A FEEDBACK QUANTIZATION STRATEGY FOR OPPORTUNISTIC BEAMFORMING SYSTEMS

by

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I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

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M. S. Thesis - Electrical and Electronics Engineering

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ABSTRACT

The presence of fading is decisive for the realization of multiuser diversity gain because it increases the probability that one user is having better channel condition at any given time. For slow fading environments multiuser diversity can be obtained by the opportunistic beamforming. To have opportunistic beamforming, multiple antennas are required at the base station to induce artificial channel fluctuations to ensure multiuser diversity in the network. Therefore transmitting through opportunistic beamforming to the user with the best channel conditions will lead to a better throughput.

The users' channel conditions have to be monitored at the base station to send the data to the user who has best channel conditions at any given time, therefore the users feed back their channel conditions all the time. It's known that the feedback channels have a limited bandwidth, so in order to use these channels with the best efficient way, feedback information needs to be quantized.

We derive the optimum quantization scheme that minimizes the probability of error by not selecting the user with the highest NSNR of the opportunistic beamforming. This schemed is proved by theoretical approaches as well as simulation results.

Keywords: Opportunistic Beamforming, Multiuser Diversity, SNR, NSNR, Throughput, Feedback Channels, Probability of error.

FIRSATÇI HÜZME OLUŞUMU SİSTEMLERİ İÇİN BİR GERİBESLEME NİCEMLENME STRATEJİSİ

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ÖZ

Herhangi belli zaman diliminde tek kullanıcı daha iyi kanal konumuna sahip olma olasılığını artırdığından çok kullanıcı çeşitlemesi kazancının gerçekleşmesi için sönümleme yapısı belirleyicidir. Uzun dönemli sönümlenme çevreleri için çok kullanıcı çeşitlemesi fırsatçı hüzme oluşumu tarafından belirlenebilir. Fırsatçı hüzme olşumuna sahip olmak, yapay kanal dalgalanmalarına neden olmak, ağ içinde çok kullanıcılı farklılığından emin olmak için baz istasyonunda çoklu antenler gerekmektedir. Bu nedenle kullanıcı için en iyi kanal konumları ile fırsatçı hüzme oluşumuna doğru yayılım daha iyi bir iş çıkarma yeteneğine ulaştıracaktır.

Herhangi belirli bir zaman diliminde, en iyi kanal konumlarına sahip bir kullanıcıya veri göndermek için, kullanıcı kanal konumları baz istasyonunda izlenmek zorundadır; bu yüzden kullanıcılar kanal konumlarını her zaman geri beslerler. Geribesleme kanallarının sınırlı bant genişliğine sahip olduğu bilinmektedir, bu nedenle en verimli şekilde bu kanalları kullanmak için, geribesleme bilgisinin nicemlenmesi gerekmektedir.

Fırsatçı hüzme oluşumunun en yüksek işaret gürültü oranı (NSNR) ile kullanıcı seçmeden hata oranını azaltan en iyi nicemleme tasarısını ispatladık. Bu tasarı teorik uygulamalar yanısıra benzetim sonuçları ile de kanıtlanmıştır.

Keywords: Fırsatçı Hüzme Oluşumu, Çok Kullanıcı Çeşitleme, İşaret Gürültü Oranı(SNR), Düzgelenmiş İşaret Gürültü Oranı (NSNR), İş Çıkarma Yeteneği, Geribesleme Kanalları, Hata Olasılığı.

DEDICATION

To my parents

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SYMBOL/ABBREVIATION	
CDMA	Code Division Multiple Access
PFS	Proportional Fair Scheduler
D	Delay Spread
W_c	Coherence Bandwidth
TDMA	Time Division Multiple Access
SNR	Signal-to-Noise Ratio
MAX-NSNR	Maximum Normalized SNR
P_e	Probability of Error
L	Number of Thresholds
T	Number of Antennas
K	Number of Users
$y_k(n)$	Received Data Vector
$q(n)$	Beamforming Vector
$h_k(t)$	Channel Coefficient
T_c	Coherence Time
$\boldsymbol{z_k(n)}$	Additive Complex white Gaussian
σ^2	Variance
$\gamma_k(n)$	SNR
$m(n)$	Max-NSNAR
PDF	Probability Distribution Function
CDF	Cumulative Distribution Functionn
$f(x)$	PDF of x.
$\mathcal{F}(x)$	CDF of x
η	Threshold
R	Throughput

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Wireless communications has been a topic of study since 1960s, lots of research papers in this area has been made in the last decade. And that's because of many reasons; one of the reasons is the demand of wireless connectivity, like cellular telephony was surpassed by wireless data applications. Another reason is the developing in signal processing algorithms, and coding techniques which ;were affected by the progress of VSLI technology in small-area and low- power implementations .The third reason is the success of wireless communications systems in particular, like second-generation (2G) digital wireless standards, the IS-95 Code Division Multiple Access (CDMA) standard, all of these standards have proved that not only this field has good theoretical ideas ,but also has a significant touch in practice.

The problem of wireless communications has two fundamental aspects, those two aspects are making the problem challenging and interesting. First aspect is what known as fading: the time variations of the channel strength due to the effect of multipath fading, as well as the effects of distance attenuation and shadowing by obstacles. The other aspect is the interference: since the users communicate over the air, there is consequential interference between them, and that can happen in various systems like in uplink of cellular system, or in downlink of a cellular system, or different cells. (Viswanath and Tse, 2005)

In a wireless system, multiuser diversity exploits of the independent changes in the strength of the channels for different users. What multiuser diversity based on is transmitting more data to users whose channel conditions are the best at any given time.

The presence of fading is decisive for the realization of multiuser diversity gain because it increases the probability that one user is having better channel conditions at any given time. For slow fading environments multiuser diversity can be obtained by the opportunistic beamforming method of (Viswanath et al. ,2002) in order to have opportunistic beamforming ,multiple antennas are required at the base station. Therefore transmitting through opportunistic beamforming to the user with the best channel conditions will lead to a better throughput. However using this technique maybe unfair to some users in the system, hence the fairness is secured by using proportional fair scheduler (PFS). In our study we will use another fairness scheduler called Maximum Normalized SNR (MAX-NSNR), which can also guarantees the fairness among the users.

The user channel conditions have to be monitored at the base station. So sending the data to the user who has better channel conditions at any given time, then the user feedback his channel conditions all the time. It’s known that the feedback channel has a limited bandwidth, so in order to use this channel with the best efficient way, feedback information needs to be quantized.

The reduction of the feedback load in multiuser diversity systems has been proposed by many researchers, most of them consider the case where the channel is fast fading.

Quantization of the feedback information has two criteria, first one is quantization in order to maximize the average throughput of the systems, and the other is quantization to minimize the probability of error. Maximizing the average throughput means increases the capacity of the system; in the other hand minimizing the probability of error depends on how we define the error. In (Tao Lau and Kschischang, 2007) they defined the error by not selecting the user with the best channel quality. However In our study we define the error by not selecting the user with the highest Normalized SNR.

Hence, the case where inducing artificial fading to the system using multiple antennas to obtain optimum probability of error, were not discussed,

In this dissertation, we derive the optimum quantization scheme for the opportunistic beamforming system that minimizes the probability of error, while ensuring the fairness among users.

1.2 CONTRIBUTIONS

The purpose of this study is to develop a feedback quantization strategy to minimize the probability of error for 2 users multiple antennas then for K users multiple antennas; supported by proof and simulation results.

1.3 THESIS ORGANISATION

This thesis is organized into five chapters. The introduction, Chapter 1, gives a brief descriptions and history of wireless communication system.

Chapter 2 is a literature survey of basics of fading, and strategies used for slow fading and fast fading environments to achieve two criterions; obtaining the maximum average throughput of the system and minimizing the probability of error.

Our strategy in minimizing the probability of error for slow fading environment and the theoretical approach is in Chapter 3.

Chapter 4 presents the simulation results Vs theoretical results and their discussions.

The conclusion drawn from the research and plans for future works were presented in Chapter 5.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

Some important papers will be discuss in the following sections 2.2 to 2.4 , those papers were responsible for the existence and the development in this field; moreover in this chapter three main papers will be discussed . However our theory will be based on these three papers.

In section 2.5 we will see what (Fredrik Flor´en et al., 2003) did to maximize the average throughput in fast fading environments. Section 2.6 shows how Ozdemir approached to maximize the average throughput in slow fading environments. Section 2.7 shows what (Tao Lau and Kschischang, 2007) achieved in order to minimize the probability of error in fast fading environments.

The last three sections have simulation results, without discussing the theoretical derivations approaches, however any of those derivations we may need in our study, will be discussed in the following chapter.

2.2 FADING CHANNELS

2.2.1 Introduction to fading:

The variations of the channel strength over time and over frequency in mobile wireless channel are defining the phenomenon of fading, and these variations can be hardly divided into two types:

- *Large-scale fading*, when the channel strength fades due to path loss as a function of distance and shadowing by large obstacles such as buildings and hills. And its frequency independent.
- *Small –scale fading*, when the channel strength fades due the constructive and destructive interference of the multiple signal paths between the transmitter and receiver. It is caused by the superposition or cancellation of multipath propagation signals, the speed of the transmitter or receiver or the bandwidth of the transmitted signal. It is also known as *Multipath fading or Rayleigh Fading*. As in figure 2.1

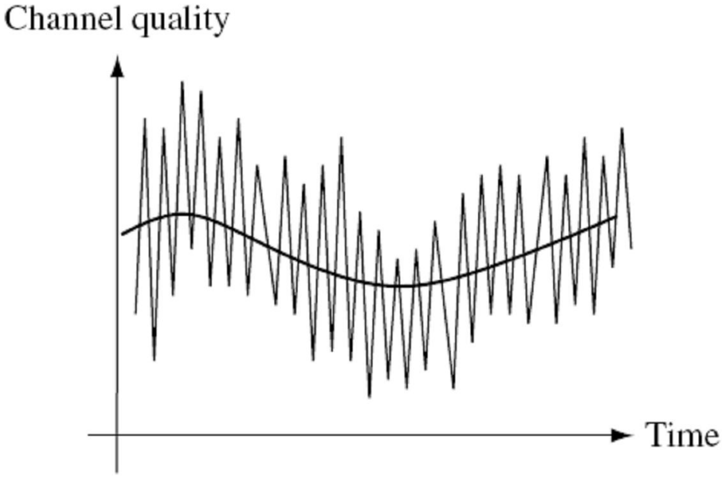

Figure 2.1 Channel quality varies over multiple time-scales. (Viswanath and Tse, 2005)

Now we will concentrate in the Multipath Fading, This type of fading experienced by a signal propagating through a channel can be determined by the nature of the transmitted signal with respect to the characteristics of the channel.

2.2.2 Time and Frequency Coherence:

2.2.2.1 Delay spread & Coherence bandwidth:

An important general parameter of a wireless system is the multipath delay spread, D, which is a type of distortion, caused when an identical signal arrives at different times at its destination. The signal usually arrives via multiple paths and with different angles of arrival. However the time difference between the arrival moment of the first multipath component (typically the Line of Sight component) and the last one is called delay spread. on the other hand Coherence bandwidth is a statistical measurement of the range of frequencies over which the channel can be considered "flat", or in other words the approximate maximum bandwidth or frequency interval over which two frequencies of a signal are likely to experience comparable or correlated amplitude fading. If the multipath time delay spread equals D seconds, then the coherence bandwidth W_c in hertz is given approximately by the equation:

$$W_c \approx \frac{1}{2\pi\ D} \tag{2.1}$$

The frequency coherence shows how quickly it changes in frequency.

2.2.2.2 Doppler spread & Coherence Time.

Another important channel parameter of a wireless systems .The coherence time of the channel is related to a quantity known as the Doppler spread of the channel. When a user is moving, the user's velocity causes a shift in the frequency of the signal transmitted along each signal path. This phenomenon is known as the Doppler shift. Signals travelling along different paths can have different Doppler shifts, corresponding to different rates of change in phase. The difference in Doppler shifts between different signal components contributing to a single fading channel tap is known as the Doppler spread.

Very important point is to recognize that the main cause in affecting time coherence is the Doppler spread, and that the larger the Doppler spread, the smaller the time coherence. The time coherence shows how quickly it changes in time.

2.2.3 Fading Types

In small scale-fading, there are different types of transmitted signals undergo different types of fading depending upon the relation between the Signal Parameters: Bandwidth, Symbol Period and Channel Parameters: Delay Spread, Doppler Spread. In any mobile radio channel a wave can be dispersed either in Time or in Frequency.

Fading effects due to Multipath Delay Spread

- Flat Fading.
- Frequency Selective Fading

Fading effects due to Doppler Spread

- Fast Fading
- Slow Fading

Table 2.1 Small-Scale Fading based on Multipath time delay spread (Viswanath and Tse, 2005)

Small -Scale Fading (Based on Multipath time delay spread)	
Flat Fading	**Frequency Selective Fading**
BW of signal < BW of channel.	BW of signal >BW of channel
Delay spread < Symbol period	Delay spread > Symbol period

Table 2.2 Small-Scale Fading based on Doppler spread (Viswanath and Tse, 2005)

Small -Scale Fading (Based on Doppler spread)	
Fast Fading	**Slow Fading**
High Doppler spread.	Low Doppler spread.
Coherence time < Symbol period	Coherence time > Symbol period
Coherence variations faster than baseband signal variations	Coherence variations slower than baseband signal variations

2.2.4 Slow Fading

The time variation of channel strengths can be demonstrated by wireless channels due to multipath fading. This variation can be either slow or fast according many categorizations in the wireless communication literature. As shown in Figure 2.2.

And as in table 2.2, fast fading channel is when the coherence time is much shorter than the delay requirement of the application, and slow fading is when the coherence time is longer.

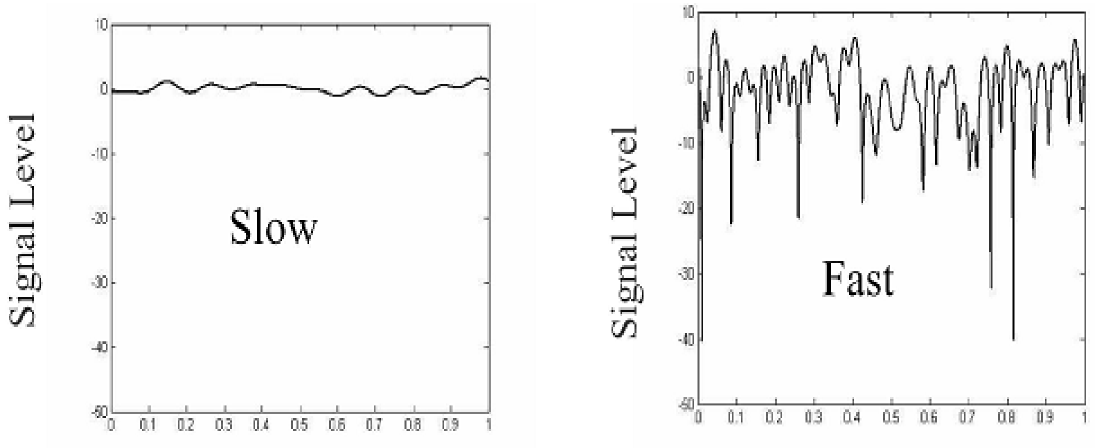

Figure 2.2 Slow and fast fading wireless channels (Ozdemir, 2007)

2.2.5 Solution for fast and slow fading channels :

Over the wireless channels, the effective transmission of the data was combated by the fading issue. Therefore, techniques have been developed to struggle that obstacle, all these techniques were depending on the diversity principle. The basic idea ruling these techniques is to transmit data in different independent dimensions, like space, time, or frequency.

