NOTES IN SOCIAL CHOICE THEORY

LEVENT KUTLU
103622011

ISTANBUL BILGI UNIVERSITY
INSTITUTE OF SOCIAL SCIENCES
MASTER OF SCIENCE IN ECONOMICS

UNDER SUPERVISION OF
PROF. DR. M. REMZİ SANVER
2007

Notes In Social Choice Theory

Sosyal Seçim Kuramı Üzerine Notlar

LEVENT KUTLU

103622011

Prof. Dr. M. Remzi Sanver : ……………………………………..

Doç. Dr. İpek Özkal Sanver: …………………………………….

Yrd. Doç. Dr. Göksel Aşan : …………………………………….

Tezin Onaylandığı Tarih : ……………………………………..

Toplam Sayfa Sayısı : 29

Anahtar Kelimeler: Keywords:

Anahtar Kelimeler	Keywords
1) Arrowcu imkansızlık	Arrovian impossibility
2) Gibbard-Satterthwaite Teoremi	Gibbard-Satterthwaite Theorem
3) Manipule edilebilirlik	Manipulability
4) Muller-Satterthwaite Teoremi	Muller-Satterthwaite Theorem
5) Maskin monotonluğu	Maskin monotonicity

ABSTRACT

We examine the domain and range results of Arrow's impossibility, Gibbard-Satterthwaite and Muller-Satterthwaite theorems. We state an impossibility result and two possibility results for Arrow's theorem depending on the range of the social choice mechanisms that we consider. We give a sufficient domain for Muller-Satterthwaite type dictatoriality to prevail. Moreover, we remark that this condition is also sufficient for superdictatoriality.

ÖZETÇE

Arrow'un imkansızlık, Gibbard-Satterthwaite ve Muller-Satterthwaite teoremleri için tanım ve değer kümelerini değiştirmenin etkileri incelenmiştir. Ele aldığımız sosyal seçim mekanizmalarının değer kümelerine bağlı olarak bir imkansızlık ve iki imkanlılık teoremi belirtilmiştir. Muller-Satterthwaite teoremi için diktatörlüğün sürdüğü yeterli bir tanım kümesi belirtilmiştir. Ayrıca, bu şart süperdiktatörlük için de yeterlidir.

ACKNOWLEDGEMENTS

I want to give my special thanks to M. Remzi Sanver. He was always more than a supervisor and a teacher to me. Without him I could not make even a single step in my academic life. Without hesitation I would say that it was one of my luckiest day in my life when I met him first. Again thank you Remzi for everything.

I want to thank Göksel Aşan for not only being in my thesis jury but also for helping me for any kind of problems that I had. Even in the times that he was very busy he always found time to anwer my questions or give his valuable advices.

Although she had a tight schedule, she was so nice to take place in my thesis jury. Thank you İpek Özkal-Sanver.

I greatly appreciate my dear friend Alper Nakkaş' efforts for helping me to get through the proofs that I made in chapter 6 of my thesis.

Table of Contents

1. INTRODUCTION

In 1951 by the revolutionary Ph.D. dissertation of Kenneth Arrow, Social Choice and Individual Values, social choice theory became a new branch of social choice theory. The most stricking part of his thesis was so called Arrow's impossibility theorem which states that no voting system can possibly aggregate individuals' preferences given some "reasonable" criterias.

Gibbard-Satterthwaite theorem was another turning point of Social Choice Theory. Gibbard (1973) and Satterthwaite (1975) separately proved that it is impossible to find a non-manipulable and non-dictatorial social choice mechanisms. A very important result of Gibbard-Satterthwaite theorem was another impossibility theorem by Muller and Satterthwaite (1977) so called Muller-Satterthwaite theorem.

It is certain that there is a close relation between all these three theorems. In all of these theorems it is assumed that individuals may have any rational preference over the alternatives they are confronting. If we drop this assumption it is well-known that impossibility results may not prevail.

Another common point of these theorems is that none of them allows ties in the choice set. In Gibbard-Satterthwaite and Muller-Satterthwaite theorems only singletons can be chosen and in Arrow's theorem only a single representative is allowed to be chosen.

My thesis examines the relation between these three theorems by not only making a related literature review of all of them but also exploring some new results that are related to this literature.

2. BASIC CONCEPTS

Given a non-empty finite set S binary relation β on S is a subset of the direct product $S^2 = S \times S$, i.e., of the set of all ordered pairs (x, y), such that $x, y \in S$. Whenever $(x, y) \in \beta$, we denote as $x\beta y$.

Definition 2.0.1. The binary relation β is *complete*, iff for all $x, y \in S$, $x\beta y$ or $y\beta x$.

Definition 2.0.2. The binary relation β is *transitive*, iff for all $x, y, z \in S$, $x\beta y$ and $y\beta z \Rightarrow x\beta z$.

Definition 2.0.3. The binary relation β is *anti-symmetric*, iff for all $x \in S$ and $y \in S \setminus \{x\}$ $x\beta y \Rightarrow$ not $y\beta x$.

Consider a society N={1,...,n} with n≥2 confronting a finite set of alternatives $A = \{a_1, a_2, ..., a_m\}$ with $m \geq 3$. We write Π for the set of complete, transitive and anti-symmetric binary relations over A. We will call any $\rho \in \Pi$ a *preference relation.* We say that an alternative x is preferred to y by individual i if $x\rho_i y$. The strict counter part of a preference relation will be denoted by $\rho^*, \rho^{*\prime}, \rho_i^*, \rho_i^{*\prime}$, etc…[1] A *preference profile* is an n-tuple $\underline{\rho} \in \Pi^N$ of individual preferences.

We let $D \subseteq \Pi$ stand for arbitrary non-empty subdomains of Π. Let Ω be the set of complete and transitive binary relations over A. A *social welfare function* (SWF) defined over D is a mapping $\alpha : D^N \to \Omega$. So, given any $\underline{\rho} \in D^N$, $\alpha(\underline{\rho})$ is a complete and transitive ordering over A. We write $\alpha^*(\underline{\rho})$ for the strict counterpart of $\alpha(\underline{\rho})$. A *social choice function* (SCF) defined over D is a mapping $f: D^N \to A$. So, at each $\underline{\rho} \in D^N$, a SCF picks a single alternative

[1] For any $x, y \in A$, we say $x\rho^* y$ iff $x\rho y$ and not $y\rho x$ .And similar for others.

from A. A *social choice correspondence* (SCC) defined over D is a mapping $\mathrm{F}:\mathrm{D}^{\mathrm{N}} \to \underline{A}$. Hence, at each $\underline{\rho} \in D^N$ a SCC picks one or more alternative from A.

3. ARROW'S IMPOSSIBILITY THEOREM

3.1 Basics of Arrow's Theorem

Definition 3.1.1. A SWF $\alpha : D^N \to \Omega$ is said to be *Pareto optimal* (PO) iff given any $x \in A$, any $y \in A \setminus \{x\}$ and any $\underline{\rho} \in D^N$ with $x\rho_i y$ for all $i \in N$, we have $x\alpha^*(\underline{\rho})y$. In other words, if everyone in the society ranks x over y, so should the SWF.

Definition 3.1.2. A SWF $\alpha : D^N \to \Omega$ is said to be *independent of irrelevant alternatives* (IIA) iff given any $x \in A$, any $y \in A \setminus \{x\}$ and any $\underline{\rho}, \underline{\rho}' \in D^N$ with $x\rho_i y \Leftrightarrow x\rho_i' y$ for all $i \in N$, we have $x\alpha^*(\underline{\rho})y \Leftrightarrow x\alpha^*(\underline{\rho}')y$. In other words, a SWF must order two distinct alternatives x and y, just by considering each individual's preference over only x and y.

