MINIMAL MASKIN MONOTONIC EXTENSION OF Q-APPROVAL FALLBACK BARGAINING WITHIN SOME FAMILY OF SOCIAL CHOICE RULES

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Minimal Maskin Monotonic Extension of q-Approval Fallback Bargaining within Some Family of Social Choice Rules

q-Onay Dönüş Pazarlığının Bazı Sosyal Seçim Kuralları Ailesindeki En Küçük Maskin Monoton Genişlemesi

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Anahtar Kelimeler (Türkçe)
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3) Maskin Monoton Genişlemeler

Anahtar Kelimeler (İngilizce)
1) Fallback Bargaining
2) Maskin Monotonicity
3) Maskin Monotonic Extensions

Abstract

Introducing the minimal Maskin monotonic extension of a social choice rule within some family of social choice rules, we define a family of social choice rules which certify to have a unique minimal Maskin monotonic extension of these social choice rules within this family. So we characterize the minimal Maskin monotonic extensions of q-Approval Fallback Bargaining (Brams and Kilgour, 2001) and a social choice rule called q-approval rule we introduce within the family.

Özet

Bir sosyal seçim kuralının bazı sosyal seçim kuralları ailesi içindeki en küçük Maskin monoton genişlemesi kavramını tanıtarak, herhangibir sosyal seçim kurallarının sadece bir tane en küçük Maskin monoton genişlemesini içeren sosyal seçim kuralları ailesini tanımlıyoruz. Böylece q-onay dönüş pazarlığının (Brams and Kilgour, 2001) ve bizim tanımladığımız q-onay kuralının, bazı sosyal seçim kuralları ailesindeki en küçük Maskin monoton genişlemesini karakterize ediyoruz.

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Contents

Chapter 1

Preface

A monotonicity condition introduced by Maskin (1999) is necessary for a social choice rule to be implementable via Nash equilibrium. However, Maskin monotonicity is a strong condition. Many social choice rules are not Maskin monotonic and therefore not Nash implementable. For example, when indifferences are ruled out, no scoring rule (Erdem and Sanver 2005). If indifferences are allowed then no Pareto optimal social choice rule (Sanver 2006) is Maskin monotonic. In particular, Muller and Satterthwaite (1977) show that Maskin monotonicity is equivalent to dictatoriality when the social choice rule is citizen sovereign and singleton-valued.

Sen (1995) proposes a method of evaluating the extent of non-monotonicity of social choice functions, by extending them minimally to social choice correspondences which are Maskin monotonic. A Maskin monotonic extension of a social choice rule F is defined to be a Maskin monotonic social choice correspondence which picks at every preference profile the alternatives that F picks. The trivial social choice correspondence which always includes all elements of the choice set in the outcome set is a Maskin monotonic extension of all social choice functions. It is known that the intersection of two Maskin monotonic extensions of a social choice function is also a Maskin monotonic extension of the social choice function. The intersection of all

Maskin monotonic extensions is therefore also a Maskin monotonic extension and is called the minimal Maskin monotonic extension of the social choice function. Thomson (1999) studies minimal Maskin monotonic extensions in economic environments by computing the minimal Maskin monotonic extensions of certain well-known allocation rules. Kara and Sönmez (1996) apply this concept to matching problems. Another application is by Erdem and Sanver (2005) who compute minimal Maskin monotonic extensions of scoring rules.

Two classes of social choice rules which fails Maskin monotonicity are q-Approval Fallback Bargaining (Brams and Kilgour 2001) and q- approval rule. For any fixed number of q, where q lies between 1 and total number of voters inclusive, q- approval rule picks the alternative(s) receiving the support of q people at the highest possible level. And q-Approval Fallback Bargaining picks the alternative(s) which gets the highest support among the alternatives which are chosen by q- approval rule. We concentrate on how to compute the minimal Maskin monotonic extension of q-Approval Fallback Bargaining. We propose a minimal Maskin monotonic extension of q-Approval Fallback Bargaining within a given family of social choice rules.

The thesis is organized as follows: Chapter 2 gives the the notations and definitions. Chapter 3 surveys some basic results on Maskin monotonicity and Maskin monotonic extensions. Chapter 4, provides a short survey on q-Approval Fallback Bargaining. Chapter 5, introduces the minimal Maskin monotonic extension of q-Approval Fallback Bargaining within some family of social choice rules. Chapter 6 concludes.

