OPTIMAL MONETARY POLICY UNDER DIFFERENT WAGE AND PRICE CONTRACTS

EMEL ALTIN
106622002

İSTANBUL BİLGİ ÜNİVERSİTESİ
SOSYAL BİLİMLER ENSTİTÜSÜ
EKONOMİ YÜKSEK LİSANS PROGRAMI

Yrd. Doç. Dr. Koray Akay

2008

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FARKLI ÜCRET VE FİYAT SÖZLEŞMELERİ ALTINDA
EN İYİ PARA POLİTİKASI

EMEL ALTIN
106622002

Tez Danışmanının Adı Soyadı	: KORAY AKAY
Jüri Üyelerinin Adı Soyadı	: EGE YAZGAN
Jüri Üyelerinin Adı Soyadı	: GÖKSEL AŞAN
Jüri Üyelerinin Adı Soyadı	: KORAY AKAY

Tezin Onaylandığı Tarih : 22.08.2008

Toplam Sayfa Sayısı: 51

Anahtar Kelimeler (Türkçe)	Anahtar Kelimeler (İngilizce)
1) Esnek ücretler ve fiyatlar	1) flexible wages and prices
2) Yapışkan fiyatlar	2) staggered prices
3) Yeni Keynesyen Model	3) New Keynesian Model
4) En iyi para politikası	4) Optimal monetary policy
5) İhtiyata karşılık Kurallar	5) Discretion versus Rules

ABSTRACT

This thesis provides a survey on optimal monetary policy under different wage and price contracts in a framework of a closed economy. We have optimal monetary policy either minimizing the loss function whose determinants are inflation and output gap or maximizing the representative agent's utility. In the thesis, we focus on the importance of nominal rigidities and New Keynesian model which we believe to give the best response to an optimal monetary policy. We will see that in recent literature, a Central Bank will generally be unable to eliminate completely the distortions caused by nominal rigidities. The optimal policy should involve balance between stabilization of three variables: the output gap, price inflation and wage inflation. Therefore we investigate monetray policy tradeoff between discretion and commitment with and without distortions. At the end, we see that optimal monetary policy under commitment is better than discretion.

ÖZET

Bu tez, kapalı bir ekonomide, farklı ücret ve fiyat sözleşmeleri altında en iyi para politikası konusunda bir literatür taramasıdır. En iyi para politikasını, belirleyici faktörleri enflasyon ve üretim açığı olan zarar fonksiyonunu en aza indirgediğimiz veya temsilci kişinin faydasını azami ölçüde arttırdığımız zaman elde edebiliriz. Tezde ücret ve fiyat yapışkanlığının önemi ve para politikasına en iyi yanıtı verdiğine inandığımız New Keynesyen Model üzerinde durduk. Yakın zamanda yapılan araştırmalara göre, Merkez Bankası nominal katılıklardan kaynaklanan bozulmaları tamamen yok edememektedir. En iyi politika üretim açığı, fiyat enflasyonu ve ücret enflasyonu arasında bir denge içermelidir. Bu yüzden, bozulmaların olduğu ve de olmadığı durumlarda, ihtiyati ve kurala dayalı para politikası arasındaki ilişkileri inceledik. Sonunda, kurala dayalı para politikasının daha iyi sonuç verdiğini gördük.

ACKNOWLEDGEMENT

My thesis is about a survey on Optimal Monetary Policy. My aim is to explain the subject in general terms with elementary plain language for undergraduate students. I only have difficulties because of the depth of the subject in the given limited time. Therefore, I only take into accountoptimal monetary policy in closed economy framework. I thank to my advisor, Yrd. Doç. Dr. Koray Akay who let me to choose this subject, showed useful references and did not spare his worthy suggestions at every stage of my thesis. Also I would like to thank to my friend Ercan Karadaş who helped me to find the books that I used. I wrote a major part of my thesis at Nesin Matematik Köyü, a wonderful place wherein the nature, so I acknowledge allowing to stay there whenever I want to Prof. Dr. Ali Nesin. Finally, I can't disregard to thank my family who never leave me alone in my hard times while I am writing my thesis.

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1 INTRODUCTION

Monetary economics investigates the relationship between real economic variables at the aggregate level - such as output, interest rate, employment and exchanges rate and nominal variables -such as the inflation rate, interest rate, exchange rate, and supply of money. We can study a variety of issues in monetary economics -the relationship between money and prices, the effects of inflation on equilibrium, and the optimal rate of inflation (Walsh 2003, chapter 2). The fundamental question of monetary economics is how we should model the demand for money. It is also concerned with the conduct monetary policy in the shadow of the debate Classical and New Keynesians. We deepen subject of optimal monetary policy in both two sides with take into account adjustment of prices and wages for closed economy.

On the Classical side, demand for money can be generated by MIU model where we used Sadrauski (1967) who analyzes the short run effects of monetary policy and monetary disturbances on real economic activity in the presence of nominal wage and price rigidities; or CIU model due to Lucas (1982) and Svensson (1985), Cooley and Hansen (1989). Walsh (2003, chapter 3) provides a detailed description of cash-in-advance models and their implications for the role of monetary policy. In a word, we face to solve social planner problem to maximize the utility of representative household for MIU model and attain Friedman rule is optimal, where nominal interest rate is zero $(i_t = 0)$.

Khan, King, and Wolman (2000) consider that If prices are flexible, it is optimal that the nominal interest rate is zero in the presence of Friedman distortion and cash-in-advance (CIA) constraint. In contrast, price stability would be optimal in the absence of the cash-in-advance (CIA) constraint. With both sticky prices and the monetary inefficiency, the optimal rate of inflation is less than zero but greater than the zero nominal interest rate. Inflation continued to depend on expected future inflation and real marginal cost, but with sticky wages, real marginal cost can no longer be measured by the gap between the household's marginal rate of substitution between leisure and consumption and the marginal product of labor.

Erceg, Henderson, and Levin (2000) show that wage stability and price stability are desirable. Because while wage stability eliminates dispersion of hours worked across households when both prices and wages are sticky, price stability eliminates price dispersion across goods. When prices are sticky but wages are flexible, optimal policy should keep the price level stable, in the direct contrary situation optimal policy should keep nominal wages stable.

On the other side, there exists New Keynesian Model which emphasizes on the role of monopolistic competition, markups, costly price adjustments (e.g.

Mankiw and Romer, 1991), and dynamic general equilibrium models where prices and wages are sticky, that come from the real business cycle literature, (e.g. Kydland and Prescott,1982; Long and Plosser, 1983; Prescott, 1986).

It basically consists of three components; the expectational IS curve, the New Keynesian Phillips Curve and a policy rule which formed in a simple way by Taylor(1993). First component is the expectational IS curve (demand side of economy) which relates the level of real activity to expected (and sometimes past) real activity and the real interest rate. Second component is the New Keynesian Phillips Curve (supply side of economy), represented by a price setting equation. There are two important key improvements while developing the New Keynesian Phillips Curve; forward-looking behavior in the inflation process, studied by (Friedman,1968; Phelps,1967; Sargent, 1971 and Lucas, 1972,1976) and the Dixit and Stiglitz's (1977) type of monopolistic model, in the tradition of Fischer (1977), Taylor (1980) and Calvo (1983). Under the assumption of quadratic costs of price adjustment, Rotemberg (1982) shows that an inflation equation identical to the new Keynesian Phillips curve can be derived. Hairault and Portier (1993) compared second moment predictions of this assumption in French and U.S. economies. Finally third component is a simple policy rule in which the interest rate responds to variations in inflation and/or the output gap. The recent literature focus on monetary rules vs discretion in the presence of a trade-off between output and inflation. Society will generally gain from commitment and such gains arise even in the absence of a classic inflation bias, i.e. even if the central bank has no desire to push output above its natural level. That result overturns an implication of the classic Barro-Gordon analysis, where the gains from commitment arise only if the central bank sets a target for output that does not correspond to its natural level. We remind that model does not involve liquidity trap. Taylor (1993) introduced the simple formula commonly known as the Taylor rule. Judd and Rudebusch (1998) and Clarida, Galí, and Gertler (2000) estimate alternative versions of the Taylor rule, and examined its (in)stability over the postwar period. Schmitt-Grohé and Uribe (2006) makes an numerical analysis of the coefficients of the interest rate rule that satisfy uniqueness of the equilibrium.

If we turn to price and wage adjustment models, we see that apart Svensson (1986), Blanchard and Kiyotaki (1987), Blanchard and Fischer (1989), Akerlof and Yellen (1991) study monopolistic competition and staggered price setting models, either static, e.g. IS-LM framework, or partial equilibrium; on the other part Goodfriend and King (1997) discuss the case of price stability. The Calvo (1983) form of price adjustment has been widely used in analyses of optimal monetary policy in models with explicit microfoundations (e.g., Goodfriend and King, 1997; Clarida et al., 1999; Woodford, 2003). Kimball (1995) and Yun (1996) are the first to introduce Calvo price setting into stochastic, optimizing-agent models. King and Wolman(1996) provides a detailed analysis of the steady state and dynamic properties of that model.

Cooley and Cho (1995) and Bénassy (1995) embedded the assumption of sticky nominal wages in a dynamic stochastic general equilibrium model, and examined its implications in the presence of both real and monetary shocks. Also Huang and Liu (2002) and Woodford (2003, chapter 3) discuss the role of wage stickiness on the persistence of the monetary shocks. Kim (2000), Smets and Wouters (2003), and Christiano, Eichenbaum and Evans (2005) study the staggered wage setting in the context of medium-scale models. Erceg, Henderson and Levin (2000) developed the new Keynesian model with both staggered price and staggered wage contracts à la Calvo. Woodford (2003, chapter 6) and Giannoni and Woodford (2003) find targeting a weighted average of wage and price inflation is optimal. Rotemberg and Woodford (1999) and Christiano, Eichenbaum and Evans (2005) discuss the effects of monetary policy shocks, and make modifications to new Keynesian model to improve the model's ability to match the estimated impulse responses. Galí (1999), Basu, Fernald and Kimball (2004) generate alternative models to find the effects of technology shocks and its implications.

As articulated by Svensson (1999), "....there is considerable agreement among academics and central bankers that the appropriate loss function both involves stabilizing inflation around an inflation target and stabilizing the real economy, represented by the output gap". Such a loss function forms a key component of "The Science of Monetary Policy" (Clarida, Galí, and Gertler 1999), and Woodford (2001a) has shown how it can be derived as an approximation to the utility of the representative agent.

The optimal policy will seek to strike a balance between stabilization of the output gap, price inflation and wage inflation, in which interest rate setting is a reaction function, responded to the output gap and (expected) inflation. An extensive literature has dealt with optimal monetary policy design in such a framework in recent years, e.g. Taylor (1999), Svensson (1999), Clarida, Galí and Gertler (1999), Woodford (2003a) and Walsh (2003). Furthermore, as we generally mentioned at the last part of the thesis, there are contributions about distortions, not eliminated completely, caused by nominal rigidities in optimal monetary policy (Gali, 2007,chapter 5). In practice, the optimizing policymaker will seek to eliminate any distortions that may exist in the economy. Khan, King, and Wolman (2000) and Woodford (1999c) analyze the role played by distortion in the design of monetary policy.

