A CHAOTIC APPROACH IN ANALYZING
THE ISE-100 INDEX

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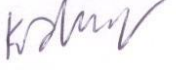INSTITUTE OF SOCIAL SCIENCES

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3) Wolf'un Algoritması	3)Wolf's Algorithm
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5) Lyapunov Katsayısı	5) Lyapunov Exponent

Acknowledgements

Abstract

This study aims to discuss the validity of the Efficient-Market Hypothesis by investigating the time evolution characteristics of ISE-100 index. Two major analysis methods are used and compared. First, the nonlinear dynamical behavior of the daily-return values of ISE-100 index (01.1990-10.2009) is explored by implementing Chaos Theory and applying recent chaotic analysis methods. Second, the same data set is used for linear time series modelling and by applying unit root tests we explore the stochastic behavior of the series. Finally, the comparison of the two major analysis is introduced. Our results uncover the chaotic behavior of the ISE-100 index and thus give more room to the predictivity analysis of daily-return values.

Keywords: *Efficient-Market Hypothesis, Nonlinear Dynamical Analysis, Wolf's Algorithm, Kantz's Algorithm, Lyapunov Exponent, Chaotic Behavior, ARIMA, Nonstationarity Analysis.*

Özet

Bu çalışma, IMKB-100 endeksinin zaman içindeki değişiminin efficient market hipotezindeki geçerliliğinin tartışmasını hedefler. İki analiz metodu uygulanmış ve karşılaştırılmıştır. İlk olarak, IMKB-100 endeksinin kapanış değerleri getirilerinin (01.1990-10.2009) doğrusal olmayan dinamik davranışını kaos teorisi uygulayarak araştırıp, halen kullanılmakta olan kaotik analiz metotları kullanarak testler yapılması oluşturmuş, ikincil olarak, aynı data setine unit root testi uygulayarak, stokastik davranışını araştırmak için doğrusal zaman serisi analiz metodu uygulanmıştır.

Son olarak iki analizin kıyaslaması yapılmış, sonuçlarımızın IMKB-100 endeksinin kaotik davranışını ortaya çıkarması ile, günlük kapanış değerleri getirilerinin tahmin edilebilirliğine daha fazla olanak sağladığı görülmüştür.

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1.   Introduction

Until relatively recently, it was more or less taken for granted that movements in stock market prices were over-whelming stochastic in nature, if not actually a random walk. The assertion seemed unchallengeable on theoretical grounds- namely, consistency with the ruling efficient-market paradigm or efficient-market hypothesis (EMP). The EMP basically says that current prices fully reflect all known information. This implies that there is little or no correlation between returns; price changes occur in a random fashion, in reaction to new information and price movements do not follow any patterns or trends. That is, past price movements cannot be used to predict the future price movements but follow what is known as a random walk, an intrinsically unpredictable pattern (Campbell et al., 1997; Fama, 1965). This assumption-which has never been conclusively proven-is the bed-rock upon which standard statistical analysis of the markets has been built. The *law of large numbers*, for example, applies only if price changes are independent (i.e., "efficient"). And it is the law of large numbers that validates statistical calculus and other linear models. Under the EMP, stock return process should be random. Therefore, it seems improbable a priori that the pattern of returns could be explained to any substantial degree by a deterministic process, given that the major cause of market movements is normally assumed to be the random flow of information. The mathematical expression of EMP is that the financial time series is characterized by a linear model and is independent and identically distributed (i.i.d.), so behave in a random manner. According to these implications, the importance of this research theme comes from the fact that chaos and EMP . As regards to the debate on the behavior of stock-index data and EMP, in this study we address two important questions that have been the focus of a substantial and still growing literature in recent years and that have been analyzed for many other stock-indexes of developed countries. Is there nonlinear dependence in ISE-100 stock-market returns (or closing values)? And, if so, is the nonlinear structure characterized by low-dimensional chaos? In other

words, is the apparent randomness of the time series pattern of returns explicable, in part at least, by a deterministic process?

2. Review of The Literature

Up to 90's, many economic studies examined U.S, U.K and Canadian markets (Kendall, 1953; Brealey, 1970; Dryden, 1970; Cunningham, 1973; Brock, 1987) and support the EMP. The studies on the United Kingdom stock market report the weak form market efficiency as conjectured by the pioneering study Fama (1965) – namely, that the linear modeling techniques have limitations as they are not sophisticated enough to capture complicated "patterns" which chartists claim to see in stock prices. In fact, linear models will prove successful only to the extent that the system being analyzed is itself linear.

If the system is nonlinear, the models will work, at best, only under "ideal" conditions and over short time periods. Thus the application of linear models to the market may be questionable, in view of recent research suggesting that the capital markets, and the economy as a whole, may be governed in part by non-linear dynamics. The main reason is briefly: when nonlinear dynamics are involved, a deterministic system can generate "random-looking" results that nevertheless exhibit persistent trends, cycles (both periodic and non-periodic) and long-term correlations.

At last in 90's, several developments that have led to serious questioning of the proposition that stock returns are inherently unpredictable have taken place. First, researchers using conventional econometric methods have uncovered several deviations from efficiency in the behavior of stock prices (Fama 1991). Second, the correlated studies in physics (Grassberger and Procaccia 1983; Wolf et al., 1985; Nychka et al., 1992; Gencay and Dechert 1992; Rosenstein et al.,1993; Kantz 1994), thermo dynamical engineering (Takens 1981; Ott E 1993), biomedical engineering (Barlow 1985; Schuster 1989; Basar 1990; Jansen 1991), and hydrodynamics (Eckamnn 1983; Eckamnn et al., 1987) have resulted in the discovery of

"random-looking or apparently random" behaviors running the dynamical systems. Moreover, the studies in physics mentioned above also involved in the development of several tests capable of detecting nonlinear as well as linear patterns in the observed data. Third, the exciting progress in the last 30 years in understanding the nonlinear dynamical systems means that we can now entertain the possibility of certain types of deterministic process in financial data (Peters 1991; Lorenz, 1993; Pesaran and Potter, 1993; Creedy and Martin, 1994; Trippe, 1995; Abhyankar et al., 1997; Hinich and Rothman, 1998; Mantegna and Stanley 2000; McKenzie, 2001; Barnett and William, 2004; Muckley, 2004; Das and Das, 2007; Hagtvedt 2009). In particular, it has become clear that many low-dimensional deterministic nonlinear systems are capable of generating output that is in most respects indistinguishable from white noise.

It is important to note that, as far as financial series are concerned, this type of process could be consistent with market efficiency if it is only forecastable at horizons too short to allow any trade-off by speculators. The unresolved issue addressed in Scheinkman and LeBaron (1989), Peters (1991), Abhyankar et al., (1997), Mandelbrot (1999) relates, therefore, to whether stock-market index returns[1] are best represented by a purely stochastic process or rather by a nonlinear deterministic structure. As can be seen from Table 1[2], there is already a substantial literature examining the questions addressed in these studies. Most of the financial research so far has concentrated on stock-market indexes, though a few have looked elsewhere (future markets, gold, and silver prices, energy prices, monetary movements, interest rates).

All of these studies address the nonlinear deterministic structure[3] (static and dynamic) of a dynamical system which is assumed to generate

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[1] Following the findings of Fama and French (1992), Hagtvedt (2009) use the dynamic characteristics of individual stocks rather than stock-index to examine anomalies in Sharpe's Capital Asset Pricing Model (CAPM) (Sharpe 1964)

[2] To conserve space, we state the studies which are mainly concerned with stock-indexes. The related studies on other types of financial data are cited in the text.

[3] The wider formal description is given at Section 4.1.

the observed time series data and which is mainly detected by applying various tests (Ott E, 1993; Kantz and Schreiber, 1997). These tests are Brock-Dechert-Scheinkman (BDS) test for independence, estimation of the correlation dimension (CD) (Grassberger and Procaccia 1983a) for quantitative description of complexity in terms of the number of degrees of freedom and Kolmogorov entropy (Grassberger and Procaccia 1983b) estimate the degree of order-disorder and thus of the complexity of a dynamic system. Finally, if nonlinear structure is observed[4], then Lyapunov Exponent (LE) should be obtained by one of the "appropriate" algorithms (Wolf et al., 1985; Nychka et al., 1992; Gencay and Dechert 1992; Rosenstein 1993; Kantz 1994).

As it can be seen from Table 1, the most commonly deployed test is Brock-Dechert-Scheinkman (BDS) test for independence, though several authors have relied on estimates of the correlation dimension (CD) and Kolmogorov Entropy estimation. While some of the authors who detect nonlinearity in observed time series data estimate LE's, some others find it sufficient to show the nonlinearity and do not estimate LE's.

Table 1

Related Researches

Authors	Dataset	Sample Info	Tests	Results
Abhyankar et al., (1997)	The S&P 500, the DAX, the Nikkei 225, and the FTSE-100	S=10000	1) BDS 2) Nychka et al.,LE test	1) Nonlinear 2) No evidence of Chaos

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[4] According to Gencay and Dechert (1992), if one of the Lyapunov Exponents is positive, then the system is nonlinear. Therefore, we do not in fact apply nonliearty test before examining chaotic behavior.

Abhyankar et al., (1995)	FTSE-100 cash	S=60000	1) Bispectral linearty test 2) BDS 3) LE	1) Nonlinear 2) No evidence of Chaos
Eldridge and Coleman (1993)	FTSE-100 cash and futures	S=1000 6.1984-9.1987	1) Correlation Dimension test 2) Wolf's LE	Not i.i.d and consistent with chaos
Hsieh (1993)	Foreign Currency spots and futures	S=1275, daily 2.1985-3.1990	Tests of linear and nonlinear predictabilities	No linear and nonlinear predictables
Philippatos et al.,(1993)	Ten major national stock indexes	S=833, weekly levels and returns 1.1976-12.1991	BDS test	Nonlinear
Van Quang T (2006)	Czech stock index PX50	S=2270, daily returns 1.1997-9.2005	1) BDS test 2) LE	1) Nonlinear 2) Evidence of chaos
Vaidyanathan and Krehbiel (1992)	S&P500 future mispricing	S=1500	1) BDS test 2)Correlation Dimension test	Nonlinear and low-dimensional chaos
Vassilicos et al., (1992)	NYSE	S=30000	1) Wolf's LE 2)Correlation Dimension test	No evidence of chaos

Brock et al.,(1991)	(1)CRSP (2) S&P 500	S=2510 daily 1.1974-12.1983	1)BDS 2)Dimension plots	Nonlinearty ; little evidence of nonlinear forecastability
Kodres and Papeil (1991)	British P, Canadian D, DM, JYEN, Swiss Franc	S=3500 daily futures 7.1973-3.1987	BDS test	Nonlinear
Frank and Stengos (1990)	Returns of (1)Gold (2) Silver prices	(1)S=2900 (2) S=3100	1) Corr. Dim. Test 2)Kolmogorov Entropy	1)dimension of 6 2)positive;low-dimensional chaos
Mayfield and Mizrach (1989)	S&P 500	S=20000, 20 seconds, 1987-1989	Correlation dimension tests	Low-dimensional chaos
Scheinkman and LeBaron (1989)	CRSP weighted index	S=5200, daily returns	BDS test on original and filtered data	Evidence of nonlinearty
Panas and Ninni (2000)	Rotterdam and Mediterranean petroleum markets (1) oil products	S=1161 daily, 1.1994-8.1998	1) Correlation dimension test 2) BDS test 3)Kolmogorov entropy 4) Wolf's LE test	Evidence of Chaos
Vraon and Jalilvand (1994)	TSE-300 index	S=3000 and 2500; daily and monthly 1.1977-12.1991	1) Correlation dimension test 2) BDS test	1) Evidence of chaos in monthly data 2)little evidence of chaos in daily returns

Serletis and Dormaar (1996)	Australlian Dollar and USD exchange rates	S=300, weekly 1.1987-6.1993	1) Nychka et al. nonparametric LE test	1)Evidence of Chaos
Matilla-Garcia M (2007)	Energy futures (1)Natural gas (2) unleaded gas (3) light crude oil	New York Mercantile Exchange (NYMEX) daily, S: 3700 3.1990-10.2005	1) BDS test 2) Wolf's LE test 3) Rosenstein LE test	1)Nonlinearity in future returns 2)Evidence of Chaos
Krager and Kugler (1992)	Exchange rates JYEN; DM; FF; Italian Lira; Swiss Franc	S=500 weekly returns 6.1980-12.1990	BDS test	Nonlinear
EE Peters (1991)	S&P 500 index	Snot indicated	1)CD 2)Wolf's LE	1)Nonlinearity 2) Evidence of chaos
E.Panas (2002)	London Metal Exchange Market	S=2987daily closing metal prices 1.1989 to 12.2000	1) Correlation Dimension test 2) LE	1) The metal commodities zinc and tin are chaotic.
Atin Das, Pritha Das (2007)	Foreign Exchange Rate of Twelve countries	S=8500 daily 01.1971-12.2005	1)Wolf's LE test	1) Nonlinearity 2) Evidence of chaos
M.Neugart (1999)	German labour market	S not indicated.	1) Correlation dimension test 2) BDS test	1) Chaos does not occur.

Following Table 1 and as regards the main conclusions of the literature, there is a broad consensus of support for the proposition that the return (or closing values) process is characterized by a pattern of nonlinear dependence. In particular, BDS test almost invariably reject the null of i.i.d process. On the other hand, the evidence on chaos is more mixed, with some evidence of low-dimensional structure in the U.S. stock-market index (Mayfield and Mizrach 1989; Vaidyanathan and Krehbiel 1992). Note, however, that these conclusions are based on CD, rather than LE estimates. Peters (1991) examine the S&P 500 by calculating CD and by estimating the largest LE as proposed by Wolf et al. (1985).

The author reports strong evidence of chaos in S&P 500 index. Moreover, for four well-known stock-index including S&P 500 index (the S&P 500; the DAX; the Nikkei 225; the FTSE-100) are analyzed by Abhyankar et al., (1997) and the authors estimate largest LE by using the method Wolf et al., (1985) and Nychka et al.,(1992). Even though Abhyankar et al., (1997) find the presence of nonlinear dependence in the returns on all indexes; the authors report no evidence of low-dimensional chaos and conclude that the data are dominated by a stochastic component. Findings of Abhyankar et al. (1997) are also consistent with the results of Scheinkman and LeBaron (1989) in sense of nonlinearity. Scheinkman and LeBaron (1989) report the existence of the nonlinearity for U.S weekly returns on the Center for Research in Security Prices (CRSP) value-weighted index, employing the BDS statistic, and find rather strong evidence of nonlinearity and some evidence of chaos. These evidences from different authors are sufficient to call into question the EMP, which underlies the linear modelling used in most capital market theory. It also lends validity to a number of investment strategies that should not work if markets are efficient, including trend analysis, market timing, value investing and tactical asset allocation. This finding is of particular importance for practitioners, because experience has shown that these strategies do work when properly applied, even though theory tells us they should not work in a random-walk environment. Conversely, strategies that depend upon efficient markets and continuous

pricing, such as portfolio insurance, Alexander filters and stop/loss orders, become suspect, because chaotic markets are neither efficient nor continuously priced. This calls into question the Capital Asset Pricing Model (CAPM) and most option-pricing theories, which are based upon normal distributions and finite variances. It is worthwhile to note that, by focusing on the chaotic properties of stocks at the firm level rather than the properties of indices, Hagtvedt (2009) obtains dominant LE and the CD for weekly returns to individual stocks and concludes that the firm characteristics size (market value) was found to exhibit chaotic characteristics of the stocks showing the anomalies in Sharpe's Capital Asset Pricing Model (Sharpe 1964). For other well-know world indexes, there are also several studies. Sewell et al. (1993) report evidence of dependency in the market index series in Japan, Hong Kong, South Korea, Singapore and Taiwan. De Lima (1995) argues that for the U.S data, nonlinear dependence is present in stock returns after the 1987 crash. Small and Tse (2003) analyze daily returns from three financial indicators: the Dow Jones Industrial Average, the London gold fixings, and the USD-JPY exchange rate and the authors conclude that all three time series are distinct from linear noise or conditional heteroskedastic models and that there therefore exists detectable deterministic nonlinearity that can potentially be exploited for prediction.

