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Computer Communication Network Reliability:

Evaluation of Two -Terminal Reliability

M. Sc. Thesis

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<u>Computer Communication Network Reliability:</u>

<u>Evaluation of Two-Terminal Reliability</u>

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ZAHRAA MUSTAFA ZAIDAN

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ABSTRACT

Computer Communication Network Reliability: Evaluation of Two- Terminal Reliability

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Reliability of computer networks plays essential role in managing and performing almost all the sensitive applications fields. Since networks dimension is increasing continually, the problem of long execution time becomes serious issue. In order to get the desired network topology, we need to evaluate the reliability at every phase quickly. Hence, fast, accurate algorithm is highly appreciated. That's way in this work we propose an algorithm to do this task efficiently. For some network reliability parameters to be basically evaluated there are two steps have to be taken. Firstly, interpreting the network topology into a reliability formula which does not considered to be a problem, since a little time is adequate to perform. Whereas, in the second step the reliability of a network is numerically computed. The proposed algorithm is a multi-stage hybrid algorithm (MHRT) based on graph reduction techniques with the minimal tie- set to measure the reliability of all commodities within a network.

Both directional and unidirectional links can be considered. There are no restrictions on the size or the shape of handled network. A new approach has been introduced for tracing all minimal paths. Compared with the well-known algorithms Backtracking, Edge Replacement, and Acyclic Path Mergence; it needs less memory. And comparing with Matrix Multiplication, and Node Removal; it does not require the application of Boolean algebra.

Keywords: reliability, multi-stage hybrid algorithm (MHRT), Minimal paths, Graph reduction technique.

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LIST OF ABBREVIATIONS

CCN	Computer Communication Network
MHRT	Multi-stage Hybrid Algorithm
GR_t	Graph Reduction Techniques
MT_s	Minimal Tie- Sets
G	Graph model
N	Node
E	Edge
PR_t	Parallel Reduction Technique
SR_t	Series Reduction Technique
Mp_s	Minimal Paths
$R_{s\,D}$	Two-terminal Reliability
Mp_H	Minimal Paths by the Proposed Algorithm
T_H(s)	Time of the proposed Algorithm
Mp_t	Minimal Paths by Tie-set Method
T_t(s)	Time of Tie-Set Method
S-D	Source-Destination

1. INTRODUCTION

1.1 GENRAL OVERVIEW ON RELIABILITY

Computer networks have continuity in both enormous spreading and massive dominative since it has come into being; noticeably, in the last few years. Thus, high reliability as a vital considerable has become inevitable consequence for various computing network-controlled applications. Particularly, applications which require high dependability, for example; military purposes, aircraft systems, banking systems, nuclear rectifier control and distributed systems [1]. Such systems should be extremely reliable, since faults may cause financial and human life losses. Telecommunication network is also tractable system that has been increased in dimension and characterized to be reliable enough to handle the existed stream, since the increasing unprecedented demand on these days leads to a de facto congestion [1]. Various reasons may outcome with a partly or wholly destroyed network for example, natural disasters as earthquakes and hurricanes, physical failures like cuts, software -hardware un matching, as well non-random failures like exposure to malefactions, electronic devices survivability [41]. Therefore, to create a sensible practical network the first stage of its structure design which is network topological problem required to be carefully planted. This stage is initial factor in deciding network reliability, however; the possibility of network components failure also directly affecting the network reliability.

There are two assumptions regarding the situation for each one of the basic graph elements (nodes and edges) must be considered before measuring network reliability [2]. The nodes may have considered to be either prefect, that is fully reliable with failure probability equal to 0 and this case helps reducing the difficulty of measuring, or imperfect with a value of failure probability more than 0. Independent edge failure indicates that the successful and the failure of its operation does not affect the operation of the remaining edges in a network since this edge may subject to random event individually.

1.2 LITRATURE REVIEW

Many valuable trials in network reliability analyses field result in a huge literature body; however; it does not indicate a complete study yet. **Manoj and R. K. in 2012** [3] defined the reliability as «the probability that a network supports a given operation». Further definitions are presented by **Musaria in 2010** [4], **Peican and Jie in 2016** [5]. There are three kinds of reliability problems have been treated. The basic one is the two-terminal reliability as stated by **Minh Lˆe in 2014** [6], **Majid and Luiz** [7].All-terminal problem is somewhat more difficult than the two-terminal reliability problem [8] as presented by **Jaime, and Teresa** [9].

K-terminal reliability is more comprehensive term depending on the value of k which can take any number from 2 to all nodes as stated by **D.K. Panda in2013** [10], and **Minh Lˆe in 2014** [11].

Wilkov in 1972 in his survey [12] classified two types of reliability measures, one finds the reliability by discrete measures which is the deterministic way. It is the measure of the worst case of the network when the connection between the source node which as transmitter side to the destination side which as receiver breaks down .in other words, when the network fails to do its functionality. Whereas, the other way of measuring computer network reliability called probabilistic method, which considers the probability of edges and nodes operation that is the probability of being up in determining network reliability. According to [13] network reliability approaches can be classified into two classes, exact approaches and approximation techniques. **Musaria** [14] has been presented an interesting survey contains explanation of different probabilistic methods for computer communication network reliability.

Cao and Zeng in 2010 [15] produced a study on reliability of computer networks which contains a review of the major achievements. Various definitions for network availability are also presented with the current problems of computer network reliability field. The major factors affecting network availability from different layers are classified in order to indicate their impact on the network availability.

Conventional Exact methods may suffer from consuming much time for every additional node or link; however, there are many efficient trials in this term such as, the fast-minimal cut set algorithm [16] of **A. Gaun, H. Renner and G. Rechberger** in **2009** which can be applied to determine the two- terminal reliability of power transmission system. This algorithm is based on graph reduction method with an adaptive recursive merge method.

S. Rajkumar and Neeraj in 2015 [17] proposed a multi-variable inversion algorithm to evaluate the reliability of omega network which is one of the most common multistage interconnection networks using path-based method which firstly enumerates minimal path sets and then evaluates reliability expression in a form of sum of disjoint products using multi-variable inversion algorithm.

The first study to implement an algorithm for estimating two-terminal reliability in parallel on multi -core processor architecture is made by **Dang and Bay in 2015** [18].When a method for computing two-terminal reliability has been proposed using logical-probabilistic calculus to convert the probability of the union to the sum of the individual event probabilities .The execution time for complex and large networks was reduced by developing a parallel version of the proposed method that takes the advantage of multi-core processor architectures. According to **Sandeep, and Aman in 2016** [19], in order to optimize the size of the paths/cuts in reliability evaluation of complex networks. Binary Decision diagram method is an effective way since reasonable consummated time is enough as shown by **Fu. Yeh and Shyue in 2002** [20], **Manoj and Girish 2010** [21]**.**

Rongsheng and Yangyang in 2016 [22] introduced an exact decision diagram method to compute the reliability of multistate flow network ,this algorithm has the advantages of taking less memory and fewer loops and smaller number of both generated nodes and variables in compared with the existing decomposition algorithms. As the reliability calculation complexity exponentially increased when increasing the number of nodes and links, researchers have been tried to overcome the problem by proposing alternative solutions [23] such as, approximation ways for example upper and lower bound, monte carlo simulation technique to evaluate network reliability in reasonable time. Recursive truncation Algorithm of **Ahmad and Omid in 2009** [24] can estimate all-terminal network reliability by computing an upper, and a lower bound, this approach is based on computing the probability of failure for a union of events in minimal cut sets by scanning all minimal cut sets. Then the computations are reduced by obtaining only the weak cut sets, which are a small portion of the scanned cut sets for highly reliable networks and ignore the rest of the cut sets.

Pierre and Gerardo in 2011 [25] proposed a new Monte Carlo method, based on dynamic importance sampling, to estimate the probability that a group of nodes are connected where each link is failed, the approximation is based on minimal cuts in subgraphs. The link states one by one are generated using a sampling strategy that approximates an ideal zero-variance importance sampling scheme. Another approximated scheme was proposed by **Jin-Myung and Fakhreddine** in **2011**[26], which is a network factoring algorithm based on special branch sets (*spanning trees and cut sets*) to calculate the all-terminal reliability of a stochastic network. They found the most reliable spanning tree and most unreliable cut set in the given network, then they used the operative and failing probabilities to update the lower and upper bounds. For each state of the spanning tree or cut set there is a corresponding sub network, for the lower or upper bound to be reached the preset of this procedure was applied to the subnetworks in a recursive manner to update the all-terminal reliability bounds at every factoring step. Genetic algorithms and neural network are other approximated methods for the network reliability evaluation and reliability optimization purpose as presented by **Baijnath, Navdeep Kaur2 in 2010** [27], **Lijie Liu in 2016** [28].

Sum of disjoint products is another approximation way presented **by Ebraham in 1979** [29], this method is based on converting the minimal path from joint events (having participated links) to disjoint ones (mutually exclusive). It has been early proposed in the form of single variable inverted which takes long execution time and terms. **Klaus D. Heidtmann in 1989** [30] proposed a new way of applying sum of disjoint products based on inverting Parts of products.

This technique results in fewer disjoint terms. However, this approximation method stills facing challenges of finding the disjoint products, thus; it has been classified as NP-hard **[31]**. The suitable arrangement ensures the accepted approximation results. There are cases of arrangements have been applied to process the minimal paths before final calculation such as; Re rearrangement based on the shortest minimal path (descending manner), shortest hope in the same path, number of participated links between the successive paths. Each one is the best for specific minimal paths cases. **Suresh in 1995 [32]** classified the existed ways with details. The most common arrangement is to put the minimal paths in order of increasing size, but **in 2013 Alexandru and Lorenzo [33]** showed that this method is not efficient in all cases as well the suitable arrangement of single variable inverted is not the same with multi inverted variables.

1.3 MOTIVATION AND THESIS ORGANIZATION

Reliable computer networks play essential role in managing and performing almost all the sensitive applications fields. Since networks dimension is increasing continually, the problem of long execution time becomes serious issue. In order to get the desired network topology, we need to evaluate the reliability at every phase quickly. Hence, fast, accurate algorithm is highly appreciated. That's way in this work we propose an algorithm to do this task efficiently.

This dissertation consists of six chapters and has been organized as the following: in chapter 1 introduction as well an interesting literature review about CCN reliability is introduced. Chapter 2 defines and explains all reliability problems with details. Chapter 3 contains an introduction with review of the existed path tracing methods. Structure of the three stages of the proposed multi-stage algorithm is also explained. In chapter 4, MHRT is implemented with a case study of different networks, Reliability is also accomplished by tie set method, and reliable topology of three communicated pairs is achieved by the two mentioned methods. Chapter 5 discusses and analyzes results gotten in chapter 4. Chapter 6 summarizes the work and suggests development ideas as a future research.

2. COMPUTER NETWORKS RELIABILITY ANALYSES

2.1 NETWORK GRAPH MODEL:

Computer and communication system is one of many physical problems those can be modeled graphically in order to be handled easily during the design and enhancement phases [13]. setting the topology of a computer communication network have direct impact on the performance as well the reliability of it. Hence, we need to treat the network graphically till we obtain the required system that is able of performing its mentioned purpose with desired characteristics in terms of reliability [41], components survivability, cost, capacity, flow etc. A computer communication network basically consists of computers communication devices, routers, switches and other components. These entities need to be capable of contacting among them, physical connection can be achieved by wired links, wireless, and even by visible light.

Graph model can be described as G (N, E), where N represents the set of contacted entities (graphically nodes), and E refers to the network edges set (graphically lines). We denote by $|N|$ the number of vertices and by $|E|$ the number of edges [40].

A subgraph of *G* is a graph whose nodes and edges are contained in *G*. That is,

G•(*N•*,*E•*) is a subgraph of *G* if *N*• ⊆ *N* and *E*• ⊆ *E*. Edges frequently have one form of ‘‘weight function’’ which refers to a physical property of the link. Properties such as the cost of an edge or the length or even the capacity of flow are often used when modelling communication networks, we consider that Pn and Pe are the probabilities of being operating correctly for nodes and edges respectively.

$$Pi = \begin{cases} 1, if \text{ ei, ni} & \text{is operational} \\ 0, & else \end{cases}$$

When two nodes can communicate through the same edge in two ways this edge is said to be undirected edge. While for example if one node can only receive data from the other transmitter one and vice versa, the edge that connects them is directed and its direction decides the path of the current. For example, if we consider a small computer system consists of four computers connected incompletely with each other by five (un directed and directed) links as in fig.1.

We can view four nodes each one represents one computer device connected with five edges where some are directed while the others are not.

This figure says that nodes (1 and 2) can communicate through edge B in two ways since B is undirected edge, while in the case of nodes (1, and 4) the stream can flow in one direction that is from node 4 to 1 (4 is transmitter,1 is receiver). For directed graphs nodes can belong to three categories [29]. Strongly connected nodes (group of nodes can be reached from any other node in the subset traversing the edges direction); transient nodes those only transmit data and cannot receive a current from any other nodes; absorbing nodes those have only ingoing edges and, thus they cannot transmit information. A walk is a sequence of nodes in which each successive two nodes indicate an edge of the Graph [2]. A path is a walk in which each variable N or E appears one time with the possible exception of the source and sink nodes. A cycle is a path that starts and ends at the same node. if a graph contains no cycles then it is an acyclic graph [2]. Edge-disjoint paths are paths with completely different edges. Node-disjoint paths are paths which share just the source and target nodes. A loop or self-edge is an edge that out goes and terminates at the same node [2]. Parallel edges appear when two or more edges are connecting the same pair of nodes. For any graph if there are no loops or parallel edges is said to be a simple graph. Usually real networks are much more connected than this minimal threshold so as these networks work in practical way.

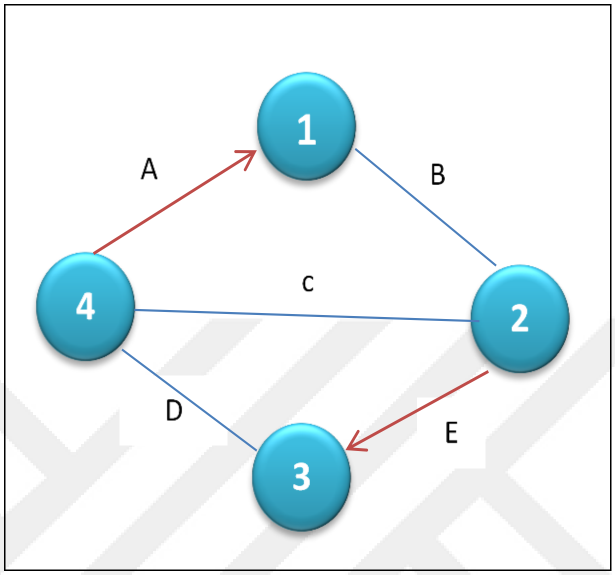


Figure 1. Simple Network with Directed and Un-Directed edges

2.2 MATRIX REPRESENTATION OF GRAPHS:

Adjacency Matrix:

Any nonempty finite graph can be represented by Boolean matrix that composes of rows and columns with either 0 or 1 elements, the value 1 indicates that there is a connection between a pair of nodes, while 0 tells the inversion. If the graph is undirected the adjacency matrix is symmetric. The diagonal elements are zeros when node self-loop is not allowed. For the network shown in Fig.1 the adjacency matrix is shown below:

	1	2	3	4
1	0	1	0	0
2	1	0	1	1
3	0	0	0	1
4	1	1	1	0

Table1. Adjacency Matrix of Fig.1

$M_{1\,,1} = 0, ther\ is\ no\ self-loop\ for\ nod\ 1\ ,M_{1,2} = 1, there\ is\ aconnection\ from\ 1\ to\ 2.$

Complete graph can be represented by symmetric adjacency matrix, where all elements are ones except the diagonal elements are zeros if there is no self-loop. zero matrix is a matrix of empty graph.

Incidence Matrix:

Incidence matrix reveals the connection between two deferent entities, where the rows describe one of them and the columns describe the other variable**.**

By considering a network comprises four nodes and five undirected edges the adjacency matrix illustrates the connection situation between nodes and edges. Where the matrix rows indicate the nodes, and the columns refer to the edges. As in Fig. 2 there is a connection between node 1 and edge A that leads to$M_{1,1} = 1$, where$M_{1,3} = 0$ tells that there is no connection between node 1 and edge 3.

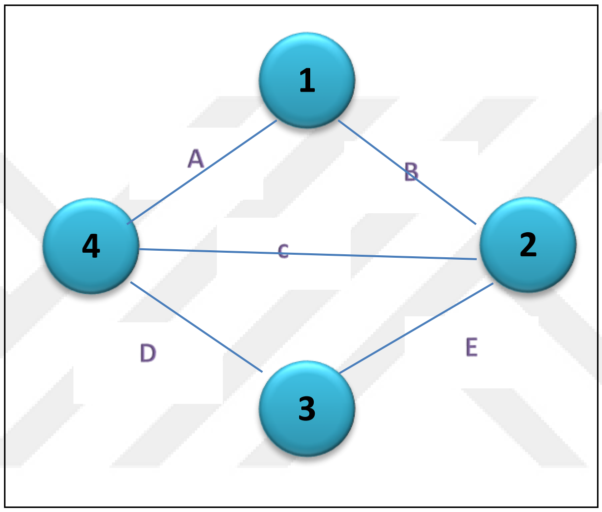


Figure2. Simple Undirected Network

Incidence matrix for Fig .2can be easily interpreted by M 4x5 as below

Table2. M4x5 Incidence Matrix of Fig.2

	A	B	C	D	E
1	1	1	0	0	0
2	0	1	1	0	1
3	0	0	0	1	1
4	1	0	1	1	0

In sense of directed edges incidence matrix, it is various of M 4x5. For example, if the undirected edges of the network shown in Fig .2 replaced by directed ones as revealed by Fig .3. The incidence matrix M 4x5 can be represented by three values the first one is 1 which means the edge runs to a node. The second value is -1 when the edge runs from a node. The last value is 0 and it indicates that there is no connection between a specific node and edge.

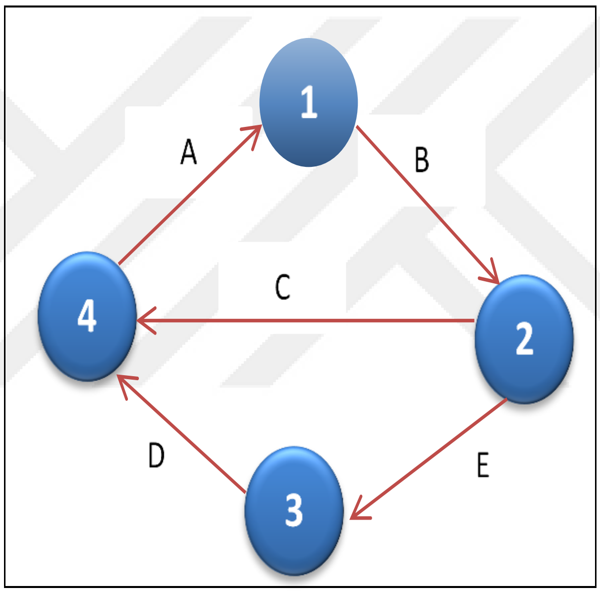


Figure3. Directed Network

Table3. M 4x5 Incidence Matrix of Fig.3

	A	B	C	D	E
1	1	-1	0	0	0
2	0	1	-1	0	0
3	0	0	0	-1	1
4	-1	0	1	1	0

2.3 COMPUTER NETWORKS RELIABILITY PROBLEMS:

The essential purpose behind the topological network design is to set networks that operate successfully in the existence of component failures.

The minimum number of nodes that have to be deleted so as to disconnect the graph or reduce it to unattached node, is the graph connectivity. It is the first factor of effective operation. In real environments, problems with pursuance are more predominant and hence more widely handled. Whereas the connectivity problems are less frequent and they are more catastrophic [29]. At the present era the issue of connectivity becomes serious problem, in order to treat and handle it, there are three points have to be considered:

- Analyzing and understanding the structure of the network.
- Classifying and ordering the reasons of components failure.
- Conditions and requirements which achieve network operation.

2.3.1 Two-Terminal reliability:

With any network in spite of the number of its nodes when the aim is to transmitting data from source node (s) to destination node (d), if there is at least one path to communicate through it, the operation is said to be successful. As in Fig .1, if node ***1*** is the source node and node ***4*** is the destination one, there are three paths to connect the two nodes ;({**A**}, {**BC**}, {**B E D**}). Two terminal reliability can be defined as the probability of successful operation between a pair of nodes [30], [31]. Two-terminal reliability can be classified as the least complex one compared with the remaining two approaches (k-terminal reliability and all terminal reliability) those will be explained in the next sections; however, the complexity increases dramatically with the increasing of the number of nodes and edges which leads to generate more paths between **s & d**.

2.3.2 All-Terminal reliability

All-terminal reliability can be defined in two ways according to two cases, the first one when the edges are un directed which leads to transmitting information in both ways, hence the presence of at least one spanning tree which connects all the network nodes ensures enough routes to connect all the pair nodes [32]. For example, if a network comprises four nodes there are six possible pairs of nodes must be able to communicate each between them for solving all-terminal connection problem as the small complete network shown in Fig .1 node **1** is able to communicate with **2**, **3** and **4** as well the other nodes **2, 3, 4**. The second case is when assuming that the edges are directed the presence of a spanning tree does not adequate condition since the connection direction is deciding in this case. Consequently, all terminal reliability can be defined as the presence of at least one path connects each node with all the remaining nodes [33]; in other words, each node in a network must be able to broadcast information to all the nodes in the network.

