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The Graduate School of Sciences and Engineering

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UNDULATOR OPTIMIZATION FOR VUV AND X-RAY FREE ELECRON LASERS

by

D. Gonca MISIR

December 2014

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D. Gonca MISIR

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APPROVAL PAGE

This is to certify that I have read this thesis written by D. Gonca MISIR and that in my opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science in Physics.

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ABSTRACT

Conventional laser mechanisms, either atomic, molecular, solid state, chemical or semiconductor lasers, all have systematic problems for lasing in the shorter wavelengths. Since the main problems being the achievement of population inversion which becomes increasingly difficult and the construction of the laser cavity becoming almost impossible for high frequency laser light, we need another mechanism to build laser-like light sources in the Vacuum Ultraviolet and X-Ray region of the spectrum. Obtaining laser light in this range of the electromagnetic spectrum is important for applications in Science, Engineering and possibly Military applications.

In this thesis we review the working principle of Free Electron Lasers (FEL) and we investigate the relevant parameters for the necessary energy range of the accelerated electrons, the undulator parameters etc. We apply the principles applied to the FEL's in the Infrared Region of the spectrum that are shown to work to shorter wavelengths of the spectrum.

Keywords: Lasers, Free electron lasers, linear accelerators, SASE.

VUV VE X-IŞINI SERBEST ELEKTRON LAZERLERİ İÇİN SALINDIRICI OPTİMİZASYONU

D. Gonca MISIR

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ÖZ

Geleneksel Lazer mekanizmalarının hepsinin, atomic, moleküler, katı hal, kimyasal ya da yarı iletken lazeri de olsa, kısa dalgaboylarında lazer ışıması yapma konusunda sistematik problemlere sahiptir. Lazer ışıması elde etmede yüksek frekanslara doğru gidildikçe temel problemler nüfus ters çevriminin elde edilmesinin gittikçe zorlaşması ve yüksek frekanslara uygun lazer kavitesi kurulumunun imkansızlaşması nedeniyle, spektrumun Vakum-Morötesi ve X-ışını bölgesinde lazer-benzeri ışık elde etmek için başka bir mekanizmaya ihtiyacımız var. Spektrumun bu bölgesinde lazer ışını elde etmek bilim, mühendislik ve muhtemel askeri uygulamalar açısından önem arz etmektedir.

Bu tezde Serbest Elektron Lazerlerinin (SEL) çalışma prensibin gözden geçirdik ve hızlandırılmış elektronların gerekli enerji seviyelerini ve undulator parametreleri gibi değişkenleri inceledik. Spektrumun kızıl-ötesi bölgesinde çalışan SEL'lerin ilkelerini spektrumun daha kısa dalga boylarına uyguladık.

Anahtar Kelimeler: Lazerler, Serbest elektron lazerleri, Lineer hızlandırıcılar, SASE.

To my parents

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LIST OF SYMBOLS AND ABBREVIATIONS

SYMBOL/ABBREVIATION

λ	Wavelength of the emitted radiation from the laser
λ_u	Undulator period
B	Magnetic Field Strength
β	Velocity normalized to the speed of light (unitless parameter)
γ	Relativistic Factor (unitless parameter)
$\Delta\nu$	Frequency Bandwidth
τ_c	Coherence length of light
K	Undulator parameter (unitless)
$\mathrm{m_0}$	Rest mass of the electron
m	Relativistic mass of an electron
E	Relativistic Energy of the electron
c	Speed of light
μ_0	Magnetic susceptibility of vacuum
ϵ_0	Dielectric constant of vacuum
$\vec{\mathrm{M}}$	Relativistic momentum of a particle
$\mathrm{v_p}$	The phase velocity of the electromagnetic wave
Ψ	Phase of the electron in an electromagnetic field
e	Electron charge
λ	Wave length

h	Planck constant
T	Temperature

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

For coherent radiation of high intensity with the advantage of tunability, Free Electron Lasers (FELs) are one of the major alternatives, since its first operation in [1] The first operation of a similar principle was in 1957 at a wavelength of 5mm, therefore a Free Electron Maser (FEM) was developed before a FEL was realized [2]. The shortest wavelengths for a FEL are obtained at DESY at Hamburg at FLASH facility. [3,4]. Worldwide, a large number of FEL user designed to produce vacuum-ultraviolet (VUV) and x-ray radiation have been proposed and some are presently under construction, such as the LINAC Coherent Light Source LCLS, using a part of the normal-conducing SLAC of linear accelerator (or “LINAC”) at the Stanford University in USA and the European X-Ray Laser FEL at DESY based on a Superconducting LINAC. A third generation of synchrotron light sources emerged in the early 1990’s with higher brilliance. A Free Electron Laser is essentially composed of three parts: an electron accelerator, a magnetic undulator structure, and an optical resonator (see Figure 1.1) The electrons are forced by the magnetic field on an oscillating trajectory, thus emitting synchrotron radiation. In the electron frame reference the process can also be seen as a scattering between the electron beam and the virtual photons of the undulator. If an external field is present, the radiation is emitted in phase with this external field. The interaction between the laser field, the static magnetic field of the undulator and the electron beam has as a final effect the spatial bunching of the electrons on the scale of the radiation wavelength, and the transfer of energy from the electron beam to the laser field. The undulator can be considered as the equivalent of the "active medium" of a conventional laser system, while the electron beam is the equivalent of the "pumping system".

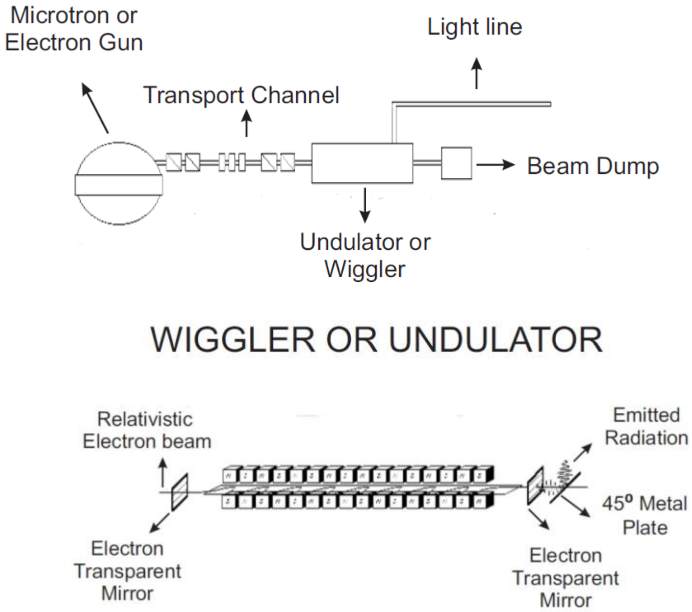


Figure 1.1 ENEA Compact Free Electron Laser Layout.

Free Electron Lasers can, in principle, cover most of the electromagnetic spectrum from the microwave region to the vacuum ultraviolet: the wavelength of the emitted radiation depends on the electron energy and on the magnitude and periodicity of the undulator magnet field according to the following relation:

$$\lambda = \frac{\lambda_u}{2\gamma^2}(1 + K^2) \tag{1.1}$$

where *a* is the radius of core, *NA* is the numerical aperture and λ is the cutoff wavelength. where λ_u is the undulator period, γ is the relativistic factor of the electrons and K is the so-called undulator parameter, proportional to the magnetic field inside the undulator. In the following chapters we will first discuss synchrotron radiation, which is a broadband radiation, and afterwards we will discuss how by means of an undulator, electrons emitting synchrotron radiation be utilized to emit coherent laser radiation through interaction with the periodic magnetic field.

CHAPTER 2

FUNDAMENTAL CONCEPTS

2.1 INTRODUCTION

In this Chapter we will discuss the fundamental concepts regarding the mechanisms of conventional Laser Radiation and what makes a laser light special. First we will discuss radiation due to heat, Black-body radiation. And then we discuss what the differences for laser light from a conventional light source are. We tabulate the X-ray laser projects worldwide. We define the term Exittance for conventional blackbody radiation. Through a short review of the Fourier series we derive the definition of coherence time and length of a laser light. We review the derivation of the general formulate relevant to radiation from relativistic particles under certain circumstances of acceleration. We derive the expressions for the total power radiated for Bremsstrahlung and Synchrotron Radiation for a comparison.

2.1.1. Black Body Radiation Spectrum

It is well known that, the first victory of the idea of a quantum of energy is to predict correctly the spectral distribution of radiation emitted form hot objects. Energy density per unit volume for given frequency and tempature, otherwise known as the frequency spectrum of an object at temperature T is units of Kelvin is given as follows.

$$\rho(\nu,\tau) = \frac{8.\pi.\nu^2}{c^3}\frac{h.\nu}{e^{\frac{h.\nu}{k_B T}} - 1} \tag{2.1}$$

And is known as Planck's radiation formula, where ν denotes the frequency and Temperature T otherwise known as the frequency spectrum of an object at Temperature T

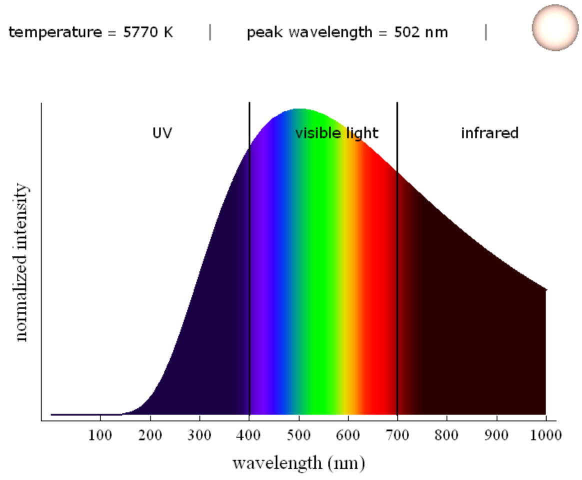


Figure 2.1 Radiation due to heat, Blackbody radiation spectrum for T=5770 K, which is the surface temperature of the Sun [5].

$$\frac{\partial}{\partial \nu}\rho(\nu,\tau) = 0 \qquad (2.2)$$

Find the maximum ν (ν_{max}), The peak frequency is

$$\nu_{peak} = \frac{c}{\lambda_{peak}} \qquad (2.3)$$

which leads to a peak wavelength of the spectral distribution of radiated energy

$$\lambda_{peak} = \frac{b}{\tau} \qquad (2.4)$$

b is Wien's constant and T is temparature in units of Kelvin

$$b = 2.8977685(51).10^{3}\text{meter Kelvin} \qquad (2.5)$$

to find the total power radiated for Blackbody radiation, integrate over all possible frequencies

$$\int_{\nu=0}^{\nu=-\infty} \rho(\nu.\mathrm{T})d\nu = M(\mathrm{T}) = \sigma\mathrm{T}^4 \tag{2.6}$$

where σ is is Stefan – Boltzman constant.

$$\sigma = 5.67.10^{-8}\frac{\mathrm{J}}{\mathrm{m}^2\ \mathrm{s}\ \mathrm{K}^4} \tag{2.7}$$

The total power radiated may be referred to as the Exittance

$$\mathrm{M(T)} = \mathrm{Exittance} \equiv \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right] \tag{2.8}$$

It is the energy radiated from the surface per second per m^2

The energy radiated by an object at tempature T in one second from $1m^2$of the surface in unitsof joules.

$$\sigma\mathrm{T}^4 = \mathrm{M(T)} = \mathrm{Exittance} \tag{2.9}$$

And the bandwidth is large. If an object radiates due essentially to heat, like your light bulb, the total power increases as the fourth power of the temperature and the spectrum of the radiation is very broad.

2.1.2. Free Electron Laser Centers Worldwide

The following is a list of the free electron laser research centers worldwide for ongoing or completed projects. As one can see, there are many facilities around the world, some are already completed, and some others are being built right now.

Table 2.1 Free Electron Laser Centers Worldwide (completed projects). [6]

CENTER	LOCATION
SACLA x-ray FEL- Riken	Japan
SLAC x – ray FEL LCLS	California (U.S.)
FLASH VUV/ X ray FEL – DESY	Germany
FERMI FEL at Elettra	Trieste
Source Development Laboratory, NSLS, Brookhaven	New York
Duke University Free Electron Laser Laboratory	U.S.
Photon Storage Ring - Ritsumeikan University	Japan
IFEL	Osaka, Japan
Vanderbilt University Free Electron Laser Center	U.S.
FELIX – FOM	Rijnhuizen, Netherlands
CLIO – LCP	Orsay, France
ENEA Compact FEL	Frascati, Italy
FEL-SUT – Science University of Tokyo	Japan
IR-FEL – S-DALINAC	Darmstadt, Germany
Beijing Free Electron Laser	China
FELBE FEL – FZR	Germany
FLARE FEL – HFML Radboud University Nijmegen	Netherlands
The UCSB Free Electron Lasers	U.S.
Tel Aviv University FEL	Israel

Table 2.1 Free Electron Laser Centers Worldwide (completed projects). [6]

CENTER	**LOCATION**
European X-ray FEL, DESY	Germany
Institute for Plasma Research	India
BESSY FEL	Germany
LEUTL APS, Argonne National Lab	U.S.
Center for Advanced Tecnology	India
University of Maryland - MIRFEL	U.S.
University of Hawaii	U.S.
Brookhaven – Accelerator Test Facility FELs	U.S.
UCLA Particle Beam Physics Lab	U.S.
Osaka University – ISIR	Japan
University of Twente Cerenkov FEL	Netherlands

Table 2.2 Free Electron Laser Centers Under Development Worldwide. [6]

CENTER	**LOCATION**
ARC-EN-CIEL	France
WIFEL University of Wisconsin, Madison	U.S.
National High Magnetic Field Laboratory	U.S.
SPARC Project – INFN	Italy
Daresbury 4GLS	U.K.
MIT Bates Lab X- ray FEL	U.S.

Table 2.3. Free Elecron Laser Proposed Centers Worlwide. [6]

CENTER	**LOCATION**
ARC-EN-CIEL	France
WIFEL University of Wisconsin, Madison	U.S.
National High Magnetic Field Laboratory	U.S.
SPARC Project – INFN	Italy
Daresbury 4GLS	U.K.
MIT Bates Lab X- ray FEL	U.S.

Table 2.4. Running X-FEL Projects. [6]

CENTER	**FEL**	**Wavelegth (Å)**
DESY	TESLA X-FEL	0.85 – 64
SLAC	LCLS	1.5-15
MIT	X-Ray FEL	3-1000
FERMI	ELETTRA	12-15
BESSY	SASE-FEL	12-600
INFN Roma	SPARX	13-135
Spring - 8	SCSS	36
Daresbury	4GLS	124
ANL	LEUTL	3850

Table 2.5. X-FELs Parameters. [6]

Parameters	**SCSS X-FEL**	**TESLA X-FEL**	**LCLS X-FEL**	**CLIC X-FEL**
Beam energy	1 GeV	10-20 GeV	14.3 GeV	15GeV
Energy dissipation	% 0.02	2.5 MeV	%0.006	150.10^{-4}
Packagers load	1 nC	1 nC	-	-
Normalized emittance	2πmm.mrd	1.4mm . mrd	1.5mm . mrad	0.6/0.01µm
Packagers length	0.15mm	1.4mm . mrad	1.5mm.mrad	35µm
Peak current	2kA	-	3.4kA	2.7kA

2.1.3. Fourier Series

In this section, we will investigate the effect of finite coherence time of the source on the spectral distribution of a monochromatic source. We first review Fourier Series briefly

He was Joseph Fourier (1768-1830) to claim that periodic signals could be written as a sum of sine and cosine functions, which may be infinite in number.

Fourier invented the series for the purpose of solving heat equations in a metal plate in his treatise; “Treatise on the propagation of heat in solid bodies” in 1822, and his work got preliminary investigation by Leonhard Euler, Jean le Rond d’Alambert and Daniel Bernoulli. Early ideas of decomposing a periodic function into a sum of simple oscillating functions date back to the 3rd century B.C., when ancient astronomers proposed an empirical model of planetary motions.. Also prior to Fourier’s work, heat equation which is a partial differential equation had no solution. With Fourier’s work, his simple solutions are sometimes referred to as eigen-solutions these days. Fourier’s idea was to model a complicated heat source as a superposition or linear combination of sines and cosines and is called the Fourier series.

From a modern point of view, Fourier’s results are somewhat informal, due to the lack of a precise notion of functions and integrals in the early nineteenth century. Later on, Dirichlet and Riemann expressed Fourier’s results with greater precision and formality. And today, the method is well established.

Although the original motivation was to solve the heat equation, it later became obvious that the same techniques could be applied to a wider array of mathematical physics problems. Especially problems involving linear differential equations with constant coefficients, for which the eigen-solutions are sinusoids. The Fourier series has many such applications in electrical engineering, vibration analysis, acoustics, optics, signal processing, image processing, quantum mechanics, econometrics, thin-walled shell theory etc.

According to Fourier, one can write a periodic function in terms of an infinite series of sines and cosines.

$$f(t) \cong \sum_{n=0}^{\infty} A_n \sin(\omega_n t + \phi_n) + \sum_{n=0}^{\infty} B_n \cos(\omega_n t + \gamma_n) \qquad (2.10)$$

The frequency spectrum of a certain function f(t) may be found by means of the transform.

$$\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)e^{-i\omega t}dt = F(\omega) \qquad (2.11)$$

Now we consider a case where we have a sinusoidal wave of a certain angular frequency of ω_0. A sine wave in nature, cannot have an infinite time extend, in other words it has a certain time span. Let us consider the case where this sine wave looses its phase information pretty much periodically with a period of time T. One might think that the frequency spectrum of this sine wave will have only one frequency ν_0, but due to the finiteness of the time span of the periodic signal, although the central frequency is ν_0, we might expect a certain broadening of the spectrum.

