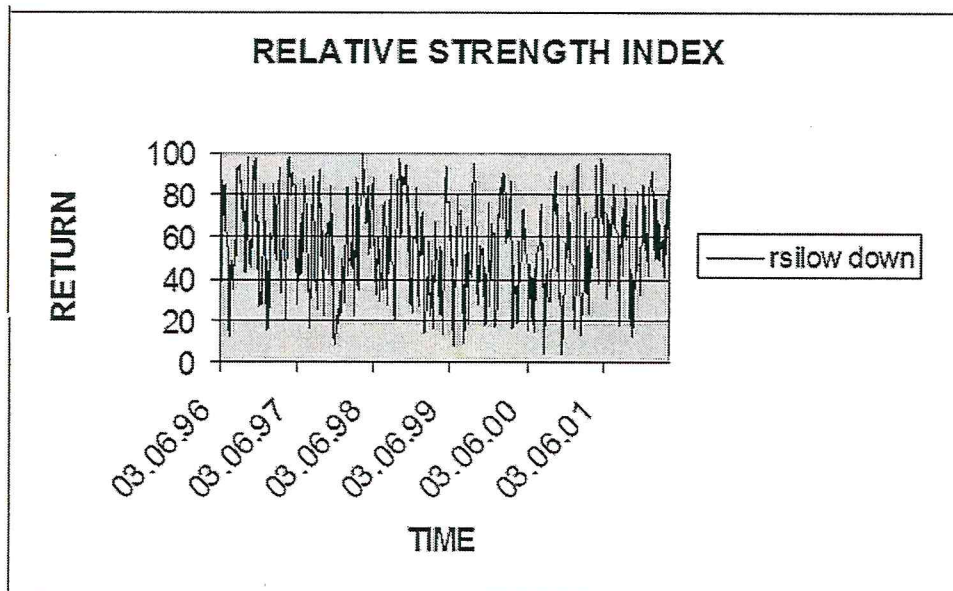
U: Average of N positive close days

D: Average of N negative close days.

RS is called the relative strength. When RS is equal to 1 then RSI becomes 50, just the center value of the RSI index. The RSI indicator ranges in value from 0 to 100, with numbers above 70 indicating overbought conditions and under 30 indicating oversold. If the RSI rises above 30, it is considered bullish, while if the RSI falls below 70, it is considered bearish. If the value of RSI is equal 80 then the value of RS is equal to 4. This means that the amount of average values of positive closing days is four times greater than average values of negative closing days and it is an over bought situation. If the value of RSI is equal 20 then the value of RS is equal to 1/4. This means that the amount of average values of positive closing days is one over four times greater than average values of negative closing days and it is an oversold situation. In our model, we choose RSI because it is one of the best anti-trend oscillator. RSI best examines the extremely expensive and extremely cheap places. When calculating the value of RSI, to find the increasing and decreasing momentum, you calculate RSI value with close prices. Instead of using close prices, if you use the lowest and highest prices you can find a much more sensitive oscillator.

In our model, we choose to calculate the value of RSI by using the lowest and highest prices to make our model much more sensitive to the examine the most expensive and cheap values.

Figure 3.2 Relative Strength Index (RSI)

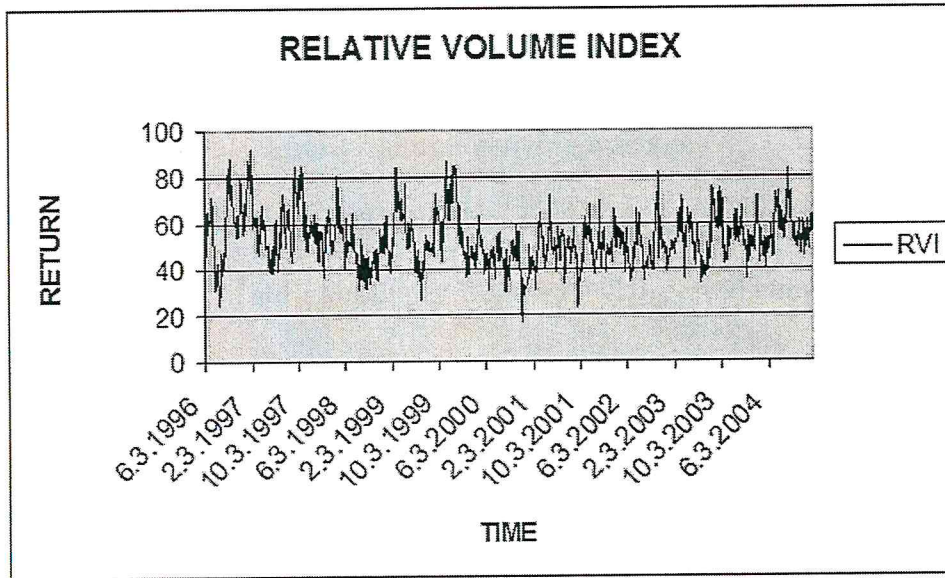


3.1.3. Relative Volatility Index (RVI)

The Relative Volatility Index was developed by Donald Dorsey. The RVI is used to measure the direction of volatility. The calculation is identical to the Relative Strength Index except that the RVI measures the standard deviation of daily price changes rather than absolute price changes. In general, traders explain the value of RVI as below:

- If $RVI > 50$, then it acts only buy signals.
- If $RVI < 50$, then it acts only sell signals.
- If $RVI > 60$ and a buy signal is ignored then take long position
- If $RVI < 40$ and a sell signal is ignored then take short position

Figure 3.3 RELATIVE VOLUME INDEX (RVI)



3.1.4. *Bollinger Bands*

Bollinger Bands are a technical trading tool created by John Bollinger in the early 1980s. They arose from the need for adaptive trading bands and the observation that volatility was dynamic, not static as was widely believed at the time.

The purpose of Bollinger Bands is to provide a relative definition of high and low. By definition prices are high at the upper band and low at the lower band. This definition can aid in rigorous pattern recognition and is useful in comparing price action to the action of indicators to arrive at systematic trading decisions.

Bollinger Bands consist of a set of three curves drawn in relation to securities prices. The middle band is a measure of the intermediate-term trend, usually a simple moving average, that serves as the base for the upper and lower bands. The interval between the upper and lower bands and the middle band is determined by volatility, typically the standard deviation of the same data that were used for the average.

The default parameters, 20 periods and two standard deviations, may be adjusted to suit your purposes:

Middle Bollinger Band = 100-period simple moving average

Upper Bollinger Band = Middle Bollinger Band + 2 * (20-period standard deviation)

Lower Bollinger Band = Middle Bollinger Band - 2 * (20-period standard deviation)

Two important tools are derived from the Bollinger Bands: BandWidth, a relative measure of the width of the bands, and %b, a measure of where the last price is in relation to the bands.

BandWidth = (Upper Bollinger Band - Lower Bollinger Band) / Middle Bollinger Band

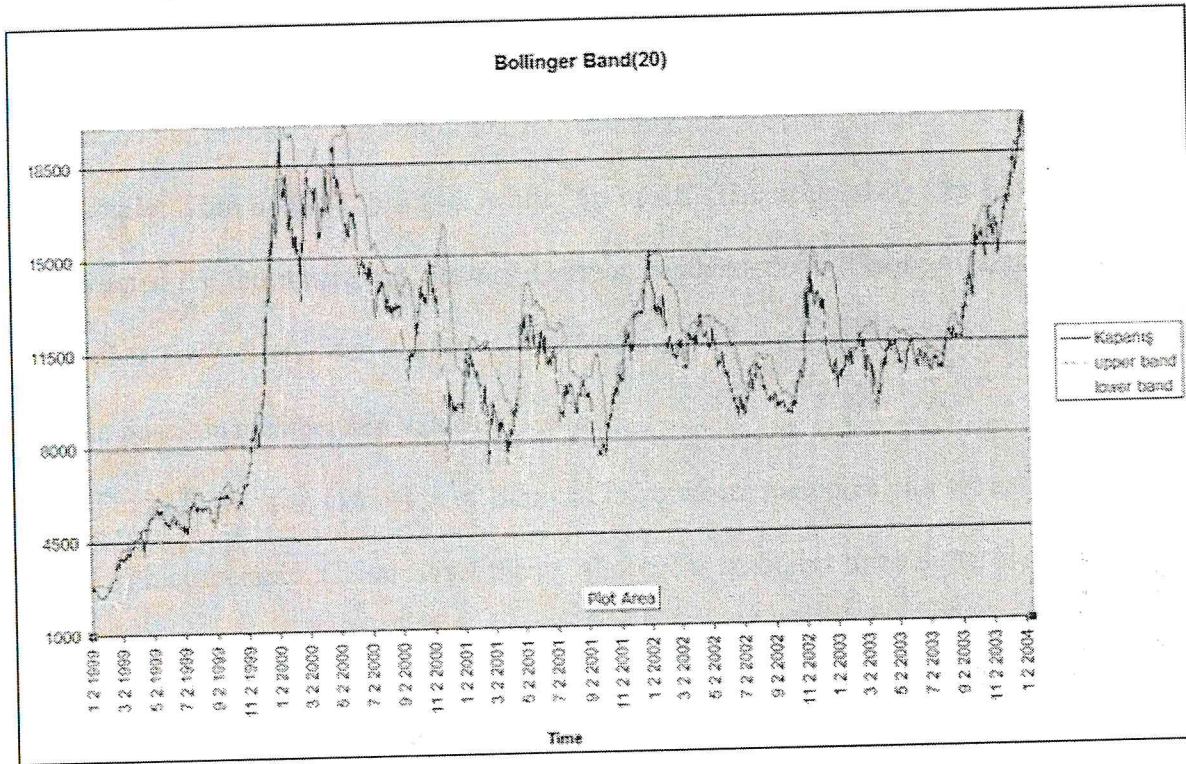
%b = (Last - Lower Bollinger Band) / (Upper Bollinger Band - Lower Bollinger Band)

Band Width is an another indicator which can be derived from the bollinger band. It is the width of the bands expressed as a percent of moving average. When the bands narrow effectively, a sharp expansion in volatility usually occurs in the very near future. The bollinger band width increase during a period of rising price volatility and it decrease during a period of low market volatility.

The indicator % b tells us where we are within the bands. Unlike stochastic, which are bounded by 0 and 100, % b can take negative values and values greater than 100 when prices are outside of the bands. When the bollinger band width is equal to 0, we are on the lower band; when it is equal to 100, we are on the upper band and if it takes values greater than 100, we are above the upper band. Shortly indicator % b lets us to compare the price actions to indicator actions.