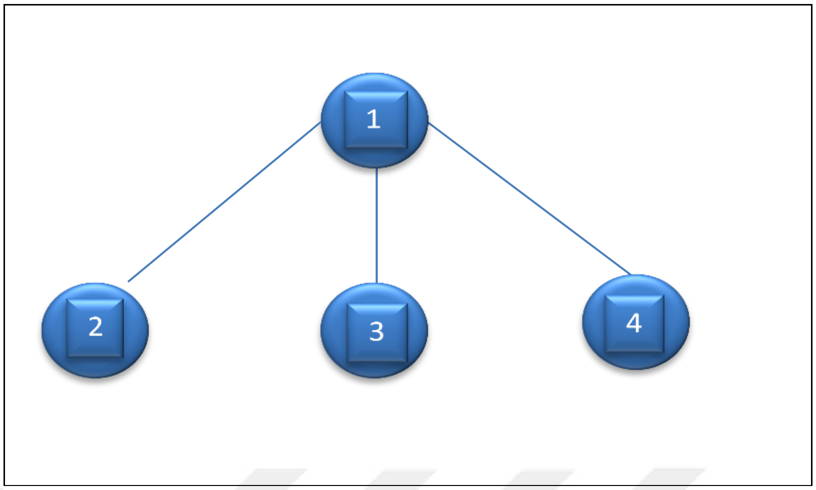


Figure 4. One Possible Spanning Tree of Fig .1

2.3.3 K-Terminal reliability

More general concept in network reliability analyses can be named as K-terminal reliability [34], where (**2 < K < n**). If k = 2, it means that it is two-terminal reliability; whereas, with k = n, (n= the order of the network) means that this approach is all terminal-reliability. Where; k = specific subset of nodes, that means all the nodes involved in the subset have to be connected with each other in order to solve the K-terminal problem [35], [42]. Consequently, the probability of successful communication among K subset nodes called K-terminal reliability.

2.4 NETWORKS RELIABILITY EVALATION METHODS:

Different classifications of methods used to find the reliability exist in many published books and papers [13]. Methods can be classified as the fallowing:

2.4.1 State Space- enumeration method

In order to calculate two-terminal reliability of a small, state space enumeration is the simplest way [13]. Because this method based on enumerate all the possible combinations of the edges those can be good or bad to results in 2^e combinations. Each one can be treated as disjoint event [42]; either, good case (operational), or bad case (failure) and the reliability of two terminal nodes basically is the union operation as in equation (1).

$$\boldsymbol{R_{sd}} = \boldsymbol{P}\,(\boldsymbol{E_1} + \boldsymbol{E_2} + \cdots + \boldsymbol{E_k}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad (\mathbf{1})$$

Since each of these events is mutually exclusive this equation can basically be treated as the sum of the individual event probabilities as in equation (2).

$$\boldsymbol{R}_{\boldsymbol{sd}} = \boldsymbol{pE_1} + \boldsymbol{pE_2} + \cdots + \boldsymbol{pE_k} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad (\mathbf{2})$$

However, it is not favorable way where the goal is to evaluate all-terminal reliability, since it demands more calculations which leads to consuming considerable time. If we consider the network in Fig .3 as an example to calculate the reliability of two-terminal which are node 1 and node 3. It is obvious that there are 2^5 = 35 (good and bad) mutually exclusive events since there are 5 edges, «consider each with probability (0.9) » and all the nodes are perfectly reliable for simplicity. Table (1) illustrates all the events, where:

Ei = mutual event

K = successful operation with probability = 0.9

$k`$= failed operation with probability = 0.1

The term good means that there is at least one route from **1** to **3** for the given combination of good and failed edges. The term bad, on the other hand, means that there are no routes. Table.1 gives the all combination of good and failed edges. Applying equation (2) to calculate the reliability:

$$R_{13} = 0.95 + 5 \times 0.94 \times 0.11 + 8 \times 0.93 \times 0.12 + 2 \times 0.92 \times 0.13$$

$$= 0.59049 + 0.32805 + 0.05832 + 0.00162 = 0.97848$$

Now if the desired is the calculation of all-terminal reliability for the same network of Fig (2), which is somewhat more difficult for a larger than this network. Some restrictions should be applied to the previous table in terms of the classification of the bad and good events.

There are three pairs (1 2), (1 3), (1 4); hence, we have to test each event if it causes complete isolation for all the pairs (bad event), or it is good event.

Note that the events from E_0 to E15 are (Good events), except the events E7 and E11 are (Bad evets) since each one causes isolation for one node away from the network. While the remaining (16) events are all bad events because they contained at least one isolated node

Substituting the terms from Table (1) into equation (4):

$$R\_all = \sum_{i \neq 7,11}^{15} p(Ei) \qquad \dots\dots\dots\dots\dots\dots\dots (3)$$

$$R_{all} = p^6 + 5\times qp^5 + 8\times q^2 p^4 \quad = (0.9)^6 + 5 \times 0.1 \times 0.9^5 + 8 \times 0.1^2 \times 0.9^4$$

$$= 0.531441 + 0.295245 + 0.052488 = 0.879174$$

It is axiomatic to expect that R _all terminal reliability value is less than two –terminal reliability.

Table 4. The Event-Space Enumeration

No failure $\binom{5}{0} = \frac{5!}{0!} = 0$	E0 = ABCDE	Good
One failure $\binom{5}{1} = \frac{5!}{1!\times 4!} = 1$	E1=A`BCDE	Good
	E2= AB`CDE	Good
	E3= ABC`DE	Good
	E4= ABCD`E	Good
	E5= ABCDE`	Good
Two failures $\binom{5}{2} = \frac{5!}{2!\times 3!} = 10\ events$	E6= A`BC`DE	Good
	E7= A`B`CDE	bad
	E8= A`BCD`E	Good
	E9= A`BCDE`	Good
	E10= AB`C`DE	Good
	E11= ABCD`E`	bad
	E12= AB`CDE`	Good
	E13= AB`CD`E	Good

	E14= ABC\`D\`E E15= ABC\`DE\`	Good Good
Three failures $\binom{5}{3} = \frac{5!}{2!\times 3!} = 10\ events$	E16= A\`B\`C\`DE  $E$17= A\`BC\`DE\` E18= A\`BC\`D\`E  $E$19= AB\`C\`D\`E E20= A\`BCD\`E\`  $E$21= A\`B\`CD\`E E22= A\`B\`CD\`E  $E$23= ABC\`D\`E\` E24= AB\`CD\`E\`  $E$25= AB\`C\`DE	bad bad Good bad bad bad bad bad bad Good
Four failures $\binom{5}{4} = \frac{5!}{1!\times 4!} = 5\ events$	E26= A\`B\`C\`D\`E E27= A\`B\`C\`DE\` E28= AB\`C\`D\`E\` E29= A\`BC\`D\`E\` E30= A\`B\`CD\`E\`	bad bad bad bad bad
Five failures=1 events	E31= A\`B\`C\`D\`E\`	bad

2.4.2 Tie-sets and Cut-Sets methods

In order to reduce the complexity of the reliability determined by state space numeration method below 2^e, one can focus on minimal tie- sets and minimal cut- set for the graph model of a network. Tie-set is a group of edges those connected serially to perform a path from source node to destination node [35]. With this path if any edge failed the path will also completely failed. But the network will work if there is at least one path from the source to destination. The term minimal refers to that no node is traversed twice. From other hand, cut set is defined as a group of edges those if removed or failed completely, the network will fail to do its functionality. If a cut set is minimal, no subset is also a cut set. However, if one edge in a cut set failed the network will operate [36]. Consequently, if there is only on edge operates of a cut set the network will never fails.

Consider the network of Fig (2) which consists of 4 nodes and 5 undirected edges if the source node is 1 and the destination node is 3, both the tie sets and the cut sets representations are illustrated in Fig (5), (6) respectively. The minimal cut sets and tie sets are found by inspection and given in Table (2)

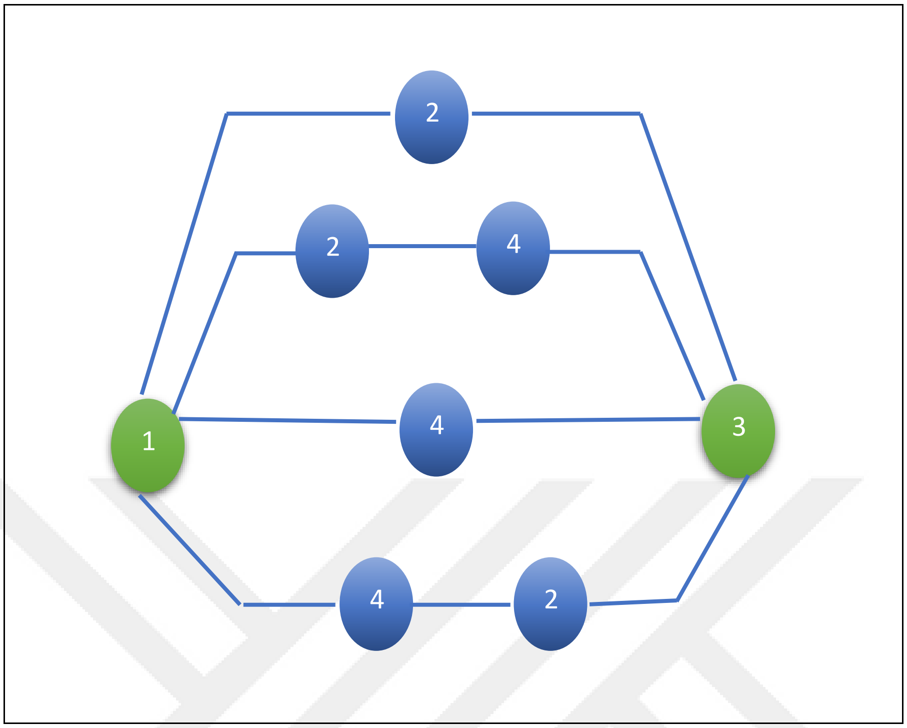


Figure 5. Tie- Sets Representation

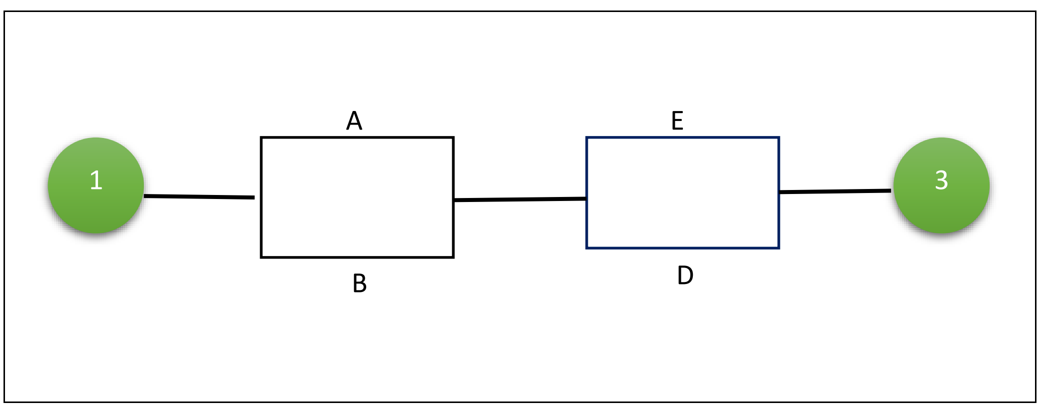


Figure 6. Cut- Sets Representation

Table 5. Minimal Tie- sets &Cut- set

Minimal Tie- sets	Minimal cut- sets
$T1 = BE$ $T2 = AD$ $T3 = BCD$ $T4 = ACE$	$C1 = A`B`$ $C2 = E`D`$

Generally, the complexity of T_s &$C_s$depends on two factors the first one is who to determine the tie sets (counting all paths from source to destination) and the second one, is the application of the inclusion exclusion expansion equation, which represents the union of all the tie sets where these tie sets are joint events; as a result, the complexity as well the time-consuming increase by increasing the number of T_s &C_s .

If there are (j) T_s between s and d, then the reliability expression is given by the expansion of:

$$Rsd = (T1 + T2 + \cdots\cdots + Tj) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

From the other hand if there are (j) Cs between s and d, the reliability expression is given by the expansion of:

$$R_{sd} = 1 - P(C1 + C2 + \cdots + Cj) \quad \dots\dots\dots\dots\dots\dots\dots (5)$$

If we apply the above theory to the network of Fig.2 the reliability is computed by the following equations:

$$R_{13} = T1 \cup T2 \cup T3 \cup T4$$

$$R_{13} = p\,(T1 + T2 + T3 + T4)$$

$$= P\,T1 + P\,T2 + P\,T3 + P\,T4 - P\,T1T2 + P\,T1T3 + P\,T1T4 + P\,T2T3$$

$$+P\,T2T4 + P\,T3T4 + P\,T1T2T3 + P\,T1T3T4 + P\,T1T2T4 + P\,T2T3T4 - P\,T1T2T3T4$$

$$= P\,(BE) + P\,(AD) + P\,(BCD) + P\,(ACE) - [\,(BEAD) + P$$

$$(BECD) + (BEAC) + P\,(ABCD) + P\,(ADCE) + P\,(ABCDE)\,] + [P$$

$$(ABCDE) + P\,(ABCDE) + P\,(ABCDE) + P\,(ABCDE)\,] - P(ABCDE)$$

$$R_{13} = 2(p^2 + p^3) - 5p^4 - p^5 + 4p^5 - p^5$$

$$= 2\,(0.9^2) + 2(0.9^3) - 5(0.9^4) + 2(0.9^5)$$

= 0.98

Once again using minimal cut-set method reliability can be found by more simple equation since the number of C_s is less than that of T_s as below:

When: $p(A) = p(B) = p(E) = p(D) = 0.9$

$$q(A) = q(B) = q(E) = q(D) = 1 - 0.9 = 0.1$$

$$R_{13} = 1 - [p(C1 + C2)]$$

$$R_{13} = 1 - [(p(C1) + p(C2)) - p(C1\,C2)]$$

$$R_{13} = 1 - [(p(A`B`) + P(E`D`) - P(A`B`E`D`)]$$

$$= 1 - [2(q^2) - q^4]$$

$$= 1 - [0.02 - 0.0001]$$

$$= 1 - 0.0199$$

$$= 0.9801$$

When the problem is the calculation of the all-terminal reliability for the same network one can made is to count node 1 as source node and the three remaining nodes as destinations in this case we ensure that there is a connection between each two nodes in the network because they are connected by node 1.

The tie-sets and the cut-sets can be found for all the possible pairs (1-2), (1-3), (1-4) as shown in tables (3), (4) respectively.

Table 6. Minimal Tie set for All-Terminal Reliability Computations

(path 12)	(path 1 3)	(path 1 4)
T1 =B T2 =AC T3 = ADE	T4 =BE T5 =AD T6 = BCD T7 = ACE	T8 =A T9 =BC T10 = BED

Table7. Minimal Cut sets for All-Terminal Reliability Computations

(path 1 2)	(path 1 3)	(path 1 4)
C1 = B\`E\`C\`  C2 = A\`B\`	C1 = A\`B\` C2 = E\`D\`	C1 = A\`D\`C\`  C2= A\`B

By applying the following equation, we can clarify the little difference between the equation of two-terminal reliability and the all-terminal reliability which just deals with more than one path resulting by multiple destination nodes.

$$Pall = P\,([path\ 1\ 2].[path\ 1\ 3].[path\ 1\ 4]) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$Pall = P\,([T1 + T2 + T3].[T4 + T5 + T6 + T7].[T8 + T9 + T10]$$

It is obvious that hand computations are starting to become Intractable since the expansion of Equation (6) involves many intersections. Furthermore, complex calculations involving expansion of the union of the resulting events (inclusion – exclusion). Similar set of equations can be written in terms of cut sets.

2.4.3 Graph transformation

For a large network to be analyzed and for its reliability to be determined, such network has to be simplified into a smaller and simpler one, so as it can be treated easily. Series - parallel reductions, delta-star transformation and edge factoring decomposition [38], [39] are possible ways those used for simplifying purposes. Let us start with the series reductions it is the most used one as it is shown in fig. (7.a). If the source is 1 and the destination is 3, then node 2 can be eliminated from the figure by applying series reductions and the two-terminal reliability can computed simply by multiplication as shown in the following equation:

$$R_{13} = p_{12} \times p_{23} \qquad \dots\dots\dots\dots\dots\dots\dots (7)$$

Parallel transformation is one that have to made for two branches those are connecting the same pair of nodes by taken the union of their probabilities. Next equation clarifies this technique:

$$R_{13} = p_a + p_b \qquad \dots\dots\dots\dots\dots\dots\dots (8)$$

Edge factoring more complex than the previous ones since it demands to consider two cases then combining them in the final equation to calculate the network reliability, take a simple example (1), which can show how to do it.

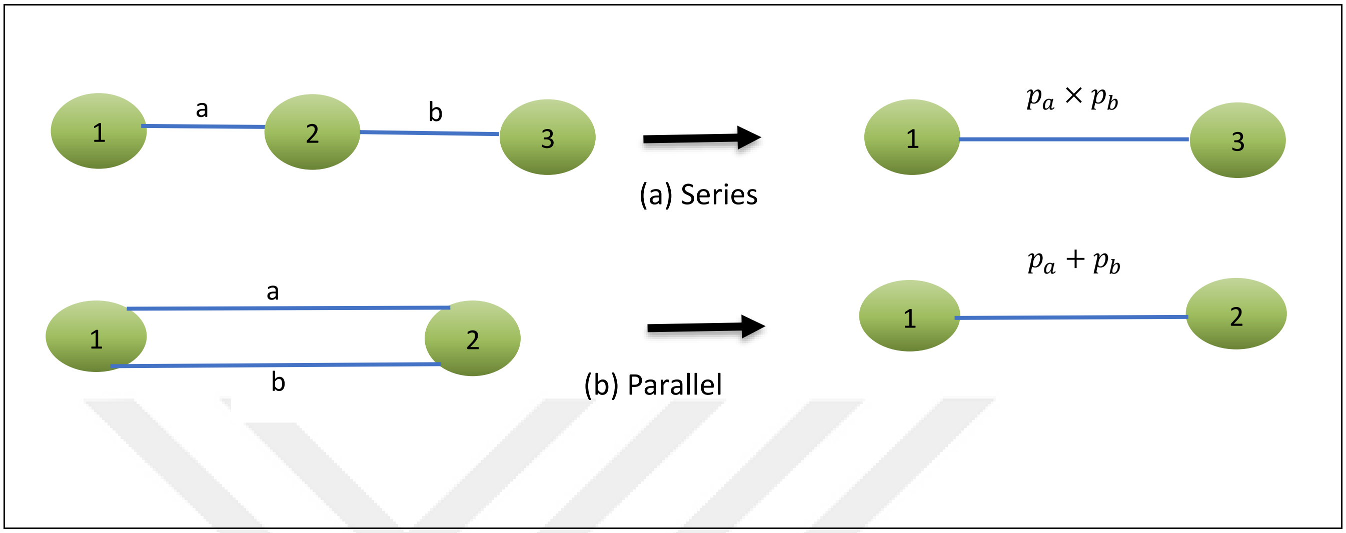


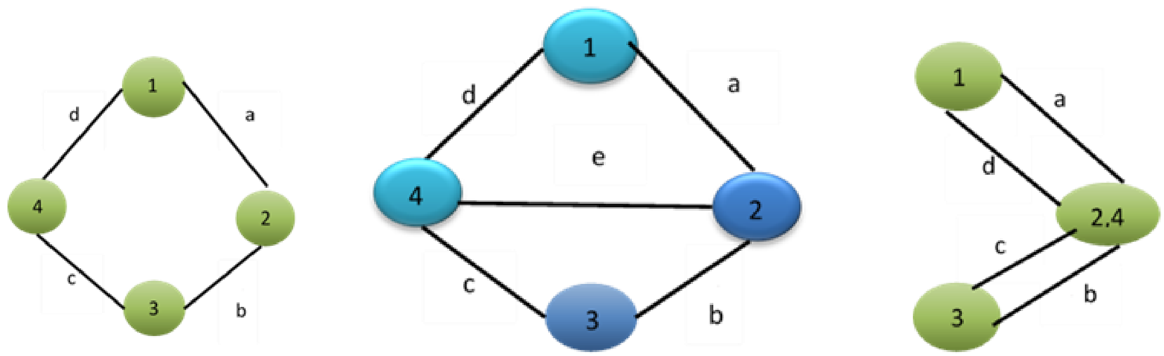
Figure7. Illustration of SR_t&PR_t for Two-Terminal

Example (1):

Consider the following simple network in Figure .8 consists of four nodes and five lines. It is obvious that there is no possible series and parallel reductions because of the being of edge **d**. In such network when we cannot apply series –parallel reduction, resorting to edge factoring is one solution to calculate the reliability. Firstly, considering **d** as short edge with probability of successful equal to 1 and call this case A_1, then take **d** as open edge with probability of successful equal to 0 and call it A_2. Combining these two cases resulting in the next equation:

$$R_{13} = [P(e)\, P(A1) + P(e`)\, P(A2)] \qquad \dots\dots\dots\dots\dots\dots\dots (9)$$

Assuming that all edges are identical and independent with probabilities of success and failure of p = 0.9 and q = 0.1.



(a)Edge (e) is open **(b) Edge (e) is short**

Figure 8. Illustration of EdgeFactoring

In case of A1 edge e is open, hence p_{13} can be determined easily by applying series reduction on edges *a and b* to get one edge the same thing in terms of edges c and d finally applying parallel reductions to get one path from *1 to 3* as in equation (9):

$$P\ A1\ = pa \cdot\ pb\ + pc\ \cdot pd - (pa \cdot\ pb\ \cdot pc\ \cdot pd\ )\ \dots \dots \dots \dots \dots \dots \dots (10)$$

While case A2 show that edge *e* is short, which just inverts the sequence of the reduction techniques. Firstly, we apply the parallel one to get one path from *1 to 2, 4* repeating the same thing to get also one path from *2,4 to 3* finally series technique results in one path from the source to destination as in the following equation:

$$P\ A2\ = (1 - q_a\ q_c) \times (1 - q_b\ q_d)\ \dots \dots \dots \dots \dots \dots \dots (11)$$

Thus equation (8) becomes:

$$P_{13}\ = P(e) \times (pa\, pb\ + pc\, pd - pa\, pb\, pc\, pd)$$
$$+\ p(e`) \times (1 - qa\, qc). (1 - qbqd)\ \dots \dots \dots \dots \dots \dots \dots (12)$$

To perform all-terminal reliability transformations, a combination of the previous three ways can applied to any network, we can take the same examples of Figs.7 & 8 to illustrate their procedures.

The first technique is series reduction which has the same term of the two-terminal reliability (intersection term), but it has to be multiplied by the probability of both the ingoing and outgoing edges liked to the eliminated node and this is the lonely difference between the two-terminal and all-terminal reliability. Whereas , the parallel transformation and node imperfection procedures don't differs of those mentioned of the two-terminal reliability where in terms of parallel transformation the equation also will be the same ; $P_{12} = P(a + b)$ (the union of the two parallel edges),the same thing in the case of edge factoring its equation doesn't affected, since the same procedure can followed to get $A_1 = P[(a + c)] \times P[(b + d)]$ $(good\ case)\ as\ well\ A_2 = P[(a \cdot b) + (c \cdot d)]$ $(bad\ case)$.

