ANALYSIS OF PUBLIC HEALTHCARE POLICIES TO IMPROVE SOCIAL UTILITY USING STOCHASTIC MODELS

Doctor of Philosophy Dissertation

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Graduate School of Sciences
Industrial Engineering Programme
Supervisor: Prof. Dr. Onur Kaya

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This thesis titled "Analysis of Public Healthcare Policies to Improve Social Utility Using Stochastic Models" has been prepared and submitted by Aydin Teymourifar in partial fullfillment of the requirements in "Anadolu University Directive on Graduate Education and Examination" for the Degree of Doctor of Philosophy (PhD) in Industrial Engineering Department has been examined and approved on 25/02/2019.

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ABSTRACT

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Aydin Teymourifar

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Supervisor: Prof. Dr. Onur Kaya
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In many health systems, public and private hospitals with different characteristics co-exist. Generally, in such systems, although the payments are low in public hospitals, the high waiting times decrease the quality level perceived by patients. In private hospitals, although the payments are high, the waiting times are low and thus the quality level perceived by the patients is also high. In this case, a more balanced system can be socially beneficial. Pricing policies have an impact on the patient's choice of hospital. Lower prices in private hospitals cause more patients to choose these hospitals. Thus, both the crowdedness of public hospitals decreases and more people can receive higher quality services. The main motivation of this thesis is to provide pricing models to provide a more balanced and socially useful system for similar situations. The prices in private hospitals are defined on the basis of contract mechanisms between the government and these hospitals. Therefore, in this thesis, the effects of different contract mechanisms between the government and private hospitals on social utility are investigated. For this aim, simulation-based and analytical models are designed. We use the parameters similar to the health system in the Eskiehir region and make a comparison between the models. The results show that contracts between the government and private hospitals are beneficial for the community. In particular, the policy of income-based pricing can be beneficial for both individuals and the government, since it leads to balanced expenses.

Keywords: Health Policies, Contract Mechanisms, Simulation, Analytical Models

ÖZET

HASTANELERE YÖNELİK SAĞLIK POLİTİKALARININ TOPLUMSAL FAYDA AÇISINDAN RASSAL MODELLER İLE ANALİZİ

Aydın Teymourifar

Endüstri Mühendisliği Anabilim Dalı

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Danışman: Prof. Dr. Onur Kaya
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Birçok sağlık sisteminde, farklı özelliklere sahip kamu ve özel hastaneler bir arada bulunmaktadır. Genelde, tür sistemlerde, devlet hastanelerinde ödemeler düük olsa da, bekleme sürelerinin yüksek olması, hastaların algıladığı kalite seviyesini düşürür. Özel hastanelerde ise, ödemelerin yüksek olmasına rağmen, bekleme süreleri düşüktür ve dolayısıyla hastalar tarafından algılanan kalite düzeyi yüksektir. Bu durumda, daha dengeli bir sistem toplumsal açıdan daha faydalı olabilir. Fiyatlandırma politikaları, hastaların hastane seçimini etkilediğinden, sistemin bu durumu üzerinde göz ardı edilemez bir etkiye sahiptir. Özel hastanelerde daha düşük fiyat, daha fazla hastanın bu hastaneleri tercih etmelerine sebep olur. Böylelikle, hem devlet hastanelerindeki yoğunluk azalır hem de daha fazla kişi daha kaliteli hizmet alabilir. Tezin temel motivasyonu, benzer durumlar için daha dengeli ve sosyal açıdan daha yararlı bir sistem sağlamak amacıyla fiyatlandırma modelleri sunmaktır. Özel hastanelerdeki fiyatlar, devlet ile bu hastaneler arasındaki kontrat mekanizmalar esasında tanımlanır. Bu nedenle, bu tezde, devlet ile özel hastaneler arasındaki farklı sözleşme mekanizmalarının toplumsal fayda üzerindeki etkileri incelenmiştir. Bu amaçla benzetim ve analitik modeller kurulmuştur. Sayısal sonuçlar kısmında, Eskişehir bölgesindeki sağlık sistemine benzer olan parametreler kullanark, modeller arasında karşılaştırma yapılmıştır. Sonuçlar devlet ile özel hastaneler arasındaki sözleşmelerin toplum için faydalı olduğu göstermektedir. Özellikle, gelire göre fiyatlandrıma politikası, dengeli ödemelere sebep olduğundan, hem bireyler hem de devlet açısından faydalı olabilir.

Anahtar Kelimeler: Sağlık Politikaları, Kontrat Mekanizmaları, Analitik Modeller, Benzetim

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Aydin Teymourifar

25/02/2019

STATEMENT OF COMPLIANCE WITH ETHICAL PRINCIPLES AND RULES

I hereby truthfully declare that this thesis is an original work prepared by me; that I have behaved in accordance with the scientific ethical principles and rules throughout the stages of preparation, data collection, analysis and presentation of my work; that I have cited the sources of all the data and information that could be obtained within the scope of this study, and included these sources in the references section; and that this study has been scanned for plagiarism with "scientific plagiarism detection program" used by Anadolu University, and that "it does not have any plagiarism" whatsoever. I also declare that, if a case contrary to my declaration is detected in my work at any time, I hereby express my consent to all the ethical and legal consequences that are involved.

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LIST OF NOTATIONS AND ACRONYMS

NOTATIONS

c_d	Cost of care for each patient in the public hospital
b_d	Amount paid by the patients to the public hospital
c_{o_i}	Cost of care for each patient in the i-th private hospital
r_i	Total price of service in the i-th private hospital
b_{o_i}	Amount paid by the patients to the i-th private hospital
s_i	Subsidy payment made by the government to the i-th private hospital for each patient
k_{o_i}	Cost of unit capacity in the private hospital
k_d	Cost of unit capacity in the public hospital
q_{o_i}	Service quality level in the i-th private hospital
q_d	Service quality level in the public hospital
λ	Total arrival rate of all hospitals' patients
p_{o_i}	Probability of selecting the i-th private hospital by a patient
p_d	Probability of selecting the public hospital by a patient
m_{o_i}	Capacity of the i-th private hospital
m_d	Capacity of the public hospital
t_{o_i}	Average examination time in the i-th private hospital
t_d	Average examination time in the public hospital
μ_{o_i}	Service rate per unit capacity in the i-th private hospital
μ_d	Service rate per unit capacity in the public hospital
N_{o_i}	Amount of service quality received by the patients of the i-th private hospital
N_d	Amount of service quality received by the patients of the public hospital
H_d	Amount of expenditure made by the government (public expenditure)
T	Financial incentive from the government
w_{o_i}	Average waiting time in the i-th private hospital
w_d	Average waiting time in the public hospital
k	Price sensitivity of a patient
U_1	Average satisfaction level of patients
U_2	Average government expenses per patient
U	Total public utility
$Z_{o_{jm}}$	The profit of the j-th private hospital in Model m

ACRONYMS

ED	Emergency department
SGK	Social security institution

1. INTRODUCTION

Public hospitals and private hospitals co-exist in the health system of many countries. Service levels in private hospitals are generally higher, waiting times are shorter, but high fees are required for this service. On the other hand, although the public hospitals claim much lower or even free service, long waiting times and low service quality decrease patient satisfaction in these hospitals. Patients make a choice between these two types of service providers, depending on their income level and quality sensitivity. Private hospitals generally serve patients with higher income levels, while those with lower income levels prefer public ones, in general. Many researchers analyze the factors affecting patients' hospital preferences (Smith et al., 2017). Qin and Prybutok (2013) analyze the factors affecting patients' satisfaction and also their behavior to choose a hospital and identify that the price is one of the most important factors affecting patients' preference. Andritsos and Tang (2014) and also Andritsos and Aflaki (2015) report that in addition to public ones, the presence of private service providers in healthcare systems could reduce patient waiting times and government spending. However, Duckett (2005) and Marchand and Schroyen (2005) state that governments should be cautious about supporting the private sector in the health system, owing to that improper supports may harm the system rather than being beneficial.

Collaboration between governments and private hospitals takes place on the basis of contract mechanisms. Therefore, these mechanisms have an important role in the design of efficient and fair health systems. In this thesis, in order to maximize the social utility under budget and other possible constraints, different mechanisms that can be proposed by a government to private hospitals are discussed and the results of these mechanisms are analyzed. As first, we design a simulation model of the emergency departments of the hospitals in Eskisehir province. In the model, the current status of the emergency services is modelled as a whole in the Rockwell Arena program by considering the demand and capacity of the emergency services, hospital preferences of the patients, the total subsidy paid by the Social Security Institution of Turkey (SGK) to these emergency services and the processes in the emergency services. The validity of the model is confirmed by comparing the outputs such as the number of patients and average waiting times with the observed system data. As stated previously, in the present state of the system, the payments in public hospitals are low, the waiting times are high and therefore the quality level perceived by the patients is low. In private hospitals, payments and service quality are high while average waiting times are low. In our research, it was determined that the price in private hospitals is one of the most important factors affecting the preference of patients in the hospital. So, it is supposed that when the fees in private hospitals decrease, more patients will prefer these hospitals and thus the total quality achieved by the society will increase. Then, the effects of various contract mechanisms affecting prices in the private hospital are analyzed using the simulation model. In addition, the impact of the increase in the capacity of public hospitals (number of doctors and nurses who examined the emergency services) is also analyzed through the simulation model.

In Chapter 4, a system similar to the system described in Chapter 3 is analytically analyzed. For this aim, all hospitals in the region are modeled as one public and one private hospital. Even though this assumption might seem restrictive at first, it is observed that the patients usually make such a choice between the nearest public hospital and the private hospital. Hence, it is assumed that hospitals in different regions are independent. So, modeling a regional system, in which all public and private hospitals in a region are unified under one public and one private hospital can adequately represent its dynamics. Also, this is a basis for the models with more than one private hospital. Thus, the analysis of such a system will provide significant contributions to the literature. In the models, the state in which there is no contract between the government and the private hospital, contract mechanism based on subsidy payments, contract mechanism based on differentiated payments by patients in the private hospital and the linear two-part tariff contract mechanism are analyzed. Then, these policies are compared in the result of the chapter.

In Chapter 5, similar to Chapter 4, the hospitals in the region are modeled as one private and one public hospital, but in addition to the pricing policies, the capacity decisions of the private hospital are taken into consideration in the models. After analyzing the case in which there is no contract between the government and the private hospital, based on a Stackelberg game approach, the decisions of the private hospital are modeled following the decisions of the government as a leader. For this aim, the models based on subsidy payment by the government as a leader, contract mechanism based on subsidy payments and the linear two-part tariff contract mechanism are compared.

In Chapter 6, the models with one private hospital are generalized to the models where two private hospitals are located near a public hospital. Similar to Chapter 4 and 5, the first model is the case where there is no contract between the government and the private hospital. In another model, which is designed based on game theory, as many real systems, each private hospital can make its own decision to accept or reject the proposed contract by the government according to its profit. In this case, the Nash equilibrium point is analyzed in different situations. In addition, the case in which the government offered a contract to only one of the private hospitals is discussed. In the numerical results of the chapter, the proposed models are compared via the concept of social utility.

2. LITERATURE REVIEW

Several methods of operations research are frequently applied in various areas of healthcare management. Health policy analysis, hospital management, appointment scheduling, operating room management, models of epidemics, transplantation models of kidney and liver, control models for cancer and similar diseases and many similar studies in the literature can be found.

2.1. General Applications of Industrial Engineering and Operations Research in Healthcare Management

Alagoz et al. (2011) present a literature review including studies on cancer control tests. Ayer et al. (2012) create a personalized mammography test model based on past test results and personal risks and also use a finite-time partially visible Markov decision process model to highlight when women should undertake this test and when they should not. Gail and Rimer (1998), Ivy (2002) and Maillart et al. (2008) present similar studies. Management and planning of operating rooms is another topic that is frequently studied. Blake and Carter (1997) and Cardeon et al. (2010) present literature reviews on this subject. Shylo et al. (2013) model and analyze the block scheduling system under some random constraints in order to maximize the expected usage level of the operating rooms by considering the uncertain operation times in operating rooms. Dexter et al. (1999), Hans et al. (2008), Denton et al. (2010), Min and Yih (2010) and Batun et al. (2011) have similar studies. Analysis of the various policies that can be applied to minimize the impact of the diseases on society and modeling epidemics is another topic in the health management literature. Ekici et al. (2013) analyze the different policies that could be applied to minimize the impact of outbreaks such as swine flu and avian influenza through simulation. Ozaltin et al. (2011) use a stochastic programming method for the planning of influenza vaccine production. Organ transplantation, especially kidney and liver transplantation, is another area where operations research techniques are frequently applied. Alagoz (2009) presents a literature review on this subject. Zenios et al. (2000), Su and Zenios (2005) emphasize the problem of transferring a large number of organs to the most appropriate persons. Ahn and

Hornberger (1996), Alagoz et al. (2004, 2007) and Sandikci et al. (2008) design a model about the decision to accept the transplant or to wait for the next organ, depending on the characteristics of the proposed organ. Stahl et al. (2005) and Kong et al. (2010) study the reorganizing of regions in the US for liver transplantation.

Appointment scheduling in hospitals is another topic that is frequently studied in the field of health management. Different appointment scheduling methods are designed in order to reduce the waiting times of patients. Cayirli and Veral (2003), and Gupta and Denton (2008) present a literature review in this area. Robinson and Chen (2003) compare systems of physician appointment. Kaandorp and Koole (2007) try to find the best appointment scheduling system and suggested a method to minimize patient waiting times and the average free time of doctors. Cayirli et al. (2006) compare different appointment scheduling systems for ambulance services using simulation.

2.2. Applications of Simulation in Health Systems

Hospital and emergency department management are one of the most applied issues in the field of healthcare management. The complexity of health systems is due to their detailed and dynamic structures. Since simulation is one of the most effective methods to deal with these complexities, it is used by many academics and managers (Chahal, 2008). For example, the use of discrete event simulation techniques is a popular approach in order to maximize patient flow and satisfaction and also to make the use of resources more efficient (Jun, 1999). Weng et al. (2011) propose a simulation-based model for the optimal assignment of resources in an emergency department. As a result of the evaluation of the different scenarios designed in the Simul8 software with the OptQuest program, the satisfaction of the patients has increased. In this study, it is showed that performance can be increased by up to 8% with correct resource assignment. In another study, Cabrera et al. (2011) perform an agent-based simulation to find the number of doctors, nurses, and other employees needed to maximize patient flow in emergency services and minimize waiting times. As stated in the study of Jacobson et al. (2006), simulation provides benefits for health sector managers to analyze systems and to make the most appropriate decisions by predicting the results of different scenarios. In addition, it is useful to analyze the relationships between factors such as patient arrival rates, service times, amount of resources, and staff assignments (Carlson et al., 1979). Baesler et al. (2003) analyze the effects of possible demand changes on the emergency department of a private hospital by simulation. Ahmed and Alkhamis (2009), combining simulation and optimization techniques, propose a decision support system for the management of operations in an emergency service. Swisher and Jacobson (2002) set up a model that benefits different statistical techniques in a visual simulation environment to improve processes of hospitals and also to improve satisfaction. They also propose a new criterion that measures the efficiency of hospitals, taking into account different objectives such as satisfaction, quality, participation and waiting times.

The majority of the simulation studies discussed in the literature deal with a single service and analyze outputs such as waiting times in this service depending on various resource allocations (Cabrera et al., 2012, Liu et al., 2017, Duguay and Chetouane, 2007, Uriarte et al., 2017). There are a limited number of simulation studies involving multiple hospitals or emergency services. Fournier and Zaric (2013) examine the effects of the number of beds in intensive care units of hospitals in a certain region. Mielczarek and Uzialko-Mydlikowska (2012) present a detailed literature review about the applications of simulation in the health sector. Gunal and Pidd (2010), note that in the last 30 years, especially since 2004, there is an increasing number of articles about the applications of discrete event simulation models in health systems, however, they are generally hospital-based and use routine methods. Brailsford (2007), reports that despite many examples of using the simulation in academic studies, only a few examples of the real application of this tool in health systems exist. Kuljis et al. (2007) summarize the potential applications of simulation in healthcare management subjects by comparing the business and production sectors.

2.3. Policy Making, Payment and Pricing Decisions in Healthcare Management

In service systems, the scheme of payments affects performance and revenue (Adida et al., 2016, Afeche, 2013). Therefore, performance-based contracts are becoming widespread in health systems (Jiang, 2012). For example, to drop off preventable readmissions, some healthcare systems have begun to apply reimbursement schemes such as pay for performance or bundled payment instead of fee for service (Andritsos and Tang, 2018). So and Tang (2000) develop a mathematical model to examine the impact of reimbursement policy for drug usage. Guo et al. (2016), examine the impact of these two reimbursement schemes on patient welfare, readmission rate, and waiting time in a public healthcare system.

Zhou et al. (2017) denote that the subsidization of private institutions by the governments is an important matter in health reform to access a safe and effective healthcare system. Governments can increase their subsidization rates, allowing more patients to go to private institutions, but this ratio must be consistent with the objectives and resources in the health system. Hoel and Saether (2003) state that in a health system involving public and private institutions, private facilities should be subsidized or public services should be limited to a definite fee if there are capacity constraints. In the case of low subsidy rates, public institutions may face over-crowd, while the capacity of private ones may remain unused. Conversely, private institutions start to become crowded and the health expenditures of the government increase due to the high subsidy payments. For these reasons, finding the best ratio of subsidy is an important research topic. Qian et al. (2017) analyze different subsidy mechanisms and report that differentiated price policies are beneficial for health systems. Qian and Zhuang (2017) state that subsidy policies can be used to direct patients with higher sensitivity to waiting time to private hospitals. Thus, the congestion in the public hospitals will decrease and the system will be more balanced and accordingly the social utility will increase.

Price differentiation is a method commonly used by airlines, hotels, insurance companies, car rental companies. It is researched in the income management and pricing literature (Borsenberger et al. 2016, Gao et al. 2015, Raza 2015, Xu et al. 2014). This approach can increase the profitability and satisfaction levels of individuals when there are different sensitivities of price and service (Wolk et al. 2010). Philips (2005) and Talluri and Van Ryzin (2004) provide detailed information about this approach.

2.4. Contract Mechanisms in Health Sector

Although there are many studies in different areas of healthcare management, the applications of contracts mechanisms in the health sector are not widely researched. As a few examples, Palmer and Short (2000) analyze health policies in Australia from various perspectives. Kreis and Schmidt (2013) survey the impact of application and evaluation of health technologies on the public through experiences in France, Germany, and the United Kingdom (UK). You and Kobayashi (2009) analyze the effect of the mandatory health insurance application on healthcare expenditures in China. Chick et al. (2008) discussed the implementation of contract mechanisms in the health sector and showed that in case of supply uncertainty, the cost-sharing contract could provide good coordination for the flu vaccine supply chain.

Despite healthcare management, in supply chain literature, contract mechanisms and their different applications are widely researched. Cachon (2003) provides a detailed study of supply chain coordination based on different contract mechanisms. Tsay et al. (1999), Lariviere (1999), Cachon and Netessine (2003) and Yano and Gilbert (2004) present literature reviews in this field. Different types of contract mechanisms are discussed in the literature, such as wholesale price contracts, repurchase contracts (Pasternack, 1985), revenue sharing contracts (Cachon and Lariviere, 2005), quantity flexibility contracts (Tsay, 1999), refund contracts (Taylor, 2002) and quantity reduction contracts (Tomlin, 2000). Kaya (2011) analyzes different contract mechanisms in a supply chain where demand depends on a costly effort and look for the contracts that yield the best results in different situations of the system. Aksin et al. (2008) research possible contract mechanisms between a service provider's with other companies for call center operations and analyze the effects of each mechanism on price and capacity decisions in the system.

3. ANALYSIS OF DIFFERENT PUBLIC POLICIES THROUGH SIMULATION TO INCREASE TOTAL SOCIAL UTILITY IN A HEALTHCARE SYSTEM

In this chapter, we consider the public and private hospitals in a certain region of Turkey and analyze the effects of public policies on the patients' preferences regarding hospital choices and the results of these choices on social utility and public spending. In order to analyze different capacity decisions, contracting and pricing mechanisms; we develop a simulation model of multiple EDs of private and public hospitals in Eskisehir region considering patient preferences among these hospitals and aim to determine the optimal system parameters under different contract mechanisms via this simulation model. After the validation and verification of the simulation model, several scenarios are designed and executed to increase social benefit, decrease government expenses, improve patient satisfaction level and decrease waiting times. We compare the proposed scenarios based on multiple objective functions and present numerical results for different scenarios in this system.

3.1. System and Model

We consider a healthcare system with multiple public and private hospitals and focus on the operations performed in emergency departments of these hospitals. A patient in this system first chooses a hospital depending on his own utility function considering the quality of the service level, proximity and the prices of the hospitals. A patient choosing a private hospital needs to pay a higher price but obtains a higher quality service and lower waiting times. On the other hand, a patient choosing a public hospital pays a lower fee but obtains a lower quality service and higher waiting times.

To overcome the problems of imbalanced demand and supply in public and private hospitals, SGK implements a policy that will encourage more patients to go to private hospitals rather than public ones. SGK offers a contract to the private hospitals setting the prices of certain procedures that will be charged by the private hospitals, and in return covers a part of this price as a subsidy payment such that the patients going to private hospitals will pay even less due to this subsidy ratio. This contract leads to more patients going to private hospitals instead of public ones, leading to less public hospital costs and lower waiting times in the public hospitals, but higher spending made by SGK for the private hospitals. The private hospitals compensate their loss due to the decrease in their prices from the additional subsidy payments made by SGK. However, even under this system, it is observed that there are much higher waiting times and lower patient satisfaction at the public hospitals compared to the private ones in Turkey. In this chapter, one of our objectives is to determine the optimal pricing mechanism that will balance the demand for public and private hospitals, considering the healthcare budget and payments of SGK, to maximize the total social utility. In addition, we also consider different strategies that can be employed by the government and analyze the results of these strategies on the system results. We aim to increase the social utility for the society as a whole considering healthcare spending as an important consideration. The total quality of the healthcare service obtained by the patients, the waiting times in the healthcare system and the payments made by the government for the healthcare system are considered to be the main components of the social utility.

We consider multiple objectives and we let H_d denote the payments made by SGK in the healthcare system and aim to decrease it in our objective function. H_d consists of fixed costs in the public hospitals, salaries of doctors and nurses and also the total amount of subsidies paid by the government for the patients going to the public and private hospitals. We also aim to decrease the waiting times at the public and private hospitals and let w_d and w_o denote the average waiting time per patient in the public and private hospitals, respectively. The total quality of the healthcare service obtained by the community is another performance measure that we consider. We let q_d and q_o denote the quality levels of the public and private hospitals, respectively. These quality levels measure the factors related to the quality of staff (nurses, doctors, etc.), quality of the devices in the hospital, cleanliness, decorations etc. that affect the patients' perceptions about the hospitals. In addition, the treatment qualities are also different between the public and private hospitals because the private hospitals generally pay more to the doctors than the public ones and more qualified doctors generally prefer to work at private hospitals. The quality levels are determined based on the patients' judgments about these hospitals, obtained through a questionnaire including more than 200 respondents in the public and private hospitals. The questionnaire designed by the Turkish Ministry of Health is used to measure emergency services patients' satisfaction coefficient. In our numerical analysis, we let the quality levels, $q_d = 0.6$ and $q_o = 0.9$ in the public and private hospitals, respectively, based on the results of our surveys. Since most of the questions in the questionnaire are related to staff and tangible facilities, it is assumed that these values denote the quality perception of the patients and are independent of the number of patients going to these hospitals. In this chapter, N_d and N_o denote the number of patients served by the public and private hospitals, respectively, and as a result, we aim to increase the value of $q_d N_d + q_o N_o$ which denotes the total quality of the healthcare service obtained by the community. Note that, even though the quality and satisfaction level in the emergency services are also related to the waiting times, these measures are separated to underline their importance. Using simulation, we analyze the results of different scenarios based on these values that will be obtained as the outputs of the simulation model and we aim to select the best scenario based on our objectives.

Since we have multiple performance measures, in order to be able to compare different scenarios, we also define a single value named as the social utility as in equation 3.1, which combines different performance measures using a weighted sum method.

$$U = \frac{q_d N_d + q_o N_o}{N_d + N_o} - \alpha_1 \frac{N_d w_d + N_o w_o}{N_d + N_o} - \alpha_2 \frac{H_d}{N_d + N_o} \tag{3.1}$$

In the above equation, the first term denotes the quality of the received healthcare service per patient, the second term denotes the average waiting time per patient and the last term denotes the average public healthcare spending per patient. We use α_1 and α_2 to denote the weights of the average waiting time and the public spending in the objective function. The values of $\alpha_1 = 0.01$ and $\alpha_2 = 0.006$ are used in our numerical analysis in order to normalize the values in the objective function such that they are in comparable units. However, the effects of different values of these coefficients are also examined in the sensitivity analysis section.

In the next sections, we first describe the current situation in the analyzed healthcare system and then explain our simulation model. We implement the simulation model using Rockwell Arena 14.5 and Process Analyzer softwares. We include all the emergency services of 3 public and 4 private hospitals in the Eskisehir province of Turkey. Then, using the simulation model, we explore different policies that can be used by the government to improve the system performance and present our numerical results.

3.2. Description of the Current State in the Emergency Services of Eskisehir Province

Eskisehir province is one of the largest health service regions in Turkey. In addition to its own population of 2.5 million, its health facilities also provide service for at least 3 neighbour states. Totally, there are 9 public hospitals in the province, serving in almost all healthcare branches such as gynecology, pediatric, surgery, etc. These hospitals have different roles in accordance with the strategic plans of Turkey and all of them have emergency services. According to the criteria of the ministry of health, based on the facilities and resources, there are 4 types of emergency services in the health system: (i) emergency units, (ii) first level, (iii) second level and (iv) third level emergency services. In this chapter, from the public hospitals, only the hospitals with second and third level emergency services are selected. In an emergency unit, there is at least one room where first aid and basic life support can be provided for emergency patients. In the first level emergency services there are 1 or 2 health officers, nurses, and technicians. There is also a medical assistant and they are managed by 24-hour uninterrupted service of medical practitioners. Owing to the lack of advanced equipment, mostly ailment, and mild injuries are treated in these units. The second level service facilities include ambulance station, triage technician (or nurse), resuscitation, observation, basic treatment, medical imaging, isolation units, triage doctor, trauma room, critical care units and examination rooms. In addition to these facilities, the third level emergency services also have specialist physicians of different branches, and have more than 800 m^2 area. There are 15 private hospitals in total with different branches in the province and only a few of them have second level or third level emergency services. 3 public and 4 private hospitals that are in the city center and have the highest demand among the hospitals are included in the model of this chapter. The other emergency clinics are much smaller and thus left out of this chapter, since the demand for these emergency services are very low compared to the others. The public hospitals are represented as Pu and the private hospitals are indicated as Pr. Pu_1 has two separate emergency units, stated as (i) adult, and (ii) pediatric and these units are denoted as $Pu_{1_{ad}}$ and $Pu_{1_{pd}}$, respectively.

