DEVELOPMENT OF MARKOWITZ APPROACH: EMPIRICAL INVESTIGATION FOR TURKISH MARKET

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MARKOWITZ YAKLAŞIMININ GELİŞİMİ :
TÜRKİYE PİYASASI İÇİN AMPİRİK BİR ARAŞTIRMA

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4) Fayda	4) Utility
5) Standart Sapma	5) Standard Deviation

Abstract

This study examines the portfolio constructing models used for investments. After a review of the Modern Portfolio Theory, the portfolios are constructed and evaluated with the use of computer techniques. The analyses are made by utilizing both compounded and daily figures of returns. In our calculations MS Excel skills supported by the data analysis and solver are applied to the various data. At the same time as an alternative method a model implemented by W.Sharpe and utilized by Elton & Gruber is applied to the data chosen. The minimum variance portfolio is tested as an optimal selection strategy separately for the selected stocks and for the composition of stock market index, foreign currency, and the gold investment. Utility is explained by a lagrange multiplier and its relation to the Modern Portfolio Theory is emphasized. The optimum point is that where the utility has been maximized by the approximate expected utility (AEU) based on mean variance behaviour.

Keywords: Markowitz, Portfolio, ISE, Utility, Standard Deviation

Özet

Bu çalışma yatırımlar için portföy oluşturma modellerini incelemek için yapılmıştır. Modern Portföy teorisine kısa bir bakıştan sonra portföyler bilgisayar teknikleri yardımı ile oluşturulmuş ve değerlendirilmiştir. Analiz hem bileşik hem de günlük getiri oranları ile yapılmıştır. Hesaplamalarımızda veri çözümleme ve çözücü ile desteklenmiş MS Excel uygulamaları çeşitli veriler için kullanılmıştır. Aynı zamanda alternatif bir yöntem olarak W.Sharpe'ın literatüre kazandırdığı ve Elton & Gruber tarafından kullanılmış olan bir model için veri uygulamaları yapılmıştır. Minimum varyanslı bir portföy seçiminin optimal bir seçim stratejisi olup olmadığı seçilen hisse senetleri için ve ayrıca hisse senedi pazar endeksi, döviz ve altın yatırımları bileşimi için test edilmiştir. Fayda lagranj çarpanı vasıtası ile açıklanmış ve Modern Portföy Teorisi ile olan ilişkisi vurgulanmıştır. Optimum noktanın ortalama-varyans analizine dayanan yaklaşık beklenen fayda (AEU) tarafından maksimize edilen fayda seviyesi olduğu gözlenmiştir.

Anahtar Kelimeler: Markowitz, Portföy, IMKB, Fayda, Standart Sapma

1. INTRODUCTION

Integration to the Global Economy resulted in the introduction of investment alternatives for developing countries. In more institutionalized countries these alternatives are numerous. Even economies of some under-developed countries are well above than what they were ten years ago in terms of investment alternatives.

People make investments according to their expectations and their risk tolerances. Unfortunately in most cases the desire of making huge profits results in loss.

Modern Portfolio Theory aims to find an optimum solution to investment alternatives. It utilizes the sets of various risks and return levels.

Modern Portfolio theory is a reasonable practice in order to analyze the historical datas. The variables in the market are dynamic and the outcomes are the functions of these dynamics.

Modern Portfolio Theory scrutinizes a social behavior in a scientific point of view. According to the findings, there should be an optimal point where the utility is maximized.

This study examines the Turkish Financial Market along with international data in the perspective of utility maximization.

2.LITERATURE REVIEW

Harry Markowitz (1952) asserts the portfolio selection process can be divided into two stages. The first stage is the observation and experience to attain a belief about the future performance of the securities. The second stage starts with the belief of the performance and ends up with the right choice of a portfolio. In his analysis Markowitz deals with the second stage[1].In his study he claims that an investor should maximize the discounted expected returns.

The Pioneer theory was introduced by J.B Williams from Harvard University who made the definition of investment and classified it apart from a simple gambling game. In "the Theory Investment Value" J.B. William proposes a stock's value is equal to the discounted income stream. Markowitz opposes this assertion by stating that as long as the dividends are uncertain this calculation only gives the expected value. Since we are not fortune tellers it should have been "expected" or "anticipated". Thus Markowitz makes the definition of the Expected Return and he indicates the importance of the risk. He states that an investor considers the expected return a desirable thing and considers the risk that is variance an undesirable thing.

The Markowitz Approach is developed in 1952. He remarks that the Portfolio Theory is normative[2], meaning that it tells investors how they should act to diversify their portfolio optimally. It is based on a small set of assumptions , including[3]:

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[1] H.Markowitz, Portfolio Selection The Journal of Finance, Mar 1952

[2]Normative means "relating to an ideal standard or model". Normative economics deals with questions of what sort of economic policies ought to be pursued, in order to achieve desired economic outcomes. (wikipedia)

[3] Brian Kettell, Financial Economics Making Sense of Information in Financial Markets, p.191 ,Prentice Hall, 2001

1. a single investment period; for example , one year;
2. no transaction costs;
3. investor preferences based only on a portfolio's expected return and risk, as measured by the variance or standard deviation.

The notations of the formulas are shown below:
Suppose there are N securities. Let r_{it} be the anticipated return, d_{it} be the rate at which the return on the i_{th} security is discounted back to the present. X_i be the weight.

With excluding a short sale the X values (X_i) for each investment is either zero or positive, but can not take a negative value. (X =1)

The Formula is,

$$R_i = \sum_{t=1}^{\infty} d_{it} r_{it} \quad (1)$$

$$R = \sum_{i=1}^{N} X_i (\sum_{t=1}^{\infty} d_{it} r_{it}) \quad (2)$$

$$R = \sum_{t=1}^{\infty} \sum_{i=1}^{N} d_{it} r_{it} X \quad (3)$$

As can be seen in the formula R is the weighted average of R_i.

Markowitz states there is a belief that an investor should diversify his funds among all those securities which give maximum expected return. The assumption was that there was a portfolio which gave a best set of expected return and minimum variance level, and that portfolio is favorable for an investor. The law of large numbers will yield a portfolio return that will be almost the same as the expected yield.

Markowitz states the returns are so correlated that diversification can not eliminate all risk. The highest expected return does not mean the minimum variance.

The idea behind this statement was to determine efficiency of the portfolio and to show the co-relation as a key factor in portfolio investment decisions.

If the probability gets into the analysis the expected return is simply the sum of probability weighted returns(1), and similarly the variance is defined to be the probability weighted deviations from the expected return, that is the mean(2).

$$E = p_1 y_1 + p_2 y_2 + ... + p_N y_N \quad (1)$$

$$E = p_1 (y_1 - E)^2 + p_2 (y_2 - E)^2 + ... + p_N (y_N - E)^2 \quad (2)$$

But in his premier study Markowitz does not deal with the probability ; rather he prefers just giving an insight about it.

Markowitz (1952) states the variance of a weighted sum is,

$$VAR(r_p) = x_1^2 VAR(r_1) + x_2^2 VAR(r_2) + 2x_1 x_2 COV(r_1, r_2)$$

Lets make a proof from Francis and Ibbitson[2002] [4]:

$VAR(r_p) = E[r_p - E(r_p)]^2$ for variance is deviation from our expectations.

$$= E\{x_1 r_1 + x_2 r_2 - [x_1 E(r_1) + x_2 E(r_2)\}^2$$

$$= E\{x_1 r_1 + x_2 r_2 - x_1 E(r_1) - x_2 E(r_2)\}^2$$

$$= E\{x_1 r_1 - x_1 E(r_1) + x_2 r_2 - x_2 E(r_2)\}^2$$

$$= E\{x_1 [r_1 - E(r_1)] + x_2 [r_2 - E(r_2)]\}^2$$

$$= x_1^2 E[r_1 - E(r_1^2)] + x_2^2 E[r_2 - E(r_2^2)] + 2x_1 x_2 E\{[r_1 - E(r_1)][r_2 - E(r_2)\}$$

$$= x_1^2 VAR(r_1) + x_2^2 VAR(r_2) + 2x_1 x_2 COV(r_1, r_2)$$

[4] Investments:A Global Perspective

Once the risk ,standard deviation, and the expected return is known it is very easy to draw a graph called "the efficient frontier" which is a reasonable set of the risk and return possibilities. The model assumes that among the same expected returns investor choose the least risky one.

Markowitz shows us, along the critical line, a point which is dominant to the others. The levels of variance determines this choice.

Diversification is a reasonable investment practice where the co-relation among the investment alternatives are weak.

If the same variance level is present for each two investment alternatives, the variance of a portfolio made up of these two assets will be less than the each asset's own.

As we mentioned before in the analysis as Markowitz states the lacking point is the probabilistic figures. Still probabilistic reformulation of security analysis is needed.

A short sale is not a field of the study in his premier article. In more institutionalized markets it is possible.

Even in the Great Depression in 1929 economists examined the treasury bills as an alternative to stock investments to cope with the risk exposures. This was for sure not quantitative.

Markowitz's article does not deal with a risk free investment. Later on Tobin made an addition to the theory simply by drawing a tangency line through the efficient frontier.

Markowits (1959) asserts “A good portfolio is more than a long list of good stocks and bonds. It is a balanced whole, providing the investor with protection and opportunities.”[5]

As long as people make security investment, there will always be uncertainty. Even though the consequences of economic conditions and the relations between the factors were understood perfectly, non-economic influences i.e. flood or some natural event can change the course of general prosperity, the level of economy and the success of a particular security.[6]

Diversification reduces risk. Because returns of different stocks are not perfectly correlated. The risk that can potentially be diversified away is known as the unique or the unsystematic risk. Additionally there is a non-diversifiable risk known as the market or the systematic risk. He states “market risk arises, because there are economy-wide hazards that threaten all businesses. This is why investors are exposed to market uncertainties and a baseline level of risk no matter how many stocks are held in a portfolio”[7].

Although we are not fully capable of predicting the future, we can make some analyses which give us an essence of thought while utilizing them as reasonable prediction methods. An important measure that we need to calculate while analyzing the various data is the correlation. For instance petroleum and natural gas are almost perfectly correlated. If there is a crisis in the economy both will move exactly in the same direction in almost the same proportions. So the diversification among these two assets is useless.
If security returns are not correlated , diversification can eliminate risk.[8]

Apart from the previous analyses Markowitz suggests a model based on a new term as a risk measuring gauge, the semi-variance. The variance used in the Modern Portfolio Theory (MPT) is a risk measurement around the

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[5] H.Markowitz ,Portfolio Selection , 1959, Edition 1990
[6] H.Markowitz ,Portfolio Selection , 1959, Edition 1990
[7] M. Rodehorst Dr. Lori Bennear , December 2007 “Evaluating Expected Electric Generation Technology Cost and Risk” Applying Modern Portfolio Theory to North Carolina Electric Power Generation Duke University
[8] H.Markowitz ,Portfolio Selection , 1959, Edition 1990

mean in either positive or negative way. It does not make a discrimination of which side the movement is. On the contrary semi-variance measures the only negative movements from the expected return. It is a measure of downward movement ,so a measure of downward risk. Semi-variance is an average of the squared deviations of outcomes that are less than the mean.

Here is the formula,

$$SV = \frac{1}{n}\sum_{r_t < Average}^{n} (Average - r_t)^2$$

Where
n= total number of observations below the mean

If there is no downward tendency, the semi-variance will be equal to the variance itself.

Markowitz (1991) states that MPT assumes an asset's return as a random variable. He himself models an optimum portfolio that is composed of two and more assets. In other words the portfolio was a weighted combination of the assets. The portfolio had an expected return and a standard deviation of its own. The risk here was gauged by the standard deviation of the whole portfolio's return[9].

There should have been a starting point. "The investor does (or should) diversify his funds among all those securities which give maximum expected return."[9] We will test the applicability of this matter in the apolications section.

As we mentioned before Markowitz delves into statistical side of the theory later on.

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[9] H.Markowitz, Portfolio Selection ,The Journal of Finance, June 1991

There is a statistical rule that, if the historical stock returns are normally distributed the portfolio risk can be explained by the standard deviation. This is one of the properties of the normal distribution[10].

Another addition to his own original study was that there were no single portfolio for everyone that was suitable for each investor, in other words there was no tailor-made suit for every investor. Everyone has different preferences of consumption, has different risk tolerence levels and also their timing is different.

Dr. Markowitz was among the first to quantify risk and demonstrate quantitatively why and how portfolio diversification can work to reduce risk, and to increase returns for investors. That's why he probably received the Nobel Prize.

He states(1991) that he was convinced with Savage that a rational agent acts under uncertainty according to the probability beliefs which are individually subjective but as their combine, they are rather objective.

According to Markowitz (1991), Kennet Arrow sought a precise and a general solution, but he sought a good approximation that can easily be implemented.

Utility is a matter of which economists always tried to find a description. Utility can be regarded as the satisfaction of an investor, of a consumer or of another entity.

Daniel Bernoulli (1738) in his famous article about the St. Petersburg Paradox[11], states that risk averse investors wants to diversify: "… it is

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[10] M. Rodehorst Dr. Lori Bennear , December 2007 "Evaluating Expected Electric Generation Technology Cost and Risk" Applying Modern Portfolio Theory to North Carolina Electric Power Generation Duke University

11 The Paradox:
In a game of chance, you pay a fixed fee to enter, and then a fair coin will be tossed repeatedly until a tail first appears, ending the game. The pot starts at 1 dollar and is

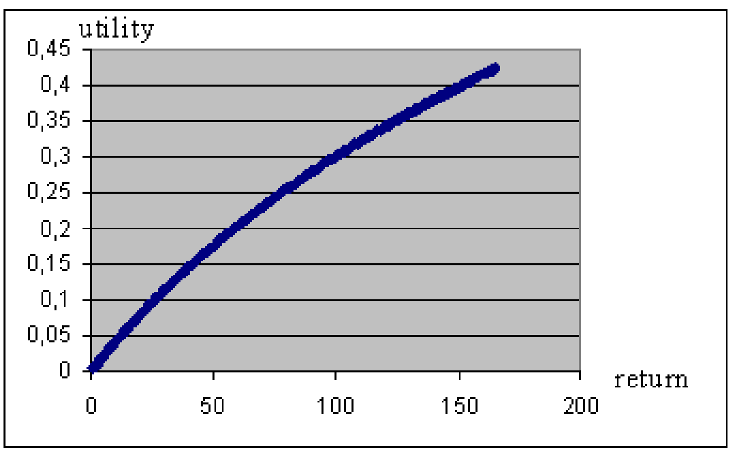
advisable to divide goods which are exposed to some small danger into several portions rather than to risk them all together."[12]

The classical resolution of the paradox involved an explicit introduction of the utility function, an expected utility hypothesis, and a presumption of diminishing marginal utility of money or an asset.

In Daniel Bernoulli's words: "The determination of the value of an item must not be based on the price, but rather on the utility it yields…. There is no doubt that a gain of one thousand ducats is more significant to the pauper than to a rich man though both gain the same amount. "

Savage states, for a multi period time scale an investor is to maximize the expected value of each period. Recite the rule "a single period utility function."

