**DOKUZ EYLÜL UNIVERSITY**

**GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES**

# VIBRATION ANALYSIS OF SYSTEMS SUBJECTED TO MOVING LOADS BY USING THE FINITE ELEMENT METHOD

**by**

**Serdar HÜGÜL**

**August, 2005**

**İZMİR**

# VIBRATION ANALYSIS OF SYSTEMS SUBJECTED TO MOVING LOADS BY USING THE FINITE ELEMENT METHOD

**A Thesis Submitted to the**

**Graduate School of Natural and Applied Sciences of Dokuz Eylül University In Partial Fulfillment of the Requirements for the Degree of Master of Science in Mechanical Engineering, Machine Theory and Dynamics Program**

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# M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled "**VIBRATION ANALYSIS OF SYSTEMS SUBJECTED TO MOVING LOADS BY USING THE FINITE ELEMENT METHOD**" completed by **Serdar HÜGÜL** under supervision of **Assist. Prof. Dr. Zeki KIRAL** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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# VIBRATION ANALYSIS OF SYSTEMS SUBJECTED TO MOVING LOADS BY USING THE FINITE ELEMENT METHOD

## ABSTRACT

In this thesis, it was purposed to comprehend the dynamic response of beams and frames which are subjected to moving point loads. The finite element method and numerical time integration method (Newmark method) are employed in the vibration analysis. The effect of the speed of the moving load on the dynamic magnification factor which is defined as the ratio of the maximum dynamic displacement at the corresponding node in the time history to the static displacement when the load is at the mid – point of the structure is investigated. The effect of the spring stiffness attached to the frame at the conjunction points of beam and columns are also evaluated. Computer codes written in Matlab are developed to calculate the dynamic responses. The results obtained from the Matlab codes are compared with the results of a commercial finite element package Ansys and good agreement is found. Dynamic responses of the engineering structures and critical load velocities can be found with high accuracy by using the finite element method.

**Keywords:** Frame vibrations, finite element method, moving load, dynamic magnification factor.

# HAREKETLİ YÜKLERE MARUZ SİSTEMLERİN SONLU ELEMANLAR YÖNTEMİ İLE TİTREŞİM ANALİZİ

## ÖZET

Bu tez çalışmasında hareketli tekil yüke maruz kiriş ve kafes yapıların dinamik cevaplarının kavranması amaçlanmıştır. Titreşim analizinde sonlu elemanlar yöntemi ve nümerik integrasyon yöntemi (Newmark metodu) kullanılmıştır. Yükün hareket hızının, yükün hareketi esnasında ilgili düğümdeki maksimum dinamik çökmenin yine aynı düğümde, yük yapının ortasında iken elde edilen statik çökmeye oranı olarak ifade edilen dinamik büyütme faktörüne etkileri incelenmiştir. Kafes tipi yapılarda kiriş ve kolonların birleşme noktalarına bağlanan yayların esnekliğinin dinamik harekete etkileri gözlemlenmiş ve değerlendirilmiştir. Dinamik analizler için Matlab kodları geliştirilmiştir. Matlab kodlarından alınan sonuçlar ticari bir sonlu elemanlar paketi olan Ansys ile karşılaştırılmış ve iyi bir uyumluluk gözlemlenmiştir. Mühendislik yapılarının dinamik cevapları ve kritik yük hızları sonlu elemanlar yöntemi kullanılarak yüksek hassasiyetle bulunabilir.

**Anahtar kelimeler**: Kafes titreşimleri, sonlu eleman yöntemi, hareketli yük, dinamik büyütme faktörü.

# CONTENTS

# CHAPTER ONE
## INTRODUCTION


## 1.1 Introduction


Beam-type structures are widely used in many branches of modern civil, mechanical and aerospace engineering. Most of the engineering structures are subjected to time and space varying loads. Moving loads have considerable effects on the dynamic behavior of the engineering structures. The dynamic analysis of a structure subjected to a moving load is an old topic of research; hence a lot of literature exists. Olsson (1991) studied the dynamics of a beam subjected to a constant force moving at a constant speed and presented analytical and finite element solutions. Thambiratnam & Zhuge (1996) studied the dynamics of beams on an elastic foundation and subjected to moving loads by using the finite element method. They investigated the effect of the foundation stiffness, traveling speed and the span length of the beam on the dynamic magnification factor, which is defined as the ratio of the maximum displacement in the time history of the mid-point to the static mid-point displacement. Wang (1997) analyzed the multi-span Timoshenko beams subjected to a concentrated moving force by using the mode superposition method and made a comparison between the Euler-Bernoulli beam and Timoshenko beam. Zheng et al. (1998) analyzed the vibration of a multi span non uniform beam subjected to a moving load by using modified beam vibration functions as the assumed modes based on Hamilton's principle. The modified beam vibration functions satisfy the zero deflection conditions at all the intermediate point supports as well as the boundary conditions at the two ends of the beam. Numerical results are presented for both uniform and non uniform beams under moving loads of various velocities. Wang & Lin (1998) studied the vibration of multi-span Timoshenko frames due to moving loads by using the modal analysis. Kadivar & Mohebpour (1998) analyzed the dynamic responses of unsymmetric composite laminated orthotropic beams under the action of moving loads. Hong & Kim (1999)

presented the modal analysis of multi span Timoshenko beams connected or supported by resilient joints with damping. The results are compared with FEM. Ichikawa et al. (2000) investigated the dynamic behavior of the multi-span continuous beam traversed by a moving mass at a constant velocity, in which it is assumed that each span of the continuous beam obeys uniform Euler-Bernoulli beam theory. The solution to this system is simply obtained by using both eigenfunction expansion or the modal analysis method and the direct integration method in combination. The effects of the inertia and the moving load velocity on the dynamic response of the continuous beam are evaluated for three kinds of continuous beams having uniform span length. Zibdeh & Hilal (2000) investigated the vibration analysis of beams with generally boundary conditions traversed by a moving force. The moving load is assumed to move with accelerating, decelerating and constant velocity type of motions. They showed the effects of type of motion, boundary conditions and damping. Chen et al. (2001) calculated the response of an infinite Timoshenko beam on a viscoelastic foundation to a harmonic moving load. Wu et al. (2001) presented dynamic responses of the structures to moving bodies using combined finite element and analytical methods including inertia effects. Savin (2001) calculated the dynamic amplification factor and the characteristic response spectrum for weakly damped beams with various boundary conditions. Michaltsos (2002) investigated the dynamic behavior of a single span beam subjected to loads moving with variable speeds including the damping effect. Oniszczuk (2003) analyzed undamped forced transverse vibrations of an elastically connected double-beam system. The problem is formulated and solved in the case of simply supported beams and the classical modal expansion method is applied. Zibdeh & Hilal (2003) investigated the random vibration of simply - supported laminated composite coated beam traversed by a random moving load. The moving load is assumed to move with accelerating, decelerating and constant velocity type of motions. Kim (2004) investigated the vibration and stability of an infinite Euler - Bernoulli beam resting on a Winkler foundation when the system is subjected to a static axial force and a moving load with either constant or harmonic amplitude variations. The effects of load speed, load frequency, damping on the deflected shape, maximum displacement and critical values of the velocity, frequency and axial force are also studied.

Kargarnovin & Younesian (2004) studied the response of a Timoshenko beam with uniform cross – section and infinite length supported by a generalized Pasternak – type viscoelastic foundation subjected to an arbitrary – distributed harmonic moving load. Bilello & Bergman (2004) presented a theoretical and experimental study on the response of a damaged Euler – Bernoulli beam traversed by a moving mass. Damage is modeled through rotational springs whose compliance is evaluated using linear elastic fracture mechanics. Law & Zhu (2004) studied the dynamic behavior of damaged reinforced concrete bridge structures under moving vehicular loads. The vehicle is modeled as a moving mass or by four - degree of freedom system with linear suspensions and tires flexibility, and the bridge is modeled as a continuous Euler-Bernoulli beam simply supported at both ends. Wu (2005) presented a technique for predicting the dynamic responses of a two dimensional (2-D) full-size rectangular plate undergoing a transverse moving line load by using the one dimensional (1-D) equivalent beam model.

In this study, the finite element method is used for constituting the element matrices. The Newmark integration method is employed for the forced vibration analysis. Computer codes written in Matlab (Hanselman, 2001) are developed to calculate the dynamic responses of the considered structures. Euler-Bernoulli beam theory is used in the finite element vibration analysis. Three different boundary conditions are considered in beam vibrations (clamped - clamped, clamped - pinned, pinned - pinned). The clamped – clamped boundary condition is considered for the vibration analysis of the frame structure. A comprehensive study including the dynamic responses of the columns of frame structure is carried out. The effects of the speed of the moving load on the dynamic magnification factor and the effect of the spring stiffness attached to the frame at the conjunction points of columns and beam are investigated. The results obtained in this study are compared with the results obtained from the commercial finite element package Ansys  (Moevani, 1999).

**1.2 Objective**

The objective of this thesis is to develop a numerical procedure for evaluating the dynamic analysis of the beam and frame structures which are subjected to moving point loads. The finite element method is used for modeling the beam and frame elements. The equations of motion of the structure subjected to the moving load are derived by the application of standard method of structural dynamic. Furthermore, the procedure applies numerical integration method, namely, Newmark integration method, to obtain the dynamic response. This study is performed to evaluate the effect of the following parameters on the dynamic behavior of the beam and frame;

1. The traveling speed of the load (dimensionless velocity parameter $\alpha$ ).
2. Boundary conditions for the beam vibrations.
3. Attached springs at the conjunction points of the beam and columns of the frame.
4. The stiffness of the springs.
5. Viscous damping.

**1.3 Overview of the Thesis**

The solution of the moving load problem is achieved by developing computer programs to calculate the dynamic displacements of the beam and frame structures subjected to moving point loads.

This chapter has presented a background of the problem. Chapter 2 presents a literature of numerical integration methods. Chapter 3 presents the theoretical development of mass and stiffness matrices and explains shortly the finite element method. The forced vibration response and the application of the Newmark method to the moving load problem are also evaluated in this chapter. Chapter 4 presents the results, discussions and interpretations of the figures. A list of the computer programs is included in the Appendices.

# CHAPTER TWO
# NUMERICAL METHODS

## 2.1 Introduction

The most general approach for the solution of the dynamic response of structural systems is the direct numerical integration of the dynamic equilibrium equations. This involves, after the solution is defined at time zero, the attempt to satisfy dynamic equilibrium at discrete points in time. Most methods use equal time intervals at $\Delta t$, $2\Delta t$, $3\Delta t$,… $N\Delta t$. Many different numerical techniques have previously been presented; however, all approaches can fundamentally be classified as either *explicit* or *implicit* integration methods.

Explicit methods do not involve the solution of a set of linear equations at each step. Basically, these methods use the differential equation at time "t" to predict a solution at time "t + $\Delta t$". For most real structures, which contain stiff elements, a very small time step is required in order to obtain a stable solution. Therefore, all explicit methods are *conditionally stable* with respect to the size of the time step.

Implicit methods attempt to satisfy the differential equation at time "t" after the solution at time "t - $\Delta t$" is found. These methods require the solution of a set of linear equations at each time step; however, larger time steps may be used. Implicit methods can be *conditionally or unconditionally* stable.

There exist a large number of accurate, higher-order, multi-step methods that have been developed for the numerical solution of differential equations. These multi-step methods assume that the solution is a smooth function in which the higher derivatives are continuous. The exact solution of many nonlinear structures requires that the accelerations, the second derivative of the displacements, are not smooth

functions. This discontinuity of the acceleration is caused by the nonlinear hysteresis of most structural materials, contact between parts of the structure, and buckling of elements. Therefore, only single-step methods will be presented in this chapter. Based on a significant amount of experience, it is the conclusion that only single-step, implicit, unconditional stable methods be used for the step-by-step analysis of practical engineering structures.

## 2.2 Newmark Family of Methods

The Newmark integration method is based on the assumption that the acceleration varies linearly between two instants of time. In 1959 Newmark presented a family of single-step integration methods for the solution of structural dynamic problems for both blast and seismic loading. During the past 45 years Newmark's method has been applied to the dynamic analysis of many practical engineering structures. In addition, it has been modified and improved by many other researchers. In order to illustrate the use of this family of numerical integration methods, we considered the solution of the linear dynamic equilibrium equations written in the following form:

$$[M]\ddot{u}_t + [C]\dot{u}_t + [K]u_t = F_t \tag{2.1}$$

where M is the mass matrix, C is the damping matrix and K is the stiffness matrix. $\ddot{u}, \dot{u}$ and u are the acceleration, velocity and displacement vectors, respectively. $F_t$ is the external loading vector. The direct use of Taylor's series provides a rigorous approach to obtain the following two additional equations:

$$u_t = u_{t-\Delta t} + \Delta t\dot{u}_{t-\Delta t} + \frac{\Delta t^2}{2}\ddot{u}_{t-\Delta t} + \frac{\Delta t^3}{6}\dddot{u}_{t-\Delta t} + ... \tag{2.2}$$

$$\dot{u}_t = \dot{u}_{t-\Delta t} + \Delta t\ddot{u}_{t-\Delta t} + \frac{\Delta t^2}{2}\dddot{u}_{t-\Delta t} + ... \tag{2.3}$$

Newmark truncated these equations and expressed them in the following form:

$$u_t = u_{t-\Delta t} + \Delta t \dot{u}_{t-\Delta t} + \frac{\Delta t^2}{2} \ddot{u}_{t-\Delta t} + \beta \Delta t^3 \dddot{u} \tag{2.4}$$

$$\dot{u}_t = \dot{u}_{t-\Delta t} + \Delta t \ddot{u}_{t-\Delta t} + \gamma \Delta t^2 \dddot{u} \tag{2.5}$$

If the acceleration is assumed to be linear within the time step, the following equation can be written:

$$\dddot{u} = \frac{\ddot{u}_t - \ddot{u}_{t-\Delta t}}{\Delta t} \tag{2.6}$$

The substitution of equation (2.6) into Equations (2.4) and (2.5) produces Newmark's equations in standard form

$$u_t = u_{t-\Delta t} + \Delta t \dot{u}_{t-\Delta t} + (\frac{1}{2} - \beta)\Delta t^2 \ddot{u}_{t-\Delta t} + \beta \Delta t^2 \ddot{u}_t \tag{2.7}$$

$$\dot{u}_t = \dot{u}_{t-\Delta t} + (1 - \gamma)\Delta t \ddot{u}_{t-\Delta t} + \gamma \Delta t \ddot{u}_t \tag{2.8}$$

## 2.3 Stability of Newmark's Method

For zero damping Newmark's method is conditionally stable if

$$\gamma \geq \frac{1}{2}, \beta \leq \frac{1}{2} \text{ and } \Delta t \leq \frac{1}{\omega_{max} \sqrt{\frac{\gamma}{2} - \beta}} \tag{2.9}$$

where $\omega_{max}$ is the maximum frequency in the structural system (Newmark, 1959). Newmark's method is unconditionally stable if

$$2\beta \geq \gamma \geq \frac{1}{2} \tag{2.10}$$

However, if $\gamma$ is greater than 1/2, errors are introduced. These errors are associated with "numerical damping" and "period elongation". For large multi-

degree of freedom structural systems the time step limit, given by equation (2.9), can be written in a more usable form as

$$\frac{\Delta t}{T_{min}} \leq \frac{1}{2\pi\sqrt{\frac{\gamma}{2} - \beta}} \tag{2.11}$$

where $T_{min}$ is the minimum time period of the structure. Computer model of large real structures normally contain a large number of periods which are smaller than the integration time step; therefore, it is essential that one select a numerical integration method that is unconditionally stable for all time steps. Table 2.1 shows the summary of the Newmark method for direct integration.

Table 2.1 The Newmark integration scheme

Initial Calculations:

dof = Total degree of freedom of the system

The initial displacement, velocity and acceleration vectors are

$$U_0 = \left.\begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix}\right\} \ \text{dof} \ , \qquad \dot{U}_0 = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix} , \qquad \ddot{U}_0 = \frac{M}{F_0(:,1) - KU_0}$$

$\Delta t = T_{20}/20; \ \beta = \dfrac{1}{4}, \ \gamma = \dfrac{1}{2}$

$a_0 = \dfrac{1}{\beta \Delta t^2}$ , $a_1 = \dfrac{\gamma}{\beta \Delta t}$ , $a_2 = \dfrac{1}{\beta \Delta t}$ , $a_3 = \dfrac{1}{2\beta} - 1$ , $a_4 = \dfrac{\gamma}{\beta} - 1$ , $a_5 = \dfrac{\Delta t}{2}(\dfrac{\gamma}{\beta} - 2)$

$a_6 = \Delta t(1 - \gamma)$ , $a_7 = \gamma \Delta t$

Form effective stiffness matrix K: $K = K + a_0 M$
Triangularize K: $K = LDL^T$

For each time step:

   Calculate effective loads at time $t + \Delta t$ :

   $^{t+\Delta t}\hat{F} = {}^{t+\Delta t}F + M(a_0 \, {}^t U + a_2 \, {}^t \dot{U} + a_3 \, {}^t \ddot{U})$

   Solve for displacements at time $t + \Delta t$ :

   $LDL^{T \ t+\Delta t}U = {}^{t+\Delta t}\hat{F}$

   Calculate accelerations and velocities at time $t + \Delta t$

   $^{t+\Delta t}\ddot{U} = a_0({}^{t+\Delta t}U - {}^t U) - a_2 \, {}^t \dot{U} - a_3 \, {}^t \ddot{U}$

   $^{t+\Delta t}\dot{U} = {}^t \dot{U} + a_6 \, {}^t \ddot{U} + a_7 \, {}^{t+\Delta t}\ddot{U}$

## 2.4 Newmark Integration Parameters

- $\beta = \dfrac{1}{6}$, $\gamma = \dfrac{1}{2}$; which lead to a linear acceleration approximation (conditionally stable scheme);

- $\beta = \dfrac{1}{4}$, $\gamma = \dfrac{1}{2}$; which lead to a constant average acceleration. This choice of parameters corresponds to a trapezoidal rule (unconditionally stable scheme in linear analyzes);

- $\beta = \dfrac{1}{12}$, $\gamma = \dfrac{1}{2}$; is the Fox-Goodwin method which is fourth order accurate (conditionally stable).
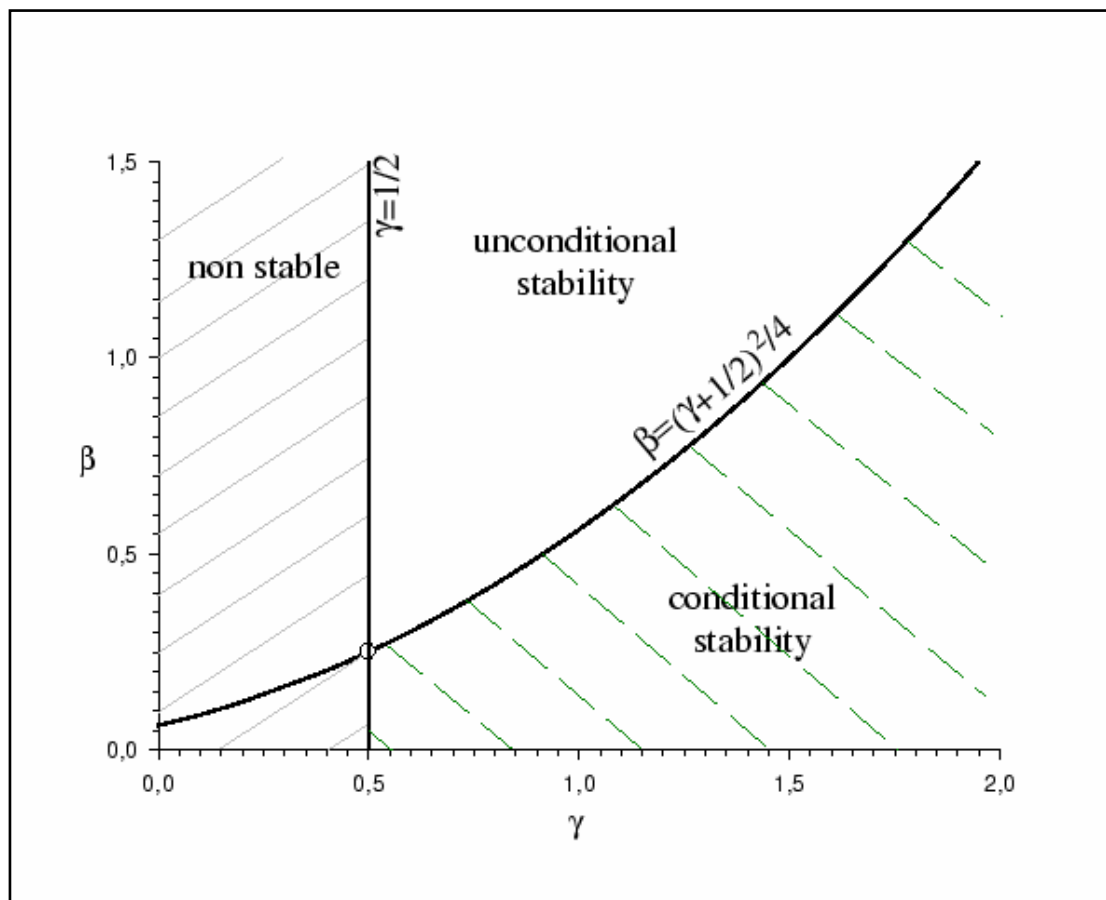


Figure 2.1 Stability of Newmark time integration schemes

It is apparent that a large number of different direct numerical integration methods are possible by specifying different integration parameters. A few of the most commonly used methods are summarized. Figure 2.1 shows stability of Newmark time integration scheme.

In this study, Newmark integration method is used with the integration parameters $\beta = 1/4$ and $\gamma = 1/2$, which lead to constant-average acceleration approximation.

