

T.C.

ALTINBAŞ UNIVERSITY

Electrical and Computer Engineering

A DIGITAL HYBRID COMMUNICATION SYSTEM USING PULSE POSITION AND WIDTH MODULATION

Lobna Watheq Mohammed Alsadoon

Master Thesis

Supervisor

Oguz Bayat

Istanbul (2019)

A DIGITAL HYBRID COMMUNICATION SYSTEM USING PULSE POSITION AND WIDTH MODULATION

by

Lobna Watheq Mohammed Alsadoon

Electrical and Computer Engineering

Submitted to the Graduate School of Science and Engineering in partial fulfillment of the requirements for the degree of Master of Science

ALTINBAŞ UNIVERSITY

2019

This is to certify that I have read this thesis and that in my opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Asst. Prof. Name SURNAME

Supervisor

Examining Committee Members

I certify that this thesis satisfies all the requirements as a thesis for the degree of

Academic Title Name SURNAME

Head of Department

Academic Title Name SURNAME

Director

Approval Date of Graduate School of

Science and Engineering: ____/____/____

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Lobna Watheq Mohammed Alsadoon

DEDICATION

First, I would like to thank Allah Almighty for the power of mind , health, strength , guidance , knowledge and skills to complete this study.

This thesis is wholeheartedly dedicated to my beloved grandfather, who have been my source of inspiration, he tells me that" every success in your life will be best gift for me". To my grandmother, who have been supporting me with the kind and pure love. To my parents , there is no words to describe what you mean to me , there is nothing that I can repay for what you have done to me. I will continue to do my best to achieve your expectations.

I dedicated this to cherished people who have meant and continue to mean so much to me . And lastly, to the family, relatives and friends who have been encouraging me during this study .

ACKNOWLEDGMENTS

I would like to thank my supervisor professor Oguz Bayat for all support during my study . It's great pleasure to express my deepest gratitude to my friends who have shared with me best moments during my study for the Master degree .

ABSTRACT

A DIGITAL HYBRID COMMUNICATION SYSTEM USING PULSE POSITION AND WIDTH MODULATION

Alsadoon,Lobna

M.Sc., Electrical and Computer Engineering, Altınbaş University Supervisor: Assoc. Prof. Dr. Oguz Bayat Date: Feb/2019 Pages: 58

Digital communications have been widely employed in data exchange among different digital system, according to the wide use of digital systems and the benefits of using digital communication. A digital value is defined using a series of bits, each of these bits can have one of two possible values. Systems that interact with such data without converting them to analogue format are known as digital systems. By defining the number of bits in a digital variable, the number of discrete values that the variable can have are known, as well as the values in the range. Thus, the use of digital data has shown significantly better resistance to noise, as well as more efficient storage.

As the digital system can only output digital values, analogue modulations techniques are not recommended in such systems to avoid the need for analogue components in the system. Pulse-Coded Modulation (PCM) has been widely used in digital communications, where the characteristic of a pulse is modified depending on the value being modulated. Pulse Amplitude Modulation (PAM) is when the amplitude of the pulse is adjusted according to the input value. However, according to the high effect of noise over the amplitude of the signal, PAM has shown limited bandwidth, defined by the number of bits the system can communicate in a second. Pules Width Modulation (PWM) adjusts the duty cycle of the outputted pulse, depending on the input value. This type of modulation has shown higher resistance to noise, as the amplitude of the pulse is not considered in the modulated value extraction. Another important, recent, pulse modulation technique is the

Pulse Position Modulation, where the position of the outputted pulse in a predefined interval is adjusted according to the input value. This modulation technique has shown significantly good noise resistance and power efficiency, as the width and amplitude of the pulse is constant regardless of the inputted value.

Most of the modern digital systems use programmable digital components, such as microprocessors and Field-Programmable Gate Array (FPGA). Increasing the bandwidth of the system by changing the modulation scheme, without the need to impose changes to the hardware, has significant importance in such systems. Thus, a hybrid modulation technique is proposed in this study, which combines the PWM and PPM modulation methods in the outputted signal. The bits in the input value are split into two groups, each modulated using one of the modulators, and each modulator outputs its own pulse, based on the value of the assigned group of bits. As the output of these modulators are binary, and two modulators exist, there are four possible states of these outputs per each time instance. Two encoding techniques are implemented to encode these possible states, so that, the receiver can detect the output of each modulator per each time instance. Depending on the characteristics of the channel, amplitude and frequency encoding can be used to establish communications using the proposed method.

The evaluation results of the proposed method have shown the ability of increasing the bandwidth of the communication, significantly, without changing the main components of the system, such as the global clock rate or pulse rate. The proposed hybrid method has been able to achieve 16Mbps bandwidth using the same system configurations that have only been able to achieve a maximum of 8Mbps using the PPM modulation, both with 10^{-4} Bit Error Rate (BER). Moreover, the results show that the use of amplitude encoding has produced less average BER, compared to the use of frequency encoding. The reason behind such difference is the distortion imposed by the encoding frequency to the timings of the output signal, especially the width of the PWM output. However, both encoding techniques have been able to increase the bandwidth to the 16Mbps rate, with a global clock of 50MHz and 1M pulse per second.

Keywords: Digital Systems; Digital Communication; Pulse Amplitude Modulation; Pulse Width Modulation; Pulse Position Modulation.

TABLE OF CONTENTS

LIST OF TABLES

Pages

LIST OF FIGURES

LIST OF ABBREVIATIONS

1. INTRODUCTION

Recently, the digital representation of data has been widely employed in different applications, which has produced the digital revolution. In digital format, a value is represented using a series of bits, where the weight of each bit depends on its position. Unlike the use of digital representation, a digital value can have only one of a set of possible values in a certain range, i.e. discrete, while such value in an analogue representation can have infinite possible values in that range. The number of possible values is recognized by the number of bits reserved for that value, where each bit can have only one of two possible values, zero or one. Hence, digital representation has shown better storage efficiency and resistance to noise, as the storage size required to store the value is defined by the number of bits, and only noise sufficient to change the transmitted value from one of the possible values to another is noticeable [1].

The storage and processing of data in digital format have encouraged communicating these data using the same format, i.e. digital. Such systems are known as digital communication systems, where the data is processed, transmitted and received in digital format, without being converted to analogue. Unlike analogue communication system, the predefined discrete values, based on the size of that value, can be used to select the nearest value to the one received by the receiver, while in analogue communication the received value is retrieved as is [2]. Thus, bandwidths of a digital communication system are measured by the number of bits the system is capable of communicating per a certain time interval, mainly bits per second (bps).