Despite the affirmative chaos test results, there are some critiques of the chaos in the literature as well (Lee et al., 1993). Hamill and Opong (1997) conclude that even though there is nonlinear dependence in Irish stock returns, there is no evidence of chaos. Willey (1992) find no evidence of chaos in the Financial Times Industrial Index.

Apart from stock-index analysis, Barnett et al. (1996) report the successful detection of chaos in the US division monetary aggregates. This conclusion is further confirmed by several authors (e.g. Hinich and Rothman 1998; Barnett and William, 2004). Frank and Stengos (1989) obtain similar results by investigating daily prices for gold and silver, using the correlation dimension and the Kolmogorov entropy. Serletis and Gogas

(1997) find the evidence consistent with a chaotic nonlinear generation process in seven East European black market exchange rates. Furthermore, foreign exchange markets are an essential domain in which chaos has been detected (Mantegna and Stanley, 2000; Das and Das, 2007). Many researchers (e.g. Granger and Newbold, 1974; Campbell et al., 1997; Lee et al., 1993; Bonilla et al., 2006) argue that financial market series exhibit nonlinearity. The terms of many financial contracts such as options and other derivative securities are also nonlinear (Mantegna and Stanley, 2000).

By examining the unemployment and the rate of change of money wage rates in the U.K. 1861-1957 Fanti and Manfredi (2007) detect chaotic behavior in wage and unemployment rates.

The studies on ISE-100 index are relatively less in number. Some studies on Turkish ISE market (e.g. Bayramoglu, 2007; Ozgen, 2007) demonstrate that the market is not efficient. Moreover, most of the studies on the behavior of ISE market prices supported the weak form market efficiency against the existence of chaos (Kenkül, 2006; Topçuoglu, 2006; Çıtak, 2003; Adali, 2006). On the other hand, in contrast with the above studies on ISE-100, by applying BDS, Hinich Bispectral, Lyapunov Exponent and NEGM tests Ozer (2010) reject the efficient market hypothesis that the ISE-100 all share equity index series is random, independent and identically distributed (i.i.d).

3. Preliminary Notions

3.1. Defining An Attractor

An attractor is a state that defines equilibrium for a specific system. As we shall see, equilibrium does not necessarily mean a static state, as econometric models define the term.

A simple attractor is a point attractor. A pendulum with friction is an example of a point attractor.

If you were to plot the velocity of the pendulum versus its position, you would obtain a graph that spirals in towards the origin, where velocity and position are zero. At this point, the pendulum has stopped.

A graph of velocity versus position is an example of the phase space of a system. Each point on the graph defines the state of the system at that time. All paths in phase space lead to a point attractor, if one exists. No matter where the motion in phase space initiates, it must end up at the origin-the point attractor. A pendulum that periodically receives energy goes back and forth with no variation in its path. Its phase-space plot becomes a closed circle, with the origin at its center. This is called a limit cycle attractor. The radius of the circle is determined by the amplitude of the pendulum's swing. As a time series, a limit cycle appears as a simple sine wave.

3.1.1. Chaotic Attractor

A third kind of attractor is a chaotic, or fractal, attractor. With a chaotic attractor, the trajectories plotted in phase space never intersect, although they wander around the same area of phase space. Orbits are always different, but remain within the same area; they are attracted to a space, but never converge to a specific point. Cycles, while they exist, are non-periodic. With a chaotic attractor, equilibrium applies to a region, rather than a particular point or orbit; equilibrium becomes dynamic. For example in economics, equilibrium is commonly defined as static. In other words, an economic system is commonly thought to tend to equilibrium (a point attractor) or to vary around equilibrium in a periodic fashion (a limit cycle). But there is no evidence that capital markets tend toward either type of equilibrium. If anything, the actual behavior of economic time series appears to be non-periodic. That is, they have cycles without well defined periods. If the markets are non-periodic, then limit cycles and point attractors cannot define their dynamics. Chaotic Attractors using a simple convection model, Edward Lorenz (1963) was able to define the first known chaotic attractor. The model is based on a fluid heated from below.

At low temperature levels, heat transfer occurs by convection; the water molecules behave independently of one another. As the heat is turned up, a convection roll starts; the water molecules behave coherently, and the warmer fluid on the bottom rises, cools as it reaches the top and falls back to the bottom. The water molecules trace a limit cycle. If the heat is turned up further, turbulence sets in; the fluid churns chaotically. At this point, the water's phase space becomes a chaotic attractor.

Chaotic attractors have an interesting property. Because of the possible nonlinearities in the underlying system, any errors in measuring current conditions eventually overwhelm any forecasting ability, even if the equations of motion are known. In other words, our ability to predict the future of a chaotic system is limited by our knowledge of current conditions. Errors in measurement grow exponentially in time, making any long-term forecast useless. This sensitive dependence on initial conditions made Lorenz conclude that any attempts at weather forecasting beyond a few days were doomed. If the economic cycle is governed by a chaotic attractor, it is easy to see why long-term econometric forecasts during the '70s and '80s were flawed.

How do we determine if a system has an underlying chaotic attractor? First, a phase space for the system must be constructed. (When the equations of motion are not known, this is not a simple task.) The resulting phase space must meet two criteria for a chaotic attractor:

(1) a fractal dimension

(2) sensitive dependence on initial conditions.

In the physics and engineering, various techniques have been developed to measure these items using experimental data.

If this attractor arises, this is because the relations between the variables that govern market movements are nonlinear. Nonlinear systems are characterized by trends and long-term correlations. If the market is

nonlinear, then the widespread use of standard statistical analysis is questionable. In particular, the Efficient Market Hypothesis and the Capital Asset Pricing Model (in its current form) are suspect as workable theories.

3.1.2. Constructing a Phase Space

A phase space consists of "m" dimensions, where each dimension is a variable involved in defining the time evolution ("motion") of the system. In a system where the equations of motion are known, constructing a phase space is simple. However, we rarely know the dynamics of the real system generating the observed time series data. By plotting one variable with different lags, one can reconstruct the original, unknown phase space with one dynamic, observable variable. This reconstructed phase space has all the characteristics of the real phase space, provided the lag time and embedding dimension are properly specified. (We discuss below how these parameters are determined.) Now the question becomes, what do we use as our single observable variable? Traditional analyses of the stock market use the percentage change in price (returns) or a logarithmic first difference. This removes the autocorrelations inherent in the original price series and makes the series suitable for linear statistical analysis. However, what standard statistical analysis considers undesirable may in fact be evidence of a nonlinear dynamic system. That is, the use of percentage changes in price may destroy any delicate nonlinear structure present in the data. In constructing attractors such as the Lorenz attractor, scientists use the actual value of the variables, not the rate of change. We apply this approach to our study of financial data. Economics, however, presents a problem that the physical sciences do not. As the economy grows, stock prices grow. Stock price data thus have to be detrended; in order to study the motion of stock prices, economic growth must be filtered out. Ping Chen, in his study of monetary aggregates, filtered out the internal rate of growth over the period.

This method has the appeal of simplicity, but it assumes a constant rate of growth, which is unrealistic. As we know, economic growth is not constant, but varies over time.

Therefore we use the deflated ISE-100 time series as our dynamic observable. From this series, we can reconstruct a phase space to obtain a fractal dimension and to measure sensitive dependence on initial conditions.

3.1.3. The Fractal Dimension

The fractal dimension of phase space gives us important information about the underlying attractor. More precisely, the next-higher integer above the fractal dimension is the minimum number of variables we need to model the dynamics of the system. This gives us a lower bound on the number of degrees of freedom in the whole system. It does not tell us what these variables are, but it can tell us something about the system's complexity. A low dimensional attractor of, say, three or four would suggest that the problem is solvable. A pure random process, such as white noise, fills whatever space it is plotted in (Recurrence Plot, Eckmann 1987). In two dimensions it fills the plane. In three dimensions, it fills the three-dimensional space, and so on for higher dimensions. In fact, white noise assumes the dimension of whatever space you place it in because its elements are uncorrelated and independent. The fractal dimension measures how an attractor fills its space. For a chaotic attractor, the dimension is fractional; that is, it is not an integer. Because a chaotic attractor is deterministic, not every point in its phase space is equally likely, as it is with white noise.

Grassberger and Procaccia (1983a) estimate the fractal dimension as the correlation dimension, D. D measures how densely the attractor fills its phase space by finding the probability that any one point will be a certain distance, R, from another point.

The correlation integral, Cm(R), is the number of pairs of points in an m-dimensional phase space whose distances are less than R.

For a chaotic attractor Cm increases at a rate R^D. This gives the following relation

$Cm = R^D$ or $\log(Cm) = D.\log(R) + c$,   where c is constant.

By calculating the correlation integral, Cm, for various embedding dimensions, *m*, we can estimate D as the slope of a log/log plot of Cm and R. Grassberger and Procaccia have shown that, as *m* is increased, D will eventually converge to its true value. Assume that for a dynamical system $D \approx 2,4$. This means that the dynamics of this particular system can be defined with a minimum of three dynamic variables. Once these variables are defined, we can model the system. Of course, the fractal dimension does not tell us what the variables are, but only how many variables we need, at a minimum, to construct the model. Our estimate is low enough to suggest that the problem is solvable.

3.1.3.1. Recurrence Plot

The method of recurrence plots (RP) was firstly introduced by Eckmannn et al., (1987) to visualize the time dependent behavior of the dynamics of systems (the recurrence of states in a phase space) which can be pictured as a trajectory in the *m*-dimensional phase space. It represents the recurrence of the phase space trajectory to a certain state, which is a fundamental property of deterministic dynamical systems. The main step of this visualization is the calculation of the distances between all points lying in the phase-space. Usually, a phase space does not have a low enough dimension (two or three) to be pictured. Higher-dimensional phase spaces can only be visualized by projection into the two or three-dimensional sub-spaces. However, Eckmann's tool enables us to investigate the *m*-dimensional phase space trajectory through a two-dimensional representation of its recurrences. Such recurrence of a state at time *i* and a different time *j* is pictured within a two-dimensional squared matrix with black and white (and grey) dots, where black dots mark a recurrence, and both axes are time (number of observed data) axes. The recurrence plot exhibits characteristic large-scale and small-scale patterns which are caused by typical dynamical behavior, e. g. diagonals (similar local evolution of different parts of the trajectory) or horizontal and vertical black lines (state does not change for some time). The RP exhibits characteristics which can

be observed as both large scale and small scale patterns). The first patterns (large scales) were denoted by Eckmann et al. (1987) as typology and the latter (small scales) as texture. The typology offers a global impression which can be characterized as homogeneous, periodic, drift and disrupted.

- *Homogeneous* RPs are typical of stationary and autonomous systems in which relaxation times are short in comparison with the time spanned by the RP. An example of such an RP is that of a random time series.

- Oscillating systems have RPs with diagonal oriented, *periodic* recurrent structures (diagonal lines, checkerboard structures). For *quasi-periodic* systems, the distances between the diagonal lines are different. However, even for those oscillating systems whose oscillations are not easily recognizable, the RPs can be used in order to find their oscillations.

- The *drift* is caused by systems with slowly varying parameters. Such slow (adiabatic) change brightens the RP's upper-left and lower-right corners.

- Abrupt changes in the dynamics as well as extreme events cause *white areas or bands* in the RP. RPs offer an easy possibility to find and to assess extreme and rare events by using the frequency of their recurrences.
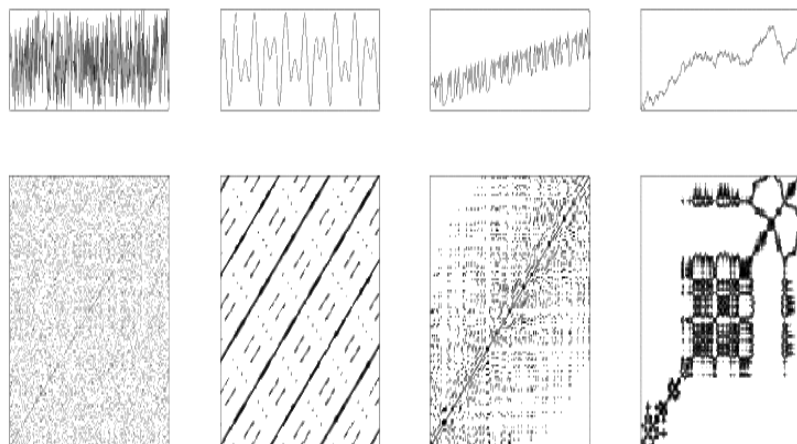
In the figure above, the upper series are the original observed time series and lower series are Recurrence Plots associated to them, respectively.

1. White Noise 2. Sine Wave 3. Lorenz System with linear time trend 4. Distrupted (Brownian motion) with rare shocks.

In addition to the large scales defined above, the closer inspection of the RPs reveals small scale structures (the texture) which are single dots, diagonal lines as well as vertical and horizontal lines (the combination of vertical and horizontal lines obviously forms rectangular clusters of recurrence points).

　　1. Single, isolated recurrence points can occur if states are rare, if they do not persist for any time or if they fluctuate heavily.

　　2. Certain points in the observed time series are parallel to each other i.e. they have same values but are placed at a different interval. A diagonal line occurs when a segment of the trajectory runs parallel to another segment, i.e. the trajectory visits the same region of the phase-space at different times. The length of this diagonal line is determined by the duration of such similar local evolution of the trajectory segments. The direction of these diagonal structures can differ. These stretches can be visualized as diagonal lines in the recurrence matrix.

These diagonal lines are called *deterministic*, because they represent a deterministic pattern in the series (the deterministic local evolution of states in the series). Therefore, Lyapunov exponent is defined as inverse of the length of these diagonal patterns.

　　a.) Diagonal lines parallel to main diagonal line represent the parallel running of the trajectories for the same time evolution.

　　b.) Diagonal lines perpendicular to main diagonal line represent the parallel running with contrary times (may show improper embedding)

　　3. A vertical (horizontal) line marks a time length in which a state does not change or changes very slowly. It seems that the state is trapped for

some time. This is a typical behavior of laminar states (intermittency) and widely used in fluid dynamics in order to model the flow of a river when its debit forced to narrow.

3.1.4. Characteristic Exponent (Lyapunov Exponent)

Lyapunov exponents, which provide a qualitative and quantitative characterization of dynamical behavior, are related to the exponentially fast divergence or convergence of nearby orbits in phase space. A system with one or more positive Lyapunov exponents is defined to be chaotic. To measure LE, we monitor the long-term growth rate of small volume elements in an attractor.

