DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

COMPLEXITY REDUCTION OF RBF MULTIUSER DETECTORS FOR DS-CDMA IN AWGN AND RAYLEIGH FADING CHANNELS USING GENETIC ALGORITHM

by
M. Uğur TORUN

July, 2007
İZMİR

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by
M. Uğur TORUN

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M.Sc. THESIS EXAMINATION RESULT FORM

We have read the thesis entitled "**COMPLEXITY REDUCTION OF RBF MULTIUSER DETECTORS FOR DS-CDMA IN AWGN AND RAYLEIGH FADING CHANNELS USING GENETIC ALGORITHM**" completed by **M. UĞUR TORUN** under supervision of **ASST. PROF. DR. DAMLA KUNTALP** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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COMPLEXITY REDUCTION OF RBF MULTIUSER DETECTORS FOR DS-CDMA IN AWGN AND RAYLEIGH FADING CHANNELS USING GENETIC ALGORITHM

ABSTRACT

The optimum detector for direct sequence code division multiple access (DS-CDMA) signals has a computational complexity which increases exponentially with the number of users. The number of users in a real-life CDMA system may become very high which makes the optimum detector impractical and expensive to implement. Thus, several suboptimal multiuser detectors (MUD) which have lower computational complexities than that of the optimum detector were proposed. Due to the decision boundary introduced by the optimum detector, nonlinear detectors outperform linear detectors. Radial basis function (RBF) MUD is a nonlinear suboptimal detector which can perfectly approximate this nonlinear decision boundary. However, RBF MUD suffers from structural complexity since the number of its centers increases exponentially with the number of users. In this thesis, a new method to reduce the number of center functions of the RBF MUD using genetic algorithm (GA) and least mean squares (LMS) algorithm is proposed. The performance of the method is tested via computer simulations which are performed in both AWGN and multipath fading channels. Simulation results showed that the proposed method immensely reduces the complexity of the RBF MUD with a negligible performance degradation.

Keywords: DS-CDMA, radial basis function network, radial basis function multiuser detector, genetic algorithm.

DD-KBÇE İÇİN RTF ÇOK KULLANICILI SEZİCİLERİN TBGG VE RAYLEIGH SÖNÜMLEMELİ KANALLARDA GENETİK ALGORİTMA İLE KARMAŞALARININ AZALTILMASI

ÖZ

Doğrudan dizili kod bölüşümlü çoklu erişim (DD-KBÇE) için tasarlanmış olan optimum sezicinin hesaplama karmaşası kullanıcı sayısı ile oranlı olarak üssel bir şekilde artmaktadır. Bu durum, bir CDMA sistemindeki kullanıcı sayısı çok fazla olabileceği için, optimum seziciyi gerçeklenmesi pahalı ve pratik olmayan bir sezici kılmaktadır. Bu yüzden, hesaplama karmaşası optimum seziciden daha az olan birçok alt-optimum çok kullanıcılı sezici (ÇKS) tasarlanmıştır. Optimum sezicinin karar sınırı doğrusal değildir. Bu nedenle, doğrusal olmayan alt-optimum sezicilerin başarımı doğrusal olanlardan daha iyidir. Radyal taban fonksiyon (RTF) ÇKS bu doğrusal olmayan karar sınırına mükemmel bir şekilde yaklaşan bir alt-optimum sezicidir. Ancak, RTF ÇKS'nin yapısındaki merkez fonksiyonların sayısı, dolayısıyla yapısal karmaşası sistemdeki kullanıcı sayısı ile oranlı olarak üssel bir şekilde artmaktadır. Bu tezde, RTF ÇKS'nin merkez sayılarını genetik algoritma (GA) ve en küçük ortalama kareler (EKOK) algoritması ile eniyileme yaparak azaltan yeni bir yöntem önerilmiştir. Önerilen yöntemin performansı, TBGG ve çokyol sönümlemeli kanallarda gerçekleştirilen benzetimler ile test edilmiştir. Benzetim sonuçları, önerilen yöntemin RTF ÇKS'nin yapısal karmaşasını, düşük bir performans kaybı ile, çok büyük oranda azaltabildiğini göstermiştir.

Anahtar Sözcükler: DD-KBÇE, radyal taban fonksiyon ağı, radyal taban fonksiyon çok kullanıcılı sezici, genetik algoritma.

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CHAPTER ONE
INTRODUCTION

Within the last decay, more people have get involved with the technology as it gets available at a lower price. Today, a cellular phone is not a luxury anymore and computers are at least ten times cheaper than they had been ten years ago. As the technology grows, not only the manufacture costs but also the physical sizes of the technological devices are decreased. This makes more people to prefer notebook personal computers and handhelds to desktop computers which are relatively bulky and heavy. As people get "mobile" they ask to be "connected". In other words, the demand for the mobile communication increases rapidly as the manufacturers make small, portable and smart devices. Today, a mobile network operator has to provide a variety of services including fax, E-mail, fast Internet, multimedia transfer etc. in addition to the telephony service. The operator also has to deal with more users who demand to transmit voice and data within a cell. From an engineering point of view, more services and users lead to a demand for more bandwidth. Since the frequency spectrum available for the commercial mobile communications is not infinite, a smarter method to multiplex the users who share a common channel is needed. It must be also possible to provide a particular user more bandwidth who shares the same channel with other users.

A promising technology is the *code division multiple access* (CDMA) which is a spread spectrum-based access method and has taken a significant role in cellular and personal communication systems in the last decades. It has already been in commercial use in 2.5G and 3G mobile communications.

CDMA assigns unique spreading codes to different users which allows multiple users to communicate simultaneously using the same frequency band. Spread spectrum communication has become popular due to its advantages like jamming and interference resistance, signal hiding, low probability of intercept, good multipath performance, secure communications, improved spectral efficiency over other access methods (Jung et al., 1993). Of the many spread spectrum-based multiple access schemes available, the most widely used one is the *direct sequence CDMA* (DS-CDMA) scheme. In DS-CDMA, the transmitter multiplies each user's transmitted signal by a unique signature waveform; the received signal is a superposition of all user's signals which overlap in time and frequency.

The *conventional detector* for DS-CDMA passes the received signal through a bank of filters matched to the corresponding users' unique signature waveform, signs the output, and decides on the information bits. Here, each user is treated separately as a signal and the others are considered as interference or noise. This interference is commonly called as *multiple access interference* (MAI). In conventional single user detection, due to MAI, there is a problem called *near-far effect* which refers to the situation that the users near the receiver supplies more power to the receiver than those far from the receiver. Hence, several power control techniques are proposed to overcome near-far effect (Babich et al., 2004).

There is a second approach where the information of multiple users are used jointly to detect the information of a particular user. This approach is called *joint multiuser detection*. Here, MAI is treated as a part of information rather than noise (Verdu, 1986). *Optimum multiuser detector* offers superior performance over the conventional detector in terms of *near-far resistance* with the cost of computational complexity which increases exponentially with the increasing number of users. In a real life CDMA system, there will probably be very large number of users which would make the optimum detector impractical and very expensive to implement. Thus, researchers tried to develop *suboptimal receivers* which have reasonable computational complexities, are near-far resistant and have performances close to that of the optimum detector. *Decorrelating detector* is one of these suboptimal receivers which is linear, near-far resistant, and has a computational complexity proportional to the number of users (Lupas & Verdu, 1989a,b). Decorrelating detector introduces performance improvement over conventional detector in terms of MAI but it suffers from noise enhancement.

It has been shown that nonlinear receivers outperform linear receivers since the optimal decision boundary in DS-CDMA is nonlinear (Mulgrew, 1996). *Multistage detector* (MSD) (Varanasi & Aazhang, 1990) is a nonlinear detector which improves each stage's estimate by subtracting the estimate of the MAI obtained by the previous stage. Performance of an MSD can reach close to that of the optimum detector but it depends highly on the initial estimate, which is usually provided by the conventional detector or decorrelating detector.

Aazhang et al. (1992) were the first who had designed a nonlinear multiuser detector based on neural networks. Two structures employing multilayer perceptrons were proposed for the demodulation of spread spectrum signals in Gaussian channels by Aazhang et al. (1992). Nonlinear detector structures based on neural networks or polynomial series may provide near-optimum performance but they also suffer from high complexity (Cruickshank, 1996, Tanner & Cruickshank, 1997). It is also possible to design a MUD based on a *radial basis function network* (RBFN) (Cruickshank, 1996). An RBFN is considered as a neural network because of its structure and is a typical model of approaching a local extremum (Hush, 1993). It has been used in nonlinear approximation, pattern recognition, and other fields such as signal processing (Chen, 1995), automation (Fabri & Kadirkamanathan, 1996), system modelling (Elanayar & Shin, 1994), and etc.

Radial basis function multiuser detector (RBF MUD) was originally introduced by Cruickshank (1996) and further investigated by Tanner & Cruickshank (1998), Sessler et al. (2000, 2001), Wei et al. (2004). Two types of RBF MUD's were introduced by Tanner & Cruickshank (1998) where one of them operates at symbol rate and the other one at chip rate, named by the authors as *preprocessing based receiver* (PPB) and *chip level based receiver* (CLB) respectively. In this thesis we have dealt with CLB RBF MUD's; and this receiver will be shortly regarded as "RBF MUD".

1.1 Motivation and Innovation

An RBF MUD needs no training since it is fully determined when the spreading codes of all users and the channel impulse response are known. However, when the number of users are large, the RBF MUD gets impractical since its structural complexity increases exponentially with the increasing number of users. A preprocessing method was proposed by Tanner & Cruickshank (1998) to reduce the complexity of the RBF MUD and the resultant RBF MUD was named as PPB RBF MUD; this work was further investigated by Ko et al. (2001). Performance analysis of CLB and PPB receivers for ULTRA-TDD were investigated by Sessler et al. (2001) and it was shown that these receivers achieve low *bit error rates* (BER) even for time-variant multipath propagation channels like pedestrian and vehicular environments.

The number of neurons in the hidden layer of an RBFN may become excessive, even equal to the number of training samples due to the training process. This problem spawned an area of research on optimization of RBFN structures. One particular tool for optimization of RBFN structures is the *genetic algorithm* (GA). The common approach is representing the network as a string and optimizing the structure by applying GA operators to these strings (Harpham et al., 2004). A different method is introduced by Whitehead & Choate (1996) where, instead of each string representing a network, the whole GA population represents one network. Many methods aiming to optimize the RBFN with GA are well documented by Harpham et al. (2004).

A method for reducing the number of neurons (centers) in the hidden layer of RBF MUD using genetic algorithm was proposed by Wei et al. (2004). By discarding the low-contribution centers, the complexity of the receiver is reduced from $O(2^{LU})$ to $O(P)$ where U is the number of users, L is the number of taps in the dispersive multipath fading channel, and P is the GA's population size. But the performance of the receiver highly depends on the training set and BER stops decreasing and remains constant with the increasing E_b/N_0 rates.

In this thesis, a new method that reduces the number of centers in RBF MUD using GA is proposed. Instead of selecting centers from supercenters (Wei et al., 2004), our method starts with a small number of centers which are randomly selected from supercenters and applies modifications to the center vectors. From another point of view, it changes the location of the centers in the space. This new method also searches for the best variance values for each center by starting with an initial value of noise variance and applying operations of GA at each generation. The resultant structure has significantly reduced number of centers in comparison with 2^U and resulting center vectors are different from the supercenter vectors. Due to the flexibility of the location and variance value of each center function, the resultant RBF structure can perfectly represent the DS-CDMA space and achieve near-optimum performance even in high E_b/N_0 rates.

1.2 Thesis Organization

The current chapter is a brief introduction into code division multiple access (CDMA), CMDA multiuser detectors (MUD), radial basis function MUD (RBF MUD), and the proposed optimization method. This chapter ends with the thesis organization.

Chapter two give information about direct sequence CDMA (DS-CDMA) system in more detail. It gives brief information about spread spectrum communications and multiple access communication and then CDMA signal model is defined in the AWGN. Then, the signal model for DS-CDMA and its vector notation in both AWGN and multipath channels are given. Autocorrelation and cross correlation properties of several spreading sequences are discussed in the chapter. Chapter ends giving a brief information about the current and past commercial systems that use CDMA.

Chapter three focuses on the detectors designed to receive CDMA signals and discriminate user's data from each other. The optimum detector and several linear and nonlinear suboptimum detectors are introduced. The detectors are considered in terms of their resistance to multiple access interference (MAI), and to near-far problem; computational complexity; and achievable system performance.

Chapter four introduces an alternative MUD to the ones introduced in chapter three called radial basis function MUD (RBF MUD) which is originally introduced by Cruickshank (1996). After a brief review about radial basis function networks, the structure of the RBF MUD in both AWGN and multipath channels are considered. It is shown that the structural complexity of the RBF MUD increases exponentially with the number of users, thus it is stated that a method is needed to reduce its complexity.

Chapter five introduces the proposed method to reduce the structural complexity of RBF MUD. After a brief review about genetic algorithm, the definition of the problem is stated and the method is introduced by the help of a simple example. Details of the method is given through the end of the chapter.

Computer simulation method and the simulation results for the effects of the number of centers, initial population, number of generations, mutation probability, and population size both in AWGN and multipath fading channels are presented in chapter six.

The last chapter summarizes and discusses the motivation and the method presented in this thesis.

CHAPTER TWO

DIRECT SEQUENCE CODE DIVISION MULTIPLE ACCESS

This chapter introduces direct sequence code division multiple access (DS-CDMA). Prior to the introduction of DS-CDMA, a brief information on spread spectrum communications and multiple access communications is given. Next, the signal model for DS-CDMA and its vector notation in both AWGN and multipath channels are given. After a review about the characteristics of several spreading sequences, a brief information about current and past commercial CDMA systems are introduced.

2.1 Spread Spectrum Communication

Spread spectrum communications are originated from military and space applications. The main distinguishing feature of the spread spectrum signals used for the transmission of digital information is that their bandwidth W is much greater than the information rate R in bits/s. In other words, the bandwidth expansion factor $B_e = W/R$ for a spread spectrum signal is much greater than unity. A second important feature introduced by the spread spectrum signals is pseudorandomness, which makes the signals appear similar to random noise and difficult to demodulate by receivers other than the intended ones (Proakis, 2001).

To be specific, spread spectrum signals are used for:

- Hiding a signal by transmitting it at low power and, thus, making it difficult for an unintended listener to detect in the presence of background noise.
- Message privacy in the presence of other listeners.
- Combating the detrimental effects of interference due to jamming.
- Suppressing the interference arising from other users of the channel.

A message may be hidden in the background noise by spreading its bandwidth with coding and transmitting the resultant signal at a low average power. Because of its low power level, the transmitted signal is said to be "covert." It has a low probability of being intercepted (detected) by a casual listener and, hence, is also called a low-probability-of-intercept (LPI) signal.

Message privacy may be obtained by superimposing a pseudorandom pattern on a transmitted message. The message can be demodulated by the intended receivers, who know the pseudorandom pattern or key used at the transmitter, but not by any other receivers who do not have knowledge of the key.

