BOLU ABANT İZZET BAYSAL UNIVERSITY

THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

DEPARTMENT OF PHYSICS

QCD SUM RULES FOR THE MASS, RESIDUE AND SELF-ENEGRY OF NUCLEON IN COLD NUCLEAR MATTER

MASTER OF SCIENCE

MEHMET ARAT

BOLU, JULY 2019

APPROVAL OF THE THESIS

QCD SUM RULES FOR THE MASS, RESIDUE AND SELF-ENEGRY OF NUCLEON IN COLD NUCLEAR MATTER submitted by **Mehmet Arat** in partial fulfillment of the requirements for the degree of **Master of Science** in **Physics Department, the Graduate School of Natural and Applied Sciences of BOLU ABANT IZZET BAYSAL UNIVERSITY in 07/31/19** by

Examining Committee Members	**Signature**
Supervisor Assoc. Prof. Dr. Nuray Er Bolu Abant İzzet Baysal University	....
Member Assist. Prof. Dr. Ali Yılmaz Giresun University	....
Member Prof. Dr. Haluk Denizli Bolu Abant İzzet Baysal University	....

Prof. Dr. Ömer ÖZYURT ....

Director, **Graduate School of Natural and Applied Sciences**

To my family

DECLARATION

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Mehmet Arat

ABSTRACT

QCD SUM RULES FOR THE MASS, RESIDUE AND SELF-ENEGRY OF NUCLEON IN COLD NUCLEAR MATTER
M.S. THESIS
MEHMET ARAT,
BOLU ABANT İZZET BAYSAL UNIVERSITY GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
DEPARTMENT OF PHYSICS
(SUPERVISOR : ASSOC. PROF. DR. NURAY ER)

BOLU, JULY 2019

In this thesis, the mass, residue and self-energies of the nucleon in cold nuclear medium are calculated in the framework of the QCD sum rules method by using the interpolating current of the nucleon, including the Ioffe value of the mixing parameter β. Firstly, the reliable regions of the auxiliary parameters required for the calculations are determined in accordance with the philosophy of this method. Then, we present these physical quantities showing stability with respect to these auxiliary parameters in their obtained working regions.

In the limit of $\rho \to 0$, we obtain our vacuum results. It is then calculated that the mass and residue of the nucleon showed negative shifts due to the cold nuclear matter. These shifts are obtained using the mean values of the auxiliary parameters and at the saturation nuclear matter density. The negative shifts on the mass is $\%31$ and on the residue is $\%11$. In addition, in the density interval $\rho = [0 - 1.5]\rho^{sat}$, the modified mass and residue of nucleon show almost linear decrease due to the increasing medium density. Another important result is obtained that our results for the vector and scalar self-energies of nucleon being a good agreement with the literature.

It is important to investigate the physical properties of the hadrons in the cold and dense nuclear medium, as opposed to vacuum, to evaluate the results of heavy ion collision experiments, and to better understand the internal structures of dense objects such as neutron stars. Our results may shed light on the planned medium experiments in the near future and may also be helpful in theoretical and phenomenological studies related to the behavior of other hadrons in the dense medium.

KEYWORDS: QCD sum rules, non-perturbative approaches, cold nuclear matter

ÖZET

SOĞUK NÜKLEER MADDE ORTAMINDA NÜKLEONUN KÜTLE, REZİDÜ VE ÖZ-ENERJİLERİ İÇİN KRD TOPLAM KURALLARI
YÜKSEK LİSANS TEZİ
MEHMET ARAT,
BOLU ABANT İZZET BAYSAL UNİVERSİTESİ FEN BİLİMLERİ ENSTİTÜSÜ
FİZİK ANABİLİM DALI
(TEZ DANIŞMANI : DOÇ. DR. NURAY ER)

BOLU, TEMMUZ 2019

Bu tezde, nükleonun ara kesim akımı keyfi karıştırma parametresinin Ioffe değerini de kapsayacak şekilde kullanılmasıyla, nükleonun nükleer madde ortamında kütle, rezidü ve öz-enerjileri KRD toplam kuralları çerçevesinde hesaplanmıştır. Öncelikle hesaplar için gerekli olan yardımcı parameterelerin yöntemin uygulanabilme felsefesine uygun olarak güvenilir bölgeleri tayin edilmiştir. Daha sonra nükleonun fiziksel büyüklükleri için hesaplanan sonuçlarının güvenilir bölgeleri için bu parametrelerden bağımsızlığı gösterilmiştir.

Yoğunluğa bağlı nümerik sonuçlarda $\rho \to 0$ limiti alınarak vakum sonuçları elde edilmiştir. Daha sonra nükleonun kütle ve rezidü değerlerinin soğuk nükleer maddeye göre negatif kaymalar gösterdiği hesaplanmıştır. Bu kayma miktarları yardımcı parametrelerin güvenilir bölgelerindeki ortalama değerleri ve nükleer maddenin doymuş yoğunluğunda kütle için yaklaşık $\%31$ ve rezidü içinse $\%11$ olarak elde edilmiştir. Ayrıca ortam yoğunluğunun $\rho = [0 - 1.5]\rho^{sat}$ değiştiği aralıkta nükleonun kütlesinin ve rezidüsünün değişiminin de yaklaşık olarak artan ortam yoğunluğuna bağlı olarak lineer bir azalma gösterdiği de bulunmuştur. Bir diğer önemli sonuç ise nükelonun vektör ve skaler öz-enerjileri için elde edilmiştir mevcut sonuçlarla uyum içinde olduğu görülmüştür.

Hadronların fiziksel özelliklerinin vakumdan farklı olarak soğuk madde ortamındaki değişimlerinin incelenmesi, ağır iyon çarpıştırma deneylerinin sonuçlarının değerlendirilmesi, ayrıca nötron yıldızları gibi yoğun cisimlerin iç yapılarının daha iyi anlaşılması açısından önemlidir. Bu çalışma sonucunda elde ettiğimiz nükleonun fiziksel özellikleri üzerinde ortam yoğunluğunun etki sonuçları, yakın zamanda yapılması planlanan yoğun madde ortamındaki hadronlarla ilgili deneylere de ışık tutabileceği düşünülmektedir. Ayrıca bu sonuçlar farklı hadronların ortamdaki davranışları ile ilgili teorik ya da fenemolojik çalışmalar açısından da önemlidir.

ANAHTAR KELİMELER: KRD toplam kuralları, pertürbatif olmayan yaklaşımlar, soğuk madde ortamı

TABLE OF CONTENTS

Page

ABSTRACT . . . v
ÖZET . . . vi
TABLE OF CONTENTS . . . vii
LIST OF FIGURES . . . viii
LIST OF ABBREVIATIONS AND SYMBOLS . . . ix

1 INTRODUCTION 1

2 STANDARD MODEL AND NUCLEONS 3
2.1 Standard Model . . . 3
2.2 Quantum Chromodynamics (QCD) . . . 4
2.3 Nucleons . . . 4

3 QCDSR FORMALISM 6
3.1 Two-point correlation function . . . 6
3.2 Phenomenological and QCD sides of the correlation function . . . 7
3.3 Borel Transformation and QCD sum rules . . . 9

4 QCD SUM RULES FOR THE MASS, RESIDUE AND SELF-ENERGY OF NUCLEON IN COLD NUCLEAR MATTER 10
4.1 Finite density correlation function . . . 10
4.2 Physical Ansatz . . . 11
4.3 QCD Ansatz . . . 12
4.4 Numerical analysis . . . 20

5 CONCLUDING REMARKS 29

REFERENCES 31

6 CURRICULUM VITAE 34

LIST OF FIGURES

Page

Figure 4.1. Variation of λ_n as a function of x. 22

Figure 4.2. Variation of λ_n^* as a function of x. 23

Figure 4.3. Variation of perturbative, two-quark condensate (QQ), two-gluon condensate (GG) and mixed condensate (QG) parts in OPE in terms of M^2 at saturation density and the average value of continuum threshold. 23

Figure 4.4. Pole contribution (PC) with respect to Borel mass parameter for the $\not{p}$ structure at nuclear matter saturation density. 24

Figure 4.5. λ_n^*/λ_n as a function of M^2. 25

Figure 4.6. Same as Fig. 4.5 for m_n^*/m_n. 26

Figure 4.7. The percentage of the shift $\frac{(\lambda_n^*/\lambda_n)}{\lambda_n} * 100$ in residue of nucleon in dense medium with respect to M^2. 26

Figure 4.8. Same as Fig. 4.7 for mass of nucleon. 27

Figure 4.9. λ_n^*/λ_n as a function of ρ^{sat}/ρ at $s_0^* = 1$ GeV^2, $M^2 = 1$ GeV^2 and $x = -0.65$. 27

Figure 4.10. Same as Fig. 9 for m_n^*/m_m. 28

Figure 4.11. The vector self-energy Σ_v versus M^2 at mean values other auxiliary parameters. 28

LIST OF ABBREVIATIONS AND SYMBOLS

$\mathcal{L}$	: QCD Lagrangian
$\mathbf{G^{a\mu\nu}}$	: Gluon field-strength tensor
$\mathbf{a, b, c}$	: Color indices
$\psi_\mathbf{q}$	: Quark field spinor
$\mathbf{q}$	: Quark flavor
$\mathbf{\Pi(p^2)}$	: Two-point correlation function
$\mathbf{\|0>}$	: Vacuum state
$\not{\mathbf{D}}$	: Covariant derivative
$\mathcal{T}$	: Time ordering operator
$\mathbf{J(x)}$	: Interpolating current
$\epsilon_\mathbf{ijk}$	: Levi-Civita tensor
$\mathbf{C}$	: Charge conjugation operator
$\mathbf{\Gamma}$	: Dirac matrices
$\mathbf{\|h(p_h, s)\rangle}$	: Hadronic state
$\mathbf{f_h}$	: Leptonic decay constant for mesons
$\mathbf{C_i(x^2)}$	: Wilson coefficients
$\mathcal{B}$	: Borel transformation
$\mathbf{M^2}$	: Borel mass
$\mathbf{s_0}$	: Vacuum continuum threshold
$\mathbf{s_0^*}$	: In-medium continuum threshold
$\mathbf{d}$	: Operator dimension
$\mathbf{\|n(p^*, s)\rangle}$	: Nucleon ground state
$\mathbf{u(p^*, s)}$	: Positive energy Dirac spinor
λ	: Vacuum residue
λ^*	: In-medium modified residue
$\mathbf{m}$	: Vacuum mass
$\mathbf{m^*}$	: In-medium modified mass
$\mathbf{\Sigma_s}$	: Scalar self-energy
$\mathbf{\Sigma}_\upsilon$	: Vector self-energy
$\mathbf{S^{ij}_{u(d)}(x)}$	: Light-quark propagators
$\mathbf{Tr}[...]$	: Tracer
ρ	: Medium density
$\rho^\mathbf{sat}$	: Saturation nuclear matter density

$\rho_{\mathbf{n}}^{\mathbf{QCD}}(\mathrm{s})$	: Two-point spectral density
QCD	: Quantum Choromodynamic
QCDSR	: Quantum Choromodynamic Sum Rules
SM	: Standard Model
OPE	: Operator Product Expansion
CF	: Correlation Function
CERN	: European Organization for Nuclear Research
BNL	: Brookhaven National Laboratory
PANDA	: The Antiproton Annihilation in Darmstadt
FAIR	: Facility for Antiproton and Ion Research
QM	: Quantum Mechanics
CBM	: The Compressed Baryonic Matter

ACKNOWLEDGEMENTS

I am hearty thankful to my supervisor, Assoc. Prof. Dr Nuray Er who has guided me with her knowledge and giving me countless advice. She always promotes me to develop my master degree studies from the final level.

I would like to thank very much Prof. Dr. Kazem Azizi and Prof. Dr. Haluk Denizli for filling me with enthusiasm for high energy physics who is always a sympathetic listener, helping me with discussions and many suggestions.

Finally the biggest thanks to each my beloved family members who supported and encouraged me a lot during my studies, their prayers and good wishes always accompanied me.

1. INTRODUCTION

Particle physics investigates the body of physical objects making up matter and radiation, and examines their interaction with each other. In the 19th century, John Dalton concluded that each chemical element consisted of a single, unique particle type and they were the elementary particles of nature and named after atoms the Greek word "atomos", which meant "indivisible". At the end of the 19th century, Dalton's idea was collapsed by the discovery of the electron in 1897 by J. J. Thomson, and this discovery is considered the beginning of particle physics. Today, we know following interactions in nature: Strong, electromagnetic, weak and gravitational force. But at the beginning of the twentieth century, scientists only knew about electromagnetism and gravity, and unfortunately this was not enough to explain the structure of the atom. After the discovery of proton (p) and neutron (n), physicists thought that physical matter made of electrons, protons, and neutrons. With the development in particle accelerators in the 1950s and the studies of cosmic rays, it became feasible to investigate inelastic scattering experiments with hundreds of MeV energy protons. Later, Murray Gell-Mann and George Zweig Gell-Mann (1964); Zweig (1964) independently predicted that p and n were composed of substructures called quarks and this prediction is then called as "the Quark Model" of subatomic particles. There are now over 400 particles, which are in the form of different combinations of several fundamental particles. The classification of these subatomic particles and investigation of their interactions provided by a theory called Standard Model (SM) and which provides successful results so far. It also can successfully explain fundamental forces other than gravitational force known in nature. This lack of SM's gravitational force is considered among the major problems of SM and this has led scientists to seek theories beyond this model.

