T. C.
BAHÇEŞEHİR ÜNİVERSİTESİ

ELECTRICITY SPOT PRICE MODELLING AND RISK-RETURN TRADE-OFF APPLICATIONS

Master of Science Thesis

Esra ADIYEKE

İSTANBUL, 2014

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Assoc. Prof. Dr. Tunç BOZBURA
Acting Director

I certify that this thesis meets all the requirements as a thesis for the degree of Master of Sciences.

Assoc. Prof. Dr. Barış SELÇUK
Program Coordinator

This is to certify that we have read this thesis and that we find it fully adequate in scope, quality and content, as a thesis for the degree of Master of Science.

Examining Commitee Members:	Signature
Asst. Prof. Dr. Ethem ÇANAKOĞLU (Supervisor)	: ....
Asst. Prof. Dr. Demet ÖZGÜR ÜNLÜAKIN	: ....
Assoc. Prof. Dr. Ebru ANGÜN	: ....

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ABSTRACT

ELECTRICITY SPOT PRICE MODELLING AND RISK-RETURN TRADE-OFF APPLICATIONS

Esra ADIYEKE

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After the liberalisation and restructuring of electricity markets, risk management has become an important objective for all the market participants. For effective risk management, modelling electricity prices has become an important issue. In this study electricity prices in the UK market has been modelled using different methodologies.Moreover, using CVaR as a risk measure, a portfolio problem for a distribution company has been defined. Different portfolio strategies for changing objectives are calculated and their performances are compared. And finally, managerial insight are provided throughout this study.

Keywords: Electricity Pricing, Forecasting, Portfolio Risk

ÖZET

Elektrik Fiyatı Modelleme ve Risk Yönetimi Uygulamaları

Esra ADIYEKE

Endüstri Mühendisliği
Tez Danışmanı: Yrd. Doç. Dr. Ethem ÇANAKOĞLU

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Elektrik piyasalarında yaşanan değişimlerle birlikte bu piyasada oyuncu pozisyonunda olan herkes için tutarlı ve anlamlı fiyat tahminleri ile maruz kalınması beklenen fiyat kaynaklı riskin yönetilmesi önemli ihtiyaçlardır. Bu çalışmada elektrik fiyatları çeşitli modelleme metodolojileri kullanılarak simüle edilmiştir. Seçilen performans testlerine göre modellerin analizleri yapılmıştır. Üretilen yapay fiyat serileri ile risk metriği olarak seçilen CVaR beraber kullanılarak sistem optimizasyon problemi olarak ifade edilmiştir. Bu yapılırken stokastik programlamadan yararlanılmıştır. Son olarak çalışma dahilinde edinilmiş bulgu ve sonuçlara dair yorumlar verilmiştir.

Anahtar Kelimeler: Elektrik Fiyatlama, Tahmin, Portföy riski

TABLE OF CONTENTS

TABLES

FIGURES

ABBREVIATIONS

Autoregressive Moving Average	:	ARMA
Conditional Value at Risk	:	CVaR
Generalized Autoregressive Conditional Heteroskedasticity	:	GARCH
Mean Reverting	:	MR
Regime switching	:	RS
Trace	:	tr
Value at Risk	:	VaR

SYMBOLS

Budget	:	B
Cost of scenario n	:	γ^n
Covariance matrix	:	Ω
Deterministic part	:	Δ
Dummy for daily prices	:	D_{day}
Error term	:	ε
Indicator matrix	:	$A_{i,t}$
Jump intensity	:	Λ
Number of contract of type i in level n	:	$\theta^n_{suc(n)}$
Mean reversion rate	:	κ
Number of contracts at time t	:	θ'_t
Probability of scenario path s	:	π^s
Probability transition matrix	:	p_{ij}
Shift	:	ϕ
Stochastic residue at time t	:	X_t
Variance	:	σ^2

1. INTRODUCTION

Electricity as a trading commodity on the power markets differs from any other asset owing to its distinctive properties such as high volatility of prices, strong seasonality of demand, and non-storability (Mayer et al. (2012)). For instance at some stage of extreme weather conditions or sudden breakdowns in one of the main plants people cannot easily utilize their “stored” electricity unless they are in need for power to make their phones or some other small home appliances work and have no chance other than purchasing electricity from market with a skyrocketed price. As a result it is clear that accurate forecasting methods that are capable of capturing the changing aspects of the electricity prices and the risk- return trade-off optimization applications that assist the companies to mitigate the destructive consequences, are crucial to survive in business world.

In the literature various forecasting methods are applied to electricity prices, i.e. fundamental models employ a holistic approach to energy systems as they need information about demands, capacity uses, maintenance hours et cetera. Game-theoretic models are interested in explaining the market settings with Nash-Cournot framework. Financial mathematical models make uses of stochastic processes like mean-reversion or Geometric Brownian Motion. Econometric models concentrate on clarifying the patterns or autocorrelations of prices and highly depend on the historical data. These models are detailed in Literature Review section of this thesis. In this study financial and econometric models are developed so as to reflect the price behavior in the UK power market in MATLAB environment.

Besides the precise price predictions, it is also a necessity for both producer and consumer groups to devise a reasonable plan for their electricity production or consumption in advance. The objective of this plan is to derive a proper using strategy for the time horizon considered using weather derivatives. In the literature, portfolio optimization applications mostly deal with deriving practical hedging strategies considering different risk measures or return functions. Following Markowitz (1952)'s mean-variance approach, which is one of the most prominent study in this framework, recent advances in risk management field are developed to define different optimization problems such as Value-at-Risk or expected shortfall minimization under some return or risk preferences. In this context , Rockafellar and Uryasev (2000) suggested a scenario based expected shortfall minimization problem which can be modelled as a linear programming problem. Altough there are more complex models that leads to nonlinear problems, e.g. (Vehlivainen and Keppo 2003), Rockafellar and Uryasev (2000)' s formulization has been used by researchers to its tractability.In our study we will also use a scenario based expected shortfall minimization model. Also, we are interested in models allow us to formulate the system as a linear programming problem are selected since these models enable the practitioners to evaluate the risk preferences with different levels and save them a significant computational burden. The scope of this study is two folds. Initially historical prices are analyzed and the formulae for simulating price scenarios are derived. Model accuracies are tested under various performance measures. Secondly several risk modelling problems are suggested and the results are evaluated.

Both manufacturers and consumers need a mechanism to handle setbacks arise from the nature of electricity and prospective losses. In this research, we aimed to provide many models which are able to emulate the dynamics of the spot prices and optimize the risk-return trade-off in order to shed light for practitioners by presenting quantitative insights about the benefits of these approaches.

1.1 ORGANIZATION OF DISSERTATION

The thesis is organized as follows. Chapter 1 includes an introduction of modeling methodologies of the electricity spot prices and the risk-return optimization framework. In Chapter 2, there is a brief literature review of electricity spot price modeling approaches and optimization applications in the risk management field. In Chapter 3, problem descriptions and models are given. This chapter is divided into two subsections. The models and numerical results are also given in this chapters. In Chapter 4, the thesis is concluded with the main insights and outcomes gained throughout this study.

2. LITERATURE REVIEW

Electricity has been attracting attention of both researchers and practitioners as a result of the significant changes in the market environment. Research devoted to understand the unique attributes of electricity, assist people to understand the characteristics of this product. The studies over electricity markets can be divided into two major streams in relation to our work. The first stream is modelling electricity price process and second is risk management of electricity portfolios. Our study closes the gap between those ares. In this section we will give a review of both areas.

2.1 PRICING MODELS

Research which are conducted towards this aim are given in the following. In Bunn and Karakatsani (2003) and Karakatsani and Bunn (2008b), price drivers are defined and analyzed for UK Power Market data. Demand, margin and scarcity are listed as price drivers. According their studies, electricity prices also exhibit trend and strong seasonality.

Moreover, Weron (2005) investigated the distinctive aspects of prices based on spot prices of Nordic Power Market. In this study, they suggested jumps are the most obvious feature of electricity and they point aout that, after the price rises up to significantly extreme values, it returns to its normal level which means this spiky movement is not permanent. These temporary movements are also the signs of severe risks over decisions incorporated with spot prices. Additionally, according to graphical and statistical observations prices display seasonality coherently with the weather changes or the daily consumption rates. Moreover, when the spiky nature of electricity is mentioned, it is also indicated that prices oscillates near its normal level. This property is named as mean reversion and could be observed in spot prices easily. Finally, the author concludes that the electricity prices exhibit heavy-tailed characteristics.

Furthermore in the literature, there exist some studies devoted to understand the specifically long-term component of electricity prices. Nowotarski et al. (2013) proposed, linear regression, monthly dummies, sinusoidal functions and wavelet smoothers to define this component of the prices in German and Nordic Markets. According to experiments wavelet based designs yielded better results.

Deterministic forecasting methods are entirely applicable to stable environments that have constant demand and balanced production, and at the time of sudden high demands inventory may help to recover the new situation. However, it is hard to integrate traditional techniques to fragile environment of the electricity prices. As a result, devising novel or hybrid methods becomes very important in modelling. In order to fill the gap between academic literature and market environment, research devoted to understand the price behavior has received much attention in the last decades. Within this scope, Möst and Keles (2010), Aggarwal et al. (2009), Higgs and Worthington (2010) and Karakatsani and Bunn (2008a)' s articles are useful for their elaborated literature surveys that are published about the design and taxonomy of the different spot price modeling studies. These papers present a detailed basis for the recent status of the literature in this area. From this point of view Möst and Keles (2010)' s survey presents a detailed outline about electricity pricing. In this paper selected models in the means of operations research and financial mathematics instances based on different decision problems and the comparison of them are given. They did not deal with operational issues, called fundamental models. Instead, they preferred to focus on research that is trying to understand the structure of the prices. Moreover they also categorized problems as short term, mid-term and long term decision models based on their impacts. They state that, researchers consider the prices either as a unique stochastic process or a stochastic process with a deterministic component. The studies show that the latter reveal better results. Deterministic part consists of trend and seasonality. Seasonality is taken into account for different periods, i.e. daily, weekly or annual. This approach is reasonable since for instance demand levels do not remain the same in summer compared to cool month or the need for electricity during the work day is obviously more than the midnight. Thus,the prices do not remain constant even at different times of a day.

To simulate the prices deterministic parts and stochastic parts are summed and the results are retransformed. Also, in some studies various exogenous factors are included such as oil, fuel price or CO_2 emission as commodities or wind power and hydropower effects on the electricity prices in order to refine the simulations.

Likewise, in the papers reviewed Karakatsani and Bunn (2008a) are limited to the time series models. After giving a brief introduction about electricity and market characteristics, they discussed the stochastic models existing in the literature.According to this study, the models have evolved in the time however these models are not adequate enough to reflect the electricity prices entirely. They are not fully capable of capturing the large deviations, also known as jumps, and these models have to be enriched with exogenous factors. In order to handle this situation regime switching models are presented. As mentioned before in Möst and Keles (2010), these regimes consist of two independent states, one is for regular movements and the other is for abnormal spikes. The other class consists of the k autoregressive processes with time-variant volatility. In this study not only historical data dependent models, but also some hybrid models that are integrated with some structural components like load or parameters of technical properties of plants such as capacity, fuel prices et cetera, are given. Finally, non-parametric modeling examples such as artificial neural network, genetic algorithms applications are mentioned.

On the other hand, in Aggarwal et al. (2009), a comprehensive outline is given about the existing pricing strategies in three categories: game theory models, time series models and simulation models. Time series has also three subcategories, parsimonious stochastic models, artificial intelligence models and regression models. Details about game theory models are also given in Möst and Keles (2010). Simulation models are actually not the same but similar with fundamental models as the system data is the key factor for this methodology. Also artificial neural network models and datamining models, as subtitles of regression models, are given as the instances of novel practices. In this paper categorization according to factors that are influencing prices are power markets are also provided.

Additionally, in Higgs and Worthington (2010), the authors prefer to focus on time series models and their variants to state the design parameters. It is assumed that, operational issues have no effect on design parameters of time series models. Firstly, they divided the existing literature into two groups as multivariate and univariate structures. In the multivariate models, relationship between power markets are examined. In the univariate models, prices consist of two components namely stochastic and deterministic parts. Instances of the vector autoregressive, ARCH, GARCH, mean reversion and regime switching methodologies are represented. This paper provides a thorough collection of previous works related with time series. Firstly they determined the model structures, solution approach, decision variables, performance measures, objective and setting under some assumptions and then they gathered this information in a table.