## 2.5 Application of Newmark Integration Method

The computer codes written in Matlab are used for the vibration analysis of a two degree of freedom system shown in Figure 2.2. Two situations are considered for the external force vector, $f_0$

- Impuls function
- Step function

Figure 2.3 shows the free body diagram of the system. Dynamic responses of this system to impulse and step loadings are also evaluated by using Ansys and Laplace method. The results are given in figures 2.4 and 2.5. The computer codes written in Matlab and Ansys for impuls and step responses of this system are given in Appendix.
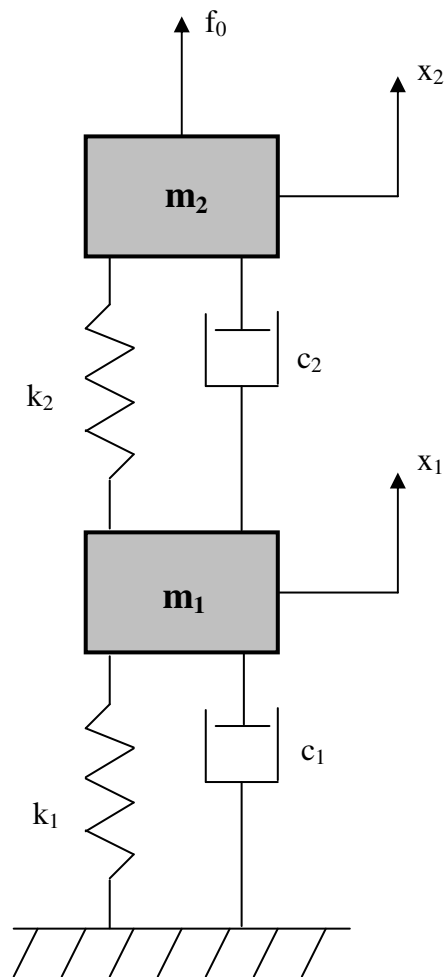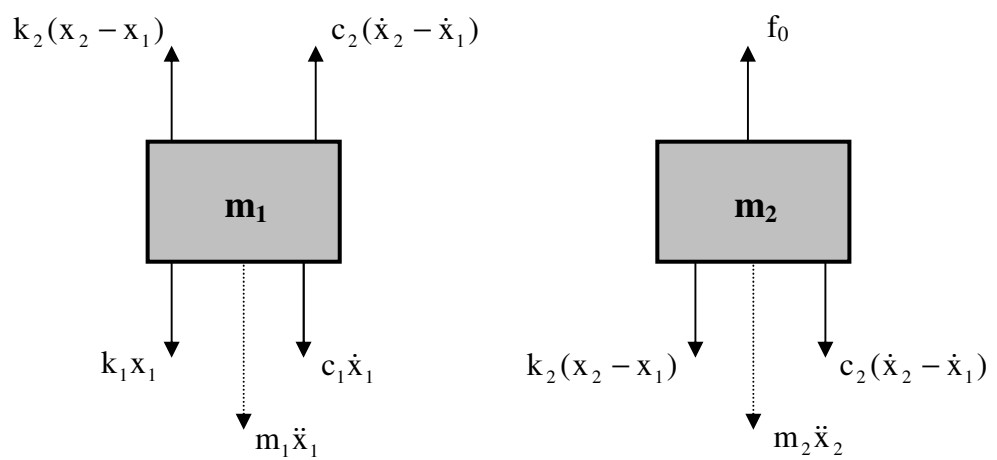
Figure 2.2 Two - degree of freedom system



Figure 2.3 Free body diagram of the system

### *2.5.1 Equations of Motion*

The application of the Newton's second law yields the following equations,

$$m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 + (k_1 + k_2)x_1 - c_2\dot{x}_2 - k_2x_2 = 0 \tag{2.12}$$

$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 - c_2\dot{x}_1 - k_2x_1 = f_0 \tag{2.13}$$

By taking the Laplace transform

$$(m_1s^2 + (c_1 + c_2)s + k_1 + k_2)X_1(s) = (c_2s + k_2)X_2(s) \tag{2.14}$$

$$(m_2s^2 + c_2s + k_2)X_2(s) = f_0(s) + (c_2s + k_2)X_1(s) \tag{2.15}$$

Rearranging Eqs. (2.14) and (2.15)

$$X_1(s) = \frac{f_0(s)(c_2s + k_2)}{(m_2s^2 + c_2s + k_2)(m_1s^2 + (c_1 + c_2)s + k_1 + k_2) - (c_2s + k_2)^2} \tag{2.16}$$

$$X_2(s) = \frac{f_0(s)(m_1s^2 + (c_1 + c_2)s + k_1 + k_2)}{(m_2s^2 + c_2s + k_2)(m_1s^2 + (c_1 + c_2)s + k_1 + k_2) - (c_2s + k_2)^2} \tag{2.17}$$

The following data are given;
$$m_1 = m_2 = 50 \text{ kg}; \quad c_1 = c_2 = 1000 \text{ Ns/m}; \quad k_1 = k_2 = 30000 \text{ N/m}$$

- *Case 1*: $f_0$ is unit impulse excitation. Figure 2.4 shows the results of three codes. newmarkimp, ansysimp, matlabimp.
- *Case 2*: $f_0$ is unit step excitation. Figure 2.5 shows the results of three codes. newmarkstep, ansysstep, matlabstep.

As shown in figures 2.4 and 2.5 newmark code is applicable to this system and there are no noteworthy differences between the newmark code, Ansys and Matlab (Laplace method) results.
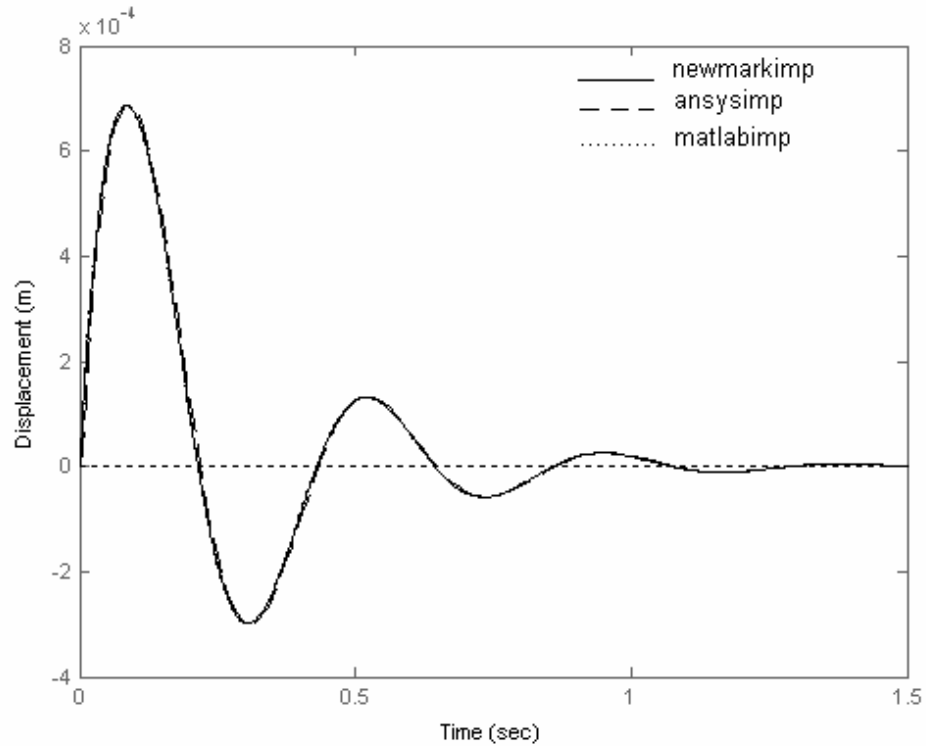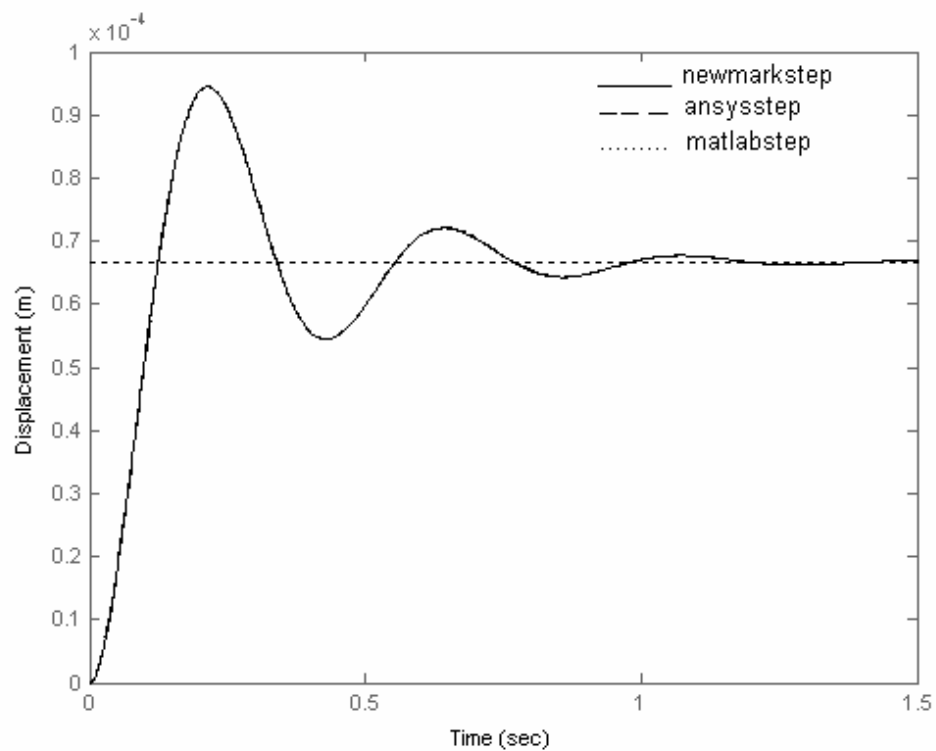


Figure 2.4 Impuls response of mass 1



Figure 2.5 Step responses of mass 2

# CHAPTER THREE
# THE FINITE ELEMENT METHOD

## 3.1 Beam Element

A straight beam element with uniform cross section is shown in Figure 3.1. The Euler-Bernoulli beam theory is used for constituting the finite element matrices. The longitudinal axis of the element lies along the x axis. The element has a constant moment of inertia I, modulus of elasticity E, density $\rho$ and length $\ell$. Two degrees of freedom per node, translation along y-axis ($y_1$, $y_2$) and rotation about z-axis ($y_1'$, $y_2'$) are considered. The beam is modeled with 20 equally sized elements.
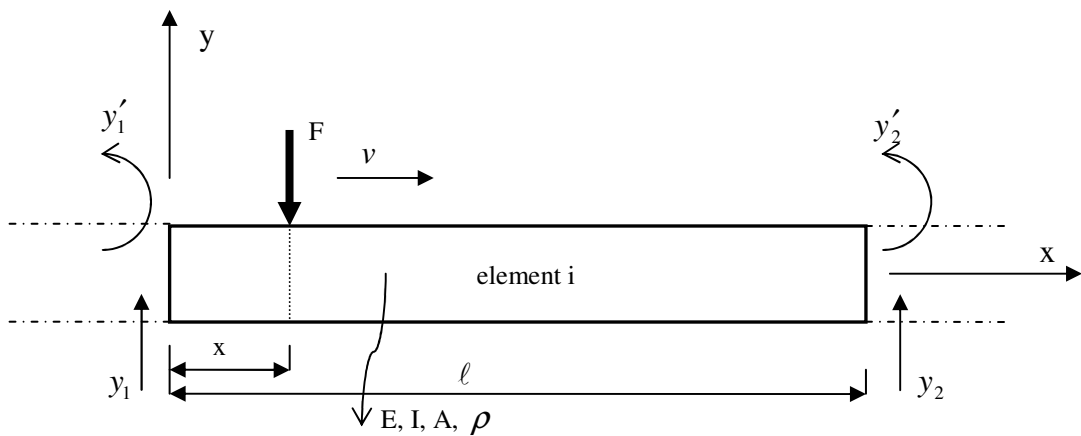


Figure 3.1 Moving load on a straight beam element

Strain energy of a single element is:

$$\text{S.E.} = \int_0^\ell \frac{EI}{2} (\frac{\partial^2 y}{\partial x^2})^2 \, dx \tag{3.1}$$

Kinetic energy of a single element:

$$\text{K.E.} = \int_0^\ell \frac{\rho A}{2} (\frac{\partial y}{\partial t})^2 \, dx \tag{3.2}$$

A cubic function, y(x,t), is assumed for the transverse displacements as

$$y(x,t) = a_0(t) + a_1(t)x + a_2(t)x^2 + a_3(t)x^3 \tag{3.3}$$

where $a_0(t)$, $a_1(t)$, $a_2(t)$ and $a_3(t)$ are obtained by applying the boundary conditions at the corresponding nodes, so the shape functions Eqs. (3.4) - (3.7) are represented as (Rao, 1995)

$$N_1(x) = 1 - 3\left(\frac{x}{\ell}\right)^2 + 2\left(\frac{x}{\ell}\right)^3 \tag{3.4}$$

$$N_2(x) = x - 2\ell\left(\frac{x}{\ell}\right)^2 + \ell\left(\frac{x}{\ell}\right)^3 \tag{3.5}$$

$$N_3(x) = 3\left(\frac{x}{\ell}\right)^2 - 2\left(\frac{x}{\ell}\right)^3 \tag{3.6}$$

$$N_4(x) = -\ell\left(\frac{x}{\ell}\right)^2 + \ell\left(\frac{x}{\ell}\right)^3 \tag{3.7}$$

at x=0  $y_1 = a_0$    $\dot{y}_1 = a_1$

at x=$\ell$  $y_2 = a_0 + a_1\ell + a_2\ell^2 + a_3\ell^3$    $\dot{y}_2 = a_1 + 2a_2\ell + 3a_3\ell^2$

$$\begin{bmatrix} y_1 \\ \dot{y}_1 \\ y_2 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & \ell & \ell^2 & \ell^3 \\ 0 & 1 & 2\ell & 3\ell^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\{y\} = [C]\{a\} \tag{3.8}$$

$$\{a\} = [C]^{-1}\{y\} \tag{3.9}$$

$$y = [x]\{a\} = \begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = [x][C]^{-1}\{y\} \tag{3.10}$$

$$\frac{\partial y}{\partial t} = [X][C]^{-1}\{\dot{y}\} \quad \text{and} \quad \left(\frac{\partial^2 y}{\partial t^2}\right) = \left\{\frac{\partial y}{\partial t}\right\}^{T}\left\{\frac{\partial y}{\partial t}\right\} \qquad (3.11)$$

$$\text{K.E.} = \frac{1}{2}\int_0^\ell \rho A\{\dot{y}\}^{T}[C]^{-T}\begin{bmatrix}1\\ x\\ x^2\\ x^3\end{bmatrix}\begin{bmatrix}1 & x & x^2 & x^3\end{bmatrix}[C]^{-1}\{\dot{y}\}dx \qquad (3.12)$$

From equation (3.12) we find the mass matrix as follow:

$$\mathbf{M} = \frac{\rho A\ell}{420}\begin{bmatrix} 156 & 22\ell & 54 & -13\ell \\ 22\ell & 4\ell^2 & 13\ell & -3\ell^2 \\ 54 & 13\ell & 156 & -22\ell \\ -13\ell & -3\ell^2 & -22\ell & 4\ell^2 \end{bmatrix} \qquad (3.13)$$

$$\text{S.E.} = \frac{1}{2}\int_0^\ell EI\left\{\frac{\partial^2 y}{\partial x^2}\right\}^{2} dx$$

From equation (3.10);

$$\frac{\partial^2 y}{\partial x^2} = \begin{bmatrix}0 & 0 & 2 & 6x\end{bmatrix}[C]^{-1}\{y\} \qquad (3.14)$$

$$\text{S.E.} = \frac{1}{2}\int_0^\ell EI\{y\}^{T}[C]^{-T}\begin{bmatrix}0\\ 0\\ 2\\ 6x\end{bmatrix}\begin{bmatrix}0 & 0 & 2 & 6x\end{bmatrix}[C]^{-1}\{y\} \qquad (3.15)$$

And from equation (3.15) we find the stiffness matrix as follow:

$$\mathbf{K} = \frac{EI}{\ell^3}\begin{bmatrix} 12 & 6\ell & -12 & 6\ell \\ 6\ell & 4\ell^2 & -6\ell & 2\ell^2 \\ -12 & -6\ell & 12 & -6\ell \\ 6\ell & 2\ell^2 & -6\ell & 4\ell^2 \end{bmatrix} \qquad (3.16)$$

The overall mass and stiffness matrices of the structure, [M] and [K], are constituted by joining 20 element matrices using Matlab code.

## 3.2 Frame Element

In this section, the moving load problem is extended to a frame structure. Axial displacement for longitudinal vibration of the frame element is assumed to be linear

$$u(x, t) = e(t) + f(t)x \qquad (3.17)$$

so the shape functions for the longitudinal vibration are,

$$N_5(x) = \left(1 - \frac{x}{\ell}\right) \qquad (3.18)$$

$$N_6(x) = \frac{x}{\ell} \qquad (3.19)$$

Three degrees of freedom per node, translation along x-axis, translation along y-axis and rotation about z-axis is assumed for frame element as shown in Figure 3.2. The coupling between bending and longitudinal vibrations is neglected. The stiffness and mass matrices for the frame element are constructed by superimposing both the axial and bending matrices.
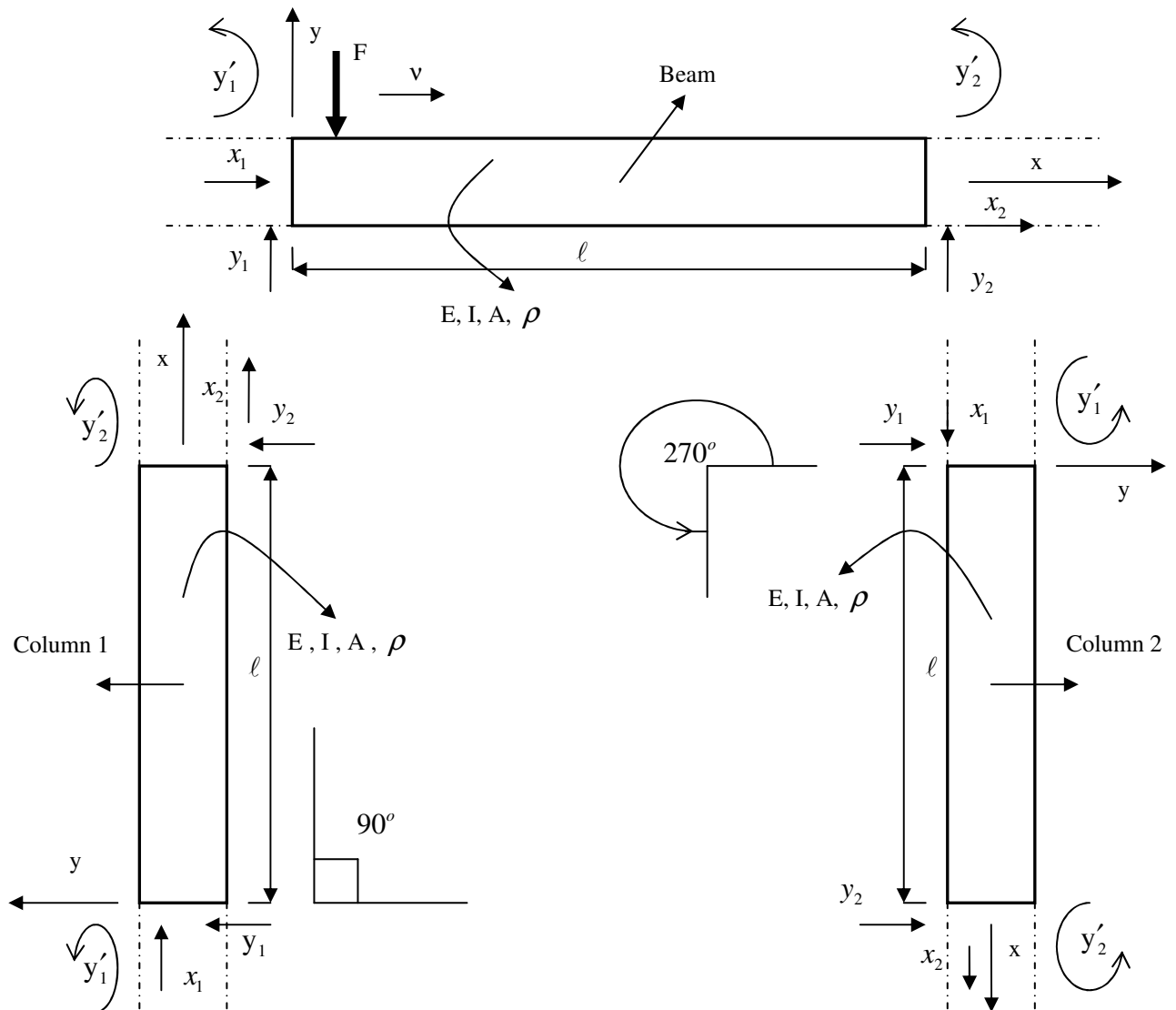
Figure 3.2 Modeling the frame elements

The transformation matrices are used to form the mass and stiffness matrices for the columns of the frame structure. Both columns and beam of the frame structure are modeled with 10 equally sized elements. All the element mass and stiffness matrices ( [K] and [M] ) are multiplied by the transformation matrix [T].

$[K] = [T]^{'}[K][T]$ and $[M] = [T]^{'}[M][T]$ where

$$[T] = \begin{bmatrix} c_\theta & s_\theta & 0 & 0 & 0 & 0 \\ -s_\theta & c_\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_\theta & s_\theta & 0 \\ 0 & 0 & 0 & -s_\theta & c_\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} , \quad c_\theta = \cos(\theta) , \ s_\theta = \sin(\theta) \tag{3.20}$$

$\theta$ for column 1 is $90^o$ and $\theta$ for column 2 is $270^o$ as shown in Figure 3.2

A spring is attached to the frame at the column and beam conjunction points in order to analyze the effect of the spring stiffness. Spring has a stiffness k in the x direction as shown in Figure 3.3.
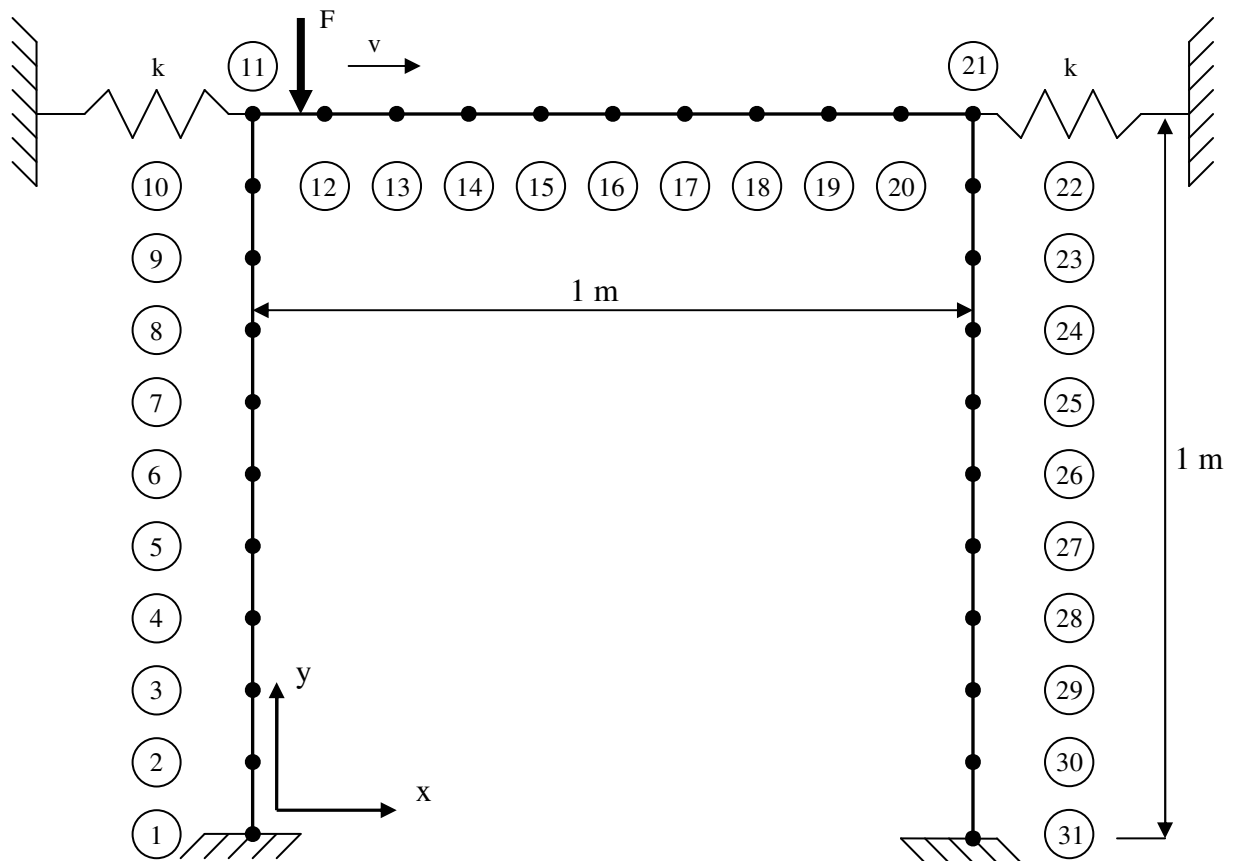


Figure 3.3 Spring attached frame

The effect of springs on the dynamic response of the frame structure is investigated. The stiffness of the spring k is added to column's stiffness matrices as a constant term at the corresponding degree of freedom.