Definition 3.1.3. A SWF $\alpha : D^N \to \Omega$ is said to be *dictatorial* over the domain D if there exists $d \in N$ such that given any given any $x \in A$, any $y \in A \setminus \{x\}$ and any $\underline{\rho} \in D^N$, we have $x\rho_d y \Rightarrow x\alpha^*(\underline{\rho})y$. The individual d is said to be the *dictator* of α. We say α is *non-dictatorial* (ND) whenever it is not dictatorial.

Definition 3.1.4. A SWF is *Arrovian* iff it satisfies PO, IIA and ND.

Theorem 3.1.1. [Arrow (1951)][2]: There exists no Arrovian SWF $\alpha : \Pi^N \to \Omega$ which is PO, IIA and ND. [3]

[2] See Geanakoplos (2001) for three recent elegant proofs.

[3] In fact antisymmetricity is not necessary for Arrow's theorem. However we will always use this property in the rest of this work.

Note that although at a first glance only PO, IIA and ND assumptions seem to be the mere reasons for dictatoriality, there are other hidden assumptions that Arrow made.

1) The *society is finite and fixed*.[4]

2) The *set A of alternatives* with $|A| \geq 3$ *has no structure*. There are studies imposing topological or algebraic structure on A.[5]

3) The *range* of the SWF is the set of *complete and transitive* orderings on A.

4) The *domain* of the SWF is *unrestricted*. In other words, every rational (complete and transitive binary relation) ordering is admissible for each individual in the society.

Throughout the paper we will concentrate on domain conditions. So it is good to review some literature about domain conditions on Arrow's impossibility theorem.

3.2 Domain Results for Arrow's Theorem

As we stated universal domain assumption is one of the important assumptions that lead to impossibility. There has been an extensive study for exploring plausible ways to escape or preserve this impossibility result by imposing some domain conditions. There seems to be two ways of restricting the universal domain. We can either apply certain restrictions to possible preference profiles or to possible preference orderings. Among the possibility results most of them were dealing the impossibility result through finding some conditions that prevents majority rule failing to be transitive such as the *single-peakedness* [Black (1948)], the *single-cavedness* [Inada (1964)] , the *value-restriction* [Sen (1966)] and *limited agreement* [Sen and Pattanaik (1969)].[6] Another common point of these conditions is that they are restrictions on the preference profile.

[4] See Fishburn (1970) for a result that Arrow's theorem does not hold with infinitely many individuals. Also, See Kirman and Sondermann (1972) for a result which is closer to Arrow's theorem for a society with infinite number of individuals.
[5] See Le Breton and Weymark (2002) for a treatment of the aggregation problem in economic environments.
[6] It is worth to mention that value restriction condition generalizes single-peakedness and single-cavedness conditions.

While finding a way to avoid intransitivity for majority rule is the centeral idea of dealing with the impossibility result, the *free triple condition* [Blau (1957) and Arrow (1963)] is its counterpart in finding *dictatorial domains*[7]. Since this condition will also be very important for our analysis it is worth to mention it. A *triple*[8] of alternatives is said to be free in a given domain of preferences if and only if this domain admits all possible orderings of these three alternatives. Hence, a triple is free in a given domain if the restriction of the domain to the triple is the full domain. After Blau (1957) and Arrow (1963) proved that free triple condition is sufficient for dictatoriality Kalai and Muller (1977) fully characterized the non-dictatorial domains with the *non-dictatorial decomposability condition*. This condition was so messy to work with that Kalai, Muller and Satterthwaite (1979) found a stronger but more effective condition, *saturation condition*, that is sufficient for dictatoriality. Ozdemir and Sanver (2006) introduced a weaker condition called *essential saturation condition*. In a different context Dogan and Sanver (2006) proved that Arrow's theorem prevails on domains that include the union of sets of leximin and leximax preferences.[9]

Lastly, we would like to mention a useful superdictatoriality result due to Ozdemir and Saver (2006).

Definition 3.2.1. A pair of alternatives $\{x, y\}$ is said to be *non-trivial* in the domain D iff there exist $R, R' \in D$ such that xRy and $yR'x$.

Theorem 3.2.1. Let D be a domain where every pair $\{x, y\}$ is non-trivial. If D is α-dictatorial then D is α-superdictatorial as well.[10]

3.3 Range Results for Arrow's Theorem

To our knowledge there is no result published in the literature that gives a dictatoriality result in a different range than that of Arrow's. First, note that completeness and transitivity plays a vital role to get the impossibility. If any one of them does not hold there are well-known examples that lead to possibility. Actually, even in the quasi-transitive and complete case

[7] Domains that prevail dictatoriality for any Arrovian SWF defined on will be called *dictatorial*.
[8] From now on, by a triple, we mean three distinct alternatives.
[9] Given any set of alternatives A. They work with the non-subsets of A instead of alternatives themselves.
[10] From now on when it is necessary we will use α, γ and μ letters as a prefix to make it clear that we are in the framework of Arrow, Gibbard-Satterthwaite and Muller-Satterthwaite respectively.

there are examples that gives possibility results. Thus, unless we impose more restrictive assumptions on SWF, impossibility result is no longer valid in this kind of ranges. As there is no research dealing with range conditions, it is our interest to investigate impossibility or possibility results due to a variety of range conditions. Although we sometimes have to choose one representative of the society this is not always the case and we might need to choose a committee instead of a single representative as it is the case for a parliament. Hence, we deal with ranges that accept a committee. First, we consider the case that number of individuals is fixed. Then we allow them to vary though it turns out that this option is not possible for our setup. The framework and proofs will be given in chapter 6.

4. GIBBARD-SATTERTHWAITE THEOREM

4.1 Basics of Gibbard-Satterthwaite Theorem

Definition 4.1.1. A SCF $f: D^N \to A$ is said to be *manipulable* iff there exist $\underline{\rho} \in D^N$, $i \in N$ and $\rho_i^* \in D$ such that $f(\rho_i', \rho_{-i}) \rho_i^* f(\rho_i, \rho_{-i})$. A SCF is *strategy-proof (SP)* if it is not manipulable.

Definition 4.1.2. A SCF $f: D^N \to A$ is said to be *unanimous* iff given any $\underline{\rho} \in D^N$ such that $\max(\rho_i; A) = \{a\}$ for all $i \in N$, $f(\underline{\rho}) = a$.[11]

Definition 4.1.3 A SCF $f: D^N \to A$ is said to be *onto* iff for every $a \in A$ there exist $\underline{\rho} \in D^N$ such that $f(\underline{\rho}) = a$.

Definition 4.1.4. A SCF $f: D^N \to A$ is said to be *dictatorial* over the domain D iff there exist $d \in N$ such that given any $\underline{\rho} \in D^N$, $f(\underline{\rho}) \in \max(\rho_d; A)$.