In our model we decided to use the bollinger band width for period 20, because also we use the same period for moving average.

Figure 3.4 The Relations Between Upper Band, Lower Band and Return



CHAPTER 4. FUZZY LOGIC MODELLING OF THE ISE

In the computational world, there are two broad areas of logic: crisp logic and fuzzy logic. Crisp logic arises out of the fundamental concepts of such people Aristotle and Pythagoras who based their work on the idea that everything in the universe can be described by numerical formulate and relationships. Crisp logic is best known as Boolean logic. In Boolean Logic, problems are simplified by reducing the possible states a variable may have. For example: True or false; black or white; on or off. The original 0, 1 or binary set theory was invented by the nineteenth century German mathematician George Cantor. The Polish philosopher Jan Lukasiewicz developed the first logic of vagueness in 1920 when he created sets with possible membership values 0, 1 and $\frac{1}{2}$.

Fuzzy logic is an extension of Boolean logic where members of a set can have varying degrees of membership. Classical set theory allows for an object to be either a member of the set, or excluded from the set. However, fuzzy logic is a multi-valued type of logic that allows intermediate values to be defined. Notions like rather warm or pretty cold can be formulated mathematically using fuzzy logic and processed by computers. It provides flexibility for representing the real world. Also, it allows logic style statements to be made about the real world using this flexible form of representation and provide methods for deriving inferences from these statements.

The mathematical modeling of fuzzy concepts was presented by Zadeh in 1965 (Fuzzy Sets, Information and Control). Zadeh's contention is that meaning unnatural language a matter of degree. In the literature, there are two kinds of justification for fuzzy system theory:

- The real world is too complicated for precise descriptions to be obtained; therefore approximation (or fuzziness) must be introduced in order to obtain a reasonable, yet trackable, model.

- As people move into the information area, human knowledge becomes increasingly important. We need a theory to formulate human knowledge in a systematic manner and put it into engineering systems.

The need for fuzzy systems is to simplify the environment and develop the most likely model. If the environment is not simplified as desired, the natural environment and fuzzy environment can not be different and fuzzy system may become useless. So the usage of fuzzy architecture depends on the capability of the simplification of the environment by fuzzy experts.

Zadeh was a well respected scholar in control theory before he was working on fuzzy theory. He developed the concept of “state”, which forms the basis for modern control theory. In the early 60’s, he thought that classical control theory had put too much emphasis on precision and therefore could not handle the complex systems. As early as 1962, he wrote to handle biological systems “*we need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not describable in terms of probability distributions*” (Zadeh, 1962). Later he formalized the ideas into his paper Fuzzy Sets. Most of the fundamentals concepts in fuzzy theory were proposed by Zadeh in the late 60’s and early in 70’s. After the introduction of fuzzy sets in 1965, he proposed the fuzzy algorithms in 1968, fuzzy decision making in 1970 (Bellman & Zadeh, [1970]) and fuzzy ordering in 1971 (Zadeh, [1971b]). In 1973, he published another seminal paper, “*Outline of a new approach to the analysis of complex systems and decision process*” (Zadeh, [1973]), which establishes the foundation for fuzzy control. In this paper, he introduced concept of linguistic variables and proposed to use if-then rules to formulate human knowledge.

In 1975, Mamdani and Assian established the basic framework of fuzzy controller and applied the fuzzy controller to control a steam engine.

They found that the fuzzy controller was very easy to construct and worked well. Later in 1978, Holmblad and Ostergaard developed the first fuzzy controller for a full-scale industrial process. In the late 70's and 80's, many researchers in fuzzy theory had to change their field because they could not find support to continue their work.

Japanese engineers, with sensitivity to new technology, quickly found that fuzzy controllers were very easy to design and worked very well for many problems. In 1983, Sugeno and Nishida began the pioneer work on a fuzzy robot, self parking car that was controlled by calling out commands.

Because fuzzy logic provides the tools to classify information into broad, coarse categorizations or groupings, it has infinite possibilities for application which have proven to be much cheaper, simpler and more effective than other systems in handling complex information. Fuzzy logic has extremely broad implications for many fields, not just for electrical engineering and computer technology. Numerous consumer goods especially household products and electronic equipment-microwaves, cameras and camcorders already incorporate fuzzy logic into their design. Also in social sciences, economy, finance, physics, psychology, religion, ethics, law, medicine, geography and anthropology that deal with complexity with the human behavior are just beginning to explore the infinite possibilities of fuzzy logic.

Fuzzy systems are used mostly for estimating, decision making and mechanical control systems such as air conditioning, automobile controls and subway system in the city of Sendai, Japan has been using a fuzzy system to keep the trains rolling, braking and accelerating without losing a second or jarring a passenger. Sony's fuzzy TV set automatically adjusts the qualities of the screen image and Nissan uses fuzzy logic in its transmission and anti-lock braking systems.

Maybe, we can think fuzzy logic as a part of a logic. However, some logicians can not believe the use of fuzzy logic. For example; Haack, a formal logician, has some criticisms about fuzzy logic. She states that there are only two areas where fuzzy logic is "needed". (But, in each case, Haack can show that ultimately classical logic can substitute for fuzzy logic.) The following are Haack's two cases that may require fuzzy logic:

- **Nature of Truth and Falsity:** Haack argues that True and False are discrete terms. In classical logic, any fuzziness that arises from a statement is due to an imprecise definition of terms. But, Haack says that if it can be shown that fuzzy values are indeed fuzzy (meaning not discrete), then a need for fuzzy logic would be demonstrated.
- **Utility of Fuzzy Logic:** Haack says if it can be shown that generalizing classic logic to include fuzzy logic would aid calculations, then fuzzy logic would be needed. But, Haack argues that data manipulation in a fuzzy system actually becomes more complex. So, fuzzy logic is not necessary.

Haack believes fuzzy logic is not necessary because the calculations are more involved and partial membership values can be eliminated by defining terms more precisely. Fox has responded to Haack's objections. He believes that the following three areas can benefit from fuzzy logic:

- **"Requisite" Apparatus-** Use fuzzy logic to describe real-world relationships that are inherently fuzzy.
- **"Prescriptive" Apparatus-** Use fuzzy logic because some data is inherently fuzzy and needs fuzzy calculus.
- **"Descriptive" Apparatus-** Use fuzzy logic because some inferencing systems are inherently fuzzy.

Besides Fox argues that fuzzy and classical logic should not be seen as competitive but as complementary. Fox also states that fuzzy logic has found its way into the world of practical applications and has proved successful there. He says this is reason enough to continue development in the field of fuzzy logic.

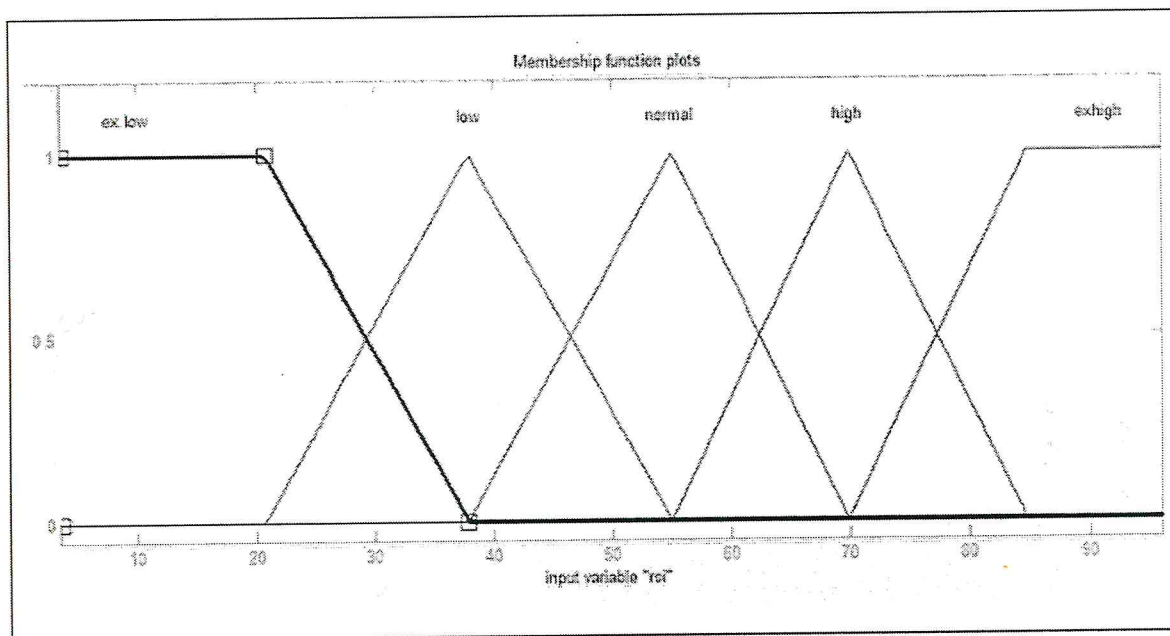
We need a fuzzy system and input and output data to apply the process. Fuzzy rule based systems apply these methods to solve many real world problems such as....

4.1. Fuzzy Sets

A fuzzy set is represented by a membership function defined on the universe of discourse. The universe of discourse is the space where the fuzzy variables are defined. The membership function gives the grade, or degree, of membership within the set, of any element of the universe of discourse. The membership function maps the elements of the universe onto numerical values in the interval $[0, 1]$. A membership function value of zero implies that the corresponding element is definitely not an element of the fuzzy set, while a value of unity means that the element fully belongs to the set. A grade of membership in between corresponds to the fuzzy membership to set. In other words, the membership function of a classical set can only take two values, zero and one, whereas the membership function of a fuzzy set is a continuous function with range $[0,1]$. So, there is nothing “fuzzy” about a fuzzy set. It is simply a set with a continuous membership function. As a result of membership functions’ the data can be converted easily. For simplicity, we use piecewise linear membership functions such as triangles and trapezoids. Also membership functions can be continuous curves. Such as gaussian membership function and bell membership function. They have the advantage of being smooth and nonzero at all points. However, they are unable to specify asymmetric membership function, which are important in many applications.

In our thesis, we have four fuzzy variables which are RSI, RVI, MA and B.BAND that we explained detailed before. They are indicators to predict the return. For example, RSI can take values in the range between 0 and 100. In some aspects the value of RSI is greater than 80 means that the prices are increasing and it is time to sell; in other words it may be a very very good increase and it is the best time to sell.

