# RISK IN FINANCIAL MARKETS AND EVALUATING THE RISK WITHIN THE CONTEXT OF INTEREST RATE LEVEL

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### İSTANBUL BİLGİ ÜNİVERSİTESİ SOSYAL BİLİMLER ENSTİTÜSÜ ULUSLARARASI FİNANS YÜKSEK LİSANS PROGRAMI

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### RISK IN FINANCIAL MARKETS AND EVALUATING THE RISK WITHIN THE CONTEXT OF INTEREST RATE LEVEL

### FİNANS PİYASALARINDA RİSK VE RİSKİ FAİZ ORANI ÇERÇEVESİNDE DEĞERLENDİRME

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#### ÖZET

Bu çalışma zaman serisi analizinde son yıllarda daha fazla kullanılmaya başlanan Uçtaki Değer Teorisi-Genelleştirilmiş Pareto Dağılımı yöntemini ile Türkiye'de İkinci El Bono Piyasasındaki beş ayrı vadedeki bonoların günlük faiz kapanış verilerini kullanarak faizin riskini ölçmeye çalışmıştır. Öncelikle, serilerin incelenen periyod -2001-2008 arası- boyunca iki kırılma noktası barındırdığı tespit edilmiş ve "birim kök" sorununu irdelemiş ve beş serinin "durağan" olmadığı hipotezini ortaya atmış ve alternatif hipotezini ilk hipotez lehine reddederek alışılagelen Normal Dağılım varsayımı yerine beş seri için Pareto Dağılımı kullanmanın uygun olduğuna karar vermiştir. Bu yöntemi kullanarak da faizler için dağılımın uçlarında VaR (Riskteki Değer) değerlerine ulaşabilmiştir.

#### ABSTRACT

This study tried to measure the risk of interest rate by using daily interest rate quotations of Goverment Bonds at different in Turkish Bounded Second Hand Bond Markets with Extreme Value Theorem-Generalised Pareto Distribution that has been started to use more. Primarily, five series are detected to comprise of two breakpoints throughout the period –between 2001-2008- and the "unit root" problem was probed and using Pareto Distribution was decided to be more suitable for five series against usual Normal Distribution by rejecting alternative hypothesis –"series are stationary"-against null hypothesis that five series are "non-stationary". VaR (Value at Risk) at tails of the distributions has been achieved by using this method.

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### LIST OF ABBREVIATIONS

ACF	Autocorrelation Function
Adf	Augmented Dickey-Fuller
EMBI	Emerging Markets Bond Index
Exp	Exponential
IMF	International Monetary Fund
ISE	İstanbul Stock Exchange
GARCH	Generalised Autoregressive Conditional Heteroskedasticity
GDP	Gross Domestic Product
GPD	Generalised Pareto Distribution
Но	Null Hypothesis
Std	Standard
Std err	Standard error
Max	Maximum
Min	Minimum
VaR	Value at Risk
Var	Variance
WTO	World Trade Organisation

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#### LITERATURE REVIEW

Risk management has been an academic issue since the mid of 1950s. At this time Harry Markwitz contributed to the issue with his article "Portfolio Selection" in 1952<sup>1</sup>. then several contributions had been followed. James Tobin with his "General Equilibrium Approach to Monetary Theory" article<sup>2</sup>. In this article Tobin argues on how the interest rate risk forms in financial markets.

In this thesis generally the recent articles and books are used. The most important one is [14] Neftçi, S and Bali, R (2001) Estimating the Term Structure of Volatility in Extreme Values, Journal of Fixed Income, March 2001. Several others are mentioned in the reference.

<sup>&</sup>lt;sup>1</sup> Harry Markowitz (1952) Portfolio Selection The Journal of Finance, Vol. 7, No. 1 (Mar., 1952), pp. 77-91

<sup>&</sup>lt;sup>2</sup> James Tobin (1969) A General Equilibrium Approach to Monetary Theory Journal of Money, Credit and Banking Feb, 1969

### 1 Introduction

Interest is defined as the rent paid for the usage of capital that was requested in the form of borrowing. Conversely, it is the amount of compensation for the lender in return for sacrificing the money disposition as the creditor. This compensation value should provide an incentive equal to an amount that backs the creditor down from using the money. Ratio of this amount is the interest rate.