Hadron consisting of quarks and gluons is the general name of the baryon and meson families. Quantum chromodynamics (QCD) named as theory of the strong interactions is a framework in which strong interactions of quarks and gluons are described. To improve knowledge about the strong interactions of standard hadrons, it is necessary to investigate the non-perturbative region of QCD well. The QCD sum rule method is an important tool of obtaining qualitative and quantitive ideas of hadronic physics from QCD inputs Cohen et al. (1995). The method was developed by Shifman, Vainshtein and Zakharov in the late 1970's to investigate the properties of mesons Shifman et al. (1979b,a) and later it was applied to baryons by Ioffe Ioffe (1981).

In order to be able to analyze the results obtained from heavy ion collision experiments such as CERN and BNL with the help of developing accelerator technologies and to better understand the internal structure of neutron stars, it is necessary to understand the modified properties of baryons and mesons in dense medium. Collaborations such as FAIR and CBM will also investigate the in-medium properties of these particles in future experimental programs. However, in near future the PANDA Collaboration is planning to focus on the medium effects on the physical properties of charmed hadrons.

In The perspective of theoretical studies, there are plenty of studies investigating modified properties of hadrons in dense medium Hayashigaki (2000); Drukarev and Levin (1990); Adami and Brown (1991); Furnstahl et al. (1992); Jin et al. (1993, 1994); Drukarev et al. (2004); Cohen et al. (1991); Hogaasen (1991); Cohen et al. (1992); Drukarev (2003); Cohen et al. (1995); Wang and Huang (2011); Wang (2012, 2011); Jin and Leinweber (1995); Hatsuda and Lee (1992); Asakawa and Ko (1993); Klingl et al. (1997); Leupold and Mosel (1998); Hilger et al. (2009); Yasui and Sudoh (2013); Thomas et al. (2007); Mallik and Sarkar (2009); Ryskin et al. (2015); Hatsuda et al. (1991). In the studies, Cohen, generally using the Ioffe current in which the mixing parameter having value $\beta = -1$, the in-medium properties of nucleons were investigated. But, in our study, we focus on the physical properties of nucleon like mass, residue and self energy propagating in dense medium using the whole interval of the auxiliary parameter $-\infty < \beta < \infty$.

The outline of of this thesis: in chapter 2 we present the SM, QCD and nucleon in more detail. Chapter 3 is reserved for the brief discussion of the QCDSR formalism both in vacuum and in medium applications. Chapter 4 contains our numerical analyzes for the physical properties of nucleons in cold nuclear medium. Finally, chapter 4 is devoted for the concluding remark of our study.

2. STANDARD MODEL AND NUCLEONS

2.1 Standard Model

The high energy physics or particle physics studies the elementary particles and their interactions between them. Elementary particles are constituents of the matter in the universe. Many of the elementary particles do not exist under normal conditions. In particle accelerators, elementary particles are created and detected by the collisions of high energetic particles. The Standard Model (SM) of the particle physics is a quantum field theory and in which all known elementary particles in the universe are classified.

In physics, there are four fundamental interactions which do not reducible to more basic forms: the gravitational, the electromagnetic, the strong and the weak interaction. The concept of field is used to describe mathematically their interactions. Three of these interactions except the gravitational interaction are used in the standard model. But the main purpose of the theoretical physics is to describe all fundamental interactions in a simple and unified theory.

The classification of the elementary particles and these fundamental interactions in SM are presented in Figure 1. Fermions are building blocks of the physical matter, have with half-integer spin. Quarks and leptons are two subgroups of fermions. Quarks carry color charge, have spin-$1/2$ and participate in strong interactions. Leptons are colorless, have spin $1/2$ spin and interact via electroweak interactions. Quarks and leptons are collected in three different generations: (u) and (d) quarks with e and ν_e are in the first generation, (c) and (s) quarks with μ and ν_μ are in second generation and (t) and (b) quarks with τ and ν_τ are in third generation. Bosons are the force carrier of the interactions, have integer spin and obey the Bose-Einstein statistics. Gluons which have eight types and force carrier of strong interaction, photons interacting via electromagnetic interaction and the W, Z bosons as a force carrier of weak interactions are spin-1 Gauge bosons and Higgs boson has spin-0.

The standard model has been proved with a great accuracy in the predictions of the interactions of quarks and leptons. But the present standard model has some weaknesses. For example, why there are only three generations of quarks and leptons can not be explained, for the masses of these particles and the strength of the various interactions it has no predictions, despite the gluons are massless the model can not explain why the strong interaction has a limited range. The quarks and gluons are not observed as free particles

2.2 Quantum Chromodynamics (QCD)

QCD is the theory of the strong force colored quarks and gluons. The color is a property which is analog of electric charge. The QCD is a non-abelian gauge theory, with symmetry group SU(3). The Lagrangian of QCD is given by

$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \sum_q \bar{\psi}_q(i\not{D} - m_q)\psi_q \tag{2.1}$$

where $G^a_{\mu\nu}$ is the gluon field-strength tensor, ψ_q are the quark field spinors with $q = u, d, c, s, t, b$ quark flavor, m_q is the quark mass and $\not{D}$ is the covariant derivative.

The characteristic properties of QCD:

- **Asymptotic Freedom:** It was discovered by David Gross and Frank Wilczek (working together) Gross and Wilczek (1973) in 1973 and all three are rewarded Nobel Prize in Physics in 2004. They found that strong force among quarks and gluons gets asymptotically weaker as the energy scale or the momentum transfer increases (in the ranges 100 $GeV - TeV$) and the corresponding distances decreases. As a result of effect, the quarks move almost as a free particles. Due to the asymptotic freedom, at high energies or short distances, perturbation theories are applicable.
- **Color Confinement**: Colored quarks can not be isolated which means that the strong force between them does not decreases as the distance increases. The effective strong coupling constant becomes larger and hence perturbation theory does not work. As a result, non-perturbative approaches are used in large distances to describe the non-perturbative effects.

2.3 Nucleons

Hadron is the general name of baryon and meson. The quarks and gluons are not observed as free particles. Hadrons are color neutral ($i.e.$ color-singlet) which means that they exit as combinations of quarks, anti-quarks and gluons. The baryons are heavier particles and consist of odd numbers of quarks, generally three quarks and the mesons are made of even number of quarks, mostly one quark and one anti-quark pair. In the baryon family, a proton and a neutron has a common name called as nucleon. They are fermion, i.e. half-integer spin particles, and composite particles which made of up (u) and down (d) quarks together by the strong interaction. Quark content of the proton is uud and the neutron is udd. The neutron is about 0.13% heavier than the proton which is the lightest baryon. In nature, although a free proton is a stable particle, a free neutron is unstable and decay into a proton via β^- radioactive decay in a half life of ten minutes. Understanding the properties of nucleons is important in this respect that particle physics and nuclear physics overlap at the boundary where nucleons exist.

3. QCDSR FORMALISM

QCDSR approach is an useful way to getting valuable information about the properties of hadrons like mass and residue from QCD parameters. It was formulated and applied to describe mesonic properties (Shifman et al., 1979b). Then, Ioffe showed how this method could be used to investigate baryons Ioffe (1981). In the application of the QCDSR method, hadrons are depicted by their interpolating currents. To derive QCDSR for the properties of the hadrons, as a starting point the correlation function is mentioned as a function of these interpolating currents. The main concept in this approach is "duality", that construct a connection between a characterization in terms of physical or hadronic degrees of freedom and QCD or operator product expansion (OPE) side based on the quark and gluon degrees of freedom. There are three main parts in QCD sum rules calculations:

- The correlation function is calculated in terms of QCD degrees of freedom via OPE. In this side, the short and long distance quark and qluon interactions are dissociated. The QCD perturbation theory is applied for the calculation of the short term effects and the long distance interactions or non-perturbative effects are parameterized in terms of the vacuum or in-medium quark, gluon and mixed condensates.
- In the physical side, the correlation function is obtained in hadronic language by inserting a complete set of hadronic state with the same quantum numbers as the interpolating current.
- These two descriptions of the approach are matched via dispersion relation and the sum rules for physical quantities is obtained. The sum rules calculated by this way let us to calculate the physical quantities of interest.

3.1 Two-point correlation function

To obtain the vacuum sum rules for the mass and residue of the hadrons, the two-point CF is used as a starting point in the following way:

$$\Pi(p^2) = i \int d^4x e^{ip\cdot x} \langle 0|\mathcal{T}[J(x)\bar{J}(0)]|0\rangle, \tag{3.1}$$

where p is the four momentum of the hadron, $|0\rangle$ is the physical non-perturbative vacuum state. T is the time ordering operator. $J(x)$ is the interpolating current The interpolating current of each particle can create the particle of interest from the vacuum with the same quantum numbers as the interpolating field. The interpolating currents for mesons are:

$$J(x) = \bar{q}_i(x)\Gamma q_j(x), \tag{3.2}$$

and for baryons are

$$J(x) = \epsilon_{ijk}\Big[q_i^a(x)\Gamma_1 q_j^b(x)\Gamma_2 q_k^c(x)\Big], \tag{3.3}$$

where i, j, k represent quark flavors. ϵ_{ijk} is Levi-Civita tensor. a, b, c are colors. $\Gamma = 1, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu}$ are the Dirac matrices. As an example, the interpolating current for the vector meson is,

$$J_\mu^V(x) = \bar{q}_1(x)\gamma_\mu q_2(x), \tag{3.4}$$

and for the spin-$3/2$ light (decuplet) baryons is

$$J_\mu = A\Bigg\{\Big[q_1^{aT}C\gamma_\mu q_2^b\Big]q_3^c + \Big[q_2^{aT}C\gamma_\mu q_3^b\Big]q_1^c + \Big[q_3^{aT}C\gamma_\mu q_1^b\Big]q_2^c\Bigg\}, \tag{3.5}$$

where A is constant, C is charge operator. T is used for the transposition.

3.2 Phenomenological and QCD sides of the correlation function

In the phenomenological or hadronic side, the correlation function in Eq. (3.1) is calculated in terms of hadronic parameters. The complete set of hadronic state with the same quantum numbers as the interpolating currents is inserted into the correlation function,

$$1 = |0\rangle\langle 0| + \sum_h \int \frac{d^4p_h}{(2\pi)^4} 2\pi\delta(p_h^2 - m_h^2)|h(p_h, s)\rangle\langle h(p_h, s)| + \text{higher states}, \tag{3.6}$$

where $|h(p_h, s)\rangle$ is the hadronic state with momentum p_h and spin s and m_h is the hadron mass. After the summation over all hadronic states $|h(p_h, s)\rangle$, the phenomenological representation of the correlation function is

$$\begin{aligned} 2\text{Im}\Pi(p^2) &= \sum_h \int \langle 0|J(0)|h(p_h, s)\rangle\langle h(p_h, s)|\bar{J}(0)|0\rangle d\tau_h (2\pi)^4\delta(p - p_h) \\ &= 2\pi f_h^2\delta(p^2 - m_h^2) + 2\pi\rho^h(p^2), \end{aligned} \tag{3.7}$$

where $\rho^h(p^2)$ comes from the higher states and contributions and f_h is the leptonic decay constant for mesons and residue for baryons of the ground state hadron and $d\tau_h$ is the volume element of the integration. The imaginary part of the correlation function is called as the spectral density, $\rho^h(s) = \mathrm{Im}\Pi(s)/\pi$. Therefore, the phenomenological side of the correlation function is

$$\Pi(p^2) = -\frac{f_h^2}{p^2 - m_h^2} + \int_{s_0^h}^{\infty} ds \frac{\rho^h(s)}{s - p^2} + \text{subtraction terms}, \tag{3.8}$$

where s_0^h is the continuum threshold.