[11] $\max(\rho_i; A)$ stands for the set of maximal elements in A according to the preference relation ρ_i

Thanks to Gibbard (1973) and Satterthwaite (1975) we know that a social choice function which is strategy-proof and onto can not be non-dictatorial given that there are at least three alternatives.[12]

Theorem 4.1.1. [Gibbard (1973), Satterthwaite (1975)][13]**:** A SCF $f : \Pi^N \to A$ is strategy-proof and onto iff *f* is dictatorial. [14]

4.2 Domain Results for Gibbard-Satterthwaite Theorem

The negative conclusion of the Gibbard-Satterthwaite theorem relies critically on the domain of preferences. It is well-known that one may get some possibility results by restricting the domain of preferences. There are conditions of domain restrictions, which let us escape the impossibility. Campbell and Kelly (2003) showed that the majority rule where Condorcet winner is chosen is the unique strategy-proof social choice function when the domain is restricted with the *value-restrictedness* condition. Also a special case of the prior condition, *single-peakedness*, ensures the existence of many non-dictatorial and strategy-proof social choice rules, which are characterized by Moulin (1980), whose results are extended in Barberà et al. (1993). There are also some results that lead to impossibility results. Barberà and Peleg (1990) showed the impossibility using a domain in which Aswal et al. (2003) call *connected*.[15] Then Aswal et al. (2003) showed that *a* strategy-proof and unanimous SCF defined on a *linked domain* should be *superdictatorial*.[16] [17] For the characterization though they note that it is not enough to consider just first two rankings of preferences. Moreover, Barberà et al. (2001) showed a possibility and an impossibility result depending on the domain they select. It is worth to mention that the possibility result that they got was not too far away from a dictatoriality result as they showed that in the corresponding domain a SCF

[12] It is enough to have at least three alternatives in the range of the SCF.
[13] See Lars-Gunnar Svensson (1999), Benoit (2000) and Sen (2001) for some simple proofs.
[14] Note that ontoness can be replaced by unanimity as under strategy-proofness (and minimal richness of the domain which will be defined later) both conditions are equivalent.
[15] Actually they did not formally claim that they proved the result using connected domains but for their proof connectedness was sufficient.
[16] It is worth to mention that in a linked domain satisfies minimal richness so under strategy-proofness ontoness and unanimity are equivalent. Moreover, a connected domain is linked.
[17] A domain D is said to be superdictatorial iff any superset $D \subseteq E$ is also dictatorial.

should be either dictatorial or *bi-dictatorial*. Also, their alternatives consist of not the usual one but the set of non-empty subsets of a given alternative set. [18]

It is very certain that Arrow's theorem and Gibbard-Satterthwaite theorem are closely related and there are not only a variety of parallel proofs for Arrow's theorem and Gibbard-Satterthwaite theorem[19] but also some proofs that generalize both theorems into one.[20] These researches bring us to the question that whether there is a logical relation between domain conditions for each theorem to hold. For this purpose we investigate a family of domains that give impossibility result for both theorems.

Lastly, we would like to mention a superdictatoriality result by Sanver (2006).

Henceforth $r_k(\rho)$ will stand for the alternative ranked k^{th} according to the preference relation ρ.

Definition 4.2.1. We qualify a domain D as *minimally rich* iff for all $x \in A$, there exists $\rho \in D$ such that $r_1(\rho) = x$.

Theorem 4.2.1. A γ-dictatorial domain D is γ-superdictatorial iff D is minimally rich.

4.3 Range Results for Gibbard-Satterthwaite Theorem

There are some "seemingly" plausible ranges like Π and $\underline{A}$ that could be investigated for validity of a Gibbard-Satterthwaite like theorem. But prevalence of Gibbard-Satterthwaite theorem for both cases is a problematic concept. The most important problem seems to stem from the very definition of strategy-proofness. For the first case we can not say anything about how individuals rank a preference given their preferences for the alternatives unless maybe in the very case that their own preference relation is chosen.[21] The situation is similar for the second case, i.e. the correspondence case. In the literature we have some solutions for

---

[18] Actually, this kind of SCF's are called social choice hyperfunction (SCHF). i.e., functions that pick a non-empty set of alternatives at each admissible preference profile over sets of alternatives.
[19] See Reny (2001) Quesada (2005) for parallel proofs Arrow's and Gibbard-Satterthwaite theorem.
[20] See Eliaz (2001).
[21] See Barberà (1977a) for a different approach to deal with a similar problem

the latter case through some additional extension axioms that show ways to order sets given the preferences over alternatives. Loosely speaking, Barberà (1977a) and Kelly (1977) showed that under certain regularity conditions, a strategy-proof social decision function necessarily makes some individuals *weak dictator*[22]. Barberà (1977b) has a dictatoriality result though he had to impose additional strong properties to the rule, unanimity and *positive responsiveness,* in addition to strategy-proofness. Using a stronger strategy-proofness assumption and a unanimity assumption which he calls *non-imposedness* Feldman (1980) showed that a SCC satisfying these properties is obliged to be either dictatorial or bi-dictatorial. Duggan and Schwartz (2000) showed that ontoness, *residually resoluteness*[23] and strategy-proofness is sufficient for dictatoriality. Then Ching and Zhou (2002) showed that given their extension axiom a strategy-proof SCC should be either dictatorial or constant. It is worth to note that their extension axiom leads to an incomplete ordering on the power set of alternatives. They also showed the same result with a domain restricted to continuous preferences. Barberà et al. (2001) considered the domains that are consistent with conditional expected utility maximization, one that considers equal probabilities and the other has no restriction on the probabilities. They showed that under these domains strategy-proofness and unanimity of SCHFs imply dictatoriality or bi-dictatoriality for the former domain and for the latter they imply just dictatoriality. Actually, their result "almost" implies the result of Ching and Zhou (2002). Except these one might also see Fishburn (1972), Gärdenfors (1976), Barberà, Bogomolnaia and Stel (1998), Benoit (2002), Taylor (2005), Ozyurt and Sanver (2006) for related literature.[24][25]

[22] An individual d is a *weak dictator* iff whenever d prefers x to y and the agenda is restricted to x versus y, x must at least tie y. Actually, Barberà (1977a) used a similar definition that he called *oligarchy*.

[23] A condition that requires the choice function to be singleton in certain cases

[24] Note that some of the results given are for social choice rules that map preference profiles to lotteries and some to sets but we did not mention the difference.

[25] See Ranking Sets of Objects, of Barberà, Bossert and Pattanaik, for a very good resourse understanding extension axioms

5. MULLER-SATTERTHWAITE THEOREM

5.1 Basics of Muller-Satterthwaite Theorem

Henceforth $L(a;\rho)$ will stand for the lower contour set of a according to the preference relation ρ. i.e. $L(a;\rho)=\{x\in A: a\rho x\}$.

Definition 5.1.1. A SCF $f: D^N \to A$ is said to be *Maskin monotonic (MM)* iff for all $\rho,\rho'\in D$, $L(a;\rho)\subseteq L(a;\rho')$ and $f(\rho)=a$ implies $f(\rho')=a$.

Theorem 5.1.1. [Muller and Satterthwaite (1977)] A SCF $f:\Pi^N \to A$ is Maskin monotonic and onto iff *f* is dictatorial.

It is worth to note that under minimal richness condition in any domain a Maskin monotonic SCF is unanimous if and only if it is onto. So we can easily replace ontoness condition with unanimity and still preserve the dictatoriality result. The importance of the theorem comes from its close relation with Nash implementability. Maskin (1999)[26] show that a SCF that satisfies Maskin monotonicity and a condition called *no veto power* then it is Nash implementable. Moreover, Nash implementibility implies Maskin monotonicity. Thus, Maskin's result is almost a characterization of Nash implementibility.

The Muller-Satterthwaite theorem was first proven as a consequence of Gibbard-Satterthwaite and Arrow theorems. After this indirect proof some direct and easier proofs given in the literature like that of Moulin (1988), Myerson (1996) and Renny (2001)[27]. There are also some results showing the equivalence between Maskin monotonicity or its variations to strategy proofness. For example, Tanaka (2001) showed equivalence between a monotonicity conditions that he calls *generalized monotonicity* and strategy-proofness. Actually, this condition was nothing but a special case of *independence of person-by-person monotonicity*

[26] The first version of this article circulated as an MIT working paper in 1977. Evidently, Maskin had some difficulties getting his paper published.
[27] See Renny (2001) for a proof of (modified) Muller-Satterthwaite and Arrow's theorem is given in a parallel fassion.

(IPM)[28] that Dasgupta et al. (1979) introduced. IPM is a condition for SCCs but if we restrict the definition to singletons it turns out that these two definitions are same. Moreover, it is well-known that Dasgupta et al. (1979) proved the equivalence of IPM with strategy proofness.