Risk on the other hand expresses the chance of occurance of an undesired event or events and non-accrual of an intended and/or planned expectation. In an economic sense risk is the probability of a monetary loss regarded with a transaction or loss resulting due to decreasing financial returns. Cyclical fluctuations and price changes can increase the risk of occurrance of the undesired situations.

Risk is divided into two as systemic and systematic risks. All securities in financial markets are subject to systematic risks, and systematic risks arise for example when fluctuations within political and economic conditions affect the behavior of assets in financial markets. As a result systematic risks are unavoidable in the sense that keeping them under control in a way is impossible. Systemic risks on the other hand are the risk related with controllable processes such as intra-firm investment risks or a risk that may be likely to occur due to a decision on a financial issue (Turanlı, Özden and Demirhan; 2002).

Interest rate risk should therefore be considered within the context of systematic risks. The fluctuations in interest rates could not totally be controlled but some measures may be taken or some tools would be developed against the interest rate risk.

Our goal in this thesis is to find a measure for interest rate risk. There are many reasons, economical as well as financial ones, why we should find a measure for interest rate risk. Measuring interest rate risk is important since it may be beneficial in taking measures before negative effects can take place in an economy (see Woodford, 1999). From the perspective of finance interest rate should be considered not only with economy but with many other factors as well. Ang and Bekaert (2001) mentioned risk hidden in the behavior of interest rates has direct effect on the functioning of markets. Duffie and Kan (1996) and Dai and Singleton (2000) had shown in their papers that interest rates not only affect the functioning of markets but also have the power to alter the structure of the markets.

There are many other perspectives as well. For example financial income perspective says that the income going to be generated in the future is effected by interest rates because today's value calculation is made by an assumed interest rate level. If there is an unexpected change in the interest rates there is a risk that the value of income would be lower than expected. From an institutional perspective, changes in interest rates affect a financial institution's market value (Carneiro and Sherris, 2008). Because the value of a financial institution's assets and liabilities on the one hand and off-balance-sheet contracts written on interest rates on the other are affected by a change in rates, the present value of future cash flows and in some cases even the cash flows themselves can change. The focal point of the present thesis is an investigation of the interest rate risk in the Turkish spot market for government bonds. We will first look at what has happened in the Turkish economy within the period under investigation (2001–2008). After this we will look at the statistical properties of changes in the daily series of interest rates.

Finally, we will derive a measure of interest rate risk based on the Generalised Pareto Distribution. This approach is similar to Neftçi and Bali (2001), who argue that the return distributions cannot be assumed to be normally distributed, and extreme value theory should be used as a model for the tails of the distributions instead, an idea which leads to the Generalised Pareto Distribution. Extreme Value Theorem is comprehensively treated by Embrechts and Chavez-Demoulin (2004), and Gilli and Kellezi (2003). Meyfredi (2005) has used the estimation of risk measures associated with fat tails for stock market returns in several countries.

Gencay, Selçuk and Ulugülyağcı (2002) applied this to ISE and derived a practically useful VaR measure in order to be considered as an alert system for the market. Gencay and Selçuk (2001) had already applied a similar methodology for overnight interest rates of Turkish money markets in order to derive a measure querying whether the ex-ante interest overnight levels are indicators of the 2001 crisis or not.

Similar to Gencay and Selçuk (2001), Neftçi and Bali (2001) are using an extreme value approach involving the Generalised Pareto Distribution to compute a VaR for interest rates for the American market. In this study we are trying to estimate with which probability the interest rates from Istanbul Stock Exchange Secondary Bond Markets go to some value tomorrow, our goal being to define an interest rate risk and to derive a measure for spot market rates concerning 91, 182, 273, 365 and 456 day-to-maturity of bonds. Our approach is similar to Neftçi and Bali (2001).

Section 2 of the study talks about the recent history of Turkish economy, Section 3 defines the data and statistical properties; Section 4 looks at the time series properties of interest rates, and Section 5 reports results concerning GPD-based interest rate risk measurement. Section 6 concludes the thesis.

### 2 Recent History of Interest Rates in Turkey

We have analysed the period between 2001-2008 for interest rates of Istanbul Stock Exchange Second Hand Bond Market. For the purpose of our analysis, we shall devide this period into three sub-periods as follows:

- period 1, from January 2001 to September 2003
- period 2, from October 2003 to May 2006
- period 3, from June 2006 to August 2008

We believe that this division is justified by economic and political events affecting Turkey. Furthermore, we shall see in Section 4 below that a statistical breakpoint analysis leads to this division. (For a somewhat finer formulation of breakpoints, see Table 2.)