Our aim, of the source is to relate this to the coherence time T_c of a laser as well the full width half maximum (FWHM) of the spectral energy distribution for light source, possibly a laser.

$$f(t) = \sin(\omega_0 t) \qquad (2.12)$$

$$\omega_0 = \frac{2\pi}{T_0} \Rightarrow \qquad (2.13)$$

$$2\pi\nu_0 = \omega_0 \Rightarrow \qquad (2.13)$$

$$f(t) \Rightarrow F(\omega) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} e^{i\omega_0} e^{-i\omega t} \qquad (2.14)$$

Spectrum is S(ω).

$$S(\omega) = F(\omega)F(\omega)^{*} \qquad (2.15)$$

$$F(\omega) = \frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{i(\omega_0-\omega)t}\,dt \tag{2.16}$$

note that

$$\int e^{\alpha t}dt = \frac{1}{\alpha}e^{\alpha t} + c \tag{2.17}$$

and also that

$$e^{-i\theta} = \cos(\theta) + i\sin(\theta) \tag{2.18}$$

$$\alpha = i(\omega_0 - \omega), \tag{2.19}$$

Let us consider a particular case where we have the period T assuming a certain value τ

$$\frac{1}{\tau}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{\alpha t}\,dt = \frac{1}{\tau}\frac{1}{\alpha}e^{\alpha t}\Big|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \tag{2.20}$$

Note

$$e^{iA} - e^{-iA} = 2i\sin(A) \tag{2.21}$$

Therefore

$$e^{i(\omega_0-\omega)\frac{\tau}{2}} - e^{-i(\omega_0-\omega)\frac{\tau}{2}} = 2i\sin((\omega_0-\omega)\tfrac{\tau}{2}), \tag{2.22}$$

$$F(\omega) = \frac{1}{\tau}\frac{2i\sin((\omega_0-\omega)\frac{\tau}{2})}{i(\omega_0-\omega)}, \tag{2.23}$$

$$F(\omega) = \frac{\sin\left((\omega_0-\omega)\frac{\tau}{2}\right)}{\frac{\tau(\omega_0-\omega)}{2}}, \tag{2.24}$$

$$S(\omega) = \frac{\sin\left((\omega_0 - \omega)\frac{\tau}{2}\right)\sin\left((\omega_0 - \omega)\frac{\tau}{2}\right)}{\frac{\tau(\omega_0 - \omega)}{2}\frac{\tau(\omega_0 - \omega)}{2}} \quad , \tag{2.26}$$

if

$$\frac{\tau(\omega_0 - \omega)}{2} = \gamma \tag{2.27}$$

$$S(\omega) = \frac{\sin^2(\gamma)}{\gamma^2} \tag{2.28}$$

$$S(\omega) = \mathrm{sinc}^2(\gamma) \tag{2.29}$$

$$S(\omega) = \frac{\sin^2((\omega_0 - \omega)\frac{\tau}{2})}{((\omega_0 - \omega)\frac{\tau}{2})^2} \tag{2.30}$$

Reference [8]

The symbol ω_0 can be interpreted as the angular frequency such that its relation to the frequency can be written as

$$\omega_0 = 2\pi\nu_0 \tag{2.31}$$

$$\nu_0 = \frac{\omega_0}{2\pi} \tag{2.32}$$

which implies that the angular frequency span is

$$\Delta\omega = 2\pi \cdot \Delta\nu \tag{2.33}$$

Or from this we can define

$$\Delta\omega = \frac{2\pi}{\tau_c} \tag{2.34}$$

where τ_c is the coherence time

$$\Delta\nu = \frac{\Delta\omega}{2\pi} \Rightarrow \tag{2.35}$$

$$\Delta\nu = 2\pi \cdot \Delta\nu \tag{2.36}$$

the coherence time τ_c and the bandwidth $\Delta\nu$ relationship can then be obtained as

$$\Delta\nu = \frac{1}{\tau_c}. \tag{2.37}$$

So now we know that frequency bandwidth is inversely proportional to the coherence time.

Blackbody radiation has a very broad spectrum, which means that its coherence time will be very short. So, temporal coherence of broad daylight is extremely short.

On the contrary, for a laser light, the main defining property is its coherence, so for a laser-like light we expect the frequency bandwidth to be very small.

2.2 TOOLS AND RESULT OF SPECİAL RELATIVITY

2.2.1. Basic Formulae of Special Relativity

In this section we make a short review of the formulation and concepts we use in the thesis.

Since speed of light is the upper limit of possible speeds for a material particle, one can define the unitless parameter of speed of a particle β, the velocity in units of the speed of light is defined as

$$\beta = \frac{v}{c} \tag{2.38}$$

One can easily notice that β assumes values between zero (for a stand-still of a particle) and almost one (but not exactly one because $\beta = 1$ is reserved for light only, or more generally, electromagnetic waves or photons travelling in vacuum).

$$0 \leq \beta < 1 \tag{2.39}$$

We may also remember that the kinetic energy of a mass is also a function of its velocity in a special relativity

$$E = mc^2 \tag{2.40}$$

In other words

$$E = \gamma\, m_0\, c^2 \tag{2.41}$$

Note that, the mass appears actually as a function of its velocity as well

$$m = \gamma\, m_0 \tag{2.42}$$

where m_0, is the rest mass of the moving particle, which in our case is an electron and

$$m_0 = m_e = 9.11 \cdot 10^{-31} \mathrm{kg} \tag{2.43}$$

and also that γ is the relativistic factor defined as

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2.44}$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \tag{2.45}$$

By this token, for a particle at rest $\beta = 0$ means $\gamma = 1$ and as $\beta \to 1$, $\gamma \to \infty$. So the energy of a particle may assume infinitely large values, namely

$$1 \leq \gamma < \infty \tag{2.46}$$

$$m_0 c^2 \leq E < \infty \tag{2.47}$$

For an electron, $m_e c^2$ is calculated to be $0.511\ \mathrm{MeV}$ where $1\ \mathrm{MeV} = 10^6 \mathrm{eV}$ and $1\ \mathrm{eV} = 1.6\ \times 10^{-19}$joules. Although this is a very small energy in the macroscopic scale, it may mean a lot for an electron. Let us take a look at how the velocity changes the relativistic factor and the energy of an electron

$$E = \gamma\, m_0 c^2 \tag{2.48}$$

$$E = \gamma \cdot 0.511\ \mathrm{MeV} \tag{2.49}$$

$$E = \frac{0.511 \cdot 10^6}{\sqrt{1-\beta^2}}\ \mathrm{eV} \tag{2.50}$$

$$E = \frac{0.511 \cdot 1.6 \times 10^{-13}}{\sqrt{1-\beta^2}}\ \mathrm{joule} \tag{2.51}$$

As β gets closer to its upper value of 1, which means $v \rightarrow c$, we coin the term, ultra-relativistic particle for the moving entity. For an ultra-relativistic electron of $\vartheta = 0.9c,\ \beta = 0.9$ and

$$\gamma = \frac{1}{\sqrt{1-0.81}} = 2.294 \tag{2.52}$$

which means, for an electron

$$E = 2.294 \cdot 0.511\mathrm{MeV} \tag{2.53}$$

$$E = 1.1723\mathrm{MeV} \tag{2.54}$$

However, the energy of the particle increases dramatically as the speed of the particle gets closer to the speed of the light. Take for example the case $\beta = 0.99$ where the speed of the electron is 99% of the speed of light, then

$$\gamma = \frac{1}{\sqrt{1-0.99^2}} = \frac{1}{\sqrt{0.0199}} = 7.088 \tag{2.55}$$

Then, for an electron

$$E = \gamma m_0 c^2 = 7.088 \cdot 0.511\text{MeV} \tag{2.56}$$

$$E = 3.622 \text{ MeV}. \tag{2.57}$$

Take the case where we go even closer to the speed of light, say $\beta = 0.9999$

Then $\gamma = 70.7125$ and $E = 36.134\text{MeV}$.

Also $\beta = 0.999999$, which is close to the speed of light by one millionth of it implies

$$\gamma = 707.1069 \tag{2.58}$$

$$E = 361.331 \text{ MeV} \tag{2.59}$$

$$E = 0.361331 \text{ GeV} \tag{2.60}$$

For general outline of E as a function of β, we may plot a graph to see how the energy of a particle is dependent on the velocity and hence, the relativistic factor γ through β

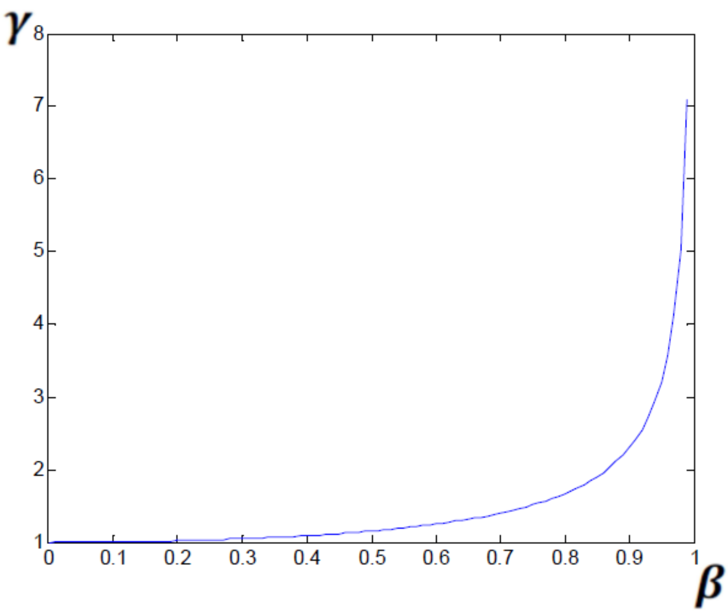


Figure 2.2 Relativistic factor γ, as a function of β, the velocity of the particle in units of the velocity of light c.

The energy of a relativistic electron as a function of β is also plotted in the Figure below

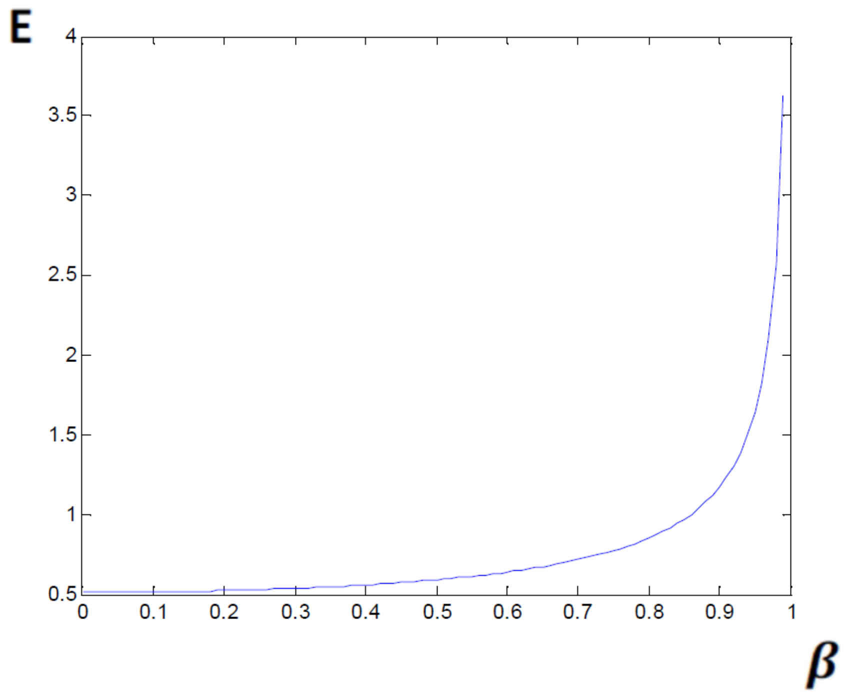


Figure 2.3 The energy of an electron as a function of the velocity factor β.

One can notice that as we reach closer to the speed of light with the electron, the relativistic factor γ and the energy of the particle E gets larger and larger.

Let us also note the relativistic momentum of a moving particle to be

$$p = \gamma m_0 v \tag{2.61}$$

For non-relativistic particles γ is usually very close to 1 and we say $p = m_0 v$, but for ultrarelativistic particle, it means that the momentum of the moving particle also gets to tremendously large values as γ gets larger since

$$p = \gamma\beta\, m_0 c \tag{2.62}$$

For an electron the momentum is

$$p = \gamma\beta\, m_e\, c \tag{2.63}$$

It also means that to charge this momentum now takes more energy, since the particle is effectively much more massive.

2.2.2. The relevance to Physics at CERN

The most recent experiments at CERN (European Center for Nuclear Research) suggest of protons to be several TeV (1TeV=10^{12}eV). Since those particles are protons their rest mass energies are noticeably larger, so having represented a proton by the symbol p^+

$$E_{proton} = m_{p^+} \cdot c^2 = 859\text{MeV} \tag{2.64}$$

For a proton of an energy $E = 7\ \text{TeV}$

The required relativistic factor for this one is

$$\gamma = \frac{E}{E_{proton-rest}} \tag{2.65}$$

$$\gamma = \frac{7 \cdot 10^{12}\text{eV}}{859 \times 10^{6}\text{eV}} = 8149 \tag{2.66}$$

For the velocity factor for this case implies

$$\beta = \frac{\sqrt{\gamma^2 - 1}}{\gamma} \tag{2.67}$$

$$\beta = \frac{8149.01416}{8149.010477} = 0.9999999925 \tag{2.68}$$

So the velocity of a proton to have $7\ \text{TeV}$ of energy, it's velocity must be $v = 0.9999999925 * c$, very close to the speed of light. One these energies are reached for the protons at the Large Hadron Collider at CERN, circulating clockwise counterclockwise, a bunch of them will be smashed head on, and the physics of the smaller constituents can be investigated. To accelerate, LHC uses large synchrotrons, and the trajectories of the protons are large circles, as large as 27 km in diameter.

However, for a free electron laser, although we don't need TeV energies, we need the electron beam to have a large quality, which means to have low emittance. We will discuss electron beam quality at the relevant section. For now, we can note that, for free electron lasers operating at shorter wavelengths, one needs much higher beam qualities for electrons, and for that reason we must not use synchrotrons as particle accelerates, but rather, we must use linear accelerators.

2.3 RADİATİON DUE TO BENT RELATİVİSTİC ELECTRON TRAJECTOİES

2.3.1. Concepts for a bent relativistic particle trajectory

We want to go through a discussion of Synchrotron Radiation, where the relativistic electron trajectory is bent, say by a magnetic field, to get a grasp of central frequency radiated by the electron. [9]

For this, we sketch the bent trajectory of the electron, for which we can define a radius of curvature vector $\vec{r}$, and its unit vector $\hat{r}$, and we can define a unit vector $\hat{n}$ form the electron position to the position of the observer, and we can define another unit vector $\hat{x}$ form the center for the supposed radius of curvature to the observer.

In this picture $\vec{\beta}$ is the relatistic velocity.

$\vec{x}$: stands for the position of the observer

$\vec{r}(t')$: denotes the position of the particle at time t'.

In this sense, $\vec{R}(t')$ is the distance between the electron and the observer.

$$\vec{R}(t') = \vec{x} - \vec{r}\,(t') \qquad (2.69)$$

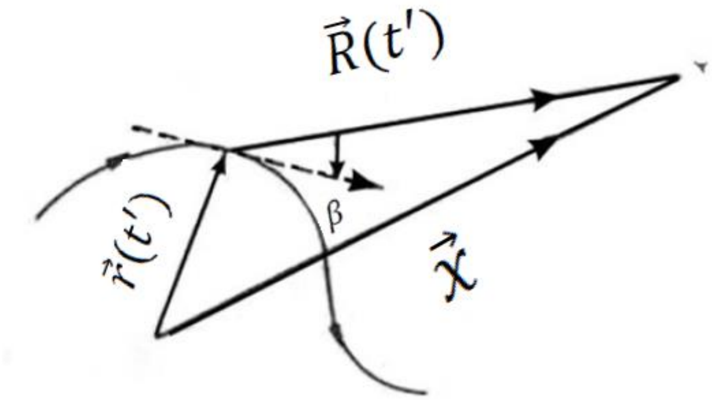


Figure 2.4 The electron trajectory and a stationary observer. [9]

$$\Delta t \cong \left(\frac{dt}{dt'}\right)\Delta t' \tag{2.70}$$

t' is the time the electron emits the radiation and t is the time the observer observes this radiation at a later time. But not only the times of the emitter and observer with respect to radiation are different, but also the time interval scales for the emitter dt' and the observer dt are different, which means that the time intervals will be transformed as well.

Note that the time the radiation emitted t' is different from the time t that the observer observes the same radiation by a certain radiation.

$$t = t' + \frac{\left|\vec{R}(t')\right|}{c} \tag{2.71}$$

$\hat{n} \cdot \vec{\beta}$ where $\vec{\beta}$ is the normalized velocity to the speed of light $\beta = \frac{\vec{v}}{c}$, leads to $\hat{n} \cdot \vec{\beta} = \beta \cos\theta$ where θ, as shown in the figure is the angle between the velocity vector of the electron and the vector from the position of the electron to the observer in the lab.