Fig.9 can illustrate the series & parallel reduction procedure for all-terminal reliability where firstly we have to choose s and d nodes as dealing with two-terminal let the source node to be 1 and destination one is 3.

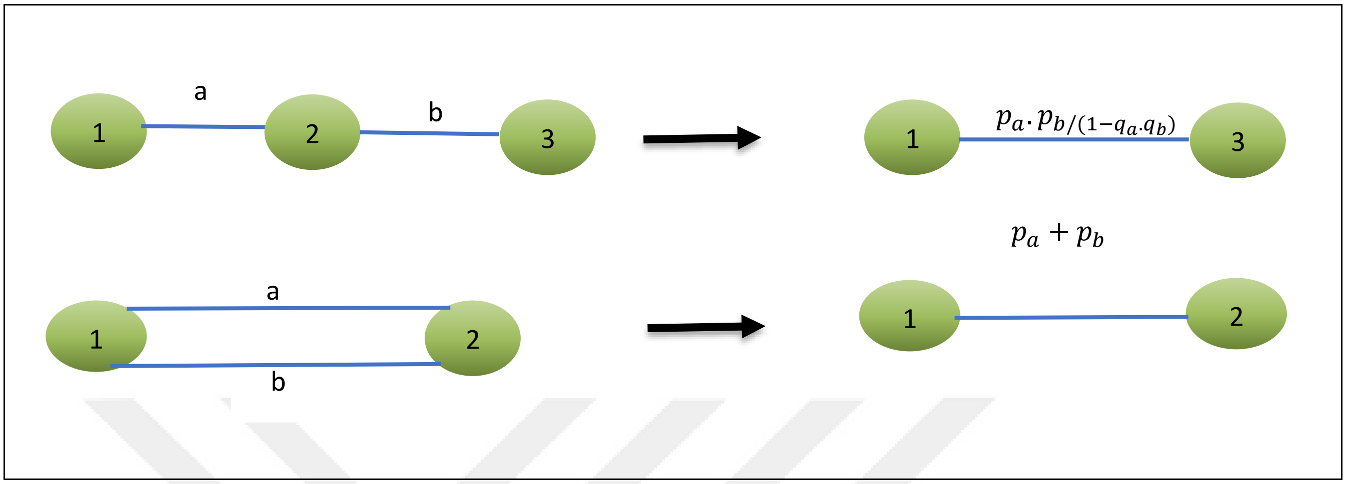


Figure 9. Illustration of $SR_t \& PR_t$ for All- Terminals

2.4.4 Approximation methods

- **<u>1-Truncation Approximations</u>**

This method used to reduce the number of terms those consist the inclusion exclusion- expansion equation. These terms are product of probabilities, so if these probabilities are small, the higher-order product terms can be neglected thus it can result in simpler formula. Reliability of the network is found between a maximum and a minimum bound, these bounds can be found based on the excepted value of the error (difference between exact and approximated value). In case of tie set probabilities, they are often high probabilities to perform a reasonable practical network.

Thus, thinking of cut set approximation as more favorable than tie set approximation, as the first one about small probability (probabilities of failure) .The value of the reliability lays between two values as below [13]:

$$Upper\ bound\ \geq\ approximated\ reliability\ \geq\ lower\ bound$$

The lower bound can found sharply by considering the union event as disjoint event, in this case the lower bound is only the summation of the cut sets; however, it is also desired to calculate the value of the upper bound this can be done by adding other terms from the basic equation of the joint events to the summation of the cut sets. We can get varied values of the upper bound by including different number of terms this thing depends on what has required.

- **<u>2-Subset Approximations</u>**

Another way that can applied to reduce the complexity of inclusion exclusion expansion equation is to drop the higher order cut sets or tie sets, more clearly the lower bound can be computed after neglecting the higher order tie sets from the reliability equation for example if there are four tie sets $(T1, T2)$ of tow hops and $(T3)$of three hops, where $(T4)$ of four hops. By eliminating T4 which is the highest order set, to get the upper bound, the equation will be as below [13]:

$$Upper\ Bound\ = p\,T1 + p\,T2 +\ pT3 \qquad \dots\dots\dots\dots\dots\dots\dots (13)$$

We can also exclude the higher order cut- sets from the reliability equation, considering the same cases of mentioned tie set, the upper bound equation is $Lower\ Bound\ = 1 - p\ (C1\ + C2\ + C3)\qquad \dots\dots\dots\dots\dots\dots\dots (14)$

Although, we can approximate the reliability by taking the midpoint of the two bounds by equation (14), The percentage error is larger than that of the truncation approximations; however, the approximation is still valid since it reduced the order of the exponential below than what was when exact calculations applied, as well the time consumed by both tie set ant cut sets algorithms also reduced since not all the sets have to be found [13].

$$\text{Midpoint } = Upper\ bound\ +\ lower\ bound\ /2 \dots\dots\dots\dots\dots\dots\dots (14)$$

3. THE PROPOSED ALGORITHM

3.1 REVIEW ON TIE SET GENERATION:

The complicity of reliability computations grows exponentially when a computer network size increases gradually; that means, more time consumed. Thus, some conventional methods may fail to measure the reliability of middle and large size network efficiently. Efficiently here means, fast and accurate way. Depending on the purpose and the functionality of the network some are needed to be analyzed and evaluated in a fully accurate way for ensuring safety, while others are preferred to be analyzed with an accepted approximation way but quickly. In other words, exact methods are usually preferred when the purpose is accurate results with statistic networks. Whereas, approximation becomes the best solution with the case of dynamic topologies, for example military networks. However, Reliability calculation of complex networks is usually accomplished by minimal path or cut enumeration. [16] cut-set method is very interesting in reliability analysis of many applications, especially when the planning & design of a system demands adequate ware of highly sensitive, dominative & exposure components for taking the required measures to ensure the optimum operation [43]. Different approaches have been introduced to conduct the minimal paths enumeration in two-terminal reliability analyses such as Modified Flooding Algorithm to deduce paths in bidirectional and unidirectional graphical representation of a communication network [4], which demands a high programming experience as well coding procedure to ensure that no node will be traversed twice. Matrix Multiplication is another approach where the connection matrix should be multiplied by itself many times until it gives a fixed result. However, this method requires the application of Boolean algebra. Path Tracing Algorithm has been introduced in [43] in order to tracing all minimal paths if both bidirectional and unidirectional links are considered. This method is suitable for two-terminal reliability and dose not demand Boolean algebra application but the problem is about programing. The hardness is likely to grow with the increasing of network size. Acyclic Paths Mergence algorithm has been proposed in [44] in the first step, the sets of paths are found. This can be done by carrying out a breadth-first search on the topology. In the second step, a recursive procedure gradually combines the paths belonging to various sets and completes the main task. Another well -Known method is Edge Replacement Search. However, the last three methods are all depend on producing all spanning trees which Requires High Memory. The proposed Tie-Set generation approach overcomes all the listed algorithms by the next three advantages:

1) Requires less memory because it traces minimal paths basing on permutation law $(Nr-1)!$ instead of generating all spanning trees$N^{(N-2)}$; ***ex:*** if N = 5, 6 ,7; NO. of spanning trees is **125, 1296, 16807** respectively, while in the proposed tie set approach $\mathrm{v}=\mathbf{24}$, $\mathbf{120}, \mathbf{720}$ respectively .

2) No need for Boolean algebra application.

3) feasible to programmed and does not need coding procedure.

3.2 MHRT DEFINITION:

The proposed multi-stage algorithm is a hybrid one which is based on two- terminal reliability evaluation methods are graph reduction techniques and tie-set method. It is composed by three stages, the first stage is, initial stage, followed by two other stages where each one is composed of sub-stages. The second stage of the algorithm concerns with producing a simpler network by reducing the number of nodes and edges by applying series and parallel techniques. While in the third stage, the minimal tie-sets (all the routs from the source node to a destination one without crossing any node more than one time are obtained. Finally, the reliability is evaluated by applying the inclusion exclusion expansion equation.

3.3 MHRT STRUCTURE:

3.3.1 Initialization Stage

As a computer network can represented graphically by nodes and edges, this graph $G\ (N, E)$ can be interpreted by three-dimensional adjacency matrix, $M\ (N\ , N\ , k)$. Depending on the maximum number of parallel links between two communicated entities, the value of K is decided.

For example, if maximally there are four parallel links, k is 4 that means, each matrix contains one parallel state and so on. However, if the network has no parallel links, two planes, $M\ (N\ , N\ , 2)$, are needed at this stage to achieve the test validity for existence of parallel edges in the next stage. In that state, the elements of $M_{N,N,1}$ represent the actual edges of the network, while all the elements of the second one $M_{N,N,2}$ are zeros.

Each element in $M_{N,N,1}$represents one edge probability of operating successfully. Whereas, the elements of $M_{N,N,2}$ describe the parallel edges probabilities of being up.

- $M_{i,j,k}$ = 1 ≤ p_{ijk}≤ 0
- If I = j, diagonal element indicates the probability of self-connection for node n_i

$$M_{i,j,k} = \begin{bmatrix} p_{11k} & \cdots & p_{N1k} \\ \vdots & \ddots & \vdots \\ p_{N1k} & \cdots & p_{NNk} \end{bmatrix}$$

3.3.2 Reduction Stage

Two approaches SR_t & PR_t are applied in order to reduce the number of nodes and edges; consequently, the reliability value can be obtained simply with acceptable processing time.

A. Parallel Reduction Procedure

The first step of this approach is to test the third dimension of the initial matrix, in order to decide with what plane, it will start the reduction. As what has been illustrated in the previous section even when there are no parallel edges the initial matrix is 3D.

Hence, the second plan has to be checked to see if there is any parallel edge or not.

If M_{ij2} is zero matrix there is no parallel links and series reduction can be directly applied. While, $p_{ij2} \neq 0$, means that parallel reduction is needed and it can be implemented as the following:

- Take the union of the two parallel edges :

  $p_{ij1*} = p_{ij1} \cup p_{ij2} = p_{ij1} + p_{ij2} = 1 - (1 - p_{ij1})(1 - p_{ij2}) = 1 - q_{ij1} \cdot q_{ij2}$
- $p_{ij2} = 0$

In the case of $k > 2$, parallel edges in the last plane $M_{i,j,k>2}$ are treated firstly, by considering plane $\boldsymbol{M_{i,j,k-1\geq 2}}$ is the base one. After applying parallel reduction procedure on the two previous matrices, the first one is deleted (all its elements become zeros), while the other one is updated to contain the new calculated values. This procedure is applied till we have got two-dimensional matrix $M`_{NxN}$ that is, with no parallel edges.

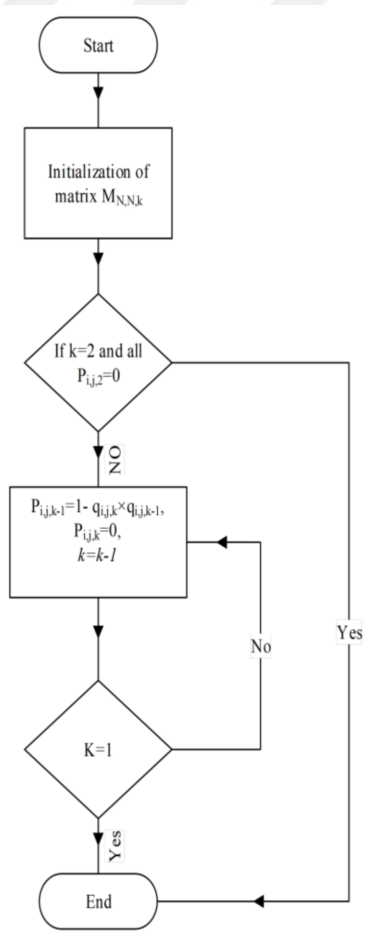


Figure 10. Flow Chart of Parallel Reduction

B. Series Reduction Procedure

Series reduction technique can be accomplished after checking the initial condition of node elimination validation that is, the node is neither a source nor destination.

If the above condition is true the second step starts by counting with how many nodes it is connected. When it is linked with just two neighbors, series reduction can easily apply as the intersection of such edges as below:

- Not; $j_1 = i_2 = a$

➢ $p_{i1j2`} = p_{i1a} \cap p_{aj2} = p_{i1a} \cdot p_{aj2}$

➢ $p_{i1a}, p_{aj2}, n_a = 0$

For a bi direction connection, the inverse rout is determined as the following equation:

➢ $p_{j2i1`} = p_{a\,i1} \cap p_{j2\,a} = p_{ai1} \cdot p_{j2a}$

➢ $p_{ai1}, p_{j2a}, n_a = 0$

For each eliminated node, there is a new position $(j_2, i_{1`})$ has to be filled with a new value, $p_{j2i1`}$, which results from applying series reduction approach and it has to be checked before filling.

If there is no saved value (p_{i1j2} = 0), the calculated value is localized directly. Whereas if the location is already filled with a value ($p_{i1j2} \neq$ 0), parallel reduction becomes inevitable step.

Union of the initial value p_{i1j2} and the new calculated one $p_{\mathbf{i1j2`}}$ results in a single edge from n_{i1} to n_{j2}:

➢ $p_{i1j2*} = p_{i1j2} + p_{i1j2`} = 1 - q_{i1j2} \cdot q_{i1j2`}$ … … … … … … … (15)

Reduction stage ends when there is no possible elimination for nodes or edges. Now a new topology has been produced with less number of edges and nodes and it can represented by R (N_r, N_r) .

- **Note:** $Remaining\ nodes\ (N_r) = Original\ nodes\ (Nor\ )$ – $Reduced\ nodes\ (Nre)$

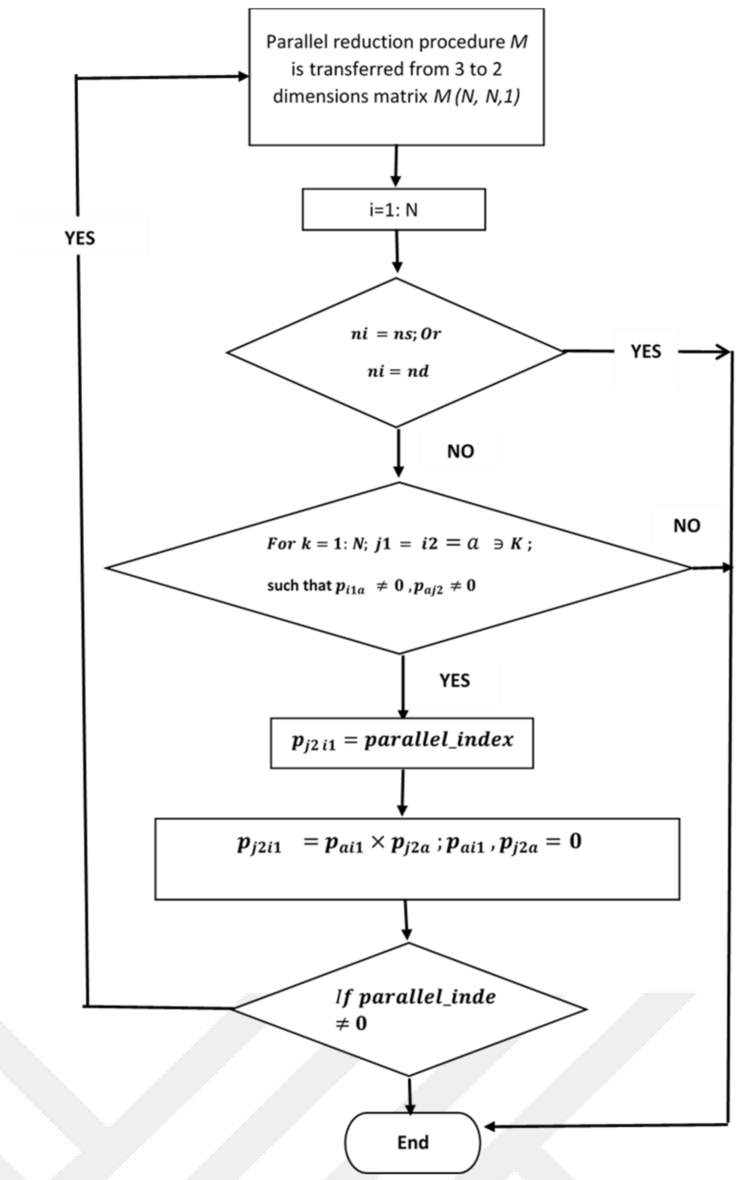


Figure 11. Flow Chart of Series Reduction

3.3.3 Tie- Sets Generation Stages:

After reduction stage matrix $M_{N,N,k}$ is converted into two-dimensional matrix $R_{Nr,Nr}$, with $Nr \leq N$. $R_{Nr,Nr}$ can be used to generate arrangement matrix, $T(v, Nr)$ where N_r is the number of nodes after reduction, and *v* is the number of passible different arrangements for $N_r - 1$ nodes.

$v = (Nr - 1)!$

The essential step for the traditional tie-set generation starts by denoting the source node (n_s) and the destination node (n_d), then seeking for all the possible minimum paths those connecting the pair (n_s, n_d). A single route may be composed of one or a group of edges and nodes.

Since the remaining nodes Nr are less than the original ones before reduction, the arrangement operation does not demand a huge memory or consuming a long processing time. Matrix *T* is formed by enumerating all possible combinations of remaining nodes (Nr) except the source node (n_s). As in (11) the number of combination is $v = (Nr - 1)!$. Elements of *T* are node numbers (giving the location of links between nodes in the matrix *R*). After that the first column is inserted with all elements equal to the number of source node (n_s). For example: $T_{3,2} = 5$, and $T_{3,3} = 3$, means that node number (5) is located at rows (3) and column number (2) at T matrix, and the element after (in the same row) is node number (3). In order to find the corresponding link probability between node (5) and node (3), element ($R[5,3] = P_{5,3}$), in matrix *R* is copied.

The ***next step*** is to check wither each successive two nodes in matrix *T* are linked or not by consulting the corresponding value of $P_{i,k}$ elements in matrix *R*. An action is made when there is no connection between two successive nodes (n_i to n_k), in one row of matrix *T*, ($P_{ik} = 0\ in\ matrix\ R$):

- Replace node n_k by n_i

This measure ensures the correctness of the connection check operation since no possible connecting case may be skipped. In order to get the correct sequence for each tie set, the following simplification steps are taken:

- Eliminate all the repeated nodes in one row,
- Each node after the destination (n_d) has to be removed,
- Eliminate identical tie-sets (redundant)

Finally, we will get a matrix TS (t_{ts},t_r), which contains the entire tie-sets, with number of rows is equal to the number of the tie sets t_{ts}, while columns number is in maximum equals to the number of nodes Nr ($t_r \leq Nr$). Figure.12 presents all steps for tie-sets procedure.

Now reliability can be measured by Appling equation (4)

$$R = P_r(T_1 + T_2 + T_3 + \cdots + T_i)$$

Since these tie sets are not disjoint events the union of them demands inclusion and exclusion of intersecting terms as below:

R= $P_r(T_1) + P_r(T_2) + P_r(T_3) + \cdots + P_r(T_i)$

= $[P_r(T_1) + P_r(T_2) + P_r(T_3) + \cdots + P_r(T_i)] - [P_r(T_1 T_2) + P_r(T_1 T_3) + \cdots + P_r(T_b T_{bb})_{b \neq bb}] + [P_r(T_1 T_2 T_3) + P_r(T_1 T_3 T_4) + \cdots + P_r(T_b T_{b2} T_{b3})_{b1 \neq b2 \neq b3}] + \ldots\ldots + (-1)^{i-1}[P_r(T_1 T_2 T_3 \ldots\ldots T_i)]$ ……………… (15)

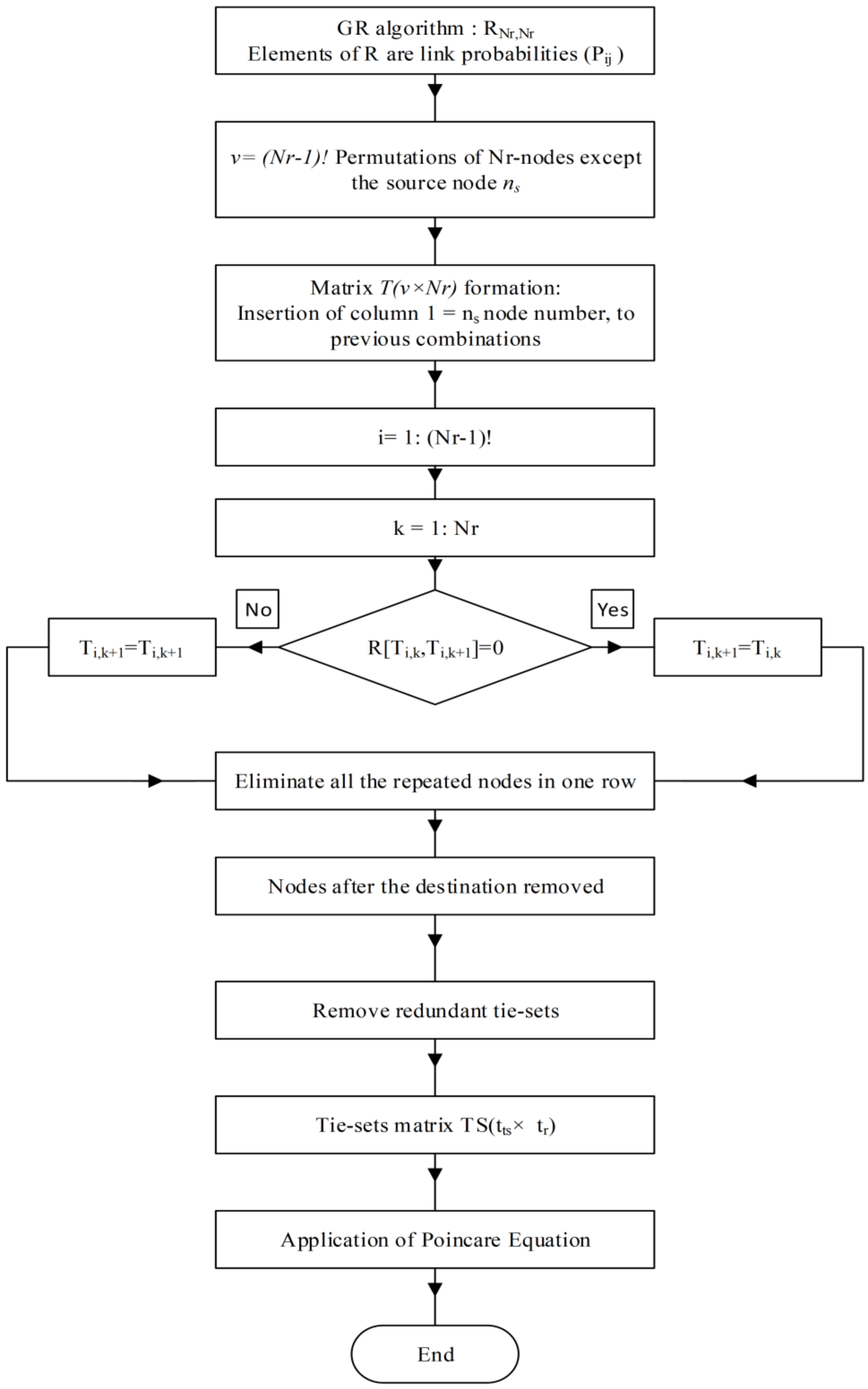


Figure 12. Flow Chart of Minimal Tie-Sets Generation

4. TWO TERMINAL RELIABILITY EVALUATION

Evaluation of two-terminal reliability by the best classical method (tie set) is the simplest problem comparing with k- terminal and all-terminal problems when the network is simple. The problem becomes more and more complicated for every additional element till the method fails to trace all minimal paths and execute the inclusion exclusion expansion equation. MHRT overcomes tie -set method since it can do the task when tie-set method cannot. Even when tie -set succeed it stills consume more time that is because the number of tie sets are more or longer than those gotten by MHRT. The case study of this chapter satisfies different situations.