The discreteness of the digital values has allowed the use of more power-efficient modulation techniques, based on pulses, than those used in analogue modulation. Each transmitted pulse can hold the value of one or more bits of the digital value, where some of the characteristics of the pulse are changed based on the modulated value. For example, in Pulse Amplitude Modulation (PAM) the number of possible amplitudes of the transmitted pulse is 2^n , where *n* is the number of bits modulated over that pulse. However, as the noise in the communication system mainly affects the amplitude of the pulse, the number of bits being modulated over the amplitude of the pulse is relatively low, compared to other techniques that modulate other characteristics of the pulse [3, 4].

One of the widely used pulse modulation technique is the Pulse Width Modulation (PWM), which has better resistance to noise than the PAM as the amplitude of the pulse is constant and irrelevant to the modulated value. In PWM, the ratio between the time a certain state of the pulse, high or low, to the total time of the pulse, i.e. the duty cycle, is manipulated depending on the value being modulated. By default, the duty cycle of the outputted pulse is 0% when the modulated value is equal to the lower limit of the rage, while a pulse with 100% duty cycle is transmitted when the modulated value is equal to the maximum of the range. The duty cycle of other discrete values in the range are calculated based on the position of the value being modulated in that range. However, as the outputted pulses can have different duty cycles, the power consumed by each of the pulses can vary depending on the value being modulated on that pulse [5, 6].

Another important pulse-based modulation technique is the Pulse Position Modulation (PPM), which also has better resistance to noise as the amplitude of the pulse is irrelevant to the modulated value. Based on the number of values being transmitted per second, the interval reserved for each value is constant and known for both the transmitter and the receiver. Depending on the size of the values being transmitted *n*, this interval is divided into $2ⁿ$ divisions. Then, a pulse of a fixed width is transmitted into one of these divisions, based on the input value being modulated. A pulse is transmitted in the first division when the modulated value is equal to the minimum of the range, while the pulse is transmitted at the last division when the value is equal to the maximum of the range. Moreover, as the amplitude and width of the pulses transmitted using PPM are fixed, the power consumption is constant regardless of the values being modulated. Thus, PPM is considered as the most efficient digital modulation technique [7, 8].

Changing more than one of the characteristics of the pulse can increase the number of bits being modulated on that pulse, where some of these bits are modulated over one characteristic and others are modulated over another. Quadrature Amplitude Modulation (QAM) is one of the most popular examples of such techniques, where two of the bits being modulated are used to define the amplitude of the outputted signal, while the remaining bits are modulated using another characteristic of the pulse, such as its width or position. The modulation of only two bits over the amplitude of the pulse is according to the high influence of the noise on the amplitude of the pulse, so that, using only two bits, the

amplitude of the pulse can have one of only four levels [9, 10]. Additionally, as another characteristic of the pulse is also manipulated depending on the value of the bits being modulated using that characteristic, the amplitude of cannot be zero for any modulated value.

1.1 PROBLEM DEFINITION

With the wide range of digital applications, many systems have been using digital communications techniques to establish links among digital systems, so that, digital information can be exchanged among these systems. Pulse modulation has been widely implemented in these systems, where one or more characteristics of the pulse are manipulated depending on the value being modulated. Hybrid pulse modulation techniques can increase the speed of the communications using the same communications channel, i.e. the same pulse rate. However, most of the existing hybrid techniques include manipulating the amplitude of the pulse based on the value of two bits from the modulated information. The significant effect of the noise on the amplitude of the pulse, compared to other characteristics, such as position and width, is the main reason behind limiting the number of bits modulated over the amplitude to only two. Hence, only four states of the amplitude are required, so that, the effect of the noise is minimized. Additionally, as another characteristic of the pulse is being manipulated, the amplitude of the pulse must be maintained above zero in all cases, so that, the other characteristics can be extracted by the receiving end to recognize the value modulated on it.

1.2 THE AIM OF THE STUDY

A hybrid digital communication scheme is proposed in this study to increase communication speed using the same channel. To minimize the effect of the noise on the communicated signals, the proposed method avoids the inclusion of the amplitude modulation and relies on the width and position pulses generated using different parts of the modulated value. However, as the same channel is being used to transmit this information, the transmitted signal is encoded to reflect the different possible combinations of the pulse width and position. Such an approach can increase the size of the digital value being modulated using the same channel, so that, more data can be communicated using the same infrastructure.

1.3 THESIS LAYOUT

The following chapters of this thesis are organized as follows:

- In chapter two, the literature related to digital communications and pulse modulation techniques, as well as the encoding techniques are reviewed.
- The proposed method is described in chapter three, where the hierarchy of the method and the encoding technique are illustrated in details.
- The experiments conducted in this study are presented in chapter four, alongside with the results that describe the improvement in the communications speed and the effect of noise on the employed encoding techniques.
- The results of the experiments are discussed in chapter five and compared to the methods exist in the literature.
- The conclusions of the study are summarized in chapter six.

2. LITERATURE REVIEW

2.1 INTRODUCTION

According to the wide employment of digital systems in different applications, a significant emphasis has been brought to communicating data in digital format among these systems, without the need to convert them to analogue format. According to the discreteness of the digital values, digital communication schemes have shown better resistance to noise, which enables the use of higher transmission rates. In this chapter, the basic components of digital systems and schemes used to exchange data among them are reviewed from the literature related to the topic discussed in this study.

2.2 DIGITAL SYSTEMS

A system is described as a digital system when the values inputted to the system, the processing of these values and the values that are produced by the system are in digital format. A digital value, also denoted as a binary value, is a set of bits where each of these bits has two possible value. Regardless of the representation of these value, such as zeros and ones or low and high voltages, logical comparisons can be executed using these values as one of these values represents True, while the other represents False. Moreover, mathematical operations on digital values are implemented using logical expressions on the bits that are in the values being processed [11, 12].

To achieve the tasks required from a digital system, two popular cores, which are the microprocessor and Field-Programmable Gate Array (FPGA), are used in these systems to process the input data and produce the required output [13, 14]. A microprocessor has a set of built-in instructions that can be employed to achieve any task required from the digital system, by cascading these tasks according to that task. Each instruction can require one or more cycles from the microprocessor, where the processor is only capable of conducting one instruction at a time instance. Thus, regardless of the topology of the communications achieved by the microprocessor, all computations must be serialized even if those computations are not related to each other, i.e. they can be executed separately [15, 16].

