MEASURING RELATIVE PERFORMANCE OF TWELVE HIGH SCHOOLS IN GAZIANTEP BY USING DATA ENVELOPMENT ANALYSIS WITH RESPECT TO OSS EXAM RESULTS.

by

Ahmet TILFARLIGİL

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I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Assist. Prof. Dr.
Ali ŞAHİN
Head of Department

This is to certify that I have read this thesis and that in my opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Assist. Prof. Dr.
Nizamettin BAYYURT
Supervisor

Examining Committee Members

Assist. Prof. Dr. Nizamettin BAYYURT ______________________

Assist. Prof. Dr. Ali ŞAHİN ______________________

Assist. Prof. Dr. Ibrahim ABU ALSHAIKH ______________________

It is approved that this thesis has been written in compliance with the formatting rules laid down by the Graduate Institute of Sciences and Engineering.

Assist. Prof. Dr.
Nurullah ARSLAN
Director

July 2006

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Ahmet TILFARLIGİL

M.S. Thesis – Mathematics Department
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Supervisor: Assist. Prof. Dr. Nizamettin BAYYURT

ABSTRACT

Relative efficiencies of twelve high schools in Gaziantep by Data Envelopment Analysis (DEA) are examined. A linear programming derived program Data Envelopment Analysis is used to evaluate efficiencies of decision making units like banks, companies, hospitals, tourism agencies with multiple inputs and outputs. Inverse of average of 11th grade students in a class, inverse of average of weekly course hour of each 11th grade teacher, number of students taking OSS exam (University Entrance Exam in Turkey), and average of attending OSS exam preparation courses are taken as inputs. Outputs of the study are several OSS results. The scores showed that any high school can determine potential increase and decrease by values. Thus DEA is powerful tool to measure relative efficiency.

Keywords: Data Envelopment Analysis, Relative Efficiency of High Schools in Gaziantep.

GAZİANTEP'TE BULUNAN ON İKİ LİSENİN ÖSS SONUÇLARINA GÖRE BAĞIL VERİMLİLİĞİNİN VERİ ZARFLAMA YÖNTEMİ İLE ÖLÇÜLMESİ

Ahmet TILFARLIGİL

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ÖZ

Bu çalışmada Gaziantep'te bulunan 12 lisenin ÖSS sonuçlarına göre bağıl verimlilikleri veri zarflama analizine (VZA) göre ölçülmüştür. Lineer programlama tabanlı bir program olan veri zarflama analizi bankalar, işletmeler, hastaneler ve turizm işletmeleri gibi karar verme birimlerinin verimliliklerini değişken girdi ve çıktılara göre hesaplamada kullanılmaktadır. Çalışmada her sınıftaki ortalama 11. sınıf öğrenci sayısı, 11. sınıf öğretmenlerinin haftalık ders yoğunlukları, ÖSS'ye giren öğrenci sayısı, 11. sınıflardan dershaneye giden sayısı girdi olarak alınmıştır. Çıktı olarak ise öğrencilerin sayısal, sözel puan ortalamaları ve okulun yerleştirme oranları alınmıştır. Çalışma sonunda herhangi bir lisenin ÖSS sonuçlarına göre potansiyel iyileştirmelerine veri zarflama analizi yardımıyla karar verilebileceği ortaya çıkmıştır. VZA bağıl verimlilik ölçmede güçlü bir araçtır.

Anahtar Kelimeler: Veri Zarflama Analizi, Okulların Bağıl Verimliliği.

To my parents and family

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SYMBOL /ABBREVIATION

STO	Number of Students taking OSS exam
1/SIC	Inverse of Average of Number of Students in Each Class.
POPC	Percentage of students attending OSS preparation courses
1/WCHET	Inverse of Average of Weekly Course Hour of Each 11th Class Teachers
SAY	Average of students' SAY Marks in OSS exam.
EA	Average of students' EA Marks in OSS exam.
PSRU	Percentage of students registering university.

CHAPTER 1

INTRODUCTION

The level of improvement of a society is properly related on the level of education. After the foundation of Republic of Turkey in 1923 Turkish people changed their educational system. This system was based on to insist all kinds of modern paradigms on education and targeted to reach the society to the level of highly developed countries. “There have been many types of innovations and changes of beliefs on education has become at each decade. Unfortunately we did nothing (Selçuk, 2006). “Currently, The Ministry of Education is trying to change the former system starting from primary schools by using constructivist approach that is, students will learn the subjects with applying daily life. According to this reform, once a young fellow graduated from a high school, s/he might have a professional job. Vocational schools are to encourage like in European countries.

But our high schools are still in the position of preparing students OSS exam (University Entrance Exam) since there is no any other valid alternative way for a Turkish teenager to have a job. It seems that at least a half of decade needs to set this new system in the society.

Imbalance between the number of students graduated from high schools in the country and the capacity of universities necessitates a widespread choosing system. OSS was emerged from this obligatory. Therefore the quality of a high school are based solely on the number of students entered the university. Private Schools and University Entrance Exam Courses (called “Dershane”) are placed at the core of notion. More OSS preparing means more education.

So what about public schools? In the light of ideas mentioned above, it is inevitable fact that they are in the same case. The unique target is to send much more students to universities. Thus to measure the level of efficiency of some public schools and some private schools is the core of this master thesis.

Education level of cities in Turkey is based on level of number of students entered university in the city. Every year OSYM publishes the results in a book and also exists in their website (www.osym.gov.tr). Especially private schools look forward to these results since they are used for advertisements of their school. You can see many types of ads on billboards such as “OSS’de 100% Başarı” (%100 Success! In OSS) unfortunately, public schools do not care much more the results because nobody asks the results to public schools’ administrations.

Therefore the questions; what is real success which billboard tells the truth, come to our minds. It would be better to differentiate schools with respect to educational level. That is which school is giving best education in best educational circumstances. Thus relative efficiency of high schools would become important in this subject.

Data Envelopment Analysis (DEA) is one of most widely used method for measuring the relative efficiency of decision-making units on the basis of multiple inputs and outputs. The efficiency of a unit is defined as a weighted sum of its outputs divided by a weighted sum of its inputs and it is measured on a bounded ratio scale. The weights for inputs and outputs are estimated in the best advantage for each unit so as to maximize its relative efficiency (Maragos, 2002). DEA is derived from linear programming. Thus, in Chapter II linear programming and its components were mentioned. Chapter III tells about what DEA is with components and with an example. And also the methodology of our thesis has landed in Chapter III. DEA in education and data collection were cited. In Chapter IV case study of the thesis is stated and conclusion existed in Chapter V.

CHAPTER 2

LINEAR PROGRAMMING

Linear Programming is a technique that helps in resource allocation decisions. Programming refers to modeling and solving a problem. In fact linear programming is a mathematical model that has two basic components (Render, 2003). First, the goal is to maximize (or minimize) some objective function such as profit. The objective function is expressed as a linear function which contains the decision variables and is called the objective function. Secondly, there are constraints that limit the degree to which the objective can be met which are also expressed as linear functions (Anderson et al., 1997). The word "optimizing" is at the core of meaning of linear programming. Programming is total of activities to optimize some specific objectives in limited constraints. If the objective is linear then the programming is simply linear (Cerit, 1996). Linear Programming is widely used technique in different types of areas like industry, business, computer science etc. Finding the optimum number of products to produce in a factory and getting the optimum (actually least) cost or optimum benefit (actually most) of goods in a business are well known examples.

2 .1 THEORY BEHIND LINEAR PROGRAMMING

As it is said before that linear programming is series of operations to find optimal point of objective functions (Render et al, 2003). The functions are linear and with more than one variables. For example $Z_{\max} = 2x_1 + 5x_1$ and constraints of the function;

$$4x_1 + 2x_2 \leq 100$$

$$2x_1 + 3x_2 \leq 300$$

$$x_1, x_2 \geq 0$$

The above inequalities must be in the form of linearly dependent vectors and elements of a convex set. Then let me first tell about vectors, linearly dependent or independent vectors, semi-plane, hyper-plane and convex sets.

2.1.1 Vectors

Any ordered combinations of n time real numbers $p_1, p_2, \cdots p_n$ and $P = (p_{1,} p_2, \cdots, p_n)$ is called n-dimensional vector, where as p_i is the i^th component of vector P. Total of two n-dimensional vectors is a vector constructed by the sum of correspondent components of two vectors;

$P = (p_{1,} p_2, \cdots, p_n)$ and $Q = (q_{1,} q_2, \cdots, q_n)$ then $P + Q = (p_1 + q_1, p_2 + q_2 \cdots p_n + q_n)$.

$P - Q$ is defined as $P + (-Q) = (p_1 - q_1, p_2 - q_2 \cdots p_n - q_n)$.

The product of an n-dimensional vector P with a scalar α is

$$A.P = \alpha(p_1, p_2, \cdots p_n) = (\alpha p_1, \alpha p_2, \cdots, \alpha p_n).$$

0 (zero vector) is a vector that all components are zero;

$$0 = (0 \cdots 0)$$

Note that an n- dimensional vector is expressed as a $(1xn)$ “row matrix” and $(nx1)$ “column matrix”. ie.

$$P = (p_{1,} p_2, \cdots, p_n) \text{ or } P = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix}$$

Let n-dimensional Euclidean space be En and $P_1, P_2, \cdots P_3$ are the elements of E_n and μ, λ are scalars then the following properties are valid.

- $P_1 + (P_2 + P_3) = (P_1 + P_2) + P_3$ (Associative Property)
- $P_1 + P_2 = P_2 + P_1$ (Commutative Property)
- $P + 0 = 0 + P$ (Existence of Zero Vector)
- For all P, there exists $-P$ where $P + (-P) = 0$. (Inverse of an Element)
- $(\mu\lambda)P = (\mu)\lambda P$
- $\lambda(P_1 + P_2) = \lambda P_1 + \lambda P_2$
- $1.P = P$

2.1.2 Linearly Independent or Dependent

Let $P_1, P_2, \cdots P_n$ be vectors and $x_1, x_2, \cdots x_n$ are scalars, $x_1 P_1 + x_2 P_2 \cdots + x_n P_n = 0$ is true if and only if when $x_1 = x_2 = \cdots = x_n = 0$ then

$P_1, P_2, \cdots P_n$ are called linearly independent vectors if not linearly dependent vectors. For example,

$$P_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, P_2 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, x_1 P_1 + x_2 P_2 = 0 \Rightarrow x_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_1 + 2x_2 = 0, \quad 3x_1 + 5x_2 = 0 .$$

The above equation system is true when $x_1 = x_2 = 0$. This means that P_1 and P_2 vectors are linearly independent. If P_1, P_2 vectors are linearly independent then the value of matrix is not equal to zero, as mentioned below;

$$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} = \text{-}1 \neq 0.$$

As a counter example, $P_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $P_3 = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ vectors are linearly dependent since $x_1 P_1 + x_3 P_3 = 0 \Rightarrow x_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_1 + 2x_3 = 0$ and $3x_1 + 6x_3 = 0$. The below system of equations is valid for $x_1 = 2, x_3 = -1$. That is, the system does not need $x_1 = 0, x_3 = 0$. And the value of matrix is zero as below;

$$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} = 0.$$

Therefore P_1 and P_3 are linearly dependent. Generally speaking, for 2-dimensional Euclidean space the vectors $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are linearly independent as it can be seen easily that;

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 \neq 0.$$

For any $P=(p_1,p_2)=p_1e_1+p_2e_2=p_1\begin{bmatrix}1\\0\end{bmatrix}+p_2\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}p_1+0\\0+p_2\end{bmatrix}=\begin{bmatrix}p_1\\p_2\end{bmatrix}$. So that $P=\begin{bmatrix}p_1\\p_2\end{bmatrix}$ can be expressed as any linear combination of vectors e_1 and e_2. For n-dimensional Euclidean space the vectors

$$e_1=\begin{bmatrix}1\\0\\.\\.\\.\\0\end{bmatrix},\ e_2=\begin{bmatrix}0\\1\\.\\.\\.\\0\end{bmatrix},\ .\ .\ .\ ,e_n=\begin{bmatrix}0\\0\\.\\.\\.\\1\end{bmatrix}$$ are linearly independent.