#### The Period of 2001 and 2008 in General

First of all, it is possible to separate this whole period into only two periods: the period until 2002; and the period from 2003 through 2008. Starting from the beginning of 2001 and ending with the end of 2002 there were three events that mainly shaped this period:

- the economic crises experienced on 28 February 2001
- September 11 2001
- Turkish General Elections in November 2002

The period was comprised of many instabilities in terms of both economy and politics throughout the period (Insel, 2003).

Between 2003 and 2008, 7% growth was seen in the economy on average. Per capita GDP had increased by 30%, domestic currency has revalued 30% as well. On the other hand a 100% set back was seen on Trade and Balance of Payments Deficit.

Inflation dropped to 12% from 40% and the interest rate level dropped to a figure of 21% from 76% of end of 2001 figure.<sup>1</sup>

#### The Period Between January 2001 and September 2003

As mentioned above the period was shaped with economic and political instabilities. The resolution that authorises the Turkish National Assembly for sending troops to Iraq was approved with 50% majority on 2003-10-06. According to the news expressed the day after this was perceived as a "political integrity" by the markets.<sup>2</sup>

It is beneficial also to mention that the inflation was explained to be the 30 years lowest before two days of voting.<sup>3</sup> Then, four days later the Treasury explained a  $\overline{\ ^{1}\text{All}}$  the figures here are taken from Banking Ragulation and Supervision Ageny (BDDK) Financial Markets Report, March-June 2006, Number 1-2. Available online at http://www.bddk.org.tr/english/Reports/Financial\_Markets\_Report/1971fprMart Haziran2006ingilizce.pdf - Accessed October 2008

<sup>2</sup>Hurriyet Online "Tezkere Geçti Asker Iraka Gidiyor, Kabul 358 Red 183", date: 2003-10-07. Available online at http://webarsiv.hurriyet.com.tr/2003/10/07/hurriyetim.asp, Accessed October 2008

<sup>3</sup>Hurriyet Online Enflasyona Eylül elmesi date: 2003-10-04 Available online at http://webarsiv.hurriyet.com.tr/2003/10/03/hurriyetim.asp, Accessed, October 2008

debt structuring in the sense of swapping the short term government bonds with longer maturities. Interest rates had dropped 200 basis points and Turkish Government is now able to borrow for longer term.<sup>4</sup>

#### The Period between October 2003 and May 2006

There were four main events shaping this period:

- WTO abolished trade barriers
- Capital flows rendered more liberalised
- Growth of developed economies had increased
- This growth brought inflation in developed countries.

It is possible to say that this period was the period of capital flows between diverse markets. Total volume of capital circulation throughout the world had reached approximately to \$15 trillion according to IMF Economic Outlook.<sup>5</sup>

Developing countries in this sense were also the beneficiaries. \$2 trillion out of this \$15 trillion had flown to them and Turkey was benefited from this with \$90 bn foreign investment according to Turkish Central Bank Inflation Report.<sup>6</sup>

<sup>&</sup>lt;sup>4</sup>Hurriyet Online "Para Kurulu Toplandı", date: 2003-10-15 Available online at http://webarsiv.hurriyet.com.tr/2003/10/15/hurriyetim.asp, Accessed October 2008

<sup>&</sup>lt;sup>5</sup>International Monetary Fund (IMF) World Economic Outlook October 2006, pp 1-6 Available

online at http://www.imf.org/external/pubind.htm , Accessed, October 2008 <sup>6</sup>Turkish Central Bank, Inflation Report 2006-IV pp. 41-46, Available online at http://www.tcmb.gov.tr/ , Accessed, October 2008

+EMBI Turkey Risk Index published by JP Morgan was explained on this date. This index as is believed gives the risk appetit of investors regarding the specific market. And according to this Index Report only the Turkeys Index figure was going compared with other developing countries.<sup>7</sup> Benchmark Bond interest rate at stanbul Stock Exchange Secondary Bond Markets was increased to 19% on this day and Central Bank followed suit by increasing gradually the overnight borrowing interest rate by 7% throughout month of June.

#### The Period between 2006-06-02 and 2008-08-29

There were four main events that shaped the period:<sup>8</sup>

- inflation fear of developed countries
- increase in interest rates
- sub-prime crises through the end of the year 2007
- Banking Crises throughout the world.

 $<sup>^7\</sup>mathrm{Ibid.}$  See graph on page 8.

