ANALYSIS OF DYNAMICAL MODELS EXTENDING THE STANDARD MODEL WITH HEAVY LEPTONS

by

A. TOLGA TAŞÇI

THESIS SUBMITTED TO

THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

OF

THE ABANT İZZET BAYSAL UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE

DEGREE OF

DOCTOR OF PHILOSOPHY

IN

THE DEPARTMENT OF PHYSICS

FEBRUARY 2008

Approval of the Graduate School of Natural Sciences

Prof. Dr. Nihat ÇELEBİ

Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Doctor of Philosophy.

Prof. Dr. Ahmet Turan ALAN

Head of Physics Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality as a thesis for the degree of Doctor of Philosophy.

Prof. Dr. Ahmet Turan ALAN

Supervisor

Examining Committee Members

1. Prof. Dr. Namık Kemal PAK ....
2. Prof. Dr. Takhmasib M. ALİEV ....
3. Prof. Dr. Ahmet Turan ALAN ....
4. Assoc. Prof. Dr. Resul ERYİĞİT ....
5. Assoc. Prof. Dr. Nurettin KARAGÖZ ....

ABSTRACT

ANALYSIS OF DYNAMICAL MODELS EXTENDING THE STANDARD MODEL WITH HEAVY LEPTONS

Taşçı, Ahmet Tolga
Ph.D., Department of Physics
Supervisor: Prof. Dr. Ahmet Turan Alan

February 2008, 106 pages

In this thesis, some analysis of models which are extending the Standard Model with heavy leptons are presented. Effects of anomalous interactions and string inspired E_6 model are investigated for heavy lepton production at future e^+e^- (ILC and CLIC), *ep* (THERA, LHeC and Linac⊗LHC) and *pp* (LHC) collisions. Hence, this thesis motivates three new aspects of new physics together, namely, string inspired E_6 model, anomalous interactions and a new generation of lepton family. Analytical expressions for the differential cross-sections are derived and numerical results are presented. The production, backgrounds and signatures of heavy leptons are analyzed.

It is shown that, with optimal choices of relevant parameters and optimal kinematical cuts, observation of heavy leptons with masses 3000 GeV at Linac⊗LHC and 2750 GeV at ILC is possible in string inspired E_6 model while heavy leptons with masses 650 GeV at LHeC, 350 GeV at ILC and 650 GeV at the LHC is possible with anomalous couplings according to the criteria of SS$\geq$3. On the other hand, with the models we used in this thesis, according to the conventional criteria of SS$\geq$3, THERA is not capable for searching heavy lepton signals due to its low luminosity.

Keywords: Models beyond the Standard Model, heavy leptons, anomalous interactions, string inspired E_6 model.

ÖZET

STANDART MODELİ GENİŞLETEN DİNAMİK MODELLERİN AĞIR LEPTONLAR İLE ANALİZİ

Taşçı, A. Tolga

Doktora, Fizik Bölümü

Tez Danışmanı: Prof. Dr. Ahmet Turan Alan

Şubat 2008, 106 sayfa

Bu tezde, Standart Model'i genişleten modellerin ağır leptonlar ile bazı analizleri sunuldu. Anormal etkileşmeler ve sicim esinli E_6 model etkileri ağır lepton için gelecek e^+e^- (ILC ve CLIC), *ep* (THERA, LHeC ve Linac⊗LHC) ve *pp* (LHC) çarpışmalarında incelendi. Bu tez bu yüzden, yeni fiziğe, sicim esinli E_6 model, anormal etkileşmeler ve yeni lepton aile kuşağı olarak adlandırılan üç yeni bakış açısını motive etmektedir. Diferansiyel tesir kesiti için analitik ifadeler türetilmiş ve nümerik sonuçlar sunulmuştur. Ağır leptonların üretim, fon ve sinyalleri analiz edilmiştir.

Uygun parametrelerin optimal seçimi ve optimal kinematik sınırlarla sicim esinli E_6 modelde, Linac⊗LHC'de 3000 GeV ve ILC'de 2750 GeV ağır leptonların gözlenebilmesi olası iken, anormal etkileşmelerde LHeC'de 650 GeV, ILC'de 350 GeV ve LHC'de 650 GeV ağır leptonların SS≥3 kriterine göre gözlenebildiği gösterilmiştir. Diğer taraftan, geleneksel kriter SS≥3'e göre, bu tezde kullandığımız modeller ile THERA, düşük ışınlığından dolayı, ağır lepton sinyalleri incelemesi için yeterli değildir.

Anahtar kelimeler: Standart model ötesi modeller, ağır leptonlar, anormal etkileşmeler, sicim esinli E_6 model.

To my parents . . .

ACKNOWLEDGEMENTS

I would like to express my gratitude to all those who supported me directly or indirectly during the Phd stage. Firstly, I like to thank my supervisor, Prof. Dr. Ahmet Turan Alan, whose guidance allowed me to have reached this stage. It was a pleasure to share my office at AIBU with A. Şenol whom I had many fruitful discussions.

Last but not least, I would like to thank my family and especially my parents, who gave me their love and continuous support during my whole life. Thank you very much.

This thesis was partially supported by Abant Izzet Baysal University Research Fund under the grant no 2005.03.02.216.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

1.1 Standard Model

The behavior of all known subatomic particles can be described within a theoretical framework called the Standard Model. The Standard Model (SM) [1-11], which describes the strong, electromagnetic and weak interactions, is remarkably successful theory of interactions of quarks and leptons which have been verified experimentally at energies up to about a few hundred GeV. Only gravity remains outside the SM because of its weakness compared to the other forces. SM is the renormalizable model of strong and electroweak interactions that describes the interactions of spin-$\frac{1}{2}$ point like fermions which are mediated by spin-1 gauge bosons, the force carrying particles.

There are three main symmetry groups in the SM. These are $SU(3)$ (color), $SU(2)$ (weak isospin) and $U(1)$ (hypercharge) symmetry groups. The SM is invariant under the following group:

$$SU(3)_{color} \times SU(2)_{left} \times U(1)_{hypercharge} \tag{1.1}$$

which describes the electromagnetic, weak and strong interactions of elementary particles. The weak and electromagnetic interactions are unified in the $SU(2)_L \times U(1)_Y$ group. Pairs from each group (one up-type quark, one down-type quarks, a lepton and its corresponding neutrino) form a generation. There are three generations of fermions. Fermions are the particles of the matter, described by

	$1^{st}Gen.$	$2^{nd}Gen.$	$3^{rd}Gen.$	Q	Color	Spin
Quarks	u	c	t	+2/3	r, b, g	1/2
	d	s	b	−1/3	r, b, g	1/2
Leptons	ν_e	ν_μ	ν_τ	0	-	1/2
	e	μ	τ	−1	-	1/2

Table 1.1: Fundamental fermions in SM.

Force	Boson	Q	Spin	Related Group
Strong	gluons	0	1	$SU(3)_C$
Electromagnetic	γ	0	1	$SU(2)_L \times U(1)_Y$
Weak	$W^{\pm}$	±1	1	$SU(2)_L \times U(1)_Y$
	Z^0	0	1	$SU(2)_L \times U(1)_Y$

Table 1.2: Fundamental gauge bosons in SM.

the SM. Leptons and quarks are the fundamental fermions. Their main properties are listed in Table 1.1.

Bosons are the force mediating particles described by the SM. Photons are the mediator of the electromagnetic force between electrically charged particles, the gauge bosons ($W^{\pm}$, Z^0) are the mediator of the weak interactions and the gluons mediate the strong interactions between quarks. Fundamental bosons and their main properties are listed in Table 1.2.

Energy, momentum, angular momentum, charge, color, baryon number and lepton numbers are conserved quantities in all interactions in SM. The parity and charge are conserved in strong and electromagnetic interactions but not conserved in weak interactions.

The SM is in good agreement with the experimental data which predicted the existence of W and Z bosons, the gluon, the t quark and the c quark before these particles had been observed. The SM has 19 free parameters whose values are determined by experiments and which are proved by theoretical calculations.

	Measured Value (GeV)	SM Prediction (GeV)
W boson mass	80.403 ± 0.029	80.390 ± 0.0180
Z boson mass	91.1876 ± 0.0021	91.1874 ± 0.0021

Table 1.3: Experimental and SM values of W and Z bosons.

Their predicted properties were experimentally confirmed with excellent precision. Table 1.3 [12], shows comparison between the measured and the predicted values of some quantities to get an idea of the success of the SM.

Although the SM provides a remarkably successful theory of the fundamental particles at energy scales of $\mathcal{O}(100)$ GeV and below, the SM is not the ultimate theory of the fundamental particles and their interactions. Some of fundamental problems that the SM cannot answer are:

- *The unclear mechanism of electroweak symmetry breaking.* The dynamics of Higgs sector is not clear yet. In the SM, the interactions of the Higgs boson are different from the interactions of the intermediate bosons, since the Higgs boson has not yet been observed and it is not clear whether it is fundamental or composite particle.

- *Large number of free parameters.* The SM contains 19 free parameters, such as particles masses, another 10 parameters are needed to include neutrino masses which, cannot be independently calculated.

- *CP violation in strong interaction.* The problem of CP violation is not well understood. QCD does not violate the CP-symmetry as easily as the electroweak theory and experiments do not indicate any CP violation in the QCD sector.

- *Number of generations are arbitrary.* In SM, quark and lepton pairs form a generation but we do not know why generations repeat or how many there are.

The answers of these problems, which SM can't answer, lies beyond the SM. Some non-standard models have the facility to go beyond the SM. There are two possible ways of going beyond the SM. The first one is to consider new interactions with same fields, which leads supersymmetry [14-16], grand unification, string theory, etc. The other way is to consider new interactions with new fields. This leads us to technicolor [17, 18], compositeness [19], extra dimensions [20-23], etc. In the case of observation of heavy leptons, the problem of finding out the true underlying model will arise. For this reason, we analyze the heavy leptons in two different models, which are; the string inspired E_6 model and an effective lagrangian description with anomalous interactions.

1.1.1 String Inspired E_6 Model

The possibility of a consistent unified theories based on the gauge group E_6 [24-34] has great interest over the last few years. Greater interest was sparked in E_6 as a grand unified theory (GUT) in the 1970's which was noted that each generations of fermions could be placed in a single **27** dimensional representation. These GUT's were only partially successful and the idea of E_6 as a GUT died. Green and Schwartz [34] showed that string theory in ten dimensions is anomaly free and the compactification of down to four dimensions can lead E_6 as an effective GUT group. New types of quarks and leptons can be predict in E_6 model. The phenomenology of E_6 is particularly rich due to the predictions of exotic fermions and new gauge bosons. Table 1.4 shows the generations of fermions lies in the **27** dimension of E_6 which contains the usual **16** fermions per generation in SO(10) as well as 11 additional two component new fields for every generation.

1.1.2 Effective Lagrangian Description

As a second way to investigate the physics beyond the SM, we use a model independent approach and formulate new physics effects in terms of an effective

SU(10)	SU(5)	$1^{st}Gen.$	$2^{nd}Gen.$	$3^{rd}Gen.$
16	10	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$
		u_L^c	c_L^c	t_L^c
		e_L^c	μ_L^c	τ_L^c
	$\bar{5}$	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu\tau \\ \tau \end{pmatrix}_L$
		d_L^c	s_L^c	b_L^c
	1	$\nu_{e_L}^c$	$\nu_{\mu_L}^c$	$\nu_{\tau_L}^c$
10	$\bar{5}$	$\begin{pmatrix} \nu'_e \\ e' \end{pmatrix}_L$	$\begin{pmatrix} \nu'_\mu \\ \mu' \end{pmatrix}_L$	$\begin{pmatrix} \nu'_\tau \\ \tau' \end{pmatrix}_L$
		$d_L^{\prime c}$	$s_L^{\prime c}$	$b_L^{\prime c}$
	5	$\begin{pmatrix} e' \\ \nu'_e \end{pmatrix}_L^c$	$\begin{pmatrix} \mu' \\ \nu'_\mu \end{pmatrix}_L^c$	$\begin{pmatrix} \tau' \\ \nu'_\tau \end{pmatrix}_L^c$
		d'_L	s'_L	b'_L
1	1	$\nu_{eL}^{\prime\prime c}$	$\nu_{\mu L}^{\prime\prime c}$	$\nu_{\tau L}^{\prime\prime c}$

Table 1.4: E_6 particle content.

lagrangian. Without specifying the detail of the new physics, the effective lagrangian which consists of the SM Lagrangian plus corrections represented by a series of effective operators is given as,

$$\mathcal{L}_{\texttt{eff}} = \mathcal{L}_{\texttt{SM}} + \frac{1}{\Lambda^2} \sum_i C_i O_i \tag{1.2}$$

where Λ is the new physics scale, O_i represents the effective operators and C_i are the constants which represent the coupling strengths of O_i.

The outline of the rest of this dissertation is as follows. We study the production, signatures and backgrounds of new heavy leptons via string inspired E_6 model at the proposed Linac$\otimes$LHC and at future linear colliders (ILC and CLIC) in chapter 2 and in chapter 3, respectively.

The phenomenology of the anomalous interactions with heavy leptons in ep collisions is studied in chapter 4.

In chapter 5 and chapter 6, we study the production of heavy leptons with anomalous couplings at linear colliders ILC and CLIC and at the Cern LHC, respectively.

All these chapters contains a detailed study for the production, signatures and backgrounds and also decays of new heavy leptons.

Chapter 7 contains our summary and conclusions.

CHAPTER 2

HEAVY LEPTON PRODUCTION AT LINAC$\otimes$LHC

In this chapter we study the possible production of new single heavy leptons suggested by string inspired E_6 model in *ep* collisions. The possibility of the existence of new heavy charged leptons is present in many extensions of the SM. The string inspired E_6 model is a well motivated one which includes extra gauge bosons and new fermions assigned to the 27-dimensional representation [24-34]. In the search for extensions of the SM the new heavy leptons play an important role. It is known that there has been no clear signal of heavy leptons found experimentally so far. The known experimental upper bounds for the heavy lepton masses were found to be 44 GeV by OPAL [35], 46 GeV by ALEPH [36], 90 GeV by H1 [37] and 100 GeV by L3 [38] Collaborations.

For the searches of new physics beyond the SM, the linac-ring type *ep* colliders have as much potential as lepton colliders [39]. Linac-ring type machines will give opportunity to investigate appropriate phenomena at higher center of mass energies and at better kinematic conditions [40].

Productions of heavy leptons in *ep* collisions have been studied in the literature [41-43].

2.1 Production of Heavy Leptons

The model that we use in the single production of a new heavy lepton is the string inspired E_6 model. We therefore assume the new heavy lepton interactions

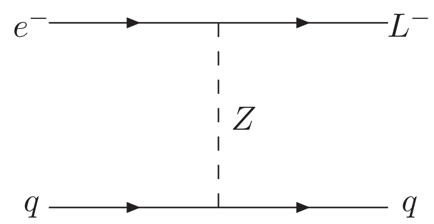

Figure 2.1: The Feynman diagram of the parton level process $eq \to Lq$ in ep collision via FCNC Z exchange in the t-channel.

in the following flavor changing neutral current (FCNC) Lagrangian:

$$\mathcal{L}_{\texttt{nc}} = g_z \sin\theta_{mix}\psi_L\gamma^\mu(1+\gamma_5)\psi_e Z^\mu + h.c. \tag{2.1}$$

and similar terms for the other leptonic families. Here θ_{mix} are the mixing angles between right handed components of the ordinary and new heavy charged leptons. From the high precision measurements of the Z properties at LEP/SLC, the upper bound for $\sin\theta_{mix}$ is found $\sin\theta_{mix} < 0.072$, at 95% confidence level [44]. This bound is more restrictive than earlier analysis [45, 46]. Throughout this chapter we will suppose an upper limit $\sin\theta_{mix} < 0.1$. We use the parameter b_{lLZ} for $\sin\theta_{mix}$ to denote the mixing in the vertex $l - L - Z$ explicitly. In E_6, the parton level process $e^-q \to L^-q$, responsible for the heavy lepton production in ep collision occurs via FCNC Z exchange in the t channel. The Feynman diagram of the parton level process $e^-(p_1)q(p_2) \to L^-(p_3)q(p_4)$ is given in Fig. 2.2.

2.2 Definition of Differential Cross Section

Using the Feynman rules, the matrix element for Z boson exchange in t channel is;

$$
\begin{aligned}
-i\mathcal{M} &= \Big[\bar{u}(p_3) g_Z b_{lLZ} \gamma^\mu (1+\gamma^5) u(p_1)\Big] \frac{-i(g_{\mu\nu} - q_\mu q_\nu / M_Z^2)}{\Big[(\hat{t} - M_Z^2) + i M_Z \Gamma_Z\Big]} \\
&\quad \times \Big[\bar{u}(p_4)\Big(\frac{-i g_Z}{2}\Big)\gamma^\nu (v_q - a_q \gamma^5) u(p_2)\Big], \qquad (2.2)
\end{aligned}
$$

which gives

$$
\begin{aligned}
\mathcal{M} &= \frac{-i b_{lLZ} g_Z^2}{2} \frac{(g_{\mu\nu} - q_\mu q_\nu / M_Z^2)}{\Big[(\hat{t} - M_Z^2) + i M_Z \Gamma_Z\Big]} \Big[\bar{u}(p_3)\gamma^\mu (1+\gamma^5) u(p_1)\Big] \\
&\quad \times \Big[\bar{u}(p_4)\gamma^\nu (v_q - a_q \gamma^5) u(p_2)\Big], \qquad (2.3)
\end{aligned}
$$

and

$$
\begin{aligned}
\overline{\mathcal{M}} &= \frac{i b_{lLZ} g_Z^2}{2} \frac{(g_{\alpha\beta} - q_\alpha q_\beta / M_Z^2)}{\Big[(\hat{t} - M_Z^2) - i M_Z \Gamma_Z\Big]} \Big[\bar{u}(p_1)\gamma^\alpha (1+\gamma^5) u(p_3)\Big] \\
&\quad \times \Big[\bar{u}(p_2)\gamma^\beta (v_q - a_q \gamma^5) u(p_4)\Big] \qquad (2.4)
\end{aligned}
$$

To obtain the spin averaged square of the transition matrix element, we must average over the spins of the incoming lepton and quark and sum over the spins of outgoing lepton and quark,

$$
\langle |\mathcal{M}|^2 \rangle = \frac{1}{4} \sum_{spins} |\mathcal{M}|^2 \qquad (2.5)
$$

ignoring the quark and ordinary lepton masses, square of the amplitude is,

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle &= \frac{b_{lLZ}^2 g_Z^4}{16\Big[(\hat{t}-M_Z^2)^2+M_Z^2\Gamma_Z^2\Big]}\Big(g_{\mu\nu}-\frac{q_\mu q_\nu}{M_Z^2}\Big)\Big(g_{\alpha\beta}-\frac{q_\alpha q_\beta}{M_Z^2}\Big)\\ &\times Tr\Big[(\not{p}_3+m_3)\gamma^\mu(1+\gamma^5)\ \not{p}_1\gamma^\alpha(1+\gamma^5)\Big]\\ &\times Tr\Big[\ \not{p}_4\gamma^\nu(v_q-a_q\gamma^5)\ \not{p}_2\gamma^\beta(v_q-a_q\gamma^5)\Big] \qquad (2.6)\end{aligned}$$

$$\begin{aligned}p_1\cdot p_2 &= \frac{\hat{s}}{2}\ ,\\ p_3\cdot p_4 &= \frac{\hat{s}-m_L^2}{2}\ ,\\ p_1\cdot p_3 &= \frac{m_L^2-\hat{t}}{2}\ ,\\ p_2\cdot p_4 &= -\frac{\hat{t}}{2}\ ,\\ p_1\cdot p_4 &= -\frac{\hat{u}}{2}\ ,\\ p_2\cdot p_3 &= \frac{m_L^2-\hat{u}}{2}\ ,\\ \hat{u} &= m_L^2-\hat{s}-\hat{t}\ , \qquad (2.7)\end{aligned}$$

using the trace theorems and the Mandelstam variables in Eq. (2.7), the differential cross section for the subprocess $e^-q\rightarrow L^-q$ in the framework of E_6 model is given by,

$$\frac{d\sigma}{dt} = \frac{1}{16\pi\hat{s}^2}\langle|\mathcal{M}|^2\rangle \qquad (2.8)$$

$$\begin{aligned}\frac{d\hat{\sigma}}{d\hat{t}} &= \frac{\pi\alpha^2 b_{lLZ}^2}{\sin^4\theta_W\cos^4\theta_W\hat{s}^2\Big[(\hat{t}-M_Z^2)^2+M_Z^2\Gamma_Z^2\Big]}\Big[(a_q+v_q)^2\hat{t}^2\\ &+(a_q+v_q)^2(2\hat{s}-m_L^2)\hat{t}+2\hat{s}(a_q^2+v_q^2)(\hat{s}-m_L^2)\Big], \qquad (2.9)\end{aligned}$$

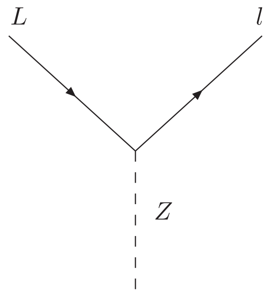

Figure 2.2: Heavy lepton decay $L \to lZ$.

where θ_W is the weak angle, a_q and v_q are vector and axial vector couplings which values are given in Table 2.1, α is the fine structure constant, $\hat{s}$ and $\hat{t}$ are the Mandelstam variables and $\hat{s} = xs$ is the square of the center of mass energy for the subprocess while x is the momentum fraction of the parton inside the proton.

f	c_v	c_a
$\nu_e,\ \nu_\mu,\ \nu_\tau$	$1/2$	$1/2$
$e^-,\ \mu^-,\ \tau^-$	$-1/2 + 2\sin^2\theta_W$	$-1/2$
$u,\ c,\ t$	$1/2 - (4/3)\sin^2\theta_W$	$1/2$
$d,\ s,\ b$	$-1/2 + (2/3)\sin^2\theta_W$	$-1/2$

Table 2.1: Vector and axial vector couplings.

2.3 $L \to lZ$ Decay

Considering the FCNC Lagrangian in Eq. (2.1), matrix element for the $L \to lZ$ heavy lepton decay, where l is an ordinary lepton (e, μ, τ), in E_6 model is written as,

$$\mathcal{M} = i b_{lLZ} g_Z \Big[\bar{u}(p_l)\gamma^\mu(1+\gamma^5)(\epsilon^\mu)^* u(p_L) \Big] \tag{2.10}$$

and

$$\overline{\mathcal{M}} = -ib_{lLZ}g_Z\Big[\bar{u}(p_L)\gamma_\nu(1+\gamma^5)(\epsilon^\nu)u(p_l)\Big] \tag{2.11}$$

ignoring the quark and ordinary lepton masses we find

$$\langle|\mathcal{M}|^2\rangle = (g_Z^2 b_{lLZ}^2)Tr\left[\not{p}_l\gamma^\mu(1+\gamma^5)(\not{p}_L + m_L)\gamma_\nu(v_q - a_q\gamma^5)\right] \tag{2.12}$$

where the relation $(\epsilon^\mu)^*(\epsilon^\nu) = [-g^{\mu\nu} + q^\mu q^\nu/M_Z^2]$ has been used. Using the relations, $g_Z = e/(\sin\theta_W\cos\theta_W)$, $e = \sqrt{4\pi\alpha}$ and

$$\begin{aligned} p_L\cdot p_l &= m_L E_l\ , \\ p_L\cdot q &= m_L^2 - m_L E_l\ , \\ p_l\cdot q &= m_L E_l\ , \\ q^2 &= m_L^2 - 2m_L E_l\ , \\ E_l &= \frac{m_L^2 - M_Z^2}{2m_L}\ , \end{aligned} \tag{2.13}$$

we obtain

$$\Gamma(L\to lZ) = \frac{b_{lLZ}^2\alpha}{\sin^2\theta_W\cos^2\theta_W}\left[\frac{m_L^3}{M_Z^2} - \frac{3M_Z^2}{m_L} + \frac{2M_Z^4}{m_L^3}\right]. \tag{2.14}$$

Where, $\theta_W(\simeq 28^\circ)$ is called weak mixing angle and m_L is the heavy lepton mass.

2.4 Numerical Calculation

The total cross section can be obtained by folding the subprocess cross section $\hat{\sigma}$ over the parton distribution functions as

$$\sigma(e^-p \to L^-qX) = \int_{x_{min}}^{1} dx f_q(x, Q^2) \int_{t_{min}}^{t_{max}} \frac{d\hat{\sigma}}{d\hat{t}} d\hat{t} \tag{2.15}$$

where $x_{min} = m_L^2/s$, $\hat{t}_{min} = -(\hat{s} - m_L^2)$ and $\hat{t}_{max} = 0$. These relations are obtained for the massless lepton and quark case. We give the production cross sections for the signal as function of the heavy lepton mass, m_L, in Fig. 2.3 for three different values of b_{lLZ}. In Fig. 2.4, we display the invariant mass distribution of the background process $e^-p \to qZe^-X$ as function of invariant mass of Ze^- system at future lepton-hadron collider Linac⊗LHC with the main parameters $\sqrt{s} = 5.3$ TeV and $L^{int} = 10^4 \text{pb}^{-1}$ [47]. We have used the COMPHEP package [48] to calculate the cross sections, decay widths and branching ratios. For the parton distribution functions we have used MRS [49].

The heavy lepton production cross sections ($\sigma \times BR$) and the number of signal events depending on the mass m_L are shown in Table 1. For decreasing values of the $l - L - Z$ couplings, b_{lLZ}, the production cross section and therefore the number of events decreases.

After their production, heavy leptons will decay via the neutral current process $L^- \to l^- Z$, where l^- is a light lepton (e^-, μ^-, τ^-). The branching ratio for these processes would be around 33% for each channel.

The backgrounds for the signal process $e^-p \to L^-qX$ with the subsequent decays $L^- \to Z\mu^-$ or $L^- \to Z\tau^-$ and $Z \to e^+e^-$ are expected to be at very low rate. By applying appropriate cuts to the final state particles this type of backgrounds can be kept at very low levels. Still we may need at least 10 signal

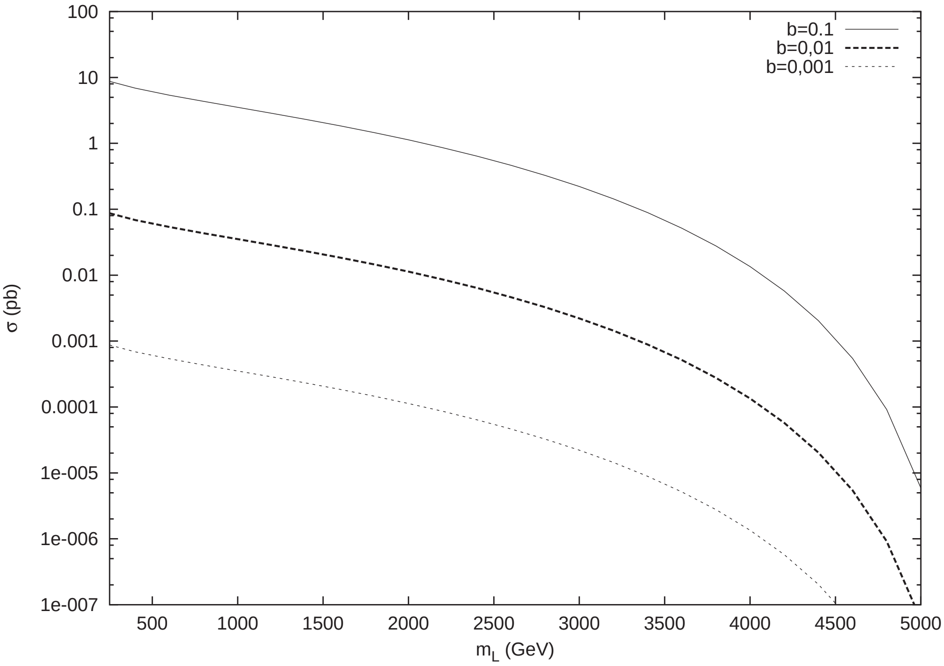

Figure 2.3: Total production cross sections for the process $e^{-}q \rightarrow L^{-}q$, as functions of the heavy lepton masses (m_L), at lepton hadron collider Linac⊗LHC ($\sqrt{s} = 5.3$ TeV, $L^{int} = 10^4 \mathrm{pb}^{-1}$) for different $l - L - Z$ couplings b_{lLZ}.

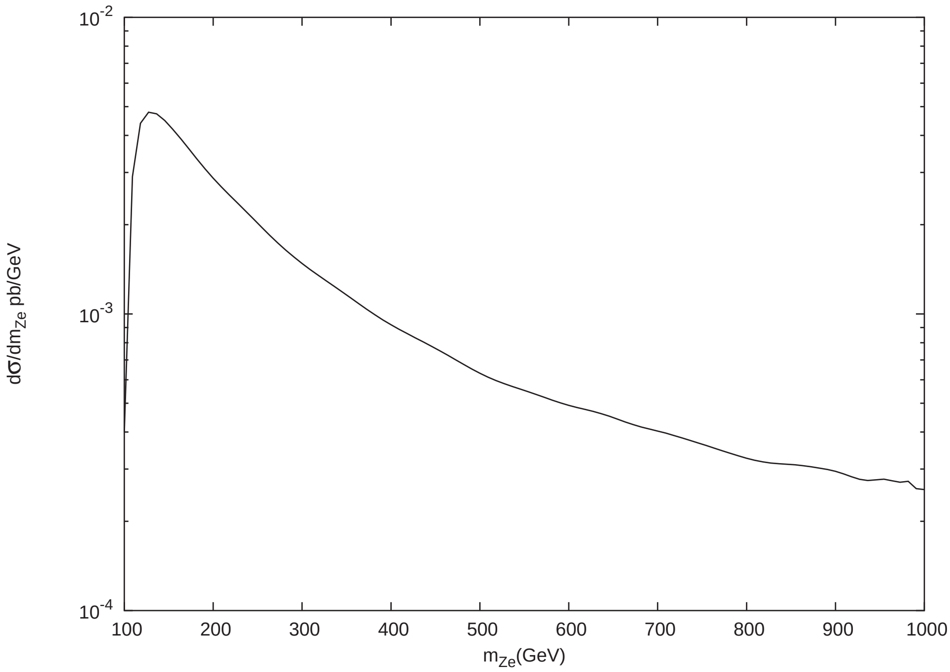

Figure 2.4: The invariant mass distribution of the Ze system for the background process $e^{-}p \rightarrow qZe^{-}X$.

m_L (GeV)	σ (pb)	$\sigma \times BR_1$ (pb)	$\sigma \times BR_1 \times BR_2$ (pb)	σ_B (pb)	$S/\sqrt{S+B}$	Γ (GeV)
200	9.423	3.109	0.105	1.21×10^{-3}	307	0.59
400	6.873	2.267	0.076	5.91×10^{-4}	262	5.18
600	5.384	1.776	0.059	3.62×10^{-4}	232	17.53
800	4.333	1.429	0.048	2.50×10^{-4}	208	41.53
1000	3.519	1.161	0.039	1.95×10^{-4}	188	81.05
2000	1.129	0.373	0.012	6.03×10^{-5}	106	647.16
3000	0.221	0.073	0.002	9.48×10^{-6}	47	2142.88

Table 2.2: Cross sections depending on the heavy lepton mass, m_L at $\sqrt{s} = 5.3$ TeV, $L^{int} = 10^4 \text{pb}^{-1}$ for $b_{lLZ} = 0.1$. The branching ratios BR_1 and BR_2 denote $BR(L^- \to Ze^-)$ and $BR(Z \to e^+e^-, \mu^+\mu^-)$, respectively. The total decay width of the heavy lepton is given in the last column.

events in the final state after all cuts. Therefore, the *ep* collider Linac⊗LHC can probe heavy lepton masses up to about 3 TeV as can be deduced from Table 2.2. For a heavy lepton with a mass of 200 GeV we expect 10^3 signal events for the coupling value of $b_{lLZ} = 0.1$.

We applied an initial cut on the electron and jet transverse momentum $p_T^{e,q} >$ 10 GeV for the signal and background analysis. These cuts reduce the background by about 20%. The total background cross section is $(\sigma \times BR)$=0.055 pb after the cuts. This improves the statistical significance,

$$SS = \frac{S}{\sqrt{S+B}} \tag{2.16}$$

where S and B denote the total signal and background events, respectively. We give the heavy lepton mass dependent SS on the Fig. 2.5. The calculated 3σ and 5σ discovery contours for heavy lepton masses and couplings, are displayed in Fig. 2.6.

We also have performed the same calculations for the THERA ($\sqrt{s} = 1$ TeV,

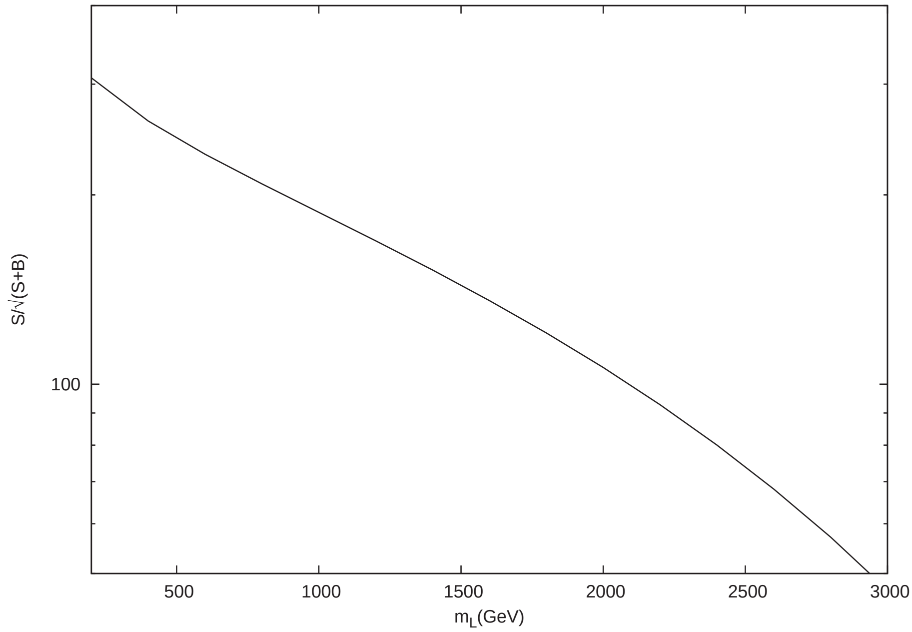

Figure 2.5: The signal significance $S/\sqrt{S+B}$ as function of heavy lepton mass.