Each positive exponent reflects a "direction" in which the system experiences the repeated stretching and folding that decorrelates nearby states on the attractor. Therefore, the long-term behavior of an initial condition that is specified with any uncertainty cannot be predicted; this is chaos. An attractor for a dissipative system with one or more positive Lyapunov exponents is said to be "strange" or "chaotic"

The magnitudes of the Lyapunov exponents quantify an attractor's dynamics in information theoretic terms.

The exponents measure the rate at which system processes create or destroy information ; thus the exponents are expressed in bits of information/s or bits/orbit for a continuous system and bits/iteration for a discrete system. For example (Wolf et al., 1985), in the Lorenz attractor the positive exponent has a magnitude of 2.16 bits/s . Hence if an initial point were specified with an accuracy of one part per million (20 bits), the future behavior could not be predicted after about 9 s [20 bits/(2.16 bits/s)], corresponding to about 20 orbits. After this time the small initial uncertainty will essentially cover the entire attractor, reflecting 20 bits of new information that can be gained from an additional measurement of the system.

3.1.4.1. Sensitive Dependence on Initial Conditions

The positive maximal Lyapunov exponent implies the sensitive dependence of the dynamical system on initial conditions. Chaotic attractors are characterized by sensitive dependence on initial conditions. An error in measuring initial conditions will grow exponentially, so that a small error could dramatically affect forecasting ability. The further out in time we look the less certain we are about the validity of our forecasts. Contrast this with the linear models used in econometric forecasts. A small change in measuring current conditions has little impact on the results of a forecast. Theoretically, linear models imply that, if we have enough variables, we can forecast an indefinite period into the future. While the certainty of linear models is desirable, the uncertainty inherent in non-linear dynamics is closer to practical experience.

Lyapunov exponents measure the loss in predictive power experienced by non-linear models over time. Lyapunov exponents measure how nearby trajectories in phase space diverge over time. Each dimension in phase space has its own Lyapunov exponent. A positive exponent measures expansion in phase space. A negative one measures contraction in phase space. For a fixed point in three dimensions, the Lyapunov exponents are negative. For a limit cycle, two exponents are negative and one equals zero. For a chaotic attractor, one is positive, one negative and one zero. A chaotic attractor is characterized by the largest Lyapunov exponent being greater than zero. It represents the divergence of points in phase space, or the sensitive dependence on the conditions represented by each point. When the equations or motion are known, Lyapunov exponents can be calculated by measuring the divergence of nearby orbits in phase space. In a linear world, points that are close together in phase space would remain close together; this reflects the linear world, where a small error in measurement has little effect on the result. In a non-linear world, things are different. Nearby points will diverge as the differences in initial conditions compound.

This method involves measuring the divergence of nearby points in the reconstructed phase space over fixed intervals of time. First, we choose two points that are at least one mean orbital period apart. The distance between the two points is measured after the fixed evolution period. If the distance is too long, one of the points is replaced. (This is necessary to ensure that we measure only the expansion of the points in phase space; if the points are too far apart, they will fold into one another.) In addition, the angle between the points is measured to keep the orientation of the points in phase space as close as possible to the original set.

In theory, with an infinite amount of noise-free data, there occurs no difficulty. However, the real world presents us with a finite amount of noisy data. This means that the embedding dimension, m, the time lag, t, and the maximum and minimum allowable distance must be chosen with care. Wolf et al. give a number of "rules of thumb" for dealing with experimental data. First, the embedding dimension should be larger than the phase space of the underlying attractor. The time lag used to reconstruct the phase space must be computed; the maximum length of growth should be no greater than 10 per cent of the length of the attractor in phase space. Finally, the evolution time should be long enough to determine stretching without including folds. However, these rules of thumb are more evaluated in recent years (Kantz and Schreiber 1997).

Once done, the calculation over a long time series should converge to a stable value of LE. If stable convergence does not occur, it is possible that the parameters have not been well chosen, there are insufficient data for the analysis or the system is not truly non-linear.

As an example, Peters (1991) use the Wolf algorithm for the detrended S&P 500 data series (monthly data from January 1950 through December 1989), with an embedding dimension of four, a time lag of 12 and an evolution time of six months. This resulted in the stable convergence to a Lyapunov exponent of 0.0241 bits per month. This means that, even if we measure initial conditions to one bit of precision (one decimal place), all predictive

power would be lost after I/L1, or 42 months. This is roughly equal to the 48-month mean orbital period obtained from rescaled range analysis. This relation would not be true of white noise, or a random walk.

4. Methodology

This part of the study aims to conceptualize the preliminary notions which constitute the basis of nonlinear dynamical analysis and which are informally introduced in Part I.

The first dynamic characteristic of interest, the largest LE measures the rate at which a system diverges or converges. If along some dimension a process diverges while as a whole converging, then the process is called chaotic, which may be defined as sensitivity to initial conditions. This is characterized by a positive LE. Although the process may be very sensitive to initial conditions, the returns process as a whole may converge to a stable space. When a process has converged, the subspace of the process moves through is called an attractor, and if the dimension of the attractor is noninteger, a strange attractor. The correlation dimension (CD) measures the dimension of the attractor. After the LE measures proposed the CD has not been applied widely. Instead, the Recurrence Plots (RP) which are also another diagnostic tool of correlation integrals have been used. In our study, also we prefer to apply RP.

There is a major difficulty in measuring these dynamic characteristics when only the dependent variable in the process is observed. The process at any one time is a vector consisting of not only the stock return itself, but also the explanatory variables. This difficulty is resolved using Takens' theorem (Takens, 1981). Adopting Takens' Theorem we state that one may measure the dynamic characteristics using a sequence of vectors constructed from the lagged dependent variable instead of the sequence of vectors of returns and explanatory variables, provided the vector of lagged returns is constructed with the lags sufficiently large to be independent (here seven weeks), and with dimension at least [2m+1], where m is the dimension of the space the

process moves through. Takens' theorem allows us to study a process with unknown independent variables. The largest LE and the CD were therefore calculated using the lagged returns vectors, but are estimates of the largest LE and the CD of the true data generating process. Although there are several measures of fractal dimension, the CD appears to be the most common in the literature (Brock et al., 1987). Grassberger and Procaccia compare the CD to other measures of fractal dimension and conclude that the CD requires far fewer observations to be accurate (Grassberger and Procaccia, 1983). Sugihara and May (1991) indicate that as few as 1000 observations can be sufficient for consistent estimation of the correlation.

The CD estimates the fractal dimension of the strange attractor while a random walk has no attractor and will fill all the available space. Therefore a test based on the difference in dimensionality of a random walk compared to the process studied is the basis for the BDS test for chaos (Scheinkman and LeBaron, 1989). This test is widely used but cannot differentiate between nonlinear noise and chaos (Brock et al., 1987), (Hsieh, 1989), (Barnett et al., 1997). Even though the stock returns may be noisy; first, there is no reason to assume the noise is simply additive and second, the noise occurs intrinsically. Therefore we do not prefer to use the LE for Noisy Nonlinear Systems (LENNS) in this study, which was developed specifically for the situations when the stochastic element of the dynamic system is intrinsic (Ellner et al., 1992).

Instead, we prefer to use Kantz (1994) algorithm which is derived from the method of Wolf et al., (1985). We know that both of these two methods are robust to nonlinear noise.

Theiler and Eubank (1993) show that prewhitening (filtering the data, e.g. using ARIMA) may remove the characteristics the analysis is meant to identify. Therefore we do not prewhiten the data in this case.

ISE-100 composite index is Market-value weighted index which is like S&P 500 index and New York Stock Exchange index.

$L_{i,t}$ = the number of i-th stock at period t. (payed /1000)

$P_{i,t}$ = the price of i-th stock at period t.

$H_{i,t}$ = the sharing percentage of the i-th stock at period t.

D_t = the index dividend at period t. (without units) If one or more of the firms are replaced by outsider firm(s), then the value of the dividend changes, otherwise it does not change.

$$ISE_t = \frac{\sum_{i=1}^{100}(L_{i,t}.P_{i,t}.H_{i,t})}{D_t}$$ is the closing value of ISE-100 index at the end

of period t

Therefore ISE_t reflects weighted-prices in TRL.

Example :

Company	Percentage of Free Float Rate	Price of Stocks (TL)	Number of Stocks
A	45%	25,000	1 million
B	95%	12,500	1,5 million
C	60%	52,000	500 thousand

$$ISE_{t=T} = \frac{(25000.1000000.0,45)+(12500.1,500000.0,95)+(52000.500000.0,60)}{156710526}$$
$$= 285$$

4.1.Introducing the Nonlinear Dynamical System

Let us denote a dynamical system, $f: R^n \rightarrow R^n$, with the trajectory,

$$x_{t+1} = f(x_t) \ , \ t = 0,1,2,....,\tag{1}$$

Definition: Lyapunov Exponent (Characteristic Exponent)

If the initial state of a time evolution (dynamical system) is slightly perturbated, the exponential rate at which the initial perturbation increases (or decreases) with time is called Characteristic Exponent or Lyapunov Exponent. Sensitive dependence on initial conditions corresponds to positive maximal Lyapunov exponent.

The Lyapunov exponents for such a dynamical system are measures of the average rate of divergence or convergence of a typical trajectory. The trajectory is also written in terms of the iterates of f.

With the convention that f^0 is the identity map, and $f^{t+1} = f^0 \circ f^t$, then we also write $x_t = f^t(x_0)$. A trajectory is also called an orbit in the dynamical system literature.

For an *n*-dimensional system as above, there are *n* exponents which are customarily ranked from largest to smallest:

$$\lambda_1 \geq \lambda_2 \geq ..... \geq \lambda_n \tag{2}$$

Associated with each exponent, $j = 1,2,...,n$, there are nested subspaces $V^j \subset R^n$ of dimension $n + 1 - j$ and with the property that

$$\lambda_j = \lim_{t \to \infty} \frac{1}{t} \ln \left\| (Df^t)_{x_0} v \right\| \qquad \text{for all } v \in V^j \backslash V^{j+1} \tag{3}$$

It is a consequence of Oseledec's Theorem (Oseledec 1968), that the limit in Eq.(3) exists for a broad class of functions. Additional properties of Lyapunov exponents and a formal definition are given in (Eckmann 83).

Notice that for $j \geq 2$ the subspaces V^j are sets of Lebesque measure zero, and so for almost all $v \in R^n$ the limit in Eq.(3) equals λ_1. This is the basis for the computational algorithm of Wolf et a., (1985) which is a method for calculating the largest LE.

Attractor

Definition: Attractor

An attractor is a set of points towards which the trajectories of f converge. More precisely, Λ is an attractor if there is an open $U \subset R^n$ with $\Lambda = \bigcap_{t \geq 0} f^t(\overline{U})$ where $\overline{U}$ is the closure of U.

An attractor can be chaotic or non-chaotic (ordinary). There is more than one definition for chaotic attractors in the dynamical systems literature (Gencay and Dechert 1992). In practice, the presence of a positive Lyapunov exponent is taken as a signal that the attractor is chaotic.

The Recurrence Plot

Let y_i be the i-th point on the orbit (trajectory) describing a dynamical system in m-dimensional phase- space, for $i = 1, ..., T$. The recurrence plot is an array of dots in a T x T square, where a dot is placed at (i, j) whenever y_j ($\varepsilon - $ neighbors of y_i) is sufficiently close to y_i, for all $i = 1, ..., T$ and $j = 1, ..., T$. In practice one proceeds as follows to obtain a recurrence plot from the observed time series $\{z_t\}$. After reconstructing m-dimensional phase-space and hence m-dimensional orbit y_i for $i = 1, ..., T$, we choose ε_i (which is the ε chosen for y_i, i-th point of the orbit) for such that the ball of radius ε_i centered at y_i in space- R^m contains a reasonable number of other points $y_j^{\varepsilon_i}$ of the orbit. In other words, we choose ε_i such that there exist at least 10 neighbors lying in the ball of y_i

with radius ε_i. Later on Kantz (1994) by using this idea, define the set U_i which is the set of $\varepsilon-$neighbors of y_i.

Finally, one plots a dot at each point (i, j) for which the point which is in the ball of radius ε_i (this point is denoted by $y_j^{\varepsilon_i}$) centered at y_i. We call this picture a *recurrence plot.*

Definition : Recurrence Plot

$$R_{i,j} = \Theta\left(\varepsilon_i - \left|y_i - y_j\right|\right) \quad \text{where } i, j = 1,....,T \tag{4}$$

where ε_i is a cut-off distance (neighborhood criteria for i-th point y_i) $|...|$ distance function (a norm , e. g. the Euclidean norm) and $\Theta(.)$ is the Heaviside function (or step-function) having the value as binary output (i.e., $\Theta(criteria.){=}1$ if the criteria is obtained; $\Theta(criteria.){=}0$, otherwise) . Eckmann et al., (1987) propose to put black dot, for all 1's and white dot for all 0 on *T* x *T* square.

A diagonal line $R_{i+k, j+k} = 1, \quad for \quad k = 1,......., L,$ where L is length of diagonal line signifies the iterated distances of initially neighbors and occurs when a segment of the trajectory runs parallel to another segment, i.e. the trajectory visits the same region of the phase-space at different times. The length of this diagonal line is determined by the duration of such similar local evolution of the trajectory segments. The direction of these diagonal structures can differ. These stretches can be visualized as diagonal lines in the recurrence matrix. These diagonal lines are called *deterministic*, because they represent a deterministic pattern in the series (the deterministic local evolution of states in the series). Therefore, Lyapunov exponent is defined as inverse of the length of these diagonal patterns.

The lengths of diagonal lines in an RP are directly related to the ratio of determinism or predictability inherent to the system. Suppose that the states at times i and j, y_i and y_j are neighbors, then the function gives $R_{i,j} = 1$ and a black dot is put on coordinate (i, j). If the system behaves predictably, similar situations will lead to a similar future: *for example, after 1 iteration* $(\tau = 1$), *the probability for* $R_{i+1, j+1} = 1,$ *is high. For perfectly predictable*

systems, this leads to infinitely long diagonal lines (like in the RP of the sine function). In contrast, if the system is stochastic, the probability for $R_{i+1,j+1} = 1$, will be small and we only find single points or short lines (the white-black signal of the no-channel program in older TV broadcasts, "Karlanma".). If the system is chaotic, initially neighboring states will diverge exponentially. Since, the lines parallel to main diagonal measures the inverse of positive Lyapunov exponents, the faster the divergence, i.e. the higher the Lyapunov exponent, the shorter the diagonals.

On the other hand, a vertical (horizontal) line $R_{i,j+k} = 1$ (for $k = 1,......., L,$ where L is the length of the vertical line). marks a time length in which a state does not change or changes very slowly. It seems that the state is trapped for some time. This is a typical behavior of laminar states (intermittency) and widely used in fluid dynamics in order to model the flow of a river when its debit forced to narrow.

The Preliminary Notations

One rarely has the advantage of observing the state of the system at any period t, x_t, and that the actual functional form, f, that generates the dynamics. The model that is widely used is the following: associated with the dynamical system in Eq.(1) there is a measurement function $h : R^n \rightarrow R$ which generates the time series observed by us,

$$z_t = h(x_t) \tag{6}$$

It is assumed that all that is available to researcher is the sequence $\{z_t\}$.