### 3.3 Equations of Motion of the Beam and Frame Structure

The equation of motion for a multiple degree of freedom undamped structural system is represented as follows

$$[M]\{\ddot{y}\} + [K]\{y\} = \{F(t)\} \tag{3.21}$$

Where $\ddot{y}$ and y are the respective acceleration and displacement vectors for the whole structure and $\{F(t)\}$ is the external force vector.

Under free vibration, the natural frequencies and the mode shapes of a multiple degree of freedom system are the solutions of the eigenvalue problem.

$$[[K] - \omega^2 [M]]\{\phi\} = 0 \tag{3.22}$$

where $\omega$ is the angular natural frequency and $\phi$ is the mode shape of the structure for the corresponding natural frequency.

The dynamic analysis is performed for two geometry, beam and frame. A beam with a length L = 1m, a uniform cross section with 0.01m x 0.01m, modulus of elasticity E = 206 x $10^9$ N/m$^2$, mass density $\rho$ = 7860 kg/m$^3$ is considered and a constant force F = -100 N is used. The same properties are also valid for the columns of the frame structure. Isotropic material properties are assumed. Two different spring stiffness, $k_1$ = 50000 N/m and $k_2$ = 200000 N/m are used in order to show the effect of the varying stiffness on the dynamic responses.

The first three natural frequencies of the beam and frame structure are given for the comparison between Matlab and Ansys results in Table 3.1. There are negligible discrepancies between the results of the Matlab and Ansys as shown in the table. A longer beam implies a smaller first natural frequency for frame structure, similarly longer columns imply smaller natural frequencies.

Table 3.1 The first three natural frequencies of the beam and the frame structure.

|  | Matlab Results (Hz) | | | Ansys Results (Hz) | | |
|---|---|---|---|---|---|---|
| Modes | 1 | 2 | 3 | 1 | 2 | 3 |
| BEAM | | | | | | |
| Clamped-Clamped | 52.62 | 145.06 | 284.39 | 52.62 | 145.07 | 284.54 |
| Clamped-Pinned | 36.26 | 117.52 | 245.21 | 36.26 | 117.52 | 245.28 |
| Pinned-Pinned | 23.21 | 92.86 | 208.93 | 23.21 | 92.85 | 280.96 |
| FRAME | | | | | | |
| Frame | 7.537 | 29.74 | 48.52 | 7.537 | 29.74 | 48.52 |
| Spring Attached ($k_1$) | 29.74 | 36.82 | 52.61 | 29.74 | 36.82 | 52.60 |
| Spring Attached ($k_2$) | 29.74 | 42.26 | 52.61 | 29.74 | 42.26 | 52.60 |

### 3.4 The Effect of Viscous Damping

A proportional damping is assumed to show the effect of damping ratio on the dynamic magnification factor. Rayleigh damping, in which the damping matrix is proportional to the combination of the mass and stiffness matrices, is used.

$$[C] = a_0 [M] + a_1 [K] \tag{3.23}$$

If the damping ratios $\xi_m$ and $\xi_n$ associated with two specific frequencies $\omega_m$, $\omega_n$, Rayleigh damping factors, $a_0$ and $a_1$ can be evaluated by the solution of the following equation (Clough, Penzien 1993).

$$\left\{\begin{matrix} a_0 \\ a_1 \end{matrix}\right\} = 2 \frac{\omega_m \omega_n}{\omega^2_n - \omega^2_m} \begin{bmatrix} \omega_n & -\omega_m \\ -1 & 1 \\ \omega_n & \omega_m \end{bmatrix} \left\{\begin{matrix} \xi_m \\ \xi_n \end{matrix}\right\} \tag{3.24}$$

The same damping ratio is applied to both control frequncies, $\omega_1$ and $\omega_2$; i.e., $\xi = \xi_1 = \xi_2$. Then the proportionality factors can be given in simplified form of Eq. 3.24

$$\left\{\begin{matrix} a_0 \\ a_1 \end{matrix}\right\} = \frac{2\xi}{\omega_1 + \omega_2} \left\{\begin{matrix} \omega_1\omega_2 \\ 1 \end{matrix}\right\} \tag{3.25}$$

Two different damping ratios, $\xi = 0.01$ and $\xi = 0.05$ are used to show the effect of the damping ratio on the dynamic magnification factor.

### 3.5 The Application of the Moving Force

The transverse point load F has a constant velocity, $v = L/\tau$, where $\tau$ is the traveling time across the beam and L is the total length of the beam.

For the forced vibration analysis an implicit time integration method, called as the Newmark integration method is used with the integration parameters $\beta = 1/4$ and $\gamma = 1/2$, which lead to constant-average acceleration approximation. The time step is chosen as $\Delta t = T_{20}/20$ during the beam and frame vibration analysis in order to ensure that all the 20 modes contribute to the dynamic response, where $T_{20}$ is the period of the 20th natural mode of the structure.

The time history of the nodal force in the transverse direction is given in Figure 3.4. The time for the load to arrive i[th] node, $\tau_i = x_i/v$, where $x_i$ is the location of ith node. The nodal force on the ith node, $F_i = 0$ except $\tau_{i-1} < t < \tau_{i+1}$. The force is applied all the nodes according to Figure 3.4. Moment effect of the force is ignored, only vertical degree of freedom is affected by this force. The Simple method in which $M_i = 0$ at any time (Wu, Whittaker & Cartmell, 2000) is used for the calculation of the

dynamic responses. A non-dimensional velocity parameter $\alpha$ is used as $\alpha = T_1 / \tau$, where $T_1$ is the period of the first natural frequency of the beam. To indicate the effect of the springs on the dynamic magnification factors of the frame structure, the same $T_1$, which belongs to the first natural period of the frame structure is used for both frame and spring attached frames while evaluating the $\alpha$ values. The computer codes written in Matlab and Ansys for the forced vibration of pinned – pinned beam (pinned_pinned.m, pinned_pinned.txt) and frame (frame.m, frame.txt) are given in Appendix.
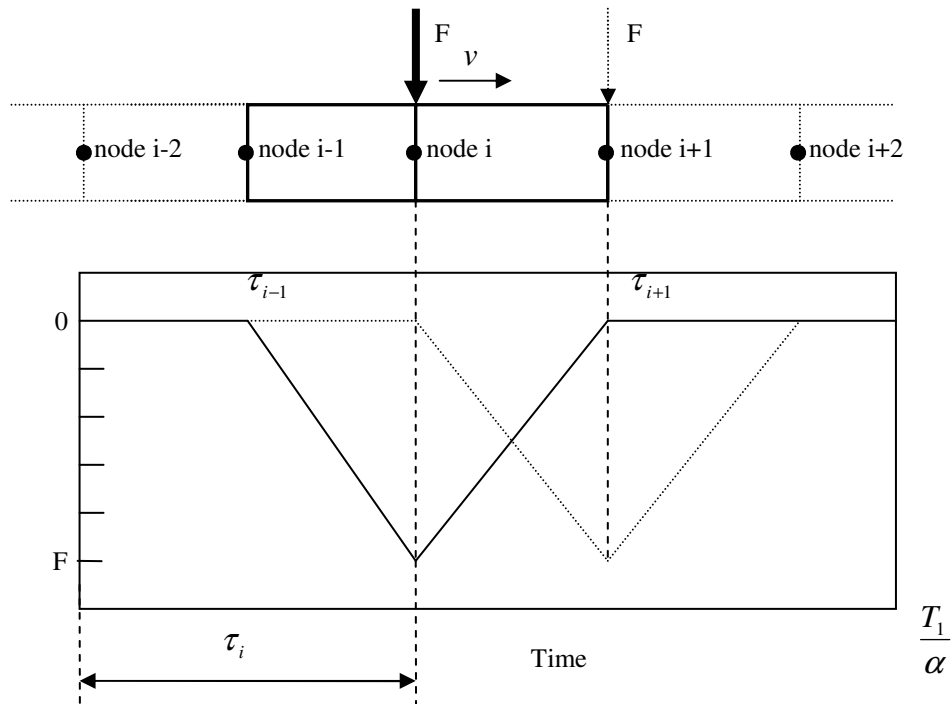


Figure 3.4 Moving load time history for the ith node

# CHAPTER FOUR
# RESULTS AND DISCUSSION

## 4.1 Results of the Dynamic Analyzes

Figure 4.1 (a-d) shows the dynamic responses of the mid-points of the beam and frame structure obtained from Matlab and Ansys for different boundary conditions and for various $\alpha$ values. The Matlab results agree well with the results obtained using Ansys regardless of $\alpha$ value as shown in this figure.

Figures 4.2-4.4 show the mid – point displacements versus the position of the moving load on the beam for different boundary conditions and various $\alpha$ values for the beam structure. With small $\alpha$ values ($\alpha$ =0.01) displacement curve is close to the static displacement curve for all boundary conditions. The moving load and the maximum dynamic displacements of the mid-point of the beam are not in the same phase at overcritical part as explained in a previous study (Kadivar and Mohebpour, 1998). The time at which the maximum mid – point displacement is observed shifts right with increasing $\alpha$ values regardless of the boundary condition of the beam. The highest dynamic deflections occur for the pinned – pinned beam as expected.

Figure 4.5 shows the dynamic magnification factor ($D_d$) of the mid - point of the beam versus $\alpha$ for three different boundary conditions. The maximum dynamic magnification occurs for $\alpha$ = 1.02 and $D_d$ = 1.632 for clamped – clamped beam. For the clamped – pinned beam the maximum value of the dynamic magnification is 1.662 and observed at $\alpha$ = 1.31. For pinned – pinned beam, the maximum $D_d$ = 1.728 and recorded at $\alpha$ = 1.22. For pinned – pinned boundary conditions the

dynamic magnification values are greater than those obtained for clamped – clamped and clamped – pinned beams for low and high moving load speeds. The clamped – clamped boundary conditions generally gives the lower dynamic magnification values except in the middle speed region.

Dynamic magnification factors for the mid – point of the beams are given for various $\alpha$ values and different boundary conditions in Table 4.1. These values are only valid in the time interval that the moving load is on the beam.

Figure 4.6 shows the first three mode shapes of the frame and spring attached frame ($k_2$). Attaching a spring to the frame at the conjunction points of the beam and columns makes the frame more rigid and shifts the mode shapes of the frame structure up.

Figures 4.7 (a-c) show the effect of the springs on the mid – point displacements of beam and columns. Three critical load speeds shown in Figures 4.8b, 4.9b and 4.10b are considered for columns and beam. $\upsilon = 4.522$ m/s for column 1 ($\alpha = 0.6$), $\upsilon = 34.67$ m/s for beam ($\alpha = 4.6$), $\upsilon = 45.22$ m/s for column 2 ($\alpha = 6$). Figure 4.7a shows that, springs have more effect on the dynamic response of the mid-point of column 1 for the critical load speed and reduce the maximum dynamic displacements. On the other hand, springs have no contribution for beam displacements as shown in Figure 4.7b and also no noteworthy contribution for the mid – point displacement of column 2 as shown in Figure 4.7c.

Figures 4.8 – 4.10 show the effect of $\alpha$ values on the dynamic magnification factor for different nodes of a normal and spring attached frame. Dynamic magnification factors are calculated for frame structure as calculated for beam. $D_d$ is the ratio of the maximum dynamic displacement to the static displacement at the considered node. The static displacements for all the nodes of the beam and columns of the frame are calculated when the force acting on the mid – point of the beam. There is no noteworthy difference between frame and spring attached frame static displacements except conjunction points. Figures 4.8 - 4.10 also show the

contribution of springs to $D_d$. It can be said that for small $\alpha$ values ($\alpha$ <1 = 7.537 m/s) springs are very effective for all nodes. In this interval, higher $D_d$ values are obtained with increasing spring stiffness. Figures 4.8 – 4.10 show the $D_d$ values only when the moving load is on the beam, so the interpretations are based on this situation. The maximum $D_d$ values occur in the neighborhood of $\alpha$ = 6 for nodes 2-5. The attachment of the spring causes higher $D_d$ values in the middle speed region. Lower $D_d$ values are obtained with increasing spring stiffness in this region. The maximum $D_d$ values for nodes 6-11 are observed close to $\alpha$ = 0.6. The springs are very effective especially in this low speed region, but higher $D_d$ values are obtained with increasing spring stiffness in this interval. For beam (nodes 12-21) not a noteworthy difference is observed both by attaching a spring or increasing spring stiffness. The maximum $D_d$ occurs at $\alpha$ = 4.5 for the mid – point of the beam. Similar to column 1, two critical moving load speeds are observed for column 2, $\alpha$ =1 and $\alpha$ = 6. The springs have no contribution to the $D_d$ values for both beam and columns at high speed region ($\alpha$ >10). Lower $D_d$ values are obtained by attaching spring for column 2 in the middle speed region.

Dynamic magnification factors for the mid – point of the beam and columns of frame and spring attached frames are given for various $\alpha$ values in Table 4.2. These values are only valid in the time interval that the moving load is on the beam.

The dynamic analyses are also performed for the case that the moving load left the beam. Figure 4.11 (a-c) shows that the maximum $D_d$ values are observed after the load leaves the beam both for columns and beam of the frame structure. The critical points for the frame structure shown in Figure 4.11 (a-c) are; $\alpha$ > 1 for column 1, $\alpha$ > 4.2 for column 2 and $\alpha$ > 5.4 for beam. For $\alpha$ values greater than these values, maximum $D_d$ occurs after the moving load left the beam. The critical $\alpha$ values at which the maximum dynamic displacements occur are; $\alpha$ = 5.1 ($D_d$ = 3.936) for column 1, $\alpha$ = 7 ($D_d$ = 4.246) for column 2 and $\alpha$ = 6 ($D_d$ = 1.571) for beam. Figures 4.11 (a-c) also shows that by attaching a spring, $D_d$ value increases both for the mid – point of column 1 and column 2 in the middle speed region.

There are no considerable discrepancies on the dynamic magnification factors for the mid-point of the beam.

### 4.1.1. Effect of the Rayleigh Damping

Dynamic analyzes are performed to show the effect of the Rayleigh damping on the magnification factors of beam and frame structures. Figure 4.12 shows the effect of damping ratio $\xi$ on the magnification factor of a clamped – clamped beam. The maximum $D_d$ value (1.632), which is observed at $\alpha = 1.02$ for the undamped case is recorded at $\alpha = 1$ ($D_d = 1.608$) for $\xi = 0.01$ and at $\alpha = 0.97$ ($D_d = 1.524$) for $\xi = 0.05$. $D_d$ values are decreased with increasing damping ratio as expected and the time at which the maximum $D_d$ occurs shifts left with increasing damping ratio.

Figures 4.13 and 4.14 (a-c) show the effect of the Rayleigh damping on the dynamic magnification factor for the mid – point of both columns and beam of frame and spring attached ($k_2$) frame. The maximum $D_d$ values for the mid – points of columns and beam of the frame are; for $\xi = 0.01$, at $\alpha = 6.1$ ($D_d = 1.849$) for column 1, at $\alpha = 4.4$ ($D_d = 1.534$) for beam and at $\alpha = 6.1$ ($D_d = 3.595$) for column 2. Similarly the maximum $D_d$ values are observed at $\alpha = 0.7$ ($D_d = 1.691$) for column 1, at $\alpha = 4.2$ ($D_d = 1.468$) for beam and at $\alpha = 6.1$ ($D_d = 3.172$) for column 2 for the damping ratio $\xi = 0.05$. Lower $D_d$ values are observed with increasing damping ratio. The time at which the maximum $D_d$ values occur shifts left for the mid – point of the beam of the frame with increasing damping ratio, but the maximum $D_d$ values are obtained with higher $\alpha$ values for the columns of the frame with increasing damping ratio. The dynamic magnification factors for the frame structure for different damping ratios are given in Table 4.3.

Table 4.4 shows the dynamic magnification factor values for the mid – points of beam and columns of spring attached ($k_2$) frame with and without damping effect. Similar to frame structure, smaller dynamic displacements are observed with increasing damping ratio. Negligible difference is observed at the occuring time of the maximum $D_d$ values for the columns of the spring attached frame. The occuring

time of maximum dynamic displacement shifts left for the beam of the spring attached frame similar to frame structure.

## 4.2 Conclusions

Moving load problem is generally studied for beam structures. In addition to the beam structures, dynamic responses of frames and spring attached frames subjected to the moving point load are also analyzed in this study. Euler-Bernoulli beam theory is used in the finite element method for constituting the element matrices. The Newmark integration method is employed for forced vibration analysis. The results obtained in this study are compared with Ansys results. The conclusions drawn can be summarized as follows:

- The moving load and the maximum dynamic displacements for the mid-point of the beam are not in the same phase at overcritical part. The time at which the maximum mid – point displacement is observed shifts right with increasing $\alpha$ values regardless of the boundary condition of the beam.

- The highest dynamic displacements occur for a pinned – pinned beam. For pinned – pinned boundary conditions the dynamic magnification values are greater than those obtained for clamped – clamped and clamped – pinned beams for low and high moving load speeds. The clamped – clamped boundary conditions generally gives the lower dynamic magnification values except the middle speed region.

- Attaching a spring to the frame at the conjunction points of beam and columns makes the frame more rigid and shifts the mode shapes of the frame structure up.

- A longer beam implies a smaller first natural frequency for frame structure, similarly longer columns imply smaller natural frequencies.

- With lower $\alpha$ values ($\alpha < 1 = 7.537$ m/s) springs are very effective for all nodes. In this interval, higher $D_d$ values are obtained with increasing spring stiffness. In the middle and high speed region, attaching a spring to the frame is not an advisable solution due to the increasing $D_d$ values.

- Maximum $D_d$ occurs after the moving load left the beam for both columns and beam of the frame structure when the $\alpha$ value is greater than some critical values.

- Lower $D_d$ values are observed with increasing damping ratio for a clamped – clamped beam. The occuring time of maximum dynamic displacement shifts left with increasing damping ratio.

- Maximum $D_d$ values are observed at smaller α values both for the beam of the frame and spring attached frame with increasing damping ratio.

clamped – clamped beam $\alpha = 0.5$



(a)

clamped – pinned beam $\alpha = 1$



(b)
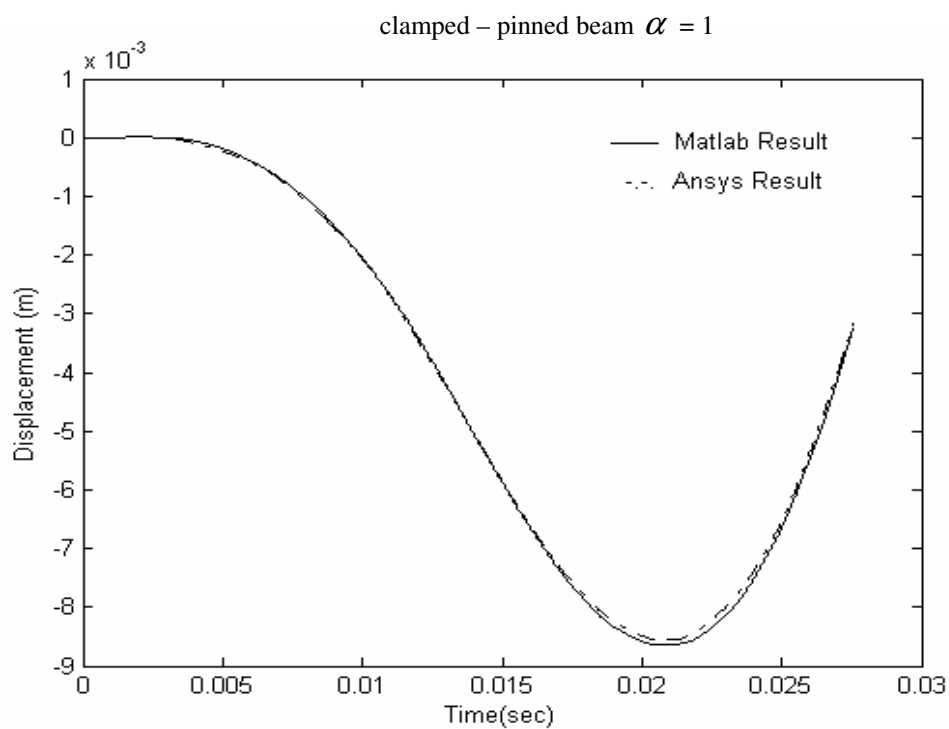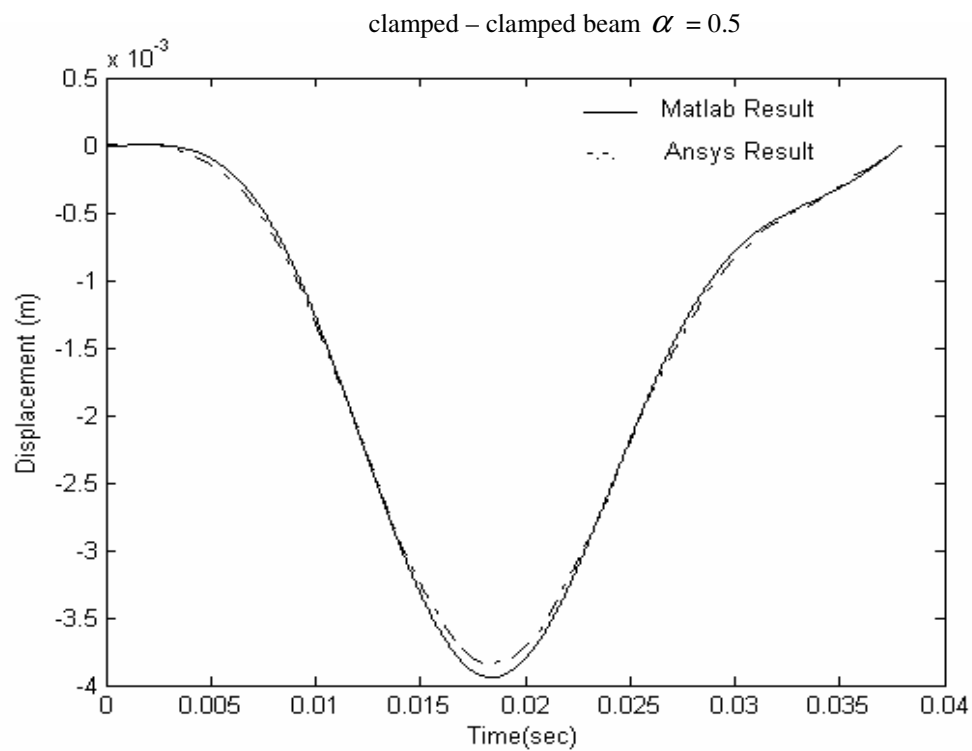
Figure 4.1 Comparison of Matlab and Ansys results for different boundary conditions and various $\alpha$ values for a) clamped – clamped beam b) clamped – pinned beam.