In order to solve the issue of fast fading, space diversity took place. Using the space diversity by placing multiple antennas at the transmitter and /or receiver to get higher data rates, has achieved significant consideration (Foschini and Gans, 1995) ,(Telatar,1995) , (Foschini,1996) ,(Tarokh et al. ,1998),(Tarokh et al. ,1999),(Gamal and Damen, 2003) ,And (Sethuraman et al. ,2003). Those studies concentrated in increasing the data rate of point to point communication system. For multiuser system, a new technique called multiuser diversity has appeared to combat fading and increase the throughput and the term multiuser diversity was coined by (Knopp and Humblet, 1995). Thus fading is seen in multiuser diversity as an important factor to be exploited in order to increase the throughput rather than a problem that needs to be solved.

The main problem of slow fading channel is that there are not enough peaks to be exploited; a new technique was developed by inducing artificial peaks using multiple antennas, and that solution known as beamforming (T. IS-856, 2000).

Multiuser diversity and beamforming will be discussed in detail within the next sections.

2.3 MULTIUSER DIVERSITY

Over fast fading channels, the role of multiuser diversity is to increase the total throughput. Multiuser diversity improves system performance by exploiting channel fluctuations of the users illustrated in Figure 2.3. The main idea is to transmit data to a user with high SNR at any given time. If the number of the users is large enough there must be a user whose channel gain is close to the peak at any time slot. (Viswanath et al., 2002)

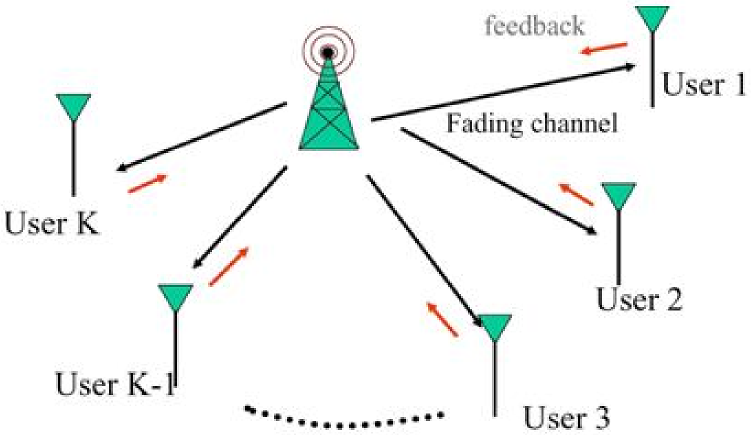

Figure 2.3 More data is transmitted to a user with high instantaneous SNR. (Ozdemir, 2007)

Selecting the users in time division multiple access (TDMA) based on round robin fashion and the conditions of the channel is not considered by the transmitter .on the other hand, selecting the users in multiuser diversity exploiting system is according to the conditions of the channel in an opportunistic way. Figure 2.4 shows what can be achieved when opportunistic selecting is used by taking in consideration the conditions of the channel. Thus users are selected in opportunistic scheduler when their channels are at their peaks.

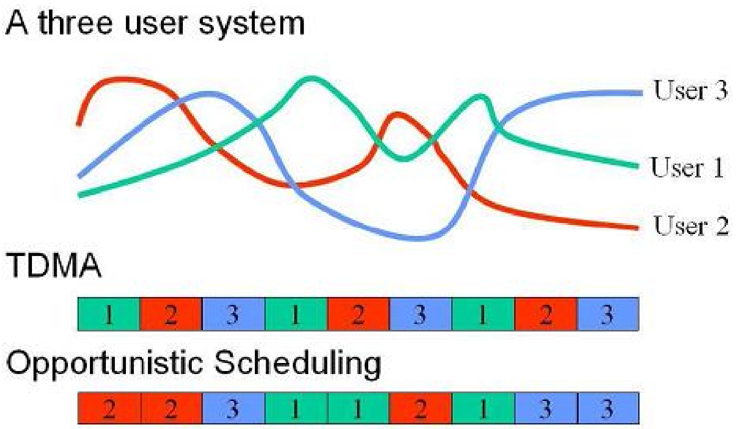

Figure 2.4 TDMA, opportunistic schedulers (Ozdemir, 2007)

Firstly, for the uplink channel, multiuser diversity technique was applied in (Knopp and Humblet, 1995), by allowing the user with the best channel to transmit .later in (Tse, 1997); it was approved for the downlink channel with similar results.

Multiuser diversity can be watched by monitoring the channel conditions of the users from the transmitter. These techniques are used in current wireless systems (such as IS-856, the third-generation data standard) (Bender et al. ,2000) where operate a feedback channel from the users to the base station .In multiuser diversity, the feedback channel sends the SNR information or the instantaneous data rate that the corresponding channel may allow.

2.4 OPPORTUNISTIC BEAMFORMING

The presence of fading is decisive for the realization of multiuser diversity gain because it increases the probability that one user is having high SNR at any given time. For slow fading environments ,since slow fading channel doesn’t has peaks to exploit by multiuser diversity ,thus the idea is to generate artificial peaks and those peaks can be achieved by allocating multiple antennas at the base station ,and sending the data with variations in the phase and the power that’s called as opportunistic beamforming method of (Viswanath et al. ,2002) as in figure 2.5 explains what happen to the channel after opportunistic beamforming , when transmit antennas change the phases and powers, a beam is randomly swept and at any time transmission is scheduled to the user currently closest to the beam. With many users, there is likely to be a user very close to the beam at any time. In order to have opportunistic beamforming, multiple antennas are required at the base station. Therefore transmitting through opportunistic beamforming to the user with the best SNR will lead to a better throughput.

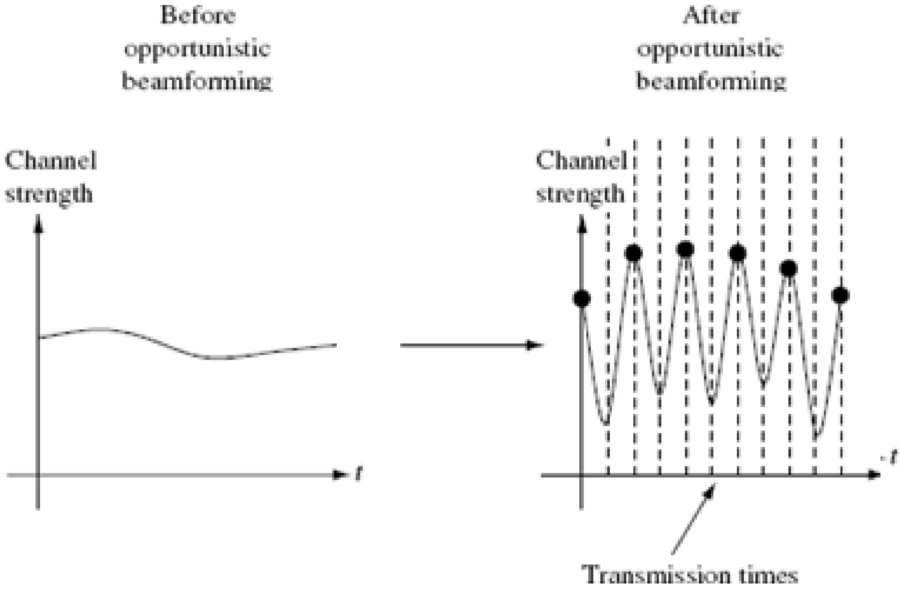

Figure 2.5 Pictorial representation of the slow fading channels of two users before (left) and after (right) applying opportunistic beamforming. (Viswanath and Tse, 2005)

However it is possible that using this technique will be unfair to some users in the system (Viswanath et al, 2002), hence the fairness is secured by using proportional fair scheduler (PFS). In our study we will use another fairness scheduler called Maximum Normalized SNR (MAX-NSNR) scheduler ,that can also guarantee the fairness among the users. In PFS, the transmitter selects the user whose ratio of instantaneous data rate to its own average data rate is the largest. The multiuser diversity concept was integrated into the downlink design of IS-856 (CDMA 2000 EV-DO) via the proportional fair scheduler by (Chaponniere et al.,2002).Opportunistic beamforming with PFS achieves the performance of true beamforming system when there are sufficient numbers of users in the system.

Figure 2.6 Plots of spectral efficiency under opportunistic beamforming as a function of the total number of users in the system. As the number of users grows, the performance approaches the performance of true beamforming.

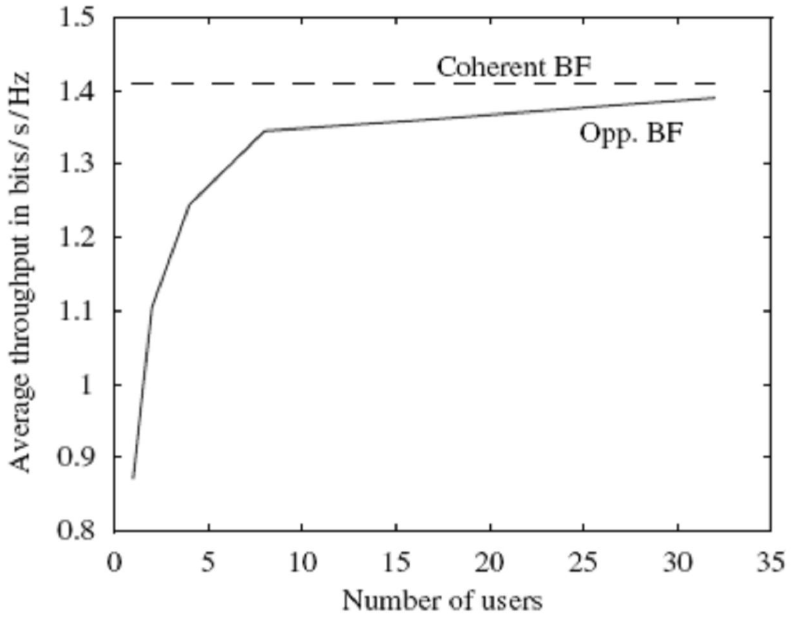

Figure 2.6 Opportunistic beamforming with PFS achieves the performance of true Beamforming system when there are sufficient numbers of users in the system. (Viswanath et al ,2002)

2.5 FEEDBACK CHANNEL QUANTIZATION

2.5.1 Introduction

The importance of feedback channels comes from that each user should return back its SNRs values to the base station in order to be check which one has the highest SNRs in anytime time, so the base station will transmit the data to that user at that given time. Since the feedback channels have limited bandwidth, therefore the data would be transmitted through it should be limited, therefore the presence of quantizing it is crucial in order to impose the limitation of its bandwidth with the best way.

The reduction of the feedback load of multiuser diversity systems, has been proposed by many approaches, most of these approaches consider the case where the channel is fast fading, hence, the case where inducing artificial fading to the system using multiple antennas were not discussed.

Quantization of the feedback channels has two methods, first one is quantization in order to maximize the average throughput of the systems, and the other is quantization to minimize the probability of error.

2.5.2 Quantization for maximizing the average throughput for fast fading environments

The impact of quantization of feedback information on the throughput of a Multiuser Diversity scheme for constant rate transmission was studied by (Fredrik Flor´en et al., 2003). In the downlink, this scheme compares the users' SNRs, and transmits to the user with the highest SNR. By only a few quantization levels can achieve a large fraction of the diversity available in the multiuser dimension. Moreover, the number of thresholds required in order to achieve a certain fraction of the throughput for the unquantized case increased with the number of users, although the difference was small for a high number of users.

The number of thresholds required in order to achieve a certain fraction of the maximum throughput increases with the number of users. For both figures 2.7 and 2.8.

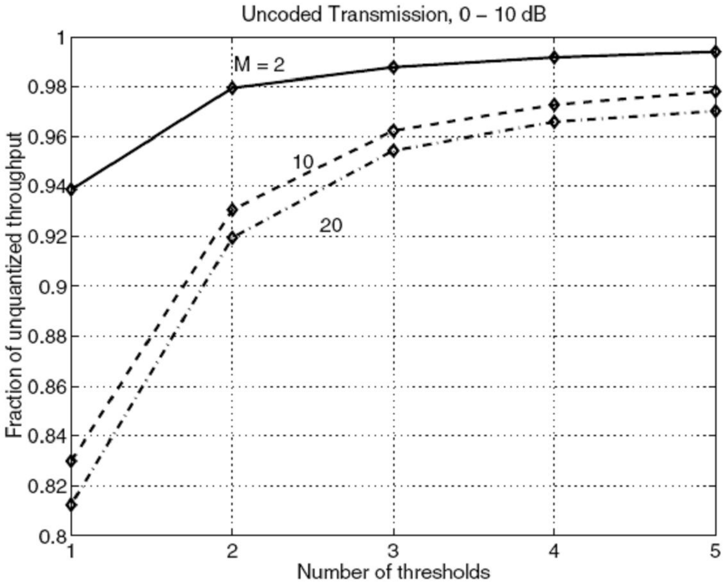

Figure 2.7 Fraction of throughput using unquantized feedback for uncoded transmission (N = 200) for 1 to 5 thresholds and for 2, 10, and 20 users. (Flor´en et al., 2003)

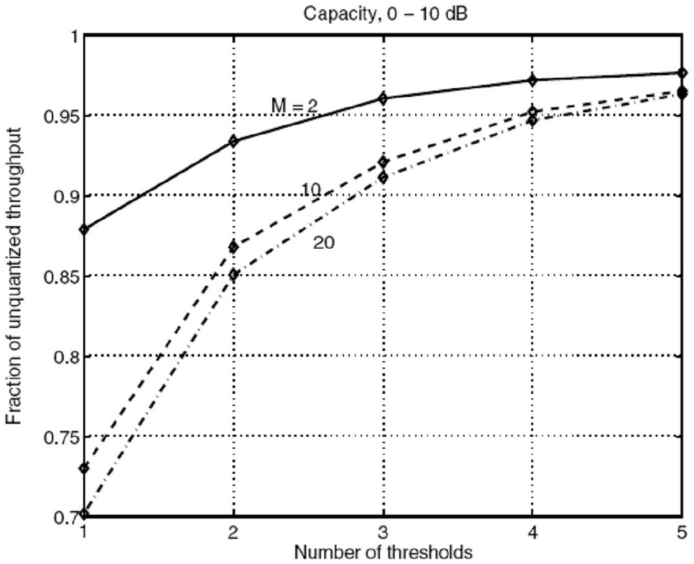

Figure 2.8 Fraction of throughput using unquantized feedback in terms of capacities for 1 to 5 thresholds and for 2, 10, and 20 users.(Flor´en et al., 2003)

For both performance measures it is seen that using quite few thresholds gives a throughput greater than 90% of that of the unquantized case.

