FATİH UNIVERSITY

The Graduate School of Sciences and Engineering

Master of Science in
Industrial Engineering

A PROPOSED ANALYTICAL MODEL FOR ORDER PICKING OPERATIONS IN MULTIPLE HORIZONTAL CAROUSEL STORAGE SYSTEM

by

Aziz AZIMOV

April 2016

Aziz AZIMOV

A PROPOSED ANALYTICAL MODEL FOR ORDER PICKING OPERATIONS IN MULTIPLE HORIZONTAL CAROUSEL STORAGE SYSTEM

M.S.
2016

A PROPOSED ANALYTICAL MODEL FOR ORDER PICKING OPERATIONS IN MULTIPLE HORIZONTAL CAROUSEL STORAGE SYSTEM

by

Aziz AZIMOV

A thesis submitted to

the Graduate School of Sciences and Engineering

of

Fatih University

in partial fulfillment of the requirements for the degree of

Master of Science

in

Industrial Engineering

April 2016
Istanbul, Turkey

APPROVAL PAGE

This is to certify that I have read this thesis written by Aziz AZIMOV and that in my opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science in Industrial Engineering.

____
Asst. Prof. Recep KIZILASLAN
Thesis Supervisor

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science in Industrial Engineering.

____
Prof. Mehmet ŞEVKLİ
Head of Department

Examining Committee Members

Assoc. Prof. Ali TÜRKYILMAZ ______________________________

Asst. Prof. Özgür UYSAL ______________________________

Asst. Prof. Ömer Faruk BEYCA ______________________________

It is approved that this thesis has been written in compliance with the formatting rules laid down by the Graduate School of Sciences and Engineering.

____
Prof. Nurullah ARSLAN
Director

April 2016

A PROPOSED ANALYTICAL MODEL FOR ORDER PICKING OPERATIONS IN MULTIPLE HORIZONTAL CAROUSEL STORAGE SYSTEM

Aziz AZIMOV

M.S. Thesis – Industrial Engineering
April 2016

Thesis Supervisor: Asst. Prof. Recep KIZILASLAN

ABSTRACT

In this study we present analytical model for order picking process in multiple horizontal carousel storage and retrieval system. Horizontal carousel systems are widely used in industrial applications. Carousel systems are kind of parts-to-picker warehousing system with high rates of return on investment, optimal storage density, full inventory control, low floor space of storing, user friendliness, modularity and easiness and swiftness of access to all stored inventories that aim efficiently and timely delivery of products to its customers. Under certain assumptions an analytical model and simulation models were developed to find the expected order picking time in multiple horizontal carousel system. Order statistics and occupancy problems are used in modeling the analytical model. The key advantage of the employed analytical model is the considerable reduction of time and money for companies. A presented model helps management to make decisions and perform analysis more easily and quickly compared to simulation models. We made analysis of the performance results of the analytical and the simulation models. Statistical analysis along with the data comparisons have been carried out. The results, showed the robustness of the analytical model versus the simulation model.

Keywords: Carousels; Multiple horizontal carousels, Expected order picking time.

BİRDEN FAZLA YATAY KARUZEL DEPOLAMA SISTEMİNDE SİPARİŞ TOPLAMA OPERASYONU İÇİN ANALİTİK MODEL ÖNERİSİ

Aziz AZİMOV

Yüksek Lisans Tezi – Endüstri Mühendisliği
Nisan 2016

Tez Danışmanı: Yrd. Doç. Dr. Recep KIZILASLAN

ÖZ

Bu çalışmada birden fazla yatay karuzel ürün depolama ve sipariş toplama sisteminde sipariş toplama işlemi için analitik model sunuyoruz. Yatay karuzel sistemleri endüstriyel uygulamalarda yaygın olarak kullanılmaktadır. Belirli varsayımları göz önünde bulundurarak birden fazla olan yatay karuzel sistemlerinde sipariş toplama işleminin beklenen değerini veren analitik model geliştirilmiştir. Ürünlerin beklenen toplama süresini bulmak için istatiksel yöntemler kullanılmıştır. İstatiksel yöntemlerden sırasıyle, karuzelin ürünleri toplama için maximum dönme süresini bulmak için sıralama istatistiği kullanılmıştır. Ayrıca, karuzel kapılarının ürün toplayıcının önünde her seferinde durması ve yeniden başka kapının gelmesi için giden zaman kaybı hesaba alınmıştır. Karuzellerin durma/kalkma süresini bulmak için olasılık istatistiğinden malum olan doluluk promlemi yöntemi kullanılmıştır. Daha sonra, analik ve simulasyon modellerini kurarar iki modelinde performans sonuçları analizleri yapılmıştır. Veri karşılaştırmaları ile birlikte istatiksel analizler yapılmıştır. Elde edilen sonuçlar analitik modelin simulasyona göre daha dayanıklı olduğunu göstermiştir.

Anahtar Kelimeler: Karuzeller, Birden fazla yatay karuzeller, Sipariş toplamanın beklenen değeri.

To my parents

ACKNOWLEDGEMENT

I express sincere appreciation to Asst. Prof. Recep KİZİLASLAN for his guidance and insight throughout the research.

I express my thanks and appreciation to my family for their understanding, motivation and patience. Lastly, but in no sense the least, I am thankful to all colleagues and friends who made my stay at the university a memorable and valuable experience.

TABLE OF CONTENTS

LIST OF TABLES

TABLE

LIST OF FIGURES

FIGURE

LIST OF SYMBOLS AND ABBREVIATIONS

SYMBOL/ABBREVIATION

AS/RS	Automated Storage and Retrieval System

CHAPTER 1

INTRODUCTION

Our main purpose in this study is to propose an analytical model which will be used in calculation of expected time of order picking activities in horizontal carousel storage and retrieval system. In this chapter information will be provided about the overall structure and problem definition of the thesis. Section 1.1 identifies the importance of the thesis subject. Section 1.2 refers to the area, limits and solution methodology of the thesis. Finally, Section 1.3 provides the basic structure of the thesis.

1.1 RESEARCH MOTIVATION

Growing product diversity and highly increasing demands of customers to receive orders in a short time and rivalry issues between companies are forcing businesses to make improvements within the company's system and supply chain. The outcomes of these challenges always contribute to system improvements.

Technology acquisition is an important factor for company's opportunity costs. Adoption of new technology makes operations of companies easier by allowing them to do things efficiently and quickly. Admittedly, this is an incremental process for manufacturing companies from the time frame and labor adoption. But the threshold in making decision for better performance results lead companies to choose modern technologies to accomplish these goals.

The recent developments in supply chain are way more progressive than they were in the past few years. A few technological breakthroughs have been achieved in supply chain by the emergence of AS/RS systems.

For supply chain, the main areas of researches though can be observed in AS/RS systems.

High operation efficiency rates could be achieved by using AS/RS systems in warehouses. Order picking process as a main operation in warehousing activities has significant impact on operating costs and efficiency. De Koster, et al. (2007) mentioned that order picking process stands for more than 50% of all warehousing costs.

Through effective design of technology and management of order picking systems, companies can achieve efficient warehousing performances. One of this highly effective order picking system which is widely used in industry is carousel storage and retrieval systems.

Carousel storage and retrieval system is parts-to-picker system commonly used in e-commerce industry and used to store and retrieve small, medium sized parts and office documents, in hospitality, pharmaceutical, biotechnology, manufacturing, finished goods, electronic, retail and apparel industries.

The main purpose of this study is to propose an analytical model which will be used to estimate the total order picking time in multiple carousel system. We developed an analytical model for order picking time procedure by adopting single carousel model of Meller and Klote (2004) to our multiple carousel system.

Analytical model and simulation models were constructed based on multiple horizontal carousel system where items identically uniformly distributed. Data comparisons and statistical analysis have been made between the analytical and the simulation models. The results of the proposed analytical model positively outperformed the simulation model by providing effective and reliable outcomes for order picking time problems in warehousing systems where decisions are made frequently and instantly.

1.2 RESEARCH TOPIC

1.2.1 Research area

Carousel storage systems are kind of parts-to-picker system which have series of bins linked vertically or horizontally and mounted to drive mechanism which rotate the mounted bins horizontally or vertically in a closed loop in order to bring the requested items to input/output station in front of a picker. In carousel system pickers do not walk through aisles to pick demanded items. Instead, items are brought to pickers by carousels.

Carousel systems have different areas of studies. Meller and Klote (2004) divided the problems of carousel system into three main areas. These are including: items placement problems within carousel bins (allocation), items optimal retrieval strategy problems (routing) and throughput of a carousel with human or robotic picker (performance problems) in carousel systems. The research area of this study considers performance problems which consider order picking time process in multiple horizontal carousel system with human pickers.

1.2.2 Research questions

It is believed that the main goal of every company is to earn higher profits by supplying high quality products to customers in low costs and short time. Though, order fulfillment in a timely and cost effective basis could be considered as a key factor in companies' financial growth. Companies have their own policy on how to fulfill the orders of customers. There are companies approaching individually to every order of customers by picking and dispatching them separately. This approach has its advantages and disadvantages and it is more suitable for enterprises where demands are diminutive. On the other hand assigning orders into groups and picking them together so as to complete sequential activities parallelly (waves of orders) is the most spread and economically reliable method for enterprises.

Having in mind customer's agility to receive the requested products in a timely manner, questions companies on how quickly and reliably they are able to supply the requested products to their customers. Companies need clear, accurate and reliable system which can produce near exact solution for customers orders picking procedure.

The reason of why order picking time is so important is that this process comprises more than half of the warehousing costs. And it has big time share and cost among other activities. Thus, this is a challenge for suppliers to propose the optimal delivery time to their customers.

The main focus of this study is to find an analytical model which gives optimal and reliable results for order picking time in multiple horizontal carousel system. The robustness of an analytical model will be tested through simulation studies.

The analytical model for order picking time will be based on multiple horizontal carousels where items are randomly and uniformly distributed. Several assumptions and operating parameters will be identified. Using assumptions and operational parameters analytical model will be constructed and several studies will be conducted.

The key advantage of the development of analytical models is the considerable reduction of time and money. In practice analytical models provide more accurate and robust results compared to heuristics. Analytical models facilitate the analysis of the performance of systems more easily and quickly compared to simulation models.

1.2.3 Solution methodology

The research stages and problem solution methods are as follows:

1.2.3.1 Analytical model

In this study we tried to build an analytical model which measures the expected order picking time of a wave of order in multiple horizontal carousel system by using statistical methods such as order statistics and occupancy probability problem.

1.2.3.2 Solution method

In horizontal carousel system there are numbers of bins (pick faces) with items which are linked together in a closed loop. There is a drive mechanism which rotates bins in clockwise or counter clockwise direction in order to bring the requested items to a picker. When order is received, a picker scans the bar code of an order and a control mechanism of a carousel identifies the locations of all items. A drive mechanism rotates pick faces to bring the requested items to a picker. Then, a picker collects all requested items from specific racks and puts them to a sortation conveyor. Order picking process continues until all requested items are retrieved from a carousel system.

In order to find the total picking time of an order the position of the last item within the bins of a horizontal carousel need to be identified. If the location of the last item can be found it would be easy to find the total rotation time of a horizontal carousel. There is a statistical method called order statistics which helps to find the location of the last item. On top of that, every time when a bin with specific items is brought in front of a picker, the system stops rotating bins in order for picker to pick the requested items from the shelves and starts moving bins to bring another bin to a picker. In this process there is a time spent for starting and stopping the bins in the system. The probability of how many bins are empty or not having the requested items could be found by employing occupancy problem.

Thus, order statistics and occupancy problem will be used in this study. Order statistics sequence the values of sample in ascending order by indicating the minimum and maximum values of the sample. We have adapted this method to find order picking time. Order picking operations completed if all items of a wave of order have been picked in all carousels. Assume that there are n items distributed in each carousel and if an analytical model is developed which gives the location of the farthest item within a carousel it would be easy to calculate the rotation time to pick items. Similarly, an analytical model which provides the location of last item within a carousel could be adapted to multiple carousels in order to find the last item picking time within a wave. In addition to order statistics occupancy problem was studied. As discussed in Section 1.1 carousels have bins or pick faces mounted in a closed loop which rotate clockwise or counter clockwise to bring the requested items to picker. Occupancy problem is about placing items into bins. There is determined start and stop time of pick faces. If requested items placed in particular pick face we will have to consider the time spent for carousel pause. By calculating start and stop time and rotation time we will be able to calculate the total order picking time for a wave of order.

1.3 THESIS OUTLINE

In this study after introduction part, in Chapter 2 a brief topic survey has been studied. In third chapter a brief literature review will be introduced. Than in fourth chapter order picking time analytical model for randomly and uniformly distributed items in multiple carousels will be proposed. In Chapter 5 the analytical model and the simulation models will be applied to multiple horizontal carousel system and data comparisons will be made to get the performance results of an analytical model. Lastly, in Chapter 6 the obtained results from the study will be analyzed and academic contribution and future works will be discussed.

CHAPTER 2

ORDER PICKING PROCESS IN CAROUSEL STORAGE AND RETRIEVAL SYSTEMS

In the literature there are many definitions given about the order picking activities in the warehouse. Frazelle (2002) defines order picking as a process of removing items from storage to meet a specific demand. Park (2012) defines order picking as the process of retrieving items from storage to meet a specific customer demand. The two definitions are same except another word used after one decade. It is clear from both definitions that order picking is the process in warehousing activity where the requested items are picked to be delivered to specific locations. Orders are picked manually, semi automatically and automatically. In this chapter we will refer to order picking process and carousel systems.

2.1 IMPORTANT FEATURES OF PICKING OPERATIONS

Order picking plays crucial role for companies in terms of having profit. Simply Tompkins et al (2003) state in their study about order picking that it is the most labor-intensive and costly process among warehouse processes which accounts for 55% of warehouse operating costs. In this regard, it is better for companies to analyze their company picking profiles. Emmet (2005) presents important features of picking operations which are might be useful companies to mention and identify the optimal way of management. The following are the important outlines for picking operations:

- Travel times.
- Product location

- Service level
- Accuracy

Travel times need to be managed in companies to cut unnecessary waste of time. Through analyzing the warehouse activities the time distribution different routes and facilitative solutions can be identified and achieved.

Allocation of products is another issue which is also facilitates to fulfill order picking process. Finding the less travel consumable products allocation policy is utmost demanded feature in order picking process.

Service level is important feature for customers. The customer satisfaction on fast delivery of the requested items can be achieved by optimizing picking, packing and dispatching operations. Companies need to find the balance between costly effective services and delivery.

Accuracy is another most important feature of order picking process. The wrong delivery of items to customers affect not only the items cost but also the reputation of company. A 1% pick error may bring 20% or more increase in cost (Emmet 2005). To avoid such mistakes companies have to analyze all activities and choose the optimal costly effective solutions.

2.2 ORDER PICKING STRATEGIES

Every warehouse facility has its own order picking strategies depending on order amount, facility layout, material handling machines, and the rate of automation. Below is the order picking strategies given in literatures:

- **Basic order picking**. In this order picking strategy picker travels line by line to pick items of individual order in the storage area using proper material handling machine.
- **Batch picking.** In this order picking strategy picker picks requested items in a batched form which consists of group of multiple orders.

- **Zone picking**. In this strategy the storage area is divided into zones with a designed order picker in each zone. When order is requested it passes through only those zones which have the items in the order list.
- **Wave picking.** In wave picking strategy all customers' orders are picked together within all picking areas and later sorted into individual orders.

2.3 ORDER PICKING METHODS

As mentioned earlier, orders can be picked manually, semi automatically and automatically. Lahmar (2008) divided order picking methods into two parts: order picking by humans and machines.

Order picking with human interventions can be grouped into three categories: picker-to-parts, put system and parts-to-picker.

Picker-to-part order picking system is the most usual system. Picking is handled by pickers by picking single orders or batch of orders. There are two variants of picker-to-part system: sort-while-pick and pick-and-sort. In sort-while-pick case parts are sorted immediately while picking. In pick-and-sort case parts are sorted after the picking process has been completed.

Parts-to-picker system is the order picking method where parts are brought to a picker by automated material handling machines. In this case picker mobility is extremely less due to no necessity of pickers move to parts. This system involves automated storage and retrieval systems, mini-loads, vertical lift modules, horizontal carousels and vertical carousels. In this study we are aimed to study performance model of horizontal carousel.

Put systems are took place between picker-to-part and parts-to-picker systems. It includes principles of both systems.

Pick-and-pass (or pick-to-box) and pick-and-sort order picking systems are most common systems which can be considered put systems. Pick-and-pass or pick-to-box system deals with the order picking process where storage area is divided into regions and these regions are linked by conveyor belts in which individual customers boxes are travel the regions where the requested items are located and be picked by pickers to fulfill customers order request.

According to Lahmar (2008) put systems are commonly used in large number of orders that have to be picked in short period of time. The schematic view of order picking methods is given in Figure 2.1.