2.2 RISK MANAGEMENT APPLICATIONS

Practitioners have to be careful at selecting the proper tool for managing the risk arises from the special structure of electricity prices. Traditional methods, i.e. Greeks, fail to reflect the large scale variations in the volatility over time. In order to deal with heteroskedasticity, exponentially moving average (EWMA) and GARCH models are proposed.(Eydeland and Wolyniec (2003)) However, using these analytical methods come with the expense of some drawbacks like making assumptions and approximations for the sake of simplicity, i.e. normally distributed prices. Also Value-at-Risk is another popular gauge for managers, yet this method also has some setbacks like lacking of subadditivity and no use of handling the extreme values. Moreover, it is hard to manage VaR in optimization applications due to non-smooth nature of it.

In Krokhmal et al. (2002), a profit expectation maximization problem under various constraints including CVaR, is proposed and compared with mean-variance approach. According to their numerical results, these two methodologies yield approximate solutions. However, this outcome is not unexpected when the distribution is near normal like in the case of designed experiment indicated in Rockafellar and Uryasev (2000).

Likewise in Hochreiter et al. (2006), authors propose a model with minimizing the cost and the average VaR in a multi-stage stochastic programming framework and compare the performance of the predefined model with hedge and forget case. A scenario tree of stochastic pot prices, is generated and a tree-based solution procedure is applied. Their experiments show that, the multi-stage stochastic programming method outperforms the hedge and forget approach.

In Yau et al. (2011), authors have defined a two-stage stochastic integer programming model in order to devise an optimal power contract portfolio from the perspective of a generation company. This model deals with the case of a generation company who has to meet its customer demands with the aid of three alternatives. Electricity could be generated in their own facility, could be bought from the spot market or contracts could be purchased. At the beginning of the first stage, the company determines the number of contracts and at the beginning of the second stage their own facility and spot market originated amounts of energy are derived. CVaR is used to model the risk preference of the company to compare with the risk neutral manner. Uncertainty stems from electricity spot prices, production capacity and customer demands.

Likewise in Eichhorn and Römisch (2006), authors proposed a stochastic programming model that takes into account a weighted risk parameter and expected cost at the same time in objective function.For a generation company heat demands, spot prices and future prices are defined as the stochastic parts of the problem.

Single period, multi-period and zero risk measure models are compared and according to experiments multi-period case outperforms. Likewise in Kettunen et al. (2010), fix mix, stochastic and periodically updated models are devised and compared. In this study, a binary scenario tree based on a specific tree generation method is given and also probabilities are defined over an existing methodology. CVaR is selected as the risk measure and applied in both stochastic and periodic cases. Also impacts of inputs on the solution are analyzed from the views of two different distributor with risk neutral and lower risk preference manners.

In Eichhorn et al. (2004), authors searched for an optimal portfolio considering with the retailer' s risk preferences. After eliminating trend and seasonal components from historical spot prices an autoregressive process is proposed. Moreover, a jump diffusion process is also incorporated into this autoregressive process in order to reflect the spiky behavior of prices. In the same fashion, load data is represented with an autoregressive process without jump diffusion component. CVaR and multi-period CVaR are selected as risk metrics. Also alternative CVaR variations are also given in Römisch et al. (2004). In this study, electricity could be bought from spot market. Also buying fixed or flexible contracts are available. It is assumed that flexible contracts allow to buy electricity within in a given interval, not a pre-fixed quantity. An objective function is stated as a summation of risk measure multiplied with a preference rate and a cost expression.

In brief, when we consider all these papers, we will see that selected studies are concentrated on statistical control methods and performance estimation is achieved through the simulation method. Statistical control mechanisms needs a lot of data collection, data storage and data analysis. Fortunately, markets allow researchers to access high quality logs of prices. Similar conditions are also binding for clarifying and analyzing these data. However, with the help of appropriate software these tasks become significantly easier. A major draw-back of these studies is, researches are concerned with just pricing methodologies. Accurate forecasting of prices has high priority however to manage the risk arises from upcoming prices in a reasonable manner is also crucial. In this dissertation, after selecting the best candidate among generated different price scenarios, several optimization models are proposed and solved explicitly. Important managerial insights on the design of portfolios are obtained with the aid of the numerical experiments.

3. ELECTRICITY PRICE MODELLING

A major part of the academic literature suggests models based on the assumption that prices consist of two components: deterministic and stochastic parts. In the context of bipartite systems, proposed stochastic processes are significant defining factors that affect the quality of the models. Besides the considerable number of benefits, the common idea of the academic literature is that traditional modeling systems are not adequate and appropriate for the real market environments. Moreover it is not necessary that a specific method is suits well for every power market. Therefore novel modeling approaches are emerged to meet the needs of a real market environment. We propose different types of approaches for spot prices in order to evaluate and select the best alternative. Various techniques have been proposed in the literature for spot prices. Among these methods financial mathematical and statistical econometric models are employed. In our study, we are interested in papers that have focused specifically on these methods and research towards this intention are given in the following.

Escribiano et al. (2002)' s study is mainly concerned with proposing models to reflect the idiosyncratic behavior of electricity prices. After defining the regular patterns, the authors have tried to explain the existence of mean reversion, jump and specific volatility properties and devise models that take into account and conducted tests about their adequacy for various power markets. Pure diffusion (AR(1)), AR(1)-GARCH(1,1), diffusion model with jump component, AR(1)-GARCH(1,1) with jump component with time dependent intensity and with constant intensity models, including the deterministic parts, are proposed. The motivation behind proposing time dependent jump intensity is, that is more likely to observe jumps in some season, i.e. summer. GARCH models with jumps have outperformed the remaining models for the markets considered.

In Guirguis and Felder (2004)' s study, in order to forecast electricity spot prices, several models are proposed; they also investigated whether the exogenous factors affect the model quality is investigated. The authors have defined a transfer function that takes into account the natural gas prices in autoregressive model with lag 1. In addition to considering the effect of the natural gas prices over the electricity prices, trend and seasonality function variants and a GARCH(1,1) model are included. Besides, the outlier effect is also questioned in this study. According to numerical results, GARCH models omitting outliers have yielded better results.

In Keles et al. (2012), authors have searched for the various models for spot prices of German Power Market for the years 2002 and 2009. In equation 3.1, X_t stands for the log-prices. X_t^{trend}, X_t^{season} and $X_t^{residue}$ stand for trend component, seasonal component and stochastic component, respectively.

$$X_t = X_t^{trend} + X_t^{season} + X_t^{residue} \tag{3.1}$$

In order to remove the trend, a linear model where X_0 stands for the log price at time zero and γ is constant, is proposed as in equation 3.2

$$X_t^{trend} = X_0 + \gamma.t \tag{3.2}$$

Similar procedures are suggested for eliminating different types seasonalities with the help of the trigonometric functions. In the first step in order to remove the weekly cycle a sinusoidal function is proposed in equation 3.3. In equation 3.3, α is a constant and β is a scalar coefficient. φ is a shift factor that adjusts the starting point to the weekly cycle' s minimum point.

$$X_t^{weekly\ cycle} = \alpha + \beta \left| sin(\frac{\pi t}{168} - \varphi) \right| \tag{3.3}$$

After eliminating the weekly cycle, averages of specific hours for four seasons are subtracted in order to deal with daily cycles. The authors have finished to work with deterministic part by taking the annual cycle, that is found by average of each month for the years considered, out of the prices. After they obtained the stochastic residues, the procedure continues with proposing four different types of stochastic processes. In mean reversion model, the differences between consecutive prices are modeled as a stochastic differential equation and with the aid of Ito formula an exact solution is found.

After removing the deterministic components from the log-prices, stochastic residues are obtained. In order to simulate these residues three types of stochastic processes are used. One of these stochastic processes is mean reversion process and prices follow the process given below. In equation 3.4, κ, μ and σ represent mean-reversion rate, long term mean and standard deviation, respectively.

$$dX_t = \kappa.(\mu - X_t.dt + \sigma.dW_t) \tag{3.4}$$

dW_t is the standard Brownian Motion and $dW_t = \varepsilon_t dt^{1/2}$. After applying Ito' s Lemma to this stochastic differential equation the following form is obtained. In equation 3.5, δ represents the time difference,i.e. 1 hour, and ε is a normally distributed random variable with given parameters.

$$X_{t+1} = X_t.e^{\kappa\delta} + \sigma\sqrt{\frac{1 - e^{-2\kappa\delta}}{2\kappa}}.\varepsilon_t \quad \varepsilon_t \sim \text{N(0,1)} \tag{3.5}$$

Another stochastic process, which is commonly employed in the literature, is the jump diffusion models which are able to reflect the sudden price movements called spikes.In equation 3.6, κ is the mean reversion rate, μ is the long term mean, σ is the standard deviation, dW_t is the standard Brownian Motion and J is the jump heights.

$$X_{t+1} = \kappa(\mu - X_t).dt + \sigma.dW_t + lnJ.dq \quad lnJ \sim \text{N}(\mu_{lnJ}, \sigma_{lnJ}) \tag{3.6}$$

dq is Poisson factor responsible for jumps arrivals. With the aid of an auxiliary variable, which represents the intensity of the spikes corresponding to the realized data, say δ, dq is 0 as long as a uniformly distributed random variable is less than or equal to δ and is 1, otherwise. This process yields better results when it is combined with regime switching component. Regime switching mechanism is based on the following assumption: At a specific time, prices follow either base regime or peak regime, and transition probabilities of regime alternations of the prices during the considered time horizon is the main driver of the price processes. Utilizing the historical data, transition probabilities are devised. For a 2-state stochastic process, the transition probability matrix is given below in equation 3.7:

$$P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \tag{3.7}$$

In addition to the given processes above, autoregressive moving average processes are widely used to model the electricity prices. In order to detail, with the assumption of weak stationary of residues and error terms' distribution is known in advance, for an ARMA(p, q) process given in equation 3.8, price pattern is consisted of weighted summation of last p prices and weighted summation of last q error terms.

$$X_t^R = \sum_{i=1}^{p} \alpha_i X_{t-i}^R + \sum_{j=1}^{q} \beta_j \varepsilon_{t-j} + \varepsilon_t \quad \varepsilon_t \sim \mathrm{N}(\mu_{residue}, \sigma_{residue}) \tag{3.8}$$

In the autoregressive models, after the autocorrelation of the residues are examined, the models ARMA(5,1) and ARMA(1,1) are proposed. MATLAB finds the coefficients of the corresponding processes via Maximum Likelihood Estimation.

Also in order to handle the spiky nature of the spot prices, GARCH(p,q) process is introduced. In this study,p and q values are selected as 1. In equation 3.9, ω is a constant, α_z

and β_z are the autoregressive and moving average coefficients, respectively. σ^2_{t-z} stands for time-variant variance and $_{t-z}$ represent normally distributed error terms.

$$\sigma^2_t = \omega + \sum_{z=1}^{p} \alpha_z \sigma^2_{t-z} + \sum_{z=1}^{q} \beta_z \varepsilon^2_{t-z} \tag{3.9}$$

Finally, as the jumps are the components that must be included in the processes so as to obtain better results, a regime-switching factor is added to the procedure. After the transition probabilities, that of directly reflect the prices' tendency to jumps or other regimes, are acquired, with the help this transition matrix and one uniform random number, jump components are derived.

After replicating sample trajectories for each scenario, the adequacy of the results are compared with historical spot prices considering different performance measures.

Similarly, Bierbrauer et al. (2007) searched for the factors that affect the sufficiency of models and examined the effects of these factors with the simulation method. In this study, German spot prices are used between the years 2000 and 2003. After eliminating the outliers with the purposes of removing the deterministic parts, a linear trend is suggested and a sinusoidal function that captures the daily patterns is proposed. The data are restructured for eliminating monthly and weekly cycles in the following manners: averages of days are obtained, i.e. averages of all wednesdays for the years, and subtracted from preprocessed data. After that, monthly averages are calculated and values are removed. In equation 3.10, α, β, τ and γ are scalar parameters, D_{day} and D_{mon} are daily and monthly dummies, respectively. t stands for time and finally, d and m are vector parameters.

$$f_{det}(t) = \alpha + \beta.t + d.D_{day} + m.D_{mon}\gamma.sin\left((t+\tau)\frac{2\pi}{365}\right) \tag{3.10}$$

In order to mimic the stochastic part several processes are derived. Mean-reverting, Ornstein-Uhlenbeck, jump-diffusion models with different jump distributions(normal, log-normal, exponential) and 3 regime models are devised and compared the results with the spot prices. The authors indicate that, normal distribution is the best alternative for jump distributions and regime switching model is outperformed the remaining models in both in-sample and out-sample tests.