The statement of the OPE for small values of x is that in the correlation function at different points of space time, the time order product of two currents are expanded:

$$\mathcal{T}[J(x)\bar{J}(0)] = \sum_i C_i(x^2)O_i, \quad i = 0, 1, 2, ..., \tag{3.9}$$

where $C_i(x^2)$ are Wilson coefficients calculated in QCD perturbative theory and O_i are the local operators ordered in dimension. The unit operator for $i = 0$ is the lowest-dimension operator which contains the perturbative contribution. Since the QCD vacuum is colorless, there is no colorless operator for the dimensions $i = 2, 3$. The operators are in higher dimensions in terms of quark field ψ, gluon field $G^a_{\mu\nu}$ and their mixture are

$$\begin{aligned} O_3 &= \bar{\psi}\psi, \\ O_4 &= G^a_{\mu\nu}G^{\mu\nu a}, \\ O_5 &= \bar{\psi}\sigma_{\mu\nu}\frac{\lambda^a}{2}G_{\mu\nu a}\psi, \\ O_6^{\psi} &= (\bar{\psi}\Gamma_r\psi)(\bar{\psi}\Gamma_s\psi), \\ O_6^{G} &= f_{abc}G^a_{\mu\nu}G^{b\nu}_{\sigma}G^{c\sigma\mu}, \\ &..., \end{aligned} \tag{3.10}$$

where λ^a are the linearly independent, traceless and Hermitian Gell-Mann color matrices, $\sigma_{\mu\nu} = \frac{1}{2}i[\gamma_\mu, \gamma_\nu]$ are the complex, Hermitian and unitary Pauli matrices in terms of Gamma matrices. The $\Gamma_{r,s}$ represents the different combinations of Lorentz and color matrices. In terms of OPE, the correlation function has the form

$$\Pi(p^2) = \sum_i C_i(x^2)\langle O_i\rangle, \tag{3.11}$$

where $\langle O_i\rangle = \langle 0|O_i|0\rangle$ are the vacuum expectation values of QCD operators or the condensates which parametrize the non-perturbative effects. In Eq. (3.10), $\langle O_3\rangle$ is the quark condensate, $\langle O_4\rangle$ is the gluon condensate, $\langle O_5\rangle$ is the mixed condensate and $\langle O_6^{\psi,G}\rangle$ is the four-quark and three-gluon condensate. The spectral density of the OPE side is $\rho^{\mathrm{OPE}}(p^2) = \frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{OPE}}(p^2)$ and the correlation function of it can be written as

$$\Pi(p^2) = \int_0^{\infty} ds \frac{\rho^{\mathrm{OPE}}(s)}{s - p^2} + \text{subtraction terms}. \tag{3.12}$$

3.3 Borel Transformation and QCD sum rules

Borel transformation is a standard mathematical method used to improve the radius of convergence of any function $f(Q^2)$. It is defined by the following formula

$$\mathcal{B}_{M^2}[f(Q^2)] = \lim_{\substack{Q^2, n \to \infty \\ Q^2/n = M^2}} \frac{(Q^2)^{n+1}}{n!} \left(\frac{-d}{dQ^2} \right)^n f(Q^2), \tag{3.13}$$

and well known formulas which are used in some QCD sum rules applications

$$\begin{aligned}
\mathcal{B}_{M^2}[(Q^2)^k] &= 0 \text{ for } k \geqslant 0, \\
\mathcal{B}_{M^2}\left[\frac{1}{(Q^2)^k}\right] &= \frac{1}{(k-1)!}\left(\frac{1}{M^2}\right)^{k-1}, \\
\mathcal{B}_{M^2}\left[\frac{1}{s+Q^2}\right] &= \exp^{-s/M^2}, \\
\mathcal{B}_{M^2}[(Q^2)^k \log(Q^2/\Lambda^2)] &= k!(-M^2)^{k+1}.
\end{aligned} \tag{3.14}$$

In the QCD sum rules method applications, after equating the hadronic and QCD side of the correlation function to suppress the contributions the higher states and continuum, the Borel transformation is applied to both sides of the equality. After Borel transformation the equated sides of the correlation function can be written as:

$$f_h^2 e^{-m_h^2/M^2} + \int_{s_0^h}^{\infty} ds \rho^h(s) e^{-s/M^2} = \int_0^{s_0} ds \rho^{\text{OPE}}(s) e^{-s/M^2} + \int_{s_0}^{\infty} ds \rho^{\text{OPE}}(s) e^{-s/M^2}, \tag{3.15}$$

where s_0 is the continuum threshold. According to the local quark hadron quality assumption in the parametrization of the contribution of higher states and continuum, The second terms on both sides of the Eq. (3.15) are taken equal to each other and ultimately cancel each other. Thus the desired sum rules have been obtained for the physical quantities

$$f_h^2 e^{-m_h^2/M^2} = \int_{s_0}^{\infty} ds \rho^{\text{OPE}}(s) e^{-s/M^2}, \tag{3.16}$$

where the Borel mass M^2. s_0 parameters are not arbitrary parameters, the physical observables show minimum dependency on them. The details how these parameters are fixed will be discussed in the next section.

4. QCD SUM RULES FOR THE MASS, RESIDUE AND SELF-ENERGY OF NUCLEON IN COLD NUCLEAR MATTER

4.1 Finite density correlation function

In this study, we calculate the behavior of the nucleons in cold nuclear medium, unlike in vacuum. To do this, we will investigate how the mass and the residue are affected by the medium and calculate the scalar and vector self-energies. These physical quantities can be extracted from the analysis of the in-medium two point correlation function,

$$\Pi(p^2) = i \int d^4x e^{ip\cdot x} \langle \psi_0 | \mathcal{T}[J(x)\bar{J}(0)] | \psi_0 \rangle, \tag{4.1}$$

where p is the four momentum of the nucleon, $|\psi_0\rangle$ is the ground state of the nucleon defined by the rest frame nucleon density ρ and by the four velocity u_μ of the medium. A colorless interpolating current $J(x)$ is made up of quark fields with the same quantum numbers of a nucleon. The interpolating currents in QCDSR method has an analogy in the role of wave functions in the QM. For the proton with the quark content uud, the interpolating current is,

$$J(x) = 2\epsilon_{abc}\Big\{\Big[u^{\mathrm{T},a}(x)Cd^b(x)\Big]\gamma_5 u^c(x) + \Big[u^{\mathrm{T},a}(x)C\gamma_5 d^b(x)\Big]\beta u^c(x)\Big\}, \tag{4.2}$$

and for the neutron with ddu is

$$J(x) = 2\epsilon_{abc}\Big\{\Big[d^{\mathrm{T},a}(x)Cu^b(x)\Big]\gamma_5 d^c(x) + \Big[d^{\mathrm{T},a}(x)C\gamma_5 u^b(x)\Big]\beta d^c(x)\Big\}. \tag{4.3}$$

In these equations a, b, c are color indices of the quark field, T denotes a transpose in Dirac space. The interpolating current with $\beta = -1$ known as to Ioffe current (Ioffe (1981); Drukarev et al. (2015); Thomas et al. (2007); Leinweber (1995); Stein et al. (1995)).

In Dirac space, the correlation function $\Pi(q^2)$ is a 4×4 square matrix, so it can be expanded using Dirac matrices as follow Cohen et al. (1995),

$$\begin{aligned}\Pi(p^2) &= \Pi_S + \Pi_p \not{p} + \Pi_u \not{u} + \Pi_1 \gamma^5 + \Pi_2 \not{p}\gamma^5 + \Pi_2 \not{p}\gamma^5 + \Pi_3 \not{u}\gamma^5 + \Pi_4(p_\mu u_\nu - p_\nu u_\mu) \\ &+ \Pi_5 \epsilon_{\mu\nu\kappa\lambda} p^\kappa u^\lambda \sigma^{\mu\nu},\end{aligned} \tag{4.4}$$

where the coefficients of each structure, i. e. $\Pi_{S,p,u,1,\ldots,5}$, are scalar functions of the invariants p^2 and $p \cdot u$. The Lorentz covariance, parity and time reversal in the rest frame of nuclear matter imply that Π_i's for i=1, 2, 3, 4, 5 vanish. As a result, the correlation function has three different structures as a function of the scalars p^2 and $p \cdot u$ in the following form,

$$\Pi(p^2) = \Pi_S + \Pi_p \not{p} + \Pi_u \not{u}. \tag{4.5}$$

In the vacuum or zero density limit, the coefficients $\Pi_u \rightarrow 0$ and Π_S and Π_p are only function of p^2.

4.2 Physical Ansatz

In the physical or hadronic side, the correlation function is saturated by a complete set of nucleon states with the same quantum numbers as the interpolating current and performing in Eq. (4.1) integral over x , we get the required expression for the physical side of the correlation function as

$$\Pi^{\text{Phys}}(p^2) = -\frac{\langle\psi_0|J(x)|n(p^*,s)\rangle\langle n(p^*,s)|\bar{J}(0)|\psi_0\rangle}{p^{*2} - m_n^{*2}} + ..., \tag{4.6}$$

where $|n(p^*,s)\rangle$ is the nucleon ground state, p^* and m_n^* are the nucleon in-medium modified momentum and mass, respectively and the dots stand for contributions of the higher resonances and continuum states. We define the coupling of the interpolating field to the physical quasi-nucleon state as the in-medium residue, λ_n^*, given by the equality

$$\langle\psi_0|J(x)|n(p^*,s)\rangle = \lambda_n^* u(p^*,s), \tag{4.7}$$

where $u(p^*,s)$ is the positive energy Dirac spinor with s stands for the spin. After inserting the parametrization in Eq. (4.7) into the Eq. (4.6), we get

$$\Pi^{\text{Phys}}(p^2) = \frac{\lambda_n^{*2}(\not{p}^* + m_n^*)}{p^{*2} - m_n^{*2}} + ... = \frac{\lambda_n^{*2}(\not{p}^* + m_n^*)}{(\not{p}^* + m_n^*)(\not{p}^* - m_n^*)} + ... = \frac{\lambda_n^{*2}}{(\not{p}^* - m_n^*)} + ..., \tag{4.8}$$

where $\not{p}^* = p^{*\mu}\gamma_\mu$. The in medium modified momentum and mass are parametrized in terms of the in-medium self energies, respectively

$$p^{*\mu} = p^\mu - \tilde{\Sigma}_v^\mu, \tag{4.9}$$

$$m_n^* = m_n + \Sigma_s, \tag{4.10}$$

where $\tilde{\Sigma}_v^\mu$ is the vector self-energy and Σ_s is the scalar self-energy. The vector self-energy can be written as

$$\tilde{\Sigma}_v^\mu = \Sigma_v u^\mu + \Sigma_v' p^\mu \tag{4.11}$$

$$\simeq \Sigma_v u^\mu. \tag{4.12}$$

Because of the small value of Σ'_v, the second term in Eq. (4.11) can be ignorable Cohen et al. (1991, 1995). Therefore, Eq. (4.8) can be written as

$$\Pi^{\text{Phys}}(p^2) = \frac{\lambda_n^{*2}}{(p^\mu - \Sigma_v u^\mu)\gamma_\mu - (m_n^* - \Sigma_s)} + ..., \tag{4.13}$$

The physical side of the correlation function, Eq. (4.6), can be written in terms of the structures $\not{p}$, $\not{u}$ and S as follows

$$\Pi^{\text{Phys}}(p^2) = \Pi_p^{\text{Phys}}(p^2, p_0)\not{p} + \Pi_u^{\text{Phys}}(p^2, p_0)\not{u} + \Pi_S^{\text{Phys}}(p^2, p_0)I + ..., \tag{4.14}$$

where $p_0 = p \cdot u$. I refers to unit matrix. The coefficients of the each structures are,

$$\begin{aligned} \Pi_p^{\text{Phys}}(p^2, p_0) &= -\frac{\lambda_n^{*2}}{p^2 - \kappa^2}, \\ \Pi_u^{\text{Phys}}(p^2, p_0) &= +\frac{\lambda_n^{*2}\Sigma_v}{p^2 - \kappa^2}, \\ \Pi_S^{\text{Phys}}(p^2, p_0) &= -\frac{\lambda_n^{*2} m_n^*}{p^2 - \kappa^2}, \end{aligned} \tag{4.15}$$

here $\kappa = \sqrt{m_n^{*2} - \Sigma_v^2 + 2p_0\Sigma_v}$. As a result, for the physical side of the correlation function, we obtain

$$\begin{aligned} \hat{\mathcal{B}}\Pi_p^{\text{Phys}}(p^2, p_0) &= -\lambda_n^{*2} e^{-\kappa^2/M^2}, \\ \hat{\mathcal{B}}\Pi_u^{\text{Phys}}(p^2, p_0) &= +\lambda_n^{*2}\Sigma_v e^{-\kappa^2/M^2}, \\ \hat{\mathcal{B}}\Pi_S^{\text{Phys}}(p^2, p_0) &= -\lambda_n^{*2} m_n^* e^{-\kappa^2/M^2}. \end{aligned} \tag{4.16}$$

where M^2 is the Borel mass.

