Impurity Effect on Spin Field Effect Transistor

by

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ABSTRACT

The two dimensional electron gas (2DEG) between two ferromagnetic contacts, known as Spin Field Effect Transistor (Spin FET), is considered theoretically in the presence of impurities. Spin precession due to Rashba spin-orbit coupling is studied using the model of Datta and Das in the one dimensional channel including impurities. Spin precession angle is found to be independent of an arbitrary impurity potential. Therefore, it is shown that the conductance modulation for purely one dimensional case is not affected by any kind of impurity potential.

Keywords: Spin FET, 2DEG, Rashba Effect, Spin Polarized Transport, Spin Precession, Spin-Orbit Coupling

KATKININ SPİN ALAN ETKİLİ TRANSİSTÖRE ETKİSİ

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Yüksek Lisans Tezi – Fizik Bölümü
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ÖZET

Spin Alan Etkili Transistör (Spin FET) olarak bilinen iki ferromanyetik malzeme arasındaki katkılı iki boyutlu elektron gazı (2DEG), Spin Alan Etkili Transistör (Spin FET), teorik olarak incelenmiştir. Rashba spin-yörünge etkileşmesinden kaynaklanan spin presesyonu bir boyutlu Data ve Das modeli kullanılarak katkılı durumda çalışılmıştır. Spin presesyon açısının herhangi bir katkı potansiyelinden bağımsız olduğu bulunmuştur. Bundan dolayı iletkenlikteki değişimin bir boyutlu sistemlerde, herhangi bir potansiyelden etkilenmediği gösterilmiştir.

Anahtar Kelimeler: Spin FET, 2 Boyutlu Elektron Gazı, Rashba etkisi, Spin Polarize Taşıma, Spin Presesyon, Spin-Yörünge Bağlaşımı

CHAPTER 1

INTRODUCTION

Spintronics is a new paradigm in electronics (I. Zutic et al., 2004). It is also known as magnetoelectronics and based on exploitation of spin, a quantum property of electron, (Fig. 1.1). Control of electrical properties and modification of information, by spin manipulation, are the two main goals of this field.

In the last 50 year period, there has been a great revolution in the technology based on charge of electrons. From the earliest transistor to the microprocessor in our computers, most electronic devices have used circuits that express data as binary digits, or bits-ones and zeroes represented by the existence or absence of electric charge. In addition to this, the communication between microelectronic devices occurs by the binary flow of electric charges. In the last two decades scientists have been eager to use another property of the electron, a characteristic known as “spin”. Spin is a purely quantum mechanical phenomenon roughly akin to the spinning of a child’s gyroscope.

The movement of spin, like the flow of charge, can also carry information among devices. One advantage of spin over charge is that spin can be easily manipulated by externally applied magnetic fields, a property already in use in magnetic storage technology. Another property of spin is its long coherence, or relaxation time, once created it tends to stay that way for a long time, unlike charge states, which are easily destroyed by scattering or collisions with defects, impurities or other charges. These characteristics open the possibility of developing devices that could be much smaller, consume less electricity and be more powerful for certain types of computations than is possible with electron charge based systems.

Spintronics is an important technology that uses quantum property of electrons to spin as well as using of their charges. Spintronics is one of the most exciting and challenging area, multidisciplinary field, important to both fundamental scientific research and industrial applications. Spintronics is playing an increasingly significant role on the high density data storage, microelectronics, sensors, quantum computing and bio-medical applications, etc. (S. D. Sarma et al., 2001)

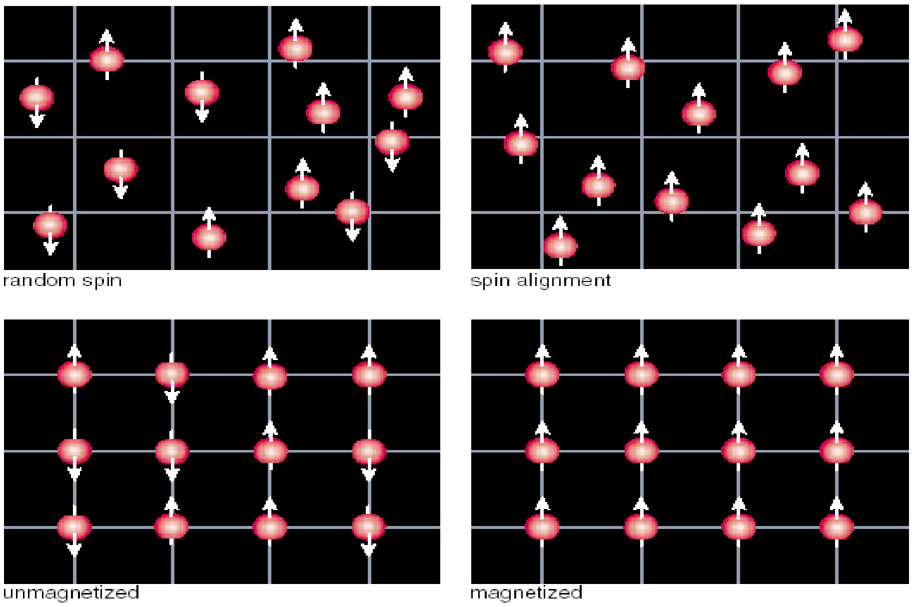

Figure 1.1. Types of Spin Alignment **[**Sarma, S. D., *et al.*,2001**]**

It is expected that the impact of spintronics to the microelectronics industry might be comparable to the development of the transistor 50 years ago. Today everyone already has a spintronics device on their desktop, as all modern computers use the spin valve (see next chapter) in order to read and write data on their hard drive. The initial driving force for spintronics has been the improvement of computer technology. At present the research has been concentrating on the fabrication of spin transistors and spin logic devices integrating magnetic and semiconductors, with the aim of improving the existing capabilities of electronic transistors and logics devices so that the future computation and thus the future computer could become faster and consume less energy. There are four main areas in spintronics:

1. Understanding the fundamental physics, such as spin-dependent transport across the magnetic/semiconductor or magnetic/nonmagnetic metal interfaces and spin coherence length in semiconductors or metals.
2. Synthesising suitable spintronic materials in the vicinity of room temperature, large spin polarization at the Fermi level and matching conductivity between the magnetic and semiconductor materials.

3. Fabricating devices with nanometer feature sizes and developing new techniques for mass production.

4. Integrating spin-devices with current microelectronics and computing.

Spin relaxation (how spins are created and disappear) and spin transport (how spins move in metals and semiconductors) are fundamentally important. Depending on the relative orientation of the magnetizations in the magnetic layers, the electrical resistance through the layers changes from small (parallel magnetizations) to large (antiparallel magnetizations) values. Scientists discovered that they could use this change in resistance to construct perfect sensitive detectors of changing magnetic fields, such as those marking the data on a computer hard-disk platter (S. A. Wolf et al., 2001).

In this thesis we focus on spin dependent transport to consider spin field effect transistor (spin FET), also named as Datta-Das field effect transitor (S. Datta and B. Das, 1990), in the presence of impuritities. It consists of two ferromagnetic electrodes and a semiconductor 2DEG channel with a gate electrode. It is basically similar in structure to a conventional FET. The source and drain electrodes are ferromagnetic metals, and two dimensional electron gas (2DEG) channel where impurities are located should have a spin-orbit interaction. We basically consider the variation in the conductance affected by the impurities.

This thesis is divided into following parts: In chapter 2 we review some spin based devices; in chapter 3 we consider the spin FET in detail; we analyze the nonballistic spin FET in chapter 4; we give an ansatz in chapter 5 concerning the our calculations; and finally in chapter 6 the thesis will be completed by the conclusions.

CHAPTER 2

SPIN BASED DEVICES

Semiconductor spintronic devices combine advantages of semiconductor with the concept of magnetoelectronics. This category of devices includes spin diodes, spin filter, and spin FET. To make semiconductor based spintronic devices, researchers need to address several problems. A first problem is the creation of inhomogeneous spin distribution. It is called spin polarization or spin injection. Spin-polarized current is the primary requirement to make semiconductor spintronics based devices. It is also very fragile state. Therefore, the second problem is achieving transport of spin-polarized electrons maintaining their spin-orientation. Final problem, related to application, is the spin relaxation time. Spins come to equilibrium by the phenomenon called spin relaxation. It is important to create long spin relaxation time for effective spin manipulation, which will allow additional spin degrees of freedom to spintronics devices with the electron charge. Utilizing spin degrees of freedom alone or add them to mainstream electronics will significantly improve the performance with higher capabilities.

The other category devices are being considered for building quantum computers (S. D. Sarma et al., 2001). Quantum information processing and quantum computation is the most ambitious goal of spintronics research. The spins of electrons and nuclei are the perfect candidates for quantum bits or qubits. Therefore, electron spin and nuclear based hardwares are some of the main candidates being considered for quantum computers.

Spin based devices offer several advantages over the conventional charge based devices. For example, as the magnetized materials maintain their spin even without power, spin based devices could be the basis of non-volatile memory device. Energy efficiency is another virtue of these devices as spins can be manipulated by low-power external magnetic field. Miniaturization is also other advantage because spintronics can be coupled with conventional semiconductor and optoelectronic devices. However, temperature is still a major bottleneck. Practical application of spintronics needs room-temperature ferromagnet in semiconductors. Making such materials represents a substantial challenge for materials scientists.

2.1. Giant Magnetoresistance (GMR)

One device already in use is the GMR, a sandwich structure, which consists of alternating ferromagnetic (permanently magnetized) and nonmagnetic metal layers (M. N. Baibich et al., 1988).

The discovery of GMR in 1988 initiated the field of spintronics. Soon after its discovery, the GMR effect found its applications in information storage and reading, for example in read heads for hard disks and magnetic read access memory (MRAM) (D. Grundler, 2002). GMR effect arises when there is a very large change in the electrical resistance that is observed in a ferromagnet/paramagnet multilayer structure when the relative orientations of the magnetic moments in alternate ferromagnetic layers change as a function of applied field. The basis of the GMR is the dependence of the electrical resistivity of electrons in a magnetic metal on the direction of the electron spin, either parallel or antiparallel to the magnetic moment of the films (see Fig. 2.1). Electrons which have a parallel spin undergo less scattering and therefore have a lower resistance. When the moments of the magnetic layers (NiFe in Fig 2.1) are antiparallel at low field, there are no electrons which have a low scattering rate in both magnetic layers, causing an increased resistance. At applied magnetic fields where the moments of the magnetic layers are aligned, electrons with their spins parallel to these moments pass freely through the solid, lowering the electrical resistance (Fig. 2.2). The resistance of the structure is therefore proportional to the cosine of the angle between the magnetic moments in adjacent magnetic layers.

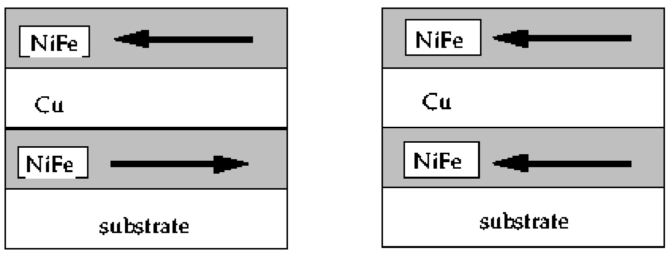

Figure 2.1. Giant Magnetoresistance. Zero-field high resistance state on the left and high-field low resistance state on the right. [Manasreh, O., 2005]

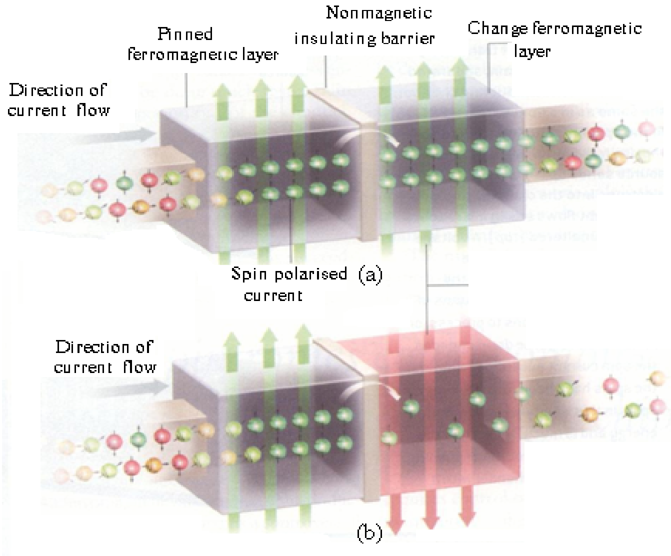

Figure 2.2. Giant magneto resistance effect; (a) electron transport takes place when magnetization direction of both ferromagnetic regions aligned parallel to each other, (b) electrons are facing high resistance and scattered away near interface when magnetization direction of both ferromagnetic regions are opposite to each other [Hammar, P. R., and M. Johnson, 2002].

Soon after its discovery, the GMR effect found its application in information storage and reading, for example in read heads for hard disks and MRAM memory (M. Dax, 1997). Researchers and developers of spintronic devices currently take two different approaches. In the first, they seek to perfect the existing GMR-based technology either by developing new materials with larger populations of oriented spins (called spin polarization) or by making improvements in existing devices to provide better spin filtering. The second effort focuses on finding novel ways both to generate and to utilize spin-polarized currents, that is, to actively control spin dynamics.