The remainder of the thesis is organized as follows. Section 2 lays out either monetary policy history or the relationships and debates between different schools of economics. It provides an motivation to see historical improvement of monetary policy. Section 3 focuses on two different model; MIU and New Keynesian Models. Section 4 gives an overlook to optimal monetary policy presence of mentioned models and investigates the trade-off between discretion and commitment with/without distortions. Section 5 includes conclusion.

2 MONETARY POLICY

Monetary policy is a process by which the government, central bank, or monetary authority of a country controls the supply of money. This process always tries to stabilize prices, financial markets, exchange markets and to obtain economic growth and fullemployment together. Interest rate, open market operations, discount rate, reserve requirements and exchange rate are the main tool of the monetary policy. There are several major macroeconomic models with different assumptions that follows different ways to conduct monetary policy.

We first discuss the fundamental relationships of Classicals, New Classicals, Keynesians, New Keynesians and Monetarists to recognize the historical dimension of macroeconomics theory and their monetary policies at the next subsection.

2.1 History of Monetary Policy

The origin of modern monetary policy came from the classical gold standard between the years 1880-1914. Under the gold standard all countries would define their currencies in terms of a fixed weight of gold and then all fiduciary money would be convertible into gold. The key role of central banks was to maintain gold convertibility. The original policy instruments were discount rate and rediscounting. After the First World War, monetary policy regime shifted towards fiat money and focused on stabilizing prices and output in the 1920s. This trend continued during 1930s and after the Second World War. Through the Great Depression in 1930s, Classical Economists were generally accepted. They made their opinion depended on Say's Laws where all prices and wages were flexible and self regulated to ensure that the economy operated at the full employment . Supply created its own demand and saving always equaled to investment, because changes in the interest rate brought saving and the investment into equality. These models suggested that there was no need to monetary or fiscal policies. In contrast with Classicals, Neoclassical Keynesian argued that Say's Law did not hold because Keynes's theory assumed some rigidities and imperfections in the markets. There were two main models represented by Neoclassical Keynesians: Hick's IS- LM model (Filho, 1996) and disequilibrium models. In IS-LM model, Keynesian involuntary unemployment was due to the existence of the liquidity trap. Hicks formalized the Keynesian and Classical IS-LM model and considered to be differences from Keynesian and Classical.

The IS curve associated interest rates and income levels in goods market, while the LM curve presented combinations of interest rates and income levels along which money market was in equilibrium.

	Classical model	Keynesian model
1	M=kl	M=L(i,l)
2	I_x=C(i)	I_x=C(i)
3	I_x=S(i,l)	I_x=S(i)

M: the total quantity of money
k: the Marshall constant in the Cambridge quantity equation,
I: the income level,
I_x: the total investment,
i: interest rate,
S: saving.

The first equation of each model (Filho,1996), defined the LM curve, while two other equations defined the IS curve. The introduction of rate of interest in Keynesian demand for money was not contradictory to the Cambridge Quantity Equation. At each of the model, demand for money depended on income levels. Consequently, Hicks argued that Keynesian involuntary unemployment persisted solely because monetary policy could not lower the interest rate sufficiently to restore the economy to its full employment income level. As we mentioned above, among disequilibrium models of Keynesians, the Patinkin's (1956) model analyzed the Keynesian disequilibrium as a result of failure to obtain short-run flexible wages in the labor market. Furthermore, Keynesian theory could be interpreted as a dynamic disequilibrium analysis of a Walrasian general equilibrium system in long run.

Barro and Grossman (1971) developed a general disequilibrium model, both for booms and depressions. The economic system would always respond differently to a specific shock, depend on how prices and wages differ from the vector of prices and wages at fullemployment equilibrium. Benassy and Malinvaud investigated the microfoundation of disequilibrium macroeconomics to explain the causes of price and wage rigidities.

Within the Great Depression in 1929, everywhere in the world, increased chronic inflation, unemployment and also diminished production caused underemployment and demand problems in the markets. Keynes' solution - expanding government spending - worked for short run. When economy fell in recession, people started to hold money in their hands and firms cut productions. As a result of this, there were a demand inadequate and unemployment. The government had to expand the economy and increased the money supply until people started to spend money again. If people still held money in their hands, this time government increased demand by consuming to achieve fullemployment.

By the 1950s and early 1960s, Keynesians used to discretionary fiscal and monetary policy to achieve full employment with only moderate amount of inflation, worked seamless to get better the economy. During mid -and late-1960s, "war on poverty" because of Vietnam furthermore The Arab Oil embargoes of the early 1970s resulted in further inflationary pressures. This inflation, however, unlike other recent periods of inflation, was accompanied by rising unemployment rates. It came to be known as "stagflation" not explained by simple Keynesian models that were in general use during this period. In addition, Keynesian did not have a monetary explanation for long run theory of unemployment. To find solutions to problems especially stagflation for development countries, Monetarist and New Classical arose in response to inadequate of Keynesian Economics. Monetarists claimed that an increase in prices would not lead to inflation unless the government increased the money supply. The most important factor which directly affected the production, fullemployment and the general price levels, was money supply. They characterized inflation as a monetary act when the rise in money supply excessed the rise in production. They found a certain link between the money supply and inflation directly. The AS curve was horizontal to AD in the relation between prices and wages versus output at fullemployment unlike Keynesian who argued that the AS curve was vertical to AD. Friedman argued that discretionary policies could have a short-run effect on the level of output, and government should rely on fixed policy rules, i.e. a long run money growth rules. Main question of Friedman was limitations of monetary policy. It was related to expectation - augmented Phillips curve which found a trade-off between rates of inflation and unemployment. According to Friedman, economic agents adapted their expectations in light of past experience and revised their expectations for each period of time.

Formally the Friedman model could be represented as follows:

$$P_t^e = f(P_{t-\lambda}) \tag{1}$$

where P_t^e was the expected rate of inflation in period t.
$\lambda \epsilon \{1, 2, ...\}$ and $P_{t-\lambda}$ was the rate of inflation which occurred in the past.

Equation (1) showed that the economic agents would learn about the inflation. Consequently, the expected rate of inflation would adjust to equal the current rate of inflation. Friedman rejected the long-run stability of the Phillips curve because monetary policy could not cause real fluctuations in an economy in long run. To sum up, Friedman and the Monetarists believed that attempts to lower the rate of unemployment below the "natural" would caused only temporary reductions in unemployment and in long run this situation produced higher inflation along with higher unemployment.

After Keynesian view lost its importance in 1970s, Lucas, Sargent, Wallace emerged as a distinctive group who relied on the concept of "rational expectations" i.e. all individuals were rational; firms maximized profits and individuals maximized utility. They made a strong emphasis on macroeconomy and the Walrasian general equilibrium framework. Complete and continuous wage and price flexibility ensured that markets continuously cleared. The quantity of money should be neutral (Lucas, 1972) and real magnitudes would be independent from nominal magnitudes. There was a positive correlation between real GDP and the nominal price level and the direct contrary relation (negative correlation) between inflation and unemployment (Phillips curve). Only relative prices mattered for optimizing decisions. Thus New Classical Economists came down on the side of rules in the "rules versus discretion' debate over the conduct of stabilization policy. Under rational expectation model, changes in monetary policy would only affected the price level but not affected unemployment in short run. They agreed with Monetarists in supporting a fixed monetary policy rule that reduce unemployment in short run by an unexpectedly large increase in the money supply, although it caused to make Fed less credible and encouraged higher future inflation. They were criticized because of foundations of their view which based on Neoclassical Economists whose theory ignored nominality and didn't try to answer problems of real worlds. Post Keynesians were developed in a context in which the real world had the following characteristics: (i) money mattered in both the short-run and long-run, (ii) the future was uncertain, (iii) contracts were denominated in money terms, (iv) money had two specific properties differentiated from the other producible goods, and (v) unemployment in a monetary or entrepreneurial economy, i.e. an economy in which fluctuations of effective demand were explained as a monetary phenomenon, was a normal result. Keynes's analysis was developed on three theoretical propositions: the theory of income determination (propensity to consume and multiplier), the theory of investment (marginal efficiency of capital), and the theory of interest rate (liquidity preference). The Hicksian interpretation of GT provided some logical misunderstandings of Keynes's theory. For example, (i) it substituted the Walrasian system of general equilibrium for Keynes's Marshall equilibrium; (ii) it dichotomized the real and monetary markets; and (iii) it did not analyze the role that expectation and uncertainty had on effective demand.

2.2 Discretion versus Rules

While searching optimal monetary policy, we need some knowledge how we associate the tools of monetary policy as a rule. And we investigate the optimal rules according to its historical development and their implications in this subsection.

At the beginning of the 1960s, central banks should achieve multiple social objectives: low inflation, high growth, low unemployment and low nominal interest rates. In addition, the Federal Reserve was expected to contribute to specific exports such as encouraging balanced payments with the rest of the world and a strong housing sector.

Monetary policy aspect interest rates, who made or lost money from its fluctuations. Therefore, the markets constantly tried to do forecasting. In the late 19th and early 20th centuries, economists were precise about the nature of the connection between money and the general price level. Irving Fisher, among others, made important contributions to monetary theory long before the Great Depression. This idea— the general price level and its rate of increase depended primarily on the level of the money stock and its rate of increase— fell out of favor with the rise of Keynesian analysis in the 1930s and 1940s. The idea was revived in the 1950s by Milton Friedman who focused inflation. The Friedman Rule was defined as the zero nominal interest rate with a deflation rate at time preference rate. However, the optimality of Friedman rule was criticized within common usage of inflation targeting as a monetary policy, which preferred to use low and positive inflation rate instead of a given deflation rate. Phelps (1973) first showed that the Friedman Rule was not optimal in the frame of Ramsey's (1927) optimal taxation by implicating the inflation, and argued that a positive nominal inflation rate was optimal when the elasticity of interest was low. Aganist to Friedman (1969) who discussed the subject of optimal interest rate in a first optimal equilibrium condition, Phelps took it consideration within the optimal taxation in a second optimal equilibrium condition. According to Phelps, Friedman ignored the fiscal efficiency. Furthermore in direct contraction to Friedman, Phelps handled a situation that government spending is met by diversionary (non-lumpsum, NLS) tax. Finally, while Friedman determines the optimal quantity of money by excluding finance of the public and decision of private sector's consumption and leisure in a partial equilibrium model, Phelps under the optimal taxation approach, determined optimal inflation rate in a general equilibrium model. Within Lucas and Stokey (1983), who carried the Ramsey's normative policy determination approach into dynamic environment, determination of optimal monetary policies were possible in short run. Allan Meltzer and Bennett McCallum had worked on variants of the Friedman rule. These were quantity-based rules that yielded a change in growth rate of the money stock or in monetary base.

Another rule which we wanted to discuss is an interest rate rule, proposed by Stanford economist John Taylor in 1993. The Taylor rule pointed out how a central bank should adjust its interest rate as an policy instrument which responded to real output and inflation rate.

