DEFAULT and PREPAYMENT PRICING in TURKEY

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DEFAULT and PREPAYMENT PRICING in TURKEY
Türkiye'de Temerrüt ve Erken Ödeme Fiyatlaması

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Binomial Tree	Binom Ağacı

ABSTRACT

The aim of this study is to provide an outlook on Mortgage Default and Prepayment Pricing and its application for Turkey. It reflects the 10 year possible behaviour of the mortgage debtor. Option pricing technique over binomial tree is used to determine how rational people behave under the condition of the repaying the mortgage loan and the price of default and prepayment as a result.

ÖZETÇE

Bu çalışmanın amacı; tut-sat (mortgage) temerrütü ve erken ödemesi ve Türkiye uygulaması hakkında bakış açısı sunmaktır. Çalışma, tut-sat borçlusunun 10 yıllık muhtemel davranışını yansıtmaktadır. Tut-sat borcunun geri ödenmesinde borçlunun nasıl davranacağını ve bunun sonucunda oluşacak temerrüt ve erken ödeme fiyalarını belirlemek için binom ağacı yöntemi üzerinden opsiyon fiyatlaması tekniği kullanılmıştır.

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1. INTRODUCTION

By its nature, mortgage loans are open to many risks because of long term agreement between parties. In our study, we will analyze how a debtor of a mortgage loan behaves under conditions of loan agreement and how and why default and prepayment behaviors occur.

In our study, we are going to price default and prepayment in mortgage loans. We applied option pricing technique using binomial trees in the excel application. Secondary information is used to completion of the study. The binomial option pricing model proceeds from the assumption that, the value of the underlying asset follows an evolution such that in each period it increases by a fixed proportion or decreases by another. We solved an optimization to infer ending node risk neutral probabilities from house prices; and finally, build the binomial tree using a recursive algorithm. We did not omit the early exercise in pricing default. While using data as secondary information, we modeled future interest rate like house price as well (see Appendix).

As being an emerging market, Turkish Government has accepted a law of the application mortgage and regulations for the mortgage market. On February 22, 2007, Turkey's Grand National Assembly passed legislation establishing a legal mortgage system that will give middle and lower income families the opportunity to become home owners. The law allows lenders to offer variable-rate mortgages that can then be turned into securities and be taken off bank's balance sheets.

Under the law, deposit banks, participation banks and leasing companies will be able to lease homes to customers. Lenders are allowed to borrow funds or create resources from institutions that operate on a wholesale basis, known as mortgage funding institutions.

The law also speeds up collection procedures in case the borrower goes bankrupt. But with interest rates on housing loans hovering around 1.49 percent in June 2008, demand for mortgages is now limited to upper income families.

Nevertheless, housing loans reached an estimated record $35 billion in 2008, or 12,3 % of all bank loans, nearly 3 times more than in 2006 that the Banks Association of Turkey reported.

In our analysis, we will assume that mortgage contract rate is fixed, although debtors have the opportunity of flexible rate ones. In addition to this assumption, LTV (loan to value) ratio is 0,8[1]

1.1 Historical Outlook to Mortgage Market in Turkey

The history of mortgage market in Turkey can broadly be divided into two periods: Before 2003 and after 2003. Even though there were many institutions and social initiatives who aimed to provide people with housing credits, the ratio of the total housing loans to GDP has never been higher than 1% of GNP before 2003. Either the LTVs were always less than 20%, or the terms were not higher than 3-4 years. This was mainly due to high interest rates and inflation. The average yield on one year government bonds was around 90%, and the average inflation was around 70% between 1992-2003. As a result, only 2.7% of all housing

[1] This is given for information. The LTV may less in Turkey and not applicable for our model.

finances in Turkey were supplied by financial institutions, or banks. Instead people preferred either to borrow from relatives, or form so-called cooperation which allowed them to own a house in 5-10 years. (Erdem, 2007)

The Association of Real Estate development Companies (GYODER), which is expecting 6 million new homes to be built in Turkey by 2016, predicts that the Turkey's annual housing loan market could double to $60 billion annually when the mortgage system is operating in full swing.

After several years of rapid price increases across the world, the housing markets in several advanced economies have been experiencing a marked turbulence. These problems began in 2006 and have accelerated considerably since summer 2008. Needless to say, problems in the housing sector have adverse implications for other sectors as well. In fact, according to research conducted by the International Monetary Fund, the spillovers from the housing sector to the rest of the economy are likely to be larger in economies like the United States, where it is easier to access mortgage credit and use homes as collateral. This is because movements in house prices, influence household spending plans through the role of housing as collateral. In the past, increases in house prices raised the value of the collateral available to households; loosened borrowing constraints, and supported spending.