Chapter 2

Preliminaries

Given any two integers $m, n \geq 2$, we consider a set of individuals $N = \{1, 2, ..., n\}$ confronting a set of alternatives A with $\#A = m$. Writing Π for the set of complete, transitive and antisymmetric binary relations over A, we attribute a *preference* $P_i \in \Pi$ to each $i \in N$. We call $P = (P_i)_{i \in \mathbf{N}} \in \Pi^{\mathbf{N}}$ a *preference profile.*

Definition 2.1 *A social choice rule (SCR) $F : \Pi^{\mathbf{N}} \longrightarrow 2^A \setminus \{\varnothing\}$ is a correspondence from $\Pi^{\mathbf{N}}$ into A, that it selects a non-empty subset of A for each possible preference profile of the society.*

Definition 2.2 *The lower contour set $L(x, P_i)$ of $x \in A$ at $P_i \in \Pi$ is defined as $L(x, P_i) = \{y \in A : x \ P_i \ y\}$.*

Definition 2.3 *The upper contour set $U(x, P_i)$ of $x \in A$ at $P_i \in \Pi$ is defined as $U(x, P_i) = \{y \in A : y \ P_i \ x\}$.*

Definition 2.4 *Given any P, $P' \in \Pi^{\mathbf{N}}$, we say that P is an improvement for $x \in A$ with respect to P' iff $L(x, P_i') \subseteq L(x, P_i)$ $\forall i \in N$.*

When P is an improvement for x with respect to P', we equivalently say that P' is a worsening for x with respect to P. Let $w_x(P) = \{P' \in \Pi^{\mathbf{N}} : P'$ is a worsening for x with respect to $P\}$.

Chapter 3

Maskin Monotonicity and Maskin Monotonic Extensions

3.1 Maskin Monotonicity

Maskin monotonicity calls for the social choice rule to satisfy the following property: if the lower contour set of a socially optimal alternative does not shrink for any agent, then this alternative must remain being socially optimal. It is satisfied by the prominent social choice rules, which are the Pareto rule when indeferences are not allowed, the Condorcet rule, and the Walrasian rule. To be more precise, let us give a small argument which explains why the Pareto rule satisfies Maskin monotonicity when indeferences are not allowed. Let $x \in A$ be a Pareto optimal alternative with respect to preference profile P, hence chosen by the Pareto rule under P. This means for any other alternative $y \in A$, there exists an agent i^* such that, $x \ P_{i^*} \ y$. If we replace the preference profile P with P' such that for all $i \in N$, $xP_i \ y$ implies xP_i' y, then $x \ P_{i^*}' \ y$ holds, therefore x is Pareto optimal with respect to P' as well, and hence it is chosen under P' by the Pareto rule, establising the monotonicity of Pareto rule.

On the other hand, some well-known social choice rules do not satisfy monotonicity. It is shown that there exists social choice problems for which no scoring rule is Maskin monotonic (Erdem and Sanver 2005).

We now give the formal definition of the Maskin monotonocity.

Definition 3.1 *A SCR F is Maskin monotonic if and only if $x \in F(P') \Longrightarrow x \in F(P)$ for any P, $P^{'} \in \Re^{\mathbf{N}}$ and any $x \in A$ with $P' \in w_x(P)$.*

3.2 Maskin Monotonic Extensions

Definition 3.2 *Given any two SCRs F and G, we say that G is a Maskin monotonic extension of F if and only if $G(P) \supseteq F(P)$ $\forall P \in \Pi^{\mathbf{N}}$ while G is Maskin-monotonic.*

Let $ME(F)$ be the set of Maskin monotonic extensions of F. As we mentioned, the trivial social choice correspondence is a monotonic extension of all social choice functions, implying $ME(F) \neq \emptyset$.

Proposition 1 *Given any two social choice rules F and G, if F and G are Maskin monotonic then $F \cap G$ is Maskin monotonic.*

Proof. Take any $x \in A$ and any $P, P' \in \Pi^{\mathbf{N}}$ such that P is an improvement for $x \in A$ with respect to $P^{'}$. Let $x \in F \cap G\ (P\ ^{'})$, implies $x \in\ F\ (P\ ^{'})$ and $x \in G(P\ ^{'})$. As F and G are Maskin monotonic $x \in\ F\ (P\ )$ and $x \in G(P\ )$, so we have $x \in F \cap G\ (P\ )$, showing the Maskin monotonicity of $F \cap G$. ■