4.1 RELIABILITY OF SIMPLE MESH CCN:

Let the network 1 of Fig.13 which comprises $|N| = 5$ & $|E| = 9$ $(directional\ edges)$.All the edges are bidirectional but E_{14} is unidirectional edge.

Consider that source node is 1 and the target is 4.

- p_{ijk} = 0.9, for all edges.
- p_{ijk} = 0, if there is no connection.
- For i= j, $p_{ijk} = 1$

According to **MHRT** the first stage is the initialization**.** Since the network shown has one parallel edge connects 1&3, there are two adjacency matrices M (6 ,6 , 2):

$$\boldsymbol{M_1} = \begin{pmatrix} 1 & 0.9 & 0 & 0.9 & 0 & 0.9 \\ 0.9 & 1 & 0.9 & 0 & 0 & 0 \\ 0 & 0.9 & 1 & 0.9 & 0 & 0 \\ 0 & 0 & 0.9 & 1 & 0.9 & 0 \\ 0 & 0 & 0 & 0.9 & 1 & 0.9 \\ 0.9 & 0 & 0 & 0 & 0.9 & 1 \end{pmatrix} \quad , \boldsymbol{M_2} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.9 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

With M_2, $both\ p_{23}.p_{32} = 0.9$, consequantly$PR_t$ is applied as below:

- $p_{231*} = p_{231} \cup p_{232} = \ \ 1 - q_{231} \cdot q_{232} = 1 - (0.1 \times 0.1) = 0.99$
- $p_{321*} = p_{321} \cup p_{322} = \ \ 1 - q_{321} \cdot q_{322} = 1 - (0.1 \times 0.1) = 0.99$

➢ $p_{232}, p_{322} = 0$. Now $M_{ij2} = [0]$ for all i, j.

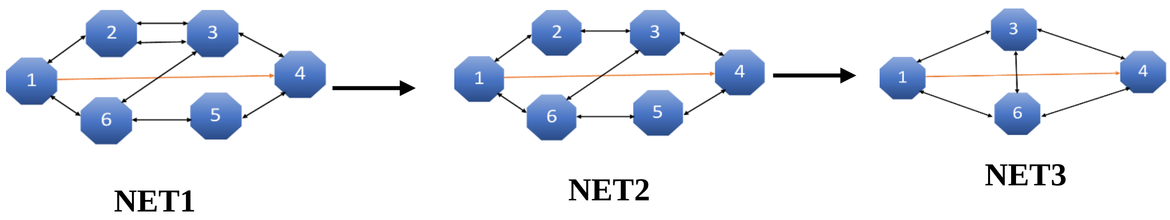


Figure 13. Simplifying of MESH Network

The initial condition of starting series reduction is to check if the node is neither source nor destination which means excluding $n_1 \& n_4$ during this stage. Looking at M_1 the rows of 2, 3, 5, and 6 are all have just two nonzero values which satisfying the condition of SR_t.

- $p_{13} = p_{13*} = p_{12} \cap p_{23} = 0.9 \times 0.99 = 0.891$, $p_{12} = 0$
- $p_{31} = p_{31*} = p_{21} \cap p_{32} = 0.9 \times 0.99 = 0.891$, $p_{21} = 0$
- $p_{46} = p_{46*} = p_{45} \cap p_{56} = 0.9 \times 0.9 = 0.81$, $p_{45} = 0$
- $p_{64} = p_{46*} = p_{54} \cap p_{65} = 0.9 \times 0.9 = 0.81$, $p_{54} = 0$

Now there is no possible reduction, thus $(N_r) = 6 - 2 = 4 = \{1,3,4,6\}$

$$R_{4,4} = \begin{pmatrix} 1 & 0.891 & 0.9 & 0.9 \\ 0.891 & 1 & 0.9 & 0.9 \\ 0 & 0.9 & 1 & 0.81 \\ 0.9 & 0.9 & 0.81 & 1 \end{pmatrix}$$

The next step is to generate $T(v\ ,\ N_r)$, with this case since $N_r - 1 = 3$ *the posible arrangementsof* $\{3,4,6\} =\ 6 = v$. Comparing with the classical methods: all spanning trees = $N^{(N-2)}$ = $6^{(6-2)}$ =1296; which is more than v in noticeable way.

$$T_{6,4} = \begin{pmatrix} 1 & 3 & 4 & 6 \\ 1 & 4 & 3 & 6 \\ 1 & 3 & 6 & 4 \\ 1 & 4 & 6 & 3 \\ 1 & 6 & 4 & 3 \\ 1 & 6 & 3 & 4 \end{pmatrix} \rightarrow T'_{6,4} = \begin{pmatrix} 1 & 3 & 4 & 0 \\ 1 & 4 & 0 & 0 \\ 1 & 3 & 6 & 4 \\ 1 & 4 & 0 & 0 \\ 1 & 6 & 4 & 0 \\ 1 & 6 & 3 & 4 \end{pmatrix}$$

After getting $T_{6\times 4}$ each row is checked successively. For the first one it is obvious that there is connection for **1→3→4.** Since destination is reached, set every node after it to be zero to get the first minimal path. The check operation of the second row is stooped directly at the second column when reaching destination and again all nodes after 4 become zeros to get the second tie set **1→4.** By the same procedure the remaining tie sets are found. Then delete the duplicated rows to get unique tie sets as follow:

$\boldsymbol{T_1} = \mathbf{1 \rightarrow 3 \rightarrow 4}$, $\boldsymbol{T_2} = \mathbf{1 \rightarrow 4}$, $\boldsymbol{T_3} = \mathbf{1 \rightarrow 3 \rightarrow 6 \rightarrow 4}$, $\boldsymbol{T_4} = \mathbf{1 \rightarrow 6 \rightarrow 4}$,

$\boldsymbol{T_5} = \mathbf{1 \rightarrow 6 \rightarrow 3 \rightarrow 4}$

By applying equation (15); **R=0.9968.**

When the task is computing tie sets for **NET2,** take for example T_{1j} = {1 6 5 3 2 4}, when it has been checked with corresponding $R_{6\times6}$; $p_{53}, p_{32}, p_{24} = 0$

$then\ T_{1j}$ = {1 6 5 5 5 4} → {1 6 5 4}.

The duplicated number means that three connection cases are checked 5→3, 5→2 & 5→4. Because there are no connections for the first two cases both 2&3 must be replaced by 5 to continue tracing the bath correctly and checking every new nod with last connected one **5.** Take another path forming case $T_{5j} = \{1\ 5\ 2\ 3\ 6\ 4\} \rightarrow \{1\ 1\ 2\ 3\ 4\ 4\} \rightarrow \{1\ 2\ 3\ 4\}$.The other cases are treated by the same manner to produce all 5 the minimal paths as listed below: $\boldsymbol{T_1} = \mathbf{1 \rightarrow 2 \rightarrow 3 \rightarrow 4}$, $\boldsymbol{T_2} = \mathbf{1 \rightarrow 4}$, $\boldsymbol{T_3} = \mathbf{1 \rightarrow 2 \rightarrow 3 \rightarrow 6 \rightarrow 5 \rightarrow 4}$, $\boldsymbol{T_4} = \mathbf{1 \rightarrow 6 \rightarrow 3 \rightarrow 4}$, $\boldsymbol{T_5} = \mathbf{1 \rightarrow 6 \rightarrow 5 \rightarrow 4}$.

For NET1 the number of minimal paths is 7 since there is parallel edge connects 2 &3.

They are same with those of Net2 but adding $\boldsymbol{T_{1p}}$ & $\boldsymbol{T_{3p}}$ resulting from the parallel edge.

4.2 № NETWORK TOPOLOGY:

Specific network topology № with a cut point node denoted as ©, that divides the network topology into completely separated sub networks can be treated efficiently by our algorithm. In order to decrease the processing time. Specific technique (₸) to the initiation stage & tie set generation stage with this technique is added. With ₸ we ensure that no undesirable node (those locate in dead portion; don't form any path) is counted while producing MPs for each commodity reliability calculation.

4.2.1 Problem Definition:

As our tie set technique is based on considering all nodes except source node when finding each tie set. With mesh topology, this does not constitute a burden, since the probability of any node to be traversed p_{tv} is 0 or 1 (no dead portion). While there are group of nodes in № have $p_{tv} = \phi$ as mentioned in the previous section. The reason behind that is, © cannot be traversed twice when tracing a MP because we concern minimal tie set method. This means, when both source and destination belong to the same sub net no other subnet nodes participate in any path.

- *Cases and measures are summarized as the following*:

a) Entering each sub net by its matrix $m_1, m_2, ...\ , m_k$. (k= number of sub nets)

b) Three cases are ordered:

1. Case one: when $(s, d) \ni m_1$,all $m_{i\neq1}$ = 0
2. Case two: $(s, d) \ni m_2$,all $m_{i\neq2}$ = 0
3. Case three: $\ni m_{k-1}$, $d \ni m_k$ or $d \ni m_{k-1}$, $s \ni m_k$.

c) Counting the reliability of the central node with all basic network nodes.

With case 1, only nodes of m1 are contribute to produce the tie sets and case 2 means that only m2 nodes are taken.

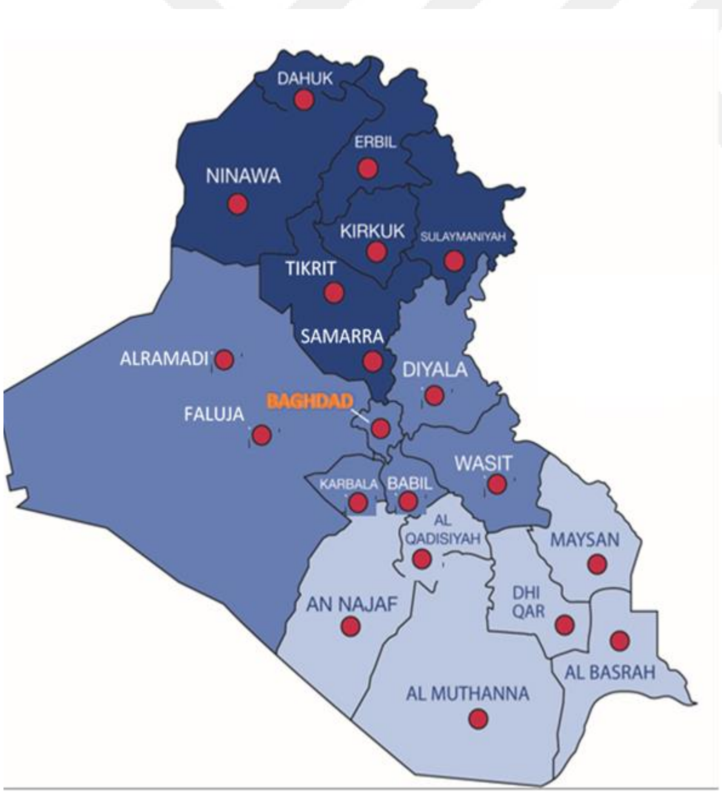
While in case 3 the reliability of two commodities can be found by the products of $R_{s©}$, $R_{©d}$.

4.2.2 WAN № Reliability:

An imaginative basic WAN № for all big cities in Iraq as shown in Fig.10 has been introduced in this section as in Fig.11, to implement the proposed algorithm with this special topology and study its efficiency in terms of the accuracy, high response and quickness. **BAGHDAD** which is the capital of Iraq is **©**, results in converting the basic network to two disconnected sub nets when it is removed. Let the northern network is **net1** and the southern is **net2**.

The basic network topology has been changed four times as illustrated in Fig.12 by adding different edges between cities and study the complexity impact of each addition.

The task is to measure all two-terminal reliability. All the next topologies are presented for comparison purpose between the tie-set method and MHRT in terms of the number of minimal paths as well as the execution time:



number	city
1	BAGHDAD
2	DAHUK
3	ERBIL
4	SULAYMANIYAH
5	KIRKUK
6	NINAWA
7	TIKRIT
8	SAMARRA
9	ALRAMADI
10	FALUJA
11	DIYALA
12	KARBALA
13	BABIL
14	ALNAJAF
15	ALQADISIYAH
16	ALMUTHANA
17	WASIT
18	MAYSAN
19	DHIQAR
20	ALBASRAH

Figure. 14 Map of Iraq

4.3 CASE STUDY

4.3.1 Basic Topology (NET1):

Consider the network shown in Fig.15 connects all big cities in Iraq and each city can communicate by two ways.

S is the source node, while **D** is the destination. **R** is the reliability which must be the same for both simulated algorithms. $\boldsymbol{M_{pH}}$**, and** $\boldsymbol{T_H(s)}$, are respectively the number of tie-sets and time required for reliability evaluation (in sec) for MHRT algorithms. $\boldsymbol{M_{pt}}$**,** and $\boldsymbol{T_t(s)}$**,** are same variables for the classical tie-sets algorithms.

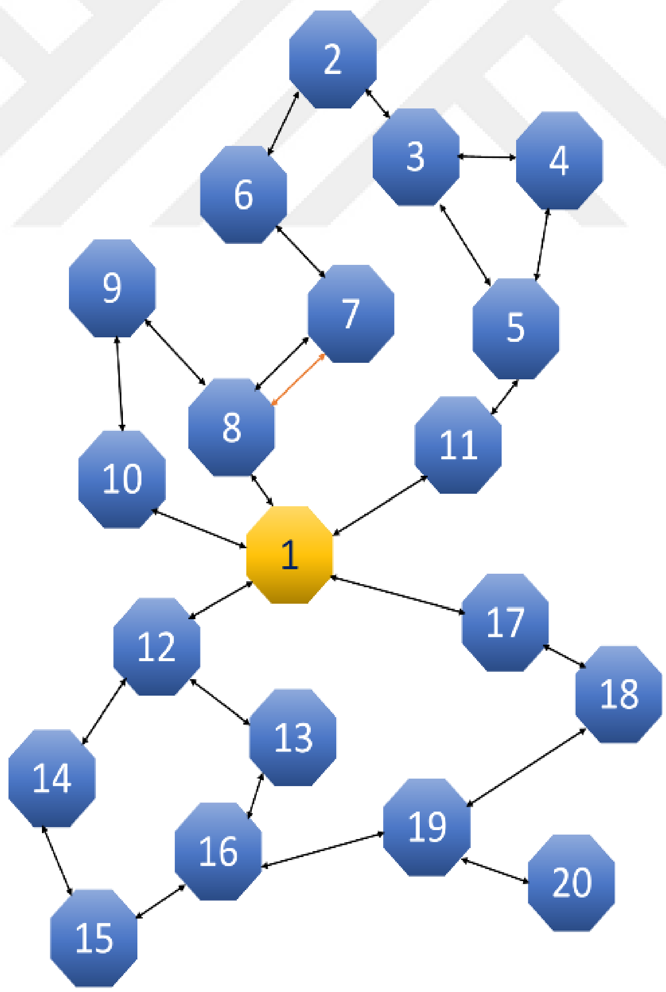


Figure15. Basic Network (NET1)

Table 8. Comparison Results of all NET1 Possible Commodities

S	D	Mp_H	T_H(s)	Mp_t	T_t(s)	R
1	2	1	0.00353	6	10.4172	0.93738
1	3	1	0.00633	6	10.5311	0.93882
1	4	4	0.0100	10	12.7721	0.93060
1	5	1	0.00361	9	10.5614	0.94087
1	6	1	0.00226	6	10.5776	0.95255
1	7	1	0.00194	6	10.2674	0.98450
1	8	1	0.00179	6	10.1429	0.98844
1	9	1	0.00180	6	10.4566	0.97371
1	10	1	0.00200	6	10.5569	0.97755
1	11	1	0.00200	9	10.4859	0.96199
1	12	2	0.00945	3	1.21065	0.96223
1	13	4	0.01544	4	1.06430	0.94680
1	14	4	0.01319	4	1.01374	0.94037
1	15	4	0.01386	4	0.95241	0.93620
1	16	2	0.00305	3	0.95085	0.94967
1	17	2	0.00276	3	1.04519	0.96223
1	18	2	0.00306	3	0.97193	0.94138
1	19	1	0.00248	3	0.94365	0.93721
1	20	1	0.00223	3	0.94788	0.84349
2	3	1	0.00271	9	10.5444	0.96199
2	4	4	0.03366	10	10.5693	0.95001
2	5	1	0.00378	6	10.0366	0.95690
2	6	1	0.00267	9	10.6401	0.96199
2	7	1	0.00266	9	10.6343	0.94087
2	8	1	0.00246	6	10.4406	0.93973
2	9	4	0.00574	8	10.5748	0.92137
2	10	4	0.00539	8	10.6508	0.92059
2	11	1	0.00234	6	10.4445	0.93882
2	12	1	0.01298	1	11.62785	0.88254
2	13	1	0.01897	1	11.48150	0.86839
2	14	1	0.01672	1	11.43094	0.86249
2	15	1	0.01739	1	11.36961	0.85866
2	16	1	0.00658	1	11.36805	0.87102
2	17	1	0.00629	1	11.46239	0.88254
2	18	1	0.00659	1	11.38913	0.86341
2	19	1	0.00601	1	11.36085	0.85959
2	20	1	0.00576	1	11.36508	0.77363
3	4	1	0.00341	6	10.9557	0.98612
3	5	1	0.00225	6	10.9009	0.99181
3	6	1	0.00212	9	12.3033	0.94087
3	7	1	0.00209	9	13.5388	0.93641
3	8	1	0.00222	6	11.5069	0.93685
3	9	4	0.00316	8	11.9060	0.91995
3	10	4	0.00383	8	11.4420	0.92060
3	11	1	0.00197	6	10.6987	0.95690
3	12	1	0.01578	1	11.74175	0.89075
3	13	1	0.02177	1	11.5954	0.87647
3	14	1	0.01952	1	11.54484	0.87051
3	15	1	0.02019	1	11.48351	0.86665
3	16	1	0.00938	1	11.48195	0.87913
3	17	1	0.00909	1	11.57629	0.89075
3	18	1	0.00939	1	11.50303	0.87145
3	19	1	0.00881	1	11.47475	0.86759
3	20	1	0.00856	1	11.47898	0.78083
4	5	1	0.00322	6	10.5393	0.98612
4	6	4	0.00579	10	10.7343	0.93060
4	7	4	0.00573	10	10.8136	0.92768
4	8	4	0.00527	8	10.9025	0.92826
4	9	8	0.08000	12	12.9745	0.91163
4	10	8	0.07369	12	12.9800	0.91241
4	11	4	0.00522	10	10.7814	0.95001
4	12	1	0.01945	1	13.98275	0.88356
4	13	1	0.02544	1	13.8364	0.86939
4	14	1	0.02319	1	13.78584	0.86348
4	15	1	0.02386	1	13.72451	0.85965
4	16	1	0.01305	1	13.72295	0.87203
4	17	1	0.01276	1	13.81729	0.88356
4	18	1	0.01306	1	13.74403	0.86441
4	19	1	0.01248	1	13.71575	0.86058
4	20	1	0.01223	1	13.71998	0.77452
5	6	1	0.00190	6	10.9638	0.93882
5	7	1	0.00209	6	10.9080	0.93738
5	8	1	0.00203	6	10.9703	0.93811
5	9	4	0.00494	10	14.3046	0.92144
5	10	4	0.00471	10	13.0196	0.92236
5	11	1	0.00203	9	10.8295	0.96199
5	12	1	0.01306	1	11.77205	0.89389
5	13	1	0.01905	1	11.6257	0.87956
5	14	1	0.01680	1	11.57514	0.87358
5	15	1	0.01747	1	11.51381	0.86971
5	16	1	0.00666	1	11.51225	0.88222
5	17	1	0.00637	1	11.60659	0.89389
5	18	1	0.00667	1	11.53333	0.87452
5	19	1	0.00609	1	11.50505	0.87064
5	20	1	0.00584	1	11.50928	0.78358
6	7	1	0.00201	9	10.6514	0.96199
6	8	1	0.00198	6	10.6730	0.95925
6	9	4	0.00472	8	10.9663	0.93912
6	10	4	0.00467	8	11.0033	0.93690
6	11	1	0.00202	6	10.9804	0.93738
6	12	1	0.01171	1	11.78825	0.88955
6	13	1	0.0177	1	11.6419	0.87529
6	14	1	0.01545	1	11.59134	0.86934
6	15	1	0.01612	1	11.53001	0.86548
6	16	1	0.00531	1	11.52845	0.87794
6	17	1	0.00502	1	11.62279	0.88955
6	18	1	0.00532	1	11.54953	0.87027
6	19	1	0.00474	1	11.52125	0.86642
6	20	1	0.00449	1	11.52548	0.77978
7	8	1	0.00189	6	10.9939	0.99564
7	9	4	0.00453	8	10.6311	0.97340
7	10	4	0.00446	8	10.8659	0.96970
7	11	1	0.00214	6	10.8615	0.95255
7	12	1	0.01139	1	11.47805	0.91187
7	13	1	0.01738	1	11.3317	0.89725
7	14	1	0.01513	1	11.28114	0.89115
7	15	1	0.01580	1	11.21981	0.88720
7	16	1	0.00499	1	11.21825	0.89997
7	17	1	0.00477	1	11.31259	0.91187
7	18	1	0.0050	1	11.23933	0.89211
7	19	1	0.00442	1	11.21105	0.88816
7	20	1	0.00417	1	11.21528	0.79934
8	9	1	0.00190	6	10.7288	0.97755
8	10	1	0.00197	6	10.5075	0.97371
8	11	1	0.00191	6	11.0320	0.95487
8	12	1	0.01124	1	11.35355	0.94975
8	13	1	0.01723	1	11.2072	0.93452
8	14	1	0.01498	1	11.15664	0.92817
8	15	1	0.01565	1	11.09531	0.92405
8	16	1	0.00484	1	11.09375	0.93735
8	17	1	0.00455	1	11.18809	0.94975