Turhan *et.al* (2008) states that, Turkey has been facing a housing shortage since the 1950s when the industrialization process led to rapid urbanization and migration from rural areas to large cities. This has led to burgeoning of squatter buildings (known as "gecekondu" built in a night) and illegal constructions. Nearly a quarter of urban population still lives in such settlements. Uncontrolled building stock aggravates taking measures against disasters;

especially against flood, earthquake and fire. Since the existing infrastructure and social services fall short of meeting the demands, the problems of urban population related with economic and social life remain unsolved and they gradually increase. Therefore, the housing problem in Turkey is a qualitative problem as well as a quantitative one. Starting from 1980, the government took important steps towards the improvement of the housing sector, and as a result investment in housing increased substantially. The establishment of the Housing Development Administration of Turkey, Housing Fund and mass housing projects through social security institutions as well as Türkiye Emlak Bankası (Emlak Bank) were few examples of initiatives taken in the 80s. Throughout that decade and mid 90s, a remarkable growth in the housing sector was observed. However, the deficiency of housing loan system arising from the macroeconomic instability was a significant problem, preventing low and middle-income families to finds means to finance their housing demand. Eventually in parallel to macroeconomic problems in late 90s, the housing sector also experienced a severe recession. In the aftermath of the Russian Crisis in 1998, of the earthquake of 17 August 1999 and of the economic and political instability, investment in this sector decreased significantly. The economic crisis in Turkey in 2001 constituted the lowest point of the housing sector. Structural reforms implemented right after the crisis, which aimed to rehabilitate and strengthen the banking sector, banks began to compete with each other to increase their market shares in credits. Furthermore, economic activity increased following the recovery from the crisis, interest rates declined as inflation fell rapidly and the elimination of fiscal dominance thanks to sound fiscal discipline created room for private borrowing through crowding-in. The restoration of economic and political stability and the accession process of Turkey to the EU decreased risk perception of global investors and hence enabled Turkish banks to borrow from the international credit markets at low rates. As a consequence of all these factors we witnessed a considerable increase in the housing activity.

Turhan *et.al* (2008) asserted that housing loans ratio to the total household credit, which used to range between 14% to 21 % from the early 2003 till mid 2005, jumped to nearly 50% of household loans in mid-2006. However, May-June 2006 turmoil negatively affected the housing sector. Increasing interest rates curbed households' willingness to buy house and as a result year over-year increase in housing loans declined from over 300 % to 25%. The development of financial systems to support housing growth in emerging markets requires a reasonable degree of diversity and efficiency, which usually involves creating specialized financial intermediaries. The parliament has recently passed the Mortgage Law, which is likely to substantially improve the framework for mortgage loans. The law will allow mortgages to carry floating interest rates and prepayment penalties. It will also facilitate mortgage securitization, thereby allowing risks to be transferred out of the banks to other parties who are willing to take them, and increasing the funding for mortgages. The law also provides for the establishment of mortgage finance companies that can raise non-deposit funds and intermediate the securitization process. Securitization will nonetheless take time to develop. Only mortgages with good titles and standard contracts will be attractive for securitizations at reasonable interest rates. Registration of real estate is not fully efficient in Turkey and represents a significant roadblock in mortgage system. Thus, the development of mortgage securitization will depend on the speed with which better titles and standard contracts develop. There is a distinct tendency for Turkish households to favor short and medium term maturities to longer dates, which is mainly a result of frequent crisis episodes. Average maturity is less than 7 years. Nearly half of the households prefer the maturity of 5 to

10 years while the share of loans with maturities longer than 10 years is 19 %, which is quite low when compared to international figures. As financial and economic stability are being enhanced and price stability is achieved the maturity structure will change in favor of longer-term maturities. Looking at the future, there are huge challenges, as well as a vast potential. According to the data from the Turkish Statistical Institute (TURKSTAT) and the State Planning Organization (SPO), Turkey's housing requirement as of today is about 2,5 million either for renewal or conversion projects or quality house production projects. Due to population growth and continuing urbanization, Turkey will require an additional 5,5 million housing units by 2015. Added to the existing housing deficit, this represents a requirement for more than 500,000 new housing units to be built each year. Furthermore, with a growing economy and rapid urban expansion, there is a need for commercial/office/professional buildings. Likewise shopping malls and retail establishments need to be built as consumer spending is getting increased. Tourism development continues to generate new construction projects. Although housing market grew significantly over time and despite the fact that home ownership ratio in Turkey is 68 %, which is exactly the same what it is in the United States, housing credit’s ratio to GDP is still at low levels compared to developed economies. For instance housing loans to GDP ratio in the United Kingdom, United States and European Union are 85 %, 75 % and 47 % respectively, whereas the corresponding figure for Turkey is a mere 4,7 % (Table 1.1).

Table 1.1 Housing Loan to GDP ratio for Turkey

Years	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
Ratio	0,13%	0,06%	0,49%	0,25%	0,17%	0,24%	0,62%	2,54%	3,85%	4,25%	4,7% (*)
Source:	Central Bank of Turkey and National Institute of Statistics										
(*)	*Source:* Capital Market Board										

These numbers imply that there is a significant potential for the housing sector and mortgage loans to grow over time and increase its share in the economy. This fact was shown by a report prepared by Real Estate Investment Trust Association in 2006. It is projected that construction sector will grow 6 % to 10 % and its contribution to GDP will range between 0.5 and 0.7 points till 2015. Also it was estimated that over an 8-years period an additional value added of nearly 200 billion US dollars would be produced. It is no surprise to expect these estimates to realize when we take into the consideration the demographic structure of the Turkey and also continuing urbanization process. It is important to note that recent regulation concerning the mortgage market would contribute to the financial deepening of Turkish markets by allowing the issuance of asset backed securities and securitization of receivables from mortgages and those developments in turn will definitely improve the financing options of banks and leasing firms.

