

**DOKUZ EYLÜL UNIVERSITY GRADUATE SCHOOL OF
NATURAL AND APPLIED SCIENCES**

**UNCERTAINTY ANALYSIS OF MODAL
PARAMETERS OBTAINED FROM SYSTEM
IDENTIFICATION METHODS**

by

Mohammad SALAVATI

June, 2012

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**UNCERTAINTY ANALYSIS OF MODAL
PARAMETERS OBTAINED FROM SYSTEM
IDENTIFICATION METHODS**

**A Thesis Submitted to the Graduate School of Natural and Applied Sciences
of Dokuz Eylül University in Partial Fulfillment of the Requirements for the
Degree of Master of Science in Civil Engineering, Structural Engineering
Program**

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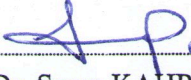
M.Sc. THESIS EXAMINATION RESULT FORM

We have read the thesis entitled “**UNCERTAINTY ANALYSIS OF MODAL PARAMETERS OBTAINED FROM SYSTEM IDENTIFICATION METHODS**” completed by **MOHAMMAD SALAVATI** under supervision of **ASS.PROF.DR. ÖZGÜR ÖZÇELİK** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.



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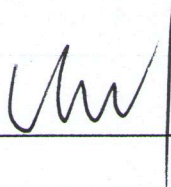
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UNCERTAINTY ANALYSIS OF MODAL PARAMETERS OBTAINED FROM SYSTEM IDENTIFICATION METHODS

ABSTRACT

In civil engineering structures, damage prognosis depends in great extent on accuracy identification of modal parameters (natural frequencies, damping ratios and mode shapes). So, it's extremely significant to investigate the factors affecting the accuracy of identified modal parameters. In order to do this, two different output-only system identification methods, namely, Natural Excitation Technique coupled with Eigensystem Realization Algorithm (NExT-ERA) and Enhanced Frequency Domain Decomposition (EFDD) are programmed in Matlab®. The uncertainty/variability of identified modal parameters due to uncertainty/variability of some input factors such as spatial sensor density, response data length and measurement noise level are investigated using an updated analytical model of a steel bridge. The experimental modal analysis applied to this calibrated model and the acceleration response acquired by sensors which placed on several different point of this model structure's body. Similarity of calibrated finite element model to their real model counterpart is realized by comparing the identified dynamic characteristic of both the actual and analytical models. As a result of this process by changing in mass and stiffness matrices of finite element model, modal assurance criteria (MAC) values is checked to approve that the analytical model is properly updated. Finally, investigation of the input factors realized by simulated data from the calibrated finite element model. Consequently, In order to accurately identify damage in structural health monitoring with non-destructive testing technology, three input factors investigated are very important and must be given utmost attention in system identification process.

Keywords: Structural health monitoring, system identification technique, Experimental and analytical modal analysis

SİSTEM TANIMLAMA YÖNTEMLERİ İLE BELİRLENMİŞ MODAL PARAMETRELERİN BELİRSİZLİK ANALİZİ

ÖZ

Son yıllarda, yapılarda yapı sağlığının gözlenmesi (structural health monitoring) işlemi, yapıların güvenli kullanım süresinin tahmini ve/veya katastrofik olaylardan sonra önemli yapılarda hasar tespitinin yapılabilmesi için önemli bir araç haline gelmiştir. Titreşim-tabanlı tahribatsız hasar değerlendirmesi, yapının modal parametrelerindeki (titreşim frekansları, mod şekilleri ve sönüm oranları) değişim takip edilerek yapılabilmektedir. Bu parametreler, deneysel modal analiz (experimental modal analysis) yöntemleriyle yapının global titreşim verileri kullanılarak tahmin edilebilir. İnşaat mühendisliği yapılarında deneysel modal analiz temelli hasar tespiti, modal parametrelerin doğru tahminine bağlıdır. Bu nedenle doğru tahmini etkileyen faktörlerin incelenmesi son derece önemlidir. Deneysel çalışmada kullanılan model köprü ST37 çelik malzemesinden yapılmış olup, 6.0 m açıklığa ve 2.05 m yüksekliğe sahiptir; döşemesiz ağırlığı ise yaklaşık 2425 N'dur. Köprü üzerinde ortamsal titreşim ve darbe testleri gerçekleştirilmiş, köprünün titreşimleri farklı noktalara yerleştirilmiş sekiz adet ivmeölçer yardımı ile kaydedilmiştir. Köprünün modal parametreleri, “doğal uyarım tekniği ve öz-sistem realizasyon algoritması” (Natural Excitation Technique with Eigensystem Realization Algorithm - NExT-ERA), “gelişmiş dekompozisyon frekans tanım alanında” (EFDD) yöntemleri kullanılarak bulunmuştur. Deneysel yolla elde edilmiş bu değerler kullanılarak köprünün kalibre edilmiş sonlu elemanlar modeli oluşturulmuş ve bu modelden elde edilen simülasyon verileri üzerinde sensör yoğunluğu, gürültü miktarı ve tepki veri uzunluğu faktörlerinin modal parametrelerin tahminine etkisi incelenmiştir.

Anahtar sözcükler: Yapı sağlığının gözlenmesi, Sistem tanımlama yöntemi, Deneysel ve analitik modal analiz

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CHAPTER ONE

INTRODUCTION

1.1 General

Engineering structure's loses their efficiency with passing time because many different events will happen during normal service life and usage duration such as an earthquake, hurricanes, explosions, progressive weakness of elements component increase due to fatigue failure and ambient/environmental interactions. Structures are designed to resist loads with considered to serviceability and the approximation of critical loading those structures may experience in their operating life duration. However, these structures may encounter with various type of loading conditions in their useful life. From experiences on the structures simulation model or supposition of some uncertain parameters, applied load to a structure can be identified. Generally, structure design towards an ultimate load intensity limit which is may happen in such a structure added by a safety factor. Designing a safest structure will cause structure so heavy; despite, the maximum design load may case once in the structure serviceability duration or never. On the other hand, extreme safety built in, particularly when the maximum design loads applied to the structure may not result in observable damage in all components of a structure and how will be the structural situation after that.

It is very important to find a way to monitor structure and knowing about their situations in those critical affecting loads and also in the normal operating conditions, because they makes structures reliable. Civil structures have acquired increasing attention in the field of structural health monitoring to get an appropriate insight about damage prognosis and assess the remaining useful life of structures. Identifying modal parameters (natural frequencies, damping ratios and mode shapes) of a structure and determines the level of confidence to these parameters plays an important role during identifying a damage occurrences in the structure. Nondestructive testing methods are more beneficial than other methods, because they do not influence the construction's performance. Visual inspection has been widely used for damage determination. This method has some considerable disabilities: (1)

it is very difficult to detect some hidden damages inside a structure and only the observable damage can be detected. These hidden damages may cause sudden collapse of the structure, for example BAYRAM Hotel collapses suddenly after the Van earthquake on 23th of October 2011 (2) assessment of the structural condition depends on the observation of inspectors and they may result different judgments (3) despite the visual inspection is a very time consuming and expensive method, it is impossible to apply continuous monitoring of a structure. Because of these reasons engineers have been researching on various state-of-the-art methods to identify structural situations simultaneously and reliably. One of common approaches is the experimental determination of modal parameters. A change in modal parameters can be used to estimate the actual performance of the construction and eventually to detect local damage zones. Also, FE-methods can be used to estimate the modal parameters of a construction in the initial state. In order to estimate the performance of a structure on the basis of measured modal parameters, the inverse problem has to be solved and the FE-model has to be adapted to correspond to the measured data.

Experimental or Operational Modal Analysis (EMA, OMA) method have been extensively used in the engineering structures to extract structural modal characteristic based on measurements vibration as well as identifying damage of structures. These modal parameters for structural damage identification and health monitoring are essential. EMA as a conventional method is based on the estimation of a set of Frequency Response Functions (FRFs) in the frequency domain and impulse frequency response functions (IFRFs) in the time domain relating the applied force and the corresponding response at several points along the structure, with enough high spatial and frequency resolution (He, 2008). However, it is very difficult to obtain FRFs or IRFs in dynamic field tests of civil structures, because in the field tests, in consequence, of the difficulty to excite large civil structures in a controlled form, only the structure dynamic response (output) can be measured. Furthermore, structures affected by various types of ambient excitation sources such as traffic, wind, variations in temperature and combinations, so it's impossible to measure input signals, but it can be measured structural response of those excitation sources. Moreover, structures may be exposed by random type of input excitation amplitudes like earthquakes with stochastic magnitude. Thus, in last year's output-

only system identification methods have received increasing attention and obtained considerable advancement between research and structural industrial communities as a potential tool to damage prognosis at the earliest possible step and assess the remaining useful serviceable life of structures (Moaveni, 2007).

1.2 Aim and Objectives

Structural Health Monitoring (SHM) objectives to give, real time monitoring of multiple component of a structure during every moment of the operating life and diagnostic insight about the “state” of the component materials of the constructed elements, and of the full installation of these elements constituting the structure as a whole by using of EMA. Non-destructive damage identification based on measurement vibrations predicts the damage according to variation of dynamic properties of a structure. EMA method has used as a technology for structure’s modal parameters identification from its measured vibration data. Accuracy of damage quantification including damage localization and identify damage intensity by EMA method strongly relate to the accuracy estimation of the structural dynamic characteristics (Moaveni, 2006).

The aim of the research study presented in this thesis is to evaluation of the accuracy and completeness of the identified modal parameters by uncertainty analysis of modal parameters obtained from different output-only system identification methods. In the first part of this thesis, some of already existing output-only system identification methods algorithms applied to numerical programming in Matlab®. In the second part of this thesis, model steel bridge is exposed to experimental modal analysis and finite element (FE) modeling, in the analytical framework of SAP2000. FE model is calibrated in order to get the updating model. In the final part, based on experimental modal analysis and analytical modal analysis, the effects of the variability/uncertainty of several input factors on the variability/uncertainty of system identification results are investigated.

1.3 Thesis Outline

This research thesis is partitioned into three topics, by particular mention: 1) output-only system identification methods programming algorithm and investigation of the accuracy of numerical programming 2) experiment design, installation of devices (e.g. Sensors, data acquisition,..) and experimental modal analysis 3) uncertainty analysis of the identified modal parameters due to variability/ uncertainty of several input factors.

Chapter 2 presents a review of the existing literature on the uncertainty analysis of identified modal parameters obtained from output-only system identification methods. In Chapter 3 presents, already exist three output-only system identification methods algorithm used to estimate the modal parameters that covers the first topic of research purpose. Chapter 4 and chapter 5 covers second and third topic and mentioned to the applied experimental modal analysis and analytical modeling of the model steel bridge that subject to dynamic tests in the DEÜ Mechanical Laboratory. Finally, Chapter 6 summarizes the work done, highlights important research findings.

CHAPTER TWO

LITERATURE REVIEW

In the last decades, Structural Health Monitoring (SHM) has been providing inexpensive, real-time monitoring systems with the non-destructive testing evaluation. When a natural or man-made event happens, the structural monitoring system provides real-time information to help the quickly identification of damaged areas, which can lead to decreases in the damage's (observable and unobservable damages) identification process duration and normal life interruption. So, structures have acquired increasing attention in the field of structural health monitoring to get an appropriate insight about damage prognosis and assess the remaining useful life of structures in different research field.

Hoon Sohn, Charles R. Farrar, Francois Hemez and Jerry Czarnecki (2001), summarized structural health monitoring studies, in the years between 1996-2001. The definition of SHM as damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics and considered to this topic as global SHM researches. In some damage detection methods, solving the inverse problem which needs to construct an analytical model is preferred. Uncertainties will arise depending on fitting between real model and new generated analytical model. Some researchers try to avoid this dependency on the numerical models. Despite, the efficiency of these approaches in identifying the onset of damage, they can only identify the existence of damage. Another way of solving inverse method is using a neural network algorithm. The neural network can inversely relate the measured response and expected modal parameters. In this method both the undamaged and damaged structure measurement response data are needed and such data are rarely available in the common structures. As well as, one of the difficulties for the extend SHM systems in the field is environmental conditions and variation of the operational conditions of structure. Advanced damage identification algorithm performance will affected by environmental and operating conditions of structures. Finally, emphasized that the statistical models will develop for damage detection applications. R.D. Nayeri; S.F.Masri; and A.G. Chassiakos (2007), working on the application of SHM techniques to monitor structural changes

in a retrofitted six story Long Beach Public Safety Building based on ambient vibration. Eigen Realization Algorithm (ERA) and the ERA with using data correlation (ERA/DC) are used, and then implement ERA by using the Natural Excitation Technique (Next) technique, to investigate modal properties of the structure. This study result: “1) low resolution sensor placement, which results in high model order reduction; and 2) the ambient excitation was so small that the higher modal displacements were in the noise level, hence not adequately excited. It was shown that these identification techniques are extremely capable to be used in online structural health monitoring schemes”.

