THE REPUBLIC OF TURKEY
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FLIGHT SCHEDULING PROBLEM

Master's Thesis

Aswif MUCYO AUNALI

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THE REPUBLIC OF TURKEY
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Name/Last Name of the Student: ASWIF MUCYO AUNALI

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The thesis has been approved by the Graduate School of Natural And Applied Sciences.

Assoc. Prof. Nafiz ARICA
Graduate School Director

I certify that this thesis meets all the requirements as a thesis for the degree of Master of Arts.

Assist. Prof. Ethem CANAKOĞLU
Program Coordinator

This is to certify that we have read this thesis and we find it fully adequate in scope, quality and content, as a thesis for the degree of Master of Arts.

Examining Comittee Members	Signatures
Thesis Supervisor Prof. Dr. Mustafa ÖZBAYRAK	-------------------------------
Member Assoc. Prof. Ahmet BEŞKESE	-------------------------------
Member Assist. Prof. Adnan ÇORUM	-------------------------------

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ABSTRACT

FLIGHT SCHEDULING PROBLEM

Aswif MUCYO AUNALI

Graduate School of Natural and Applied Sciences

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Supervised by Prof. Dr. Mustafa ÖZBAYRAK

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With an increasingly competitive environment of the air transportation sector, several airlines are compelled to design strategies in order to cope with large and perplexing optimization problems at planning and operations levels, especially the fleet management. In the airline industry, the Fleet Assignment Model designates fleet-types to a series of destinations while satisfying an assortment of constraints and minimizing the operation cost. The assignment has to meet a large variety of requirements and has to deal with the complementary objectives of minimizing all costs over the operated network.

Setting a good flight schedule, for amalgamated airlines, intensifies their operating efficiency and it helps the management in their decision-making. In this dissertation, we introduce an integrated model in which facilities with constraining capacities have to meet the amount of fleet-types and destinations available so as to produce a better schedule that best assigns an appropriate fleet-type to the appropriate flight leg.

We have developed a fleet assignment model to assign the best available aircraft type to the desired destination with a minimum cost satisfying the imposed constraint such as seat capacity; flight destination range; the number of aircraft in the fleet and so on. The model successfully assigned the right plane to the right destination at the right time with a minimum cost. The model has been solved using GAMS, which is an optimization software specifically designed to solve LP models, which is an open source program on the web.

Keywords: Flight Scheduling, Fleet Assignment, Optimization Model.

ÖZET

UÇUŞ PROGRAMLAMA SORUNU

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Hava taşımacılığı sektörinde artan rekabet oranıyla beraber, çeşitli hava yolları başta filo yönetimi olmak üzere, planlama ve işletim aşamalarında büyük ve karmaşık optimizasyon sorunlarıyla karşılaşmaktadırlar. Havayolu endüstrisinde, işletim maliyetlerini enaza indirerek hem filo'dan uygun uçağın uygun bir destinasyona atanmasını hemde pekçok çeşit kısıtın karşılanması gerekmektedir. Atama, farklı ihtiyaçlar, karşılamak ve maliyetleri minimize etmek durumundadır.

İyi bir uçuş programı,havayolu işletim etkinliğini artırdığı gibi,yonetimide karar verme süreçlesinde yardimci olur. Bu tezde, mevcut uçaklar arasından uygun bir uçağın uygun yine mevcut destinasyonlar arasından, bir destinasyona atamasını yapacak ve mevcut kısıtlarıda karşılayacak entegre bir modelin sunumunu yapmış bulunuyoruz.

Geliştirmiş olduğumuz bu uçak atama modelli, koltuk kapasitesi, uçuş yapılacak mesafe, filo'daki uçak sayısı gibi kısıtlarıda gözömüne olarak en düşük maliyetle mevcut en uygun uçağı en uygun destinasyona atamaktadır. Model,doğru uçağı,doğru destinasyona minimum maliyetle atamaktadır. Model özellikte Lineer Programlama modellerini çözmek için geliştirilmiş olan ve erişmi serbest olan bir optimizasyon yazılım olan GAMS kullanılarak çözülmiştir.

Anahtar Kelimeler: Uçuş Programlama, Uçak Atama, Optimizasyon Modeli

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ACRONYMS

ASM	: Available Seat Miles
ASK	: Available Seat Kilometers
ATW	: Arrival Time Window
CASM	: Cost per Available Seat Mile
CRS	: Central Reservation System
DDR	: Demand Driven Reflecting
DTW	: Departure Time Window
EA	: Evolutionary Algorithm
FAA	: Federal Aviation Administration
FAM	: Fleet Assignment Model
FAP	: Fleet Assignment Problem
GAP	: General Assignment Problem
GAMS	: General Algebraic Modeling System
GDS	: Global Distribution System
LP	: Linear Programming
O & D	: Origin and Destination
QSI	: Quantitative Service Index
RASK	: Revenue per Available Seat Kilometers
RASM	: Revenue per Available Seat Miles
TA	: Turkish Airlines
THY	: Turk Hava Yollari

1. GENERAL INTRODUCTION

1.1. INTRODUCTION TO THE STUDY

We recognize the critical role played by the transportation sector in the economy of most developed countries and this inspires the use of operation research and management methodology to reorganize the effectiveness of transportation. The implementation of mathematical programming methods to solve transportation intricacies differs depending on the mode that is air, rail, land or sea and also depending on what is being transported, in this case people or cargo.

A specific characteristic of the airline sector is that it involves high operational costs and firm regulations. Many bid companies have been inspired by this to sponsor and to lay the ground work in the growth of new and better methods to make the planning process more effectual. Flight scheduling is the most important component of an airline and it is the core service on which all the other services are based. It is however one of the most challenging and complex classes of problems. Different destinations, distances, planes, seat capacities, the maximum range distance that can be travelled by the plane (fuel capacity) and the number of flights can make the problem very complex.

Airline scheduling aims to determine when and where the airplane would fly. In order to minimize operational cost and or to maximize profits, schedules are built. These schedules help in the right assignment of a plane to the right destination under certain constraints while minimizing the total cost of operation. In this project, using the Turkish Airlines flight data, we focus on portion of the available data of flight scheduling which is fleet planning; we model how best to assign the applicable fleet-type to the appropriate destination while minimizing all operation costs.

In this research we are principally interested in the airline problem of fleet-type assignment. The fleet architecture involves selecting an exemplary set of fleet-types to be included in the schedule based on conjectured demand, and assign them to flight legs while maximizing profits and or minimizing operational costs. We assess and substantiate existing fleet problems and propose a new approach to adapt the airline fleet assignment problems.

1.2. BACKGROUND

The new vision for Turkish Airlines that goes *“To become the preferred leading European air carrier with a global network of coverage… whilst maintaining its identity as the flag carrier of the Republic of Turkey in the civil air transportation industry”* adopted in 2003 is still practiced. Despite the world economy has been in serious turmoil since 2008, this has little or no effect on Turkish Airlines because of the new way of structuring the management of the company as well as the operations of airlines, which was started in 2003. The new way of structuring the company has paid back very quickly and while many major airlines are either in very difficult position in terms of profitability or even facing a bankruptcy, Turkish Airlines has managed to expand its fleet size and the number of destinations it flies and has managed to increase the number of passengers use Turkish Airlines and the more importantly the profitability; Airlink (2014) [1].

Turkish Airlines has managed to grow, 2 percent in 2006 and 12 percent in 2008, 7 percent in 2012, while almost all the major airlines in Europe and America were struggling to preserve their current situation forgetting the growth. Turkish Airlines is now one of Europe’s most profitable carriers, behind only Ryan Air and EasyJet which can be classified in a category of low cost airlines.

Turkish Airlines has also been growing and expanding in the routes. TA has been opening new routes since 2005 and reaching approximately 300 routes in 2015 while strategically positioning itself for becoming global transfer airliner using Istanbul as a hub in between Asia, Africa, Europe and America; CAPA Centre for aviation (2016) [2].

In addition, TA has been acknowledged in terms of its service quality by the several major Airline auditing organizations. For example, Skytrax, a site which registers customer evaluations, has presented many awards on TA in recent years. Turkish Airlines has been awarded as the best airline in Europe and South Europe since 2011 in succeeding five years. Additionally, it is recognized as one of the best airlines in terms of cabin and seat quality as well as catering and positioned itself in the top 10 major airlines in last 7 years. Furthermore, passengers of Turkish Airlines have been recognizing it as one of the world’s best airlines. Hence, TA is categorized as a 4-star airline, a member of a small and exclusive group of carriers to gain this title; Airlink (2014) [1]. Turkish Airlines has the world’s second largest network by number of international destinations, eighth by number of international seats.

Figure 1.1: The new and old international destinations of THY around the world

Source: CAPA Centre for Aviation, 2016

1.3. PROBLEM STATEMENT

Flight scheduling problem is one of the most difficult problems in the class of scheduling problems and comprises finding out which market to serve with what frequency and deciding how to schedule flights to be able to meet these frequencies while satisfying the constraints imposed naturally by the type of problem dealt with.

Table 1.1: Simple example of a particular flight itinerary

Flight	***Origin***	***Departure***	***Destination***	***Arrival***	***Days***
205	IST	06:30 am	SFO	10:45 pm	123456_
706	ESB	02:15 pm	ATY	03:30 pm	1234___
1001	IST	12:05 pm	CAI	04:30 pm	1_3_567

Fleet Assignment Problem deals with what type of aircraft should be assigned to a particular destination satisfying all the imposed constraints such as range, seat, and customer demand while minimizing the overall cost.

To be able to operate an airline smoothly, a large number of decision making and optimization are needed at different stages of the airline operations alongside with a very large number of reliable data. Among the problems that need to be tackled and solved by the operational teams of an airline are predicting passenger demand, assigning aircraft and crew to all flights they operate, securing and maintaining aircraft, handling luggage and cargo, assisting passengers at check-in/the gate and managing re-accommodation of passengers and crew in case of disruptions. On the contrary of fleet assignment problem, the airline scheduling problem deals with assigning aircraft/fleet types -each having a different capacity- to the scheduled flights, based on equipment capabilities/availabilities, operational costs and potential revenues, Sherali et al. (2005) [40].

Airline fleeting revenues are highly affected by the decisions made. For example assigning a larger aircraft to a destination which has much less demand than the seat capacity of the assigned aircraft will result in unsold seats and most probably higher operating costs.

Whereas assigning a smaller aircraft than needed on a flight will result in lost customers due to inadequate capacity. Thus this makes the aircraft scheduling problem very vulnerable, since it comprises of an essential component of an airline's overall scheduling process.

Since a large number of flights planed each day, which may even reach hundreds for a major airline, it is vital for airlines to have a very reliable airline scheduling system considering the other difficulties encountered in airline processes such as schedule design, crew scheduling, aircraft routing, maintenance planning and revenue management; solving the FAP has always been a challenging task for the airlines, Sherali et al. (2005) [40].

Another hardship in this sector is the enormous size of the optimization models created since the problem type is typically combinational. This kind of problems grow very fast with varying constraints such as fleet number, number of destinations, passenger frequency demand growth, etc. This makes the whole process too complex to be treated globally, especially because of modeling and computational limitations, Sherali et al. (2005) [40].

The need to find optimal solution by minimizing costs and trying to be competitive in the market has motivated many researchers and a large amount of research results in airline optimization problems have been published for over the past 50 years. This is so because of the complexity of the problem.

1.4. RESEARCH QUESTION

From the above mentioned problems of airlines scheduling, researcher came to formulate the following research question: Which airplane should be assigned to which destination under the several constraints?

1.5. RESEARCH ASSUMPTION

In this project, Turkish Airlines routes, passenger demands, fleets types and flight times are taken as a basis to build an optimization model and solve it by using a commercial software package called *“GAMS”*, The General Algebraic Modeling System.

The approach has been applied to the daily flight scheduling of the international flight destinations from Istanbul Ataturk airport. The objective is to determine a particular aircraft type to assign to a particular destination while satisfying all the constraints imposed as well as minimizing assigned operation costs.

1.6. AIM OF THE RESEARCH

The aim of this project is to build a hub-and-spoke optimization model for Turkish Airlines and solving the problem of assigning the most adequate airplane to the most appropriate destination considering all the constraints.

1.7. OBJECTIVES OF THE RESEARCH

The overall objective of this project is to develop a model that Turkish Airlines would adapt to minimize operating costs while assigning the suitable fleet to the fitting destination with the given constraints that include different destinations, maximum range distance, aircraft seat capacities, the number of airplanes available and number of hours flew. This objective will be achieved by defining a state of the art model and a methodology, with original contributions to solve the fleet assignment problem.

This project would help to overcome the airline fleet scheduling problem and other setbacks that come along with it. To be able to do that, we will construct a mathematical model that will incorporate all the imposed constraints and then use GAMS as a solver to help to obtain an optimum solution. Thereby determine the best matching planes and destinations.

1.8. PROJECT SCOPE

To attain the research objectives, we start with sequential approach of Fleet Assignment Problem. In doing so, we propose a mathematical model and an algorithm that solves the Fleet Scheduling Problem. Basing on the set of fleet types (Boeing and Airbus family), the project shall consider the several destinations in Europe, Asia, Africa and America as limitation from/to Istanbul Ataturk airport in order to match in accordance with the tolerable scenario. An overview of the fleet assignment rank will be underscored and cost assessments will be submitted in relation with the proposed design. The scope will comprehend the following:

- ***i.*** Destination range
- ***ii.*** Different fleet-types
- ***iii.*** Number of planes available on the ground at the time decision is made
- ***iv.*** Aircraft seat capacities
- ***v.*** Number of hours flew as duty

1.9. CONCLUSION

As stated above, the project's aim is to solve aircraft assignment problems of Turkish Airlines while minimizing the flight operation costs under given constraints. To achieve this, a model is to be built and solved using GAMS solver, hence a minimum operating cost would be obtained and the best match between planes and destinations are to be determined.

After such a brief introduction, the rest of the thesis will be organized as follows:

Chapter TWO will tackle the Research Process which briefly explains how the research about this project was conducted; chapter THREE will focus on Conceptual and Theoretical Framework; which introduces you to the Flight Scheduling Problems; Chapter FOUR will focus on the Airline Fleet Assignment Problem at large; giving the literature review, and different models. Chapter FIVE will focus on Fleet Assignment Problem for Turkish Airlines as our case study, chapter SIX will give the Computational Results, and finally, chapter SEVEN will give Conclusions of the research done in the thesis.

2. RESEARCH PROCESS

2.1. THE OVERVIEW

This dissertation aims to investigate better ways of assigning an appropriate fleet type to an appropriate flight while minimizing the operation costs.

This chapter will enclose the methods that are used to research and collect data required for this study, and the limitations which have been encountered during this research.

The objective of our project as already been stated in chapter one, is to be able to develop a model that Turkish Airlines can adapt to so as to be able to minimize operation costs while assigning the suitable fleet to the fitting destination with the given constraints that include different destinations, maximum range distance, seat capacities airplane, the number of aircraft available and number of hours flew.

This project would help to contribute to overcome the airline fleet scheduling problem difficulties and other setbacks that come along with it.

The method which is a mathematical modeling which, were used to reach this goal were chosen carefully so that they can fit the research. Readings were done from Turkish Airlines annual reports, notes, books, publications and open sources of internet resources.

2.2. DOCUMENTS READING

The searchers are the read publication, papers from the internet regarding fleet scheduling problems; its solutions and setback and methods to use to solve such problems. They also read from the library which helped a lot in attaining beneficial data.

Literature reading has also helped to know and understand the different heuristics available in flight and fleet scheduling, each of these technologies is applied and its outcomes. This contributed a lot on the decision of which heuristic to use and how to use it.

2.3. INTERVIEWS

Researchers used interviews to improve the quality of the information gathered. Interview is an indispensable research method used when a researcher asks prepared questions to whom he needs information from either face to face or by other means. Interviews are classified into three categories which are structured, semi-structured and unstructured.

As Yan Zhang et al. (2009) defined, a *structured interview* is an interview that has a set of predefined questions and the questions would be asked in the same order for all respondents. This standardization is intended to minimize the effects of the instrument and the interviewer on the research results. And, a *semi-structured interview* is like a structured one but more flexible [3].

Minichiello V et al. (1990) described an *unstructured interview* as an interview in which neither the question nor the answer categories are predetermined. Instead, they rely on social interaction between the researcher and the informant [4].

In this research, we used unstructured interview to ask various people who are experts in flight scheduling department in Turkish Airlines some information on the existing flight schedules and network schedules. The questions asked during the different interview included the nature of such scheduling algorithms, heuristics used, problems faced before and the ones still occurring due to such methodologies, proposed solutions, among others. The answers given were so helpful in constructing this thesis paper.

2.4. LIMITATIONS OF METHODOLOGY

In this thesis, we limited ourselves on several things. We don’t consider the whole entire flight scheduling problem, which we believe it is even beyond a PhD level study as size and complexity; we limit our research only on Fleet Assignment Problem. We also focus on Turkish Airlines data, but since Turkish Airlines fly the largest number of destinations in the world, the size of the problem is beyond a MSc study, therefore, we consider only a small sample of fleet types and flight legs to build the adapted model, which is well enough for a MSc study as size and complexity.

The number of destination considered is seventy out of approximately 300 destinations and the number of aircraft from different fleet types is also 70 out of approximately 300 aircrafts.

An implementation and simulation of the project are provided, though we are restrained from some information from Turkish Airlines, which made our work laborious as a result of lack of appropriate material.

2.5. SOFTWARE USED

In this section we refer to the software and hardware products that we will use by their commercial name. We will hence use the General Algebraic Modeling System (GAMS) as a solver which is powerful enough to tackle the problem we considered in this study.

3. CONCEPTUAL AND THEORICAL FRAMEWORK

3.1. FLIGHT AND NETWORK SCHEDULE

In order to facilitate the comprehension of this massive decision making process, we will first provide some definitions, background information, and some key words that help understanding the airline scheduling and fleet assignment subjects better for the audience who have little or no knowledge on the subject.

3.1.1. Conceptual Definitions

Network effect*;* Refers to the problem that occur in the in the basic Fleet Assignment Problem arising from assumptions made on unpredictable passengers flow.

Fleet-type (aircraft type)*:* This is the categorization of certain model of aircraft with the same crew qualification requirements, maintenance requirements, seat capacity and destination range; Hanif D Sherali et al. (2004) [5].

Fleet-family (aircraft family)*:* A set of aircraft types, each having the same cockpit configuration and crew qualification requirements. Thus, the same crew can fly any aircraft type of the same family. An example of an aircraft family is the Airbus A320/321 family, which consists of multiple aircraft types, having capacity ranges between 180 and 225 passengers; Hanif D Sherali et al. (2004) [5].

Leg (flight leg)*:* An airport-to-airport flight segment that starts at a specific departure time and connects two stops of a flight, i.e., a leg measures the time that is needed for a journey from the time an aircraft takes off until it lands; Hanif D Sherali et al. (2004) [5].

Path (itinerary)*:* A sequence of one or more flight legs between a specific origin and destination, starting at a specific departure time. Thus, there can be multiple paths between each origin–destination pair; Hanif D Sherali et al. (2004) [5]. However, in this study, as it is practiced by Turkish Airlines, a concept of duty is implemented. The duty is a typical return journey. When an aircraft is assigned to a particular destination, the very same aircraft is used for the return flight.

Through-flight*:* Two or more legs those are desirable to be flown by the same aircraft. It would be very beneficial for customers who fly multiple legs between their origins and destinations. Though the aircraft makes intermediate stops, the passengers can stay on board until they reach their final destination; Hanif D Sherali et al. (2004) [5].

Fare class*:* Particular type of fare restriction. Example, F fare is the unrestricted fare (i.e. after purchase, the departure day can be changed with no penalty), G fare is more restricted (i.e. the departure day can be changed only by incurring a penalty, and the ticket should be purchased at least 2 weeks in advance of flight departure); Hanif D Sherali et al. (2004) [5].

