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CAPITAL ASSET PRICING MODEL AND BANKING SECTOR APPLICATION IN ISTANBUL STOCK EXCHANGE MARKET (1999-2009)

Master Thesis

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ABSTRACT

Capital Asset Pricing Model and Banking Sector Application in Istanbul Stock Exchange Market

Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Model are two main asset pricing models that explain capital asset pricing and stock returns in financial theory.

The purpose of this study is to search the validity of CAPM, which is also known as the capital market equilibrium model, in our country.

In the application of the research daily closing prices of 11 banks stocks that are traded in Istanbul Stock Exchange Market (ISE) are evaluated over the 1999-2009 period.

There are two main reasons while choosing banking sector in order to test the validity of CAPM;

Banking sector in the ISE is subject to more controls and audits compared to other sectors. Therefore, assets chosen from the banking sector have less possibility of overpricing, under-pricing or manipulation risks over the prices. Thus, probability of reflecting the right prices of the assets is higher for the banking sector assets.

Secondly, changes in the interest rates are immediately reflected in banking sector, since their affairs based on interest rates banks are highly sensitive to interest rate changes. This sensitivity played a crucial role to consider banking sector assets. In the research performed by Copeland, Koller and Murrin (1994) it is observed that beta of banking sector is the most close to the market beta. Thereupon, it is thought that market representation likelihood of banking sector is high.

ÖZET

Finansal Varlık Fiyatlama Modeli ve İstanbul Menkul Kıymetler Borsası Bankacılık Sektörü Uygulaması

Finansal Varlık Fiyatlama Modeli ve Arbitraj Fiyatlama Modeli, finans teorisinde finansal varlıkların fiyatlanması ve bu varlıkların getirilerini açıklamaya yönelik temel fiyatlama modelleri olarak yer almaktadırlar.

Bu çalışmanın amacı sermaye pazarı denge modeli olarak da bilinen Finansal Varlık Fiyatlama Modeli'nin ülkemizde geçerliliğini araştırmaktır.

Araştırma gerçekleştirilirken ülkemizde bankacılık alanında faaliyette bulunan ve sermaye piyasası pazarı olarak bilinen İstanbul Menkul Kıymetler Borsası (IMKB)'nda hisselerinin alım satımı yapılan 11 adet banka hissesinin 1999–2009 dönemindeki günlük hisse kapanış fiyat verileri dikkate alınmıştır.

Finansal Varlık Fiyatlama Modeli'nin geçerliliği test edilirken bankacılık sektörünün tercih edilmesinin iki temel nedeni bulunmaktadır;

Bankacılık sektörü İMKB'de işlem gören diğer sektörlere nispeten daha fazla denetime ve kontrollere tabi olan bir sektör olarak yer almaktadır. Bu nedenle ilgili sektörden seçilen hisse fiyatları üzerinde olası yüksek veya düşük değerlemelerin ve manipülasyonların daha az olması beklenmektedir. Bu da fiyat verilerinin gerçeği yansıtma olasılığını arttırmaktadır.

Bankacılık sektörünün faaliyetlerinin faizlerde meydana gelecek değişiklikleri anında yansıtması, faiz değişikliklerine karşı duyarlılığın yüksek olması bankacılık sektörü verilerinin tercih edilmesinde önemli olmuştur. Copeland, Koller ve Murrin (1994) tarafından yapılan çalışmalarda da bankacılık sektörü betasının piyasa betasına en yakın olduğu gözlenmiştir. Bu gözlem üzerine bankacılık sektörünün piyasayı temsil etme olasılığının yüksek olduğu düşünülmektedir.

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LIST of ABBREVIATIONS

APT	:	Arbitrage Pricing Theory
B/M	:	Book/Market Ratio
CAPM	:	Capital Asset Pricing Model
CML	:	Capital Market Line
ISE	:	Istanbul Stock Exchange Market
IMKB	:	İstanbul Menkul Kıymetler Borsası
MPT	:	Modern Portfolio Theory
MRS	:	Marginal Rate of Substitution
NYSE	:	New York Stock Exchange
SML	:	Security Market Line
SCL	:	Security Characteristics Line

Stocks and Indices

AKBNK	:	Akbank T.A.Ş.
ALNTF	:	Alternatifbank A.Ş.
FINBN	:	Finansbank A.Ş.
GARAN	:	T.Garanti Bankasi A.Ş.
ISBTR	:	T.İş Bankasi A.Ş. - İş Bankası B
ISCTR	:	T.İş Bankasi A.Ş. - İş Bankası C
SKBNK	:	Şekerbank T.A.Ş.
TEKST	:	Tekstil Bankasi A.Ş.
TKBNK	:	T.Kalkinma Bankasi A.Ş.
TSKB	:	T.Sinai Kalkinma Bankasi A.Ş.
YKBNK	:	Yapi Ve Kredi Bankasi A.Ş.
XU100	:	ISE 100 Index
XU30	:	ISE 30 Index
XU50	:	ISE 50 Index
XBANK	:	ISE Banks Index
XUTUM	:	ISE All Index

PART 1 INTRODUCTION

The predictability of stock returns is a particular issue of debate in financial literature. It is a subject that attracts enormous focus of researchers because of its theoretical significance and practical implications. Predictability is associated with the possibility of generating an excess returns in using past information. A large number of studies in the finance literature have confirmed the evidence of the predictability of stock returns.

The bases for the improvement of asset pricing models were set by Markowitz (1952) and Tobin (1958). Early theories suggested that the risk of an individual security is the standard deviation of its returns as an assessment of return volatility. In consequence, the larger the standard deviations of a security return the greater the risk is. But, an investor's main concern is about the risk of all of his wealth; which, actually, is a portfolio composed of different securities. Markowitz surveyed that; when two risky assets are combined, their standard deviations are not additive, if the returns from the two assets are not perfectly positively correlated. On the other hand, when a portfolio of risky assets is formed, the standard deviation of the portfolio is less than the sum of standard deviations of its constituents if they are not positively perfectly correlated.

Markowitz established a specific measure of portfolio risk and to derive the expected return and risk of a portfolio. The Markowitz portfolio selection model generates an efficient frontier of portfolios and the investors are expected to select a portfolio from the frontier. That is, all investors behave rationally in their investment decisions and aim to maximize their utility by choosing the portfolio with the highest reward-to-risk ratio.

Later, Sharpe (1964) developed a computationally efficient method, the single index model CAPM, where return on an individual security is related to the return on a common index. According to Sharpe's theory, when analyzing the risk of an individual asset, the individual asset risk must be considered in relation to other assets in the portfolio. Especially, the risk of

an individual asset must be measured in terms of the extent to which it adds risk to the investor's portfolio. Consequently, an asset's contribution to portfolio risk is different from the risk of the individual asset. In other words; risk should not simply be defined as the volatility of a stock's return but as the stock's contribution to a well diversified portfolio's risk. The single index model can be extended to portfolios as well. This is achievable because the expected return on a portfolio is a weighted average of the expected returns on individual assets. This implies a portfolio's risk should be appraised as its involvement to a well diversified portfolio.

As it is known that investors demand a premium for carrying risk; that is, the higher the riskiness of an asset, the higher the expected return required to encourage investors to purchase that asset. However, if investors are primarily concerned with portfolio risk rather than the risk of the individual assets in the portfolio, Capital Asset Pricing Model provides an important measurement tool for the risk contribution of an individual asset to a portfolio. According to Brigham (1994), the primary conclusion of the CAPM is that; "The relevant riskiness of an individual stock is its contribution to the riskiness of a well- diversified portfolio."

Actually, CAPM gives an accurate estimation of the relationship that should be recognized between the risk of an asset and its expected return. This relationship supplies two important functions. First, it contributes a benchmark rate of return for assessing feasible investments. Second, the model facilitates to make a good prediction as to the expected return on assets that have not yet been traded in the marketplace such as pricing initial public offering stocks.

CAPM has attracted attention of academic environment as well as professionals since starting of its foundation. The theory has been experienced in many of the developed markets and concerned many critiques due to its over-simplifying assumptions. However, the predictability power of the model is important in investment decisions.

The aim of the study is to test the predictability power of CAPM for the Istanbul Stock Exchange. As a relatively immature market compared to developed markets, there exist only a few studies regarding CAPM application in Istanbul Stock Exchange (ISE). Hence, this study tries to fill in this gap by testing the predictability power of CAPM for Istanbul Stock Exchange.

As an outline of the paper, which including seven parts, main concepts about risk and the Markowitz's portfolio selection theory was discussed firstly after this introduction part. Consideration of Markowitz's theory has great importance since it is the fundamental assumption of CAPM. Then, CAPM theory and its formulation were explained theoretically including its assumptions and formulation. A detailed literature review in a separate section follows the theory and provides information about the testing methods performed to date as well as the critiques forwarded to the model. Finally, CAPM's prediction power for ISE was tested for a specific sample by re-performance of Sharpe's single index model. The last section contains the concluding explanations.

Daily, weekly and monthly stock returns during the 1999-2009 periods were used in order to evaluate the predictability power of CAPM. Therefore, the study may be regarded as an important practice which analyzes such a large number of observations for the CAPM testing in ISE.

As the testing methodology, a time-series regression analysis was carried out in order to predict beta value of each asset which measuring the risk contribution of the asset to the market.

PART 2 MAIN CONCEPTS OF RISK

2.1 Risk and Fundamental Types of Risk

Risk definitions inundate the literature. Identification of risk is likely to involve an aspect of subjectivity, depending upon the nature of the risk and to what it is applied. As such there is no all encompassing definition of risk. Chicken and Posner (1998) acknowledge this, and instead present their understanding of what a risk components:

$$Risk = Hazard \times Exposure$$

They describe hazard as "the way in which a thing or situation can cause harm," and exposure as "the extent to which the likely recipient of the harm can be influenced by the hazard". Harm is taken to imply injury, damage, loss of performance and finances, whilst exposure imbues the notions of frequency and probability.

Stenchion (1997) notes that risk might be defined simply as the probability of the occurrence of an undesired event but be better described as the probability of a hazard contributing to a potential disaster importantly; it involves consideration of vulnerability to the hazard.

Risk is the probability of a loss, and this depends on three elements, hazard, vulnerability and exposure. If any of these three elements in risk increases or decreases, then risk increases or decreases respectively. (Crichton, 1999)

Additionally, Sayers et al. (2002) defines the risk as a combination of the chance of a particular event, with the impact that the event would cause if it occurred. Risk therefore has two components – the chance (or probability) of an event occurring and the impact (or consequence) associated with that event. The consequence of an event may be either desirable or undesirable. In some, but not all cases; therefore a convenient single measure of the importance of a risk is given by:

$$\text{Risk} = \text{Probability} \times \text{Consequence}$$

Upon these definitions, it could be summarized that risk is most commonly conceived as reflecting variation in the distribution of possible outcomes, their likelihoods, and their subjective values. Risk is measured either by nonlinearities in the revealed utility for money or by the variance of the probability distribution of possible gains and losses associated with a particular alternative (Shapira and March, 1987).

On the other hand, in a financial context risk essentially refers the uncertainty associated with any investment. That is, risk is the possibility that the actual return on an investment will be different from its expected return. And risk usually is measured by calculating the standard deviation of the historical returns or average returns of a specific investment.

Traditionally, the risk of an alternative has primarily been associated with the dispersion of the corresponding random variable of monetary outcomes. Then it is common to measure the riskiness of an alternative by its variance σ^2 or its standard deviation σ. If an alternative's future value is characterized by a continuous random variable $\tilde{x}$ with density $f = f_{\tilde{x}}$, distribution $F = F_{\tilde{x}}$, and expectation

$$\mu = E(\tilde{x}) = \int_{-\infty}^{+\infty} x\, f(x)\, dx \qquad \text{Equation (2.1)}$$

these risk measures are defined by

$$\sigma^2 = Var(\tilde{x}) = \int_{-\infty}^{+\infty} (x - \mu)^2\, f(x)\, dx \qquad \text{Equation (2.2)}$$

and

$$\sigma = \left[\int_{-\infty}^{+\infty} (x - \mu)^2 f(x) dx \right]^{1/2}$$

Equation (2.3)

In the finance context the standard deviation of continuous growth rates usually is called volatility. Similar standardized risk measures are the expected absolute deviation around μ

$\int_{-\infty}^{+\infty} |x - \mu| f(x) dx$ and the expected absolute deviation around zero $\int_{-\infty}^{+\infty} |x| f(x) dx$ (Neftçi, 1996).

Markowitz (1952) proposed to measure the risk associated to the return of each investment by means of the deviation from the mean of the return distribution, the variance, and in the case of a combination (portfolio) of assets, to gauge the risk level via the covariance between all pairs of investments, for instance:

$$Cov[i, j] = E[i, j] - E[i]E[j]$$

Equation (2.4)

where i and j are random returns. The main innovation introduced by Markowitz is to measure the risk of a portfolio via the multivariate distribution of returns of all assets. Multivariate distributions are characterized by statistical properties of all component random variables and by their dependence structure. Markowitz expressed the former by the first two moments of the univariate distributions, the asset returns, and the latter through the linear correlation coefficient between each pair of random returns;

$$\rho(i, j) = Cov[i, j] / (\sigma_i^2 \sigma_j^2)^{1/2}$$

Equation (2.5)

where σ denotes the standard deviations of the assets – i and j- returns.

The measure of dispersion could be accepted as a measure of risk only if the relevant distribution is symmetric. The correlation coefficient is strictly related to the slope parameter

of a linear regression of the random variable "j" on the random variable "i", and it measures only on the co-dependence between the linear components of "i" and "j" while the coefficient allows to fully describe a multivariate distribution by taking into account the dependence structure among all pairs of components, Alexander (2001). Indeed,

$$\rho(i,j)^2 = \sigma_j^2 - \min E\left[(j-(ai+b))^2\right]/\sigma_j^2$$
Equation (2.5)

that is the relative variation of σ_j^2 by linear regression on "i". It could be proved that for all vectors "z" and random vectors "i", the variance of the linear combination $z^T i$, satisfies the relationship

$$\sigma^2(z^T i) = z^T Cov(i) z$$
Equation (2.6)

which is essential in Markowitz portfolio theory. The Markowitz model proceeds together with appropriate utility functions that allow a subjective preference ordering of assets and their combinations. In the case of non-normal distributions the utility functions must be quadratic. In practice this limitation restricts the use of this model to portfolios characterized by normal joint return distribution.

Multivariate normal distribution-based models are very appealing, because the association between any two random variables can be fully expressed by their marginal distributions and the linear correlation coefficient. Application and presentation of multivariate based models have been impeded by the lack of proper theoretical framework. And probabilistic models for univariate returns have bee explored and extended to the multivariate case under the assumption that all the combined returns and their dependence structure have the same probabilistic structure.

In the 1960s the concept of β (volatility) was introduced. Beta is a measure of a stock's volatility in relation to the market. By definition, the market has a beta of 1.0, and individual

stocks are ranked according to how much they deviate from the market. A stock that swings more than the market over time has a beta above 1.0. If a stock moves less than the market, the stock's beta is less than 1.0. High-beta stocks are supposed to be riskier but provide a potential for higher returns; low-beta stocks pose less risk but also lower returns. The complexity of the mean variance approach was considered too high and the β based portfolio method was the insufficient data to compute the variance – covariance matrix.

The measure of the linear dependence between the return of each asset and that of the market, β, led to the development of the main pricing models, Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Theory (APT).

CAPM decomposes a portfolio's risk into systematic and specific risk. Systematic risk is the risk of holding the market portfolio. As the market moves, each individual asset is more or less affected. To the extent that any asset participates in such general market moves, that asset entails systematic risk. Specific risk is the risk which is unique to an individual asset. It represents the component of an asset's return which is uncorrelated with general market moves. Let's take a look at the two basic types of risk and some of other risk examples: currency risk-exchange rate risk, inflation risk, principal risk, country risk, economic risk, liquidity risk, market risk, opportunity risk, income risk, interest rate risk, prepayment risk, counterparty risk, business risk, financial risk, default risk.

2.1.1 Systematic Risk

Systematic risk influences a large number of assets. A significant political event, for example, could affect several of the assets in your portfolio. It is virtually impossible to protect yourself against this type of risk, the risk inherent to the entire market or entire market segment. Also known as "un- diversifiable risk" or "market risk". Interest rates, recession and wars all represent sources of systematic risk because they affect the entire market and cannot be avoided through diversification. Whereas this type of risk affects a broad range of securities, unsystematic risk affects a very specific group of securities or an

individual security. Systematic risk can be mitigated only by being hedged. Even a portfolio of well-diversified assets may not escape all sorts of risk because it is also subject to systematic risk. (Kealhofer and Bohn, 2001)

2.1.2 Unsystematic Risk

Unsystematic risk is sometimes referred to as "specific risk". This kind of risk affects a very small number of assets. An example is news that affects a specific stock such as a sudden strike by employees. Diversification is the only way to protect investors from unsystematic risk. Company or industry specific risk that is inherent in each investment. The amount of unsystematic risk can be reduced through appropriate diversification. Also it is known as "specific risk", "diversifiable risk" or "residual risk". For example, news that is specific to a small number of stocks, such as a sudden strike by the employees of a company you have shares in, is considered to be unsystematic risk. (Sharpe and others, 1993)[1]

Now that we have determined the fundamental types of risk, let's look at more specific types of risk, particularly when we talk about stocks and bonds.

2.1.3 Credit or Default Risk

Credit risk is the risk that a company or individual will be unable to pay the contractual interest or principal on its debt obligations. This type of risk is of particular concern to investors who hold bonds in their portfolios. Government bonds, especially those issued by the federal government, have the least amount of default risk and the lowest returns, while corporate bonds tend to have the highest amount of default risk but also higher interest rates. Bonds with a lower chance of default are considered to be investment grade, while bonds with higher chances are considered to be junk bonds. Bond rating services, such as Moody's, allows investors to determine which bonds are investment-grade, and which bonds are junk.

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[1] Alexander, Gordon J. ve William F. Sharpe. Fundemantals of Investments. London: Prentice-Hall, 1993

2.1.4 Country Risk

Country risk refers to the risk that a country won't be able to honor its financial commitments. When a country defaults on its obligations, this can harm the performance of all other financial instruments in that country as well as other countries it has relations with. Country risk applies to stocks, bonds, mutual funds, options and futures that are issued within a particular country. This type of risk is most often seen in emerging markets or countries that have a severe deficit.

2.1.5 Foreign-Exchange Risk

When investing in foreign countries you must consider the fact that currency exchange rates can change the price of the asset as well. Foreign-exchange risk applies to all financial instruments that are in a currency other than your domestic currency.

2.1.6 Interest Rate Risk

Interest rate risk is the risk that an investment's value will change as a result of a change in interest rate.

2.1.7 Political Risk

Political risk represents the financial risk that a country's government will suddenly change its policies.

2.1.8 Market Risk

This is the most familiar of all risks. Also referred to as volatility, market risk is the the day-to-day fluctuations in a stock's price. Market risk applies mainly to stocks and options. As a whole, stocks tend to perform well during a bull market and poorly during a bear market - volatility is not so much a cause but an effect of certain market forces. Volatility is a measure of risk because it refers to the behavior, or "temperament", of your investment rather than the reason for this behavior. Because market movement is the reason why people can make money

from stocks, volatility is essential for returns, and the more unstable the investment the more chance there is that it will experience a dramatic change in either direction.

As it is seen, there are several types of risk that a smart investor should consider and pay careful attention to.
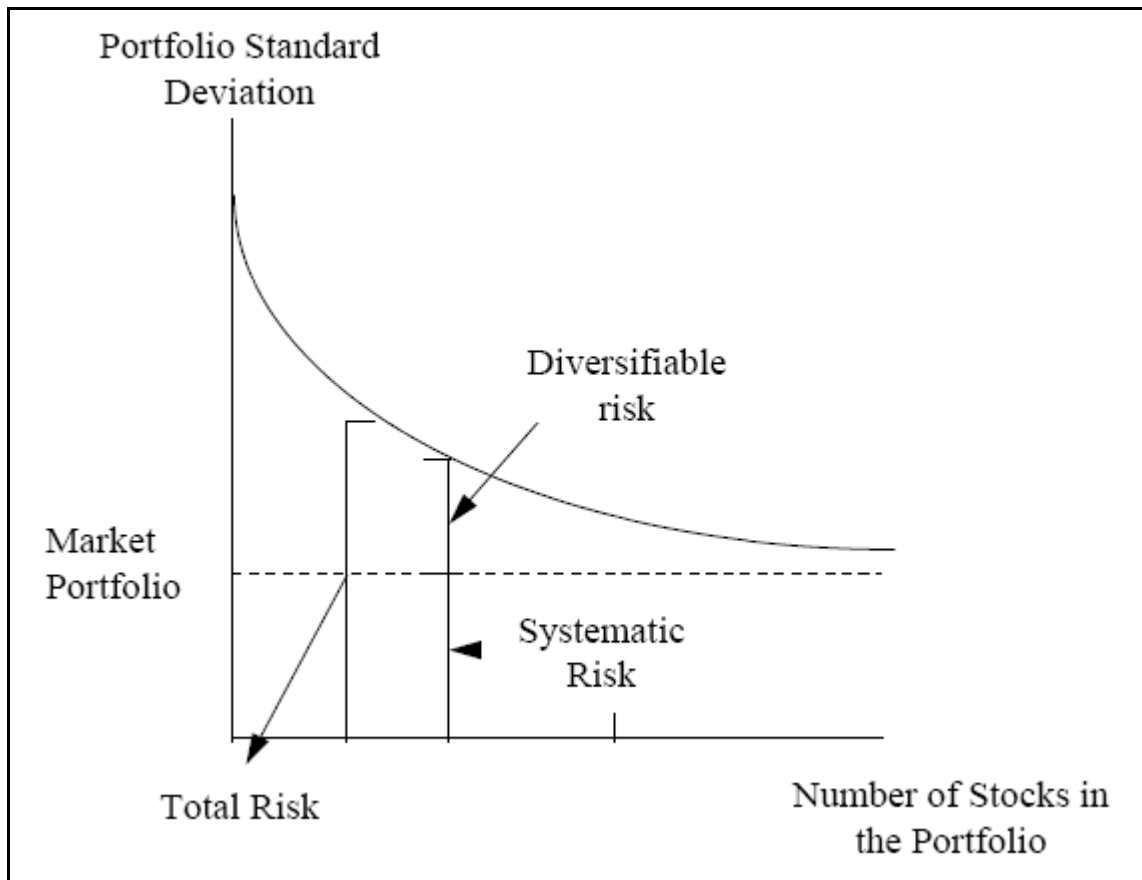
Figure 2.1 The standard deviation of portfolio return as a function of the number of securities in the portfolio. (Fama, E.F., *Foundation of Finance*, 1976, 253-254.)

The figure represents how a portfolio risk reduces with the increasing number of stocks in a portfolio. In fact, adding a new stock eliminates the firm specific risk of other stocks and thus reduces the diversifiable risk Fama (1976) has illustrated this above result empirically, as it is shown shortly graph above. He randomly selected 50 New York Stock Exchange (NYSE) listed securities and calculated their standard deviations using monthly data from July 1963 to June 1968. After a single security was chosen randomly, its standard deviation of return was

approximately 11%. Then this security combined with another randomly selected security to form an equally weighted portfolio of two securities. The result was decrease of standard deviation to around 7 %. Next, step by step more randomly selected securities were added to the portfolio until 50 securities included. All the diversifications was obtained after the first 10-15 securities selected, also the portfolio standard deviation quickly approached a limit that is almost equal to the average covariance of all securities.

On the other hand, adding new stocks do not help in reducing the portfolio risk after a certain number of stocks. The remaining risk, which cannot be eliminated, is the non-diversifiable risk, which mainly depends on the macro factors that affect all the firms whose stocks are included in the portfolio.

Risk is formulized by William Sharpe as;

$$\sigma^2(r_p) = \beta_p^{\ 2}\sigma^2(r_m) + \sigma^2(e_p)$$

Equation (2.7)

$\sigma^2(r_p) =$ Total Risk

$\beta_p^{\ 2}\sigma^2(r_m) =$ Systematic risk in other words non-diversifiable risk or market risk

$\sigma^2(e_p) =$ Unsystematic risk, diversifiable risk or specific risk

β_p = The beta coefficient, the relative volatility of a stock, fund, or other security in comparison with the market as a whole.

2.2 Measuring Risk for a Single Asset and Portfolio Risk Composed of Two Assets

Risks of a single asset and portfolio are measured differently. The total risk of a single asset is measured by the standard deviation of return on asset. Standard deviation is the square root of variance. In simple terms, standard deviation shows how much variation there is from the "average" (mean). It may be thought of as the average difference of the scores from the mean

of distribution, how far they are away from the mean. To measure variance, we must have some distribution/ possibility of asset returns or expected returns.

The variance of a random variable or distribution is the expected, or mean, value of the square of the deviation of that variable from its expected value or mean. It is calculated as follows:

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N-1}$$

Equation (2.8)

where, x_i is random variable μ is the expected value or the mean and N is the size or observation. And standard deviation is shown as below:

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N-1}}$$

Equation (2.9)

For a single asset that its probability distribution is given or known, risk or standard deviation is calculated as follow:

$$\sigma_i = \sqrt{\sum_1^n P_i [R_i - E(R_i)]^2}$$

Equation (2.10)

σ_i = standard deviation of asset (i)

P_i = probability

R_i = return

$E(R_i)$ = Expected return of asset (i) under given probabilities and it is formulized as;

$$E(R_i) = \sum P_i * R_i$$

Equation (2.11)

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On the other hand calculation of a portfolio risk is different from calculation of a single asset risk. To make an easier explanation, assume a portfolio composed of two assets; asset 1 (i) and asset 2 (j) and a proportion denoted by W_i is invested in the asset (i) and the rest 1- W_i denoted by Wj is invested in the asset (j). The rate of return on this portfolio, Rp, is calculated as follow:

$$R_p = W_i * R_i + W_j * R_j \qquad \text{Equation (2.12)}$$

In general terms, for a portfolio composed of "k" assets; expected returns for that portfolio could be expressed as;

$$E[R_p] = \Sigma W_k * E[R_k] \qquad \text{Equation (2.13)}$$

where $k = 1, 2, \ldots k$

And variance of two assets portfolio is;

$$\sigma_p^{\ 2} = W_i^{\ 2}\sigma_i^{\ 2} + W_j^{\ 2}\sigma_j^{\ 2} + 2W_iW_j Cov(R_i, R_i) \qquad \text{Equation (2.14)}$$

As it is shown formula above a new term "*Cov*"- *covariance* which is a measure of how much two variables change together. In other words the extent to which two variables vary together (co-vary) could be measured by their covariance.

