STEREOTYPE FORMATION AS TRAIT AGGREGATION

Burak Can
105622004

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Burak Can
105622004

Tez Danışmanı: M. Remzi Sanver	: ....
Jüri Üyesi: Göksel Aşan	: ....
Jüri Üyesi: Nicholas Baigent	: ....
Jüri Üyesi: M. Remzi Sanver	: ....

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Abstract

The problem of understanding how stereotypes are formed is still valid today. Our approach proposes a multi-disciplinary approach to social psychology literature by means of social choice theory familiar to those in welfare economics. According to the model we propose, when confronted a trait profile of a fixed society, an individual observer aggregates the trait profile into a stereotype through what we call "a perception function". Regarding the possibility of prejudice and individual subjectivity, we extend our model to subjective majority rules in which individual opinions about the representativeness of each subgroup within a society is also encaptured.

Özetçe

Basmakalıpların nasıl oluştuğu sorunsalı günümüzde halen geçerlidir. Bizim yaklaşımımız; sosyal psikoloji literatürüne, refah iktisadı çalışanlara yakın bir konu olan sosyal seçim kuramı araçları kullanarak, çok-disiplinli bir yaklaşım önerir. Önerdiğimiz modele göre, bir gözlemci birey, sabit bir topluluğun belirli bir karakter profiline sahip bireyleriyle karşılaştığında, bu profili “algı fonksiyonu” dediğimiz metodla bir basmakalıba dönüştürür. Önyargı ihtimalini ve bireysel öznelliği de gözönünde bulundurarak; modelimizi, toplumun alt kümelerinin temsiliyet gücü hakkındaki bireysel fikirleri de yansıtan, öznel çoğunluk kurallarına genişletiyoruz.

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[1] "Stereotype Formation as Trait Aggregation", Burak Can and M. Remzi Sanver, April 2007. By the time this dissertation is written, the paper was submitted to a journal and awaiting for editorial referee.

".... thou shalt see with thine own eyes and not through the eyes of others, and shalt know of thine own knowledge and not through the knowledge of thy neighbour."

(Mírzá Ḥusayn-‘Alí, Tablets of Baha’u’llah, p. 36)

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Introduction

Stereotyping is a very particular way of categorizing. Individuals do categorize people regarding their attributes or stereotype them so as to perceive them internally "consistent" bodies (Judd, Ryan & Park 1991) and as simplified portrays. It is very common that individuals either subjectively or via social interaction form stereotypes about groups of people or societies. These stereotypes may be *ex ante* beliefs and judgements due to society which the individual attributes himself to. On the other hand, the stereotype may be formed by individual experiences. Individual may confront a group of objects and considering the trait in question, the individual may form a stereotype as an aggregation of the individual traits in the society.

The term "stereotype" comes from printing typos. It was first Lippmann (1922) who conceptualized the metaphor, calling a stereotype a "picture in our heads"[1]. A stereotype,thus, is an overall judgment brought over a given group of objects, such as “Princeton students are smart”, “French food is delicious” or “Muslim women wear scarf”. Understanding the formation of stereotypes is a central question of social psychology. As Krueger et al. (2003) eloquently discuss, a main strand of the literature rests on the *attribution hypothesis* which assumes a direct association between traits and groups. Under the attribution hypothesis, an observer judges a group according to the traits he observes in that group. For example, he looks at Muslim women; sees that some wear a scarf and some do not; his mental processing of that observation leads to some kind of a general judgment about Muslim women such as “Muslim women wear scarf” or as “Muslim women do not

[1] http://en.wikipedia.org/wiki/Stereotype

wear scarf". Of course, bringing no judgment hence avoiding a stereotype is also possible. According to the attribution hypothesis, a trait which is "sufficiently prevalent" in a given group is associated with that group. To quote Zawadski (1948), "The popular conception of a group characteristic seems to be a characteristic which is present in the majority of the members of the group. According to this concept, it is a necessary and sufficient condition for a group characteristic to be represented in at least 51 per cent of the members of the group".

The problem of understanding how stereotypes are formed is still valid today. Our approach proposes a multi-disciplinary approach to social psychology literature by means of social choice theory. In chapter 1, we shortly review the stereotype literature in social psychology and overview basics of social choice theory afterwards. Having visited the preliminaries of both disciplines, we come up, in chapter 2, with our model to understand the stereotype formation in particular cases, e.g. we neglect the ex ante stereotypes such as inherited or imposed ones. We propose, then, some reasonable axioms for stereotype formation and then characterize the rules that satisfy those certain axioms. chapter 2 distinguishes between formation of stereotypes under perfect information and of those under imperfect information. By the former, we mean an observer trying to bring an overall judgment about a society which she can see all the members and hence has complete information about the trait profile. By imperfect information in chapter 2 we mean a situation in which the observer is aware of the existence of the members which she cannot see. In chapter 3, we propose a brief discussion for our model and conclude with the contribution of our studies with further possible extensions.

1 Literature Review

The literature on stereotypes is mainly framed within social psychology. There has been ongoing research in the discipline both in terms of theoretical basis and of empirical. The studies on different types of stereotypes are present; i.e. gender stereotypes, ethnic stereotypes etc. Since stereotype is "a picture in our heads", how this picture is formed, is a very hot topic in social psychology. On the other hand there seems to be few that is directly related to stereotype issue in social choice theory. Nevertheless the choice theory, we believe, a lot to offer in terms of aggregation traits into a stereotype over a given group of people.

Before we propose our model in the next chapter, we would like to devote the first part of this chapter to have a look at the history of the literature in social psychology. We proceed, then, to the findings in social choice theory which, at first glance, may seem irrelevant, yet reveals herself quite useful for an axiomatic approach to the problem. Thereafter we propose the choice-theoretic model to stereotype formation

1.1 Stereotyping and Social Psychology

Stereotypes have been studied since early 20th century. Lippmann (1922) laid down the theoretical basis for stereotypes and was followed by Katz and Braly's (1933) empirical works. The latter two developed their checklist paradigm in which the observers were given a set of traits and asked to associate the traits with a list of ethnic groups, and the most of the empirical studies today rests on this paradigm. Regarding the methodology of the stereotype studies, there has been two hypothesis that arose: attribution hypothesis and categorization hypothesis. The attribution hypothesis (Krueger, Hasman, Acevedo 2003) suggests that stereotypes are derived from the typicality of traits that are observed whereas for categorization hypothesis stereotypes are assumed to be derived by comparing at least two groups of people. The latter rests on the assumption that a stereotype over a group of people is formed by comparing the availability or typicality of the same trait in another group. Krueger (1996) shows in his empirical work that this assumption which is actually defined as "contrast" (Zawadski 1948) seems irrelevant. On the other hand attribution hypothesis has found more attention by most of the researchers which according to Krueger et al (2003) assumes "a simple associationist process by which people learn and encode the properties of social groups"

Throughout the next chapter we formalize our model using the attribution hypothesis as we do investigate stereotypes when the observer confronts only one group of people. Despite its complex structure, the categorization hypothesis does not constitute a proper workspace for us as there need not be another group of people so as to compare the typicality of the trait for the observer. Thus we neglect the possible effect of comparative trait typicality in stereotyping and lay our work on the attribution hypothesis. This is how our work can benefit from social choice theory via some aggregation concepts which we mention in the next part.

1.2 Majority Decision Rule and Social Choice Theory

Our model which we propose in the Chapter 2 benefits a lot from social choice theory particularly from majority decision rule. In 1951, Kenneth J. Arrow argued that when aggregating individual preferences over a set of alternatives with at least 3 elements, there cannot be any non-dictatorial aggregation rule which satisfies PO and IIA axioms and still gives a transitive and complete order as a social outcome. It was May (1952) who characterized the majority decision rule when individuals confront two alternatives. The discourse of the majority decision rule can be applied to the case where a group of individuals vote for or against a given nominee. Regarding the anonymous and neutral aggregation rules, Maskin (1995) shows that, majority remains the best among various types of aggregation rules. In fact a decision rule (when there are two alternatives) is anonymous, neutral and monotonic if and only it is simple majority decision rule. This characterization à la May, despite the critics against the demanding feature of monotonicity, has been extended to various forms. Among these studies there are different characterizations of the majority rules by Campbell and Kelly (2000),Yi (2005), Woeginger (2003).