Turn-time*:* The minimum time an aircraft needs between its landing time and the next take-off time. This includes the time for some minor inspections, cleaning, tiding up, preparation of the aircraft for its next trip, and its movement on the runway. The turn-time is aircraft-and airport-dependent, and generally it requires around 60 minutes for domestic flights, and even more for the large aircrafts and the busy airports; Hanif D Sherali et al. (2004) [5].

Distribution*:* It is the process of taking the airline products and putting them on the shelf for sale. The store front for the airline industry is primarily *central reservation systems* (CRS) and *global distribution systems* (GDS). A CRS allows an airline's reservation agents to book their own flights and fares. CRSs are relatively expensive to develop and maintain. Large airlines typically have their own CRS, while second and third tier carriers tend to rent space in another carrier's (often their competitor's) system; Timothy L Jacobs et al. (2011) [6].

Operation Costs*:* These are flight and fleet detailed costs resulting from the operation of the flight with a given aircraft type. These costs, as well as the minimum amount of fuel, gate rental and takeoff and landing costs are independent of the number of passengers on board; Barnhart C et al. (2002) [7].

Spill Costs*:* This is the overall routes of the estimate of revenue spilled from the journeys due to insufficient capacity; Barnhart C et al. (2002) [7].

Recaptured Revenue*;* It is the ratio of the spill costs that are recuperated by transporting passengers on routes other than their desired journeys. Here, if spill is only approximated as in basic FAM models, then recapture is at best approximate. In some basic FAM models, recapture is estimated as some ratio of the approximated spill cost, independent of whether or not capacity exists to transport these passengers; Barnhart C et al. (2002) [7].

Carrying Costs*:* They depend on the number of passengers flown, including; but not limited to, the costs of extra fuel, baggage handling, reservation systems processing, and meals. Because the number of passengers on a given flight is a function of the capacity assigned to that flight and to other flights due to network efforts. Hence in the basic fleet assignment model, estimates of carrying costs by flight leg for each assigned fleet type cannot be meticulous; Barnhart C et al. (2002) [7].

ASM (ASK): ***Available Seat Mile/Kilometer*** represents the annual airline capacity, or supply of seats, and refers to the number of seats available for passengers during the year multiplied by the number of miles/kilometers that those seats are flown; Bazargan M (2010) [8].

RASM (RASK): ***Revenue per Available Seat Mile/Kilometer*** or "*unit revenue"* represents how much an airline made across all the available seats that were supplied. RASM (RASK) is calculated by dividing the total operating revenue by available seat miles/kilometers or ASM (ASK); Bazargan M (2010) [8].

CASM (CASK): ***Cost per Available Seat Mile/Kilometer*** or *"unit cost"* is the average cost of flying one seat for a mile/kilometer. CASM (CASK) is calculated by dividing the total operating cost by ASM (ASK); Bazargan M (2010) [8].

3.1.2. Airline Planning Process

Airline companies have become very sophisticated businesses working with the advanced computer technologies and software engineering such as artificial intelligence and artificial agent technologies from being simple passenger and cargo carriers in several decades. The modern airline industry is very dynamic and has to maintain its productivity.

Airline companies need to make sophisticated planning and control, marketing and operations considering the major competitors in industry. Since, even the smaller airlines have the access to the high technology and supporting services easily in the industry the market has become very competitive and becoming a profitable airline carrier is becoming increasingly difficult.

In an airline company, demand constitutes to the type of flight schedule and route network an airline should build. Demand also dictates the composition of nonstop flights, origins and destination markets and the departure and arrival times of certain flight legs.

There are basically two kinds of networks, a hub-and-spoke network; and a point-to-point network.

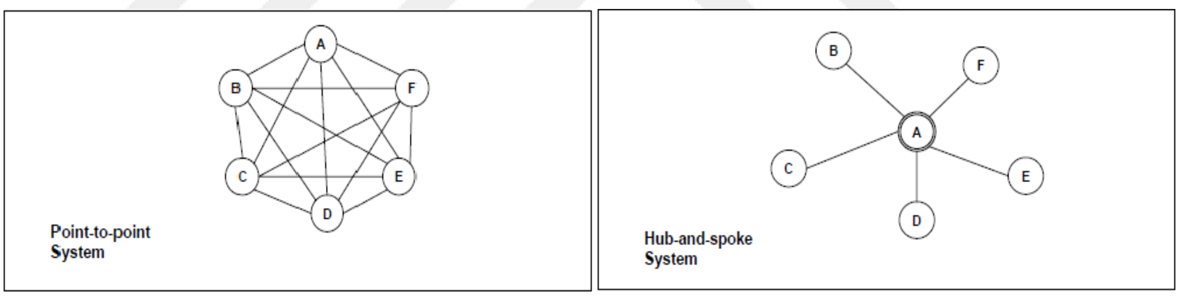
In hub-and-spoke network systems for example, passengers flow interdependently between hubs and spokes. This helps an airline centralize hub operations and lower operating unit costs by serving more markets with higher frequencies, Ki-Hwan Bae (2010) [17].

Major advantages of the airlines adopting hub-and-spoke operations include higher revenues, higher efficiency, and lower number of aircrafts needed as explained by Bazargan (2010) [8]. This system is the same as that we used in our thesis case where Istanbul is our main hub and other airports serve as different spokes.

In point-to-point network both hubs and spokes serve the same purpose.

The figure 3.1 below describes the functioning of a point-to-point network and a hub-and-spoke system.

Figure 3.1: The Point-to-Point and the Hub-and-Spoke System

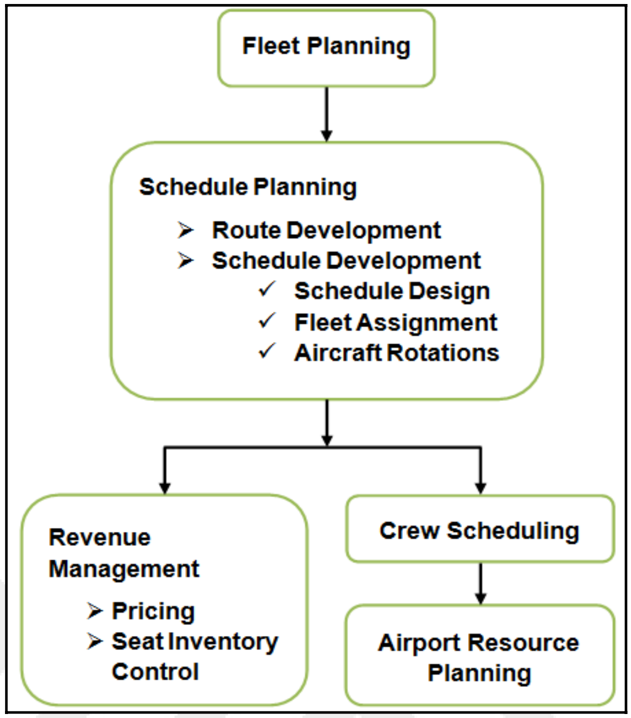


Source: Ki-Hwan Bae (2010)

The next figure 3.2 demonstrates the steps that are observed in an airline planning process which include time horizon activities and decision-making activities. These steps vary as per airline though these are main activities tackled.

As Timothy L Jacobs et al. (2011) relate; with the high application of operation research by the airline sector, other industries have been heavily motivated to take caution on this issue. The real time solution of optimization has played a great and considerably important role in framing today's airline industry. To minimize customer inconvenience and costs to the airline company; real time changes to planned schedules must be determined; and this is through scheduling problem [6].

Figure 3.2: Hierarchy of an Airline Scheduling



3.1.2.1. Fleet Planning

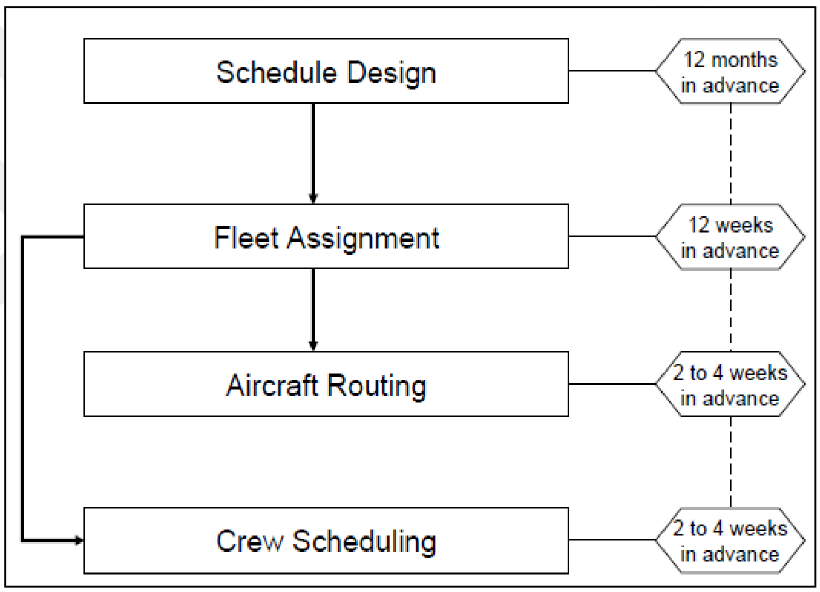
Better planning leads to better execution. The first step in an operational airline schedule is *fleet planning*. It is a collaboration of fleet processes and other fleet departments. A successful fleet planning also incorporates other areas of the company like treasury and strategy. In order to minimize operational costs and to maximize long-term profits, airline companies have built schedules which they follow.

For Timothy L Jacobs et al. (2011), Fleet planning is more of a process that encompasses steps like scheduling, marketing and distribution. And these steps are done occasionally, and each decision made has a long lasting effect on the airline. Other activities that are involved in fleet planning could include revenue management, crew scheduling, and airport resource planning. These activities help an airline determine its boundaries by which the airline schedules will be operated and managed [6].

3.1.2.2. Schedule Planning

Schedule planning is the opening step of an airline development. It normally begins 12 months before the schedule goes into operation lasting at most 9 months. Airlines put a lot of focus and time in Schedule Planning in order to survive in the competitive environment and to make fruitful profits. The costs of operating the schedule depend on the flight legs, which drive the number and type of aircraft used. The schedule must reflect the cost and availability of cabin and flight deck crews, as well as the requirement that aircraft cycle through maintenance bases at regular intervals. Competent schedules which match supply and demand are a key to airline profitability. Airlines address many scheduling issues from assigning aircraft and crews to flights, routing aircraft to maintenance bases with large-scale combinatorial optimization techniques, Timothy L Jacobs (2011) [6].

Figure 3.3: Planning stage of an aircraft



Source: Ki-Hwan Bae (2010) [17]

Referring to Teodorovic D (1989), Schedule Planning begins with the airline determining the flight legs/routes. The airlines search for what market to handle and this is defined by origins and destinations; it wants to serve basing on the wide demand information. Normally, most of the schedule planning steps which include route development and schedule development; begin from an already existing schedule but with a more developed route network in which alterations are made that reflect on changing demand and environment [9].

This can be defined as Schedule Development which includes the steps below:

i. Schedule design

ii. Fleet Assignment

iii. Aircraft Rotations

3.1.2.2.1. Schedule design

The schedule design is the most sophisticated step; and also known as the core of all other airline activities and operations. Typically also known as *flight scheduling*, it is divided into two sequential steps:

i. **Frequency planning:** In this phase, schedule designers determine the best service frequency in the market. Planners make sure that daily and weekly frequencies are matched to the forecasted demand in every market. They make sure the anticipated demand and the frequency are well balanced. This depends on many factors like market type, length of haul, among others. If the airline is met with a long international haul, the airline might offer daily flights while if the haul is short of domestic market, the airline might hourly flights. For example; Turkish Airline might assign only a two day in week flight to Kigali, while offer an every one hour flight to Ankara.

Teodorovic D et al. (1989) present a procedure that maximizes total profit and market share and minimizes the total schedule delay of all passengers on the network by determining the optimal flight frequencies on a network [9].

ii. **Timetable Development**: The airline needs to define at what time the scheduled flight will be performed. This is done immediately after the airline has determined how many flights it wants to offer in a particular market. The challenging driving factors of this step are the characteristics of the market and the constraints enforced on the schedule.

For example; in a market that comprises of business flyers, their desire is to have available, reliable and flexible hourly flights. This creates a constraint on the airline since the departure times have to be integrated in such a way that the schedule is accommodated to the available aircrafts.

A sub-timetable has been developed by Berge (1994); which takes as input a set of candidate flight legs that are determined by the origin, destination, arrival and departure times. This sub-timetable is optimized and enhanced in the already existing timetable. This timetable covers a maximum market range; including the frequent business flyers [26].

As of present, it is not feasible to optimize a full-scale timetable and implement it because of its complexity and size; Belobaba (1999) [15].

3.1.2.2.2. *Fleet assignment*

The fleet assignment purpose is to allocate an aircraft to every flight in such a way that the seating capacity strictly matches the demand for every flight in the network.

In a situation where the demand is small and a big fleet is assigned to such flight leg, many empty seats which can hypothetically be utilized more usefully elsewhere are flown.

On the other hand, if the frequency is vast and a small aircraft is assigned, it leads to many promising passengers being rejected, or spilled. In either case, they culminate into potential revenue loss. The distribution of aircraft to flight legs has to respect preservation of aircraft flow. The airline cannot allocate more aircraft than are available. If the schedule cannot be fleeted with the available number of aircraft, minor changes must be made to the schedule; Teodorovic D et al. (1989) [9].

Assignment process incorporates one of the most important and well-studied applications of operations research in the airline industry. Fleeting process embodies the complexities and computational difficulties characteristics of many aspects of the airline industry; Timothy L Jacobs (2011) [6].

Abara J (1989) [11], Hane C et al. (1995) [12], and others revise the fleet assignment problem from many facets. The fleet assignment with time windows model by Rexing B et al. (2000) [13], allow minor re-timing of flight legs, hence allowing otherwise infeasible schedules to be fleeted. The route-based fleet assignment model by Barnhart C [14], develops upon the basic assignment model by Hane C et al. (1995) [12], by making an allowance for network effects and recapture.

3.1.2.2.3. *Aircraft rotations*

Aircraft rotation aims at identifying which distinctive aircraft from a particular fleet type is assigned to serve what particular flight leg in the network. A rotation is collection of connected flight legs that are allocated to a specific aircraft, starting and ending at the same location, over a specified period of time. Teodorovic D et al. (1989) view on aircraft rotation step is to find a preservation possible rotation of aircraft, provided with the available number of aircraft of each type and a fleeted schedule [9].

3.1.2.3. Revenue Management

The revenue management step in the airline planning process aims at maximizing revenue. Belobaba P (1999) describes two separate but narrowly related components of an ideal revenue management system: *Differential pricing* and *Seat inventory control*.

3.1.2.3.1. Differential Pricing

Since 1978, the airline has changed from a simple industry with very stable pricing to one with inconsistent and complex pricing structure.

As Belobaba (1999) explains; most airlines now practice a pricing strategy which offers "*fare products*" with different restrictions at different prices [15].

This concept targets the "*passenger inclination to pay*" where the same product can be sold for different prices to different consumers based on the values that the consumers associate with the product. The differential price strategy encourages discount seekers and captures the willingness to pay of high fare passengers. Fare restrictions attempt to avoid demand intensity from diversion; the array of existing high fare passengers opt to take in favor of low fare offering.

3.1.2.3.2. Seat Inventory Control

It determines the number of seats on a flight reserved to a specific fare product. The main idea is to reserve as many seats as possible for high fare passengers and limit the seats for discount seekers as much as possible; Belobaba (1992). Airlines employ a set of tools to achieve this objective [27].

These include:

a. ***Overbooking:*** strategy to minimize empty seats on board by allowing bookings in excess of capacity.
b. ***Fare class mix:*** restraining the availability of seats sold on a flight leg.
c. ***Route control:*** discerning among passengers travelling on diverse leg routes.

Overbooking and fare class mix have been the emphasis of early revenue yield management. In order to determine any seat inventory control practice demand estimation is taken into consideration. The level of variation in the data differs from each other, depending on leg-based fare classes to origin-destination-based fare classes, Belobaba (1987) [28].

3.1.2.4. Crew scheduling

Crew members in an airline company may include pilots and/or flight attendants. The main aim of crew scheduling process in an airline planning process is to minimize the cost of assigning crew members to fight legs respecting certain restrictions that abide crew members; Jian Liu (2003) [16].

Crew scheduling is usually divided into two steps; crew pairing problem and crew assignment problem; Barnhart et al. (1996) [29].

Crew pairing simulates crew schedules and finds a set of work pairings that minimizes total crew costs while covering each flight the appropriate number of times needed. Crew pairing problems are commonly formulated in such a way that each pair is made up of duties which are separated by rest periods. A duty is a classification of flight legs to be flown uninterruptedly in one day that satisfies all work rules, Jian Liu (2003) [16].

Crew assignment simulates crew pairings that are formed in crew pairing above and create extended work schedules according to rest periods, vacations, etc. The main objective of crew assignment problem is to minimize the cost of assigning such crew members to these work schedules.

In some cases, deadheading which is a situation whereby flight crews being repositioned by flying as passengers, is allowed. Deadheading can be beneficial, especially in long haul crew pairing problems as shown in Bernhart C et al. (1995) [12].

Recognizing Barnhart C et al. research they did on crew scheduling, they demonstrate two approaches for crew assignment: *roster making* and *bid-line generation*.

With roster making a mutual practice in Europe, schedules are constructed for specific individuals. A subset of schedules is selected so that each individual is allotted to a schedule and all pairings in the crew pairing problem solution are contained in the appropriate number of schedules. While with bid line generation, which is so common in North America, cost minimizing subset of scheduling is done without acknowledgement of specific individuals. Through a bidding, employees reveal their preferences for the schedules and the airline assigns employees to the schedules according to the individual priority rankings; Barnhart C et al. (2004) [19].

3.1.2.5. Aircraft routing

With the help from the solution retrieved from the fleet assignment problem, aircraft routing process identifies which respective aircraft from a particular fleet type will be assigned to what flight leg in the network. Aircraft Rotation Problem also known as Aircraft Routing Problem was first tackled by Daskin M et al. (1989)where most solution methods gathered are mainly heuristic in nature [20]. In their research, they take as input fleeted schedules, and the available number of aircrafts in the fleet type. Kabbani N and Patty B (1992) then modeled the aircraft routing problem as a set separate problem, where they disregarded maintenance constraints [21]. Also, Clarke L et al. (1997) used a Lagrangian relaxation solution tactic that added up sub tour-elimination and maintenance constraints when disrupted to present a fight-based model [22].

Later on, Bartholomew-Biggs M et al. (2003) analyzed the aircraft routing problem and created direct search, deterministic, and stochastic optimization methods which discovered an optimal flight path crossing a minimal distance between a given origin and destination pair while avoiding hindrances in a geographical sense [23].