For the current system, we collected data about the number of arrivals to the emergency services in different time periods of the day and in different days of the week. In addition, the information about the number of patients, arrival patterns, costs, etc. was taken by the hospital administration of the public hospitals. Although there are some changes between the seasons, the presented data about the current state is generally representative of the system for all times. In **Table 3.1.**, based on the real-life data in the current system, we present the percentages of the yearly number of patients going to emergency services of the hospitals in the Eskisehir region. We observe that about 80% of the patients prefer public hospitals, while only 20% of the patients prefer to go to private hospitals for emergency cases.

Based on the arrival numbers, on the weekdays, patient arrivals are divided into three time intervals as 02:00-08:59, 09:00-16:59 and 17:00-01:59, while they are divided into two time intervals as 02:00-08:59, 09:00-01:59, on the weekends. In the current system, in weekdays polyclinics are closed after 17:00. They also closed in weekends, hence most of the outpatient treatments are done in the green zone of emergency services. Therefore, between 17:00 and 01:59 there is a crowd in the emergency services, however, it decreases gradually after 2:00. Since many patients who use these emergency services come for outpatient treatment, they do not prefer to take service between 02:00 and 08:59. Arrivals in these intervals are found to be best fitted to exponential distributions with mean times as presented in **Table 3.2.**, based on the results of the Input Analyzer tool of the Rockwell Arena 14.5.

We observe 3 types of arrivals to the emergency services: (i) from patient admission unit, (ii) as consultation and (iii) by ambulance. Arrivals from the patient admission unit are about 90% of all arrivals. Consultation is the state, in which the patient is sent from other branches of hospitals to the emergency area. In the public hospitals emergency services, excluding $Pu_{1_{pd}}$, the patients are tagged for their priority identifications. The tags are (i) red tag (emergent, immediate): patients whose life is threatened and lifesaving measures must be applied immediately, (ii) yellow tag (urgent, delayed): patients whose lives might be in a threatening condition if medical attention is not applied in hours, (iii) green tag (non-urgent, minor): patients who don't have any life-threatening condition and can be delayed while other patients are treated, and (iv) black tag: patients who are dead or medically unexpected to survive and have no priority. On the other hand, in the private hospitals' emergency services, patients are categorized as medical (minor) and emergency types. As shown in **Figure 3.1.**, based on these tags, several routes are followed by the patients. General steps of emergency services can be summarized as (i) patient admission, (ii) immediate treatment, (iii) waiting room, (iv) triage, (v) green examination unit, (vi) critical care unit (vii) premier patients care unit, (viii) trauma room, (ix) treatment units, (x) yellow examination units, (xi) observation unit, (xii) medical imaging units and (xiii) hospitalization. Patients are directed on different routes depending on their requirements. Some of these steps contain more than one process, for example, yellow examination unit includes consultation and secondary triage. The absence of secondary triage tends to increase patients waiting time because doctors have to examine the patients first before ordering clinical tests if they were needed. More detailed figures related to the simulation model can be found in the appendix.

Generally, red zone patients are brought to the emergency services by ambulance. These patients are instructed by the emergency command center, which selects hospitals, based on a coordinated system that shows available resources, which is updated once per hour. When a patient is entered into the emergency service by ambulance, according to the instruction, it is necessary to be examined as a critical patient, even if medically he isn't in the critical state. If the patient is in the critical state, he has the highest priority and should be treated without any waiting. Usually, the process of resuscitation is at most 45 minutes. After the initial treatment, the patients are generally dispatched to the intensive care units. In most hospitals, this unit isn't a part of the emergency service and sometimes the patients are dispatched to other provinces for intensive care. Usually, in each shift (8 hours), there is one doctor, who is the emergency medicine specialist and responsible for the emergency service, and one nurse in the red zone. When needed, additional doctors and nurses are added to the red zone personnel. Because of the highest priority of the red zone patients, if it is required, doctors of the other units help in these processes.

Consultation patients are directly passed to doctors for examination, medical tests, and imaging units. Other patients, making an entrance in the admission unit, wait for the triage process. In the triage zone, they are tagged as red, yellow and green zone patients. Red zone patients have the highest priority and are dispatched to red zone treatment quickly, similar to the patients that come with an ambulance. Yellow tagged patients have the second priority and green tagged patients have the least priority. Some patients are observed to be directly sent to the immediate treatment rooms before admission and without triage. Staffs of this section are health technicians and about 5% of patients are treated in this area.

Examination of each yellow tagged patient is done by one doctor and one nurse (assistant). After the first examination, the patients are generally dispatched to different medical analysis and imaging units according to their medical condition. During this process, they may get back to the doctor and be dispatched to other units. If they don't receive observation care, their treatment process is generally under 30 minutes. Green zone patients generally leave the emergency service after a basic examination. In addition to the red, yellow and green zone patients, some patients use emergency services for ordinary services like the injection.

Table 3.1. *Demands of the emergency departments*

	$Pu_{1_{ad}}$	$Pu_{1_{pd}}$	Pu_2	Pu_3	Pr_1	Pr_2	Pr_3	Pr_4
Percentage of arrivals	24.2%	10.7%	20.4%	24.7%	6.0%	6.7%	0.3%	7.0%

Table 3.2. *The average time between arrivals (in seconds)*

		Hospitals							
	Time intervals	$Pu_{1_{ad}}$	$Pu_{1_{pd}}$	Pu_2	Pu_3	Pr_1	Pr_2	Pr_3	Pr_4
Weekdays	02:00-08:59	373	845	391	360	408	420	8500	431
	09:00-16:59	94	212	106	99	1435	1436	28800	1435
	17:00-01:59	78	176	82	75	226	227	4600	227
Weekdays	02:00-08:59	368	826	402	347	432	403	8321	408
	09:00-01:59	85	192	92	82	377	379	7650	378

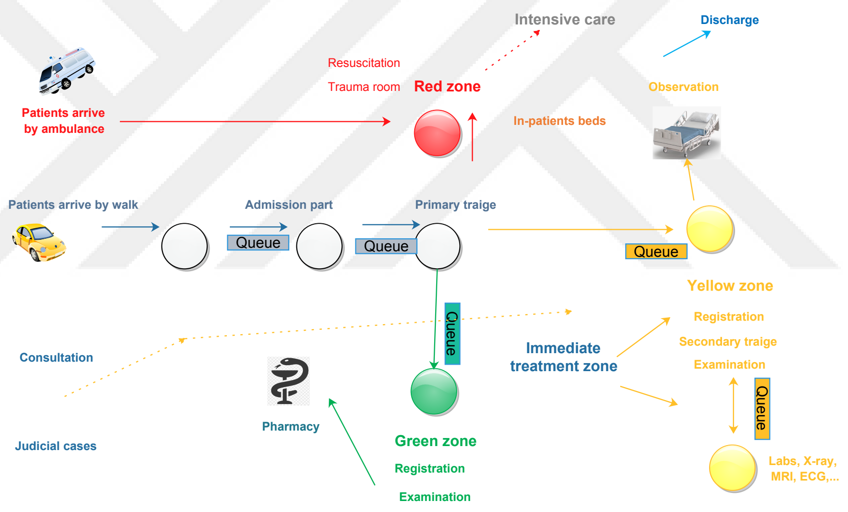


Figure 3.1. *Several routes in ED*

In the current system, about 75% of the patients are tagged as yellow zone patients in the public hospitals, while the red zone patients are about 3%, and the rest are green zone patients. In the private hospitals, green and yellow zone patients are generally combined and tagged as medical, while red zone patients are tagged as urgent patients, which are about 96% and 4% of all patients, respectively. This difference between public and private hospitals is because of the payment policies of the Turkish Social Security Institution (SGK) about the yellow and green zone patients. According to their interpretation, patients of the green zone aren't urgent, so they can be only examined and other items like injection cannot be billed and thus will not be covered by SGK. However, this application in the public hospitals, especially after 17:00 is impossible. Even though some patients should be tagged as green zone patients, many of the patients are categorized in the yellow zone in order for the doctors to be able to apply some basic outpatient treatments due to SGK regulations.

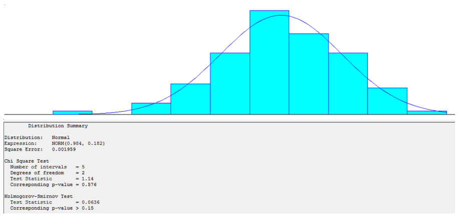
We observe that about 95% of the patients of the emergency services in both the public and private hospitals are treated as outpatients and only about 5% of them take place in the inpatient treatment category. Most of the inpatient treatments are in the pediatric units, which is about 9% at most.

Resources of the emergency services for a typical time of the week are presented in **Table 3.3.**, however, it is also observed that resources might change during the week depending on the number of patients in the emergency service. For example, in each shift, generally there are 1 doctor and 7 nurses in $Pu_{1_{pd}}$ and 1 doctor and 3 nurses in the private hospitals' emergency services, while 1 doctor and 1 nurse are added to these staffs in the crowded periods. In our simulation model, we reflect this situation by considering a critical level, which changes the resources.

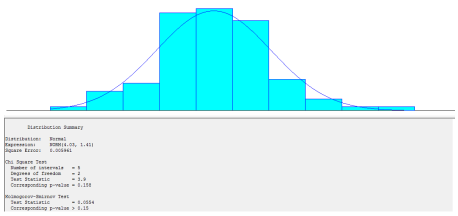
The distributions for process times are fitted using the Input Analyzer tool of the Rockwell Arena 14.5 simulation software, which are summarized in **Table 3.4.**. Input data for these processes are mostly collected by observation. For each process, over 100 data are gathered at different times of the day.

Table 3.3. *Resources of the emergency services in each shift*

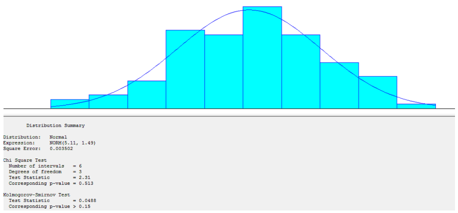
		Triage	Green zone		Immediate treatment	Examination		Observation	Resuscitation	
		Nurse	Doctor	Nurse	Technician	Doctor	Nurse	Nurse	Doctor	Nurse
Hospitals	$Pu_{1_{ad}}$	1	1	1	2	2	2	3	1	1
	$Pu_{1_{pd}}$	1	-	1	1	-	2	3	1	1
	Pu_2	1	1	1	2	2	2	3	1	1
	Pu_3	1	1	1	2	2	2	3	1	1
	Pr_1	-	-	-	1	1	1	2	1	1
	Pr_2	-	-	-	1	1	1	2	1	1
	Pr_3	-	-	-	1	-	-	2	1	1
	Pr_4	-	-	-	1	1	1	2	1	1



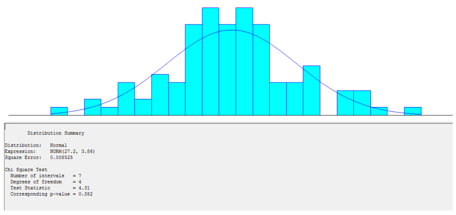
(a) *Fitted distribution for the admission and triage processes*



(b) *Fitted distribution for the green zone patients' examination time*



(c) *Fitted distribution for the yellow zone patients' examination time*



(d) *Fitted distribution for the outpatient treatment time*

Figure 3.2. *Fitted distributions*

Table 3.4. *Process times distribution (in minutes)*

		Distribution
Processes	Admission and triage processes times	Normal (0.984,0.182)
	Green zone patients' examination time	Normal (4.03, 1.41)
	Yellow zone patients' examination time	Normal (5.11, 1.49)
	Outpatient treatment time	Normal (27.2, 3.86)
	Observation time	30 + 225 × Gamma(3.13, 1.7)

Graphs of the fitted distribution for examination time in the green and yellow zones and also admission and triage processes are given in **Figure 3.2.**. These processes are the ones that lead to waiting periods before an examination, which is mostly focused on in this chapter. The input analyzer finds the best-fitted distribution doing Chi-square, and Kolmogorov-Smirnov tests. In these tests, if the p-value is greater than 0.10, then the null hypothesis, i.e., that the data is well fitted to a certain distribution, cannot be rejected. The p-values of the tests are written under the figures.

3.2.1. Cost and payment in the emergency services

The written notification of health services (SUT), issued by SGK, defines treatment payments in Turkish hospitals. According to this declaration, yellow and red zone patients don't pay any price for the services they get, either in the public or private hospitals emergency services. In the public hospitals' green zone, patients pay 5 Turkish Lira (TL) (To provide a perspective about these values, we note that the minimum wage was around 1200 TL per month and 1 TL was about 0.37 USD at the time). The average cost of green zone patients' examination is found to be 19.57 TL based on the receipts obtained from the public hospitals and SGK pays $19.57 - 5 = 14.57$ TL on average per green zone patient to the public hospitals. According to the contract between SGK and the private hospitals, the prices of examination and its services (red and yellow zones examinations as emergency cases aren't in this category) can be 3 times the price in the public hospitals. In contrast to the public organizations, that don't pay tax, there is also 8% tax in the private sector. According to this contract system, there are two types of pricing for the green zone patients in the private hospitals. If a patient is only examined without using the other medical services and analysis, the price is calculated as $(19.57 + 8\%\ \text{tax}) \times 3 = 63.4$ TL. 54 TL of this amount is paid by the patient and the remaining amount, 9.4 TL, is covered by SGK and paid to the private hospitals as a subsidy payment per patient. If in addition to the examination, a patient uses services like Rontgen or medical imaging, they will have additional costs. The distribution for the total price per patient in the private hospitals is found to have a Normal distribution with parameters (380, 90) based on the data obtained from the private hospitals. About 14% of this amount is paid by SGK while the rest is paid by the patients. Under this payment schedule, since the prices are generally much higher in the private hospitals, it is generally not suitable for many patients and due to this high cost of private hospitals, most patients do not prefer private hospitals. However, in the public hospitals, the costs are much cheaper and the cost a green zone patient is fixed at 19.57 which only includes an examination by the doctor. However, the costs of yellow and red zone patients, who require additional procedures at the public hospitals, are best fitted by the random function 15 +

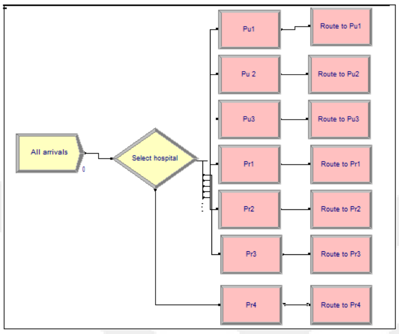
$Weibull(38.2, 0.68)$ using Arena's Input Analyzer function, based on the billing data obtained from public hospitals. The cost of red zone patients in the public hospitals is defined as $1.25 \times (15 + Weibull(38.2, 0.68))$, based on the data obtained from the provincial organization of health ministry. The differences between the costs of different patients are mainly because of the uses of different physical equipment, devices of medical tests, imaging and different operations performed. According to the obtained information, the cost of red zone patients in the private hospitals is approximately twice the values in the public ones. In **Table 3.5.**, we present the costs and the usage percentages for different procedures applied for the patients coming to the emergency services.

Table 3.5. *Emergency services general expenses in the public hospitals*

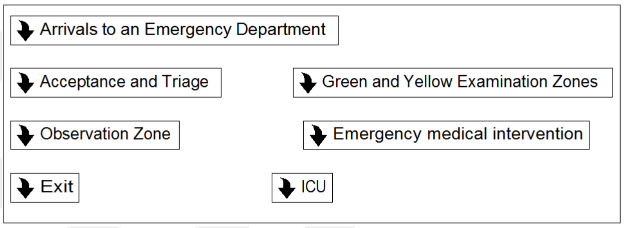
		Percentage	Average amount in the invoices (TL)
Treatment type	Surgery	7%	128.94
	Anaesthesia	8%	18.64
	Biochemistry	8%	10.63
	Electrocardiography (ECG)	10%	3.33
	Echo-cardiography (Echo)	<1%	26.2
	Hormone test	1%	6.5
	Drug	40%	7.84
	Magnetic resonance imaging (MRI)	<1%	65
	Examination	100%	19.57
	Immediate treatment	45%	27.36
	Psychological support	<1%	17.5
	Radiologic diagnosis	4%	6.88
	Rontgen	22%	16.79
	Other Expenses	27%	2.98
	Medical analysis	38%	29.67
	Treatment	<1%	8.5
	Tomography	13%	80.58
	Ultrasonography	1%	23.8
	Bed	29%	5.84

3.3. Outputs of the Current State Simulation and Validation of Results

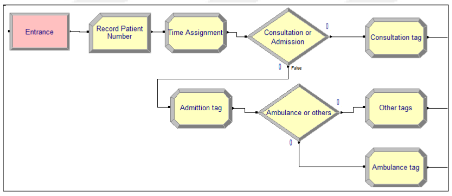
The described system in the previous section is simulated in the Rockwell Arena 14.5 simulation software on a system with Pentium dual-core, 2.20 GHz processor and 3.00 GB RAM. Some screenshots of the Arena model are shown in **Figure 3.3.**.



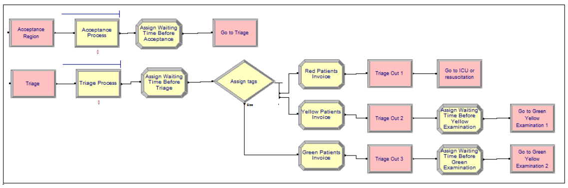
(a) *Arrivals to the region and patients' preferences to select an emergency department*



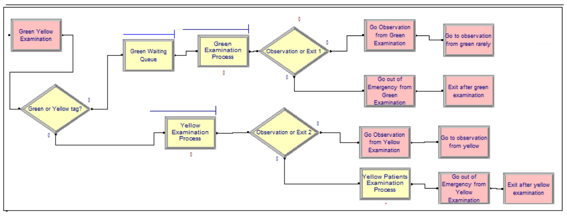
(b) *Different units within each emergency department*



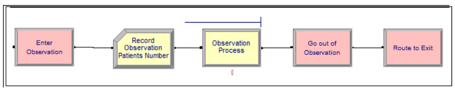
(c) *Arrivals to an emergency department*



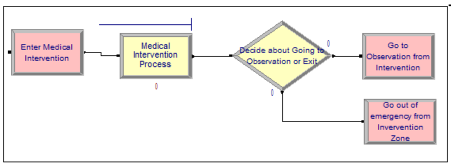
(d) *Acceptance and triage sub-units*



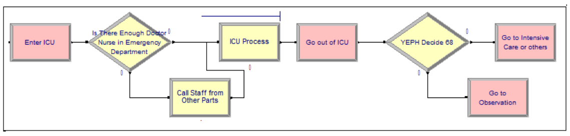
(e) *Green and yellow examination sub-units*



(f) *Observation unit*



(g) *Medical intervention unit*



(h) ICU

Figure 3.3. *Some screenshots of the Arena model*

Replication length of one simulation run is 30 days and the average of 30 simulations are considered in the results. The warm-up period is defined as the first day in each simulation run since the waiting times are seen to reach their steady-state situation after this time. Queues in the simulation model are (i) waiting queue in the admission part, (ii) waiting queue before triage and (iii) waiting queue after triage and before examination (for yellow and green patients). Other queues are either neglected or included with the other processes. For example, waiting queues for medical tests and imaging units are included in the yellow zone treatment time. Patients' total average waiting time in the emergency service is the average of total waiting times in the green and yellow zones calculated as *waiting time in the admission part queue + waiting time before triage queue + waiting time after triage and before examination queue.* Outputs of our simulation study are summarized in the following tables. The number of patients and costs (in TL) are monthly values and the time unit is a minute.

As shown in **Table 3.6.**, the average total monthly cost of the patients in the emergency services is approximately 8,420,521 TL. About 4,768,598.44 TL of this cost is paid by SGK, while the patients pay about 3,651,923 TL.

Validation of the simulation is tested based on the number of patients and their average waiting times in the real system and in the simulation outputs. The number of patients and their waiting times at each hospital in real-life and in our simulation results are seen to be very close to each other as presented in **Table 3.7.**. We also construct a 95% confidence interval for the mean of these values. Since all the confidence intervals include the mean values of the real-life system, we can state that the number of patients and their waiting times in the simulation outputs are not significantly different than the real system. In addition, the government expenses of SGK in the real-life are seen to be consistent with the government expenses observed in our simulation results. Thus, we can state that the simulation model is validated and we can conclude that the simulation model results are consistent with the actual system observations.

Lastly, observe that the ratio of the half-widths based on the 95% confidence intervals to the value of the means in **Table 3.7.** for all statistics are less than 10% for 30 replications, i.e. for n=30, the ratio $\frac{z_{(1-\alpha/2)}\frac{S(n)}{\sqrt{n}}}{\bar{X}(n)} \leq 0.1$, where

$\alpha = 0.95$, $S(n)$ denotes the standard deviation and $\bar{X}(n)$ denotes the mean of the simulation outputs with n replications, presented at each row of **Table 3.7.**. Thus, we conclude that $n = 30$ replications provide enough precision on our results.

Table 3.6. *Outputs for the current system*

	$Pu_{1_{ad}}$	$Pu_{1_{pd}}$	Pu_2	Pu_3	Pr_1	Pr_2	Pr_3	Pr_4
Number of patients	22042.8	9442.5	18696.4	21986.3	5533.3	5997.7	276.5	6202.1
Waiting time per patient	20.34	4.09	18.38	19.99	1.61	1.70	0.02	1.72
Maximum waiting time	45.72	10.34	40.45	43.18	5.47	5.78	0.18	5.19
Total cost	1204135	512874	996723	1193457	1382324	1502083	64235	1564690
Total spending of SGK	1179974	502583	976724	1164809	289585	314407	3784	336731

Table 3.7. *Comparison between the real system and the outputs of the simulation model*

	Number of Patients				Waiting Times			
	Real system	Simulation model			Real system	Simulation model		
	Average	Average	Std. dev.	95% Conf. Int.	Average	Average	Std. dev.	95% Conf. Int.
$Pu_{1_{ad}}$	21776	22042	1342	[21562, 22522]	21.18	20.34	3.81	[18.98, 21.70]
$Pu_{1_{pd}}$	9636	9442	723	[9183, 9701]	3.86	4.09	1.12	[3.69, 4.49]
Pu_2	18324	18696	1128	[18292, 19099]	19.33	18.38	3.35	[17.18, 19.58]
Pu_3	22239	21986	1454	[21465, 22506]	21.29	19.99	3.79	[18.63, 21.35]
Pr_1	5419	5533	625	[5309, 5756]	1.52	1.61	0.37	[1.48, 1.74]
Pr_2	6034	5997	687	[5751, 6243]	1.79	1.70	0.43	[1.55, 1.85]
Pr_3	268	276	35	[263, 289]	0.02	0.02	0.005	[0.018, 0.022]
Pr_4	6345	6202	712	[5947, 6457]	1.81	1.72	0.47	[1.55, 1.89]

3.4. Definitions of Different Scenarios and Their Outcomes

In this section, we explain different policies that can be implemented by the government in order to improve the healthcare system. We aim to come up with strategies that will decrease the waiting times at the public hospitals that will lead to a better-balanced system between public and private hospital utilization. However, we also have a budget constraint such that we don't want to increase the government expenses. Thus, we aim to increase N_o while decreasing H_d, N_d, W_d, and W_o. Note that some of these objectives conflict with each other and we aim to find a system that balances these values.

In addition to considering these values separately, we also consider a weighted sum of these values, defined as the social utility, U, as given in Equation 3.1. In addition to evaluating different parameters defined in each group of scenarios below, we also determine the best values of these parameters based on the objective of maximizing U.

We propose different scenarios in the following sections and analyze their results based on the simulation outputs. We make 30 replications for each scenario, each of length 30 days and the warm-up period of each scenario is the first 3 hours since the waiting times are seen to reach their steady-state situation after this time. In order to eliminate the effect of random number generation when comparing different scenarios, we use the same random number seeds among the runs of different scenarios. Thus, the same random numbers will be used in different scenarios, the number of patients generated will be the same, and the only difference between different scenarios will exist only because of the differences of the applied policies. The results of all the scenarios are summarized in **Table 3.8.**. The improvement percentages obtained by the scenarios according to the current state is shown in the last column of this table.

3.4.1. Scenario 1: Policies based on the resource capacity decisions

We first consider the capacity decisions in the public hospitals and analyze the system results for different amounts of resources. In this scenario, the number of examination teams has been increased in the public hospitals. We note that the waiting times at the private hospitals are significantly lower and the number of resources in the private hospitals is thought to be sufficient. In scenario 1-1, we add one more doctor and one more nurse to all shifts of all emergency services of the public hospitals. This change results in the average waiting time at the public hospitals to be 3.28 minutes, however, it leads to about 6% increase in the public spending due to the cost of additional resources. In case of adding two (scenario 1-2), three (scenario 1-3), four (scenario 1-4) and five (scenario 1-5) examination teams to all shifts of all emergency services of the public hospitals, the average total waiting times decrease to 2.02, 1.81, 1.73 and 1.72 minutes, while the government expenses increase by 11%, 17%, 23%, and 28%, respectively. The results of these scenarios can be seen in **Table 3.8.** in more detail. As seen in the table, adding more resources to the emergency services of the public hospitals will lead to more costs and the healthcare budget needs to be increased significantly, which might not be possible. In addition, it is generally not possible to increase the number of resources, not only because of the costs but also because of the infeasibility of such changes. Since the number of doctors and nurses are limited in the healthcare system, it might not be possible to find new doctors to employ in the system. Adding more doctors or nurses into the emergency system might require the number of doctors or nurses in the other parts of the hospitals to be decreased which will cause more significant problems in those areas. In addition, we also consider the option of opening an additional public hospital in the region which will decrease the waiting times, however we do not analyze that option in our simulation experiments since opening an additional public hospital requires an extensive amount of investment which is out of the budget constraint of the government and thus said to be infeasible. Because of these reasons, we search for other policies which will not require additional healthcare budget within the given resource constraints and analyze different pricing strategies in the next sections in order to better utilize the public and private hospital resources.

3.4.2. Scenario 2: Policies based on financial incentives for private hospitals

Directing some of the public hospital patients to the private ones might benefit the system as a whole. However, when a patient chooses to go to the private hospital instead of the public one, the healthcare spending of the government will also change. In the current system, about 86% of the private hospital expenses are paid by the patients, while 14% of these expenses are paid by the government as subsidy payments. For example, the green zone examination price is 63 TL in total and 54 TL is paid by the patients while 9 TL is paid by the government to the private hospital. The other expenses are also paid based on these ratios. Changing these ratios might result in improvements in the system by changing the patient preferences and the demand pattern for the hospitals. In this section, we analyze the results of applying different subsidy payment ratios for the private hospitals using the simulation model and aim to determine the optimal subsidy value.

Recall that in the current system, the price of just an examination at the private hospitals is 63 TL and the government pays 9 TL (14%) of this price while the patients pay 54 TL. We analyze four scenarios in which the ratio of the payments made by the government as subsidy payments for the patients going to the private hospitals are increased to 20%, 28%, 36% and 45%, such that the patients need to pay 50, 45, 40 and 35 TL, respectively. All the other prices for additional operations will also be paid in the proposed payment ratios. For this purpose, we first prepare a survey to understand the patient preferences between public and private hospitals based on different payment values for the private hospitals. In this questionnaire, 200 patients in various hospitals were asked about their maximum willingness to pay in order to prefer the private hospitals rather than public ones for emergency departments. Results of this survey demonstrate that most of the patients going to the public hospitals might prefer the private hospitals if the price they need to pay to the private hospital is decreased. Using the survey results, we estimate the percentage of patients that will prefer the private hospitals under different price values. We present the estimated demand structure for the private hospitals under different subsidy payment ratios based on our survey results in **Figure 3.4.**. In the next scenarios, we analyze the system results for varying subsidy payments and patient preferences.