Asan and Sanver (2002) characterized majority decision rule by dropping monotonicity and using another two conditions instead. Furthermore they also show in another work (2005) that using Maskin-monotonicity instead of monotonicity characterizes absolute qualified majority rules. The literature in majority decision rules has, thus, been quite explored and in the next chapter we benefit of these findings to apply to the behaviour of stereotypes. Since the two alternative world can be read as admitting a trait or not, we can use the social choice literature when a group of individuals (a society) exhibits a trait profile (a voting profile) and when aggregation of this profile is read as a stereotype formation.

2 Axiomatic Approach

2.1 Modelling A Stereotype through Social Choice Perspective: Perception Functions

We propose the following model: Take some group, e.g., Turkish citizens, and a certain trait, e.g., smoking. Some of the members of the group do and some do not possess this trait and a judgement such as "Turks do smoke" is an aggregation of individual traits into a social one. So we can speak of a *perception function* that maps individual traits into a subjective stereotype about the society. More formally, we have a finite set N of individuals with $\#N \geq 2$, to which we refer as a *group*. There is a trait which the members of the group may or may not possess. We write $t_i = 1$ when $i \in N$ possesses this trait and $t_i = -1$ otherwise. We let $T = \{-1, 1\}^N$ stand for the set of *trait profiles*. There is an observer[2] who looks at the group which exhibits a trait profile $t = (t_1, ..., t_{\#N}) \in T$. Not necessarily all members of the group are visible to the observer. We write $V \subseteq N$ for the members of the group that are visible to the observer. An observer who sees $V \subset N$ is aware of the existence of the unobserved $N \backslash V$. On the other hand, we rule out the possibility of "wrong observation", i.e., the trait of every visible member of the group is observed as it truly is. We let $T_V = \{-1, 1\}^V$ stand for the set of trait profiles of the observable members. The observer has a subjective perception of the group as a function of the trait profile he is able to observe, which we express through a *(subjective) perception function* $\psi_V : T_V \to \{1, 0, -1\}$.

[2]To avoid confusion, we assume that the observer is not a member of the group. Although this has no effect to our model, belongingness of the observer to the observed group seems to actually matter, according to our intepretation of Krueger et al. (2003).

So given any non-empty set $V \subseteq N$ of observed members and any prevailing trait profile $t \in T_V$ of these observed members, we write $\psi_V(t) = 1$ when the observer globally perceives the group N as possessing the trait in question. Similarly, we write $\psi_V(t) = -1$ when the observer globally perceives the group as not possessing the trait in question and $\psi_V(t) = 0$ refers to the observer's abstention of reaching a global perception of the group. We refer to the case $V = N$ as *perfect observation* and to $V \subset N$ as *imperfect observation.* Under perfect observation, we write ψ instead of ψ_N.

What kind of perception functions are used? We approach the problem axiomatically by considering the cases of perfect and imperfect observation separately.

2.2 Stereotype Formation Under Perfect Observation

We propose the following model: Take some group, e.g., Turkish citizens, and a certain trait, e.g., smoking. Some of the members of the group do and some do not possess this trait and a judgement such as "Turks do smoke" is an aggregation of individual traits into a social one. So we can speak of a *perception function* that maps individual traits into a subjective stereotype about the society. More formally, we have a finite set N of individuals with $\#N \geq 2$, to which we refer as a *group.* There is a trait which the members of the group may or may not possess. We write $t_i = 1$ when $i \in N$ possesses this trait and $t_i = -1$ otherwise. We let $T = \{-1, 1\}^N$ stand for the set of *trait profiles.* There is an observer[3]

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who looks at the group which exhibits a trait profile $t = (t_1, ..., t_{\#N}) \in T$. Not necessarily all members of the group are visible to the observer. We write $V \subseteq N$ for the members of the group that are visible to the observer. An observer who sees $V \subset N$ is aware of the existence of the unobserved $N \backslash V$. On the other hand, we rule out the possibility of "wrong observation", i.e., the trait of every visible member of the group is observed as it truly is. We let $T_V = \{-1, 1\}^V$ stand for the set of trait profiles of the observable members. The observer has a subjective perception of the group as a function of the trait profile he is able to observe, which we express through a *(subjective) perception function* $\psi_V : T_V \rightarrow \{1, 0, -1\}$. So given any non-empty set $V \subseteq N$ of observed members and any prevailing trait profile $t \in T_V$ of these observed members, we write $\psi_V(t) = 1$ when the observer globally perceives the group N as possessing the trait in question. Similarly, we write $\psi_V(t) = -1$ when the observer globally perceives the group as not possessing the trait in question and $\psi_V(t) = 0$ refers to the observer's abstention of reaching a global perception of the group. We refer to the case $V = N$ as *perfect observation* and to $V \subset N$ as *imperfect observation*. Under perfect observation, we write ψ instead of ψ_N.

What kind of perception functions are used? We approach the problem axiomatically by considering the cases of perfect and imperfect observation separately.

2.2.1 Axioms for Perception Functions

Being sensitive to individual traits is incorporated in the concept of a perception function. So, we wish to rule out imposed perceptions that are independent of individual traits such as "Muslims do not drink alcohol because this is what the Quran says". Hence we posit that under perfect observation, the observer would say "Muslims do not drink alcohol" if no Muslim drinks alcohol and "Muslims do drink alcohol" if every Muslim drinks alcohol. We express these through the following axiom:

Non-imposedness: A perception function $\psi : T \to \{1, 0, -1\}$ satisfies *non-imposedness* iff $\psi(1, 1, ..., 1) = 1$ and $\psi(-1, -1, ..., -1) = -1$.

The non-imposedness axiom is a weak unanimity requirement which rules out imposed perceptions while it does not exclude biased ones such as saying "Muslims eat pork" if and only if every Muslim eats pork and saying "Muslims do not eat pork" even when there exists a single Muslim who does not eat pork. It is clear that such a perception is based on an unequal treatment of traits. Of course this may happen but when we wish to rule it out, we use the following axiom:

Impartiality: A perception function $\psi : T \to \{1, 0, -1\}$ satisfies *impartiality* iff $\psi(-t) = -\psi(t)\ \forall t \in T$.

Remark that given a trait profile t, the trait profile $-t$ stands for the reversal of every individual trait. So impartiality is an adaptation of the usual neutrality condition of social choice theory which ensures the equal treatment of alternatives. An observer with an impartial perception function is not prejudiced about the group's possessing or not possessing the trait: If the trait of every observed individual is reversed then so is the perception.

In contrast to what impartiality requires, one can perceive a society under an unequal treatment of the traits. For example, it is possible that the observer has a bias towards thinking that the society exhibits the trait in question. Such a bias is formally expressed through the following axiom:

Positive Prejudice: We say that a perception function $\psi : T \to \{1, 0, -1\}$ admits *positive prejudice* iff

(i) $\exists\ t \in T$ such that $\psi(t) = 1$ and $\psi(-t) \in \{0, 1\}$

and

(ii) $\psi(t) \in \{0, -1\} \implies \psi(-t) = 1\ \forall t \in T$.

So under a perception function admitting positive prejudice, there is a trait profile t such that the trait is rejected neither at t nor at $-t$. Moreover, there exists no trait profile t such that the observer rejects the trait or is indecisive both at t and $-t$.

Similarly, as expressed below, the observer can have a bias towards thinking that the society does exhibit the trait in question:

Negative Prejudice: We say that a perception function $\psi : T \to \{1, 0, -1\}$ admits *negative prejudice* iff

(i) $\exists\ t \in T$ such that $\psi(t) = -1$ and $\psi(-t) \in \{-1, 0\}$

and

(ii) $\psi(t) \in \{0, 1\} \implies \psi(-t) = -1\ \forall t \in T$

Another axiom we borrow from the social choice literature is a monotonicity condition: If a trait profile changes so that some individuals who did not possess the trait now possess it while this is the only change, then the perception should not change in the opposite direction. We express this formally as follows:

Monotonicity: A perception function $\psi : T \to \{1, 0, -1\}$ is *monotonic* iff $\psi(t) \geq \psi(t')\ \forall t, t' \in T$ with $t_i \geq t'_i\ \forall i \in N$.

These axioms pave the way to the characterization of a class of perception functions which we call *subjective majority rules*. We have three main characterization results where we use the conjunction of non-imposedness and monotonicity with one of impartiality, positive prejudice and negative prejudice.[4] We close the section by establishing the logical independence of the axiom triples that we use.