The Taylor Rule:

$$i - i^* = \theta_\pi(\pi - \pi^*) + \theta_q(q - q^*) \tag{2}$$

where i was the short term nominal interest rate.

i^* was a baseline path in proportion to deviations of target variables which are nominal income while the other targeted inflation and real output.

Taylor (1999b) emphasized that the coefficient of inflation deviation was greater than 1. If the coefficient was below 1, then an increase in inflation would call for an increase in the nominal interest rate that was smaller than an increase in inflation. Besides this, the interest rate is rising when the coefficient of output deviation is rising. Taylor developed a "hypothetical but representative policy rule" (p. 214) by using the sum of the equilibrium or natural rate of interest, r^*, and inflation, π, for i^* and setting the inflation target and equilibrium real interest equal to two and the response parameters to one half. The result was what became known as the classic Taylor rule:

$$i = 2 + \pi + \frac{1}{2}(\pi - 2) + \frac{1}{2}(q - q^*) \tag{3}$$

The stabilization properties of this rule and its usefulness for understanding historical monetary policy in a period generally accepted by central banks to provide guidance in policy decisions. By linking interest rate decisions directly to inflation and economic activity, Taylor rules offered a convenient tool for studying monetary policy while abstracting from a detailed analysis of the demand and supply of money. This allowed the development of simpler models (see the survey in Clarida, Gali, and Gertler, 1999 and papers in Taylor, 1999) and the replacement of the "LM curve" with a Taylor rule in treatments of the Hicksian IS-LM apparatus. (It should be noted, however, that this abstraction was overly simplistic when the short-term interest rate approached zero. At the zero bound, the stance of monetary policy could no longer be measured or communicated with a short-term interest rate instrument; (see, for example, Orphanides and Wieland 2000). Subsequent research (see Orphanides, 2003b, for a survey) suggested that a generalized form of Taylor's classic rule could provide a useful common basis both for econometric policy evaluation across diverse families of models and for historical monetary policy analysis over a broad range of experience.

If we compare these two rule (Friedman and Taylor) under assumption of constant growth of money supply, we had Taylor Rule could be derived from the quantity equation (MV=PQ) (Taylor 1999c). Therefore interest rate fluctuated where the coefficient of output and inflation deviation were turning out positive.

3 THE MODEL

In this section we construct two general models (MIU model and New Keynesian model) which start with flexible wage and price contracts then maintain with rigidities of wage and prices. In the first subsection, utility depends directly on agents' consumption of goods and their holdings of money. (Money is seen in the budget constraint and utility function in the form of the real money balances.) Then we turn to models under the price of nominal rigidities in which monetary policy and monetary disturbances have important short run effects on real economy. Nominal wages or price rigidities mean that they fail to adjust immediately and completely to changes in the nominal quantity of money, is used to describe the short run real effects of monetary disturbances. We discuss why the price stickiness is important that the change in price rate affects the rate of inflation and what we gain in a manner of monetary policy by analyzing New Keynesian Model which allows that wages and prices are sticky together.

3.1 MIU Model

3.1.1 Flexible Wages and Prices

We will ignore uncertainty and any labor leisure choice focusing instead on the implications of the model for money demand, the value of money, and the costs of inflation. When money enters the utility, it helps to reduce the time needed to purchase consumption goods.

Utility of the representative household takes the form without money

$$U_t = u(c_t, z_t) \tag{4}$$

z_t is the flow of services yielded by money holding
c_t is time t per capita consumption.
Utility is strictly concave and continuously differentiable.
The demand for monetary services will always be positive if assuming that $\lim_{z\to 0} u_z(c,z) = \infty$ for all c, where $u_z = \partial u(c,z)/\partial z$ equal real per capita money holdings:
$z_t = \frac{M_t}{P_t N_t} \equiv m_t$.

To ensure that a monetary equilibrium exists, it is often assumed that, for all c, there exists a finite $\overline{m} > 0$ such that $u_m(c,m) \leq 0$ for all $m > \overline{m}$. This means that the marginal utility of money eventually becomes negative for sufficiently high money balances. The role of this assumption will be made clear when a steady state exists.

The representative household total utility

$$W = \sum_{t=0}^{\infty} \beta^t u(c_t, m_{t),} \tag{5}$$

where 0<$\beta < 1$ is a subjective rate of discount.

Household can hold money, that bonds pay a nominal interest rate i_t, and physical capital. Physical capital produces output according to a standard neoclassical production function. Given its current income, its assets, and any net transfers received from the government (τ_t).

The household allocates its resources between consumption, gross investment in physical capital, and gross accumulation of real money balances and bonds. If the rate of depreciation of physical capital is δ, the aggregate economy-wide budget constraint of the household sector takes form:

$$Y_t + \tau_t N_t + (1-\delta)K_{t-1} + \frac{(1+i_{t-1})B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} = C_t + K_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} \tag{6}$$

where Y_t is aggregate output
K_{t-1} is aggregate stock of capital at the start of period t,
$\tau_t N_t$ is the aggregate real value of any lump-sum transfers (taxes if negative).

The aggregate production function

$$Y_t = F(K_{t-1}, N_t) \tag{7}$$

where Y_t is output
K_{t-1} is the available capital stock
N_t is employment

The production function is homogeneous with constant returns to scale, and output per capita at time t will be a function of per capita capital stock: $y_t = f(\frac{k_{t-1}}{1+n})$,
n is the constant population growth rate
Output is produced in period t using capital carried over from period t-1.
The production function is assumed to continuously differentiable and to satisfy the usual Inada conditions: ($f_k \geq 0$, $f_{kk} \leq 0$, $\lim_{k\to\infty} f_k(k) = 0$).

Now we divide the both sides of the budget constraint (6) by the population N_t, per capita version becomes

$$\omega_t \equiv f(\frac{k_{t-1}}{1+n}) + \tau_t + (\frac{1-\delta}{1+n})k_{t-1} + \frac{(1-i_{t-1})b_{t-1} + m_{t-1}}{(1+\pi_t)(1+n)} = c_t + k_t + m_t + b_t, \tag{8}$$

where τ_t is the rate for inflation,

$b_t = \frac{B_t}{D_t N_t}$

$m_t = \frac{M_t}{P_t N_t}$,

The household's problem maximize (5) subject to (6). This problem is a problem in dynamic optimization,(Sargent, 1987; Lucas and Stokey 1989; Dixit 1990; Chiang 1992; Obstfield and Rogoff,1996 or Ljungquist and Sargent, 2000). We rearrange (5) as a value function define as the present discounted value of utility if the household optimally chooses consumption, capital holdings, bond holdings, and money balances,

$$V(\omega_t) = max\{u(c_t, m_t) + \beta V(\omega_{t+1})\} \tag{9}$$

where the maximization is subject to the budget constraint (6) and

$$\omega_{t+1} \equiv \frac{f(k_t)}{1+n} + \tau_{t+1} + (\frac{1-\delta}{1+n})k_t + \frac{(1-i_t)b_t + m_t}{(1+\pi_{t+1})(1+n)} \tag{10}$$

using (8) to express $k_t = \omega_t - c_t - m_t - b_t$ and making use of the definition of ω_{t+1}, can be written as

$$V(\omega_t) = max\{u(c_t, m_t) + \beta V(\frac{f(\omega_t - c_t - m_t - b_t)}{1+n} + \tau_{t+1} + (\frac{1-\delta}{1+n})(\omega_t - c_t - m_t - b_t) + \frac{(1-i_t)b_t + m_t}{(1+\pi_{t+1})(1+n)})\} \tag{11}$$

unconstrained one over c_t, m_t and b_t.

As a result, we have Fisher relationship (Fisher, 1896) which equates the nominal interest rate to the real interest rate plus the expected rate of inflation and money demand, a positive function of consumption also a negative function interest rate from the first order conditions (see details from Walsh 2003, p. 50).

$$i_t = r_t + \pi_{t+1} \tag{12}$$

$$m_t = \varphi(c_t, i_t) \tag{13}$$

where φ is a money depend function.

We can calculate the steady-stade value of m_t from the money demand relationship

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = \frac{i}{1+i} \tag{14}$$

where $u_m(c_t, m_t)$ is the first derivative of the utility function respects to m_t, $u_c(c_t, m_t)$ is the first derivative of the utility function respects to c_t,

and $C_t = Y_t$

In the model as we mentioned above, money is either neutral and superneutral in the steady-state where neutrality means that changes in the level of M do not affect real variables: C = Y where Y is exogenously given and does not depend on M. Furthermore, superneutrality of money means that the steady-state values of real variables are all independent of the rate of inflation (and the rate of monetary growth). In the steady state nominal rate of interest is given by $[(1 + \pi)/\beta] - 1$ and varies approximately one for one with inflation. Outside of the steady state, the nominal rate can still be written as the sum of the expected real rate plus the expected rate of inflation, but there is no longer any presumption that short-run variations in inflation will leave the real rate unaffected.

MIU model (Sdrauski, 1967) lets us examine the welfare cost of inflation and determine the optimal inflation rate. Friedman conclusion is that the inflation rate is optimal when it produces the zero rate of nominal interest rate.

About the welfare cost of inflation, money holdings yield direct utility and higher inflation reduces real money balances, as a result, inflation generates a welfare loss. The question we will try to answer is whether there is an optimal rate of inflation that maximizes the steady-state welfare of the representative household or not. Government chooses its policy instrument π to achieve the steady-state optimal value of M=P.

Lucas (1972) simplifies the same MIU model, we used, to illustrate how variations in the nominal quantity of money can have real effects when the information is imperfect. First capital is ignored. Second there is only money as an available asset. Finally, agents view the monetary transfers associated with changes in the nominal quantity of money.

Lucas's basic result is that aggregate monetary shocks have real effects on employment (and therefore output) if and only if there is an imperfect information. Contrast to publicly announced changes, i.e. predictable changes, in the money supply, unanticipated changes have real effects on output.

In most works in monetary economics imperfect information no longer plays a major role as the source of monetary nonneutrality. Instead, the assumption of flexible prices is dropped and prices and/or wages are assumed to be sticky.

3.1.2 Staggered Price and Wage Adjustments

Now we can clearly see in this subsection that how we put price rigidities into a model. Several authors have argued that nominal rigidities arise because of small menu costs, essentially fixed costs, associated with changing wages or prices. Mankiw, Akerlof and Yellen (1985) show that small costs of changing prices "menu costs" produce large nominal rigidities. Any sort of nominal rigidity naturally raises the question of who is setting wages and prices. Once we need to address the issue of price setting, we must examine the model that incorporate some aspect of imperfect competition, such as monopolistic competition. Chari, Kehoe, and McGrattan (2000) introduce price stickiness into their model by following Taylor (1979, 1980), who argued that an unexpected, permanent increase in the nominal money supply produces a rise in output with a slow adjustment and a gradual rise in the price level as a symmetry according to a horizontal line response to price level and output(Walsh ,2003: Figure 5.1, p.222). Though the model assumes that prices are set for only two periods, the money shock leads to a persistent, long-lasting effect on output. Chari, Kehoe, and McGrattan assume that employment must be consistent with household labor supply choices, and they show that γ i.e. it depends on the elasticity of labor supply with respect to the real wage, is a function of the parameters of the representative agent's utility function. They argue that a very high labor-supply elasticity is required to obtain a value of γ on the order of 0.05. With a low labor-supply elasticity, as seems more plausible, γ will be greater than or equal to 1. If this is the case, the Taylor model is not capable of capturing realistic adjustment to monetary shocks. Ascari (2000) reaches similar conclusions in a model that is similar to the framework in Chari, Kehoe, and McGrattan (2000) but that follows Taylor's original work in making wages sticky rather than prices.