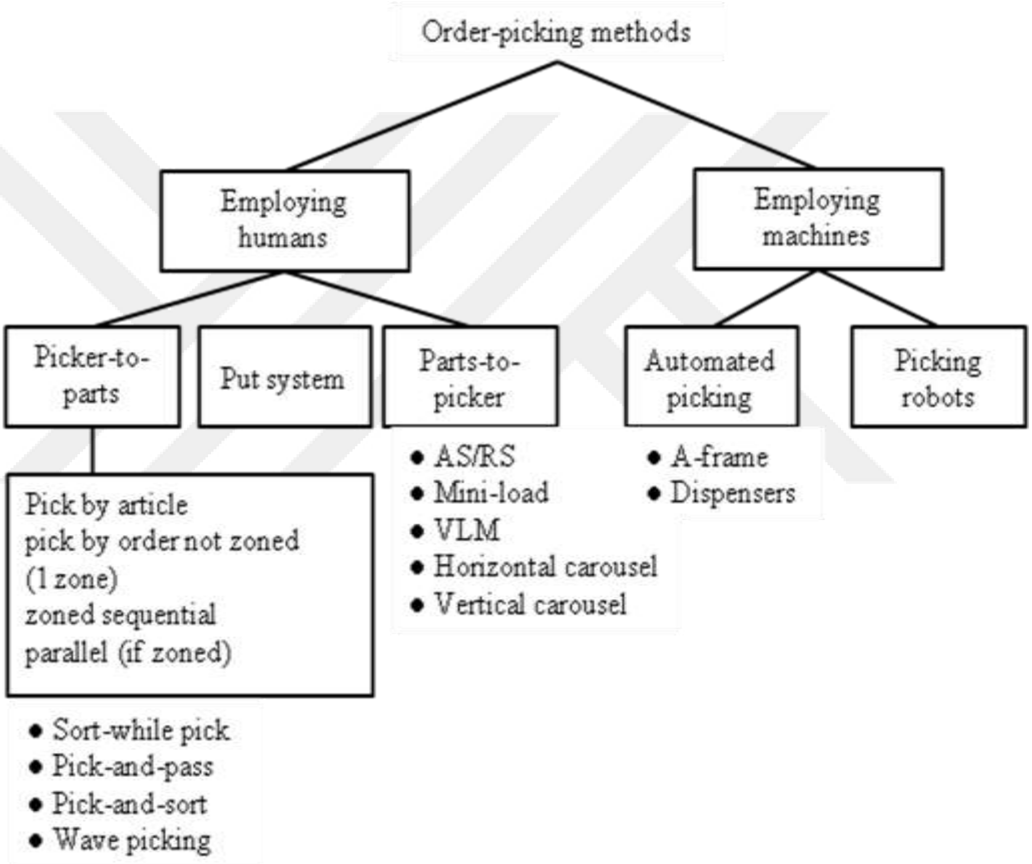

Figure 2.1 Order picking methods (de Koster, 2008).

Order picking with machines can be grouped into two categories: automated picking systems and system with robotic pickers.

Order picking system which do not include human intervention are called automated order picking systems. Orders are picked by automated machines instead of human pickers. They are controlled by computers which include sophisticated software programs. A-frame and dispensers examples of automated order picking machines (See Figure 2.2).

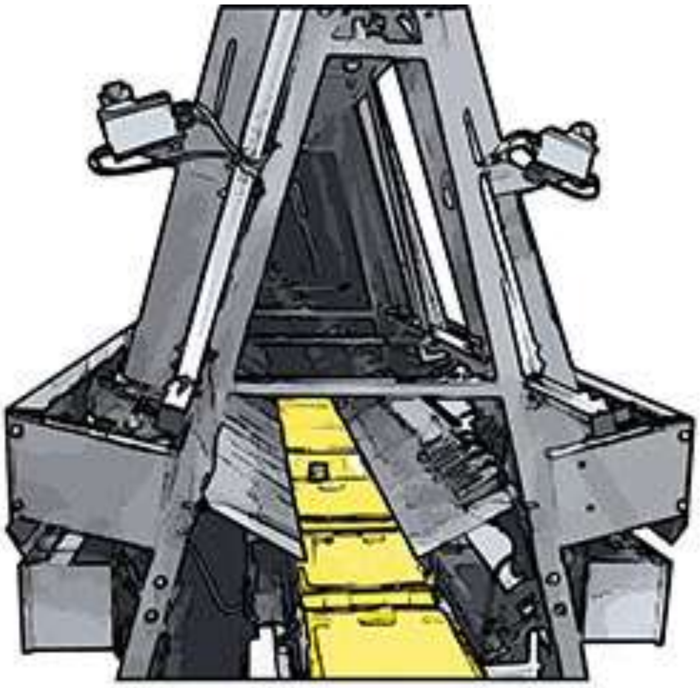

Figure 2.2 A-frame order picking machine (Source: http://mhwebportal.org/node/52).

Similarly to automated order picking systems there are order-picking systems which employ robotic pickers. As an example of order picking with robots can be carousel systems with robot picker.

2.4 CAROUSEL STORAGE AND RETRIEVAL SYSTEMS

In this chapter we will discuss about parts-to-picker order picking systems which are widely being used in industry nowadays. The aim of this chapter is to familiarize with current parts-to-picker order picking systems. These systems will be the base of our study in the following chapters. Vertical lift modules and carousel storage systems are called parts-to-picker order picking systems. Before introduction of parts-to-picker systems we would like bring arguments about the necessity of such systems.

On the one hand advancements in warehouse technologies brought new opportunities to manufacturers to respond quickly to customers need.

On the other hand, with the emergence of e-commerce the need for more flexible and reliable warehouse technologies has raised. Fisher (2000) lists the operational advantages of technology acquisition as follows:

- Quickness and convenience of taking orders for customers
- Quickly and profitably fulfillment of orders.

Lastly, based on second argument which mentions the importance of immediate order dispatch, the travelling distance becomes important criteria for suppliers in order to fulfill demands of customers in a timely manner. Tompkins (2003) have studied order-picker time distribution by activities and found that 50% of order-picker time goes to travelling among other activities within warehouse for picking requested items and consequently by acquisition of parts-to-picker systems these losses will be reduced rapidly (Figure 2.3).

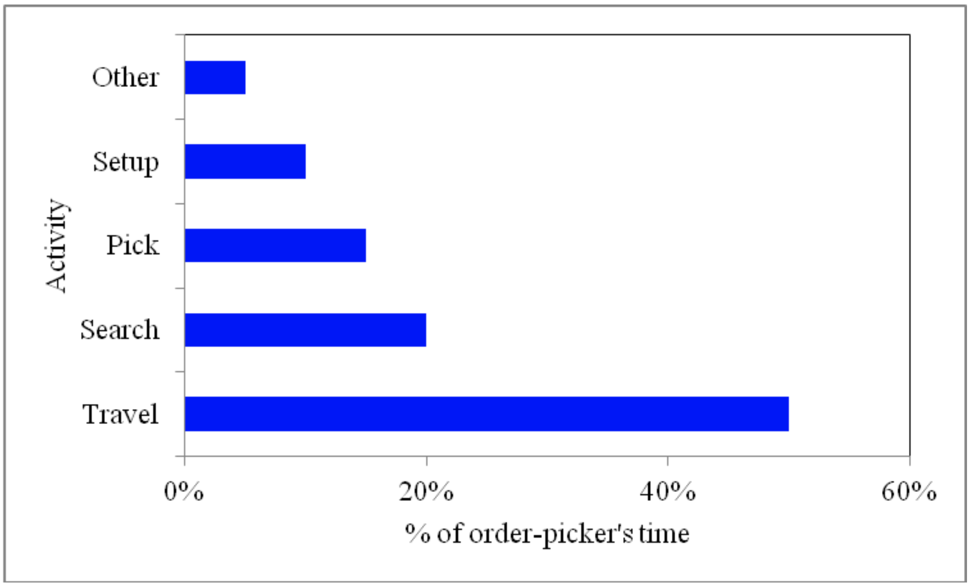

Figure 2.3 Order picker time distribution (Tompkins, 2003)

Coyle et al. (1996) provide another important statistic which states that order picking cost averagely is 65% of the total operating costs of the distribution center. Technology acquisition to warehouse is the solution to eliminate these costs and losses and to increase the efficiency of system.

The following automated storage and retrieval systems are one of those technologies which can reduce travel times, order picking times and related costs accordingly. Also we want to note that VLMs and Carousel AS/RS systems are designed to store small and mid size items. The advantages of carousel systems include better floor space utilization, better organization of parts as well as tracking and sequencing, substantial labor savings, reduced number of injuries, high productivity enhancement and improved accuracy in picking, among others (Trunk, 1996).

2.4.1 Vertical Lift Modules (VLM)

Vertical Lift Modules are AS/RS systems designed around a vertical module with center aisle lift which is used to store large inventories in a unit load basis. VLMs are widely used in production facilities and warehouses. One of the primary advantages of VLMs is better space utilization by using overhead space and service level. Figure 2.4 shows the basic illustration of VLM. It has extractor or mini-load in between two racks which moves vertically up and down to bring stored tray where the requested order items are placed. There is one pick position in VLM where operators handle both retrieval and storage processes.

VLMs maximize space utilization with minimal labor.

Computer controlled system allows to minimize order picking errors significantly.

With its modular design it is capable to store items with different sizes. Thus it allows storing a large number of inventories with less space and quick and easy access to pick items.

Benefits of VLM:

- Storage space efficiency
- Reduced labor costs

- High rate of order picking speed
- User friendly
- Modular
- Travel time diminution
- Minimized order picking errors due to computer controlled systems
- Performance accretion

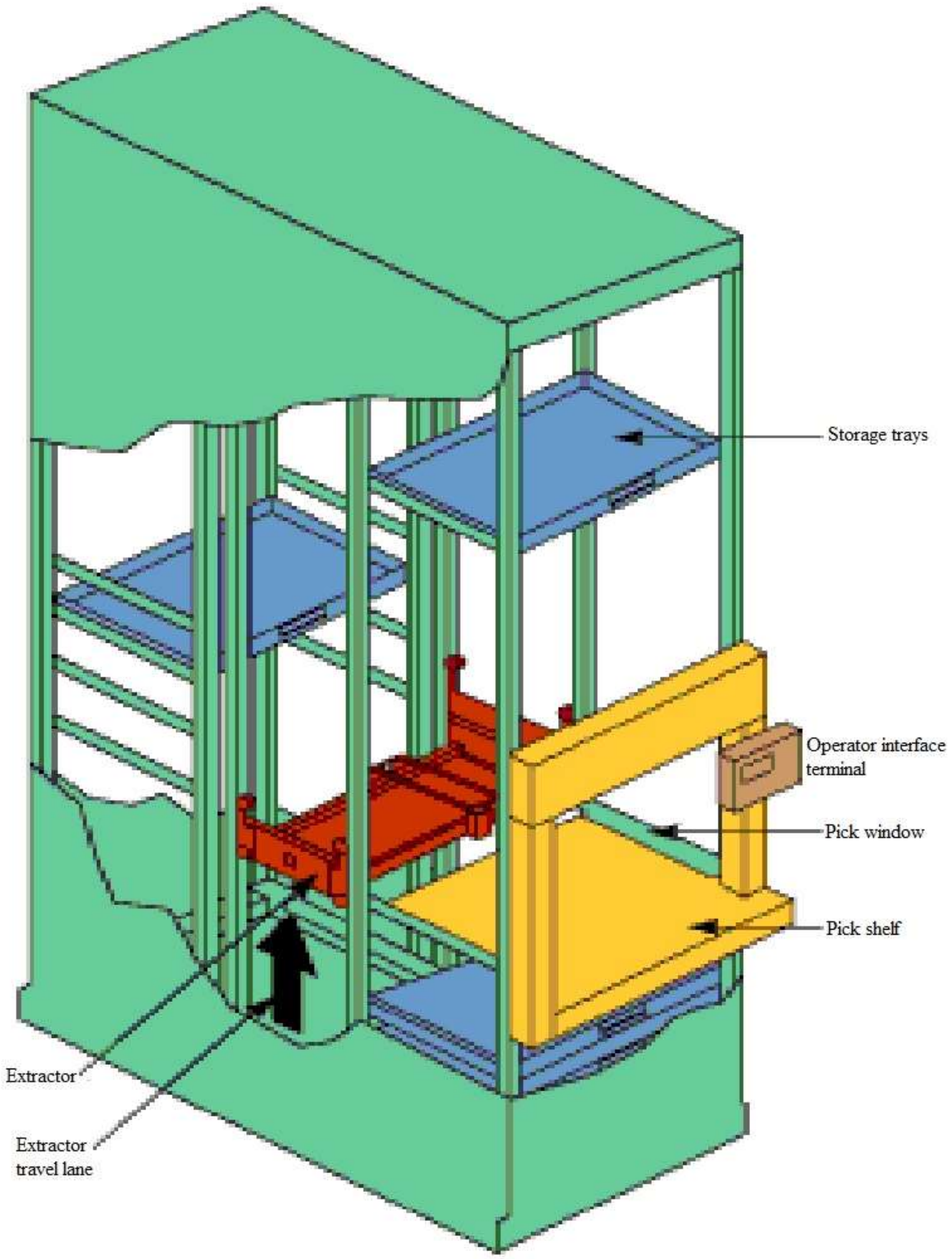

Figure 2.4 Vertical Lift Module Illustration (Source: www.mh-partners.com).

The most important feature of VLM systems is the increase of order picking performance. Although VLMs have many benefits in warehousing system there are some limitations too. It is restrained to store small sized parts within its modules.

2.4.2 Vertical Carousel Storage and Retrieval System

In Section 2.3 we presented order picking methods grouped into two groups which employed human interaction and machines. We also mentioned that Vertical Carousel Storage and Retrieval System or vertical carousels are categorized as parts-to-picker or stock-to-picker order picking systems which are used store and retrieve small and medium sized products.

A vertical carousel is AS/RS system which consist horizontally linked series of bins with drive mechanism which rotates vertically in a closed loop and brings the requested bins automatically to the input/output station in front of a picker (Figure 2.5). Vertical carousels rotate both clockwise and counter clockwise. Bidirectional (two way rotating) types of carousels are the most common used in industry (Hassini & Vickson, 2003). The main advantage of the vertical carousel is that a picker does not travel to collect products through storage area rather it brings the requested items to a picker.

Vertical carousels are suitable for storage and picking of textile products, small, medium sized parts and office documents in industries such as hospitality industry, medical sector, vehicle manufacturing, mechanical engineering, electronic industry, finished goods industries and retails.

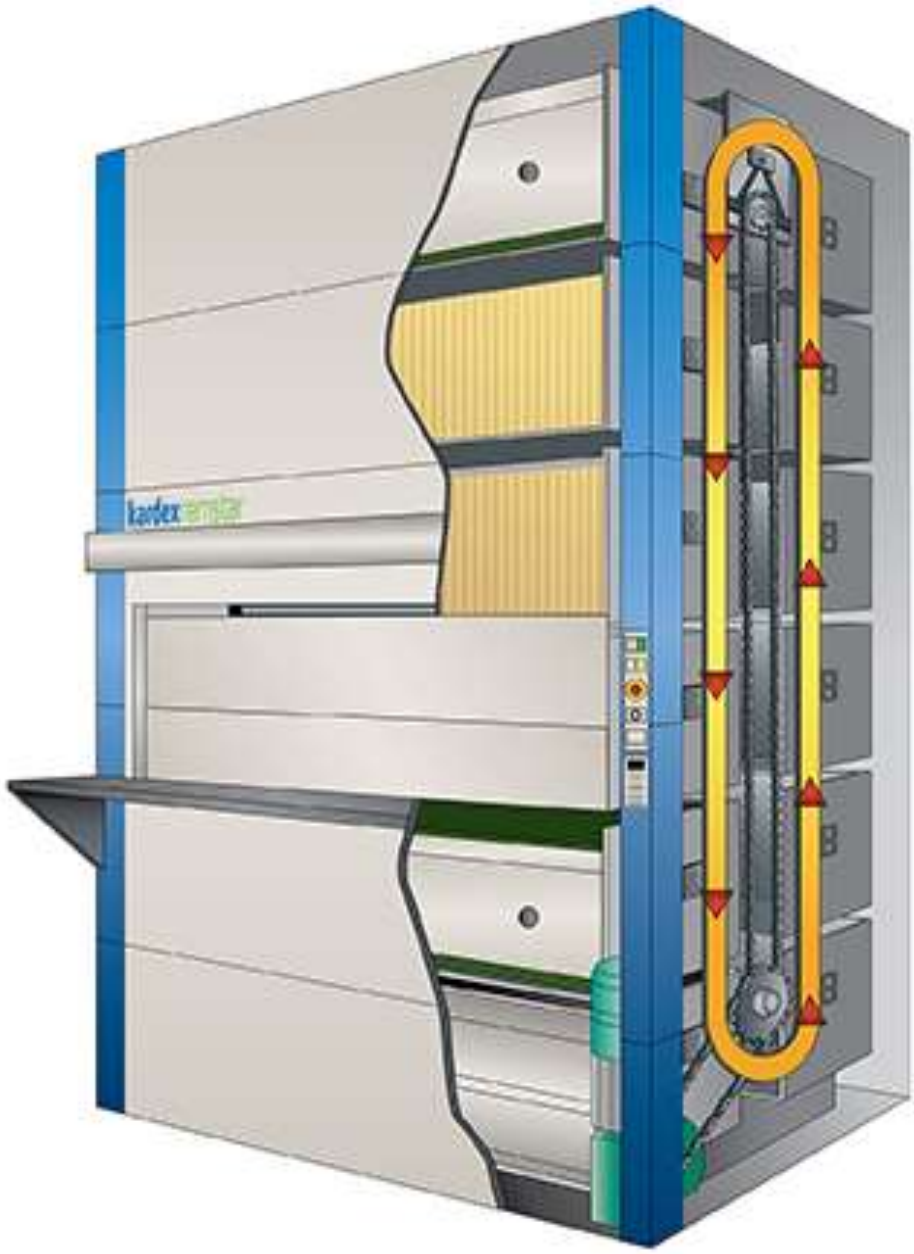

Figure 2.5 Vertical Carousel (Source: www.kardex-remstar.co.uk).

