DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

BAYESIAN AGGREGATION METHODS FOR ANALYTIC HIERARCHY PROCESS AND ANALYTIC NETWORK PROCESS IN GROUP DECISION MAKING

by
Zeynep Filiz EREN DOĞU

March, 2012
İZMİR

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Ph.D. THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"BAYESIAN AGGREGATION METHODS FOR ANALYTIC HIERARCHY PROCESS AND ANALYTIC NETWORK PROCESS GROUP DECISION MAKING"** completed by **ZEYNEP FİLİZ EREN DOĞU** under supervision of **PROF. DR. CAN CENGİZ ÇELİKOĞLU** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy.

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BAYESIAN AGGREGATION METHODS FOR ANALYTIC HIERARCHY PROCESS AND ANALYTIC NETWORK PROCESS IN GROUP DECISION MAKING

ABSTRACT

The problems whose objective is to search the best alternative or to rank the alternatives in terms of a number of conflicting criteria are the multi-criteria decision making (MCDM) problems. As the interdisciplinary teams, composed of different scientists developed in different sectors, group decision making in MCDM problems gains more importance and necessity. The increasing complexity of the group decision problems requires the use of more flexible approaches.

The Analytic Hierarchy Process (AHP) and the Analytic Network Process (ANP) are widely used approaches for solving complex MCDM problems. The AHP group decision making (AHP-GDM) method decomposes a complex MCDM problem into a system of hierarchies and selects the best alternative in terms of some criteria by making pairwise comparisons. The extension of AHP-GDM to the case of dependence and feedback is called the ANP group decision making (ANP-GDM).

In order to aggregate the individual's group judgements in a group setting "the aggregation of individual judgements (AIJ)" and "the aggregation of individual priorities (AIP)" methods are used. However these classical methods have some assumptions such as: the pairwise comparison matrices of decision makers are complete and consistent. In real life problems, it is hard to satisfy these assumptions due to the complexity of the problem or inexperience of the decision makers.

This research proposes Bayesian aggregation procedures for AHP-GDM and ANP-GDM which do not require intermediate filters for the decision makers' initial judgements. The weights of the decision makers are inversely proportional to their consistency levels. The proposed procedures are extended to the analysis of incomplete pairwise comparison matrices where they provide more robust manner than classical methods in terms of the priorities and have lower values of the mean square errors. The methodology has been illustrated with case studies and compared with the conventional aggregation method.

Keywords: Group decisions and negotiations, Multi-criteria decision making (MCDM), Analytic Hierarchy Process (AHP), Analytic Network Process (ANP), Bayesian prioritization procedure (BPP).

COK KRİTERLİ KARAR PROBLEMLERİNİN ÇÖZÜMÜNDE BAYESCİ ÖNCELİKLENDİRME METODUNA DAYALI ANALİTİK HİYERARŞİ SÜRECİ VE ANALİTİK SERİM SÜRECİ

ÖZ

Karar vericinin mevcut alternatifler arasından birbiriyle çelişen kriterler doğrultusunda bir seçim, sıralama ya da değerlendirme yaptığı problemlere Çok Kriterli Karar Verme (ÇKKV) problemleri denir. Günümüzün koşulları gereği değişik sektörlerde bir grup bilim adamı ve/veya araştırmacıdan oluşan disiplinler arası ekiplerin hızla çoğalması, ÇKKV problemlerinin grup kararı ile çözümünün önemini ve gerekliliğini arttırmaktadır. Grup kararı gerektiren problemlerin karmaşıklığı arttıkça problemi çözmek için daha esnek yöntemlere ihtiyaç duyulmaktadır.

Analitik Hiyerarşi Süreci (AHS) ve Analitik Serim Süreci (ASS), karmaşık ÇKKV problemlerinin grup kararıyla çözümü gereken durumlarda sıklıkla kullanılan yöntemlerdir. AHS ile grup kararı verme (AHS-GKV) yöntemi, ÇKKV problemini hiyerarşiler sistemine dönüştürerek belirlenen kriterlere göre alternatiflerin ikili karşılaştırmasını yapar ve sonunda en iyi alternatifi seçer. ASS-GKV yöntemi ise AHS-GKV yönteminin bağımlılık ve geri bildirim yapılarının olduğu durumlar için genelleştirilmiş halidir.

Literatürde grup AHS kararlarının birleştirilmesi için kullanılan klasik yöntemler mevcuttur. Bunlar “Bireysel kararların birleştirilmesi (BKB) ve bireysel önceliklerin birleştirilmesi (BÖB)” metotlarıdır. Bu klasik yöntemlerin birtakım yetersizlikleri bulunmaktadır. Literatürde bu tür yetersizlikleri gidermeye yönelik metodolojik yenilikler yer almaktadır.

Eksikliklerden birisi bu klasik yöntemlerin bazı varsayımlarından ve gerekliliklerinden kaynaklanmaktadır. Klasik metotlar karşılaştırma matrislerinin eksiksiz olduğunu varsayar ve tutarlı olmasını şart koşar. Halbuki gerçek hayatta bu koşulların sağlanması genellikle mümkün olmamaktadır.

Bu çalışmada, AHS-GKV ve ASS-GKV yöntemleri için Bayesci önceliklendirme yöntemi önerilmektedir. Önerilen yöntem, bireysel kararların ön elemesini gerektirmemektedir. Önerilen yöntem eksik veya tutarsız cevapları olduğu durumlarda da kullanılabilmekte ve bu problemli durumlarda klasik yöntemlere göre daha tutarlı ağırlıklar ve daha düşük hata kareler ortalaması vermektedir. Yöntem, örnek olgu çalışmaları ile desteklenerek; AHS-GKV ve ASS-GKV yöntemlerinde grup kararlarının birleştirilmesinde kullanılan klasik metotlarla karşılaştırılmaktadır.

Anahtar Sözcükler: Grup kararları ve uzlaşma, Çok Kriterli Karar Verme (ÇKKV), Analitik Hiyerarşi Süreci (AHS), Analitik Serim Süreci (ASS), Bayesci önceliklendirme yöntemi.

CONTENTS

CHAPTER ONE
INTRODUCTION

1.1 Introduction

To be a person is to be a decision maker (Saaty, 2001). No matter who, how old or how educated a person is; he or she always has to decide about something. The process in which a decision maker selects, ranks or evaluates the alternatives depending on at least one objective can be defined as decision making process. According to this definition, decision making process is composed of a decision maker or decision makers, alternatives, criteria and the results of the decision process.

Decision theory is an interdisciplinary area of study which is studied by many practitioners and researchers in all branches of science, engineering and in all human social activities. In classical decision models, an optimal solution is selected from a set of alternatives according to a certain objective function. However, the decision problems in real life have conflicting objectives. As a result, the goal of decision-making process becomes finding some satisfactory solutions rather than selecting a single optimal solution. In order to handle such kind of decision making problems, the methodologies for solving multiple criteria decision-making (MCDM) problems have been emerged. Since 1960's many theories and methods have been developed in order to search for an optimal decision or solution.

1.2 Multi-Criteria Decision Making (MCDM)

The problems whose objective is to search the best alternative or to rank the alternatives in terms of a number of conflicting criteria are called multi-criteria decision making (MCDM) problems. It is hard to solve such kind of problems. Generally, no optimal solution exists for these problems, i.e. none of the alternatives can be concluded as the best one in terms of each criterion. An alternative can be the best one in terms of one criterion, where it can be worse in terms of the other criteria.

MCDM methodologies have been developed for more than fifty years; nevertheless, those methodologies appear to be quite diversified due to many changes in decision concepts. The MCDM techniques can be categorized into multiobjective decision making (MODM) and multiattribute decision making (MADM).

In the MODM, an alternatives set is defined with a set of constraints to be satisfied, which result in a large set of decision choices for the decision maker. As a result, the MODM models study decision problems with continuous solution spaces (Levy, 2005). It assumes that the problem can be modeled as a mathematical programming model. That is why it sometimes is referred as multiple objective mathematical programming (MOMP). These problems are often formulated and then solved as linear, integer, or nonlinear mathematical programming problems. However, most of the real world MCDM problems cannot be solved by MODM models.

On the other hand, MADM is applied to a discrete set of explicit alternatives (finite and usually small) (Levy, 2005). In MADM problems, the highest objective is usually a broadly defined goal which may be broken down into a hierarchy of criteria or objectives, with the lower levels becoming more detailed and measurable, but more conflicting. Performance indicators (also referred as criteria or attributes) measure the degree of which these objectives are achieved.

MADM does not try to compute an optimal solution but tries to determine via various ranking procedures (Brito et al., 2007). The aim here is to evaluate and rank performance of a finite set of alternatives in terms of a number of decision criteria. The problem is how to rank the alternatives when all decision criteria are considered simultaneously.

The most widely used methods, such as the SAW (Simple Additive Weighting) method (Fishburn, 1967), the WPM (Weighted Product Model) method (Bridgman, 1922; Miller, 1969), the ELECTRE (Elimination Et Choix Traduisant la Realite) method (Benayoun et al., 1966), the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method (Hwang & Yoon, 1980), the PROMETHEE (Preference Ranking Organization methods for Enrichment Evaluations) methods (Brans et al. 1984, 1985), the VIKOR (Vlse Kriterijumska Optimizacija Kompromisno Resenje) method (Opricovic, 1998), the Analytic Hierarchy Process (AHP) (Saaty, 1980), and the Analytic Network Process (ANP) (Saaty, 1996) are described in the following chapter.

Out of those methods, the Analytic Hierarchy Process (AHP) and the Analytic Network Process (ANP), proposed by Thomas L. Saaty (1980, 1996) are widely used descriptive approaches in multi-criteria group decision making. They both allow multiple actors, criteria and scenarios to be involved in the analysis. However, the conventional procedures used in group decisions for these methods have some limitations. They assume the pairwise comparison matrices containing the decision makers' judgements are complete and accurate. However, especially for large problems (including large numbers of clusters and elements) there might be incomplete matrices including empty positions due to various reasons. Some methodological developments have been aroused in the literature in order to overcome the limitations of classical aggregation methods in group judgements. For example, aggregation methods with linear programming (Mikhailov, 2004) and Bayesian approach (Altuzarra et al., 2007) have been proposed in order to make a decision even when comparisons are missing, for example when a stakeholder does not feel to have the expertise to judge a particular comparison (Ishizaka & Labib, 2011).

1.3 Objective of the Dissertation

Bayesian approaches allow the treatment of missing data or incomplete information using data augmentation techniques (Tanner and Wong, 1987). The integration of high-dimensional functions was the major limitation towards the wide application of Bayesian analysis before Markov Chain Monte Carlo (MCMC) methods have been introduced. Up to time, the Bayesian analyses have not been widely used in the AHP literature. Alho and Kangas (1997) provided a Bayesian extension of their regression formulation of the AHP. Basak (1998) used MCMC methods to calculate the posterior distributions of judgements and estimated the vector of priorities and the most likely rankings. Altuzarra et al. (2007) provided a Bayesian prioritization procedure (BPP) for AHP group decision making.

In this dissertation, our aim is to apply the Bayesian prioritization approach for the AHP in a multi-criteria complex group decision problem and then to extend the Bayesian prioritization approach to a more general approach to decisions, which is a generalization of hierarchies to networks with dependence and feedback, the ANP.

This method also can be extended to the case of incomplete and inconsistent pairwise comparison matrices, which are the common problems in complex decision making problems. The methodology is illustrated by the analysis of two case studies and compared with one of the conventional prioritization procedures.

The Bayesian method could also be applied for any other MCDM approaches applied in group decision making (GDM), such as ELECTRE, TOPSIS etc., but they are not presented in this dissertation since other methods are out of our scope.

The remainder of this dissertation is as follows: In the following chapter, the most widely used MCDM methods are briefly described and some comparisons will be given. In the third chapter, theoretical background of the AHP-GDM is briefly given and the Bayesian prioritization procedure (BPP) based AHP is presented. A real life example illustrates the methodology and the main results of this chapter are given with the concluding remarks. In chapter four, the theoretical background of the ANP-GDM is briefly given and the proposed methodology, Bayesian prioritization procedure (BPP) based ANP is presented. The proposed methodology is illustrated using a practical case study and the main results of the chapter are given with the concluding remarks. In the final chapter, the conclusions which include total results and future research directions are provided. Each chapter is organized to include its own literature review, statistical model, formulas, a complex group decision problem, the simulated dataset, the statistical results and the conclusions.

In this dissertation, Microsoft Office Excel 2007 and R, which is a free software environment for statistical computing and graphics, are used for all of the calculations and the graphics.

CHAPTER TWO
MULTI-CRITERIA DECISION MAKING METHODS

2.1 Introduction

Multi-criteria decision making analysis aims to search the best alternative or to rank the alternatives in terms of a number of conflicting criteria. It is usually hard to solve such kind of problems and no optimal solution exists for these problems, i.e. none of the alternatives can be concluded as the best one in terms of each criterion. One of the alternatives can be the best in terms of one criterion, where it can be worse in terms of the other criteria (Yaralıoğlu, 2010).

MCDM methodologies have been developed for more than fifty years; nevertheless, those methodologies appear to be quite diversified due to many changes in decision concepts. The MCDM techniques can be classified as: multiobjective decision making (MODM) and multiattribute decision making (MADM).

The processes involved in the multiple criteria decision making can be characterized as making preferenced decisions through evaluation, prioritization or selection of alternatives in the presence of multiple, usually conflicting criteria.

A wide variety of MCDM techniques have been developed. The most widely used methods are: the SAW (or WSM), the WPM, the ELECTRE, the TOPSIS, the PROMETHEE, the VIKOR, the AHP, and the ANP methods (Yaralıoğlu, 2010).

Independent of the method being used, the multi-criteria decision making process usually involves a numerical analysis of the alternatives that can be classified into four main steps (Trianthaphyllou, 2000). These steps typically comprise:

(i) Determination of the relevant criteria,
(ii) Assignment of numerical measures to the alternative's performance and relative importance of the criteria,
(iii) Aggregation process and finally
(iv) Determination of ranking for each alternative on the basis of numerical values obtained from the previous steps.

Regarding this issue, Chen and Hwang (1991) provided a taxonomy of the MCDM methods suitable for application under certain conditions, as described in the Figure 2.1.

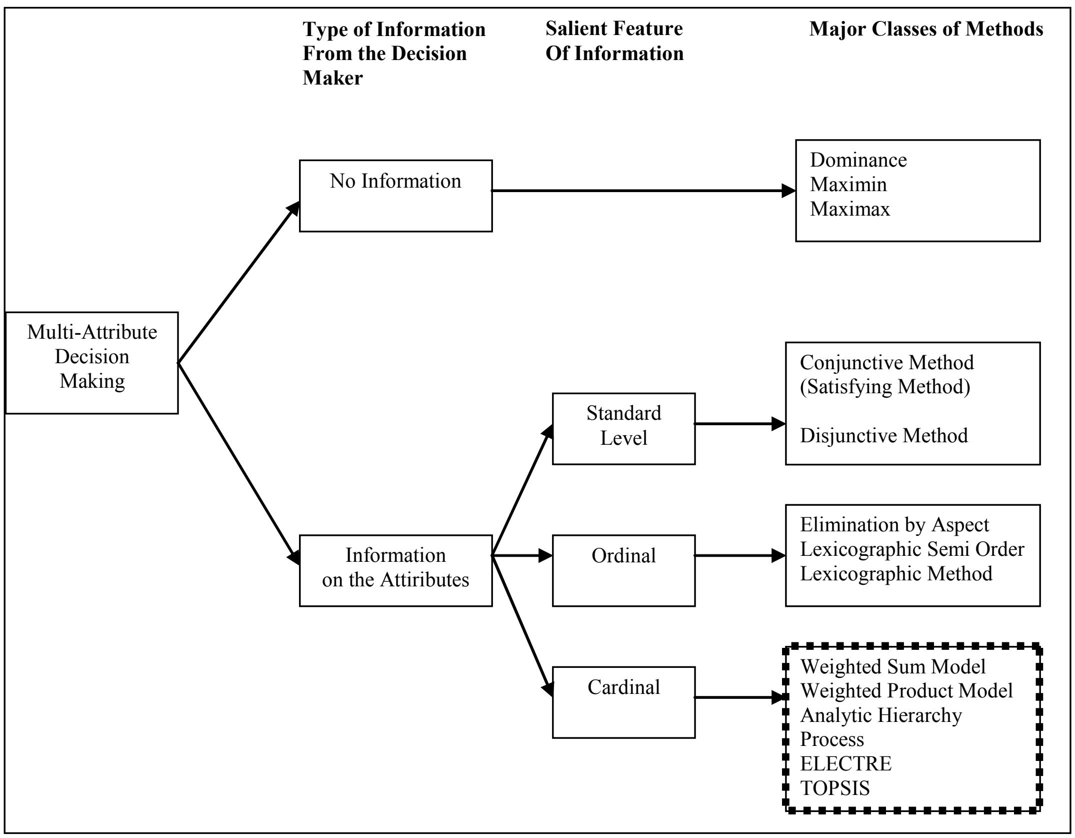


Figure 2.1 A taxonomy of MCDM methods.

We are dealing with the decision problems whose attributes could be identified and in which the alternatives could be attained a cardinal score out of those predefined attributes. The methods used in these types of decision problems are given in the bottom of the figure.

Basically, these methods work with the same fundamental tool: the decision matrix. Table 2.1 shows a decision matrix used in a situation involving three alternatives and five different criteria. In a decision matrix, the a_{ij} is the performance of alternative i according to criterion j. The problem of MCDM is how to rank alternatives when all the decision criteria are to be considered simultaneously. In principle, once the aggregated scores are determined, the ranking order of alternatives can be automatically decided (Trianthaphyllou, 2000).

Table 2.1 Decision matrix.

	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5
Alternative 1	a_{11}	a_{12}	a_{33}	a_{14}	a_{15}
Alternative 2	a_{12}	a_{22}	a_{23}	a_{24}	a_{25}
Alternative 3	a_{13}	a_{32}	a_{33}	a_{34}	a_{35}

Methods like the members of ELECTRE family only provide the sorting of the alternatives (in this case, a dominance principles based ranking). Others methods also provide performance measurements for all alternatives according every criterion and alternatives sorting based on these performances.

2.2 Multi-Criteria Decision Making Methods

In this section, some commonly used MCDM methods representing different evaluation principles will be briefly reviewed. These evaluation principles consist of the selection of an alternative which has the largest utility value (SAW and WPM), the arrangement of a set of overall preference rankings which best satisfy a given concordance measure (ELECTRE), the selection of an alternative which has the maximum value of linear preference function (PROMETHEE), the selection of an alternative which has the largest relative closeness to the ideal solution (TOPSIS, VIKOR), and prioritization of the alternatives by making paired comparisons in terms of each alternative (AHP and ANP).

2.2.1 The Simple Additive Weighted Method (SAW)

This method is also known as Weighted Sum Method (WSM) (Fishburn, 1967) and is probably the most commonly used MCDM approach, particularly in dealing with a single dimensional problem. This approach is based on the hypothesis that in any decision problem, there exists a real utility function defined by the set of feasible actions, which the decision maker wishes to evaluate.

The method is characterized by the additive utility assumption, referring to the total value of each alternative being equal to the sum of the products of the criteria ratings and their weight from the respective alternatives. To determine the best alternatives among a discrete number of alternatives, the steps are as follows:

(i) The weights of each attribute are determined.
(ii) Each alternative is given a score in terms of each criteria
(iii) The total value of each alternative is calculated by taking the sum of the products of the criteria ratings and their weight from the respective alternatives.

The simplicity of this method makes it widely used by practitioners. However there are some limitations:

(i) It can be used only in single dimensional problems.
(ii) It requires that the attribute values and the corresponding weight must both be numerical and comparable.
(iii) The attributes are preferentially independent, meaning that the contribution of an individual attribute to the global score is independent of another attribute's values.

2.2.2 The Weighted Product Method (WPM)

The WPM (Bridgman, 1922; Miller, 1969) method is similar to WSM but it uses multiplicative model instead of additive and could be used both in single and multi-dimensional decision problems.

The basic steps of this method can be given as follows:

(i) The weights of each attribute are determined.
(ii) Each alternative is given a score in terms of each criterion which must be greater than 1.
(iii) The total value of each alternative is calculated by taking the products of the alternatives' criteria ratings where the weights of the corresponding criteria are their exponents.

One of the advantages of applying this method is its structure which eliminates any unit of measurement by employing the relative value in terms of the ratio of the respective criteria to the ideal value instead of the actual value. Last two limitations are the same as WSM method.

2.2.3 The ELECTRE Method

The basic concept of the ELECTRE (Elimination and Choice Translating Reality) (Benayoun et al., 1966) method is how to deal with outranking relation by using pairwise comparisons among alternatives under each of the criteria separately. It compares two alternatives at a time and selects one over the other if one alternative is better in most criteria and not acceptably worse in the remaining criteria. An alternative is dominated if there is another alternative that outranks it at least in one criterion and equals it in the remaining criteria.

The ELECTRE method consists of a pairwise comparison of alternatives based on the degree to which evaluation of the alternatives and preference weight confirms or contradicts the pairwise dominance relationship between the alternatives. The decision maker may declare that s/he has a strong, weak, indifference or even be unable to express his or her preference between two compared alternatives.

The steps of this method can be written as:

(i) Calculate the normalized decision matrix.
(ii) Calculate the weighted normalized decision matrix.
(iii) Determine the concordance and discordance set.
(iv) Calculate the concordance matrix.
(v) Calculate the discordance matrix.
(vi) Determine the concordance and discordance dominance matrix.
(vii) Determine the aggregate dominance matrix.
(viii) Eliminate the less favorable alternatives. The best alternative is the one that dominates all the other alternatives in this manner.

First the normalized decision matrix is calculated and then a partial preference ordering of the alternative can be derived from the aggregate dominance matrix. The best alternative is the one that dominates all the other alternatives.

2.2.4 The TOPSIS Method

Hwang and Yoon in 1980 first developed a Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) as an alternative to the laborious ELECTRE method. The logic and basic principle behind this concept are that the most preferred alternative is not only the shortest Euclidean distance from the 'ideal' solution, but also the farthest from the undesirable solution (nadir point), across all criteria simultaneously.

The assumption of this method is that each attribute involved in decision making takes either monotonically increasing or monotonically decreasing utility. The method is simple and comprehensible. The method is able to measure the relative performance of the decision alternatives with a high computational efficiency, due to a minimum numerical calculation. Furthermore, the TOPSIS method delivers performance ratings and the weights of the criteria in the form of crisp values that facilitate a comparison of the available alternatives.

The steps of TOPSIS can be described as follows:

(i) Calculating the normalized and weighted normalized decision matrix.
(ii) Determining the ideal and negative ideal solution for each criterion.
(iii) Calculating the separation measures.
(iv) Calculating the relative closeness of an alternative to the ideal solution.

The ranking of the alternatives can be obtained by ordering the performance index in descending order. The larger the performance index, the more preferred the alternative is.

2.2.5 The PROMETHEE Method

The PROMETHEE (Preference Ranking Organization Methods for Enrichment Evaluations) method is developed by Brans et al. (1984, 1985) for solving multi-criteria problems. PROMETHEE methods belong to the outranking methods consisting in enriching the dominance order. They include five phases:

(i) Composing the evaluation matrix which presents the performance of each alternative in relation to each criterion.
(ii) Comparing the alternatives pairwisely with respect to every single criterion. (Here, the results are expressed by the preference functions, which are calculated for each pair of options and can range from 0 to 1. 0 means that there is no difference between the pair of options, 1 indicates a big difference).
(iii) Assigning a preference function.
(iv) Estimating the outranking degree of the options.
(v) Determining weights to criteria and choosing a preference function by decision makers.