Calculation of *covariance,* for example between asset (*i)* and asset (*j*) is shown below;

$$Cov_{i,j} = \Sigma P_k (R_i - E(R_i)) * (R_j - E(R_j)) \qquad \text{Equation (2.15)}$$

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where expected returns $= E(R_i) = \sum P_k * R_i$ and P_k is different probabilities of expected situations.

And also

$$Cov_{i,j} = \sigma_i * \sigma_j * \rho_{i,j}$$ Equation (2.16)

where $\rho_{i,j}$ is the correlation between assets or variables (i) and (j).

The value of the covariance, *correlation coefficient,* is interpreted as follows:

If the result of the covariance calculation is zero then the two variables are not related to one another. When the result is positive then the two variables have moved in the same direction. The larger the result the more strongly related the two variables are. If the result is negative then the two variables have a negative relationship with one another, that is are moving in opposite directions.

Thus,

If cov(i, j) is < 0, then i and j move in opposite direction

If cov(i, j) is > 0, then i and j move in same direction

If cov(i, j) is = 0, then i and j have no systematic co-movement
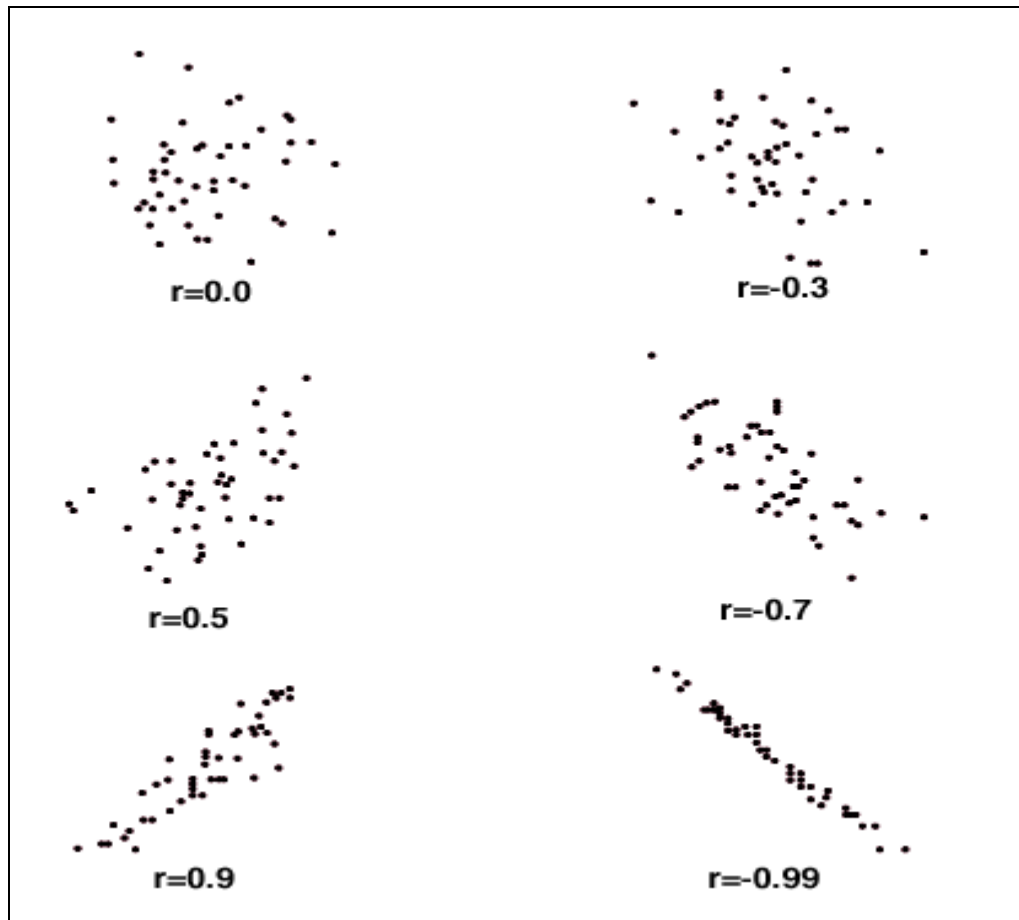
Figure 2.2 Scattered correlation graphs between two variables (http://www.gseis.ucla.edu/courses/ed230bc1/notes1/var1.html)

The more tightly the points are clustered together the higher the correlation between the two variables and the higher the ability to predict one variable from another -the symbol r generally is used to stand for the correlation-. Also it is known as, the Pearson correlation coefficient or just the correlation coefficient.

Correlation coefficients can take on any value between -1 and +1, with + and - 1 representing perfect correlations between the variables. And a correlation of zero represents no relationship between the variables.

A rule of thumb for interpreting correlation coefficients:

Correlation	Interpretation
0 to 0.1	trivial
0.1 to 0. 3	small
0.3 to 0.5	moderate
0.5 to 0.7	large
0.7 to 0.9	very large

Correlations are interpreted by squaring the value of the correlation coefficient. The squared value represents the proportion of variance of one variance that is shared with the other variable, in other words, the proportion of the variance of one variable that can be predicted from the other variable.

2.3 Measuring Risk for a Portfolio Composed of "n" Number of Assets

The beta coefficient is a relevant risk measure if the market model has had wide acceptance. Theoretical justification for this risk measure can be derived from the portfolio approach which makes the basic assumption that investors evaluate the risk of a portfolio as a whole rather than the risk of each asset individually. Also, the portfolio approach concludes that the risk of a portfolio should be measured by the covariability of its return of the market portfolio. Thus the market risk of an individual security, as it contributes to the portfolio covariability, is the appropriate measure of risk for that security since nonmarket related risk can be eliminated by the aggregation of securities in a portfolio (Klemkosky and Martin, 1975).

Risks in investment means that future returns are unpredictable. The spread of possible outcomes is usually measured by standard deviation. Most individual stocks have higher standard deviations, but much of their variability represents unique risks that can be eliminated by diversification. Diversification cannot eliminate market risk. Diversified portfolios are

exposed to variations in the general level of the market. A security's contribution to the risk of a well-diversified portfolio depends on how the security is reliable to be affected by a general market decline. This sensitivity to market movements is known as beta. Beta measures the amount that investors expect the stock price to change for each additional 1 percent change in the market.

Previously we have mentioned measure of risk of a portfolio composed of two assets, and here we will make our explanations for three assets in order to show variance and covariance of a portfolio then we will shortly explain the "beta" which represents the risk, formula of portfolio.

Suppose a three assets portfolio w_1, w_2, w_3 to be the percentages invested; $E(R_1)$, $E(R_2)$, $E(R_3)$ to be the expected returns; σ_1^2, σ_2^2, σ_3^2 to be the variances; σ_{12}, σ_{23}, σ_{13} to be the covariance and finally R_1, R_2, R_3 to be the random returns of a three assets portfolio respectively. The formulation of the portfolio mean return, as also mentioned previously, will be;

$$E(R_p) = E[w_1 R_1 + w_2 R_2 + w_3 R_3]$$ Equation (2.17)

or reversing the formula as

$$E(R_p) = w_1 E(R_1) + w_2 E(R_2) + w_3 E(R_3)$$ Equation (2.18)

The expected portfolio return is simply a weighted average of the expected return on individual assets, which can be written shortly;

$$E(R_p) = \sum_{i=1}^{3} w_i E(R_i)$$ Equation (2.19)

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The definition of portfolio variance for three assets is the expectation of the sum of the mean differences squared;

$$Var(R_p) = E\{[(w_1 R_1 + w_2 R_2 + w_3 R_3) - (w_1 E(R_1) + w_2 E(R_2) + w_3 E(R_3))]^2\}$$

$$= E\{[w_1(R_1 - E(R_1)) + w_2(R_2 - E(R_2)) + w_3(R_3 - E(R_3))]^2\}$$

$$= \left\{\begin{array}{l} E w_1^2(R_1 - E(R_1))^2 + w_2^2(R_2 - E(R_2))^2 + w_3^2(R_3 - E(R_3))^2 \\ + 2w_1 w_2(R_1 - E(R_1))(R_2 - E(R_2)) \\ + 2w_1 w_3(R_1 - E(R_1))(R_3 - E(R_3)) \\ + 2w_2 w_3(R_2 - E(R_2))(R_3 - E(R_3)) \end{array}\right\}$$

$$= w_1^2 Var(R_1) + w_2^2 Var(R_2) + w_3^2 Var(R_3) + 2w_1 w_2 COV(R_1, R_2)$$
$$+ 2w_1 w_3 COV(R_1, R_3) + 2w_2 w_3 COV(R_2, R_3)$$

Equation (2.20)

Shortly the portfolio variance is a weighted sum of variance and covariance terms and could be rewritten as;

$$Var(R_p) = \sum_{i=1}^{3} \sum_{j=1}^{3} w_i w_j \sigma_{ij}$$

Equation (2.21)

where w_i and w_j are the percentages invested in each asset, and σ_{ij} is the covariance of asset i with asset j.

When we replace the three assets with N, Equations (2.20) and (2.21) could be used as a general representation of the mean and variance of a portfolio with N assets. These equations could be also rewritten in matrix form, which for two assets looks like this:

$$E(R_p) = [E(R_1) E(R_2)]\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = R'W$$

Equation (2.22)

$$Var\left(R_p\right)=\begin{bmatrix}w_1 w_2\end{bmatrix}\begin{bmatrix}\sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22}\end{bmatrix}\begin{bmatrix}w_1 \\ w_2\end{bmatrix}=W'\,\Sigma\,W$$

<div align="right">Equation (2.23)</div>

The expected portfolio return is the $(1\,x\,N)$ row vector of expected returns, $[E(R_1), E(R_2)]=R'$ postmultiplied by the $(N\,x\,1)$ column vector of weights held in each asset, $[w_1\,w_2]=W$. The variance is the $(N\,x\,N)$ variance-covariance matrix, Σ, premultiplied by the vector of weights, W. As it seen that the matrix definition of the variance is identical to Equation (2.21) when the variance-covariance matrix is firstly postmultiplied by the column vector of weights in order to get

$$Var\left(R_p\right)=\begin{bmatrix}w_1 & w_2\end{bmatrix}\begin{bmatrix}w_1\sigma_{11}+w_2\sigma_{12} \\ w_1\sigma_{21}+w_2\sigma_{22}\end{bmatrix}$$

<div align="right">Equation (2.24)</div>

then postmultiplied the second vector times the first,

$$Var\left(R_p\right)=w_1^2\sigma_{11}+w_1 w_2\sigma_{12}+w_2 w_1\sigma_{21}+w_2^2\sigma_{22}$$

<div align="right">Equation (2.25)</div>

Finally terms are collected and seen that this is equal to

$$Var\left(R_p\right)=\sum_{i=1}^{N}\sum_{j=1}^{N}w_i w_j\sigma_{ij} \text{ where } N=2$$

<div align="right">Equation (2.26)</div>

This shows that the matrix definition of variance is equivalent to equation (2.21).

2.4 Types of Investors under Different Risk Preferences

A central idea underlying most microeconomic theory is that agents optimize subject to constraints. The optimizing principle applied to consumer choice begins from the notion that

agents have preferences over consumption bundles and will always choose the most preferred bundle subject to applicable constraints. Preferences are relationships between alternative consumption "bundles". These can be represented graphically using "indifference curves", as illustrated below in Figure (2.3) Focusing now on the preferences of a single consumer, the indifference curve is a line which connects all combinations of two goods x and y between which the consumer is indifferent. As this curve is drawn, we have represented an agent with "well-behaved preferences": at any allocation, more is better (monotonicity), and averages are preferred to extremes (convexity). Exactly one such indifference curve goes through each positive combination of x and y. Higher indifference curves lie to the "north-east".
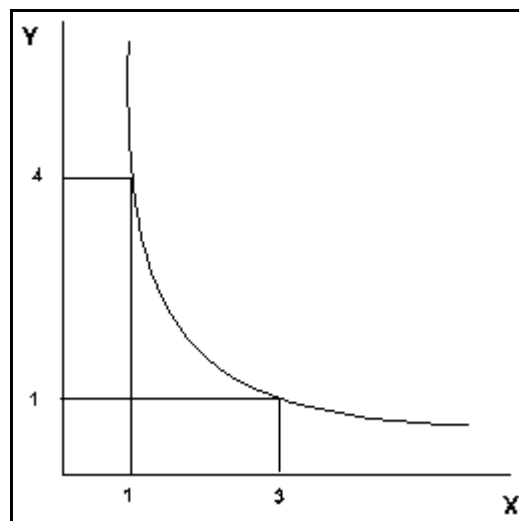
Figure 2.3 Indifference curve

If we wish to characterize an agent's preferences, the "marginal rate of substitution" (MRS) is a useful point of reference. At a given combination of x and y, the marginal rate of substitution is the slope of the associated indifference curve. As drawn, the MRS increases in magnitude as we move to the northwest and the MRS decreases as we move to the south east. The intuitive understanding is that the MRS measures the willingness of the consumer to trade off one good for the other. As the consumer has greater amounts of x, she will be willing to trade more units of x for each additional unit of y.

An important feature of the optimal portfolio that investors choose in order to maximize their utility is that the marginal rate of substitution between their preference for risk and return represented by the indifference curves must equal the marginal rate of transformation offered by the minimum variance opportunity set.

Selecting the most desirable portfolio involves the use of *indifference curves*. These curves represent an investor's preferences for risk and return. It can be drawn on a two-dimensional graph, where the horizontal axis usually indicates risk as measured by variance or standard deviation and the vertical axis indicates reward as measured by expected return. Using variance as relevant risk measure comes from Markowitz's (1952), and is always used in practice, although other possibilities have been considered. This definition gives us the following properties, assuming we have a rational investor:

All portfolios that lie on the same indifference curve are equally desirable to the investor (even though they have different expected returns and variance.)

An investor will find any portfolio that is lying on an indifference curve that is further northwest to be more desirable than any portfolio lying on indifference curve that is not as far northwest.

Generally it is assumed that investors are *risk averse*, which means that the investor will choose the portfolio with the smaller variance given the same return. Risk-averse investors will not want to take fair gambles (where the expected payoff is zero). These two assumptions of nonsatiation and risk aversion cause indifference curves to be positively sloped and convex. Risk aversion which, intuitively, implies that when facing choices with comparable returns, agents tend chose the less-risky alternative (Friedman and J. Savage, 1948).

Risk aversion (the symbol A is used) is measured as the additional marginal reward an investor requires to accept additional risk. Formulized as;

$$A = \frac{dE(R)}{d\sigma}$$ Equation (2.27)
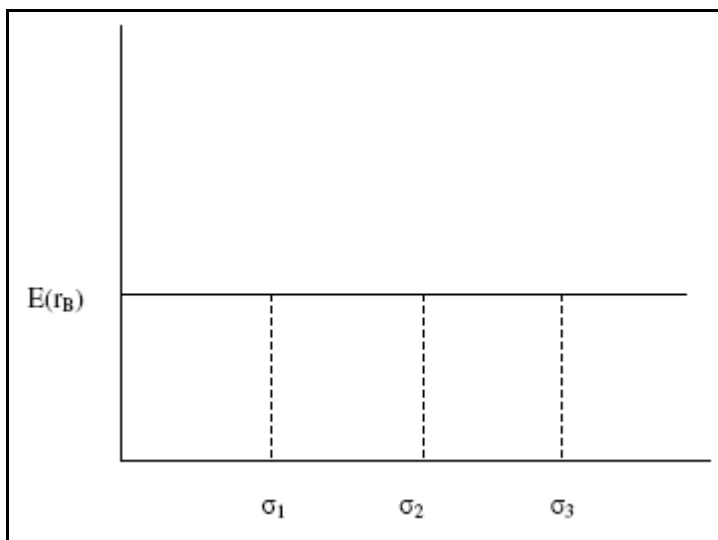
Figure 2.4 Indifference curve of absolutely risk insensitive investor

Indifference curves of an investor who is absolutely insensitive against risk are parallel to the horizontal axis and slope of the line is equal to zero. For these kinds of investors, return level should be fixed on a certain point regardless of level of risk. Indifference curve of an investor who is absolutely insensitive against risk as shown above whatever the level of risk is expected return is fixed at the point of $E(r_B)$.
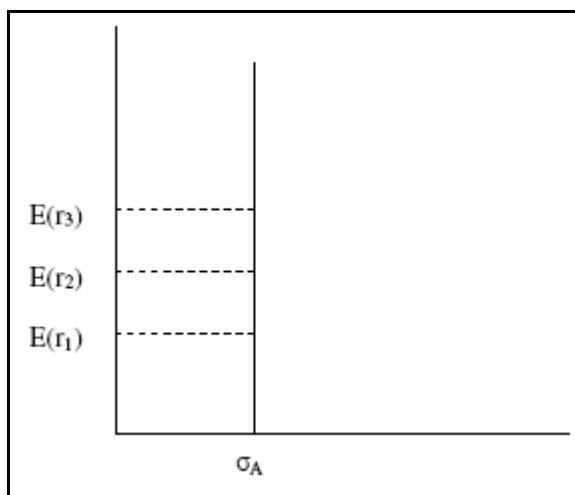
Figure 2.5 Indifference curve of absolutely risk sensitive investor

Indifference curve of this type of investors is parallel to the vertical axis. For these kinds of investors, risk level should be fixed on a certain point regardless of level of return. They do not like to take risk and they want to make their investments in safer instruments. Indifference curve of an investor who is absolutely sensitive against risk as shown above whatever the level of return is acceptable risk level for that investor is σ_A .

In real life, it is unlikely to come across such investors who are absolutely sensitive or insensitive against risk. In real life these types of investors are available who emphasize a little more on risk compared to return or regard return more important than risk.
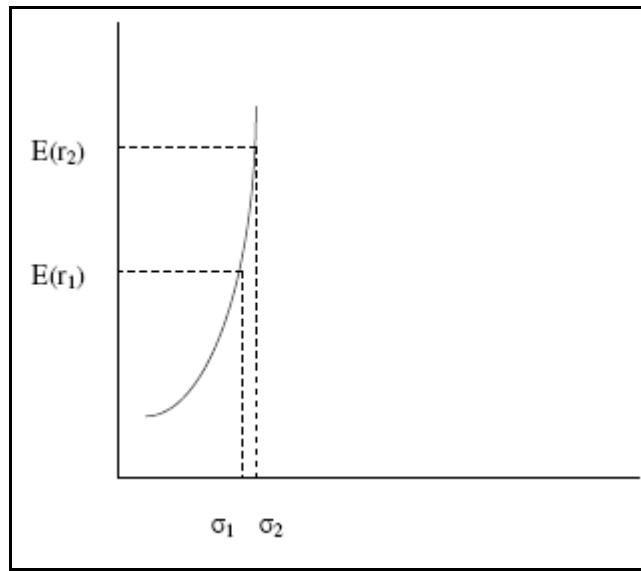
Figure 2.6 Indifference curve of risk averse investor

The above risk indifference curve specifies the type of investors who do not like to take risk. Slope of this curve is high. As it is shown on the graph, a small increase in the risk would be acceptable only a greater growth in return. These types of investors do not behave recklessly and they escape from risk. Therefore these types of investors make their investments in securities which are less risky and bring more stable return.
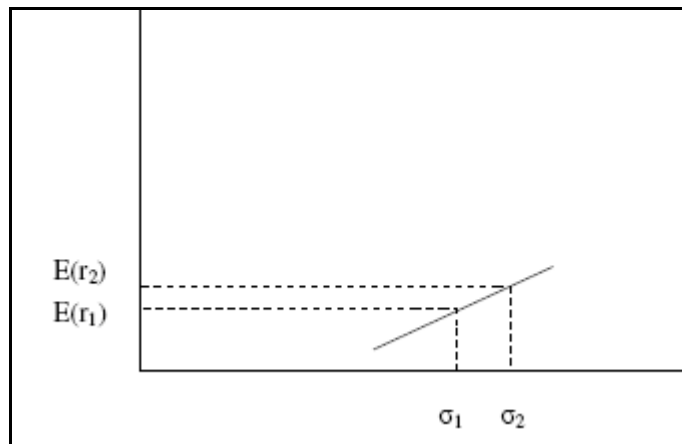
Figure 2.7 Indifference curve of risk lover investor

This figure shows the type of investors who like to take risks. A small increase in return is satisfactory for these types of investors compared with a greater increase in risk. They do not avoid making investment in riskier securities. As shown above, risk lover investor accepts to take a higher risk σ_2 in order to achieve $E(r_1)$ return which is a little higher than $E(r_2)$.

As a result, attitudes and preferences of capital owners determine making investment in which securities according to risk and return levels.

2.5 Return and Its Calculation

Investors take risk in order to get a satisfactory gain for themselves. In other words; risk is endured for return. However return may always not be gain but also loss for investors. Therefore, return could be defined as the gain or loss of a security in a particular period. It consists of the income and the capital gains relative on an investment.

Attention on returns rather than prices is focused because for an average investor, financial markets may be considered close to perfectly competitive, so that size of the investment does not affect price changes. Returns also have more attractive statistical properties than prices Lucas (1978).

Calculation of return may vary. Arithmetic mean and geometric mean could be used or logarithmic functions could be taken into account while return calculation. In our analysis log returns will be taken into consideration.

Arithmetic average is formulized as;

$$= \frac{\sum_{i=1}^{n} R_i}{n} = r_1 + r_2 + r_3 + ... + r_n / n \qquad \text{Equation (2.28)}$$

R_i = Rate of return

n = Number of period

Geometric mean = $\sqrt[n]{(1+R_1)(1+R_2)\cdots(1+R_n)} - 1$ 　　　　　　　　Equation (2.29)

where R_i is again defined as the rate of return.

Return simply formulized as;

Percentage of return= (Value at ending period – Value at beginning period)/ Value at beginning period; which is the price of percentage change for a period of time. The return, R_i, -for an asset priced P_t at the date t - on the asset between dates $t-1$ and t is defined as;

$$R_t = \frac{P_t}{P_{t-1}} - 1$$ 　　　　　　　　Equation (2.30)

The simple gross return on the asset is just one plus the net return, $1+R_t$. And from this equation it is apparent that the asset's gross return over the most recent k periods from date $t-k$ to date t, or $1+R_t(k)$ will simply be equal to the product of the k single period returns from $t-k+1$ to t which is formulized as;

$$1+R_t(k) = (1+R_t) \cdot (1+R_{t-1}) \cdots (1+R_{t-k+1})$$

$$= \frac{P_t}{P_{t-1}} \cdot \frac{P_{t-1}}{P_{t-2}} \cdot \frac{P_{t-2}}{P_{t-3}} \cdots \frac{P_{t-k+1}}{P_{t-k}} = \frac{P_t}{P_{t-k}}$$ 　　　　　　　　Equation (2.31)

and its net return over the most recent k periods, $R_t(k)$, is equal to its k period gross return minus one. These multiperiod returns are called compound returns. Although returns are scale free, they are defined with respect to some intervals.

The continuously compounded return or log return, r_t of an asset is defined to be the natural logarithm of its gross return $(1 + R_t)$;

$$r_t = \log(1 + R_t) = \log \frac{P_t}{P_{t-1}} \text{ or } \log P_t - \log P_{t-1} \qquad \text{Equation (2.32)}$$

where "P" represents price of a stock.

The advantages of continuously compounded returns become clear when we consider multiperiod returns, since

$$
\begin{aligned}
r_t(k) = \log(1 + R_t(k)) &= \log\left((1 + R_t) \cdot (1 + R_{t-1}) \cdots (1 + R_{t-k+1})\right) \\
&= \log(1 + R_t) + \log(1 + R_{t-1}) + \cdots + \log(1 + R_{t-k+1}) \\
&= r_t + r_{t-1} + \cdots + r_{t-k+1}, \qquad \text{Equation (2.33)}
\end{aligned}
$$

and hence the continuously compounding multiperiod return is simply the sum of continuously compounded single-period returns. Compounding, a multiplicative operation, is converted to an additive operation by taking logarithms. However, this simplification is not just in reducing multiplication to addition, but more in the modeling of the statistical behavior of asset returns over time.

On the other hand, continuously compounded returns do have disadvantages such as not linear in portfolio return and unrealistic approach. The simple return on a portfolio of assets is a weighted average of the simple returns on the assets themselves, where the weight on each asset is the share of the portfolio's value invested in that asset. In empirical applications this problem is usually minor. When returns are measured over short intervals of time, and are therefore close to zero. The continuously compounded return on a portfolio is close to the weighted average of the continuously compounded returns on the individual assets (Campbell and others, 1996)

The return of portfolio is the simply weighted mathematical average of a series of returns generated over a period of time. For example, you invest in several asset classes and get different return from each other, so your overall investment portfolio is;

$$\text{Return of portfolio} = (W_1 * R_1) + (W_2 * R_2) + \cdots + (W_n * R_n)$$ Equation (2.34)

where W_1, W_2 and W_n stand for the weightings in % of assets, for asset 1 to n, in the portfolio. Whereas, R_1, R_2 and R_n are returns for the respective assets, 1 to n, in the portfolio. More than half is invested in real estates.

Table 2.1 Overall portfolio return

Investment	Market Value	Weight (%)	Return	Weighted Return
Stocks	$50.000	17,06%	25%	4,27%
Unit Trust	$13.000	4,44%	10%	0,44%
Fixed Deposit	$80.000	27,30%	3,70%	1,01%
Real Estates	$150.000	51,20%	8%	4,10%
Total	$293.000	100,00%	Portfolio Return	9,82%

Weight of stocks = 50k/293k = 17.06%

Weighted return of stocks = 25% x 17.06% = 4.27%

Overall portfolio return = 4.27 + 0.44 + 1.01 + 4.10 = 9.82%

When probability distribution is known expected return for a single asset as formulized previously will be;

$$E(R_i) = \sum_{k=1}^{n} P_k * R_{ik}$$ Equation (2.35)

where P_k is the occurrence probability of case k and R_{ik} is the return of asset (i) when case (k) is realized.

And expected return of the portfolio will be defined weighted average of the assets invested in that portfolio which is shown as;

$$E(R_p) = \sum_{i=1}^{n} W_i \cdot E(R_i)$$
Equation (2.36)

where W_i is the weight of asset (i) in the portfolio.

PART 3 MODERN PORTFOLIO THEORY

3.1 The Theory and Its Assumptions

Modern portfolio theory (MPT) was pioneered by Harry Markowitz in his paper "Portfolio Selection," published in 1952 and it was considered an important advance in the mathematical modeling of finance which tries to maximize return and minimize risk by choosing different assets.