4.3 QCD Ansatz

The QCD side of the sum rules $\Pi^{\text{QCD}}(p^2)$ can be obtained in deep Euclidean space $p^2 \to -\infty$. After inserting the explicit form of the interpolating current $J(x)$ in the correlation function of Eq. (4.1) and by contracting out all pairs via Wick's theorem, we have an expression for the sum rules $\Pi^{\rm QCD}(p^2)$,

$$\begin{aligned}\Pi^{QCD}(p^2) = &- 4i\epsilon_{abc}\epsilon_{a'b'c'}\int d^4x e^{ipx}\Big\langle\psi_0\Big|\Big\{\Big(\gamma_5 S_u^{cb'}(x)\tilde{S}_d^{ba'}(x)S_u^{ac'}(x)\gamma_5 \\ &- \gamma_5 S_u^{cc'}(x)\gamma_5 Tr\Big[S_u^{ab'}(x)\tilde{S}_d^{ba'}(x)\Big]\Big) + \beta\Big(\gamma_5 S_u^{cb'}(x)\gamma_5\tilde{S}_d^{ba'}(x)S_u^{ac'}(x) \\ &+ S_u^{cb'}(x)\tilde{S}_d^{ba'}(x)\gamma_5 S_u^{ac'}(x)\gamma_5 - \gamma_5 S_u^{cc'}(x)Tr\Big[S_u^{ab'}(x)\gamma_5\tilde{S}_d^{ba'}(x)\Big] \\ &- S_u^{cc'}(x)\gamma_5 Tr\Big[S_u^{ab'}(x)\tilde{S}_d^{ba'}(x)\gamma_5\Big]\Big) + \beta^2\Big(S_u^{cb'}(x)\gamma_5\tilde{S}_d^{ba'}(x)\gamma_5 S_u^{ac'}(x) \\ &- S_u^{cc'}(x)Tr\Big[S_d^{ba'}(x)\gamma_5\tilde{S}_u^{ab'}(x)\gamma_5\Big]\Big)\Big\}\Big|\psi_0\Big\rangle, \end{aligned} \tag{4.17}$$

where $S^{ij}_{u(d)}(x)$ are the light-quark propagators, $\tilde{S}^{ij}_{u(d)}(x) = CS^{ij\mathrm{T}}_{u(d)}(x)C$, γ_5 is the gamma or Dirac matrix. It is best to work in the coordinate space and then to transform into momentum space. In the coordinate-space in the presence of the background fields, they have the following forms in the fixed-point gauge Reinders et al. (1985)

$$\begin{aligned} S_q^{ab}(x) &\equiv \langle\psi_0|T[q^a(x)\bar{q}^b(0)]|\psi_0\rangle_\rho \\ &= \frac{i}{2\pi^2}\delta^{ab}\frac{1}{(x^2)^2}\not{x} - \frac{m_q}{4\pi^2}\delta^{ab}\frac{1}{x^2} + \chi_q^a(x)\bar{\chi}_q^b(0) \\ &- \frac{ig_s}{32\pi^2}F_{\mu\nu}^A(0)t^{ab,A}\frac{1}{x^2}\left[\not{x}\sigma^{\mu\nu} + \sigma^{\mu\nu}\not{x}\right] + ..., \end{aligned} \tag{4.18}$$

where a, b are Lorentz indices and i, j are Dirac indices and ρ is the medium density. The first two terms in Eq. (4.18) are obtained in the expansion of the free quark propagator to first order in the quark mass. They are named as the perturbative part of the propagator. According to the background-field method, Grassmann background quark fields $\chi_q^a(x)$ and $\bar{\chi}_q^b(0)$ and a classical background gluon field $F_{\mu\nu}^A$ in Eq. (4.18) are used to parametrize non-perturbative quark and gluon condensates, respectively Reinders et al. (1985); Hubschmid and Mallik (1982); Novikov et al. (1984); Shifman (1980). The gluonic contribution to light quark propagator comes from a single gluon interaction keeping only the leading term in the short-distance expansion of the gluon field. Contributions coming from derivatives of the gluon field tensor as well as additional gluon insertion are ignored. After inserting the light quark propagators seen in Eq. (4.18) into Eq. (4.17), we obtain the prodects of Grassmann background quark fields and classical background gluon fields. They correspond to ground-state matrix elements of the ersatz quark and gluon operators:

$$\begin{aligned}\text{two-quark} &: \quad \chi^q_{a\alpha}(x)\bar{\chi}^q_{b\beta}(0) = \langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_\rho, \\ \text{two-gluon} &: \quad F^A_{\kappa\lambda}F^B_{\mu\nu} = \langle G^A_{\kappa\lambda}G^B_{\mu\nu}\rangle_\rho, \\ \text{mixed quark-gluon} &: \quad \chi^q_{a\alpha}\bar{\chi}^q_{b\beta}F^A_{\mu\nu} = \langle q_{a\alpha}\bar{q}_{b\beta}G^A_{\mu\nu}\rangle_\rho, \\ \text{four-quark} &: \quad \chi^q_{a\alpha}\bar{\chi}^q_{b\beta}\chi^q_{c\gamma}\bar{\chi}^q_{d\delta} = \langle q_{a\alpha}\bar{q}_{b\beta}q_{c\gamma}\bar{q}_{d\delta}\rangle_\rho, \end{aligned} \tag{4.19}$$

where the fields are evaluated at the point $x = 0$ unless otherwise stated Cohen et al. (1995). In the right hand side of Eqs. (4.19), the matrix elements are named as condensates. To make progress, these condensates are needed to be defined in nuclear medium.

Since nuclear matter is colorless and the ground state is assumed to be parity and time reversal invariant, the Dirac and color structure of the matrix element $\langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_\rho$ can be projected out as

$$\langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_\rho = -\frac{\delta_{ab}}{12}\Bigg[\langle\bar{q}(0)q(x)\rangle_\rho\delta_{\alpha\beta} + \langle\bar{q}(0)\gamma_\lambda q(x)\rangle_\rho\gamma^\lambda_{\alpha\beta}\Bigg], \tag{4.20}$$

and using the Taylor series expansion of the quark field $q(x)$ at short distances, we have

$$\begin{aligned}\langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_\rho = -\frac{\delta_{ab}}{12}\Bigg[&\Bigg(\langle\bar{q}q\rangle_\rho + x^\mu\langle\bar{q}D_\mu q\rangle_\rho + \frac{1}{2}x^\mu x^\nu\langle\bar{q}D_\mu D_\nu q\rangle_\rho + ...\Bigg)\delta_{\alpha\beta} \\ &+\Bigg(\langle\bar{q}\gamma_\lambda q\rangle_\rho + x^\mu\langle\bar{q}\gamma_\lambda D_\mu q\rangle_\rho + \frac{1}{2}x^\mu x^\nu\langle\bar{q}\gamma_\lambda D_\mu D_\nu q\rangle_\rho + ...\Bigg)\gamma^\lambda_{\alpha\beta}\Bigg],\end{aligned} \tag{4.21}$$

where all fields and their derivatives in the above condensates are evaluated at $x = 0$ point Cohen et al. (1995). To proceed, we need to evaluate the condensates seen in Eq. (4.21). In the vacuum, these condensates can only be denominated using the metric tensor $g_{\mu\nu}$ and the anti-symmetric metric tensor $\varepsilon_{\kappa\lambda\mu\nu}$. On the other hand, in-medium condensates can also be expressed in terms of the four velocity of the medium, u_μ which causes to exit new condensates and new Lorentz structures. The various in-medium condensates appearing in

Eq. (4.21) can be expressed as:

$$
\begin{aligned}
\langle\bar{q}\gamma_\mu q\rangle_\rho &= \langle\bar{q}\,\not{u}q\rangle_\rho u_\mu, \\
\langle\bar{q}D_\mu q\rangle_\rho &= \langle\bar{q}u\cdot Dq\rangle_\rho u_\mu = -im_q\langle\bar{q}\,\not{u}q\rangle_\rho u_\mu, \\
\langle\bar{q}\gamma_\mu D_\nu q\rangle_\rho &= \frac{4}{3}\langle\bar{q}\,\not{u}u\cdot Dq\rangle_\rho(u_\mu u_\nu-\frac{1}{4}g_{\mu\nu})+\frac{i}{3}m_q\langle\bar{q}q\rangle_\rho(u_\mu u_\nu-g_{\mu\nu}), \\
\langle\bar{q}D_\mu D_\nu q\rangle_\rho &= \frac{4}{3}\langle\bar{q}u\cdot Du\cdot Dq\rangle_\rho(u_\mu u_\nu-\frac{1}{4}g_{\mu\nu})-\frac{1}{6}\langle g_s\bar{q}\sigma\cdot Gq\rangle_\rho(u_\mu u_\nu-g_{\mu\nu}), \\
\langle\bar{q}\gamma_\lambda D_\mu D_\nu q\rangle_\rho &= 2\langle\bar{q}\,\not{u}u\cdot Du\cdot Dq\rangle_\rho\left[u_\lambda u_\mu u_\nu-\frac{1}{6}(u_\lambda g_{\mu\nu}+u_\mu g_{\lambda\nu}+u_\nu g_{\lambda\mu})\right] \\
&\quad -\frac{1}{6}\langle g_s\bar{q}\,\not{u}\sigma\cdot Gq\rangle_\rho(u_\lambda u_\mu u_\nu-u_\lambda g_{\mu\nu}),
\end{aligned}
\tag{4.22}
$$

where the equations of motion have been used and because of their small contributions, $O(m_q^2)$ terms have been ignored. In his way, the expansion of the condensate $\langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_\rho$ up to dimension five contains quark condensates and quark-gluon condensates.

Not only $\langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_\rho$ is the source of the quark-gluon condensates, another one comes from the contributions of the form $\chi^q_{a\alpha}\bar{\chi}^q_{b\beta}F^A_{\mu\nu}$ in the light quark propagator. The corresponding quark-gluon matrix element can be decomposed as:

$$
\begin{aligned}
\langle g_s q_{a\alpha}\bar{q}_{b\beta}G^A_{\mu\nu}\rangle_\rho = &- \frac{t^A_{ab}}{96}\Bigg\{\langle g_s\bar{q}\sigma\cdot Gq\rangle_\rho\left[\sigma_{\mu\nu}+i(u_\mu\gamma_\nu-u_\nu\gamma_\mu)\,\not{u}\right]_{\alpha\beta} \\
&+ \langle g_s\bar{q}\,\not{u}\sigma\cdot Gq\rangle_\rho\left[\sigma_{\mu\nu}\,\not{u}+i(u_\mu\gamma_\nu-u_\nu\gamma_\mu)\right]_{\alpha\beta} \\
&- 4\Big(\langle\bar{q}u\cdot Du\cdot Dq\rangle_\rho+im_q\langle\bar{q}\,\not{u}u\cdot Dq\rangle_\rho\Big) \\
&\times \left[\sigma_{\mu\nu}+2i(u_\mu\gamma_\nu-u_\nu\gamma_\mu)\,\not{u}\right]_{\alpha\beta}\Bigg\},
\end{aligned}
\tag{4.23}
$$

where t^A_{ab} are Gell-Mann matrices and $D_\mu = \frac{1}{2}(\gamma_\mu\,\not{D}+\,\not{D}\gamma_\mu)$. The matrix element in Eq. (4.23) is calculated by projecting out color, Dirac and Lorentz structure by taking suitable traces.

In the light quark propagator (Eq. (4.18)), the dimension-four gluon condensates comes from factors of $F^A_{\kappa\lambda}F^B_{\mu\nu}$ in which the matrix element $\langle G^A_{\kappa\lambda}G^B_{\mu\nu}\rangle_\rho$ can also be written as

$$
\langle G^A_{\kappa\lambda}G^B_{\mu\nu}\rangle_\rho = \frac{\delta^{AB}}{96}\left[\langle G^2\rangle_\rho(g_{\kappa\mu}g_{\lambda\nu}-g_{\kappa\nu}g_{\lambda\mu})+O(\langle\mathbf{E}^2+\mathbf{B}^2\rangle_\rho)\right], \tag{4.24}
$$

where **E** and **B** are the color-electric and color magnetic fields, respectively. The contributions coming from $O(\langle\mathbf{E}^2+\mathbf{B}^2\rangle_\rho$ are neglected because of small contributions in further calculations. For six-dimension, the four-quark condensate is not well-known is nuclear medium. Therefore, it is factorized into two dimension-three condensates. It is considered for the same quarks as follow,

$$\langle \bar{u}_{a\alpha} u_{b\beta} \bar{u}_{c\gamma} u_{d\delta} \rangle_\rho \simeq \langle \bar{u}_{a\alpha} u_{b\beta} \rangle_\rho \langle \bar{u}_{c\gamma} u_{d\delta} \rangle_\rho - \langle \bar{u}_{a\alpha} u_{d\delta} \rangle_\rho \langle \bar{u}_{c\gamma} u_{b\beta} \rangle_\rho, \tag{4.25}$$

and for the different quarks as well Cohen et al. (1995)

$$\langle \bar{u}_{a\alpha} u_{b\beta} \bar{d}_{c\gamma} d_{d\delta} \rangle_\rho \simeq \langle \bar{u}_{a\alpha} u_{b\beta} \rangle_\rho \langle \bar{d}_{c\gamma} d_{d\delta} \rangle_\rho. \tag{4.26}$$

The necessary density dependent QCDSR of nucleon can be obtained by equating the same structures in both $\Pi^{Phys}(p^2)$ and $\Pi^{QCD}(p^2)$. After inserting in-medium light quark propagator and using condensates presented for different dimensions above into Eq. (4.17), the function $\Pi^{\mathrm{QCD}}(p^2)$ can be decomposed over the Lorentz structures:

$$\Pi^{QCD}(p^2) = \Pi_p^{QCD} \not{p} + \Pi_u^{QCD} \not{u} + \Pi_S^{QCD} I, \tag{4.27}$$

where the invariant amplitude Π_n^{QCD} with $n = p, u, S$ corresponding to each structure in Eq. (4.27) can be represented as the dispersion integral,

$$\Pi_n^{QCD} = \int_0^\infty \frac{\rho_n^{QCD}(s)}{s - p^2} ds, \tag{4.28}$$

where $\rho_n^{QCD}(s) = \frac{1}{\pi}\mathrm{Im}\big[\Pi_n^{QCD}\big]$ is the two-point spectral density. Now, our main aim is to calculate these spectral densities. Eq. (4.17) is in coordinate space, the following Schwinger parametrization is applied to transform the calculations into the momentum space,

$$\frac{1}{[A^2]^n} = \frac{1}{\Gamma[n]} \int_0^\infty t^{n-1} e^{-tA^2}\, dt. \tag{4.29}$$