But this implies that $\frac{dt}{dt'} = 1 - \beta\cos(\theta)$ will be a very small quantity where the electron is ultra-relativistic ($\beta \sim 1$) and where observer is the located at a small angle to the instantaneous velocity vector of the electron

$$\frac{dt}{dt'} = 1 - \vec{n}(t').\vec{\beta}(t') \qquad (2.72)$$

$$\Delta t \cong \left(\frac{dt}{dt'}\right)\Delta t' \qquad (2.73)$$

$$\rightarrow \beta = 1 - \frac{1}{2\gamma^2} \qquad (2.74)$$

If $0 \ll 1$, then cos(θ)$\sim 1 - \frac{\theta^2}{2}$ and

$$\frac{dt}{dt'} = 1 - \beta\cos(\theta) \qquad (2.75)$$

$$\frac{dt}{dt'} = 1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 - \frac{\theta^2}{2}\right) \qquad (2.76)$$

$$\frac{dt}{dt'} = 1 - \left\{1 - \frac{1}{2\gamma^2} - \frac{\theta^2}{2} + \frac{\theta^2}{4\gamma^2}\right\} \qquad (2.77)$$

Since the last term is extremely small in compared to others

$$\frac{dt}{dt'} = \frac{1}{2}\left(\frac{1}{\gamma^2} + \theta^2\right) \qquad (2.78)$$

Taking a look at the arc-length at the figure, we can, by the info that the opening angle for synchrotron radiation is of the order of $\frac{1}{\gamma}$

$$AB = \rho \cdot \frac{2}{\gamma} \qquad (2.79)$$

$$AB = \frac{2\rho}{\gamma} \qquad (2.80)$$

The vertical displacement Δx during this motion is

$$\Delta x = \rho \cdot (1 - \cos(\theta)) \qquad (2.81)$$

by $1 - \cos(\theta) = 1 - \left(1 - \frac{\theta^2}{2}\right) = \frac{\theta^2}{2}$ where $\theta \longrightarrow \frac{1}{\gamma}$ leads to

$$\Delta x = \frac{\rho}{2\gamma^2} \tag{2.82}$$

For the electron the time it takes to transverse the arc AB with velocity $c \cdot \beta$ is

$$\Delta t' = \frac{AB}{\beta c} = \frac{\frac{2\rho}{\gamma}}{\left(1 - \frac{1}{2\gamma^2}\right) \cdot c} \tag{2.83}$$

$$\Delta t' \cong \frac{2\rho}{\gamma c}\left(1 + \frac{1}{2\gamma^2}\right) \tag{2.84}$$

using Taylor expansion again

$$\Delta t \cong \frac{\Delta t'}{\gamma^2} \rightarrow \tag{2.85}$$

$$\Delta t = \frac{2\rho}{\gamma^3 \cdot c} \tag{2.86}$$

So we can have an estimate of the acceleration

$$a = \frac{\Delta x}{(\Delta t)^2} \tag{2.87}$$

$$a \cong \gamma^4 \frac{c^2}{8\rho} \tag{2.88}$$

$$v_{typ} \cong \left(\sim \frac{1}{\Delta t}\right) \tag{2.89}$$

Since the radiation is emitted in a narrow cone of angular aperture it is highly directional especially for ultra-relativistic electron of high γ and has a large bandwidth.

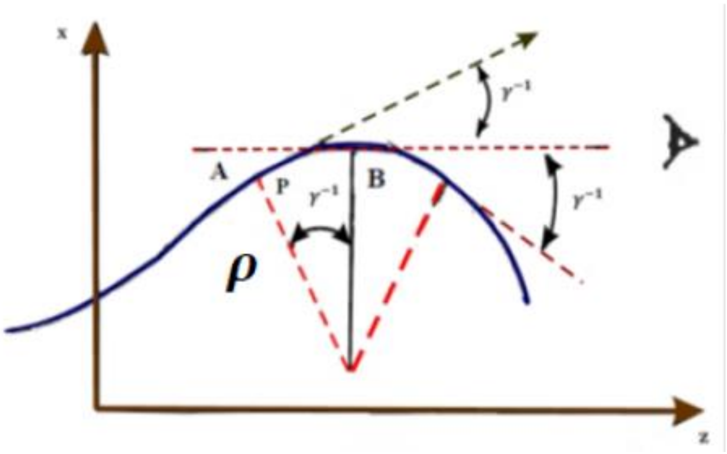


Figure 2.5. Sample electron trajectory, for the case of synchrotron radiation.

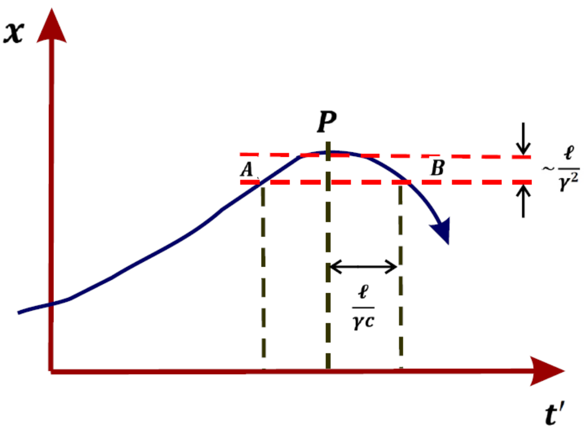


Figure 2.6 Electron trajectory as a function of the electron time t′.

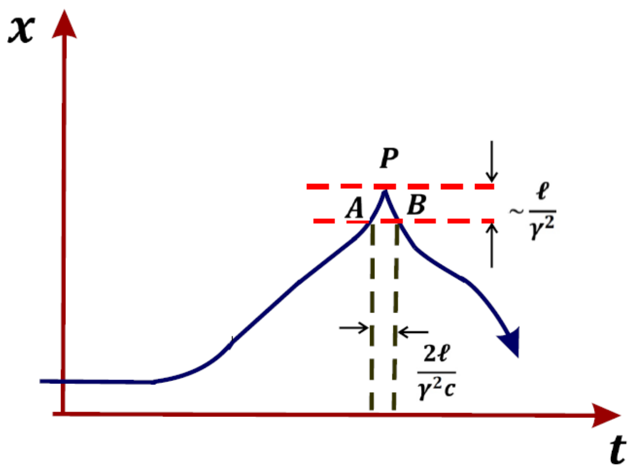


Figure 2.7 Electron trajectory for the observer time t. [9]

2.3.2. Undulator : Sequence of N bending structures

In this section we go through a heuristic derivation of such radiation but this time instead of just one, N bending structures of alternating polarity with a period of λ_u (undulator period) as shown in the Figure 2.8

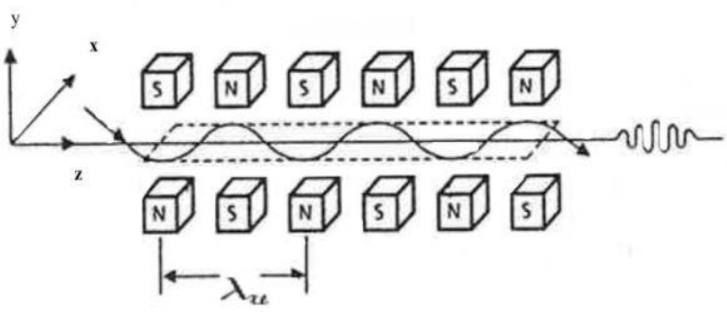


Figure 2.8. Basic components of the Undulator FEL.

λ_1: received by the observer

$$\frac{dt}{dt'} = \frac{1}{2}\left(\theta^2 + \frac{1}{\gamma^2}\right) \rightarrow \tag{2.91}$$

$$\frac{\lambda_1}{\lambda_u} = \frac{1}{2}\frac{1}{\gamma^2}\left(\gamma^2 < \theta^2 > +1\right) \tag{2.92}$$

$< \theta^2 >$: average over one period of trajectory.

$$\lambda_1 = \frac{\lambda_u}{2\gamma^2}\left(1 + \gamma^2 < \theta^2 >\right) \tag{2.93}$$

This is called central emission frequency. The undulator is expected to emit a series of sharp peaks at the harmonics of this fundamental wavelength, which leads to a central angular frequency of

$$\omega_1 = \frac{2\pi c}{\lambda_1} \tag{2.94}$$

with an estimated broadening of the central angular frequency of emission of the order of

$$\Delta\omega \sim \frac{\pi}{\Delta t} \tag{2.95}$$

where Δt is the pulse duration which is related to the time difference of the time of flight of electron and photons throughout the undulator as follows.

$$\Delta t = (1-\beta)\frac{N \cdot \lambda_u}{c} \approx \frac{N \cdot \lambda_u}{2 \cdot \gamma^2 \cdot c} \tag{2.96}$$

So we can calculate the relative broadening of the central frequency of the undulator.

$$\frac{\Delta\omega}{\omega_1} = \frac{\frac{\pi}{\Delta t}}{\frac{2\pi c}{\lambda_1}} = \frac{\lambda_1}{2 \cdot \Delta t \cdot c} \Rightarrow \tag{2.97}$$

$$\frac{\Delta\omega}{\omega_1} = \frac{1}{2} \cdot \frac{2 \cdot \gamma^2 \cdot c}{N \cdot \lambda_u \cdot c}\lambda_1 \tag{2.98}$$

$$\frac{\gamma^2}{N \cdot \lambda_u} \cdot \lambda_1 = \frac{\gamma^2}{N \cdot \lambda_u} \cdot \frac{\lambda_u}{2 \cdot \gamma^2} \cdot [1 + \gamma^2 + < \theta^2 >] \tag{2.99}$$

so

$$\frac{\Delta\omega}{\omega_1} = \frac{\Delta\nu}{\nu_1} = \frac{1}{2N} \tag{2.100}$$

which is also called the homogeneous bandwidth of the undulator. So, as the number of the periods of oscillator in an undulator increases, the sharpness of the frequency spectrum of the frequency spectrum of the central frequency also is expected to increase.

2.4 INTRODUCTION THE ANGULAR DISTRIBUTION OF THE RADIATED POWER FOR ACCELERATED RELATIVISTIC CHARGED PARTICLES

2.4.1. The Angular Distribution of the Radiated Power For the case where $\vec{\mathbf{v}}$ (velocity) and $\vec{\mathbf{a}}$ (acceleration) are parallel

We want to find the angular distribution of the emitted power, in the other words, the power radiated into solid angle dΩ for the case where the instantaneous velocity vector and the instantaneous acceleration vector are collinear. The General Formula is derived in the Electrodynamics book written by Griffiths as [10]

$$\frac{dP}{d\Omega} = \frac{q^2}{16\pi^2\epsilon_0} \cdot \frac{|\hat{n} \times (\vec{u} \times \vec{a})|^2}{(\hat{n} \cdot \vec{u}\,)^5} \tag{2.101}$$

In this section and the next section, we will derive the angular distribution of the power radiated from acceleration (or decelerated electron including the case where the motion is ultra-relativistic.)

where

$$\vec{u} = c\hat{n} - \vec{v} \;\; \Rightarrow \; \vec{u} \times \vec{a} = c\hat{n} \times \vec{a} - \; \vec{v} \times \; \vec{a} \tag{2.102}$$

since for the case where the velocity and the acceleration are collinear.

$$\vec{v} \,/\!/\, \vec{a} \Rightarrow \vec{v} \times \vec{a} = 0 \Rightarrow \vec{u} \times \vec{a} = c\hat{n} \, \times \, \vec{a} \tag{2.103}$$

$$\frac{dP}{d\Omega} = \frac{q^2c^2}{16\pi^2\epsilon_0} \cdot \frac{|\hat{n} \times (\hat{n} \times \vec{a})|^2}{\left(\hat{n} \cdot (c\hat{n} - \vec{v})\right)^5} \tag{2.104}$$

As shown it the Figure, to define the direction where a part of radiation is emitted into . We define a unit vector $\hat{n}$

$$\hat{n} = \sin(\theta)\cos(\phi)\hat{x} + \sin(\theta)\sin(\phi)\hat{y} + \cos(\theta)\hat{z} \tag{2.105}$$

using back-cab rule

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b}) \tag{2.106}$$

$$\hat{n} \times (\hat{n} \times \vec{a}) = \hat{n}(\hat{n} \cdot \vec{a}) - \vec{a}(\hat{n} \cdot \hat{n}) = \hat{n}(\hat{n} \cdot \vec{a}) - \vec{a} \tag{2.107}$$

$$|\hat{n} \times (\hat{n} \times \vec{a})|^2 = (\hat{n}(\hat{n} \cdot \vec{a}) - \vec{a}) \cdot (\hat{n}(\hat{n} \cdot \vec{a}) - \vec{a}) \cdot (\hat{n}(\hat{n} \cdot \vec{a}) - \vec{a}) = (\hat{n} \cdot \vec{a})^2 - 2(\hat{n} \cdot a)^2 + \vec{a} \cdot \vec{a} = a^2 - (\hat{n} \cdot \vec{a})^2 \tag{2.108}$$

$$\frac{dP}{d\Omega} = \frac{q^2c^2}{16\pi^2\epsilon_0} \cdot \frac{a^2 - (\hat{n} \cdot \vec{a})^2}{(c - \hat{n} \cdot \vec{v})^5} \tag{2.108}$$

Since

$$(\hat{n} \cdot (c\hat{n} - \vec{v}))^2 = (c - \hat{n} \cdot \vec{v})^5 \tag{2.109}$$

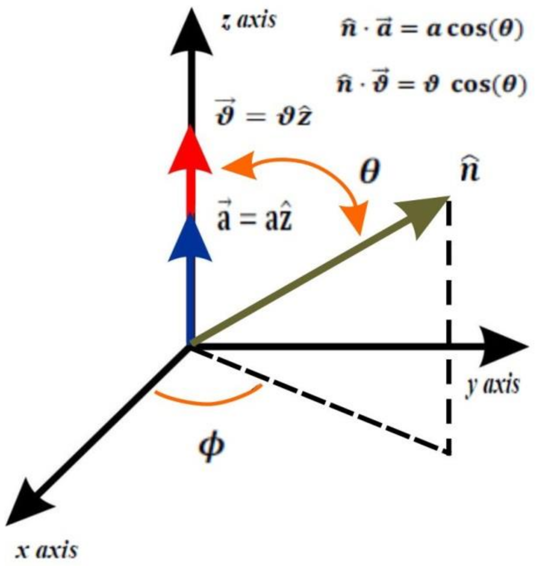


Figure 2.9. The case where the acceleration and the velocity of the electron are parallel.

$$\frac{dP}{d\Omega} = \frac{q^2c^2}{16\pi^2\epsilon_0} \cdot \frac{a^2 - (\hat{n} \cdot \vec{a})^2}{(c - \hat{n} \cdot \vec{v})^5} \tag{2.110}$$

$$\frac{dP}{d\Omega} = \frac{q^2c^2}{16\pi^2\epsilon_0} \cdot \frac{1}{c^5} \cdot \frac{a^2 - (\hat{n} \cdot \vec{a})^2}{\left(1 - \hat{n} \cdot \frac{\vec{v}}{c}\right)^5} \tag{2.111}$$

$$\frac{dP}{d\Omega} = \frac{q^2c^2}{16\pi^2\epsilon_0} \cdot \frac{1}{c^5} \cdot \frac{a^2\left(1-\cos^2(\theta)\right)}{(1-\beta\cos(\theta))^5} \tag{2.112}$$

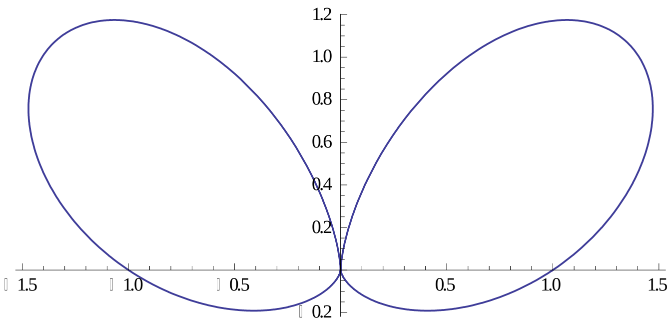
$$\frac{dP}{d\Omega} = \frac{q^2a^2}{16\pi^2c} * \mu_0 * \frac{sin^2(\theta)}{(1-\beta\cos(\theta))^5} \tag{2.113}$$

since

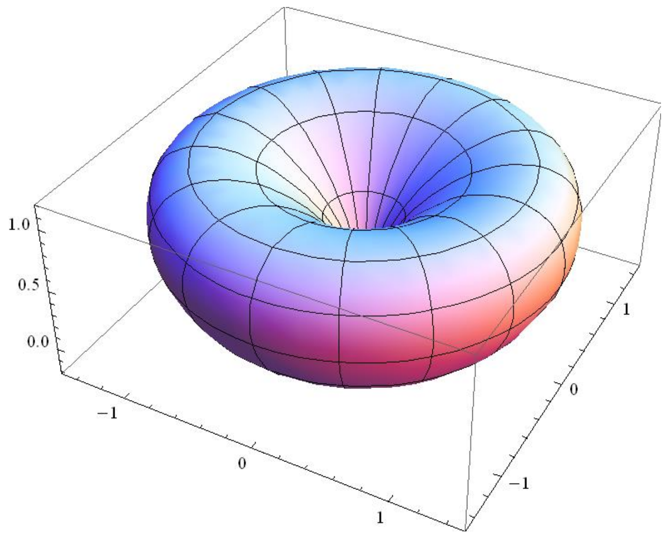
$$\frac{1}{c^2} = \mu_0\epsilon_0 \tag{2.114}$$

The form of the distribution of the Radiated Power depends on the relativistic velocity factor β. When, $\beta = 1$, the electron has negligibly small velocity, but still a certain acceleration. This is a simple representation of an electron moving up and down at its location, which actually is nothing but an antenna. In the figure we can see the cross section of the angular distribution of the Power radiated, as well as its 3 dimensional representation in space.