Various PROMETHEE tools and modules (such as PROMETHEE-I for partial ranking, PROMETHEE-II for complete ranking and GAIA plane for visualisation) have been developed so far.

2.2.6 The VIKOR Method

The VIKOR method (Vlse Kriterijumska Optimizacija Kompromisno Resenje) was proposed by Opricovic (1998), which is a multi-attribute decision making method for complex system based on ideal point method. VIKOR method uses linear normalization and it proposes a compromise solution with an advantage rate.

(i) The compromise ranking algorithm VIKOR has the following steps:
(ii) Representation of Normalized Decision Matrix.
(iii) Determination of Ideal and Negative-Ideal Solutions.
(iv) Calculation of Utility Measure and Regret Measure.
(v) Computation of VIKOR Index.

The alternative having smallest VIKOR value is determined to be the best solution. This method is a distance-based method like the TOPSIS method.

2.2.7 The AHP and the ANP Methods

The AHP (Saaty, 1980) method simplifies the problem of constructing hierarchic structures which comprise a goal, criteria, and alternatives. It assumes that the factors presented in the hierarchical structure are independent. However, many decision problems involve the interaction and dependence of higher-level elements in a hierarchy on lower-level elements and dependence of elements within a level therefore cannot be structured hierarchically (Saaty, 1999).

In the case of dependence and feedback, the ANP (Saaty, 1996) is used instead of the AHP. It is the generalized version of the AHP. It allows interactions and feedback within and between clusters. It generalizes on the supermatrix approach proposed by Saaty (1980). With the ANP, one constructs feedback networks, then makes judgments or performs measurements on pairs of elements with respect to a controlling element to derive relative absolute scales that are then synthesized throughout the structure to select the best alternative. The criteria are pairwise compared with respect to the goal, the subcriteria with respect to their parent criterion, and the alternatives of choice with respect to the last level of subcriteria above them (the covering criteria). Each set of comparisons yields an absolute scale of priorities. An absolute scale is a special instance of a ratio scale with a constant multiplier equal to one.

In the ANP, not only does the importance of the criteria determine the importance of the alternatives as in a hierarchy, but also the importance of the alternatives themselves determines the importance of the criteria (Saaty, 2005). These two methods both derive ratio scale priorities for elements and clusters of elements by making pairwise comparisons of elements on a common criterion or property.

In our study, our focus is the AHP and its generalized form, the ANP. The main reason why we are focusing on these two approaches and more detailed information about these two methods will be given in the following sections. Our aim is to search for the limitations of these two methods and search for some statistical configuration and treatment for them.

2.3 Comparative Studies of the MCDM Methods

In this part, three comparative studies for the mentioned methods are given. These studies are the ones which deal with many methods at a time for one certain problem.

Santana (1996) conducted a comparative study on the methods AHP, ELECTRE and TOPSIS for choosing a new automobile plant in the Brazilian state of Santa Catarina. The alternatives were the cities of Joinville, Blumenau and Imbituba; the criteria were the conditions of infrastructure, transportation facilities, local labour capability, basis industries potential and installed capacity expansion potential. In order to apply the ELECTRE and TOPSIS methods, experts from Regional Development Banks were consulted and for every criterion weights were obtained as

0.20, 0.25, 0.20, 0.25 and 0.10. Those experts have also provided the values showed on Table 2.2.

According to a concordance principle the ELECTRE method concludes that the alternative Joinville dominates the others. But, it does not make any other consideration regarding the other alternatives. Here, the only definitive conclusion is that the option Joinville is the most attractive one.

For the TOPSIS method the location to be selected is the one with the lower distance to the ideal solution (A+) and, simultaneously, the bigger distance to undesirable solution (A–). In Table 2.2 it can be observed that:

A+ = [2, 5, 5, 3, 5]
A– = [1, 3, 2, 1, 3]

Table 2.2 Decision matrix for ELECTRE and TOPSIS.

	Infrastructure	Transportation	Labour capability	Basis industries	Capacity expansion
Joinville	2	3	5	3	3
Blumenau	2	3	4	2	3
Imbituba	1	5	2	1	5

The alternative that provides the higher prioritization coefficient must be selected. The prioritization coefficient is the ratio of the distance to the ideal solution to the sum of two distances (distance to the ideal solution + distance to the undesirable solution).

Table 2.3 verifies that, according to TOPSIS, Joinville will be the most attractive location, because it assures the lower distance to ideal solution and, simultaneously, the bigger distance to the undesirable solution. Blumenau will be the second choice given the set of criteria.

Table 2.3 Prioritization of alternatives according to TOPSIS.

	Distance to ideal solution	Distance to ideal solution+ Distance to undesirable solution	Prioritization coefficient
Joinville	0.70	1.40	0.67
Blumenau	1.05	1.05	0.50
Imbituba	1.40	0.70	0.33

In the AHP utilization, there is a single expert. The criteria (infrastructure, transportation facilities, local labour capability, basis industries potential, installed capacity expansion potential) have obtained the following weights: 0.14, 0.34, 0.14, 0.34 and 0.04. The consistency index from the judgments among the criteria was 0.0386 and since it is less than 0.10, it is accepted as consistent. This way the prioritization of the alternatives was configured as shown in Table 2.4, i.e., again Joinville will be the most attractive location. However, this time followed by Imbituba.

Table 2.4 Prioritization of alternatives according to AHP.

	Global Priority
Joinville	0.39
Blumenau	0.27
Imbituba	0.34

Santana (1996) had considered that “by the fact of the AHP assure the consistency analysis of the judgments, the Saaty’s model means, a priori, more robust than the others two”. The TOPSIS was considered the simplest of the studied methods.

Zanakis et al. (1998) also made a comparative study in which they compared the performances of five methods: ELECTRE, TOPSIS, WPM, SAW, and four versions of AHP (original vs. geometric scale and right eigenvector vs. mean transformation solution). They took the SAW method as the basis to compare the other methods, because of its simplicity and acceptability. According to their results, all versions of the AHP method behave similarly and closer to SAW than the other methods. ELECTRE is the least similar to SAW (except for closer matching the top-ranked alternative), followed by WPM. TOPSIS behaves closer to AHP and differently from ELECTRE and WPM, except for problems with few criteria.

Chu et al. (2007) made another comparison study for three methods: TOPSIS, VIKOR and SAW. According to their results, TOPSIS and SAW had identical rankings overall, but TOPSIS had better distinguishing capability. TOPSIS and VIKOR had almost the same success setting priorities by weight. However, VIKOR produced different rankings than those from TOPSIS and SAW. They also concluded that choosing appropriate strategies with VIKOR is easy.

Brans et al. (1986) also showed that PROMETHEE is more stable than ELECTRE. Macharis et al. (2004) made a comparison between PROMETHEE and AHP, which showed PROMETHEE has some strength of various approaches. A number of papers combined PROMETHEE with AHP. The final ranking of alternatives in this integration was done by PROMETHEE and the importance of criteria was determined by AHP.

We have conducted a literature comparison study and searched ISI Web of Knowledge, which is an academic citation indexing and searching service. It is combined with web linking and provided by Thomson Reuters. The following figures are obtained through its database. The figures present the frequency of published items on the left side and the frequency of citations on the right since year 2000 for the methods AHP-ANP, TOPSIS, ELECTRE, PROMETHEE and VIKOR respectively. We should think the AHP and the ANP methods as the same method here. Because if there is no dependence structure in the problem, the practitioner would use the AHP; whereas if there exist a dependence or feedback structure between the clusters he would then use the ANP, which is the general case of the AHP. The Figures 2.2-2.6 indicate that the AHP-ANP methods have been attracting popularity throughout the years.

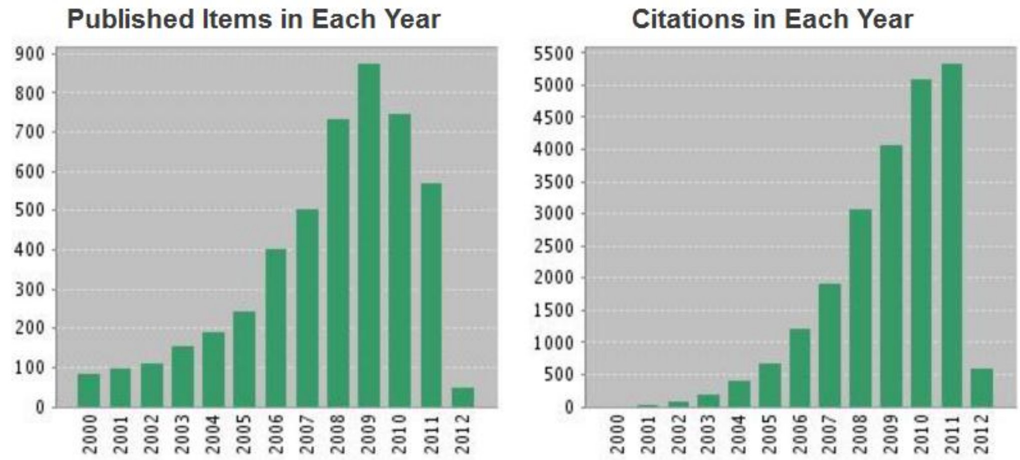


Figure 2.2 Published items and citations of the AHP and ANP since 2000.

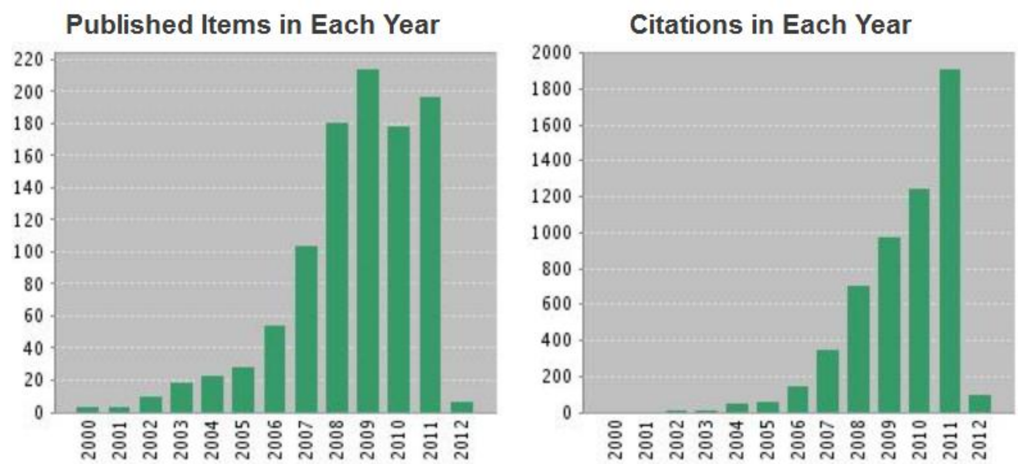


Figure 2.3 Published items and citations of the TOPSIS method since 2000.

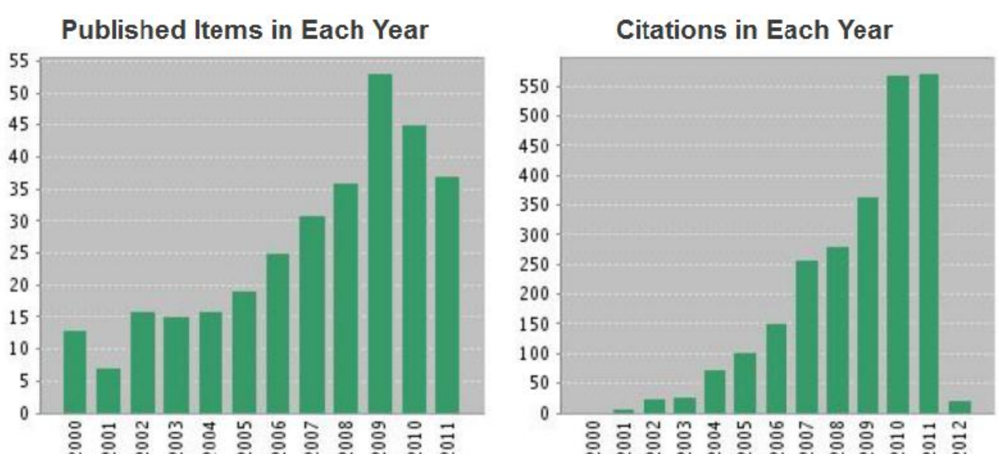


Figure 2.4 Published items and citations of the ELECTRE method since 2000.

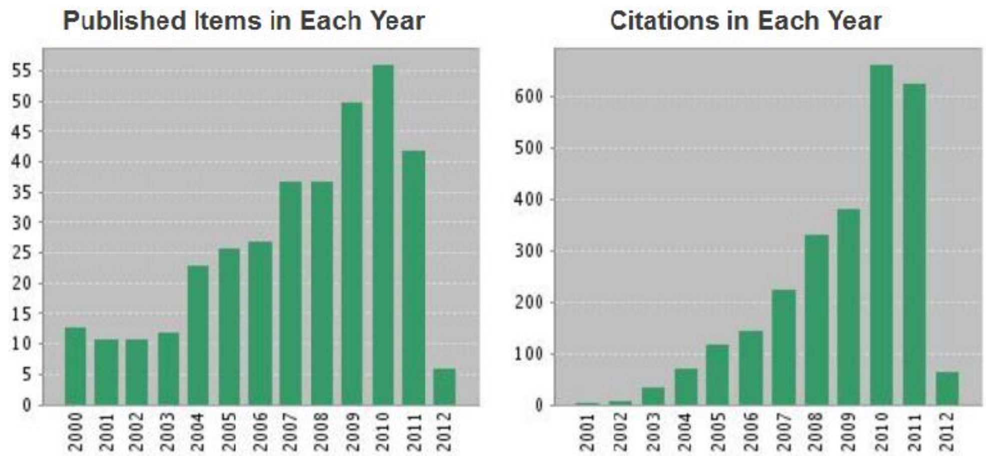


Figure 2.5 Published items and citations of the PROMETHEE method since 2000.

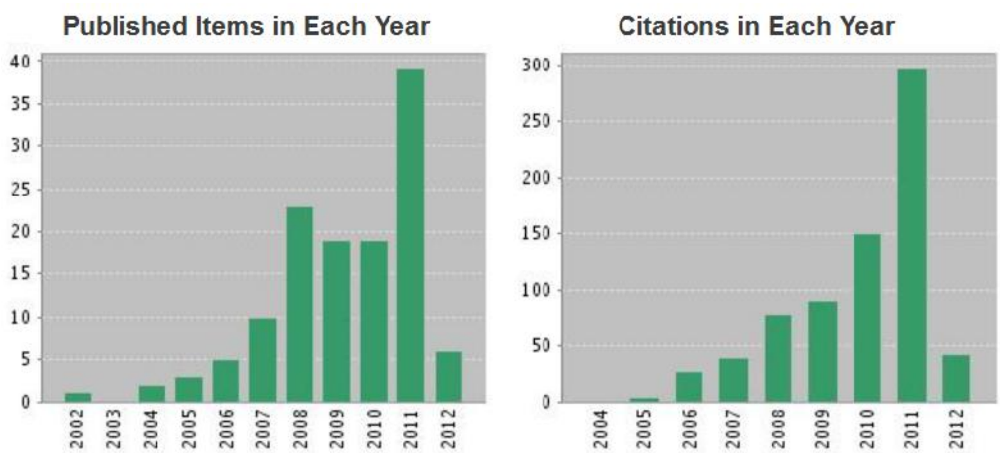


Figure 2.6 Published items and citations of the VIKOR method since 2000.

It is not argued that the AHP-ANP methods are the best MCDM methods but it is obvious that they are the most popular ones. The authors might have cited the methods

(i) positively because of their superiorities,
(ii) negatively because of their limitations or
(iii) they might just have used the method for their purpose.

Whatever the reason is, the AHP together with the ANP methods have more than 5000 citations for each of the last two years (2010 and 2011).

In our study, our focus is the AHP and its generalized form, the ANP. The main reason why we are focusing on these two approaches is that the AHP is one of the most widely used MADM tool in the literature. The superiority of these two methods is that the consistency of the decision makers can be calculated. These methods capture all kinds of comparisons which makes them a more comprehensive approach than others. Moreover, the ANP is a powerful approach since it is the only method that can capture dependencies and the feedback in the problem, if exist.

CHAPTER THREE
BAYESIAN PRIORITIZATION PROCEDURE BASED ANALYTIC HIERARCHY PROCESS

3.1 Introduction

The Analytic Hierarchy Process (AHP) proposed by Thomas L. Saaty (1980) is a widely used descriptive approach in multi-criteria decision making. It deals with problems which involve consideration of multiple criteria simultaneously. It allows multiple actors, criteria and scenarios to be involved in the analysis. It has been extensively applied in complex decision-making problems of choice, prioritization and evaluation. Its ability to synthesize both tangible and intangible characteristics, to accommodate both shared and individual values and monitor the consistency with which a decision-maker makes his judgements made the AHP a widely used multiple criteria decision making (MCDM) tool (Dyer & Forman, 1992).

The AHP simplifies the problem of constructing hierarchic structures which comprise a goal, criteria, and alternatives. It assumes that the factors presented in the hierarchical structure are independent. The AHP has particular applications in individual and group decision making. According to many researchers AHP is an effective and flexible tool for structuring and solving complex group decision problems (Altuzarra et al., 2007; Ramanathan & Ganesh, 1994; Dyer & Forman, 1992).

There are different methods to accommodate the judgements of decision makers in a group setting. Saaty (1989) suggests one of two methods to proceed:

(i) Decision makers make each paired comparison individually.
(ii) The group is required to achieve consensus on each paired comparison.

If individual's paired comparison ratio judgments are gathered, the literature describes different methods for the prioritization and synthesis procedures (Saaty, 1989; Crawford & Williams, 1985; Aguarón & Moreno-Jiménez, 2000). The two conventional procedures to obtain group priorities are “the aggregation of individual judgements (AIJ)” and “the aggregation of individual priorities (AIP)”.

Based on individual judgements, a new judgement matrix is constructed for the group as a whole in AIJ procedure and the priorities are computed from the new matrix. If the individuals are experts, they may not wish to combine their judgements but only their final outcomes obtained by each from their own network (Saaty, 2008). In that case, the AIP method can be used, and the total priorities are obtained on the basis of individual priorities using an aggregation procedure. Synthesis of the model can be done using any of the aggregation procedures. The weighted geometric mean method is the most commonly used technique for both (Saaty, 1983).

One of the limitations of these conventional procedures is that they assume the pairwise comparison matrices containing the decision makers’ judgements are complete and accurate. However, especially for large problems (including large numbers of attributes and alternatives) there might be incomplete matrices including empty positions. According to Kim and Ahn (1997), the reasons to the incomplete information are as follows:

(i) A decision might be made under time pressure and lack of data,
(ii) Many of the attributes might be intangible or non-monetary because they reflect social and environmental impacts,
(iii) Decision maker might have limited attention and information processing capabilities,
(iv) All participants might not have equal expertise about the problem domain in group settings.

Methodological developments have been emerged in the literature in order to overcome the limitations of classical aggregation methods in group judgements. For instance, Mikhailov (2004) proposed aggregation methods with linear programming and Altuzarra et al. (2007) proposed a Bayesian approach in order to make a decision even when comparisons are missing, for example when a stakeholder does not feel to have the expertise to judge a particular comparison (Ishizaka & Labib, 2010).

Bayesian approaches allow the treatment of missing data or incomplete information using data augmentation techniques (Tanner & Wong, 1987). The integration of high-dimensional functions was the major limitation towards the wide application of Bayesian analysis before Markov Chain Monte Carlo (MCMC) methods have been introduced.

There are very few references to Bayesian analysis in the AHP literature. Alho and Kangas (1997) provided a Bayesian extension of their regression formulation of the AHP. Basak (1998) used MCMC methods to calculate the posterior distributions of judgements and estimated the vector of priorities and the most likely rankings. Altuzarra et al. (2007) provided a Bayesian prioritization procedure (BPP) for AHP group decision making that does not require filters for the initial judgements of the decision makers. Contrary to the conventional prioritization methods applied in AHP-GDM (Saaty, 1989; Ramanathan & Ganesh, 1994; Forman & Peniwati, 1998) this technique does not require intermediate filters for decision makers' initial judgements. This approach provides more efficient and robust estimates than the classical prioritization methods applied in AHP-GDM.

In this chapter, we aim at providing an effective and practical group decision mechanism to prioritize the alternatives. We propose using BPP based AHP-group decision making (GDM) for complex multi-criteria decision problems, which allows a group of people to participate in the analysis. This approach provides flexibility to the group of participants, when expressing their judgements, and to the AHP practitioner, who may not be professional, by treating incomplete or inconsistent judgements properly. This technique can be used alone or with any other decision support systems.

This chapter is organized as follows: Section 3.2 gives the relevant theoretical background of the AHP-GDM approach and the Bayesian prioritization procedure for the AHP-GDM. In Section 3.3, an illustrative example is provided to show how the proposed method can be implemented in an information security risk assessment problem. The main results of the illustrative example are also given here. Finally, Section 3.4 summarizes the conclusions obtained from this study.

3.2 The AHP Group Decision Making (AHP-GDM)

The AHP was developed by Saaty (1980) in order to deal with problems which involve consideration of multiple criteria simultaneously. It has been extensively applied in complex decision-making problems of choice, prioritization and evaluation. Its ability to synthesize both tangible and intangible characteristics, to accommodate both shared and individual values and monitor the consistency with which a decision-maker makes his judgements made the AHP a widely used multiple criteria decision making (MCDM) tool (Dyer & Forman, 1992). The AHP has particular applications in individual and group decision making (Basak & Saaty, 1993). According to many researchers AHP is an effective and flexible tool for structuring and solving complex group decision situations (Altuzarra et al., 2007; Ramanathan & Ganesh, 1994; Dyer & Forman, 1992).

3.2.1 Steps of the AHP Analysis

The AHP comprises of four stages: modeling, valuation, prioritization and synthesis. In the modeling stage, a hierarchy which describes the problem is constructed. As presented in Figure 3.1, the overall goal or mission is placed at the top of the hierarchy. The main attributes, criteria and subcriteria are placed in the subsequent levels below. Finally, the alternatives are placed at the bottom of the hierarchy.

A hierarchy does not have to be complete, i.e., an element in a given level does not have to function as an attribute or a criterion for all the elements in the level below (Saaty, 1990). Similarly, there can be a hierarchy which does not have any alternatives layer. According to the type of the problem the model given in Figure 3.1 can be developed.

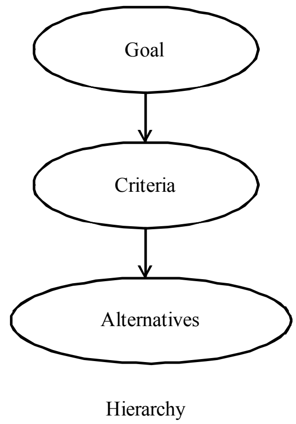


Figure 3.1 The AHP structure.

The Analytic Hierarchy Process (AHP) derives relative scales using decision makers' judgments or data from a standard scale, and performs the subsequent arithmetic operation on those scales. The judgments are given in the form of paired comparisons. Decision makers compare all the criteria with regard to goal and then all the alternatives with respect to each criterion in the evaluation stage. Their preferences are included as pairwise comparison matrices in the analysis and they are based on the fundamental scale (given in Table 3.1), proposed by Saaty (1980).

Table 3.1 The fundamental scale for pairwise comparisons.