S	D	Mp_H	T_H(s)	Mp_t	T_t(s)	R
8	18	1	0.00485	1	11.11483	0.92917
8	19	1	0.00427	1	11.08655	0.92505
8	20	1	0.00402	1	11.09078	0.83255
9	10	1	0.00178	6	10.7795	0.97755
9	11	4	0.00184	10	11.3765	0.93931
9	12	1	0.01125	1	11.66725	0.93607
9	13	1	0.01724	1	11.5209	0.92107
9	14	1	0.01499	1	11.47034	0.91480
9	15	1	0.01566	1	11.40901	0.91075
9	16	1	0.00485	1	11.40745	0.92386
9	17	1	0.00470	1	11.50179	0.93607
9	18	1	0.00468	1	11.42853	0.91579
9	19	1	0.00442	1	11.40025	0.91173
9	20	1	0.00417	1	11.40448	0.82056
10	11	4	0.00424	10	12.8039	0.94167
10	12	1	0.01145	1	11.76755	0.94022
10	13	1	0.01744	1	11.6212	0.92514
10	14	1	0.01519	1	11.57064	0.91886
10	15	1	0.01586	1	11.50931	0.91478
10	16	1	0.00505	1	11.50775	0.92795
10	17	1	0.00476	1	11.60209	0.94022
10	18	1	0.00506	1	11.52883	0.91984
10	19	1	0.00448	1	11.50055	0.91577
10	20	1	0.00423	1	11.50478	0.82419
11	12	1	0.0115	1	11.69655	0.92566
11	13	1	0.0174	1	11.5502	0.91082
11	14	1	0.0152	1	11.49964	0.90463
11	15	1	0.0159	1	11.43831	0.90062
11	16	1	0.0051	1	11.43675	0.91358
11	17	1	0.0048	1	11.53109	0.92566
11	18	1	0.0051	1	11.45783	0.90560
11	19	1	0.0045	1	11.42955	0.90159
11	20	1	0.0042	1	11.43378	0.81143
12	13	1	0.01183	3	1.0271	0.98001
12	14	1	0.01082	3	1.0127	0.97470
12	15	1	0.01058	3	1.0254	0.96770
12	16	1	0.00294	3	1.0266	0.97891
12	17	1	0.00365	3	1.0167	0.94138
12	18	1	0.00308	3	1.0281	0.93721
12	19	1	0.00220	3	1.0327	0.94967
12	20	1	0.00217	3	1.0303	0.85471
13	14	4	0.02650	4	1.0105	0.96190
13	15	4	0.00523	4	1.0054	0.96190
13	16	1	0.0102	3	1.0273	0.98001
13	17	4	0.00344	4	1.0171	0.93029
13	18	4	0.00227	4	1.0287	0.93029
13	19	4	0.0120	4	1.0365	0.94680
13	20	4	0.0120	4	1.0148	0.85212
14	15	1	0.00193	3	1.0076	0.97470
14	16	1	0.01001	3	1.0296	0.96770
14	17	4	0.00181	4	1.0044	0.92261
14	18	4	0.00194	4	1.0121	0.92122
14	19	4	0.0113	4	1.0091	0.93620
14	20	4	0.01227	4	1.0142	0.84258
15	16	1	0.010	3	1.0270	0.97470
15	17	4	0.00177	4	0.96700	0.92122
15	18	4	0.00210	4	1.01050	0.92261
15	19	4	0.01193	4	0.92719	0.94037
15	20	4	0.0120	4	0.93676	0.84633
16	17	1	0.0028	3	0.92236	0.93721
16	18	1	0.00305	3	0.91771	0.94138
16	19	1	0.0020	3	0.91075	0.96223
16	20	1	0.00203	3	0.93445	0.86601
17	18	1	0.00281	3	0.91547	0.96223
17	19	1	0.00192	3	0.90964	0.94137
17	20	1	0.00217	3	0.90766	0.84724
18	19	1	0.00202	3	0.90114	0.96223
18	20	1	0.00210	3	0.90737	0.86600
19	20	1	0.00842	1	0.90204	0.9000

4.3.2 NET 2

If we add bidirectional link between Ninawa and Erbil 6 & 3, only the northern part will be complicated. In other words, when R is measured if s& d ∋ net2 $R_{NET2} = R_{NET1}$, else $R_{NET2} \neq R_{NET1}$.

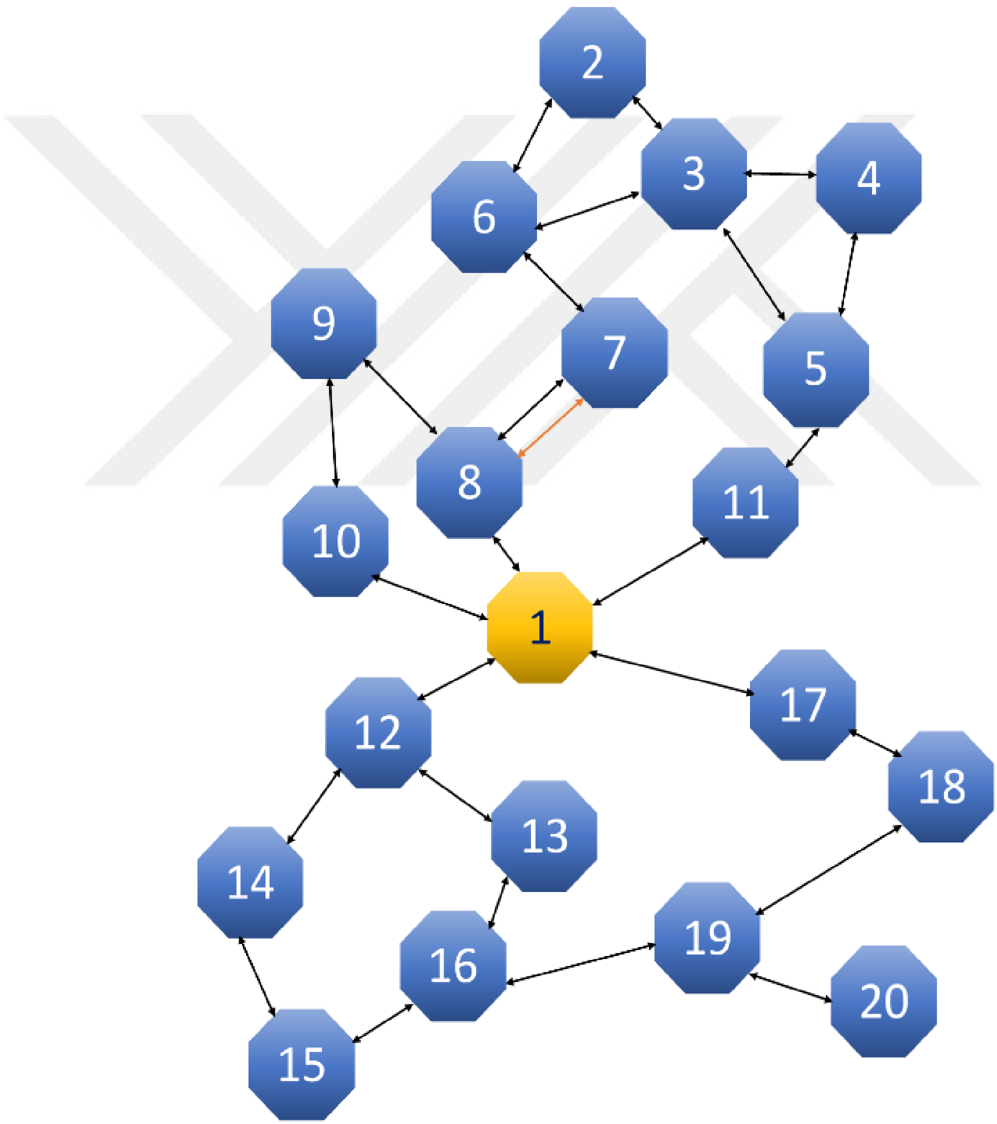


Figure16. NET 2

Table 9. Comparison Results of all NET2 Possible Commodities

S	D	Mp_H	T_H(s)	Mp_t	T_t(s)	R
1	2	4	0.03073	12	12.2098	0.96037
1	3	1	0.0205	10	10.8404	0.96927
1	4	4	0.0112	18	258.4359	0.95931
1	5	1	0.0037	17	106.4485	0.96850
1	6	1	0.0036	8	10.3717	0.97064
1	7	1	0.0049	8	10.0965	0.98901
1	8	1	0.0026	10	10.5054	0.99172
1	9	1	0.0024	10	10.7846	0.97578
1	10	1	0.0028	10	10.77833	0.97853
1	11	1	0.0024	17	90.3348	0.97508
1	12	2	0.0106	3	1.21065	0.96223
1	13	4	0.0169	4	1.06430	0.94680
1	14	4	0.0156	4	1.01374	0.94037
1	15	4	0.0151	4	0.95241	0.93620
1	16	2	0.0037	3	0.95085	0.94967
1	17	2	0.0040	3	1.04519	0.96223
1	18	2	0.0042	3	0.97193	0.94138
1	19	1	0.0039	3	0.94365	0.93721
1	20	1	0.0053	3	0.94788	0.84349
2	3	1	0.28114	10	13.8967	0.98720
2	4	7	0.08842	16	54.3239	0.97470
2	5	4	0.01179	12	14.6475	0.98165
2	6	1	0.00376	10	13.0568	0.98720
2	7	4	0.00641	18	72.5330	0.96418
2	8	4	0.00623	12	11.7406	0.96293
2	9	8	0.08770	16	43.4201	0.94400
2	10	8	0.08115	16	43.1858	0.94315
2	11	4	0.00729	12	11.6900	0.96195
2	12	1	0.0413	1	13.42045	0.92410
2	13	1	0.0476	1	13.27410	0.90928
2	14	1	0.0463	1	13.22354	0.90310
2	15	1	0.0458	1	13.16221	0.89910
2	16	1	0.0344	1	13.16065	0.91204
2	17	1	0.0347	1	13.25499	0.92410
2	18	1	0.0349	1	13.18173	0.90407
2	19	1	0.0346	1	13.15345	0.90007
2	20	1	0.0360	1	13.15768	0.81006
3	4	1	0.00400	10	10.7594	0.98720
3	5	1	0.00939	10	10.8090	0.99409
3	6	1	0.00252	10	10.8462	0.99409
3	7	1	0.00223	10	10.7848	0.97252
3	8	1	0.00229	8	10.6171	0.97142
3	9	4	0.00488	12	11.5769	0.95246
3	10	4	0.00470	12	11.4666	0.95175
3	11	1	0.00204	10	10.8525	0.97252
3	12	1	0.0311	1	12.05105	0.93266
3	13	1	0.0374	1	11.9047	0.91771
3	14	1	0.0361	1	11.85414	0.91147
3	15	1	0.0356	1	11.79281	0.90743
3	16	1	0.0242	1	11.79125	0.92049
3	17	1	0.0245	1	11.88559	0.93266
3	18	1	0.0247	1	11.81233	0.91245
3	19	1	0.0244	1	11.78405	0.90841
3	20	1	0.0258	1	11.78828	0.81757
4	5	1	0.00346	10	10.4039	0.98720
4	6	4	0.00606	12	1.70843	0.98165
4	7	4	0.00582	12	11.7464	0.96195
4	8	4	0.00595	12	12.4004	0.96102
4	9	8	0.08210	20	long	0.94239
4	10	8	0.07842	20	long	0.94183
4	11	4	0.00552	18	246.666	0.96418
4	12	1	0.0218	1	259.6465	0.92308
4	13	1	0.0281	1	259.5002	0.90828
4	14	1	0.0268	1	259.4496	0.90211
4	15	1	0.0263	1	259.3883	0.89811
4	16	1	0.0149	1	259.3867	0.91103
4	17	1	0.0152	1	259.4810	0.92308
4	18	1	0.0154	1	259.4078	0.90308
4	19	1	0.0151	1	259.3795	0.89907
4	20	1	0.0165	1	259.3838	0.80917
5	6	1	0.00204	8	10.7754	0.98879
5	7	1	0.00212	8	10.8672	0.97057
5	8	1	0.00205	10	10.5908	0.96979
5	9	1	0.00492	18	240.2615	0.95114
5	10	4	0.00470	18	242.1131	0.95071
5	11	4	0.00215	17	89.4507	0.97508
5	12	1	0.0143	1	107.6591	0.93192
5	13	1	0.0206	1	107.512	0.91698
5	14	1	0.0193	1	107.4622	0.91075
5	15	1	0.0188	1	107.4009	0.90671
5	16	1	0.0074	1	107.3993	0.91976
5	17	1	0.0077	1	107.4936	0.93192
5	18	1	0.0079	1	107.4204	0.91173
5	19	1	0.0076	1	107.3921	0.90769
5	20	1	0.0090	1	107.3963	0.81692
6	7	1	0.00211	17	91.7443	0.97508
6	8	1	0.00210	10	10.5416	0.97366
6	9	4	0.00485	12	11.7330	0.95438
6	10	4	0.00462	12	12.0573	0.95338
6	11	1	0.00203	8	10.7005	0.97057
6	12	1	0.0142	1	11.5824	0.93398
6	13	1	0.0205	1	11.4360	0.91901
6	14	1	0.0192	1	11.3854	0.91276
6	15	1	0.0187	1	11.3241	0.90871
6	16	1	0.0073	1	11.3226	0.92179
6	17	1	0.0076	1	11.4169	0.93398
6	18	1	0.0078	1	11.3436	0.91374
6	19	1	0.0075	1	11.3154	0.90969
6	20	1	0.0089	1	11.3196	0.81872
7	8	1	0.00192	10	10.7231	0.99683
7	9	4	0.00458	12	11.8028	0.97556
7	10	4	0.00451	12	11.8119	0.97296
7	11	1	0.00207	8	10.6706	0.97064
7	12	1	0.0155	1	11.3072	0.95166
7	13	1	0.0218	1	11.1608	0.93640
7	14	1	0.0205	1	11.1102	0.93003
7	15	1	0.0200	1	11.0489	0.92591
7	16	1	0.0086	1	11.0474	0.93924
7	17	1	0.0089	1	11.1417	0.95166
7	18	1	0.0091	1	11.0684	0.93103
7	19	1	0.0088	1	11.0402	0.92691
7	20	1	0.0102	1	11.0444	0.83422
8	9	1	0.00210	10	10.8825	0.97853
8	10	1	0.00188	10	10.8617	0.97578
8	11	1	0.00193	10	11.8505	0.97160
8	12	1	0.0132	1	11.7161	0.95427
8	13	1	0.0195	1	11.5697	0.93897
8	14	1	0.0182	1	11.5191	0.93258
8	15	1	0.0177	1	11.4578	0.92845
8	16	1	0.0063	1	11.4563	0.94181
8	17	1	0.0066	1	11.5506	0.95427

S	D	Mp_H	T_H(s)	Mp_t	T_t(s)	R
8	18	1	0.0068	1	11.4773	0.93359
8	19	1	0.0065	1	11.4491	0.92945
8	20	1	0.0079	1	11.4533	0.83650
9	10	1	0.00186	10	11.2676	0.97853
9	11	4	0.00464	18	244.222	0.95447
9	12	1	0.0130	1	11.71605	0.93892
9	13	1	0.0193	1	11.56970	0.92387
9	14	1	0.0180	1	11.51914	0.91759
9	15	1	0.0175	1	11.45781	0.91352
9	16	1	0.0061	1	11.45625	0.92667
9	17	1	0.0064	1	11.55059	0.93892
9	18	1	0.0066	1	11.47733	0.91857
9	19	1	0.0063	1	11.44905	0.91451
9	20	1	0.0077	1	11.45328	0.82305
10	11	4	0.00456	18	240.4324	0.95562
10	12	1	11.8095	1	11.98898	0.94157
10	13	1	0.0197	1	11.84263	0.92647
10	14	1	0.0184	1	11.79207	0.92017
10	15	1	0.0179	1	11.73074	0.91609
10	16	1	0.0065	1	11.72918	0.92928
10	17	1	0.0068	1	11.82352	0.94157
10	18	1	0.0070	1	11.75026	0.92116
10	19	1	0.0067	1	11.72198	0.91708
10	20	1	0.0081	1	11.72621	0.82537
11	12	1	0.0130	1	91.5455	0.93825
11	13	1	0.0193	1	91.3991	0.92321
11	14	1	0.0180	1	91.3485	0.91693
11	15	1	0.0175	1	91.2872	0.91287
11	16	1	0.0061	1	91.2857	0.92601
11	17	1	0.0064	1	91.3800	0.93825
11	18	1	0.0066	1	91.3067	0.91792
11	19	1	0.0063	1	91.2785	0.91385
11	20	1	0.0077	1	91.2827	0.82247
12	13	1	0.01283	3	1.0271	0.98001
12	14	1	0.01072	3	1.0127	0.97470
12	15	1	0.01050	3	1.0254	0.96770
12	16	1	0.00284	3	1.0266	0.97891
12	17	1	0.00310	3	1.0167	0.94138
12	18	1	0.00289	3	1.0281	0.93721
12	19	1	0.00211	3	1.0327	0.94967
12	20	1	0.00216	3	1.0303	0.85471
13	14	4	0.07249	4	1.0105	0.96190
13	15	4	0.07251	4	1.0054	0.96190
13	16	1	0.01058	3	1.0273	0.98001
13	17	4	0.07201	4	1.0171	0.93029
13	18	4	0.07207	4	1.0287	0.93029
13	19	4	0.01236	4	1.0365	0.94680
13	20	4	0.01212	4	1.0148	0.85212
14	15	1	0.07045	3	1.0076	0.97470
14	16	1	0.01037	3	1.0296	0.96770
14	17	4	0.07229	4	1.0044	0.92261
14	18	4	0.07227	4	1.0121	0.92122
14	19	4	0.01195	4	1.0091	0.93620
14	20	4	0.01190	4	1.0142	0.84258
15	16	1	0.01019	3	1.0270	0.97470
15	17	4	0.07155	4	0.96700	0.92122
15	18	4	0.07217	4	1.01050	0.92261
15	19	4	0.01203	4	0.92719	0.94037
15	20	4	0.01206	4	0.93676	0.84633
16	17	1	0.00290	3	0.92236	0.93721
16	18	1	0.00286	3	0.91771	0.94138
16	19	1	0.00203	3	0.91075	0.96223
16	20	1	0.00210	3	0.93445	0.86601
17	18	1	0.00282	3	0.91547	0.96223
17	19	1	0.00191	3	0.90964	0.94138
17	20	1	0.00194	3	0.90766	0.84724
18	19	1	0.00187	3	0.90114	0.96223
18	20	1	0.00202	3	0.90737	0.86601
19	20	1	0.01319	1	0.90204	0.90000

4.3.3 NET 3

Now consider the bidirectional link between Kirkuk and Tikrit 7 & 5 only the northern part will complicated.in other words, When R is measured if s& d ∋ net2 $R_{NET3} = R_{NET2}$,else $R_{NET3} \neq R_{NET2}$.