(c)



(d)
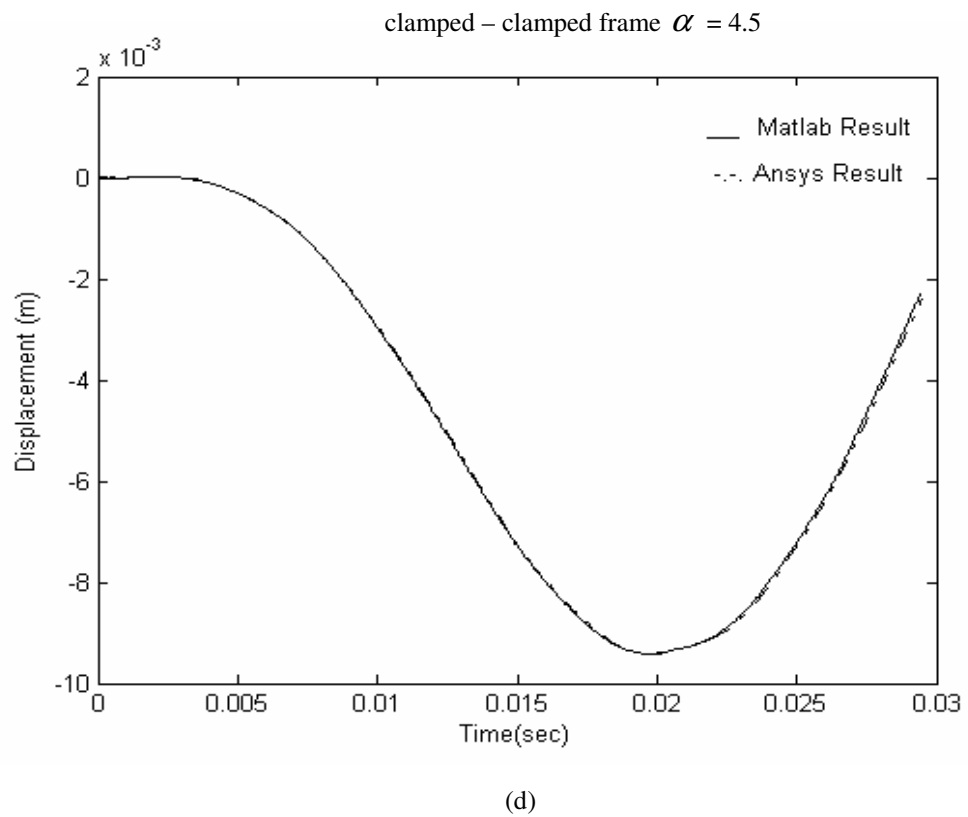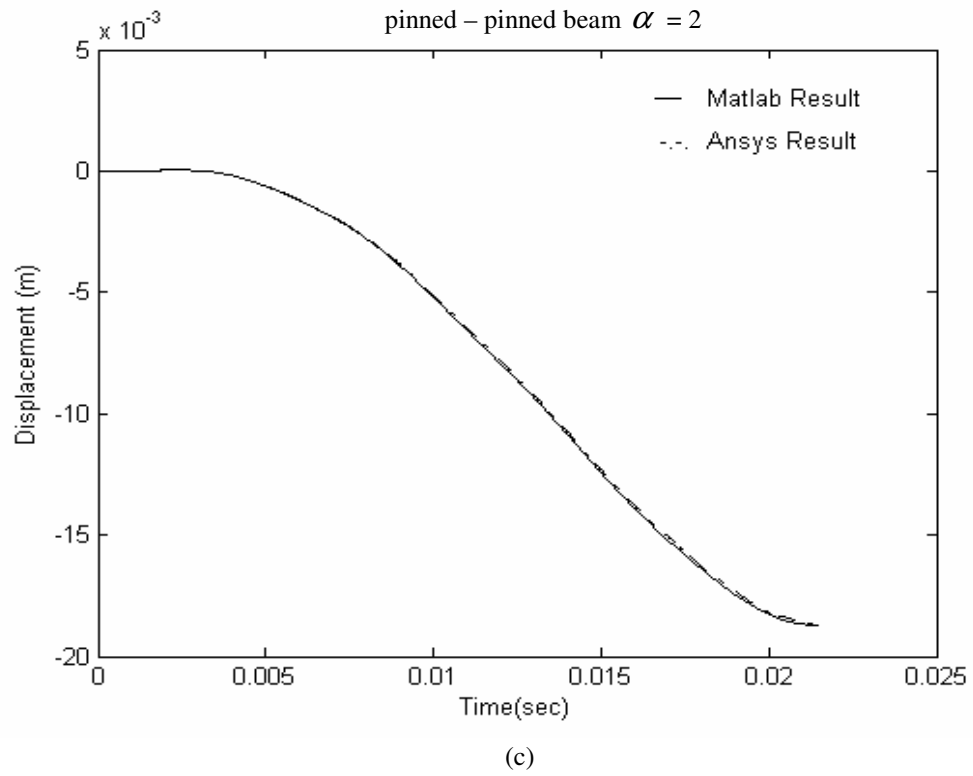
Figure 4.1 (continued) Comparison of Matlab and Ansys results for different boundary conditions and various $\alpha$ values for c) pinned – pinned beam d) clamped – clamped frame.
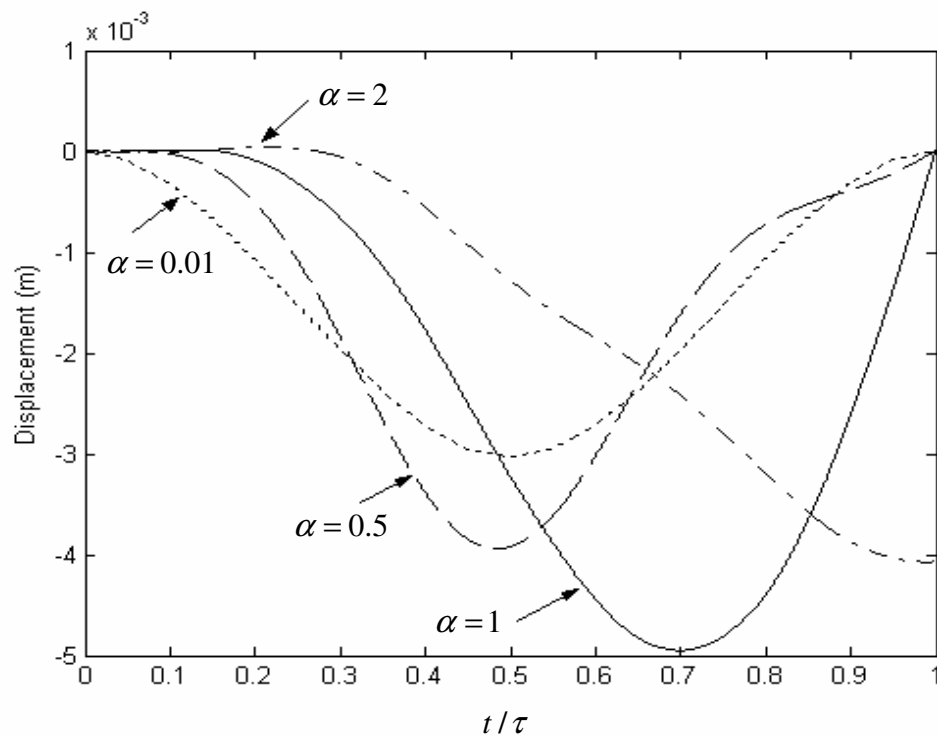
Figure 4.2 Dynamic displacements of the mid – point of the clamped – clamped beam versus the position of the moving load on the beam for various $\alpha$ values.
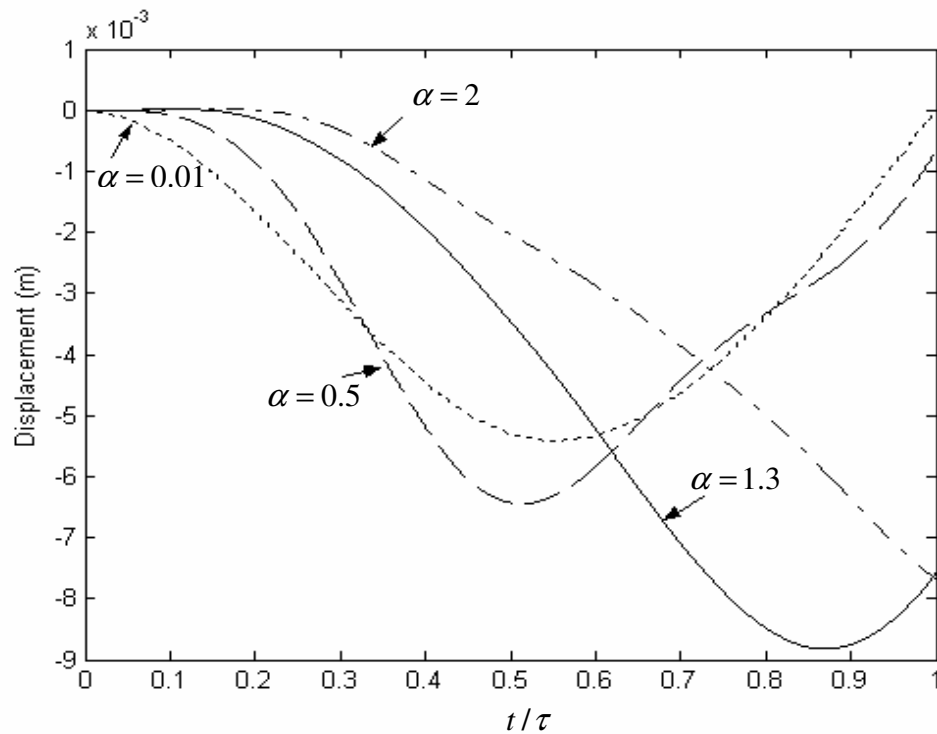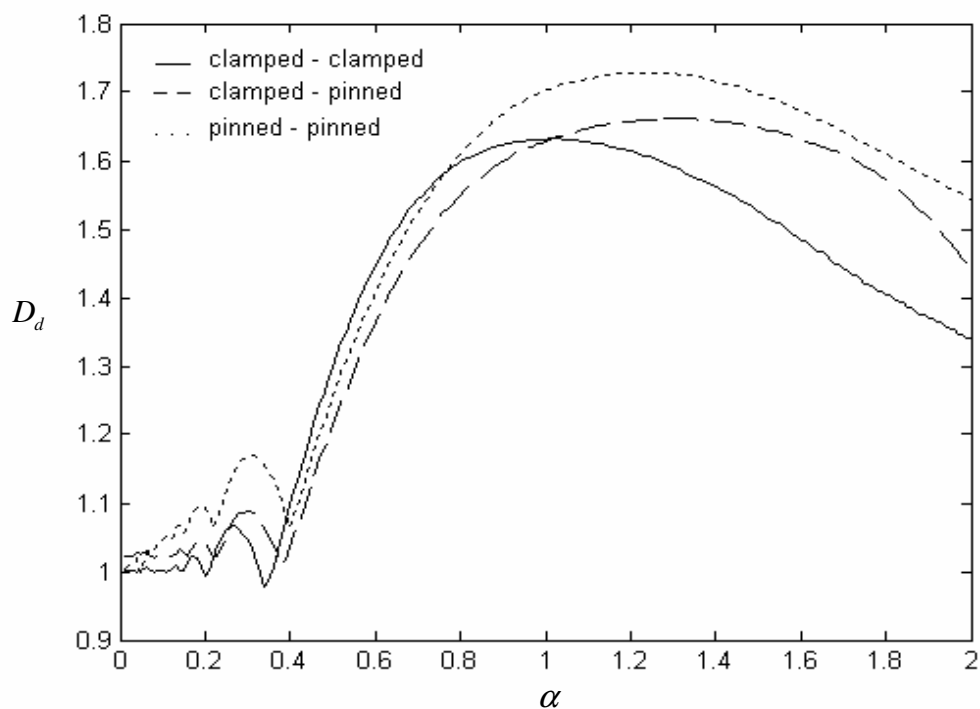


Figure 4.3 Dynamic displacements of the mid – point of the clamped – pinned beam versus the position of the moving load on the beam for various $\alpha$ values.
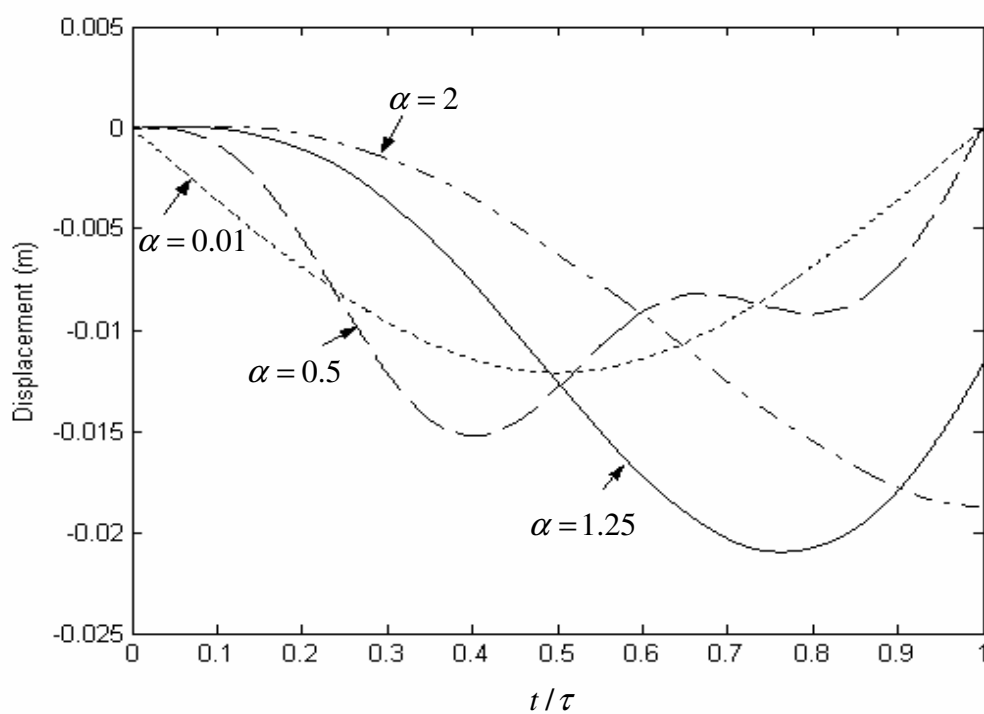
Figure 4.4 Dynamic displacements of the mid – point of the pinned – pinned beam versus the position of the moving load on the beam for various $\alpha$ values.



Figure 4.5 . Dynamic magnification factors for the mid – point of the beam versus $\alpha$ for three different boundary conditions.

Table 4.1 Dynamic magnification factors for the mid – point of the beams for various $\alpha$ values.(*$\alpha$ values which makes $D_d$ maximum)

| BEAM | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | *1.02 |
| Clamped-Clamped | 1.000 | 0.993 | 1.045 | 1.097 | 1.299 | 1.445 | 1.543 | 1.598 | 1.623 | *1.632 |
| Clamped-Pinned | 1.022 | 1.042 | 1.087 | 1.035 | 1.213 | 1.365 | 1.474 | 1.549 | 1.599 | 1.632 |
| Pinned-Pinned | 1.046 | 1.094 | 1.167 | 1.064 | 1.257 | 1.409 | 1.524 | 1.610 | 1.667 | 1.707 |
| **BEAM** | | | | | | | | | | |
| $\alpha$ | 1.1 | *1.23 | *1.31 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2 |
| Clamped-Clamped | 1.627 | 1.608 | 1.589 | 1.561 | 1.526 | 1.484 | 1.444 | 1.404 | 1.369 | 1.342 |
| Clamped-Pinned | 1.647 | 1.659 | *1.662 | 1.658 | 1.648 | 1.631 | 1.610 | 1.572 | 1.515 | 1.443 |
| Pinned-Pinned | 1.719 | *1.728 | 1.726 | 1.716 | 1.697 | 1.672 | 1.642 | 1.608 | 1.576 | 1.542 |

Frame

$f_1 = 7.537$ Hz.

Spring Attached $k_2$

$f_1 = 29.74$ Hz.

Frame

$f_2 = 29.74$ Hz.

Spring Attached $k_2$

$f_2 = 42.26$ Hz.

Frame

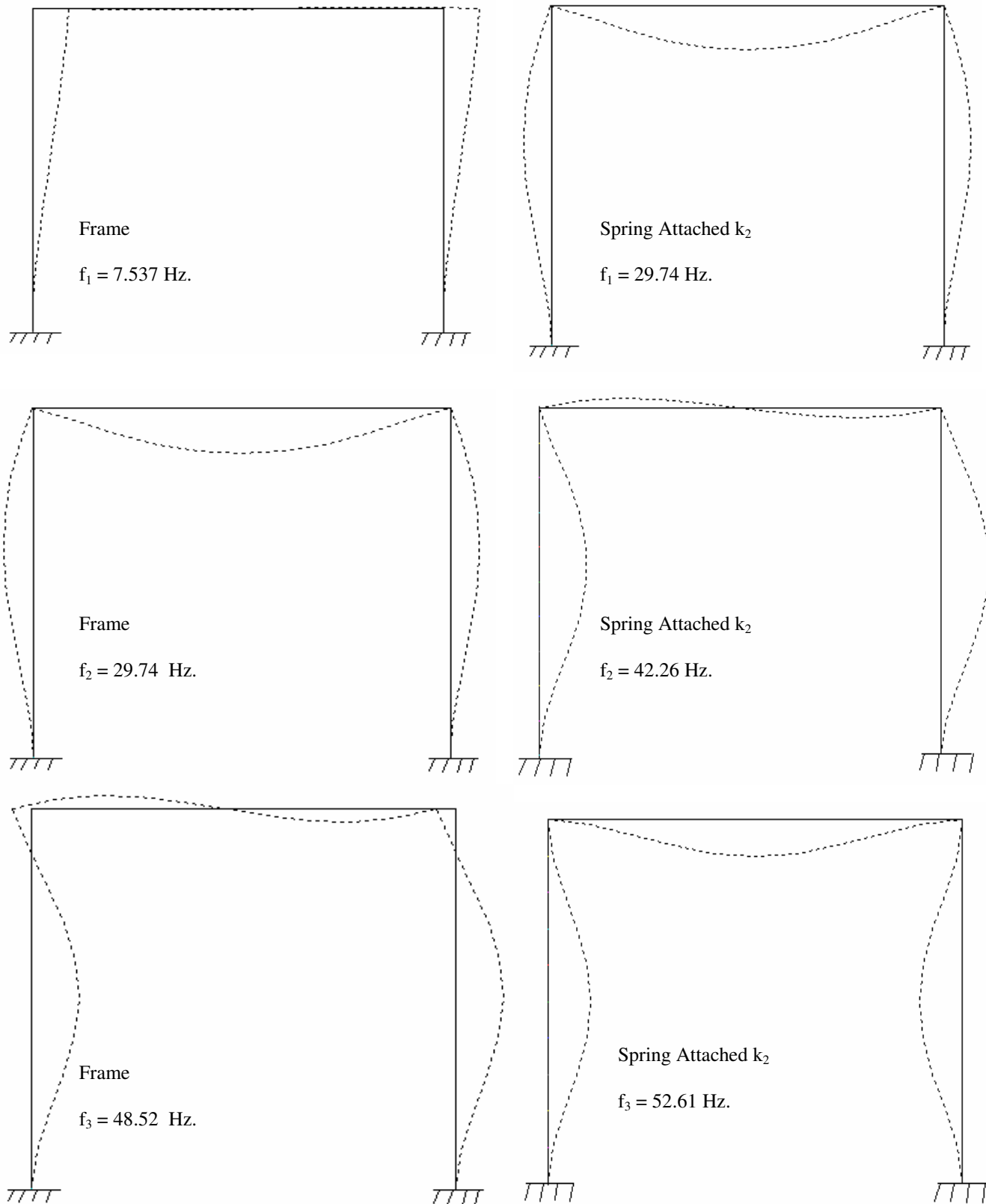$f_3 = 48.52$ Hz.
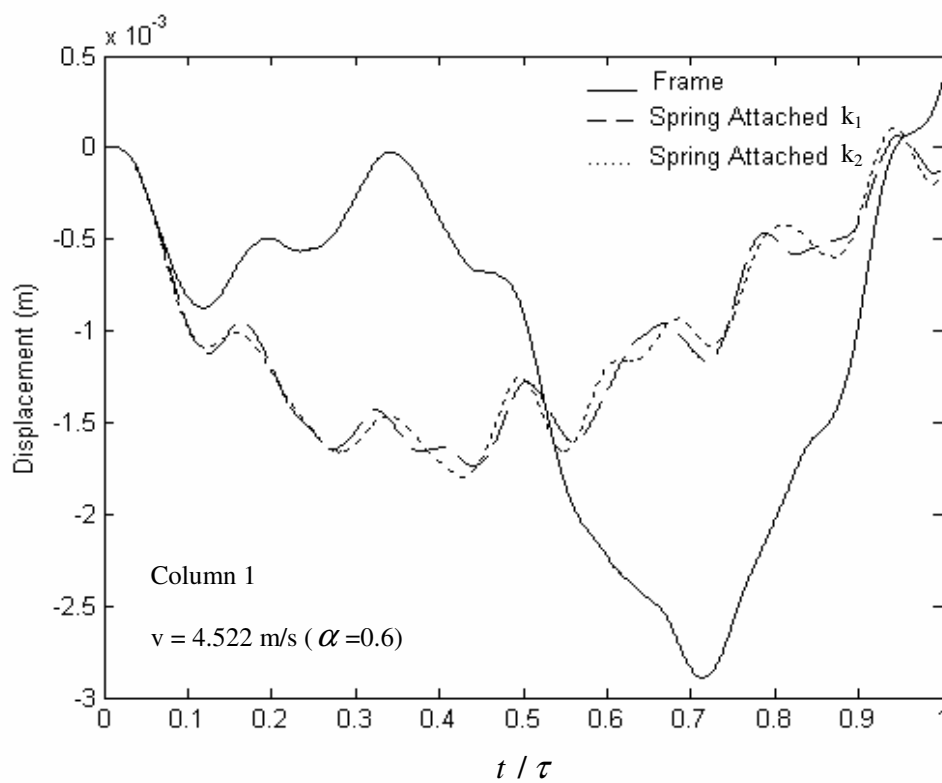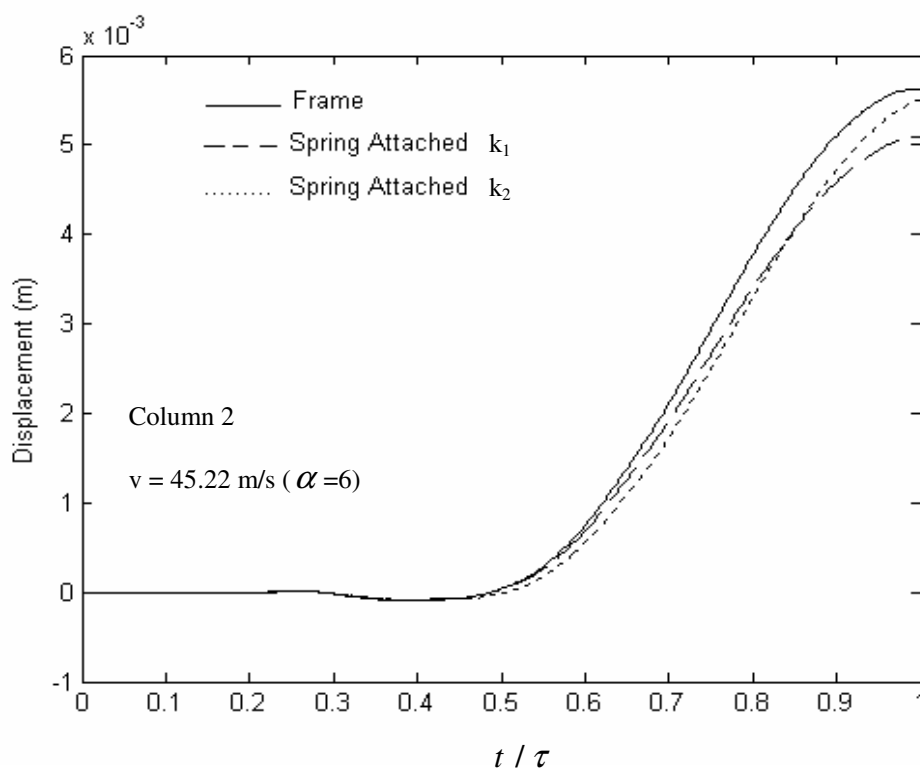
Spring Attached $k_2$

$f_3 = 52.61$ Hz.