Cartea and Figueroa (2005) proposed a model such that can capture mean reverting characteristics of electricity. In order to refine their model, a jump component is also introduced. Once the spot price model is derived, closed form of future prices are also obtained. In this study UK Power Market data are used. Initially, the log-prices are deseasonalized in a straightforward manner, mean of hours are subtracted from each hour month by month, $g(t)$ and then a preprocessing procedure applied in order to define a jumps class J. In equation 3.11, S_t, $g(t)$ and Y_t represent the spot prices, the deterministic and the stochastic parts.

$$lnS_t = g(t) + Y_t \tag{3.11}$$

This preprocessing consists of an algorithm that separates the outliers that exceeds the mean value more than three times the standard deviation, from the original series. After that, a mean reversion process with the jump component is given in equation 3.12. In equation 3.12, α is mean reversion rate and σ_t is time- varying volatility.

$$dY_t = -\alpha Y_t + \sigma(t)dZ_t + lnJdqt \tag{3.12}$$

However, it needs some explanation about its terms. In equation 3.13 φ is a normally distributed error term. In this process, Z_t is Brownian motion, dq_t is a Poisson process that has the same character with the jump controller in Keles et al. (2012), and J is responsible for the jump size which is defined as in equation 3.13, equation 3.14 and equation 3.15. In equation 3.14 and equation 3.15, ρ represents mean reverting level.

$$J = e^{\phi} \quad \phi \sim \mathrm{N}(-\sigma_J^2, \sigma_J^2) \tag{3.13}$$

$$dS_t = \alpha(\rho_t - lnS_t)S_t dt + \sigma(t)S_t dZ_t + S_t(J-1)dq_t \tag{3.14}$$

$$\rho = \frac{1}{\alpha}(\frac{dg(t)}{dt} + \frac{1}{2}\sigma^2(t)) + g(t) \tag{3.15}$$

Mean reversion rate is found by linear regression differences of historical log-prices with log-prices. Time-varying deviation is acquired from rolling horizons consisted of 30-day time windows. Once mean reversion rate α and $\sigma(t)$ is found, these values are plugged in the equation above for simulating price process.

Likewise, in Mayer et al. (2012), spot prices of German, French, Nordic and British Power Markets between 2004 and 2009 are used. To eliminate the daily, weekly and annual patterns least square estimation is employed with dummies corresponding time periods. For deseasonalized log-prices, two processes are combined: one is for normal part and the other is for extreme values. The latter process is defined with a jump component $dI(t)$. This $dI(t)$ jump process is a compound Poisson processes whose arrival rates (in this context jump intensities) are λ^+ and λ^- and given in equation 3.16.

$$I^+ = \sum_{I=1}^{N_t^+} J_i^+ \tag{3.16}$$

Jump sizes are calculated in the same way given in Cartea and Figueroa (2005) with a slight difference. The authors also categorized the down jumps and assigned I^- for this type of price movements. In equation 3.17, Λ_{det} is deterministic and $Z(t)$ is the stochastic parts of price F_{price}.

$$F_{price} = \Lambda_{det} + Z(t) \tag{3.17}$$

$$dZ(t) = dX(t) + dY(t) \tag{3.18}$$

After discretizing the price process equation 3.18 is obtained and using linear regression mean reversion rate is found which helps to obtain a residue term represents the effects of jumps. In equation 3.19 γ is a constant parameter and $\varepsilon(t)$ represent the error term.

$$Z(t) = \gamma Z(t)\Delta t + \varepsilon(t) \tag{3.19}$$

This error term is refined according to the following assumption: if a value is in the interval between 2.57 times of standard deviation, then this value would be considered as ‘normal’ value. Rest of the data is classified as extreme values. This classification helps to find the mean reversion rates for base process and jump process separately using Maximum Likelihood Estimation. Since X and Y are different processes, each process has its own mean reversion rate, α_x and α_y. In this representation, also dB and dI are responsible for changes caused by mean reversion represented with equation 3.20:

$$X(t+1) = X(t) + dX(t), \quad Y(t+1) = Y(t) + dY(t) \tag{3.20}$$

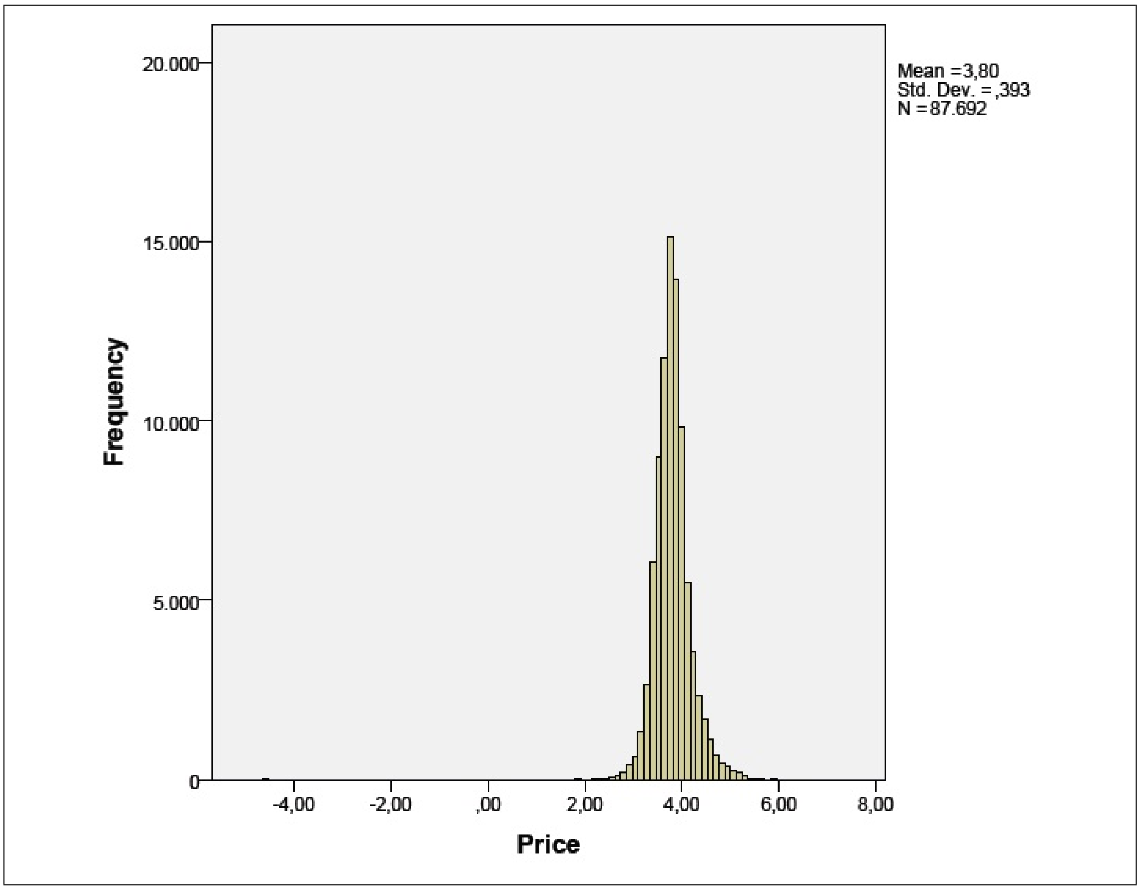
Replacing with increment definitions of X and Y equation 3.21 below are obtained.

$$X(t+1) = (1-\alpha_X)X(t) + \sigma dB(t), \quad Y(t+1) = (1-\alpha_Y)Y(t) + dI(t) \tag{3.21}$$

In this study for the different market data, different modeling approaches outperform for p-value and rejection tests.

The study of Weron and Misiorek (2008) investigated whether the exogenous inputs such as load or temperature, have effect over model adequacy or not for several methods. The datasets are provided from California (1999-2000) and Nordic Power Markets(1998-1999, 2003-2004). All modeling procedures start with deseasonalization. In this study, treatment of outliers is also investigated. An autoregressive model is derived for preprocessed prices. Moreover regime-switching and mean reverting models are also devised. Besides several time series models, authors also extend their tests with semi-parametric models which are beyond the scope of this dissertation. According to the performance tests, semi-parametric models have yielded better results than the Gaussian competitors. In our study, half-hourly based electricity prices of the British power market, known as APX, for a period of time from 1 January 2008 to 31 December 2012 are selected to analyze and test. Some appealing features of this market, such as relative maturity of the market environment, high quality records of the historical prices, and data accessibility for research intends, make us to select this alternative. Figure 3.1 indicates that the historical prices do not exhibit normal distribution.

Figure 3.1: Histogram of the log-prices



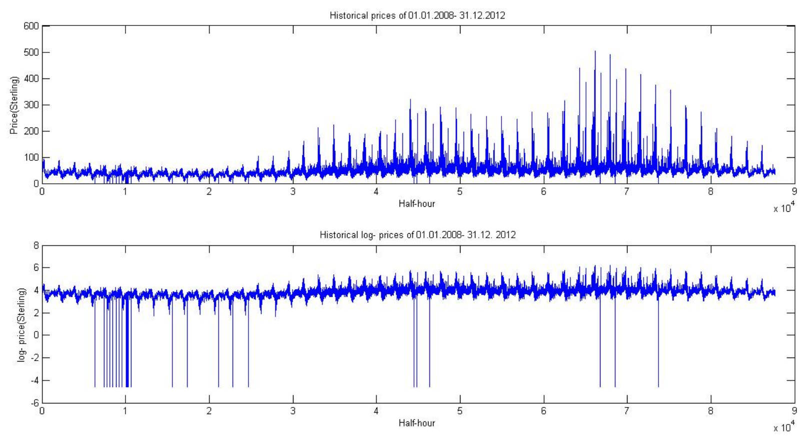
In the previous parts of this study, it is stressed that, the importance of the accurate estimation of the deterministic and stochastic parts of the historical data, is required in order to understand and emulate the price behavior. Historical data is our critical element to detect patterns, make statistical inferences and obtain the parameters. To start with, it is assumed that the electricity prices S_t consist of deterministic F_t and stochastic X_t parts in equation 3.22.

$$S_t = e^{F_t + X_t} \tag{3.22}$$

$$logS_t = F_t + X_t \tag{3.23}$$

In order to obtain the variance stability, logarithms of the prices are calculated and equation 3.23 is obtained. In Figure 3.2 it is clearly seen that, the transformation of the historical prices reduces the variance notoriously compared to the original values.

Figure 3.2: Price and log price series between 2008-2012



In Janczura et al. (2013), it is investigated that how the deterministic components and their parameters are responsive to the outliers. In this study, they have revealed that, handling these extreme values using with appropriate filtering techniques like threshold based or recursive filters, have yielded more robust parameter estimations. Likewise in Janczura et al. (2013), an outlier elimination procedure is also applied to the log-prices. The highest 2.5% of the log-prices are considered as the outliers and they are replaced with the average of the two neighbor log-prices with the help of a computational routine.

Deterministic part F_t is assumed as a summation of the linear trend and the cycles given in equation 3.24.

$$F_t^{det} = f_t^{trend} + f_t^{weekly} + f_t^{annual} + f_t^{daily} \tag{3.24}$$

First and foremost the existence of trend is investigated. A slight trend is observable in Figure 3.3. A linear trend is proposed as in equation 3.25. Least-square error estimation is employed to determine the parameters. In equation 3.25, a is constant, b is a scalar coefficient and t represents half-hour t.