Assume U(R)= log(1+R), The utility function will be as shown below [13]



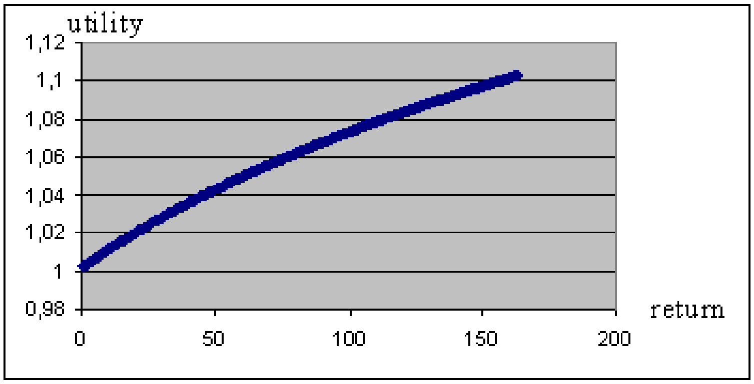
doubled every time a head appears. You win whatever is in the pot after the game ends. Thus you win 1 dollar if a tail appears on the first toss, 2 dollars if on the second, 4 dollars if on the third, 8 dollars if on the fourth, etc. In short, you win 2k–1 dollars if the coin is tossed k times until the first tail appears.

What would be a fair price to pay for entering the game? To answer this we need to consider what would be the average payout: With probability 1/2, you win 1 dollar; with probability 1/4 you win 2 dollars; with probability 1/8 you win 4 dollars etc. The expected value is thus is infinite.

[12] Mark Rubinstein, Journal of Finance, 2002

[13] Mean Variance versus Direct Maximization, Kroll,Levy,Markowitz, Journal of Finance,Vol.39 1984

Assume $U(R) = (1+R)^{0.1}$, The utility curve will be



The second derivation of the utility function results in a negative solution which demonstrates a diminishing marginal utility. The data of the two diagrams are different. The common property of the two graphs is that for higher return levels the utility increase starts diminishing. In other words as we move along the curve utility increase diminishes for an additional return. Obviously the first derivation is positive and the second derivation is negative.

The modern portfolio theory aims to maximize the utility. A utility function can be better understood by utilizing a Lagrange Multiplier (λ). A maximized utility is a point where it is no longer possible to increase the utility. The partial derivations vis a vis the independent variables and vis a vis the Lagrange multiplier itself is zero. The first derivation equals to zero means that it's not possible to utilize more by increasing any variable. It is the point of efficient portfolio which we are going to examine in the applications section utilizing the solver facility of Excel.

Below X and Y are the weights and P stands for prices or returns.

$$I = P_X X + P_Y Y$$

$$V = f(X, Y) + \lambda(I - P_X X - P_Y Y)$$

To maximize utility, all the partial derivatives must be equal to zero[14].

$$\partial V / \partial X = \partial f / \partial X - \lambda P_X = 0$$

$$\partial V / \partial Y = \partial f / \partial Y - \lambda P_Y = 0$$

$$\partial V / \partial \lambda = I - P_X X - P_Y Y = 0$$

In his studies Markowitz (1979 and 1991) analyzed "the expected utility" with the help of Taylor series. We can say that almost the same outcomes have been found. Close figures of Expected Utility (EU) and Approximate Expected Utility (AEU) were prevailed. Returns were on logarithmic scales and g (geometric mean) was applied. In any case the EU or the AEU were maximized. That meant the expected utility could have been explained by mean-variance analysis. Utility maximization rule was applicable.

Let's make the analysis again:

$$EU = (1/T)\sum_{t=1}^{T} U(R)$$

$AEU = U(\mu) + (1/2)\sigma^2 U^{II}(\mu)$ where U^{II} is the second derivation

(The term in the middle $U^{I}(\mu)(R-\mu)$ is zero after calculating the expected value that is the mean)

For $LN^{I}(X) = 1/X$, $LN^{II}(X) = -1/X^2$

Then the utility functions can be re-written in terms of variance terms in Taylor Series,

$$AEU = LN(1+\mu) - \frac{(1/2)\sigma^2}{(1+\mu)^2},$$

$$AEU = (1+\mu)^a + (1/2)\sigma^2 a(a-1)(1+\mu)^{(a-2)}$$

According to Fischer Black the forward price of any asset is the current price plus accrued interest calculated till the end of the period. The excess return is the fractional difference between the asset's payoff and its forward

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[14]http://www.mhhe.com/economics/mcconnell15e/graphics/mcconnell15eco/common/dothemath/utilitymaximizingrule.html

price. Maximizing expected utility is equivalent to maximizing the mean excess return for a given variance. In other words it is equivalent to minimized variance of excess return for a given mean[15].

Envelope[16] and Siegel paradox show us an investors will want some risk for more return levels and the result would not be a 100% hedge[17].

After his description in 1959, Markowitz states (1991) there experienced no case semi-variance is dominant to variance analysis[18].

The importance of semi-variance shows itself in investor's dislike of downside movements.

As we mentioned before Markowitz made a discrimination between the systematic and unsystematic risk. During a bear market every stock has a downward tendency. Though you do not put your eggs in the same market, you may end up loosing. The market risk can not be eliminated by diversification. The graph below shows the risks for the Standard deviation and number of stock sets[19]. The unique risk decreases until the bold line as the number of stocks increases, while the market risk (below the bolded line) remains the same as it is independent of the number of stocks in the portfolio.

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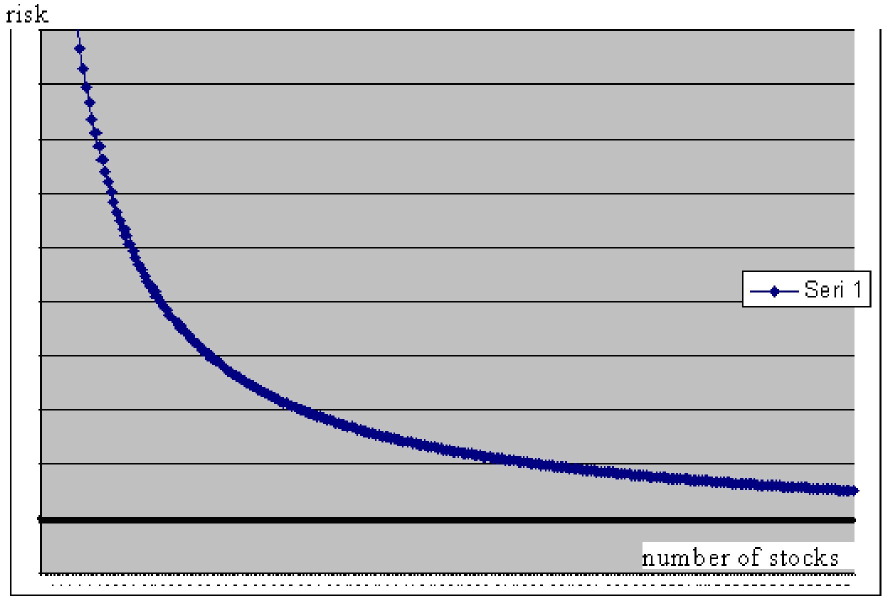
15 Fischer Black, Equilibrium Exchange Rate Hedging, The Journal of Finance ,July 1990

16 Frieder Bolle , The Envelope Paradox, the Siegel Paradox, and the Impossibility of Random Walks in Equity and Financial Markets , February 2003

17 Fischer Black, Equilibrium Exchange Rate Hedging, The Journal of Finance ,July 1990

18 H.Markowitz, Portfolio Selection ,The Journal of Finance, June 1991

19 Adaptation from Brealey and Myers, 2000



Is it desirable to balance a portfolio between long positions in securities considered underpriced and short positions in securities considered overpriced. By this means is the market risk completely be eliminated? Or should one strive to diversify a portfolio so completely that only market risk remains? [20]

It is asserted that from among a given set of investment alternatives, the most reasonable set is obtained by discarding those investments with a lower mean and a higher variance[21]. Eliminating the extreme values may be the first step of the study. But is it that simple? We will get more into it in the applications section of our study.

Markowitz was the first person to get into the concept of an *efficient portfolio*, defined as "one that has the smallest portfolio risk for a given level of expected return, or the largest expected return for a given level of risk"[22].

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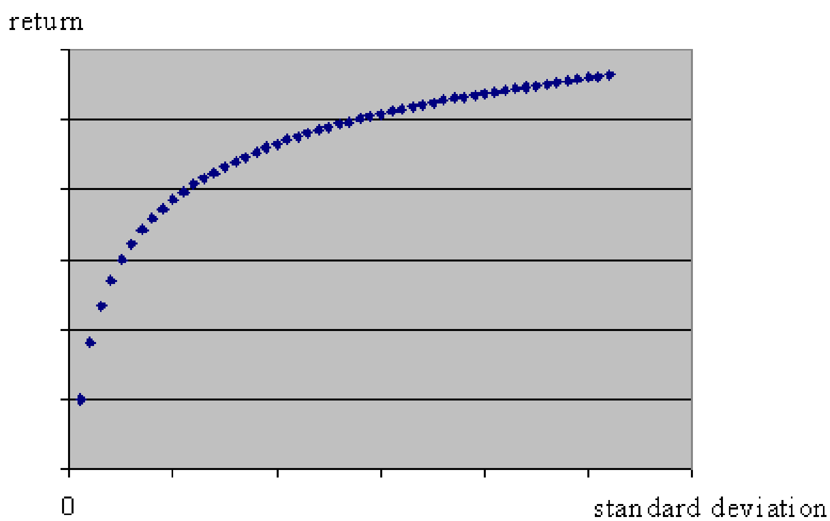
[20] Jack L. Treynor; Fischer Black, How to Use Security Analysis to Improve Portfolio Selection *The Journal of Business*, Vol. 46, No. 1. (Jan., 1973)

[21] Vijay S. Bawa, Admissible Portfolios for All Individuals, The Journal of Finance, Vol. 31, No. 4. (Sep., 1976)

[22] Brian Kettell, Financial Economics Making Sense of information in financial markets,

A line created from the risk-reward graph, comprised of optimal portfolios is drawn below.The risk and return increases as we move along the curve. The portfolio selection on the graph depends on the risk tolerence of the investor.

In the above mentioned graph shown below the return is displayed on the y axis. The standard deviation is displayed on the x axis, the left part of the curve is a low risk - low return sets of data. As we move along the graph as long as we know the property of diminishing marginal utility, the new added proportions for the higher sets differ from the previous values in terms of relativeness though nominal higher risk and higher return choices are observed.



The optimal portfolios should lie on this curve known as the "efficient frontier". Portfolios below the curve are not efficient. For the same risk level one could achieve a greater return. On the other hand a portfolios above the curve are impossible.

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p.192 ,Prentice Hall, 2001

A portfolio's expected return is the weighted average of the expected returns of the assets that make up the portfolio[23].

$E(r_p) = \sum x_i E(r_i)$

Let's assume the Standard deviation is 4% and the return is 9% and it is a medium risk and medium return portfolio which lie on the curve above, say on the efficient frontier line. Can we accept it as an optimal portfolio? For the two-thirds of the outcomes we would expect returns to fall between 5 % and 13%. (9%+4%=13% and 9%-4%=5%). It is the first Sigma Rule that we recognize from the theory of statistics. Any other portfolio will carry greater variability of return and thus will have greater risk[24].

As we continue our study, we can see the same insight from another outstanding person namely from Cervantes. In his book Don Quijote we recall "Don't put your eggs in one basket".

Or in "Merchant of Venice" written by Shakespeare:
"…I thank my fortune for it,
My ventures are not in one bottom trusted,
Nor to one place; nor is my whole estate
Upon the fortune of this present year…"[25]

As we stated before Markowitz's paper is the first mathematical formalization of diversification: the financial version of "the whole is greater than the sum of its parts, in other words the synergy effect": Through diversification risk can be reduced but not totally eliminated without changing expected portfolio return. [25]

A.D. Roy (July 1952) independently sets down the same equation relating portfolio variance of return to the variances of return of the constituent

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[23] Francis and Ibbotson ,Investments "A global perspective" p.403 ,Prentice Hall 2002
[24] Brian Kettell, Financial Economics Making Sense of information in financial markets, p.191-192 ,Prentice Hall, 2001
[25] Mark Rubinstein, Journal of Finance, 2002

securities. He develops a similar mean- variance efficient set. Whereas Markowitz left it to the investors what to choose along the efficient sets they would invest, Roy advised choosing a single portfolio in the mean-variance efficient set.

According to him the Formula is $(\mu P - d)/\sigma P$,where d is a "disaster level return" a critical point in constructing a portfolio. That is where the investor should place a high priority not falling below it.

Lets see what Markowitz says for Roy,
"On the basis of Markowitz (1952), I am often called the father of modern portfolio theory (MPT), but Roy (1952) can claim an equal share of this honor."[26]

"Markowitz lays out how to solve the multi-period expected utility of the consumption problem by using the backwards recursive technique of dynamic programming, used subsequently by Phelps (1962) and then by many others to solve the multi-period problem."[26]

Markowitz's portfolio model focused only on the choice of risky assets. Tobin(1958) , motivated by Keynes' theory of liquidity preference[27], extended the model to include a riskless asset. In doing so, he discovered a surprising fact. With the inclusion of a risk free asset , the set of efficient risk-return combinations turned out to be a straight line.[28] Risk free rate has a Standard deviation of zero and a constant return, so it can be represented by a straight line touching the set of efficient portfolios. Because an investor

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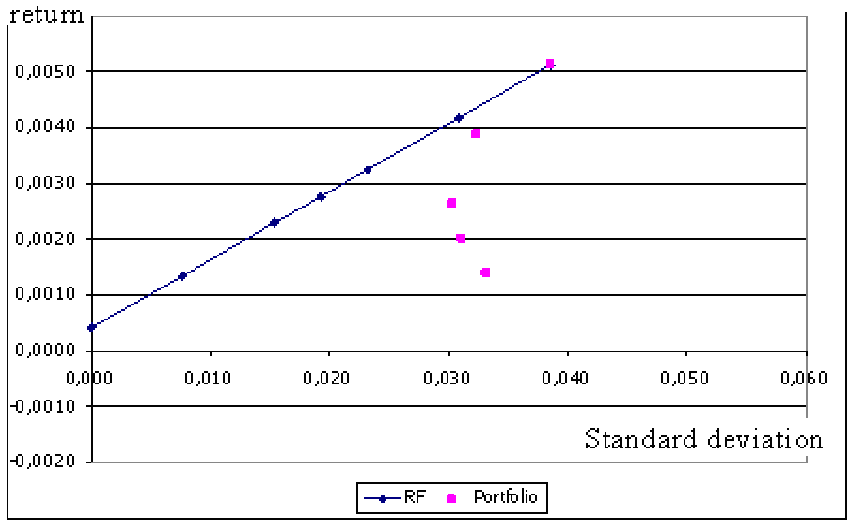
[26] Mark Rubinstein, Journal of Finance, 2002

[27] F.S.Mishkin in his book The Economics of Money, Banking and Financial Markets " Keynes's liquidity preference theory states that the demand for Money in real terms M^d/P depends on income Y(aggregare output) and interest rates i. The demand for Money is positively related to income for two reasons. First, a rise in income raises the level of transactions in the economy, which in turn raises the demand for Money because it is used to carry out these transactions. Second a rise in income increases the demad for Money because it increases the wealth of individuals who want to hold more assets, one of which is Money. The opportunity cost of holding Money is the interest sacrificed by not holding other assets (such as bonds) instead. As interest rates rise, the opportunity cost of holding Money rises, and the demand for Money falls. According to the liquidity preference theory, the demad for Money is positively related to aggregate output and negatively related to interest rates."

[28] Hal Varian, **A Portfolio of Nobel Laureates: Markowitz, Miller and Sharpe**
The Journal of Economic Perspectives, 1993

who purchases a risk-free asset at the beginning of a holding period knows what exactly the value of the asset will be; Thus how much he will be earned at the end of the holding period.

In the graph below x axis is the Standard deviation and the y axis is the return.