2.5.3 Quantization for maximizing the average throughput for slow fading environments

Designing the optimum SNR quantizer in opportunistic beamforming to maximize the average throughput of the system was achieved by (Ozdemir, 2007).

A closed form solution for the quantization levels that maximize average throughput is difficult to obtain even in the simplest case. However, numerical methods used to compute the optimum quantization levels. Furthermore, by increasing the number of users to reach infinity the performance of the opportunistic beamforming approaches to that of the true beamforming. Therefore, when there are sufficient number of users in the system opportunistic beamforming can still be operated.

As the number of users increases the optimum threshold level increases. Asymptotically they both go to one which is the highest value of the normalized SNR metrics. For a given number of users, the optimum threshold level is higher for a system with 2 antennas than that with 3 antennas. As in figure 2.9

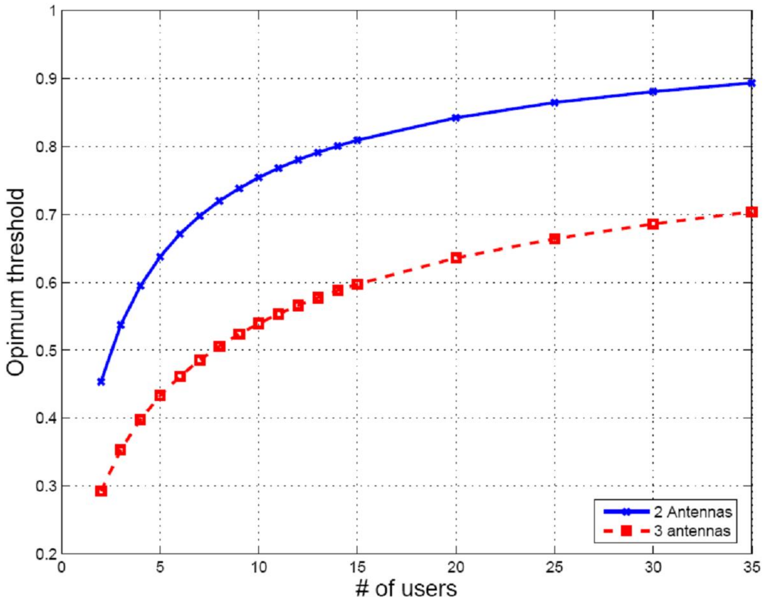

Figure 2.9 Optimum threshold level (1-bit feedback) as number of users grows for both 2 and 3 transmitting antennas (Ozdemir, 2007)

2.5.4 Quantization for minimizing the probability of error for fast fading environment

In this paper they (Tao Lau and Kschischang, 2007) based their theory on the quality of the channel, and therefore they developed strategies in order to define the error according to the incorrectly received signals.

They developed a strategy to obtain optimum minimization of the probability of P_e using the channel quality in order to identifying the user with the best channel quality.

As can be noticed from figure 2.10 that optimal quantization scheme minimizes the probability of error more than it is in uniform quantization scheme

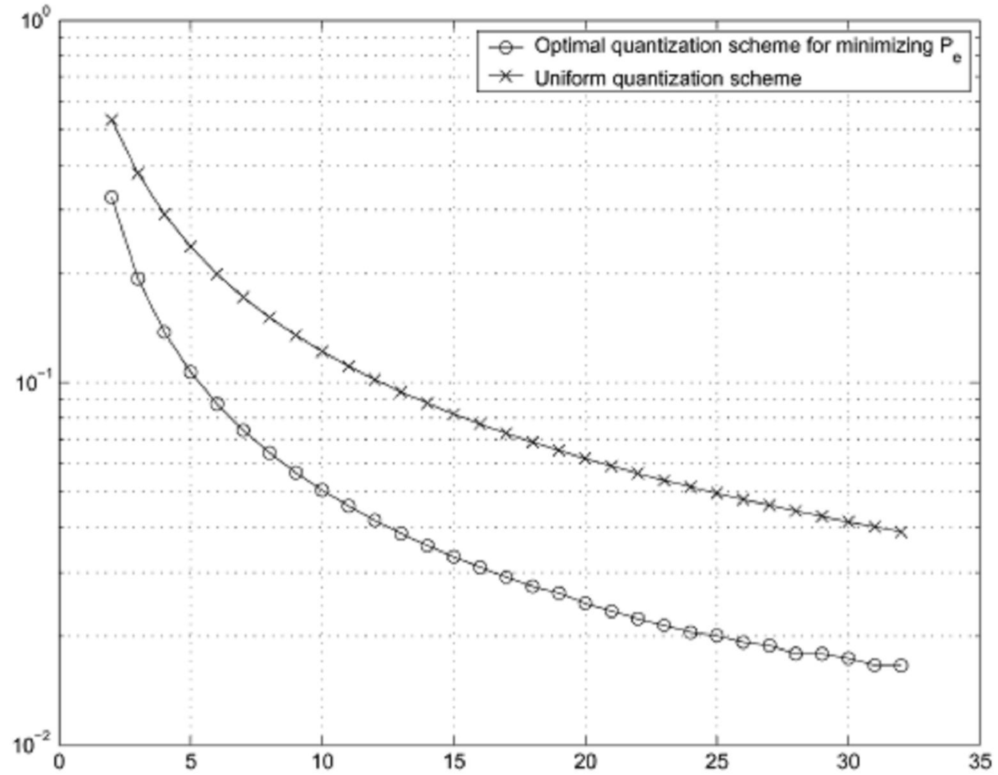

Figure 2.10 P_e For the uniform quantization scheme and the optimal quantization scheme for a system of five users and various values of L.

(Tao Lau and Kschischang, 2007)

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CHAPTER 3

QUANTIZATION FOR MINIMIZING THE PROBABILITY OF ERROR IN SLOW FADING ENVIRONMENTS

3.1 INTRODUCTION

In a wireless system, multiuser diversity makes use of the independent changes in the strength of the channels for different users. What multiuser diversity based on is transmitting more data to user whose channel condition is better at any given time.

The presence of fading is decisive for the realization of multiuser diversity gain because it increases the probability that one user is having high SNR at any given time. For slow fading environments multiuser diversity can be obtained by the opportunistic beamforming method of (Viswanath et al ,2002) in order to have opportunistic beamforming ,multiple antennas are required at the base station. Therefore transmitting through opportunistic beamforming to the user with the best SNR will lead to a better throughput. However using this technique maybe will be unfair to some users in the system (Viswanath et al, 2002), hence the fairness is secured by using proportional fair scheduler (PFS). Or maximum normalized SNR (Max-NSNR) scheduler (Ozdemir, 2007). Here we will use Max-NSNR scheduler.

Since the user channel conditions have to be monitored at the base station in order to send the data according to the principle of transmitting data to the user who has better channel conditions at any given time, then feedback channels send their channel conditions all the time, but as it’s known that the feedback channels have a limited bandwidth, so in order to use these channels with the best efficient way, feedback channel need to be quantized.

The reduction of the feedback load of multiuser diversity systems, has been proposed by many approaches, most of these approaches consider the case where the channel is fast fading, hence, the case where inducing artificial fading to the system using multiple antennas were not discussed.

Quantization of the feedback channels has two criteria, first one is quantization in order to maximize the average throughput of the systems, and the other is quantization to minimize the probability of error.

Maximizing the average throughput it means increases the capacity of the system, in the other hand minimizing the probability of error it depends on how we define the error. In (Tao Lau and Kschischang, 2007) they defined the error by not selecting the user with the best channel quality. However In our study we define the error by not selecting the user with the highest Normalized SNR.

In this chapter, we derive the optimum quantization scheme for the opportunistic beamforming system that minimizes the probability of error, while ensuring the fairness among users. In section 3.2 we discuss the system model we will build our theory on. Scheduling scheme we use will be discussed in 3.3. feedback channels discussed in section 3.4.In sections 3.5 quantization to minimize the probability of error in a simple case of 2 users with 2 thresholds with multiple antennas, quantization to minimize the probability of error for multi user with multi thresholds with multiple antennas, will be discussed theoretically in section 3.6, however the simulation results will be followed in the next chapter.

3.2 SYSTEM MODEL

We consider the downlink channel in a cellular radio system where a base station with T transmit antennas communicate with K single antenna active users, Assume Transmissions are time slotted and the channels between the base station and each user are slowly time varying, and the channel coefficient $h_k(t)$ from the tth transmit antenna to the kth user keeps constant during the coherence time T_c of the channel.

Since this system is for slow fading environments, opportunistic beamforming technique is used in order to obtain the multiuser diversity gain, therefore the base station has to induce fluctuations to the channel by using a random $T \times 1$ beamforming vector q(n). Under this setup, the data vector $y_k(n)$ received by kth user can be written as:

$$\boldsymbol{y_k(n) = (q^H(n)h_k)x(n) + z_k(n)} \tag{3.1}$$

where $\boldsymbol{h_k}$ is a T × 1 vector that contains the channel coefficients, and $\boldsymbol{z_k(n)}$ is the additive complex white Gaussian noise with zero mean and variance $\sigma^2 = 1$. We assume that $P_x = E\{x^*(n)\, x(n)\} = 1$, $h_k(t)$ is complex Gaussian with zero mean and variance $\bar{\gamma}$, and the path loss and other powers are lumped into the channel process. We assume a block fading channel model. Note that in this model the effective channel observed by the user k is $\boldsymbol{q^H(n)h_k}$ which changes with $q(n)$ for each time slot n. Fig. 3.1 illustrates our system model for opportunistic downlink communications.

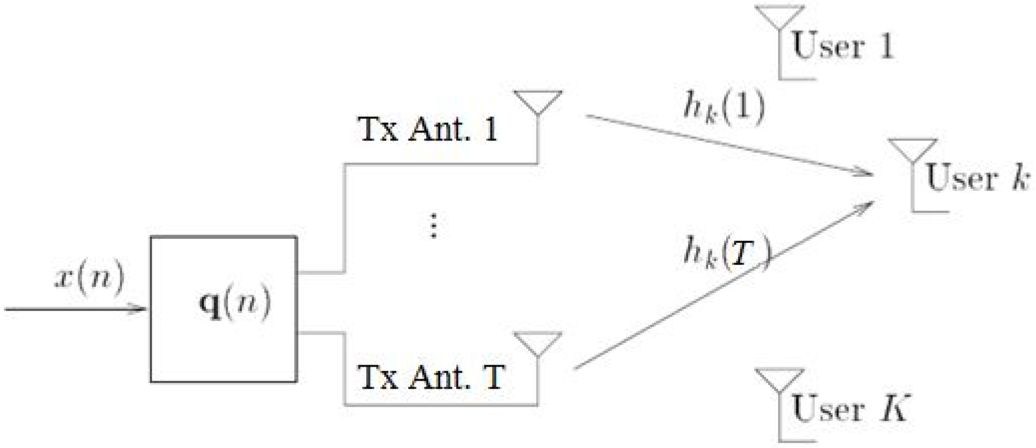

Figure 3.1 the system model for opportunistic downlink communications

3.3 SCHEDULING ALGORITHMS

The simplest scheduling method that the transmitter may use is the round robin scheduler which gives the users access to the channel periodically without favoring any user. The maximum SNR scheduler, referred to as Max-SNR scheduler in the literature, on the other hand, favors the user with the maximum SNR over each transmission slot without any fairness concern. However In order to achieve fairness among users the proportional fair scheduler (PFS) (Viswanath et al, 2002) was developed and Max-SNR Scheduler (Sharma and Ozarow, 2005). In this dissertation we will focus on Max - NSNR. $\gamma_k(n)$ is defined as the instantaneous SNR of the user k

$$\gamma_k(n) = \frac{P_x}{\sigma^2}\, \boldsymbol{q}^H(n)\boldsymbol{h}_k\boldsymbol{h}_k^H\boldsymbol{q}(n) \tag{3.2}$$

where $\frac{P_x}{\sigma^2} = \mathbf{1}$.

Previous studies proved that this technique is using multi user diversity that allows us to use it. However, this technique is based on transmitting to the user whose normalized SNR is the highest in any given time, and it can be written as:

$$m_k(n) = \frac{q^H(n)h_k h_k^H q(n)}{h_k^H h_k} \tag{3.3}$$

And selecting the user $k^*$with the largest normalized SNR can be written as:

$$k^* = \mathrm{arg}_{\mathrm{k}=\{1,2,\ldots\ldots,\mathrm{K}\}}\, \mathrm{max}\ \ m_k(n) \tag{3.4}$$

Therefore throughput R of the system can be calculated as follows:

$$R = log_2\left(1 + q^H(n)\, h_{k^*}\, h_{k^*}^H(n)\right) \tag{3.5}$$

Since it is shown in (Ozdemir, 2007) that all users can be selected with equal probability. The Max-NSNR scheduler as PFS guarantees fairness among users. Therefore we will use Max-NSNAR scheduler, first we will use the probability distribution functions (PDFs) and cumulative distribution functions (CDFs) that will be used here and throughout the work. Since h_k is a zero mean complex Gaussian vector process, The PDF and CDF of m_k are given by

$$f\mathrm{M_k(m_k)} = \begin{cases} (\mathrm{T}-1)(1-\mathrm{m_k})^{\mathrm{T}-1} & 0 \le \mathrm{m_k} \le 1 \\ 0 & \text{otherwise} \end{cases} \tag{3.6}$$

And

$$\mathcal{F}\mathrm{M_k(m_k)} = \begin{cases} 0 & \mathrm{m_k} \le 0 \\ 1 - (1-\mathrm{m_k})^{\mathrm{T}-1} & 0 \le \mathrm{m_k} \le 1 \\ 1 & \mathrm{m_k} \ge 1 \end{cases} \tag{3.7}$$

Respectively as in (Sharma and Ozarow, 2005).

3.4 FEEDBACK CHANNELS

The importance of feedback channels comes from that each user should return back its SNRs values to the base station in order to be check which one has the highest SNRs in anytime time, so the base station will transmit the data to that user at that given time. Since the feedback channels have limited bandwidth, therefore the data would be transmitted through it should be limited, therefore the presence of quantizing it is crucial in order to impose the limitation of its bandwidth with the best way.

The reduction of the feedback load of multiuser diversity systems, has been proposed by many approaches, most of these approaches consider the case where the channel is fast fading, hence, the case where inducing artificial fading to the system using multiple antennas were not discussed.

Quantization of the feedback channels has two methods, first one is quantization in order to maximize the average throughput of the systems, and the other is quantization to minimize the probability of error.