$L^{int} = 40$ pb^{-1}) collider [50]. Unfortunately, we have seen that at this collider it is not possible to observe a heavy lepton with mass greater than 200 GeV. Taking the $l - L - Z$ coupling value of 0.1 for a heavy lepton with a mass of 200 GeV, the production rate is 100 events per year.

2.5 Discussion

This chapter shows that some of future high energy lepton-hadron colliders can test the existence of heavy leptons. Linac$\otimes$LHC has very promising discovery potential for heavy leptons with masses up to 3 TeV at 3σ significance, that is, it offers the opportunity of the manifestations of new physics beyond the SM, while at THERA, it does not seem to be likely to achieve masses greater than 200 GeV.

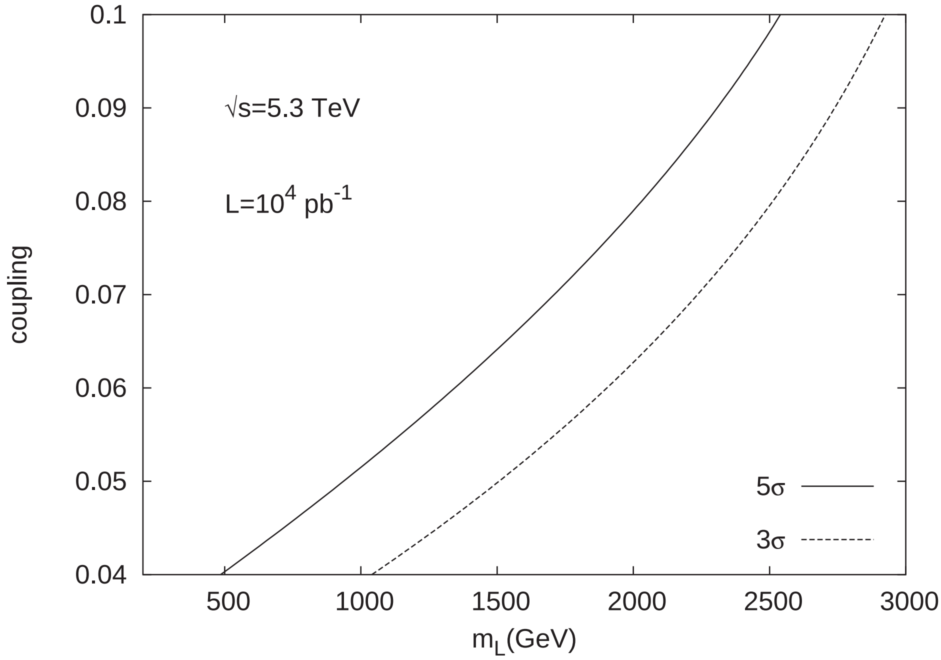

Figure 2.6: Attainable mass limits depending on the coupling b for heavy leptons at Linac$\otimes$LHC.

CHAPTER 3

SINGLE AND PAIR PRODUCTION OF HEAVY LEPTONS IN STRING INSPIRED E_6 MODEL

We consider both the single and the pair production of heavy leptons separately by using the same model in Chapter 2, taking into account the signal and background events at future linear colliders. The main parameters of these collider options were taken from Refs. [51, 52] and displayed in Table 3.1. Several model independent studies of heavy lepton production have been appeared in the literature in e^+e^- colliders [53-60].

e^+e^- Colliders	E_{e^+} (TeV)	E_{e^-} (TeV)	$\sqrt{s_{e^+e^-}}$ (TeV)	$L^{int}_{e^+e^-}$(pb^{-1})
ILC	0.25	0.25	0.5	10^5
ILC	0.5	0.5	1.0	10^5
CLIC	1.5	1.5	3.0	10^5

Table 3.1: The main parameters of the future e^-e^+ colliders.

3.1 Single Production of Heavy Leptons

The single production of heavy leptons L, in e^-e^+ collisions occur through the s and t channel processes $e^-e^+ \to L^-e^+$ caused by the flavor changing neutral current (FCNC) Lagrangian:

$$\mathcal{L}_{\mathtt{nc}} = g_z \sin\theta_{mix}\psi_L\gamma^\mu(1+\gamma_5)\psi_e Z^\mu + h.c. \tag{3.1}$$

where $\sin\theta_{mix}$ are the mixing angles between right handed components of the ordinary and new heavy charged leptons. The Feynman diagrams of the s and t channel processes $e^-e^+ \rightarrow L^-e^+$ are given in Fig. 5.1. The order of the mix-

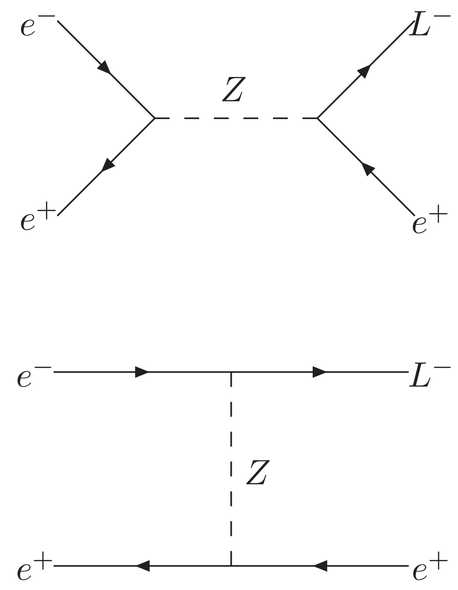

Figure 3.1: The Feynman diagrams of the s and t channel Z exchange processes in ep collision.

ings of the ordinary and heavy leptons are known to be $\sin^2\theta_{mix} \approx 10^{-2} - 10^{-3}$, which comes from low energy phenomenological calculations and the high precision measurements of the Z properties at linear colliders [44-46]. We use the parameter b_{lLZ} to denote the mixing angles and take 0.1 as an upper value in the numerical calculations.

3.1.1 Definition of Differential Cross Section

The total differential cross section for the process $e^+e^- \rightarrow L^-e^+$ is obtained as

$$\frac{d\sigma_{tot.}}{dt} = \frac{1}{16\pi s^2}\langle|\mathcal{M}_{tot.}|^2\rangle \quad (3.2)$$

where

$$\langle|\mathcal{M}_{tot.}|^2\rangle = \langle|\mathcal{M}_s|^2\rangle + \langle|\mathcal{M}_t|^2\rangle + \langle\mathcal{M}_s\overline{\mathcal{M}_t}\rangle + \langle\overline{\mathcal{M}}_s\mathcal{M}_t\rangle \tag{3.3}$$

s channel matrix elements are,

$$\begin{aligned}\mathcal{M}_s &= \Big[\bar{v}(p_2)\frac{g_z}{2}\gamma^\mu(v_e - a_e\gamma^5)u(p_1)\Big] \\ &\times\Big[\frac{-ig_{\mu\nu} - q_\mu q_\nu/M_Z^2}{(s-M_Z^2) + iM_Z\Gamma_Z}\Big]\Big[\bar{u}(p_3)g_Z b_{lLZ}\gamma^\nu(1+\gamma^5)v(p_4)\Big]\end{aligned} \tag{3.4}$$

and

$$\begin{aligned}\overline{\mathcal{M}_s} &= \Big[\bar{v}(p_1)\frac{g_Z}{2}\gamma^\alpha(v_e - a_e\gamma^5)u(p_2)\Big] \\ &\times\Big[\frac{ig_{\alpha\beta} - q_\alpha q_\beta/M_Z^2}{(s-M_Z^2) - iM_Z\Gamma_Z}\Big]\Big[\bar{u}(p_4)g_Z b_{lLZ}\gamma^\beta(1+\gamma^5)v(p_3)\Big]\end{aligned} \tag{3.5}$$

ignoring the ordinary lepton masses we find,

$$\begin{aligned}\langle|\overline{\mathcal{M}_s}|^2\rangle &= \frac{g_Z^4 b_{lLZ}^2}{16\Big[(s-M_Z)^2 + \Gamma_Z^2 M_Z^2\Big]}\Big(g_{\mu\nu} - q_\mu q_\nu/M_Z^2\Big)\Big(g_{\alpha\beta} - q_\alpha q_\beta/M_Z^2\Big) \\ &\times Tr\Big[(\not{p}_2\gamma^\mu(v_e - a_e\gamma^5)\ \not{p}_1\gamma^\alpha(c_V - c_A\gamma^5)\Big] \\ &\times Tr\Big[(\not{p}_3 + m_3)\gamma^\nu(1+\gamma^5)\ \not{p}_4\gamma^\beta(1+\gamma^5)\Big]\end{aligned} \tag{3.6}$$

t channel matrix elements are,

$$\begin{aligned}\mathcal{M}_t &= \Big[\bar{u}(p_3)g_Z b_{lLZ}\gamma^\mu(1+\gamma^5)u(p_1)\Big] \\ &\times\Big[\frac{-ig_{\mu\nu} - q_\mu q_\nu/M_Z^2}{(t-M_Z^2) - iM_Z\Gamma_Z}\Big]\Big[\bar{v}(p_2)\frac{g_Z}{2}\gamma^\nu(v_e - a_e\gamma^5)v(p_4)\Big]\end{aligned} \tag{3.7}$$

and

$$\begin{aligned}\overline{\mathcal{M}_t} &= \Big[\bar{u}(p_1) g_Z b_{lLZ} \gamma^\alpha (1+\gamma^5) u(p_3)\Big] \\ &\times \Big[\frac{i g_{\alpha\beta} - q_\alpha q_\beta / M_Z^2}{(t - M_Z^2) + i M_Z \Gamma_Z}\Big] \Big[\bar{v}(p_4) \frac{g_Z}{2} \gamma^\beta (v_e - a_e \gamma^5) v(p_2)\Big] \end{aligned} \tag{3.8}$$

ignoring the ordinary lepton masses we find,

$$\begin{aligned}\langle |\overline{\mathcal{M}_t}|^2 \rangle &= \frac{g_Z^4 b_{lLZ}^2}{16\Big[(t - M_Z)^2 + \Gamma_Z^2 M_Z^2\Big]} \Big(g_{\mu\nu} - q_\mu q_\nu / M_Z^2\Big)\Big(g_{\alpha\beta} - q_\alpha q_\beta / M_Z^2\Big) \\ &\times Tr\Big[(\not{p}_3 + m_3)\gamma^\mu (1+\gamma^5)\ \not{p}_1 \gamma^\alpha (1+\gamma^5)\Big] \\ &\times Tr\Big[\ \not{p}_2 \gamma^\nu (v_e - a_e \gamma^5)\ \not{p}_4 \gamma^\beta (v_e - a_e \gamma^5)\Big]. \end{aligned} \tag{3.9}$$

The interference terms of the s and t channel processes are given by,

$$\begin{aligned}\langle \mathcal{M}_s \overline{\mathcal{M}_t} \rangle &= \frac{g_Z^4 b_{lLZ}^2 \Big(g_{\mu\nu} - q_\mu q_\nu / M_Z^2\Big)\Big(g_{\alpha\beta} - q_\alpha q_\beta / M_Z^2\Big)}{16\Big[(s - M_Z^2) + i\Gamma_Z M_Z\Big]\Big[(t - M_Z^2) + i\Gamma_Z M_Z\Big]} \\ &\times Tr\Big[\ \not{p}_2 \gamma^\mu (v_e - a_e \gamma^5)\ \not{p}_1 \gamma^\beta (1+\gamma^5) \\ &\times (\not{p}_3 + m_L)\gamma^\nu (1+\gamma^5)\ \not{p}_4 \gamma^\alpha (v_e - a_e \gamma^5)\Big] \end{aligned} \tag{3.10}$$

and

$$\begin{aligned}\langle \overline{\mathcal{M}}_s \mathcal{M}_t \rangle &= \frac{g_Z^4 b_{lLZ}^2 \Big(g_{\alpha\beta} - q_\alpha q_\beta / M_Z^2\Big)\Big(g_{\mu\nu} - q_\mu q_\nu / M_Z^2\Big)}{16\Big[(s - M_Z^2) - i\Gamma_Z M_Z\Big]\Big[(t - M_Z^2) - i\Gamma_Z M_Z\Big]} \\ &\times Tr\Big[\ \not{p}_1 \gamma^\alpha (v_e - a_e \gamma^5)\ \not{p}_2 \gamma^\nu (v_e - a_e \gamma^5) \\ &\times\ \not{p}_4 \gamma^\beta (1+\gamma^5)(\not{p}_3 + m_L)\gamma^\mu (1+\gamma^5)\Big] \end{aligned} \tag{3.11}$$

$$
\begin{aligned}
p_1 \cdot p_2 &= \frac{s}{2} \ , \\
p_3 \cdot p_4 &= \frac{s - m_L^2}{2} \ , \\
p_1 \cdot p_3 &= \frac{m_L^2 - t}{2} \ , \\
p_2 \cdot p_4 &= -\frac{t}{2} \ , \\
p_1 \cdot p_4 &= -\frac{u}{2} \ , \\
p_2 \cdot p_3 &= \frac{m_L^2 - u}{2} \ , \\
u &= m_L^2 - s - t \ ,
\end{aligned} \tag{3.12}
$$

using the trace theorems and the Mandelstam variables in Eq. (3.12), the differential cross section takes the form,

$$
\begin{aligned}
\frac{d\sigma}{dt} &= \frac{\pi \alpha^2 b_{lLZ}^2}{\sin^4\theta_w \cos^4\theta_w s^2} \\
&\times \Bigg\{ \frac{4(m^2 - s - t)(s+t)(a_e - v_e)^2 \Big((s - M_Z^2)(t - M_Z^2) + \Gamma_Z^2 M_Z^2 \Big)}{\Big[(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2 \Big] \Big[(t - M_Z^2)^2 + M_Z^2 \Gamma_Z^2 \Big]} \\
&- \frac{(m^2 - s - t)(s+t)(a_e - v_e)^2 + t(m^2 - t)(a_e + v_e)^2}{\Big[(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2 \Big]} \\
&+ \frac{\Big[\Big((s+t)^2 + s^2 - (2s+t)m^2 \Big)(a_e^2 + v_e^2) + 2t(m^2 - 2s - t) a_e v_e \Big]}{\Big[(t - M_Z^2)^2 + M_Z^2 \Gamma_Z^2 \Big]} \Bigg\}
\end{aligned} \tag{3.13}
$$

where $a_e = -\frac{1}{2}$ and $v_e = -\frac{1}{2} + 2\sin^2\theta_w$, Γ_Z is the decay width and M_Z is the mass of Z boson, s and t are Mandelstam variables.

3.1.2 Numerical Results

In Fig. 3.2, we display the total cross sections as function of the heavy lepton masses, for the three center of mass energies of the proposed options. After their production, the heavy leptons will decay via the neutral current process $L \rightarrow lZ$, where $l = e, \mu, \tau$. The branching ratios for these processes would be around 33% for each channel.

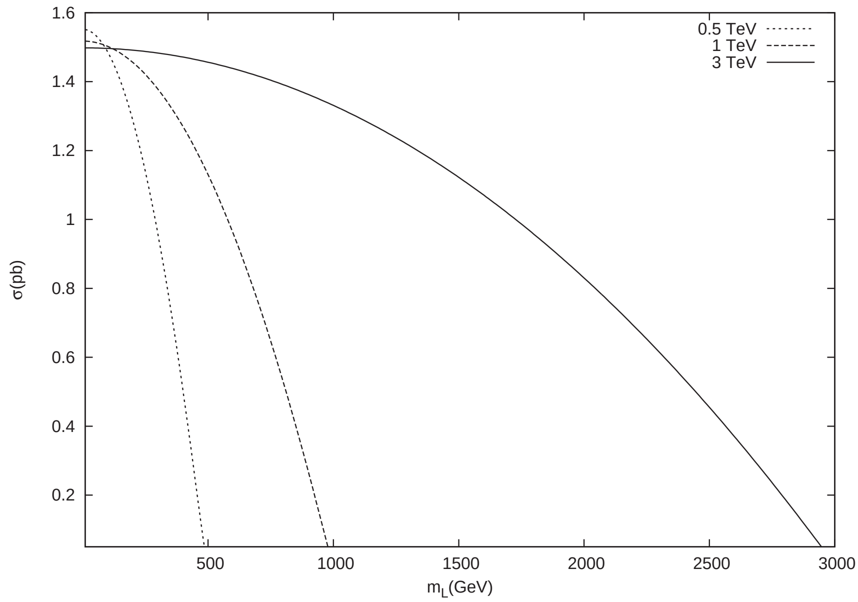

Figure 3.2: The total cross sections for the process $e^- e^+ \rightarrow L^- e^+$, as function of the heavy lepton masses, for linear colliders ILC ($\sqrt{s}$ =0.5 and 1 TeV, $L^{int} = 10^5 \mathrm{pb}^{-1}$) and CLIC ($\sqrt{s}$ =3 TeV, TeV, $L^{int} = 10^5 \mathrm{pb}^{-1}$) for single production of heavy lepton. Solid line is at CLIC with $\sqrt{s}$ =3 TeV, dashed line is at ILC with $\sqrt{s}$ =1 TeV and dotted line is at ILC with $\sqrt{s}$ =0.5 TeV.

In Tables 3.2, 3.3 and 3.4, we presented the single production cross sections ($\sigma \times \mathrm{BR}_1$), signal and background cross sections depending on the heavy lepton mass m_L, for 0.5, 1 and 3 TeV energy $e^- e^+$ colliders, respectively. The branching ratios BR_1 and BR_2 refer to $\mathrm{BR}(L \rightarrow Ze)$ and $\mathrm{BR}(Z \rightarrow e^+ e^-,\ \mu^+ \mu^-)$. The significance of signal and background is defined as $S/\sqrt{S+B}$, here S and B are the signal and background number of events. The total decay widths of the heavy

m_L (GeV)	σ (pb)	$\sigma\times\mathrm{BR}_1$ (pb)	$\sigma\times\mathrm{BR}_1\times\mathrm{BR}_2$ (pb)	$\sigma_B\times10^{-3}$ (pb)	$S/\sqrt{S+B}$	Γ_{Total} (GeV)
100	1.41	0.46	0.015	0.63	38	0.008
200	1.22	0.40	0.013	1.25	35	0.589
300	0.91	0.30	0.010	1.34	30	2.159
400	0.48	0.16	0.005	1.76	20	5.183

Table 3.2: The signal and background cross sections and SS depending on the heavy lepton masses for ILC with $\sqrt{s}=0.5$ TeV, $L^{int}=10^5\mathrm{pb}^{-1}$ for the single production of heavy lepton. The branching ratios BR_1 and BR_2 denote $BR(L^-\to Ze^-)$ and $BR(Z\to e^+e^-,\mu^+\mu^-)$, respectively. The total decay width of the heavy lepton is given in the last column.

m_L (GeV)	σ (pb)	$\sigma\times\mathrm{BR}_1$ (pb)	$\sigma\times\mathrm{BR}_1\times\mathrm{BR}_2$ (pb)	$\sigma_B\times10^{-4}$ (pb)	$S/\sqrt{S+B}$	Γ_{Total} (GeV)
100	1.47	0.49	0.016	0.99	40	0.008
300	1.35	0.45	0.015	2.89	38	2.159
500	1.11	0.37	0.012	2.83	35	10.146
700	0.74	0.25	0.008	3.33	28	27.834
900	0.26	0.09	0.003	5.39	16	59.108

Table 3.3: The signal and background cross sections and SS depending on the heavy lepton masses for ILC with $\sqrt{s}=1$ TeV, $L^{int}=10^5\mathrm{pb}^{-1}$ for the single production of heavy lepton. The branching ratios BR_1 and BR_2 denote $BR(L^-\to Ze^-)$ and $BR(Z\to e^+e^-,\mu^+\mu^-)$, respectively. The total decay width of the heavy lepton is given in the last column.

m_L (GeV)	σ (pb)	$\sigma\times\mathrm{BR}_1$ (pb)	$\sigma\times\mathrm{BR}_1\times\mathrm{BR}_2$ (pb)	$\sigma_B\times10^{-5}$ (pb)	$S/\sqrt{S+B}$	Γ_{Total} (GeV)
250	1.48	0.49	0.016	21.06	40	1.22
750	1.40	0.46	0.015	3.97	39	34.23
1250	1.23	0.41	0.013	3.32	37	158.19
1750	0.98	0.32	0.011	4.50	33	433.67
2250	0.65	0.21	0.007	6.35	26	921.23
2750	0.24	0.08	0.003	10.05	16	1681.41

Table 3.4: The signal and background cross sections and SS depending on the heavy lepton masses for CLIC with $\sqrt{s}=3$ TeV, $L^{int}=10^5\mathrm{pb}^{-1}$ for the single production of heavy lepton. The branching ratios BR_1 and BR_2 denote $BR(L^-\to Ze^-)$ and $BR(Z\to e^+e^-,\mu^+\mu^-)$, respectively. The total decay width of the heavy lepton is given in the last column.

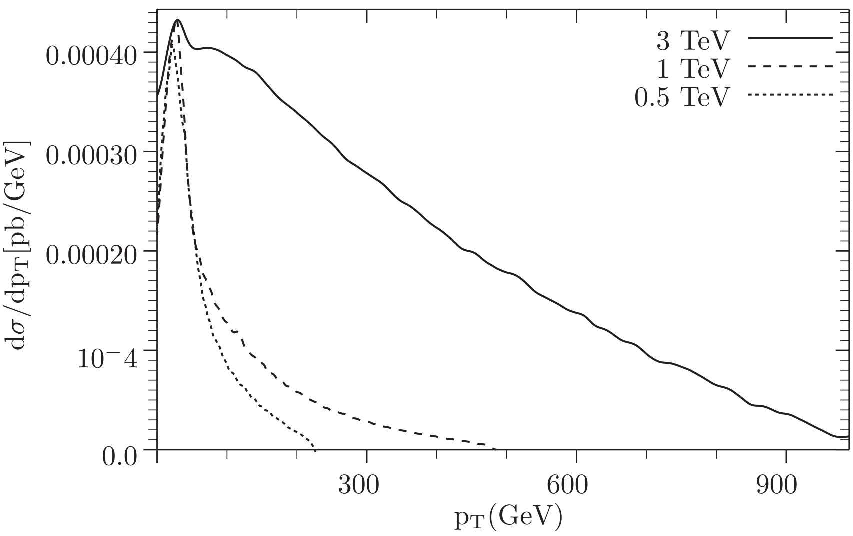

Figure 3.3: p_T distribution of the background process $e^-e^+ \to e^-Ze^+$ for linear colliders ILC and CLIC for single production of heavy lepton. Solid line is at CLIC with $\sqrt{s}$ =3 TeV, dashed line is at ILC with $\sqrt{s}$ =1 TeV and dotted line is at ILC with $\sqrt{s}$ =0.5 TeV.

leptons are given in the last column of the tables. As seen from these tables, the $S/\sqrt{S+B}$ values are higher than five, which is enough for observability, up to the center of mass energies of the colliders. Single production of heavy lepton is feasible up to the center of mass energies of the e^-e^+ colliders even with smaller mixing coupling values. For example, if we take at least 10 signal events and $S/\sqrt{S+B} \geq 5$ as discovery criteria, the ILC with $\sqrt{s}$ =0.5 TeV can probe mixing values of $b_{lLZ} = 0.032$ for 350 GeV leptons. The same couplings can be probed at $\sqrt{s}$ =1 and $\sqrt{s}$ =3 TeV for even greater masses such as 800 and 2750 GeV. We applied a cut of $|m_{Ze^-} - m_{L^-}| < 10$ GeV in order to form the signal and reduce the background for the SM background process $e^-e^+ \to e^-Ze^+$. Fig. 3.3 shows the p_T distributions at three different linear colliders. In Figs. 3.4, 3.5 and 3.6, we give the invariant mass distributions m_{Ze^-} with cut $p_T^{e^-,j} > 10$ GeV at $\sqrt{s}$ =0.5, 1 and 3 TeV, respectively. Figs. 3.4 and 3.5 have an increasing character for a cut

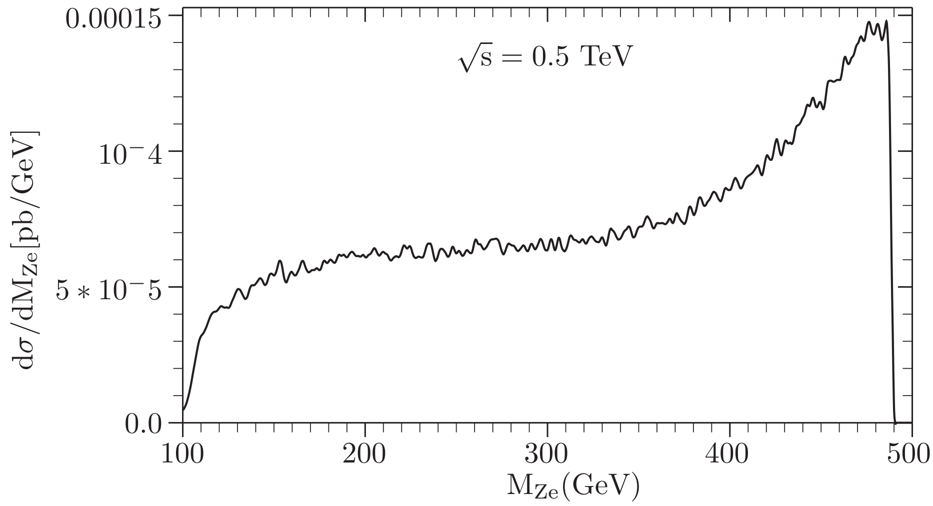

Figure 3.4: The invariant mass distribution of the Ze^- system for the background at ILC ($\sqrt{s}$ =0.5 TeV).

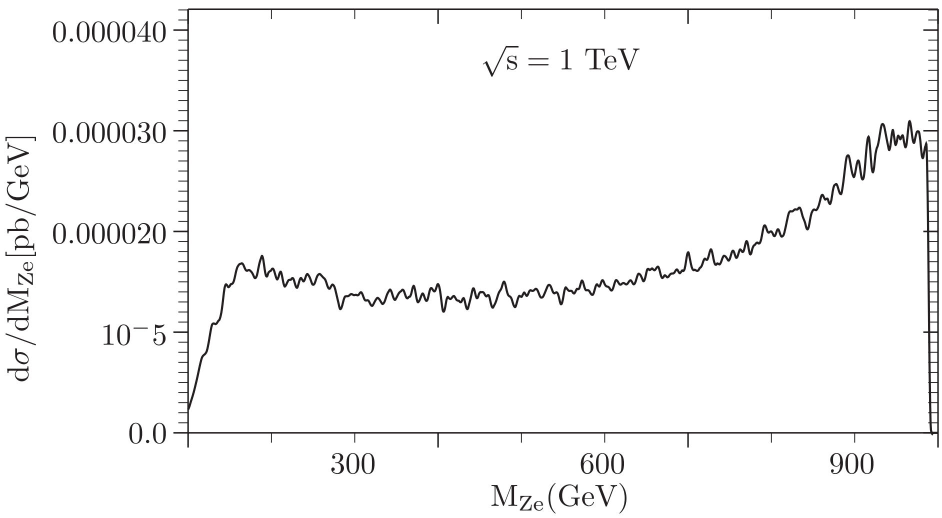

Figure 3.5: The invariant mass distribution of the Ze^- system for the background at ILC ($\sqrt{s}$ =1 TeV).

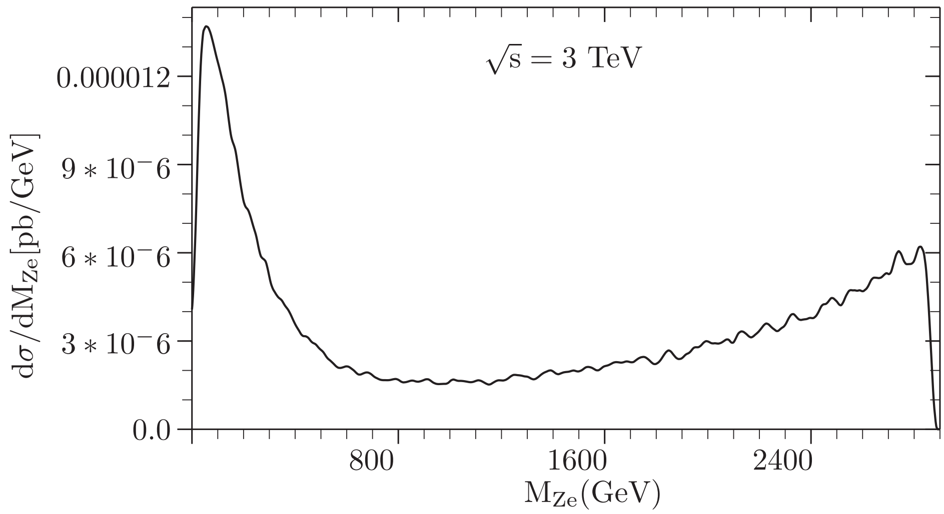

Figure 3.6: The invariant mass distribution of the Ze^- system for the background at CLIC ($\sqrt{s}$ =3 TeV).

10 GeV interestingly, but lose this character for higher cuts. For instance, all of the three distributions in Figs. 3.4-3.6 have decreasing characters with Jacobian peaks around 150-200 GeV for a cut value of 80 GeV.

3.2 Pair Production of Heavy Leptons

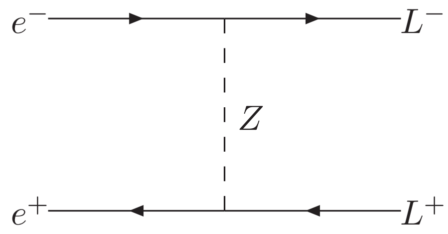

Figure 3.7: The Feynman diagrams of the s and t channel processes $e^-e^+ \to L^-L^+$.

Pair production of heavy leptons in E_6 occur through the t-channel flavor changing neutral current process $e^-e^+ \to L^-L^+$. The Feynman diagram of the t

channel process $e^-e^+ \rightarrow L^-L^+$ is given in Fig. 5.15.

3.2.1 Definition of Differential Cross Section

We can write the matrix elements from the FCNC Lagrangian given in Eq. (3.1) as,

$$\begin{aligned}\mathcal{M} &= \frac{g_Z^2 b_{lLZ}^2}{\Big[(t-M_Z^2)+iM_Z\Gamma_Z\Big]}\Big(g_{\mu\nu}-q_\mu q_\nu/M_Z^2\Big)\\ &\quad\times\Big[\bar{u}(p_3)\gamma^\mu(1+\gamma^5)u(p_1)\bar{v}(p_2)\gamma^\nu(1+\gamma^5)v(p_4)\Big] \end{aligned}\tag{3.14}$$

and

$$\begin{aligned}\overline{\mathcal{M}} &= \frac{g_Z^2 b_{lLZ}^2}{\Big[(t-M_Z^2)-iM_Z\Gamma_Z\Big]}\Big(g_{\alpha\beta}-q_\alpha q_\beta/M_Z^2\Big)\\ &\quad\times\Big[\bar{u}(p_1)\gamma^\alpha(1+\gamma^5)u(p_3)\bar{v}(p_4)\gamma^\beta(1+\gamma^5)v(p_2)\Big] \end{aligned}\tag{3.15}$$

ignoring the lepton masses,

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle &= \frac{g_Z^4 b_{lLZ}^2}{4\Big[(t-M_Z^2)^2+M_Z^2\Gamma_Z^2\Big]}\Big(g_{\mu\nu}-q_\mu q_\nu/M_Z^2\Big)\Big(g_{\alpha\beta}-q_\alpha q_\beta/M_Z^2\Big)\\ &\quad\times Tr\Big[(\not{p}_3+m_L)\gamma^\mu(1+\gamma^5)\ \not{p}_1\gamma^\alpha(1+\gamma^5)\Big]\\ &\quad\times Tr\Big[\ \not{p}_2\gamma^\nu(1+\gamma^5)(\not{p}_4-m_L)\gamma^\beta(1+\gamma^5)\Big] \end{aligned}\tag{3.16}$$

Mandelstam variables for the pair production of heavy leptons are,

$$\begin{aligned} p_1\cdot p_2 &= \frac{s}{2}\ , \\ p_3\cdot p_4 &= \frac{s-2m_L^2}{2}\ , \\ p_1\cdot p_3 &= \frac{m_L^2-t}{2}\ , \end{aligned}$$

$$
\begin{aligned}
p_2 \cdot p_4 &= \frac{m_L^2 - t}{2} \,, \\
p_1 \cdot p_4 &= \frac{m_L^2 - u}{2} \,, \\
p_2 \cdot p_3 &= \frac{m_L^2 - u}{2} \,, \\
u &= 2m_L^2 - s - t \,. \qquad (3.17)
\end{aligned}
$$

Using the trace theorems and the Mandelstam variables in Eq. (3.17), the differential cross section for this process is obtained as,

$$
\begin{aligned}
\frac{d\sigma}{dt} &= \frac{4\pi\alpha^2 b_{lLZ}^4}{s^2 M_Z^4\Big[(t - M_Z^2)^2 + M_Z^2\Gamma_Z^2\Big]} \\
&\times\Big[(m_L^2 - t)^2 m_L^4 + 4M_Z^2 m^4 s + 4(s + t - m_L^2)^2 M_Z^4\Big]. \qquad (3.18)
\end{aligned}
$$

3.2.2 Numerical Results

The total cross sections as functions of heavy lepton masses m_L, are displayed in Fig. 3.8.