Borrowing and lending options transform the efficient frontier into a straight line[29].

A linear relationship between the expected return and the standard deviation prevails.

If the aim is having a 50% equal of risky and risk-free portfolio, we have to go a half way from (0,RF) point through the tangency point.

Lets say the tangency point on the efficient frontier has a 10% return and 6% Standard deviation. If we place one-half of available funds in the

[29] Brian Kettell, Financial Economics Making Sense of information in financial markets, p.191 ,Prentice Hall, 2001

riskless asset an one-half in the risky portfolio, the resulting combined risk-return measures for our mixed portfolio, can be found from Equations (1) and (2) [30].

$$R_P = X\ R_M + (1-X)\ R_F \qquad (1)$$

Where
R_P= expected return on portfolio
X= percentage of funds invested in risky portfolio
(1-X)= percentage of funds invested in riskless asset
R_M= expected return on risky portfolio
R_F= expected return on riskless asset

And since the Standard deviation of the risk free asset is zero,

$$\sigma_P = X\ .\ \sigma_M \qquad (2)$$

From Equations (1) and (2) we can calculate risk-return measures for portfolio M as:
R_P= (1/2) (0.10) + (1/2) (0.05) =7.5%
σ_P= (1/2) (0.06) + (1/2) (0.00) = 3%

The result indicates that our return and risk have been reduced from each (10 % and 6%).

Let's assume borrowing and the lending costs are the same in value; Then we can re-write the equation (1) as follows[30]:

$$R_P = X\ R_M - (X-1)\ R_B \qquad (3)$$

Where R_B is the cost of borrowing.

[30] Brian Kettell, Financial Economics Making Sense of information in financial markets, p.191 ,Prentice Hall, 2001

As long as we know X= 1.25 , that would indicate that the investor borrows an amount equal to 25 per cent of his or her investment wealth. The investor's net return on the investment would become[31]:

$R_p = (1.25)(0.10) - (0.25)(0.05) = 11.25\%$

The associated risk would become[31]:

$\sigma_p = X \cdot \sigma_M = (1.25)(0.06) = 7.5\%$

The optimum portfolio involves selecting securities that yields the best combination of expected return and risk which, of course, depends on the investor's utility function[31].

"Random diversification does not use the full information set available to investors and does not, in general , lead to optimal diversification"[32].

If a portfolio P consisting of n securities and each security is weighted on a percentage basis, the sum of all weight equals one. Note that these weights can be negative, indicating a short sale. A short sale is where an investor borrows a share of stock from a broker and sells it hoping the price will decrease so that the investor can buy it back later at a lower price.[33] Markowitz in his early article did not mention a short sale.

In the course of time Markowitz gets into probabilistic figures , especially in 1959.

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[31] T. H. Naylor; Francis Tapon The Capital Asset Pricing Model: An Evaluation of Its Potential As a Strategic Planning Tool *Management Science*, Vol. 28, No. 10. (Oct., 1982)
[32] Brian Kettell, Financial Economics Making Sense of information in financial markets,
[33] Beste, Leventhal, Williams , Dr.Qin Lu, The Markowitz Model, Selecting an Efficient Investment Portfolio, Lafayette College 2002.

Let Y be a random variable, i.e., a variable whose value is decided by chance. Let the probability that $Y= y_1$ be p_1; that $Y=y_2$ be p_2 etc. The expected value (or mean) of Y is defined to be[34]

$$E= p_1 y_1 + p_2 y_2+...+ p_n y_n \quad \textbf{(1)}$$

The variance of Y is defined to be

$$V = p_1(y_1 - E)^2 + p_2(y_2 - E)^2 + ...P_N(y_N - E)^2 \quad \textbf{(2)}$$

V is the average squared deviation from its expected value. V is a commonly used measure of dispersion. Other measures of dispersion closely related to V are the standard deviation $\sigma = \sqrt[2]{V}$ and the coefficient of variation σ/E.

The next contribution to portfolio theory was a more simplified way related to Markowitz's type of computation[35].

In Markowitz model the weights can take negative numbers. But the rule is the sum of the weights must be equal to one. Tobin and Sharpe with the introduction of borrowing and lending options state that the weights say $w_i \geq 0$ rule is not a must.

It was Sharpe's contribution that all securities bear a common relationship with some "underlying base factor". That factor could be a stock market index, a gross national product, or some other price index. Using Sharpe's theory, an analyst would only need to measure the relationship of the security to the dominant base factor. Sharpe means that to some extent each stock is derived from some factor, i.e. the Market itself. He has greatly simplified Markowitz's approach[36].

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[34] H.Markowitz, Portfolio Selection The Journal of Finance, June 1991
[35] Hal Varian, A Portfolio of Nobel Laureates: Markowitz, Miller and Sharpe The Journal of Economic Perspectives, 1993
[36] Robert G.Hagstrom "The Essentail Buffett"

Sharpe's volatility measure is the "beta factor". Beta is described as the degree of correlation between two seperate price movements: the market as a whole and the individual stock as the part of it. Stocks that rise and fall in exactly the same proportions as the market are assigned beta of 1.0. If a stock rises and falls twice as far as the market, its beta is 2.0; if a stock's motion is only 80 percent of the market's motion , the beta is 0.8. The conclusion is that any portfolio with a beta greater than 1.0 is assigned to be more risky than the market, and a portfolio beta less than 1.0 is less risky[37].

Sharpe states that there is a linear relationship between each stock and the market. That can be explained by a linear regression line.

$St = (Rt - Rf) / \sigma t$

St= Sharpe index
Rt= Average return of portfolio t
Rf= Risk free rate
σt= Standard deviation of portfolio returns

Sharpe's performance criteria explains the extra return of a portfolio over the risk free return in relation to its total risk.

Sharpe expained the Markowitz Model by a utility function :

$U = E\,(Ew, \sigma w)$

Where Ew indicates expected future wealth and σw the predicted standard deviation of the possible divergence of actual future wealth from Ew.
Investors are assumed to prefer a higher expected future wealth to a lower value, ($dU/dEw > 0$). Moreover, they exhibit risk-aversion, choosing an investment offering a lower value of σw to one with a greater..

The second derivation is negative ($dU/d\sigma w, < 0$).

These assumption simply shows that the indifference curves relating Ew and σ_W are upward-sloping[37].

Sharpe considers on the possibility of borrowing. The investor can borrow at the pure rate of interest. Disinvesting is possible. The effect of borrowing to purchase a stock is simply letting the weight take on negative values. We will test the negative weights in the application section.

In the following section we will examine how to find an optimum portfolio and to draw a line which is tangent to the optimum portfolio. This will give the combinations of optimum portfolio and the risk free asset placements.

Treynor index gauges the risk premium of a portfolio. Risk premium equals to the portfolio return minus the risk free rate. As we mentioned before the slope of the characteristic line is the β coefficient. It is an indicator that shows the sensitivity of the portfolio to the changes in the over all market. The greater β , greater the risk.

Treynor's performance index is as it is shown below:

Tn= (Rn-Rf) / β

Tn= Treynor index
Rn=Average return of the portfolio n
Rf= Risk free rate
B= Beta coefficient

In the length of one decade, two academicians had defined two important elements of what would later be called "The Modern Portfolio Theory": Markowitz with his idea of the proper reward/risk balance which depends on diversification, and Sharpe with his definition of risk.

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[37] William Sharpe, Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk,The Journal of Finance ,September 1964

Optimum portfolio is described as “for a given level of risk, a portfolio which has the greatest return”. An analysis can be made by calculating the Sharpe Ratios of every possible portfolio combinations.

An important assetion came from a young assistant professor of finance at the University of Chicago, Eugene Fama[38].
“Fama’s message was very clear: Stock prices are not predictable, because the market is too efficient. In an efficient market, as information becomes available, many smart people (Fama called them “rational profit maximizers”) aggressively apply that information in a way that causes prices to adjust instantaneously, before anyone can profit”[38].

The Efficient Market Hypothesis (EMH) contradicts the technical analysis, by stating historical values cannot be used to predict the future prices.

Eugene Fama published a paper on the EMH in the Journal of Finance in 1970, and said in short “the evidence in support of the Efficient Markets Model is extensive, and (somewhat uniquely in economics) contradictory evidence is sparse."

EMH advocates state that if prices quickly reflect all relevant information, no method, including technical analysis, can "beat the market." The market will adjust to the information and there is no space for an investor to process the new information to reach outstanding returns.

And there is an addition to this view which came from Peter Bernstein. “The market disaster of 1974 convinced me that there had to be a better way to manage investment portfolios” [38].

Bachelier (1900) asserted that the historical price data were useless for predicting future price changes. The movement resembles a “Random Walk”.

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[38] Robert G.Hagstrom “The Essential Buffett” ,2001

Samuelson states that the risk taking in terms of mean and variance alone is applicable only if the returns are Gaussian, or where the utility function which we will deal with in the rest of this study is quadratic. As wealth increases the risk taking is reduced[39]. The utility curve is not linear and can not be.

Meir Statman claims that a well diversified porfolio must consist of at least 30-40 randomly selected stocks[40]. He states that there is a contradiction with the traditional 10 stocks belief. According to him diversification should be increased as long as the marginal benefits exceeded the marginal costs. The benefit is risk reduction , the cost is the transaction cost. As we mentioned before the theory of Markowitz based on there were no transaction costs. But in practice there are.. The usual argument for limited diversification is that marginal costs increase faster than marginal benefits as diversification increases.

It is clear that the risk is more diversified in 30-40 stocks.

Evans and Archer states that although the risk is more diversified the economic benefit is exhausted with 10 stocks or so. 8 to 16 stocks would resemble the market itself.

In Elton – Gruber's model , the linear regression model is used in the Sharpe's single index model, but the variables are different. Single index model assumes there is a linear relationship between a stock's return and the market return.

According to Elton and Gruber this kind of relationship is as it is shown below;

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[39] Paul A. Samuelson, The Fundamental Approximation Theorem of Portfolio Analysis in terms of Means, Variances and Higher Moments , The Review of Economic Studies, Vol. 37, No. 4. (Oct., 1970)

[40] Meir Statman, How Many Stocks Make a Diversified Portfolio?
The Journal of Financial and Quantitative Analysis, Vol. 22, No. 3. (Sep., 1987)

$R_i = \alpha_i + \beta_i RM + e_i$

R_i = real return of the stock i

RM=return of the market index

β_i=beta coefficient

e_i=error term

Elton and Gruber proposes a model based on the single index model. A single criteria in developing the optimum portfolio makes the application more easy. The criteria here is the Excess return/ Beta coefficient. Let's say Ci is the result of the formula. Excess Return is the return of an asset over the risk free interest rate.

And a Cut Off Rate is calculated (say C_i) . This is a reference ratio for the stocks which are to be included in the portfolio. Include if the Ci rate is greater than the Treynor Ratio.

$$C_i = \frac{\sigma_m^2 \sum_{i=1}^{n} \frac{(R_i - R_F)\beta_i}{\sigma_{ei}^2}}{1 + \sigma_m^2 \sum_{i=1}^{n} \frac{\beta_i^2}{\sigma_{ei}^2}}$$

Elton and Gruber's work shows that 51 percent of a portfolio standard deviation is eliminated as diversification increases from 1 to 10 securities. Adding 10 more securities eliminates an additional 5 percent of the standard deviation. Increasing the number of securities to 30 eliminates only an additional 2 percent of the standard deviation[41].

[41] Meir Statman, How Many Stocks Make a Diversified Portfolio?
The Journal of Financial and Quantitative Analysis, Vol. 22, No. 3. (Sep., 1987)

"The benefits of diversification are achieved with a surprisingly small number of securities."[42]

Statman calculates a 1,502 per cent return difference between a 10-stocks portfolio and a 500-stock portfolio through his data analysis based on the years between 1926 and 1984. The 500-stock portfolio has a greater return. And the return difference gets smaller as the course of time of the data changes .

Through 1979-1984 the figure is 0,49 Per Cent. This is smaller than 1,502 due to the difference of the time scale. The return is higher in a 500 stock portfolio.

As we mentioned before Sharpe explored an approach today known as the "market model" or the "single factor" model. It assumes that the return on each security is linearly related to a single index, usually a return on some stock market index such as the S&P500.

Thus the random return on asset a at time t can be written as

$$R_{at} = c + bR_{mt} + \epsilon_{at},$$ [43]

Where R_{mt} is the return on the S&P 500, and ϵ_{at} is an error term with expected value of zero. In this equation c is the expected return of the asset if the market is expected to have a zero return, while parameter *b* measures the sensitivity of the asset to "market conditions".

Problems that took half an hour of computer time using Markowitz model took only 30 seconds with Sharpe's model.[45]

Since the Markowitz Model is composed of quadratic calculations, it is more easy for an investor to use the Sharpe's model in predicting the tendency of the portfolio.

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[42] M.J Gruber "Modern Portfolio Theory"

[43] Hal Varian, A Portfolio of Nobel Laureates: Markowitz, Miller and Sharpe The Journal of Economic Perspectives, 1993

Chen, Roll and Ross (1986) states there are some factors that have influence over the stocks returns. This model is named the multi-factor model. Its a multi-regression model. The common properties are the dependent variable (the stock return) and the independent variable (the market return)[44].

Here is another criticism made by market researchers to the Modern Portfolio theory. It is claimed that the return distribution is not Gaussian. As a rule the risk can be explained by variance and standard deviation if the returns are normally distributed. We can test the skewness in order to understand whether the returns are normally distributed or not. The Central Limit theorem states as the frequency increases the distribution gets a bell shape. As a snap-shot it may not be distributed normally. It is almost impossible to make a perfect measurement and Markowitz himself states the theorem as an approximate calculation.

Ross (1976) initiated "The Arbitrage Pricing Theory". According to him expected return is a linear function of various macroeconomic factors. The sensitivity of return is explained by specific beta coefficients.

Samuelson states that the Tobin-Markowitz analysis is applicable only the statistical distributions are Gaussian or where the utility function to be maximized is quadratic.

Levy-Markowitz claim that mean-variance analysis' outcomes are almost identical whether expected utility or various utility functions are examined[45].

In recent years there has been a re-birth of random walk theory about the main subject. It claims return are independent and identically distributed. The advocates of the random walk return distribution are Bachalier, Kendall and the subject is also discussed by Fama.

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[44] İlhan EGE, http://www.tuik.gov.tr/ias/ias06/oturumI-3/ilhaneged%C3%BCzlt.doc (Korkmaz; Pekkaya 2005, 557).
[45] Mean-Variance Versus Direct Utility Maximization, Kroll,Levy,Markowitz

Kroll is not fully convinced of taking Standard deviation as measure of risk and Elton&Gruber point to the difficulty of estimating a numerous correlations within a matrix.

Konno and Yamazaki (1991) in their article utilize the mean absolute deviation in order to skip the computational difficulties of covariance matrices[46]. But this is not the subject of our thesis.

"Markowitz's model of portfolio selection shows how quadratic programming can be used to generate the set of portfolios which are efficient according to the mean-variance criterion. From the efficient sets, the investor selects that portfolio which best satisfies his preferences with respect to risk and return levels. Markowitz's analysis assumes that the investor is choosing only among risky securities. Following Tobin, Litner's analysis has shown that in a market with homogenous expectations, if investors had the option of investing in a riskless asset yielding the pure rate of interest, and if the costs of transactions, information, and management were all zero, then there shall be a unique optimal portfolio of risky securities for all. In such a situtation, the investment decision can be seperated in two steps: First find the portfolio that maximizes the ratio between the expected value and the standard deviation of the excess return over the pure rate of interest and second decide how to allocate the funds between the riskless asset and the portfolio of risky securities. Since every risky security has to be held, this unique optimal portfolio must include many different risky securities.