Maximizing the average throughput it means increases the capacity of the system, in the other hand minimizing the probability of error depends on the way we define the error. In (Tao Lau and Kschischang, 2007) they defined the error by not selecting the user with the best channel quality. However In our study we define the error P_e by not selecting the user with the highest Normalized SNR.

3.5 MINIMIZATION OF P_e FOR A SYSTEM WITH 2 USERS AND 2 THRESHOLDS

In this section, we will discuss quantization schemes that minimize the probability P_e that the system incorrectly identifies the user with the highest normalized SNR. These schemes are jointly designed and are dependent on the number of active users K in the system and number of antennas and number of quantization levels L allowed for each user's NSNR. We will first show the optimal scheme in minimizing P_e when $K = 2$ followed by the solution for a general K.

Consider the normalized SNR values are independent and none uniformly distributed, as we have T number of antennas.

In order to clarify the problem we consider a simple scenario where T=2, K=2, L=2, where T is the number of antennas, K= number of users, L= number of quantization thresholds.

We have one threshold for each user $\eta_1$and $\eta_2$for k_1 and k_2 respectively.

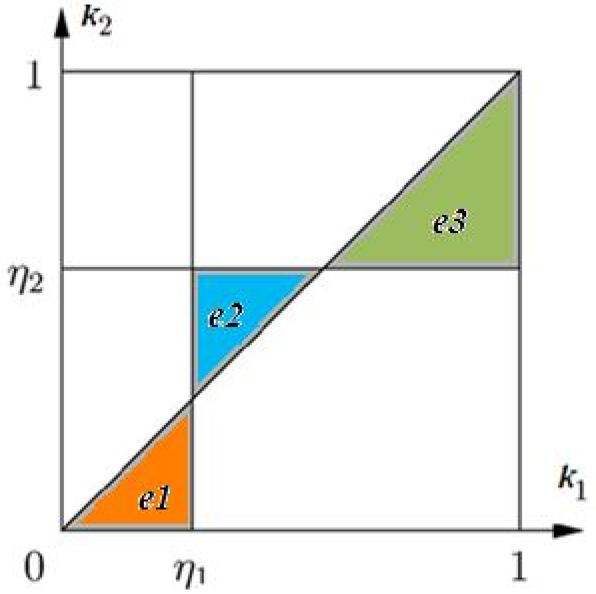

Figure 3.2 Error regions for decision rule for 2 users

Shaded regions are the region of error according to decision rule, where in the region e1, where users $k_1$and k_2 are less than their thresholds $\eta_1$and η_2 respectively, however user k_2 is selected because it has higher thresholds although k_1 should be selected since it has greater unquantized value than userk_2 . In the second region e2, k_1 is greater than its threshold η_1, and k_2 is less than its threshold η_2 ,thus k_1 is selected although k_2 should be selected cause it has higher unquantized value than k_1. In the last region e3 both $k_1$and k_2 are greater than their thresholds η_1 and η_2 respectively, k_2 is selected since it has the highest threshold, although k_1 has higher unquantized value than k_2. This technique used to map the quantization levels.

Note that this scheme is two dimensional plot since the PDF of uniform distribution is equal to 1, thus it's required to calculate the areas to find the probability of error and that reduces to (Tao Lau and Kschischang, 2007). Instead of using $k_1$and k_2 , they used the channel quality indices ($c_1$and c_2).

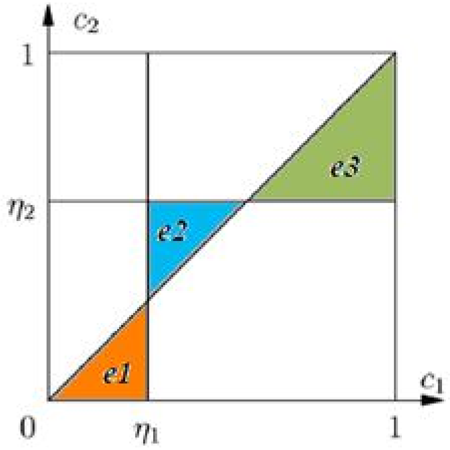

Figure 3.3 Error regions for channel quality indices

Therefore P_e is equal to:

$$P_e(\eta_1,\eta_2) = \frac{\eta_1^2}{2} + \frac{(\eta_2 - \eta_1)^2}{2} + \frac{(1-\eta_2)^2}{2} \tag{3.8}$$

Then they considered a system with L thresholds and after some calculations they achieved this formula for 2 users case with L thresholds and T=2

$$P_e(\eta) = \sum_{j=1}^{2L-1} \frac{1}{2} (\eta_j - \eta_{j-1})^2 \tag{3.9}$$

Therefore To find the optimal quantization scheme that minimizes P_e , they solved the partially differentiated equations $\frac{\partial P_e}{\partial \eta_i} = 0\,, for\ i\ \in \{1,2\}$, and obtained

$$\eta_i = \frac{i}{2L-1} \tag{3.10}$$

In our case we are using multiple antennas at the base station, that’s means we are going to use three dimensional plots since the PDF of non-uniform distribution is not 1, PDF as mentioned in (3.6), thus it’s required to calculate the volumes to find the probability of error. Instead of using $c_1$and c_2, we use the Normalized SNR indices ($m_1$and m_2).

When T=2, L=2, and K=2. In this case the PDF from (3.6) = 1, then the same results will be achieved as in (Tao Lau and Kschischang, 2007).

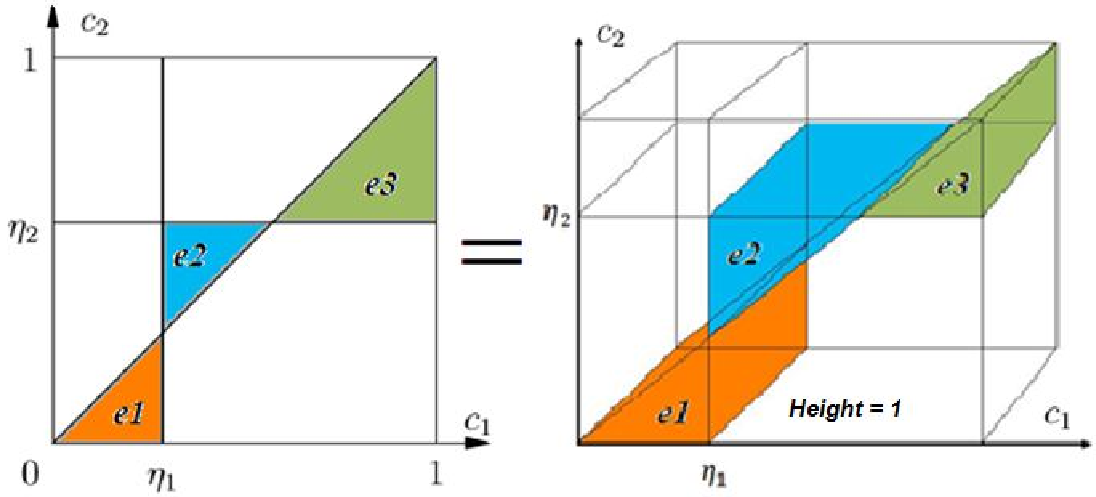

Figure (3.4) When T=2 then the PDF = 1, then reduces to (Tao Lau and Kschischang, 2007).

For a system with K = 2, L = 2 and T=3, so in order to find the whole error region, then we have three regions to cover as shown in figure 3.5

Since the PDF of our NSNR values for two users ($m_1$and m_2) are (3.6):

$$\not{f}\mathrm{M}_1(\mathrm{m}_1) = (\mathrm{T}-1)(1-\mathrm{m}_1)^{\mathrm{T}-1}$$
$$\not{f}\mathrm{M}_2(\mathrm{m}_2) = (\mathrm{T}-1)(1-\mathrm{m}_2)^{\mathrm{T}-1}$$

And since we have T=3, then those PDF can be written as:

$$\not{f}\mathrm{M}_1(\mathrm{m}_1) = 2(1-\mathrm{m}_1)^2$$
$$\not{f}\mathrm{M}_2(\mathrm{m}_2) = 2(1-\mathrm{m}_2)^2$$

Using the PDF values is because we have non uniform distribution function for the NSNR values since we have multiple antennas in the system.

Now we are thinking about finding the volume under the error regions and that can be done by using double integrals, and the boundaries we use is according to the decision rule and this selection of the boundaries for integrals will be used for the rest of this work.

The total error region P_e can be written for three regions since we have three different sets of boundaries.

$$
\begin{aligned}
P_e = &\int_0^{\eta 1}\int_{m2}^{\eta 1} 2(1-\mathrm{m}_1)^2 * 2(1-\mathrm{m}_2)^2 \, dm_1 \, dm_2 \\
&+ \int_{\eta 1}^{\eta 2}\int_{\eta 1}^{m2} 2(1-\mathrm{m}_1)^2 * 2(1-\mathrm{m}_2)^2 \, dm_1 \, dm_2 \\
&+ \int_{\eta 2}^{1}\int_{m2}^{1} 42(1-\mathrm{m}_1)^2 * 2(1-\mathrm{m}_2)^2 \, dm_1 \, dm_2 \qquad (3.11)
\end{aligned}
$$

$$
\begin{aligned}
P_e = &\int_0^{\eta 1}\int_{m2}^{\eta 1} 4[1-m_1-m_2+(m_1 * m_2)] \, dm_1 \, dm_2 \\
&+ \int_{\eta 1}^{\eta 2}\int_{\eta 1}^{m2} 4[1-m_1-m_2+(m_1 * m_2)] \, dm_1 \, dm_2 \\
&+ \int_{\eta 2}^{1}\int_{m2}^{1} 4[1-m_1-m_2+(m_1 * m_2)] \, dm_1 \, dm_2 \qquad (3.12)
\end{aligned}
$$

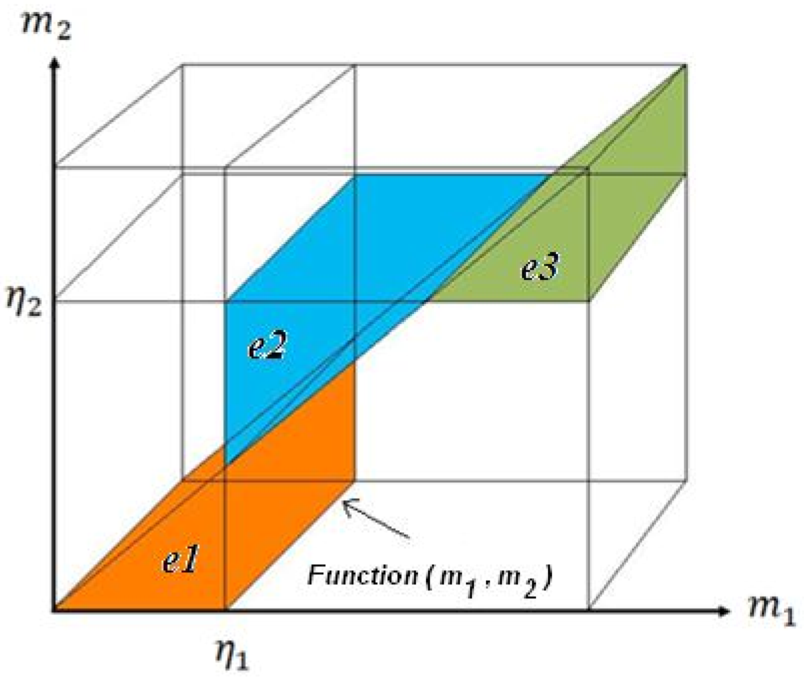

Figure 3.5 : The corresponding quantization regions. The three shaded regions, labeled by e1, e2, e3, are the error regions resulting from the decision rule for NSNR

After some calculations:

$$
\begin{aligned}
P_e \quad = {} & \frac{\eta_1^2}{2}\,[\eta_1 - 2]^2 \\
& + \frac{\eta_2^2}{2}\,[\eta_2 - 2]^2 - (\eta_1\eta_2)\,(\eta_1 - 2)(\eta_2 - 2) + \frac{\eta_1^2}{2}\,[\eta_1 - 2]^2 \\
& + \frac{1}{2}\,(1 - \eta_2)^4 \qquad (3.13)
\end{aligned}
$$

Let's consider general T and L=2, K=2 as number of the Transmitting antennas is general value, then

$$
\begin{aligned}
P_e = {} & \int_0^{\eta 1}\int_{m2}^{\eta 1}(T-1)^2\,(1-m_1)^{T-2}\ \ (1-m_2)^{T-2}\ \ dm_1\,dm_2 \\
& + \int_{\eta 1}^{\eta 2}\int_{\eta 1}^{m2}(T-1)^2\,(1-m_1)^{T-2}\ \ (1-m_2)^{T-2}\ \ dm_1\,dm_2 \\
& + \int_{\eta 2}^{1}\int_{m2}^{1}(T-1)^2\,(1-m_1)^{T-2}\ \ (1-m_2)^{T-2}\ \ dm_1\,dm_2 \qquad (3.14)
\end{aligned}
$$

After some calculations and derivations we achieved this formula that can allow us to determine the Probability of error in case of having T antennas and L thresholds for tow users systems.

$$
P_e = \sum_{j=1}^{2L-1}\left(\frac{(\eta_j - 1)^{2T-2}}{2} - \left(\eta_{j-1} - 1\right)^{T-1}\left(\eta_j - 1\right)^{T-1} + \frac{(\eta_{j-1} - 1)^{2T-2}}{2}\right) \qquad (3.15)
$$

For $\eta_{2L-1} = 1$ & $\eta_0 = 0$.

3.6 MINIMIZATION OF P_e FOR A SYSTEM WITH GENERAL K USERS AND GENERAL L THRESHOLDS

Finding the probability of error for K users system achieved with some equations as follows:

After understanding theorems provided by Lau and Kschischang in their paper, we will start from these equations they achieved:

$$P_e = 1 - \sum_{i=1}^{(L-1)K} \int_{\eta_{i-K+1}}^{\eta_{i+1}} \prod_{j-1}^{K-1} \Gamma\big(c, 0, \eta_{i-j}\big) dc \tag{3.16}$$

Since

$$P\big(M_i, H_q\big) = \int_{L_i}^{U_i} \prod_{\substack{j\in K \\ j\neq i}} \Gamma\big(c_i, L_j, U_j\big) dc_i \tag{3.17}$$

And therefore probability of error is:

$$P_e = 1 - \int_{L_i}^{U_i} \prod_{\substack{j\in K \\ j\neq i}} \Gamma\big(c_i, L_j, U_j\big) dc_i \tag{3.18}$$

Where

$$\Gamma\big(c_i, L_j, U_j\big) = \min(c_i, U_i) - \min(c_i, L_i) \tag{3.19}$$

Therefore the final formula in finding the probability of error is:

$$P_e = 1 - \frac{1}{K} - \sum_{j=1}^{K} \frac{1}{K-j} \sum_{i=1}^{(L-1)K} \eta_i \eta_{i-1} \dots \eta_{i-j+1} ({\eta_{i+1}}^{K-j} - {\eta_i}^{K-j}) \tag{3.20}$$

For our study we will make some modifications in these algorithms, as follows; Suppose we have M_i be the event that max $\{m_1,m_2, m_3,\dots\dots\dots m_i\}= m_i$. Suppose that,

$$\Gamma\big(mk_i, L_j, U_j\big) = f(mk)\ (\mathcal{F}\ (\min(m_i, U_i)) - \mathcal{F}(\min(m_i, L_i))) \tag{3.21}$$

Where PDF and CDF functions provided respectively;

$$f(m) = (T-1)(1-m)^{T-2} \tag{3.22}$$

$$\mathcal{F}(m) = 1 - (1-m)^{T-1} \tag{3.23}$$

And after some calculations we achieved that by changing the values in of Γ in the origin equation, there would be no changes in the mathematical procedures were taken in order to reach this final form.