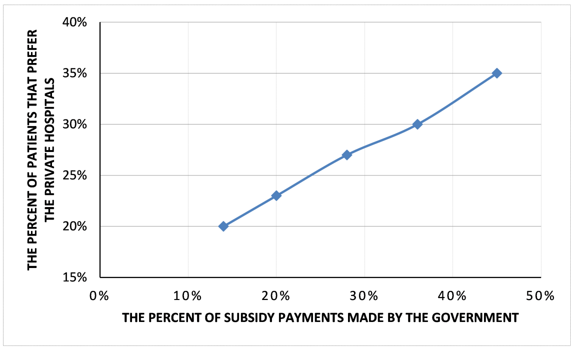


Figure 3.4. *Estimated demand for the private hospitals under different subsidy payment*

Scenario 2.1: The government subsidy payment for the private hospitals is increased to 20%

In this scenario, we analyze the system results when the subsidy payments made by the government is increased by 6% such that 80% of the total expenses are paid by the patients and 20% is paid by the government. According to the survey results, about an additional 3% of the population will now prefer the private hospitals instead of the public ones as a result of this decrease in patient payments. As a result of this change, based on the simulation outputs as seen in **Table 3.8.**, the expenses of the SGK will increase by 6%. In this case, the total average waiting time of the patients in the public hospitals is seen to decrease to 11.14 minutes, while the average waiting time at the private hospitals increases to 1.85 minutes.

Scenario 2.2: The government subsidy payment for the private hospitals is increased to 28%

Now, we assume that the patients need to pay 72% while the government pays 28% of the total private hospital expenses. In this case, about an additional 7% of the population will now prefer the private hospitals instead of the public ones, compared to the current system. As a result of the change in this demand pattern, expenses of SGK are seen to increase about 17%. The total average waiting time of the patients in the public hospitals is seen to decrease to 7.13 minutes, while the average waiting time at the private hospitals, increases to 2.36 minutes. As a result of this scenario, the average waiting time at the public hospitals is decreased even more, but 17% increase in government expenses brings a significant cost to the government.

Scenario 2.3: The government subsidy payment for the private hospitals is increased to 36%

In this scenario, we assume that the patients need to pay 64% while the government pays 36% of the total private hospital expenses. In this case, about an additional 10% of the population will now prefer the private hospitals instead of the public ones. As a result of this scenario, expenses of SGK increase by 30%, while the average waiting times at the public and private hospitals are 4.69 and 3.57 minutes respectively. Even though the average waiting times are better balanced in this scenario, the additional 30% increase in government expenses is not seen acceptable.

Scenario 2.4: The government subsidy payment for the private hospitals is increased to 45%

Now, we assume that the patients need to pay 55% while the government pays 45% of the total private hospital expenses. In this case, about an additional 15% of the population will now prefer the private hospitals instead of the public ones. As a result of this scenario, expenses of SGK are seen to increase by 45%, while the average waiting times at the public and private hospitals are 3.18 and 5.72 minutes, respectively. In this case, the average waiting times at the private hospitals are now more than the public ones since the capacity of the private hospitals is not enough to handle the increased demand in this scenario. As a result, the private hospitals might not be preferable as before, which is not an acceptable situation for the managers of the private hospitals. In addition, the 45% increase in government expenses is significantly more than the acceptable values. Thus, this scenario is not seen to be an acceptable one.

Scenario 2-opt: The government subsidy payment for the private hospitals is determined to maximize U

When we analyze different scenarios as explained below, each case provides different and contradicting results in terms of our objectives. For example, as we decrease the payment ratio made by the patients in the second group of scenarios, the number of patients going to public hospitals and thus the average waiting time at the public hospitals decrease while the number of patients going to private hospitals and thus the average waiting time at the private hospitals and the government expenses increase. With the development of the social utility function U as given in Equation 3.1, we combine contradicting results into a single value and aim to maximize that value by the optimal choice of the ratio of the payments made by the patients. We find that the maximum value of U is obtained when the ratio of the subsidy payment for the private hospitals is increased to 27%. To find this value, using the Process Analyzer program, the graph of U is drawn according to possible subsidies paid by the government and then the highest value is determined. This method is also applied for 3-opt, 4-opt and 5-opt scenarios. The last row of the second group of scenarios in **Table 3.8.** presents the resulting values when we apply this subsidy ratio with a value of $U = 0.254$. In this case, the waiting times at the public and private hospitals turn out to be 7.21 and 2.26 minutes, while the government expenses increase by about 15%. We note that as the weights of different objectives change, the resulting optimal subsidy ratio will also change. For example, as the weight of government expenses increase, a smaller subsidy ratio would be optimal. On the other hand, as the weight of waiting times at the public hospitals rises, the optimal subsidy ratio would also increase.

3.4.3. Scenario 3: Differentiated two-price policies in the private hospitals based on the patient income level

We observe that the waiting times at the public hospitals can be decreased when some of the public hospital patients are directed to private hospitals by decreasing the patients' payment ratios at the private hospitals. However, such changes lead to increased public spending which is not acceptable by the government. To overcome this problem, we propose a differentiated pricing strategy by dividing the population into two segments such that two different prices are applied to these segments.

In this scenario, we propose that the government increase the subsidy payment ratio, only for the patients with an income lower than a certain level, since the patients with a higher income are already choosing to go to the private hospitals at the current payment ratio. With this mechanism, additional patients with lower incomes (who are more sensitive to price) would prefer to go to private hospitals. Since patients with higher incomes are less sensitive to price, they will still prefer to go to private hospitals and the effect of this change would be very small on higher income patients. This differentiated pricing policy would induce more people in total to go to private hospitals and the higher compensation payments made by SGK for lower-income patients can be balanced with the lower compensation payments made for the higher income patients.

The use of price discrimination in healthcare might be thought to pose some ethical concerns. However, In Turkey, patients with very low income already have a card called green card that allows them to obtain healthcare service free of charge from any public institution, while the rest of the population needs to pay a certain amount for those services. This application can also be considered as a form of price discrimination, and is widely accepted by the community. Since a similar system was previously in Turkey, the proposed differentiated mechanism is also considered to be applicable. Price discrimination is also applied in various other systems in different forms, such as student discounts, senior discounts etc. In addition, the tax systems of the governments are based on the discrimination of the population based on their incomes. A fair tax system is considered to be the one which obtains higher tax from the people with higher income. The price discrimination in the healthcare system that is applied based on the income levels of the patients can be seen to be a similar application as the tax system and people with lower incomes getting higher subsidies can even be thought to be more fair than providing the same subsidy for everyone.

In this model, we assume that there is no cannibalization effect such that the current patients preferring the private hospitals at the current price are among the high-income patients and will still pay the current price and only the additional patients with lower income levels will benefit from the differentiated price and pay the decreased price. Based on our survey results we determine that most of the patients that prefer the private hospitals at the current price are among the high-income population and using the income level in order to determine who can benefit from the decreased price is an efficient strategy to segment the population with negligible cannibalization effects. In our models, we allow only the patients whose income levels are below a certain level to benefit from the decreased price in order to generate new demand for private hospitals without affecting the payments of the current demand. However, we also note that, in the worst case, even if all the patients who benefit from the increased subsidy ratios are among the current patients who pay the current subsidy ratio in the current system (i.e when the cannibalization ratio is 100%), the system results will be the same as the results of the second group of scenarios stated above. For cannibalization ratios less than 100%, the system results will always be better than the second group scenarios. The third group of scenarios is designed considering this differentiated pricing policy.

Scenario 3.1: The differentiated subsidy payment ratios are 14% and 20%

In our first scenario, we use the new subsidy payment ratio as 20% in addition to the current ratio of 14%. The patients whose income levels are above a certain level obtains 14% subsidy payment as before and these are the same patients who prefer the private hospitals at the current system. However, we also provide 20% subsidy ratio for those patients whose income levels are below a certain level and based on the survey results we observe that this new pricing strategy will generate an additional 3% demand in the population for the private hospitals. As a result of this scenario, based on our simulation results we observe that the average waiting times at the public hospitals decrease to 11.14 minutes while the average waiting times at the private hospitals increase to 1.85 minutes. The government expenses are seen to increase by only 1% as a result of this differentiated pricing policy which is much better than the government expenses in Scenario 2.1. In this scenario, we assume that the number of patients among the lower income population, who currently prefer the private hospitals but can benefit from the decreased price, are negligible and do not cannibalize the system. We note that as the number of such patients increases the government expenses get closer to the results in Scenario 2.1 and becomes the same as the one in that scenario in the worst case when all the patients are in that segment. Recall that, we use the same random number seeds in order to eliminate the effect of randomness in comparing different scenarios. As a result of this, the values of N_d, N_o, W_d and W_o are the same as in Scenario 2.1, since the only difference between scenarios 2.1 and 3.1

is the ratio of subsidy payments made by the government for the patients in the private hospitals, everything else remaining the same.

This policy also provides an improvement over the current system in terms of the patients' utility since an additional 3% of the population now choose to go to the private hospitals and obtain a better service based on their own utility and preference, which they did not prefer before. In addition, this change in the demand pattern has a positive externality effect on the patients going to the public hospitals. Even though these patients did not change their preference and still go to the public hospitals since the number of patients going to the public hospitals is decreased, the waiting times at the public hospitals are decreased and the patients need to wait shorter times at the public hospitals, increasing their utility as well.

Scenario 3.2: The differentiated subsidy payment ratios are 14% and 28% of the total price

In this scenario, we apply the same strategy as in Scenario 3.1 but we use 28% as the new subsidy payment ratio differentiated from the current ratio of 14% in order to increase the demand for the private hospitals. In this case, about an additional 7% of the population will now prefer the private hospitals instead of the public ones and the average waiting times at the public and private hospitals are seen to be 7.13 and 2.36 minutes, respectively. The expenses of SGK are seen to increase by only 2% in this case.

Scenario 3.3: The differentiated subsidy payment ratios are 14% and 36% of the total price

Now, we use 36% as the new subsidy payment ratio differentiated from the current ratio of 14% in order to further increase the demand for the private hospitals. In this case, the demand for the private hospitals is found to increase about 10% as a result of our survey. The average waiting times at the public and private hospitals are seen to be better balanced in this case as 4.69 and 3.57 minutes, respectively. The expenses of SGK are seen to increase by only 6% as a result of this scenario.

Scenario 3.4: The differentiated subsidy payment ratios are 14% and 45% of the total price

When we use 45% as the new subsidy payment ratio differentiated from the current ratio of 14%, we find that an additional 15% of the population will now prefer the private hospitals, as a result of our survey. The average waiting times at the public and private hospitals are seen to be 3.18 and 5.72 minutes, respectively, while the expenses of SGK are seen to increase by 12%.

Scenario 3-opt: The second subsidy payment ratio, in addition to 14%, for the private hospitals is determined to maximize U

We find that the maximum value of U is obtained when the increased subsidy payment ratio for the private hospitals is 34%, in addition to the subsidy payment ratio of 14% that currently exists in the system. The last row of the third group of scenarios in **Table 3.8.** presents the resulting values when we apply these differentiated subsidy ratios, and the resulting value of U will be 0.311. In this case, the waiting times at the public and private hospitals turn out to be 5.13 and 3.17 minutes, while the government expenses increase by about 4.5%.

Table 3.8. *Outputs of the scenarios, where waiting times and expenses are in minutes and TL*

	H_d (TL)	N_d	N_o	W_d	W_o	U	$Imp\%$
Cu	4768598	72168	18009.60	17.60	1.65	0.198	
S-1-1	5043598	72168	18009.6	3.28	1.65	0.295	48.99%
S-1-2	5318598	72168	18009.6	2.02	1.65	0.287	44.95%
S-1-3	5593598	72168	18009.6	1.81	1.65	0.270	36.36%
S-1-4	5868598	72168	18009.6	1.73	1.65	0.252	27.27%
S-1-5	6143598	72168	18009.6	1.72	1.65	0.234	18.18%
S-2-1	5050144	69471.7	20705.9	11.14	1.85	0.243	22.73%
S-2-2	5566532	66131.2	24046.4	7.13	2.36	0.251	26.77%
S-2-3	6184395	62790.5	27387.1	4.69	3.57	0.236	19.19%
S-2-4	6896159	59384.2	30793.4	3.18	5.72	0.203	2.53%
S-2-opt	5498400	66489.1	23688.5	7.21	2.26	0.254	28.28%
S-3-1	4779214	69471.7	20705.9	11.14	1.85	0.261	31.82%
S-3-2	4867137	66131.2	24046.4	7.13	2.36	0.298	50.51%
S-3-3	5062797	62790.5	27387.1	4.69	3.57	0.310	56.57%
S-3-4	5342757	59384.2	30793.4	3.18	5.72	0.306	54.55%
S-3-opt	4986376	63587.2	26590.4	5.13	3.17	0.311	57.07%
S-4-1	4318924	69471.7	20705.9	11.14	1.85	0.291	46.97%
S-4-2	4489712	66131.2	24046.4	7.13	2.36	0.323	63.13%
S-4-3	4747653	62790.5	27387.1	4.69	3.57	0.332	67.68%
S-4-4	5071426	59384.2	30793.4	3.18	5.72	0.325	64.14%
S-4-opt	4773836	62362.3	27815.3	4.40	3.78	0.333	68.18%
S-5-1	4707101	69471.7	20705.9	11.14	1.85	0.266	34.34%
S-5-2	4747219	66131.2	24046.4	7.13	2.36	0.306	54.55%
S-5-3	4911707	62790.5	27387.1	4.69	3.57	0.321	62.12%
S-5-4	5226943	59384.2	30793.4	3.18	5.72	0.314	58.59%
S-5-opt	4948070	62203.6	27974	4.34	4.00	0.322	62.63%

3.4.4. Scenario 4: A complete price differentiation policy in the private hospitals based on the patient income level

Instead of using just two different subsidy ratios, in the extreme, the government can apply a completely differentiated pricing policy based on the income levels of the patients such that each patient needs to pay a different ratio based on his/her income level. For example, some patients who have the highest income level obtains no subsidy payment and need to pay 100% of the private hospitals' expenses, while some others with lower income levels obtain positive subsidy percentages. We assume that the price sensitivities of the patients decrease as their income levels increase such that the patients with the highest income levels will still prefer the private hospitals even if no subsidy payment is given to them, while more subsidy payments need to be given to lower income patients. In these scenarios, we define a maximum subsidy payment ratio s_{max}, and each patient obtains a different ratio between 0 and this defined maximum level based on their income level. For example, the patients with an income level higher than a certain value I_{max} gets 0% while the patients with an income level lower than a certain value I_{min} gets s_{max}. Then, the patients with an income level I such that $I_{min} < I < I_{max}$, is assumed to obtain $\frac{s_{max}(I_{max}-I)}{I_{max}-I_{min}}$.

We assume a uniformly distributed willingness to pay distribution for the patients in the population and thus a linear demand function for the private hospitals. We also assume that the patients' willingness to pay for the private hospital services is linearly increasing with the patients' income levels. Even though the income levels do not completely and exactly characterize the patients' willingness to pay distribution, these scenarios allow us to observe the best results that could be obtained through differentiated pricing and provide the extreme solutions.

Scenario 4.1: The subsidy payment ratios are continuously differentiated between 0 and 20% of the total price

In this scenario, we use $s_{max} = 20\%$ and the patients going to the private hospitals obtain different subsidy payment ratios uniformly distributed between 0 and 20% based on their income levels. As a result of this scenario, based on our simulation results, we observe that the average waiting times at the public hospitals decrease to 11.14 minutes while the average waiting times at the private hospitals increase to 1.85 minutes. The government expenses are seen to decrease by 9% as a result of this differentiated pricing policy. The additional expenses of the patients going to private hospitals as a result of the increased subsidy ratios between 14% to 20% are now financed by the decreased subsidy ratios of the higher income patients obtaining subsidy ratios between 14% and 0. As a result of this strategy, the average waiting times at the public hospitals and the government expenses are both decreased.

Scenario 4.2: The subsidy payment ratios are continuously differentiated between 0 and 28% of the total price

In this scenario, we use $s_{max} = 28\%$ and based on our simulation results, we observe that the average waiting times at the public and private hospitals are 7.13 and 2.36 minutes. The government expenses are now seen to decrease by 6% as a result of this differentiated pricing policy.

Scenario 4.3: The subsidy payment ratios are continuously differentiated between 0 and 36% of the total price

In this scenario, we use $s_{max} = 36\%$ and based on our simulation results the average waiting times at the public and private hospitals are better balanced as 4.69 and 3.57 minutes. The government expenses are now almost the same as the government expenses in the current system. We observe in this scenario that the waiting times at the public hospitals can be significantly decreased without any increase in the government expenses via differentiated pricing.

Scenario 4.4: The subsidy payment ratios are continuously differentiated between 0 and 45% of the total price

In this scenario, we use $s_{max} = 45\%$ and based on our simulation results, we observe that the average waiting times at the public and private hospitals are 3.18 and 5.72 minutes. The government expenses now increase by 6%. As a result of this scenario, we can state that increasing the subsidy payment ratios this much does not benefit the system since it leads to both increased government expenses and increased waiting times at the private hospitals.

Scenario 4-opt: The subsidy payment ratios are continuously differentiated between 0 and an upper bound which is determined to maximize U

We find that the maximum value of U is obtained when $s_{max} = 37\%$, such that the subsidy payment ratios are continuously differentiated between 0 and 37%. In this case, the value of U becomes 0.333, and the average waiting times at the public and private hospitals are 4.40 and 3.78 minutes. The government expenses now increase by only 0.1%. Thus, we can state that this scenario reaches to a system with a much better balanced waiting times at the public and private hospitals with almost the same amount of public spending.

3.4.5. Scenario 5: A linear two-part tariff contract

In this section, we analyze the effect of applying a linear two-part tariff (a fixed fee) contract that the government can make with the private hospitals. Under this contract scenario, the government agrees to pay a fixed fee to the private hospitals in addition to the prices paid per patient. The government expects the private hospitals to decrease their prices in return for the fixed fee payment such that the private hospitals will again obtain the same profit as in the current system. When the private hospitals decrease their prices, their profit will decrease, thus in the current system they don't accept to decrease their prices, however when the government offers to pay a fixed fee if they decrease their prices, the private hospitals might accept this agreement since their loss due to decreased prices will be compensated by the government's fixed payment.

We analyze the linear two-part tariff contracts in this section using different amounts of fixed fee payments to be offered to the private hospitals. As a result of different fixed fee payments, the private hospitals will accept different percentages of decreases in their prices. We observe that the private hospitals are not willing to decrease their prices without an additional payment since their profits are seen to decrease as a result of a decrease in their prices. However, if these decreases in the profits are compensated by the fixed fee payments, they will be willing to accept such changes. We analyze these results in detail below.

Scenario 5.1: 7% decrease in prices in return for a fixed fee payment

In this scenario, we propose that the private hospitals decrease their prices by 7%. Note that this decrease in price combined with the 14% subsidy payment of the government will result in the same payment requirement for the patients as in Scenario 2.1. The patients need to pay 80% of the initial cost since (1-0.07)(1-0.14)=0.8 and the patient preferences will be the same as in Scenario 2.1. We observe that when the private hospitals decrease their prices by about 7%, their profits decrease by 10,026 TL, based on our simulation results. In order for the private hospitals to accept this contract, an amount that is at least equal to 10,026 TL needs to be paid as a fixed fee to the private hospitals. In this case, more patients prefer the private hospitals due to the decrease in prices and as a result, the average waiting times at the public and private hospitals turn out to be 11.14 and 1.85 minutes, respectively. The government expenses also decrease to 4,707,101 TL, even after making the 10,026 TL fixed fee payment to the private hospitals. The total government expenses are 1.3% less than the government expenses in the current system. We note that even though higher fixed fee amounts might be asked by the private hospitals in order for them to accept these contracts, we assume that an amount that increases their profit to the level of their profit in the current system, is enough for them to accept this contract. As a result of this contract, the government expenses and the average waiting times at the public hospitals are both seen to decrease.

Scenario 5.2: 16% decrease in prices in return for a fixed fee payment

If the private hospitals agree to decrease their prices by 16%, combined with the 14% subsidy payment of the government, the patients now need to pay 72% of the initial cost since (1-0.16)(1-0.14)=0.72, similar to the case in Scenario 2.2. In this case, the profits of the private hospitals are seen to decrease by 109,286 TL and this amount needs to be paid as a fixed fee in order for the private hospitals to accept this contract. In this case, the average waiting times at the public and private hospitals turn out to be 7.13 and 2.36 minutes, respectively. The government expenses increase to 4,747,219 TL after making the 109,286 TL fixed fee payment to the private hospitals, which is 0.5% more than the government expenses in the current system. Under this contract, the private hospitals make the same profit as in the current system and the average waiting times at the public hospitals can be significantly decreased with almost the same amount of public spending.

Scenario 5.3: 25% decrease in prices in return for a fixed fee payment

If the private hospitals agree to decrease their prices by 25%, combined with the 14% subsidy payment of the government, the patients now need to pay 64% of the initial cost since (1-0.25)(1-0.14)=0.64, similar to the case in Scenario 2.3. In this case, the profits of the private hospitals decrease by 374,289 TL and this amount is paid as the fixed fee by the government such that the private hospitals will be willing to accept this contract. The average waiting times at the public and private hospitals turn out to be 4.69 and 3.57 minutes, respectively. The government expenses increase to 4,911,707 TL after making the 374,289 TL fixed fee payment, which is 3% more than the government expenses in the current system. In this scenario, the average waiting times at the public hospitals are significantly decreased, however, the government expenses need to increase due to the high amount of the fixed fee payments required for this contract to be acceptable.

Scenario 5.4: 36% decrease in prices in return for a fixed fee payment

If the private hospitals decrease their prices by 36%, combined with the 14% subsidy payment of the government, the patients now need to pay 55% of the initial cost since (1-0.36)(1-0.14)=0.55, similar to the case in Scenario 2.4. In this case, the profits of the private hospitals are seen to decrease by 828,794 TL and this amount is paid as the fixed fee by the government. The government expenses increase to 5,226,943 TL after making the fixed fee payment, which is 9.6% more than the government expenses in the current system. The average waiting times at the public and private hospitals turn out to be 3.18 and 5.72 minutes, respectively.

Scenario 5-opt: The decrease in price in return for a fixed fee payment is determined to maximize U

We find that the maximum value of U is obtained when the private hospitals decrease their prices by 28% in return for a fixed fee payment at the amount of 429,755 TL. In this setting, the private hospitals obtain the same amount of profit as they obtain in the current system. As a result, the value of U becomes 0.322, and the average waiting times at the public and private hospitals are 4.34 and 4.00 minutes. The government expenses are seen to increase by 3.8% compared to the current system.

3.4.6. A general comparison of scenarios

As a result of our numerical experiments, we observe that increasing the capacity of the public hospitals help in significantly decreasing the waiting times at the public hospitals, however increasing the capacities are generally not feasible due to a limited amount of resources. In addition, increasing the capacities require significant increases in public spending which are generally not acceptable by the governments due to budget constraints.

When we consider the strategies affecting the payment structures at the private hospitals, we observe that as more subsidy payments are offered by the government, significant decreases can be obtained in the waiting times at the public hospitals. However, these strategies also require significant increases in public spending values. To overcome this issue, we propose differentiated pricing strategies that will lead to similar decreased waiting times without the additional public spending amounts. We observe that differentiated pricing can benefit the system as a whole. Finally, when we consider the linear two-part tariff contract, we observe that significant improvements can be obtained compared to the current system. We observe that fourth group of scenarios give the best results with Scenario 4-opt providing the highest value of $U = 0.333$. After the fourth group of scenarios, Scenario 5-opt gives the next best value of $U = 0.322$. Then Scenario 3-opt provides $U = 0.311$, Scenario 1-1 provides $U = 0.295$ and Scenario 2-opt provides $U = 0.254$.

3.4.7. Sensitivity analysis

Note that the simulation results provided above are obtained for the emergency departments in Eskisehir region using real life data, however we believe that similar structures exist in most of the healthcare systems in which public and private hospitals coexist. Even though the data and the parameters related to these systems might be different from each other and thus values of the obtained results and benefits might differ, we believe that similar benefits can be obtained through the use of proposed scenarios in different healthcare settings. In order to analyze the effects of changing parameter values on the system results, we provide a sensitivity analysis on various parameter values, namely on different values of q_d, q_o, α_1, α_2, c_d (average cost of green zone examination at public hospitals) and b_o (price sensitivity of patients for going to private hospitals). We present the best utility levels obtained in each group of scenarios for varying parameter values in **Table 3.9.**.

Recall that the initial values used for the analyzed parameters are $q_d = 0.6$, $q_o = 0.9$, $\alpha_1 = 0.01$, $\alpha_2 = 0.006$, $c_d = 19.57$ and $b_o = 0.5$ (slope of the line in **Figure 3.4.**). The first row of **Table 3.9.** summarizes the results for these base case parameters. In the following rows, the first column denotes the new value of the parameter that has been changed from its initial value, while all others remain the same. Note that as q_d, q_o or b_o decrease, or α_1, α_2 or c_d increase, lower utility values are obtained, however, in all cases we observe that the proposed scenarios still provide similar and significant improvements in the system results. The quality levels at the public hospitals seem to affect the results more than the quality levels at the private ones. Thus, an investment that increases the quality levels at the public hospitals might have a significant effect on the utility level of the society. If the government spending have a lower weight in the utility function, in other words, if the government focuses more on patient satisfaction rather than the money spent, it is observed that significant improvements can be obtained in social utility. Finally, when the slope of the function given in **Figure 3.4.** is decreased, meaning that the patients are less sensitive to price, we observe that the benefits of the proposed mechanisms decrease. When $b_o = 0.25$, observe that S-1-opt provides the maximum utility, denoting that increasing the capacity has higher benefits than the pricing mechanisms. The main reason for this result is that if the patients are less sensitive to price, more subsidies need to be given in order to direct the patients from public to private hospitals, leading to much higher government spending. The same amount of subsidies given in the initial results will now lead less patients to move to private hospitals, leading to decreased utilities. Note that people may prefer the public system for long care treatments (chronic diseases) due to the smaller long-term cost while preferring the private system for emergency care, which is a one-time event. For other health services like long-care treatments, the price sensitivities of the patients might be lower and much higher discounts might need to be made in order for them to select the private hospitals. In such cases, as observed in **Table 3.9.**, the improvements obtained through the proposed mechanisms might be lower than the results obtained for the emergency departments. However, in either case, we observe that some improvement is still possible and a better system can still be obtained through the proposed mechanisms.