At this step, to take x integral, we need to go to Euclidean space by using the replacement $ip{\cdot}x \to -ip_E{\cdot}x_E$ and variable changing $x_E \to y_E - \frac{i}{2t} p_E$ and then Gaussian integration can be easily performed over p_E leading to a Dirac Delta. After making use of the replacement $p_E^2 \to -p^2$, we come back to Minkowski space to take derivative $x_\mu \to i\frac{\partial}{\partial p_\mu}$. The resultant Dirac Delta function is used to perform second four-integral. Borel transformation on the variable p^2 is applied to physical side of the correlation function and we subtract the contributions of the higher resonances and continuum states by carrying out the assumption of the quark hadron duality. The invariant amplitudes of each structures obtained after these calculations as follows:

$$
\begin{aligned}
\hat{\mathcal{B}}\Pi_p^{\rm QCD}(s_0^*,M^2) = & - \frac{1}{256\pi^4}\int_0^{s_0^*} ds e^{-s/M^2} s^2 \Big[5+\beta(2+5\beta)\Big] \\
& + \frac{1}{72\pi^2}\int_0^{s_0} ds e^{-s/M^2}\Bigg\{ p_0\Big[15\langle q^\dagger q\rangle_\rho + 3\beta(2 \\
& + 5\beta)\langle q^\dagger q\rangle_\rho\Big] - 8\Big[5+\beta(2+5\beta)\Big] m_q \langle\bar{q}q\rangle_\rho \\
& + +9(-1+\beta)\Big[3(1+\beta)m_d + 2m_u + 4\beta m_u\Big]\langle\bar{q}q\rangle_\rho \\
& + 5\Big[5+\beta(2+5\beta)\Big]\langle q^\dagger i D_0 q\rangle_\rho \Bigg\} \\
& - \frac{\langle g_s^2 G^2\rangle_\rho}{1024\pi^4}\int_0^{s_0} ds e^{-s/M^2}(6+\beta+5\beta^2) \\
& + \frac{1}{192 M^2\pi^2}\Bigg\{(-1+\beta)\Bigg[-\Big(40(1+\beta)m_d \\
& + (26+43\beta)m_u\Big)M^2 + 8\Big(3(1+\beta)m_d \\
& + 2m_u + 4\beta m_u\Big)p_0^2\Bigg]\Bigg\}\langle\bar{q}g_s\sigma G q\rangle_\rho \\
& + \frac{p_0}{576 M^2\pi^2}\Bigg\{-3\Big(1+3\beta(2+\beta)\Big)M^2 \\
& + 8\Big(5+\beta(2+5\beta)\Big)p_0^2\Bigg\}\langle q^\dagger g_s\sigma G q\rangle_\rho \\
& - \frac{1}{48 M^2\pi^2}\Bigg\{(-1+\beta)\Bigg[(1+5\beta)m_u M^2 \\
& - 32(1+2\beta)m_u p_0^2 - 4(1+\beta)m_d \\
& \times (M^2+12p_0^2)\Bigg]\Bigg\}\langle\bar{q} i D_0 i D_0 q\rangle_\rho \\
& - \frac{p_0}{12 M^2\pi^2}\Bigg\{\Bigg[5+\beta(2+5\beta)\Bigg](M^2-2p_0^2)\Bigg\}\langle q^\dagger i D_0 i D_0 q\rangle_\rho \\
& - \frac{1}{144\pi^2}\Bigg\{\Bigg[3(\beta-1)m_q\Big(4(1+\beta)m_d \\
& - (1+5\beta)m_u\Big) + 16\Big(5+\beta(2+5\beta)\Big)p_0^2\Bigg]\Bigg\}\langle q^\dagger i D_0 q\rangle_\rho \\
& + \frac{1}{36\pi^2}\Bigg\{\Bigg[5+\beta(2+5\beta)\Bigg]m_q p_0^2\Bigg\}\langle\bar{q}q\rangle_\rho \\
& - \frac{p_0}{4\pi^2}\Bigg\{(\beta-1)m_q\Bigg[3(1+\beta)m_d + (2+4\beta)m_u\Bigg]\Bigg\}\langle q^\dagger q\rangle_\rho .
\end{aligned}
\tag{4.30}
$$

$$
\begin{aligned}
\hat{\mathcal{B}}\Pi_u^{\mathrm{QCD}}(s_0^*, M^2) = \; & + \; \frac{1}{72\pi^2}\int_0^{s_0^*} ds e^{-s/M^2}\Bigg\{-3\Big(5+\beta(2+5\beta)\Big)\langle q^\dagger g_s \sigma G q\rangle_\rho \\
& - \; 9(-1+\beta)m_q\Big[3(1+\beta)m_d + 2m_u(1+2\beta)\Big]\langle q^\dagger q\rangle_\rho \\
& + \; 3s\langle q^\dagger q\rangle_\rho\Bigg\} + \frac{1}{128\pi^2}\int_0^{s_0} ds e^{-s/M^2} 5(1+\beta^2) \\
& \times \; \langle q^\dagger g_s \sigma G q\rangle_\rho + \frac{1}{24\pi^2}\Bigg[5+\beta(2+5\beta)\Bigg] p_0^2 \langle q^\dagger g_s \sigma G q\rangle_\rho \\
& + \; \frac{1}{2\pi^2}\Bigg[5+\beta(2+5\beta)\Bigg] p_0^2 \langle q^\dagger i D_0 i D_0 q\rangle_\rho \\
& + \; \frac{p_0}{72\pi^2}\int_0^{s_0^*} ds e^{-s/M^2}\Bigg[5\Big(5+\beta(2+5\beta)\Big)m_q\langle \bar{q} q\rangle_\rho \\
& + \; 2\Big(5+\beta(2+5\beta)\Big)(-10\langle q^\dagger i D_0 q\rangle_\rho \\
& + \; \frac{1}{96\pi^2}\Bigg\{(\beta-1)\Bigg[8(1+\beta)m_d + 3(3+7\beta)m_u\Bigg]\Bigg\}\langle \bar{q} g_s \sigma G q\rangle_\rho \\
& + \; \frac{1}{12\pi^2}\Bigg\{(\beta-1)\Bigg[8(1+\beta)m_d + 3(3+7\beta)m_u\Bigg]\Bigg\}\langle \bar{q} i D_0 i D_0 q\rangle_\rho \\
& + \; \frac{1}{12\pi^2}\Bigg\{(\beta-1)m_q\Bigg[4(1+\beta)m_d - (1+5\beta)m_u\Bigg]\Bigg\}\langle q^\dagger i D_0 q\rangle_\rho,
\end{aligned}
\tag{4.31}
$$

$$
\begin{aligned}
\hat{\mathcal{B}}\Pi_S^{\rm QCD}(s_0^*,M^2) &= -\frac{1}{64\pi^4}\int_0^{s_0^*} ds e^{-s/M^2} s^2\Bigg[(\beta-1)^2 m_d + 6(\beta^2-1)m_u\Bigg] \\
&- \frac{1}{32\pi^2}(\beta-1)\int_0^{s_0^*} ds e^{-s/M^2}\Bigg\{\Big((5+7\beta)\langle\bar{q}g_s\sigma Gq\rangle_\rho\Big) \\
&+ 4m_q\Bigg[(\beta-1)m_d + 6(\beta+1)m_u\Bigg]\langle\bar{q}q\rangle_\rho - 2(5+7\beta)s\langle\bar{q}q\rangle_\rho\Bigg\} \\
&+ \frac{\langle g_s^2G^2\rangle_\rho}{512\pi^4}(\beta-1)\int_0^{s_0} ds e^{-s/M^2}\Bigg[\beta m_d - 6(1+\beta)m_u\Bigg] \\
&+ \frac{1}{128\pi^4}(\beta-1)\beta\int_0^{s_0^*} ds e^{-s/M^2}\langle\bar{q}g_s\sigma Gq\rangle_\rho \\
&+ \frac{1}{192\pi^2}(\beta-1)(20+29\beta)p_0^2\langle\bar{q}g_s\sigma Gq\rangle_\rho \\
&- \frac{1}{24\pi^2}\Bigg[20+(9-29\beta)\beta\Bigg]p_0^2\langle\bar{q}iD_0iD_0q\rangle_\rho \\
&+ \frac{1}{12\pi^2}(\beta-1)m_q\Bigg[(\beta-1)m_d + 6(\beta+1)m_u\Bigg]p_0^2\langle\bar{q}q\rangle_\rho \\
&- \frac{p_0}{32\pi^2}(\beta-1)\int_0^{s_0^*} ds e^{-s/M^2}\Bigg\{4\Bigg[m_q(5+7\beta) + m_d(1-\beta) \\
&- 6(1+\beta)m_u\Bigg]\langle q^\dagger q\rangle_\rho\Bigg\} \\
&+ \frac{1}{192M^2\pi^2}(\beta-1)\Bigg\{3 + (8+7\beta)m_uM^2 + 48(1+\beta)m_up_0^2 \\
&+ 4m_d\Bigg[M^2(1-4\beta) + 2(\beta-1)p_0^2\Bigg]\Bigg\}\langle q^\dagger g_s\sigma Gq\rangle_\rho \\
&+ \frac{1}{4M^2\pi^2}(\beta-1)\Bigg[(\beta-1)m_d + 6(\beta+1)m_u\Bigg](M^2+2p_0^2)\langle q^\dagger iD_0iD_0q\rangle_\rho \\
&- \frac{1}{24\pi^2}(\beta-1)\Bigg[\beta m_q - 8m_d(1-\beta) + 48(1+\beta)m_u\Bigg]\langle q^\dagger iD_0q\rangle_\rho.
\end{aligned}
\tag{4.32}
$$

After matching the invariant amplitudes of different structures form the physical and QCD sides of the correlation function, we drive the sum rules for the mass, residue and self-energies of nucleon:

$$
\begin{aligned}
-\lambda_n^{*2}e^{-\kappa^2/M^2} &= \hat{\mathcal{B}}\Pi_p^{\rm QCD}(s_0^*,M^2), \\
+\lambda_n^{*2}\Sigma_v e^{-\kappa^2/M^2} &= \hat{\mathcal{B}}\Pi_u^{\rm QCD}(s_0^*,M^2), \\
-\lambda_n^{*2}m_n^* e^{-\kappa^2/M^2} &= \hat{\mathcal{B}}\Pi_S^{\rm QCD}(s_0^*,M^2).
\end{aligned}
\tag{4.33}
$$

Using the above sum rules , we derive following expressions for κ^2, the vector self-energy Σ_v, the modified in-medium mass m_n^* and residue λ_n^* of nucleon, respectively:

$$
\begin{aligned}
\kappa^2(s_0^*, M^2, \beta, \rho) &= \frac{\frac{\partial}{\partial(-1/M^2)}\Big[\hat{\mathcal{B}}\Pi_p^{\rm QCD}(s_0^*, M^2)\Big]}{\hat{\mathcal{B}}\Pi_p^{\rm QCD}(s_0^*, M^2)}, \\
\Sigma_v(s_0^*, M^2, \beta, \rho) &= -\frac{\hat{\mathcal{B}}\Pi_u^{\rm QCD}(s_0^*, M^2)}{\hat{\mathcal{B}}\Pi_p^{\rm QCD}(s_0^*, M^2)}, \\
m_n^*(s_0^*, M^2, \beta, \rho) &= \sqrt{\kappa^2 + \Sigma_v^2 - 2p_0\Sigma_v}, \\
\lambda_n^*(s_0^*, M^2, \beta, \rho) &= \sqrt{\frac{e^{-\kappa^2/M^2}}{\hat{\mathcal{B}}\Pi_p^{\rm QCD}(s_0^*, M^2)}}.
\end{aligned}
\tag{4.34}
$$

These expressions will be used in numerical calculation with the necessary numerical inputs in the following section.

4.4 Numerical analysis

In the previous subsection, the sum rules' expressions obtained for the physical quantities of nucleon contain the vacuum and in-medium expectation values of different operators which are used as an input parameters in the numerical calculations. Before taking into account the in-medium condensates, we need to estimate vacuum condensate. Just spin-0 operators can have non-vanishing vacuum expectation values, due to the Lorentz invariance of the vacuum state $|0\rangle$. Therefore, we have the following the lowest mass dimension, d, of vacuum condensates:

$$d = 3 \qquad \langle\bar{u}u\rangle_0, \langle\bar{d}d\rangle_0, \langle\bar{s}s\rangle_0, \tag{4.35}$$

$$d = 4 \qquad \Big\langle\frac{\alpha_s}{\pi}G^2\Big\rangle_0, \tag{4.36}$$

$$d = 5 \qquad \langle\bar{u}g_s\sigma Gu\rangle_0, \langle\bar{d}g_s\sigma Gd\rangle_0, \langle\bar{s}g_s\sigma Gs\rangle_0, \tag{4.37}$$

where $\sigma G = \sigma_{\mu\nu}G^{\mu\nu}$ with $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$ and $G^2 = G^A_{\mu\nu}G^{A\mu\nu}$. The notation used in above the expressions for vacuum condensate is $\langle\hat{O}\rangle_0 = \langle 0|\hat{O}|0\rangle$. The numerical values of vacuum condensates are well known for the quark and mixed condensates. For the generalization, the in-medium condensates are expanded in terms of the rest-frame nucleon density where $u_\mu = (1, 0)$. Up to first order in this expansion, we have

$$\langle\hat{O}\rangle_\rho = \langle\hat{O}\rangle_0 + \langle\hat{O}\rangle_n\rho + ..., \tag{4.38}$$

where ... represents the correction terms coming from higher order in nucleon density and the spin-averaged nucleon matrix element is

$$\langle\hat{O}\rangle_n = \int_V d^3x\Big[\langle n|\hat{O}|n\rangle - \langle 0|\hat{O}|0\rangle\Big], \tag{4.39}$$

where the state vector of a nucleon $|n\rangle$ is normalized to unity at rest over a volume V Cohen et al. (1995). To proceed, we need some numerical values of input parameters, such as: masses of quark and baryon, saturated nuclear matter density, the quark, gluon and mixed condensates of vacuum and in-medium. These input parameters are presented in Table 4.1

Table 4.1: Input parameters used in calculations Tanabashi et al. (2018); Jin et al. (1994, 1993); Cohen et al. (1995, 1992).