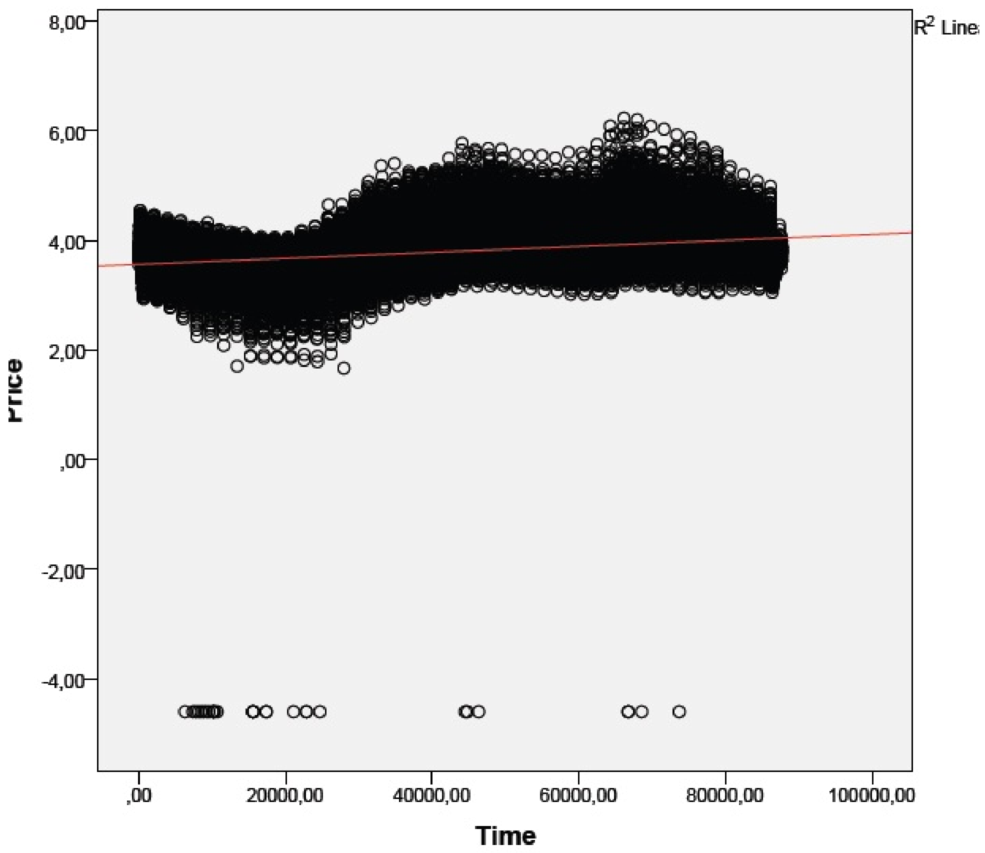
$$f_t^{trend} = a + b.t \tag{3.25}$$

Once the trend is eliminated from the log-prices, another significant cycle is detected. It is reasonable to propose a pattern that reflects the daytimes and midnights of the weekdays and week-ends price relations and similarities with each other caused by business activities. These special time slots of the days are called as peak and off-peak times. The time interval between 8:00 and 20:00 are considered as peak and the time interval between 20:00 and 8:00 are assumed as off-peak times of a day. In order to remove the 'weekly cycle' fitting a trigonometric function is common in the literature. For details cf Enders (2010). In equation 3.26, c is a constant, d is a scalar coefficient, φ is the shift parameter that adjusts the starting point to the weekly cycle' s minimum point.

$$f_t^{weekly} = c + d \left| \sin\left(\frac{\pi.t}{336} - \phi \right) \right| \tag{3.26}$$

In order to obtain the parameters of the weekly periodicity, a least square error estimation

Figure 3.3: Trend curve of the prices



procedure is proposed. After subtracting the weekly cycle from de-seasonalized log-prices, daily periodicity still exists. Likewise in Keles et al. (2012), once the year is categorized as winter, spring, summer and fall, averages of the corresponding season's each hour is calculated and subtracted. To eliminate this deterministic component the following function given in equation 3.27 is proposed.

$$f_t^{daily} = \frac{1}{7}\sum_{i=1}^{7} h_{ijk} \quad j = 1, .., 48 \qquad k = 1, \dots, 4 \tag{3.27}$$

The only deterministic component still need to be removed is the annual cycle. Due to the high level weather dependency of the electricity prices, it is inevitable to taking into account weather effect on prices. A monthly average elimination procedure is employed as in equation 3.28.

$$f_t^{annual} = \frac{1}{4}\sum_{j=1}^{4} m_{i,j} \quad j = 1,..,12 \tag{3.28}$$

Up to now, deterministic parts are handled. Removing all of these components yields a stochastic component. However, we also need to reflect the spiky nature of the electricity prices in our pricing methods in order to obtain accurate results. In order to deal with the stochastic parts, evaluate the performances and select the best pricing method that exhibits the original series characteristics, a variety of pricing procedures are given. For details cf. the Literature Review part of the dissertation.

Mean-reversion is a distinctive property of the electricity prices (Cartea and Figueroa (2005), Bunn and Karakatsani (2003)) and stochastic processes help to handle this issue. A mean-reversion process, called Ornstein-Uhlenbeck process, is given as follows. In equation 3.29, κ is the speed of the mean-reversion and μ is the long-term mean.

$$dX_t = \kappa(\mu - X_t).dt + \sigma.dW_t \tag{3.29}$$

In equation 3.29, dW_t is the standard Brownian Motion and $dW_t = \varepsilon_t dt^{1/2}$.

$$dW_t = \varepsilon_t dt^{1/2} \qquad \varepsilon_t \sim \text{N(0,1)} \tag{3.30}$$

With the aid of Ito' s Lemma (Neftci (2000)), the following solution form is obtained. In equation 3.31, δ represents the time difference,i.e. 1 half-hour, and ε is a normally distributed random variable with given parameters.

$$X_{t+1} = X_t.e^{-\kappa\delta} + \mu(1 - e^{-\kappa\delta}) + \sigma.(\frac{1 - e^{2\kappa\delta}}{2\kappa})^{1/2}\varepsilon_t \quad \varepsilon_t \sim \text{ N(0,1)} \tag{3.31}$$

For the sake of brevity, the equation above would be discretized using the time period δ

as 1 and re- written as follows. In equation 3.32 g is scalar coefficient and h is a constant parameter. Using with the Maximum Likelihood procedures, the parameters are obtained.

$$X_{t+1} = g.X_t + h + \varepsilon_t \quad \varepsilon_t \sim \mathrm{N}(\mu_\varepsilon, \sigma_\varepsilon) \tag{3.32}$$

Another modeling approach is autoregressive moving average process, called ARMA. These methods are interested in incorporating the weighted last p prices and q error terms in a recursive manner as it can be seen clearly in equation 3.33:

$$X_t = \sum_{i=1}^{p} \alpha_i X_{t-i} + \sum_{j=1}^{q} \beta_j \varepsilon_{t-j} + \varepsilon_t \quad \varepsilon_t \sim \mathrm{N}(\mu_\varepsilon, \sigma_\varepsilon) \tag{3.33}$$

To identify the parameters of ARMA(p,q) process, Box-Jenkins methodology is employed. Box-Jenkins methodolgy is given in Appendix A. (Enders (2010)).

Figure 3.4: Residual autocorrelation of ARMA(1,1) for 2010

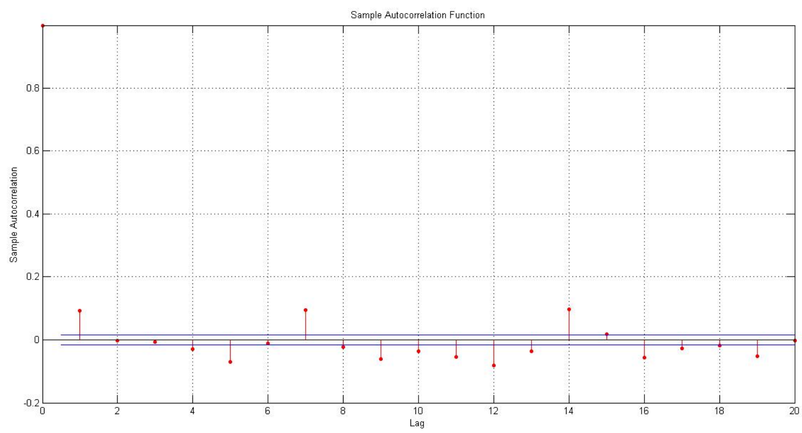


Figure 3.5: Residual autocorrelation of ARMA(5,1) for 2010

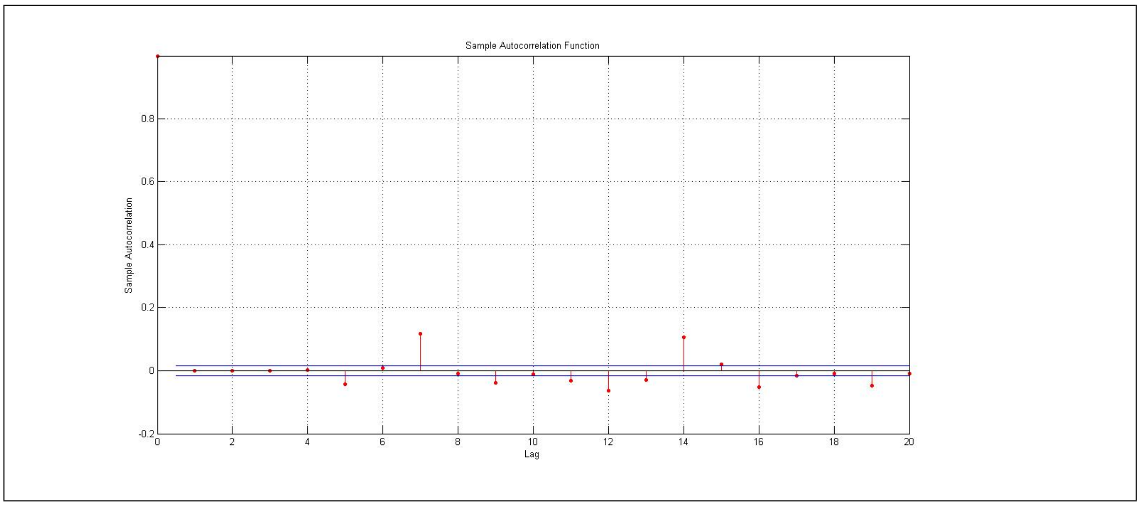
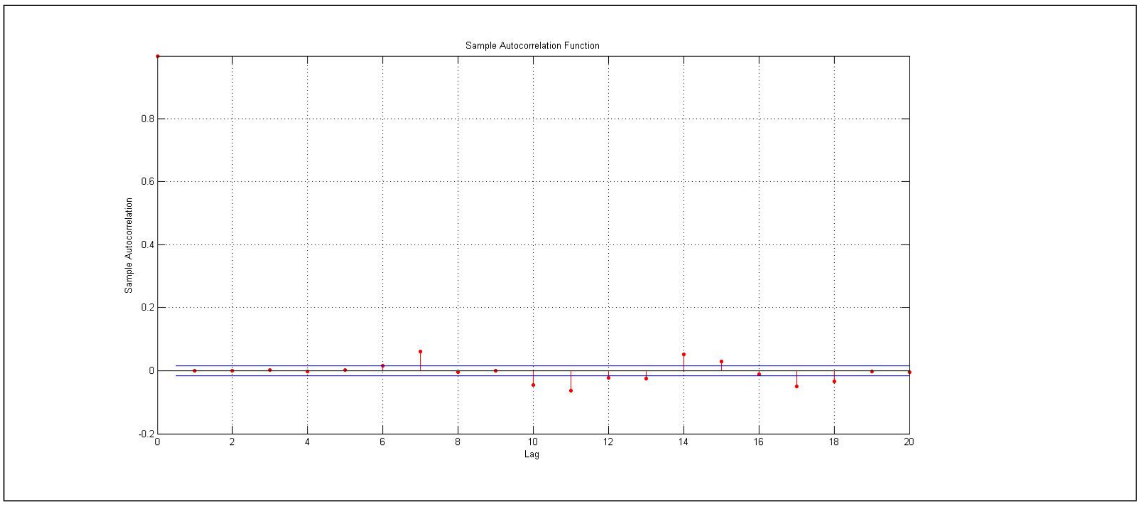


Figure 3.6: Residual autocorrelation of ARMA(7,3) for 2010



After analyzing the autocorrelation diagram of the residues and comparing the error term quality of several models, an ARMA(1,1), ARMA(5,1) and ARMA(7,3) are selected. Residual autocorrelations are given in Figures 3.4, 3.5, and3.6. MATLAB' s garchset and garchfit commands are performed in order to find parameters using with Maximum Likelihood Estimation.

With the purpose of combining time-varying variance with the stochastic processes generalized autoregressive conditional heteroskedasticity, called GARCH, class is proposed.

In the literature, it is common to use GARCH(1,1) process that has the formula given in equation 3.34. Also GARCH class can capture the spikes owing to its heteroskedastic variance(Keles et al. (2012)). In equation 3.34, ω is constant parameter, α and β are scalar coefficients.σ_t^2 stands for time varying variance and ε_t represent normally distributed error terms.

$$\sigma_t^2 = \omega + \alpha.\sigma_{t-1}^2 + \beta.\varepsilon_{t-1}^2 \tag{3.34}$$

Due to ARMA(p,q) processes have a homoskedastic structure, researchers are utilized integrating a regular ARMA(p,q) with a jump component, for details cf Cartea and Figueroa (2005), Keles et al. (2012), Mayer et al. (2012). In this regime-switching context, it is assumed that the electricity prices follow independent price regimes, i.e. base and jump regimes. The procedure starts with classifying the regime transitions. In this study prices that exceed the mean value three times of standard deviation are categorized as jumps and a positive jump class is created. Likewise, if a price is less than three times standard deviation from mean value then it is categorized as a negative jump. Using these classes positive and negative jumps' error terms' means and standard deviations are estimated. With the aid of piece of codes written in MATLAB environment, the procedure is performed.