5.2 Domain Results for Muller-Satterthwaite Theorem

A similar close relation between Arrow's theorem and Gibbard-Satterthwaite theorem prevails between Muller-Satterthwaite and Gibbard-Satterthwaite theorem so it is a plausible question to ask for a domain condition that leads all these theorems to hold at a time. But it is unfortunate that to our knowledge there is no literature that deals with the domain conditions of Muller-Satterthwaite theorem. Moreover, although we can state a small theorem to give some rough idea about sufficient domains for μ-dictatoriality and a superdictatoriality result, this result does not help us much to make such a connection. Thus, this question is still waiting for a satisfactory answer.

Definition 5.2.1. A SCF $f:\Pi^N \to A$ is said to satisfy *independent person-by-person monotonicity (IPM)* iff $\forall \underline{\rho} \in \Pi^N$, $\forall i \in N$, $\forall \rho_i \in \Pi$, $\forall \{a,b\} \subseteq A$, if $f(\rho_i, \rho_{-i}) = a$ and $a\rho_i b \Rightarrow a\rho_i{}^* b$ then $f(\rho_i{}', \rho_{-i}) \neq b$.

Definition 5.2.2. For a set of alternatives A, the domain $D \subseteq \Pi$ is said to be *rich* iff $\forall \rho, \rho' \in D$ and $\forall \{a,b\} \subseteq A$ such that $a\rho b \Rightarrow a\rho' b$ and $a\rho^* b \Rightarrow a\rho^{*\prime} b$, then there exists $\rho'' \in D$ such that $L(a;\rho) \subseteq L(a;\rho'')$ and $L(b;\rho') \subseteq L(b;\rho'')$.

Definition 5.2.3. A pair of alternatives a_j, a_k is said to be connected, denoted $a_j \sim a_k$, if there exists $\rho, \rho' \in D$ such that $r_1(\rho) = a$, $r_2(\rho) = b$, $r_1(\rho') = b$ and $r_2(\rho') = a$.

Definition 5.2.4. Let $B \subset A$ and $a_j \notin B$. Then a_j is linked to B if there exists $a_k, a_r \in B$ such that $a_k \sim a_j$ and $a_r \sim a_j$.

[28] IPM will be defined in next section.

Definition 5.2.5. The domain D is linked if there exists a one to one function $\sigma : A \to A$ such that

i) $a_{\sigma(1)} \sim a_{\sigma(2)}$

ii) $a_{\sigma(j)}$ is linked to $\{a_{\sigma(1)},...,a_{\sigma(j-1)}\}$, $j = 3,...,m$

Lemma 5.1.1. [Theorem 4.3.1. Dasgupta et al. (1979)] Let $D \subseteq \Pi$. A SCF $f : D^N \to A$ is strategy-proof iff it is IPM.

Lemma 5.1.2. [Corollary 3.2.3. Dasgupta et al. (1979)] Let $D \subseteq \Pi$ be a rich domain. Then Maskin monotonicity and IPM are equivalent in a SCF .

Lemma 5.1.3. [Theorem 3.1. Aswal et al. (2003)] Let $D \subseteq \Pi$ be a linked domain then D is μ-dictatorial. [29]

Theorem 5.1.1. Let $D \subseteq \Pi$ be a linked and rich domain then D is μ-dictatorial.

Proof: Result directly follows from the lemmas above.

Corollary 5.1.2. Let $D \subseteq \Pi$ be a linked and rich domain then D is μ-superdictatorial.

Proof: Follows directly from Theorem 5.1.1 and Theorem 4.2.1

5.3 Range Results for Muller-Satterthwaite Theorem

Like the Gibbard-Satterthwaite case the results are centered at SCCs instead of SWFs. As the structure of SCCs are distinct from that of SCFs a new kind of monotonicity definition needed. The results relating this part are generally concentrated on finding a monotonicity condition that is equivalent to strategy-proofness conditions that were made in the literature. There is not much study going on related to this session we just mention Nehring (1998) and Tanaka (2003) as some reference and leave the rest to the reader.

[29] As a linked domain is also minimally rich it is no harm to replace unanimity assumption of Aswal et al. (2003) with ontoness.

6. CHOOSING A COMMITTEE

Arrow's impossibility theorem is one of the most disappointing theorems of social choice. But it only considers the special case of choosing a single "opinion" to represent the society as a whole. What happens if it could be chosen more than a single opinion? One reason for dealing with this question could be that decision mechanism may not give a precise answer but instead recommend some possible candidate opinions. Another possibility is that it could be the case that we need to choose an opinion from each part of a region and gather a set of members to represent the whole region as a whole. One important example for this is choosing the parliament's members to represent all the society. So it seems this kind of generalizations of Arrow's theorem has meaningful applications. But there seems to be more than one way of generalizing Arrow's theorem in this sense and we examine all possible cases that we can think of. Our analysis consists of three parts.

6.1 Range as a Vector of Individuals: A Dictatoriality Result

In this part we give a series definitions and a dictatoriality theorem that follows directly from Arrow's theorem itself.

Definition 6.1.2. A *generalized social welfare function (GSWF)* is a mapping $\alpha : \Pi^N \to \bigcup_{t=1}^{T} \Omega^t$.

Definition 6.1.2. A GSWF $\alpha : \Pi^N \to \bigcup_{t=1}^{T} \Omega^t$ is said to be *Pareto optimal (PO)* iff whenever $\forall \{x, y\} \subseteq A$, $\forall i \in N$, $x \rho_i y$ implies $\alpha_{\{x,y\}}(\underline{\rho}) = \begin{pmatrix} x & x & \dots & x \\ y & y & & y \end{pmatrix}$ where $\alpha_{\{x,y\}}$ is the restriction of α to x and y.[30]

---

[30] By $\begin{matrix} x \\ y \end{matrix}$ we mean x is preferred to y and by xy we mean x and y are indifferent. For technical simplicity we will not use "," to separate preferences in our notation.

Definition: 6.1.3. A GSWF $\alpha:\Pi^N \to \bigcup_{t=1}^{T}\Omega^t$ is said to satisfy *independence of irrelevant alternatives condition (IIA)* if for any pair of alternatives $\{x,y\}\subseteq A$ and for any pair of preference profiles $\underline{\rho},\underline{\rho}'\in\Pi^N$ with the property that, $\forall i\in N$, $x\rho_i y \Leftrightarrow x\rho_i^{'} y$ we have that $\alpha_{\{x,y\}}(\underline{\rho})=\alpha_{\{x,y\}}(\underline{\rho}')$.

Theorem 6.1.1. Let $\alpha:\Pi^N \to \bigcup_{t=1}^{T}\Omega^t$ be a GSWF satisfying IIA. Then $\exists K\in\{1,...,T\}$ and a mapping $\widehat{\alpha}:\Pi^N\to\Omega^K$ such that $\alpha=\widehat{\alpha}$. i.e. Imposing IIA condition prevents number of committee members to vary.

Proof: Let $\{x,y,z\}\subseteq A$ be any triple of alternatives and $\underline{\rho}\in\Pi^N$ be an arbitrary but fixed preference profile. Now, let $\underline{\rho}',\underline{\rho}''\in\Pi^N$ be any preference profiles with the property that, $\forall i\in N$, $x\rho_i y\Leftrightarrow x\rho_i^{'} y$, $z\rho_i x\Leftrightarrow z\rho_i^{''} x$, and $z\rho_i y\Leftrightarrow z\rho_i^{'} y\Leftrightarrow z\rho_i^{''} y$. It can be easily checked that such preference profiles always exist. Then by IIA we have that $\alpha_{\{x,y\}}(\underline{\rho})=\alpha_{\{x,y\}}(\underline{\rho}')$, $\alpha_{\{x,z\}}(\underline{\rho})=\alpha_{\{x,z\}}(\underline{\rho}'')$ and $\alpha_{\{y,z\}}(\underline{\rho})=\alpha_{\{y,z\}}(\underline{\rho}')=\alpha_{\{y,z\}}(\underline{\rho}'')$. Let $\left\langle\alpha(\underline{\rho})\right\rangle$ stand for the number of components of α given that the preference profile is $\underline{\rho}$. Thus, we conclude that $\left\langle\alpha_{\{x,y\}}(\underline{\rho})\right\rangle=\left\langle\alpha_{\{x,z\}}(\underline{\rho})\right\rangle=K$ for some $K\in\{1,...,T\}$. Hence, by IIA result follows immediately.