In contrast, an FPGA consists of an enormous number of Configurable Logic Blocks (CLB) that can be configured to achieve any logical task by defining a lookup table for all possible input combinations. These blocks are connected to each other and the inputs/outputs of the FPGA using a set of programmable interconnections network, so that, the inputs and outputs of each block can be directed according to the requirements of the implemented task, as shown in Figure 2.1. Thus, unlike microprocessors, irrelevant computations can be conducted using parallel paths in the FPGA, so that, multiple LBs can output their values in the same cycle [17].

Figure 2.1: Generic architecture of an FPGA [17].

As illustrated earlier, both the microprocessor and the FPGA require a global clock to define the frequency of execution, and data flow. Hence, the use of higher global clock rate can increase the processing speed in these devices. However, the maximum clock rate is limited by the capabilities of the processing device, which are defined by the minimum cycle time required by the microprocessor and the FPGA to complete a single instruction. Increasing the rate of the global clock behind these capabilities can produce errors as the microprocessor and FPGA are instructed to output their values before accomplishing the process. Moreover, this clock is also used to control the flow of the data among the different parts of the system, so that, the data is maintained unchanged until the process is finished [18, 19]. Thus, a system with a higher clock rate requires higher processing power,

which imposes the need for more expensive processing units. One of the widely used global clock rate, in both the microprocessor and FPGA, is the 50MHz, as this frequency is sufficient for most applications and the cost range of processors with such clock range is low, compared to processors with higher capabilities [20-23].

2.3 DIGITAL COMMUNICATIONS

As the use of digital systems has been growing rapidly, establishing communications among these systems without the need for embedding analogue components has become essential. As the digital values are discrete, the generation and use of analogue signals as carries cannot be implemented in digital systems. This limitation has encouraged the use of Pulse-Coded Modulation (PCM), where pulses with discrete levels are used instead of the continuous carrier signals, such as sine waves [24, 25]. According to the discreteness of the digital values, the received values are quantized according to the predefined possible values, based on the size of the value being transmitted. Thus, digital communication schemes have shown higher resistance to noise, reflected by higher Noise to Signal Ratio (SNR) [26, 27]. To modulate the value over the transmitted pulse, different techniques are proposed to manipulate one or more of the pulse's characteristics, such as its amplitude, width and position.

2.3.1 Pulse Amplitude Modulation (PAM)

In PAM, the amplitude of the transmitted pulse is manipulated depending on the value being modulated over that pulse. Depending on the size (*n*) of the values modulated over each pulse, defined in bits, the output pulse can have one of only 2^n possible amplitude level. Upon arrival, the received level is quantized into these 2^n level to reduce the effect of noise. In order for the noise to affect the received value, the level of the pulse must be transitioned from one level to another, i.e. the effect of the noise must be greater than the difference between two consequent levels. However, as the noise mainly affect the amplitude of the signal, PAM is considered as the most affected PCM communications scheme [28, 29]. Moreover, as shown in Figure 2.2, as the width of the pulse is constant, defined by the pulse rate, while the amplitude of the pulse is changed according to the modulated value, the power consumed by the modulator is relative to the value being modulated [30].

Figure 2.2: PAM output for sample input values.

Different techniques are proposed to allow the digital system of outputting different levels of the pulses' amplitudes. The method proposed by Xi et al. [31] uses a 4×4 Light Emitting Diode (LED) array, shown in Figure 2.3, to transmit digital video data from one digital system to another, using Visual Light Communication (VLC). As the array consists of 16 LEDs, the proposed system has been able to modulate four bits per each pulse, where the number of LEDs turned on per a pulse is equal to the value being modulated. The system also filters background noise using a software procedure that quantizes the received value. Although the system has been able to achieve 40Mbps, using an FPGA as the core of the digital system, the study does not report the rate of the global clock of the FPGA.

Figure 2.3: The 4×4 LED array used to produce different pulse levels for PAM [31].

Moreover, the Micro-LED, shown in Figure 2.4, designed by Ferreira et al. [32] consists of multiple light emitting junctions that can be turned on and off separately. The aim of this implementation is to allow digital systems to establish VLC by only outputting digital signals and using the same space, instead of an LED array. By controlling the number of junctions being turned on per each pulse, the intensity of the transmitted light varies according to that number, where increasing the number of forwardly-biased junctions increases the intensity of the transmitted light. By measuring the intensity of the received light, the receiver can recognize the modulated value. Despite the fact that this LED is designed for high-speed optical communications, it illustrates the idea behind controlling the level of the output pulse using a digital system, with only on and off possible states.

Figure 2.4: A high-speed micro-LED for VLC (All measurements are in μ m) [32].

2.3.2 Pulse Width Modulation (PWM)

The amplitude of the outputted signal in PWM has a constant value, which is easier for the digital system to output. However, the duty cycle of the outputted signal is manipulated according to the value being modulated over that pulse, so that, the receiver can retrieve that value by measuring the duty cycle of the received pulse. As the noise mainly affect the amplitude of the pulse, PWM has better resistance to such noise because the amplitude of the pulse has no value at the receiving end [33, 34]. However, as shown in Figure 2.5, despite the constant amplitude, the width of the positive portion of the pulse is relative to the input value. Hence, the power consumed by the modulator to transmit the pulse is dependent on the value that the pulse holds.

Figure 2.5: PWM output for sample input values.

The VLC system proposed by Pradana et al. [35] uses PWM to exchange text data from one digital system to another. Different scenarios are evaluated in the study using different distances between the transmitter and receiver, different view angles and multiple numbers of bits per each pulse. The results of the study show that the system has been able to achieve 920bps bandwidth with BER of 10^{-4} when a single bit is modulated per each pulse. Increasing the number of bits per each pulse has increased the bandwidth but the BER has also increased, according to the limited capabilities of the hardware used in the proposed system. Such behavior reflects the importance of including another modulations technique, in a single digital communications system, so that, the same components can produce higher bandwidths using the same equipment.

2.3.3 Pulse Position Modulation (PPM)

Pulse position modulation is one of the recent PCM techniques that have shown significant improvement in the communications among digital systems, according to its resistance to noise and high power efficiency. To modulate a value in PPM, the position of the pulse in a predefined interval is adjusted, so that, by measuring the delay between the

beginning of the interval and the occurrence of the pulse, the receiver can recognize that value [36, 37]. As shown in Figure 2.6, the width of the interval reserved for a pulse to occur T_v is defined by the transmission rate, i.e. the number of pulses per seconds, as shown in Equation 2.1. Moreover, the width of the pulse T_p is defined by the number of discrete values that the pulse can hold, i.e. the size of the input value in bits n , as shown in Equation 2.2. The time between the beginning of the interval until the pulse is outputted by the transmitter T_d is calculated using the modulated value *I*, which has a maximum value of 2^n , as shown in Equation 2.3.