Jian-Huang Weng, Chin-Hsiung Loh, Jerome P. Lynch, Kung-Chun Lu, Pei-Yang Lin, Yang Wang (2008), represent a study on output-only modal identification of a long span (240 m), cable-stayed bridge using wireless monitoring systems. They used two output-only system identification methods consist of: Stochastic Subspace Identification (SSI) method and Frequency Domain Decomposition (FDD) method, to identify first ten modes of the bridge in the frequency range of 0-7 Hz. An updated finite element model used to assess the operational condition of structure, in order to do this, an analytical model of the Bridge had been extended using a Matlab based computer program and the identified modal frequencies and mode shapes of both experimental and analytical approaches are compared quite well. SSI method can eliminate the uncorrelated noise while preserving vibration information and FDD can clarify the close mode despite contaminated by noise, with priority of dominant mode. The first mode's damping ratios are estimated approximately 2.5% on average and, in the higher mode the damping ratios decreased less than 1.0%. A research study suggested, in order estimating accurate damping ratios. Enjoying Yu, Ertugrul Taciroglu and John W. Wallace (2006), investigate on finite element model-updating methods, in order to parameter identification of framed structures which using these methods. In this study, comparison of two common methods and one proposed methods subjected to improve the numerical difficulties when the various physical parameters subjected to identification, from experimental data. O. Ozcelik, J.E. Luco, J.P. Conte, T.L. Trombetti and J.I. Restrepo (2007), studied a simple conceptual mathematical model for identification of model parameters by using a wide range of experimental data, experimental hysteresis loops and response during periodic

sinusoidal and triangular excitations. Various sets of dynamic tests are performed on the system and validate their proposed conceptual model by comparing the identified result from both, analytical and experimental model. Dionysius M. Siringoringo, YozoFujino (2008), represent a research on the system identification of a suspension bridge with the total length of 1380 m, consists of 720 m center span and two symmetric side spans of 330 m, from ambient vibration response. Two output-only system identification methods, including: the Ibrahim Time Domain (ITD) method and the Natural Excitation Technique (NExT) combined with the Eigensystem Realization Algorithm (ERA) are used to extract modal parameters of the structure and accuracy and efficiency of both methods compared with the results from a Finite Element Model. The result of this investigation approved the accuracy and reliability of the identified modal parameters of bridge from those used output-only system identification methods. Babak Moaveni, Andre R. Barbosa, Joel P.Conte, and François M.Hemez (2006), provide a research on uncertainty analysis of identified modal parameters of the seven-story R/C building due to variability of four significant input factors which affected the accuracy of identified modal parameters that parameter's accuracy plays an important role in accuracy of damage prognosis process. The Calibrated Finite Element model used to investigate effects of these uncertainties parameters. The measurement acceleration response of the experimental test data and simulated data recorded from updated analytical model exposed to three output-only system identification methods like as: (1) Natural Excitation Technique combined with the Eigensystem Realization Algorithm (NExT-ERA), (2) Data-driven Stochastic Subspace Identification (SSI-DATA), and (3) Enhanced Frequency Domain Decomposition (EFDD). They show some of predicted input factors have a significant effect on accurate identification of modal parameters and consequently effected accurate identification of damage identification results.

The purpose of this represented study covers output-only system identification procedure such as: NExT-ERA, Multiple NExT-ERA, ERA and EFDD methods also, some of these methods subjected to numerical programming. According to previous experience and other research work, variability/uncertainty of modal parameters due to variability of some important input factors such as: 1) spatial sensor density 2) recorded response data length 3) measurement/sensor noise level, are considered for

investigation by using two out-put only system identification methods consist of: NExT-ERA and EFDD. The propose research study first, validate in a simple shear frame model and then mentioned in the applied experimental modal analysis and analytical modeling of the steel bridge model that subject to dynamic tests in the DEÜ Structural Mechanic Laboratory.

CHAPTER THREE

SYSTEM IDENTIFICATION TECHNIQUES

3.1 Introduction

Experimental modal analysis (EMA) has been drawing significant attention in the mechanical, aerospace and structural civil engineering research communities, based on vibration measurements to identify modal parameters such as natural frequencies, damping ratios and mode shapes. The traditional experimental modal analysis makes use of measured input excitation as well as output response. However, for large civil structures, structures' excitations through the control of the amplitude of input excitations are a very difficult issue. It is also impossible to measure all the inputs under operational conditions, especially those from ambient sources. EMA has been extensively used in order to extract dynamic parameters of those structures and consequently, identifying damage in the structures based on vibration measurement data. These modal parameters are indispensably necessary for structural damage identification and health monitoring.

The conventional EMA is based on the estimation of a set of Frequency Response Functions (FRFs) in the frequency domain and impulse response functions (IRFs) in the time domain concerned with applied force and corresponding response at different points on the structure, with enough spatial sensor density with high-frequency resolution. Practically, in the dynamic field tests of civil structures, this is very difficult to gain FRFs or IRFs, in consequence of the difficulty to excite those structures in a controlled form; consequently, only the response of the excited structure can be measured. Also, Structures affected by various types of ambient excitation sources such as traffic density, wind loads, variations in temperature and combinations of these environmental factors, so measurement of input signals are impossible but structural response (output-response) can be measured. Moreover, structures may be exposed by random type of input excitation amplitudes like earthquakes with stochastic magnitude. Thus, in last year's output-only system identification methods have received increasing attention and obtained considerable advancement between research and structural industrial communities as a potential

tool to identify damage at the first possible stage and assessment of remaining useful life of structures. These output-only system identification techniques are necessary and essential for developing a real-time continuous vibration-based structural health monitoring system for continuously monitoring a structure during its service life.

Output-only system identification methods are in the frequency or time domains. The main frequency domain methods include the classical peak picking (PP), the frequency domain decomposition (FDD) and the enhanced frequency domain decomposition (EFDD) (Brincker et al., 2000, 2001). These methods are extended based on response cross-spectral density matrices. Moreover, the time domain output-only system identification methods include two and one stage methods. In the two-step methods, first random decrement functions and response correlation functions are gained as a free vibration response estimates, and then modal parameters are obtained in the second step by using any classical system identification algorithm based on free estimates of the response function (impulse response function). Conversely, in the one-step system identification methods like as the data-driven stochastic subspace identification (SSI-DATA) method, modal parameters can be identified by output-only measurements directly. (Van Overschee and De Moor, 1996) On the other hand, the development of EMA in the time domain can be divided into three main approaches: Natural Excitation Technique (NExT) based approaches, Autoregressive Moving Average (ARMA) model based approaches, and Stochastic Subspace Identification (SSI) based approaches (He, 2008, Hong 2006).

Already existing output-only system identification techniques are provided in following sub-sections. These methods include: (1) the Natural Excitation Technique (James et al., 1993) combined with ERA (NExT-ERA), and Multiple-Reference Natural Excitation Technique (Moaveni et al, 2005) combined with ERA (MNExT-ERA) (2) Enhanced Frequency Domain Decomposition (EFDD). The first method belongs to two-step time domain system identification methods as long as in order to improve the reliability and accuracy of identified modal parameters using NExT-ERA, the Multiple-reference NExT-ERA (MNExT-ERA) is applied as an extension

of the NExT-ERA, and the second methods are frequency domain non-parametric methods.

3.2 Input-Output Eigensystem Realization Algorithms (ERA)

State-space model applied to system analysis and design in control and systems research programs. In order to design controls for a dynamic system it is necessary to have a mathematical model that will adequately describe the system's motion. The process of constructing a state space representation from experimental data is called system realization. It is known from control theory, that a system with repeated eigenvalues and independent mode shapes is not identifiable by single input and single output. Methods which allow only one initial condition (input) at a time will miss repeated eigenvalues. Also, if the realized system is not of minimum order and matrix inversion is used for constructing an oversized state matrix, numerical errors may become dominant (Juang and Pappa, 1985).

There are several methods available to compute a state-space model and many types of inputs that are particularly important in theory as well as in practice. The pulse input is frequently used to generate a sequence of the pulse response for modal parameter identification, including system frequencies, damping and mode shapes. The Markov parameters are commonly used as the basis for identifying mathematical models for linear dynamical systems. Markov parameters can be obtained from time-domain experimental data. The sequence of Markov parameter is simply the pulse response of the system and it must be unique for a given system. From Markov parameters, the embedded system matrices can be extracted.

State-space representation of a linear time invariant system of order n with finite dimensional in the discrete-time is given by

$$\begin{aligned}\mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) \quad (3.1)\end{aligned}$$

$\mathbf{A} \in \mathbb{R}^{n \times n}$, $\mathbf{B} \in \mathbb{R}^{n \times 1}$, $\mathbf{C} \in \mathbb{R}^{m \times n}$, $\mathbf{D} \in \mathbb{R}^{m \times 1}$ = state space matrices

$\mathbf{x}(k) \in \mathbb{R}^n$ = state vector, $\mathbf{u}(k) \in \mathbb{R}^l$ = arbitrary input signal or load vector

$\mathbf{y}(k) \in \mathbb{R}^m = [y_1(k)y_2(k) \dots y_m(k)]^T$, size of vector (m) is the number of output channels; shows the measured response of a system between the m measured degrees of freedom (DOFs) at discrete time of $t = k(\Delta t)$. The free vibration response of the system can be obtained as

$$\mathbf{y}(0) = \mathbf{C}\mathbf{x}(0), \mathbf{y}(1) = \mathbf{C}\mathbf{A}\mathbf{x}(0), \mathbf{y}(2) = \mathbf{C}\mathbf{A}^2\mathbf{x}(0), \dots, \mathbf{y}(k) = \mathbf{C}\mathbf{A}^k\mathbf{x}(0) \quad (3.2)$$

Based on the free response vector, the following two $(m \times s) \times s$ Hankel matrices (block data matrices) are formed

$$H(0) = \begin{bmatrix} y(1) & y(2) & \dots & y(s) \\ y(2) & y(3) & \dots & y(s+1) \\ \vdots & \vdots & \ddots & \vdots \\ y(s) & y(s+1) & \dots & y(2s-1) \end{bmatrix}_{(m \times s \times s)} \quad (3.3)$$

$$H(1) = \begin{bmatrix} y(2) & y(3) & \dots & y(s+1) \\ y(3) & y(4) & \dots & y(s+2) \\ \vdots & \vdots & \ddots & \vdots \\ y(s+1) & y(s+2) & \dots & y(2s) \end{bmatrix}_{(m \times s \times s)} \quad (3.4)$$

s = an integer that determines the Hankel matrix size

The matrix $H(0)$ is of rank n (the order of the system) if $m > n$. To verify this point, substituting the system free vibration response (Markov parameters of the system) from

Eqs.(3.2) into Eqs.(3.3) and decomposing $H(0)$ into two matrices

$$H(0) = \begin{bmatrix} \mathbf{C}\mathbf{A}\mathbf{x}(0) & \mathbf{C}\mathbf{A}^2\mathbf{x}(0) & \dots & \mathbf{C}\mathbf{A}^s\mathbf{x}(0) \\ \mathbf{C}\mathbf{A}^2\mathbf{x}(0) & \mathbf{C}\mathbf{A}^3\mathbf{x}(0) & \dots & \mathbf{C}\mathbf{A}^{s+1}\mathbf{x}(0) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}\mathbf{A}^s\mathbf{x}(0) & \mathbf{C}\mathbf{A}^{s+1}\mathbf{x}(0) & \dots & \mathbf{C}\mathbf{A}^{2s-1}\mathbf{x}(0) \end{bmatrix} = \mathcal{P}\mathcal{Q} \quad (3.5)$$

Where \mathcal{P} and \mathcal{Q} are

$$\mathcal{P} = \begin{bmatrix} \mathbf{C} \\ \mathbf{C}\mathbf{A} \\ \vdots \\ \mathbf{C}\mathbf{A}^{s-1} \end{bmatrix} \quad \mathcal{Q} = [\mathbf{A}\mathbf{x}(0) \quad \mathbf{A}^2\mathbf{x}(0) \quad \dots \quad \mathbf{A}^s\mathbf{x}(0)] \quad (3.6)$$

The block matrices \mathcal{P} and \mathcal{Q} are the observability and controllability matrices respectively. The order of the system is n , and then the minimum dimension of the state matrix is $n \times n$. If the system is controllable and observable, the rank of block matrices \mathcal{P} and \mathcal{Q} is n . Therefore, the Hankel matrix $H(0)$ is in rank of n .

In the same way substituting Eq (3.2) into Eq(3.4), the Hankel matrix $H(1)$ can be obtained in terms of system matrices A and C

$$H(1) = \begin{bmatrix} \mathbf{CA}^2\mathbf{x}(0) & \mathbf{CA}^3\mathbf{x}(0) & \cdots & \mathbf{CA}^{s+1}\mathbf{x}(0) \\ \mathbf{CA}^3\mathbf{x}(0) & \mathbf{CA}^4\mathbf{x}(0) & \cdots & \mathbf{CA}^{s+2}\mathbf{x}(0) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{CA}^{s+1}\mathbf{x}(0) & \mathbf{CA}^{s+2}\mathbf{x}(0) & \cdots & \mathbf{CA}^{2s}\mathbf{x}(0) \end{bmatrix} = \mathcal{P}\mathbf{A}\mathcal{Q}$$

$$H(1) = \mathcal{P}\mathbf{A}\mathcal{Q} \quad (3.7)$$

Where \mathcal{P} and \mathcal{Q} are as defined as Eq(3.6). There exist \mathcal{P}^\dagger and \mathcal{Q}^\dagger are a left inverse and a right inverse respectively, such that

$$\begin{aligned} \mathcal{P}^\dagger\mathcal{P} &= I_{n \times n} \quad \text{and} \quad \mathcal{Q}\mathcal{Q}^\dagger = I_{n \times n} \\ \mathcal{P}^\dagger &= [\mathcal{P}^T \quad \mathcal{P}]^{-1}\mathcal{P}^T \quad \text{and} \quad \mathcal{Q}^\dagger = \mathcal{Q}^T[\mathcal{Q} \quad \mathcal{Q}^T]^{-1} \end{aligned} \quad (3.8)$$

Therefore the state space system matrices A and C can be obtained

$$A = \mathcal{P}^\dagger H(1)\mathcal{Q}^\dagger$$

$$C = E_m^T \mathcal{P} \quad (3.9)$$

Where $E_m^T = [I_m \quad 0]$, and I_m is the $m \times m$ unit matrix.