2.2. Magnetic Tunnel Junctions (MTJ)

Quantum mechanical tunneling is the phenomenon in which the wave nature of particles enables them to pass through an energy barrier that an otherwise classical object could not. One of the most fascinating phenomena in condensed matter physics is spin dependent tunneling. Spin dependent tunneling arises when one or both the electrodes separated by the tunneling barrier are ferromagnetic or otherwise capable of producing spin-polarized carriers. The number of electrons with spin-up and spin-down are not the same, and the magnetization direction in ferromagnetic materials can be expressed as the spin orientation of the majority electrons. When the magnetization of the two electrodes are parallel then majority spin state or minority spin state will have the same spin orientation for both the electrodes, whereas, for antiparallel orientation of the magnetization majority spin state in one electrode is the minority spin state of the other electrode. Hence, the corresponding resistances are different for parallel and antiparallel magnetization orientation of the electrodes. This is the basis of tunneling magnetoresistance (TMR) (M. Johnson, 1994).

MTJ (M. Johnson, 1994) are particularly important since they are already solving the problem of volatility of logic and memory components that ordinarily need constant refreshing power to function properly. By replacing capacitor-based charge storage elements, MTJ are providing much needed non-volatility of information storage.

2.3. All-Metal Spin Transitor

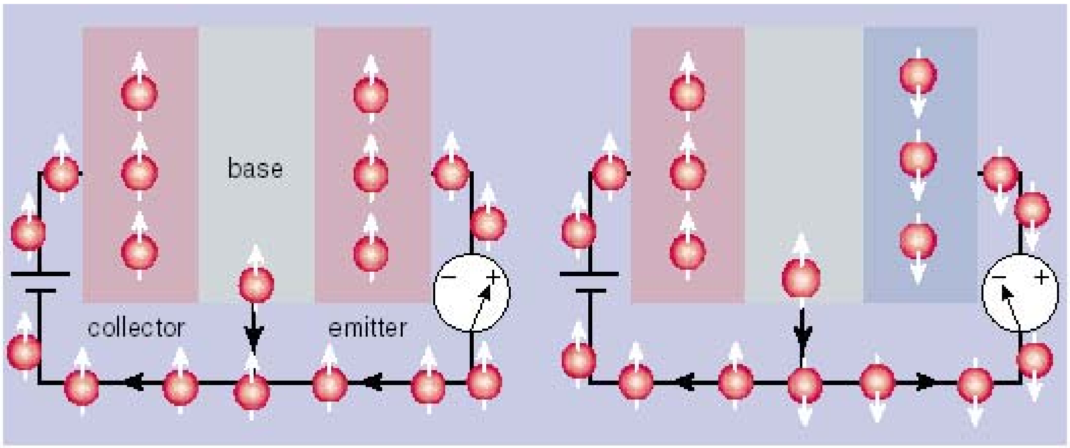

Figure 2.3. All Metal Spin Transistor. Spin transistor invented by Mark Johnson. [M. Johnson,1994].

Another interesting concept is the all-metal spin transistor (Fig 2.3), konown as Johnson transistor, developed by Mark Johnson (M. Johnson, 1994). It is a trilayer structure consisting of a nonmagnetic metallic layer sandwiched between two ferromagnets. The all-metal transistor has the same design philosophy as do GMR devices: The current flowing through the structure is modified by the relative orientation of the magnetic layers, which in turn can be controlled by an applied magnetic field. In this scheme, a battery is connected to control circuit (emitter-base), while the direction of the current in the working circuit (base-collector) is effectively switched by changing the magnetization of the collector. The current is drained from the base in order to allow for the working current to flow under the *reverse* base-collector bias (antiparallel magnetizations). Neither current nor voltage is amplified, but the device acts as a switch or spin valve to sense changes in an external magnetic field. A potentially significant feature of the Johnson transistor is that it can in principle be made extremely small using nanolithographic techniques (M. Johnson, 1994). An important disadvantage of Johnson transistor is that it will be difficult to integrate this spin transistor device into existing semiconductor microelectronic circuitry. Besides that metal-based spintronic devices do not amplify signals.

2.4. Spin Polarized p-n Junction

Motivated by the possibility of having both spin polarization and amplification, S. D. Sarma and his group have studied a prototype device, the spin polarized *p-n* junction (S. D. Sarma, 2001). In their scheme they illuminate the surface of the *p*-type region of a gallium arsenide (GaAs) *p-n* junction with circularly polarized light to optically orient the minority electrons. By performing a realistic device-modeling calculation they have discovered that the spin can be effectively transferred from the *p* side into the *n* side, via what we call spin pumping through the minority channel. In effect, the spin gets amplified going from the *p* to the *n* region through the depletion layer. One possible application of their proposed spin-polarized p-n junction (see Fig. 2.4) is something called the spin-polarized solar cell.

2.5. Magnetic Field Effect Transitor

In a magnetic field effect transistor proposed by I. Zutic (I. Zutic et al., 2004), electrodes of an external circuit are placed perpendicular to the *p-n* junction. The current is determined by the amount of available electrons in the region of the junction around the electrodes. If the depletion layer is wider than the electrodes, no (or very small) electric current flows. As the width decreases, more and more electrons come into contact with the electrodes and the current rapidly increases. Traditionally field effect transistors operate with an applied electric field (voltage) along the junction, as the width of the depletion layer is sensitive to the voltage. They proposed to use electric field instead a magnetic field. If the *n* or *p* region (or both) is doped with magnetic impurities, an external magnetic field produces a physical effect equivalent to applying an external voltage and could effectively tailor the width of the junction. (At the same time, this affects spin-up and spin-down electrons differently: A spin-polarized current results as well). Such a device could be used in magnetic sensor technology such as magnetic read heads or magnetic memory cells.

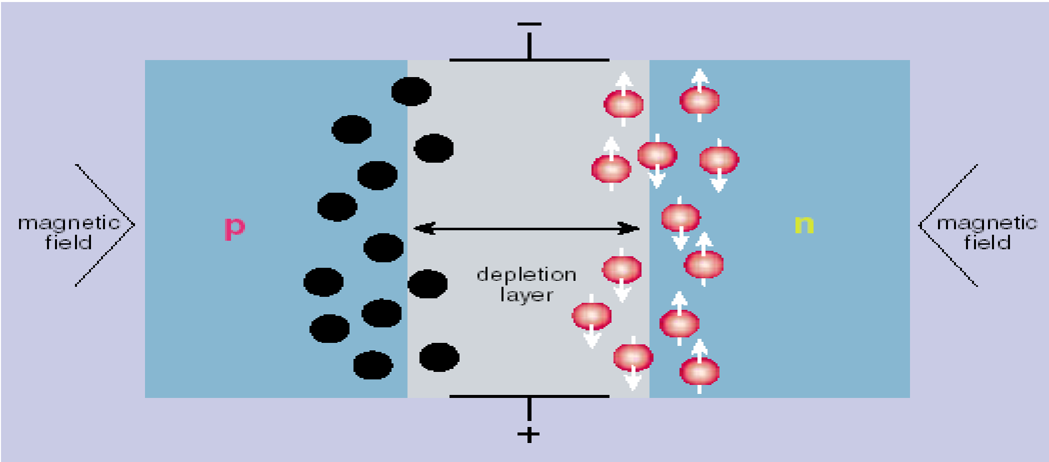

Figure 2.4. Magnetic Field Effect Transistor [Sarma, S. D, 2001].

2.6. Spin Valve

A spin valve (R. Jansen, 2003) is a device consisting of two or more conducting magnetic materials, shown in Fig 2.5, that alternates its electrical resistance (from low to high or high to low) depending on the alignment of the magnetic layers, in order to exploit the GMR effect. The magnetic layers of the device align *up* or *down* depending on an external magnetic field. Layers are made of two materials with different hysteresis curves, so one layer (*soft* layer) changes polarity while the other (*hard* layer) keeps its polarity. In Fig. 2.5, the top layer is soft and the bottom layer is hard.

Spin valves work, based on a quantum property of electrons and other particles. When a magnetic layer is polarized, the unpaired carrier electrons align their spins to the external magnetic field. When a potential exists across a spin valve, the spin-polarized electrons keep their spin alignment as they move through the device. If these electrons encounter a material with a magnetic field pointing in the opposite direction, they have to flip spins to find an empty energy state in the new material. This flip requires extra energy which causes the device to have a higher resistance than when the magnetic materials are polarized in the same direction.

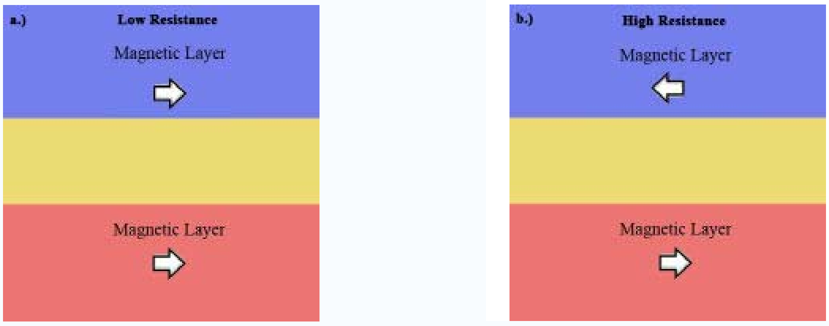

Figure 2.5. Spin valve: Resistance alternates depending on the external magnetic field [Potz, W., *et al.*,2007].

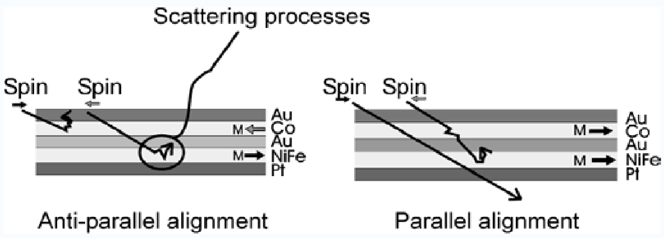

Figure 2.6. Scattering in a spin valve .

Fig. 2.6 shows a multilayer spin valve that exhibits the GMR effect. In the figure spin valve consists of two magnetic metal layers separated by a non-magnetic metal layer. On the left of the figure, the magnetizations of the two magnetic layers are antiparallel aligned. Here the majority spin electrons of the top magnetic layer are the minority spin electrons of the bottom magnetic layer. Equally, the minority electrons of the top magnetic layer are the majority electrons of the bottom magnetic layer. Either way, the electron experiences a layer with high scattering conditions and a large resistance is obtained in the anti-parallel alignment.

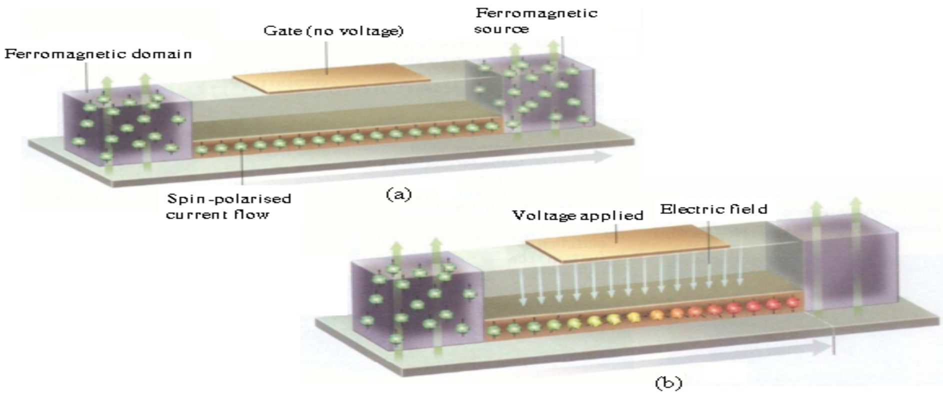

Figure 2.7. Datta-Das or spin FET [Datta, S., and B. Das, 1990]

2.7. Spin Field Effect Transistor (Spin FET)

The first scheme for a spintronic device based on the metal-oxide-semiconductor technology familiar to microelectronics designers was the field effect spin transistor proposed in 1990 by Supriyo Datta and Biswajit Das (S. Datta and B. Das, 1990). In a conventional field effect transistor, electric charge is introduced via a source electrode and collected at a drain electrode. A third electrode, the gate, generates an electric field that changes the size of the channel through which the source-drain current can flow. This results in a very small electric field being able to control large currents.

In the spin FET, or Datta-Das device, a structure made from, for instance, InAlAs and InGaAs provides a channel for two-dimensional (2D) electron transport between two ferromagnetic electrodes. One electrode acts as an emitter, the other as a collector (similar, in effect, to the source and drain, respectively, in a FET). The emitter emits electrons with their spins oriented along the direction of the electrode's magnetization, while the collector (with the same electrode magnetization) acts as a spin filter (W. Potz et al, 2007) and accepts electrons with the same spin orientation only. In the absence of any changes to the spins during transport, every emitted electron enters the collector. In this device, the gate electrode produces a field that forces the electron spins to precess, just like the precession of a spinning top under the force of gravity. The electron current is modulated by the degree of precession in electron spin introduced by the gate field: An electron passes through the collector if its spin is parallel, and does not if it is antiparallel, to the magnetization. The Datta-Das effect is the most visible for narrow band-gap semiconductors such as InGaAs, which have relatively large spin-orbit interactions (that is, a magnetic field introduced by the gate current has a relatively large effect on electron spin) (see Fig. 2.7 and Fig. 2.8). Spin FET will be considered in detail in the next chapter.