So the shape of the new equation is going to be as in this general formula:

$$P_e = 1 - \sum_{i=1}^{(L-1)K} \int_{\eta_{i-K+1}}^{\eta_{i+1}} \prod_{j-1}^{K-1} \Gamma\left(m, 0, \eta_{i-j}\right) dm \tag{3.24}$$

$$\begin{aligned} P_e = 1 - \sum_{i=1}^{(L-1)K} \Bigg[& \int_{\eta_{i-K+1}}^{\eta_{i-K+2}} f(m)\, \mathcal{F}(m)^{K-1}\, dm \\ & + \int_{\eta_{i-K+2}}^{\eta_{i-K+3}} f(m)\, \mathcal{F}(\eta_{i-K+2}) \mathcal{F}(m)^{K-1}\, dm + \cdots \\ & + \int_{\eta_{i-K+3}}^{\eta_{i-K+4}} f(m)\, \mathcal{F}(\eta_i)\, \mathcal{F}(\eta_{i-1}) \mathcal{F}(\eta_{i-2}) \ldots \mathcal{F}\left(\eta_{i-(k-2)}\right) dm \Bigg] \end{aligned} \tag{3.25}$$

$$\begin{aligned} P_e = 1 - \frac{1}{K} - \frac{1}{K-1} & \sum_{i=1}^{(L-1)K} \mathcal{F}(\eta_i)\, \left(\mathcal{F}(\eta_{i+1})^{K-1} - \mathcal{F}(\eta_i)^{K-1}\right) \\ & - \frac{1}{K-2} \sum_{i=1}^{(L-1)K} \mathcal{F}(\eta_i) \mathcal{F}(\eta_{i-1})\, \left(\mathcal{F}(\eta_{i+1})^{K-2} - \mathcal{F}(\eta_i)^{K-2}\right) - \cdots \\ & - \sum_{i=1}^{(L-1)K} \mathcal{F}(\eta_i) \mathcal{F}(\eta_{i-1}) \ldots \mathcal{F}\left(\eta_{i-(K-2)}\right) \left(\mathcal{F}(\eta_{i-1}) - \mathcal{F}(\eta_i)\right) \end{aligned} \tag{3.26}$$

So the final form is:

$$P_e = 1 - \frac{1}{K} - \sum_{j=1}^{K} \frac{1}{K-j} \sum_{i=1}^{(L-1)K} \mathcal{F}(\eta_i) \mathcal{F}(\eta_{i-1}) \ldots \mathcal{F}\left(\eta_{i-j+1}\right) \left(\mathcal{F}(\eta_{i+1})^{K-j} - \mathcal{F}(\eta_i)^{K-j}\right) \tag{3.27}$$

Where

$$\mathcal{F}(\eta_i) = 1 - (1 - \eta_i)^{T-1} \tag{3.28}$$

$$\mathcal{F}(\eta_{i-1}) = 1 - (1 - \eta_{i-1})^{T-1} \tag{3.29}$$

$$\mathcal{F}(\eta_{i+1}) = 1 - (1 - \eta_{i+1})^{T-1} \tag{3.30}$$

Note that when the case is T=2 in (3.27), then it will lead directly to (3.20) .In order to find the optimum minimization of the probability of error; we need to find first derivative and make it equal to zero and solve that equation.

Since the partial differentiated equations for all cannot be solved analytically in general, the sequential quadratic programming (SQP) method of (Fletcher and Powell, 1963), (Goldfarb, 1970) is used in the minimization of and the results will be shown in the following chapter.

CHAPTER 4

SIMULATION RESULTS AND DISCUSSION

4.1 INTRODUCTION

Since the partial differential equations for the previous equation (3.27) cannot be solved analytically in general, the sequential quadratic programming (SQP) method of (Fletcher and Powell, 1963), (Goldfarb, 1970) is used to minimize the probability of error.

Verifying this strategy can be achieved if simulation results are approximately equal to theoretical results .Simulation results achieved using Monte Carlo simulation method. Simulation results were under the same conditions of the theoretical approach. All of this will be discussed in section 4.2 with figures.

Section 4.3, by using random threshold values, can the optimum minimum probability of error be achieved? By using simulation results for 100,000 trials and using numerical methods for theoretical results.

In section 4.4 the optimum threshold values are calculated for different L, K, and T values.

In section 4.5 the effect of adding multiple antennas to the system on the probability of error will be discussed.

In section 4.6 the effect of adding multiple antennas to the system on the average throughput will be discussed.

In section 4.7 includes discussion of the presented results.

4.2 SYSTEM VERIFICATION

Assume that we have a system that consists of T number of antennas, L number of thresholds, and K number of users, in this section we will fix two of them and change the other and see what is the behavior of the system, and see if simulation results by counting the errors -according to the decision rule discussed before- will be close enough to the results by computing the probability of error to the same variables or not.

4.2.1 Fixing T = 6 and L=5 and changing K=2:10

By fixing T = 6 and L=5, and changing K, in order to find P_e . Theoretical results (drawn as line) highly match the simulation results (drawn as stars). For the simulation results we assumed the same characteristic conditions of theoretical part , we just counted down the error according to the same decision rule we used in building the theory, calculating the errors in the system with 10,000 trials for simulation ,the results are shown figure 4.1.

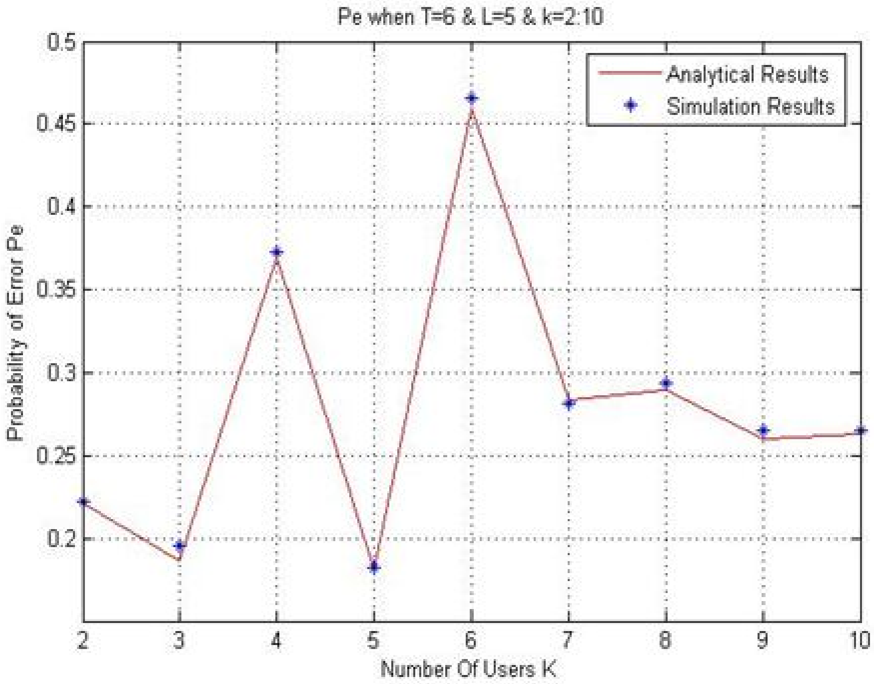

Figure 4.1 Plot of P_e for Simulation results (*) Vs Theoretical results (-), when T=6, L=5, and K=2:10 for random threshold values

4.2.2 Fixing T = 6 and K=4 and changing L=2:10

Again by fixing T = 6 and K=4, and changing L, in order to find P_e . Theoretical results (drawn as line) highly match the simulation results (drawn as stars). For the simulation results we assumed the same characteristic conditions of theoretical part , we just counted down the error according to the same decision rule we used in building the theory, calculating the errors in the system with 10,000 trials for simulation ,the results are shown in figure 4.2.

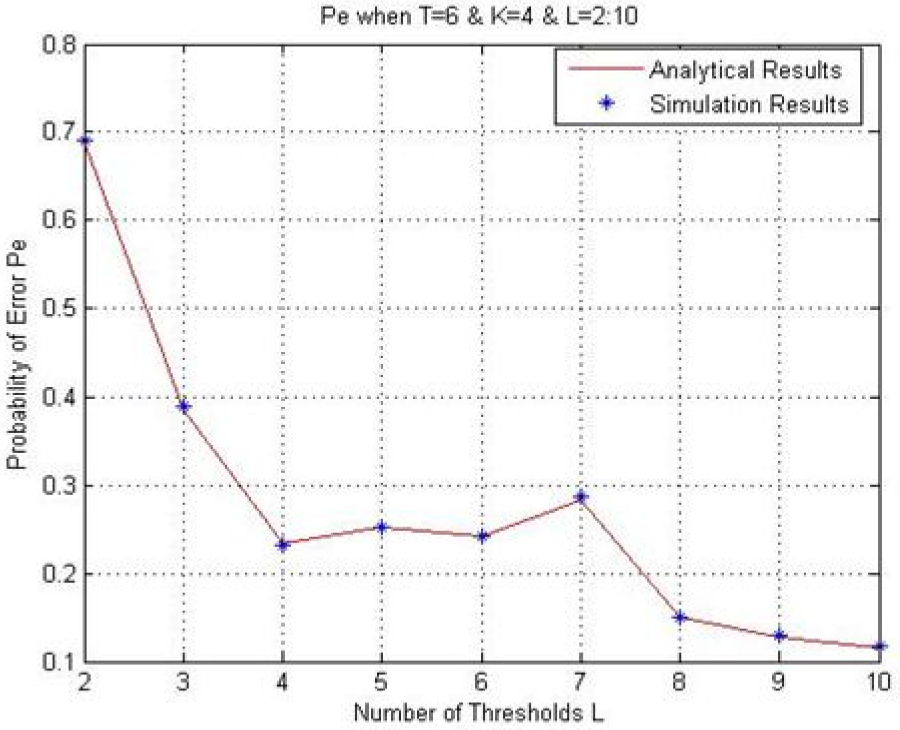

Figure 4.2 Plot of P_e for simulation results (*) Vs theoretical results (-),when T=6,K=4,and L=2:10 for random threshold values

4.2.3 Fixing K = 4 and L=5 and changing T=2:10

Finally by fixing L =5 and K=4, and changing T, in order to find P_e . Theoretical results (drawn as line) highly match the simulation results (drawn as stars). For the simulation results we assumed the same characteristic conditions of theoretical part , we just counted down the error according to the same decision rule we used in building the theory, calculating the errors in the system with 10,000 trials for simulation ,the results are shown in figure 4.3.

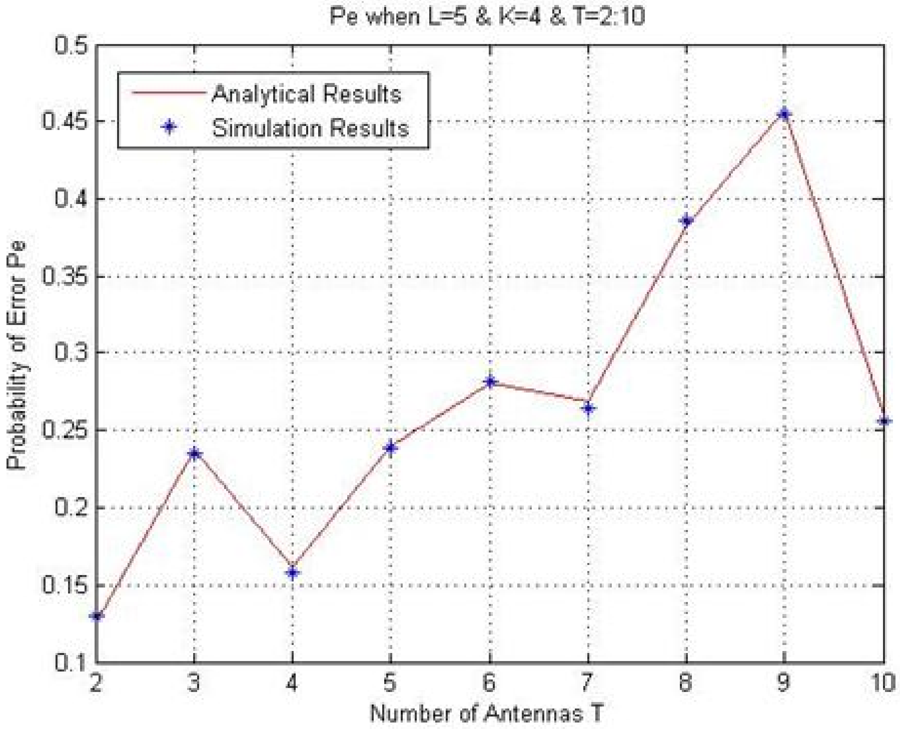

Figure 4.3 Plot of P_e for simulation results (*) Vs theoretical results (-), when K = 4 ,L=5 and T=2:10 for random threshold values

4.2.4 Discussion of Results:

In figures 4.1, 4.2, and 4.3, we first showed that the probability of error equation in (3.27) is valid by using Monte Carlo simulations. In these simulations we are not minimizing the probability of error therefore the thresholds are selected randomly and (3.27) is evaluated for these thresholds values. In order to show the validity of (3.27) Monte Carlo simulations with 10,000 trials is conducted with the same system parameters.

The results are shown in figures 4.1, 4.2, and figure 4.3. For both cases the simulation results and theoretical results are very close to each other validating the probability of error expression in (3.27).

Therefore this strategy mathematically is functioning appropriately .and this approach can be used in order to study the effects of adding multiple antennas to the system.

4.3 OPTIMUM PROBABILITY OF ERROR

Here, approaching the optimum minimum P_e if random thresholds are used to run the system is going to be studied.

4.3.1 Fixing T = 6 and K=4 and changing L=2:15

In this part we will just check with one case by fixing T = 6 and K=4 and changing L=2:15.