Modern portfolio theory states that the risk for individual asset returns has two components, systematic and unsystematic risk which details explained in part 2.

Markowitz's (1952) theory says that it is not enough to look at the expected risk and return of one particular asset. Through investing in more than one asset, an investor can achieve the benefits of diversification.

According to the theory, expected return of the investors and risk values are the two basic variables which determine their portfolio choice. Markowitz explains that main purpose of portfolio creation is to minimize risk while maximizing return. In this theory, number of assets included in the portfolio has as great importance as correlation between these asset returns with each other. For example marginal benefits of putting two assets in a portfolio which their returns are moving in same direction will not be high as explained in part 2.2 correlation sections.

In modern portfolio theory expected return and risk for any portfolio is formulized as following:

$$E(r_p) = \sum_{i=1}^{n} E(r_i) \cdot w_i \qquad \text{Equation (3.1)}$$

$$\sigma^2{}_p = \sum_{i=1} \sum_{j=1} Cov_{ij} \cdot w_i \cdot w_j \qquad \text{Equation (3.2)}$$

3.1.1 Assumptions of modern portfolio theory

- The theory assumes that investors are risk averse, meaning that given two assets that offer the same expected return, investors will prefer the less risky one. Thus, an investor will take on increased risk only if compensated by higher expected returns. Conversely, an investor who wants higher returns must accept more risk. The exact trade-off will differ by investor based on individual risk aversion characteristics. The implication is that a rational investor will not invest in a portfolio if a second portfolio exists with a more favorable risk-return profile – i.e., if for that level of risk an alternative portfolio exists which has better expected returns.

- Investors take into consideration only risk and return while making their investment decisions. MPT further assumes that the investor's risk / reward preference can be described via a quadratic utility function. The effect of this assumption is that only the expected return and the volatility (i.e., mean return and standard deviation) matter to the investor. The investor is indifferent to other characteristics of the distribution of returns, such as its skew or kurtosis.

- Main aims of the investors are to maximize expected utilities.

- There is no transaction cost.

- Expectations of investors are homogeneous.

- Capital markets are efficient. Market efficiency states that the price already reflects the all available information. It assumes that all news and information which effecting the price of the assets are reflected to the market immediately and accurately. Therefore a market cannot be outperformed because all available information is already built into all asset prices. And also it states that forecasting for future in other words technical analysis through looking at past date is not possible.

3.2 Efficient Frontier

Markowitz has brought the concept of "efficient frontier" to the portfolio theory. One of its largest contributions was its demonstration of the power of diversification. It is assumed that data for a collection of assets which the return rates and standard deviations are graphed for these assets, and for all portfolios it could be gotten by allocating among them. Markowitz showed that through this way it could be achieved a region bounded by an upward-sloping curve, which he called the efficient frontier.
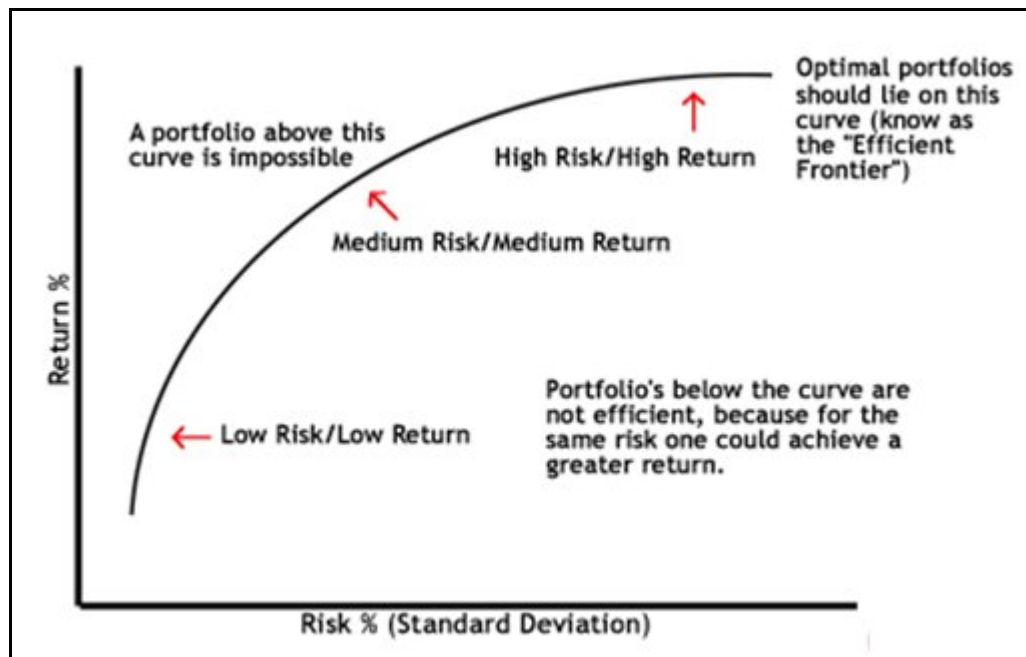
Figure 3.1 Efficient frontier (http://www.investopedia.com/terms/e/efficientfrontier.asp)

It's obvious that for any given value of standard deviation, investors would like to choose a portfolio that gives the greatest possible rate of return; so investors always want a portfolio that lies up along the efficient frontier, rather than lower down, in the interior of the region. This is the property of the efficient frontier that it is where the best portfolios are lie on the curve. The other important property of the efficient frontier is that it is *curved*, not straight. This curve is the key to how diversification lets investors improve reward-to-risk ratio. The curved shape of the efficient frontier is formed because there is a diminishing marginal return

to risk. Each unit of risk added to a portfolio gains a smaller and smaller amount of return. The smaller the covariance between the assets included in portfolio- the more out of sync they are -- the smaller the standard deviation of a portfolio that combines them. The ultimate would be to find assets with negative covariance.

In Markowitz model, investors should make "n" number of expected returns, "n" number of standard deviation, " n (n-1) / 2 " number of covariance , in total " n (n+3) / 2 " number of data predictions which it is difficult practically in order to get efficient frontier.

3.3 Determination of Optimal Portfolio on Efficient Frontier with the Help of Indifference Curves

The efficient frontier consists of the set of all efficient portfolios that yield the highest return for each level of risk. The curve showing the expected return and risk preferences of investors is called indifference curve. Optimal portfolio could be determined with the help of these two curves. As it is known all portfolios located on the efficient frontier line is optimum.

For the investors choosing which of the portfolio on this line depends on their preferences and risk attitudes that determines their indifference curves. For an investor optimal portfolio is the portfolio, lying on the efficient frontier, which provides the highest utility. This appropriate portfolio selection realizes on the point where the indifference curve of the investor is tangent to the efficient frontier.
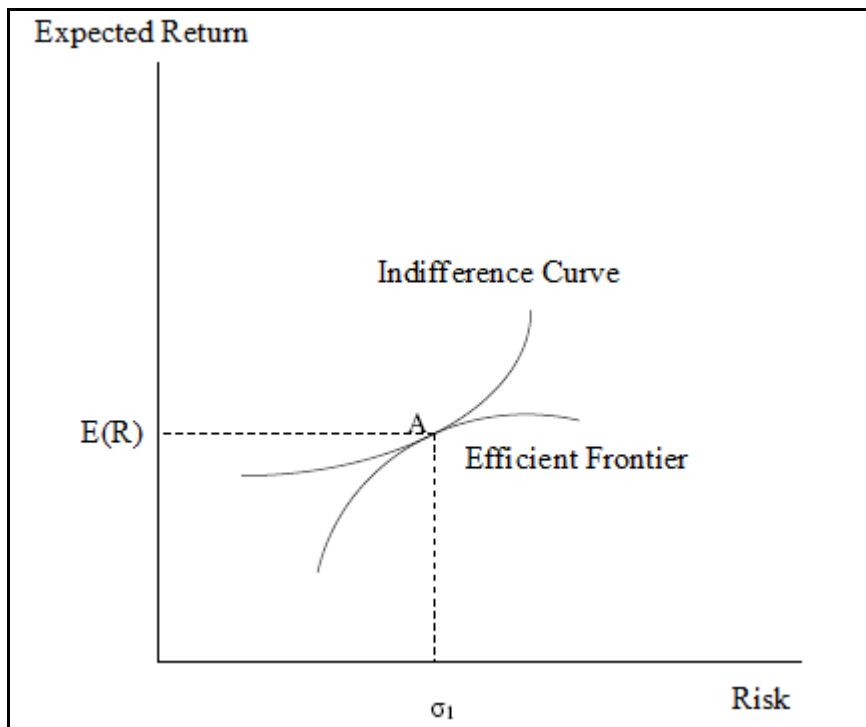
Figure 3.2 Determination of optimal portfolio on efficient frontier

As shown figure above, optimal portfolio is on the point of "A" where the indifference curve is tangent to the efficient frontier. For instance, at the point of "A" risk level is σ_1 and expected return is $E(R)$ and for an investor who has these risk and return expectation levels portfolio "A" is the optimal portfolio for that investor.
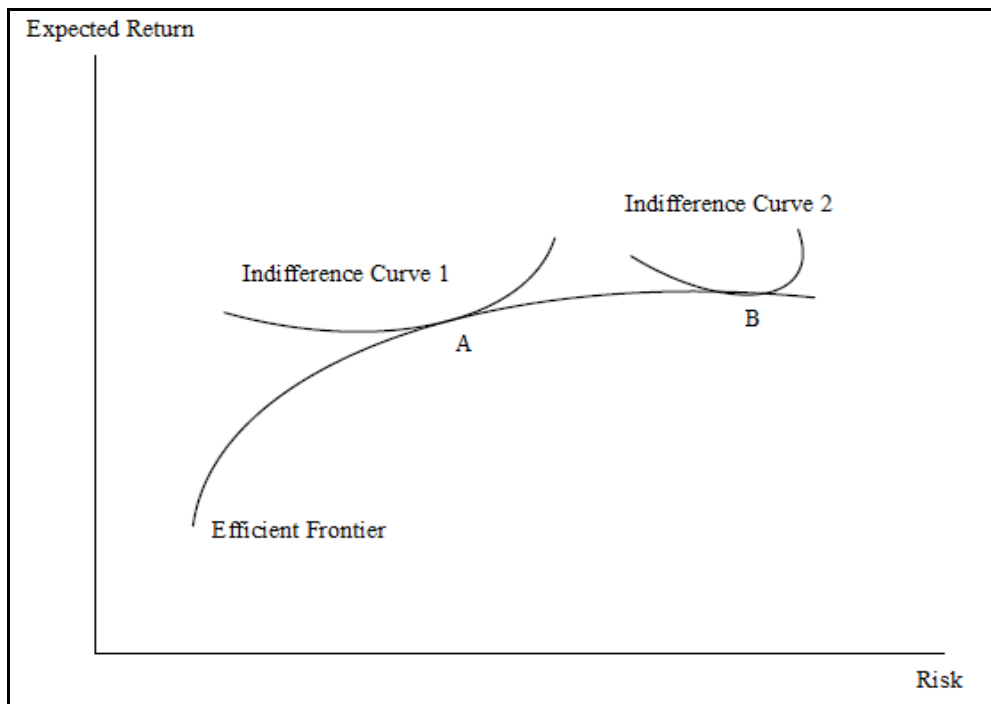
Figure 3.3 Optimal portfolio determination under different risk preferences of investors

Investors choose the most appropriate portfolio at the any point on the Markowitz's efficient frontier depending on their risk aversion degrees. Let us assume "a" and "b" two kind of investors, investor "a" dislike risk compared to investor "b". For these investors points A and B are the optimal portfolios respectively graphed on the figure above. Investor "b" is more risk lover than investor "a", therefore risk and expected return of his portfolio are higher than portfolio "A".

PART 4 CAPITAL MARKET THEORY

The capital market theory builds upon the Markowitz portfolio model. In Harry Markowitz's efficient frontier hypothesis just risky assets are taken into consideration in order to build efficient portfolio set, however in this model a new efficient set is constructed by adding risk free assets. The prominent characteristic of this model is concept of risk-free asset.

Assets like treasury bills and bonds which have a certain future return are considered to be risk-free because they are backed by the government. These securities are considered to be risk-free because the likelihood of governments defaulting is extremely low. Investing in these kinds of financial instruments eliminates default risk. However, these instruments could carry other types of risk such as inflation risk (Dimson and others, 2002).

The theory figure outs the rates of return for efficient portfolios depending on the risk-free rate of return and the level of risk for a particular portfolio. It is derived by drawing a tangent line from the intercept point on the efficient frontier to the point where the expected return equals the risk-free rate of return. The line is created in a graph of all possible combinations of risky and risk-free assets.

4.1 Assumptions of Capital Market Theory

- All investors are efficient investors who follow Markowitz idea of the efficient frontier and choose to invest in portfolios along the frontier.

- Investors could borrow and lend for any amount of money at the risk-free rate

- All investors have homogeneous expectations and have the same probability for outcomes, in other words all investors have the same probability for outcomes while determining the expected returns.

- The investment time horizon is equal for all investors, investors have equal time horizons for the chosen investments.

- All investment alternatives have the property of divisibility and assets are infinitely divisible. This indicates that fractional shares could be purchased and the stocks could be infinitely divisible.

- There are no taxes and transaction costs which in turn investors' results are not affected by taxes and transaction costs.

- There is no change in inflation and interest rates or all modifications are fully predictable. Inflation does not exist and returns are not affected by the inflation rate in a capital market as none exists in capital market theory.

- There is no mispricing within the capital markets and the markets are efficient.

(Copeland and others, 2004)

4.2 The Capital Market Line (CML)

According to the capital market assumptions, rational investors would like to make their investment on capital market line. In capital market theory relation between risk and expected return for efficient portfolios is explained by capital market line. This line is shown and formulized together with an example below:
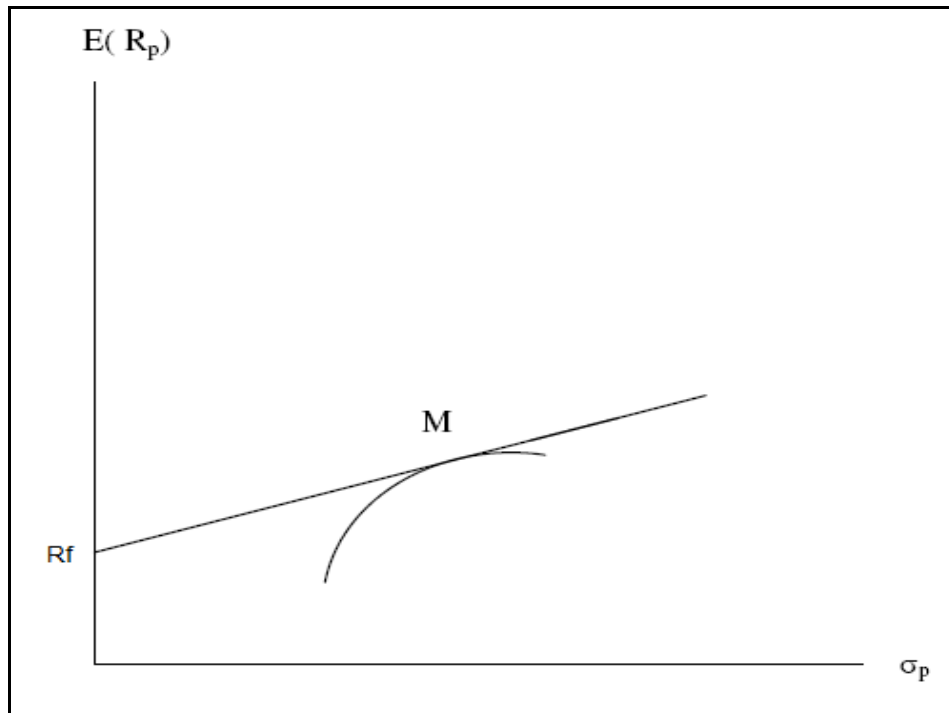
Figure 4.1 Capital market line

$$E(r_p) = R_f + \frac{E(r_m) - r_f}{\sigma_m} \cdot \sigma_p \qquad\qquad \text{Equation (4.1)}$$

$E(r_p) =$ Expected return of the portfolio

$R_f =$ Return of risk-free asset

$E(r_m) =$ Expected return of market

$\sigma_m =$ Standard deviation-risk- of the market

$\sigma_p =$ Standard deviation-risk- of the portfolio

For instance; return of risk-free asset (r_f) is 15%, expected return of the market $[E(r_m)]$ is 30% with a standard deviation (σ_m) 0,1 and standard deviation of efficient portfolio (σ_p) is

0,08 in this case according to the capital market line formulation expected return of efficient portfolio is calculated as below;

$$E(r_p) = 0,15 + \frac{0,3 - 0,15}{0,1} \cdot 0,08 \qquad \qquad \text{Equation (4.2)}$$

$$E(r_p) = 0,27 = 27\%$$
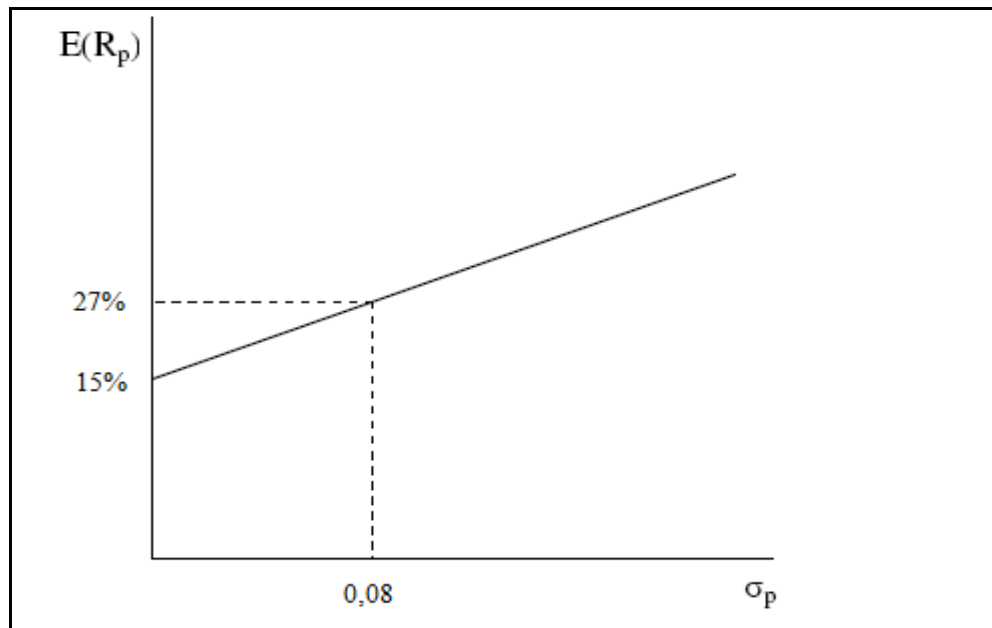
Figure 4.2 Expected return calculation of a portfolio on capital market line

CML consists of the market portfolio and risk-free combination. Together with adding risk-free asset to model curve shape of the Markowitz's efficient frontier is replaced by a linear line. CML is a line, extended from risk-free rate to the market portfolio, which shows the relationship between the return and standard deviation of the portfolio and only efficient portfolios are located on the capital market line.

In Figure 4.1 which showing the CML the letter "M" represents the optimal risky portfolio. At this point CML is tangent to efficient frontier which consists of risky portfolios. All investors prefer market portfolio "M" as optimal risky portfolio but investment weights of the investors

in risky and risk-free assets will be different from each other. This market portfolio represents the most appropriate combination of every accessible asset in the market available to investors, which is consisted proportionally weighted sum of their market values.

On the capital market line only market portfolio "M" which is consisted of risky assets is efficient, whereas in the Markowitz's model large number of risky portfolios could be optimal portfolios. Market portfolio is superior to all other portfolios. In market portfolio, all assets such as Treasury bill, government bonds, stocks, options and etc. are included. And it could be claimed that market portfolio contains only systematic risk since it is a very well diversified portfolio (Konuralp, 2001)

Shortly, an investment in market portfolio M (the tangency point) and the riskless asset is an optimal strategy for all investors. Investors will only differ in the relative proportions invested in the two components. The following figure illustrates that both investor "A" and investor "B" will prefer this strategy to an investment in risky assets alone. Investor "A", who is more averse towards risk, invests a higher proportion of his wealth in the riskless asset than does investor "B". From the figure, investor "A" invests about half his wealth in the riskless asset and half in the portfolio of risky assets "M". Investor "B" invests all his wealth in the portfolio of risky assets "M", then borrows at the riskless rate, r_f, and invests this in the portfolio of risky assets as well.
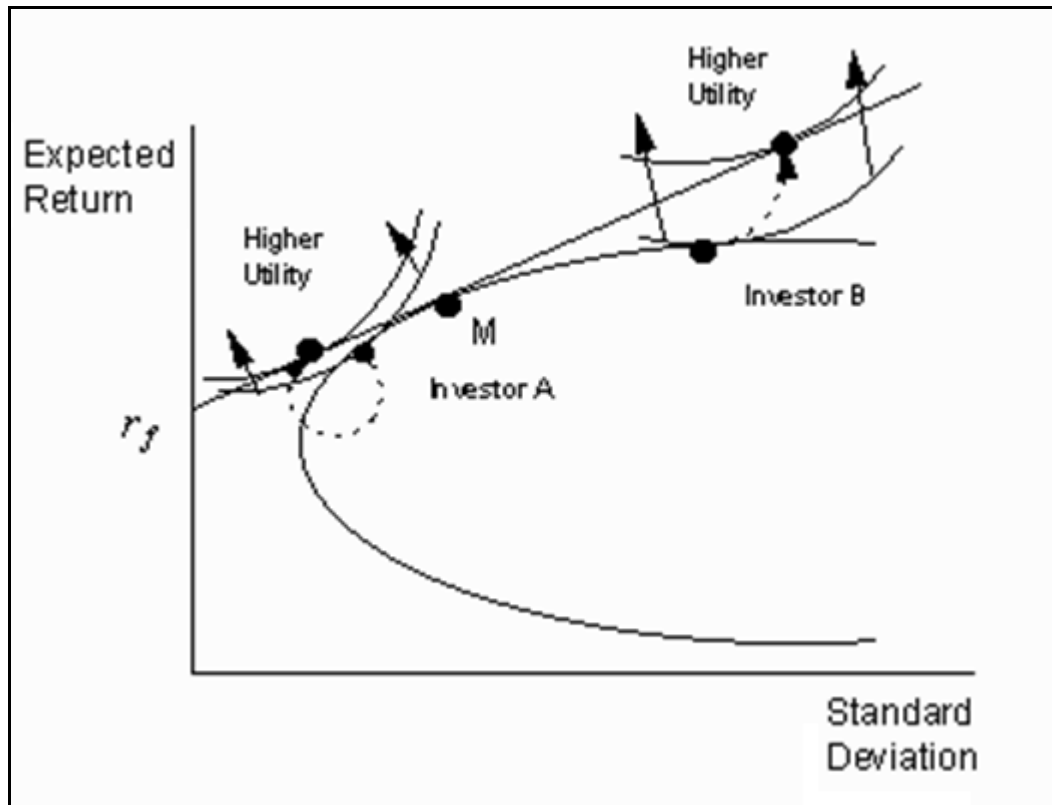
Figure 4.3 Portfolio choice of investors on the capital market line

As mentioned before inefficient financial assets and portfolios do not exist on capital market line, only efficient portfolios are located on CML. The results which CML equation reveals are that standard deviation is an appropriate risk measurement for efficient portfolios and expected return is positively correlated with standard deviation when investment is made in efficient portfolio. Slope of the CML, $\left[\left(E(R_m) - r_f\right)/\sigma_m\right]$ equals the market price of risk. The slope is a calculation of the equilibrium market price of risk and it determines the additional expected return needed to compensate for a unit change in risk. Equilibrium market price of the risk, the slope, is always positive because rational investors are generally risk averse, they do not like to take risk and they demand extra return in exchange for additional risk they take.

As a result, expected return of an optimal portfolio located on the capital market line depends on the risk-free rate, standard deviation of the portfolio and market price of the risk.

4.3 Expected Return and Risk of A Portfolio Consisted of Risk-free and Risky Assets

The most important concept of the capital market theory is the risk-free asset. Expected return and realized future return of the risk-free asset are equal. Treasury bills and government bonds are generally accepted as risk-free assets.

Expected return and risk of a portfolio which is composed of a risk-free asset and a risky asset are simply formulized as:

$$E(R_p) = w \cdot E(R_i) + (1 - w) \cdot R_f \qquad \text{Equation (4.3)}$$

$$\sigma_p^2 = w^2 \cdot \sigma_i^2 + (1 - w)^2 \cdot \sigma_f^2 + 2w(1 - w) \cdot \rho_{if} \cdot \sigma_i \cdot \sigma_f \qquad \text{Equation (4.4)}$$

In the equation showing the expected return of portfolio, $E(R_p)$, w represents the weight invested in risky asset and $E(R_i)$ denotes expected return of this risky asset.

The variance of the portfolio could be rearranged as shown below since the covariance between risky asset and variance of the risk-free asset is zero.

$$\sigma_p^2 = w^2 \cdot \sigma_i^2 \qquad \text{Equation (4.5)}$$

In other words the standard deviation of the portfolio, risk, is $\sigma_p = w \cdot \sigma_i$ the multiplication of the risky asset's standard deviation and its weights in the portfolio.