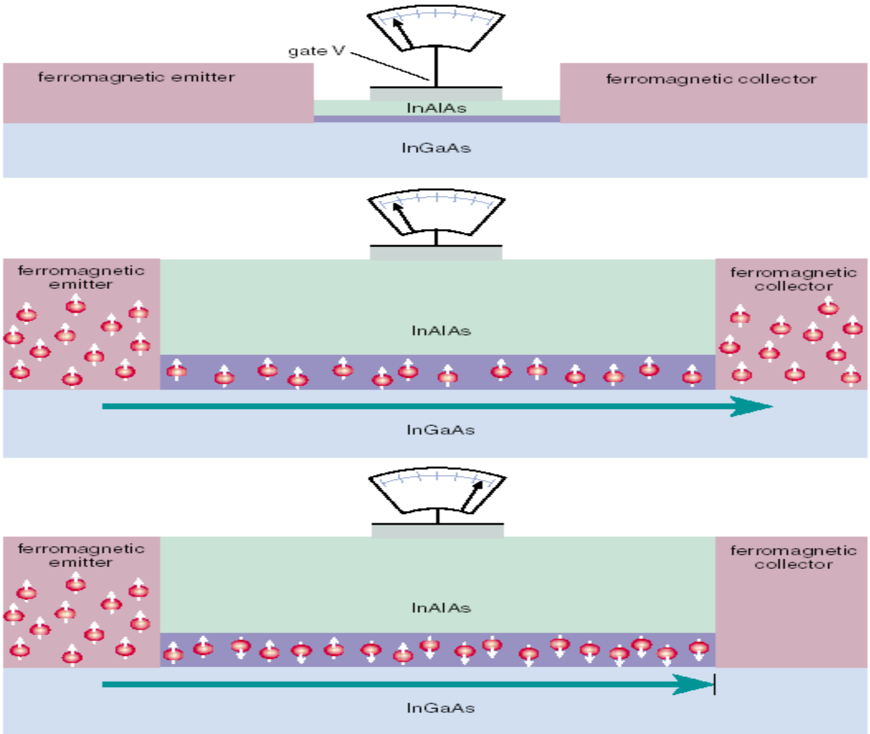

Figure 2.8. Direction of Spins in Datta-Das FET [Sarma, S. D., *et al.*, 2001].

If spintronic devices are ever to be practical, we need to understand how spins move through materials and how to create large quantities of aligned spins. Pioneering experiments on spin transport were performed by Paul M. Tedrow and Robert Meservey on ferromagnet/superconductor sandwiches to demonstrate that current across the interface is spin-polarized (P. M. Tedrow and R. Meservey, 1973). Today, the range of materials we can study has significantly increased, including novel ferromagnetic semiconductors, high-temperature superconductors and carbon nanotubes (O. Manasreh, 2005). As devices decrease in size, the scattering from interfaces plays a dominant role. In these hybrid structures the presence of magnetically active interfaces can lead to spin-dependent transmission (spin filtering) and strongly influence operation of spintronic devices by modifying the degree of spin polarization. One way to test these ideas is by directly injecting spins from a ferromagnet, where the spins start out in alignment, into a nonmagnetic semiconductor. Understanding this kind of spin injection is also required for hybrid semiconductor devices, such as the Datta-Das spin transistor. But this situation is very complicated, and a complete picture of transport across the ferromagnetic-semiconductor interface is not yet available.

Experiments on spin injection (A. T. Hanbicki et al., 2002) into a semiconductor indicate that the obtained spin polarization is substantially smaller than in the ferromagnetic spin injector, spelling trouble for spintronic devices. In this case, where spins diffuse across the interface, there is a large mismatch in conductivities, and this presents a basic obstacle to achieving higher semiconductor spin polarization with injection. An interesting solution has been proposed by inserting tunnel contacts, a special kind of express lane for carriers, scientists found that they could eliminate the conductivity mismatch (V. F. Motsnyi et al., 2002). Moreover, to reduce significant material differences between ferromagnets and semiconductors, one can use a magnetic semiconductor as the injector (J. H. Davies, 1993). While it was shown that this approach could lead to a high degree of spin polarization in a nonmagnetic semiconductor, it only worked at low temperature. For successful spintronic applications, future efforts will have to concentrate on fabricating ferromagnetic semiconductors in which ferromagnetism will persist at higher temperatures. The issues involving spin injection in semiconductors, as well as efforts to fabricate hybrid structures, point toward a need to develop methods to study fundamental aspects of spin-polarized transport in semiconductors.

CHAPTER 3

SPIN POLARIZED TRANSPORT AND SPIN FET

As explained in the previous chapter the first model of transistor using active control of electron spin was proposed by Datta and Das (S. Datta and B. Das, 1990). In the Datta-Das field effect transistor or spin FET, the non-magnetic layer acts as a gate while two ferromagnetic layers act as source and drain respectively. The gate plays an important role in spin FET. The gate voltage modifies direction of the electron spin by generating electric field, as a result of relativistic effect, giving rise to effective magnetic field and thereby switching the transistor. In the proposed device the electrons are ballistically transported in the channel and enter the drain (collector) if its spin is parallel to the magnetization direction of drain (spin detector), otherwise, it is scattered away.

The control of charge current in spin FET is similar to the conventional transistors (S. D. Sarma et al., 2001), but the spin FET possesses advantages over conventional transistors. It is smaller in size, and consumes less power. Still, spin FET exists in prototype model because of theoretical limitation related to spin behavior in different materials. Now consider how Datta-Das spin FET works.

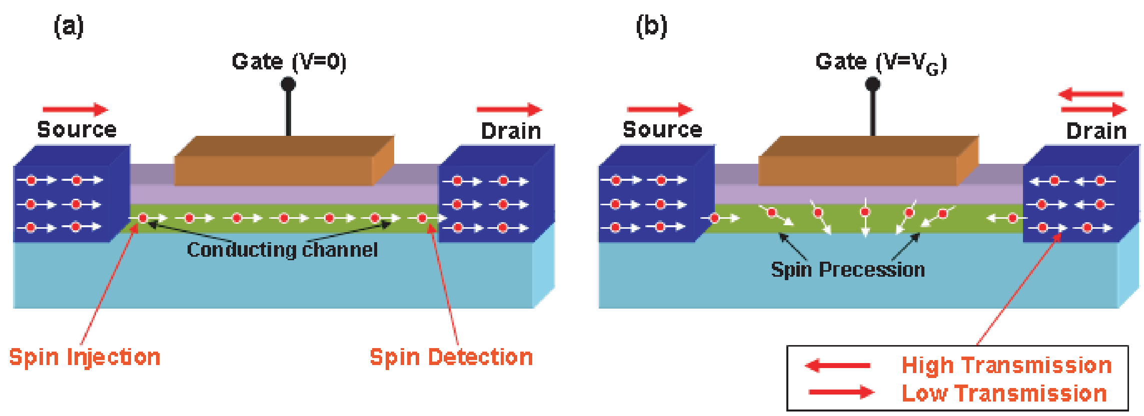

Figure 3.1. Spin FET or Datta-Das FET.

The proposed device, by Datta and Das, spin FET utilizes the spin of the electrons while they are travelling through a one dimensional (1D) channel without being scattered. The spin FET is an electronic device where spin-polarized current could be created, manipulated and detected by means of an electric field. Using ferromagnetic materials as source and drain, one can create a spin-polarized current, whereas spin manipulation is done by the gate electrode. An electric field applied via the gate electrode creates a conducting channel from source to drain in conventional FET devices. In spin FET devices, the electric field applied via gate electrode can also be used to control the orientation of the spin of electrons travelling through the conducting channel. To understand how an electric field can control the orientation of spins, we have to look back at relativistic effects on the spin of the electrons (P. Grundler, 2002). When an electric field is applied perpendicularly to the transport direction, an electron in its moving frame of reference feels an effective magnetic field directed perpendicularly to both the electric field and the transport direction. Hence, the spin of the injected electron can precess about this effective magnetic field while it is travelling through the conducting channel. This phenomenon is similar to the spin-orbit interaction in an atomic system, where electrons orbit around the nucleus in the presence of an electric field created by the positively charged nucleus. The underlying spin-orbit interaction in spin FET devices is called *Rashba* effect or Rashba spin-orbit coupling (E. I. Rashba, 2004). The output current will be proportional to the projection of the spin orientation of the electrons carrying current to the magnetization direction of the ferromagnetic drain electrode. Therefore, the source-drain current can be manipulated, i.e. spin direction of the current carrying electrons can be rotated willingly, by utilizing the Rashba spin-orbit effect via an applied electric field through the gate electrode. This basic principle of a spin FET is shown schematically in Fig. 3.1. The spin precession and output current modulation are controlled via the field effect by applying an external gate voltage. The spin precession between source and drain electrodes is a function of gate voltage via Rashba spin-orbit interaction parameter.

3.1. Two Dimensional Electron Gas (2DEG)

2DEG is a 2D system arising at the interfaces of a heterostructure. It is trapped at a heterojunction and the most important low-dimensional system for the electronic transport. It forms the core of a FET, which goes by many acronyms including modulation-doped field-effect transistor (MODFET) and high electron mobility transistor (HEMT).

GaAs-AlGaAs heterojunctions have importance in mesoscopic conductors. In these heterojunctions, a thin two dimensional conducting layer is formed at the interface between GaAs and AlGaAs. The reason is that, considering the conduction and valance band line up in one direction, the Fermi Energy (ε_f) in the widegap AlGaAs layer is higher than that in the narrow gap GaAs layer. According to this difference electrons spill over from the *n*-AlGaAs leaving behind positively charged donors. This space changes give rise to an electrostatic potential that causes the bands to bend (see Fig. 3.3). The electron density is sharply peaked near the GaAs-AlGaAs interface forming a thin conducting layer which is called as 2DEG.

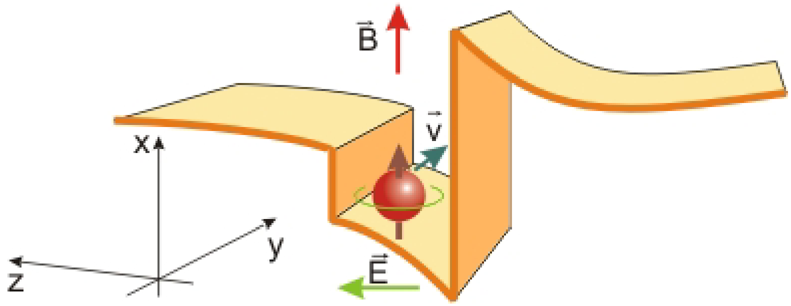

Figure 3.2. An electron, subject to electric and magnetic field, in 2DEG.

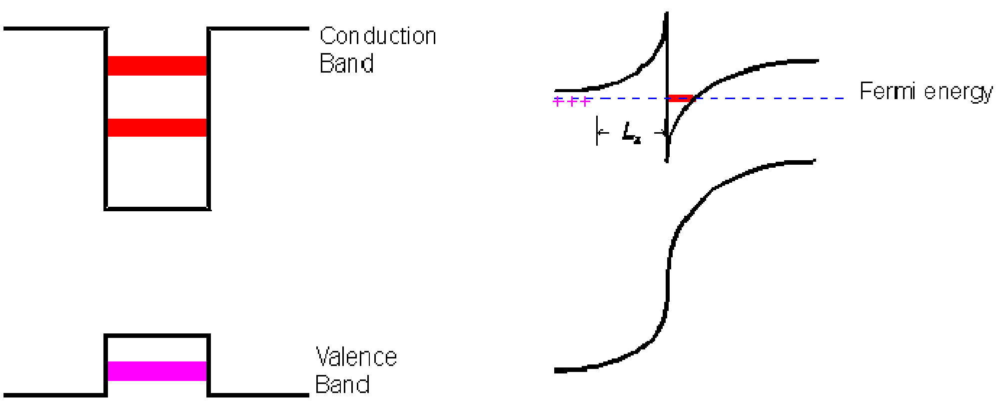

Figure 3.3. Band Diagram.

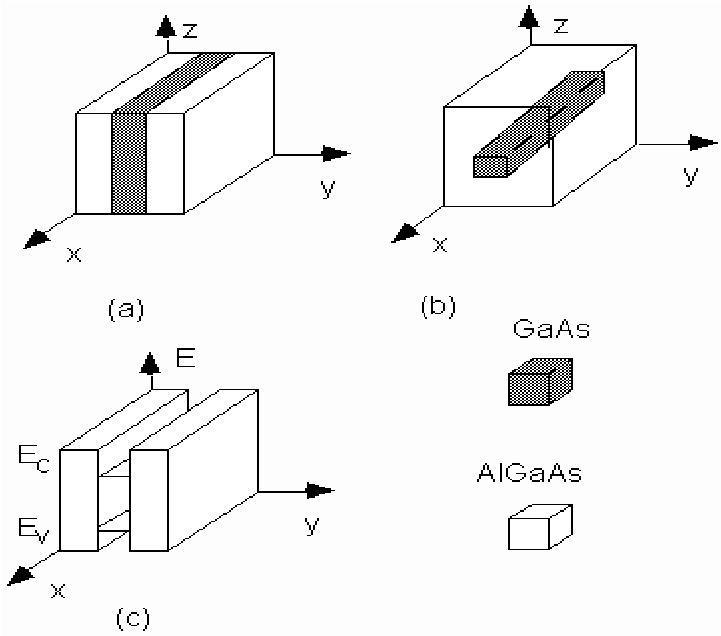

Figure 3.4. (a) GaAs layer sandwiched between AlGaAs layers, (b) GaAs region surrounded by AlGaAs, and (c) Corresponding band [Manasreh, O., 2005].