Let y_t be a vector,

$$y_t = (z_{t+m-1}, z_{t+m-2}, ..., z_t) \tag{7}$$

Takens' Theorem (Taken 1981):

Under general conditions, it is shown in (12) that if the set $\overline{U}$ is a compact manifold then for $m \geq 2n+1$

$$J^m(x) = y_t = \left(h\left(f^{m-1}(x)\right), h\left(f^{m-2}(x)\right), \ldots, h\left(f^0(x)\right)\right) \tag{8}$$

Is an embedding of $\overline{U}$ onto $J^m\left(\overline{U}\right)$. Generically for $m \geq 2n+1$, there can be found a function $g : R^m \to R^m$ such that

$$y_{t+1} = g(y_t) \tag{9}$$

where $y_{t+1} = \left(z_{t+m}, z_{t+m-1}, \ldots, z_{t+1}\right)$ \hfill (10)

Moreover, notice that

$$y_{t+1} = J^m(x_{t+1}) = J^m\left(f(x_t)\right) \tag{11}$$

Hence from (9) and (11)

$$J^m\left(f(x_t)\right) = g\left(J^m(x_t)\right). \tag{12}$$

Under the assumption that J^m is a homeomorphism, f is topologically conjugate to g.

This implies that certain dynamical properties such as Correlation dimension, Fractal dimension and Lyapunov exponents are identical. Therefore, if we obtain a positive Lyapunov exponent in R^m, which is our phase-space, this reflect the chaotic characteristic of f and hence the original dynamical system.

Construction of Phase-Space R^m

From observed time series (sequence) $\{z_t\}$, data vector $y_t = \left(z_{t+(m-1).d}, z_{t+(m-2).d}, \ldots, z_t\right)$ is generated, where d is the time delay; this vector indicates one point of a *m*-dimensional reconstructed phase space

R^m, where m is call embedding dimension. Therefore a trajectory can be drawn in the m-dimension reconstructed phase space by changing t. Note that in *Preliminary notation* subsection above, d the time delay, is taken as equal to 1, (d=1) for notational easiness.

Assume that the target system is a deterministic dynamical system as given in Eq.(1), and that the observed time series is obtained through an measurement function as given in Eq.(6) . Then, the reconstructed trajectory is an embedding of the original trajectory when the m value is sufficiently large. If any attractor has appeared in the original dynamical system, another attractor, which retains the phase structure of the first attractor, will appear in the reconstructed state space. In order that such reconstruction achieves embedding, it has been proven that the dimension m should satisfy the condition given by Eq.(8)   However, this is a sufficient condition and upper-worst case. Depending on the data, embedding can be established even when m is less than 2n+1.

In the embedding method, there are two parameters, embedding dimension and time delay.

Abarbanel (1995) gives us a good suggestion on how to select those two parameters. Time delay does not strongly affect reconstruct phase space and Lyapunov exponents estimation. One approach to estimating this value is to select the frequency (1/time scale) that corresponds to a dominant power spectral feature. The reconstructed phase consists of points in m-dimension phase space.

Given the observed time series data set $z_1, z_2, ..., z_N$, where N is the length of the observed time series (sequence $\{z_t\}$. While embedding those data to m-dimension phase space, the time delay d can be used to get $(N - (m-1).d)$ points in phase-space (or the length of the orbit). Let us denote a point in the orbit by y_i. Then formally we obtain

$$y_i = \left(z_i, z_{i+d}, ..., z_{i+(m-1).d}\right) \qquad \text{for all } i \in \left(N - (m-1).d\right) \tag{13}$$

If this is applied for all $i \in \left(N - (m-1).d\right)$, we obtain the orbit, or trajectory and hence phase-space.

From now on, since we analyze a discrete time series (ISE-100) the time delay d, is taken to equal to 1.

4.2. Computing Lyapunov exponents

In this subsection, we obtain the algorithms designed to estimate Lyapunov exponents of a dynamical system and widely used in Chaos Theory, by using Takens' Theorem and Eqs. [6-12].

As we have indicated, Eq.(3) is the basis of Wolf's algorithm. Now, let us focus on it more closely.

4.2.1. Wolf's Algorithm

The first algorithm to compute Lyapunov exponent for an observed time series is introduced by Wolf et al., (1985). The method proposed by Wolf et al. (1985) not only estimates maximal Lyapunov exponent, but also computes all Lyapunov exponents of a dynamical system. In delay coordinates of appropriate dimension we look for a point of the time series which is closest to its first point, y_1. This is considered as the beginning of a neighboring trajectory, given by the consecutive delay vectors. Then we compute the increase in the distance between these two trajectories in time (iteration). When the distance exceeds some a priori determined threshold, for this point of the series a new neighboring trajectory is searched for, such that its distance is as small ias possible under the constraint that the new difference vector points more or less into the same direction as the old one. This is the direction of as the old one. This the direction of the local eigenvector associated to λ_1 (maximal LE). The logarithms of the stretching

factors of the difference vectors are averaged in time to yield the maximal Lyapunov exponent. In principle one should be able to compute all Lyapunov exponents by looking at the time evolutions of not distances but areas of (hyper) – surfaces.

Given a continuous dynamical system in an m-dimensional phase space, we monitor the long-term evolution of an infinitesimal *m*-sphere of initial conditions; the sphere will become an *m*-ellipsoid due to the locally deforming nature of the flow. The *i*-th one-dimensional Lyapunov exponent is then defined in terms of the length of the ellipsoidal principal axis

$$y_t(i): \quad \lambda_i = \lim_{t \to \infty} \frac{1}{t} \log_2 \frac{y_t(i)}{y_0(i)} \tag{14}$$

where the λ_i are ordered from largest to smallest as indicated by Eq.(2).

Notice that the linear extent of the ellipsoid grows as $2^{\lambda_1 t}$, the area defined by the first two principal axes grows as $2^{(\lambda_1 + \lambda_2)t}$, the volume defined by the first three principal axes grows as $2^{(\lambda_1 + \lambda_2 + \lambda_3)t}$, and so on. This property yields another definition of the spectrum of exponents: the sum of the first *j* exponents is defined by the long-term exponential growth rate of a *j*-volume element.

4.2.2. Gencay and Dechert's Algortihm

Gencay and Dechert (1992) use a result of (Gencay-Dechert 1990) which shows that the n largest Lyapunov exponents of a diffeomorphism which is topologically conjugate to the data generating process are the n Lyapunov exponents of the data generating process. The authors employ multilayer forward networks as a nonparametric estimation technique.

Gencay and Dechert show that Eq.(3) suggests a more direct approach to calculating the exponents. Since

$$\left(Df^t\right)_{x_0} = \left(Df\right)_{x_t}\left(Df\right)_{x_{t-1}}....\left(Df\right)_{x_0} \tag{15}$$

All of the Lyapunov exponents can be calculated by evaluating the Jacobian of the function f along a trajectory, $\{x_t\}$. The authors use QR decomposition for extracting the eigenvalues from $\left(Df^t\right)_{x_0}$ and hence function g.

From Eq.(9) the mapping g may be taken to be

$$g : \begin{pmatrix} z_{t+m-1} \\ z_{t+m-2} \\ \vdots \\ z_t \end{pmatrix} \rightarrow \begin{pmatrix} v(z_{t+m-1}, z_{t+m-2},......, z_t) \\ z_{t+m-1} \\ \vdots \\ z_{t+1} \end{pmatrix} \tag{16}$$

And this reduces to estimating

$$z_{t+m} = v(z_{t+m-1}, z_{t+m-2},....., z_t) \tag{17}$$

The derivative of g is the matrix

$$\left(Dg\right)_{y_t} = \begin{pmatrix} v_m & v_{m-1} & v_{m-2} & .. & . & v_1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \tag{18}$$

where $v_m = \dfrac{\partial v}{\partial z_{t+m-1}}$ for all $m \geq 1$.

Gencay-Dechert (1990) demonstrate that, if $\lambda_1^f \geq \lambda_2^f \geq .... \geq \lambda_n^f$ are the Lyapunov exponents of f and $\lambda_1^g \geq \lambda_2^g \geq .... \geq \lambda_n^g$ are the Lyapunov exponents of g. Then generically $\lambda_i^f = \lambda_i^g$ for $i = 1,2,...,n$.

Now, we have to estimate the function g based on the data sequence $\{J^m(x_t)\}$, and compute the Lyapunov exponents of g. As m increases there is value between n and 2n+1 at which the n largest exponents remain

constant and the remaining m-n exponents diverge to $-\infty$ as the number of observations increases.

4.2.3. Kantz's Algorithm

Kantz (1994) propose a robust algorithm in order to compute maximal Lyapunov exponent directly. As we show below, the method proposed by Kantz (1994) basis on the algorithm introduced by Wolf et al., (1985). Let us rewrite Eq.(3) in a different manner. Let y_t be the time evolution of some initial condition y_0 in our phase-space, R^m.

Then maximal Lyapunov exponent is found with probability 1 by,

$$\lambda_{max} = \lim_{t \to \infty} \lim_{\varepsilon \to 0} \frac{1}{t} \ln\left(\frac{\left|y_t - y_t^{(\varepsilon)}\right|}{\varepsilon}\right),$$

$$\left|y_0 - y_0^{(\varepsilon)}\right| = \varepsilon \tag{19}$$

For almost all difference vectors $\left(y_0 - y_0^{(\varepsilon)}\right)$. To compute λ_{max} one can either apply Eq.(13) in R^m by searching for pairs of neighboring trajectories and follow how they diverge or one can evaluate Eq.(13) in the tangent space.

Similar to the algorithm of Wolf et al.,(1985), Kantz (1994) use the fact that the distance between two trajectories typically increases with a rate given by the maximal Lyapunov exponent. Of course this is true only asymptotically.

$$\lambda_\tau(t) = \lim_{\varepsilon \to 0} \frac{1}{\tau} \ln\left(\frac{\left|y_{t+\tau} - y_{t+\tau}^{(\varepsilon)}\right|}{\varepsilon}\right) \tag{20}$$

$$\left(y_t - y_t^{(\varepsilon)}\right) = \varepsilon.w_u(t)$$

where $\left(y_t - y_t^{(\varepsilon)}\right) = \varepsilon . w_u(t)$ signifies the difference vector lying between the point y_t and its one of the neighbor(s) $y_t^{(\varepsilon)}$ in the phase-space.

And, where the operator $|...|$ shows the magnitude (norm) of the difference vector (distance between), lying between any two points in the phase-space. Thus we define the initial distance between a point y_t and its one of the ε − neighborhood point(s) $y_t^{(\varepsilon)}$ by ,

$$\left|y_t - y_t^{(\varepsilon)}\right| = \left|\varepsilon . w_u(t)\right| = \varepsilon . \left|w_u(t)\right| = \varepsilon \qquad (21)$$

where $w_u(t)$ is the local eigenvector associated with $\lambda_\tau(t)$ and its magnitude is $\left|w_u(t)\right| = 1$. This local eigenvector can be considered as the *base vector* of the phase-space.

$\lambda_\tau(t)$ depends on the structure in tangent space and thus is position dependent. It is approximately the same for all trajectories inside a small neighborhood. By definition the average of $\lambda_\tau(t)$ along the trajectory is the true Lyapunov exponent.

All delay vectors of the series falling into the ε − neighborhood U_t of y_t will be considered as the beginning of neighboring trajectories, which are simply given by the points of the time series consecutive in time. If we measured the distance between neighboring trajectories in their true phase space, we would see exactly the fluctuations of the divergence rate described by the distribution of effective Lyapunov exponents (at least after some transient time, when the difference vector points into the most unstable direction). Starting from a univariate time series, we could realize this situation by measuring the distance in the embedding space. But apart from the fact that we have to fix the dimension in which we search for neighbors we do not want to distinguish any particular embedding dimension.

Therefore we define the distance between a reference trajectory y_t and one of its ε − neighbor(s) $y_t^{(\varepsilon)}$ after the relative time (iteration) τ by a function

$$K(.): R^m \to R$$

$$K\left(y_t, y_t^{(\varepsilon)}; \tau\right) = \left| y_{t+\tau} - y_{t+\tau}^{(\varepsilon)} \right| \tag{22}$$

As we have denoted y_t as a vector in Eq.(7) and composed it with the lagged values of observed time series (sequence) $\{z_t\}$ embedding with appropriate dimension, we can rewrite

$$K\left(y_t, y_t^{(\varepsilon)}; \tau\right) = \left| y_{t+\tau} - y_{t+\tau}^{(\varepsilon)} \right| \tag{23}$$

$$= \sqrt{\left((z_{t+m-1+\tau} - z_{t+m-1+\tau}^{(\varepsilon)})^2 + \left(z_{t+m-2+\tau} - z_{t+m-2+\tau}^{(\varepsilon)}\right)^2 + \ldots + \left(z_{t+\tau} - z_{t+\tau}^{(\varepsilon)}\right)^2 \right)}$$

where using Eq.(7),

$$y_{t+\tau} = \left(z_{t+m-1+\tau}, z_{t+m-2+\tau}, \ldots, z_{t+\tau} \right)$$

and

$$y_{t+\tau}^{(\varepsilon)} = \left(z_{t+m-1+\tau}^{(\varepsilon)}, z_{t+m-2+\tau}^{(\varepsilon)}, \ldots, z_{t+\tau}^{(\varepsilon)} \right)$$

$K\left(y_t, y_t^{(\varepsilon)}; \tau\right) = \left| y_{t+\tau} - y_{t+\tau}^{(\varepsilon)} \right|$ gives us the magnitude of the difference vector $\left(y_{t+\tau} - y_{t+\tau}^{(\varepsilon)} \right)$ lying between the point $y_{t+\tau}$ (which is the τ-times iterated value of point y_t) and the point $y_t^{(\varepsilon)}$ (which is τ-times iterated value of point its neighbor $y_t^{(\varepsilon)}$) in the phase-space.

As given in Eq.(6), these distances are (generally nonlinear) projections of the difference vectors in the true phase space onto a one dimensional subspace spanned by the observable (in the simplest case the observable itself is one coordinate of the phase space).

In order to measure the maximal Lyapunov exponent we fix t, search for all neighbors $y_t^{(\varepsilon)}$ of y_t , inside an $\varepsilon-$neighborhood U_t and compute the average of the distances between all neighboring trajectories and the reference trajectory y_t as a function of τ (by increasing τ).

As defined in Eq.(23) , τ is the relative time (iteration) depends on t, therefore we may need to smooth the output of function $K(.)$ by taking the logarithm of it. We perform this calculation for all t=1,2,..,T where T is the number of points in phase-space.

Thus we compute

$$S(\tau) = \frac{1}{T} \sum_{t=1}^{T} \ln \left(\frac{1}{|U_t|} \sum_{y_t^{(\varepsilon)} \in U_t} K\left(y_t, y_t^{(\varepsilon)}; \tau\right) \right) \tag{24}$$

where $|U_t|$ denotes the number of elements of set of $\varepsilon-$neighbors of y_t.

The first sum in the RHS of the Eq.(24) composed of the distance between y_t and every $\varepsilon-$neighbor of y_t (for all $y_t^{(\varepsilon)} \in U_t$). Then we obtain its average and take logarithm. Since we should perform this calculation for all $t = 1,2,.....,T$, we take its average to obtain average magnitude of the principal axis of the m-dimensional ellipsoid.