Weight components of two risky assets for an optimal portfolio could be formulized as (Bodie and Merton, 2000):

$$w_1 = \frac{\left[E(r_1 - r_f)\right] \cdot \sigma_2^2 - \left[E(r_2 - r_f)\right] \cdot \rho_{1,2}\sigma_1\sigma_2}{\left[E(r_1) - r_f\right] \cdot \sigma_2^2 + \left[E(r_2) - r_f\right] \cdot \sigma_1^2 - \left[E(r_1) - r_f + E(r_2) - r_f\right] \cdot \rho_{1,2}\sigma_1\sigma_2}$$ Equation (4.6)

$$w_2 = 1 - w_1$$ Equation (4.7)

4.4 Optimal Portfolio Selection

In the capital market theory there is a concept of risk-free asset and optimal portfolio does not depend on investors' attitudes and preferences. In this theory only one portfolio is efficient and investors choose the same optimal portfolio under similar expectations. Investors just construct their individual portfolios from risk-free assets and market portfolio according to their degrees of risk aversion. Creation of optimal portfolio and individual portfolio operations are disassociated from each other.
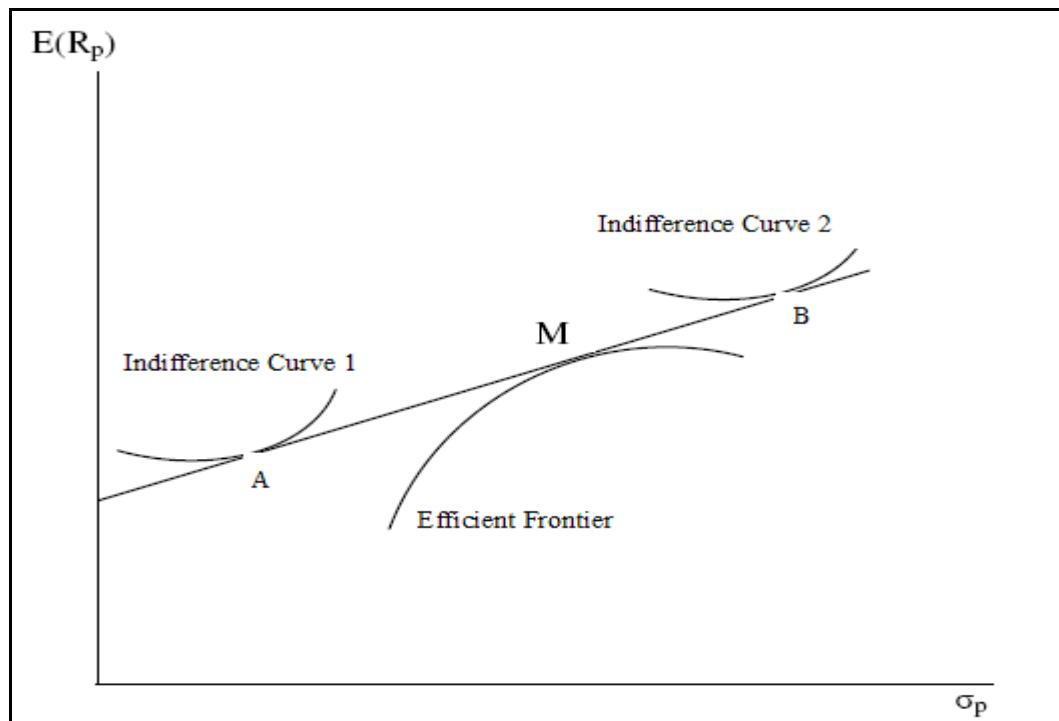
Figure 4.4 Optimal portfolio selection along CML

As illustrated figure above, in the capital market model each investor's optimal portfolio occurs on the points along the CML where the line is tangent to the investors' indifference curves. And at these points investors maximize their utilities. In the figure investor "a" who is more risk averse than investor "b" has an indifference curve 1. In this situation optimal portfolio for investor "a" is the point A where the indifference curve 1 is tangent to the capital market line. On the other hand investor "b" that is willing to take a bit more risk has an indifference curve 2 and his optimal portfolio is on the point B where his indifference curve is tangent to the capital market line. In both cases, investors own utilities are maximized at the points A and B respectively.

PART 5 CAPITAL ASSET PRICING MODEL

Capital asset pricing model (CAPM) is one of the economic models to determine the market price for risk and the appropriate measure of risk for a single asset. The model was developed almost simultaneously by Sharpe (1963, 1964) and Treynor (1961, 1962), and then further developed by Mossin (1966), Lintner (1965, 1969), and Black (1972). It shows that the equilibrium rates of return on all risky assets are a function of their covariance with the market portfolio.

Every investment carries two distinct risks, the CAPM explains. One is the risk of being in the market, which Sharpe called systematic risk. This risk, later denominated by "beta," cannot be diversified away. The other—unsystematic risk—is specific to a company's characteristics. Since this uncertainty could be mitigated through appropriate diversification, Sharpe figured that a portfolio's expected return depends solely on its beta—its relationship to the overall market. The CAPM helps measure portfolio risk and the return an investor expects for taking that risk.

The capital asset pricing model is a ceteris paribus model which is valid within a special set of assumptions. The model is developed in a hypothetical world where the following assumptions are made about investors and the opportunity set:

5.1 Assumptions of CAPM

- Investors are risk-averse individuals who maximize the expected utility of their wealth.

- Investors are price taker and have homogeneous expectations about asset returns that have a joint normal distribution: This assumption requires a perfect competition that wealth of each investor is small compared to the total wealth traded in the market. So investors are price takers, that is, they are not able to affect the security prices by their own trades. Moreover, all investors analyze assets in the same way and share the same economic view of the world.

They consider the same probability distribution of cash flows for the same assets in other words they use the same input data while deriving the Markowitz model.

- There exists a risk-free asset such that investors may borrow or lend unlimited amounts at a risk-free rate.

- The quantities of assets are fixed. Also, all assets are marketable and perfectly divisible. The model assumes that investments are limited to a universe of publicly traded financial assets. This assumption excludes investments in non traded assets like human capital, government social investments, and etc.

- Asset markets are frictionless, and information is costless and simultaneously available to all investors.

- There are no market imperfections such as taxes, regulations, or restrictions on short selling. Investors pay no tax for their profit on asset returns and pay no commission for their trade transactions shortly there is no cost associated with the trade decisions.

- All investors are rational mean variance optimizers, meaning that they all use the same Markowitz's portfolio selection model. Indifference curves in other words utility function of the investors determine where to invest on the efficient frontier set.

Main idea CAPM is based on is that investors demand additional expected return, called the risk premium, if they are asked to accept additional risk. It is worthwhile to discuss some of the implications of the assumptions which have not been mentioned earlier. For instance, if markets are frictionless, the borrowing rate equals the lending rate so it is possible to develop a linear efficient set called the capital market line. Another important assumption is that investors have homogeneous beliefs. They all make decisions based on an identical opportunity set. In other words, no one can be fooled because everyone has the same information at the same time. Also, since all investors maximize the expected utility of their

end-of-period wealth, the model is implicitly a one-period model in other words all investors plan for one identical holding period and they ignore everything that may happen after the holding period.

Although not all these assumptions conform to reality, which will be discussed more detailed in section 5.6, they are simplifications that allow the development of the CAPM which is useful to create a simple arena where market equilibrium could be constructed in a hypothetical world.

Firstly, borrowing and lending would cancel out each other so that the aggregate risky portfolio equals the entire wealth of economy; which is the market portfolio "M" mentioned in capital market model. The proportion of each asset in this portfolio equals the market value of the asset divided by the sum of entire wealth of market. CAPM implies that as individuals attempt to optimize their portfolios, they each arrive at the same portfolio with weights on each asset equal to those of the market portfolio.

Based on the CAPM assumptions, investors will desire to hold identical risky portfolios. That is; if all investors use identical Markowitz analysis applied to the same assets for the same holding period and use the same input list, they all must arrive at the same optimal risky portfolio which is figured below.
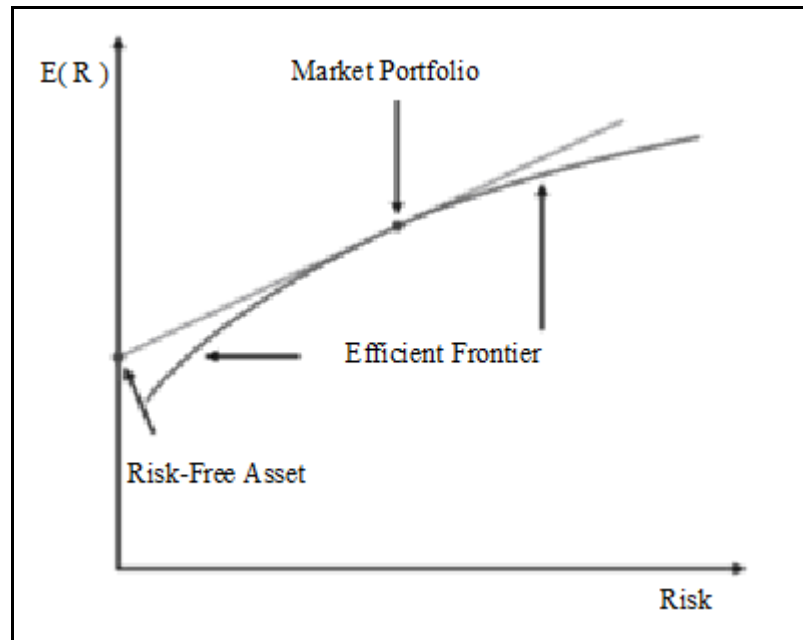
Figure 5.1 Optimal risk portfolio-market portfolio

With complete agreement about the distribution of returns, all investors see the same opportunity set and they combine the same risky tangency portfolio-market portfolio- with risk-free lending or borrowing. Since all investors hold the same portfolio "M" of risky assets, this should be the value weighted market portfolio of risky assets. Moreover, each risky asset's weight in the tangency portfolio must be the total market value of all outstanding units of the asset divided by total market value of all risky assets. In addition, the risk-free rate must be set to clear the market for risk-free borrowing and lending. In other words, CAPM assumptions imply that the market portfolio must be on the minimum variance frontier if the asset market is to clear.

5.2 Derivation of the CAPM

CAPM is based on the view that risk premium on an asset is obtained by its contribution to the risk of investors' overall portfolio. So rather than risk of a single asset but risk of portfolio is concern for investors according to CAPM theory. Therefore, determining the amount of risk that an asset contributes to a portfolio should be taken in account in order to derive CAPM.

Assume a portfolio which includes "n" different assets. According to the Equation (2.14); the variance of the portfolio is a weighted sum of covariance, and each weight is the product of the portfolio proportions of the pair of assets in the covariance term.

The risk contribution of an asset "j" to the portfolio could be determined as;

$$w_j \left[w_1 Cov(R_1, R_j) \right] + w_2 \left[w_2 Cov(R_2, R_j) \right] + \ldots + w_n \left[w_n Cov(R_n, R_j) \right] \qquad \text{Equation (5.1)}$$

The equation explains the risk participation of asset "j" to the portfolio as the weighted sum of covariance, where each weight is the product of the portfolio proportions of the pair of assets in the covariance term. The covariance of a particular asset with all other assets dominates contribution of asset "j" to total market portfolio, which symbolized with "M", could shortly be formulized as;

$$\text{Risk contribution of asset "j" to the market} = w_j \, Cov(R_j, R_M) \qquad \text{Equation (5.2)}$$

Corresponding to the Equation (2.13), the rate of return on the market portfolio could be rearranged as follows:

$$R_M = \sum w_i \cdot R_i \quad \text{where i = 1, 2,.....,n} \qquad \text{Equation (5.3)}$$

Also, the covariance of the return on asset "j" with the market portfolio could be written as;

$$Cov(R_j, R_M) = Cov\left(R_j, \sum w_i \cdot R_i\right) = \sum w_i \, Cov(R_j, R_i) \quad \text{where i = 1,2,....,n} \qquad \text{Equation (5.4)}$$

When the last terms in equations (5.2) and (5.4) are compared, it could be concluded that the covariance of asset "j" with the market portfolio is proportional to the contribution of "j" to the variance of the market portfolio. If the covariance between asset "j" and market is positive,

asset "j" generates a positive contribution to overall portfolio risk. On the contrary, if the covariance is negative asset "j" stabilizes the return on overall portfolio.

Risk is measured in percent squared as the variance of market return that is risk measure of asset "j" is its covariance with the market portfolio. This ratio could also be called as the market price of risk that defines how much extra return should be accepted per unit of portfolio risk.

Suppose an investor invested all his wealth in the market portfolio and aiming to increase his position in the market portfolio with a fraction of δ through borrowing at the risk free rate. So the new portfolio will be a combination of three assets: the initial position in the market with a return of R_M, plus a short position in risk free asset, $-\delta R_f$, and plus a long position in market with a return of δR_M. When all returns are summed, the portfolio return will be

$$R_M + \delta(R_M - R_f) \hspace{4cm} \text{Equation (5.5)}$$

And taking expectations and comparing with the initial expected return, the incremental expected rate of return with the new positions will be;

$$\Delta E[R] = E[R_M + \delta(R_M - R_f)] - E[R_M]$$
$$\Delta E[R] = \delta[E(R_M) - R_f] \hspace{3cm} \text{Equation (5.6)}$$

In order to measure the impact of new position on the market price of risk, the relevant change on the portfolio variance should also be calculated. The new portfolio has a weight of $(1+\delta)$ in the market and $-\delta$ in the risk free asset. And, the variance of the adjusted portfolio is;

$$\sigma_p^2 = (1+\delta)^2 \cdot \sigma_M^2 = (1 + 2\delta + \delta^2) \cdot \sigma_M^2 = \sigma_M^2 + (2\delta + \delta^2)\sigma_M^2 \hspace{2cm} \text{Equation (5.7)}$$

The fraction δ has a small value, less than 1, so the term δ^2 is negligible and could be disregarded in portfolio variance. And the variance of the new portfolio could be stated as $\sigma_M^2 + 2\delta\sigma_M^2$, consequently increase in the variance of new portfolio could be determined as;

$$\Delta\sigma_p^2 = \sigma_M^2 + 2\delta\,\delta_M^2 - \sigma_M^2$$
$$= 2\delta\,\sigma_M^2 \qquad\qquad \text{Equation (5.8)}$$

Furthermore, the marginal price of risk which is the trade off between the incremental risk premium and incremental risk could be formulized as;

$$\frac{\Delta E[R]}{\Delta\sigma^2} = \frac{\left[E(R_M) - R_f\right]}{2\sigma_M^2} \qquad\qquad \text{Equation (5.9)}$$

On the other hand, consider the same investor chooses to invest δ proportion in asset "j" instead of investing in market portfolio by financed again through borrowing at the risk free rate. In this new situation, mean excess return increase will be as follows;

$$\Delta E[R] = E\left[R_M - \delta\,R_f + \delta\,R_j\right] - E[R_M]$$
$$\Delta E[R] = \delta\left[E(R_j) - R_f\right] \qquad\qquad \text{Equation (5.10)}$$

This new portfolio will have a weight of 1 in the market, δ in asset "j" and $-\delta$ in the risk free asset. And variance of the new portfolio and increase in the variance will be respectively as;

$$\sigma_M^2 + \delta^2\,\sigma_j^2 + \left[2\delta\,Cov(R_j, R_M)\right]$$
$$\Delta\sigma_p^2 = \sigma_M^2 + \delta^2\,\sigma_j^2 + \left[2\delta\,Cov(R_j, R_M)\right] - \sigma_M^2$$
$$\Delta\sigma^2 = \delta^2\sigma_M^2 + 2\delta\,Cov(R_j, R_M) \qquad\qquad \text{Equation (5.11)}$$

The marginal price of risk of asset "j" will be $\left[E(R_j) - R_f\right] \big/ 2\,Cov(R_j, R_M)$ when the negligible term is ignored.

The marginal price of risk of asset "j" equals that of the market portfolio in order to provide market equilibrium. Because, if the marginal price of risk of asset "j" is greater than the market's, rational investors -according to CAPM assumptions- will increase their portfolio reward for carrying risk through increasing the weight of "j" in their portfolio till the price of asset "j" rises relative to the market. Investors will keep buying asset "j" and the process will run until asset prices adjust thus the marginal price of risk "j" equals the risk of market.

Equating the Equation (5.9) and marginal price of risk of asset "j"

$\left[E(R_M) - R_f\right]\big/ 2\sigma_M^2 = \left[E(R_j) - R_f\right] \big/ 2\,Cov(R_j, R_M)$ and rearranging the equation

$$E(R_j) - R_f = \left[Cov(R_j, R_M)\big/\sigma_M^2\right] \cdot \left[E(R_M) - R_f\right] \qquad \text{Equation (5.12)}$$

The ratio of $Cov(R_j, R_M)\big/\sigma_M^2$, which is also called as beta, measures the contribution of asset "j" to the variance of the market portfolio as a fraction of the total variance of the market portfolio. This measurement could be expressed as CAPM expected return-beta relationship which is formulized as;

$$E(R_j) = R_f + \beta_j \left[E(R_M) - R_f\right] \qquad \text{Equation (5.13)}$$

5.3 Security Market Line and Comparison with Capital Market Line

The beta of a security is the appropriate measure of its risk because beta is proportional to the risk that the security contributes to the optimal portfolio. The security market line (SML) graphs the relationship between risk and return. The SML straight line touches the efficient

frontier and passes though the risk free rate of return and lies above the efficient frontier. The point at which the securities market line touches the efficient frontier shows the risk and return on the market portfolio. The expected return and beta relationship could be portrayed graphically as;
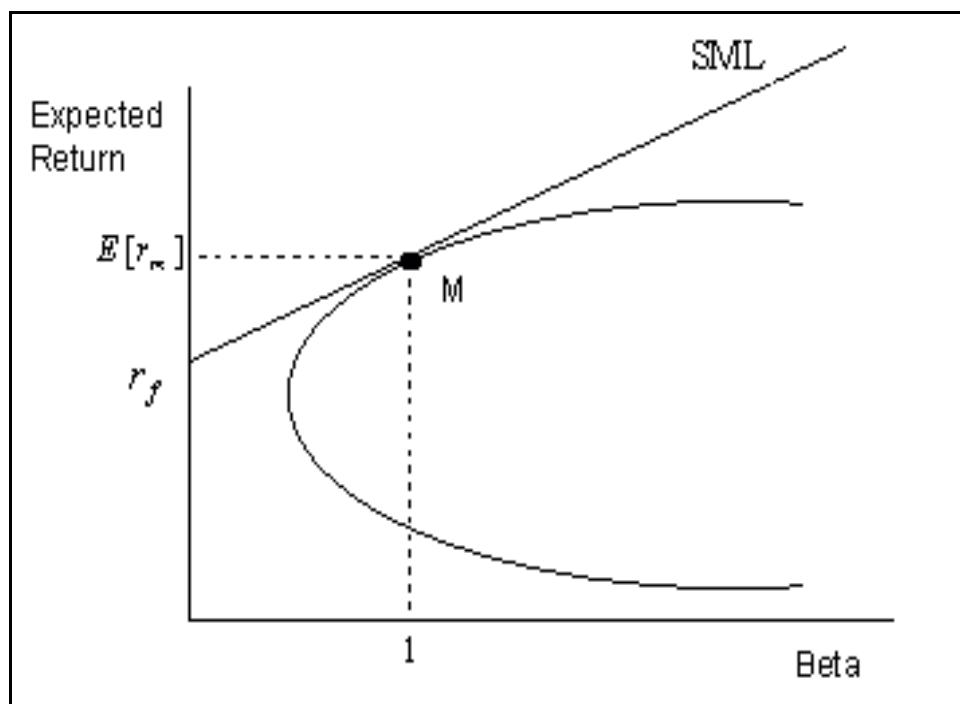
Figure 5.2 Graph of Security Market Line

On the other hand, capital market line illustrates the risk premiums of efficient portfolios which composed of risky assets and risk free assets as a function of portfolio standard deviation. The security market line, in contrast, graphs individual asset risk premiums as a function of asset risk.

We know that if market equilibrium is to exist, the prices of all assets must adjust until all are held by investors. There can be no excess demand in other words prices must be established so that the supply of all assets equals the demand for holding them. Consequently, in equilibrium the market portfolio will consist of all marketable assets held in proportion to their value weights. The equilibrium proportion of each asset in the market portfolio must be;

$$w_i = \frac{\textit{market value of the individual asset}}{\textit{market value of all assets}}$$

Equation (5.14)

A portfolio consisting of *a%* invested in risky asset and (1-*a*) % in the market portfolio will have the following mean and standard deviation:

$$E(R_p) = aE(R_i) + (1-a)E(R_m),$$

Equation (5.15)

$$\sigma(R_p) = \left[a^2\sigma_i^2 + (1-a)^2\sigma_m^2 + 2a(1-a)\sigma_{im}\right]^{1/2},$$

Equation (5.16)

where

σ_i^2 = the variance of risky asset

σ_m^2 = the variance of the market portfolio

σ_{im} = the covariance between risky asset and the market portfolio

As mentioned previously, Figure 4.3 shows the expected return and standard deviation of the market portfolio, *M*, the risk free asset, R_f, and risky asset. The straight line in the Figure 4.3 connecting the risk-free asset and the market portfolio is the capital market line. It could be said that the market portfolio already contains risky asset held according to its market value weight. In fact the definition of the market portfolio is that it consists of all assets held according to their market value weights. The opportunity set provided by various combinations of the risky asset and the market portfolio is the line of opportunity set in the figure.

The change in the mean and the standard deviation with respect to the percentage of the portfolio, *a*, invested in risky asset is determined as follows:

$$\frac{\partial E(R_p)}{\partial a} = E(R_i) - E(R_m),$$

Equation (5.17)

$$\frac{\partial \sigma(R_p)}{\partial a} = \frac{1}{2}\left[a^2\sigma_i^2 + (1-a)^2\sigma_m^2 + 2a(1-a)\sigma_{im}\right]^{-1/2}$$

$$\text{x }\left[2a\sigma_i^2 - 2\sigma_m^2 + 2a\sigma_m^2 + 2\sigma_{im} - 4a\sigma_{im}\right]$$

Equation (5.18)

Sharpe's and Treynor's insight, which permitted them to use the above equations to determine a market equilibrium price for risk, was that in equilibrium the market portfolio already has the values weight, w_i percent, invested in the risky asset. Therefore the percentage a in the above equations is the excess demand for an individual risky asset. However, as it is known that in equilibrium the excess demand for any asset must be *zero*. Prices will adjust until all assets are held by someone. That is why, if the equations 5.17 and 5.18 are evaluated where excess demand, a, equals zero, then it could be determined the equilibrium price relationships at point M in Figure 4.3. This will gives the equilibrium price of risk. After revising the equations 5.17 and 5.18 where $a= 0$, it is obtained;

$$\left.\frac{\partial E(R_p)}{\partial a}\right|_{a=0} = E(R_i) - E(R_m),$$

Equation (5.19)

$$\left.\frac{\partial \sigma(R_p)}{\partial a}\right|_{a=0} = \frac{1}{2}\left(\sigma_m^2\right)^{-1/2}\left(-2\sigma_m^2 + 2\sigma_{im}\right) = \frac{\sigma_{im} - \sigma_m^2}{\sigma_m}$$

Equation (5.20)

The slope of the risk return trade-off assigned at point M, in market equilibrium, is

$$\left.\frac{\partial E(R_p)/\partial a}{\partial \sigma(R_p)/\partial a}\right|_{a=0} = \frac{E(R_i) - E(R_m)}{\left(\sigma_{im} - \sigma_m^2\right)/\sigma_m}$$

Equation (5.21)

The distinctive perception is to notice that the slope of the opportunity set provided by the relationship between the risky asset and the market portfolio at point M must also be equal to the slope of the capital market line, R_f M.

The capital market line is also an equilibrium relationship, the tangency portfolio, M, must be the market portfolio where all assets are held according to their market value weights. And the slope of the market line is:

$$\frac{E(R_m) - R_f}{\sigma_m}$$
Equation (5.22)

where σ_m is the standard deviation of the market portfolio. When this formulation is equated with the slope of the opportunity set at the point M, it will be gotten;

$$\frac{E(R_m) - R_f}{\sigma_m} = \frac{E(R_i) - E(R_m)}{\left(\sigma_{im} - \sigma_m^2\right)/\sigma_m}$$
Equation (5.23)

The equation could be rearranged to solve for $E(R_i)$ as follows:

$$E(R_i) = R_f + \left[E(R_m) - R_f\right]\frac{\sigma_{im}}{\sigma_m^2}$$
Equation (5.24)

This equation is known as the capital asset pricing model. It is presented graphically in Figure 5.3 below where it is also called the security market line.
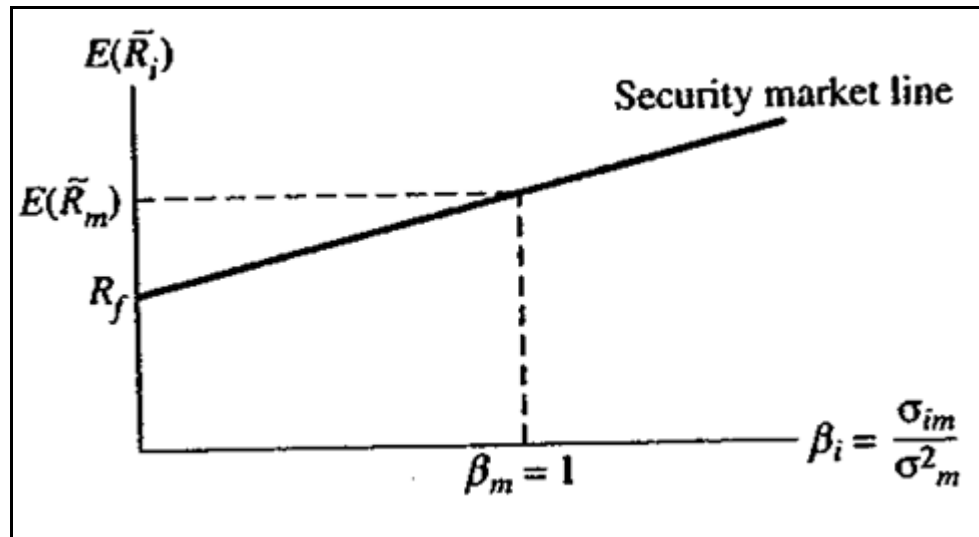
Figure 5.3 Security market line (Copeland, Thomas E; Weston, J. Fred; Shastri, Kuldeep, Financial Theory and Corporate Policy, Fourth Edition, Pearson Addison Wesley, pg 151)

The required rate of return on any asset, in Equation (5.24), is equal to the risk-free rate of return plus a risk premium. The risk premium is the price of risk multiplied by the quantity of risk which it is σ_{im}/σ_m^2 . In the terminology of the CAPM, the price of risk is the slope of the line, the difference between the expected rate of return on the market portfolio and the risk-free rate of return. On the other hand, the quantity of the risk is in fact the beta, β_i which it is the covariance of the stock's return with the market returns divided by the market variance which it is formulized as:

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} = \frac{Cov(R_i, R_m)}{Var(R_m)}$$ Equation (5.25)

The equation above shows the covariance between returns on the risky asset (i) and the market portfolio, (M), divided by the variance of the market portfolio. The risk-free asset has a beta of zero because its covariance with the market portfolio is zero. The market portfolio has a beta of one because the covariance of the market portfolio itself is identical to the variance of the market portfolio which is formulized as;

$$\beta_m = \frac{Cov(R_i, R_m)}{Var(R_m)} = \frac{Var(R_m)}{Var(R_m)} = 1$$

Equation (5.26)

5.4 Characteristic of the CAPM

In equilibrium, every asset must be priced so that its risk adjusted required rate of return falls exactly on the straight line in the security market line which is shown in Figure 5.3, because not all the variance of an asset's return is of concern to risk-averse investors. Investors could always diversify away all risk except the covariance of an asset with the market portfolio that is they can diversify away all risk except the risk of the economy as a whole. The only risk that investors will pay a premium to abstain from is the covariance risk.