The jammer that is trying to disrupt the communication may have prior knowledge of the overall channel bandwidth and the type of modulation (PSK, FSK, etc.). More sophisticated jammers may mimic the signal emitted by the transmitter if the signal is encoded using block and convolutional codes and confuse the receiver. It is possible to overcome this problem by introducing a randomness in the transmitter. The spectrum of the information signal is spread using a pseudorandom code which is known by the intended receiver but not by the jammer. Thus, the jammer must synthesize and transmit an interfering signal without the knowledge of the pseudorandom pattern.

Interference from the other users arises in multiple access communication systems in which a number of users share a common channel bandwidth. It is possible to overcome this *multiple access interference* (MAI) by spreading each users information data with uncorrelated codes (Proakis, 2001). This method will be explained in more detail throughout the chapter.

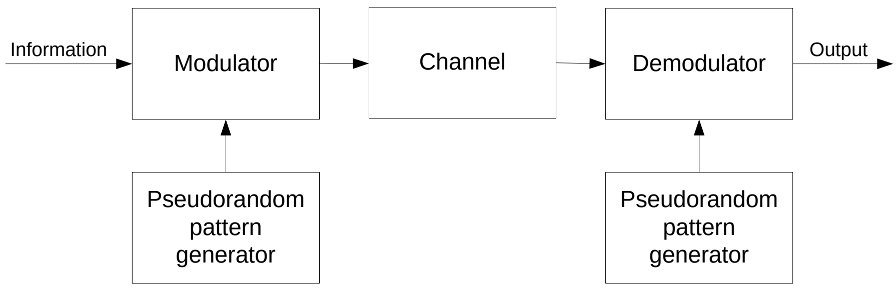


Figure 2.1: Basic elements of a spread spectrum digital communication system.

The block diagram of the basic elements of a spread spectrum digital communication system is given in Fig. 2.1. The pseudorandom pattern generators at both the modulator and the demodulator side are identical to each other and they generate a pseudonoise (PN) binary-valued sequence. This sequence is impressed on the transmitted signal at the modulator and removed from the received signal at the demodulator. Spread spectrum digital communication systems are mainly categorized by the modulation technique used. There are many types of spread spectrum digital communication systems of which two commonly used ones are considered: If the modulation type used is *phase shift keying* (PSK) and the phase of the carrier is shifted pseudorandomly according to the pattern from the PN generator in the modulation stage, the resulting modulated signal is called a *direct sequence* (DS) spread spectrum signal. If the modulation types used is frequency shift keying (FSK) and the frequency of the carrier signal is changed according to the pattern from the PN generator, the resultant modulated signal is called a *frequency hopped* (FH) spread spectrum signal.

2.2 Multiple Access Communication

Multiple access communication is a type of multiuser communication system in which a large number of users share a common channel bandwidth. At any given time, a subset of these users may transmit information simultaneously over the common channel to corresponding receivers. There are several ways to accommodate multiple users to send information through the same channel. The simplest way is subdividing the available channel bandwidth into a number of subchannels which do not overlap in frequency. This method is called *frequency division multiple access* (FDMA) and is used in voice and data transmission over wireline channels. Fig. 2.2 depicts FDMA in time, frequency, and power domain. It can be seen from Fig. 2.2 that a subchannel is assigned to a user and the transmission bandwidth is limited to the bandwidth of that subchannel.

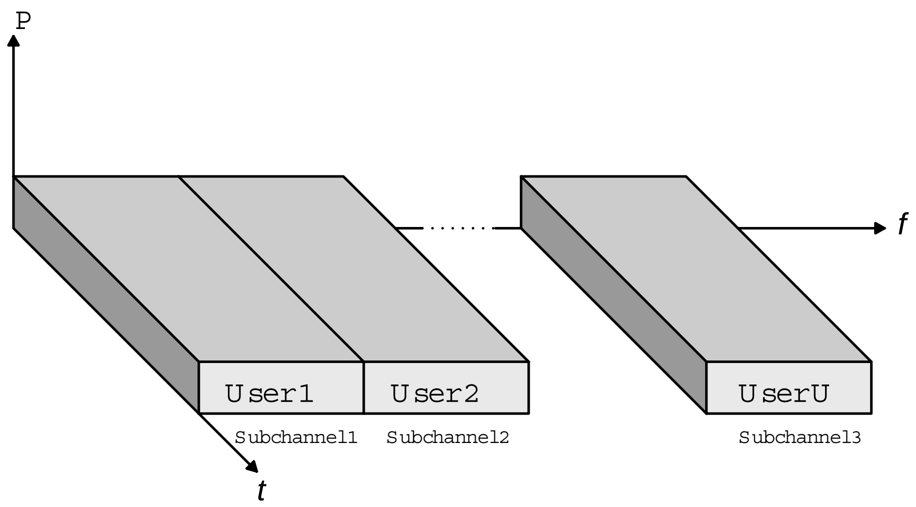


Figure 2.2: Frequency division multiple access (FDMA).

Another multiple access technique is subdividing the frame duration, T_f, into time durations which do not overlap in time. This method is called *time division multiple access* (TDMA) and is used for digital data and voice transmission. In TDMA, a user transmits information during the particular time slot which is assigned to that user. A TDMA system supporting U users in each channel is shown Fig. 2.3.

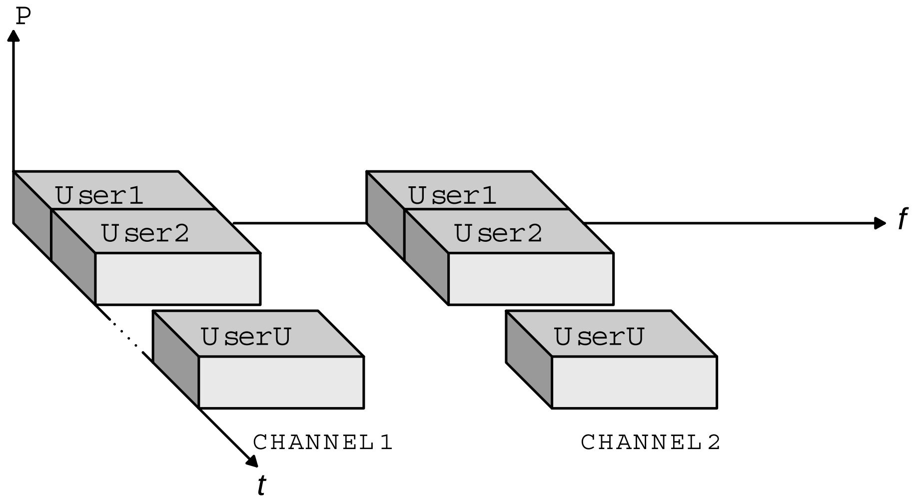


Figure 2.3: Time division multiple access (TDMA).

In both FDMA and TDMA, since the channel is successfully partitioned into subchannels, each user may transmit and receive data without interfering other users. But in some systems like computer communication networks and mobile cellular communication networks the possibility of a user to go silent (not transceiving any data) is high. In such networks, FDMA and TDMA tend to be inefficient since some of the available frequency or time slots do not carry information when the user is inactive. Inefficiently designed FDMA and TDMA systems also limits the number of users that can transmit and receive information simultaneously over a channel. An alternative method to FDMA and TDMA is *code division multiple access* (CDMA).

In CDMA, information from multiple users is not separated by different frequency or time slots but by different (unique for the user) spreading codes. Two main types of CDMA are the *direct sequence code division multiple access* (DS-CDMA) and *frequency hopped code division multiple access* (FH-CDMA). DS-CDMA is considered in this thesis and interested reader is referred to Proakis (2001) for more information about FH-CDMA.

2.3 Direct Sequence Code Division Multiple Access

Direct sequence code division multiple access (DS-CDMA) is a type of multiple access communication technique. In DS-CDMA each user share a common channel or sub-channel by transmitting signals which overlap in both time and frequency as it can be seen from Fig. 2.4. Each user is assigned a unique spreading sequence which spreads the data from that particular user to the entire frequency band of the channel. The spreading sequences also allow the receiver to separate the information transmitted by multiple users from each other. In this section the CDMA signal model in both non-dispersive AWGN and multipath channels will be introduced.

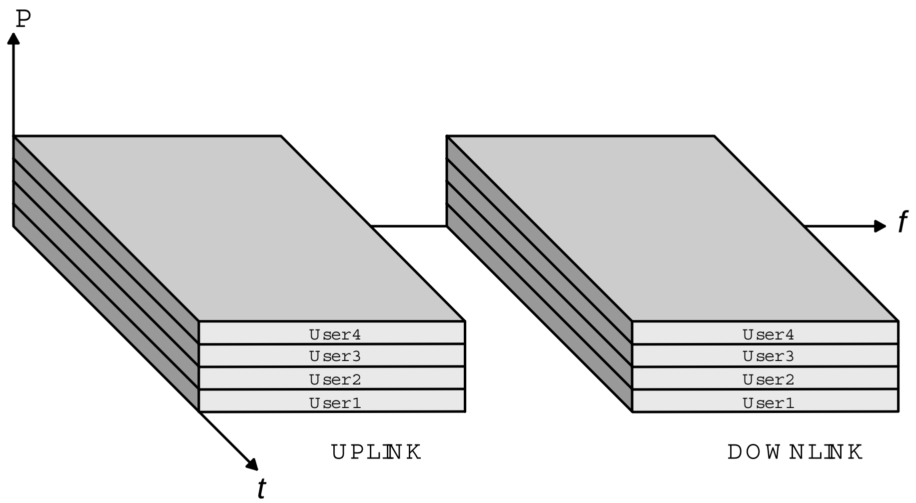


Figure 2.4: Code division multiple access (CDMA).

2.3.1 DS-CDMA Signal Model

We will limit our discussion to synchronous CDMA transmission. Let the CDMA channel to be shared by U independent users. Each user is assigned a signature waveform $s_u(t)$,

$$s_u(t) = \sum_{n=0}^{N-1} S_{u,n} p(t - nT_c), \qquad 0 \leq t \leq T \tag{2.3.1}$$

where N is the length of the spreading sequence for user u, $S_{u,n} \in (-1, +1)$ is the nth chip of the spreading sequence, T is the bit interval, $p(t)$ is a pulse of duration T_c, and $T_c = T/N$ is the chip interval. It is assumed that all U spreading sequences have the unit energy, i.e.,

$$\int_0^T s_u^2(t)dt = 1 \tag{2.3.2}$$

The cross correlation between spreading sequences plays an important role on the performance of the detectors and is defined as

$$\rho_{ij}(\tau) = \int_{\tau_c}^{T} s_i(t) s_j(t-\tau) dt \tag{2.3.3}$$

where τ is the delay. For synchronous transmission $\tau = 0$, thus Eqn. 2.3.3 becomes

$$\rho_{ij}(0) = \int_{0}^{T} s_i(t) s_j(t) dt \tag{2.3.4}$$

Let the kth data bit which is transmitted by user u be denoted by $D_u(k) \in (-1, +1)$, then the equivalent low-pass signal $x(t)$ for user u may be expressed as

$$x_u(t) = \sqrt{\varepsilon_u} \sum_{k=-\infty}^{\infty} D_u(k) s_u(t - kT), \qquad 1 \leq u \leq U \tag{2.3.5}$$

where ε_u is the signal energy per bit. The transmitted superposition signal for the U users may be expressed as

$$x(t) = \sum_{u=1}^{U} x_u(t) = \sum_{u=1}^{U} \sqrt{\varepsilon_u} \sum_{k=-\infty}^{\infty} D_u(k) s_u(t - kT) \tag{2.3.6}$$

and in the non-dispersive additive white Gaussian noise (AWGN) channel the received signal will become

$$y(t) = x(t) + g(t) \tag{2.3.7}$$

where $g(t)$ is the Gaussian noise with double sided power spectral density $N_0/2$.

2.3.2 DS-CDMA Signal Model in Vector Notation

Since sampled signals are used in the signal processing task, it will be more convenient to rewrite the Eqn. 2.3.7 in a vector notation (Tanner, 1998). In defining the radial basis function multiuser detector (RBF MUD) in chapter four and in the simulation environment in chapter six it is assumed that the DS-CDMA signals transmitted by U independent users to be bit and chip synchronous with equal power which is normalized to 1 which is a situation that may be realized in a downlink (base to mobile) scenario in a cellular mobile communication system. Thus, in this section, it is found convenient to define the vector notations for DS-CDMA signals under these assumptions.

2.3.2.1 Non-Dispersive AWGN Channel

Let us denote the kth data bit transmitted by user u by $D_u(k) \in (-1, +1)$, the unique spreading code of length N which is assigned to user u by $\mathbf{S}_u$, and each chip in the spreading code by $S_{u,n} \in (-1, +1),\ n = 1, 2, ..., N$. Then the received signal at chip rate in the presence of AWGN may be expressed as

$$y(kN + n) = \sum_{u=1}^{U} D_u(k) S_{u,n} + g(kN + n) \tag{2.3.8}$$

where $g(kN + n)$ is the added noise component with the variance $\sigma_n^2 = N_0/2$ and $N_0/2$ is the double-sided noise power spectral density.

Let us give an example of a CDMA system that is supporting four users. First ten binary bits of four users and each user's spreading sequence of length eight chips are shown in Fig. 2.5.a and Fig. 2.5.b respectively. Each bit of the user data is spread into eight chips with the corresponding spreading sequence and the resultant signal forms are given in Fig. 2.6.a. The DS-CDMA signal is shown in Fig. 2.6.b and it can be seen that it is simply the sum of all the signals that are given in Fig. 2.6.a. However, the signal shown in Fig. 2.6.b is the output of the transmitter. The form of the DS-CDMA signal that is corrupted with AWGN is straightforward and will not be considered.