As for the case where β tends to be significant (close to 1), then the charged particle is moving with relativistic velocities. In the following figures we show the case where β=0.3 and β=0.6



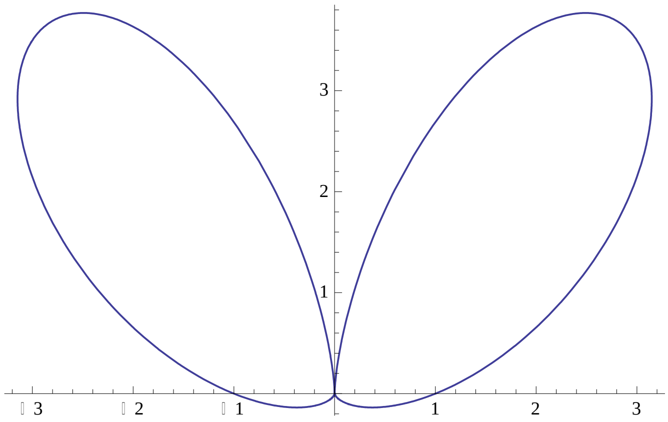
(a)



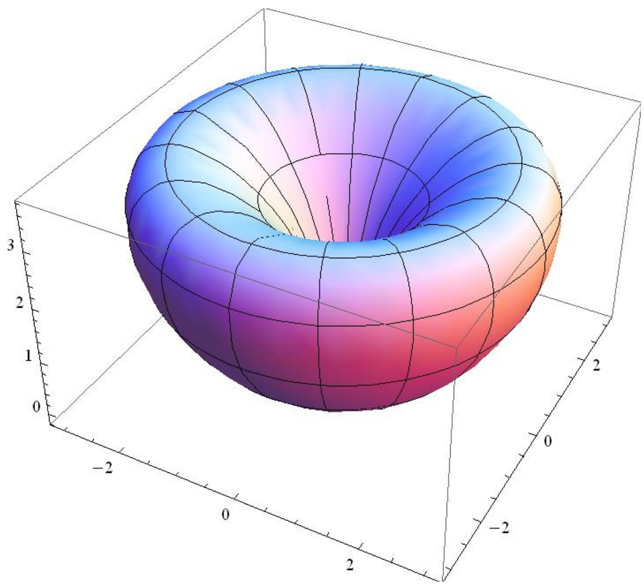
(b)

Figure 2.10 The angular power distribution of a charged particle moving up and having an acceleration collinear (a) Cross sectional distribution (b) 3 Dimensional distribution.

When a relativistic particle is stopped drastically with a very large deceleration, the particle will radiate what is known as *Bremsstrahlung*, or what is called literally “stopping radiation”. It is a powerful radiation as soon as the acceleration gets larger and as such, as the charged particle becomes ultra-relativistic as β tends to 1, it becomes pretty directional.



(a)



(b)

Figure 2.11 The angular power distribution of a charged particle moving up and having an acceleration collinear with its velocity for β=0.5 (a) Cross sectional distribution (b) 3 Dimensional distribution.

2.4.2. Angular Distribution of Radiated Power: The case where $\vec{a}$ is perpendicular to the velocity $\vec{\vartheta}$

As we already know the general expression for the power radiated within a solid angle dΩ to be given by

$$\frac{dP}{d\Omega} = \frac{q^2}{16\pi^2\epsilon_0} \cdot \frac{|\hat{n} \times (\vec{u} \times \vec{a})|^2}{(\hat{n} \cdot \vec{u})^5} \quad (2.115)$$

where $\vec{u} = c\hat{n} - \vec{v}$, we may proceed to solve for the particular case where

$$\vec{v} = v \cdot \hat{z} \quad (2.116)$$

and

$$\vec{a} = a \cdot \hat{x} \quad (2.117)$$

The instantaneous acceleration and instantaneous acceleration and instantaneous velocity are perpendicular.

For a general case remember that

$$\hat{n} = \sin(\theta) \times \cos(\phi)\,\hat{x} + \sin(\theta) \times \sin(\phi)\,\hat{y} + \cos(\theta)\hat{z} \quad (2.118)$$

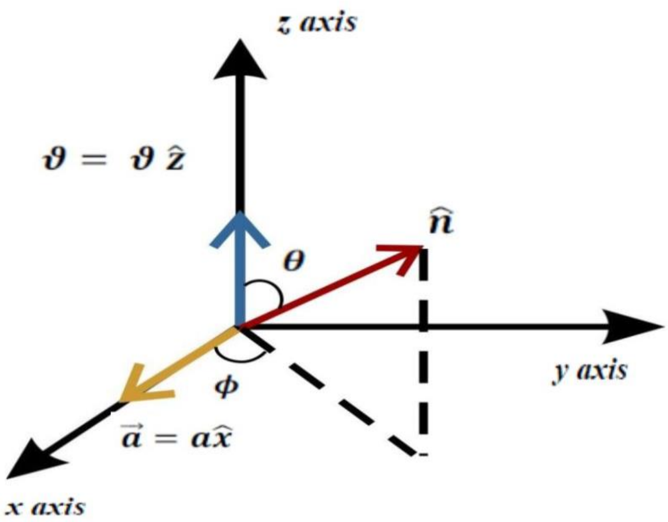


Figure 2.12: Case where $\vec{a} \perp \vec{v}$.

Remembering bac-cap rule for the triple cross product

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{a} \cdot \vec{c}) - \vec{c} \cdot (\vec{a} \cdot \vec{b}) \tag{2.119}$$

which implies for

$$\hat{n} \times (\vec{u} \times \vec{a}) = \vec{u} \cdot (\hat{n} \cdot \vec{a}) - \vec{a} \cdot (\hat{n} \cdot \vec{u}) \tag{2.120}$$

Since

$$\vec{a} = a\hat{x} \tag{2.121}$$

then

$$\hat{n} \cdot \vec{a} = a \sin(\theta) \cos(\phi) \tag{2.122}$$

Also since

$$\vec{u} = c\hat{n} - \vec{v} \tag{2.123}$$

$$\hat{n} \cdot \vec{u} = c(\hat{n} \cdot \hat{n}) - \hat{n} \cdot \vec{v} \tag{2.124}$$

But

$$\vec{v} = v\hat{z} \implies \tag{2.125}$$

$$n \cdot \vec{v} = v\cos(\theta) \tag{2.126}$$

$$\hat{n} \times (\vec{u} \times \vec{a}) = \vec{u}(a \sin(\theta) \cos(\phi)) - \vec{a}(c - v\cos(\theta)) \tag{2.127}$$

$$= (c\hat{n} - \vartheta\hat{z}) \cdot (a \sin(\theta) \cos(\phi)) - a\hat{x}(c - v\cos(\theta)) \tag{2.128}$$

$$= ac\{(\hat{n} - \beta\hat{z})\sin(\theta) \cos(\phi) - \hat{x}(1 - \beta\cos(\theta))\} \tag{2.129}$$

Therefore

$$|\hat{n} \times (\vec{u} \times \vec{a})|^2 = (\hat{n} \times (\vec{u} \times \vec{a})) \cdot (\hat{n} \times (\vec{u} \times \vec{a})) \tag{2.130}$$

$$|\hat{n} \times (\vec{u} \times \vec{a})|^2 \\ = a^2c^2\left(\vec{A}\sin(\theta)\cos(\phi) - \hat{x}(1-\beta\cos(\theta))\right) \\ \cdot \left(\vec{A}\sin(\theta)\cos(\phi) - \hat{x}(1-\beta\cos(\theta))\right) \tag{2.131}$$

$$|\hat{n} \times (\vec{u} \times \vec{a})|^2 = a^2c^2\{\sin^2(\theta)\cos^2(\phi)(\vec{A}\cdot\vec{A}) - 2(\vec{A}\hat{x})(1 \\ - \beta\cos(\theta))\sin(\theta)\cos(\phi) + (\hat{x}\cdot\hat{x})(1-\beta\cos(\theta))^2\} \tag{2.132}$$

$$(\hat{n} - \beta\hat{z})\cdot(\hat{n} - \beta\hat{z}) = \vec{A}\cdot\vec{A} = \hat{n}\cdot\hat{n} - 2\beta n\cdot\hat{z} + \beta^2\hat{z}\cdot\hat{z} \tag{2.133}$$

$$= 1 - 2\beta\cos(\theta) + \beta^2 \tag{2.134}$$

$$\vec{A}\cdot\hat{x} = \sin(\theta)\ \cos(\phi) \tag{2.135}$$

$$\Rightarrow |\hat{n} \times (\vec{u} \times \vec{a})|^2 \tag{2.136}$$

$$= a^2c^2\{sin^2(\theta)cos^2(\phi)(1 - 2\beta\cos(\theta) + \beta^2) - 2sin^2(\theta)cos^2(\phi)(1 - \beta\cos(\theta)) \\ + (1-\beta cos(\theta))^2\} \tag{2.137}$$

$$= a^2c^2[\{\sin^2(\theta)\cos^2(\phi)\{1 - 2\beta\cos(\theta) + \beta^2 - 2 + 2\beta\cos(\theta)\} \\ + (1-\beta\cos(\theta))^2] \tag{2.138}$$

$$= a^2c^2\{(1-\beta)^2 - (1-\beta^2)\sin^2(\theta)\cos^2(\phi)\} \tag{2.139}$$

Likewise

$$\hat{n}\cdot\vec{u} = \hat{n}\cdot(c\hat{n} - \vec{v}) = c(\hat{n}\cdot\hat{n}) - \hat{n}\cdot v\hat{z} \tag{2.140}$$

$$= c - v\cos(\theta) \tag{2.141}$$

$$= c(1-\beta\cos(\theta)) \tag{2.142}$$

Note that

$$\frac{1}{c^2} = \mu_0\epsilon_0 \tag{2.143}$$

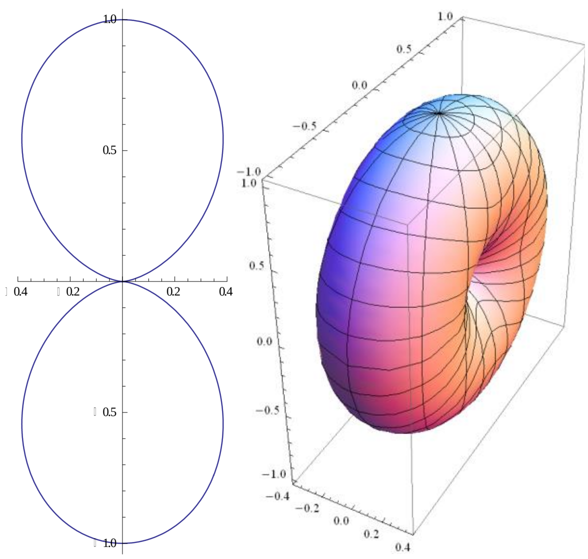
Then

$$\frac{dP}{d\Omega} = \frac{q^2}{16\pi^2\epsilon_0}\cdot\frac{a^2c^2\ [(1-\beta\cos(\theta))^2 - (1-\beta^2)\sin^2(\theta)\cos^2(\phi)]}{c^5(1-\beta\cos(\theta))^5} \tag{2.144}$$

$$\frac{dP}{d\Omega} = \frac{q^2 a^2}{16\pi^2} \cdot \frac{\mu_0}{c} \cdot \frac{[- \ - \ - \ -]}{(\quad)^5} \tag{2.145}$$

$$\frac{dP}{d\Omega} = \frac{q^2 a^2 \mu_0}{16\pi^2} \cdot \frac{[(1-\beta\cos(\theta))^2(1-\beta)sin^2(\theta)cos^2(\phi)]}{(1-\beta\cos(\theta))^5} \tag{2.146}$$

This is the expression for the angular distribution of the radiated power in space as a function of the spherical coordinates, θ and ϕ. Remember that the acceleration is perpendicular to the direction of velocity. In the following Figures we plot the pattern of the radiated power distribution in space given the case where the velocity is upwards and the acceleration is perpendicular to the velocity.



(a) (b)

Figure 2.13 The case where the charged particle accelerates on the horizontal axis.

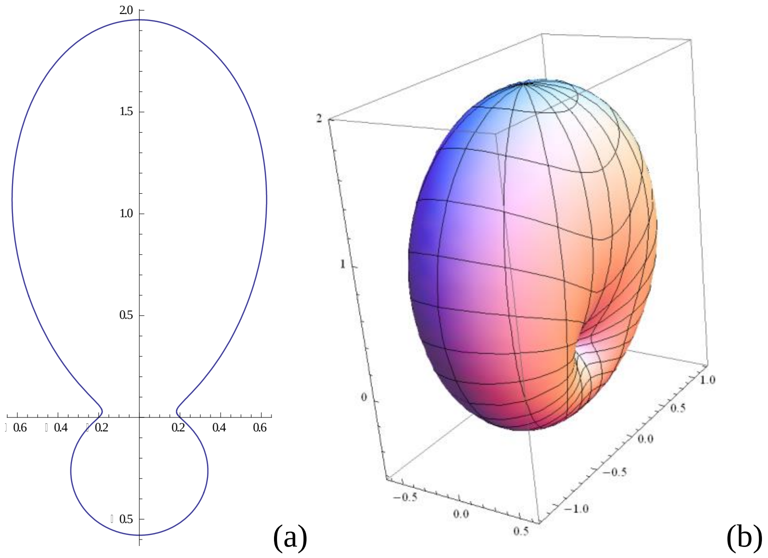


Figure 2.14 The angular distribution pattern of the radiated power for the case of Synchrotron Radiation, where the velocity is in the z-direction.

As we can see from the plots for the case of the instantaneous velocity of the charged particle being perpendicular to the instantaneous acceleration of the particle, the pattern of the radiation depends very strongly on the relativistic velocity factor β. The pattern is simply identical to that of a dipole antenna for a stationary particle with β=0. But as the particle velocity becomes more significant the radiation pattern for the Laboratory observer becomes more and more distorted, it becomes more directed to the side where the charged particle moves as β=0.2 and for more and more relativistic cases like say that of β=0.8, the pattern is very strongly directed in the direction of the instantaneous velocity of the particle. For ultra-relativistic particles β tends to 1, it can assume values very close to 1 but 1 itself. For such ultra-relativistic velocities, the radiation for the laboratory observer is very much directed in the direction of the velocity of the charged particle, in a very narrow cone. This identity of the Synchrotron radiation spells directionality of the emitted radiation just like the case of a laser. But remember that, at this stage, the emitted radiation is not coherent, so it is not a laser light just yet.

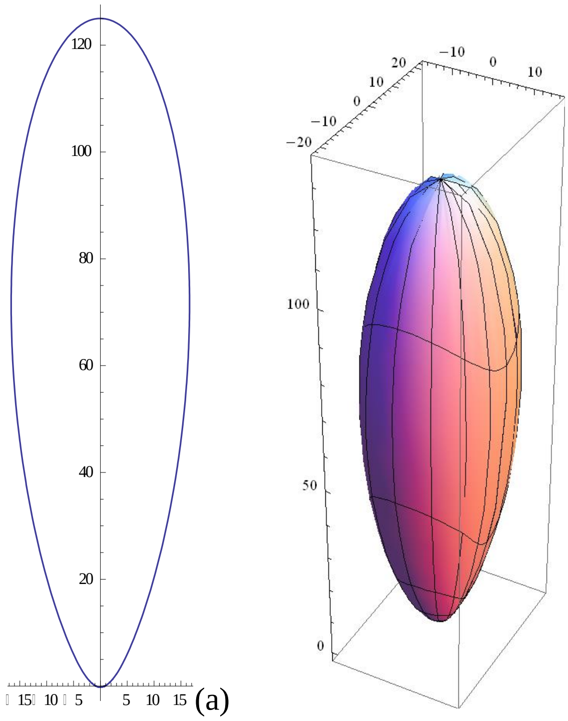


Figure 2.15 The angular distribution pattern of the radiated power for the case of Synchrotron Radiation where β=0.8.

2.4.3. The power radiated Total Power for Bremsstrahlung

$$\frac{dP}{d\Omega} = \frac{\mu_o q^2 a^2}{16\pi^2 c} \frac{\sin^2(\theta)}{(1-\beta\cos(\theta))^5} \tag{2.147}$$

with

$$\frac{\mu_0 q^2 a^2}{16\pi^2 c} = K : \text{constant} \Rightarrow \tag{2.148}$$

$$P = \int \frac{dP}{d\Omega} d\Omega \qquad (2.149)$$

$$P = K \int \frac{\sin^2(\theta)}{(1-\beta\cos(\theta))^5} \sin(\theta)\, d\theta\, d\phi \qquad (2.149)$$

substuting

$$\cos(\theta) = u \tag{2.150}$$

$$-\sin(\theta)\,d\theta = du \tag{2.151}$$

$$\sin(\theta) = \sqrt{1-u^2} \tag{2.152}$$

$$\sin^2(\theta) = 1 - u^2 \tag{2.153}$$

$$P = K\int_{\substack{\theta=0\\u=1}}^{\substack{u=-1\\\theta=\pi}} \frac{1-u^2}{(1-\beta\,u)^5}\quad du\cdot\left(-1\int_{\theta=0}^{\phi=2\pi} d\phi\right) \tag{2.154}$$

$$P = 2\pi K\int_{u=-1}^{u=1} \frac{1-u^2}{(1-\beta u)^5}\,du \tag{2.155}$$

$$\Rightarrow P = \frac{2\pi\cdot\mu_0 q^2 a^2}{16\pi^2 c}\cdot\frac{4}{3}\cdot\frac{1}{(1-\beta^2)^3} \tag{2.156}$$

since

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{2.157}$$

$$\gamma^6 = \frac{1}{(1-\beta^2)^3} \quad\Rightarrow \tag{2.158}$$

$$P = \frac{\mu_0 q^2 a^2 \gamma^6}{6\pi c} \tag{2.159}$$

This is the expression for the total power radiated for Bremsstrahlung.

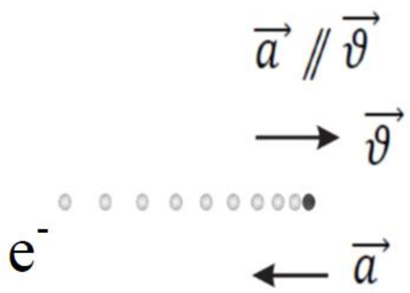


Figure 2.16 Schematic representation of stopping of an electron, i.e.,.