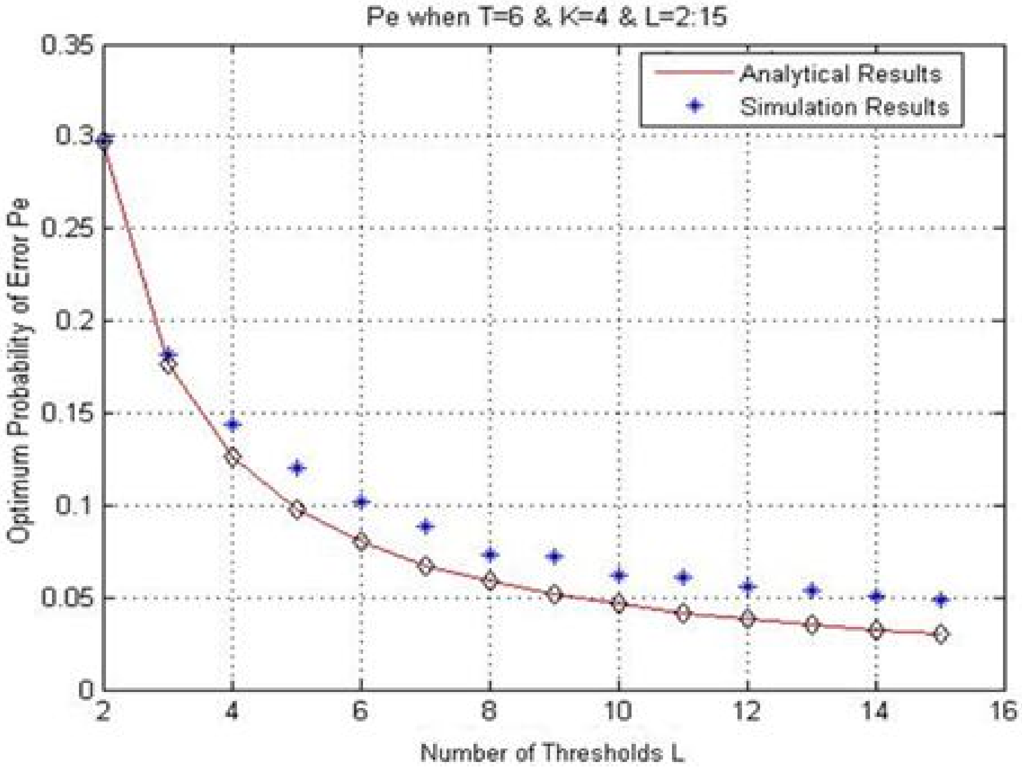

Figure 4.4 Plot of P_e simulation results (*) Vs Theoretical results, when T=6,K=4,and L=2:10

4.3.2 Fixing T = 6 and L=5 and changing K=2:15

In this part we will just check with one case by fixing T = 6 and L=5 and changing K=2:15.

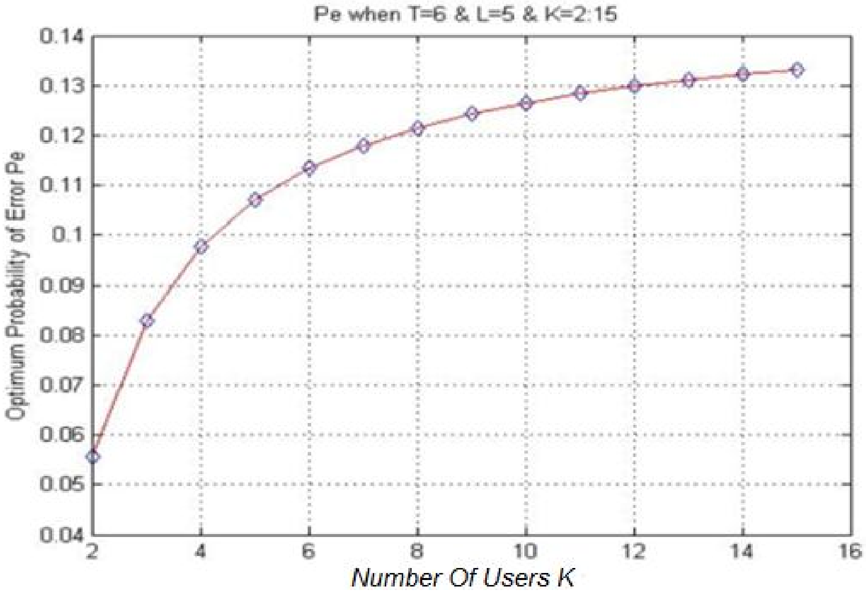

Figure 4.5 Plot of Theoretical results of P_e , when T=6, L=5, and K=2:15

4.3.3 Fixing K = 4 and L=5 and changing T=2:15

In this part we will just check with one case by fixing K = 4 and L=5 and changing T=2:15.

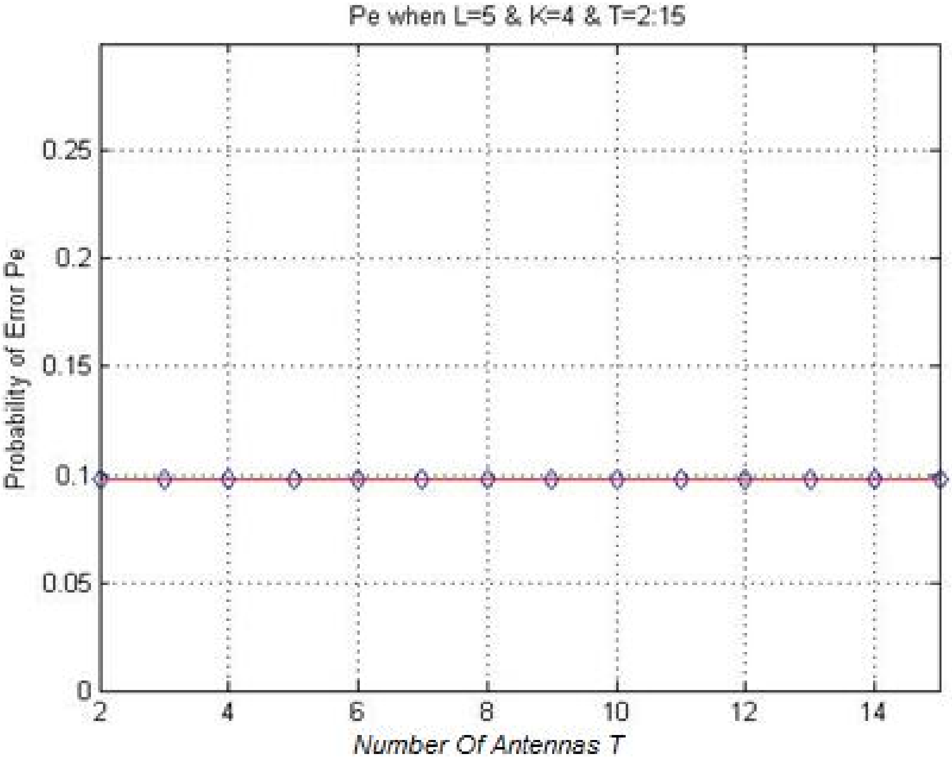

Figure 4.6 Plot of Theoretical results of P_e , when K=4, L=5, and T=2:15

4.3.4 Discussion of Results:

Once we validated the probability of error expression in (3.27), we are interested in minimizing this error for a given T, K, and L values, by finding the optimum threshold values. The optimum threshold values are calculated by (fmincon) function in MATLAB. Note that in figure 4.4 the optimum threshold values have been calculated and optimum probability of error has been found for T=6, K=4, and various values of L. in order to demonstrate that these threshold values are actually optimizes the probability of error, we calculated the probability of error for the same K, T, and L values but with random threshold values for 100,000 times. This figure demonstrated the validity of our optimization.

Figure 4.5 and figure 4.6 showed the probability of error for different values of K, L, and T. Note that the changing of the number of antennas didn't change the optimum probability of error. This will be investigated in the following sections.

4.4 OPTIMUM THRESHOLDS VALUES

4.4.1 Optimum thresholds for 2 users :

For two user system with variable number of thresholds L=2-4 , and variable number of antennas T=2-4. This is simple table to show how it the values of thresholds are decreased when the number of antennas increases.

Table 4.1 thresholds changes with changes of number of antennas

		T= 2	T=3	T=4
L=2	**User1**	0.3333	0.1834	0.1264
	User2	0.6667	0.4226	0.3066
L=3	**User 1**	0.2000	0.1056	0.0717
		0.4000	0.2255	0.1566
	User2	0.6000	0.3676	0.2633
		0.8000	0.5528	0.4156
L=4	**User1**	0.1427	0.0742	0.0502
		0.2856	0.1549	0.1062
		0.4285	0.2440	0.1705
	User2	0.5714	0.3455	0.2464
		0.7142	0.4656	0.3419
		0.8571	0.6221	0.4777

Since the number of the thresholds for one user is given by $Q = K(L-1)$ so for each user has one threshold figure 4.5, however in figure 4.6 each user has two thresholds.

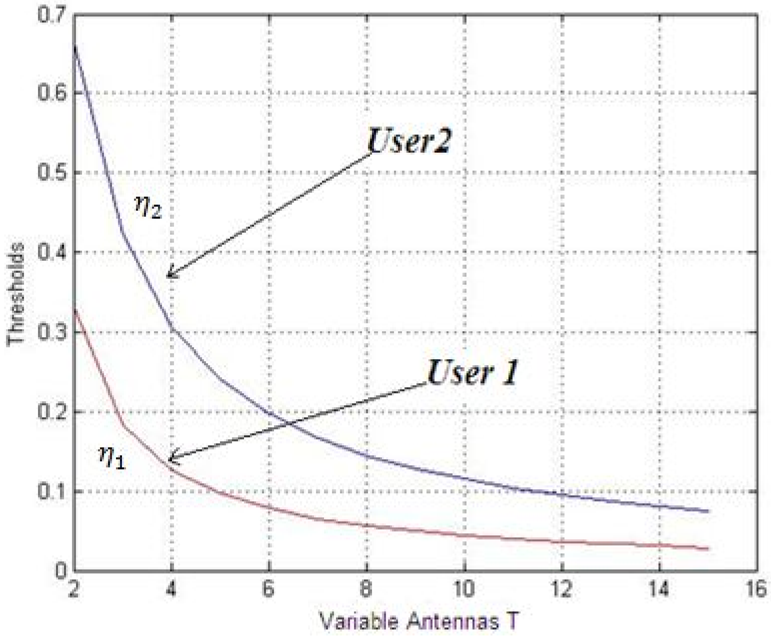

Figure 4.7 Plot the threshold values as a function of T for L=2,and K=2.

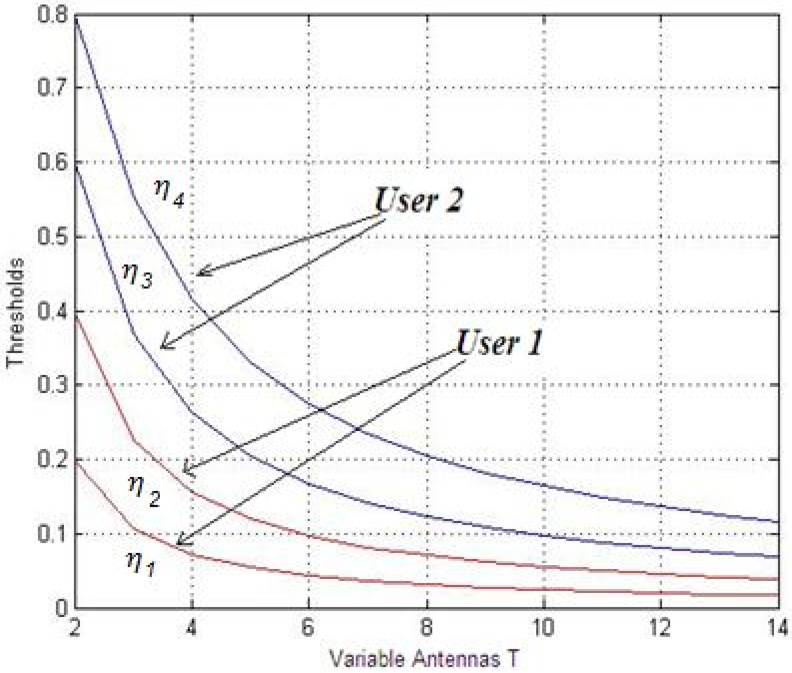

Figure 4.8 Plot the threshold values as a function of T for L=3,and K=2.

4.4.2 Optimum thresholds for 3 users:

For three user system with variable number of thresholds L=3, and variable number of antennas T=2-14,so each user has two thresholds.

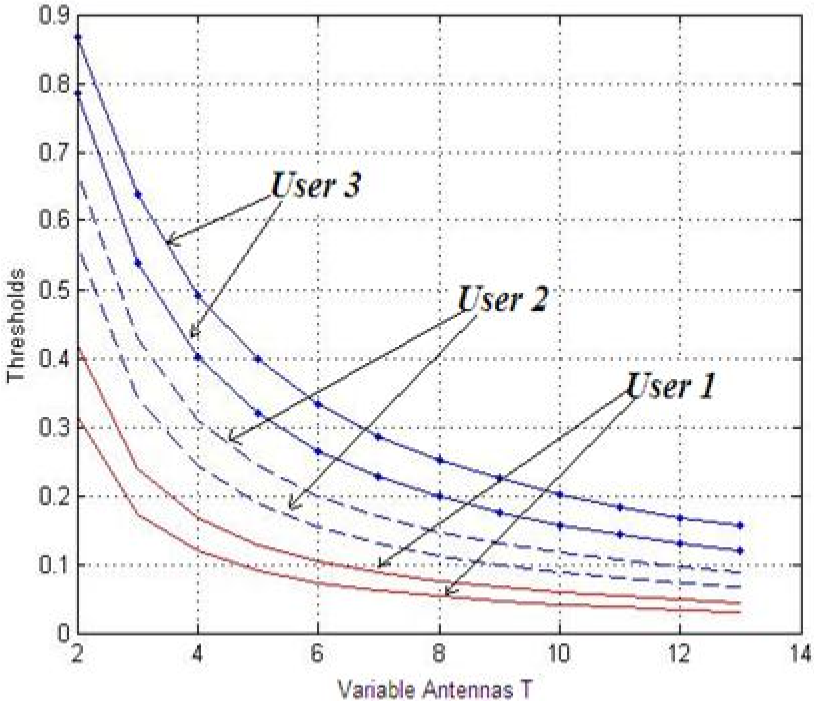

Figure 4.9 Plot the threshold values as a function of T for L=3,and K=3

4.4.3 Discussion of Results:

It can be noticed that for figure 4.5 thresholds start at high values, then by increasing the number of antennas in the system, theses values decreases. And that also can be noticed in figure 4.6 and figure 4.7.

In other hand this means that optimum thresholds can be achieved when multiple antennas is added.

4.5 THE EFFECT OF MULTIPLE ANTENNAS ON THE PROBABILITY OF ERROR

This strategy guarantees that the system reaches the optimum minimum value of P_e. However, studying the effect of adding multiple antennas to the system need to be clarified. In this section plots of P_e for two systems , first as in figure 4.8 when uniform quantization scheme using 2 antennas, is used to calculate the P_e for 20 users, on the other hand figure 4.9 plot of non uniform quantization scheme using 10 antennas to calculate P_e for 20 users.