An important item of the agenda should be to provide affordable housing to every segment of the society. Under current economical conditions, poor people or low-income groups cannot acquire quality houses with favorable terms and conditions. According to research by the Central Bank of Turkey, only those which take the biggest share from the income distribution (the top 10 %) are able to use the mortgage credit, based on the assumption that households will spend 30 % of their annual income on housing credit payments. When the payment capacity of households related to housing credits for various interest rates and maturities is analyzed, it is found that the number of households who can use housing credits is only 1.8 million. If monthly interest rate (which is about 1.5 % today) diminishes to 0.5 % and the maturity (which is about 7 years today) extends to 30 years, this figure would increase to 9 million. Currently, the Government Housing Administration (TOKI) addresses this problem, by constructing and selling homes, mainly to families in the lower 40 % of the income distribution. TOKI contributes significantly to the supply of housing in Turkey, but there is risk that it may dominate the mortgage market and crowd out private lenders. Another challenge is how to increase the quality of existing dwellings. According to the "Housing Demand Research of Turkey" carried out by the Prime Ministry Housing Undersecretariat, illegal construction of buildings in Turkey has reached 40 %. In the urban areas, 62 % of the housing stock on the average are licensed and authorized. When permits to use buildings are taken instead of the licenses, this number falls to 33 %. In other words, 67 out of 100 buildings are illegal. There is a clear need to renew the existing stock of infrastructures and buildings.

1.2 Literature Review

The field of finance has investigated the pricing of options with multiple-state variables. Boyle *et.al* (1988) extends the one-state binomial model of Cox, Ross and Rubinstein *et.al* (1979) to one that involves two state variables and uses a trinomial (three-jump) approach rather than a binomial approach. Leung and Sirmans *et.al* (1990) and Ho, Stapleton and Subrahmanyam *et.al* (1993) apply the Boyle model to the pricing of fixed-rate mortgages.

The binomial model was first proposed by Cox, Ross and Rubinstein *et.al* (1979). Essentially, the model uses a "discrete-time" model of the varying price over time of the underlying financial instrument. Option valuation is then computed via application of the risk neutrality assumption over the life of the option, as the price of the underlying instrument evolves.

The model reduces possibilities of price changes, removes the possibility for arbitrage, assumes a perfectly efficient market, and shortens the duration of the option. Under these simplifications, it is able to provide a mathematical valuation of the option at each point in time specified.

Hilliard, Schwartz and Tucker *et.al* (1994) extended the procedures developed by Nelson and Ramaswamy *et.al* (1990) and Hull and White *et.al* (1990) to accommodate more generalized diffusions and two possible correlated state variables thus yielding a bivariate binomial options pricing technique. The advantage this technique offers is the ability to price American style options, thereby accommodating early exercise, despite the existence of two correlated underlying state variables. They illustrate the technique with an application to American futures options where the futures price and the short-rate of interest are stochastic.

Third other approach in building tree for two variables involves first assuming no correlation and then adjusting the probabilities at each node to reject the correlation. This last method is suggested by Hull and White *et.al* (1994). Hilliard *et al.*(1996), derive a bivariate binomial option pricing technique by extending the Nelson and Ramaswamy *et.al* (1990)'s technique, who demonstrate that nearly all diffusion processes can be approximated in the sense of convergence in distribution by a binomial process, to two correlated state variables. In their study, the procedure takes place in a three dimensional binomial lattice in which there are four possibilities stemming from each possible node at each time. Hilliard *et.al* (1998) use this technique to price the default and prepayment in a fixed-rate mortgage environment and use interest rates and real estate values as two state variables.

In their study of "A Lattice Approach to Pricing Fixed Rate Mortgages with default and Prepayment Options", Leung and Sirmans *et.al* (1989) provide an application of Boyle lattice model to price secured debt with two risky assets. They estimated the option values under a range of assumptions about the underlying parameters.

Cossin and Lu *et.al* (2004) used discrete model to price commercial and industrial loan prepayment option. They first propose that prepayment penalties should be viewed as fees in arrears reflecting the value of a call option at origination. They then rationalize business prepayment behavior with both refinancing and non-refinancing incentives. The model is developed in discrete time rather than in continuous time. Their preference is the American style of the option which requires the optimal stopping rules, time varying transaction costs, and more accurate description of multiple interest payments.