Figure 4.1 MEMBERSHIP FUNCTION OF RSI



4.1.1. Linguistic Variables and Fuzzy If-Then Rules

If a variable can take words in natural languages as its values, it is called a linguistic variable, where the words are characterized by fuzzy sets defined in universe of discourse in which the variable is defined. In other words, a linguistic variable is characterized by (X, T, U, M) where:

- X is the name of linguistic variable,
- T is the set of linguistic values that X can take,

- U is the actual physical domain in which the linguistic variable X takes its crisp values
- M is a semantic rule that relates each linguistic value in T with a fuzzy set in U.

The concept of linguistic variable is so important because they are the most fundamental elements in human knowledge representation. When we use sensors to measure a variable, they give us numbers; when we ask human experts to evaluate a variable, they give us words. Now, we are able to formulate vague descriptions in natural languages in precise mathematical terms.

4.2. Fuzzification & Rule Generation

Fuzzification is the process of decomposing a system input and/or output into one or more fuzzy sets (It is the first step for the fuzzy inference system). Many types of curves can be used, but triangular or trapezoidal shaped membership functions are the most common ones, because they are easier to represent in embedded controllers. Each fuzzy set spans a region of input (or output) value graphed with the membership. Any particular input is interpreted from this fuzzy set and a degree of membership is interpreted. The membership functions should overlap to allow smooth mapping of the system. The process of fuzzification allows the system inputs and outputs to be expressed in *linguistic terms* so that rules can be applied in a simple manner to express a complex system. There are five fuzzy sets for our work. They are; extremely low, low, normal, high, and extremely high. Fuzzification of the five crisp variables, causes spreading of variables with a distribution profile. For example, RSI takes value 71.069 at 01.07.1996, the figure of the membership function help us to identify. In the membership function figure, when x axis is equal to 71.069 and a vertical line plotted crossing 71.069 at the x-axis, the line intersects with the membership function with the “extremely high” and membership function “high” at different points.

All the vertical lines intersects at one or two points. If it intersects at only one point, this means it will be in the range only extremely low, low, normal, high, extremely high and the y- value is equal to one. In other words, it is belongs the %100 to that fuzzy set. If it intersects at two points; a and b. The sum of the values of y-axis (a+b) is equal to 1. This means that, it belongs to one of the fuzzy sets with % a percentage and with % b percentage to the other set.

The purpose of the fuzzification process is to allow a fuzzy condition in a rule to be interpreted. For example the condition 'RSI = 65' in a rule can be true for all values of 'RSI', however, the confidence factor or membership value of this condition can be derived from membership function graph. An indicator which has a value of 65 with a confidence factor of 0.5 (membership value of the club 'high'). It is the gradual change of the membership value of the condition 'high' with height that gives fuzzy logic its strength.

In fuzzy problems, the rules are produced by experience. Concerning problems that deal with fuzzy control or fuzzy engines, all possible fuzzy input output relationships should be in fuzzy terms. For my project, I decided to cover all the bases and write rules for every single possible set of inputs. For any output, the number of rules needed can be found by multiplying together the number of membership sets for all inputs.

In a fuzzy rule-based system, the rules can be represented such as:

If x is in A and y is in B and ... then z is in Z

where x , y and z represent variables; A , B and Z are linguistic variables such as extremely low, low, normal, high and extremely high (A, B are fuzzified inputs and Z is the action for each variable).

For two variables with five membership functions for each, the total number of rules should be 25 (5×5).

For example, if we have two linguistic variables (inputs) such as RSI and RVI which each have five membership functions such as ex.low, low, normal, high, ex.high and think as they were numbers such as 1, 2, 3, 4, 5; respectively. Then apply the rule as $\text{round}((2\text{RSI}+\text{RVI})/3)$ as the membership functions of the output below.

Table 4.1 RULES BETWEEN RSI AND RVI

		RSI				
		Ex. Low	Low	Normal	High	Ex. High
RVI	Ex. Low	Ex. Low	Low	Low	Normal	High
	Low	Ex. Low	Low	Normal	Normal	High
	Normal	Low	Low	Normal	High	High
	High	Low	Normal	Normal	High	Ex. High
	Ex. High	Low	Normal	High	High	Ex. High

Table 4.2 RULES BETWEEN MA AND BBAND

		B. BAND				
		Ex. Low	Low	Normal	High	Ex. High
MA	Ex. Low	Ex. Low	Ex. Low	Low	Low	Low
	Low	Low	Low	Low	Normal	Normal
	Normal	Low	Normal	Normal	Normal	High
	High	Normal	Normal	High	High	High
	Ex. High	High	High	High	Ex. High	Ex. High

If we have more than two inputs and more than five membership functions, it is difficult to develop the fuzzy model.

For example; if we have three variables and six membership functions, number of rules increase to 216 rules ($6 \times 6 \times 6$). However, we use fuzzy model to simplify the environment and develop the most useful model. If the environment can not be simplified as we want, it is meaningless to use fuzzy system. In the case of highly complex systems, fuzzy logic is the only way to solve the system. Fuzzy logic can not be used for unsolvable problems. This seems fairly reasonable, but its perception of being a guessing game may lead people to believe that it can be used for anything. An obvious drawback a fuzzy logic is that it is not always accurate. The results perceived as agues, so it may not be as widely trusted as an answer from classical logic. Generally fuzzy logic, confused with probability theory. In some way they are similar concepts but they do not say the same things. Probability is likelihood that something is true, fuzzy logic is the degree to which something is true.

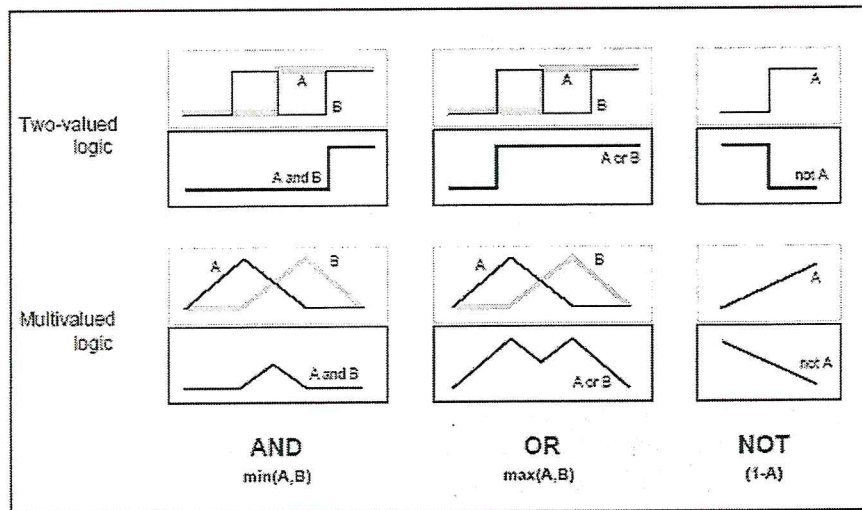
4.3. Operations on Fuzzy Sets

Applying the fuzzy operators can be thought as a second step of the fuzzy inference system.

Let A and B be two fuzzy sets. We say that A and B equal if and only if $\mu_A(x) = \mu_B(x)$ for all $x \in U$. We say that $A \subset B$ if and only if $\mu_A(x) \leq \mu_B(x)$ for all $x \in U$.

The complement \bar{A} of A is a fuzzy set in U whose membership function is defined as $\mu_{\bar{A}} = 1 - \mu_A(x)$. The union of A and B is a fuzzy set $A \cup B$ in U whose membership function is defined as $\mu_{A \cup B}(x) = \max[\mu_A(x), \mu_B(x)]$. The intersection of A and B is a fuzzy set $A \cap B$ in U with membership function $\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)]$.

Table 4.3 OPERATIONS ON FUZZY SETS (AND, OR, NOT)



The fuzzy propositions are interpreted as fuzzy relations. The key question is how to interpret the “if then” rule operation. Interpreting an if-then rule involves distinct parts: first evaluating the antecedent (which involves fuzzifying the input and applying any necessary fuzzy operators) and second applying that result to the consequent (known as implication). It is the third part of the fuzzy inference system.

In classical propositional calculus, the expression if p then q is written as $p \rightarrow q$ with implication \rightarrow regarded as a connective defined as below.

Table 4.4 TRUTH TABLE FOR p implies q

P	q	$p \rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	0

Note that, $p \rightarrow q$ is also equivalent to $\neg p \vee q$ or $(p \wedge q) \vee \neg p$ (*).

In other words, they share the same truth table. Now, we can interpret the fuzzy “if then rules” by replacing \neg , \vee and \wedge operators in (*) with fuzzy complement, fuzzy union, fuzzy intersection, respectively. Since there is a wide variety of fuzzy complement, fuzzy union, fuzzy intersection operators, a number of different interpretations of fuzzy “if then” rules were proposed in the literature. Like Zadeh, Godel, Mamdani, Lukasiewicz, Dienes-Rescher implications...

For example in Dienes-Rescher implication; they replace the logic operators \neg and \vee in $\neg p \vee q$ by the basic fuzzy complement and the basic fuzzy union, respectively. In other words, the fuzzy “if then” rule If A then B is interpreted as a fuzzy relation Q in $U \times V$ with the membership function $\mu_D(x, y) = \max[1 - \mu_A(x), \mu_B(y)]$.

In Mamdani implication, the fuzzy “if then” rule is interpreted as a fuzzy relation Q in $U \times V$ with the membership function $\mu_D(x, y) = \min[\mu_A(x), \mu_B(y)]$.