A singular value decomposition of Hankel matrix $H(0)$ is performed as

$$H(0) = U\Sigma V^T = \begin{bmatrix} U_n & U_p \end{bmatrix} \begin{bmatrix} \Sigma_n & 0 \\ 0 & \Sigma_p \end{bmatrix} \begin{bmatrix} V_n^T \\ V_p^T \end{bmatrix} \quad (3.10)$$

n = corresponding to the order of realized system

p = corresponding to computational error or noise

Then the matrices of \mathcal{P} and \mathcal{Q} can be obtained

$$\mathcal{P} = U_n \Sigma_n^{1/2} ; \mathcal{Q} = \Sigma_n^{1/2} V_n^T \quad (3.11)$$

One obvious solution for state space matrix A and C can be estimated by

$$A = \Sigma_n^{-\frac{1}{2}} U_n^T H^s(1) V_n \Sigma_n^{-1/2}$$

$$C = E_m^T U_n \Sigma_n^{1/2} \quad (3.12)$$

Modal parameters can be obtained from state-space matrices.

$$\omega_i = |\ln(\lambda_{2i-1})/\Delta t|$$

$$\xi_i = -\cos(\text{angel}(\ln(\lambda_{2i-1}))), \quad i = 1, 2, 3, \dots, N \quad (3.13)$$

$$\Phi_i = C \cdot T_{2i-1}$$

Δt = sampling time and $T_i = i^{th}$ eigenvector of matrix A.

$N = n/2$, $\lambda_i = i^{th}$ eigenvalue of matrix A (He, 2008, Moaveni 2007).

3.3 Output-Only System Identification Methods

3.3.1 Natural Excitation Technique Combined with Eigensystem Realization Algorithm (NExT-ERA) and Multiple-Reference Natural Excitation Technique Combined with Eigensystem Realization Algorithm (MNEExT-ERA)

The theory behind the NExT is that the theoretical cross-correlation function between two response measurements made along two degrees of freedom (DOF) collected from an ambient excited structure has the same analytical form as the free vibration response of the structure. In order to analyze the randomly excited systems, correlation functions commonly are in used and can be expressed as summations of decaying sinusoids. Each of them has a damped natural frequency and damping ratio and that is identical of a corresponding structural mode shapes (James et al. 1993). After acquiring acceleration response from the operating structure, first an estimation of the output cross-correlation vector is acquired for a selected reference

channel. Selecting the reference channel is a significant issue, because of avoiding missing mode vectors due to adjacency of a reference channel to a modal node.

Consider the differential equation of motion of an N DOF, linear time-invariant system

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{P}\mathbf{f}(t) \quad (3.14)$$

With initial condition $\mathbf{x}(0) = \mathbf{x}_0$ and $\dot{\mathbf{x}}(0) = \dot{\mathbf{x}}_0$

\mathbf{M} , \mathbf{C} and \mathbf{K} = mass, damping and stiffness matrices, respectively

$\mathbf{x}(t)$, $\dot{\mathbf{x}}(t)$ and $\ddot{\mathbf{x}}(t)$ = Nodal displacement, velocity and acceleration vectors

\mathbf{P} = load distribution matrix; $\mathbf{f}(t)$ = load vector function.

By assuming that the ambient excitation function and measured structural responses are stationary stochastic processes, then the equation (3.14) can be revised to

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{P}\mathbf{F}(t) \quad (3.15)$$

$\mathbf{X}(t)$, $\dot{\mathbf{X}}(t)$ and $\ddot{\mathbf{X}}(t)$ = Displacement, velocity and acceleration stochastic vector processes, respectively

$\mathbf{F}(t)$ = stochastic excitation vector process

By multiplying each term of the equation (3.15) expressed at time $t = t + \tau$ with the scalar reference response quantity $X_r(t)$ and taking the mathematical expectation (E) yields

$$\mathbf{M}\mathbf{E}[X_r(t)\ddot{\mathbf{X}}(t + \tau)] + \mathbf{C}\mathbf{E}[X_r(t)\dot{\mathbf{X}}(t + \tau)] + \mathbf{K}\mathbf{E}[X_r(t)\mathbf{X}(t + \tau)] = \mathbf{P}\mathbf{E}[X_r(t)\mathbf{F}(t + \tau)] \quad (3.16)$$

Under the condition that future input forces are uncorrelated with the current structural response in the reference channel r, equation (3.16) reduces to

$$\mathbf{M}\mathbf{E}[X_r(t)\ddot{\mathbf{X}}(t + \tau)] + \mathbf{C}\mathbf{E}[X_r(t)\dot{\mathbf{X}}(t + \tau)] + \mathbf{K}\mathbf{E}[X_r(t)\mathbf{X}(t + \tau)] = 0 \quad (3.17)$$

This is equivalent to the homogeneous form (Lutes and Sarkani, 1997)

$$\mathbf{M}\ddot{\mathbf{R}}_{X_r X}(\tau) + \mathbf{C}\dot{\mathbf{R}}_{X_r X}(\tau) + \mathbf{K}\mathbf{R}_{X_r X}(\tau) = 0 \quad (3.18)$$

$\mathbf{R}_{X_r X}(\tau)$ is the cross-correlation vector between $X_r(t)$ and $\mathbf{X}(t)$. It is observed that the above differential equation governing the cross-correlation vector function $\mathbf{R}_{X_r X}(\tau)$ is identical to the equation of motion (3.14) under free vibration condition.

The multiple-reference NExT-ERA (MNExT-ERA) can be used as an extension of NExT-ERA, to improve the accuracy of identified dynamic characteristics of a structure (He et al. 2006). In order to do this instead of using a single reference response channel as in NExT-ERA, multiple reference channels are used to obtain an output cross correlation matrix between an N-DOF response vector $\mathbf{X}(t)$ and a subset of this response vector, $X^r(t)$ and defined as

$$\mathbf{R}_{X^r X}(\tau) = \begin{bmatrix} \mathbf{R}_{X_1^r X}(\tau) & \mathbf{R}_{X_2^r X}(\tau) & \dots & \mathbf{R}_{X_{N_r}^r X}(\tau) \end{bmatrix}_{N \times N_r} \quad (3.19)$$

$\mathbf{R}_{X^r X}(\tau)$ = cross-correlation column vector matrix which each column vector represents the cross-correlation between a single reference response and the system response vector. In order to identify modal parameters by using ERA, cross-correlation matrix exchange to form of Hankel matrices

$$\mathbf{H}^s(0) = \begin{bmatrix} \mathbf{R}(1) & \mathbf{R}(2) & \dots & \mathbf{R}(s) \\ \mathbf{R}(2) & \mathbf{R}(3) & \dots & \mathbf{R}(s+1) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}(s) & \mathbf{R}(s+1) & \dots & \mathbf{R}(2s-1) \end{bmatrix}_{(m \times s) \times (N_r \times s)} \quad (3.20)$$

$$\mathbf{H}^s(1) = \begin{bmatrix} \mathbf{R}(2) & \mathbf{R}(3) & \dots & \mathbf{R}(s+1) \\ \mathbf{R}(3) & \mathbf{R}(4) & \dots & \mathbf{R}(s+2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}(s+1) & \mathbf{R}(s+2) & \dots & \mathbf{R}(2s) \end{bmatrix}_{(m \times s) \times (N_r \times s)} \quad (3.21)$$

The basic idea behind the use of multiple reference channels instead of single reference channel is to avoid missing modes in the NExT-ERA identification process due to the approximation of the reference channel to nodes of these modes. In MNExT-ERA, the ERA as an input-output method is applied in its multiple-input,

multiple-output form and the Hankel matrix form to apply ERA, exchange to block Hankel matrix (He, 2008, Moaveni 2007).

3.3.2 Enhanced Frequency Domain Decomposition

One of the frequency domain approaches is the Frequency Domain Decomposition (FDD). FDD is an extension of the Basic Frequency Domain approach and referred to as peak picking technique. In the FDD identification, the first step is to estimate the power spectral density (PSD) matrix. The estimate of the output PSD known at discrete frequencies is then decomposed by taking the singular value decomposition (SVD) of the matrix. Through taking the (SVD) of the spectral matrix, the spectral matrix is decomposed into a set of cross spectral density (CSD) (auto spectral density) functions, each corresponding to a single degree of freedom (SDOF) system. By decomposing CSD functions into single degree of freedom (SDOF) CSD functions, each corresponding to a single vibration mode of the dynamic system. By using this decomposition technique close modes can be identified with high accuracy even in the case of strong noise contamination of the signals.

In order to identify the natural frequency and damping ratio of a vibration mode through correspond SDOF of CSD function, the SDOF function turned back to the time domain using inverse Fourier transformation, and the frequency and damping ratio are estimated from the crossing times and the logarithmic decrement, respectively. CSD functions are estimated based on the Welch-Bartlett's method using Hanning windows with 50 percent overlap. Estimated CSD matrices are decomposed to singular values at each discrete frequency (Brincker et al. 2001).

CHAPTER FOUR
UNCERTAINTY ANALYSIS OF MODAL PARAMETERS OBTAINED
FROM SYSTEM IDENTIFICATION METHODS FOR SIMPLE SHEAR
FRAME

4.1 Introduction

Modal properties of engineering structures will change with time due to different reasons and structures may encounter with various types of loading conditions in their useful life. When the maximum design load applied to a structure, it may cause some observable and often unobservable damages in the structures.

It is so important to find a way to monitor structure and knowing about their situations in critical affecting loads and also in the normal operating conditions, because they make structures reliable. Civil structures have acquired increasing attention in the field of structural health monitoring to get an appropriate insight about damage prognosis and assess the remaining useful life of structures. Identifying modal parameters (e.g., natural frequencies, damping ratios and mode shapes) of a structure and determining the level of confidence to these parameters plays an important role during identifying a damage occurrence in the structure. So, in this research, a simple shear frame subjected to finite-element modeling and investigation of variability of modal parameters due to variability of some input factors. Two state-of-the-art system identification algorithms based on output-only data were used to estimate the modal parameters of the structure. The level of accuracy of identified modal parameters investigated as a function of uncertainty/variability in the following input factors: (1) spatial density of measurements (considered at 3 levels), (2) measurement noise (considered at 4 levels), and (3) length of response data used in the identification process (considered at 4 levels). To do this, the assumption that the structure F-E model calibrated in the optimized way with its real modal condition, we jumped to the second step to simply showing the theoretical effect of these input factors on the variability of the modal parameters, we will discuss about the full set of research (experimental analysis and then analytical assessments) in the next chapter. Uncertainty analysis is performed

based on the acceleration response of the simulated model, excited with 50 % RMS white noise. Analytical model generated in the analysis framework Sap2000 (Moaveni, Barbosa, Conte, and Hemez, 2007).

4.2 Analytical Model of Tested Structure

Simple shear frame subjected to modeling in analysis framework Sap 2000 and shown in the Figure (4.1). This model composed of 8 nodes and 9 beam-column element. Constant damping 0.02 assigned to the model. During the analysis, model subjected to time history and modal analysis. The structure restricted to analysis in x-z plane.

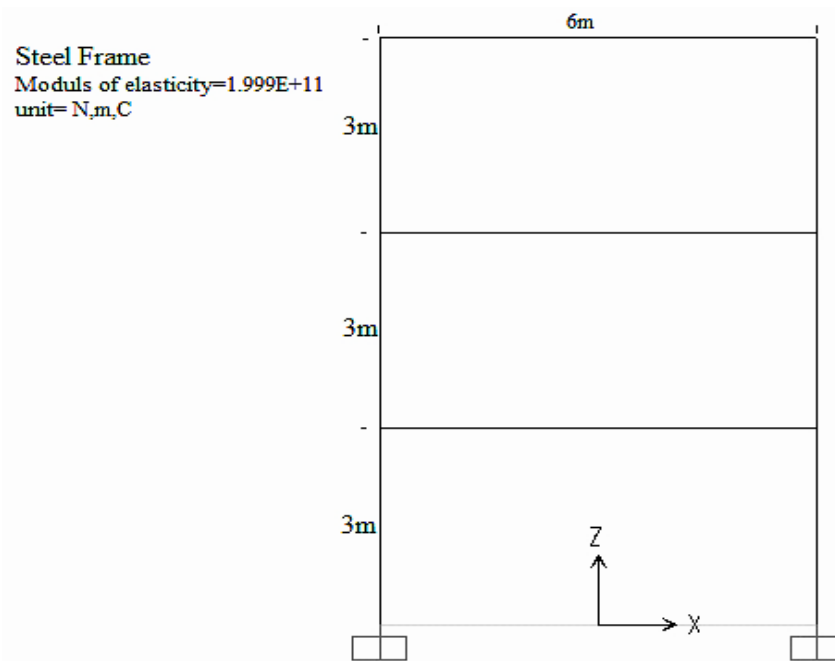


Figure 4.1 Finite element model of shear frame

The gravity columns are assumed to remain linear elastic during the analyses, so they are modeled as linear elastic elements. During the analysis, the gravity loads are first applied to the model quasi-statically followed by the rigid-base excitation, which is applied dynamically. As base acceleration records, four records are generated as Gaussian banded white noise processes (between 0.5Hz and 50Hz) with a root mean square acceleration of 0.05g, where g denotes the acceleration of gravity. Time-step of 1/200sec is used as time-stepping scheme. The longitudinal acceleration

response histories are recorded at 8 different locations. The first three longitudinal mode shapes together with their corresponding natural frequencies, damping ratios and modal participating mass ratio (on x direction) are shown in Figure (4.2).