As a price-level adjustment models Taylor (1979, 1980), Calvo (1983), and Fuhrer and Moore (1995a) are developed by Roberts (1995). Taylor (1979, 1980) originally developed his model in terms of nominal wage-setting behavior. With prices assumed to be a constant markup over wage costs, the adjustment of wages translates directly into an adjustment equation for prices. In the Fuhrer-Moore (1995a) specification, the backward-looking nature of the inflation process implies that reductions in the growth rate of money will be costly in terms of output. They argue that their specification fits U.S. data better than the Taylor model does. There are two reason while we are choosing Calvo's model in detail. First it shows how the coefficient on output in the inflation equation depends on the frequency with which prices are adjusted. A rise in ω, causes $[\frac{(1-\omega)(1-\omega\beta)}{\omega}]\gamma$ to decrease. Output movements have a smaller impact on current inflation, holding expected future inflation constant. Second reason is we use this model in New Keynesian model.

Here we mention the price adjustment model which based on Calvo (1983) who assumes that firms adjust their prices infrequently and that opportunity of adjusting prices occurs randomly. Besides, we can construct our model taking into consideration only wage rigidity as shown below.

Calvo Model (1983)

Calvo assumes that firms adjust their prices infrequently and that opportunities to adjust arrived as an exogenous Possion process. Each period, there is a constant probability 1-ω that firm can adjust its price; the expected time between price adjustments is 1/(1-ω). Because these adjustment opportunities occur randomly, the interval between price changes for an individual firm is a random variable. Following Rotemberg (1987), suppose the representative firm i sets its price to minimize a quadratic loss function that depends on the difference between the firm's actual price in period t, $p_{it,}$and its optimal price, p^*_t. This latter price might denote the profit- maximizing price for firm i in the absence of any restrictions or costs associated with price adjustment. If the firm can adjust at time t, it will set its price to minimize

$$\frac{1}{2}E_t\sum_{j=0}^{\infty}\beta^i(p_{it+j}-p^*_{t+j})^2 \tag{15}$$

subject to the assumed process for determining when the firm will next be able to adjust. Equation becomes

$$\sum_{j=0}^{\infty} \omega^i \beta^i E_t (p_{it} - p_{t+j}^*)^2 \tag{16}$$

ω^i is the probability that the firm has not adjusted after i periods so that the price set at t still holds in t+i.

x_t denote the optimal price set at t by all firms adjusting their prices:

$$x_t = (1 - \omega\beta) \sum_{j=0}^{\infty} \omega^i \beta^i E_t p_{t+i}^* \tag{17}$$

(17) is written again

$$x_t = (1 - \omega\beta) p_t^* + \omega\beta E_t x_{t+i} \tag{18}$$

The price set by the firm at time t is a weighted average of current and expected future values of the target price p^* depends on the aggregate price level and output, we can replace p_t^* with $p_t + \gamma y_t + \varepsilon_t$, where ε is a random disturbance to capture other determinants of p^*.the firm's optimal price will be shown to be a function of its marginal cost, which in turn, can be expressed as

$$p_t = (1 - \omega) x_t + \omega p_{t-1} \tag{19}$$

To obtain an expression for aggregate inflation,

$$\pi_t = p_t - p_{t-1} \tag{20}$$

$$\pi_t = \beta E_t \pi_{t+1} + [\frac{(1-\omega)(1-\omega\beta)}{\omega}](\gamma y_t + \varepsilon_t) = \beta E_t \pi_{t+1} + \gamma^{'} y_t + \varepsilon_t \tag{21}$$

Wage Rigidity

Here we construct our model with wage rigidity. First we take a flexible-price MIU model (Walsh, 2003, chapter 5) in which households prefer to adjust their prices every period.

Utility of the representative household takes the form

$$U_t = u(c_t, z_t) \tag{22}$$

z_t is the flow of services yielded by money holding

c_t is time t per capita consumption.

Utility is strictly concave and continuously differentiable.

The demand for monetary services will always be positive if assuming that $\lim_{z\to 0}$ $u_z(c,z) = \infty$ for all c, where $u_z = \partial u(c,z)/\partial z$ equal real per capita money holdings: $z_t = \frac{M_t}{P_t N_t} \equiv m_t$.

To ensure that a monetary equilibrium exists, it is often assumed that, for all c, there exists a finite $\overline{m}$ >0 such that $u_m(c,m) \leq 0$ for all m>$\overline{m}$. This means that the marginal utility of money eventually becomes negative for sufficiently high money balances. The role of this assumption will be made clear when a steady state exists.

The representative household total utility

$$W = \sum_{t=0}^{\infty} \beta^t u(c_t, m_{t)}, \tag{23}$$

where 0<$\beta < 1$ is a subjective rate of discount.

Household can hold money, that bonds pay a nominal interest rate i_t, and physical capital. Physical capital produces output according to a standard neoclassical production function.

Given its current income, its assets, and any net transfers received from the government (τ_t).

The household allocates its resources between consumption, gross investment in physical capital, and gross accumulation of real money balances and bonds. If the rate of depreciation of physical capital is δ, the aggregate economy-wide budget constraint of the household sector takes form:

$$Y_t + \tau_t N_t + (1-\delta)K_{t-1} + \frac{(1+i_{t-1})B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} = C_t + K_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} \tag{24}$$

where Y_t is aggregate output
K_{t-1} is aggregate stock of capital at the start of period t,
$\tau_t N_t$ is the aggregate real value of any lump-sum transfers (taxes if negative).

The aggregate production function

$$Y_t = F(K_{t-1}, N_t) \tag{25}$$

where Y_t is output
K_{t-1} is the available capital stock
N_t is employment

The production function is homogeneous with constant returns to scale, and output per capita at time t will be a function of per capita capital stock: $y_t = f(\frac{k_{t-1}}{1+n})$,

n is the constant population growth rate

Output is produced in period t using capital carried over from period t-1.

The production function is assumed to continuously differentiable and to satisfy the usual Inada conditions: ($f_k \geq 0$, $f_{kk} \leq 0$, $\lim_{k\to\infty} f_k(k) = 0$).

Now we divide the both sides of the budget constraint (24) by the population N_t, per capita version becomes

$$\omega_t \equiv f(\frac{k_{t-1}}{1+n}) + \tau_t + (\frac{1-\delta}{1+n})k_{t-1} + \frac{(1-i_{t-1})b_{t-1} + m_{t-1}}{(1+\pi_t)(1+n)} = c_t + k_t + m_t + b_t, \tag{26}$$

where τ_t is the rate for inflation,
$b_t = \frac{B_t}{D_t N_t}$ and $m_t = \frac{M_t}{P_t N_t}$,

The household's problem maximize (23) subject to (24). We rearrange (23) as a value function defined as the present discounted value of utility if the household optimally chooses consumption, capital holdings, bond holdings, and money balances,

$$V(\omega_t) = max\{u(c_t, m_t) + \beta V(\omega_{t+1})\} \tag{27}$$

where the maximization is subject to the budget constraint (24) and

$$\omega_{t+1} \equiv \frac{f(k_t)}{1+n} + \tau_{t+1} + (\frac{1-\delta}{1+n})k_t + \frac{(1-i_t)b_t + m_t}{(1+\pi_{t+1})(1+n)} \tag{28}$$

using (26) to express $k_t = \omega_t - c_t - m_t - b_t$ and making use of the definition of ω_{t+1},can be written as

$$V(\omega_t) = max\{u(c_t,m_t) + \beta V(\frac{f(\omega_t - c_t - m_t - b_t)}{1+n} + \tau_{t+1} + (\frac{1-\delta}{1+n})(\omega_t - c_t - m_t - b_t) + \frac{(1-i_t)b_t + m_t}{(1+\pi_{t+1})(1+n)})\} \tag{29}$$

unconstrained one over c_t, m_t and b_t.

The MIU model focuses on steady state properties. Now we are interested in understanding the implications of the model for the dynamic process the economy follows as it adjust in response to exogenous disturbances. From the linearization we have eight equations that have solved for capital stock, money holdings, output, consumption, employment, the real rate of interest, the nominal interest rate, and the inflation rate.

The equations are written in terms:

$$y_t = (1-\alpha)n_t + z_t \tag{30}$$

$$y_t = c_t \tag{31}$$

$$y_t - n_t = w_t - p_t \tag{32}$$

$$\Phi E_t[\Omega(c_{t+1} - c_t) - r_t = 0 \tag{33}$$

$$w_t - p_t = \eta(\frac{n^{ss}}{1-n^{ss}})n_t + \Phi c_t \tag{34}$$

$$m_t - p_t = c_t - (\frac{1}{b})i_t \tag{35}$$

$$i_t = r_t + E_t p_{t+1} - p_t \tag{36}$$

$$m_t = \gamma m_{t-1} + s_t \tag{37}$$

The system is written in terms of the price level p rather than the inflation rate. m represents the nominal stock of money. (30) represents the production function in which output deviations from the steady state are a linear function of the deviations of labor supply from the steady state and a productivity shock. (31) represent the resource constraint derived from the condition that, in the absence of investment or government purchases. Labor demand is derived from the condition that labor is employed up to the point where the marginal product of labor equals the real wage. (32) derived from the Cobb- Douglas production function is written in terms of percentage deviations from the steady state. (33) and (34) are derived from the representative household's first order conditions for consumption, leisure, and money holdings. (34) is the Fisher equation linking the nominal and real rates of interest. (35) gives the exogenous process for the nominal money supply. (30) and (34) form a system of equations that can be solved for the equilibrium time paths of output, labor, consumption, the real wage,and the real rate of interest when the prices are flexible. (35) and (37) determine the evolution of real money balances, the nominal interest rate, and the price level. The monetary disturbance s_t have no effect on output when prices are flexible.