As mentioned in section 2 main concepts, total risk of any individual asset could be partitioned into two parts; systematic risk which is a measure of how the asset covaries with the economy, and unsystematic risk which is independent of the economy;

Total risk=systematic risk + unsystematic risk

The return on any asset is a linear function of market return plus a random error term ε_j, which is independent of the market:

$$R_j = a_j + b_j R_m + \varepsilon_j$$

Equation (5.27)

The formulation has three concepts: a constant, a_j which has no variance; a constant times a random variable, $b_j R_m$; and a second random variable, ε_j which has zero covariance with R_m. And variance of this relationship could be written as

$$\sigma_j^2 = b_j^2 \sigma_m^2 + \sigma_\varepsilon^2$$

Equation (5.28)

The variance represents total risk. It could be divided into systematic risk which is the first portion of the equation, $b_j^2 \sigma_m^2$, and unsystematic risk, σ_ε^2. It proves that b_j in the simple linear relationship between individual asset return and market return is the same as β_j in the CAPM. Furthermore, as it is known that if investors are risk averse, there should be a positive trade off between risk and return. When the standard deviation is used as a measure of risk for individual security in comparison with a well diversified portfolio, it led to make the inappropriate observation that the security with higher risk has a lower return. It is a wrong measure of risk because it cannot be compared the variance of return on a single asset with the variance for a well diversified portfolio. The variance of the portfolio will almost always be smaller. Therefore the appropriate measure of risk for a single asset is beta which its covariance with the market divided by the variance of the market. This risk is nondiversifiable and is linearly related to the rate of return.

The measure of risk for individual assets is linearly additive when the assets are combined into portfolios. For instance, if someone puts a% of his wealth into asset X, with a systematic risk of β_x, and b% of his wealth into asset Y, with a systematic risk of β_y, then the beta of the resulting portfolio, β_p, is simply the weighted average of the betas of the individual assets which is shown below:

$$\beta_p = a\beta_x + b\beta_y \qquad\qquad \text{Equation (5.29)}$$

Evidence of this follows from the definition of covariance and the properties of the mean and variance. The definition of the portfolio beta is

$$\beta_p = \frac{E\{[a\,X + b\,Y - aE(X) - bE(Y)][R_m - E(R_m)]\}}{Var(R_m)}, \text{ rearranging the terms;}$$

$$\beta_p = \frac{E\{[a[X - E(X)] + b[Y - E(Y)]][R_m - E(R_m)]\}}{Var(R_m)}$$

$$\beta_p = a \frac{E\left[(X - E(X))(R_m - E(R_m))\right]}{Var(R_m)} + b \frac{E\left[(Y - E(Y))(R_m - E(R_m))\right]}{Var(R_m)} \qquad \text{Equation (5.30)}$$

and using the definition of beta,

The equation equals to $\beta_p = a\beta_x + b\beta_y$ that is formulated in the Equation (5.29). As shown from the equation the portfolio betas are linearly weighted combinations of individual asset betas. Therefore, to be able to measure the systematic risk of portfolios the betas of individual assets are needed to known. Furthermore, the correct definition of a particular asset's risk is its contribution to portfolio risk which is also explained in Equation (2.21) at part 2. Referring the equation again variance of the portfolio return is;

$$Var(R_p) = \sigma^2(R_p) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} \quad \text{which can be rewritten as}$$

$$\sigma^2(R_p) = \sum_{i=1}^{N} \left(\sum_{j=1}^{N} w_j \sigma_{ij} \right) = \sum_{i=1}^{N} w_i Cov(R_i, R_p) \qquad \text{Equation (5.31)}$$

Portion $w_i Cov(R_i, R_p)$ of the equation could be interpreted as the risk of security "i" in portfolio "p". However, the change in the contribution of asset "i" to portfolio risk is simply the covariance of the asset with the portfolio that is symbolized as $Cov(R_i, R_p)$. Therefore covariance risk is the appropriate definition of risk since it measures the change in portfolio risk as the weighting of an individual asset in the portfolio varied.

5.5 Beta Estimation

There are some basic methods available in order to predict the beta. The first is to apply the historical data on market prices for individual investments. The other way is to estimate the

betas from the essential characteristics of the investment. And lastly, the third approach is to use the accounting records.

5.5.1 Historical Market Betas

The common way for estimating the beta of an investment is a regression of the historical returns on the investments against the historical returns on a market index (Jones, 1991). For assets traded in a stock exchange market for an effectual period of time, it is relatively simple to predict returns that an investor would have made on investing in stock in intervals (i.e. weekly, monthly or annually) over that period. In theory, these stock returns on the assets should be correlated with returns on a market portfolio. On the other hand, in practice, one might be likely to use a stock index, such as the XU-100, XU-BANK etc. as a proxy for the market portfolio, and might predict betas for stocks against the index. In this study betas are calculated over historical data by accepting XUTUM as a market proxy.

Regressing stock returns, which denoted as R_i, against market returns, R_M is a common practice to predict betas;

$$R_i = a + b\,R_m \hspace{4cm} \text{Equation (5.32)}$$

where "a" represents the intercept from regression and "b" represents the slope of the regression which equals $Cov(R_i, R_m)/\sigma_M^2$.

The slope of the regression equals to the beta of the stock and quantifies the riskiness of the stock. Moreover, the intercept of the regression assures an easy measure of performance on the investment during the regression period when returns are measured against the expected returns from the capital asset pricing model. This performance measurement could be emphasized by rearranging the CAPM equation as follows:

$$R_i = R_f + \beta \left(R_M - R_f \right) = R_f \left(1 - \beta \right) + \beta \, R_M \qquad \qquad \text{Equation (5.33)}$$

Comparing equation (5.33) of the return on investment to the return equation from the regression in equation (5.32) which is $R_i = a + b \, R_m$:

So that a comparison of the intercept "a" and $R_f \left(1 - \beta \right)$ will provide a measure of stock's performance. Briefly;

If $a > R_f \left(1 - \beta \right)$, stock did better than expected during the regression period

If $a = R_f \left(1 - \beta \right)$, stock did as well as expected during the regression period

If $a < R_f \left(1 - \beta \right)$, stock did worse than expected during the regression period

The difference between "a" and $R_f \left(1 - \beta \right)$ is called Jensen's alpha and provides a measure of whether the investment earned a return greater than or less than its required return under given market performance and risk (Jensen, 1968)

On the other hand, instead of historical market betas adjusted betas are usually preferred. In this model, the beta coefficients of stocks move towards one over time (Jones, 1991). Adjusted beta approach bears both intuitive and statistical explanations. In statistical terms, the best estimation of a stock beta would be unity since the average beta of all stocks is one. When the beta coefficient is predicted over a sample period, some unknown sampling errors are sustained. The greater the difference between the beta prediction and unity, the greater is the possibility that there will be huge prediction error incurred. For the sample period, historical market beta is good estimate but if the beta coefficient tends to move to one in the long run, a forecast of the future beta should be adjusted. Moreover, for intuitive approach, assume two companies one is more conventional and the other is unconventional. As firms becomes more conventional, it starts to resemble the rest of the economy even more, thus its beta coefficient will tend to close in the direction of one.

Betas are adjusted with the following formulation especially by many investment professionals;

$$Adjusted\ beta = (2/3)\cdot(Historical\ beta) + (1/3)\cdot(1)$$ Equation (5.34)

Data providers and investment companies generally use the stated weights (2/3 and 1/3) which in fact totally subjective numbers in their computations. This aims to adjust beta forecast toward 1 which should also be the average beta of all securities.

5.5.2 Fundamental Beta

Considering the fundamental financial variables of a company is another method to estimate betas (Jones, 1991). In this method, beta of a firm may be estimated from a regression which taking into account firm's size, financial leverage, debt ratio and other financial indicators.

Rosenberg and Guy (1976) listed following beta factor variables to estimate betas; variance of cash flows, variance of earnings, market capitalization, dividend yield, growth in earnings per share and debt to asset ratio.

The beta forecast regression could be established as follows under these beta determinants:

$$Current\ beta = a + b_1(Historical\ beta) + b_2(var\,iance\ of\ cash\ flows) + \cdots\cdot b_6(debt\ ratio)$$

Equation (5.35)

By predicting b_1, b_2, ...b_6, future betas could be estimated.

5.5.3 Accounting Beta

Another approach is to estimate the market risk variables from accounting earnings rather than from traded prices (Jones, 1991). So earning changes could be related to the changes in earnings for the whole market.

Many of the earlier studies examine the relationship between market betas and several accounting variables including debt to equity ratio, dividends, growth, and accounting beta empirically. For instance, Ball and Brown (1969) evaluate the ability of accounting measures of risk (operating income, net income, and earnings per share) to convey information about the risk of the firm to the market. Applying regression analysis on a sample of 261 firms over the period 1946-66, they conclude that at approximately 35 to 40 percent of the cross-sectional variability in the systematic risk can be explained by the co-movement in the accounting income of firms. Beaver, Kettler, and Scholes (1970), another empirical study, examine whether accounting variables (payout ratio, growth, leverage, liquidity, size, earning variability, and accounting beta) can be viewed as surrogates for the total variability of market return. The Beaver, *et. al* (1970) study, based on a sample of 307 firms for two periods 1947-56 and 1957-65, find that the degree of association between accounting and market betas is 44 and 23 percent within the two periods respectively. The results are greatly increased when analyzed on a portfolio basis (i.e., the association increases to 68 and 46 percent respectively). Gonedes (1973) examines whether the evidence provided by the studies of Ball and Brown (1969) and Beaver, et al (1970), regarding the correlation between market-based and accounting-based estimates of systematic risk, is valid. Applying regression analysis to a sample of 99 firms randomly chosen from those listed on the New York Stock Exchange, Gonedes finds a statistically significant relationship between market-based and accounting-based estimates of systematic risk but at a much lower level. Gonedes explains the differences to the market deflator of accounting earnings used. The studies of Ball and Brown (1969) and Beaver, et al (1970) use the market value of common equity to deflate accounting earnings versus his usage of the book value of total assets.

However, there are some drawbacks of this approach. Accounting earnings tend to be smooth relative to the underlying value of the firms because income and expenses are spread to multiple periods. Thus, this will result in betas that are "biased down" for risky firms and "biased up" for safer firms. So, betas are likely to be closer to one for all firms using accounting data (Beaver and Mangold, 1975)

Furthermore, accounting earnings are affected by non-operating factors like changes in depreciation methods or allocating expenses at the divisional level. And finally, accounting earnings are measured at most every quarter and often once a year, therefore limited number of observation supports the regression model which could not provide a much explanatory power such as high standard errors.

5.6 Unrealistic Assumptions and Limitations of CAPM

The assumptions mentioned in the 5.1 section oversimplify the real world. In reality, investors cannot borrow and lend at the risk-free rate. Additionally, CAPM ignores taxes and transaction costs which usually exist and influence investment decisions. Recent researches have questioned the assumption of CAPM by finding systematic deviations from the CAPM and the portfolio theory. Empirical confirmation of CAPM has been debated for years by Black, Jensen and Scholes, (1972); Fama and MacBeth, (1973); Roll, (1977); Gibbons, (1982); Fama and French, (1992). These new evidences and unrealistic assumptions will be examined in the following.

First, CAPM states that investors could borrow and lend at the risk free rate. However, this is not the case in reality. This problem was solved by Black (1972). His argument is illustrated in Figure 5.4 below:
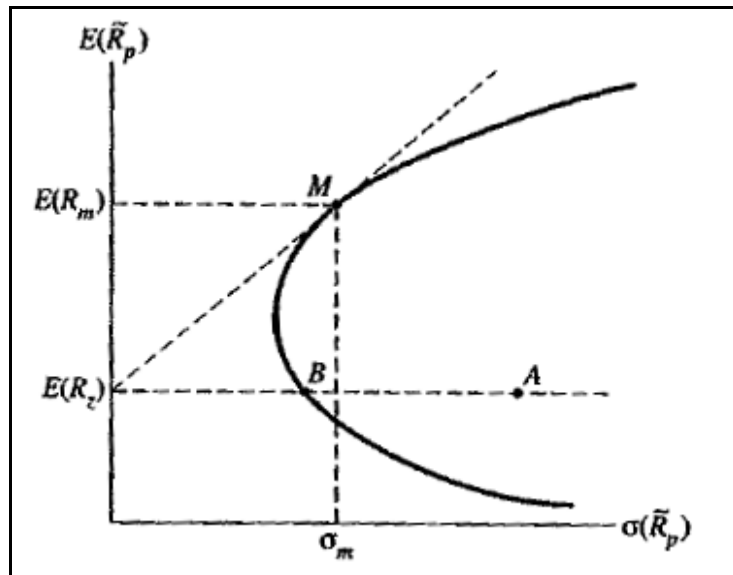
Figure 5.4 Capital market line with no risk free rate

Portfolio M is identified by the investors as the market portfolio that lies on the efficient set. It could be supposed that all portfolios that are uncorrelated with the true market portfolio. This means that their returns have zero covariance with the market portfolio, and they have the same systematic risk. Also each portfolio must have the same expected return since they have the same systematic risk. Portfolios A and B in Figure 5.4 are both uncorrelated with the market portfolio M and have the same expected return, $E(R_z)$. However, only portfolio B lies on the opportunity set which is the minimum-variance zero-beta portfolio. Portfolio A also has zero betas, but it has a higher variance and therefore does not lie on the minimum-variance opportunity set.

The slope of the line $E(R_Z)$ M can be driven by forming a portfolio with a% in the market portfolio and (1-a) % in the minimum-variance zero-beta portfolio. The mean and standard deviation of such a portfolio can be written as;

$$E(R_p) = a\,E(R_m) + (1-a)E(R_Z),$$

$$\sigma(R_p) = \left[a^2\sigma_m^2 + (1-a)^2\sigma_Z^2 + 2a(1-a)r_{Zm}\sigma_Z\sigma_m\right]^{1/2}$$

<div align="right">Equation (5.36)</div>

However, the last term drops out since the correlation, r_{Zm}, between the zero-beta portfolio and the market portfolio is zero. The slope of a line tangent to the efficient set at point M, where 100% of the investor's wealth is invested in the market portfolio, could be driven by taking the partial derivatives of the equation 5.36 and evaluating them where a=1. The partial derivative of the mean portfolio return and the standard deviation are respectively,

$$\frac{\partial E(R_p)}{\partial a} = E(R_m) - E(R_Z),$$

$$\frac{\partial \sigma(R_p)}{\partial a} = \frac{1}{2}\left[a^2\sigma_m^2 + (1-a)^2\sigma_Z^2\right]^{-1/2}\left[2a\sigma_m^2 - 2\sigma_Z^2 + 2a\sigma_Z^2\right]$$

<div align="right">Equation (5.37)</div>

Taking the ratio of these partials and evaluating where a=1, the slope of the line $E(R_Z)$ M could be obtained;

$$\frac{\partial E(R_p)/\partial a}{\partial \sigma(R_p)/\partial a} = \frac{E(R_m) - E(R_Z)}{\sigma_m}$$

<div align="right">Equation (5.38)</div>

Additionally, since the line must pass through the point $[E(R_m), \sigma(R_m)]$, the intercept of the tangent line must be $E(R_Z)$. Consequently, the equation of the line must be;

$$E(R_p) = E(R_Z) + \left[\frac{E(R_m) - E(R_Z)}{\sigma_m}\right]\sigma_p$$

<div align="right">Equation (5.39)</div>

Given the above result, the expected rate of return on any risky asset must be a linear combination of the rate of return on the zero-beta portfolio and the market portfolio. To illustrate this, recall that in equilibrium the slope of a line tangent to a portfolio composed of the market portfolio and any other asset at the point represented by the market portfolio must be equal to equation 5.21 which is shown in section 5.2- Derivation of the CAPM-;

$$\frac{\partial E(R_p)/\partial a}{\partial \sigma(R_p)/\partial a}\bigg|_{a=0} = \frac{E(R_i) - E(R_m)}{(\sigma_{im} - \sigma_m^2)/\sigma_m}$$ if two definitions of the slope of a line tangent to point M

are equated, that is if the equations (5.21) and (5.38) are equated, it will be obtained;

$$\frac{E(R_m) - E(R_z)}{\sigma_m} = \frac{[E(R_i) - E(R_m)]\sigma_m}{\sigma_{im} - \sigma_m^2}$$ and solving the equation for the required rate of return

on asset "i";

$$E(R_i) = (1 - \beta_i)E(R_z) + \beta_i E(R_m)$$ Equation (5.40)

where $\beta_i = \sigma_{im}/\sigma_m^2 = Cov(R_i, R_m)/\sigma_m^2$

Equation (5.40) shows that the expected rate of return on any asset can be written as a linear combination of the expected rate of return of any two assets- the market portfolio and the unique minimum-variance zero-beta portfolio. Interestingly, the weight to be invested in the market portfolio is the beta of the ith asset. If equation (5.40) is rearranged, it will be seen that it is exactly equal to the CAPM except that the expected rate of return on the zero-beta portfolio has replaced the rate of return on the risk-free asset:

$$E(R_i) = E(R_z) + [E(R_m) - E(R_z)]\beta_i$$ Equation (5.41)

The inference of this proof is that the main results of the CAMP do not require the existence of a pure riskless asset.

Second subject inapplicable with CAPM assumption is the existence of nonmarketable assets. Assume that the cost of transacting in an asset is infinite or the asset is not marketable. For instance such an asset is human capital which cannot be traded, slavery is forbidden. This has the effect of introducing a non-diversifiable asset into the portfolio- human asset capital, because human skills cannot be divided up and cannot be sold to different investors.

If there are no transaction costs and if all assets are perfectly divisible, every investor regardless of the shape of indifference curve will hold one of two assets: the risk-free asset or the market portfolio. However, casual empiricism tells that this is not what actually happens since people hold different portfolios of risky assets. Mayers (1972) exhibits that when investors are restrained to hold nonmarketable assets that have risky rates of return, denoted as R_C, the CAPM takes the following form:

$$E(R_j) = R_f + \lambda \left[V_m Cov(R_j, R_m) + Cov(R_f, R_C) \right]$$ Equation (5.42)

where

$$\lambda = \frac{E(R_m) - R_f}{V_m \sigma_m^2 + Cov(R_m, R_C)} \ ,$$

V_m = the current market value of all marketable assets

R_C = the total risky asset return on all nonmarketable assets

It can be inferred from this equation; first market equilibrium price of a risky asset may be determined independently of the shape of the individual's indifference curves, that is, the market price of risk is independent of individual attitudes and preferences toward risk. In Equation (5.41) no variable is subscripted for the preference of the ith individual, both the price of risk and the amount of risk depend only on properties of the jth asset, the portfolio of all

marketable assets and the portfolio of aggregated nonmarketable assets. Furthermore, individuals will hold different portfolios of risky assets because their human capital has differing amounts of risk. Lastly, the appropriate measure of risk is still the covariance, but it must be considered the covariance between the jth risky asset and two portfolios, one composed of marketable and a second of nonmarketable assets, Fama and Schwert (1977).

On the other hand, Merton (1973) has derived a version of CAPM which assumes that trading occurs continuously over time, and asset returns are distributed lognormally. If the risk free rate of interest is non-stochastic over time, the equilibrium returns must comprise the following equation regardless of individual preferences, distribution of their wealth or their time horizon;

$$E(R_i) = R_f + \left[E(R_m) - R_f\right] \beta_i \qquad \text{Equation (5.43)}$$

The equation is continuous time analogy to the CAPM, it is exactly the same as the CAPM except that instantaneous rates of return have replaced rates of return over discrete intervals of time and the distribution of returns is lognormal instead of normal. If the risk-free rate is stochastic, investors are subject of another kind of risk, which is the risk of unfavorable shifts in the investment opportunity set. Merton points out that investor will hold portfolios consisted of three funds; the riskless asset, the market portfolio, and a portfolio chosen so that its returns are perfectly negatively correlated with the riskless asset. And the third fund is used to hedge against unforeseen changes in the future risk free rate. In Merton's model, the required rate of return on the jth asset is formulated as;

$$E(R_j) = R_f + \gamma_1 \left[E(R_m) - R_f\right] + \gamma_2 \left[E(R_K) - R_f\right] \qquad \text{Equation (5.44)}$$

where

R_K = the instantaneous rate of return on a portfolio that has perfect negative correlation with the riskless asset

$$\gamma_1 = \frac{\beta_{jm} - \beta_{jK}\,\beta_{Km}}{1 - \rho_{Km}^2}, \quad \gamma_2 = \frac{\beta_{jK} - \beta_{jm}\,\beta_{Km}}{1 - \rho_{Km}^2}$$

ρ_{Km} = the correlation between portfolio K and the market portfolio M.

Merton claims that the sign of γ_2 will be negative for high beta assets and positive for low beta assets. And empirical tests of CAPM confirm Merton's argument.

The other arguments of unrealistic assumption of CAPM are the existence of heterogeneous expectations and taxes. If investors do not have the same information about the distribution of future returns, they will perceive different opportunity sets and will obviously prefer different portfolios. Lintner (1969) has shown that the existence of heterogeneous expectations does not critically divert the CAPM except that expected return and covariance are represented as complex weighted averages of investor expectations. But, if investors have heterogeneous expectations, the market portfolio is not necessarily efficient, which makes the CAPM non-testable.

Furthermore, Brennan (1970) has investigated the effect of different tax rates on capital gains and dividends. His model includes an extra term which causes the expected return on an asset to depend on dividend yield as well as systematic risk. The model is formulized as follows;

$$E\!\left(R_j\right) = \gamma_1 R_f + \gamma_2 \beta_j + \gamma_3 DY_j \qquad\qquad \text{Equation (5.45)}$$

where
DY_j = the dividend yield on asset j.

Shortly, Brennan's model predicts that higher rates of return will be required on assets with higher dividend yields. In other words, investors do not like dividends because they must pay

ordinary income tax rates on dividends but not only capital gains rates on stock price increases.

5.6.1 Consumption

The CAPM assumes no consumption. If consumption is added to the model, the optimal portfolio might look quite different because the concept of risk as introduced above will change. The new risk will be the financing risk: Investors are interested in having the necessary funds available at the appropriate time to finance their consumption. As most of the consumption takes place in the investor's country and is paid in his home currency, he will tend to avoid assets that pay off in foreign currencies in order to lower the exchange rate risk. (Cornell, 1981)

5.6.2 Portfolio Restrictions

Many investors would like to or have to avoid short sales and might be restricted to a minimum and a maximum percentage for the contribution of each asset. In addition, riskless borrowing might not be possible. Let us first consider the case that short sales are forbidden. Then the minimum-variance frontier does not extend to infinity and is further to the right than the original one as seen in Figure 5.5 below.
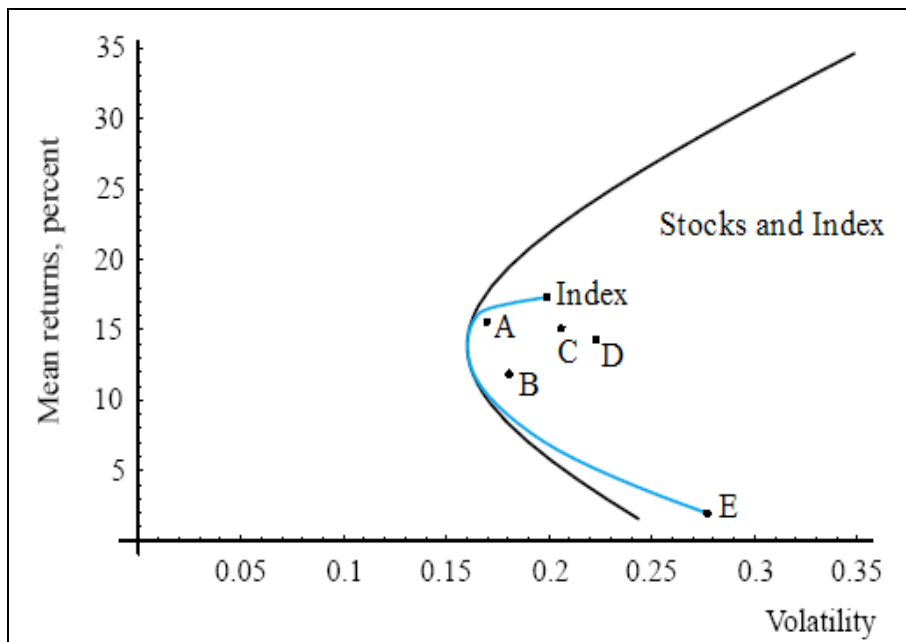
Figure 5.5 Minimum variance frontier without short sales

Introducing such restrictions comes along with a lower utility for the investors. The portfolio theory presented previously might imply that the market-portfolio consists of a considerable amount of only one stock. However, an investor normally prefers to distribute his budget more equally among the available assets in order to avoid depending on one firm only. Additionally, investment funds often have to make their investment decisions under investment restrictions.
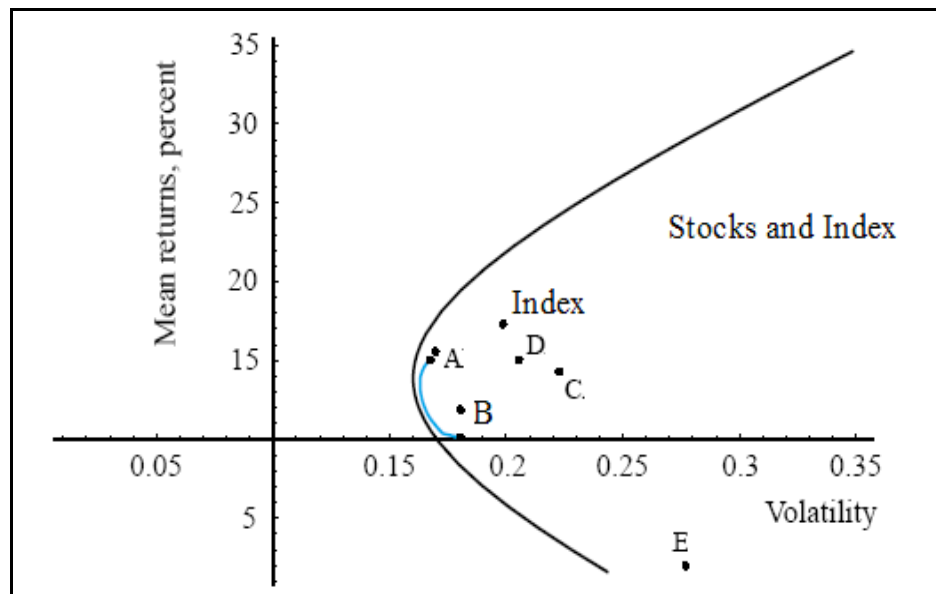
Figure 5.6 A minimum-variance frontier with restrictions: the fraction of each asset in the
portfolio is between 5% and 30%.