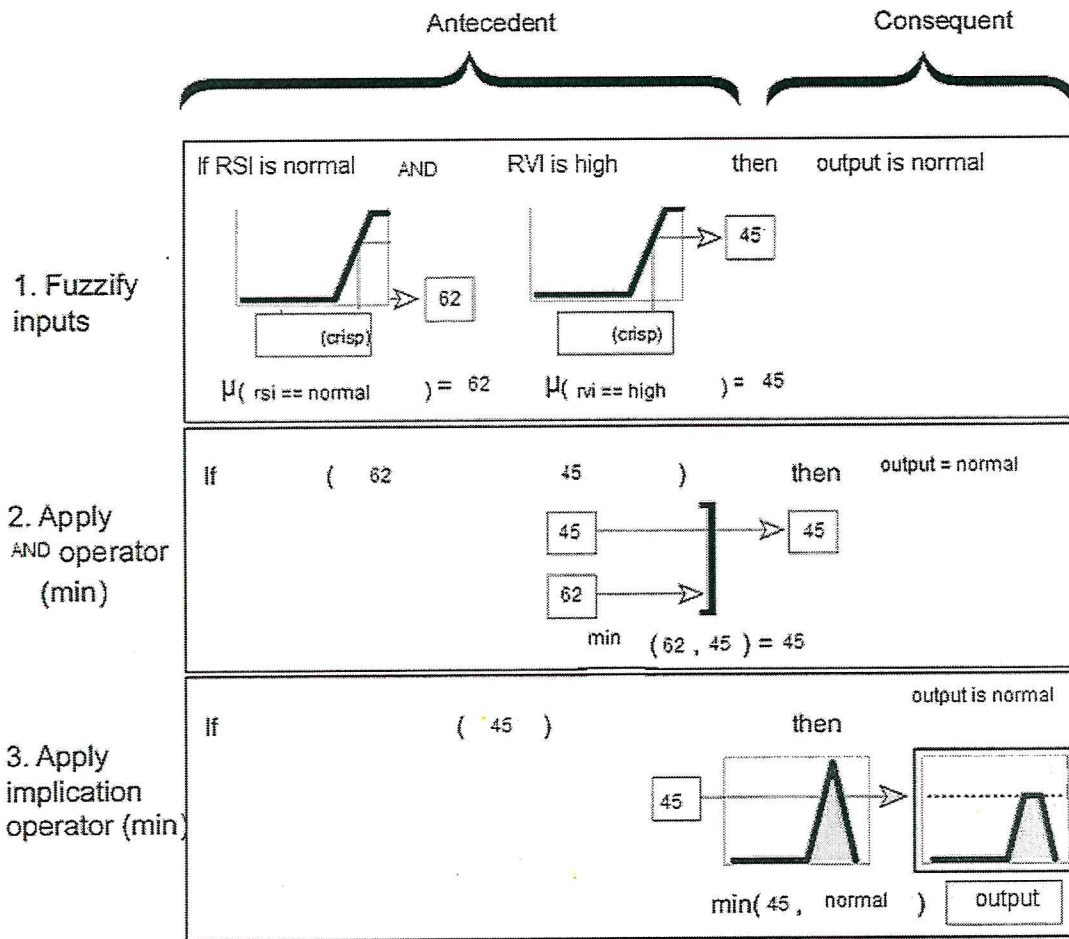
Mamdani implications are the most widely used implications in fuzzy systems and fuzzy control. They are supported by the argument that fuzzy if then rules are local. Mamdani’s method was among the first control system’s built using fuzzy set theory. It was proposed in 1975 by Elbrahim Mamdani as an attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators. Mamdani's effort was based on Lotfi Zadeh’s 1973 paper on fuzzy algorithms for complex system and decision processes.

Takagi-Sugeno-Kong Method is similar to the Mamdani method in many respects. The first two parts of the fuzzy inference process; fuzzifying the inputs and applying the fuzzy operator are exactly the same. The main difference between Mamdani and Sugeno is that Sugeno output membership functions are either linear or constant. We can think that they are the types of the fuzzy inference systems.

Only Mamdani and Takagi-Sugeno-Kong Method can be used in matlab. Takagi-Sugeno-Kong Method is more compact and computationally efficient representation than Mamdani Method, so Sugeno system lends itself to the use of adaptive techniques for constructing fuzzy models. These adaptive techniques can be used to customize the membership function so that fuzzy systems best models the data. However, we did not use the Takagi-Sugeno-Kong Method because in Takagi-Sugeno-Kong Method, the output membership function would be constant or linear. It does not let us to define the value of the output. In other words, we can not say in what percentage the output belongs to each fuzzy set and we can not identify the each membership function as we want.

Aggregation is the combination of the consequents of each rule in a Mamdani fuzzy inference system in preparation for defuzzification. Note that as long as the aggregation method is commutative (which it always should be), then the order in which the rules are executed is unimportant. There exists three aggregation methods such as *maximum* (max), *probabilistic* (algebraic sum; $\text{probor}(a, b) = a + b - ab$), and *sum* (simply the sum of each rule's output set). After the aggregation process, there is a fuzzy set for each output variable that needs defuzzification and defuzzification is the final process of the fuzzy inference systems.

Figure 4.2 SUMMARY OF FUZZY IF THEN RULES



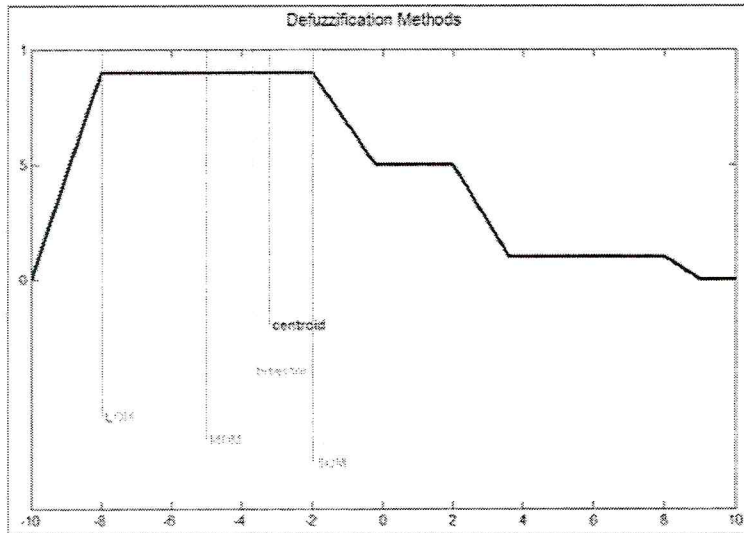
The figure above is the summary of the topics that we mentioned above. It shows that the effect of one fuzzy if then rule when calculating the output. If the rule is with and implication or something different, it is not important, we should know what this means in mathematical sense. After finding the output as a fuzzy set we should defuzzy it to understand what this really means.

4.4 Defuzzification

If the conclusion of the fuzzy rule set involves fuzzy concepts, then these concepts will have to be translated back into objective terms before they can be used in practice. The process of converting the fuzzy output to a crisp number is called defuzzification.

Before an output is defuzzified, all the fuzzy outputs of the system are aggregated with a union operator. The union is the maximum of the set of given membership functions and can be expressed as a fuzzy set. Then, by choosing a defuzzification method we need to convert the fuzzy value to an objective term.

Figure 4.3 DEFUZZIFICATION METHODS

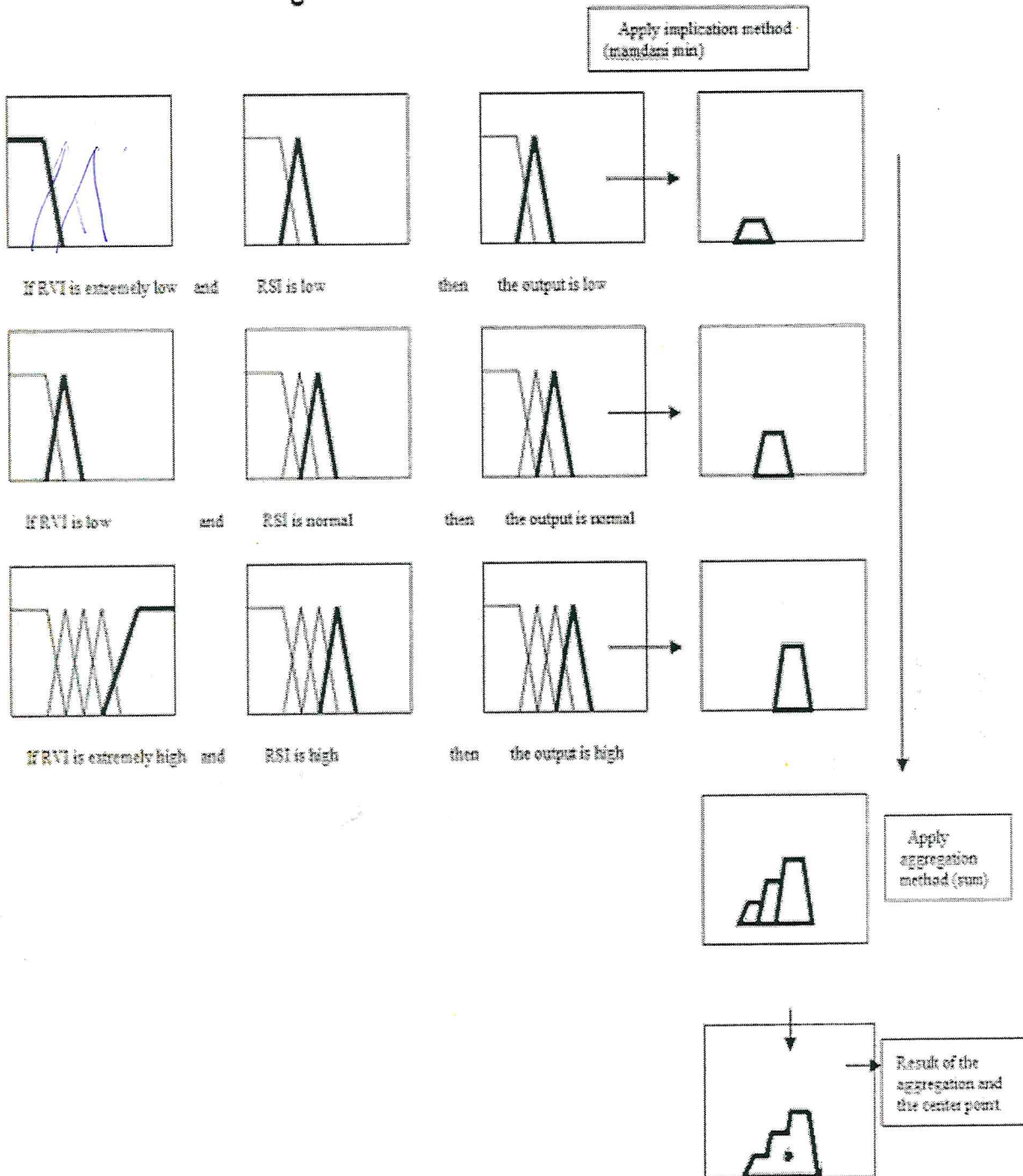


There are many defuzzification methods (at least thirty) but primarily only three of them in common use. Maximum Defuzzification Method, Centroid Defuzzification Method, Weighted Average Defuzzification Technique. In the Centroid method, the crisp value of the output variable is computed by finding the variable value of the center of gravity of the membership function for the fuzzy value. In the Maximum method, one of the variable values at which the fuzzy subset has its maximum truth value is chosen as the crisp value for the output variable (Mizumoto). Also, in matlab fuzzy designer only supports five defuzzification methods which are centroid method, bisector method, mean of maximum method, smallest of maximum method and the largest of maximum method that can be seen in the above graph. Centroid Average and Maximum Center average methods belongs to continuous ones and are frequently used in decision making and pattern recognition application for selecting the alternatives.