Figure 4.6 First three mod shapes of the frame and spring attached frame
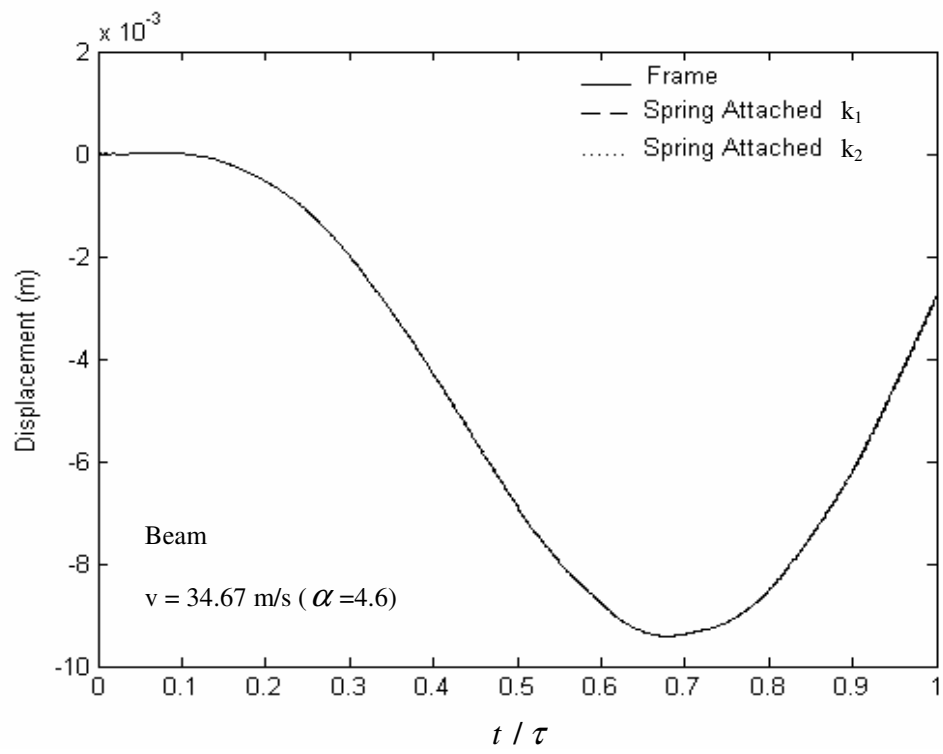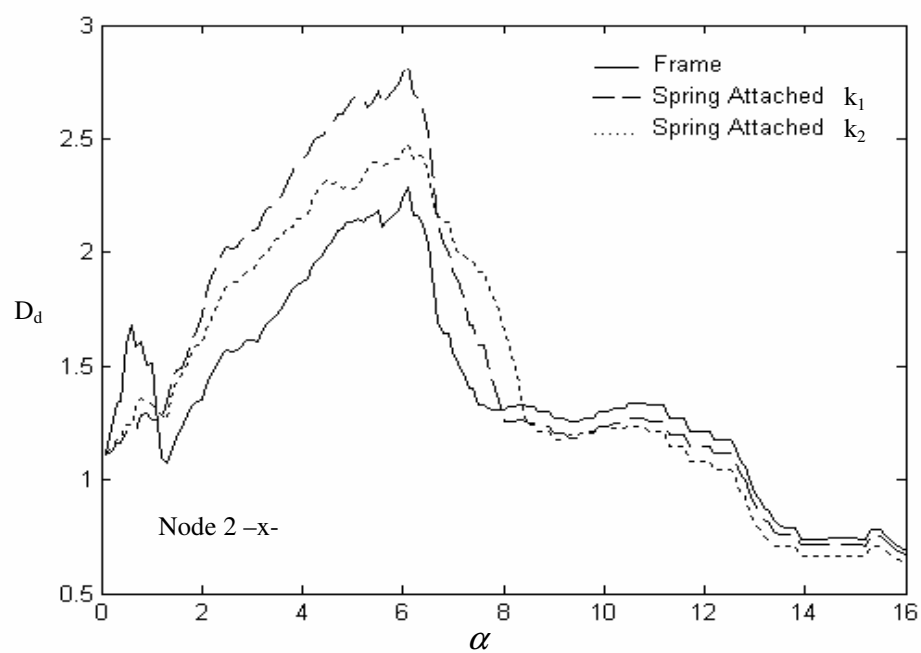
(a)



(b)

Figure 4.7 The effect of the springs on the mid – point displacements of the a) column 1
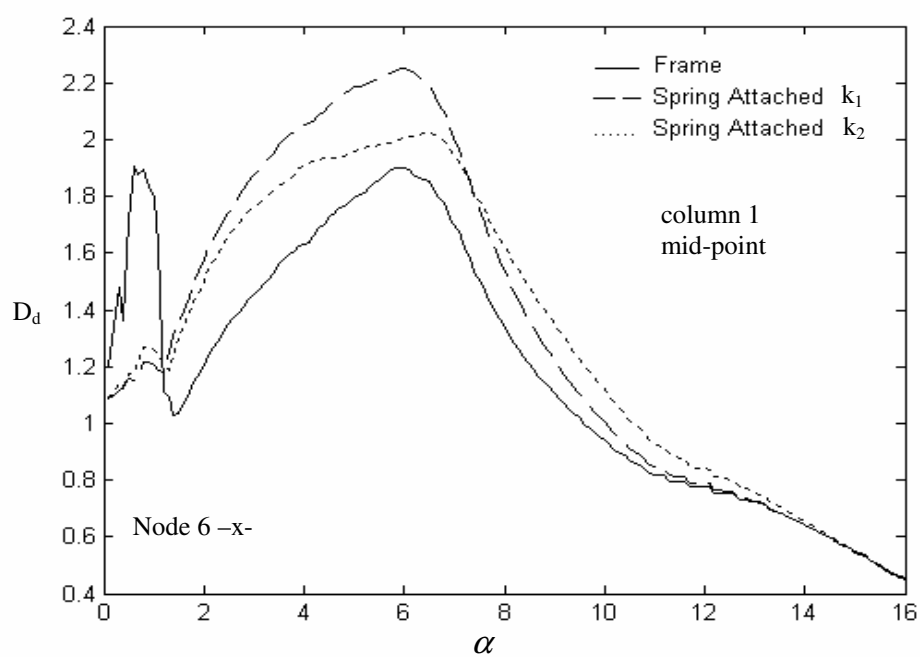
b) column 2 of the frame structure.

(c)

Figure 4.7 (continued) The effect of the springs on the mid – point displacements of the
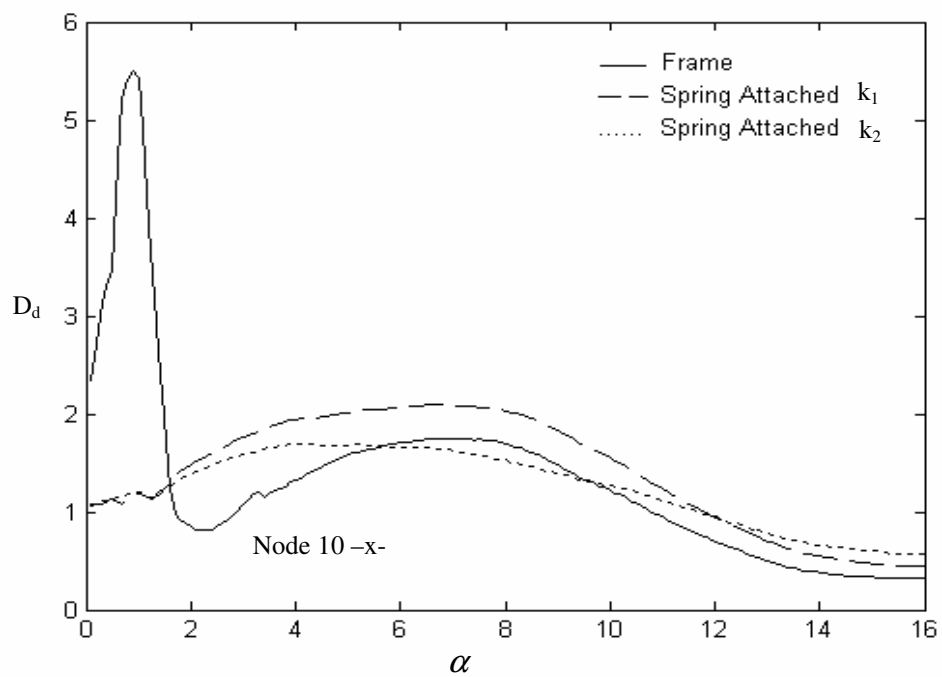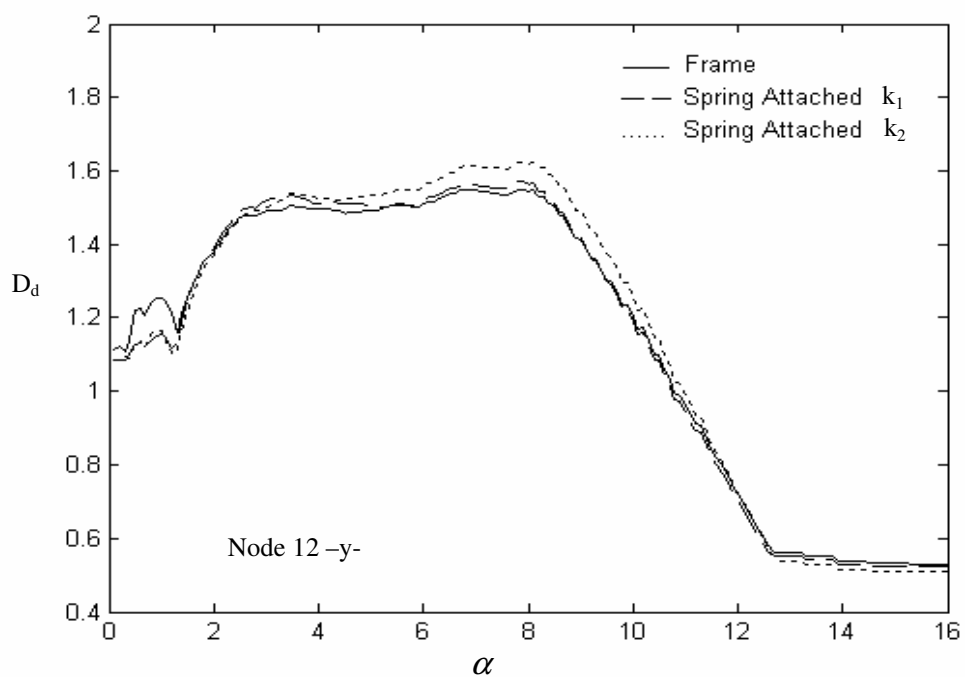
c) beam of the frame structure.

(a)



(b)

Figure 4.8 The effect of $\alpha$ on the dynamic magnification factor of column 1 for normal and spring attached frame at different nodes.
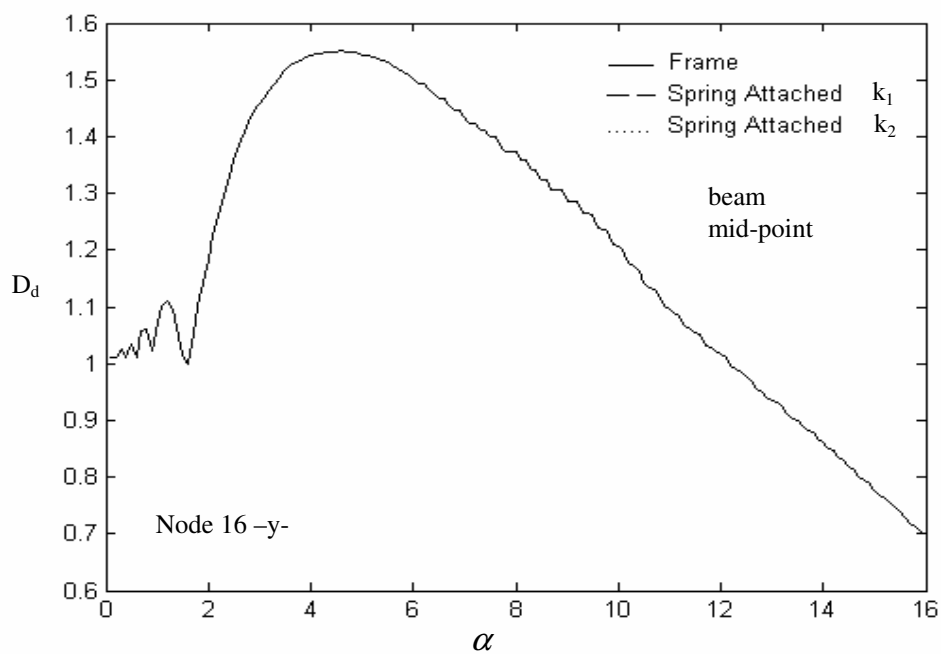
(c)

Figure 4.8 (continued) The effect of $\alpha$ on the dynamic magnification factor of column 1 for normal and spring attached frame at different nodes.
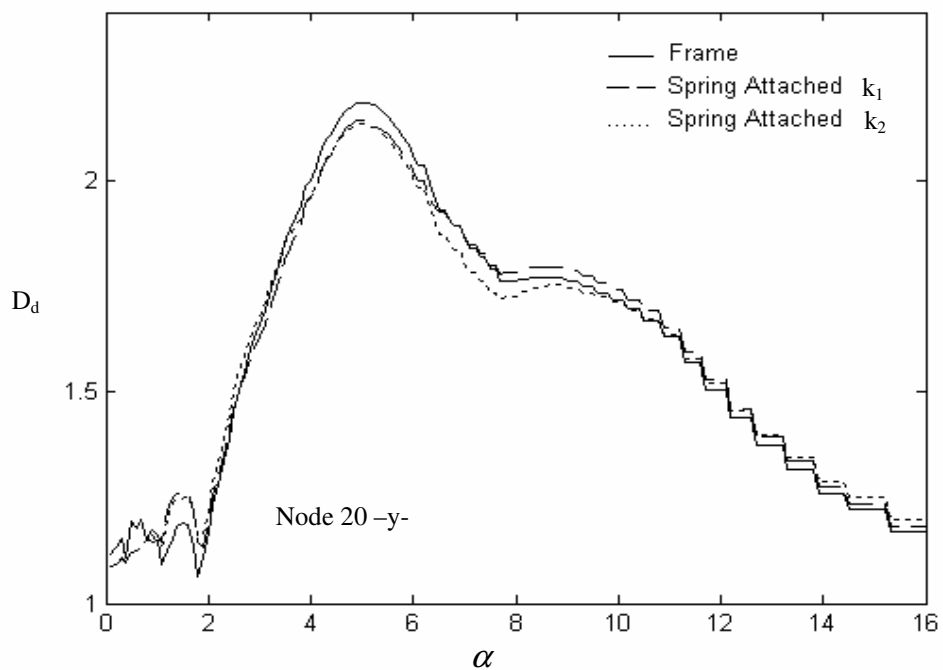
(a)



(b)

Figure 4.9 The effect of $\alpha$ on the dynamic magnification factor of the beam for normal and spring

attached frame at different nodes

(c)

Figure 4.9 (continued) The effect of $\alpha$ on the dynamic magnification factor of the beam for normal and spring attached frame at different nodes.
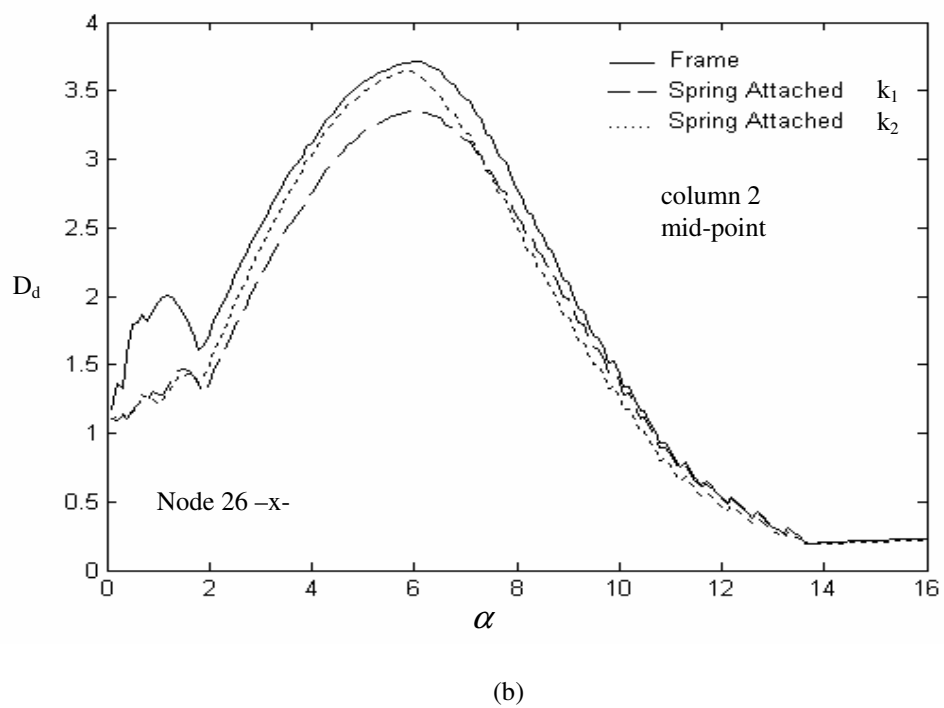
(a)



(b)

