DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

COMPARISON OF EFFICIENCY OF TWO-LAYER MEDIAN RANKED SET SAMPLING USING CONCOMITANT VARIABLES

by
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August, 2015
İZMİR

COMPARISON OF EFFICIENCY OF TWO-LAYER MEDIAN RANKED SET SAMPLING USING CONCOMITANT VARIABLES

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M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"COMPARISON OF EFFICIENCY OF TWO-LAYER MEDIAN RANKED SET SAMPLING USING CONCOMITANT VARIABLES"** completed by **BEGÜM KARA** under supervision of **ASSIST. PROF. DR. NESLİHAN DEMİREL** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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COMPARISON OF EFFICIENCY OF TWO-LAYER MEDIAN RANKED SET SAMPLING USING CONCOMITANT VARIABLES

ABSTRACT

In ranked set sampling (RSS), ranking is done on the basis of a concomitant variable which is functionally related with the response variable. Chen and Shen (2003) developed two-layer RSS procedure with two concomitant variables. In addition, Muttlak (1997) proposed on modifications to RSS procedure which is called median ranked set sampling (MRSS). In this study, we extend median ranked set sampling design in terms of concomitant variables and proposed the two-layer median ranked set sampling (TMRSS). The performance of MRSS and TMRSS are evaluated for different distributions in terms of mean estimator and regression estimators by simulation studies. Also, the efficiency of TRSS method is compared with respect to corresponding estimators in simple random sampling (SRS) and RSS methods.

Keywords: Ranked set sampling, median ranked set sampling**,** two-layer ranked set sampling, concomitant variables, mean estimator, relative effciency.

İKİ AŞAMALI MEDYAN SIRALI KÜME ÖRNEKLEMESİNDE YARDIMCI DEĞİŞKENLER KULLANILARAK ETKİNLİĞİN KARŞILAŞTIRILMASI

ÖZ

Sıralı küme örneklemesinde, sıralama yanıt değişkeni ile fonksiyonel olarak ilişkili olan bir yardımcı değişkene bağlı olarak yapılır. Chen ve Shen (2003) iki yardımcı değişkenli iki aşamalı sıralı küme örneklemesini önermiştir. Ayrıca, Muttlak (1997) sıralı küme örneklemesi prosedürünün modifikasyonu olan medyan sıralı küme örneklemesini önermiştir. Bu çalışmada medyan sıralı küme örneklemesi yardımcı değişkenler açısından genişletilmiştir. Çalışmanın temel amacı tek yardımcı değişken kullanılan medyan sıralı küme örneklemesinde, iki yardımcı değişken kullanarak iki aşamalı medyan sıralı küme örneklemesini göstermektir. Medyan sıralı küme örneklemesi ve iki aşamalı medyan sıralı küme örneklemesinin etkinlikleri ortalama kestiricisi ve regresyon kestiricileri açısından farklı dağılımlar için simülasyon çalışmaları ile değerlendirilmiştir. Ayrıca iki aşamalı sıralı küme örneklemesi ile elde edilen kestiricilerin etkinliği açısından basit rasgele örnekleme ve sıralı küme örneklemesi kestiricileri ile karşılaştırılmıştır.

Anahtar kelimeler: Sıralı küme örneklemesi, medyan sıralı küme örneklemesi**,** iki aşamalı sıralı küme örneklemesi, yardımcı değişkenler, ortalama kestiricisi, göreli etkinlik.

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CHAPTER ONE
INTRODUCTION

Sample is the subset of population that allows researchers to make statistical inference about a population, without having to investigate every unit. If a sample is to be used, it is critical that the units chosen are representative of the entire population. Consequently, there are several sampling techniques accessible in literature.

The most common method of sampling is known as simple random sampling (SRS). But SRS sometimes does not represent the whole population. Moreover, in certain practical sampling issues the measurement of a survey variable for a sampled item is costly or time-consuming. However, the ranking of a set of items related to the variable can be easily done by certain means without actual measurement. A sampling scheme known as ranked ret sampling (RSS) can be applied in such situation to reduce cost and to increase efficiency (Chen, 2000).

Ranked set sampling was first proposed by McIntyre (1952) for estimation of a population mean pasture yield. Halls and Dell (1966) applied the method to estimating forage yields. Takahashi and Wakimoto (1968) developed the mathematical theory of RSS. They indicated that with perfect ranking, the mean of the simple random sample had a higher variance than the mean of the ranked set sample and RSS estimator was unbiased. As a result of these studies, it was found that RSS is more efficient than SRS.Dell and Clutter (1972) and David and Levine (1972) showed that the same results are true when ranking is imperfect and also the effect of the ranking error.