Intensity of Importance	**Definition**
1	Equal Importance
2	Weak
3	Moderate importance
4	Moderate plus
5	Strong importance
6	Strong plus
7	Very strong
8	Very, very strong
9	Extreme importance

The hierarchy allows decision makers to focus on their judgments separately on each of several criteria one by one by making them take a pair of elements and compare them on that single criterion without any concern for other criteria or other elements.

The pairwise comparisons comprise a set of matrices called “pairwise comparison matrices”. There are $n\,(n-1)/2$ judgments required to develop the set of matrices. Reciprocals are automatically assigned in each pair-wise comparison.

After all the pairwise comparisons are done, the consistency is determined by using the eigenvalue, $\lambda_{\max}$, to calculate the consistency index, CI, as follows:

$$CI = \frac{\lambda_{\max} - n}{n-1},$$

where n is the matrix size.

Judgment consistency can be checked by taking the consistency ratio (CR) of CI, with the appropriate value in Table 3.2. The CR is acceptable, if it does not exceed 0.10. If it is more, the judgment matrix is inconsistent. To obtain a consistent matrix, judgments should be reviewed and improved.

Table 3.2 Average random consistency.

Size of matrix	1	2	3	4	5	6	7	8	9	10
Random consistency	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

In the prioritization stage, the local priorities are derived by calculating the eigenvalues of the comparison matrix of each element. Then the sum is taken over all weighted eigenvector entries corresponding to those in the next lower level of the hierarchy and global priorities are derived using the hierarchic composition principle. In the last stage, the global priorities for each alternative are synthesized in order to get their total priorities.

3.2.2 Incomplete Pairwise Comparison Matrices

Most MCDM methods are based on the assumption that complete information about the model parameters (scores, attribute weights) are elicited as 'exact' point estimates (Salo & Hämäläinen, 2010).

However, in real life decision makers sometimes might provide only incomplete or inconsistent information. The reasons for the incomplete or inconsistent information in the AHP pairwise comparison matrices can be summarized as follows:

(i) Decision makers might have limited attention or limited time so they might skip some questions or might provide inconsistent answers.
(ii) Decision makers might have limited experience or information about the subject so they might hesitate to give exact answers.
(iii) Decision makers might have limited knowledge about the AHP assumptions or requirements.
(iv) As the number of elements increase in the model, it becomes a hard task to provide complete and consistent answers

The practitioner may also prefer to ignore the inconsistent or opposing judgements while keeping the consistent or homogeneous ones in order to increase the consistency or consensus among decision makers.

As a consequence, all of the decision makers may not express the $n(n-1)/2$ possible judgements in the reciprocal pairwise comparison matrix or may express inconsistent judgements. There are some methods proposed to overcome this problem (see Salo and Hämäläinen (2010) for more information).

3.2.3 Inconsistency in Pairwise Comparison Matrices

Compared to other MCDM methods, one of the superior characteristics of AHP is that it allows for the quantitative assessment of the decision makers' inconsistency when they are eliciting their judgements.

Escobar et al. (2004) discusses about inconsistency in group decision making and mentions that less attention has been given to this topic. In a group setting, the inconsistency of the group is smaller than the largest individual inconsistency, i.e., if judgement matrices given by each decision-maker have an acceptable inconsistency, then so has their aggregated complex judgement matrix (Escobar et al., 2004). The opposite condition does not hold so one has to be careful calculating the consistency index. The two most commonly used procedures in the AHP literature are the Consistency Ratio (Saaty, 1980) and the Geometric Consistency Index (Crawford & Williams, 1985; Aguarón et al., 2003).

Alho et al. (1996) provides a regression formulation of the AHP which allows the statistical decomposition of the variation in the judgements into three parts: The amount of variation between individuals, the inconsistency of the judgments and the residual error. They indicate that there is considerable variation in the judgements of various experts, and also considerable internal inconsistencies in individual judgements. Their results show that the expert judgements must be used with caution in the decision-making process.

3.3 Bayesian prioritization procedure (BPP) for AHP-GDM

Before the introduction of Markov Chain Monte Carlo (MCMC) methods, the integration of high-dimensional functions has been the major limitation towards the wide application of Bayesian analysis. Nowadays Bayesian approaches are widely used in the treatment of missing data or incomplete information.

One of the limited Bayesian studies in the AHP literature is the Bayesian prioritization procedure (BPP) by Altuzarra et al. (2007). They provided a for AHP group decision making that does not require filters for the initial judgements of the decision makers. This procedure is based on the prior assumption of the existence of consensus among the decision makers. Unlike the AIJ and the AIP methods, this process uses weightings that are inversely proportional to the decision makers' levels of inconsistency and is more efficient when compared to them. This method also can be extended to the case of incomplete pairwise comparison matrices, which is a common problem in complex decision making problems. For such cases, Altuzarra et al. (2007) showed that BPP performs much more robust manner than the conventional methods, especially with regard to consistency.

3.3.1 Statistical model for complete and consistent judgements

Assuming a single criterion, and a set of n alternatives, $\{A_1,...,A_n\}$, let $\mathbf{D}=\{D_1,...,D_r\}, r \geq 2$ be a group of r decision makers, each express individual pairwise comparisons with regard to the criterion considered, resulting in r reciprocal judgement matrices, $\{\mathbf{R}^{(k)}, k=1...,r\}$. Their preferences are based on the fundamental scale proposed by Saaty (1980). $\mathbf{R}^{(k)}=(r_{ij}^{(k)})$ is a positive square matrix $(n \times n)$ which validates:

$$(r_{ii}^{(k)})=1, \quad (r_{ij}^{(k)})=1\big/(r_{ji}^{(k)})>0 \text{ for } i,j=1...,n.$$

The judgements $r_{ij}^{(k)}$ represent the preference of the decision maker, D_k, when a comparison between A_i and A_j is required

Let

$$\{v_1^G,...,v_n^G\}, v_i^G \geq 0$$

and

$$\{w_1^G,...,w_n^G\}, w_i^G = v_i^G \Big/ \sum_{j=1}^{n} v_i^G$$

be the group's unnormalized and normalized priorities for the alternatives, respectively.

As traditionally employed in stochastic AHP (Crawford & Williams, 1985; Alho & Kangas, 1997), a multiplicative model with log-normal errors is applied in the Bayesian analysis of the model. If the decision makers express all possible judgements, the model will be:

$$r_{ij}^{(k)} = \frac{v_i^G}{v_j^G} e_{ij}^{(k)}, \quad i, j = 1,\ldots,n, \quad k = 1,\ldots,r, \text{ with } e_{ij}^{(k)} \sim LN\left(0, \sigma^{(k)2}\right) \quad i < j.$$

Taking the logarithms and eliminating the reciprocal judgements, a regression model with normal errors is obtained given by:

$$y_{ij}^{(k)} = \mu_i^G - \mu_j^G + \varepsilon_{ij}^{(k)}, \quad i = 1,\ldots,n-1, \;\; j = 1,\ldots,n, \quad k = 1,\ldots,r, \quad \varepsilon_{ij}^{(k)} \sim N\left(0, \sigma^{(k)2}\right).$$

Here, A_n is established as the benchmark alternative ($\mu_n = 0 \Leftrightarrow v_n = 0$). In matrix notation, model can be written as:

$$\mathbf{y}^{(k)} = \mathbf{X}\boldsymbol{\mu}^G + \boldsymbol{\varepsilon}^{(k)}, \text{ with } \boldsymbol{\varepsilon}^{(k)} \sim N_t\left(0, \sigma^{(k)2}\mathbf{I}_t\right),$$

where

$$\mathbf{y}^{(k)} = (y_{12}^{(k)}, y_{13}^{(k)}, \ldots, y_{n-1n}^{(k)})', \text{ and } \mathbf{X}_{t \times n-1} = (x_{pq}),$$

with

(i) $x_{pi} = 1$, $x_{pj} = -1$ and $x_{p\lambda} = 0$, if $\lambda \neq i, j$, $\lambda = 1,\ldots,n-1$ and $p = \frac{2n-i}{2}(i-1) + (j-1)$ with $1 \leq i < j \leq n$,

(ii) $x_{pi} = 1$, $x_{p\lambda} = 0$, if $\lambda \neq i$, $\lambda = 1,\ldots,n-1$ and $p = \frac{2n-i}{2}(i-1) + (n-i)$,

and

$$\boldsymbol{\mu}^G = (\mu_1^G, \mu_2^G, ..., \mu_{n-1}^G)', \; k = 1,...,r, \; \boldsymbol{\varepsilon}^{(k)} = (\varepsilon_{12}^k, \varepsilon_{13}^k, ..., \varepsilon_{n-1n}^k)' \text{ and } t = n(n-1)/2.$$

With a constant non-informative distribution as the prior distribution for the vector of log-priorities, $\boldsymbol{\mu}^G$, the posterior distribution of $\boldsymbol{\mu}^G$ for complete and precise information is given by:

$$\boldsymbol{\mu}^G \mid \mathbf{y} \sim \mathrm{N}_{\text{n-1}}\left(\hat{\boldsymbol{\mu}}_B, \hat{\boldsymbol{\Sigma}}_B\right),$$

where

$$\hat{\boldsymbol{\mu}}_B = \frac{\sum_{k=1}^{r} \tau^{(k)} \hat{\boldsymbol{\mu}}^{(k)}}{\sum_{k=1}^{r} \tau^{(k)}},$$

$$\hat{\boldsymbol{\Sigma}}_B = \left(\sum_{k=1}^{r} \tau^{(k)}\right)^{-1} (\mathbf{X'X})^{-1} \begin{pmatrix} 2/n & 1/n & ... & 1/n \\ 1/n & 2/n & ... & 1/n \\ ... & ... & ... & ... \\ 1/n & 1/n & ... & 2/n \end{pmatrix},$$

$$\tau^{(k)} = 1/\sigma^{(k)2} \text{ and } \mathbf{y} = (\mathbf{y}^{(1)'}, \mathbf{y}^{(2)'}, ..., \mathbf{y}^{(r)'})'.$$

For the conventional procedure, AIP, the most commonly used method to aggregate group judgements is the geometric mean method. It can be presented as:

$$\hat{\boldsymbol{\mu}}_{AIP} = \frac{1}{r} \sum_{k=1}^{r} \hat{\boldsymbol{\mu}}^{(k)},$$

where

$$\hat{\boldsymbol{\mu}}^{(k)} = (\hat{\mu}_1^{(k)}, ..., \hat{\mu}_{n-1}^{(k)}) \text{ with } \hat{\mu}_i^{(k)} = \overline{y}_{i.}^{(k)} - \overline{y}_{n.}^{(k)}.$$

The other conventional procedure, AIJ, is not mentioned in this study since Altuzarra et al. (2007) showed that it gives almost the same results with the AIP method. Further information and theorems can also be found in their study.

3.3.2 Statistical model for incomplete or inconsistent judgements

Assuming the same conditions in the previous section, the model can be written as:

$$\mathbf{y}^{(k)} = \mathbf{X}\boldsymbol{\mu}^{G} + \boldsymbol{\varepsilon}^{(k)}$$

with

$$\boldsymbol{\varepsilon}^{(k)} \sim N_{t_k}\left(0, \sigma^{(k)2}\mathbf{I}_{t_k}\right),\ k = 1,\ldots,r\ ;$$

In the matrix form it can be expressed as:

$$\mathbf{y} = \mathbf{X}\left(\mathbf{1_r} \otimes \mathbf{I_{n-1}}\right)\boldsymbol{\mu}^{G} + \boldsymbol{\varepsilon}$$

with

$$\boldsymbol{\varepsilon} \sim N_t\left(0, \mathbf{D}\right),$$

where

$$\mathbf{y} = (\mathbf{y}^{(1)'}, \mathbf{y}^{(2)'}, \ldots, \mathbf{y}^{(r)'})',$$

$$\mathbf{X} = diag(\mathbf{X}^{(1)}, \mathbf{X}^{(2)}, \ldots, \mathbf{X}^{(r)}),$$

$$\boldsymbol{\varepsilon} = (\boldsymbol{\varepsilon}^{(1)'}, \boldsymbol{\varepsilon}^{(2)'}, \ldots, \boldsymbol{\varepsilon}^{(r)'})',$$

$\mathbf{D} = diag(\sigma^{(1)2}\mathbf{I}_{t_k},...,\sigma^{(r)2}\mathbf{I}_{t_k})$.

Here,

$\mathbf{1}_r = (1,1,...,1)'$, t_k is the number of judgements issued by each decision maker,

D_k, $t = t_1 + ... + t_r$ is the total number of judgements by all decision makers, and

$\otimes$ denotes the Kronecker product.

With a constant non-informative distribution as the prior distribution for the vector of log-priorities $(\boldsymbol{\mu}^G)$, the posterior distribution of $\boldsymbol{\mu}^G$ for incomplete and precise information is given by:

$$\boldsymbol{\mu} \mid \mathbf{y} \sim \mathrm{N}_{\text{n-1}}\left(\hat{\boldsymbol{\mu}}_B, \hat{\boldsymbol{\Sigma}}_B\right),$$

where

$$\begin{aligned}\hat{\boldsymbol{\mu}}_B &= \left(\sum_{k=1}^{r} \tau^{(k)} \mathbf{X}^{(k)}{}' \mathbf{X}^{(k)}\right)^{-1} \left(\sum_{k=1}^{r} \tau^{(k)} \mathbf{X}^{(k)}{}' \mathbf{y}^{(k)}\right)^{-1} \\ &= \left\{(\mathbf{1}_r \otimes \mathbf{I}_{n-1})\left(\mathbf{X}'\mathbf{D}^{-1}\mathbf{X}\right)(\mathbf{1}_r \otimes \mathbf{I}_{n-1})\right\}^{-1} (\mathbf{1}_r \otimes \mathbf{I}_{n-1})\left(\mathbf{X}'\mathbf{D}^{-1}\mathbf{y}\right),\end{aligned}$$

$$\hat{\boldsymbol{\Sigma}}_B = \left(\sum_{k=1}^{r} \tau^{(k)} \mathbf{X}^{(k)}{}' \mathbf{X}^{(k)}\right)^{-1}.$$

The estimator of μ^G obtained by means of the AIP procedure is given by:

$$\hat{\boldsymbol{\mu}}_{AIP} = \frac{1}{r}\sum_{k=1}^{r} \hat{\mu}^{(k)} = \frac{1}{r}\sum_{k=1}^{r} \left(\mathbf{X}^{(k)}{}'\mathbf{X}^{(k)}\right)^{-1} \left(\mathbf{X}^{(k)}{}'\mathbf{y}^{(k)}\right) \frac{1}{r}(\mathbf{1}_r \otimes \mathbf{I}_{n-1})(\mathbf{X}'\mathbf{X})^{-1}(\mathbf{X}'\mathbf{y}).$$

3.4 A Real World Example

In this section, both the AIP and the BPP methods are applied in AHP-GDM analysis of the same example and the results are compared.

3.4.1 Introduction

Information security risk management is a recurrent process of identification, assessment and prioritization of risks, where risk could be defined as a possibility that a threat exploits a particular vulnerability in an asset and causes damage or loss to the asset. Risk management has two primary activities, risk assessment and risk control. Risk assessment is a very important decision mechanism which identifies the information security assets that are vulnerable to threats, calculates the quantitative or qualitative value of risk (or expected loss), and prioritizes risk incidents.

In an organization, in the past, a single manager was used to be the responsible staff to protect information systems where, nowadays, a group of managers could take the responsibility of this task or participate in the risk analysis process. As risk analysis becomes a cross-functional decision making process, researchers seek ways to develop new risk analysis methods which allow a group of people to participate.

Although risk is well defined and practical for decision making, it is often difficult to calculate a priori (Sommestad et al., 2010). Due to the difficulty in adapting complex risk analysis tools in today's information systems, researchers have proposed new techniques which are capable of analyzing information security risk properly. A number of quantitative and qualitative risk analysis methods have been developed.

The quantitative approaches use mathematical and statistical tools to represent risk as a function of the probability of a threat and the expected loss due to the vulnerability of the organization to this threat (Bodin et al., 2008; Feng & Li, 2011). Due to the shortage of reliable data on incidents (probabilities and impacts), quantitative approaches may not yield reliable results. Consequently, security or risk management professionals mostly prefer qualitative methods rather than quantitative ones. In qualitative methods, estimated risk is calculated using only the estimated potential loss instead of the probability data. These approaches depend on the ideas of the analyst so they are subjective and might yield inconsistent results (Karabacak & Soğukpınar, 2005).

There is not a single risk evaluation method which is best under all circumstances and for all purposes. Some researchers claimed that neither of the quantitative and qualitative approaches could properly model the assessment process alone. Alternatively, some of them developed comprehensive approaches combining both the quantitative and the qualitative approaches (Bodin et al., 2008; Feng & Li, 2011; Zhao et al., 2009).

The Analytic Hierarchy Process (AHP) is one of the most widely used multi-criteria decision technique which can combine qualitative and quantitative factors for prioritizing, ranking and evaluating alternatives. It allows multiple actors, criteria and scenarios to be involved in the analysis. So it can be used to evaluate and prioritize the risk incidents with a group of experts.

Previously, AHP analysis was used as support for an organization's information security system to evaluate the weights of risk factors by Guan et al. (2003); to determine the optimal allocation of a budget by Bodin et al. (2005); to evaluate the weighting factors needed to combine risk measures by Bodin et al. (2008); to obtain the indices' weights with respect to the final goal of the security evaluation by Cuihua & Jiajun (2009); to select information security policy by Syamsuddin and Hwang (2010); and to establish e-commerce information security evaluation by Huang (2011). Xinlan et al. (2010) proposed calculating a relative risk value with AHP-GDM instead of calculating the actual value of the risk. They mentioned that the loss could be measured by the value of assets; and probability of risk could be described in an equation with the danger degree of threat and vulnerability as its two variables.

The AHP method is operable and efficient as it prioritizes and orders risk incidents, which could also satisfy the aim of risk management. However, there might be some complexities when using AHP-GDM for information security risk evaluation. For instance, in AHP-GDM, it is assumed that the pairwise comparison matrices containing the judgements expressed by decision makers are complete and accurate.

For the specified problem, decision makers might provide only incomplete information or sometimes inconsistent answers due to following situations:

(i) Some of the experts may have limited expertise about the risk management problem domain or the AHP analysis.
(ii) Decision makers participated in the analysis would prefer to concentrate on the risk assessment itself rather than the AHP tool being implemented in the risk analysis.
(iii) They may have difficulties in making pairwise comparisons efficiently as the number of assets, threats and vulnerabilities increase.

Altuzarra et al. (2007) proposed a Bayesian prioritization approach for AHP-GDM which can naturally be extended to the case of incomplete pairwise comparison matrices. Contrary to the conventional prioritization methods applied in AHP-GDM (Saaty, 1989; Ramanathan & Ganesh, 1994; Forman & Peniwati, 1998), this technique does not require intermediate filters for decision makers' initial judgements. This approach provides more efficient and robust estimates than the classical prioritization methods applied in AHP-GDM in terms of the mean square error (MSE).

We aim at providing an effective and practical group decision mechanism to prioritize the risk incidents. We propose using BPP based AHP-GDM for information security risk evaluation, which allows a group of people to participate in the analysis. This approach provides flexibility to the group of participants, when expressing their judgements, and to the risk analysts, who may not be professional

AHP practitioners, by treating incomplete or inconsistent judgements properly. Other advantages of this technique can be listed as follows: it can easily be adapted to any information security standard by updating the groups of assets, threats and vulnerabilities, and can be used alone or with any other information security risk analysis methods as a support.

3.4.2 Application

Let us consider the group decision analysis situation on information security risk assessment taken from Xinlan et al. (2010). They defined 3 criteria, which are assumed to have same weights; confidentiality, integrity and availability, and 3 key factors conducting the security risk assessment:

- Assets $(\{A_1,...,A_m\},\, m=5)$,
- Threats $(\{T_1,...,T_s\},\, s=6)$,
- Vulnerabilities $(\{V_1,...,V_h\},\, h=6)$

based on GB/T20984: Risk Assessment Specification for Information Security. The criteria and factors we used can be seen in Table 3.3.

Table 3.3 List of assets, threats and vulnerabilities.

Assets	Threats	Vulnerabilities
A_1-Service A_2-Data A_3-Software A_4-Hardware A_5-People	T_1-Physical environment influences T_2-Hardware & software breakdowns T_3-Malicious code T_4-Ultra vires T_5-Cyber attacks T_6-Management problems	V_1-Physical damages V_2-Network vulnerabilities V_3-Operating systems vulnerabilities V_4-Application systems vulnerabilities V_5-Application middleware vulnerabilities V_6-Problems in technique and organization management

We previously have noted that the AHP-GDM analysis could contain some complexities for this problem. Here, there are 3 criteria, 5 assets, 6 threats and 6 vulnerabilities in the model which make 180 comparisons for each decision maker, and expressing complete and consistent judgements for 180 different comparisons is quite difficult.

Consequently, we aimed to solve this problem with the AHP-GDM based on BPP in order to show that it would present a more practical and flexible way of information security risk assessment. It would also provide more efficient results for risk analysis even with incomplete pairwise comparison matrices.

In this study, there are three AHP models to be analyzed. The first AHP model given in Figure 3.2 is established for calculating the priorities of assets ($a_1,...,a_m$), with respect to the attributes: confidentiality, integrity and availability. The importance of these three factors might be different for each organization so we assumed that all three factors have the same importance in this study. The overall goal is placed at the top of the hierarchy. The attributes are placed in the second layer, and the assets are in the third layer, which is the "alternatives" layer.

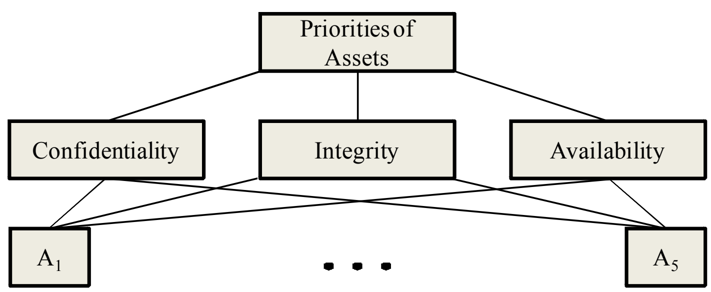


Figure 3.2 AHP decision tree for asset prioritization.

The second and the third AHP models are constructed in order to calculate the danger degree of threats ($t_1,...,t_s$) and vulnerabilities ($v_1,...,v_h$) in terms of each asset respectively. Figure 3.3 shows the decision tree for the danger of threats model. A similar model is prepared for the vulnerabilities.

Since we assumed that the attributes in the first AHP model are equal, we did not require any comparisons for them. So, for each AHP models, we had 3, 5 and 5 different set of pairwise comparisons to be completed by each decision makers respectively. We assumed that there is a cross-functional team composed of 5 decision makers from various departments, who are not forced to give complete answers to the pairwise comparison matrices.

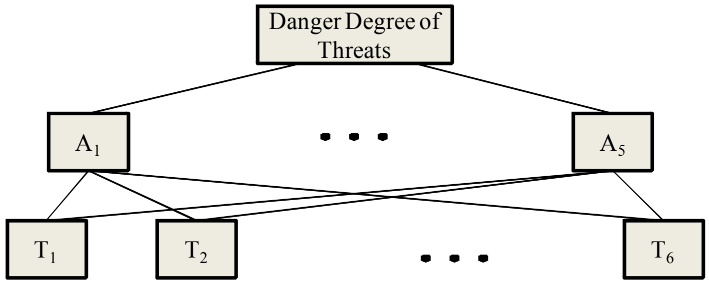


Figure 3.3 AHP decision tree for danger degree of threat prioritization.