Sharp, Johnson, Newton and Duck *et.al* (2008) stated that, their new model simulates a delay in prepayment by the borrower (beyond the time simple ruthless prepayment dictates), thus increasing the value of the mortgage to the lender compared to the value gained using more basic models. Their new model of prepayment presented and analyzed in their paper provides a parsimonious structural means of modeling a borrower's termination behavior that appears 'irrational' according to the results of a basic optimal exercise model, i.e. results can be obtained outside the scope of simple rational models. By incorporating an occupation-time derivative into the valuation framework a more advanced borrower decision process is developed (compared with the previous simple ruthless approach to prepayment modeling). Sharp, Newton and Duck *et.al* (2007), in their paper of "Fixed rate Mortgage Valuation Methodology with Interacting Prepayment and Default Options", consider in detail realistic mortgage valuation model (including the potential for early prepayment and the risk of default), based on house-price and interest rate models. They presented that perturbation theory is a very efficient and effective tool in the solution of contingent claims mortgage valuation models.

2. BASIC CONCEPTS and ASSUMPTIONS

2.1 Put Option

A **put option** (sometimes simply called a "put") is a ***financial contract*** between two parties, the seller (writer) and the buyer of the ***option***. The put allows its buyer the *right but not the obligation* to sell a ***commodity*** or ***financial instrument*** (the ***underlying instrument***) to the writer (seller) of the option at a certain time for a certain price (the ***strike price***). The writer (seller) has the obligation to purchase the underlying asset at that strike price, if the buyer exercises the option.

Note that the writer of the option is agreeing to *buy* the underlying asset if the buyer exercises the option. In exchange for having this option, the buyer pays the writer (seller) a fee (the premium). (Note: Although option writers are frequently referred to as sellers, because they initially sell the option that they create, thus taking a short ***position*** in the option, they are not the only sellers. An option holder can also sell his long position in the option. However, the difference between the two sellers is that the option writer takes on the legal obligation to buy the underlying asset at the strike price, whereas the option holder is merely selling his long position, and is not contractually obligated by the sold option.)

If an option can be exercised only on the expiration date, it is called a ***European-type*** option. If it can be exercised any time up to and including the expiration date, it is called an ***American-type*** option.

If K is the strike price and S_T is the final price of the underlying asset, the payoff from a long position (holding) in a put option is;

$$\textbf{max}\ (K - S_T\ ;\ \textbf{0}) \qquad \textbf{(2.1.1)}$$

This means that the option will be exercised if $S_T < K$ and will not be exercised if S_T bigger or equals K

2.2 Call Option

A **call option** is a financial contract between two parties, the buyer and the seller of this type of ***option***. Often it is simply labeled a "call". The buyer of the option has the *right, but not the obligation* to buy an agreed quantity of a particular ***commodity*** or ***financial instrument*** (the ***underlying instrument***) from the seller of the option at a certain time (the expiration date) for a certain price (the ***strike price***). The seller (or "writer") is obligated to sell the commodity or financial instrument should the buyer so decide. The buyer pays a fee (called a premium) for this right.

The buyer of a call option wants the price of the underlying instrument to rise in the future; the seller either expects that it will not, or is willing to give up some of the upside (profit) from a price rise in return for the premium (paid immediately) and retaining the opportunity to make a gain up to the strike price

If K is the strike price and S_T is the final price of the underlying asset, the payoff from a long position (holding) in a put option is;

$$\textbf{max}\ (S_T - K\ ;\ \textbf{0}) \qquad \textbf{(2.2.2)}$$

2.3 The Default Risk

Default is the failure to fulfill the terms of a loan agreement. For example, a borrower is in default if he or she does not make scheduled interest payments on a loan or fails to pay off the loan at the agreed time. Judging the likelihood of default is a crucial part of pricing a loan. Interest rates are set so that, on average , a portfolio of loans will be profitable to the creditor , even if some individual loans are loss-making as a result of borrowers defaulting.

The mortgagor defaults when the sum of the value of his house H_t ; and transaction costs f_1, becomes less than her remaining mortgage balance B_{t-1} (e.g. for a sufficiently low value of the house, it is not worth to continue). If the mortgagor defaults at the beginning of any time t, this means that he exercises his default option, D_t and he rejects to pay his mortgage balance B_{t-1}. Since he loses the opportunity to prepay, the value of the prepayment option, P_t, becomes zero. In this case, the value of the options is;

$$D_t = \max (B_{t-1} - H_t ; 0); \quad \textbf{(2.2.3)}$$

$$P_t = 0 \quad \textbf{(2.2.4)}$$

Note that the default is a put option whose underlying asset (house) is H_t and exercise price is B_{t-1}. It gives the holder the right to "sell" the house to the lender for the current mortgage balance.

2.4 The Prepayment Risk

In the case of prepayment we follow the same view and assume that the mortgagor prepays when the present value of his mortgage balance PV_t, is higher than the sum of his unpaid mortgage balance B_{t-1}, the prepayment penalty $f_2\, B_{t-1}$ where $0 < f_2 < 1$. If the borrower prepays at the beginning of time t, this means that he exercises his prepayment option, P_t, and loses the opportunity to default. Then the value of the default option D_t becomes zero. In this case the value of the options is;

$$P_t = \max\,(PV_t - (1 + f_2)\,B_{t-1};\, 0); \qquad \textbf{(2.2.5)}$$

$$D_t = 0 \qquad \textbf{(2.2.6)}$$

Note that prepayment is a call option whose underlying asset is PV_t and exercise price is $(1 + f_2)\,B_{t-1}$: It gives the mortgagor the right to call the loan back at any future date for an exercise price determined now.