$$\begin{vmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&0&1\end{vmatrix}=1\neq 0$$

Base; n-dimensional linearly independent n vectors of E_n form a basement in E_n. That is, any element of E_n can be expressed as linear combination of other vectors in E_n. Let me explain the above definition with the help of mathematical expressions. If P_1, P_2 . . . P_n are linearly independent vectors that is form a basement in En. Thus vector Q Є E_n can be written in linear combinations of $P_1,P_2,\cdots P_n$;

$$x_1P_1+x_2P_2\cdots+x_nP_n=Q$$

or

$$x_1(p_{11},p_{21},\cdots p_{n1})+x_2(p_{12}+p_{22},\cdots p_{n2})+\cdots+x_n(p_{1n},p_{2n},\cdots,p_{nn})=(p_1,p_2,\cdots,p_n)$$

$$x_1p_{11}+x_2p_{12}+\cdots+x_np_{1n}=q_1$$

$$x_1p_{21}+x_2p_{22}+\cdots+x_np_{2n}=q_2$$

....

$$x_1p_{n1}+x_2p_{n2}+\cdots+x_np_{nn}=q_n \qquad (2.1)$$

The above system has one only one solution satisfying $x_1,x_2,\cdots,x_n$. It would be better to give a simple example the above formula. For example $P_1=\begin{bmatrix}2\\3\end{bmatrix}$, $P_2=\begin{bmatrix}-1\\5\end{bmatrix}$ be two vectors arbitrarily chosen from E_2. Since $\begin{bmatrix}2&-1\\3&5\end{bmatrix}=13\neq 0$ then P_1 and P_2 are linearly independent and form a basement in E_2. For instance take a vector $Q = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$ that can be expressed in linear combination of P_1 and P_2.

$$x_1P_1 + x_2P_2 = Q \Rightarrow \; x_1\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \; x_2\begin{bmatrix} -1 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix} \Rightarrow 2x_1 - x_2 = 3 \Rightarrow \text{ and } \; 3x_1 + 5x_2 = 11$$

solved $x_1 = 2, x_2 = 1$. The above expression says us that the sum of "2" times of P_1 and "1" time of P_2 gives the result as Q. If we choose base vectors $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ then we get that $P_1 = 2e_1 + 3e_2, P_2 = -e_1 + 5e_2, Q = 3e_1 + 11e_2$.

2.1.3 Semi Plane and Semi Hyper Plane

It is easily seen that graph of any linear equation like $a_1x_1 + a_2x_2 = b$ divides the coordinate plane into two parts. One part of the plan is solution set of $a_1x_1 + a_2x_2 - b > 0$, another is $a_1x_1 + a_2x_2 - b < 0$. Let's take the equation $2x_1 + 3x_2 = 6$. The numbers satisfying $2x_1 + 3x_2 - 6 > 0$ are on the upper part of the graph and $2x_1 + 3x_2 - 6 < 0$ is at down.

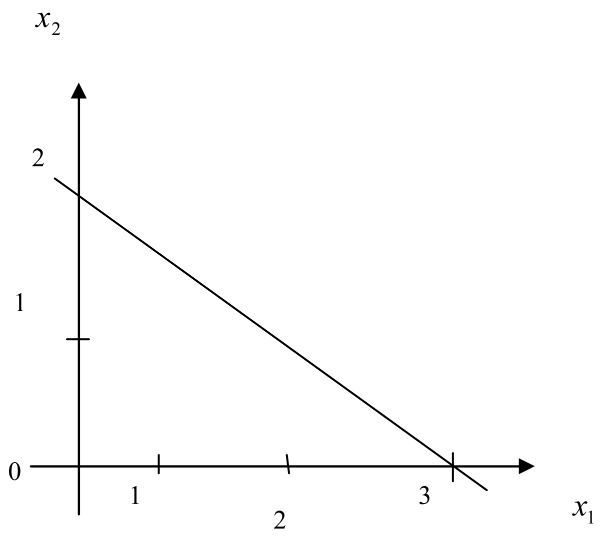

Figure 2.1 Hyper Plane

Generally, $a_1, a_2, \cdots, a_n$ and b are constants in E_n. $a_1x_1 + a_2x_2 \cdots a_nx_n = b$ defines a hyper plane and E_n divides this plane into two parts. One of the parts $a_1x_1 + a_2x_2 - b < 0$ and the other is $a_1x_1 + a_2x_2 - b > 0$.

2.1.4 Convex Combination and Convex Sets

$\alpha_i \geq 0$, $\sum_{i=1}^{n} \alpha_i = 1$ such that $U = \alpha_1 U_1 + \alpha_2 U_2 + \cdots \alpha_n U_n$ are called convex combination of $U_1, U_2, \cdots, U_n$. (2.2)

E. g. $U_1 = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$, $U_2 = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$, $U_3 = \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$ and $\alpha_1 = 1/5, \alpha_2 = 2/5, \alpha_3 = 2/5$. The vector satisfying the following equation;

$$U = \alpha_1 U_1 + \alpha_2 U_2 + \alpha_3 U_3 = 1/5 \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} + 2/5 \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix} + 2/5 \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 7/5 \\ 11/5 \end{bmatrix}$$

is an convex combination of $U_{1,} U_2, U_3$. Convex Set; if convex combination of two points of set K is element of K then K is called a convex set or If $U_1, U_2 \in K$ and $\alpha_1, \alpha_2 \geq 0,\quad \alpha_1 + \alpha_2 = 1 \Rightarrow \alpha_1 U_1 + \alpha_2 U_2 = U \in K$ then K is a convex set. Let me try to explain relationship between semi-plane and convexity of sets.

Theorem 2.1: A semi plane is a convex set.

Proof: (x_1, x_2) plane is divided into two parts by line; $a_1 x_1 + a_2 x_2 = b$. Let points $U_1(\alpha_1, \alpha_2)$ and $U_2(\beta_1, \beta_2)$ be in part of the plane where $a_1 x_1 + a_2 x_2 > b$.

We will show that any $U\ (\gamma_1, \gamma_2)$ which is any convex combination $U_1(\alpha_1, \alpha_2)$ and $U_2(\beta_1, \beta_2)$ lands on the where U_1 and U_2 live. U can be written in terms of U_1 and U_2: $0 \leq \alpha \leq 1$ such that

$U = \alpha U_1 + (1 - \alpha\ ) U_2 \Rightarrow (\gamma_1, \gamma_2) = \alpha(\alpha_1, \alpha_2) + (1 - \alpha)(\beta_1, \beta_2),$

$(\gamma_1, \gamma_2) = [\alpha\alpha_1 + (1-\alpha)\beta_1, \alpha\alpha_2 + (1-\alpha)\beta_2] \Rightarrow \gamma_1 = \alpha\alpha_1 + (1-\alpha)\beta_1, \gamma_2 = \alpha\alpha_2 + (1-\alpha)\beta_2$

Now let see that γ_1, γ_2 satisfies $a_1[\alpha\alpha_1 + (1-\alpha)\beta_1] + a_2[\alpha\alpha_2 + (1-\alpha)\beta_2]$

$= \alpha[a_1\alpha_1 + a_2\alpha_2] + (1-\alpha)(a_1\beta_1 + a_2\beta_2) \geq \alpha b + (1-\alpha)b = b.$ so the proof is complete.

Theorem 2.2 is generalized by another theorem that If K is convex set then any point formed by any number of convex combinations of points of K is the element of K. That is the convex combination of n points is also belonged to K.

Theorem 2.3: Any point on line segment from E_n can be defined in terms of convex combination of end points of the line segment.

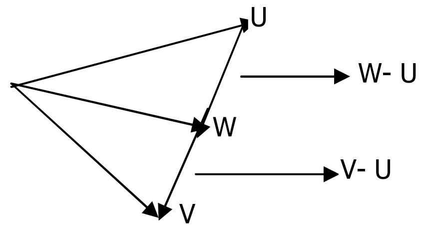

Figure 2.2 Convex Combination

Proof: Let U and V are end points of line segment and W be any point on the line segment (Figure 2.2). V-U and W-U vector have same directions. By $0\leq \alpha \leq 1$, W-U = α (V-U) => W=$(1-\alpha)$U + αV. This means that W is suitable convex combination of U and V. Some convex and non-convex (concave) sets are shown in Figure 2.3

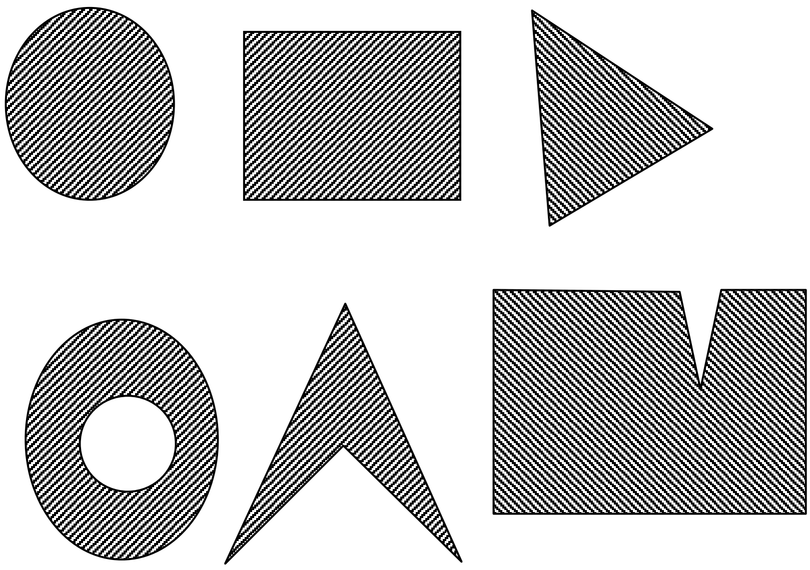

Figure 2.3 Convex and Concave Sets

Generally, we say that a linear programming problem is to find a column vector $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ which satisfies the following objective (maximize or minimize) function with constraints for $m < n$ and $c_j \ a_{ij}, b_i$ $(i = 1, 2, \cdots, m, \quad j = 1, 2, \cdots, n)$ are constants.

$$Z_{\max} = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

$$x_1a_{11} + x_2a_{12} + \cdots + x_na_{1n} = b_1$$

$$x_1a_{21} + x_2a_{22} + \cdots + x_na_{2n} = b_2$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$x_1a_{m1} + x_2a_{m2} + \cdots + x_na_{mn} = b_n$$

$$x_1, x_2, \cdots, x_n \geq 0 \qquad (2.3)$$

The above form is called standard form of linear programming.

2.2 SIMPLEX METHOD

Simplex method is the first generated method to solve linear programming problem. Before explain the simplex method it is better to give some basic theorems in which the simplex method emerged.

Theorem 2.4 A collection of solution sets of a linear programming problem is a convex set.

Proof: Let solution K be set. K has one element which is X. Any linear combination of X for $0 \leq \alpha \leq 1$, $\alpha X + (1-\alpha)X = X \in K$ so the proof is complete for K has one element. Let K has at least two solutions like X_1 and X_2. $X_1 \geq 0$ and $AX_1 = b$, $X_2 \geq 0$ and $AX_2 = b$ For $0 \leq \alpha \leq 1$, let any linear convex combination of X_1 and X_2 is $\alpha X_1 + (1-\alpha)X_2 = X$. It is easily seen that $X \geq 0$.

Let's show hat $AX = b$ $AX = A[\alpha X_1 + (1-\alpha)X_2] = \alpha A + (1 - \alpha AX_2) = \alpha b + (1-\alpha)b = b$.

Therefore the equation above i.e. the convex combination of any two solutions is also a solution. This means that solution set K is a convex set. Generally, solution set K of linear programming problem $AX = b$, $X \geq 0$ with linear constraints is defined by hyper planes. There are three cases for K. K is a null set. K is a convex polygon with the element of En. K is a half-open convex region belonged to En. The following examples show three cases of K.