In order to illustrate this case, we simulated data based on the fundamental scale proposed by Saaty (1980) for each set of pairwise comparison matrices that are presented in Tables 3.4-3.6.

Table 3.4 Simulated pairwise comparisons of *5* assets in terms of *3* attributes.

pairs		1-2	1-3	1-4	1-5	2-3	2-4	2-5	3-4	3-5	4-5
Confidentiality	D_1	1/4	2	5	2	7	9	5	2	1	1/3
	D_2	1/3	4	7	3	5	8	5	1	NA	1/2
	D_3	1/2	3	7	5	9	9	4	3	3	**2**
	D_4	**2**	5	9	2	7	6	2	1	1	1/4
	D_5	1/2	3	6	**1/3**	9	5	5	**1/3**	2	1/3
	$oc_{ij}\geq 8$	8			15				9		8
Integrity	D_1	1/3	2	5	2	7	9	5	2	1	1/3
	D_2	1/4	3	9	2	7	6	2	1	1	1/4
	D_3	1/2	3	7	5	9	9	4	3	3	1/3
	D_4	1/3	3	6	**1/3**	9	5	5	2	2	1/3
	D_5	1/2	4	7	3	5	8	5	**1/3**	NA	**2**
	$oc_{ij}\geq 8$				15				9		8
Availability	D_1	1/4	2	5	2	7	7	5	2	1	1/3
	D_2	1/4	3	6	2	7	6	2	1	1	1/4
	D_3	1/2	4	7	5	9	7	3	3	3	1/3
	D_4	1/2	3	8	4	9	5	3	**8**	2	1/3
	D_5	1/3	4	7	3	**1**	8	4	3	2	**3**
	$oc_{ij}\geq 8$					9			8		9

Table 3.5 Simulated pairwise comparisons of *6* threats in terms of *5* assets.

pairs		1-2	1-3	1-4	1-5	1-6	2-3	2-4	2-5	2-6	3-4	3-5	3-6	4-5	4-6	5-6
T_j of A_1	D_1	1/5	1/3	1/3	1/7	2	3	3	1/4	5	2	1/3	7	1/4	3	9
	D_2	1/4	1/3	1/2	1/5	1	3	3	1/2	6	1	1/4	3	1/3	3	9
	D_3	1/5	NA	1/5	1/5	1	5	6	1/3	5	1	1/4	4	1/2	3	**1**
	D_4	1/4	1/2	1/2	NA	2	2	3	**3**	7	1	1/7	**1/2**	1/5	2	7
	D_5	1/5	**3**	1/2	1/3	1/2	3	4	1/3	4	1	1/3	5	1/6	2	7
T_j of A_2	D_1	1/4	1/7	1/9	1/9	1/2	1/3	1/5	1/4	3	1/2	1/3	2	1	5	9
	D_2	1/3	1/6	1/9	1/9	1/2	1/2	1/3	1/5	1	1/2	1/3	2	1/2	5	7
	D_3	1/3	1/7	1/8	1/8	1	1/3	1/3	1/5	2	1	1/4	3	1/2	6	8
	D_4	1/5	1/8	**1**	1/7	2	1/2	1/4	1/3	1	1/3	1/5	1	1	**1/3**	7
	D_5	1/2	NA	1/7	1/9	1/2	1/2	**2**	1/4	3	**4**	1/5	2	1/2	4	9
T_j of A_3	D_1	1/9	1/7	1/2	1/5	1/2	2	6	3	9	4	2	6	1/2	1	3
	D_2	1/7	1/7	1	1/4	1/3	1	7	4	9	5	3	3	1/2	1	2
	D_3	1/5	**2**	1/2	1/5	NA	4	5	3	7	3	5	3	1	2	3
	D_4	1/9	1/5	1/3	1/8	1/2	**1/2**	8	2	8	7	2	5	1/3	1	3
	D_5	1/6	1/6	3	1/7	1/2	3	6	3	9	6	3	**1/3**	1	2	1
T_j of A_4	D_1	1/4	3	6	3	1/4	9	7	5	2	2	1/2	1/7	1/4	1/9	1/6
	D_2	1/5	4	7	2	1/2	9	9	4	2	1	1/3	1/5	1/4	1/9	1/3
	D_3	1/5	2	5	2	1/3	7	8	6	3	3	NA	1/6	1/3	1/7	**2**
	D_4	1/4	3	6	4	1/2	1	8	**1/2**	3	2	1/4	1/7	1/3	1/9	1/7
	D_5	1/6	**1/2**	6	3	1/3	7	9	5	2	3	1/3	**2**	1/4	1/7	1/5
T_j of A_5	D_1	5	4	1/2	5	1/3	1	1/8	1	1/9	1/5	1	1/9	**1/2**	1/4	1/9
	D_2	4	4	1	6	1/3	1	1/7	2	1/9	1/3	2	1/7	6	1/2	1/8
	D_3	4	6	1/2	7	1/4	1/2	1/5	1/2	1/6	1/4	3	1/9	7	1/3	1/7
	D_4	8	5	1/3	5	1/2	1/2	1/9	2	1/8	1/8	2	1/8	4	1	1/7
	D_5	**1**	7	1	4	1/3	2	1/8	1	1/7	**1**	3	1/9	8	1/3	1/9

In Table 3.4, the simulated pairwise comparisons for the first AHP model are given, where 5 assets are compared by 5 decision makers in terms of confidentiality, integrity and availability attributes. In the first model, D_2 did not compare A_3 with A_5, and D_5 did not compare A_3 with A_5 in the second one, which resulted in incomplete judgement situations.

Table 3.6 Simulated pairwise comparisons of *6* vulnerabilities in terms of *5* assets.

	pairs	1-2	1-3	1-4	1-5	1-6	2-3	2-4	2-5	2-6	3-4	3-5	3-6	4-5	4-6	5-6
V_k of A_1	D_1	1/4	1/3	1/9	1/8	5	1	1/3	1/2	3	1/4	1/2	2	2	8	4
	D_2	1/4	1/3	1/9	1/6	3	1	1/7	2	2	1/3	1/2	3	3	8	5
	D_3	1/3	**3**	1/8	1/5	5	1/2	1/5	1/2	4	1/4	1	1	1	9	6
	D_4	1/3	1/2	1/7	1/7	5	1/2	1/4	2	3	1/8	1/5	1/2	2	5	3
	D_5	1/5	1/4	1/7	1/8	3	2	1/8	1	**1/3**	1/3	1/3	4	3	1	5
V_k of A_2	D_1	1	1/2	1/9	1/8	1/2	1	1/3	1/2	1/2	1/2	1/2	1/2	2	4	3
	D_2	1/2	1/3	1/9	1/7	1/2	1	1/3	1/2	1/2	1/5	1/2	1/2	3	4	3
	D_3	1/2	1/3	1/8	1/8	NA	1/2	1/5	1/2	1/3	NA	1	1	1	5	4
	D_4	1	1/2	1/9	1/9	3	1/2	1/4	1/3	1/4	**3**	1/5	1/2	2	3	3
	D_5	2	1/4	1/9	1/9	1/2	2	1/8	1	1	1/3	1/3	1/8	3	**1/3**	2
V_k of A_3	D_1	1/4	1/6	1/8	1/4	1/2	1/4	1/7	1/3	1/2	1	2	3	2	4	1
	D_2	1/2	1/7	1/9	1/5	1/3	1/4	1/7	1/2	1/2	1/2	3	3	3	4	2
	D_3	NA	1/9	1/8	1/3	1/2	1/3	1/6	1/4	1	NA	1	4	4	5	3
	D_4	1/3	1/7	1/9	NA	1/5	**4**	1/8	1/3	1/2	1/2	2	**1/5**	3	6	3
	D_5	**3**	1/6	1/7	1/4	1/3	1/4	1/6	1/2	**4**	1/3	3	**1/3**	3	2	2
V_k of A_4	D_1	5	2	3	8	1/2	1	2	1	1/5	2	1	1/2	1	1/7	1/8
	D_2	4	2	5	6	1/2	1/2	2	2	1/5	3	3	1/2	1	1/9	1/9
	D_3	3	3	3	8	1	1	3	1	1/4	1	2	1/3	2	1/6	1/8
	D_4	8	1	5	7	1/3	1/2	1	3	**3**	3	3	1/4	1/2	1/8	1/6
	D_5	**1**	NA	5	9	1/3	1/3	1	2	1/2	4	**1/3**	1	1	1/9	1/9
V_k of A_5	D_1	2	1/3	2	1/2	1/6	1/5	1/4	1	1/9	2	5	1/2	3	1/3	1/9
	D_2	2	1/3	2	2	1/7	1/4	1/5	2	1/8	3	4	1/3	3	1/4	1/8
	D_3	4	**3**	4	1	1/8	1/3	1/4	2	1/8	2	6	1/2	4	1/5	1/7
	D_4	3	1/2	1	1/2	1/6	1	1/2	1	1/7	1	3	1	5	1/3	1/9
	D_5	3	1/2	**1/2**	1/2	1/7	1	NA	NA	1/9	2	7	1/3	1	**4**	1/8

The opening coefficients (oc_{ij}) reflect the variability of judgements expressed by decision makers, and are calculated by: $Max_k r_{ij}^k / Min_k r_{ij}^k$, $k = 1,...,5,\ 1 \le i < j \le n$. In this study we omitted the most inconsistent judgements which cause oc_{ij} to be large. In Table 3.4, the oc_{ij} line is presented to illustrate the omitting procedure, but the same procedure is applied for each matrix.

Table 3.5 and 3.6 gives the simulated pairwise comparisons for the second and third AHP models, where 6 threats and 6 vulnerabilities are compared by 5 decision makers in terms of 5 different assets respectively.

In Tables 3.4-3.6, the incomplete judgements are written as "NA" and the judgements which are selected to be omitted are given in bold. It can be concluded that five decision makers have a consensus in general, where D_1 and D_2 are the most consistent ones and D_5 is the most inconsistent one. Some decision makers, especially D_3 and D_5 preferred not to express some of the pairwise comparisons. Looking at the oc_{ij} line, it can also be noted that D_4 and D_5 seem to pay less attention compared to others since the omitted judgements mostly belong to them.

It is assumed that consensus exists among the decision makers with regard to the priorities for each alternative. The degree of inconsistency for each decision maker $\left(\sigma^{(k)2}\right)$ is assumed to be known and below the threshold. We used the inconsistency levels $\sigma^{(k)2} = (0.127, 0.043, 0.243, 0.272, 0.431)$ extracted from the first AHP model.

3.4.3 Statistical Results for the Example

Both the AIP method and the BPP have been applied for aggregating judgements in group AHP analysis respectively. After omitting the judgements given in bold, the methods are repeated and are named as AIP* and BPP*. Tables 3.7-3.9 show the priorities of assets, threats and vulnerabilities with the mean square errors (MSE). The MSE values for each method are calculated by:

$$MSE = \sum_{1 \le i < j}^{n} \sum_{k=1}^{r} \varepsilon_{ij}^{(k)} \Bigg/ \sum_{k=1}^{r} t_k$$

for each method.

Table 3.7 Group priorities for assets estimated by each method.

		A_1	A_2	A_3	A_4	A_5
A_i of Con.	AIP	0.263	0.489	0.083	0.057	0.108
	Bayesian	0.263	0.496	0.081	0.055	0.106
	AIP*	0.332	0.418	0.079	0.068	0.104
	Bayesian*	0.305	0.448	0.078	0.061	0.108
		A_1	A_2	A_3	A_4	A_5
A_i of Int.	AIP	0.241	0.511	0.091	0.052	0.105
	Bayesian	0.239	0.506	0.090	0.051	0.114
	AIP*	0.274	0.480	0.090	0.048	0.108
	Bayesian*	0.263	0.484	0.088	0.048	0.118
		A_1	A_2	A_3	A_4	A_5
A_i of Ava.	AIP	0.272	0.467	0.114	0.050	0.098
	Bayesian	0.257	0.480	0.104	0.050	0.110
	AIP*	0.271	0.467	0.100	0.049	0.112
	Bayesian*	0.256	0.479	0.094	0.050	0.120

For different AHP models, each method gives similar weights and almost the same ranking but the Bayesian estimates reflect more robust results since the priorities (w_i) does not change too much after omitting the inconsistent judgements.

Out of the assets, “data” is the most important one, which is followed by “service”. The order for the priorities of assets in terms of the main criteria is: $A_2 > A_1 > A_5 > A_3 > A_4$. The priorities of threats and vulnerabilities change for each asset. For example, T_2 (hardware and software breakdowns) is the most dangerous threat for A_1, A_3 and, A_4 (service, software and hardware respectively), where it is the least dangerous threat for A_5 (people).

Table 3.8 Group priorities for threats estimated by each method.

		T_1	T_2	T_3	T_4	T_5	T_6
T_j of A_1	AIP	0.082	0.309	0.120	0.180	0.234	0.076
	Bayesian	0.072	0.293	0.121	0.159	0.297	0.058
	AIP*	0.084	0.278	0.146	0.229	0.177	0.085
	Bayesian*	0.074	0.275	0.141	0.182	0.268	0.059
		T_1	T_2	T_3	T_4	T_5	T_6
T_j of A_2	AIP	0.041	0.110	0.158	0.305	0.314	0.072
	Bayesian	0.037	0.098	0.155	0.287	0.359	0.065
	AIP*	0.040	0.099	0.211	0.316	0.260	0.075
	Bayesian*	0.036	0.092	0.185	0.305	0.319	0.063
		T_1	T_2	T_3	T_4	T_5	T_6
T_j of A_3	AIP	0.048	0.427	0.244	0.073	0.136	0.072
	Bayesian	0.046	0.422	0.259	0.070	0.135	0.069
	AIP*	0.050	0.473	0.201	0.103	0.108	0.065
	Bayesian*	0.047	0.457	0.227	0.085	0.116	0.068
		T_1	T_2	T_3	T_4	T_5	T_6
T_j of A_4	AIP	0.142	0.411	0.062	0.034	0.108	0.243
	Bayesian	0.144	0.412	0.054	0.033	0.101	0.257
	AIP*	0.154	0.395	0.040	0.040	0.127	0.243
	Bayesian*	0.148	0.397	0.040	0.034	0.108	0.273
		T_1	T_2	T_3	T_4	T_5	T_6
T_j of A_5	AIP	0.187	0.047	0.057	0.226	0.042	0.442
	Bayesian	0.190	0.047	0.057	0.222	0.042	0.442
	AIP*	0.203	0.044	0.070	0.189	0.059	0.435
	Bayesian*	0.197	0.044	0.064	0.212	0.047	0.437

Table 3.10 shows the mean square errors (MSE) of different prioritization methods for each of the AHP models, in which the value of assets and then the danger degree of threats and vulnerabilities are evaluated. $WMSE$ is the weighted average of MSE's, which could be calculated as:

$$WMSE = \sum_{i=1}^{m} w_i MSE_i \Big/ m,$$

Table 3.9 Group priorities for vulnerabilities estimated by each method.

		V_1	V_2	V_3	V_4	V_5	V_6
V_k of A_1	AIP	0.059	0.129	0.107	0.432	0.225	0.049
	Bayesian	0.056	0.131	0.111	0.443	0.213	0.045
	AIP*	0.056	0.128	0.147	0.469	0.137	0.062
	Bayesian*	0.054	0.131	0.139	0.465	0.160	0.051
V_k of A_2		V_1	V_2	V_3	V_4	V_5	V_6
	AIP	0.054	0.103	0.129	0.365	0.221	0.129
	Bayesian	0.051	0.101	0.122	0.378	0.221	0.127
	AIP*	0.054	0.099	0.156	0.354	0.240	0.097
	Bayesian*	0.051	0.098	0.139	0.369	0.234	0.108
V_k of A_3		V_1	V_2	V_3	V_4	V_5	V_6
	AIP	0.044	0.077	0.294	0.337	0.149	0.100
	Bayesian	0.041	0.073	0.292	0.353	0.144	0.097
	AIP*	0.036	0.092	0.368	0.254	0.141	0.109
	Bayesian*	0.035	0.082	0.337	0.311	0.135	0.100
V_k of A_4		V_1	V_2	V_3	V_4	V_5	V_6
	AIP	0.276	0.114	0.128	0.060	0.080	0.342
	Bayesian	0.273	0.100	0.131	0.056	0.062	0.377
	AIP*	0.277	0.115	0.117	0.056	0.094	0.342
	Bayesian*	0.273	0.100	0.123	0.054	0.069	0.381
V_k of A_5		V_1	V_2	V_3	V_4	V_5	V_6
	AIP	0.114	0.071	0.216	0.155	0.080	0.364
	Bayesian	0.105	0.060	0.211	0.126	0.059	0.440
	AIP*	0.113	0.076	0.228	0.119	0.131	0.333
	Bayesian*	0.105	0.064	0.220	0.112	0.082	0.418

where w_i is the weight of the attribute and MSE_i is the MSE of the group when comparing the alternatives in terms of the i^{th} attribute. Among four approaches, BPP* generally provided the minimum $WMSE$, and conventional approaches did not provide lower values of $WMSE$ than BPP*. Consequently, BPP* results are selected for further implementation of risk evaluation.

Table 3.10 MSE values for each method.

		AIP	BPP	AIP*	BPP*
A_i	Con.	0.391	0.392	0.281	0.235
	Int.	0.348	0.351	0.183	0.188
	Ava.	0.348	0.352	0.265	0.281
	WMSE	0.362	0.365	0.243	0.235
T_j	A_1	0.578	0.414	0.510	0.317
	A_2	0.450	0.402	0.321	0.199
	A_3	0.382	0.379	0.312	0.222
	A_4	0.431	0.435	0.286	0.211
	A_5	0.293	0.293	0.330	0.266
	WMSE	0.466	0.394	0.377	0.245
V_k	A_1	0.486	0.478	0.301	0.190
	A_2	0.740	0.736	0.661	0.602
	A_3	0.510	0.514	0.513	0.396
	A_4	0.388	0.340	0.371	0.245
	A_5	0.516	0.452	0.645	0.378
	WMSE	0.604	0.593	0.568	0.461

Table 3.11 reflects the final value of all risk incidents which is

$$R_{ijk} = \left(a_i \times \sqrt{t_{ij} \times v_{ik}}\right)^{1/2},$$

and the danger degree of threats for each asset which is:

$$R_{ij} = \sum_{k=1}^{h} R_{ijk} \Big/ h,$$

can be drawn from Table 3.11 for the remaining cases.

The danger degree order of all assets (R_i), which could be determined by maximum, minimum or average value of R_{ij} for each asset. Here we followed the mathematical concept and theorems of Altuzarra et al. (2007).

Table 3.11 Risk values of R_{ijk} , R_{ij} and R_i calculated by BPP*.

		V_1	V_2	V_3	V_4	V_5	V_6	R_{ij}	R_i
A_1	T_1	0.125	0.154	0.220	0.215	0.175	0.162	0.175	
	T_2	0.173	0.214	0.305	0.299	0.242	0.225	0.243	0.243(max)
	T_3	0.147	0.181	0.258	0.253	0.205	0.190	0.206	0.166(min)
	T_4	0.156	0.193	0.275	0.269	0.219	0.203	0.219	0.208(aver)
	T_5	0.172	0.212	0.303	0.297	0.241	0.223	0.241	
	T_6	0.118	0.146	0.208	0.204	0.165	0.153	0.166	
A_2	T_1	0.139	0.164	0.178	0.228	0.203	0.168	0.180	
	T_2	0.175	0.206	0.225	0.287	0.256	0.211	0.227	0.310
	T_3	0.209	0.246	0.268	0.342	0.305	0.252	0.270	0.180
	T_4	0.237	0.278	0.304	0.388	0.346	0.285	0.306	0.250
	T_5	0.240	0.282	0.307	**0.392**	0.350	0.288	0.310	
	T_6	0.160	0.188	0.205	0.262	0.234	0.193	0.207	
A_3	T_1	0.063	0.078	0.079	0.107	0.082	0.062	0.078	
	T_2	0.111	0.138	0.140	0.189	0.145	0.109	0.139	0.139
	T_3	0.093	0.116	0.117	0.159	0.122	0.091	0.116	0.078
	T_4	0.073	0.091	0.092	0.124	0.095	0.072	0.091	0.101
	T_5	0.079	0.098	0.099	0.134	0.103	0.077	0.098	
	T_6	0.069	0.086	0.087	0.118	0.090	0.068	0.086	
A_4	T_1	0.111	0.086	0.091	0.074	0.079	0.121	0.093	
	T_2	0.142	0.110	0.116	0.094	0.101	0.154	0.120	0.120
	T_3	0.080	0.062	0.066	0.053	0.057	0.087	0.067	0.065
	T_4	0.077	0.060	0.063	**0.051**	0.055	0.084	0.065	0.090
	T_5	0.102	0.080	0.084	0.068	0.073	0.111	0.086	
	T_6	0.129	0.101	0.106	0.086	0.092	0.140	0.109	
A_5	T_1	0.125	0.110	0.150	0.127	0.117	0.176	0.134	
	T_2	0.086	0.076	0.103	0.087	0.081	0.121	0.092	0.164
	T_3	0.094	0.083	0.113	0.096	0.088	0.133	0.101	0.092
	T_4	0.127	0.112	0.153	0.129	0.119	0.179	0.137	0.120
	T_5	0.087	0.077	0.105	0.088	0.082	0.123	0.093	
	T_6	0.152	0.134	0.183	0.155	0.143	0.215	0.164	

According to Table 3.11, the risk incidents can be ordered as: $R_{254} > R_{244} > R_{255} > ... > R_{445} > R_{434} > R_{444}$, with the highest value, 0.392 and the lowest, 0.051.

It can be concluded that risk incidents associated with A_2 and A_1 have higher values, where the ones associated with A_4 have lower values.

For A_2, the danger degree of T_j in descending order is: $T_5 > T_4 > T_3 > T_2 > T_6 > T_1$, which means that the "cyber attacks" and "ultra vires" are the most dangerous threat for "data". For A_3, the order is: $T_2 > T_3 > T_5 > T_4 > T_6 > T_1$, which means that the "hardware and software breakdowns" and "malicious code" are the most dangerous threats for "software". Similar conclusions

For the whole system, the danger degree order of assets can also derived by comparing the maximum, minimum or average value of R_{ij} for each asset. Consequently, the danger degree order of A_i is: $A_2 > A_1 > A_5 > A_3 > A_4$, which means that the assets need to take precautionary measure could be ranked in this order. The outputs given in this table could support the company efficiently when making the information security management decisions.

In order to be fair in comparison, pairs of plots are provided to show the performance of AIP vs. BPP and AIP* vs. BPP*. As an example, Figures 3.4-3.6 demonstrates the MSE and weighted MSE (WMSE) values for the results of three AHP models given in Table 3.10. The blue lines represent the output of the Bayesian method where the red ones are the classical method.