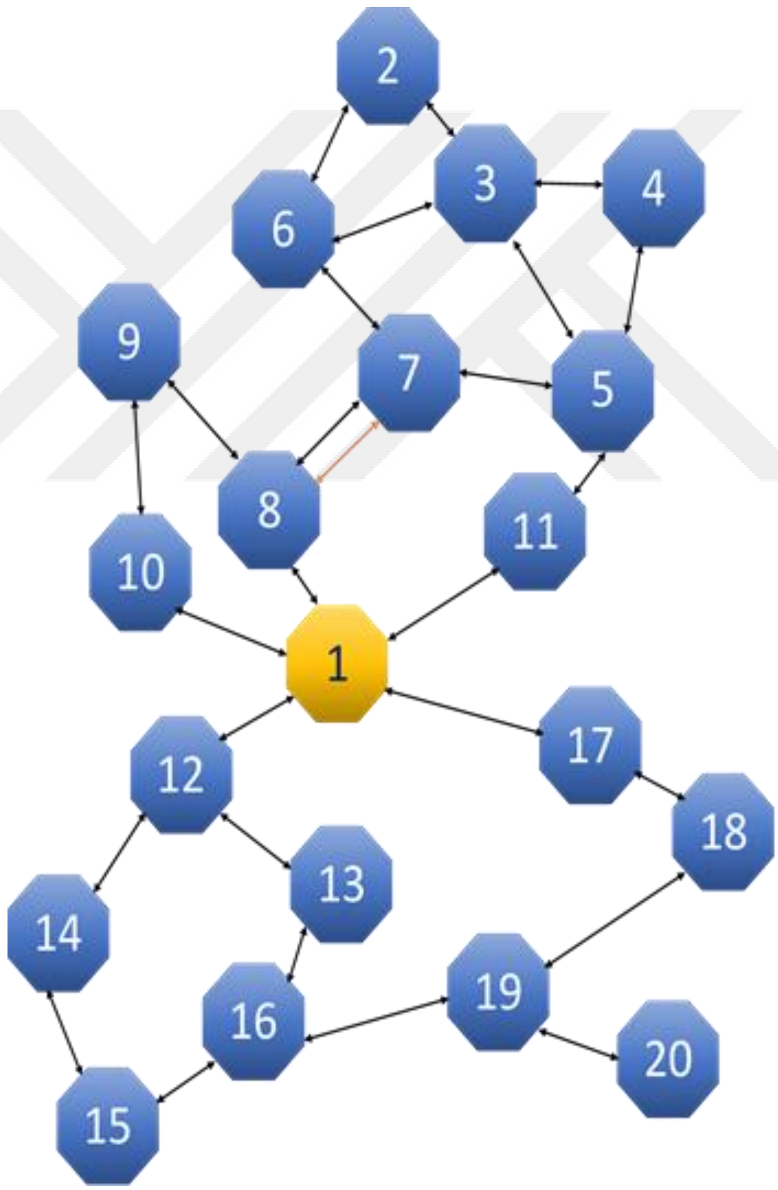


Figure17. NET3

Table10. Comparison Results of all NET3 Possible Commodities

S	D	Mp_H	T_H(s)	Mp_t	T_t(s)	R
1	2	8	0.0424	30	long	0.97819
1	3	4	0.0141	20	long	0.98864
1	4	7	0.0408	28	long	0.97975
1	5	1	0.0176	21	Long	0.99055
1	6	4	0.0079	25	long	0.98740
1	7	1	0.0028	9	10.596	0.99260
1	8	1	0.0027	12	13.5710	0.99434
1	9	1	0.0029	12	12.7389	0.97743
1	10	1	0.0019	12	12.5267	0.97931
1	11	1	0.0017	21	long	0.98552
1	12	2	0.0099	3	1.21065	0.96223
1	13	4	0.0145	4	1.06430	0.94680
1	14	4	0.0129	4	1.01374	0.94037
1	15	4	0.0142	4	0.95241	0.93620
1	16	2	0.0027	3	0.95085	0.94967
1	17	2	0.0027	3	1.04519	0.96223
1	18	2	0.0026	3	0.97193	0.94138
1	19	1	0.0024	3	0.94365	0.93721
1	20	1	0.0023	3	0.94788	0.84349
2	3	1	0.3155	12	13.5680	0.98877
2	4	7	0.4314	19	894.000	0.97783
2	5	4	0.0293	15	19.2411	0.98654
2	6	1	0.0136	11	14.8627	0.98877
2	7	4	0.0234	22	long	0.98479
2	8	8	0.1385	24	long	0.98278
2	9	16	39.8852	36	long	0.96276
2	10	16	40.7743	36	long	0.96124
2	11	8	0.1352	30	long	0.97274
2	12	1	0.0521	1	long	0.94125
2	13	1	0.0574	1	long	0.92616
2	14	1	0.0560	1	long	0.91986
2	15	1	0.0558	1	Long	0.91578
2	16	1	0.0452	1	long	0.92897
2	17	1	0.0452	1	long	0.94125
2	18	1	0.0451	1	long	0.92085
2	19	1	0.0450	1	long	0.91677
2	20	1	0.0442	1	Long	0.82509
3	4	1	0.0061	12	13.4372	0.98877
3	5	1	0.0076	12	14.2270	0.99741
3	6	1	0.0043	12	14.3646	0.99741
3	7	1	0.0034	12	13.2411	0.99525
3	8	4	0.0053	16	53.7950	0.99324
3	9	8	0.0687	24	Long	0.97302
3	10	8	0.0689	24	Long	0.97149
3	11	4	0.0055	20	long	0.98329
3	12	1	0.0239	1	long	0.95130
3	13	1	0.0291	1	long	0.93605
3	14	1	0.0277	1	long	0.92969
3	15	1	0.0275	1	Long	0.92556
3	16	1	0.0169	1	long	0.93889
3	17	1	0.0170	1	long	0.95130
3	18	1	0.0168	1	long	0.93068
3	19	1	0.0167	1	long	0.92656
3	20	1	0.0159	1	Long	0.83391
4	5	1	0.0054	11	12.3056	0.98877
4	6	4	0.0213	14	17.1922	0.98654
4	7	4	0.0155	14	16.3258	0.98625
4	8	7	0.0419	20	long	0.98427
4	9	14	7.1935	32	long	0.96424
4	10	14	7.1787	32	long	0.96274
4	11	7	0.0393	28	long	0.97462
4	12	1	0.0494	1	long	0.94275
4	13	1	0.0546	1	long	0.92764
4	14	1	0.0532	1	long	0.92133
4	15	1	0.0530	1	Long	0.91724
4	16	1	0.0424	1	long	0.93045
4	17	1	0.0425	1	long	0.94275
4	18	1	0.0423	1	long	0.92232
4	19	1	0.0422	1	long	0.91823
4	20	1	0.0414	1	Long	0.82641
5	6	1	0.0033	9	10.2002	0.99549
5	7	1	0.0094	9	10.2608	0.99706
5	8	1	0.0024	12	13.2899	0.99507
5	9	4	0.0047	22	long	0.97484
5	10	4	0.0045	22	long	0.97334
5	11	1	0.0023	21	long	0.98552
5	12	1	0.0259	1	long	0.95314
5	13	1	0.0312	1	long	0.93786
5	14	1	0.0298	1	long	0.93148
5	15	1	0.0296	1	Long	0.92735
5	16	1	0.0190	1	long	0.94070
5	17	1	0.0191	1	long	0.95314
5	18	1	0.0189	1	long	0.93248
5	19	1	0.0188	1	long	0.92835
5	20	1	0.0180	1	Long	0.83552
6	7	1	0.0032	21	long	0.99412
6	8	4	0.0057	20	long	0.99208
6	9	8	0.0695	30	long	0.97185
6	10	8	0.0705	30	Long	0.97030
6	11	4	0.0053	25	long	0.98173
6	12	1	0.0170	1	long	0.95011
6	13	1	0.0223	1	long	0.93488
6	14	1	0.0209	1	long	0.92852
6	15	1	0.0207	1	Long	0.92440
6	16	1	0.0100	1	long	0.93771
6	17	1	0.0101	1	long	0.95011
6	18	1	0.0099	1	long	0.92952
6	19	1	0.0098	1	long	0.92540
6	20	1	0.0091	1	Long	0.83286
7	8	1	0.0023	12	long	0.99777
7	9	4	0.0046	14	18.3205	0.97728
7	10	4	0.0045	14	18.2222	0.97556
7	11	1	0.0022	9	11.4114	0.98508
7	12	1	0.0126	1	11.8067	0.95511
7	13	1	0.0179	1	11.6603	0.93980
7	14	1	0.0165	1	11.6097	0.93341
7	15	1	0.0162	1	11.5484	0.92927
7	16	1	0.0056	1	11.5469	0.94265
7	17	1	0.0057	1	11.6412	0.95511
7	18	1	0.0055	1	11.5679	0.93441
7	19	1	0.0054	1	11.5397	0.93028
7	20	1	0.0047	1	11.5439	0.83725
8	9	1	0.0020	12	15.3534	0.97931
8	10	1	0.0020	12	14.4464	0.97743
8	11	1	0.0021	12	15.2109	0.98496
8	12	1	0.0127	1	14.7817	0.95679
8	13	1	0.0180	1	14.6353	0.94145
8	14	1	0.0166	1	14.5847	0.93505
8	15	1	0.0164	1	14.5234	0.93090
8	16	1	0.0057	1	14.5219	0.94430
8	17	1	0.0058	1	14.6162	0.95679

S	D	Mp_H	T_H(s)	Mp_t	T_t(s)	R
8	18	1	0.0056	1	14.5429	0.93605
8	19	1	0.0055	1	14.5147	0.93190
8	20	1	0.0048	1	14.5189	0.83871
9	10	1	0.0020	12	12.9396	0.97931
9	11	4	0.0042	22	long	0.96656
9	12	1	0.0117	1	13.9496	0.94051
9	13	1	0.0170	1	13.8032	0.92543
9	14	1	0.0156	1	13.7526	0.91914
9	15	1	0.0154	1	13.6913	0.91507
9	16	1	0.0048	1	13.6898	0.92824
9	17	1	0.0048	1	13.7841	0.94051
9	18	1	0.0047	1	13.7108	0.92013
9	19	1	0.0046	1	13.6826	0.91605
9	20	1	0.0038	1	13.6868	0.82445
10	11	4	0.0043	22	long	0.96675
10	12	1	0.0125	1	13.7373	0.94232
10	13	1	0.0178	1	13.591	0.92721
10	14	1	0.0164	1	13.5404	0.92091
10	15	1	0.0161	1	13.4791	0.91683
10	16	1	0.0055	1	13.4775	0.93002
10	17	1	0.0056	1	13.5718	0.94232
10	18	1	0.0054	1	13.4986	0.92190
10	19	1	0.0053	1	13.4703	0.91782
10	20	1	0.0046	1	13.4745	0.82603
11	12	1	0.0116	1	long	0.94830
11	13	1	0.0169	1	long	0.93310
11	14	1	0.0155	1	long	0.92676
11	15	1	0.0152	1	long	0.92265
11	16	1	0.0046	1	long	0.93593
11	17	1	0.0047	1	long	0.94830
11	18	1	0.0045	1	long	0.92775
11	19	1	0.0044	1	long	0.92364
11	20	1	0.0037	1	long	0.83128
12	13	1	0.0118	3	1.0271	0.98001
12	14	1	0.0114	3	1.0127	0.97470
12	15	1	0.0107	3	1.0254	0.96770
12	16	1	0.0033	3	1.0266	0.97891
12	17	1	0.0034	3	1.0167	0.94138
12	18	1	0.0031	3	1.0281	0.93721
12	19	1	0.0023	3	1.0327	0.94967
12	20	1	0.0022	3	1.0303	0.85471
13	14	4	0.0733	3	1.0105	0.96190
13	15	4	0.0733	4	1.0054	0.96190
13	16	1	0.0106	4	1.0273	0.98001
13	17	4	0.0723	3	1.0171	0.93029
13	18	4	0.0736	4	1.0287	0.93029
13	19	4	0.0119	4	1.0365	0.94680
13	20	4	0.0122	4	1.0148	0.85212
14	15	1	0.0727	4	1.0076	0.97470
14	16	1	0.0106	3	1.0296	0.96770
14	17	4	0.0735	3	1.0044	0.92261
14	18	4	0.0718	4	1.0121	0.92122
14	19	4	0.0114	4	1.0091	0.93620
14	20	4	0.0117	4	1.0142	0.84258
15	16	1	0.0099	4	1.0270	0.97470
15	17	4	0.0706	3	0.96700	0.92122
15	18	4	0.0710	4	1.01050	0.92261
15	19	4	0.0113	4	0.92719	0.94037
15	20	4	0.0116	4	0.93676	0.84633
16	17	1	0.0028	4	0.92236	0.93721
16	18	1	0.0028	3	0.91771	0.94138
16	19	1	0.0019	3	0.91075	0.96223
16	20	1	0.0020	3	0.93445	0.86601
17	18	1	0.0031	3	0.91547	0.96223
17	19	1	0.0019	3	0.90964	0.94138
17	20	1	0.0024	3	0.90766	0.84724
18	19	1	0.0020	3	0.90114	0.96223
18	20	1	0.0021	3	0.90737	0.86601
19	20	1	0.0135	3	0.90204	0.90000

4.3.4 NET 4

We turn now to **net2** be adding bidirectional link between Babil and Al qadisyah 13 &15

when R is measured if $\mathbf{s}\&\ \mathbf{d} \ni$ **net1** $R_{NET4} = R_{NET3}$,else $R_{NET4} \neq R_{NET3}$.

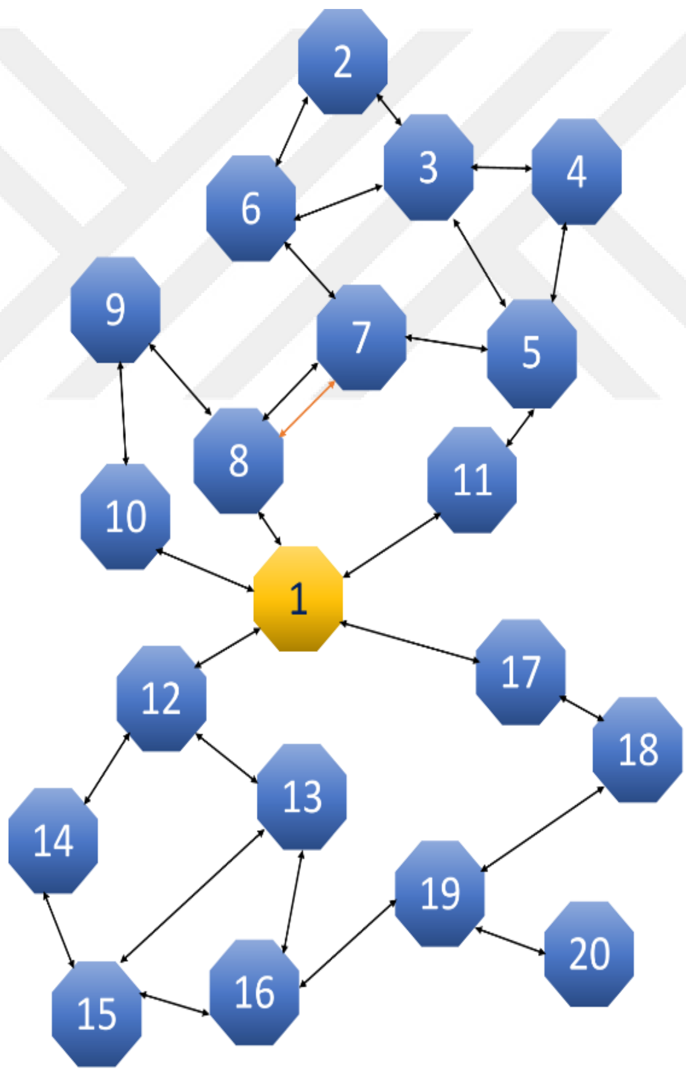


Figure18.NET 4

Table 11. Comparison results of all NET4 possible commodities

S	D	Mp_H	T_H(s)	Mp_t	T_t(s)	R
1	2	8	0.0424	30	long	0.97819
1	3	4	0.01410	20	long	0.98864
1	4	7	0.04020	28	long	0.97975
1	5	1	0.01584	21	long	0.99055
1	6	4	0.00731	25	long	0.98740
1	7	1	0.00238	9	10.596	0.99260
1	8	1	0.00275	12	13.5710	0.99434
1	9	1	0.00202	12	12.7389	0.97743
1	10	1	0.00193	12	12.5267	0.97931
1	11	1	0.00174	21	long	0.98552
1	12	5	0.01648	5	1.15542	0.9635
1	13	6	0.02252	6	1.15862	0.9577
1	14	7	0.04015	7	1.21870	0.9507
1	15	6	0.01645	6	1.15307	0.9566
1	16	5	0.00960	5	1.03094	0.9559
1	17	5	0.01597	5	1.13866	0.9635
1	18	5	0.01628	5	1.15561	0.9442
1	19	5	0.01581	5	0.99856	0.9416
1	20	5	0.07640	5	1.29883	0.84752
2	3	1	0.31320	12	13.5680	0.98877
2	4	7	0.45417	19	894.000	0.97783
2	5	4	0.03026	15	19.2411	0.98654
2	6	1	0.01433	11	14.8627	0.98877
2	7	4	0.02399	22	long	0.98479
2	8	8	0.14248	24	long	0.98278
2	9	16	36.2003	36	long	0.96276
2	10	16	37.5283	36	long	0.96124
2	11	8	0.14078	30	long	0.97274
2	12	1	0.05888	1	long	0.94256
2	13	1	0.06492	1	long	0.93686
2	14	1	0.08255	1	long	0.93000
2	15	1	0.05885	1	long	0.93582
2	16	1	0.05200	1	long	0.93515
2	17	1	0.05837	1	long	0.94256
2	18	1	0.05868	1	long	0.92362
2	19	1	0.05821	1	long	0.92115
2	20	1	0.11880	1	long	0.82904
3	4	1	0.00670	12	13.4372	0.98877
3	5	1	0.01330	12	14.2270	0.99741
3	6	1	0.00453	12	14.3646	0.99741
3	7	1	0.00354	12	13.2411	0.99525
3	8	4	0.00602	16	53.7950	0.99324
3	9	8	0.07414	24	Long	0.97302
3	10	8	0.07335	24	Long	0.97149
3	11	4	0.00567	20	long	0.98329
3	12	1	0.03058	1	Long	0.95263
3	13	1	0.03662	1	Long	0.94687
3	14	1	0.05425	1	Long	0.93993
3	15	1	0.03055	1	long	0.94582
3	16	1	0.02370	1	Long	0.94513
3	17	1	0.03006	1	Long	0.95263
3	18	1	0.03038	1	Long	0.93348
3	19	1	0.02991	1	long	0.93099
3	20	1	0.09049	1	Long	0.83789
4	5	1	0.00565	11	12.3056	0.98877
4	6	4	0.02861	14	17.1922	0.98654
4	7	4	0.01785	14	16.3258	0.98625
4	8	7	0.04429	20	long	0.98427
4	9	14	8.02158	32	long	0.96424
4	10	14	7.56328	32	long	0.96274
4	11	7	0.03961	28	long	0.97462
4	12	1	0.05668	1	long	0.94406
4	13	1	0.06272	1	Long	0.93836
4	14	1	0.08036	1	Long	0.93148
4	15	1	0.05665	1	Long	0.93732
4	16	1	0.04981	1	long	0.93664
4	17	1	0.05617	1	Long	0.94406
4	18	1	0.05648	1	Long	0.92509
4	19	1	0.05602	1	Long	0.92262
4	20	1	0.11660	1	Long	0.83036
5	6	1	0.00317	9	10.2002	0.99549
5	7	1	0.00916	9	10.2608	0.99706
5	8	1	0.00547	12	13.2899	0.99507
5	9	4	0.00980	22	long	0.97484
5	10	4	0.00455	22	long	0.97334
5	11	1	0.00272	21	long	0.98552
5	12	1	0.03232	1	long	0.95447
5	13	1	0.03836	1	long	0.94870
5	14	1	0.05599	1	long	0.94175
5	15	1	0.03229	1	Long	0.94765
5	16	1	0.02544	1	long	0.94696
5	17	1	0.03180	1	long	0.95447
5	18	1	0.03212	1	long	0.93529
5	19	1	0.03165	1	long	0.93279
5	20	1	0.09223	1	Long	0.83951
6	7	1	0.00320	21	long	0.99412
6	8	4	0.00610	20	long	0.99208
6	9	8	0.07184	30	long	0.97185
6	10	8	0.07135	30	Long	0.97030
6	11	4	0.00543	25	long	0.98173
6	12	1	0.02379	1	long	0.95143
6	13	1	0.02983	1	long	0.94568
6	14	1	0.04746	1	long	0.93875
6	15	1	0.02376	1	Long	0.94463
6	16	1	0.01691	1	long	0.94395
6	17	1	0.02328	1	long	0.95143
6	18	1	0.02359	1	long	0.93231
6	19	1	0.02312	1	long	0.92983
6	20	1	0.08371	1	Long	0.83684
7	8	1	0.00229	12	long	0.99777
7	9	4	0.00454	14	18.3205	0.97728
7	10	4	0.00443	14	18.2222	0.97556
7	11	1	0.00212	9	11.4114	0.98508
7	12	1	0.01886	1	11.7514	0.95644
7	13	1	0.02490	1	11.7546	0.95066
7	14	1	0.04253	1	11.8147	0.94370
7	15	1	0.01883	1	11.7491	0.94961
7	16	1	0.01198	1	11.6269	0.94892
7	17	1	0.01835	1	11.7347	0.95644
7	18	1	0.01866	1	11.7516	0.93722
7	19	1	0.01819	1	11.5946	0.93472
7	20	1	0.07878	1	11.8948	0.84125
8	9	1	0.00230	12	15.3534	0.97931
8	10	1	0.00201	12	14.4464	0.97743
8	11	1	0.00226	12	15.2109	0.98496
8	12	1	0.01923	1	14.7264	0.95812
8	13	1	0.02527	1	14.7296	0.95233
8	14	1	0.04290	1	14.7897	0.94535
8	15	1	0.01920	1	14.7241	0.95127
8	16	1	0.01235	1	14.6019	0.95058
8	17	1	0.01871	1	14.7097	0.95812

S	D	Mp_H	T_H(s)	Mp_t	T_t(s)	R
8	18	1	0.01903	1	14.7266	0.93886
8	19	1	0.01856	1	14.5696	0.93636
8	20	1	0.07914	1	14.8698	0.84272
9	10	1	0.00239	12	12.9396	0.97931
9	11	4	0.00421	22	long	0.96656
9	12	1	0.01850	1	13.8943	0.94182
9	13	1	0.02454	1	13.8975	0.93613
9	14	1	0.04218	1	13.9576	0.92927
9	15	1	0.01847	1	13.8920	0.93509
9	16	1	0.01163	1	13.7698	0.93441
9	17	1	0.01799	1	13.8776	0.94182
9	18	1	0.01830	1	13.8945	0.92289
9	19	1	0.01784	1	13.7375	0.92043
9	20	1	0.07842	1	14.0377	0.82839
10	11	4	0.00417	22	long	0.96675
10	12	1	0.01842	1	13.6821	0.94363
10	13	1	0.02446	1	13.6853	0.93793
10	14	1	0.04209	1	13.7454	0.93106
10	15	1	0.01839	1	13.6798	0.93689
10	16	1	0.01154	1	13.5576	0.93621
10	17	1	0.01790	1	13.6654	0.94363
10	18	1	0.01822	1	13.6823	0.92467
10	19	1	0.01775	1	13.5253	0.92220
10	20	1	0.07833	1	13.8255	0.82998
11	12	1	0.01823	1	long	0.94962
11	13	1	0.02427	1	long	0.94388
11	14	1	0.04190	1	long	0.93697
11	15	1	0.01820	1	long	0.94284
11	16	1	0.01135	1	long	0.94215
11	17	1	0.01771	1	long	0.94962
11	18	1	0.01803	1	long	0.93054
11	19	1	0.01756	1	long	0.92806
11	20	1	0.07814	1	long	0.83525
12	13	5	0.08344	5	1.59801	0.99090
12	14	5	0.74117	5	1.37675	0.98510
12	15	5	0.07762	5	1.36591	0.98967
12	16	5	0.07665	5	1.32673	0.98727
12	17	5	0.77117	5	1.39046	0.94421
12	18	5	0.72107	5	1.33836	0.94169
12	19	5	0.07777	5	1.38657	0.95599
12	20	5	0.07729	5	1.32612	0.86039
13	14	6	0.70958	6	1.39478	0.98383
13	15	5	0.07920	5	1.35031	0.99619
13	16	5	0.07763	5	1.30053	0.99421
13	17	6	0.77669	6	1.16335	0.94159
13	18	6	0.77305	6	1.20885	0.94224
13	19	6	0.09856	6	1.27788	0.95972
13	20	6	0.10934	6	1.24478	0.86375
14	15	5	0.72632	5	1.35340	0.98510
14	16	6	0.72753	6	1.32137	0.98079
14	17	7	8.48363	7	1.46193	0.93321
14	18	7	8.22172	7	1.19874	0.93236
14	19	7	0.77505	7	1.28256	0.94816
14	20	7	0.78046	7	1.24373	0.85334
15	16	5	0.07644	5	1.05699	0.99368
15	17	6	0.72721	6	1.01072	0.94068
15	18	6	0.72909	6	1.01050	0.94147
15	19	6	0.08316	6	0.98219	0.95908
15	20	6	0.08356	6	0.99676	0.86317
16	17	5	0.72251	5	0.98936	0.94169
16	18	5	0.71264	5	0.98771	0.94421
16	19	5	0.09181	5	0.97075	0.96357
16	20	5	0.07781	5	0.98445	0.86721
17	18	5	0.79956	5	0.97547	0.96357
17	19	5	0.72190	5	0.97964	0.94421
17	20	5	0.72493	5	0.97766	0.84978
18	19	5	0.71914	5	0.97114	0.96357
18	20	5	0.83526	5	0.97737	0.86721
19	20	1	0.07580	1	0.99204	0.90000

4.3.3 NET 5

Again, complicating net2 be adding bidirectional link between Maysan and Al basrah (18,20) **if R** is measured if $\mathbf{s}\&\ \mathbf{d} \ni$ **net1** $R_{NET5} = R_{NET4}$ **,else** $R_{NET5} \neq R_{NET4}$.