Investors with different expectations may invest in different numbers of securities. Moreover since the costs of transactions, information, and management are not zero but a positive number, it is economical to limit the number of securities in the portfolio."[47]

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[46] Hiroshi Konno and Hiroaki Yamazaki, Management Science, Vol. 37, No. 5, (May, 1991)
[47] Essentials of Portfolio Diversification Strategy , James C. T. Mao
The Journal of Finance, Vol. 25, No. 5. (Dec., 1970)

If an international diversification is to be made, it would be more meaningful to make the analysis based on Purchasing Power Parity, that is the unique price[48]. The measure can possibly be the U.S.Dollar denominated. But there is a need for an exchange rate adjustment due to the exchange rate movements. The Formula is,

$$\frac{1+R_S}{1+R} = (1+\Delta e)$$

Where,

R_S is the asset's rate of return in terms of US Dollars

Δe is the value change of domestic currency vis a vis the US Dolar.

R is Rate of Return of the Domestic Market/Asset.

Erdoğan concluded in his work that an international optimal portfolio selection should contain a 5% ISE share[49].

[48] Uluslar arası Portföy Yatırımları Analizi ve Fiyatlandırma Modeli , Oral Erdoğan, IMKB Araştırma Yayınları no:2 , November 1994

3. DATA AND METHODOLOGY

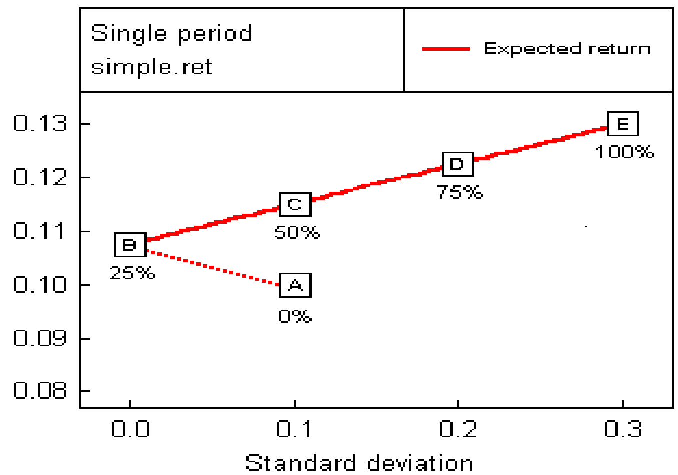
First of all we will be examining a simple two stock model and later on we will delve into some anaylses utilizing numerous stock datas while building up a portfolio. Along with simple mathematical equations, we will be utilizing Excel-Solver for more complex calculations. Statistical measure of skewness will be used in the decision of whether the distribution is Gaussian or not.

The Data provided through web sites mainly came from www.analiz.com. The year 2002 is chosen as data's beginning period. We will be utilizing the historical figures beginning from that date. For the multi-firm example the historical figures especially between 2005 and 2008 will be processed.

Below we can find expected return and standard deviation figures of two assets. Asset 1 is a low risk placement and Asset 2 is a risky one. The means and variances of the two assets and their correlation matrix are shown below

Asset	Expected return	Standard deviation	Correlation matrix	
			Asset 1	Asset 2
Asset 1	10.0%	10.0%	1.0	-1.0
Asset 2	13.0%	30.0%	-1.0	1.0

In the graph below x axis is the standard deviation and the y axis is the return. By taking into account the combination of the two asset, a graph like the one below can be plotted.



In the graph we start from Portfolio A (100% Asset 1) and begin to include some Asset 2. The expected return increases and the risk actually decreases until we reach Portfolio B at 25% of Asset 2. This "minimum variance portfolio" has almost zero risk (this is possible because the assets are 100% negatively correlated) [49].

The efficient frontier is displayed from Portfolio B, the minimum variance portfolio, to Portfolio E, the maximum return portfolio. The investor should select a portfolio on the efficient frontier in accordance with his/her risk tolerance.

Note that the maximum return portfolio consists 100% of the highest returning asset (in this case Asset 2). This is a general feature of single period mean variance optimization; while it is often possible to decrease the risk below that of the lowest risk asset, it is not possible to increase the expected return beyond that of the highest return asset[52].

But one stil needs to determine just which stocks, and which proportions of stocks shall be chosen as the compositions of the magic portfolio. That is a difficult and costly computation.

For an efficient choice we have to find a set of stocks which has a weak correlation. The intuition behind this can be explained as "during times of

[49] http://www.effisols.com/basics/MVO.htm#SinglePeriodMVO

drawback in the overall market we make sure we are standing on a balanced pair of stocks". More important than their individual standard deviations , the key point is the standard deviation of the pairs together and the correlation or more explicitly the standard deviation of the portfolio.

After a reasonable choice of the pairs we have to to find the standard deviation and examine our optimum selection by taking into account a risk free asset.

At first choice of the pairs is usually based on the historical volatility of the pairs and personal expectation about a stocks performance . If we believe a stock will do well in the future, we will be desiring to acquire the shares. On the other hand some accounting figures has to support our belief. If the asset size of the company increases the stock will do well. If a company plans an investment which will make a positive contribution to its profitability, the shares will go up. This kind of information according to Fama would already be priced by the market. If it is not priced it happens a transmission of information, insider trading or asymmetric information. In the text books an insider trader is defined as "anyone who has access to information that is both materially related to the value of a corporation's securities and unavailable to the general public". Asymmetric information happens where one party has more information than the other. A strong – form of efficient market would be one in which it is almost impossible to earn abnormal profits by using any kind of information. A weak form of efficient market is where it is almost impossible to do it by looking at the past records, the historical values. And it would give some an advantage over the others.

But it would be quite assertive if we say the Turkish Stock market is efficient. High tax rates on one hand is a drawback for efficiency and difference between the returns of domestic and foreign investors in terms of tax applications is on the other are among the main reasons of inefficiency.

We can talk about efficiency of a capital market if every sector in the economy is represented in the capital markets[50]. Erdoğan (1996) states although the shipping sector is important in terms of trading volume, there is no maritime company in ISE. The NYSE is one of the largest and the most efficent markets in the world.

The capital markets need long term funds for a less volatile system[51]. The more corporate the investors , the less volatile the market. Berk, Erdoğan (2004) examined the insurance sector and structured a model where the insurance premium over the GNP is an independent variable and the stock market volatility is a dependent variable. The inverse relation between these two variables is experienced. And the study concludes that towards the recent years portfolio investments in ISE securities decreased to minimal levels. The ratio of fixed asset securities to stocks changes. After 1994 it has been experienced deep decline in this ratio. The two reasons of this event are the precautionary attitudes of investors towards volatility and the high return levels of goverment debt instruments (treasury bills). The 1999-2001 period shows that the performance of treasury bill investments dominate ISE investnents in terms of return performances. This is one of the negative effect of the high levels of interest rates in Turkey.

After the financial crises of 1999-2001 period, since the economy started to recover, the asset prices made an adjustment. Before financial crises generally a boom is experienced in the asset prices. That is why we choose the year 2002 as a starting period of the historical data in our study.

As we gave the insight before, one of the drawbacks in our analysis is the weak form of efficiency of the Turkish Market, and its dependence to the world economy or in other words its fragility to the global shocks. Global

[50] Comparable Approach to "The Theory of Efficient Markets" A Modified Capital Assets Pricing Model For Maritime Firms, Dr.Oral Erdoğan ,November 1996 -Capital Markets Board

[51] Menkul Kıymetler Piyasasında Kurumsal Yatırımlar ve Performansı: Türkiye Örneği, Niyazi Berk, Oral Erdoğan ,İzmir İktisat Kongresi 2004

shocks affect the Turkish Market for it looses its plausibility of investing in the eyes of the investors.

And as long as there are not numerous stocks in the market, it is really hard to find a set that has a low level of correlation. Thus a medium level of correlation that is between 30-40% is regarded as low levels.

Let's start with some stocks quoted in ISE namely Bağfaş, İş C, Hürriyet and Adana Çimento. To go on with our analysis, we need a pair which has a weak correlation level. Below we can see the standard deviation, expected return and the correlation values of the pairs calculated from the date of 01.01.2002 until 19.03.2008.

2002-2008 data

BAĞFAŞ			HÜRRİYET	
average ret	0,001454		Average ret	0,000575
st dev	0,027278		st dev	0,030608
Correlation		0,439649		

HÜRRİYET			İŞ C	
average ret	0,000575		Average ret	0,000325
st dev	0,030608		st dev	0,032622
Correlation		0,601591		

HÜRRİYET			ADANA	
average ret	0,000575		Average ret	0,001195
st dev	0,030608		st dev	0,02827
Correlation		0,433564		

ADANA			İŞ C	
average ret	0,001195		Average ret	0,000325
st dev	0,02827		st dev	0,032622
Correlation		0,476115		

BAĞFAŞ			ADANA	
average ret	0,001454		Average ret	0,001195
st dev	0,027278		st dev	0,02827
Correlation		0,387603		

3.1 A two stock case

We need a weak correlation pair. Examining the final pair Bağfaş and Adana will be reasonable.

The standard deviation of stock returns tends to increase with the square root of time. If the standard deviation of daily returns is 2.72% and as long as there are 250 trading days in a year (T = 250), the monthly standard deviation is represented by the formula below:

$$\sigma_{annual} = \sigma_{daily} * \sqrt{T} = 2{,}72\% * \sqrt{250} \qquad (1)$$

We need to find an annual rate of return since the average rate of return is a daily figure. We need to convert it into a yearly basis. As we ommit the week-ends in our analysis we need to calculate the compound the annual rates as 250 times to reach a yearly base.

The Formula is, $R\ annual = (1 + Rdaily)^{250} - 1$ (2)

The annualized figures are shown below.

2002-2008 data

BAĞFAŞ			ADANA		
		Annualized			Annualized
Average ret	0,001454	0,437958	Average ret	0,001195	0,347859
st dev	0,027278	0,431309	Annual st dev	0,02827	0,44699
correlation		0,387603			

The TRT050809 refrenced bond has a 18,56% simple rate of interest and a 17,97% compounded interest rate. This bond will be our risk free asset as long as it is one of the main indicators of riskless interest rate.

In the literature review section, the calculation of the portfolio's standard deviation and the expected return were displayed.

But how to calculate the optimal combination is a matter of question. Here is the formula:

Optimal Combination of Risky Assets. Using the notation that
E1= E(r1)-rf and E2=E(r2)-rf,

Then the formula for the optimal proportion in the first asset is

$$w_1 = (E_1 \sigma^2 - E_2 p \sigma_1 \sigma_2)/(E_1 \sigma^2 + E_2 \sigma^2 - (E_1 + E_2) p \sigma_1 \sigma_2)$$

2002-2008 data

BAĞFAŞ				ADANA		
		Annualized				annualized
average ret	0,001454	0,437958		average ret	0,001195	0,347859
St dev	0,027278	0,431309		annual st dev	0,02827	0,44699
Correlation		0,387603				

ANNUAL CALCULATION

	E.Return	S.Deviation				
Inputs(annual)						
Riskless Rate	17,97%	0%				
Risky Asset 1	43,80%	43,13%				
Risky Asset 2	34,79%	44,70%				
Correlation		0,387603				
Outputs	Proportion in Risky Asset 1 or Opt. Comb	(x-axis) Standard Deviation	Risk-Ret Trade-off Curve Expected Return	E1 E(r1)-rf	E2 E(r2)-rf	Opt.W1
Trade-off Curve	-60%	0,659533	29,38%	25,83%	16,82%	0,7651099
Trade-off Curve	-50%	0,619651	30,28%			
Trade-off Curve	-40%	0,581102	31,18%			
Trade-off Curve	-30%	0,544167	32,08%			
Trade-off Curve	-20%	0,5092	32,98%			
Trade-off Curve	-10%	0,476633	33,88%			
Trade-off Curve	0%	0,44699	34,79%			
Trade-off Curve	10%	0,420891	35,69%			
Trade-off Curve	20%	0,399031	36,59%			
Trade-off Curve	30%	0,382138	37,49%			
Trade-off Curve	40%	0,370892	38,39%			
Trade-off Curve	50%	0,365814	39,29%			
Trade-off Curve	60%	0,36716	40,19%			
Trade-off Curve	70%	0,374861	41,09%			
Trade-off Curve	80%	0,388538	41,99%			
Trade-off Curve	90%	0,407592	42,89%			
Trade-off Curve	100%	0,431309	43,80%			
Trade-off Curve	110%	0,458968	44,70%			
Trade-off Curve	120%	0,489901	45,60%			
Trade-off Curve	130%	0,523528	46,50%			

Trade-off Curve	140%	0,559364	47,40%
Trade-off Curve	150%	0,597011	48,30%
Trade-off Curve	160%	0,636148	49,20%
Trade-off Curve	170%	0,676515	50,10%
Trade-off Curve	180%	0,717906	51,00%
Trade-off Curve	190%	0,760153	51,90%
Trade-off Curve	200%	0,803122	52,81%

	Expected Return	Standard Deviation			
Inputs (daily)					
Riskless Rate	0,07%	0%			
Risky Asset 1	0,15%	2,73%			
Risky Asset 2	0,12%	2,83%			
Correlation		0,387603			
Outputs	Proportion in Risky Asset 1 or Opt. Comb	(x-axis) Standard Deviation	Risk-Ret Trade-off Curve Expected Return	E1 E(r1)-rf	E2 E(r2)-rf
Trade-off Curve	-60%	0,041712331	0,10%	0,08%	0,05%
Trade-off Curve	-50%	0,039190009	0,11%		
Trade-off Curve	-40%	0,036751935	0,11%		
Trade-off Curve	-30%	0,034416018	0,11%		
Trade-off Curve	-20%	0,032204495	0,11%		
Trade-off Curve	-10%	0,030144757	0,12%		
Trade-off Curve	0%	0,02827	0,12%		
Trade-off Curve	10%	0,026619337	0,12%		
Trade-off Curve	20%	0,025236777	0,12%		
Trade-off Curve	30%	0,024168377	0,13%		
Trade-off Curve	40%	0,023457101	0,13%		
Trade-off Curve	50%	0,023135911	0,13%		
Trade-off Curve	60%	0,023221	0,14%		
Trade-off Curve	70%	0,023707994	0,14%		
Trade-off Curve	80%	0,024573009	0,14%		
Trade-off Curve	90%	0,025778018	0,14%		
Trade-off Curve	100%	0,027278	0,15%		
Trade-off Curve	110%	0,029027264	0,15%		
Trade-off Curve	120%	0,030983616	0,15%		
Trade-off Curve	130%	0,033110368	0,15%		
Trade-off Curve	140%	0,035376803	0,16%		
Trade-off Curve	150%	0,037757775	0,16%		
Trade-off Curve	160%	0,040232954	0,16%		

Trade-off Curve	170%	0,042785994	0,16%
Trade-off Curve	180%	0,045403762	0,17%
Trade-off Curve	190%	0,048075686	0,17%
Trade-off Curve	200%	0,050793221	0,17%

The results of the daily based figures would be in in line with the analysis made with compounded figures. But is decompounding of a risk free assets to be made on 250 days or 365 days? If we use 365 days in our decompounding calculations it makes quite a difference. Utilizing the yearly compounded rates based on 365 days is accurate.

Since the optimal selection is made (that is %76,5 of W_1) we can calculate the expected return of this selection.

Outputs	Proportion in Risky Asset 1 or Opt. Comb	(x-axis) Standard Deviation	Risk-Ret Trade-off Curve Expected Return
Opt.selection	0,765	0,3831	41,68%

The result of the expected return is 41,68%.