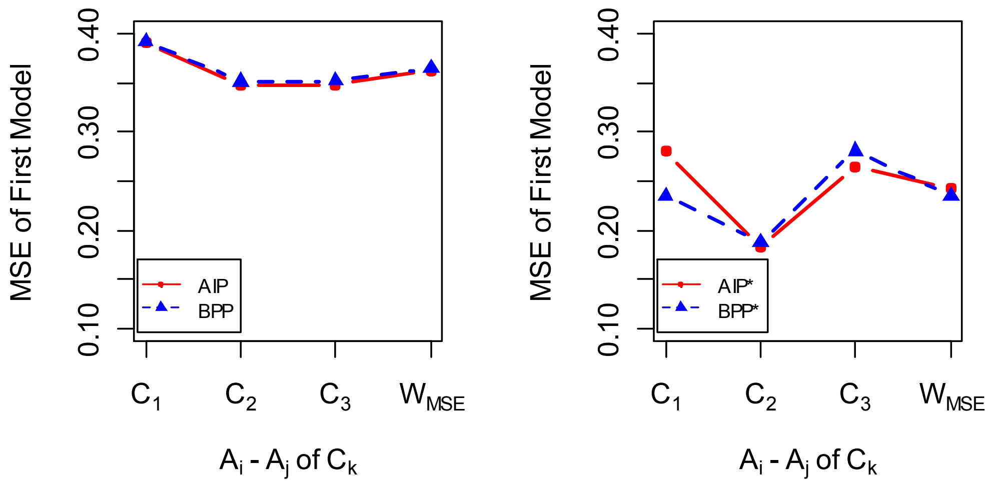


Figure 3.4 MSE values of the first model, comparing the assets in terms of 3 criteria.

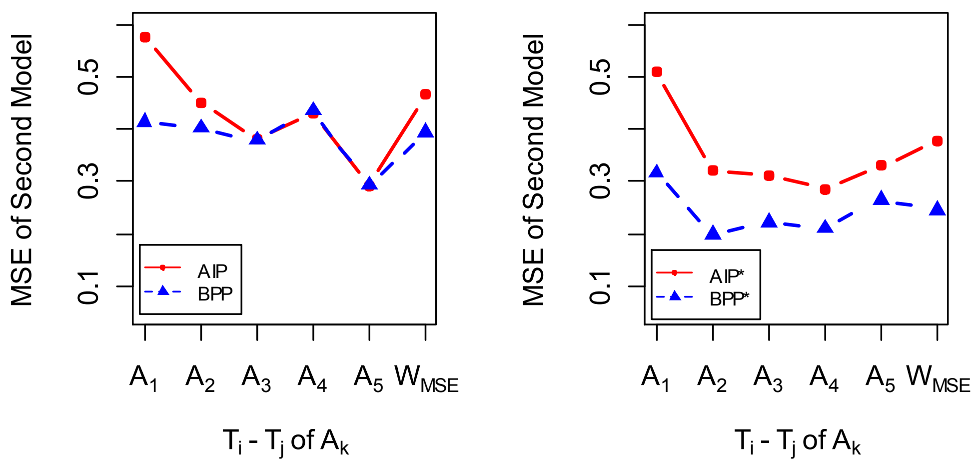


Figure 3.5 MSE values of the second model, comparing 6 threats in terms of 5 assets.

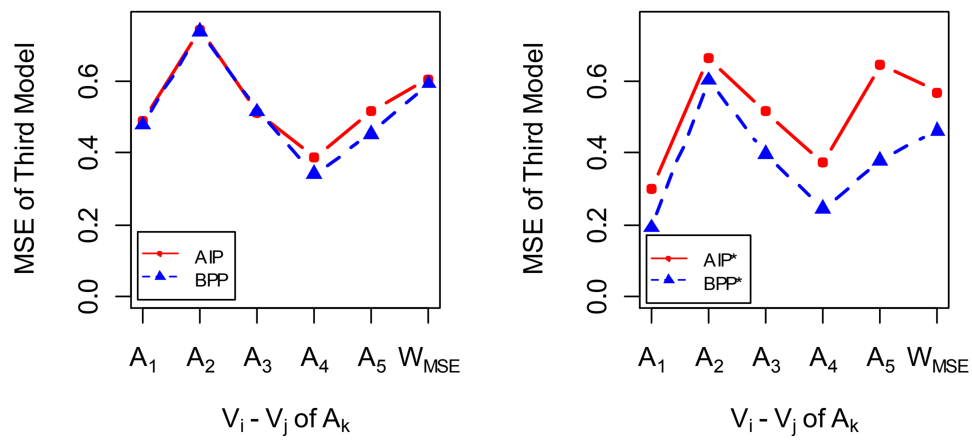


Figure 3.6 MSE values of the third model, comparing 6 vulnerabilities in terms of assets.

The results of each AHP model show that the Bayesian method has smaller MSE and WMSE values than the conventional method, especially after omitting the inconsistent judgements. It can be concluded that the performance of the Bayesian method is higher when the aggregating the group's judgements and estimating the priorities of the alternatives.

3.5 Conclusions

In this chapter, the case study was about the information security risk assessment problem. Risk assessment requires the use of more flexible approaches to measure information security risk. The AHP-GDM offers a technical support for risk analysis by obtaining the judgements of managers and systematically calculating the relative risk values.

The AHP-GDM is a powerful technique that is easy to understand and simple to operate. It is a flexible and practical tool for any organization to prioritize the risk incidents recurrently. However, there might be some complexities to use the AHP-GDM in risk evaluation. For example, decision makers participated in the analysis may have limited expertise about the problem domain or the AHP analysis. Also, they may have difficulties to make pairwise comparisons efficiently because of the large number of assets, threats and vulnerabilities which could result in incomplete or inconsistent judgements.

Considering the problems mentioned above, we propose using BPP based AHP for information security risk assessment. It is assumed that consensus exists among the decision makers with regard to the priorities for each element in this decision system. The multiplicative model with log-normal errors is applied to the problem and the Bayesian analysis is used. This is a process of weighted aggregation of individual priorities and the weights are inversely proportional to the decision makers' levels of inconsistency. We compared the method with the conventional approaches used in the AHP-GDM.

The results show that the proposed methodology is more efficient than the conventional prioritization based AHP approaches. It can treat incomplete or inconsistent judgements properly. It provides managers a flexible way to express their judgements, without forcing them to give complete and consistent judgements and letting them completely focus on the risk management itself. Moreover, it serves the practitioner since the judgements of decision makers directly enter the analysis without any reducing or filtering process.

Any organization can easily adapt this method to their information security system by updating all the elements in the illustrative model, i.e., list of most valuable information assets, threats and vulnerabilities. This technique could be used alone or with any other information security risk analysis methods as a support; and can easily be adapted to any information security standard.

In this study, we applied BPP based AHP to prioritize and order risk incidents which could satisfy the aim of risk management. This approach can also be used for many multiple criteria group decision making problems such as project selection, facility location selection, supplier selection or evaluation, diagnosis and treatment selection for disease management, financial decision making and crisis forecasting, and evacuation selection for emergency management.

Our study is based on the model from a non-informative Bayesian standpoint, where the variances of error terms represented by the inconsistency levels of decision makers are assumed to be known. In the future, this approach can be extended by taking the variances of error terms as additional parameters, or by implementing an informative Bayesian model in which a good estimate of prior distribution for the vector of log-priorities is used.

Our study has some limitations that need to be addressed. This study is based on two assumptions. The first assumption is that there is a consensus among the decision makers. Gargallo et al. (2007) proposed a Bayesian estimation procedure to determine the priorities where a prior consensus among them is not required. Altuzarra et al. (2010) proposed some procedures to search for consensus between the actors involved in the decision making process. The second assumption is that there is no interaction or dependence between the elements in the decision system.

In the next chapter, the situation where the independence assumption is not satisfied is covered.

CHAPTER FOUR
BAYESIAN PRIORITIZATION PROCEDURE BASED ANALYTIC NETWORK PROCESS FOR GROUP DECISION MAKING

4.1 Introduction

The Analytic Hierarchy Process (AHP) and the Analytic Network Process (ANP), proposed by Thomas L. Saaty (1980, 1996) are widely used descriptive approaches in multi-criteria decision making. They both allow multiple actors, criteria and scenarios to be involved in the analysis.

The AHP simplifies the problem of constructing hierarchic structures which comprise a goal, criteria, and alternatives. It assumes that the factors presented in the hierarchical structure are independent. However, many decision problems involve the interaction and dependence of higher-level elements in a hierarchy on lower-level elements and dependence of elements within a level therefore cannot be structured hierarchically (Saaty, 1999).

The ANP extends the AHP to the case of dependence and feedback. It allows interactions and feedback within and between clusters. It generalizes on the supermatrix approach proposed by Saaty (1980). In the ANP, not only does the importance of the criteria determine the importance of the alternatives as in a hierarchy, but also the importance of the alternatives themselves determines the importance of the criteria (Saaty, 2005). These two methods both derive ratio scale priorities for elements and clusters of elements by making pairwise comparisons of elements on a common criterion or property.

ANP can solve real-world multi-criteria problems based on the following motivations:

- ANP has a systematic approach to set priorities and trade off among goals and criteria (Mishra et al., 2002).

- Criteria weights or priorities established by ANP are based on the use of a ratio scale by human judgment instead of arbitrary scales (Saaty, 1999);
- ANP can measure all tangible and intangible criteria in the model (Saaty, 2000).
- ANP is a relatively simple, intuitive approach that can be accepted by managers and other decision-makers (Presley & Meade, 1999).
- ANP can easily be used to solve multicriteria decision problems involving multiactors or group decision making with multiactors (Karabacak & Soğukpınar).
- ANP enables a better communication, leading to a clearer understanding and consensus among actors so that they will commit to the selected alternative more likely (Torkkeli & Tuominen, 2002).
- As a general form of AHP, ANP allows for more complex interrelationships among the decision levels and attributes (Raisinghani, 2001).
- ANP incorporates dependences and feedback using a multilevel (or hierarchical) decision network which can adequately model dependence (or interdependence) relations among components, represent and analyze interactions, and synthesize their mutual effects by a single logical procedure (Sarkis & Sundarraj, 2002).

In this chapter, our aim is to extend the Bayesian prioritization procedure (BPP) to a more general approach to decisions, which is a generalization of hierarchies to networks with dependence and feedback, the ANP. Bayesian prioritization approach is used for deriving the ratio scale priorities and building the supermatrix. This method also can be extended to the case of incomplete pairwise comparison matrices, which is a common problem in complex decision making problems. The methodology is illustrated by the analysis of a case study and compared with one of the conventional prioritization procedures.

The remainder of this chapter is as follows: the theoretical background of the ANP-GDM is briefly given in Section 4.2. Section 4.3 presents the Bayesian prioritization procedure (BPP) based ANP. Section 4.4 illustrates the proposed methodology using a practical case study. Finally, Section 4.5 summarizes the main results of this chapter and offers concluding remarks.

4.2 ANP Group Decision Making (ANP-GDM)

The ANP is an effective tool for structuring and solving complex group decision problems and provides accurate results according to many researchers (Tohumcu and Karasakal, 2010; Ho et al., 2010). The ANP provides a way to elicit judgments of decision maker(s) and uses measurements to obtain the ratio scale priorities for the distribution of influence among the factors and groups of factors in the decision process.

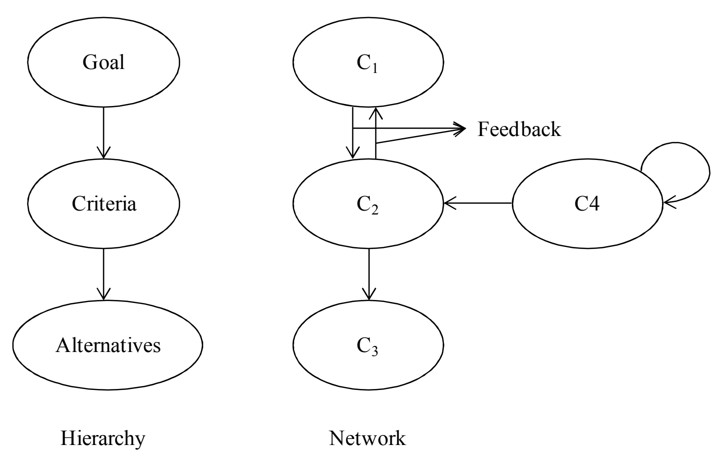


Figure 4.1. Hierarchy and Network structures.

The ANP is a network system, which is composed of clusters and their elements (nodes). It involves interactions and feedback within clusters (inner dependence) and between clusters (outer dependence). A sample network is given in Fig. 4.1. Arc from cluster C_4 to C_2 indicates outer dependence of the elements in C_2 on the elements of C_4. Elements in C_2 affect elements in C_4. Loop in the cluster C_4

indicates inner dependence of the elements in that cluster. Elements affect other elements of the same cluster.

4.2.1 Pairwise Comparisons in the ANP

When making pairwise comparisons in an ANP model the questions are formulated in terms of dominance or influence: "Given a parent element, which of two elements being compared with respect to it has greater influence (is more dominant) with respect to that parent element?" Or, "which is influenced more with respect to that parent element?" You want to avoid changing perspective. For example, in comparing A to B with respect to a criterion, you ask whether the criterion influences A or B more. Then if for the next comparison involving A and C you ask whether A or C influences the criterion more, this would be a change in perspective that would undermine the whole exercise. You must keep in mind whether the influence is flowing from the parent element to the elements being compared, or the other way around.

Use one of the following two questions throughout an application:

1. Given a parent element and comparing elements A and B under it, which element has greater influence *on* the parent element?
2. Given a parent element and comparing elements A and B, which element is influenced more *by* the parent element?

In the ANP-GDM, each decision maker makes paired comparisons on the clusters, elements, and alternatives based on the fundamental scale proposed by Saaty (1980) which previously were given in Table 3.1. The ANP allows decision makers to focus their judgments separately on each of several elements one by one by making the decision makers take a pair of elements and compare them on that single element without any concern for other criteria or other elements.

The pairwise comparisons comprise a set of matrices called "pairwise comparison matrices". There are $n(n-1)/2$ judgments required to develop each set of comparison matrices. Reciprocals are automatically assigned in each pair-wise comparison.

After all the pairwise comparisons are done, the consistency is determined by using the eigenvalue, $\lambda_{\max}$, to calculate the consistency index, CI, as follows:

$$CI = \frac{\lambda_{\max} - n}{n-1}$$

where n is the matrix size.

Judgment consistency can be checked by taking the consistency ratio (CR) of CI, with the appropriate value in Table 3.2

The CR is acceptable, if it does not exceed 0.10. If it is more, the judgment matrix is inconsistent. To obtain a consistent matrix, judgments should be reviewed and improved.

4.2.2 The Supermatrix: Construction and Calculation

In the prioritization stage, the local priorities are derived by calculating the eigenvalues of the comparison matrix of each element. The obtained priorities are entered in the appropriate columns of a matrix, which is called the "supermatrix". The entries in the supermatrix represent the influence priority of an element at the left of the matrix on an element at the top of the matrix. If the element of a cluster has no influence on an element in another cluster, then the corresponding entry is zero.

The columns of the supermatrix are normalized such that they sum up to 1 and the "weighted supermatrix" is obtained. The weighted supermatrix has to be column stochastic (Saaty, 1996), i.e., its columns must sum to one.

In order to capture the overall influence, the weighted supermatrix is raised to a limiting power to converge into a stable supermatrix, which is called the "limit supermatrix". This matrix shows the global priorities of the elements with respect to the goal.

4.2.3 Methods for Aggregating Group Judgements

According to Saaty (1989), after the model for the problem is established, there are two methods to accommodate the judgements of decision makers in a group setting. One of the alternatives is that decision makers make each paired comparison individually, or the group is required to achieve consensus on each paired comparison.

If individual's paired comparison ratio judgments are gathered, the literature describes different methods for the prioritization and synthesis procedures (Saaty, 1989; Crawford & Williams, 1985; Aguarón & Moreno-Jiménez, 2000). The two conventional procedures to obtain group priorities are "the aggregation of individual judgements (AIJ)" and "the aggregation of individual priorities (AIP)". Based on individual judgements, a new judgement matrix is constructed for the group as a whole in AIJ procedure and the priorities are computed from the new matrix. If the individuals are experts, they may not wish to combine their judgements but only their final outcomes obtained by each from their own network (Saaty, 2008). In that case the AIP method can be used, in which the total priorities are obtained on the basis of individual priorities using an aggregation procedure. Synthesis of the model can be done using any of the aggregation procedures. The weighted geometric mean method is the most commonly used technique for both (Saaty, 1983).

One of the limitations of these conventional procedures is that they assume the pairwise comparison matrices containing the decision makers' judgements are complete and accurate. However, especially for large problems (including large numbers of clusters and elements) there might be incomplete matrices including empty positions which would result in wrong conclusions

4.2.4 Incomplete and inconsistent judgements

For the ANP- GDM problems, the incomplete information and the inconsistency problems could occur more frequently than the other MCDM problems such as the AHP-GDM. The reasons can be summarized as follows:

(i) The ANP has a complex network model with dependence and feedback so it is harder for a decision maker to completely understand the model.

(ii) Because of the network structure, the number of pairwise comparisons is large so it becomes hard for decision makers to provide complete and inconsistent answers.

The practitioner may also prefer to ignore the inconsistent or opposing judgements while keeping the consistent or homogeneous ones in order to increase the consistency or consensus among decision makers.

4.3 Bayesian prioritization procedure (BPP) for ANP-GDM

Methodological developments have been aroused in the literature in order to overcome the limitations of classical aggregation methods in group judgements. Aggregation methods with linear programming (Mikhailov, 2004) and Bayesian approach (Altuzarra et al., 2007) have been proposed in order to make a decision even when comparisons are missing, for example when a stakeholder does not feel to have the expertise to judge a particular comparison (Ishizaka & Labib, 2010).

Bayesian approaches allow the treatment of missing data or incomplete information using data augmentation techniques (Tanner & Wong, 1987). The integration of high-dimensional functions was the major limitation towards the wide application of Bayesian analysis before Markov Chain Monte Carlo (MCMC) methods have been introduced. As have been mentioned in the previous chapter, there are very few references to Bayesian analysis in the AHP literature. And up to now, we have not reached any paper which is dealing with Bayesian methods for the ANP. As a consequence, our study can be considered as a pioneering one.

4.3.1 Statistical model for complete and consistent judgements

In this paper, we adopt the BPP to ANP-GDM by using the theorems which are previously provided by Altuzarra et.al. (2007). Assume a network composed of m clusters, $\{C_1,...,C_m\}$ and T sets of paired comparisons required for the network at total, $(t=1,...,T)$. Let $\mathbf{DM}=\{DM_1,...,DM_r\}, r\geq 2$ be a group of r decision makers, who will answer the generic questions by making individual pairwise comparisons. The questions are in the form of: given a control criterion (subcriterion), a component (element) of the network and given a pair of components (elements), how much more does a given member of the pair influence that component (element) with respect to the control criterion (subcriterion) than the other member? Their preferences are based on the fundamental scale proposed by Saaty (1980).

Assuming that there are n_t number of elements $E_{ti},(t=1,...,T), i=1,...,n_t$ to be compared in each comparison set, the paired judgements $r_{tij}^{(k)}$ represent the preference of the decision maker, DM_k, when a comparison between E_{ti} and E_{tj} is required for set t.

As a result of different pairwise comparisons, there are $(T\times r)$ sets of reciprocal judgement matrices:

$$\{\mathbf{R}_t^{(k)}, k=1...,r,\ t=1,...,T\}.$$

Here $\mathbf{R}_t^{(k)} = (r_{tij}^{(k)})$ is a positive square matrix $\left(n_t \times n_t\right)$ which validates:

$$(r_{tii}^{(k)}) = 1, \quad (r_{tij}^{(k)}) = 1 \big/ (r_{tji}^{(k)}) > 0 \text{ for } i, j = 1 \ldots, n_t, \ t = 1, \ldots T .$$

Each judgement matrix will have its associated priority vector. These priority vectors compose the supermatrix. For each set of comparisons, let

$$\{v_{t1}^G, \ldots, v_{tn}^G\}, \, v_{ti}^G \geq 0$$

and

$$\{w_{t1}^G, \ldots, w_{tn}^G\}, \, w_{ti}^G = v_{ti}^G \Big/ \sum_{j=1}^{n} v_{ti}^G$$

be the group's unnormalized and normalized priorities for the elements, respectively.

As previously employed in stochastic AHP (Crawford & Williams, 1985; Alho & Kangas, 1997), a multiplicative model with log-normal errors is applied in the Bayesian analysis of the model. If the decision makers express all possible judgements, the model will be:

$$r_{tij}^{(k)} = \frac{v_{ti}^G}{v_{tj}^G} e_{tij}^{(k)}, \quad i, j = 1, \ldots, n_t, \quad k = 1, \ldots, r, \quad t = 1, \ldots, T,$$

with

$$e_{tij}^{(k)} \sim LN\left(0, \sigma^{(k)2}\right), \ i < j.$$

Taking the logarithms and eliminating the reciprocal judgements, a regression model with normal errors is obtained:

$$y_{tij}^{(k)} = \mu_{ti}^{G} - \mu_{tj}^{G} + \varepsilon_{tij}^{(k)}, \quad i = 1,..., n_t - 1, \quad j = 1,..., n_t, \quad k = 1,..., r, \quad t = 1,..., T,$$

and

$$\varepsilon_{tij}^{(k)} \sim N\left(0, \sigma^{(k)2}\right)$$

Here, for each set, E_{tn_t} is established as the benchmark alternative:

$$\mu_{tn_t} = 0 \Leftrightarrow v_{tn_t} = 0.$$

In matrix notation, model can be written as:

$$\mathbf{y}_t^{(k)} = \mathbf{X}_t \boldsymbol{\mu}_t^G + \boldsymbol{\varepsilon}_t^{(k)}$$

with

$$\boldsymbol{\varepsilon}_t^{(k)} \sim N_{s_t}\left(0, \sigma^{(k)2} \mathbf{I}_{s_t}\right),$$

where

$s_t = n_t(n_t - 1)/2$ is the total number of judgements by all decision makers in set t,

$$\mathbf{y}_t^{(k)} = (y_{t12}^{(k)}, y_{t13}^{(k)}, ..., y_{tn-1n}^{(k)})', \quad k = 1,..., r;$$

$$\boldsymbol{\mu}_t^G = (\mu_{t1}^G, \mu_{t2}^G, ..., \mu_{tn-1}^G)',$$

$$\boldsymbol{\varepsilon}_t^{(k)} = (\varepsilon_{t12}^k, \varepsilon_{t13}^k, ..., \varepsilon_{tn-1n}^k)'$$

and

$$\mathbf{X}_{t(s_t \times n_t - 1)} = (x_{tpq})$$

with

(i) $x_{tpi} = 1$, $x_{tpj} = -1$ and $x_{tp\lambda} = 0$, if $\lambda \neq i, j$ $\lambda = 1,\ldots,n_t - 1$ and $p = \frac{2n_t - i}{2}(i-1) + (j-1)$ with $1 \leq i < j \leq n_t$,

(ii) $x_{tpi} = 1$ and $x_{tp\lambda} = 0$, if $\lambda \neq i$, $\lambda = 1,\ldots,n_t - 1$ and $p = \frac{2n_t - i}{2}(i-1) + (n_t - i)$.