Figure 5.6 shows a minimum variance frontier that respects the restriction, that each asset must have a share between 5% and 30% of the portfolio. The implementation of these restrictions has the effect that high risk respectively returns portfolios cannot be achieved any longer. Additionally, the new minimum variance frontier lies further to the right and thus leads to a worse result than the one without restrictions.

5.6.3 Market Proxy Problem

Roll (1977) argues that the CAPM has never been tested and probably never will be. The problem is that the market portfolio at the heart of the model is theoretically and empirically elusive. It is not theoretically clear which assets (for example, human capital) can legitimately be excluded from the market portfolio, and data availability substantially limits the assets that are included. As a result, tests of the CAPM are forced to use proxies for the market portfolio, in effect testing whether the proxies are on the minimum variance frontier. Roll claims that such proxies are not good enough to replace the true market portfolio.

The empirical tests performed by Black, Jensen and Scholes (1972) suggest that there is at least another factor besides the market that systematically affects the returns on securities. Despite Roll's critique, according to which the CAPM cannot be tested statistically and thus cannot be rejected, the idea has been accepted that there are additional risk factors that affect asset prices. Consequently, the trend in financial markets research goes towards multifactor models.

5.6.4 Size Effect

The first anomaly in financial markets is the "size-effect" (Banz, 1981). In average, stocks of small companies (measured by their market value) tend to perform better than stocks of big companies. The data point in the top-right corner (high-beta) in Figure 5.7 is the portfolio consisting of the smallest firms; the lower left point represents the treasury bills. The CAPM demonstrates itself by the solid line which goes through the market proxy and the treasury bill rates. The dashed line shows the ordinary least square cross-sectional regression of the data points. Statistically, the small firms earn a higher return than expected by the CAPM. This is a significant failure of the CAPM, although its general prediction fits and a higher beta earns a higher return.
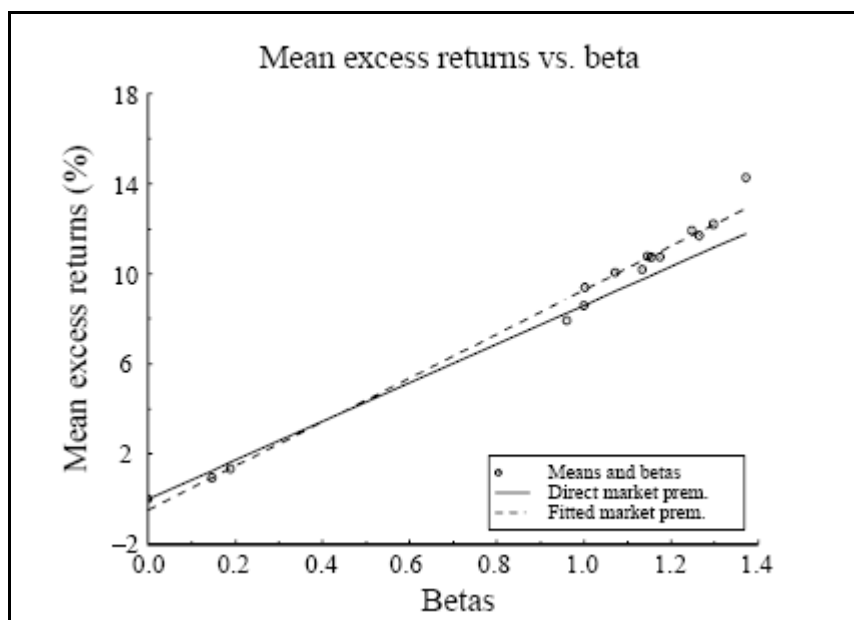

Figure 5.7 Stocks grouped by size (Cochrane, 1999).

In the above graph, ten portfolios of stocks grouped by size are shown in a $\mu - \beta$ diagram together with corporate and government bonds. The solid line represents the theoretical CAPM relation and the dashed line the empirical one, found by a cross-sectional regression.

Haugen (1995) finds similar results- below Figure 5.8-; the positive relationship between beta and the return only holds if all stocks are assessed together. After adjusting for the size effect, however, the relationship turns to negative since stocks from small companies have high beta.
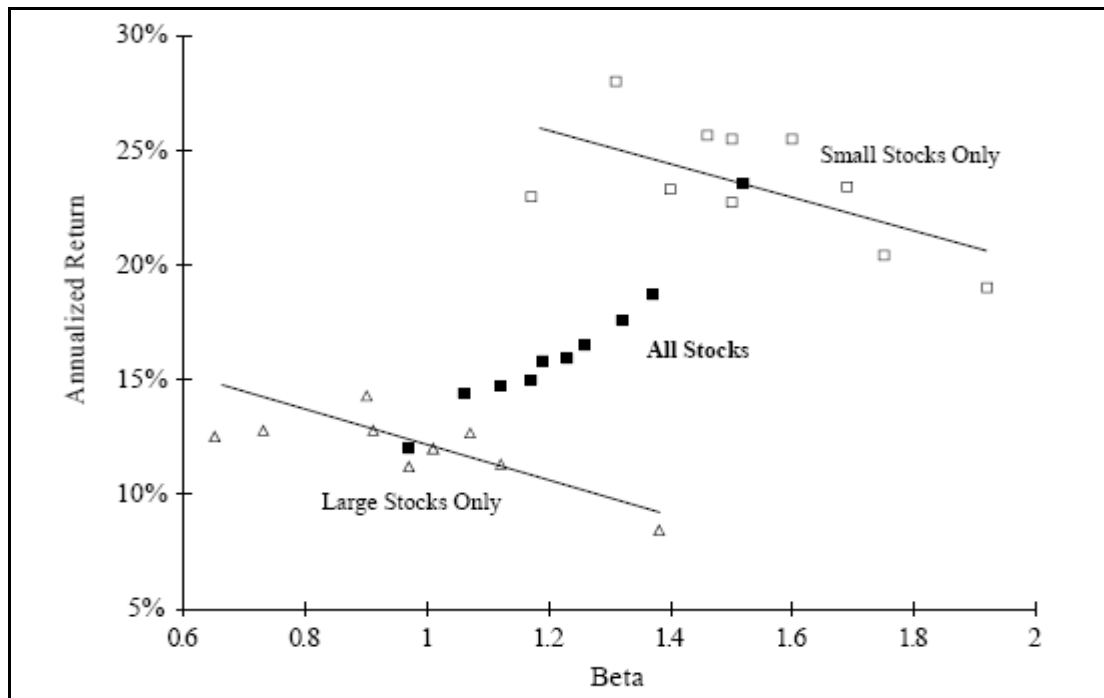
Figure 5.8 Size effect for a positive slope of SML (Haugen ,1995).

The figure evidences that the size-effect rather than betas are responsible for a positive slope of the security market line. After adjusting for the size-effect, the SML is even negative.

5.6.5 Value Effect

Another observed anomaly on financial markets is called the "value effect": Empirical evidence suggests that on average, value stocks outperform growth stocks. Value stocks are stocks with low valuations compared to their assets, measured by the book/market ratio (B/M

ratio). Growth stocks on the other hand have very high valuations relative to their assets, respectively a low B/M-ratio. They tend to be stocks of companies with very high earnings growth over the previous years, (Cochrane, 1999).

In Figure 5.9, the relationship between the size-effect and the value effect can be examined more in detail. In the left diagram, assets with similar B/M-ratios are connected by lines. For every category of similar B/M-ratios, the mean excess return increases with decreasing size. In the right diagram, in every category of similar size (connected by lines), the mean excess return increases with increasing B/M ratio. This justifies the results from Haugen (1995) in Figure 5.8 above. After adjusting for the size effect (i.e. looking at each line in the right diagram separately), it is obtained a negative relationship between the beta and the mean excess return (i.e. a negative slope of all the lines).
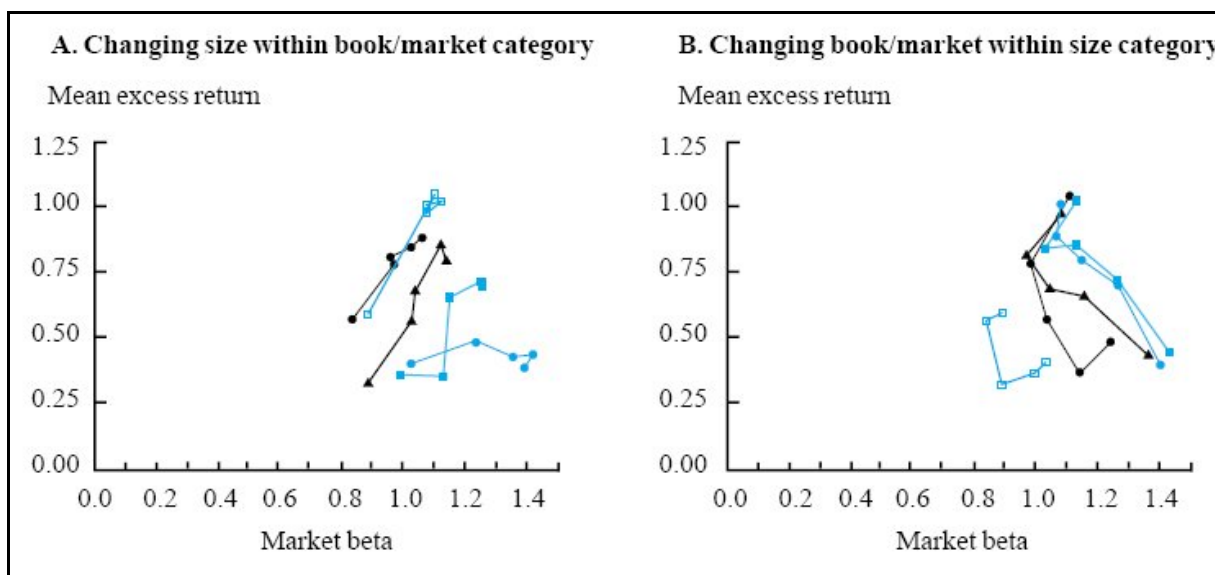

Figure 5.9 Mean excess returns against market-beta

On the graph, mean excess returns are plotted against market-beta. On the left, the lines connect changing sizes within categories of similar B/M ratio. On the right, the lines connect changing B/M ratios within categories of similar size.

5.6.6 Momentum Effect

The momentum-effect implies positive serial correlation of returns and appears primarily over short horizons of about one to twelve months, Jegadeesh and Titman (1993). An economic interpretation is that the market only after some time incorporates news into asset prices that is also called underreaction to news. An investor who knows that the asset price will only gradually adjust might want to profit from this phenomenon. He could follow the momentum strategy which means buying stocks with a good performance in the past one to twelve months and adjusting this portfolio over time. Then, on average, the investor with his portfolio of past winners will earn a superior return because the winners will continue to win and the losers will continue to lose. It should be noted that the momentum anomaly is not explained by the size- or value effect, because those effects predict high average returns for past losers. It is not yet clear whether the momentum effect is big enough to be exploitable after transaction costs since frequent changing of the portfolio is necessary to capture the momentum effect.
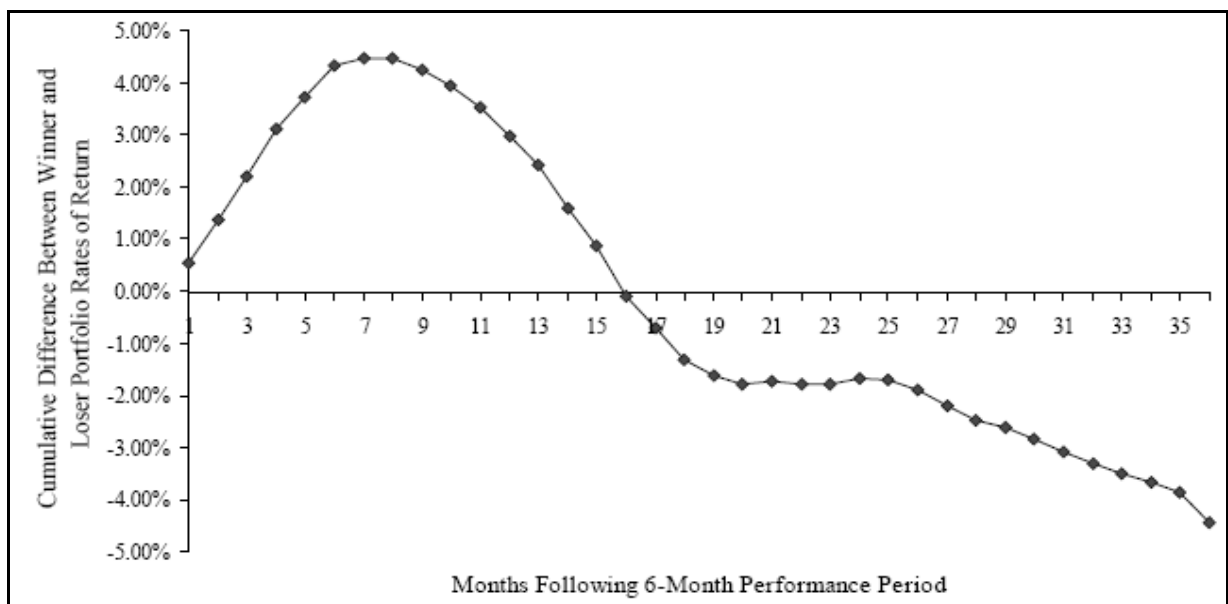

Figure 5.10 Momentum in the short run and mean-reversion in the long run

Figure 5.10, which is an example for momentum in the short run and mean reversion in the long run, from Haugen (1995) presents empirical evidence of the momentum effect. The winner portfolios which were constructed in a six month formation period continue to

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outperform the loser portfolios in the following 16 months. Thereafter, however, the loser portfolios perform better. This effect is called mean-reversion.

5.6.7 Mean Reversion Effect

Mean-reversion indicates negative serial correlation of asset returns, i.e. past loser will outperform past winners and vice versa. This effect emerges over longer horizons, about three to five years, DeBondt and Thaler (1985). The economic explanation is that investors overreact to good respectively bad news. A strategy that aims to profit from this phenomenon would be to buy stocks with a poor performance in the past. In contrast to the momentum-strategy, this is a long-term strategy.

5.6.8 Horizon Effects

Samuelson and Merton (1969) observe that if assets are independently and identically distributed, an investor who rebalances his portfolio optimally should choose the same asset allocation, regardless of investment horizon. In the case of predictable returns, however, the investor's investment horizon may no longer be irrelevant. If returns are predictable, mean and variance no longer can be predicted in an easy way because of positive or negative serial correlation. In the case of positive serial correlation, the variance increases faster over time than the return does and stocks are less attractive for the long run. On the other hand, in the case of mean reversion, the opposite is true and stocks are more attractive over long horizons. Barberis (2000) estimates significant mean reversion in financial markets. This suggests that stocks are safer in the long run and that an investor with a long investment horizon holds a greater proportion of stocks in his portfolio. Thus, predictability does have an effect on portfolio choice.
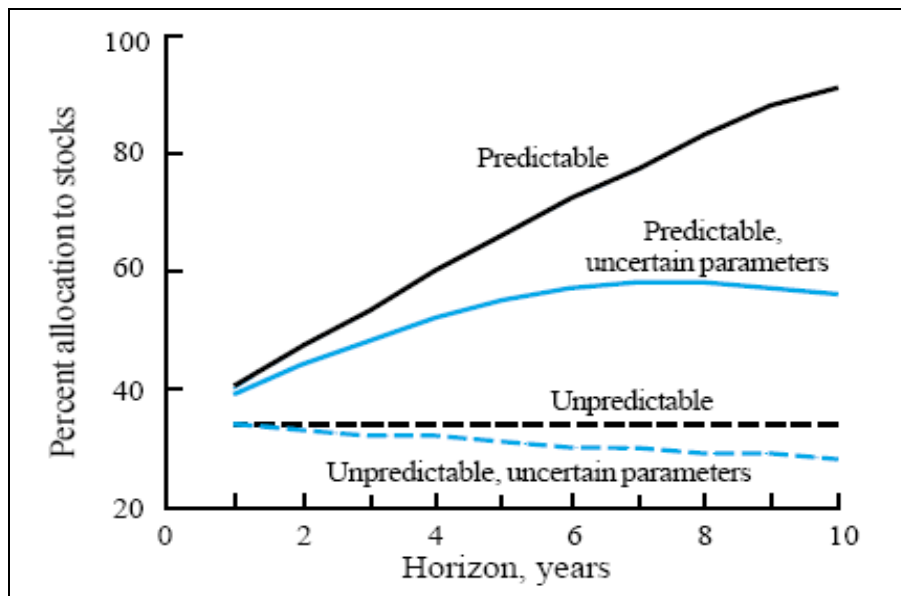
Figure 5.11 The effect of estimation risk and the investment horizon

The graph illustrates the effect of estimation risk and the investment horizon on the optimal allocation of stocks. It is important to take into account the estimation risk of predictability. There is some standard error that leads to uncertainty which is harmful to risk-averse investors. Thus, this uncertainty lowers the optimal allocation to stocks compared to the case with fully predictable returns. The longer the horizon, the smaller the proportion of stocks an investor will choose to hold. In Figure 5.11, shows that for an investment horizon of ten years, the optimal allocation to stocks with uncertain predictability is only half as much as in the case of full certainty, (Barberis, 2000). But there is still enough predictability in returns to make investors allocate substantially more to stocks, the longer their horizon. However, the difference is not as large compared to predictability and full certainty. A long-horizon investor who ignores estimation risk may over-allocate to stocks by a sizeable amount.

PART 6 TESTING CAPM FOR BANKING SECTOR

6.1 Test Methodology and Data

For the purpose of single index CAPM testing, one of the important considerations which should be regarded during the data establishment process is the length of the estimation period. A longer estimation period will certainly provide much data, but the companies or industries or even countries might have changed in their risk features over the time period.

Generally, a much more longer estimation time period were used for instance Sharpe's (1964) study covers period of 1927 – 1963; on the contrary rating and investment institutions like Standard & Poors and Bloomberg use shorter period of data such as five and two-three years of data.

When we look at the dynamics of Turkey, many firms and holdings invested in different sectors and many companies also changed their ways of doing business with the developments of both technology and know-how during the last decade. Therefore, investment environment in Turkey could be seen very dynamic for institutions and individual investors who trade stocks as well as it is dynamic for other stakeholders. To be able to cover the impact of the changes, caused by the dynamic environment, in risk character of establishments, the test period is determined as August 1999 and August 2009 as a recent date.

The length of the testing period could be seen very long to make this research for the ISE. Actually, this is a period during which the economic, political and financial environment changed a great deal. Currency crises, high inflation, budget and balance of payment deficits as well as unemployment were the major problems challenging economic stability during the mentioned period. Furthermore; high growth rates, participation to customs union and increasing regulatory standards were some factors that had positive impact on risk characters of firms.

Secondly, return intervals could be regarded as an important concern for the estimation issue.

Using daily or intra-day returns would increase the number of observations in the regression. When approximately ten years of sample period is considered monthly returns and risk free rates were assumed to be significant enough to be used for the purpose of this study.

The other estimation concern is the choice of a market index to be used in the regression. The general practice used in literature is to estimate the betas of stocks relative to the index of the market in which the stocks are traded. The crucial problem in selection of the market index is that the indices which measure market returns in small markets like Istanbul Stock Exchange tend to be dominated by a few large companies or companies of a holding or a group. Hence, using XU-030 or XU-050 indices could provide biased results in beta estimates of the firms not included in the index calculation. Hence, it was decided to use the XUTUM index which considers more firms as the market proxy for the testing purpose. XUTUM index is also calculated since establishment of the Stock Exchange Market; and hence, the only one providing more clue as the market proxy. The index is a kind of weighted average index and is calculated with the formulation below:

$$E_t = \frac{\sum F_{it} * \overset{n}{\underset{i=1}{N}}_{it} * H_{it}}{B_t}$$

Equation (6.1)

where,

E_t is the index value at time t

n is the number of stocks in the XUTUM index

F_{it} is the price of stock i at time t

N_{it} is the total number of issued stocks of I

H_{it} is the ratio of public offer of stock i at time t

B_t is the adjusted base market capitalization (overall value)

Indices are used to depict the overall health of the economy as well. Since, the XUTUM index is calculated with the weighted average principle it could be assumed that no particular company or a group will have a dominant impact on beta regressions.

Daily closing prices of the stocks were obtained from the ISE electronic database. Over the stock prices received necessary price adjustments, which caused by dividend distributions, share splits, and etc., are applied to the data. Then stock returns calculated by using logarithmic function with the formula mentioned previously.

$$r_t = \log \frac{P_t}{P_{t-1}} \text{ or } \log P_t - \log P_{t-1}$$ Equation (6.2)

where,

P_t is the price of the stock at time t

P_{t-1} is the price of the stock at time $t-1$

Only the banking shares which have been traded during the observation period of the research (August 1999-2009) are included in our analysis. Due to the technical difficulties only daily closing prices could be achieved rather than intra-day prices. Moreover, the days stocks were not traded also excluded from the analysis. Totally 11 banking sector stocks, 5 indices and approximately 40,000 daily returns are considered in the analysis. Over these daily data, weekly and monthly data were calculated.

To calculate risk premiums of the market and stocks, interest rates were obtained from the Treasury's electronic database and converted to continuous compounding rates. Rates of the index government bonds are taken benchmark interest and accepted as risk free interest rate. As it is clearly seen risk free rate fluctuations are high which fluctuates between among 10,51 % and even almost 1400,00 % with a mean of 53,90 % and a high standard deviation of 66 % which proves the magnitude of the fluctuations. These fluctuations indicate how economic conditions have changed during the testing period.

Risk premium of a stock is the expected excess return of a stock over the risk-free rate. Hence; once obtaining the stock returns and the risk free rate, it is easy to measure the risk premium of any stock for any period of time by using the following equation.

Risk premium of stock k in time $t = r_{kt} - r_{ft}$
Equation (6.3)

where r_{kt} and r_{ft} are the returns of stock k and risk-free rate at the time of t respectively.

In a similar way, the market risk premium is calculated for August 1999 - 2009 period as presented in table 6.1 below. In the table, average of monthly, weekly and daily returns and their risk premiums are also indicated. The average risk premium of the market or a stock represents how well or worse it has performed over risk free rate during the research period. In other words, one may anticipate that stock (a) will have over or under perform the risk free rate at the average risk premium rate.

Table 6.1 Risk premiums of stocks and indices

Stocks and Indices	Average Monthly Returns	Risk Premiums of Monthly Returns	Average Weekly Returns	Risk Premiums of Weekly Returns	Average Daily Returns	Risk Premiums of Daily Returns
GARAN	2,64%	-0,65%	0,60%	-0,10%	0,12%	0,02%
AKBNK	2,45%	-0,84%	0,56%	-0,14%	0,11%	0,01%
ALNTF	1,93%	-1,37%	0,44%	-0,26%	0,09%	-0,02%
FINBN	3,74%	0,45%	0,85%	0,15%	0,17%	0,07%
ISBTR	0,79%	-2,51%	0,18%	-0,52%	0,04%	-0,07%
SKBNK	1,16%	-2,14%	0,24%	-0,46%	0,05%	-0,05%
TEKST	1,23%	-2,07%	0,28%	-0,42%	0,06%	-0,05%
TKBNK	1,17%	-2,12%	0,27%	-0,43%	0,05%	-0,05%
TSKB	2,80%	-0,49%	0,64%	-0,06%	0,13%	0,02%
YKBNK	1,21%	-2,08%	0,28%	-0,42%	0,06%	-0,05%
ISCTR	1,88%	-1,41%	0,43%	-0,27%	0,09%	-0,02%
XU100	1,80%	-1,49%	0,41%	-0,29%	0,08%	-0,02%
XU30	1,78%	-1,51%	0,42%	-0,28%	0,08%	-0,02%
XU50	0,79%	-2,50%	0,21%	-0,49%	0,04%	-0,07%
XUTUM	1,85%	-1,44%	0,42%	-0,27%	0,08%	-0,02%
XBANK	2,08%	-1,21%	0,48%	-0,22%	0,09%	-0,01%
R(F) Rates	3,29%	0,00%	0,70%	0,00%	0,10%	0,00%

It is seen that the market, XUTUM, risk premiums calculated over monthly returns vary between -43,17 % and 64,52 % with an average of -1.44 % presented in the table above and a standard deviation of 14 %. This is a huge range which is due to the speculative nature of ISE. It should be noticed that the lowest market risk premium (-43,17%) occurred in the last periods of 2000 and beginning of 2001, during which a great international portfolio capital outflow took place because of the currency crises in emerging markets started in Eastern Asia. And the positive market risk premiums are observed during the period of 2003-2004 when the number of accounts in the clearing system more tripled with the participation of small investors. It is thought that irrational investment decisions and greedy behaviors of investors in that period caused over pricing of stocks and then continued by a bear market in following few years.

Average risk premium of each stock is also calculated by using the equation 6.3. Results of the average risk premiums showed an important notice regarding the risk premiums is that all of the stocks (except only FINBN) and indices analyzed have negative risk premiums, which does not support the hypothesis of positive market risk premium predicted by the CAPM.

These remarks contradict with the fundamental of Markowitz portfolio selection theory. According to the theory, estimated return of stocks must exceed the risk free rate since there is a risk associated with the stock investment. It should also be noticed that the previous performance of stock is used as the proxy of expected risk premiums of the stocks in the regression methodology. The average risk premiums might be negative because the previous realized returns are used in the testing methodology whereas a negative risk premium should not be expected according to the theoretical model of CAPM, that is if the expected future risk premium is zero for a stock, then all the investors would sell it short which will provide the risk return equilibrium. In any case, average risk premiums of stocks should tend to converge at the market premium according to the theory.