After the development of the theory of RSS, applications of this approach increased. Evans (1967) applied RSS to regeneration surveys in areas direct-seeded to longleaf pine. He noticed that the means based on both of RSS and SRS methods are not basically distinctive, but the computed variances of the means are altogether different. Cobby et al. (1985) conducted four experiments at Hurley (UK) during

1983 to research the performance of RSS with respect to SRS for estimation of herbage mass in immaculate grass swards, and of herbage mass and clover content in blended grass-clover swards. Mode et al. (1999) explored under which conditions RSS becomes a cost-effective sampling method for natural and ecological field studies where the rough but cheap measurement has an expense. They presented formula for the aggregate cost for both RSS and SRS, and cost ratios for a real data set consisting of judgment estimated and physically measured stream. Al-Saleh and Al-Shrafat (2001) studied the performance of RSS in estimating milk yield based on 402 sheep.

Stokes (1976) studied the estimation of the population variance, interval estimation, and the use of a concomitant variable for ranking the sample. Stokes (1980a, 1980b) estimated the variance using judgment ordered ranked set samples and correlation coefficient using ranked set samples in bivariate normal distribuiton.

Patil, Sinha and Taillie (1993) showed that RSS estimator is more efficient than regression estimator if the correlation between the concomitant variable and the actual variable is low.

Muttlak (1997) proposed median ranked set sampling (MRSS) to reduce ranking error. Al-Saleh and Al-Omari (2002) suggested the multistage RSS to increase efficiency of the estimator of population mean and estimate the average of Olives yields in a field in West of Jordan. Al-Omari (2008) also studied double percentile RSS method.

Chen and Shen (2003) proposed two-layer RSS which has two concomitant variables. In the first layer of the procedure, sampling units are ranked with respect to one concomitant variable, and in the second layer, the sampling units are ranked with respect to another concomitant variable. Since the two-layer RSS falls into the scheme of the general RSS, they claim that all the features of the general RSS can be applied for the two-layer RSS without any details of the proofs. The results of the simulations illustrate the superiority of the two-layer RSS over the marginal RSS (Al-Omari and Bouza, 2014).

The aim of this study is to extend the median ranked set sampling design in terms of concomitant variables. This study is arranged as follows. In Chapter 2, the motivations of simple random sampling, ranked set sampling and two-layer ranked Set sampling are discussed and the details of these methods are described. In Chapter 3, the motivation of median ranked set sampling is discussed and some definition about two-layer median ranked set sampling is given. The simulation results are presented in Chapter 4. These results include the comparison of SRS and RSS with TRSS, and the comparison of MRSS with TMRSS. Finally, Chapter 5 provides concluding remarks.

CHAPTER TWO
SIMPLE RANDOM SAMPLING, RANKED SET SAMPLING AND TWO-LAYER RANKED SET SAMPLING

2.1 Simple Random Sampling

The most common method of sampling is known as simple random sampling (SRS). SRS is defined as a method of selecting n units from a population of size N so that each of the $\binom{N}{n}$ certain samples is equally likely to be chosen by Cochran (1977).

The method involves making a list of all members of the population and using random tables or a computer program that produces such a table to throw up numbers for inclusion.

A simple random sample of size n is drawn with replacement from the population having mean μ and variance σ^2; say $X_1, X_2, ..., X_n$; then the sample mean can be estimated by Equation 2.1.

$$\bar{X}_{SRS} = \frac{1}{n}\sum_{i=1}^{n} X_i \quad (2.1)$$

The variance of the sample mean estimator is defined as in Equation 2.2.

$$V(\bar{X}_{SRS}) = \frac{\sigma^2}{n} \quad (2.2)$$

2.2 Ranked Set Sampling

Ranked set sampling is proposed that enables more accurate representation of the true population to be provided to the collected sample items. This method was first published in a paper by McIntyre (1952) for situations where measuring the sample observations is not easy or it is costly, destructive and time consuming. In his paper, McIntyre was interested in finding a cheap way to improve the precision in estimation of average yield from large plots of arable crops without a huge number of samples from which detailed costly and tedious measurements needed to be taken.

The goal of RSS is illustrated in Figure 2.1. The sample which is selected by SRS design is signified by the curve with the solid line and the curve with dotted line signified sample which is selected by RSS design. As we can see in Figure 2.1, the selection of observations by RSS from a population is more likely to span the full range of values in the population than observations obtained by simple random sampling. In this manner, the estimator of a parameter can provide better information about the parameter (Wolfe, 2012).

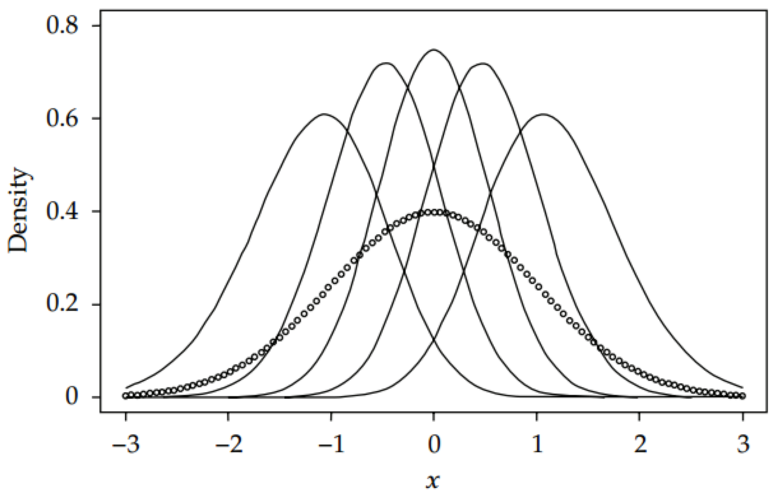

Figure 2.1 How RSS can provide better information about the parameter

Let k be a positive integer which denotes the set size. The steps of RSS method are given below:

1. First, randomly select k^2 units by simple random sampling from the target population.
2. Divide the k^2 selected units as randomly as possible into k sets, each of size k.
3. Without yet knowing any values for the variable of interest, rank the units within each set with respect to variable of interest. This ranking process may be based on personal professional judgment or a concomitant variable correlated with the variable of interest. The measurement of this concomitant variable is much cheaper than the measurement of the variable of interest.

4. Select a sample for actual quantification by including the smallest ranked unit from the first set, the second smallest ranked unit from the second set, the procedure is proceeded in this way until the largest ranked unit is selected from the last set.
5. Repeat the previous steps for m cycles to obtain a sample of size $n=km$ for actual quantification.

As an illustration in Table 2.1, consider the set size $k=4$ with $m=3$ cycles. This situation is illustrated in Table 2.1 where each row denotes a judgment-ordered sample within a cycle, and the units selected for quantitative analysis are bolded. Note that 48 units have been randomly selected in four cycles; however, only 12 units are actually selected to obtain the ranked set sample of measurements.

Table 2.1 RSS procedure for $k=4$ and $m=3$

Cycle 1
$\boldsymbol{X_{[1]1}} \leq X_{[2]1} \leq X_{[3]1} \leq X_{[4]1} \rightarrow \boldsymbol{X_{[1]1}}$
$X_{[1]1} \leq \boldsymbol{X_{[2]1}} \leq X_{[3]1} \leq X_{[4]1} \rightarrow \boldsymbol{X_{[2]1}}$
$X_{[1]1} \leq X_{[2]1} \leq \boldsymbol{X_{[3]1}} \leq X_{[4]1} \rightarrow \boldsymbol{X_{[3]1}}$
$X_{[1]1} \leq X_{[2]1} \leq X_{[3]1} \leq \boldsymbol{X_{[4]1}} \rightarrow \boldsymbol{X_{[4]1}}$
Cycle 2
$\boldsymbol{X_{[1]2}} \leq X_{[2]2} \leq X_{[3]2} \leq X_{[4]2} \rightarrow \boldsymbol{X_{[1]2}}$
$X_{[1]2} \leq \boldsymbol{X_{[2]2}} \leq X_{[3]2} \leq X_{[4]2} \rightarrow \boldsymbol{X_{[2]2}}$
$X_{[1]2} \leq X_{[2]2} \leq \boldsymbol{X_{[3]2}} \leq X_{[4]2} \rightarrow \boldsymbol{X_{[3]2}}$
$X_{[1]2} \leq X_{[2]2} \leq X_{[3]2} \leq \boldsymbol{X_{[4]2}} \rightarrow \boldsymbol{X_{[4]2}}$
Cycle 3
$\boldsymbol{X_{[1]3}} \leq X_{[2]3} \leq X_{[3]3} \leq X_{[4]3} \rightarrow \boldsymbol{X_{[1]3}}$
$X_{[1]3} \leq \boldsymbol{X_{[2]3}} \leq X_{[3]3} \leq X_{[4]3} \rightarrow \boldsymbol{X_{[2]3}}$
$X_{[1]3} \leq X_{[2]3} \leq \boldsymbol{X_{[3]3}} \leq X_{[4]3} \rightarrow \boldsymbol{X_{[3]3}}$
$X_{[1]3} \leq X_{[2]3} \leq X_{[3]3} \leq \boldsymbol{X_{[4]3}} \rightarrow \boldsymbol{X_{[4]3}}$

In SRS the sampler must increase the sample size to increase the possibility of coverage of the entire range of conceivable observations values. On the other hand, in RSS, one can increase the representativeness based on a specific number of sample observations. Consequently, there is a significant saving on the measurement costs. Thus, based on the measured ranked set sample, we can obtain unbiased estimators of population parameters, such as the mean, and for more than one cycle, the population variance (Al-Omari and Bouza, 2014).

Additionally, as it is understood from the name ranked set sampling, there is an ordering mechanism. This implies that the ranked data are order statistics if the following assumptions are satisfied,

1. sets are independent,
2. judgment order statistics are independent,
3. consistent ranking mechanism is used

2.2.1 *The Mean Estimator of RSS*

Let $X_{[1]1}$, …, $X_{[k]m}$ be a ranked set sample. RSS estimator $\bar{X}_{RSS}$ is an unbiased estimator for the population mean μ regardless of whether the judgment rankings are perfect or imperfect. This can be seen from Equation 2.3.

$$\bar{X}_{RSS} = \frac{1}{k}\sum_{i=1}^{k}\frac{1}{m}\sum_{j=1}^{m}X_{[i]j} \tag{2.3}$$

The variance of $\bar{X}_{RSS}$ is given in Equation 2.4.

$$Var(\bar{X}_{RSS}) = Var(\bar{X}_{SRS}) - \frac{1}{k^2}\sum_{i=1}^{k}\left(\mu_{[i]} - \mu\right)^2 \tag{2.4}$$

It is clear that

$$Var(\bar{X}_{RSS}) \leq Var(\bar{X}_{SRS}).$$

The variance of RSS estimator is equal or smaller than variance of SRS estimator. Inequality becomes equality when the ranking is completely random.