A linear approximation was used to examine the time-series implications of an MIU model. Wages and prices were assumed to adjust to ensure market equilibrium,and, as a consequence, the behavior of the money supply mattered only to the extent that anticipated inflation was affected. A positive disturbance to the growth rate of money would, assuming that the growth rate of money was positively serially correlated, raise the expected rate of inflation, leading to a rise in the nominal rate of interest that affects labor supply and output. These last effects depended on the form of the utility function; if utility was separable in money, changes in expected inflation had no affect on labor supply or real output. We modify the model as we mentioned above by adding a one period nominal wage rigidity to illustrate the effect on the impact of monetary disturbances. Since workers and firms are assumed to have a real wage target in mind, the nominal wage will adjust fully to reflect expectations of price-level changes held at the time the nominal wage is set. The equilibrium level of employment and real wage with flexible prices can be obtained by equating labor supply and labor demand. From (30), (31), (32) and (34), we obtain,

$$n_t^* = [\frac{1-\Phi}{1+\overline{\eta}+(1-\alpha)(\Phi-1)}]z_t = b_0 z_t \tag{38}$$

and

$$\omega_t^* = [\frac{\overline{\eta} + \Phi}{1 + \overline{\eta} + (1-\alpha)(\Phi - 1)}]z_t = b_1 z_t \tag{39}$$

where η^* is the flex-price equilibrium employment,
ω^* is the flex-price equilibrium real wage,

$$\overline{\eta} \equiv \eta n^{ss}/(1 - n^{ss}) \tag{40}$$

The contract nominal wage w^c will satisfy

$$w_t^c \equiv E_{t-1}\omega_t^* + E_{t-1}p_t \tag{41}$$

using production function then we have

$$E_{t-1}\omega_t^* = -\alpha E_{t-1}n_t^* + E_{t-1}z_t \tag{42}$$

$$n_t = E_{t-1}n_t^* + (\frac{1}{\alpha})(p_t - E_{t-1}p_t) + (\frac{1}{\alpha})\varepsilon_t \tag{43}$$

where $\varepsilon_t \equiv (z_t - E_{t-1}z_t)$

(42) shows that employment deviates from the expected flexible price equilibrium level in the face of unexpected movements in prices. An unanticipated increase in prices reduces the real value of the contract wage and leads firms to expand employment. An unaccepted productivity shock ε_t raises the marginal product of labor and leads to an employment increase. If prices are unexpectedly low, the actual real wage will exceed the level expected to clear the labor market, and firms will reduce employment. By substituting (42) into production function we obtain

$$y_t = (1-\alpha)[E_{t-1}n_t^* + (\frac{1}{\alpha})(p_t - E_{t-1}p_t) + (\frac{1}{\alpha})\varepsilon_t] + e_t \tag{44}$$

which implies that

$$y_t - E_{t-1}y_t^* = a(p_t - E_{t-1}p_t) + (1+a)\varepsilon_t \tag{45}$$

where $E_{t-1}y^* = (1-\alpha)E_{t-1}n_t^* + E_{t-1}z_t$ is expected equilibrium output under flexible prices and a=$\frac{1-\alpha}{\alpha}$.

Innovations to output are positively related to price innovations. Thus monetary shocks that produce unanticipated price movements directly affect real output. Imperfect competition can lead to aggregate demand externalities, Blanchard and Kiyotaki (1987), equilibria in which output is inefficiently low, and multiple equilibria (Ball and Romer 1991, Rotemberg and Woodford 1995) but it alone does not lead to monetary nonneutrality. If prices are free to adjust one period, permanent changes in the level of the money supply induce proportional changes in all prices, leaving the real equilibrium unaffected. Now we add price stickiness by assuming that intermediate goods producers engage in multi-period, staggered price setting. After we explain what staggered wage and price adjustment mean, we will see New Keynesian Model wherein both wages and prices are staggered.

3.2 A New Keynesian Model

The second model is New Keynesian Model (i.e. both of prices and wages are sticky) whose elements are nominal rigidities and imperfect competition in a dynamic general equilibrium models. Equilibrium conditions for aggregate variables are derived from optimal individual behavior on the part of consumers and firms, and are consistent with the simultaneous clearing of all markets. Before New Keynesian economics, DGE models largely relate to Real Business Cycle paradigm which analyzes the relation between money, inflation, and the business cycle.

The name "New Keynesian Theory" is first introduced by Michael Parkin (1982). One of the earliest using of the term "New Keynesian Economics" is in article by Ball, Mankiw and Romer (1988). New is used instead of "Neo" to distinguish from "Neoclassical Synthesis Keynesian Economics" and also to show that it is the counter- argument to the New Classical Economics. The foundations of the Keynesian Economics are usually attributed in Stanley Fischer, Edmund Phelps, and John Taylor.

We prefer to study with New Keynesian models to provide a tractable framework for analysis of optimal monetary policy design for a closed economy. Optimality tells us the perspective of representative agent's -household or consumer - utility function. We suppose that objective of monetary authority, central bank or government, sets policy to maximize the utility of the representative agents or to minimize the loss function. The fundamental assumptions of this model concern who the agents are, their preferences and endowments, the technology which they have accessed, and the market structure.

The problem setup:

1-Specify how households make optimal choices

2-Specify how firms make optimal choices and how production occurs.

3-Consider simultaneously the optimal choices of both households and firms along the resource constraint of the economy.

4-Together setup the equilibrium of the economy.

5-Evaluate the welfare of any given policy by simply inserting the resulting equilibrium levels of consumption (and/or leisure) into the representative agents' utility functions.

The New Keynesian models bring a new perspective on the nature of inflation dynamics, the concept of output gap, the form of the working of policy instruments. In addition to being a source of monetary non-neutralities, the presence of sticky prices may also have implications for the economy's response to non monetary shocks. Optimal monetary policy requires that the central bank respond to a simple policy rule that has the central bank adjusted (sufficiently) the interest rate in response to variations in inflation. The output gap generally provides a good approximation to the optimal rule (with the implied welfare losses being small). The coexistence of staggered price setting has important implications for monetary policy. An extensive literature has dealt with optimal monetary policy design in such a framework in recent years, e.g. Taylor (1999), Svensson (1999), Clarida Gali and Gertler (1999), Woodford (2003a) and Walsh (2003).

As Walsh (2006) noted: "Today . . . [c]entral banks employ DSGE models for policy analysis. Policy makers think in terms of rules. They recognize the value of credibility and commitment. They try to reduce uncertainty in markets by providing information about the likely future path of interest rates... ".

3.2.1 Model Description

The model consists of household and firms. Household supply labor, purchases goods for consumption, and hold money and bonds. Firms hire labor, produce and sell differentiated products in monopolistically competitive goods markets. The basic model of monopolistic competition is drawn from Dixit and Striglitz (1977). Each firm sets the price of the good it produces, but not all firms reset their price in each period. Households and firms behave optimally; households maximize the expected present value of utility, and firms maximize the profits. There is also a central bank who controls the nominal rate of interest. The central bank, in contrast to households and firms, is not assumed to behave optimally.

The preferences of representative household are defined over a composite consumption good C_t, Real money balances $\frac{Mt}{P_t}$,

Leisure 1-N_t, where N_t is the time devotes to market employment.

Households maximize the expected present discounted value of utility:

$$E_t \sum_{i=0}^{\infty} \beta^i [\frac{C_{i+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} (\frac{M_{t+i}}{P_{t+i}})^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta}] \tag{46}$$

The composite consumption good consists of differentiated products produce by monopolistically competitive final goods producers (firms). The composite consumption good which enters the household's utility function

$$C_t = [\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj]^{\frac{\theta}{\theta-1}} \tag{47}$$

where $\theta > 1$ and θ govern the price elasticity of demand for the individual goods. The household first minimizes the cost of the composite good, then given the cost of achieving any given level of C_t, choose C_t, N_t, and M_t optimally. Dealing first with the problem of minimizing the cost of buying C_t, the household's decision problem is to

$$min_{c_{jt}} \int p_{jt} c_{jt} dj \tag{48}$$

subjects to

$$[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj]^{\frac{\theta}{\theta-1}} > C_t \tag{49}$$

where p_{jt} is the price of good j and the consumption index given by C_t.

Aggregated price index for consumption:

$$P_t \equiv [\int_0^1 p_{jt}^{1-\theta} dj]^{\frac{1}{1-\theta}} \tag{50}$$

the demand for good j can be written as

$$c_{jt} = (\frac{p_{jt}}{P_t})^{-\theta} C_t \tag{51}$$

The price elasticity of demand for good j is equal to θ. As $\theta \to \infty$, the individual goods become closer and closer substitutes, and, as a consequence, individual firms have less market power.

Given the definition of the aggregate price index, the budget constraint of the household is in real terms,

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = (\frac{W_t}{P_t})N_t + \frac{M_{t-1}}{P_t} + (1+i_{t-1})(\frac{B_{t-1}}{P_t}) + \Pi_t \tag{52}$$

where $M_t(B_t)$ was the household's nominal holdings of money (one-period bonds).
i_t : Nominal interest rate paid for bonds.
Π_t : Real profits received from firms.

Now we maximize the household's utility (46) subject to budget constraint (52). The Euler Conditions where the budget constraint (52) has to hold in equilibrium.

Optimal intertemporal allocation of consumption

$$C_t^{-\sigma} = \beta(1+i_t)E_t(\frac{P_t}{P_{t+1}})C_{t+1}^{-\sigma}; \tag{53}$$

The marginal rate of substitution between money and consumption equal to the opportunity cost of holding money.

$$\frac{\gamma(\frac{M_t}{P_t})^{-b}}{C_t^{-\sigma}} = \frac{i_t}{1+i_t} \tag{54}$$

The marginal rate of substitution between leisure and consumption equal to the real wage.

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t} \tag{55}$$

.

Firms maximize profits, subject to production function summarizing the available technology, demand curve each firm faces, and the price stickiness due to Calvo(1983). The other price adjustment models are state dependent pricing models (Dotsey, King and Wolman, 1999; Kiley ,2000) and endogenous price stickiness model (Haubrich and King,1991).

Before the firm's pricing decision, we minimize its cost

$$min_{N_t}(\frac{W_t}{P_t})N_t + \varphi_t(c_{jt} - Z_t N_{jt}) \tag{56}$$

where φ_t is firm's real marginal cost: $\varphi_t = \frac{W_t/P_t}{Z_t}$.
then we return to the pricing decision problem

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{t,t+i}[(\frac{p_{jt}}{P_{t+i}})^{1-\theta} - \varphi_{t+i}(\frac{p_{jt}}{P_{t+i}})^{-\theta}]C_{t+i} \tag{57}$$

where $\Delta_{t,t+i} = \beta^i (C_{t+i}/C_t)^{-\sigma}$ is the discount factor

Individual firms who produce differentiated products, all have the same production technology and face demand curves with constant and equal demand elasticities. Let p* the optimal price is chosen by all firms adjusting at time t and the definition of the discount factor we obtain

$$(\frac{p_t^*}{P_t}) = (\frac{\theta}{\theta - 1})\frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+1}(\frac{P_{t+i}}{P_t})^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma}(\frac{P_{t+i}}{P_t})^{\theta-1}} \tag{58}$$

If $\omega = 0$, all firms are able to adjust their prices every period then, (58) reduces to (59).

$$(\frac{p_t^*}{P_t}) = (\frac{\theta}{\theta - 1})\varphi_t = \mu\varphi_t \tag{59}$$

In a standard monopolistic competition model, each firm sets its price p_t^* equal to a markup $\mu > 1$ over its nominal marginal cost $P_t\varphi_t$. When prices are flexible, all firms charge the same price. In this case p_t^*=P_t and $\varphi_t = 1/\mu$.