In the next chapter we will deal with risk management applications in the electricity markets.

4. RISK MANAGEMENT APPLICATIONS

4.1 MOTIVATION

The extraordinary nature of the electricity, i.e. lacking the opportunity of utilizing the inventory, spikes or high weather- dependency (Escribiano et al. (2002)), makes the market players shape and manage their risk constantly if they want to survive. To exemplify it, in the aftermath of the failure of two major plants in the US in 1998, have made the prices escalated up to $7000 per MWatt, noting that the normal price level oscillates in the interval of $ 30- $ 60 at that time of the year (Deng (2000)). Taking action against the exposure of extreme spikes in advance, would provide the shareholder from catastrophic outcomes such as bankruptcy. To give an idea about the relative price volatility of the electricity and some other assets are represented in Table 4.1 .(Weron (2005)).

Table 4.1: Volatility examples of German Spot Market

Daily Prices	Volatility
Treasury Bills and Notes	0.5%
Stock Indices	1-1.5%
Commodities (crude oil or natural gas)	1.5- 4
Electricity	up to 50%

4.2 MEAN VARIANCE MODEL

In Huisman et al. (2009), authors investigate the optimal selection of electricity derivatives for a one- period planning term. In this study, authors show that categorizing the forward contracts into two classes (peak and off-peak) has an impact on hedging strategy. In this study, Markowitz (1952)'s mean-variance model is employed. In equation 4.1,

$C(T)$ represents cost function, N_o and N_p are number of off-peak and peak hours at day T, respectively. θ_o and θ_p are the number of contract decisions. $f_o(t,T)$ and $f_p(t,T)$ are market prices of off-peak and peak types, respectively. $s(h,T-1)$ stands for the electricity spot price and $v(h,T)$ represents the demand during hour h on day T.

$$\begin{aligned} C(T) = N_o\theta_o f_o(t,T) + N_p\theta_p f_p(t,T) + \sum_{h\in H_o}(v(h,T)-\theta_o)s(h,T-1) + \\ \sum_{h\in H_p}(v(h,T)-\theta_p)s(h,T-1) \end{aligned} \tag{4.1}$$

After defining the cost function, with the assumption of independent loads and spot prices a more tractable system is obtained and 'dividing decision matrix into two' case is investigated. In equation 4.2, $E_t\ C(T)$ is the expected cost and N is the number of off-peak and peak hours at a day. $\theta_t^{'}$ is contract decision vector and f_t is forward price vector. B is an indicator vector and, $\mu_v^{'}$ and μ_s are the hourly spot price means and the hourly demand means, respectively.

$$E_t\ C(T) = \theta_t^{'}(N.f_t - B^{'}\mu_s) + \mu_v^{'}\mu_s \tag{4.2}$$

In equation 4.3, Ω_s and Ω_v represent the covariance matrices of the hourly spot prices and hourly energy demand, respectively.

$$var_t(C(T)) = \theta_t^{'}B^{'}\Omega_s B\theta_t + tr(\Omega_s\Omega_v) + \mu_s^{'}\Omega_v\mu_s + \mu_v^{'}\Omega_s\mu_v + 2\theta_t^{'}(B^{'}\mu_s)(\mu_v^{'}\mu_s) \tag{4.3}$$

Practitioners use various tools for measuring the risk, i.e. Greeks, standard deviation, variance, VaR, and CVaR. Greeks fails to provide adequate information since they need a lot of parameters when the manager deals with portfolios with great number of components. Standard deviation and variance are interested in the dispersion of the values. These quantifiers are symmetrical and they have nothing to do with extreme values. Additionally, variance is indifferent to positive and negative deviations and for investor and produces sides these deviations are not considered the same. Moreover, mean-variance approach works properly when the underlying distribution of loss or return is symmetrical. It is impractical to make decision considering with this risk measure in electricity portfolios because electricity prices exhibit heavy tails (Weron (2005)).

4.3 COHERENT RISK MEASURES

VaR is a threshold value given with a probability level and actually tells nothing about beyond of itself. Additionally VaR is not a coherent risk measure which makes it impractical to use it in optimization applications. A coherent risk measure satisfies the following four properties (Hull (2009)):

i. Monotonicity: If one portfolio always produces a worse outcome than another its risk measure should be greater.

ii. Translation Invariance: If we add an amount of cash K to a portfolio, its risk measure should go down by K.

iii. Homogeneity: Changing the size of a portfolio by l should result in the risk measure being multiplied by l.

iv. Subadditivity: The risk measures for two portfolios after they have been merged should be no greater than the sum of their risk measures before they were merged.

VaR is not a coherent risk measure since it does not meet the last property which makes it contradicts the principle of risk reduction the by diversification. Also it does not provide a convex structure which results computationally intractable optimization models. VaR is formulated as follows:

$$\alpha_\beta = min \left\{ \alpha \in \Re : \Psi(x, \alpha)) \geq \beta \right\} \tag{4.4}$$

With the advent of the Conditional Value-at-Risk, defined as the average of values that exceed VaR, requirement for a subadditive measure that takes into account losses exceed VaR is satisfied. Since CVaR meets the coherency conditions, it is widely used by practitioners. CVaR formula is given in equation 4.5:

$$\varphi_\beta(x) = (1-\beta)^{-1} \int_{f(x,y) \geq \alpha_\beta(x)} f(x,y)p(y)dy \tag{4.5}$$

In Rockafellar and Uryasev (2000), a linear expression that minimizes VaR and CVaR simultaneously is given to utilize it in the optimization applications. In this study, instead of dealing with traditional CVaR formula, another closed form contains Var and CVaR is proposed which is given in the following equation. The authors show in their work that minimizing this convex function, given in equation 4.6, is equal to minimizing corresponding expected shortfall and their approach is widely employed in various studies.

$$F_\beta(x,a) = \alpha + (1-\beta)^{-1} \int_{y \in \Re^m} [f(x,y) - \alpha]^+ \, p(y)dy \tag{4.6}$$

In Krokhmal et al. (2002), a profit expectation maximization problem under various constraints including CVaR, is proposed and compared with mean-variance approach. According to their numerical results, these two methodologies yield approximate solutions. However, this outcome is not unexpected when the distribution is near normal like in the case of designed experiment indicated in Rockafellar and Uryasev (2000).

Likewise in Hochreiter et al. (2006), authors propose a model with minimizing the cost and the average VaR in a multi-stage stochastic programming framework and compare the performance of the predefined model with hedge and forget case. A scenario tree of stochastic pot prices, is generated and a tree-based solution procedure is applied. Their experiments show that, the multi- stage stochastic programming method outperforms the hedge and forget approach.

In Yau et al. (2011), authors have defined a two- stage stochastic integer programming model in order to devise an optimal power contract portfolio from the perspective of a generation company. This model deals with the case of a generation company who has to meet its customer demands with the aid of three alternatives. Electricity could be generated in their own facility, could be bought from the spot market or contracts could be purchased. At the beginning of the first stage, the company determines the number of contracts and at the beginning of the second stage their own facility and spot market originated amounts of energy are derived. CVaR is used to model the risk preference of the company to compare with the risk neutral manner. Uncertainty stems from electricity spot prices, production capacity and customer demands.

In order to manage the risk stems from the characteristics of the electricity, precise forecasting is the initial step to devise a plan in advance. After drawing the contours of the upcoming horizon's price structure clearly, risk management tools enable market participants to hedge themselves in reasonable manners. Financial products such as futures, forwards or options are some instances of instruments that allow people to make decisions in accordance with their risk preferences. For details cf Hull (2009), also Deng and Oren (2006) provides a broad review of electricity derivatives that are commonly in use. In this study forward contracts are preferred. Forwards are contracts between buyers and seller which makes them selling and buying an asset or commodity, in this context electricity, with pre-specified price in a predefined future time.

Considering with the delivery period of the electricity, there are three types of forwards which are in use: peak, off-peak and base load. Peak time delivery is usually higher than the others and a peak time forward provides a pre-defined amount of electricity between 8:00 and 20:00. The time period between 20:00 and 8:00 are accepted as off-peak time and the corresponding forward prices are relatively lower than peak hours. Base load contracts supplies the consumers some desired amount of electricity 24-hour day time constantly.

In this context, a retailer company who has to fulfill electricity demand of its customers constantly over a three-month planning horizon is considered. This retailer company has two energy procurement alternatives; they may buy forward contracts or purchase the electricity directly from the spot market. Forward contracts help its writers to hedge themselves against the market price risk, however this risk averse action comes with an expense since the forward contracts' prices are usually higher than the spot prices. Moreover, since forwards oblige their writers' to buy the energy in a predefined amount, one needs to be careful with the hedged and non-hedged parts of the energy need. In order to devise an optimal energy portfolio the following decision environments are considered.

In the first case, it is assumed that the company determines the contracted and non-contracted amounts at the beginning of the planning horizon and is allowed to buy only three-month contracts. In the second case the company makes the policy decisions at beginning of each month and the contract durations are also limited with one month. In the third case, the company decides the portfolio structures at the beginning of the months and has no limits over delivery durations of the forward contracts. Furthermore, in order to understand how uncertainty and risk preferences affect the optimal solution, each case is defined with two different systems. One of them is interested in minimizing CVaR, and the other one minimizes the total expenditures of energy demand fulfillment considering with a given budget which is limited with the CVaR value. Optimization models of these models are given as follows. In this study, it is assumed that all transactions take place at the beginning of the months, transaction costs are omitted, and the company cannot sell the excess energy in the market. And lastly, it is assumed that the only uncertainty of a defined system stems from the electricity spot prices.

Let the planning horizon is indexed with half hour t=1,...,T. The scenarios and forward contract types are indexed with s=1,...,S, and i=1,..., I, respectively. D^t and p_t^s denote the demand at time t, and the electricity spot prices at time t under scenario s, respectively. Additionally, π^s and f^i represent the probability of scenario s, and market price of long position in forward contract i, respectively. Since a forward contract delivers 1

MWh of electricity all the times t if it includes, the system needs an indicator $A_{i,t}$ states if the contract type i is active at time t and consists of zeros and ones. To exemplify the block structure of $A_{i,t}$, all the elements of $A_{i,t}$ related with base contracts consist of ones. The incorporation of risk aversion is achieved with linear programming formula of CVaR given in Rockafellar and Uryasev (2000). This model allows its users to calculate VaR represented with α and CVaR simultaneously in a β probability level. By introducing θ^i as the number of contract type i bought as decision variables and u^s as auxiliary variables, the optimization problem of the first case becomes the following:

$$
\begin{aligned}
& Min \;\; \alpha + \frac{1}{1-\beta}\sum_{s=1}^{S}\pi^s u^s \\
& s.t. \\
& \alpha + u^s \geq \sum_{i=1}^{I}\theta^i f^i \sum_{t=1}^{T} A_{i,t} + \sum_{t=1}^{T} p_t^s (D_t - \sum_{i=1}^{I}\theta^i A_{i,t}) \quad \forall s \in S \\
& \theta^i \geq 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \forall i \in I \\
& u^s \geq \; 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \forall s \in S
\end{aligned}
\tag{4.7}
$$

Case 2 is a multi stage version of the previous problem and is given as in the following. In this case, we need to state the K intermediate steps indexed with scenario set N whose elements are consisted of n=1,...,N. Then the optimization problem becomes:

$$
\begin{aligned}
& Min \;\; \alpha + \frac{1}{1-\beta} \sum_{n \in N_K} \pi^n u^n \\
& s.t. \\
& \gamma^{suc(n)} = \gamma^n + \sum_{i=1}^{I} \theta_n^i f_n^i \sum_{t=1}^{T} A_{i,t} + \sum_{t=T_{k(n)+1}}^{T_{k(n)}} p_t^n (D_t - \sum_{i=1}^{I} \theta_n^i A_{i,t}) \quad \forall n \in N \setminus \{N_K\} \\
& u^n \geq \gamma^n \qquad \forall n \in N_K \\
& u^n \geq 0 \qquad \forall n \in N_K \\
& \theta_n^i \geq 0 \qquad \forall n
\end{aligned} \tag{4.8}
$$

Case 3 is differs from case 2 as the company is allowed to purchase forward contracts for different delivery periods, say 1-month contract, 2-month contract, 3-month contracts.