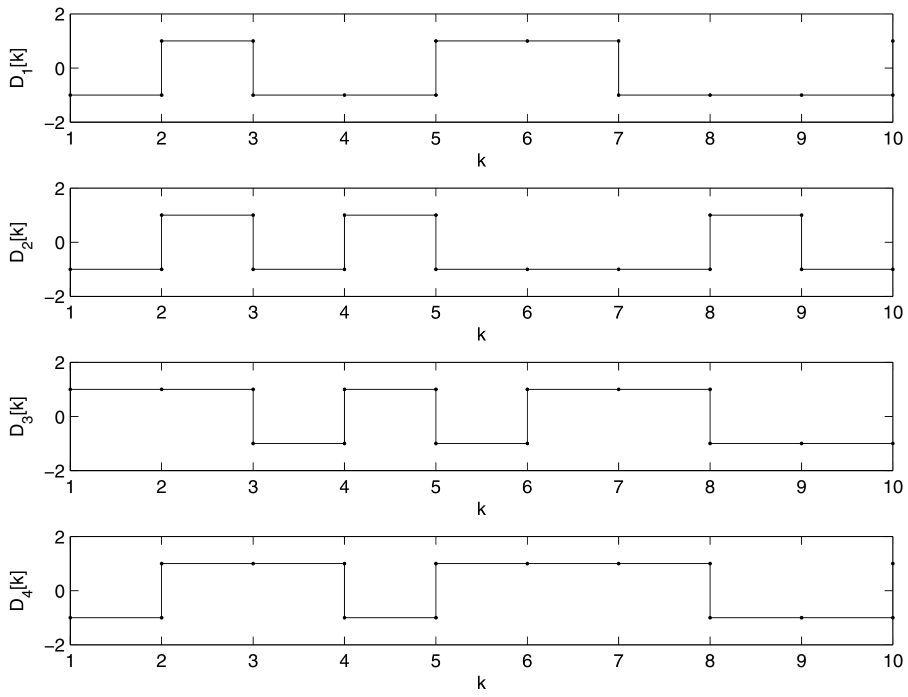
Since the user's transmitted bits are synchronized, we may write the vector representation of chip level expression $y(kN + n)$ of the received signal by

$$\mathbf{y}(k) = \begin{bmatrix} y(kN+1) & y(kN+2) & \cdots & y(kN+n) \end{bmatrix}^T \tag{2.3.9}$$

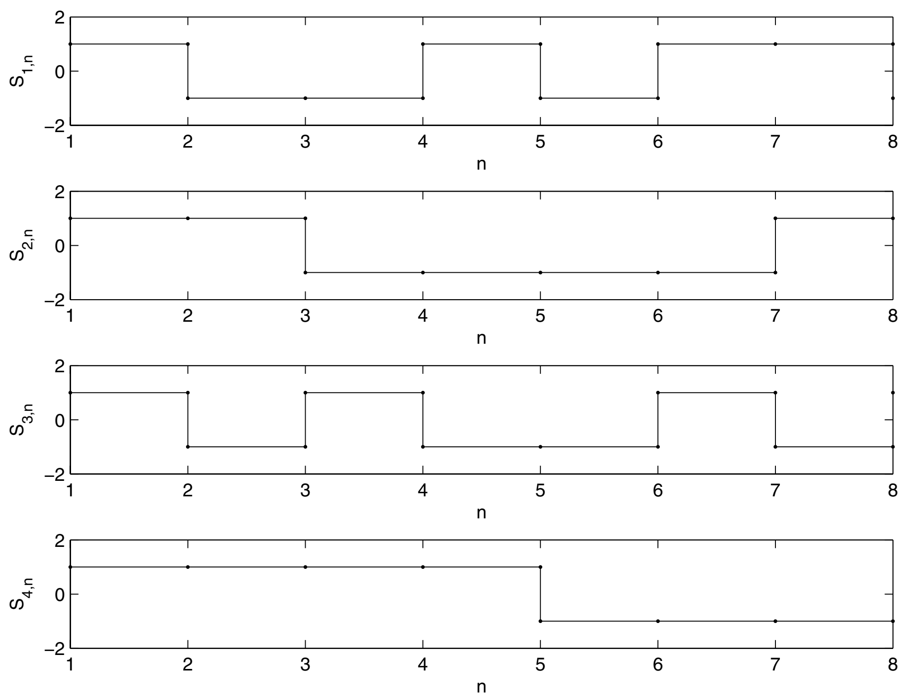
where $\mathbf{y}(k)$ is a vector of length N.

2.3.2.2 Multipath Channel

Prior to the introduction of the vector notation of the DS-CDMA signals in multipath channel, we will discuss the term called *interchip interference* (ICI). It may seen from Fig. 2.7 that a number of $L - 1$ head chips of a sequence in the multipath environment is affected by the previous transmitted sequence, and a number of $L - 1$ tail chips will affect the next

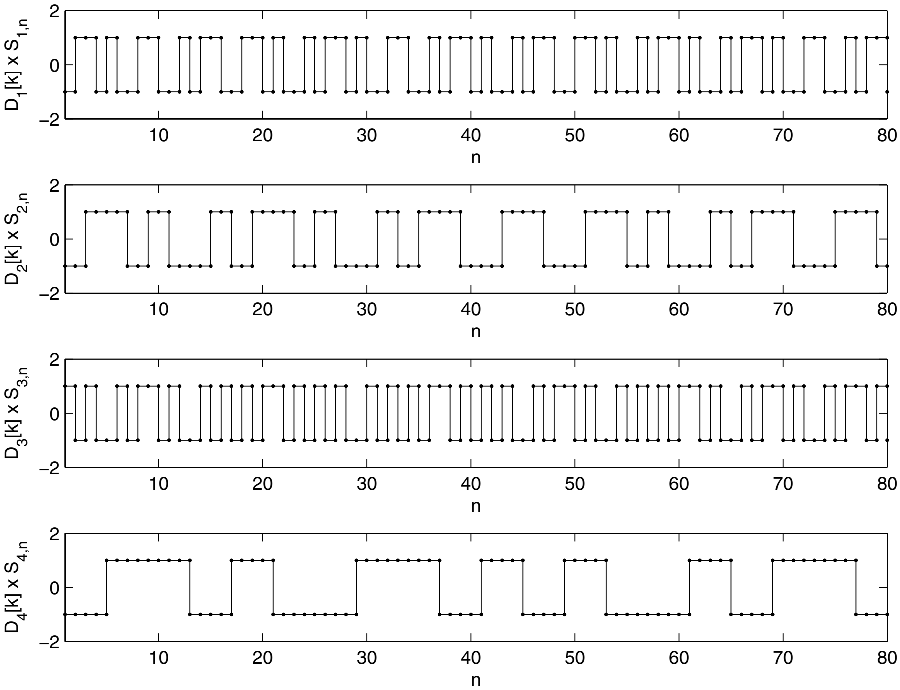


(a)

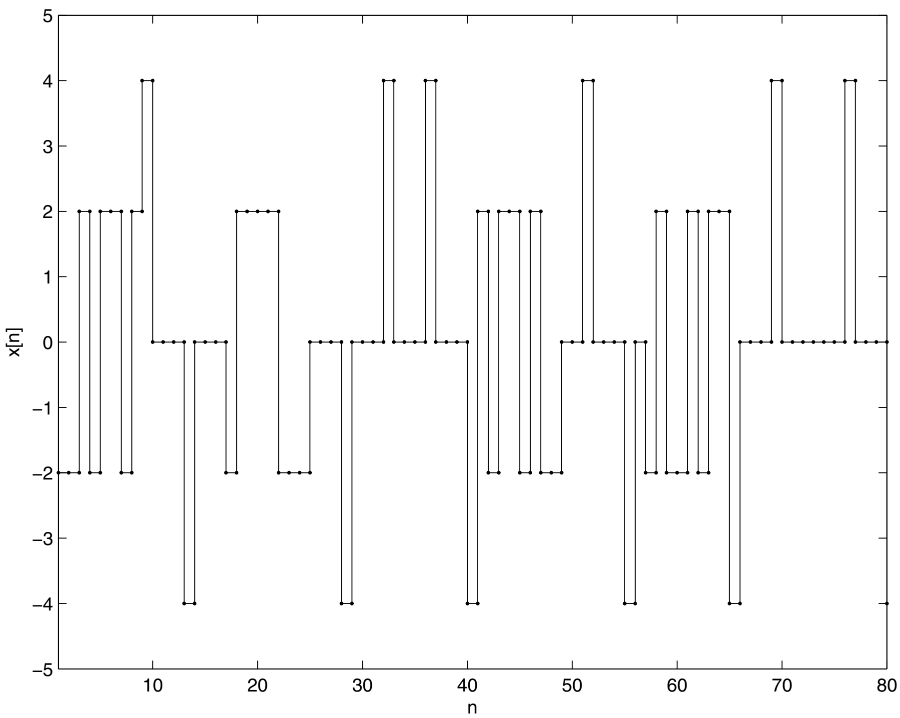


(b)

Figure 2.5: (a) User data, (b) spreading sequences.



(a)



(b)

Figure 2.6: (a) Spread data, (b) CDMA data.

transmitted sequence. This problem is called *interchip interference* (ICI) and in commercial CDMA systems RAKE receivers are (Mohr & Kottkamp, 1996) used in combatting with the ICI. RAKE's can be considered as FIR filters where the weights are convolution of the spreading sequence of the user and the channel impulse response H_{ch}.

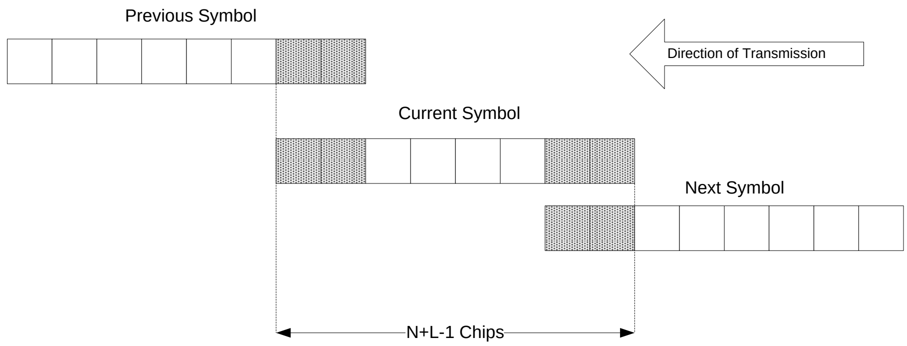


Figure 2.7: Interchip interference (ICI).

It is possible to model the multipath channel using a finite impulse response (FIR) structure with L taps (Proakis, 2001). In conventional CDMA systems, base station transmits a pilot tone and the receiver estimates the channel response by monitoring this tone.

Let the channel be a stationary L tap with the impulse $H_{ch}(z) = h_1 + h_2 z^{-1} + \cdots + h_L z^{-L+1}$, then the received signal at chip rate becomes

$$\begin{aligned} y(kN+n) = \; & h_1 \sum_{u=1}^{U} D_u(k) S_{u,n} + h_2 \sum_{u=1}^{U} D_u(k) S_{u,n-1} + \cdots \\ & + h_L \sum_{u=1}^{U} D_u(k) S_{u,n-L+1} + g(kN+n) \end{aligned} \tag{2.3.10}$$

where $g(kN+n)$ is the added noise component with the variance $\sigma_n^2 = N_0/2$ and $N_0/2$ is the double-sided noise power spectral density. The vector representation of chip level expression $y(kN+n)$ of the received signal becomes

$$\mathbf{y}(k) = \Big[y(kN-L+2) \quad \cdots \quad y(kN+1) \quad y(kN+2) \quad \cdots \quad y(kN+n) \Big]^T \tag{2.3.11}$$

where $\mathbf{y}(k)$ is a vector of length $N + (L-1)$.

2.4 Spreading Sequences

In DS-CDMA, each user's data is spread by a spreading sequence prior to transmission and the received data is correlated with the same spreading sequence in order to detect the user's transmitted data. The autocorrelation and cross correlation properties of different spreading sequences play an important role in terms of system performance, thus these properties of several spreading sequences will be considered in this section.

2.4.1 PN-Sequences

Pseudo-Noise (PN) sequences are noise-like binary sequences. Most common and well known PN sequences include maximal length sequences or *m* sequences in short, Gold sequences, and Kasami sequences. Information on other important PN sequences can be found in (Simon et al., 1994).

2.4.1.1 Maximal Length Sequences

Maximal length sequences (*m* sequences) are generated using *m*-stage linear feedback shift registers as illustrated in Fig. 2.8. An *m* sequence has length $n = 2^{m-1}$ bits and is periodic with period n. Each period of the sequence contains 2^{m-1} ones and $2^{m-1} - 1$ zeros.

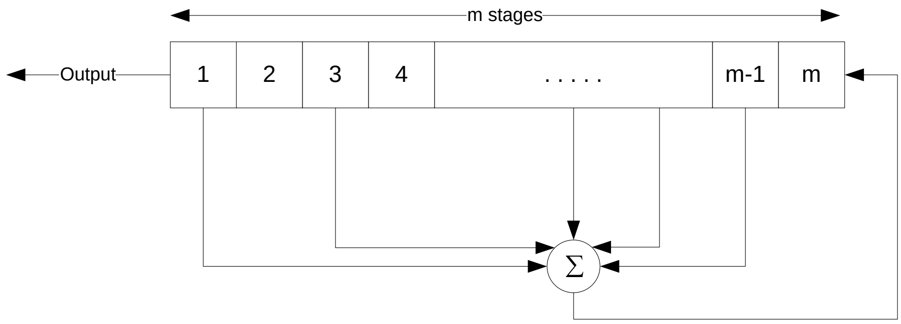


Figure 2.8: M-stage linear feedback shift register.

In CDMA, the autocorrelation function properties of the spreading sequences are exploited in order to optimally combine the multipath signals of a particular user (Hanzo et al., 2003). The periodic autocorrelation function of a periodic PN sequence is defined as

$$R_a(j) = \sum_{i=1}^{i=n} S_i S_{i+j} \tag{2.4.1}$$

where n is the period of the sequence. The autocorrelation function of an ideal pseudorandom sequence would have the properties that $R_a(0) = n$ and $R_a(j) = 0$ for $1 \leq j \leq n-1$ (Proakis, 2001). The periodic autocorrelation function of an *m* sequence is given by

$$R_a(j) = \begin{cases} n & (j = 0) \\ -1 & 1 \leq j \leq n-1 \end{cases} \tag{2.4.2}$$

Off-peak values of $R_a(j)$ relative to the peak value $R_a(j)/\ R_a(0) = -1/n$ is small for large values of n which makes the *m* sequences to be considered as ideal in terms of autocorrelation function.

In CDMA, cross correlation properties of a spreading sequence are as important as the autocorrelation properties since the multiple access interference diminishes as the spreading sequences gets mutually orthogonal. The periodic cross correlation function between pairs of *m* sequences can have relatively high peaks which leads to an undesirable situation in CDMA (Proakis, 2001).

2.4.1.2 Gold Sequences

Gold sequences have better cross correlation properties than *m* sequences and are generated by linearly combining a pair of *m* sequences which are called *preferred sequences* and exhibit a three-valued cross correlation function with values $\{-1, -t(m), t(m) - 2\}$, where

$$t(m) = \begin{cases} 2^{(m+1)/2} + 1 & (\text{odd } m) \\ 2^{(m+2)/2} + 1 & (\text{even } m) \end{cases} \tag{2.4.3}$$

A number of n Gold sequences can be generated by taking the modulo-2 sum of one of the preferred sequences shifted n times and the other sequence itself (Hanzo et al., 2003). Since the original sequences can be included in the set we may generate $n + 2$ Gold sequences from a pair of *m* sequences. Gold sequences exhibit lower peak cross-correlations than *m* sequences which leads to a desirable situation in CDMA.

2.4.1.3 Kasami Sequences

Kasami sequences have optimal cross correlation values, reaching the Welch lower bound (Simon et al., 1994, Proakis, 2001). The lower bound on the cross-correlation between any pair of binary sequences of period n in a set of M sequences is:

$$R_{\max} \geq n\sqrt{\frac{M-1}{Mn-1}} \tag{2.4.4}$$

The procedure for the generation of the Kasami sequences is similar to that for the generation of Gold codes and is given in (Simon et al., 1994).