2.4.4. Calculation of the Total power radiated for Synchrotron Radiation

Total Power Radiated For the case of radiation where $\vec{\vartheta}$ and $\vec{a}$ are perpendicular. We already know the power radiated within a solid angle $d\Omega = \sin(\theta)\, d\theta\, d\phi$ as follows.

$$\frac{dP}{d\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \frac{[(1-\beta\cos(\theta))^2 - (1-\beta^2)\sin^2(\theta)\cos^2(\phi)]}{[1-\beta\cos(\theta)]^5} \tag{2.160}$$

Integration through θ and ϕ we can find the total radiated power

$$P = \iint \frac{dP}{d\Omega} d\Omega = \iint K_\mu \frac{[(1-\beta\cos(\theta))^2 - (1-\beta^2)\sin^2(\theta)\cos^2(\phi)]}{[1-\beta\cos(\theta)]^5} \sin(\theta)\, d\theta\, d\phi \tag{2.161}$$

Where we have defined the constant K_μ as follows

$$K_\mu = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \tag{2.162}$$

Taking K_μ out of the integral this is

$$\int_{\theta=0}^{\pi} \frac{\sin(\theta) d\theta}{(1-\beta\cos(\theta))^5} \cdot \int_0^{2\pi} d\phi - \int_0^{2\pi} \cos^2(\phi) d\phi \cdot \int_0^{\pi} \frac{(1-\beta^2)\sin^3(\theta)\, d\theta}{[1-\beta\cos(\theta)]^5} \tag{2.163}$$

Using Mathematica we can find these integrals depending on the variable β

$$\frac{2}{(1-\beta^2)^2} \cdot 2\pi - \pi \cdot \frac{4\pi}{3(1-\beta^2)^2} = \frac{4\pi}{(1-\beta^2)^2} - \frac{4\pi}{3(1-\beta^2)^2} \tag{2.164}$$

$$= \frac{8\pi}{3} \gamma^4 \tag{2.165}$$

since

$$\gamma^2 = \frac{1}{1-\beta^2} \tag{2.166}$$

Total power

$$P = K_\mu \cdot \frac{8\pi}{3}\gamma^4 = \frac{\mu_0 q^2 a^2}{16\pi^2 c}\cdot\frac{8\pi}{3}\gamma^4 \tag{2.167}$$

$$P = \frac{\mu_0 q^2 a^2}{16\pi^2 c}\gamma^4 \tag{2.168}$$

This the expression for the total power radiated for the Synchrotron Radiation.

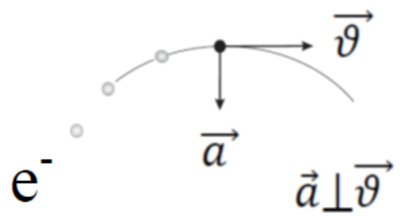


Figure 2.17 Electron trajectory bent.

2.5 NON-RELATIVISTIC FREE ELECTRON DEVİCES OF RADİATION

Much before first FEL emerged there were other free electron devices to generate electromagnetic waves. Among them what is called Klystron is particularly important, because it utilizes similar properties of electron dynamics for increasing the emitted power and the coherence of the electromagnetic waves. The fist FEL amplifier was built in 1970's [11] But the fist klysron is built in 1930's. [12] But both these devices use electron bunching to obtain an increase of coherence properties by means of constructive interference.

In fact the term Klystron comes from the German world "klystern" where Klyster means Cluster or Bunch. So basically a Klysron is a bunching device for free electrons upon which we can obtain amplification of Radio Frequency signals.

Radio Frequency signals can be obtained by electronic means. But Klystron and similar devices like magnetron can be used amplify the amplitude of the RF signals. Klystron also increases the partial coherence of the signals. The degree of coherence of the signals is about 60% [13] The idea is as follows [14] See Figure below.

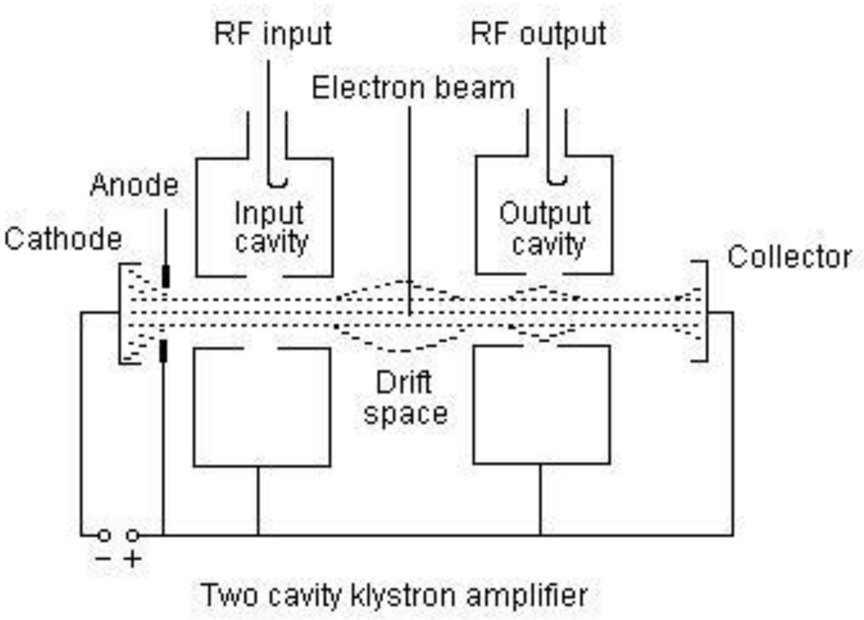


Figure 2.18 Working principle of a Klystron. [15]

There is a Cathode that emits electrons and anode on the other side of the device that collects the free electrons. As the electrons are emitted from the Cathode, they enter the region of the first Radio Frequency cavity, such that the oscillating electromagnetic field of this cavity, causes the beam of electrons to converge to each other in the following Drift Space. As the bunched electron beam reaches the second RF Cavity, this time, the Phase of the RF cavity is adjusted such that, the Electric Field of the RF cavity would be in the opposite direction to the direction of motion of the electrons. Since the coupling between the electric field of the RF cavity $\vec{E}$ and the velocity vector of the electrons $\vec{v}$ is given in terms of $\vec{E} \cdot \vec{v}$, as this scalar product is negative, it means that there will be an energy transfer from the electrons to the RF Electromagnetic field.

So the important thing to notice here is, bunched electron beams have been used to amplify Electromagnetic Radiation even before the IInd World War.

CHAPTER 3

PARTICLE DYNAMICS

3.1 INTRODUCTION

In this Chapter we will discuss the fundamental concepts regarding the mechanisms of conventional Laser Radiation and what makes a laser light special. First we will discuss radiation due to heat, Black-body radiation. And then we discuss what the differences for laser light from a conventional light source are. We tabulate the X-ray laser projects worldwide. We define the term Exittance for conventional blackbody radiation. Through a short review of the Fourier series we derive the definition of coherence time and length of a laser light. We review the derivation of the general formulate relevant to radiation from relativistic particles under certain circumstances of acceleration. We derive the expressions for the total power radiated for Bremsstrahlung and Synchrotron Radiation for a comparison.

3.1.1. Quadrupole Magnets and Focusing of Electron Beams: Optical Analogy

Let us start with review of what we already know from about the behavior of lens combinations in optics. Assuming that, we have two different lenses placed with a certain distance d apart, and let us suppose that they have optical powers (Dioptries) of D_1 and D_2 respectively. Remember that Optical Power or Dioptry is defined as the reciprocal of the focal distance of a lens given in units of meters. [16]

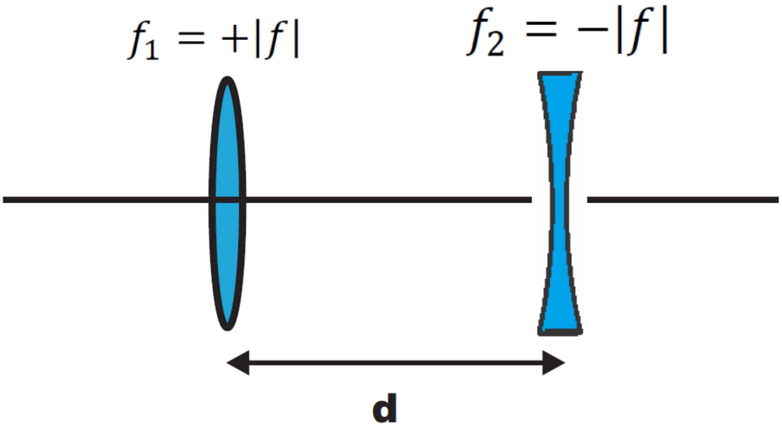


Figure 3.1 Representative lens combination for two lenses separated by a distance d.

From basic optics, we know that, the total effective Dioptry of the combined lens system is given by

$$D = D_1 + D_2 - d\, D_1\, D_2 \tag{3.1}$$

Or written in terms of focal distances, the total effective total distance of a lens combination is

$$\frac{1}{f_{total}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \tag{3.2}$$

Remember that focal distance of a lens could be positive or negative depending on the type of a lens; converging or diverging.

Now let us assume this time that, there may be a case where there are two lenses, both with the same absolute value for the focal distance, but one converging and the other one a diverging lens placed a part by a distance d, as shown by the two cases in the following figure

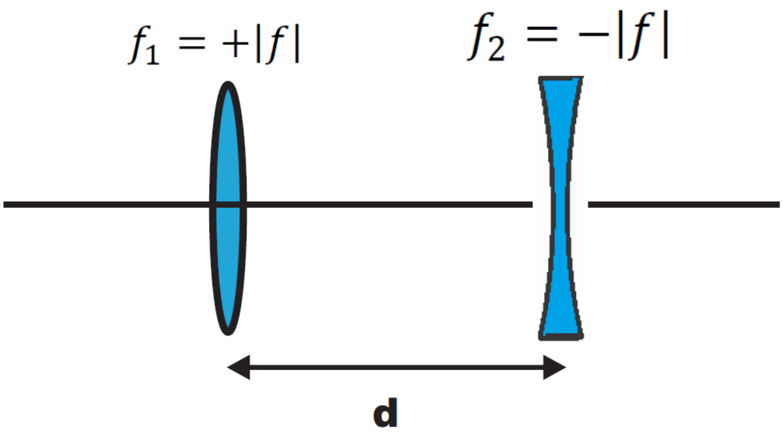


Figure 3.2 Lens combination where the light meets a converging lens first and then a diverging lens a distance d apart. Both have the same absolute value for the focal distance though.

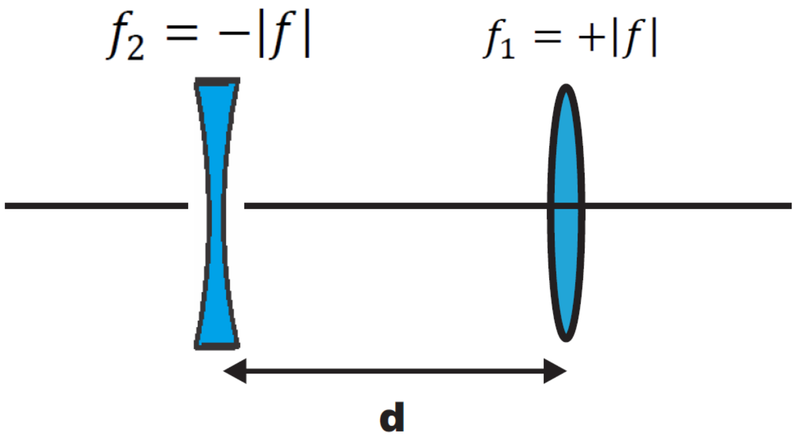


Figure 3.3 Lens combination where the light meets a diverging lens first and then a converging lens a distance d apart. Both have the same absolute value for the focal distance.

In the first instance where $f_1 = +|f|$ and $f_2 = -|f|$ The effective total focal length is given according to formula

$$\frac{1}{f_{total}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \tag{3.3}$$

As

$$\frac{1}{f_{total}} = \frac{1}{|f|} - \frac{1}{|f|} + \frac{d}{|f|^2} \tag{3.4}$$

$$\frac{1}{f_{total}} = \frac{d}{|f|^2} \tag{3.5}$$

Implying

$$\frac{1}{f_{total}} = \frac{|f|^2}{d} \tag{3.6}$$

A positive value for f_{total} hence a converting lens. The same idea applies for the other case where we have $f_1 = -|f|$ and $f_2 = +|f|$ and it yields the same result. Also take note of the special cases where $d = |f|$. Then, for both cases, regardless of the ordering of the lenses.

$$f_{total} = |f| \tag{3.7}$$

The combination of the lenses then act as a system as an effective converging lens.

The same idea applies to the case of Quadrupole, sextupole and octupole focusing magnets structures.

The problem with electron beams is that electrons repel each other with significant forces. As the electrons get faster and faster, this repelling effect becomes less pronounced, but those kinds of energies correspond to GeV range energies. Before reaching that range of energies, electrons must be accelerated from a few MeV where they are ripped off from electron guns, and in the meantime, they need to be refocused repeatedly to keep them together. This is achieved by the application of the above mentioned idea to the electron by means of electron optics. Electron optics utilizes the electron charge and its interaction with the magnetic field and the electrostatic fields as well.

A quadrupole consists of four magnets, where they could either be electromagnets or solid magnets of high intensity. It is also possible to play around with this distances and the strength of the magnetic fields so as to tune the focusing properties of these structures.

3.1.2. Working Principle of Lorentz Force in a Quadrupole Focusing Device

Now that we have an electron beam, if it goes directly through a quadrupole magnet, based on the Lorentz force that describes the force on a charged particle moving in a magnetic field

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right) \tag{3.8}$$

Also note that the charge is negative if the moving particle is an electron, so the direction of the cross product is reversed for an electron. In the following Figure we show the direction of the forces on an electron moving in a direction represented by a velocity vector directed into the page. The forces are focusing in one perpendicular axis while defocusing in the other perpendicular axis.

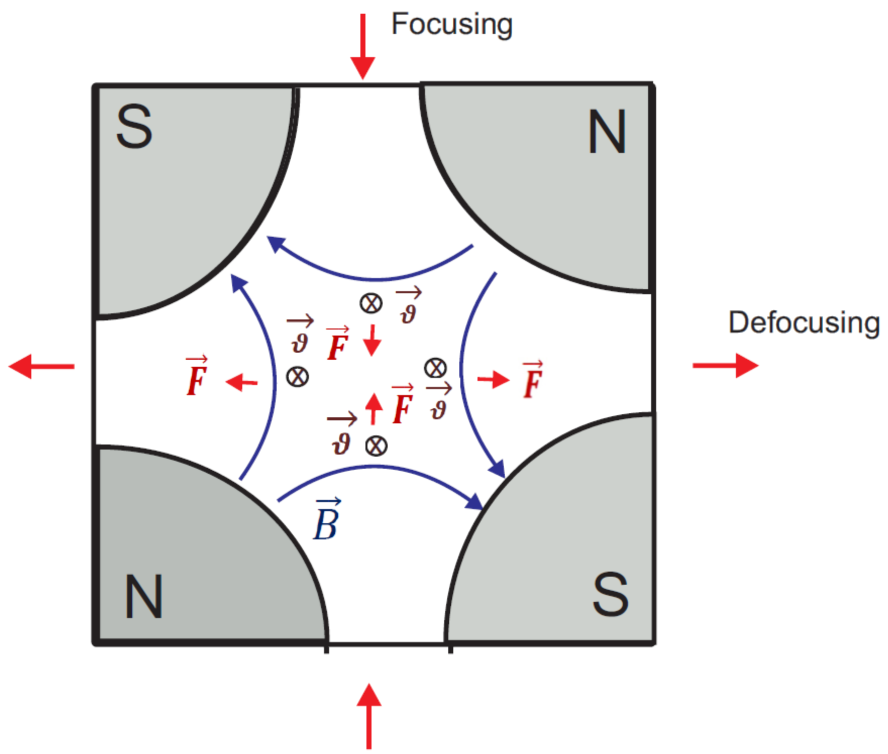


Figure 3.4 The forces acting on an electron moving in the velocity vector of an electron moving into the page according to the Lorentz Force.

Now notice that if the entire Quadrupole structure is rotated by 90^o, this time the Quadrupole will lead to focusing in the horizontal direction and defocusing in the vertical direction as shown in the following Figure

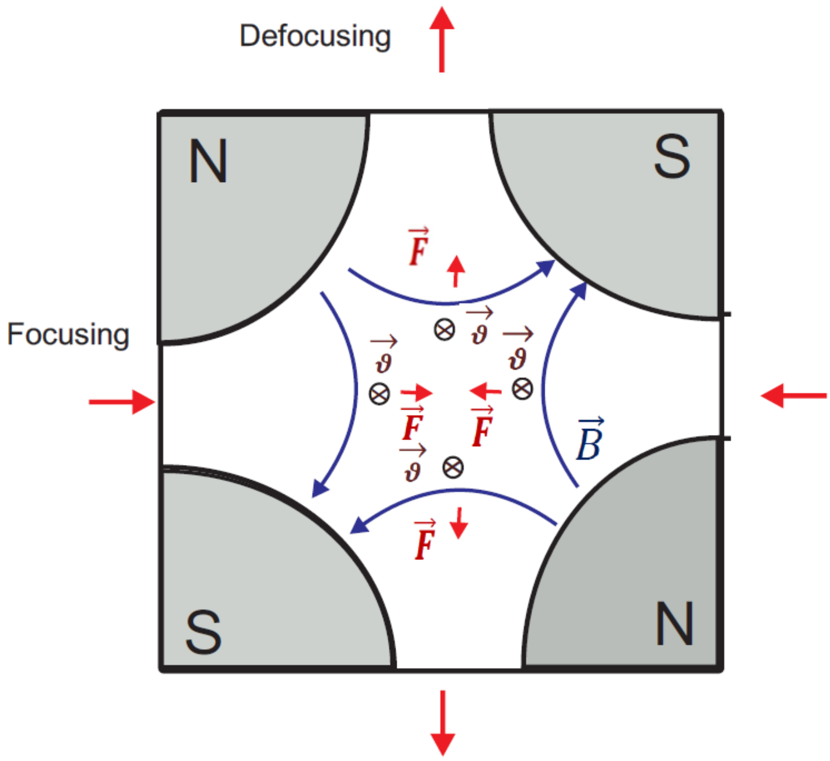


Figure 3.5 The forces acting on an electron moving in the velocity vector of an electron moving.