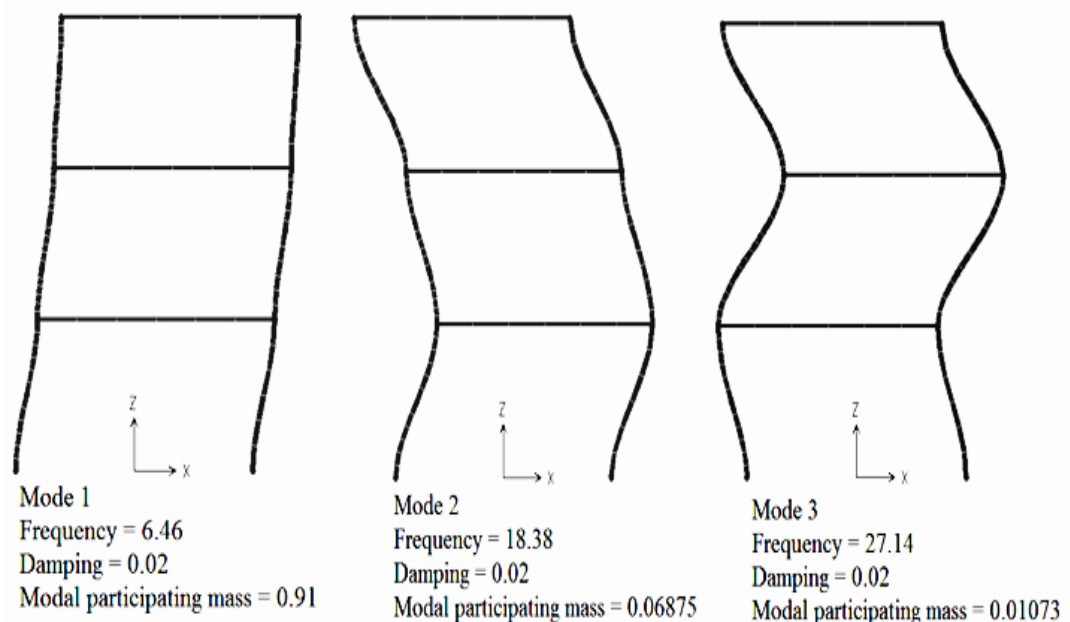


Figure 4.2 First three longitudinal mode shapes of FE model

4.3 Applied System Identification Methods and Numerical Programming Algorithm

Two different state-of-art output-only system identification methods subjected to programming in Matlab, to estimate the modal parameters of the FE structural model. Acceleration response data acquired from 8 special joint from the model in order to use as an output response which shown in the Figure (4.3). The system identification methods used consist of: (1) Natural Excitation Technique combined with the Eigensystem Realization Algorithm (NExT-ERA), in the time domain and (2) Enhanced Frequency Domain Decomposition (EFDD), in the frequency domain. The acceleration responses before subjected to system identification methods, all acceleration response time histories exposed filtering by band-pass filter with frequency passing range of 0.5 Hz - 50 Hz and using high order (1024) FIR filter. Also, The interested modal frequency in this study is <30Hz, but the acceleration responses are simulated at a rate of 200 Hz and then down sampled to the rate of 175 Hz .

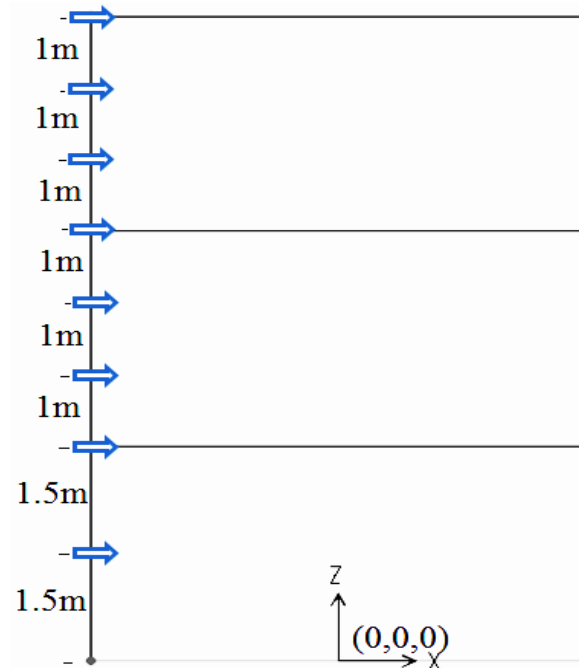


Figure 4.3 Simulated sensors places

In the Next-ERA identification, the basic principle behind NExT is that the theoretical cross-correlation function between two response channels from an ambient excited structure has the same analytical form as the free vibration response of the structure. The response cross-correlation vector is obtained for a given reference channel, selecting the reference channel play an important role to accurately identify modal parameters, because of avoiding missing mode vectors due to adjacency of a reference channel to a modal node. The first step is to estimate the cross spectral density matrix (CSD), based on Welch Bartlett's method with 50% overlap of hamming window. The cross power spectral density is the distribution of power per unit frequency. CSD of the first channel of acceleration response is shown in the Figure (4.4), and then obtained CSD will back again in the time domain by using Inverse Fast Fourier Transform (IFFT). The cross-correlation of the first channel is shown in the Figure (4.5). Then cross-correlation functions are used to form Hankel matrices for applying ERA in the second step of the modal identification process. Modal parameters can be obtained from state-space matrices, which extracted by ERA.

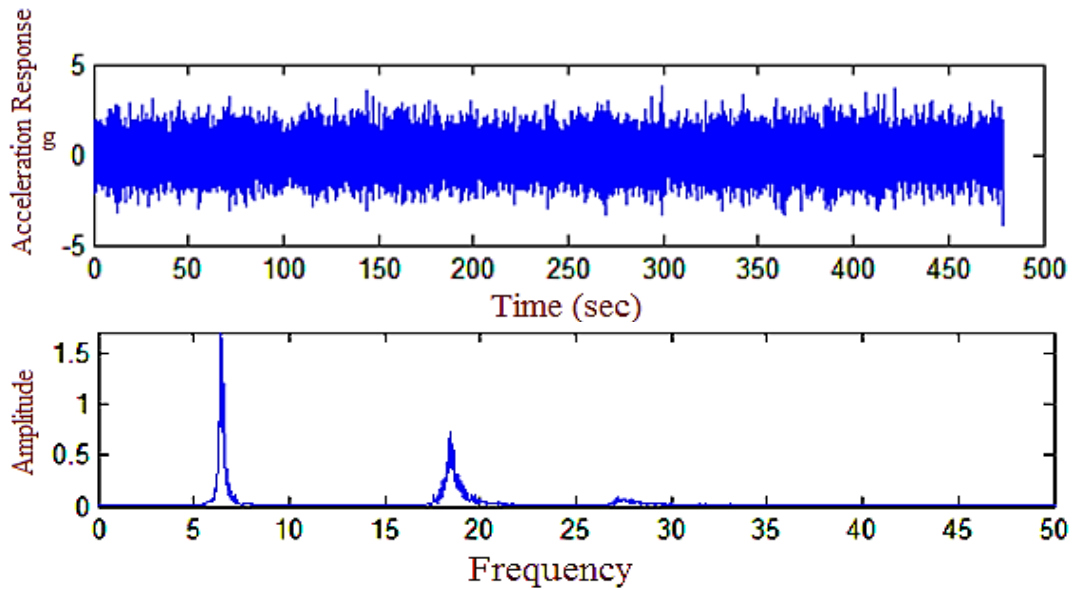


Figure 4.4 Acceleration response and CSD of the first channel

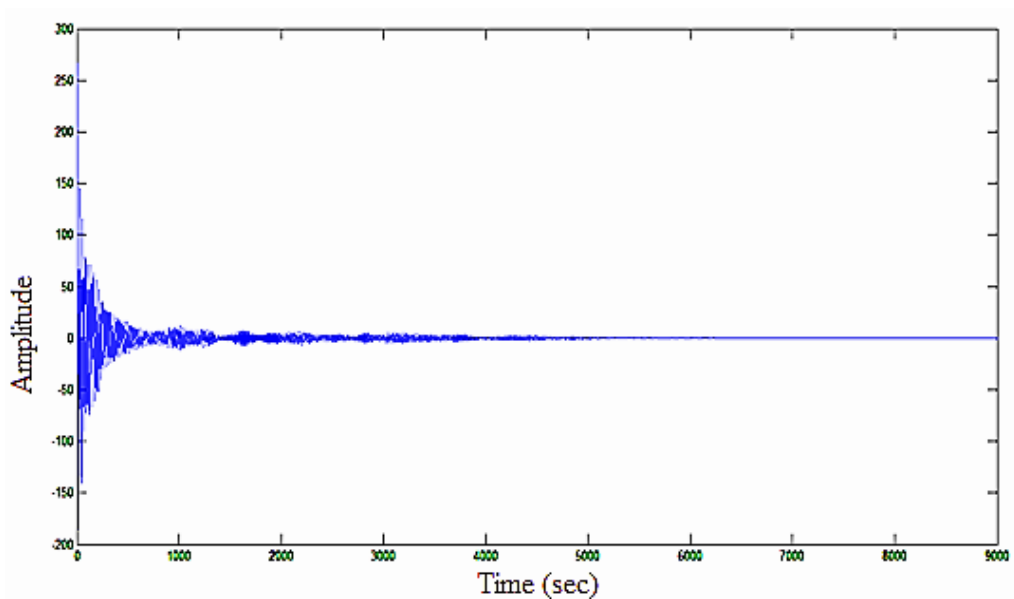


Figure 4.5 Cross-correlation of the first channel with reference channel

In the EFDD identification, the Frequency Domain Decomposition (FDD) is an extension of the basic frequency domain approach also referred to as the classical approach where the modal parameters are estimated by simple peak picking technique. In this method applied, the PSD functions are estimated based on the Welch-Bartlett's method using Hanning windows. The EFDD technique estimates the vibration modes using singular value decomposition (SVD) of the PSD matrices at all discrete frequencies, for this study, singular values are shown as Figure (4.6).

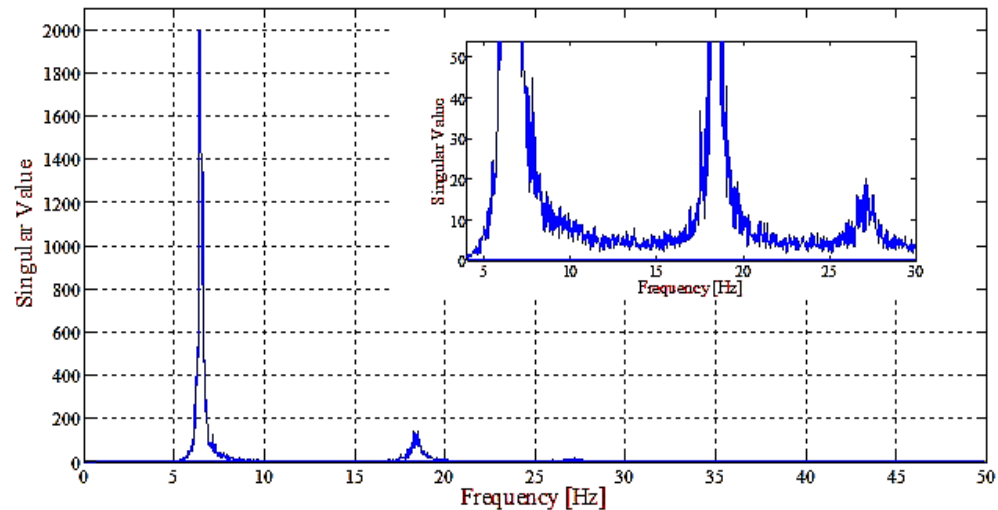


Figure 4.6 Singular values of the PSD matrix

Single-degree-of-freedom (SDOF) systems are estimated, each corresponding to a single vibration mode of the dynamic system, and then each vibration mode's corresponding SDOFs clarifying by MAC criteria. From the piece of the SDOF density function obtained around the peak of the PSD, the natural frequency and the damping ratio can be obtained. In order to do this, the estimated PSD function corresponding to each mode is taken back to the time domain by inverse Fourier transformation, and the natural frequencies and corresponding damping ratios are estimated from the crossing times and the logarithmic decrement of the corresponding SDOFs auto-correlation function, respectively.

4.4 Uncertainty Analysis of Shear Frame Model and System Identification Results

The purpose of this study is to analyze the variability/uncertainty of the modal parameters obtained using two system identification methods due to the variability of three input factors: (1) spatial density of the sensors, (2) level of measurement noise, and (3) length of structural response records used for system identification. Selection of these three factors is based on previous experience (Moaveni et al 2007). System identification and analytical FE analysis results are considered in the Table (4.1).

Table 4.1 Identified modal parameters from analytical and experimental modal analysis

Method	Natural Frequency			Damping Ratio			MAC		
	ω [Hz]			ζ [%]					
Mode No	1	2	3	1	2	3	1	2	3
SAP Analysis	6.46	18.38	27.14	0.02	0.02	0.02	-	-	-
NExT-ERA	6.46	18.41	27.15	0.019	0.018	0.02	0.9999	0.9990	0.9724
EFDD	6.68	18.32	27.29	0.017	0.02	0.01	0.9999	0.9647	0.7768

4.4.1 Spatial Sensors Density

An instrumentation array of 8 acceleration channels is simulated for excitation level using the FE model of the structure in Sap2000. This array of 8 acceleration channels is shown in Figure (4.3). To study the variability of modal parameters due to variation of the spatial density of the sensor array (i.e., number of sensors), three different subsets of the 8 sensor array are considered. The three configurations of simulated accelerometers consist of: (1) 2 accelerometers (2) 4 accelerometers, (3) full array of 8 accelerometers and shown in the Figure (4.7), when the other input factors remain fixed. For this purpose, a set of 60 ($2 \times 3 \times 10$) identification process runs and the mean value of the identified modal parameters results gave as a bar plot in Figure (4.8.a and b) to investigate variability of modal parameters.