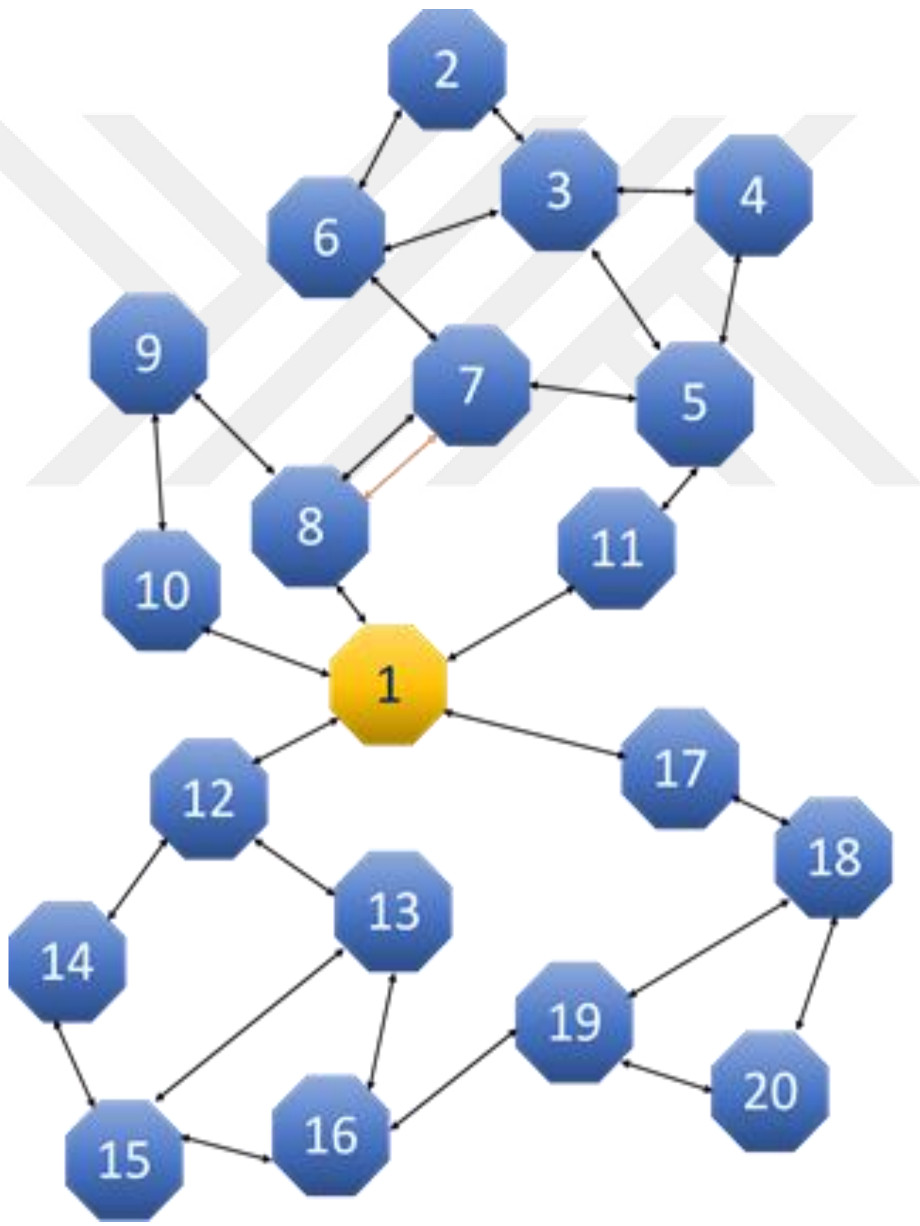


Figure19.NET 5

Table 12. Comparison Results of all NET5 Possible Commodities

S	D	Mp_H	T_H(s)	Mp_t	T_t(s)	R
1	2	8	0.0424	30	long	0.97819
1	3	4	0.0135	20	long	0.98864
1	4	7	0.0408	28	long	0.97975
1	5	1	0.0155	21	long	0.99055
1	6	4	0.0072	25	long	0.98740
1	7	1	0.0024	9	10.596	0.99260
1	8	1	0.0027	12	13.5710	0.99434
1	9	1	0.0018	12	12.7389	0.97743
1	10	1	0.0025	12	12.5267	0.97931
1	11	1	0.0017	21	long	0.98552
1	12	5	0.0173	9	1.37719	0.96929
1	13	6	0.0226	9	1.37186	0.96461
1	14	7	0.0382	11	1.68986	0.95696
1	15	6	0.0161	9	1.20744	0.96359
1	16	5	0.0093	6	1.28843	0.96355
1	17	5	0.0156	9	1.3135	0.96929
1	18	5	0.0163	9	1.20444	0.95628
1	19	5	0.0152	6	1.18251	0.95581
1	20	10	1.0175	10	1.38426	0.94662
2	3	1	0.3052	12	13.5680	0.98877
2	4	7	0.4354	19	894.000	0.97783
2	5	4	0.0290	15	19.2411	0.98654
2	6	1	0.0131	11	14.8627	0.98877
2	7	4	0.0222	22	long	0.98479
2	8	8	0.1352	24	long	0.98278
2	9	16	38.2968	36	long	0.96276
2	10	16	38.5796	36	long	0.96124
2	11	8	0.1274	30	long	0.97274
2	12	1	0.0597	1	long	0.94816
2	13	1	0.0650	1	long	0.94357
2	14	1	0.0806	1	long	0.93609
2	15	1	0.0585	1	long	0.94258
2	16	1	0.0517	1	long	0.94254
2	17	1	0.0580	1	long	0.94816
2	18	1	0.0587	1	long	0.93543
2	19	1	0.0576	1	long	0.93496
2	20	1	1.0599	1	long	0.92597
3	4	1	0.0063	12	13.4372	0.98877
3	5	1	0.0073	12	14.2270	0.99741
3	6	1	0.0041	12	14.3646	0.99741
3	7	1	0.0032	12	13.2411	0.99525
3	8	4	0.0056	16	53.7950	0.99324
3	9	8	0.0702	24	Long	0.97302
3	10	8	0.0695	24	Long	0.97149
3	11	4	0.0056	20	long	0.98329
3	12	1	0.0308	1	long	0.95828
3	13	1	0.0361	1	long	0.95365
3	14	1	0.0517	1	long	0.94609
3	15	1	0.0296	1	long	0.95264
3	16	1	0.0228	1	long	0.95260
3	17	1	0.0291	1	long	0.95828
3	18	1	0.0298	1	long	0.94542
3	19	1	0.0287	1	long	0.94495
3	20	1	1.0310	1	long	0.93586
4	5	1	0.0055	11	12.3056	0.98877
4	6	4	0.0214	14	17.1922	0.98654
4	7	4	0.0149	14	16.3258	0.98625
4	8	7	0.0412	20	long	0.98427
4	9	14	7.2260	32	long	0.96424
4	10	14	7.2974	32	long	0.96274
4	11	7	0.0385	28	long	0.97462
4	12	1	0.0581	1	long	0.94967
4	13	1	0.0633	1	long	0.94508
4	14	1	0.0789	1	long	0.93759
4	15	1	0.0568	1	long	0.94408
4	16	1	0.0501	1	long	0.94404
4	17	1	0.0564	1	long	0.94967
4	18	1	0.0570	1	long	0.93692
4	19	1	0.0560	1	long	0.93645
4	20	1	1.0583	1	long	0.92745
5	6	1	0.0031	9	10.2002	0.99549
5	7	1	0.0096	9	10.2608	0.99706
5	8	1	0.0025	12	13.2899	0.99507
5	9	4	0.0047	22	long	0.97484
5	10	4	0.0046	22	long	0.97334
5	11	1	0.0025	21	long	0.98552
5	12	1	0.0329	1	long	0.96014
5	13	1	0.0381	1	long	0.95549
5	14	1	0.0537	1	long	0.94792
5	15	1	0.0316	1	long	0.95449
5	16	1	0.0249	1	long	0.95444
5	17	1	0.0312	1	long	0.96014
5	18	1	0.0318	1	long	0.94725
5	19	1	0.0308	1	long	0.94678
5	20	1	1.0331	1	long	0.93767
6	7	1	0.0031	21	long	0.99412
6	8	4	0.0059	20	long	0.99208
6	9	8	0.0685	30	long	0.97185
6	10	8	0.0675	30	Long	0.97030
6	11	4	0.0051	25	long	0.98173
6	12	1	0.0245	1	long	0.95708
6	13	1	0.0298	1	long	0.95245
6	14	1	0.0454	1	long	0.94490
6	15	1	0.0233	1	long	0.95145
6	16	1	0.0165	1	long	0.95141
6	17	1	0.0228	1	long	0.95708
6	18	1	0.0235	1	long	0.94424
6	19	1	0.0224	1	long	0.94377
6	20	1	1.0247	1	long	0.93469
7	8	1	0.0021	12	long	0.99777
7	9	4	0.0042	14	18.3205	0.97728
7	10	4	0.0043	14	18.2222	0.97556
7	11	1	0.0021	9	11.9732	0.98508
7	12	1	0.0198	1	11.9679	0.96212
7	13	1	0.0250	1	12.2859	0.95747
7	14	1	0.0406	1	11.8034	0.94988
7	15	1	0.0185	1	11.8844	0.95646
7	16	1	0.0118	1	11.9095	0.95642
7	17	1	0.0180	1	11.8004	0.96212
7	18	1	0.0187	1	11.7785	0.94921
7	19	1	0.0177	1	11.9803	0.94874
7	20	1	1.0199	1	11.9732	0.93961
8	9	1	0.0020	12	15.3534	0.97931
8	10	1	0.0020	12	14.4464	0.97743
8	11	1	0.0021	12	15.2109	0.98496
8	12	1	0.0201	1	14.948	0.96381
8	13	1	0.0253	1	14.943	0.95915
8	14	1	0.0409	1	15.261	0.95154
8	15	1	0.0188	1	14.778	0.95814
8	16	1	0.0121	1	14.859	0.95809
8	17	1	0.0183	1	14.885	0.96381

S	D	Mp_H	T_H(s)	Mp_t	T_t(s)	R
8	18	1	0.0190	1	14.775	0.95087
8	19	1	0.0180	1	14.754	0.95040
8	20	1	1.0202	1	14.955	0.94126
9	10	1	0.0020	12	12.9396	0.97931
9	11	4	0.0044	22	long	0.96656
9	12	1	0.0191	1	14.1161	0.94741
9	13	1	0.0244	1	14.1108	0.94283
9	14	1	0.0399	1	14.4288	0.93536
9	15	1	0.0178	1	13.9463	0.94184
9	16	1	0.0111	1	14.0273	0.94180
9	17	1	0.0174	1	14.0524	0.94741
9	18	1	0.0181	1	13.9433	0.93470
9	19	1	0.0170	1	13.9214	0.93423
9	20	1	1.0193	1	14.1232	0.92525
10	11	4	0.0045	22	long	0.96675
10	12	1	0.0198	1	13.9039	0.94924
10	13	1	0.0250	1	13.8986	0.94465
10	14	1	0.0406	1	14.2166	0.93716
10	15	1	0.0185	1	13.7341	0.94365
10	16	1	0.0118	1	13.8151	0.94361
10	17	1	0.0181	1	13.8402	0.94924
10	18	1	0.0187	1	13.7311	0.93650
10	19	1	0.0177	1	13.7092	0.93603
10	20	1	1.0200	1	13.9110	0.92703
11	12	1	0.0190	1	long	0.95526
11	13	1	0.0242	1	long	0.95064
11	14	1	0.0398	1	long	0.94311
11	15	1	0.0177	1	long	0.94964
11	16	1	0.0110	1	long	0.94960
11	17	1	0.0173	1	long	0.95526
11	18	1	0.0180	1	long	0.94244
11	19	1	0.0169	1	long	0.94197
11	20	1	1.0192	1	long	0.93291
12	13	5	0.0154	7	1.59801	0.99090
12	14	5	0.0152	7	1.44922	0.98510
12	15	5	0.0081	7	1.45555	0.98967
12	16	5	0.0080	6	1.49289	0.98727
12	17	5	0.0153	9	1.45342	0.94421
12	18	5	0.0147	9	1.38681	0.94169
12	19	5	0.0146	6	1.16442	0.95599
12	20	10	0.9956	10	1.63542	0.95270
13	14	6	0.0221	9	1.39971	0.98383
13	15	5	0.0081	7	1.37037	0.99619
13	16	5	0.0084	7	1.33291	0.99421
13	17	6	0.0220	9	1.40194	0.94159
13	18	6	0.0219	9	1.41846	0.94224
13	19	6	0.0216	9	1.28832	0.95972
13	20	12	2.1836	12	2.42693	0.95493
14	15	5	0.0149	7	1.40460	0.98510
14	16	6	0.0214	8	1.43808	0.98079
14	17	7	0.0947	11	1.62250	0.93321
14	18	7	0.0938	11	1.65218	0.93236
14	19	7	0.0956	10	1.29758	0.94816
14	20	14	7.8757	14	10.9271	0.94411
15	16	5	0.0085	7	1.29980	0.99368
15	17	6	0.0213	9	1.11396	0.94068
15	18	6	0.0253	9	1.10689	0.94147
15	19	6	0.0361	9	1.11401	0.95908
15	20	12	2.6525	12	3.63252	0.95423
16	17	5	0.0144	6	0.98994	0.94169
16	18	5	0.0145	6	0.99239	0.94421
16	19	5	0.0147	9	1.10721	0.96357
16	20	10	1.0526	10	1.30889	0.95791
17	18	5	0.0720	9	1.10441	0.96357
17	19	5	0.0711	6	1.06119	0.94421
17	20	10	7.7812	10	1.60463	0.95791
18	19	5	0.0743	6	1.21344	0.96357
18	20	6	0.7048	6	1.17872	0.98672
19	20	6	0.6981	6	1.15226	0.98672

4.4 DESIGN OF RELIABLE TOPOLOGY:

Network design is composed of three phases [13]:

(a) Backbone design (BD),

(b) Local access design (LAD),

(c) Local area network within the building (LAN).

We will focus our study to the BD, by considering LAD and LAN are well designed (technologies very well acquired). Assume also that BD can be broken into two phases: B1 and B2. Phase B1 is the choice of an initial connected configuration (backbone design), whereas phase B2 is augmentation with additional arcs to improve performance under constraint(s). Each new arc in general increases the reliability, cost, and throughput and may decrease the delay.

4.5 ENHANCEMENT PHASE:

Assume that the task is to design a reliable network that allows communication between Baghdad 1 and the farthest northern city Dahuk 2, as well between Bagdad and the farthest southern city Albasrah 20. The communication between Dahuk and Albasrah is also considered, in this case the reliability is calculated using technique ($\bar{T}$). two phases are required to get the optimum design. In our study we will focus on the enhancement phase and consider NET1 as the base of this task.

As shown in table 1 the reliabilities R_{12}=0.93738, R_{120} = 0.84349, and R_{220} = 0.79067 are not adequate when the desired is high reliable connection.

This phase includes addition of more links in the way of increasing the reliability.

Redundancy case must be avoiding in order for optimum design, that's why we did not add more than one link during each phase.

Both MHRT & tie-set are applied to show how each one behaves when increasing the complexity of the basic network and to show when does tie-set fail. Simulated results are briefly illustrated in Fig.22& Fig.23 for R_{12}, R_{120}, and R_{220}.

- **Phase A**

Consider that NET2 is the first enhancement for the pairs 1&2 which includes the adding of Ninawa Erbil link $e_{3\ 6}$ $R_{1\ 2A}$ = 0.96037, G_{RA} = 0.96037 - 0.93738 = 0.02299.

NET4 represents the first enhancement phase for (1-20) when a slight increasing has been noticed $R_{1\ 20A}$ = 0.84752, G_{RA} = 0.84752 – 0.84349 = 0.00403.

$R_{2\ 20A}$ = 0.81393, G_{RA} = 0.81393 – 0.79067 = 0.0232.

- **Phase B**

When providing additional link between Tikrit and Kirkuk as in **NET3** there will be 4 additional routs those achieving acceptable gain, but less than the previous phase which adds three new paths**. G_{RB} = 0.97819 – 0.96037 = 0.01782.**

With NET5, $R_{1\ 20B}$ has been increased when providing five new paths to achieve good gain **G_{RB} = 0.94662 – 0.84752 = 0.09910.**

$R_{2\ 20B}$ = 0.92597, **G_{RB} = 0.92597 – 0.81393 = 0.11204.**

- **Phase C**

Direct connection e_{120} has been provided at **NET6** to achieve highly reliable connection $R_{1\ 2c}$ = 0.99781, G_{RC} = 0.99781 – 0.97819 = 0.01961.

NET6 represent the best net yet for the case of 1 & 20, $R_{1\ 20c}$ = 0.9947, G_{RC} = 0.99470 – 0.97820 = 0.0165. $R_{2\ 20c}$ = 0.9925. G_{RC} = 0.992597 = 0.06653.

- **Phase D**

Networks of **phase C** are reliable enough to be applied in real life; however, in this phase we will get a slight increasing in R, but we consider that this increment is still acceptable and does not indicate the redundancy case.

When adding $e_{2\ 9}$ & $e_{1\ 19}$ as in NET7, $R_{1\ 2D} = 0.999$ & $R_{1\ 20D} = 0.9985 \cong 0.999$, $R_{2\ 20E} = 0.998$

- **Phase E (Redundancy)**

As in **NET 8** the new parallel link $e_{2\ 3}$ achieves $R_{1\ 2E} = 0.999$ and another parallel $e_{20\ 19}$ achieves $R_{1\ 20E} = 0.999$, while e_{1219} results in $R_{120E} = 0.999$, $R_{2\ 20E} = 0.998$.