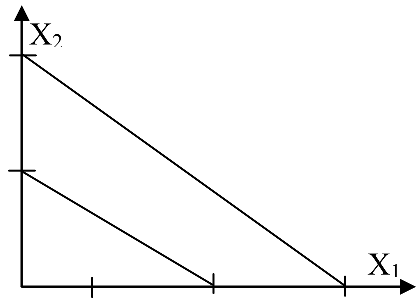

Figure 2.4 K is a null set

$x_1 + 2x_2 \le 2$

$2x_1 + 3x_2 \ge 6$

$x_1, x_2 \ge 0.$ The above inequalities have no solution set.

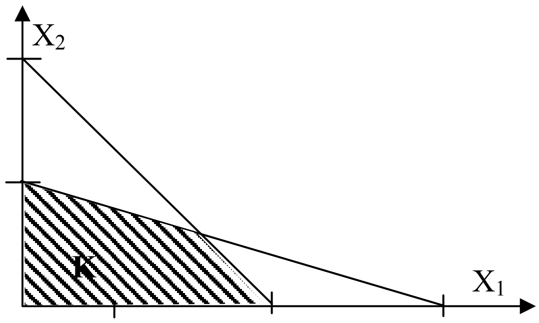

Figure 2.5 K is convex

$x_1 + x_2 \le 2$

$x_1 + 3x_2 \le 3$

$x_1, x_2 \ge 0.$ The above inequalities have solution which is convex polygon K.

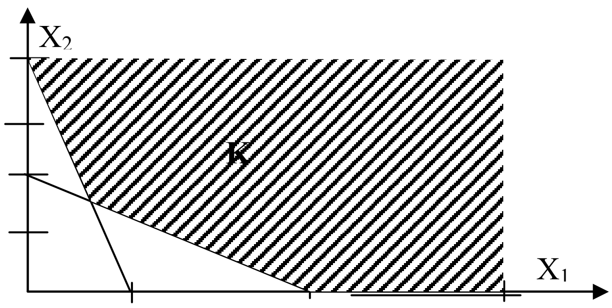

Figure 2.6 K is half open

$x_1 + x_2 \ge 2$

$4x_1 + x_2 \ge 4$

$x_1, x_2 \ge 0.$

The above inequalities have solutions which is half open convex polygon K.

Another theorem says that the objective function has minimum the corner points of K. The last theorem reaching the method is that; if the point $X=(x_1,x_2,\cdots,x_n)$ is the corner point of K which is solution set then vectors corresponded positive x_i's are linearly independent. This can be easily seen in the following example.

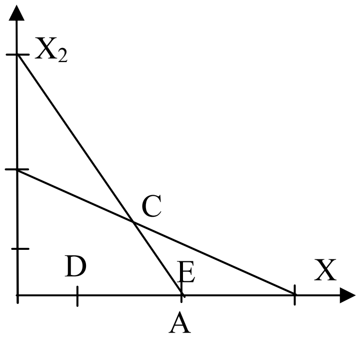

Figure 2.7 End Points

$Z_{\max}=2x_1+5x_2$

$3x_1+2x_2\le 6$

$2x_1+3x_2\le 6$, $x_1,x_2\ge 0$. Let rewrite the problem in standard form.

$$Z_{\max}=2x_1+5x_2+0x_3+0x_4$$
$$3x_1+2x_2+x_3 \quad =6$$
$$2x_1+3x_2+ \quad x_4=6$$
$$x_1,x_2,x_3,x_4\ge 0. \tag{2.4}$$

In the above problem if we construct vectors. We see that these vectors are linearly independent. This idea is basis of corner point method of linear programming. But the Simplex method is the systematic way of corner point method. So let me give an example for simplex method.

2.2.1 An Example for Simplex Method

The problem maximizing $Z=4x_1+3x_2$ subject to the constraints

$$4x_1+x_2\le 3$$
$$2x_1+3x_2\le 4$$
$$x_1,x_2\ge 0.$$

2.2.1.1 Slack Variables

Slack variables represent unused capacity in the constraints:

$$4x_1 + x_2 + s = 3$$

$$2x_1 + 3x_2 + t = 4$$

$$x_1, x_2 \geq 0.\ s, t \geq 0.$$

Rewrite the objective to get the (ideal) objective function (i.e. all excess capacity used)

$$4x_1 + 3x_2 + 0s + 0t = Z$$

$$Z - 4x_1 - 3x_2 - 0s - 0t = 0.$$

2.2.1.2 Simplex Table

This application of the simplex algorithm uses tables, to represent calculations and intermediate steps to completing a problem. The first step is to form an initial table. We have four variables: x, y, s, t, and two equations. Therefore, we have to set two variables to zero to solve them. The variables set to zero are called non-basic variables. The variables not set to zero are called basic variables. For the initial table, the basic variables are always the slack variables. The initial table is set up as follows:

Table 2.1 Initial Table

Basic variable	x_1	x_2	s	t	Value
S	4	1	1	0	3
T	2	3	0	1	4
Z	-4	-3	0	0	0

The first row represents the first constraint. The second row represents the second constraint. The third row represents the objective function Notice that the basic variables appear only in one row, and only with a coefficient of one, and in their column, all other entries read zero. This table represents the solution $x_1 = 0$, $x_2 = 0$, s = 3, t = 4, Z = 0. To find these values, the first and last columns are read off: s = 3, t = 4, Z = 0. We know x and y are zero because they are non-basic variables.

A table represents an optimal solution if the objective row contains zero entries in the columns of basic variables. The objective row contains no negative entries in the columns of non-basic variables. Table 2.1 does not satisfy this condition so the solution is not optimal. We therefore modify this table to give a second one corresponding to another vertex in the linear programming space.

We need to find a non-basic variable to become a basic variable and also the basic variable to replace with this non-basic variable. To find the new basic variable, select the column with the smallest entry in the objective row. In Table 2.1 this is the x_1 column with -4. This is called the pivotal column. In order to find the basic variable being replaced, we calculate θ values for each row (except the objective row). θ is given by the entry in the value column divided by the entry in the pivotal column. For the first row: θ = 3/4. For the second row: θ = (4/2) = 2.

The pivotal row is then taken to be the smallest of these θ values. The pivot is found at the intersection of the pivotal row and the pivotal column. It is usually ringed or shown in bold:

Table 2.2 Table with Pivot

Basic variable	x_1	x_2	s	t	Value
S	**4**	**1**	**1**	**0**	**3**
T	**2**	3	0	1	4
Z	**-4**	-3	0	0	0

The first step to Table 2.2 is to divide the pivotal row by the pivot so the pivot now reads 1:

Table 2.3 Dividing Pivotal Row

Basic Variable	x_1	x_2	s	t	Value
s	1	1/4	1/4	0	3/4
t	2	3	0	1	4
Z	-4	-3	0	0	0

The next step is to add or subtract multiples of this new row 2 from the other rows in the tables, so that they read zero, as they should for a basic variable. For the second row (the t row), subtract 2 x Row 1 (the s row)

Table 2.4 Adding or Subtracting 1

Basic Variable	x_1	x_2	s	t	Value
s	1	1/4	1/4	0	3/4
t	0	2.5	-0.5	1	2.5
Z	-4	-3	0	0	0

For the third row (the Z row), add 4 x Row 1 (the s row):

Table2.5 Adding or Subtracting 2

Basic Variable	x_1	x_2	s	T	Value
s	1	1/4	1/4	0	3/4
t	0	2.5	-0.5	1	2.5
Z	0	-2	1	0	3

The final step is to replace the old basic variable with the new:

Table 2.6 Adding or Subtracting 3

Basic variable	x_1	x_2	s	T	Value
x_1	1	1/4	1/4	0	3/4
t	0	2.5	-0.5	1	2.5
Z	0	-2	1	0	3

This table represents the solution where t = 2.5, x_1 = 3/4, x_2 = 0, s = 0, Z = 3. However there is still a negative value in the objective row, so this solution is not optimal either. Therefore a third tableau is needed. Repeat the pivoting process. The pivotal column is the one with −2 in the objective row, which is 'y'. The pivotal row is the one with the smallest θ value:

For the first row: $\theta = \dfrac{\frac{3}{4}}{\frac{1}{4}} = 3$ For the second row: $\theta = \dfrac{\frac{5}{2}}{\frac{5}{2}} = 1$

Therefore, the pivot is the intersection of the t-row and the y-column. Now divide the pivotal row by the pivot.

Table 2.7 Adding or Subtracting 4

Basic variable	x_1	x_2	s	T	Value
x_1	1	1/4	1/4	0	3/4
t	0	1	-0.2	0.4	1
Z	0	-2	1	0	3

Then add or subtract multiples of this row to the remaining rows:
Row 1: Subtract 1/4 × Row 2, Row 3: Add 2 × Row 2 then, replace the basic variable:

Table 2.8 the Final

Basic variable	x_1	x_2	s	T	Value
x_1	1	0	0.3	-0.1	0.5
x_2	0	1	-0.2	0.4	1
Z	0	0	0.6	0.8	5

This table represents the solution where $x_1 = 0.5$, $x_2 = 1$, s = 0, t = 0, Z = 0. It is an optimal solution, as there exists no negative basic variables and all non-basic variables are zero. This method works with any number of variables, but it may take longer. (Ottery, 2006)

CHAPTER 3

DATA ENVELOPMENT ANALYSIS

In this chapter, theory behind data envelopment analysis and methodology of the study are given. Data envelopment analysis (DEA) was first introduced in the literature in 1978 (Charnes et al. 1978). It is an empirically based methodology that eliminates the need for some of the assumptions and limitations of traditional efficiency measurement approaches. It was originally intended for use as a performance measurement tool for organizations that lacked a profit motivation, e.g., not-for-profit and governmental organizations. However, since its introduction, it has been developed and expanded for a variety of uses in for-profit as well as not-for-profit situations. (Seiford, 1995), and other sources provide good bibliographies of DEA which include applications to hospitals, education, military, airlines, and other areas.

3.1 THEORY OF DEA

Data envelopment analysis (DEA) is a linear programming based technique for measuring the relative performance of organizational units where the presence of multiple inputs and outputs makes comparisons difficult. Also it is defined as methodology that has been used to evaluate the efficiency of entities (e.g., programs, organizations etc.) which are responsible for utilizing resources to obtain outputs of interest. Linear Programming is the first keyword to be stressed in the definition, since as it is mentioned in Chapter II data envelopment analysis is application of linear programming. Efficiency or relative performance is crucial in DEA that is productivity of an organization. Productivity is measured with respect to others values. To measure efficiency there must be data in our hands so these are inputs and outputs of organization. Inputs and outputs are multiple i.e. different. Thus data envelopment analysis is a method based by linear programming, to measure productivity of an organization with respect to inputs and outputs.

3.2 WHERE USED

There is an increasing concern with measuring and comparing the efficiency of organizational units such as local authority departments, schools, hospitals surgical units, shops, public schools, business companies, banks, tourism sector, and real-property maintenance for the U.S. Air Force and bank branches and similar instances where there is a relatively homogeneous set of units. (Bowlin, 2002)

3.3 LITERATURE REVIEW

Several models have been proposed in the literature on justification of DEA. DEA has become within 20 years a central in productivity and efficiency analysis and can safely be considered one the recent success stories in operational research. (Charnes et al. 1978) After the first appearance of data envelopment analysis in 1978 by Charnes, Cooper ad Rhodes two basic models are CCR (Charnes, Cooper and Rhodes) and BCC (Banker Charnes and Cooper) exercised. However DEA has application mostly on business areas like industry, tourism it is used education past years. “Guiding schools to improved performance using data envelopment analysis” (Thannasoulis et al., 1994) is one of the earliest studies on schools performance in the world. Maragos (2006) measured efficiency of high schools Greece found that socio economic background of students affect productivity. Wooton (2002) has considered DEA as a tool for ranking and benchmarking for secondary schools. She cited that by using DEA performance of any school district in USA can be measured with respect to national exam. Efficiency potential and efficiency variation in Norwegian lower secondary schools was measured that assessment grades as outputs and teacher hours (certified and non-certified) as inputs (Borge et al. 2002). In our country Baysal (1999) and Atan’s (2002) studies are the first appearance of measuring performance schools. A wide variety studies must be done.