Initially, the difference vectors in the true phase space are pointing in any direction.

Thus for small τ we cannot expect a scaling behavior. For an intermediate range of τ , $S(\tau)$ increases linearly with the slope λ which is our estimate of the maximal Lyapunov exponent. This is the scaling range, where on the one hand τ is large enough such that nearly all distance vectors point into the unstable direction and on the other hand the corresponding distances $K(\tau)$ are smaller than the size of the attractor. When they reach the latter size, $S(\tau)$ asymptotically tends towards a constant (which is $<$ ln 1/4 if the diameter of the attractor is normalized to 1) since the distances

cannot grow any more. If the data are noisy, the typical distance between two nearby trajectories is of the order of the noise level. If we choose e smaller than the noise amplitude and if we find neighbors for this value, $S(\tau)$ jumps from a value smaller than $(\ln \varepsilon)$ to a value given by the noise level at $\tau = 1$.

If this value is not too large, one can still find a scaling range, and the exponent thus found is not affected by the noise.

Finally, the slope of the curve $S(\tau)$ gives us the maximal Lyapunov exponent :

$$\frac{\partial S(\tau)}{\partial \tau} \cong \lambda_\tau(t) \text{ for any } \tau \text{ in the scaling region.} \qquad (25)$$

In summary, our numerical value for the maximal Lyapunov exponent is the slope of the curve $S(\tau)$ in the scaling region. It does not contain the embedding dimension explicitly, but nevertheless it enters. We have to fix a dimension m for the delay embedding space in which we search for neighboring trajectories. If m is too small there is a nonzero probability that two trajectories are close in the embedding space but not in the original space, R^n.

4.2.4. The Comparison of the Effectiveness of Wolf's, Gencay Dechert's and Kantz's Algorithms

The main difference between these three methods is the space that the embedded points examined. While Gencay and Dechert (1992) propose to work on tangent space, Wolf et al., (1985) and Kantz (1994) prefer real space. Therefore, Gencay and Dechert (1992) computes the product of the Jacobians along the trajectory.

The second difference lies between the method of Kantz (1994) and both the method of Gencay -Dechert and Wolf. That is, while Kantz (1994)

only computes the maximal Lyapunov exponents, both Gencay-Dechert and Wolf computes the spectrum of Lyapunov exponents.

1.The method proposed by Wolf et al., (1985) is limited to maximal exponent one. The experiences show that even for the maximal exponent this algorithm does not yield very precise results. One reason is that if no optimal new neighboring trajectory is found, one has to use a bad one.

One could think of disregarding parts of the trajectory where only "bad" neighbors are found but then the information about neighbors are found but then the information about the unstable direction also gets lost.

If the data are noisy one has to demand that the initial distance between the trajectory and a new neighbor is larger than the noise level, otherwise we would interpret fluctuations due to noise as if they were deterministic divergence.

2. Since the method proposed by Gencay and Dechert (1992) yields production of Jacobians along the trajectory in order to calculate the spectrum of Lyapunov exponents, the function f which drives the dynamical system and which is given by Eq.(1) must at least approximately be known.

3. Another drawback for Wolf's method and Gencay-Dechert's method is that the embedding dimension is an important parameter. For too small a dimension the exponent is severely overestimated, since trajectories may diverge simply because they are not neighbors in the true phase space. While too large a dimension does not in principle cause problems, it does in practice. Typically the initial distance between the neighboring trajectories increases with the embedding dimension. Therefore the small length scales are less well explored and the number of time steps until a new neighboring trajectory has to be searched for decreases. This induces larger errors due to the deviations of directions. Kantz's method does not need the exact determination of embedding parameter.

4. All of the three models suffer when the data is set is not long enough. For short data sets, higher embedding dimension and smaller ε very few neighboring trajectories generally can be found.

Therefore the curves may fluctuate due to low statistics. This is not a serious problem unless no neighbor is found at all for many points y_t. In such a situation only the parts of the attractor are explored where the natural measure is highest.

If the spatial distribution of the effective Lyapunov exponents is non-homogeneous this may lead to a wrong estimate of the average. Note that in Eq. (24) the averages are taken in such a way that exponents are averaged according to the natural measure only if all points of the time series have neighbors.

Thus when working with fixed ε-neighborhoods, one has to keep track of how many points have no neighbors and compares the results with different ε. Fixed mass neighborhoods will lead to a mixture of length scales and reduce the scaling range.

5. In Kantz's method; first of all, an exponential divergence sets in only after some transient time, since an arbitrary difference vector has to turn into the most unstable direction. Eventually, this will happen with probability one. Secondly, the divergence rate of trajectories naturally fluctuates along the trajectory, with the fluctuations given by the spectrum of effective Lyapunov exponents.

According to the comparison of the three models and our main goal which is to determine the maximal Lyapunov exponent of ISE-100, we choose to implement the method proposed by Kantz (1994).

5. Time Series Analysis of   ISE -100 Composite Index

Understanding stock market price fluctuations plays an important role in economic policy and in corporate investment and financing strategies.

The proposition that nonlinear processes studied in Chaos theory play an important role in these fluctuations. Studies in the application of non-linear modelling to examine the behaviour of asset returns in Turkey .

The ISE-100 consists of 100 stocks which are selected amongst the companies, except investment trusts, traded on National Market in accordance with the criteria set by ISE.

ISE 100 Index automatically covers ISE 30 and ISE 50 stocks. The study uses daily prices data from the base period of index series.

The data used in the study covers the period from January 1990 to October 2009 . First , each of the ISE-100 index series transformed into logarithmic return, log_return.

Parameter	Value
Number of observations	4922
Mean	0,000676
Median	0,000000
Std. Deviation	0,012624
Maximum	0,077190
Minimum	-0,086766
Skewness	-0,044823
Jarque-Bera	2280,135
Probability	0,000000

Figure 1. Statistical Properties of *logarithmic return* ISE-100

The Figure 1 shows that the distribution of the daily return on the ISE 100 index is right-skewed. The results of the study also suggest that nonlinear processes play a significant role in stock market behavior.

5.1. Time Series Analysis

Let y_t be a time series. Let us denote the expected value (mean) of y_t by $\mu = E(y_t)$ then the variance $Var(y_t)$ of y_t is given by $Var(y_t) = E\left[(y_t - \mu)^2\right]$.

Definition 1(Hamilton 1994): A stochastic process y_t is stationary (Hamilton 1994) if and only if

1. $E(y_t) = \mu$ for $\forall t$ μ ε R is constant.

2. $Var(y_t) = \sigma^2$ for $\forall t$ σ^2 is constant.

3. $Cov(y_t, y_{t-h}) = \gamma(h)$ $\forall t$,h certain function of $\gamma(h)$ in which

$E(.)$ is expected value operator, $Var(.)$ signifies variance and $Cov(.)$ signifies covariance.

If one of these properties can not be reached, we can say that the stationarity of the process y_t is violated. The more general definition of the non-stationary processes has been given in Banarjee et al.(1993) and these general processses are called Integrated processes. A knowledge of the fundamental properties of integrated processes is essential for understnading of tests for both non stationary and the existence of long-run equilibrium relationships.

5.1.1. Unit Root and Integrated Processes

Let a vector autoregressive process order $p, Var(p)$ process, be

$y_t = A_1 y_{t-1} + ... + A_p y_{t-p} + u_t$ where y_t is vector of interested variables,

and $A_i, i = 1,......p$ coefficient matrix, is stable if the polynomial defined

by $\det(Ip - A_1 z - .... - A_p z^p)$ has no unit roots in and on the complex

unit circle.

On the other hand, for univariate a autoregressive order one, AR(1)

process, $y_t = p y_{t-1} + u_t$,this property means that $1 - pz \neq 0$ for $|z| \leq 1$

or equivalently, $|p| < 1$.

For univariate AR(1) process $y_t = p y_{t-1} + u_t$ consider the borderline case,

where $p = 1$. The resulting process $y_t = p y_{t-1} + u_t$ is called *random walk*.

Starting the process at $t = 0$ with some fixed y_0 , it is easy to see by

successive substitution for lagged y_t's, that

$y_t = p y_{t-1} + u_t = y_{t-2} + u_{t-1} = ... = y_0 + \sum_{i=1}^{t} u_i$

Thus, y_t consist of the sum of all disturbances or innovations of the previous

periods so that each disturbance has a lasting impact on the process. If u_t is

white noise with variance $Var(u_t) = \sigma_u^2$ then expected value of the signal

y_t is $E(y_t) = y_0$ and the variance of it will be calculated by $Var(y_t) = t.$

$Var(u_t) = t\sigma_u^2$. Hence, the variance of a random walk tends to infinity.

Futhermore, the correlation is given by

$$Corr(y_t, y_{t+h}) = \frac{\left[\left(\sum_{i=1}^{t} u_i\right)\left(\sum_{i=1}^{t} ui\right)\right]}{\left[t\sigma u^2 (t+h)\sigma u^2\right]\frac{1}{2}} = \frac{t}{(t^2 + t.h)\frac{1}{2}} \xrightarrow[t\to\infty]{} 1$$

for any integer h.

This latter property of a random walk means that y_t and y_s are strongly correlated even if they are far apart in time. If $\left(1 - p_1 z - .... - p_p z^p\right)$ has a root for $z = 1$ where μ is called as drift, then a behavior similar to that of a random walk is also observed for higher order AR(p) processes such as :

$$y_t = \mu + p_i y_{t-1} + ... + p_p y_{t-p} + u_t$$

Note that $\left(1 - p_1 z - ...... - p_p z^p\right) = \left(1 - \lambda_1 z\right)\left(1 - \lambda_2 z\right).....\left(1 - \lambda_p z\right)$

where $\lambda_1, \lambda_2, ... \lambda_p$ are the reciprocals of the roots of the polynomial. If the process has just one unit root (a root equal to 1) and all other roots are outside the complex unit circle, its behavior is similar to that of the random walk .

That is, its variances increase linearly ,the correlation between variables h periods aparts tends to 1 and the process has linear trend in mean if µ≠0.

The univariate processes with d unit roots (d roots equal to 1) in their AR operators are called "integrated of order d" and are shown by I(d) . Therefore, the ARMA(p,q) process with d unit roots may also be expressed as ARIMA(p,d,q). As one can easily derive, the stationary processes can be shown by I(0) series and the series containing one unit root can be shown by I(1). The properties of series integrated of strictly positive orders differ

substantially from those of I(0) series. Consider a series containing a single unit root $y_t = py_{t-1} + u_t$; where $y_0 = 0$ and ut $\sim$I (0) or ,after integrating, $y_t = S_t$ where $S_t \equiv \sum_{i=1}^{t-1} p^i u_{t-i}$. If $|p \geq 1|$ then y_t is non stationary, and if p=1, then it is integrated of order 1 , I(1).The series y_t is then the sum of all previous errors or innovations.To compare these types of processes let us consider two special cases of the process:

$$y_t = py_{t-1} + u_t \tag{1}$$

The first alternative of Eq.(1) is $y_t = py_{t-1} + u_{1t}$, where $|p| < 1$ and u_{1t} is independently, identically distributed process shown by u_{1t} $\sim$i.i.d $(0, \sigma_1^2)$. The second one is $y_t = y_{t-1} + u_{2t}$ where $p = 1, y_0 = 0$ and $u_{2t} \sim$ i.i.d$(0, \sigma_2^2)$. The sequence u_t need not be an innovation sequence; u_t may itself follow a stationary ARMA process. In the first case, to ensure stationarity, let us assume that y_0 is drawn from the unconditional distribution of y, that is, $y_0 \sim$ i.i.d $\left[(0, \sigma_1^2)/(1 - p^2) \right]$. Table 2 compares the properties of these two processes. Specifically, in order to determine whether there exists unit root (non-stationarity) in the ISE-100 values or not, we apply the test methods which are described and reported by Fuller (1976); Dickey and Fuller (1979) and which are called as Dickey-Fuller test.

We give a detailed description of the test procedure that we employ. For the remainder of the study we may interchangeably use the terms "non-stationary time series" , "I(d) variable" and "ARIMA(p,d,q) process" as well as the terms "stationary time series" and "I(0) variable".

Table 2:

Some Properties of Stationary and Integrated Process

Processes:	$y_t = \rho y_{t-1} + u_{1t}$;	$y_t = y_{t-1} + u_{2t}$;
	Where $|\rho|<1$; $u_{1t} \sim$ i.i.d$(0,\sigma_1^2)$	Where $y_0=0$; $u_{2t} \sim$ i.i.d $(0,\sigma_2^2)$
Integration order :	(I(0))	(I(1))
Variance	Finite $\left(\sigma_1^2\left(1-\rho^2\right)^{-1}\right)$	Unbounded (grows as $t\sigma_2^2$)
Conditional variance	σ_1^2	σ_2^2
Autocorrelation function at lag i	$\rho_i = \rho^i$	$\rho_i = \sqrt{1-\left(i/t\right)} \to 1 \ \forall i$ as $t \to \infty$
Expected time between crossings of	$y = 0$ Finite	Infinite
Memory[5]	Temporary	Permanent

source Banarjee et.al.(1993)

5.1.1.1 Unit Root Analysis

In time series analysis literature various tests (Fuller 1976 and Hall 1990) are defined for the unit root detection which can validly be applied to series that follow AR(p) processes containing no more than one unit root.

However, for the processes which may have unusual characteristics we find it useful to perform statistical tests which were designed for further class of series, ideally in such a way as to allow exogenous variables to enter the process as well. (Said and Dickey 1984) provide a test procedure valid for a general ARMA process in the errors; (Phillips 1987a) and (Perron and Phillips1988) offer a still more general procedure. These procedures totally called as "Augmented" Dickey-Fuller (ADF) tests.

While the Said-Dickey approach does present a generalization of the

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[5] We say that a process has a permanent memory if the effect of an innovation or shock does not disappear as $t \to \infty$.

Dickey-Fuller procedure, it again yields test statistics within the same asymptotic critical values as those tabulated by Dickey and Fuller. The particular advantage of Said-Dickey test is that we can apply it not only to models with MA parts in the errors, but also to models for which the orders of the AR and MA polynomials in the error process are unknown. The method involves approximating the true process by an autoregression in which the number of lags increases with sample size. Said-Dickey approach begins by assuming that the data generating process follows:

$$y_t = \mu + ct + \rho y_{t-1} + u_t, \text{ where } \mu, c \in R \text{ and } u_t \sim ARMA(p,q) \tag{2}$$

or

$$\delta(L)u_t = \mu(L)\varepsilon_t \qquad \text{where } \varepsilon_t \sim N(0,R) \text{ and } L \text{ is lag operator.}$$

so that the error term in the autoregression follows an $ARMA(p,q)$ process, presumed to be stationary and invertible. Then the data generating process in Eq.(2) can be rewritten as

$$y_t - y_{t-1} = \mu + ct + (\rho-1)y_{t-1} + \sum_{i=1}^{p} \gamma_i (y_{t-i} - y_{t-i+1}) + \varepsilon_t \qquad where \qquad \varepsilon_t \sim N(0,\sigma) \qquad \text{or} \tag{3}$$

$$\Delta y_t = \mu + ct + (\rho-1)y_{t-1} + \sum_{i=1}^{p} \gamma_i (y_{t-i} - y_{t-i+1}) + \varepsilon_t \qquad where \qquad \varepsilon_t \sim N(0,\sigma)$$ and Δ is the first difference operator, μ is drift and c is time trend and where p is large enough to allow a good approximation to the $ARMA(p,q)$ process u_t, so that ε_t is approximately white noise. The null hypothesis $(H_0 : \rho = 1)$ is $\rho = 1$.