Using the definition of real marginal cost (φ_t), we have

$$\frac{W_t}{P_t} = \frac{Z_t}{\mu} \tag{60}$$

in a flexible- price equilibrium. However, the real wage has to also equal the marginal rate of substitution between leisure and consumption to be consistent with household optimization. From (55)

$$\frac{W_t}{P_t} = \frac{Z_t}{\mu} = \frac{\chi N_t^{\eta}}{C_t^{-\sigma}} \tag{61}$$

Around the steady state:

$$\eta \widehat{n}_t^f + \sigma \widehat{c}_t^f = \widehat{z}_t \tag{62}$$

$$\widehat{y}_t^f = \widehat{n}_t^f + \widehat{z}_t \tag{63}$$

$$\widehat{y}_t^f = \widehat{c}_t^f. \tag{64}$$

where the superscript f denotes the flexible-price equilibrium. Combining (62), (63), (64) the flexible price equilibrium output $\widehat{y}_t^f$ can be expressed as

$$\widehat{y}_t^f = (\frac{1+\eta}{\sigma+\eta})\widehat{z}_t \tag{65}$$

when prices are sticky (w>0), output can differ from the flexible-price equilibrium level. Because it will not adjust its price every period. The aggregate price index is an average of the price charged by the fraction 1-ω of firms setting their price in period t and the average of the remaining fraction ω of all firms setting their price in earlier periods.

The average price in period t satisfies

$$P_t^{1-\theta} = (1-\omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta} \tag{66}$$

aggregate inflation is obtained from (58) and (66) by approximating around a zero average inflation at a steady-state equilibrium

$$\pi_t = \beta E_t \pi_{t+1} + \widetilde{\kappa}\widehat{\varphi}_t \tag{67}$$

where $\widetilde{\kappa} = \frac{(1-\omega)(1-\beta\omega)}{\omega}$ is an increasing function of the fraction of firms able to adjust each period.

$\widehat{\varphi}_t$ is real marginal cost, expressed as a percentage deviation around its steady-state value.

(67) is often referred to as the New Keynesian Phillips Curve. It implies that the inflation process is forward-looking with current inflation as a function of expected future inflation. When a firm sets its price, it has to be concerned with inflation in future because it may be unable to adjust its price for several periods. Solving (67) forward, we have $\pi_t = \widehat{\kappa} \sum_{i=0}^{\infty} \beta^i E_t \widehat{\varphi}_{t+i}$, which shows that inflation is a function of the present discounted value of current and future real marginal costs. (67) implies that inflation depends on real marginal cost and not directly on a measure of the output gap between actual and potential output or on a measure of unemployment relative to the natural rate, as in typical in traditional Phillips curves. The firm's real marginal cost equals the real wage divides by the marginal product of labor. In a flexible price equilibrium, all firms set the same price so the real marginal cost will equal its steady state value $1/\mu$. Because nominal wages have been assumed to be completely flexible, the real wage has to equal the marginal rate of substitution between leisure and consumption. Expressed in terms of percentage deviations of marginal cost around its the steady state (55) implies that

$$\widehat{w_t} - \widehat{p}_t = \eta \widehat{n}_t + \sigma \widehat{y}_t. \tag{68}$$

Recalling that $\widehat{c}_t = \widehat{y}_t$ and $\widehat{y}_t = \widehat{n}_t + \widehat{z}_t$, (68) becomes

$$\widehat{\varphi}_t = (\widehat{w}_t - \widehat{p}_t) - (\widehat{y}_t - \widehat{n}_t) = (\sigma + \eta)[\widehat{y}_t - (\frac{1+\eta}{\sigma+\eta})\widehat{z}_t] \tag{69}$$

But from (65), this can be written as

$$\widehat{\varphi}_t = \gamma(\widehat{y}_t - \widehat{y}_t^f) \tag{70}$$

where $\gamma = \sigma + \eta$. Using this result, the inflation adjustment equation (67) becomes

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{71}$$

where $\kappa = \gamma\widetilde{\kappa} = \gamma(1-\omega)(1-\beta\omega)/\omega$ and $x_t \equiv \widehat{y}_t - \widehat{y}_t^f$ is the output gap between actual output and flexible- price equilibrium output.

Each firm's production function under the assumption of existence of constant return to scale (0<a $\leq$1) is

$$c_{jt} = Z_t N_{jt}^a \tag{72}$$

where 0<a$\leq$ 1,

When a<1, firms with different production function levels face different marginal cost is derived in terms of deviations around the steady-state for firm j is

$$\widehat{\varphi}_{jt} = \widehat{\varphi}_t - [\frac{\theta(1-a)}{a}](\widehat{p}_{jt} - \widehat{p}_t) \tag{73}$$

Firms with relatively high prices (and therefore low output) have relatively low real marginal costs.

In the case of constant returns scale (a=1), all firms face the same marginal cost. According to Sbordone (2002) and Gali, Gertler,and Lopez-Salido (2001), the New Keynesian inflation adjustment equation is

$$\pi_t = \beta E_t \pi_{t+1} + \widetilde{\kappa}[\frac{a}{a+\theta(1-a)}]\widehat{\varphi}_t \tag{74}$$

The labor market equilibrium condition under flexible prices:

$$\frac{W_t}{P_t} = \frac{aZ_t N_t^{a-1}}{\mu} = \frac{\chi N_t^{\eta}}{C_t^{-\sigma}}$$

and flexible-price output is

$$\widehat{y}_t^f = [\frac{1+\eta}{1+\eta+a(\sigma-1)}]\widehat{z}_t. \tag{75}$$

when a=1, this reduces to (65).

(71) relates output, in the form of the deviation around the level of output that will occur in the absence of nominal price rigidity, to inflation and a linearized version of the household's Euler condition (53). They form key components of an optimizing model that can be used for monetary policy analysis.

Esteralla and Fuhrer (2002) write the inflation adjustment equation (71) as $\beta E_t\pi_{t+1} - \pi_t = -\kappa x_t$. If we let u_{t+1} denote the error in forecasting future inflation, this can be written as $\beta\pi_{t+1} - \pi_t = -\kappa x_t + \beta(\pi_{t+1} - E_t\pi_{t+1}) = -\kappa x_t + \beta u_{t+1}$. Since $\beta \approx 1$ in quarterly data according to U.S. data, $\pi_{t+1} - \pi_t \approx - -\kappa x_t + u_{t+1}$. An increase in the output gap should lead to a fall in future inflation. Unemployment is direct proportion with inflation. (71) doesn't successful to fit with quarterly U.S. data. The estimated coefficient on the gap measure in quarterly U.S. data is actually negative (Gali and Getler, 1999; Sbordone 2001), although Roberts (1995) found a small positive coefficient using annual data. Fuhrer (1997b) finds little role for future inflation once lagged inflation is added to the inflation adjustment equation under the persistence inflation. Gali and Gertler (1999) test model of inflation adjustment by using real marginal cost rather than using an output gap variable. They conclude that lagged inflation is much less important than suggested by Rudebusch and Fuhrer if real marginal cost is used in place of an output gap measure. Sbordone (2002) also implies that there is a dependence of inflation on expected future inflation and real marginal cost. These results suggest that the problem is the link between marginal cost and output rather than the link between marginal cost and inflation.

3.2.2 General Equilibrium

(53), (58) and (66) provide to determine output, nominal quantity of money in equilibrium and the aggregate price level depends on the nominal rate of interest.

$$x_t = E_t x_{t+1} - (\frac{1}{\sigma})(\widehat{i}_t - E_t\pi_{t+1}) + u_t \tag{76}$$

(76) represented the demand side of the economy which was expectational, fordward-looking IS curve and where u$_t \equiv E_t\widehat{y}^f_{t+1} - \widehat{y}^f_t$ depended only on the exogenous productivity disturbance. New Keynesian Phillips Curve (71) corresponded to the supply side derived from the pricing decisions of individual firm. Combining (76) with (71) gave a simple two equation, forward-looking,

rational-expectations model for inflation and the output gap measure x_t. (71) and (76) contained the output gap, inflation, and the nominal interest rate. The central bank controlled the nominal interest rate to implement monetary policy. Let us assume that the central bank follows the Taylor rule.

$$i_t = \rho + \phi_\pi \pi_t + \phi_x x_t \tag{77}$$

where ϕ_π and ϕ_x are satisfied the condition for uniqueness

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_x > 0 \tag{78}$$

The rule could minimize the deviations from the optimal path by choosing sufficiently large values of ϕ_π and ϕ_x. A Taylor rule with very high inflation or output gap coefficients would potentially lead to huge instrument-instability: any small deviation of inflation or the output gap from zero would imply infinite changes in the rate.

For more detail, we seek to Taylor (1993, 1999) and Judd and Rudebusch (1988). Clarida, Galí, and Gertler (1998, 2000) estimate a forward looking version of that rule, in which the interest rate is assumed to respond to anticipated inflation and output gap, instead of the realized values. Orphanides (1999) discusses the difficulties and perils of implementing a Taylor-type rule in real time.

3.2.3 Economic Disturbances

There were two commonly objectives of monetary policy that maintain a low and stable average rate of inflation and to stabilize output around full employment. A supply shock, such as an increase in oil prices, increases inflation and reduces output. To keep inflation constant, central bank use contractional policies that would exacerbate the decline in output; and to keep output at a same level, they apply expansionary policies that would worsen inflation. However, if the output objective is interpreted as meaning that output should be stabilized around its flexible-price equilibrium level, then (71) implies that the central bank can always achieve a zero output gap. and keep inflation equal to zero. Solving (71) forward yielded,

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i.} \tag{79}$$

Current and expected future output equal to the flexible-price equilibrium level, $E_t x_{t+i} = 0$ for all i and inflation remains equal to zero. If we added an error term to inflation adjustment equation (71) becomes

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t \tag{80}$$

then

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} + \sum_{i=0}^{\infty} \beta^i E_t e_{t+i.} \tag{81}$$

As long as $\sum_{i=0}^{\infty} \beta^i E_t e_{t+i} \neq 0$, maintaining $\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i}$ is not sufficient to ensure that inflation always remains equal to zero. Disturbances terms in the inflation adjustment equation are often called cost shocks or inflation shocks. Since shocks, unless they are permanent, ultimately affect only the price level, they are also called price shocks. Clarida Gali and Gertler (2001) add the shochastic wage markup to shock in the inflation adjustment equation to represent deviations between the marginal rate of substitution between leisure and consumption and the real wage. The labor supply (55) becomes

$$\frac{W_t}{P_t} = (\frac{\chi N_t^{\eta}}{C_t^{-\sigma}}) e^{\mu_t^w} \tag{82}$$

where μ_t^w is a random disturbance. If labor markets are imperfectly competitive, it could arise from shochastic shifts in the markup of wages over the marginal rate of substitution (Clarida Gali and Gertler 2002). When linearized around the steady state, we obtain,

$$\eta \widehat{n}_t + \sigma \widehat{c}_t + \mu_t^w = \widehat{w}_t - \widehat{p}_t \tag{83}$$

The real marginal cost variable becomes

$$\varphi_t = (\eta \widehat{n}_t + \sigma \widehat{c}_t) - (\widehat{y}_t - \widehat{n}_t) + \mu_t^w \tag{84}$$

then, the inflation adjustment equation became

$$\pi_t = \beta E_t \pi_{t+1} + \gamma \widetilde{\kappa} x_t + \widetilde{\kappa} \mu_t^w \tag{85}$$

In (85), we used μ_t^w as a source of inflation shocks. If μ_t^w is a markup due to imperfect competition in the labor market, then μ_t^w also effects the flexible-price equilibrium level of output.