Hence, from now on we will denote a GSWF as a mapping $\alpha:\Pi^N\to\Omega^K$ by implicitly assuming that it satisfies IIA.

Definition 6.1.4. A GSWF $\alpha:\Pi^N\to\Omega^K$ is said to be *dictatorial for* j^{th} *component* iff for j^{th} component of α, $\exists\ d\in\left\{1,2,3,...,N\right\}$ such that $x\rho_d y\Rightarrow x\alpha_j{}^*(\underline{\rho})y$

Definition 6.1.5. A GSWF $\alpha:\Pi^N\to\Omega^K$ is said to be *dictatorial* iff for any $j\in\left\{1,2,3,...,N\right\}$, α is dictatorial for j^{th} component.

Theorem 6.1.2. Any GSWF $\alpha:\Pi^N\to\Omega^K$ satisfying PO and IIA has to be dictatorial.

Proof: Let j be an arbitrary component of α. By Arrow's impossibility theorem we conclude that α is dictatorial for j^{th} component. As j were arbitrary dictatoriality of α follows immediately.

6.2 Range as a Set of Individuals: An Oligarchy Result

We give a possibility result for Arrow's theorem. Our result can be thought as an Arrow's theorem version of the result of Tanaka (2003) in which he gave an oligarchy result for SCCs in Gibbard-Satterthwaite environment.

Definition 6.2.1. A *social welfare correspondence (SWC)* is a mapping $\alpha : \Pi^N \to 2^{\Omega} \setminus \{\varnothing\}$.

Definition 6.2.2. A SWC $\alpha : \Pi^N \to 2^{\Omega} \setminus \{\varnothing\}$ is said to be *Pareto optimal (PO)* iff whenever $\forall \{x, y\} \subseteq A$, $\forall i \in N$, $x\rho_i y$ implies $\alpha_{\{x,y\}}(\underline{\rho}) = \begin{Bmatrix} x \\ y \end{Bmatrix}$.

Definition: 6.2.3. A SWC $\alpha : \Pi^N \to 2^{\Omega} \setminus \{\varnothing\}$ is said to satisfy *independence of irrelevant alternatives condition (IIA)* if for any pair of alternatives $\{x, y\} \subseteq A$ and for any pair of preference profiles $\underline{\rho}, \underline{\rho}' \in \Pi^N$ with the property that, $\forall i \in N$, $x\rho_i y \Leftrightarrow x\rho_i^{'} y$ we have that $\alpha_{\{x,y\}}(\underline{\rho}) = \alpha_{\{x,y\}}(\underline{\rho}')$.

Remark 6.2.1. By PO it is trivial that the only possible setup for a committee that has a fixed number of members is the case of choosing singleton which is nothing but Arrow's case. Thus, we will only be interested in SWCs.

Definition 6.2.4. A SWC $\alpha : \Pi^N \to 2^{\Omega} \setminus \{\varnothing\}$ is said to be *dictatorial* iff there exist a set $K \subseteq N$ such that for any preference profile $\underline{\rho} \in \Pi^N$ and for any individual $i \in K$ there exists an ordering $\rho' \in \alpha(\underline{\rho})$ with, $\forall \{x, y\} \subseteq A$, $x\rho_i y \Rightarrow x\rho^{*\prime} y$.

First, we give an example of a SWC that is not dictatorial.

For simplicity consider a three person and three alternative world and following SWC: $\alpha(\underline{\rho}) = \bigcup_{i=1}^{N} \{\rho_i\}$ for $\underline{\rho} \neq \begin{Bmatrix} a & a & b \\ b & c & c \\ c & b & a \end{Bmatrix}$ and for $\underline{\rho} = \begin{Bmatrix} a & a & b \\ b & c & c \\ c & b & a \end{Bmatrix}$ let $\alpha(\underline{\rho}) = \begin{Bmatrix} a & c \\ b & b \\ c & a \end{Bmatrix}$ then we see that α is not dictatorial.

Definition 6.2.5. A SWC $\alpha : \Pi^N \to 2^{\Omega} \setminus \{\varnothing\}$ is said to be *pairwise oligarchical* iff there exists $\varnothing \neq K \subseteq N$ such that $\forall \underline{\rho} \in \Pi^N$, $\forall \{x, y\} \subseteq A$, $\forall i \in K$, $x \rho_i y \;\Rightarrow \begin{matrix} x \\ y \end{matrix} \in \alpha_{\{x,y\}}(\underline{\rho})$. The maximal K that makes α pairwise oligarchical is said to be *pairwise oligarchy*.

Definition 6.2.6. Given a SWC $\alpha : \Pi^N \to 2^{\Omega} \setminus \{\varnothing\}$, we say that a subset of agents $S \subseteq N$ is

i) *Decisive for x over y* if whenever every agent in S prefers x *to* y and every agent not in S prefers y to x we have $\begin{matrix} x \\ y \end{matrix} \in \alpha_{\{x,y\}}(\underline{\rho})$

ii) *Decisive* if for any pair $\{x, y\} \subseteq A$, S is decisive for x over y.

iii) *Completely decisive for x over y* if whenever every agent in S prefers x *to* y we have $\begin{matrix} x \\ y \end{matrix} \in \alpha_{\{x,y\}}(\underline{\rho})$

iv) *Completely decisive* if for any pair $\{x, y\} \subseteq A$, S is completely decisive for x over y.

Lemma 6.2.1. Let $\alpha : \Pi^N \to 2^{\Omega} \setminus \{\varnothing\}$ be a SWC that is PO and IIA. Then there is an $h \in N$ such that S={h} is completely decisive.

Proof: We will show our lemma in 5 steps.

Step1: If for some $\{x, y\} \subseteq A$, $S \subseteq N$ is decisive for x over y, then for any $z \neq x$, S is decisive for x over z. Similarly, we have for any $z \neq y$, S is decisive for z over y.

If $z = y$ there is nothing to prove. So assume that $z \neq y$. By IIA it is enough to find a profile of preferences $\underline{\rho} \in \Pi^N$ such that $\begin{matrix} x \\ z \end{matrix} \in \alpha_{\{x,z\}}(\underline{\rho})$. Now consider the following profile of preferences $\underline{\rho} \in \Pi^N$ where

$x\rho_i y\rho_i z \quad \forall \mathrm{i} \in \mathrm{S}$

$y\rho_i z\rho_i x \quad \forall \mathrm{i} \in N \setminus \mathrm{S}$

We know that S is decisive x over y so $\begin{matrix} x \\ y \end{matrix} \in \alpha_{\{x,y\}}(\underline{\rho})$. Also by PO we have that $\alpha_{\{y,z\}}(\underline{\rho}) = \begin{Bmatrix} y \\ z \end{Bmatrix}$. Then we should have $\begin{matrix} x \\ z \end{matrix} \in \alpha_{\{x,z\}}(\underline{\rho})$. This shows that S is decisive for x over z, $\forall \mathrm{z} \neq x$. Note that "=" in $\alpha_{\{y,z\}}(\underline{\rho}) = \begin{Bmatrix} y \\ z \end{Bmatrix}$ was very important for us to get our transitivity like result.

Similarly, one can show that for any $z \neq y$, S is decisive for z over y.