$$
\begin{aligned}
& Min \;\; \alpha + \frac{1}{1-\beta} \sum_{n \in N_K} \pi^n u^n \\
& s.t. \\
& \gamma^{suc(n)} = \gamma^n + \sum_{i=1}^{I} \theta_n^i f_n^i \sum_{t=1}^{T} A_{i,t} + \ldots \\
& \qquad \sum_{t=T_{k(n)+1}}^{T_{k(n)}} p_t^n (D_t - \sum_{i=1}^{I} \tilde{\theta_n^i} A_{i,t}) \qquad \forall n \in N \setminus \{N_K\} \\
& \tilde{\theta}^i_{suc(n)} = \tilde{\theta_n^i} + \theta^n_{suc(n)} \qquad \forall n \in N \setminus \{N_K\} \\
& u^n \geq \gamma^n \qquad \forall n \in N_K \\
& u^n \geq 0 \qquad \forall n \in N_K \\
& \theta_n^i \geq 0 \qquad \forall n
\end{aligned} \tag{4.9}
$$

All of these problems deal with minimizing CVaR subject to system's constraints. However, in order to evaluate how budget B constraint affect the optimal the decision, these problems are reformulated such that minimizes total expected expenditures subject to previous constraints and an extra budget constraint. Considering with this adjustment first case becomes:

$$
\begin{aligned}
& Min \quad \sum_{\forall s} \pi^s \left(\sum_{i=1}^{I} \theta^i f^i \sum_{t=1}^{T} A_{i,t} + \sum_{t=1}^{T} p_t^s (D_t - \sum_{i=1}^{I} \theta^i A_{i,t}) \right) \\
& \quad s.t. \\
& \quad \alpha + \frac{1}{1-\beta} \sum_{\forall s} \pi^s u^s \leq B \\
& \alpha + u^s \geq \sum_{i=1}^{I} \theta^i f^i \sum_{t=1}^{T} A_{i,t} + \sum_{t=1}^{T} p_t^s (D_t - \sum_{i=1}^{I} \theta^i A_{i,t}) \qquad \forall s \in S \\
& \quad u^s \geq 0 \qquad \forall s \in S \\
& \quad \theta \geq 0
\end{aligned}
\tag{4.10}
$$

Case 2 is introduced as following:

$$
\begin{aligned}
& Min \sum_{n \in N_K} \pi^n \gamma^n \\
& \quad s.t. \\
& \quad \alpha + \frac{1}{1-\beta} \sum_{\forall s} \pi^n u^n \leq B \\
& \gamma^{suc(n)} = \gamma^n + \sum_{i=1}^{I} \theta_n^i f_n^i \sum_{t=1}^{T} A_{i,t} + \sum_{t=T_{k(n)+1}}^{T_{k(n)}} p_t^n (D_t - \sum_{i=1}^{I} \tilde{\theta_n^i} A_{i,t}) \quad \forall n \in N \setminus \{N_K\} \\
& \quad u^n \geq \gamma^n \qquad \forall n \in N_K \\
& \quad u^n \geq 0 \qquad \forall n \in N_K \\
& \quad \theta^n \geq 0
\end{aligned}
\tag{4.11}
$$

Finally case 3 is stated as following:

$$
\begin{aligned}
& Min \quad \sum_{n \in N_K} \pi^n \gamma^n \\
& s.t. \\
& \alpha + \frac{1}{1-\beta} \sum_{\forall s} \pi^n u^n \leq B \\
& \gamma^{suc(n)} = \gamma^n + \sum_{i=1}^{I} \theta_n^i f_n^i \sum_{t=1}^{T} A_{i,t} + ... \\
& \sum_{t=T_{k(n)+1}}^{T_{k(n)}} p_t^n (D_t - \sum_{i=1}^{I} \tilde{\theta}_n^i A_{i,t}) \qquad \forall n \in N\{N_K\} \\
& \tilde{\theta}_{suc(n)}^i = \tilde{\theta}_n^i + \theta_{suc(n)}^i \qquad \forall n \in N \setminus \{N_K\} \\
& u^n \geq \gamma^n \qquad \forall n \in N_K \\
& u^n \geq \qquad \forall n \in N_K \\
& \theta^n \geq 0
\end{aligned}
\tag{4.12}
$$

In the next section we will analyze different models using simulations done for the UK market.

5. RESULTS AND DISCUSSIONS

In this part efficiency of modeling methodologies and optimization models are discussed.After estimating parameters for each model, the performances of the simulations are evaluated. Three error performance measures are selected: mean average percentage value, root mean squared error and coefficient of determination. The formulae of these performance measures are given in equations 5.1 and 5.2. In these equations, N, T, and P represents the number of simulations, the number of simulated hours and the prices, respectively.

$$E(MAPE) = \frac{1}{N}\sum_{k=1}^{N}\frac{1}{T}\sum_{t=1}^{T}\frac{|P_{k,t,generic} - P_t|}{P_t} \tag{5.1}$$

$$E(RMSE) = \frac{1}{N}\sum_{k=1}^{N}\left(\frac{1}{T}\sqrt{\sum_{t=1}^{T}(P_{k,t,generic})^2) - P_t}\right) \tag{5.2}$$

After running 100 simulations, overall performances are given in the following table. These tests are performed using sorted prices since we are not interested in pinpointing the jumps, which is impossible, instead we are trying to predict the sizes of shocks because overall expected loss or gain does not depend on the time point of shocks. Different models have outperformed according to different performance measures. Table 5.1 and Table 5.2 shows the coefficients of ARMA(7,3) and Mean Reverting processes. Coefficients of the ARMA(p,q) processes are given in Appendix B. Table 5.3 indicates the test results for 2010.

In addition to numerical tests, in order to observe the accuracy of simulations graphs of the pricing methodologies are obtained and given in Figure 5.1 and Figure 5.2.

Table 5.1: Model parameters of the ARMA(7,3) for different years

Model	Parameter	2008	2009	2010	2011	2012	2008-2012
ARMA(7,3)	μ	1.027	0.689	0.626	2.4988	0.2272	2.360
	σ	0.224	-0.668	-0.5689	-2.7433	0.30	-2.454
	α_i	-0.277	0.935	0.10206	1.471	0.338	1.186
		0.0372	-0.212	-0.1286	-0.187	-0.073	-0.0466
		-0.023	0.0026	-0.057	-0.036	0.0174	-0.059
		0.0898	0.075	-0.0004	0.035	0.046	0.0516
		-0.080	0.135	0.0316	-0.04	0.11	-0.0378
		-0.080	0.135	0.0316	-0.04	0.11	-0.0378
	β_i	0.930	0.371	-0.465	-2.103	0.175	-2.138
		-0.219	0.615	0.588	0.2	-0.146	2.089
		0.195	-0.785	-0.897	-0.8508	-0.309	-0.883

Table 5.2: Estimated model parameters of the MR process for different years

Model	Parameter	2008	2009	2010	2011	2012	2008-2012
MR	μ	3.2902E-5	3.290E-5	-8.31E-6	-1.03E-4	1.2E-3	3.9152E-6
	σ	0.499	0.256	0.330	0.247	0.1888	0.312
	κ	0.881	0.355	0.471	0.2459	0.198	0.43

Table 5.3: Test results of pricing methodologies for 2010

Model	RMSE	MAPE	R
ARMA(1,1) w/o RS	6.94	28.79	0,93
ARMA(1,1)- GARCH(1,1)	6.72	28.7	0.93
ARMA(5,1) w/o RS	7.37	29.66	0.93
ARMA(7,3) w/o RS	7.33	29.04	0.93
Mean Reverting w/o RS	9.08	34.551	0.87
ARMA(1,1) w/ RS	11.57	19.17	0.88
ARMA(5,1) w/ RS	12.45	21.97	0.87
ARMA(7,3) w/ RS	14.76	28.18	0.85
Mean Reverting w/RS	6.54	37	0.92

In order to make further inferences about our model, out- of- sample tests are performed and given in Table 5.4. As it is expected, out-of-sample tests have not yielded not as good as in-sample tests, nevertheless results are still acceptable.

As it is mentioned before, a three-month planning horizon is considered and since all t' s represent half- hours, we have actually T=4320 half hours. Also there exist three types of contracts which makes i values are limited with I=3. We have the demand information

Figure 5.1: Historical and simulated price curves(Mean Reverting)

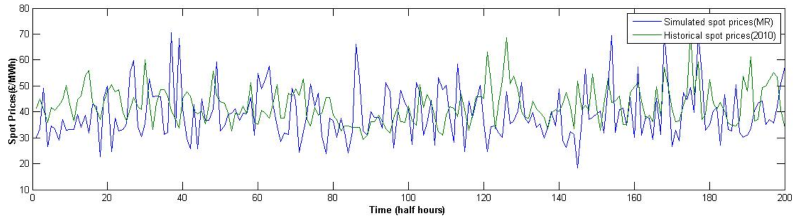


Figure 5.2: Historical and simulated price curves (ARMA(5,1) w/ RS)

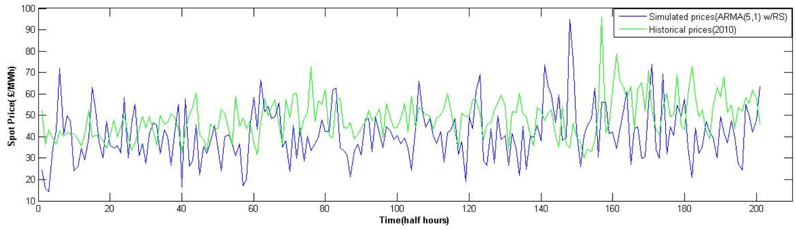


Table 5.4: Test results for 2011 and 2012 (out-of-sample)

Model	R	RMSE	MAPE
ARMA(1,1) w/o RS	0,97	10	27,9
ARMA(1,1)- GARCH(1,1)	0,96	47,2	47,6
ARMA(5,1) w/o RS	0,97	7,3	27,7
ARMA(7,3) w/o RS	0,96	10,1	28,6
Mean Reverting w/o RS	0,95	7,2	28,4
ARMA(5,1) w/ RS	0,89	30,9	41,0
ARMA(7,3) w/ RS	0,96	48,9	48,3
Mean Reverting w/RS	0,88	51,3	39,7

for upcoming planning term in advance. Prices of the forward contracts are also known. In multi stage cases, we have limited the number of stages with three. Finally it is assumed that the only uncertainty arises from spot prices.

In order to simulate the price paths, the mean reversion process is employed and total scenarios are limited with 1000 simulations. After obtaining the price paths, optimization models are solved in MATLAB R2010b environment. YALMIP and GUROBI 5.6.2 are selected as modeling language and solver, respectively.

We have tested each case with respect to changing β levels in order to evaluate the CVaR values. It can be observed that, in Figure 5.3, Figure 5.4 and Figure 5.5, as the decision maker behaves more conservative; the expected shortfall and the expected cost values both increase which in accordance with price of risk aversion concept. It is reasonable because when β level increases, CVaR value shifts toward extreme values. Also, Case 1 and Case 2 are actually subproblems of Case 3 and it can be observed in Figure 5.5 it is possible to devise a hedging strategy with a lower cost by updating at the beginning of each month rather than Case 1 and Case 2.