3.2.4 Sticky Wages and Prices

The model of inflation adjustment based on the Calvo specification (Erceg, Henderson and Levin, 2000) implies that inflation depends on real marginal cost. In terms of deviations from the flexible-price equilibrium, real marginal cost equaled the gap between the real wage and the marginal product of labor (mpl). Other models incorporating both wage and price stickiness include those of Guerrieri (2000), Ravenna (2000), Christiano, Eichenbaum, and Evans (2001), and Sbordone (2001, 2002). Erceg, Henderson, and Levin assume that a randomly drawn fraction of households optimally set their wage each period, just as the models of price stickiness assume that only a fraction of firms adjust their price each period. Thus letting ω_t denote the real wage,

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(\omega_t - mpl_t) \tag{86}$$

Wage inflation responds the appropriate gap depends on a comparison between the real wage and the households marginal rate of substitution between leisure and consumption. With flexible wages and price stickiness, workers were always on their labor supply curves; despite price stickiness, nominal wages can adjust to ensure that the real wage equals the marginal rate of substitution between leisure and consumption (mrs). when wages are also sticky, this means that $\omega_t < mrs_t$ workers will want to raise their nominal wage when the opportunity to adjust arises.

Erceg, Henderson, and Levin showed that

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \kappa^w(mrs_t - \omega_t) \tag{87}$$

where π_t^w is the rate of nominal wage inflation,
From the definition of real wage,

$$\omega_t = \omega_{t-1} + \pi_t^w - \pi_t \qquad (88)$$

(86) and (88) constitute the inflation adjustment block of an optimizing model with both wage and price rigidities.

Christiano, Eichenbaum, and Evans (2001) report that wage rigidity, not price rigidity, is the key in accounting for the observed dynamics of inflation and output according to U.S. data. However, a model with sticky wages and flexible prices implies that real wages should move countercyclically; a monetary policy expansion raises the price level, and the resulting decline in real wages induces firms to increase employment and output. Huang and Liu (2002) argue that wage stickiness is more important than price stickiness for generating output persistence. In contrast, Goodfriend and King (2001), while accepting that nominal wages are sticky, argue that the long-term nature of employment relationships means that nominal wage rigidity has little implication for real resource allocation.

4 OPTIMAL MONETARY POLICY

We have two key components; (71) and (76) while conducting optimal monetary policy. Kerr and King (1996) discuss how the evaluation of interest-rate policy rules can be asaffected by the role of expected future output, while McCallum and Nelson (1999) conduct an empirical evaluation of alternative policy rules using a small model in which current aggregate demand depends on expectations of future output. However Fuhrer (1997b) ignores the role of expected future inflation in the inflation-adjustment equations, the actual policy instrument i_t becomes

$$i_t = (1 - \frac{B(1 - a_3\gamma)}{a_3})\pi_t + \frac{a_1}{a_3}y_t + \frac{a_2}{a_3}y$$

where the optimal decision rule will be in the form $\theta_t = B\pi_t$ and $\beta\lambda\gamma B^2 + (\beta\lambda - \lambda - \gamma^2)B - \gamma = 0.$

where stability of the inflation process requires $|1+\gamma B|<1$, so we take into consideration the negative solution of B, and

$\theta_t \equiv \frac{a_1 y_t + a_2 y_{t-1} - a_3(i_t - \pi_t)}{1 - a_3\gamma}$

For the parameter values of $a_1 = 1.53, a_2 = -0.55, a_3 = 0.35, \gamma = 0.002$ and $\beta = 0.989$. The coefficients of inflation and output for this optimal policy rule are defined as a function of λ, assume that $\lambda = 1$, then the policy rule becomes

$$i_t = 1.50\pi_t + 4.37y_t - 1.57y_{t-1}$$

Ball (1997) obtains for a similar exercise,

$$i_t = 1.48\pi_t + 0.8y_t$$

Ball's model has only one lag of output in the aggregate spending equation, unlike Fuhrer. He views his model as appropriate for annual data, so the numbers reported are based on $\beta = 0.96$. His other parameter values are $a_1 = 0.8, a_3 = 1.0, \gamma = 0.4$. Note that this implies a much stronger response of inflation to output (γ) and of spending to the interest rate (a_3). These changes affect mainly the coefficient on output. Ball also assumes that output enters with a lag in the inflation equation, so it should actually be y_{t-1} in the policy rule. Both of these rules for adjusting the nominal interest rate are similar to Taylor rules. Taylor (1993a) has shown that $i_t = 1.5\pi_t + 0.5y_t$ which provides a good fit to the behavior of the federal funds rate in the United States. According to the Taylor rule, the nominal rate is increased linearly proportional to inflation. This policy ensures a real rate response that will act to lower inflation. For a given inflation rate, the real rate is also increased in response to increase in output. Fuhrer and Moore's and Ball's parameter values have the basic form of a Taylor rule, they both imply relative more weight on output than characterizes the best fit to actual U.S. policy, a point emphasized by Ball (1999).

4.1 The Case Without Distortions

When nominal rigidities coexist with real imperfections, the flexible price equilibrium allocation was inefficient and no longer optimal for the central bank. We analyzed the optimal monetary policy problem where the presence of some real imperfections generate a time-varying gap between output and its efficient counterpart, even in the absence of price rigidities. The representative household's welfare losses are

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \alpha_x x_t^2 \right) \right\} \quad (89)$$

where $x_t \equiv y_t$-y_t^* denotes the welfare-relevant output gap, i.e. the deviation between (log) output y_t and its efficient level y_t^*.

$\pi_t \equiv p_t - p_{t-1}$ denotes the rate of inflation between periods t -1 and t.

α_x represents the weight of output gap fluctuations (relative to inflation) in the loss function, and is given by $\alpha_x = \frac{\kappa}{\epsilon}$

where is the coefficient on x_t in the new Keynesian Phillips curve (NKPC), and ϵ is the elasticity of substitution between goods.

The central bank will seek to minimize (89) subject to

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa x_t + u_t \quad (90)$$

the disturbance $u_t \equiv \kappa(y_t^* - y_t^n)$

Time variations in the gap between the efficient and natural levels of output generate a trade-off for the monetary authority, since they make it impossible to attain simultaneously zero inflation and an efficient level of activity. We refer to disturbance u_t which is AR(1) process in (90) as a cost-push shock.

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \quad (91)$$

where $\rho_u \in [0,\ 1)$ and $\{\varepsilon_t^u\}$ is a white noise process with constant variance σ_u^2.

While (90) is the only constraint needed in order to determine the equilibrium path for output and inflation under the optimal policy, implementation of that policy requires an additional condition linking those variables with the monetary policy instrument, i.e. the interest rate. This condition can be obtained from the dynamic IS curve in terms of the welfare-relevant output gap,

$$x_t = -\frac{1}{\sigma}(i_t - E_t \{\pi_{t+1}\} - r_t^e) + E_t \{x_{t+1}\} \quad (92)$$

where $r_t^e \equiv \rho + \sigma E_t \left\{ \Delta y_{t+1}^e \right\}$ is the interest rate that supports the efficient allocation and which is invariant to monetary policy. We refer to r_t^e as the efficient interest rate.

The forward-looking nature of constraint (90) in the policy problem, requires that we specify the extent to which the central bank can credibly commit in advance to future policy actions. The following two sections characterize the optimal monetary policy under discretionary and commitment. We compare them in Conclusion section.

4.1.1 Optimal Discretionary Policy

Each period the monetary authority was assumed to choose (x_t, π_t) in order to minimize the period losses

$$\pi_t^2 + \alpha_x x_t^2 \tag{93}$$

subject to the constraint

$$\pi_t = \kappa x_t + \upsilon_t \tag{94}$$

where the term $\upsilon_t \equiv \beta E_t \{\pi_{t+1}\} + u_t$ is taken as given.

By the monetary authority, since u_t is exogenous and $E_t \{\pi_{t+1}\}$ is a function of expectations about future output gaps (as well as future u_t's) which by assumption, cannot be currently influenced by the policymaker.

The optimality condition was given by

$$x_t = -\frac{\kappa}{\alpha_x}\pi_t \tag{95}$$

for t = 0, 1, 2,...

In the face of inflationary pressures resulting from a cost-push shock the central bank must respond by driving output below its efficient level–thus creating a negative output gap– with the objective of dampening the rise in inflation. The condition (95) describes a relation between target variables that the discretionary central bank will seek to maintain at all times and it is in that sense that may be labeled a "targeting rule." Using (95) to substitute for x_t in (90), we obtain an expression for equilibrium inflation under the optimal discretionary policy:

$$\pi_t = \alpha_x \frac{1}{\kappa^2 + \alpha_x(1 - \beta\rho_u)} u_t \tag{96}$$

and expression for the output gap

$$x_t = -\kappa \frac{1}{\kappa^2 + \alpha_x(1 - \beta\rho_u)} u_t \tag{97}$$

Thus, under the optimal discretionary policy, the central bank lets the output gap and inflation deviate from their targets in proportion to the current value of the cost-push shock. Finally, we see that the implied response of inflation leads naturally to a permanent change in the price level, whose size is increasing in the persistence of the shock. The analysis above implicitly assumes that the monetary authority can choose its desired level of inflation and the output gap at each point in time. One possible approach to implementing that policy is to adopt an interest rate rule that guarantees that the desired outcome is attained.

$$i_t = r_t^e + \phi_\pi \pi_t \tag{98}$$

where $\phi_\pi \equiv (1 - \rho_u)\frac{\kappa\sigma}{\alpha_x} + \rho_u$,

A rule of the form (98) leads to a determinate equilibrium (corresponding to the desired outcome) if and only if the inflation coefficient is greater than one (Taylor Principle) or, equivalently, if and only if $\kappa\sigma > \alpha_x$.

In practice, interest rate rules like (98) are not easy to implement, they require knowledge of the model's parameters, and real-time observation of variations in the cost-push shock and the efficient interest rate. Those difficulties have led some authors to emphasize "targeting rules" like (95) as practical guides for monetary policy, as opposed to "instrument rules" like (98). Under a targeting rule, the central bank would adjusts its instrument until a certain optimal relation between target variables is satisfied. In our example, however, following such a targeting rule requires that the efficient level of output y_t^e be observed in real time, in order to determine the output gap x_t.