To defuzzify the Mamdani style inference system, one can choose centroid, bisector (is the vertical line that will divide region into two sub-regions of equal area), middle of maximum, smallest of maximum or largest of maximum methods. For the Sugeno style inference system one can choose weighted average or weighted sum because the output membership function has the form of linear or constant.

“Which defuzzification method is the right one to use?” has no simple answer but the center of area method has mostly used one. We use centroid average method in our model. This method was developed by Sugeno in 1985. It is the most commonly used and very accurate to apply. The only disadvantage of this method is that it is computationally difficult for complex membership functions.

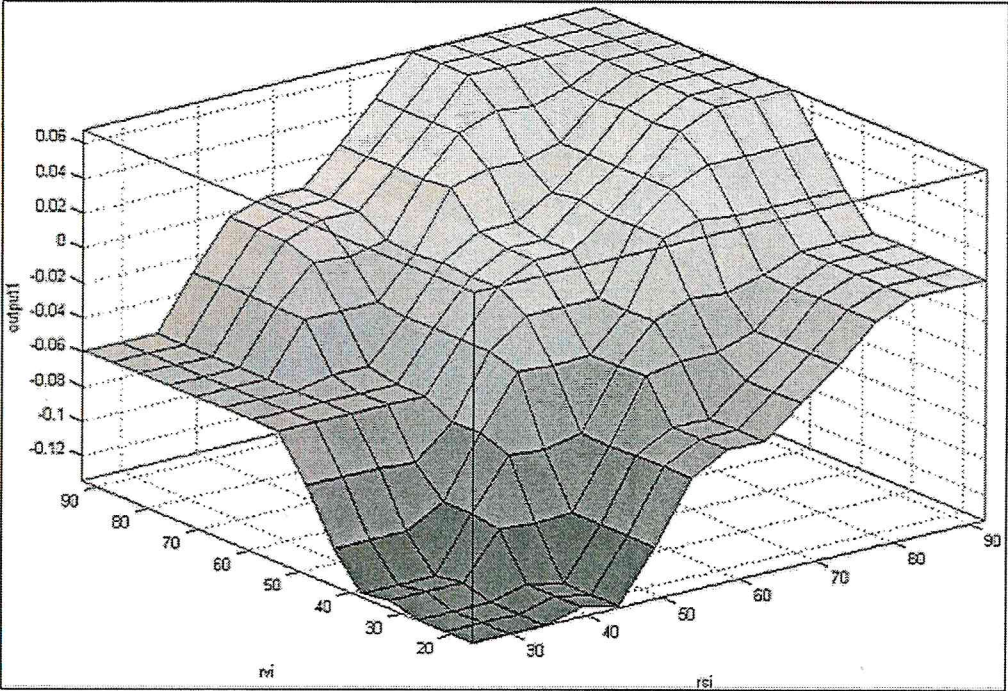
Figure 4.4 SUMMARY OF FUZZY SYSTEM



4.5. Graphs of the Relations between Inputs

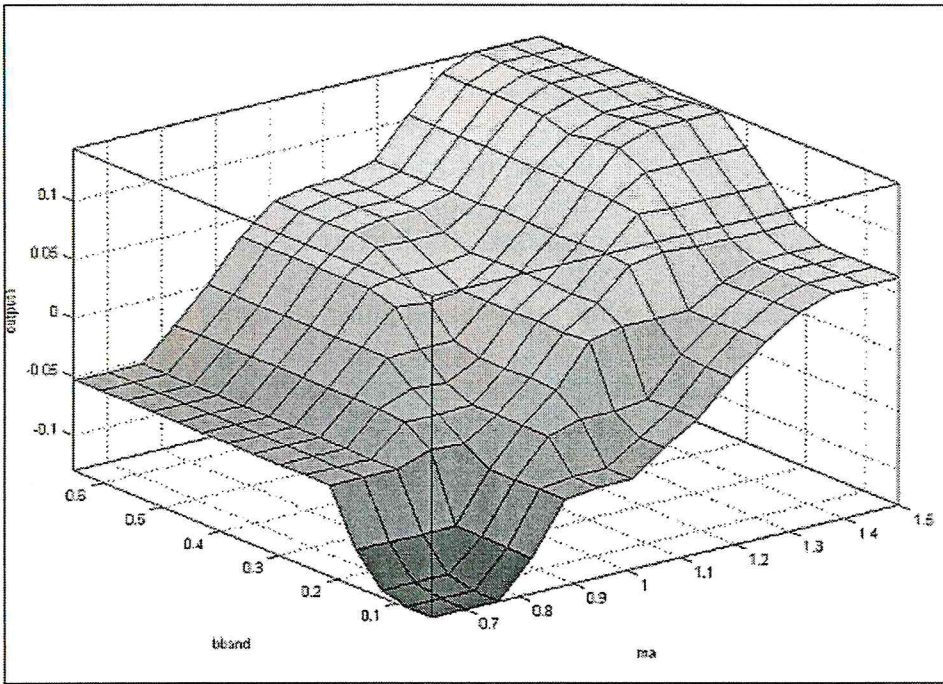
Below Surfaces are used to display the dependency of one of the outputs on any one or two of the input, that is, it generates and plots an output surface map for your system.

Figure 4.5 SURFACE THAT SHOWS RELATION BETWEEN RSI & RVI



By looking above figure, we can easily examine the relations between RSI and RVI with the output space. So, after investigate the figure, you can change up your mind and decide to use another indicator that best fits the output.

Figure 4.6 SURFACE THAT SHOWS RELATION BETWEEN MA & BBAND



Above figure shows the relation between MA and BBAND with the output space. By comparing each figures, we can see that they fits the different values of the output space so that our inputs is a good choice for the model.

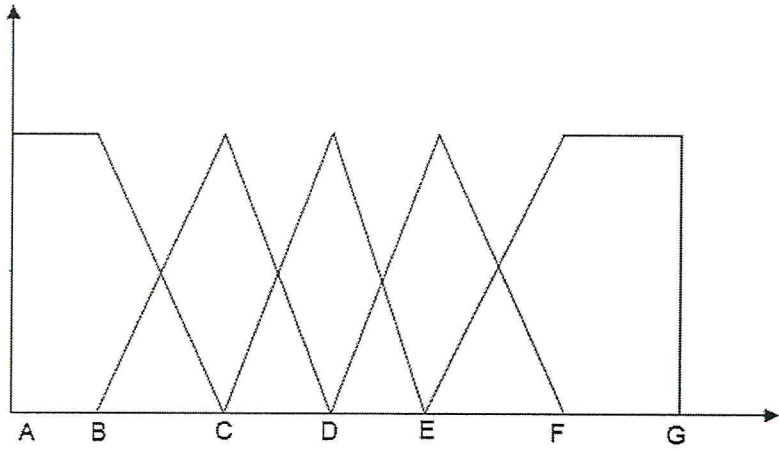
4.6. MODELLING

We have four inputs and four each input, we have five membership functions that two of them is trapezoids and three of them is triangles. We need to characterize the each membership functions. In other words, we need to find the border points of the each membership function. First, we find the maximum, minimum, standard deviation and the mean of the each input. By using these values and expert opinion, we decide the borders of the each membership functions as seen in the following table:

Table 4.5 RANGE OF THE VARIABLES

	Max	Min	Mean	St.Dev	A	B	C	D	E	F	G
RSI	99.58	3.62	55.01	21.04	3.62	20.75	37.8	55.01	69.86	84.72	99.58
RVI	92.6	16.59	52.93	10.93	16.59	28.71	40.82	52.93	65.98	79.02	92.06
MA	1.50	0.64	1.02	0.09	0.64	0.77	0.89	1.02	1.18	1.34	1.50
BBAND	0.65	0.04	0.21	0.11	0.04	0.0988	0.15	0.21	0.35	0.50	0.65
OUTPUT	0.19	-0.18	0.002	0.03	-0.18	-0.12	-0.05	0.00	0.06	0.13	0.19

TABLE 4.5.A RANGE OF VARIABLES

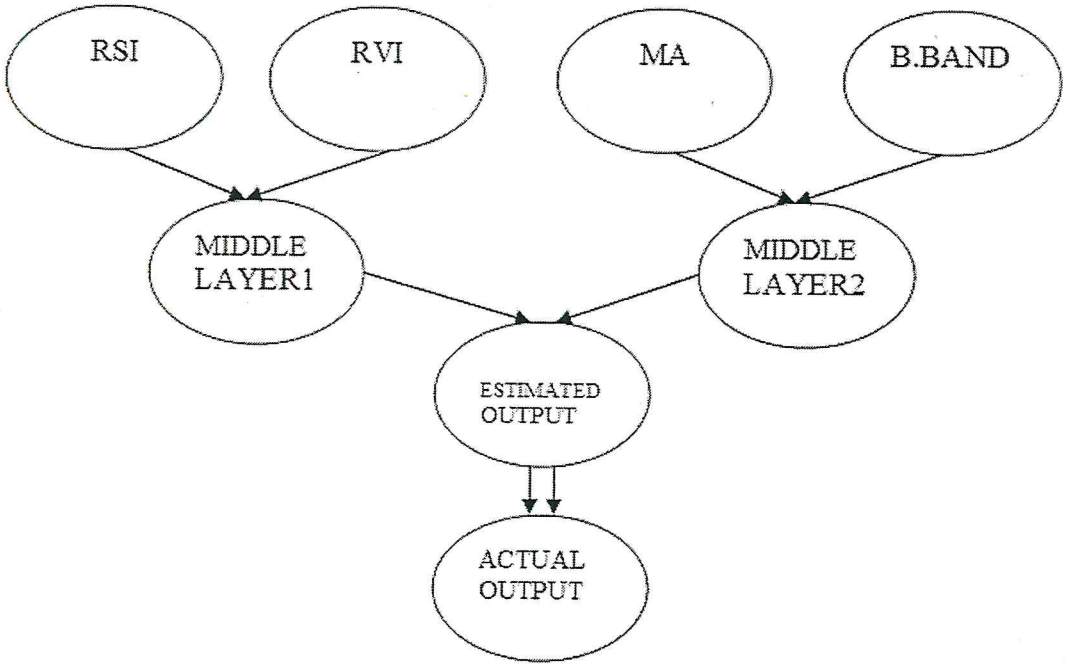


In the tables above; A values are the minimum values of the each input and the G values are the maximum values of the each input and D values are the average values of the each input.