With a constant non-informative distribution as the prior distribution for the vector of log-priorities, $\boldsymbol{\mu}_t^G$, the posterior distribution of $\boldsymbol{\mu}_t^G$ for complete and precise information is given by:

$$\boldsymbol{\mu}_t^G \mid \mathbf{y}_t \sim \mathrm{N}_{\mathrm{n_t\text{-}1}}\left(\hat{\boldsymbol{\mu}}_{tB}, \hat{\boldsymbol{\Sigma}}_{tB}\right),$$

where

$$\hat{\boldsymbol{\mu}}_{tB} = \frac{\sum_{k=1}^{r} \tau^{(k)} \hat{\boldsymbol{\mu}}_t^{(k)}}{\sum_{k=1}^{r} \tau^{(k)}},$$

$$\hat{\boldsymbol{\Sigma}}_{tB} = \left(\sum_{k=1}^{r} \tau^{(k)}\right)^{-1} \left(\mathbf{X}_t{}'\mathbf{X}_t\right)^{-1} \begin{pmatrix} 2/n_t & 1/n_t & \ldots & 1/n_t \\ 1/n_t & 2/n_t & \ldots & 1/n_t \\ \ldots & \ldots & \ldots & \ldots \\ 1/n_t & 1/n_t & \ldots & 2/n_t \end{pmatrix},$$

$\tau^{(k)} = 1/\sigma^{(k)2}$ and $\mathbf{y}_t = (\mathbf{y}_t^{(1)'}, \mathbf{y}_t^{(2)'}, \ldots, \mathbf{y}_t^{(r)'})'$.

For the conventional procedure, AIP, the most commonly used procedure to aggregate group judgements is the geometric mean method. It can be presented as:

$$\hat{\boldsymbol{\mu}}_{AIP} = \frac{1}{r}\sum_{k=1}^{r} \hat{\boldsymbol{\mu}}_t^{(k)} ,$$

where

$$\hat{\boldsymbol{\mu}}_t^{(k)} = (\hat{\mu}_{t1}^{(k)},...,\hat{\mu}_{tn_T-1}^{(k)}) \text{ with } \hat{\mu}_{ti}^{(k)} = \bar{y}_{ti.}^{(k)} - \bar{y}_{tn_t.}^{(k)} .$$

For both methods, the priorities of the group, which will constitute the columns of the supermatrix can then be computed as

$$\hat{w}_{ti}^{G} = \exp\left(\hat{\boldsymbol{\mu}}_{ti}^{G}\right) \Big/ \sum_{j=1}^{5} \exp\left(\hat{\boldsymbol{\mu}}_{tj}^{G}\right).$$

4.3.2 Statistical model for incomplete or inconsistent judgements

BPP can be extended to the case of incomplete information, where it performs a more robust manner compared to the conventional methods in terms of variability. In such case, the matrix notation could be expressed as:

$$\mathbf{y}_t^{(k)} = \mathbf{X}_t \boldsymbol{\mu}_t^G + \boldsymbol{\varepsilon}_t^{(k)}$$

with

$$\boldsymbol{\varepsilon}_t^{(k)} \sim N_{s_{tk}}\left(0, \sigma^{(k)2} \mathbf{I}_{s_{tk}}\right),\ k = 1,...,r,\ t = 1,...,T;$$

and in the matrix form it can be expressed as:

$$\mathbf{y}_t = \mathbf{X}_t \left(\mathbf{1_r} \otimes \mathbf{I}_{\mathbf{n}_t - \mathbf{1}}\right) \boldsymbol{\mu}_t^G + \boldsymbol{\varepsilon}_t$$

with

$$\boldsymbol{\varepsilon}_t \sim N_{s_{tk}}\left(0, \mathbf{D}\right),$$

where s_{tk} is the number of judgements issued by k^{th} decision maker (DM_k) in t^{th} set of comparisons,

$$\mathbf{y}_t = (\mathbf{y}_t^{(1)'}, \mathbf{y}_t^{(2)'}, ..., \mathbf{y}_t^{(r)'})',$$

$$\mathbf{X}_t = diag(\mathbf{X}_t^{(1)}, \mathbf{X}_t^{(2)}, ..., \mathbf{X}_t^{(r)}),$$

$$\boldsymbol{\varepsilon}_t = (\boldsymbol{\varepsilon}_t^{(1)'}, \boldsymbol{\varepsilon}_t^{(2)'}, ..., \boldsymbol{\varepsilon}_t^{(r)'})',$$

$$\mathbf{1}_r = (1,1,...,1)',$$

$$\mathbf{D} = diag(\sigma^{(1)2}\mathbf{I}_{s_{t1}}, ..., \sigma^{(r)2}\mathbf{I}_{s_{tr}}),$$

$s_t = s_{t1} + ... + s_{tr}$ is the total number of judgements in set t by all decision makers

and

$\otimes$ denotes the Kronecker product.

With a constant non-informative distribution as the prior distribution for the vector of log-priorities $(\boldsymbol{\mu}_t^G)$, the posterior distribution of $\boldsymbol{\mu}_t^G$ for incomplete and precise information is given by:

$$\boldsymbol{\mu}_t^G \mid \mathbf{y}_t \sim \mathrm{N}_{n_t-1}(\hat{\boldsymbol{\mu}}_{tB}, \hat{\boldsymbol{\Sigma}}_{tB}),$$

where

$$\hat{\boldsymbol{\mu}}_{tB} = \left(\sum_{k=1}^{r} \tau^{(k)} \mathbf{X}_t^{(k)}{'} \mathbf{X}_t^{(k)}\right)^{-1} \left(\sum_{k=1}^{r} \tau^{(k)} \mathbf{X}_t^{(k)}{'} y_t^{(k)}\right)^{-1}$$

$$= \left\{ \left(\mathbf{1}_r \otimes \mathbf{I}_{n_t-1}\right) \left(\mathbf{X}_t{'} \mathbf{D}^{-1} \mathbf{X}_t\right) \left(\mathbf{1}_r \otimes \mathbf{I}_{n_t-1}\right) \right\}^{-1} \left(\mathbf{1}_r \otimes \mathbf{I}_{n_t-1}\right) \left(\mathbf{X}_t{'} \mathbf{D}^{-1} \mathbf{y}_t\right),$$

$$\hat{\boldsymbol{\Sigma}}_{tB} = \left(\sum_{k=1}^{r} \tau^{(k)} \mathbf{X}_t^{(k)}{'} \mathbf{X}_t^{(k)}\right)^{-1}.$$

The estimator of $\boldsymbol{\mu}_t^G$ obtained by means of the AIP procedure is given by:

$$\hat{\boldsymbol{\mu}}_{t\,AIP} = \frac{1}{r} \sum_{k=1}^{r} \hat{\boldsymbol{\mu}}_t^{(k)} = \frac{1}{r} \sum_{k=1}^{r} \left(\mathbf{X}_t^{(k)}{'} \mathbf{X}_t^{(k)}\right)^{-1} \left(\mathbf{X}_t^{(k)}{'} y_t^{(k)}\right) \frac{1}{r} \left(\mathbf{1}_r \otimes \mathbf{I}_{n-1}\right) \left(\mathbf{X}_t^{(k)}{'} \mathbf{X}_t^{(k)}\right)^{-1} \left(\mathbf{X}_t^{(k)}{'} y_t\right).$$

4.4 A Real World Example

In this section, both the AIP method and the proposed BPP methods are applied in ANP-GDM analysis of the same example and the results are compared.

4.4.1 Introduction

Let us consider the group decision analysis problem on diagnosis with dependent symptoms taken from (Saaty & Vargas, 1998). They defined a medical diagnosis problem with dependent symptoms, which makes determining the disease from which they originate difficult. The treatment is based on the diagnoses which in turn depend on how well physicians can isolate the relationships between symptoms and diseases. The patient has 7 symptoms (anemia, low platelet counts, blood clots, elevated PTT reflecting clotting abnormalities, abnormal liver tests reflecting inflammation, elevated ANA and ACA titers reflecting the presence of abnormal antibodies against the patient's own cellular component) and 4 possible diagnoses (SLE, TTP, HELLP, ACA). They are given with the 2 alternative treatments in Table 4.1.

Table 4.1 List of symptoms, diseases and treatments.

Symptoms	Diseases	Treatments
S_1- Anemia S_2- Low platelet S_3- AB Liver S_4- Blood clots S_5- APTT-H S_6- ANA-H S_7- ACA-H	D_1- Lupus D_2- TTP D_3- HELLP D_4- ACA	T_1-Terminate T_2-Not terminate

4.4.2 Application

For the same scenario, we assume a case study where five physicians needed to decide whether or not to terminate the pregnancy as part of the therapy for the patient.

We note that the application of the ANP-GDM analysis could contain some complexities for this problem. First of all, it is a very hard task to make a decision about a human's life. The physicians participated in the analysis might have a limited experience on the relationships of some symptoms or combinations of symptoms and diseases. They might hesitate to answer all of the questions in the analysis completely for some reasons. So they might express incomplete judgements. They also might not have sufficient information about the ANP analysis and its requirements or they might have limited attention which may result in inconsistent situations. Moreover, expressing complete and consistent judgements is difficult with a large number of clusters and elements. In this model, there are 7 dependent symptoms, 4 probable diseases and 2 alternative treatments that were given in Table 4.1.

Consequently, we aimed to solve this problem with the proposed method in order to investigate if it would provide a more efficient and flexible way of diagnostic analysis and treatment selection even with incomplete pairwise comparison matrices.

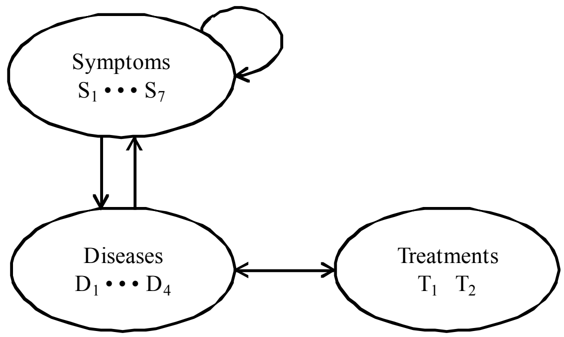


Figure 4.2. The ANP diseases-treatments network.

The network model to solve this problem is given in Figure 4.2. For each decision maker (DM_k), the supermatrix corresponding to this network is given by:

$$W^{(k)} = \begin{array}{r} \\ Diseases \\ Symptoms \\ Treatments \end{array} \begin{array}{c} \begin{array}{ccc} D & S & T \end{array} \\ \begin{pmatrix} 0 & \alpha_1 W_{12}^{(k)} & 0 \\ \beta_1 W_{21}^{(k)} & \alpha_2 W_{22}^{(k)} & 0 \\ \beta_2 W_{31}^{(k)} & 0 & I \end{pmatrix} \end{array}$$

The problem can be solved in two parts if we reduce the supermatrix $W^{(k)}$ to a simpler supermatrix, that is:

$$Q^{(k)} = \begin{pmatrix} 0 & \alpha_1 W_{12}^{(k)} \\ W_{21}^{(k)} & \alpha_2 W_{22}^{(k)} \end{pmatrix}$$

In the first part of the analysis, the supermatrix $Q^{(k)}$ is obtained and in the second part, the composite priorities of the alternatives are calculated. We essentially deal with the first part of the analysis which includes the dependence structure.

Here, $W_{12}^{(k)}$ corresponds to the likelihoods of the diseases producing the symptom observed, $W_{21}^{(k)}$ corresponds to the likelihoods of the symptoms associated with a given disease, and $W_{22}^{(k)}$ corresponds to the strengths of the relationships among symptoms. α_1 and α_2 give the weights of the clusters of diseases and symptoms, respectively. In this study they are assumed to be equal $(\alpha_1 = \alpha_2 = 0.5)$.

In order to obtain the columns of $W_{12}^{(k)}$, $W_{21}^{(k)}, W_{22}^{(k)}$, (Saaty & Vargas, 1998) mentioned that the physicians have to answer the following questions respectively to obtain the required pairwise comparison matrices:

(i) Given a symptom and two diseases, which disease is more likely to exhibit this symptom, and how much more likely is it?
(ii) Given a disease and two symptoms, which symptom is more characteristic of the disease, and how strongly is it?
(iii)Given a symptom, which of the symptoms is more likely to be associated with or occur jointly with the given symptom and how much more likely is it?

We assume that there is a group of physicians, who are not forced to give complete answers to the pairwise comparison matrices. In order to illustrate this case, we simulated data based on the fundamental scale proposed by Saaty (1990) for each set of paired comparisons. Table 4.2 presents the simulated data, where 5 physicians, i.e. decision makers (DM_k) compared 4 diseases in terms of 7 symptoms by answering the first question above. DM_4 did not compare S_1 with S_6, and S_2 with S_6 in the first set of comparisons. DM_5 did not compare S_1 with S_7, and S_4 with S_7 in the second set which resulted in incomplete judgement situations. Similar situations can be noticed for the remaining sets of comparisons.

Table 4.2 Simulated pairwise comparisons of 7 symptoms in terms of 4 diseases.

S_i–S_j of D_1	1-2	1-4	1-5	1-6	1-7	2-4	2-5	2-6	2-7	4-5	4-6	4-7	5-6	5-7	6-7
DM_1	2	3	1/3	1	1	1/2	1/9	1/3	1/5	1/7	1/4	1/7	1/2	5	5
DM_2	6	5	1/5	3	1/2	2	1/7	1	1	1/9	1/2	1/9	1/3	9	9
DM_3	3	**1/2**	1/4	5	1/4	1	1/8	1	1	1/9	1/2	1/9	1/4	9	9
DM_4	5	4	1/3	NA	**7**	1	1/9	NA	1/2	1/6	1/3	1/9	1	9	9
DM_5	**1/2**	5	1/5	2	2	1/2	1/9	1/5	1	1/5	**3**	1/9	1/2	4	4
$oc_{tij} \geq 7$	12	10			28						12				

S_i–S_j of D_2	1-2	1-4	1-5	1-7	2-4	2-5	2-7	4-5	4-7	5-7
DM_1	1/2	5	5	5	9	5	8	1/2	1/2	2
DM_2	1/2	7	7	9	7	9	8	2	2	**1/3**
DM_3	1/5	6	4	5	9	6	7	1	1/3	1
DM_4	1/3	3	3	**1**	9	3	1	**5**	1	1
DM_5	1/5	5	**1**	NA	9	9	9	2	NA	3
$oc_{tij} \geq 7$			7	9			9	10		9

S_i–S_j of D_3	1-2	1-3	1-5	2-3	2-5	3-5
DM_1	1/2	1/2	3	1	5	5
DM_2	2	1	7	1	8	7
DM_3	1	2	5	2	2	3
DM_4	1/2	**4**	7	2	9	5
DM_5	**4**	1/2	**1**	1/2	3	5
$oc_{tij} \geq 7$	8	8	7			

S_i–S_j of D_4	1-2	1-4	1-5	1-7	2-4	2-5	2-7	4-5	4-7	5-7
DM_1	1/2	1/3	1/3	1/4	1/2	1/3	1	2	2	**3**
DM_2	1/2	1/5	1/5	1/4	1/4	1	1/3	1	1/2	1/2
DM_3	1/5	1/5	1/5	1/4	1/4	1	1/3	1	1	1/3
DM_4	1	1/2	NA	1/2	**3**	NA	1/2	2	**1/5**	1/2
DM_5	**3**	1/4	3	1/3	1/2	1/2	1	3	2	1
$oc_{tij} \geq 7$	15				12				10	9

Table 4.3 Simulated pairwise comparisons of 4 diseases in terms of 7 symptoms.

D_i–D_j of S_1	1-2	1-3	1-4	2-3	2-4	3-4
DM_1	1/6	1/7	1/6	1	1	1
DM_2	1/7	1/9	1/8	1/2	1/2	2
DM_3	1/5	1/5	1/7	1	2	3
DM_4	1/5	1/5	**1**	1/2	1	2
DM_5	1/4	1/2	1/5	NA	**5**	1/2
$oc_{tij}\geq 7$			8		10	

D_i–D_j of S_2	1-2	1-3	1-4	2-3	2-4	3-4
DM_1	1/2	1/2	3	1	5	5
DM_2	2	1	7	1	8	7
DM_3	1	2	5	2	2	3
DM_4	1/2	**4**	7	2	9	5
DM_5	**4**	1/2	**1**	1/2	3	5
$oc_{tij}\geq 7$	8	8	7			

D_i–D_j of S_4	1-2	1-3	1-4	2-3	2-4	3-4
DM_1	3	1	1/3	1	1/7	1/4
DM_2	5	4	1	1/2	1/5	1/6
DM_3	4	3	**3**	1/2	1/8	1/3
DM_4	NA	3	1/2	1/3	1/6	**3**
DM_5	1	**1/3**	1/2	1/2	1/4	**5**
$oc_{tij}\geq 7$		12	9			30

D_i–D_j of S_5	1-2	1-3	1-4	2-3	2-4	3-4
DM_1	3	5	1/4	1	1/9	1/8
DM_2	7	3	1/2	1/2	1/8	1/9
DM_3	5	6	1/3	2	NA	NA
DM_4	5	5	**3**	1	1/9	1/7
DM_5	4	3	1/3	1/3	**1**	**2**
$oc_{tij}\geq 7$			9		9	18

D_i–D_j of S_6	1-2	1-3	1-4	2-3	2-4	3-4
DM_1	9	9	5	1	1	1
DM_2	7	7	7	1	1/2	1/2
DM_3	5	6	9	2	2	**4**
DM_4	7	9	5	1/2	1	1
DM_5	**1**	5	**1**	1/3	1/3	1/4
$oc_{tij}\geq 7$	9		9			16

D_i–D_j of S_7	1-2	1-3	1-4	2-3	2-4	3-4
DM_1	1	3	1/6	1	1/9	1/8
DM_2	NA	4	NA	2	1/8	1/5
DM_3	NA	5	0	1	1/9	NA
DM_4	1	3	NA	1/2	1/8	1/5
DM_5	3	5	1/8	1	1/8	1/5
$oc_{tij}\geq 7$						

Table 4.3 presents the simulated data, where *5* physicians compared related symptoms in terms of *4* diseases by answering the second question. Table 4.4 presents the simulated data, where *5* physicians compared related symptoms in terms of *6* symptoms by answering the third question. Diseases are not compared for S_3 since that symptom is only related with one disease, and symptoms are not compared for S_1 since that symptom is not related with others. There are 16 comparison sets at total.

Table 4.4 Simulated pairwise comparisons of 7 symptoms in terms of 7 symptoms.

S_i–S_j of S_2	3-6
DM_1	1/2
DM_2	1
DM_3	1
DM_4	1/3
DM_5	1/3
oc_{tij}≥7	

S_i–S_j of S_4	2-5	2-6	2-7	5-6	5-7	6-7
DM_1	1/4	7	1/4	8	1	1/8
DM_2	1/3	**1/2**	1/4	6	1/2	1/5
DM_3	1/2	3	1/5	4	2	1/4
DM_4	1	4	1/2	7	**4**	1/9
DM_5	1/4	2	1/5	NA	1	**2**
oc_{tij}≥7		14			8	18

S_i–S_j of S_3	1-2
DM_1	1/4
DM_2	1/2
DM_3	1/5
DM_4	1/2
DM_5	1/6
oc_{tij}≥7	

S_i–S_j of S_5	2-4	2-6	2-7	4-6	4-7	6-7
DM_1	1/4	3	1/4	9	1	1/9
DM_2	1/2	5	1/2	9	5	1/2
DM_3	1/7	3	1/5	9	5	1/5
DM_4	1/3	2	**2**	7	1	1/4
DM_5	**1**	1	1/7	7	1	1/4
oc_{tij}≥7	7		14			

S_i–S_j of S_6	1-2	1-4	1-7	2-4	2-7	4-7
DM_1	1	1	2	1	2	**1/2**
DM_2	4	5	4	3	2	2
DM_3	3	7	9	5	3	3
DM_4	1	**1/3**	9	1	3	5
DM_5	2	NA	2	2	3	4
oc_{tij}≥7		21				8

S_i–S_j of S_7	1-2	1-4	1-5	1-6	2-4	2-5	2-6	4-5	4-6	5-6
DM_1	1	1/5	1/4	1	1/5	1/9	1/2	1/2	3	2
DM_2	3	1	1/2	3	1/9	1/8	1/4	1	NA	**7**
DM_3	3	1	1/4	4	1/7	1/7	1/3	1	5	3
DM_4	**7**	1	1/2	**1/3**	1/9	1/8	1/4	3	1	1/2
DM_5	2	1/5	1/4	2	1/9	**1**	1/5	3	4	2
oc_{tij}≥7	7			12		9				14

Applying the model and the equations given in Section 4, simulated data is used to construct the columns of $W_{12}^{(k)}$, $W_{21}^{(k)}$ and $W_{22}^{(k)}$ respectively.

The opening coefficients (oc_{tij}) reflect the variability of judgements expressed by decision makers, and are calculated by:

$$Max_k r_{tij}^k \big/ Min_k r_{tij}^k, \quad k = 1,\ldots,5,\ 1 \le i < j \le n_t,\ t = 1,\ldots,16.$$

In this study we omitted the most inconsistent judgements which cause oc_{tij} to be large. In Tables 4.2-4.4, the oc_{tij} lines are presented to illustrate the omitting procedure.

In Tables 4.2-4.4, the incomplete judgements are written as “NA” and the judgements which are selected to be omitted are given in bold. It can be concluded that five decision makers have a consensus in general, where DM_1 and DM_2 are the most consistent ones and DM_4 and DM_5 are the most inconsistent ones. Looking at the oc_{tij} line, it can also be concluded that DM_4 and DM_5 seem to pay less attention while expressing their judgements compared to others so the omitted judgements mostly belong to them.

It is assumed that consensus exists among the decision makers with regard to the priorities for each alternative. The degree of inconsistency for each decision maker is assumed to be known, $\sigma^{(k)2} = (0.03, 0.06, 0.14, 0.20, 0.36),$ and below the threshold.

4.4.3 Statistical Results for the Example

Both the AIP method and the BPP have been applied for aggregating judgements in group ANP analysis respectively. After omitting the inconsistent judgements given in bold, the methods are applied again and are named as AIP* and BPP*. Tables 4.5-4.8 show the supermatrices obtained using the AIP, BPP, AIP* and BPP* methods for the group priorities respectively.

Table 4.5 AIP-Q block of the supermatrix.

$Q=$

	D_1	D_2	D_3	D_4	S_1	S_2	S_3	S_4	S_5	S_6	S_7
D_1	0.000	0.000	0.000	0.000	0.032	0.160	0.000	0.140	0.159	0.317	0.092
D_2	0.000	0.000	0.000	0.000	0.156	0.157	0.000	0.054	0.038	0.056	0.033
D_3	0.000	0.000	0.000	0.000	0.162	0.148	0.500	0.125	0.043	0.058	0.088
D_4	0.000	0.000	0.000	0.000	0.151	0.035	0.000	0.181	0.261	0.070	0.288
S_1	0.185	0.267	0.319	0.095	0.500	0.000	0.145	0.000	0.000	0.215	0.075
S_2	0.060	0.510	0.314	0.134	0.000	0.000	0.355	0.072	0.073	0.145	0.025
S_3	0.000	0.000	0.296	0.000	0.000	0.279	0.000	0.000	0.000	0.000	0.000
S_4	0.069	0.071	0.000	0.310	0.000	0.000	0.000	0.000	0.246	0.089	0.185
S_5	0.095	0.068	0.071	0.210	0.000	0.000	0.000	0.201	0.000	0.000	0.150
S_6	0.484	0.000	0.000	0.000	0.000	0.221	0.000	0.038	0.032	0.000	0.065
S_7	0.107	0.085	0.000	0.251	0.000	0.000	0.000	0.190	0.150	0.052	0.000

Table 4.6 BPP-Q block of the supermatrix.