2.4.2 Orthogonal Sequences

In contrast to PN sequences orthogonal sequences have zero cross correlation. However they are only attractive in perfectly synchronized environments such as in the down-link of mobile communications since they exhibit zero cross correlation only when there is no offset between the sequences. In fact, their cross correlation functions exhibit higher peak values at non-zero offsets than that of PN sequences. Their autocorrelation properties are usually not attractive either (Hanzo et al., 2003).

The best-known orthogonal sequences are probably Walsh sequences which are generated by the method called the Hadamard transform (Peterson et al., 1995). Orthogonal Gold sequences show reasonable cross correlation and off-peak autocorrelation values, while providing perfect orthogonality in the zero-offset case (Hanzo et al., 2003). In this section we will consider the generation of the Walsh codes. The methods and information about other types of orthogonal sequences can be found in (Hanzo et al., 2003).

2.4.2.1 Walsh Sequences

Generation of the Walsh sequences is easy. First, let us define a one by one dimensional zero matrix as

$$\mathbf{H}_1 = \begin{bmatrix} 0 \end{bmatrix} \tag{2.4.5}$$

and apply Hadamard transform (Peterson et al., 1995) to $\mathbf{H}_1$ to generate $\mathbf{H}_2$

$$\mathbf{H}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{2.4.6}$$

where the Hadamard transform is defined as

$$\mathbf{H}_{2n} = \begin{bmatrix} \mathbf{H}_n & \mathbf{H}_n \\ \mathbf{H}_n & \bar{\mathbf{H}}_n \end{bmatrix} \tag{2.4.7}$$

Each column or row of an n by n dimensional Hadamard matrix corresponds to a Walsh sequence of length n. Every Walsh sequence is orthogonal to all other Walsh sequences that are formed up from the same Hadamard matrix.

Let us give an example for the case $n = 3$. By applying the Hadamard transform to $\mathbf{H}_1$ for 2 times we will end up with 8 by 8 Hadamard matrix which is

$$\mathbf{H}_8 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{2.4.8}$$

2.5 Commercial DS-CDMA Systems

The first notable commercial applications of CDMA were initiated in the mobile satellite systems, e.g. Iridium, Globalstar (Doany, 1998, Lyons et al., 1998). Iridium and Globalstar are low Earth orbit (LEO) satellite constellation for telephone and low-speed data communications. Later, CDMA has also found applications in digital cellular mobile communications.

2.5.1 IS-95

Interim Standard 95 (IS-95) is a 2G mobile telecommunications standard and it is the first CDMA-based digital cellular standard pioneered by Qualcomm and approved by the US Telecommunications Industry Association. The brand name for IS-95 is "cdmaOne". IS-95 is also known as TIA-EIA-95.

In the downlink of IS-95 the user data is convolutional coded and spread, which leads to a channel chip rate of 1.2288 Mchip/s. The modulation scheme is QPSK. User data is spread with a Walsh code of length 64 and in order to ensure privacy it is encrypted by a long PN sequence with a period of $2^{42} - 1$. The downlink channel consists of a pilot, a synchronization, up to 7 paging and up to 63 traffic channels. Of special interest is the pilot channel, which allows a mobile to acquire the timing of the channel, provides a phase reference for coherent demodulation, and provides each mobile with a means for signal strength comparisons between base stations to determine when to handoff. The pilot channel is also modulated by the base station with a specific short PN code (Tanner, 1998).

In the uplink of IS-95 the modulation scheme is orthogonal QPSK (OQPSK) in order to operate in the optimal range of the mobile's power amplifier. The uplink channel consists of access and traffic channels. The access channel is used by the mobile to initiate communication with the base station and to respond to paging messages. All data transmitted is convolutionally encoded, block interleaved, OQPSK modulated, and spread with a short PN code with period of $2^{15} - 1$ prior to transmission (Tanner, 1998). In both the uplink and the downlink the base station and the mobile RAKE receivers (Proakis, 2001) are used.

IS-95 is used in the USA, South Korea, Canada, Mexico, India, Israel, Australia, Sri Lanka, Venezuela, Brazil and China. It is now being supplanted by IS-2000 (CDMA2000), a latter CDMA-based standard.

2.5.2 CDMA2000

CDMA2000 is a hybrid 2.5G / 3G protocol of mobile telecommunication standards that use CDMA. The CDMA2000 is a direct successor to 2G CDMA, IS-95 (cdmaOne). There are several CDMA2000 standards called CDMA2000 1xRTT, CDMA2000 EV-DO, and CDMA2000 EV-DV.

1xRTT is a standard also known as IS-2000 and it almost doubles the capacity of IS-95 by adding 64 more traffic channels to the downlink, orthogonal to (in quadrature with) the original set of 64. Today, CDMA2000 is in use in many countries.

CHAPTER THREE
DS-CDMA DETECTORS

This chapter reviews detector structures for DS-CDMA. Chapter starts with an introduction of optimum detector, and continues with the reviews on suboptimum detectors. Conventional detector and decorrelating detector as linear receivers and multistage detector (MSD) as a nonlinear receiver are introduced. Finally, the chapter ends with a comparison of the detectors introduced throughout the chapter.

3.1 Optimum Detector

The optimum detector is defined as the detector that selects the most probable sequence of bits $\{D_u(k), -\infty \leq k \leq \infty, 1 \leq u \leq U\}$ given the received signal $y(t)$ (Proakis, 2001). In additive white Gaussian noise, it is sufficient to consider the signal received in one signal interval, say $0 \leq t \leq T$, to determine the optimum detector. From Eqns. 2.3.6 and 2.3.7 we may write the received signal when $k = 0$ in AWGN as

$$y(t) = \sum_{u=1}^{U} \sqrt{\varepsilon_u} D_u(0) s_u(t) + g(t) \tag{3.1.1}$$

The optimum detector computes the log-likelihood function

$$\Lambda(\mathbf{D}) = \int_0^T \left[y(t) - \sum_{u=1}^{U} \sqrt{\varepsilon_u} D_u(0) s_u(t) \right]^2 dt \tag{3.1.2}$$

and selects the information sequence $\{D_u(0), 1 \leq u \leq U\}$ that minimizes $\Lambda(\mathbf{D})$. If we expand the integral in Eqn. 3.1.2 we obtain

$$\begin{aligned} \Lambda(\mathbf{D}) = \quad & \int_0^T y^2(t) dt - 2 \sum_{u=1}^{U} \sqrt{\varepsilon_u} D_u(0) \int_0^T y(t) s_u(t) dt \\ & + \sum_{v=1}^{U} \sum_{u=1}^{U} \sqrt{\varepsilon_v \varepsilon_u} D_v(0) D_u(0) \int_0^T s_v(t) s_u(t) dt \end{aligned} \tag{3.1.3}$$

The cross correlation of the received signal with each of the U spreading sequences is defined as

$$r_u = \int_0^T y(t) s_u(t) dt, \qquad 1 \leq u \leq U \tag{3.1.4}$$

The first term in Eqn. 3.1.3 is same for any information sequence $D_u(0)$ and does not provide any relevance thus may be neglected. Using Eqns. 3.1.4 and 2.3.4 we may rewrite the Eqn.

3.1.3 in the form of correlation metrics as

$$C(\mathbf{r},\mathbf{D}) = 2\sum_{u=1}^{U}\sqrt{\varepsilon_u}D_u(0)r_u - \sum_{v=1}^{U}\sum_{u=1}^{U}\sqrt{\varepsilon_v\varepsilon_u}D_v(0)D_u(0)\rho_{uv}(0) \qquad (3.1.5)$$

These correlation metrics may be expressed in the vector notation as

$$C(\mathbf{r},\mathbf{D}) = 2\mathbf{D}^T\mathbf{r} - \mathbf{D}^T\mathbf{R}\mathbf{D} \qquad (3.1.6)$$

where

$$\mathbf{r} = \begin{bmatrix} r_1 & r_2 & \cdots & r_U \end{bmatrix}^T \qquad (3.1.7)$$

and

$$\mathbf{D} = \begin{bmatrix} \sqrt{\varepsilon_1}D_1(0) & \sqrt{\varepsilon_2}D_2(0) & \cdots & \sqrt{\varepsilon_U}D_U(0) \end{bmatrix}^T \qquad (3.1.8)$$

and $\mathbf{R}$ is the correlation matrix

$$\mathbf{R} = \begin{bmatrix} \rho_{11}(0). & \rho_{12}(0) & \cdots & \rho_{1v}(0) \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{u1}(0) & \rho_{u2}(0) & \cdots & \rho_{uv}(0) \end{bmatrix} \qquad (3.1.9)$$

In optimum detector, the received signal is passed through U matched filters (correlators) and the correlator outputs are fed into a detector which computes the 2^U possible correlation metrics for each sequence and selects the sequence that gives the largest correlation metric. It is observed that the optimum detector requires the knowledge of energies of the received signal and its computational complexity grows exponentially with the increasing number of users.

3.2 Suboptimum Detectors

The computational complexity of the optimum detector grows exponentially with the increasing number of users since it calculates 2^U metrics for U users sharing the channel. In a real life CDMA system, there will probably be very large number of users which would make the optimum detector impractical and very expensive to implement. Thus, a lot of effort is given to develop suboptimum detectors which have reasonable computational complexities, are near-far resistant and have performances close to that of the optimum detector.

3.2.1 Conventional Detector

Conventional detector simply match-filters the received signal with the spreading waveform of the user and makes a decision based on the single correlator output. Thus, conventional detector neglects the presence of other users in the channel. Multiple access interference (MAI) and the channel noise are assumed to be additive white Gaussian.

If the received signal is in the form of Eqn. 3.1.1 than the match filter output for user u will be given as

$$\begin{aligned} r_u &= \int_0^T y(t)s_u(t)dt \\ &= \sum_{v=1}^{U} \sqrt{\varepsilon_v} D_v(0)\rho_{uv}(0) + \int_0^T g(t)s_u(t)dt \end{aligned} \tag{3.2.1}$$

To simplify the equations the noise component for user u in bit interval k is defined as

$$g_u(k) = \int_{kT}^{(k+1)T} g(t)s_u(t)dt \tag{3.2.2}$$

Since the noise $g(t)$ is white and Gaussian with the power spectral density $N_0/2$, from Eqn. 2.3.2 it follows that the variance of the noise component is

$$E\left[g_u^2(k)\right] = \frac{1}{2}N_0 \int_0^T s_u^2(t)dt = \frac{1}{2}N_0 \tag{3.2.3}$$

We may rewrite Eqn. 3.2.1 by fragmenting the first term as

$$r_u = \sqrt{\varepsilon_u} D_u(0) + \sum_{\substack{v=1 \\ v\neq u}}^{U} \sqrt{\varepsilon_v} D_v(0)\rho_{uv}(0) + g_u(0) \tag{3.2.4}$$

The middle term in Eqn. 3.2.4 represents the interference from the other users and it equals to zero if all spreading sequences $\{s_u(t), 1 \leq u \leq U\}$ are orthogonal to each other. Thus, the conventional detector is optimum and its complexity grows linearly with the number of users. But, if one or more of the spreading sequences are not orthogonal, the multiple access interference may not be neglected. The interference from a user may become excessive if that user is transmitting at a higher power level than the other users. This situation is called *near-far problem* and power control techniques are required to combat this problem.

3.2.2 Decorrelating Detector

Let us define the signal vector $\mathbf{r}$ that represents the output of the U matched filters as

$$\mathbf{r} = \mathbf{RD} + \mathbf{g} \tag{3.2.5}$$

where

$$\mathbf{D} = \left[\sqrt{\varepsilon_1}D_1(0) \quad \sqrt{\varepsilon_2}D_2(0) \quad \cdots \quad \sqrt{\varepsilon_U}D_U(0)\right]^T \tag{3.2.6}$$

and the noise vector is

$$\mathbf{g} = \begin{bmatrix} g_1(0) & g_2(0) & \cdots & g_U(0) \end{bmatrix}^T \tag{3.2.7}$$

with a covariance

$$E(\mathbf{g}\mathbf{g}^T) = \frac{N_0}{2}\mathbf{R} \tag{3.2.8}$$

Since the noise is Gaussian, the PDF of r is a U-dimensional Gaussian with mean $\mathbf{RD}$ and covariance $\mathbf{R}$. That is,

$$p(\mathbf{r} \mid \mathbf{D}) = \frac{1}{\sqrt{(N_0\pi)^U \det \mathbf{R}}} \exp\left[-\frac{1}{N_0}\left(\mathbf{r} - \mathbf{RD}\right)^T \mathbf{R}^{-1}\left(\mathbf{r} - \mathbf{RD}\right)\right] \tag{3.2.9}$$

The decorrelating detector calculates the best linear estimate of $\mathbf{D}$ which is the value of $\mathbf{D}$ that minimizes the likelihood function

$$\Lambda(\mathbf{D}) = \left(\mathbf{r} - \mathbf{RD}\right)^T \mathbf{R}^{-1}\left(\mathbf{r} - \mathbf{RD}\right) \tag{3.2.10}$$

where the optimization yields

$$\mathbf{D}^0 = \mathbf{R}^{-1}\mathbf{r} \tag{3.2.11}$$

Finally, the detected symbols are obtained by taking the sign of each element of $\mathbf{D}^0$

$$\hat{\mathbf{D}} = \mathrm{sgn}(\mathbf{D}^0) \tag{3.2.12}$$

Eqn. 3.2.11 represents a linear transformation on the signal vector which consists of the outputs of U matched filters. Thus, computational complexity of the decorrelating detector increases linearly with U.

Now let us consider a scenario where two users are sharing the same channel. Then the correlation matrix of the spreading sequences and its inverse would be

$$\mathbf{R} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \tag{3.2.13}$$

$$\mathbf{R^{-1}} = \frac{1}{1-\rho^2}\begin{bmatrix} 1 & -\rho \\ -\rho & 1 \end{bmatrix} \tag{3.2.14}$$

Recall from Eqn. 2.3.4 that the cross-correlation between two spreading sequences is defined as

$$\rho = \int_0^T s_1(t)s_2(t)dt \tag{3.2.15}$$

Let the received signal be in the form

$$r(t) = \sqrt{\varepsilon_1}D_1s_1(t) + \sqrt{\varepsilon_2}D_2s_2(t) + g(t) \tag{3.2.16}$$

Then the signal vector $\mathbf{r}$ defined by Eqn. 3.2.5 becomes

$$\mathbf{r} = \begin{bmatrix} \sqrt{\varepsilon_1} D_1 + \rho\sqrt{\varepsilon_2} D_2 + g_1 \\ \rho\sqrt{\varepsilon_1} D_1 + \sqrt{\varepsilon_2} D_2 + g_2 \end{bmatrix} \tag{3.2.17}$$

where g_1 and g_2 are the noise components at the output of the matched filters. The best linear estimate of $\mathbf{D}$ calculated by the decorrelating detector is

$$\begin{aligned} \mathbf{D}^0 = &\ \mathbf{R}^{-1}\mathbf{r} \\ = &\ \begin{bmatrix} \sqrt{\varepsilon_1} D_1 + (g_1 - \rho g_2)/(1-\rho^2) \\ \sqrt{\varepsilon_2} D_2 + (g_2 - \rho g_1)/(1-\rho^2) \end{bmatrix} \end{aligned} \tag{3.2.18}$$

It can be seen from Eqn. 3.2.18 that the decorrelating detector successfully eliminated the interference terms between two users and it does not need the information of the power levels of the users.