Figure 2.6: PPM output for sample input values.

$$
T_v = \frac{1}{pulserate} \tag{2.1}
$$

$$
T_p = \frac{T_v}{2^n} \tag{2.2}
$$

$$
T_d = I \times T_p \tag{2.3}
$$

As the figure and equations show, the amplitude of the transmitted pulse, as well as its width, are irrelative to the value modulated over that pulse. Additionally, a single pulse is outputted per each interval, defined by the transmission rate, with the same width and magnitude, for any modulated value. Thus, the power consumption in PPM is constant, regardless of the input values, which is not the case in both the PAM and PWM. Moreover, as the amplitude of the pulse has no effect on the calculated value, PPM has high resistance to noise, as the noise mainly affects the amplitude of the communicated signal.

The performance of a digital communication system that uses PPM over a blue-green laser to establish VLC is evaluated by Xu et al. [38]. This evaluation shows that the use of PPM has been able to achieve 5Mpbs, with a pulse rate of 1MHz and a 32MHz timer to calculate the timings required by the PPM, with a BER of 10^{-4} . Increasing the communications speed beyond this bandwidth requires reducing the width of the pulse, by reducing the interval reserved per each pulse or increasing the number of positions in the same interval, i.e. increasing the size of the modulated value. Narrower pulses are beyond the capabilities of such system, according to the limited capabilities of the laser transmitter and the timer's clock. Thus, the only possible way to improve the bandwidth of such system is to maintain these configurations, while embedding another modulation scheme in the same pulse interval.

2.3.4 Quadrature Amplitude Modulation (QAM)

Recently, QAM has been one of the widely used hybrid digital modulation schemes, where the amplitude of the pulse is used to increase the number of bits being modulated over the pulse. According to the high sensitivity of the Amplitude Modulation (AM) to the noise and to maintain acceptable error rates, only four levels are used for the amplitude of the pulse, which represents the possible values of two bits. The VLC system proposed by Sarwar et al. [39] uses QAM with a blue LED and a solar panel receiver, where two of the four modulated bits are recognized based on the amplitude of the pulse. As there are 16 possible states for the received value, i.e. four-bit value, such system is denoted as 16- QAM. This system has been able to achieve a bandwidth of 15Mbps with BER of 1.6883×10^{-3} . As shown in Figure 2.7, in QAM the received values are split into four regions, where the number of possible values in each split is defined by the number of the remaining bits modulated over the pulse. By projecting the received value on that split, it is possible to select the nearest possible discrete value, in order to reduce the effect of noise

on the modulation. Thus, using QAM, the number of discrete values a pulse can hold is multiplied by four. To increase the number of bits a pulse can hold, it is important to modify the encoding method, so that, more bits can be encoded using these four levels.

Figure 2.7: Constellation map of the 16-QAM communication in [39].

3. METHODOLOGY

To increase the communications speed, the proposed method aims to increase the number of bits being modulated over the time interval reserved for each pulse, defined by the communications channel. To accomplish such a task, the time interval transmits digital data modulated using PWM and PPM in each of these intervals. However, to recognize the characteristics of each of the transmitted pulses, the proposed method encodes these pulses in the output channel. Two encoding techniques are employed in the proposed method, depending on the characteristics of the communication channel. In this chapter, the proposed method and the employment of the encoding techniques are described in details.

3.1 THE HYBRID PWM-PPM METHOD

In the proposed method, the bits of the digital input are equally split into two separate sets. Each set is fed to a separate modulator, i.e. one set to the PWM and the other to the PPM. The output signals from these modulators are then fed to a multiplexer, in order to select the appropriate encoding from a set of waveforms with different characteristics, depending on the communication system. As the output of each of these systems is limited to two states, high or low, there are only four possible combinations between these outputs. Thus, four signals of different characteristics are required as inputs to the multiplexer, while the channel-select bits are controlled by the outputs of the modulators, as shown in Figure 3.1.

The topology of the transmitter, in the proposed method, is not affected by the type of encoding used to represent the four possible states. However, it is important to recognize the characteristics of each of these signals at the receiving end, as the scheme used to distinguish each of the encoded relies on the characteristic that changes in the transmitted signal. Two types of encoding are employed in the proposed method, amplitude and frequency encoding. The types of encoding are selected depending on the communication channel and how the signal is transmitted from one end to another.

Figure 3.1: The transmitter of the proposed hybrid PWM-PPM communication system.

3.2 SIGNAL ENCODING

As illustrated earlier, there are four possible states of the output from the transmitter, depending on the outputs of the PWM and PPM modulators. Per each time instance, the encoded signal holds the states of the PPM and PWM modulators' outputs. Upon arrival, the receiver retrieves the values of these outputs and forwards them to a PPM and PWM demodulators, respectively. Each modulator outputs the original value being transmitted using the corresponding modulation technique. Then, these retrieved bits are combined into a single value that represents the original input value to the transmitter. Depending on the channel used in the communications, one of two types of encoding are proposed to be used in the hybrid technique.

3.2.1 Amplitude Encoding

The amplitude of the output signal is adjusted according to the combination of the value outputted from the modulators. As there are only four possible combinations of these outputs, four different levels are selected for the output signal. As the amplitude is adjusted states, to improve the power efficiency of the system. However, it is also possible to use any other predefined four amplitude levels to encode these values, in case the communication system requires a signal to be maintained in the channel. Upon arrival, the amplitude of the signal is quantized into the predefined four levels, in order to recognize the state of each modulator at that time instance. Based on the predefined encoding values, the state corresponding to each modulation scheme is delivered to a demodulator. As there are only two possible states per each demodulator, namely True or False, an or gate is used with the output of the level detector, so that, a True value is delivered to the modulator whenever the encoded signal indicates that the corresponding value or both values are True. For the sample encoding values shown in Table 3.1, the receiver can retrieve the transmitted value using the topology shown in Figure 3.2.

PWM State	PPM State	Amplitude Level
False	False	
True	False	
False	True	
True	True	

Table 3.1: A sample amplitude encoding values.

Figure 3.2: Receiver of the amplitude-encoded hybrid PWM-PPM communication scheme.

As shown in Figure 3.2, the output of the level that represents the absence of both outputs from the modulators, zero in the previous example, is neglected in the logical comparisons. However, when the amplitude that represents the absence of these outputs is not set to zero, it is possible to use its value to indicate the existence of the communications signal, without any role in the computations of the received value. As the addition of a false value to the or gate does not affect its output, the inclusion of the amplitude level that indicates the absence of the outputs increases the complexity of the model without achieving any useful task. This type of encoding can be employed with radio communications, as the amplitude of the transmitted signal can be easily modified. It is also possible to employ this encoding scheme in optical communications but a more complex circuitry is required to control the intensity of the transmitted light.