There are different types of vertical carousels being used in industries depending on requirements of users. Due to its modularity, flexible design and better space utilization it can be set up anywhere.

Below are the benefits of Vertical Carousels:

- High cycle speed
- Ease and fast access to all stored inventories

- Highly dense storage spaces within the carriers
- Maximum use of space in facility
- High rate of picking accuracy
- Energy saving technology
- User friendly
- Modularity
- Performance accretion

Vertical Carousel enables to track stored items accurately by inventory management system. It is mostly suitable for storing office documents and subsequently facilitates the document management process in companies.

2.4.3 Horizontal Carousel Storage and Retrieval System

A horizontal carousel has the same principles of vertical carousel as discussed in Section 2.3.2 except vertically linked and mounted bins position.

A horizontal carousel is AS/RS system which has vertically linked series of bins with drive mechanism which rotates horizontally in a closed loop and brings the requested bins automatically to the input/output station in front of a picker. Horizontal carousels can rotate both clockwise and counter clockwise. Bidirectional (two way rotating) types of carousels are the most commonly used in industry (Hassini & Vickson, 2003). The main advantage of the horizontal carousel is that pickers do not travel through storage are to collect products rather it brings the requested items to a picker.

Horizontal carousels are used to store and retrieve small and medium sized parts, in industries such as pharmaceutical, biotechnology, manufacturing, finished goods, electronic industry, finished goods industries, retails and apparels.

Horizontal carousels provide high pick accuracy, picking rate, pick density, accurate storage location, and good segregation of goods.

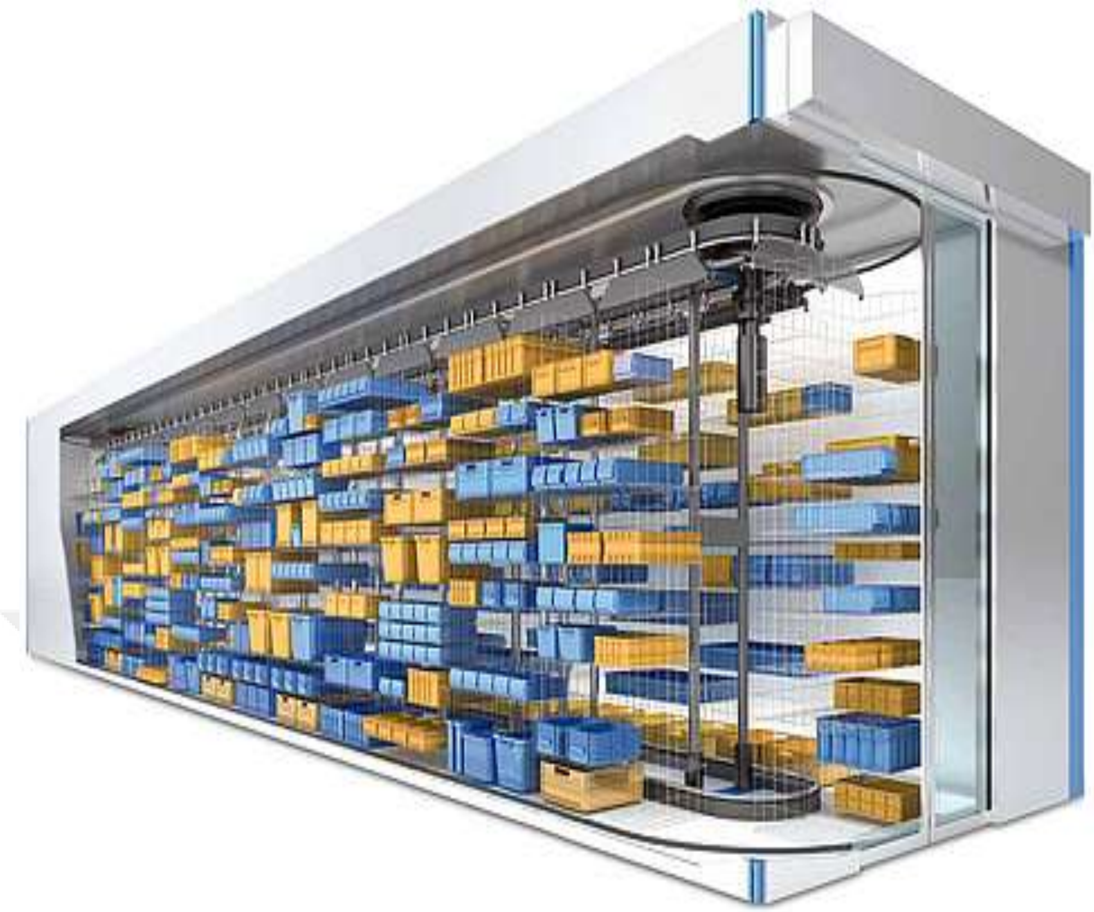

Figure 2.6 Horizontal Carousel (Source: www.shelfplus.com).

In single carousels one order is processed but in multiple carousels more than one order can be processed at the same time.

In multiple parts-to-picker and pick-and-pass (pick-to-box) order picking methods can be combined.

In parts-to-picking method batch of orders are picked from horizontal carousels and dispatched to sortation conveyor.

In combination of both parts-to-picking and pick-and-pass order picking method items are picked from horizontal carousels and sorted by pickers to designated boxes immediately. Then picked and sorted orders are dispatched to packing process for order delivery.

Below is how horizontal carousel operates order picking process.

Order picking process works as follows in multiple carousels: orders are received by purchasing department. Then batch of orders are assigned to carousels on randomly or uniformly basis. A control system operates carousels to bring the requested items to input/output point. When specific orders are arrived to I/O point pickers take and put them to sortation conveyor lane or sort them immediately to designated boxes of customers and dispatch them to packing process.

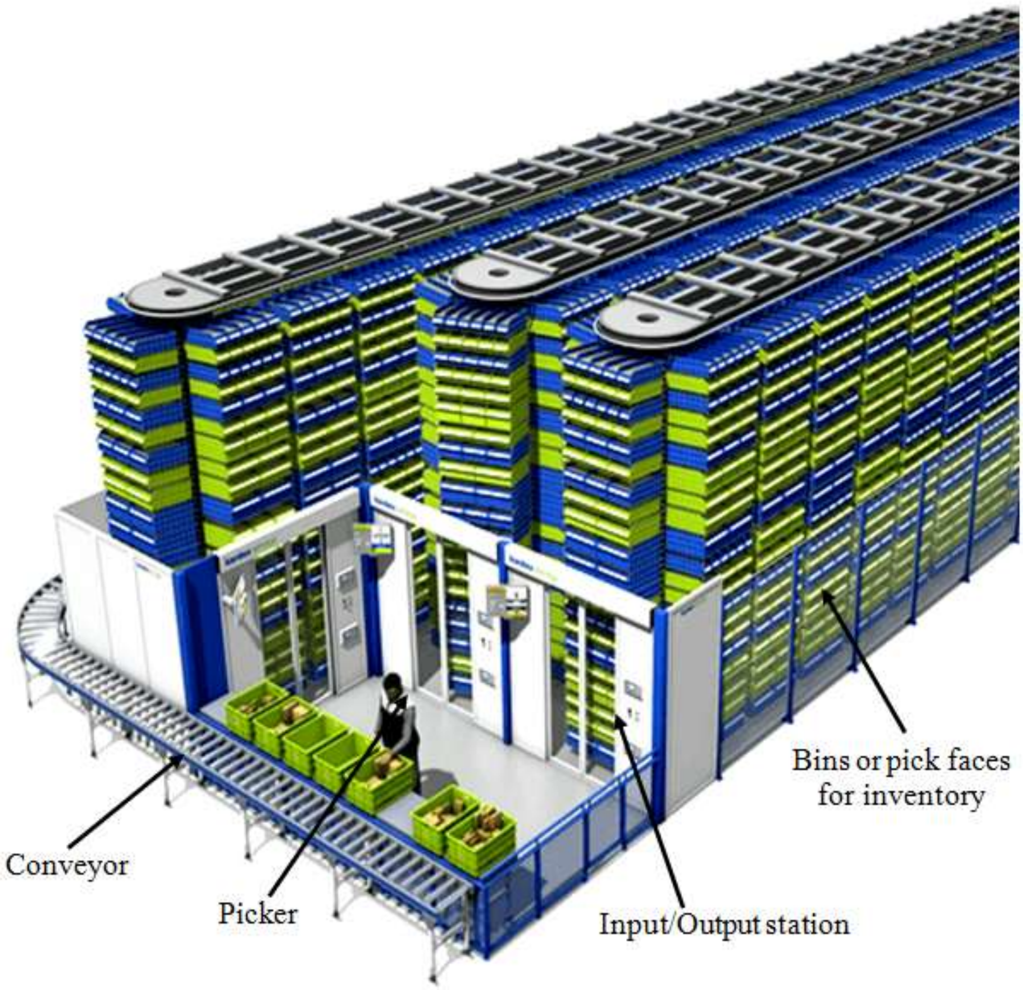

Figure 2.7 Sample Multiple Horizontal Carousels
(Source: www.kardexhandlingsolutions.com)

Modern horizontal carousels have pick and put-to-light displays with vertical shelf LED lights which show the exact position and quantity of items to be retrieved. Pick to light system increases picking accuracy, traceability of SKUs and obviously decreases items search time. Pick-to-light work as follows: containers are placed in a special pick to light tray, an operator connects to the system to get order requests with individual barcodes and pairs by scanning order request barcodes with containers barcode. All lights of designated containers illuminate with respective pick quantities. Work indicators will display containers identification. Then required items are placed in relevant containers. Operators pushes button of pick to light system to confirm the pick. Upon completion of all items, the work indicator will show that picks have been finished. For more information one may visit material handling equipments companies' websites.

Below are the benefits of Horizontal Carousels:

- Productivity
- Ease and fast access to all stored inventories
- High payloads
- Optimal space utilization
- Batch picking
- High rate of picking accuracy
- Energy saving technology
- User friendly
- Modularity
- Picking performance

In conclusion we can highlight important features of horizontal carousels as storage density, picking performance, accuracy, high throughput, efficient inventory management and decreased labor hours.

In the following chapter we will discuss about academic overview of carousel systems.

CHAPTER 3

LITERATURE REVIEW

3.1 OVERVIEW OF ORDER PICKING OPERATIONS

Carousel storage and retrieval system literature is extensive. The basic forms of the related works goes to the late 80th due to wide use of the technology in industries especially in warehousing. In this part of the thesis we will refer to some older publications which are considered as ground works in carousel storage and retrieval systems. Further we will refer to the recent literature review in this field by referring to some distinguished authors publications. Meller and Klote (2004) classified carousel storage and retrieval warehousing system literature into three categories:

1. Rotation strategies (retrieval strategies for a set of items)
2. Assignment of the storage locations (assigning items to locations in the carousel)
3. Performance models (throughput problems)

Initial examples of research on carousel storage systems derive from Bartholdi and Platzman (1986). They wrote the seminar paper that present an algorithm that finds the optimal retrieval sequence in linear time (linear in the number of items to be picked) for the single order problem. They also propose simple heuristics, some of which showed good performance bounds and promise to be near optimal when an order is spread over most locations on the conveyor.

For the multiple order situations, Bartholdi and Platzman make the assumption that the orders may be retrieved in any sequence. Based on that premise and a second assumption that an individual order is always retrieved along the "minimum spanning interval" they develop a near-optimal scheme with an excellent performance guarantee.

They proceed to offer once again simple heuristics, and argue that the performance of these heuristics will approach optimality as the number of items in an order becomes large.

Ghosh and Wells (1991) have developed algorithms of optimal retrieval strategies for both single and multiple orders. They demonstrate a dynamic programming solution to the multiple order case with a FIFO procedure, and provide a bounding scheme that improves the computational efficiency of the solution.

Vickson and Fujimoto (1996) present product assignment problem in carousel storage and retrieval system which considered as an optimal scheme. They bring a heuristic which constitutes positive correlation of a few of the largest demands due to the fact of keeping high demands the items in close proximity. Briefly organ-pipe storage assignment policy aims to minimize retrieval time of single items for a fixed sequence of items. The organ-pipe storage policy assigns high demanded items first and groups them according to the size of bins. The most-active group is assigned arbitrarily to a bin then the next-most-active groups assigned next to that bin. The third group of items is placed in the opposite side of first group. The next fourth group is assigned to the next of the second group and so forth. Authors mention that organ-pipe arrangement is not an optimal storage policy when items are batched.

Hwang et al. (1999) measure analytically the throughput model by applying double shuttle storage and retrieval machine in the standard and double carousel storage system. The results showed the advantage of double shuttle (DS) over single shuttle in four command cycle in double carousel system.

Jacobs et al. (2000) presented a simple heuristic that enjoins the number of cases needed for each item in the load. According to Jacobs et al. the performance of the carousel depends on it method of how it was loaded and the way products distributed on carousels. They developed a heuristic that helps to determine the storage capacity of a carousel and the amount of cases that are needed to load on it.

Litvak et al (2000) proceeded the works of Bartholdi and Platzman (1986), Stern(1986) and Rouwenhorst (1996) providing a tight upper bound for the travel time and developed a procedure to get the mean travel time of carousel by using nearest item heuristic. In their model they consider only travel time in order to minimize the total pick time in a single carousel machine.

Din-Horng Yeh (2002) considers Jacob et al. (2000) carousel storage system and proposed a simpler approach that maximizes the service level of carousel storage system. An iterative heuristic is proposed to maximize the number of groups of items that helps to retrieve items with minimum stock outage without encountering and which gave more accurate results compared to Jacobs et al (2000).

Hassini and Vickson (2003) developed formulation of a problem of storing products in two carousels with single operator and proposed heuristics which minimize rotation time in a retrieval operation. It is mentioned that the efficiency of a carousel depends on layout, number of layers and direction of rotation. They developed four heuristics solutions namely generic algorithm, alternating, zigzagging and greedy, which helps to partition 2n items into two carousels. In each heuristic items are sorted in decreasing order according to their frequencies. In this paper as the result it is shown and proposed that for less utilized carousel system the zigzagging heuristic is advisable while for more utilized ones generic algorithm heuristic is more suitable.

Meller and Klote (2004) developed throughput analytical models for single carousel, a pod of carousel and vertical lift modules with human picker picking batch of orders. The model of Meller and Klote suggest carousel storage system with human picker. The results of testing of the carousel and VLM throughput models performed well under different system configurations.

Zhang et al. (2004) developed the execution time model for the problem of pick sequencing in the rotary rack S/R machine (PPS-RRS) by proposing different scheduling policies. In addition several sequencing algorithms proposed for sequencing problem. As a rotary rack they mention multiple level horizontal carousels with their own drive units which rotate independently. They considered only the pick sequencing process and prescribed the routing optimization process as a combinatorial problem of PPR-RRS system. A set of optimization algorithms developed namely conventional sequence method (CS), local search (LS), generic algorithm (GA) and a new hybrid generic algorithm (HGA) to obtain a solution for sequencing and total execution time. Authors proposed the more advisable algorithms as the local search algorithm which provided better results for medium-scale PPR-RRS where HGA gained advantage over other sequencing algorithms by easily obtaining the optimal or near-optimal solution sequences within the solution space.

Kim (2005) considered a carousel storage problem proposed by Jacobs et al (2000) for maximizing service of carousel storage and showed Yeh’s (2002) heuristic insufficient in getting optimal solution and provides a modified algorithm of Yeh which is shown to give optimal solution in polynomial time (i.e. a polynomial function of the dimension of the problem (for example, on the amount of its input)).

Park and Rhee (2005) present the performance model of organ-pipe storage policy. In this study the performance measures of the model are system capacity which considers organ-pipe storage policy, job sojourn time - the time from job request until job completion (Park and Rhee (2005) and rotation time. For carousel storage and retrieval system they used floating dwell point strategy rather than non-mandatory fixed dwell point strategy. By presenting the performance model they show that wittily assignment of the totes can increase the throughput up to 30%.

Litvak (2006) presents an analytical model with non-uniform continuously distributed items throughout carousel. Prior to this study there is no works related to the travel time model with non-uniform items location. The aim of the study is to propose algorithms which help to find travel time to collect a single order in carousels with m pick faces with fixed-dwell point strategy. Mathematical expressions and simulation of the model were provided with different allocation rules. The results of the simulations have shown that the average travel time under non-uniform item distribution with cleverly allocation provide better results compared to uniformly distributed items in carousel systems.