A figure of merit for the performance of a multiuser detector is the probability of bit error which is defined by

$$P_u(\gamma_u) = Q\left(\sqrt{2\gamma_u}\right) \tag{3.2.19}$$

where $\gamma_u = \varepsilon_u/N_0$, ε_u is the signal energy per bit, and $N_0/2$ is the power spectral density of the AWGN. Since the interference of the other users are completely eliminated in the decorrelating detector we may write the probability of error of a particular user u as

$$P_u = Q\left(\sqrt{\varepsilon_u/\sigma_u^2}\right) \tag{3.2.20}$$

where σ_u^2 is the variance of the noise in the uth element of the estimate $\mathbf{D}^0$. For the example of two user case, the noise component is

$$g = \frac{g_1 - \rho g_2}{1-\rho^2} \tag{3.2.21}$$

and the variance of the noise is

$$\sigma_1^2 = \frac{E[(g_1 - \rho g_2)]^2}{E[(1-\rho^2)]^2} = \frac{E[g_1^2] - E[\rho^2] - E[g_2^2]}{(1-\rho^2)^2} = \frac{(N_0/2)(1-\rho^2)}{(1-\rho^2)^2} = \frac{1}{1-\rho^2}\frac{N_0}{2} \tag{3.2.22}$$

and the probability of error for $u = 1$ is

$$P_1 = Q\left(\sqrt{\frac{2\varepsilon_1}{N0}(1-\rho^2)}\right) \tag{3.2.23}$$

It can be seen from Eqn. 3.2.23 that the noise variance is increased by a of factor $(1-\rho^2)^{-1}$ which can be considered as a disadvantage of the decorrelating detector.

3.2.3 Multistage detector

Multistage detector employs multiple iterations in detecting the user bits and cancelling the interference. It starts with the output of any of the suboptimum detectors for the first stage. If we suppose that the multistage detector starts with the output of the decorrelating detector, then the estimate in the first stage is

$$\begin{aligned} \hat{b}_{11} &= \text{sgn}(r_1 - r_2\rho) \\ \hat{b}_{21} &= \text{sgn}(r_2 - r_1\rho) \end{aligned} \tag{3.2.24}$$

and the second stage would be

$$\begin{aligned} \hat{b}_{12} &= \text{sgn}(r_1 - \sqrt{\varepsilon_2}\hat{b}_{21}\rho) \\ \hat{b}_{22} &= \text{sgn}(r_2 - \sqrt{\varepsilon_1}\hat{b}_{11}\rho) \end{aligned} \tag{3.2.25}$$

and the third stage

$$\begin{aligned} \hat{b}_{13} &= \text{sgn}(r_1 - \sqrt{\varepsilon_2}\hat{b}_{22}\rho) \\ \hat{b}_{23} &= \text{sgn}(r_2 - \sqrt{\varepsilon_1}\hat{b}_{12}\rho) \end{aligned} \tag{3.2.26}$$

where $\hat{b}_{ij}$ is the estimate in the jth stage for the ith user in the above equations.

The computation is generally terminated when the average change between iterations is zero or less than a predefined value. It can be seen that the overall performance of the detector highly depends on the initial estimate which is provided by any other suboptimum detector. Multistage detector has received considerable attention by many researchers (Varanasi & Aazhang, 1990, Buehrer et al., 1999).

3.3 Comparison of the Detectors

The optimum detector selects the most probable sequence of bits given the received signal but its computational complexity increases exponentially with the number of users in the channel. In a real life CDMA system, there will probably be very large number of users which would make the optimum detector impractical and very expensive to implement. Thus, researchers tried to develop suboptimal detectors which have reasonable computational complexities, are near-far resistant and have performances close to that of the optimum detector.

The conventional detector passes the received signal through a bank of filters matched to the corresponding users' unique spreading sequences, signs the output, and decides on the information bits. If the cross correlations between the spreading sequences are zero, that is they are orthogonal, the conventional detector is optimum and its computational complexity increases linearly with the number of users. But if the spreading sequences are not orthogonal to each other its performance considerably decreases due to multiple access interference (MAI). It's performance gets even worse if one or more of the users in the channel are transmitting relatively at high power. This problem is generally called "near-far effect."

Decorrelating detector is a suboptimal detector which is linear, near-far resistant, and has a computational complexity proportional to the number of users (Lupas & Verdu, 1989a,b). Decorrelating detector introduces performance improvement over conventional detector in terms of MAI but it suffers from noise enhancement.

Multistage detector (MSD) is a nonlinear detector which improves each stage's estimate by subtracting the estimate of the MAI obtained by the previous stage (Varanasi & Aazhang, 1990). Performance of an MSD can reach close to that of the optimum detector but it depends highly on the initial estimate, which is usually provided by the conventional detector or decorrelating detector.

CHAPTER FOUR
RADIAL BASIS FUNCTION MULTIUSER DETECTOR

This chapter introduces the usage of radial basis function networks (RBFN) as multiuser detectors for DS-CDMA. First section gives a brief review about radial basis function networks, and the structure of the RBF MUD in both AWGN and multipath channels are considered in the following section. Final section discusses the complexity of the RBF MUD.

4.1 Radial Basis Function Networks

A Radial Basis Function Network (RBFN) is a type of neural network which uses radial basis functions as the activation functions. Originally, the RBFN was developed for data interpolation in multi-dimensional space (Micchelli, 1986, Powell, 1987, Dyn, 1987) and it has been used in a wide range of areas like time series prediction, function approximation, control theory, communications, and etc.

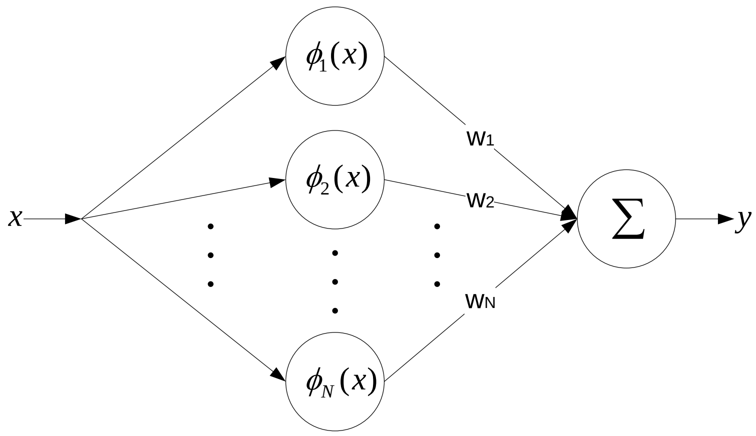


Figure 4.1: The structure of an radial basis function network.

An RBFN consists of three layers as shown in Fig. 4.1. The input layer connects the network to the environment while the second layer applies a nonlinear transformation from the input space to the hidden space. In most applications, the hidden space has a higher dimension than the input space. The output layer sums the output of the basis functions after suitable weighting. The equation that defines an RBFN is

$$y = \sum_{i=1}^{N} w_i \phi(||\mathbf{x} - \mathbf{c}_i||) \tag{4.1.1}$$

where $\mathbf{x}$ is the input vector, w_i is the weight of the i^{th} basis function output's path, N is the number of neurons in the hidden layer, and $\phi(\cdot)$ is a radially symmetric function with $\mathbf{c}_i$ as its center. Hence, the vector $\mathbf{c}_i$ is usually called as the *center*. The most common basis function used in the RBFN's is the Gaussian kernel

$$\phi(\zeta) = \exp\left(-\frac{\zeta^2}{2\sigma^2}\right) \tag{4.1.2}$$

where σ^2 is the variance that controls the radius of the influence of the basis function, and ζ^2 is the euclidean distance between the input vector and the center vector.

There have been many methods proposed to determine the parameters of an RBFN. The most common method is grouping the training samples with *k*-means algorithm, selecting the centers from the means, and using the *least mean squares* (LMS) algorithm to determine the weights in the third layer.

4.2 Radial Basis Function Multiuser Detector (RBF MUD)

An alternative DS-CDMA detector to the ones introduced in the previous chapter is the *radial basis function multiuser detector* (RBF MUD) which is introduced by Cruickshank (1996). It's structure is based on radial basis function network and is fully determined when the spreading sequences of all users and the channel impulse response are known. Hence, it needs no training (Tanner & Cruickshank, 1998). The structure of the RBF MUD and construction of the center vectors of it in both AWGN and multipath channels are considered in this section.

4.2.1 RBF MUD for AWGN Channel

4.2.1.1 The structure of the detector

The structure of the RBF MUD is shown in Fig. 4.2. The RBF MUD needs a set of M basis functions (centers). The most common basis function used in the RBFN is the Gaussian kernel

$$\phi_m(\mathbf{y}(k)) = \exp\left(-\frac{||\mathbf{y}(k) - \mathbf{c}_m||^2}{2\sigma^2}\right) \tag{4.2.1}$$

where, $\mathbf{c}_m,\ m = 1, 2, ..., M$ are the center vectors of length N, M is the number of center vectors that are introduced by the RBF MUD for each 2^U possible received signal where U is the number of users in the Gaussian channel. Since the vector set $\mathbf{c}_m,\ m = 1, 2, ..., M$ contains all of the possible received signal vectors, $\mathbf{y}(k)$, these centers are also called as *supercenters*. Variance of the Gaussian center function, σ^2, equals to variance of the added noise component, σ_n^2. It worths mentioning that the noise power and the spreading code of all users must be known at the detector to form an RBF MUD structure.

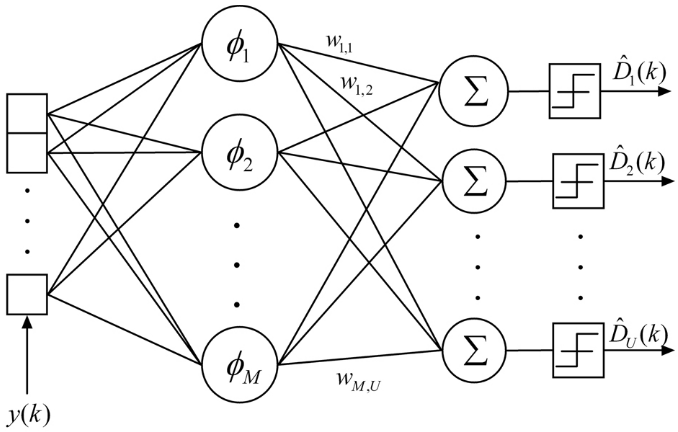


Figure 4.2: The structure of the radial basis function multiuser detector.

The output layer of the RBF MUD consists of linear weights which are denoted by $w_{m,u},\ m = 1, 2, ..., M$. The outputs of the center functions are linearly weighted by $w_{m,u}$, summed up and fed into sign operator, resulting the detected symbol for user u, $\hat{D}_u$

$$\hat{D}_u(k) = sgn\left(\sum_{m=1}^{M} w_{m,u}\phi_m(\mathbf{y}(k))\right) \tag{4.2.2}$$

where $\mathbf{y}(k)$ is the vector of length N containing the DS-CDMA signal of U users distorted by AWGN. The weights, $w_{m,u}$, in the output layer of the RBF MUD are chosen from the *code matrix*. Construction of the *code matrix* which comprises all combinations of all users and *supercenter matrix* which has supercenter vectors as its rows will be explained in the following section.

4.2.1.2 Construction of Supercenter and Code Matrices

The supercenter matrix, $\mathbf{C}$, contains all possible received DS-CDMA signals of U users in AWGN channel as its rows, and is derived using the formula

$$\mathbf{C} = \mathbf{DS} \tag{4.2.3}$$

where $\mathbf{S}$ is the $U \times N$ matrix, comprising the spreading codes of length N of all U users and expressed as

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_1^T \\ \mathbf{S}_2^T \\ \vdots \\ \mathbf{S}_{U-1}^T \\ \mathbf{S}_U^T \end{bmatrix} \tag{4.2.4}$$

where S_u is the $N \times 1$ spreading code vector of user u.

In Eqn. 4.2.3 $\mathbf{D}$ is the $M \times U$ code matrix which contains all possible bit combinations as its rows where $M = 2^U$ and is expressed as

$$\mathbf{D} = \begin{bmatrix} -1 & -1 & \cdots & -1 \\ -1 & -1 & \cdots & +1 \\ \vdots & \vdots & \ddots & \vdots \\ +1 & +1 & \cdots & -1 \\ +1 & +1 & \cdots & +1 \end{bmatrix} \tag{4.2.5}$$

Thus, Eqn. 4.2.3 can be written in the expanded form

$$\mathbf{C} = \mathbf{DS} \ = \begin{bmatrix} -\mathbf{S}_1^T - \mathbf{S}_2^T - \cdots - \mathbf{S}_{U-1}^T - \mathbf{S}_U^T \\ -\mathbf{S}_1^T - \mathbf{S}_2^T - \cdots - \mathbf{S}_{U-1}^T + \mathbf{S}_U^T \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ +\mathbf{S}_1^T + \mathbf{S}_2^T + \cdots + \mathbf{S}_{U-1}^T - \mathbf{S}_U^T \\ +\mathbf{S}_1^T + \mathbf{S}_2^T + \cdots + \mathbf{S}_{U-1}^T + \mathbf{S}_U^T \end{bmatrix} \tag{4.2.6}$$

Each row in Eqn. 4.2.6 represents a center vector of the RBF MUD and the weights $w_{m,u}$ in Eqn. 4.2.2 are selected from m^{th} row and u^{th} column of matrix $\mathbf{D}$ (Cruickshank, 1996).

4.2.2 RBF MUD for Multipath Channel

4.2.2.1 The structure of the detector

The structure of the RBF MUD for multipath channel is the same with the structure for the AWGN channel which is shown in Fig. 4.2. The basis function used is again the Gaussian kernel

$$\phi_m(\mathbf{y}(k)) = \exp\left(-\frac{||\mathbf{y}(k) - \mathbf{c}_m||^2}{2\sigma^2}\right) \tag{4.2.7}$$

where, $\mathbf{c}_m,\ m = 1, 2, ..., M$ are the center vectors of length $N + (L - 1)$, N is the length of the spreading sequences, L is the number of taps of the multipath channel, M is the number of center vectors that are introduced by the RBF MUD for each 2^{3U} possible received signal where U is the number of users in the channel. As in AWGN, the vector set $\mathbf{c}_m,\ m = 1, 2, ..., M$ contains all of the possible received signal vectors, $\mathbf{y}(k)$, these centers are called as *supercenters*. Variance of the Gaussian center function, σ^2, equals to variance of the added noise component, σ_n^2.