3.4 EFFICIENCY AND WEIGHTS

DEA is a fractional programming model that can include multiple outputs and inputs without recourse to a priori weights relations between inputs and outputs. It computes a scalar measure of efficiency and determines efficient levels of inputs and outputs for the organizations under evaluation (Emrouznejad, 2001).

Usually efficiency is measured as:

$$efficiency = \frac{output}{input}$$

This equation is mostly inadequate because of multiple inputs and outputs related to different resources, activities and environmental factors. Therefore efficiency must be measured relatively as:

$$Efficiency = \frac{Weighted\ sum\ of\ outputs}{Weighted\ sum\ of\ inputs}$$

Which

$$Efficiency \quad of \quad unit \quad j = \frac{u_1 y_{1j} u_2 y_{2j} + \cdots}{v_1 x_{1j} v_2 x_{2j} + \cdots}$$

Where

$$u_1 = the \quad weight\ given \quad to \quad output \quad 1$$
$$y_{1i} = amount \quad of \quad ouput \quad 1 \quad from \quad unit \quad j$$
$$v_1 = the \quad weight\ given \quad to \quad input \quad 1$$
$$x_{1i} = amount \quad of \quad input \quad 1 \quad from \quad unit \quad j$$

Efficiency is always between [0, 1]. The highest scoring DMU (Decision Making Units) then is considered the most efficient unit and all others are rated in comparison to this unit. It is important to note that relative efficiency compares all DMU's.

According to the above set, we need weights of inputs and outputs to measure efficiency. So how can it be measured? There are two problems in that. First it may simply be difficult to value the inputs or outputs. That is values of outputs may relate each other so it is difficult to measure. On the other hand different units may choose to organize their operations differently so that the relative values of the different outputs may legitimately be different. That is units may value outputs differently. For example in a school, a success in sports (for instance winning city cup in football) may differ from another school. Becoming in three degrees may be good result for one school but may not be another. Therefore inputs and outputs need different weights. This measure of efficiency coupled with the assumption that a single common set of weights is required is thus unsatisfactory. (Emrouznejad, 2002)

Traditionally, when calculating relative efficiency based strictly on the equation, the decisions for the values of the weights are arbitrarily set by the analyst and can lead to biases against DMU' s. (Wooton, 2003). Hatch (2002) published weights and decision variables for ranking colleges and universities. So an analyst decides a priori the weights of decision variables.

Charnes, Cooper and Rhodes (1978) realized to find common set of weights that each unit should be allowed to adopt a set of weights which shows it in the most favorable light in comparison to the other units. Under these circumstances, efficiency of a target unit j_0 can be obtained as a solution to the following problem. Maximize the efficiency of unit j_0, subject to the efficiency of all units being smaller than equal to 1. The variables of the above problem are the weights and the solution produces the weights most favorable to unit j_0, and also produces a measure of efficiency.

The algebraic model is as follows:

$$Max \quad h_0 = \frac{\sum_r u_r y_{rj_o}}{\sum_i v_i x_{irj_o}} \qquad (3.1)$$

Subject to

$$\frac{\sum_r u_r y_{rj}}{\sum_i v_i x_{ij}} \le 1 \quad for \quad each \quad unit \quad j.$$

$$u_r, v_i \ge \varepsilon$$

Therefore DEA may be appropriate where units can properly value inputs or outputs differently, or where there is a high uncertainty or disagreement over the value of some input or outputs. The above equation (3.1) is not linear so it has to be changed into linear program form so that the methods of linear programming can be applied. By using a few mathematical operations it is easier to get linear form in (3.2) as follows;

$$Max \quad h_0 = \sum_r u_r y_{rj_o} \qquad (3.2)$$

Subject to

$$\sum_i v_i x_{ij_0} = 1$$

$$\sum_r u_r y_{rj} - \sum_i v_i x_{ij} \le 0, \quad j = 1,2,\ldots$$

$$u_r, \quad v_i \quad \ge \varepsilon$$

3.5 DEA MODEL FORMULATION

The formulation described above is the CCR (Charnes, Cooper and Rhodes) Input-Oriented Model. There are four basic DEA model formulations: CCR Input-Oriented, CCR Output-Oriented, BCC (Banker, Charnes and Cooper) Input-Oriented, and BCC Output-Oriented. These models can be classified based on two factors; a) How the efficient frontier is produced, b) How an inefficient DMU is projected onto the efficient frontier.

The efficient frontier is set of DMU's which are considered to be efficient. The efficient frontier can be built assuming a constant returns-to-scale (CRS) or variable returns-to-scale (VRS). The CRS is described by Cooper et al. as "if an activity (x,y) is feasible, then, for every positive scalar t, the activity, (tx, ty) is also feasible. " For example, as the number of tellers in a bank double; we would expect the number of services also double. The CCR model assumes a CRS. In the VRS case, the frontier is produced creating a convex hull with the outermost efficient DMU's. The efficient frontier is built by connecting these relatively efficient DMU's with linear segments. The name variable returns-to-scale results from the fact that these different linear segments may have decreasing, increasing, or constant returns-to-scale. For example, as the number of teachers doubles in a school district, we would not expect to see the number of students graduating from school doubling.

The second classification factor is how an inefficient DMU is "projected onto the frontier". In other words, the focus the model takes for making an inefficient DMU efficient. Model formulations can have an input or output orientation. These two formulations are the inverse of each other. Input orientation focuses on using fewer inputs to produce the same output while output orientation focuses on producing more output with the given inputs. Output oriented formulation is excellent for evaluating DMU's that have little control over the amount of input, but should seek to maximize their outputs given the level of input available. Lack of control over inputs happens in service organizations like school districts where they cannot determine the number of children on free and reduced lunch, their budget, etc. A further explanation of these models can be found by Cooper. (1978)

3.6 DUAL MODEL

For any linear program (LP) it is possible to formulate a partner LP using the same data, and the solution to either the original LP (the primal) or the partner (the dual) provides the same information about the problem being modeled. DEA is no exception to this. The dual model is constructed by assigning a variable (dual variable) to each constraint in the primal model and constructing a new model on these variables. This is shown below.

Primal Model: Dual variables

$$Max \quad h_0 = \sum_r u_r y_{rj_o} \qquad Z \qquad (3.3)$$

Subject to

$$\sum_i v_i x_{ij_0} = 1 \qquad \lambda_0$$

$$\sum_r u_r y_{rj} - \sum_i v_i x_{ij} \leq 0, \quad j = 1,2,\ldots n$$

$$-v_i \leq -\varepsilon \qquad i = 1,2,\ldots m \qquad s_1^+$$

$$-u_r \leq -\varepsilon \qquad r = 1,2,\ldots t \qquad s_1^-$$

Dual Model

$$Min \quad Z_0 - \varepsilon \sum_r s_r^+ - \varepsilon \sum s_i^- \qquad (3.4)$$

Subject to

$$x_{ij_0} Z_0 - s_i^- - \sum_j x_{ij} \lambda_j = 0 \qquad i = 1,\cdots\cdots,m$$

$$-s_i^+ + \sum_j y_{rj} \lambda_j = y_{rj_0} \qquad r = 1,\cdots\cdots,t$$

$$s_r^+, s_i^- \lambda_j \geq 0, \quad Z_0 \quad unconstrained.$$

The first thing to note is that the primal model has n + t + m + 1 constraints while the dual model has m + t constraints. As n, the number of units, is usually considerably larger than t + m, the number of inputs and outputs, it can be seen that the primal model will have many more constraints than the dual model. For linear programs in general the more constraints the more difficult a problem is to solve. (Emrouznejad, 2001) Hence for this reason it is usual to solve the dual DEA model rather than the primal. From the theory of linear programming it is known that the values of the dual variables as a result of solving a dual model are identical to the shadow prices in the primal model. The dual variables λj's are thus also the shadow prices related to the constraints limiting the efficiency of each unit to be no greater than 1. It is also known that where a constraint is binding, a shadow price will be positive normally and where the constraint is non-binding the shadow price will be zero. If a constraint is a non-binding this means that it does not affect the solution and vice versa i.e. if binding than it is limiting factor to the solution. In the solution to the primal model therefore a binding constraint implies that the corresponding unit has an efficiency of 1 and there will be a positive shadow price or dual variable. Shadow price is defined as the change in the objective value of the optimal solution of an optimization problem obtained by relaxing the right hand side of the constraint by one unit. This is also referred to as a dual variable. Hence positive shadow prices in the primal, or positive values for the λ_j in the dual, correspond to and identify the peer group for any inefficient unit. The dual model can also be interpreted in terms of the composite unit introduced in the previous section. Rearranging the previous equations we get:

$$Min \quad Z_0 \quad -\varepsilon\sum_r s_r^+ - \varepsilon\sum s_i^- \qquad (3.5)$$

Subject to

$$\sum_j x_{ij}\lambda_j = x_{ij}Z_0 - s_i^- \qquad i = 1,\cdots\cdots,m$$

$$\sum_j y_{rj}\lambda_j = y_{rj_0} + s_r^+ \qquad r = 1,\cdots\cdots,t$$

$$s_r^+, s_i^- \lambda_j \geq 0, \quad Z_0 \qquad unconstrained.$$

The solution to this model seeks value of λ_j to form a composite unit with outputs $\sum_j \lambda_j y_{r_j}, r = 1,\cdots\cdots,t$ and inputs $\sum_j \lambda_j y_{r_j}, \quad r = 1,\cdots\cdots,m$ more efficient than j_0.

Note that since the slack variables are non-negative and Z_0 cannot exceed l, the composite unit has input levels that do not exceed those of unit j_0 and output levels that are at least as high). If unit j_0 is indeed efficient, the slacks will equal 0 and Z_0 will equal 1, i.e. it has proved impossible to find a composite unit outperforming unit j_0. If j_0 is not efficient Z_0 will be less than l and some slacks may be positive, i.e. it has proved possible to find a more efficient composite unit. The Lambda (j) 's form an efficient composite unit providing targets for j_0, and Z_0 represents the proportion of the input levels of j_0 that the efficient composite unit would require to produce at least the output levels of j_0. Z_0 is thus a measure of the efficiency of j_0. The composite unit thus provides a set of targets for an inefficient unit.

3.7 RANKING METHODS IN DEA

In general ranking is considered to be a well founded approach for analyzing units. Although Data Envelopment Analysis differentiates decision making units with respect to their efficiencies or inefficiencies, there would need to improve ranking capabilities of DEA. A fully ranking techniques, i.e. which is more efficient than the others, exist like super-efficiency, cross-efficiency, benchmarking, multivariate statistics. (Adler et al., 2002), canonical correlation analysis (Friedman et al. 1997)

It would be better to tell why ranking techniques emerged; lack of discrimination in DEA applications, in particular when a) there are insufficient DMU's. Actually number of DMU's must be equal to at least two times the sum of inputs and outputs.
b) the number of inputs and outputs is too high relative to the number of units.

Super-efficiency model developed by Andersen and Petersen in 1993 enables an extreme efficient unit *k* to achieve an efficiency score greater than one by removing the k^{th} constraint in the primal formulation.