Litvak and Vlasiou (2010) present a review paper which includes performance evaluation and design of carousel storage and retrieval systems. This paper discusses problems related to picking strategies and throughput in a single and two carousels respectively. The paper presented with introduction which briefly refers carousel systems and classifies them into groups. The reasonable order picking strategies and the challenges of the performance models of multiple carousels are described. A broad literature review with good taxonomy provides a better overview in carousel storage and retrieval systems.

Pazour and Meller (2013) proposed analytical and simulation model by analyzing the impact of a batch processing strategy on the throughput performance of a complex order fulfillment technology, a carousel system with an S/R machine. This work is actually the further advancement of Meller and Klote (2004) models with robotic picker. Through validation of their analytical model they provide good estimates that batch processing has the ability to increase the throughput of a carousel system. By testing the batch retrievals they showed that the impact of batching on throughput performance resulted an average improvement of 20.03% over the case of sequential sequencing.

Wang et al (2013) present a single order picking problem in pod of carousels with one picker. The aim of the study is to minimize total order picking time. Several heuristics were developed namely dynamic programming algorithm is proposed for two carousel storage and retrieval system and improved nearest items heuristic. Experiment has been conducted following the given heuristic rules to make comparison with existing algorithms such that conventional search (CS) and nearest item heuristics (NIH). The results of the experiment have shown that the proposed algorithms outperform the existing heuristics in terms of getting swift and steady optimal solutions. General overview on carousel storage and retrieval systems are summarized in Table 3.1.

Table 3.1 Literature Review about Carousel storage systems

Writer	Method	Objective/decision problem
Bartholdi and Platzman (1986)	Heuristic	Retrieval strategies for a carousel conveyor. An algorithm that finds the optimal retrieval sequence in linear time (linear in the number of items to be picked) for the single order problem.
Wen and Chang (1988)	Analytical	Picking rules for a carousel conveyor in an automated warehouse.
Ghosh and Wells (1991)	Heuristic	A dynamic programming solution to the multiple order case with a FIFO procedure in a carousel storage system.
Vickson and Fujimoto (1996)	Heuristic	Optimal storage locations in carousel storage and retrieval systems.
Hwang, et al.(1999)	Simulation	Performance analysis of carousel systems with double shuttle.
Jacobs et al. (2000)	Heuristic	A simple heuristic that prescribes how many cases of each item type should be loaded.
Litvak et al. (2000)	Heuristic	They proposed nearest item heuristic for carousel systems which minimizes the total pick time in a single carousel machine
Din-Horng Yeh (2002)	Heuristic	A simpler approach for the works of Jacobs et al. (2000).
Hassini and Vickson (2003)	Analytical, Heuristics	A two-carousel storage location problem with single operator and proposed heuristics which minimize rotation time in a retrieval operation.

Table 3.1 (cont.)

Meller and Klote (2004)	Analytical, Simulation	A throughput model for carousel/VLM pods.
Zhang et al. (2004)	Heuristic, simulation	Pick sequencing optimization problem in the rotary rack S/R system.
Kim (2005)	Heuristic	Maximizing service of carousel storage. Modified solution to Yeh (2002) algorithm.
Park and Rhee (2005)	Heuristic, Simulation	They presented the performance model of organ-pipe storage policy
Litvak (2006)	Analytical	Optimal picking of large orders in carousel systems
Litvak and Vlasiou (2010)	Literature review	A Survey on Performance Analysis of Warehouse Carousel Systems
Pazour and Meller (2013)	Analytical, Simulation	The impact of batch retrievals on throughput performance of a carousel system serviced by a storage and retrieval machine
Wang et al. (2013)	Heuristic , simulation	Order picking optimization in carousels storage system. They presented a single order picking problem in pod of carousels with one picker the aim of which was to minimize total order picking time.

CHAPTER 4

ANALYTICAL MODEL OF ORDER PICKING OPERATION

The main goal of this chapter is to search for analytical formula which gives an answer to the question of how the independently and identically distributed items of waves of order operate in carousel storage and retrieval system.

Statistical methods which describe order picking process in horizontal carousel systems are order statistics and occupancy problem. These methods will be used in formation of the analytical model.

This chapter gives information about the goals, assumptions, model parameters and solution methodology of this study.

4.1 INTRODUCTION

Order picking operations is one of the most important operations within the warehouse. Because of its importance both in academic and business fields there are many studies have been done on this topic. Information about the studies in this area was provided in previous chapter.

There are four basic system parameters that affect the performance of order picking operation (Kizilaslan, 2014):

1. Product parameters. The sizes of different products, sizes of space within the shelves.
2. Picker parameters. Velocity and number of pickers.

3. Operational parameters. Sequencing rules according to which orders are picked, the location of products in storage area etc.
4. Physical parameters. Number of shelves, height, sizes, length of horizontal carousels, number of carousels, width etc.

These parameters are important in designing, implementing and using both carousel systems and other systems in warehouses.

4.2 PROBLEM STATEMENT

In this section an analytical formula will be presented which will be used to find order picking time in multiple carousels with different amount of wave of items.

4.3 ASSUMPTIONS

In this analytical model the following assumptions have been implied:

1. Items are uniformly distributed throughout all carousels.
2. Opportunistic replenishment policy is implied.
3. There is one order picker in each carousel.
4. Order picker's picking time is known and constant.
5. Carousels speed is known and constant.
6. The lengths of all carousels are same.
7. There is equal number of bins in all carousels.
8. Start and stop time of bins are known and constant.
9. Acceleration and deceleration of carousels are ignored.
10. A no-reversal strategy is applied to all carousels.
11. Items size is suitable to all shelves of carousels.

4.4 PARAMETERS OF THE MODEL

Operational and design parameters of an analytical model are given in Table 4.1:

Table 4.1 Operational and design parameters of order picking time

Notation	Explanation
C	*number of carousels (pcs)*
w	*number of items within a wave (pcs)*
n	*number of items within an order (pcs)*
T_r	*total time to rotate the carousel one complete revolution (sec)*
T_s	*carousel start and stop time for order-picking (sec)*
T_p	*picking time of one item (sec)*
m	*pick faces (bins)(pcs)*
l	*length of the carousel (m)*
v	*speed of the carousel (m/s)*
k	*pick faces (bins) at which carousel does not stop in retrieving of n items(pcs)*
i	*carousel index*
R	*number of rows within a carousel(pcs)*
S	*total storage space, size(pcs)*
$T_c(i)$	*total picking time of all items in carousel i (sec)*
T_w	*total picking time of all items in a wave (sec)*

4.5 SOLUTION METHODOLOGY

In this section we will present a solution method that used to estimate the total order picking time of waves of orders in multiple carousels. Below is the concise description of the methodology.

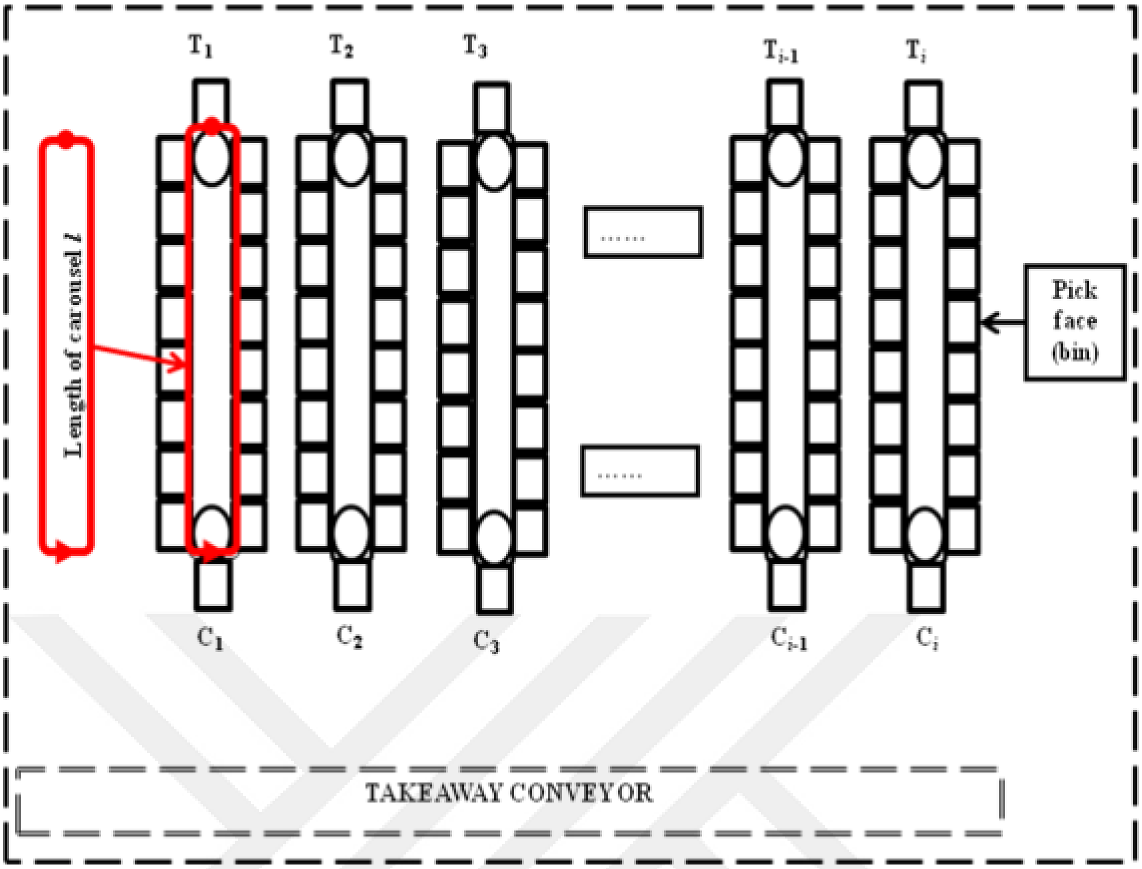

Figure 4.1 Top view of order picking operation in multiple horizontal carousels.

Items are stored in *i* number of carousels. The length of each carousel is *l* and the rotation speed of a carousel is constant. Each carousel has *m* number of pick faces (bins) and assigned picker with determined picking time. Pickers collect the requested items only in their own responsible carousel and put them to a takeaway conveyor.

In Section 1.2.3.2 we have discussed about order picking process operations in horizontal carousels. As we mentioned at that section, there is time spent for a horizontal carousel to rotate bins either clockwise or counter clockwise direction in order to bring specific bins which have requested items to input/output point to a picker.

Additionally, in horizontal carousels there is start and stop time of bins to move to another bin to collect items.

Consequently, there are three important components of order picking process in horizontal carousels (Meller & Klote, 2004):

1. Rotation time of the carousel, E[R (i,n,m)],
2. Start and stop time of each carousel, E[S (i,n,m)],
3. Picker's picking time of each item, E[P (i,n,m)].

The summation of these three components will give the order picking time in a single carousel.

The total time to pick all requested items from each carousel equals to $T_c(i)$ (ex: the first carousel total picking time is $T_c(1)$). Total order picking time includes rotation time, start/stop time and pickers picking time. Order picking process considered as finished if all items of a wave of order are picked from the all relevant carousels.

In order to find the rotation time, start/stop time of a carousel, the location of the farthest item of a wave of order and the number of occupied pick faces must be detected. For instance assume that the items of a wave of order are located in 2nd, 3rd, 5th, 7th and 9th carousels with 30 pick faces each. Assuming the constant speed of carousels and pickers picking time we may say that the total order picking time of a wave of order within the multiple carousels would be equal to the total order picking time a carousel with the maximum rotation, start/stop and picking time summation. Briefly let's assume above 2nd, 3rd, 5th, 7th and 9th carousels have total order picking time of 71, 121, 141, 86, 99 seconds respectively. In this case we may say that order picking of a wave of order would be finished if all items in the 5th carousel are picked.

Accordingly, we will use order statistics to find the rotation time of carousels and occupancy problems to determine the number of pick faces that have items.

4.6 ORDER STATISTICS

Order statistics sequence the values of random samples in ascending form by indicating the minimum and maximum values of the sample.

Suppose, there are five numbers are observed as a random sample size of 5. The sample values are x_1=6, x_2=1, x_3=7, x_4=2 and x_5=3. The sample order statistics would be the sample values in ascending order which satisfy,

$$X_{(1)} \leq X_{(2)} \ldots\ldots X_{(n)}$$

Thus, the above samples will be sequenced as $X_{(1)} = 1$, $X_{(2)} = 2$, $X_{(3)} = 3$, $X_{(4)} = 6$ and $X_{(5)} = 7$ according to order statistics.

The samples are independently and identically distributed (iid). Order statistics are dependently distributed because of order restriction.

There are first (minimum) order statistics and largest order statistics (maximum) of the sample of size *n*.

$X_{(1)}$ is the minimum of the sample n

$$X_{(1)} = \min \{X_{1\ldots} X_{n}\}$$

$X_{(n)}$ is the maximum of the sample n

$$X_{(n)} = \max \{X_{1\ldots} X_{n}\}$$

For $X_{1}, X_{2} \ldots X_{n}$ iid continuous random variables with probability density function *(f)* and cumulative density function *(F)* the first and last order statistics are as follows:

$$F_{x_{(1)}}(x) = 1 - (1 - F(x))^{n} \tag{4.1}$$

$$F_{x_{(n)}}(x) = F(x)^{n} \tag{4.2}$$

$$f_{x_{(1)}}(x) = nf(x)(1 - F(x))^{n-1} \tag{4.3}$$

$$f_{x_{(n)}}(x) = nf(x)F(x)^{n-1} \tag{4.4}$$

The derivation of the above given Equations is given in Appendix A.

The following notations were derived by Meller and Klote (2004) for single carousel system. In horizontal carousel system, items location positioned between 0 and

1. It is assumed that cumulative distribution function *(F)* of items locations in a bidirectional horizontal carousel (carousel rotates either clockwise or counter clockwise) is based on Uniform distribution (0, 1). In this case the probability density function *(f)* and cumulative density function *(F)* of the Uniform distribution will be as follows:

$$f(x) = 1 \text{ and } \quad F(x) = x \qquad (4.5) \text{ and } (4.6)$$

By applying Equations 4.5 and 4.6 to order statistics, the following cumulative density function CDF and probability density function PDF are obtained.

For bidirectional carousel system Meller and Klote (2004) considered a special case where the number of item n=1. In this case the CDF is calculated as follows:

$$F_1\ (x) = \begin{cases} 2x & if \quad x \leq 0.5 \\ 1 & otherwise \end{cases} \qquad (4.7)$$

For other cases where the number of items is greater than one (n>1), CDF is found as follows:

$$F_n\ (x) = 1 - (1 - x^n)^2 \qquad (4.8)$$

By taking the derivative of the Equation 4.7, for cases when n=1 the PDF is as follows:

$$f_1(x) = 2 \qquad (4.9)$$

By taking the derivative of the Equation 4.8, for cases when n>1 the PDF is as follows:

$$f_n\ (x) = 2nx^{n-1} - 2nt^{2n-1} \qquad (4.10)$$

4.6.1 Expected rotation time of a single carousel

Meller and Klote (2004) have developed the expected rotation time for a bidirectional single horizontal carousel.

Expected values have been calculated considering the special cases in which n=1 and n>1. The expected rotation is obtained by taking the integral of the probability density functions of the first and last order statistics of a bidirectional horizontal carousel which are given in Equations 4.9 and 4.10.

$$E[X_n] = \begin{cases} 1/4 & if\ n = 1, \\ \frac{2n}{n+1} - \frac{2n}{2n+1} & otherwise. \end{cases} \tag{4.11}$$

In order to find the rotation time of a carousel the time to rotate the carousel one complete revolution need to be found. This is found by fraction of the length and the velocity of a carousel.

$$T_r = \frac{l}{v} \tag{4.12}$$

By using Equations 4.11 and 4.12 the rotation times of a carousel will be as follows:

$$E[R(n,m)] = \begin{cases} 1/4 \times T_r & if\ n = 1, \\ \left(\frac{2n}{n+1} - \frac{2n}{2n+1} \right) \times T_r & otherwise. \end{cases} \tag{4.13}$$

4.7 OCCUPANCY PROBLEM

The occupancy problem is a part of probability theory about placing *n* balls into *m* cells. It is used to find the probability that exactly *k* cells are occupied through *m* cells in randomly placing *n* balls into *m* cells. Additionally it helps to find the expected number of occupied cells (or expected number of empty cells). Below is the probability of occupied cells with *n* items by Feller (1968):

$$p_k(m,n) = \binom{m}{k} \sum_{j=0}^{m-k} (-1)^j \binom{m-k}{j} \left(1 - \frac{j+k}{m}\right)^n \tag{4.14}$$

The detailed derivation of the occupancy problem probability is provided in Appendix B.

Occupancy problem is used in the analytical model in order to find the number of empty bins or bins that have no requested items. By employing occupancy problem into our model we will be able to find the number of bins which have requested items of orders. This will help us to find the starting and stopping time spent by horizontal carousel.

4.7.1 Expected start and stop time

In horizontal carousel case there are *m* bins in which *n* items can be stored. It is a common interest that how many bins are occupied with items. The probability that carousel will not stop at *k* of *m* bins is given in Section 4.7. By using this probability the expected number of occupied bins can be calculated.