$Q=$

	D_1	D_2	D_3	D_4	S_1	S_2	S_3	S_4	S_5	S_6	S_7
D_1	0.000	0.000	0.000	0.000	0.028	0.150	0.000	0.141	0.149	0.334	0.080
D_2	0.000	0.000	0.000	0.000	0.149	0.165	0.000	0.049	0.036	0.052	0.032
D_3	0.000	0.000	0.000	0.000	0.184	0.153	0.500	0.089	0.041	0.053	0.063
D_4	0.000	0.000	0.000	0.000	0.140	0.033	0.000	0.222	0.274	0.062	0.326
S_1	0.181	0.285	0.299	0.083	0.500	0.000	0.145	0.000	0.000	0.212	0.071
S_2	0.058	0.506	0.329	0.135	0.000	0.000	0.355	0.068	0.071	0.141	0.025
S_3	0.000	0.000	0.306	0.000	0.000	0.279	0.000	0.000	0.000	0.000	0.000
S_4	0.061	0.065	0.000	0.310	0.000	0.000	0.000	0.000	0.250	0.089	0.177
S_5	0.083	0.068	0.066	0.227	0.000	0.000	0.000	0.198	0.000	0.000	0.165
S_6	0.505	0.000	0.000	0.000	0.000	0.221	0.000	0.032	0.029	0.000	0.063
S_7	0.113	0.077	0.000	0.245	0.000	0.000	0.000	0.203	0.150	0.059	0.000

Table 4.7 AIP*-Q block of the supermatrix.

Q=	D_1	D_2	D_3	D_4	S_1	S_2	S_3	S_4	S_5	S_6	S_7
D_1	0.000	0.000	0.000	0.000	0.030	0.145	0.000	0.121	0.138	0.298	0.092
D_2	0.000	0.000	0.000	0.000	0.147	0.215	0.000	0.042	0.026	0.051	0.033
D_3	0.000	0.000	0.000	0.000	0.185	0.102	0.500	0.112	0.041	0.070	0.088
D_4	0.000	0.000	0.000	0.000	0.139	0.039	0.000	0.226	0.295	0.083	0.288
S_1	0.201	0.323	0.289	0.076	0.500	0.000	0.145	0.000	0.000	0.222	0.062
S_2	0.068	0.453	0.429	0.178	0.000	0.000	0.355	0.087	0.101	0.154	0.036
S_3	0.000	0.000	0.203	0.000	0.000	0.279	0.000	0.000	0.000	0.000	0.000
S_4	0.070	0.064	0.000	0.260	0.000	0.000	0.000	0.000	0.209	0.067	0.182
S_5	0.197	0.088	0.079	0.224	0.000	0.000	0.000	0.140	0.000	0.000	0.161
S_6	0.342	0.000	0.000	0.000	0.000	0.221	0.000	0.050	0.061	0.000	0.059
S_7	0.122	0.072	0.000	0.262	0.000	0.000	0.000	0.224	0.130	0.058	0.000

Table 4.8 BPP*-Q block of the supermatrix.

Q=	D_1	D_2	D_3	D_4	S_1	S_2	S_3	S_4	S_5	S_6	S_7
D_1	0.000	0.000	0.000	0.000	0.027	0.142	0.000	0.122	0.134	0.326	0.080
D_2	0.000	0.000	0.000	0.000	0.149	0.193	0.000	0.038	0.029	0.052	0.032
D_3	0.000	0.000	0.000	0.000	0.187	0.132	0.500	0.081	0.037	0.057	0.063
D_4	0.000	0.000	0.000	0.000	0.138	0.035	0.000	0.260	0.300	0.066	0.327
S_1	0.194	0.320	0.283	0.073	0.500	0.000	0.145	0.000	0.000	0.217	0.061
S_2	0.065	0.470	0.385	0.155	0.000	0.000	0.355	0.087	0.086	0.147	0.032
S_3	0.000	0.000	0.263	0.000	0.000	0.279	0.000	0.000	0.000	0.000	0.000
S_4	0.064	0.062	0.000	0.287	0.000	0.000	0.000	0.000	0.224	0.078	0.174
S_5	0.133	0.081	0.069	0.210	0.000	0.000	0.000	0.151	0.000	0.000	0.179
S_6	0.417	0.000	0.000	0.000	0.000	0.221	0.000	0.043	0.036	0.000	0.053
S_7	0.128	0.067	0.000	0.275	0.000	0.000	0.000	0.220	0.155	0.059	0.000

Raising these matrices to powers, limiting priorities for each are obtained and then normalized. The limiting and normalized priorities are given in Table 4.9. For different sets, each method gives similar weights and almost same ranking but the Bayesian estimates reflect more robust results since the priorities (w_{ti}) does not change too much after omitting the inconsistent judgements.

Table 4.9 Limiting and normalized priorities for AIP, AIP*, BPP, BPP*.

	Limiting priorities					Normalized priorities			
	w_{AIP}	w_{BPP}	w_{AIP^*}	w_{BPP^*}		w_{AIP}	w_{BPP}	w_{AIP^*}	w_{BPP^*}
D_1	0.074	0.075	0.075	0.069	D_1	0.225	0.220	0.224	0.207
D_2	0.063	0.064	0.075	0.066	D_2	0.193	0.188	0.226	0.197
D_3	0.105	0.110	0.110	0.103	D_3	0.321	0.323	0.329	0.310
D_4	0.085	0.092	0.099	0.095	D_4	0.260	0.269	0.296	0.286
S_1	0.203	0.206	0.216	0.198	S_1	0.311	0.302	0.324	0.297
S_2	0.125	0.131	0.153	0.136	S_2	0.192	0.193	0.230	0.205
S_3	0.066	0.070	0.065	0.065	S_3	0.101	0.103	0.098	0.098
S_4	0.068	0.071	0.068	0.067	S_4	0.104	0.104	0.102	0.101
S_5	0.059	0.063	0.073	0.064	S_5	0.091	0.093	0.109	0.096
S_6	0.072	0.075	0.071	0.068	S_6	0.110	0.110	0.107	0.102
S_7	0.060	0.064	0.069	0.068	S_7	0.092	0.094	0.104	0.102

Table 4.10 shows the individual and group mean square errors (MSE_t) of different prioritization methods for each set of comparisons. The MSE_t is calculated for each method with the following formula:

$$MSE_t = \sum_{1\le i<j}^{n}\sum_{k=1}^{r}\varepsilon_{tij}^{(k)} \Big/ \sum_{k=1}^{r} s_{tk} \ .$$

Here, the comparison sets which include only 1 comparison are not included since all methods would give the same MSE values for them.

In order to be fair in comparisons, pairs of plots are provided to show the performance of AIP vs. BPP and AIP* vs. BPP*. For the most consistent decision makers $(DM_1$ and $DM_2)$ the BPP estimates provide smaller MSE_t values than the AIP method.

Table 4.10 Individual and group MSE values for AIP, BPP, AIP*, BPP*.

	AIP	BPP	AIP*	BPP*		AIP	BPP	AIP*	BPP*
	DM_1					*DM_2*			
S_i–S_i of D_1	0.533	0.485	0.719	0.524		0.704	0.663	1.091	0.873
S_i–S_i of D_2	0.477	0.412	0.326	0.318		0.625	0.603	0.596	0.546
S_i–S_i of D_3	0.467	0.403	0.610	0.421		0.460	0.457	0.755	0.568
S_i–S_i of D_4	0.567	0.514	0.453	0.426		0.559	0.526	0.532	0.465
D_i–D_i of S_1	0.186	0.174	0.213	0.177		0.587	0.482	0.506	0.468
D_i–D_i of S_2	0.467	0.403	0.61	0.421		0.460	0.457	0.755	0.568
D_i–D_i of S_4	0.711	0.482	0.542	0.383		0.847	0.599	0.781	0.607
D_i–D_i of S_5	0.443	0.384	0.426	0.348		0.363	0.349	0.235	0.283
D_i–D_i of S_6	0.302	0.221	0.449	0.264		0.357	0.328	0.431	0.339
D_i–D_i of S_7	0.844	0.631	0.844	0.631		1.085	0.901	1.085	0.901
S_i–S_i of S_4	0.628	0.533	0.873	0.766		0.651	0.670	0.493	0.426
S_i–S_i of S_5	0.447	0.413	0.941	0.541		0.696	0.700	0.779	0.698
S_i–S_i of S_6	0.746	0.679	0.748	0.638		0.582	0.582	0.525	0.544
S_i–S_i of S_7	0.586	0.526	0.456	0.429		0.608	0.59	0.737	0.666
	DM_3					*DM_4*			
S_i–S_i of D_1	0.725	0.725	1.018	0.829		0.729	0.677	0.857	0.601
S_i–S_i of D_2	0.489	0.470	0.537	0.522		0.912	0.975	0.432	0.447
S_i–S_i of D_3	0.507	0.564	0.483	0.535		0.747	0.743	0.488	0.410
S_i–S_i of D_4	0.666	0.621	0.597	0.536		0.992	1.028	0.567	0.545
D_i–D_i of S_1	0.524	0.47	0.475	0.462		0.733	0.712	0.292	0.309
D_i–D_i of S_2	0.507	0.564	0.483	0.535		0.747	0.743	0.488	0.410
D_i–D_i of S_4	0.848	0.743	0.553	0.337		0.864	0.987	0.515	0.397
D_i–D_i of S_5	0.583	0.550	0.654	0.564		0.524	0.560	0.278	0.212
D_i–D_i of S_6	0.848	0.81	0.827	0.61		0.369	0.33	0.428	0.333
D_i–D_i of S_7	1.085	0.885	1.085	0.885		0.690	0.574	0.690	0.574
S_i–S_i of S_4	0.460	0.493	0.613	0.557		0.803	0.800	0.676	0.580
S_i–S_i of S_5	0.664	0.641	1.051	0.795		0.624	0.648	0.513	0.200
S_i–S_i of S_6	0.797	0.844	0.747	0.796		1.006	1.047	0.868	0.795
S_i–S_i of S_7	0.579	0.561	0.655	0.598		0.870	0.918	0.834	0.847
	DM_5					*Group*			
S_i–S_i of D_1	0.834	0.846	0.972	0.745		0.506	0.475	0.887	0.532
S_i–S_i of D_2	0.812	0.812	0.751	0.712		0.462	0.465	0.288	0.264
S_i–S_i of D_3	0.957	0.976	0.996	0.741		0.432	0.439	0.452	0.287
S_i–S_i of D_4	0.833	0.906	0.489	0.454		0.533	0.540	0.281	0.235
D_i–D_i of S_1	0.893	0.980	0.746	0.803		0.384	0.370	0.215	0.216
D_i–D_i of S_2	0.957	0.976	0.996	0.741		0.432	0.439	0.452	0.287
D_i–D_i of S_4	1.047	1.294	0.585	0.629		0.757	0.751	0.375	0.235
D_i–D_i of S_5	1.374	1.420	0.408	0.465		0.582	0.600	0.168	0.143
D_i–D_i of S_6	1.206	1.291	0.788	0.918		0.506	0.517	0.346	0.260
D_i–D_i of S_7	0.859	0.691	0.859	0.691		0.820	0.531	0.820	0.531
S_i–S_i of S_4	1.085	1.172	0.612	0.567		0.549	0.571	0.455	0.356
S_i–S_i of S_5	0.811	0.839	0.922	0.732		0.434	0.439	0.755	0.406
S_i–S_i of S_6	0.525	0.539	0.653	0.588		0.574	0.590	0.507	0.463
S_i–S_i of S_7	0.778	0.808	0.629	0.642		0.485	0.490	0.398	0.384

Figures 4.3-4.20 point out that for the consistent decision makers the Bayesian methods provide smaller MSE_t values. As the inconsistency increases, the BPP method and the AIP method provide same values. Nevertheless, for any decision maker, if the inconsistent judgements are deleted, the BPP* provides smaller MSE_t values than the AIP* method. As an example, Figures 4.3-4.5 give the results for DM_1 and indicate that Bayesian method has smaller MSE_t values than the conventional method, especially after omitting the inconsistent judgements.

On the other hand, for the most inconsistent decision maker (DM_5), Figures 4.15-4.17 point out that the difference between BPP and AIP methods is not significant (see Table 4.10) where BPP* still provides smaller MSE_t values than the AIP* method. Figures 4.18-4.20 are the results of the group as a whole and the similar conclusions are obtained for the group.

The omission of these judgements raises the consistency levels of decision makers and also increases the degree of consensus among all of the decision makers for every comparison set. After omitting the inconsistent judgements, the Bayesian estimates generally have the lower levels of individual MSE_t values and best reflect the opinions of the group, as shown by the smaller MSE_t (see Table 4.10). The results show that the proposed methodology is more efficient than the conventional approach especially after omitting the inconsistent judgements.

In the second part of the analysis, $W_{31}^{(k)}$ is obtained where five physicians compare two alternatives for four different diseases answering the question: "which alternative treatment would be most appropriate given the disease in question?". Since there are only two alternatives to compare, each doctor has to give only one pairwise comparison for every disease. We simulate data assuming that physicians give complete answers and the variability is small ($oc_{tij} < 7$). Consequently, there are no judgements to be omitted. We applied the AIP method and BPP for this part of the analysis. Since there is only one entry in the comparison matrices, both methods provided nearly the same results in terms of the priorities and the level of consistencies.

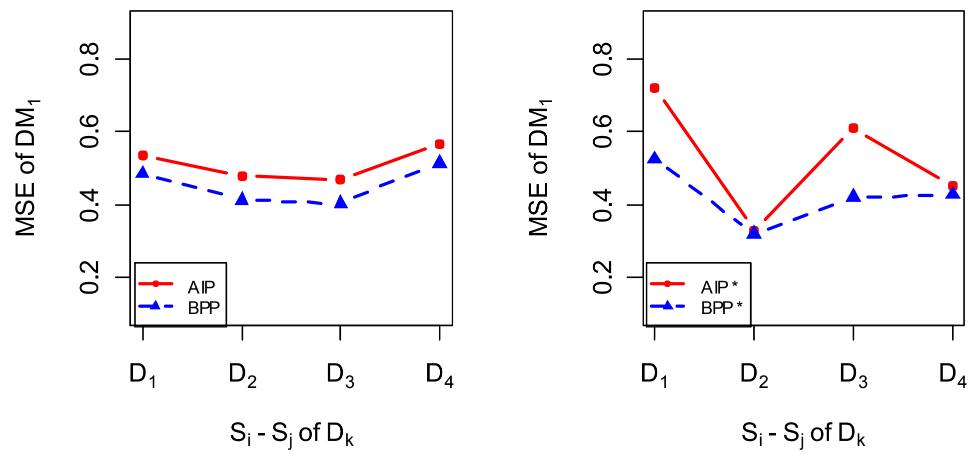


Figure 4.3 MSE values of DM_1, comparing the related symptoms in terms of 4 diseases.

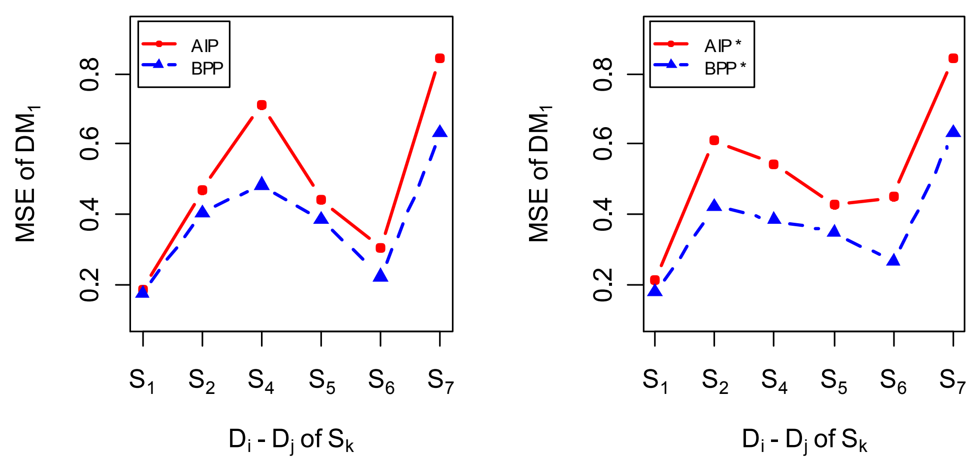


Figure 4.4 MSE values of DM_1, comparing the related diseases in terms of 6 symptoms.

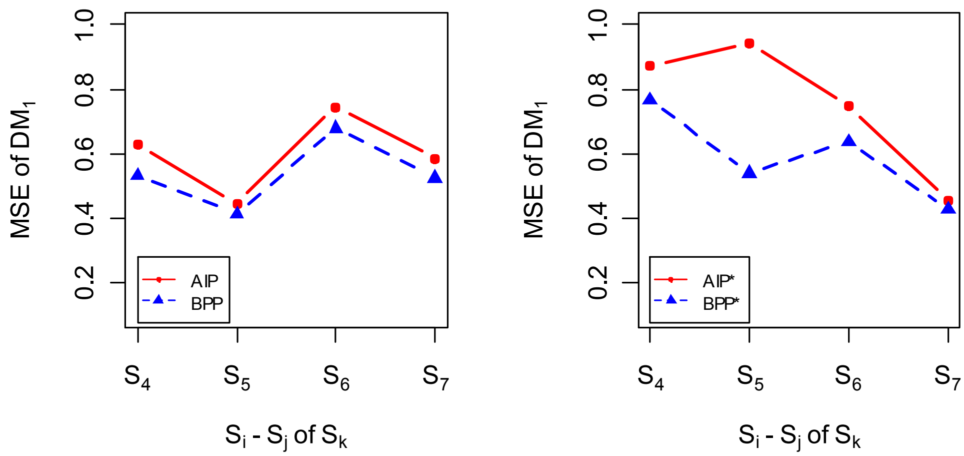
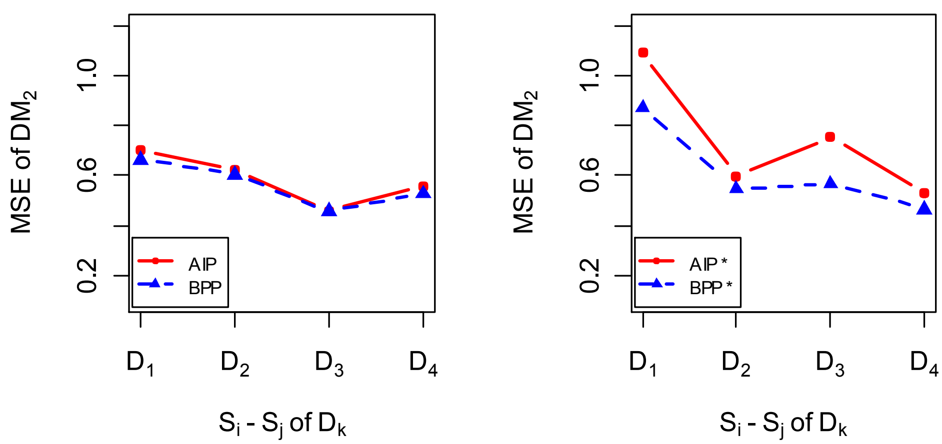


Figure 4.5 MSE values of DM_1 , comparing the related symptoms in terms of 4 symptoms.



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gure 4.6 MSE values of DM_2 , comparing the related symptoms in terms of 4 diseases.

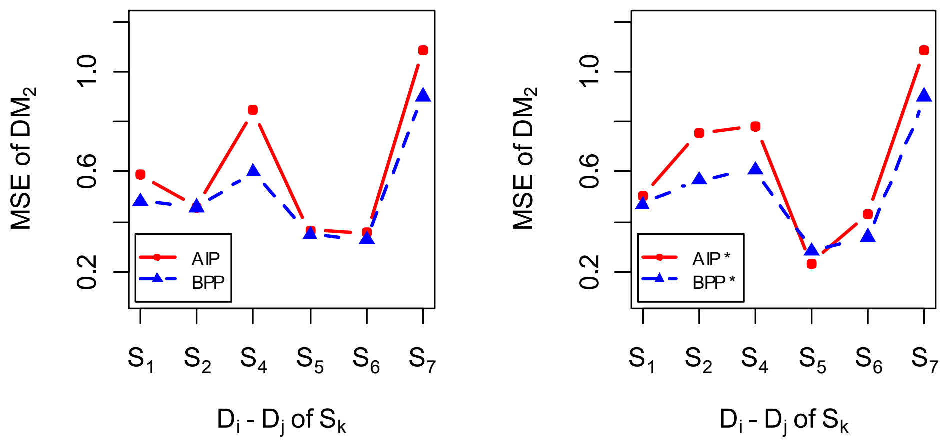

Figure 4.7 MSE values of DM_2, comparing the related diseases in terms of 6 symptoms.

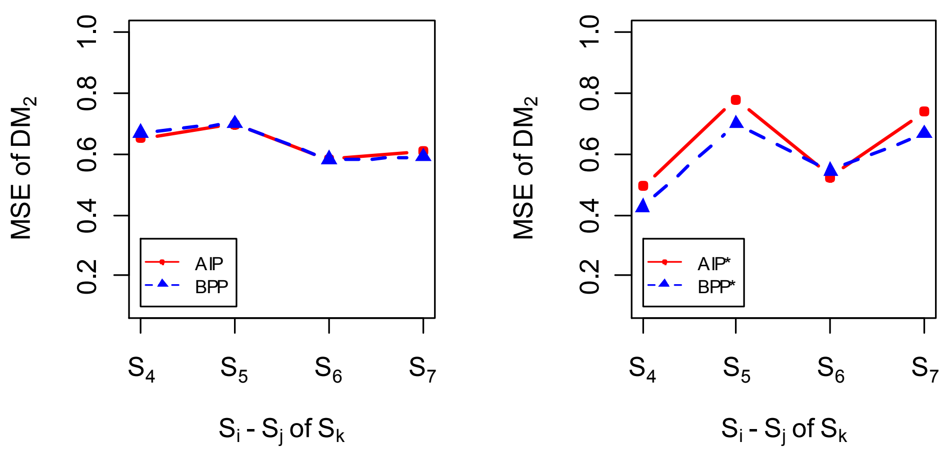

Figure 4.8 MSE values of DM_2, comparing the related symptoms in terms of 4 symptoms.

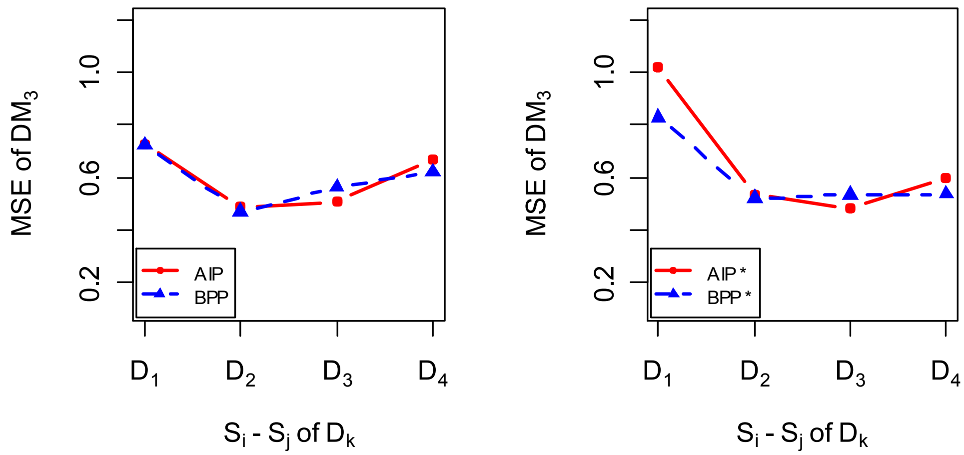

Figure 4.9 MSE values of DM_3, comparing the related symptoms in terms of 4 diseases.