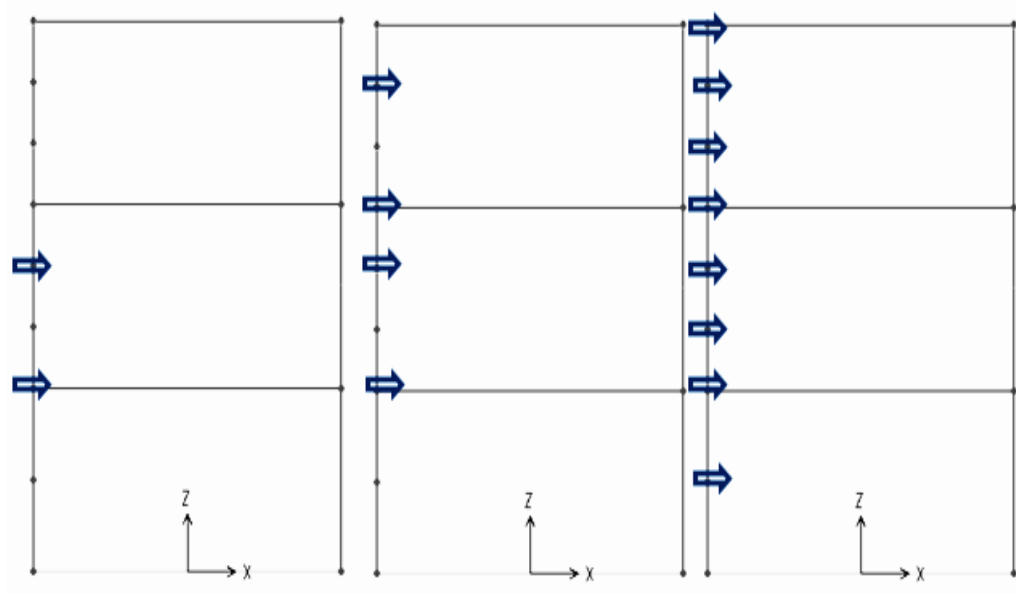
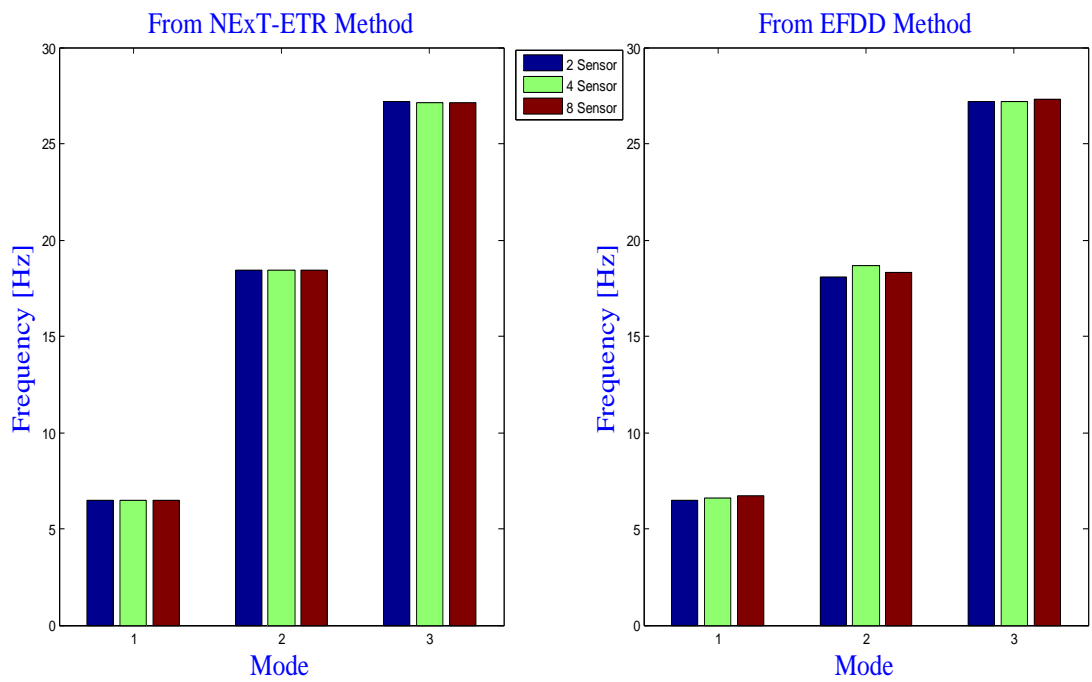
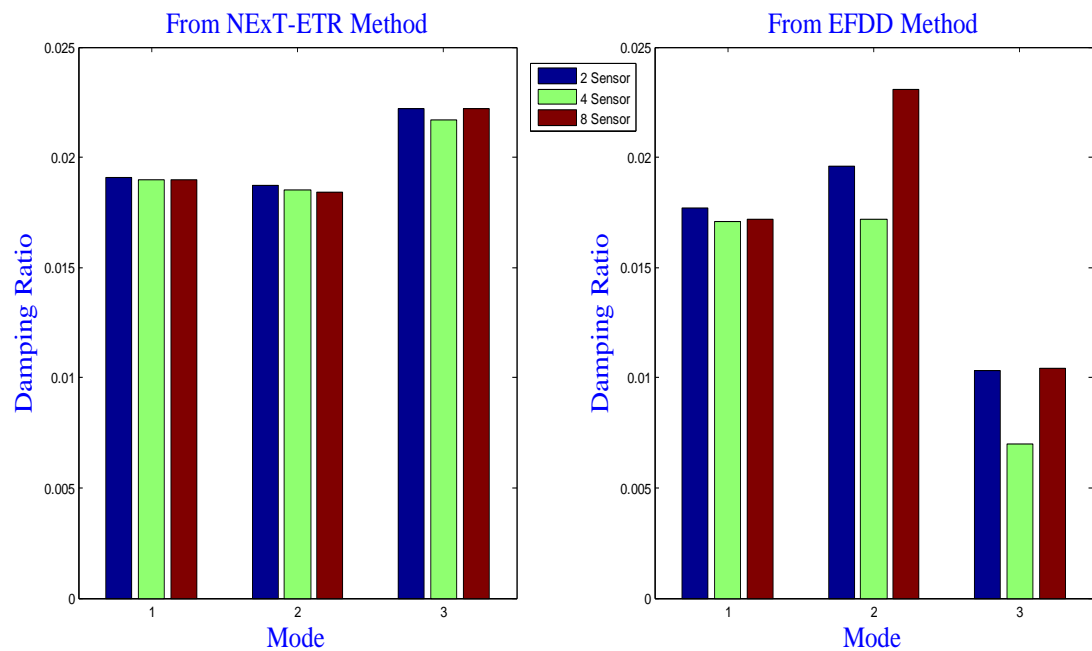


Figure 4.7 Sensor densities



a) Uncertainty/variability of natural frequencies



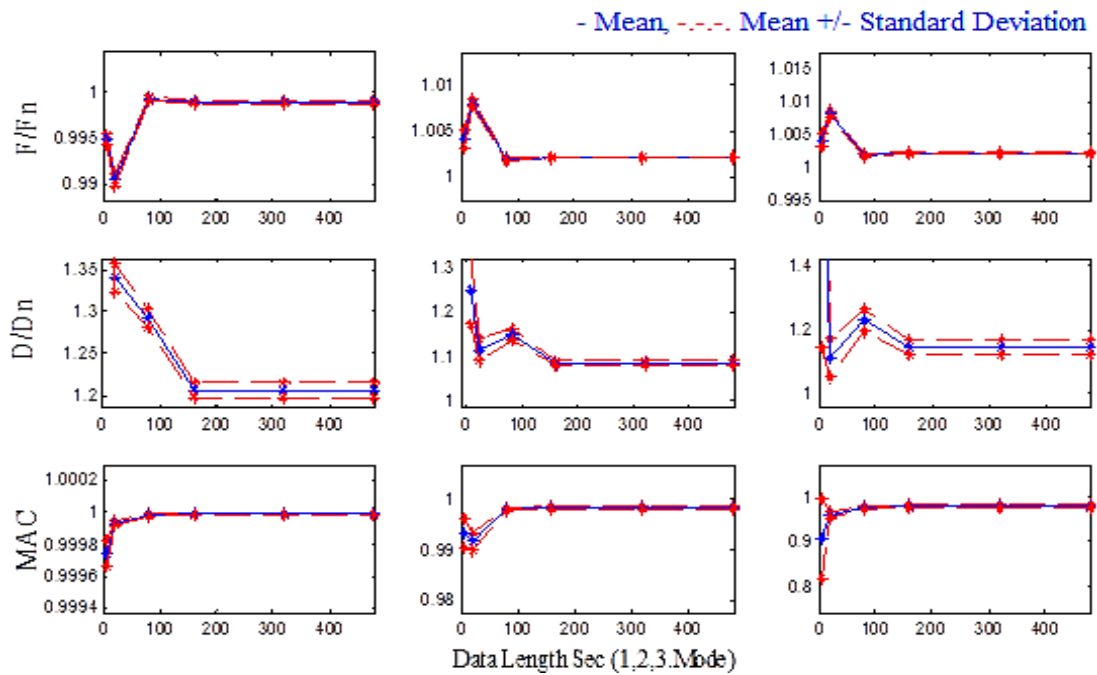
b)Uncertainty/variability of damping ratios

Figure 4.8.a.b Uncertainty/variability of modal parameters due to variability of sensor density

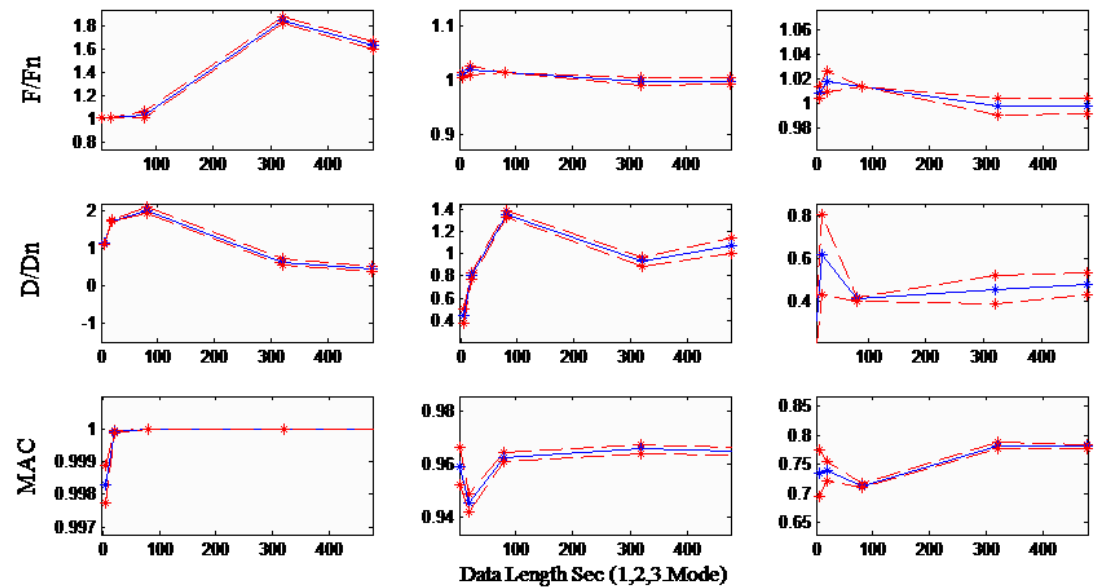
According to this figure result, in both system identification methods estimation of natural frequencies was not affected by the variability of sensor density but damping ratios affected by this input factor.

4.4.2 Response Data Length

In order to investigate variability of modal parameters due to variations of input data length, the 8 sensor array, a measured noise level, simulates as a Gaussian white noise with RMS 20% and five different length of data: 5 , 20 , 40 , 80, 320,480 (seconds), used in this study. Two identification methods used to identify modal parameters. A set of 120 identification process runs and Variability of the mean and standard deviation statistics of the identified modal parameters due to variability of data length input factor for these 120 identification trials is shown in Figure (4.9.a.b).



a)Using NEXt-ERA technique



b)Using EFDD technique

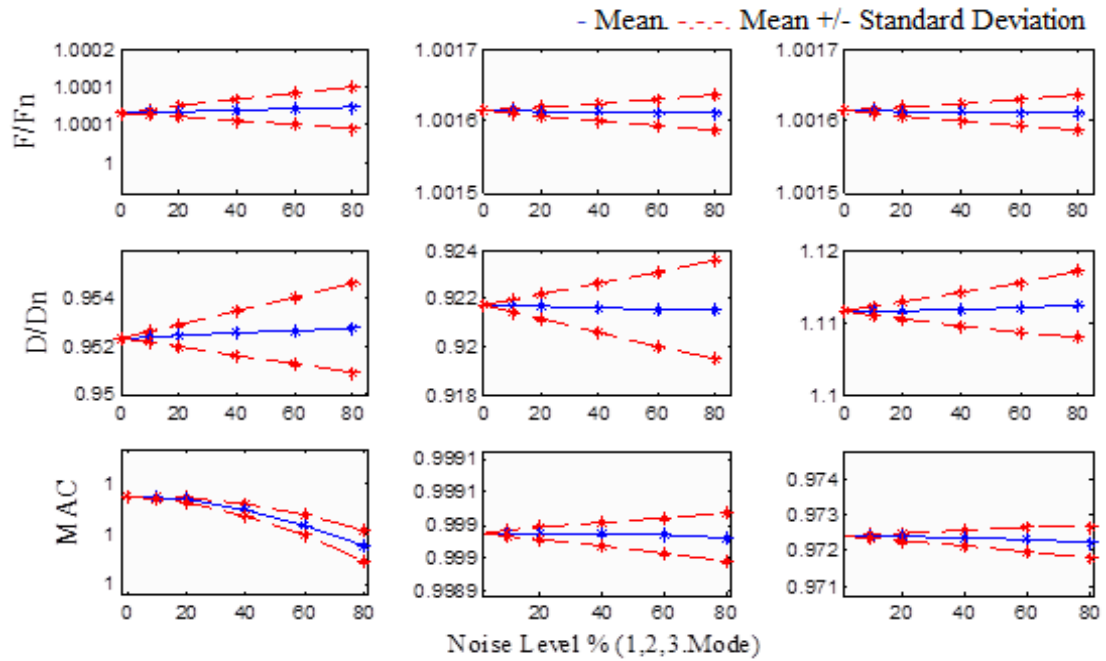
Figure 4.9.a.b Variability of the identified modal parameters due to variability of response data length