5. 1.1.2. Application of Time Series Modelling

Worldwide Stock Exchanges (SE)

Most of the past studies on the behavior of security prices have utilised linear modelling techniques in their analysis. Fama (1965) admits that linear modelling techniques have limitations as they are not sophisticated enough to

capture complicated patterns which chartists claim to see in stock prices. Damodaran (1993) estimates the price adjustment coefficients for firms listed various stock markets in the U.S. and reports evidence of lagged adjustment to new information in shorter return intervals.

5.2. Application of Time Series Modelling to ISE-100 Stock Exchange

In this study we search for unit root (non-stationarity) in each of the sub-segments and we particularly apply the Said-Dickey method. We consider that, the ISE -100 data obtained can be examined by linear time series model which has two general forms shown by Eqs. (4) and (5) respectively.

$$y_t = \mu + c.t + u_t, \text{ where } \mu, c \in R \text{ and } u_t \sim ARMA(p, q) \tag{4}$$

or $\delta(L)u_t = \mu(L)\varepsilon_t$ where ε_t $\sim$ $N(0, R)$ and L is lag operator.

$$\phi(L)\Delta y_t = \zeta(L)\varepsilon_t, \text{ where } \varepsilon_t \sim N(0, R) \tag{5}$$

These two different structures, enable us to apply the non-stationarity tests to the control ISE-100 data. For determining the existence of unit root, we use ADF statistic tests by following the algorithm which requires three steps procedure:

i.)   By testing them individually we research which one of the two alternatives Eq. (4) or Eq. (5) characterizes our ISE-100 data them.

ii.) We assume that there may be drift and a time trend $(\mu + c.t)$ in these ISE-100 data and we construct our null hypothesis according to this assumption. We apply Non-standard *t*-statistics by Ordinary Least Squares (OLS) regression method.

iii.) We explore whether the errors or noise u_t, to the system are independently, identically distributed (i.i.d) process or they have Autoregressive properties (AR(p)). If they have autoregressive character, then we must first, determine the exact order of the noise in the process by using

both Phillips – Perron Test (Phillips and Perron 1988) and Hannan Test (Banarjee et al. 1993; Hamilton 1994) and second, we have to transform to the white noise by using the Said- Dickey method (Said and Dickey 1984). This approach is called "whitening the noise". After this transformation we are eligible to perform the ADF non-stationary tests.

When we realize the first step, from the results of the OLS regressions we find out that both the control ISE-100 data have no drift and have no independent time trend.

Thus we reject both the null hypothesis that the process contains drift and time trend ($H_0 : \mu \neq 0$) and ($H_0 : c \neq 0$). Thus we focus on (5) to characterize ISE-100 data.

Our test equation becomes:

$$\Delta y_t = (\rho - 1)y_{t-1} + \sum_{i=1}^{p} \gamma_i (y_{t-i} - y_{t-i+1}) + \varepsilon_t \qquad where \qquad \varepsilon_t \sim$$

$$N(0, \sigma) \tag{6}$$

and p (order of the error in ISE-100 data) defines the dependence of the its own history. When the second and third steps indicated above were realized, we obtained the results shown in the following section, Section 4.3.

5.2.1. Results of The Analysis

The first step in testing for nonlinearity and chaos is to test for the presence of a stochastic trend (a unit root) in the autoregressive representation of the series. In what follows we test the null hypothesis of a stochastic trend against the trend-stationary alternative by estimating by ordinary least squares (OLS) the following Augmented Dickey-Fuller (ADF) regression (Dickey and Fuller ,1981). In what follows we test the null hypothesis of a stochastic trend against the trend-stationary alternative by estimating by ordinary least squares (OLS) the following Augmented Dickey-Fuller (ADF) regression (Dickey and Fuller ,1981).

- Results of Eq.(3) $\rightarrow \Delta\left(\log\left(\dfrac{y_t}{y_{t-1}}\right)\right) = 0{,}0038 + 0{,}9218.\log\left(\dfrac{y_t}{y_{t-1}}\right) + \varepsilon_t$

The serie Log_return, has no unit root . We rejected the Null Hypothesis.
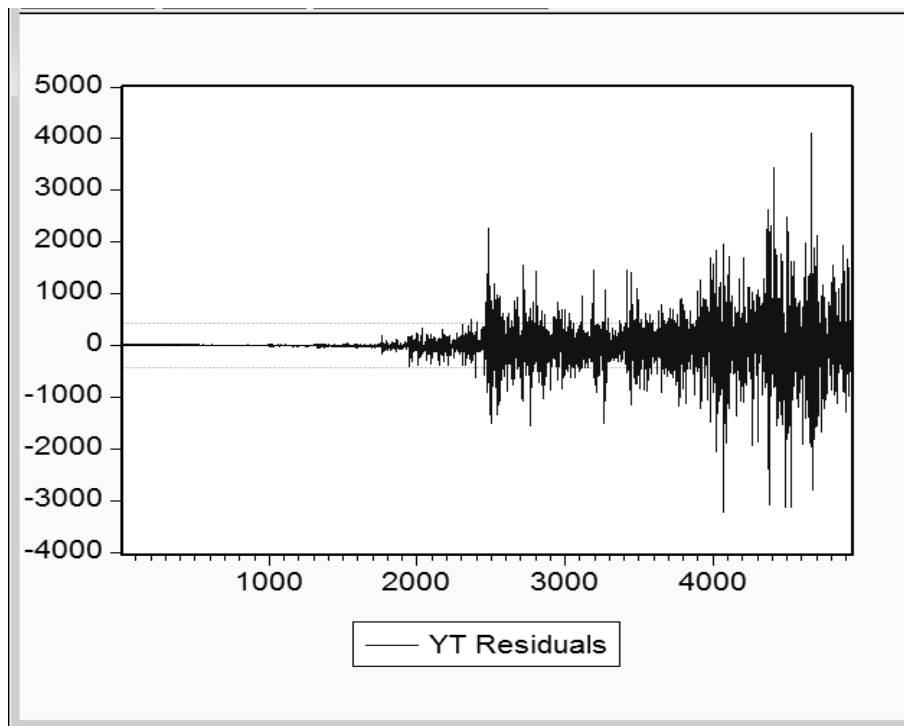
The serie is stationary and has no stochastic trend also.
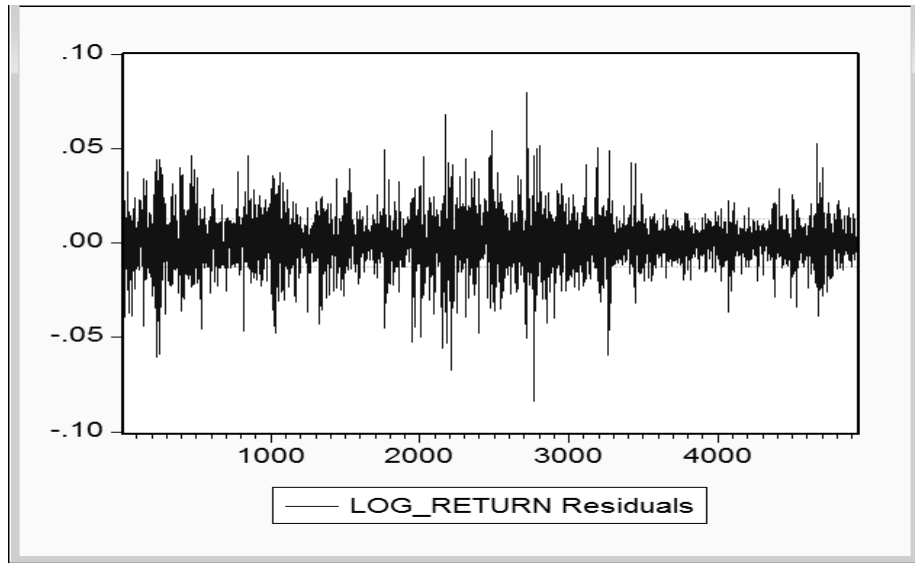

Figure 2. Time Plot of y_t Residuals

Figure 3. Time Plot of $\log\left(\dfrac{y_t}{y_{t-1}}\right)$ Residuals

Table 3

Results of the ADF Test statistics of $\log\left(\dfrac{y_t}{y_{t-1}}\right)$ Residuals

Significance Level	Critical Values
% 1	-3,95
% 5	-3,41
% 10	-3,12

(*) Note that the ADF test statistic critical value is -64,85.

Table 4

Correlation Matrix of Index Serie

No root lies outside the unit circle.
ARMA model is invertible.
The correlation matrix of the daily closed values and the first lagged of that serie is ;

Variable	y_t	y_{t-1}
y_t	1	0.99963
y_{t-1}	0.99963	1
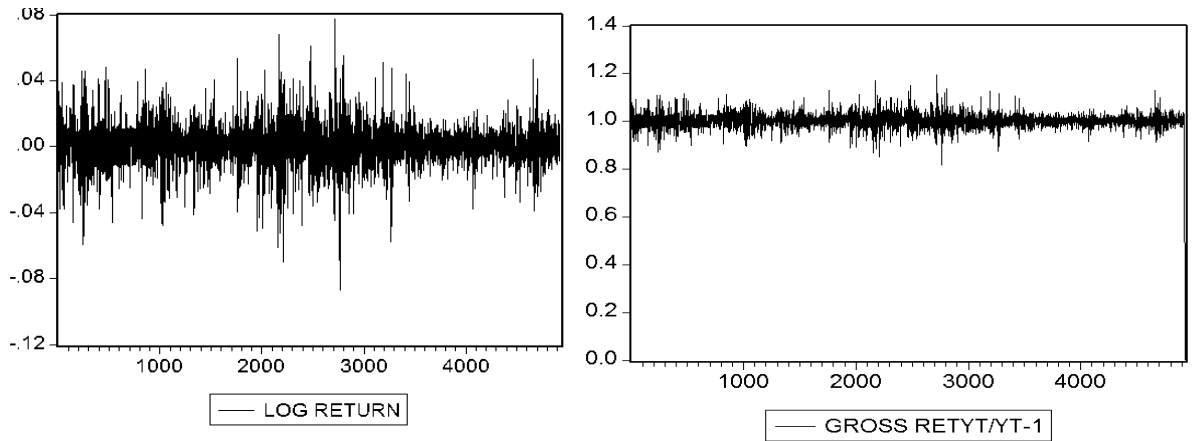
Figure 4. Time Plot of $\log\left(\dfrac{y_t}{y_{t-1}}\right)$

- Results for Eq.(6)

$$\Delta(\log(r_t)) = (\rho - 1).(\log(r_{t-1})) + (\sum_{i=1}^{p} \gamma_i.(\log(r_{t-i}) - \log(r_{t-(i+1)}))) + \varepsilon_t$$

$$\Delta y_t = 0,0266\Delta y_{t-1} + 0,01\Delta y_{t-2} - 0,0131\Delta y_{t-3} + 0,031.\Delta y_{t-4} - 0,0062\Delta y_{t-5} -$$
$$0,0323\Delta y_{t-6} - 0,01\Delta y_{t-7} - 0,33\Delta y_{t-8} + 0,039\Delta y_{t-9} + 0,065\Delta y_{t-10}$$

According to the test results, the closing values (Δy_t) of ISE-100 index has a unit root, and non-stationary.

For the model selection criteria the Akaike Information Criterion (AIC) (Akaike, 1974) was used, and the order of the model was determined by relying on the minimum Akaike Information Criterion estimated as MA (1).

Eq. of AR modelling $\quad y_t = \mu + p_i y_{t-1} + ... + p_p y_{t-p} + u_t$

Eq. of MA modelling $\quad y_t = \mu + u_t - k_i y_{t-1} - .k_k y_{t-2} + ...$

Variable	Akaike info criterion	Std.error
AR(1)	-5,538	0.012
MA(1)	-5,905	0.0013

5. 3. Comparison Between ARIMA Modelling and Chaos Modelling of ISE-100

It appears very interesting to use chaos for modelling. Indeed, it is a non-explosive system (no trend); it is an aperiodic system (no seasonality); it is a stationary system (invariant distribution-ergodicity). Thus, if we are able to know in which space the attractor lies, by determining the phase space using the embedding dimension for instance, and if we are able to re-build the orbits, then we can make predictions.

Open problems

This study discussed the approaches followed by some economists and financial researchers when they apply chaos theory to real data sets. The main difference between physicists and economics is due to the fact that in economy, we only have a unique trajectory (and not the possibility to repeat the experience as in physics) and also to presence of measurement noise in the time series.

The presence of noise in real data sets is a brake for the use of chaos theory in practice. Thus, robust deconvolution techniques need to be developed more.

Estimation theory is very often used under strong assumptions like independence of the data sets or Gaussianity of the observations: assumptions which are not verified in practice. Thus, new developments need to be considered in particular for data sets characterized by skewness or kurtosis.

Another problem which is important is the existence of an invariant measure for chaotic systems. We can question if this assumption is realistic and we refer to Guégan (in press) for a discussion on this subject.

Concerning predictions, no good solution has been proposed for medium-term predictions. We conjecture that to predict at a medium-term horizon necessitates taking into account the divergence of the orbits if we are in a region of the attractor where the Lyapunov exponent is positive.

6. Results of Chaotic Analysis

Table 5

Average Log Distance of daily reel-return values. Maximal Lyapunov Exponent for ISE-100 , daily reel-return values (detrended by Consumer Price Index 87=100 i.e., TUFE 87=100). The curves differs from each other in sense of the embedding dimension (*m*) and epsilon chosen (*eps*). Tho τ , signifies iteration. The negative values derives from logarithm of the distances lying the interval (0,1].
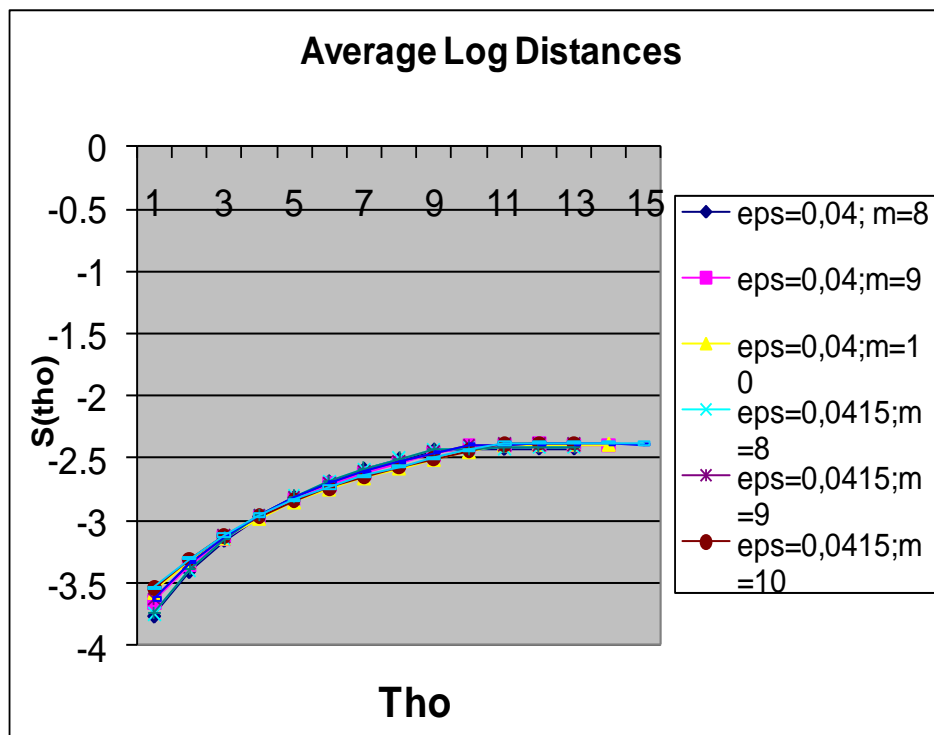
The slope of the curves give the maximal Lyapunov Exponent, $\lambda_\tau^{max}(t)$ a la Kantz.