Signal and background cross sections depending again on the heavy lepton masses, are presented in Tables 3.5, 3.6 and 3.7 at 0.5, 1 and 3 TeV, respectively. For the pair production of heavy leptons at linear colliders we expect of order of $10^2 - 10^3$ signal events for 1250 GeV leptons for the coupling value of $b_{lLZ} = 0.1$. On the other hand, the lower limit of the coupling which can be probed by pair production at linear colliders is found to be 0.05.

We applied an initial cut on the electron and jet transverse momentum $p_T^{e^-,j} >$ 20 GeV for the signal and background analysis. Fig. 3.9 shows the p_T distributions of the background at the colliders. The distribution of invariant mass m_{Ze^-} is presented in Figs. 3.10, 3.11 and 3.12 at $\sqrt{s}$ =0.5, 1 and 3, respectively.

m_L (GeV)	σ (pb)	$\sigma \times \mathrm{BR}_1$ (pb)	$\sigma \times \mathrm{BR}_1 \times \mathrm{BR}_2$ (pb)	$\sigma_B \times 10^{-8}$ (pb)	$S/\sqrt{S+B}$	Γ_{Total} (GeV)
100	0.36	0.12	0.0039	0.005	20	0.01
150	0.35	0.12	0.0038	2.49	20	0.13
200	0.37	0.12	0.0040	5.03	20	0.59
240	0.27	0.09	0.0029	1.25	17	1.22

Table 3.5: The signal and background cross sections and SS depending on the heavy lepton masses for ILC with $\sqrt{s} = 0.5$ TeV, $L^{int} = 10^5\mathrm{pb}^{-1}$ for the pair production of heavy lepton. The branching ratios BR_1 and BR_2 denote $BR(L^- \to Ze^-)$ and $BR(Z \to e^+e^-, \mu^+\mu^-)$, respectively. The total decay width of the heavy lepton is given in the last column.

m_L (GeV)	σ (pb)	$\sigma \times \mathrm{BR}_1$ (pb)	$\sigma \times \mathrm{BR}_1 \times \mathrm{BR}_2$ (pb)	$\sigma_B \times 10^{-7}$ (pb)	$S/\sqrt{S+B}$	Γ_{Total} (GeV)
100	0.43	0.14	0.0047	0.004	22	0.01
200	0.44	0.15	0.0048	1.51	22	0.59
300	0.56	0.18	0.0061	3.57	25	2.16
400	0.94	0.31	0.0102	5.12	32	5.18

Table 3.6: The signal and background cross sections and SS depending on the heavy lepton masses for ILC with $\sqrt{s} = 1$ TeV, $L^{int} = 10^5\mathrm{pb}^{-1}$ for the pair production of heavy lepton. The branching ratios BR_1 and BR_2 denote $BR(L^- \to Ze^-)$ and $BR(Z \to e^+e^-, \mu^+\mu^-)$, respectively. The total decay width of the heavy lepton is given in the last column.

m_L (GeV)	σ (pb)	$\sigma \times \mathrm{BR}_1$ (pb)	$\sigma \times \mathrm{BR}_1 \times \mathrm{BR}_2$ (pb)	$\sigma_B \times 10^{-6}$ (pb)	$S/\sqrt{S+B}$	Γ_{Total} (GeV)
250	0.47	0.16	0.0051	0.90	23	1
500	0.61	0.20	0.0066	4.04	26	10
750	1.51	0.50	0.0164	3.92	41	34
1000	3.96	1.31	0.0431	1.18	66	81
1250	7.79	2.57	0.0848	16.20	92	158

Table 3.7: The signal and background cross sections and SS depending on the heavy lepton masses for CLIC with $\sqrt{s} = 3$ TeV, $L^{int} = 10^5\mathrm{pb}^{-1}$ for the pair production of heavy lepton. The branching ratios BR_1 and BR_2 denote $BR(L^- \to Ze^-)$ and $BR(Z \to e^+e^-, \mu^+\mu^-)$, respectively. The total decay width of the heavy lepton is given in the last column.

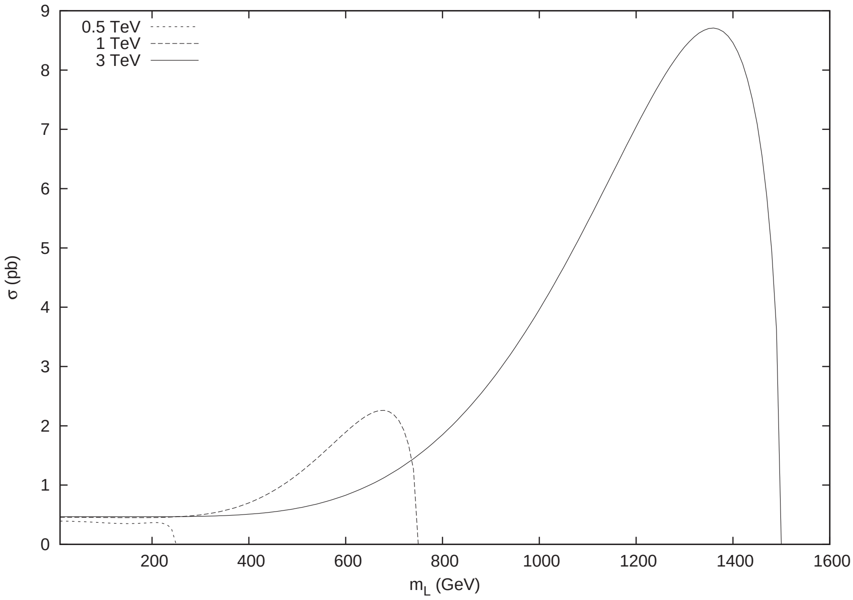

Figure 3.8: The total cross sections for the process $e^-e^+ \to L^-L^+$, as function of the heavy lepton masses, for linear colliders ILC ($\sqrt{s}$ =0.5 and 1 TeV, $L^{int} = 10^5 \mathrm{pb}^{-1}$) and CLIC ($\sqrt{s}$ =1 TeV, TeV, $\mathcal{L}^{int} = 10^5 \mathrm{pb}^{-1}$) for pair production of heavy lepton.

3.3 Discussion

The production of a single heavy lepton is more relevant than the pair production. Namely, in the case of $\sqrt{s} = 3$ TeV option, we expect 257 single events for $b_{lLZ} = 0.1$ and 26 single events for $b_{lLZ} = 0.032$ for 2750 GeV (which is the upper bound) leptons. In the case of pair production, for $b_{lLZ} = 0.1$ we expect $10^2 - 10^3$ events for 1250 GeV (upper value) leptons, while no pair event can be observed for $b_{lLZ} = 0.032$, since the pair production cross section is suppressed by the fourth power of mixing couplings.

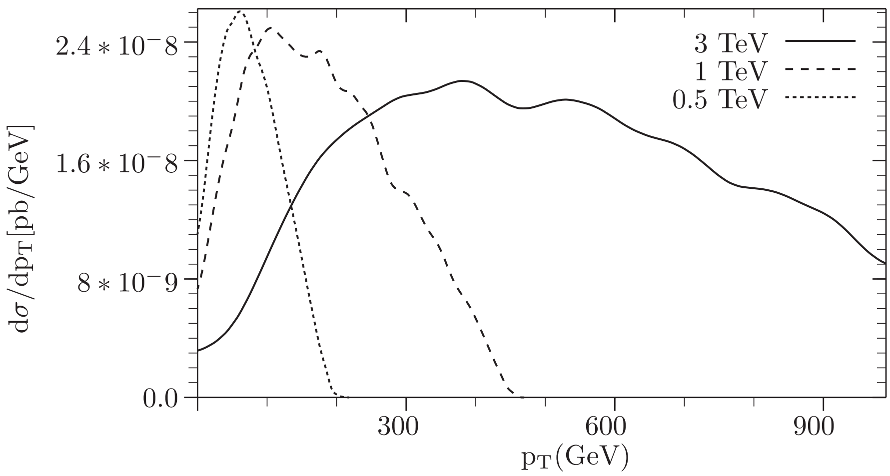

Figure 3.9: p_T distribution of the background process $e^-e^+ \to ZZe^-e^+$ for linear colliders ILC and CLIC for pair production.

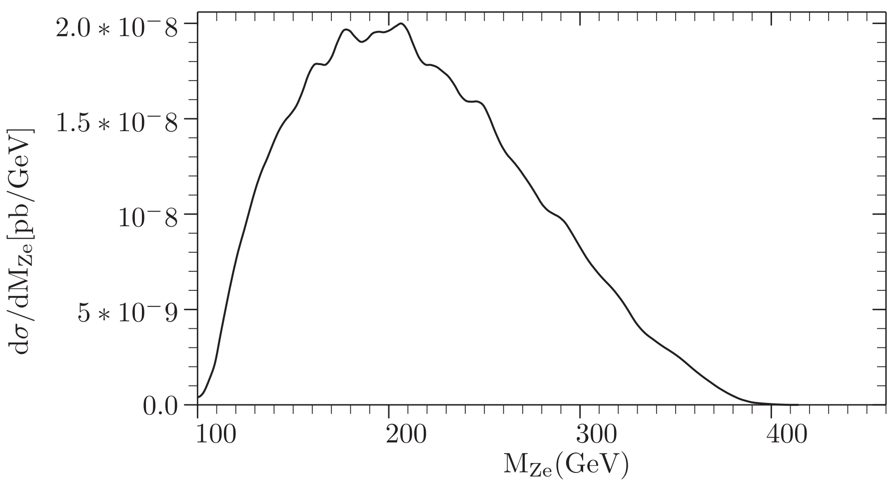

Figure 3.10: The invariant mass distribution of the Ze system for the background for ILC at $\sqrt{s}$ =0.5 TeV for pair production.

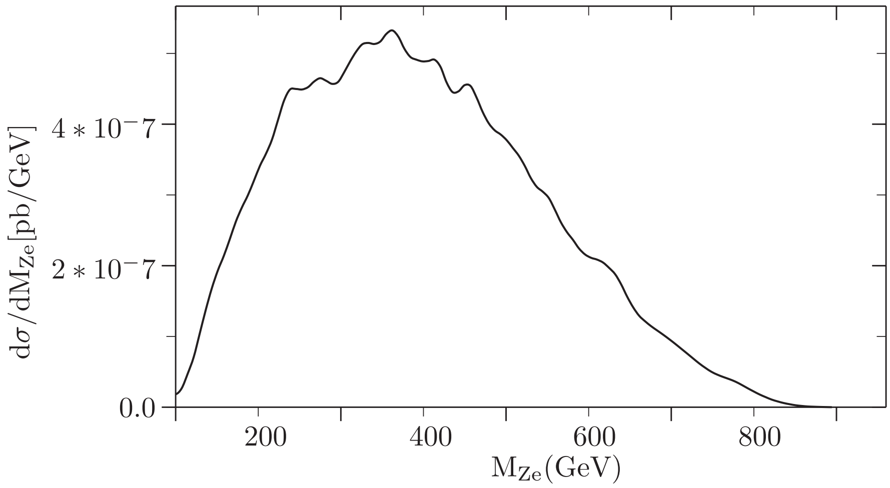

Figure 3.11: The invariant mass distribution of the Ze system for the background for ILC at $\sqrt{s}$ =1 TeV for pair production.

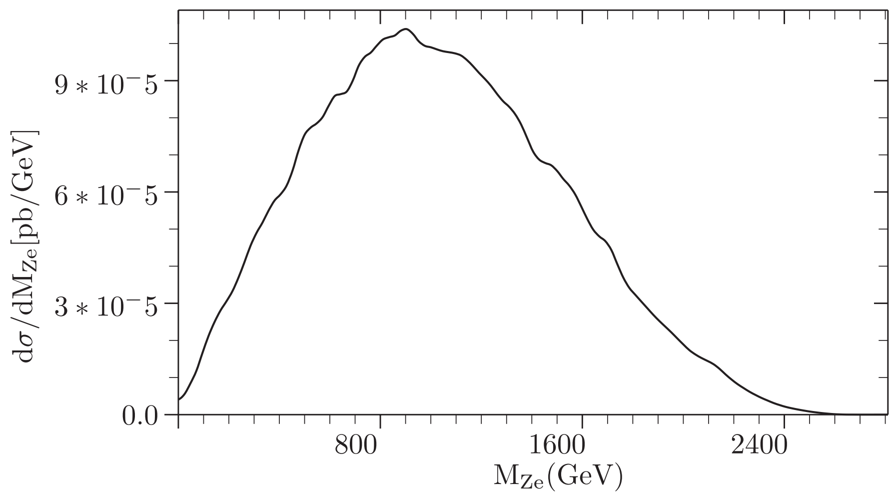

Figure 3.12: The invariant mass distribution of the Ze system for the background for CLIC at $\sqrt{s}$ =3 TeV for pair production.

CHAPTER 4

EP ANALYSIS OF ANOMALOUS INTERACTIONS WITH HEAVY LEPTONS

Many models extending the standard theory of quarks and leptons predict the existence of new generations of fermions. Any signal for the production of such fermions will play a milestone role in the discovery of new physics. In this chapter, we analyze the possible production and decay of new heavy leptons in lepton hadron collisions via some anomalous interactions at CERN Large Hadron Electron Collider (LHeC) and at Desy THERA. These high energy *ep* collider options are ideal places to investigate the production of new leptons which are heavy compared to the standard ones. In this work, we present the anomalous single production of heavy leptons at future *ep* colliders THERA [50] and LHeC [61]. The main parameters of these colliders are given in Table 4.1.

Collider	E_e (TeV)	E_p (TeV)	$\sqrt{s_{ep}}$ (TeV)	$L^{int}(pb^{-1})$
THERA	0.25	1	1	40
LHeC	0.07	7	1.4	10^4

Table 4.1: The main parameters of the *ep* colliders, L^{int} denotes the integrated luminosity for one year.

4.1 Production of Heavy Leptons

In the Standard Model, Flavor Changing Neutral Current (FCNC) processes receive contributions from only higher order corrections [62-75]. Here we offer the following effective Lagrangian describing these interactions via anomalous magnetic transition moment type vertices;

$$\mathcal{L}_{\texttt{eff}} = \frac{ie\kappa_\gamma}{\Lambda} L\sigma_{\mu\nu}q^\nu l A^\mu + \frac{g}{2\cos\theta_W} L\left[\gamma_\mu(c_v - c_a\gamma_5) + \frac{i\sigma_{\mu\nu}q^\nu}{\Lambda}\kappa_Z\right] l Z^\mu + h.c., \tag{4.1}$$

where κ_γ and κ_Z are the anomalous magnetic dipole moment factors, c_v and c_a are the corresponding anomalous non-diagonal Z couplings which are zero in the SM, q is the momentum of the exchanged gauge boson, θ_W is the Weinberg angle, e and g denote the gauge couplings relative to $U(1)$ and $SU(2)$ symmetries respectively, A^μ and Z^μ the fields of the photon and Z boson and Λ is the new physics scale.

The parton level process $e^-q \to L^-q$, responsible for the heavy lepton production in ep collision occurs via FCNC γ and Z exchange in the t channel. The Feynman diagram of the parton level process $e^-(p_1)q(p_2) \to L^-(p_3)q(p_4)$ is given in Fig. 4.1.

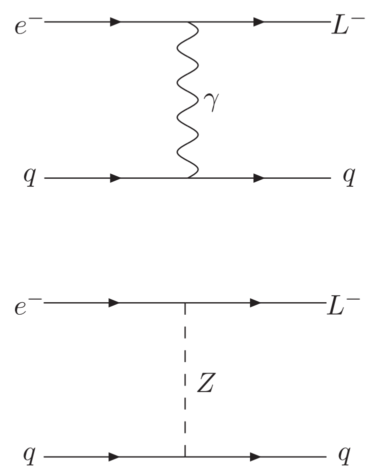

Figure 4.1: The Feynman diagrams of the t channel γ and Z exchange processes $e^-q \to L^-q$.

4.2 Definition of Differential Cross Section

Considering the effective Lagrangian in Eq. (4.1), we find the amplitudes for the γ exchange in the t channel as,

$$\mathcal{M}_\gamma = -\frac{e_q e^2 \kappa_\gamma}{\Lambda \hat{t}} \Big[\bar{u}(p_3)\sigma^{\mu\rho} q_\rho u(p_1)\Big]\Big[\bar{u}(p_4)\gamma_\mu u(p_2)\Big] \tag{4.2}$$

and

$$\overline{\mathcal{M}}_\gamma = -\frac{e_q e^2 \kappa_\gamma}{\Lambda \hat{t}} \Big[\bar{u}(p_1)\sigma^{\alpha\delta} q_\delta u(p_3)\Big]\Big[\bar{u}(p_2)\gamma_\alpha u(p_4)\Big] \tag{4.3}$$

ignoring the quark and electron masses we find,

$$\begin{aligned} \langle|\mathcal{M}_\gamma|^2\rangle &= \frac{e_q^2 e^4 \kappa_\gamma^2}{4\Lambda^2 \hat{t}^2} \times Tr\Big[(\not{p}_3 + m_L)\sigma^{\mu\rho} q_\rho \ \not{p}_1 \sigma^{\alpha\delta} q_\delta\Big] \\ &\quad \times Tr\Big[\ \not{p}_4 \gamma_\mu \ \not{p}_2 \gamma_\alpha\Big] \end{aligned} \tag{4.4}$$

and we find the amplitudes for the Z exchange in the t channel as,

$$\begin{aligned} \mathcal{M}_Z &= \frac{-ig_Z^2}{4} \frac{(g_{\mu\nu} - q_\mu q_\nu / M_Z^2)}{(\hat{t} - M_Z^2) + iM_Z\Gamma_Z} \\ &\quad \times \Big[\bar{u}(p_3)\Big(\gamma^\mu (v_L - a_L\gamma^5) + \frac{i\kappa_Z}{\Lambda}\sigma^{\mu\rho} q_\rho\Big) u(p_1)\Big] \\ &\quad \times \Big[\bar{u}(p_4)\gamma^\nu (v_q - a_q\gamma^5) u(p_2)\Big], \end{aligned} \tag{4.5}$$

and

$$\begin{aligned} \overline{\mathcal{M}}_Z &= \frac{ig_Z^2}{4} \frac{(g_{\alpha\beta} - q_\alpha q_\beta / M_Z^2)}{(\hat{t} - M_Z^2) - iM_Z\Gamma_Z} \\ &\quad \times \Big[\bar{u}(p_1)\Big(\gamma^\alpha (v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\alpha\delta} q_\delta\Big) u(p_3)\Big] \\ &\quad \times \Big[\bar{u}(p_2)\gamma^\beta (v_q - a_q\gamma^5) u(p_4)\Big], \end{aligned} \tag{4.6}$$

which gives,

$$
\begin{aligned}
\langle|\mathcal{M}_Z|^2\rangle &= \frac{g_Z^4}{64\Big[(\hat{t}-M_Z^2)^2+M_Z^2\Gamma_Z^2\Big]}\Big[g_{\mu\nu}-q_\mu q_\nu/M_Z^2\Big]\Big[g_{\alpha\beta}-q_\alpha q_\beta/M_Z^2\Big] \\
&\times Tr\Big[(\not{p}_3+m_L)\Big(\gamma^\mu(v_L-a_L\gamma^5)+\frac{i\kappa_Z}{\Lambda}\sigma^{\mu\rho}q_\rho\Big) \\
&\times \not{p}_1\Big(\gamma^\alpha(v_L-a_L\gamma^5)-\frac{i\kappa_Z}{\Lambda}\sigma^{\alpha\delta}q_\delta\Big)\Big] \\
&\times Tr\Big[\not{p}_4\gamma^\nu(v_q-a_q\gamma^5)\not{p}_2\gamma^\beta(v_q-a_q\gamma^5)\Big] \qquad (4.7)
\end{aligned}
$$

The interference terms are given by,

$$
\begin{aligned}
\langle\mathcal{M}_\gamma\overline{\mathcal{M}}_Z\rangle &= \Big\{-\frac{ie^2e_q\kappa_\gamma g_Z^2}{16\Lambda\hat{t}}\Big\}\Big[\frac{g_{\alpha\beta}-q_\alpha q_\beta/M_Z^2}{(\hat{t}-M_Z^2)-iM_Z\Gamma_Z}\Big] \\
&\times Tr\Big[(\not{p}_3+m_L)\sigma^{\mu\rho}q_\rho\not{p}_1\Big(\gamma^\alpha(v_L-a_L\gamma^5)-\frac{i\kappa_Z}{\Lambda}\sigma^{\alpha\delta}q_\delta\Big)\Big] \\
&\times Tr\Big[\not{p}_4\gamma_\mu\not{p}_2\gamma^\beta(v_q-a_q\gamma_5)\Big] \qquad (4.8)
\end{aligned}
$$

and

$$
\begin{aligned}
\langle\overline{\mathcal{M}}_\gamma\mathcal{M}_Z\rangle &= \Big\{\frac{ie^2e_q\kappa_\gamma g_Z^2}{16\Lambda\hat{t}}\Big\}\Big[\frac{g_{\mu\nu}-q_\mu q_\nu/M_Z^2}{(\hat{t}-M_Z^2)+iM_Z\Gamma_Z}\Big] \\
&\times Tr\Big[\not{p}_1\sigma^{\alpha\delta}q_\delta(\not{p}_3+m_L)\Big(\gamma^\mu(v_L-a_L\gamma^5)+\frac{i\kappa_Z}{\Lambda}\sigma^{\mu\rho}q_\rho\Big)\Big] \\
&\times Tr\Big[\not{p}_2\gamma_\alpha\not{p}_4\gamma^\nu(v_q-a_q\gamma_5)\Big] \qquad (4.9)
\end{aligned}
$$

The total differential cross section for the process $e^-q\to L^-q$ is obtained as

$$
\frac{d\hat{\sigma}_{tot.}}{d\hat{t}} = \frac{1}{16\pi s^2}\Big[\langle|\mathcal{M}_\gamma|^2\rangle+\langle|\mathcal{M}_Z|^2\rangle+\langle\mathcal{M}_\gamma\overline{\mathcal{M}}_Z\rangle+\langle\overline{\mathcal{M}}_\gamma\mathcal{M}_Z\rangle\Big] \qquad (4.10)
$$

using the trace theorems and the Mandelstam variables in Eq. (2.7), the differential cross section for the subprocess $e^-q\to L^-q$, mediated by γ and Z in the t

channel, takes the form,

$$\begin{aligned}
\frac{d\hat{\sigma}(eq\to Lq)}{d\hat{t}} &= \frac{2\kappa_\gamma^2 e_q^2\pi\alpha^2}{\Lambda^2\hat{s}^2\hat{t}}\Big[(2\hat{s}+\hat{t})m_L^2-2\hat{s}(\hat{s}+\hat{t})-m_L^4\Big] \\
&+\frac{\pi\alpha^2}{8\Lambda^2\hat{s}^2\sin^4\theta_W\cos^4\theta_W\Big[(\hat{t}-M_Z^2)^2+M_Z^2\Gamma_Z^2\Big]} \\
&\times\Big\{2\kappa_Z\Lambda v_L({a_q}^2+{v_q}^2)(m_L^2-\hat{t})m_L\hat{t} \\
&+4\Lambda v_q a_q a_L(\Lambda v_L-m_L\kappa_Z)(m_L^2-2\hat{s}-\hat{t})\hat{t} \\
&-\Lambda^2({a_q}^2+{v_q}^2)({a_L}^2+{v_L}^2)\Big((2\hat{s}+\hat{t})m_L^2-2\hat{s}^2-2\hat{s}\hat{t}-\hat{t}^2\Big) \\
&-\kappa_Z^2({a_q}^2+{v_q}^2)\Big(m_L^4-(2\hat{s}+\hat{t})m_L^2+2\hat{s}(\hat{s}+\hat{t})\Big)\hat{t}\Big\} \\
&+\frac{\kappa_\gamma e_q\pi\alpha^2(\hat{t}-M_Z^2)}{\Lambda^2\hat{s}^2\sin^2\theta_W\cos^2\theta_W\Big[(\hat{t}-M_Z^2)^2+M_Z^2\Gamma_Z^2\Big]} \\
&\times\Big\{\kappa_Z v_q\Big(m_L^4-(2\hat{s}+\hat{t})m_L^2+2\hat{s}(\hat{s}+\hat{t})\Big) \\
&+\Lambda m_L\Big(a_L a_q(m_L^2-2\hat{s}-\hat{t})+v_L v_q(\hat{t}-m_L^2)\Big)\Big\}, \qquad (4.11)
\end{aligned}$$

where e_q is quark charge in units of e, Γ_Z and M_Z are the decay width and mass of mediator Z, m_L is the mass heavy lepton, $\hat{s}$ and $\hat{t}$ are the Mandelstam variables.

4.3 Heavy Lepton Decays

After their production, heavy leptons decay via the neutral current processes $L\to\gamma l$ and $L\to Zl$ due to the anomalous couplings in Eq. (4.1), where l denotes one of the ordinary charged leptons.

4.3.1 $L\to\gamma l$ Decay

Matrix element for the γ decay is written as,

$$\mathcal{M}=\bar{u}(p_l)\Big(-\frac{\kappa_\gamma e}{\Lambda}\sigma_{\mu\nu}q^\nu\Big)u(p_L)\epsilon^{\mu*} \qquad (4.12)$$

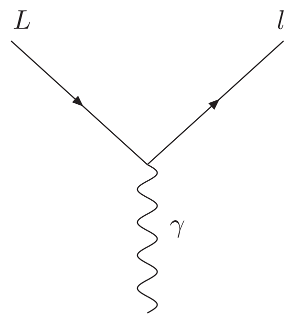

Figure 4.2: Heavy lepton decay $L \to \gamma l$.

and

$$\overline{\mathcal{M}} = \bar{u}(p_L)\Big(-\frac{\kappa_\gamma e}{\Lambda}\sigma_{\alpha\beta}q^\beta\Big)u(p_l)\epsilon^\alpha \tag{4.13}$$

squaring, we have

$$|\mathcal{M}|^2 = \Big\{\frac{\kappa_\gamma^2 e^2}{\Lambda^2}\Big\}\Big[\bar{u}(p_l)\sigma_{\mu\nu}q^\nu u(p_L)\Big]\Big[\bar{u}(p_L)\sigma_{\alpha\beta}q^\beta u(p_l)\Big](-g^{\mu\alpha}) \tag{4.14}$$

and we find

$$\langle|\mathcal{M}|^2\rangle = \Big\{-\frac{\kappa_\gamma^2 e^2}{\Lambda^2}\Big\} \times Tr\Big[(\not{p}_l + m_l)\sigma_{\mu\nu}q^\nu(\not{p}_L + m_L)\sigma^\mu_\beta q^\beta\Big] \tag{4.15}$$

Neglecting ordinary lepton masses the decay width is obtained as,

$$\Gamma(L \to l\gamma) = \frac{\alpha\kappa_\gamma^2 m_L^3}{2\Lambda^2}, \tag{4.16}$$

4.3.2 $L \to Zl$ Decay

Matrix element for the Z decay is written as,

$$\mathcal{M} = \bar{u}(p_l)\Big[\frac{ig_Z}{2}\Big(\gamma_\mu(v_L - a_L\gamma_5) + \frac{i\kappa_Z\sigma_{\mu\nu}q^\nu}{\Lambda}\Big)\Big]u(p_L)\epsilon^{\mu*} \tag{4.17}$$

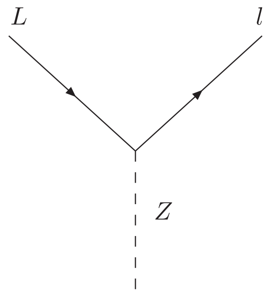

Figure 4.3: Heavy lepton decay $L \rightarrow Zl$.

and

$$\overline{\mathcal{M}} = \bar{u}(p_L)\Big[\frac{-ig_Z}{2}\Big(\gamma_\alpha(v_L - a_L\gamma_5) - \frac{i\kappa_Z\sigma_{\alpha\beta}q^\beta}{\Lambda}\Big)\Big]u(p_l)\epsilon^\alpha \tag{4.18}$$

squaring, we have

$$\begin{aligned} |\mathcal{M}|^2 &= \Big\{\frac{g_Z^2}{4}\Big\}\Big[\bar{u}(p_l)\Big(\gamma_\mu(v_L - a_L\gamma_5) + \frac{i\kappa_Z\sigma_{\mu\nu}q^\nu}{\Lambda}\Big)u(p_L)\Big] \\ &\times\Big[\bar{u}(p_L)\Big(\gamma_\alpha(v_L - a_L\gamma_5) - \frac{i\kappa_Z\sigma_{\alpha\beta}q^\beta}{\Lambda}\Big)u(p_l)\Big] \\ &\times\Big(-g^{\mu\alpha} + \frac{q^\mu q^\alpha}{M_Z^2}\Big) \end{aligned} \tag{4.19}$$

and we find

$$\begin{aligned} \langle|\mathcal{M}|^2\rangle &= \Big\{\frac{g_Z^2}{4}\Big\}\Big(-g^{\mu\alpha} + \frac{q^\mu q^\alpha}{M_Z^2}\Big) \\ &\times Tr\Big[(\not{p}_l + m_l)\Big(\gamma_\mu(v_L - a_L\gamma_5) + \frac{i\kappa_Z\sigma_{\mu\nu}q^\nu}{\Lambda}\Big) \\ &\times(\not{p}_L + m_L)\Big(\gamma_\alpha(v_L - a_L\gamma_5) - \frac{i\kappa_Z\sigma_{\alpha\beta}q^\beta}{\Lambda}\Big)\Big] \end{aligned} \tag{4.20}$$

Neglecting ordinary lepton masses the decay width is obtained as,

$$\Gamma(L \rightarrow lZ) = \frac{\alpha(m_L^2 - M_Z^2)^2}{16\Lambda^2 m_L^3 M_Z^2 \sin^2\theta_W \cos^2\theta_W}\Big[\kappa_Z^2 M_Z^4$$

$$+2M_Z^2\Big[\Lambda^2({v_L}^2+{a_L}^2)+(\kappa_Z^2 m_L^2-3c_v\kappa_Z\Lambda m_L)\Big]$$
$$+\Lambda^2({v_L}^2+{a_L}^2)m_L^2\Big], \qquad (4.21)$$

where α is the electromagnetic coupling constant, M_Z and m_L refer to masses of Z boson and decaying lepton, respectively.

4.4 Numerical Results

The total production cross section is obtained by the integration of differential cross section over the parton distributions in the proton as;

$$\sigma(ep\to LqX)=\int_{x_{min}}^{1}dx f_q(x,Q^2)\int_{t_{min}}^{t_{max}}\frac{d\hat{\sigma}}{d\hat{t}}d\hat{t}, \qquad (4.22)$$

where $x_{min}=m_L^2/s$, $\hat{t}_{min}=-(\hat{s}-m_L^2)$ and $\hat{t}_{max}=0$. For the parton distribution functions $f_q(x,Q^2)$ we have used the CTEQ5 parametrization [76] providing the dependence on momentum transfer which have been taken as $Q=m_L$ and for illustration, values of $\kappa_\gamma=\kappa_Z=0.02$ have been taken for anomalous magnetic moment couplings.

We give the production cross sections for the signal as functions of the heavy lepton masses, m_L, in Fig. 4.4 at LHeC and THERA energies, namely $\sqrt{s}=1.4$ and $\sqrt{s}=1$ TeV, respectively. Since the non-diagonal axial and vector couplings a_L and v_L are additive with anomalous couplings κ_γ and κ_Z in the lagrangian (4.1), for compatible contributions, the values of $a_L=v_L=0.02$ were used in parallel with the conventional values of $\kappa_\gamma=\kappa_Z=0.02$ in numerical calculations. Furthermore, the values of $\Lambda=m_L$ were used for the scale parameter. With the integrated luminosity of 10^4 pb^{-1}, LHeC would yield substantial event rates compared with THERA ($L^{int.}=40$ pb^{-1}). Fig. 4.5 shows the p_T distributions of

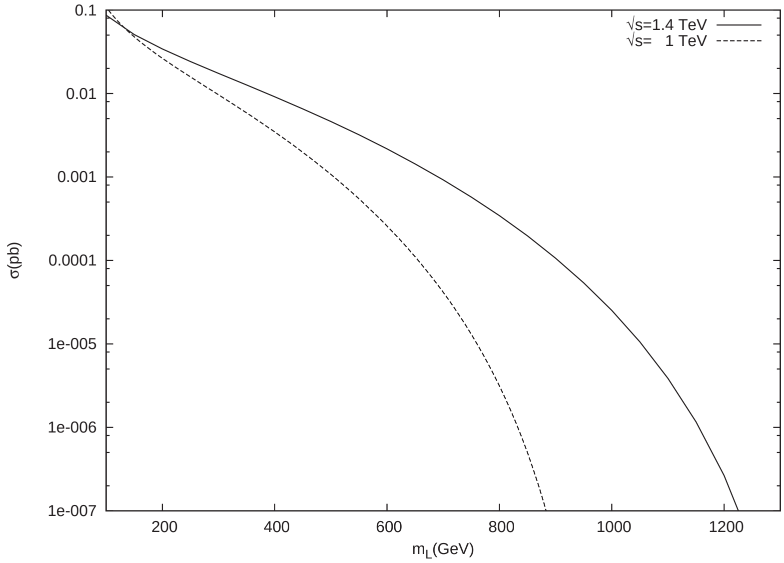

Figure 4.4: The total cross sections for the process $eq \to Lq$, as function of the heavy lepton masses. Solid line is for LHeC with $\sqrt{s}$ =1.4 TeV and $L^{int} = 10^4$pb^{-1}, dotted line is for THERA $\sqrt{s}$ =1 TeV and $L^{int} = 40$pb^{-1}.

the final state particles for the relevant background process $eq \to eZq$, related to the signal reaction $ep \to LqX$ in ep collisions. These p_T distributions have peaks around 50 GeV and suppressed at higher values. We applied an optimal cut of $|M_{Ze} - m_L| < 50$ GeV for the considered heavy lepton mass range, in order to contain all signal events that are smeared by the experimental resolution. Fig. 4.6 shows the invariant mass distributions of Ze system, after a cut of $p_T^{e,j} > 50$ GeV.