3.2.2 Frequency Encoding

In communication systems where the manipulation of the signal's amplitude is not applicable, it is possible to use the proposed hybrid system by encoding the outputs of the modulators through adjusting the frequency of the output signal. Similar to the amplitude encoding, when the communication system allows the absence of the signal in the channel, it is possible to use only three frequencies to encode the three values that represent the existence of one or both outputs from the modulator. However, the use of a fourth frequency can be used to detect the existence of the communications and distinguish the loss of communications from the absence of the outputs. To decode the received signal, three bandpass filters are used to detect the frequencies required to determine the state of each demodulator. For the sample encoding frequencies shown in Table 3.2, the transmitted value can be retrieved using the receiver topology shown in Figure 3.3.

PWM State	PPM State	Signal's Frequency
False	False	Freq1
True	False	Freq2
False	True	Freq3
True	True	Freq4

Table 3.2: A sample frequency encoding values.

Figure 3.3: Receiver of the frequency-encoded hybrid PWM-PPM communication scheme.

4. EXPERIMENTAL RESULTS

To measure the improvement in the bandwidth of the communication system when the proposed method is used and compare it to the use of the PPM and PWM methods, solely, a communication system is implemented in Matlab's Simulink environment. To illustrate this improvement, and according to the ability of the proposed method to handle twice the bandwidth using the same channel, the performance of these systems is evaluated while modulating 8 and 16 bits per each pulse, or pulse interval. Time computations in the system are implemented using a 50MHz crystal, as such frequency is widely used in microcontroller- and FPGA-based digital systems. The performances of the simulated models are illustrated by measuring the BER between the transmitted and received data.

4.1 USING PWM MODULATION

Two models are simulated that use PWM to communicate data at 8 and 16 bits per pulse. The implemented models are shown in Appendices A and B, where a random value generator is used to generate values for the transmitter, which transmits them using PWM. Upon arrival, the signal is passed through a PWM demodulator in order to retrieve the received data. A BER calculator is used to calculate the bit error between the values generated by the random integer generator and those outputted by the PWM demodulator. The block diagram shown in Figure 4.1 summarizes the topology of the implemented model, where the measured BERs, the pulse rate and the bandwidth per each simulated scenario, using this topology, are summarized in Table 4.1.

Figure 4.1: Block diagram of the simulated PWM digital communication system.

Pulse Rate	Transmission Rate (bps)	Bit Error Rate
1 _K	8K	0.83%
1K	16K	0.96%
5K	40K	0.83%
5K	80K	0.84%
10K	80K	0.00%
10K	160K	1.73%
50K	400K	0.84%
50K	800K	1.47%
100K	800K	0.84%
100K	1.6M	3.38%
500K	4M	0.33%
500K	8M	2.11%
1 _M	8M	0.72%
1 _M	16M	2.61%
5M	40M	8.04%
5M	80M	10.29%
10M	80M	7.94%
10M	160M	11.83%

Table 4.1: Summary of the implemented PWM scenarios and their BERs.

These results are illustrated visually in Figure 4.2, which shows that the BER in both input sizes increases dramatically when 1M pulse per second, and above, are used to transmit data in the communication system. Such rapid increment indicates that the computations required to achieve the PWM at that pulse rate require higher crystal frequency. Moreover, beside the modulation of eight bits per each of the 10K pulse per

second, the system has not been able to establish communications with BER less than 0.01%, while the highest bandwidth achieved with less than 1% BER is 4Mbps.

Figure 4.2: BER versus pulse rate for the PWM modulation system.

A sample from the output signal of the modulator in the implemented model, alongside with the input values of the modulator, generated by the random generator, are shown in Figure 4.2.

Figure 4.3: Output signal of the PWM modulator in the implemented system.

4.2 USING PPM MODULATION

Similar to the previous experiment, two models are implemented to evaluate the bandwidths of a digital system that uses PPM modulation to establish communications. One of these systems modulates eight bits per pulse, shown in Appendix C, and the other modulates 16 bits per a pulse, shown in Appendix D. The BER is measured between each of the values generated by the random integer generator, with a range of values depending on the number of bits modulated per a pulse, and the value retrieved by the PPM demodulator. The block diagram shown in Figure 4.4 summarizes the models implemented for this experiment, where the BER and the parameters of each simulated scenario are shown in Table 4.2.

Figure 4.4: Block diagram of the simulated PPM digital communication system.

Pulse Rate	Transmission Rate (bps)	Bit Error Rate
1 _K	8K	0.00%
1K	16K	0.00%
5K	40K	0.00%
5K	80K	0.00%
10K	80K	0.00%
10K	160K	0.00%
50K	400K	0.01%
50K	800K	0.01%
100K	800K	0.01%
100K	1.6M	0.01%
500K	4M	0.08%
500K	8M	0.01%
1M	8M	0.42%
1 _M	16M	0.61%
5M	40M	1.56%
5M	80M	2.34%
10M	80M	2.61%
10M	160M	3.80%

Table 4.2: Summary of the implemented PPM scenarios and BERs.

The results show that the use of PPM has significantly improved the performance of the system with lower BERs, as shown in Figure 4.5, compared to the use of PWM. Although both modulation schemes have shown significant increment at 1M pulse per second, in both sizes, the PPM has shown less overall BER prior to that pulse rate. Moreover, even in higher pulse rates, the PPM has lower BER than the PWM, which indicates that the PPM requires less-complex computations than the PWM, so that, the same crystal frequency used to derive the system has been able to perform better in PPM. The results of this experiment also show that the use of PPM has been able to achieve 8Mbps bandwidth with a BER of 0.01%.

Figure 4.5: BER versus pulse rate for the PPM modulation system.

A sample of the signal outputted from the PPM modulator implemented for this experiment and the input values per each of them are shown in Figure 4.6.

Figure 4.6: Output signal of the PPM modulator in the implemented system.

4.3 AMPLITUDE-ENCODED HYBRID PWM-PPM MODULATION

The communications method proposed in this study, when the amplitude-encoding is used to distinguish the position of each modulation technique, is simulated for both the 8 and 16 bits per pulse, as shown in Appendices E and F. The same scenarios simulated in the previous experiments, pulse rates and bits per pulse, as implemented in this experiment, using the proposed PWM-PPM method with amplitude encoding. The implemented model is summarized in the block diagram shown in Figure 4.7 in order to measure the BER per each simulated scenario, as summarized in Table 4.3.