3.3 The Minimal Maskin Monotonic Extension

Definition 3.3 *The minimal Maskin monotonic extension $\mu(F)$ of a SCR F is defined by $\mu(F) = \cap\{G : G \in ME(F)\}$.*

Proposition 2 *Every social choice rule F admits a unique minimal Maskin monotonic extension $\mu(F)$.*

Proof. Suppose for a contradiction, there exist another minimal Maskin monotonic extension $\alpha(F)$, such that $\alpha(F) \neq \mu(F)$. By Proposition 1, we know that $\alpha(F) \cap \mu(F)$ is Maskin monotonic and is also an extension of F since $\alpha(F)$ and $\mu(F)$ are both Maskin monotonic extensions of F. Therefore $(\alpha(F) \cap \mu(F)) \subset \mu(F)$ contradics with the minimality of $\mu(F)$. ■

We now introduce a minimal Maskin monotonic extension of a social choice rule F within a given family of social choice rules.

Definition 3.4 *Let $\mathcal{F}$ be some family of SCRs such that $\exists G \in \mathcal{F}$ with $G(P) = A \; \forall P \in \Pi^{\mathbf{N}}$ and given any two social choice rules F, G; if F, $G \in \mathcal{F}$ then $F \cap G \in \mathcal{F}$ .Then $ME_{\mathcal{F}}(F) = \{H \in \mathcal{F} : H$ is a Maskin monotonic extension of $F\}$ is the set of Maskin monotonic extensions of F within the family $\mathcal{F}$.*

Remark 1 $ME_{\mathcal{F}}(F) \neq \emptyset$.

Definition 3.5 *The minimal Maskin monotonic extension of F within the family $\mathcal{F}$ is the SCR $\mu_{\mathcal{F}}(F) = \cap\{H : H \in ME_{\mathcal{F}}(F)\}$.*

Proposition 3 (Sen (1995)) *Every social choice rule F admits a unique minimal Maskin monotonic extension $\mu_{\mathcal{F}}(F)$.*

Proof. Suppose for a contradiction, there exist another minimal Maskin monotonic extension $\alpha_{\mathcal{F}}(F)$ within the family $\mathcal{F}$, such that $\alpha_{\mathcal{F}}(F) \neq \mu_{\mathcal{F}}(F)$. By construction of $\mathcal{F}$, we have $\alpha_{\mathcal{F}}(F) \cap \mu_{\mathcal{F}}(F) \in \mathcal{F}$ and is also Maskin monotonic extension of F within the family $\mathcal{F}$. Therefore $(\alpha_{\mathcal{F}}(F) \cap \mu_{\mathcal{F}}(F)) \subset \mu_{\mathcal{F}}(F)$ contradics with the minimality of $\mu_{\mathcal{F}}(F)$. ■

Chapter 4

q-Approval Fallback Bargaining

Fallback Bargaining, introduced by Brams and Kilgour (2001), is an approach to bargaining that produces a prediction about the bargaining outcome. People are seen as beginning by insisting on their most preferred alternatives, then falling back, in lockstep, to less preferred alternatives until there is an alternative with sufficient support (i.e. majority or supermajority support, or unanimity, as appropriate). The outcome of Fallback Bargaining is a subset of alternatives called the Compromise Set, which may be compared to the product of a social choice rule.

Fallback Bargaining has many variants. Brams and Kilgour show that Unanimity Fallback Bargaining leads to the alternative(s) receiving unanimous support at the highest possible level. In Unanimity Fallback Bargaining, the Compromise Set consists of exactly those alternatives that maximize the minimum satisfaction among all people. If a decision rule other than unanimity is adopted, the outcome of Fallback Bargaining may be different from the Unanimity Fallback Bargaining outcome. If preferences are strict, any Fallback Bargaining outcome is Pareto-optimal, but need not be unique; the Unanimity Fallback Bargaining outcome is at least middling in everybody's ranking. Fallback Bargaining does not necessarily select a Condorcet alternative, or even the first choice of a majority of bargainers. However, it maximizes the satisfaction of the most dissatisfied individual.

For any fixed number of q, where q lies between 1 and total number of voters inclusive, q- approval rule picks the alternative(s) receiving the support of q people at the highest possible level. And q-Approval Fallback Bargaining picks the alternative(s) which gets the highest support among the alternatives which are chosen by q- approval rule. Majoritarian Compromise, introduced by Sertel (1986), and Unanimity Fallback Bargaining are particular cases of q-Approval Fallback Bargaining when q is equal to majority and unanimity, respectively. Moreover, q-Approval Fallback Bargaining coincides with the plurality rule when q = 1.