Proposition 1 *Non-imposedness, monotonicity and impartiality are logically independent.*

Proof. To see that impartiality and non-imposedness do not imply monotonicity, let $\#N = 3$ and consider $\psi : T \to \{1, 0, -1\}$ which is defined for each $t \in T$ as $\psi(t) = 1$ when $\#\{i \in N : t_i = 1\} \in \{1, 3\}$ and $\psi(t) = -1$ otherwise. To see that impartiality and monotonicity do not imply non-imposedness, take $\psi(t) = 0$ for all $t \in T$. Finally, to see that non-imposedness and monotonicity do not imply impartiality, let $\psi(t) = -1$ if $t_i = -1$ $\forall i \in N$ and $\psi(t) = 1$ otherwise. ■

Proposition 2 *Non-imposedness, monotonicity and positive prejudice are logically independent.*

Proof. To see that positive prejudice and non-imposedness do not imply monotonicity, let $\#N = 3$ and consider $\psi : T \to \{1, 0, -1\}$ which is defined as $\psi(-1, -1, -1) = -1$, $\psi(1, 1, -1) = 0$ and $\psi(t) = 1$ otherwise. To see that positive prejudice and monotonicity do not imply non-imposedness, take $\psi(t) = 1$ for all $t \in T$. Finally, to see that non-imposedness and monotonicity do not imply positive prejudice, let $\psi(t) = 1$ if $t_i = 1$ $\forall i \in N$ and $\psi(t) = -1$ otherwise. ■

Proposition 3 *Non-imposedness, monotonicity and negative prejudice are logically independent.*

[4] Remark that impartiality, positive prejudice and negative prejudice are pairwise logically incompatible.

Proof. To see that negative prejudice and non-imposedness do not imply monotonicity, let $\#N = 3$ and consider $\psi : T \to \{1, 0, -1\}$ which is defined as $\psi(1, 1, 1) = 1$, $\psi(-1, -1, 1) = 0$ and $\psi(t) = -1$ otherwise. To see that negative prejudice and monotonicity do not imply non-imposedness, take $\psi(t) = -1$ for all $t \in T$. Finally, to see that non-imposedness and monotonicity do not imply negative prejudice, let $\psi(t) = -1$ if $t_i = -1$ $\forall i \in N$ and $\psi(t) = 1$ otherwise. ■

2.2.2 A Solution: Subjective Majority Rules

We first define a *(subjective) weight distribution* as a mapping $\omega : 2^N \to [0, 1]$ such that $\omega(K) + \omega(N \backslash K) = 1$ for all $K \in 2^N$ while $\omega(N) = 1$. So ω expresses the subjective opinion of the observer about the representation weight of each subgroup of N. A weight distribution ω is *monotonic* iff $\omega(K) \leq \omega(L)$ for all $K, L \in 2^N$ with $K \subseteq L$. For the rest of the paper, we embed monotonicity into the definition of a weight distribution.

Given a weight distribution ω and any $q \in (0, 1)$, *a subjective* $(\omega, q)-$*majority rule* is a perception function $\psi^{\omega,q} : T \to \{1, 0, -1\}$ defined for each $t \in T$ as follows:

$$\psi^{\omega,q}(t) = \left\{ \begin{array}{c} 1 \quad \text{if } \omega(\{i \in N : t_i = 1\}) > q \\ -1 \quad \text{if } \omega(\{i \in N : t_i = -1\}) > 1 - q \\ 0 \qquad \text{otherwise} \end{array} \right\}$$

So the observer looks at the group with some subjective opinion about how representative the subgroups are. If, according to this subjective opinion, the weight of those who possess the trait exceeds q, then the observer concludes that the group globally possesses that trait. Similarly, if the (subjective) weight of those who possess the trait is below q, then the observer concludes that the group globally does not possess that trait.[5] If neither of these two cases holds then no conclusion is derived.

Theorem 1 *A perception function $\psi : T \to \{1, 0, -1\}$ satisfies non-imposedness, monotonicity and impartiality iff ψ is a subjective $(\omega, \frac{1}{2})-$majority rule.*

Proof. We leave the "if" part to the reader. To see the "only if" part, take any perception function $\psi : T \to \{1, 0, -1\}$ that satisfies non-imposedness, monotonicity and impartiality. We define $W_\psi = \{K \in 2^N : \psi(t) = 1$ for $t \in T$ with $t_i = 1$ $\forall i \in K$ and $t_i = -1$ $\forall i \in N \backslash K\}$ and $L_\psi = \{K \in 2^N : \psi(t) = -1$ for $t \in T$ with $t_i = 1$ $\forall i \in K$ and $t_i = -1$ $\forall i \in N \backslash K\}$. As ψ satisfies non-imposedness, $N \in W_\psi$ and $\emptyset \in L_\psi$, hence W_ψ and L_ψ are each non-empty. Let $O_\psi = 2^N \backslash (W_\psi \cup L_\psi)$ be the (possibly empty) set of coalitions which are neither in W_ψ nor in L_ψ. Now consider a function $\omega : 2^N \to [0, 1]$ defined for each $K \in 2^N$ as $\omega(K) = 1$ if $K \in W_\psi$, $\omega(K) = 0$ if $K \in L_\psi$ and $\omega(K) = \frac{1}{2}$ if $K \in O_\psi$. As ψ is impartial, for each $K \in 2^N$, we have $K \in L_\psi \iff N \backslash K \in W_\psi$ which implies $K \in O_\psi \iff N \backslash K \in O_\psi$. Thus $\omega(K) + \omega(N \backslash K) = 1$ $\forall K \in 2^N$ while $\omega(N) = 1$. Moreover, the monotonicity of ψ implies $\omega(K) \leq \omega(L)$ for all $K, L \in 2^N$ with $K \subseteq L$. So ω is a weight distribution. We complete the proof by showing that the subjective $(\omega, \frac{1}{2})-$majority rule $\psi^{\omega, \frac{1}{2}} : T \to \{1, 0, -1\}$ coincides with ψ. To see this, take any $t \in T$. If $\psi(t) = 1$, then $K = \{i \in N : t_i = 1\} \in W_\psi$, implying $\omega(K) = 1 > \frac{1}{2}$, which establishes $\psi^{\omega, \frac{1}{2}}(t) = 1$. If $\psi(t) = -1$, then $K = \{i \in N : t_i = 1\} \in L_\psi$, implying

[5] Remark that $\omega(\{i \in N : t_i = -1\}) > 1 - q$ and $\omega(\{i \in N : t_i = 1\}) < q$ are equivalent requirements. However, we use the former statement to be coherent with our definition in Section 3, where we consider subjective majority rules under imperfect observation.

$\omega(K) = 0$, hence $\omega(N \backslash K) = 1 > \frac{1}{2}$, which establishes $\psi^{\omega,\frac{1}{2}}(t) = -1$. If $\psi(t) = 0$ then $K \in O_\psi$, implying $\omega(K) = \frac{1}{2}$, which establishes $\psi^{\omega,\frac{1}{2}}(t) = 0$. ■

Theorem 2 *A perception function $\psi : T \to \{1, 0, -1\}$ satisfies non-imposedness, monotonicity and positive prejudice iff ψ is a subjective $(\omega, q)-$majority rule with $q \in (0, \frac{1}{2})$ and $\omega(K) \in [q, 1-q]$ for some $K \in 2^N$.*