Figure 4.10 The effect of $\alpha$ on the dynamic magnification factor of column 2 for normal and spring attached frame at different nodes
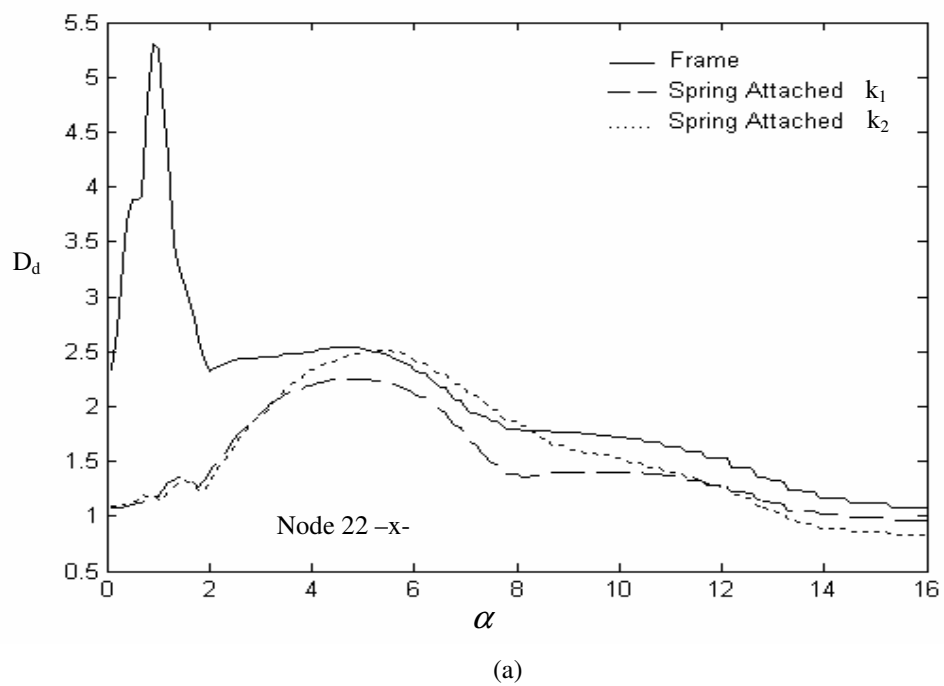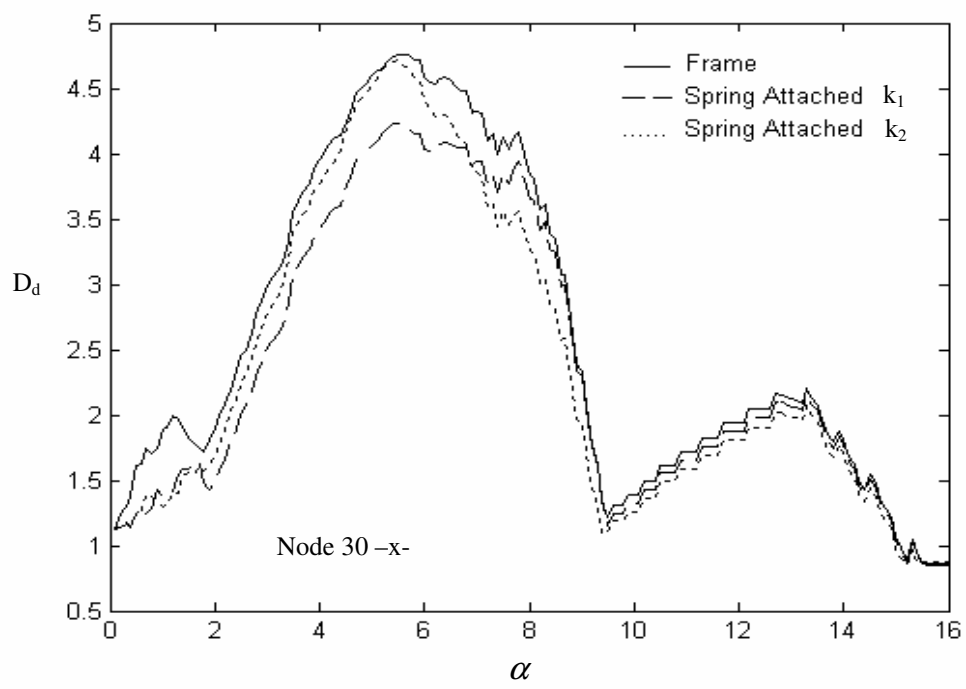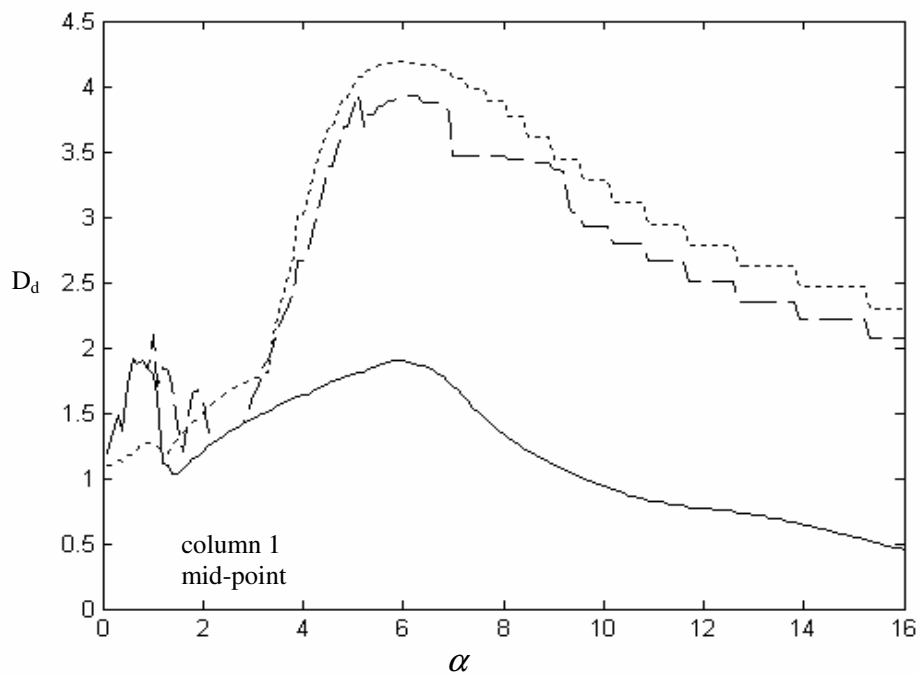
(c)

Figure 4.10 (continued) The effect of $\alpha$ on the dynamic magnification factor of column 2 for normal and spring attached frame at different nodes

(a)
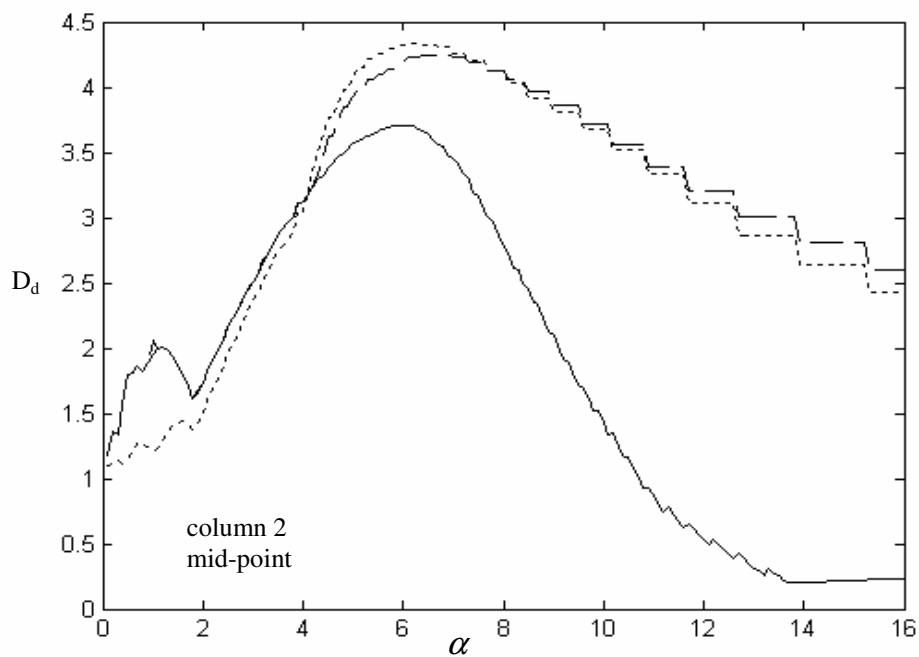


(b)

Figure 4.11 Comparison of the time at which the maximum $D_d$ values occur for the mid – points of

the a) column 1 b) column 2 of the frame and spring attached frame.

————— $D_d$ values for the frame when the moving load is on the beam

—  —  — $D_d$ values for the frame after the moving load leaves the beam

………. $D_d$ values after the moving load leaves the beam for the spring attached frame with spring

constant $k_2$

(c)

Figure 4.11 (continued) Comparison of the time at which the maximum $D_d$ values occur for the mid

– points of the c) beam of the frame and spring attached frame.

———— $D_d$ values for the frame when the moving load is on the beam

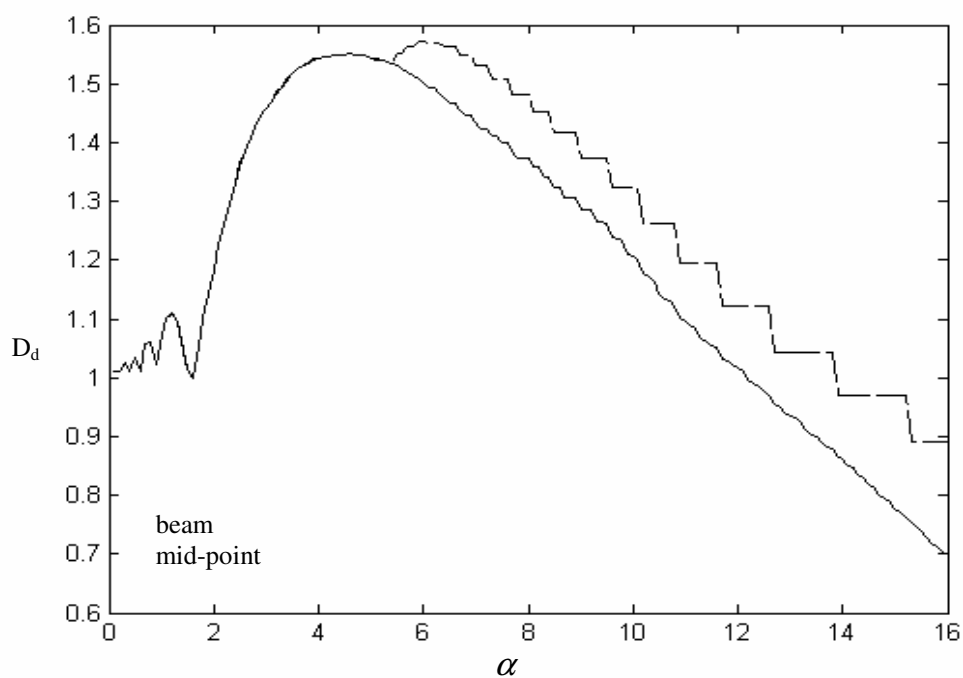__ __ __ $D_d$ values for the frame after the moving load leaves the beam

………. $D_d$ values after the moving load leaves the beam for the spring attached frame with spring

constant $k_2$

Table 4.2 Dynamic magnification factors for the mid – points of the beam and columns of the frame and spring attached frame for different $\alpha$ values. (*$\alpha$ values which makes $D_d$ maximum when the moving load is on the beam

| FRAME (Column 1) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | *0.6 | 1 | 1.5 | 2 | 4 | *4.5 | *6 | 8 | 10 | 16 |
| Frame | *1.908 | 1.792 | 1.032 | 1.201 | 1.632 | 1.729 | 1.902 | 1.341 | 0.942 | 0.455 |
| Spring Attached($k_1$) | 1.149 | 1.202 | 1.358 | 1.576 | 2.052 | 2.125 | 2.253 | 1.540 | 1.005 | 0.451 |
| Spring Attached($k_2$) | 1.183 | 1.257 | 1.295 | 1.497 | 1.905 | 1.936 | 2.003 | 1.625 | 1.120 | 0.448 |
| FRAME (Beam) | | | | | | | | | | |
| $\alpha$ | *0.6 | 1 | 1.5 | 2 | 4 | *4.5 | *6 | 8 | 10 | 16 |
| Frame | 1.011 | 1.073 | 1.015 | 1.184 | 1.542 | *1.550 | 1.502 | 1.373 | 1.206 | 0.698 |
| Spring Attached($k_1$) | 1.011 | 1.073 | 1.015 | 1.184 | 1.542 | 1.550 | 1.502 | 1.373 | 1.206 | 0.698 |
| Spring Attached($k_2$) | 1.011 | 1.073 | 1.015 | 1.184 | 1.542 | 1.550 | 1.502 | 1.373 | 1.206 | 0.698 |
| FRAME (Column 2) | | | | | | | | | | |
| $\alpha$ | *0.6 | 1 | 1.5 | 2 | 4 | *4.5 | *6 | 8 | 10 | 16 |
| Frame | 1.804 | 1.955 | 1.873 | 1.730 | 3.123 | 3.388 | *3.710 | 2.770 | 1.433 | 0.230 |
| Spring Attached($k_1$) | 1.214 | 1.284 | 1.470 | 1.372 | 2.754 | 3.025 | 3.358 | 2.571 | 1.370 | 0.226 |
| Spring Attached($k_2$) | 1.203 | 1.209 | 1.424 | 1.504 | 3.028 | 3.312 | 3.630 | 2.486 | 1.256 | 0.220 |

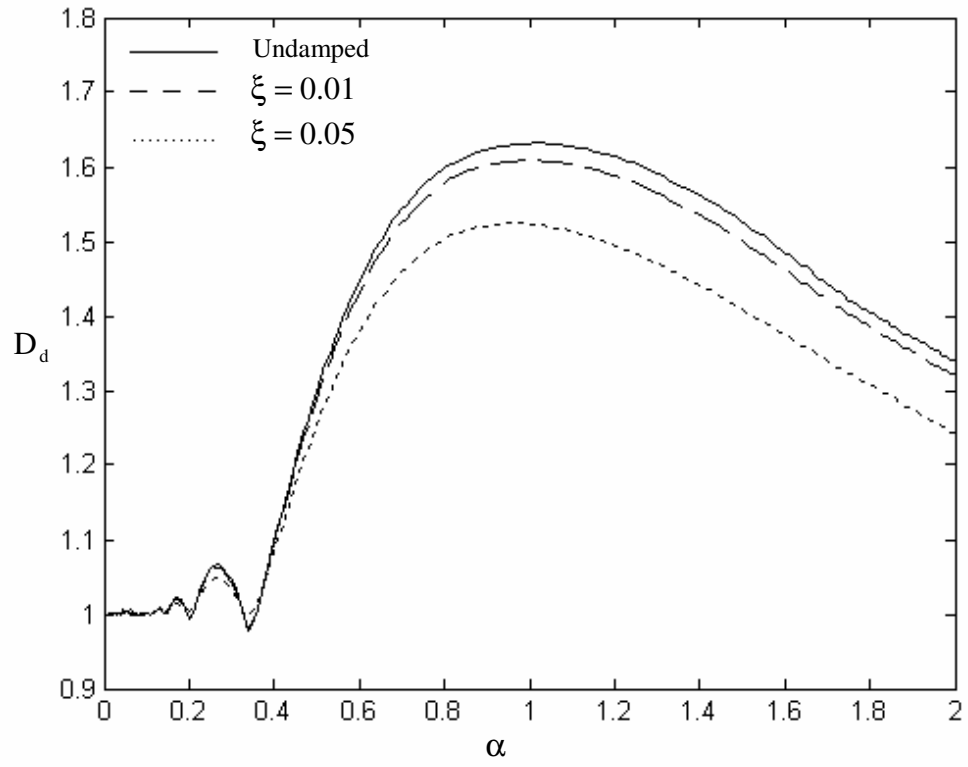Figure 4.12 The effect of Rayleigh damping on the magnification factor of clamped – clamped beam
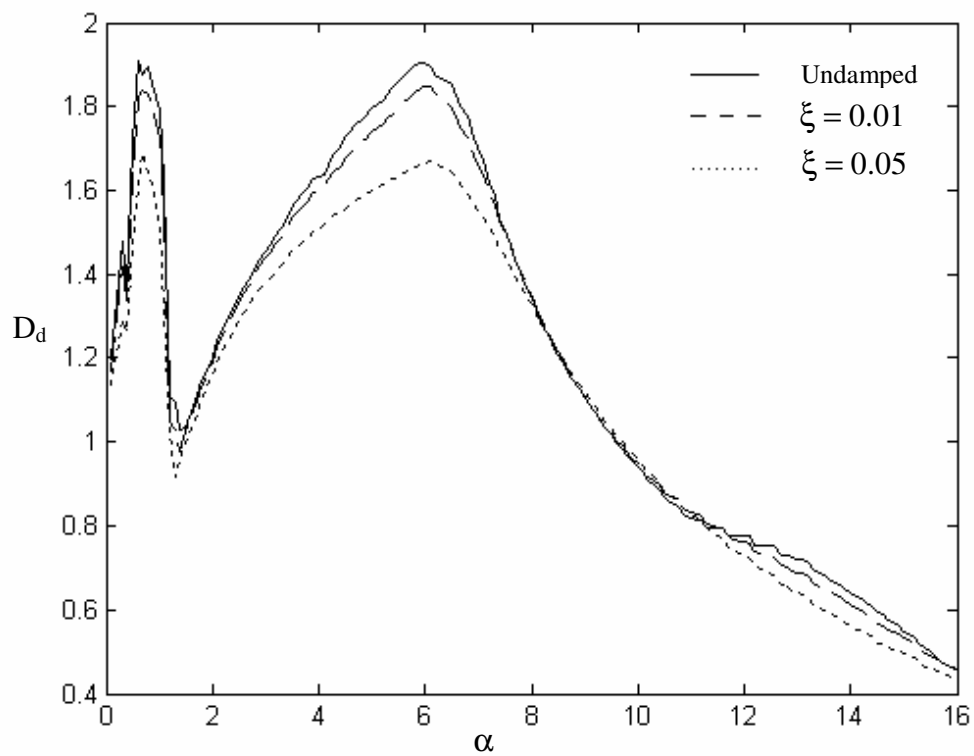
(a)
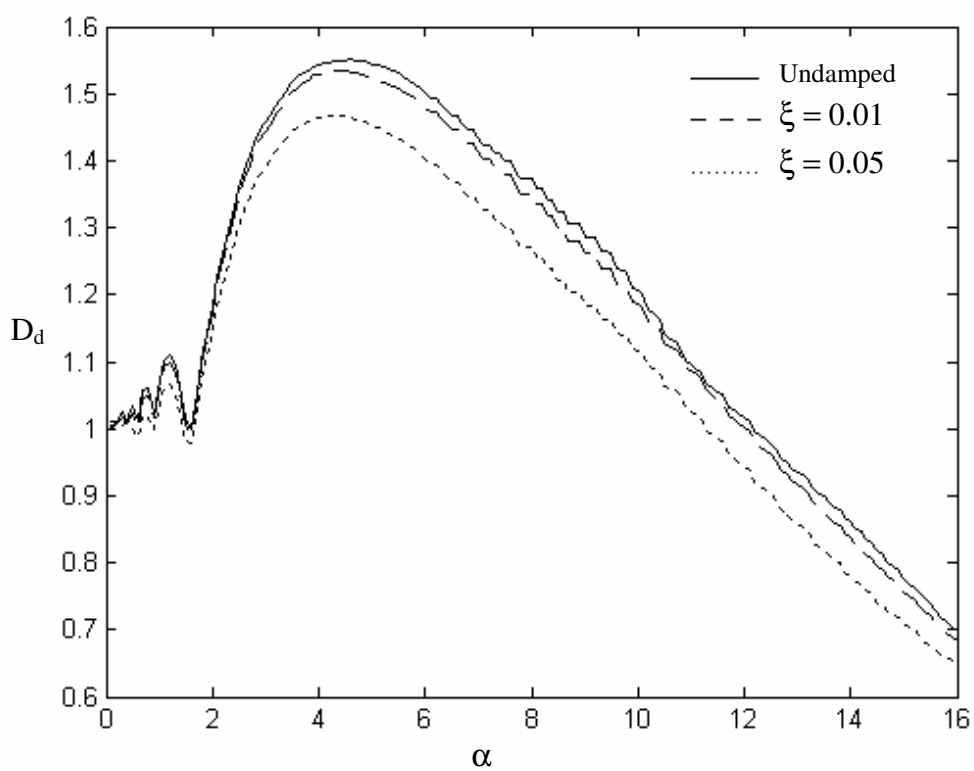


(b)

Figure 4.13 The effect of Rayleigh damping on the dynamic magnification factor for the midpoint of a) column 1  b) beam of the frame structure.
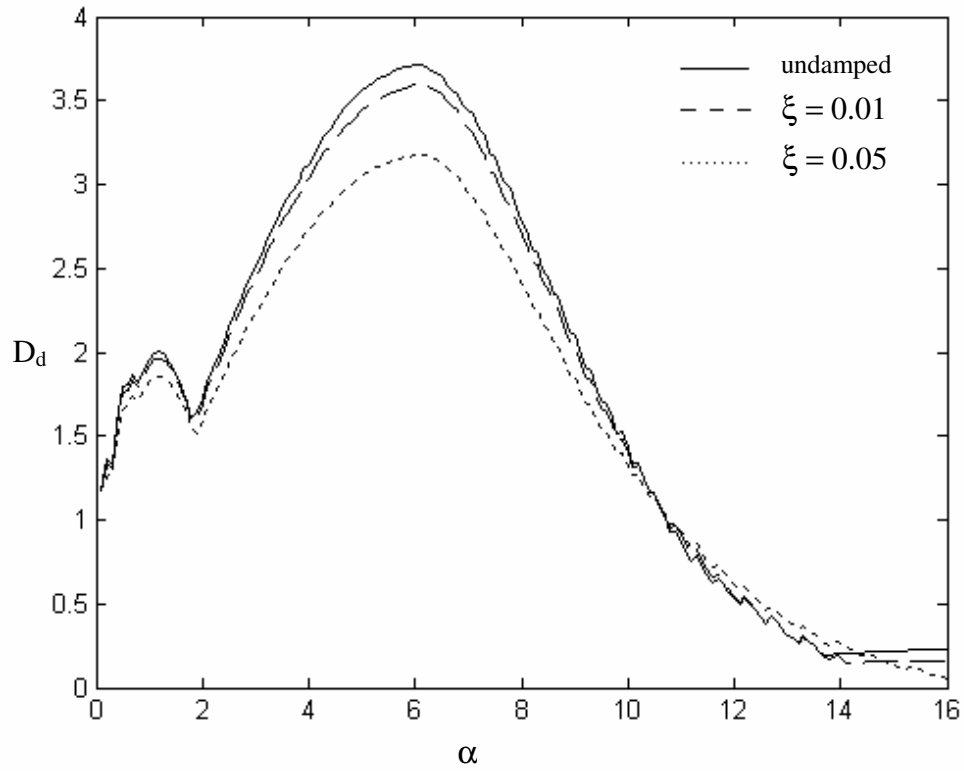
(c)

Figure 4.13 (continued) The effect of Rayleigh damping on the dynamic magnification factor for the midpoint of c) column 2 of the frame structure.

(a)



(b)

Figure 4.14 The effect of Rayleigh damping on the dynamic magnification factor for the midpoint of a) column 1 b) beam of the spring attached ($k_2$) frame structure.

(c)

Figure 4.14 (continued) The effect of Rayleigh damping on the dynamic magnification factor for the midpoint of c) column 2 of  the spring attached ($k_2$) frame structure.