Step2: If $S \subseteq N$ is decisive for x over y for some pair $\{x, y\} \subseteq A$, then S is decisive.

Let z be an alternative which is distinct from x and y also let w be an alternative which is distinct from z. By step1 we know that S is decisive for z over y. So applying step1 we have that S is decisive for z over w. Now, let $\{a,b\} \subseteq A$ be arbitrary. If a=z or b=z the result follows immediately. So assume that $\mathrm{a} \neq \mathrm{z}$ and $\mathrm{b} \neq \mathrm{z}$. Since S is decisive for z over w, we have S is decisive for z over a. But then S is decisive for b over a. As $\{a,b\}$ was arbitrary we conclude that S is decisive.

Step3: There exists a decisive set.

Let $S \subseteq N$ be arbitrary. Take an arbitrary triple $\{x, y, z\} \subseteq A$ and consider a profile of preferences $\underline{\rho} \in \Pi^N$ where

$x\rho_i z\rho_i y \quad \forall \mathrm{i} \in \mathrm{S}$

$y\rho_i x\rho_i z \quad \forall \mathrm{i} \in N \setminus \mathrm{S}$

By PO we know that $\alpha_{\{x,z\}}(\underline{\rho}) = \begin{Bmatrix} x \\ z \end{Bmatrix}$. If $\begin{matrix} x \\ y \end{matrix} \in \alpha_{\{x,y\}}(\underline{\rho})$ S is decisive so we are done. If $\begin{matrix} x \\ y \end{matrix} \notin \alpha_{\{x,y\}}(\underline{\rho})$ then we have that $\begin{matrix} y \\ z \end{matrix} \in \alpha_{\{y,z\}}(\underline{\rho})$ but this means $N \setminus \mathrm{S}$ is decisive. So for any set $S \subseteq N$ we have either S or $N \setminus \mathrm{S}$ is decisive. So there exists a decisive set.

Step4: If $S \subseteq N$ is decisive then, S is completely decisive.

Let $\{x, y\} \subseteq A$ be an arbitrary pair. All we need is to show that $\forall T \subseteq N \setminus \mathrm{S}$ we have $\begin{matrix} x \\ y \end{matrix} \in \alpha_{\{x,y\}}(\underline{\rho})$ whenever every agent in S and T prefers x to y and every other agent prefers y to x. In order to show this consider a profile of preferences $\underline{\rho} \in \Pi^N$ such that

$x\rho_i z\rho_i y \quad \forall \mathrm{i} \in \mathrm{S}$

$z\rho_i x\rho_i y \quad \forall \mathrm{i} \in T$

$z\rho_i y\rho_i x \quad \forall \mathrm{i} \in N \setminus (S \cup T)$

Since S is decisive $\begin{matrix} x \\ z \end{matrix} \in \alpha_{\{x,z\}}(\underline{\rho})$. Also by PO we have $\alpha_{\{y,z\}}(\underline{\rho}) = \begin{Bmatrix} z \\ y \end{Bmatrix}$ so we have $\begin{matrix} x \\ y \end{matrix} \in \alpha_{\{x,y\}}(\underline{\rho})$. Hence S is completely decisive for x over y. But x and y were arbitrary so S is completely decisive.

Step5: There exist a completely decisive set which is a singleton.

Let $S \subseteq N$ be decisive. We know that there exists such a set by step3. Let $h \in S$ be arbitrary. If $S \setminus \{h\}$ is decisive we are done. So assume not. Consider a profile of preferences $\underline{\rho} \in \Pi^N$ such that

$z\rho_i x\rho_i y \quad \forall \mathrm{i} \in \mathrm{S} \backslash \{\mathrm{h}\}$

$x\rho_i y\rho_i z \quad \forall \mathrm{i} \in \{h\}$

$y\rho_i z\rho_i x \quad \forall \mathrm{i} \in N$

Since S is decisive $\begin{matrix} x \\ y \end{matrix} \in \alpha_{\{x,y\}}(\underline{\rho})$. If $\begin{matrix} z \\ y \end{matrix} \in \alpha_{\{y,z\}}(\underline{\rho})$ then $S \setminus \{h\}$ is decisive so we would get a contradiction. So $\begin{matrix} z \\ y \end{matrix} \notin \alpha_{\{y,z\}}(\underline{\rho})$. But in any case we will have $\begin{matrix} x \\ z \end{matrix} \in \alpha_{\{x,z\}}(\underline{\rho})$. So $\{h\}$ is decisive. Thus we conclude that if S is decisive then $\{h\}$ or $S \setminus \{h\}$ is also decisive. If $\{h\}$ is decisive we are done. So assume not. Then $S \setminus \{h\}$ is decisive. Now, let $h' \in S \setminus \{h\}$ be arbitrary and apply same argument above. Hence, $\{h'\}$ or $S \setminus \{h, h'\}$ is decisive. Continuing this way we conclude that there exist a completely decisive set which is a singleton as to be shown.

Theorem 6.2.1. Any SWC $\alpha : \Pi^N \to 2^\Omega \setminus \{\varnothing\}$ satisfying PO and IIA has to be pairwise oligarchical.

Proof: Let $\alpha : \Pi^N \to 2^\Omega \setminus \{\varnothing\}$ be an arbitrary SWC that satisfies IIA and PO. Consider all decisive sets and we know that for each of these sets there is at least one completely decisive singleton set. Take the union of these singleton sets and call this union K. It is obvious that K is a pairwise oligarchy which completes the proof.

Remark 6.2.2. Note that for any given SWC $\alpha : \Pi^N \to 2^\Omega \setminus \{\varnothing\}$ satisfying PO and IIA it is always possible to find another SWC $\hat{\alpha} : \Pi^N \to 2^\Omega \setminus \{\varnothing\}$, with $\forall \underline{\rho} \in \Pi^N \ \left|\hat{\alpha}(\underline{\rho})\right| \leq 2$, that is *IIA equivalent* to α. Loosely speaking, we say that two SWCs are IIA equivalent if IIA can not detect the difference between them.

6.3 Range as a Multiset of Individuals: An Oligarchy Result

Although the definitions and the results are almost the same as that of section 6.2, for the sake of completeness we state them.

A *multiset* can be formally defined as a set pairs $M = \{(a, \psi(a)) : a \in S\}$ where S is some set and $\psi : S \to \mathbb{N}^+$ is a function from S to the set $\mathbb{N}^+$. The set S is called the *underlying set of elements*. For each $a \in S$ the *multiplicity* (that is, number of occurrences) of a is the number $\psi(a)$. We denote a multiset just like a set that allows repeatation of its elements. For example, the multiset $\{(a,1),(b,2)\}$ is written as $\{a,b,b\}$. An element $a \in S$ is said to be an element of a multiset M, denoted like usual set notation $a \in M$, if $\exists (a,t) \in M$ with $t \geq 1$. We denote a multiset that has t elements as M_t.

Definition 6.3.1. A *social welfare multiset function (SWMF)* is a mapping $\alpha : \Pi^N \to \bigcup_{t=1}^{T} M_t$.

Definition 6.3.2. A SWMF $\alpha : \Pi^N \to \bigcup_{t=1}^{T} M_t$ is said to be *Pareto optimal (PO)* iff whenever $\forall \{x, y\} \subseteq A$, $\forall i \in N$, $x \rho_i y$ implies $\alpha_{\{x,y\}}(\underline{\rho}) = \begin{Bmatrix} x & x & \dots & x \\ y & y & & y \end{Bmatrix}$.