In Tables 4.2 and 4.3 we present signal cross sections in association with Fig. 4.4, background cross sections and corresponding statistical significance (SS) numbers for some mass values of heavy leptons in the ranges of heavy lepton masses. Table 4.3 shows that the LHeC provides clean signatures with quite small backgrounds for the heavy lepton signals. As usual, we use the criteria

$$\frac{\mathrm{S}}{\sqrt{\mathrm{S}+\mathrm{B}}} \geq 3, \tag{4.23}$$

as the limit of possible observability, where S and B are the numbers of signal and

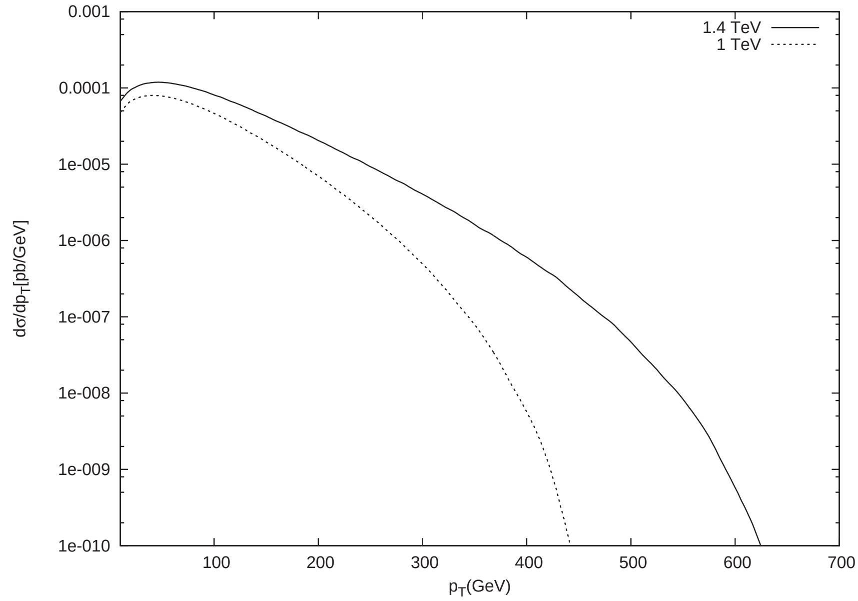

Figure 4.5: p_T distributions of the background process $eq \to eZq$ at LHeC (solid line) and THERA (dotted line).

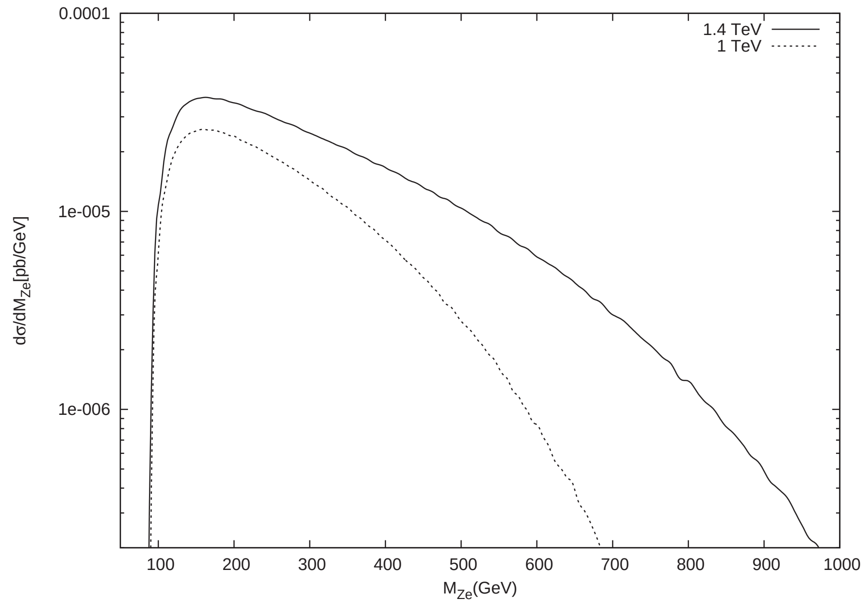

Figure 4.6: The invariant mass distributions of the Ze system for the background for lepton-hadron colliders LHeC and THERA.

m_L (GeV)	σ (pb)	σ_B(pb)	$S/\sqrt{S+B}$
100	1.08×10^{-1}	4.53×10^{-5}	2
150	4.77×10^{-2}	2.54×10^{-4}	1
200	2.64×10^{-2}	2.39×10^{-4}	1

Table 4.2: The signal and background cross sections and SS depending on the heavy lepton masses for THERA with $\sqrt{s} = 1$ TeV.

m_L (GeV)	σ (pb)	σ_B(pb)	$S/\sqrt{S+B}$
100	8.66×10^{-2}	2.01×10^{-5}	29
300	1.74×10^{-2}	6.40×10^{-5}	13
500	4.61×10^{-3}	2.91×10^{-5}	7
650	1.44×10^{-3}	1.37×10^{-5}	4

Table 4.3: The signal and background cross sections and SS depending on the heavy lepton masses for LHeC with $\sqrt{s} = 1.4$ TeV, $L^{int} = 10^4 \text{pb}^{-1}$.

background events, respectively. Hence, the expected number of signal events at LHeC are about 15 and 865 for 650 and 100 GeV heavy leptons respectively. On the other hand according to the conventional criteria of SS$\geq$3 THERA is not capable for searching heavy lepton signals due to its low luminosity.

In Figs. 4.7 (4.8) the cross sections are plotted as functions of anomalous coupling κ_γ (κ_Z) by taking κ_γ (κ_Z)=0.02 for various mass values.

4.5 Discussion

In conclusion, the present work gives an analysis of possible production of heavy leptons via anomalous interactions in *ep* collisions at future *ep* collider options THERA and LHeC. It is shown that, after kinematical cuts a statistical significance of SS ≥ 3 can be achieved for only LHeC. LHeC, with an integrated luminosity of 10^4 pb^{-1} yields 15 events per year for 650 GeV leptons, hence a good place to search for heavy leptons. On the other hand THERA seems capable for only about four 100 GeV leptons with SS$\simeq$ 2 which is not sufficient for observation. Therefore LHeC will play an important role in searching for new

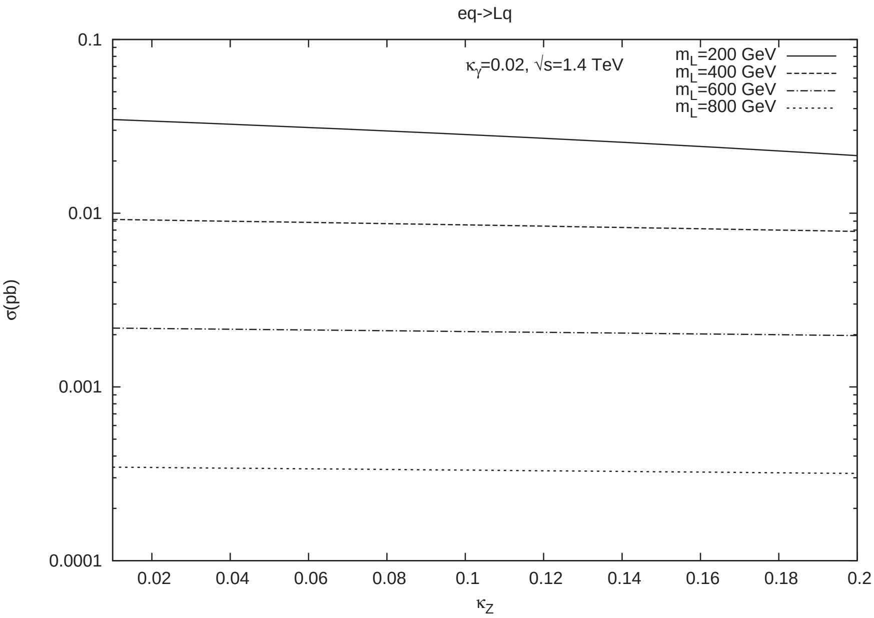

Figure 4.7: Total cross sections as functions of κ_Z ($\kappa_\gamma = 0.02$) for various masses of heavy leptons for the subprocess $eq \to Lq$.

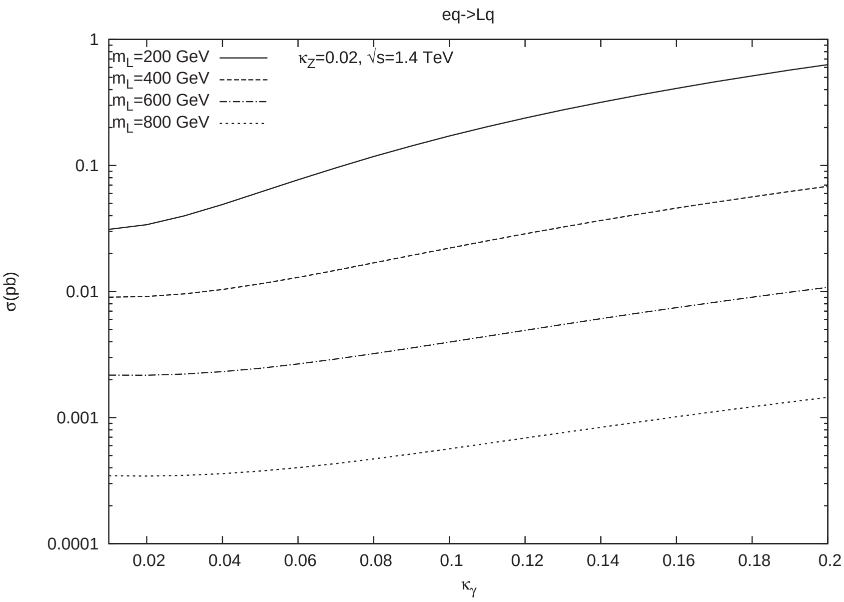

Figure 4.8: Total cross sections as functions of κ_γ ($\kappa_Z = 0.02$) for various masses of heavy leptons for the subprocess $eq \to Lq$.

physics beyond the SM through the anomalous interactions.

CHAPTER 5

SINGLE AND PAIR PRODUCTION OF HEAVY LEPTONS WITH ANOMALOUS COUPLINGS

In this chapter we study the single and pair production of heavy leptons via anomalous interactions at linear colliders ILC [51] and CLIC [52]. These linear colliders are excellent places to produce the heavy leptons with their energies and luminosities, which are given in Table 5.1

e^+e^- Colliders	E_{e^+} (TeV)	E_{e^-} (TeV)	$\sqrt{s_{e^+e^-}}$ (TeV)	$L^{int}_{e^+e^-}$ (pb^{-1})
ILC	0.25	0.25	0.5	10^5
ILC	0.5	0.5	1.0	10^5
CLIC	1.5	1.5	3.0	10^5

Table 5.1: The main parameters of the future e^-e^+ colliders, L^{int} denotes the integrated luminosity for one year.

5.1 Single Production of Heavy Leptons

The single production of heavy leptons L, in e^-e^+ collisions occur through the s and t channel, γ and Z exchange processes $e^-e^+ \to L^-e^+$ caused by the following FCNC Lagrangian describing interactions via anomalous magnetic transition moment type vertices;

$$\begin{aligned}\mathcal{L}_{\texttt{eff}} &= \frac{ie\kappa_\gamma}{\Lambda} L\sigma_{\mu\nu}q^\nu l A^\mu \\ &+ \frac{g}{2\cos\theta_W} L\left[\gamma_\mu(c_v - c_a\gamma_5) + \frac{i\sigma_{\mu\nu}q^\nu}{\Lambda}\kappa_Z\right] lZ^\mu + h.c.,\end{aligned} \tag{5.1}$$

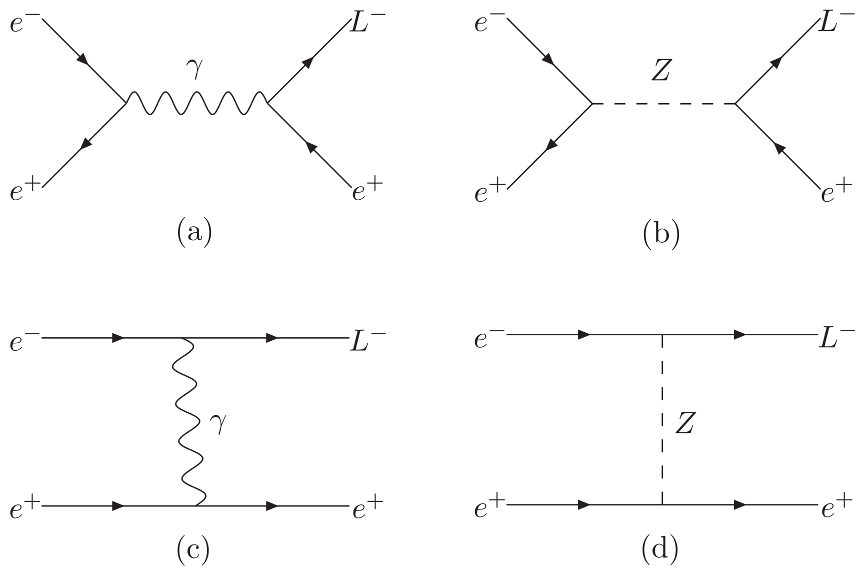

Figure 5.1: The Feynman diagrams of the s and t channel processes $e^-e^+ \to L^-e^+$.

where κ_γ and κ_Z are the anomalous magnetic dipole moment factors, q is the momentum of the exchanged gauge boson, θ_W is the Weinberg angle, e and g denote the gauge couplings relative to $U(1)$ and $SU(2)$ symmetries respectively, A^μ and Z^μ the fields of the photon and Z boson and Λ is the new physics scale. The Feynman diagrams of the s and t channel, γ and Z exchange processes $e^-(p_1)e^+(p_2) \to L^-(p_3)e^+(p_4)$ are given in Fig. 5.1. The total cross section is the sum of the cross sections of s and t channel, γ and Z exchange and their interference processes.

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} < |M_{tot.}|^2 > \tag{5.2}$$

$$\begin{aligned} |M_{tot.}|^2 &= |M_a|^2 + |M_b|^2 + |M_c|^2 + |M_d|^2 \\ &+ \overline{M}_a M_b + M_a \overline{M}_b + \overline{M}_a M_c + M_a \overline{M}_c \end{aligned}$$

$$+\overline{M}_a M_d + M_a \overline{M}_d + \overline{M}_b M_c + M_b \overline{M}_c$$
$$+\overline{M}_b M_d + M_b \overline{M}_d + \overline{M}_c M_d + M_c \overline{M}_d. \tag{5.3}$$

Details are given in Appendix B. The corresponding total cross section is obtained as follows;

$$\begin{aligned}
\frac{d\sigma}{dt} = & \frac{\pi\alpha^2}{8\Lambda^2 s^2 \sin^4\theta_W \cos^4\theta_W \Big[(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2\Big]\Big[(t-M_Z^2)^2 + M_Z^2\Gamma_Z^2\Big]} \\
& \times\Big\{2\Lambda\kappa_Z st(t-s)m_L\Big(v_L({a_l}^2+{a_l}^2)(s+t-2m_L^2) - 2a_l v_l(2m_L^2 - 3(s+t))\Big) \\
& -\kappa_Z^2({a_l}^2+{v_l}^2)st\Big((s+t)m_L^4 - 2(s^2+t^2)m_L^2 + 2(s^3+t^3)\Big) \\
& +\Lambda^2({a_l}^2+{v_l}^2)({a_L}^2+{v_L}^2)\Big(2(s^4-t^2s^2+t^4) - (s+t)(2s^2-3st+2t^2)m_L^2\Big) \\
& +4a_l v_l a_L v_L \Lambda^2 st\Big(m_L^2(s+t) - 2st\Big) + M_Z^2\Big[2\kappa_Z^2({a_l}^2+{v_l}^2) \\
& \times\Big(2m_L^4 - 2(s+t)m_L^2 + (s+t)^2\Big)st + 2\kappa_Z\Lambda(s-t)m_L \\
& \times\Big((s^2+t^2-(s+t)m_L^2\Big)({a_l}^2 v_L + 2a_l v_l a_L + {v_l}^2 v_L) + 8a_l v_l a_L st) \\
& -2\Lambda^2\Big(({a_l}^2+{v_l}^2)({a_L}^2+{v_L}^2) - 4a_l v_l a_L v_L\Big)\Big(s^3+t^3-(s^2+t^2)m_L^2\Big)\Big] \\
& -(M_Z^2+\Gamma_Z^2)\Big[\kappa_Z^2({a_l}^2+{v_l}^2)(s+t)\Big(m_L^4-(s+t)m_L^2+2st\Big) \\
& +\Lambda^2\Big(({a_l}^2+{v_l}^2)({a_L}^2+{v_L}^2) - 4a_l v_l a_L v_L\Big)\Big((s+t)m_L^2 - s^2 - t^2\Big)\Big]\Big\} \\
& -\frac{2\pi\alpha^2\kappa_\gamma^2}{\Lambda^2 s^3 t}\Big\{(s+t)m_L^4 - 2(s^2+t^2)m_L^2 + 2(s^3+t^3)\Big\} \\
& +\frac{\pi\alpha^2\kappa_\gamma M_Z\Gamma_Z}{\Lambda^2 s^2\sin^2\theta_W\cos^2\theta_W\Big[(t-M_Z^2)^2+M_Z^2\Gamma_Z^2\Big]} \\
& \times\Big\{\kappa_Z v_l m_L^4 - 2\Lambda(a_l a_L + v_l v_L)m_L^3 - 2\kappa_Z v_l m_L^2 s \\
& +\Lambda\Big(a_l a_L(3s+2t) + v_l v_L(s+2t)\Big)m_L + \kappa_Z v_l(2s-t)(s+t)\Big\} \\
& +\frac{\pi\alpha^2\kappa_\gamma M_Z\Gamma_Z}{\Lambda^2 s^2\sin^2\theta_W\cos^2\theta_W\Big[(s-M_Z^2)^2+M_Z^2\Gamma_Z^2\Big]} \\
& \times\Big\{\kappa_Z v_l\Big(m_L^4 - 2tm_L^2 - (s-2t)(s+t)\Big)
\end{aligned}$$

$$+\Lambda\Big((a_l a_L + v_l v_L)(2m_L^2 - 2s - t) - 2a_l a_L t\Big)m_L\Big\}. \tag{5.4}$$

5.1.1 Numerical Results

In Fig. 5.2, we plot the corresponding cross sections for the single productions at ILC and CLIC energies with $\sqrt{s} = 0.5$ and 1 TeV, and $\sqrt{s} = 3$ TeV, respectively. In plotting this figure we have used $\kappa_\gamma = \kappa_Z = 0.02$ and $a_L = v_L = 0.02$ for the coupling parameters. Figs. 5.3, 5.4 and 5.5 shows the p_T distributions of the final state particles at these linear colliders. The characters of p_T distributions at these linear colliders are similar to that of ep colliders in the case of the same cut, $|M_{Ze} - m_L| < 50$ GeV imposed. In Figs. 5.6, 5.7 and 5.8, we give the invariant mass distributions M_{Ze}, with cut $p_T^{e,j} > 50$ GeV at $\sqrt{s}$ =0.5, 1 and 3 TeV, respectively.

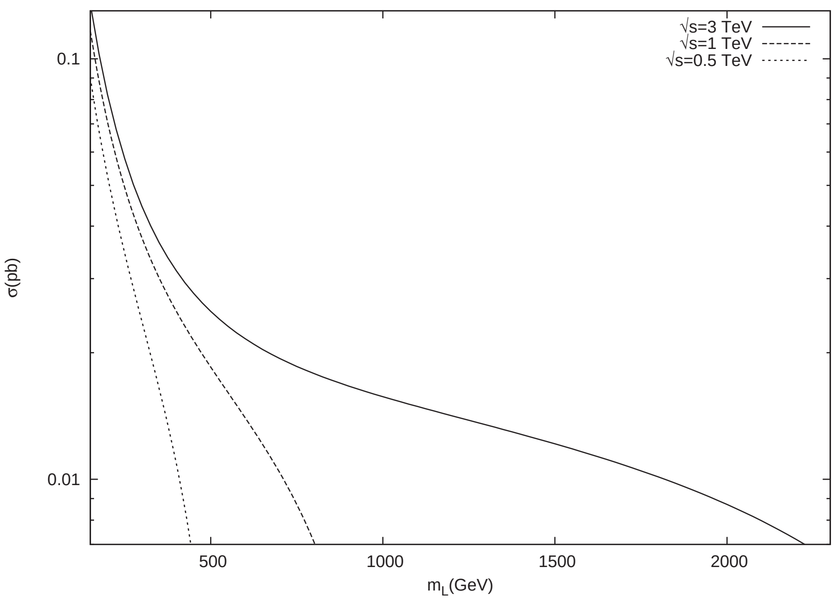

Figure 5.2: Cross sections for the process $e^-e^+ \rightarrow L^-e^+$, as function of the heavy lepton masses, for linear colliders ILC ($\sqrt{s}$ =0.5 and 1 TeV, $L^{int} = 10^5\text{pb}^{-1}$) and CLIC ($\sqrt{s}$ =3 TeV, $L^{int} = 10^5\text{pb}^{-1}$) for single production of heavy lepton. Solid line is at CLIC with $\sqrt{s}$ =3 TeV, dashed line is at CLIC with $\sqrt{s}$ =1 TeV and dotted line is at ILC.

m_L (GeV)	σ (pb)	σ_B(pb)	$S/\sqrt{S+B}$
150	9.02×10^{-2}	7.74×10^{-3}	91
250	3.44×10^{-2}	3.29×10^{-3}	56
350	1.64×10^{-2}	1.14×10^{-3}	39
450	6.30×10^{-3}	7.71×10^{-6}	25

Table 5.2: The signal and background cross sections and SS depending on the heavy lepton masses at $\sqrt{s} = 0.5$ TeV, $L^{int} = 10^5\text{pb}^{-1}$ for single production in e^+e^- collisions.

m_L (GeV)	σ (pb)	σ_B(pb)	$S/\sqrt{S+B}$
100	2.47×10^{-1}	1.43×10^{-2}	153
300	3.75×10^{-2}	4.60×10^{-3}	58
600	1.40×10^{-2}	1.64×10^{-3}	35
900	3.90×10^{-3}	1.81×10^{-4}	19

Table 5.3: The signal and background cross sections and SS depending on the heavy lepton masses at $\sqrt{s} = 1$ TeV, $L^{int} = 10^5\text{pb}^{-1}$ for single production in e^+e^- collisions.

m_L (GeV)	σ (pb)	σ_B(pb)	$S/\sqrt{S+B}$
250	5.78×10^{-2}	6.37×10^{-3}	72
750	1.86×10^{-2}	3.55×10^{-3}	39
1250	1.38×10^{-2}	3.04×10^{-3}	34
1750	1.05×10^{-2}	2.44×10^{-3}	29

Table 5.4: The signal and background cross sections and SS depending on the heavy lepton masses at $\sqrt{s} = 3$ TeV, $L^{int} = 10^5\text{pb}^{-1}$ for single production in e^+e^- collisions.

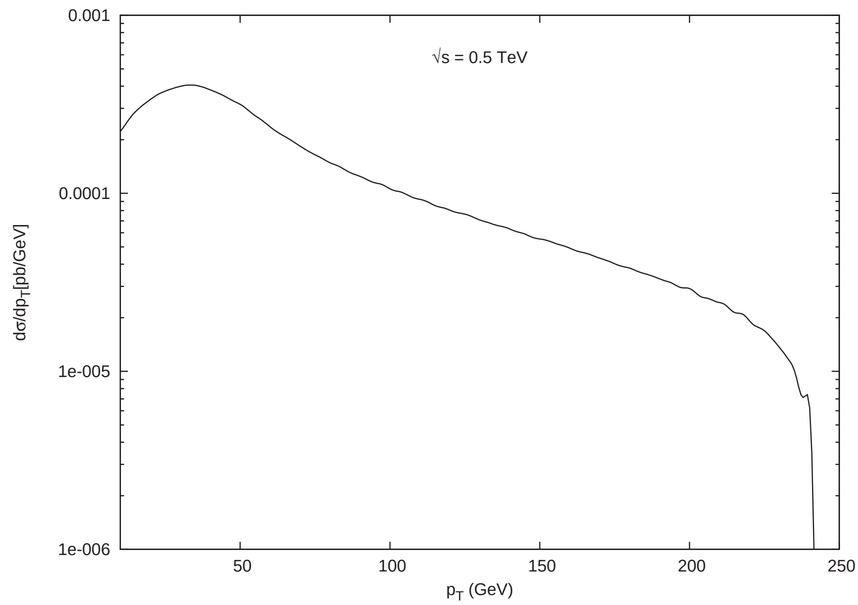

Figure 5.3: p_T distribution of the background process $e^-e^+ \to e^-Ze^+$ for ILC ($\sqrt{s} = 0.5$ TeV) for single production.

In Tables 5.2, 5.3 and 5.4, we presented the signal and total background cross sections for the single production of heavy leptons depending on their masses, for 0.5, 1 and 3 TeV center of mass energies, respectively. As seen from these tables, the SS values are high enough for possible observation of these leptons at linear colliders, even for mass values up to the center of mass energies of these collider options. The expected number of events per year are about 630 and 390 for 450 and 900 GeV leptons at ILC with $\sqrt{s} = 0.5$ TeV and $\sqrt{s} = 1$ TeV, respectively. At CLIC ($\sqrt{s} = 3$ TeV), this number is about 1050 events per year for 1750 GeV leptons.

In Figs. 5.9, 5.10 and 5.11 the cross sections are plotted as functions of anomalous coupling κ_Z by taking κ_γ=0.02 for various mass values for 0.5, 1 and 3 TeV center of mass energies, respectively. In Figs. 5.12, 5.13 and 5.14 the cross sections are plotted as functions of anomalous coupling κ_Z by taking κ_γ=0.02 for various mass values of heavy leptons for 0.5, 1 and 3 TeV center of mass energies, respectively.

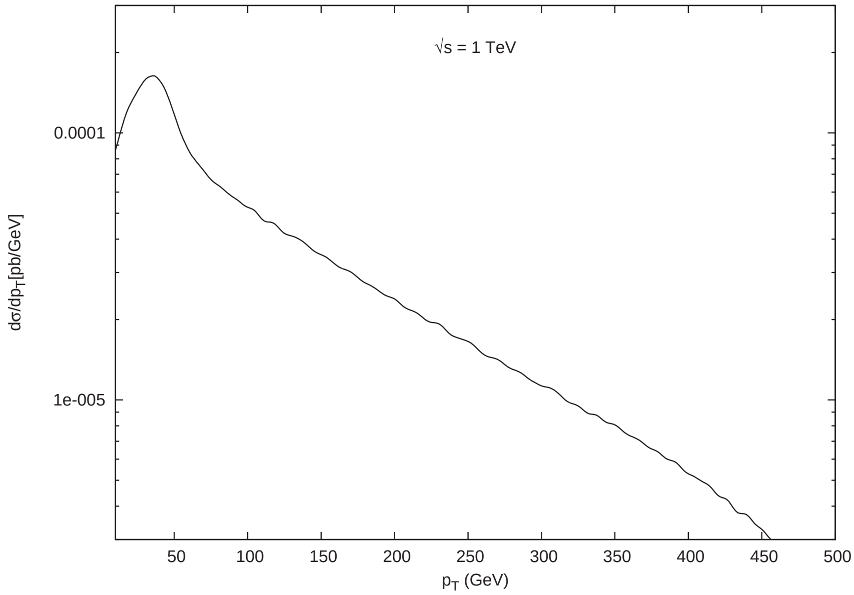

Figure 5.4: p_T distribution of the background process $e^- e^+ \to e^- Z e^+$ for ILC ($\sqrt{s} = 1$ TeV) for single production.

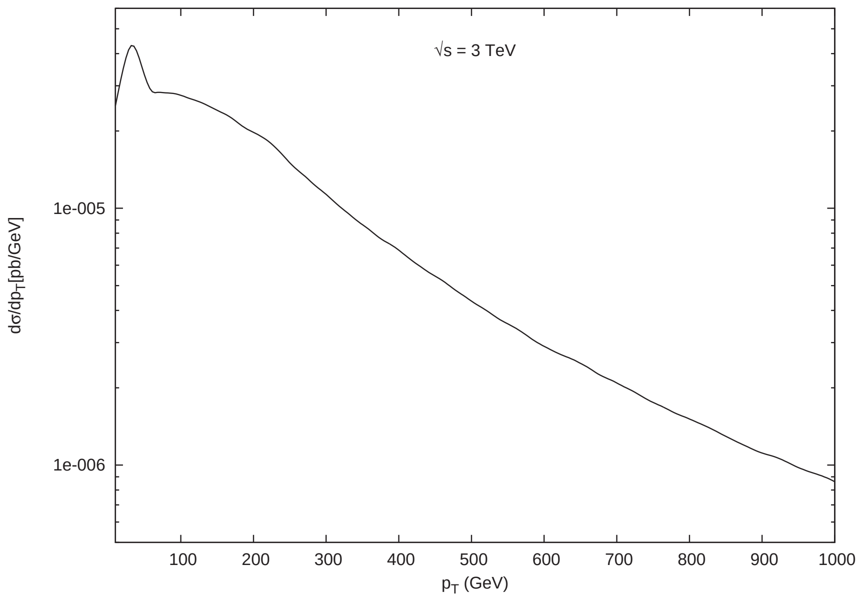

Figure 5.5: p_T distribution of the background process $e^- e^+ \to e^- Z e^+$ for CLIC ($\sqrt{s} = 3$ TeV) for single production.

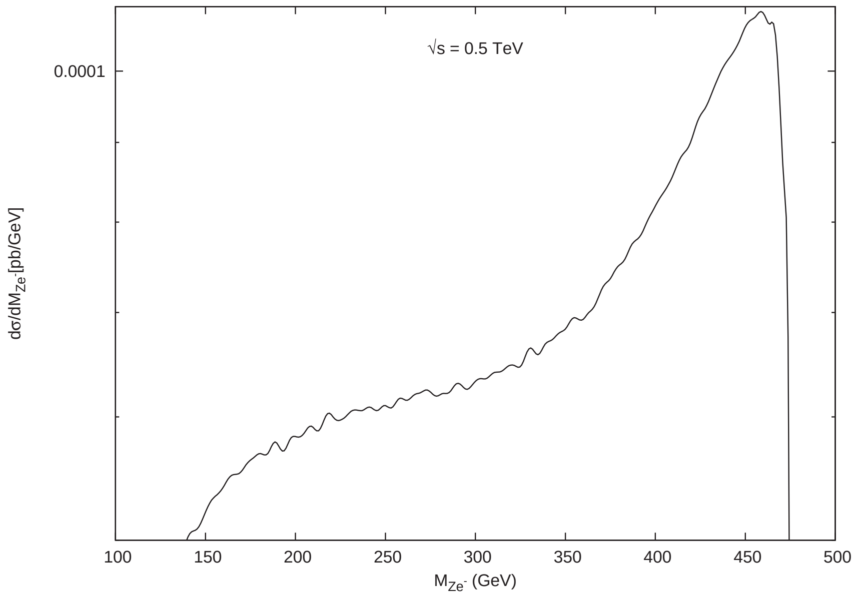

Figure 5.6: The invariant mass distribution of the Ze system for the background for ILC ($\sqrt{s} = 0.5$ TeV) for single production.

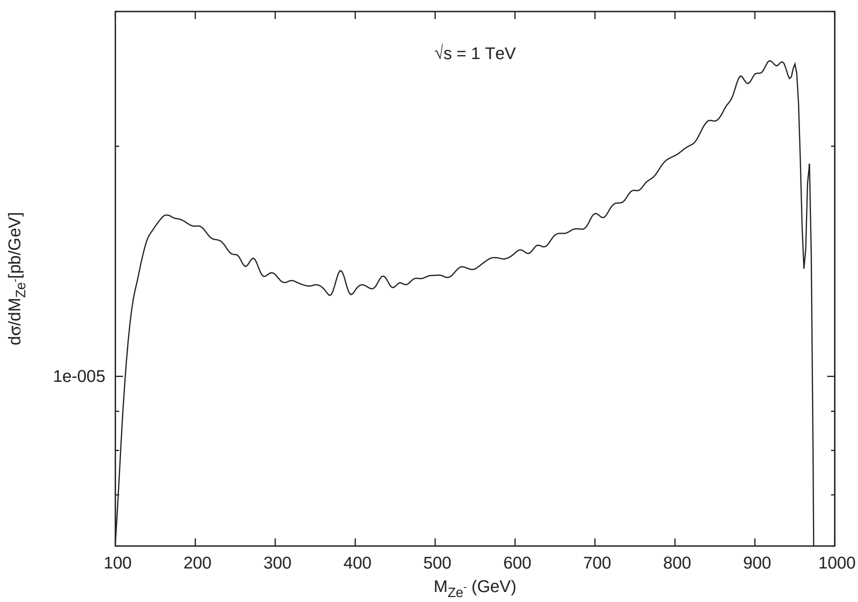

Figure 5.7: The invariant mass distribution of the Ze system for the background for ILC ($\sqrt{s} = 1$ TeV) for single production.