The result could be shown in a different way.

$$Var(\bar{X}_{SRS}) = Var(\bar{X}_{RSS}) + \frac{1}{k^2}\sum_{i=1}^{k}\left(\mu_{[i]} - \mu\right)^2$$

Another important issue is the efficiency of the estimators.

$$eff\left(Var(\bar{X}_{SRS}), Var(\bar{X}_{RSS})\right) = \frac{Var(\bar{X}_{SRS})}{Var(\bar{X}_{RSS})} = \frac{Var(\bar{X}_{RSS}) + \frac{1}{k^2}\sum_{i=1}^{k}\left(\mu_{[i]} - \mu\right)^2}{Var(\bar{X}_{RSS})} > 1$$

It is clear that the mean estimator of RSS is more efficient then SRS.

2.3 Two-layer Ranked Set Sampling

Chen, Bai and Sinha (2004) described the concomitant variable as follows. A sampling unit can be associated with several variables, one of which is expensive to measure but the others can be obtained easily and cheaply. The "expensive" variable is either the variable of main interest or the response variable in the setting of a regression model. For the sake of convenience, they refered to the "expensive" variable as the response variable in all cases and to the others as concomitant variables and concentrate on consistent ranking mechanism susing multiple concomitant variables. They developed a two-layer RSS procedure with two concomitant variables. The procedure can be applied whether or not the relationship between the variable of interest and the concomitant variables are linear (Chen and Shen, 2003).

Let $X^{\{1\}}$and $X^{\{2\}}$denote the concomitant variables. The steps of two-layer RSS method are given below:

1. Identify kl^2elements from the target population and divide these elements randomly into l^2 sets each of size k.
2. The units in each of these sets are ranked by $X^{\{1\}}$.
3. Then, for kl^2 ranked sets, the units with $X^{\{1\}}$-rank 1 are selected, for another l^2 ranked sets, the units with $X^{\{1\}}$-rank 2 are selected, and so on.
4. This completes the first layer of the procedure.
5. In the second layer, the units are separated – haphazardly or systematically - into *l* subsets, each of size *l*.
6. The units in each of these subsets are ranked by $X^{\{2\}}$.
7. At that point, for the first ranked subset, the unit with $X^{\{2\}}$-rank 1 are chosen and its value on *Y* will be measured, for the second ranked subset, the unit with $X^{\{2\}}$-rank 2 is chosen and its value on *Y* will be measured, and so on.
8. These steps are referred to as a cycle.

Table 2.2 The two-layer ranked set sample

$$\left(Y_{[1]1}, X_{[1]1}^{\{1\}}, X_{[1]1}^{\{2\}}\right), \left(Y_{[1]2}, X_{[1]2}^{\{1\}}, X_{[1]2}^{\{2\}}\right), \ldots, \left(Y_{[1]l^2}, X_{[1]l^2}^{\{1\}}, X_{[1]l^2}^{\{2\}}\right)$$
$$\left(Y_{[2]1}, X_{[2]1}^{\{1\}}, X_{[2]1}^{\{2\}}\right), \left(Y_{[2]2}, X_{[2]2}^{\{1\}}, X_{[2]2}^{\{2\}}\right), \ldots, \left(Y_{[2]l^2}, X_{[2]l^2}^{\{1\}}, X_{[2]l^2}^{\{2\}}\right)$$
$$\vdots$$
$$\left(Y_{[k]1}, X_{[k]1}^{\{1\}}, X_{[k]1}^{\{2\}}\right), \left(Y_{[k]2}, X_{[k]2}^{\{1\}}, X_{[k]2}^{\{2\}}\right), \ldots, \left(Y_{[k]l^2}, X_{[k]l^2}^{\{1\}}, X_{[k]l^2}^{\{2\}}\right)$$

9. After repeating the cycle *m* times, the data set

$$\{ Y_{[r][s]j} : r = 1, \ldots, k;\ s = 1, \ldots, l;\ j = 1, \ldots, m\}$$

where $Y_{[r][s]j}$is the measurement of *Y* in the j^{th} cycle on the unit with $X^{\{1\}}$- rank *r* and $X^{\{2\}}$-rank *s* (Chen and Shen, 2003).