Before formally define q-Approval Fallback Bargaining, we introduce q-approval rule which picks all the alternatives receiving the support of q people at the highest possible level.

For any positive integer $l \in \{1, ..., m\}$, we write $s_l(x; P) = \#\{i \in N : \#U(x, P_i) \leq l\}$ for the l-level support of $x \in A$ at $P \in \Pi^{\mathbf{N}}$, which is the number of voters who rank x among their l best alternatives at P. For any $P \in \Pi^{\mathbf{N}}$ and any $q \in \{1, ..., n\}$, let $l(q, P) \in \{1, ..., m\}$ be the smallest integer satisfying $s_{l(q,P)}(x; P) \geq q$ for some $x \in A$. So $s_l(x; P) < q$ for all $x \in A$ and for all $l < l(q, P)$.

Definition 4.1 *Picking some $q \in \{1, ..., n\}$, a SCR F_q is the q-approval rule if and only if at each $P \in \Pi^{\mathbf{N}}$, we have $F_q(P) = \{x \in A : s_{l(q,P)}(x; P) \geq q\}$.*

Definition 4.2 *Picking some* $q \in \{1, ..., n\}$*, a SCR* F_q^* *is q-Approval Fallback Bargaining if and only if at each* $P \in \Pi^{\mathbf{N}}$*, we have* $F_q^*(P) = \{x \in A : s_{l(q,P)}(x; P) \geq s_{l(q,P)}(y; P) \ \forall y \in A\}$.

Remark 2 $F_q^*(P) \subseteq F_q(P)$ *for all* $P \in \Pi^{\mathbf{N}}$.

We illustrate that q-Approval Fallback Bargaining fails Maskin monotonicity when q is equal to majority via an example:

Example 1 *Let* $\#N = 3$, $A = \{a, b, c\}$ *and take* $P, P' \in \Pi^{\mathbf{N}}$ *as follows:*

	P'				P		
1 voter:	a	b	c	*1 voter:*	a	c	b
1 voter:	c	a	b	*1 voter:*	c	a	b
1 voter:	b	c	a	*1 voter:*	b	c	a

We read the prefence orderings from left to right, i.e. in profile P', the first voter's best alternative is a, and second best is b, etc.

As we can see a is chosen by q-Approval Fallback Bargaining when q is equal to majority in profile P'. However, a is not selected in profile P while, a has not deteriorated in any voter's preference when passing from P' to P.

Remark 3 *Example 1 can be extented to show that q-Approval Fallback Bargaining fails Maskin monotonictiy for any* q.

Chapter 5

The Minimal Maskin Monotonic Extension of q-Approval Fallback Bargaining

First, we propose some propositions to construct the minimal Maskin monotonic extension of q-Approval Fallback Bargaining and the q-approval rule within the family that we will define.

For each $(q, l) \in \{1, 2, ..., n\} \times \{1, 2, ..., m\}$, define a mapping $F_{q,l} : \Pi^{\mathbf{N}} \longrightarrow 2^A$ where for each $P \in \Pi^{\mathbf{N}}$ we have $F_{q,l}(P) = \{x \in A : s_l(x; P) \geq q\}$. The non-emptiness of $F_{q,l}(P)$ is not ensured. Note that we have $F_{q,l}(P) \subseteq F_{q,l'}(P)$ whenever $l \leq l'$. On the other hand, as Brams and Kilgour (2001) show, for every $q \in \{1, ..., n\}$, there exists $l(q) = \left\lfloor \frac{mq-m+n}{n} \right\rfloor$ such that $l(q; P) \leq l(q)$ for all $P \in \Pi^{\mathbf{N}}$.

Theorem 1 (*Brams and Kilgour (2001)*) *:* $l(q) = \left\lfloor \frac{mq-m+n}{n} \right\rfloor$.

Proof. *The pigeonhole principle shows that some alternative must appear at least* $\left\lfloor \frac{nd}{m} \right\rfloor$ *times in the first* d *entries of all* n *rows of a preference profile . If* $d > \frac{m(q-1)}{n}$*, then* $\frac{nd}{m} > q - 1$*, which implies that* $d \geq l(q, P)$*. If* $\frac{m(q-1)}{n}$ *is integral, this proves that* $l(q, P) \leq \frac{m(q-1)}{n} + 1$*; if not, if proves that* $l(q, P) \leq \left\lceil \frac{m(q-1)}{n} \right\rceil$*. The conclusion now follows directly.* ■

Let $W = \{(q,l) \in \{1,2,...,n\} \times \{1,2,...,n\} : \ l \geq l(q)\}$.