Figure 3.4: A Two-Day cyclic routing

	Flight	***Origin***	***Destination***	***Departure Time***	***Arrival Time***
Day 1	TK15	Istanbul	Sao Paulo	09:30	16:55
	TK15	Sao Paulo	Buenos Aires	18:30	21:20
Day 2	TK16	Buenos Aires	Sao Paulo	00:10	02:50
	TK16	Sao Paulo	Istanbul	04:25	23:10

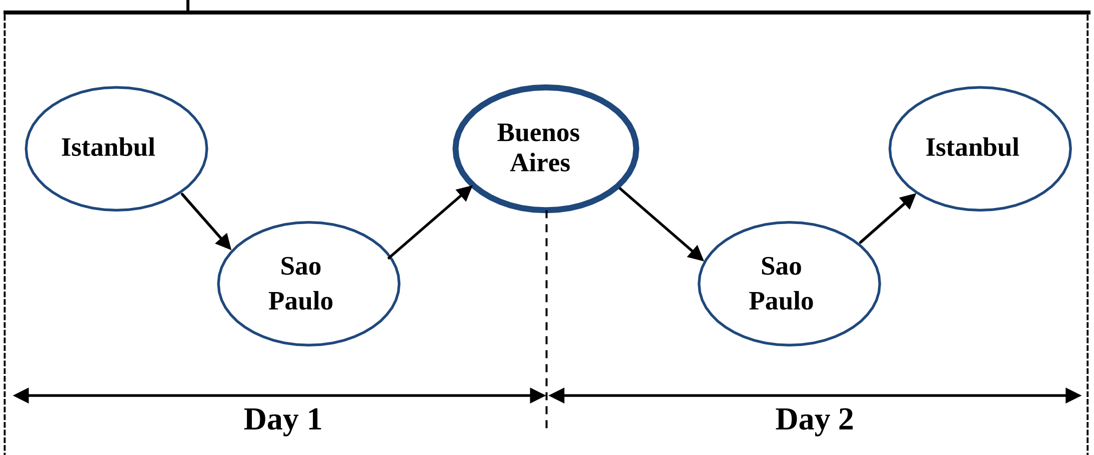


Figure 3.4 presented by Bazargan M (2010) shows a two-day-cyclic routing of a fleet-type B777-300ER that starts its journey from Istanbul to Buenos Aires via Sao Paulo and then come back to Istanbul where it started its journey. This is a journey that starts at one airport and after 2 days the same aircraft is routed back to its origin station [8].

While using a string-based approach, Cohn and Barnhart (2004) integrated maintenance routing decisions within the crew scheduling problem which resulted in the aircraft routing model considering maintenance [19].

3.1.2.6. Airport resource planning

Gate allocation, slot allocation and ground personnel scheduling as explained by Mangoubi Ret al. in their research (1984) are fundamental tasks accomplished in aircraft resource planning step [24].

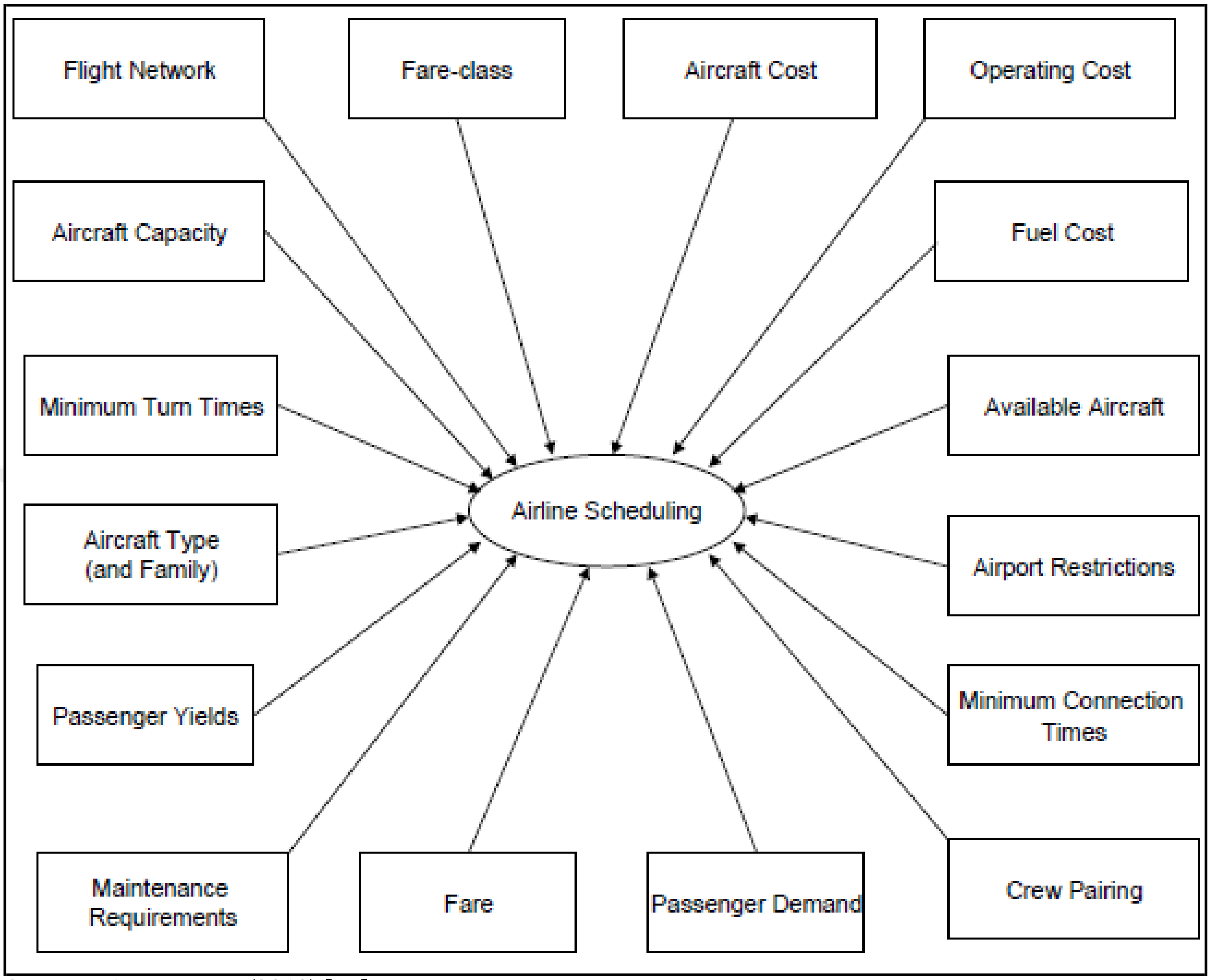
Gate allocation is a process where all flight legs are covered and passenger connections made within a minimum given time slot by assigning available gates at the airports to arriving and departing aircrafts.

Slot allocation can be compared to gate allocation only that it applies to slot controlled airports. These are airport where the number of takeoffs and landings is controlled by regulatory agencies. At these airports, before airline carriers land or take-off, slots are allocated to them. Airlines have to schedule according to the slots allocated, which must be synchronized with gate allocation and incorporated into the schedule building process in order to avoid potential infeasibilities or violations; Buch I (1994) [25]. This is done in order to minimize flight delay problems resulting from congestion.

Ground personnel scheduling as explained by Buch I (1994) on the other hand involves scheduling airline personnel to different positions like luggage handling agents, check-in agents, and ground personnel among others [25].

Recognizing that the main target of airline planning process is to maximize profits and minimize operating costs, proper schedules need to be made. Schedules that match demand capacity and at the same time these schedules need to minimize costs incurred while allocating resources. Below is a figure that elaborates some of the activities related to airline planning process; Ki-Hwan B (2010) [17].

Figure 3.5: Constituents of Airline Scheduling



Source: Ki-Hwan Bae (2010) [17]

Flight scheduling is a sophisticated and complex problem but very important in practice since commotions are costly in terms of rescheduling issues. A bad schedule can lead to delaying or canceling flights, changing aircraft among flights or using spare aircraft, which in turn affect future deployment of aircraft and crews. As soon as commotions occur, carriers usually adjust the planed schedules. This is so stressful and no little or no time is available to analyze cost-effective scheduling alternatives; Ki-Hwan B (2010) [17].

Normally, airlines are free to outline their own timetable. This is however complex in a sense that airline planning and scheduling departments have to meet all the safety and legal governances and the availability of slots at airports while balancing the availability of resources such as staff, facilities and equipment with economic and marketing targets; Ki-Hwan B (2010) [17].

The scheduled times of competitors can be taken into account, though operational variables starting with aircraft types may not be identical. The schedules of competitors will therefore usually only be used as a marketing yardstick rather than a practical input; Ki-Hwan B (2010) [17].

In such a case, an optimum schedule is built. This is a schedule that balances the needs of passengers, shippers and the aircraft operator by best echoing the expected travel periods between two airports. An optimum schedule should not be either too short or too long. If it is too short, arrival delays are created which causes rigid delays that might spread throughout the entire network. If the optimum schedule is too long; because departing flight will arrive at their destinations before their scheduled time and this might upset the destination airport planning. Both cases result in an overestimation of airline resources (aircraft, staff, etc.) required to execute the scheduled program; Jian Liu (2003) [16].

Predictability is crucial information in schedule development. In order to deliver sound services to their customers with on-time accomplishments, airlines include time-buffers which can be integrated in the ground phase or in the flight phase. Airlines can also go extra miles risking their profitability by tempering their resource utilization for reserved aircraft and crew; Jian Liu (2003) [16].

Though it is important to have a well balance schedule, there are some setbacks towards it. Some of the difficulties airlines faces are:

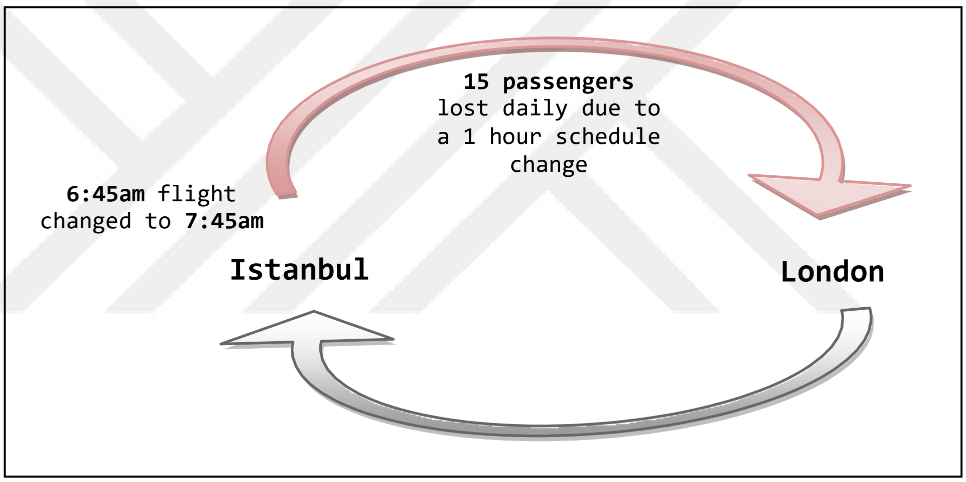
i. **Traffic Flow:** the study of originating and or connecting passenger on a flight is complicated to understand as Wensveen G (2015) explains [30]. Due to its complexity, traffic flow varies from case to case, depending on geography, route structure, season, economic developments, etc., and alternative service available. Some cities, because of favorable geography, obtain maximum benefit from traffic flow while others do not. Traffic flow does not only differ from city to city, but also in the same cities depending on the time of the year; let's say traffic flow differs in the same city but from summer periods to fall to winter to spring seasons.

ii. **Schedule salability:** this is a process that involves making a schedule that generates profits, rather than generating losses. Schedule salability is a major problem of airline scheduling in the sense that it is highly discrete to even minor differences in departure time or other factors.

It has been observed that different organizations spend time and concern differently, on schedule building. Some companies will spend days while others will spend few minutes to come up with a schedule. For an airline company, this is not time wastage; experiments have shown that even such lesser modifications can significantly affect the success of a flight as explained by Wensveen G (2015) [30].

Schedule salability is a big deal for an airline, because flight schedules affect the customer's choice of an airline. Despite the fact that a customer is a frequent flyer, he/she would not wish to miss an important business meeting due to delays created by the airline.

Figure 3.6: Schedules are sensitive to changes (hypothetical case)



Source: Wensveen G (2015) [30]

Every schedule an airline administers is an independent product, having its own special market and salability. And as competition gets even more fierce, the emphasis of even lesser schedule changes becomes fairly greater, making the job of scheduling more difficult.

iii. **Schedule Adjustments**: An airline has every right to alter its departing and arriving schedule. An airline's total schedule sequence represents a solidly and highly integrated structure. Most schedules are associated with other schedules because of connections, equipment routing, or other factors. Many aspects of flight scheduling are solidly governed by specific regulatory or constitutional requirements, such as those relating to maintenance of equipment, and working conditions of flight crews etc., Wensveen G (2015) [30].

iv. **Time Zones;** Another factor critical to schedule actions is the *time zone effect*. The fact that we lose five to eight hours on the clock while going westbound and gain five to nine hours going eastbound from Istanbul; has a major impact on scheduling a plane fleet.

An eastbound nonstop plane from Istanbul to Japan takes twelve hours "on the clock": nine hours of flight time plus three hours lost crossing time zones. Most customers do not like to arrive at their destination close to or after 11:00 p.m. They usually prefer to travel overnight and arrive early in the morning. With the twelve-hour clock time, any Istanbul departure at or after 16:00 means a Japan arrival at or after midnight. For all practical purposes, therefore, the period from 3:00 p.m. on is impracticable for salable eastbound nonstop departures.

Then, beginning at about 23:00 Istanbul time, the plane fleet can schedule the overnight flights. And after that, it will again have an impracticable period, lasting for about 8:00 the next morning. Thus, the plane's choices of salable eastbound departure times are dramatically limited to the period from about 8:00 am to about 15:00 and then at about 23:00. Allowing time to service and turn the equipment (maintenance among others services), the planes become available for return trips at about 14:00. This, together with the other factors, determines the pattern of service that can economically be operated.

v. **Load-Factor Influence;** this is the ratio of utilization of an aircraft. An airline sells available seat miles despite the fact that it produces revenue passenger miles. The parameters affecting load factor include flight times, frequency, type of service and fare levels, seasons, particular characteristics of the destination.

An airline company is a time perishable industry. Unlike other companies that can predict an estimated demand of a product and keep the excess in the inventory in cases of overestimation, airline companies do not have similar convenience. It may be convinced that, a given non-stop jet to Los Angeles from Istanbul, if it operates the schedule, it must fly a seat-mile including 230 seats.

And, of course, once it produces the empty seat-miles, they are irretrievably lost. It should be noted however that a higher load-factor does not necessarily translate into higher revenues for the airlines. Costs of operating a schedule vary only slightly as load factor changes, whereas revenue varies in direct proportion to changes in load factor. Thus, a shift in load factor of only a few percentage points can make all the difference between a money loser and a profitable trip; Wensveen G (2015) [30].

3.2. THE GENERAL ASSIGNMENT PROBLEM

General Assignment Problem (GAP) can be described as optimally assigning the available tasks to available processors, given the profit and the amount of resources corresponding to the assignment of a particular task.

The problem instance has a number of agents and a number of tasks. Any agent can be assigned to perform any task, incurring some cost that may vary depending on the agent-task assignment. It is required to perform all tasks by assigning exactly one agent to each task and exactly one task to each agent in such a way that the total cost of the assignment is minimized; Rainer B et al (2009) [31].

For example GAP can be viewed as a scheduling problem on parallel machines, where each machine has a capacity (or a maximum load) and each job has a size (or a processing time) and a profit, each possibly dependent on the machine to which it is assigned, and the objective is to find a feasible scheduling which maximizes the total profit

The problem we are dealing with in this thesis can be classified as GAP because our problem shows the general assignment problem characteristics in the sense that it deals with assigning the right aircraft type among a set of fleet-type to the right destination satisfying several constraints. We have a set of aircrafts in fleet types; these can be considered agents, and a set of destinations that can be considered as tasks. Aircrafts are assigned to perform a destination incurring some cost that varies according to the type of aircraft used and where it is being assigned to, in which in this thesis we aim at reducing.

4. FLIGHT SCHEDULING MODELS

4.1. INTRODUCTION

In this chapter, we present some planning and control activities in Turkish Airlines. The literature is divided activities as day by day activities, week by week activities, and dated booking activities. In the day by day issue it is expected that the calendar rehashes each day, for instance; the same flights are scheduled each day. Numerous aircrafts in Turkish Airlines work the same timetable on every weekday and a subsection of flights at the weekend.

Normally, the fleet assignment model takes into reflection the accessible sorts and quantities of airplanes and a given timetable with scheduled flight times. The fundamental motivation behind the fleet assignment issue is to discover the commitment assignment of aircraft types to flight legs such that:

i. Each flight route is covered absolutely one aircraft type;

ii. The number of flights allocated to a particular fleet type into and out of a location are equal or balanced; and

iii. The total number of scheduled aircrafts is less or equal to the total number of available aircrafts in such a particular fleet type.

Additional constraints such as destination range, different plane types, number of planes available, seat capacities and number of hours flew can also be included. The fleeting contribution is defined as the total passenger revenue less the total flight costs, disregarding the aircraft ownership costs, overhead costs, and many more. Another purpose function for the fleet assignment model is to minimize assignment costs, which is the abstract of total flight operation costs and expected spill costs.

4.2. LITERATURE REVIEW

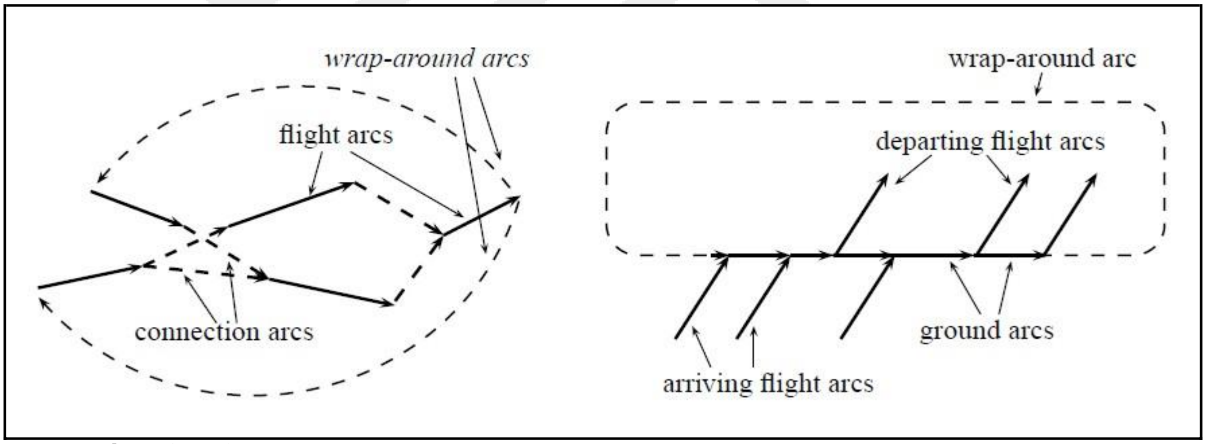
In the early 1954, Dantzig B et al. introduced the use of linear programming to solve fleet assignment problems. They assumed the itineraries for fleet assignment problems are non-stop. In order to achieve fractional solutions, they set the problem as a linear program.

Nevertheless, fractional solutions might not be vital if the assignment is considered over some period of time. In most cases, planners would be able to find integer solutions for various sub-period intervals that yield these fractional averages [32].

Remarkable achievements have been observed in this sector throughout the years. The most recent developments include Daskin et al. (1989) [20], Hane et al. (1995) [12], Rexing et al. (2000) [13]. Daskin et al. (1989) presented an integer programming model that assigns aircrafts to itineraries, used Lagrangian relaxation to get lower bounds on the optimum objective value and developed heuristics to acquire possible solution [20].

Abara (1989) presents a model that leads an outburst of numerous variables. In his model, he uses applies the underlying connection arcs as decision variables but does not allow different turn times for various fleet types at various locations [11].

Figure 4.1: Connection network with 5 flights and timeline network for single airport



Source: Abara (1989)

Hane C et al. (1995) presented a model which has been seen as a milestone for most of the basis for several fleet assignment models that are in use today in the airline industry.

The fleet assignment problems were considered by several people as multi commodity network flow problem, where using the available number of aircrafts, fleet types were assigned to flight legs in the network only once. Several problem complexity reduction techniques are devised such as node consolidation and island construction to reduce the number of nodes by separating a consolidated series of arrival nodes from departing nodes. Island construction is employed mostly at spoke stations where flight connections occur scarcely during the day [12].