We have to go on with our analysis taking a risk free asset in our portfolio as a riskless placement.

Since the risk is zero, the standard deviation of the risk free placement is zero. Standard deviation in this time is quite a short formula,

$\sigma = w\,\sigma_T$ where σ_T is the Standard deviation of the "optimal combination of Risky Assets (or Tangent Portfolio).

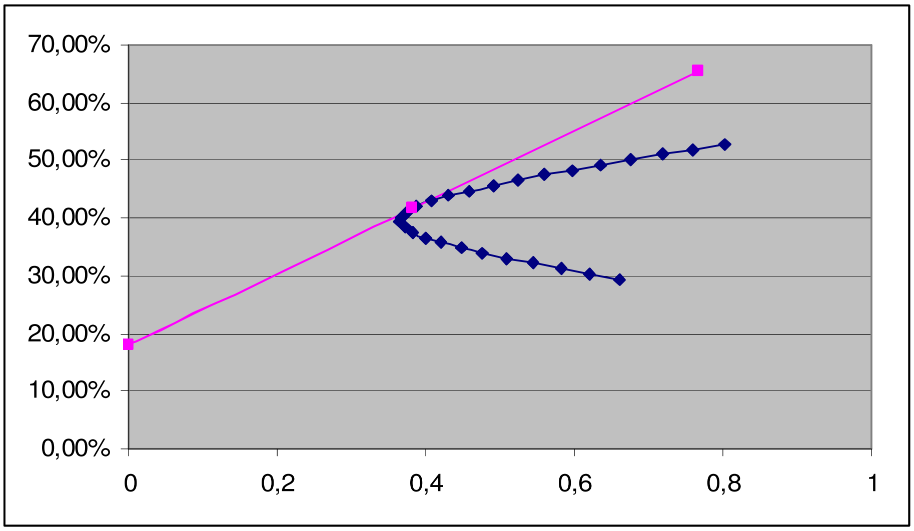
	Proportion in Risky Asset 1 or Opt. Comb	(x-axis) Standard Deviation
eff trade off line	0%	0
eff trade off line	100%	0,383121
eff trade off line	200%	0,766243

Since, the risk free rate is known as the 17,97%,

The expected returns of the efficient trade-off line can be calculated as follows,

	Proportion İn Risky Asset 1 or Opt. Comb	(x-axis) Standard Deviation	Efficient Trade – off Line Expected Return
Opt.sel.	**76,5%**	**0,383121**	**41,68%**
eff trade off line	0%	0	17,97%
eff trade off line	100%	0,383121	41,68%
eff trade off line	200%	0,766243	65,39%

We can now create a graph; on the y axis the expected return and on the x axis the standard deviation are displayed.



In practice is the optimal portfolio is meaningful i.e. within the following 1 year term? We are going to take historical values between 2002-2006 as data , and later will examine the actual figures for the year 2007.

2002-2006 years

Hürriyet			Ereğli		
		Annualized			annualized
average ret	0,001053257	0,437958	Average ret	0,00142	0,347859
st dev	0,03108753	0,431309	annual st dev	0,026831	0,44699
correlation		0,545697			

	Expected Return	Standard Deviation
Inputs(annual)		
Riskless Rate	17,97%	0%
Risky Asset 1	43,80%	43,13%
Risky Asset 2	34,79%	44,70%
correlation		0,545697

Outputs	Proportion in Risky Asset 1 or Opt. Comb	(x-axis) Standard Deviation	Risk-Ret Trade-off Curve Expected Return	Efficient Trade – off Line Expected Return
Trade-off Curve	-60%	0,613566	29,38%	
Trade-off Curve	-50%	0,581592	30,28%	
Trade-off Curve	-40%	0,550947	31,18%	
Trade-off Curve	-30%	0,521866	32,08%	
Trade-off Curve	-20%	0,494626	32,98%	
Trade-off Curve	-10%	0,469546	33,88%	
Trade-off Curve	0%	0,44699	34,79%	
Trade-off Curve	10%	0,427359	35,69%	
Trade-off Curve	20%	0,41107	36,59%	
Trade-off Curve	30%	0,398536	37,49%	
Trade-off Curve	40%	0,390117	38,39%	
Trade-off Curve	50%	0,386082	39,29%	
Trade-off Curve	60%	0,38657	40,19%	
Trade-off Curve	70%	0,391563	41,09%	
Trade-off Curve	80%	0,400893	41,99%	
Trade-off Curve	90%	0,414267	42,89%	
Trade-off Curve	100%	0,431309	43,80%	
Trade-off Curve	110%	0,451604	44,70%	
Trade-off Curve	120%	0,474735	45,60%	
Trade-off Curve	130%	0,500308	46,50%	
Trade-off Curve	140%	0,527969	47,40%	
Trade-off Curve	150%	0,557408	48,30%	
Trade-off Curve	160%	0,588357	49,20%	
Trade-off	170%	0,62059	50,10%	

Curve				
Trade-off Curve	180%	0,653918	51,00%	
Trade-off Curve	190%	0,688182	51,90%	
Trade-off Curve	200%	0,723248	52,81%	
Optimum portfolio	**89%**	**0,413034**		42,82%
eff trade off line	0%	0		17,97%
eff trade off line	100%	0,413034		42,82%
eff trade off line	200%	0,826068		67,67%

E(r1)-rf	E(r2)-rf	opt.sel
25,83%	16,82%	0,76511

The optimum portfolio for two assets is %89 of Hürriyet and %11 of Ereğli. (Based on historical values from 2002 to 2006)

As an outcome an 11% inclusion of Asset 2 we will decrease the annual standard deviation almost by 2% .

Lets analyse now what happened in the year 2007.

2007	Hürriyet	Ereğli
average	-0,0002	0,000645
st dev	0,024626	0,051198

The figures for 2007 occured differently. As a reason the standard deviation is not a guarantee of price movements. So it only can give us an insight.

3.2 A three stock case

We will examine a three stock case quoted in ISE 100 Index.

a efes			eregli			İş c		
return	-0,0002	-0,0463	return	0,0012	0,3494	Return	0,0003	0,0727
st dev	0,0445	0,7034	st dev	0,0315	0,4979	St dev	0,0326	0,5158
hürriyet			sahol					
return	0,0005	0,1461	return	0,0001	0,0255			
st dev	0,0306	0,4838	st dev	0,0290	0,4581			

We choose Ereğli - İş C -Hürriyet from a pool of five stocks whose individual returns and pair of correlation levels are at desirable levels.

The variance-covariance matrix is shown below:

Variance - Covariance Matrix			
	Ereğli	**iş c**	**hürriyet**
ereğli	0,2479	0,1256	0,1089
İşç	0,1256	0,2661	0,1501
hürriyet	0,1089	0,1501	0,2341

Assumption: We assume a composition as shown below

composition of stocks	
Ereğli	0,1500
İşç	0,3000
Hürriyet	0,5500
Total	**1,0000**

The easiest way to compute portfolio variance is to use the matrix multiplication method. Let's take the three in a matrix. Weights and variance-covariance matrix are displayed below.

Matrix Multiplication

0,15	0,30	0,55	0,2479	0,1256	0,1089		0,15
			0,1256	0,2661	0,1501		0,3
			0,1089	0,1501	0,2341		0,55

0,13	0,18	0,19	0,4
			0,35
			0,25

The portfolio variance is

0,16

The portfolio Standard Deviation of the portfolio

0,4060

The Expected Return of The Portfolio

0,1023

As a consequence the standard deviation of the portfolio is less than any of the three firms' individual standard deviations. And there is a reasonable portfolio expected return level of 10,23%.

That means the theory holds for a three company case. But when the number of stocks in a portfolio increases the computation becomes complicated and the calculations may consume a great deal of time.

The expected return of the portfolio above is calculated by the summation of the weights of each stock which were multiplied by their individual expected returns.

For a simple explanation we can apply a linear regression model in predicting the future returns;

We need to know how a linear regression model is to be build up.

Example:

What is the least squares regression line for the data set of X and Y axis {(1,1), (2,3), (4,6), (5,6)}?

$\bar{x}$ = (1+2+4+5)/4 = 3
$\bar{y}$ = (1+3+6+6)/4 = 4
SS_xx = ((1-3)^2+(2-3)^2+(4-3)^2+(5-3)^2) = 10
SS_yy = ((1-4)^2+(3-4)^2+(6-4)^2+(6-4)^2) = 18
SS_xy = ((1-3)(1-4)+(2-3)(3-4)+(4-3)(6-4)+(5-3)(6-4)) = 13
B_1 = 13/10 = 1.3

Since we know the means we can calculate

B_0 = 4 - 1.3 × 3 = 0 .1
$\hat{y}$ = 0.1 + 1.3x

Since this kind of computation is a basic one, it is useful in showing us how to calculate the Alpha and the Beta figures. This gives usd an insight about setting a linear regression model based on the market performance.

As we know from the literature review section we recall that W.Sharpe introduced the Beta Coefficient which was a derivation of the whole market.

The formula for the Beta of an asset within a portfolio is
$\beta a = Cov(r_a, r_p) / Var(r_p)$

Below we will use the Beta as a risk performance criteria and we will make a comparison with the Treynor Ratio. As we can see below "Ereğli" is more favorable than the other two assets.

	annual return(Ri)	Rf	Beta	(Ri-Rf)/Beta
ereğli	0,349406738	0,1797	0,948639	0,1789
iş c	0,072720031	0,1797	1,271096	-0,0842
hürriyet	0,146070498	0,1797	1,052552	-0,0320

Diversification is sucessful if the Beta of the combination is less than the Beta of the individual assets.

Here is the return and the standart deviation figures of the ISE 100 index. Portfolio theory holds where the overall market has lower risk while the expected return levels are similar.

Thus between the years of 2002-2008 a 100 firm placement would be less risky and more profitable.

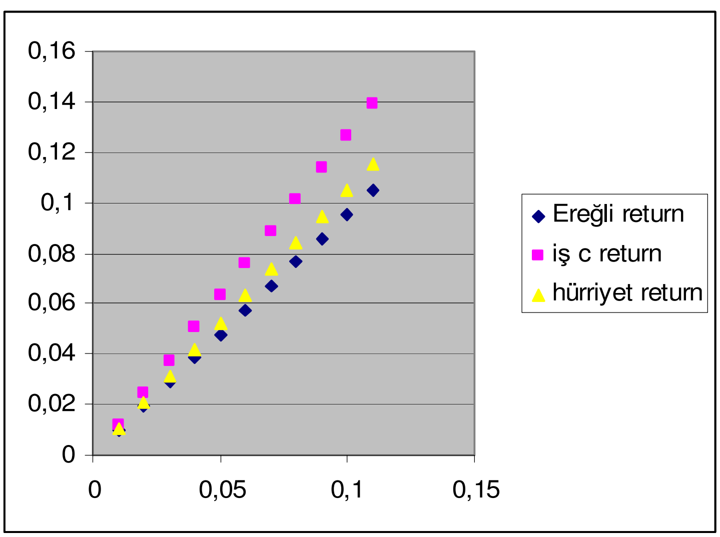
İSE	Daily	Annualized
Return	0,0007	0,1914
st dev	0,0217	0,3425

Covariance ISE-Ereğli	0,000445
Covariance ISE-iş c	0,000597
Covariance ISE-hürriyet	0,000494

Beta ISE-Ereğli	0,948639
Beta ISE-iş c	1,271096
Beta ISE-hürriyet	1,052552

The graph below shows the characteristic lines (regression line) of the three stocks; the x axis is the market return and the y axis is the expected return (in relation to a potential market return).

The choice of the assets will differ due to the risk tolerance.



Since the composition of the stocks are:

composition of stocks	
Ereğli	0,15
İşç	0,30
Hürriyet	0,55
Total	**1,00**

The Portfolio Beta is:

BETA of the PORTFOLIO
1,102528021

The portfolio's Beta is slightly higher than the Beta of the first and the third stock. As long as the Beta is a risk measuring indicator a risk aversed person will not choose these three firms together. Because it has a risk level more than the market.

Here are the Alpha values that puts each stock in a linear model with ISE 100.

	covariance with ISE	Stdev of ISE	variance of ISE	BETA	ALPHA
Ereğli	0,000445	0,021665	0,000469	0,948639	0,000535
iş c	0,000597	0,021665	0,000469	1,271096	-0,000610
hürriyet	0,000494	0,021665	0,000469	1,052552	-0,000192

In line with the Beta Calculation the portfolio's alpha is;

ALPHA OF p
-0,000208295

3.3 Single Index Model : A five stock case

We are now in the aim of constructing a portfolio made up of five stocks. According to the Single Index Model we will examine if the stocks are to be included in that portfolio or not.

First of all we have to calculate the error term utilizing a linear regression between each individual stock and the market. SSR here is the "sum of square of residuals".

Data: 02.01.2002 – 20.03.2008

	a efes	eregli	iş c	hürriyet	Sahol
Return	-0,00019	0,001199	0,000281	0,000546	0,000101
st dev	0,044484	0,031487	0,032623	0,030598	0,028975
Cov with ISE	0,000317	0,000445	0,000597	0,000494	0,000495
Var ISE	0,000469	0,000469	0,000469	0,000469	0,000469
Beta	0,676406	0,948639	1,271096	1,052552	1,055683
Alpha	-0,00066	0,000535	-0,00061	-0,00019	-0,00064
SSR	2,617463	0,843616	0,45245	0,61669	0,468607

We have to find the Cut-Off Rates by appling the figures in the formula and then comparing each result with its Treynor Ratio. We take the stock in the portfolio if it has a higher level of Treynor Ratio over the Cut off rate.

	Return i	Rf	Rf decomp.	Beta	SSR	(Av. Ret.-Rf)	(Av. Ret.-Rf) * beta	(Av. Ret.-Rf) * beta /SSR	Cumul.	Cumul. * Var(M)
a efes	-0,0002	0,1797	0,0005	0,6764	2,6175	-0,0006	-0,0004	-0,0002	-0,0002	-0,00000008
eregli	0,0012	0,1797	0,0005	0,9486	0,8436	0,0007	0,0007	0,0008	0,0007	0,00000032
iş c	0,0003	0,1797	0,0005	1,2711	0,4524	-0,0002	-0,0002	-0,0005	0,0002	0,00000009
hürriyet	0,0005	0,1797	0,0005	1,0526	0,6167	0,0001	0,0001	0,0002	0,0003	0,00000016
sahol	0,0001	0,1797	0,0005	1,0557	0,4686	-0,0004	-0,0004	-0,0008	-0,0004	-0,00000021

Beta^2	Nominator	Cumul.	cumul.* Var(M)	denom.	(nom./denom.) FORMULA RESULT	Treynor Ratio	(Treynor-result) Comparison	**decision**	
0,4575	0,1748	0,1748	0,0001	1,0001	-0,00000008	-0,0009	Less	exclude	a efes
0,8999	1,0667	1,2415	0,0006	1,0006	0,00000032	0,0008	More	include	eregli
1,6157	3,5710	4,8125	0,0023	1,0023	0,00000009	-0,0001	Less	exclude	iş c
1,1079	1,7965	6,6090	0,0031	1,0031	0,00000016	0,0001	More	include	hürriyet
1,1145	2,3783	8,9872	0,0042	1,0042	-0,00000021	-0,0003	-less	exclude	sahol

3.4 Markowitz Model with Computer Techniques: A five stock case

Data: 02.01.2002 – 20.03.2008

	Variance level	st deviation	Return	AEU
Min variance	0,000618	0,024865	0,000204	- 0,000105
P1	0,000622	0,024939	0,000204	- 0,000107
P2	0,000625	0,025000	0,000205	- 0,000108
P3	0,000632	0,025139	0,000206	- 0,000110
P4	0,000636	0,025215	0,000206	- 0,000112
P5	0,000641	0,025313	0,000207	- 0,000114
P6	0,000670	0,025884	0,000211	- 0,000124
P7	0,000700	0,026462	0,000214	- 0,000136

Diversification is as shown below where Approximate Expected Utility (AEU) is maximized.

a. efes	0,18
Eregli	-
iş c	0,13
hürriyet	0,30
Sahol	0,39

In our cases we see the Elton Gruber's Model is 30% consistent with Markowitz's Model.