Just as two optical lenses with different signs of focal distances placed en tandem to function as a lens combination, a Quadrupole magnet pair also may act as a magnetic lens combination. Considering a certain focal distance $+f$ for the focusing axis, say x-axis, the same Quadrupole will act as a defocusing lens of focal length $-f$ for the perpendicular axis, this time it would be the y-axis. Now if we place yet another but the same exact Quadrupole with its poles rotated by 90^o, the x-axis will become the defocusing axis with focal length $-f$, and the y-axis will become the focusing axis with focal distance, $+f$. For the particular case where the distance between the Quadrupoles d turns out to be equal to this particular focal distance the effective focal distance of the combination reads

$$f_{total} = |f| \tag{3.9}$$

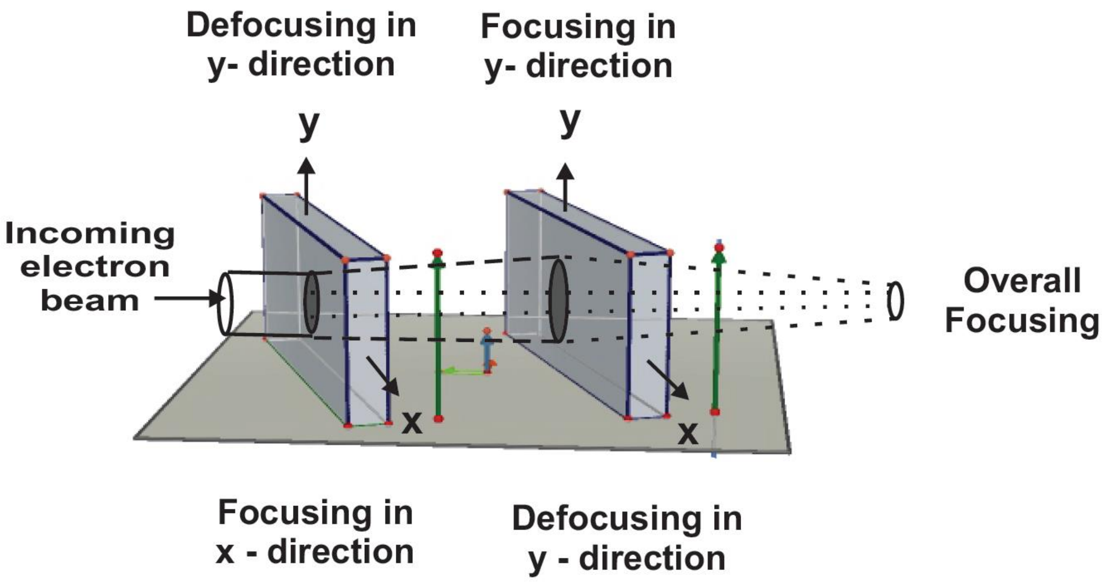


Figure 3.6 Quadrupole lenses in combination with rotated axes by 90^{o}.

So a series of Quadrupole magnets will lead to focusing-and-defocusing of the beam in conjuction with the Quadrupole structure such that in the overall sense the beam is kept focused.

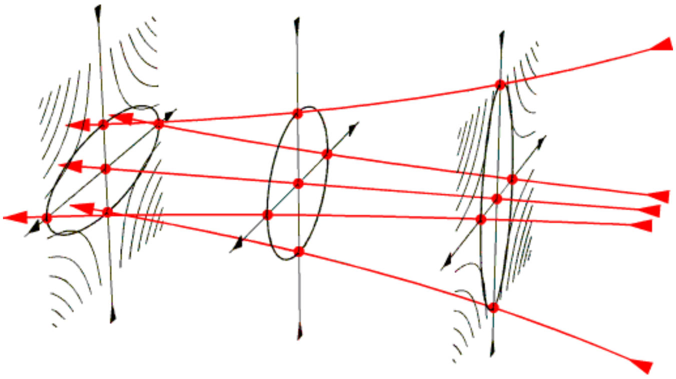


Figure 3.7 A series of Quadrupole structures will keep the beam keep its original symmetrical beam structure [17].

The idea of Quadrupole magnets in quadrature spaced apart by electrical insulators to cause strong focusing is first introduced by Nicholas Constantine Christofilos in 1949 [18].

For Linear Accelerators or Synchrotrons, Quadrupoles as well as Sextupoles are used for beam focusing. The same idea of lens combinations equally apply to Sextupoles but with 60^{o} rotation.

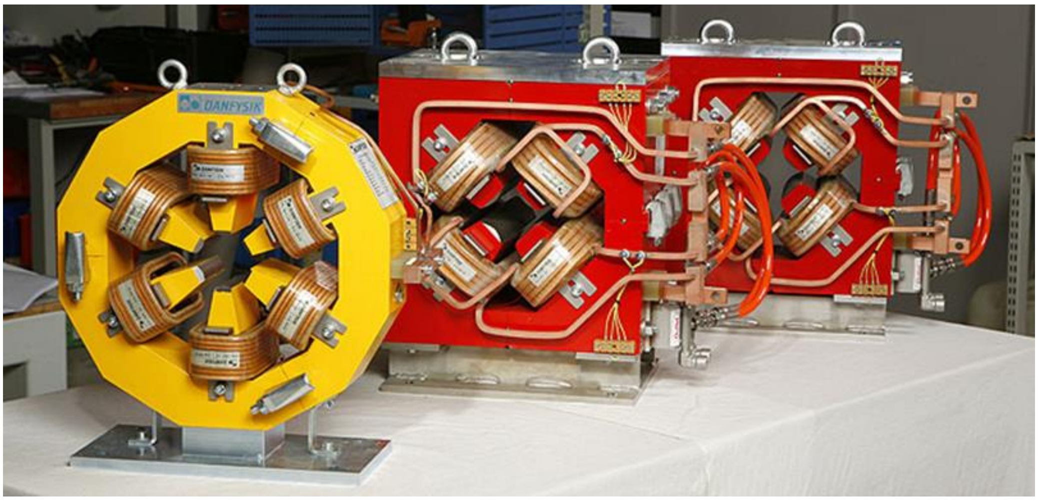


Figure 3.8 Dipole, quadrupole and sextupole magnets for the ELETTRA Storage Ring at Sincrotrone Trieste, Italy [19].

3.1.3. Bending Magnets and Synchrotron Radiation

It is also noteworthy that dipole structures are widely used in some accelerator structures called Synchrotrons. These are mainly bending magnets, magnets that bend the trajectory of electrons with strong uniform magnetic fields in a direction perpendicular to the direction of motion of the electrons as shown in the Figure below. Also note that when the trajectories of relativistic charged particles are bent, it is well known that they emit a broadband radiation with a certain spectrum within a small cone directed in the instantaneous velocity of the charged particle. This radiation is named Synchrotron Radiation since it was first observed at a synchrotron right after the IIndW.W. When it was first observed in a Synchrotron, which is built as an accelerator, it was considered as an energy loss mechanism and was not welcome. However, later on it was discovered that it has peculiar spectroscopic properties and could be utilized for spectroscopy as well, and dedicated Synchrotrons for obtaining and Synchrotron radiation and using it in the research laboratories were built around the world.

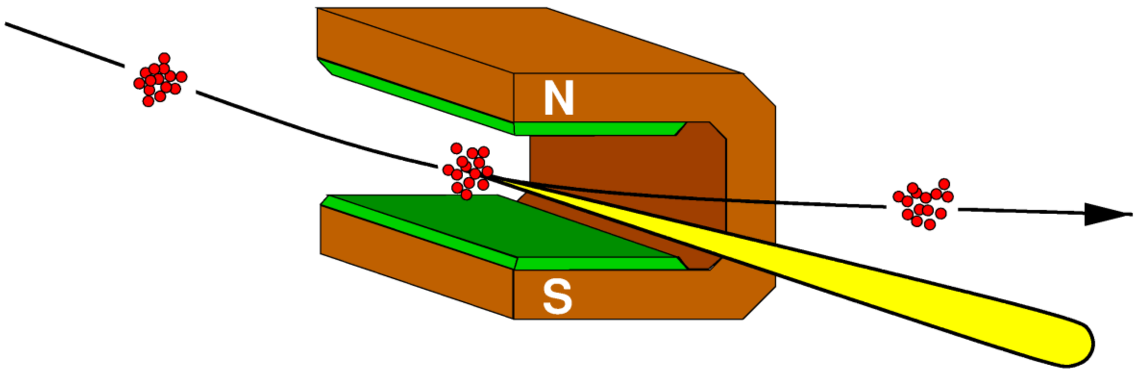


Figure 3.9 Trajectories of relativistic electrons bent by magnetic fields resulting in the emission of Synchrotron Radiation in a narrow cone[20].

The motion of an electron whose trajectory is bent in an external uniform magnetic field in the electrons own reference frame is like an oscillating charge up and down. So an electron treats its own motion like an electron in that of an antenna, therefore radiating in the well-known Dipole radiation pattern as shown in the following figure. However, for an observer in the Laboratory frame, this radiation is very much directional due to Lorentz Transformations of the field itself. The radiation is emitted within a cone of opening angle of the order of $\frac{1}{\gamma}$, where γ is measured in units of radians. For example, since the electron rest mass is like 0.511 MeV, it means that for an energy range of 1 GeV, the relativistic factor γ would be of the order of 2000, and that means the opening angle would be about 10^{-3} radians. And since one radian is about 57.3^{o}, this would imply an opening angle of about 0.05^{o}, which implies a very high spectral brightness, and great directionality. Directionality is one of the main properties of any laser light. Yet this emitted radiation is far from being a laser, although it is very bright, at this stage the radiated spectrum of the Synchrotron Radiation is of very broad spectrum, but in a very narrow cone. So, this solves the directionality problem of the light, but does not build the coherence property of a laser, since the emission is not stimulated emission and the radiated spectrum is very broad.

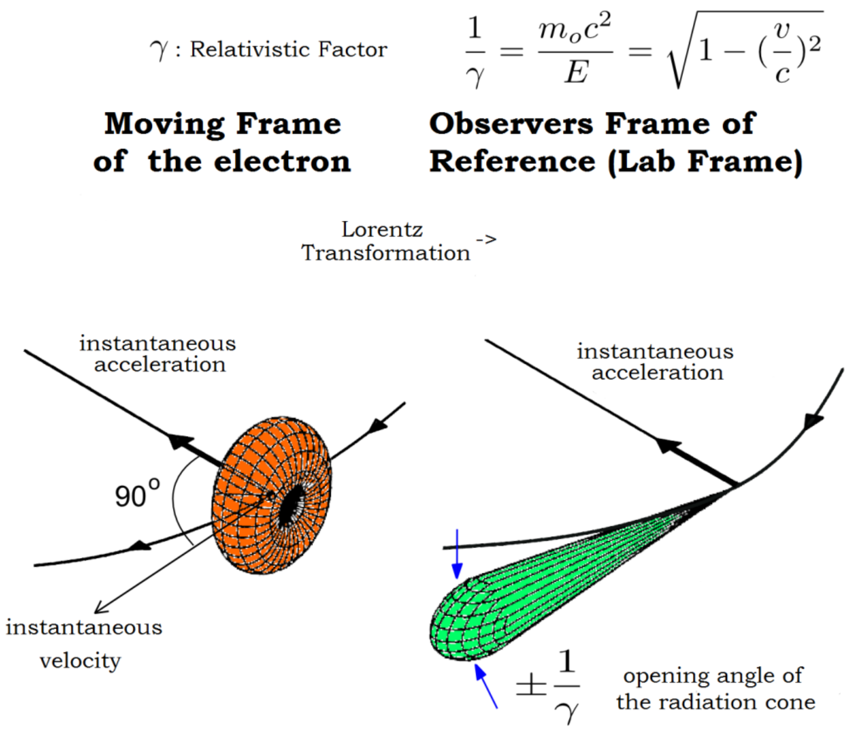


Figure 3.10 After Lorentz transformations.[20]

3.2 EMERGENCE OF COHERENCE IN AN UNDULATOR

3.2.1. Constructive interference in an undulator

Now let us consider the trajectory of an electron as shown in the Figure below, where alternating magnetic fields make the electron wiggle as it moves through and in the meantime the electron radiates a continuous spectrum of Synchrotron Radiation. At a first approximation, the electron follows almost a sinusoidal path which is periodic with a period we may name as the undulator period λ_u.

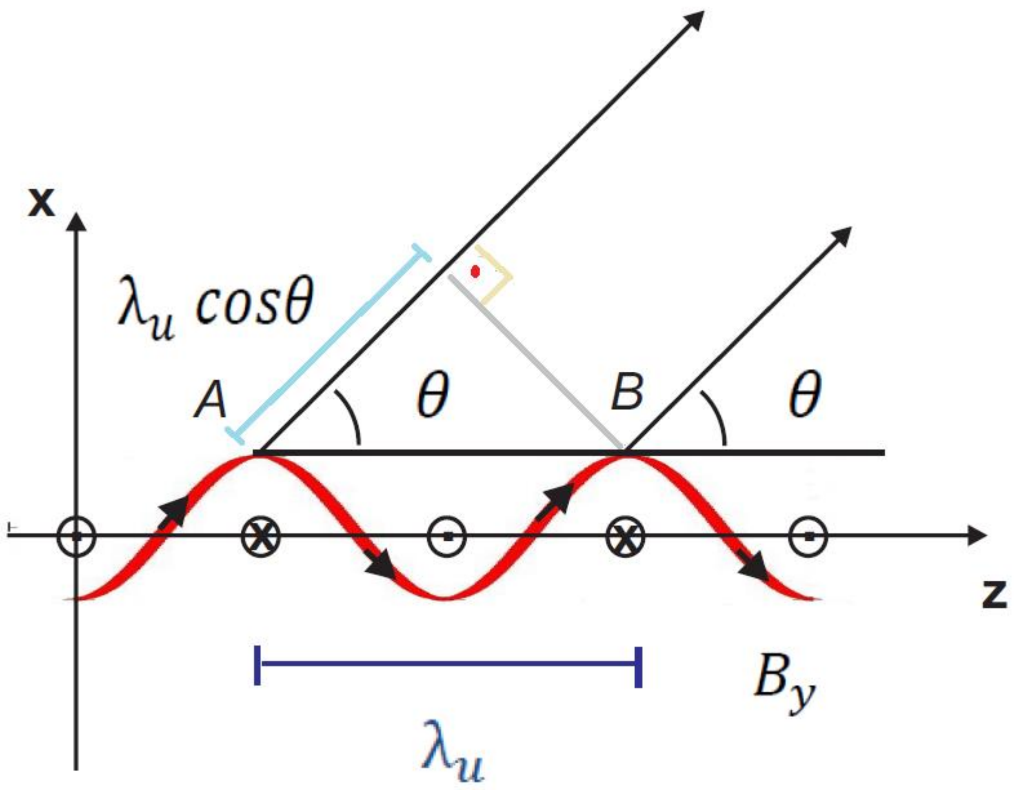


Figure 3.11 Constructive interference scheme of the emitted radiation in an undulator in conjunction with the electron trajectory. [21]

The magnetic field as a function of the z-coordinate on the plane of the motion (only on the plane close to the axis which is an approximation, because taken at its face value, it does not satisfy Maxwell's divergence Equation.) is described as

$$B_y = B_o cos\left(\frac{2\pi}{\lambda_u} z\right) \tag{3.10}$$

$$B_y = B_o \cos(k_w z) \tag{3.11}$$

Whenever the electron trajectory is bent, it emits a broad spectrum of radiation. However, we now presume the particular case that, the electron emitting at point A, after traversing the path in between and reaching point B and emitting there, should be in phase with the former radiation. This is a case of continuity of the flow of electrons and constructive interference of the emitted radiation considered simultaneously.