<sup>&</sup>lt;sup>8</sup>International Monetary Fund (IMF) World Economic Outlook, October 2008,Financial Stress, Downturns and Recoveries pp1-46Available online athttp://www.imf.org/external/pubs/ft/weo/2008/02/pdf/text.pdf, Accessed, October 2008

### **3** Data and Their Statistical Properties

#### 3.1 The Data and Their Origin

We use daily closing quotations of interest rates of at ISE Bounded Bond Purchasing Market 90, 182, 273, 365 and 456 days to maturity government bonds. This data is available upon request from ISE. A plot of the series is shown in Figure 1. for the three periods under investigation. There are no corporate bonds in this market. The Turkish Bond Market is dominated by Treasury Bonds. As mentioned in the beginning, we are looking for a measure which is capable of showing the risk in this market.

The rates comprise the period between 2001 and 2008 and can be treated as time series. This type of data is critised as they are being lagged values and required to be collected retrospectively and they need to be processed before their message about the economy as a whole can be distilled. However as this data comprised of past values we believe it will reflect the effect of lagging situation in the analysis to be done below.

### 3.2 Statistical Properties of Daily Interest Rate Changes

Let  $(i_t)$  designate any of the five interest rate series (t indicates the day). In this section, we are interested in the behaviour of the changes in this series, that is, in the series

$$r_t = \frac{i_t - i_{t-1}}{i_{t-1}} \cdot 100\%.$$
(1)



Figure 1: The faiz series, three periods

faiz		faiz182	faiz273	faiz365	faiz456	
period 1: 2001-01-02 – 2003-10-06 (692 observations)						
mean	-0.06	-0.00	0.07	0.14	0.20	
var	21.04	30.37	47.56	66.06	83.40	
std deviation	4.59	5.51	6.90	8.13	9.13	
skewness	5.92	9.46	13.13	14.67	15.22	
std error	3.37	5.56	6.93	7.75	7.16	
kurtosis	102.35	184.96	283.45	322.97	334.72	
std error	33.87	75.08	111.22	128.63	132.83	
min	-38.20	-45.74	-48.25	-48.66	-48.13	
median	-0.10	-0.13	-0.16	-0.15	-0.14	
max	70.25	102.17	144.86	176.72	200.37	
day of min	2001-02-26	2001-02-26	2001-02-26	2001-02-26	2001-02-26	
day of max	2001-02-20	2001-02-23	2001-02-23	2001-02-23	2001-02-23	
period 2: 2003	3-10-07-2006	6-06-01 (666 c	observations)			
mean	-0.10	-0.10	-0.10	-0.09	-0.09	
var	0.73	0.91	1.29	1.76	2.24	
std deviation	0.85	0.96	1.14	1.33	1.50	
skewness	-1.62	-0.52	0.16	0.42	0.50	
std error	0.73	0.82	0.75	0.68	0.62	
kurtosis	13.76	11.54	11.01	10.71	10.09	
std error	4.38	2.43	2.10	2.31	2.24	
min	-7.09	-6.73	-7.05	-8.59	-9.70	
median	0.00	0.00	-0.07	-0.07	-0.12	
max	4.58	6.20	7.34	8.56	9.81	
day of min	2004-09-08	2004-12-20	2004-05-12	2004-05-12	2004-05-12	
day of max	2004-05-10	2004-05-10	2004-05-10	2004-09-20	2004-09-20	
period 3: 2006	5-06-02 - 2008	8-08-29 (569 c	observations)			
mean	0.05	0.04	0.04	0.04	0.03	
var	0.71	0.80	1.04	1.35	1.70	
std deviation	0.84	0.90	1.02	1.16	1.30	
skewness	1.10	2.41	3.00	3.00	2.81	
std error	0.84	1.23	1.39	1.38	1.19	
kurtosis	11.04	24.82	30.16	29.24	26.35	
std error	6.18	11.57	12.82	12.93	10.80	
min	-3.80	-4.55	-5.17	-5.84	-6.32	
median	0.00	0.00	0.00	0.00	-0.05	
max	7.24	9.48	11.16	12.49	13.55	
day of min	2006-07-04	2006-07-04	2006-07-04	2006-07-04	2006-07-04	
day of max	2006-06-26	2006-06-26	2006-06-26	2006-06-26	2006-06-26	

Table 1: Statistical properties of interest rate changes, three periods

Table 1 gives an analysis of the distributional properties of the percent point changes in the five series for the three periods in terms of mean, variance and standard deviation, skewness, kurtosis, minimum, median, and maximum.