Table 6.1 also indicates that the average risk premium of stocks and indices vary between -2,50 % and 0,45 % during the sample period with an -1,44 % average market risk premium (XUTUM) . So it could be anticipated that ISBTR stock performance will be 2,51% less than the risk free rate whereas stock FINBN's performance 0,45 % more than the risk free rate. However, future expected stock returns may not be known accurately beforehand as required by CAPM because these predictions are based on previous stock performance. From the table, one may expect that XBANK indices will not achieve a good performance but perform least worst under the risk free rate compared to other indices.

6.2 Calculation of Security Characteristic Line (SCL)

If the expected return and beta relationship holds, the expected rate of return on any stock will be;

$$E(r_i) = R_f + \beta_i \left[E(R_m - R_f) \right]$$ Equation (6.4)

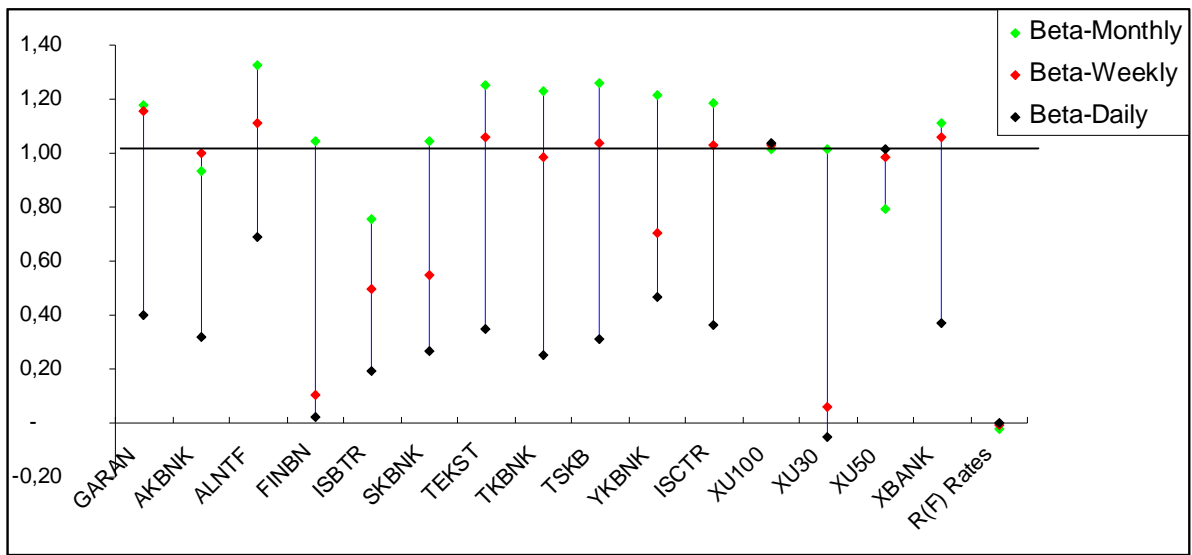where the β_i is defined as $Cov(r_i, r_m) / \sigma_m^2$

Figure 6.1 Beta distribution

Holding period excess return on the stock is expressed as;

$$r_{it} - r_{ft} = a_i + \beta_i \left(r_{mt} - r_{ft} \right) + e_{it}$$ Equation (6.5)

The model could be established in terms of excess returns over r_f rather than in terms of total returns. A higher return of market compared to the risk free rate in any period could be considered as good news in contrast a lower rate of market return would be disappointing

signals for macro economic conditions. For instance; market was offering on the average 27,43 % return and risk free rate was 93,73 % in 2001 which alerts bad news in the economy.

The equation (6.5) provides the formula of the market and firm specific risks measurements. By use of this equation the security line could be drawn via regression, once the risk premium of a stock and market is calculated. The diagram below mentions the methodology and concept.
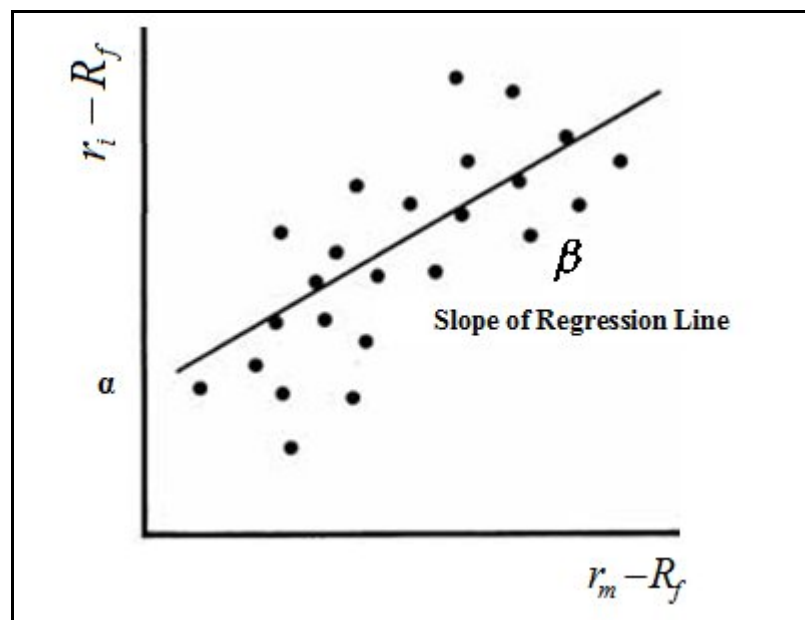

Figure 6.2 Security characteristic regression line

The vertical axis measures the excess return over the risk free rate on the stock (i) and the horizontal axis shows the excess return on the market index. These excess return pairs represent points on the scatter diagram.

The figure above is a simple single variable regression equation, the dependent variable visualize a straight line with an intercept α and a slope β. The diversions from the straight line that is symbolized as e_i, are considered both mutually unrelated and uncorrelated with independent variable.

The sensitivity of the stock to the market measured by β which is the slope of regression line. The intercept,"α", of the regression line points out the stock's average return when the market's excess return is zero.

GARAN

ALNTF

AKBNK

FINBN

ISBTR

SKBNK
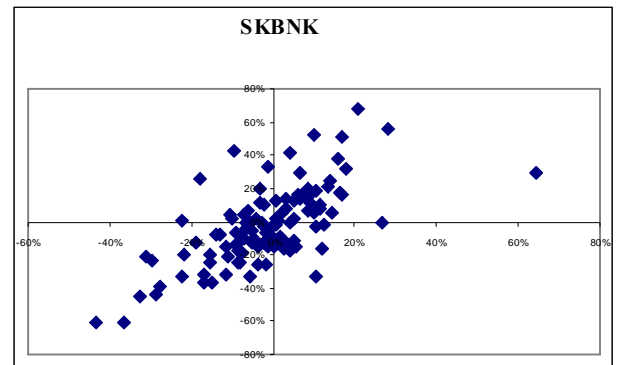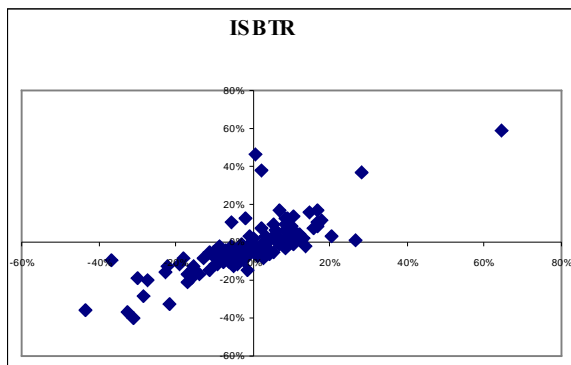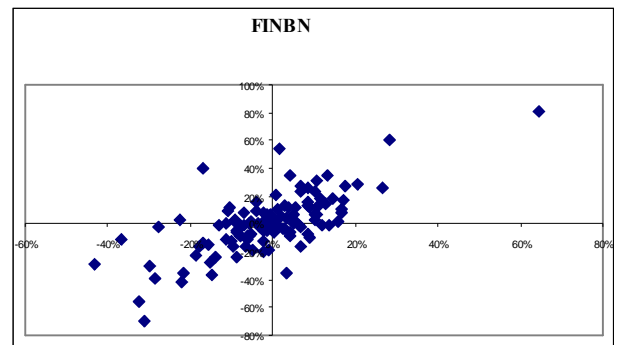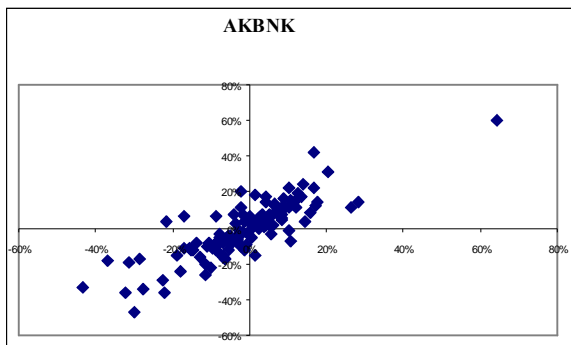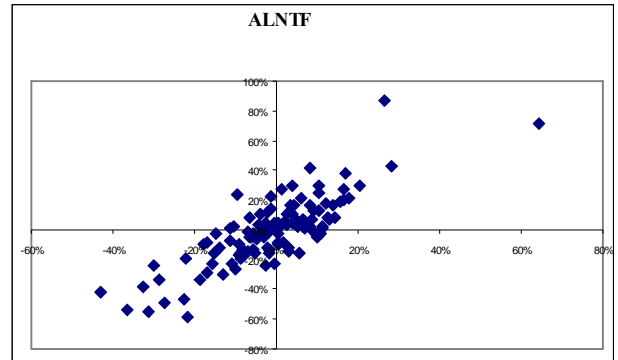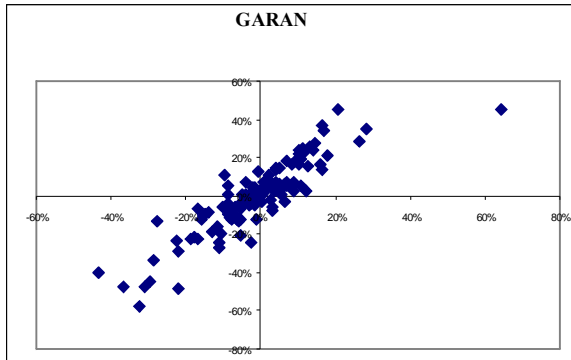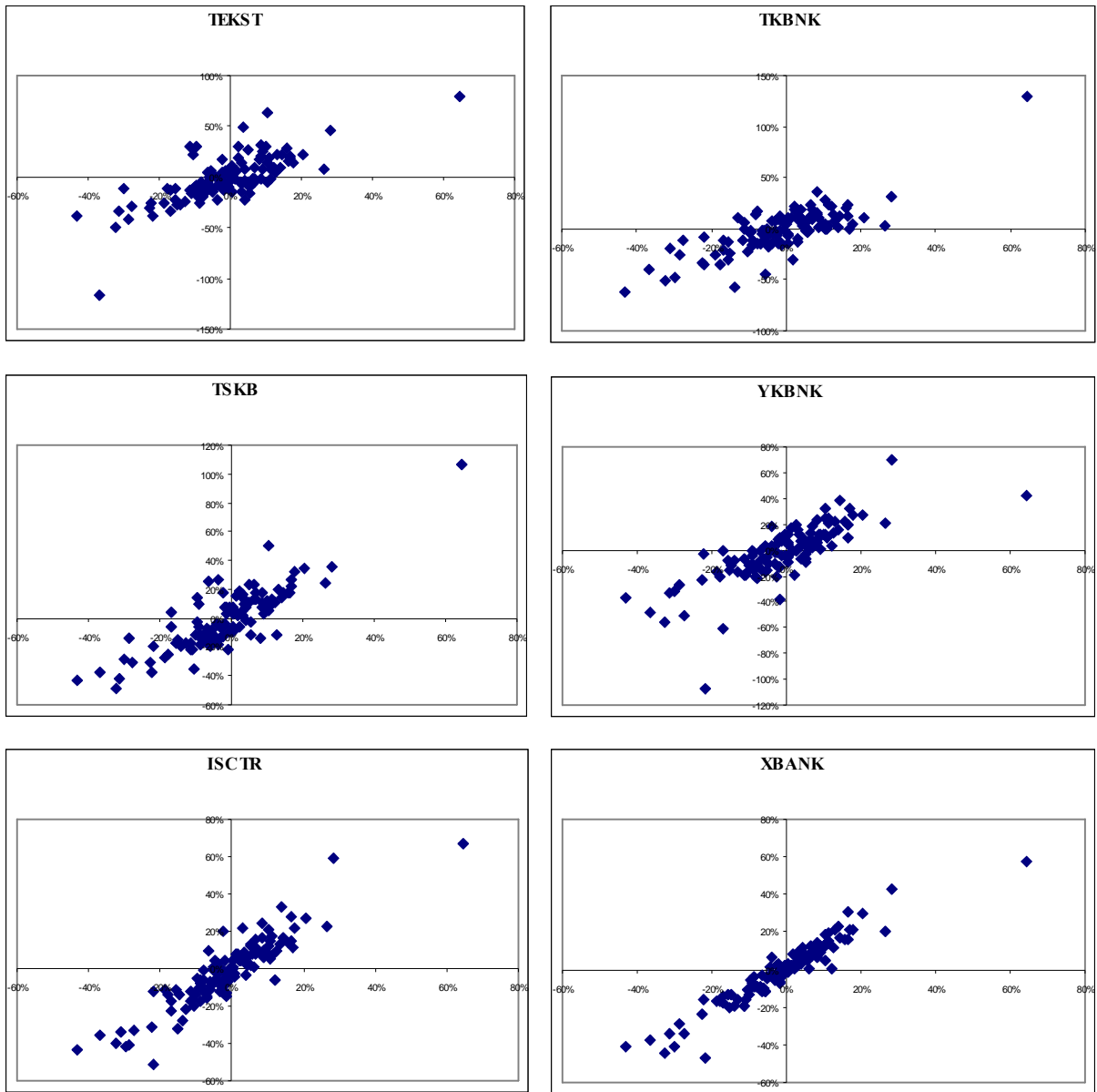
Figure 6.3 Excess returns over the risk free rate on the stock and on the market index

The points on the diagrams show the deviations from the regression line in period (t). These points symbolized as e_i and called residuals which are the vertical distances of each point from the regression line. Residuals, in other words, are the differences between the realized stock returns and the returns to be estimated from the regression equation and they also give clue investors about firm specific risks.

CAPM theory describes the expected return and beta relationship which is formulized as;

$$E(r_i) = R_f + \beta_i \left[E(R_m - R_f) \right]$$
Equation (6.6)

If the market index (XUTUM) identifies the accurate market portfolio, an investor could prefer the expectation of each side in the equation 6.5 ($r_{it} - r_{ft} = a_i + \beta_i(r_{mt} - r_{ft}) + e_{it}$).

$$E(r_i) - R_f = \alpha_i + \beta_i \left[E(R_m - R_f) \right] + E(e_{it})$$
Equation (6.7)

From the derivation of CAPM, as it is mentioned in part 5.2, the risk-free asset has a beta of zero because its covariance with the market portfolio is zero.

Table 6.2 Beta calculation of risk free asset over different indices

	XUTUM	XU100	XU50	XU30	XUBANK
Covariance	-0,0005	-0,0005	-0,0006	-0,0005	-0,0007
Market Variance	0,0187	0,0192	0,0151	0,0193	0,0256
Beta	-0,0246	-0,0244	-0,0399	-0,0248	-0,0288

In order to determine the market portfolio indices, each index is assumed as market portfolio then beta of risk free asset is calculated by using monthly returns. Results of this calculation are presented in the table above. According to the CAPM derivation risk free asset should have a beta of zero. The results show that calculated betas do not have zero betas but they are close to zero. XUTUM index is accepted as a market portfolio because it includes more assets and has better ability to cover whole market than others.

A comparison of the index model relationship to the CAPM expected return and beta relationship tells that the CAPM predicts that α_i should be zero for all assets. The alpha of a stock is shortly a measurement indicator between actual return and expected return based on

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risk level predicted by the CAPM formulation. If a stock is fairly priced, its alpha must be zero.

$$\alpha = R_p - E(R_i)$$ Equation (6.8)

R_p is actualized return calculated over given prices of stocks by use of formulation $\log P_t - \log P_{t-1}$, where "P" is denoted for price of the stock.

According to the CAPM assumptions in an efficient market, the expected value of the alpha coefficient is zero. Therefore the alpha coefficient indicates how an investment has performed when the risk is taken into account and it could be summarized that if;

$\alpha_i < 0$; the investment has earned too little for its risk (or, was too risky for the return)

$\alpha_i = 0$; the investment has earned a return adequate for the risk taken

$\alpha_i > 0$; the investment has a return in excess of the reward for the assumed risk

According to this statement about expected returns for stocks, some stocks will perform better or worse than expected and some will bring higher or lower returns than predicted by the CAPM. In other words, stocks will show positive or negative alphas over the examination period however these better and worse performances could not have been predicted in advance.

When the index model is predicted for all stocks using the equation 6.7, the realized alphas for the stocks in the sample should center on zero. This representation model of the CAPM grasps the ex post value of alphas should average out to zero whereas the CAPM says that the expected value of alpha is zero for all stocks for the sample period.

Table 6.3 Alpha distribution of stocks and indices

Stocks & Indices	Max. α Values	Min. α Values	Average α Values	Standard Deviation
GARAN	19,15%	-15,20%	1,42%	0,07
AKBNK	27,42%	-16,52%	0,51%	0,08
ALNTF	30,85%	-18,21%	-0,07%	0,11
FINBN	52,36%	-39,56%	3,18%	0,16
ISBTR	35,76%	-13,61%	-1,17%	0,08
SKBNK	43,99%	-28,78%	1,72%	0,15
TEKST	52,01%	-76,65%	1,06%	0,19
TKBNK	27,75%	-36,72%	-0,39%	0,13
TSKB	38,53%	-23,16%	1,28%	0,10
YKBNK	19,59%	-22,52%	0,13%	0,08
ISCTR	18,05%	-8,45%	0,01%	0,06
XU100	1,02%	-1,27%	-0,07%	0,01
XU30	2,92%	-3,05%	-0,07%	0,01
XU50	6,03%	-10,61%	-0,09%	0,03
XBANK	12,80%	-7,23%	0,62%	0,04

According to the CAPM theory predicted firm specific risks should also be zero on average. Actually the predicted firm specific risk is zero because of the nature of risk definition since the firm specific risk is the result of unexpected firm specific incidents.

Equation 6.4, $E(r_i) = R_f + \beta_i [E(R_m - R_f)]$, is evaluated as a security market line and the second pass regression equation established with the estimates β_i from the first pass regression as the independent variable;

$$Average(R_i - R_f) = \gamma_0 + \gamma_1 \beta_i + \gamma_2 \sigma^2(e_i)$$ Equation (6.9)

When the equations 6.4 and 6.9 are compared it should be resulted that, γ_0 γ_1 and γ_2 must fulfill the following requirements provided that CAPM is valid:

$$\left. \begin{array}{l} \gamma_0 = 0 \\[2em] \gamma_1 = Average\big(R_m - R_f\big) \\[2em] \gamma_2 = 0 \end{array} \right\} \qquad \text{Equation (6.10)}$$

The hypothesis that $\gamma_2 = 0$ is consistent with the idea that nonsystematic risk should not be priced which is there is no risk premium earned for taking nonsystematic risk. Furthermore, $\gamma_2 = 0$ should also hold that the realized alphas average out to zero. Eventually, if the index is a good proxy of the market, average of all stocks' risk premium must be equal to the market premium in order to deduct that CAPM is acceptable theory to predict stock returns in the market.
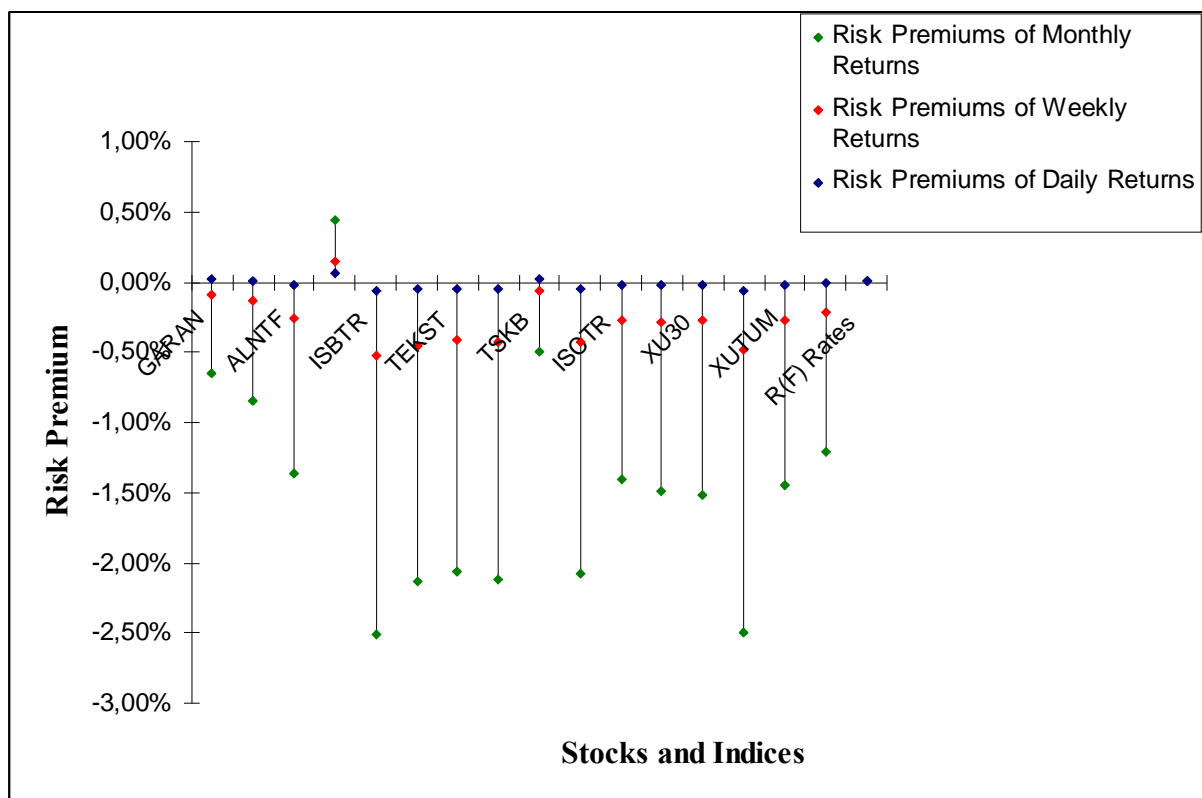

Figure 6.4 Risk premium distribution

When the Figure above 6.4 and Table 6.1 (risk premiums of stocks and indices) are evaluated, it is seen that monthly average risk premiums of 11 banking sector stocks is -1,38 % while risk premium of the market (XUTUM) is -1,44 %. On the other hand, these stocks' risk premiums of weekly and daily returns are in line with the risk premium of the market return. Weekly average risk premium of these stocks is -0, 27%, the market's risk premium is also -0, 27% and daily average risk premium of the stocks is -0, 02 % and the market has same risk premium rate. In this respect, CAPM could be acceptable theory to estimate stock returns in the market. However, doing calculation again after adding all other stocks in the market may not lead to same acceptable result. These stocks may carry the market risk or their representation power of the market is higher.

According to the CAPM, the risk premium depends only on beta therefore any additional right hand side variable in the Equation (6.9) except beta should have a coefficient that is insignificantly different from zero in the second pass regression.

In the regression model, the intercept points indicate the estimation of risk premium of stocks when the market risk premium and company specific risk is zero. In other words, if the beta of stock is null ($\beta_i = 0$) and there is no company specific risk ($e_{it} = 0$); then $a_i = r_{it} - r_{ft}$ should hold as stated in Equation (6.5). Actually, index model given in this equation, a_i should be zero for all assets. The value of a_i of an asset is the difference of its expected return from the fair expected return as predicted by the CAPM. And if the asset is fairly priced a_i should be zero. However, it should be noticed that statement about expected returns may not reflect the reality because some stocks will do better or worse than expected and will have returns higher or lower than estimated by the CAPM, that is stock returns will may exhibit positive or negative alphas over the sample period. On the other hand, the most important notice is that this superior or inferior performance could not have been forecasted in advance.

That is why, it should be found that the ex-post alphas, regression intercepts, for the firms in the sample center around zero through estimating the index model for all the stocks in the market by using the equation, $E(r_m) = r_f + \beta_m [E(r_m) - r_f]$ where the expected return of

market portfolio is $E(r_m)$ and the beta of this portfolio is β_m. Figure below presents the alpha distribution of the securities.
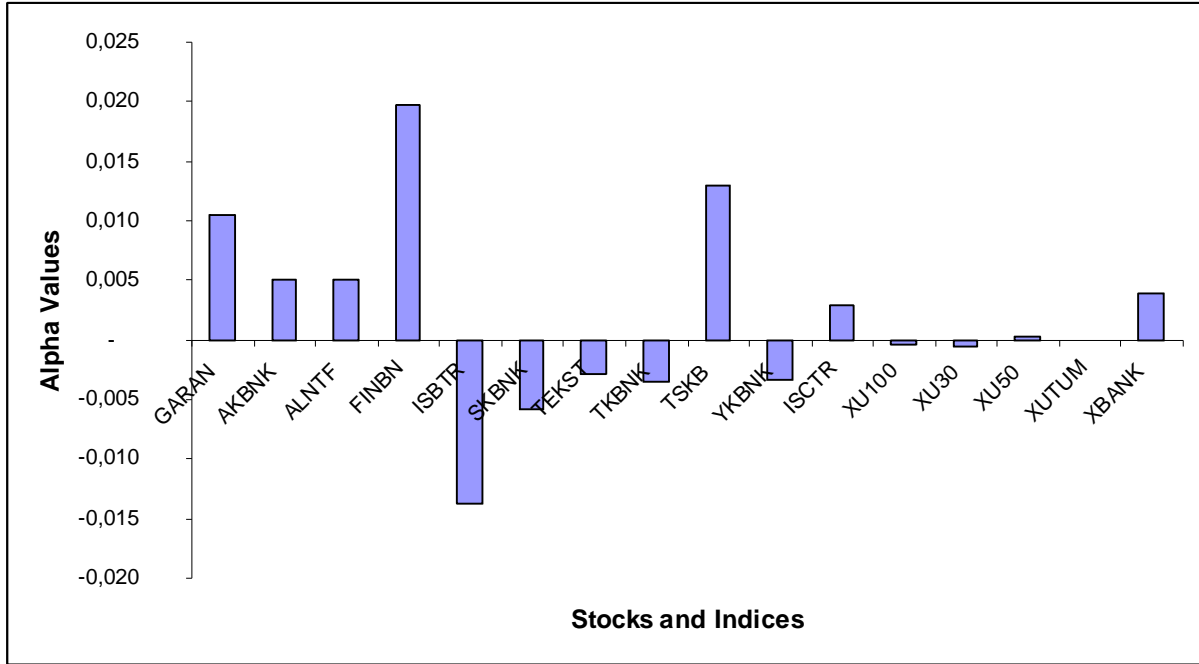

Figure 6.5 Alpha distribution of stocks and indices

The CAPM mentions that the expected value of alpha is zero for all stocks whereas the index model representation of CAPM holds that the realized value should average out to zero for the sample. The sample alphas should be unpredictable, that is, independent from one sample period to the next. When the alpha values are analyzed, as shown on the graph above there is a range between -1, 38% and 1, 97% which is accumulated around zero with an average value of 0, 18%. Also p-values of alpha estimate are higher then the confidence probability, 5%, therefore it could be concluded that the regression model is not to be a very good fit to estimate risk premium in the absence of market related risk factors because alpha predictions for the stocks and indices are not significant enough in statistical terms.