If we assume the average velocity in the z-direction to be denoted by $< v_z >$ then the time it takes for an electron to reach from point A to point B will be

$$\tau_u = \frac{\lambda_u}{< v_z >} \tag{3.12}$$

Also note that, The condition to have constructive interference between the radiation emitted at point A and point B is that the transit distances is an integral multiple of the emitted radiation. For the special case where this transit distance corresponds to one wavelength of the emitted radiation

With this at hand and an estimate of the average velocity we can proceed. We write the kinetic energy of the relativistic particle as follows, since the total energy is merely $\gamma m_o c^2$

$$\lambda = c \frac{\lambda_u}{<v_z>} - \lambda_u \cos(\theta) \tag{3.13}$$

With this at hand and an estimate of the average velocity we can proceed. We write the kinetic energy of the relativistic particle as follows, since the total energy is merely $\gamma m_o c^2$

$$\text{K.E.} = (\gamma - 1) m_o c^2 \tag{3.14}$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3.15}$$

Also note that the relativistic momentum is defined as

$$\vec{M} = m\vec{v} = \gamma m_o \vec{v} \tag{3.16}$$

Since the Force is defined as the time rate of change of momentum

$$\frac{d\vec{M}}{dt} = \frac{d}{dt}(\gamma m_o \vec{v}) = q(\vec{E} + \vec{v} \times \vec{B}) \tag{3.17}$$

When Newton's second law is written for relativistic charged particles under the effect of Lorentz Force in a uniform magnetic field . The coupling between electric field, usually referred to as the optical component, say of an electromagnetic wave, and the Kinetic Energy of the particle is defined through the velocity of the charged particle. The energy interchange formula reads

$$\frac{d\,K.E.}{dt} = \frac{d}{dt}(\gamma m_o c^2) = q\vec{v} \cdot \vec{E} \tag{3.18}$$

So, to have an increase of the Kinetic Energy of the particle, the dot product needs to be positive on the average, and to have an increase of the radiation field at the expense of the kinetic energy of the particle, the dot product needs to be negative on the average. From the above equations it could be inferred that

$$\frac{d}{dt}\vec{v} = \frac{q}{\gamma m_o}\left(E + \frac{\vec{v}}{c^2}(\vec{v} \cdot \vec{E}) + \vec{v} \times \vec{B}\right) \tag{3.19}$$

In this equation, taking into account the first order approximation of the undulator magnetic field to be

$$B_y = B_o \cos(k_w z) \tag{3.20}$$

One reaches for the undulating velocity of the electron to be

$$\frac{d}{dt}v_x = \frac{e\,B_o}{\gamma m_o} v_z \cos(k_w z) \tag{3.21}$$

Also utilizing

$$v_z = \frac{dz}{dt} \tag{3.22}$$

One readily solves

$$v_x = \frac{e\ B_o}{\gamma m_o k_w} \sin(k_w z) \tag{3.23}$$

Remembering that

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3.24}$$

One gets

$$v_z = \sqrt{v^2 - {v_x}^2} \tag{3.25}$$

$$v_z = c\sqrt{1 - \frac{1}{\gamma^2}(1 + K^2 \sin(k_w z))} \tag{3.26}$$

Here we can refer to K as the undulator parameter and we have just defined it as

$$K = \frac{e\ B_o}{m_o k_w c} \tag{3.27}$$

Such that, assuming a very large relativistic factor, i.e., $\gamma \gg 1$

$$v_z \approx c\left(1 - \frac{1}{2\gamma^2}(1 + K^2 \sin^2(k_w z))\right) \tag{3.28}$$

Averaging this over one period of the undulator λ_u, gives our needed average velocity in the propagation direction of the electron to be

$$< v_z >= c\left(1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)\right) \tag{3.29}$$

So the wavelength with the constructive interference condition we have obtained above

$$\lambda = c\frac{\lambda_u}{<v_z>} - \lambda_u\cos(\theta) \tag{3.30}$$

With the assumption of small angles and large relativistic factor, will turn out to be

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{1}{2}K^2 + (\gamma\theta)^2\right) \tag{3.31}$$

In the exact forward direction this constructive interference condition would imply

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{1}{2}K^2\right) \tag{3.32}$$

Or in more complete terms

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{1}{2}\left(\frac{e\,B_o}{m_o k_w c}\right)^2\right) \tag{3.33}$$

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{1}{2}\frac{e^2\,B_o{}^2}{m_o{}^2 k_w{}^2 c^2}\right) \tag{3.34}$$

$$k_w = \frac{2\pi}{\lambda_u} \tag{3.35}$$

This is simply

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{e^2\,B_o{}^2\lambda_u{}^2}{8\pi^2 m_o{}^2 c^2}\right) \tag{3.36}$$

The only variables in this form of the undulator equation turns out to be the relativistic factor γ, the undulator period λ_u , and the magnetic field strength B_o. This is the wavelength of the laser like radiation emitted from the undulator, and the good thing is that it could be tuned by tuning the Energy of the electron, The architecture of the undulator through the undulator period, or the field strength by adjusting the gap between the Halbach undulator magnet arrays.

So as a quick check, we can insert the MKS values of the constant and get a more handy formula, by inserting $e = 1.60217657 \cdot 10^{-19}$ Coulomb, $m_o = 9.10938291 \cdot 10^{-31}$kg, and $c = 2.99\,792\,458 \cdot 10^{8}\,\frac{m}{s}$, and $\pi = 3.14159265359$

will lead to

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{e^2\, B_o{}^2 \lambda_u{}^2}{8\pi^2 m_o{}^2 c^2}\right) = \frac{\lambda_u}{2\gamma^2}\left(1 + \; C \cdot B_o{}^2 \lambda_u{}^2\right) \qquad (3.37)$$

Where the constant C is defined as

$$C = \frac{e^2}{8\pi^2 m_o{}^2 c^2} \qquad (3.38)$$

$$C = \; 4359.25 \qquad (3.39)$$

$$\lambda = \frac{\lambda_u[m]}{2\gamma^2}\left(1 + \; 4359.25 \cdot B_o[T]^2 \lambda_u[m]^2\right) \qquad (3.40)$$

3.2.2 Estimation of Parameters for X-ray Laser Undulators

We are now at a position to estimate the radiated wavelength of the laser-like radiation, if we have the values for the magnetic field strength in units of Tesla, the undulator wavelength in units of meters, and the relativistic factor γ, which has no units.

Let us now assume an ultra-relativistic electron of relativistic factor about 2000, which would correspond to an electron accelerated to energies as h,gh as 1GeV, a Magnetic field strength of $1\mathrm{T}$, which is a pretty high magnetic field strength for our daily lives, and an undulator period of $10\ \mathrm{cm} = 0.1\ \mathrm{m}$, which is relatively easy to construct.

Then this yields,

$$\lambda = \frac{0.1[\mathrm{m}]}{8\cdot 10^{6}}(1 + \ 4359.25\cdot 1^{2}[\mathrm{T}]^{2}0.1^{2}[\mathrm{m}]^{2}) \tag{3.40}$$

$$\lambda = 5574\cdot 10^{-10}\mathrm{m} = 5574\ \mathrm{\mathring{A}} \tag{3.41}$$

Which is visible blue-green visible light. Now if we proceed with an undulator period of 1cm=0.01m then

$$\lambda = \frac{0.01[\mathrm{m}]}{8\cdot 10^{6}}(1 + \ 4359.25\cdot 1^{2}[\mathrm{T}]^{2}0.01^{2}[\mathrm{m}]^{2}) \tag{3.42}$$

$$\lambda = 17.949\cdot 10^{-10}$$

$$\mathrm{m}=17.949\ \mathrm{\mathring{A}} \tag{3.43}$$

So one can obtain in principle a laser-like radiation with a above parameters to a obtain a coherent radiation of wavelength about $17.949\ \mathrm{\mathring{A}}$. This is a region of hard X-rays. In principle, it should be possible to obtain even a wavelength as small as $1\mathrm{\mathring{A}}$.

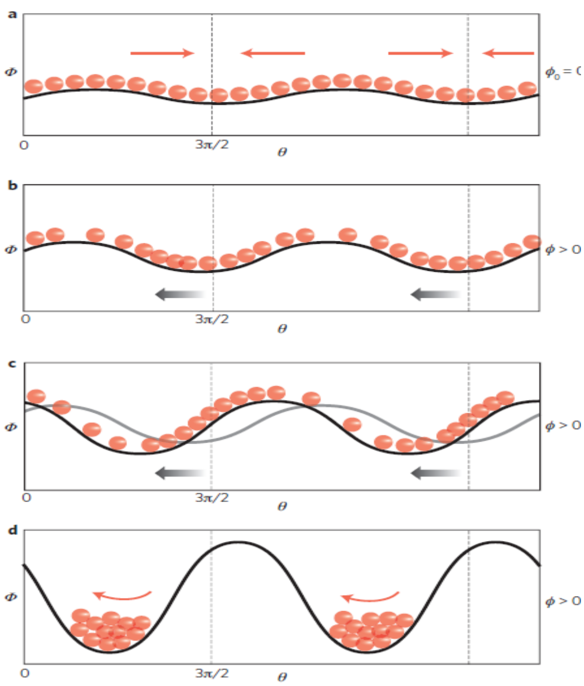


Figure 3.12 The rest frame of the electron beam, which is propogating left to right. [23]

3.2.3 Single Particle Approach of Free Electron Laser Theory

As shown in Figure, we can consider a two- dimensional model composed of a dielectric plane of infinite extent and a uniform linear distribution of electrons moving with constant velocity in the z direction. [21] Due to the 2-D geometry, each electron in Figure, is regarded as a sample electron extracted from a uniform linear distribution of electrons of infinite extent in the x direction. For simplicity, we also assume that an infinite magneto-static field is applied on the electron beam to restrict the motion of electrons only to z direction. In the 2-D model of Figure we calculate the energy transfer from a linear distribution of electrons to the electromagnetic wave propagating along a dielectric-coated conducting plane waveguide, neglecting the space- charge effect and the thermal motion of the electrons.

The relation describing the energy coupling between the e^- and the EM field, the time rate of change of the kinetic energy for relativistic particles is given as

$$\dot{K} = \frac{dK}{dt} = \frac{d}{dt}\left(\gamma m_0 c^2\right) \tag{3.44}$$

Which is also described as the work done by the optical field on the moving charged particle

$$\frac{dK}{dt} = q\vec{v} \cdot \vec{E} \tag{3.45}$$

Since the velocity is entirely in the z-direction

$$\dot{K} = -e\, v_z\, E_z \tag{3.46}$$

The rest mass energy subtracted from the relativistic energy of the electron is simply the kinetic energy of the electron

$$K = m_0\, c^2\, (\gamma - 1) \tag{3.47}$$

where

$$\gamma = \frac{1}{\sqrt{1-(v_z\,/c)^2}} \tag{3.48}$$

For which the time derivative implies $\dot{\gamma} = \gamma^3 v_z\, \dot{v}_z$ and plugging back in (3.46) leads to what one might as well name the equation of motion

$$\frac{dv_z}{dt} = -\frac{e}{\gamma^3 m_0} E_z \tag{3.49}$$

Let the z component of the electric field has a general form of

$$E_z = E_0 \cos(wt - k_z z + \phi), \tag{3.50}$$

where, E_0 is the amplitude of the electric field (constant), and ϕ stands for a phase denoting the position of a particular electron relative to the electric field.

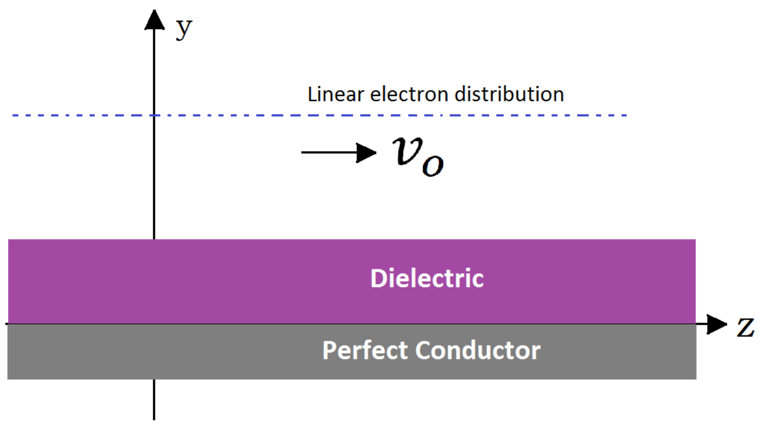


Figure 3.13 Perfect conductor with a dielectric coating acts as a waveguide for electron-field interaction.

Let the z-coordinate of the electron as a function of time t be expressed as $z = v_0 t$ for the zeroth-order approximation. Then, the equation of motion for the electron (2) reduces, with in the fist-order approximation, to

$$\frac{dv_z}{dt} = v_0 - \frac{e}{\gamma_0{}^3\, m_0\, \Omega} [\sin(\Omega t + \phi) - \sin\phi] \tag{3.51}$$

where

$$\Omega = \omega - v_0 k_z, \tag{3.52}$$

Ω is the effective frequency measured at the position of the electron. Integrating the equation of the electron (3.51) with respect to time from zero to t, the velocity reads,

$$v_z(t,\phi) = -\frac{e\,E_0}{\gamma_0{}^3\, m_0 \Omega}\left[\sin(\Omega t+\phi) - \sin\phi\,\right] \tag{3.53}$$

Integrating further (3.53) from 0 to t with respect to time, we obtain the first-order value of the electron position at time t,

$$z(t,\phi) = v_0 t + \frac{eE_0}{\gamma_0{}^3 m_0 \Omega^2}\left[\cos(\Omega t+\phi) - \cos\phi + \Omega t \sin\phi\,\right] \tag{3.54}$$

The second- order value for the z component of electric field acting upon the electron at time t is obtained by inserting (3.54) in (3.50) as

$$E_z(t,\phi) = E_0 \cos(\Omega t+\phi) + \frac{e^2 k_z E_0{}^2}{\gamma_0{}^3 m_0 \Omega^2}\sin(\Omega t+\phi)\left[\cos(\Omega t+\phi) - \cos\phi + \Omega t \sin(\phi)\right] \tag{3.55}$$

The energy equation for the electron, which is correct to the second-order with respect to small-signal fields, can be found by substituting (3.53) and (3.55) in (3.48) and leaving the resultant terms to the second-order in E_0:

$$\frac{dK}{dt} = -e\, v_0\, E_0 \cos(\Omega t+\phi) - \frac{e\, v_0 k_z E_0{}^2}{\gamma_0{}^3 m_0 \Omega^2}\sin(\Omega t+\phi)\left[\cos(\Omega t+\phi) - \cos\phi + \Omega t \sin\phi\right] + \frac{e^2 E_0{}^2}{\gamma_0{}^3 m_0 \Omega^2}\cos(\Omega t+\phi)\left[\sin(\Omega t+\phi) - \sin\phi\right] \tag{3.56}$$

Averaging (3.56) over electrons with phase ϕ (which is uniformly distributed form 0 to 2π relative to the electric field,

$$\left\langle\frac{dK}{dt}\right\rangle_\phi \equiv \frac{1}{2\pi}\int_0^{2\pi}\frac{dK}{dt}\,d\phi = \frac{\omega e^2 E_0{}^2}{2\gamma_0{}^3 m_0 \Omega^2}\left[\sin\Omega t - \cos\Omega t\right], \tag{3.57}$$

assuming

$$|\Omega| \ll \omega \tag{3.58}$$

If it is positive, the average amount of the energy is gained by the electron, and is lost by the electron if it is negative. Hence we have for the average variation of energy carried by the electron by the electron when it has traveled the distance L,

$$\langle \Delta K \rangle_{\phi} \equiv \int_0^{L/2} \langle \frac{dK}{dt} \rangle_{\phi} = \frac{\omega e^2 \tau^3 {E_0}^2}{2{\gamma_0}^3 m_0} \, g(\Omega\tau) \tag{3.59}$$

with

$$g(\Omega\tau) = \frac{2(1 - \cos\Omega\tau) - \Omega\tau \, \sin \Omega\tau}{(\Omega\tau)^3} \tag{3.60}$$

where $\tau = L / \ v_0$ and $g(\Omega\tau)$ denotes the gain function, which is an antisymetric function, and its absolute value takes the maximum value 0.135 at $\Omega\tau = \mp 2.6$. As the average increment of kinetic energy of electrons becomes negative for $g(\Omega\tau) < 0$, while it becomes positive for $g(\Omega\tau) > 0$. In other words, in the region where $g(\Omega\tau) < 0$, the ensemble of electrons loses energy to the electromagnetic wave interacting with it, as a result of which the latter is amplified. On the other hand, in the region where $g(\Omega\tau) < 0$, the ensemble of electrons loses energy to the electromagnetic wave interacting with it, as a result of which the latter is amplifield. On the other hand, in the region where $g(\Omega\tau) > 0$, the ensemble of electrons gains energy from the electromagnetic wave getting it damped. The main gain for the electromagnetic wave occurs for, $\Omega\tau < 0$, which means

$$\omega - v_0 k_z < 0, \tag{3.61}$$

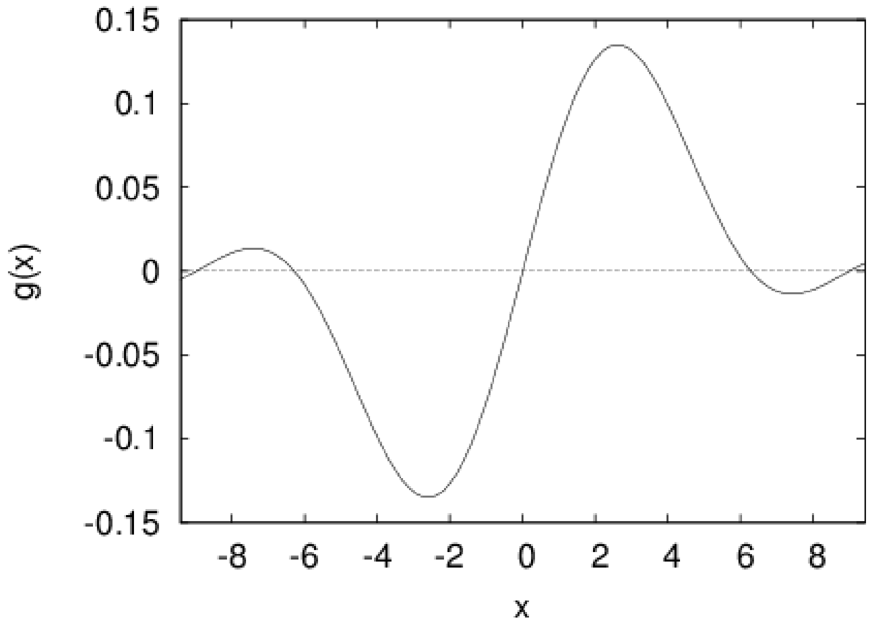


Figure 3.14 Gain Function $g(x)$ where $x = \Omega\tau$.

$$v_p < v_0, \tag{3.62}$$

$$v_p = \omega / k_z, \tag{3.63}$$

where v_p denotes the phase velocity of the electromagnetic wave. Equation (3.61) is just the condition for the Cherenkov radiation to occur, which has been clarified in the foregoing discussion. The maximum amount of energy, which the electromagnetic wave gains from the electron beam occurs when at $\Omega\tau$ = - 2.6.