Consider the electrons in a narrow gap semiconductor layer, such as shown in Fig.3.4a. If this layer is thin enough, the motion of carriers in the direction perpendicular to the heterointerfaces is quantized, meaning that this motion involves discrete (quantum) energy levels. In this case, electrons propagating in the narrow gap semiconductor are referred to as 2DEG. Electrons in an unrestricted semiconductor are sometimes called a three dimensional (3D) electron gas. Electrons propagating in the structure shown in Fig.3.4b (often called a quantum wire) are called a 1D electron gas.

Quantum well (a 2D system) structures are also important applications in novel semiconductor devices. In such structures, a thin region of a narrow gap semiconductor is sandwiched between layers of a wide band gap semiconductor or surrounded by a wide band gap semiconductor.

3.2. Rashba Effect

Heterostructures provide an inversion layer channel for 2D electron transport between two ferromagnetic electrodes, acting as an emitter (polarizer or source) and collector (analyzer or detector). The emitter emits electrons with their spins oriented along the direction of the electrode's magnetization (along the transport direction), while the collector (with the same electrode magnetization) acts as a spin filter and accepts electrons with the same spin orientation only. In the absence of spin relaxation and spin dependent processes during transport, every emitted electron enters the collector.

For a 2DEG carrier moving, for instance, in the $x(y)$ direction, perpendicular electric field, E_z transforms, in the rest frame of the carrier, as an effective magnetic field B_y (B_x). This effective field interacts with the spin of the carriers, and is therefore called a spin-orbit interaction. The coupling results in the spin eigenstates described in Fig 3.5(a) and Fig.3.5(b). The spin-orbit coupling can be characterized by a strength that is proportional to momentum and *g* value. It adds a term to the Hamiltonian and the corresponding field is known as Rashba field leading to the spin precession of the electrons (see Fig 3.6).

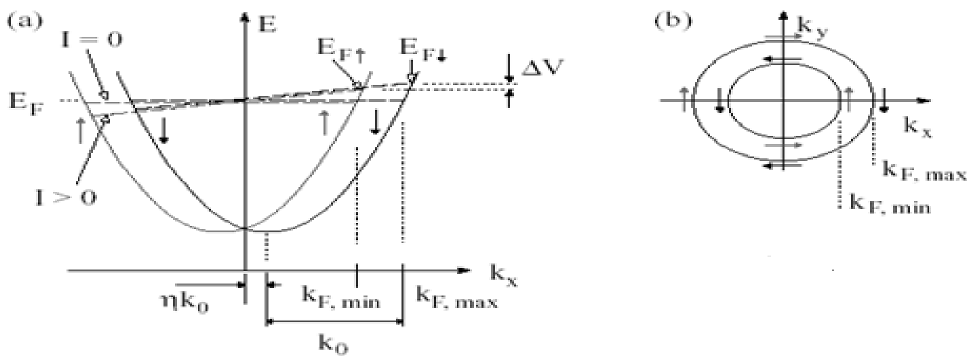

Figure 3.5. Energy dispersion and Fermi surface diagrams describing Rashba effect and current-induced nonequilibrium spin magnetization. (a) Energy dispersion along k_x; showing spin-split subbands. (b) Fermi surface in equilibrium [Potz, W., 2007].

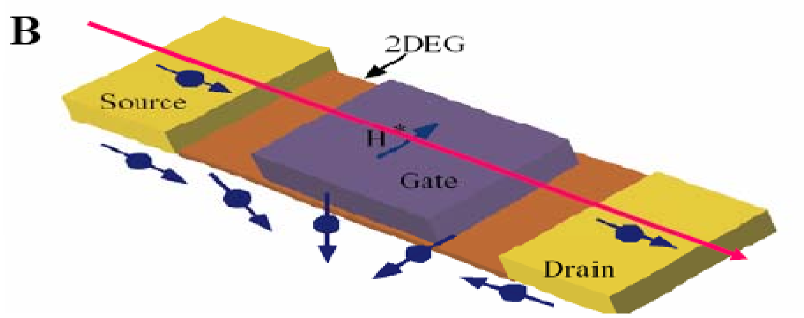

Figure 3.6. Rashba Field [Potz, W., 2007]

Depending on the direction of the electron spin (when entering into the collector) relative to the collector magnetization, the electron current is modulated: an electron passes through if its spin is parallel and does not if it is antiparallel to the magnetization. The current is in effect modulated by the external electric field induced spin-orbit field naturally existing in asymmetric zinc blende semiconductor structures (Rashba effect) (E. I. Rashba, 2004). The Rashba field can be, in turn, modulated by the applied perpendicular field at the gate. The Hamiltonian for the Rashba interaction is written as:

$$H_R = -\alpha\left(k_x\sigma_y - k_y\sigma_x\right) \tag{3.1}$$

where α is the Rashba interaction constant which is linearly dependent on electric field through the energy gap and effective mass, σ_x and σ_y are Pauli spin matrices corresponding to the electron spin. k_x and k_y are the wave vectors along the *x* and *y* axis, respectively.

The Rashba spin-orbit effect has several important consequences. First, if a carrier is injected into such a 2DEG with a known spin orientation, the orientation will randomize very rapidly. The spin precesses rapidly around the Rashba field because the magnitude of the effective field is large, but the direction of this field changes with any scattering event that alters momentum. The result is that spin relaxation is rapid. Second, the Rashba effect can be modulated by varying an externally applied gate voltage. This permits novel studies of the Fermi surface of a 2DEG (Nitta et al., 1997) and is the concept that underlies the original device idea of Datta and Das. Third, the unusually strong spin-orbit effect can be used to generate a population of nonequilibrium spin polarized electrons. In turn, this population can be detected using a ferromagnetic film as a spin-sensitive electrode. The result is a technique, called *current-induced nonequilibrium magnetization*, that can characterize the spin transport properties of an individual ferromagnetic-2DEG interface (P. R. Hammar et al, 1999; P. R. Hammar and M. Johnson, 2002). Given the difficulties associated with the experimental development of a spin FET, studies of the component parts of the device are very useful.

The other type of spin-orbit interaction in semiconductor heterostructures is Dresselhaus spin-orbit interaction which appears as a result of the asymmetry in a certain lattice (G. Dresselhaus, 1955). For 2D heterostructures with appropriate growth geometry, the Dresselhaus spin-orbit interaction is of the form:

$$H_D = -\beta\left(k_x\sigma_x - k_y\sigma_y\right) \tag{3.2}$$

where β is the corresponding interaction constant. Here the Hamiltonian term comes from the bulk inversion asymmetry.

Enhancement of spin-orbit coupling in solids come from two basic sources. First, this coupling originates from fast electron motion in a strong electric field near the nuclei. Second, the symmetry of microstructure is essentially lower than the symmetry of a vacuum. As a result new terms that critically change spin dynamics appear in electron Hamiltonians. There is no simple way to calculate Rashba constant. Because it depends both on the field applied and boundary conditions at the interfaces. For InAs based quantum wells, typical values of Rashba constant are about 10^{-9} eV cm, however, values as large as 6×10^{-9} eV cm have been also reported (L. J. Cui et al., 2002). This is important, because Rashba constant can be controllably changed by the gate voltage.

Other possible sources of spin-orbit interaction are nonmagnetic impurities, phonons (V. F. Gantmakher and Y. B. Levinson, 1987), sample inhomogenity, surfaces and interfaces. In some situations these could play a role in spin transport and spin relaxation dynamics.

3.3. Spin Matrices and Spin Direction

The spin direction in any system is defined through spin matrices by spherical coordinates and is given by the following parameters:

$$\eta = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta) \tag{3.3}$$

where θ is the angle between the vector and z axis, φ is the azimuthal angle. Upon representing the corresponding vector in unit vector notation as $\vec{\eta} = \eta_x\hat{\imath} + \eta_y\hat{\jmath} + \eta_z\hat{k}$, in terms of its components becomes $\vec{\eta} = (\sin\theta\cos\varphi\hat{\imath} + \sin\theta\sin\varphi\hat{\jmath} + \cos\theta\hat{k})$.

The direction of spin is given by scalar product of Pauli spin matrices and $\vec{\eta}$: $S_n = \vec{S}.\vec{\eta}$, where $\vec{S} = \sigma_x\hat{\imath} + \sigma_y\hat{\jmath} + \sigma_z\hat{k}$ and Pauli spin matrices are:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{3.4}$$

which gives rise to:

$$S_n = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \sin\theta\cos\varphi + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \sin\theta\sin\varphi + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cos\theta \tag{3.5}$$

After taking the summation of matrices, the above equation can simply be written as:

$$S_n = \begin{pmatrix} \cos\theta & \sin\theta\cos\varphi - i\sin\theta\sin\varphi \\ \sin\theta\cos\varphi + i\sin\theta\sin\varphi & -\cos\theta \end{pmatrix} \tag{3.6}$$

Using the definitions $\cos\varphi - i.\sin\varphi = e^{-i\varphi}$ and $\cos\varphi + i.\sin\varphi = e^{i\varphi}$ the equation above takes the following form:

$$S_n = \begin{pmatrix} \cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta\, e^{i\varphi} & -\cos\theta \end{pmatrix} \tag{3.7}$$

There are two eigenstates for an electron spin: $\left|\frac{1}{2}\,\frac{1}{2}\right\rangle$ (spin up) and $\left|\frac{1}{2}\frac{-1}{2}\right\rangle$ (spin down). These eigenstates form the spinor part of any wave function describing the motion of an electron in a material. Using these as basis vectors, the electron wave function can be expressed in terms of orbital (ϕ) and spinor part (X) as a two element column matrix: $\psi = \phi X$, where the spinor contains the terms realted to spin up and down:

$$X = \begin{pmatrix} a \\ b \end{pmatrix} = aX_{\downarrow} + bX_{\uparrow} \tag{3.8}$$

here a and b are coefficients. In terms of these coefficients S_n can be written in matrix form as:

$$S_n \begin{pmatrix} a \\ b \end{pmatrix} = C_n \begin{pmatrix} a \\ b \end{pmatrix} \tag{3.9}$$

where C_n is the eigenvalue. Substituting (3.7) into the equation above:

$$\begin{pmatrix} \cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta\, e^{i\varphi} & -\cos\theta \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = C_n\begin{pmatrix} a \\ b \end{pmatrix} \tag{3.10}$$

Then this expression will result in the following equations:

$$a(\cos\theta - C_n) + b\sin\theta e^{-i\varphi} = 0 \tag{3.11}$$

$$a\sin\theta\, e^{i\varphi} - b(\cos\theta + C_n) = 0 \tag{3.12}$$

which will yield:

$$\begin{vmatrix} \cos\theta - C_n & \sin\theta e^{-i\varphi} \\ \sin\theta\, e^{i\varphi} & -\cos\theta - C_n \end{vmatrix} = 0 \tag{3.13}$$

and it becomes

$$(\cos\theta - C_n)(-\cos\theta - C_n) - \sin\theta\, e^{i\varphi}\sin\theta e^{-i\varphi} = 0 \tag{3.14}$$

Solving it for C_n gives: $C_n^2 = 1 \rightarrow C_n = \mp 1$. For $C_n = 1 \rightarrow a(\cos\theta - 1) + b\sin\theta e^{-i\varphi} = 0$ which result in:

$$\frac{a}{b} = \frac{\sin\theta e^{-i\varphi}}{1-\cos\theta} = \frac{2\sin\left(\theta/2\right)\cos(\theta/2)e^{-i\varphi}}{1-\left[1-2\sin^2\left(\frac{\theta}{2}\right)\right]} = \cot\left(\frac{\theta}{2}\right)e^{-i\varphi} \tag{3.15}$$

For $C_n = -1 \rightarrow a(\cos\theta + 1) + b\sin\theta e^{-i\varphi} = 0$ which leads to:

$$\frac{a}{b} = \frac{-\sin\theta e^{-i\varphi}}{1+\cos\theta} = \frac{-2\sin\left(\theta/2\right)\cos(\theta/2)e^{-i\varphi}}{1+\left[2\cos^2\left(\frac{\theta}{2}\right)-1\right]} = -\tan\left(\frac{\theta}{2}\right)e^{-i\varphi} \tag{3.16}$$

Since the spinor part of the wave function includes two components, in (3.8), two values of C_n must be associated with both the spin up and spin down parts: Positive and negative values correspond to spin up and spin down components, respectively. Employing (3.15) and (3.16), they can be expressed in matrix form:

$$X_{\uparrow} = \begin{pmatrix} \cos\left(\theta/_2\right) e^{-i\left(\frac{\varphi}{2}\right)} \\ \sin\left(\theta/_2\right) e^{i\left(\frac{\varphi}{2}\right)} \end{pmatrix} \qquad X_{\downarrow} = \begin{pmatrix} \sin\left(\theta/_2\right) e^{-i\left(\frac{\varphi}{2}\right)} \\ -\cos\left(\theta/_2\right) e^{i\left(\frac{\varphi}{2}\right)} \end{pmatrix} \tag{3.17}$$

The above equation gives the eigenfunctions for up and down spin. They describe the direction of spin in terms of spherical angles while electrons are traveling in a system. Actually these angles give the amount of deviation form the *z* axis by θ and *x* axis by φ. Hence they define the spin precession angles which are fundamental in Rashba effect.

CHAPTER 4

BALLISTIC AND NONBALLISTIC SYSTEMS

A ballistic conductor is a conductor where electrons propagate freely, and resistance arises from the contacts. The conductance of such a conductor is indeed independent of its length. The ohmic length dependence of the conductance comes from scattering processes within the conductor. Comparing mean free path ℓ with characteristic dimensions of the system, L, one can discriminate between diffusive, $\ell << L$ and ballistic, $\ell \geq L$, transport (see Fig. 4.1).