Proof. To see the "if" part, take any subjective $(\omega, q)-$majority rule $\psi^{\omega,q}$ with $q \in (0, \frac{1}{2})$ and $\omega(K) \in [q, 1-q]$ for some $K \in 2^N$. It is straightforward to check that $\psi^{\omega,q}$ satisfies non-imposedness and monotonicity. To show that $\psi^{\omega,q}$ satisfies positive prejudice, take some $K \in 2^N$ with $\omega(K) \in (q, 1-q]$. Remark that $\psi^{\omega,q}(t) = 1$ and $\psi^{\omega,q}(-t) \in \{0, 1\}$ for $t \in T$ with $t_i = 1$ $\forall i \in K$ and $t_i = -1$ $\forall i \in N \backslash K$. Now take any $t \in T$ with $\psi^{\omega,q}(t) \in \{0, -1\}$. Thus, letting $K = \{i \in N : t_i = 1\}$, we have $\omega(K) \leq q$, hence $\omega(N \backslash K) > q$, implying $\psi^{\omega,q}(-t) = 1$, showing that $\psi^{\omega,q}$ satisfies positive prejudice. To see the "only if" part, take any perception function $\psi : T \to \{1, 0, -1\}$ that satisfies non-imposedness, monotonicity and positive prejudice. Let W_ψ, L_ψ and O_ψ be defined as in the proof of Theorem 1. Note that $N \in W_\psi$ and $\emptyset \in L_\psi$ while O_ψ may be empty. Now pick some $q \in (0, \frac{1}{2})$ and consider a function $\omega : 2^N \to [0, 1]$ defined for each $K \in 2^N$ as $\omega(K) = 0$ if $K \in L_\psi$, $\omega(K) = q$ if $K \in O_\psi$. Moreover, if $K \in W_\psi$, then let $\omega(K) = 1$ when $N \backslash K \in L_\psi$; $\omega(K) = 1 - q$ when $N \backslash K \in O_\psi$ and $\omega(K) = \frac{1}{2}$ when $N \backslash K \in W_\psi$. As ψ satisfies positive prejudice, for each $K \in L_\psi \cup O_\psi$ we have $N \backslash K \in W_\psi$. Thus $\omega(K) + \omega(N \backslash K) = 1$ $\forall K \in 2^N$ while $\omega(N) = 1$. Moreover, the monotonicity of ψ implies $\omega(K) \leq \omega(L)$ for all $K, L \in 2^N$ with $K \subseteq L$. So ω is a weight distribution. Note also that by positive prejudice, $\exists K \in W_\psi$ such that $N \backslash K \in W_\psi \cup O_\psi$, implying $\omega(K) \in [q, 1-q]$. We complete the proof by showing that the subjective $(\omega, q)-$majority rule $\psi^{\omega,q} : T \to \{1, 0, -1\}$ coincides with ψ. To see this, take any $t \in T$. If $\psi(t) = 1$, then $K = \{i \in N : t_i = 1\} \in W_\psi$ and $\omega(K) \in \{\frac{1}{2}, 1-q, 1\}$ implying $\omega(K) > q$, which establishes $\psi^{\omega,q}(t) = 1$. If $\psi(t) = 0$, then $K = \{i \in N : t_i = 1\} \in O_\psi$ and $\omega(K) = q$, which establishes $\psi^{\omega,q}(t) = 0$. If $\psi(t) = -1$, then $K = \{i \in N : t_i = 1\} \in L_\psi$ and $\omega(K) = 0$, hence $\omega(N \backslash K) = 1 > 1 - q$, which establishes $\psi^{\omega,q}(t) = -1$. ■

Theorem 3 *A perception function $\psi : T \to \{1, 0, -1\}$ satisfies non-imposedness, monotonicity and negative prejudice iff ψ is a subjective $(\omega, q)-$majority rule with $q \in (\frac{1}{2}, 1)$ and $\omega(K) \in [1-q, q]$ for some $K \in 2^N$.*

Proof. To see the "if" part, take any subjective $(\omega, q)-$majority rule $\psi^{\omega,q}$ with $q \in (\frac{1}{2}, 1)$ and $\omega(K) \in [1-q, q]$ for some $K \in 2^N$. It is straightforward to check that $\psi^{\omega,q}$ satisfies non-imposedness and monotonicity. To show that $\psi^{\omega,q}$ satisfies negative prejudice, take some $K \in 2^N$ with $\omega(K) \in (1-q, q]$. Remark that $\psi^{\omega,q}(t) = -1$ and $\psi^{\omega,q}(-t) \in \{-1, 0\}$ for $t \in T$ with $t_i = -1$ $\forall i \in K$ and $t_i = 1$ $\forall i \in N \backslash K$. Now take any $t \in T$ with $\psi^{\omega,q}(t) \in \{0, 1\}$. Thus, letting $K = \{i \in N : t_i = 1\}$, we have $\omega(K) \geq q > 1-q$, implying $\psi^{\omega,q}(-t) = -1$ showing that $\psi^{\omega,q}$ satisfies negative prejudice. To see the "only if" part, take any perception function $\psi : T \to \{1, 0, -1\}$ that satisfies non-imposedness, monotonicity and negative prejudice. Let W_ψ, L_ψ and O_ψ be defined as in the proof of Theorem 1. Note that $N \in W_\psi$ and $\emptyset \in L_\psi$ while O_ψ may be empty. Now pick some $q \in (\frac{1}{2}, 1)$ and consider a function $\omega : 2^N \to [0, 1]$ defined for each $K \in 2^N$ as $\omega(K) = 1$ if $K \in W_\psi$, $\omega(K) = q$ if $K \in O_\psi$. Moreover, if $K \in L_\psi$, then let $\omega(K) = 0$ when $N \backslash K \in W_\psi$; $\omega(K) = 1 - q$ when $N \backslash K \in O_\psi$ and $\omega(K) = \frac{1}{2}$ when $N \backslash K \in L_\psi$. As ψ satisfies negative prejudice, for each $K \in W_\psi \cup O_\psi$ we have

$N \backslash K \in L_\psi$. Thus $\omega(K) + \omega(N \backslash K) = 1 \; \forall K \in 2^N$ while $\omega(N) = 1$. Moreover, the monotonicity of ψ implies $\omega(K) \leq \omega(L)$ for all $K, L \in 2^N$ with $K \subseteq L$. So ω is a weight distribution. Note that by negative prejudice, $\exists K \in L_\psi$ such that $N \backslash K \in O_\psi \cup L_\psi$, implying $\omega(K) \in [1-q, q]$. We complete the proof by showing that the subjective $(\omega, q)-$majority rule $\psi^{\omega,q} : T \rightarrow \{1, 0, -1\}$ coincides with ψ. To see this, take any $t \in T$. If $\psi(t) = -1$, then $K = \{i \in N : t_i = 1\} \in L_\psi$ and $\omega(K) \in \{0, 1-q, \frac{1}{2}\}$ implying $\omega(K) < q$, hence $\omega(N \backslash K) > 1-q$, which establishes $\psi^{\omega,q}(t) = -1$. If $\psi(t) = 0$, then $K = \{i \in N : t_i = 1\} \in O_\psi$ and $\omega(K) = q$, which establishes $\psi^{\omega,q}(t) = 0$. If $\psi(t) = 1$, then $K = \{i \in N : t_i = 1\} \in W_\psi$ and $\omega(K) = 1 > q$, which establishes $\psi^{\omega,q}(t) = -1$. ■

Remark 1 *Monotonicity is a normatively appealing condition for perception functions and this is why we are keeping it throughout our analysis. However, it is clear from their proofs that Theorems 1, 2 and 3 can be stated by simultaneously dispensing with the monotonicity of the perception function and the monotonicity condition incorporated into the definition of a weight distribution.*

Until now, we did not bring any requirement for an equal treatment of individuals by the weight distribution ω. In fact, at one extreme, it is possible to have an observer who believes that a group is fully represented in the personality of one of its members $d \in N$ which would correspond to a weight distribution $\omega(K) = 1$ for all $K \in 2^N$ with $d \in K$. At the other extreme, we have $\omega^{=}(K) = \frac{\#K}{\#N}$ for all $K \in 2^N$ where all individuals are thought of having equal weight. Given the subjective nature of weight distributions (hence of stereotype formation), we do not think that an equal treatment of individuals should be required. However, we wish to explore the effects of imposing such a requirement. A perception function $\psi : T \rightarrow \{1, 0, -1\}$ is *anonymous* iff given any $t = (t_1, ..., t_{\#N}) \in T$ and any bijection $\pi : N \longleftrightarrow N$, we have $\psi(t_1, ..., t_{\#N}) = \psi(t_{\pi(1)}, ..., t_{\pi(\#N)})$. Given some $\alpha \in (0, 1)$, a weight distribution $\omega : 2^N \rightarrow [0, 1]$ is $\alpha-$*anonymous* iff given any $K, L \in 2^N$ with $\#K = \#L$ we have $\omega(K) > \alpha \iff \omega(L) > \alpha$ and $\omega(K) < \alpha \iff \omega(L) < \alpha$.[6]

Theorem 4 *A perception function $\psi : T \rightarrow \{1, 0, -1\}$ satisfies non-imposedness, monotonicity, impartiality and anonymity iff ψ is a subjective $(\omega, \frac{1}{2})-$majority rule for some $\frac{1}{2}-$anonymous ω.*

Proof. To show the "if" part, let $\psi : T \rightarrow \{1, 0, -1\}$ be a subjective $(\omega, \frac{1}{2})-$majority rule where ω is $\frac{1}{2}-$anonymous. We know by Theorem 1 that ψ satisfies non-imposedness, monotonicity and impartiality. To see the anonymity of ψ, take any $t = (t_1, ..., t_{\#N}) \in T$. Let $K = \{i \in N : t_i = 1\}$. Take any bijection $\pi : N \longleftrightarrow N$. Let $\pi(t) = (t_{\pi(1)}, ..., t_{\pi(\#N)})$ and $\pi(K) = \{\pi(i)\}_{i \in K}$. As π is a bijection, $\#K = \#\pi(K)$. Moreover, $\{i \in N : t_{\pi(i)} = 1\} = \pi(K)$. Thus $\psi(t_1, ..., t_{\#N}) = \psi(t_{\pi(1)}, ..., t_{\pi(\#N)})$ holds by the $\frac{1}{2}-$anonymity of ω.