Proposition 4 *$F_{q,l}(P) \neq \varnothing$ for all $P \in \Pi^{\mathbf{N}} \Leftrightarrow (q,l) \in W$.*

Proof. To show the "if" part, take any $(q,l) \in W$ and any $P \in \Pi^{\mathbf{N}}$. By definition of $l(q;P)$, $s_{l(q,P)}(x;P) \geq q$ for some $x \in A$. Hence $x \in F_{q,l(q,P)}(P)$. As $l(q;P) \leq l(q)$ and $l(q) \leq l$ by construction of W, we have $l(q;P) \leq l$, hence $x \in F_{q,l}(P)$, establishing $F_{q,l}(P) \neq \varnothing$. To show the "only if" part, let $(l,q) \notin W$. So $l < l(q)$. Pick $m = n$ and let, without loss of generality, $A = \{a_0, a_1, ..., a_{m-1}\}$. Consider $P \in \Pi^{\mathbf{N}}$ such that for every $i \in N$ we have $a_{k \operatorname{mod} m} \ P_i \ a_{(k \operatorname{mod} m)+1} \ \forall k \in \{i-1, ..., i+m-3\}$. By construction of P, we have $s_{l'}(x;P) = l' \ \forall l' \in \{1,...m\}$, $\forall x \in A$. Thus $l(q,P) = q$. As $m = n$, we have $l(q) = q$, implying $l(q,P) = l(q)$. As $l < l(q)$, $s_l(x;P) = q$ holds for no $x \in A$. ■

Now we ensure the non-emptiness of $F_{q,l}(P)$ by picking any (q,l) from the family W.

Now we will define two families;

Let $\Phi = \{F_{q,l}\}_{(q,l) \in W}$ and $\Phi^* = \{F_{q,l} \in \Phi : l = l(q)\}$.

Proposition 5 *Every $F_{q,l} \in \Phi$ is Maskin monotonic.*

Proof. Take any $F_{q,l} \in \Phi$, any $P, P' \in \Pi^{\mathbf{N}}$ and any $x \in F_{q,l}(P)$ with $P \in w_x(P')$. As $x \in F_{q,l}(P)$, we have $s_l(x;P) \geq q$. As $P \in w_x(P')$, we have $s_l(x;P') \geq \ s_l(x;P)$, implying $s_l(x;P') \geq q$, hence $x \in F_{q,l}(P')$, establishing the Maskin monotonicity of $F_{q,l}$. ■

Proposition 6 *Given any* $\overline{q} \in \{1, 2, ..., n\}$ *and any* $F_{q,l} \in \Phi$ *we have,*

(i) $F_{\overline{q}}(P) \subseteq F_{q,l}(P) \subseteq F_{\overline{q},l(\overline{q})}(P)\ \forall P \in \Pi^{\mathbf{N}} \Longrightarrow \overline{q} = q.$

(ii) $F^*_{\overline{q}}(P) \subseteq F_{q,l}(P) \subseteq F_{\overline{q},l(\overline{q})}(P)\ \forall P \in \Pi^{\mathbf{N}} \Longrightarrow \overline{q} = q.$