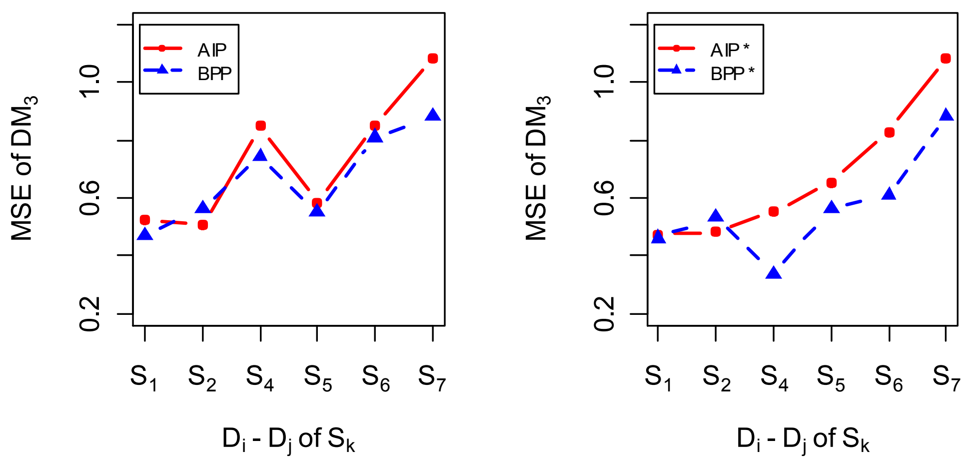

Figure 4.10 MSE values of DM_3, comparing the related diseases in terms of 6 symptoms.

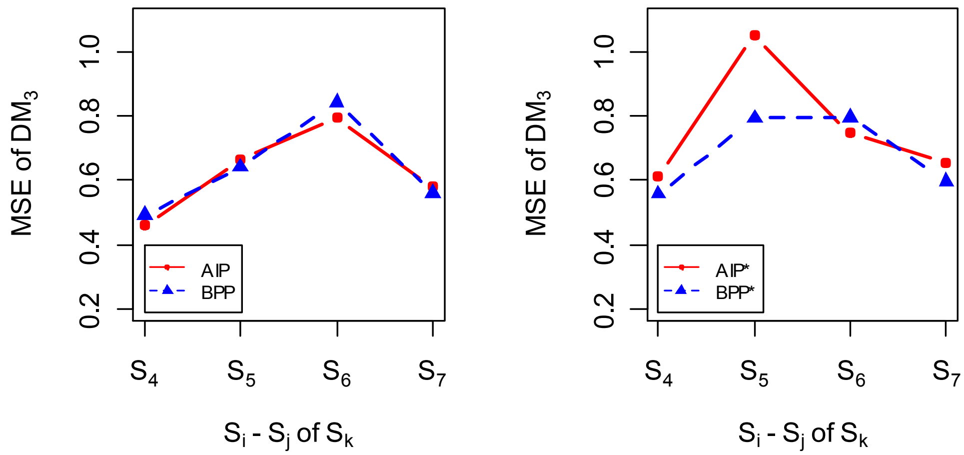

Figure 4.11 MSE values of DM_3 , comparing the related symptoms in terms of 4 symptoms.

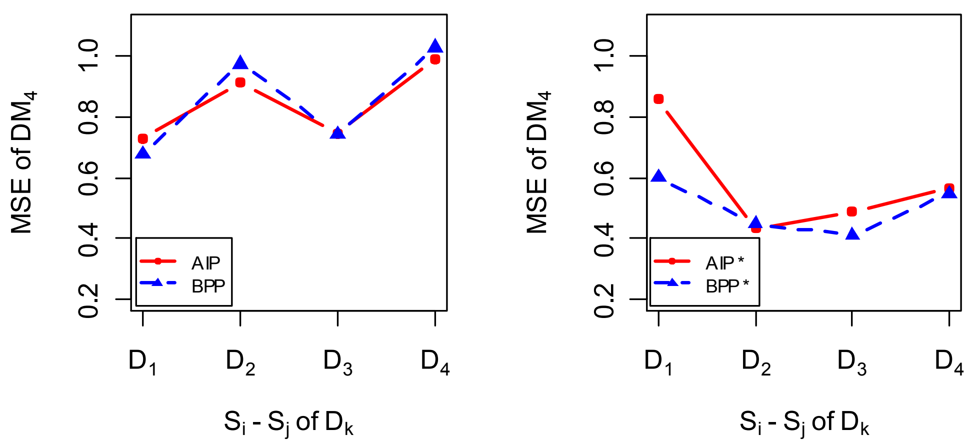

Figure 4.12 MSE values of DM_4, comparing the related symptoms in terms of 4 diseases.

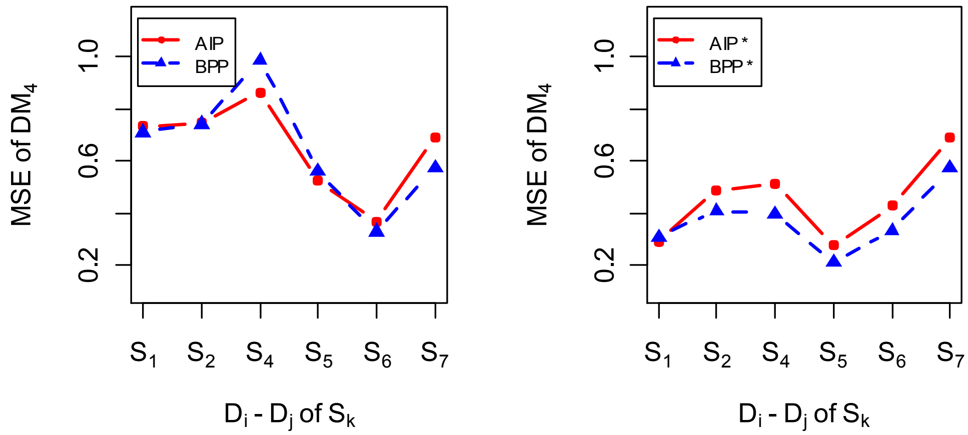

Figure 4.13 MSE values of *DM_4*, comparing the related diseases in terms of 6 symptoms.

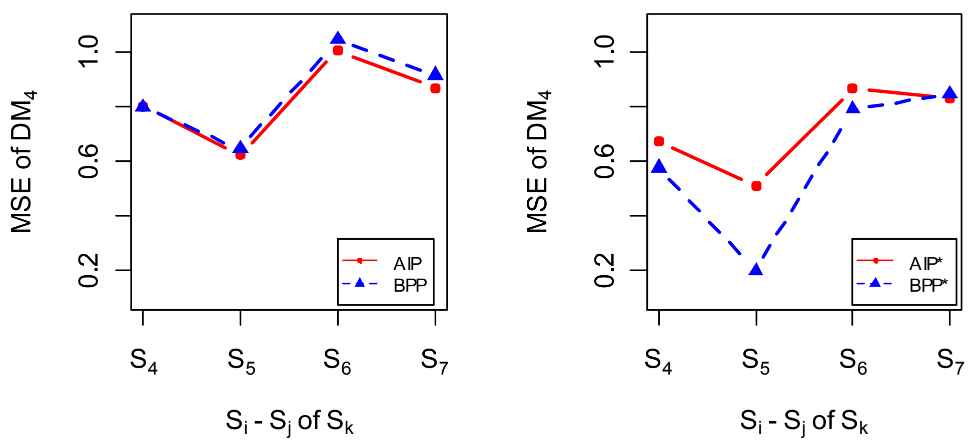

Figure 4.14 MSE values of *DM_4*, comparing the related symptoms in terms of 4 symptoms.

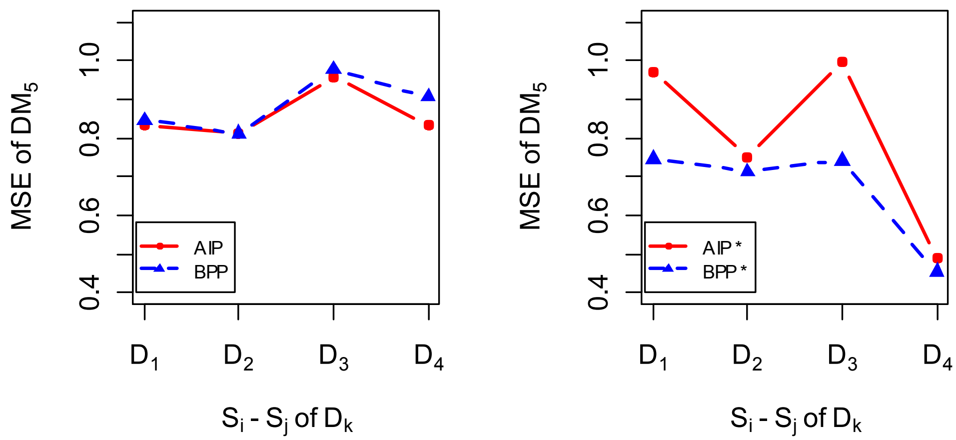

Figure 4.15 MSE values of DM_5, comparing the related symptoms in terms of 4 diseases.

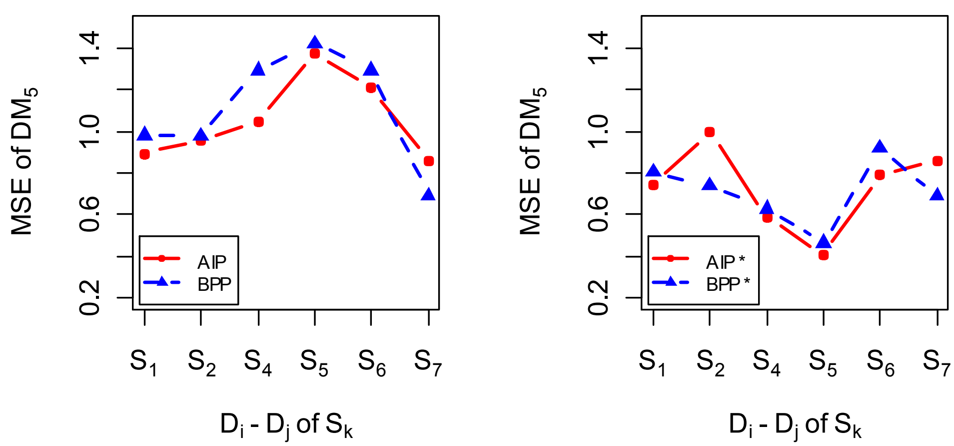

Figure 4.16 MSE values of DM_5, comparing the related diseases in terms of 6 symptoms.

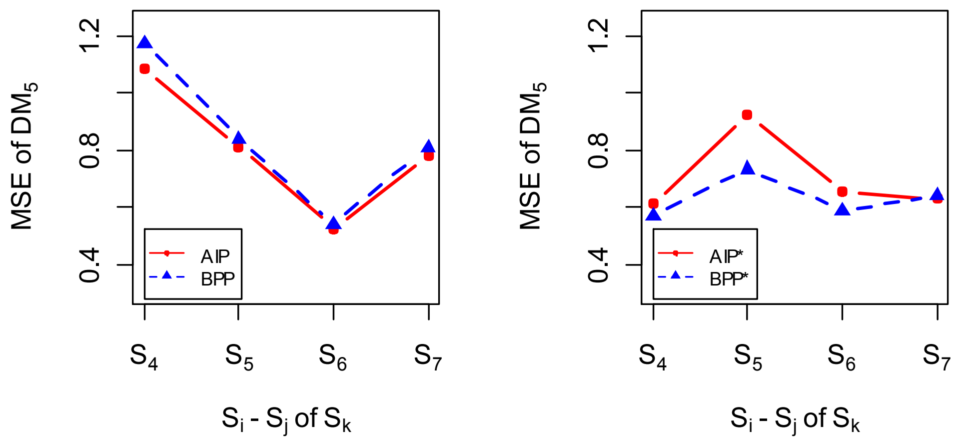

Figure 4.17 MSE values of DM_5 , comparing the related symptoms in terms of 4 symptoms.

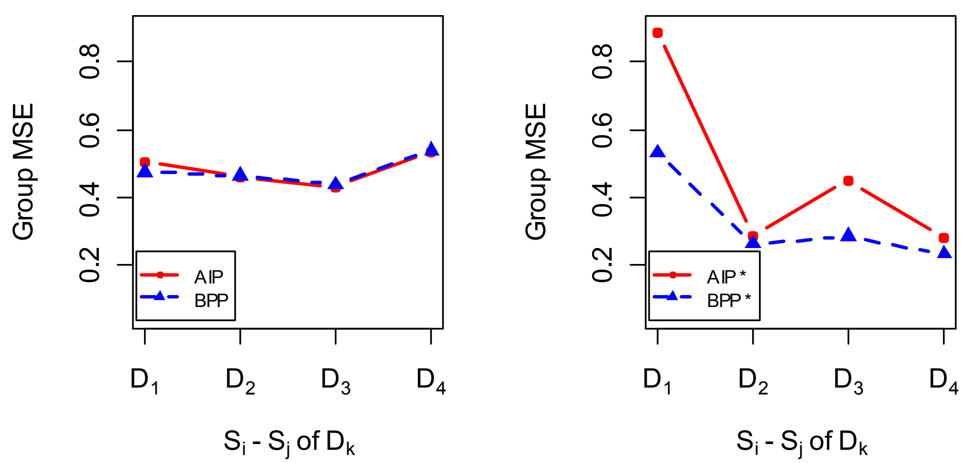

Figure 4.18 MSE values of the group , comparing the related symptoms in terms of 4 diseases.

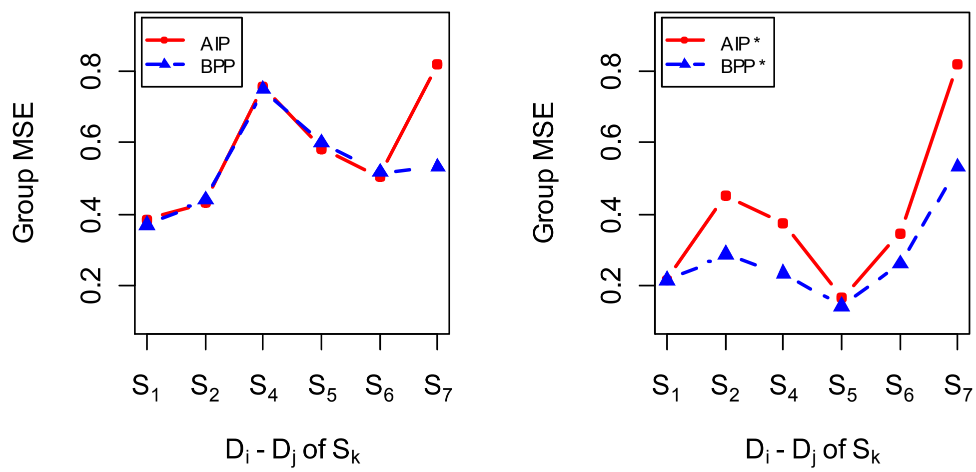

Figure 4.19 MSE values of the group, comparing the related diseases in terms of 6 symptoms.

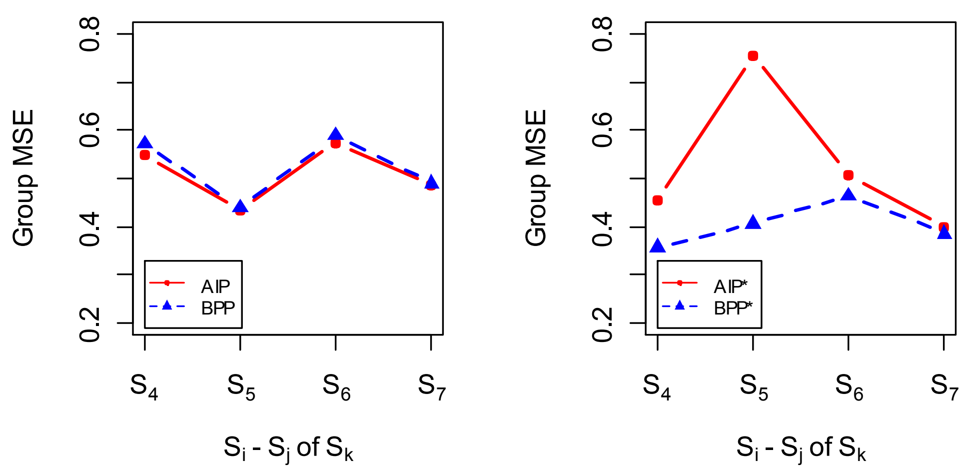

Figure 4.20 MSE values of the group, comparing the related symptoms in terms of 4 symptoms.

Table 4.11 gives the final result, the priorities of the two alternatives. For the AIP method, omitting the inconsistent judgements resulted in a change of $w_{AIP} - w_{AIP*} = 0.663 - 0.739 = -0.076$ in the first alternative's weight, where the change is only $w_{BPP} - w_{BPP*} = 0.692 - 0.684 = 0.008$ for the Bayesian method. The absolute change in the BPP is less than 1/9 of the AIP method. It can be concluded that the Bayesian method performs more robust manner in the overall priorities.

Table 4.11 Alternatives' priorities.

	w_{AIP}	w_{BPP}	w_{AIP*}	w_{BPP*}
A_1	0.663	0.692	0.739	0.684
A_2	0.337	0.308	0.261	0.316

4.5. Conclusions

The ANP-GDM is a widely used technique in multiple criteria group decision making problems. When applying the ANP in a group decision, some problems might occur: The formation of pairwise comparison matrices becomes a complex task for the decision makers as the number of elements in the analysis increase. Also, decision makers participated in the analysis may have limited expertise about the problem domain or the ANP analysis. The practitioner may also prefer to ignore the inconsistent or opposing judgements while keeping the consistent or homogeneous ones in order to increase the consistency or consensus among decision makers. These situations could result in incomplete or inconsistent judgements, which may give wrong results.

Considering the problems mentioned above, we proposed a Bayesian prioritization procedure for the ANP-group decision making which can be extended to the case of incomplete pairwise comparison matrices. The multiplicative model with log-normal errors is applied to the problem and the Bayesian analysis is used. This is a process of weighted aggregation of individual priorities and the weights are inversely proportional to the decision makers' levels of inconsistency.

We illustrated this methodology with a case study and compared it with the conventional technique, the AIP.

With regard to variability, the results show that the proposed methodology generally performs more robust and efficient manner than the conventional approach especially after omitting the incomplete judgements in the pairwise comparison matrices.

In this research, the proposed methodology is illustrated with a medical diagnosis problem which is a very common problem in healthcare. We selected this example since there is a wide application potential of the ANP in healthcare problems.

The proposed procedure can be used to deal with many problems like: financial crisis management, risk management, selecting projects that offer return on investment, education, resource planning, common clinical problems, sustainable forest management, customer requirement management, requirement prioritization, measuring potential ethnic conflict, selecting the power units for appropriate price allocation in a competitive power environment, location selection and etc.

In the previous chapter, the limitations of the proposed Bayesian aggregation procedure were addressed. In this chapter we have removed the independence assumption by applying the ANP-GDM analysis.

Other than that, our study is based on the model from a non-informative Bayesian standpoint, where the variances of error terms represented by the inconsistency levels of decision makers are assumed to be known. In the future, this approach can be extended by taking the variances of error terms as additional parameters, or by implementing an informative Bayesian model in which a good estimate of prior distribution for the vector of log-priorities is used.

CHAPTER FIVE
CONCLUSIONS

The problems whose objective is to search the best alternative or to rank the alternatives in terms of a number of conflicting criteria are the multi-criteria decision making problems. It is hard to solve such kind of problems. Generally, no optimal solution exists for these problems, i.e. none of the alternatives can be concluded as the best one in terms of each criteria. An alternative can be the best one in terms of one criteria, where it can be worse in terms of the other criteria. A wide variety of Multiple criteria decision making (MCDM) techniques have been developed.

The Analytic Hierarchy Process (AHP) and the Analytic Network Process (ANP) are powerful techniques widely used in multiple criteria group decision making problems. They are flexible and practical tools for both the researchers and the practitioners. However, there might be some complexities when applying the AHP and the ANP in complex group decision making problems:

(i) The formation of pairwise comparison matrices becomes a complex task for the decision makers as the number of elements in the analysis increase,

(ii) Decision makers participated in the analysis may have limited expertise about the problem domain or the AHP-ANP analysis and their requirements,

(iii) The practitioner may also prefer to ignore the inconsistent or opposing judgements while keeping the consistent or homogeneous ones in order to increase the consistency or consensus among decision makers.

These three situations could result in incomplete or inconsistent judgements, which may give wrong results.

In order to overcome the problems and the complex situations mentioned above, we proposed Bayesian prioritization procedure (BPP) for the AHP and the ANP group decision making analyses which can also be extended to the case of incomplete pairwise comparison matrices.

In this research, it is assumed that consensus exists among the decision makers with regard to the priorities for each element in this decision system. For both the AHP and the ANP, the multiplicative model with log-normal errors is applied to the problem and the Bayesian analysis is used for both techniques respectively. What we do in these approaches is obtaining the weighted aggregation of individual priorities where the weights are inversely proportional to the decision makers' levels of inconsistency. We compared the Bayesian methods with the conventional approach, AIP, used in the AHP-ANP group decision analysis, with one case study for each.

The superiorities and the advantages of the proposed methodology can be summarized as follows: First of all, it provides decision makers a flexible and comfortable way to express their judgements, without forcing them to give complete and consistent judgements and letting them completely focus on their area of interest. Because in the proposed method, there is no assumption or requirement that the pairwise comparison matrices must be complete and each decision makers' judgements must be consistent. The proposed methodology can treat incomplete or inconsistent judgements properly.

Second, it serves the practitioner since the judgements of decision makers directly enter the analysis without any reducing or filtering process. The practitioner does not have to make any further tests or any rearrangements for the pairwise comparison matrices. The proposed method can be used directly whether the individuals' pairwise comparison matrices in the AHP or the ANP group decision making analyses are complete or incomplete; consistent or inconsistent.

Third, the proposed method automatically gives weights to each of the decision makers, which are inversely proportional to the decision makers' levels of inconsistency. So the consistent decision makers have a higher effect in the final decision.

Fourth, when the inconsistent judgements are omitted, the change in the priorities of the alternatives is smaller when the proposed method is applied instead of the conventional method. This result indicates that the BPP method generally performs more robust manner in terms of the weights of the alternatives than the conventional approach.

Finally, with regard to the variability, the proposed method calculates the final priorities with smaller MSE values. So it can be concluded that the proposed methodology aggregate the individuals' judgements more effectively than the conventional method, especially after omitting the inconsistent judgements in the pairwise comparison matrices.

In this study, we first applied BPP based AHP to prioritize and order information security risk incidents which could satisfy the aim of risk management and then applied BPP based ANP for diagnosis and best treatment selection. These approaches can also be used for many multiple criteria group decision making problems such as project selection, facility location selection, supplier selection or evaluation, diagnosis and treatment selection for disease management, financial decision making and crisis forecasting, and evacuation selection for emergency management.

For any kind of MCDM problems, BPP based AHP or BPP based ANP techniques could be used alone or with any other decision support tools.

In each chapter the limitations of our studies were given in detail. To sum up, it is assumed that (i) there is a consensus among the decision makers with regard to the priorities for each element in this decision system, (ii) both of the case studies are based on the model from a non-informative Bayesian standpoint, which can be extended to an informative Bayesian model.

This research only captures the AHP and the ANP out of many multi-criteria decision making (MCDM) approaches. In the future, the Bayesian prioritization process could also be applied to any other MCDM approaches such as TOPSIS, VIKOR, etc. as a remedy for the incomplete and/or inconsistent judgement situations. This could help us to achieve more general conclusions in terms of the affects of Bayesian methods in MCDM analysis.

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