Figure 5.3: Results with respect to changing β- Case 1

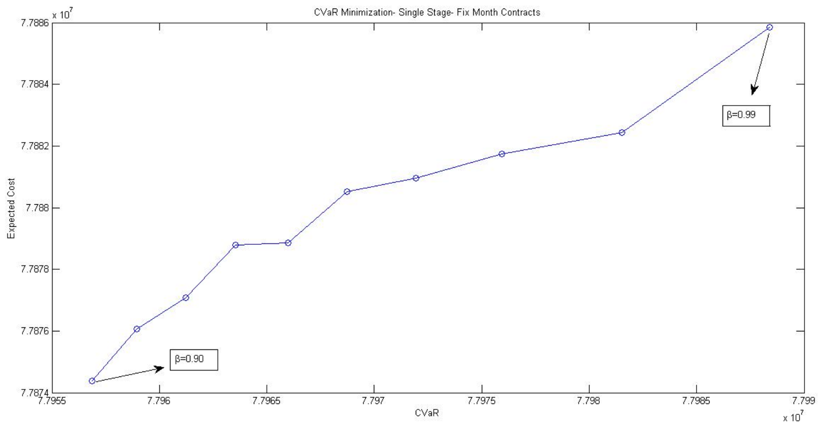


Figure 5.4: Results with respect to changing β- Case 2

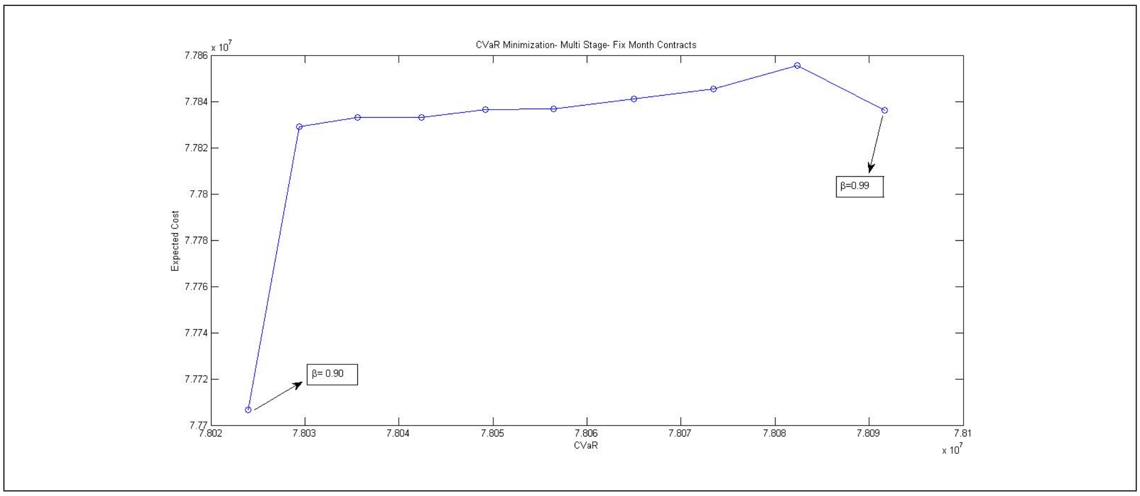
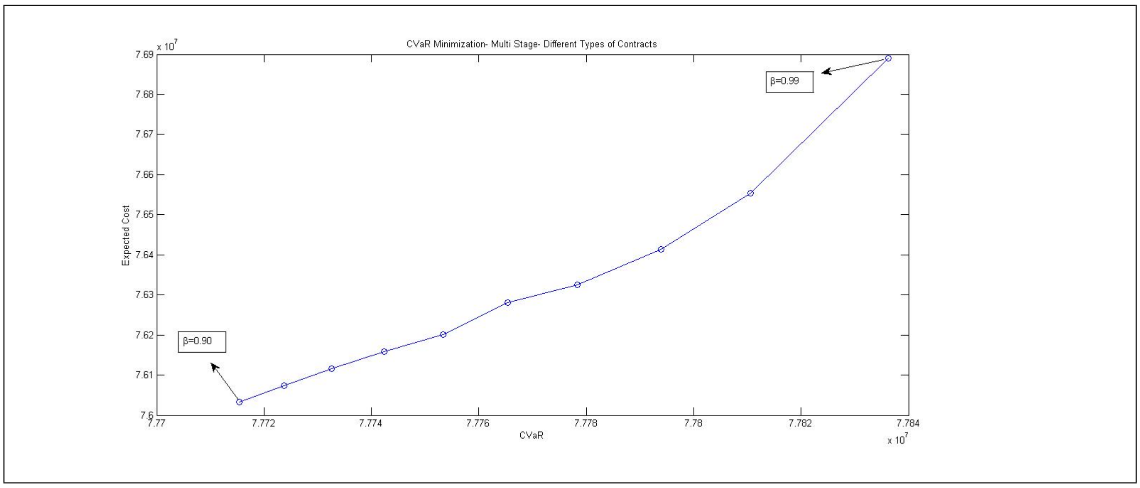


Figure 5.5: Results with respect to changing β- Case 3



According to Figure 5.6, Figure 5.7 and Figure 5.8, generally the optimal expected cost value is higher when CVaR is low which means the system cannot adapt itself into more conservative one with a lower budget.

Figure 5.6: Results with respect to changing budgets- Case 1

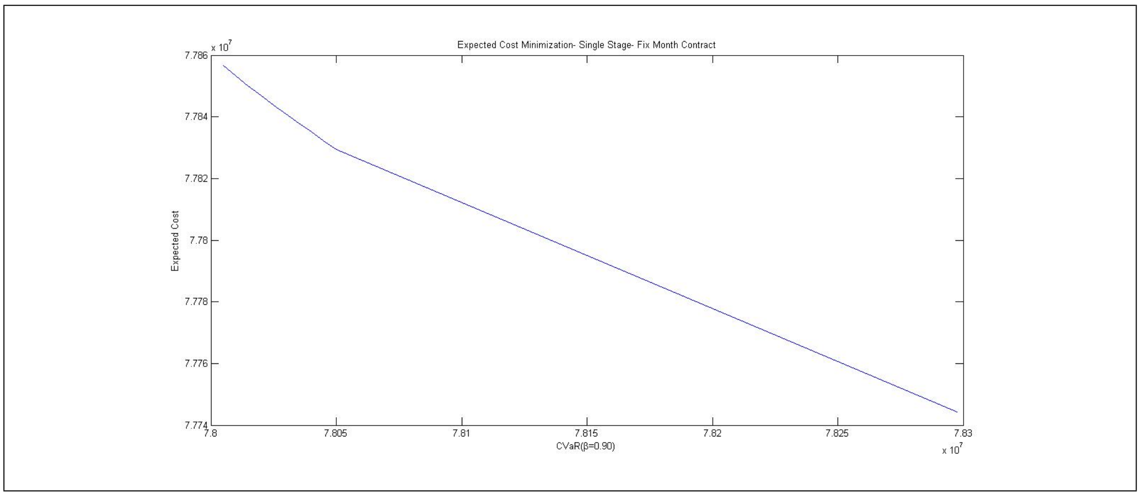


Figure 5.7: Results with respect to changing budgets- Case 2

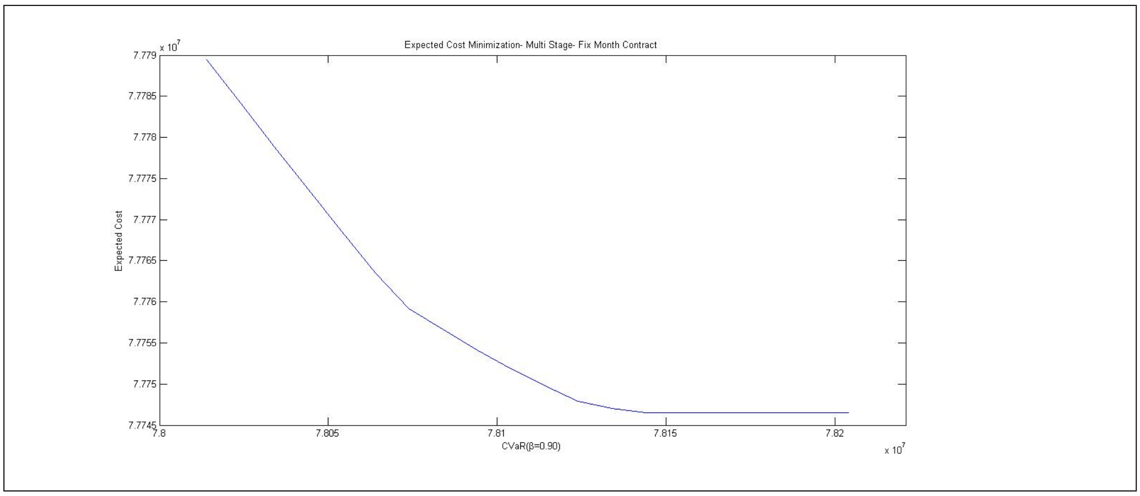
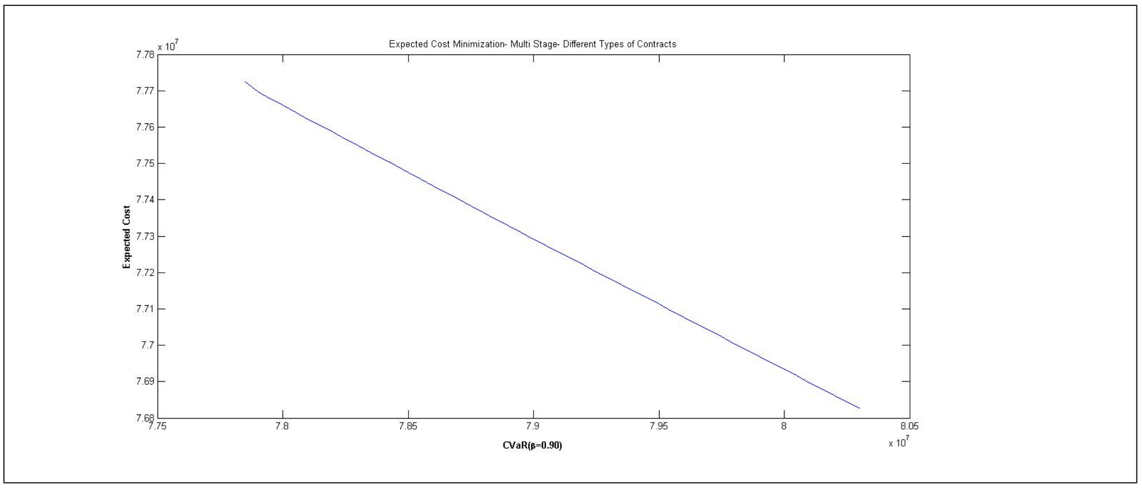


Figure 5.8: Results with respect to changing budgets- Case 3



6. CONCLUSIONS

Liberalization of the electricity market in a country is an important breakpoint for the market participants and this importance originates from the nature of the electricity itself. It cannot be effectively stored which means actors cannot make use of the inventory when there exist a gap between the supply and demand amounts. From the consumer view, electricity is not such a regular requirement, it must be delivered in the time of need since almost all of the manufacturing, trading even smaller scale activities highly depend on electricity as a result of today' s technology. Likewise, for an electricity generation company, an imbalance between production and transfer rates would lead undesirable outcomes.

Another interesting thing about electricity comes from its production conditions. There are several ways of producing electricity such as via hydropower, solar power, wind power, fuel or nuclear plants. Except the nuclear or the fuel based production cases, it is obvious that the electricity production highly depends on weather conditions. Also for end user side, excluding the manufacturing companies that have fixed and predetermined electricity usage schedules, the consumption is mainly formed according to the climate conditions. For instance in moderate conditions consumption level decreases, however in the times of extreme temperatures, need for electricity also escalates.

The combination of all of these issues given above, makes modeling the electricity prices attractive for the researchers. In the literature a lot of effort devoted to model the electricity prices using with different methodologies such as game theoretical, financial mathematical or econometric models which are detailed in the previous sections of this dissertation. In this dissertation financial mathematical and econometric approaches are employed. After analyzing the historical time series, firstly models are proposed and afterwards maximum likelihood or linear regression estimations are performed in order to find the relevant parameters which are also specified in the previous chapters of this thesis. Qualities of the fitted models are measured with several performance metrics and according to these tests, performance levels are adequate enough.

After comparing the models' price trajectories, mean-reverting model is selected to construct the optimization problems. In these optimization problems, a customer who has a predetermined schedule of electricity demand case is considered. In order to meet the expected future consumption, the consumer may directly buy electricity from spot market and make forward contracts include predefined amounts of electricity for a specific period of time. Forward contracts are usually more expensive than spot prices, however these forward contracts prepares a safer position in the fragile market environment against price shocks.

As it is mentioned before, if the market risk is not managed properly and a hedging strategy is not prepared in advance, then the market players face the devastating aftereffects of the price shocks. In this stage, decision on the selection of the risk metrics is up to the planner, however variance, VaR, and CVaR are frequently used risk quantifiers. The former two quantifiers have some drawbacks such as working with variance means a serious computational burden or VaR is not a coherent risk measure which does not allow to evaluate the portfolio construction cases properly. Nevertheless, with the aid of recent contributions of Rockafellar and Uryasev (2000), CVaR as coherent risk measure is expressed, in linear form and used in defining the risk of various cases. In our study, CVaR is preferred.

According to our results, models for Turkish power market can be also derived. However, due to the infancy of Turkish power market, it will be challenging to estimate model parameters and structures. Both econometric and financial models rely on relatively long time series, which makes the researchers to devise some other techniques for Turkish power market case. Additionally, these models could be tested on different countries' market data such as Australian case because Australian power market is also based on a half-hour structure likewise British case. Nevertheless, refining the parameters makes it possible to adapt the models for different markets.

Moreover, in this dissertation only financial and econometric models are considered. As it is known that the failure of a major supplier may lead to unusual price movements. As a result, taking into account the physical conditions of generation plants would be useful. However, these models are investigated under the category of fundamental models and they are criticized because the scale of information requirements.

Additionally, in our dissertation the exogenous factors that affect the electricity prices such as renewable energy, load or fuel prices are omitted. For example, governments promotes the companies to use renewable sources or solar power and wind power has no marginal cost which decreases prices indirectly.