The effect of a data length input factor in the accuracy of modal parameters identification in both system identification methods summarized in the Figure (4.9.a.b). In order to conveniently track the results identified modal parameters normalized by attention to the nominal values (Table 4.1) of these parameters. In these figures modal parameters affected by increasing the data length of the response

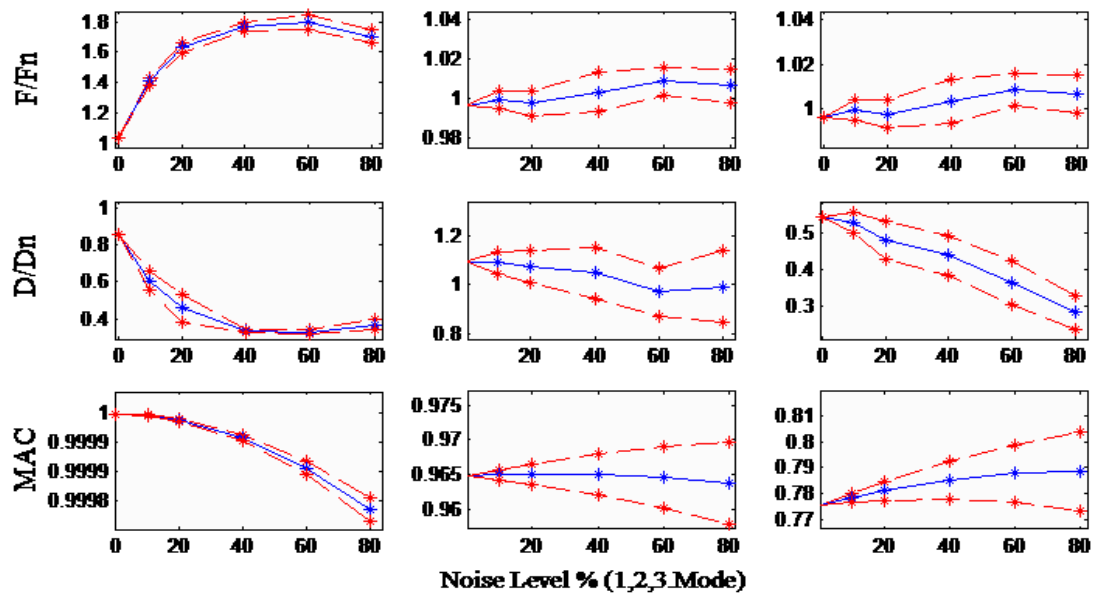
and show that by increasing in the length of data, the variances of the identified modal parameters decreases and the identified parameters converge to their nominal value by converge to 1 value but it will be variable in the upper modes.

4.4.3 Measurement/Sensor noise

In order to estimate modeling errors, zero-mean Gaussian white noise commonly adds as measurement/sensor noise to the output acceleration response in research. To do this, the probability measurement/sensor noise is modeled as a zero-mean Gaussian white noise process that is added to all channels of simulated acceleration response. Five levels of measurement noise, namely 0%, 20%, 40%, 60% and 80%, are considered here to study the effect of measurement noise on the variability of the identified modal parameters. The noise level is defined as the ratio of the RMS of the noise process to the RMS of the acceleration response process at each channel. This ratio is kept constant for all channels for a given noise level. The noise process added to each acceleration channel is statistically independent from the noise processes added to the other channels. Due to the random characteristics of the added noise vector processes, a set of 100 ($2 \times 1 \times 1 \times 5 \times 10$) identification runs. Variability of the mean and standard deviation statistics of the identified modal parameters due to variability of measurement noise input factors in these 100 identification trials is shown in Figure (4.10.a.b).



a) NExT-ERA



b) EFDD

Figure 4.10.a.b Uncertainty of the identified modal parameters due to variability of measurement noise

Identified modal parameters affect by increasing noise level and its result summarized in the figure (4.10.a.b). In order to conveniently track the results identified modal parameters normalized by their nominal values (Table 4.1). In this figure increasing in the noise level caused increasing of variances in all identified modes.

CHAPTER FIVE
UNCERTAINTY ANALYSIS OF MODAL PARAMETERS OBTAINED
FROM SYSTEM IDENTIFICATION METHODS FOR MODEL STEEL
BRIDGE TESTED ON THE DEÜ STRUCTURAL MECHANIC
LABORATORY

5.1 Introduction

Structural health monitoring (SHM) has become an important tool that can be used in the evaluation of existing structures in earthquake zone countries, damage assessment and strengthening of a reinforced structure. In this study, NEXT-ERA and EFDD methods were used for the SHM.NEXT technique based on the theoretical cross-correlation function between two response measurements made along two degrees of freedom (DOF) collected from an ambient (broad-band) excited structure has the same analytical form as the free vibration response of the structure (James et al, 1993).The cross-correlation function values has analytically same as free vibration response of structure. These values are particularly suitable for the use in light-damping systems, by entering to the ERA method, as appropriate modal identification method, which results to obtain the reduced dynamic model. By using this model, the dynamic (modal) parameters are estimated (Juang and Pappa, 1985).In the EFDD identification, based on the classical peak picking methods, first, estimate the power spectral density (PSD) matrix. Then the spectral matrix is decomposed into a set of cross spectral density (CSD) (auto spectral density) functions, each corresponding to a single degree of freedom (SDOF) system. In order to identifying the natural frequency and damping ratio of a vibration mode from the SDOF CSD function corresponding to that mode, the SDOF CSD function is taken back to the time domain by inverse Fourier transformation, and the frequency and damping ratio of the mode considered are estimated from the crossing times and the logarithmic decrement, respectively, of the corresponding SDOF auto-correlation function (Brincker et al.2001).

The purpose of this study, investigate uncertainty/variability of identified modal parameters due to uncertainty/variability of three significant input factors. These

input factors consist of spatial sensor density, measurement/ sensor noise and response data length. To do this, first, the modal parameters of a model steel bridge identified by NExT-ERA and EFDD, then the finite element model of the bridge which, modeled on an analysis framework of SAP2000, is subjected to calibration and update by attention to the identified modal parameters. In the next step three input factors investigate by using this calibrated FE model.

5.2 Applied Devices and Installation

Non slab steel bridge model with the span of 6.30 m, 2.05 m height and 2453 N weight, exposed to testing in the D.E.Ü. Eng. Faculty. Civil Eng. Department of Structural Mechanics Laboratory, shown in Figure (5.1). Additionally, eight concrete blocks approximately 1275 N cumulative weight that, represents the model bridge deck weight, are shown. Deck weights was selected particularly high, In this way, fundamental vibration frequencies of the bridge have been reduced, and accelerometers and data-acquisition devices will work on an appropriate frequency range. In this study, +/- 4.0g power range, DC-100 Hz frequency band-width and noise ratio of 10 mg RMS, 8 numbers of capacitive accelerometers and 8-channel PCI-type 12-bit resolution data-acquisition card and as a data acquisition software, LabVIEW software, was used.





Figure 5.1 Different views of model steel bridge

Bridge elements jointed to each other with bolts; however, this bolt joints so as not to form as classical truss elements, elements connect on the connection joints will be bearing moment. These details are considered in the creating of a FE model of the bridge. Bolts tightened with a torque wrench to 30 Nm torque. All bolts shall be tightened in a controlled way; it was important in terms of control of the dynamic properties of the bridge and allowing to follow for any changes and the determination of the reason, in those properties over time.

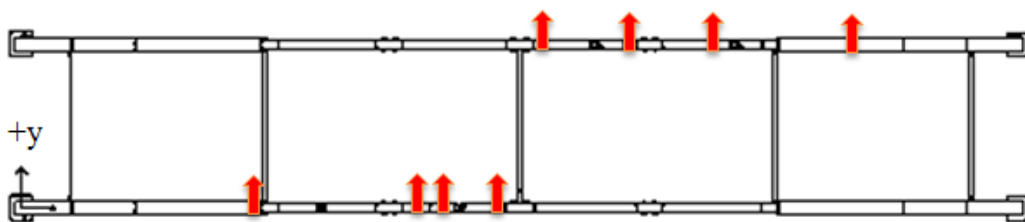


Figure 5.2 Positive directions of sensors in the experiments

In this study, the lateral motion of the bridge studied as a basic direction of movement of the bridge, because of this reason, sensors placed as recording in lateral direction which shown in the Figure (5.2). In order to identified modal parameters of the model, measured data from ambient vibrations and impulse responses of bridges subjected to output-only system identification methods which shown in the Table (5.1).

Table 5.1 Identified modal parameters using ambient vibration and impulse response

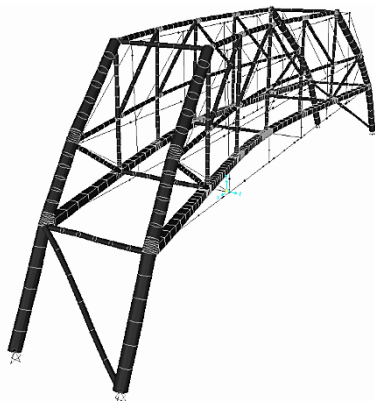
Method	Natural Frequency			Damping Ratio			MAC		
	ω [Hz]			ζ [%]					
Mode No	1	2	3	1	2	3	1	2	3
SAP Analysis	2.98	8.20	15.50	0.020	0.020	0.020	-	-	-
NEExT-ERA	2.85	8.23	15.01	0.027	0.024	0.022	0.99	0.80	0.82
EFDD	2.65	8.29	15.56	0.083	0.003	0.0001	0.99	0.86	0.84

5.3 Analytical Model of Tested Structure

In order to systematically study of the uncertainty/variability analysis of modal parameters due to uncertainty/variability of the input factors, are needed to calibrated/updated Finite Element (FE) model. First, the primary model formed by considered to the geometry of the bridge, material and geometric properties of the elements that constitute the bridge. Analytical model of bridge elements is chosen as elastic beam-column elements. Concrete blocks used as a representing deck weight in the respective point of the model as a point mass source.

The supports of analytical model modeled as fixed supports, damping ratio of the model is selected with attention to the experimental data analysis results and previously system identification studies for the steel bridge model, so, 2.0 % constant damping used as analytical model's damping ratio (Salavati et al.2011).Mode shapes and natural frequencies corresponding to the mode shapes of analytical models, matched by an iterative manner with real bridge mode shapes and natural frequencies obtained by a system identification process with considered to stay within certain physical limits, changes are needed. To evaluate that mode shapes obtained from the analytical method with those obtained by experimental method,

whether or not compatible with each other, The Modal Assurance Criteria (MAC) can be determined; MAC value can take values ranging between 0 and 1 (Allemang ve Brown, 1982).The high MAC value between the two modes shows their similarity to each other. In order to compare the suitability of mode shapes, during the calibration phase (iteratively), mode shapes were found from output-only system identification method by using recorded acceleration response that acquired from a certain point of the model which, obtained from a 0.5Hz-50Hz frequency band range, 8.0%g RMS band-limited Gaussian white noise, excited in a lateral direction. Calculated MAC values, between this mode shapes and mode shapes obtained from experimental data, other identified modal parameters and three maximum modal participating mass ratios are shown in Figure (5.2).



Mode 1

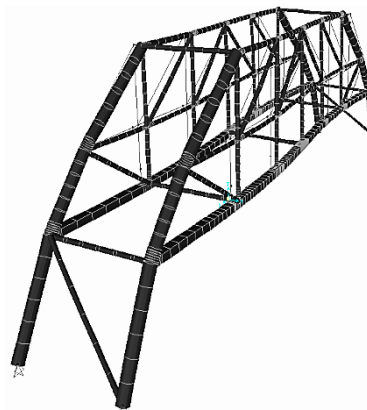
$\omega = 2.98$

$\zeta = 0.02$

MAC (NExT-ERA&SAP) = **0.99**

MAC (EFDD&SAP) = **0.99**

Modal Participating Mass Ratio=
0.664



Mode 2

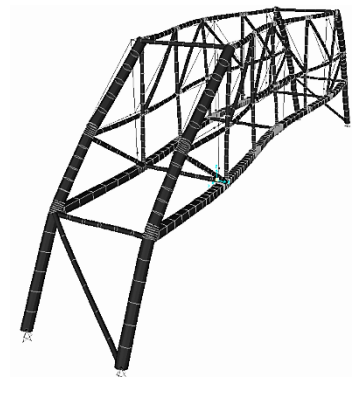
$\omega = 8.20$

$\zeta = 0.02$

MAC (NExT-ERA&SAP) = **0.80**

MAC (EFDD&SAP) = **0.86**

Modal Participating Mass Ratio=
0.015



Mode 3

$\omega = 15.50$

$\zeta = 0.02$

MAC (NExT-ERA&SAP) = **0.82**

MAC (EFDD&SAP) = **0.84**

Modal Participating Mass Ratio=
0.123

Figure 5.2 The first three modes obtained from the calibrated finite element model

Modal parameters obtained from the calibrated FE model (shown in Figure (5.1)) shall be regarded as nominal values. The high MAC values between nominal mode vectors from FE model and corresponding mode vectors obtained from system identification methods, indicate that mode shapes are matched.