4.2.2.2 Construction of Supercenter and Code Matrices

In order to construct the supercenter matrix for the multipath channel, the L-tap impulse response H_{ch} of the channel has to be known at the detector. As it is discussed in Section 2.3.2.2, we have to deal with interchip interference (ICI) in a multipath environment. It is possible to realize the convolution of the spreading sequences with the channel impulse response using matrix algebra in order to combat with ICI while constructing the RBF MUD that operates in the multipath environment.

The supercenter matrix for the multipath channel is defined by Tanner (1998) as

$$\mathbf{C}^{MP} = \mathbf{S}^{MP}\mathbf{H}^T \tag{4.2.8}$$

where $\mathbf{H}$ is an $(N + L - 1) \times 3N$ Toeplitz matrix constructed using the channel impulse response H_{ch}, and N is the length of the spreading sequence. The first $N - L + 1$ columns in $\mathbf{H}$ is zero. As an example for the case $L = 3$ and $N = 4$, the matrix $\mathbf{H}$ would be

$$\mathbf{H} = \begin{bmatrix} 0 & 0 & h_3 & h_2 & h_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & h_3 & h_2 & h_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_3 & h_2 & h_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & h_3 & h_2 & h_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h_3 & h_2 & h_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_3 & h_2 & h_1 & 0 & 0 \end{bmatrix} \tag{4.2.9}$$

The $\mathbf{S}^{MP}$ in Eqn. 4.2.8 is the Hadamard product of extended code matrix $\mathbf{D}^{MP}$ and $U \times 3N$ matrix comprising the spreading sequences of length N of all U users for the previous, current, and next symbols, thus

$$\mathbf{S}^{MP} = \mathbf{D}^{MP} \bullet \begin{bmatrix} \mathbf{S} & \mathbf{S} & \mathbf{S} \end{bmatrix} \tag{4.2.10}$$

where $\mathbf{S}$ is defined in Eqn. 4.2.4 and the Hadamard product of two matrices $\mathbf{A}$ and $\mathbf{B}$ is defined as

$$(\mathbf{A} \bullet \mathbf{B})_{ij} = a_{ij} b_{ij} \tag{4.2.11}$$

The extended code matrix $\mathbf{D}^{MP}$ is a $2^{3U} \times 3U$ matrix which contains all possible bit combinations of previous, current, and next symbols for the U users as it rows and it is in the form

$$\mathbf{D}^{MP} = \begin{bmatrix} -1 & -1 & \cdots & -1 & -1 & -1 & \cdots & -1 & -1 & -1 & \cdots & -1 \\ -1 & -1 & \cdots & +1 & -1 & -1 & \cdots & +1 & -1 & -1 & \cdots & +1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ +1 & +1 & \cdots & -1 & +1 & +1 & \cdots & -1 & +1 & +1 & \cdots & -1 \\ +1 & +1 & \cdots & +1 & +1 & +1 & \cdots & +1 & +1 & +1 & \cdots & +1 \end{bmatrix} \tag{4.2.12}$$

and it can be partitioned into three sub-matrices in order to simplify the notation

$$\mathbf{D}^{MP} = \begin{bmatrix} \mathbf{D}_P & \mathbf{D}_C & \mathbf{D}_N \end{bmatrix} \tag{4.2.13}$$

where $\mathbf{D}_P$, $\mathbf{D}_C$, and $\mathbf{D}_N$ are $2^{3U} \times U$ matrices representing the previous, current, and next code matrices respectively. Substituting Eqns. 4.2.13 and 4.2.4 into Eqn. 4.2.10 we have

$$\begin{aligned} \mathbf{S}^{MP} &= \begin{bmatrix} \mathbf{D}_P & \mathbf{D}_C & \mathbf{D}_N \end{bmatrix} \bullet \begin{bmatrix} \mathbf{S} & \mathbf{S} & \mathbf{S} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{D}_P \begin{bmatrix} \mathbf{S}_1^T \\ \vdots \\ \mathbf{S}_U^T \end{bmatrix} & \mathbf{D}_C \begin{bmatrix} \mathbf{S}_1^T \\ \vdots \\ \mathbf{S}_U^T \end{bmatrix} & \mathbf{D}_N \begin{bmatrix} \mathbf{S}_1^T \\ \vdots \\ \mathbf{S}_U^T \end{bmatrix} \end{bmatrix} \end{aligned} \tag{4.2.14}$$

Each row in the matrix $\mathbf{C}^{MP}$ represents a center vector of the RBF MUD for the multipath channel and the weight $w_{m,u}$ in Eqn. 4.2.2 must be selected from m^{th} row and u^{th} column of matrix $\mathbf{D}c$ (Cruickshank, 1996).

4.3 Complexity of the RBF MUD

Although RBF MUD needs no training and it is fully defined when the spreading sequences of all users and the channel impulse response are known, its structural complexity increases exponentially with the number of users. Its structural complexity gets even worse in the multipath environment since the number of centers needed to span the received signal space of a U user DS-CDMA equals to 2^{3U}. In a real-life CDMA system there will probably be a large number of users in the channel thus, RBF MUD is impractical to realize both in AWGN and multipath channels. Thus, a noticeable number of researchers have tried to reduce the structural complexity of the RBF MUD (Tanner & Cruickshank, 1998, Sessler et al., 2001, Wei et al., 2004). In this thesis, a new method that uses genetic algorithm to reduce the structural complexity of the RBF MUD is proposed where the method is introduced in detail in the next chapter.

The reader should realize that RBF MUD whose structure is given in this chapter and our optimization method which will be introduced in the next chapter is valid under the assumption that the DS-CDMA signals transmitted by U independent users to be bit and chip synchronous with equal power which is normalized to 1 which is a situation that may be realized in a downlink (base to mobile) scenario in a cellular mobile communication system.

CHAPTER FIVE
COMPLEXITY REDUCTION OF RBF MUD USING GA

This chapter introduces the proposed method to reduce the structural complexity of the radial basis function multiuser detector. First section briefly reviews the genetic algorithm, then the definition of the problem is stated in the next section. The method is introduced by the help of a simple example in the following section and then details of the method are given.

5.1 Genetic Algorithm (GA)

The *genetic algorithm* (GA) (Goldberg, 1989, Tang et al., 1996) is a stochastic search method based on the laws of natural selection, biological evolution, and genetics which operates as an entirely different optimization procedure among other optimization methods (like calculus based techniques, enumerative techniques, etc.) In general, a basic GA consists of three operations: *Selection*, *Genetic Operation,* and *Replacement*. Fig. 5.1 shows the flow diagram of a simple GA.

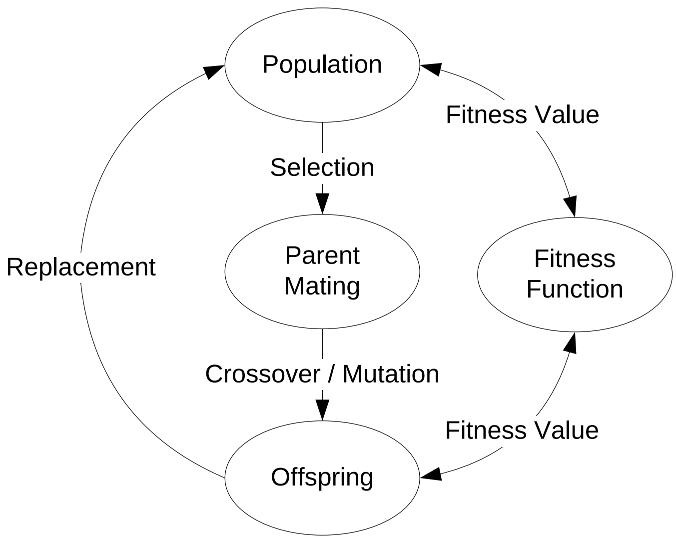


Figure 5.1: Simple genetic algorithm cycle.

In the GA, the population consists of a group of chromosomes where each of them represents a solution to the problem. A chromosome is a string of numbers, usually it is a vector of binary digits. Initial population may be generated randomly or manually if there is an initial guess about the solution. At each iteration, all of the chromosomes are evaluated and their fitness values are calculated. According to their fitness values, a probability of selection is assigned to each one of them. A particular group of chromosomes (parents) are selected and genetic recombination (crossover) is applied to pairs of parents to generate the offsprings. Some of the offsprings are mutated with a pre-defined probability and a new population whose chromosomes would be the parents of the next generation is created.

The GA cycle ends when a desired criterion is satisfied, where the criterion may be defined as the number of generations, fitness value, and etc. Due to this simulated evolution, the chromosome with the best fitness value in the final population can become a highly evolved solution to the problem.

5.2 Definition of the problem

The RBF MUD uses all the centers in the supercenter matrix $\mathbf{C}$ (see Eqns. 4.2.3 and 4.2.8) and acts as a *maximum likelihood symbol detector* (MLSD) in AWGN, so it reaches to the optimum performance (Tanner & Cruickshank, 1998). However, the number of rows in matrix $\mathbf{C}$ (which is also the number of centers in the RBF MUD) is equal to 2^U in non-dispersive AWGN channel and 2^{3U} in multipath channels where U is the number of users in the channel. When the number of users in the channel is large, the structure of RBF MUD gets too complicated since its number of centers increases exponentially with the increasing number of users, U. Thus, a need for structure optimization of this detector arises, especially when the number of users in the channel is large.

The input space of a DS-CDMA system has N dimensions in AWGN channels and $N+L-1$ dimensions in multipath channels where N is the length of the spreading codes that are assigned to each user in the system and L is the number of paths in the channel. Low dimensional views of the input space of a DS-CDMA system can be insightful using multidimensional scaling techniques one of which is the *self organizing map* (SOM). SOM is a subtype of artificial neural networks and it is trained using unsupervised learning to produce low dimensional representation of the training samples while preserving the topological properties of the input space. In Fig. 5.2, plot of the SOM for user one, $u = 1$, is shown for a particular DS-CDMA system where six users are in the AWGN channel, $U = 6$, and each user has a spreading code of length eight, $N = 8$. Two dimensional representation of the original eight

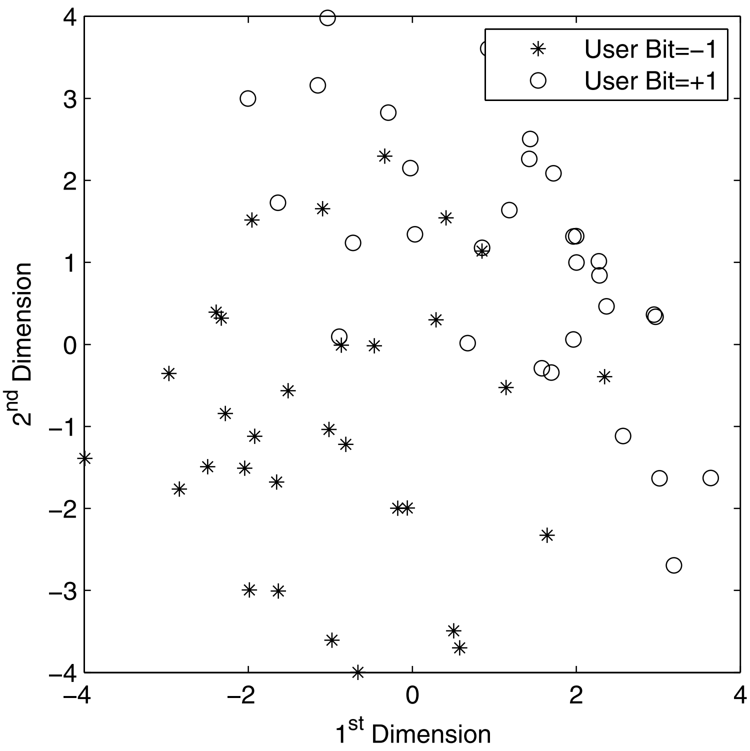


Figure 5.2: SOM for a user in a DS-CDMA system of 6 users.

dimensional DS-CDMA space suggests that the space can be represented by less than 2^U basis functions. The problem is to find the best center locations and variation values of the basis functions. Another problem is determining the minimum number of centers to be used. These optimization problems can be solved by the GA. In other words, the structure of the RBF MUD can be optimized using the GA. The proposed method will be explained with the help of an example in detail.

5.3 A Simple Example

In the traditional RBF MUD, there are $M = 2^U$ centers in AWGN, and each center has a variance which is equal to the variance of the added noise, σ_n^2. In the proposed RBF MUD, the center vectors are not chosen from the set of supercenter vectors and the variance of each center can take any value. This RBF MUD structure shall be further explained with the help of the following example of the simplest case:

The supercenter matrix of an RBF MUD receiving a DS-CDMA signal which is a superposition of signals from two users sharing the same AWGN channel, $U = 2$, and having

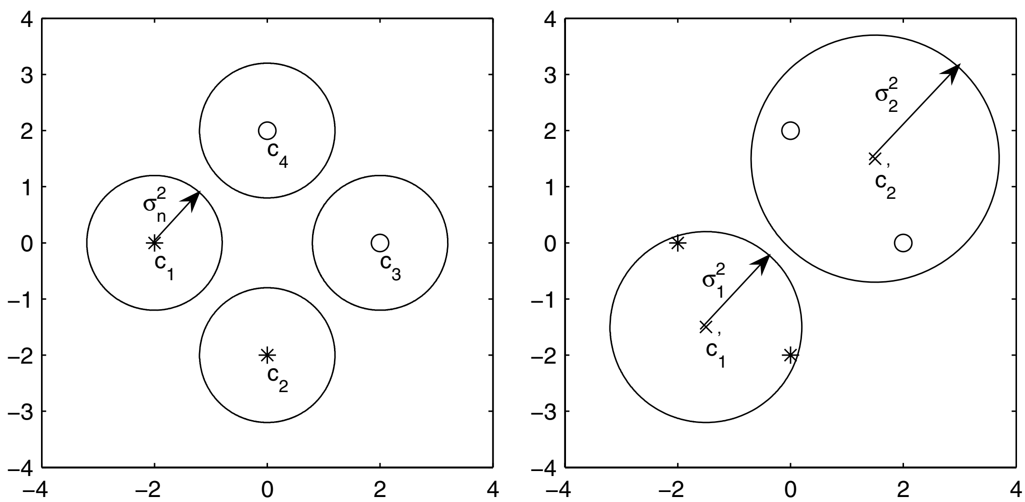
Walsh spreading codes of length two, $N = 2$, and the spreading codes are $\mathbf{S}_1 = \begin{bmatrix} +1 & +1 \end{bmatrix}$ and $\mathbf{S}_2 = \begin{bmatrix} +1 & -1 \end{bmatrix}$ for the first and second user respectively, can be calculated using Eqn. 4.2.6, as

$$\mathbf{C} = \mathbf{DS} = \begin{bmatrix} -1 & -1 \\ -1 & +1 \\ +1 & -1 \\ +1 & +1 \end{bmatrix} \begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \\ 0 & +2 \\ +2 & 0 \end{bmatrix} \tag{5.3.1}$$

These four supercenter vectors, $\mathbf{c}_m,\ m = 1, 2, 3, 4,$ Gaussian basis functions, ϕ_m where $m = 1, 2, 3, 4,$ and their variations, σ_m^2 where $m = 1, 2, 3, 4$ are shown in Fig. 5.3.a. In the original definition of RBF MUD by Cruickshank (1996) and in related works by Tanner & Cruickshank (1998), Sessler et al. (2001), Wei et al. (2004), the variation of all the centers has been chosen to be constant and equal to the noise variance, σ_n^2

$$\sigma_m^2 = \sigma_n^2 \ \ \forall m,\ \ m = 1, 2, ..., M \tag{5.3.2}$$

where $M = 2^U$ is the number of centers and U is the number of users in the channel.