Cross-efficiency (Sexton et al. 1986) calculates the efficiency score of each DMU *n* times, using the optimal weights evaluated by *n* LPs. The results o all the DEA cross efficiency scores can be summarized in a matrix which is called the Cross Efficiency Matrix (CEM). (Friedman et al. 1998) The cross efficiency matrix (CEM) provides information how well a DMU is performed with respect to the optimal DEA weights of other DMU's (Talluri et al., 2000)

Benchmarking technique measures the importance of DMU's as a benchmark for inefficient DMU's. (Torgersen et al. 1996) It occurs in two stages; a) evaluating the value of slacks for which the set of efficient units is identified as their slacks are zero b) applying this to all DMU's. Multivariate statistics techniques is emerged from the idea that gap between data envelopment analysis and classical statistical approaches. It is known that DEA is a methodology directed towards frontiers rather than central tendencies. In DEA the value of weights differ from unit to unit, while regression searches through center of data and different weights can not be used for ranking efficiently. Therefore classical statistical techniques are needed. (Adler et al., 2002)

The canonical correlation analysis (CCA) is utilized to provide a full rank scaling for all units rather than a categorical classification (for efficient and inefficient units. When there is only one output y_j and multiple inputs x_j the regression analysis provides weights for all inputs, common to all the observations (units): $\hat{}y_j=x_j.v$ In CCA we rank the units in decreasing order of the ratios $y_j/\hat{}y_j$ thus all the units are ranked on the same scale – ratio between the single output and the composite unit. For fully ranking a new scaling T as a ratio of linear combinations of the outputs (W) and inputs (Z) is defined in CCA. Then we utilize the common weights for the linear combinations that come from the largest eigenvalue of the CCA method:

$$T_j = \frac{W_j}{Z_j} = \frac{\sum_{r=1}^{s} U_{1r} y_{rj}}{\sum_{i=1}^{m} V_{1i} x_{ij}} \qquad j=1\ldots n. \tag{3.6}$$

If the first eigenvector (V_1) is not strictly positive then the variable the negative weight is deleted. After deleting the variable then the new significant eigenvalue is taken if its weights are strictly positive. If not we can not find common weights for CCA/DEA method. Note that while the DEA efficiency ratio is bounded by 1, but the scaling ratio T of the CCA/DEA unbounded can exceed 1. (Friedman et al. 1997) After we obtain a ranking scale, we can continue the analysis by combining the CCA with DEA.

3.8 WEIGTHING LIMITATIONS OF DEA

The goal of performance evaluation can be classified into two major parts: efficiency and effectiveness. Efficiency has to do with the relationship of the ratio between the input of the resources into the system and the output from the system. Effectiveness usually indicates the degree of fulfillment of the predetermined goal and highest effectiveness by using the most efficient method. (Liu et al., 2005) DEA has capabilities of mentioned properties but it has fitness and limitation problem. There are three basic approaches on limiting weights in DEA:

1. The Absolute Range: The values of the weighting variables in the DEA model are confined by the upper and lower bounds, however those bounds are obtained through historical data or experts' opinion. As shown below

$$L_{Ii} \leq v_i \leq U_{Ii}$$
$$L_{Or} \leq u_r \leq U_{Or}$$

2. The Assurance Region (AR): In DEA model sometimes the ratio of two factors has some kind of relationship, and under this condition it sis required to confine the corresponding ratio for the weights of these two factors to a range. Actually for

DEA problems with a finite number of DMU's and a well-defined data domain, an AR is a subset of all virtual multipliers. (Thompson et al. 1997)

$$L_{Ii} \leq (v_i)/(v_1) \leq U_{Ii}$$
$$L_{Or} \leq (u_r)/(u_1) \leq U_{Or}$$

3. The Polyhedral Cone-Ratio: it requires the weighting variables to satisfy the restriction of a multiple-face cone as shown below:

$$u_r \in U \; v_i \in V$$
$$U = c_1u_1 + c_2u_2 + c_3u_3 + . \; . \; . + c_su_s \geq 0$$
$$V = d_1v_1 + d_2v_2 + d_3v_3 + . \; . \; . + d_mv_m \geq 0 \qquad (3.7)$$

3.9 AN EXAMPLE FOR DEA

Let's we have DMU's as P_1, P_2 . . . P_6 with each unit consuming the same amount of a single resource and producing different amounts of outputs, y_1 and y_2. For a given amount of resource input, units providing greater amounts of the outputs will be the efficient ones. (Figure 3.1)Applying the DEA approach to this set of units will identify units P_1, P_2 P_3 and P_4 as efficient and they provide an envelope round the entire data set units P_5 and P_6 are within this envelope and are inefficient. The data envelopment analysis has been notionally extended to the axes by the lines P_1 $y`_2$ and $P_4$ $y`_1$ to enclose the data set. P_5 could become efficient and move to the efficient frontier at point P_1, by increasing its outputs or decreasing its input. P_1 is the closest "efficient peer" of P_5, and in fact it is the model unit for the inefficient unit P_5. This is another distinct characteristic of DEA. Apart from the efficiency scores, it also provides guidelines for improvement and specific targets for the inefficient DMU's. For unit P_5 the peer group consists of the units P_1 and P_2 and a set of targets for P_5 is provided at $P`_5$. These targets are obtained by a pro rata increase in the outputs of unit P_5. Clearly there are other possible targets for P_5 and for example if the output level y_2 could not be increased for P_5 then a target $P``_5$ could be set which would rely entirely on increasing output y_1. For unit P_6 the pro rata increase leads to the set of targets $P`_6$. However $P`_6$ is clearly dominated by P_4 which produces the same amount of output y_1 but more output y_2.

In this case the pro rata increase needs to be supplemented by a further increase in the output of y_2 to provide an efficient target. Returning to unit P_5 the set of targets P_5 can be obtained from a weighted average of the peer units P_1 and P_2. Thus P_5 can be thought of as a composite unit made up of a weighted average of the peer units and this composite unit provides a target for the inefficient unit.

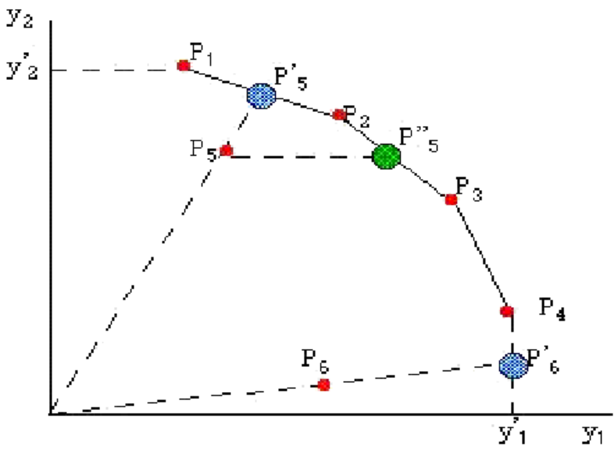

Figure 3.1 Data Envelope

3.10 COMPARING TO REGRESSION ANALYSIS

In the past, regression approaches have been commonly used for measuring efficiency. Consequently, in this section we present some of the differences between regression and DEA in order to highlight DEA's characteristics. Unlike traditional regression approaches, DEA does not require explicit specification of the functional forms relating inputs to outputs. (Bowlin, 2002) More than one function (e.g., more than one production function) is admitted, and the DEA solution can be interpreted as providing a local approximation to whatever function is applicable in the neighborhood of the coordinate values formed from the outputs and inputs of the DMU o being evaluated. Thus, DEA is more flexible in recognizing differences in production functions between DMUs. Secondly, DEA is oriented toward individual decision making units which are regarded as responsible for utilizing inputs to produce the outputs of interest. It therefore utilizes n optimizations, one for each DMU, in place of the single optimization that is usually associated with the regressions used in traditional efficiency analyses. Hence, the DEA solution is unique for each DMU under evaluation. Third, a deficiency of all of the regression approaches is their inability to identify sources and estimate the inefficiency amounts associated with these sources. Hence, no clue as to corrective action is provided even when the dependent variable shows that inefficiencies are present. DEA provides both the sources (input and output) and amounts of any inefficiency. (Bowlin, 2002)

3.11 METHODOLOGY OF THE STUDY

The study is a mathematics based project related with education. It aroused from application of linear programming, data envelopment analysis uses multiple inputs and outputs of a decision making unit to measure efficiency decision making units.

3.11.1 DEA in Education

Although the development of evaluation schemes for educational units is dated back to the 50's, the quantitative approach to school evaluation has been established in the past two decades (Bradley et al.,2001) Measuring the efficiency of schools is first done by Rhodes in 1978 , it was DEA' s original application on education. Due to the unknown nature of the educational production function data envelopment analysis determines the production possibility frontier non-parametrically and, hence, it evaluates the technical efficiency of schools. (Hanushek, 1986) Efficiency located the relationship among frontiers in a model with multilevel structure, with purpose of evaluating the influence of the school and of the school type in the students' performance (Thannasoulis et al. 1992). In Bonilha's study efficient frontiers – equilibrium conditions of teachers in schools in the short run- and their relationship are analyzed under the perspective of the economic comparative static theory and used in the evaluation of the qualification policies of the primary education teachers that teach Portuguese, Mathematics and Science simultaneously.

Although a ranking system that is based on the percentage of students passing exams does not accurately reflect the transformational process of education according to experts on learning skills test measurement is still better way than classical methods (like exams show all your work). (Wooton, 2003) In this thesis, twelve high schools in Gaziantep are analyzed efficiencies by using Data Envelopment Analysis.

3.11.2 Data Collection

Someone may think that inputs of education are students (resources) and outputs are educated citizens. This idea is a bit true but specifically in a school inputs and outputs are completely different. Multiple inputs and outputs must be chosen in the problem. Therefore students taking OSS, average of students in 11th classes, percentage of attending OSS preparation courses, average of weekly course hour of each 11th class teacher in a school was taken as inputs. The outputs are average of “SAY” (Marks of Science Courses) and “EA” (Marks of Social Science Courses) marks, percentage of students registering a university. Correct relation among inputs and outputs is absolutely high.

In another study on measuring relative performance of 22 Anatolian High Schools in Ankara by Atan, inputs are number of students, teachers, classes, grades, labs and computers and number of graduated students, registering university, passing OSS exam and passing classes are taken as outputs. (Atan et al., 2002) No high school student behind left as graduated so this output may not affect solution. In the same manner there is no almost drop out in high schools in Turkey, therefore the same thing occurs also for these outputs. For inputs, there is very tiny correlation between number labs (unfortunately nobody uses in public schools) and passing OSS exam (no question from lab works in OSS)

Schools were determined with idea that “equality” of school circumstances like teachers experience years, students’ socio-economic status, facilities of schools. Different types of schools were taken since sample space may reflect well Gaziantep’s educational media.

Three Anatolian High Schools, Eight High Schools Educating In Foreign Languages (Three of them are Private Schools) and one Normal High School. One might think that Private Schools’ facilities are so high then this disrupts equality of opportunity, but according to 2005 OSYM (Appendix-A) have same results.

Data about schools was collected with the help of Administration of Gaziantep School Districts of Ministry of Education (Appendix-B). Purpose of this thesis is to measure relative performance of twelve high schools in Gaziantep with respect to OSS exams.

3.11.3 How Does DEA Work in the Problem?

A excel based program XLDEA is used for calculations. Inputs and outputs are read by program. (www.prodtools.com) DEA models; input-oriented, output-oriented and constant returns to scale, variable returns to scale are chosen by menu.

In an input oriented model, the calculations to find most favorable weights and the efficiency of DMU are equivalent to improving performance of this DMU by minimizing its inputs while producing at least the observed output levels. This is for example as in the case of decision maker who seeks opportunities cost reduction. So the inefficient DMU should adopt to reach efficient ones as in the example. But in output oriented model the calculations aim at improving the performance of a DMU by maximizing outputs while consuming at most the observed input levels.

Constant returns-to-scale (also called CRR name from Charnes, Cooper and Rhodes) was first used and applied for several years. This model does not differentiate between pure "technical" inefficiencies and inefficiencies due to non-constant (increasing-decreasing) returns-to-scale effects, for example due to constraints in finance, competition etc. Therefore the result produced input and output oriented is the same in this model.