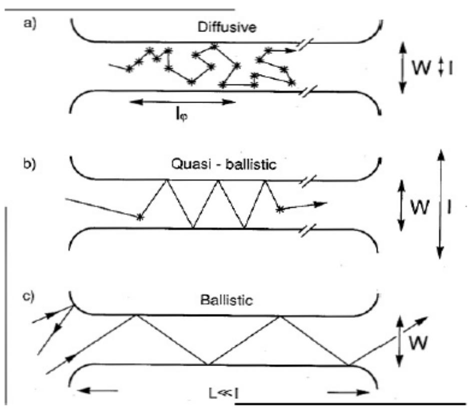

Figure 4.1. Ballistic and non ballistic conductors.

4.1. Ballistic (Impurity Free) Systems

Ballistic conduction is the characteristic of a material, known as a ballistic conductor, which has crystalline properties that allow electrons to flow through the material without collisions. The material must be free of impurities that the electrons will be capable of colliding with. In ordinary conductors, flowing electrons continually collide with the atoms making up the material, slowing down the electrons and causing the material to heat, effectively creating resistance in the material. Despite ballistic conductivity is really connected with very low resistance of particular material, the major importance isn't energy efficient transport of power on the long distance but nanoscience and information processing electronic applications.

Ballistic conduction enables utilization of quantum mechanical properties of electron wave function. The ballistic transport is coherent in terms of wave mechanics. For a ballistic system in the presence of Rashba effect, one can describe Hamiltonian in 2D as following:

$$\widehat{H} = \frac{p_x^2+p_z^2}{2m} + \alpha\frac{(\sigma_z p_x - \sigma_x p_z)}{ħ} \tag{4.1}$$

When the width (w) of the 2DEG is sufficiently narrow, $\ll \hbar^2/\alpha m$, the intersubband mixing can be neglected (S. Datta and B. Das, 1990). Moreover the number of available channels reduces to two, including the spin degeree of freedom, for sufficiently small w. In such a situation the transport via two available channels is given by a simple 1D Hamiltonian which describes 1DEG. Assuming electrons are traveling along the x direction, $k_z = 0$, the reduced Hamiltonian becomes:

$$\widehat{H} = \frac{p_x^2}{2m} + \alpha\frac{(\sigma_z p_x)}{ħ} = -\frac{ħ}{2m}\frac{d^2}{dx^2} + \alpha\sigma_z\frac{p_x}{ħ} \tag{4.2}$$

Wave functions, describing the motion of electrons in 1DEG for up and down spins can be written as:

$$\psi_\uparrow = e^{ik_\uparrow x}\begin{pmatrix}1\\0\end{pmatrix} \qquad \psi_\downarrow = e^{ik_\downarrow x}\begin{pmatrix}0\\1\end{pmatrix} \tag{4.3}$$

Time independent Schrodinger equation is for both spin directions is given by $\widehat{H}\psi_{\uparrow\downarrow} = E\psi_{\uparrow\downarrow}$. Upon substituting (4.3) into this expression, energy eigenvalues and eigenstates can be found by solving the following matrix:

$$\begin{vmatrix}\frac{ħ^2k^2}{2m} + \alpha k - E & 0\\ 0 & \frac{ħ^2k^2}{2m} - \alpha k - E\end{vmatrix} = 0 \tag{4.4}$$

Upon diagonalizing the matrix above we obtain the energy eigenstates for up and down spins:

$$E_{\downarrow} = \frac{\hbar^2 k_{\downarrow}^2}{2m} - \alpha k_{\downarrow} \quad (4.5)$$

$$E_{\uparrow} = \frac{\hbar^2 k_{\uparrow}^2}{2m} + \alpha k_{\uparrow} \quad (4.6)$$

where $k_{\uparrow}$ and $k_{\downarrow}$ are the wave vectors for up spin and down spin, respectively. From these equations it is possible to recover the momentum difference because the two energies are fixed to the Fermi energy (see Fig. 4.2):

$$k_{\downarrow} - k_{\uparrow} = \frac{2m\alpha}{\hbar^2} \quad (4.7)$$

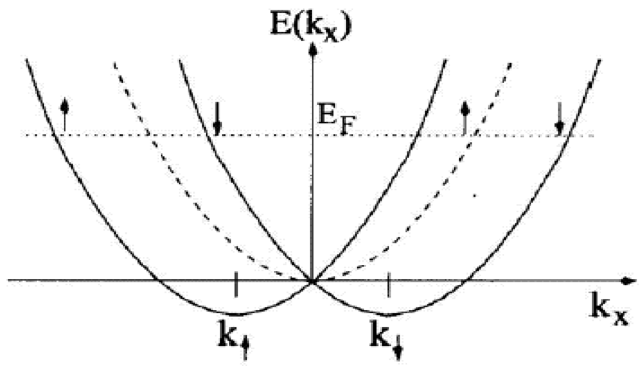

Figure 4.2. Energy dispersion for up and down spins.

Phase shift ($\Delta\theta$) corresponding to (4.7) is described as the difference between down spin and up spin multiplied by L, where L is the length of the system that we consider, is proportional Rashba parameter α:

$$(k_{\downarrow} - k_{\uparrow})L = \frac{2m\alpha}{\hbar^2} L = \Delta\theta \quad (4.8)$$

Total wave function (ψ) of an electron is defined as the superposition or linear combination of two waves, : $\psi = C_1\psi_{\uparrow} + C_2\psi_{\downarrow}$

$$\psi = c_1 e^{ik_{\uparrow}x} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{ik_{\downarrow}x} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (4.9)$$

To set up a relation between coefficients C_1, C_2 and spin precession angles one can introduce the following definitions: $C_1 = |c_1|e^{i\gamma_1}$, $C_2 = |c_2|e^{i\gamma_2}$. After putting them into the (4.9), it becomes:

$$\begin{pmatrix} C_1 e^{ik_\uparrow x} \\ C_2 e^{ik_\downarrow x} \end{pmatrix} = \begin{pmatrix} |c_1| e^{i\gamma_1 + ik_\uparrow x} \\ |c_2| e^{i\gamma_2 + ik_\downarrow x} \end{pmatrix} \tag{4.10}$$

here γ_1 and γ_2 are the angles, indicating deflections from the phase. From the matrix expression (4.10) we will get:

$$\tan(\theta/2) = \left|\frac{c_2}{c_1}\right| \quad \text{and} \quad \varphi = \gamma_2 - \gamma_1 + (k_\downarrow - k_\uparrow)x \tag{4.11}$$

In the equation above we see that spin precession angle (θ) and the azimuthal spin precession angle φ depend on the ratio of coefficients and $(k_\downarrow - k_\uparrow)$, respectively. Hence from the equation (4.7) φ is also dependent on Rashba Coefficient. It means that as the electrons are traveling in 1DEG the deflection from the *z* axis and the *x* axis are given by this ratio and the parameters γ_1, γ_2 and $(k_\downarrow - k_\uparrow)$.

4.2. Nonballistic Systems

A perfect crystal consists only of intrinsic (host) atoms and stoichiometric vacancies occupying intrinsic sites in the crystal lattice. Any deviation from crystal perfection is known as a *defect*, and the process that has brought it into life is termed as defect formation. In generally accepted classification, impurities and vacancies are referred to as point defects. In a spin FET the conductance modulation in the field of impurities has been studied (S. Caliskan, 2006). In this section, first of all we will examine those systems which include a single impurity, and then examine effect of more than one impurity and different kind of potentials on the system.

4.2.1. Systems of One Impurity

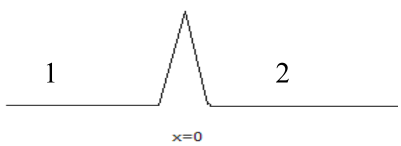

Figure 4.3. A system including one impurity

Take a *Dirac* potential at x = 0 as an impurity in the system..In the presence of that impurity one can rewrite the Hamiltonian in (4.2) as following:

$$\hat{H} = -\frac{\hbar}{2m}\frac{d^2}{dx^2} + \alpha\sigma_z\frac{p_x}{\hbar} + \Gamma\delta(\text{x}) \tag{4.12}$$

where $\Gamma\delta(\text{x})$ is the Dirac Delta potential term. As a result of one impurity at x = 0, the wavefunctions before and after the impurity can simply be written for up spin:

$$\psi_\uparrow = \begin{Bmatrix} e^{ik_\uparrow x}\begin{pmatrix}1\\0\end{pmatrix} + r_\uparrow e^{-ik_\downarrow x}\begin{pmatrix}1\\0\end{pmatrix} & \text{for } x < 0 \\ t_\uparrow e^{ik_\uparrow x}\begin{pmatrix}1\\0\end{pmatrix} & \text{for } x > 0 \end{Bmatrix} \tag{4.13}$$

and for down spin:

$$\psi_\downarrow = \begin{Bmatrix} e^{ik_\downarrow x}\begin{pmatrix}0\\1\end{pmatrix} + r_\downarrow e^{-ik_\uparrow x}\begin{pmatrix}0\\1\end{pmatrix} & \text{for } x < 0 \\ t_\downarrow e^{ik_\downarrow x}\begin{pmatrix}0\\1\end{pmatrix} & \text{for } x > 0 \end{Bmatrix} \tag{4.14}$$

As a linear combination of wave functions of up and down spin, the total wave function becomes:

$$\psi = \begin{Bmatrix} \begin{pmatrix} C_1 r_\uparrow e^{ik_\downarrow x} \\ C_2 r_\downarrow e^{ik_\uparrow x} \end{pmatrix} & \text{for } x < 0 \\ \\ \begin{pmatrix} C_1 t_\uparrow e^{ik_\uparrow x} \\ C_2 t_\downarrow e^{ik_\downarrow x} \end{pmatrix} & \text{for } x > 0 \end{Bmatrix} \tag{4.15}$$

Transmission coefficients for both spin directions can be introduced in a similar fashion as done in the previous section:

$$t_\uparrow = |t_\uparrow| e^{i\xi_\uparrow} \tag{4.16}$$

$$t_\downarrow = |t_\downarrow| e^{i\xi_\downarrow} \tag{4.17}$$

Using these definitions for $x > 0$, the total wave function will take the following form:

$$\psi = \left\{\begin{pmatrix} |c_1||t_\uparrow|e^{i\gamma_1+ik_\uparrow x+i\xi_\uparrow} \\ |c_2||t_\downarrow|e^{i\gamma_2+ik_\downarrow x+i\xi_\downarrow} \end{pmatrix}\right\} \tag{4.18}$$

The spin precession angles θ and φ can be found in terms of coefficients in a similar manner as done for systems with no impurity:

$$\tan\left(\frac{\theta}{2}\right) = \left|\frac{c_2 t_\downarrow}{c_1 t_\uparrow}\right| \text{ and } \varphi = \gamma_2 - \gamma_1 + \xi_\uparrow - \xi_\downarrow + (k_\downarrow - k_\uparrow)x \tag{4.19}$$

Wave functions for both spin orientations, before and after the impurity taken place at $x = 0$, can be written as:

$$\psi_{1\uparrow} = e^{ik_\uparrow x}\begin{pmatrix}1\\0\end{pmatrix} + r_\uparrow e^{-ik_\downarrow x}\begin{pmatrix}1\\0\end{pmatrix}$$

$$\psi_{1\downarrow} = e^{ik_\downarrow x}\begin{pmatrix}0\\1\end{pmatrix} + r_\downarrow e^{-ik_\uparrow x}\begin{pmatrix}0\\1\end{pmatrix}$$

$$\psi_{2\uparrow} = t_\uparrow e^{ik_\uparrow x}\begin{pmatrix}1\\0\end{pmatrix}$$

$$\psi_{2\downarrow} = t_\downarrow e^{ik_\downarrow x}\begin{pmatrix}0\\1\end{pmatrix}$$

At the boundary, $x = 0$, the wave functions must be equal to each other, and the difference in the derivatives of them is given by the corresponding potential strength:

$$\psi_{1\downarrow}(0) = \psi_{2\downarrow}(0)$$

$$\psi_{1\uparrow}(0) = \psi_{2\uparrow}(0)$$

$$\frac{\partial\psi_{1\uparrow}(0)}{\partial x} - \frac{\partial\psi_{2\uparrow}(0)}{\partial x} = \frac{2m\Gamma}{\hbar^2}$$

$$\frac{\partial\psi_{1\downarrow}(0)}{\partial x} - \frac{\partial\psi_{2\downarrow}(0)}{\partial x} = \frac{2m\Gamma}{\hbar^2}$$

After setting and employing boundary conditions, we get the following equalities for the polarized transmission and reflection coefficients:

$$1 + r_\uparrow = t_\uparrow \qquad \text{and} \qquad 1 + r_\downarrow = t_\downarrow \tag{4.20}$$

and upon applying the derivatives, the following expression is obtained:

$$t_\uparrow = t_\downarrow = \frac{i(k_\uparrow + k_\downarrow)}{i(k_\uparrow + k_\downarrow) - \frac{2m\Gamma}{\hbar^2}} \tag{4.21}$$

It is obtained that the polarized transmission coefficients for up and down spin, after passing through the impurity are equal to each other. It implies that an electron with down spin can have up spin after passing the impurity. Hence a single impurity in the type of Dirac Delta Potential does not affect the spin precession angle and thus the behaviour of the transmission coefficients for up and down spin travelling in the system. We have found these results for those electrons having positive energies. One can also find a relationship for negative energies, $E < 0$:

For systems of negative energy we can follow the same procedures: Define the wave functions for up and down spins before and after the impurity; set and employ the boundary conditions:

$$\psi_{1\uparrow} = e^{ikx}\begin{pmatrix}1\\0\end{pmatrix} + r_\uparrow e^{ik'x}\begin{pmatrix}1\\0\end{pmatrix} \tag{4.22}$$

$$\psi_{1\downarrow} = e^{-ik'x}\begin{pmatrix}0\\1\end{pmatrix} + r_\downarrow e^{-ikx}\begin{pmatrix}0\\1\end{pmatrix} \tag{4.23}$$

$$\psi_{2\uparrow} = t_\uparrow e^{ikx}\begin{pmatrix}1\\0\end{pmatrix} \tag{4.24}$$

$$\psi_{2\downarrow} = t_\downarrow e^{-ik'x}\begin{pmatrix}0\\1\end{pmatrix} \tag{4.25}$$

$$\psi_{1\downarrow}(0) = \psi_{2\downarrow}(0) \xrightarrow{yields} 1 + r_\uparrow = t_\uparrow \tag{4.26}$$

$$\psi_{1\uparrow}(0) = \psi_{2\uparrow}(0) \xrightarrow{yields} 1 + r_\downarrow = t_\downarrow \tag{4.27}$$

and utilizing the difference in derivatives of wave functions, we again find that even at negative energies the polarized transmission coefficients become equal to each other:

$$t_{\uparrow} = t_{\downarrow} \frac{1}{1+\dfrac{i2m\Gamma}{\hbar^2 (k-k^{'})}} \tag{4.28}$$

which implies that spin precession angle will be inert to impurity at negative energies.