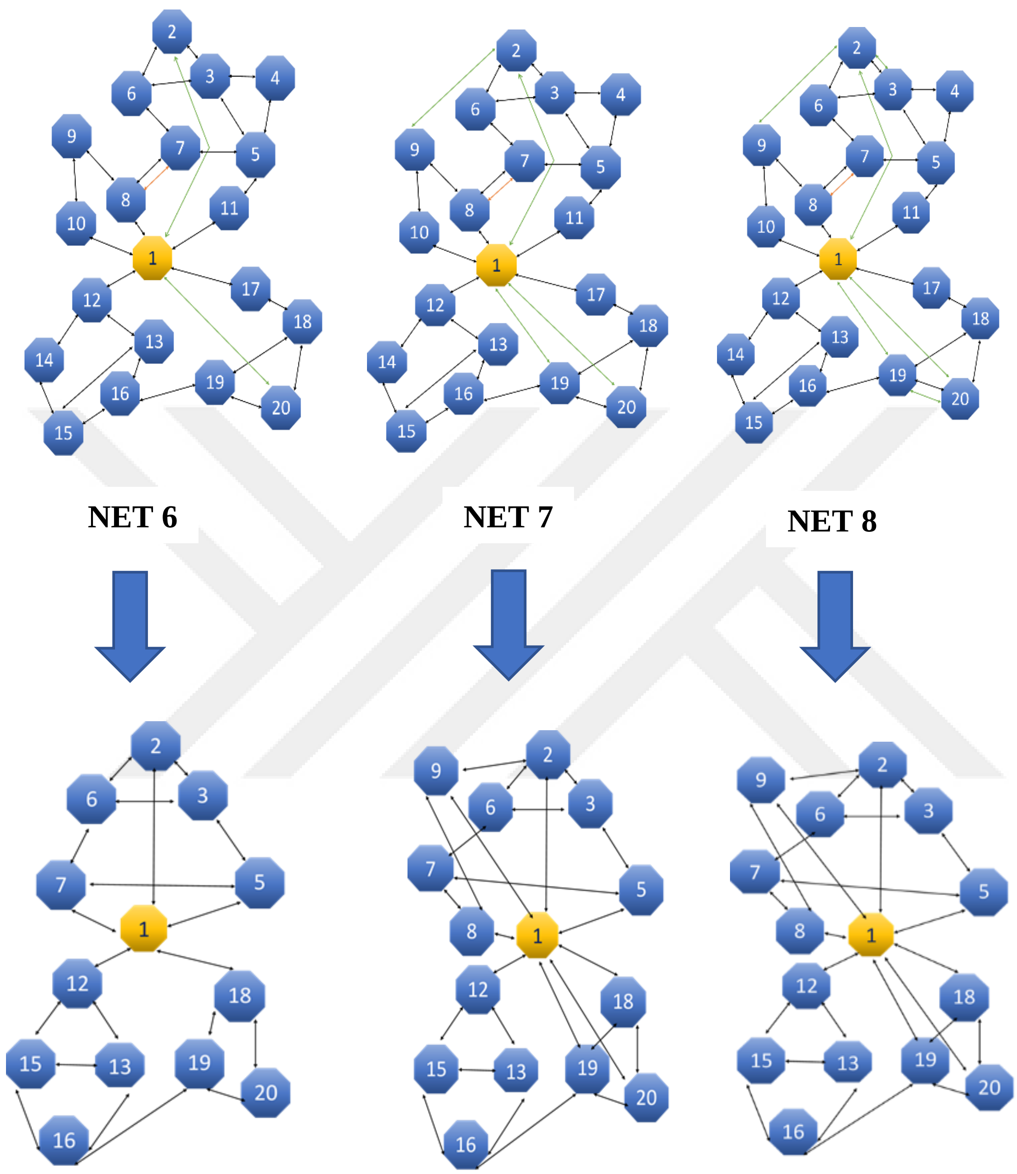
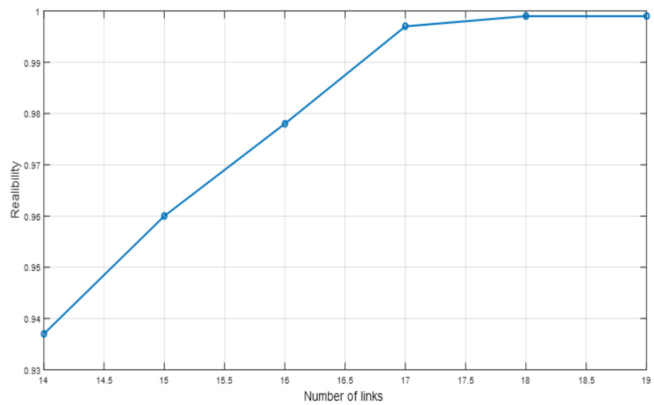


Figure 20. Networks of Phases C&D&E before and after Reduction



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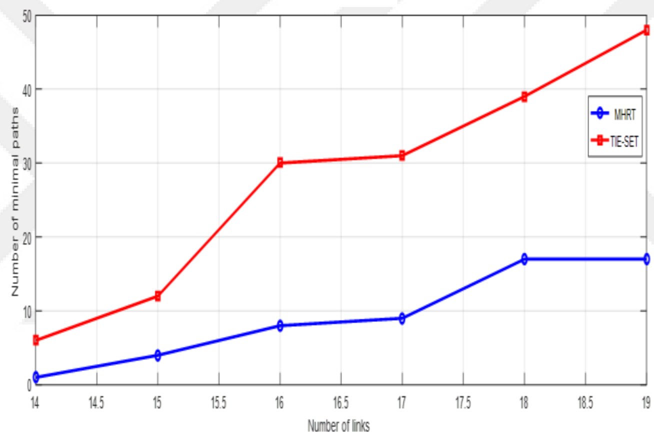

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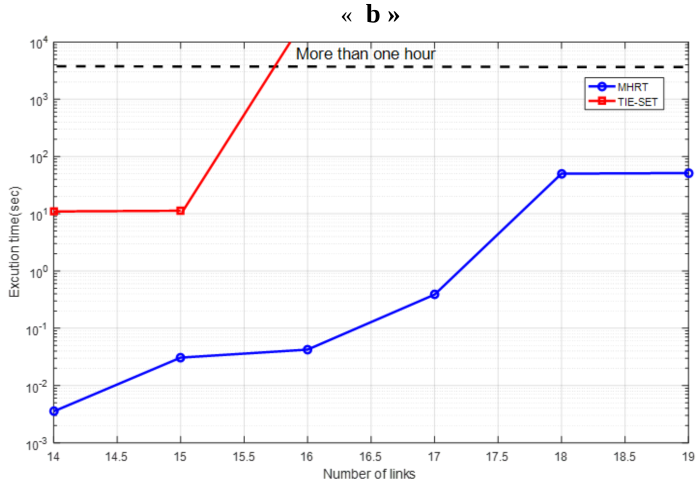

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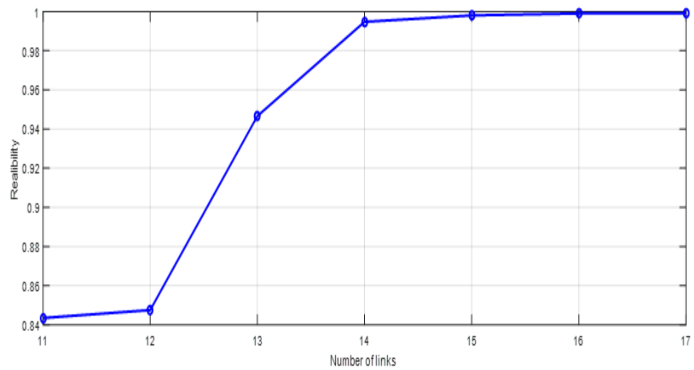
Figure.21. $R_{1\,2}\,case$;**(a) Relation between of** $R_{1\,2}$ **and the number of links, (b) Relationship between No. of MPs by MHRT & tie-set and No. of links, (c)Execution time of MHRT & tie-set**



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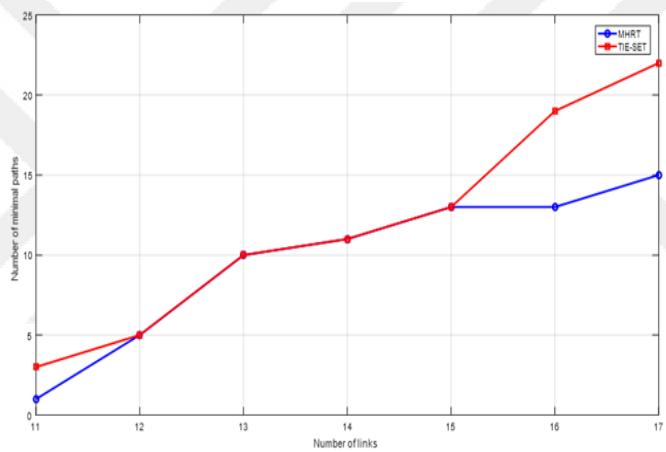

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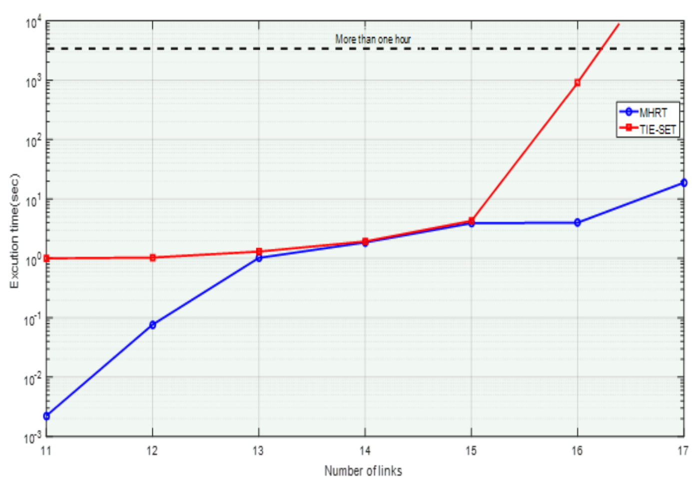

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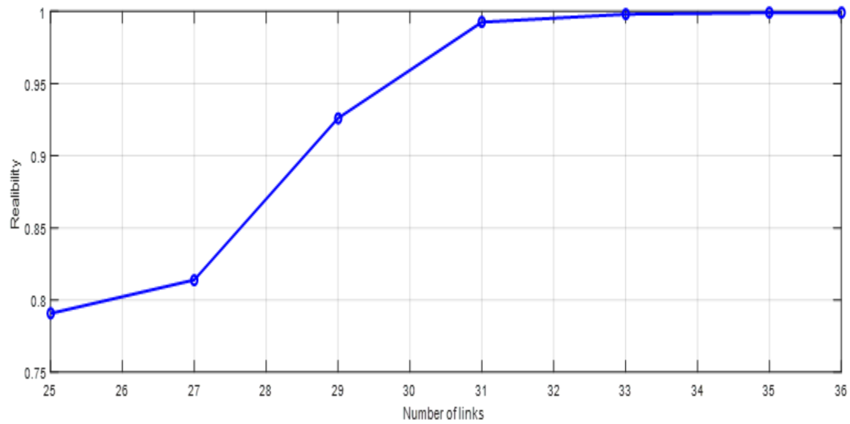
Figure22. $R_{1\,20}$ $case$;**(a) Relation between** $R_{1\,20}$ **and the number of links, (b) Relationship between No. of MPs by MHRT & tie-set and No. of links, (c)Execution time of MHRT& tie-set**



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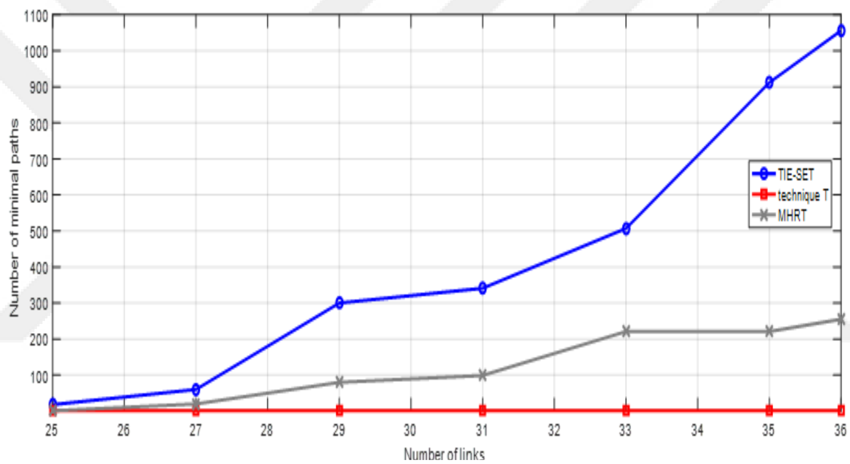

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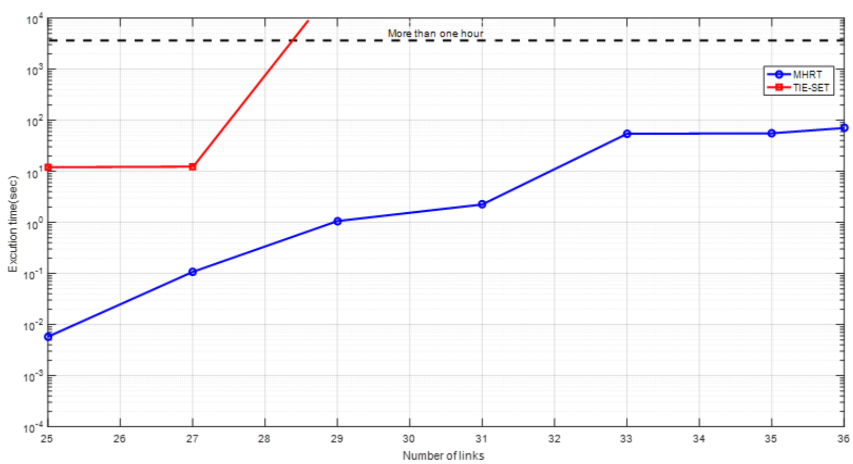

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Figure23. $R_{2\,20}$ $case$;(a) Relation between $R_{2\,20}$ and the number of links, (b) Relationship between

No. of MPs by MHRT & tie-set and No. of links, (c)Execution time of MHRT& tie-set

5.RESULTS ANALYSES AND DISCUSSION

5.1 FACTORS AFFECTING RELIABILITY:

- **Example1:**

Let the cases (13,16), (13,17) of NET4, there are five minimal paths for the first one:

$\boldsymbol{T1}$ =13 →12 →15 →16 = 0.9, 0.8, 0.9

$\boldsymbol{T_2}$ =13→ 15→ 12 →16 =0.9, 0.81, 0.5905

$\boldsymbol{T_3}$ = 13 →15 →16 =0.9, 0.9

$\boldsymbol{T_4}$ =13 →12 →16 =0.9, 0.5905

$\boldsymbol{T_5}$ =13 →16 = 0.9

$\boldsymbol{R_{13\,16}} = [[P_r(T_1) + P_r(T_2) + P_r(T_3) + P_r(T_4) + P_r(T_5)\ ] - [P_r(T_1\,T_2) + P_r(T_2\,T_3) +$

$P_r(T_1\,T_4) + P_r(T_2\,T_4) + P_r(T_1\,T_3) + P_r(T_1\,T_5) + P_r(T_2\,T_5) + P_r(T_4\,T_5) P_r(T_5\,T_3)] + [P_r(T_1\,T_2\,T_3) +$
$P_r(T_1\,T_2\,T_4) + P_r(T_5\,T_2\,T_3) + P_r(T_1\,T_2\,T_5) + P_r(T_5\,T_4\,T_3) + P_r(T_4\,T_2\,T_3) + P_r(T_1\,T_5\,T_4) +$
$P_r(T_1\,T_5 T_3) + P_r(T_1\,T_2\,T_3) + P_r(T_4\,T_2\,T_5)]$-[$P_r(T_1\,T_2\,T_3\,T_4) + P_r(T_1\,T_2\,T_3\,T_5) +$
$P_r(T_1\,T_4\,T_3\,T_5) + P_r(T_4\,T_2\,T_3\,T_5) + P_r(T_1\,T_2\,T_4\,T_5)]$+[$P_r\left(T_4\,T_2\,T_1\,T_3\,T_5\right)]$

$= 3.3280 - 4.6509 + 3.4813 - 1.4184 + 0.2542$

=0.99421

While the last one has more paths but less reliability:

$\boldsymbol{T_1}$ =13 →12 →15 →16 →17 = 0.9, 0.8, 0.9, 0.7292

$\boldsymbol{T_2}$ =13→ 16→ 15→ 12 →17 =0.9, 0.9, 0.81, 0.81

$\boldsymbol{T_3}$ = 13 →15 →12→17 =0.9, 0.81, 0.81

$\boldsymbol{T_4}$ =13 →15 →16→17 =0.9, 0.9, 0.729

$\boldsymbol{T_5}$ =13 →16→17 = 0.9, 0.729

$\boldsymbol{T_6}$ =13 →12→17 =0.9, 0.81

$\boldsymbol{R_{13\,17}} =$ $[[P_r(T_1) + P_r(T_2) + P_r(T_3) + P_r(T_4)] - [P_r(T_1 T_2) + P_r(T_1 T_3) + P_r(T_1 T_4) + P_r(T_2 T_3) + P_r(T_2 T_4) + P_r(T_4 T_3)] + [P_r(T_1 T_2 T_3) + P_r(T_1 T_3 T_4) + P_r(T_1 T_2 T_4) + P_r(T_2 T_3 T_4)] - [P_r(T_1 T_2 T_3 T_4)]]$

$= 3.5758 - 6.1893 + 6.3411 - 4.0089 + 1.4517 - 0.2288$

$= 0.9416 < R_{13\,16}$

This example shows that many factors contribute in increasing the reliability and increasing the number of paths must be taking with considerations.

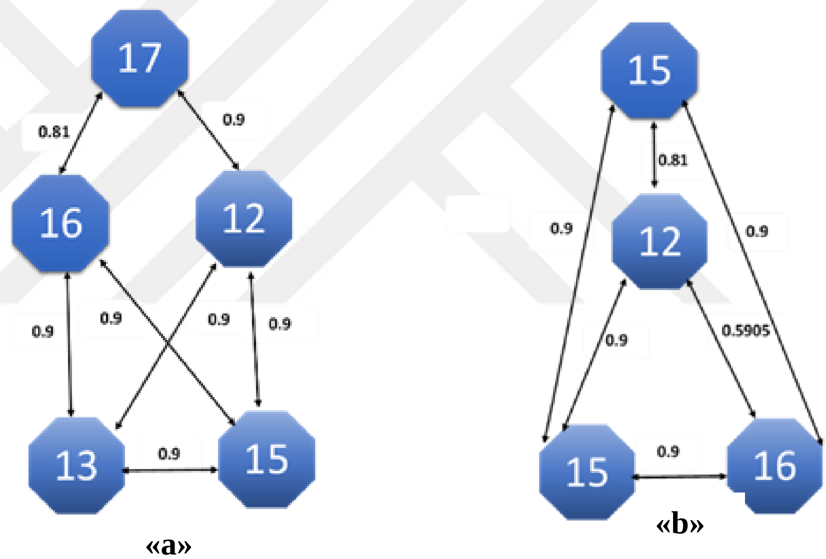

Figure 24. The Simplified NET4 for $a. R_{13\,16}$; b.$R_{13\,17}$

Example2:

Let the cases (15, 16), (16, 20) of NET4, $R_{15\ 16}$ has five paths:

$\boldsymbol{T1} =$ 15→12→13→16 0.81, 0.9, 0.9

$\boldsymbol{T2} =$ 15→13→12→16 0.9, 0.9, 0.5909

$\boldsymbol{T3} =$ 15→12→16 0.81, 0.5909

$\boldsymbol{T4} =$ 15→13→16 0.9, 0.9

$\boldsymbol{T5} =$ 15→16 0.9

$$\boldsymbol{R_{15\ 16} = 3.3227 - 4.6775 + 3.5723 - 1.5063 + 0.2824 = 0.9937}$$

As well $\boldsymbol{R_{20\ 16}}$:

$\boldsymbol{T1}$ =16→13→15 →12→19→20 0.9, 0.9, 0.81, 0.6565, 0.9

$\boldsymbol{T2}$ =16→15 →13→12→19→20 0.9, 0.9, 0.9, 0.6565 0.9

$\boldsymbol{T3}$ =16→15→12→19→20 0.9, 0.81, 0.6561 0.9

$\boldsymbol{T4}$ =16→13→12→19→20 0.9, 0.9, 0.6561 0.9

$\boldsymbol{T5}$ =16→19→20 0.9, 0.9

$$\boldsymbol{R_{15\ 16} = 2.5367 - 3.6186 + 3.0506 - 1.3557 + 0.2542 = 0.8672}$$

It is obvious that both cases have five paths but the last is more reliable. The reasons behind that are the lengths of paths are shorter as well the terms of even power union is greater.

The previous Examples show that the greater number of minimal paths does not always result in greater reliability, because the arrangement of each path as well the length of shortest path have direct impact on reliability. Common links of even power united paths and odd power united ones are also affecting Reliability. Less common number of even case produce less term, on contract with the odd case. The direct connection between source and destination also makes deference, it always increases Reliability in noticeable way as in case $R_{15\,16}$ and $R_{20\,16}$ $of\ table\ 4$.

5.2 DISCUSSION:

1. **NET1:** This network represents the base of designing a reliable network, it is the simplest one. both MHRT and tie-set method success to evaluate all the pairs in reasonable time. However, time consumed by MHRT does not comparable with that of tie-set method. Longest execution time was of ($R_{4\,9}$, M_p= 8; T= 0.08 s) compared with ($R_{4\,12}$; T=13.98275 s). least reliability values are of node 20 that's because node 20 is connected via just one link; hence the probability of failure is more than the others connected via more than one link. From the other hand, parallel link cause highest reliability values as in $\boldsymbol{R_{7\,8}}$.
2. **NET2 & NET3:** The northern part of both networks becomes more reliable as well the pairs compress different area nodes reveal noticeable increment. However, pairs belong to the Sothern part don't make difference because node 1 cannot be traversed twice. MHRT succeed to evaluate all pairs. $R_{2\,10}$ $R_{2\,9}$ both with M_p= 16 spent the longest time of 40.7743 s and 39.8852 s respectively. While tie set failed to measure $R_{4\,10}$&$R_{4\,11}$ in NET2 and **83** pairs in NET3 those have more than 20 minimal paths.
3. **NET4 & NET5:** These networks reveal the opposite increment case of the previous paragraph. In other words, all pairs reliability of northern part didn't change for the same illustrated reason. Both MHRT and tie set method succeed to evaluate all pairs. In NET4 all the minimal paths gotten by both methods are the same but they treated by MHRT with less time than that of tie set. The difference between number of minimal paths appeared in NET5. However, time comparison stills interesting.

4.NET 6 & NET 7& NET 8: All these networks represent highly reliable topologies for three pairs belongs to different areas .Net 6 can be conserved as the optimum design for this task. However, we developed the design through Net7 & 8 to show the impact of adding redundant link. From Fig. 21 it is obvious that tie-set method failed to do the final 4 phases of $R_{1\,2}$,because they involved number of minimal paths more than that it can handle. While MHRT achieves all the phases in sensible time. For $R_{1\,20}$ both methods succeed to complete the enhancement phase but even for 12, 13, 14, 15 links the number of minimal paths is the same for MHRT and tie set but tie set spent more time; furthermore, it failed to simulate the redundancy phase when $M_p = 22$. The last pair is measured by $\overline{\mathrm{T}}$. Therefore, the number of minimal paths has not be taken in account and the time consumed was that spent to find $R_{1\,20}$ and $R_{2\,20}$ of each phase. However, tie set failed when the number of links became 31 because it is already failed to calculate $R_{1\,20}$ during this phase as well the following phases.

6. CONCLUSION AND FUTURE WORK

6.1 SUMMARY

Tie set method can only treat small and simple networks; while reduction techniques used in the proposed algorithm make it efficient enough for RELIABILITY evaluation when tie set method fails. It is obvious that when both methods success in evaluation, MHRT stills consumed less time even when the number of minimal paths is the same. The reason behind that is the length of the minimal paths is less than that of tie set; hence they don’t require long execution time as in tie set case.

Tables in (chapter 4) illustrate adequate information for comparison purpose, and results are analyzed with details in (chapter 5) when the factors effecting reliability are discussed. Sometimes the larger number of paths does not achieve greater reliability as in Net1 when $\boldsymbol{R_{2\,9\,,Mp=8} < R_{2\,5\,,Mp=6}}$. The additional links do not always result with the same increment in reliability as in the following cases of NET2 $\boldsymbol{G_{3\,9} = 0.00228 < G_{6\,9} = 0.01518 < G_{4\,9} = 0.03076}$, because the number of minimal paths increased by 4 for the first two cases and by 8 for the last case. The position of placing new link is also makes great deference. Direct connection between source and destination always increasing reliability.

6.2 FUTURE WORK

The following two proposed ways are efficient for complex computer networks reliability:

1. A new technique to convert the complex mesh computer network to a network with cut points. In order to use technique for evaluate two-terminal reliability when the pairs belong to different sub networks for sensitive applications purposes.

2. When the priority is for fast calculations rather than exact results; for example, dynamic large network. Reliability of complex networks can be evaluated using accepted approximation method depends on eliminating special terms of union operation of the inclusion exclusion expansion equation to reduce the execution time.

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