There are obvious differences between the periods: The range of daily changes is widest for period 1; the variance and the kurtosis are largest for period 1. The behaviour of the five series within the periods gives insight into the characteristics of the different maturities, but also reveals further differences between the periods. In particular, some of the characteristics resulting from Table 1 are:

- The arithmetic mean of the daily changes in the faiz series increases from faiz091 through faiz456 in period 1, but not in the other two periods. An explanation may be that period 1 was regarded as risky by many investors in the sense that the Turkish financial market's risk premium is still high. As a consequence, investors demanded high long-maturity interest rates as a compensation for risks in future periods.
- The variance increases from faiz091 through faiz456 throughout all periods, in other words: The interest rate risk increases with maturity.
- A similar observation which is in line with our remarks about the variance can be made for the kurtosis, which becomes larger as maturity increases. This means: The tails of the distribution of interest rate fluctuations are becoming heavier the longer the maturity is. This points again to an elevated risk for higher maturities.

- The ratio between minimum and maximum percentage point change is increasing with maturity.
- The days when minima (maxima) occurred is always the same or very close in periods 1 and 3. This is not the case in period 2. This may have to do with the exceptionally low and stable volatility in period 2: There were no identifiable spikes occurring simultaneously in all five series.

Our goal in the present thesis is an evaluation of the interest rate risk. Therefore, the two most important items in the previous list are the variance and the kurtosis. The results of Table 1 point to a high risk in period 1 and lower (and similar) risks in periods 2 and 3. The kurtosis points to very heavy tails in period 1, relatively light tails in period 2, and moderately heavy tails in period 2.

### 4 Time Series Properties of Interest Rates

#### 4.1 Structural Changes in the Interest Rate Series

It was argued in Section 2 that, due to economic and political events in Turkey, it is justified to divide the time period 2001 through 2008 into three sub-periods. We shall now approach this question more formally and apply a statistical test for structural changes to the time series of daily interest rates. This will provide further arguments for a separate risk analysis in the three sub-periods. In addition, we will clearly see the limitations of regression models when applied to the interest rate series.

The method we use will find breakpoints in a regression relationship, with interest rates as dependent variable and time (i.e. day) as independent variable. This method is based on Bai and Perron [?]; its implementation is described in Zeileis et al. [?]. Breakpoints are computed with the objective of minimizing the residual sum of squares under the constraint that no segment should be shorter than 15% of total time period considered. (Our time series is 1930 days long.) The number of breakpoints is not predetermined, but results from the procedure.

The test for structural changes finds four breakpoints in the series faiz091, which we chose for this purpose to represent interest rate evolution. The results of the breakpoint analysis are displayed in Figure 2. In our subsequent analysis, we shall ignore the first breakpoint and form period 1 with 2003-10-06 as last day. This is justified because of the relative homogeneity of circumstances and events in this



Figure 2: Breakpoint analysis of faiz091

period. We are therefore led to a definition of sub-periods and their characterization as shown in Table 2.

### 4.2 Stylized Facts

Concerning the time series properties of the interest rate, we shall investigate stationarity properties in the interest rate series and look for autocorrelation in the series

	starts	ends	characteristics
period 1	2001-01-02	2003-10-06	high interest rates, decreasing rapidly af-
			ter peak; large daily fluctuations
period 2	2003-10-07	2006-06-01	more moderate interest rates, decreasing;
			small daily fluctuations
period 3	2006-06-02	2008-08-29	moderate interest rates at a relatively sta-
			ble level; moderate daily fluctuations

Table 2: Dividing 2001 – 2008 into sub-periods

	faiz091	faiz182	faiz273	faiz365	faiz456
period 1	0.016	0.010	0.010	0.010	0.015
period 2	0.933	0.924	0.842	0.732	0.653
period 3	0.346	0.518	0.572	0.577	0.561

Table 3: p-values of  $H_0$ : series is non-stationary (adf test)

of changes; the presence of volatility clusters is examined using the autocorrelation in squared changes. We analyze the five series for the three periods separately.

Results of an augmented Dickey-Fuller (adf) test for a unit root are reported in Table 3. This table gives the p-value of the null hypothesis  $H_0$ : "The series has a unit root" (that is: "The series is non-stationary") against the alternative  $H_1$ : "The series is stationary". The null hypothesis is rejected for each series only for period 1: There is evidence for stationarity in period 1.