3.5 Markowitz Model with Computer Techniques : A ten stock case

Here is a table composed of risk and return combinations of ten ISE quoted stocks from the beginning of 2005 until the end of March 2008.

	AKBANK	ARÇELİK	ASELSAN	ECZACI	GARANTİ	HÜRRİYET	PINAR	ŞİŞE CAM	TOFAŞ	ÜNYE ÇİM
RETURN	0,0016	0,0014	0,0017	0,0017	0,0018	0,0018	0,0021	0,0014	0,0012	0,0018
STDEV	0,0380	0,0384	0,0414	0,0394	0,0417	0,0433	0,0393	0,0405	0,0397	0,0316

We calculate the values in the correlation matrix with the help of Excel-Tools-Data Analysis[52].

CORRELATION MATRIX

	AKBANK	ARÇELİK	ASELSAN	ECZACI	GARANTİ	Hurriyet	PINAR	ŞİŞE CAM	Tofaş	ÜNYE ÇİM
AKBANK	1,0000	0,5627	0,4519	0,4937	0,5983	0,5132	0,3915	0,5224	0,5327	0,4474
ARÇELİK	0,5627	1,0000	0,4758	0,5178	0,5521	0,5246	0,4334	0,5408	0,5589	0,5055
ASELSAN	0,4519	0,4758	1,0000	0,4963	0,4501	0,4828	0,4360	0,4506	0,4698	0,4486
ECZACI	0,4937	0,5178	0,4963	1,0000	0,5095	0,5001	0,4327	0,5041	0,5166	0,4963
GARANTİ	0,5983	0,5521	0,4501	0,5095	1,0000	0,5173	0,3934	0,5167	0,5182	0,4495
HÜRRİYET	0,5132	0,5246	0,4828	0,5001	0,5173	1,0000	0,4123	0,4823	0,4816	0,4546
PINAR	0,3915	0,4334	0,4360	0,4327	0,3934	0,4123	1,0000	0,4273	0,4362	0,4149
ŞİŞE CAM	0,5224	0,5408	0,4506	0,5041	0,5167	0,4823	0,4273	1,0000	0,5244	0,4956
TOFAŞ	0,5327	0,5589	0,4698	0,5166	0,5182	0,4816	0,4362	0,5244	1,0000	0,4791
ÜNYE ÇİM	0,4474	0,5055	0,4486	0,4963	0,4495	0,4546	0,4149	0,4956	0,4791	1,0000

As long as we know the Standard deviations,

	AKBANK	ARÇELİK	ASELSAN	ECZACI	GARANTİ	HÜRRİYET	PINAR	ŞİŞE CAM	TOFAŞ	ÜNYE ÇİM
STDEV	0,0380	0,0384	0,0414	0,0394	0,0417	0,0433	0,0393	0,0405	0,0397	0,0316

We can calculate a covariance matrix by simply multiplying the correlation coefficients with the multiplication of two stock's standard deviations.

COVARIANCE MATRIX

	AKBANK	ARÇELİK	ASELSAN	ECZACI	GARANTİ	HÜRRİYET	PINAR	ŞİŞE CAM	TOFAŞ	ÜNYE ÇİM
AKBANK	0,0014	0,0008	0,0007	0,0007	0,0009	0,0008	0,0006	0,0008	0,0008	0,0005
ARÇELİK	0,0008	0,0015	0,0008	0,0008	0,0009	0,0009	0,0007	0,0008	0,0009	0,0006
ASELSAN	0,0007	0,0008	0,0017	0,0008	0,0008	0,0009	0,0007	0,0008	0,0008	0,0006
ECZACI	0,0007	0,0008	0,0008	0,0015	0,0008	0,0009	0,0007	0,0008	0,0008	0,0006
GARANTİ	0,0009	0,0009	0,0008	0,0008	0,0017	0,0009	0,0006	0,0009	0,0009	0,0006
HÜRRİYET	0,0008	0,0009	0,0009	0,0009	0,0009	0,0019	0,0007	0,0008	0,0008	0,0006
PINAR	0,0006	0,0007	0,0007	0,0007	0,0006	0,0007	0,0015	0,0007	0,0007	0,0005
ŞİŞE CAM	0,0008	0,0008	0,0008	0,0008	0,0009	0,0008	0,0007	0,0016	0,0008	0,0006
TOFAŞ	0,0008	0,0009	0,0008	0,0008	0,0009	0,0008	0,0007	0,0008	0,0016	0,0006
ÜNYE ÇİM	0,0005	0,0006	0,0006	0,0006	0,0006	0,0006	0,0005	0,0006	0,0006	0,0010

[52] G.Küçükkocaoğlu, Optimal Portföyün Seçimi ve İMKB Ulusal-30 Endeksi Üzerine Bir Uygulama

We assume an equal weight for each stock in a portfolio. That makes 10% weight for each asset. We multiply the values on the rows and columns with the covariance values.

COVARIANCE MATRIX

		AKBANK	ARÇELİK	ASELSAN	ECZACI	GARANTİ	HÜRRİYET	PINAR	ŞİŞE CAM	TOFAŞ	ÜNYE ÇİM
		0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
AKBANK	0,1	0,0014	0,0008	0,0007	0,0007	0,0009	0,0008	0,0006	0,0008	0,0008	0,0005
ARÇELİK	0,1	0,0008	0,0015	0,0008	0,0008	0,0009	0,0009	0,0007	0,0008	0,0009	0,0006
ASELSAN	0,1	0,0007	0,0008	0,0017	0,0008	0,0008	0,0009	0,0007	0,0008	0,0008	0,0006
ECZACI	0,1	0,0007	0,0008	0,0008	0,0015	0,0008	0,0009	0,0007	0,0008	0,0008	0,0006
GARANTİ	0,1	0,0009	0,0009	0,0008	0,0008	0,0017	0,0009	0,0006	0,0009	0,0009	0,0006
HÜRRİYET	0,1	0,0008	0,0009	0,0009	0,0009	0,0009	0,0019	0,0007	0,0008	0,0008	0,0006
PINAR	0,1	0,0006	0,0007	0,0007	0,0007	0,0006	0,0007	0,0015	0,0007	0,0007	0,0005
ŞİŞE CAM	0,1	0,0008	0,0008	0,0008	0,0008	0,0009	0,0008	0,0007	0,0016	0,0008	0,0006
TOFAŞ	0,1	0,0008	0,0009	0,0008	0,0008	0,0009	0,0008	0,0007	0,0008	0,0016	0,0006
ÜNYE ÇİM	0,1	0,0005	0,0006	0,0006	0,0006	0,0006	0,0006	0,0005	0,0006	0,0006	0,0010

The results of the multiplications are,

RESTRICTED COVARIANCE MATRIX

		AKBANK	ARÇELİK	ASELSAN	ECZACI	GARANTİ	HÜRRİYET	PINAR	ŞİŞE CAM	TOFAŞ	ÜNYE ÇİM
		0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
AKBANK	0,1	0,000014	0,000008	0,000007	0,000007	0,000009	0,000008	0,000006	0,000008	0,000008	0,000005
ARÇELİK	0,1	0,000008	0,000015	0,000008	0,000008	0,000009	0,000009	0,000007	0,000008	0,000009	0,000006
ASELSAN	0,1	0,000007	0,000008	0,000017	0,000008	0,000008	0,000009	0,000007	0,000008	0,000008	0,000006
ECZACI	0,1	0,000007	0,000008	0,000008	0,000015	0,000008	0,000009	0,000007	0,000008	0,000008	0,000006
GARANTİ	0,1	0,000009	0,000009	0,000008	0,000008	0,000017	0,000009	0,000006	0,000009	0,000009	0,000006
HÜRRİYET	0,1	0,000008	0,000009	0,000009	0,000009	0,000009	0,000019	0,000007	0,000008	0,000008	0,000006
PINAR	0,1	0,000006	0,000007	0,000007	0,000007	0,000006	0,000007	0,000015	0,000007	0,000007	0,000005
ŞİŞE CAM	0,1	0,000008	0,000008	0,000008	0,000008	0,000009	0,000008	0,000007	0,000016	0,000008	0,000006
TOFAŞ	0,1	0,000008	0,000009	0,000008	0,000008	0,000009	0,000008	0,000007	0,000008	0,000016	0,000006
ÜNYE ÇİM	0,1	0,000005	0,000006	0,000006	0,000006	0,000006	0,000006	0,000005	0,000006	0,000006	0,000010
TOTAL	1	0,000082	0,000086	0,000085	0,000085	0,000091	0,000092	0,000074	0,000087	0,000086	0,000063

The final row is the sum of the weighted covariances while each row shows the stock's risk in the portfolio the sum of them will show the overall riskiness explicitly "the variance of the portfolio".

The sum of the final row is 0,00083.

Portfolio variance	0,00083	
Portfolio Std deviation	0,028833194	Daily
Portfolio R	0,001653295	Daily

If we compare the portfolio's standard deviation to the standard deviations of each stock, we can see that portfolio diversification has worked well for our portfolio application. The risk is lower than each of the individual assets and the return of the portfolio is reasonable.

	AKBANK	ARÇELİK	ASELSAN	ECZACI	GARANTİ	HÜRRİYET	PINAR	ŞİŞE CAM	TOFAŞ	ÜNYE ÇİM
STDEV	0,038	0,038	0,041	0,039	0,042	0,043	0,039	0,041	0,040	0,032

Portfolio Std deviation	0,0288

Next we will calculating a composition of the assets with a higher return level and with the same level of risk. In this phase of our analysis "a short selling" gets in our calculations. The figures below are calculated through the Solver facility of Excel. Solver will be used for quadratic programming[53].

We realize that we are not allowed for a short sale in ISE. With the assumption of neglecting a short sale, the efficient composition provides a 0,00074 of portfolio variance and a 0,00175 of daily return. The optimum portfolio in this example would not be a short sold one even if we were allowed to.

[53] Jackson, Staunton, Programming Applications in Finance Using Excel, The Journal of the Operating Research Society, Vol 50 No 12, December 1999

RESTRICTED COVARIANCE MATRIX

		AKBANK	ARÇELİK	ASELSAN	ECZACI	GARANTİ	HÜRRİYET	PINAR	ŞİŞE CAM	TOFAŞ	ÜNYE ÇİM
		0,1376668	0,053786384	0,066615629	0,060609575	0,02244123	0,007220584	0,165013483	0,02968507	0,050153411	0,406807834
AKBANK	0,1376668	0,000027	0,000006	0,000007	0,000006	0,000003	0,000001	0,000013	0,000003	0,000006	0,000030
ARÇELİK	0,053786384	0,000006	0,000004	0,000003	0,000003	0,000001	0,000000	0,000006	0,000001	0,000002	0,000013
ASELSAN	0,066615629	0,000007	0,000003	0,000008	0,000003	0,000001	0,000000	0,000008	0,000001	0,000003	0,000016
ECZACI	0,060609575	0,000006	0,000003	0,000003	0,000006	0,000001	0,000000	0,000007	0,000001	0,000002	0,000015
GARANTİ	0,02244123	0,000003	0,000001	0,000001	0,000001	0,000001	0,000000	0,000002	0,000001	0,000001	0,000005
HÜRRİYET	0,007220584	0,000001	0,000000	0,000000	0,000000	0,000000	0,000000	0,000001	0,000000	0,000000	0,000002
PINAR	0,165013483	0,000013	0,000006	0,000008	0,000007	0,000002	0,000001	0,000042	0,000003	0,000006	0,000035
ŞİŞE CAM	0,02968507	0,000003	0,000001	0,000001	0,000001	0,000001	0,000000	0,000003	0,000001	0,000001	0,000008
TOFAŞ	0,050153411	0,000006	0,000002	0,000003	0,000002	0,000001	0,000000	0,000006	0,000001	0,000004	0,000012
ÜNYE ÇİM	0,406807834	0,000030	0,000013	0,000016	0,000015	0,000005	0,000002	0,000035	0,000008	0,000012	0,000166
TOTAL	1	0,000102	0,000040	0,000049	0,000045	0,000017	0,000005	0,000123	0,000022	0,000037	0,000302

Portfolio variance	0,00074	
Portfolio Std deviation	0,02726	Daily
Portfolio R	0,00175	Daily

We simply can say than with a lower level of risk exposure , a higher expected return prevails in our application above. The outcome of 0,175% indicated a more efficient selection related tothe previous equally weighted scenario.

Now we are going to examine the year 2006. The calculation is based on the historical values from the beginning of 2002 through the end of 2005.

Here are the historical values,

	AKBANK	ARÇELİK	ASELSAN	ECZACI	GARANTİ	HÜRRİYET	PINAR	ŞİŞE CAM	TOFAŞ	ÜNYE ÇİM
RETURN	0,0017	0,0011	0,0016	0,0022	0,0016	0,0017	0,0018	0,0013	0,0005	0,0012
ST DEV	0,0292	0,0299	0,0359	0,0302	0,0319	0,0314	0,0312	0,0278	0,0285	0,0283

The correlation matrix is,

	AKBANK	ARÇELİK	ASELSAN	ECZACI	GARANTİ	HÜRRİYET	PINAR	ŞİŞE CAM	TOFAŞ	ÜNYE ÇİM
AKBANK	1,0000	0,6598	0,3248	0,4818	0,6941	0,6309	0,3656	0,6472	0,6132	0,4088
ARÇELİK	0,6598	1,0000	0,3009	0,4910	0,6358	0,6308	0,3824	0,6194	0,6316	0,4301
ASELSAN	0,3248	0,3009	1,0000	0,3304	0,3197	0,3156	0,2555	0,3105	0,3119	0,2823
ECZACI	0,4818	0,4910	0,3304	1,0000	0,5254	0,4741	0,3100	0,5081	0,4884	0,4012
GARANTİ	0,6941	0,6358	0,3197	0,5254	1,0000	0,6450	0,3814	0,6733	0,6398	0,4349
HÜRRİYET	0,6309	0,6308	0,3156	0,4741	0,6450	1,0000	0,3660	0,5948	0,5775	0,3991
PINAR	0,3656	0,3824	0,2555	0,3100	0,3814	0,3660	1,0000	0,4221	0,4106	0,3608
ŞİŞE CAM	0,6472	0,6194	0,3105	0,5081	0,6733	0,5948	0,4221	1,0000	0,6357	0,4389
TOFAŞ	0,6132	0,6316	0,3119	0,4884	0,6398	0,5775	0,4106	0,6357	1,0000	0,4473
ÜNYE ÇİM	0,4088	0,4301	0,2823	0,4012	0,4349	0,3991	0,3608	0,4389	0,4473	1,0000

Since we know the standard deviations,

	AKBANK	ARÇELİK	ASELSAN	ECZACI	GARANTİ	HÜRRİYET	PINAR	ŞİŞE CAM	TOFAŞ	ÜNYE ÇİM
STDEV	0,0292	0,0299	0,0359	0,0302	0,0319	0,0314	0,0312	0,0278	0,0285	0,0283