Variable-returns-to-scale model (BCC Banker, Charnes, Cooper) is commonly used today and allows the decomposition of the "global" (that is CCR efficiency) into a "local" pure technical efficiency. (Charnes et al. 1978)

Input and output oriented models result differently in this model. Scores, scores chart, frequencies, frequencies chart, peers, virtual inputs and outputs, slacks are calculated by XLDEA (www.prodtools.com)

CHAPTER 4

CASE STUDY

Gaziantep had potential improvements on industry and commerce but it could not respond same thing in education. (Çetin et al. 2006) Therefore percentage of students attending university from the city is too low. There is a great progress to improve OSS level of city. Therefore the efficiencies of high schools are stressfully concerned by authorities. A commonly accepted measure of a well-performing school is based on the records of its students in the national matriculation examinations (Kirjavainen et al. 1998)

Twelve schools in Gaziantep were chosen:

1. Atatürk High School
2. Gaziantep High School
3. Mimar Sinan High School
4. 19 Mayıs High School
5. Gaziantep Anatolian High School
6. Akınal Anatolian High School
7. Tekerekoglu Anatolian High School
8. Bayraktar High School
9. Private Seçkin High School
10. Private Mutafoğlu High School
11. Private Sanko High School
12. Private Kolej Vakfı High School

Atatürk HS, Gaziantep HS, Mimar Sinan HS, 19 Mayıs HS, Bayraktar HS are educated in foreign languages. Gaziantep, Akınal, Tekerekoglu are anatolian high schools (more courses on foreign languages) and the rest are private schools in educated in froreign languages type except Mutafoglu. These 12 schools have almost equal level of education and they land same localities having equal socio-economic-status. Since schools from rural areas have poor education unfortunately. Actually there are not great educational and income differences between parents of private and public schools twelve schools. The results of study are shown below.

4.1 INPUTS AND OUTPUTS

In the study (Atan et al., 2002) twenty-two Anatolian High Schools in Ankara number of students, number of teachers, number of classes and grades, and number of labs were inputs. Number of graduated students, number of students passing classes, number registering university and percentage of OSS success were outputs. First number of labs and number of students have little meaning in OSS i.e. small weight. And number students passing class has also small weight since every student graduate easily from a high school in Turkey.

Table 4.1: Inputs

	TYPE	*NAME OF SCHOOLS*	S T O	1/SIC	POPC	1/WCHET
1	EFLHS	ATATÜRK HS	46	4,3	80,4	4,5
2	EFLHS	GAZIANTEP HS	105	3,8	84,8	4,8
3	EFLHS	MIMAR SINAN HS	50	4,0	54,0	2,7
4	EFLHS	19 MAYIS HS	59	5,1	76,3	5,0
5	AHS	GAZIANTEP ANA. HS (GAL)	114	4,4	100,0	5,0
6	AHS	AKINAL ANA. HS	110	3,6	100,0	4,8
7	AHS	TEKEREKOGLU ANA. HS	82	4,9	100,0	4,0
8	EFLHS	BAYRAKTAR HS	45	5,3	77,8	4,5
9	EFLHS	PRIVATE SECKIN HS	18	5,6	88,9	5,0
10	HS	PRIVATE MUTAFOGLU HS	97	5,2	99,0	4,0
11	EFLHS	PRIVATE SANKO HS	61	8,2	93,4	5,0
12	EFLHS	PRIVATE KOLEJ VAKFI HS	98	6,1	100,0	3,3

Table 4.1 shows the inputs which abbreviated as follows;

- STO: Number of students taking OSS exam. Besides most of students graduated from high schools take OSS exam, I took STO as input since there is big correlation with outputs i.e. the weight is much more than the other inputs.
- 1/SIC: Inverse of average of number of students in each class. Since negative influence of efficiency caused to take the input how many classes are needed to accommodate 100 students. Crowded classes are the first shortcoming problem to our mind for Turkish education. Actually it has real impact on problems.
- POPC: Percentage of students attending OSS preparation courses. Another extraordinary special aspect of our educational system. There is no any other alternative way for a high school students to have a ob (to go to university) and this way pass on OSS preparation courses, unfortunately POPC positively influence number attending university.

- 1/WCHET: Like in SIC inverse of average of weekly course hour of each 11^{th} class teachers is taken. How many teachers in 100 hours courses. That is the input shows number of teachers needed for 100 hours courses.

The school type was abbreviated as;

- EFLHS : Educates In Foreign Languages
- AHS : Anatolian High Schools
- HS : High School

Table 4.2 Outputs

	TYPE	*NAME OF SCHOOLS*	*S A Y*	*EA*	*PSRU*
1	EFLHS	ATATÜRK HS	184,654	199,241	19,57
2	EFLHS	GAZIANTEP HS	205,842	219,814	45,71
3	EFLHS	MIMAR SINAN HS	200,173	207,969	30,00
4	EFLHS	19 MAYIS HS	181,843	200,747	27,12
5	AHS	GAZIANTEP ANA. HS (GAL)	245,3	243,386	68,42
6	AHS	AKINAL ANA. HS	228,103	232,561	63,64
7	AHS	TEKEREKOGLU ANA. HS	226,753	233,892	56,10
8	EFLHS	BAYRAKTAR HS	193,803	212,184	57,78
9	EFLHS	PRIVATE SECKIN HS	208,076	216,017	77,78
10	HS	PRIVATE MUTAFOGLU HS	183,939	205,135	42,27
11	EFLHS	PRIVATE SANKO HS	195,41	215,6	80,33
12	EFLHS	PRIVATE KOLEJ VAKFI HS	176,44	192,848	48,98

Table 4.2 shows the outputs abbreviated as:

- SAY : Average of students' SAY marks in OSS exam. Three outputs show exact result for achievement. Marks from OSYM pages. (Appendix A).
- EA : Average of students' EA marks in OSS exam.
- PSRU: Percentage of students registering university. It is the single indicator for any school in OSS success. Especially some private schools choose a few brilliant students and grow them well so these few take great marks in OSS. They make great advertisements. But they do not have extended success. PSRU will be sufficient indicator for success.

4.2 THE OBJECTIVE FUNCTION AND CONSTRAINTS

Let me give the objective function of Atatürk High School is;

$Max\ Z = 184.654y_1 + 199.241y_2 + 19.57y_3$ (Table 4.1 and Table 4.2)

Subject to

$46x_1 + 4.3x_2 + 80.4x_3 + 4.5x_4 = 1$

$184.654y_1 + 199.241y_2 + 19.57y_3 - 46x_1 - 4.3x_2 - 80.4x_3 - 4.5x_4 \leq 1$

$205.842y_1 + 219.814y_2 + 45.71y_3 - 105x_1 - 26.3x_2 - 84.8.4x_3 - 22x_4 \leq 1$

$200.173y_1 + 207.969y_2 + 30y_3 - 50x_1 - 4x_2 - 54.4x_3 - 2.7x_4 \leq 1$

$181.843y_1 + 200.747y_2 + 27.12y_3 - 59x_1 - 5.1x_2 - 76.3.4x_3 - 5x_4 \leq 1$

$245.3y_1 + 243.386y_2 + 68.42y_3 - 114x_1 - 4.4x_2 - 100x_3 - 5x_4 \leq 1$

$228.103y_1 + 232.561y_2 + 63.64y_3 - 110x_1 - 3.6x_2 - 100x_3 - 4.8x_4 \leq 1$

$226.753y_1 + 233.892y_2 + 56.10y_3 - 82x_1 - 4.9x_2 - 100x_3 - 4x_4 \leq 1$

$193.803y_1 + 212.184y_2 + 57.78y_3 - 45x_1 - 5.3x_2 - 77.8x_3 - 4.5x_4 \leq 1$

$208.076y_1 + 216.017y_2 + 77.78y_3 - 18x_1 - 5.6x_2 - 88.9x_3 - 5x_4 \leq 1$

$183.939y_1 + 205.135y_2 + 42.27y_3 - 97x_1 - 5.2x_2 - 99x_3 - 4x_4 \leq 1$

$195.41y_1 + 2215.6y_2 + 80.33y_3 - 61x_1 - 8.2x_2 - 93.4x_3 - 5x_4 \leq 1$

$176.44y_1 + 192.848y_2 + 48.98y_3 - 98x_1 - 6.1x_2 - 100x_3 - 3.3x_4 \leq 1.$

All calculations are made by writing the above formula for 12 twelve schools

4.2 EFFICIENCY SCORES

Two type efficiency scores were calculated, former is basic, the latter is advanced. In basic type, the problem is solved by the models inputs-oriented and constant-returns-to scale (CCR). Since the inputs of problem would be changed. Table 4.3 shows relative efficiencies of twelve high schools in Gaziantep, 4 out of 12 are inefficient. For example the value for 19 Mayıs HS means that its inputs can simultaneously be reduced by a factor of 1-0,7772, i.e. 22,28%. This corresponds to moving this unit to the efficient frontier radially, that is without altering the proportion of its inputs.

Table 4.3 Efficiency Scores

ATATÜRK HS	0,9382
GAZIANTEP HS	0,9678
MIMAR SINAN HS	1,0000
19 MAYIS HS	0,7772
GAZIANTEP ANA. HS (GAL)	1,0000
AKINAL ANA. HS	1,0000
TEKEREKOGLU ANA. HS	1,0000
BAYRAKTAR HS	1,0000
PRIVATE SECKIN HS	1,0000
PRIVATE MUTAFOGLU HS	0,8024
PRIVATE SANKO HS	1,0000
PRIVATE KOLEJ VAKFI HS	1,0000

As mentioned earlier inefficient DMU must approach to efficient frontier. So we form a virtual DMU as a weighted combination of some efficient DMU's. These are equivalent ways to express improvement. For inefficient DMU, the set of suitable efficient units is called its reference set, or simply the of its efficient peers. The table 4.4 shows efficient peers of inefficient schools. For example Gaziantep HS is to become 35.24 % of Mimar Sinan HS and 63.01% of Akınal HS. This means that Gaziantep HS has to adopt methods and practices from Mimar Sinan and Akınal HS's.

Table 4.4 Efficient Peers and Weights

	MIMAR SINAN	AKINAL	TEKEREKOGLU	PRIVATE SECKIN
ATATÜRK HS	0,8128			0,1398
GAZIANTEP HS	0,3524	0,6301		
19 MAYIS HS	0,8915			0,0710
PRIVATE MUTAFOGLU HS	0,3489	0,0004	0,5665	

The virtual inputs and outputs for each DMU can be calculated based on its efficient peers and weights. Table 4.5 showed that how much one inefficient school must adopt its inputs (increase or decrease). For example Gaziantep HS has peers (Mimar Sinan HS and Akınal HS) so the virtual inputs for Gaziantep HS are:

-STO: 0,3524(Input-1 for Mimar Sinan HS) + 0,6301 (Input-1 for Akınal) = 86,93

i.e. 0,3524x50 + 0,6301x110 = 86,93. This means that Gaziantep HS has to decrease its number of students from 105 to 87 by 17,21 % as shown in next column of STO column.