To show the "only if" part, take any $\psi : T \rightarrow \{1, 0, -1\}$ satisfying non-imposedness, monotonicity, impartiality and anonymity. We know, by Theorem 1 that ψ is a subjective $(\omega, \frac{1}{2})-$majority rule. To see that ω is $\frac{1}{2}-$anonymous, take any $K, L \in 2^N \backslash \{\emptyset\}$ with $\#K = \#L$. Take $t = (t_1, ..., t_{\#N}) \in T$ with $\{i \in N : t_i = 1\} = K$. Take also some bijection $\pi : N \longleftrightarrow N$ such that $\{\pi(i)\}_{i \in K} = L$. Now let $\omega(K) > \frac{1}{2}$. So $\psi(t) = 1$. As ψ is anonymous, $\psi(t_{\pi(1)}, ..., t_{\pi(\#N)}) = 1$ as well, implying $\omega(\{i \in N : t_{\pi(i)} = 1\}) = \omega(L) > \frac{1}{2}$.

[6] Hence we also have $\omega(K) = \alpha \iff \omega(L) = \alpha$. Note that α-anonymity is weaker than a more standard anonymity condition which would require $\omega(K) = \omega(L)$ for all $K, L \in 2^N$ with $\#K = \#L$.

One can similarly establish that letting $\omega(K) < \frac{1}{2}$ implies $\omega(L) < \frac{1}{2}$, thus showing the $\frac{1}{2}-$anonymity of ω. ■

Remark 2 *The mathematics of our model belongs to the literature on majority characterizations, which goes back to May (1952). This allows to make a remark about Theorem 4. Consider a set $A = \{x, y\}$ of alternatives and let each $i \in N$ have a preference $p_i \in \{{}^x_y, {}^y_x\}$ over A.*[7] *Denoting xy for indifference between x and y, we conceive a social choice rule as a mapping $f : \{{}^x_y, {}^y_x\}^N \rightarrow \{{}^x_y, {}^y_x, xy\}$. Let n^* be the lowest integer exceeding $\frac{\#N}{2}$. Picking any $\mu \in \{n^*, ..., n\}$, we define a $\mu-$majority rule as a social choice rule $f_\mu : \{{}^x_y, {}^y_x\}^N \rightarrow \{{}^x_y, {}^y_x, xy\}$ where for any $p = (p_1, ..., p_{\#N}) \in \{{}^x_y, {}^y_x\}$ we have $f_\mu(p) = {}^x_y \iff \#\{i \in N : p_i = {}^x_y\} \geq \mu$ and $f_\mu(p) = {}^y_x \iff \#\{i \in N : p_i = {}^y_x\} \geq \mu$.*[8] *Theorem 3.2 of Asan and Sanver (2006) characterizes the set of Pareto optimal, anonymous, neutral and Maskin monotonic aggregation rules in terms of $\mu-$majority rules. In that abstract setting, Pareto optimality, anonymity and neutrality respectively coincide with our non-imposedness, anonymity and impartiality. On the other hand Maskin monotonicity is stronger than our monotonicity. So by Theorem 4, we can deduce that the class of $\mu-$majority rules is a subset of the class of subjective $(\omega, \frac{1}{2})-$majority rules with ω being $\frac{1}{2}-$anonymous. In other words, every aggregation rule that gives every coalition in the society its "objective" weight (i.e., letting the weight of $K \in 2^N$ be $\frac{\#K}{\#N}$) but possibly qualifies the required majority can alternatively be expressed by fixing majority as usual (i.e., as any coalition whose cardinality exceeds its complement) but assigning monotonic and $\frac{1}{2}-$anonymous (subjective) weights to coalitions.*

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[7] where ${}^x_y$ is interpreted as x being preferred to y and ${}^y_x$ is interpreted as y being preferred to x. So individual preferences do not admit indifference between x and y.

[8] Thus $f_\mu(p) = xy \iff \#\{i \in N : p_i = {}^x_y\} < \mu$ and $\#\{i \in N : p_i = {}^y_x\} < \mu$.

Remark 3 *As is for Theorem 1, Theorems 2 and 3 can be stated by simultaneously adding anonymity to the perception function and the corresponding $\alpha-$anonymity with $\alpha = q$ to the weight distribution.*

2.3 Stereotype Formation Under Imperfect Observation

In this section we model the behaviour of the perception functions under imperfect observation. Throughout the section, we fix some non-empty set $V \subsetneq N$ of visible group members and consider the perception function $\psi_V : T_V \to \{1, 0, -1\}$. The existence of invisible group members entails a revision of the non-imposedness axiom. For, an observer who fails to observe some members of the group may be cautious to bring a global perception of the group, even when the prevailing trait profile is unanimous We revisit our axioms from the perfect observation

2.3.1 Axioms for Perception Functions

Non-imposedness: A perception function $\psi_V : T_V \to \{1, 0, -1\}$ satisfies *non-imposedness* iff $\psi_V(1, 1, ..., 1) \in \{0, 1\}$ and $\psi_V(-1, -1, ..., -1) \in \{-1, 0\}$.

Remark that the imperfect information version of non-imposedness is neither weaker nor stronger than its perfect information version. For, it is weakened by allowing the refusal of judgements but strengthened by being imposed over the profiles where unanimity is reached among the members of V.

Monotonicity, impartiality, positive prejudice and negative prejudice exhibit a strengthening of the similar spirit, as we now impose them when the related changes in the trait profiles occur in the visible part of the group.

Monotonicity: A perception function $\psi_V : T_V \to \{1, 0, -1\}$ is *monotonic* iff

$\psi_V(t) \geq \psi_V(t') \ \forall \ t, t' \in T_V$ with $t_i \geq t'_i \ \forall i \in V$.

Impartiality: A perception function $\psi_V : T_V \to \{1, 0, -1\}$ satisfies *impartiality* iff $\psi_V(t') = -\psi_V(t) \ \forall t, t' \in T_V$ such that $t'_i = -t_i \ \forall i \in V$.

Positive Prejudice: We say that a perception function $\psi_V : T_V \to \{1, 0, -1\}$ admits *positive prejudice* iff

(i) $\exists \ t \in T_V$ such that $\psi_V(t) = 1$ and $\psi_V(-t) \in \{0, 1\}$

and

(ii) $\psi_V(t) \in \{-1, 0\} \Longrightarrow \psi_V(-t) = 1 \ \forall \ t \in T_V$.

Negative Prejudice: We say that a perception function $\psi_V : T_V \to \{1, 0, -1\}$ admits *negative prejudice* iff

(i) $\exists \ t \in T_V$ such that $\psi_V(t) = -1$ and $\psi_V(-t) \in \{-1, 0\}$

and

(ii) $\psi_V(t) = \{0, 1\} \Longrightarrow \psi_V(-t) = -1 \ \forall \ t \in T_V$

To characterize perception under imperfect observation, we use the conjunction of non-imposedness and monotonicity with one of impartiality, positive prejudice and negative prejudice. The following proposition establishes the logical relationship between these axioms[9]:

Proposition 4 *(i) Monotonicity and impartiality imply non-imposedness.*

(ii)Monotonicity and impartiality are logically independent.

(iii)Non-imposedness, monotonicity and positive prejudice are logically independent.

(iv)Non-imposedness, monotonicity and negative prejudice are logically independent.

[9] As is in the perfect observation case (see Footnote 2), impartiality, positive prejudice and negative prejudice are pairwise logically incompatible.