Table 3.9.*Sensitivity analysis according to different values of the parameters*

	Cu	S-1-opt	S-2-opt	S-3-opt	S-4-opt	S-5-opt
Base case	0.198	0.295	0.254	0.311	0.333	0.322
$q_d = 0.5$	0.118	0.215	0.180	0.244	0.264	0.254
$q_d = 0.7$	0.279	0.375	0.328	0.382	0.403	0.390
$q_o = 0.8$	0.179	0.275	0.228	0.282	0.303	0.290
$q_o = 1$	0.218	0.315	0.280	0.344	0.364	0.354
$\alpha_1 = 0.005$	0.271	0.310	0.288	0.334	0.355	0.343
$\alpha_1 = 0.02$	0.054	0.267	0.200	0.271	0.291	0.281
$\alpha_2 = 0.005$	0.251	0.351	0.315	0.369	0.386	0.377
$\alpha_2 = 0.007$	0.146	0.239	0.193	0.256	0.280	0.267
$c_d = 10$	0.209	0.305	0.263	0.322	0.337	0.330
$c_d = 30$	0.183	0.274	0.234	0.296	0.325	0.304
$b_o = 0.25$	0.198	0.295	0.224	0.290	0.294	0.256
$b_o = 0.75$	0.198	0.295	0.273	0.322	0.354	0.363

4. PRICING MODELS FOR A SYSTEM INCLUDING ONE PRIVATE AND ONE PUBLIC HOSPITAL

In this chapter, the system discussed in Chapter 3, is analyzed analytically. We noted that, in public hospitals of the system, waiting times are high while, the perceived quality of service and payments are low. In private hospitals, the payments are high, the waiting times are low and the perceived quality are high. Although most of the patients in the current state prefer public hospitals, based on the results of the surveys it has been determined that the majority of them prefer private hospitals if they pay less for their treatment. Choosing private hospitals by the patients, the level of quality they perceived increases, while expenses of the government decrease. As the public policies, possible contract mechanisms and their effects on the strategic patients' choices, satisfaction level, public expenditure and social utility are explored to find a more beneficial healthcare system. For this aim, the interaction between the private and public hospitals' of a city in Turkey are modeled, considering parameters as the arrival rates and numbers of patients, service times, waiting periods in queues, quality of services, pricing policies based on the contract between the government and private hospitals, patients and government payments. For analyzing new policies, analytical models are designed based on the fact that in a particular region, strategic patients make their choice between a public hospital and a private hospital in their vicinity. Thus, as a new approach, all hospitals in the city are unified under one public and one private hospital, which not only represents the real system properly but also it provides a basis for the multi-hospital models. The effects of different contract mechanisms are analytically analyzed based on the defined social utility concept.

4.1. Description of the System

In the system, where all hospitals of a region are unified under one public and one private hospital, the patients arrive with a rate equal to λ and make a decision to choose one of the hospitals. As seen in **Figure 4.1.**, if a patient goes to a private hospital, he pays b_o while if he selects the public hospital his payment is b_d.

In this case, his perceived quality level and the average waiting time in the private hospital are q_o and w_o while in the public hospital they are q_d and w_d. We define the utility that a patient achieves in the public and private hospitals as $\frac{q_d}{w_d} - kb_d^2$ and $\frac{q_o}{w_o} - kb_o^2$ respectively. It is clear that a patient chooses the private hospital if it provides a higher utility for him, which is expressed in Equation 4.1.

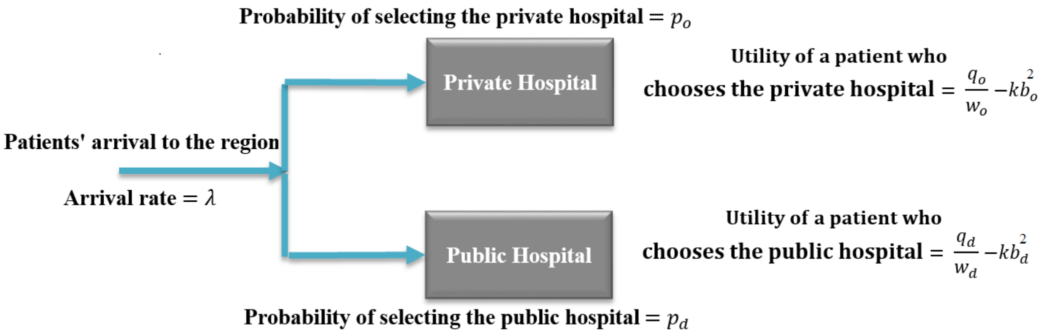


Figure 4.1. *A strategic patients chooses one of the hospitals based on the utility he gets.*

$$\frac{q_o}{w_o} - kb_o^2 \geq \frac{q_d}{w_d} - kb_d^2 \tag{4.1}$$

k is the price sensitivity of a patient, which is distributed between 0 and $\bar{k}$. If the value of k is low for a patient, the quality is more important than the price for him. Also, we assume that as seen in Figure **Figure 4.2.**, there is an inverse relationship between k and the income level of patients. The patient chooses the private hospital if k is as in Equation 4.2.

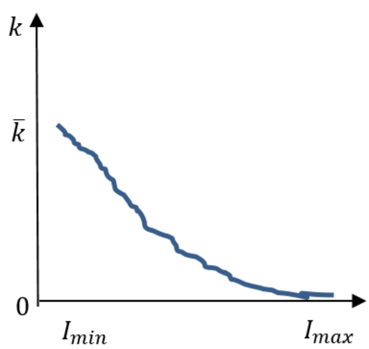


Figure 4.2. *The relationship between k and the income level of patients*

$$k \leq \frac{\frac{q_o}{w_o} - \frac{q_d}{w_d}}{b_o^2 - b_d^2} \tag{4.2}$$

p_o, the probability of choosing the private hospital by a patient can be written as in Equation 4.3, where $F_k(x)$ denotes the cumulative probability function of k.

$$p_o = F_k(\frac{\frac{q_o}{w_o} - \frac{q_d}{w_d}}{b_o^2 - b_d^2}) \tag{4.3}$$

The average waiting times in the hospitals are expressed based on the M / M / 1 queuing system with the service rates of μ_o and μ_d in the private and public hospital, as in Equations 4.4 and 4.5.

$$w_o = \frac{1}{m_o\mu_o - \lambda p_o} \tag{4.4}$$

$$w_d = \frac{1}{m_d\mu_d - \lambda p_d} \tag{4.5}$$

Using Equations 4.4 and 4.5, Equation 4.3 can be rewritten more clearly as in Equation 4.6.

$$p_o = F_k(\frac{q_o(m_o\mu_o - \lambda p_o) - q_d(m_d\mu_d - \lambda(1 - p_o))}{b_o^2 - b_d^2}) \tag{4.6}$$

We assume that k is uniformly distributed between 0 and $\bar{k}$. Then, Equation 4.6 can be written as in Equation 4.7.

$$p_o = \begin{cases} 1, & if \quad \frac{q_o(m_o\mu_o - \lambda p_o) - q_d(m_d\mu_d - \lambda + \lambda p_o)}{(b_o^2 - b_d^2)\bar{k}} > 1 \\ \frac{q_o(m_o\mu_o - \lambda p_o) - q_d(m_d\mu_d - \lambda + \lambda p_o)}{(b_o^2 - b_d^2)\bar{k}}, & if \quad 0 \leq \frac{q_o(m_o\mu_o - \lambda p_o) - q_d(m_d\mu_d - \lambda + \lambda p_o)}{(b_o^2 - b_d^2)\bar{k}} \leq 1 \\ 0, & if \quad \frac{q_o(m_o\mu_o - \lambda p_o) - q_d(m_d\mu_d - \lambda + \lambda p_o)}{(b_o^2 - b_d^2)\bar{k}} < 0 \end{cases} \tag{4.7}$$

From Equation 4.7, p_o can be calculated as in Equation 4.8.

$$p_o = \begin{cases} 1, & if \;\; \frac{q_d(\lambda - m_d\mu_d) + q_o m_o \mu_o}{\lambda(q_o+q_d) + \bar{k}(b_o^2 - b_d^2)} > 1 \\ \frac{q_d(\lambda - m_d\mu_d) + q_o m_o \mu_o}{\lambda(q_o+q_d) + \bar{k}(b_o^2 - b_d^2)}, & if \;\; 0 \leq \frac{q_d(\lambda - m_d\mu_d) + q_o m_o \mu_o}{\lambda(q_o+q_d) + \bar{k}(b_o^2 - b_d^2)} \leq 1 \\ 0, & if \;\; \frac{q_d(\lambda - m_d\mu_d) + q_o m_o \mu_o}{\lambda(q_o+q_d) + \bar{k}(b_o^2 - b_d^2)} < 0 \end{cases} \quad (4.8)$$

It is clear that the probability of choosing the public hospital by a patient is as in Equation 4.9.

$$p_d = 1 - p_o \quad (4.9)$$

U_1 is the expected utility received by all patients that choose the private or the public hospital.

$$U_1 = \int_{k=0}^{A_1} \left(\frac{q_o}{w_o} - kb_o^2\right) f(k)dk + \int_{A_1}^{\bar{k}} \left(\frac{q_d}{w_d} - kb_d^2\right) f(k)dk \quad (4.10)$$

k is uniformly distributed between 0 and $\bar{k}$, then $f(k) = \frac{1}{\bar{k}}$. A_1, which is the critical value of k for choosing the private hospital, is calculated as in Equation 4.11.

$$A_1 = \bar{k}p_o \quad (4.11)$$

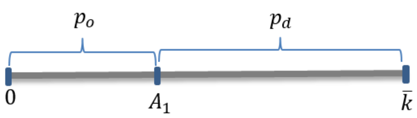


Figure 4.3. A_1 *is the critical values of* k *for the patients to select the hospitals.*

After evaluating the integrals, Equation 4.10 is written as in Equation 4.12.

$$U_1 = \frac{q_o}{w_o}p_o - b_o^2\frac{\bar{k}p_o^2}{2} + b_d^2\frac{\bar{k}(p_o^2 - 1)}{2} + \frac{q_d}{w_d}p_d \quad (4.12)$$

According to Equation 4.13, H_d consists of the total amount paid by the government as subsidy for the patients and the total cost of the capacity.

$$H_d = \lambda p_d(c_d - b_d) + \lambda p_o s + k_d m_d^2 \tag{4.13}$$

The social utility, defined in Equation 4.15 is defined as in Equation 4.13. α_1 and α_1 are the coefficients to adjust the value of U_1 and U_2.

$$U_2 = \frac{H_d}{\lambda} \tag{4.14}$$

$$U = \alpha_1 U_1 - \alpha_2 U_2 \tag{4.15}$$

The profit function of the private hospital is defined as in Equation 4.16.

$$Z_o = (r - c_o)p_o\lambda - k_o m_o^2 \tag{4.16}$$

4.2. Model NC: No Contract Between the Government and the Private Hospital

In this model, which is summarized in **Figure 4.4.**, it has been assumed that there is no contract between the government and the private hospital. In this case, only the patients that care about the quality of service prefer the private hospital. So the decision of a strategic patient to choose this hospital is done according to Equation 4.17.

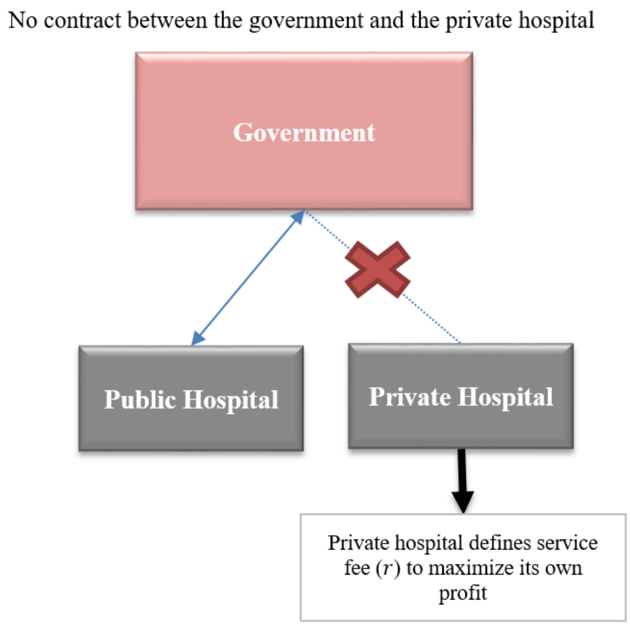


Figure 4.4. *Model NC*

$$\frac{q_o}{w_o} - kr^2 \geq \frac{q_d}{w_d} - kb_d^2 \tag{4.17}$$

where r that is the service fee in the private hospital is paid totally by the patients and the government does not pay any subsidies. In this model, if the value of k for a patient is as in Equation 4.18, he chooses the private hospital.

$$k \leq \frac{\frac{q_o}{w_o} - \frac{q_d}{w_d}}{r^2 - b_d^2} \tag{4.18}$$

Similar to Equations 4.3 and 4.6, p_o is as in Equations 4.19 and 4.20.

$$p_o = F_k\left(\frac{q_o(m_o\mu_o - \lambda p_o) - q_d(m_d\mu_d - \lambda p_d)}{r^2 - b_d^2}\right) \tag{4.19}$$

$$p_o = \begin{cases} 1, & if \quad \frac{q_d(\lambda - m_d\mu_d) + q_o m_o \mu_o}{\lambda(q_o+q_d) + \bar{k}(r^2 - b_d^2)} > 1 \\ \frac{q_d(\lambda - m_d\mu_d) + q_o m_o \mu_o}{\lambda(q_o+q_d) + \bar{k}(r^2 - b_d^2)}, & if \quad 0 \leq \frac{q_d(\lambda - m_d\mu_d) + q_o m_o \mu_o}{\lambda(q_o+q_d) + \bar{k}(r^2 - b_d^2)} \leq 1 \\ 0, & if \quad \frac{q_d(\lambda - m_d\mu_d) + q_o m_o \mu_o}{\lambda(q_o+q_d) + \bar{k}(r^2 - b_d^2)} < 0 \end{cases} \tag{4.20}$$

When $k \sim U[0, \bar{k}]$, in the optimal solution, p_o satisfies Equation 4.21.

Because, if the value of $\frac{q_d(\lambda - m_d\mu_d)+q_o m_o \mu_o}{\lambda(q_o+q_d)+\bar{k}(r^2\_b_d^2)} > 1$, then $p_o = 1$ and decreasing m_o slightly will not change the value of p_o and it will still be equal to 1 while $Z_{o_{NC}}$, which is the objective function of this model, will increase. Thus, such a solution can not be optimal. Similarly, if the value of $\frac{q_d(\lambda - m_d\mu_d)+q_o m_o \mu_o}{\lambda(q_o+q_d)+\bar{k}(r^2\_b_d^2)} < 0$, then $p_o = 0$ and $Z_{o_{NC}} = 0$. The same value can be obtained when $\frac{q_d(\lambda - m_d\mu_d)+q_o m_o \mu_o}{\lambda(q_o+q_d)+\bar{k}(r^2\_b_d^2)} = 0$. Thus, $p_o = \frac{q_d(\lambda - m_d\mu_d)+q_o m_o \mu_o}{\lambda(q_o+q_d)+\bar{k}(r^2\_b_d^2)}$ will always be satisfied in the optimal solution.

$$p_o = \frac{q_d(\lambda - m_d\mu_d) + q_o m_o \mu_o}{\lambda(q_o + q_d) + \bar{k}(r^2 - b_d^2)} \tag{4.21}$$

In this model, since the government does not pay any subsidy to the patient going to the private hospital, H_d is as in Equation 4.22.

$$H_d = \lambda p_d(c_d - b_d) + k_d m_d^2 \tag{4.22}$$

Similar to Equation 4.10, U_1 is as in Equation 4.23.

$$\begin{aligned} U_1 &= \int_{k=0}^{A_1} (\frac{q_o}{w_o} - kr^2) f(k)dk + \int_{A_1}^{\bar{k}} (\frac{q_d}{w_d} - kb_d^2) f(k)dk \\ &= \frac{q_o}{w_o} p_o - r^2 \frac{\bar{k}p_o^2}{2} + b_d^2 \frac{\bar{k}(p_o^2 - 1)}{2} + \frac{q_d}{w_d} p_d \end{aligned} \tag{4.23}$$

Also, U_2 and U are as in Equations 4.14 and 4.15. The model, in which r is a decision variable is defined as in Equation 4.24.

$$Max_r \;\; Z_{o_{NC}} = \lambda p_o(r - c_o) - k_o m_o^2 \tag{4.24}$$

s.t.

$$p_o \leq 1 \tag{4.25}$$

$$p_o \geq 0 \tag{4.26}$$

$$\lambda p_o \leq m_o \mu_o \tag{4.27}$$

$$\lambda(1 - p_o) \leq m_d \mu_d \tag{4.28}$$

$$r \geq 0 \tag{4.29}$$

Solution method

As a solution method, we design an algorithm that searches on different prices to maximize the profit.

4.3. Model SC: Contract Mechanism Based on Subsidy Payments

In this model, which is summarized in **Figure 4.5.**, it is assumed that there is a contract mechanism based on the fixed prices between the private hospital and the government. In this case, according to Equation 4.30, r is the sum of subsidy that the government pays for each patient of this hospital (s), and the payment of each patient (b_o).

$$r = s + b_o \tag{4.30}$$

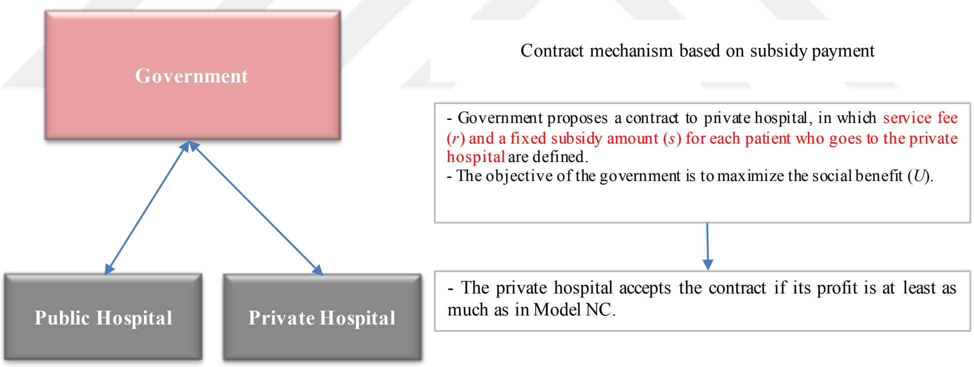


Figure 4.5. *Model SC*

The decision of a strategic patient is made through Equation 4.1 and the value of k is as in Equation 4.2 to choose the private hospital. In this case, p_o, U_1, H_d, U_2 and U are as in Equations 4.6, 4.10, 4.13, 4.14 and 4.15 respectively. The profit of the private hospital, which is given Equation 4.16, should be at least as much as $Z_{o_{NC}}$, in which there is no contract with the government. Otherwise, it is not reasonable to make a contract for the private hospital. This model, in which s and r are the decision variables is defined as in Equation 4.31.

$$Max_{r,s} \ U \tag{4.31}$$

s.t.

$$Z_{o_{SC}} \geq Z_{o_{NC}}$$

$$Constraints \ \ 4.25 - 4.29$$

As a solution method, we design an algorithm that searches on different prices and subsidies to maximize the social utility.

4.4. Model DP-2p: Contract mechanism based on differentiated payments by patients in the private hospital

We suppose that based on the Model DP-2p, which is summarized in **Figure 4.6.**, the payments of the patients in the private hospital are made according to 2 different prices. In this case, in the private hospital if the income level of a patient is above a certain amount he pays b_{o_1} and the rest pay b_{o_2}. Like the other models, patients in the public hospital pay b_d. Decision of a strategic patient to select the private hospital is made according to Equation 4.32.

$$\frac{q_o}{w_o} - kb_{o_2}^2 \geq \frac{q_d}{w_d} - kb_d^2 \tag{4.32}$$

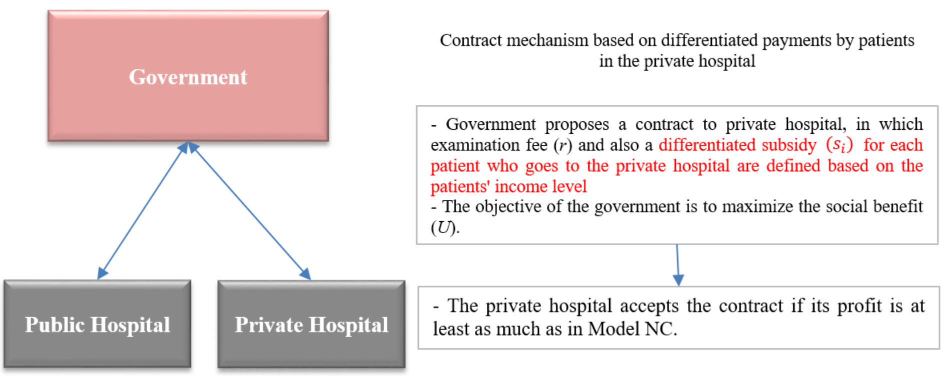


Figure 4.6. *Model DP*

Similar to Model SC, we assume that there is an inverse linear relationship between the income level of a patient and the value of k. If the income level of a patient is high, the value of k is low and vice versa. To select the private hospital, the value of k for the patient is as in Equation 4.33.

$$k \leq \frac{\frac{q_o}{w_o} - \frac{q_d}{w_d}}{b_{o_2}^2 - b_d^2} \tag{4.33}$$

So, p_o is as in Equation 4.34.

$$p_o = F_k\left(\frac{q_d(\lambda - m_d\mu_d) + q_o m_o \mu_o}{\lambda(q_o + q_d) + b_{o_2}^2 - b_d^2}\right) \tag{4.34}$$

In a similar way, the decision of a patient who pays b_{o_1} in the private hospital is made according to Equation 4.35 and the value of k is as in Equation 4.36.

$$\frac{q_o}{w_o} - kb_{o_1}^2 \geq \frac{q_d}{w_d} - kb_d^2 \tag{4.35}$$

$$k \leq \frac{\frac{q_o}{w_o} - \frac{q_d}{w_d}}{b_{o_1}^2 - b_d^2} \tag{4.36}$$

Therefore, the probability of choosing the private hospital by these patients is as in Equation 4.37.

$$p_{o_1} = F_k\left(\frac{q_d(\lambda - m_d\mu_d) + q_o m_o \mu_o}{\lambda(q_o + q_d) + b_{o_1}^2 - b_d^2}\right) \tag{4.37}$$

Hereby, p_{o_2} which corresponds to the patients who pay b_{o_2} is equal to $p_o - p_{o_1}$.

In this model, U_1 is as in Equation 4.38. U_2 and the social utility are as in Equations 4.14 and 4.15.

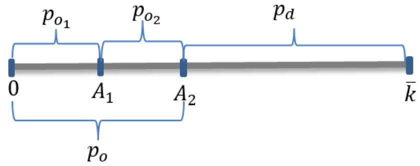


Figure 4.7. *The probability of choosing the hospital by the patients in Model DP-2p*

As seen in **Figure 4.7.**, in Model DP-2p, the probability of choosing the private hospital by the patients is equal to p_{o_2}. In private hospitals, patients' payments are according to a dual price based on their income level. The government pays a subsidy equal to s_1 to the patients whose income levels are above a certain threshold. In the figure, the probability of these patients to choose the private hospital is indicated by p_{o_2}. For these patients, the value of k is between 0 and A_1. The government pays a subsidy equal to s_1 for the other patients going to the private hospital.

$$U_1 = \int_0^{A_1} (\frac{q_o}{w_o} - kb_{o_1}^2) f(k)dk + \int_{A_1}^{A_2} (\frac{q_o}{w_o} - kb_{o_2}^2) f(k)dk + \int_{A_2}^{\bar{k}} (\frac{q_d}{w_d} - kb_d^2) f(k)dk \quad (4.38)$$

where, $A_1 = \bar{k}p_{o_1}$ and $A_2 = \bar{k}p_o$ because $k \sim U[0, \bar{k}]$.

Calculating the integrals of Equation 4.38, it can be written as in Equation 4.39.

$$U_1 = \frac{q_o}{w_o} p_{o_1} - b_{o_1}^2 \frac{\bar{k} p_{o_1}^2}{2} + \frac{q_o}{w_o}(p_o - p_{o_1}) - b_{o_2}^2 \frac{\bar{k}(p_o^2 - p_{o_1})^2}{2} + b_d^2 \frac{\bar{k}(p_o^2 - 1)}{2} + \frac{q_d}{w_d} p_d \quad (4.39)$$

Owing to that the government gives two kinds of subsidies for the patients of the private hospital, H_d is as in Equation 4.40.

$$H_d = \lambda p_d (c_d - b_d) + \lambda p_{o_1} s_1 + \lambda p_{o_2} s_2 + k_d m_d^2 \quad (4.40)$$

$$Max_{r, s_1, s_2} \ U \quad (4.41)$$

s.t.

$$Z_{o_{DP}} \geq Z_{o_{NC}}$$

$$Constraints \ 4.25 - 4.29$$

Model DP-np

There are similar cases in Model DP-np, but the payments in the private hospital are made based on n prices. In this state, U_1 is calculated as in Equation 4.42.

$$U_1 = \int_0^{A_1} (\frac{q_o}{w_o} - kb_{o_1}^2) f(k)dk + \int_{A_1}^{A_2} (\frac{q_o}{w_o} - kb_{o_2}^2) f(k)dk + ... \\ \int_{A_{n-1}}^{A_n} (\frac{q_o}{w_o} - kb_{o_n}^2) f(k)dk + \int_{A_n}^{\bar{k}} (\frac{q_d}{w_d} - kb_d^2) f(k)dk \tag{4.42}$$

As seen in **Figure 4.8.**, in this model, the payment of patients in the private hospital is done according to n price because the government pays in different subsidies based on the patients' income level. The thresholds of the k values of these patients are shown as A_n.

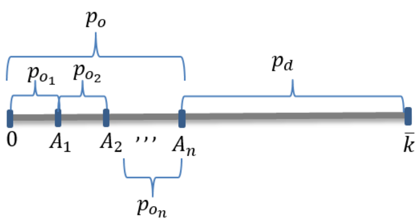


Figure 4.8. *The probability of choosing the hospitals by the patients in Model DP-np*

In Model DP-np, H_d is as in Equation 4.43.

$$H_d = \lambda p_d(c_d - b_d) + \lambda p_{o_1} s_1 + \lambda p_{o_2} s_2 + ... + \lambda p_{o_n} s_n + k_d m_d^2 \tag{4.43}$$

The objective function of Model DP-np is as in Equation 4.44.

$$Max_{r,s_1,s_2,...,s_n} \ U \tag{4.44}$$

As a solution method, we design an algorithm that searches on different prices and subsidies to maximize the social utility.