3.2.4 Trapping of Electrons in the Electric Field

In the foregoing section, we have discussed the interaction of a relativistic electron beam and electromagnetic wave on the basis of the single-particle approach, together with quasi-linear approximations. Finding that, a strong beam-wave interaction occurs for the drift velocity of the electron beam nearly equal to the phase velocity of the electromagnetic wave. In this section, let us consider the beam-wave interaction in a more general manner which allows a nonlinear treatment of the problem. How do a uniform distribution of electrons along the z-axis behave as they interact with the electromagnetic field. Let us first assume that a uniform distribution of electrons is drifting with the velocity equal to the phase velocity of the electromagnetic wave at the initial state. Then, as time passes, electrons in the accelerating phases, electrons appear alternately at the same period, as that of the electromagnetic wave, In other words, electrons are density-modulated or bunched along the electron beam. For the case where the electron beam and the electromagnetic wave traveling with the same velocity, the amount of accelerated electrons turns wave are traveling with the same velocity, the amount of accelerated electrons turns out be equal to that of decelerate electrons. Hence no net transfer of energy occurs between the electron beam and the electromagnetic wave. On the other occurs between the electron beam and the electromagnetic wave. On the other hand, if a distribution of electrons is drifting with a velocity slightly greater than the phase velocity of the electrons, it is drifting with a velocity slightly greater than the phase velocity of the electromagnetic wave, the dense parts in the distribution of electrons shift toward the decelerating phases of the electric field, while the

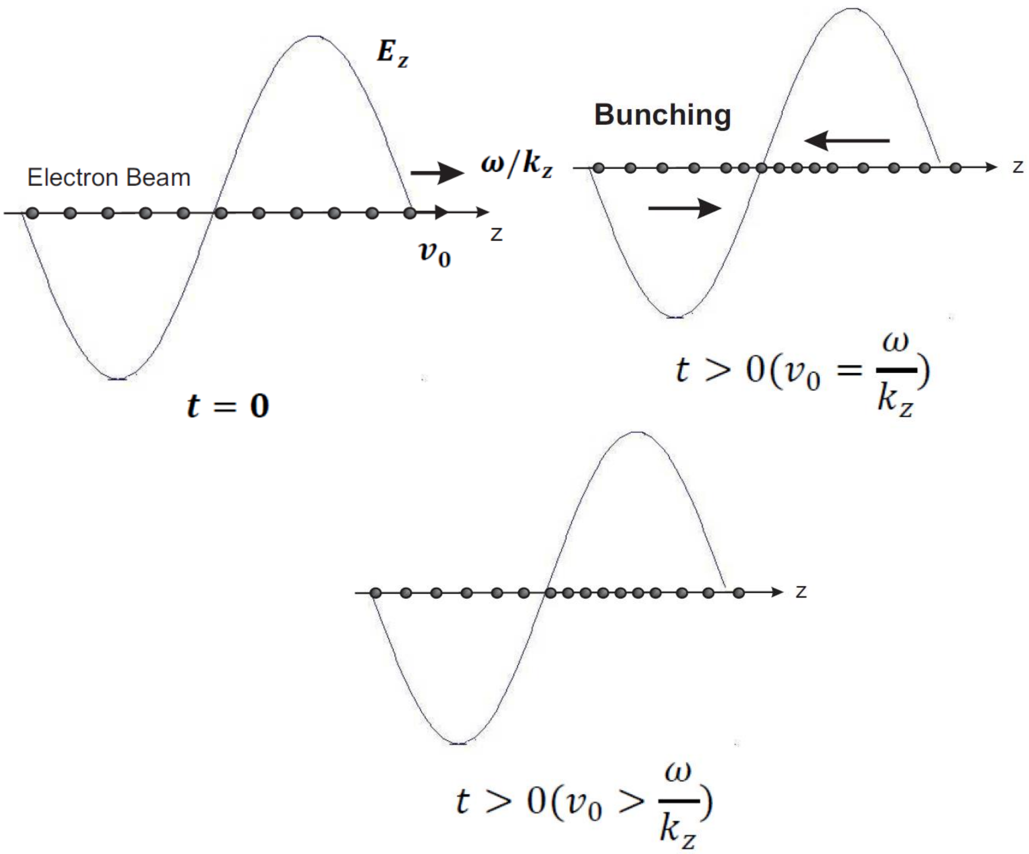


Figure 3.15 Bunching of electrons (Stimulated Cherenkov Effect).

lower density parts shift toward the accelerating phases, as symbolized by the Figures. Then more electrons are decelerated, and thus the net energy is transferred from the electron beam to the electromagnetic wave.

Let us generally discuss the bunching and trapping of electrons in the electric field. The basic equations for the following discussion are the energy relation for the electron (1) and the relativistic equation of motion for the electron (2). First, we rewrite the left-hand side of (1) as

$$\frac{dK}{dt} = \frac{dK}{dz}\frac{dz}{dt} = v_z \frac{dK}{dz} \tag{3.64}$$

where we must insert the value of k_z, which satisfies of which satisfies the dispersion relation, for a given value of ω. In the Figure, the frequency dependence of the power gain for the TEM_{00} mode is illustrated

we get the relation

$$\frac{dK}{dz} = -eE_z = -eE_0 \cos(k_z z - \omega t - \phi). \tag{3.65}$$

Similarly,

$$\frac{dv_z}{dz} = -\frac{e}{\gamma^3 v_z m_0} E_z = -\frac{e}{\gamma^3 v_z m_0} E_0 \cos(k_z - \omega t - \phi). \tag{3.66}$$

In order to describe the behavior of a particular electron in the electric field, we define a new phase Ψ as

$$\Psi = k_z z - \omega t - \phi + \frac{\pi}{2}, \tag{3.67}$$

from which we find the following relation:

$$\frac{d\Psi}{dz} = k_z - \frac{\omega}{v_z}. \tag{3.68}$$

Introduction the resonant velocity for the electron v_r , we rewrite v_z as

$$v_z = v_r + \Delta v_z, \tag{3.69}$$

$$v_r = v_p = \frac{\omega}{k_z}, \tag{3.70}$$

where v_p is the phase velocity of the electromagnetic wave, and we assume that $|\Delta v_z| \ll v_r$. Inserting (3.69) (3.70) in (3.68), reduces to

$$\frac{d\Psi}{dz} = \frac{\omega}{v_r}\frac{\Delta v_z}{v_r} = k_z \frac{\omega}{k_z}, \tag{3.71}$$

In the view of the radiation $\Delta\gamma = \gamma^3 v_z / c^2 \Delta v_z$, Eq (3.71) can also be expressed as

$$\frac{d\Psi}{dz} = k_z \frac{c^2}{\gamma_r^{\,2} v_r^{\,2}} \frac{\Delta\gamma}{\gamma_r}, \tag{3.72}$$

$$\gamma_r = \frac{1}{\sqrt{1-\left(\frac{v_r}{c}\right)^2}}. \tag{3.73}$$

Similarly, from (3.68), (3.67), (3.66) and (3.69),(3.70), we get the following relation:

$$\frac{d^2\Psi}{dz^2} = -k_\Psi^{\;2}\sin\Psi, \tag{3.74}$$

where

$$k^2 = \frac{\omega e E_0}{\gamma_r^{\,3} v_r^{\,3} m_0}. \tag{3.75}$$

Equation (3.74) is referred to as the pendulum equation, because it is of the same from as the equation for describing the motion of a pendulum swinging in a gravitational field. So some electrons will swing back and forth around a resonant electron.

3.2.5 Magnetic Field of a Planar Undulator

The motion of an electron in a planar undulator magnet is shown schematically in the Figure below. The undulator axis is along the direction of the beam (z direction), the magnetic field points in the y direction (vertical). Let us assume for simplicity that the period λ_u of the magnet arrangement is at the order of 30 mm, and also assume that the horizontal width of the pole shoes is larger than the undulator period λ_u, then one can neglect the x-dependence of the field in the close proximity of the tightly collimated electron beam. In the vacuum chamber of the electron beam we have

$$\vec{\nabla} \times \vec{B} = 0 \tag{3.76}$$

Hence the magnetic field can be written as the gradient of a so called scalar magnetic potential

$$\vec{B} = -\vec{\nabla}\,\phi_{mag} \tag{3.77}$$

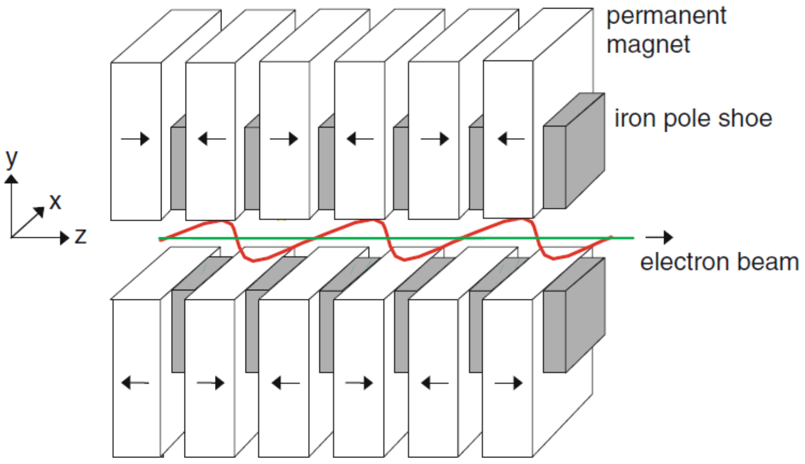


Figure 3.16 Schematic view of a hybrid planar undulator magnet. [25]

3.2.6 Undulator Radiation

The potential Φ_{mag} satisfies the Laplace equation

$$\nabla^2 \Phi_{mag} = 0. \tag{3.78}$$

The field on the axis is approximately harmonic, so making the ansatz that the y and the z-components are separable

$$\Phi_{mag}(y,z) = f(y)\sin(k_u z) \tag{3.79}$$

$$\Rightarrow \frac{d^2 f}{dy^2} - k^2{}_u = 0 \tag{3.80}$$

$$k_u = \frac{2\pi}{\lambda_u} \tag{3.81}$$

We obtain for the general solution

$$f(y) = c_1 \sinh(k_u y) + c_2 \cosh(k_u y). \tag{3.82}$$

The vertical field is

$$B_y(y,z) = -\frac{\partial \Phi_{mag}}{\partial y} \tag{3.83}$$

$$B_y(y,z) = -k_u(c_1 \cosh(k_u y) + c_2 \sinh(k_u y))\sin(k_u z) \tag{3.84}$$

There, B_y has to be symmetric with respect to the plane $y = 0$ so obviously, $c_2 = 0$.

We set

$$k_u c_1 = B_0 \tag{3.85}$$

and obtain

$$B_y(0,z) = -B_0 \sin(k_u z). \tag{3.86}$$

So the potential is

$$\Phi_{(x,y,z)} = \frac{B_0}{k_u}\sinh(k_u y)\sin(k_u z). \quad (3.87)$$

For $y \neq 0$ the magnetic field has also a longitudinal component B_z.

$$B_x = 0 \quad (3.88)$$

$$B_y = -B_0\cosh(k_u y)\sin(k_u z) \quad (3.89)$$

$$B_z = -B_0\sin(k_u y)\cos(k_u z). \quad (3.90)$$

On the symmetry plane $y = 0$ and using the idealized field as

$$\vec{B} = -B_0\sin(k_u z)\,\hat{j} \quad (3.91)$$

$\hat{j}$ is the unit vector in y-direction

3.3 EMERELECTRON MOTİON IN AN UNDULATOR

3.3.1 Trajectory in First Order

We call

$$W = W_{kin} + m_e c^2 = \gamma m_e c^2 \quad (3.92)$$

The total relativistic energy of the electron. The transverse acceleration by the Lorentz force is

$$\gamma m_e \dot{\vec{v}} = -e\vec{v} \times \vec{B}. \quad (3.93)$$

This results in two coupled equations

$$\ddot{x} = \frac{e}{\gamma m_e} B_y \dot{z} \quad (3.94)$$

$$\ddot{z} = -\frac{e}{\gamma m_e} B_y \dot{x} \tag{3.95}$$

The equations of motion of an electron in an undulator are solved iteratively. To obtain the first-order solution note that

$$vz = \dot{z} \approx v = \beta c = \text{constant} \tag{3.96}$$

and

$$v_x \ll v_z \tag{3.97}$$

then $\ddot{z} \approx 0$ and the solutions for $x(t)$ and $z(t)$ are

$$x(t) \approx \frac{e\,B_0}{\gamma m_e k_u} \sin(k_u \beta c t) \tag{3.98}$$

$$z(t) \approx \beta c t \tag{3.99}$$

if the initial conditions are set as

$$x(0) = 0, \tag{3.100}$$

$$\dot{x}(0) = \frac{eB_0}{\gamma m_e k_u} \tag{3.101}$$

And are realized by a suitable beam steering system in front of the undulator (the undulator magnet starts at $z = 0$). The electron travels on the sine-like trajectory

$$x(z) = \frac{K}{\beta \gamma k_u} \sin(k_u z). \tag{3.102}$$

In this equation we have introduced the important dimensionless undulator parameter that we came across in the former sections as well by a different reasoning

$$K = \frac{eB_0}{m_e c k_u} \tag{3.103}$$

$$K = \frac{eB_0 \lambda_u}{2\pi m_e c} \tag{3.104}$$

$$K = 0.934 \cdot B_0[T] \cdot \lambda_u[cm]. \tag{3.105 a}$$

$$K = 93.4 \cdot B_0[T] \cdot \lambda_u[m]. \tag{3.105 b}$$

The transverse velocity reads

$$v_x(z) = \frac{K\,c}{\gamma} \cos(k_u z). \tag{3.106}$$

3.3.2 Condition for the emission of Coherent FEL Radiation

It is a general property of the radiation emitted by relativistic electrons in a magnetic field that at large distances most of the intensity is concentrated in a narrow cone of opening angle $1/\gamma$, The cone is centered around the instantaneous tangent to the particle trajectory. The direction of the tangent varies along the sinusoidal orbit in the undulator magnet, the maximum angle with respect to the axis being

$$\theta_{max} \approx \left[\frac{dx}{dz}\right]_{max} \tag{3.107}$$

$$\theta_{max} = \frac{K}{\beta\gamma} \approx \frac{K}{\gamma} \tag{3.108}$$

If this directional variation is less than $1/\gamma$ the radiation field contributions from various sections of the trajectory overlap in space and interfere with each other and interact collectively with the electron beam in front of them. Consequently, asa result of constructive interference, the radiation spectrum in the forward direction is not really a continuously wide spectrum but rather it is nearly monochromatic (more precisely, it is composed of a narrow spectral line at a well-defined wavelength. [26]

The condition for this constructive interference condition to hold is

$$\theta_{max} \leq \frac{1}{\gamma} \tag{3.109}$$

which then implies in return that

$$\Rightarrow K \leq 1. \tag{3.110}$$

If the maximum angle θ_{max} exceeds the radiation cone angle $1/\gamma$ by a considerably large factor, which means $K \gg 1$, it is termed a wiggler magnet. Wiggler radiation consists of many densely spaced spectral lines forming a quasi-continuous spectrum which resembles the spectrum of ordinary synchrotron radiation in bending magnets, only with a comb-like structure and also brighter than ordinary Synchrotron Radiation as well.

However, if one needs a laser–like light with significant coherence, we need to construct an undulator where the undulator parameter $K \leq 1$.

CHAPTER 4

CONCLUSION

4.1 CONCLUSION

It appears that for the time being, the only possible way to build and X-ray laser is a Free Electron Laser. But, to have an X-FEL, one needs to have very high electron energies of the order of a few GeV, very high electron beam quality, and very well designed Undulator structures in vacuum. All of that requires a lot of investment. One needs to build very special electron guns, then pre-accelerator stage, focusing elements in the form of Quadrupoles and Sextupoles, bending magnets in the form of Dipoles, very high quality Radio Frequency Cavities made of Helium cooled superconductors, with very smooth surfaces made of Neodymium alloys, specially designed Undulators and all of this huge structures must operate under high vacuum conditions.

So the costs, as well as engineering professionalism must be at its peak. Astronomical costs force us to think of an alternative method if possible.

Recent advances in novel accelerator methods make us think of alternative accelerating methods. One if them have been proposed by Tajima and Dawson in 1979, [27] as the plasma Wakefield accelerator. It has been widely studied by theoretical physicists. This system uses two lasers of different frequencies, to obtain a wake-field in few years ago and very recently it is reported that 4.3 GeV is reached by this system on a tabletop accelerator. It is very important, because, this system, if it can be realized, is a much cheaper way of obtaining very high energy electrons necessary for the X ray FEL [28].

Another problem with the X ray FEL is that it requires undulators with very short undulator period and magnets with very high intensity. This is very demanding on the design of the undulators from the perspective of current technology. To build a cheaper and a more practical undulator structure, recently Tantawi and Shumail and coworkers at SLAC National Accelerator Laboratory proposed a Microwave Undulator in a microwave waveguide. The tunable microwave undulator realized at SLAC is 1.09 meters long, so it saves a lot of space and money from vacuum equipment [29].

Our proposal is that the future work must focus on Novel Accelerator Methods like the above mentioned plasma wakefield accelerator technology and Microwave Undulator structures at SLAC.

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