Figure 4.7: Block diagram of the simulated PWM-PPM digital communication system using amplitude encoding.

Pulse Rate	Transmission Rate (bps)	Bit Error Rate
1 _K	8K	0.00%
1 _K	16K	0.00%
5K	40K	0.00%
5K	80K	0.01%
10K	80K	0.00%
10K	160K	0.00%
50K	400K	0.00%
50K	800K	0.00%
100K	800K	0.00%
100K	1.6M	0.01%
500K	4M	0.00%
500K	8M	0.01%
1M	8M	0.01%
1 _M	16M	0.01%
5M	40M	0.80%
5M	80M	0.91%
10M	80M	1.03%
10M	160M	2.17%

Table 4.3: Summary of the implemented amplitude-encoded PWM-PPM scenarios and their BERs.

The results show that the proposed method has been able to maintain low error rates up to 1M pulse per second, which both the PPM and PWM methods have not been able to achieve solely. This improvement in the performance is according to the split of the bits received by the modulator to both modulators, so that, each modulator has to handle only half of the bits being modulated in the same scenario, solely. This reduction in the size of the data inputted to the modulator reduces the number of possible discrete values, significantly, where for four bits there are 16 discrete values, 256 for eight bits and 65,536 for the 16 bits. Thus, using the proposed method, the PWM and PPM modulators have to divide the time reserved for a pulse to only 256, instead of 65,536 when used solely. Such reduction increases the resistance to noise and the time available for the computations, as the time reserved per each discrete value is significantly longer. The proposed method has also been able to achieve a higher bandwidth of 16Mbps with only 0.01% BER, which is higher than any of the modulators when used solely. Figure 4.8 shows the BER versus the pulse rate of the proposed method when the amplitude encoding is used.

Figure 4.8: BER versus pulse rate for the amplitude-encoded PWM-PPM modulation system.

As shown in the sample output signal of the proposed method, using amplitude modulation, in Figure 4.9, the signal has four possible values. However, as shown in the block diagram in Figure 4.7, only three of these levels are used, as the level that represents the absence of the outputs from both modulators have no logical value in the or gates used to detect the state of each modulator at any time instance.

Figure 4.9: Output signal of the PPM, PWM and the amplitude-encoded PWM-PPM method in the implemented system.

4.4 FREQUENCY-ENCODED HYBRID PWM-PPM MODULATION

In this experiment, the frequency of the transmitted signal is adjusted to represent the states of the modulators in the proposed method per each time instance. As shown in the block diagram in Figure 4.10, similar to the amplitude encoding, only three of the four frequencies are detected using bandpass filters. The state that indicates the absence of the output from both modulators is neglected, according to the no effect it has in the logical comparisons, conducted using the or gates. The same communication scenarios are simulated using this model, where the BER per each of these scenarios is stated in Table 4.4.

Figure 4.10: Block diagram of the frequency-encoded PWM-PPM digital communication system using amplitude encoding.

Pulse Rate	Transmission Rate (bps)	Bit Error Rate
1 _K	8K	0.00%
1 _K	16K	0.00%
5K	40K	0.00%
5K	80K	0.01%
10K	80K	0.00%
10K	160K	0.00%
50K	400K	0.00%
50K	800K	0.00%
100K	800K	0.01%
100K	1.6M	0.01%

Table 4.4: Summary of the implemented frequency-encoded PWM-PPM scenarios and their BERs.

The results of this experiment show that the frequency-encoding has also been able to lower the BER in the simulated scenarios and maintain a 0.01%, at 1M pulse per second, and achieve 16Mbps bandwidth at that pulse rate with 16 bits input size, as shown in Figure 4.11. However, the results of this experiment show that the use of frequency-encoding has slightly higher BERs. According to the sensitivity of the PWM to the width of the received signals, the frequency of the encoding signal can impose changes in the received values, which increases the BER. When the signal is absent by the end of the PWM output, according to the absence of the selected signal at the multiplexer, the receiver calculates the value based on the width of the received signal, regardless of the reason of absence, as shown in Figure 4.12. Although increasing the frequency of the encoding signals can reduce such effect, the range of available frequencies is limited by the characteristics of the communications channel.

Figure 4.11: BER versus pulse rate for the frequency-encoded PWM-PPM modulation system.

Figure 4.12: Output signal of the PPM, PWM and the frequency-encoded PWM-PPM method in the implemented system.

5. DISCUSSION

The comparison among the BERs of the evaluated methods, shown in Figure 5.1, illustrates that the proposed PWM-PPM method has been able to achieve higher bandwidths than the use of the PWM and PPM method separately. The use of amplitudeencoding has been able to outperform all other methods, including the frequency-encoded hybrid method, while the PWM method has the lowest performance among them, even in lower bandwidths. However, the performance of the PWM did not reduce the performance of the proposed hybrid method, as the number of bits being modulated by the PWM modulator in the proposed method is half the number of bits modulated when the PWM is used solely. Such reduction reduces the number of discrete values significantly, which increases the time reserved of these values per each pulse, i.e. more time is available for the computations and higher flexibility can be achieved. Moreover, the relatively lower performance of the PWM modulator has also affected the performance of the proposed method when frequency encoding is used. Such effect is imposed by the possibility of changing the width of the outputted pulse by the encoding frequency, rather than the actual modulation. Thus, higher encoding frequencies are recommended, within the allowable range limited by the communication channel.

Figure 5.1: Comparison of the BERs of the evaluated methods. Top: with 8-bit input values; Bottom: With 16-bit input values.

Moreover, Figure 5.2 shows the maximum bandwidth achieved by the simulated systems with BER less than or equal to 10^{-4} , which is the resolution of the BER measurement in the environment. The results show that the proposed method has been able to achieve a minimum of twice the bandwidths achieved using the PWM and PPM separately. However, as the digital communication systems are simulated using a computer with high processing power, the additional computations required by the proposed hybrid system, which is a combination of the computations required by the PWM and PPM, such superiority cannot be guaranteed in digital systems that use microprocessors. As FPGAs have the ability to parallelize the processes of each modulation scheme, the proposed method can be guaranteed to achieve a minimum of double the bandwidth but serializing the computations in the microprocessors can require more processing time, which can impose undesired changes in the shape of the outputted signal, which eventually manipulates the received value and increases the BER.

Figure 5.2: Summary of the maximum bandwidths achieved by the evaluated modulation methods.