The estimator of population mean μ using TRSS method and a sample of size *n*= *klm* with *m* cycles is given by Equation 2.5.

$$\bar{X}_{TRSS} = \frac{1}{m}\sum_{j=1}^{m}\frac{1}{l}\sum_{s=1}^{l}\frac{1}{k}\sum_{r=1}^{k} Y_{[r][s]j} \tag{2.5}$$

CHAPTER THREE
MEDIAN RANKED SET SAMPLING AND TWO-LAYER MEDIAN RANKED SET SAMPLING

3.1 Median Ranked Set Sampling

Several researchers propose of modifications to ranked set sampling procedure. The basic purpose of all modified methods is using not every order statistic in each cycle; rather the researcher picks which ones would be most useful. One of the modified methods is MRSS, which was proposed by Muttlak (1997).

MRSS can be performed with less error in ranking in practical applications. Since all we have to do is to find the element in the middle of the sample and measure it, RSS method can be easily employed in the field and will save some time in performing the ranking of the units with respect to the variable of interest (Hajighorbani and Saba, 2012).

RSS method can be summarized as follows:

1. Choose randomly select k^2 units by simple random sample from the population.
2. Share the k^2 selected units as randomly as possible into k sets, each of size k and rank the units within each sample with respect to a variable of interest.
3. If the sample size k is odd, from each sample select for measurement the $((k+1)/2)^{th}$ smallest rank which is the median of the sample.
4. If the sample size k is even, select for measurement from the first $k/2$ samples the $(k/2)^{th}$ smallest rank and from the second $k/2$ samples the $((k+2)/2)^{th}$ smallest rank.
5. The cycle may be repeated m times to get km units.

As an illustration, consider the set size k=4 with m=3 cycles. This situation is illustrated in Table 3.1 where each row denotes a judgment-ordered sample within a cycle, and the units selected for quantitative analysis are bolded

Table 3.1 MRSS procedure for k=4 and m=3

Cycle 1
$X_{[1]1} \le \boldsymbol{X_{[2]1}} \le X_{[3]1} \le X_{[4]1} \rightarrow \boldsymbol{X_{[1]1}}$
$X_{[1]1} \le \boldsymbol{X_{[2]1}} \le X_{[3]1} \le X_{[4]1} \rightarrow \boldsymbol{X_{[2]1}}$
$X_{[1]1} \le X_{[2]1} \le \boldsymbol{X_{[3]1}} \le X_{[4]1} \rightarrow \boldsymbol{X_{[3]1}}$
$X_{[1]1} \le X_{[2]1} \le \boldsymbol{X_{[3]1}} \le X_{[4]1} \rightarrow \boldsymbol{X_{[4]1}}$
Cycle 2
$X_{[1]2} \le \boldsymbol{X_{[2]2}} \le X_{[3]2} \le X_{[4]2} \rightarrow \boldsymbol{X_{[1]2}}$
$X_{[1]2} \le \boldsymbol{X_{[2]2}} \le X_{[3]2} \le X_{[4]2} \rightarrow \boldsymbol{X_{[2]2}}$
$X_{[1]2} \le X_{[2]2} \le \boldsymbol{X_{[3]2}} \le X_{[4]2} \rightarrow \boldsymbol{X_{[3]2}}$
$X_{[1]2} \le X_{[2]2} \le \boldsymbol{X_{[3]2}} \le X_{[4]2} \rightarrow \boldsymbol{X_{[4]2}}$
Cycle 3
$X_{[1]3} \le \boldsymbol{X_{[2]3}} \le X_{[3]3} \le X_{[4]3} \rightarrow \boldsymbol{X_{[1]3}}$
$X_{[1]3} \le \boldsymbol{X_{[2]3}} \le X_{[3]3} \le X_{[4]3} \rightarrow \boldsymbol{X_{[2]3}}$
$X_{[1]3} \le X_{[2]3} \le \boldsymbol{X_{[3]3}} \le X_{[4]3} \rightarrow \boldsymbol{X_{[3]3}}$
$X_{[1]3} \le X_{[2]3} \le \boldsymbol{X_{[3]3}} \le X_{[4]3} \rightarrow \boldsymbol{X_{[4]3}}$

Let $X_{[\boldsymbol{i}]\boldsymbol{j}}$ denote a median ranked set sample unit in the j^{th} cycle on rank i. For the chosen ranked set sample, the estimator of μ is given by Equation 3.1.

$$\bar{X}_{MRSS} = \frac{1}{m}\sum_{j=1}^{m}\frac{1}{k}\sum_{i=1}^{k} X_{[\boldsymbol{i}]\boldsymbol{j}}. \tag{3.1}$$

3.2 Two-Layer Median Ranked Set Sampling

In this section, we proposed a method which is a combination of MRSS and TRSS methods to obtain mean estimator and it is called as two-layer median ranked set sampling (TMRSS).

Let $X^{\{1\}}$ and $X^{\{2\}}$ denote the concomitant variables. A two-layer median RSS procedure goes as follows.