Rexing B et al. (2000) [13] presents an integrated model that was introduced by Hane C et al. (1995). Rexing's model allows re-timing with small time window. Jacobs T et al. (1999) [33] present a route method for solving the fleet assignment model that enhances the spill estimation process, which is static in the basis of Hane C et al. (1995) model [8]. A special linear programming relaxation of the model is used on an estimated passenger flow, and the algorithm is iterated until a terminating state is achieved. From the integer solution obtained, the passenger flow in a network is studied.

Numerous integrated models have been developed to find better solutions for an airline operation planning process which is inter-linked. In order to increase revenues by allowing improved flight connection opportunities in the FAP, a link between fleet assignment and schedule design models takes place.

Airlines with large aircrafts cannot easily determine the actual revenue clearly especially if many passengers are likely to fly on such fleet types. In order to solve such a problem, Kniker T et al. (1998) considered to model the origin-destination fleet assignment models in which passenger revenue for each flight leg were modeled using passenger mix model. This also enabled the airline to determine the level of booking by passengers on each fixed-seat aircraft [34].

Desaulniers G et al. (1997) [44], and Rexing B et al. (2000) [13] made assumptions where the flight origin and destinations data are provided and deterministic but the flight departure times vary. They doubted the set of departure times for each destination within the specified time window; hence they provided more choices for assigning flights in order to improve the fleet assignment problem.

Desaulniers G et al. (1997) again used the branch-and-price schemes with departure time windows enforced in the sub problem to be able to specify the sequence of flights and to assign these sequences to fleet types [44]. In the like manner though with a twist, Barnhart et al. (1998) suggested a better way to integrate fleet assignment and aircraft routing problems, and solve them weekly by using the a sequence of connected flights that begin and end at maintenance stations [14]. A very large scale neighborhood search algorithm that initially starts from the fleet assignment solution then determines the through-flights was developed by Ahuja R et al. (2002) [35].

Yan S et al. (2002) included itinerary based demand in their model hence combining the scheduling process with fleet assignment. In their model, all destinations are considered as optional and are selected independently when needed to be included in the schedule. This model led to passenger paths [36]. Yan S et al. (2005) formulated a nonlinear mixed integer program that was solved by a heuristic. This model was intended to solve an integrated scheduling and routing model that Yan et al formulated [37].

A combination of schedule design, aircraft routing, and crew scheduling was devised by Klabjan D et al. (2002). He altered the older solution sequence, in a way that, the routing problem was solved. They added plane-count constraint to the crew scheduling problem and solved the crew scheduling problem hence maintaining the feasibility of aircraft routing [38].

Cordeau J et al. (2001) solved the aircraft routing problem at an early stage and the crew pairing problem at a later stage by using the Bender’s decomposition model [39]. Contrariwise, Mercier et al. (2005) [40] reversed the approach where crew scheduling problem is solved at the early stage and routing problem solved at a later stage.

Cohn A et al. (2003) established a more developed model from the one by Cordeau et al. (2001) [39] where they integrate crew scheduling and aircraft routing problem and then incorporated some maintenance routing decisions into existing model. Due to the complexity of the model, the authors suggested that step by step approach in solving a series of aircraft routing problems are more practical [41].

Sometimes, some changes need to be made on schedules that were previously made. This leads to changes in resources such as aircrafts, crews, etc. These kinds of turmoil lead to changes in the execution of originally made flight plans, regular operations, and on a high extent the entire network as explained by Ki-Hwan B (2010) [17].

In order to solve the aircraft recovery problem, Jarrah et al. (1993), offered two network flow models and used flight delays, cancellations, aircraft swapping, and replacement aircraft as recovery strategies. In their model, they first they don’t allow flight cancellations and then they do not consider flight delays. The authors then suggest an approach of combining delays and cancellations and constructed a model that used the total number of passengers, the number of passengers with a downstream connection, lost crew time, and disruption of maintenance as their emphasis [36].

In order to maximize flight revenues without including swapping fleet types and flight delay costs, Cao and Kanafani (1997), offered a model which took into consideration the delays and cancellations, ferrying, flying a deadhead aircraft to a station for the next operation, and numerous aircraft type swapping [37]. To improve this formulation, Clarke (1997) [16], proposed a complete framework for reallocating operational aircraft to scheduled flights in the aftermath of irregularities. A combination of crew availability, flight delays and cancellations, aircraft swapping and air traffic management initiatives were used as constraints in the model.

In cases when crew members don't return to their bases planned in a schedule and there is a need to alter the schedule, Teodorovic and Stojkovic (1995) suggest an approach to handle such cases. In their approach, flight legs are integrated into crew rotations and these crew rotations integrated into aircraft rotations [45].

Maintenance is another factor that affects airline companies, normally tackled at successive stages while solving the problem of aircraft routing. Many others like Klabjan et al. (2002) suggested that in order to minimize maintenance problems, there was a need to introduce short-connections. Here the same aircraft would be reused again after it had serves a short itinerary. This is feasible in the case of a hub-and-spoke network as long as the number of short-connections was kept low [38].

Sandhu R et al. (2007) nearly solved the aircraft routing problem by developing a model that considered fleeting, aircraft routing and crew scheduling with certain appropriate plane-count constraints. The proposed integrated model was solved using Lagrangian relaxation and a Benders' decomposition procedure [42]. As Mercier A et al. (2007) further explains, the main issue considered in solving aircraft routing problems; is to determine an adjustable schedule, set of network and crew pairings that minimize the operation cost of each fleet type. Bender's decomposition and the addition of a dynamic constraint were used to solve this problem [43].

Following a hub closure, Thengvall et al. (2000) used a framework regarding time-space network with flight arcs, ground arcs, and added on ferry arcs, shifted flight arcs; and modeled a hub closure problem. The objective functions for this model was to minimize delay costs, cancellation costs and to keep the aircraft routing schedule as unchanged as much as possible [18].

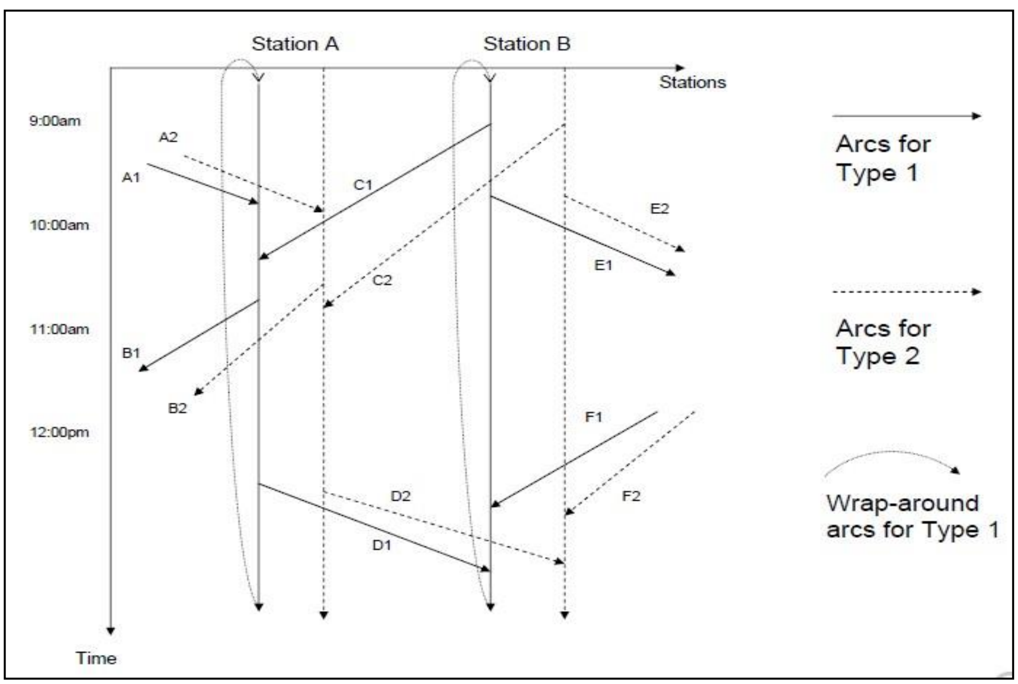
4.3. ASSUMPTIONS IN AIRLINE FLEET ASSIGNMENT MODELS

In order to resolve a fleet assignment problem, while obeying the operational requirements, maintenance rules and crew restrictions the assembly of a flight schedule that relates to the flight network must be done.

The main concern in this first stage of airline planning process is the fleet type. A fleet assignment problem was first modeled by Hane C et al. (1995) who assumed that each flight is flown each week [12]. This assumption was made to facilitate the model hence smoothing the complexity of the problem. As a result, Clark L et al. (1996) included maintenance and crew scheduling to the model to make a general FAM. Any violations of the maintenance constraint created serious penalties in the model's objective function [46].

Hane C et al. (1995) used a time-space network arrangement in order to ease the fleet assignment problem. This is a structure that allows the assignment of fleets to flights by imposing a set of time-space network, where each fleet type has its own node, arrival and departure time, ground arcs, flight and wrap-around arcs (Ref. Figure 4.2) [12]. This has now become the primary framework for formulating the fleet assignment problem. However, this formulation by Hane C et al. is limited to some situations because it does not differentiate among known fleet-types on the ground as Rushmeier R et al. (1997) view [47].

Figure 4.2: The Arc Types



Source: Hane C et al. (1995)

Federal Aviation Administration (FAA) protocols and union contracts demand that fleet assignment requirements be made well in advance of departures, even though demand is highly indeterminate at this point as Ki-Hwan (2010) explains in his dissertation [17].

Airlines have used a consecutive framework involving an initial fleet assignment, a re-fleeting process, and finally, certain limited swapping procedures in order to provide flexibility in supply management. In the same manner, a mixed-integer programming Demand-Driven Re-fleeting (DDR) model considering path-level demands along with a polyhedral analysis to boost problem solvability was proposed Sherali et al. (2005) [40].

The benefit of the DDR approach is that it gives a clear and detailed passenger demand forecast before departure times and aircraft assignment is limited to fleet type and to flights assigned to the same fleet family.

4.4. REVIEW OF AIRLINE FLEET ASSIGNMENT MODELS

A typical airline conducts its fleet assignment in a periodic fashion, mostly on a daily basis. The factors that influence the assignment of fleet types include seating capacity, operational costs, number of available aircrafts, various technical and FAA requirements. The objective is either to maximize the profits or equivalently minimize the total costs.

The following models are presented as the samples of the most widely studied integrated airline fleet assignment and scheduling problems. These models are good examples of the related problems and are given as a comparison to model created.

4.4.1. Basic Fleet Assignment Model (FAM)

Under this subdivision, we define the basic Fleet Assignment Model which we borrow focus on to build our own model. The basic FAM solves the fleet assignment problem which involves decision making of which fleet type best fit a certain flight leg in a network. The Fleet Assignment Model either maximizes profits or minimizes operating costs as objective function. The FAM was first modelled by Abara J et al. (1989) as a mixed integer program considering an airline’s flight network but the current fleet assignment model used by most airlines was formulated by Hane C et al. (1995) as Sherali H et al. (2005) explain in their research [48].

For every fleet assignment model to be formulated, an estimated demand on each flight leg is required, Manoj L (2002) [49]. However, determining demand is a complex task to accomplish; hence assumptions need to be made. A basic Fleet Assignment Model covers three main constraints, which include:

i. ***Flight coverage***; whereby every flight leg in the network needs to be covered.
ii. ***Balance***; the number of flights covered by a fleet type to and fro are equal.
iii. ***Fleet size***; the number of aircrafts assigned are less or equal to the total number of aircraft in that particular fleet family available.

Extra constrains are added on basing on the requirements being modeled. The objective function of a basic FAM is to either maximize revenue or minimize assignment costs. The basic formulation of the basic FAM is as follow;

> *"Given a flight schedule with fixed departure times and costs, find the minimum cost assignment of aircraft types to flight legs, such that first; Each flight is covered once by a fleet type, secondly; Flow of fleet types is conserved at each airport, and lastly; Only available numbers of fleet types are used",* Manoj L (2002) [49].

Model Description and Notation;

Precedent to defining the model, a small description is given below:

Sets;

A: the set of airports indexed by o.

L: the set of flight legs in the flight schedule indexed by i.

K: the set of different fleet types indexed by k.

T: the sorted set of all event (departure or availability) times at all airports, indexed by t_j. The event at time t_j occurs before the event at time t_{j+1}. $|T| = m$.

N: the set of nodes in the timeline network indexed by $\{k, o, t_j\}$.

N_{ki}: the set of copies of flight leg $i \in L$ for fleet type $k \in K$.

$CL(k)$: the set of flight legs that pass the count time when flown by fleet type k.

$I(k, o, t)$: the set of inbound flight legs to node $\{k, o, t_j\}$.

$O(k, o, t)$: the set of outbound flight legs from node $\{k, o, t_j\}$.

Parameters;

N_k: the number of aircraft in fleet type k, $\forall k \in K$.

$C_{k,i}$: the assignment cost when fleet type $k \in K$ is assigned to flight leg $i \in L$.

$C_{n,k,i}$: the assignment cost when fleet type $k \in K$ is assigned to copy $n \in N_{ki}$ of flight leg $i \in L$.

Decision variables;

$$f_{k,i}: = \begin{cases} 1 & \text{if flight leg } i \in N \text{ is assigned to fleet type } k \in K; \\ 0 & \text{otherwise.} \end{cases}$$

$$f_{n,k,i}: = \begin{cases} 1 & \text{if copy } n \in N_{ki} \text{ of flight leg } i \in N \text{ is assigned to fleet type } k \in K; \\ 0 & \text{otherwise.} \end{cases}$$

y_{k,o,t_j^+}: the number of fleet type $k \in K$ aircraft that are on the ground at airport $o \in A$ immediately after time $t_j \in T$.

y_{k,o,t_j^-}: the number of fleet type k aircraft that are on the ground at airport $o \in A$ immediately before time $t_j \in T$. If t_1 and t_2 are the times associated with adjacent events, then $Y_{k,o,t_1^+} = Y_{k,o,t_2^-}$.

Hence, the model is formulated as follows:

$$Min \sum_{i \in L} \sum_{k \in K} C_{k,i} f_{k,i} \tag{4.1}$$

Subject to:

$$\sum_{k \in K} f_{k,i} = 1, \forall i \in L \tag{4.2}$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} f_{k,i} = 0, \forall k, o, t \tag{4.3}$$

$$\sum_{o \in A} y_{k,o,t_m} + \sum_{i \in CL(k)} f_{k,i} \leq N_k, \forall k \in K \tag{4.4}$$

$$f_{k,i} \in \{0,1\}, \forall k \in K, \forall i \in L \tag{4.5}$$

$$y_{k,o,t} \geq 0, \forall k, o, t \tag{4.6}$$

The objective function in equation (4.1) which aims at minimizing assignment costs, has a coefficient $C_{k,i}$ which is the summation of the operating costs, carrying costs, spill costs and recapture revenue. Constraint (4.2) is the coverage constraint that ensures that every flight leg in the network is covered; Constrain (4.3) is the aircraft balance constraint that makes sure that an assigned aircraft comes back at a certain point of time; Constraint (4.4) is the aircraft count constraint that limit the number of assigned aircraft; it makes sure that they do not exceed the total number of aircraft in a particular fleet family. Constrain (4.5) and (4.6) are binary and non-negativity constraints respectively; Manoj L (2002) [49].

4.4.2. The Fleet Assignment with Time Windows Model (FAMTW)

After the formulation of the basic model, Rexing B et al. (2000) was prompted to add a twist to this model by including time windows for which each flights can depart, hence creating the Fleet Assignment with Time Windows model (FAMTW), with a more cost effective fleeting and schedule [13].

The FAMTW reduces the fleet assignment costs by;

i. Allowing more fleet connections and making it more convenient to assign a more suitable aircraft type to serve a flight leg in the network.
ii. More utilization of aircraft since few aircrafts is used in the network. This is possible because of the re-timings in the flight schedule.

The time window concept is such that if one aircraft can be used to serve two or more flight legs (Ref. Table 4.1). Assume the flights X and Y has the same demand; it is more suitable to assign one aircraft to serve these flights. However this might seem challenging the departing time of these flights is not the same. Flight X has a ready time that is later than flight Y. Hence to be able to assign one aircraft to serve these two flight legs, either we allow flight Y to leave a little late or allow flight X leave earlier; Manoj L (2002) [49].

Table 4.1: Example of flight connection

Flight	***Origin***	***Destination***	***Departure Time***	***Arrival Time***	***Mean Demand***
X	Istanbul	Kigali	18:20	00:20	140
Y	Kigali	Istanbul	03:20	09:20	96

The FAMTW uses the same variables as those of a basic FAM with an exception that a binary variable $f_{n,k,i}$ takes the value of 1 if n flight I is served by fleet-type k, and zero otherwise; Rexing B et al. (2000) [13]. A modification in the objective function and constraints are made from the basic FAM by replacing $\sum_{k\in K} f_{k,i}$ by $\sum_{k\in K}\sum_{n\in N_{ki}} f_{n,k,i}$.

In the objective function, another summation is added to incorporate the times a certain aircraft will be flown to serve several flight legs. The final FAMTW can hence be formulated as below;

$$Min \sum_{i \in L} \sum_{k \in K} \sum_{n \in N_{ki}} C_{n,k,i} f_{n,k,i} \tag{4.7}$$

Subject to:

$$\sum_{k \in K} \sum_{n \in N_{ki}} f_{n,k,i} = 1 \quad \forall i \in \quad L \tag{4.8}$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} \sum_{n \in N_{ki}} f_{n,k,i} - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} \sum_{n \in N_{ki}} f_{n,k,i} = 0 \quad \forall k, o, t \tag{4.9}$$

$$\sum_{o \in O} y_{k,o,t_n} + \sum_{i \in CL(k)} \sum_{n \in N_{ki}} f_{n,k,i} \leq N_k \quad \forall k \in \quad K \tag{4.10}$$

$$f_{n,k,i} \in \{0,1\} \tag{4.11}$$

$$y_{k,o,t} \geq 0 \tag{4.12}$$

4.4.3. Passenger Mix Model (PMM)

Kniker T (1998) [34] proposes the Passenger Mix Model (PPM) that aims at maximizing the fleeting contribution and/or minimizes the assignment costs. The PMM takes as input an already made feet schedule, unconstrained route demand; and finds the flow of passengers over that route. The main objective of the model is to find a best mix of passengers from different routes by spilling off passengers on the less profitable routes; which secures seats for others on more profitable routes. The PMM can be simplified as:

> *"Given a fleeted flight schedule and the unconstrained itinerary demands, find the flow of passengers over the network, minimizing carrying plus spill cost, such that (1) the total number of passengers on each flight does not exceed the capacity of the flight, and (2) the total number of passengers on each itinerary does not exceed the unconstrained demand of that itinerary";* Manoj L (2002) [49].

The Passenger Mix Model does not encounter spill costs from the basic Fleet Assignment Model because;

i. Spill costs from all flight legs are all equal

ii. There are no fare allocations among flight legs

The passenger Mix Model is also used to model recaptures; which is an absurd in leg-based spill models since spill are mare approximations. In order to formulate the PMM, some notations need to be explained.

Sets:

P: the set of itineraries in a market indexed by p or r.

A: the set of airports, or stations, indexed by o.

L: the set of flight legs in the flight schedule indexed by i.

K: the set of fleet types indexed by k.

T: the sorted set of all event (departure or availability) times at all airports, indexed by t_j. The event at time t_j occurs before the event at time t_{j+1}. Suppose $|T| = m$; therefore t_1 is the time associated with the first event after the count time and t_m is the time associated with the last event before the next count time.

N: the set of nodes in the timeline network indexed by $\{k, o, t_j\}$.

$CL(k)$: the set of flight legs that pass the count time when flown by fleet type k.

$I(k, o, t)$: the set of inbound flight legs to node $\{k, o, t_j\}$.