Parameter	Numeric Values	Unit
m_u	2.3	MeV
m_d	4.8	MeV
p_0	1	GeV
ρ^{sat}	$(0.11)^3$	GeV^3
m_q	$0.5(m_u + m_d)$	GeV
σ_N	0.045	GeV
m_0^2	$0.8\,GeV^2$	GeV^2
$\langle q^\dagger q\rangle_\rho$	$\frac{3}{2}\rho$	GeV
$\langle \bar{q}q\rangle_0$	$(-0.241)^3$	GeV^3
$\langle \bar{q}q\rangle_\rho$	$\langle \bar{q}q\rangle_0 + \frac{\sigma_N}{2m_q}\rho$	GeV^3
$\langle q^\dagger g_s\sigma Gq\rangle_\rho$	$-0.33\,GeV^2\rho_N$	GeV^5
$\langle q^\dagger iD_0 q\rangle_\rho$	$0.18\,GeV\,\rho$	GeV^4
$\langle \bar{q}iD_0 q\rangle_\rho$	$\frac{3}{2}m_q\rho \simeq 0$	GeV^4
$\langle \bar{q}g_s\sigma Gq\rangle_0$	$m_0^2\langle \bar{q}q\rangle_0$	GeV^5
$\langle \bar{q}g_s\sigma Gq\rangle_\rho$	$\langle \bar{q}g_s\sigma Gq\rangle_0 + 3\,GeV^2\rho$	GeV^5
$\langle \bar{q}iD_0iD_0q\rangle_\rho$	$0.3\,GeV^2\rho - \frac{1}{8}\langle \bar{q}g_s\sigma Gq\rangle_\rho$	GeV^5
$\langle q^\dagger iD_0iD_0q\rangle_\rho$	$0.031\,GeV^2\rho_N - \frac{1}{12}\langle q^\dagger g_s\sigma Gq\rangle_\rho$	GeV^5
$\langle \frac{\alpha_s}{\pi}G^2\rangle_0$	$(0.33 \pm 0.04)^4\,GeV^4.$	GeV^4
$\langle \frac{\alpha_s}{\pi}G^2\rangle_\rho$	$\langle \frac{\alpha_s}{\pi}G^2\rangle_0 - 0.65\,GeV\,\rho.$	GeV^4

Additionally, in the sum rules presented in Eqs. (4.33) for the physical quantities of nucleon, we need three auxiliary parameters to be fixed at this step: the in-medium continuum threshold s_0^*, the Borel mass parameter M^2 and the mixing parameter β. According to the standard prescription of QCD sum rules, the physical quantities show weak dependency on these auxiliary parameters within their working regions.

The continuum threshold s_0^* depends on the energy of the first excited state with the same quantum numbers, because it is not completely an arbitrary parameter. While the mass of the nucleon m_n is the ground state energy, the energy for the first excited state of the nucleon is $\sqrt{s_0^*} - m$. We choose the value of continuum threshold in the interval as

$$1.5 \text{ GeV} \leqslant s_0^* \leqslant 2 \text{ GeV}. \tag{4.40}$$

Our numerical results show that the physical quantities like mass and residue weakly depend on the continuum threshold in this interval. To obtain the working region of the Borel mass parameter M^2, the condition used for the upper bound of this parameter that the pole contribution is larger than the contribution of the higher states and continuum. Therefore, the following integral describing the contributions of the continuum and pole should be satisfied,

$$R = \frac{\int_0^{s_0^*} \rho(s) e^{-s/M^2}}{\int_0^{\infty} \rho(s) e^{-s/M^2}}, \tag{4.41}$$

where the restriction for the ratio is $R > 1/2$. For the lower bound condition on M^2 is that the non-perturbative contribution should be smaller than the perturbative contribution. Under these requirements, the interval for the Borel mass parameter is:

$$0.8 \ \text{GeV}^2 \leqslant M^2 \leqslant 1.2 \ \text{GeV}^2. \tag{4.42}$$

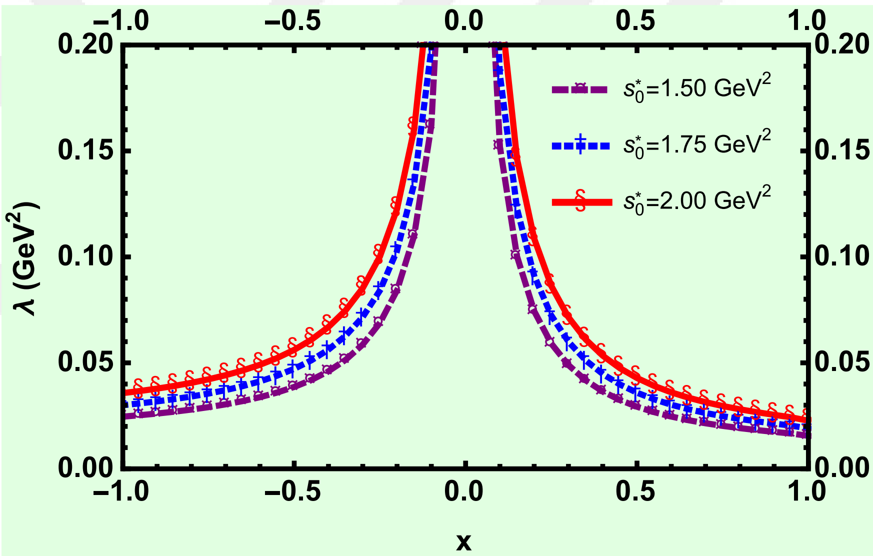

Figure 4.1: Variation of λ_n as a function of x.

And also, we need to find the working region of the mixing parameter β in which the physical quantities should be independent of this parameter. The notation $x = \cos\theta$ with $\beta = \tan\theta$ is used to explore the whole region $-\infty < \beta < +\infty$ by varying in the region $-1 \leqslant x \leqslant +1$. In Fig. 4.1 and Fig. 4.2, to obtain the reliable region of the parameter β, we plot the vacuum residue λ_n and in-medium residue λ_n^* of nucleon with respect to the parameter x for three different continuum threshold values, $s_0^* = 1.50, 1.75, 2.00$ GeV^2, and the mean value of Borel mass $M^2 = 1$ GeV^2. As seen in this figure, in the following intervals:

$$-1 \leqslant x \leqslant -0.5 \ \text{ and } \ +0.5 \leqslant x \leqslant +1, \tag{4.43}$$

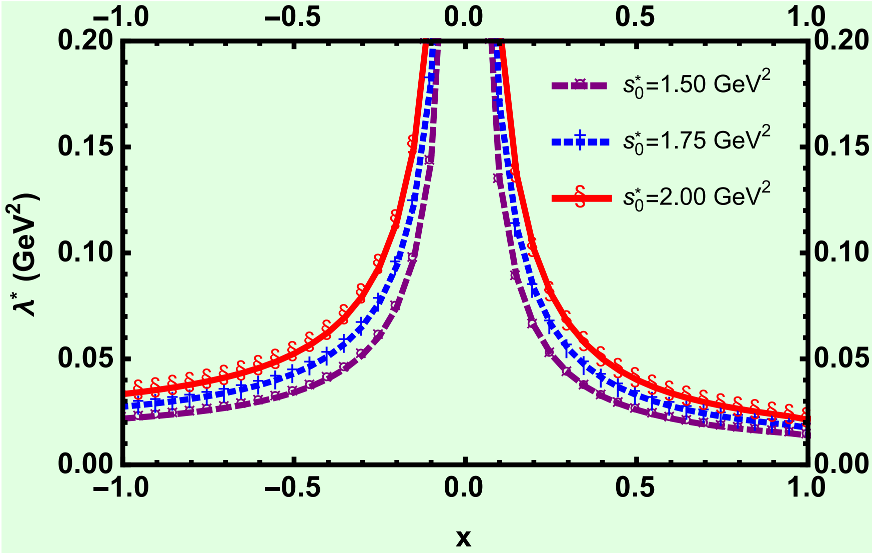

Figure 4.2: Variation of λ_n^* as a function of x.

the residues λ_n and λ_n^* show particularly stability with respect to x and the variation with due to the continuum threshold is very weak. Moreover, the current corresponding with $\beta = -1$ or $x \simeq -0.71$ named as Ioffe current is included by the above intervals.

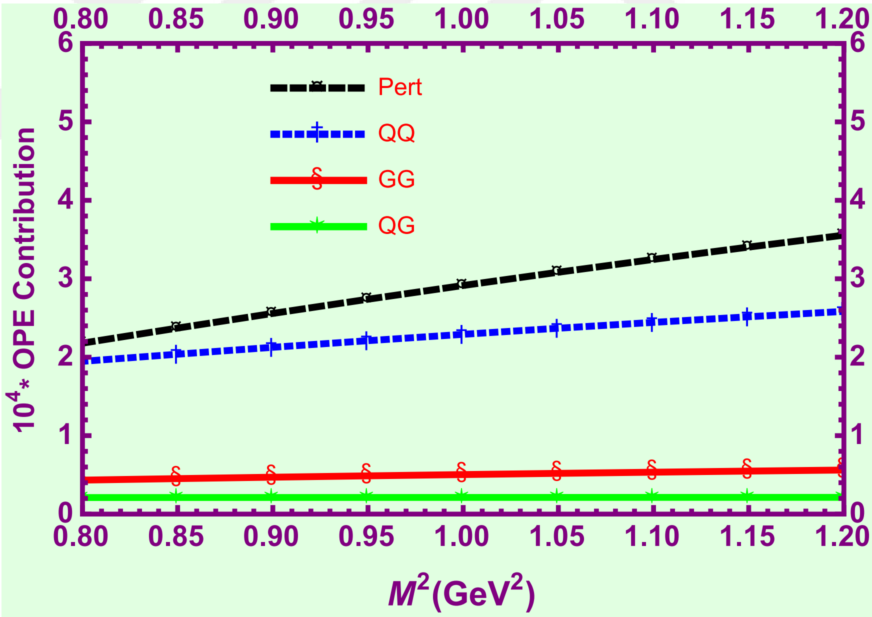

Figure 4.3: Variation of perturbative, two-quark condensate (QQ), two-gluon condensate (GG) and mixed condensate (QG) parts in OPE in terms of M^2 at saturation density and the average value of continuum threshold.

In Fig 4.3, in order to show the convergency of OPE in our calculations, we plot the variations of perturbative, two-quark condenate (QQ), two-gluon condensate (GG) and mixed condensate (QG) parts in OPE in terms of M^2 at saturation density and the average

Table 4.2: The average in-medium masses, residues and their vacuum values with the comparison of other studies.

	m_n^* (GeV)	m_n (GeV)	λ_n^{*2} (GeV^6)	λ_n^2 (GeV^6)
PS	0.743 ± 0.102	1.056±0.065	0.0010 ± 0.0004	0.0012 ± 0.0005
Ioffe (1981)	-	0.985	-	0.0012 ± 0.0006
Nasrallah and Schilcher (2014)	-	0.990 ± 0.050	-	-

value of continuum threshold for the structure $\not{p}$. We see that OPE nicely converges, i.e. the contribution of perturbative part is larger than the non-perturbative parts contributions and also contributions of decreases with increasing dimension of condensates. Note that it as is seen the contribution of mixed condensate is almost zero.

We plot the pole contribution (PC) for the $\not{p}$ structure as a function of M^2 at three fixed values of the continuum structure $s_0^* = [2.0, 2.25, 2.5]$ GeV^2 and at saturation nuclear matter density and at the parameter $x = -0.65$ in Fig 4.4. At the lower limit of the Borel parameter, we obtained PC= 0.57 and at the higher limit of it, this value decreases to PC= 0.31. Another important result of our analyses show that in the obtained working regions of auxiliary parameters, the series of sum rules have a nice convergency.