As shown in Table 5.1, despite the higher bandwidth achieved by the PAM method proposed by Xi et al. [31], the BER is not reported in the study, which makes the comparison unfair for the proposed method, as the proposed method could reach up to 80Mbps with less than 1% BER. The similar bandwidths and BER of the evaluated PWM and PPM with the previous studies indicate that the use of the proposed hybrid modulation scheme can significantly improve the bandwidth of these systems. The proposed communication scheme has been able to double the bandwidth, compared to the use of PPM, and significantly improve the bandwidth of the PWM system. Thus, the implementation of the hybrid method in these systems is expected to have the same effect on their bandwidths, which is a significant increment for such systems.

Study	Modulation	Bandwidth (bps)	BER
Xi et al. [31]	PAM	40M	
Pradana et al. [35]	PWM	920	10^{-4}
Xu et al. [38]	PPM	5M	10^{-4}
	PWM	80K	0
This Study	PPM	8M	10^{-4}
	AE PWM-PPM	16M	10^{-4}
	FE PWM-PPM	16M	10^{-4}

Table 5.1: A comparison with the bandwidths and BERs for systems implemented in earlier studies.

6. CONCLUSION

The use of digital systems is growing rapidly in recent years, according to the benefits of these systems compared to the use of analogue systems. A digital value is represented using a series of bits, where each of these values can have one two possible values. Digital systems store and process these values directly, without the need to convert them into analogue format. This growth has imposed the need of communicating these values without using any analogue components in the system. As digital systems can only output a series of two-state values, pulse-coded modulation has been widely employed to establish communication among these systems. However, as these systems are being implemented in a wide range of applications, increasing the bandwidth of a communication system without the need of additional, more expensive, equipment is of significant importance.

As digital values are discrete, digital communication schemes have shown better resistance to noise, compared to analogue communications. Upon arrival, the value extracted from the signal is quantized into the possible discrete values that the transmitted digital information can have. However, as noise mainly affect the amplitude of the communicated signal, pulse amplitude modulation has shown more sensitivity to noise, which limits the number of bits that can be modulated over the pulse's amplitude. Pulse width and position modulation schemes have shown better resistance to noise, as the amplitude of the pulse has no effect over the demodulated value. The number of bits modulated over the width or the position of the pulse is limited by the capabilities of the digital system and the communications channel. Thus, a new hybrid modulation technique is proposed in this study that combines both the PWM and PPM schemes to increase the number of bits modulated per each pulse.

The proposed method splits the bits of the input values into two sets, one modulated using PWM and the other using PPM. As the outputs of these modulators have only two possible states, and there are two modulators, the output signal has four possible states. These outputs are encoded in the signal using one of two possible methods, amplitude and frequency, depending on the channel being used to establish the communications. Despite the sensitivity of the pulse's amplitude to the noise, the existence of only four possible states reduces this sensitivity and is being widely used in different applications. However,

as the proposed method uses these states to encode the output, instead of modulating input bits, more bits from the input can be modulated using the proposed method. Moreover, a frequency encoding method is also evaluated to be used in channels that do not allow different pulse amplitudes.

The evaluation results show that the proposed method has been able to significantly increase the bandwidth of communication without increasing the BER of the system, using the same systems. A minimum of twice the number of bits has been achieved using the proposed method, compared to the use of PWM and PPM schemes, solely. When such bandwidth is implemented using PWM or PPM, the BER has increased significantly, which reduces the reliability of the communications. A digital communication system is simulated using Matlab's Simulink, where all timings in these systems are implemented based on a 50MHz global clock. The proposed hybrid communication scheme has been able to achieve 16Mbps bandwidth at 10-4 BER, compared to a maximum of 8Mbps bandwidth at the same BER achieved by the PPM. Moreover, the amplitude encoding has shown lower average BER, compared to the use of frequency encoding. This difference occurs according to the distortion imposed by the encoding frequencies to the output signal, especially on the width of the pulse. However, the use of the frequency encoding has also been able to achieve the same bandwidth with the same BER.

In future work, the ability of using PAM with the PPM is going to be evaluated, so that, the effect of the distortion imposed by the encoding frequency is eliminated. Such a scheme can be used with communication channels that do not allow amplitude encoding. If this combination shows similar bandwidth without increasing the BER, it is possible to use the amplitude encoding with PWM-PPM with systems that allow amplitude encoding, while the frequency encoding is used with the PAM-PPM in a system that does not allow amplitude encoding.