$O(k, o, t)$: the set of outbound flight legs from node $\{k, o, t_j\}$.

Decision Variables;

t_p^r: the number of passengers requesting itinerary p but redirected by the model to itinerary r.

$$f_{k,i}: = \begin{cases} 1 & \text{if flight leg } i \in N \text{ is assigned to fleet type } k \in K; \\ 0 & \text{otherwise.} \end{cases}$$

y_{k,o,t_j^+}: the number of fleet type $k \in K$ aircraft that are on the ground at airport $o \in A$ immediately after time $t_j \in T$.

y_{k,o,t_j^-}: the number of fleet type k aircraft that are on the ground at airport $o \in A$ immediately before time $t_j \in T$. If t_1 and t_2 are the times associated with adjacent events, then $y_{k,o,t_1^+} = y_{k,o,t_2^-}$.

Parameters/Data;

CAP_i: the number of seats available on flight leg i (assuming fleeted schedule).

$SEATS_k$: the number of seats available on aircraft of fleet type k.

N_k: the number of aircraft in fleet type k, $\forall k \in K$.

D_p: the unconstrained demand for itinerary p, i.e., the number of passengers requesting itinerary p.

$fare_p$: the fare for itinerary p.

$\widetilde{fare}_p$: the carrying cost adjusted fare for itinerary p.

b_p^r: recapture rate from p to r; the fraction of passengers spilled from itinerary p that the airline succeeds in redirecting to itinerary r.

$$\delta_i^p: = \begin{cases} 1 & \text{if itinerary } p \in P \text{ includes flight leg } i \in N; \\ 0 & \text{otherwise.} \end{cases}$$

And the Passenger Mix Model can finally be formulated as below;

$$Min \sum_{p \in P} \sum_{r \in P} (\widetilde{fare}_p - b_p^r \widetilde{fare}_r) t_p^r \tag{4.13}$$

Subject to:

$$\sum_{p \in P} \sum_{r \in P} \delta_i^p t_p^r - \sum_{r \in P} \sum_{p \in P} \delta_i^p b_r^p t_r^p \geq Q_i - CAP_i \quad , \forall i \in L \tag{4.14}$$

$$\sum_{r \in P} t_p^r \leq D_p \quad , \forall p \in P \tag{4.15}$$

$$t_p^r \geq 0 \quad , \forall p, r \in P \tag{4.16}$$

The objective function, i.e. (4.13) minimizes the assignable costs which include passenger carrying costs and spill costs by finding the best mix of passengers from each destination on each flight leg. Constraint (4.14) manages spilled passenger on one flight leg and recovers them on another. Constraint (4.15) makes sure that the spilled passenger does not exceed the total demand on a flight leg. Constraint (4.16) is the non-negativity constraint that makes sure that the variable t_p^r is positive and an integer since it deals with passenger demand.

A set of variables that were first proposed by Barnhart et al. (1995) [10] are exploited in model above. Passengers are assigned to their respective journeys, and if the journey is less profitable, the model finds a way to spill passengers off from these flights and minimizes the total spill. The model combines recaptures using a set of Quantitative Service Index (QSI) based parameters called *recapture rates*, b_p^r, which is defined as the recapture rate from itinerary *p* to itinerary *r*; Manoj L (2002) [49].

From the above model, the objective function will be zero if passengers redirected on flight leg *r* are the same as those spilled on flight leg *p*. This is possible even though most of the time the fare of recovering passengers on leg *r* is higher than spilling them on leg *p*.

4.4.4. Itinerary-Based Fleet Assignment Model (IFAM)

Since the Fleet Assignment Model couldn't accommodate revenues that it ignored from the interaction between flight legs and demands; Farkas A (1996) was motivated to formulate a combinatorial model that consisted of spilled costs for each fleet type and flight leg, hence leading to the Itinerary-Based Fleet Assignment Model (IFAM) [50]. In IFAM, fleet assignment and passenger mix problems are modeled synchronously and are solved using two procedures:

i. *Column generation approach;* this approach requires the repeated FAM because each decision variable counts for a fleet.

ii. *Partition of flight legs into sub-networks;* in this approach multi-leg routes are used.

Erdmann A et al. (2001) suggested another approach to solve the itinerary-based fleet assignment model by first solving the fleet assignment model, then using the results obtained to solve the passenger mix model. Among the suggested approach is the use of the Lagrangian relation [51].

Jacobs T et al. (1999) on the other hand recommend another way to deal with origin and destination fleet assignment problems where a combination of fleet assignment model and passenger mix model are solved repetitively, hence leading to the IFAM [33].

In the IFAM, the decisions that are based on are spill costs, recapture costs and the related costs. IFAM is an integrated model that combines the basic FAM and PMM. The IFAM improves the FAM by covering a wider network of capacity and revenue.

Using the same set of data, parameter and variables as in PMM above, the IFAM is formulated as follows:

$$Min \sum_{i \in L} \sum_{k \in K} \tilde{c}_{k,i} f_{k,i} + \sum_{p \in P} \sum_{r \in P} (\widetilde{fare}_p - b_p^r \widetilde{fare}_r) t_p^r \tag{4.17}$$

Subject to:

$$\sum_{k \in K} f_{k,i} = 1 \ , \forall i \in L \tag{4.18}$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} f_{k,i} = 0 \ , \forall k, o, t \tag{4.19}$$

$$\sum_{o \in A} y_{k,o,t_m} + \sum_{i \in CL(k)} f_{k,i} \leq N_k \ , \forall k \in K \tag{4.20}$$

$$\sum_{k \in K} SEATS_k f_{k,i} + \sum_{r \in P} \sum_{p \in P} \delta_i^p t_p^r - \sum_{r \in P} \sum_{p \in P} \delta_i^p b_r^p t_r^p \geq Q_i \ , \forall i \in L \tag{4.21}$$

$$\sum_{r \in P} t_p^r \leq D_p \ , \forall p \in P \tag{4.22}$$

$$f_{k,i} \in \{0, 1\} \ , \forall k \in K, \forall i \in L \tag{4.23}$$

$$y_{k,o,t} \geq 0 \ , \forall k, o, t \tag{4.24}$$

$$t_p^r \geq 0 \ , \forall p, r \in P \tag{4.25}$$

The objective function of the IFAM, equation (4.17) minimizes the assignment costs. However a slight change is mad in the variables where CAP_i is replaced by fleet type k capacity variable $SEAT_k$.

Constraints (4.18) to (4.20) are the same constraints as those used in the basic FAM that deal with flight coverage, flight balance and fleet balance respectively. While constraints (4.21) and (4.22) are the same as those in the PMM dealing with the spilled demand from one flight leg being recovered on another flight leg respectively; Manoj L (2002) [49].

Generalizing FAM;

The IFAM formulation above could be considered as an "enhancement" to the fleet assignment model but with a twist of including carrying costs; recapture and passenger flow conservation. Consider the IFAM case where $\sum_{i \in L} fare_p(i) \delta_i^p = fare_p$ for all p Є P, carrying cost equals zero and b_p^r= 0 for all p, r Є P; Manoj L (2002) [49].

Hence the IFAM becomes:

$$Min \sum_{i \in L} \sum_{k \in K} \tilde{c}_{k,i} f_{k,i} + \sum_{i \in L} \sum_{p \in P} \sum_{r \in P} fare_p(i) \delta_i^p t_p^r(i) \tag{4.26}$$

Subject to:

$$\sum_{k \in K} f_{k,i} = 1 \text{ ,} \forall i \in L \tag{4.27}$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} f_{k,i} = 0 \text{ ,} \forall k, o, t \tag{4.28}$$

$$\sum_{o \in A} y_{k,o,t_m} + \sum_{i \in CL(k)} f_{k,i} \leq N_k \text{ ,} \forall k \in K \tag{4.29}$$

$$\sum_{k \in K} SEATS_k f_{k,i} + \sum_{p \in P} \sum_{r \in P} \delta_i^p t_p^r(i) \geq Q_i \text{ ,} \forall i \in L \tag{4.30}$$

$$\sum_{r \in P} \delta_i^p t_p^r(i) \leq D_p \text{ ,} \forall p \in P, \ \forall i \in L \tag{4.31}$$

$$t_p^r(i) - t_p^r = 0 \text{ ,} \forall p, r \in P, \ \forall i \in L \tag{4.32}$$

$$f_{k,i} \in \{0,1\} \text{ ,} \forall k \in K, \ \forall i \in L \tag{4.33}$$

$$y_{k,o,t} \geq 0 \text{ ,} \forall k, o, t \tag{4.34}$$

$$t_p^r(i) \geq 0 \text{ ,} \forall p, r \in P, \ \forall i \in L \tag{4.35}$$

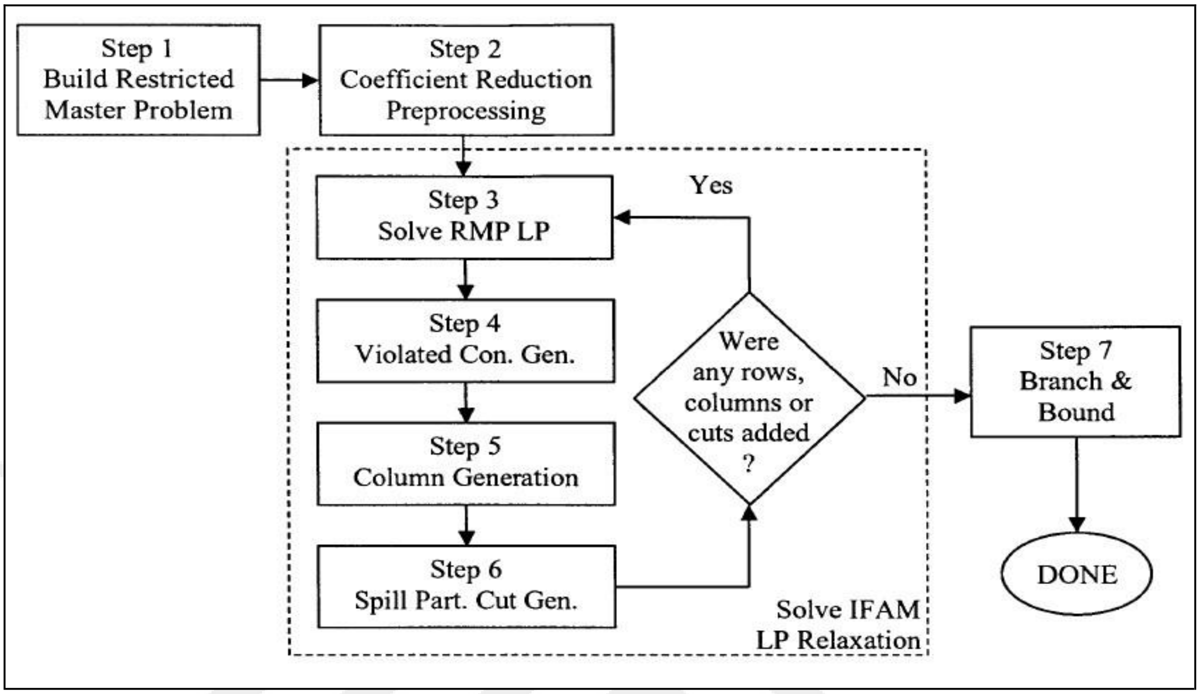
Altering constraint (4.32), will lead to the greedy spill estimation procedure used to determine the optimum spill decisions.

The objective function in the Generalized FAM can be used to find the spilled costs, and the demand recovered on another flight leg by eliminating Constraint (4.30), (4.31) and (4.35). Thus, the optimum solutions to basic FAM are the same as those for (4.26) – (4.35) with no carrying costs or recapture. IFAM is hence an improved basic FAM with captured network effects as well as recapture and carrying costs; Manoj L (2002) [49].

Solution Approach

As illustrated in the figure above, a Restricted Master Problem (RPM) model is built without considering the itinerary demand and spill variables, i.e. in the FAM. Then the Coefficient Reduction Pre-processer is applied to tighten the IFAM LP relaxation. The LP relaxation obtained in the restricted master problem is solved using column and row generation.

Figure 4.3: The Approach for the mixed fleet assignment and passenger mix model



Source: Manoj L (2002)

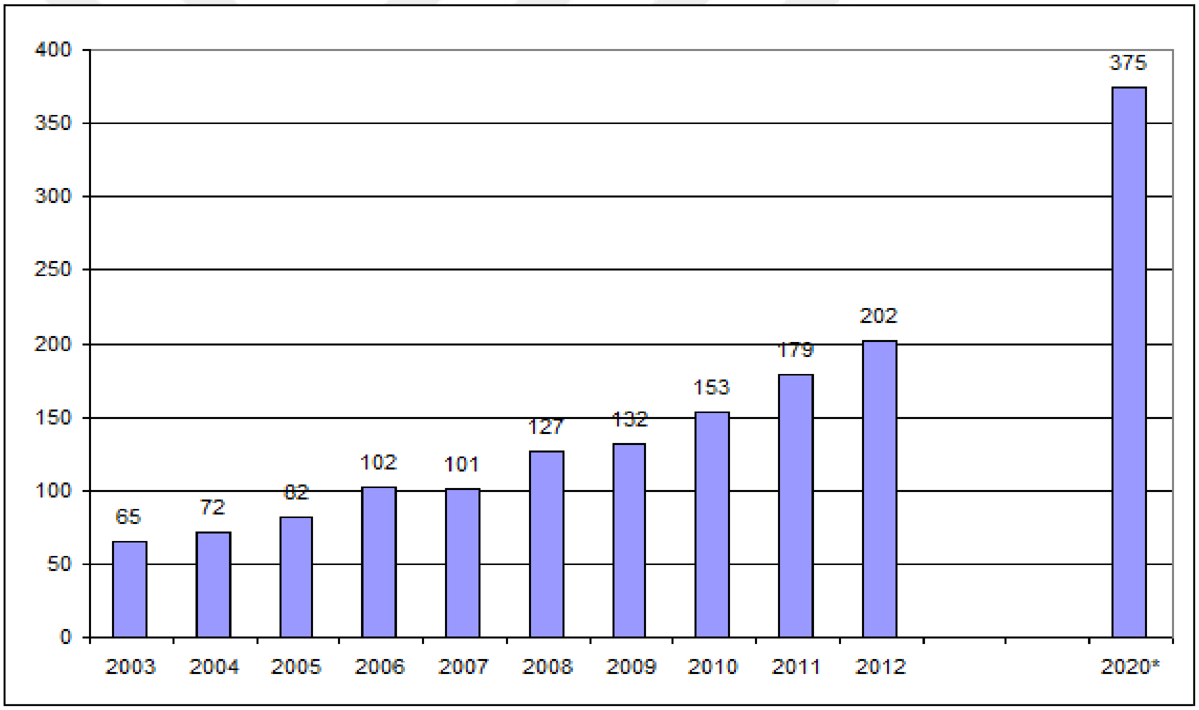
Negative reduced costs from the spills are added to the RMP and resolved until the IFAM relaxation is resolved. The IFAM LP is solved by branch and bound and an integer solution obtained; Manoj L (2002) [49].

5. OPTIMIZATION MODEL FOR TURKISH AIRLINES

5.1. TURKISH AIRLINES IN BRIEF

Turkish Airlines (TA) was established in 1933with only 5 airplanes. But that fleet size has grown with the years to over a number of 300 aircrafts. TA has the youngest fleet in Europe and the only carrier that continuously upgrades service quality while enlarging their aircraft orders; CAPA-Centre for Aviation and Turkish Airlines [2].

Figure 5.1: Turkish Airline's fleet development 2003-2012 and the plan for 2020



Source: CAPA - Centre for Aviation and Turkish Airlines

By 2012 Turkish Airlines was considered the youngest fleet with a 7 percent operating margin that places it behind Ryan Air and Easy Jet. Its profit had increased roughly to threefold, with a revenue of 26 percent on passengers and a capacity growth of 18 percent; Airlinkflight [1].

TA has also one of the most successful airlines in Europe in terms of financial strength and has a very productive work force. TA has expanded its fleet in recent years and now it has over 300 aircrafts in its fleet.

The chart (Figure 5.1) shows TA's fleet development from 2003 to 2012 and the plan for 2020; CAPA-Centre for Aviation and Turkish Airlines [2].

A fleet assignment model has been established for Turkish Airlines using the data obtained from TA open sources using a linear integer programming.

In order to determine the objective function of the model constructed, operation costs and other costs including spill costs are calculated for each fleet type with consideration of the demand and standard deviation for each flights leg. In order to attain the objective function, a fleet assignment model has been set up according to given constraints; which include fleet size, destination range, fleet type, number of hours flown, etc.

5.2. PROBLEM DESCRIPTION

Generally, fleet assignment problems are modeled in order to assign the appropriate aircrafts available to scheduled flight legs such that the revenue is maximized and simultaneously operational costs minimized. Different aircrafts have different flying performances; for instance flying altitude ceiling, maximum takeoff weight, range, climbing ability etc. and this affirms the fact that different models of aircraft performance might serve different routes as elaborated by Yaohua Li et al. (2013) [52].

Furthermore, Yaohua Li et al. (2013) explain that as fleet types differ according to seating layout, so do their operating costs. For example, an A340-300 fleet type has a capacity of 270 seats and its direct operational cost is more than 100,000 dollars per hour, whereas the seat capacity of B737-700 fleet is 149 and its direct operation cost is between 20 and 30 thousand dollars per hour [52].

They go on explaining that the route development of such problem is;

- ***i.*** The limitations imposed on flight legs by fleet types,
- ***ii.*** Each fleet's crew distribution,
- ***iii.*** Operation costs incurred by each fleet type on a given route and
- ***iv.*** passenger demand forecasts on each flight

Such kind of formulated problem aims to minimize the operating costs of flight leg as much as possible [52].

Table 5.1: THY Fleet-Type Characteristics

Fleet n	*Fleet Types Total: 10*	*Number of Aircraft Total= 70*	*Seat Capacity (Seats)*	*Max Range (Miles)*	*CASM ($)*
1	**A319-100** 	2	126	4200	$0.046
2	**B737-700** 	1	149	3750	$0.045
3	**B737-900ER** 	3	151	3700	$0.049
4	**A320-200** 	8	153	3500	$0.046
5	**B737-800** 	9	165	3400	$0.047
6	**A321-200** 	27	188	3450	$0.048
7	**A330-200** 	3	250	7600	$0.048
8	**A340-300** 	2	270	8400	$0.049
9	**A330-300** 	7	289	6500	$0.051
10	**B777-300ER** 	8	337	9100	$0.052

Source: www.skylife.com

In this dissertation, the data that we work with comprises of; 2 aircrafts for A319-100 fleet type, 1 for B737-700 fleet type, 3 for B737-900ER fleet type, 8 for A320-200 fleet type, 9 for B737-800 fleet type, 27 for A321-200 fleet type, 3 for A330-200, 2 for A340-300 fleet type, 7 for A330-300 fleet type and 8 for B777-300ER fleet type. The objective function represents the *Total Cost* fleet assignment to a flight, which we seek to minimize.

In order to determine this cost, we need two costs namely *Operating Costs* and *Passenger-Spill Costs*. The Operating Costs for a flight mainly depend on the type of the fleet assigned to that flight and inspiring ourselves from the formulas presented by Bazargan M (2010), these costs are calculated as follows [8]:

Operating Cost of flight = Distance × Seat Capacity of the aircraft × CASM of the fleet

For example, we will focus on two fleet types such as A319-100 and B737-700. The seating capacities are 126 and 149 respectively. In addition, we have the following information about these fleet types:

Revenue per available seat mile (RASM) is \$0.17 (17 cents).

Cost per available seat mile (CASM) for A319-100 and B737-700 are \$0.046 and \$0.045 respectively (Ref. Table 5.1).