The mortgagor decides which option to exercise considering the higher payoff between the two. If neither default nor prepayment option is in-the-money (i.e. their values are zero), then mortgage holder continues holding the loan.

In our model, the debtors' behavior in the case of default is like the behavior of the put option holder. So that; in the pricing of default, we priced put option. In the case of the prepayment pricing, we priced call option by using binomial tree option pricing technique over excel application. In other words, it could be said that default is put option and prepayment is the call option.

The rest of article is organized as follows. In the second section, we introduce literature briefly. In the third section, we mention the methodology of the system. In the fourth part, we display our data and provide empirical findings. Finally, in the last section, we present our conclusion together with the summary of our results.

3. METHODOLOGY

Binomial trees are often used in the pricing of financial derivatives. The price of the asset underlying the derivative (for instance, the stock price in the case of a stock option) is assumed to follow an evolution such that, in each period in time, it increases by a fixed proportion or decreases by another fixed proportion. These fixed proportions are labeled above as the "up factor" and the "down factor". The tree traces out all possible price histories of the underlying asset.

The binomial option pricing model proceeds from the assumption that the value of the underlying asset follows an evolution such that in each period it increases by a fixed proportion (the up factor) or decreases by another (the down factor). Using a binomial tree one can project all possible values of the underlying asset at the option's expiration date, and from them, all possible final values for the option. To find the current value of the option, we need to work backwards through the tree starting with the known final option values.

The key is to recognize that it is always possible to create a portfolio made up of a position in the underlying asset combined with a position in the lending market that will have the same next period value as the option. The restricted assumptions about the movements in the value of the underlying asset imply that there is enough information to determine the portfolio weights and thus the value of the replicating portfolio. Under the assumption of no-arbitrage, the replicating portfolio must have the same value as the option.

The binomial model has proved over time to be the most flexible, intuitive and popular approach to option pricing. It is based on the simplification that over a single period (of possibly very short duration), the underlying asset can only move from its current price to two possible levels. Among other virtues, the model embodies the assumptions of no riskless arbitrage opportunities and perfect markets. Neither does it rely on investor risk aversion or rationality, nor does its use require estimation of the underlying asset expected return.

Tom Arnold, Timothy Falcon Crack and Adam Schwartz *et.al* (2006) implemented a Rubinstein-type (1994) implied binomial tree (IBT) using an Excel spreadsheet, but without using VBA (Visual Basic Application). They demonstrate both the optimization needed to generate implied ending risk-neutral probabilities from a set of actual option prices and the backwards recursion needed to solve for the entire implied tree. By using only standard Excel spreadsheet functions, and not resorting to VBA.

An IBT is a generalization of the Cox, Ross, and Rubinstein *et.al* binomial tree (CRR) for option pricing (CRR [1979]). IBT techniques, like the CRR technique, build a binomial tree to describe the evolution of the values of an underlying asset. An IBT differs from CRR because the probabilities attached to outcomes in the tree are inferred from a collection of actual option prices, rather than simply deduced from the behavior of the underlying asset. These option-implied risk-neutral probabilities (or alternatively, the closely related risk-neutral state-contingent claim prices) are then available to be used to price other options[2].

Jackwerth *et.al* (1999) reviews two inter-related strands of the literature: how to infer probability distributions from option prices, and how to build IBTs. The best known practical methods for implementing IBTs include Rubinstein *et.al* (1994), Derman and Kani *et.al* (1994), and Jackwerth *et.al* (1997). They compare and contrast these three in Table 3.1

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[2] Stephen Ross asserts that options should be spanned by state-contingent claims (Ross [1976]). One implication is that with sufficient structure, we should be able to infer state-contingent claim prices or a probability density from options prices (Rubinstein [1994, p779]).

Table 3.1

Properties of Competing Implied Binomial Tree Models	Derman/Kani 1994	Rubinstein 1994	Jackwerth 1997
IBT Constructed backwards from ending nodes?	No	Yes	Yes
Ability to use intermediate-maturity options in IBT constructions?	Yes	No	Yes
Ability to use other than Europeanstyle options in IBT construction?	No	No	Yes
Requires extrapolation and interpolation in IBT construction?	Yes	No	No
Assumes all paths leading to given node are equally likely?	No	Yes	No
Approximately lognormal distribution of ending nodal probability	No	No	No

Comparison of Three Approaches to Binomial Tree

Rubinstein *et.al* (1994) is conceptually easier to implement and more stable than Derman and Kani *et.al* (1994), while only slightly more mathematically restrictive than Jackwerth *et.al* (1997). Rubinstein's 1994 IBT is thus the ideal candidate for their exposition.