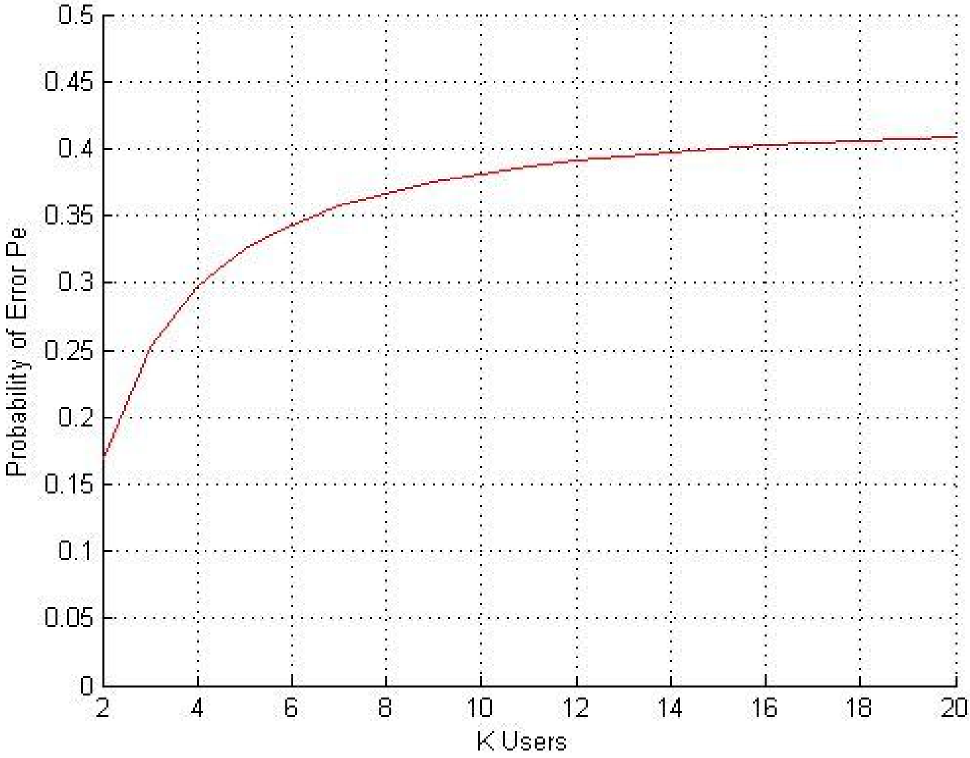

Figure 4.10 P_e when T=2, L=2 K=2:20

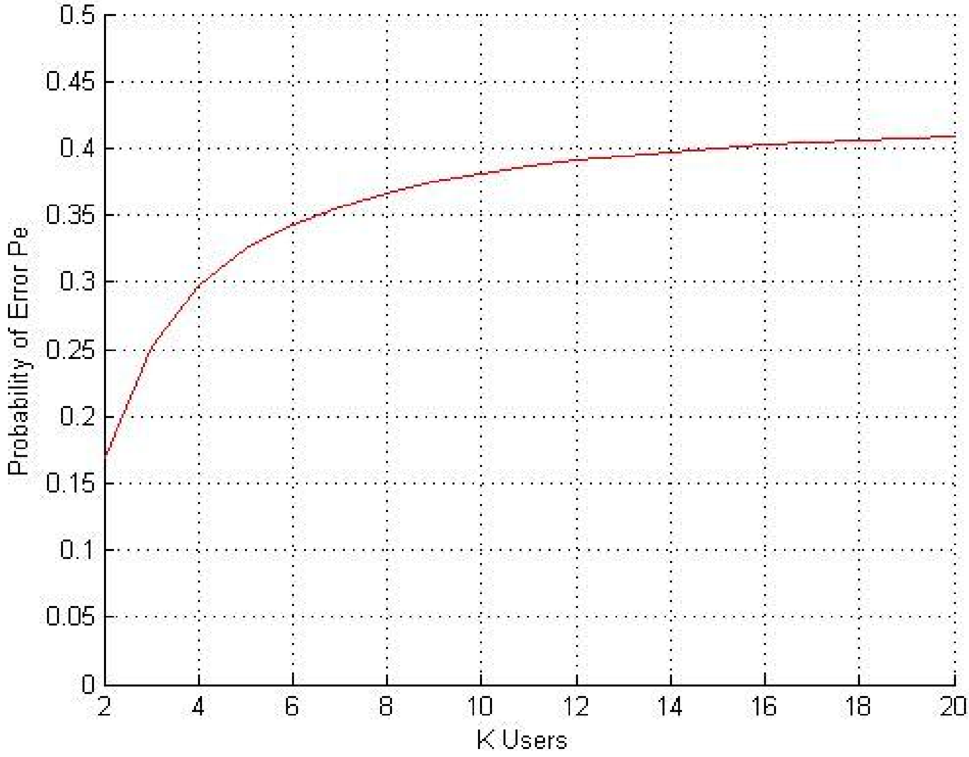

Figure 4.11 P_e when T=10, L=2 and K=2:20

Discussion:

Note that it can be seen from both of these plots that they are trying to reach the point 0.41, and that mean one thing; that even this strategy is guarantees optimum minimized P_e . However the results of P_e before adding antennas and after it are the same.

So what is the benefit of adding multiple antennas to the system if the P_e is not going to be reduced?

First lets discuses the following section, then the answer of this question will be offered.

4.6 THE EFFECT OF MULTIPLE ANTENNAS ON AVERAGE THROUGHPUT

In this section we will study the effect of adding multiple antennas. The main point for any addition to any system is to improve it and increase its efficiency.

The throughput used here is (3.5):

$$R = log_2 \left(1 + q^H(n)\, h_{k^*}\, h_{k^*}^H(n)\right)$$

In the first plot , figure 4.10 ,average throughput is found for 4 users and variable thresholds from 4 to 15 , however , the upper draw is average throughput for non uniform scheme using 6 antennas, the lower draw is average throughput for uniform scheme using 2 antennas.

In the second plot, figure 4.11, average throughput is found for 4 thresholds and variable users from 4 to 15. And the upper section is for non uniform scheme 6 antennas used, and the lower is for uniform, 2 antennas used.

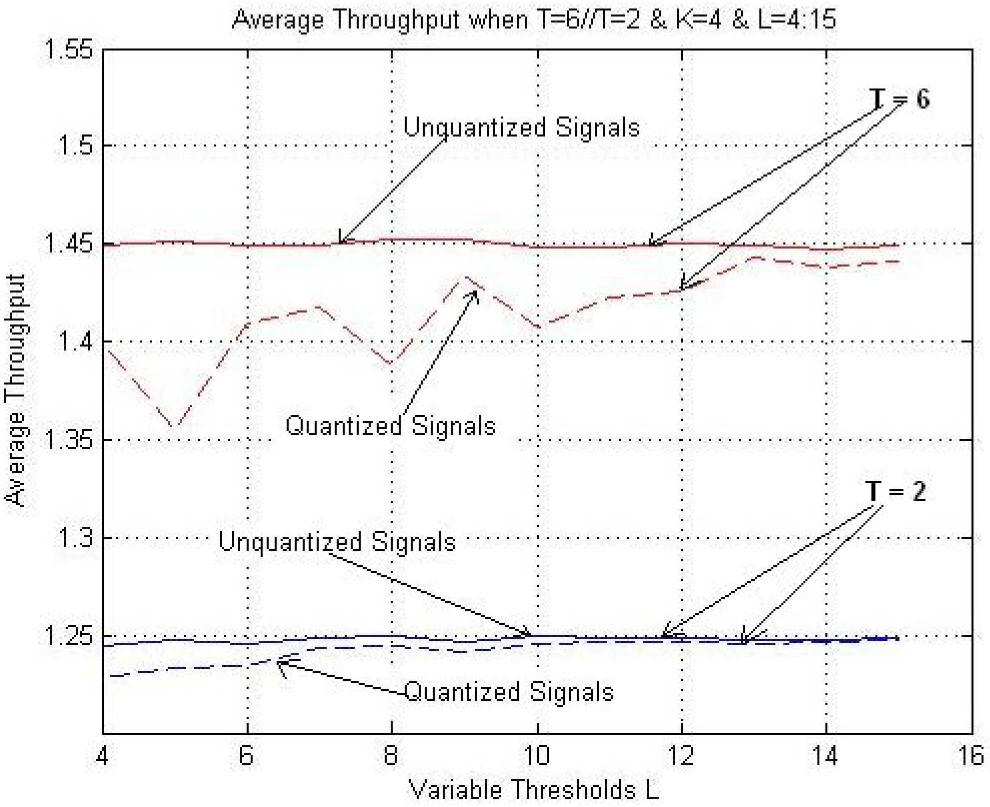

Figure 4.12 Average Throughput when T=6 & T=2, K=4 and L=4:15

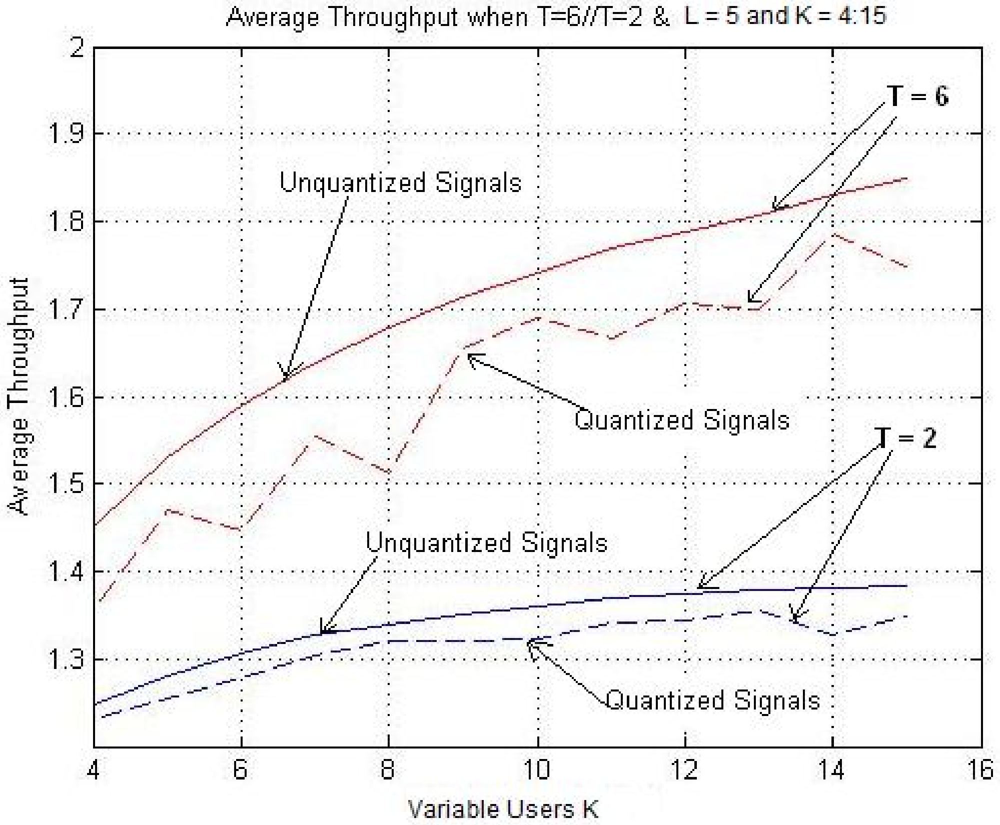

Figure 4.13 Average Throughput when T=6 & T=2, L=4 and K=4:15

Discussion of Results:

It can be noticed here that by increasing the number of antennas, the average throughput increases for both, quantized and unquantized values.

In the first figure by changing the values of thresholds from 4 to 15, the improvement in average throughput is low

It can be noticed that in figure 4.11 by increasing the number of users from 4 to 15. The improvement in average throughput is approximately is higher.

Therefore increasing the number of users in opportunistic beamforming systems using multiuser diversity will lead to a better performance.

4.7 DISCUSSION OF RESULTS

It was shown and proved that simulated results are almost equal to theoretical results. Therefore this strategy mathematically is functioning appropriately .and this approach can be used in order to study the effects of adding multiple antennas to the system.

Also it was shown and proved that using this strategy can achieve the optimum thresholds when multiple antennas are added.

On the other hand, Even though this strategy guarantees optimum minimized probability of error, the results of P_e before adding antennas and after it are the same. So what is the benefit of adding multiple antennas to the system if the is not going to be reduced?

Increasing the number of antennas, increases the throughput for both, quantized and unquantized schemes, that’s shown in section 4.6.

Therefore increasing the number of users in opportunistic beamforming systems using multiuser diversity will lead to a better performance.

Although increasing the number of antennas is not changing the minimum probability of error, the average throughput increases. Thus optimum thresholds can be achieved to minimize the probability of error, and increase the throughput in the system, and this is the answer of the left question.

CHAPTER 5

CONCLUSIONS

This chapter concludes the thesis by considering all the other chapters. In section 5.1 thesis outlines that describes the general flow of this work. In section 5.2 suggestions and future studies are discussed, and finally a brief summery that summarizes this dissertation.

5.1 THESIS HIGHLIGHTS

In chapter 1, General introduction to the wireless communication systems, described the main challenging problems of transmitting data over the wireless channels. In addition to the contribution of this study, the organization of this dissertation also mentioned there.

In chapter 2, a literature review is presented about the essential topics of this study, which are fading channels, Multiuser diversity, Opportunistic beamforming, Quantization strategies of fast fading channels.

In chapter 3, derivation of our strategy that minimizes the probability of error for Opportunistic beamforming systems was discussed.

Chapter 4 covered the simulation results, the numerical results, and their interpretations which are revealed by this thesis. Verifying the system proved first by matching the simulation results with the numerical results. Finding optimum threshold values obtained as well. Optimum probability of error are presented and achieved also. Observations were taken in two cases, first when adding antennas to the system, the results of minimum probability of error are the same as without adding any antennas. Second, although increasing the number of antennas is not changing the minimum probability of error, it is increases the average throughput in the system. And finally, the results are compared and discussed.

The appendix contained in this thesis must not be overlooked. Appendix A provides programming codes for calculating minimum probability of error for both numerical and simulation methods, also for calculating average throughput. These codes are written by MATLAB Programming.

5.3 SUGGESTIONS FOR FUTURE STUDIES

In this thesis, optimum thresholds that minimize the probability of error for slow fading channels are achieved. By using these thresholds the average throughput of the system increases. Finding the optimum thresholds that maximizes the average throughput using the same channel condition (Max-NSNR) for slow fading channels can be done in the future.