Proof. *Proof of (i): Let* $\psi_V : T_V \to \{-1, 0, 1\}$ *satisfy impartiality and fail non-imposedness. We have* $\psi_V(1, 1, ...1) = -1$ *or* $\psi_V(-1, -1, ..., -1) = 1$ *by the failure of non-imposedness which, by impartiality, implies* $\psi_V(1, 1, ...1) = -1$ *and* $\psi_V(-1, -1, ..., -1) = 1$, *contradicting monotonicity.*

Proof of (ii): Define $\psi_V : T_V \to \{-1, 0, 1\}$ *as* $\psi_V(1, 1, ..., 1) = -1$, $\psi_V(-1, -1, ..., -1) = 1$ *and* $\psi_V(t) = 0$ $\forall t \in T_V$ *with* $t_i = 1$, $t_j = -1$ *for some* $i, j \in V$. *Check that* ψ_V *is impartial but not monotonic. Now let* $\psi_V(t) = 1$ $\forall t \in T_V$ *and check that* ψ_V *is monotonic but not impartial.*

Proof of (iii): To see that non-imposedness and monotonicity do not imply positive prejudice, let $\psi_V(1, 1, ..., 1) = 1$ *and* $\psi_V(t) = -1$ *for any* $t \in T_V$ *with* $t_i \in \{-1, 0\}$ *for some* $i \in V$. *To see that non-imposedness and positive prejudice do not imply monotonicity, let* $\#V = 3$ *and let* $\psi_V(t) = 1$ *if* $\#\{i \in V : t_i = 1\} \in \{1, 3\}$; $\psi_V(t) = -1$ *if* $\#\{i \in V : t_i = 1\} = 2$ *and* $\psi_V(-1, -1, -1) = 0$. *To see that monotonicity and positive prejudice do not imply non-imposedness let* $\psi_V(t) = 1$ $\forall t \in T_V$.

Proof of (iv): To see that non-imposedness and monotonicity do not imply negative prejudice, let $\psi_V(-1, -1, ..., -1) = -1$ *and* $\psi_V(t) = 1$ *for any* $t \in T_V$ *with* $t_i \in \{0, 1\}$ *for some* $i \in V$. *To see that non-imposedness and negative prejudice do not imply monotonicity let* $\#V = 3$ *and let* $\psi_V(t) = -1$ *if* $\#\{i \in V : t_i = 1\} \in \{0, 2\}$; $\psi_V(1, 1, 1) = 0$ *and* $\psi_V(t) = 1$ *if* $\#\{i \in V : t_i = 1\} = 1$. *To see that monotonicity and negative prejudice do not imply non-imposedness let* $\psi_V(t) = -1$ $\forall t \in T_V$. ■

2.3.2 Subjective Majority Rules Revisited

A *(subjective) weight distribution* as an ordered pair $\sigma = (\omega, p)$ where $\omega : 2^V \to [0,1]$ is a mapping satisfying

(i) $\omega(K) + \omega(V \backslash K) = 1$ for all $K \in 2^V$

(ii) $\omega(V) = 1$

(iii) $\omega(K) \leq \omega(L)$ for all $K, L \in 2^V$ with $K \subseteq L$

and $p \in [0,1]$ reflects the weight of V in N.[10]

Given a weight distribution $\sigma = (\omega, p)$ and any $q \in (0,1)$, *a subjective* $(\sigma, q)-$*majority rule* is a perception function $\psi_V^{\sigma,q} : T_V \to \{1, 0, -1\}$ defined for each $t \in T_V$ as follows:

$$\psi_V^{\sigma,q}(t) = \left\{ \begin{array}{c} 1 \quad \text{if } p.\omega(\{i \in V : t_i = 1\}) > q \\ -1 \quad \text{if } p.\omega(\{i \in V : t_i = -1\}) > 1 - q \\ 0 \qquad \text{otherwise} \end{array} \right\}$$

So the observer looks at V with some subjective opinion about how representative its subgroups are. Moreover, he has a subjective opinion about the representativeness of V within the whole society. If, according to these subjective opinions, the weight of those who possess the trait exceeds q, then the observer concludes that the group globally possesses that trait. Similarly, if the (subjective) weight of those who do not possess the trait exceeds $1 - q$, then the observer concludes that the group globally does not possess that trait.[11] If neither of these two cases holds then no conclusion is derived.

[10] Remark that under perfect observation, we used ω to express the weight distribution within N but now it expresses the (monotonic) weight distribution within V coupled with the parameter p which reflects the weight of V in N. Of course when $p = 1$, V can be conceived as the whole society, bringing us back to the case of perfect observation.

[11] Remark that $p.\omega(\{i \in V : t_i = -1\}) > 1 - q$ and $p.\omega(\{i \in V : t_i = 1\}) < q$ are equivalent requirements if and only if $p = 1$. See Footnote 3.

Theorem 5 *A perception function $\psi_V : T_V \to \{1, 0, -1\}$ satisfies monotonicity and impartiality iff ψ_V is a subjective $(\sigma, \frac{1}{2})-$majority rule for some subjective weight distribution $\sigma = (\omega, p)$.*

Proof. *We leave the "if" part to the reader. To see the "only if" part, take any $\psi_V : T_V \to \{-1, 0, 1\}$ that satisfies monotonicity and impartiality. Let $W_\psi = \{K \in 2^V : \psi_V(t) = 1$ for $t \in T_V$ with $t_i = 1\ \forall i \in K$ and $t_i = -1\ \forall i \in V \backslash K\}$ and $L_\psi = \{K \in 2^V : \psi_V(t) = -1$ for $t \in T_V$ with $t_i = 1\ \forall i \in K$ and $t_i = -1\ \forall i \in V \backslash K\}$. We set $O_\psi = 2^V \backslash (W_\psi \cup L_\psi)$. Note that the perception function ψ_V defined as $\psi_V(t) = 0$ at each $t \in T_V$ is monotonic and impartial. So W_ψ and L_ψ can both be empty. However, by the impartiality of ψ_V, we have $W_\psi = \emptyset \Longleftrightarrow L_\psi = \emptyset$. In fact, $W_\psi = \emptyset \Longleftrightarrow L_\psi = \emptyset \Longleftrightarrow \psi_V(t) = 0\ \forall t \in T_V$. First consider the case where $W_\psi = \emptyset$ and $L_\psi = \emptyset$. So $\psi_V(t) = 0\ \forall t \in T_V$. Take any subjective weight distribution $\sigma = (\omega, p)$ with $p \in [0, \frac{1}{2})$. It is straightforward to check that the subjective $(\sigma, \frac{1}{2})-$majority rule coincides with ψ_V. Now consider the case where neither W_ψ nor L_ψ is empty. Thus, $V \in W_\psi$ and $\emptyset \in L_\psi$. Consider the function $\omega : 2^V \to [0, 1]$ where $\omega(K) = 1\ \forall K \in W_\psi$, $\omega(K) = 0\ \forall K \in L_\psi$ and $\omega(K) = \frac{1}{2}$ $\forall K \in O_\psi$. The impartiality of ψ_V ensures $K \in W_\psi \Longleftrightarrow V \backslash K \in L_\psi\ \forall K \in 2^V$ and thus $K \in O_\psi \Longleftrightarrow V \backslash K \in O_\psi\ \forall K \in 2^V$. Hence $\omega(K) + \omega(V \backslash K) = 1\ \forall K \in 2^V$ while $\omega(V) = 1$. Moreover, the monotonicity of ψ_V implies $\omega(K) \leq \omega(L)$ for all $K, L \in 2^V$ with $K \subseteq L$. Thus, any $p \in [0, 1]$ induces a subjective weight distribution (ω, p). Pick $p = 1$ and let $\sigma = (\omega, 1)$. We claim that the subjective $(\sigma, \frac{1}{2})-$majority rule $\psi_V^{\sigma, \frac{1}{2}} : T_V \to \{-1, 0, 1\}$ coincides with ψ_V. To see this, take any $t \in T_V$. If $\psi_V(t) = 1$, then $K = \{i \in V : t_i = 1\} \in W_\psi$ and $\omega(K) = 1$, implying $p.\omega(K) = 1 > \frac{1}{2}$, which establishes $\psi_V^{\sigma, \frac{1}{2}}(t) = 1$. If $\psi_V(t) = -1$, then $K = \{i \in V : t_i = 1\} \in L_\psi$ and $\omega(K) = 0$, hence $\omega(V \backslash K) = 1$, implying $p.\omega(V \backslash K) = 1 > \frac{1}{2}$, which* *establishes* $\psi_V^{\sigma,\frac{1}{2}}(t) = -1$. *If* $\psi_V(t) = 0$, *then* $K = \{i \in V : t_i = 1\} \in O_\psi$ *and* $\omega(K) = \frac{1}{2}$, *hence* $\omega(V \backslash K) = \frac{1}{2}$. *Thus neither* $p.\omega(K) > \frac{1}{2}$, *nor* $p.\omega(V \backslash K) > \frac{1}{2}$ *holds, which establishes* $\psi_V^{\sigma,\frac{1}{2}}(t) = 0$. ∎

Theorem 6 *Given any* $V \subsetneq N$, *a perception function* $\psi_V : T_V \to \{1, 0, -1\}$ *satisfies weak non-imposedness, monotonicity and positive prejudice iff* ψ_V *is a subjective* $(\sigma, q)-$*majority rule with* $q \in (0, \frac{1}{2})$ *while* $\sigma = (\omega, p)$ *is a weight distribution such that* $p > \max\{2q, 1-q\}$ *and* $\omega(K) \in [\frac{q}{p}, \frac{1-q}{p}]$ *for some* $K \in 2^V$.[12]