Definition: 6.3.3. A SWMF $\alpha : \Pi^N \to \bigcup_{t=1}^{T} M_t$ is said to satisfy *independence of irrelevant alternatives condition (IIA)* if for any pair of alternatives $\{x, y\} \subseteq A$ and for any pair of preference profiles $\underline{\rho}, \underline{\rho}' \in \Pi^N$ with the property that, $\forall i \in N$, $x \rho_i y \Leftrightarrow x \rho_i' y$ we have that $\alpha_{\{x,y\}}(\underline{\rho}) = \alpha_{\{x,y\}}(\underline{\rho}')$.

Theorem 6.3.1. Let $\alpha : \Pi^N \to \bigcup_{t=1}^{T} M_t$ be a SWMF satisfying IIA. Then $\exists K \in \{1, \dots, T\}$ and a mapping $\widehat{\alpha} : \Pi^N \to M_K$ such that $\alpha = \widehat{\alpha}$. i.e. Imposing IIA condition prevents number of committee members to vary.

Proof: The proof is omitted as it is very similar to that of Theorem 6.1.1.

Hence, from now on we will denote a SWMF as a mapping $\alpha : \Pi^N \to M_K$ by implicitly assuming that it satisfies IIA.

Definition 6.3.4. A SWMF $\alpha : \Pi^N \to M_K$ is said to be *dictatorial* iff there exist a set $K \subseteq N$ such that for any preference profile $\underline{\rho} \in \Pi^N$ and for any individual $i \in K$ there exists an ordering $\rho' \in \alpha(\underline{\rho})$ with, $\forall \{x, y\} \subseteq A$, $x\rho_i y \Rightarrow x\rho^{*\prime} y$.

First, we give an example of a SWMF that is not dictatorial.

For simplicity consider a three person and three alternative world and following SWC: $\alpha(\underline{\rho}) = \{\rho_1, \rho_2, \rho_3\}$ for $\underline{\rho} \neq \begin{Bmatrix} a & a & b \\ b & c & c \\ c & b & a \end{Bmatrix}$ and for $\underline{\rho} = \begin{Bmatrix} a & a & b \\ b & c & c \\ c & b & a \end{Bmatrix}$ let $\alpha(\underline{\rho}) = \begin{Bmatrix} a & a & c \\ b & b & b \\ c & c & a \end{Bmatrix}$ then we see that α is not dictatorial.

Definition 6.3.5. A SWMF $\alpha : \Pi^N \to M_K$ is said to be *pairwise oligarchical* iff there exists $\varnothing \neq K \subseteq N$ such that $\forall \underline{\rho} \in \Pi^N$, $\forall \{x, y\} \subseteq A$, $\forall i \in K$, $x\rho_i y \Rightarrow \begin{matrix} x \\ y \end{matrix} \in \alpha_{\{x,y\}}(\underline{\rho})$. The maximal K that makes α pairwise oligarchical is said to be *pairwise oligarchy*.

Definition 6.3.6. Given a SWMF $\alpha : \Pi^N \to M_K$, we say that a subset of agents $S \subseteq N$ is:

i) *Decisive for* x *over* y if whenever every agent in S prefers x *to* y and every agent not in S prefers y to x we have $\begin{matrix} x \\ y \end{matrix} \in \alpha_{\{x,y\}}(\underline{\rho})$

ii) *Decisive* if for any pair $\{x, y\} \subseteq A$, S is decisive for x over y.

iii) *Completely decisive for* x *over* y if whenever every agent in S prefers x *to* y we have $\begin{matrix} x \\ y \end{matrix} \in \alpha_{\{x,y\}}(\underline{\rho})$

iv) *Completely decisive* if for any pair $\{x, y\} \subseteq A$, S is completely decisive for x over y.

Theorem 6.3.2. If a SWMF $\alpha : \Pi^N \to M_K$ is PO and IIA then α is pairwise oligarchical.

Proof: The proof is omitted as it is a replication of Theorem 6.2.2.

7. CONCLUSION

I tried to give some literature review for some of the most important impossibility theorems and while doing this I also tried to make some comments to express the close relation between them.

To our knowlegde there is no literature dealing with *social welfare correspondences (SWC)* or *social welfare multiset functions (SWMF)* . It is usual that society choose more than one representative hence it is interesting to see the consequences of this setup. Moreover, Arrow's theorem is a corollary to our setup as one representative case is just a special case of at least one individual case. We explore that Arrow's impossibility holds in the case that we chose representatives for a specific place. For example, if we are to choose a committee and at the begingging if we know that each member of the committee will be assigned to a specific city (or say a specific chair) then by the vector case we conclude that Arrow's impossibility still prevails. But when we are to assign committee member to anonymous places (or say send them to the same place as a group) then things reverse and we get a possibility result in the sense that we do not need to choose dictators. There is still a disturbing oligarchy result, but it is much weaker then dictatoriality result. Actually, our oligarchy result is very natural due to our IIA assumption. As a conclusion, we conclude that if we are to choose a committee of representatives we see that Arrow's negative result is not that disturbing for it only holds if our committee consists of one individual.

REFERENCES

Arrow, K. (1951), *Social Choice and Individual Values*, John Wiley, New York

Arrow, K. (1963), *Social Choice and Individual Values*, John Wiley, New York

Asan, G. and M. R. Sanver. (2002), Another Characterization of the Majority Rule, *Economics Letters* 75(3), 409-413.

Aswal, N., Chatterji, S. and Arunava Sen (2003), Dictatorial Domains, *Economics Theory*, 22, 45-62

Barberà, S. (1977a), The Manipulation of Social Choice Mechanizms That Do Not Leave "Too Much" Chance, *Econometrica*, 45, 1573-1588.

Barberà, S. (1977b), The Manipulation of Social Choice Decision Functions, *Economic Theory*, 15, 266-278.

Barberà, S. (1983), Strategy Proofness and Pivotal Voters: A Direct Proof of the Gibbard-Satterthwaite Theorem, *International Economic Review*, 24, 413-418.

Barberà, S. and B. Peleg (1990), Strategy-Proof Voting Schemes with Continuous Preferences, *Social Choice and Welfare*, 7, 31-38.

Barberà, S., H. Sonnenschein and L. Zhou (1991), Voting by Committees , *Econometrica*, 59, 595-609.

Barberà, S., A. Bogomolnaia and H. Van der Stel (1998), Strategy-Proof Probability Rules For Expected Utility Maximizers, *Mathematical Social Sciences*, 35, 89-103.

Barberà, S., Gul, F. And E. Stacchetti (1993), Generalized Median Voter Schemes and Committees, *Journal of Economic Theory*, 61, 262-289.

Barberà, S., B. Dutta, and A. Sen (2001), Strategy-Proof Social Choice Correspondences, *Journal of Economic Theory*, 101: 374-394.

Barberà, S., W. Bossert, and P. K. Pattanaik (2004), Ranking Sets of Objects, in S. Barberà, P. J. Hammond and C. Seidl (Eds), Handbook of Utility Theory, Volume II Extensions, Kluwer Academic Publishers, Dordrecht. 893-977

Beja, A. (1993), Arrow and Gibbard-Satterthwaite Theorem Revisited: Extended Domains and Shorter Proofs, *Mathematical Social Sciences*, (25) 3, 281-286

Benoit, J. P. (2000), The Gibbard-Satterthwaite Theorem: A Simple Proof, *Economic Letters* 60(3), 319,322.

Benoit, J. P. (2002), Strategic Manipulation in Voting Games When Lotteries and Ties are Permitted, *Journal of Economic Theory*, 102, 421-436.

Black, D. (1948), On the Rationale of Group Decision Making, *Journal of Political Economy*, 56, 23-34.

Blau, J. H. (1957), The Existence of Social Welfare Functions, *Econometrica*, 25, 302-313.

Bordes, G. A. and M. le Breton (1990), Arrovian Theorems for Economic Domains: The Case where there are Simultaneously Private and Public Goods, *Social Choice and Welfare*, 7, 1-17.