5.4 Applied System Identification Methods and Numerical Programming Algorithm

In this study, two output-only system identification methods are used to estimation of modal parameters. NExT-ERA as a time domain method by using theoretical cross- correlation function between two response channels from an ambient excited structure has the same analytical form as the free vibration response of the structure. The response cross-correlation vector is obtained for a given reference channel, selecting the reference channel plays a significant role to accurately identify modal parameters, because of avoiding missing mode vectors due to adjacency of a reference channel to a modal node (James et al.1993).In the EFDD identification, the Frequency Domain Decomposition (FDD) is an extension of the basic frequency domain approach also referred to as the classical approach where the modal parameters are estimated by simple peak picking technique (Brincker et al.2001).

5.5 Dynamic Tests of Bridge Model and Assessment of uncertainty of System Identification Results

5.5.1 Spatial Sensors Density

In the structural health monitoring procedure, accurate estimation of modal parameters is extremely important. Depending on the estimated modal parameters, the undamaged or damaged conditions of structures can be evaluated. Therefore, it is very important investigating the factors having an influence on modal parameter estimation. The factors investigated in the literature consist of: sensor density, measurement/sensor noise, response data length, applied system identification techniques, amplitude of input excitation (Moaveni et al.2007).In this sensor density, measurement/sensor noise and response data length are investigating using two

different output-only system identification techniques, namely: NExT-ERA and EFDD.

The calibrated FE model is used to investigate effects of spatial sensor density on modal parameters. The FE model is calibrated by using identified modal parameters obtained experimentally from a real steel bridge model. Calibrated model excited by, zero mean Gaussian with noise (WN) band-limited between 0.5 Hz-50 Hz, 80% RMS amplitude. The WN is applied to the FE model along the lateral direction. Three different sensor densities consisting of: 4,8,16 virtual sensors, placed on the deck recorded the responses from the analytical model. The simulation results are used with the aforementioned system identification methods. Three different sensor arrays used to assess the effect of sensor density are shown in the Figure (5.3).

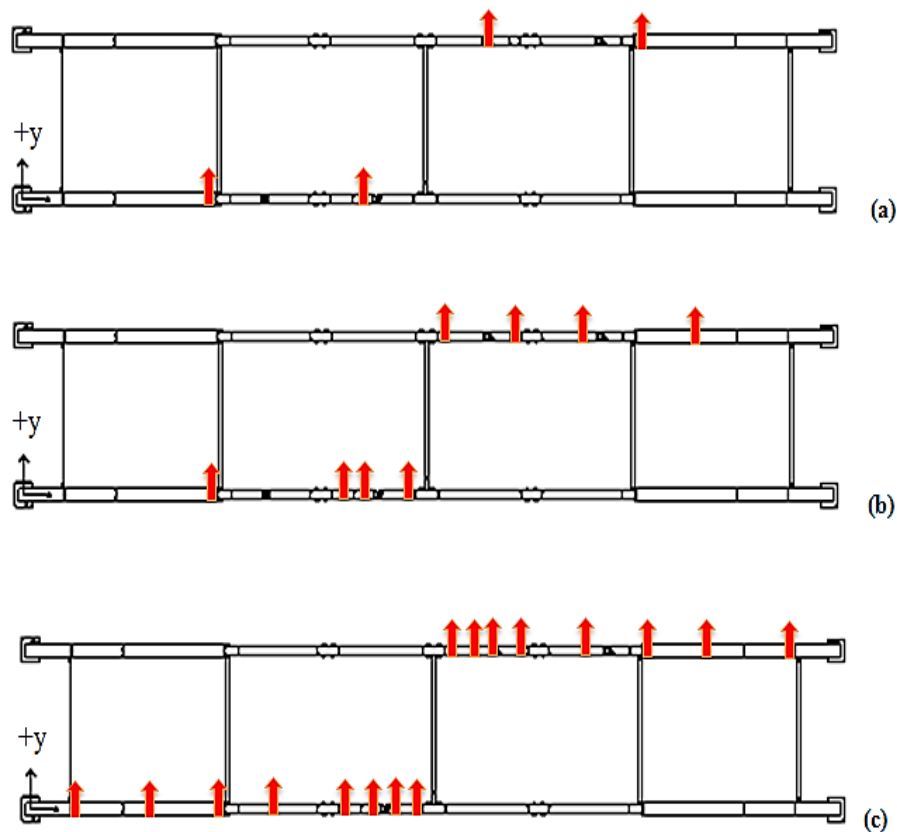
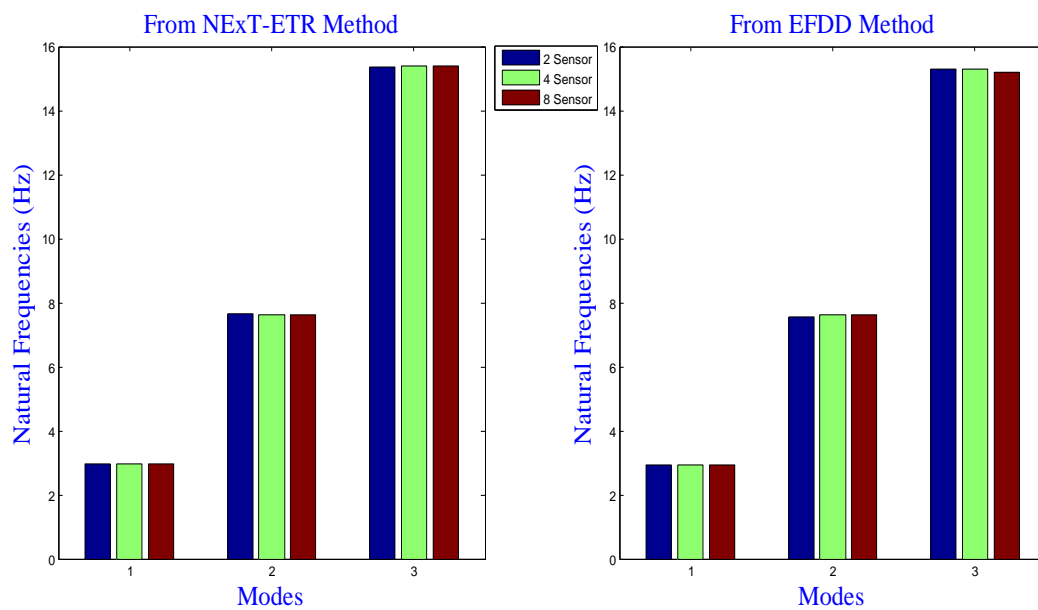
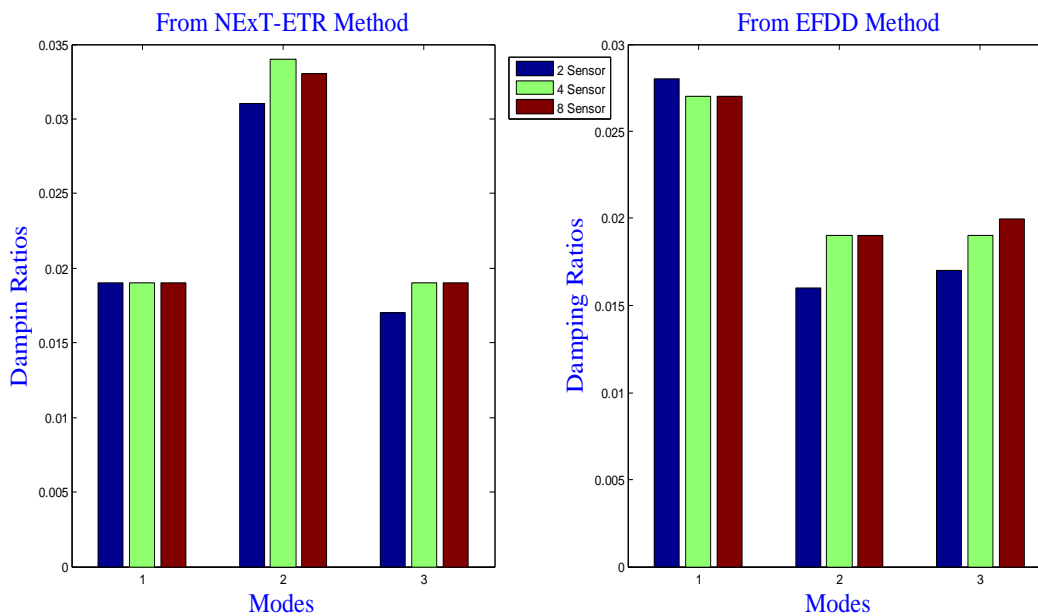


Figure 5.3 Sensor densities: (a) 4 sensors, (b) 8 sensors and (c) 16 sensors

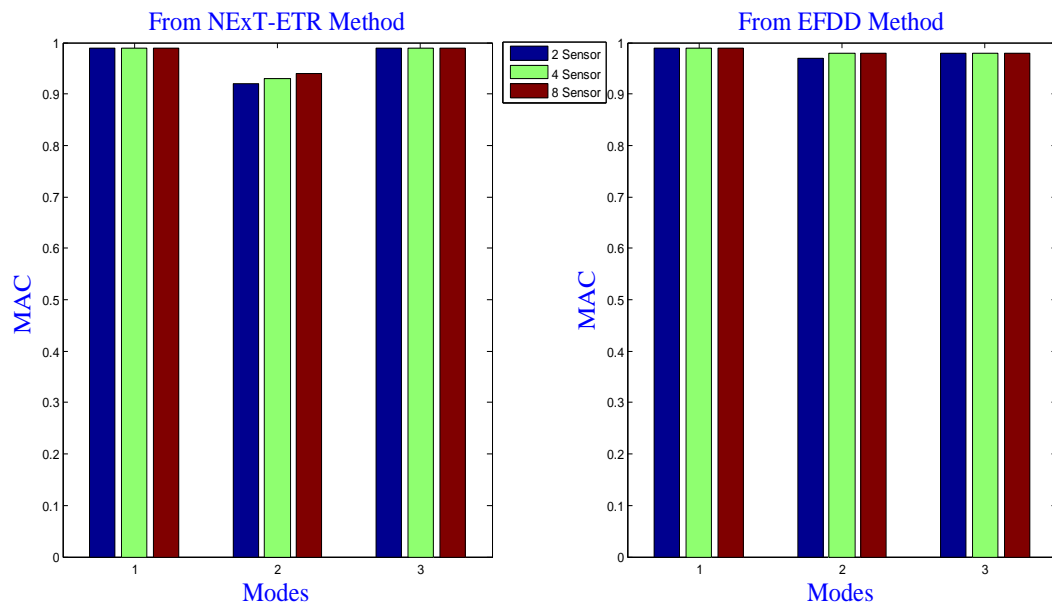
The mean value of the identified modal parameters for three different sensor configurations is shown in the Figure (5.4.a.b.c).



a) Natural frequencies



b) Dampin Ratios



c) MAC values

Figure 5.4.a.b Uncertainty/variability of modal parameters due to variability of spatial sensor density

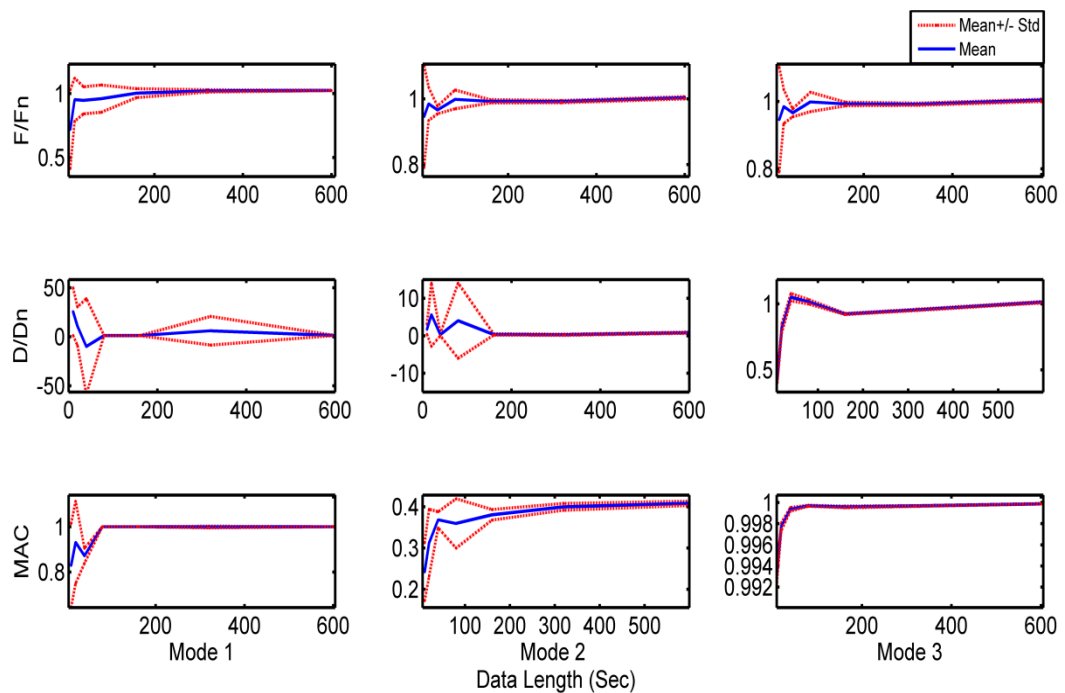
Sensor density input factor results shows that this factor does not affect the estimation accuracy of natural vibration frequencies both for NExT-ERA and EFDD. On the other hand, damping ratio estimated using NExT-ERA estimation for the second and third modes are affected by this factor. In the second mode, for all the sensor configuration damping estimates are higher than its nominal values (Figure 5.2). For the third mode, it is clear that by increasing the sensor number (density increases), damping ratio is estimated more accurately. Damping ratios identified using EFDD method are affected by the sensor density factor, for the first mode in all sensor arrays estimated damping values are larger than its nominal value. For the other two modes by increasing the sensor density, damping ratios for all modes correctly identified. MAC values found by both of the methods are not affected because of similarity of identified mode shapes to their nominal counterparts.