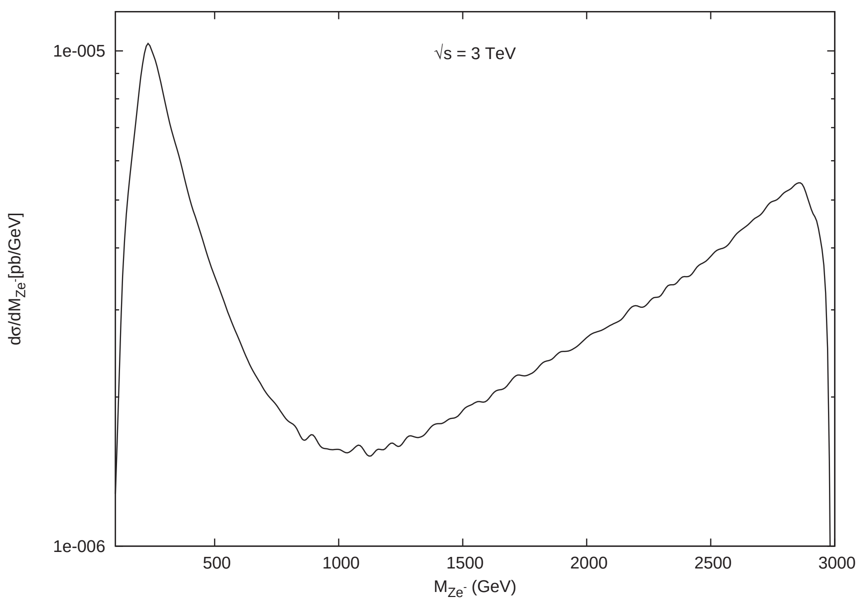

Figure 5.8: The invariant mass distribution of the Ze system for the background for CLIC ($\sqrt{s} = 3$ TeV) for single production.

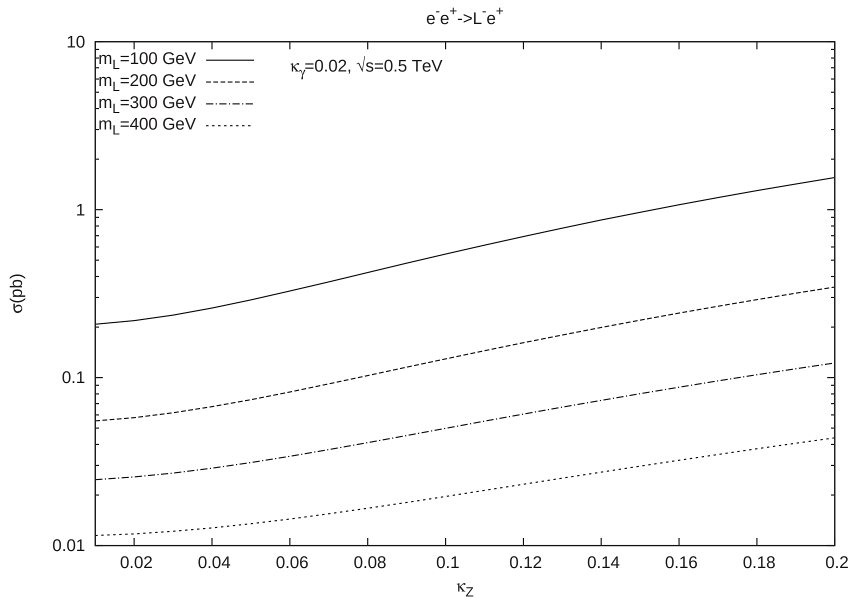

Figure 5.9: Total cross sections as functions of κ_Z ($\kappa_\gamma = 0.02$) for various masses of heavy leptons for the reaction $e^+e^- \to L^-e^+$.

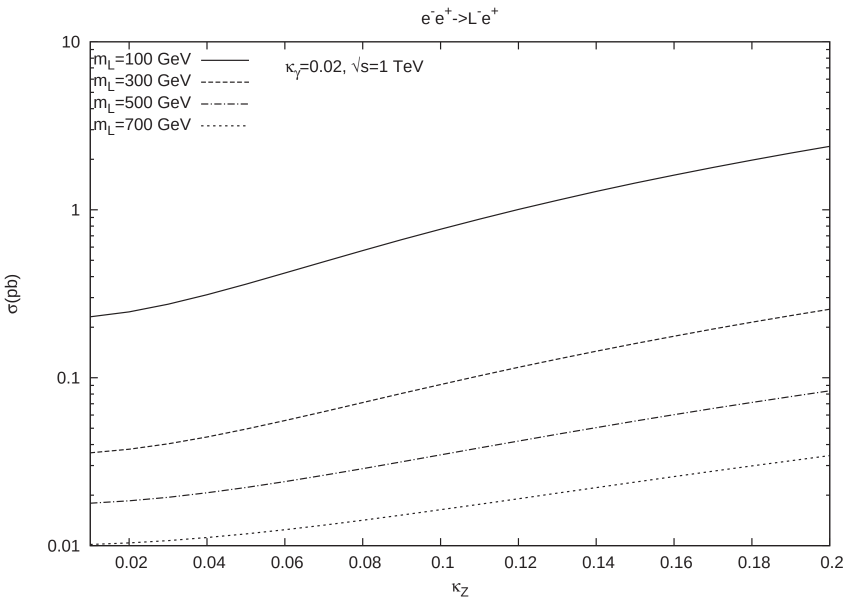

Figure 5.10: Total cross sections as functions of κ_Z ($\kappa_\gamma = 0.02$) for various masses of heavy leptons for the reaction $e^+e^- \to L^-e^+$.

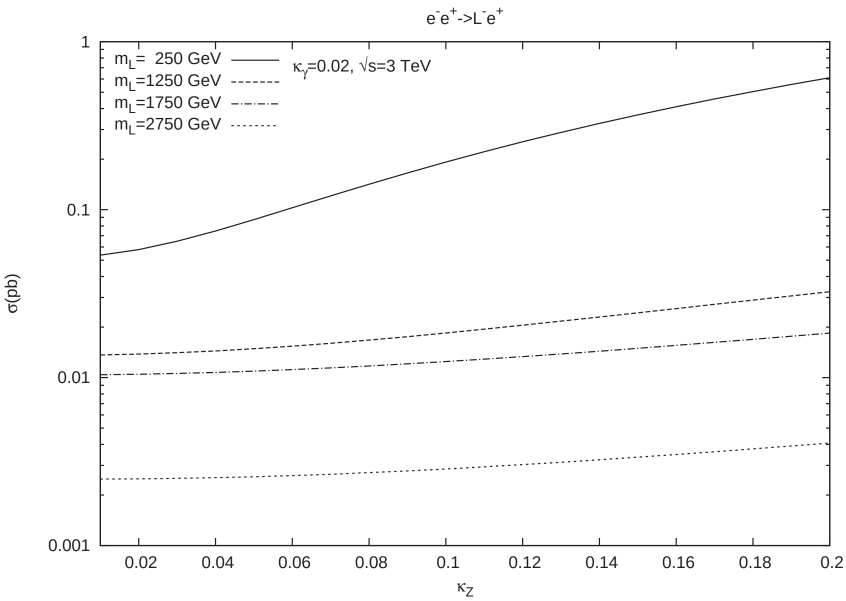

Figure 5.11: Total cross sections as functions of κ_Z ($\kappa_\gamma = 0.02$) for various masses of heavy leptons for the reaction $e^+e^- \to L^-e^+$.

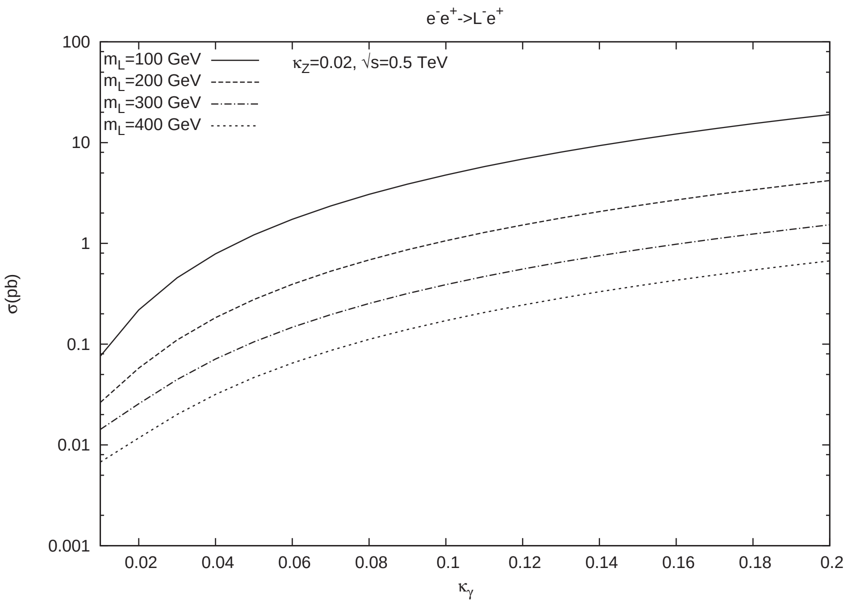

Figure 5.12: Total cross sections as functions of κ_γ ($\kappa_Z = 0.02$) for various masses of heavy leptons for the reaction $e^+e^- \to L^-e^+$.

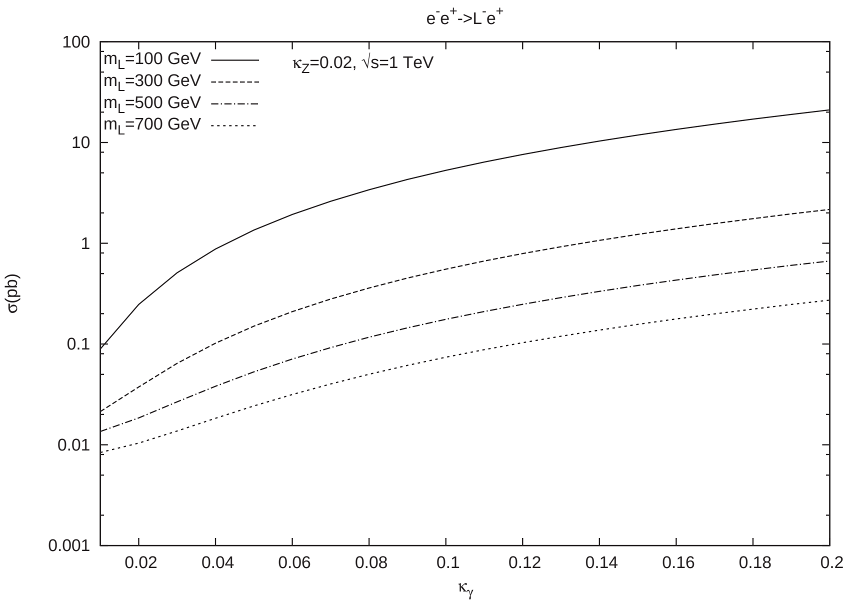

Figure 5.13: Total cross sections as functions of κ_γ ($\kappa_Z = 0.02$) for various masses of heavy leptons for the reaction $e^+e^- \to L^-e^+$.

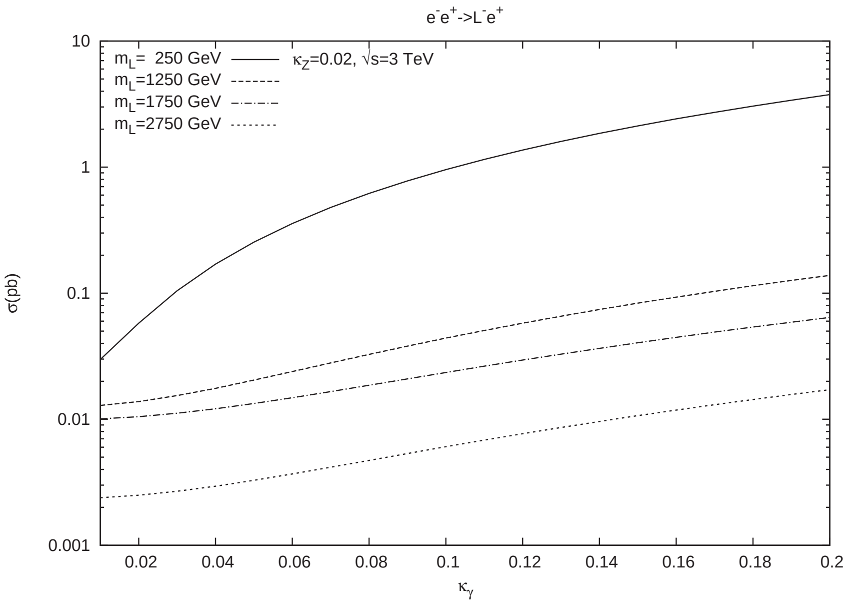

Figure 5.14: Total cross sections as functions of κ_γ ($\kappa_Z = 0.02$) for various masses of heavy leptons for the reaction $e^+e^- \to L^-e^+$.

5.2 Pair Production of Heavy Leptons

Pair production of heavy leptons with anomalous couplings occur through the t-channel flavor changing neutral current process $e^-e^+ \to L^-L^+$. The Feynman diagram of the t channel process $e^-e^+ \to L^-L^+$ is given in Fig. 5.15.

5.2.1 Definition of Differential Cross Section

Considering the FCNC Lagrangian in Eq. (5.1), we find the amplitudes for the γ exchange in the t channel as,

$$\mathcal{M}_\gamma = -\frac{e^2\kappa_\gamma^2}{\Lambda^2 t}\Big[\bar{u}(p_3)\sigma^{\mu\lambda}q_\lambda u(p_1)\Big]\Big[\bar{v}(p_2)\sigma_\mu^\rho q_\rho v(p_4)\Big] \tag{5.5}$$

and

$$\overline{\mathcal{M}}_\gamma = -\frac{e^2\kappa_\gamma^2}{\Lambda^2 t}\Big[\bar{u}(p_1)\sigma^{\alpha\omega}q_\omega u(p_3)\Big]\Big[\bar{v}(p_4)\sigma_\alpha^\delta q_\delta v(p_2)\Big] \tag{5.6}$$

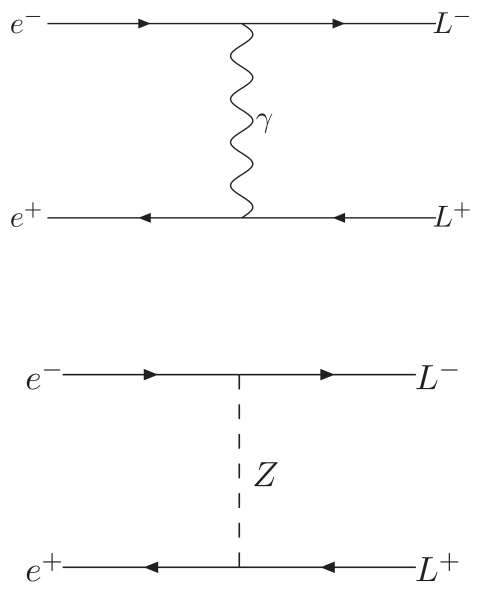

Figure 5.15: The Feynman diagrams of the t channel γ and Z exchange process $e^-e^+ \to L^-L^+$.

ignoring the quark and electron masses we find,

$$\langle|\mathcal{M}_\gamma|^2\rangle = \frac{e^4\kappa_\gamma^4}{4\Lambda^4 t^2} \times Tr\Big[(\not p_3 + m_L)\sigma^{\mu\lambda}q_\lambda \not p_1 \sigma^{\alpha\omega}q_\omega\Big] \times Tr\Big[\not p_2 \sigma_\mu^\rho q_\rho (\not p_4 - m_L)\sigma_\alpha^\delta q_\delta\Big] \quad (5.7)$$

and we find the amplitudes for the Z exchange in the t channel as,

$$\mathcal{M}_Z = \frac{g_Z^2}{4}\frac{(g_{\mu\nu} - q_\mu q_\nu/M_Z^2)}{(t - M_Z^2) + iM_Z\Gamma_Z} \times\Big[\bar{u}(p_3)\Big(\gamma^\mu(v_L - a_L\gamma^5) + \frac{i\kappa_Z}{\Lambda}\sigma^{\mu\lambda}q_\lambda\Big)u(p_1)\Big] \times\Big[\bar{v}(p_2)\Big(\gamma^\nu(v_L - a_L\gamma^5) + \frac{i\kappa_Z}{\Lambda}\sigma^{\nu\rho}q_\rho\Big)v(p_4)\Big], \quad (5.8)$$

and

$$\overline{\mathcal{M}}_Z = \frac{g_Z^2}{4}\frac{(g_{\alpha\beta} - q_\alpha q_\beta/M_Z^2)}{(t - M_Z^2) - iM_Z\Gamma_Z}$$

$$\times\Big[\bar{u}(p_1)\Big(\gamma^\alpha(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\alpha\omega}q_\omega\Big)u(p_3)\Big]$$
$$\times\Big[\bar{v}(p_4)\Big(\gamma^\beta(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\beta\delta}q_\delta\Big)v(p_2)\Big], \quad (5.9)$$

which gives,

$$\begin{aligned}
\langle|\mathcal{M}_Z|^2\rangle &= \frac{g_Z^4}{64\Big[(t-M_Z^2)^2 + M_Z^2\Gamma_Z^2\Big]}\Big(g_{\mu\nu} - q_\mu q_\nu/M_Z^2\Big)\Big(g_{\alpha\beta} - q_\alpha q_\beta/M_Z^2\Big) \\
&\times Tr\Big[(\not{p}_3 + m_L)\Big(\gamma^\mu(v_L - a_L\gamma^5) + \frac{i\kappa_Z}{\Lambda}\sigma^{\mu\lambda}q_\lambda\Big) \\
&\times \not{p}_1\Big(\gamma^\alpha(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\alpha\omega}q_\omega\Big)\Big] \\
&\times Tr\Big[\not{p}_2\Big(\gamma^\nu(v_L - a_L\gamma^5) + \frac{i\kappa_Z}{\Lambda}\sigma^{\nu\rho}q_\rho\Big) \\
&\times(\not{p}_4 - m_L)\Big(\gamma^\beta(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\beta\delta}q_\delta\Big)\Big] \quad (5.10)
\end{aligned}$$

The interference terms are given by,

$$\begin{aligned}
\langle\mathcal{M}_\gamma\overline{\mathcal{M}}_Z\rangle &= \Big\{-\frac{e^2\kappa_\gamma^2 g_Z^2}{16\Lambda^2 t}\Big\}\Big[\frac{g_{\alpha\beta} - q_\alpha q_\beta/M_Z^2}{(t-M_Z^2) - iM_Z\Gamma_Z}\Big] \\
&\times Tr\Big[(\not{p}_3 + m_L)\sigma^{\mu\lambda}q_\lambda\ \not{p}_1\Big(\gamma^\alpha(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\alpha\omega}q_\omega\Big)\Big] \\
&\times Tr\Big[\ \not{p}_2\sigma^\rho_\mu q_\rho(\not{p}_4 - m_L)\Big(\gamma^\beta(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\beta\delta}q_\delta\Big)\Big] \quad (5.11)
\end{aligned}$$

and

$$\begin{aligned}
\langle\overline{\mathcal{M}}_\gamma\mathcal{M}_Z\rangle &= \Big\{-\frac{e^2\kappa_\gamma^2 g_Z^2}{16\Lambda^2 t}\Big\}\Big[\frac{g_{\mu\nu} - q_\mu q_\nu/M_Z^2}{(t-M_Z^2) + iM_Z\Gamma_Z}\Big] \\
&\times Tr\Big[\ \not{p}_1\sigma^{\alpha\omega}q_\omega(\not{p}_3 + m_L)\Big(\gamma^\mu(v_L - a_L\gamma^5) + \frac{i\kappa_Z}{\Lambda}\sigma^{\mu\lambda}q_\lambda\Big)\Big] \\
&\times Tr\Big[(\not{p}_4 - m_L)\sigma^\delta_\alpha q_\delta\ \not{p}_2\Big(\gamma^\nu(v_L - a_L\gamma^5) + \frac{i\kappa_Z}{\Lambda}\sigma^{\nu\rho}q_\rho\Big)\Big] \quad (5.12)
\end{aligned}$$

The total differential cross section for the process $e^-e^+ \to L^-L^+$ is obtained as

$$\frac{d\sigma_{tot.}}{dt} = \frac{1}{16\pi s^2}\Big[\langle|\mathcal{M}_\gamma|^2\rangle + \langle|\mathcal{M}_Z|^2\rangle + \langle\mathcal{M}_\gamma\overline{\mathcal{M}}_Z\rangle + \langle\overline{\mathcal{M}}_\gamma\mathcal{M}_Z\rangle\Big] \quad (5.13)$$

using the trace theorems and the Mandelstam variables in Eq. (2.7), the differential cross section for the subprocess $e^-e^+ \to L^-L^+$, mediated by γ and Z in the t channel takes the form,

$$\begin{aligned}
\frac{d\sigma}{dt} = & \frac{\pi\alpha^2\kappa_\gamma^2}{\Lambda^4 s^2 t^2}\Big\{2m_L^8 - 4tm_L^6 + t(4s+3t)m_L^4 - 2t^2(4s+t)m_L^2 + t^2(2s+t)^2\Big\} \\
& + \frac{\pi\alpha^2}{16M_Z^4\Lambda^4 s^2 \sin^4\theta_W \cos^4\theta_W\Big[(t-M_Z^2)^2 + M_Z^2\Gamma_Z^2\Big]} \\
& \times\Big\{\Lambda^4({a_L}^2 + {v_L}^2)m_L^4\Big((m_L^2 - t)^2 + 4M_Z^2 s\Big) \\
& -2\Lambda^2 M_Z^2\kappa_Z^2 {v_L}^2 m_L^2 t[m_L^4 + t(t+2s-2m_L^2)] \\
& +M_Z^4\Big[2\Lambda^4\Big(({a_L}^4 + {v_L}^4)\Big(m_L^4 - 2(2s+t)m_L^2 + 2s^2 + t^2 + 2st\Big) \\
& +2{a_L}^2{v_L}^2\Big(3m_L^4 - 2(2s+3t)m_L^2 + 2s^2 + 3t^2 + 6st\Big)\Big) \\
& -2\kappa_Z^2\Lambda^2\Big(4({a_L}^2 + {v_L}^2)s(m_L^4 + st + t^2) + ({a_L}^2 + 2{v_L}^2) \\
& \times(m_L^6 - 2tm_L^4 + t^2m_L^2) - 10{v_L}^2 tsm_L^2\Big) \\
& +\kappa_Z^2\Big(2m_L^8 - 4tm_L^6 + tm_L^4(4s+3t) - 2t^2(4s+t)m_L^2 + t^2(2s+t)^2\Big)\Big]\Big\} \\
& + \frac{\pi\alpha^2\kappa_\gamma^2(t-M_Z^2)}{2M_Z^2\Lambda^4 s^2 t\sin^2\theta_W\cos^2\theta_W\Big[(t-M_Z^2)^2 + M_Z^2\Gamma_Z^2\Big]} \\
& \times\Big\{\kappa_Z^2 M_Z^2\Big(2m_L^8 - 4tm_L^6 + t(4s+3t)m_L^4 - 2t^2(4s+t)m_L^2 \\
& +t^2(2s+t)^2\Big) - 2\Lambda^2 M_Z^2 m_L^2\Big(({a_L}^2 + {v_L}^2)(m_L^4 + t^2 - 2tm_L^2) \\
& +st(2{a_L}^2 - {v_L}^2)\Big) - {v_L}^2\Lambda^2 tm_L^2\Big(m_L^4 - 2tm_L^2 + (2s+t)t\Big)\Big\} \quad (5.14)
\end{aligned}$$

m_L (GeV)	σ (pb)	σ_B(pb)	$N_S/\sqrt{N_S+N_B}$
100	2.99×10^{-4}	4.61×10^{-8}	5
125	1.39×10^{-4}	1.50×10^{-7}	4
150	7.93×10^{-5}	1.78×10^{-7}	3

Table 5.5: The signal cross sections for the process $e^-e^+ \rightarrow L^-L^+$, total background cross sections for the process $e^-e^+ \rightarrow e^-Ze^+Z$ and SS depending on the heavy lepton masses at $\sqrt{s} = 0.5$ TeV, $L^{int} = 10^5\text{pb}^{-1}$ for pair production in e^+e^- collisions.

5.2.2 Numerical Results

The total cross sections as functions of heavy lepton masses m_L, are displayed in Fig. 5.16. Signal and background cross sections depending on the heavy lepton

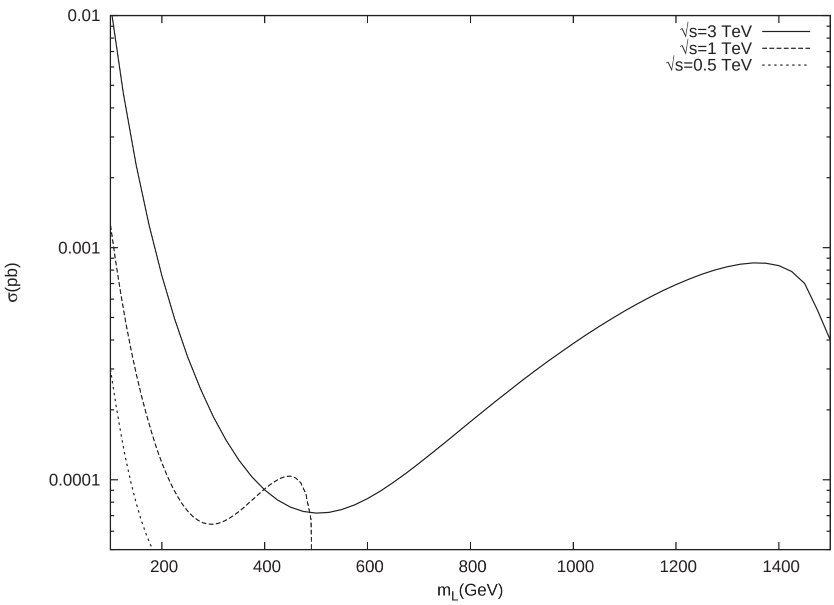

Figure 5.16: The total cross sections for the process $e^-e^+ \rightarrow L^-L^+$, as function of the heavy lepton masses, for linear colliders ILC ($\sqrt{s}$ =0.5 TeV, $L^{int} = 10^5\text{pb}^{-1}$) and CLIC ($\sqrt{s}$ =1 and 3 TeV, $L^{int} = 10^5\text{pb}^{-1}$) for pair production of heavy lepton.

masses, are presented in Tables 5.5, 5.6 and 5.7 at 0.5, 1 and 3 TeV, respectively.

We applied an initial cut on the electron and jet transverse momentum $p_T^{e,j} >$ 50 GeV for the background analysis. Figs. 5.17, 5.18 and 5.19 shows the p_T

m_L (GeV)	σ (pb)	σ_B(pb)	$N_S/\sqrt{N_S+N_B}$
100	1.25×10^{-3}	3.35×10^{-7}	11
140	3.63×10^{-4}	1.52×10^{-6}	6
170	1.90×10^{-4}	1.78×10^{-6}	4
200	1.18×10^{-4}	1.89×10^{-6}	3

Table 5.6: The signal cross sections for the process $e^-e^+ \to L^-L^+$, total background cross sections for the process $e^-e^+ \to e^-Ze^+Z$ and SS depending on the heavy lepton masses at $\sqrt{s} = 1$ TeV, $L^{int} = 10^5\text{pb}^{-1}$ for pair production in e^+e^- collisions.

m_L (GeV)	σ (pb)	σ_B(pb)	$N_S/\sqrt{N_S+N_B}$
100	1.12×10^{-2}	4.85×10^{-6}	33
150	2.27×10^{-3}	3.56×10^{-5}	15
250	3.39×10^{-4}	5.54×10^{-5}	5
350	1.21×10^{-4}	7.88×10^{-5}	3

Table 5.7: The signal cross sections for the process $e^-e^+ \to L^-L^+$, total background cross sections for the process $e^-e^+ \to e^-Ze^+Z$ and SS depending on the heavy lepton masses at $\sqrt{s} = 3$ TeV, $L^{int} = 10^5\text{pb}^{-1}$ for pair production in e^+e^- collisions.

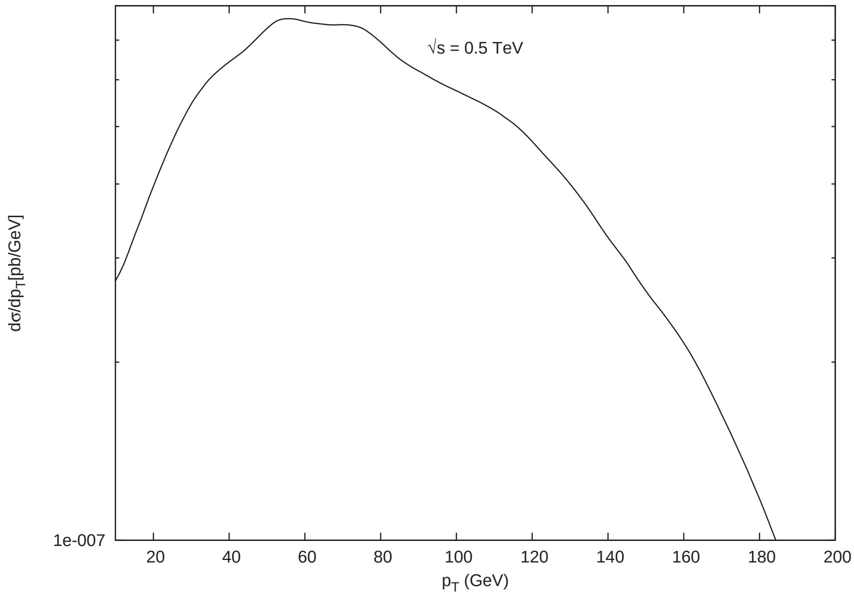

Figure 5.17: p_T distribution of the background process $e^-e^+ \to ZZe^-e^+$ for ILC ($\sqrt{s} = 0.5$ TeV) for pair production.

distributions of the final state particles at the colliders. The distribution of invariant mass M_{Ze} is presented in Figs. 5.20, 5.21 and 5.22 at $\sqrt{s}$ =0.5, 1 and 3, respectively.

In Figs. 5.23, 5.24 and 5.25 the cross sections are plotted as functions of anomalous coupling κ_γ by taking κ_γ=0.02 for various mass values for 0.5, 1 and 3 TeV center of mass energies, respectively. In Figs. 5.26, 5.13 and 5.28 the cross sections are plotted as functions of anomalous coupling κ_Z by taking κ_Z=0.02 for various mass values of heavy leptons for 0.5, 1 and 3 TeV center of mass energies, respectively.

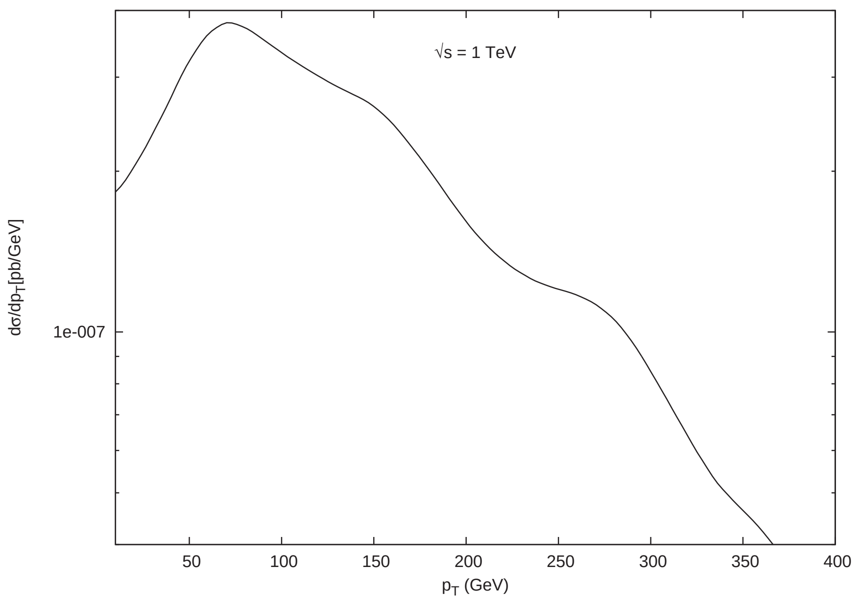

Figure 5.18: p_T distribution of the background process $e^-e^+ \to ZZe^-e^+$ for ILC ($\sqrt{s} = 1$ TeV) for pair production.

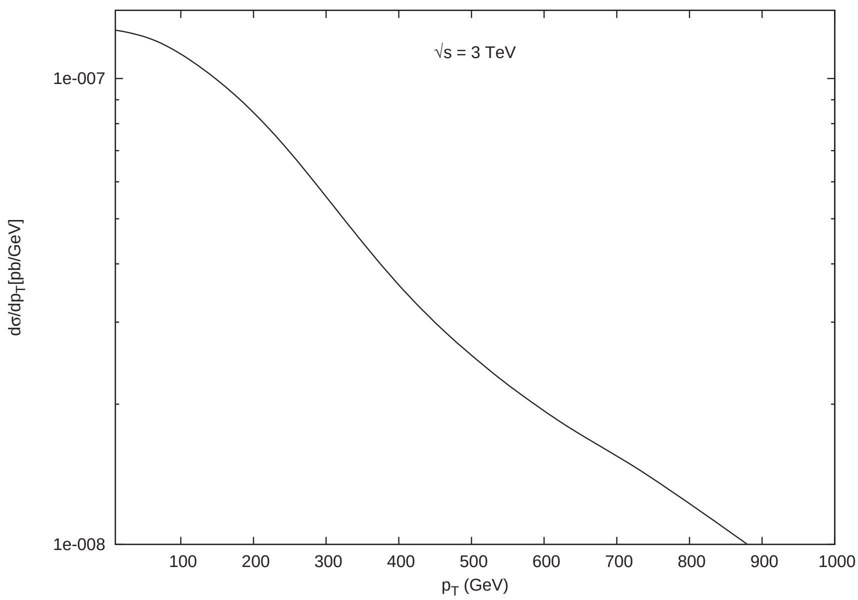

Figure 5.19: p_T distribution of the background process $e^-e^+ \to ZZe^-e^+$ for CLIC ($\sqrt{s} = 3$ TeV) for pair production.