3.1 The Method of Application

We review these steps: build a traditional CRR tree to provide priors; solve an optimization to infer ending node risk neutral probabilities from house prices; and finally, build the binomial tree using a recursive algorithm. We implement the model in Excel using actual house prices.

We can generalize the argument above by considering a house whose price is S_0 and an option on the house whose current price is f. We suppose that the option lasts time T and that during the life of the option the house price can either move up from S_0 to a new level, $S_0 u$, or down from S_0 to a new level, $S_0 d$ (u>1; d<1).

The proportional increase in the house price when there is an up movement is u-1, the proportional decrease when there is a down movement is 1-d. If the house price moves up to $S_0 u$, we suppose that the payoff from the option is f_u; if the house price moves down to $S_0 d$, we suppose the payoff from the option is f_d .

If there is an up movement in the house price, the value of the house at the end of the life of the option is;

$$S_0 u - f_u \qquad \textbf{(3.1.1)}$$

If there is a down movement in the house price, the value becomes;

$$S_0 d - f_d \qquad \textbf{(3.1.2)}$$

The two equations are equal when;

$$S_0 u - f_u = S_0 d - f_d \qquad \textbf{(3.1.3)}$$

In this case, the portfolio is riskless and must earn the risk-free interest rate.

If we donate the risk-free interest rate by r , the present value of the house is

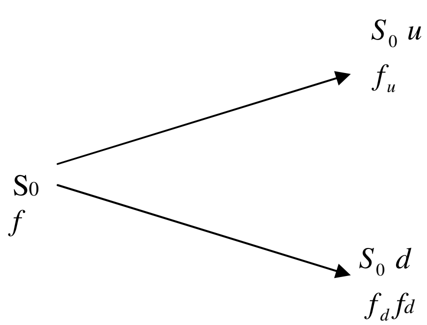
$$(S_0 u - f_u) e^{-rT} \qquad \textbf{(3.1.4)}$$

The cost of getting up the house is

$$S_0 - f \qquad \textbf{(3.1.5)}$$

It follows that;

$$S_0 - f = (S_0 u - f_u) e^{-rT} \qquad \textbf{(3.1.6)}$$



(House and option prices in a general one-step tree)

3.2 Risks-Neutral Valuation

It is natural that up and down movements has their own probabilities. From the equation just above;

$$f = S_0 - (S_0 u - f_u) e^{-rT} \quad \textbf{(3.2.1)}$$

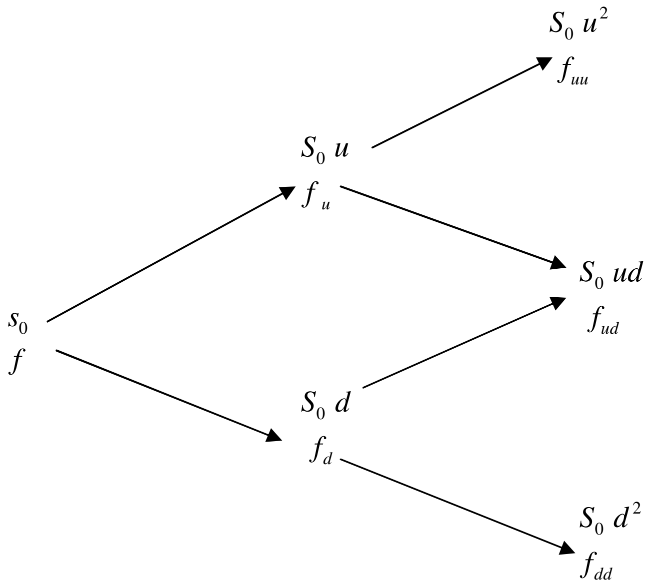
or

$$f = e^{-rT} (p f_u + (1 - p) f_d) \quad \textbf{(3.2.2)}$$

where;

$$p = (e^{rT} - d) / (u - d) \quad \textbf{(3.2.3)}$$

Finally one-step binomial tree is extended to the two-step binomial tree analysis. This analysis constitutes the building block of our analysis.



(House and option prices in a general two-step tree)

We design the tree to represent the behavior of the house price in a risk-neutral world. The parameters pu and d must give correct values for the mean and variance of the house price during a time interval δ_t. The expected return from a house is the risk-free rate r. Hence; expected value of the house price at the end of a time interval δ_t is $S\ e^{r\delta t}$, where S is the house price at the beginning of the time interval. It follows that;

$$S\ e^{r\delta t} = S_u\ p + (1-p)\, S_d \qquad \textbf{(3.2.4)}$$

or

$$e^{r\delta t} = pu + (1-\ p)\, d \qquad \textbf{(3.2.5)}$$

$$\Theta^2 \delta t = pu^2 + (1-p)\, d^2 - (pu + (1-p)\, d) \qquad \textbf{(3.2.6)}$$

A condition used by Cox, Ross and Rubinstein is

$$u = 1/d \qquad \textbf{(3.2.7)}$$

It can be shown that provided that δt is small

$$p = (a - d)/(u - d) \qquad \textbf{(3.2.8)}$$

$$u = e^{\Theta \sqrt{\delta t}} \qquad \textbf{(3.2.9)}$$

$$d = e^{-\Theta \sqrt{\delta t}} \qquad \textbf{(3.2.10)}$$

$$a = e^{r \delta t} \qquad \textbf{(3.2.11)}$$

4. EMPRICAL RESULTS

In this section, we illustrate the result of the analysis we have done. In the following subsection, we mention about the properties of the data. In the second subsection, we represent the results of the data.