The variance-covariance matrix is,

COVARIANCE MATRIX										
	AKBANK	ARÇELİK	ASELSAN	ECZACI	GARANTİ	HÜRRİYET	PINAR	ŞİŞE CAM	TOFAŞ	ÜNYE ÇİM
AKBANK	0,0009	0,0006	0,0003	0,0004	0,0006	0,0006	0,0003	0,0005	0,0005	0,0003
ARÇELİK	0,0006	0,0009	0,0003	0,0004	0,0006	0,0006	0,0004	0,0005	0,0005	0,0004
ASELSAN	0,0003	0,0003	0,0013	0,0004	0,0004	0,0004	0,0003	0,0003	0,0003	0,0003
ECZACI	0,0004	0,0004	0,0004	0,0009	0,0005	0,0004	0,0003	0,0004	0,0004	0,0003
GARANTİ	0,0006	0,0006	0,0004	0,0005	0,0010	0,0006	0,0004	0,0006	0,0006	0,0004
HÜRRİYET	0,0006	0,0006	0,0004	0,0004	0,0006	0,0010	0,0004	0,0005	0,0005	0,0004
PINAR	0,0003	0,0004	0,0003	0,0003	0,0004	0,0004	0,0010	0,0004	0,0004	0,0003
ŞİŞE CAM	0,0005	0,0005	0,0003	0,0004	0,0006	0,0005	0,0004	0,0008	0,0005	0,0003
TOFAŞ	0,0005	0,0005	0,0003	0,0004	0,0006	0,0005	0,0004	0,0005	0,0008	0,0004
ÜNYE ÇİM	0,0003	0,0004	0,0003	0,0003	0,0004	0,0004	0,0003	0,0003	0,0004	0,0008

Here are the figures of the equally weighted portfolio,

COVARIANCE MATRIX

		AKBANK	ARÇELİK	ASELSAN	ECZACI	GARANTİ	HÜRRİYET	PINAR	ŞİŞE CAM	TOFAŞ	ÜNYE ÇİM
		0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
AKBANK	0,1	0,000009	0,000006	0,000003	0,000004	0,000006	0,000006	0,000003	0,000005	0,000005	0,000003
ARÇELİK	0,1	0,000006	0,000009	0,000003	0,000004	0,000006	0,000006	0,000004	0,000005	0,000005	0,000004
ASELSAN	0,1	0,000003	0,000003	0,000013	0,000004	0,000004	0,000004	0,000003	0,000003	0,000003	0,000003
ECZACI	0,1	0,000004	0,000004	0,000004	0,000009	0,000005	0,000004	0,000003	0,000004	0,000004	0,000003
GARANTİ	0,1	0,000006	0,000006	0,000004	0,000005	0,000010	0,000006	0,000004	0,000006	0,000006	0,000004
HÜRRİYET	0,1	0,000006	0,000006	0,000004	0,000004	0,000006	0,000010	0,000004	0,000005	0,000005	0,000004
PINAR	0,1	0,000003	0,000004	0,000003	0,000003	0,000004	0,000004	0,000010	0,000004	0,000004	0,000003
ŞİŞE CAM	0,1	0,000005	0,000005	0,000003	0,000004	0,000006	0,000005	0,000004	0,000008	0,000005	0,000003
TOFAŞ	0,1	0,000005	0,000005	0,000003	0,000004	0,000006	0,000005	0,000004	0,000005	0,000008	0,000004
ÜNYE ÇİM	0,1	0,000003	0,000004	0,000003	0,000003	0,000004	0,000004	0,000003	0,000003	0,000004	0,000008
TOTAL	1	0,000051	0,000052	0,000042	0,000046	0,000057	0,000054	0,000040	0,000049	0,000049	0,000039

Portfolio variance	0,00048	
Portfolio Std deviation	0,021905122	daily
Portfolio R	0,001461792	daily

If the restrictions are $\sum w_i = 1$ and $w_i \geq 0$, shortselling is not allowed, the efficient set will be as it is shown below. It actually is more efficient than the outcomes of the equally weighted scenario.

COVARIANCE MATRIX

		AKBANK	ARÇELİK	ASELSAN	ECZACI	GARANTİ	HÜRRİYET	PINAR	ŞİŞE CAM	TOFAŞ	ÜNYE ÇİM
		0,083413693	0,031860353	0,129392872	0,13231076	0	0,021275607	0,178015983	0,108767678	0,083294903	0,231668151
AKBANK	0,083413693	0,000006	0,000002	0,000004	0,000005	-	0,000001	0,000005	0,000005	0,000004	0,000007
ARÇELİK	0,031860353	0,000002	0,000001	0,000001	0,000002	-	0,000000	0,000002	0,000002	0,000001	0,000003
ASELSAN	0,129392872	0,000004	0,000001	0,000022	0,000006	-	0,000001	0,000007	0,000004	0,000003	0,000009
ECZACI	0,13231076	0,000005	0,000002	0,000006	0,000016	-	0,000001	0,000007	0,000006	0,000005	0,000010
GARANTİ	0	-	-	-	-	-	-	-	-	-	-
HÜRRİYET	0,021275607	0,000001	0,000000	0,000001	0,000001	-	0,000000	0,000001	0,000001	0,000001	0,000002
PINAR	0,178015983	0,000005	0,000002	0,000007	0,000007	-	0,000001	0,000031	0,000007	0,000005	0,000013
ŞİŞE CAM	0,108767678	0,000005	0,000002	0,000004	0,000006	-	0,000001	0,000007	0,000009	0,000005	0,000009
TOFAŞ	0,083294903	0,000004	0,000001	0,000003	0,000005	-	0,000001	0,000005	0,000005	0,000006	0,000007
ÜNYE ÇİM	0,231668151	0,000007	0,000003	0,000009	0,000010	-	0,000002	0,000013	0,000009	0,000007	0,000043
TOTAL	1	0,000037	0,000014	0,000057	0,000058	-	0,000009	0,000078	0,000048	0,000037	0,000102

Portfolio variance	0,00044	
Portfolio Std deviation	0,020945406	daily
Portfolio R	0,001480825	daily

Now we are going to examine the result of the model with the actual figures in 2006. Here are two tables of "equally weighted" and "optimum" portfolios respectively. The optimum portfolio is favorable since its 2006 return score is higher than the equally weighted portfolio.

Actual performance of the stocks in 2006 are,

Equally weighted return	- 0,00016
Diversified portfolio return	- 0,00008

Though 2006 was a bad year for performances, our diversified portfolio is dominant to the equally weighted portfolio.

The return and risk figures of the year 2007 is as follows,

	AKBANK	ARÇELİK	ASELSAN	ECZACIBAŞI İLAÇ	GARANTİ BANKASI	HÜRRİYET	PINAR SÜT	ŞİŞE CAM	TOFAŞ	ÜNYE ÇİM
RETURN	- 0,0001	- 0,0004	0,0003	- 0,0002	0,0039	- 0,0001	0,0030	- 0,0040	0,0013	0,0020
STDEV	0,0331	0,0262	0,0281	0,0278	0,0330	0,0245	0,0342	0,0635	0,0279	0,0257

Though the test for 2007 with the same figures may be meaningful,

Equally weighted return	0,00057
Diversified portfolio return	0,00064

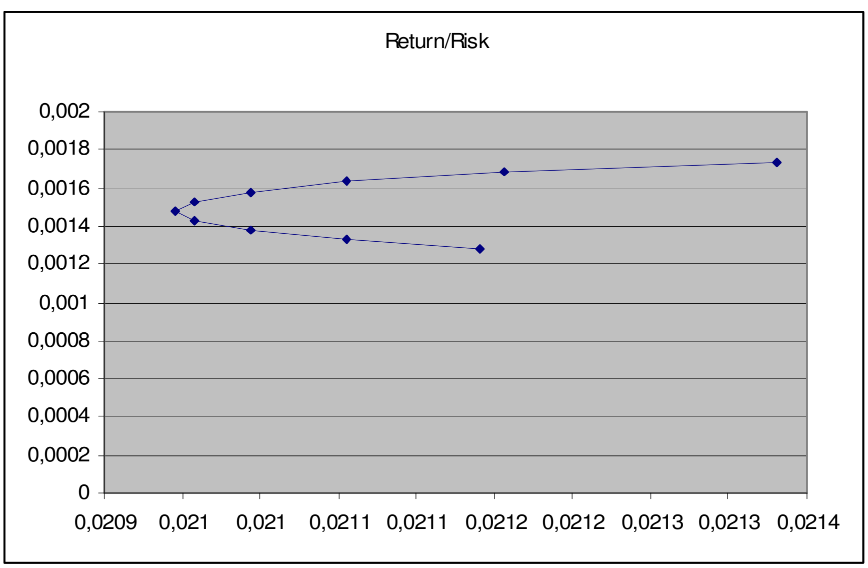
It is not favorable. Because the information is changed in 2006. What we should make for a better analysis was taking the 2006 figures in our analysis.

Here are the various combinations of standard deviation and return pairs. The figures are calculated through the Excel Solver property by simply setting limitations through various return levels and minimizing the variance.

The standard deviations are the square roots of the each outcome.

st dev	Return
0,02114053	0,0012808
0,02105526	0,0013308
0,02099433	0,0013808
0,02095767	0,0014308
0,02094543	0,0014808
0,02095757	0,0015308
0,02099415	0,0015808
0,02105494	0,0016308
0,02115663	0,0016808
0,02133038	0,0017308

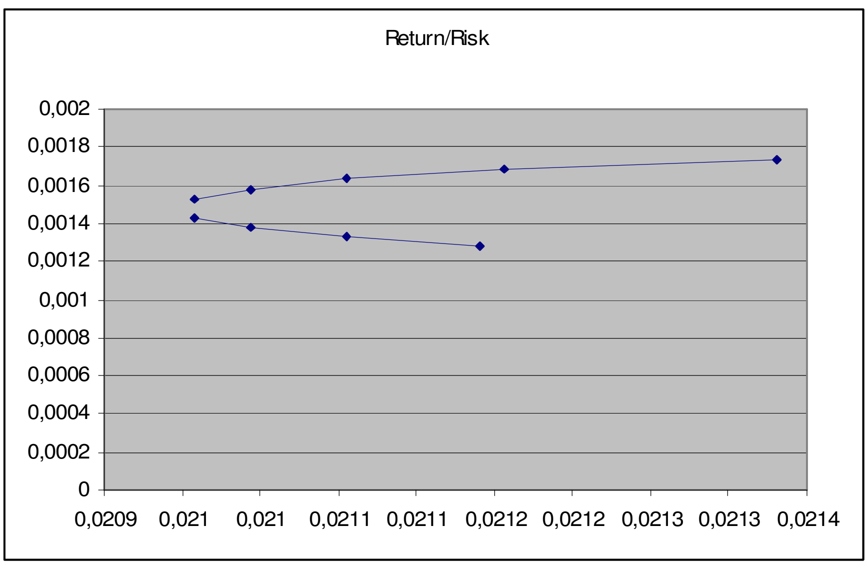
If we plot an efficient frontier line based on the chart above where Y axis is the return and the X axis is the standard deviation.



Here we can see the breaking point is the one that is closer to the y axis. It is the point the minimum variance prevails as a limitation. A tangency point with a risk free asset would simply be the point as shown below.(the deleted point)

st dev	Return
0,02114053	0,0012808
0,02105526	0,0013308
0,02099433	0,0013808
0,02095767	0,0014308
0,02094543	Deleted
0,02095757	0,0015308
0,02099415	0,0015808
0,02105494	0,0016308
0,02115663	0,0016808
0,02133038	0,0017308

Y axis is the return and the X axis is the standard deviation.



In our example we are able to make a selection of an efficient portfolio among a thousand of investment alternatives. As an alternative we can go further taking the risk free asset as a part of the investment mixture[54] or

[54] David J Buckle , **Some aspects of active portfolio management** ,January 2003

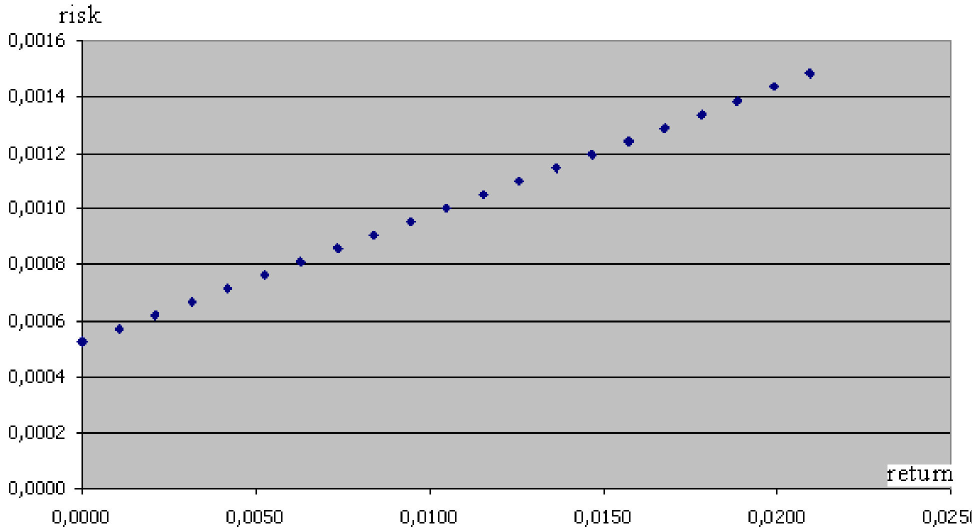
simply can decide on an efficient set and adding up a risk free asset after that.

($S_t = (R_t - R_f) / \sigma_t$) On all over the line the Sharpe Ratio will be the same as long as there is a linear relationship between the return and the standard deviation.

Here is the return and standard deviation statistics of the efficient portfolio we have found based on the 2002-2005 historical values along with inclusion of 14% risk free rate.

	Portfolio	Rf
Return	0,00148	0,00052
STD	0,02095	0,00000

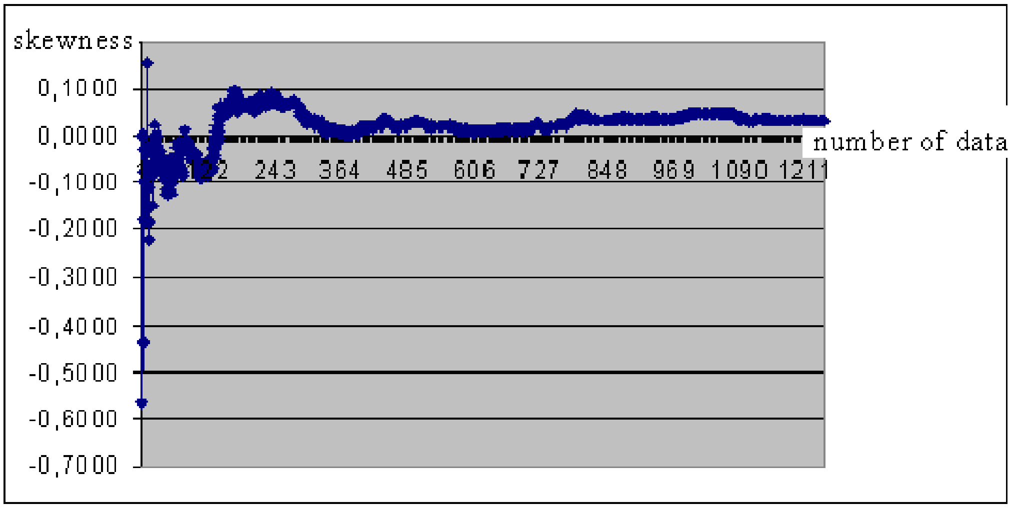
Portfolio Weight		Return	Risk	SHARPE RATIO
100%	0%	0,0015	0,0209	0,04568
95%	5%	0,0014	0,0199	0,04568
90%	10%	0,0014	0,0189	0,04568
85%	15%	0,0013	0,0178	0,04568
80%	20%	0,0013	0,0168	0,04568
75%	25%	0,0012	0,0157	0,04568
70%	30%	0,0012	0,0147	0,04568
65%	35%	0,0011	0,0136	0,04568
60%	40%	0,0011	0,0126	0,04568
55%	45%	0,0011	0,0115	0,04568
50%	50%	0,0010	0,0105	0,04568
45%	55%	0,0010	0,0094	0,04568
40%	60%	0,0009	0,0084	0,04568
35%	65%	0,0009	0,0073	0,04568
30%	70%	0,0008	0,0063	0,04568
25%	75%	0,0008	0,0052	0,04568
20%	80%	0,0007	0,0042	0,04568
15%	85%	0,0007	0,0031	0,04568
10%	90%	0,0006	0,0021	0,04568
5%	95%	0,0006	0,0010	0,04568
0%	100%	0,0005	0,0000	



In the graph above x axis represents the return and the y axis represents the risk.