Figure 5 shows 15 correlograms of the five change in interest rate series in three periods. It is remarkable that significant autocorrelation is found in each series in periods 1 and 3, but autocorrelation is almost absent in period 2. Also, for periods 1 and 3 the level of autocorrelation seems to increase with maturity. The sign of the observed autocorrelation is always alternating, as if an over-adjustment of the interest rate on one day had to be corrected on the next day.

Figure 6 shows evidence of volatility clustering for all five series, throughout all periods, with the notable exception of faiz091 during period 2. This may indicate a very smooth control policy of short-time maturity interest rates in that period.

### 5 GPD-Based Interest Rate Risk Measurement

#### 5.1 The Generalized Pareto Distribution (GPD)

The GPD is a model for excesses of a random variable. The rationale behind using the GPD is a limit theorem which states<sup>9</sup>: Let  $R_1, \ldots, R_n$  be iid random variables, and let R be distributed like  $R_i$ . Then, for large n and u, there are  $\xi$  and  $\sigma$  such that the distribution function of the excess

$$R-u$$
, conditional on  $R>u$ ,

is approximately given by

$$F(x;\xi,\sigma) = \begin{cases} 1 - \left(1 + \xi\frac{x}{\sigma}\right)^{-1/\xi} & \text{if } \xi \neq 0, \\ \\ 1 - \exp\left(-\frac{x}{\sigma}\right) & \text{if } \xi = 0. \end{cases}$$

Here,  $\sigma > 0$  is a scale parameter; it depends on the threshold and on the probability density function of  $R_i$ . The shape parameter  $\xi$  is called the tail index, since it characterizes the tail of the density function:

- The case  $\xi > 0$  corresponds to fat-tailed distributions; in this case, the GPD reduces to the Pareto distribution.
- The case  $\xi = 0$  corresponds to thin-tailed distributions; the GPD then reduces to the exponential distribution with mean  $\sigma$ .

<sup>&</sup>lt;sup>9</sup>For example, see Coles [?].

The case ξ < 0 corresponds to distributions with no tail (i.e. finite distributions). When ξ = -1, the GPD becomes a uniform distribution on the interval [0, σ].</li>

#### 5.2 Empirical Results

A typical example of fitting the GPD to the upper tail of one of our data series is shown in Figure 3. The histogram represents the upper tail of the empirical distribution of daily changes in the series faiz456 during period 2, where we used the 80% quantile as cutoff point. (This quantile was used as cutoff point throughout our study.) The red line is the density of the normal distribution with the same mean and variance as faiz456 in period 2, and the green line is the density of the GPD fitted to the data. It is obvious that the normal distribution overestimates the probability of moderate changes and underestimates the probability of large changes. This makes it inappropriate for risk analysis.

Similar plots for all the cases we consider in the present thesis are shown in Figure 4. The estimation results are reported in Table 4. In our context of risk measurement, the estimated tail index  $\hat{\xi}$  is more important than  $\hat{\sigma}$ . As stated above, a positive tail index indicates that the distribution of interest rate changes has a heavy upper tail. Table 4 shows that none of the fitted  $\hat{\xi}$ 's is negative. The distribution of changes in period 2 does have tails, but no heavy ones. According to the estimated  $\hat{\xi}$ , the behaviour of relative risk in periods 1 and 3 is similar, although the absolute risk has decreased very much in period 3.



Figure 3: Fitting the GPD to data

The estimated GPD can also be used to compute quantiles within the upper tail, to represent the VaR of an investment. Table 4 gives the 5% and the 1% VaR for each faiz series in each period. For example, the 5% VaR can be computed as follows. The 75% quantile of the GPD (corresponding to the 95% quantile of the entire distribution, since our cutoff point is the 80% quantile,  $q_{0.8}$ ) as  $q_{0.8} + x$ , where x (the excess) solves the relation

$$1 - \left(1 + \xi \frac{x}{\sigma}\right)^{-1/\xi} = 0.75 \quad \text{if } \xi \neq 0,$$
$$1 - \exp\left(-\frac{x}{\sigma}\right) = 0.75 \quad \text{if } \xi = 0.$$

Table 4 compares the VaRs thus obtained to the corresponding empirical quantiles of the observed series. They should have a similar magnitude if the estimation procedure is reliable, which is the case here. For purposes of risk assessment, we would rely on the VaRs computed on the basis of the GPD rather than on the empirical quantiles, because the latter can be quite shaky and erratic, as can be seen from the plots in Figure 4.