4.5. Model LTC: The Linear Two Part Tariff Contract Mechanism

In this model, which is summarized in **Figure 4.9.**, it is assumed that the private hospital receives a financial incentive from the government if it makes a discount for r. The strategic patient makes a decision according to Equation 4.1 and therefore the value of k for choosing the private hospital and p_o are as in Equations 4.2 and 4.3. As it is in Equation 4.45, the financial incentive (T), is added to the public expenses and subtracted from the social utility.

$$H_d = \lambda p_d(c_d - b_d) + \lambda p_o s + T \tag{4.45}$$

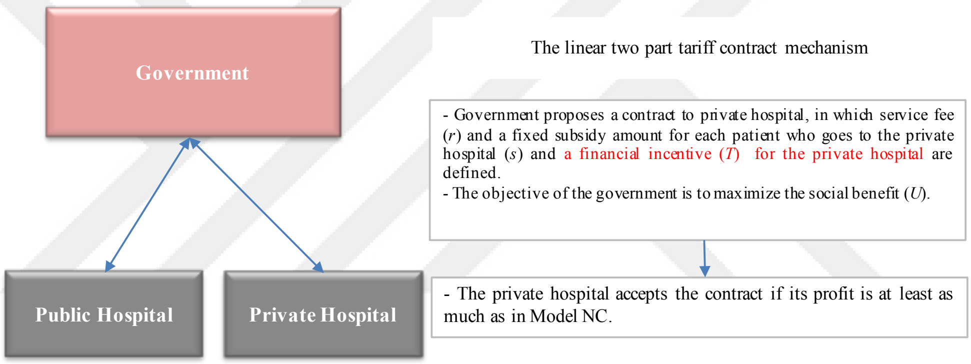


Figure 4.9. *Model LTC*

Also, T, is added to the profit of the private hospital.

$$Z_{o_{LTC}} = (r\_c_o)\lambda p_o - k_o m_o^2 + T \tag{4.46}$$

The model is as in Equation 4.47, where b_o, r and T are the decision variables. In this model, U, U_1 and U_2 are as in Equations 4.15, 4.10 and 4.14.

$$Max_{r,s,T}\ U \tag{4.47}$$

s.t.

$$Z_{o_{LTC}} \geq Z_{o_{NC}}$$

$$Constraints\ 4.25 - 4.29$$

As a solution method, we design an algorithm that searches on different prices, subsidies and financial incentives to maximize the social utility.

4.6. Experimental Results Based on a Case Study from Turkey

Models of this chapter are designed to analyze the emergency services of private and public hospitals of a region. The parameters are taken from a real system in a city in Turkey. Where there are three private and three public hospitals in the city. Assuming that there are one private and one public hospitals in the models, the patients' choices between these hospitals and the effects of different contracting mechanisms on this selection and therefore on the social utility are analyzed. The used parameters are summarized in **Table 4.1.**.

Table 4.1. *Values of base case parameters*

λ	c_o	c_d	k_o	k_d	q_o	q_d	t_o	t_d	b_d	m_o	m_d	$\bar{k}$	α_1	α_2
70000	20	10	50000	10000	0.7	0.5	8	3	5	3.5	5	1	0.005	1

The obtained results are summarized in **Table 4.2.** and **Figure 4.10.**. As seen, Model DP has the best value of the social utility, while its worst value is in Model NC. Also, the private hospital gains the highest profit, in Model DP.

Table 4.2. *Results obtained by the models with the base case parameters*

	r	s	p_o	w_o	w_d	H_d	U_1	U_2	U	T
Model NC	310	0	0.068	0.05	0.11	576229.35	3586.14	8.23	9.70	-
Model SC	208	27	0.105	0.06	0.08	761340.12	4833.98	10.88	13.29	-
Model DP-2p	165	$s_1 = 0$ $s_2 = 88$	$p_{o_1} = 0.110$ $p_{o_2} = 0.026$	0.076	0.062	712868.28	5674.52	10.18	18.19	-
Model DP-3p	160	$s_1 = 0$ $s_2 = 52$ $s_3 = 106$	$p_{o_1} = 0.112$ $p_{o_2} = 0.016$ $p_{o_3} = 0.012$	0.079	0.060	705464.52	5784.09	10.08	18.84	-
Model DP-4p	158	$s_1 = 0$ $s_2 = 37$ $s_3 = 75$ $s_4 = 117$	$p_{o_1} = 0.112$ $p_{o_2} = 0.012$ $p_{o_3} = 0.011$ $p_{o_4} = 0.008$	0.081	0.060	703034.98	58298.5	10.04	19.11	-
Model DP-5p	157	$s_1 = 0$ $s_2 = 29$ $s_3 = 58$ $s_4 = 89$ $s_5 = 124$	$p_{o_1} = 0.113$ $p_{o_2} = 0.009$ $p_{o_3} = 0.008$ $p_{o_4} = 0.008$ $p_{o_5} = 0.006$	0.082	0.060	700501.64	5851.61	10.00	19.25	-
Model LTC	200	22	0.106	0.06	0.08	772182.92	4865.46	11.03	13.30	46350.09

In Models SC, DP-2p and LTC, 32.9%, 46.74%, and 35.48% discounts are made on the examination price in the private hospital, according to Model NC. Hence, in these models, the number of patients choosing the private hospital increases by 54.41%, 100%, and 55.88%. As a result, the average waiting time of all patients decreases. Also, owing to that more people are treated in private hospitals, the quality level that they receive increases by 34.79%, 58.23% and 34.80% in Models SC, DP-2p and LTC, respectively. In these models, the average expenditure of the government increases and also the social utility increases by 37%, 87.5%, and 37%. Although in Model LTC the profit of the private hospital is the same as Model NC, in Models SC and DP-2p, the profit of private hospital increases by 26.88 TL and 2047.26 TL. In Model DP, the government spends the least amount, while the private hospital and also the social utility are the highest.

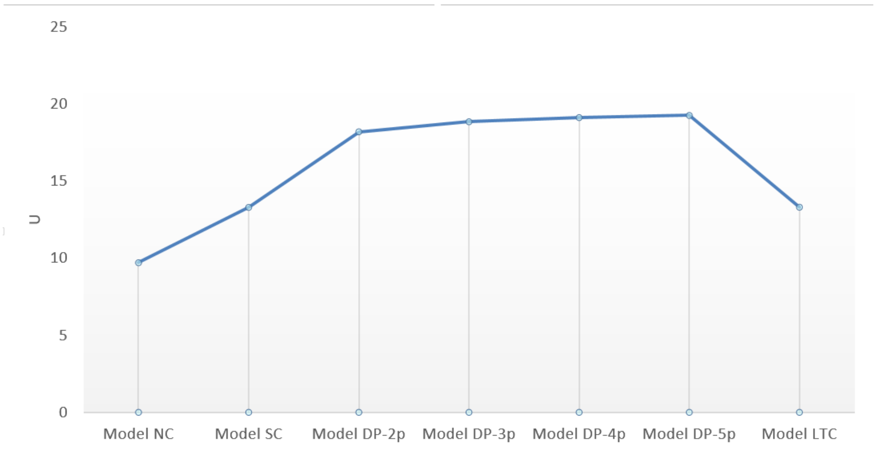


Figure 4.10. The values of social utility in the models

In Models SC, DP-2p and LTC, the expenses of the government are more than Model NC. However, in real life, there are generally several constraints on the budget. Therefore, the results are also analyzed by adding a constraint as $H_d \leq \epsilon$ to the models. For the upper limit of H_d, the values between 600,000 TL and 780,000 TL are used, which are the amounts close to the values in Models 1 and 4. As seen in **Table 4.3.**, it is usually possible to achieve better U by raising the budget.

Table 4.3. *Obtained U when there is a constraint to increase H_d* $(\times 10,000)$

	U						
	If $H_d \leq 60$	*If* $H_d \leq 63$	*If* $H_d \leq 66$	*If* $H_d \leq 69$	*If* $H_d \leq 72$	*If* $H_d \leq 75$	*If* $H_d \leq 78$
Model SC	11.82	12.41	12.83	13.08	13.21	13.27	13.29
Model DP-2p	16.89	17.63	18.00	18.17			
Model DP-3p	17.42	18.26	18.65	18.82			
Model DP-4p	17.71	18.51	18.91	19.09			
Model DP-5p	17.88	18.70	19.09	19.23			
Model LTC	11.83	12.49	12.87	13.09	13.22	13.28	13.30

The used parameters affect the results, so the sensitivity analysis is done according to the different values of the parameters and the results are summarized in **Table 4.4.**. When $\lambda = 75000$, in all models the price of the examination in the private hospital increases. More patients choose the private hospital. In this case, although the profit of the private hospital increases, the social utility falls. As the amount of subsidies increase in Models NC, DP-2p and LTC expenses of the government also increase. If $\lambda = 65000$, in all models, patients pay less in the private hospital, but fewer people choose this hospital, so the profit of private hospital decreases. If the cost of care for each patient in the private hospital increases, the private hospital earns less profit. If the cost of care for each patient in the public hospital rises, the social utility decreases as the government spends more money. Decreases in these costs also cause to the opposite of these effects. If the cost of capacity increases in the private hospital, profit decreases but the social utility does not change. The increase in the capacity cost in the public hospital negatively affects the social utility. An increase in the perceived quality level in the private hospital has a positive effect on both the private hospital's profits and the social utility. When the level of perceived quality in the public hospital increases, the social utility improves, and the private hospital's profit reduces. The increase in the average examination time in both of the private and public hospitals reduces social utility and also the profit of private hospital. A slight increase in the payment of patients to the public hospital improves the social utility. If the price sensitivity of patients increases, fewer patients choose this hospital even with fewer prices, which decreases social utility and also the profit of private hospital.

Table 4.4. *Results obtained by the models according to different parameters*

	Model NC	Model SC	Model DP-2p	Model DP-3p	Model DP-4p	Model DP-5p	Model LTC
$\lambda = 75000$	0.29	4.73	10.40	12.72	12.92	13.02	4.74
$\lambda = 65000$	19.05	21.87	25.95	26.51	26.73	26.85	21.88
$c_o = 25$	9.45	13.10	18.04	18.57	18.85	19.00	13.11
$c_o = 15$	9.91	13.47	18.34	19.10	19.35	19.49	13.48
$c_d = 15$	5.04	8.82	13.87	14.55	14.82	14.97	8.83
$c_d = 5$	14.36	17.77	22.51	23.14	23.39	23.53	17.77
$k_d = 15000$	7.91	11.51	16.40	17.06	17.32	17.47	11.51
$k_d = 5000$	11.48	15.08	19.97	20.63	20.89	21.04	15.08
$q_o = 0.8$	10.79	14.46	19.50	20.94	21.15	21.27	14.46
$q_o = 0.6$	8.46	11.94	16.62	16.30	16.63	22.31	11.95
$q_d = 0.6$	11.64	16.06	21.49	21.81	22.14	22.67	16.06
$q_d = 0.4$	7.49	10.24	14.43	15.35	15.54	15.65	10.24
$t_o = 9$	7.98	11.16	15.61	16.21	16.45	16.59	11.17
$t_o = 7$	11.96	16.07	21.50	23.29	23.50	23.62	16.07
$t_d = 4$	-6.31	-6.47	-6.02	-5.77	-5.66	-5.60	-6.31
$b_{td} = 10$	14.18	17.59	22.35	22.97	23.23	23.37	17.60
$b_{td} = 0$	5.06	8.87	13.93	14.60	14.88	15.02	8.88
$k_1 = 5$	8.32	15.14	19.08	19.62	19.80	19.89	15.17
$\alpha_1 = 0.006$	13.29	18.24	23.90	24.65	24.95	25.11	18.24
$\alpha_1 = 0.004$	6.11	8.52	12.57	13.09	13.31	13.43	8.53

As seen in **Table 4.4.** with different parameters, from Model NC to Model DP, the social utility and the profit of the private hospital increase.

5. MODELS FOR PRICING AND CAPACITY DECISIONS FOR A SYSTEM INCLUDING ONE PRIVATE AND ONE PUBLIC HOSPITAL

In this chapter, capacity and pricing models for the system mentioned in the previous chapters are proposed. As in Chapter 4, hospitals in the region are unified under one private and one public hospital. We propose bilevel solution approaches based on the models of subsidy payment by the government based on a committee, the contract mechanisms proposed by the government based on the subsidy payments and also based on fixed financial payments made by the government in addition to subsidy payments. In the models, the government is a leader, while the private hospital is a follower, as in the Stackelberg models. We present our results with a detailed sensitivity analysis about the parameters.

5.1. Description of the System

We consider a healthcare system consisting of one public and one private hospital, where the strategic patients choose one of these hospitals that has a higher utility for themselves considering the payments, the expected waiting times and the service quality they are going to receive. As seen in **Figure 4.1.**, we define the utility of a patient going to the public hospital as $\frac{q_d}{w_d} - kb_d^2$, where q_d denotes the service quality level, w_d denotes the expected waiting time and b_d denotes the payment that the patient needs to make at the public hospital. k denotes the price sensitivity of the patient, which is assumed to have a certain distribution among the whole population. Also, as seen in **Figure 4.2.**, it is supposed that there is an inverse relationship between k and the income level of patients. On the other hand, when a patient chooses to go to a private hospital, her utility is defined as $\frac{q_o}{w_o} - kb_o^2$, where q_o denotes the service quality level, w_o denotes the expected waiting time and $b_o > b_d$ denotes the payment that the patient needs to make at the private hospital. Then, the patient chooses to go to the hospitals that will provide her the higher utility. In other words, a patient prefers to go the private hospital if Equation 5.1 is satisfied, and she prefers to go to the public hospital, otherwise.

$$\frac{q_o}{w_o} - kb_o^2 \geq \frac{q_d}{w_d} - kb_d^2 \quad (5.1)$$

Based on Equation 5.1, we can state that a patient prefers to go to the private hospital if her price sensitivity, k, satisfies Equation 5.2.

$$k \leq \frac{\frac{q_o}{w_o} - \frac{q_d}{w_d}}{b_o^2 - b_d^2} \quad (5.2)$$

We let $F_k(x)$ denote the cumulative probability function for the price sensitivities, k, of the patients in the population. Thus, the probability of a patient going to the private hospital, denoted as p_o, can be written as the solution of the Equation 5.3. Also, the percentage of the population going to the public hospitals is $p_d = 1 - p_o$.

$$p_o = F_k(\frac{\frac{q_o}{w_o} - \frac{q_d}{w_d}}{b_o^2 - b_d^2}) \quad (5.3)$$

We let λ denote the total arrival rate of the patients per unit time and assume that the patient arrivals follow an exponential distribution. Thus, the arrival rate of patients to the private and public hospitals are both exponentially distributed with rates λp_o and λp_d, respectively. The service rates at these hospitals are also assumed to be exponentially distributed with rates $m_o \mu_o$ and $m_d \mu_d$ at the private and public hospitals, respectively. In order to estimate the expected waiting times at these hospitals, we use the results of M/M/1 queuing model. Thus, the expected waiting times at the private and public hospitals are defined as in Equations 4.4 and 4.5, respectively.

Then, combining Equation 5.3 with Equations 4.4 and 4.5, we obtain the value of p_o as the solution of the Equation 5.4.

$$p_o = F_k(\frac{q_o(m_o \mu_o - \lambda p_o) - q_d(m_d \mu_d - \lambda + \lambda p_o)}{b_o^2 - b_d^2}) \quad (5.4)$$

We assume in our models that the capacity of the private hospital, m_o, is a decision variable decided by the private hospital owners in order to maximize their own profit function. However, the capacity of the public hospitals, m_d, is assumed to be fixed and given since public healthcare capacities are generally fixed in many countries in the short term and providing additional capacity is generally difficult due to public budget limitations and bureaucratic reasons. In addition, providing new public facilities, public hospitals etc. requires long term governmental decisions and increased budgets. On the other hand, private hospital capacities are easier to change through private financing options, if they provide an additional profit. Thus, we assume that the value of m_d is fixed in our models, while m_o is a decision of the private healthcare providers.

In our model, N_o and N_d, denote the total service quality obtained by the whole population in the healthcare system and they are defined as in Equations 5.5 and 5.6, where λ denotes the total number of patients.

$$N_o = \lambda p_o q_o \tag{5.5}$$

$$N_d = \lambda p_d q_d \tag{5.6}$$

The public expenses of the government, denoted as H_d, consist of the cost of capacity in the public hospitals, the costs of the patients going to the public hospitals and the subsidy payments made to the private hospitals, as stated in Equation 5.7.

$$H_d = \lambda p_d(c_d - b_d) + \lambda p_o s + k_d m_d^2 \tag{5.7}$$

In our models, the government aims to maximize the total service quality obtained in the system, thus, we try to increase the value of U_1, where U_1 represents the average service quality obtained in the system per patient.

$$U_1 = \frac{1}{\lambda}(\frac{N_o}{w_o} + \frac{N_d}{w_d}) \tag{5.8}$$

However, in many healthcare systems, the government has a certain budget for healthcare operations and the government does not want to spend too much for these operations. Thus, we also aim to decrease the government spending for healthcare operations. Thus we denote U_2 as the average payment made by the government per patient and we aim to decrease U_2 in our models.

U_1 and U_2 are two conflicting objectives in our model. In order to increase the service quality obtained by the population in this system, the government spending also need to be increased. In order to combine these two conflicting objectives, we employ a weighted summation of U_1 and U_2 and aim to maximize the value of U that denotes the total public utility as in Equation 5.9, where α denotes the relative weight of U_1 with respect to U_2.

$$U = \alpha U_1 - U_2 \tag{5.9}$$

As an alternative approach, instead of combining U_1 and U_2 in order to maximize U, we consider the possible budget constraints in the system and try to maximize U_1 under the constraint $H_d \leq \epsilon$. In our alternative approach, we aim to determine the maximum service quality that can be obtained without spending more than a certain limit. We analyze both models and present the numerical results of our models in Section 5.6..

In this system, the profit function for the private hospital can be written as in Equation 5.10, where c_o denotes the cost of service per patient at the private hospital and $k_o m_o^2$ denotes the fixed cost of using capacity m_o.

$$Z_o = \lambda p_o(r - c_o) - k_o m_o^2 \tag{5.10}$$

In the next sections, we first analyze the system in which there is no contract between the government and the private hospitals. Then, we analyze different contract mechanisms that can be offered by the government in order to improve the system performance.

5.2. Model NC: No Contract Between the Government and the Private Hospital

In our base case model, we assume that there are no contracts between the government and the private hospital. In this model, which is summarized in **Figure 5.1.**, the private hospital aims to maximize its own profit by deciding on his own price r, and the capacity level m_o, and the government does not provide any subsidy payment to the private hospital. Thus, in this model, $s = 0$ and $b_o = r$.

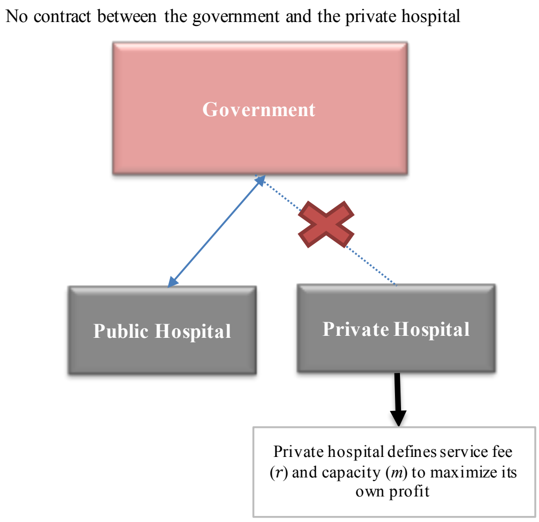


Figure 5.1. *Model NC*

Then, the optimal values of m_o and r are found through the solution of the following optimization problem.

$$Max_{r,m_o} \ Z_{o_{NC}} = \lambda p_o(r - c_o) - k_o m_o^2 \tag{5.11}$$

s.t.

$$p_o = F_k(\frac{q_o(m_o\mu_o - \lambda p_o) - q_d(m_d\mu_d - \lambda + \lambda p_o)}{r^2 - b_d^2}) \tag{5.12}$$

$$p_o \leq 1 \tag{5.13}$$

$$p_o \geq 0 \tag{5.14}$$

$$\lambda p_o \leq m_o\mu_o \tag{5.15}$$

$$\lambda(1 - p_o) \leq m_d\mu_d \tag{5.16}$$

$$r \geq 0 \tag{5.17}$$

$$m_o \geq 0 \tag{5.18}$$

In the above model, the objective function is composed of the profit per patient multiplied with the number of patients minus the cost of capacity. The first constraint denotes the percentage of patients going to the private hospitals and Constraints 5.13 and 5.14 force this value to be between 0 and 1. Then, constraints 5.15 and 5.16 are required for the capacity of the private and public hospitals to be enough to serve the arriving patients. If these two constraints are not satisfied, then the queue lengths will go to infinity at these hospitals. The last two constraints are the nonnegativity constraints.

After determining the optimal r and m_o values for the private hospital, we calculate the total base case utility level of the society through Equation 4.15 with these parameters.

5.2.1. A Special case: Uniformly distributed price sensitivities

In this section, as a special case of our model, for the sake of simplicity, we assume that the price sensitivity of the patients in the whole population is uniformly distributed between 0 and $\bar{k}$. Then, Equation 5.4 can be written as in Equation 5.19.

$$p_o = \begin{cases} 1, & if \ \frac{q_o(m_o\mu_o-\lambda p_o)-q_d(m_d\mu_d-\lambda+\lambda p_o)}{(b_o^2-b_d^2)\bar{k}} > 1 \\ \frac{q_o(m_o\mu_o-\lambda p_o)-q_d(m_d\mu_d-\lambda+\lambda p_o)}{(b_o^2-b_d^2)\bar{k}}, & if \ 0 \leq \frac{q_o(m_o\mu_o-\lambda p_o)-q_d(m_d\mu_d-\lambda+\lambda p_o)}{(b_o^2-b_d^2)\bar{k}} \leq 1 \\ 0, & if \ \frac{q_o(m_o\mu_o-\lambda p_o)-q_d(m_d\mu_d-\lambda+\lambda p_o)}{(b_o^2-b_d^2)\bar{k}} < 0 \end{cases} \tag{5.19}$$

If Equation 5.19 is solved for p_o, we can obtain a closed form solution for p_o as in Equation 5.20.

$$p_o = \begin{cases} 1, & if \ \frac{q_d(\lambda-m_d\mu_d)+q_o m_o\mu_o}{\lambda(q_o+q_d)+\bar{k}(b_o^2-b_d^2)} > 1 \\ \frac{q_d(\lambda-m_d\mu_d)+q_o m_o\mu_o}{\lambda(q_o+q_d)+\bar{k}(b_o^2-b_d^2)}, & if \ 0 \leq \frac{q_d(\lambda-m_d\mu_d)+q_o m_o\mu_o}{\lambda(q_o+q_d)+\bar{k}(b_o^2-b_d^2)} \leq 1 \\ 0, & if \ \frac{q_d(\lambda-m_d\mu_d)+q_o m_o\mu_o}{\lambda(q_o+q_d)+\bar{k}(b_o^2-b_d^2)} < 0 \end{cases} \tag{5.20}$$

Since the value of k is assumed to be uniformly distributed between 0 and $\bar{k}$, we can write Constraint 5.12 clearer. First we provide the following lemma about the value of p_o when $k \sim U[0, \bar{k}]$.

Lemma 5.1. *Even though p_o was defined as in Equation 5.20 when $k \sim U[0, \bar{k}]$, in the optimal solution of this problem, it will satisfy the following equation.*

$$p_o = \frac{q_d(\lambda - m_d\mu_d) + q_o m_o\mu_o}{\lambda(q_o + q_d) + \bar{k}(r^2 - b_d^2)} \tag{5.21}$$

Proof. Note that the objective function of the private hospital is defined as $Z_{o1} = \lambda p_o(r - c_o) - k_o m_o^2$. If the value of $\frac{q_d(\lambda - m_d\mu_d) + q_o m_o \mu_o}{\lambda(q_o+q_d) + \bar{k}(r^2 - b_d^2)} > 1$, then $p_o = 1$ and decreasing m_o slightly will not change the value of p_o and it will still be equal to 1 while Z_{o1} will increase. Thus, such a solution can not be optimal. Similarly, if the value of $\frac{q_d(\lambda - m_d\mu_d) + q_o m_o \mu_o}{\lambda(q_o+q_d) + \bar{k}(r^2 - b_d^2)} < 0$, then $p_o = 0$ and $Z_{oNC} = 0$. The same value can be obtained when $\frac{q_d(\lambda - m_d\mu_d) + q_o m_o \mu_o}{\lambda(q_o+q_d) + \bar{k}(r^2 - b_d^2)} = 0$. Thus, $p_o = \frac{q_d(\lambda - m_d\mu_d) + q_o m_o \mu_o}{\lambda(q_o+q_d) + \bar{k}(r^2 - b_d^2)}$ will always be satisfied in the optimal solution $\square$

As a result of Lemma 5.1, Constraint 5.12 in Model 5.4. can be written more clearly as below.

$$p_o = \frac{q_d(\lambda - m_d\mu_d) + q_o m_o \mu_o}{\lambda(q_o + q_d) + \bar{k}(r^2 - b_d^2)}$$

Solution method

As a solution method, we design an algorithm that searches on different prices and capacities to maximize the profit.

5.3. Model SP-GL: Subsidy Payment by the Government as a Leader

In this model, which is summarized in **Figure 5.2.**, we assume that the government determines the amount of subsidy for each patient going to the private hospital, indicated as s. Thus, the payment of each patient in the private hospital is equal to $b_o = r - s$.

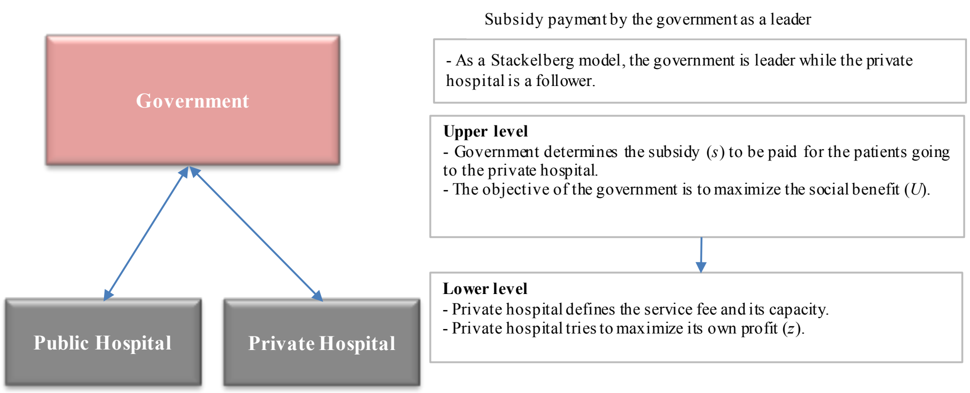


Figure 5.2. *Model SP-GL*

In this model, the private hospital defines the price of service, r, and its own capacity, m_o. The model is defined as a bilevel optimization problem, where in the first level, the government defines s to maximize the social utility, U, while in the second level, the private hospital decides on the price of service and its capacity level. To solve the problem, at first, the second stage of the problem is solved, which is stated in Equation 5.23. In this level, the private hospital decides on r and m_o for the given s. We state the second stage problem as below:

$$Max_{r,m_o}\ Z_{o_{SP-GL}} = \lambda p_o(r - c_o) - k_o m_o^2 \tag{5.22}$$

$$s.t.$$

$$p_o = F_k\left(\frac{q_o(m_o\mu_o - \lambda p_o) - q_d(m_d\mu_d - \lambda + \lambda p_o)}{(r - s)^2 - b_d^2}\right)$$

$$p_o \leq 1$$

$$p_o \geq 0$$

$$\lambda p_o \leq m_o\mu_o$$

$$\lambda(1 - p_o) \leq m_d\mu_d$$

In the first level, the government determines the optimal s to maximize the social utility. The values of r and m_o are obtained from the solution of the second stage problem. Thus, the first stage problem is stated as below:

$$Max_s\ U = \alpha U_1 - U_2 \tag{5.23}$$

s.t.

$$
\begin{aligned}
U_1 &= \frac{1}{\lambda}(\frac{N_o}{W_o} + \frac{N_d}{W_d}) && (5.24)\\
U_2 &= \frac{H_d}{\lambda} && (5.25)\\
p_o &= F_k(\frac{q_o(m_o\mu_o - \lambda p_o) - q_d(m_d\mu_d - \lambda + \lambda p_o)}{(r\_s)^2 - b_d^2}) && (5.26)\\
W_o &= \frac{1}{m_o\mu_o - \lambda p_o} && (5.27)\\
W_d &= \frac{1}{m_d\mu_d - \lambda p_d} && (5.28)\\
N_o &= \lambda p_o q_o && (5.29)\\
N_d &= \lambda p_d q_d && (5.30)\\
H_d &= \lambda p_d(c_d - b_d) + \lambda p_o s + k_d m_d^2 && (5.31)\\
p_o &\leq 1 && (5.32)\\
p_o &\geq 0 && (5.33)\\
\lambda p_o &\leq m_o\mu_o && (5.34)\\
\lambda(1 - p_o) &\leq m_d\mu_d && (5.35)\\
r\ and\quad m_o &\quad are\ the\ outputs\ of\ Model\ 5.23 && (5.36)
\end{aligned}
$$

Solution method

The solution method is summarized in **Figure 5.3.**, where the government as a leader proposes the amount of subsidy, foreseeing the decision of the private hospital. As a Stackelberg model, in the upper level, the government by searching on the subsidy amount tries to maximize the social utility, while in the lower level the private hospital by searching on capacity and service price objects to maximize its own profit.

```
Search on s
foreach s do
    Find Best r and m_o that maximize Z_o
    if constraints are not violated then
        Calculate U(r, s, m_o)
    end
end
Find Best s that maximizes U;
Find r and m_o that correspond to Best s;
Calculate Z_o;
```

Figure 5.3. *Solution method for Model SP-GL*

5.4. Model SC: Contract Mechanism Based on Subsidy Payments

In order to improve the healthcare system and provide a more balanced supply and demand, the government may propose certain contracts to the private hospitals. One of such contracts is based on subsidy payments made to the private hospitals for each patient in return for a decrease in prices. In this model, which is summarized in **Figure 5.4.**, we assume that the government offers a contract with parameters r and s where r denotes the price of service at the private hospital per patient and s denotes the subsidy payment made by the government to the private hospital for each patient. As a result of this contract, each patient pays an effective amount equal to $b_o = r - s$ when she goes to the private hospital and the government pays an amount equal to s resulting in a total payment of r made to the hospital.