Proof. *To see the "if" part, let* ψ_V *be a subjective* $(\sigma, q)-$*majority rule as in the statement of the theorem. It is straightforward to check* $\psi_V^{\sigma,q}$ *satisfies weak non-imposedness and monotonicity. To show that* $\psi_V^{\sigma,q}$ *satisfies positive prejudice, take some* $K \in 2^V$ *with* $\omega(K) \in (\frac{q}{p}, \frac{1-q}{p}]$. *So* $\psi_V^{\sigma,q}(t) = 1$ *for* $t \in T_V$ *with* $t_i = 1$ $\forall i \in K$ *and* $t_i = -1$ $\forall i \in V \backslash K$. *Moreover, as* $p.\omega(K) \leq 1-q$, *we have* $\psi_V^{\sigma,q}(-t) \in \{0, 1\}$. *Now take any* $t \in T_V$ *with* $\psi_V^{\sigma,q}(t) \in \{-1, 0\}$ *and let* $K = \{i \in V : t_i = 1\}$. *If* $\psi_V^{\sigma,q}(t) = -1$ *then* $p.\omega(V \backslash K) > 1-q > q$, *implying* $\psi_V(-t) = 1$. *If* $\psi_V^{\sigma,q}(t) = 0$ *then* $p.\omega(K) \leq q$. *As* $p > 2q$, *we have* $\omega(K) < \frac{1}{2}$, *thus* $\omega(V \backslash K) > \frac{1}{2}$ *and* $p.\omega(V \backslash K) > 2q$, *implying* $\psi_V(-t) = 1$ *which shows that* $\psi_V^{\sigma,q}$ *satisfies positive prejudice. To see the "only if" part, take any* $\psi_V : T_V \to \{1, 0, -1\}$ *that satisfies weak non-imposedness, monotonicity and positive prejudice. We define* W_ψ, O_ψ *and* L_ψ *as in Theorem 5. Note that* $V \in W_\psi$. *Moreover, while one of* O_ψ *and* L_ψ *may be empty,* $O_\psi \cup L_\psi$ *is non-empty. Now pick some* $q \in (0, \frac{1}{2})$ *and consider the function* $\omega : 2^V \to [0, 1]$ *defined for each* $K \in 2^V$ *as* $\omega(K) = \omega(V \backslash K) = \frac{1}{2}$ *when* K, $V \backslash K \in W_\psi$; $\omega(K) = 0$ *and* $\omega(V \backslash K) = 1$ *when* $K \in L_\psi$ *and* $V \backslash K \in W_\psi$; $\omega(K) = q$ *and* $\omega(V \backslash K) = 1-q$ *when* $K \in O_\psi$ *and* $V \backslash K \in W_\psi$. *Note*

[12] Note that $p > \max\{2q, 1-q\}$ ensures $\frac{q}{p}, \frac{1-q}{p} \in (0, 1)$. Moreover, $q \in (0, \frac{1}{2})$ ensures $\frac{q}{p} < \frac{1-q}{p}$.

that positive prejudice ensures $K \in L_\psi \cup O_\psi \implies V\backslash K \in W_\psi$ *for each* $K \in 2^V$. *Thus* $\omega(K) + \omega(V\backslash K) = 1$ $\forall K \in 2^V$ *with* $\omega(V) = 1$, *while the monotonicity of* ψ_V *implies* $\omega(K) \leq \omega(L)$ *for all* $K, L \in 2^V$ *with* $K \subseteq L$. *Thus, any* $p \in [0,1]$ *induces a subjective weight distribution* (ω, p). *Take any* $p \in [0,1]$ *with* $p > \max\{2q, 1-q\}$. *We will show that* $\omega(K) \in [\frac{q}{p}, \frac{1-q}{p}]$ *for some* $K \in 2^V$. *Recall that* $O_\psi \cup L_\psi$ *is non-empty. First let* O_ψ *be non-empty and take some* $S \in O_\psi$. *So* $V\backslash S \in O_\psi$ *and by construction of* ω *we have* $\omega(V\backslash S) = 1-q$, *thus* $\omega(V\backslash S) \leq \frac{1-q}{p}$. *Moreover,* $1-q > q$ *and* $p > 2q$, *thus* $\omega(V\backslash S) = 1-q > \frac{q}{p}$, *establishing* $\omega(V\backslash S) \in (\frac{q}{p}, \frac{1-q}{p}]$. *By definition of* O_ψ, *we have* $p.\omega(S) \leq q$, *thus* $\omega(S) \leq \frac{q}{p} < \frac{1}{2}$ *implying* $\omega(V\backslash S) \geq 1 - \frac{q}{p} > \frac{1}{2} > \frac{q}{p}$. *Again by definition of* O_ψ, *we have* $p.\omega(V\backslash S) \leq 1-q$. *Thus* $\omega(V\backslash S) \in (\frac{q}{p}, \frac{1-q}{p}]$. *Now let* O_ψ *be empty. By positive prejudice,* $\exists K \in W_\psi$ *such that* $V\backslash K \in W_\psi$. *Thus* $\omega(K) = \frac{1}{2} \in [\frac{q}{p}, \frac{1-q}{p}]$, *by the choice of* p. *Writing* $\sigma = (\omega, p)$, *we complete the proof by showing that the* $(\sigma, q)-$*majority rule* $\psi_V^{\sigma;q}$ *coincides with* ψ_V. *To see this, take any* $t \in T_V$. *If* $\psi(t) = 1$, *then* $K = \{i \in N : t_i = 1\} \in W_\psi$ *and* $\omega(K) \in \{\frac{1}{2}, 1-q, 1\}$. *Moreover,* $p > 2q$. *Thus,* $p.\omega(K) > q$, *establishing* $\psi_V^{\sigma;q}(t) = 1$. *If* $\psi(t) = 0$, *then* $K = \{i \in N : t_i = 1\} \in O_\psi$ *and* $\omega(K) = q$, *hence* $\omega(V\backslash K) = 1-q$. *Thus, neither* $p.\omega(K) > q$, *nor* $p.\omega(V\backslash K) > 1-q$ *holds, which establishes* $\psi_V^{\sigma;q}(t) = 0$. *If* $\psi(t) = -1$, *then* $K = \{i \in N : t_i = 1\} \in L_\psi$ *and* $\omega(K) = 0$, *hence* $\omega(V\backslash K) = 1$ *implying* $p.\omega(V\backslash K) = p > 1-q$, *which establishes* $\psi_V^{\sigma;q}(t) = -1$. ■

Theorem 7 *Given any* $V \subsetneq N$, *a perception function* $\psi_V : T_V \to \{1, 0, -1\}$ *satisfies weak non-imposedness, monotonicity and negative prejudice iff* ψ_V *is a subjective* $(\sigma, q)-$*majority rule with* $q \in (\frac{1}{2}, 1)$ *while* $\sigma = (\omega, p)$ *is a weight distribution such that* $p > \max\{2q, 1-q\}$ *and* $\omega(K) \in [\frac{1-q}{p}, \frac{q}{p}]$ *for some* $K \in 2^V$.[13]

[13]Note that $p > \max\{2q, 1-q\}$ ensures $\frac{q}{p}, \frac{1-q}{p} \in (0,1)$. Moreover, $q \in (\frac{1}{2}, 1)$ ensures $\frac{1-q}{p} < \frac{q}{p}$.