Table 4.3 Maximum $D_d$ values for the mid – points of the columns and beam of the frame with the effect of damping
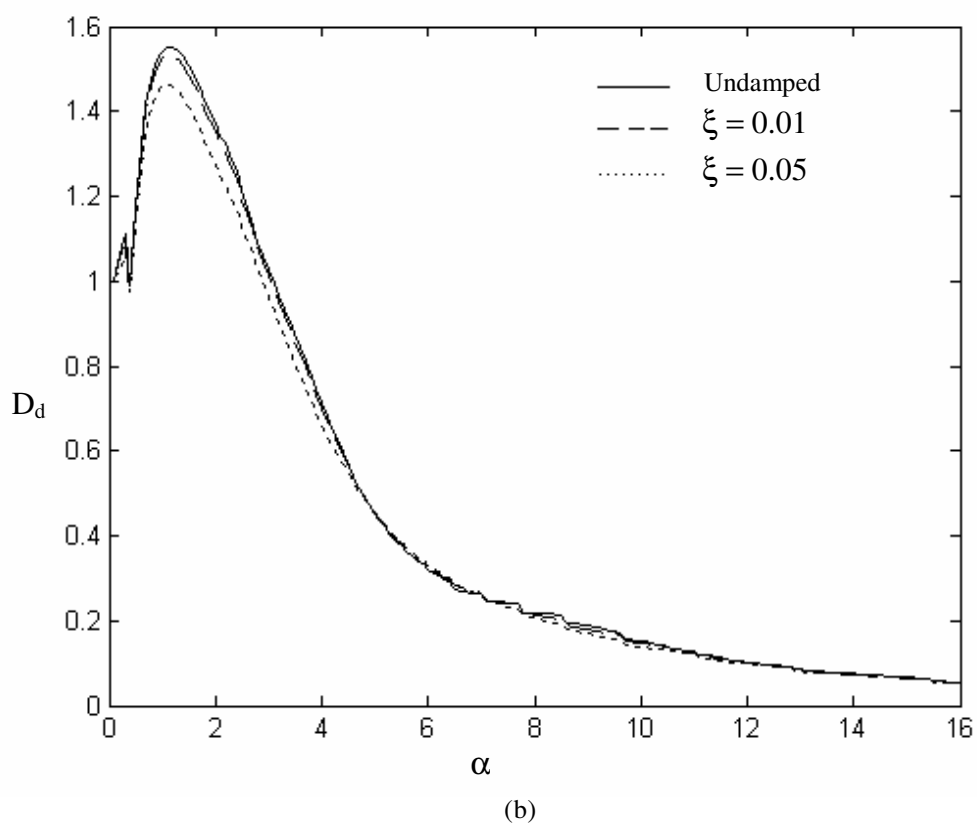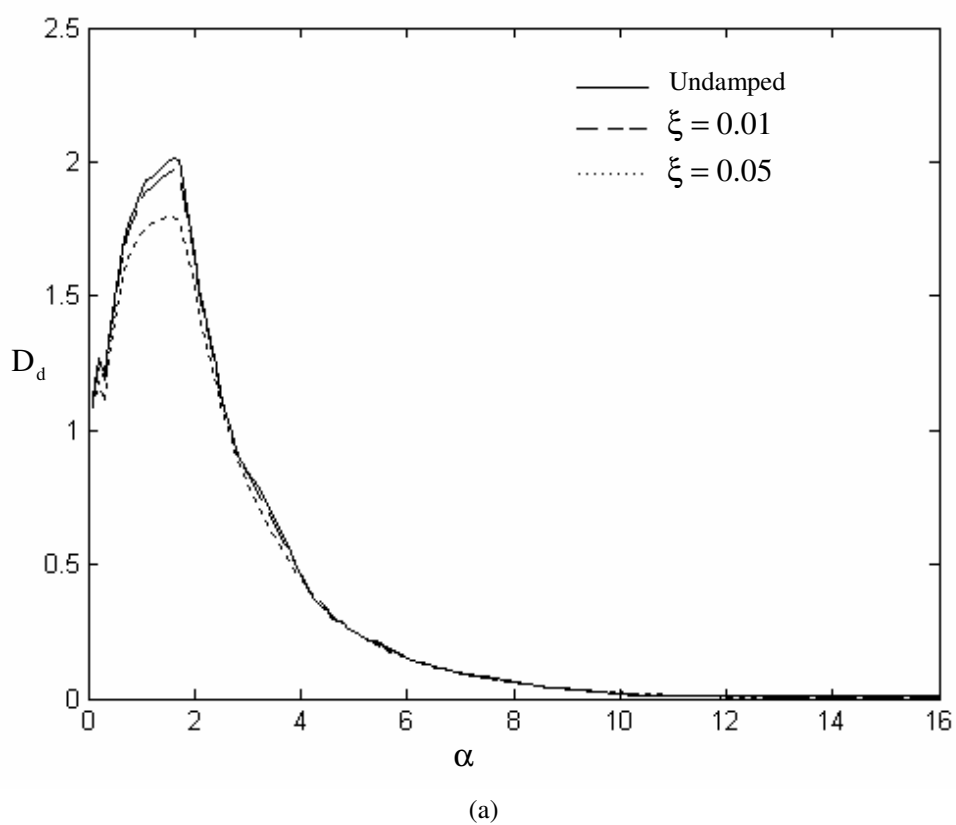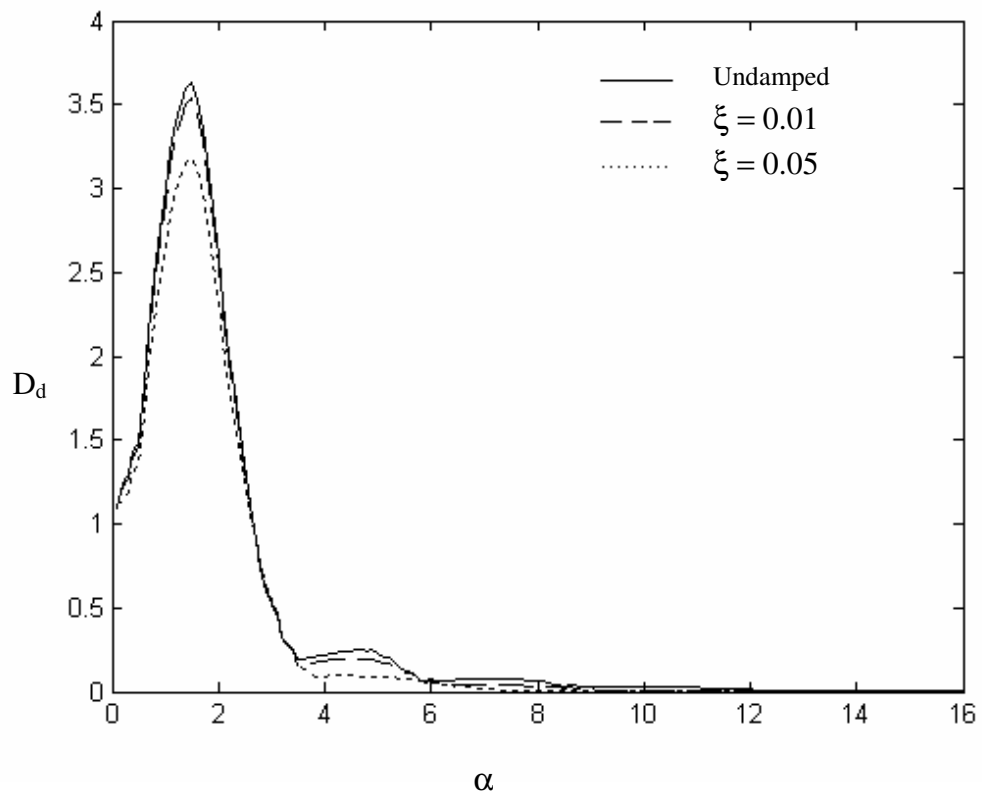
|  | Undamped |
| --- | --- |
| Column 1 | $D_d$=1.908 at $\alpha$=0.6 |
| Beam | $D_d$=1.550 at $\alpha$=4.5 |
| Column 2 | $D_d$=3.710 at $\alpha$=6.0 |
|  | $\xi$=0.01 |
| Column 1 | $D_d$=1.849 at $\alpha$=6.1 |
| Beam | $D_d$=1.534 at $\alpha$=4.4 |
| Column 2 | $D_d$=3.595 at $\alpha$=6.1 |
|  | $\xi$=0.05 |
| Column 1 | $D_d$=1.691 at $\alpha$=0.7 |
| Beam | $D_d$=1.468 at $\alpha$=4.2 |
| Column 2 | $D_d$=3.172 at $\alpha$=6.1 |

Table 4.4 Maximum $D_d$ values for the mid – points of the columns and beam of the spring attached ($k_2$) frame with the effect of damping

|  | Undamped |
| --- | --- |
| Column 1 | $D_d$=2.016 at $\alpha$=1.6 |
| Beam | $D_d$=1.550 at $\alpha$=1.2 |
| Column 2 | $D_d$=3.635 at $\alpha$=1.5 |
|  | $\xi$=0.01 |
| Column 1 | $D_d$=1.969 at $\alpha$=1.6 |
| Beam | $D_d$=1.532 at $\alpha$=1.1 |
| Column 2 | $D_d$=3.538 at $\alpha$=1.5 |
|  | $\xi$=0.05 |
| Column 1 | $D_d$=1.795 at $\alpha$=1.6 |
| Beam | $D_d$=1.465 at $\alpha$=1.1 |
| Column 2 | $D_d$=3.189 at $\alpha$=1.5 |

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## APPENDICES

**Nomenclature**

| | |
|---|---|
| A | Cross-section area of beam and frame element |
| $a_{0,1}$ | Rayleigh damping factors |
| $\alpha$ | $T_1/\tau$ (Non-dimensional velocity parameter) |
| $c_\theta$ | $Cos(\theta)$ |
| C | Proportional damping matrix |
| $\Delta t$ | Time step for dynamic analysis |
| E | Modulus of Elasticity |
| F | Moving force with constant velocity |
| $F_0$ | Force vector at time $t = 0$ |
| $F_i$ | Force on the $i^{th}$ node |
| I | Moment of Inertia about the z axis |
| $k_{1,2}$ | Longitudinal stiffness of the springs attached to frame |
| K | Stiffness matrix of the overall system |
| K.E. | Kinetic energy of a single beam element |
| $\ell$ | Length of a single beam element |
| L | Total length of the beam and columns of the frame |
| M | Mass matrix of the overall system |
| $M_i$ | Moment on the $i^{th}$ node |
| $N_i(i=1-6)$ | Shape Functions |
| $s_\theta$ | $Sin(\theta)$ |
| S.E. | Strain energy of a single beam element |

| | |
|---|---|
| T | Transformation matrix for the column elements of the frame |
| $T_1$ | Period of first natural frequency |
| $T_{20}$ | Period of $20^{th}$ natural frequency |
| u(x,t) | Assumed displacement function for the longitudinal vibration of the frame |
| $U_0$ | Displacement vector at time t = 0 |
| $\dot{U}_0$ | Velocity vector at time t = 0 |
| $\ddot{U}_0$ | Acceleration vector at time t = 0 |
| $\upsilon$ | Velocity of the moving force |
| $x_1$ | Longitudinal degree of freedom of the first node of a frame element |
| $x_2$ | Longitudinal degree of freedom of the second node of a frame element |
| $x_i$ | Location of the $i^{th}$ node |
| $y_1$ | Vertical degree of freedom of the first node of a single beam element |
| $y'_1$ | Rotational degree of freedom of the first node of a single beam element |
| $y_2$ | Vertical degree of freedom of the second node of a single beam element |
| $y'_2$ | Rotational degree of freedom of the second node of a single beam element |
| y(x,t) | Assumed displacement function for the bending vibration of the beam |
| y | Displacement vector for the whole structure |
| $\ddot{y}$ | Acceleration vector for the whole structure |
| $\omega_{m,n}$ | Two specific control frequencies for Rayleigh damping |
| $\beta$ | Integration parameter for Newmark Integration method |
| $\gamma$ | Integration parameter for Newmark Integration method |

$\theta$          Rotation angle for the element of column 1 and 2

$\phi$          Eigenvectors of the structure

$\rho$          Mass density of the beam and frame element

$\tau$          Traveling time of the F on a single beam element

$\tau_i$          The time for the load to arrive i[th] node

$\xi_{m,n}$          Damping ratios for viscous damping

# LIST OF COMPUTER PROGRAMS

## newmarkimp.m

```
dt=0.001; tson=2;
 dof=2;
 beta=1/4; gama=0.5;


 m1=50;
 c1=1000;
 k1=30000;


 m2=50;
 c2=1000;
 k2=30000;


 MM=[m1 0;0 m2];
 CC=[c1+c2 -c2;-c2 c2];
 KK=[k1+k2 -k2;-k2 k2];
 adimsay=tson/dt+1;
 adimsay=int32(adimsay);
 adimsay=double(adimsay);


 dep=zeros(adimsay,dof);
 hiz=zeros(adimsay,dof);
 ivm=zeros(adimsay,dof);
 zaman=zeros(adimsay,dof);


 F0=[0;1/dt];
 u0=[0;0];
 ud0=[0;0];
 udd0=MM\(F0(:,1)-CC*ud0-KK*u0);
```

```
dep(1,1:dof)=u0';
hiz(1,1:dof)=ud0';
ivm(1,1:dof)=udd0';
zaman(1,1:dof)=0;
sayac=2;
t=dt;

a0=1/(beta*dt^2); a1=gama/(beta*dt); a2=1/(beta*dt); a3=1/(2*beta)-1;
a4=gama/beta-1;
a5=(dt/2)*(gama/beta-2); a6=dt*(1-gama); a7=gama*dt;

KK=KK+a0*MM+a1*CC;
[L,U]=lu(KK);
ss=2;
  while t<=tson;
F0=zeros(dof,adimsay);
F0(2,2)=0.5/dt;
F=F0(:,ss)+MM*(a0*u0+a2*ud0+a3*udd0)+CC*(a1*u0+a4*ud0+a5*udd0);
z=inv(L)*F;
u1=inv(U)*z;

udd1=a0*(u1-u0)-a2*ud0-a3*udd0;
ud1=ud0+a6*udd0+a7*udd1;

for i=1:dof
  dep(sayac,i)=u1(i);
  hiz(sayac,i)=ud1(i);
  ivm(sayac,i)=udd1(i);
end

zaman(sayac,1)=t;
```

```
    sayac=sayac+1;
   t=t+dt;


   u0=u1;
   ud0=ud1;
   udd0=udd1;
   ss=ss+1;
  end


  plot(zaman(:,1),dep(:,2),'black')
```

# matlabimp.m

```
m1=50;
c1=1000;
k1=30000;

m2=50;
c2=1000;
k2=30000;

s=sym('s');
num1=(c2*s+k2);
den1=(m2*s^2+c2*s+k2)*(m1*s^2+(c1+c2)*s+k1+k2)-(c2*s+k2)^2;

num2=(m1*s^2+(c1+c2)*s+k1+k2);
den2=(m2*s^2+c2*s+k2)*(m1*s^2+(c1+c2)*s+k1+k2)-(c2*s+k2)^2;

num1;
den1;
num2;
den2;
num1=expand(num1);
den1=expand(den1);
num2=expand(num2);
den2=expand(den2);

num1=[1000 30000];
den1=[2500 150000 5500000 60000000 900000000];
num2=[50 2000 60000];
den2=den1;
impulse(num2,den2)
%impulse(num1,den1)
```

```
/prep7
et,1,mass21
et,2,combin14
m=50
c=1000
k=30000

r,1,0,m
r,2,k,c

n,1,0,0,0
n,2,0,1,0
n,3,0,2,0

type,1
real,1
e,2
e,3

type,2
real,2

e,1,2
e,2,3

eplot

/solu
antype,2
modopt,lanb,2
```

```
d,all,ux,0
d,all,uz,0
d,1,uy,0

solve

*get,f1,mode,1,freq
*get,f2,mode,2,freq

finish

dt=1/(20*f2)
tson=2
f0=1/dt

/solu
antype,4
outres,all,all
kbc,1
deltim,dt

f,3,fy,f0
time,dt/100000
solve

f,3,fy,f0
time,dt
solve

f,3,fy,0
time,dt+dt/100000
solve
```

```
f,3,fy,0
time,tson
solve

finish
/post26
nsol,2,3,u,y
plvar,3
```

# newmarkstep.m

```matlab
beta=1/4; gama=0.5;
dt=0.001; tson=2.2;
dof=2;

m1=50;
c1=1000;
k1=30000;

m2=50;
c2=1000;
k2=30000;

MM=[m1 0;0 m2];
CC=[c1+c2 -c2;-c2 c2];
KK=[k1+k2 -k2;-k2 k2];
adimsay=tson/dt+1;
adimsay=int32(adimsay);
adimsay=double(adimsay);

dep=zeros(adimsay,dof);
hiz=zeros(adimsay,dof);
ivm=zeros(adimsay,dof);
zaman=zeros(adimsay,dof);
F0=[0;1];
u0=[0;0];
ud0=[0;0];

udd0=MM\(F0-CC*ud0-KK*u0);

dep(1,1:dof)=u0';
```

```
hiz(1,1:dof)=ud0';
ivm(1,1:dof)=udd0';
zaman(1,1:dof)=0;
sayac=2;
t=dt;



a0=1/(beta*dt^2); a1=gama/(beta*dt); a2=1/(beta*dt); a3=1/(2*beta)-1;
a4=gama/beta-1;
a5=(dt/2)*(gama/beta-2); a6=dt*(1-gama); a7=gama*dt;


KK=KK+a0*MM+a1*CC;
[L,U]=lu(KK);
ss=2;
  while t<=tson;


F=F0+MM*(a0*u0+a2*ud0+a3*udd0)+CC*(a1*u0+a4*ud0+a5*udd0);
z=inv(L)*F;
u1=inv(U)*z;


udd1=a0*(u1-u0)-a2*ud0-a3*udd0;
ud1=ud0+a6*udd0+a7*udd1;


for i=1:dof
  dep(sayac,i)=u1(i);
  hiz(sayac,i)=ud1(i);
  ivm(sayac,i)=udd1(i);
end


zaman(sayac,1)=t;


 sayac=sayac+1;
```

```
    t=t+dt;

  u0=u1;
  ud0=ud1;
  udd0=udd1;
  ss=ss+1;
end
%subplot(2,1,1),plot(zaman(:,1),dep)
%subplot(2,1,2),plot(zaman(:,2),dep)

plot(zaman(:,1),dep(:,2),'black')
```

# matlabstep.m

```
m1=50;
c1=1000;
k1=30000;

m2=50;
c2=1000;
k2=30000;

s=sym('s');
num1=(c2*s+k2);
den1=(m2*s^2+c2*s+k2)*(m1*s^2+(c1+c2)*s+k1+k2)-(c2*s+k2)^2;

num2=(m1*s^2+(c1+c2)*s+k1+k2);
den2=(m2*s^2+c2*s+k2)*(m1*s^2+(c1+c2)*s+k1+k2)-(c2*s+k2)^2;

num1;
den1;
num2;
den2;
num1=expand(num1);
den1=expand(den1);
num2=expand(num2);
den2=expand(den2);

num1=[1000 30000];
den1=[2500 150000 5500000 60000000 900000000];
num2=[50 2000 60000];
%den2=[2500 150000 5500000 60000000 900000000];
den2=den1;
step(num2,den2)
```

```
/prep7
et,1,mass21
et,2,combin14
m=50
c=1000
k=30000

r,1,0,m
r,2,k,c

n,1,0,0,0
n,2,0,1,0
n,3,0,2,0

type,1
real,1
e,2
e,3

type,2
real,2

e,1,2
e,2,3
eplot

/solu
antype,2
modopt,lanb,2

d,all,ux,0
```

```
d,all,uz,0
d,1,uy,0

solve

*get,f1,mode,1,freq
*get,f2,mode,2,freq

finish

dt=1/(20*f2)
tson=2
f0=1

/solu
antype,4
outres,all,all
kbc,1
deltim,dt

f,3,fy,f0
time,dt/100000
solve

f,3,fy,f0
time,tson
solve

finish
/post26
nsol,2,3,u,y
plvar,
```

# pinned_pinned.m

```matlab
clc;clear
tic;
lenL=1;  %input('Kirisin Boyunu Giriniz=');
b=0.01; %input('Kirisin genisligini giriniz=');
h=0.01; %input('Kirisin yüksekligini giriniz=');
n=20;    %input('Kiriste kullanilacak eleman sayisi=');
%Ro=input('Kirisin yogunlugunu giriniz=');
%E=input('Kirisin elastisite modülünü giriniz=');
ksi1=0.05;%Birinci Doğal frekansın sönüm oranı
ksi2=0.06;%İkinci doğal frekansın sönüm oranı

Ro=7860;
E=206e9;
A=b*h;
I=b*h^3/12;
L=lenL/n;
mcof=(Ro*A*L)/420;
kcof=(E*I)/L^3;

alfa=2;%input('alfa degerini giriniz=');

M=zeros(2*(n+1));
K=zeros(2*(n+1));

ki1=[12 6*L -12 6*L;
    6*L 4*L^2 -6*L 2*L^2;
    -12 -6*L 12 -6*L;
    6*L 2*L^2 -6*L 4*L^2];

k1=ki1(1:2,1:2);
k2=ki1(1:2,3:4);
```

```matlab
k3=ki1(3:4,1:2);
k4=ki1(3:4,3:4);


 K(1:2,1:2)=k1;
K(1:2,3:4)=k2;


 for i=1:n
   K(2*i+1:2*i+2,2*i-1:2*i)=k3;
end
 for i=1:n-1
   K(2*i+1:2*i+2,2*i+3:2*i+4)=k2;
   K(2*i+1:2*i+2,2*i+1:2*i+2)=k1+k4;
end
K(2*n+1:2*n+2,2*n+1:2*n+2)=k4;


K;


mi1=[156 22*L 54 -13*L;
    22*L 4*L^2 13*L -3*L^2;
    54 13*L 156 -22*L;
    -13*L -3*L^2 -22*L 4*L^2];


 m1=mi1(1:2,1:2);
m2=mi1(1:2,3:4);
m3=mi1(3:4,1:2);
m4=mi1(3:4,3:4);


M(1:2,1:2)=m1;
M(1:2,3:4)=m2;


for i=1:n
   M(2*i+1:2*i+2,2*i-1:2*i)=m3;
```

```matlab
end

for i=1:n-1
    M(2*i+1:2*i+2,2*i+3:2*i+4)=m2;
    M(2*i+1:2*i+2,2*i+1:2*i+2)=m1+m4;
end
M(2*n+1:2*n+2,2*n+1:2*n+2)=m4;

M;

K=K*kcof;
M=M*mcof;

KK=K([2:2*n 2*n+2],[2:2*n 2*n+2]);
MM=M([2:2*n 2*n+2],[2:2*n 2*n+2]);

fr=sqrt(eig(KK,MM));

r=(2*fr(4)*fr(2)/(fr(4)^2-fr(2)^2))*[fr(4) -fr(2);-1/fr(4) 1/fr(2)]*[ksi1;ksi2];
CC=r(1)*MM+r(2)*KK;
for i=1:2*n
    ksi(i)=r(1)/(2*fr(i))+r(2)*fr(i)/2;
end
disp('Her Modun Sönüm Oranı')
ksi'

fr=sort(sqrt(eig(KK,MM)))/(2*pi);
disp('Hertz cinsinden Doğal frekanslar')
vpa(fr,5)

CC=0;
```

```matlab
% Newmark Integration Method

beta=1/4; gama=0.5;
T1=1/fr(1);
dt=1/(fr(20)*20); tson=T1/alfa;
dof=2*n;

adimsay=tson/dt+1;
adimsay=int32(adimsay);
adimsay=double(adimsay);

dep=zeros(adimsay,dof);
hiz=zeros(adimsay,dof);
ivm=zeros(adimsay,dof);
zaman=zeros(adimsay,dof);

u0=zeros(dof,1);
ud0=zeros(dof,1);

% kuvvetler üretiliyor

F0=zeros(dof,adimsay);
FG=-100;
dtnod=tson/n;

adimsay1=dtnod/dt;
adimsay1=int32(adimsay1);
adimsay1=double(adimsay1);

%Hareket eden yük tanımlanıyor
zz=1;
for i=2:2:dof-2;
```

```matlab
k=1;
for stp=(zz*adimsay1-adimsay1)+1:zz*adimsay1+1;
    F0(i,stp)=FG*(adimsay1-(adimsay1-(k-1)))/adimsay1;
    k=k+1;
end
k=1;
for stp=zz*adimsay1+1:zz*adimsay1+adimsay1+1;
    F0(i,stp)=FG*(adimsay1-(k-1))/adimsay1;
    k=k+1;
end
zz=zz+1;
end


udd0=MM\(F0(:,1)-CC*ud0-KK*u0);


dep(1,1:dof)=u0';
hiz(1,1:dof)=ud0';
ivm(1,1:dof)=udd0';
zaman(1,1:dof)=0;
sayac=2;
t=dt;


a0=1/(beta*dt^2);      a1=gama/(beta*dt);      a2=1/(beta*dt);      a3=1/(2*beta)-1;
a4=gama/beta-1;
a5=(dt/2)*(gama/beta-2); a6=dt*(1-gama); a7=gama*dt;


KK=KK+a0*MM+a1*CC;


[L,U]=lu(KK);
ss=2;
while t<=tson;
F=F0(:,ss)+MM*(a0*u0+a2*ud0+a3*udd0)+CC*(a1*u0+a4*ud0+a5*udd0);
```

```matlab
 z=inv(L)*F;
u1=inv(U)*z;
 udd1=a0*(u1-u0)-a2*ud0-a3*udd0;
ud1=ud0+a6*udd0+a7*udd1;

 for i=1:dof
  dep(sayac,i)=u1(i);
  hiz(sayac,i)=ud1(i);
  ivm(sayac,i)=udd1(i);
 end

 zaman(sayac,1)=t;
 sayac=sayac+1;
t=t+dt;

 u0=u1;
 ud0=ud1;
 udd0=udd1;
 ss=ss+1;
end

minx=min(dep(:,n))*1000;
maxx=max(dep(:,n))*1000;

plot(zaman(:,1),dep(:,n),'red')
xlabel('Time(sec)')
ylabel('Displacement (m)')

minx
bu=-12.13592233010;
minx/bu
toc
```

```
/config,nres,20000
b=0.01
h=0.01
a=b*h
iz=b*h*h*h/12

/prep7
et,1,beam3
r,1,a,iz,h
mp,ex,1,2.06e11
mp,dens,1,7860
mp,nuxy,1,0.3

l1=1
n1=10
ntoplam=n1+1
dx1=l1/n1

!Birinci Kısım Oluşturuluyor
k=1
*do,x,0,l1,dx1
n,k,x,0
k=k+1
*enddo

type,1
real,1
*do,k,1,n1,1
e,k,k+1
k=k+1
```

```
*enddo
eplot

/solu
antype,2
modopt,lanb,10
d,1,ux,0
d,1,uy,0
d,ntoplam,ux,0
d,ntoplam,uy,0
solve

*get,f1,mode,1,freq
*get,f2,mode,2,freq
*get,f3,mode,3,freq
*get,f4,mode,4,freq
*get,f5,mode,5,freq
*get,f6,mode,6,freq
*get,f7,mode,7,freq
*get,f8,mode,8,freq
*get,f9,mode,9,freq
*get,f10,mode,10,freq

FINISH

f0=-100
alfa=2
v=alfa*l1*f1
tson=l1/v
t10=1/f10
dt=t10/10
```

```
/solu
antype,4
outres,all,all
kbc,0
deltim,dt

nbas=1
f,nbas,fy,f0
time,dt/100
solve

nbit=nbas+n1
nort=nbas+(n1/2)

dtnod=tson/n1

*do,nd,nbas+1,nbit,1
f,nd-1,fy,0
f,nd,fy,f0
time,(nd-1)*dtnod
solve
*enddo


finish
/post26
nsol,2,nort,u,y
plvar,2
```