Table 4.5: Virtual Inputs

	STO		1/SIC		POPC		1/WCHET	
ATATÜRK HS	43,16	*6,18%*	4,03	*6,18%*	56,32	*29,98%*	2,89	*35,70%*
GAZIANTEP HS	86,93	*17,21%*	3,68	*3,22%*	82,04	*3,22%*	3,98	*17,17%*
MIMAR SINAN HS	50,00	*0,00%*	4,00	*0,00%*	54,00	*0,00%*	2,70	*0,00%*
19 MAYIS HS	45,85	*22,28%*	3,96	*22,28%*	54,45	*28,61%*	2,76	*44,76%*
GAZIANTEP ANA. HS (GAL)	114,00	*0,00%*	4,40	*0,00%*	100,00	*0,00%*	5,00	*0,00%*
AKINAL ANA. HS	110,00	*0,00%*	3,60	*0,00%*	100,00	*0,00%*	4,80	*0,00%*
TEKEREKOGLU ANA. HS	82,00	*0,00%*	4,90	*0,00%*	100,00	*0,00%*	4,00	*0,00%*
BAYRAKTAR HS	56,00	*0,00%*	5,30	*0,00%*	62,50	*0,00%*	4,50	*0,00%*
PRIVATE SECKIN HS	18,00	*0,00%*	5,60	*0,00%*	88,89	*0,00%*	5,00	*0,00%*
PRIVATE MUTAFOGLU HS	63,94	*34,09%*	4,17	*19,76%*	75,52	*23,69%*	3,21	*19,76%*
PRIVATE SANKO HS	61,00	*0,00%*	8,20	*0,00%*	93,44	*0,00%*	5,00	*0,00%*
PRIVATE KOLEJ VAKFI HS	98,00	*0,00%*	6,10	*0,00%*	100,00	*0,00%*	3,30	*0,00%*

Table 4.6: Virtual Outputs

	SAY		EA		PSRU	
ATATÜRK HS	191,79	*3,87%*	199,24	*0,00%*	35,26	*80,21%*
GAZIANTEP HS	214,26	*4,09%*	219,81	*0,00%*	50,67	*10,83%*
MIMAR SINAN HS	200,17	*0,00%*	207,97	*0,00%*	30,00	*0,00%*
19 MAYIS HS	193,23	*6,26%*	200,75	*0,00%*	32,27	*18,99%*
GAZIANTEP ANA. HS	245,30	*0,00%*	243,39	*0,00%*	68,42	*0,00%*
AKINAL ANA. HS	228,10	*0,00%*	232,56	*0,00%*	63,64	*0,00%*
TEKEREKOGLU ANA. HS	226,75	*0,00%*	233,89	*0,00%*	56,10	*0,00%*
BAYRAKTAR HS	193,80	*0,00%*	212,18	*0,00%*	46,43	*0,00%*
PRIVATE SECKIN HS	208,08	*0,00%*	216,02	*0,00%*	77,78	*0,00%*
PRIVATE MUTAFOGLU HS	198,37	*7,85%*	205,13	*0,00%*	42,27	*0,00%*
PRIVATE SANKO HS	195,41	*0,00%*	215,60	*0,00%*	80,33	*0,00%*
PRIVATE KOLEJ VAKFI HS	176,44	*0,00%*	192,85	*0,00%*	48,98	*0,00%*

The potential improvements (increase or decrease) of some high schools are listed below;

- Ataturk HS reaches the efficient schools by increasing its SAY marks just 3,87%(From 191,79 to 199,24). But it needs much more effort for PSRU by %80.21. Atatürk HS is from rural area so registering to university is low because of expensive private universities.
- Gaziantep HS has to decrease number of students in a class by %17.21. It does not need much more to change number students attending university preparation courses.
- Private Mutafoglu has to impact students to register a university. Students from public schools can not go to private universities due to fee. But private high schools help students in guiding and choosing best universities.

Table 4.7: Input and Output Slacks

	STO	POPC	WCHET	SAY	EA	PSRU
ATATÜRK HS	0,00	19,14	1,33	7,14	15,69	0,00
GAZIANTEP HS	14,70	0,00	0,67	8,42	4,95	14,70
MIMAR SINAN HS	0,00	0,00	0,00	0,00	0,00	0,00
19 MAYIS HS	0,00	4,82	1,12	11,39	5,15	0,00
GAZIANTEP ANA. HS (GAL)	0,00	0,00	0,00	0,00	0,00	0,00
AKINAL ANA. HS	0,00	0,00	0,00	0,00	0,00	0,00
TEKEREKOGLU ANA. HS	0,00	0,00	0,00	0,00	0,00	0,00
BAYRAKTAR HS	0,00	0,00	0,00	0,00	0,00	0,00
PRIVATE SECKIN HS	0,00	0,00	0,00	0,00	0,00	0,00
PRIVATE MUTAFOGLU HS	13,90	3,89	0,00	14,43	0,00	13,90
PRIVATE SANKO HS	0,00	0,00	0,00	0,00	0,00	0,00
PRIVATE KOLEJ VAKFI HS	0,00	0,00	0,00	0,00	0,00	0,00

Table 4.7 shows slack values which direct number of change for particular input or output for inefficiency units. Therefore Atatürk HS has to increase SAY marks by 4,03 points and has to increase number of students registering university by 45. Gaziantep HS has to decrease STO by 35,66 and SIC by 8,61. Also Gaziantep HS has to increase SAY marks by 3,16 points and has to PSRU by 14,91. Akınal Anatolian HS has to decrease STO 2,61 and SIC by 4,83. Tekerekoglu AHA would not need much more change but Mutafoglu and Kolej Vakfı have to decrease STO consecutively by 18,76 and 28,7. It is easily seen that inefficiency of schools is due to having much more

students taking OSS exam (STO). A slight change in schools will bring efficiency to our schools.

Table 4.8 Advanced Efficiency Scores

	VRS	Scale Eff.	Returns-to-scale	CCR
ATATÜRK HS	0,9465	0,9912	decreasing	0,8451
GAZIANTEP HS	0,9755	0,9921	decreasing	0,9961
MIMAR SINAN HS	1,0000	1,0000	constant	1,0000
19 MAYIS HS	0,9126	0,8517	decreasing	0,9914
GAZIANTEP ANA. HS (GAL)	1,0000	1,0000	constant	1,0000
AKINAL ANA. HS	1,0000	1,0000	constant	0,9418
TEKEREKOGLU ANA. HS	1,0000	1,0000	constant	0,9183
BAYRAKTAR HS	1,0000	1,0000	constant	1,0000
PRIVATE SECKIN HS	1,0000	1,0000	constant	1,0000
PRIVATE MUTAFOGLU HS	0,8782	0,9137	decreasing	0,7966
PRIVATE SANKO HS	1,0000	1,0000	constant	1,0000
PRIVATE KOLEJ VAKFI HS	1,0000	1,0000	constant	0,7665

Let us look Advanced Efficiency scores of our schools. In table 4.8 8 out of 12 schools were found inefficient (grayed schools). The inefficients are Atatürk High Schools,(scored 0.8451) Gaziantep High Schools(scored 0,9961), 19 Mayıs HS (scored 0,9914) Akınal Anatolian High School(scored 0,9418), Tekerekoğlu Ana. HS (scored 9183), Private Mutafoğlu High Schools (scored 0, 7966), Private Kolej Vakfı High Schools (0,7665). Scores from inefficient schools may lie near among each other, this is reason by almost same type of schools chosen by the analyst. As it is mentioned earlier the constant returns-to-scale (CCR score) is a kind of global measurement in which inefficiencies due to pure technical reasons are confounded by inefficiencies due to the scale of operations. But VRS (variable returns-to scale) is more strict model, since it devoids the scale effect, therefore it is always larger. Thus it is decomposed the global CCR efficiency as;

CCR score = VRS score **x** scale efficiency

Therefore in the study for instance Atatürk High School's scores;

0, 8451 = 0, 9465 x 0, 9912

And in returns-to-scale column is written "decreasing" this means that the linear segment of DMU is decreasing. ie inefficiency of unit is decreasing. If constant then DMU's are fully efficient. Table of peer sheet, virtual inputs/outputs, slacks for advanced scores showed in Appendices C-D-E.

CHAPTER 5

CONCLUSIONS

Data Envelopment Analysis (DEA) is a powerful method widely used in the evaluation of performance of Decision Making Units (Bowlin, 2002). In this relative performance of some high schools in Gaziantep are determined by using DEA. Although the mathematics of DEA is difficult for general user, the assumptions and the underlying idea of the algorithm are quite simple.

The relative efficiency of some high schools in Gaziantep is measured by data envelopment analysis. It was seen that although educational productivity is non-measurable phenomenon efficiency of a school with respect to an exam can be measured easily. The results showed that a slight difference in some particular inputs would change the efficiency of schools. Crowded classes and lack of coaching of teachers to students are two basic problems of our education. That is tired teachers in crowded classes could not give sufficient interest to their children. Therefore less-educated teenagers spread out to society.

Productivity is inalienable target of each decision making unit in every branch of life. Time is money. There is no time to waste. Therefore if we want to catch developed countries, we have to increase our productivity. DEA is simple but very practicable tool to find relative performances- i. e. efficiencies of every unit consisting inputs and outputs. By the help of simple but influential program DEA, performances of our schools can be measured easily.