A, B and C are the borders of the first trapezoid; B, C and D are the borders of the first triangular; C, D and E are the borders of the second triangular; D, E and F are the borders of the third triangular and E, F and G are the borders of the last triangular. C, D and E are the head values of the triangular and the others are the bottom values. Since, to define a trapezoid we need five borders, for the first trapezoid the borders are A, A, B and C and for the last trapezoid the borders are E, F, G, G.

For the next step of the modeling, we need to define the rules. We have four input variables and two middle variables which are produced from these four variables. RSI and RVI produced the first middle variable. MA and BBAND produced the second middle variable. Then, the first middle variable with the second middle variable produce the estimated output. We want to approximate the estimated output to the actual output by changing the ranges of membership functions. For each fuzzy process, we have 25 rules as we defined earlier. We have three fuzzy process, so the total number of rules is 75.

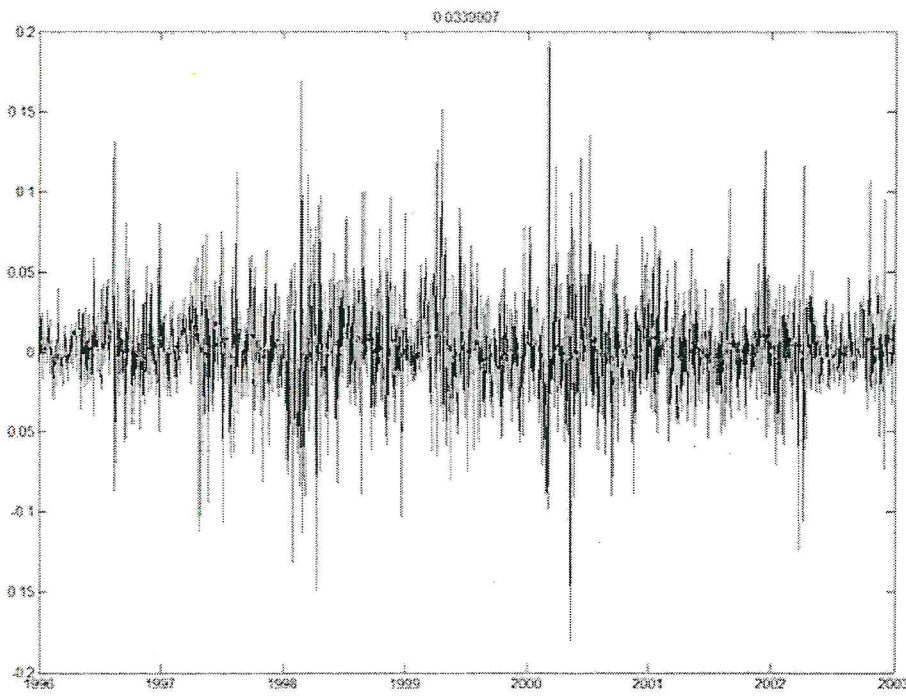
FIGURE 4.7 FUZZY LOGIC MODEL



The above figure is the final model that we use. Firstly, we thought that the four inputs (variables that we decided to use) are combined to produce the estimated output. However, if we use the model such that the number of rules will be increased to 625 that we do not want because of the simplicity. We decide to combine the indicators in a way that they can best predict the same side of the stock market.

In our model, we have a %3.39 R square value as seen in the following table between the periods 1996 to 2004. Thus, the next work is to find the optimum stock market model, which can be developed by changing the border points of the membership functions by an optimization method. We choose to use the steepest descent method to optimize the model.

Figure 4.8 R square is equal to 3.39 % before optimization

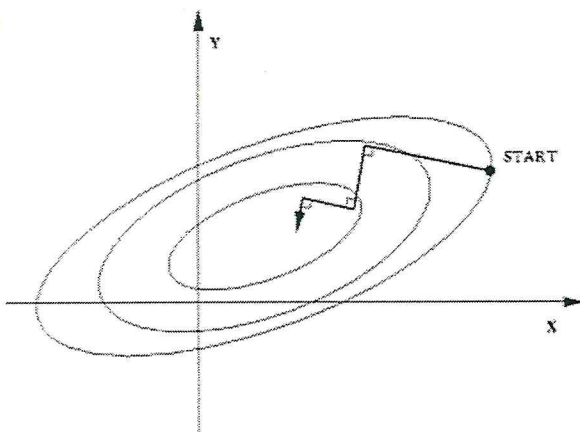


Steepest Descent Method

Steepest Descent Method is the oldest and simplest method. It is actually more theoretical, rather than practical in some ways (because it works much more slowly than the other techniques). However, it is easy to understand and easy to use because it does not require gradients, no matrix inversion or no Hessian computation. Also, it works even if the function is not twice differentiable. In fact if we have no closed form for the gradient, it is possible to use an approximate gradient. This implies that we do not even need the gradient very nice. It always progresses from step to step. For example, in Newton method, we can jump over a solution and not make progress. In addition, it avoids saddle points of the function that will be minimized.

Steepest Descent Method is generally used in optimization problems and it approaches the minimum in a zig-zag manner, where the new search direction is orthogonal to the level sets, as seen in the below graph. The search starts at any point and then slides down to the minimum until we are close enough to the solution.

Figure 4.9 CONVERGENCE OF THE STEEPEST DESCENT



In other words, the gradient of the objective function gives the direction $(-\nabla f)$ for the steepest descent. Here, the objective function is ratio of the mean square error of the difference of actual output and the estimated output.

The gradient will be:

$$\nabla f(\bar{x})' = \lim_{\Delta \rightarrow 0^+} \frac{f(\bar{x} + \Delta \bar{y}) - f(\bar{x})}{\Delta} \quad \text{where } \bar{y} \text{ is the}$$

direction.

The steepest descent algorithm moves along the direction \bar{y} with

$$\|\bar{y}\| = 1 \quad \text{and} \quad \bar{y} = \frac{-\nabla f(\bar{x})}{\|\nabla f(\bar{x})\|},$$

that minimizes the above limit. Here \bar{y} represents the changes in the borders of the membership functions. As a result the algorithm will be;

1. Given x^0 , set $\epsilon \rightarrow 0$.
2. $\bar{y}^\epsilon = -\nabla f(\bar{x}^\epsilon)$. If $\bar{y}^\epsilon = 0$, then stop.
3. By using one dimensional search, choose Δ^ϵ .
 - a. Set $\Delta^\epsilon = 1$.
 - b. Calculate $f(\bar{x}^\epsilon)$, $f(\bar{x}^\epsilon + (\Delta^\epsilon / 2)\bar{y}^\epsilon)$ and $f(\bar{x}^\epsilon + \Delta^\epsilon \bar{y}^\epsilon)$.
 - c. If $\min \{f(\bar{x}^\epsilon), f(\bar{x}^\epsilon + (\Delta^\epsilon / 2)\bar{y}^\epsilon), f(\bar{x}^\epsilon + \Delta^\epsilon \bar{y}^\epsilon)\} = f(\bar{x}^\epsilon)$,
 - i. Set counter,
 - ii. Change $\Delta^\epsilon \rightarrow \Delta^\epsilon / 2$,
 - iii. If counter is less than 100, go to Step 3.c.,
 - iv. Else go to Step 4.
 - d. Else if $\min \{f(\bar{x}^\epsilon), f(\bar{x}^\epsilon + \Delta^\epsilon / 2)\bar{y}^\epsilon, f(\bar{x}^\epsilon + \Delta^\epsilon \bar{y}^\epsilon)\} = f(\bar{x}^\epsilon + (\Delta^\epsilon / 2)\bar{y}^\epsilon)$,

$$\text{i. Calculate } \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} (\Delta^\epsilon)^2 & \Delta^\epsilon & 1 \\ (\Delta^\epsilon/2)^2 & \Delta^\epsilon/2 & 1 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} f(\bar{x}^\epsilon + \Delta^\epsilon \bar{y}^\epsilon) \\ f(\bar{x}^\epsilon + (\Delta^\epsilon/2) \bar{y}^\epsilon) \\ f(\bar{x}^\epsilon) \end{bmatrix}$$

ii. Change $\Delta^\epsilon \rightarrow -b/2a$,

d. Else if

$$\min \{f(\bar{x}^\epsilon), f(\bar{x}^\epsilon + \Delta^\epsilon/2 \bar{y}^\epsilon), f(\bar{x}^\epsilon + \Delta^\epsilon \bar{y}^\epsilon)\} = f(\bar{x}^\epsilon + \Delta^\epsilon \bar{y}^\epsilon)$$

i. Set counter.

ii. Change $\Delta^\epsilon \rightarrow 2\Delta$,

iii. If counter is less than 100, go to Step 3.e.,

iv. Else go to Step 4

4. Set $(\bar{x}^\epsilon + \Delta^\epsilon \bar{y}^\epsilon) \rightarrow \bar{x}^{\epsilon+1}, \epsilon \rightarrow \epsilon + 1$.