4.1.2 Optimal Policy under Commitment

A central bank is assumed to be able to commit, with full credibility, to a policy plan. In the context of our model such a plan consists of a specification of the desired levels of inflation and the output gap at all possible dates and states of nature, current and future. More specifically, the monetary authority is assumed to choose a state-contingent sequence that minimizes

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t(\pi_t^2+\alpha_x x_t^2) \tag{99}$$

subject to the sequence of constraints:

$$\pi_t=\beta E_t\{\pi_{t+1}\}+\kappa x_t+u_t \tag{100}$$

"targeting rule" which the central bank must follow period by period in order to implement the optimal policy under commitment like as in (95) is

$$x_t=-\frac{\kappa}{\alpha_x}\widehat{\rho}_t \tag{101}$$

for t=0,1,2,... where $\widehat{p}_t\equiv p_t-p_{-1}$ is the (log) deviation between the price level and an "implicit target" given by the price level prevailing once period before the central bank chooses its optimal plan.

(101) is viewed as a "targeting rule" which the central bank must follow period by period in order to implement the optimal policy under commitment. The difference between (95) and (101) is that the optimal discretionary policy requires that the central bank keeps output below (above) its efficient level as long as inflation is positive (negative). By way of contrast, under the optimal policy with commitment the central bank sets the sign and size of the output gap in proportion to the deviations of the price level from its implicit target.

$$\widehat{p}_t=\delta\widehat{p}_{t-1}+\frac{\delta}{(1-\delta\beta\rho_u)}u_t \tag{102}$$

for t=0,1,2,... where $\delta\equiv\frac{1-\sqrt{1-4\beta a^2}}{2a\beta}\in(0,1)$, *and* $a\equiv\frac{\alpha_x}{\alpha_x(1+\beta)+\kappa^2}$.

and output gap is

$$x_t = \delta x_{t-1} - \frac{\kappa\delta}{\alpha_x(1-\delta\beta\rho_u)} u_t \tag{103}$$

and the forward looking inflation

$$\pi_t = \kappa x_t + \kappa \sum_{k=1}^{\infty} \beta^k E_t \{x_{t+k}\} + u_t \tag{104}$$

The central bank can offset the inflationary impact of a cost push shock by lowering the current output gap x_t, but also by committing to lower future output gaps (or, equivalently, future reductions in the price level). If credible, such "promises" will bring about a downward adjustment in the sequence of expectations $E_t\{x_{t+k}\}$ for k=1,2,3,... As a result, and in response to a positive realization of the cost-push shock u_t, the central bank may achieve any given level of current inflation π_t with a smaller decline in the current output gap x_t.

Equilibrium nominal rate under the optimal policy with commitment where serially uncorrolated cost push shocks ($\rho_u = 0$).

$$i_t = r_t^e - (1-\delta)(1 - \frac{\sigma\kappa}{\alpha_x})\widehat{p}_t \tag{105}$$

4.2 The Case With Distortions

Uncorrelated real imperfections generate a permanent gap between the natural and the efficient levels of output, which is reflected in an inefficient steady state. The size of the steady state distortion by a parameter Φ representing the wedge between the marginal product of labor and the marginal rate of substitution between consumption and hours, both evaluated at the steady state.

The representative household's welfare losses in a neighborhood of the zero inflation steady state is expressed as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \alpha_x x_t^2 \right) - \Lambda \widehat{x}_t \right] \tag{106}$$

where $\Lambda \equiv \Phi\frac{\lambda}{\epsilon} > 0$ (i.e. $\Phi \equiv 1 - \frac{1}{\zeta} > 0$, where ζ is the steady state gross markup) and $\widehat{x}_t = x_t - x$ represents the deviation of the welfare-relevant output gap from its value x < 0 in the zero inflation steady state.

$\widehat{x}_t$ captures the fact that any marginal increase in output has a positive effect on welfare (thus increasing welfare losses), since output is assumed that to be below its efficient level.

The central bank will seek to minimize (106) subject to

$$\pi_t = \beta E_t\left\{\pi_{t+1}\right\} + \kappa\widehat{x}_t + u_t \tag{107}$$

where the disturbance $u_t \equiv \kappa(\widehat{y}_t^e - \widehat{y}_t^n)$

4.2.1 Optimal Discretionary Policy

The monetary authority is assumed to choose (x_t, π_t) in order to minimize the period losses

$$\frac{1}{2}(\pi_t^2 + \alpha_x\widehat{x}_t^2) - \Lambda\widehat{x}_t \tag{108}$$

subject to the constraint

$$\pi_t = \kappa\widehat{x}_t + \upsilon_t \tag{109}$$

where the term $\upsilon_t \equiv \beta E_t\left\{\pi_{t+1}\right\} + u_t$ is taken as given.

The optimality condition is given by

$$x_t = \frac{\Lambda}{\alpha_x} - \frac{\kappa}{\alpha_x}\pi_t \tag{110}$$

for t = 0, 1, 2,...

(108) implies for any given level of inflation, a more expansionary policy than in the absence of a steady state distortion. This is a consequence of the desire by the central bank to partly correct for the inefficiently low average level of activity.

We obtain an expression for equilibrium inflation under the optimal discretionary policy:

$$\pi_t = \frac{\Lambda\kappa}{\kappa^2 + \alpha_x(1-\beta)} + \alpha_x\Psi u_t \qquad (111)$$

and expression for the output gap

$$\widehat{x}_t = \frac{\Lambda(1-\beta)}{\kappa^2 + \alpha_x(1-\beta)} - \kappa\Psi u_t \qquad (112)$$

Thus, we see that the presence of a distorted steady state does not affect the response of the output gap and inflation to shocks under the optimal policy. It has, however, an effect on the average levels of inflation and the output gap around which the economy fluctuates. In particular, when the natural level of output and employment are inefficiently low ($\Lambda > 0$) the optimal discretionary policy leads to positive average inflation, as a consequence of the central bank's incentive to push output above its natural steady state level. That incentive increases with the degree of inefficiency of the natural steady state, which explains the fact that the average inflation is increasing in (and hence in), giving rise to the classical inflation bias phenomenon.

4.2.2 Optimal Policy under Commitment

As in the case of an efficient steady state the equilibrium price level for the optimal policy under commitment

$$\widehat{p}_t = \delta\widehat{p}_{t-1} + \frac{\delta}{(1-\delta\beta\rho_u)}u_t + \frac{\delta\kappa\Lambda}{1-\delta\beta} \qquad (113)$$

for t=0,1,2,... where $\delta \equiv \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \in (0,1)$, *and* $a \equiv \frac{\alpha_x}{\alpha_x(1+\beta)+\kappa^2}$.
and output gap is

$$\widehat{x}_t = \delta\widehat{x}_{t-1} - \frac{\kappa\delta}{\alpha_x(1-\delta\beta\rho_u)}u_t + \Lambda\left[1 - \delta(1 + \frac{\kappa^2}{\alpha_x(1-\delta\beta)})\right] \qquad (114)$$

Presence of a distorted steady state response to a cost-push shock didn't affect under the optimal policy with commitment as in the discretionary policy characterized by an identical stabilization bias. Within distortion an additional difference arises between the discretionary and commitment policies, unrelated to the response to shocks: it has to do with the deterministic component of inflation and its evolution over time. In the case of discretion that component takes the form a constant positive mean, resulting from the period-by-period incentive to close the gap between output and its efficient level, which results in inflation. In the case of commitment, however, we see that the price level converges asymptotically to a constant, given by $\lim_{T\to\infty} p_T = p_{-1} + \frac{\delta\kappa\Lambda}{(1-\delta\beta)(1-\delta)}$.Hence, after displaying a positive value at the beginning of the optimal plan's implementation, the deterministic component of inflation (around which actual inflation fluctuates in response to shocks) declines gradually over time, following the path $\frac{\delta^{t+1}\kappa\Lambda}{1-\delta\beta}$. Hence, under the optimal plan the economy eventually converges to an equilibrium characterized by zero average inflation, and in that sense observationally equivalent to that of an economy with an efficient steady steady state. The central bank's ability to commit avoids (at least asymptotically) the inflation bias that characterizes the outcome of the discretionary policy.

5 CONCLUSION

We have two results about this thesis that involve only interest rate as an instrument in a manner of monetary policy in a closed economy. The first result is about MIU model, the other result is about New Keynesian Model. As we mentioned above, under the Friedman Rule ($\mathrm{i}_t = 0$) the economy experiences a deflation in the long run. This may create indeterminacy in the price level and central bank avoid it by using the rule $i_t = \phi(r_{t-1} + \pi_t)$ for some ϕ >1. On the other side, the stationary solution is only satisfied when $\mathrm{i}_t = 0$. So $\pi_t = -r_{t-1}$. Friedman (1969) concludes that the optimal inflation rate must be negative to make nominal rate of interest zero. At the New Keynesian side, we decide which policy instrument equation gives better results, discretion or commitment. And what the coefficients of inflation and output implies in the different versions of Taylor rules. We decide whether the rule gives optimal results or not according to fit the U.S. data, (Walsh, 2003). In general, Taylor Rule tries to minimize the deviations from the optimal path by choosing sufficient values of ϕ_π and/or ϕ_x, inflation and/or output gap coefficients. The large values would potentially lead to huge instrument-instability: any small deviation of inflation or the output gap from zero would imply infinite changes in the rate.

Finally, when we compare optimal discretionary and commitment monetary policy to understand what we gain from commitment. While under the optimal discretionary policy, central bank keeps output below (above) its efficient level as long as inflation is positive (negative), under commitment the sign and size of the output gap is determined in proportion to the deviations of the price level from its implicit target.

Discretion and commitment are identical if the cost shock is serially uncorrelated ($\rho_u = 0$). If $0 < \rho_u < 1$, there is a stabilization bias under discretion relative to the case of committing to a simple rule. When $\rho_u > 0$, there is no average inflation bias. The increase in inflation and decline of output resulting from a unit cost-push shock is smaller under commitment than under discretion. Under discretion, the output gap returns to zero once the shock dies out. By way of contrast, under commitment the output gap remains negative well after the direct effects of the shock have vanished, and returns to its initial level only asymptotically. The welfare losses associated with the optimal policy with commitment (0.17 %) are smaller than in the discretion (0.22 %) case. We have a standard deviations of the output gap (%3.17) and yearly inflation 1.39 % with commitment, whereas 3.80 % of output gap and 1.38 % of inflation deviation under discretion (according to calibration of Gali, 2001).

The distortion caused an inflation bias under discretion. Kydland and Prescott (1977), Barro and Gordon (1983), and Rogoff (1985) indicates the central bank's inability to commit a low inflation policy when there is a persistence rising inflation. Besides Persson and Tabellini (1990) discuss in-depth this literature. Many of the most important papers are collected in Persson and Tabellini (1994a). Without such a bias an efficient outcome characterized by zero output and zero inflation can be attained under discretion. In other words, in the absence of an inflation bias there would not be any gains from commitment. In Clarida, Galí, and Gertler (1999) and Woodford (1999b), they get potential welfare gains response to shocks that generate a trade-off between output and inflation without an inflation bias.

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