The number of *m* bins that horizontal carousels need to stops to retrieve *n* items could be found by the following formula (Meller and Klote 2004):

$$. E[X_m] = \left(\frac{m-1}{m}\right)\left[\sum_{k}^{m-1}(m-k)\,p_k(m,n)\right] \tag{4.15}$$

Equation 4.15 represents the total probability that *k* events with a known probability itself will happen out of *m* events.

$$E\ [S(n,m)] = T_s\left(\frac{m-1}{m}\right)\left[\sum_{k}^{m-1}(m-k)\,p_k(m,n)\right] \tag{4.16}$$

Subsequently, in order to find the carousels expected start/stop time of carousel, the predetermined bins stopping and moving time must be multiplied with the total probability of events (Eq.4.16).

4.8 EXPECTED TIME OF A PICKER

The expected time of a picker to pick items from shelves is equal to the pick time spent for retrieval of *n* items in an assigned carousel.

$$E[P(n,m)] = nT_p \tag{4.17}$$

4.9 EXPECTED ORDER PICKING TIME IN A SINGLE CAROUSEL

As we mentioned in Section 4.5 a single carousel order picking process comprises three components rotation time, start/stop time and pickers picking time which are need to be summed. The expected time to retrieve *n* items in a carousel with *m* bins is as follows:

$$E[T_c] = E[R(n,m)] + E[S(n,m)] + E[P(n,m)] \tag{4.18}$$

The above given equations are subject to form an order picking time model in multiple carousels.

4.10 ANALYTIC MODEL OF ORDER PICKING TIME OF A WAVE OF ORDER IN MULTIPLE CAROUSELS

As we discussed in previous sections multiple carousel storage system is the storage and retrieval system which consists of *C* number of horizontal carousels rotating independently from each other and where orders are dispersed randomly.

In order to find the total order picking time of a wave of order in multiple carousel storage system, the carousel with the longest rotation time, start/stop time and pick time need to found. We can consider order picking process finished if carousel with maximum picking time is identified among a group of carousels. Consequently, T_w the time required to collect all items from multiple carousel system is as follows:

$$T_w = \max(\mathrm{T_c}(1), \mathrm{T_c}(2), \mathrm{T_c}(3), \ldots.., \mathrm{T_c}(i)) \tag{4.19}$$

Assume that items of a wave order are distributed randomly in four carousels. The expected total order picking of carousels are $E[T_c(1)]=45$ seconds, $E[T_c(2)]=40$ seconds, $E[T_c(3)]=56$ seconds and $E[T_c(4)]=50$ seconds, respectively. In this case, the total order picking time of a wave of order will be the third carousel’s order picking time. If the picking operation of the third carousel is finished, it would be considered that items of a wave order were already picked.

Based on formula given in Section 4.9 the total order picking time of a wave of order in multiple carousel system is as follows:

$$E[T_w] = E[R(i,n,m)] + E[S(i,n,m)] + E[P(i,n,m)] \tag{4.20}$$

By applying Equations 4.13 and 4.14 we can calculate the total items retrieval time in a multiple carousel storage system.

Expected rotation time, start/stop time and picking time of a carousel which has maximum order picking time are given in Equations 4.15, 4.16 and 4.18 respectively:

$$E[R(i,n,m)] = \begin{cases} 1/4 \times T_r & \text{if } n = 1, \\ \left(\frac{2n}{n+1} - \frac{2n}{2n+1} \right) \times T_r & \text{otherwise.} \end{cases} \tag{4.21}$$

$$E\ [S(i,n,m)] = T_r \left(\frac{m-1}{m} \right) \left[\sum_{k}^{m-1} (m-k)\, p_k\,(i,n,m) \right] \tag{4.22}$$

where, $$p_k\,(i,n,m) = \binom{m}{k} \sum_{j=0}^{m-k} (-1)^j \binom{m-k}{j} \left(1 - \frac{j+k}{m} \right)^n \tag{4.23}$$

$$E[P(i,n,m)] = nT_p \tag{4.24}$$

In summary, we conclude that in order to find the total order picking time of a wave of order in multiple horizontal carousel system firstly all respective carousels order picking times need to be calculated then a carousel with the longest rotation time, start/stop time and picking time must be identified. The carousel with the maximum order picking time among multiple carousels will define the order picking time completion of a wave of order.

CHAPTER 5

APPLICATION AND RESULTS

In this chapter application will be carried out to implement the analytical model which was presented in previous chapter and the results hence are explained in this chapter. Additionally, simulation studies will be done in order to make comparisons between analytical and simulation models. The results of analysis of the analytical model and the simulation model will be presented. This chapter is divided into four sections.

The first section describes the parameters used within the analytical and simulation models.

The second and third sections present the results obtained from the analytical model and simulated models.

Lastly in the fourth section the comparison of analytical and simulated models are presented. Additionally, statistical inference presented to show the reliability of the analytical model.

5.1 PARAMETERS AND CHARACTERISTICS OF THE ANALYTICAL AND SIMULATION MODELS.

Parameters used in analytical and simulation models are given in Table 5.1.

Table 5.1 Parameters used in analytical and simulation models.

Notation	Explanation	Value
C	number of carousels (pcs)	10
w	number of items within a wave (pcs)	?
T_r	total time to rotate the carousel one complete revolution (sec)	60
T_s	carousel start and stop time for order-picking (sec)	3
T_p	picking time of one item (sec)	5
m	pick faces (bins) (pcs)	30
l	length of the closed-loop conveyor (m)	50
v	speed of the carousel (m/s)	0.83
R	number of rows within a carousel (pcs)	5
S	total storage space, size (pcs)	1500
T_w	total picking time of all products within an order (sec)	?

5.2 ANALYTICAL MODEL

In developing the analytical model, assumptions that are made in Section 4.3 are used to generate analytical model results. To illustrate the analytical model results we considered 10 carousels (C=10) with 30 pick faces (m=30) in each carousel where the one revolution of each carousel is 60 seconds (T_r=60), start and stop time is 3 seconds (T_s=3) and picking time of an item is 5 seconds (T_p=5). Analytical model is not considering the storage characteristics of carousels such as number of storage space and number of rows.

5.2.1 Results

For this example, Table 5.2 shows the order picking time for different sizes of waves of orders. For analytical model experimentation Microsoft Office Excel is used.

Table 5.2 Total order picking time for different waves of orders in the analytical model.

w	Total picking time (sec) E[Tw(a,n,m)]	Time spent to pick one item (sec/item)	Throughput/hour
10	79.2	7.9	454.7
20	87.7	4.4	820.8
30	109.9	3.7	982.4
40	102.9	2.6	1399.0
50	116.0	2.3	1552.1
60	125.0	2.1	1728.1
70	125.0	1.8	2016.2
80	146.2	1.8	1970.0
90	146.0	1.6	2219.3
100	166.5	1.7	2162.4
110	166.6	1.5	2377.0
120	173.4	1.4	2490.8
130	186.9	1.4	2504.7
140	193.1	1.4	2609.8
150	186.8	1.2	2891.5
160	199.9	1.2	2882.1
170	199.8	1.2	3063.8
180	212.7	1.2	3045.8
190	244.2	1.3	2801.2
200	225.4	1.1	3195.0
210	238.0	1.1	3176.7
220	244.1	1.1	3244.8

Tabel 5.2 (cont.)

230	262.6	1.1	3153.7
240	250.3	1.0	3451.2
250	262.5	1.0	3429.2

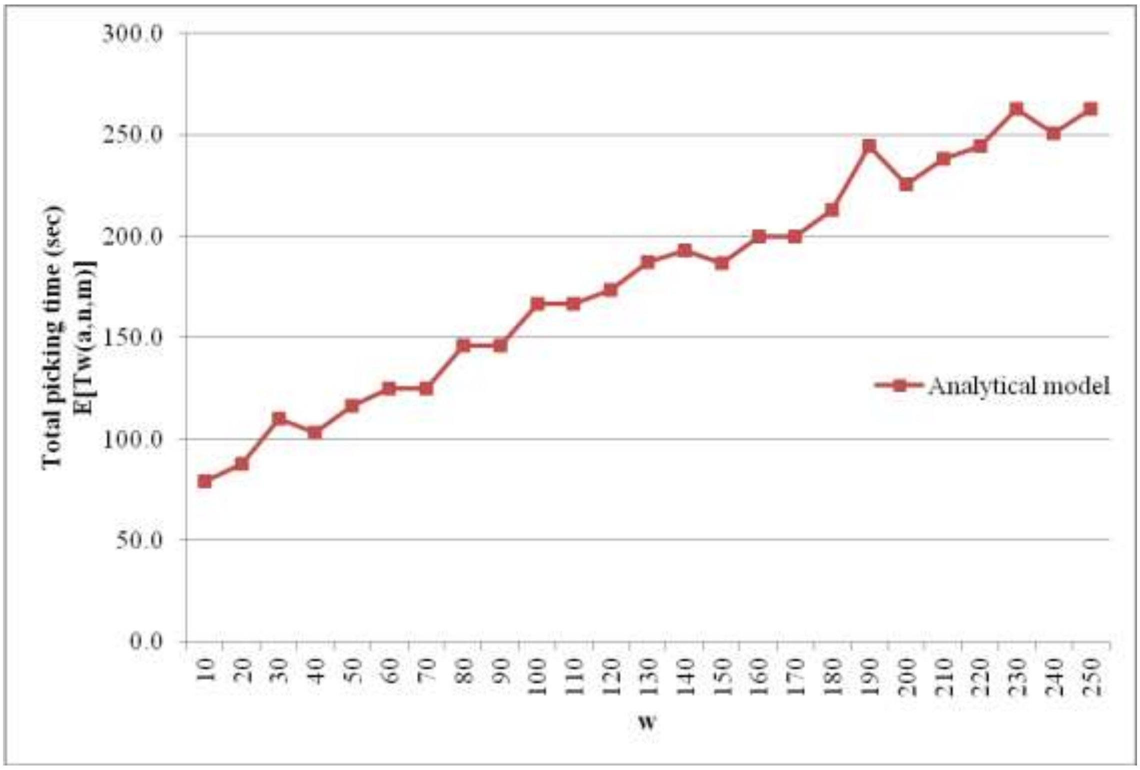

Figure 5.1 Total order picking time for different waves of orders in analytical model.

The line graph illustrates the total order picking time of an analytical model for waves of orders ranging from 10 to 250. Units are measured in seconds.

Overall, the total order picking time rose as the number of items increased. There was considerable grown in cases to peak of 109.9 seconds when the number of items reached 30 after which it experienced a slight fall to 102.9 seconds when the number of items was 40. The similar scenario observed when the number items were190 and 230 with the total order picking time of 244.2 and 262.6 seconds respectively.

The reason behind the sudden leap in order picking time when the wave size reached 30 could be defined by randomly distributed items to carousels in one of which a big amount of items were dispersed. The same scenario happened in other situation where order picking time soared.

Similarly, when the wave size reached 190, it has been dispersed randomly to 10 carousels in order to be colected. Out of ten carousels one had bigger amount of items to be collected. A significant rise of the order picking time could be characterized by picker's collection time since it has strong positive linear relationship against the amount of items. The fluction of the line graph can be described by probability values of each pick face given in start/stop time equation.

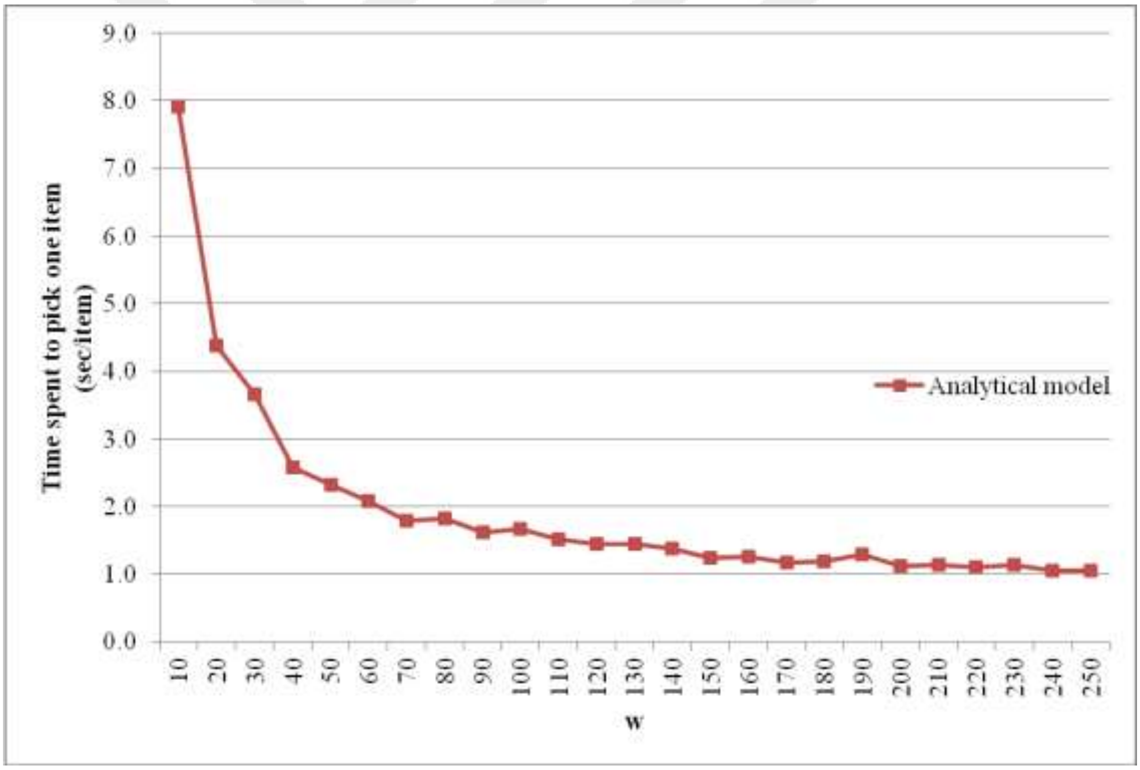

Figure 5.2 Time spent to pick one item (sec/item) in the analytical model.

The line graph presents the time spent to pick one item from multiple carousels for different sizes of waves.

Overall, the time spent to collect a single item decreased rapidly as the size of the waves increased. The system spent almost 8 seconds to pick one item when the size of the wave 10.

In sharp contrast to this, picking time decreased significantly until the wave size of 70. Single item picking time leapt when the wave size reached 190. This happened because of the total order picking time growth in the system. The reason of the order picking time leap can be characterized by a higher number of items dispersion in one of the ten horizontal carousels. Since, the picker's picking time have a strong linear correlation with wave size, it can be concluded that sudden leap has happened because of this factor.

The graph then levelled of after wave size of 240. It is fairly likely that total picking time remains constant after the wave size of 240.

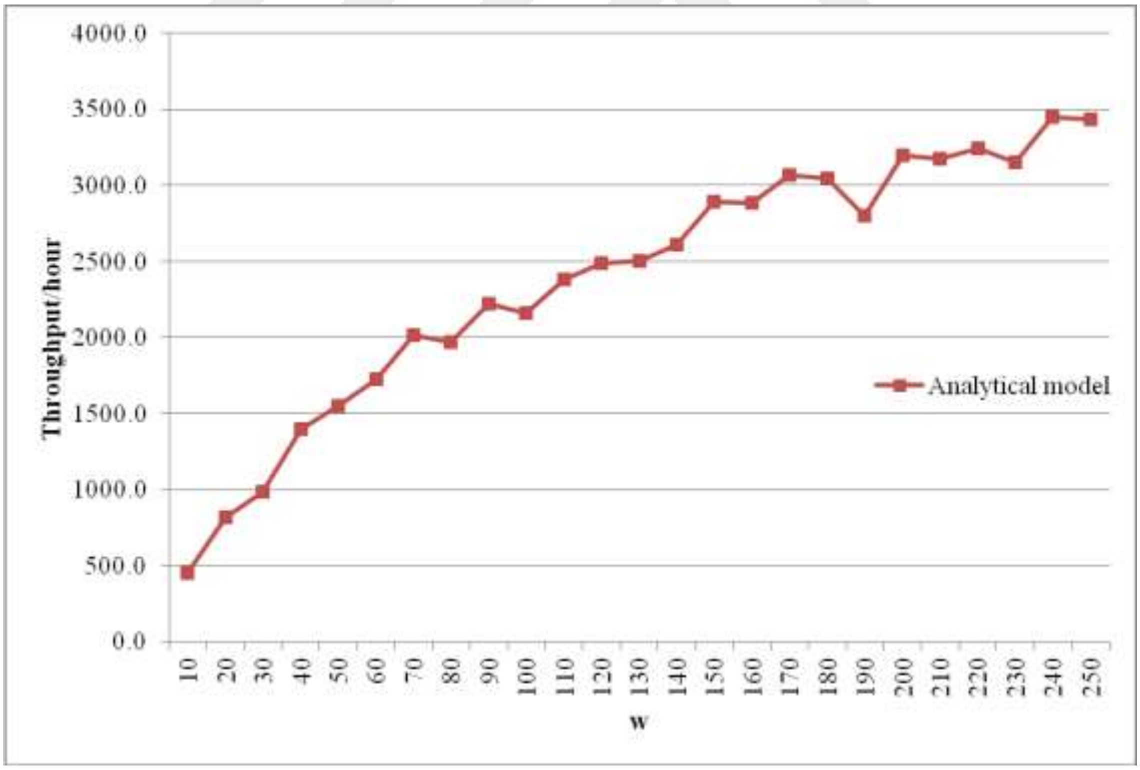

Figure 5.3 Throughput for different sizes of waves in the analytical model.