We have an afore mentioned debate related to the Central Limit Theorem. Does the distribution of returns turn out to be a bell shape as the number of trials increases. Or does it just follow a random walk.

With an increase in the number of data, it would be reasonable to get closer mode, median and mean values. We can make an application with any share quoted in ISE 100 Index. With inclusion of more data from 2002 towards the end of 2006 the skewness follows a tendency which systematically gets closer to zero. It can be a reference that the shares can be valued as normally distributed.



3.6 Markowitz Model with Computer Techniques: FX,Gold,ISE 30

We will analyze some figures from 2005 towards 17 april 2008 for USD/YTL, EUR/YTL, Gold Investments, Rf and ISE 30 index.

If we take a risk free asset in our variance covariance matrix as long as its standard deviation is zero, the negative relation between the assets make the variance negative. Variance can not take negative values. We have made a calculation by Excel-Solver with a zero variance restrictment. We have expected a return equal to Rf; but it actually was not. So it will be reasonable to exclude the Rf first from our efficient portfolio calculation. We can later calculate efficient points along the capital market line if needed.

2005		USD	EUR	GOLD	ISE 30
	Weights	0,411651	0,436632	-	0,151719
USD	0,411651	0,000008	0,000006	-	- 0,000000
EUR	0,436632	0,000006	0,000009	-	- 0,000001
GOLD	-	-	-	-	-
ISE 30	0,151719	- 0,000000	- 0,000001	-	0,000006
TOTAL	1,00000	0,00001	0,00001	-	0,00001

VARIANCE					0,00003
RETURN	0,00001	- 0,00022	-	0,00029	0,00008

2005-2006		USD	EUR	GOLD	ISE 30
	Weights	0,405243	0,370518	0,035799	0,188441
USD	0,405243	0,000012	0,000009	0,000001	- 0,000001
EUR	0,370518	0,000009	0,000010	0,000001	- 0,000001
GOLD	0,035799	0,000001	0,000001	0,000000	0,000000
ISE 30	0,188441	- 0,000001	- 0,000001	0,000000	0,000012
TOTAL	1,00000	0,00002	0,00002	0,00000	0,00001

VARIANCE					0,00005
RETURN	0,00006	0,00003	0,00003	0,00016	0,00028

2005-2007		USD	EUR	GOLD	ISE 30
	Weights	0,343295	0,476217	-	0,180489
USD	0,343295	0,000010	0,000011	-	- 0,000001
EUR	0,476217	0,000011	0,000018	-	- 0,000001
GOLD	-	-	-	-	-
ISE 30	0,180489	- 0,000001	- 0,000001	-	0,000012
TOTAL	1,00000	0,00002	0,00003	-	0,00001

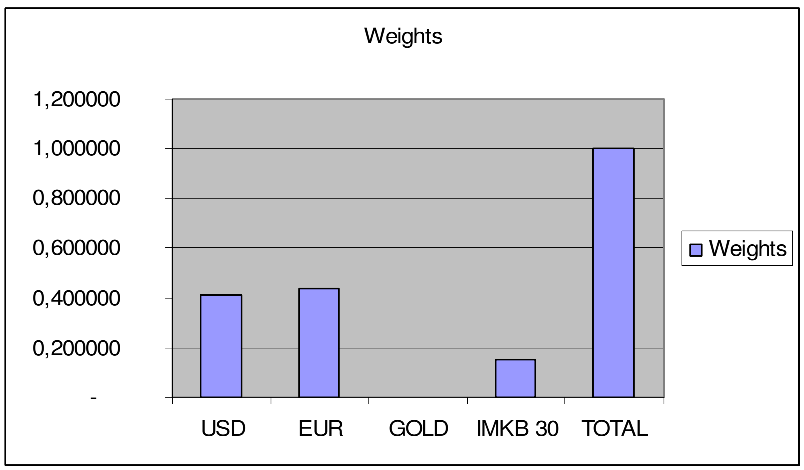
VARIANCE					0,00006
RETURN	- 0,00004	- 0,00006	-	0,00021	0,00011

2005-04.2008		USD	EUR	GOLD	ISE 30
	Weights	0,400033	0,426146	0,002524	0,171296
USD	0,400033	0,000014	0,000013	0,000000	- 0,000000
EUR	0,426146	0,000013	0,000016	0,000000	- 0,000000
GOLD	0,002524	0,000000	0,000000	0,000000	0,000000
ISE 30	0,171296	- 0,000000	- 0,000000	0,000000	0,000012
TOTAL	1,00000	0,00003	0,00003	0,00000	0,00001

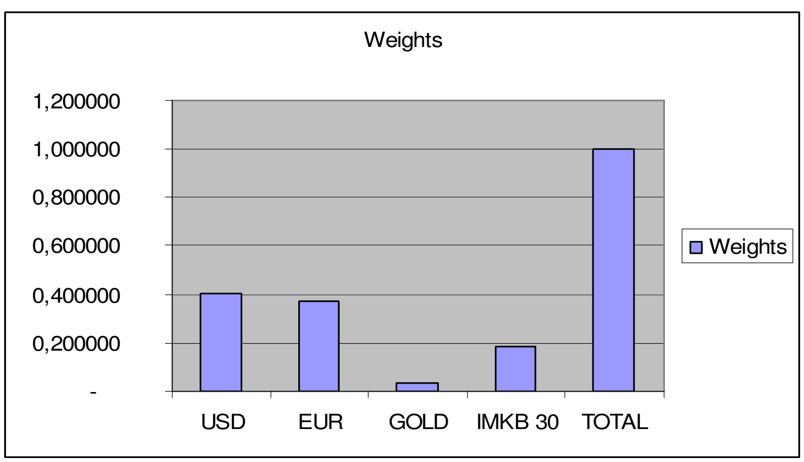
VARIANCE					0,00007
RETURN	0,00004	0,00007	0,00000	0,00011	0,00023

Here are the minimum variance portfolio graphs.

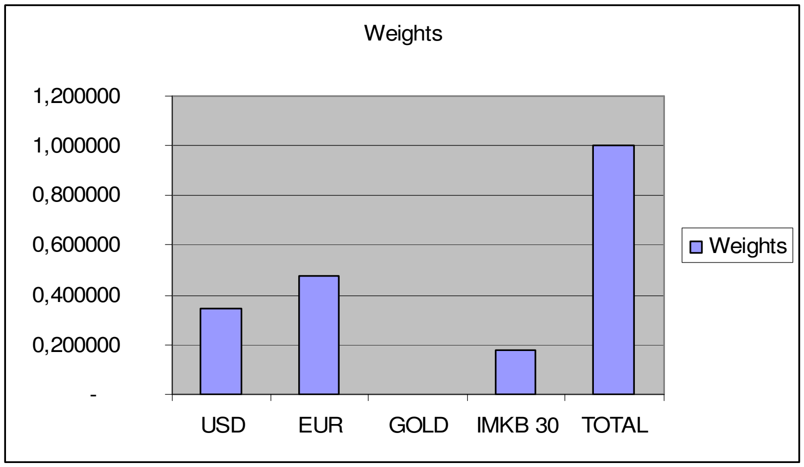
2005



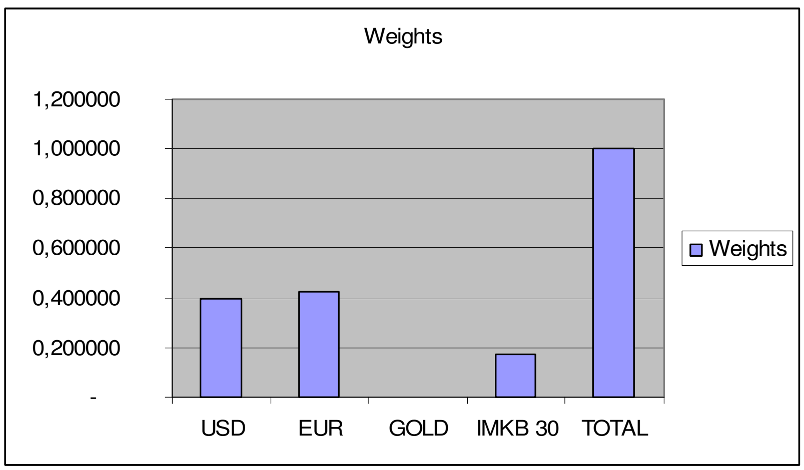
2005-2006



2005-2007



2005- 04.2008



Return of gold is not proportionate to its standard deviation. That is why the Solver takes just a pinch of gold in our portfolio.

If we were in the aim of maximizing return whatever the risk was, the result would be 100% of gold.

2005-04.2008		USD	EUR	GOLD	ISE 30
	Weights	-	-	1,000000	-
USD	-	-	-	-	-
EUR	-	-	-	-	-
GOLD	1,000000	-	-	0,000208	-
ISE 30	-	-	-	-	-
TOTAL	1,00000	-	-	0,00021	-

VARIANCE					0,00021
RETURN	-	-	0,00102	-	0,00102

Would it be an optimum portfolio with sacrification of high returning gold. It probably would not. In our portfolio more than a pinch of gold should prevail. As we mentioned before the minimum variance portfolio selection is not a single choice that determines the optimum portfolio.

The minimum variance portfolios neglect the gold investment due to its huge level of variance though it a has high level of return. This can be a way of explanation why Markowitz's idea of semi-variance would be a more plausible approximation in the case of drastic increases of any asset.

Here we have calculated the various variance levels based on Excel Solver Application. In AEU calculations LOG(1+R) is chosen as a utility function.

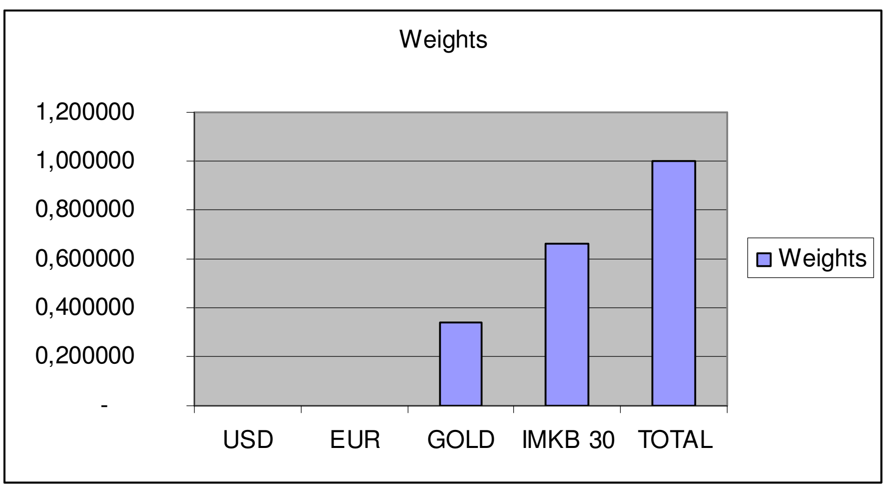
	variance level	st deviation	Return	AEU
Min variance	0,0000653	0,0080807	0,0002254	0,000065
P2	0,0000694	0,0083330	0,0001980	0,000051
P3	0,0000799	0,0089394	0,0005048	0,000179
P4	0,0000896	0,0094649	0,0003086	0,000089
P5	0,0000942	0,0097064	0,0003205	0,000092
P6	0,0000991	0,0099557	0,0003320	0,000095
P7	0,0001507	0,0122765	0,0004192	0,000107
P8	0,0001807	0,0134427	0,0004566	0,000108
P9	0,0002006	0,0141645	0,0004787	0,000108
P10	**0,0002197**	**0,0148219**	**0,0007692**	**0,000224**
P11	0,0002249	0,0149982	0,0007713	0,000223
P12	0,0002307	0,0151880	0,0007660	0,000217
P13	0,0002357	0,0153512	0,0007616	0,000213
P14	0,0002408	0,0155162	0,0007572	0,000209
P15	0,0002508	0,0158379	0,0005279	0,000104

Utilizing LN (1+R) ,

				LOG(1+R)	LN(1+R)
	variance level	st deviation	Return	AEU	AEU
Min variance	0,0000653	0,0080807	0,0002254	0,000065	0,000193
P2	0,0000694	0,0083330	0,0001980	0,000051	0,000163
P3	0,0000799	0,0089394	0,0005048	0,000179	0,000465
P4	0,0000896	0,0094649	0,0003086	0,000089	0,000264
P5	0,0000942	0,0097064	0,0003205	0,000092	0,000273
P6	0,0000991	0,0099557	0,0003320	0,000095	0,000282
P7	0,0001507	0,0122765	0,0004192	0,000107	0,000344
P8	0,0001807	0,0134427	0,0004566	0,000108	0,000366
P9	0,0002006	0,0141645	0,0004787	0,000108	0,000378
P10	**0,0002197**	**0,0148219**	**0,0007692**	**0,000224**	**0,000659**
P11	0,0002249	0,0149982	0,0007713	0,000223	0,000659
P12	0,0002307	0,0151880	0,0007660	0,000217	0,000651
P13	0,0002357	0,0153512	0,0007616	0,000213	0,000644
P14	0,0002408	0,0155162	0,0007572	0,000209	0,000637
P15	0,0002508	0,0158379	0,0005279	0,000104	0,000403

The result is the same.

P10 is the optimum level where utility function is maximized. The weights are depicted in the chart below.



We have built a variance covariance matrix with more than 250 shares as it was in the ten stock case and let the Excel calculate a minimum variance portfolio.

Excel Solver in this case was not successful. It can not calculate huge numbers. It is tested that it can not determine a minimum variance portfolio with so many covariances.

4. CONCLUSION

The aim of this study was to examine the models used for investments. A drawback for Markowitz Model is calculating numerous covariances. Utilizing the computer techniques, on the contrary, makes the model somehow a practical application. The analyses are made by utilizing both compounded and daily rate of returns. The empirical findings show us that the results are in line with each other. In our calculations MS Excel skills are used. “Data Analysis” and “Solver” are applied to the various data. The conclusion of this application showed us that MS Excel was not a sufficient tool in the case of including numerous data in a covariance matrix. Data of different time scales, due to some factors, do not always result in best choices. The results of the Elton & Gruber Model were not completely in line with the empirical findings of the Markowitz Model. The last but not the least, the minimum variance portfolio is tested as an optimal selection strategy. The applications based on ISE 30 Index, Exchange Rate and Gold Investments have shown us that the minimum variance portfolio was not the optimum point in our investments. The optimum point was that where the utility has been maximized. Applications based on mean-variance analysis are made through AEU maximization.

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