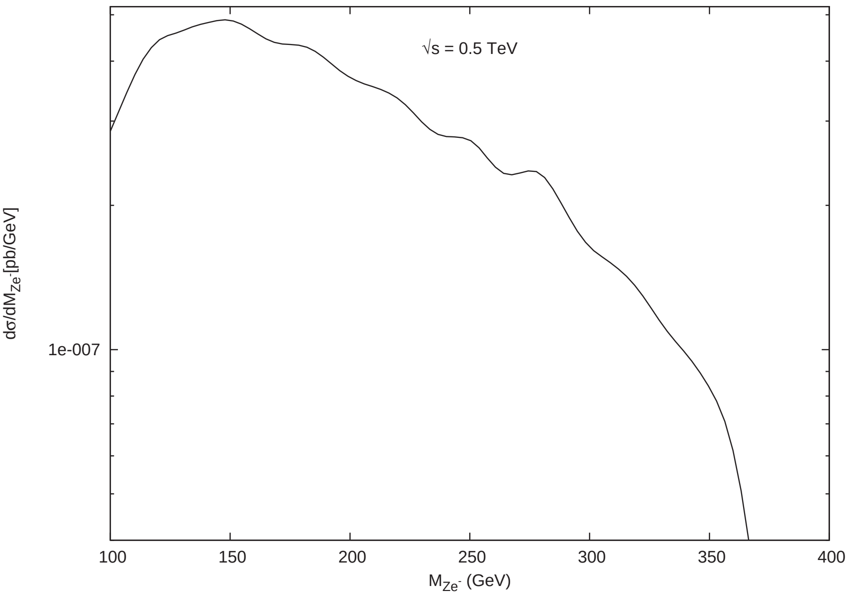

Figure 5.20: The invariant mass distribution of the Ze system for the background background process $e^-e^+ \to ZZe^-e^+$ for ILC ($\sqrt{s} = 0.5$ TeV) for pair production.

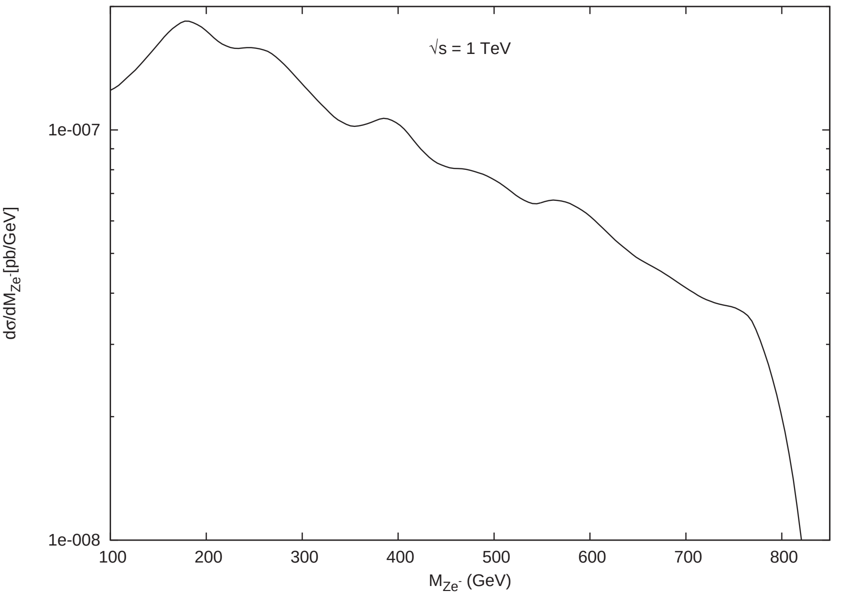

Figure 5.21: The invariant mass distribution of the Ze system for the background background process $e^-e^+ \to ZZe^-e^+$ for ILC ($\sqrt{s} = 1$ TeV) for pair production.

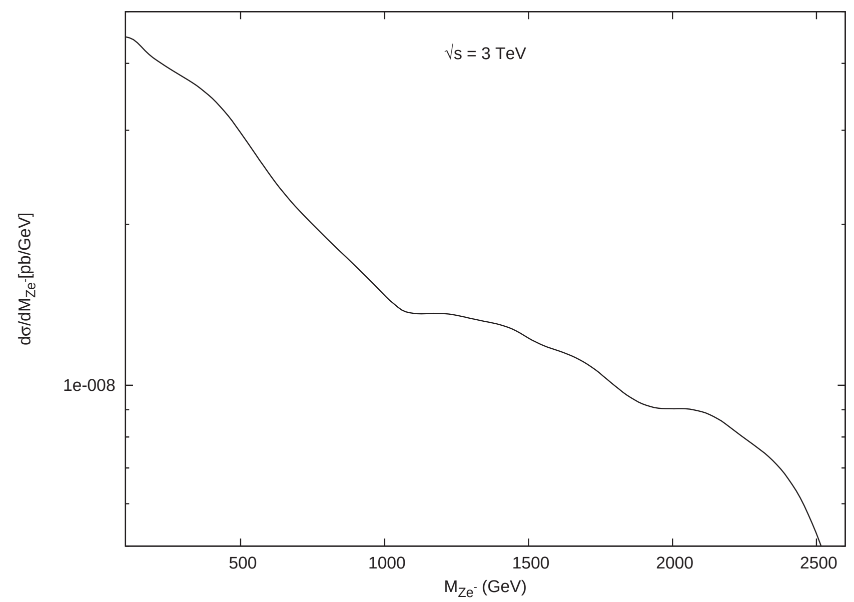

Figure 5.22: The invariant mass distribution of the Ze system for the background background process $e^-e^+ \to ZZe^-e^+$ for CLIC($\sqrt{s} = 3$ TeV) for pair production.

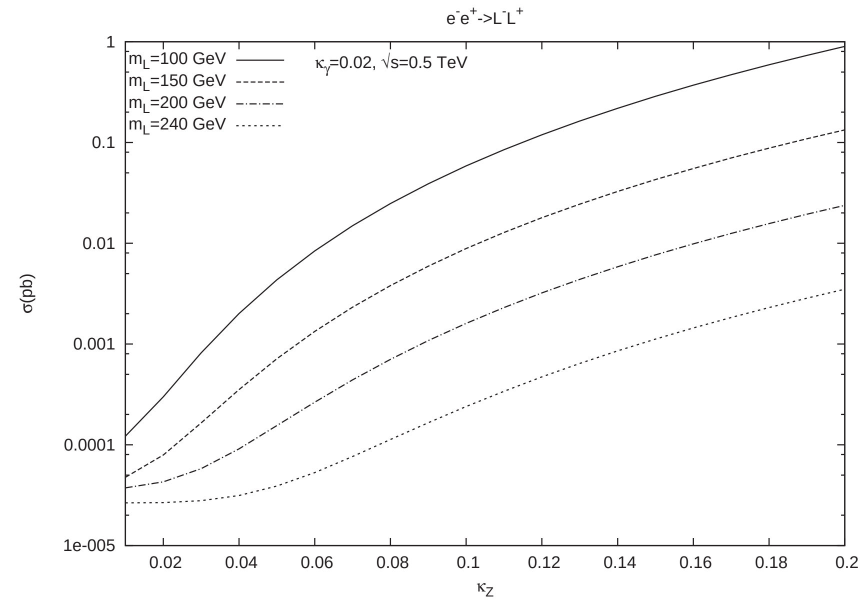

Figure 5.23: Total cross sections as functions of κ_Z ($\kappa_\gamma = 0.02$) for various masses of heavy leptons for the reaction $e^+e^- \to L^-L^+$.

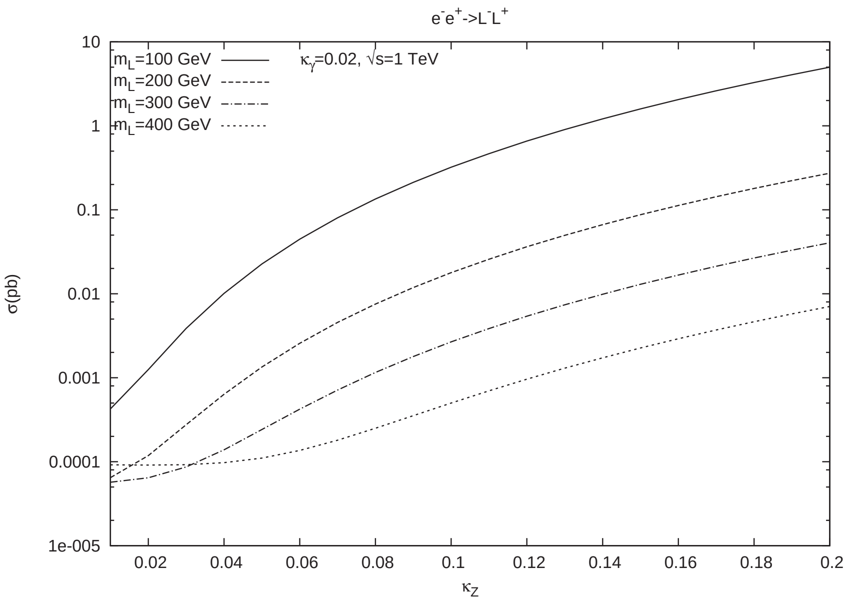

Figure 5.24: Total cross sections as functions of κ_Z ($\kappa_\gamma = 0.02$) for various masses of heavy leptons for the reaction $e^+e^- \to L^-L^+$.

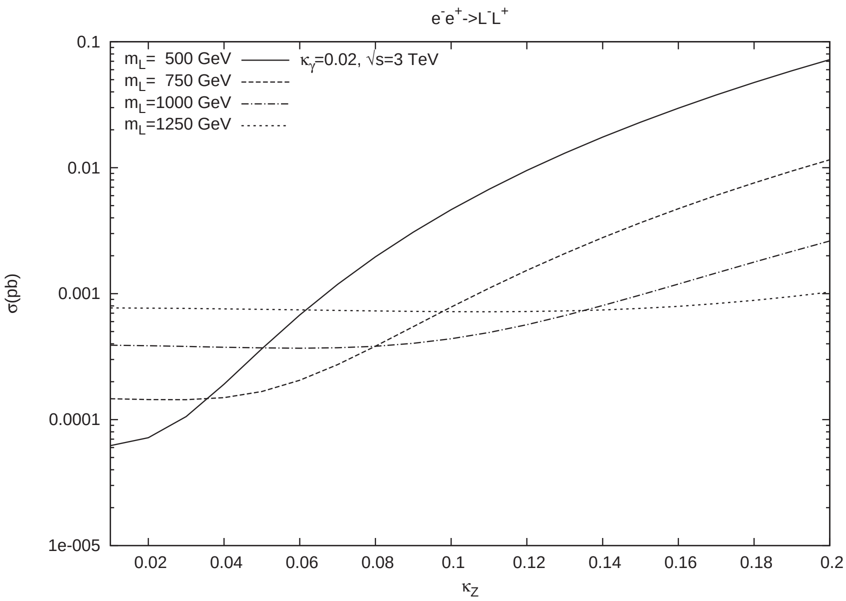

Figure 5.25: Total cross sections as functions of κ_Z ($\kappa_\gamma = 0.02$) for various masses of heavy leptons for the reaction $e^+e^- \to L^-L^+$.

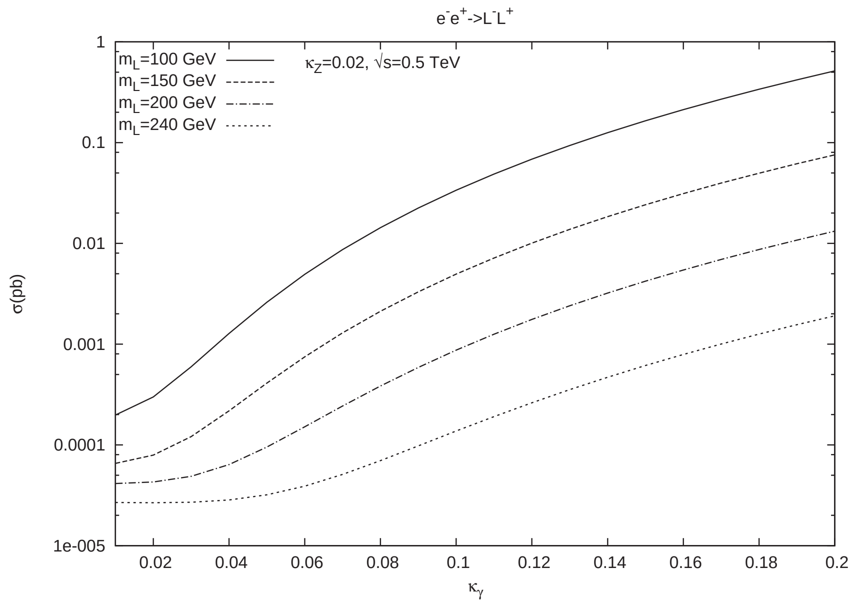

Figure 5.26: Total cross sections as functions of κ_γ ($\kappa_Z = 0.02$) for various masses of heavy leptons for the reaction $e^+e^- \to L^-L^+$.

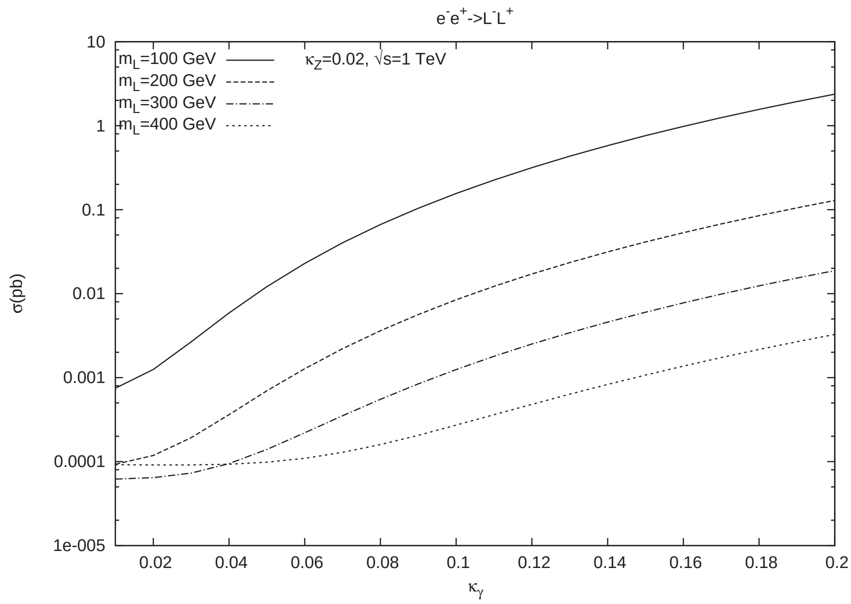

Figure 5.27: Total cross sections as functions of κ_γ ($\kappa_Z = 0.02$) for various masses of heavy leptons for the reaction $e^+e^- \to L^-L^+$.

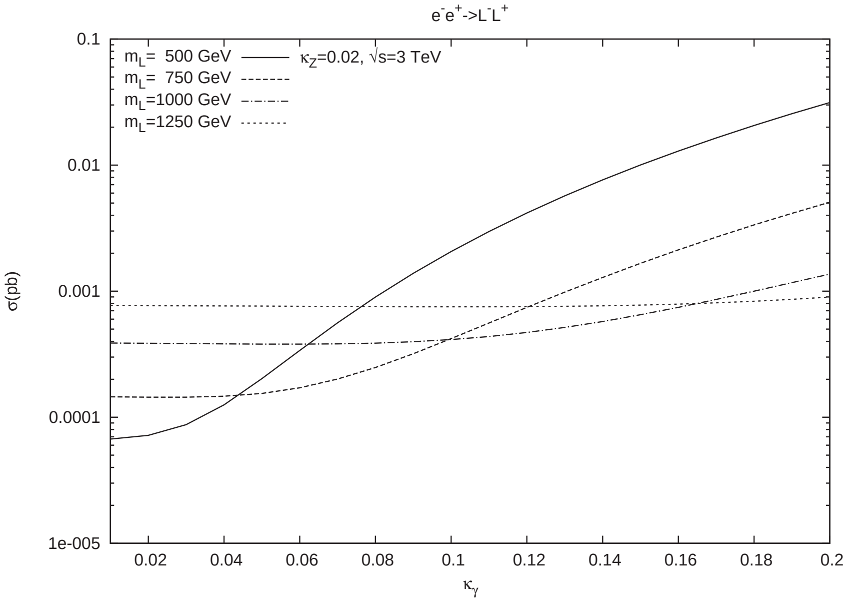

Figure 5.28: Total cross sections as functions of κ_γ ($\kappa_Z = 0.02$) for various masses of heavy leptons for the reaction $e^+e^- \to L^-L^+$.

5.3 Discussion

In the pair production case we expect 8 events per year for 150 GeV leptons, 12 events for 200 GeV leptons and 12 events for 350 GeV leptons at 0.5, 1 and 3 TeV, respectively. Hence, future linear colliders, with their clean environments, are good platforms for searching single and pair productions of heavy leptons.

CHAPTER 6

SINGLE PRODUCTION OF HEAVY LEPTONS WITH ANOMALOUS COUPLINGS AT THE CERN LHC

In this chapter, we study the single production and decays of charged heavy leptons in hadronic collisions via some anomalous interactions at LHC [77, 78]. These interactions can include additional leptons which are expected to have TeV scale masses. LHC has enough energy to produce and detect these leptons with the total energy $\sqrt{s} = 14$ TeV and $L = 10^5 \text{pb}^{-1}$ at which two 7 TeV proton beams collide each other. With this energy LHC will be the ideal machine to answer all the basic questions at present open through the wonderful success of the SM and by its obvious lack of predictive power for understanding the mass range of heavy leptons. Many analysis have been done for the production of heavy leptons with different models through the pp collisions in [79-89].

6.1 Production of Heavy Leptons

In the Standard Model, Flavor Changing Neutral Current (FCNC) processes receive contributions from only higher order corrections. Here we offer the following effective Lagrangian describing these interactions via anomalous magnetic transition moment type vertices;

$$\begin{aligned}\mathcal{L}_{\texttt{eff}} &= \frac{ie\kappa_\gamma}{\Lambda} L\sigma_{\mu\nu}q^\nu l A^\mu \\ &+ \frac{g}{2\cos\theta_W} L\left[\gamma_\mu(c_v - c_a\gamma_5) + \frac{i\sigma_{\mu\nu}q^\nu}{\Lambda}\kappa_Z\right] lZ^\mu + h.c., \end{aligned} \tag{6.1}$$

where κ_γ and κ_Z are the anomalous magnetic dipole moment factors, q is the momentum of the exchanged gauge boson, θ_W is the Weinberg angle, e and g denote the gauge couplings relative to $U(1)$ and $SU(2)$ symmetries respectively, A^μ and Z^μ the fields of the photon and Z boson and Λ is the new physics scale.

The parton level process $qq \rightarrow Ll$, responsible for the heavy lepton production in pp collision occurs via FCNC, which takes place through the exchange of Z boson and γ in the in the s channel. The Feynman diagram of this process is given in Fig. 6.1.

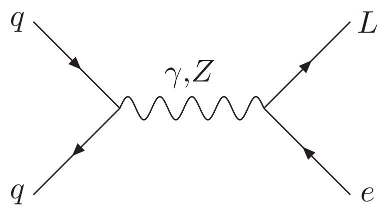

Figure 6.1: The Feynman diagrams of the s channel γ and Z exchange processes $qq \rightarrow Ll$.

6.2 Definition of Differential Cross Section

Considering the effective Lagrangian in Eq. (6.1), we find the amplitudes for the γ exchange in the s channel as,

$$\mathcal{M}_\gamma = -\frac{ee_q\kappa_\gamma}{\Lambda\hat{s}}\left[\bar{v}(p_2)\gamma^\mu u(p_1)\right]\left[\bar{u}(p_3)\sigma^\rho_\mu q_\rho v(p_4)\right] \tag{6.2}$$

and

$$\overline{\mathcal{M}}_\gamma = -\frac{ee_q\kappa_\gamma}{\Lambda\hat{s}}\left[\bar{u}(p_1)\gamma^\alpha v(p_2)\right]\left[\bar{v}(p_4)\sigma^\delta_\alpha q_\delta u(p_3)\right] \tag{6.3}$$

ignoring the quark and electron masses we find,

$$\langle|\mathcal{M}_\gamma|^2\rangle = \frac{e^2 e_q^2 \kappa_\gamma^2}{4\Lambda^2 \hat{s}^2} \times Tr\Big[\not{p}_2 \gamma^\mu \not{p}_1 \gamma^\alpha \Big] \times Tr\Big[(\not{p}_3 + m_L)\sigma_\mu^\rho q_\rho \not{p}_4 \sigma_\alpha^\delta q_\delta \Big] \quad (6.4)$$

and we find the amplitudes for the Z exchange in the s channel as,

$$\mathcal{M}_Z = \frac{g_Z^2}{4} \frac{(g_{\mu\nu} - q_\mu q_\nu / M_Z^2)}{\Big[(\hat{s} - M_Z^2) + iM_Z\Gamma_Z\Big]} \Big[\bar{v}(p_2)\gamma^\mu(v_q - a_q\gamma^5)u(p_1)\Big] \times \Big[\bar{u}(p_3)\Big(\gamma^\nu(v_L - a_L\gamma^5) + \frac{i\kappa_Z}{\Lambda}\sigma^{\nu\rho}q_\rho v(p_4)\Big)\Big], \quad (6.5)$$

and

$$\overline{\mathcal{M}}_Z = \frac{g_Z^2}{4} \frac{(g_{\alpha\beta} - q_\alpha q_\beta / M_Z^2)}{\Big[(\hat{s} - M_Z^2) - iM_Z\Gamma_Z\Big]} \Big[\bar{u}(p_1)\gamma^\alpha(v_q - a_q\gamma^5)v(p_2)\Big] \times \Big[\bar{v}(p_4)\Big(\gamma^\beta(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\beta\delta}q_\delta u(p_3)\Big)\Big], \quad (6.6)$$

which gives,

$$\langle|\mathcal{M}_Z|^2\rangle = \frac{g_Z^4}{64\Big[(\hat{s} - M_Z^2)^2 + M_Z^2\Gamma_Z^2\Big]} \Big(g_{\mu\nu} - q_\mu q_\nu / M_Z^2\Big)\Big(g_{\alpha\beta} - q_\alpha q_\beta / M_Z^2\Big) \times Tr\Big[\not{p}_2 \gamma^\mu (v_q - a_q\gamma^5) \not{p}_1 \gamma^\alpha (v_q - a_q\gamma^5)\Big] \times Tr\Big[(\not{p}_3 + m_L)\Big(\gamma^\nu(v_L - a_L\gamma^5) + \frac{i\kappa_Z}{\Lambda}\sigma^{\nu\rho}q_\rho\Big) \times \not{p}_4 \Big(\gamma^\beta(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\beta\delta}q_\delta\Big)\Big] \quad (6.7)$$

The interference terms are given by,

$$\langle\mathcal{M}_\gamma\overline{\mathcal{M}}_Z\rangle = \Big\{ - \frac{e e_q \kappa_\gamma g_Z^2}{16\Lambda\hat{s}} \Big\} \Big[\frac{g_{\alpha\beta} - q_\alpha q_\beta / M_Z^2}{(\hat{s} - M_Z^2) - iM_Z\Gamma_Z} \Big]$$

$$\times Tr\Big[\not{p}_2\gamma_\mu \not{p}_1\gamma^\alpha(v_q - a_q\gamma_5)\Big]$$
$$\times Tr\Big[(\not{p}_3 + m_L)\sigma^\rho_\mu q_\rho \not{p}_4\Big(\gamma^\beta(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\beta\delta}q_\delta\Big)\Big] \quad (6.8)$$

and

$$\langle\overline{\mathcal{M}}_\gamma\mathcal{M}_Z\rangle = \Big\{-\frac{ee_q\kappa_\gamma g_Z^2}{16\Lambda\hat{s}}\Big\}\Big[\frac{g_{\mu\nu} - q_\mu q_\nu/M_Z^2}{(\hat{s} - M_Z^2) + iM_Z\Gamma_Z}\Big]$$
$$\times Tr\Big[\not{p}_1\gamma_\alpha \not{p}_2\gamma^\mu(v_q - a_q\gamma_5)\Big]$$
$$\times Tr\Big[\not{p}_4\sigma^\delta_\alpha q_\delta(\not{p}_3 + m_L)\Big(\gamma^\nu(v_L - a_L\gamma^5) + \frac{i\kappa_Z}{\Lambda}\sigma^{\nu\rho}q_\rho\Big)\Big] \quad (6.9)$$

We evaluate the total differential cross section for the process $qq \to Ll$ obtaining:

$$\frac{d\hat{\sigma}_{tot.}}{d\hat{t}} = \frac{1}{16\pi s^2}\Big[\langle|\mathcal{M}_\gamma|^2\rangle + \langle|\mathcal{M}_Z|^2\rangle + \langle\mathcal{M}_\gamma\overline{\mathcal{M}}_Z\rangle + \langle\overline{\mathcal{M}}_\gamma\mathcal{M}_Z\rangle\Big]. \quad (6.10)$$

Using the trace theorems and the Mandelstam variables in Eq. (2.7), the form of the differential cross section is as follows;

$$\frac{d\hat{\sigma}(q\bar{q} \to Ll)}{d\hat{t}} = \frac{2e_q^2\kappa_\gamma^2\pi\alpha^2}{\Lambda^2\hat{s}^3}\Big\{(\hat{s} + 2\hat{t})m_L^2 - m_L^4 - 2\hat{t}(\hat{s} + \hat{t})\Big\}$$
$$+\frac{\pi\alpha^2}{8\Lambda^2\hat{s}^2\sin^4\theta_W\cos^4\theta_W\Big[(\hat{s} - M_Z^2)^2 + M_Z^2\Gamma_Z^2\Big]}$$
$$\times\Big\{\kappa_Z^2({a_q}^2 + {v_q}^2)\hat{s}\Big((\hat{s} + 2\hat{t})m_L^2 - 2\hat{t}(\hat{s} + \hat{t}) - m_L^4\Big)$$
$$+2\kappa_Z\Lambda\hat{s}m_L\Big(v_L({a_q}^2 + {v_q}^2)(\hat{s} - m_L^2) - 2a_La_qv_q(m_L^2 - \hat{s} - 2\hat{t})\Big)$$
$$+\Lambda^2\Big[({a_L}^2 + {v_L}^2)({a_q}^2 + {v_q}^2)\Big(\hat{s}^2 + 2\hat{t}^2 + 2\hat{s}\hat{t} - (\hat{s} + 2\hat{t})m_L^2\Big)$$
$$+4a_Lv_La_qv_q\hat{s}(\hat{s} + 2\hat{t} - m_L^2)\Big]\Big\}$$
$$+\frac{e_q\pi\alpha^2\kappa_\gamma M_Z\Gamma_Z}{\Lambda^2\hat{s}^2\sin^2\theta_W\cos^2\theta_W\Big[(\hat{s} - M_Z^2)^2 + M_Z^2\Gamma_Z^2\Big]}$$
$$\times\Big\{\kappa_Zv_q\Big(m_L^4 - (\hat{s} + 2\hat{t})m_L^2 + 2\hat{t}(\hat{s} + \hat{t})\Big)$$

$$+\Lambda m_L(a_L a_q(m_L^2-\hat{s}-2\hat{t})+v_L v_q(m_L^2-\hat{s}))\Big\} \quad (6.11)$$

where Γ_Z and M_Z are the decay width and mass of mediator Z, $\hat{s}$ and $\hat{t}$ are the Mandelstam variables.

6.3 Heavy Lepton Decays

After their production, heavy leptons decay via the neutral current processes $L \to \gamma l$ and $L \to Zl$ due to the anomalous couplings in Eq. (6.1), where l denotes one of the ordinary charged leptons. The detailed analytical calculation of decay widths for each channel can be found in chapter 4.3.1 and 4.3.2. Numerical results for total decay widths of the heavy leptons depending on their masses for $\kappa_\gamma, \kappa_Z = 0.02$ and $\Lambda = m_L$ are given in Table 6.1. The branching ratio values would be around 33% for each channel concerning with the leptons e, μ and τ.

m_L (GeV)	$\Gamma(L \to \gamma e)$(GeV)	$\Gamma(L \to Ze)$(GeV)	Γ_{Total} (GeV)
250	7.30×10^{-4}	3	3
500	1.46×10^{-3}	21	21
750	2.19×10^{-3}	72	72
1000	2.92×10^{-3}	171	171
1250	3.65×10^{-3}	333	333

Table 6.1: Decay widths of heavy leptons.

6.4 Numerical Results

The partonic cross section is given by,

$$\hat{\sigma}(\hat{s}) = \int_{t_{min}}^{t_{max}} \frac{d\hat{\sigma}}{d\hat{t}} d\hat{t}, \quad (6.12)$$

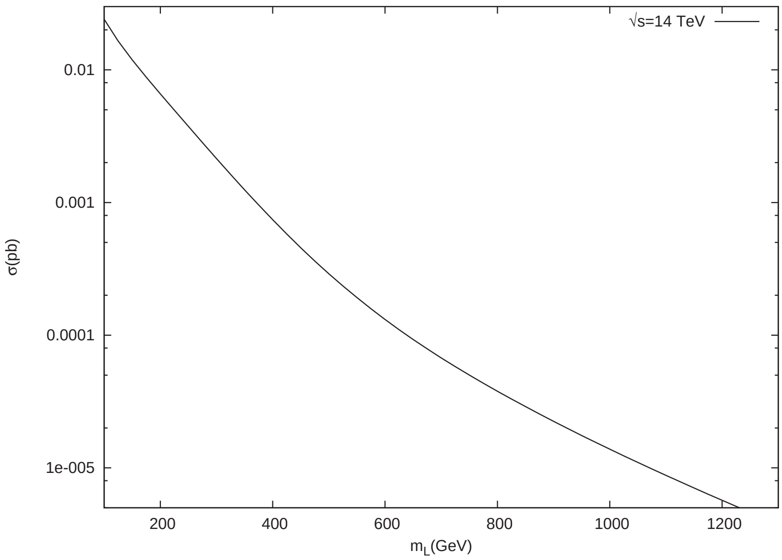

Figure 6.2: The total cross sections for the process $q\bar{q} \to L^- e^+$, as function of the heavy lepton masses, for LHC ($\sqrt{s}$ =14 TeV, $L^{int} = 10^5$pb^{-1}).

The total production cross section is obtained by the integration over the parton distributions as;

$$\begin{aligned}\sigma(q\bar{q} \to Ll) &= \int_{m_L^2/s}^{1} d\tau \int_{\tau}^{1} \frac{dx}{x}[f_{q/p}(x, Q^2) f_{\bar{q}}(x/\tau, Q^2) \\ &\quad + f_q(x/\tau, Q^2) f_{\bar{q}}(x, Q^2)]\hat{\sigma}(\hat{s}) \end{aligned} \tag{6.13}$$

where $x_{min} = m_L^2/s$, $\hat{t}_{min} = -(\hat{s} - m_L^2)$ and $\hat{t}_{max} = 0$. Here we use the CTEQ5 [76] parametrization with $Q = m_L$ and we take $\kappa_\gamma = \kappa_Z = 0.02$ and $\Lambda = m_L$.

We give the production cross sections for the signal as functions of the heavy lepton masses, m_L, in Fig. 6.2 for the center of mass energy of $\sqrt{s} = 14$ TeV. We applied an optimal cut of $|M_{Ze} - m_L| < 50$ GeV for the considered heavy lepton mass range. Fig. 6.3 shows the p_T distribution of the final state particles for the relevant background process $q\bar{q} \to eZl$ in pp collisions. This p_T distribution has a peak around 50 GeV and suppressed at higher values. Fig. 6.4 shows the invariant mass distributions of Ze system, after a cut of $p_T^{e,j} > 50$ GeV.

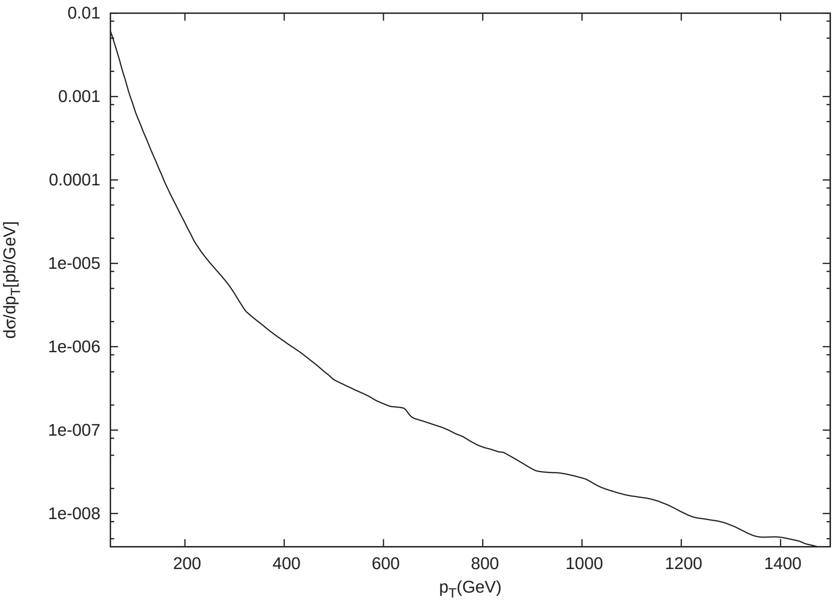

Figure 6.3: p_T distribution of the background process $q\bar{q} \rightarrow e^- Z e^+$ for LHC.