1. First, kl^2 independent sets, each of size k, are drawn from the target population.
2. The units in each of these sets are ranked according to $X^{\{1\}}$.
3. If the sample size k is odd, from each sample select for measurement the $((k+1)/2)^{th}$ smallest rank which is the median of the sample.
4. If the sample size k is even, for l^2ranked sets, select for measurement from the first $k/2$ samples the $(k/2)^{th}$ smallest rank and from the second $k/2$ samples the $((k+2)/2)^{th}$ smallest rank, and so on. This completes the first layer of the procedure.
5. In the second layer, every sets are separated – haphazardly or systematically - into l subsets, each of size l.
6. The units in each of these subsets are ranked by $X^{\{2\}}$.
7. If the sample size l is odd, from each sample select for measurement the $((l+1)/2)^{th}$ smallest rank which is the median of the sample.
8. If the sample size l is even, for l^2 ranked sets, select for measurement from the first $l/2$ samples the $(l/2)^{th}$ smallest rank and and its value on Y will be measured, for another l^2 ranked sets, from the second $l/2$ samples the $((l+2)/2)^{th}$ smallest rank, and its value on Y will be measured, and so on. This completes the one cycle of the procedure.

Table 3.2 The two-layer median ranked set sample

$$\left(Y_{[1]1}, X^{\{1\}}_{[1]1}, X^{\{2\}}_{[1]1}\right), \left(Y_{[1]2}, X^{\{1\}}_{[1]2}, X^{\{2\}}_{[1]2}\right), \ldots, \left(Y_{[1]l^2}, X^{\{1\}}_{[1]l^2}, X^{\{2\}}_{[1]l^2}\right)$$
$$\left(Y_{[2]1}, X^{\{1\}}_{[2]1}, X^{\{2\}}_{[2]1}\right), \left(Y_{[2]2}, X^{\{1\}}_{[2]2}, X^{\{2\}}_{[2]2}\right), \ldots, \left(Y_{[2]l^2}, X^{\{1\}}_{[2]l^2}, X^{\{2\}}_{[2]l^2}\right)$$
$$\vdots$$
$$\left(Y_{[k]1}, X^{\{1\}}_{[k]1}, X^{\{2\}}_{[k]1}\right), \left(Y_{[k]2}, X^{\{1\}}_{[k]2}, X^{\{2\}}_{[k]2}\right), \ldots, \left(Y_{[k]l^2}, X^{\{1\}}_{[k]l^2}, X^{\{2\}}_{[k]l^2}\right)$$

9. The cycle may be repeated m times to get klm units. The data set

$$\{ Y_{[r][s]j} : r = 1, \ldots, k; s = 1, \ldots, l; j = 1, \ldots, m \}$$

where $Y_{[r][s]j}$ is the measurement of *Y* in the *j*th cycle on the unit with $X^{\{1\}}$- rank *r* and $X^{\{2\}}$- rank *s* (Chen and Shen, 2003).

The estimator of population mean μ using TMRSS method and a sample of size *n*= *klm* with *m* cycles is given by Equation (3.2)

$$\bar{X}_{TMRSS} = \frac{1}{m}\sum_{j=1}^{m}\frac{1}{l}\sum_{s=1}^{l}\frac{1}{k}\sum_{r=1}^{k} Y_{[r][s]j} . \tag{3.2}$$

CHAPTER FOUR
SIMULATION STUDY

In this chapter, we present two simulation studies. The efficiency of the TRSS method is evaluated with respect to corresponding estimators in SRS and RSS methods by the first simulation study. In the second simulation study, we are concerned with the comparison between the TMRSS and RSS.

In addition, we discuss and compare mean square errors of regression models, relative efficiencies of sample mean estimators and the amount of biases of regression coefficients of five methods. In simulation results, comparison of SRS and RSS with TRSS, and comparison of MRSS with TMRSS are given.

Several probability distribution functions are considered: normal (0,1), uniform (0,1), exponential (1), gamma (5,1) and lognormal (0,1). The skewness of distributions are seen in Figure 4.1.