4.2.2. Systems of Two Impurities

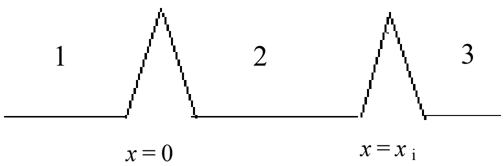

Figure 4.4. Systems of two impurities

In the presence of two impurities, we follow the same way as done in the previous section. Firstly we define the wave functions for each region (see Fig.4.4). The first two regions contain both transmission and reflection coefficients, whereas the third region will consist of only polarized transmission coefficients:

$$\psi_{1\uparrow} = e^{ik_{\uparrow}x}\begin{pmatrix}1\\0\end{pmatrix} + r_{\uparrow}e^{-ik_{\downarrow}x}\begin{pmatrix}1\\0\end{pmatrix} \tag{4.29}$$

$$\psi_{1\downarrow} = e^{ik_{\downarrow}x}\begin{pmatrix}0\\1\end{pmatrix} + r_{\downarrow}e^{-ik_{\uparrow}x}\begin{pmatrix}0\\1\end{pmatrix} \tag{4.30}$$

$$\psi_{2\uparrow} = t_{\uparrow}e^{ik_{\uparrow}x}\begin{pmatrix}1\\0\end{pmatrix} + r_{i\uparrow}e^{-ik_{\downarrow}x}\begin{pmatrix}1\\0\end{pmatrix} \tag{4.31}$$

$$\psi_{2\downarrow} = t_{\downarrow}e^{ik_{\downarrow}x}\begin{pmatrix}0\\1\end{pmatrix} + r_{i\downarrow}e^{-ik_{\uparrow}x}\begin{pmatrix}0\\1\end{pmatrix} \tag{4.32}$$

$$\psi_{3\uparrow} = t_{i\uparrow}e^{ik_{\uparrow}x}\begin{pmatrix}1\\0\end{pmatrix} \tag{4.33}$$

$$\psi_{3\downarrow} = t_{i\downarrow}e^{-ik_{\downarrow}x}\begin{pmatrix}0\\1\end{pmatrix} \tag{4.34}$$

Upon indicating the wave functions for each region, now we can set boundary conditions at the location of the impurities. The impurity potential at $x = 0$ gives rise to the similar results as in the previous section, except that there are now polarized reflection coefficients due to other impurity:

$$1 + r_{\uparrow} = t_{\uparrow} + r_{i\uparrow} \tag{4.35}$$

$$ik_{\uparrow}t_{\uparrow} - ik_{\downarrow}r_{i\uparrow} - (ik_{\uparrow} - ik_{\downarrow}r_{\uparrow}) = \frac{2m\Gamma}{\hbar^2}(t_{\uparrow} + r_{i\uparrow}) \tag{4.36}$$

Similarly, by utilizing the boundary conditions for the impurity potential at $x = x_i$ the following equalities can easily be obtained:

$$t_{i\uparrow}e^{ik_{\uparrow}x_i} = t_{\uparrow}e^{ik_{\uparrow}x_i} + r_{i\uparrow}e^{-ik_{\downarrow}x_i}$$

$$ik_{\uparrow}t_{i\uparrow}e^{ik_{\uparrow}x_i} - ik_{\uparrow}t_{\uparrow}e^{ik_{\uparrow}x_i} + ik_{\downarrow}t_{i\uparrow}e^{-ik_{\downarrow}x_i} = \frac{2m\Gamma}{\hbar^2}t_{i\uparrow}e^{ik_{\uparrow}x_i}$$

$$t_{i\uparrow} = \frac{(k_{\uparrow}+k_{\downarrow})^2}{e^{i(k_{\uparrow}+k_{\downarrow})x_i\left(\frac{2m\Gamma}{\hbar^2}\right)-\left(\frac{2m\Gamma}{\hbar^2}-i(k_{\uparrow}+k_{\downarrow})\right)^2}} = t_{i\downarrow} \tag{4.37}$$

We again found that the polarized transmission coefficients for up and down spin are equal. Therefore even the presence of two impurities don't influence the behavior of spin precession angle. It means that the corresponding current is inert to scatterings due to impurities.

4.2.3. Systems of Potential Well

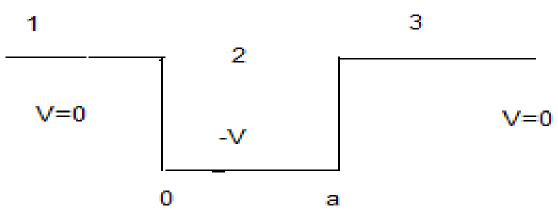

Figure 4.5 A potential well

In this section we will introduce a potential well of width a to the Hamiltonian. The Hamiltonian including the potential well type impurity will take the following form:

$$\hat{H} = -\frac{\hbar}{2m}\frac{d^2}{dx^2} + \alpha\sigma_z\frac{p_x}{\hbar} - |V| \tag{4.38}$$

Similar to the previous sections, first of all we must define the wavefunctions for both spin directions in each region (Fig.4.5):

$$\psi_{1\uparrow} = e^{ik_\uparrow x}\begin{pmatrix}1\\0\end{pmatrix} + r_\uparrow e^{-ik_\downarrow x}\begin{pmatrix}1\\0\end{pmatrix} \tag{4.39}$$

$$\psi_{1\downarrow} = e^{ik_\downarrow x}\begin{pmatrix}0\\1\end{pmatrix} + r_\downarrow e^{-ik_\uparrow x}\begin{pmatrix}0\\1\end{pmatrix} \tag{4.40}$$

$$\psi_{2\uparrow} = t_\uparrow e^{iq_\uparrow x}\begin{pmatrix}1\\0\end{pmatrix} + r_{a\uparrow} e^{-iq_\downarrow x}\begin{pmatrix}1\\0\end{pmatrix} \tag{4.41}$$

$$\psi_{2\downarrow} = t_\downarrow e^{iq_\downarrow x}\begin{pmatrix}0\\1\end{pmatrix} + r_{a\downarrow} e^{-iq_\uparrow x}\begin{pmatrix}0\\1\end{pmatrix} \tag{4.42}$$

$$\psi_{3\uparrow} = t_{a\uparrow} e^{ik_\uparrow x}\begin{pmatrix}1\\0\end{pmatrix} \tag{4.43}$$

$$\psi_{3\downarrow} = t_{a\downarrow} e^{-ik_\downarrow x}\begin{pmatrix}0\\1\end{pmatrix} \tag{4.44}$$

where $q_\uparrow$ and $q_\downarrow$ are the electron wave vectors for up and down spin inside the potential well. Upon applying the Schrodinger equation and diagonilizing the Hamiltonian, the following energy eigenvalues are found:

$$E = \hbar^2\frac{q_\uparrow^2}{2m} + \alpha q_\uparrow - |V| = \hbar^2\frac{q_\downarrow^2}{2m} - \alpha q_\downarrow - |V| \tag{4.45}$$

inside the potential well and

$$E = \hbar^2\frac{k_\uparrow^2}{2m} + \alpha k_\uparrow = \hbar^2\frac{k_\downarrow^2}{2m} - \alpha k_\downarrow \tag{4.46}$$

outside the well. The above two expressions give:

$$q_\uparrow - k_\uparrow = q_\downarrow - k_\downarrow \tag{4.47}$$

In this system the boundary conditions can be set as:

$$\psi_{1\downarrow}(0) = \psi_{2\downarrow}(0) \quad \text{and} \quad \psi_{1\uparrow}(0) = \psi_{2\uparrow}(0)$$

$\psi_{2\downarrow}(a) = \psi_{3\downarrow}(a)$ and $\psi_{2\uparrow}(a) = \psi_{3\uparrow}(a)$

These relations together with their derivatives give rise to the following equations:

$$1 + r_\uparrow = t_\uparrow + r_{a\uparrow}$$

$$ik_\uparrow - ik_\downarrow r_\uparrow = iq_\uparrow t_\uparrow - iq_\downarrow r_{a\uparrow}$$

$$t_\uparrow e^{iq_\uparrow a} + r_{a\uparrow} e^{-iq_\downarrow a} = t_{a\uparrow} e^{ik_\uparrow a}$$

$$iq_\uparrow t_\uparrow e^{iq_\uparrow a} - iq_\downarrow r_{a\uparrow} e^{-iq_\downarrow a} = ik_\uparrow t_{a\uparrow} e^{ik_\uparrow a}$$

From the expressions above and utilizing (4.31), the polarized transmission coefficients for electrons passing through the potential well are found to be:

$$t_{a\uparrow} = t_{a\downarrow} = -\frac{e^{ia(q_\uparrow - k_\downarrow)(q_\downarrow - k_\uparrow)}(k_\uparrow + k_\downarrow)(q_\uparrow + q_\downarrow)}{e^{ia(q_\uparrow + q_\downarrow)}(q_\uparrow - k_\downarrow)(q_\downarrow - k_\uparrow) - (k_\uparrow + q_\uparrow)(k_\downarrow + q_\downarrow)} \tag{4.48}$$

which are equal to each other. Even if we take a different kind of impurity potential the ratio of polarized transmission coefficients again becomes unity, meaning that the spin precession angle will not be influenced either by a potential well. It implies that electron with down spin before the well can have up spin after the well.

4.3. Effect of Magnetic Field

In this section we will examine what happens when the system is under an external magnetic field (S. Caliskan and M. Kumru, 2007). We choose two different directions for the magnetic field:

4.3.1. Parallel Magnetic Field

In this part the external magnetic field is taken to be along the *z* direction, $\vec{B} = B\hat{z}$, i.e. parallel to the Rashba Field, $\vec{B}_R = B_R\hat{z}$. The analogous magnetic vector potential $\vec{A}$ can be chosen using the *Landau Gauge* which may yield $\vec{A} = (-\vec{B}y, 0, 0)$.

The quantum mechanical description of electron motion in a magnetic field is obtained by solving the Schrödinger equation:

$$\hat{H}\psi_{\uparrow\downarrow}(x, y, z) = E\psi_{\uparrow\downarrow}(x, y, z).$$

with Hamiltonian:

$$\hat{H} = \frac{p_x^2}{2m} + \frac{eB_y p_x}{m} + \frac{e^2B^2y^2}{2m} + \alpha\left[\left(\frac{p_x}{\hbar} + eB_y\right)\sigma_z\right] + \mu_B B_z \tag{4.49}$$

In the expression above since we may write:

$$\frac{eB_y p_x}{m} + \frac{e^2B^2y^2}{2m} \to \frac{eBh}{m}\langle y\rangle k_x \to 0$$

the Hamiltonian in (4.33) reduces to:

$$\hat{H} = \frac{\hbar^2 k_x^2}{2m} + \alpha k_x \sigma_z + \mu_B B\sigma_z \tag{4.50}$$

Replacing k_x by k and diagonalizing the Hamiltonian gives:

$$\begin{vmatrix} \frac{\hbar^2k^2}{2m} + \alpha k + \mu_B B - E & 0 \\ 0 & \frac{\hbar^2k^2}{2m} - \alpha k - \mu_B B - E \end{vmatrix} = 0 \tag{4.51}$$

Solving this expression for energy gives the eigenvalues for each spin direction:

$$E_+ = \frac{\hbar^2 k_\uparrow^2}{2m} + \alpha k_\uparrow + \mu_B B = E_\uparrow \tag{4.52}$$

$$E_{-}=\frac{\hbar^2 k_{\downarrow}^2}{2m}-\alpha k_{\downarrow}-\mu_B B=E_{\downarrow} \tag{4.53}$$