(a) (b)

Figure 5.3: Centers and variances of (a) traditional RBF MUD and (b) GA assisted RBF MUD.

It is possible to reduce the number of centers by allowing the center locations to be anywhere in the DS-CDMA space and setting different variance values to different basis functions. By doing so, the RBF MUD can cover the whole space with a less number of centers, with a very little performance degradation in terms of BER, and with less structural complexity. In Fig. 5.3.b, the center vectors, Gaussian basis functions, and associated variations of each basis function are shown. As it is seen in Fig. 5.3.b, the center vectors of the proposed RBF MUD are not chosen from the vector set of supercenters

$$\mathbf{c}'_k \neq \mathbf{c}_m;\ k = 1, 2, ..., K;\ \ m = 1, 2, ..., M \tag{5.3.3}$$

where $\mathbf{c}'_k$ is the k^{th} center vector of the prosed RBF MUD and $\mathbf{c}_m$ is the m^{th} supercenter vector of the traditional RBF MUD. As it is also seen in Fig. 5.3.b, the variances of Gaussian basis functions of the proposed RBF MUD are different for each center

$$\sigma_i^2 \neq \sigma_j^2,\ i \neq j,\ 1 \leq i < j \leq K \tag{5.3.4}$$

This flexibility of the proposed method lets a center to represent more than one supercenter which leads to a considerable amount of performance increase in terms of computational complexity, especially when the number of users, U, is large.

5.4 Optimizing the RBF MUD Structure in AWGN with GA

Optimizing the structure of the RBF MUD starts with a randomly selected small subset of supercenter vector set. In other words, the $K \times N$ matrix $\mathbf{C}'$ is generated by selecting from the rows of $M \times N$ matrix $\mathbf{C}$ in Eqn. 4.2.6. Initial variances of centers are set to be equal to the noise variance, σ_n^2. Then, center vector locations and variances of each center function are optimized to get better BER's by using a GA.

Each member of the population is formed as

$$\mathbf{I}_{p,i} = \begin{bmatrix} c'_{1,i} & c'_{2,i} & \cdots & c'_{K,i} & \sigma_{1,i}^2 & \sigma_{2,i}^2 & \cdots & \sigma_{K,i}^2 \end{bmatrix} \tag{5.4.1}$$

where $p = 1, 2, ..., P$ is the member number, P is the population size, and i is the generation number. As it is seen in Eqn. 5.4.1, each member of the population represents a different RBF MUD structure. At each generation of the GA, the RBF MUD structures defined by each member of the population are formed up and tested with as input set of a predefined size. The fitness function of each member is defined as

$$f = 1 - BER \tag{5.4.2}$$

where BER is the bit error rate of the RBF MUD whose structure is defined by the associated member. Thus, the GA minimizes BER though maximizing the fitness function. At each iteration, the members with the best fitness function are selected. Then GA operators like crossover and mutation are applied to these members and a new population is generated. The optimization is terminated when the number of iterations reaches to a predefined value.

Since the optimized locations of the center vectors are different from the locations of the supercenter vectors, the weight values in the output layer of the RBF MUD, $w_m, m = 1, 2, .., M$, can not be determined from the columns of the code matrix $\mathbf{D}$. Thus, the weight values are calculated by using the *least mean squares* (LMS) algorithm for each member at each iteration of the GA. When the algorithm terminates, the member with the best BER is selected to be the centers and variances of the RBF MUD and associated weights calculated by the LMS become the weights of the output layer of the RBF MUD.

5.5 Optimizing the RBF MUD Structure in Multipath Channel with GA

The method for optimizing the RBF MUD structure in multipath channel is similar to the one in AWGN. It is possible to generate $K \times (N + L - 1)$ matrix $\mathbf{C}'$ again by randomly selecting from the rows of $M \times (N + L - 1)$ matrix $\mathbf{C}^{MP}$ in Eqn. 4.2.8. But since $M = 2^{3U}$ and N can be very large, dimensions of the matrix $\mathbf{C}^{MP}$ can get very high, thus it would be cumbersome to generate this matrix. Instead, a number of P integers can be randomly selected in the interval $1 \leq p \leq 2^{3U}$. Then, it is straightforward to generate only pth rows of the code matrix $\mathbf{D}^{MP}$ given in Eqn. 4.2.12 and then calculating P rows of the matrix $\mathbf{C}^{MP}$.

Since the dimension of the input space is greater than it is in AWGN, it is recommended to run the GA with a larger population size and number of iterations. Again, the weight values are calculated by using the LMS algorithm for each member at each iteration of GA.

CHAPTER SIX
SIMULATION RESULTS

This chapter gives the simulation method and the results of the several tests defined to measure the performance of the proposed method. First section gives the performance criteria used in the simulation results and next section defines the method used in the simulations. Following sections give bit error rate plots and comments on the corresponding plots for the several tests defined.

6.1 Performance Criteria

The performance criteria used in our simulations is the bit error probability of the multiuser detectors. Since the evaluation of the probability of error for the optimum detector is extremely difficult, we may use the probability of error of a single user detector in the absence of the other users as a lower bound which is defined as

$$P_e(\gamma) = Q\left(\sqrt{2\gamma}\right) \tag{6.1.1}$$

where $\gamma = \varepsilon/N_0$, ε is the signal energy per bit, $N_0/2$ is the power spectral density of the AWGN, and the $Q(x)$ is the Q-function defined as

$$Q(x) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right) \tag{6.1.2}$$

Thus the probability of error for a single user detector in AWGN is

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{\varepsilon}{N_0}}\right) \tag{6.1.3}$$

For multipath channels the probability of error for a single user detector (Proakis, 2001) is defined as

$$P_e = Q\left(\sqrt{\frac{2\varepsilon\left(\sum_{k=1}^{L} h_k\right)}{LN_0}}\right) \tag{6.1.4}$$

where L is the number of paths, and h_k is the impulse response of the kth path of the channel. Due to the complexity of the radial basis function multiuser detector, a time-invariant multipath channel used in our simulations with the channel impulse response given as

$$H_{ch}(z) = 0.3482 + 0.8704z^{-1} + 0.3482z^{-2} \tag{6.1.5}$$

Thus, the received DS-CDMA signal at chip rate becomes

$$\begin{aligned} y(kN+n) = \quad & 0.3482 \sum_{u=1}^{U} D_u(k) S_{u,n} + 0.8704 \sum_{u=1}^{U} D_u(k) S_{u,n-1} \\ & + 0.8704 \sum_{u=1}^{U} D_u(k) S_{u,n-2} + g(kN+n) \end{aligned} \tag{6.1.6}$$

In Fig. 6.1 plots for the probability of error of single user detectors in both AWGN channel and 3-tap multipath fading channel with an impulse response of Eqn. 6.1.5 is given. This curves represent a lower bound on the performance of the detectors tested in our simulations.

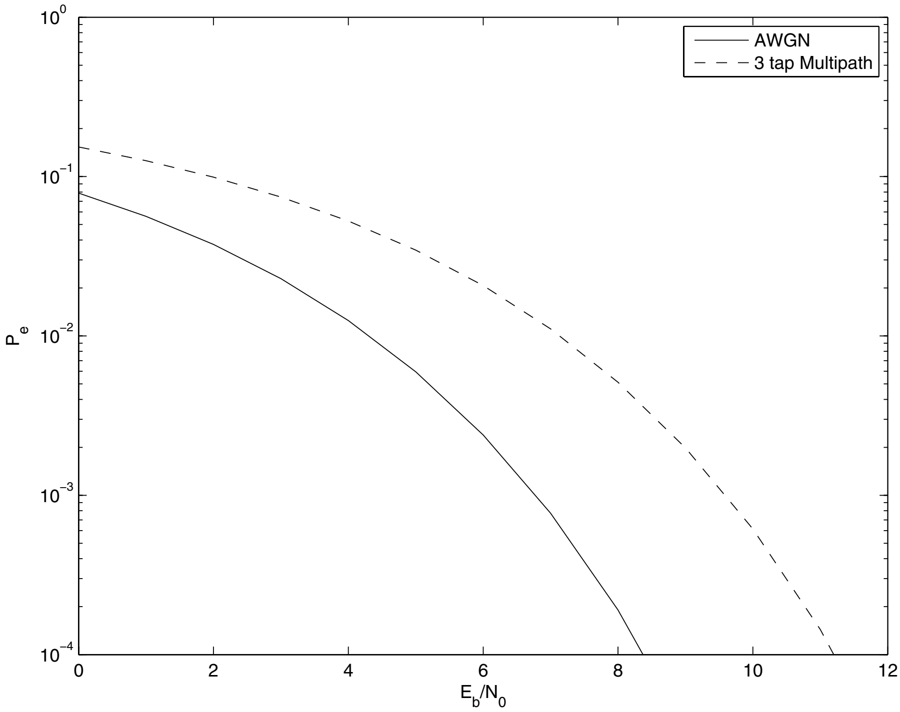


Figure 6.1: Probability of error for a single user detector.

6.2 Simulation Method

A DS-CDMA system with 20 users having Walsh spreading sequences of length 32 in non-dispersive channel distorted by AWGN is simulated. Since the number of supercenters is very high for 3-tap multipath fading channel, the number of users in the system is limited to 10 for multipath case. A number of 10^4 equiprobable bits for training and 10^7 bits for testing are generated for each user. As explained in chapter four, code matrix $\mathbf{D}$ and supercenter matrix $\mathbf{C}$ are generated for $U = 20$ and $N = 32$. Each member (chromosome) in the initial population of GA is formed up as follows:

$$\mathbf{I}_{p,0} = \begin{bmatrix} \mathbf{c}'_{1,0} & \mathbf{c}'_{2,0} & \cdots & \mathbf{c}'_{K,0} & \sigma^2_{1,0} & \sigma^2_{2,0} & \cdots & \sigma^2_{K,0} \end{bmatrix} \tag{6.2.1}$$

Table 6.1: Genetic Algorithm parameters used in different tests.

	Test for Number of Centers	Test for Number of Generations	Test for Mutation Probability	Test for Population Size
Number of Centers	20, 60 and 80	40	40	40
Population Type	Double Vector	Double Vector	Double Vector	Double Vector
Population Size	40	40	40	20, 40 and 60
Selection Function	Stoc. Uniform	Stoc. Uniform	Stoc. Uniform	Stoc. Uniform
Mutation Function	Gaussian	Gaussian	Uniform	Gaussian
Crossover Function	Scattered	Scattered	Scattered	Scattered
Mutation Probability	Shrink:0.75; Scale:0.5	Shrink:0.75; Scale:0.5	0.001, 0.1 and 0.5	Shrink:0.75; Scale:0.5
Crossover Probability	0.8	0.8	0.8	0.8
Number of Generations	50	50 and 200	50	50

where $\mathbf{c}_{x,0},\ x = 1, 2, ..., K$, vectors are selected randomly from the rows of supercenter matrix $\mathbf{C}$. Each row of matrix $\mathbf{C}$ is selected only once in the same chromosome. Initial variance values in Eqn.6.2.1, $\sigma^2_{y,0},\ y = 1, 2, ..., K$, are set to be equal to the variance of the added noise component, σ^2_n.

Not only the transmitted data of a selected user but also the transmitted data of all the users in the channel are detected in both the train and test stages of our simulations. This is done by updating the weights of the RBF MUD which are calculated at each iteration of the training stage and in the beginning of the testing stage by the LMS algorithm for that particular user.

Of many parameters which may effect the GA RBF MUD's performance, the parameters that may have significant effect on the results are tested for different values. These parameters are as follows: number of centers used in the structure, initial population, number of generations produced by the GA, mutation probability of the GA and population size (number of members in the population) of the GA. RBF MUD and GA parameters used in the tests are presented in Table 1. Effects of these parameters on the performance of the detector will be explained in the following subsections in the light of the simulation results.

6.3 Results in AWGN Channel

6.3.1 Test for the Effect of Number of Centers

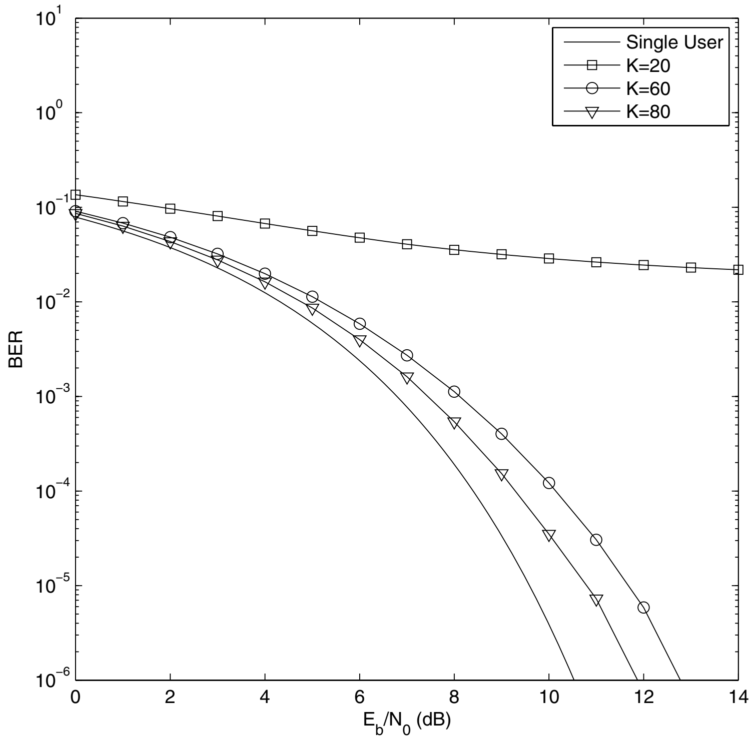


Figure 6.2: Simulations resuts of test for number of centers.

Three GA assisted RBF MUD's with different number of centers are simulated. Numbers of centers are selected to be 20, 60, and 80. In order to operate at optimum performance, an RBF MUD needs 2^{20} centers to support 20 users. Thus, increasing the number of centers would improve the performance of the detector. BER versus E_b/N_0 plot for the GA assisted RBF MUD having different number of centers is given in Fig. 6.2, where K is the number of centers of the GA RBF MUD. As can be seen in Fig. 6.2, $K = 80$ gives the best performance where this result meets the theoretical assumption stating the performance improvement due to the increase of the number of centers. When $K = 20$, the detector is unable to provide a near-optimum performance. However, due to the flexible structure of the proposed detector, even in the case of 20 centers, its BER performance is about 1 dB better than the performance of the detector proposed by Wei et al. (2004) which forms up an RBF MUD by selecting the most influential supercenters as the centers.