To sum up, in this study uncertain electricity prices are modeled and generated a series of possible outcomes which are expressed as scenarios. After simulating the price trajectories, a risk quantifier is selected and the portfolio decision problem is expressed as a linear programming problem. And finally, the optimization problem is solved using with commercial software.

The main contribution of this study would be expressed in the following. In the literature, existing studies are focused on only price modeling or on decision optimization parts. Most of the studies which are concerned about price modeling totally ignore the strategies about making decision about how to manage market price risk. On the other hand, studies that deal with constructing portfolio towards hedging against the fluctuations, use the scenario trees that are not generated in ways sophisticated enough that the complex structure of the electricity prices require. These stressed drawbacks would be eliminated with an integrated approach that uses both of adequate modeling techniques and proper problem definition simultaneously. In this study, both of these sides of the system are considered as their importance is equally-weighted. To consider one side as trivial would mislead the decision makers.

Moreover, in order to reduce the complexity of the price modeling, it is common to aggregate the prices in for instance 4-hour blocks. In short, instead of working with time series point by point, it is preferred to perform on time slots for the sake of simplicity. In our study, half- hourly data structure is protected, besides all of the experiments and estimations are carried on using with this original form which makes the estimations and experiments more reliable.

This study would serve as a useful gauge to the decision makers in the electricity markets and is not only supportive on the task of pricing the spot and devising hedging strategies, but also would be beneficial in pricing some financial instruments, such as options or other derivatives based on electricity. Pricing of commodity derivatives widely studied in the literature, however power portfolios are relatively novel they need to be investigated more.

REFERENCES

Books

Enders, W.: 2010, *Applied Macroeconometric Time Series*, Vol. 3, Wiley& Sons.

Eydeland, A. and Wolyniec, K.: 2003, *Energy and Power Risk Management*, John Wiley & Sons.

Hull, J.: 2009, *Options, Futures and Other Derivatives*, Vol. 7, Pearson International Edition.

Neftci, S.: 2000, *An Introduction to the Mathematics of Financial Derivatives*, Vol. 2, Academic Press Advanced Finance.

Periodicals

Aggarwal, S. K., Saini, L. M. and Kumar, A.: 2009, Electricity price forecasting in deregulated markets: A review and evaluation, *Electrical Power and Energy Systems* **31**, 13–22.

Bierbrauer, M., Menn, C., Rachev, T. S. and Trück, S.: 2007, Spot and derivative pricing in the EEX power market, *Journal of Banking and Finance* **31**, 3462– 3485.

Cartea, A. and Figueroa, G. M.: 2005, Pricing in electricity markets: a mean reverting jump diffusion model with seasonality, *Applied Mathematical Finance* **12**(10), 313–335.

Deng, S. J. and Oren, S. S.: 2006, Electricity derivatives and risk management, *Energy* **31**, 940–953.

Eichhorn, A., Gröve-Kuska, N., Liebscher, A., Römisch, W., Spangardt, G. and Wegner, I.: 2004, Mean- risk optimization of electricity portfolios, *Proceedings of Applied Mathematical Mechanics* (4), 3–6.

Guirguis, S. H. and Felder, A. F.: 2004, Further advances in forecasting day- ahead electricity prices using time series models, *KIEE International Transactions on PE* **4-A**(3), 159–166.

Higgs, H. and Worthington, C. A.: 2010, Modelling spot prices in deregulated wholesale electricity markets: A selected empirical review, *Energy Studies Review* **17**(1), 1–25.

Hochreiter, H., Pflug, G. and Wozabal, D.: 2006, Multi- stage stochastic electricity portfolio optimization in liberalized energy markets, *IFIP International Federation for Information Processing* **199**.

Huisman, R., Mahieu, R. and Schlichter, F.: 2009, Electricity portfolio management: Optimal peak/ off- peak allocations, *Energy Economics* **31**(1), 169– 174.

Janczura, J., Trück, S., Weron, R. and Wolff, R. C.: 2013, Identifying spikes and seasonal components in electricity spot price data: A guide to robust modeling, *Energy Economics* **38**, 96– 110.

Karakatsani, V. N. and Bunn, D.: 2008a, Forecasting electricity prices: The impact of fundamentals and time-varying coefficients, *International Journal of Forecasting* **24**, 764– 785.

Karakatsani, V. N. and Bunn, D.: 2008b, Intra-day and regime- switching dynamics in electricity price formation, *Energy Economics* **30**, 1776– 1797.

Keles, D., Genoese, M., Möst, D. and Fichtner, W.: 2012, Comparison of extended mean-reversion and time series models for electricity spot price simulation considering negative prices, *Energy Economics* **34**, 1012– 1032.

Kettunen, J., Salo, A. and Bunn, W. D.: 2010, Optimization of electricity retailer's contract portfolio subject to risk preferences, *IEEE Transactions On Power Systems* **25**(1), 117–128.

Krokhmal, P., Palmquist, J. and Uryasev, S.: 2002, Portfolio optimization with conditional value-at-risk objective and constraints, *Journal of Risk* **4**, 11–27.

Markowitz, H.: 1952, Portfolio selection, *The Journal of Finance* **7**, 77–91.

Mayer, K., Schmid, T. and Weber, F.: 2012, Modeling electricity price: combining mean reversion, spikes, and stochastic volatility, *The European Journal of Finance* **Accepted for publication**, 1–24.

Möst, D. and Keles, D.: 2010, A survey of modelling approaches for liberalised electricity markets, *European Journal of Operational Research* **207**(2), 543– 556.

Nowotarski, J., Tomczyk, J. and Rafal, W.: 2013, Robust estimation and forecasting of the long-term seasonal component of electricity spot prices, *Energy Economics* **39**, 13–27.

Rockafellar, T. and Uryasev, S.: 2000, Optimization of conditional value-at-risk, *Journal of Risk* **2**, 21–41.

Römisch, W., Eichhorn, A. and Wegner, I.: 2004, Polyhedral risk measures in electricity portfolio optimization, *Proceedings of Applied Mathematical Mechanics* (4), 7–10.

Vehlivainen, I. and Keppo, J.: 2003, Managing electricity market price risk, *European Journal of Operational Research* **145**, 136–147.

Weron, R. and Misiorek, A.: 2008, Forecasting spot electricity prices: A comparison of parametric and semiparametric time series models, *International Journal of Forecasting* **24**, 744–763.

Yau, S., Kwon, R., Rogers, S. and Wu, D.: 2011, Financial and operational decisions in the electricity sector: Contract portfolio optimization with conditional value at risk criterion, *International Journal of Production Economics* **134**, 67–77.

Other Publications

Bunn, D. and Karakatsani, N.: 2003, Forecasting electricity prices, EMG Working Paper, London Business School.

Deng, S.: 2000, Pricing electricity derivative under alternative stochastic spot price models, Proceedings of the 33rd Hawaii International Conference on System Sciences.

Eichhorn, A. and Römisch, W.: 2006, Mean- risk optimization models for electricity portfolio management, Proceedings of PMAPS.

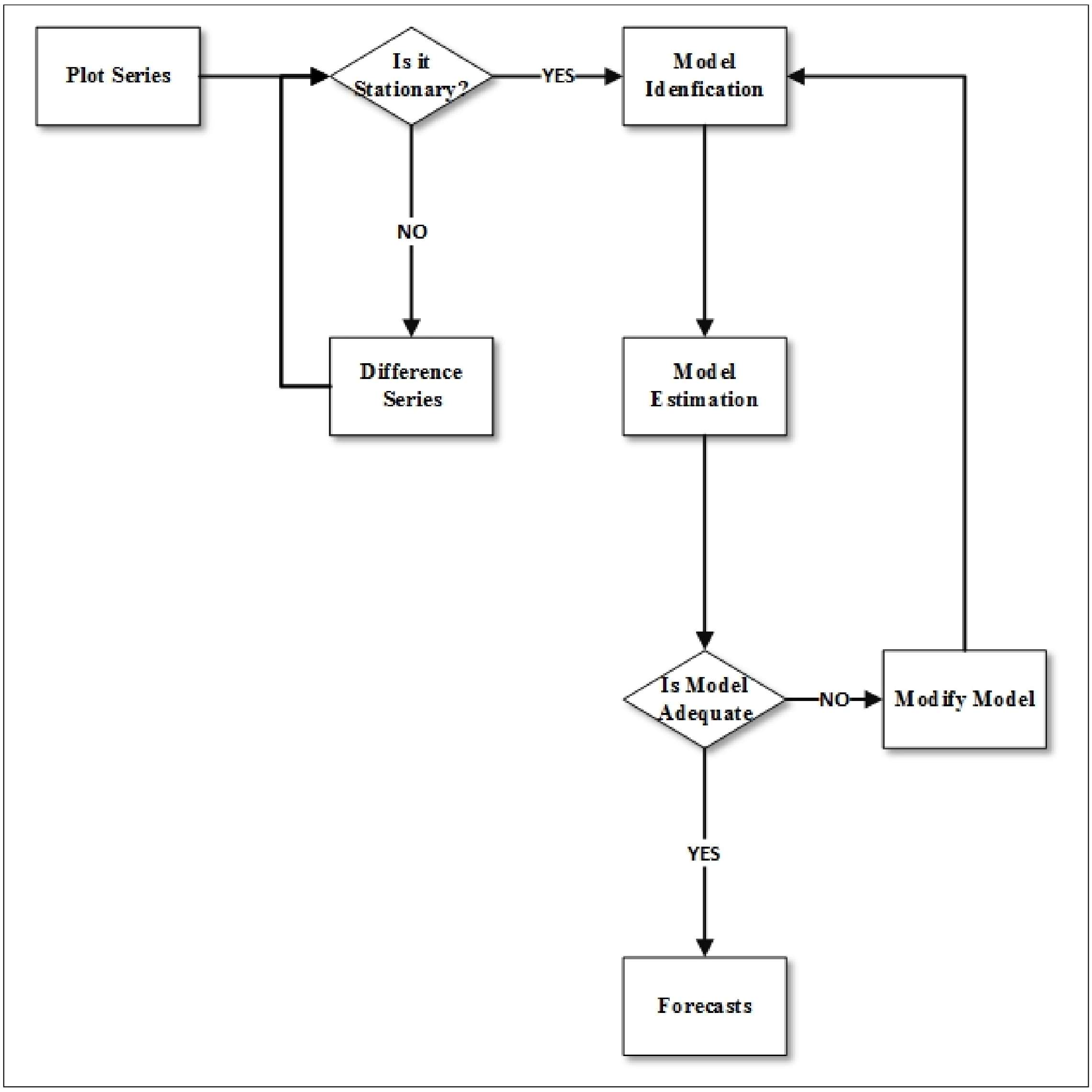
Escribiano, A., Pena, Juno, P. and Pablo, V.: 2002, Modelling electricity prices: International evidence, Working Paper 02-27, Economic Series 08, Universidad Carlos 3 De Madrid.

Weron, R.: 2005, Heavy tails and electricity prices, HSC/05/02, HSC Research Reports, Hugo Steinhaus Center, Wroclaw University of Technology.

APPENDICES

APPENDIX A. FIGURES

Figure A.1: Box- Jenkins Methodology (Enders (2010))



APPENDIX B. TABLES

Table B.1: Model parameters of the ARMA(1,1) for different years

Model	Parameter	2008	2009	2010	2011	2012
ARMA(1,1)	α_i	0.994	0.994	0.992	0.976	0.985
	β_i	-0.919	-0.866	-0.863	-0.681	-0.705

Table B.2: Model parameters of the ARMA(5,1) for different years

Model	Parameter	2008	2009	2010	2011	2012
ARMA(5,1)	α_i	1.02	1.172	1.136	1.224	1.205
		0.01	-0.266	-0.114	-0.268	-0.2405
		-0.068	0.038	0.004	-0.067	0.006
		0.045	-0.02	-0.0297	0.006	-0.0158
		-0.014	0.079	0.0005	0.0932	0.0375
	β_i	-0.927	-0.864	-0.917	-0.782	-0.803

Table B.3: Model parameters of the AR(1) for different years

Model	Parameter	2008	2009	2010	2011	2012
AR(1)	α_i	0.414	0.7005	0.623	0.781	0.819
	Constant	-3.627E-4	1.33E-1	-3.243E-4	2.365E-4	1.039E-4

CV

Name Surname	:	Esra Adıyeke
Languages	:	Turkish (native), English (fluent)
B.S.	:	Yıldız Technical University - Mathematical Engineering
M.S.	:	Bahçeşehir University
Institute	:	The Graduate School of Natural and Applied Sciences
Program	:	Industrial Engineering
Work Experience	:	Bahcesehir University Industrial Engineering Dept.
		Research and Teaching Assistant (2012-Present)