Figure 5.3 shows the analytical model's hourly thoughput results for different sizes of wave. From the graph it can be observed that hourly throughput results of horizontal carousels have upward pattern over the wave increment.

There were significant drops in throughput rates of the analytical model when the wave size was 190 and 230. This sudden falls happened due to the higher amount of time spent to pick waves of orders.

The minimum hourly rate of items collection within multiple horizontal carousel system was 454 items for wave size of 10; the maximum hourly rate of items collection was 3451 items when wave size was 240.

There is a drop in the number of hourly throughput, to just about 2801 items and 3153 items in wave sizes of 190 and 230 respectively. These abrupt decreases can be explained by the total order picking increase in these ranges of wave sizes.

In an average current multiple horizontal carousel system can pick 2385 items within an hour.

5.3 SIMULATION MODEL

In this simulation study order picking results are obtained for different size of order waves. The main goal of this simulation study is to observe order picking time change over the various operational and design parameters in regard to wave size. Simulation model is carried in order to make comparison and analysis with the analytical model results.

In our simulation model, we considered the assumptions given in Section 4.3 and the parameters given in Table 5.1.

To illustrate the simulation model results we considered 10 carousels (C=10) with 30 pick faces (m=30) in each carousel where the one revolution of each carousel is 60 seconds (T_r=60). Start and stop time is 3 seconds (T_s=3) and horizontal carousels acceleration and deceleration times were ignored.. Picking time of an item is 5 seconds

(T_p=5). As for design parameters of the carousel we considered 5 rows (R=5) in each bin of a carousel and 1500 bays in whole storage space (S=1500).

5.3.1 Results

Table 5.3 shows simulation results of the order picking time for different sizes of waves of orders. In this thesis a replication size of 1000 is chosen for each wave size simulation experiment. Simulation model experimentation was done in Microsoft Office Excel with the help of macros developed in Visual Basic for Applications.

Table 5.3 Total order picking time for different waves of orders in simulation model.

w	Total picking time E[Tw(a,n,m)]	Time spent to pick one item (sec/item)	Throughput/hour
10	71.7	7.2	501.9
20	86.8	4.3	829.7
30	99.3	3.3	1087.9
40	111.0	2.8	1296.8
50	121.0	2.4	1487.4
60	131.4	2.2	1644.3
70	141.0	2.0	1787.0
80	150.8	1.9	1909.3
90	159.4	1.8	2033.3
100	169.2	1.7	2127.5
110	176.7	1.6	2240.5
120	185.2	1.5	2332.2
130	194.6	1.5	2405.1
140	203.3	1.5	2479.2
150	210.6	1.4	2564.1
160	218.7	1.4	2634.3
170	225.9	1.3	2709.0
180	233.2	1.3	2778.2
190	241.5	1.3	2832.2
200	248.7	1.2	2894.7
210	257.2	1.2	2939.8
220	262.9	1.2	3012.4
230	270.8	1.2	3057.4
240	277.4	1.2	3114.2
250	284.7	1.1	3161.0

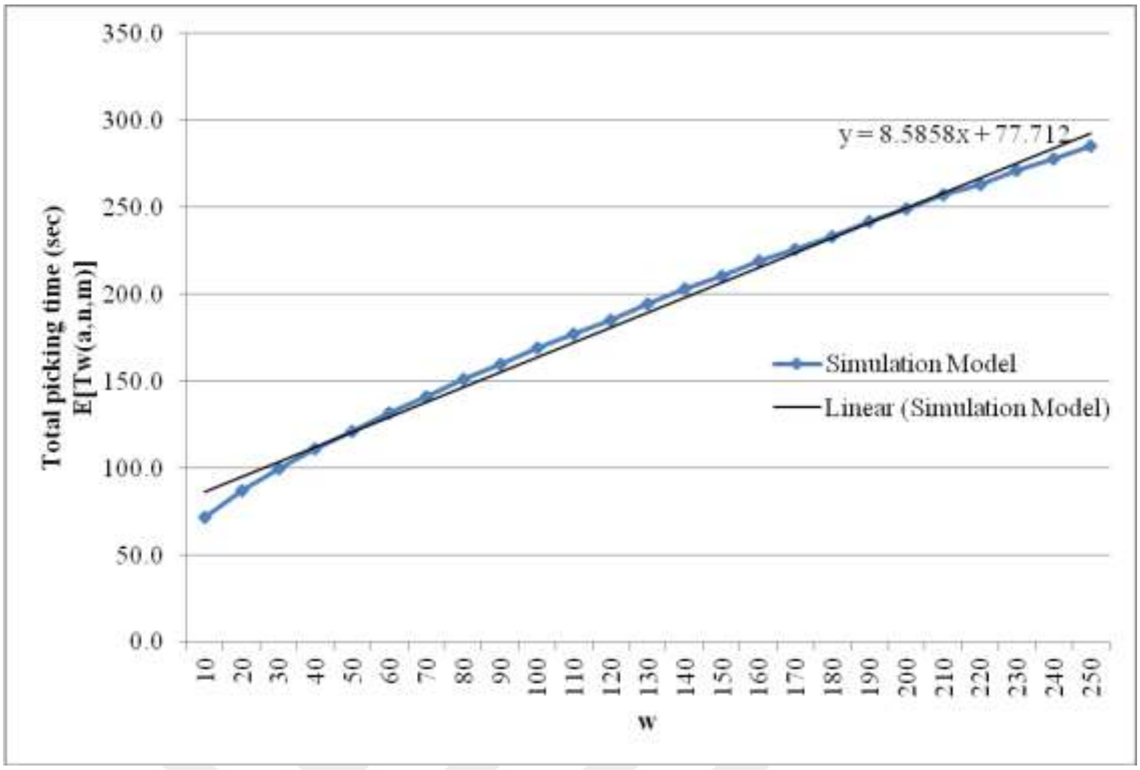

Figure 5.4 Total order picking time of for different waves of orders in simulation model.

The curvilinear line shows the total order picking time of the simulation model for different waves of orders ranging from 10 to 250. Units are measured in seconds. 1000 replications have been done for every size of waves.

Overall, upward trend can be observed from the graph with a steady growth in the total order picking time with repect to the size of the wave.

The reason of the steady growth of the total order picking time for different sizes of waves, is the averaging of 1000 iteration of the order picking time for every size of waves. The line graph become smoother as the number of replications increase.

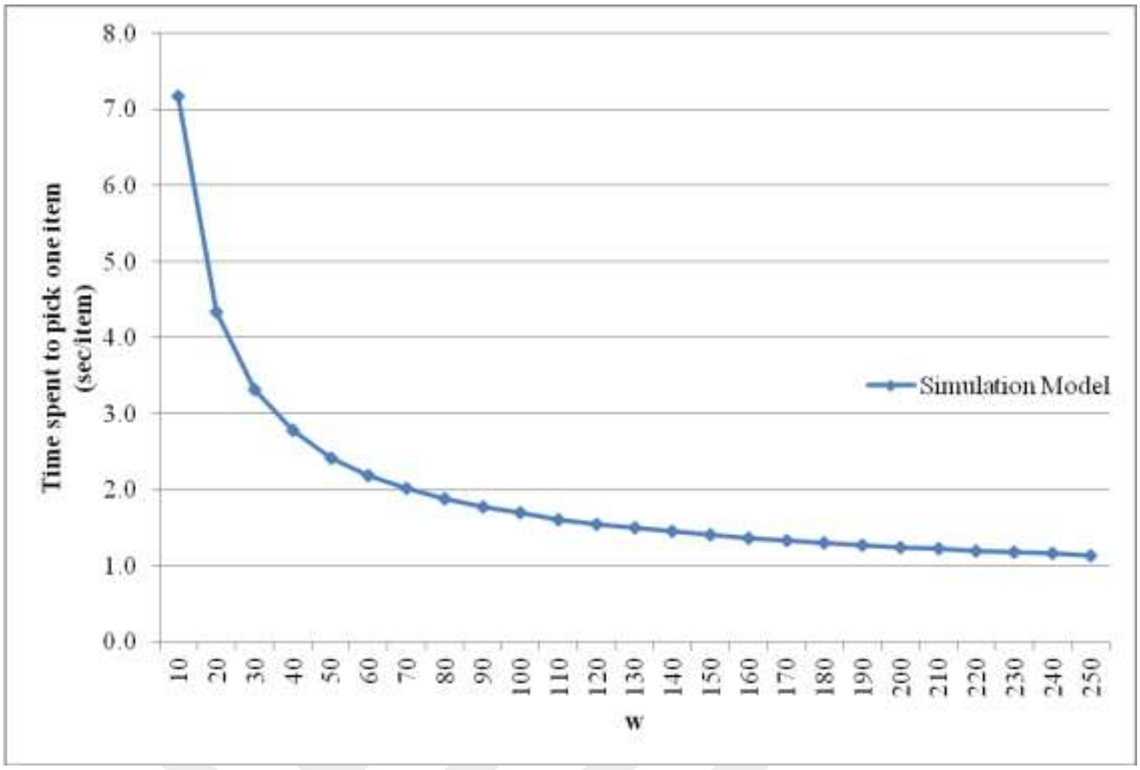

Figure 5.5 Time spent to pick one item (sec/item) in simulation model.

The time spent to pick one item in multiple horizontal carousels experienced a dramatic fall with regard to the wave size. This graph is derived from the total order picking time data obtained from simulation study with 1000 replications for every wave size.

The system spent 7.2 seconds to pick one item when the size of the wave 10 while this number was almost 1 second when wave size reached 250.

The time spent to pick an item remained plateau more frequently between the wave sizes of 200-240.

From the graph it is clear that the system spends more time to pick one item when the wave size is small. It can be concluded that it is more effective for companies to give an order to start operating horizontal carousels when there is a large number of waves.

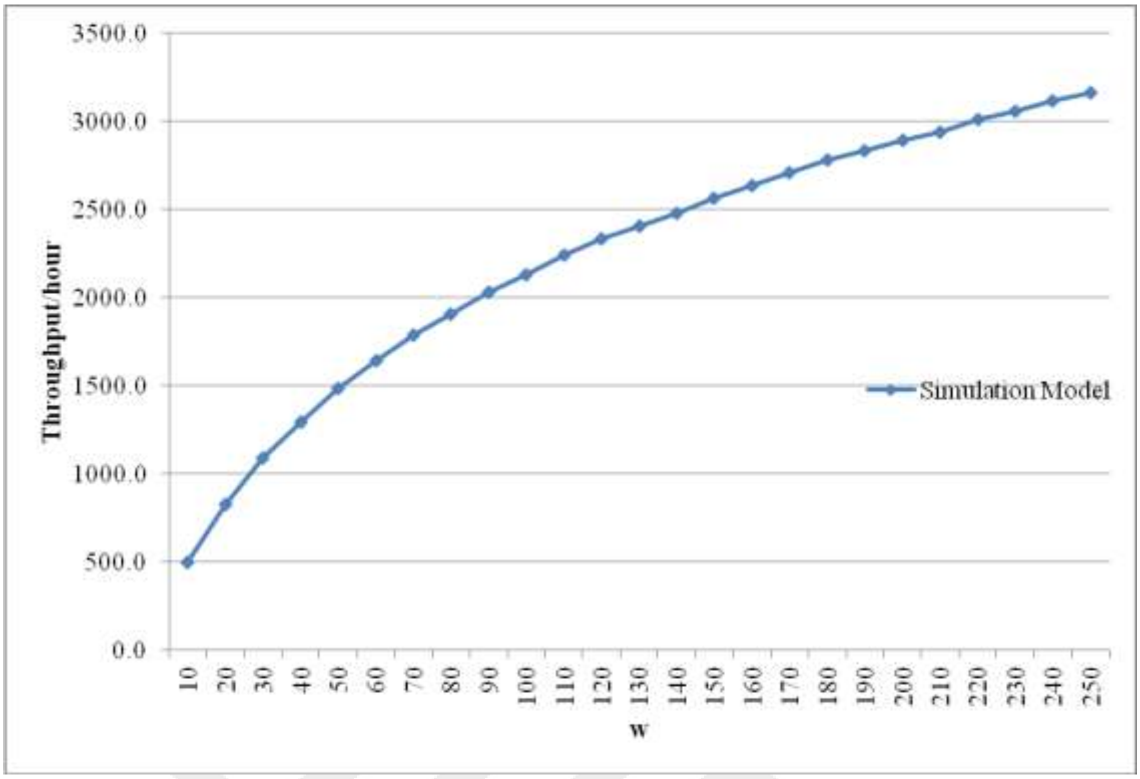

Figure 5.6 Throughput results in the simulation model.

Figure 5.6 shows the simulation model's hourly thoughput results for different sizes of wave. From the graph it can be observed that hourly throughput results of horizontal carousels have upward trend over the wave increment.

The minimum hourly rate of items collection within multiple horizontal carousel system was 501 items for wave size of 10; the maximum hourly rate of items collection was 3161 items when wave size was 250.

Overall, the hourly pick rate rose steadily over the increase of the wave size. The reason of the steady growth of the hourly throughput for different sizes of waves, can be characterized by the averaging of 1000 iteration of the order picking time for every sizes of waves. The line graph become smoother as the number of replications increase.

In an average current multiple horizontal carousel system can pick 2234 items in one hour.

5.4 COMPARISON OF THE ANALYTICAL MODEL TO SIMULATION MODEL

This section investigates whether the analytical model can take place of the simulation model. One of the most important feature of the analytical model is the capabilty to save amount of time and money. In previous sections the analytical model and the simulation models were constructed. Thus, data given in Tables 5.2 and 5.3 will used for making comparisons in this section. This section is divided into two sub sections: the first section is devoted to comparisons of graphs of two models; and the last section contains the statistical analysis.

5.4.1 Comparison of the total order picking time of a wave of order in the analytical and the simulation models

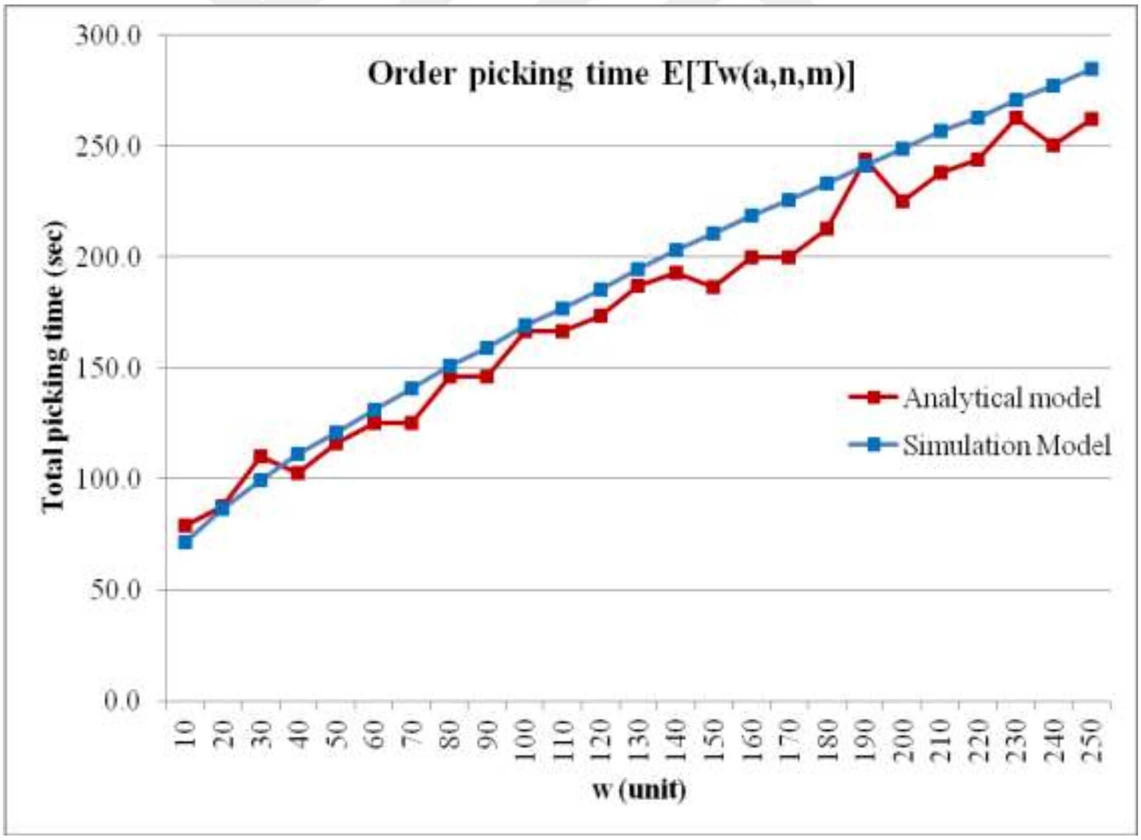

Figure 5.7 The comparison of the analytical and the simulation models total order picking time of a wave of order.