Using the above information we developed a case study where we investigate the operating costs of these two fleet types being assigned on two flight legs in the Turkish Airlines schedule. For example, for flight TK1806; TLS, Toulouse and flight TK 1356; LUX, Luxemburg have a distance of 1410 miles and 1245 miles respectively from Istanbul.

Hence the operating costs of flights are:

Operating cost for flight TK1806 with fleet type A319 = \$0.046×1410×126 = \$8172.36

Operating cost for flight TK1356 with fleet type B737 = \$0.045×1245×149 = \$8347.72

Then we calculate the Passenger-Spill Cost is. This is the degree of average demand, which may exceed the capacity offered. These costs are so crucial since they determine which fleet types to assign to what flights. This is so because assigning large capacity aircrafts lead to low utilization and equally low load-factor. On the other hand, assigning a small aircraft to flight legs with a dense demand of passengers, leads to passenger spills. The spill cost hence leads to lost revenue from spilled passengers resulting from insufficient aircraft capacity.

Bazargan M (2010) had shown the way to calculate the expected spill costs using the formula as follows:

Expected Spill Cost for a fleet* = *Expected number of Passenger Spill* × *RASM* × *Distance

The expected number of passenger spill is computed as follows [8]:

$$\textbf{\textit{Expected number of Passenger Spill}} = \int_{c}^{\infty} (x-c) f(x) dx$$

From the equation above, the fleet capacity is denoted by "***c***" and the probability distribution function of the demand is denoted by ***f(x)***. Mathematical software or some special calculators are used to obtain the integral, but it is advisable to use Spreadsheets since it is much easier to calculate the integral using Spreadsheets. The data obtained from the open sources indicate that the demand follows a normal distribution with a certain average and standard deviation in each destination; Bazargan M (2010) [8]. Excel is a very convenient medium to calculate this kind of probabilistic values when the averages and standard deviations are provided.

From the above calculations and results from Excel spreadsheets, the expected number of passenger spill for fleet types B737-700 with 149 seats and A319-100 with 126 seats will be:

Fleet type B737–700with 149 seat capacity has an expected passenger spill of = 9.34
Fleet type A319–100 with 126 seat capacity has an expected passenger spill of = 11.46

After determining the expected passenger spill on these fleet types, we now can calculate the respective expected spill costs as follows:

B737–700 = 9.34 × 0.17 × 3700 =$5874.86
A319–100 = 11.46 × 0.17 × 4200 =$8182.44

Recapture Rate, which is more related topic to passenger spill cost, is determined after finding the expected spill costs. Considering a particular schedule, the percentage of passengers that are spilled on a less profitable journey and then recovered later on other more profitable journey by the same fleet; is represented by the recapture rate; Bazargan M (2010). Due to high flight frequencies offered by some major airlines, the recapture rate is normally high. This is due to some marketing reasons like frequency-flyer programs [8].

Expected spill costs for fleets considering recapture rate is given by; Bazargan M (2010) [8]:

Expected Spill Cost*= *Expected Spill Cost* × *(1 - Recapture Rate)

Using the above example of 2 fleet types i.e. B737-700 with 149 seats and A319-100 with 126 seats, we make an assumption that the recapture rate is 25 percent for this airline.

This implies that 75 percent of passengers who reserve to fly on Turkish Airlines are turned down and hence lost.

Hence the expected spill costs from the two fleet types are calculated as:

Expected spill costs for B737-700 =$5874.86 × 0.75 = $4406.145
Expected spill costs for A319-100 = $8182.44 × 0.75 = $6136.83

We can hence find the total cost of assigning a fleet type to a flight leg using the formula presented by Bazargan M (2010) [8] as:

Total Cost = Operating Cost + Passenger Spill Costs

Total assignment cost of B737-700 to flight TK1356 =$8347.725 +$4406.145 =$12753.83
Total assignment cost of A319-100 to flight TK1806 =$8172.36 + $6136.83= $14309.19

In the same manner, the operating costs of all other flights in a given schedule are calculated. This is done by GAMS.

5.3. OPTIMIZATION MODEL BUILDING

In order to develop the fleet assignment model that we propose for Turkish Airlines, we had a set of data and parameter that we used.

Among the data set, we had:

i. Different flights; 70 in our case; from Istanbul to different International airports. For example, flight TK0015 to Buenos Aires, flight TK0052 to Tokyo, flight TK0003 to New York, flight TK0605 to Entebbe, flight TK1631 to Munich, etc.(Ref. Table 5.2)

ii. Different fleet-types; in our case 10 fleet types i.e. A319-100 fleet-type, B737-700 fleet-type, B737-900ER fleet-type, A320-200 fleet-type, B737-800 fleet-type, the A321-200 fleet-type, A330-200, A340-300 fleet-type, A330-300 fleet-type and the B777-300ER fleet-type.

iii. Available number of aircrafts in each fleet-type, i.e. 2 for A319-100 fleet-type, 1 for B737-700 fleet-type, 3 for B737-900ER fleet type, etc. (Ref. Table 5.1).

Among the Parameters we had:

i. The cost of assigning a certain fleet type to perform a certain flight and this particular cost was obtaining by summing up the Total Operating Cost and the Total Passenger Spill Costs. i.e. ***Total Cost = Operating Cost + Passenger Spill Costs***

The Total operating cost is obtained by multiplying the CASM; which is the *unit cost* of flying a passenger a mile, this cost encompasses all the other costs like fuel, crew cost, catering etc. it is multiplied to flight Distance and the Fleet Seat Capacity. This cost varies according to fleet type (Ref. Table 5.1)

i.e., ***Operating Cost of flight = Distance*** × ***Seat Capacity of the aircraft*** × ***CASM of the fleet***

The other cost included in the Total costs is the Total Passenger Spill Cost. It is obtained by multiplying RASM; which is *unit revenue* an airline makes across all the available seats that were supplied, this cost for Turkish Airlines is fixed to 17 Cents, by the flight distance and the fleet seat capacity i.e.

Expected Spill Cost for a fleet = Expected number of Passenger Spill × ***RASM*** × ***Distance***

ii. The other parameter we had; the Mean Demand per Fight. For example a mean of 302 passengers to Buenos Aires, a mean of 228 passengers to Tokyo, a mean of 217 passengers to New-York, a mean of 142 passengers to Entebbe, a mean of 167 passengers to Munich, etc.(Ref. Table 5.2).

iii. Seat capacities of each fleet types. i.e. 126 seats for A319-100 fleet type, 149 seats for B737-700 fleet type, 151 seats for B737-900ER fleet type. (Ref. Table 5.1)

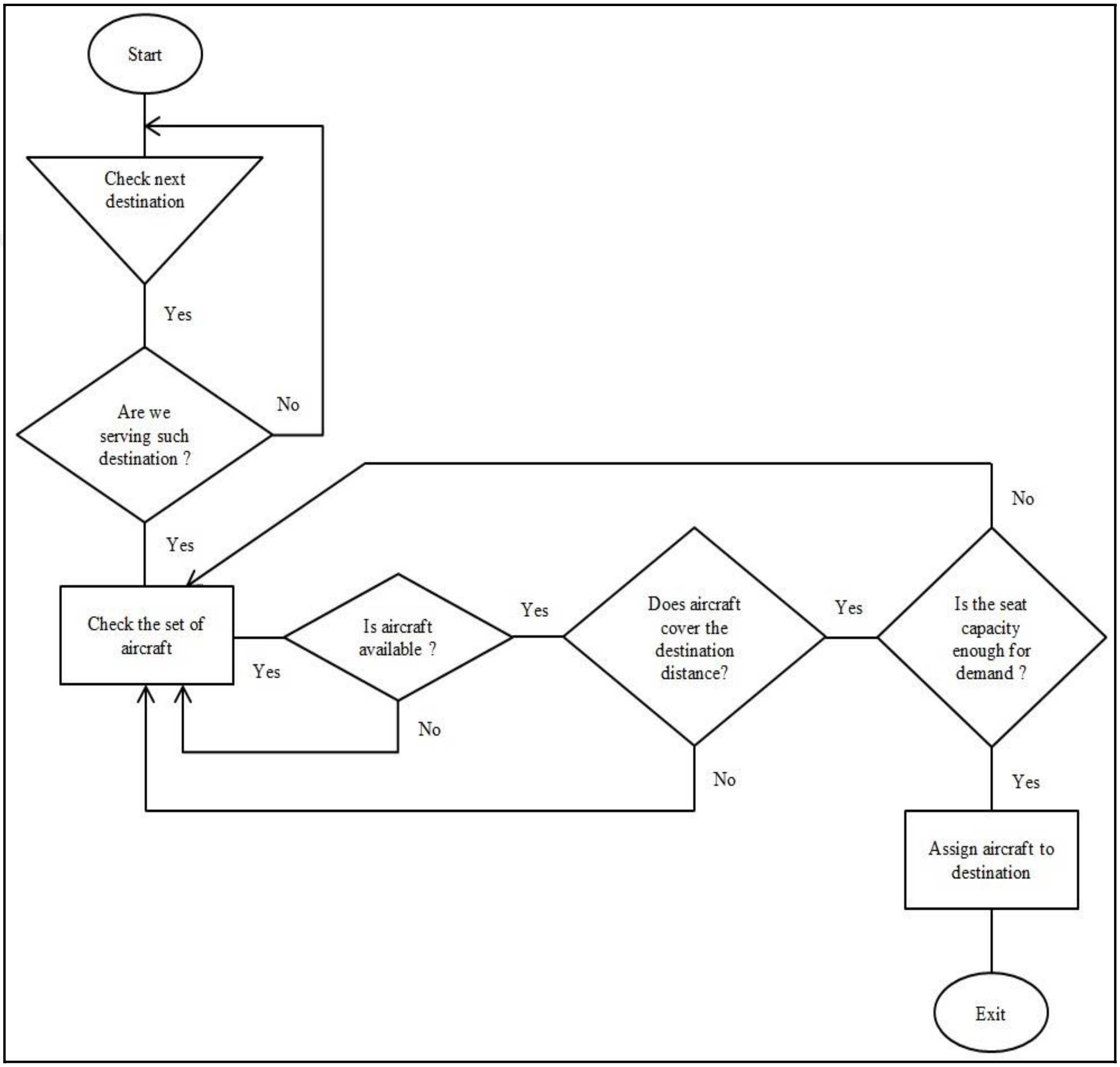
iv. Destination Range in miles of each flight i.e.7624 miles to Buenos Aires, 5596 miles to Tokyo, 5008 miles to New York, 2840 miles to Entebbe, 978 miles to Munich, etc.(Ref. Table 5.2)

v. Maximum Range in miles of each fleet type. i.e., 4200 miles for A319-100 fleet type, 3750 miles for B737-700 fleet type, 9100 for B737-900ER fleet type, 6500 miles for A330-300 fleet type. (Ref. Table 5.1)

vi. Duty Duration of each flight; which is the time it takes to leave Istanbul the main hub to a particular airport and back to Istanbul. i.e. 44.4 hours for Buenos Aires, 26 hours Tokyo, 22.5 hours New York, 15.8 hours for Entebbe, 6.3 hours for Munich.

To clearly explain what our model does, we built a process flow chart shown in Figure 5.2. The process starts with deciding which destination to serve, and Turkish Airlines decides whether or not to serve such a destination.

Figure 5.2: The Process Flow-Chart Diagram



If the decision is to serve it, we check among the set of aircrafts; which include 10 fleet-types to choose from; which one to assign to that destination. Then we ask the question; "is the selected aircraft to serve that destination available?" if the aircraft is available, we check whether it can cover the destination range and the passenger demand. If it does not we go back and check among the set of aircrafts the appropriate airplane in different fleet-type and its availability. Otherwise, we assign it to cover the flight.

With the above information at hand, we developed the following model:

Data Sets;

$\boldsymbol{M}$: daily outbound flights from Istanbul ($\boldsymbol{m} = 1, 2, \ldots, 70$)

$\boldsymbol{N}$: different fleet-types ($\boldsymbol{n} = 1, 2, \ldots, 10$)

$\boldsymbol{T}$: time-slot in hours ($\boldsymbol{t} = 1, 2, \ldots, 48$)

Parameters;

$\boldsymbol{C_{m,n,t}}$: cost of assigning fleet type $\boldsymbol{n \in N}$ to perform flight leg $\boldsymbol{m \in M}$ at time $\boldsymbol{t}$

$\boldsymbol{A_n}$: number of available aircraft in fleet-type $\boldsymbol{n}$

$\boldsymbol{p_m}$: passengers mean demand of flight $\boldsymbol{m \in M}$

$\boldsymbol{S_n}$: seat capacity of fleet-type $\boldsymbol{n \in N}$

$\boldsymbol{d_m}$: destination distance in miles of flight $\boldsymbol{m \in M}$

$\boldsymbol{R_n}$: maximum range in miles of fleet-type $\boldsymbol{n \in N}$

$\boldsymbol{q_m}$: the duty duration of flight $\boldsymbol{m \in M}$

Decision Variables;

$$\pi_{m,n,t} = \begin{cases} 1, & \text{if fleet type } \boldsymbol{n \in N} \text{ is assigned to flight } \boldsymbol{m \in M} \text{ at time } \boldsymbol{t} \\ 0, & \text{otherwise} \end{cases}$$

$$\omega_{n,t} = \text{number of available aircraft in fleet type } \boldsymbol{n \in N} \text{ on the ground at time } \boldsymbol{t}$$

$$\min \sum_{m \in M} \sum_{n \in N} \sum_{t \in T} C_{m,n,t}\, \pi_{m,n,t} \qquad (5.0)$$

Model;

$$\sum_{n \in N} \sum_{t \in T} \pi_{m,n,t} = 1 \qquad \forall\, \boldsymbol{m} \in \boldsymbol{M} \qquad (5.1)$$

$$\omega_{n,t} = \omega_{n,t-1} - \sum_{m \in M} \pi_{m,n,t} + \sum_{m \in M} \pi_{m,n,t-q_m} \qquad \forall\, \boldsymbol{n} \in \boldsymbol{N}, \forall\, \boldsymbol{t} \in \boldsymbol{T} \qquad (5.2)$$

$$\omega_{n,t} \leq A_n \qquad \forall\, \boldsymbol{n} \in \boldsymbol{N}, \forall\, \boldsymbol{t} \in \boldsymbol{T} \qquad (5.3)$$

$$\sum_{n \in N} \sum_{t \in T} S_n\, \pi_{m,n,t} \geq p_m \qquad \forall\, \boldsymbol{m} \in \boldsymbol{M} \qquad (5.4)$$

$$\sum_{n \in N} \sum_{t \in T} R_n\, \pi_{m,n,t} \geq d_m \qquad \forall\, \boldsymbol{m} \in \boldsymbol{M} \qquad (5.5)$$

$$\pi_{m,n,t} \in \{0, 1\} \qquad \forall\, \boldsymbol{m} \in \boldsymbol{M}, \forall\, \boldsymbol{n} \in \boldsymbol{N}, \forall\, \boldsymbol{t} \in \boldsymbol{T} \qquad (5.6)$$

$$\omega_{n,t} \geq 0 \qquad \forall\, \boldsymbol{n} \in \boldsymbol{N}, \forall\, \boldsymbol{t} \in \boldsymbol{T} \qquad (5.7)$$

5.4. MODEL DESCRIPTION

Objective Function;

From the created model, constraint (5.0) which is our objective function is to minimize the total costs required to assign various fleet types to all flights in the given schedule.

We need to adopt some decision variables which are stated above. In the binary decision variable $\boldsymbol{\pi_{m,n,t}}$, index ***m*** represents flight legs while index ***n***; fleet-types and ***t***; the time.

Constraints;

There are seven constraints in the above model. They are discussed as follows:

- ***i.*** Constraint (5.1) indicates the flight coverage. Here, every flight leg is operated by one and only one fleet-type. To cover a flight leg at a needed time, the sum of all the decision variables which represent that flight must add up to 1.
- ***ii.*** Constraint (5.2) represents the inventory equation which ensures the compatibility between the aircraft landing and the aircrafts taking off. The number of aircraft for any fleet-type at time *t* is the number of aircraft of that fleet-type just before that time minus the departure flights plus the arrival flights after a previous duty. This is the balance constraint.
- ***iii.*** Constraint (5.3) addresses the fleet-type size. It prevents for each fleet-type from surpassing the available aircrafts.
- ***iv.*** Constraint (5.4) is about the traffic passenger demand. This constraint makes sure that the aircraft seat capacity is enough to meet the seat demand for that particular flight leg.
- ***v.*** Constraint (5.5) makes sure that the destination flight distance is within the limit of the aircraft maximum range.
- ***vi.*** Lastly, constraints (5.6) and (5.7) lay the numerical domains of decision variables. The variables associated with the fleet-types assignments are binary and the variables associated with fleet-type availability are non-negative. Note that, in any feasible solution, fleet-type availability is an integer.

6. COMPUTATIONAL RESULTS

The model created (Ref. Chapter 5) is solved using the open access optimization software called GAMS, (Ref. 3.2.5) utilizing the data gathered from open access data provided by Turkish Airlines. However, this data set (Ref. 6.3) is rather provided in an aggregated term and we had to make some assumptions to obtain the ones needed for the picked destinations which were selected among over 260 routes.

6.1. DATA SET USED

The Table 6.1 gives the flight program for 70 flights selected among over 260 destinations. These are the daily program and as it is recognized from the table, some of the destinations are close to hub station (Istanbul) such as Sofia, Athens, and Kiev, and others are long haul flights that take more than 24 hours with return and boarding as well as disembarking. As it can be realized, some of the destinations have a high demand such as London, Paris, Berlin, Beijing etc. and some have relatively low demand such as Kigali, Rome, Entebbe, etc. which drives what sort of plane should be assigned for the flight.

The flight time is also a main determiner for the assignment of the plane. Since planes can only fly to limited destinations because of technical as well as fuel capacity limits, this is another constraint to assigning every plane to every destination. For example, B737-800 aircrafts can fly maximum 3500 miles; these aircrafts could be assigned only to the destinations which are less than 3500 miles such as, Baku, Cairo, Moscow, etc.

6.2. SOLUTION METHODOLOGY AND PROCEDURE

The optimization software GAMS is easy to use and handling the data entry as well as running the program is standard. We have considered 30 destinations worldwide ranging from 303 air miles to 7624 air miles, which requires assigning 5-38 hours timeslot for one duty which is the established time slot by Turkish Airlines for any destination including return flights plus boarding, disembarking and preparing the aircraft for flight such as cleaning, tiding up, and very minor visual maintenance as shown in Table 6.1.