4.1 The Data

We analyze monthly data of number of the flats and their total value in TL. The value and number of flats gives us the average value of a flat in months of the concerning month of the year. So that we see how the house prices followed in the past periods. However, that data may not reflect possible house price behavior in Turkey because there is not a house price index calculated by a statistics institute or etc. Standard deviation of the house price is calculated using that data. In addition to this, for future interest rate, we modeled interest rate using past interest rates starting from January, 2003 ending July, 2008 by binomial tree method. Standard deviation of interest rate is calculated by that data. (See Appendix). Risk free rate is taken as of end of June 2008. Calculations are made according to the equations in subsections of 3.1 and 3.2 but that estimation may not be eligible when the volatile economic condition of Turkey and its reflections considered.

In addition to this, because the put option holder has the opportunity of early exercise, we analyzed the early exercise and then added it into default pricing. By assumption, transaction cost is excluded pricing both default and prepayment.

The House Price Data starts from January, 2002 and ends at the end of March 2008. The past house price and interest rate are collected from Turkish Republic Central Bank (TCMB) (***www.tcmb.gov.tr***). So that the out-layer effects of the economic crisis are eliminated and the result becomes more confident.

4.2 Results

In the case of default pricing we indexed house prices according to the up and down factors. After each node house price goes up or down according with the parameters we calculated (see table 4.1). At the final node we maximized possible house price minus the remaining loan balance (see equation 2.3.1) and carry the value to the present with respect to up and down probabilities and reached default price. However; because the debtor has the opportunity of early exercise, we calculated equation (2.3.1) for each node. Then we combined both calculation and reached our default price which is 13 % of the loan. That means; in the case of a loan agreement made between debtor and creditor, 13 % of the loan should be paid by the debtor as an insurance of the loan in order her risk to be eliminated. The debtors are asked by the creditors for the cost of default by adding that cost into the interest and other payment (see conclusion).

In addition to this, for the case of prepayment, we indexed house price and applied the equation 2.5.1 to our prepayment pricing over binomial tree with the parameters we calculated and used (see table 4.1 and 4.2). In prepayment pricing, we needed future interest rates. We modeled future interest rates over binomial trees using past interest rates. Starting from the final node in which we maximized present value of the remaining loan balance minus mortgage balance which carries legal prepayment penalty for Turkey. Then the results at the final nodes were carried to the present value and we reached the price of prepayment which is 1% of the loan. In that case, the prepayment risk premium is the 1 % of the loan balance. To cover the risk of prepayment, the creditor should ask for the 1% of the loan balance from the debtor. This is existed in market conditions by adding that cost into the interest or any other balance (see conclusion) into the total payment made by debtor.

5. CONCLUSION

Long term housing loans are associated with many risks like default risk and prepayment risk since both parties have a long-term relationship which may last up to thirty or forty years. The ratio of the total housing credits is 4,7 % of GNP in 2008 (Table 1.1). This is very low rate compared to USA and EU countries. The rates were always less than 20%, or the terms were not higher than 3-4 years. This was mainly due to high interest rates and inflation. The average inflation was around 70% between 1992-2003 (Erdem, 2007).

In Turkey mortgage loan suppliers are the banks and when they supply loan, they ask for some additional costs to the debtor. Some of those costs are reflected to the debtor within the loan interest rate or fee paid by the debtor while acquiring the loan. Loan exercise fee varies from 0,5% to 2% depending on the risk perception of the bank. In addition to this fee, the bank ask for " File Fee (Dosya Parası) ", "Information Fee", "Application Fee", "Operation Fee", "Attorney Fee" or any other additional commissions. In actual fact, those fee and costs are the assurances for the bank who may encounter in the future. In addition to this, in global practice, there is mortgage insurance that covers the lender against some of the losses incurred as a result of a default on a home loan. However; in Turkey there is not such insurance, there is only life insurance for the debtor in the case of death, instead and there is prepayment penalty. The prepayment penalty, which is 2% for Turkey, is considered while pricing the prepayment but Erdem (2007) states that calculation shows that the rate is low for Turkey's condition

In that case, all those costs, fees, penalty and insurance can be defined as the premiums asked by banks in order to cover the default and prepayment risks.

To decrease the negative effect of the inflation that together with interest rate and the changes in house price, we used data after 2003 in order to make our findings more realistic when the today's position of the Turkey is thought.