Table 6

The Average Distances in phase space of daily closing values. Maximal Lyapunov Exponent for ISE-100 , daily closing values detrended by Consumer Price Index 87=100 (TUFE 87=100). The curves differs from each other in sense of the embedding dimension (m) and epsilon chosen (eps). Tho τ , signifies iteration.
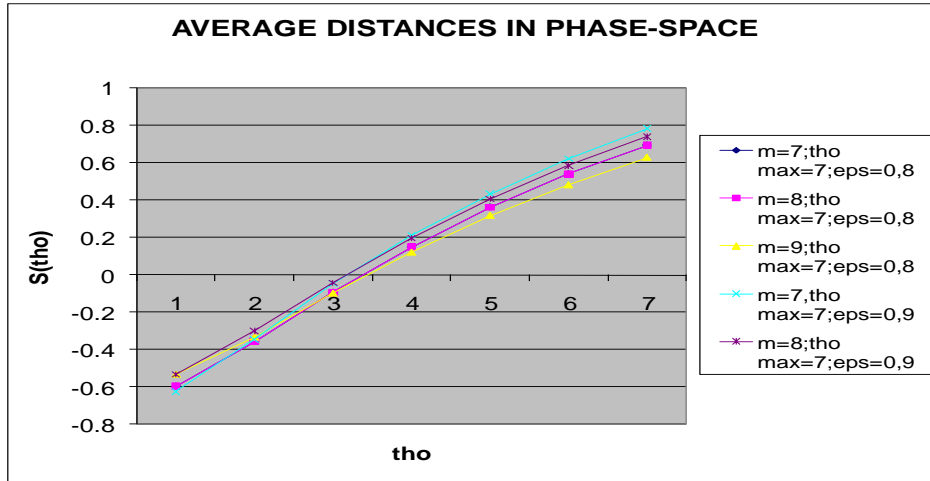
The slope of the curves give the maximal Lyapunov Exponent, $\lambda_\tau^{max}(t)$ a la Kantz.

Table 7

 Maximal Lyapunov Exponent for ISE-100 , daily closing values detrended by Consumer Price Index 87=100 (TUFE 87=100). Tho max is 35, which shows the limiting case of the curves given in   Table 5.2.
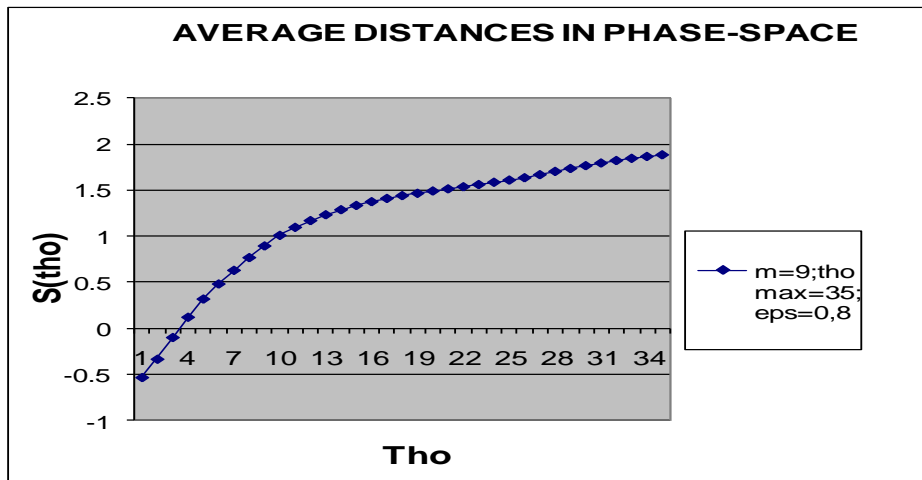
6.1. Implications

We have seen that the ISE-100 has an underlying "chaotic" attractor with a positive Lyapunov exponent. All this indicates that there is an underlying, non-linear mechanism to the ISE-100. The exact form of this mechanism is unknown at this time. A likely candidate, however, is a mixed feed-back system coupling the various capital markets. Earlier work has shown that the capital markets are non-linear. Now we have additional evidence that the ISE-100 has an underlying chaotic attractor, which may involve the other capital markets.

The next step is to develop a model that explains the nonlinear mechanism that generates the chaotic attractor. We leave that for further research.

6.2. Practical Considerations

The existence of a chaotic attractor and non-periodic cycles vindicates a number of strategies that have been "disproved" using random-walk-based mathematics. Most studies of market timing and trend analysis, for example, make a Gaussian assumption about probability distributions. As we have noted, fractal distributions are infinite-variance distributions and require quite different analysis. Strategies based on "mean reversion," including value-based stock selection and market-timing techniques, also need to be updated. Because equilibrium is dynamic, rather than static, mean-reversion models of value need to be dynamic as well. A "chaotic" market also means, however, that market timing, value in-vesting and tactical asset allocation may all have relevance, and if properly done, should be able to capitalize on the market cycle. For the practitioner, this means that care should be taken when formulating an opinion about how the market works. For too long, we seem to have been divided between "technicians" who believe that the market follows a regular cycle and "quants" who believe there is no cycle at all. The truth lies somewhere in between. There are cycles, but they are not regular, hence may be invisible to standard statistical techniques. Investment professionals should begin relying on quantitative techniques that do not rely on random-walk assumptions but are more dynamic. These techniques

are just beginning to be developed. Long-range market forecasting is impossible and should not be attempted, or even considered.

6.3.Final Considerations

On a broader scale, we can see why standard econometric methods have failed in the past. Attempts to "whiten" data in order to make it suitable for linear-based forecasting technique, cover up the conclusions that non-linear dynamics gives us.

Surprisingly, these conclusions tie in with experience.

(1) The market has cycles and trends.

(2) A small change in an indicator can have a major impact on the future.

(3) The further out in time we go, the less reliable our forecasts are. The last point is particularly important. It means that, as with the weather, accurate, long-range economic and market forecasting is not feasible from a practical standpoint.

7. Conclusion

In this study we test the Chaotic behaviour of ISE-100 returns with Lyapunov Exponent. According to the test results we have seen that chaos existed in the ISE-100 and has an underlying "chaotic" attractor with a positive Lyapunov exponent. All this indicates that there is an underlying, non-linear mechanism to the ISE-100. The exact form of this mechanism is unknown at this time. A likely candidate, however, is a mixed feed-back system coupling the various capital markets. Earlier work has shown that the capital markets are non-linear. Now we have additional evidence that the ISE-100 has an underlying chaotic attractor, which may involve the other capital markets. We were able to determine the equations of motion underlying the ISE-100, we would still not be able to forecast beyond a short time frame, because we are never able to measure current conditions with significant accuracy. The future in quantitative economic theory should

thus revolve around estimating the current probability distribution, which is not normal, so that we can better analyze the risks. Once that is done, viable economic and investment decisions can be made. Finally, the Efficient Market Hypothesis conforms to none of these observations. Neither does standard econometrics, which is based on static equilibrium assumptions. If the social sciences are to grow, consideration should be given to defining equilibrium in a more dynamic fashion. The next phase involves models that explain these dynamics and statistical techniques for estimating risk. There is still much work to be done.

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RP Analysis With Deflated (Real) Values
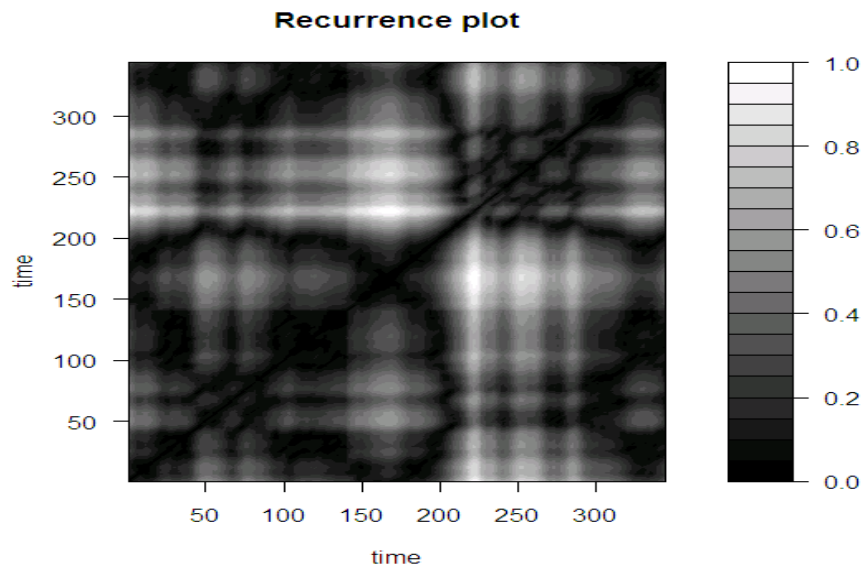
Graph A. 1.

 [1-2000] points, 01.90- 01.98

The RP shows that, the ISE-100 closing deflated values show a *checkerboard* caharacteristic which is already stated in literature (Eckmann et al., 1987), implying chaotic behavior. This can be identified by irregular periodic oscillating dynamical system (Marwan 2002).
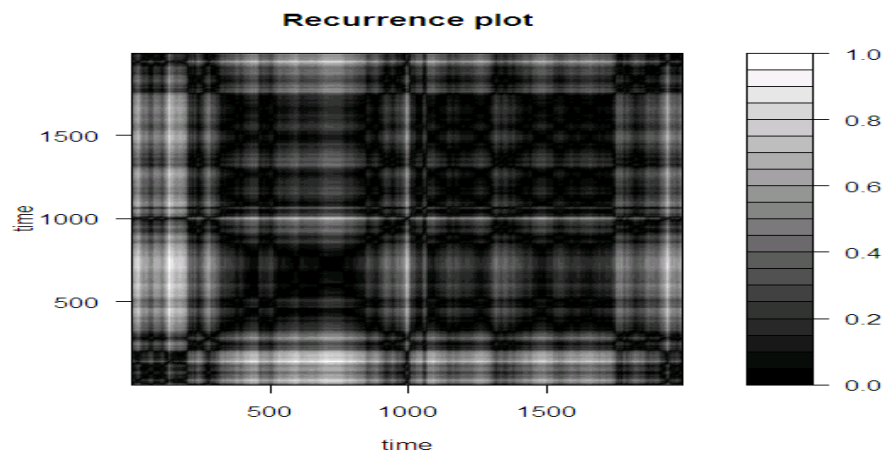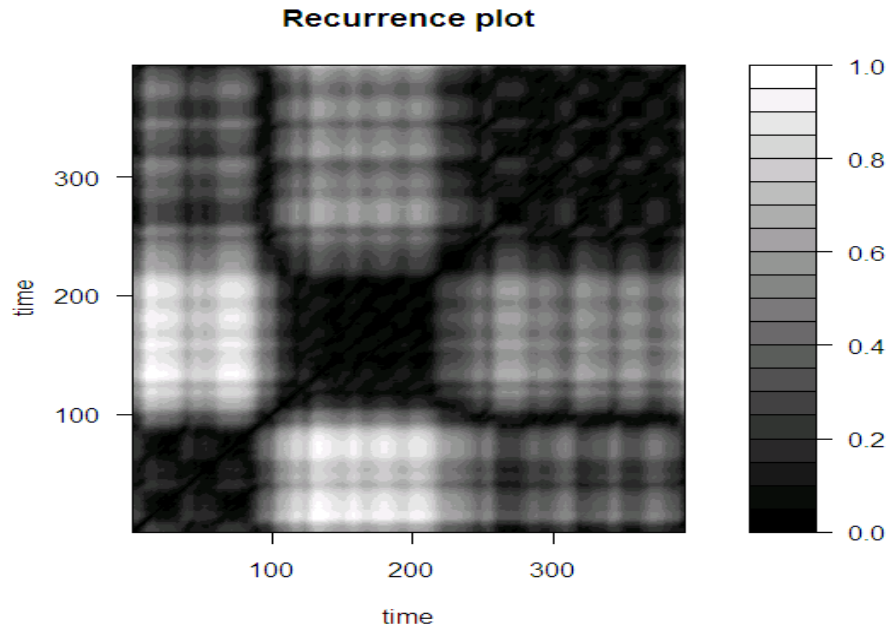

Graph A.2.

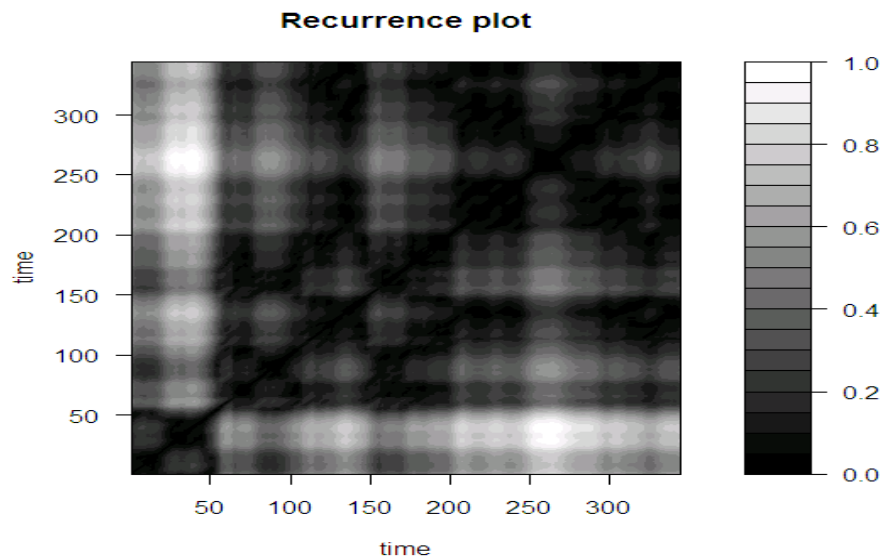 94 crisis. 94Q1-95Q1 Vertical lines are observed.

Graph A.3.

 98Q1- 99Q2, 98 Crisis

Recurrence plot

Graph A.4.