# frame.m

```
clc
clear
tic;
        %Çubuklara ait Bilgiler--------------------------------
len1=1;
len2=1;
len3=1;

%len1=input('Birinci cubugun Boyunu Giriniz=');
%len2=input('Ikinci cubugun Boyunu Giriniz=');
%len3=input('Üçüncü cubugun Boyunu Giriniz=');
%n1=input('Ilk çubukta kullanilacak eleman sayisi=');
%n2=input('Ikinci çubukta kullanilacak eleman sayisi=');
%n3=input('Üçüncü çubukta kullanilacak eleman sayisi=');
%Ro1=input('Ilk çubugun yogunlugunu giriniz=');
%Ro2=input('Ikinci çubugun yogunlugunu giriniz=');
%Ro3=input('Üçüncü çubugun yogunlugunu giriniz=');
%E1=input('Ilk çubugun elastisite modülünü giriniz=');
%E2=input('Ikinci çubugun elastisite modülünü giriniz=');
%E3=input('Üçüncü çubugun elastisite modülünü giriniz=');
Ro1=7860;
Ro2=7860;
Ro3=7860;
n1=10;
n2=n1*len2/len1;

if rem(n2,2)~=0,

disp('n2 kirisinin orta noktasinin olabilmesi için n1 için girdiginiz degeri
degistiriniz!');
```

```matlab
break;
else
n2=n2;

n3=n1*len3/len1;
E1=206e9;
E2=206e9;
E3=206e9;

alfa=4.5;%input('alfa degerini giriniz=');

%b1=input('Ilk çubugun genisligini giriniz=');
%b2=input('Ikinci çubugun genisligini giriniz=');
%b3=input('Üçüncü çubugun genisligini giriniz=');
b1=0.01;
b2=0.01;
b3=0.01;
%t1=input('Ilk çubugun yüksekligini giriniz=');
%t2=input('Ikinci çubugun yüksekligini giriniz=');
%t3=input('Üçüncü çubugun yüksekligini giriniz=');
t1=0.01;
t2=0.01;
t3=0.01;

A1=b1*t1;
I1=b1*t1^3/12;

%A1=(pi*0.04^2/4);
%I1=(pi*0.04^4/64);

A2=b2*t2;
I2=b2*t2^3/12;
```

```matlab
%A2=(pi*0.04^2/4);
%I2=(pi*0.04^4/64);

A3=b3*t3;
I3=b3*t3^3/12;

%A3=(pi*0.04^2/4);
%I3=(pi*0.04^4/64);

q1=90;
q2=0;
q3=270;
%q1=input('Ilk çubugun yatayla yaptigi açiyi giriniz(Derece olarak)=');
%q2=input('Ikinci çubugun yatayla yaptigi açiyi giriniz(Derece olarak)=');
%q3=input('Üçüncü çubugun yatayla yaptigi açiyi giriniz(Derece olarak)=');

c1=cos(q1*pi/180);
s1=sin(q1*pi/180);
c2=cos(q2*pi/180);
s2=sin(q2*pi/180);
c3=cos(q3*pi/180);
s3=sin(q3*pi/180);

%1. Çubugun  rijitlik,kütle, ve geometrik matrisleri-----------------------------

%E2=E1;
%E3=E1;
%Ro2=Ro1;
%Ro3=Ro1;

l1=len1/n1;
```

```matlab
mcof1=(Ro1*A1*l1)/420;
kcof1=(E1*I1)/l1^3;
R1=A1*l1^2/I1;

m1=zeros(3*(n1+1));
k1=zeros(3*(n1+1));

ki1=[R1 0 0 -R1 0 0;
    0 12 6*l1 0 -12 6*l1;
    0 6*l1 4*l1^2 0 -6*l1 2*l1^2;
   -R1 0 0 R1 0 0;
    0 -12 -6*l1 0 12 -6*l1;
    0 6*l1 2*l1^2 0 -6*l1 4*l1^2];

 db1=[c1 s1 0 0 0 0;
  -s1 c1 0 0 0 0;
  0 0 1 0 0 0 ;
  0 0 0 c1 s1 0;
  0 0 0  -s1 c1 0;
  0 0 0 0 0 1];

kd1=db1'*ki1*db1;

k11=kd1(1:3,1:3);
k12=kd1(1:3,4:6);
k13=kd1(4:6,1:3);
k14=kd1(4:6,4:6);

k1(1:3,1:3)=k11;
k1(1:3,4:6)=k12;
```

```
 for i=1:n1-1
   k1(3*i+1:3*i+3,3*i-2:3*i)=k13;
   k1(3*i+1:3*i+3,3*i+1:3*i+3)=k11+k14;
   k1(3*i+1:3*i+3,3*i+4:3*i+6)=k12;
 end

k1(3*n1+1:3*n1+3,3*n1-2:3*n1)=k13;
k1(3*n1+1:3*n1+3,3*n1+1:3*n1+3)=k14;

k1;

mi1=[140 0 0 70 0 0;
  0 156 22*l1 0 54 -13*l1;
  0 22*l1 4*l1^2 0 13*l1 -3*l1^2;
  70 0 0 140 0 0;
  0 54 13*l1 0 156 -22*l1;
  0 -13*l1 -3*l1^2 0 -22*l1 4*l1^2];

md1=db1'*mi1*db1;
m11=md1(1:3,1:3);
m12=md1(1:3,4:6);
m13=md1(4:6,1:3);
m14=md1(4:6,4:6);

m1(1:3,1:3)=m11;
m1(1:3,4:6)=m12;

 for i=1:n1-1
   m1(3*i+1:3*i+3,3*i-2:3*i)=m13;
   m1(3*i+1:3*i+3,3*i+1:3*i+3)=m11+m14;
   m1(3*i+1:3*i+3,3*i+4:3*i+6)=m12;
 end
```

```matlab
m1(3*n1+1:3*n1+3,3*n1-2:3*n1)=m13;
m1(3*n1+1:3*n1+3,3*n1+1:3*n1+3)=m14;

m1;

k1=k1*kcof1;
m1=m1*mcof1;

%2. Çubugun  rijitlik,kütle, ve geometrik matrisleri------------------------------

l2=len2/n2;
mcof2=(Ro2*A2*l2)/420;
kcof2=(E2*I2)/l2^3;
R2=A2*l2^2/I2;


m2=zeros(3*(n2+1));
k2=zeros(3*(n2+1));

ki2=[R2 0 0 -R2 0 0;
  0 12 6*l2 0 -12 6*l2;
  0 6*l2 4*l2^2 0 -6*l2 2*l2^2;
  -R2 0 0 R2 0 0;
  0 -12 -6*l2 0 12 -6*l2;
  0 6*l2 2*l2^2 0 -6*l2 4*l2^2];

db2=[c2 s2 0 0 0 0;
  -s2 c2 0 0 0 0;
  0 0 1 0 0 0 ;
  0 0 0 c2 s2 0;
  0 0 0 -s2 c2 0;
```

```matlab
     0 0 0 0 0 1];

kd2=db2'*ki2*db2;

k21=kd2(1:3,1:3);
k22=kd2(1:3,4:6);
k23=kd2(4:6,1:3);
k24=kd2(4:6,4:6);

k2(1:3,1:3)=k21;
k2(1:3,4:6)=k22;

 for i=1:n2-1
   k2(3*i+1:3*i+3,3*i-2:3*i)=k23;
   k2(3*i+1:3*i+3,3*i+1:3*i+3)=k21+k24;
   k2(3*i+1:3*i+3,3*i+4:3*i+6)=k22;
 end

k2(3*n2+1:3*n2+3,3*n2-2:3*n2)=k23;
k2(3*n2+1:3*n2+3,3*n2+1:3*n2+3)=k24;

k2;

mi2=[140 0 0 70 0 0;
   0 156 22*l2 0 54 -13*l2;
   0 22*l2 4*l2^2 0 13*l2 -3*l2^2;
   70 0 0 140 0 0;
   0 54 13*l2 0 156 -22*l2;
   0 -13*l2 -3*l2^2 0 -22*l2 4*l2^2];

md2=db2'*mi2*db2;
m21=md2(1:3,1:3);
```

```matlab
m22=md2(1:3,4:6);
m23=md2(4:6,1:3);
m24=md2(4:6,4:6);

m2(1:3,1:3)=m21;
m2(1:3,4:6)=m22;

 for i=1:n2-1
   m2(3*i+1:3*i+3,3*i-2:3*i)=m23;
   m2(3*i+1:3*i+3,3*i+1:3*i+3)=m21+m24;
   m2(3*i+1:3*i+3,3*i+4:3*i+6)=m22;
 end

m2(3*n2+1:3*n2+3,3*n2-2:3*n2)=m23;
m2(3*n2+1:3*n2+3,3*n2+1:3*n2+3)=m24;

m2;

k2=k2*kcof2;
m2=m2*mcof2;

%3. Çubugun  rijitlik,kütle, ve geometrik matrisleri------------------------------

l3=len3/n3;
mcof3=(Ro3*A3*l3)/420;
kcof3=(E3*I3)/l3^3;
R3=A3*l3^2/I3;

m3=zeros(3*(n3+1));
k3=zeros(3*(n3+1));

ki3=[R3 0 0 -R3 0 0;
```

```matlab
   0 12 6*l3 0 -12 6*l2;
   0 6*l3 4*l3^2 0 -6*l3 2*l3^2;
   -R3 0 0 R3 0 0;
   0 -12 -6*l3 0 12 -6*l3;
   0 6*l3 2*l3^2 0 -6*l3 4*l3^2];

db3=[c3 s3 0 0 0 0;
   -s3 c3 0 0 0 0;
   0 0 1 0 0 0 ;
   0 0 0 c3 s3 0;
   0 0 0 -s3 c3 0;
   0 0 0 0 0 1];

 kd3=db3'*ki3*db3;

k31=kd3(1:3,1:3);
k32=kd3(1:3,4:6);
k33=kd3(4:6,1:3);
k34=kd3(4:6,4:6);

k3(1:3,1:3)=k31;
k3(1:3,4:6)=k32;

 for i=1:n3-1
   k3(3*i+1:3*i+3,3*i-2:3*i)=k33;
   k3(3*i+1:3*i+3,3*i+1:3*i+3)=k31+k34;
   k3(3*i+1:3*i+3,3*i+4:3*i+6)=k32;
 end

k3(3*n3+1:3*n3+3,3*n3-2:3*n3)=k33;
k3(3*n3+1:3*n3+3,3*n3+1:3*n3+3)=k34;
```

```
k3;

mi3=[140 0 0 70 0 0;
   0 156 22*l3 0 54 -13*l3;
   0 22*l3 4*l3^2 0 13*l3 -3*l3^2;
   70 0 0 140 0 0;
   0 54 13*l3 0 156 -22*l3;
   0 -13*l3 -3*l3^2 0 -22*l3 4*l3^2];

md3=db3'*mi3*db3;

m31=md3(1:3,1:3);
m32=md3(1:3,4:6);
m33=md3(4:6,1:3);
m34=md3(4:6,4:6);

m3(1:3,1:3)=m31;
m3(1:3,4:6)=m32;

 for i=1:n3-1
   m3(3*i+1:3*i+3,3*i-2:3*i)=m33;
   m3(3*i+1:3*i+3,3*i+1:3*i+3)=m31+m34;
   m3(3*i+1:3*i+3,3*i+4:3*i+6)=m32;
 end

m3(3*n3+1:3*n3+3,3*n3-2:3*n3)=m33;
m3(3*n3+1:3*n3+3,3*n3+1:3*n3+3)=m34;

m3;

k3=k3*kcof3;
m3=m3*mcof3;
```

```
%Sistem matrisleri olusturuluyor---------------------------------

K=zeros(3*(n1+n2+n3+1));
M=zeros(3*(n1+n2+n3+1));

K(1:3*(n1+1),1:3*(n1+1))=k1;
K(3*(n1+1)-2:3*(n1+n2+1),3*(n1+1)-2:3*(n1+n2+1))=k2;
K(3*(n1+n2+1)-2:3*(n1+n2+n3+1),3*(n1+n2+1)-2:3*(n1+n2+n3+1))=k3;
K(3*(n1+1)-2:3*(n1+1),3*(n1+1)-2:3*(n1+1))=k1(3*(n1+1)-2:3*(n1+1),3*(n1+1)-
2:3*(n1+1))+k2(1:3,1:3);
K(3*(n1+n2+1)-2:3*(n1+n2+1),3*(n1+n2+1)-2:3*(n1+n2+1))=k2(3*(n2+1)-
2:3*(n2+1),3*(n2+1)-2:3*(n2+1))+k3(1:3,1:3);

M(1:3*(n1+1),1:3*(n1+1))=m1;
M(3*(n1+1)-2:3*(n1+n2+1),3*(n1+1)-2:3*(n1+n2+1))=m2;
M(3*(n1+n2+1)-2:3*(n1+n2+n3+1),3*(n1+n2+1)-2:3*(n1+n2+n3+1))=m3;
M(3*(n1+1)-2:3*(n1+1),3*(n1+1)-2:3*(n1+1))=m1(3*(n1+1)-2:3*(n1+1),3*(n1+1)-
2:3*(n1+1))+m2(1:3,1:3);
M(3*(n1+n2+1)-2:3*(n1+n2+1),3*(n1+n2+1)-2:3*(n1+n2+1))=m2(3*(n2+1)-
2:3*(n2+1),3*(n2+1)-2:3*(n2+1))+m3(1:3,1:3);

K;
M;
KK=K(4:3*(n1+n2+n3+1)-3,4:3*(n1+n2+n3+1)-3);
MM=M(4:3*(n1+n2+n3+1)-3,4:3*(n1+n2+n3+1)-3);

[V,D]=eig(KK,MM);

fr=sort(sqrt(eig(KK,MM)))/(2*pi);
vpa(fr,5)
CC=0;
```

```
% Newmark Integration Method

beta=1/4; gama=0.5;
T1=1/fr(1);
dt=1/(fr(20)*20); tson=T1/alfa;
dof=3*(n1+n2+n3)-3;

adimsay=tson/dt+1;
adimsay=int32(adimsay);
adimsay=double(adimsay);

dep=zeros(adimsay,dof);
hiz=zeros(adimsay,dof);
ivm=zeros(adimsay,dof);
zaman=zeros(adimsay,dof);

u0=zeros(dof,1);
ud0=zeros(dof,1);

% kuvvetler üretiliyor

F0=zeros(dof,adimsay);
FG=-100;
dtnod=tson/n2;

adimsay1=dtnod/dt;
adimsay1=int32(adimsay1);
adimsay1=double(adimsay1);

%Hareket eden yük tanımlanıyor
k=1;
```

```matlab
  for stp=1:adimsay1+1;
     F0(3*n1-1,stp)=FG*(adimsay1-(k-1))/adimsay1;
     k=k+1;
  end
  zz=1;
  for i=3*n1+2:3:3*(n1+n2)-4;
   k=1;
   for stp=(zz*adimsay1-adimsay1)+1:zz*adimsay1+1;
      F0(i,stp)=FG*(adimsay1-(adimsay1-(k-1)))/adimsay1;
      k=k+1;
   end
   k=1;
   for stp=zz*adimsay1+1:zz*adimsay1+adimsay1+1;
      F0(i,stp)=FG*(adimsay1-(k-1))/adimsay1;
      k=k+1;
   end
   zz=zz+1;
  end

  k=1;
  for stp=zz*adimsay1+1:zz*adimsay1+adimsay1+1;
    F0(3*(n1+n2)-1,stp)=FG*(adimsay1-(adimsay1-(k-1)))/adimsay1;
     k=k+1;
  end
  udd0=MM\(F0(:,1)-CC*ud0-KK*u0);

  dep(1,1:dof)=u0';
  hiz(1,1:dof)=ud0';
  ivm(1,1:dof)=udd0';
  zaman(1,1:dof)=0;
  sayac=2;
  t=dt;
```

```
a0=1/(beta*dt^2); a1=gama/(beta*dt); a2=1/(beta*dt); a3=1/(2*beta)-1;
a4=gama/beta-1;
a5=(dt/2)*(gama/beta-2); a6=dt*(1-gama); a7=gama*dt;

KK=KK+a0*MM+a1*CC;

[L,U]=lu(KK);
ss=2;

while t<=tson;

 F=F0(:,ss)+MM*(a0*u0+a2*ud0+a3*udd0)+CC*(a1*u0+a4*ud0+a5*udd0);
 z=inv(L)*F;
 u1=inv(U)*z;

 udd1=a0*(u1-u0)-a2*ud0-a3*udd0;
 ud1=ud0+a6*udd0+a7*udd1;

 for i=1:dof
   dep(sayac,i)=u1(i);
   hiz(sayac,i)=ud1(i);
   ivm(sayac,i)=udd1(i);
 end

 zaman(sayac,1)=t;

 sayac=sayac+1;
 t=t+dt;

 u0=u1;
 ud0=ud1;
```

```
 udd0=udd1;
 ss=ss+1;
end
on=3*n1-3+((n2+1)*3+1)/2;
minx=min(dep(:,on))*1000;
maxx=max(dep(:,on))*1000;

plot(zaman(:,1),dep(:,on),'red')
 %subplot(3,1,1),plot(zaman(:,1),dep(:,on)*1000,'b.-')
%xlabel('Time(sn)')
%ylabel('Displacement (mm)')
%subplot(3,1,2),plot(zaman(:,1),hiz(:,on)*1000,'b.-')
%xlabel('Time(sn)')
%ylabel('Velocity (mm/sn)')
%subplot(3,1,3),plot(zaman(:,1),ivm(:,on)*1000,'b.-')
%xlabel('Time(sn)')
%ylabel('Acceleration (mm/sn^2)')

end

toc
```

```
/config,nres,10000
b=0.01
h=0.01
a=b*h
iz=b*h*h*h/12

/prep7
et,1,beam3
r,1,a,iz,h
mp,ex,1,2.06e11
mp,dens,1,7860
mp,nuxy,1,0.3

l1=1
l2=1
l3=1

n1=10
n2=10
n3=10
ntoplam=n1+n2+n3+1

dx1=l1/n1
dx2=l2/n2
dx3=l3/n3

!Birinci Kısım Oluşturuluyor
k=1
*do,y,0,l1,dx1
n,k,0,y
k=k+1
```

```
*enddo
!İkinci Kısım Oluşturuluyor
*do,x,dx2,l2-dx2,dx2
n,k,x,l1
k=k+1
*enddo
!Üçüncü Kısım Oluşturuluyor
*do,y,l3,0,-dx3
n,k,l2,y
k=k+1
*enddo

type,1
real,1
*do,k,1,n1+n2+n3,1
e,k,k+1
k=k+1
*enddo
eplot

/solu
antype,2
modopt,lanb,10
d,1,all,0
d,ntoplam,all,0
solve

*get,f1,mode,1,freq
*get,f2,mode,2,freq
*get,f3,mode,3,freq
*get,f4,mode,4,freq
*get,f5,mode,5,freq
```

```
*get,f6,mode,6,freq

*get,f7,mode,7,freq

*get,f8,mode,8,freq

*get,f9,mode,9,freq

*get,f10,mode,10,freq


FINISH


f0=-100

alfa=1

v=alfa*l2*f1

tson=l2/v

t10=1/f10

dt=t10/20


/solu

antype,4

outres,all,all

kbc,0

deltim,dt


nbas=n1+1

f,nbas,fy,f0

time,dt/100

solve


nbit=nbas+n2

nort=nbas+(n2/2)


dtnod=tson/n2


*do,nd,nbas+1,nbit,1
```

```
f,nd-1,fy,0
f,nd,fy,f0
time,(nd-(n1+1))*dtnod
solve
*enddo
finish
/post26
nsol,2,nort,u,y
plvar,2
```