Campbell, D. E. and J. S. Kelly (2003), A Strategy-Proofness Characterization of Majority Rule, *Economic Theory*, 22, 557-568

S. Ching and L. Zhou (2002), Multi-Valued Strategy-Proof Social Choice Rules, *Social Choice and Welfare*, 19, 569-580.

Dasgupta P., Hammond P. and Maskin E. S. (1979), The Implementation of Social Choice Rules: Some General Results on Incentive Compatibility , *The Review of Economic Studies*, (46)2, 185-216.

Dogan, E. and M. R. Sanver (2006), Arrovian Impossibilities in Aggregating Preferences over Sets, unpublished manuscript.

J. Duggan and T. Schwartz (2000), Strategic Manipulability Without Resoluteness or Shared Beliefs: Gibbard-Satterthwaite Generalized, *Social Choice and Welfare*, 17, 85-93.

Eliaz, K. (2001), Arrow's Theorem and the Gibbard-Satterthwaite Theorem as a Special Case of a Single Theorem, working paper

A. Feldman (1979a), Nonmanipulable Multi-valued Social Decision Functions, *Public Choice,* 34, 177-188.

A. Feldman (1979b), Manipulation and the Pareto Rule, *Journal of Economic Theory,* 21, 473-482.

A. Feldman (1980), Strongly Nonmanipulable Multi-valued Collective Choice Rules, KYKLOS, *Int. Rev. Social Sci.,* 33, fasc. 2.

Fishburn, P. C. (1970), Even-chance Lotteries in Social Choice Theory, *Journal of Economic Theory,* 2, 103-106.

Fishburn, P. C. (1972), Arrow's Impossibility Theorem: Concise Proof and Infinite Voters, *Theory and Decision,* 3, 18-40.

Fishburn, P. C. and J. S. Kelly (1997), Super Arrovian Domains with Strict Preferences, *SIAM Journal of Discrete Mathematics,* 10 (1), 83-95.

Gaertner, W. (2001), *Domain Conditions in Social Choice Theory*, Cambridge University Press, Cambridge.

Gibbard, A. (1973), “Manipulation of Voting Schemes: A General Result”, *Econometrica*, 41, 587-601.

Gibbard, A. (1977), “Manipulation of Schemes that Mix Voting with Chance”, *Econometrica*, 45, 665-681.

Gärdenfors, P. (1976), Manipulation of Schemes That Mix Voting with Chance, *Econometrica*, 45, 665-681.

Gärdenfors, P. (1978), “On Definitions of Manipulation of Social Choice Functions”, in J.J. Laffont (ed.), Aggregation and Revelation of preferences,. Amsterdam: North Holland.

Geanakoplos, J. (2001), Three Brief Proofs of Arrow's Impossibility Theorem, Yale Cowles Foundation Discussion Paper No.1123RRR

Inada, K. (1964), A Note on the Simple Majority Decision Rule, *Econometrica*, 32, 525-531

Kalai, E. and E. Muller (1977), Characterization of Domains Admitting Non-Dictatorial Social Welfare Functions and Nonmanipulable Voting Procedures, *Journal of Economic Theory* 16, 457-469.

Kalai, E., E. Muller, M.A. Satterthwaite (1979), Social Welfare Functions when Preferences are Convex, Strictly Monotonic and Continuous, *Public Choice* 34, 87-97.

Kannai Y. and B. Peleg (1984), “A Note on the Extension of an Order on a Set to the Power Set”, *Journal of Economic Theory*, 32, 172-175

Kaymak, B. and M. R. Sanver (2003), “Sets of Alternatives as Condorcet Winners”, *Social Choice and Welfare*, 20 (3), 477-494.

Kelly, J. S. (1977), Strategy-proofness and social choice functions without single-valuedness. *Econometrica*, 45, 439-446.

Kelly, J. S. (1994a), The Free Triple Assumption, *Social Choice and Welfare*, 11, 97-101.

Kelly, J. S. (1994b), The Bordes-Le Breton Exceptional Case, *Social Choice and Welfare*, 11, 273-281.

Le Breton, M. and J. A. Weymark. (2003), Arrovian Social Choice Theory on Economic Domains, in: K. J. Arrow, A. K. Sen, and K. Suzumura, eds., *Handbook of Social Choice and Welfare: Volume 2*, (Amsterdam, North-Holland).

Mas-Colell, A., Whinston, M. D. and J.R. Green (1995) *Microeconomic Theory,* Oxford Univesity Press, New York.

Maskin, E. S. (1999), Nash Equilibrium and Welfare Optimality, *Review of Economic Studies*, (66)1, 23-38

May, K. (1952), A Set of Independent, Necessary and Sufficient Conditions for Simple Majority Decision, *Econometrica* 20, 680-684.

Myyerson, R. B. (1996), Fundementals of Social Choice Theory, Center of Mathematical Studies in Economics and Management Science Discussion Paper No. 1162

Moulin, H. (1980), On Strategy-Proofness and Single Peakedness, *Public Choice* 35, 437-455.

Nehring, K. (1997), Monotonicity implies strategy proofness for correspondences, working paper.

Ozdemir, U. and M.R. Sanver (2006), Dictatorial Domains in Preference Aggregation, *Social Choice and Welfare*, forthcoming.

Ozyurt, S. and M.R. Sanver (2006), A General Impossibility Result on Strategy Proof Social Choice Hyperfunctions, unpublishes manuscript.

Ozkal-Sanver, I, and M. R. Sanver (2006), Nash Implementation via Hyperfunctions, *Social Choice and Welfare*, 26 (3), 607-623.

P. K. Pattanaik (1973), On the Stability of Sincere Voting Situations, *Journal of Economic Theory*. 6, 558-574.

P. K. Pattanaik (1978), Strategy and Group Decision, North Holland, Amsterdam.

Quesada, A. (2005), Parallel Proofs of Arrow's and Gibbard- Satterthwaite Theorem, unpublished manuscript.

Reny, Philip J. (2001), Arrow's Theorem and the Gibbard-Satterthwaite Theorem : A Unified Approach, *Economics Letters*, (70)1, Pages 99-105

Roth, A. and M. A. O. Sotomayor (1990), "Two-Sided Matching: A Study in Game Theoretic Modelling and Analysis", Cambridge University Press.

M. R. Sanver (2006), A Characterization of Superdictatorial Domains For Strategy-proof Social Choice Functions, unpublished manuscript.

Satterhwaite, M. (1975), "Strategy-proofness and Arrow's Conditions : Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions", *Journal of Economic Theory*. 10, 187-217.

Sen, A. K. (1966), A Possibility Theorem on Majority Decisions, *Econometrica*, 34, 491-499.

Sen, A. K. and P. K. Pattanaik (1969), Necessary and Sufficient Conditions for Rational Choice under Majority Decision, *Journal of Economic Theory,* 1, 178-202.

Sen, A. K. (2001), Another Direct Proof of the Gibbard-Satterthwaite Theorem, *Economic Letters,* (70)3, 381-385.

Serizawa, S. (1995), Power of Voters and Domain of Preferences Where Voting Committees is Strategy-proof, *Journal of Economic Theory*, 67, 599-608.

Svensson, L. G. (1999), The Proof of Gibbard-Satterthwaite Theorem Revisited, unpublished manuscript.

Tanaka, Y. (2001), Generalized Monotonicity and Strategy-proofness for Non-resolute Social Choice Correspondences, *Economics Bulletin,* (4)12, 1-8.

Tanaka, Y. (2003), Oligarchy for Social Choice Correspondences and Strategy-proofness, *Theory and Decision,* 55, 273, 287.

Taylor, A. D. (2005), Social Choice and the Mathematics of Manipulation, Cambrigde England, Cambridge University Press.

Woeginger, G. (2003), A New Characterization of the Majority Rule, *Economics Letters*, 81, 89-94.