To see the effect of magnetic field we focus on, for instance, those systems including two impurities (see Fig. 4.4). As it was done in the previous sections, first of all we describe the wavefunctions, then set the boundary conditions and finally solve for polarized transmission coefficients in the presence of external magnetic field. The wave functions in each region are:

$$\psi_{1\uparrow}=e^{ik_{\uparrow}x}\begin{pmatrix}1\\0\end{pmatrix}+r_{\uparrow}e^{-ik'_{\downarrow}x}\begin{pmatrix}1\\0\end{pmatrix} \tag{4.54}$$

$$\psi_{1\downarrow}=e^{ik_{\downarrow}x}\begin{pmatrix}0\\1\end{pmatrix}+r_{\downarrow}e^{-ik'_{\uparrow}x}\begin{pmatrix}0\\1\end{pmatrix} \tag{4.55}$$

$$\psi_{2\uparrow}=t_{\uparrow}e^{ik_{\uparrow}x}\begin{pmatrix}1\\0\end{pmatrix}+r_{i\uparrow}e^{-ik'_{\downarrow}x}\begin{pmatrix}1\\0\end{pmatrix} \tag{4.56}$$

$$\psi_{2\downarrow}=t_{\downarrow}e^{ik_{\downarrow}x}\begin{pmatrix}0\\1\end{pmatrix}+r_{i\downarrow}e^{-ik'_{\uparrow}x}\begin{pmatrix}0\\1\end{pmatrix} \tag{4.57}$$

$$\psi_{3\uparrow}=t_{i\uparrow}e^{ik_{\uparrow}x}\begin{pmatrix}1\\0\end{pmatrix} \tag{4.58}$$

$$\psi_{3\downarrow}=t_{i\downarrow}e^{ik_{\downarrow}x}\begin{pmatrix}0\\1\end{pmatrix} \tag{4.59}$$

The first impurity located at $x = 0$ yields:

$$1+r_{\uparrow}=t_{\uparrow}+r_{i\uparrow} \tag{4.60}$$

$$(ik_{\uparrow}t_{\uparrow}-ik'_{\downarrow}r_{i\uparrow})-(ik_{\uparrow}-ik'_{\downarrow}r_{i})=\frac{2m\Gamma}{\hbar^2}(t_{\uparrow}+r_{i\uparrow}) \tag{4.61}$$

The second impurity located at $x = x_i$ results in:

$$t_{i\uparrow}e^{ik_{\uparrow}x_i}+r_{i\uparrow}e^{-ik'_{\downarrow}x_i}=t_{i\uparrow}e^{ik_{\uparrow}x_i} \tag{4.62}$$

$$\left(ik_{\uparrow}t_{i\uparrow}e^{ik_{\uparrow}x_i}\right)-\left(ik_{\downarrow}\ t_{i\uparrow}e^{ik_{\uparrow}x_i}-ik'_{\downarrow}r_{i\uparrow}e^{-ik'_{\downarrow}x}\right)=\frac{2m\Gamma}{\hbar^2}\left(t_{\uparrow}+r_{i\uparrow}\right)t_{i\uparrow}e^{ik_{\uparrow}x_i} \tag{4.63}$$

The above relations give the following expression:

$$t_{i\uparrow}=\frac{(k_{\uparrow}+k'_{\downarrow})^2}{e^{i(k_{\uparrow}+k'_{\downarrow})x_i}\left(\frac{2m\Gamma}{\hbar^2}\right)-\left(\frac{2m\Gamma}{\hbar^2}-i(k_{\uparrow}+k'_{\downarrow})\right)^2}=t_{i\downarrow} \tag{4.64}$$

which indicates that the spin precession angle again is not influenced due to the ratio of polarized transmission coefficients.

4.3.2. Perpendicular Magnetic Field

In this part the external magnetic field is taken to be along the *y* direction, $\vec{B}=B\hat{y}$, i.e. perpendicular to the Rashba Field, $\vec{B}_R=B_R\hat{z}$. The corresponding Hamiltonian may reduce the following form, upon choosing an appropriate *Landau Gauge*:

$$\hat{H}=\frac{\hbar^2k^2}{2m}+\alpha k\sigma_z+\mu_B B\sigma_z \tag{4.65}$$

Diagonalizing the Hamiltonian in the presence of perpendicular magnetic field gives the following determinant:

$$\begin{vmatrix}\frac{\hbar^2k^2}{2m}+\alpha k-E & -i\mu_B B\\ i\mu_B B & \frac{\hbar^2k^2}{2m}-\alpha k-E\end{vmatrix}=0 \tag{4.66}$$

which gives rise to following energy eigenvalues for both spin orientations:

$$E_{\pm}=\frac{\hbar^2k^2}{2m}\pm\sqrt{(\alpha k)^2+(\mu_B B)^2} \tag{4.67}$$

and corresponding eigenstates can simply be found by solving the matrix equation:

$$\begin{pmatrix} \frac{\hbar^2 k^2}{2m} + \alpha k - E_{\pm} & -i\mu_B B \\ i\mu_B B & \frac{\hbar^2 k^2}{2m} - \alpha k - E_{\pm} \end{pmatrix} \begin{pmatrix} a_{\pm} \\ b_{\pm} \end{pmatrix} = 0 \tag{4.68}$$

which yields, after obtaining *a* and *b*:

$$\psi_{\pm} = e^{ikx} \begin{pmatrix} \frac{i\mu_B B}{\alpha k \pm \sqrt{(\alpha k)^2 + (\mu_B B)^2}} \\ 1 \end{pmatrix} \tag{4.69}$$

This expression can be written in more compact form:

$$\psi_{\pm} = e^{ikx} \begin{pmatrix} \eta_{\pm} \\ 1 \end{pmatrix} \tag{4.70}$$

where $\eta_{\pm} = \frac{i\mu_B B}{\alpha k \pm \sqrt{(\alpha k)^2 + (\mu_B B)^2}}$. (4.71)

The wave functions for up and down spin in each region (see Fig. 4.4) can be described as following, using (4.66):

$$\psi_{1+} = e^{ik_+ x} \begin{pmatrix} \eta_+ \\ 1 \end{pmatrix} + r_+ e^{-ik_+ x} \begin{pmatrix} \eta'_+ \\ 1 \end{pmatrix} + r_- e^{-ik_- x} \begin{pmatrix} \eta'_- \\ 1 \end{pmatrix} \tag{4.72}$$

$$\psi_{1-} = e^{ik_- x} \begin{pmatrix} \eta_- \\ 1 \end{pmatrix} + r'_+ e^{-ik_+ x} \begin{pmatrix} \eta'_+ \\ 1 \end{pmatrix} + r'_- e^{-ik_- x} \begin{pmatrix} \eta'_- \\ 1 \end{pmatrix} \tag{4.73}$$

$$\psi_{2+} = t_+ e^{ik_+ x} \begin{pmatrix} \eta_+ \\ 1 \end{pmatrix} + t_- e^{ik_- x} \begin{pmatrix} \eta_- \\ 1 \end{pmatrix} + r_{i+} e^{-ik_+ x} \begin{pmatrix} \eta'_+ \\ 1 \end{pmatrix} + r_{i-} e^{-ik_- x} \begin{pmatrix} \eta'_- \\ 1 \end{pmatrix} \tag{4.74}$$

$$\psi_{2-} = t'_{i+} e^{ik_+ x} \begin{pmatrix} \eta_+ \\ 1 \end{pmatrix} + t_- e^{ik_- x} \begin{pmatrix} \eta_- \\ 1 \end{pmatrix} + r'_{i+} e^{-ik_+ x} \begin{pmatrix} \eta'_+ \\ 1 \end{pmatrix} + r_{i-} e^{-ik_- x} \begin{pmatrix} \eta'_- \\ 1 \end{pmatrix} \tag{4.75}$$

$$\psi_{3+} = t_{i+} e^{ik_+ x} \begin{pmatrix} \eta_+ \\ 1 \end{pmatrix} + t_{i-} e^{ik_- x} \begin{pmatrix} \eta_- \\ 1 \end{pmatrix} \tag{4.75}$$

$$\psi_{3-} = t'_{i+} e^{ik_+ x} \begin{pmatrix} \eta_+ \\ 1 \end{pmatrix} + t_{i-} e^{ik_- x} \begin{pmatrix} \eta_- \\ 1 \end{pmatrix} \tag{4.76}$$

As in the case of previous section, upon setting the boundary conditions at $x = 0$ and $x = x_i$, we again find that the polarized transmission coefficients in the presence of perpendicular magnetic field becomes equal to each other.

In this section we investigated the effect of parallel and perpendicular external magnetic field, with respect to built in Rashba field. In both situations the ratio of transmission coefficients for up and down spin again goes to unity. Therefore even the external magnetic field does not modify this ratio, and so spin precession angle. It results in only a phase shift.

This chapter is devoted to effect of different kind of impurity potentials and of external magnetic field in different directions. In all cases we see that the polarized transmission coefficients are equal to each other for down spin and up spin. Since any type of impurity potential together with external field don't change this equality, there must be a mapping between up and down spin: Up spin may be transformed to down. In the next chapter we will consider in detail this mapping and generalize these results, by introducing a *gauge* transformation, using an operator..

4.4. Inhomogeneous Rashba Coefficient

Lastly we consider briefly the case where the Rashba coefficient α is not homogeneous. An example is the case where the gate potential covers only a part of the quantum wire, so that α becomes an x-dependent function $\alpha(x)$. To address the effects of its x dependence we consider the following Hamiltonian:

$$H^{inh} = \frac{p_x^2}{2m} + V(x) + \frac{\sigma_z}{2\hbar}[\alpha(x)p_x + p_x\alpha(x)] \tag{4.77}$$

Note that the Rashba term in the Hamiltonian is replaced by its symmetric combination, so that Hamiltonian remains Hermitian. The scattering state in homogenous system can be converted to a new state associated with inhomogenous system, by an analogous gauge transformation:

$$\tilde{\psi} = e^{i(m/\hbar^2)\sigma_z \int d\acute{x}\alpha(\acute{x})}\psi \tag{4.78}$$

Under this transformation the Schrodinger equations for $\tilde{\psi}_+$ and $\tilde{\psi}_-$ become identical:

$$\tilde{h}^{inh}\tilde{\psi}_{+,-} = \tilde{E}\tilde{\psi}_{+,-} \quad (4.79)$$

where $\tilde{h}^{inh} = \frac{p_x^2}{2m} + V^{inh}(x)$. Here the effective potential $V^{inh}(x)$:

$$V^{inh}(x) = V(x) - \frac{m\propto^2(x)}{2\hbar^2} \quad (4.80)$$

is renormalized by $\alpha(x)$. Thus the inhomogeneous α can induce backscattering and reduce the mean free path l just as an inhomogeneous $V(x)$ does. Note that for a given slope $d\alpha/dx$ this backscattering effect becomes stronger as the average α value gets larger since $\alpha(x)$ affects $V^{inh}(x)$ quadratically. Once this reduction of l by the inhomogeneous $\alpha(x)$ is taken into account, the rest of the analysis is the same as for the homogeneous α case and the results will be similar.

CHAPTER 5

NONBALLISTIC SPIN FET and ANSATZ

In the previous chapter we have dealed with those systems which contain different number and kind of impurities. Even if these systems inserted in an external magnetic field of different directions, we have seen that in all situations polarized transmission coefficients have been found to be equal for down and up spin. Since the impurities that we have considered does not affect the ratio of polarized transmission coefficients one can imply that the spin precession angle must be independent of impurity potential. Therefore we may also make a comment that any *arbitrary potential* would not change our results. To see whether this comment is really true or not can be given by an *ansatz*. We are going to take an impurity potential of arbitrary shape and examine the ratio of transmission coefficients for up spin and for down spin and see whether it is gonna be unity or not. It will give the behaviour of the spin precession angle, and so the polarized transmission coeffcients or polarized conductance influenced by impurities . If the ratio goes to unity our ansatz that we suggest will be true.

Ansatz: Spin precession angle is independent of potential. Let us take an arbitrary potential of $V(x)$ and write corresponding Hamiltonian:

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{\alpha}{i}\sigma_z\frac{\partial}{\partial x} + V(x) \tag{5.1}$$

and energy eigenvalues for up and down spin can be found as:

$$E = \frac{\hbar^2 k_\uparrow^2}{2m} + \alpha k_\uparrow = \frac{\hbar^2 k_\downarrow^2}{2m} - \alpha k_\downarrow$$

thus,

$$\frac{\hbar^2\left(k_\uparrow^2 - k_\downarrow^2\right)}{2m} + \alpha k_\uparrow = -\alpha\left(k_\uparrow + k_\downarrow\right) \Rightarrow k_\uparrow - k_\downarrow = -\frac{2m\alpha}{\hbar^2} \tag{5.2}$$

There are two possibilities for this case:

- There may exist a *mapping* betwen ↑ and ↓ solutions,
- There may exist a *dynamic symmetry*, eigenvalue of which represents spin precession angle.