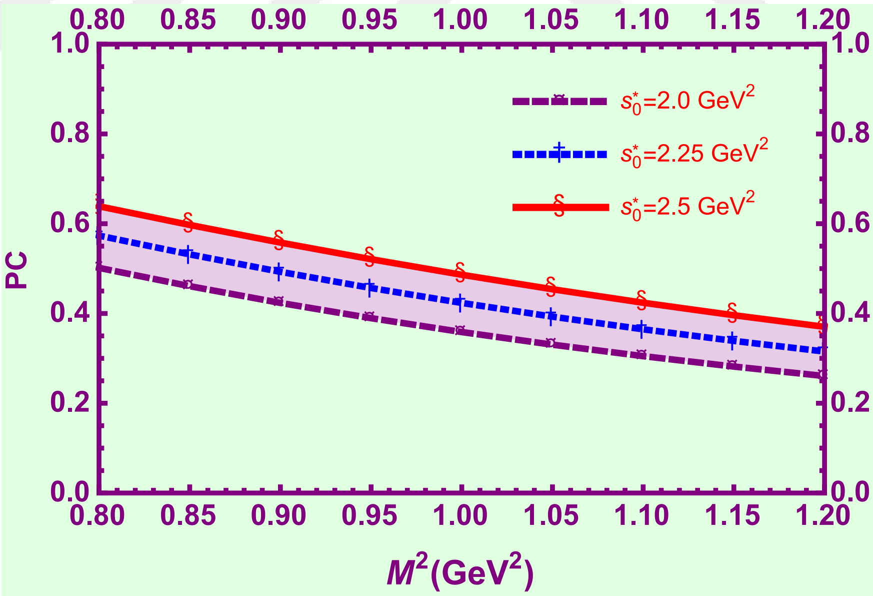

Figure 4.4: Pole contribution (PC) with respect to Borel mass parameter for the $\not{p}$ structure at nuclear matter saturation density.

In the above equations, the mass sum rule is the ratio of two sum rules and thus their unstable points in nominator and denominator can easily cancel each other.

In table 4.2, we present our average results of the in-medium and vacuum obtained in the limit $\rho \to 0$ masses and residue squares of nucleon (PS for used for present study) and compare them with existing results of Ioffe (1981) and Nasrallah and Schilcher (2014) in the literature. From this table, it can be concluded that our vacuum results are in good agreement within the errors with numerical values obtained using Ioffe current in the vacuum sum rules calculations. It can be also seen that the average values of those physical quantities shifts considerably due to nuclear medium.

We plot the variations of ratios λ_n^*/λ_n and m_n^*/m_n as a function of M^2 in the Figs. 4.5-6, respectively to show the weak dependency of the physical quantities under consideration with respect to the above auxiliary parameters and to see clearly how the in-medium results deviate from the vacuum values. We observe that the shifts of the physical quantities due to medium are negative. While the shifts in residue roughly increase with the increasing value of M^2, but the shift in mass changes considerably with respect to the same parameter.

To show the percentage of the shift $\frac{(\lambda_n^*/\lambda_n)}{\lambda_n} * 100$ in residue and mass $\frac{(m_n^*/m_n)}{m_n} * 100$

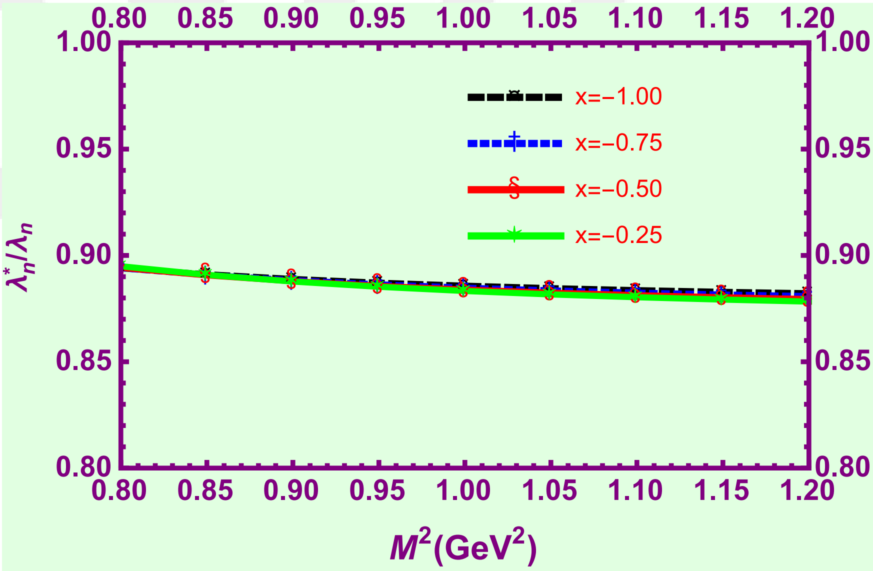

Figure 4.5: λ_n^*/λ_n as a function of M^2.

of nucleon in dense medium, we plot the Figs. 4.7-8, respectively. Our calculations present that the value of λ_n^* decreases when compared with the value of λ_n by approximately 11% and the percentage of the shift in the mass of nucleon in cold nuclear matter is nearly 31%.

In our calculations for the saturation density of nuclear medium $\rho^{sat} = 0.11^3$ is used. To investigate the density dependency of physical observables, λ_n^* and m_n^* of nucleon, we plot the in-medium residue ratio to vacuum residue in Fig. 4.9 and the in-medium mass ratio to vacuum mass in Fig. 4.10 as a function of $(\rho^{sat}/\rho) = [1, 1.5]$ at the input parameters

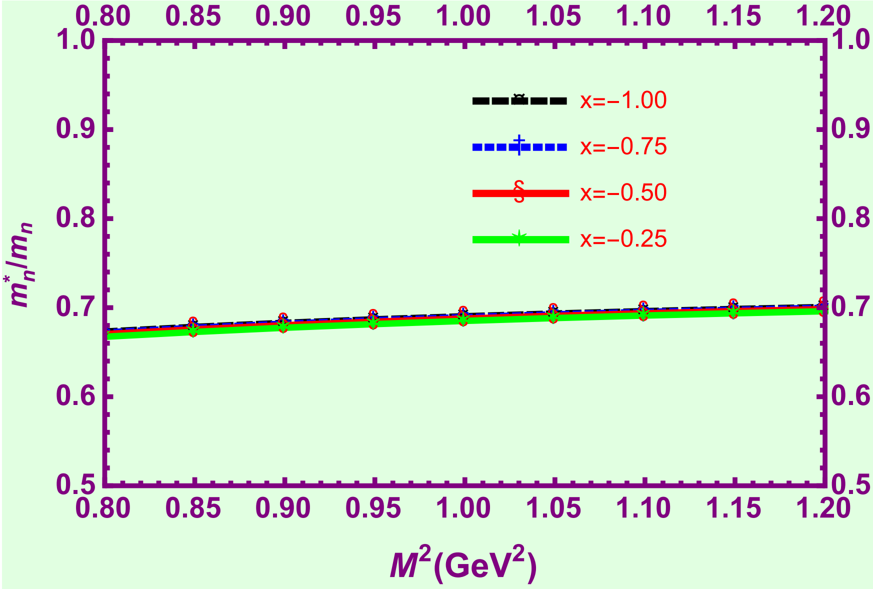

Figure 4.6: Same as Fig. 4.5 for m_n^*/m_n.

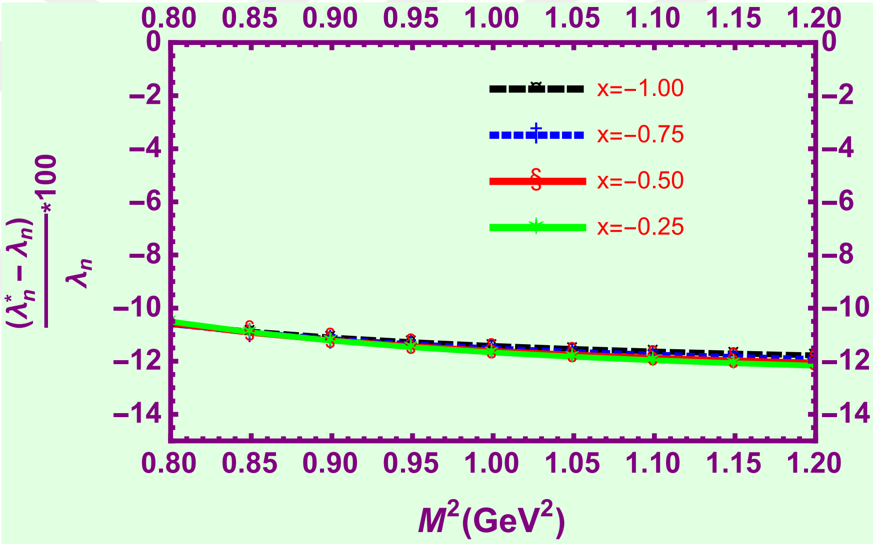

Figure 4.7: The percentage of the shift $\frac{(\lambda_n^*/\lambda_n)}{\lambda_n} * 100$ in residue of nucleon in dense medium with respect to M^2.

$s_0^* = 1$ GeV^2, $M^2 = 1$ GeV^2 and $x = -0.65$. In Fig. 4.9, it can be seen that the behaviour of λ_n^*/λ_n has a exactly linear response, but the density effect on in-medium mass is approximately linear as seen in Fig 4.10. At $(\rho^{sat}/\rho) = 1.5$ value of density ratio, these ratios are 0.86 for modified residue and 0.64 for modified mass of nucleon. The last analyze for the nucleon in dense medium is its vector self-energy and scalar self-energy. Using the obtained sum rules, in Fig. 4.11, we show the vector self-energy dependency of the nucleon on the auxiliary parameter Borel mass at mean values of other auxiliary parameters. As seen in this figure, it shows stability in the reliable region of the parameter M^2. Using the results of

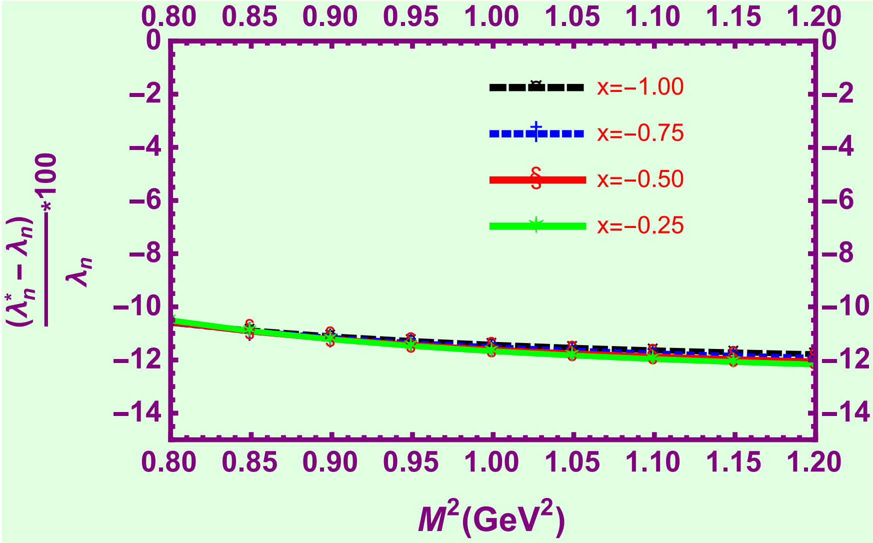

Figure 4.8: Same as Fig. 4.7 for mass of nucleon.

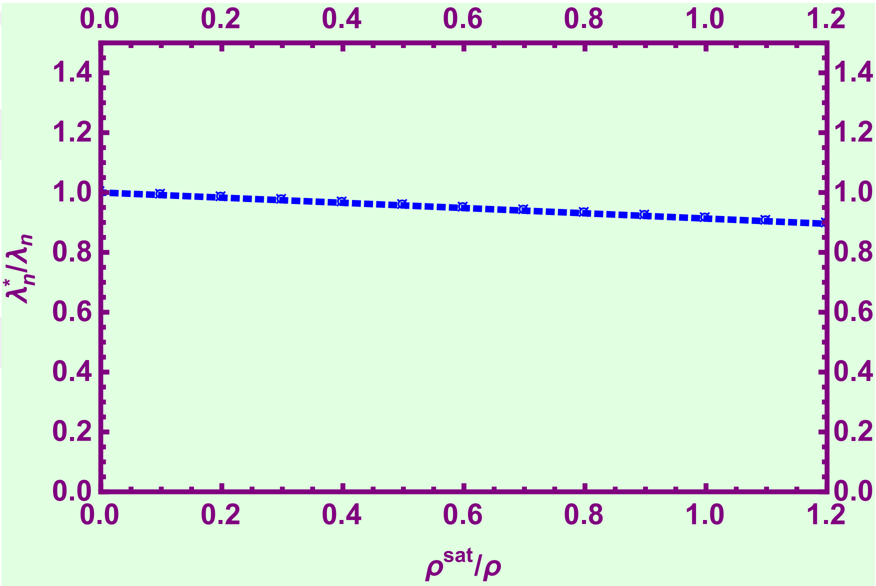

Figure 4.9: λ_n^*/λ_n as a function of ρ^{sat}/ρ at $s_0^* = 1$ GeV^2, $M^2 = 1$ GeV^2 and $x = -0.65$.

our analysis the average values of vector Σ_v and scalar Σ_s self-energies of nucleon in cold nuclear matter presented in Table. 4.3.