Proof. As $F^*_{\overline{q}}(P) \subseteq F_{\overline{q}}(P)\ \forall P \in \Pi^{\mathbf{N}}$, (ii) implies (i). To show (ii), let $\overline{q} = n$, and consider the case $q < \overline{q}$. Let $m = 2n$ and $n \geq 3$. Pick some $x \in A$, some $K \subseteq N$ with $\#K = n-1$ and construct a profile $P \in \Pi^{\mathbf{N}}$ such that $F^*_{\overline{q}}(P) \subseteq F_{q,l}(P)$, $x\ P_i\ z\ \forall z \in A, \forall i \in K$ and $z\ P_i x\ \forall z \in A, \forall i \in N \backslash K$. As $q < n-1$, we have $x \in F_{q,l}(P)$. As $m = 2n$, we have $l(\overline{q}) = m - 1$, implying $x \notin F_{\overline{q},l(\overline{q})}(P)$, giving a contradiction. Now let $\overline{q} \in \{1, 2, ..., n-1\}$. First consider the case where $q < \overline{q}$. Let $m = n \geq 3$. Pick some $x \in A$, some $K \subseteq N$ with $\#K = q$ and construct a profile $P \in \Pi^{\mathbf{N}}$ such that $F^*_{\overline{q}}(P) \subseteq F_{q,l}(P)$, $x\ P_i\ z\ \forall z \in A$, $\forall i \in K$ and $z\ P_i x\ \forall z \in A$, $\forall i \in N \backslash K$. Note that $x \in F_{q,l}(P)$. As $q < \overline{q}$, if $x \in F_{\overline{q},l(\overline{q})}(P)$ then $l(\overline{q}) = m$, which implies $\overline{q} = m$, giving a contradiction. Now consider the case where $q > \overline{q}$. Let $m = n + 1$. Let, without loss of generality $A = \{a_1, a_2, ..., a_{n+1}\}$. Pick some $a_n \in A$, construct a profile $P \in \Pi^{\mathbf{N}}$ such that $F^*_{\overline{q}}(P) \subseteq F_{q,l}(P)$, $s_{l(\overline{q})+1}(a_n; P) = n$ and put all alternatives different from a_n in a cycling way: each $a_k \in A \backslash \{a_n\}$appears exactly once in each line. As $m = n+1$, we have $l(q) = q$ for any q and $l(q) > l(\overline{q})$, implying $l \geq l(q) > l(\overline{q})$. So $a_n \in F_{q,l}(P)$ while $a_n \notin F_{\overline{q},l(\overline{q})}(P)$, giving a contradiction.

■

Proposition 5 conjoined with Proposition 6 implies for any $\overline{q} \in \{1, 2, ..., n\}$

that the minimal monotonic extension of both $F_{\overline{q}}$ and $F^*_{\overline{q}}$ within the family Φ coincides with $F_{\overline{q},l(\overline{q})}$.

We state this formally below.

Theorem 2 $\mu_{\Phi}(F_{\overline{q}}) = \mu_{\Phi}(F^*_{\overline{q}}) = F_{\overline{q},l(\overline{q})}$ *for every* $\overline{q} \in \{1, 2, ..., n\}$*.*

As $\Phi^* \subseteq \Phi$, Theorem 1 immediately implies the following corrollary.

Corollary 1 $\mu_{\Phi^*}(F_{\overline{q}}) = \mu_{\Phi^*}(F^*_{\overline{q}}) = F_{\overline{q},l(\overline{q})}$ *for every* $\overline{q} \in \{1, 2, ..., n\}$*.*

Chapter 6

Conclusion

We have calculated the minimal Maskin Monotonic extension of q-Approval Fallback Bargaining within the family Φ, family of compromise sets. If we consider the minimal Maskin Monotonic extension of q-Approval Fallback Bargaining within all social choice rules, then it fails to be a $F_{q,l}$ rule for some $q \in \{1, ..., n\}$ and $l \in \{1, 2, ..., m\}$ for all $P \in \Pi^{\mathbf{N}}$.

We illustrate this below :

Let $\#N = 6$, $A = \{a, b, c, d, e, f\}$ and take $P \in \Pi^{\mathbf{N}}$ as follows:

	P					
1 voter:	a	b	c	d	e	f
1 voter:	b	f	d	a	c	e
1 voter:	c	e	b	a	d	f
1 voter:	d	f	b	e	c	a
1 voter:	e	f	d	c	b	a
1 voter:	f	c	d	b	a	e

As Erdem and Sanver (2005) propose, for any $x \in A$ and $P \in \Pi^{\mathbf{N}}$, we have $x \in \mu_{\Phi}(F^{*}_{\overline{q}}(P))$ when Φ is the family of all social choice rules if and only if there exists some $P' \in w_x(P)$ such that $x \in F^{*}_{\overline{q}}(P')$.(see proposition 3.1

in Erdem and Sanver (2005)). So $\mu_{\Phi}(F_{\overline{q}}^{*}(P)) = \{b, c, f\}$.

Let $q = 1$ there exists no l such that $F_{q,l} = \{b, c, f\}$. That holds for any q where $1 < q \leq n$. So There exists no (q, l) pairs that $\mu_{\Phi}(F_{\overline{q}}^{*}(P)) = F_{q,l}$.

Since Φ can be interpreted as the family of compromise rules, it thus appears that the minimal Maskin Monotonic extension of a specific compromise rule, namely the q-Approval Fallback Bargaining , fails to be itself a compromise rule. Hence, there might exist a trade off between extending a compromise in order to ensure Maskin Monotonicity on the one hand, and preserving the spirit of this compromise on the other hand.

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