Proof. *To see the "if" part, let ψ_V be a subjective $(\sigma, q)-$majority rule as in the statement of the theorem. It is straightforward to check $\psi_V^{\sigma;q}$ satisfies weak non-imposedness and monotonicity. To show that $\psi_V^{\sigma;q}$ satisfies negative prejudice, take some $K \in 2^V$ with $\omega(K) \in (\frac{1-q}{p}, \frac{q}{p}]$. So $\psi_V^{\sigma;q}(t) = -1$ for $t \in T_V$ with $t_i = -1$ $\forall i \in K$ and $t_i = 1$ $\forall i \in V \backslash K$. Moreover, as $p.\omega(K) \leq q$, we have $\psi_V^{\sigma;q}(-t) \in \{-1, 0\}$. Now take any $t \in T_V$ with $\psi_V^{\sigma;q}(t) \in \{0, 1\}$ and let $K = \{i \in V : t_i = 1\}$. If $\psi_V^{\sigma;q}(t) = 1$ then $p.\omega(K) > q > 1 - q$, implying $\psi_V(-t) = -1$. If $\psi_V^{\sigma;q}(t) = 0$ then $p.\omega(V \backslash K) \leq 1 - q$. As $p > 2q$ and $q > 1 - q$, we have $p > 2(1 - q)$. So $\omega(V \backslash K) < \frac{1}{2}$, thus $\omega(K) > \frac{1}{2}$ and $p.\omega(K) > 1 - q$, implying $\psi_V(-t) = -1$ which shows that $\psi_V^{\sigma;q}$ satisfies negative prejudice. To see the "only if" part, take any $\psi_V : T_V \to \{1, 0, -1\}$ that satisfies weak non-imposedness, monotonicity and negative prejudice. We define W_ψ, O_ψ and L_ψ as in Theorem 5. Note that $\emptyset \in L_\psi$. Moreover, while one of W_ψ and O_ψ may be empty, $W_\psi \cup O_\psi$ is non-empty. Now pick some $q \in (\frac{1}{2}, 1)$ and consider the function $\omega : 2^V \to [0, 1]$ defined for each $K \in 2^V$ as $\omega(K) = \omega(V \backslash K) = \frac{1}{2}$ when $K, V \backslash K \in L_\psi$; $\omega(K) = 0$ and $\omega(V \backslash K) = 1$ when $K \in L_\psi$ and $V \backslash K \in W_\psi$; $\omega(K) = q$ and $\omega(V \backslash K) = 1 - q$ when $K \in O_\psi$ and $V \backslash K \in L_\psi$. Note that negative prejudice ensures $K \in W_\psi \cup O_\psi \Longrightarrow V \backslash K \in L_\psi$ for each $K \in 2^V$. Thus $\omega(K) + \omega(V \backslash K) = 1$ $\forall K \in 2^V$ with $\omega(V) = 1$, while the monotonicity of ψ_V implies $\omega(K) \leq \omega(L)$ for all $K, L \in 2^V$ with $K \subseteq L$. Thus, any $p \in [0, 1]$ induces a subjective weight distribution (ω, p). Take any $p \in [0, 1]$ with $p > \max\{2q,\ 1 - q\}$. We will show that $\omega(K) \in [\frac{1-q}{p}, \frac{q}{p}]$ for some $K \in 2^V$. Recall that $W_\psi \cup O_\psi$ is non-empty. First let O_ψ be non-empty and take some $S \in O_\psi$. By construction of ω we have $\omega(S) = q$, thus $\omega(S) \leq \frac{q}{p}$. Moreover, $q > 1 - q$ and $p > 2(1 - q)$, thus $\omega(S) = q > \frac{1-q}{p}$, establishing $\omega(S) \in (\frac{1-q}{p}, \frac{q}{p}]$. Now let O_ψ be empty. By negative prejudice, $\exists K \in L_\psi$ such that $V \backslash K \in L_\psi$. Thus $\omega(K) =$*

$\frac{1}{2} \in [\frac{1-q}{p}, \frac{q}{p}]$, by the construction of ω and the choice of p. Writing $\sigma = (\omega, p)$, we complete the proof by showing that the $(\sigma, q)-$majority rule $\psi_V^{\sigma;q}$ coincides with ψ_V. To see this, take any $t \in T_V$. If $\psi(t) = 1$, then $K = \{i \in N : t_i = 1\} \in W_\psi$ and $\omega(K) = 1$, implying $p.\omega(K) = p > q$, which establishes $\psi_V^{\sigma;q}(t) = 1$. If $\psi(t) = 0$, then $K = \{i \in N : t_i = 1\} \in O_\psi$ and $\omega(K) = q$, hence $\omega(V \backslash K) = 1 - q$. Thus, neither $p.\omega(K) > q$, nor $p.\omega(V \backslash K) > 1 - q$ holds, which establishes $\psi_V^{\sigma;q}(t) = 0$. If $\psi(t) = -1$, then $K = \{i \in N : t_i = 1\} \in L_\psi$ and $\omega(K) \in \{0, 1-q, \frac{1}{2}\}$, hence $\omega(V \backslash K) \in \{\frac{1}{2}, q, 1\}$. As $p > 2q$, $p.\omega(V \backslash K) > q > 1-q$, establishing $\psi_V^{\sigma;q}(t) = -1$.

■

3 Conclusion

3.1 Discussion

The contribution of the of this work can be thought as axiomatization of the ongoing research in the field of stereotypes. One of the important features of the model we present is that it distinguishes between the subjective opinion of the observer on how much representative the subgroups of a society is and the possible prejudice one might have. At first glance it may sound as an ex ante prejudice to assign more representativeness to a subgroup with respect to another subgroup with the same number of members within. However, the prejudice, we suggest, does not lie in the representativeness of a subgroup. Instead one can find it in the outcome of the perception when the same subgroup completely changes their traits and when that profile is aggregated.

Second interesting result in our interpretation of stereotypes is that it allows, under imperfect observation, hesitation of the observer to bring a global judgment over the society. One might argue that in such a case where there is so few information about the society, the prejudice of the observer may not hold. We think, however, that the prejudice of an observer is trivial in such a scenario. Nevertheless the lack of information about the society does not change the nature of the perception function of an observer. It is true that the prejudice reveals itself when there is enough information about the society. Yet, the subjective majority rule is still the same and it reveals the prejudice as long as the visible set is representative enough formally speaking when our parameter p is large enough. It is also worth to note that this parameter p is inversely proportional to prudence of the observer. As we have shown in the previous chapter when $p = 1$ our model of imperfect observation becomes exactly the same as perfect observation. So the essence of stereotyping is also encaptured in the prudence of the observer as it is almost impossible for one to form an ethnic stereotype under perfect observation unless the society is composed of only a bunch of individuals or is about to extinct.

Another way of making benefit of the works in stereotyping is to understand possible perception of immigrants[14] by the natives in a country. In such a case the visible set (V) is obviously those who immigrated and hence, united with their citizens in their home country they constitute a certain fixed set of individuals (N). The problem of stereotyping here turns into one of an imperfect observation we mentioned in our worked. Furthermore it is important to underline that not only the weight (representativeness) of the coalitions of immigrants are crucial here but also the prudence of the natives who observe the immigrants from a country and bring an overall judgement over the whole individuals of the country.

3.2 Further Extensions

The model we propose in this work can be extended to various forms. One particular way of future research can be analyzing the behaviour of perceptions via a sequence of observation i.e. *the observer meets with members of the society in a particular sequence and updates his stereotype.* This would be interesting in two aspects. One of them is that this approach would involve a dynamic setting and hence would allow the observer to adjust her stereotype, second aspect is that although the trait vector of the society is fixed, the order -sequence- that the individuals are observed would probably matter in terms of stereotype outcome.

[14] I would ,here, particularly thank to Nicholas Baigent for his comments and examples about perception and stereotyping over immigrants.

This encaptures a wider explanation for stereotypes. Through this approach, the learning literature can also be encaptured in the picture. This entails the essence of stereotyping by experiencing.

Another extension of our model could be analyzing the traits. From the very beginning we assumed the effect of the trait intangible so as to say "what the trait is" did not really matter. Yet, in various scenarios the meaning of the trait could matter. Although this could violate the trait neutrality, it is already violated in many real life scenarios.

4 References

Arrow, K.J., 1951. Social Choice and Individual Values. John Wiley, New York.

Asan, G., Sanver, M.R., 2002. Another characterization of the majority rule. Economics Letters 75 (3), 409–413.

Asan, G., Sanver, M.R., 2006, Maskin monotonic aggregation rules. Economics Letters 91 (2), 179-183.

Campbell, D.E., Kelly, J.S., 2000. A simple characterization of majority rule. Economic Theory 15, 689–700.

Judd, C. M., Ryan, C.S., & Park, B. 1991. Accuracy in the judgment of in-group and outgroup variability. Journal of Personality and Social Psychology, 61, 366-379.

Katz, D., & Braly, K. 1933. Racial stereotypes on one hundred college students. Journal of Abnormal and Social Psychology, 28, 280-290.

Krueger, J., 1996. Probabilistic National Stereotypes. European Journal of Social Psychology. 26 (6), 961-980.

Lippmann, W. 1922. Public opinion. New York: MacMillan.

May, K., 1952. A set of independent, necessary and sufficient conditions for simple majority decision. Econometrica 20,680–684.

Woeginger, G., 2003. A new characterization of the majority rule. Economics Letters 81, 89–94.

Yi, J., 2005. A complete characterization of majority rules. Economics Letters 87, 109–112.