In order to measure the predictability power of the regression 95 % confidence interval for t-test was applied and any beta estimate with a p-value of less than 5% is accepted as significant

beta estimate. After the application the tests for beta predictions, it is seen that beta provides significant predictability power for the model that is regression model is a good fit for the beta estimation. Also betas of each stock and indices are calculated through the use of following equation, 6.11, and then these betas compared with the results of regression model's betas.

$$\beta_i = \sigma_{i,m}/\sigma^2_m = Cov(R_i, R_m)/\sigma^2_m \qquad\qquad \text{Equation (6.11)}$$

Table 6.4 Beta comparison

Stocks & Indices	Beta Results of Regression	Beta calculated by use of Eq.(6.11)
GARAN	1,18	1,17
AKBNK	0,94	0,94
ALNTF	1,31	1,32
FINBN	1,06	1,04
ISBTR	0,78	0,76
SKBNK	1,08	1,04
TEKST	1,25	1,25
TKBNK	1,23	1,23
TSKB	1,25	1,26
YKBNK	1,22	1,22
ISCTR	1,18	1,18
XU100	1,01	1,01
XU30	1,01	1,01
XU50	1,02	0,79
XUTUM	1,00	1,00
XBANK	1,12	1,11

As it is seen from the table, regression model is in fact good fit for the beta estimation, approximately beta results of the regression fit with the calculation results, except beta for XU50. The number of observations for this index is relatively less than the others therefore the prediction of the model is not significant which could be accepted as the reason of model's failure in beta estimates.

A prominent point which could be noted in this study is that p-value decreases with increasing number of observations so the model provides much more significant estimates for the stocks

in the long time. Total of 114 observations from September 1999 were used in the regressions for each stock and indices.

On the other hand, the best prediction for the beta could be accepted as one when there is no sign about firm specific risk of a stock. CAPM assumptions require that investors could sell short the over priced stocks and buy long the under priced stocks rationally. Hence, price fluctuations of stocks look like similar to market movements. Based on this assumption as shown the graph below beta distribution accumulates near one. The beta at the 15th point on the X axis representing the beta of XUTUM which is assumed as the market portfolio is exactly 1.
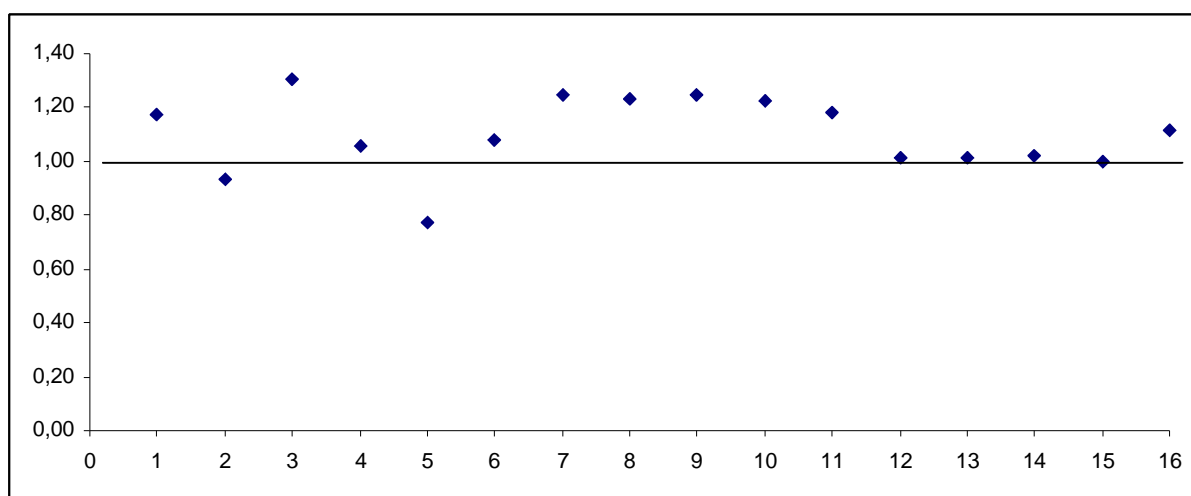
Figure 6.6 Beta distribution of the regression

It is observed that the beta values of the selected banking stocks vary between a range of 0,78 and 1,31 with an average of 1,13. Therefore beta estimation of SCL regression could be considered as a good proxy for the actual betas. Also both these banks and banking sector indices, XBANK, have beta estimates of on the average a larger or closer to one. This may mean that banking sector has more sensitivity to economical and financial risk factors in the market.

This observation leads to the point that banking sector is closely correlated with the market. Improvement of the banking sector depends of the success of all other sectors in the market because profit of other sectors in the market provides funds for banks.

In the SCL regression methodology, the dependent variable plots around the SCL with an intercept alpha and a slope beta. The deviations from the line, e_{it}, are supposed to be uncorrelated with each other and the independent variable. Divergence of observations from the regression is symbolized as e_{it} that is regression residual. Residuals are the difference between the actual stock return and the return predicted from the regression. They measure the impact of firm specific risk events and their variance shows this risk during the particular period.
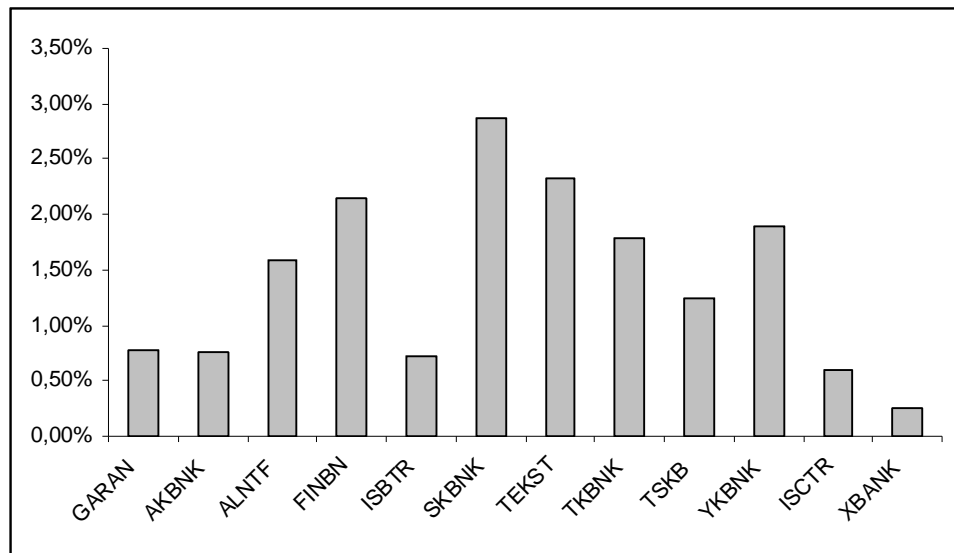
Figure 6.7 Residual variances

Residual variances which representing the company specific risks vary within a range of 0,60 % and 2,87% for the sample stocks and for the banking sector index XBANK is 0,25 %. Therefore it could be concluded that return fluctuations due to company specific risk is much lower for the selected banking sector stocks. Firm specific risk is generally described as the unanticipated risk that a firm faces due to the characteristic of its business.

Adjusted R-Square in the table 6.5 below shows the square of the correlation between risk premiums of stocks and the market ($r_i - r_f$ and $r_m - r_f$). Adjusted R square also called coefficient of determination represents the fraction of the variance of the dependent variable - the risk premium of stocks- which is clarified by the changes in the independent variable – the return on the market indices-.

Table 6.5 Adjusted R Squares

Stocks & Indices - Adjusted R Square			
GARAN	78,1%	TSKB	71,4%
AKBNK	69,9%	YKBNK	61,2%
ALNTF	68,1%	ISCTR	82,4%
FINBN	50,9%	XU100	99,9%
ISBTR	62,7%	XU30	99,1%
SKBNK	44,5%	XU50	99,6%
TEKST	57,1%	XBANK	90,8%
TKBNK	62,7%	XUTUM	100,0%

The variance of a stock's risk premium constitutes of the variance caused by the market return and company specific risk. The variance could be formulized as follows:

$$\sigma_i^2 = \beta_i^2\, \sigma_m^2 + \sigma_i^2(e_i)$$

Equation (6.12)

Adjusted R square is systematic variance over total variance, which informs about what fraction of a company's volatility is attributable to market movements which are shown in formulation below:

$$Adjsuted\ \ R\ Square = \left(\beta_i^2\, \sigma_m^2\right)\big/\sigma_i^2$$

Equation (6.13)

When the results of adjusted R squares are analyzed, it is seen that volatility of the stocks are attributable to market fluctuations. For instance; variance of stock ISCTR (82,4%) return depend on the market movement more than the stock SKBNK (44,5 %). That is ISCTR carries

more market risk than others. And attribute of market variance on SKBNK stock is due to company specific risk rather than market risk. When banking sector indices XBANK is also investigated it could be concluded that variance of banks is highly due to the market variance.

6.3 Security Market Line Regression

The security market line shows the changes in expected return, due to changes in the beta coefficient. On the horizontal axis beta of assets and on the vertical axis expected rate of returns are located.
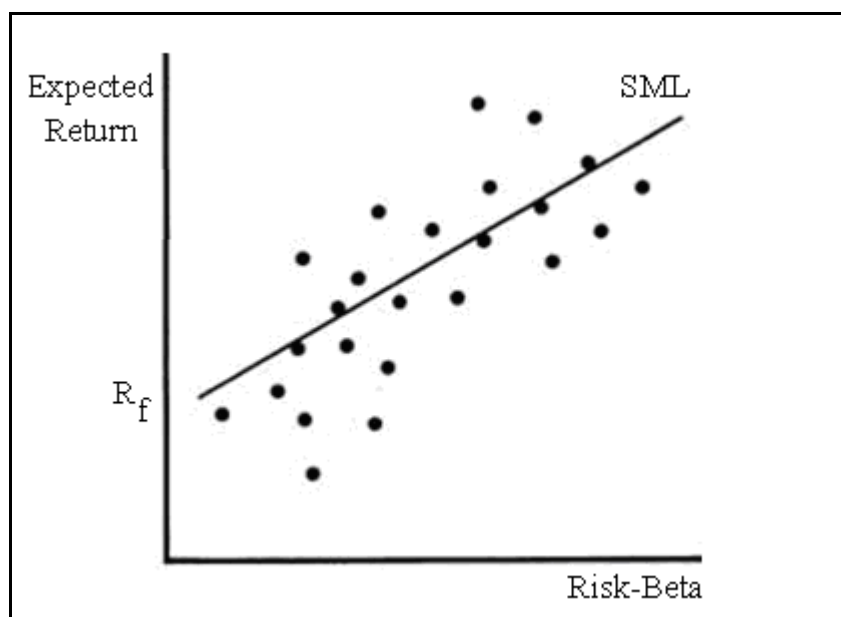

Figure 6.8 Security market regression line

SML is the graphical demonstration of the expected return and beta relationship, correctly priced assets plot exactly on the SML; that is, their expected returns are proportional with their risk. To test the validity of CAPM assumptions, all assets must lie on the SML in market equilibrium. If an asset is underpriced, which plots above SML, in the market, it will provide an expected return in excess of the return predicted by SML. On the other hand, any over-priced asset plotted below SML will provide a less return than estimated by SML. However, any under or over-priced asset has to move to the equilibrium in the long run because investors

will tend to buy under-priced assets and the expected return of these assets will go down getting closer to their appropriate return estimated by SML. Likewise, investors will sell over-priced assets, causing their expected return to increase and move towards their fair return on the security market line.

The regression model is constructed on;

- Beta estimation in the SCL regression is used as an input to find betas for SML regression.
- Returns are calculated percentage change between value at the beginning and ending period which is formulized in Equation (2.30).
- Regression with betas (independent variable) and returns (dependent variable) is applied within a 95 % of confidence interval.
- Regression is run for both monthly and annual data.

Table 6.6 P-Values from SML regression over monthly data

P-Values of Independent Variable	
GARAN	0,8523
AKBNK	0,7245
ALNTF	0,1372
FINBN	0,1131
ISBTR	0,2908
SKBNK	0,9474
TEKST	0,4687
TKBNK	0,0864
TSKB	0,2183
YKBNK	0,9600
ISCTR	0,9542
XBANK	0,8525

The table 6.6 summarizes the statistical results of the SML regression within a 95% confidence interval. P values of the estimation are higher than the confidence probability, 5%, therefore the regression statistically does not provide significant predictability power for the

model that is to predict the asset returns based on beta of assets. In other words the results are not reliable enough to test the validity of CAPM model.

Furthermore, regression is applied over 10 year-annual data, for the period of 1999-2009. Totally 110 annual betas predicted for 11 stocks and regressed with their returns. The results of the regression with a 0, 84 p-value expresses that the model is not good fit for the estimation. The results are also shown graphically in figure 6.9 demonstrating that the returns are not in line with the expected from the regression.
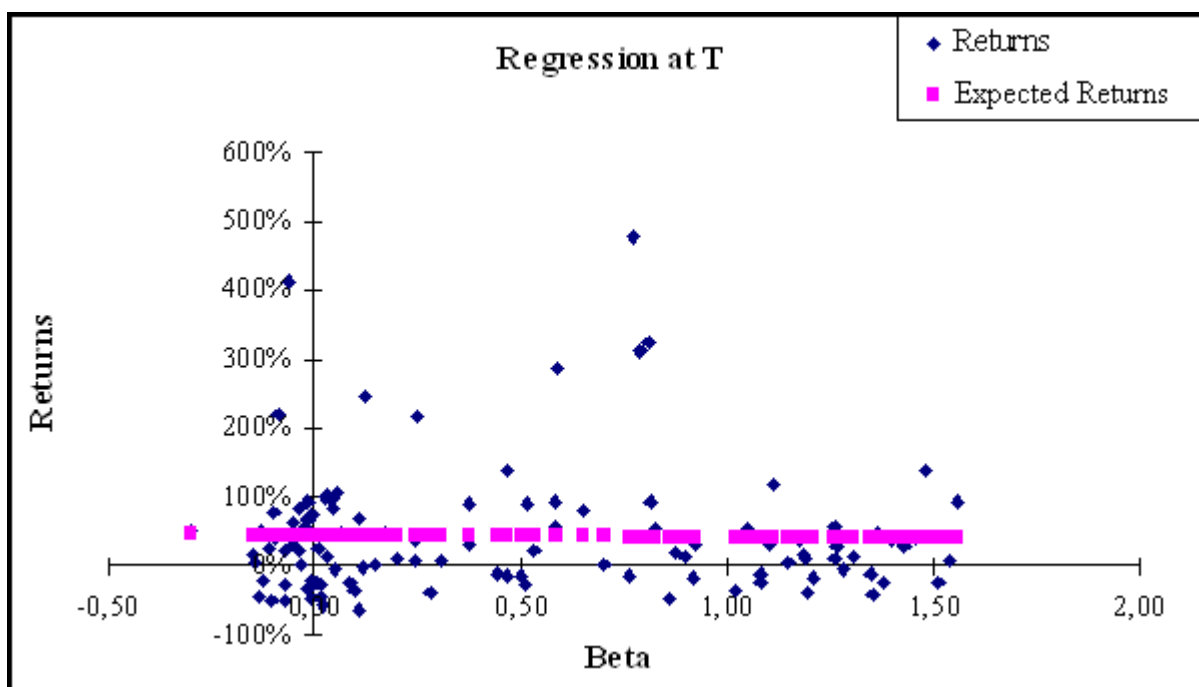

Figure 6.9 SML regression over annual data

Also an additional regression is applied that is; estimated beta at "t" time is regressed with the return at "t+1" time. In this regression model, the main concern is to estimate the future returns with ex post betas. The results of this regression are shown in figure 6.10.
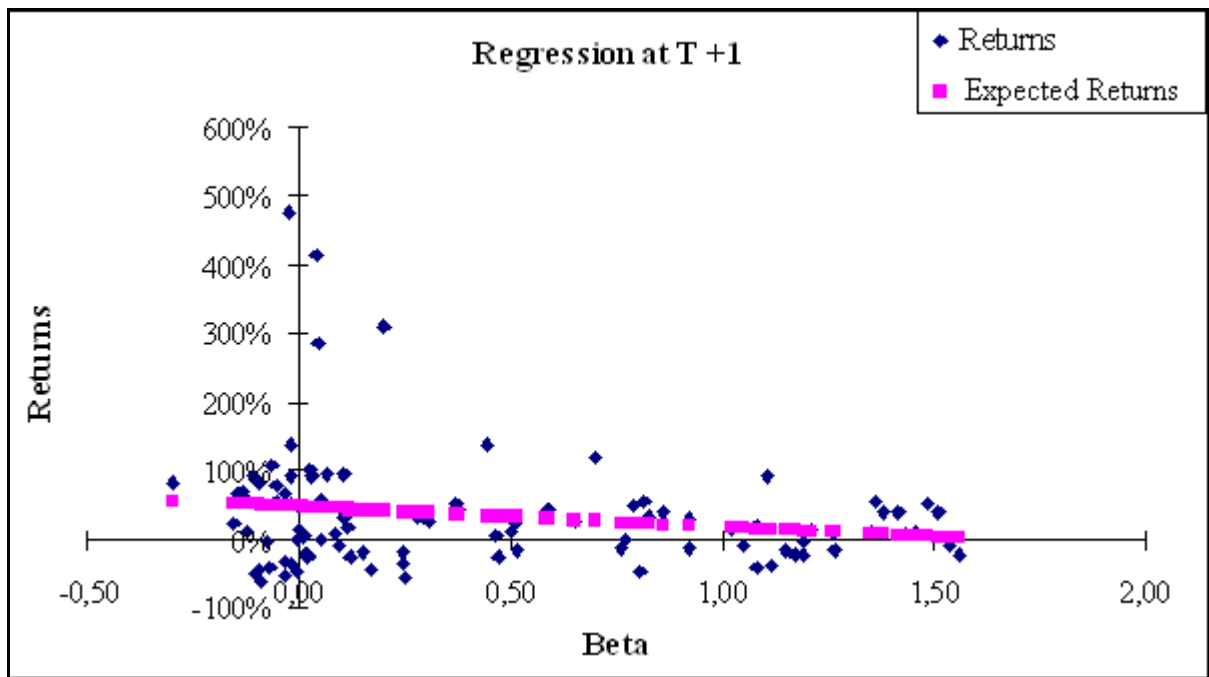
Figure 6.10 SML regression over annual data for t +1 period

The regression at "t+1" has a p-value of 0,065 and it has higher predictability power when it is compared the p-value of regression at "t" period, 0, 84. However, the p-value of 0,065 which is higher than the confidence probability, 5 %, is not significant enough in statistical terms.

As a result, when the regression has been applied, the statistically significant relationship between beta and return has not been found in all periods.

PART 7 CONCLUSION

CAPM form developed by Sharpe (1964) and Lintner (1965) has never been an empirical success. In the early empirical work, the Black (1972) version of the model, which can accommodate a flatter tradeoff of average return for market beta, has some success. However, in the late 1970s, studies arisen to uncover variables like size, price ratios, and momentum that add to the explanation of average returns provided by beta. The problems are serious enough to make invalid most applications of the CAPM. For instance;

The application is to predict a stock's market beta and combine it with the risk free interest rate and the average market risk premium to construct an estimate of the cost of equity. The typical market portfolio in these exercises includes just common stocks. But empirical works, say that the relation between beta and average return is flatter than predicted by the Sharpe – Lintner version of the CAPM. As a result, CAPM estimates of the cost of equity for high-beta stocks are too high (relative to historical average returns) and estimates for low-beta stocks are too low (Friend and Blume, 1970). Similarly, if the high average returns on value stocks (with high book-to-market ratios) imply high expected returns, CAPM cost of equity estimates for such stocks are too low.

Besides, CAPM is used to measure the performance of mutual funds and other managed portfolios. The approach, dating to Jensen (1968), is to estimate the CAPM time-series regression for a portfolio and use the intercept (Jensen's alpha) to measure abnormal performance. The problem is that, because of the empirical failings of the CAPM, even passively managed stock portfolios generate abnormal returns if their investment strategies involve propensities toward CAPM problems (Elton, Gruber, Das and Hlavka, 1993). For example, funds that include low beta stocks, small stocks, or value stocks will tend to produce positive abnormal returns relative to the predictions of the Sharpe – Lintner CAPM.

In this research the model testing was based on regression analysis. First, beta value of each stock was estimated simply by using a first step regression (SCL regression). The hypothesis

was that; any coefficient on the regression other than betas must be zero. Otherwise, the variable with non-zero coefficient would also contribute to the estimated stock return and that would be in contradiction with CAPM's argument.

Results of this study could be summarized shortly as;

Stocks in banking sectors on the average are less profitable than the average risk free rate during the research period. Both these stocks and other indices have negative risk premiums. This situation exemplifies worse economic conditions in the country and expresses government's interest burden. However, as mentioned in view of the theoretical model of CAPM, a negative risk premium should not be anticipated in other words if the expected risk premium is zero for an asset, then all the investors will sell it short and will obtain the risk return equilibrium.

Also the results show that XBANK indices will not achieve a good performance but present least worst performance under the risk free rate compared to other indices.

According to the CAPM theory risk free asset should have a beta of zero. The results illustrate that calculated betas do not have zero betas but they are nearly zero. CAPM theory states that predicted firm specific returns should also be zero on average. Actually the predicted firm specific risk is zero because of the nature of risk definition since the firm specific risk is the result of unexpected firm specific incidents. However, as presented in the study these are not zero.

Additionally, if the index is a good proxy of the market, average of all stocks' risk premium must be equal to the market premium in order to suppose that CAPM is appropriate theory to estimate stock returns in the market. When the results are assessed, it is observed that monthly average risk premiums of 11 banking sector stocks is -1,38 % while risk premium of the market (XUTUM) is -1,44 %. But, these stocks' risk premiums for weekly and daily returns are consistent with the risk premium of the market return. Weekly average risk premium of

these stocks is -0, 27 %, the market's risk premium is also -0, 27 % and daily average risk premium of the stocks is -0, 02 % and the market has same risk premium rate. Thus, CAPM could be acceptable theory to predict stock returns in the market. However, making calculations and observations again after including all other stocks in the market may not lead to same acceptable results.

The study also points out that the regression model is a proper method for the beta prediction, approximately beta results of the regression proportional to the calculation results. Both these banks and banking sector indices, XBANK, have beta estimates of on the average a larger or closer to one which may interpret that banking sector is more sensitive to economical and financial risk factors in the market. In other words it could be seen that banking sector is closely correlated with the market.

When the residuals -the differences between the actual stock return and the return predicted from the regression and measuring the impact of firm specific risk events- are observed, their variances which representing the company specific risks vary within a range of 0,60 % and 2,87% for the sample stocks and for the banking sector index XBANK is 0,25 %. Therefore, it could be resulted that return fluctuations due to company specific risk is much lower for the selected banking sector stocks. In other words, variance of banks is highly due to the market variance.

On the other hand, the regression results were confusing. Beta estimates attained by the SCL regression were statistically significant. It was also observed that signification increases with the increasing number of observations, and beta estimates of stocks included in XUTUM indices are generally close to one within an acceptable range.

SML regression is resulted in statistically insignificant relationship between the expected return and beta of assets.

Furthermore, the XUTUM index used in the test is definitely not the "Market Portfolio" as required by CAPM. The index includes stocks of companies operating in many sectors but it, of course, excludes some non-traded assets such as education (human capital), private enterprises, and investments financed by government. These remarks lead to the idea that XUTUM index may be a biased proxy of the macro-economic factors. This, indeed, is a general criticism against CAPM. Further research may be performed by separating and using each macro-economic factor (inflation, economic growth, unemployment rate, etc.) as a regression input in order to eliminate the bias in the index. And XUTUM index may be questioned to measure how well it acts as a proxy of all macroeconomic factors and non-traded assets.

Failure of the model may also be due to speculative structure of ISE, correlation of international markets, sampling errors or inability of XUTUM index to reflect market portfolio as well as CAPM's over-simplifying assumptions. Moreover, the return on the index not only depends on national macro economic factors but also depends on the movement of international capital. The capital flow can also be investigated to measure its effect on return predictability power.

Finally, it was assumed that investors can borrow at a risk free rate and there exists no cost associated with trading. These are un-realistic assumptions apart from the real world. Despite simplicity of the CAPM, its empirical problems probably invalidate its use in applications. Further research may be re-performed for the ISE by including trade costs and real interest rates in the case of borrowing.

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RESUME

I graduated from Bosphorus University, School of Applied Disciplines, BA Degree at Department of International Trade in 2005. I have attended many management, finance, economics and accounting courses during my education.

As well as academic courses I also attended a lot of various technical and business training programs and courses such as treasury-trading simulations, tax and legal issues. Additionally, I worked on several technical reports and presentations like a subject of "Development of E-Commerce and Effects on Competition in Turkey" which was rewarded and certified by the Undersecretariat of the Prime Ministry for Foreign Trade Export Promotion Center in October 2003.

I accomplished the program successfully on time with an honor degree of 3, 26 GPA over 4.0. I have been working in audit area in private sector since graduation. In the career path I plan to take part in an academic position in the future. Thus, I started this master program and aim to attend a Ph.D program in finance department.