5.5.2 Response Data Length

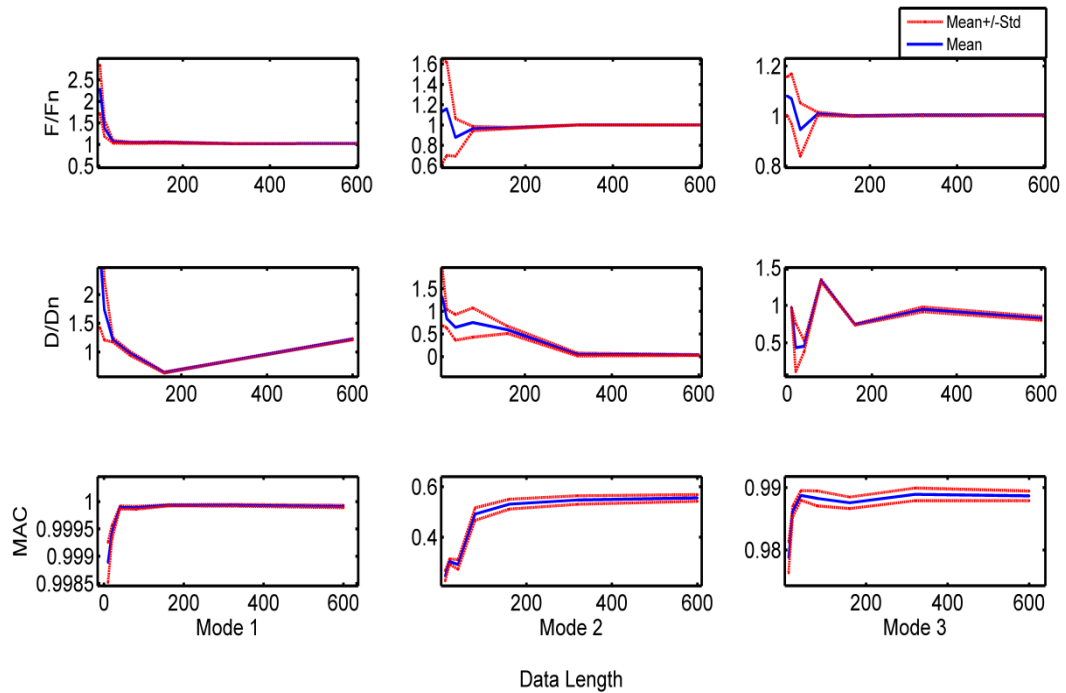
In order to investigate variability of modal parameters due to variability of input data length, the FE model with 16 sensor points is used. A Gaussian white noise with

20% g RMS amplitude is used for input excitation along lateral direction. Five different response data length of: 5, 20, 40, 80, 320, 480 (seconds) are used to investigate the effect of data length.

From previous experience (Salavati et al.2011), this factor is expected to play an important role in the variability of the identified modal parameters. Two identification methods mentioned earlier are used to identify modal parameters. A set of 120 different identification runs have been conducted and variability in modal parameters is investigated using the mean and standard deviation statistics of the identified modal parameters. The normalized results in terms of mean and $\pm 1\sigma$ are shown in Figure (5.5.a.b).



a) NExT-ERA



b) EFDD

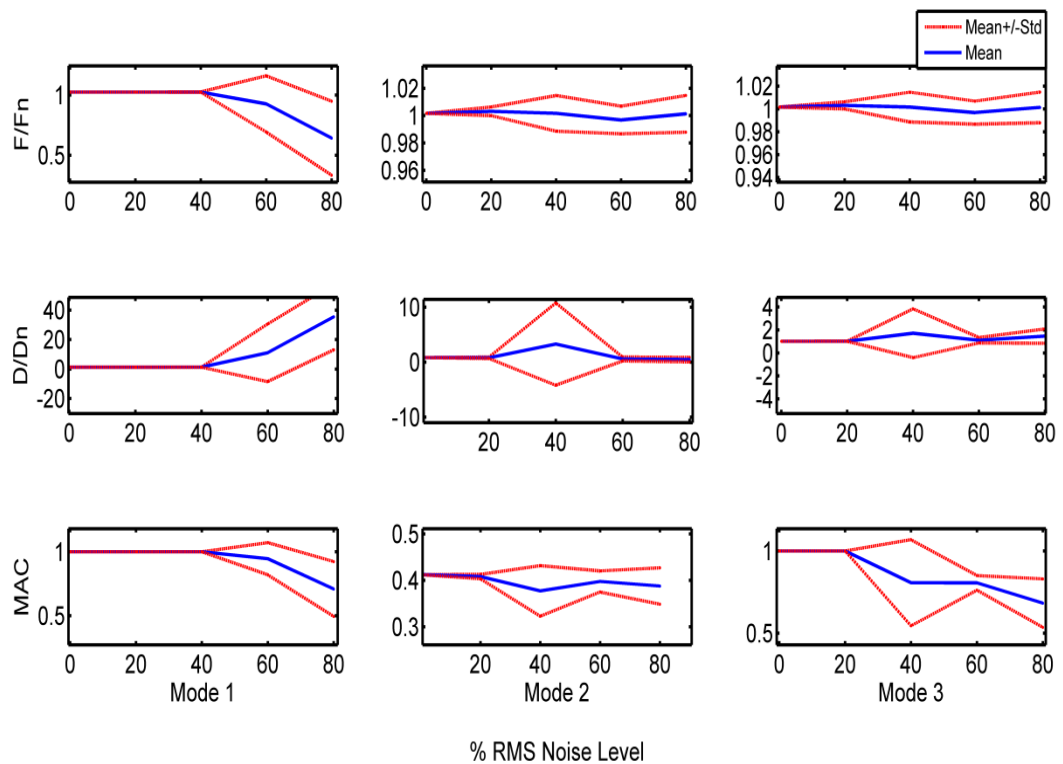
Figure 5.5.a.b Variability of the mean and standard deviation ($\pm 1\sigma$) statistics of the identified modal parameters due to variability of data length input factor

Mean and standard deviation values of the identified modal parameters normalized by their nominal values are summarized in Figure (5.5.a.b) in increasing values of response data length. It is clear from the results that identified modal parameters are affected by increasing the length namely increasing the data length results in decreasing uncertainty (lower variations).

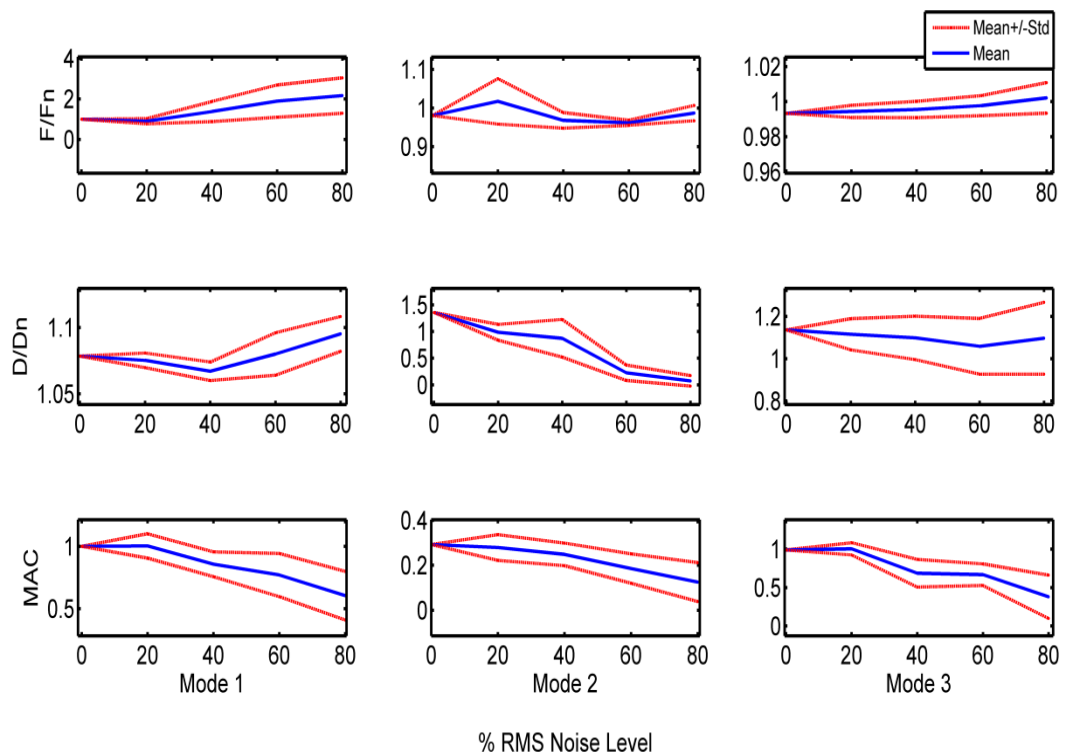
5.5.3 Measurement/Sensor Noise

Measurement noise is modeled as zero-mean Gaussian white noises and is added to the output acceleration response. To do this, the measurement/sensor noise is modeled as a zero-mean Gaussian white noise process that is added to all channels of simulated acceleration response. Five levels of measurement noise, namely 0%, 20%, 40%, 60% and 80%, are considered to study the effect of measurement noise on the variability of the identified modal parameters. The noise level is defined as the ratio of the RMS of the noise process to noise process added to each acceleration channel is made statistically independent by changing the seed number of the random number

generator in Matlab® from the noise processes added to the other channels. A set of 100 (2 system identification methods \times 1 sensor density \times 1 response data length \times 5 noise levels \times 10 random seed number) identification process runs using independent Gaussian white noise vector processes when the other input factors remain fixed. Variability of the mean and standard deviation statistics of the identified modal parameters due to variability of measurement noise level input factor for these 100 identifications trials are shown in Figure 5.6.a. b the RMS of the acceleration response process at each channel. This ratio is kept constant for all channels for a given noise level. The



a)NexT-ERA



b) EFDD

Figure 5.6 Uncertainty/Variability of the mean and standard ($\pm 1\sigma$) deviation statistics of the identified modal parameters due to variability of measurement noise

Effect of noise factor is shown in the Figure (5.6) which shows that the accuracy of the identified modal parameters is affected by increasing noise level. Based on the results presented variances of identified parameters in different level noise decrease by decreasing noise levels, This means that identified parameters are greatly affected by increasing noise level.

CHAPTER SIX

CONCLUSIONS

In civil engineering structures, damage prognosis and damage quantification (localization and intensity identification) depend in great extent on the accuracy of the estimated modal parameters. Modal parameter identification is affected by input factors relevant to the identification process. In this research study, the accuracy of identified modal parameters are investigated and effects of input factors such as spatial sensor density, data length and measurement noise level are investigated. Also, the effect of two different system identification methods, namely NExT-ERA and EFDD on the modal parameter identification are investigated. Both of the methods are programmed in Matlab® environment. Matlab programs are checked through using simulation results from a simple shear frame model. Then more complex real-life problem is used to realistically assess the input factors affecting the modal parameter estimations. In this stage of the study, a steel bridge currently housed in the structural mechanics laboratory of Dokuz Eylul University is used for experimental modal analysis.

Based on the analysis results, for both system identification techniques spatial sensor density does not affect the identified natural frequencies and MAC values of identified three modes. Damping ratios are affected by this factor, it is observed that: 1) In the NExT-ERA method damping ratio of the second mode in all sensor configuration (different sensor densities) is higher than their nominal counterparts. Moreover, damping ratios of the first and the third modes are approximately equal to their nominal values as sensor density increases; this means that damping ratio may be accurately identified by increasing sensor number. 2) For EFDD method, identified first mode damping ratios are higher than their nominal values, and for the second and the third modes, nominal values were approximately obtained. For both methods, it is clear that for modes with adequate response contributions as the sensor density increases better estimates of damping ratios can be obtained.

The other significant input factor that affects the accuracy of estimated modal parameters is response data length. The investigation of this factor in modal

parameter identification revealed that, the variability of this factor affects the accuracy of estimated modal parameters of structure greatly. For both NExT-ERA and EFDD methods with increasing response data length, estimation variances decrease and identified modal parameters converge to their nominal values. Rate of convergence is related to the performance of system identification methods and selecting appropriate values used for in different identification methods, such as: window length in Welch-Bartlett method to estimate the power spectral density, an appropriate resampling frequency before structuring the Hankel matrices, band-pass and low-pass filters used to pre-process the raw data.

The third important input factor affecting the accuracy of identified modal parameters is measurement noise level. In the identification of modal parameters using NExT-ERA method in increasing level of measurement noise, can be summarized as follows: 1) increasing noise level up to a certain level of noise (60%) was not affecting the accuracy of natural frequency estimations. 2) Damping ratios significantly affected by high levels of noise, meaning that, variances progressively increase. 3) For MAC values decreasing trends are observed, with increasing noise levels. For the identification of modal parameters using EFDD method, the effects of this input factor can be summarized as follow: 1) there was not a real clear trend in identified natural frequencies using EFDD. 2) For the second, variances suddenly decrease after a certain amount of noise level (80%). For the other modes, with the increasing the noise level, variances increase as well. 3) A decreasing trend in the MAC values is observed as the noise level increases. Overall it can be concluded that, increasing trend in variance values can clearly be seen for both identification techniques in all identified modal parameters with increasing noise level. Finally, it is possible to say that the most sensitive modal parameter to the noise level is damping ratio.

In order to accurately identify damage in structural health monitoring with non-destructive testing technology, three input factors investigated are very important and must be given utmost attention in system identification process.

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