#### 5.3 Using the GPD: Conclusions

The normal distribution is not appropriate to measure the risk associated with interest rates in Turkey. The GPD, derived as an explicit model for distribution tails, fits very well and provides a close fit between the theoretical VaRs and empirical quantiles.

Our GPD analysis lends itself also to a further distinction between the three periods with respect to the tail behavior of the distribution of relative percent-point changes in the interest rate series. We found that periods 1 and 3 are surprisingly (according to the plots, see Figure 1) similar. We also found that the five series were not very different from each other with respect to their tail behavior within the periods, in spite of the big differences in their kurtosis.

### 6 Summary and Conclusions

The focus of this thesis is an assessment of the risk associated with interest rates in Turkey. We used data from Istanbul Stock Exchange (ISE) Second Hand Bond Market, Government Bond interest rate closing quotations, for the time period 2001 through 2008. A risk analysis is important in this context because of several aspects:

- risk as a measure of portfolio risk,
- risk as a measure of financial risk,
- risk as a measure of decisional risk.

There are several approaches to measuring the interst rate risk: using the yield curve; using GARCH models; or one based on the Generalised Pareto distribution (GPD). We undertook our risk assessment efforts based on the latter one, leading to a value at risk at the 5% and 1% levels. This is in line with research documented in scientific literature, for example, Neftçi and Bali (2001).

We found economic as well as statistical arguments for dividing the period under investigation into three sub-periods, period 1 reaching from January 2001 through September 2003, period 2 from October 2003 through May 2006, and period 3 beginning in Juen 2006 and ending in August 2008. Estimating GPDs to the data resulted in a good fit between the model and our data for all periods and maturities. Periods 1 and 3 turned out to be similar with respect to the kurtosis of the distribution of interest rate changes as well as with respect to the tail properties, analyzed on the basis of the GPD. Our results can be used for a detailed assessment of the interest rate risk in Turkey.

	$\hat{\xi}$	std.err. $\hat{\xi}$	$\hat{\sigma}$	std.err. $\hat{\sigma}$	$q_{.95}$	5% VaR	<i>q</i> .99	1% VaR
period 1	(2001-0	1-02 - 2003	8-10-06):					
faiz091	0.6229	0.1413	0.0121	0.0019	0.0352	0.0347	0.1189	0.1139
faiz182	0.5160	0.1283	0.0145	0.0021	0.0409	0.0405	0.0998	0.1147
faiz273	0.4898	0.1274	0.0169	0.0025	0.0521	0.0476	0.1029	0.1290
faiz365	0.4202	0.1173	0.0210	0.0029	0.0575	0.0554	0.1046	0.1420
faiz456	0.3996	0.1138	0.0237	0.0033	0.0614	0.0610	0.1153	0.1541
period 2	2 (2003-1	0-07 - 2006	6-06-01):					
faiz091	0.0000	0.0546	0.0046	0.0004	0.0103	0.0106	0.0199	0.0180
faiz182	0.0000	0.0437	0.0059	0.0006	0.0116	0.0124	0.0214	0.0219
faiz273	0.0000	0.0432	0.0075	0.0009	0.0148	0.0154	0.0277	0.0275
faiz365	0.0000	0.0439	0.0090	0.0010	0.0182	0.0187	0.0335	0.0331
faiz456	0.2579	0.1078	0.0075	0.0010	0.0210	0.0200	0.0373	0.0415
period 3	6 (2006-0	6-02 - 2008	8-08-29):					
faiz091	0.4867	0.0899	0.0094	0.0010	0.0273	0.0273	0.0682	0.0725
faiz182	0.4946	0.0957	0.0100	0.0011	0.0314	0.0300	0.0774	0.0788
faiz273	0.5173	0.0961	0.0105	0.0011	0.0338	0.0332	0.0767	0.0871
faiz365	0.5222	0.0947	0.0113	0.0012	0.0379	0.0366	0.0849	0.0952
faiz456	0.5070	0.0927	0.0124	0.0013	0.0410	0.0398	0.0957	0.1021

Table 4: Parameters of fitted GPDs



Figure 4: Fitting the GPD to change data



Figure 5: Correlograms of interest rate change series



Figure 6: Correlograms of squared interest rate change series

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