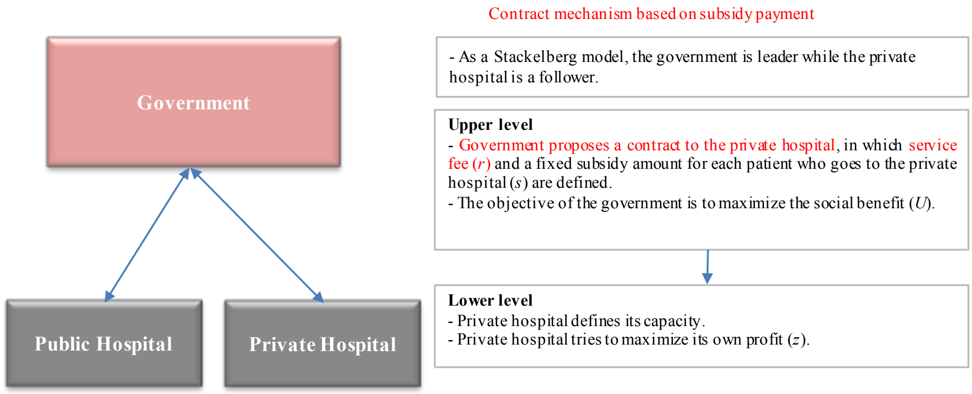


Figure 5.4. *Model SC*

We note that the private hospital is free to accept or reject this contract based on its own profit function and if it rejects the contract, the system will be operated as in Model NC, as explained above, in which there are no contracts and the private hospital obtains a profit equal to $Z_{o_{NC}}$. However, if the private hospital accepts the contract, then it decides on its own capacity m_o. When the government is proposing the contract parameters, he needs to make sure that the private hospital makes a profit that is at least equal to $Z_{o_{NC}}$ as a result of accepting this contract. Otherwise, the contract will be rejected. Thus, we have a bilevel optimization problem such that in the first level, the government needs to decide on r and s in order to maximize the total utility, U, and in the second stage, the private hospital decides whether to accept the contract and its capacity level.

In order to solve this problem, we first need to solve the second stage problem in which the private hospital needs to decide on his capacity level m_o for the given contract parameters r and s.

We state the second stage problem as below:

$$Max_{m_o} \ Z_{o_{SC}} = \lambda p_o(r - c_o) - k_o m_o^2 \tag{5.37}$$

$$s.t.$$

$$p_o = F_k\left(\frac{q_o(m_o\mu_o - \lambda p_o) - q_d(m_d\mu_d - \lambda + \lambda p_o)}{(r-s)^2 - b_d^2}\right)$$

$$p_o \leq 1$$

$$p_o \geq 0$$

$$\lambda p_o \leq m_o \mu_o$$

$$\lambda(1 - p_o) \leq m_d \mu_d$$

The above model is almost the same as in Model , except that r and s are assumed to be given in the contract and we only aim to determine the value of the capacity m_o. Then, in the first stage problem, we need to determine the optimal contract parameters r and s that will maximize the total social utility. Note that the value of m_o in this model will be obtained from the solution of the second stage problem. In addition, the contract needs to satisfy the constraint that the profit of the private hospital as a result of this mechanism should be as least equal to the profit of the private hospital without the contract.

Thus, the first stage problem can be stated as below:

$$Max_{r,s}\ U = \alpha U_1 - U_2 \tag{5.38}$$

s.t.

$$U_1 = \frac{1}{\lambda}\left(\frac{N_o}{W_o} + \frac{N_d}{W_d}\right) \tag{5.39}$$

$$U_2 = \frac{H_d}{\lambda} \tag{5.40}$$

$$p_o = F_k\left(\frac{q_o(m_o\mu_o - \lambda p_o) - q_d(m_d\mu_d - \lambda + \lambda p_o)}{(r_{-}s)^2 - b_d^2}\right) \tag{5.41}$$

$$W_o = \frac{1}{m_o\mu_o - \lambda p_o} \tag{5.42}$$

$$W_d = \frac{1}{m_d\mu_d - \lambda p_d} \tag{5.43}$$

$$N_o = \lambda p_o q_o \tag{5.44}$$

$$N_d = \lambda p_d q_d \tag{5.45}$$

$$H_d = \lambda p_d(c_d - b_d) + \lambda p_o s + k_d m_d^2 \tag{5.46}$$

$$p_o \leq 1 \tag{5.47}$$

$$p_o \geq 0 \tag{5.48}$$

$$\lambda p_o \leq m_o\mu_o \tag{5.49}$$

$$\lambda(1 - p_o) \leq m_d\mu_d \tag{5.50}$$

$$m_o \quad is\ the\ solution\ of\ Model\ 5.37 \tag{5.51}$$

$$Z_{o_{SC}} \geq Z_{o_{NC}} \tag{5.52}$$

In the above model, the objective function is composed of a weighted sum of the average quality obtained per patient and the average government spending per patient. The first eight constraints denote the expressions for U_1, U_2, p_o, W_o, W_d, N_o, N_d and H_d as stated before. The constraints 5.47 and 5.48 require p_o to be between 0 and 1. The constraints 5.49 and 5.50 are required for the waiting times at the private and public hospitals to be finite. The last two constraints are for the solutions of the second stage problem. The constraint 5.51 denotes that the value of m_o will be determined as a solution of the second stage problem and constraint 5.52 is required for the voluntary acceptance of the contract by the private hospital. The last constraint denotes that the profit of the private hospital as a result of this contract mechanism after the second stage decision should be at least equal to the profit value without the contract, otherwise the private hospital will not accept this contract.

We also note that, as an alternative approach, instead of maximizing U, we consider the possible budget constraints in the system and try to maximize U_1 under the budget constraint $H_d \leq \epsilon$ in addition to the other constraints. Thus, we also solve the following model.

$$Max_{r,s} \ U_1 \tag{5.53}$$

s.t.

$$H_d \leq \epsilon$$

$$Constraints \ 5.39 - 5.52$$

In order to solve these models, we employ an algorithm that does a two-dimensional search over r and s and solve the second stage problem 5.37 using a commercial solver, for each combination of r and s. Then, we calculate the value of the objective function of the first stage problem using the value of m_o obtained from the second stage model solution and pick the best combination of r and s values that are feasible and provide the maximum objective value. We present the numerical results of both models in Section 5.6..

Solution method

The solution method is summarized in **Figure 5.5.**, where the government as a leader proposes the service price and the amount of subsidy, foreseeing the decision of the private hospital. As a Stackelberg model, in the upper level, the government by searching on the service price and the subsidy amount tries to maximize the social utility, while in the lower level the private hospital by searching on capacity objects to maximize its own profit.

```
Search on r and s
foreach r and s do
    Find Best m_o that maximizes Z_o
    if Z_o > Z_oNC then
        The private hospital accepts the contract;
        if constraints are not violated then
            Calculate U(r, s, m_o)
        end
    end
end
Find Best r and s that maximizes U;
Find m_o that corresponds to Best r and s;
Calculate Z_o;
```

Figure 5.5. *Solution method for Model SC*

5.5. Model LTC: The Linear Two Part Tariff Contract Mechanism

In this model, which is summarized in **Figure 5.6.**, we analyze a contract mechanism that includes a fixed payment, T, in addition to the decisions regarding r and s values. In this contract, the government makes a fixed payment T, in expectation of a decrease in private hospital prices. The problem under this contract is similar to the previous one and we model it as two stage problem as before.

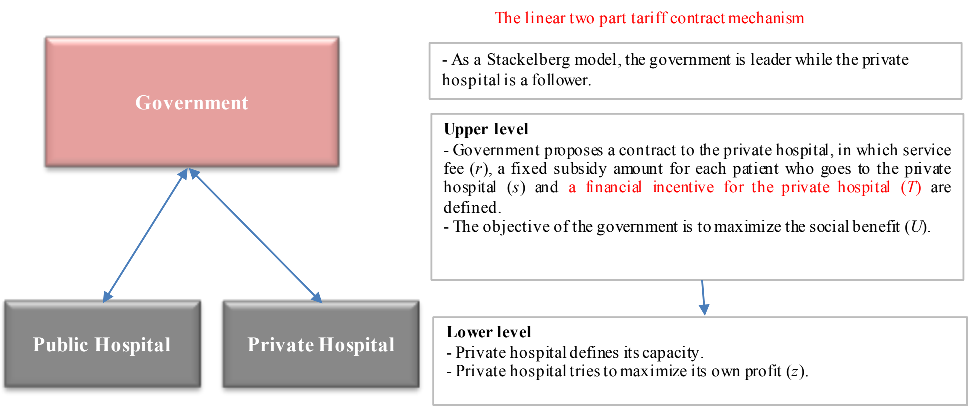


Figure 5.6. *Model LTC*

In order to solve this problem, we first need to solve the second stage problem in which the private hospital needs to decide on his capacity level m_o for the given contract parameters r, s and T. We state the second stage problem as below:

$$Max_{m_o}\ Z_{o_{LTC}} = \lambda p_o(r - c_o) - k_o m_o^2 + T \tag{5.54}$$

$$s.t.$$

$$p_o = F_k(\frac{q_o(m_o\mu_o - \lambda p_o) - q_d(m_d\mu_d - \lambda + \lambda p_o)}{(r_{-}s)^2 - b_d^2})$$

$$p_o \leq 1$$

$$p_o \geq 0$$

$$\lambda p_o \leq m_o\mu_o$$

$$\lambda(1 - p_o) \leq m_d\mu_d$$

Then, in the first stage problem, we determine the optimal contract parameters r, s and T that will maximize the total social utility. Again, the value of m_o in this model will be obtained from the solution of the second stage problem stated above. In addition, the contract needs to satisfy the constraint that the profit of the private hospital as a result of this mechanism should be as least equal to the profit of the private hospital without the contract.

Thus, the first stage problem can be stated as below:

$$Max_{r,s,T}\ U = \alpha U_1 - U_2 \tag{5.55}$$

s.t.

$$\begin{aligned}
U_1 &= \frac{1}{\lambda}\left(\frac{N_o}{W_o} + \frac{N_d}{W_d}\right) && (5.56)\\
U_2 &= \frac{H_d}{\lambda} && (5.57)\\
p_o &= F_k\left(\frac{q_o(m_o\mu_o - \lambda p_o) - q_d(m_d\mu_d - \lambda + \lambda p_o)}{(r-s)^2 - b_d^2}\right) && (5.58)\\
W_o &= \frac{1}{m_o\mu_o - \lambda p_o} && (5.59)\\
W_d &= \frac{1}{m_d\mu_d - \lambda p_d} && (5.60)\\
N_o &= \lambda p_o q_o && (5.61)\\
N_d &= \lambda p_d q_d && (5.62)\\
H_d &= \lambda p_d(c_d - b_d) + \lambda p_o s + k_d m_d^2 + T && (5.63)\\
p_o &\leq 1 && (5.64)\\
p_o &\geq 0 && (5.65)\\
\lambda p_o &\leq m_o\mu_o && (5.66)\\
\lambda(1-p_o) &\leq m_d\mu_d && (5.67)\\
& m_o \quad is\ the\ solution\ of\ Model\ 5.54 && (5.68)\\
Z_{o_{LTC}} &\geq Z_{o_{NC}} && (5.69)
\end{aligned}$$

The first and second stage models are very similar to the models in the previous section. However, note that in the above problem, the expenditures made by the government is increased by T and the profit function of the private hospital is also increased by T. As a result of this fixed payment, the private hospital is expected to obtain the critical level of profit in order to accept this contract with a lower unit price, r.

Lemma 5.2. *In the optimal solution of the above problem,* $Z_{o_{LTC}}^{optimal} = Z_{o_{NC}}$*. Thus,* T *can be written as a function of* s *and* r*.*

Proof. Assume that for a given contract with parameters r, s and T, the resulting solution satisfies $Z_{o_{LTC}}^{optimal} > Z_{o_{NC}}$. Then, there exists another solution with parameters r, s and $T - \epsilon$, for some small enough $\epsilon > 0$, and this solution will provide an increase in U and decrease in $Z_{o_{LTC}}$, since T is a fixed payment that does not effect the other decision variables. Thus, such a solution can not be optimal and in the optimal solution $Z_{o_{LTC}}^{optimal} = Z_{o_{NC}}$ should be satisfied. □

As a result of the above Lemma, we can find the value of T as a function of r and s through the equation $Z_{o_{LTC}} = Z_{o_{NC}}$. Then, in order to solve the above problem, we write a similar algorithm as in the previous case, that does a two-dimensional search over r and s and solve the second stage problem 5.54 using a commercial solver, for each combination of r and s. We calculate the value of $T = Z_{o_{NC}} - Z_{o_{LTC}}$ and U using the value of m_o obtained from the above solution. Then, we pick the best combination of r and s values that are feasible and provide the maximum value of U. Note that some values of r and s that are not feasible in Model SC will be feasible in Model LTC with the help of the fixed payment T and this is expected to lead to better solutions as will be seen in our numerical results section.

We also note that, as an alternative approach, instead of maximizing U, we also consider the model that maximizes U_1 under the constraint $H_d \leq \epsilon$ in addition to the other constraints as stated below.

$$Max_{r,s,T} \; U_1 \tag{5.70}$$

s.t.

$$H_d \leq \epsilon$$

$$Constraints \; 5.56 - 5.69$$

Solution method

The solution method is summarized in **Figure 5.7.**, where the government as a leader proposes the service price, the amount of subsidy and the financial incentive, foreseeing the decision of the private hospital. As a Stackelberg model, in the upper level, the government by searching on the service price, the subsidy amount and the financial incentive tries to maximize the social utility, while in the lower level the private hospital by searching on capacity objects to maximize its own profit.

Search on r, s and T
foreach r, s *and* T **do**
    Find Best m_o that maximizes Z_o
    if $Z_o > Z_{o_{NC}}$ **then**
        The private hospital accepts the contract;
        if *constraints are not violated* **then**
            Calculate $U(r, s, T, m_o)$
        end
    end
end
Find Best r, s and T that maximizes U;
Find m_o that corresponds to Best r, s and T;
Calculate Z_o;

Figure 5.7. *Solution method for Model LTC*

5.6. Experimental Results Based on a Case Study from Turkey

The numerical analyses are designed based on a regional health system in Turkey. We consider the capacities of the hospitals and the arrival rates of the patients to these hospitals in the Eskisehir region of Turkey. In this system, more than 70,000 patients are treated in the emergency services of the hospitals per month. In order to satisfy the total demand, the average capacity level, defined as the average number of examination teams of doctors and nurses, is seen to be 5 in the public hospitals. The cost of the basic examinations in the public and the private hospitals are stated to be 10 TL and 20 TL on average respectively. The unit costs of capacity, which are defined based on the monthly salary of the employed doctors and nurses are assumed to be 10,000 TL and 50,000 TL in the public and private hospitals, respectively. The levels of service quality, perceived by the patients in the public and the private hospitals have been defined as 0.5 and 0.7 respectively based on the results of a questionnaire done to the patients of these hospitals. Examination fee paid by the patients at the public hospital is 5 TL. All the parameters used in our base case study are summarized in **Table 5.1.**.

Table 5.1. *Values of base case parameters*

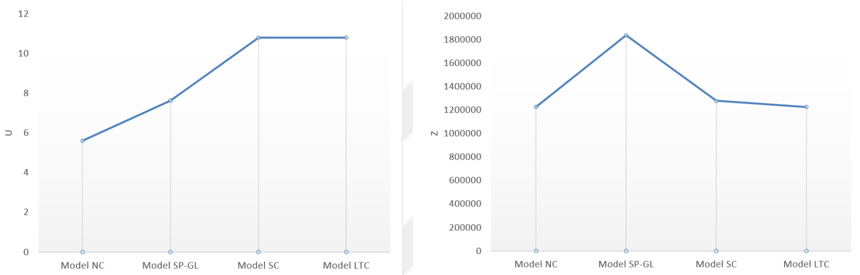
λ	m_d	c_o	c_d	k_o	k_d	q_o	q_d	t_o	t_d	b_d	$\bar{k}$	α
76000	5	20	10	50000	10000	0.7	0.5	8	3	5	1	0.005

We present the results of our models with the above parameters in **Table 5.2.**. We observe that the total public utility can be significantly improved through the contract mechanisms used in Models 2 and 3. The payment of the patients to the private hospital in Model SC is 126 TL less than Model NC. The percentage of patients going to the private hospitals, p_o, also increases by 0.052 and the private hospital managers decide to increase the capacity leading to decreased waiting times and better service. Even though the expenditure of the government is increased by about 52.10% in Model SC compared to Model NC, which is due to the subsidy payments made to the private hospitals per patient, the total utility level, U, increases by more than 92.5%. The waiting times are also seen to decrease significantly in the public hospitals, from 0.21 in Model NC to 0.10 in Model SC. Model LTC provides even better results with decreased government expenditure, and increased utility level, compared to Model SC.

Table 5.2. *Results obtained by the models with the base case parameters*

	r	s	m_o	p_o	w_o	w_d	H_d	U_1	U_2	U	Z_{o_i}	T
Model NC	323	0	4.50	0.097	0.04	0.21	593050.37	2681.14	7.80	5.60	1226647.43	-
Model SP-GL	326	67	5.6	0.146	0.04	0.10	1319816.60	5000.16	17.37	7.63	1836578.71	-
Model SC	226	29	4.60	0.149	0.05	0.10	902062.04	4530.82	11.87	10.78	1277199.16	-
Model LTC	219	26	4.50	0.148	0.06	0.10	866661.29	4440.59	11.40	10.80	1226647.43	412.93

In both Models SC and LTC, we observe that the total utility of the society can increase significantly, however, the expense of government increases as well. In real life, there might exist certain budget constraints about healthcare expenditures, which generally limit the total utility level. Thus, we also analyze the model under this budget constraint, as stated in problems 5.53 and 5.70. For this constraint, we use varying government expenditure values, H_d, between 650,000 and 1,300,000 TL, which are close to the values of H_d in the Models NC and SP-GL. As seen in **Table 5.3.**, a significant increase in the total utility levels can be obtained with a small increase in the public expenditures. The increased government expenditures are seen to have decreasing contributions to the total utility level. With a contract mechanism as proposed in Model SC, it is possible to improve the value of U, even with very similar government expenditures as in Model NC. Model LTC provides even better results than Model SC for most of the budget level, with a higher difference for more strict budget constraints.



(a) (b)

Figure 5.8. *The values of (a) social benefit and (b) private hospital profit in the models*

Table 5.3. *Obtained U when there is a constraint to increase H_d* ($\times$10000)

	U							
	If $H_d \leq 60$	*If* $H_d \leq 70$	*If* $H_d \leq 80$	*If* $H_d \leq 90$	*If* $H_d \leq 100$	*If* $H_d \leq 110$	*If* $H_d \leq 120$	*If* $H_d \leq 130$
Model SP-GL	5.60	6.00	6.60	7.07	7.21	7.58	7.61	
Model SC	7.15	10.13	10.67	10.78				
Model LTC	7.28	10.13	10.72	10.80				

5.6.1. Sensitivity Analysis

The parameters of the system also affect the results significantly, thus we present a sensitivity analysis to understand the effects of the parameters on the system results. In order to understand the effects of the parameters individually, we change one of the parameters at a time, while the other parameters are fixed. The results of our sensitivity analysis are provided in **Table 5.4.**.

Table 5.4. *Results obtained by the models according to different parameters*

	Model NC	**Model SP-GL**	**Model SC**	**Model LTC**
$m_d = 5.5$	17.63	20.00	22.16	22.16
$m_d = 4.5$	1.80	1.80	3.68	3.71
$\lambda = 80000$	-0.79	1.46	5.14	5.14
$\lambda = 70000$	15.12	17.07	19.37	19.37
$c_o = 25$	4.72	6.92	10.08	10.08
$c_o = 15$	5.99	8.37	11.55	11.55
$c_d = 15$	1.09	3.37	6.55	6.55
$c_d = 5$	10.12	11.92	15.05	15.07
$k_o = 55000$	2.79	5.00	8.16	8.16
$k_o = 45000$	7.93	11.11	14.13	14.13
$k_d = 15000$	3.96	5.99	9.14	9.15
$k_d = 5000$	7.25	9.28	12.43	12.44
$q_o = 0.8$	9.89	12.75	15.72	15.72
$q_o = 0.6$	1.28	2.98	6.16	6.16
$q_d = 0.6$	4.69	7.99	11.36	11.36
$q_d = 0.4$	5.67	7.26	10.09	10.09
$t_o = 9$	0.36	1.63	4.70	4.71
$t_o = 7$	13.56	17.51	20.26	20.26
$t_d = 4$	10.27	10.27	16.26	16.37
$t_d = 2$	72.73	88.73	73.32	73.32
$b_{td} = 10$	10.19	11.96	15.08	15.09
$b_{td} = 0$	1.09	3.36	6.53	6.53
$k_1 = 5$	-6.61	-2.77	-1.89	-1.89
$\alpha_1 = 0.006$	8.28	13.12	15.72	15.72
$\alpha_1 = 0.004$	2.92	3.36	6.49	6.49

If the capacity of the public hospital increases, in all models, more patients choose this hospital, so the profit of the private hospital decreases. In this case, the private hospital also increases its own capacity and higher prices are defined in this hospital. As the average waiting time of patients decreases, the public benefit increases. When $\lambda = 80000$, in all models the price of the examination in the private hospital increases. Also, the private hospital capacity raises. More people choose the private hospital and the average waiting times increases. In this case, the expenses of the government increase and so the public benefit decreases. Also, the profit of private hospital increases. If the cost of care for each patient in the private hospital increases, the private hospital reduces its own capacity slightly as in Model SC and LTC. In addition, in all models, the price of the examination in the private hospital increases. In this case the private hospital gets less profit. If the cost of care for each patient in the public hospital increases, the public benefit reduces because the government spends more. A decrease in this costs also causes the opposite of these effects. If the cost of the capacity in private hospital increases, its capacity reduces in all models. Also, the profit of this hospital reduces but there is no change in the public benefit. The increase in the cost of capacity in the public hospital negatively affects the public benefit. The increase in the perceived quality level in the private hospital has a positive effect on both the profit of private hospital and the public benefit. If the perceived quality level in the public hospital increases, the prices in the private hospital decrease in all models. With the exception of Model LTC, the capacity of the private hospital reduces in the other models. This also increases the profit of the private hospital even though it reduces the public benefit in Models SC and LTC. The increase in the average examination time in both of the private and public hospitals reduces the public benefit and the profit of private hospital. A slight increase in the payments of patients in the public hospital improves the public benefit. If the price sensitivity of patients increases, fewer patients choose this hospital even with fewer prices, which decreases public benefits and also the profit of private hospital.

6. PRICING MODELS FOR A SYSTEM INCLUDING TWO PRIVATE HOSPITALS AND ONE PUBLIC HOSPITAL

In this chapter, it is assumed that in a region there are two private hospitals with different characteristics near a public hospital. The hospitals interact with each other and their decisions about pricing and quality level affect the preferences of patients to choose them. The state in which the government proposes a contract to both of the private hospitals and also the model where the contract is recommended to only one of them are discussed. In these situations, private hospitals may accept or reject the contract. In case of rejection, they make their own pricing decisions. In the model where the government proposes a contract to both private hospitals, the Nash equilibrium points for the private hospitals are searched using the game theory techniques. After the introduction of the models, in the results subsection, a comparison is made between the models based on the social utility.

6.1. Description of the System

Let us assume that there is a contract based on fixed prices between the government and private hospitals, and also the government pays a definite subsidy to the patients that receive service from a private hospital. In this case, as seen in **Figure 6.1.**, the service fee, the average waiting time and the perceived quality level in the i-th private hospitals are denoted as b_{o_i}, w_{o_i} and q_{o_i}. We define the utility of the patients in the i-th private hospital as $\frac{q_{o_i}}{w_{o_i}} - k{b_{o_i}}^2$, and it is $\frac{q_d}{w_d} - kb_d^2$ in the public hospital. k is the price sensitivity of the patients and we suppose that if it is low for a patient, she gives more importance to quality than price and vice versa. In addition, we assume that there is an inverse relationship between k and the income level of patients, as seen in **Figure 4.2.**.

We suppose that a strategic patient makes a choice between the hospitals according to the utility she will receive in the chosen hospital.