However, as being an emerging market Turkey's economy is still open to many risks and in the future any volatility in interest rate and house prices will affect the behaviors of the debtors and creditors. Depending on our results, we can say that, in Turkey, because of high interest rate and volatility, prepayment and default risk may be higher accordingly. In addition to this, because of low prepayment penalty rate, it can be derived that, creditors will add risk premiums into loan by different ways states in result section. In that case, additional charges a on the loan increases and the maturity of the loan decreases accordingly and in order to decrease their risks creditors asks low LTV ratios.

Our study is aiming to provide an outlook for pricing default and prepayment in Turkey's conditions using past data. We can conclude that, because of high volatility and interest rate and not having a stable economy and an established housing sector, prepayment and default risk is higher for Turkey. In order to cover their risks, creditors asks higher interest rates or operation costs.

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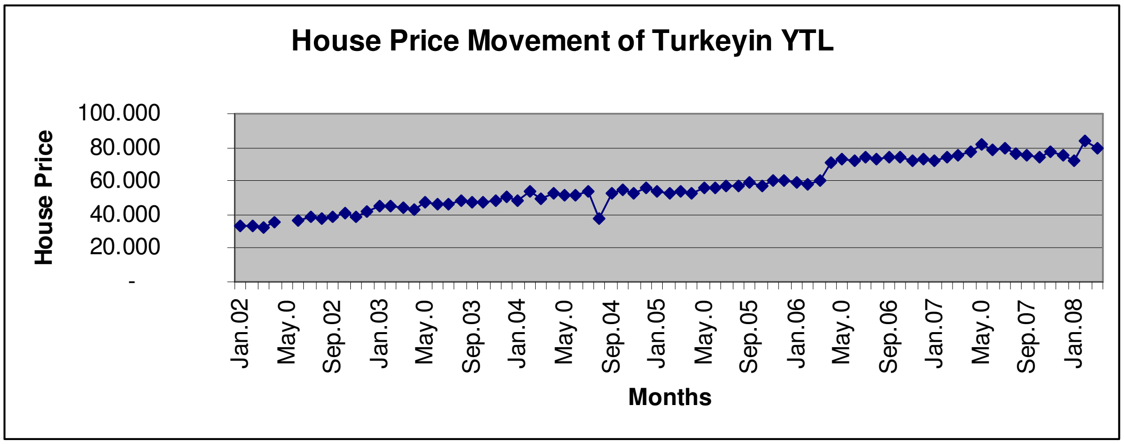
APPENDIX

Graph 4.1



Graph 4.2

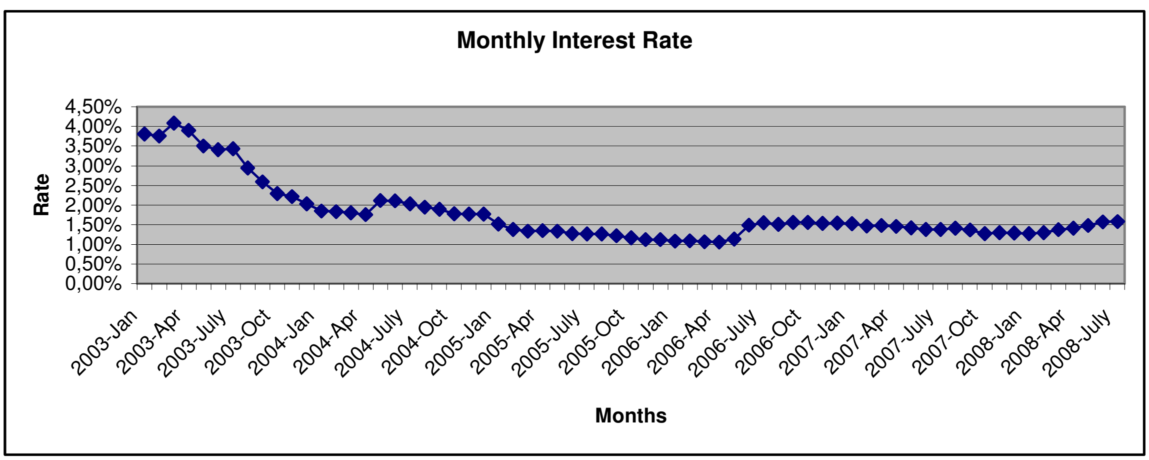


Table 4.1

H(0)	100.000
SD(per month)	0,070
risk free rate	0,015
t	1
u	1,020
d	0,980
e(rf*t)	1,001
p	0,526
q	0,474
delta time as taken 1	1
T	120

The Parameters Used for Default Pricing

Table 4.2

H(0)	100.000
SD(per month)	6,70%
risk free rate	0,01
t	1,0000
u	1,0693
d	0,9352
e(rf*t)	1,0150
p	0,5952
q	0,4048
delta time as taken 1	1
T	120

The Parameters Used for Interest Rate Modeling

Table 4.3

H(0)	100.000
SD(per month)	0,07
risk rate	0,0149
t	1
u	1,07251
d	0,93239
e(rf*t)	1,01501
p	0,58965
q	0,41035
delta time as taken 1	1,00000
Prepayment penalty	0,02
T	120

The Parameters Used for Prepayment Pricing