Table 6.1: Flight Schedule in Turkish Airlines Network

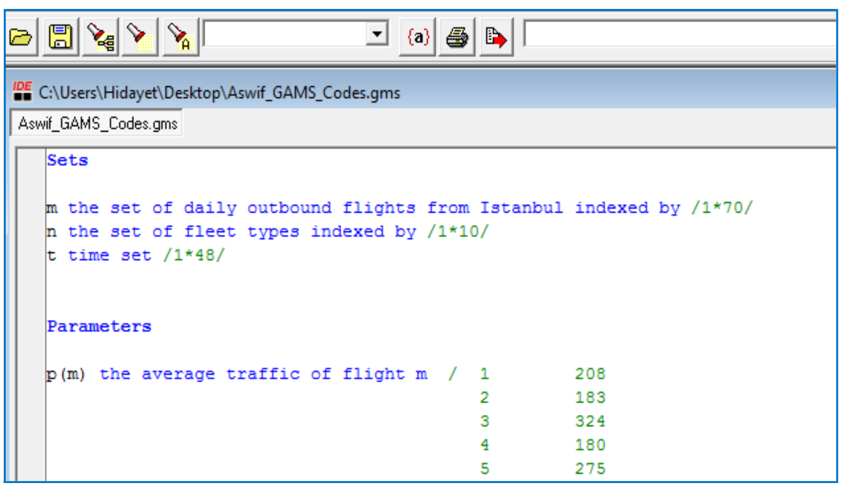
	Flight m	*Origin*	*Destination*	*Departure Time*	*Arrival Time*	*Mean Deman d p_m*	*St. Dev.*	*Distance (miles) d_m*	*Duration of Duty q_m*
1.	TK762	IST, Istanbul	DXB, Dubai	00:20	05:45	208	22	1871	10.1
2.	TK338	IST, Istanbul	GYD, Baku	00:25	05:15	183	45	1111	6.8
3.	TK20	IST, Istanbul	PEK, Beijing	00:35	15:05	324	11	4400	21.5
4.	TK692	IST, Istanbul	CAI, Cairo	00:40	01:50	180	24	764	5.3
5.	TK72	IST, Istanbul	CAN, Guangzhou	00:45	16:00	275	11	4905	23.7
6.	TK40	IST, Istanbul	CPT, Cape Town via Johannesburg	00:55 10:50	09:40 13:10	287	57	5224	26.6
7.	TK52	IST, Istanbul	NRT, Tokyo	01:00	18:30	228	46	5596	26
8.	TK690	IST, Istanbul	CAI, Cairo	06:45	07:55	169	24	764	5.3
9.	TK3	IST, Istanbul	JFK, New York	07:20	11:20	217	40	5008	22.5
10.	TK1027	IST, Istanbul	SOF, Sofia	07:25	08:45	165	25	303	3.4
11.	TK1821	IST, Istanbul	CDG, Paris	07:35	10:15	255	39	1391	8.2
12.	TK1845	IST, Istanbul	ATH, Athens	07:40	09:05	128	25	334	3.7
13.	TK1629	IST, Istanbul	MUC, Munich	07:45	09:30	167	26	978	6.3
14.	TK1857	IST, Istanbul	MAD, Madrid	07:50	11:30	169	39	1688	9.6
15.	TK457	IST, Istanbul	KBP, Kiev	07:55	09:55	182	17	656	5.1
16.	TK1979	IST, Istanbul	LHR, London	08:00	10:05	234	52	1563	9.4
17.	TK1793	IST, Istanbul	STO, Stockholm	08:15	10:45	165	47	1375	7.9
18.	TK1861	IST, Istanbul	FCO, Rome	08:20	10:00	146	27	861	6.2
19.	TK1951	IST, Istanbul	AMS, Amsterdam	08:25	11:00	135	22	1375	8.1
20.	TK1721	IST, Istanbul	TXL, Berlin	08:30	10:20	142	32	1083	6.5
21.	TK413	IST, Istanbul	VKO, Moscow	08:35	12:20	164	28	1082	6.9
22.	TK332	IST, Istanbul	GYD, Baku	08:45	13:40	172	42	1111	6.8
23.	TK15	IST, Istanbul	EZE, Buenos Aires via Sao Paulo	09:30 18:30	16:55 21:15	302	5	7624	44.4
24.	TK617	IST, Istanbul	CMN, Casablanca	10:25	13:25	270	66	2058	11.1
25.	TK1805	IST, Istanbul	TLS, Toulouse	10:55	13:40	120	22	1410	8.2
26.	TK417	IST, Istanbul	VKO, Moscow	11:00	14:55	147	27	1082	6.9
27.	TK1957	IST, Istanbul	AMS, Amsterdam	11:05	13:40	171	19	1375	8.1
28.	TK455	IST, Istanbul	KBP, Kiev	11:10	13:10	153	14	656	5.1
29.	TK1823	IST, Istanbul	CDG, Paris	11:15	13:55	175	25	1391	8.2
30.	TK1725	IST, Istanbul	TXL, Berlin	12:05	13:50	159	37	1083	6.5
31.	TK1631	IST, Istanbul	MUC, Munich	12:10	13:40	129	36	978	6.3
32.	TK108	IST, Istanbul	MED, Medina	12:15	15:35	233	24	1299	8.8
33.	TK1849	IST, Istanbul	ATH, Athens	12:40	14:05	162	6	334	3.7
34.	TK9	IST, Istanbul	LAX, Los Angeles	12:45	16:30	256	36	6864	28.5
35.	TK1865	IST, Istanbul	FCO, Rome	12:50	14:25	123	24	861	6.2
36.	TK1985	IST, Istanbul	LHR, London	13:10	15:15	225	41	1563	9.4
37.	TK1	IST, Istanbul	JFK, New York	13:15	16:50	298	6	5008	22.5
38.	TK1859	IST, Istanbul	MAD, Madrid	13:20	16:50	126	35	1688	9.6
39.	TK1355	IST, Istanbul	LUX, Luxemburg	13:35	15:55	120	18	1245	7.5
40.	TK1795	IST, Istanbul	STO, Stockholm	14:00	16:30	155	35	1375	7.9
41.	TK1827	IST, Istanbul	CDG, Paris	14:25	17:10	289	59	1391	8.2
42.	TK1953	IST, Istanbul	AMS, Amsterdam	14:40	17:15	151	39	1375	8.1
43.	TK334	IST, Istanbul	GYD, Baku	15:00	19:45	144	44	1111	6.8
44.	TK1635	IST, Istanbul	MUC, Munich	16:05	17:45	173	45	978	6.3
45.	TK625	IST, Istanbul	LOS, Lagos	16:10	21:00	194	31	2851	14.3
46.	TK1723	IST, Istanbul	TXL, Berlin	16:15	18:00	173	21	1083	6.5
47.	TK1863	IST, Istanbul	FCO, Rome	16:25	18:05	127	35	861	6.2
48.	TK415	IST, Istanbul	VKO, Moscow	16:40	20:25	160	1	1082	6.9
49.	TK50	IST, Istanbul	NRT, Tokyo	17:10	10:25/1	326	12	5596	25.3
50.	TK694	IST, Istanbul	CAI, Cairo	18:05	19:25	187	39	764	5.3
51.	TK459	IST, Istanbul	KBP, Kiev	18:25	20:15	152	32	656	5.1
52.	TK605	IST, Istanbul	EBB, Entebbe via Kigali	18:30 00:55	23:55 02:50	142	16	2840	15.8
53.	TK1727	IST, Istanbul	TXL, Berlin	19:05	20:55	118	25	1083	6.5
54.	TK1983	IST, Istanbul	LHR, London	19:15	21:15	169	34	1563	9.4
55.	TK1357	IST, Istanbul	MAD, Madrid	19:20	22:55	132	4	1688	9.6

56.	TK1843	IST, Istanbul	ATH, Athens	19:25	20:50	157	34	334	3.7
57.	TK1955	IST, Istanbul	AMS, Amsterdam	19:30	22:05	135	8	1375	8.1
58.	TK720	IST, Istanbul	BOM, Mumbai	19:35	04:30/1	322	20	2997	14.7
59.	TK1029	IST, Istanbul	SOF, Sofia	19:40	20:25	165	23	303	3.4
60.	TK1637	IST, Istanbul	MUC, Munich	19:45	21:30	125	38	978	6.3
61.	TK1829	IST, Istanbul	CDG, Paris	19:50	22:30	149	34	1391	8.2
62.	TK760	IST, Istanbul	DXB, Dubai	19:55	01:30/1	191	35	1871	10.1
63.	TK423	IST, Istanbul	OVB, Novosibirsk	20:05	05:55/1	151	40	2599	13.2
64.	TK1987	IST, Istanbul	LHR, London	20:30	22:30	161	37	1563	9.4
65.	TK336	IST, Istanbul	GYD, Baku	20:55	01:50/1	177	46	1111	6.8
66.	TK1791	IST, Istanbul	STO, Stockholm	21:00	23:30	123	26	1375	7.9
67.	TK1361	IST, Istanbul	FCO, Rome	22:15	23:50	102	23	861	6.2
68.	TK98	IST, Istanbul	MED, Medina	23:10	02:30/1	223	45	1299	8.7
69.	TK419	IST, Istanbul	VKO, Moscow	23:15	03:00/1	139	36	1082	6.9
70.	TK1841	IST, Istanbul	ATH, Athens	23:20	00:40/1	127	37	334	3.7

The Turkish Airline practice is to assign a plane to any destination and use the very same plane for the return flight assigning a convenient time slot in order to prevent seeking a separate plane for the return flights, which is called a *duty*.

The time is assumed to be 48 hours for the long haul flights. The flight to Los Angeles, Singapore, Sao Paolo, Buenos Aires etc., take more than 24 hours for a round trip back to the hub station Istanbul. The data set is entered in a platform provided by GAMS following the syntax and the procedure of GAMS; see Figure 6.1 below.

Figure 6.1: GAMS platform



The model created is entered into the GAMS using the very same platform, which is used to store the data used for the model, Figure 6.2-3. After entering the model and the data set, one need to compile the model using the icon Run , and if there is a syntax error the program generates a syntax error and indicates the line which has an error. And if the model has a logical error, this is understood from the output generated by an experienced modeler.

Figure 6.2: Data Sets

C o m p i l a t i o n

Sets

m daily outbound flights from Istanbul /1*70/

n the set of different fleet types /1*10/

t time set in hour /1*48/

Parameters

p(m) the passenger mean demand of flight m /	1	208
	2	183
	…	
	70	127 /
d(m) the distance to destination of flight m /	1	1871
	2	1111
	…	
	70	334 /
q(m) the time used for the duty of flight m /	1	10.1
	2	6.8
	…	
	70	3.7 /
A(n) the number of aircraft ineach fleet type n /	1	2
	2	1
	…	
	10	8 /
S(n) the seat capacity of fleet type n /	1	126
	2	149
	…	
	10	337 /
R(n) the max range in miles of fleet type n /	1	4200
	2	3750
	…	
	10	9100 /
CASM(n) max cost for fleet type n per seat /	1	0.046
	2	0.045
	…	
	10	0.052 /

Figure 6.3: Data Input

```
Table
nps(m,n) expected number of passenger spill
                        1       2       …       10
                1       9.34    10.08   …       8.47
                2       11.46   9.34    …       6.78
                …
                70      10.56   11.29   …       7.57  ;
Parameter
            opc(m,n,t) cost of assigning fleet n to perform flight m ;
            opc(m,n,t) = CASM(n) * d(m) * S(n);
            scf(m,n,t) passenger spill cost for a fleet;
            scf(m,n,t) = 0.17 * nps(m,n) * d(m) ;
            psc(m,n,t) passenger spill cost ;
            psc(m,n,t) = scf(m,n) * (1-0.25);
            differ(m,t) difference between q(m) and t ;
            differ(m,t) = ord(t)-q(m) ;
variable pi(m,n,t) flight m is performed by fleet n at time t ;
variable omega(n,t) number of available aircraft in fleet n at time t ;
variable cost objective funtion
binary variable pi ;
positive variable omega ;
Equations
            Obj     Minimize the Operation Cost
            const1  Each flight is covered by a fleet type
            const2  Inventory Equation
            const3  Availability aircraft constraint
            const4  Fleet assigned to the flight seat capacity requirement
            const5  Fleet assigned to the flight destination distance
obj..cost=e=sum((m,n,t), ((opc(m,n,t)+psc(m,n,t))*pi(m,n,t)));
const1(m)..sum((n,t), pi(m,n,t))=e=1;
const2(n,t)$(ord(t) ne1)..omega(n,t)=e=omega(n,t-1)-sum(m,pi(m,n,t))+(sum(m,pi(m,n,t-(z(m)))));
const3(n,t)..omega(n,t)=l=A(n);
const4(m)..sum((n,t), pi(m,n,t)*S(n))=g=p(m);
const5(m)..sum((n,t), pi(m,n,t)*R(n))=g=d(m);
Model fleet_assignment /all/ ;
Solve fleet_assignment using mip minimizing cost ;
```

6.3. RESULTS AND DISCUSSIONS

The program generates the output on the same screen in a classified manner. Our model seeks to minimize the total assignable costs that are incurred while assigning a particular fleet type to a particular flight leg in the given schedule.

From the computations performed by GAMS, it could be observed that the objective value $\boldsymbol{z}$ is 1,439,446.6670 US Dollars. For the sake of assigning each fleet to flight, we would discuss about some fleet-type and their corresponding costs. For example, the cost of assigning fleet 1 to flight 1 at time $\boldsymbol{t}$ equals to 1 is 10,844.316 US Dollars.

We would also explain why a certain fleet is assigned to a certain flight and the concept behind it. Data has been entered to GAMS; this data set includes passenger demand, range of the travel and the total time of duty by flight. It was supposed to assign the correct fleet to flight considering the constraints as well as the data provided. We base our discussion on range and seat capacity, because certain fleets are assigned to certain flights according to distance; time and the passenger mean demand.

According to the results, it has been observed that flights with small passenger demand are assigned fleets with small seat capacity and vice versa. For example, fleet type A321-200 was assigned to Cairo due to its passenger demand. Assigning a bigger fleet-type like a B777-300ER would be a loss since passenger spill costs would be high. Passenger mean demand for Cairo is 180 which is less than the seat capacity of an A321-200 fleet that can accommodate 188 passengers. From this observation, two or three flights can be assigned per day to serve this journey, instead of assigning one bigger flight per day. Another fleet-type that is assigned to a short distance destination is a B737-700 or a B737-800 that was assigned to Athens depending on the aircraft availability in a given time-slot.

Table 6.2: Assigned Fleet-Type

	Flight m	*Origin*	*Destination*	*Mean Demand* p_m	*Distance (miles)* d_m	*Duration of Duty* q_m	*Assigned Time*	*Assignment Cost*	*Assigned Fleet Type*
1.	TK762	IST, Istanbul	DXB, Dubai	208	1871	10.1	1	22,452.000	**A330-200**
2.	TK338	IST, Istanbul	GYD, Baku	183	1111	6.8	47	10,025.664	**A321-200**
3.	TK20	IST, Istanbul	PEK, Beijing	324	4400	21.5	1	77,105.600	**B777-300ER**
4.	TK692	IST, Istanbul	CAI, Cairo	180	764	5.3	45	6,894.336	**A321-200**
5.	TK72	IST, Istanbul	CAN, Guangzhou	275	4905	23.7	1	72,294.795	**A330-300**
6.	TK40	IST, Istanbul	CPT, Cape Town via Johannesburg	287	5224	26.6	1	76,996.536	**A330-300**
7.	TK52	IST, Istanbul	NRT, Tokyo	228	5596	26	24	67,152.000	**A330-200**
8.	TK690	IST, Istanbul	CAI, Cairo	169	764	5.3	47	6,894.336	**A321-200**

9.	TK3	IST, Istanbul	JFK, New York	217	5008	22.5	1	60,096.000	**A330-200**
10.	TK1027	IST, Istanbul	SOF, Sofia	165	303	3.4	1	2,349.765	**B737-800**
11.	TK1821	IST, Istanbul	CDG, Paris	255	1391	8.2	47	18,402.930	**A340-300**
12.	TK1845	IST, Istanbul	ATH, Athens	128	334	3.7	35	2,239.470	**B737-700**
13.	TK1629	IST, Istanbul	MUC, Munich	167	978	6.3	8	8,825.472	**A321-200**
14.	TK1857	IST, Istanbul	MAD, Madrid	169	1688	9.6	1	15,232.512	**A321-200**
15.	TK457	IST, Istanbul	KBP, Kiev	182	656	5.1	44	5,919.744	**A321-200**
16.	TK1979	IST, Istanbul	LHR, London	234	1563	9.4	31	18,756.000	**A330-200**
17.	TK1793	IST, Istanbul	STO, Stockholm	165	1375	7.9	22	10,663.125	**B737-800**
18.	TK1861	IST, Istanbul	FCO, Rome	146	861	6.2	7	5,773.005	**B737-700**
19.	TK1951	IST, Istanbul	AMS, Amsterdam	135	1375	8.1	26	9,219.375	**B737-700**
20.	TK1721	IST, Istanbul	TXL, Berlin	142	1083	6.5	9	7,261.515	**B737-700**
21.	TK413	IST, Istanbul	VKO, Moscow	164	1082	6.9	46	8,390.910	**B737-800**
22.	TK332	IST, Istanbul	GYD, Baku	172	1111	6.8	45	10,025.664	**A321-200**
23.	TK15	IST, Istanbul	EZE, Buenos Aires via Sao Paulo	302	7624	44.4	1	133,600.552	**B777-300ER**
24.	TK617	IST, Istanbul	CMN, Casablanca	270	2058	11.1	47	27,227.340	**A340-300**
25.	TK1805	IST, Istanbul	TLS, Toulouse	120	1410	8.2	1	8,172.360	**A319-100**
26.	TK417	IST, Istanbul	VKO, Moscow	147	1082	6.9	11	7,254.810	**B737-700**
27.	TK1957	IST, Istanbul	AMS, Amsterdam	171	1375	8.1	1	12,408.000	**A321-200**
28.	TK455	IST, Istanbul	KBP, Kiev	153	656	5.1	1	4,616.928	**A320-200**
29.	TK1823	IST, Istanbul	CDG, Paris	175	1391	8.2	1	12,552.384	**A321-200**
30.	TK1725	IST, Istanbul	TXL, Berlin	159	1083	6.5	48	8,398.665	**B737-800**
31.	TK1631	IST, Istanbul	MUC, Munich	129	978	6.3	18	6,557.490	**B737-700**
32.	TK108	IST, Istanbul	MED, Medina	233	1299	8.8	46	15,588.000	**A330-200**
33.	TK1849	IST, Istanbul	ATH, Athens	162	334	3.7	47	2,590.170	**B737-800**
34.	TK9	IST, Istanbul	LAX, Los Angeles	256	6864	28.5	1	90,810.720	**A340-300**
35.	TK1865	IST, Istanbul	FCO, Rome	123	861	6.2	35	4,990.356	**A319-100**
36.	TK1985	IST, Istanbul	LHR, London	225	1563	9.4	1	18,756.000	**A330-200**
37.	TK1	IST, Istanbul	JFK, New York	298	5008	22.5	1	87,760.192	**B777-300ER**
38.	TK1859	IST, Istanbul	MAD, Madrid	126	1688	9.6	27	9,783.648	**A319-100**
39.	TK1355	IST, Istanbul	LUX, Luxemburg	120	1245	7.5	17	7,216.020	**A319-100**
40.	TK1795	IST, Istanbul	STO, Stockholm	155	1375	7.9	1	10,663.125	**B737-800**
41.	TK1827	IST, Istanbul	CDG, Paris	289	1391	8.2	1	20,501.949	**A330-300**
42.	TK1953	IST, Istanbul	AMS, Amsterdam	151	1375	8.1	23	9,677.250	**A320-200**
43.	TK334	IST, Istanbul	GYD, Baku	144	1111	6.8	39	7,449.255	**B737-700**
44.	TK1635	IST, Istanbul	MUC, Munich	173	978	6.3	1	8,825.472	**A321-200**
45.	TK625	IST, Istanbul	LOS, Lagos	194	2851	14.3	41	34,212.000	**A330-200**
46.	TK1723	IST, Istanbul	TXL, Berlin	173	1083	6.5	19	9,772.992	**A321-200**
47.	TK1863	IST, Istanbul	FCO, Rome	127	861	6.2	32	5,773.005	**B737-700**
48.	TK415	IST, Istanbul	VKO, Moscow	160	1082	6.9	48	8,390.910	**B737-800**
49.	TK50	IST, Istanbul	NRT, Tokyo	326	5596	25.3	1	98,064.304	**B777-300ER**
50.	TK694	IST, Istanbul	CAI, Cairo	187	764	5.3	48	6,894.336	**A321-200**
51.	TK459	IST, Istanbul	KBP, Kiev	152	656	5.1	6	4,616.928	**A320-200**
52.	TK605	IST, Istanbul	EBB, Entebbe via Kigali	142	2840	15.8	33	19,042.200	**B737-700**
53.	TK1727	IST, Istanbul	TXL, Berlin	118	1083	6.5	11	6,277.068	**A319-100**
54.	TK1983	IST, Istanbul	LHR, London	169	1563	9.4	1	14,104.512	**A321-200**
55.	TK1357	IST, Istanbul	MAD, Madrid	132	1688	9.6	16	11,318.040	**B737-700**
56.	TK1843	IST, Istanbul	ATH, Athens	157	334	3.7	1	2,590.170	**B737-800**
57.	TK1955	IST, Istanbul	AMS, Amsterdam	135	1375	8.1	1	9,219.375	**B737-700**
58.	TK720	IST, Istanbul	BOM, Mumbai	322	2997	14.7	1	52,519.428	**B777-300ER**
59.	TK1029	IST, Istanbul	SOF, Sofia	165	303	3.4	1	2,349.765	**B737-800**
60.	TK1637	IST, Istanbul	MUC, Munich	125	978	6.3	1	5,668.488	**A319-100**
61.	TK1829	IST, Istanbul	CDG, Paris	149	1391	8.2	24	9,326.655	**B737-700**
62.	TK760	IST, Istanbul	DXB, Dubai	191	1871	10.1	46	22,452.000	**A330-200**
63.	TK423	IST, Istanbul	OVB, Novosibirsk	151	2599	13.2	36	18,291.762	**A320-200**
64.	TK1987	IST, Istanbul	LHR, London	161	1563	9.4	15	12,121.065	**B737-800**
65.	TK336	IST, Istanbul	GYD, Baku	177	1111	6.8	5	10,025.664	**A321-200**
66.	TK1791	IST, Istanbul	STO, Stockholm	123	1375	7.9	41	7,969.500	**A319-100**
67.	TK1361	IST, Istanbul	FCO, Rome	102	861	6.2	45	4,990.356	**A319-100**
68.	TK98	IST, Istanbul	MED, Medina	223	1299	8.7	1	15,588.000	**A330-200**
69.	TK419	IST, Istanbul	VKO, Moscow	139	1082	6.9	1	7,254.810	**B737-700**
70.	TK1841	IST, Istanbul	ATH, Athens	127	334	3.7	46	2,239.470	**B737-700**

The B737-700 and B737-800 fleet-type accommodates 149 passengers and 165 passengers respectively, which is almost the same as the passenger demand for Athens; 159. Here, two or three aircrafts can be assigned per day instead of assigning one big aircraft per day. One more observation made, it was that these small fleet-types were assigned to destinations with low time of flights such as 2.1 hours and 1.4 hours for Cairo and Athens respectively.