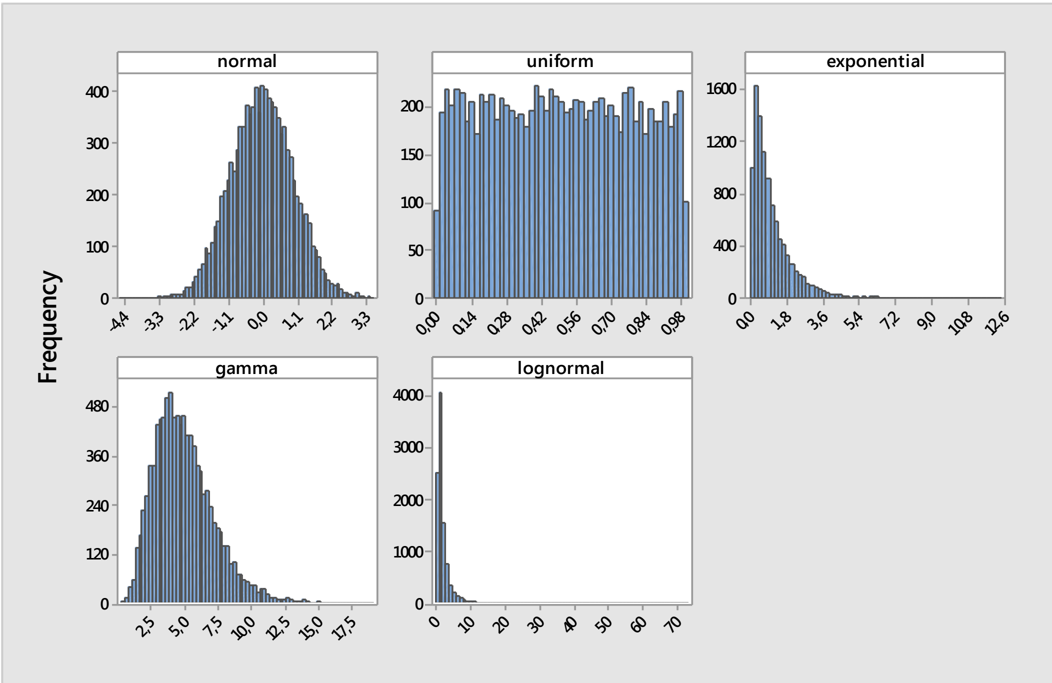

Figure 4.1 The histograms of distributions which are used in the simulation study

The simulation studies are performed with R Project using the process given below:

1. Generate 10,000 observations which come from five different distributions such as: normal (0, 1), uniform (0, 1), exponential (1), gamma (5, 1) and lognormal (0, 1) for X_1 and X_2.
2. Generate random error term (ε) from normal distribution with parameter Z(0, σ_ε^2), and σ_ε^2 is 0.25, 0.5 and 1.
3. Y_i is calculated by the following regression model

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i,$$

where $\beta_0 = \beta_1 = \beta_2 = 1$ and $i = 1, \dots, N$.

4. Run the simulation 2,000 times for simple random sampling, ranked set sampling, two-layer ranked set sampling, median ranked set sampling and two-layer median ranked set sampling.
5. Set sizes and cycle sizes of methods should be determined as in the Table 4.1.

Table 4.1 Set sizes and cycle sizes of methods

SRS	Sample size	*n*
RSS	Set size	*kl*
	Cycle size	*m*
TRSS	Set size of first layer	*k*
	Set size of second layer	*l*
	Cycle size	*m*
MRSS	Set size	*kl*
	Cycle size	*m*
TMRSS	Set size of first layer	*k*
	Set size of second layer	*l*
	Cycle size	*m*

In this simulation study, the performance of the estimators will be investigated for *k*=3 and 4, *l*=2 and *m*= 3, 5 and 10. For all possible *k*, *l* and m satisfying the sample size *n*= 18, 24, 30, 40, 60 and 80 as in the Table 4.2.

Table 4.2 Set sizes and cycle sizes which are used in simulation study

SRS	***RSS***		***TRSS (l=2)***		***MRSS***		***TMRSS (l=2)***	
n	***k***	***m***	***k***	***m***	***k***	***m***	***k***	***m***
18		3		3		3		3
30	6	5	3	5	6	5	3	5
60		10		10		10		10
24		3		3		3		3
40	8	5	4	5	8	5	4	5
80		10		10		10		10

6. The two-layer RSS is compared with RSS and the two-layer median RSS is compared with RSS using X_1 as the ranking variable.
7. Bias values of regression coefficients are computed by Equation 4.1.

$$\text{Bias} = \beta_j - \hat{\beta}_j, \text{ where } j = 0, 1, 2. \quad (4.1)$$

8. In order to calculate Mean Square Error (MSE) of regression model, after bias is calculated, MSE iscomputed by Equation 4.2

$$\text{MSE} = \text{bias}^2 + \text{variance} \quad (4.2)$$

9. In the first part of simulation study, the efficiency of population mean estimator using TRSS method is calculated with respect to corresponding estimators in SRS and RSS methods by Equation 4.3 and Equation 4.4, respectively.

$$RE_1 = \frac{MSE_{\bar{Y}_{SRS}}}{MSE_{\bar{Y}_{TRSS}}} \quad (4.3)$$

$$RE_2 = \frac{MSE_{\bar{Y}_{RRS}}}{MSE_{\bar{Y}_{TRSS}}} \quad (4.4)$$

10. In the secondpart of simulation study, we compare the performance of population mean estimator using TMRSS method with MRSS method by Equaiton 4.5.

$$RE_3 = \frac{MSE_{\bar{Y}_{MRSS}}}{MSE_{\bar{Y}_{TMRSS}}} \quad (4.5)$$

4.1 Simulation Results

The results of two simulation studies are shown in this section. First, the result of simulation according to SRS, RSS and TRSS is given. And then we present the results of simulation process according to MRSS and TMRSS.

4.1.1 Simulation Result of Comparing TRSS with SRS and RSS

The results from simulation of comparing TRSS with SRS and RSS are analyzed. The results of MSE of regression model and the relative efficieny of sample mean and the bias values of regression coefficients are given by Table 4.3-4.7.