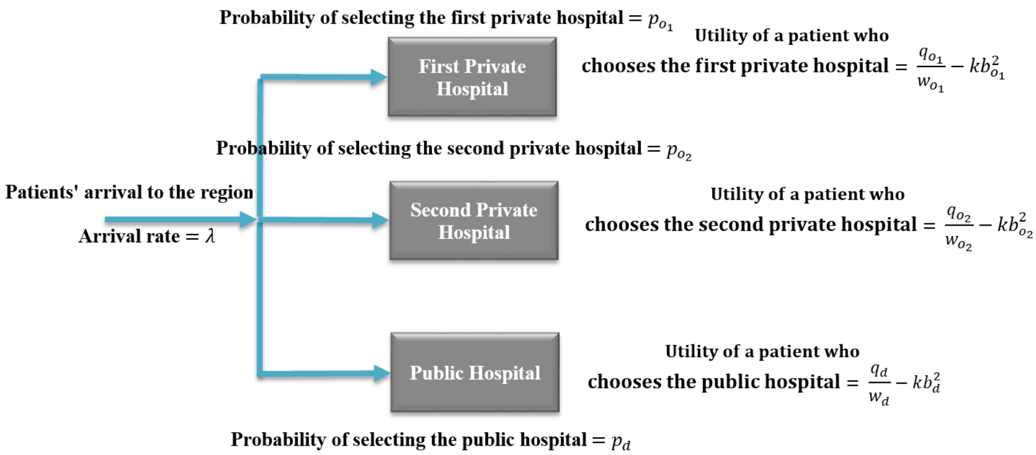


Figure 6.1. *A strategic patient chooses one of the hospitals based on the utility she gets.*

A strategic patient selects the first hospital if Equation 6.1 is valid.

$$\frac{q_{o_1}}{w_{o_1}} - kb_{o_1}^2 \geq \frac{q_d}{w_d} - kb_d^2 \quad and \quad \frac{q_{o_1}}{w_{o_1}} - kb_{o_1}^2 \geq \frac{q_{o_2}}{w_{o_2}} - kb_{o_2}^2 \tag{6.1}$$

From Equation 6.1, if k for a patient is as in Equation 6.2, she chooses the first private hospital.

$$k \leq \frac{\frac{q_{o_1}}{w_{o_1}} - \frac{q_d}{w_d}}{b_{o_1}^2 - b_d^2} \quad and \quad k \leq \frac{\frac{q_{o_1}}{w_{o_1}} - \frac{q_{o_2}}{w_{o_2}}}{b_{o_1}^2 - b_{o_2}^2} \tag{6.2}$$

We can assume that $b_{o_1} > b_{o_2} > b_d$.

Lemma 6.1. *Considering the assumption that $b_{o_1} > b_{o_2} > b_d$, it can be concluded that $\frac{q_{o_1}}{w_{o_1}} > \frac{q_{o_2}}{w_{o_2}} > \frac{q_d}{w_d}$.*

Proof. If $\frac{q_{o_1}}{w_{o_1}} < \frac{q_{o_2}}{w_{o_2}}$ then we can conclude that $p_{o_1} \to 0$ and accordingly $w_{o_1} \to 0$. Because based on our assumption to select the first one among the private hospitals by a patient her utility in this hospital would be more than the other, means $\frac{q_{o_1}}{w_{o_1}} - kb_{o_1}^2 \geq \frac{q_{o_2}}{w_{o_2}} - kb_{o_2}^2$ and it is obvious that this is not possible for many patients. On the other hand if $w_{o_1} \to 0$ then $\frac{q_{o_1}}{w_{o_1}} \to \infty$ and accordingly $\frac{q_{o_1}}{w_{o_1}} > \frac{q_{o_2}}{w_{o_2}}$ which contradicts $\frac{q_{o_1}}{w_{o_1}} < \frac{q_{o_2}}{w_{o_2}}$. So we can conclude that $\frac{q_{o_1}}{w_{o_1}} > \frac{q_{o_2}}{w_{o_2}} > \frac{q_d}{w_d}$.

□

In addition, we also assume that the intersection of $\frac{q_{o_1}}{w_{o_1}}$ and $\frac{q_{o_2}}{w_{o_2}}$ according to k is in the order shown in **Figure 6.2.**a. If the intersections are in different order,

$\frac{q_{o_1}}{w_{o_1}} > \frac{q_{o_2}}{w_{o_2}} > \frac{q_d}{w_d}$ becomes violated. For example, if the order is as in **Figure 6.2**.c, then the second private hospital is not much preferred, $w_{o_2} \to 0$ then $\frac{q_{o_2}}{w_{o_2}} \to \infty$ which contradicts $\frac{q_{o_1}}{w_{o_1}} > \frac{q_{o_2}}{w_{o_2}} > \frac{q_d}{w_d}$.

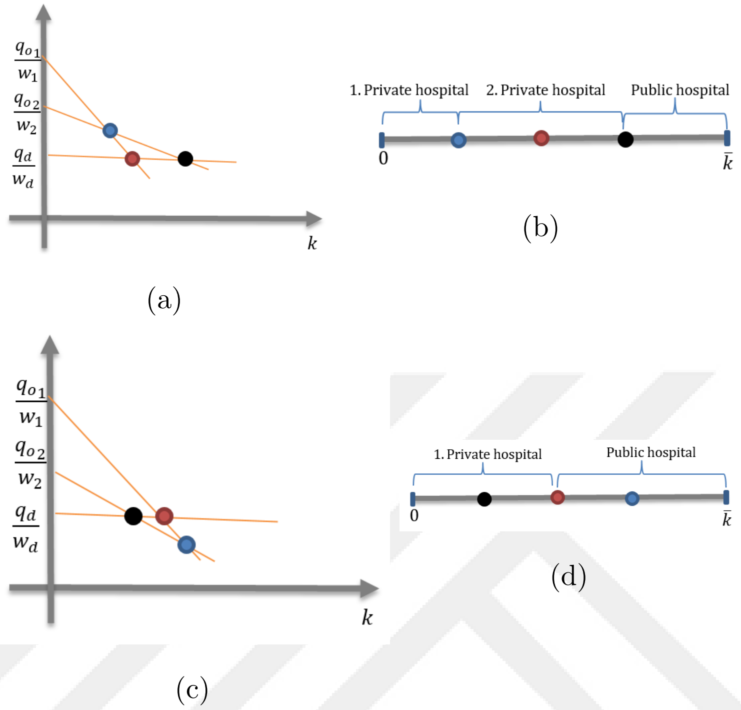


Figure 6.2. *We suppose that the intersection of* $\frac{q_{o_i}}{w_{o_i}}$ *according to* k *is according (a)*

So, p_{o_1} is as in Equation 6.3.

$$p_{o_1} = F_k\left(\frac{q_{o_1}(m_{o_1}\mu_{o_1} - \lambda p_{o_1}) - q_{o_2}(m_{o_2}\mu_{o_2} - \lambda p_{o_2})}{b_{o_1}^2 - b_{o_2}^2}\right) \tag{6.3}$$

In a similar way, p_{o_2} is calculated as in Equation 6.4.

$$\begin{aligned} p_{o_2} &= F_k\left(\frac{q_{o_2}(m_{o_2}\mu_{o_2} - \lambda p_{o_2}) - q_d(m_d\mu_d - \lambda p_d)}{b_{o_2}^2 - b_d^2}\right) \\ &F_k\left(\frac{q_{o_1}(m_{o_1}\mu_{o_1} - \lambda p_{o_1}) - q_{o_2}(m_{o_2}\mu_{o_2} - \lambda p_{o_2})}{b_{o_1}^2 - b_{o_2}^2}\right) \end{aligned} \tag{6.4}$$

U_1, the total utility received by all patients is calculated as in Equation 6.5.

$$U_1 = \int_0^{A_1} \left(\frac{q_{o_1}}{w_{o_1}} - kb_{o_1}^2\right) f(k)dk + \int_{A_1}^{A_2} \left(\frac{q_{o_2}}{w_{o_2}} - kb_{o_2}^2\right) f(k)dk + \int_{A_2}^{\bar{k}} \left(\frac{q_d}{w_d} - kb_d^2\right) f(k)dk \tag{6.5}$$

Since we supposed k is uniformly distributed between 0 and $\bar{k}$. As seen in **Figure 6.3.**, A_1 and A_2 are the critical values of k for the patients to select the hospitals, which are calculated as $A_1 = \bar{k}p_{o_1}$ and $A_2 = \bar{k}(p_{o_1} + p_{o_2}) = \bar{k}p_o$.

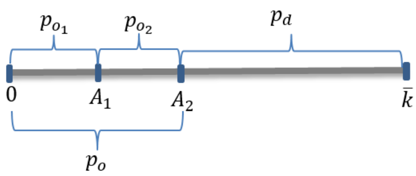


Figure 6.3. *A_1 and A_2 are the critical values of k for the patients to select the hospitals.*

The social utility, which is defined as in Equation 4.15 consists of U_1 and U_2, defined in Equations 4.14, which is the average public expenditure.

The profit functions of the private hospitals are as in Equations 6.6 and 6.7.

$$Z_{o_1} = (r_1 - c_{o_1})p_{o_1}\lambda - k_{o_1}m_{o_1}^2 \tag{6.6}$$

$$Z_{o_2} = (r_2 - c_{o_2})p_{o_2}\lambda - k_{o_2}m_{o_2}^2 \tag{6.7}$$

H_d is as in Equation 6.8.

$$H_d = \lambda p_d(c_d - b_d) + \lambda(p_{o_1}s_{o_1} + p_{o_2}s_{o_2}) + k_d m_d^2 \tag{6.8}$$

6.2. Model NC: No Contract Between the Government and the Private Hospital

In this model, which is summarized in **Figure 6.4.**, each private hospital determines its own examination fees to maximize its profit. The examination fee, the average waiting time and the perceived quality level are r_i, w_{o_i} and s_{o_i} in the i-th private hospital, while in the public hospital they are b_d, w_d and s_d. Thus, the utility of the patients is $\frac{q_{o_i}}{w_{o_i}} - kr_i^2$ in the i-th private hospital and $\frac{q_d}{w_d} - kb_d^2$ in the public hospital. As it is described in Equation 6.9, a strategic patient selects the first private hospitals if she earns more utility there.

$$\frac{q_{o_1}}{w_{o_1}} - kr_1^2 \geq \frac{q_d}{w_d} - kb_d^2 \quad and \quad \frac{q_{o_1}}{w_{o_1}} - kr_1^2 \geq \frac{q_{o_2}}{w_{o_2}} - kr_2^2 \tag{6.9}$$

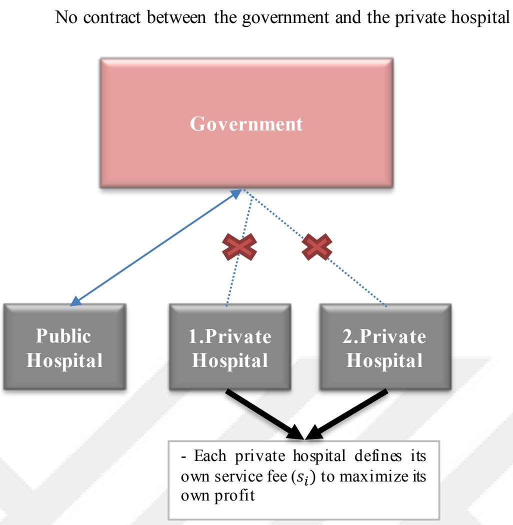


Figure 6.4. *Model NC*

So, the patient chooses a private hospital, if the value of k is as in Equation 6.10.

$$k \leq \frac{\frac{q_{o_1}}{w_{o_1}} - \frac{q_d}{w_d}}{r_1^2 - b_d^2} \quad and \quad k \leq \frac{\frac{q_{o_1}}{w_{o_1}} - \frac{q_{o_2}}{w_{o_2}}}{r_1^2 - r_2^2} \tag{6.10}$$

p_{o_1} is as in Equation 6.11, where $F_k(x)$ is the cumulative probability function of k.

$$p_{o_1} = F_k\left(\frac{q_{o_1}(m_{o_1}\mu_{o_1} - \lambda p_{o_1}) - q_{o_2}(m_{o_2}\mu_{o_2} - \lambda p_{o_2})}{r_1^2 - r_2^2}\right) \tag{6.11}$$

In a similar way, p_{o_2} is written as in Equation 6.12.

$$\begin{aligned} p_{o_2} = F_k\left(\frac{q_{o_2}(m_{o_2}\mu_{o_2} - \lambda p_{o_2}) - q_d(m_d\mu_d - \lambda p_d)}{r_2^2 - b_d^2}\right) \\ F_k\left(\frac{q_{o_1}(m_{o_1}\mu_{o_1} - \lambda p_{o_1}) - q_{o_2}(m_{o_2}\mu_{o_2} - \lambda p_{o_2})}{r_1^2 - r_2^2}\right) \end{aligned} \tag{6.12}$$

We assume that k is uniformly distributed between 0 and $\bar{k}$.

It is clear that the probability of choosing the public hospital by a patient is $p_d = 1 - p_{o_1} - p_{o_2}$.

$$H_d = \lambda p_d(c_d - b_d) + k_d m_d^2 \tag{6.13}$$

In this model, U_1 is defined in Equation 6.14.

$$U_1 = \int_0^{A_1} (\frac{q_{o_1}}{w_{o_1}} - kr_1^2) f(k)dk + \int_{A_1}^{A_2} (\frac{q_{o_2}}{w_{o_2}} - kr_2^2) f(k)dk + \int_{A_2}^{\bar{k}} (\frac{q_d}{w_d} - kb_d^2) f(k)dk \tag{6.14}$$

Also, U_2 and U are as in Equations 4.14 and 4.15.

In this model, private hospitals attempt to maximize their profits by defining appropriate examination fees. So this model actually consists of two problems; in the first problem, the profit of the first hospital and in the second one the profit of the second hospital are to be maximized. The problem for the first hospital is defined as in Equation 6.15.

$$Max_{r_1} \;\; Z_{o_{1_{nc}}} = \lambda p_{o_1}(r_1 - c_{o_1}) - k_{o_1} m_{o_1}^2 \tag{6.15}$$

The problem of the second hospital is defined in Equation 6.16.

$$Max_{r_2} \;\; Z_{o_{2_{nc}}} = \lambda p_{o_2}(r_2 - c_{o_2}) - k_{o_2} m_{o_2}^2 \tag{6.16}$$

The constraints of the model are defined in Equations 6.17 to 6.26.

$$p_{o_1} \leq 1 \tag{6.17}$$

$$p_{o_1} \geq 0 \tag{6.18}$$

$$\lambda p_{o_1} \leq m_{o_1}\mu_{o_1} \tag{6.19}$$

$$p_{o_2} \leq 1 \tag{6.20}$$

$$p_{o_2} \geq 0 \tag{6.21}$$

$$\lambda p_{o_2} \leq m_{o_2}\mu_{o_2} \tag{6.22}$$

$$\lambda p_d \leq m_d \mu_d \quad (6.23)$$

$$r_1, r_2 \geq 0 \quad (6.24)$$

$$r_1 \geq r_2 \quad (6.25)$$

$$\frac{q_{o_1}}{w_{o_1}} \geq \frac{q_{o_2}}{w_{o_2}} \quad (6.26)$$

As seen in **Figure 6.5.**, the service fees in the private hospitals begin from their minimum points, which are the cost of care for each patient and when one of the private hospitals grows the fee, the other one also raises its own fee. The equilibrium occurs at the point where the fees intersect after they become stable.

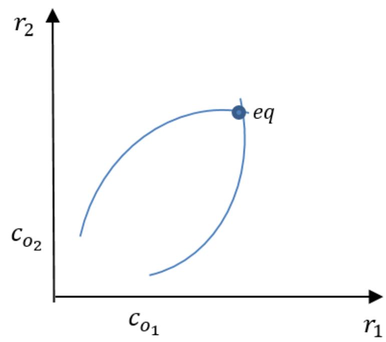


Figure 6.5. *Equilibrium point of* r_1 *and* r_2.

Solution method

The designed solution method is summarized in **Figure 6.6.**.

Initialize Best r_2 and r_2
while $Best\ r_2 \neq r_2$ **do**
    $r_2 \leftarrow$ Best r_2

    Search on r_1
    Find Best r_1 that maximizes $Z_{o_1}(r_1, r_2)$

    Search on r_2
    Find Best r_2 that maximizes $Z_{o_2}(Best\ r_1, r_2)$
end
Calculate $Z_{o_1}(Best\ r_1, Best\ r_2)$;
Calculate $Z_{o_2}(Best\ r_1, Best\ r_2)$;

Figure 6.6. *Solution method for Model NC*

6.3. Model SC: Contract Mechanism Based on Subsidy Payments

In this model, which is summarized in **Figure 6.7.**, the private hospitals decide to accept or reject the contract proposed by the government according to their own profits. The contract includes the price of examination and a subsidy. In this model, the choice of hospital for a strategic patient is made based on Equations 6.1, 6.2, 6.3 and 6.4. Therefore, H_d, U_1, U_2, U, Z_{o1} and Z_{o2} are defined as in Equations 6.8, 6.5, 4.14, 4.15, 6.6 and 6.7. In this model, first of all, the profits of the private hospitals are calculated for the case that none of them accepts the contract, which is indicated as Reject- Reject (RR) in **Table 6.1.**. Then, the profits of the private hospitals are calculated according to the price and subsidy proposed by the government. Each private hospital compares this profit with RR state; if it makes more profit, it accepts the contract. Otherwise, by rejecting the contract, it defines its price.

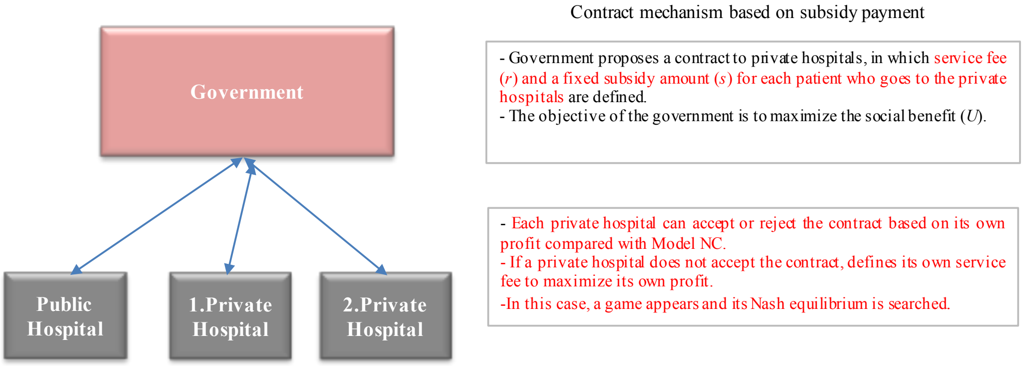


Figure 6.7. *Model SC*

Table 6.1. *Profit of private hospitals and social utility in different regions*

		Second private hospital	
		Reject	Accept
First private hospital	Reject	$Z_{o1_{RR}}, Z_{o2_{RR}}, U_{RR}$	$Z_{o1_{RA}}, Z_{o2_{RA}}, U_{RA}$
	Accept	$Z_{o1_{AR}}, Z_{o2_{AR}}, U_{AR}$	$Z_{o1_{AA}}, Z_{o2_{AA}}, U_{AA}$

In the algorithm written for this model by using the MATLAB software, as described previously, the profit that the two private hospitals obtain when they reject the contract and define their own price is calculated as $Z_{o1_{RR}}$ and $Z_{o2_{RR}}$. Then, the profit of each hospital is calculated for the cases of AA, RA and AR. Then the Nash equilibrium point is searched over the situations summarized below.

Reject-Reject (RR) state: if $Z_{o1_{AA}} > Z_{o1_{RA}}$ and $Z_{o2_{AA}} > Z_{o2_{AR}}$ then both hospitals reject the contract.

Accept-Accept (AA) state: if $Z_{o1_{RR}} > Z_{o1_{AR}}$ and $Z_{o2_{RR}} > Z_{o2_{RA}}$ then both hospitals accept the contract.

Reject-Accept (RA) state: if $Z_{o1_{RA}} > Z_{o1_{AA}}$ and $Z_{o2_{RA}} > Z_{o2_{RR}}$ then the first hospital rejects and the second one accepts the contract.

Accept-Reject (AR) state: $Z_{o1_{AR}} > Z_{o1_{RR}}$ and $Z_{o2_{AR}} > Z_{o2_{AA}}$ then the second hospital rejects and the first one accepts the contract.

After determining the Nash equilibrium point, the relevant social utility is also calculated.

Solution method

The solution method is summarized in **Figure 6.8.**.

Search on r and s, and Calculate U and Z_{o_i} in states of RA, AR, AA as:
foreach r *and* s **do**
  RA (1. private hospital rejects, 2. private hospital accepts)
  Find Best r_1 that maximizes $Z_{o_1}(r_1, r, s)$
  Calculate $Z_{o_{1_{RA}}}(Best\ r_1, r, s)$
  Calculate $Z_{o_{2_{RA}}}(Best\ r_1, r, s)$
  Calculate $U_{RA}(Best\ r_1, r, s)$
  AR (1. private hospital accepts, 2. private hospital rejects)
  Find Best r_2 that maximizes $Z_{o_2}(r, s, r_2)$
  Calculate $Z_{o_{2_{AR}}}(Best\ r, s, r_2)$
  Calculate $Z_{o_{1_{AR}}}(Best\ r, s, r_2)$
  Calculate $U_{AR}(Best\ r, s, r_2)$
  AA (both private hospital accept)
  Calculate $Z_{o_{2_{AA}}}(Best\ r, s)$
  Calculate $Z_{o_{1_{AA}}}(Best\ r, s)$
  Calculate $U_{AA}(Best\ r, s)$
  if $Z_{o_{1_{AA}}} > Z_{o_{1_{RA}}}$ *and* $Z_{o_{2_{AA}}} > Z_{o_{2_{AR}}}$ **then**
    Equilibrium point is AA;
    Calculate U_{AA};
  else if $Z_{o_{1_{NC}}} > Z_{o_{1_{AR}}}$ *and* $Z_{o_{2_{NC}}} > Z_{o_{2_{RA}}}$ **then**
    Equilibrium point is RR;
    Calculate U_{RR};
  else if $Z_{o_{1_{RA}}} > Z_{o_{1_{AA}}}$ *and* $Z_{o_{2_{RA}}} > Z_{o_{2_{NC}}}$ **then**
    Equilibrium point is RA;
    Calculate U_{RA};
  else
    $Z_{o_{1_{AR}}} > Z_{o_{1_{NC}}}$ and $Z_{o_{2_{AR}}} > Z_{o_{2_{AA}}}$
    Equilibrium point is AR;
    Calculate U_{AR};
  end
end
Find Best the U, the corresponding fees and related equilibrium point;
Calculate Z_{o_1} and Z_{o_2};

Figure 6.8. *Solution method for Model SC*

6.4. Model NC-SC: The Government Recommends a Contract to Only One of the Private Hospitals

In this model, it is assumed that the government offers a contract to only one of the private hospitals, which may be refused or accepted based on the profit of the hospital. If no contract is recommended to a hospital or when it rejects the contract, it defines its own service fee.

6.4.1. Model NC-SC-1: The government recommends a contract only to the second private hospital

In this model, which is summarized in **Figure 6.9.**, the contract is not recommended to the first private hospital, so this hospital determines the amount of r_1 to maximize its profit. A strategic patient chooses the first private hospital if Equation 6.27 is valid.

$$\frac{q_{o_1}}{w_{o_1}} - kb_{o_2}^2 \geq \frac{q_d}{w_d} - kb_d^2 \quad and \quad \frac{q_{o_1}}{w_{o_1}} - kr_1^2 \geq \frac{q_{o_2}}{w_{o_2}} - kb_{o_2}^2 \tag{6.27}$$

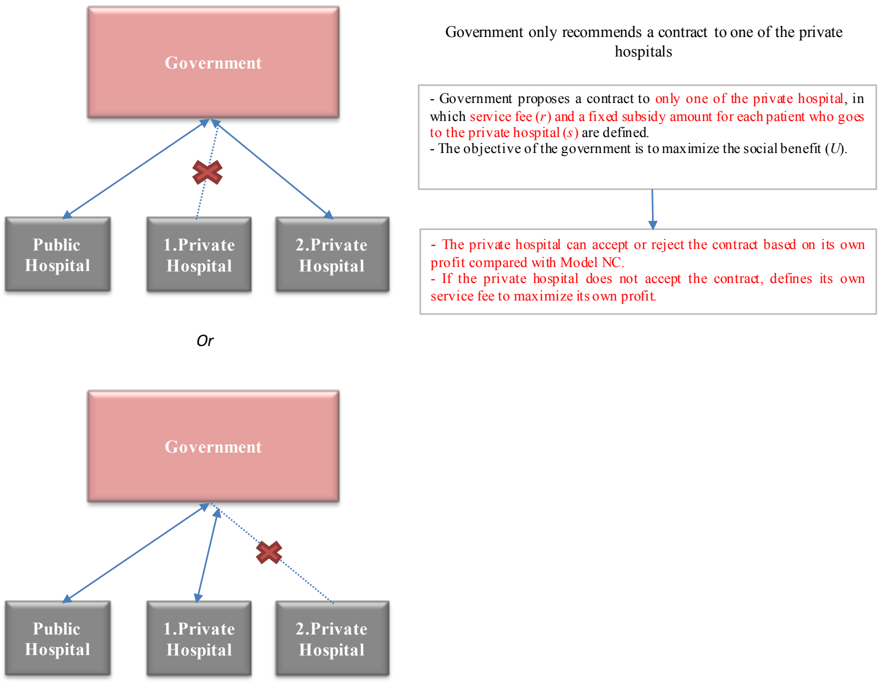


Figure 6.9. *Model NC-SC*

$$k \leq \frac{\frac{q_{o_1}}{w_{o_1}} - \frac{q_d}{w_d}}{r_1^2 - b_d^2} \quad and \quad k \leq \frac{\frac{q_{o_1}}{w_{o_1}} - \frac{q_{o_2}}{w_{o_2}}}{r_1^2 - b_{o_2}^2} \tag{6.28}$$

p_{o_1} is written as in Equation 6.30.

$$p_{o_1} = F_k(\frac{q_{o_1}(m_{o_1}\mu_{o_1} - \lambda p_{o_1}) - q_{o_2}(m_{o_2}\mu_{o_2} - \lambda p_{o_2})}{r_1^2 - b_{o_2}^2}) \tag{6.29}$$

$$\begin{aligned} p_{o_2} = F_k(\frac{q_{o_2}(m_{o_2}\mu_{o_2} - \lambda p_{o_2}) - q_d(m_d\mu_d - \lambda p_d)}{b_{o_2}^2 - b_d^2}) \\ F_k(\frac{q_{o_1}(m_{o_1}\mu_{o_1} - \lambda p_{o_1}) - q_{o_2}(m_{o_2}\mu_{o_2} - \lambda p_{o_2})}{r_1^2 - b_{o_2}^2}) \end{aligned} \tag{6.30}$$

In this model, H_d is defined as in Equation 6.31.

$$H_d = \lambda p_d(c_d - b_d) + k_d m_d^2 \tag{6.31}$$

In this model, U_1 is as in Equation 6.32.

$$U_1 = \int_0^{A_1} (\frac{q_{o_1}}{w_{o_1}} - kr_1^2)f(k)dk + \int_{A_1}^{A_2} (\frac{q_{o_2}}{w_{o_2}} - kb_{o_2}^2)f(k)dk + \int_{A_2}^{\bar{k}} (\frac{q_d}{w_d} - kb_d^2)f(k)dk \tag{6.32}$$

The objective function is as in Equation 6.33, because the government attempts to maximize the social utility.

$$Max_{r_2,s_2} \ U \tag{6.33}$$

In addition, the first private hospital tries to maximize its profit.

$$Max_{r_1} \ Z_{o_{1_{ncc}}} = p_{o_1}\lambda(r_1 - c_{o_1}) - k_{o_1}m_{o_1}^2 \tag{6.34}$$

The constraints defined in Model 6.3. are valid in this model.