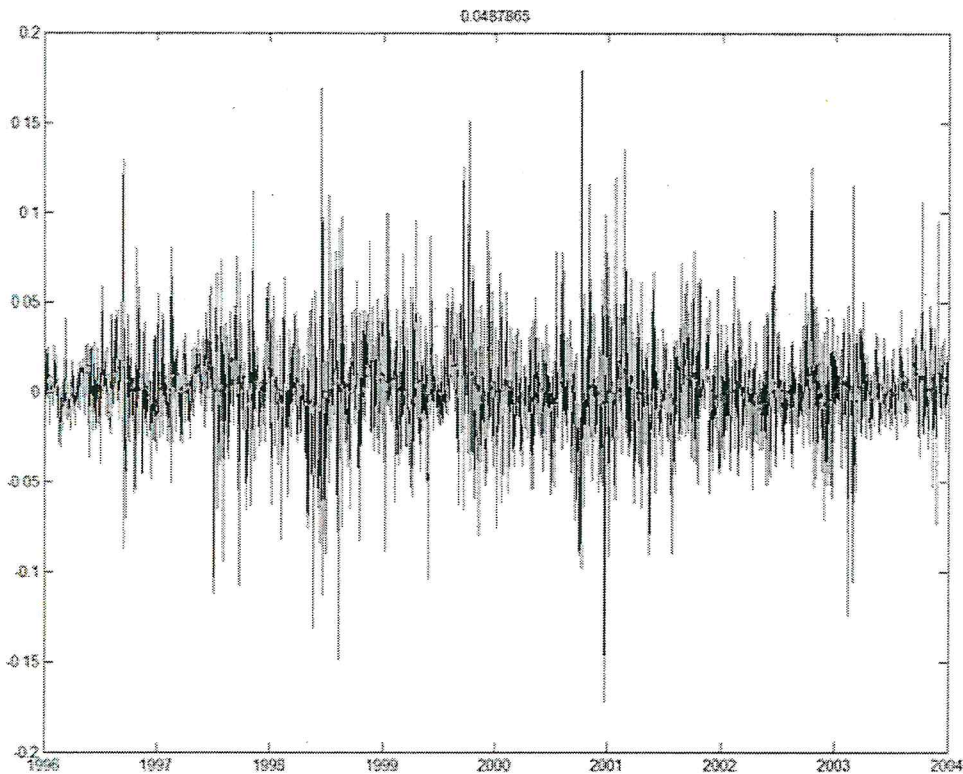
5. Go to Step 2.

1.6.2. Results of the Optimisation

We want to minimize the square of the error, which is the difference of the real market stock returns with the estimated stock market returns that we found from the fuzzy logic process. First, we did this process to the middle layer 1 and middle layer 2 because in another view they are the outputs of the fuzzy processes that we did not decide their ranges. When doing this, the membership functions of the RSI, RVI, MA and BBAND are also changed. So the second step is to do the same process to these variables. Lastly, the same process done to the middle layer 1 and middle layer 2 again.

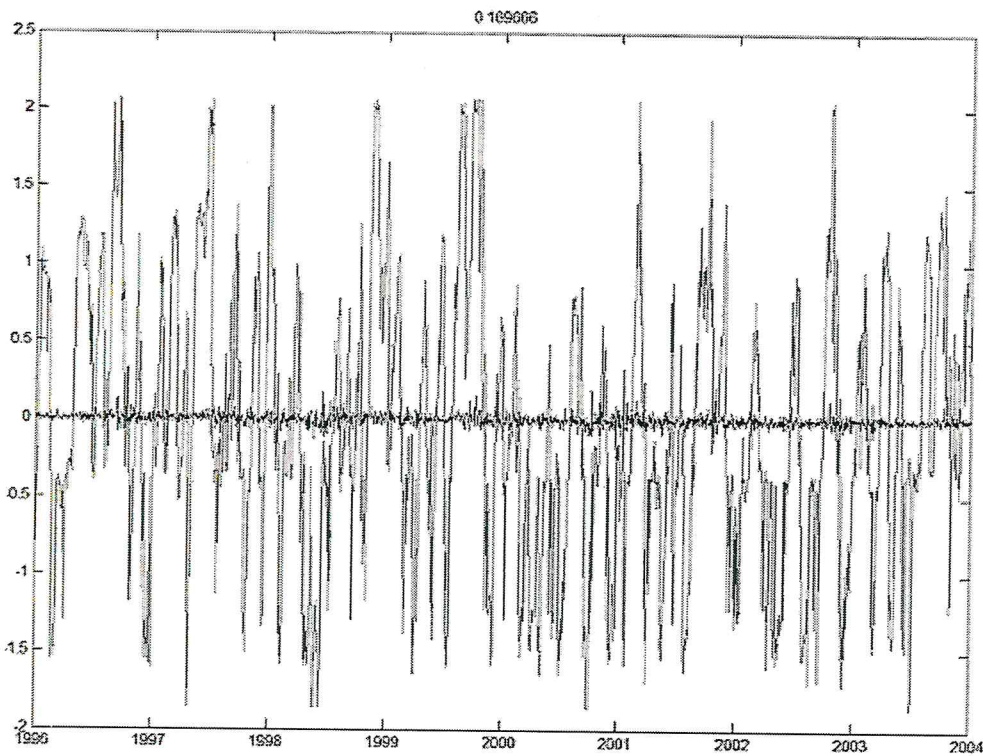
Finally, the new model is different from the model that we start. All the membership functions changed but of course we did not change the rules.

Figure 4.10 R square is equal to 4.87 % after optimization



When we optimize the model, we find a greater R square, which is the sum of the squares of the difference of the actual outputs and estimated outputs divided by the sum of the squares of the actual outputs of the stock market. We mean real returns with the actual outputs. R square is not as high as we want but we have a better correlation for this model as seen in the following figure.

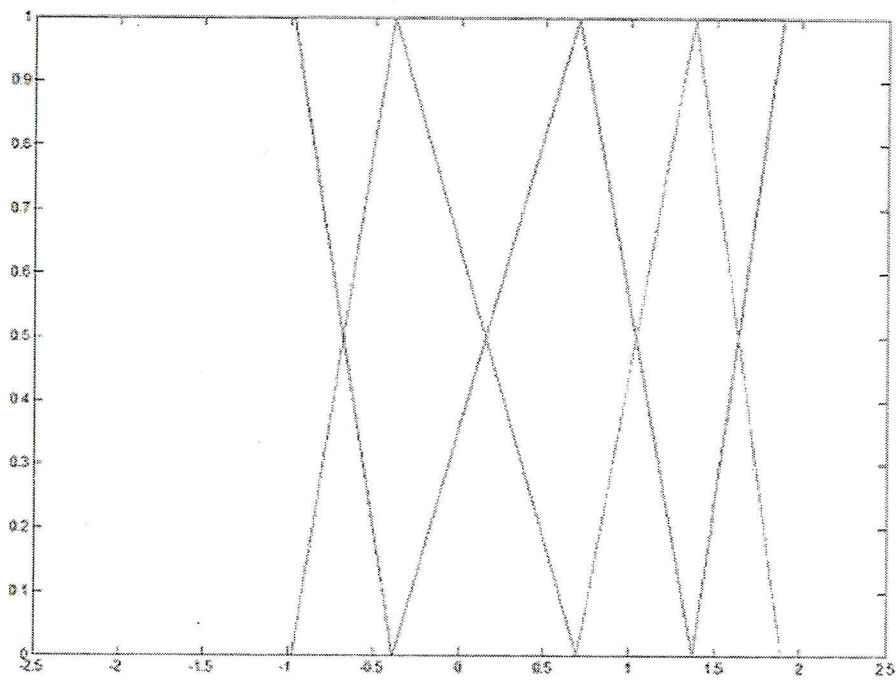
Figure 4.11 CORRELATION OF ACTUAL AND ESTIMATED OUTPUT



The above graph shows the correlation of the actual and estimated output; from the graph we can say that the model seems to be successful with % 16.9 correlation.

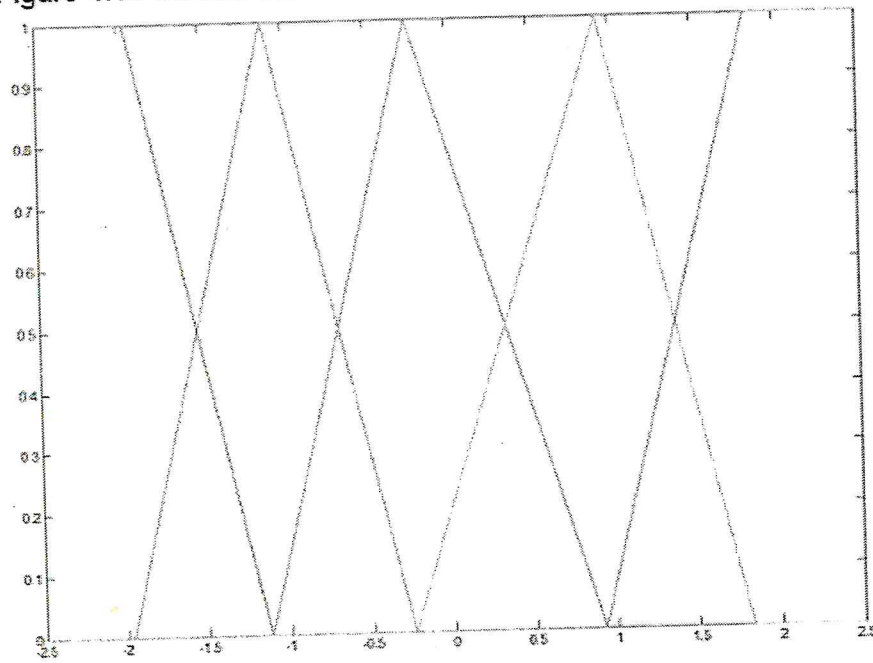
The membership function of the middle layer 1, middle layer 2 and the output variables are changed after the optimisation process that we did by steepest descent method. The following three graphs are ranges of the each fuzzy sets of the middle layer 1, middle layer 2 and output, respectively.

Figure 4.12 MEMBERSHIP FUNCTION OF THE MIDDLE LAYER 1



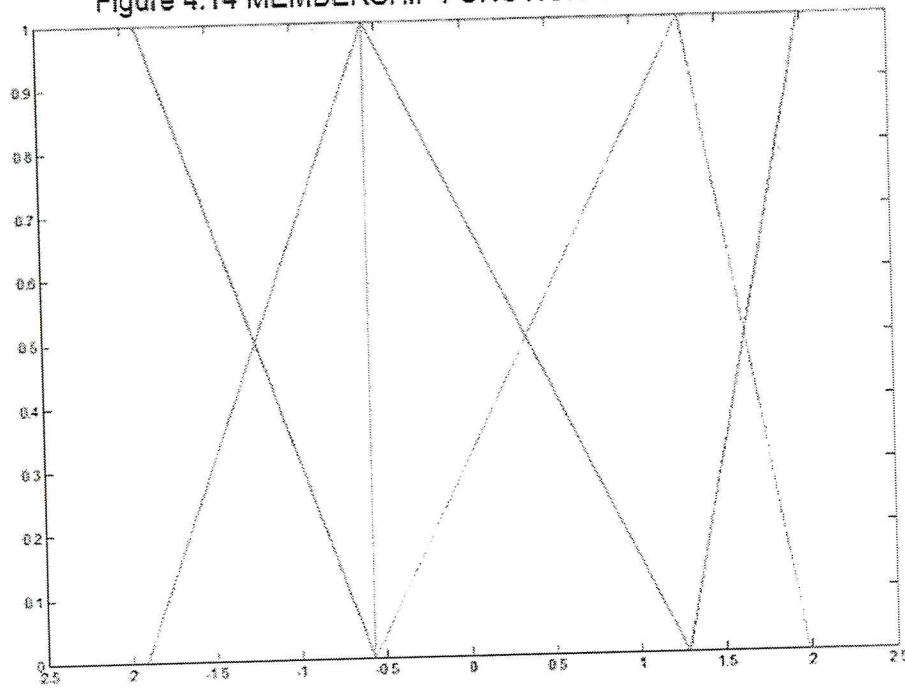
For the middle layer 1, it is seen that the extremely low membership border is enlarged most and the extremely high gets smaller by this time, which means that the variable is not sensitive. The normal membership border has the biggest x-axis. The symmetries of low and normal membership borders are changed much more than the high membership border.