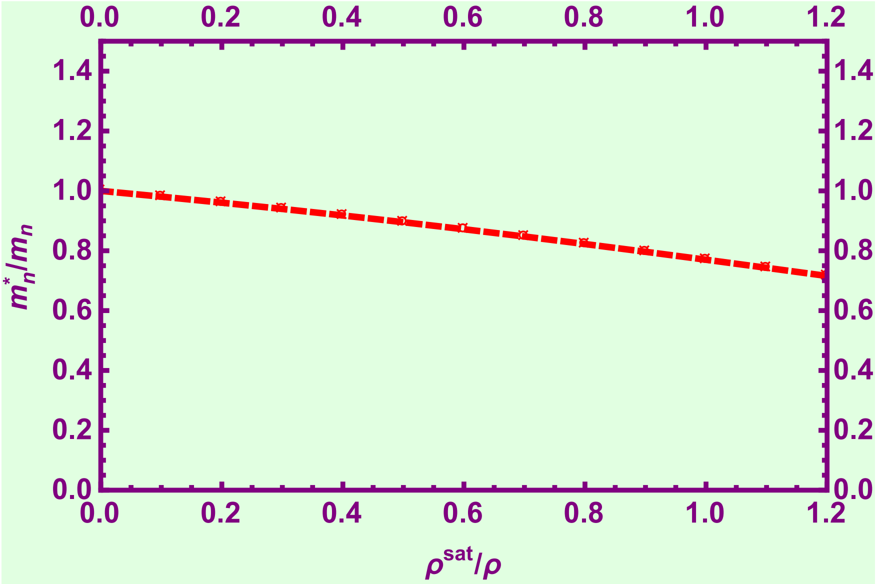

Figure 4.10: Same as Fig. 9 for m_n^*/m_m.

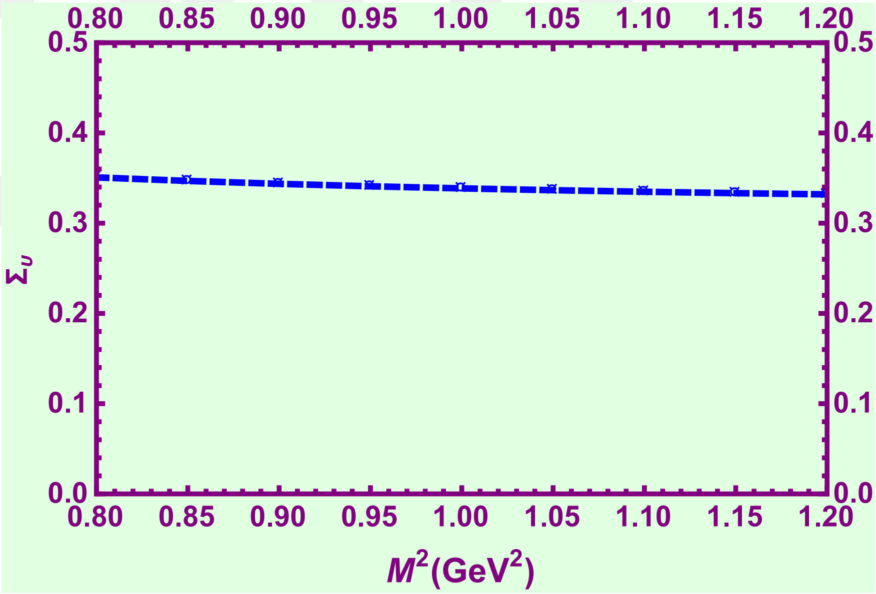

Figure 4.11: The vector self-energy Σ_υ versus M^2 at mean values other auxiliary parameters.

Table 4.3: The average values of vector and scalar self-energies of nucleon.

	Σ_υ (GeV)	Σ_s (GeV)
PS	0.345 ± 0.058	-0.308 ± 0.049
Plohl et al. (2006a,b)	$(0.350 - 0.400)$	$-(0.400 - 0.450)$

5. CONCLUDING REMARKS

In this thesis, we investigated the physical properties of nucleon in cold dense medium within the framework of QCDSR formalism as a powerful and reliable non-perturbative theoretical application. In the literature, plenty of theoretical studies are devoted for the investigation of the vacuum properties of nucleon. In addition, using the Ioffe current for the medium, it was studied how these physical properties deviated from the vacuum due to the nuclear matter. Theoretical and phenomenological investigation of those properties and their comparison with the existing experimental results can give more essential idea about the nature of nucleon.

Initially, we used the interpolating current with an arbitrary mixing parameter in the correlation function and inserted the light quark propagator with its density dependent condensates. After applying the some tools of the QCD sum rules formalism, we derived in-medium sum rules for the mass, residue and self-energies of the nucleon. In the $\rho \to 0$ limit, we obtained our vacuum results to compare the other existing vacuum results in the literature.

In the sum rules of the nucleon, there are three auxiliary parameters: arbitrary mixing parameter of the propagator β, in-medium continuum threshold s_0^* and Borel mass parameter M^2. First of all, we determined the reliable regions of these parameters in accordance with the philosophy QCD sum rules. We showed that working region of the β parameter contains Ioffe current value. After entering the numerical input parameters such as quark, gluon, mixed condensates of the nuclear medium and quark masses, we obtained the numerical values of the physical observables for vacuum and medium. Since we know that physical observables should be independent of the above auxiliary parameters in QCD sum rules method, we have demonstrated our numerical results showing good stability with respect to these parameters.

We observed that the in-medium mass and residue have considerable negative shifts due to cold nuclear matter at the saturation density of nuclear matter and the average values of the in-medium continuum threshold and Borel mass parameters. The shift in the mass of nucleon is by an amount of 31% and in the residue it is 11%. Depending on the change in the density of the medium ρ^{sat}/ρ, the ratios in the mass m^*/m and residue λ^*/λ are almost linear. At the end of our calculations, we investigated the nucleon vector and scalar self-energies in dense medium. Our results are consistent with the model independent results in Plohl et al. (2006a,b).

As a result, we can conclude that the results of vacuum in the literature for the physical properties of the nucleon are in good agreement with our results. In the previous studies, using Ioffe current the effects of the medium on the physical observables of the nucleon were investigated. However, in our study, we extended these studies using whole region of the β parameter in the current which includes the Ioffe value and also obtained working region of β for the observables of nucleon. These results may be used as a guide for the near future experiments.

REFERENCES

Adami C and Brown G E (1991) “Isospin breaking in nuclear physics: The nolen-schiffer effect”, Zeitschrift für Physik A Hadrons and Nuclei, 340(1):93–100.

Asakawa M and Ko C (1993) “Qcd sum rules for a rho meson in dense matter”, Nuclear Physics A, 560(1):399 – 410.

Cohen T D, Furnstahl R J, and Griegel D K (1991) “From qcd sum rules to relativistic nuclear physics”, Phys. Rev. Lett., 67:961–964.

Cohen T D, Furnstahl R J, and Griegel D K (1992) “Quark and gluon condensates in nuclear matter”, Phys. Rev. C, 45:1881–1893.

Cohen T, Furnstahl R, Griegel D, and Jin X (1995) “Qcd sum rules and applications to nuclear physics”, Progress in Particle and Nuclear Physics, 35:221 – 298.

Drukarev E (2003) “Finite density qcd sum rules for nucleons”, Progress in Particle and Nuclear Physics, 50(2):659 – 675.

Drukarev E and Levin E (1990) “The qcd sum rules and nuclear matter”, Nuclear Physics A, 511(3):679 – 700.

Drukarev E G, Ryskin M G, and Sadovnikov V A (2015) “Nucleon QCD sum rules in instanton medium”, J. Exp. Theor. Phys., 121(3):408–421.

Drukarev E G, Ryskin M G, Sadovnikova V A, Gutsche T, and Faessler A (2004) “Nucleon qcd sum rules in nuclear matter including four-quark condensates”, Phys. Rev. C, 69:065210.

Furnstahl R J, Griegel D K, and Cohen T D (1992) “Qcd sum rules for nucleons in nuclear matter”, Phys. Rev. C, 46:1507–1527.

Gell-Mann M (1964) “A schematic model of baryons and mesons”, Physics Letters, 8(3):214 – 215.

Gross D J and Wilczek F (1973) “Ultraviolet behavior of non-abelian gauge theories”, Phys. Rev. Lett., 30:1343–1346.

Hatsuda T, Høgaasen H, and Prakash M (1991) “Qcd sum rules in medium and the okamoto-nolen-schiffer anomaly”, Phys. Rev. Lett., 66:2851–2854.

Hatsuda T and Lee S H (1992) “Qcd sum rules for vector mesons in the nuclear medium”, Phys. Rev. C, 46:R34–R38.

Hayashigaki A (2000) “Mass modification of d-meson at finite density in qcd sum rule”, Physics Letters B, 487(1):96 – 103.

Hilger T, Thomas R, and Kämpfer B (2009) “Qcd sum rules for d and b mesons in nuclear matter”, Phys. Rev. C, 79:025202.

Hogaasen H (1991) “An Application of QCD sum rules in nuclear physics: The Nolen-Schiffer anomaly”, Acta Phys. Polon., B22:1123–1129.

Hubschmid W and Mallik S (1982) “Operator expansion at short distance in qcd”, Nuclear Physics B, 207(1):29 – 42.

Ioffe B (1981) “Calculation of baryon masses in quantum chromodynamics”, Nuclear Physics B, 188(2):317 – 341.

Jin X, Cohen T D, Furnstahl R J, and Griegel D K (1993) “Qcd sum rules for nucleons in nuclear matter ii”, Phys. Rev. C, 47:2882–2900.

Jin X and Leinweber D B (1995) “Valid qcd sum rules for vector mesons in nuclear matter”, Phys. Rev. C, 52:3344–3352.

Jin X, Nielsen M, Cohen T D, Furnstahl R J, and Griegel D K (1994) “Qcd sum rules for nucleons in nuclear matter iii”, Phys. Rev. C, 49:464–477.

Klingl F, Kaiser N, and Weise W (1997) “Current correlation functions, qcd sum rules and vector mesons in baryonic matter”, Nuclear Physics A, 624(4):527 – 563.

Leinweber D B (1995) “Nucleon properties from unconventional interpolating fields”, Phys. Rev. D, 51:6383–6393.

Leupold S and Mosel U (1998) “Qcd sum rules for vector mesons in nuclear medium”, Phys. Rev. C, 58:2939–2957.

Mallik S and Sarkar S (2009) “Qcd sum rule for nucleon in nuclear matter”, The European Physical Journal C, 65(1):247.

Nasrallah N F and Schilcher K (2014) “New sum rule determination of the nucleon mass and strangeness content”, Phys. Rev. C, 89:045202.

Novikov V A, Shifman M A, Vainshtein A I, and Zakharov V I (1984) “Calculations in external fields in quantum chromodynamics. technical review”, Fortschritte der Physik, 32(11):585–622.

Plohl O, Fuchs C, and Dalen E N E v (2006a) “Model-independent study of the dirac structure of the nucleon-nucleon interaction”, Phys. Rev. C, 73:014003.

Plohl O, Fuchs C, and Faessler A (2006b) “Model independent study of the nucleon self-energy in matter”, in “Proceedings, 44th International Winter Meeting on Nuclear Physics (Bormio 2006): Bormio, Italy, January 29-February 4, 2006,” .

Reinders L, Rubinstein H, and Yazaki S (1985) “Hadron properties from qcd sum rules”, Physics Reports, 127(1):1 – 97.

Ryskin M G, Drukarev E G, and Sadovnikova V A (2015) “Nucleon qcd sum rules in the instanton medium”, Journal of Experimental and Theoretical Physics, 121(3):408–421.

Shifman M A (1980) “Wilson loop in vacuum fields”, Nuclear Physics B, 173(1):13 – 31.

Shifman M, Vainshtein A, and Zakharov V (1979a) “Qcd and resonance physics. applications”, Nuclear Physics B, 147(5):448 – 518.

Shifman M, Vainshtein A, and Zakharov V (1979b) “Qcd and resonance physics. theoretical foundations”, Nuclear Physics B, 147(5):385 – 447.

Stein E, Górnicki P, Mankiewicz L, Schäfer A, and Greiner W (1995) “Qcd sum rule calculation of twist-3 contributions to polarized nucleon structure functions”, Physics Letters B, 343(1):369 – 376.

Tanabashi et al. (2018) “Review of particle physics”, Phys. Rev. D, 98:030001.

Thomas R, Hilger T, and Kämpfer B (2007) “Four-quark condensates in nucleon qcd sum rules”, Nuclear Physics A, 795(1):19 – 46.

Wang Z G (2011) “Analysis of the λqbaryons in the nuclear matter with the qcd sum rules”, The European Physical Journal C, 71(12):1816.

Wang Z G (2012) “Analysis of Σ_Q baryons in nuclear matter with qcd sum rules”, Phys. Rev. C, 85:045204.

Wang Z G and Huang T (2011) “In-medium mass modifications of the D_0 and B_0 mesons from qcd sum rules”, Phys. Rev. C, 84:048201.

Yasui S and Sudoh K (2013) “$\overline{\mathrm{d}}$ and B mesons in a nuclear medium”, Phys. Rev. C, 87:015202.

Zweig G (1964) “An SU(3) model for strong interaction symmetry and its breaking. Version 2”, in D Lichtenberg and S P Rosen, editors, “DEVELOPMENTS IN THE QUARK THEORY OF HADRONS. VOL. 1. 1964 - 1978,” pages 22–101.

CURRICULUM VITAE

Name Surname : Mehmat Arat
Place and Date of Birth : İSKENDERUN, 12.09.1988
University
Bachelor's Degree : Bolu Abant İzzet Baysal University, Department of physics
e-mail : mmehmetarat@gmail.com