The line graph compares the total order picking time of the analytical model and the simulation for waves of orders ranging from 10 to 250. Units are measured in seconds.

Both graphs follow upward trent with respect to the increase of the wave of order. Overall, the total order picking time rose steadily in the simulated model in contrast to the analytical model where several fluctuations are observed. The reason of fluctuation of analytical model is due to the high number of items dispersed randomly in one of horizontal carousels which affected the increase of rotation time, start/stop time and picking time respectively.

The minimum values of the total order picking time of both analytical and simulation models are about 79 and 72 seconds respectively.

The maximum values of the total order picking time of both analytical and simulation models are 263 and 285 seconds respectively.

It can seen from the graph that the simulation model results are greater than the analytical model. The reason of this is that in simulation model, the rotation time, start/stop time and picking times surpassed the results of the analytical model.

In conclusion, if the size of the wave increases, the analytical model's total order picking time results become smaller than the simulation model results.

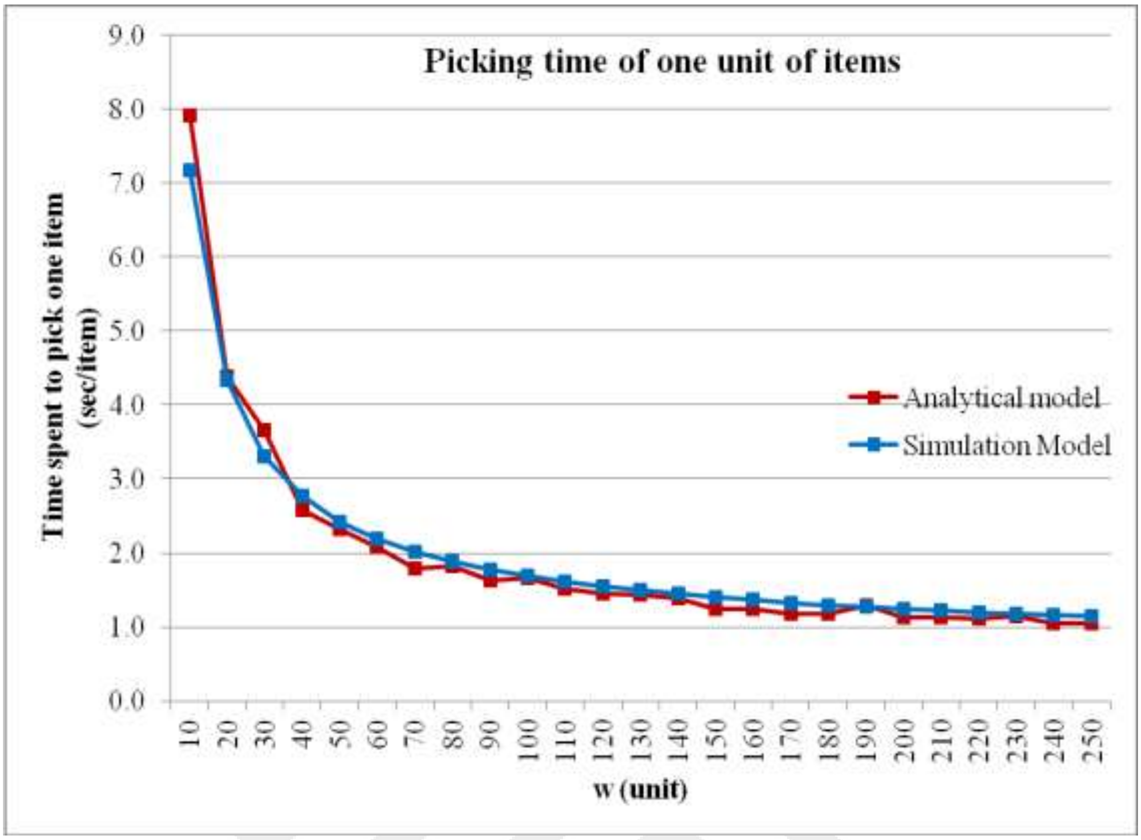

Figure 5.8 Time spent to pick one item (sec/item) in the analytical and simulation model.

The line graph presents the picking time of one item from multiple horizontal carousels for different sizes of waves.

Overall, the time spent to collect a single item decreased rapidly in both models as the size of the waves increased.

A significant decrease of single item picking time can be observed until the wave size of 70 in both analytical and simulation model.

The minimum picking time of one item from multiple carousel system was 1.0 second in the analytical model and 1.1 seconds in the simulation model when the wave size was 250 for both models.

The maximum picking time of one item from multiple carousel system was 7.9 second in the analytical model and 7.2 seconds in the simulation model when the wave size was 10 for both models.

The time spent to pick an item remained stable in both models after the wave size of 240.

The simulation model showed higher results than the analytical model after the wave size of 40. The reason of this is that in simulation model, there are additional time spent. The simulation model spent more time for rotating, start/stoping and picking compared to the analytical model.

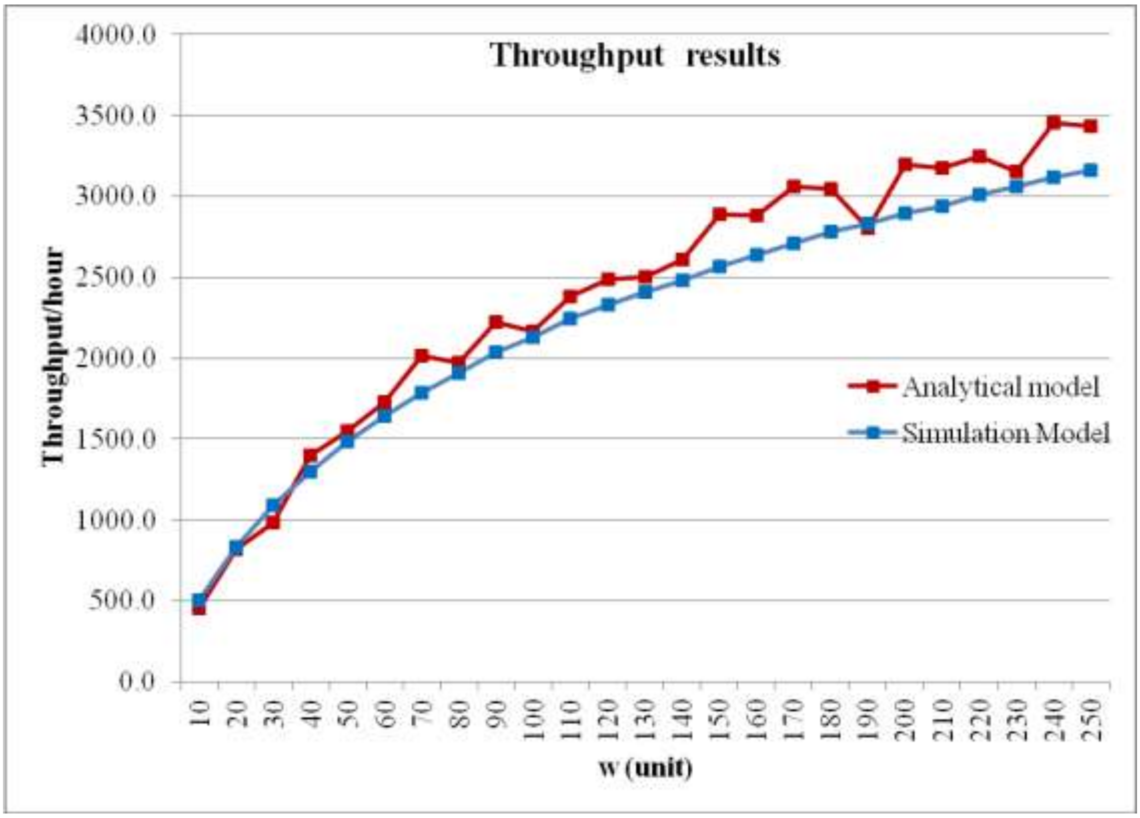

Figure 5.9 Throughput for different sizes of waves in the analytical model.

Figure 17 shows the analytical and the simulation models thoughput results for different sizes of wave. From the graph it can be observed that hourly throughput rate of horizontal carousels have upward trend in both models with regard to the wave size increase.

It can be seen from the line graph that there were significant drops in throughput rates of the analytical model when the wave size was 190 and 230. This sudden falls happened due to the higher amount of time spent to pick waves of orders. The smoothness of the simulation model graph happened because of the replications that have been done for each wave sizes.

The minimum hourly rate of items collection within the multiple horizontal carousel system was 454 items in the analytical model and 502 items in the simulation model when wave size was 10; the maximum hourly rate of items collection for analytical and simulation models were 3451 items and 3161 items respectively.

In an average current multiple horizontal carousel system can pick 2385 items based on the analytical model and 2234 items based on the simulation model in one hour.

5.4.2 Statistical analysis of the analytical and the simulation models

In this subsection statistical analysis of the total order picking time will be performed for both analytical and simulation models. The data analysis is based on the total order picking times of both models (Table 5.4). Firstly summary statistics of both data will be presented. Then two sample t-test will be performed to see the significance of two sample data.

Table 5.4 Total order picking time of the analytical model and simulation models for different sizes of waves.

w	Total picking time (sec)E[Tw(a,n,m)]	
	Analytical model	Simulation Model
10	79.2	71.7
20	87.7	86.8
30	109.9	99.3
40	102.9	111.0
50	116.0	121.0
60	125.0	131.4
70	125.0	141.0
80	146.2	150.8
90	146.0	159.4
100	166.5	169.2
110	166.6	176.7
120	173.4	185.2
130	186.9	194.6
140	193.1	203.3
150	186.8	210.6
160	199.9	218.7
170	199.8	225.9
180	212.7	233.2
190	244.2	241.5
200	225.4	248.7
210	238.0	257.2
220	244.1	262.9
230	262.6	270.8
240	250.3	277.4
250	262.5	284.7

5.4.2.1 Summary statistics of order picking times.

The sample data are analyzed using MINITAB 14 statistical software. The summary statistics for both analytical and simulation models are made for wave size 250 with 1000 replications each:

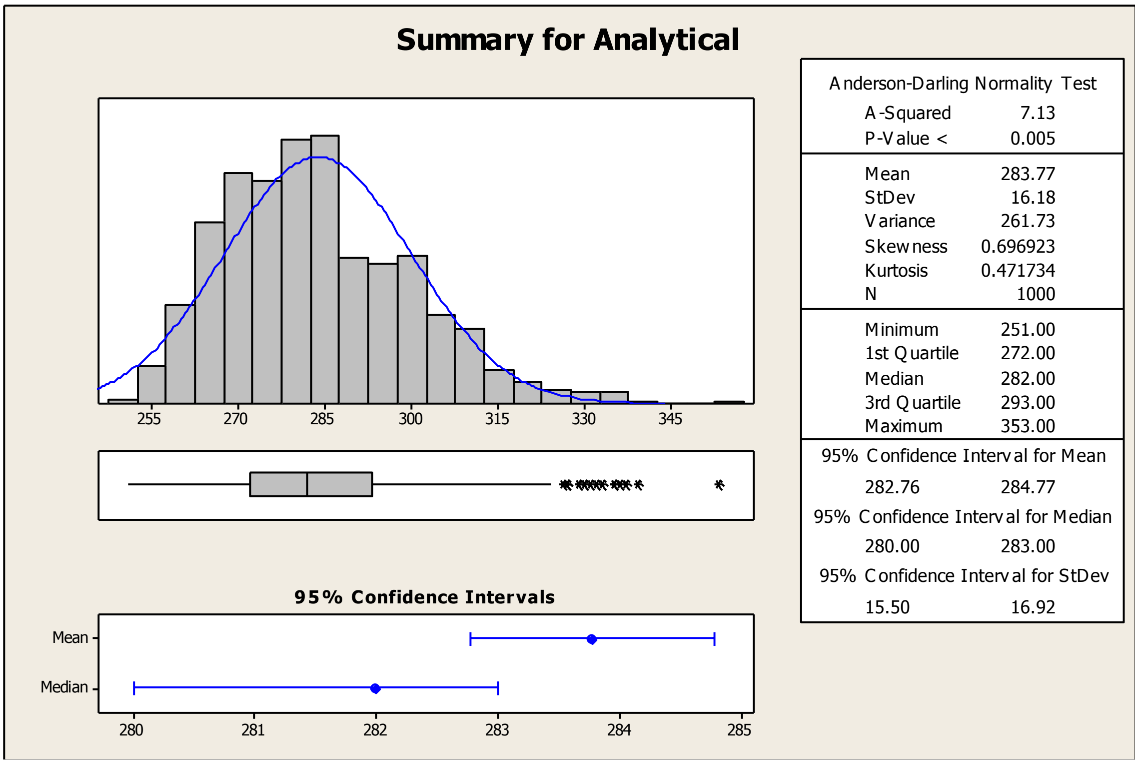

Figure 5.10 Graphical summary of the total order picking time of the analytical model.

The mean of the analytical model's total order picking time is 283.77 (95% confidence intervals of 282.76 and 284.77). The standard deviation is 16.18 (95% confidence intervals of 15.50 and 16.92). Using a significance level of 0.05, the Anderson-Darling Normality Test A-Squared = 7.13, P-Value = 0.005) indicates that the analytical model's total order picking time data do not follow a normal distribution.

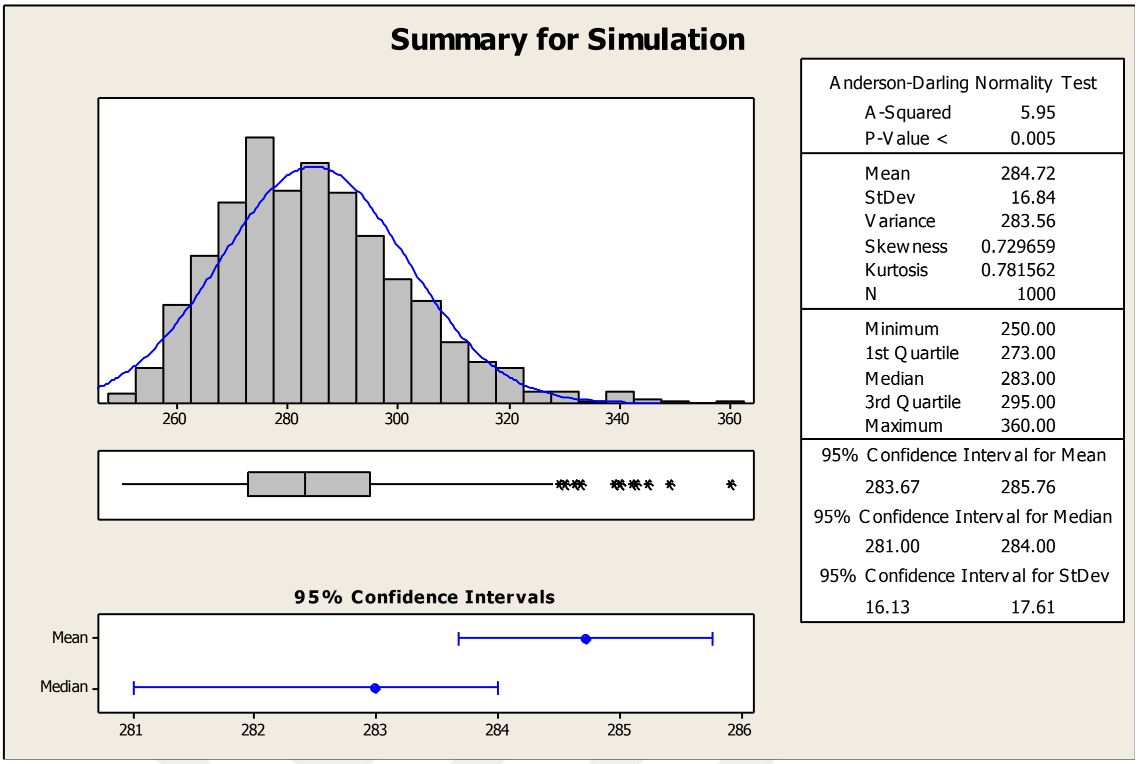

Figure 5.11 Graphical summary of the total order picking time of the simulation model.

The mean of the simulation model's total order picking time is 284.72 (95% confidence intervals of 283.67 and 285.76). The standard deviation is 16.84 (95% confidence intervals of 16.13 and 17.61). Using a significance level of 0.05, the Anderson-Darling Normality Test A-Squared = 5.95, P-Value = 0.005) indicates that the analytical model's total order picking time data do not follow a normal distribution.

5.4.2.2 Two sample t-test for analytical and simulation models results.

The two sample t-test is commonly applied in conducting hypothesis tests. It is used to compare whether or not the average difference of two samples is significant. It helps us to answer the question like whether the test results of the analytical model are significantly different than test results of the simulation model.

Below is the results obtained from MINITAB for wave size of 250 for both models which we replicated 1000 times to conduct two-sample t test.