2000.06 -2002Q1 data 2001 crisis

Recurrence plot

RP Analysis With Nominal Closing Values

B1. Data: 08.93- 01.95

   Data Interval [90,110] and [155-165] are white, implying that these points are far from other points. Before and after these intervals, we have black regions, implying deterministic movement. [90,100] and [150-160] may imply regime change. The 90-th point dated as [01.94,02.94] and [03.94, 04.94]. After [155-165] points region , after 04.94, we have wide white region, implying that the points becoming far from previous points. However, there occurs a deterministic dependence to the point [90,110], these points are where initailly far from all their previous (in time) points.
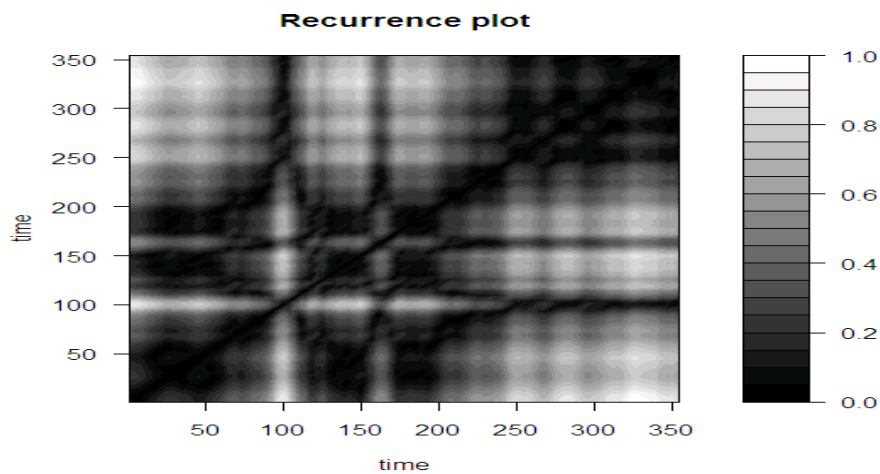
B2. Data: 08.97 - 02.99 the points [1900-2270]

Between the points [150-250], corresponding to the dates [03.98-09.98] the system losts its dependence and diverges, implying rare shocks. This can be seen from the white region at the middle-bottom of the plot. This divergence is such that, the points lying in [150-250], do not show nay neighborhood to the first 150 points.
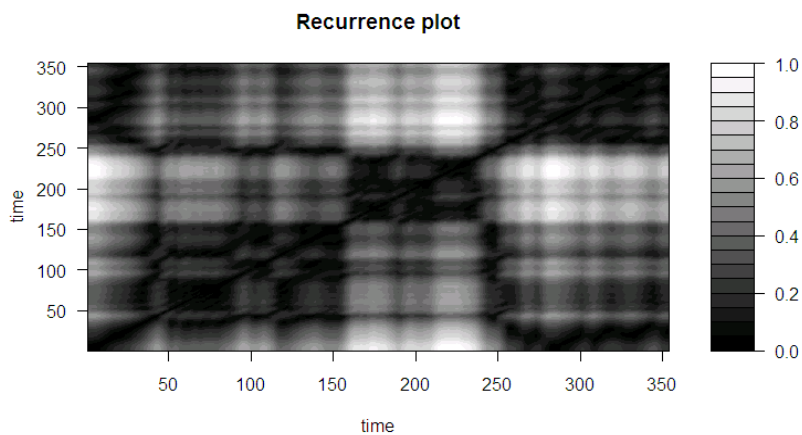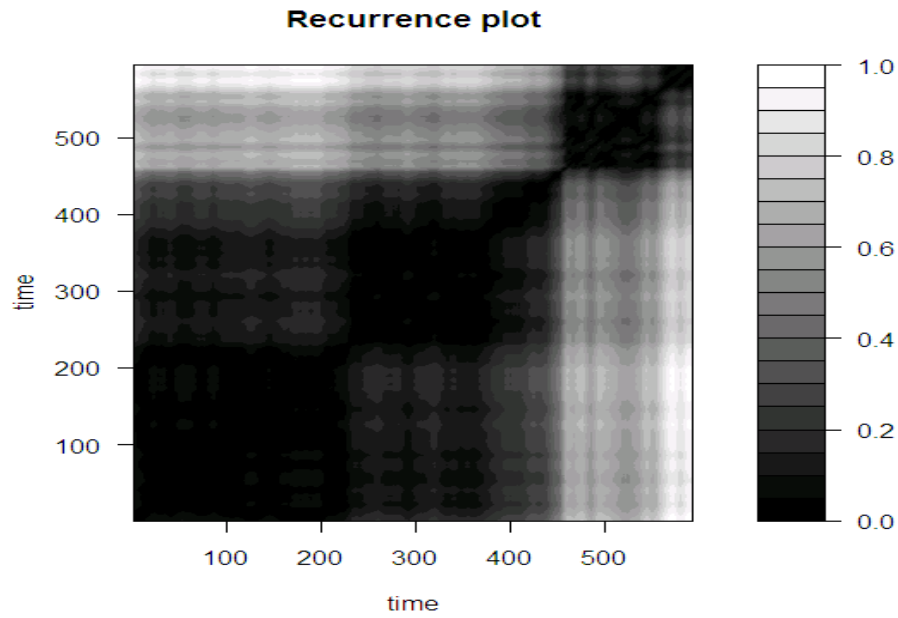
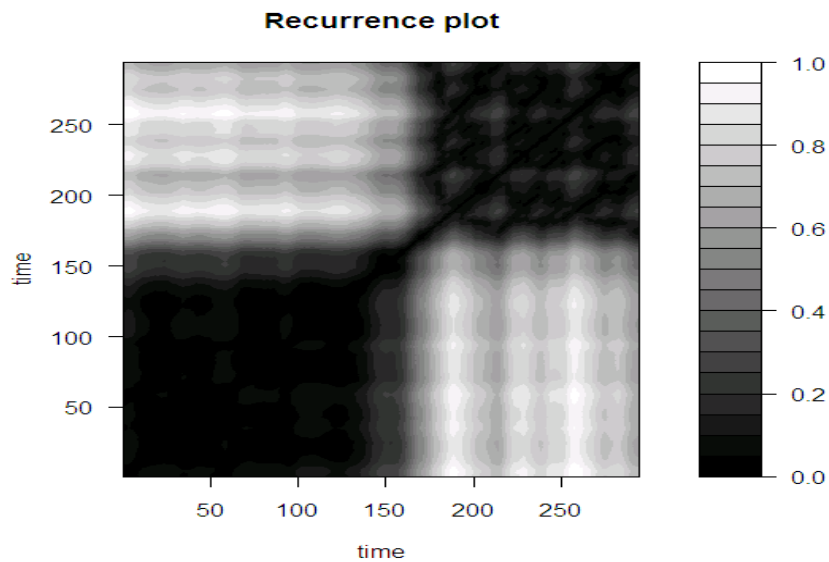B3. Data: 10.2000 - 07.2001 the points [2669-2900]
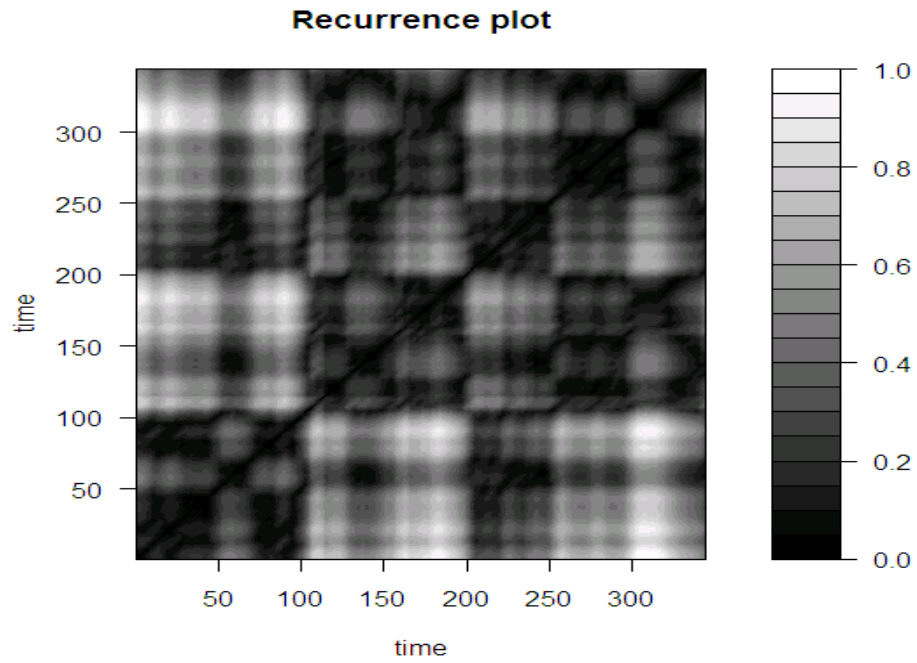
Recurrence plot

B4.Data: 01.2003- 02.2006
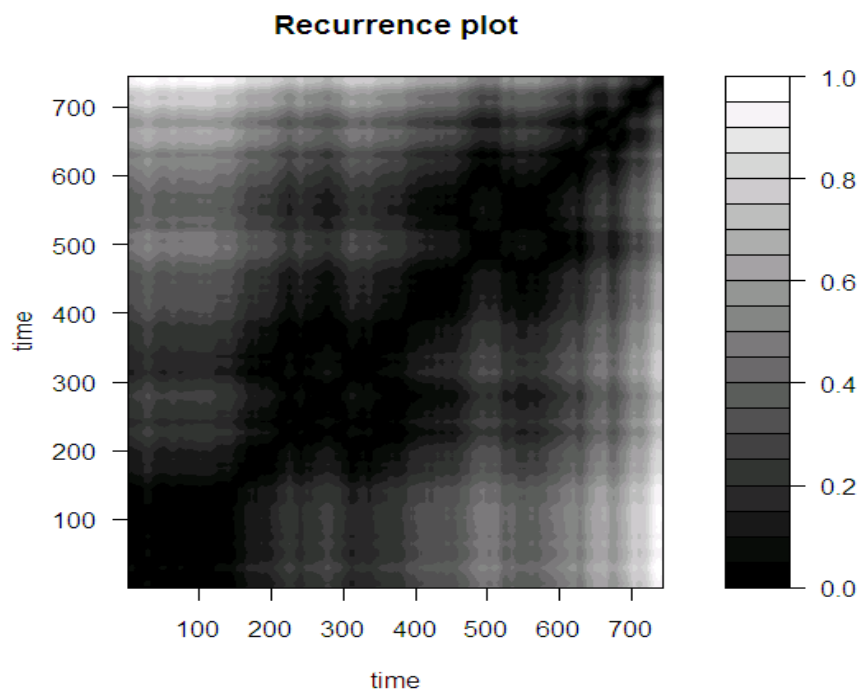
Recurrence plot

B5.Data: 03.95- 08.97 between two crisis

Recurrence plot

B6.Data: 99Q1-10.2000 between two crisis

Recurrence plot

Recurrence Plot Analysis With Reel-Return

The RP shows that, the ISE-100 closing deflated values show a *checkerboard* caharacteristic which is already stated in literature (Eckmann et al., 1987), implying chaotic behavior. This can be identified by irregular periodic oscillating dynamical system (Marwan 2002).
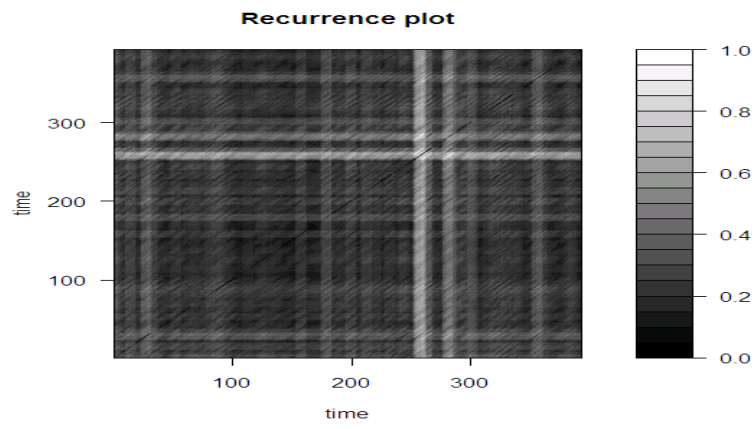
C1. Between 1000-1350.

Recurrence plot
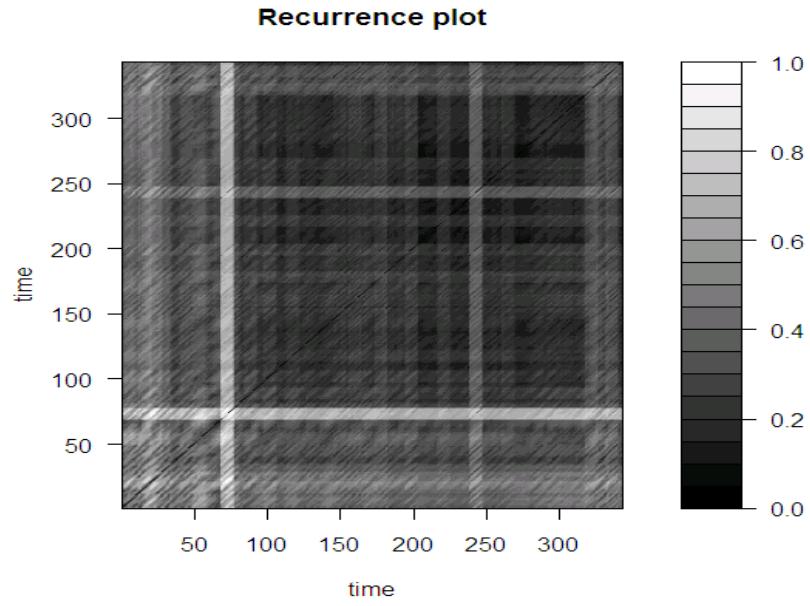
C2. Between 1900-2300

Recurrence plot

70
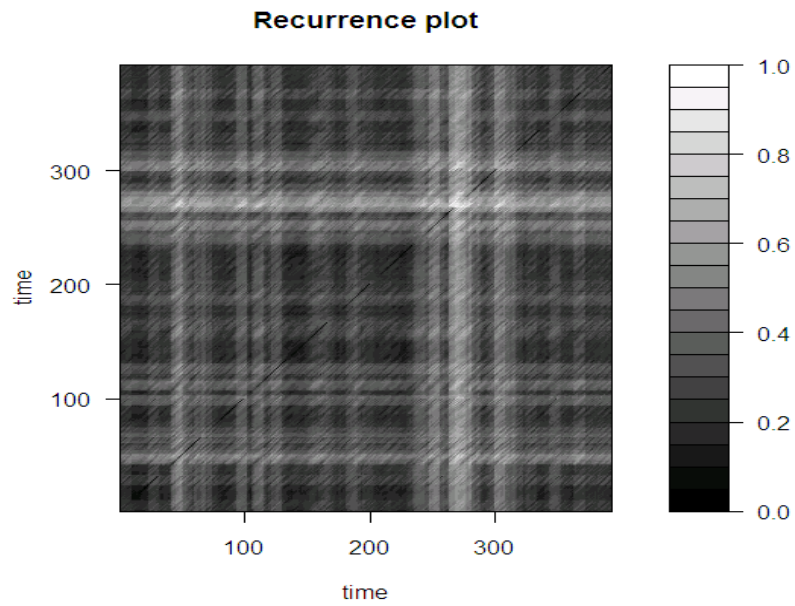
C3. Between 1500-1900 , Between crisis 94 and crisis 98 and without crisis.

Recurrence plot

Between 2600-2900

Recurrence plot