Figure 4.13 MEMBERSHIP FUNCTION OF THE MIDDLE LAYER 2



For the figure of the middle layer 2 all functions seems to be symmetric and we can say that this variable is much more sensitive than the others. The border of the extremely high membership increase little, by the time the border of the extremely low decrease little.

Figure 4.14 MEMBERSHIP FUNCTION OF THE OUTPUT

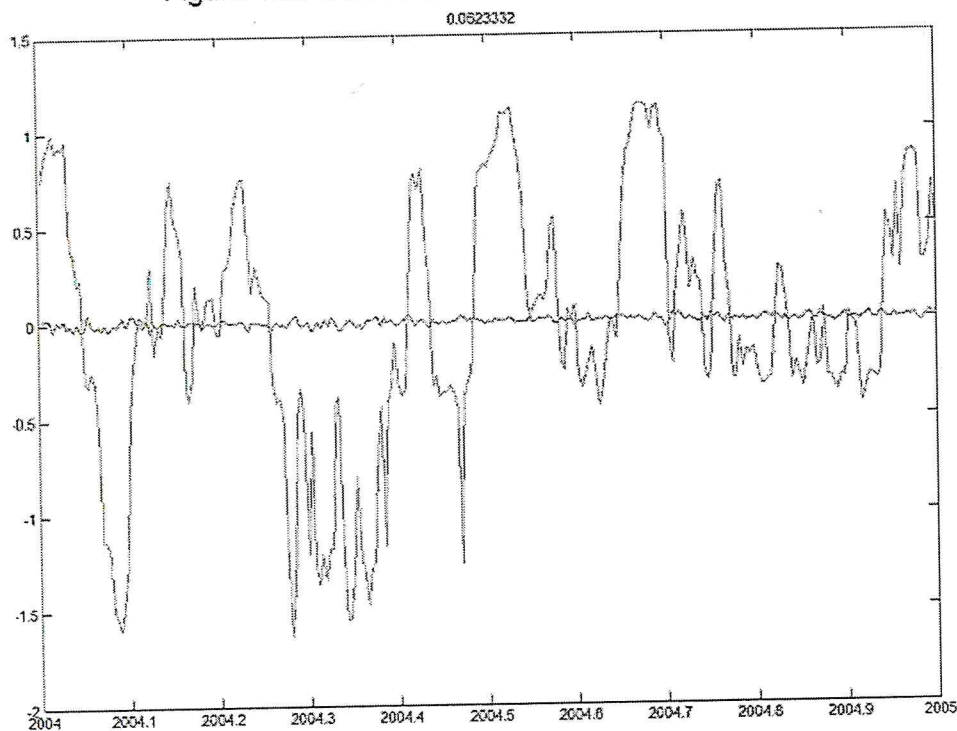


The membership function for the output change much more than the others. The borders for the extremely low and high are enlarged mostly and triangular denote the low and normal ranges seems to be symmetric to each other by the y-axis. The borders for the extremely high gets smaller by this time.

4.6.3. Testing The Model

Now, we need to compare the model with the data that are not used in the optimisation process to test that if it works. To test the model we used the data between 2004 and 2005. The results of the fuzzy process can be observed in the following graphs. The correlation of the output is % 6.2. The correlation is not so near that we find after optimisation. However, it is positive and the difference of the values are not surprising.

Figure 4.15 OUTPUT OF THE TESTING MODEL



Summary of the algorithm for the fuzzy & optimisation processes

We use an algorithm which was written in matlab to apply the fuzzy and optimization processes. We have some variables such as mssvardata, datax, datay, totalrulenum, mprule, output, datev, loopnum and bloopnum. Datax is the number of the input variables (RSI, RVI, MA, BBAND). Datay is the number of data (1876-from 1997 to 2004). rulenum is three because we have three fuzzy process in our model. Mssvardata is a matrix with six columns which are the date, the value of each indicator or input and the value of the output or the actual return, respectively. In addition, it is a matrix with datay rows. So, we define these variables and we need some rules to apply fuzzification or namely the fuzzy inference system. For the borders of membership functions that will be changed with optimization, we define another matrix with five rows and seven columns. Columns shows the borders of the membership functions that we decide by looking buy and sell strategies. So, we have five fuzzy sets that we describe before, we have seven borders. Each row shows the inputs or the output that can be described by these borders. Then, we need to define the rules for the each fuzzy process. Firstly, we should use a code that make the fuzzification and defuzzification process for the each data. Then we can find the estimated output for the each data to compare with the actual data. The next step is to use an optimization technique to make the difference of the actual and the estimated returns smaller and smaller or near to zero. We choose to use steepest descent method and apply it to each step of the fuzzy process, starting the process that we did not agree the borders of the membership functions. After optimization the borders of the membership functions changed and the difference gets smaller as it can be. So, we develop a model that can predict the actual return as it can be possible. The final step is to test it with data that we did not use when developing the model. In other words, do the same things without using optimization.

- Define the data
- Define the borders of the membership functions
- Define the model that develop the output (RSI and RVI develops middle layer 1)
- Define the rules
- Fuzzification
- Defuzzification
- Apply optimization methods for the each process

CHAPTER 5. CONCLUSION

In this study, we used the most extensive index the ISE100, to make our model much more general. We want to show the serviceability of the fuzzy logic. Generally, fuzzy logic is used in control system theory and we try to show that it has also some applications in finance. For the inputs of the model, we examined the indicators as looking their means, standard deviations and in which sides of the market that they can best explain. In addition we try to find indicators that have different properties such as one of them can find the areas that the prices are extremely expensive or extremely high, on the other side one can examine the volatility of the stock market. In finance application of fuzzy logic, also in application, one should construct the rules of the model. So, we need to know in all ways of our inputs and also how they are good in explaining the stock market together. Then, by expert experience you can construct the rules to explain the market. Fuzzy logic modeling may have some disadvantage because of this subjective expert experience. However, you can easily change the rules of the system by comparing the estimated outputs with the actual output. We construct the model with the period 1997 to 2004 and optimize the model with the steepest descent method and find new parameters for the indicators that we used in the model or new borders for our membership functions. Their ratio of the explained sum of squares to the total sum of squares, to estimate the daily return of the ISE100 Index is calculated as %16.9. We need extra data to test the in_sample, beginning of the 2004 up to beginning of the 2005. We use our model with new borders, which gives the best output before (estimated outputs near the actual outputs), and compare the outputs for the period 2004 to 2005 that models develop with the actual outputs. Their ratio of the explained sum of squares to the total sum of squares, to estimate the out_of_sample of the ISE100 Index is calculated as % 6.2.

However, when we change the period of the data that we use to test, such as six months, we find greater results, which is nearer the models result.

In addition, we test the weak form efficiency of the Istanbul stock exchange and observed that the day of the week, the month of the year (January), the turn of the month and the turn of the year effects exists in our country, too.

Consequently, a fuzzy logic model is constructed through technical analysis. This model combines the technical analysis rules together with the expert's experience.

REFERENCES

- han, E. and K. Kunter (1996) "Financial Market Efficiency in a Developing Economy: Turkish Case," **Discussing Paper No. 9611**, Research Department, The Central Bank of the Republic of Turkey, (March).
- is, L. (1986) "A Transaction Data Study of Weekly and Intradaily Patterns in Stock Markets," **Journal of Financial Economics**, 16, 99-117.
- R. W. and L. T. Starks (1995) "The Day-of-the Week Anomaly: The Role of the Institutional Investors," **Financial Analysts Journal**, (May-June), 58-67.
- ci, B. and G. Muradolu "Dissemination of Stock Recommendations and Small Investors: Benefits?,"
- in, Y. and E. M. Yucel (2003) "The Day of the Week Effect on Stock Market Volatility: Evidence from Emerging Markets,"
- an, E. and B. Hamarat (2004) "Do Cloudy Days Affect Stock Exchange Returns: Evidence from Istanbul Stock Exchange," **Jornal of Naval Science and Engineering**, 117-
- ia, E.F., "Efficient Capital Markets: A Review of Theory and Empirical Work", **The Journal of Finance**, Volume 25, Issue 2, pp. 383-417, 1970.

F. and A. Vasquez, "Simple Technical Trading Rules of Stock Returns: evidence from Chile", Emerging Markets Review, Issue 1, pp. 152-164, 2000.

va, C. W. (1995). Intelligent control: Fuzzy Logic Applications. CRC.

makos, E. (1979). "Application of Fuzzy Concepts to Power Demand Forecasting," Transactions on Systems, Man and Cybernetics, Vol: 9. pp: 651-657

E. F. (1991). "Efficient Capital Markets II.," Journal of Finance, Vol:46. pp: 1575-

ler, E. and Kandel, A. (1989). "The Use of Fuzzy Set Theory for Forecasting Corporate revenues," Fuzzy Sets and Systems, Vol:31. pp: 187-204.

o, M. and Kang, G. T. (1988). "Structure Identification of Fuzzy Model," Fuzzy Sets and Systems, Vol:28. pp: 15-33.

, B. (1997). Fuzzy Engineering, Prentice Hall

s, A.(1997). IMKB Uygulamali Modern Teknik Analiz, Dunya Yayincilik

, L. (1997). A Course in Fuzzy Systems and Control, Prentice Hall

in, J. F. (1996). Fuzzy Logic, Wiley

Altrock, C. (1997). Fuzzy Logic and neuroFuzzy Applications in Business and
ance, Prentice Hall

yen, H. T. and E. A. Walker, (2000). A first course in Fuzzy Logic, Chapman & Hall

lemer, H. and S. Gottwald, (1995). Fuzzy sets, fuzzy logic , fuzzy methods with
lications, Wiley

aidono, M. (2001). Fuzzy Logic for Beginners, World Scientific