Table 5.5 Two-sample T test for Analytical model vs Simulation Model

Two-sample T for Analytical model vs Simulation Model				
	N	Mean	StDev	SE Mean
Analytical model	1000	283.8	16.2	0.51
Simulation Model	1000	284.7	16.8	0.53

Difference = mu (Analytical model) - mu (Simulation Model)

Estimate for difference: -0.950000

95% CI for difference: (-2.398179, 0.498179)

T-Test of difference = 0 (vs not =): T-Value = -1.29

P-Value = 0.198 DF = 1994

Table 5.5 provides the sample sizes, sample means, standard deviations, and standard errors for the two samples.

In this example, a 95% confidence interval is (-2.398179, 0.498179) which includes zero and estimated mean difference of the analytical and the simulation models, thus suggesting that there is no substantial difference. Next is the hypothesis test result. The test statistic is -1.29, with p-value of 0.198, and 1994 degrees of freedom.

Since the p-value is greater than commonly chosen α-levels, there is no evidence for a significant difference in the total order picking times when using the analytical model versus a simulation model.

Based on t-test results we can conclude that the analytical model can be used instead of simulation model due to insignificant relationship between both models. The analytical model is very helpful in getting the total order picking time easily and quickly. The free adjustment of the operating parameters of the system allows making performance analyses effortlessly. These features of the analytical model help companies to make fast and reliable management decisions.

CHAPTER 6

CONCLUSIONS AND FUTURE WORKS

In this chapter, we will briefly refer to the importance of this master thesis. Information will be provided about the solution methodology along with the research summary. In Section 6.2 the contribution of this study will be discussed. Finally in Section 6.3 the possible future researches will be introduced.

6.1 RESEARCH SUMMARY

In this thesis an analytical model for order picking time of a wave of order is developed for multiple horizontal carousels system with similar configuration.

As the reason of focusing in order picking time in the horizontal carousel systems was the carousel system itself by highly efficient technology usage in industries and optimization of the order picking time due to the fact that it accounts for more than 55% of the total warehousing operational cost (De Koster, et al. 2007). From the technical approach to the problem we have been motivated by carousel systems which are kind of parts-to-picker warehousing system with high rates of return on investment, optimal storage density, full inventory control, low floor space of storing, user friendliness, modularity and easiness and swiftness of access to all stored inventories the aim which is to deliver products efficiently and timely to customers.

The aim of this thesis was to propose an analytical model for order picking time estimation in multiple carousels. The key advantage of the employment of the analytical model is the considerable reduction of time and money.

By approaching the problem analytically we have studied literatures about the carousel systems in order to understand specifically the work of the system.

After literature review we proposed the analytical model and results to them. Order statistics and occupancy problems were used in development of the analytical model. Additionally, simulation studies have been carried out in order to make comparisons with the analytical model. In obtaining the results of the analytical model and the simulation models random storage policy have been applied along with the several assumptions made in Section 4.3.

According to the results, there was no significant difference in obtaining the total order picking times in multiple horizontal carousel system using the analytical model versus a simulation model. Thus, the proposed analytical model provides decision makers timely and cost-effective model to determine the total order picking time in multiple horizontal carousel systems.

6.2 THE CONTRIBUTION TO THE LITERATURE

In Chapter 3 literature review was introduced with recent research studies in carousel systems. It has been mentioned that the literature for carousel systems could be divided into three parts. These were grouped as storage assignments problems, routing problems and performance problems. By researching the literature the conclusion can be drawn on the basis of the evidence that analytical studies have been done by several authors while heuristics studies were studied by many authors. Consequently, the followings are contributions to the literature:

- The proposed analytical model is useful for order picking operations in obtaining realistic and easier results in less time.
- The user is free to decide on operating parameters of the system. This helps to analyze easily and quickly the performance of the system with different parameters.
- The analytical model is appropriate for real cases.

6.3 FUTURE RESEARCH

Future researches could be done in the following topics:

- In designing the simulation model class based storage policy could be applied.
- In this study picker utilization is not mentioned since assumed that all carousels have assigned pickers. Thus in future researches picker utilization issues could be taken into consideration.
- One of the most important performance measurements of the warehousing is system is obviously order sortation systems. In this study sortation procedure is not mentioned. Thus, the integration of both horizontal carousel order picking system and sortation systems could be studied.

REFERENCES

Bartholdi, J.J. and Platzman, L.K., "Retrieval strategies for a carousel conveyor", *IIE Transactions*, Vol.18 (2), pp. 166–173, 1986.

Coyle, J.J., Bardi, E.J. and Langley, C.J., "The Management of Business Logistics", West Publishing, St. Paul, MN, 1996.

Dallari, F., Marchet, G., & Melacini, M., "Design of order picking system", *The International Journal* of *Advanced Manufacturing Technology*, *42*(1-2), 1-12. doi:10.1007/s00170-008-1571-9, 2008.

de Koster, R., Le-Duc, T., & Roodbergen, K. J., "Design and control of warehouse order picking": A literature review. *European Journal of Operational Research,* Vol.182 (2), pp. 481-501. doi:10.1016/j.ejor.2006.07.009, 2007.

Emmett, S., "Excellence in Warehouse Management", *John Wiley & Sons, Ltd*(1), pp. 316, 2005.

Farahani, R. Z., Rezapour, S., & Kardar, L., "Logistics Operations and Management". *Elsevier,* Vol.1(1), 475. doi:10.1016/B978-0-12-385202-1.X0001-1, 2011.

Feller, W., "An Introduction to Probability Theory and Its Applications", *John Wiley & Sons, Inc,* Vol.1, pp.1-512. 1968.

Fisher, M., "Using e-commerce to deliver high productivity", *Work Study,* Vol.49(2), pp.59-62. doi:10.1108/00438020010311188, 2000.

Ghosh, J. B., & Wells, C. E. (1992). Optimal retrieval strategies for carousel conveyors. *Mathl. Comput. Modelling,* Vol.16(1), pp.59-70, 1992.

Hassini, E., & Vickson, R. G., "A two-carousel storage location problem", *Computers & Operations Research*, Vol.30(2), pp.527–539, 2003.

Hwang, H., Kim, C.-s., & Ko, K.-h., "Performance analysis of carousel systems with double shuttle", *Computers & Industrial Engineering,* Vol.36(1), pp.473-485, 1999.

Jacobs, D. P., Peck, J. C., & Davis, J. S., "A simple heuristic for maximizing service of carousel storage", *Computers & Operations Research,* Vol. 27(2), pp.1351-1356, 2000.

Kim, B., “Maximizing service of carousel storage”, *Computers & Operations Research,* Vol.32(4), pp.767-772. doi:10.1016/j.cor.2003.08.016, 2005.

Kizilaslan, R., “A proposed integrated analytical model for order picking and sortation operations in warehouses” (Phd thesis), Istanbul Technical University, İstanbul, 2014.

Lahmar, M., “Facility Logistics Approaches and Solutions to Next Generation Challenges”, *Taylor & Francis Group*, 2008.

Litvak, N., Adan, J., I. , & Wessels, W. H. M. Z., “The Nearest Item Heuristic for Carousel Systems”, pp.12, 2000.

Litvak, N., & Adan, I., “The travel time in carousel systems under the nearest item heuristic”, *J.Appl.Prob,* Vol.38(1), pp.45-54, 2001.

Litvak, N., “Optimal picking of large orders in carousel systems”, *Operations Research Letters,* Vol.34(2), pp.219-227. doi:10.1016/j.orl.2005.05.001, 2006.

Litvak, N., & Vlasiou, M., “A Survey on Performance Analysis of Warehouse Carousel Systems”, 2010.

Manzini, R., “Warehousing in the Global Supply Chain”, *Springer London Dordrecht Heidelberg New York,* pp.506. doi:10.1007/978-1-4471-2274-6, 2012.

Meller, R. D., & Klote, J. F., “A throughput model for carousel/VLM pods”, *IIE Transactions,* Vol.36(8), pp.725-741. doi:10.1080/07408170490458472, 2004.

Park, B. C., & Rhee, Y., “Performance of carousel systems with ‘organ-pipe’ storage”, *International Journal of Production Research,* Vol.43(21), pp.4685-4695. doi:10.1080/00207540500185182, 2005.

Pazour, J. A., & Meller, R. D., “The impact of batch retrievals on throughput performance of a carousel system serviced by a storage and retrieval machine”, *International Journal of Production Economics,* Vol.142(2), pp.332-342. doi:10.1016/j.ijpe.2012.12.005, 2013.

Tompkins, J. A., “Facilities Planning”: J. Wiley, 2003.

Tompkins, J. A., White, J. A., Bozer, Y. A., & Tanchoco, J. M. A., “Facilities Planning”: J. Wiley, 2010.

Trunk, C. “Horizontal carousels tackle the big jobs”, Material Handling Engineering, Vol.51 (7), pp.47–48, 1996a.

Vickson, R. G. and Fujimoto, A., “Optimal storage locations in a carousel storage and retrieval system”, *Location Science,* Vol.4(2), pp.237-245, 1996.

Wang, X.-z., Wang, G.-q., & Li, X.-c., “Order Picking Optimization in Carousels Storage System”, *Mathematical Problems in Engineering, 2013*, 1-8. doi:10.1155/2013/692701, 2013.

Yeh, D.-H., “A note on a simple heuristic for maximizing service of carousel storage”. *Computers & Operations Research*, Vol.29(2), pp.1605–1608, 2002.

Zhang, P., Jia, L., & Tian, G. (2004). Pick sequencing optimization problem in the rotary rack S/R system. *Journal of Control Theory and Applications*, Vol.3(1), pp.229–238, 2004.

APPENDIX A

Order statistics sequence the values of random samples in ascending form by indicating the minimum and maximum values of the sample.

Suppose, there are five numbers are observed as a random sample size of 5. The sample values are x_1=6, x_2=1, x_3=7, x_4=2 and x_5=3. The sample order statistics would be the sample values in ascending order which satisfy,

$$X_{(1)} \leq X_{(2)} \ldots\ldots X_{(n)}$$

Since $P(X_{(i)=} X_{(j)})=0$ where $i \neq j$ for continuous variable. Thus, the above samples will be sequenced as $X_{(1)} = 1$, $X_{(2)} = 2$, $X_{(3)} = 3$, $X_{(4)} = 6$ and $X_{(5)} = 7$ according to order statistics.

The samples are independently and identically distributed (iid). Order statistics are dependently distributed because of order restriction.

There are first (minimum) order statistics and largest order statistics (maximum) of the sample of size *n.*

$X_{(1)}$ is the minimum of the sample n

$$X_{(1)} = \min \{X_{1\ldots} X_n\}$$

$X_{(n)}$ is the maximum of the sample n

$$X_{(n)} = \max \{X_{1\ldots} X_n\}$$

For continuous random variables probability density function *f(x)* can be seen as follows (https://www2.stat.duke.edu/courses/Spring12/sta104.1/Lectures/Lec15.pdf):

$$
\begin{aligned}
& f(x)\varepsilon \approx P(x \leq X \leq x+\varepsilon) = P(X \in [x, x+\varepsilon]) \\
& \lim_{\varepsilon \to 0} f(x)\varepsilon = \lim_{\varepsilon \to 0} P(X \in [x, x+\varepsilon]) \\
& f(x) = \lim_{\varepsilon \to 0} P(X \in [x, x+\varepsilon]) / \varepsilon
\end{aligned} \tag{A.1}
$$

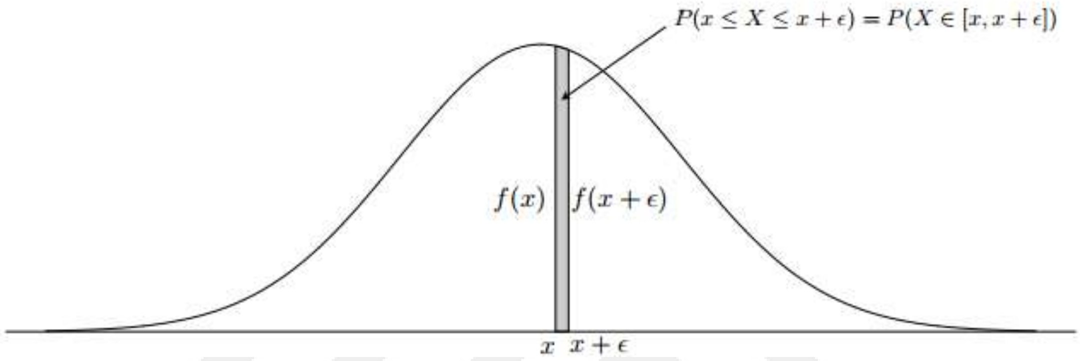

Figure A.1 Probability distribution functions for continuous random variables.

For $X_1, X_2 \ldots X_n$ iid continuous random variables the minimum probability density is as follows:

$$
\begin{aligned}
P(X_{(1)} \in [x, x+\varepsilon]) &= P(\text{one of the X's} \in [\text{x}, \text{x}+\varepsilon] \text{ and all others} > x) \\
&= \sum_{i=1}^{n} P(X_i \in [\text{x}, \text{x}+\varepsilon] \text{ and all others} > \text{x}) \\
&= nP(\text{X}_1 \in [\text{x}, \text{x}+\varepsilon] \text{ and all others} > \text{x}) \\
&= nP(\text{X}_1 \in [\text{x}, \text{x}+\varepsilon]) \; P(\text{all others} > \text{x}) \\
&= nP(\text{X}_1 \in [\text{x}, \text{x}+\varepsilon]) \; P(\text{X}_2 > \text{x}) \cdots P(\text{X}_n > \text{x}) \\
&= nf(x)\varepsilon(1-F(x))^{\text{n-1}}
\end{aligned} \tag{A.2}
$$

$$
f_{x_{(1)}}(x) = nf(x)(1-F(x))^{n-1} \tag{A.3}
$$

For $X_{1}, X_{2} \dots X_{n}$ iid continuous random variables the maximum probability density function *(f)* is as follows:

$$
\begin{aligned}
P(X_{(n)} \in [x, x+\varepsilon]) = P(&\text{one of the X's} \in [\text{x, x} + \varepsilon] \text{ and all others} < x) \\
&= \sum_{i=1}^{n} P(X_i \in [\text{x, x} + \varepsilon] \text{ and all others} < \text{x}) \\
&= nP(\text{X}_1 \in [\text{x, x} + \varepsilon] \text{ and all others} < \text{x}) \\
&= nP(\text{X}_1 \in [\text{x, x} + \varepsilon]) \; P(\text{all others} < \text{x}) \\
&= nP(\text{X}_1 \in [\text{x, x} + \varepsilon]) \; P(\text{X}_2 < \text{x}) \cdots P(\text{X}_n < \text{x}) \\
&= nf(x)\varepsilon F(x)^{\text{n-1}}
\end{aligned}
\tag{A.4}
$$

$$
f_{x_{(n)}}(x) = nf(x)F(x)^{n-1} \tag{A.5}
$$

For $X_{1}, X_{2} \dots X_{n}$ iid continuous random variables the cumulative density functions *(F)* of the *k*th order statistics are as follows:

$$
\begin{aligned}
F_{x_{(1)}}(x) &= P(X_{(1)} < x) = 1 - P(X_{(1)} > x) \\
&= 1 - P(X_1 > x, ..., X_n > x) = 1 - P(X_1 > x) ....P(X_n > x) \\
&= 1 - (1 - F(x))^n
\end{aligned}
\tag{A.6}
$$

$$
\begin{aligned}
F_{x_{(n)}}(x) &= P(X_{(1)} < x) = 1 - P(X_{(1)} > x) \\
&= 1 - P(X_1 < x, ..., X_n < x) = P(X_1 < x) ....P(X_n < x) \\
&= F(x)^n
\end{aligned}
\tag{A.7}
$$

APPENDIX B

The occupancy problem is probability theory about placing *n* balls into *m* cells. It is used to find the probability that exactly *k* cells are occupied through *m* cells in randomly placing *n* balls into *m* cells. Additionally it helps to find the expected number of occupied cells (or expected number of empty cells). Below is the derivation of probability of occupied cells with *n* items by Feller (1968);

Let A_k be the event that cell number k is empty (k=1, 2, … , n).

In this event all *n* balls are placed in the remaining *m-1* cells. This can be done $(m-1)^n$ ways. Thus,

$$p_i = \frac{(m-1)^n}{m^n} = \left(1 - \frac{1}{m}\right)^n \quad \text{(B.1)}$$

Similarly, there are $(m-2)^n$ arrangements, leaving two assigned cells empty. Thus,

$$p_i = \frac{(m-2)^n}{m^n} = \left(1 - \frac{2}{m}\right)^n \quad \text{(B.2)}$$

For $j \leq m$,

$$S_j = \binom{m}{j}\left(1 - \frac{j}{m}\right)^n \quad \text{(B.3)}$$

The probability of having no empty cells is as follows:

$$p_0(n,m) = \sum_{j=0}^{m}(-1)^j \binom{m}{j}\left(1 - \frac{j}{m}\right)^n \quad \text{(B.4)}$$

The following is the probability of having exactly k cells empty:

$$p_k(m,n) = \binom{n}{m}\left(1-\frac{k}{m}\right)^n p_0(n,m-k) =$$

$$= \binom{m}{k}\sum_{j=0}^{m-k}(-1)^j \binom{m-k}{j}\left(1-\frac{j+k}{m}\right)^n \tag{B.5}$$