REFERENCES

- [1] D. M. Berry, "Binary Code," *The International Encyclopedia of Communication Theory and Philosophy,* pp. 1-5, 2016.
- [2] S. W. Smith, "The scientist and engineer's guide to digital signal processing," 1997.
- [3] G. A. Haidar, R. Achkar, and H. Dourgham, "A comparative simulation study of the real effect of PCM, DM and DPCM systems on audio and image modulation," in *2016 IEEE International Multidisciplinary Conference on Engineering Technology (IMCET)*, 2016, pp. 144-149.
- [4] D. J. Barros, S. K. Wilson, and J. M. Kahn, "Comparison of orthogonal frequencydivision multiplexing and pulse-amplitude modulation in indoor optical wireless links," *IEEE Transactions on Communications,* vol. 60, pp. 153-163, 2012.
- [5] N. Ginot, M. A. Mannah, C. Batard, and M. Machmoum, "Application of power line communication for data transmission over PWM network," *IEEE Transactions on Smart Grid,* vol. 1, pp. 178-185, 2010.
- [6] Y. Liu, H. Abu-Rub, B. Ge, F. Blaabjerg, O. Ellabban, and P. C. Loh, "Modulation Methods and Comparison," 2016.
- [7] A. Pradana, N. Ahmadi, T. Adiono, W. A. Cahyadi, and Y.-H. Chung, "VLC physical layer design based on Pulse Position Modulation (PPM) for stable illumination," in *2015 International Symposium on Intelligent Signal Processing and Communication Systems (ISPACS)*, 2015, pp. 368-373.
- [8] C. Gabriel, M.-A. Khalighi, S. Bourennane, P. Léon, and V. Rigaud, "Investigation of suitable modulation techniques for underwater wireless optical communication," in *Proc. IWOW*, 2012, pp. 1-3.
- [9] N. Kikuchi and S. Sasaki, "Highly sensitive optical multilevel transmission of arbitrary quadrature-amplitude modulation (QAM) signals with direct detection," *Journal of Lightwave Technology,* vol. 28, pp. 123-130, 2010.
- [10] I. Fatadin, D. Ives, and S. J. Savory, "Laser linewidth tolerance for 16-QAM coherent optical systems using QPSK partitioning," *IEEE Photonics Technology Letters,* vol. 22, pp. 631-633, 2010.
- [11] S. H. Hall, G. W. Hall, and J. A. McCall, *High-speed digital system design: a handbook of interconnect theory and design practices*: Citeseer, 2000.
- [12] C. L. Phillips and H. T. Nagle, *Digital control system analysis and design*: Prentice Hall Press, 2007.
- [13] Z. Hajduk, "An FPGA embedded microcontroller," *Microprocessors and Microsystems,* vol. 38, pp. 1-8, 2014.
- [14] R. Brzoza-Woch and P. Nawrocki, "FPGA-Based Web Services--Infinite Potential or a Road to Nowhere?," *IEEE Internet Computing,* vol. 20, pp. 44-51, 2016.
- [15] A. P. Chandrakasan, W. J. Bowhill, and F. Fox, *Design of high-performance microprocessor circuits*: Wiley-IEEE press, 2000.
- [16] J. R. Burch and D. L. Dill, "Automatic verification of pipelined microprocessor control," in *International Conference on Computer Aided Verification*, 1994, pp. 68-80.
- [17] E. Monmasson and M. N. Cirstea, "FPGA design methodology for industrial control systems—A review," *IEEE transactions on industrial electronics,* vol. 54, pp. 1824- 1842, 2007.
- [18] D. K. Soni and A. Hiradhar, "A Review on Existing Clock Gating," *International Journal of Computer Science and Mobile Computing,* vol. 4, pp. 371-382, 2015.
- [19] A. Arora, S. Dolev, and M. Gouda, "Maintaining digital clocks in step," in *International Workshop on Distributed Algorithms*, 1991, pp. 71-79.
- [20] L. Mintzer, "Digital filtering in FPGAs," in *Signals, Systems and Computers, 1994. 1994 Conference Record of the Twenty-Eighth Asilomar Conference on*, 1994, pp. 1373-1377.
- [21] R. W. Linderman, M. H. Linderman, and C.-S. Lin, "FPGA acceleration of information management services," AIR FORCE RESEARCH LAB ROME NY INFORMATION DIRECTORATE2005.
- [22] B. J. Benschneider, W. J. Bowhill, E. Copper, M. N. Gavrielov, P. E. Gronowski, V. K. Maheshwari*, et al.*, "A pipelined 50-MHz CMOS 64-bit floating-point arithmetic processor," *IEEE Journal of Solid-State Circuits,* vol. 24, pp. 1317-1323, 1989.
- [23] T. D. Burd, T. A. Pering, A. J. Stratakos, and R. W. Brodersen, "A dynamic voltage scaled microprocessor system," *IEEE Journal of solid-state circuits,* vol. 35, pp. 1571-1580, 2000.
- [24] S. Faruque, "Pulse Code Modulation (PCM)," in *Radio Frequency Source Coding Made Easy*, ed: Springer, 2015, pp. 65-90.
- [25] S. Weinstein, "Introduction to digital transmission," in *Academic Press Library in Mobile and Wireless Communications*, ed: Elsevier, 2016, pp. 1-48.
- [26] Z. Jako and G. Kis, "Application of noise reduction to chaotic communications: A case study," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications,* vol. 47, pp. 1720-1725, 2000.
- [27] G. Kolumbán and M. P. Kennedy, "Overview of digital communications," in *Chaotic Electronics in Telecommunications*, ed: CRC Press, 2000, pp. 131-149.
- [28] R. Y. Chang, S.-J. Lin, and W.-H. Chung, "Symbol and bit mapping optimization for physical-layer network coding with pulse amplitude modulation," *IEEE Transactions on Wireless Communications,* vol. 12, pp. 3956-3967, 2013.
- [29] T. N. Davidson, "Efficient design of waveforms for robust pulse amplitude modulation," *IEEE Transactions on Signal Processing,* vol. 49, pp. 3098-3111, 2001.
- [30] R. Dubé-Demers, S. LaRochelle, and W. Shi, "Ultrafast pulse-amplitude modulation with a femtojoule silicon photonic modulator," *Optica,* vol. 3, pp. 622-627, 2016.
- [31] C. Xi, A. Mirvakili, and V. J. Koomson, "A visible light communication system demonstration based on 16-level pulse amplitude modulation of an LED array," in *2012 Symposium on Photonics and Optoelectronics*, 2012, pp. 1-4.
- [32] R. X. Ferreira, E. Xie, J. J. McKendry, S. Rajbhandari, H. Chun, G. Faulkner*, et al.*, "High bandwidth GaN-based micro-LEDs for multi-Gb/s visible light communications," *IEEE Photonics Technology Letters,* vol. 28, pp. 2023-2026, 2016.
- [33] G. Ntogari, T. Kamalakis, J. Walewski, and T. Sphicopoulos, "Combining illumination dimming based on pulse-width modulation with visible-light communications based on discrete multitone," *Journal of Optical Communications and Networking,* vol. 3, pp. 56-65, 2011.
- [34] M. Barr, "Pulse width modulation," *Embedded Systems Programming,* vol. 14, pp. 103-104, 2001.
- [35] A. Pradana, N. Ahmadi, and T. Adionos, "Design and implementation of visible light communication system using pulse width modulation," in *2015 International Conference on Electrical Engineering and Informatics (ICEEI)*, 2015, pp. 25-30.
- [36] Y. Fan and R. J. Green, "Comparison of pulse position modulation and pulse width modulation for application in optical communications," *Optical Engineering,* vol. 46, p. 065001, 2007.
- [37] U. Sethakaset and T. A. Gulliver, "Differential amplitude pulse-position modulation for indoor wireless optical communications," *EURASIP Journal on Wireless Communications and Networking,* vol. 2005, pp. 3-11, 2005.
- [38] T. Xu, X. Chen, D. Wen, and X. Sun, "Design of transmitter and receiver for experimental blue-green laser communication system," in *Ninth International Symposium on Precision Engineering Measurement and Instrumentation*, 2015, p. 944628.
- [39] R. Sarwar, B. Sun, M. Kong, T. Ali, C. Yu, B. Cong*, et al.*, "Visible light communication using a solar-panel receiver," in *2017 16th International Conference on Optical Communications and Networks (ICOCN)*, 2017, pp. 1-3.

Appendix C 8-bit PPM Model

Appendix D 16-bit PPM Model

Appendix E 8-bit Amplitude-Encoded Hybrid Model

Appendix F 16-bit Amplitude-Encoded Hybrid Model

Appendix G 8-bit Frequency-Encoded Hybrid Model

Appendix H 16-bit Frequency-Encoded Hybrid Model