5.4 SUMMARY

In order to observe multiuser diversity, the transmitter needs to monitor the channel strength changes of the users. This requires each user to feed back their instantaneous SNR measurements. The original approach in (Viswanath et al ,2002) assumes unquantized feedback sent from the receiver to the transmitter to show the potential of opportunistic beamforming gain. However current wireless protocols require having feedback channel with a limited bandwidth. The quality of the SNR feedback such as the degree of SNR quantization is essential for opportunistic beamforming because the base station selects the best receiving user based on the SNR measurements sent by the users.

In this dissertation we have derived the optimum SNR quantization scheme that minimizes the probability of error of the opportunistic beamforming system. It was shown and proved that this strategy is mathematically functioning appropriately. Also it was shown and proved that using this strategy can achieve the optimum thresholds when multiple antennas are added.

Even though this strategy guarantees optimum minimized probability of error, the results of probability of error are the same before and after adding antennas.

Although increasing the number of antennas is not changing the minimum probability of error, it increases the average throughput. Thus optimum thresholds can be achieved to minimize the probability of error, and increase the throughput in the system, and this is the answer of the left question.

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APPENDIX A

MATLAB CODES:

FINDING THE PROBABILITY OF ERROR
NUMERICAL METHOD:

```
%-------------------------------------------------------------------
% Changing K and Fixed L and T
clear all
global L
global k
global T
L=2;
T=2;

for k=2:20
 x0= ones(k*(L-1)+2*k,1);
 % x0 is random variables...
 x0=0.5.*x0;
x0(1:k)=zeros(k,1);
x0(k*(L-1)+k+1:k*(L-1)+2*k)=ones(k,1);

% ----Generate A
%L= number of thresholds.....
A=zeros(((L-1)*k)+1,(L-1)*k+2*k);

for i=1:((L-1)*(k))+1
 A(i,k-1+i)=1;
  A(i,k+i)=-1;
end

%------Generate B
B=zeros(((L-1)*(k))+1,1);

% %----------Generate Aeq
Aeq= zeros(2*k,(L-1)*k+2*k);
for i=1:k
Aeq(i,i)=1;
Aeq(k+i,(L-1)*k+k+i)=1;
end
%
% %--------Generate Beq
Beq=[zeros(k,1); ones(k,1)];
% [x,fval] = fmincon(@myfun35,x0,A,B)
```

```
%[x,fval] = fmincon(@myfun55,x0,A,B,Aeq,Beq);
options = optimset('display','iter','TolFun',1e-12,'TolX',1e-8,'TolCon',1e-
12,'MaxFunEvals',30000);
[x,fval] = fmincon(@myfun55,x0,A,B,Aeq,Beq,[],[],[],options);
oppemin(k)=fval;
end
AXIS([2 20 0 .5])
hold on
sx=1:20;
plot(sx(2:20),oppemin(2:20),'r')
grid
xlabel ('K Users ');
ylabel ('Probability of Error Pe');


% --------------------myfunc55.m
% L= the number of thresholds;
% T= the number of Antennas;

function f = myfun55(x)
%f = ((((x(1))^2)/2)*(x(1)-2)^2)+((((x(2))^2)/2)*(x(2)-2)^2)-(x(1)*x(2)*(x(1)-2)*(x(2)-
2))+((((x(1))^2)/2)*(x(1)-2)^2)+0.5*(x(2)-1)^4;
global k
global L
global T

f1=0;
for j=1:k-1
  f2=0;
 for i=1:(L-1)*k


     %f2 = f2 + prod([(1-(1-(x(i+1+k-1:-1:i-j+2+k-1))).^T-1)]) * (  (1-(1-(x(i+2+k-
1))).^T-1)^(k-j) - (1-(1-(x(i+1+k-1))).^T-1)^(k-j)  );
     Fx=1-(1-x).^(T-1);
     f2 = f2 + prod([Fx(i+1+k-1:-1:i-j+2+k-1)]) * ((Fx(i+2+k-1)^(k-j)) - (Fx(i+1+k-
1)^(k-j)));

   end
   f1 = f1 + f2/(k-j);
end
f= 1 - (1/k) - f1;

%-----------------------------------------------------------------------
```

SIMULATION METHOD:

```
clear all
clc
jay=sqrt(-1);
M=10000;
```

```
l=5;
%k=4;
T=6;
global k

for k=2:10
   thrsh=(l-1)*k;
   qq= sort(rand(1,thrsh));
   err=0;
   for n=1:M
      %step 1
      % Fix All the Following Variables

      gb=1;
      rowh=0;
      users=k;
      h=zeros(users,T);
      t=zeros(users,T);

      %----------------------------------------------------------------------
      %Step 2
      % Generate Cahnnels Of the Users
      % h1,h2,.....,hk
      for k=1:users
         for i=1:T;
            h(k,i)=((sqrt(gb)/sqrt(2)).*randn(1,1) + jay*(sqrt(gb)/sqrt(2)).*randn(1,1));
         end
      end
      %---------------------------------------
      %Step 3
      %Slect User C
      c = randint(1,1,[1,users]);
      hc=h(c,:);
      %------------------------------
      %step 4
      %Generate Beamformaing Vector
      %q= rowh*hc+sqrt(1-(rowh)^2/)t....
      q=zeros(1,T);
      t=zeros(1,T);
      for i=1:T;
         t(1,i)=((sqrt(gb)/sqrt(2)).*randn(1,1) + jay*(sqrt(gb)/sqrt(2)).*randn(1,1));
      end
      for i=1:T
         q(1,i)=rowh.*hc(1,i)+(sqrt(1-(rowh)^2)).*t(1,i);
      end
      q=q/norm(q);
      %----------------------------------------------------------------
      % Step 5
      %Find m1,....,mk
```

```
  %mk= (qh*hk*hkh*q)/(hkh*hk)
  q=q.';
  qh=q';
  h=h.';
  hh=h';

  for k=1:users
     mk(k)=(qh*h(:,k)*hh(k,:)*q)/(hh(k,:)*h(:,k));
  end
  mk=real(mk);
  mk=mk.';

  %------------------------------------------------------------------
  % Step 6 choose the quantizing threshold
  % Quantizaing Step
  % Creating Quantizing Regions...
  %     qq=[qq ones(1,k)];
  % Separating the thresholds for each user
  for i=1:k
     qqq(i,:)=qq(i:k:(l-1)*k);
  end

  for i=1:k
     for j=1:l-1
        if mk(i)>=max(qqq(i,:))
           x(i)=l-1;
        elseif mk(i)<=min(qqq(i,:))
           x(i)=0;
        elseif mk(i)>=qqq(i,j)& mk(i)<=qqq(i,j+1)
           x(i)=j;
        end
     end
  end




  %------------------------------------------------------------------
  % Quantization level 2
  % Slecting the Highest Quantized User....

  cc=find(x==max(x));

  %Slecting with the highest threshld
  quser= max(cc);
  %finiding the highest mk
  suser=find(mk==max(mk));
%-------------------------------------------------------
  % Error Checking
```

```
        if quser~=suser;
            err=err+1;
        end
    end

    errors(k)=err/M;

    % Theoretical probability of error

    global L
    global T


    L=5;

    T=6;

    qqqq=[zeros(1,k) qq ones(1,k)];
    therrors(k)=myfun55(qqqq);
end
sx=1:10;
plot(sx(2:10),therrors(2:10),'r')
hold on
plot(sx(2:10),errors(2:10),'*')
grid
xlabel ('Varibale Users K');
ylabel ('Probability of Error Pe');
title('Pe when T=6 & L=5 & k=2:10');
save vK
```

FINDING THE AVERAGE THROUGHPUT

```
clear all
clc
jay=sqrt(-1);
M=100000;
%l=5;
k=4;
T=6;
TT=2;

for l=2:15
    thrsh=(l-1)*k;
    qq= sort(rand(1,thrsh));
    err=0;
    err2=0;
    uathr=0;
```

```
uathr2=0;
qathr=0;
qathr2=0;

qqq=zeros(k,l-1);;
for n=1:M
   %step 1
   % Fix All the Following Variables

   gb=1;
   rowh=0;
   users=k;
   h=zeros(users,T);
   h2=zeros(users,TT);
   t=zeros(users,T);
   t2=zeros(users,TT);
   %-------------------------------------------------------------------
   %Step 2
   % Generate Cahnnels Of the Users
   % h1,h2,.....,hk
   for k=1:users
      for i=1:T;
         h(k,i)=((sqrt(gb)/sqrt(2)).*randn(1,1) + jay*(sqrt(gb)/sqrt(2)).*randn(1,1));
      end
   end
   %-------------------------------------
   %Step 3
   %Slect User C
   c = randint(1,1,[1,users]);
   hc=h(c,:);
   %----------------------------
   %step 4
   %Generate Beamformaing Vector
   %q= rowh*hc+sqrt(1-(rowh)^2/)t....
   q=zeros(1,T);
   t=zeros(1,T);
   for i=1:T;
      t(1,i)=((sqrt(gb)/sqrt(2)).*randn(1,1) + jay*(sqrt(gb)/sqrt(2)).*randn(1,1));
   end
   for i=1:T
      q(1,i)=rowh.*hc(1,i)+(sqrt(1-(rowh)^2)).*t(1,i);
   end
   q=q/norm(q);
   %-------------------------------------------------------------
   % Step 5
   %Find m1,....,mk
   %mk= (qh*hk*hkh*q)/(hkh*hk)
   q=q.';
   qh=q';
```

```
h=h.';
hh=h';

for k=1:users
   mk(k)=(qh*h(:,k)*hh(k,:)*q)/(hh(k,:)*h(:,k));
end
mk=real(mk);
mk=mk.';

%-------------------------------------------------------------------
% Step 6 choose the quantizing threshold
% Quantizaing Step
% Creating Quantizing Regions...
%    qq=[qq ones(1,k)];
% Separating the thresholds for each user
for i=1:k
   qqq(i,:)=qq(i:k:(l-1)*k);
end

for i=1:k
   for j=1:l-1
      if mk(i)>=max(qqq(i,:))
         x(i)=l-1;
      elseif mk(i)<=min(qqq(i,:))
         x(i)=0;
      elseif mk(i)>=qqq(i,j)& mk(i)<=qqq(i,j+1)
         x(i)=j;
      end
   end
end

%-------------------------------------------------------------------
% Quantization level 2
% Slecting the Highest Quantized User....

cc=find(x==max(x));

%Slecting with the highest threshld
quser= max(cc);
%finiding the highest mk
suser=find(mk==max(mk));

%-----------------------------------------------------

% Checking errors

if quser~=suser;
   err=err+1;
end
```

```
 %-----------------------------------------------------
% throughput of unquantized users:
 uathr= uathr+log2(1+ (qh*h(:,suser)*h(:,suser)'*q) );
%-------------------------------------
% throughput of quantized users:

qathr= qathr+log2(1+ (qh*h(:,quser)*h(:,quser)'*q) );
%--------------------------------------------------
%-----------------------------------------------------------------
%-----------------------------------------------------------------
%----------------------------------------------------------------
% for T=2;

%Step 2
   % Generate Cahnnels Of the Users
   % h1,h2,.....,hk
   for k=1:users
      for i=1:TT;
         h2(k,i)=((sqrt(gb)/sqrt(2)).*randn(1,1) + jay*(sqrt(gb)/sqrt(2)).*randn(1,1));
      end
   end
   %-------------------------------------
   %Step 3
   %Slect User C
   c2 = randint(1,1,[1,users]);
   hc2=h2(c2,:);
   %-----------------------------
   %step 4
   %Generate Beamformaing Vector
   %q= rowh*hc+sqrt(1-(rowh)^2/)t....
   q2=zeros(1,TT);
   t2=zeros(1,TT);
   for i=1:TT;
      t2(1,i)=((sqrt(gb)/sqrt(2)).*randn(1,1) + jay*(sqrt(gb)/sqrt(2)).*randn(1,1));
   end
   for i=1:TT
      q2(1,i)=rowh.*hc(1,i)+(sqrt(1-(rowh)^2)).*t(1,i);
   end
   q2=q2/norm(q2);
   %----------------------------------------------------------
   % Step 5
   %Find m1,....,mk
   %mk= (qh*hk*hkh*q)/(hkh*hk)
   q2=q2.';
   q2h=q2';
   h2=h2.';
   h2h=h2';

   for k=1:users
```

```
     mk2(k)=(q2h*h2(:,k)*h2h(k,:)*q2)/(h2h(k,:)*h2(:,k));
  end
  mk2=real(mk2);
  mk2=mk2.';

  %---------------------------------------------------------------
  % Step 6 choose the quantizing threshold
  % Quantizaing Step
  % Creating Quantizing Regions...
  %     qq=[qq ones(1,k)];
  % Separating the thresholds for each user
  for i=1:k
     qqq(i,:)=qq(i:k:(l-1)*k);
  end

  for i=1:k
     for j=1:l-1
        if mk2(i)>=max(qqq(i,:))
           x2(i)=l-1;
        elseif mk2(i)<=min(qqq(i,:))
           x2(i)=0;
        elseif mk2(i)>=qqq(i,j)& mk2(i)<=qqq(i,j+1)
           x2(i)=j;
        end
     end
  end
  %--------------------------------------------------------
  % Quantization level 2
  % Slecting the Highest Quantized User....

  cc2=find(x2==max(x2));

  %Slecting with the highest threshld
  quser2= max(cc2);
  %finiding the highest mk
  suser2=find(mk2==max(mk2));

  %----------------------------------------------------
  % Checking errors

  if quser2~=suser2;
     err2=err2+1;
  end

 %  throughput of unquantized 2users:
uathr2= uathr2+log2(1+ (q2h*h2(:,suser2)*h2(:,suser2)'*q2) );
%-----------------------------------
%  throughput of quantized 2users:
```

```
    qathr2= qathr2+log2(1+ (q2h*h2(:,quser2)*h2(:,quser2)'*q2) );




    end
    %----------------------------------------------------
    % Checking TOTAL errors for 6 antennas
    errors(l)=err/M;
    %----------------------------------------------------
    % Average throughput of unquantized  6 antennas
    unavg(l)=real(uathr)/M;
    %-----------------------------------------------------
    % Average throughput of quantized for 6 antennas
    qnavg(l)=real(qathr)/M;

    %-----------------------------------------
    %-----------------------------------------

    % Checking TOTAL errors for 2 antennas
    errors2(l)=err2/M;
    %----------------------------------------------------
    % Average throughput of unquantized  2 antennas
    unavg2(l)=real(uathr2)/M;
    %-----------------------------------------------------
    % Average throughput of quantized for 2 antennas
    qnavg2(l)=real(qathr2)/M;

end


sx=1:15;
AXIS([4 15 0 1.6])
plot(sx(4:15),unavg(4:15),'r')
hold on
plot(sx(4:15),qnavg(4:15),'--r')
%title('Pe when T=6 & K=4 & L=2:15');
hold on
plot(sx(4:15),unavg2(4:15),'b')
hold on
plot(sx(4:15),qnavg2(4:15),'--b')
grid
xlabel ('Variable Thresholds L');
ylabel ('Average Throughput');
title('Average Throughput when T=6//T=2 & K=4 & L=4:15');
```