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APPENDIX-A

GAZİANTEP	ADAY SAYILARI			SAYISAL								SÖZEL								ÖSS PUANLARI					
OKULUN ADI	ÖSS GİR 160≤PN<185 / PN≥185	LİSANS ÖNLİSANSA AÖF		MATEMATİK		FİZİK		KİMYA		BİYOLOJİ		TÜRKÇE		TARİH		COĞRAFYA		FELSEFE		ÖSS-SAY		ÖSS-SÖZ		ÖSS -EA	
		SINAVSIZ	SINAVLI	GNL GRP TÜR	ADAY ORT SAP	GNL GRP TÜR	ADAY ORT SAP	GNL GRP TÜR	ADAY ORT SAP	GNL GRP TÜR	ADAY ORT SAP	GNL GRP TÜR	ADAY ORT SAP	GNL GRP TÜR	ADAY ORT SAP	GNL GRP TÜR	ADAY ORT SAP	GNL GRP TÜR	ADAY ORT SAP	GNL GRP TÜR	ADAY ORT SAP	GNL GRP TÜR	ADAY ORT SAP	GNL GRP TÜR	ADAY ORT SAP
ATATÜRK LİSESİ	46	0	8	1138	46	1157	46	954	46	993	46	1610	46	1797	46	3008	46	2039	46	1443	46	1602	46	1349	46
TOPLAM BAŞVURAN: 46	8	0	0	1043	16,11	1011	3,85	860	3,08	880	2,23	1449	28	1466	5,61	24458	3,85	1601	4,2	1046	184,654	1432	202,774	1251	199,241
	37	0	1	451	12,58	425	5,82	336	4,29	346	3,22	685	10,53	585	4,92	698	4,15	574	3,87	454	37,56	672	33,049	598	30,933
GAZİANTEP LİSESİ	104	0	46	591	104	518	104	548	104	725	104	652	104	1781	104	2010	104	2112	104	570	104	815	104	619	104
TOPLAM BAŞVURAN: 104	2	0	2	525	22,81	452	6,57	482	4,48	637	2,79	562	33,11	1452	5,63	1622	4,58	1664	4,15	500	205,842	711	218,201	539	219,814
	101	0	0	95	13,22	70	6,65	94	4,6	192	3,45	189	7,84	577	6,16	559	5,07	588	4,36	90	36,85	250	27,633	110	24,215
MİMAR SİNAN LİSESİ	49	0	14	705	49	601	49	526	49	545	49	1579	49	3525	49	3910	49	3442	49	701	49	1611	49	994	49
TOPLAM BAŞVURAN: 49	4	0	1	830	21,18	530	6,01	463	4,62	477	3,39	1424	28,18	2865	3,9	3185	3,15	2733	3,34	619	200,163	1440	202,608	898	207,969
	45	0	0	170	11,78	117	6,25	81	4,61	96	3,3	672	8,58	767	5,35	760	4,52	740	4,3	166	35,252	677	28,605	354	24,718
19 MAYIS LİSESİ	55	0	14	1339	55	1166	55	1442	55	1166	55	1134	55	1334	55	1762	55	1275	55	1244	55	1265	55	1289	55
TOPLAM BAŞVURAN: 55	8	0	0	1232	14,26	1020	3,82	1033	2,62	1029	1,92	1015	30,86	1082	6,23	1422	4,79	970	4,75	1145	181,843	1134	209,85	1192	200,747
	46	0	2	565	13,81	431	5,86	436	3,99	437	3,37	457	9,83	439	6,19	511	4,66	405	4,33	519	38,585	517	32,761	560	27,824
GAZİANTEP ANADOLU LİSESİ	114	0	78	154	114	119	114	127	114	121	114	511	114	1308	114	2490	114	4011	114	135	113	343	113	170	113
TOPLAM BAŞVURAN: 114	0	0	0	148	33,24	116	12,37	124	8,51	119	6,84	433	33,8	1059	6,27	2026	4,21	3150	2,95	132	245,3	277	228,112	162	243,386
	113	0	2	70	8,43	34	6	40	4,5	37	3,71	195	9,56	301	6,41	349	5,25	382	3,93	53	30,58	157	32,253	87	25,366
AKINAL ANADOLU LİSESİ	110	0	70	266	110	253	110	297	110	285	110	805	110	1722	110	2647	110	3729	110	282	110	668	110	340	110
TOPLAM BAŞVURAN: 110	2	0	0	250	29,83	238	9,7	272	6,31	261	5,04	703	37,47	1404	5,7	2155	4,1	2947	3,15	261	228,103	570	220,598	195	232,561
	108	0	1	151	10,56	136	6,74	164	4,77	156	4,1	272	8,81	352	6,49	361	5,14	365	4,02	163	34,958	271	33,165	188	29,408
TEKEREKOĞLU ANADOLU LİSESİ	82	0	46	305	82	254	82	302	82	319	82	514	82	912	82	1633	82	2526	82	290	82	422	82	312	82
TOPLAM BAŞVURAN: 82	2	0	0	281	28,78	239	9,66	276	6,27	288	4,75	436	33,79	724	6,89	1313	4,89	2008	3,89	266	226,753	346	226,131	273	233,892
	82	0	0	175	12,42	137	7,05	167	4,85	179	4,01	197	7,2	233	6,24	269	5,45	263	4,21	168	39,006	195	27,976	173	26,09
BAYRAKTAR LİSESİ	56	0	25	914	56	957	56	778	56	797	56	668	56	1016	56	1727	56	1249	56	879	56	842	56	842	56
TOPLAM BAŞVURAN: 56	0	0	0	829	18,45	845	4,48	692	3,58	703	2,61	577	33,05	811	6,73	1392	4,81	949	4,77	793	193,803	735	217,585	750	212,184
	56	0	1	305	14,78	329	5,88	230	4,45	236	3,58	201	8,57	311	6	503	4,55	396	4,16	284	40,136	267	27,923	249	27,686
ÖZEL SEÇKİN LİSESİ	18	0	14	584	18	374	18	456	18	539	18	1251	18	1787	18	2745	18	3515	18	533	18	1169	18	725	18
TOPLAM BAŞVURAN: 18	2	0	0	520	22,93	334	8,01	400	4,99	471	3,42	1128	30,33	1457	5,62	2235	4,03	2789	3,3	465	208,076	1043	211,626	638	216,017
	16	0	0	91	12,19	16	7,21	50	4,62	93	4,2	524	9,42	581	5,77	668	4,56	746	3,81	71	43,084	461	33,491	174	33,108
ÖZEL MUTAFOĞLU LİSESİ	97	0	35	1335	97	1162	97	1092	97	1108	97	838	97	734	97	890	97	457	97	1165	96	855	96	1095	96
TOPLAM BAŞVURAN: 97	9	0	2	1228	14,3	1016	3,84	988	2,74	983	2,02	733	32,32	574	7,18	681	5,74	324	5,69	1068	183,939	747	217,3	998	205,135
	83	0	4	134	14,83	83	6,37	82	4,64	74	3,78	63	8,8	70	5,26	86	4,53	34	3,55	99	46,555	70	31,763	89	36,297
ÖZEL SANKO LİSESİ	61	0	43	802	61	1049	61	909	61	915	61	608	61	1022	61	748	61	1058	61	827	61	682	61	741	61
TOPLAM BAŞVURAN: 61	0	0	3	722	19,84	926	4,19	817	3,19	810	2,37	521	33,31	816	6,72	565	5,94	799	4,93	742	195,41	583	220,378	652	215,6
	61	0	2	234	12,37	374	6	309	4,56	300	3,38	168	8,01	312	5	169	4,21	345	3,89	243	37,04	164	23,853	183	23,354
ÖZEL KOLEJ VAKFI LİSESİ	98	0	44	911	98	590	98	639	98	651	98	1078	98	2371	98	2985	98	2570	98	815	98	1278	98	989	98
TOPLAM BAŞVURAN: 98	0	0	3	826	18,47	519	6,07	564	4,03	569	3,08	960	31,16	1943	4,97	2443	3,36	2047	3,87	730	195,747	1146	209,647	893	208,138
	0	0	0	304	14,36	110	6,67	149	4,58	151	3,76	425	8,41	687	5,61	696	4,41	657	3,64	238	42,78	526	27,386	351	27,501

APPENDIX-B

GAZİANTEP İLİNE BAĞLI BAZI OKULLARIN 2005 YILINA AİT BİLGİLERİ									
			11. SINIF ÖĞRENCİ SAYISI		ÖSS'YE GİREN SAYISI	11. SINIF DERSLİK SAYISI	DERSHANEYE GİDEN ÖĞRENCİ SAYISI	11. SINIF ÖĞRETMEN SAYISI	11. SINIF ÖĞRETMEN DERS YOĞUNLUĞU
SIRA	OKUL ADI	OKUL TÜRÜ	SAY	EA					
1	ATATÜRK LİSESİ	YAB. DİL. AĞ. LİS.	25	21	46	2	37	20	22
2	GAZİANTEP LİSESİ	YAB. DİL. AĞ. LİS.	78	27	105	4	89	14	21
3	MİMAR SİNAN LİSESİ	YAB. DİL. AĞ. LİS.	33	17	50	2	27	11	37
4	19 MAYIS LİSESİ	YAB. DİL. AĞ. LİS.	24	35	59	3	45	24	20
5	GAZİANTEP ANADOLU LİSESİ	ANADOLU LİS.	60	54	114	5	114	35	20
6	AKINAL ANADOLU LİSESİ	ANADOLU LİS.	81	29	110	4	110	27	21
7	TEKEREKOĞLU ANADOLU LİSESİ	ANADOLU LİS.	59	23	82	4	82	12	25
8	BAYRAKTAR LİSESİ	YAB. DİL. AĞ. LİS.	30	20	56	3	40	23	22
9	ÖZEL SEÇKİN LİSESİ	YAB. DİL. AĞ. LİS.	12	5	17	2	10	3	20
10	ÖZEL MUTAFOĞLU LİSESİ	NORMAL LİS.	45	52	97	5	96	12	25
11	ÖZEL SANKO LİSESİ	YAB. DİL. AĞ. LİS.	30	31	61	5	57	12	20
12	ÖZEL KOLEJ VAKFI LİSESİ	YAB. DİL. AĞ. LİS.	60	36	96	6	96	18	30

APPENDIX-C

	MIMAR SINAN HS	GAZIANTEP ANA. HS	AKINAL ANA. HS	TEKEREK ANA. HS	BAYRAKTAR HS	PRIVATE SECKIN HS	PRIVATE SANKO HS	PRIVATE KOLEJ VAKFI HS
ATATÜRK HS	0,7644		0,0385			0,1971		
GAZIANTEP HS	0,3313	0,0844	0,5844					
MIMAR SINAN HS	1,0000							
19 MAYIS HS	0,4499	0,2771				0,2730		
GAZIANTEP ANA. HS		1,0000						
AKINAL ANA. HS			1,0000					
TEKEREKOGLU ANA. HS				1,0000				
BAYRAKTAR HS					1,0000			
PRIVATE SECKIN HS						1,0000		
PRIVATE MUTAFOGLU HS	0,0224	0,0291		0,9485				
PRIVATE SANKO HS							1,0000	
PRIVATE KOLEJ VAKFI HS								1,0000

APPENDIX-D

	STO		1/SIC		POPC		1/WCHET		SAY		EA		PSRU	
ATATÜRK HIGH SCHOOL	46,00	*0,00%*	4,30	*0,00%*	62,65	*22,12%*	3,23	*28,13%*	202,81	*9,83%*	210,50	*5,65%*	40,71	*108,08%*
GAZIANTEP HIGH SCHOOL	90,46	*13,85%*	3,80	*0,00%*	84,76	*0,00%*	4,12	*14,14%*	220,30	*7,02%*	225,33	*2,51%*	52,90	*15,71%*
MIMAR SINAN HIGH SCHOL	50,00	*0,00%*	4,00	*0,00%*	54,00	*0,00%*	2,70	*0,00%*	200,17	*0,00%*	207,97	*0,00%*	30,00	*0,00%*
19 MAYIS HIGH SCHOOL	59,00	*0,00%*	4,55	*10,83%*	76,27	*0,00%*	3,97	*20,70%*	214,84	*18,14%*	219,98	*9,58%*	53,69	*97,98%*
GAZIANTEP ANATOLIAN HIGH SCHOOL	114,00	*0,00%*	4,40	*0,00%*	100,00	*0,00%*	5,00	*0,00%*	245,30	*0,00%*	243,39	*0,00%*	68,42	*0,00%*
AKINAL ANATOLIAN HIGH SCHOOL	110,00	*0,00%*	3,60	*0,00%*	100,00	*0,00%*	4,80	*0,00%*	228,10	*0,00%*	232,56	*0,00%*	63,64	*0,00%*
TEKEREKOGLU ANATOLIAN HIGH SCHOOL	82,00	*0,00%*	4,90	*0,00%*	100,00	*0,00%*	4,00	*0,00%*	226,75	*0,00%*	233,89	*0,00%*	56,10	*0,00%*
BAYRAKTAR HIGH SCHOOL	56,00	*0,00%*	5,30	*0,00%*	62,50	*0,00%*	4,50	*0,00%*	193,80	*0,00%*	212,18	*0,00%*	46,43	*0,00%*
PRIVATE SECKIN HIGH SCHOOL	18,00	*0,00%*	5,60	*0,00%*	88,89	*0,00%*	5,00	*0,00%*	208,08	*0,00%*	216,02	*0,00%*	77,78	*0,00%*
PRIVATE MUTAFOGLU HIGH SCHOOL	82,22	*15,24%*	4,87	*6,44%*	98,97	*0,00%*	4,00	*0,00%*	226,70	*23,25%*	233,59	*13,87%*	55,87	*32,18%*
PRIVATE SANKO HIGH SCHOOL	61,00	*0,00%*	8,20	*0,00%*	93,44	*0,00%*	5,00	*0,00%*	195,41	*0,00%*	215,60	*0,00%*	80,33	*0,00%*
PRIVATE KOLEJ VAKFI HIGH SCHOOL	98,00	*0,00%*	6,10	*0,00%*	100,00	*0,00%*	3,30	*0,00%*	176,44	*0,00%*	192,85	*0,00%*	48,98	*0,00%*

APPENDIX-E

	STO	SIC	POPC	WCHET	SAY	EA	PSRU
ATATÜRK HIGH SCHOOL	0,00	0,00	17,79	1,27	7,72	0,00	20,04
GAZIANTEP HIGH SCHOOL	14,54	0,00	0,00	0,68	9,30	0,00	6,04
MIMAR SINAN HIGH SCHOL	0,00	0,00	0,00	0,00	0,00	0,00	0,00
19 MAYIS HIGH SCHOOL	0,00	0,55	0,00	1,03	15,57	0,00	23,97
GAZIANTEP ANATOLIAN HIGH SCHOOL	0,00	0,00	0,00	0,00	0,00	0,00	0,00
AKINAL ANATOLIAN HIGH SCHOOL	0,00	0,00	0,00	0,00	0,00	0,00	0,00
TEKEREKOGLU ANATOLIAN HIGH SCHOOL	0,00	0,00	0,00	0,00	0,00	0,00	0,00
BAYRAKTAR HIGH SCHOOL	0,00	0,00	0,00	0,00	0,00	0,00	0,00
PRIVATE SECKIN HIGH SCHOOL	0,00	0,00	0,00	0,00	0,00	0,00	0,00
PRIVATE MUTAFOGLU HIGH SCHOOL	14,78	0,33	0,00	0,00	17,25	0,00	7,74
PRIVATE SANKO HIGH SCHOOL	0,00	0,00	0,00	0,00	0,00	0,00	0,00
PRIVATE KOLEJ VAKFI HIGH SCHOOL	0,00	0,00	0,00	0,00	0,00	0,00	0,00