Consider the following eigenstates for polarized spins before and after an arbitrary potential located at $x = 0$:

$$\psi_\uparrow = \begin{cases} e^{ik_\uparrow x}\begin{pmatrix}1\\0\end{pmatrix} + r_\uparrow e^{-ik_\downarrow x}\begin{pmatrix}1\\0\end{pmatrix} & \text{for } x < 0 \\ \\ t_\uparrow e^{ik_\uparrow x}\begin{pmatrix}1\\0\end{pmatrix} & \text{for } x > 0 \end{cases}$$

$$\psi_\downarrow = \begin{cases} e^{ik_\downarrow x}\begin{pmatrix}0\\1\end{pmatrix} + r_\downarrow e^{-ik_\uparrow x}\begin{pmatrix}0\\1\end{pmatrix} & \text{for } x < 0 \\ \\ t_\downarrow e^{ik_\downarrow x}\begin{pmatrix}0\\1\end{pmatrix} & \text{for } x > 0 \end{cases}$$

Concerning the mapping between up and down solutions, an operator $\hat{L}$ can be introduced which transforms up (down) eigenstate to down (up) as:

$$\hat{L}\left\{e^{ik_\uparrow x}\begin{pmatrix}1\\0\end{pmatrix}\right\} = e^{ik_\downarrow x}\begin{pmatrix}0\\1\end{pmatrix} \tag{5.3}$$

$$\hat{L}\left\{e^{-ik_\downarrow x}\begin{pmatrix}1\\0\end{pmatrix}\right\} = e^{-ik_\uparrow x}\begin{pmatrix}0\\1\end{pmatrix} \tag{5.4}$$

By utilizing (5.2) one can define this operator as following:

$$\hat{L} = e^{i\frac{2m\alpha}{\hbar^2}x}\begin{pmatrix}0 & 0\\1 & 0\end{pmatrix} \tag{5.5}$$

where the exponent on the right side is related to phase shift and so spin precession angle. Substituting (5.5) into the (5.3) and (5.4) yields:

$$\begin{pmatrix} 0 & 0 \\ e^{i\frac{2m\alpha}{\hbar^2}x} & 0 \end{pmatrix}\begin{pmatrix} e^{ik_\uparrow x} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ e^{i\left(k_\uparrow + \frac{2m\alpha}{\hbar^2}\right)x} \end{pmatrix} = \begin{pmatrix} 0 \\ e^{ik_\downarrow x} \end{pmatrix} \tag{5.6}$$

$$\begin{pmatrix} 0 & 0 \\ e^{i\frac{2m\alpha}{\hbar^2}x} & 0 \end{pmatrix}\begin{pmatrix} e^{-ik_\downarrow x} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ e^{-i\left(k_\downarrow - \frac{2m\alpha}{\hbar^2}\right)x} \end{pmatrix} = \begin{pmatrix} 0 \\ e^{-ik_\uparrow x} \end{pmatrix} \tag{5.7}$$

which satisfy (5.3) and (5.4). According to the definition of the $\hat{L}$ operator, (5.5) can also be written in the form of Pauli spin matrices as:

$$\hat{L} = e^{i\frac{2m\alpha}{\hbar^2}x}\,\frac{1}{2}\left(\sigma_x - i\sigma_y\right) \tag{5.8}$$

$\hat{L}$ operator, to be a valid operator for any system, must commute with the Hamiltonian. Hence, now let's investigate the commutation of $\hat{H}$ and $\hat{L}$:

$$\left[\hat{H}, \hat{L}\right] = 0 \ \text{(if they commute)} \tag{5.9}$$

To find whether it is zero or not, we can examine each component of the Hamiltonian, (5.1), with $\hat{L}$. It is readily seen that the arbitrary potential and $\hat{L}$ operator are commutative:

$$\left[V(x), \hat{L}\right] = 0 \tag{5.10}$$

The first term of the Hamiltonian includes $\hat{p}_x^2$. Begin with commutation of momentum operator $\hat{p}_x$ with $\hat{L}$ operator:

$$\left[\hat{p}_x, \hat{L}\right] = \left[\frac{\hbar}{i}\frac{\partial}{\partial x}, e^{i\frac{2m\alpha}{\hbar^2}x}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right] = \frac{\hbar}{i}\left[\frac{\partial}{\partial x}, e^{i\frac{2m\alpha}{\hbar^2}x}\right]\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

which results in:

$$\frac{2m\alpha}{\hbar}\hat{L} \tag{5.11}$$

The relationship between $\frac{\hat{p}_x^2}{2m}$ and $\hat{L}$ will take the following form, upon employing the commutation properties together with (5.11):

$$\frac{1}{2m}\left(\hat{p}_x 2m\frac{\alpha}{\hbar}\hat{L} + 2m\frac{\alpha}{\hbar}\hat{L}\hat{p}_x\right) = \frac{\alpha}{\hbar}\left(\hat{p}_x\hat{L} + \hat{L}\hat{p}_x\right) \tag{5.12}$$

By utilizing (5.8), the commutation of the second term and $\hat{L}$ operator can be written as:

$$\left[\alpha\sigma_z\frac{\hat{p}_x}{\hbar}, \hat{L}\right] = \left[\alpha\sigma_z\frac{\hat{p}_x}{\hbar}, e^{i\frac{2m\alpha}{\hbar^2}x}\ \frac{1}{2}\left(\sigma_x - i\sigma_y\right)\right] \tag{5.13}$$

$$= \frac{\alpha}{\hbar}\sigma_z\left[\hat{p}_x, e^{i\frac{2m\alpha}{\hbar^2}x}\ \frac{1}{2}\left(\sigma_x - i\sigma_y\right)\right] + \left[\frac{\alpha}{\hbar}\sigma_z, e^{i\frac{2m\alpha}{\hbar^2}x}\ \frac{1}{2}\left(\sigma_x - i\sigma_y\right)\right]\hat{p}_x$$

$$= \frac{\alpha}{\hbar}\sigma_z\left[\hat{p}_x, \hat{L}\right] + \frac{\alpha}{\hbar}\left[\sigma_z, e^{i\frac{2m\alpha}{\hbar^2}x}\ \frac{1}{2}\left(\sigma_x - i\sigma_y\right)\right]\hat{p}_x$$

which gives:

$$\frac{\alpha}{\hbar}\sigma_z\left(2m\frac{\alpha}{\hbar}\hat{L}\right) + \frac{\alpha}{\hbar}\left\{\left[\sigma_z, e^{i\frac{2m\alpha}{\hbar^2}x}\ \frac{1}{2}\left(\sigma_x - i\sigma_y\right)\right] + e^{i\frac{2m\alpha}{\hbar^2}x}\left[\sigma_z, \frac{1}{2}\left(\sigma_x - i\sigma_y\right)\right]\right\}\hat{p}_x \tag{5.14}$$

$\left[\sigma_z, e^{i\frac{2m\alpha}{\hbar^2}x}\ \frac{1}{2}\left(\sigma_x - i\sigma_y\right)\right] = 0$ and the third term in the above equation can easily be evaluated by obtaining the commutation relations of the Pauli spin matrices:

$$[\sigma_z, \sigma_x] = \sigma_z\sigma_x - \sigma_x\sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} - \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = 2i\sigma_y$$

In a similar manner,

$$\left[\sigma_z, \sigma_y\right] = -2i\sigma_x$$

Then:

$$\left[\sigma_z, \frac{1}{2}\left(\sigma_x - i\sigma_y\right)\right] = \frac{1}{2}\left(2i\sigma_y - i(-2i)\sigma_x\right)$$

$$= -\sigma_x + i\sigma_y = -(\sigma_x - i\sigma_y) \tag{5.15}$$

and (5.14) becomes:

$$\left[\alpha\sigma_z\frac{\hat{p}_x}{\hbar},\hat{L}\right]=\frac{2m\alpha^2}{\hbar^2}\sigma_z\hat{L}-\frac{\alpha}{\hbar}e^{i\frac{2m\alpha}{\hbar^2}x}\left(\sigma_x-i\sigma_y\right)\hat{p}_x$$

In the expression above the second term $\frac{\alpha}{\hbar}e^{i\frac{2m\alpha}{\hbar^2}x}\left(\sigma_x-i\sigma_y\right)=2\hat{L}$. As a result (5.13) is found to be:

$$\left[\alpha\sigma_z\frac{\hat{p}_x}{\hbar},\hat{L}\right]=\frac{2m\alpha^2}{\hbar^2}\sigma_z\hat{L}-2\frac{\alpha}{\hbar}\hat{L}\hat{p}_x \tag{5.16}$$

Finally the commutaion of the Hamiltonian with $\hat{L}$ operator can be obtained by adding (5.12) to (5.16):

$$\left[\hat{H},\hat{L}\right]=\frac{\alpha}{\hbar}\left(\hat{p}_x\hat{L}+\hat{L}\hat{p}_x\right)+\frac{2m\alpha^2}{\hbar^2}\sigma_z\hat{L}-2\frac{\alpha}{\hbar}\hat{L}\hat{p}_x$$

$$=\frac{\alpha}{\hbar}\left(\hat{p}_x\hat{L}-\hat{L}\hat{p}_x\right)+2m\frac{\alpha^2}{\hbar^2}\sigma_z\hat{L}$$

$$=\frac{\alpha}{\hbar}\left[\hat{p}_x,\hat{L}\right]-2m\frac{\alpha^2}{\hbar^2}\hat{L}$$

$$=\frac{\alpha}{\hbar}2m\frac{\alpha}{\hbar}\hat{L}-2m\frac{\alpha^2}{\hbar^2}\hat{L}=0 \tag{5.17}$$

We have found that, $\left[\hat{H},\hat{L}\right]=0$, $\hat{H}$ and $\hat{L}$ are commutative, meaning that the introduced operator $\hat{L}$ can be an appropriate operator to define our systems. In the following steps, we apply the *Gauge* transformation in employing ansatz. Gauge theories are a class of theories based on the idea that symmetry transformations can be performed locally as well as globally. The operator $\hat{L}$ can should transform the wave function ψ to a new function $\acute{\psi}$:

$$\hat{L}\psi=\acute{\psi}$$

The Schrodinger equation is $\hat{H}\psi=E\psi$. For the new function it becomes:

$$\hat{H}\acute{\psi}=\hat{H}\left(\hat{L}\psi\right)=\hat{L}\hat{H}\psi=\hat{L}E\psi=E\hat{L}\psi=E\acute{\psi}$$

Since $\hat{H}\acute{\psi}=E\acute{\psi}$, the same Hamiltonian describes the system for the new wave function as well. Applying the operator on wave function ψ for up (down) spin will give rise to $\acute{\psi}$ for down (up) spin:

$$\psi_\uparrow = \begin{cases} e^{ik_\uparrow x}\begin{pmatrix}1\\0\end{pmatrix} + r_\uparrow e^{-ik_\downarrow x}\begin{pmatrix}1\\0\end{pmatrix} & \text{for } x < 0 \\ t_\uparrow e^{ik_\uparrow x}\begin{pmatrix}1\\0\end{pmatrix} & \text{for } x > 0 \end{cases}$$

$$\hat{L}\psi_\uparrow = \acute{\psi}_\downarrow = \begin{cases} e^{ik_\downarrow x}\begin{pmatrix}0\\1\end{pmatrix} + r_\uparrow e^{-ik_\uparrow x}\begin{pmatrix}0\\1\end{pmatrix} & \text{for } x < 0 \\ t_\uparrow e^{ik_\downarrow x}\begin{pmatrix}0\\1\end{pmatrix} & \text{for } x > 0 \end{cases}$$

similarly,

$$\psi_\downarrow = \begin{cases} e^{ik_\downarrow x}\begin{pmatrix}0\\1\end{pmatrix} + r_\downarrow e^{-ik_\uparrow x}\begin{pmatrix}0\\1\end{pmatrix} & \text{for } x < 0 \\ t_\downarrow e^{ik_\downarrow x}\begin{pmatrix}0\\1\end{pmatrix} & \text{for } x > 0 \end{cases}$$

$$\hat{L}\psi_\downarrow = \acute{\psi}_\uparrow = \begin{cases} e^{ik_\uparrow x}\begin{pmatrix}1\\0\end{pmatrix} + r_\downarrow e^{-ik_\downarrow x}\begin{pmatrix}1\\0\end{pmatrix} & \text{for } x < 0 \\ t_\downarrow e^{ik_\uparrow x}\begin{pmatrix}1\\0\end{pmatrix} & \text{for } x > 0 \end{cases}$$

From the above expressions we see that for any arbitrary impurity potential polarized refelection and transmission coefficients for up and down spin become equal to each other. Therefore their ratio always go to unity, indicating that ansatz is really valid and our generalization is true. Thus spin precession angle will be inert to the impurity potentials located in the system.

CHAPTER 6

CONCLUSIONS

The two dimensional electron gas between two ferromagnetic contacts known as spin field effect transistor is considered theoretically in the presence of impurities. Spin precession due to Rashba spin orbit coupling is studied using the model of Datta and Das in the one dimensional channel. We assume that the channel is narrow so that there is only a single channel in the system. In this way we are able to neglect the subband mixture. We have taken a few systems for which we examined the effect of one impurity, two impurities, potential well together with the external magnetic field parallel and perpendicular to the Rashba field. We mainly concentrated on the ratio of polarized transmission coefficients for up and down spin since one of the spin precession angle (θ) depends on this ratio. In the field of impurities equality of this ratio means that the θ will not be influenced by potential. The other azimuthal spin precession angle φ gives rise to only small fluctuations in the variation of the polarized transmission coefficients or current.

For all systems including an arbitrary potential we have found that transmission coefficients are equal to each other for up and down spin. Hence we get the result that the spin precession angle θ giving the deflection from the z axis is obtained to be independent of the any type of impurity potential. In order to generalize this result we have made an ansatz. We have taken an arbitrary impurity potential and defined an operator compatible with the ansatz. This operator relates the wave functions for up and down spin and maps them onto each other. The introduced operator can transform the wave function for up spin to that for down and vice versa. As a result of the proposed ansatz we have found that in general spin precession angle is independent of the any type of impurity potential. Therefore, it is shown that the conductance modulation for purely one dimensional case is not affected by an arbitrary potential but only with small fluctuations due to azimuthal angle. As the real systems always include defects or impurities, the results we get will have a significant impact on the applications of spin field effect transistor. However note that for such applications the channel width must be sufficiently narrow.

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