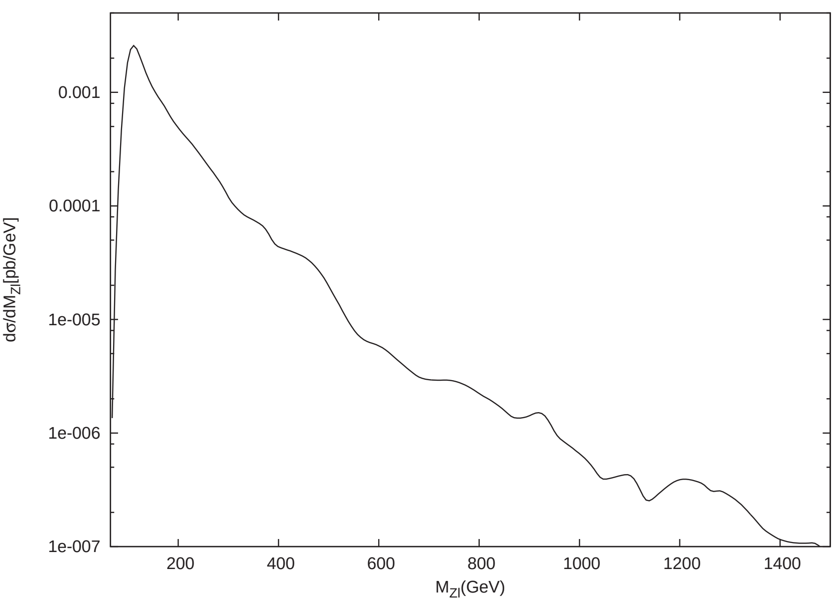

Figure 6.4: The invariant mass distribution of the Ze system for the background for LHC.

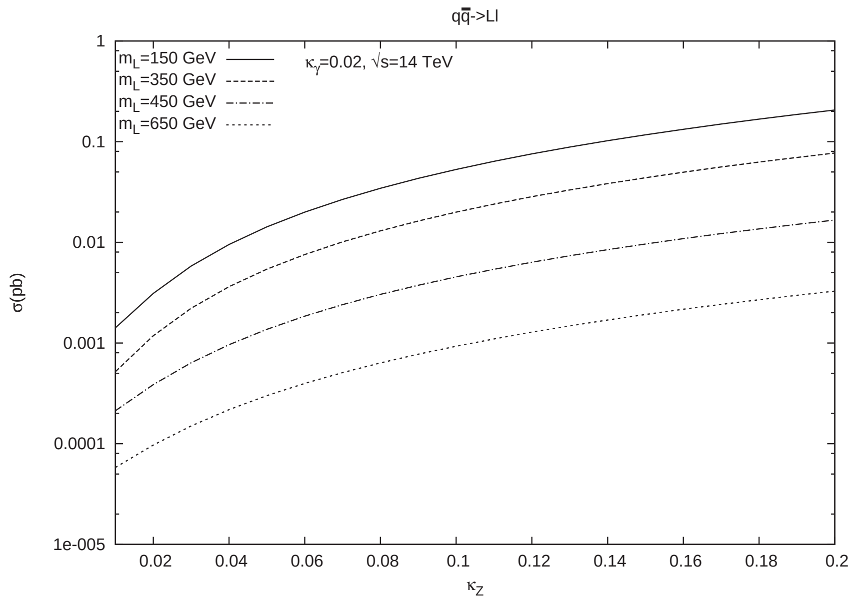

Figure 6.5: Total cross sections as functions of κ_Z ($\kappa_\gamma = 0.02$) for various masses of heavy leptons for the subprocess $q\bar{q} \to L^- e^+$.

In Table 6.2, we present the signal cross sections for the subprocess $q\bar{q} \to L^- e^+$ and background cross sections for the subprocess $q\bar{q} \to e^- Z e^+$ depending on the heavy lepton masses for a suitable range. We applied an optimal cut of $|M_{Ze} - m_L| < 50$ GeV for the considered heavy lepton mass range. Production of heavy leptons provides a clean signature, as shown in this table. The expected number of signal events for 600 GeV leptons at the LHC is about 13 with SS number of 4.

In Figs. 6.5 (6.6) the cross sections are plotted as functions of anomalous coupling κ_γ (κ_Z) by taking κ_γ (κ_Z)=0.02 for various mass values. From these figures, we can see the production cross sections increase as the values of couplings, κ_γ, κ_Z increases and as the heavy lepton mass decreases. By varying the couplings, the cross sections changes up to three orders of magnitude and the event number changes accordingly.

We have used the high energy package CompHEP for calculations of background cross sections reported in this study [48].

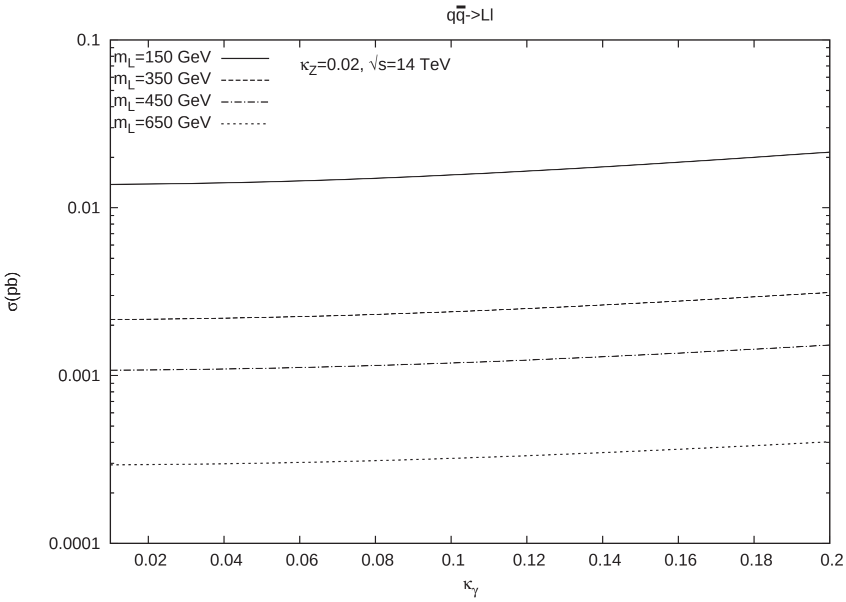

Figure 6.6: Total cross sections as functions of κ_γ ($\kappa_Z = 0.02$) for various masses of heavy leptons for the subprocess $q\bar{q} \to L^- e^+$.

m_L (GeV)	σ(pb)	σ_B(pb)	$S/\sqrt{S+B}$
100	2.40×10^{-2}	2.65×10^{-6}	49
200	6.58×10^{-3}	3.64×10^{-6}	26
400	7.40×10^{-4}	5.75×10^{-7}	9
600	1.30×10^{-4}	1.69×10^{-7}	4

Table 6.2: The signal and background cross sections and SS depending on the heavy lepton masses for the LHC ($\sqrt{s} = 14$ TeV, $L^{int} = 10^5\text{pb}^{-1}$).

6.5 Discussion

This work was motivated by some of the extensions of the Standard Model which includes some anomalous interactions. It is shown that, after kinematical cuts a statistical significance of SS ≥ 3 can be achieved. 11 signal events for 650 GeV leptons are possible. Therefore LHC will play an important role in searching for new physics beyond the SM through the anomalous interactions.

CHAPTER 7

CONCLUSION

In this chapter we briefly present the conclusions. Detailed summaries of chapter 2, 3, 4, 5 and 6 are presented in sections 2.5, 3.3, 4.5, 5.3 and 6.5, respectively.

This thesis gives an analysis of possible production of heavy leptons via two non-standard models which are string inspired E_6 model and the effective Lagrangian description via anomalous magnetic transitions at lepton-hadron, linear and hadron-hadron colliders.

After some phenomenological calculations it is shown that heavy leptons with masses between 100-3000 GeV can be observed. When we compare these results with the experimental ones which are given in [35], [36], [37] and [38], we can talk about the new physics beyond the Standard Model. Hence, these collider options seems to be capable of probing new physics in the case of the two models analyzed in this thesis.

REFERENCES

[1] D. Griffiths, "Introduction to Elementary Physics," (1987, Willey).

[2] F. Halzen, A. D. Martin, "Quarks and Leptons: An Introductory Course in Modern Particle Physics," (1984, Willey).

[3] D. H. Perkins, "Introduction to High Energy Physics," 3rd ed. (1987, Addison-Wesley).

[4] W. B. Rolnick, "The Fundamental Particles and Their Interactions," (1994, Addison-Wesley).

[5] W. Greiner, B. Müller, "Gauge Theory of Weak Interactions," 3rd rev. ed. (2000, Springer).

[6] W. E. Burcham and M. Jobes, "Nuclear and Particle Physics," (1995, Longman Group).

[7] S. L. Glashow, "Partial Symmetries Of Weak Interactions," Nucl. Phys. **22**, 579 (1961).

[8] S. Weinberg, "A Model Of Leptons," Phys. Rev. Lett. **19**, 1264 (1967).

[9] A. Salam, *in*: "Elementary Particle Theory," W. Svartholm, ed., Almquist and Wiksell, Stockholm (1968).

[10] H. D. Politzer, "Reliable Perturbative Results For Strong Interactions?," Phys. Rev. Lett. **30**, 1346 (1973).

[11] D. J. Gross and F. Wilczek, "Ultraviolet Behavior Of Non-Abelian Gauge Theories," Phys. Rev. Lett. **30**, 1343 (1973).

[12] W. M. Yao *et al.* [Particle Data Group], "Review of particle physics," J. Phys. G **33**, 1 (2006).

[13] L. Susskind, "Dynamics Of Spontaneous Symmetry Breaking In The Weinberg-Salam Theory," Phys. Rev. D **20**, 2619 (1979).

[14] H. P. Nilles, "Supersymmetry, Supergravity And Particle Physics," Phys. Rept. **110**, 1 (1984).

[15] M. F. Sohnius, "Introducing Supersymmetry," Phys. Rept. **128**, 39 (1985).

[16] H. E. Haber and G. L. Kane, "The Search For Supersymmetry: Probing Physics Beyond The Standard Model," Phys. Rept. **117**, 75 (1985).

[17] E. Eichten and K. D. Lane, "Dynamical Breaking Of Weak Interaction Symmetries," Phys. Lett. B **90**, 125 (1980).

[18] E. Farhi and L. Susskind, "Technicolor," Phys. Rept. **74**, 277 (1981).

[19] E. Eichten, K. D. Lane and M. E. Peskin, "New Tests For Quark And Lepton Substructure," Phys. Rev. Lett. **50**, 811 (1983).

[20] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, "The hierarchy problem and new dimensions at a millimeter," Phys. Lett. B **429**, 263 (1998), [arXiv:hep-ph/9803315].

[21] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, "New dimensions at a millimeter to a Fermi and superstrings at a TeV," Phys. Lett. B **436**, 257 (1998), [arXiv:hep-ph/9804398].

[22] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, "Phenomenology, astrophysics and cosmology of theories with sub-millimeter dimensions and TeV scale quantum gravity," Phys. Rev. D **59**, 086004 (1999), [arXiv:hep-ph/9807344].

[23] T. Appelquist, H. C. Cheng and B. A. Dobrescu, "Bounds on universal extra dimensions," Phys. Rev. D **64**, 035002 (2001), [arXiv:hep-ph/0012100].

[24] F. Gursey, P. Ramond and P. Sikivie, "A Universal Gauge Theory Model Based On E6," Phys. Lett. B **60**, 177 (1976).

[25] F. Gursey and M. Serdaroglu, "Basic Fermion Masses And Mixings In The E(6) Model," Lett. Nuovo Cim. **21** (1978) 28.

[26] F. Gursey and M. Serdaroglu, "E6 Gauge Field Theory Model Revisited," Nuovo Cim. A **65** (1981) 337.

[27] T. G. Rizzo, "Phenomenology Of Exotic Particles In E(6) Theories," Phys. Rev. D **34** (1986) 1438.

[28] J. L. Rosner, "E_6 And Exotic Fermions," Comments Nucl. Part. Phys. **15**, 195 (1986).

[29] P. K. Mohapatra, R. N. Mohapatra and P. B. Pal, "Implications Of E6 Grand Unification," Phys. Rev. D **33**, 2010 (1986).

[30] M. Serdaroglu, "E(6) Phenomenology, a 3 x 3 matrix approach," Int. J. Mod. Phys. A **2** (1987) 1617.

[31] J. L. Hewett and T. G. Rizzo, "Low-Energy Phenomenology of Superstring Inspired E(6) Models," Phys. Rept. **183** (1989) 193.

[32] T. G. Rizzo, “Phenomenology of E(6) superstring-inspired models,” MAD/PH/526.

[33] E. Ma, “New Low-Energy Realization of the Superstring-Inspired E_6 Model,” arXiv:hep-ph/9601238.

[34] M. B. Green and J. H. Schwarz, “Anomaly Cancellation In Supersymmetric D=10 Gauge Theory And Superstring Theory,” Phys. Lett. B **149**, 117 (1984).

[35] M. Z. Akrawy *et al.* [OPAL Collaboration], “A Direct Search For New Charged Heavy Leptons At Lep,” Phys. Lett. B **240**, 250 (1990).

[36] D. Decamp *et al.* [ALEPH Collaboration], “Searches for new particles in Z decays using the ALEPH detector,” Phys. Rept. **216**, 253 (1992).

[37] T. Ahmed *et al.* [H1 Collaboration], “A Search for heavy leptons at HERA,” Phys. Lett. B **340**, 205 (1994).

[38] P. Achard *et al.* [L3 Collaboration], “Search for heavy neutral and charged leptons in e+ e- annihilation at LEP,” Phys. Lett. B **517**, 75 (2001), [arXiv:hep-ex/0107015].

[39] S. Sultansoy, “Linac-ring type colliders: Second way to TeV scale,” Eur. Phys. J. C **33** (2004) 1064, [arXiv:hep-ex/0306034].

[40] S. Sultansoy, “Four ways to TeV scale,” Turk. J. Phys. **22** (1998) 575.

[41] F. M. L. Almeida, J. A. Martins Simoes and A. J. Ramalho, “Heavy Lepton Production In E P Collisions,” Nucl. Phys. B **347** (1990) 537.

[42] T. G. Rizzo, “Production Of Exotic E(6) Leptons In E P Collisions,” Phys. Lett. B **188**, 95 (1987).

[43] A. T. Alan, A. T. Tasci and O. Cakir, “Heavy lepton production at Linac x LHC,” Acta Phys. Polon. B **35**, 2199 (2004), [arXiv:hep-ph/0402184].

[44] F. M. L. Almeida, Y. A. Coutinho, J. A. Martins Simoes and M. A. B. do Vale, “On a signature for heavy Majorana neutrinos in hadronic collisions,” Phys. Rev. D **62**, 075004 (2000), [arXiv:hep-ph/0002024].

[45] E. Nardi, E. Roulet and D. Tommasini, “Limits on neutrino mixing with new heavy particles,” Phys. Lett. B **327**, 319 (1994), [arXiv:hep-ph/9402224].

[46] P. Bamert, C. P. Burgess and I. Maksymyk, “New physics and recent high precision electroweak measurements,” Phys. Lett. B **356**, 282 (1995), [arXiv:hep-ph/9505339].

[47] R. W. Assmann *et al.*, "A 3-TeV e+ e- linear collider based on CLIC technology," CERN-2000-008, http://www.slac.stanford.edu/spires/find/hep/www?r=cern-2000-008.

[48] A. Pukhov *et al.*, "CompHEP: A package for evaluation of Feynman diagrams and integration over multi-particle phase space.", [arXiv:hep-ph/9908288].

[49] A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, "Parton distributions: A new global analysis," Eur. Phys. J. C **4** (1998) 463, [arXiv:hep-ph/9803445].

[50] H. Abramowicz *et al.* [TESLA-N Study Group], "TESLA: The superconducting electron positron linear collider with an integrated X-ray laser laboratory. Technical design report. Pt. 6: Appendices. Chapter 2: THERA: Electron proton scattering at s**(1/2) approx. 1-TeV," DESY-01-011.

[51] International Linear Collider-Technical Review Report, ILC-TRC/2003 Report (2003), http://www.slac.stanford.edu/xorg/ilc-trc/2002/index.html.

[52] E. Accomando *et al.* [CLIC Physics Working Group], "Physics at the CLIC multi-TeV linear collider", [arXiv:hep-ph/0412251].

[53] F. M. L. Almeida, J. H. Lopes, J. A. Martins Simoes and C. M. Porto, "Looking For New Single Heavy Leptons In Electron - Positron Collisions," Phys. Rev. D **44**, 2836 (1991).

[54] F. M. L. Almeida, J. H. Lopes, J. A. Martins Simoes, P. P. Queiroz Filho and A. J. Ramalho, "Single heavy lepton production in high-energy electron positron collisions," Phys. Rev. D **51**, 5990 (1995), [arXiv:hep-ph/9410348].

[55] F. M. L. Almeida, Y. A. Coutinho, J. A. Martins Simoes and M. A. B. do Vale, "Neutral heavy lepton production at next high energy e+ e- linear colliders," Phys. Rev. D **63**, 075005 (2001), [arXiv:hep-ph/0008201].

[56] F. M. L. Almeida, Y. A. Coutinho, J. A. Martins Simoes, S. Wulck and M. A. B. do Vale, "New heavy charged leptons at future high energy electron positron colliders," Eur. Phys. J. C **30**, 327 (2003), [arXiv:hep-ph/0305314].

[57] J. E. Cieza Montalvo, "Pair production of heavy-exotic-fermions at e^+e^- colliders," Phys. Rev. D **59**, 095007 (1999), [arXiv:hep-ph/9807213].

[58] E. De Pree and M. Sher, "Single Production of Heavy Charged Leptons at the ILC," [arXiv:0709.3305 [hep-ph]].

[59] C. X. Yue, N. Zhang and S. H. Zhu, "Pair production of the heavy leptons in future high energy linear e^+e^- colliders," Eur. Phys. J. C **53**, 215 (2008), [arXiv:0707.0729 [hep-ph]].

[60] A. T. Alan, A. T. Tasci and N. Karagoz, "Single and pair production of heavy leptons in E(6) model," Mod. Phys. Lett. A **21**, 1869 (2006), [arXiv:hep-ph/0601027].

[61] J. B. Dainton, M. Klein, P. Newman, E. Perez and F. Willeke, DESY-06-006, "Deep inelastic electron nucleon scattering at the LHC," arXiv:hep-ex/0603016.

[62] R. D. Peccei, S. Peris and X. Zhang, "Nonstandard Couplings Of The Top Quark And Precision Measurements Of The Electroweak Theory," Nucl. Phys. B **349**, 305 (1991).

[63] T. Han, R. D. Peccei and X. Zhang, "Top Quark Decay Via Flavor Changing Neutral Currents At Hadron Colliders," Nucl. Phys. B **454**, 527 (1995), [arXiv:hep-ph/9506461].

[64] T. Han, K. Whisnant, B. L. Young and X. Zhang, "Searching for t $\to$ c g at the Fermilab Tevatron," Phys. Lett. B **385**, 311 (1996), [arXiv:hep-ph/9606231].

[65] T. G. Rizzo, "Searching for anomalous tau nu W couplings," Phys. Rev. D **56**, 3074 (1997), [arXiv:hep-ph/9704337].

[66] T. Tait and C. P. Yuan, "Anomalous t-c-g coupling: The connection between single top production and top decay," Phys. Rev. D **55**, 7300 (1997), [arXiv:hep-ph/9611244].

[67] T. Han, M. Hosch, K. Whisnant, B. L. Young and X. Zhang, "Single top quark production via FCNC couplings at hadron colliders," Phys. Rev. D **58**, 073008 (1998), [arXiv:hep-ph/9806486].

[68] S. Bar-Shalom and J. Wudka, "Flavor changing single top quark production channels at e+ e- colliders in the effective Lagrangian description," Phys. Rev. D **60**, 094016 (1999), [arXiv:hep-ph/9905407].

[69] T. Huang, Z. H. Lin and X. Zhang, "Effects of anomalous couplings of the tau lepton," [arXiv:hep-ph/0009353].

[70] T. G. Rizzo, "The Exotic Fermions In E6 And The Anomalous Magnetic Moments Of Leptons," Phys. Rev. D **33**, 3329 (1986).

[71] P. M. Ferreira, R. B. Guedes and R. Santos, "Lepton flavour violating processes at the international linear collider," Phys. Rev. D **75**, 055015 (2007), [arXiv:hep-ph/0611222].

[72] T. Han and J. L. Hewett, "Top charm associated production in high energy e+ e- collisions," Phys. Rev. D **60**, 074015 (1999), [arXiv:hep-ph/9811237].

[73] K. J. Abraham, K. Whisnant and B. L. Young, “Searching for an anomalous anti-t q gamma coupling via single top quark production at a gamma gamma collider,” Phys. Lett. B **419**, 381 (1998), [arXiv:hep-ph/9707476].

[74] E. Malkawi and T. Tait, “Top-Charm Strong Flavour-Changing Neutral Currents at the Tevatron,” Phys. Rev. D **54**, 5758 (1996), [arXiv:hep-ph/9511337].

[75] N. Kidonakis and A. Belyaev, “FCNC top quark production via anomalous t q V couplings beyond leading order,” JHEP **0312**, 004 (2003), [arXiv:hep-ph/0310299].

[76] H. L. Lai *et al.* [CTEQ Collaboration], “Global QCD analysis of parton structure of the nucleon: CTEQ5 parton distributions,” Eur. Phys. J. C **12**, 375 (2000), [arXiv:hep-ph/9903282].

[77] D. Boussard et al., [LHC Study Group], “The Large Hadron Collider: Conceptual Design,” CERN-AC-95-05-LHC (1995).

[78] J. G. Branson, D. Denegri, I. Hinchliffe, F. Gianotti, F. E. Paige and P. Sphicas [ATLAS and CMS Collaborations], “High transverse momentum physics at the Large Hadron Collider: The ATLAS and CMS Collaborations,” Eur. Phys. J. direct C **4**, N1 (2002).

[79] P. H. Frampton, D. Ng, M. Sher and Y. Yuan, “Search for heavy leptons at hadron colliders,” Phys. Rev. D **48**, 3128 (1993), [arXiv:hep-ph/9210270].

[80] Y. A. Coutinho, J. A. Martins Simoes, C. M. Porto and P. P. Queiroz Filho, “Single heavy lepton production in hadron hadron collisions,” Phys. Rev. D **57**, 6975 (1998).

[81] J. E. Cieza Montalvo and P. P. de Queiroz Filho, “Production of a single and a pair of heavy-exotic leptons through the pp collisions,” [arXiv:hep-ph/9811521].

[82] J. E. Cieza Montalvo and P. P. de Queiroz Filho, “Exotic heavy leptons predicted by extended models at the CERN LHC,” Phys. Rev. D **66**, 055003 (2002).

[83] J. E. Cieza Montalvo and M. D. Tonasse, “Pairs of charged heavy leptons from an SU(3)L x U(1)N model at CERN LHC,” Nucl. Phys. B **623**, 325 (2002), [arXiv:hep-ph/0008196].

[84] M. M. Boyce, M. A. Doncheski and H. Konig, “Charged heavy lepton production in superstring inspired E6 models,” Phys. Rev. D **55**, 68 (1997), [arXiv:hep-ph/9607376].

[85] G. Bhattacharya, P. Kalyniak and K. A. Peterson, "Photon and Z induced heavy charged lepton pair production at a hadron supercollider," Phys. Rev. D **53**, 2371 (1996), [arXiv:hep-ph/9512255].

[86] V. D. Barger, T. Han and J. Ohnemus, "Heavy Leptons At Hadron Supercolliders," Phys. Rev. D **37**, 1174 (1988).

[87] H. Baer, V. D. Barger and R. J. N. Phillips, "Fourth Generation Charged Leptons And Neutrinos Via Z0," Phys. Rev. D **32**, 688 (1985).

[88] J. E. Cieza Montalvo and P. P. de Querioz Filho, "Heavy-exotic leptons at LHC," [arXiv:hep-ph/0112263].

[89] G. Bhattacharya, P. A. Kalyniak and K. A. Peterson, "Heavy charged lepton pair production through photon fusion at hadron supercolliders," [arXiv:hep-ph/9310223].

[90] T. Williams, L. Hecking, and H.-B. Broeker, gnuplot, version 4 (2004), based on the original version released by Thomas Williams and Colin Kelley in 1986. http://www.gnuplot.info/.

APPENDIX A

DIRAC MATRICES AND TRACE THEOREMS

We first prove some useful properties of Dirac matrices. We start with the definitions of the γ matrices:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} i = 1, 2, 3. \tag{A.1}$$

These matrices satisfy the basic anti-commutation relation

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \tag{A.2}$$

In terms of the metric

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

note that $g^{\mu\nu} g_{\mu\nu} = 4$, we have,

$$\begin{aligned} \gamma_\mu \gamma^\mu &= 4 \\ \gamma_\mu \gamma^\nu \gamma^\mu &= -2\gamma^\nu \\ \gamma_\mu \gamma^\nu \gamma^\lambda \gamma^\mu &= 4g^{\nu\lambda} \\ \gamma_\mu \gamma^\nu \gamma^\lambda \gamma^\sigma \gamma^\mu &= -2\gamma^\sigma \gamma^\lambda \gamma^\nu, \end{aligned} \tag{A.3}$$

γ^5 is defined as

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{A.4}$$

so that the following relations are satisfied;

$$\begin{aligned} (\gamma^5)^2 &= I \\ \gamma^5\gamma^\mu &= -\gamma^\mu\gamma^5. \end{aligned} \tag{A.5}$$

The trace theorems are (using the notation $\not{a} = \gamma_\mu a^\mu$):

$$\begin{aligned} Tr[1] &= 4 \\ Tr[\not{a}\ \not{b}] &= 4a \cdot b \\ Tr[\not{a}\ \not{b}\ \not{c}\ \not{d}] &= 4[(a \cdot b)(c \cdot d) - (a \cdot c)(b \cdot d) + (a \cdot d)(b \cdot c) \\ Tr[\gamma_5] &= 0 \\ Tr[\gamma_5\ \not{a}\ \not{b}] &= 0 \\ Tr[\gamma_5\ \not{a}\ \not{b}\ \not{c}\ \not{d}] &= 4i\epsilon_{\mu\nu\lambda\sigma}a^\mu b^\nu c^\lambda d^\sigma, \end{aligned} \tag{A.6}$$

where $\epsilon_{\mu\nu\lambda\sigma} = +1(-1)$ for μ, ν, λ, σ an even (odd) permutation of 0, 1, 2, 3; and 0 if two indices are the same.

If A and B are any two matrices and α is any number,

$$\begin{aligned} Tr(A+B) &= Tr(A) + Tr(B) \\ Tr(\alpha A) &= \alpha Tr(A) \\ Tr(AB) &= Tr(BA) \\ Tr(ABC) &= TR(CBA) = Tr(BCA) \end{aligned} \tag{A.7}$$

APPENDIX B

CROSS SECTIONS FOR SINGLE HEAVY LEPTON PRODUCTION AT LINEAR COLLIDERS WITH ANOMALOUS COUPLINGS

$$\langle|\mathbf{M_a}|^2\rangle :$$

$$\mathcal{M}_a = \frac{e^2\kappa_\gamma}{\Lambda s}\Big[\bar{v}(p_2)\gamma^\mu u(p_1)\Big]\Big[\bar{u}(p_3)\sigma^\rho_\mu q_\rho v(p_4)\Big]$$

$$\overline{\mathcal{M}}_a = \frac{e^2\kappa_\gamma}{\Lambda s}\Big[\bar{u}(p_1)\gamma^\alpha v(p_2)\Big]\Big[\bar{v}(p_4)\sigma^\delta_\alpha q_\delta v(p_3)\Big]$$

$$\langle|M_a|^2\rangle = \frac{e^4\kappa_\gamma^2}{4\Lambda^2 s^2}\times Tr\Big[\not{p}_2\gamma^\mu \not{p}_1\gamma^\alpha\Big]\times Tr\Big[(\not{p}_3+m_L)\sigma^\rho_\mu q_\rho \not{p}_4\sigma^\delta_\alpha q_\delta\Big]$$

$$\langle|M_a|^2\rangle = \frac{32\pi^2\alpha^2\kappa_\gamma^2}{\Lambda^2 s}\Big[(s+2t)m_L^2-2t(s+t)-m_L^4\Big] \quad \text{(B.1)}$$

$$\langle|\mathbf{M_b}|^2\rangle :$$

$$\mathcal{M}_b = \frac{g_Z^2}{4}\frac{(g_{\mu\nu}-q_\mu q_\nu/M_Z^2)}{\Big[(s-M_Z^2)+iM_Z\Gamma_Z\Big]}\Big[\bar{v}(p_2)\gamma^\mu(v_l-a_l\gamma^5)u(p_1)\Big] \times\Big[\bar{u}(p_3)\Big(\gamma^\nu(v_L-a_L\gamma^5)+\frac{i\kappa_Z}{\Lambda}\sigma^{\nu\rho}q_\rho\Big)v(p_4)\Big]$$

$$\overline{\mathcal{M}}_b = \frac{g_Z^2}{4}\frac{(g_{\alpha\beta}-q_\alpha q_\beta/M_Z^2)}{\Big[(s-M_Z^2)-iM_Z\Gamma_Z\Big]}\Big[\bar{u}(p_1)\gamma^\alpha(v_l-a_l\gamma^5)v(p_2)\Big]$$

$$\times\Big[\bar{v}(p_4)\Big(\gamma^\beta(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\beta\delta}q_\delta\Big)u(p_3)\Big]$$

$$\begin{aligned}
\langle|\mathcal{M}_b|^2\rangle &= \frac{g_Z^4\Big(g_{\mu\nu} - q_\mu q_\nu/M_Z^2\Big)\Big(g_{\alpha\beta} - q_\alpha q_\beta/M_Z^2\Big)}{64\Big[(\hat{s} - M_Z^2) + iM_Z\Gamma_Z\Big]\Big[(\hat{s} - M_Z^2) - iM_Z\Gamma_Z\Big]} \\
&\times Tr\Big[\not{p}_2\gamma^\mu(v_l - a_l\gamma^5)\not{p}_1\gamma^\alpha(v_l - a_l\gamma^5)\Big] \\
&\times Tr\Big[(\not{p}_3 + m_L)\Big(\gamma^\nu(v_L - a_L\gamma^5) + \frac{i\kappa_Z}{\Lambda}\sigma^{\nu\rho}q_\rho\Big) \\
&\times \not{p}_4\Big(\gamma^\beta(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\beta\delta}q_\delta\Big)\Big]
\end{aligned}$$

$$\begin{aligned}
\langle|M_b|^2\rangle &= \frac{2\pi^2\alpha^2}{\Lambda^2\sin^4\theta_W\cos^4\theta_W\Big[(s - M_Z^2) + \Gamma_Z^2M_Z^2\Big]} \\
&\times\Big[\Lambda^2\Big[(s+t)(s+t-m_L^2)\left((a_L^2+v_L^2)(a_l^2+v_l^2) + 4a_La_lv_Lv_l\right) \\
&+t(t-m_L^2)\left((a_L^2+v_L^2)(a_l^2+v_l^2) - 4a_La_lv_Lv_l\right)\Big] \\
&+\kappa_Z s\Big[2\Lambda m_L\left(v_L(a_l^2+v_l^2)(s-m_L^2) - 2a_L(m_L^2 - s - 2t)\right) \\
&-\kappa_Z(a_l^2+v_l^2)\left(m_L^4 - (s+2t)m_L^2 + 2t(s+t)\right)\Big]\Big] \qquad \text{(B.2)}
\end{aligned}$$