It also seems that medium fleet-types were assigned to flights with medium passenger demand. For example, an A330-200 fleet-type with seat capacity of 250 passengers was assigned to Lagos and Dubai with passenger demands of 194 and 191 respectively. The cost of assigning this particular fleet-type to Lagos is $34,212.336 while assigning it to Dubai it costs $22,452.000. There exists a difference in cost even though it is the same fleet-type; it is assigned because the passenger demand for these two cities is different. Another fleet-type in this category was an A340-300 of seat capacity 270 that was assigned to Los Angeles that has a passenger demand of 256. The cost of assigning this fleet-type has been observed as $90,810.720. These fleets also cover a medium to long time flight in the range of 8-14 hours.

Another observation that is made according to fleet assignment was that fleet types with large seat capacities are assigned to flights with high passenger demands and high ranges. For instance; fleet type B777-300ER was assigned to Beijing that has a range of 4400 miles and a passenger demand of 335 people per flight, while the same fleet was also assigned to Buenos Aires with a range of 7624 miles and passenger demand of 302 people per flight.

The cost of assigning such a fleet to Beijing is $77,105.600 while assigning it to Buenos Aires is $133,600.552, which is higher than the cost of assigning it to Beijing. This is so because Buenos Aires is farther from Istanbul as compared to Beijing. This fleet-type also covers longer flight hours obviously of 11 hours and 17 hours for Beijing and Buenos Aires respectively. Assumptions normally made by many airlines is that; once the problem is formulated and modeled, it is easy to obtain the optimum solution using software. Quite the opposite and very unfortunate, most of the airline problems involve millions of decision variables. These huge models cannot be solved using standard software packages. For example, it takes approximately 5 seconds to solve a linear program with 100 variables and 50 constraints using a specific software package, it is unimaginable how long it would take to solve a similar model with 1,000 variables and 500 constraints on the same computer and using the same software.

Hence, it is advisable to use some intelligent algorithms that present a general overview of the complexity of the problem, as the size of the problem grows instead of making computational time dependent on the computer.

Again, the fact that Turkish Airlines is a developed and big airline company; it schedules almost thousands of flights each day. Solving the fleet assignment problem has always been a challenging task for the airlines especially if the airline relies on the aircraft scheduling problem on other airline processes such as schedule design, crew scheduling, aircraft routing, maintenance planning, and revenue management.

There has been a constant need to classify and compare most real-world problems and their solution methodology basing on their computational tractability.

Another characteristic of the raised problem in fleet assignment is the enormous size of the optimization models generated because of the combinatorial nature of the problem. These kinds of problems grow fast when varying constraints such as fleet number, number of destinations, passenger frequency demand grow. This makes the whole process too complex to be treated globally, especially because of computational limitations. Nevertheless, the global costs resulting in this consecutive approach might not be optimal ones.

Lastly, it is almost impossible for Turkish Airlines to get the optimal solutions for their crew scheduling or aircraft routing in a reasonable computational time since the algorithms and methodologies that are adopted to solve integer linear programming models all belong to non-deterministic polynomial (NP) time algorithms.

However, the computational complexity lies within the solution algorithms, and not the mathematical models or the way they have been formulated.

In the beginning of this study, as we have repeatedly stated that the aim of this study is to solve the problem of fleet and flight assignment that Turkish Airlines has been facing and assigning the desired fleet to a desired flight in consideration of passenger demand, range and flight time. With the help of optimization software, we are able to assign the perfect fleet to a perfect flight while minimizing all operational costs that might be involved in it. The main objective value z, which is to minimize the operational cost, was found to be $1,439,446.667. In this study, different costs according to assignment of fleet to flight are also obtained as shown in table 6.2; which helped identify if such a fleet should be assigned to the flight.

7. GENERAL CONCLUSION

7.1. RESEARCH WRAP-UP

Flight scheduling problems are classified as one of the hardest problems among the optimization problems because of their very complex nature. The number of constraints involved is enormous and satisfying all the hindrances is most of the times impractical. Therefore, the optimization model has to be partial and modeler should compromise on satisfying all the constraints.

The general expectation of airline companies from such an optimization model is to find a lowest cost under several constraints. In this study, it is aimed to minimize the total cost of assigning a certain aircraft type to any destination. Turkish Airlines (TA) Flight Operation is taken as a basis and it is assumed that the single flight normally covers the round trip. In another term, when the TA assigns an aircraft to any destination, the very same aircraft is used for the return flight. The allocated time for the entire flight is simply the summation of flight-time to and from destination, the waiting time and the boarding time i.e. *Duty duration*.

Seven constraints are considered in this model, they are sensible and primary constraints in any flight scheduling model.

The first constraint assumes that when any aircraft is assigned to any destination, that flight must be realized at a given time in the time-slot. Hence, every flight leg is covered by one and only one fleet-type. The second is the balance equation in inventory manner which ensures the compatibility between the aircraft landing and the aircrafts taking off. The third ensures the compatibility between the fleet-type availability and its size.

The fourth constraint considers the seat capacity. If there is a certain demand for a certain destination, the assigned aircraft must have enough seat capacity to satisfy this demand. In another term, if there are 200 passengers for a certain destination, any aircraft that has less than 200 seats capacity cannot be assigned to this destination. The fifth constraint guarantees that the assigned flight is technically capable of fleeing the assigned destination, it makes sure that the destination flight distance is within the limit of the aircraft maximum range.

Which means, if an aircraft has a maximum 4500 miles fleeing destination range, the airline cannot assign an aircraft to a destination which has a range more than 4500 miles.

Lastly, the sixth and seventh constraints lay the numerical domains of decision variables. The variables associated with the fleet-types assignments are binary and the variables associated with fleet-type availability are non-negative. The created model is a linear-integer programming model that meets the expectations of assigning the right aircraft to right destination at the right time while satisfying the imposed constraints. It generates a minimized operating cost for an assigned aircraft to any destination. The model is solved using a commercial software package, GAMS, freely available on open access platforms and capable of solving large scale optimization problems.

The model created is sensible and covers some of the major issues concerned by airlines, therefore, it is realistic. Because of the complexity and impracticality of the one single large model that covers all the issues concerned by airlines, it is always advisable to create small-scale models and then integrate the smaller models through a common database, so that all the models share the same common database, which enables the users to compare different parts and easier to draw a conclusion.

The data used for the case study conducted as a test bed are taken some open sources of the Turkish Airlines and the results found are compared with some of the actual practices of Turkish Airlines. The model output proves that the results are highly compatible with the Turkish Airlines practices. For example, model has generated the results such that the narrow body, short-to-medium range aircrafts such as B737-800 or A320-200, are assigned to close destinations such as Athens, Cairo, Sofia, Thessaloniki, Kiev, etc., to hub station, which is Istanbul, and wide-body, long-haul aircrafts such as B777-300R, A330-200, A340, etc., are assigned to long range destinations such as New York, Los Angeles, Chicago, Peking, Tokyo, etc. This is indeed the practice of Turkish Airlines and this validates the model is right and realistic.

7.2. RECOMMENDATION TO STAKEHOLDERS

The model created is a relatively limited model that is enough to satisfy a master's degree standard. This study concerns with satisfying only the most important constraints such as aircraft seat capacity and the destination range, which means guaranteeing the right aircraft to right destination while satisfying the demand. However, there are number of other constraints that need to be satisfied such as situations where not all the aircrafts are in service (some may be under planned maintenance; some aircraft may be out of service because of any unexpected event, etc.). This could be included to the current model, so that a more realistic model would be created.

During high seasons, almost all airline companies put more flights to some popular destinations and the ticket prices are generally higher than low seasons, which heavily affect the company's financial situation. Yet again, the number of aircraft in air is expected to be higher than during low seasons and the aircrafts maximum flight time is reached much quickly. The immediate result of this more frequent flight is that the time between two planned maintenance becomes shorter and maintenance is done more frequently. Since the seat capacity of the airline company does not change in very short time, unless it has been already planned before the season starts, the company faces much pressure in terms of availability of the suitable aircraft. Therefore, this seasonally changing capacity would be forecasted and the more realistic seat capacity should be introduced in the database.

Since no company can compromise on safety standards and the further imposed constraints, the airline companies need a more sophisticated flight planning. Almost all the airline companies use a base airport and plan the flight scheduling using this airport as a hub. If this is the center of the model built then this problem is more suitable for network scheduling modeling and a more comprehensive airline scheduling model should be built using the network optimization model.

The number of destinations and the aircraft considered in this project is the fraction of the fleet size of TA and the number of destination TA flees. A more comprehensive model should consider the entire fleet size and the destinations to be a more suitable model.

The cost calculation covers the major cost items involved in any flight. There are many more cost items involved in any airline operation and these are mostly classified under the group of overhead, which may not be very adequate for large airline companies. A more sophisticated cost accounting and calculation methodology, such as activity based costing, should be used to identify which operation component contributes to overall cost and at what level, so that a better financial planning for profiting and non-profiting destinations can easily be identified.

REFERENCES

[1] Airlinkflight.org

[2] Centreforaviation.com

[3] Yan Zhang (2009), Barbara M. Wildemuth, *Unstructured interviews*, Applications of Social Research Methods to Questions in Information and Library Science p222-231.

[4] Minichiello V (1990), Aroni R, Timewell E, Alexander L, *In-depth Interviewing: Researching people.* Hong Kong: LongmanCheshire Pty Limited.

[5] Hanif D Sherali (2004), Ebru K Bish, Xiaomei Zhu, *Airline fleet assignment concepts, models, and algorithms*, European Journal of Operational Research, Vol 172, p1-30.

[6] Timothy L Jacobs (2011), Laurie A Garrow, ManojLohatepanont, Frank S Koppelman, Gregory M Coldren, HadiPurnomo, *Quantitative Problem Solving Methods in the Airline Industry*, Vol169 of the series International Series in Operations Research & Management Science p35-99.

[7] Barnhart C (2002), Kniker T, Lohatepanont M, *Itinerary-based fleet assignment*. Transportation Science, 36(2) p199-217.

[8] Bazargan M (2010), *Airline Operations and Scheduling*, 2nd ed., Ashgate, USA, p40-58.

[9] Teodorovic D (1989), Krcmar-NozicE, *Multicriteria Model to Determine Flight Frequencies on an Airline Network under Competitive Conditions*, Transportation Science, 23(1), p14-25.

[10] Berge E Matthew (2007), John C Haws, Craig A Hopperstad, Aslaug Haraldsdottir, *Future airline schedules for air traffic management concept analysis*, 26th Digital Avionics Systems Conference.

[11] Abara J (1989), *Applying Integer Programming to the Fleet Assignment Problem*, Interfaces, 19, p20-28.

[12] Hane C (1995), Barnhart C, Johnson E, Marsten R, Nemhauser G, and Sigismondi G, *The Fleet Assignment Problem: Solving a Large-scale Integer Program,* Mathematical Programming, 70, p211-232.

[13] Rexing B (2000), Barnhart C, Kniker T, Jarrah T, and Krishnamurthy N, *Airline Fleet Assignment with Time Windows*, Transportation Science, 34, p1-20.

[14] Barnhart C, Shenoi R(1998), *An Approximate Model and Solution Approach for the Long-haul Crew Pairing Problem*, Transportation Science, 32(3), p221-231.

[15] Belobaba P (1999), *Airline Planning System: Economic and Models*, Airline Management, Massachusetts Institute of Technology.

[16] Jian L (2003), *Solving real-life transportation scheduling problems*, Florida University.

[17] Ki-Hwan Bae (2010), Integrated Airline Operations: Schedule Design, Fleet Assignment, Aircraft Routing, and Crew Scheduling, Blacksburg, Virginia

[18] Thengvall B (2000), Bard J, and Yu G, *Balancing User Preferences for Aircraft Schedule Recovery during Irregular Operations*, IIE Transactions, 32, p. 181193.

[19] Barnhart C (2004), Cohn A, *Airline Schedule Planning: Manufacturing & Service Operations Management (INFORMS)*, 6(1), p3-22.

[20] Daskin M (1989), and Panayoyopoulos N ,*Lagrangian relaxation approach to assigning aircraft to routes in hub and spoke networks*, Transportation Science, Vol 23p91-99.

[21] Kabbani N (1992) and Patty B, *Aircraft routing at American airlines*, Proceedings of the 32nd Annual Symposium of AGIFORS, Budapest, Hungary.

[22] Clarke L (1997), Johnson E, Nemhauser G, Zhu Z, *The Aircraft Rotation Problem*, Annals of Operations Research, Mathematics of Industrial Systems II, 33-46.

[23] Bartholomew-Biggs M (1997) and Parkhurst S, *Global optimization approach to solve multiaircraft routing problems*, Numerical Optimization Centre, University of Hertfordshire, United Kingdom.

[24] Mangoubi R (1984), Mathaisel D, *Optimizing Gate Assignment at Airport Terminals*, AGIFORS proceeding, p419-450.

[25] Buch I (1994), *Airport Ground Staff Scheduling at El Al*, AGIFORS Proceeding p116.

[26] Berge M (1994), *Timetable Optimization: Formulation, Solution Approaches, and Computational Issues*, AGIFORS proceeding, p341-357.

[27] Belobaba P (1992), *The Revenue Enhancement Potential of Airline Yield Management Systems*, ASTAIR Proceedings, p143-164, London.

[28] Belobaba P (1987), *Application of a Probabilistic Decision Model to Airline Seat Inventory Control*, Operations Research, Vol. 37, p. 183-197.

[29] Barnhart C (1996**)**, Talluri K, *Airline Operations Research, Chapter 10.* Working paper

[30] Wensveen G (2015) *Air Transportation: A Management Perspective*, eighth edition. Head of Aviation Technology, Purdue University.

[31] Rainer B (2009), Mauro D, Silvano M, *Assignment Problems*, Society for Industrial and Applied Mathematics, Philadelphia.

[32] Dantzig B (1954) and Ferguson A, The Problem of Routing **-** *A mathematical Solution*, Government Res. Report, Federal Clearinghouse, AD604395.

[33] Jacobs T(1999), Smith B, and Johnson E,*O&DFAM: Incorporating Passenger Flow Into the Fleeting Process*, AGIFORS Presentation.

[34] Kniker T (1998*), Itinerary-based Airline Fleet Assignment*, Ph.D. Dissertation, Massachusetts Institute of Technology, Cambridge, MA.

[35] Ahuja R (1993), Magnanti T, and Orlin J, *Network Flows: Theory, Algorithms, and Applications*, Prentice Hall, New Jersey.

[36] Yan S (2002), Lai W, Chen M, *Production Scheduling and Truck Dispatching of Ready Mixed Concrete*. Transportation Research Part E 44, p164–179.

[37] Yan S (2005), and Tseng C, *An integrated Model and Solution Algorithm for Fleet Routing and Flight Scheduling*, Working Paper, National Central University, Taiwan.

[38] Klabjan D (2002), Johnson E., Nemhauser G, Gelman E, and Ramaswamy S, *Airline crew scheduling with time windows and plane-count constraints.* Transportation Science, 36 (3), p337–348.

[39] Cordeau J (2001), Stojkovic G, Soumis F, and Desrosiers, *Benders Decomposition for Simultaneous Aircraft Routing and Crew Scheduling*, TransportationScience, 35(4), p375-388.

[40] Mercier A (2005), Cordeau J, and Soumis F, *A Computational Study of Benders Decomposition for the Integrated Aircraft Routing and Crew Scheduling Problem*; *Computers and Operations Research*, 32, p1451-1476.

[41] Cohn A (2003), and Barnhart C, *Improving Crew Scheduling by Incorporating Key Maintenance Routing Decisions; Operations Research*, 51(3), p387-396.

[42] Sandhu R (2007), and Klabjan D, *Integrated Airline Fleeting and Crew-Pairing Decisions; Operations Research*, 55, p439-456.

[43] Mercier A(2007), and Soumis F, *An Integrated Aircraft Routing, Crew Scheduling and Flight Retiming Model*, Computers and Operations Research, 34 p2251-2265

[44] Desaulniers G (1997), Desrosiers J, Dumas, Y, Solomon M, and Soumis, F, *Daily Aircraft Routing and Scheduling*, Management Science, 43, p841-855.

[45] Teodorovic D (1995) and Stojkovic G, *Model to reduce airline schedule disturbances,* journal of Transportation Engineering,121, p324-331.

[46] Clarke L (1996), Hane C, Johnson E and Nemhauser G, *Maintenance and Crew Considerations in Fleet Assignment*, Transportation Science, 30, p249-260.

[47] Rushmeier R (1997) and Kontogiorgis S, *Advances in the Optimization of Airline Fleet Assignment*, Transportation Science, 31, p159-169.

[48] Sherali H (2005), Bish E, and Zhu X, *Polyhedral Analysis and Algorithms for a Demand Driven Re-fleeting Model for Aircraft Assignment*, Transportation Science, p349-366.

[49] Manoj Lohatepanont (2002*), Airline Fleet Assignment and Schedule Design: Integrated Models and Algorithms*, Massachusetts Institute of Technology.

[50] Farkas A (1996), *The Influence of Network Effects and Yield Management on Airline Fleet Assignment Decisions*, Doctoral Thesis, Massachusetts Institute of Technology, Cambridge, MA.

[51] Erdmann A (2001), Nolte A, Noltemeier A and Schrader R, *Modeling and Solving an Airline Schedule Generation Problem,* Annals of Operations Research, 107, p117-142.

[52] Yaohua Li (2013) and Na Tan, *Study on Fleet Assignment Problem Model and Algorithm*, Research Article. Aeronautical Engineering College, Civil Aviation University of China, Tianjin 300300, China.