6.4.2. Model NC-SC-2: The government only recommends a contract to the first private hospital

This model is similar to Model NC-SC-1, but the government hereby offers a contract only to the first private hospital.

Solution method

The solution method is summarized in **Figure 6.10.**.

Search on r and s as:
foreach r *and* s **do**
  Find Best r_1 that maximizes $Z_{o_1}(r_1, r, s)$
  Calculate $Z_{o_1}(Best\ r_1, r, s)$
  Calculate $Z_{o_2}(Best\ r_1, r, s)$
  if $Z_{o_2} > Z_{o2_{NC}}$ **then**
    The second private hospital accepts the contract;
    Calculate $U(Best\ r_1, r, s)$
  end
end
Find Best (r, s) that maximizes U;
Calculate Z_{o_1} and Z_{o_2};
if $Best\ U \not> U_{NC}$ **then**
  The government does not propose the contract;
end

Figure 6.10. *Solution method for Model NC-SC*

6.5. Experimental Results Based on a Case Study from Turkey

In this section, the numerical results of the models are presented. All the parameters used in our base case study are summarized in **Table 6.2.**.

Table 6.2. *Values of base case parameters*

$\lambda = 70,000$	$\alpha_1 = 0.01$	$\alpha_2 = 1$	$b_d = 5$	$\bar{k} = 1$
$c_{o_1} = 20, c_{o_2} = 15, c_d = 10$	$k_{o_1} = 50,000, k_{o_2} = 40,000, k_d = 10,000$	$q_{o_1} = 0.7, q_{o_2} = 0.6, c_d = 0.5$	$t_{o_1} = 8, t_{o_2} = 8, t_d = 3$	$m_{o_1} = 3, m_{o_2} = 3, m_d = 4$

The obtained results are presented in **Table 6.3.**.

Table 6.3.*Results obtained by the models with the base case parameters*

Model	Hospital	r	b_o	p_o	w_o	Z	w_d	H_d	U_1	U_2	U
Model NC	First private hospital	230	0	0.081	0.07	739281.16	5.69	447367.13	516.82	6.39	-1.22
	Second private hospital	176	0	0.098	0.08	744996.25					
Model SC-RR region	First private hospital	230	0	0.081	0.07	739281.16	5.69	447367.13	516.82	6.39	-1.22
	Second private hospital	176	0	0.098	0.08	744996.25					
Model SC-RA region	First private hospital	**272**	**0**	**0.070**	**0.06**	**784625.27**	**0.22**	**799935.97**	**1979.49**	**11.43**	**8.37**
	Second private hospital	**117**	**34**	**0.155**	**0.13**	**745935.01**					
Model SC-AR region	First private hospital	117	34	0.141	0.11	508820.00	0.19	765520.00	2029.50	10.94	9.36
	Second private hospital	134	0	0.089	0.07	381080.00					
Model SC-AA region	First private hospital	117	34	0.137	0.11	480970.00	0.13	1034700.00	2999.70	14.78	15.21
	Second private hospital	117	34	0.121	0.09	506710.00					
Model NC-SC-1	First private hospital	272	0	0.070	0.06	784625.27	0.22	799935.97	1979.49	11.43	8.37
	Second private hospital	117	34	0.155	0.13	745935.01					
Model NC-SC-2	First private hospital	230	0	0.081	0.07	739281.16	5.69	447367.13	516.82	6.39	-1.22
	Second private hospital	176	0	0.098	0.08	744996.25					

Using the parameters summarize in **Table 6.2.**, the obtained results are presented in **Table 6.3.**. In Model NC, where there is no contract between the government and the private hospitals, the examination prices in these hospitals are defined as 230 TL and 176 TL respectively. Compared to Model NC, the social utility increases by 786% by Model SC, in which the second private hospital made a discount of approximately 33%, while the payment of patients decreases by 47% in the second hospital and as a result the number of patients who select the second private hospital increases by about 6%. With decreasing the average waiting time in the public hospital, U_1 improves 383%. Although the expenditure of the government increases, but Model SC provides beneficial results for society. Even the profits of both private hospitals increase.

It is possible to raise both of the social utility and the profits of private hospitals applying suitable contract mechanisms.

6.5.1. Sensitivity Analysis

The results of our sensitivity analysis are provided in **Table 6.4.**.

Table 6.4. *Results obtained by the models according to different parameters*

	Model NC	Model SC		Model NC-SC-1	Model NC-SC-2
	U	U	Equilibrium Region	U	U
$\lambda = 75000$	-1.44	-0.79	RA	-0.79	-1.44
$\lambda = 65000$	13.18	21.12	RA	21.12	13.18
$c_{o_1} = 25$	-1.64	7.97	RA	7.97	-1.64
$c_{o_2} = 20$	-1.58	8.12	RA	8.12	-1.58
$c_{o_2} = 10$	-0.87	8.63	RA	8.63	-0.87
$c_d = 15$	-5.33	4.49	RA	4.49	-5.33
$c_d = 5$	2.88	12.24	RA	12.24	2.88
$k_d = 15000$	-2.37	7.22	RA	7.20	-2.37
$k_d = 5000$	-0.08	9.51	RA	9.51	-0.08
$q_{o_1} = 0.8$	0.01	9.72	RA	9.72	0.01
$q_{o_1} = 0.6$	-2.07	7.32	AA	6.75	-2.07
$q_{o_2} = 0.7$	-1.34	8.97	AA	8.67	-1.34
$q_{o_2} = 0.5$	-1.38	7.85	RA	7.85	-1.38
$q_d = 0.6$	-1.65	9.50	RA	9.50	-1.65
$t_{o_1} = 9$	-2.05	5.31	RA	5.31	-2.05
$t_{o_1} = 7$	3.05	12.01	RA	12.01	3.05
$t_{o_2} = 9$	-2.21	5.45	RA	5.45	-2.21
$t_{o_2} = 7$	0.74	11.93	RA	11.93	0.74
$t_d = 4$	-2.50	-3.40	RA	-3.40	-2.50
$m_{o_1} = 3.5$	3.85	12.62	RA	12.62	3.85
$m_{o_1} = 2.5$	-2.20	3.96	RA	3.96	-2.20
$m_{o_2} = 2.5$	-2.24	4.00	RA	4.00	-2.24
$m_d = 4.5$	18.90	26.44	RA	26.44	18.90
$m_d = 3.5$	-1.08	-1.08	RR	-1.08	-1.08
$b_{td} = 10$	2.56	4.58	RA	4.58	2.56
$b_{td} = 0$	-5.22	4.59	RA	4.59	-5.22
$k_1 = 5$	-26.69	10.68	RA	10.68	-26.69
$\alpha_1 = 0.02$	3.95	41.33	AA	30.70	3.95
$\alpha_1 = 0.005$	-3.81	-0.56	RA	-0.56	-1.90

If $\lambda = 65000$, in the equilibrium point, the social utility increases but the profit of private hospitals decreases. When $\lambda = 75000$, the social utility decreases. In this case, while the profit of the first private hospital increases, the profit of the second private hospital reduces. In both states of $\lambda = 65000$ and 75000, the equilibrium point for Model SC is in the RA area, i.e. the first hospital rejects the contract and the second accepts it. The same equilibrium point (RA) is also obtained with parameters $c_{o_1} = 25$, $c_{o_2} = 10$ and $m_{o_1} = 2.5$. However, if $m_d = 3.5$ and $q_{o_1} = 0.6$, the equilibrium point is in the RR and AA areas, respectively.

In general, the highest social utility with the parameters summarized in **Table 6.2.** is provided by Model SC, where all decisions are made by the government. The profit of both private hospitals in this model is higher than Model NC.

7. CONCLUSIONS

In this thesis, we analyze the healthcare systems in which public and private hospitals co-exist and the demand for the public hospitals is seen to be much higher than the capacities of these hospitals. Thus, the waiting times at the public hospitals are high, while the service quality and satisfaction levels are low. On the other hand, the private hospitals have ample capacity and they are underutilized since the demand is low for these hospitals, mainly due to their high price. We provide new contract mechanisms in order to improve the total utility level of the society, which can be applied between the government and the private hospital owners in similar systems. We develop simulation-based and mathematical models for these mechanisms and compare them. We show that how the government can improve the utility level of the society with a proper contract mechanism. Because in this case, it is possible to change prices, which changes the situation of the system. More patients prefer the private hospital if the fee of examination in the private hospital is less. Then, the patients are more satisfied because, in the private hospital, although the payments are higher than the public hospitals, the average waiting times of the patients are low and the quality of services they receive are high. On the other hand, when the number of patients in the private hospital increases, the average waiting times increase and accordingly, the satisfaction level decreases.

In Chapter 3, emergency services of public and private hospitals in a city are simulated with Rockwell Arena 14.5 software, in which service qualities, waiting times and payments are different from each other. We propose and evaluate new policies that will impact patient choices and thus system status. We first analyze the situation in which the resource capacities at the public hospitals are increased. In this case, even though the waiting times can be significantly decreased, it brings an additional cost to the government. More importantly, capacity increases are seen to be infeasible due to limited resources in the healthcare system, which cannot be increased in the short run. Secondly, since the patients' preferences are significantly related to the payments they need to make at the hospitals, we analyze different pricing policies, including single and differentiated pricing strategies. As the government provides more subsidies for private hospitals, which in turn decreases the payment requirements of the patients, more patients will prefer private hospitals and obtain higher quality service, improving their satisfaction. In addition, since fewer patients will go to public hospitals, the heavy load on the public hospitals will be decreased. However, government expenses are seen to significantly increase as the subsidy payment ratio increases, which is an important problem for the government. To overcome this problem, we propose differentiated pricing strategies which will provide similar patient preferences and waiting times without significantly increasing the government expenses. Finally, we propose a linear two-part tariff contract and observe that it also provides significant improvements in the system results. We also provide the optimal parameters for each of these scenarios based on a social utility definition composed of the multiple objectives of decreasing government payments and waiting times while increasing the obtained total service quality.

Note that this part of thesis is applied in the emergency departments of the hospitals and the applications in different departments of the hospitals or in different parts of the healthcare systems might lead to different results. We acknowledge that people may prefer the public system for long care treatments due to the smaller long-term cost while preferring the private system for emergency care, which is a one-time event. However, after suitable modifications, considering the adjusted patient preferences, we believe that the proposed settings can still be useful for outpatient clinics, long-term care units or similar health systems. We also note that outpatient services of the hospitals have similar structures as the emergency services and similar models can be proposed for outpatient clinics. We leave the analysis of such different healthcare units for further studies. We assume that the quality levels are fixed and independent of the number of patients going to those hospitals. However, as more patients move from public to private hospitals, these quality levels might also change. When some of the patients move from public to private hospitals, the quality levels of the private hospitals are expected to decrease while the quality levels of the public ones are expected to increase. As a result, we expect less number of patients to move from public to private hospitals compared to the results in this paper. Even though we expect to obtain slightly lower utilities, we believe that the main results of the paper will still hold. We think that such modifications can be analyzed in more detail in the future studies.

We propose price differentiation based on a segmentation of the population. However, perfect segmentation is never possible and cannibalization effects might decrease the potential benefits of the suggested mechanisms. Analysis of different segmentation mechanisms, a more detailed analysis on how to segment the population and how much discount to apply to each segment might be another extension of this work. We also note that the pricing mechanisms or the contracts offered by the government may not be accepted by some of the private hospitals if they don't want to handle the additional demand under the given price. When more patients move from public to private hospitals, they may want to increase their prices as well.

In Chapter 4, we develop pricing models for health services. We consider all the hospitals in a city as an associated system contains one public and one private hospital and investigate the effects of different contract mechanisms on the social utility. Based on this approach, the state in which there is no contract between the government and the private sector, policies based on the financial incentive by the government and payment according to the patients' income level models are analyzed. The results show that all contracting mechanisms have significant effects on the social utility and in all of the new mechanisms this value increases. The most social utility is obtained by applying the differentiated pricing according to patients' incomes. This mechanism, which has positive effects on patient satisfaction and public expenditures, creates a fairer system of payments. The contract based on the government financial incentive to the private sector is also useful because, in these policies, the reductions in the costs of the private hospital in addition to the payment of patients.

In Chapter 5, we develop models for capacity decisions and pricing in a health system that contains one public and one private hospital. The state in which, there is no contract between the government and the private hospital and the model, where after defining the price, subsidy and financial incentives in the private hospital as a contract by the government the private hospital defines its own capacity are discussed. We observe that the government can improve the utility level of society significantly at the expense of small increases in healthcare spending. When the government proposes a certain subsidy payment for each patient to the private hospital, the total utility level of the society can increase while the waiting times at the public hospitals decrease. Even under certain budget constraints that limit government expenditures, the healthcare system can improve significantly. In addition to subsidy payments per patient, when the government also employs a fixed fee payment mechanism, even better results can be obtained. We observe that better quality healthcare service can be obtained with much lower waiting times via new contract mechanisms that can balance the supply and demand by a better allocation of demand among the public and private hospitals. The results show that it is possible to obtain a better system in terms of the government, society and also the private sector if the decision variables in the system are optimally defined based on the appropriate contract mechanisms.

In Chapter 6, it is assumed that there are one public and two private hospitals in the system. The model in which, there is no contract mechanism between the government and the private hospitals, the state that the government offers a contract to only one private hospital and the model, where a contract is recommended to both of the private hospitals are analyzed. When a contract is offered to either of the private hospitals, they decide whether to accept it or not according to their profits. Therefore, in the case that a contract is offered to both private hospitals, a game occurs and its Nash equilibrium is searched. In general, we observe that if an appropriate contract mechanism is defined for a health system, the social utility increases.

[1] Adida, E., Mamani, H., & Nassiri, S. Bundled payment vs. fee-for-service: Impact of payment scheme on performance. Management Science, 63(5), 1606-1624, (2016).

[2] Afeche, P. Incentive-compatible revenue management in queueing systems: optimal strategic delay. Manufacturing & Service Operations Management, 15(3), 423-443, (2013).

[3] Ahmed, M. A. and T. M. Alkhamis. Simulation optimization for an emergency department healthcare unit in Kuwait. European Journal of Operational Research, 198(3), 936-942, (2009).

[4] Ahn, J. H., J. C. Hornberger. Involving patients in the cadaveric kidney transplant allocation process: A decision-theoretic perspective. Management Sci. 42(5) 629641, (1996).

[5] Aksin, O.Z., de Vericourt, F., Karaesmen, F. Call Center Outsourcing Contract Analysis and Choice, Management Science, 54(2), 354-368, (2008).

[6] Alagoz, O., A. J. Schaefer, M. S. Roberts. Optimizing organ allocation and acceptance. P. M. Padalos, H. E. Romeijn, eds. Handbook of Optimization in Medicine. Springer, New York, 121, (2009).

[7] Alagoz, O., L. M. Maillart, A. J. Schaefer, M. S. Roberts. Determining the acceptance of cadaveric livers using an implicit model of the waiting list. Oper. Res. 55(1) 2436, (2007).

[8] Alagoz, O., T. Ayer, F. S. Erenay. Operations research models for cancer screening. J. J. Cochran, L. A. Cox Jr., P. Keskinocak, J. P. Kharoufeh, J. C. Smith, eds. Wiley Encyclopedia of Operations Research and Management Science. John Wiley & Sons, Hoboken, NJ, (2011).

[9] Andritsos, D. A., & Tang, C. S. Incentive Programs for Reducing Readmissions when Patient Care is CoProduced. Production and Operations Management, 27(6), 999-1020, (2018).

[10] Andritsos, D. A., & Tang, C. S. Introducing competition in healthcare services: The role of private care and increased patient mobility. European Journal of Operational Research, 234(3), 898-909, (2014).

[11] Andritsos, D. A., & Aflaki, S. Competition and the operational performance of hospitals: The role of hospital objectives. Production and Operations Man-

agement, 24(11), 1812-1832, (2015).

[12] Ayer, T., O. Alagoz, N. K. Stout. A POMDP Approach to Personalize Mammography Screening Decisions, Operations Research, evrimii olarak yayndadr, (2013)

[13] Baesler, F. E., H. E. Jahnsen, and M. DaCosta. Emergency departments i: the use of simulation and design of experiments for estimating maximum capacity in an emergency room. In Proceedings of the 35th conference on Winter simulation: driving innovation, Winter Simulation Conference, pages 1903-1906, (2003).

[14] Batun S, Denton BT, Huschka TR, Schaefer AJ. Operating room pooling and parallel surgery processing under uncertainty. INFORMS J. Comput., 23, 220237, (2011).

[15] Blake JT, Carter MW. Surgical process scheduling: A structured review. J. Soc. Health Systems 5:1730, (1997).

[16] Borsenberger, C., H. Cremer, P. De Donder, and D. Joram. Differentiated pricing of delivery services in the e-commerce sector. In The Future of the Postal Sector in a Digital World, Springer, pages 191-211. (2016).

[17] Brailsford, Sally C. Advances and challenges in healthcare simulation modeling: tutorial. Proceedings of the 39th conference on Winter simulation. IEEE Press, (2007), 1436-1448.

[18] Cabrera, Eduardo and Taboada, Manel and Iglesias, Ma Luisa and Epelde, Francisco and Luque, Emilio. Optimization of healthcare emergency departments by agent-based simulation. 1880-1889, (2011).

[19] Cabrera, E., M. Taboada, M. L. Iglesias, F. Epelde, and E. Luque. Simulation optimization for healthcare emergency departments. Procedia Computer Science, 9, 1464-1473, (2012).

[20] Cachon G.P. Supply chain coordination with contracts. In: Graves S, de Kok T, editors. Handbooks in operations research and management science: supply chain management. Amsterdam: North-Holland, (2003).

[21] Cachon GP, Netessine S. Game theory in supply chain analysis. In: Simchi-Levi D, Wu SD, Shen ZM, editors. Supply chain analysis in the eBusiness Era; (2003).

[22] Cachon, G. P., Lariviere, M. A. Supply chain coordination with revenue-sharing

contracts:Strengths and limitations. Management Science 51 (1), 30-44, (2005).

[23] Carlson, R. C., J. C. Hershey, and D. H. Kropp. Use of optimization and simulation models to analyze outpatient health care settings. Decision Sciences, 10(3), 412-433, (1979).

[24] Chahal, Kirandeep and Eldabi, Tillal. Applicability of hybrid simulation to different modes of governance in UK healthcare. 2008 Winter Simulation Conference. IEEE, 1469-1477 (2008).

[25] Chick, S. E., Mamani, H., Simchi-Levi, D. Supply chain coordination and influenza vaccination. Operations Research 56 (6), 14931506, (2008).

[26] Duckett, S. J. Private care and public waiting. Australian Health Review, 29(1), 87-93, (2005).

[27] Duguay, C., & Chetouane, F. Modeling and improving emergency department systems using discrete event simulation. Simulation, 83(4), 311-320, (2007).

[28] Fournier, D.L., G.S. Zaric. Simulating neonatal intensive care capacity in British Columbia. Socio-Economic Planning Sciences, 47(2), 131-141, (2013).

[29] Gao, P., X. Wang, Y. Jing, and L. Deng. Differentiated pricing policy for new, remanufactured and refurbished products. Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 229(11), 2063-2075, (2015).

[30] Gunal, Murat M and Pidd, Michael. Discrete event simulation for performance modelling in health care: a review of the literature. Journal of Simulation, (2010), 42-51.

[31] Guo, P., Tang, C. S., Wang, Y., & Zhao, M. The impact of reimbursement policy on patient welfare, readmission rate and waiting time in a public healthcare system: Fee-for-service vs. UCLA Anderson School of Management Working Paper, (2016).

[32] Hoel, M., & Sther, E. M. Public health care with waiting time: the role of supplementary private health care. Journal of Health Economics, 22(4), 599-616, (2003).

[33] Jacobson, S.H., S.N. Hall, and J.R. Swisher. Discrete event simulation of health care systems. In Patient flow: Reducing delay in healthcare delivery, pages 211-252. Springer, (2006).

[34] Jiang, H., Pang, Z., & Savin, S. Performance-based contracts for outpatient medical services. Manufacturing & Service Operations Management, 14(4), 654-669, (2012).

[35] Jun, JB and Jacobson, Sheldon H and Swisher, JR. Application of discrete-event simulation in health care clinics: A survey. Journal of the operational research society, 109-123, (1999).

[36] Kaya, O. Outsourcing vs. in-house production: a comparison of supply chain contracts with effort dependent demand. Omega (39), 168-178, (2011).

[37] Kong, H. Y., Zhu, S. M., Wang, L. Q., He, Y., Xie, H. Y., & Zheng, S. S. (2010). Sevoflurane protects against acute kidney injury in a small-size liver transplantation model. American journal of nephrology, 32(4), 347-355.

[38] Kreis, J., H. Schmidt. Public Engagement in Health Technology Assessment and Coverage Decisions: A Study of Experiences in France, Germany, and the United Kingdom. Journal of Health Politics, Policy and Law, 38(1), 89-122, (2013).

[39] Kuljis, Jasna and Paul, Ray J and Stergioulas, Lampros K. Can health care benefit from modeling and simulation methods in the same way as business and manufacturing has? Proceedings of the 39th conference on Winter simulation. IEEE Press, (2007), 1449-1453.

[40] Lariviere MA. Supply chain contracting and coordination with stochastic demand. Boston: Kluwer Academic Publishers, (1999).

[41] Liu, Z., Rexachs, D., Epelde, F., & Luque, E. A simulation and optimization based method for calibrating agent-based emergency department models under data scarcity. Computers & Industrial Engineering, 103, 300-309, (2017).

[42] Marchand, M., & Schroyen, F. Can a mixed health care system be desirable on equity grounds? Scandinavian Journal of Economics, 107(1), 1-23, (2005).

[43] Mielczarek, B., and Uzialko-Mydlikowska, J. Application of computer simulation modeling in the health care sector: a survey. Simulation, 88(2), 197-216, (2012).

[44] Ozaltin, O. Y., Prokopyev, O. A., Schaefer, A. J., & Roberts, M. S. Optimizing the societal benefits of the annual influenza vaccine: A stochastic programming approach. Operations research, 59(5), 1131-1143, (2011).

[45] Palmer, G.R., S.D. Short. Health care and public policy: an Australian analysis. MacMillan Publishers Australia, (2000).

[46] Pasternack, B. Optimal pricing and returns policies for perishable commodities. Marketing Science 4 (2), 166176, (1985).

[47] Phillips, R.L. Pricing and revenue optimization. Stanford University Press, (2005).

[48] Qian, Q., Guo, P., & Lindsey, R. Comparison of subsidy schemes for reducing waiting times in healthcare systems. Production and Operations Management, 26(11), 2033-2049, (2017).

[49] Qian, Q., & Zhuang,W. Tax/subsidy and capacity decisions in a two-tier health system with welfare redistributive objective. European Journal of Operational Research, 260(1), 140-151, (2017).

[50] Qin, H. and V.R. Prybutok. A quantitative model for patient behavioral decisions in the urgent care industry. Socio-Economic Planning Sciences, 47(1), 50-64, (2013).

[51] Raza, S. A. Optimal fare price differentiation with demand leakage in airline industry. International Journal of Revenue Management, 8(2), 99-129, (2015).

[52] Smith, H., Currie, C., Chaiwuttisak, P., and Kyprianou, A. Patient choice modelling: how do patients choose their hospitals?. Health Care Management Science, 1-10, (2017).

[53] So, K. C., & Tang, C. S. (2000). Modeling the impact of an outcome-oriented reimbursement policy on clinic, patients, and pharmaceutical firms. Management Science, 46(7), 875-892.

[54] Stahl, J. E., Kong, N., Shechter, S. M., Schaefer, A. J., & Roberts, M. S. A methodological framework for optimally reorganizing liver transplant regions. Medical Decision Making, 25(1), 35-46, (2005).

[55] Swisher, James R and Jacobson, Sheldon H. Evaluating the design of a family practice healthcare clinic using discrete-event simulation. Health Care Management Science, (2002), 75-88.

[56] Talluri, K.T. and G. J. Van Ryzin. The theory and practice of revenue management. international series in operations research and management science, vol. 68, (2004).

[57] Taylor, T. A. Supply chain coordination under channel rebates with sales effort effects. Management Science 48 (8), 9921007, (2002).

[58] Tomlin, B. Capacity investments in supply chain: sharing-the-gain rather than sharing-the-pain. University of North Carolina working paper, (2000).

[59] Tsay AA, Nahmias S, Agrawal, N. Modeling supply chain contracts: a review. Quantitative Models for Supply Chain Management, 299336, (1999).

[60] Tsay, A. A. The quantity flexibility contract and supplier-customer incentives. Management Science 45 (10), 13391358, (1999).

[61] Uriarte, A. G., Zuniga, E. R., Moris, M. U., & Ng, A. H. How can decision makers be supported in the improvement of an emergency department? A simulation, optimization and data mining approach. Operations Research for Health Care, 15, 102-122, (2017).

[62] Weng, Shao-Jen and Cheng, Bing-Chuin and Kwong, Shu Ting and Wang, Lee-Min and Chang, Chun-Yueh. Simulation optimization for emergency department resources allocation. Proceedings of the 2011 winter simulation conference (WSC). Phoenix, AZ: IEEE, 1231-1238, (2011).

[63] Wolk, A. and C. Ebling. Multi-channel price differentiation: An empirical investigation of existence and causes. International Journal of Research in Marketing, 27(2), 142-150, (2010).

[64] Xu, X., R. Zhao, and L. Xu. The differentiated pricing strategy or hotel under dual-channel reservation forms. Open Journal of Social Sciences, 2(04), 352, (2014).

[65] Yano CA, Gilbert SM. Coordinated pricing and production/procurement decisions: a review. In: Chakravarty AK, Eliashberg J, editors. Managing business interfaces: marketing, engineering, and manufacturing perspectives. Boston, MA: Kluwer Academic Publishers, (2004).

[66] You, X., Kobayashi, Y. The new cooperative medical scheme in China. Health Policy, 91(1), 1-9, (2009).

[67] Zhou, W., Wan, Q., & Zhang, R. Q. Choosing among hospitals in the subsidized health insurance system of China: A sequential game approach. European Journal of Operational Research, 257(2), 568-585, (2017).

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Teymourifar, A., Bahadir, O., & Ozturk, G. (2018). Dynamic Priority Rule Selection for Solving Multi-objective Job Shop Scheduling Problems. Universal Journal of Industrial and Business Management, 6(1), 11-22.

Teymourifar, A., & Ozturk, G. (2018). New dispatching rules and due date assignment models for dynamic job shop scheduling problems. International Journal of Manufacturing Research, 13(4), 302-329.

Teymourifar, A., Ozturk, G., Ozturk, Z. K., & Bahadir, O. (2018). Extracting New Dispatching Rules for Multi-objective Dynamic Flexible Job Shop Scheduling with Limited Buffer Spaces. Cognitive Computation, 1-11.

Teymourifar, A., & Ozturk, G. (2018). A Neural Network-based Hybrid Method to Generate Feasible Neighbors for Flexible Job Shop Scheduling Problem. Universal Journal of Applied Mathematics, 6(1), 1-16.