$\langle|\mathbf{M_c}|^\mathbf{2}\rangle$:

$$\mathcal{M}_c = \frac{e^2\kappa_\gamma^2}{\Lambda t}\Big[\bar{u}(p_3)\sigma_\mu^\rho q_\rho u(p_1)\Big]\Big[\bar{v}(p_2)\gamma^\mu v(p_4)\Big]$$

$$\overline{\mathcal{M}}_c = \frac{e^2\kappa_\gamma^2}{\Lambda t}\Big[\bar{u}(p_1)\sigma_\alpha^\delta q_\delta u(p_3)\Big]\Big[\bar{v}(p_4)\gamma^\alpha v(p_2)\Big]$$

$$\langle|M_c|^2\rangle = \frac{e^4\kappa_\gamma^2}{4\Lambda^2t^2}\times Tr\Big[(\not{p}_3 + m_L)\sigma_\mu^\rho q_\rho\not{p}_1\sigma_\alpha^\delta q_\delta\Big]\times Tr\Big[\not{p}_2\gamma^\mu\not{p}_4\gamma^\alpha\Big]$$

$$|M_c|^2 = \frac{32\pi^2\alpha^2\kappa_\gamma^2}{\Lambda^2t^2}\Big[(2s+t)m_L^2 - 2s(s+t) - m_L^4\Big] \qquad \text{(B.3)}$$

$$\langle|\mathbf{M_d}|^2\rangle :$$

$$\mathcal{M}_d = \frac{g_Z^2}{4}\frac{(g_{\mu\nu}-q_\mu q_\nu/M_Z^2)}{\Big[(t-M_Z^2)+iM_Z\Gamma_Z\Big]}$$
$$\times\Big[\bar{u}(p_3)\Big(\gamma^\mu(v_L-a_L\gamma^5)+\frac{i\kappa_Z}{\Lambda}\sigma^{\mu\rho}q_\rho\Big)u(p_1)\Big]$$
$$\times\Big[\bar{v}(p_2)\gamma^\nu(v_l-a_l\gamma^5)v(p_4)\Big]$$

$$\overline{\mathcal{M}}_d = \frac{g_Z^2}{4}\frac{(g_{\alpha\beta}-q_\alpha q_\beta/M_Z^2)}{\Big[(t-M_Z^2)-iM_Z\Gamma_Z\Big]}$$
$$\times\Big[\bar{u}(p_1)\Big(\gamma^\alpha(v_L-a_L\gamma^5)-\frac{i\kappa_Z}{\Lambda}\sigma^{\alpha\delta}q_\delta\Big)u(p_3)\Big]$$
$$\times\Big[\bar{v}(p_4)\gamma^\beta(v_l-a_l\gamma^5)v(p_2)\Big]$$

$$\langle|\mathcal{M}_d|^2\rangle = \frac{g_Z^4}{64\Big[(t-M_Z^2)^2+M_Z^2\Gamma_Z^2\Big]}\Big[g_{\mu\nu}-q_\mu q_\nu/M_Z^2\Big]\Big[g_{\alpha\beta}-q_\alpha q_\beta/M_Z^2\Big]$$
$$\times Tr\Big[(\not{p}_3+m_L)\Big(\gamma^\mu(v_L-a_L\gamma^5)+\frac{i\kappa_Z}{\Lambda}\sigma^{\mu\rho}q_\rho\Big)$$
$$\times\not{p}_1\Big(\gamma^\alpha(v_L-a_L\gamma^5)-\frac{i\kappa_Z}{\Lambda}\sigma^{\alpha\delta}q_\delta\Big)\Big]$$
$$\times Tr\Big[\not{p}_2\gamma^\nu(v_l-a_l\gamma^5)\not{p}_4\gamma^\beta(v_l-a_l\gamma^5)\Big]$$

$$\langle|M_d|^2\rangle = \frac{2\pi^2\alpha^2}{\Lambda^2\sin^4\theta_W\cos^4\theta_W\Big[(t-M_Z^2)^2+\Gamma_Z^2M_Z^2\Big]}$$
$$\times\Big[\Lambda^2\Big[(a_L^2+v_L^2)(a_l^2+v_l^2)\Big((2s+t)m_L^2-2s^2-t^2-2st\Big)$$
$$+4a_Lv_La_lv_lt(m_L^2-2s-t)\Big]$$
$$-2m_L\Lambda\kappa_Zt\Big(v_L(a_l^2+v_l^2)(m_L^2-t)+2a_La_lv_l(m_L^2-2s-t)\Big)$$
$$+\kappa_Z^2t(a_l^2+v_l^2)\Big(m_L^4-(2s+t)m_L^2+2s(s+t)\Big)\Big] \quad \text{(B.4)}$$

$$\langle \overline{\mathbf{M}}_{\mathbf{a}} \mathbf{M}_{\mathbf{b}} \rangle :$$

$$\overline{\mathcal{M}}_a = \frac{e^2 \kappa_\gamma}{\Lambda s} \Big[\bar{u}(p_1) \gamma^\alpha v(p_2) \Big] \Big[\bar{v}(p_4) \sigma^\delta_\alpha q_\delta v(p_3) \Big]$$

$$\mathcal{M}_b = \frac{g_Z^2}{4} \frac{(g_{\mu\nu} - q_\mu q_\nu / M_Z^2)}{\Big[(s - M_Z^2) + i M_Z \Gamma_Z \Big]} \Big[\bar{v}(p_2) \gamma^\mu (v_l - a_l \gamma^5) u(p_1) \Big] \times \Big[\bar{u}(p_3) \Big(\gamma^\nu (v_L - a_L \gamma^5) + \frac{i \kappa_Z}{\Lambda} \sigma^{\nu\rho} q_\rho \Big) v(p_4) \Big]$$

$$\langle \overline{\mathcal{M}}_a \mathcal{M}_b \rangle = \frac{e^2 g_Z^2 \kappa_\gamma \Big(g_{\mu\nu} - q_\mu q_\nu / M_Z^2 \Big)}{16 \Lambda s \Big[(s - M_Z^2) + i M_Z \Gamma_Z \Big]} \times Tr \Big[\not{p}_1 \gamma^\alpha \not{p}_2 \gamma^\mu (v_l - a_l \gamma^5) \Big] \times Tr \Big[\not{p}_4 \sigma^\delta_\alpha q_\delta (\not{p}_3 + m_L) \Big(\gamma^\nu (v_L - a_L \gamma^5) + \frac{i \kappa_Z}{\Lambda} \sigma^{\nu\rho} q_\rho \Big) \Big]$$

$$\langle \overline{M}_a M_b \rangle = \frac{8 i \pi^2 \alpha^2 \kappa_\gamma}{\Lambda^2 \sin^2 \theta_W \cos^2 \theta_W \Big[(s - M_Z^2) + i \Gamma_Z M_Z \Big]} \times \Big[\kappa_Z v_l \Big(m_L^4 - (s + 2t) m_L^2 + 2t(s+t) \Big) + m_L \Lambda \Big(a_L a_l (m_L^2 - s - 2t) + v_L v_l (m_L^2 - s) \Big) \Big] \qquad \text{(B.5)}$$

$$\langle \mathbf{M}_{\mathbf{a}} \overline{\mathbf{M}}_{\mathbf{b}} \rangle :$$

$$\mathcal{M}_a = \frac{e^2 \kappa_\gamma}{\Lambda s} \Big[\bar{v}(p_2) \gamma^\mu u(p_1) \Big] \Big[\bar{u}(p_3) \sigma^\rho_\mu q_\rho v(p_4) \Big]$$

$$\overline{\mathcal{M}}_b = \frac{g_Z^2}{4} \frac{(g_{\alpha\beta} - q_\alpha q_\beta / M_Z^2)}{\Big[(s - M_Z^2) - i M_Z \Gamma_Z \Big]} \Big[\bar{u}(p_1) \gamma^\alpha (v_l - a_l \gamma^5) v(p_2) \Big]$$

$$\times\Big[\bar{v}(p_4)\Big(\gamma^{\beta}(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\beta\delta}q_{\delta}\Big)u(p_3)\Big]$$

$$\begin{aligned}
\langle \mathcal{M}_a \overline{\mathcal{M}}_b \rangle &= \frac{e^2 g_Z^2 \kappa_\gamma \Big(g_{\mu\nu} - q_\mu q_\nu / M_Z^2\Big)}{16\Lambda s\Big[(s - M_Z^2) - iM_Z\Gamma_Z\Big]} \\
&\times Tr\Big[\not{p}_2\gamma^{\mu}\not{p}_1\gamma^{\alpha}(v_l - a_l\gamma^5)\Big] \\
&\times Tr\Big[(\not{p}_3 + m_L)\sigma^{\rho}_{\mu}q_{\rho}\not{p}_4\Big(\gamma^{\beta}(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\beta\delta}q_{\delta}\Big)\Big]
\end{aligned}$$

$$\begin{aligned}
\langle M_a \overline{M}_b \rangle &= -\frac{8i\pi^2\alpha^2\kappa_\gamma}{\Lambda^2 \sin^2\theta_W \cos^2\theta_W\Big[(s - M_Z^2) - i\Gamma_Z M_Z\Big]} \\
&\times\Big[\kappa_Z v_l\Big(m_L^4 - (s + 2t)m_L^2 + 2t(s + t)\Big) \\
&+ m_L\Lambda\Big(a_L a_l(m_L^2 - s - 2t) + v_L v_l(m_L^2 - s)\Big)\Big] \qquad \text{(B.6)}
\end{aligned}$$

$$\langle \overline{\mathbf{M}}_{\mathbf{a}} \mathbf{M}_{\mathbf{c}} \rangle :$$

$$\overline{\mathcal{M}}_a = \frac{e^2\kappa_\gamma}{\Lambda s}\Big[\bar{u}(p_1)\gamma^{\alpha}v(p_2)\Big]\Big[\bar{v}(p_4)\sigma^{\delta}_{\alpha}q_{\delta}v(p_3)\Big]$$

$$\mathcal{M}_c = \frac{e^2\kappa_\gamma}{\Lambda t}\Big[\bar{u}(p_3)\sigma^{\rho}_{\mu}q'_{\rho}u(p_1)\Big]\Big[\bar{v}(p_2)\gamma^{\mu}v(p_4)\Big]$$

$$\langle \overline{M}_a M_c \rangle = \frac{e^4\kappa_\gamma^2}{4\Lambda^2 st} \times Tr\Big[\not{p}_1\gamma^{\alpha}\not{p}_2\gamma^{\mu}\not{p}_4\sigma^{\delta}_{\alpha}q_{\delta}(\not{p}_3 + m_L)\sigma^{\rho}_{\mu}q'_{\rho}\Big]$$

$$\langle \overline{M}_a M_c \rangle = \frac{32\pi^2\alpha^2\kappa_\gamma^2}{\Lambda^2}\Big[s + t - m_L^2\Big] \qquad \text{(B.7)}$$

$$\langle \mathbf{M}_{\mathbf{a}} \overline{\mathbf{M}}_{\mathbf{c}} \rangle :$$

$$\mathcal{M}_a = \frac{e^2\kappa_\gamma}{\Lambda s}\Big[\bar{v}(p_2)\gamma^{\mu}u(p_1)\Big]\Big[\bar{u}(p_3)\sigma^{\rho}_{\mu}q_{\rho}v(p_4)\Big]$$

$$\overline{\mathcal{M}}_c = \frac{e^2\kappa_\gamma^2}{\Lambda t}\Big[\bar{u}(p_1)\sigma^\delta_\alpha q'_\delta u(p_3)\Big]\Big[\bar{v}(p_4)\gamma^\alpha v(p_2)\Big]$$

$$\langle M_a\overline{M}_c\rangle = \frac{e^4\kappa_\gamma^2}{4\Lambda^2 st}\times Tr\Big[\not{p}_2\gamma^\mu\not{p}_1\sigma^\delta_\alpha q'_\delta(\not{p}_3+m_L)\sigma^\rho_\mu q_\rho\not{p}_4\gamma^\alpha\Big]$$

$$\langle M_a\overline{M}_c\rangle = \frac{32\pi^2\alpha^2\kappa_\gamma^2}{\Lambda^2}\Big[s+t-m_L^2\Big] \tag{B.8}$$

$$\langle\overline{\mathbf{M}}_\mathbf{a}\mathbf{M}_\mathbf{d}\rangle:$$

$$\overline{\mathcal{M}}_a = \frac{e^2\kappa_\gamma}{\Lambda s}\Big[\bar{u}(p_1)\gamma^\alpha v(p_2)\Big]\Big[\bar{v}(p_4)\sigma^\delta_\alpha q_\delta v(p_3)\Big]$$

$$\begin{aligned}\mathcal{M}_d = {} & \frac{g_Z^2}{4}\frac{(g_{\mu\nu}-q'_\mu q'_\nu/M_Z^2)}{\Big[(t-M_Z^2)+iM_Z\Gamma_Z\Big]} \\ & \times\Big[\bar{u}(p_3)\Big(\gamma^\mu(v_L-a_L\gamma^5)+\frac{i\kappa_Z}{\Lambda}\sigma^{\mu\rho}q'_\rho\Big)u(p_1)\Big] \\ & \times\Big[\bar{v}(p_2)\gamma^\nu(v_l-a_l\gamma^5)v(p_4)\Big]\end{aligned}$$

$$\begin{aligned}\langle\overline{M}_a M_d\rangle = {} & \frac{e^2\kappa_\gamma g_Z^2}{16\Lambda s}\frac{(g_{\mu\nu}-q'_\mu q'_\nu/M_Z^2)}{\Big[(t-M_Z^2)+iM_Z\Gamma_Z\Big]} \\ & \times Tr\Big[\not{p}_1\gamma^\alpha\not{p}_2\gamma^\nu(v_l-a_l\gamma_5)\not{p}_4\sigma^\delta_\alpha q_\delta(\not{p}_3+m_L) \\ & \times\Big(\gamma^\mu(v_L-a_L\gamma^5)+\frac{i\kappa_Z}{\Lambda}\sigma^{\mu\rho}q'_\rho\Big)\Big]\end{aligned}$$

$$\begin{aligned}\langle\overline{M}_a M_d\rangle = {} & -\frac{8i\pi^2\alpha^2\kappa_\gamma}{\Lambda^2\sin^2\theta_W\cos^2\theta_W\Big[t-M_Z^2+i\Gamma_Z M_Z\Big]} \\ & \times(m_L^2-s-t)\Big[\Lambda m_L(a_La_l+v_Lv_l)-\kappa_Z v_l t\Big]\end{aligned} \tag{B.9}$$

$$\langle\mathbf{M}_\mathbf{a}\overline{\mathbf{M}}_\mathbf{d}\rangle:$$

$$\mathcal{M}_a = \frac{e^2\kappa_\gamma}{\Lambda s}\Big[\bar{v}(p_2)\gamma^\mu u(p_1)\Big]\Big[\bar{u}(p_3)\sigma^\rho_\mu q_\rho v(p_4)\Big]$$

$$\overline{\mathcal{M}}_d = \frac{g_Z^2}{4}\frac{(g_{\alpha\beta} - q'_\alpha q'_\beta/M_Z^2)}{\Big[(t - M_Z^2) - iM_Z\Gamma_Z\Big]} \times\Big[\bar{u}(p_1)\Big(\gamma^\alpha(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\alpha\delta}q'_\delta\Big)u(p_3)\Big] \times\Big[\bar{v}(p_4)\gamma^\beta(v_l - a_l\gamma^5)v(p_2)\Big]$$

$$\langle M_a\overline{M}_d\rangle = \frac{e^2\kappa_\gamma g_Z^2}{16\Lambda s}\frac{(g_{\alpha\beta} - q'_\alpha q'_\beta/M_Z^2)}{\Big[(t - M_Z^2) - iM_Z\Gamma_Z\Big]} \times Tr\Big[\not{p}_2\gamma^\mu\not{p}_1\Big(\gamma^\alpha(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\alpha\delta}q'_\delta\Big) \times(\not{p}_3 + m_L)\sigma^\rho_\mu q_\rho\not{p}_4\gamma^\beta(v_l - a_l\gamma^5)\Big]$$

$$\langle M_a\overline{M}_d\rangle = \frac{8i\pi^2\alpha^2\kappa_\gamma}{\Lambda^2\sin^2\theta_W\cos^2\theta_W\Big[(t - M_Z^2) - i\Gamma_Z M_Z\Big]} \times(m_L^2 - s - t)\Big[\Lambda m_L(a_La_l + v_Lv_l) - \kappa_Z v_l t\Big] \quad \text{(B.10)}$$

$$\langle\overline{\mathbf{M}}_\mathbf{b}\mathbf{M}_\mathbf{c}\rangle :$$

$$\overline{\mathcal{M}}_b = \frac{g_Z^2}{4}\frac{(g_{\alpha\beta} - q_\alpha q_\beta/M_Z^2)}{\Big[(s - M_Z^2) - iM_Z\Gamma_Z\Big]}\Big[\bar{u}(p_1)\gamma^\alpha(v_l - a_l\gamma^5)v(p_2)\Big] \times\Big[\bar{v}(p_4)\Big(\gamma^\beta(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\beta\delta}q_\delta\Big)u(p_3)\Big]$$

$$\mathcal{M}_c = \frac{e^2\kappa_\gamma^2}{\Lambda t}\Big[\bar{u}(p_3)\sigma^\rho_\mu q'_\rho u(p_1)\Big]\Big[\bar{v}(p_2)\gamma^\mu v(p_4)\Big]$$

$$\langle\overline{M}_b M_c\rangle = \frac{e^2\kappa_\gamma g_Z^2}{16\Lambda t}\frac{(g_{\alpha\beta} - q_\alpha q_\beta/M_Z^2)}{\Big[(s - M_Z^2) - iM_Z\Gamma_Z\Big]} \times Tr\Big[\not{p}_1\gamma^\alpha(v_l - a_l\gamma^5)\not{p}_2\gamma^\mu\not{p}_4 \times\Big(\gamma^\beta(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\beta\delta}q_\delta u(p_3)\Big)(\not{p}_3 + m_L)\sigma^\rho_\mu q'_\rho\Big]$$

$$\langle\overline{M}_b M_c\rangle = -\frac{8i\pi^2\alpha^2\kappa_\gamma}{\Lambda^2\sin^2\theta_W\cos^2\theta_W\Big[(s-M_Z^2)-i\Gamma_Z M_Z\Big]} \times(m_L^2-s-t)\Big[\Lambda m_L(a_L a_l+v_L v_l)+\kappa_Z v_l s\Big] \quad \text{(B.11)}$$

$\langle\mathbf{M_b}\overline{\mathbf{M}}_\mathbf{c}\rangle$:

$$\mathcal{M}_b = \frac{g_Z^2}{4}\frac{(g_{\mu\nu}-q_\mu q_\nu/M_Z^2)}{\Big[(s-M_Z^2)+iM_Z\Gamma_Z\Big]}\Big[\bar{v}(p_2)\gamma^\mu(v_l-a_l\gamma^5)u(p_1)\Big] \times\Big[\bar{u}(p_3)\Big(\gamma^\nu(v_L-a_L\gamma^5)+\frac{i\kappa_Z}{\Lambda}\sigma^{\nu\rho}q_\rho\Big)v(p_4)\Big]$$

$$\overline{\mathcal{M}}_c = \frac{e^2\kappa_\gamma^2}{\Lambda t}\Big[\bar{u}(p_1)\sigma_\alpha^\delta q'_\delta u(p_3)\Big]\Big[\bar{v}(p_4)\gamma^\alpha v(p_2)\Big]$$

$$\langle\overline{M}_a M_d\rangle = \frac{e^2\kappa_\gamma g_Z^2}{16\Lambda t}\frac{(g_{\mu\nu}-q_\mu q_\nu/M_Z^2)}{\Big[(s-M_Z^2)+iM_Z\Gamma_Z\Big]} \times Tr\Big[\not{p}_2\gamma^\mu(v_l-a_l\gamma^5)\not{p}_1\sigma_\alpha^\delta q'_\delta(\not{p}_3+m_L) \times\Big(\gamma^\nu(v_L-a_L\gamma^5)+\frac{i\kappa_Z}{\Lambda}\sigma^{\nu\rho}q_\rho\Big)\not{p}_4\gamma^\alpha\Big]$$

$$\langle M_b\overline{M}_c\rangle = \frac{8i\pi^2\alpha^2\kappa_\gamma}{\Lambda^2\sin^2\theta_W\cos^2\theta_W\Big[s-M_Z^2+i\Gamma_Z M_Z\Big]} \times(m_L^2-s-t)\Big[\Lambda m_L(a_L a_l+v_L v_l)+\kappa_Z v_l s\Big] \quad \text{(B.12)}$$

$\langle\overline{\mathbf{M}}_\mathbf{b}\mathbf{M_d}\rangle$:

$$\overline{\mathcal{M}}_b = \frac{g_Z^2}{4}\frac{(g_{\alpha\beta}-q_\alpha q_\beta/M_Z^2)}{\Big[(s-M_Z^2)-iM_Z\Gamma_Z\Big]}\Big[\bar{u}(p_1)\gamma^\alpha(v_l-a_l\gamma^5)v(p_2)\Big] \times\Big[\bar{v}(p_4)\Big(\gamma^\beta(v_L-a_L\gamma^5)-\frac{i\kappa_Z}{\Lambda}\sigma^{\beta\delta}q_\delta\Big)u(p_3)\Big]$$

$$\mathcal{M}_d = \frac{g_Z^2}{4}\frac{(g_{\mu\nu} - q_\mu q_\nu/M_Z^2)}{\Big[(t - M_Z^2) + iM_Z\Gamma_Z\Big]}$$
$$\times\Big[\bar{u}(p_3)\Big(\gamma^\mu(v_L - a_L\gamma^5) + \frac{i\kappa_Z}{\Lambda}\sigma^{\mu\rho}q'_\rho\Big)u(p_1)\Big]$$
$$\times\Big[\bar{v}(p_2)\gamma^\nu(v_l - a_l\gamma^5)v(p_4)\Big]$$

$$\langle\overline{M}_b M_d\rangle = \frac{g_Z^4}{64}\frac{(g_{\alpha\beta} - q_\alpha q_\beta/M_Z^2)}{\Big[(s - M_Z^2) - iM_Z\Gamma_Z\Big]}\frac{(g_{\mu\nu} - q'_\mu q'_\nu/M_Z^2)}{\Big[(t - M_Z^2) + iM_Z\Gamma_Z\Big]}$$
$$\times Tr\Big[\not{p}_1\gamma^\alpha(v_l - a_l\gamma^5)\not{p}_2\gamma^\nu(v_l - a_l\gamma^5)\not{p}_4$$
$$\times\Big(\gamma^\beta(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\beta\delta}q_\delta\Big)(\not{p}_3 + m_L)$$
$$\times\Big(\gamma^\mu(v_L - a_L\gamma^5) + \frac{i\kappa_Z}{\Lambda}\sigma^{\mu\rho}q'_\rho\Big)\Big]$$

$$\langle\overline{M}_b M_d\rangle = \frac{2\pi^2\alpha^2}{\Lambda^2\sin^4\theta_W\cos^4\theta_W\Big[(s - M_Z^2) - i\Gamma_Z M_Z\Big]\Big[(t - M_Z^2) + i\Gamma_Z M_Z\Big]}$$
$$\times\Big[\kappa_Z^2 st(a_l^2 + v_l^2)(s + t - m_L^2)$$
$$+\Lambda^2(s+t)(m_L^2 - s - t)\Big((a_L^2 + v_L^2)(a_l^2 + v_l^2) + 4a_L a_l v_L v_l\Big)$$
$$-\Lambda m_L\kappa_Z(s - t)(s + t - m_L^2)\Big(2a_L a_l v_l + v_L(a_l^2 + v_l^2)\Big)\Big] \quad \text{(B.13)}$$

$$\langle\mathbf{M_b}\overline{\mathbf{M}}_\mathbf{d}\rangle :$$

$$\mathcal{M}_b = \frac{g_Z^2}{4}\frac{(g_{\mu\nu} - q_\mu q_\nu/M_Z^2)}{\Big[(s - M_Z^2) + iM_Z\Gamma_Z\Big]}\Big[\bar{v}(p_2)\gamma^\mu(v_l - a_l\gamma^5)u(p_1)\Big]$$
$$\times\Big[\bar{u}(p_3)\Big(\gamma^\nu(v_L - a_L\gamma^5) + \frac{i\kappa_Z}{\Lambda}\sigma^{\nu\rho}q_\rho\Big)v(p_4)\Big]$$

$$\overline{\mathcal{M}}_d = \frac{g_Z^2}{4}\frac{(g_{\alpha\beta} - q'_\alpha q'_\beta/M_Z^2)}{\Big[(t - M_Z^2) - iM_Z\Gamma_Z\Big]}$$
$$\times\Big[\bar{u}(p_1)\Big(\gamma^\alpha(v_L - a_L\gamma^5) - \frac{i\kappa_Z}{\Lambda}\sigma^{\alpha\delta}q'_\delta\Big)u(p_3)\Big]$$

$$\times\Big[\bar{v}(p_4)\gamma^\beta(v_l - a_l\gamma^5)v(p_2)\Big]$$

$$\begin{aligned}
\langle M_b\overline{M}_d\rangle &= \frac{g_Z^4}{64}\frac{(g_{\mu\nu}-q_\mu q_\nu/M_Z^2)}{\Big[(s-M_Z^2)+iM_Z\Gamma_Z\Big]}\frac{(g_{\alpha\beta}-q'_\alpha q'_\beta/M_Z^2)}{\Big[(t-M_Z^2)-iM_Z\Gamma_Z\Big]} \\
&\times Tr\Big[\not{p}_2\gamma^\mu(v_l-a_l\gamma^5)\not{p}_1\Big(\gamma^\alpha(v_L-a_L\gamma^5)-\frac{i\kappa_Z}{\Lambda}\sigma^{\alpha\delta}q'_\delta\Big) \\
&\times(\not{p}_3+m_L)\Big(\gamma^\nu(v_L-a_L\gamma^5)+\frac{i\kappa_Z}{\Lambda}\sigma^{\nu\rho}q_\rho\not{p}_4\gamma^\beta(v_l-a_l\gamma^5)\Big)
\end{aligned}$$

$$\begin{aligned}
\langle M_b\overline{M}_d\rangle &= \frac{2\pi^2\alpha^2}{\Lambda^2\sin^4\theta_W\cos^4\theta_W\Big[(t-M_Z^2)-i\Gamma_Z M_Z\Big]\Big[(s-M_Z^2)+i\Gamma_Z M_Z\Big]} \\
&\times\Big[\kappa_Z^2 st(a_l^2+v_l^2)(s+t-m_L^2) \\
&+\Lambda^2(s+t)(m_L^2-s-t)\Big((a_L^2+v_L^2)(a_l^2+v_l^2)+4a_La_lv_Lv_l\Big) \\
&-\Lambda m_L\kappa_Z(s-t)(s+t-m_L^2)\Big(2a_La_lv_l+v_L(a_l^2+v_l^2)\Big)\Big] \qquad \text{(B.14)}
\end{aligned}$$

$\langle \mathbf{\overline{M}_c M_d}\rangle$:

$$\overline{\mathcal{M}}_c = \frac{e^2\kappa_\gamma^2}{\Lambda t}\Big[\bar{u}(p_1)\sigma_\alpha^\delta q_\delta u(p_3)\Big]\Big[\bar{v}(p_4)\gamma^\alpha v(p_2)\Big]$$

$$\begin{aligned}
\mathcal{M}_d &= \frac{g_Z^2}{4}\frac{(g_{\mu\nu}-q_\mu q_\nu/M_Z^2)}{\Big[(t-M_Z^2)+iM_Z\Gamma_Z\Big]} \\
&\times\Big[\bar{u}(p_3)\Big(\gamma^\mu(v_L-a_L\gamma^5)+\frac{i\kappa_Z}{\Lambda}\sigma^{\mu\rho}q_\rho\Big)u(p_1)\Big] \\
&\times\Big[\bar{v}(p_2)\gamma^\nu(v_l-a_l\gamma^5)v(p_4)\Big]
\end{aligned}$$

$$\begin{aligned}
\langle\overline{M}_c M_d\rangle &= -\frac{e^2\kappa_\gamma g_Z^2}{16\Lambda t}\frac{(g_{\mu\nu}-q_\mu q_\nu/M_Z^2)}{\Big[(t-M_Z^2)+iM_Z\Gamma_Z\Big]} \\
&\times Tr\Big[\not{p}_1\sigma_\alpha^\delta q_\delta(\not{p}_3+m_L)\Big(\gamma^\mu(v_L-a_L\gamma^5)+\frac{i\kappa_Z}{\Lambda}\sigma^{\mu\rho}q_\rho\Big)\Big] \\
&\times Tr\Big[\not{p}_4\gamma^\alpha\not{p}_2\gamma^\nu(v_l-a_l\gamma^5)\Big]
\end{aligned}$$

$$\begin{aligned}
\langle \overline{M}_c M_d \rangle &= \frac{8i\pi^2\alpha^2\kappa_\gamma}{\Lambda^2\sin^2\theta_W\cos^2\theta_W\Big[(t-M_Z^2)+i\Gamma_Z M_Z\Big]} \\
&\times\Big[\kappa_Z v_l\left(m_L^4-(2s+t)m_L^2+2s(s+t)\right) \\
&-m_L\Lambda\left(a_L a_l(m_L^2-2s-t)+v_L v_l(m_L^2-t)\right)\Big] \qquad \text{(B.15)}
\end{aligned}$$

$$\langle \mathbf{M_c}\overline{\mathbf{M}}_\mathbf{d} \rangle :$$

$$\mathcal{M}_c = \frac{e^2\kappa_\gamma^2}{\Lambda t}\Big[\bar{u}(p_3)\sigma_\mu^\rho q_\rho u(p_1)\Big]\Big[\bar{v}(p_2)\gamma^\mu v(p_4)\Big]$$

$$\begin{aligned}
\overline{\mathcal{M}}_d &= \frac{g_Z^2}{4}\frac{(g_{\alpha\beta}-q_\alpha q_\beta/M_Z^2)}{\Big[(t-M_Z^2)-iM_Z\Gamma_Z\Big]} \\
&\times\Big[\bar{u}(p_1)\Big(\gamma^\alpha(v_L-a_L\gamma^5)-\frac{i\kappa_Z}{\Lambda}\sigma^{\alpha\delta}q_\delta\Big)u(p_3)\Big] \\
&\times\Big[\bar{v}(p_4)\gamma^\beta(v_l-a_l\gamma^5)v(p_2)\Big]
\end{aligned}$$

$$\begin{aligned}
\langle M_c\overline{M}_d \rangle &= -\frac{e^2\kappa_\gamma g_Z^2}{16\Lambda t}\frac{(g_{\alpha\beta}-q_\alpha q_\beta/M_Z^2)}{\Big[(t-M_Z^2)-iM_Z\Gamma_Z\Big]} \\
&\times Tr\Big[(\not{p}_3+m_L)\sigma_\mu^\rho q_\rho\ \not{p}_1\Big(\gamma^\alpha(v_L-a_L\gamma^5)-\frac{i\kappa_Z}{\Lambda}\sigma^{\alpha\delta}q_\delta\Big)\Big] \\
&\times Tr\Big[\ \not{p}_2\gamma^\mu\ \not{p}_4\gamma^\beta(v_l-a_l\gamma^5)v(p_2)\Big]
\end{aligned}$$

$$\begin{aligned}
\langle M_c\overline{M}_d \rangle &= -\frac{8i\pi^2\alpha^2\kappa_\gamma}{\Lambda^2\sin^2\theta_W\cos^2\theta_W\Big[(t-M_Z^2)-i\Gamma_Z M_Z\Big]} \\
&\times\Big[\kappa_Z v_l\left(m_L^4-(2s+t)m_L^2+2s(s+t)\right) \\
&-m_L\Lambda\left(a_L a_l(m_L^2-2s-t)+v_L v_l(m_L^2-t)\right)\Big] \qquad \text{(B.16)}
\end{aligned}$$

CURRICULUM VITAE

Ahmet Tolga Taşçı

Address:

Abant Izzet Baysal University

Department of Physics,

14280 Bolu, Turkey

Phone: +90 374 2541000

Fax: +90 374 2534642

Email: tolgatasci@gmail.com

Personal Details:

Gender: Male

Date of birth: 23rd of September, 1975

Place of birth: Bolu, Turkey

Education:

1994-1998 Undergraduate Studies in Physics at the Gazi University

1999-2002 MSc in Physics at the Abant Izzet Baysal University

Thesis: CP Violation in and Beyond The Standard Model.

2002-2008 PhD in Physics at the Abant Izzet Baysal University

Thesis: Analysis of Dynamical Models Extending The Standard Model With Heavy Leptons.

Specialization: High Energy Physics.

Publications:

1. A. T. Tasci and A. T. Alan, "ep analysis of anomalous interactions with heavy leptons," [arXiv:hep-ph/0608045].

2. A. T. Alan, A. T. Tasci and N. Karagoz, "Single and pair production of heavy leptons in E(6) model," Mod. Phys. Lett. A **21**, 1869 (2006) [arXiv:hep-ph/0601027].

3. A. T. Alan, A. T. Tasci and O. Cakir, "Heavy lepton production at Linac x LHC," *Acta Phys. Polon. B* **35**, 2199 (2004) [arXiv:hep-ph/0402184].

4. A. T. Alan, A. Senol and A. T. Tasci, "CP violating asymmetries in the flavor changing single top quark *J. Phys. G* **29**, 279 (2003) [arXiv:hep-ph/0205195].

Presentations:

1. "Single and Pair Production of Heavy Leptons in E(6) Model," A. T. Alan,

A. T. Tasci and N. Karagoz, APS April Meeting, Jacksonville, Florida, USA, April 14-17, 2007.

2. “Heavy Lepton Production at LINAC $\otimes$ LHC” A.T. Alan, A.T. Tasci, O. Cakir, Workshop on Recent Developments at High Energy Physics , University of Abant Izzet Baysal, Bolu, 23-25 June 2004.

3. “Heavy Lepton Production at Future Lepton-Hadron Colliders” A.T.Alan and A.T.Tasci, Fifth General Conference of the Balkan Physical Union, BPU-5, Vrnjacka Banja, Serbia and Montenegro, August 25-29, 2003.

4. “Single Top Production at High Energy Lepton Colliders”, Ankara High Energy Physics Workshop, Middle East Technical University, May 2002.

Conferences and Summer Schools Attended:

1. “Central European School in Particle Physics”, Institute of Particle and Nuclear Physics, Faculty of Mathematics and Physics Charles University, Prague-Czech Republic, 12-20 September 2007.

2. “International Summer School on High Energy Physics: Standard Model and Beyond”, Akyaka, Mugla-Turkey, 25-30 September 2006.

3. “Italio-Hellenic School of Physics 2006, The Physics of LHC: theoretical tools and experimental challenges”, Martignano (Lecce), Italy, June 12-18, 2006.

4. “International Workshop on Physics Beyond the Standard Model”, Mugla, Turkey, 22-26 September 2005 (http://fizik.mu.edu.tr/yef2005.html).

5. “23^{rd} International Physics Conference”, Mugla, Turkey, 13-16 September 2005.

6. “Quantization, Dualities & Integrable Systems IV” University of Abant Izzet Baysal, Bolu, 1-4 February 2005.

7. “Workshop on Recent Developments at High Energy Physics” , University of Abant Izzet Baysal, Bolu, 23-25 June 2004.

8. “Fifth General Conference of the Balkan Physical Union”, BPU-5, Vrnjacka Banja, Serbia and Montenegro, August 25-29, 2003.

9. “National Particle Accelerators and Applications Conference”, TAEK (Turkish Atomic Energy Agency), Ankara, Turkey, October 2001.

10. “1st Hellenic Turkish International Physics Conference”, Bodrum-Turkey and Kos-Greece, 10-15 September 2001.

11. “CP Violation and B Physics Workshop”, University of Ankara, Turkey, 18-21 September 2000.

Projects:

1. "Models extending the Standard Model and Searching for New particles at this Models " AIBU Research Fund, Grant No.2005.03.02.216 (Researcher).

2. "Flavor Changing Reactions at Lepton-Hadron Colliders" AIBU Research Fund, Grant No.2001.03.02.93 (Researcher)

Working Experience:

1998-2008 Working in Abant Izzet Baysal University, Physics Department as an expert.