The RBF MUD needs 2^{20} centers to operate while the method proposed in this thesis reduces this number to 80. The complexity reduction ratio is a considerable amount which is about $1/13000$ on a rough calculation. Performance loss in the new detector is about 0.6 dB for a BER of 10^{-3}.

6.3.2 *Test for the Effect of Initial Population and Number of Generations*

The performance of the GA depends on the choice of initial population. GA would provide better performance if the algorithm is started with an initial population whose members are close to the solution to the optimization problem. In our case, the members in the population represent the center vectors and variance values of the basis functions which forms up the RBF MUD. Thus, the method would converge to optimum structure faster if the initial population were set close to the final structure. Hence, we start the GA with an initial population whose members are formed up by randomly selecting the centers from a number of 2^{20} supercenters.

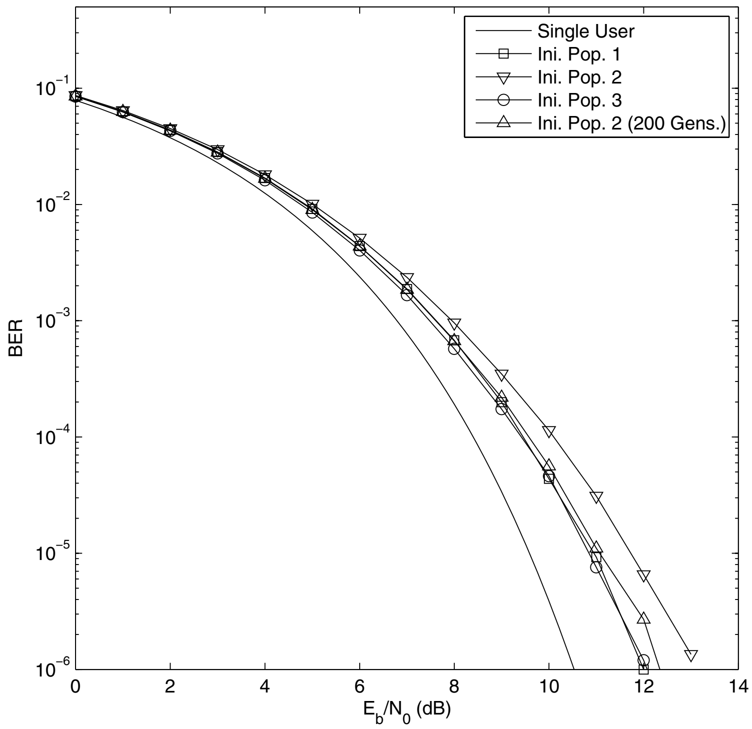


Figure 6.3: Simulations resuts of test for initial population.

In Fig. 6.3, BER versus E_b/N_0 plots are shown for the RBF MUD's which are optimized by the GA starting with different initial populations. In this test, the GA is terminated when the number of generations reached to 50. It is seen in Fig. 6.3 that starting with the first and third initial populations has lead the GA to generate RBF MUD's which have identical and better performances than the RBF MUD which was generated by the GA started with the second initial population. But starting with the same initial population and terminating the

GA at 200 generations instead of 50, the resultant optimized RBF MUD provided the identical performance with the other RBF MUD's. Thus, the effect of the initial population on the performance of the resultant RBF MUD can be eliminated by letting the algorithm to generate more populations.

6.3.3 Test for the Effect of Mutation Probability

The recommended mutation probability range in the literature (Goldberg, 1989) is $10^{-3} - 10^{-2}$. Since our strings (members in the population) are represented by real numbers and no encoding is used, the space that is needed to be scanned is real valued and there are infinite number of locations for a center to be located. Hence, we would need a high mutation probability to search the space effectively. According to Fig. 6.4, the GA with a mutation probability close to the upper recommended limit, 0.05, generated an RBF MUD with the best performance. A probability that is less than the recommended lower limit, 0.0005, or a probability that is much greater than the upper limit, 0.5, ends up with an RBF MUD of worse performance.

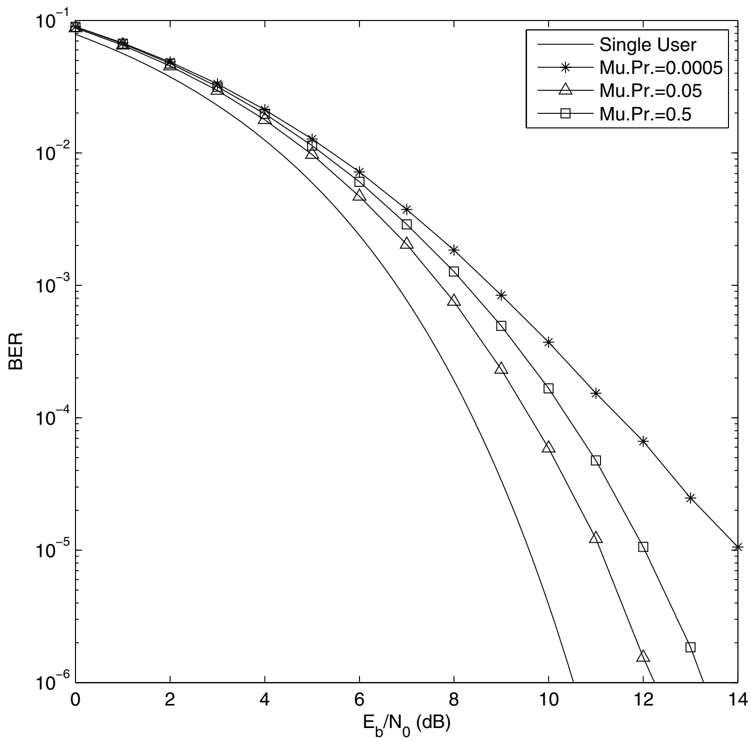


Figure 6.4: Simulations resuts of test for mutation probability.

6.3.4 Test for the Effect of Population Size

In GA, increasing the number of members in the population to be evolved decreases the probability of algorithm to be stuck at local maxima. In this test, populations of different sizes are generated and optimized via the GA. The algorithm is again stopped at the same number of generations for each population. Observing Fig. 6.5, we may conclude that a GA evolving a population of greater size would generate an RBF MUD with better performance.

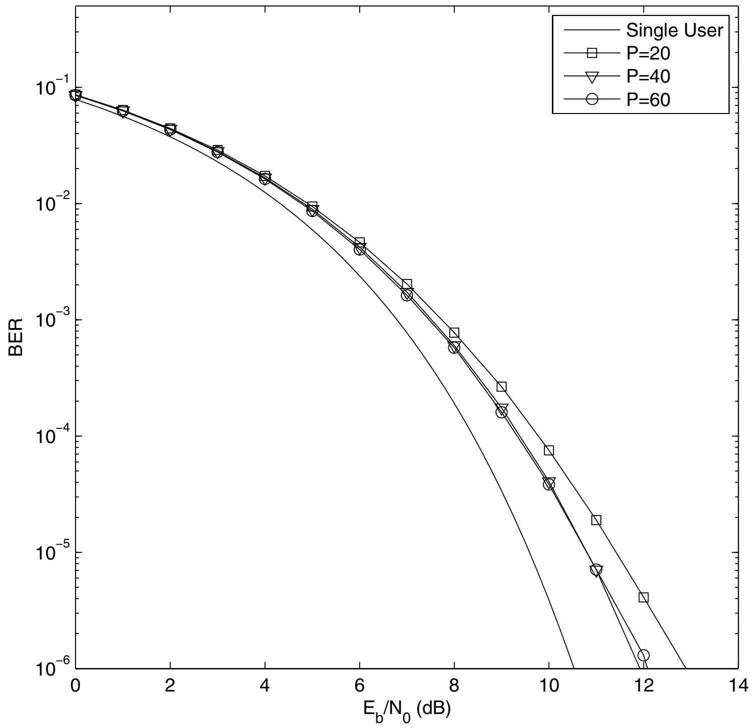
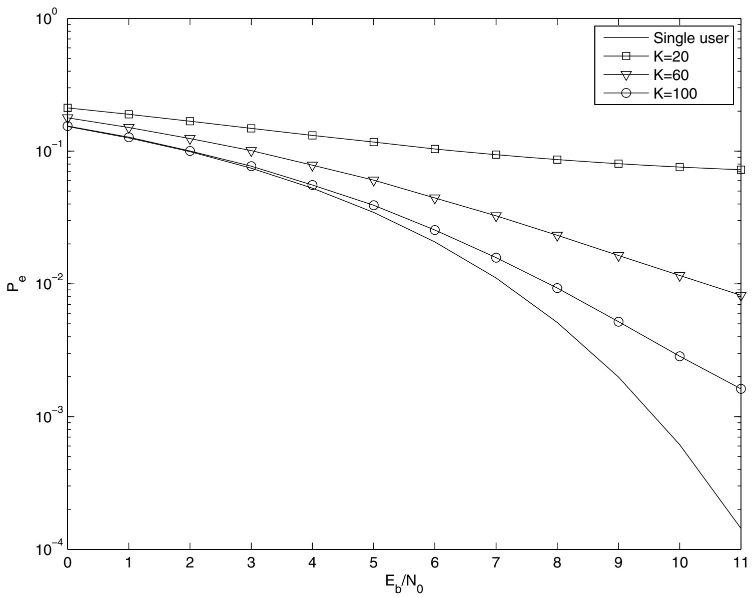


Figure 6.5: Simulations resuts of test for population size.

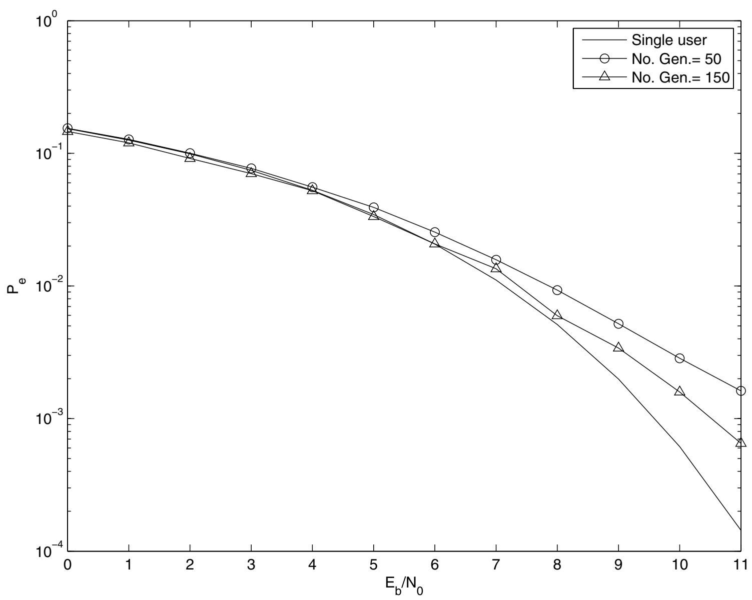
6.4 Results in Multipath Fading Channel

The number of centers for the RBF MUD supporting 10 users in a 3-tap multipath fading channel are reduced to 20, 60, and 100. It can be seen in Fig. 6.6.a that the proposed method reduced the number of centers from 2^{30} to 100 with a slight performance degradation. When $K = 20$ and $K = 60$ the detector is unable to provide a near-optimum performance. If we consider the case for $K = 100$, the complexity reduction ratio is a considerable amount which is about $1/10^7$ on a rough calculation. Performance loss in the new detector is about 1 dB for a BER of 10^{-3}.

In Fig. 6.6.b bit error rate performances of two RBF MUDS where one of them is generated after 50 iterations of GA and the other is generated after 150 iterations are shown. We may conclude that the performance of the optimized RBF MUD may be increased further if we let the genetic algorithm run more iterations.



(a)



(b)

Figure 6.6: Simulations resuts of (a) test for number of centers and (b) test for number of generations in multipath fading channel.

CHAPTER SEVEN
CONCLUSIONS

Optimum detector for DS-CDMA selects the most probable sequence of transmitted bits given the received signal and it provides superior performance. On the other hand its computational complexity increases exponentially with the number of users in the channel. Since the number of users in a real-life CDMA system may become very high, the implementation of the optimum detector would be very impractical and expensive. Several linear and nonlinear suboptimal detectors are proposed in the literature. The main goal of the researchers while developing a suboptimum detector is to achieve a computational complexity that is less than that of the optimum detector.

Radial basis function multiuser detector proposed by Cruickshank (1996) is a suboptimum DS-CDMA detector. It is based on radial basis function networks and is fully determined when the spreading sequences of the users and the channel impulse response are known. However, it has a structural complexity that increases exponentially with the number of users. The number of centers needed to realize an RBF MUD equals to 2^U to support U users. The situation in multipath environment is even worse since the number of centers needed to span the input space equals to 2^{3U} for an RBF MUD supporting U users.

Using a multidimensional scaling technique, self organizing map (SOM) in this case, a two dimensional representation of the original N dimensional DS-CDMA space is obtained and it is shown that the input space of the detector can be represented by an RBF MUD which has a fewer number of centers than 2^U. Based on this idea, a new method is proposed which reduces the number of centers of the RBF MUD by using genetic algorithm (GA) as the optimization tool.

The proposed optimization method, starts with a predefined number of centers which are selected from the supercenters and sets the initial variance of these centers to noise variance. This predefined number is very small compared to the number of supercenters. It is expected that the genetic algorithm will evolve an RBF MUD structure which gives the minimum bit error rate (BER), by applying small changes to the locations of the centers and center variances at each iteration. The proposed method is tested with a variety of parameters via computer simulations and promising results are obtained.

The structural complexity of an RBF of MUD in AWGN channel is decreased by a ratio of about $1/13000$. This ratio is about $1/10^7$ for an RBF MUD in a 3-tap multipath fading channel. It is observed that by increasing the number of centers, population size, and number of generations, the proposed method generates an RBF MUD with a performance that is even closer to the single user performance.

Consequently, the proposed method successfully reduces the complexity of the RBF MUD for DS-CDMA by minimizing the number of center functions using GA. By determining the optimal values of the centers and the variances of the radial basis functions through GA, the complexity of the RBF MUD is reduced from $O(2^U)$ to $O(K)$, where K is a predetermined number of centers, in the expense of negligible performance degradation compared to the single user detector.

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APPENDIX A
PUBLICATIONS

- Torun, M. U.; Kuntalp, D., "Genetic algorithm assisted radial basis function multi-user detector for DS-CDMA", *15th Signal Processing and Communications Applications*, 2007 IEEE June, 11th 2007.
- Torun, M. U.; Kuntalp, D., "Complexity reduction of RBF based multiuser receiver for DS-CDMA using genetic algorithms", manuscript is submitted for review to IEEE Transactions on Communications.