Table 4.3 shows that MSE of the regression models under five distributions for SRS, RSS and TRSSdesings. In terms of this estimator, we can assume that these three designs have equivalent efficiency since mean square errors of all desings are very close to each other. As you can see in Table 4.3, when the variance of the residuals decreases, MSE of the all methods also decrease in all distributions. Especially, in asymmetric distributions, when set size increases, MSE of the TRSS decreases.

The simulation results of the relative efficiencies of the sample mean estimator are shown in Table 4.4. RE_1 and RE_2 are calculated by Equaiton 4.3 and Equaiton 4.4, respectively.According to Table 4.4, it can be seen that almost every value of relative efficiency is over than 1 and it indicates that the sample mean estimator in TRSS design is more efficient than the sample mean estimators in SRS and RSS designs. In addition, TRSS with gamma distribution is observed more effective than the other distributions. In more detail, decreasing variance of residuals or increasing set size *k* increases efficiency of TRSS by SRS. Also, RE_2 does not seem to be affected by set size or cycle size.

Based on Table 4.5, Table 4.6 and Table 4.7 which are tables of biases value, the values on these tables are very close to 0. It indicates that the estimators for regression coefficients in SRS, RSS and TRSS designs are unbiased estimator.

Table 4.3 Mean square errors (MSE) of the regression models for SRS, RSS and TRSS desings

			NORMAL			UNIFORM			EXPONENTIAL			GAMMA			LOGNORMAL		
k	***m***	σ_ε^2	MSE_{SRS}	MSE_{RSS}	MSE_{TRSS}	MSE_{SRS}	MSE_{RSS}	MSE_{TRSS}	MSE_{SRS}	MSE_{RSS}	MSE_{TRSS}	MSE_{SRS}	MSE_{RSS}	MSE_{TRSS}	MSE_{SRS}	MSE_{RSS}	MSE_{TRSS}
		1	1.000	1.001	1.001	1.003	1.008	0.998	1.009	1.004	1.014	0.976	0.986	0.981	1.006	1.011	1.011
	3	0.5	0.503	0.507	0.502	0.507	0.505	0.510	0.498	0.500	0.504	0.495	0.497	0.497	0.506	0.507	0.505
		0.25	0.246	0.251	0.249	0.251	0.250	0.251	0.249	0.248	0.251	0.253	0.254	0.256	0.254	0.250	0.252
		1	1.003	1.006	0.998	1.008	1.000	1.002	1.013	1.012	1.031	0.972	0.981	0.980	1.012	1.014	1.008
3	5	0.5	0.505	0.511	0.509	0.506	0.503	0.506	0.501	0.499	0.502	0.492	0.502	0.494	0.508	0.510	0.505
		0.25	0.248	0.250	0.252	0.251	0.253	0.253	0.247	0.248	0.251	0.252	0.253	0.250	0.255	0.252	0.252
		1	1.001	1.003	1.000	1.004	1.008	1.007	1.010	1.010	1.023	0.982	0.982	0.980	1.010	1.006	1.014
	10	0.5	0.509	0.508	0.510	0.509	0.507	0.512	0.500	0.499	0.494	0.496	0.499	0.492	0.513	0.511	0.510
		0.25	0.247	0.249	0.249	0.250	0.249	0.251	0.248	0.247	0.247	0.251	0.251	0.252	0.254	0.253	0.253
		1	1.013	1.001	1.002	0.991	1.000	1.005	1.018	0.991	1.007	0.989	0.991	0.983	1.009	1.027	1.006
	3	0.5	0.508	0.508	0.507	0.511	0.502	0.510	0.491	0.506	0.507	0.501	0.494	0.489	0.503	0.514	0.504
		0.25	0.250	0.248	0.252	0.251	0.250	0.252	0.248	0.250	0.249	0.249	0.255	0.251	0.253	0.252	0.252
		1	0.997	1.004	1.003	1.003	1.005	0.997	1.020	1.017	1.017	0.987	0.971	0.977	1.013	1.007	1.007
4	5	0.5	0.509	0.503	0.511	0.505	0.503	0.508	0.503	0.499	0.496	0.498	0.497	0.490	0.504	0.508	0.506
		0.25	0.248	0.249	0.249	0.250	0.249	0.251	0.247	0.249	0.251	0.253	0.253	0.252	0.253	0.251	0.252
		1	0.996	0.998	1.000	1.003	1.000	1.002	1.017	1.013	1.022	0.983	0.980	0.984	1.018	1.022	1.016
	10	0.5	0.508	0.509	0.509	0.503	0.507	0.506	0.499	0.497	0.496	0.496	0.491	0.496	0.501	0.502	0.499
		0.25	0.251	0.250	0.249	0.253	0.251	0.251	0.249	0.248	0.247	0.252	0.252	0.252	0.252	0.252	0.253

Table 4.4 Relative efficiencies (RE) of the sample mean estimator for SRS, RSS and TRSS desings

			NORMAL			UNIFORM			EXPONENTIAL			GAMMA			LOGNORMAL		
k	***m***	σ_ε^2	**MSE_{SRS}**	**MSE_{RSS}**	**MSE_{TRSS}**	**MSE_{SRS}**	**MSE_{RSS}**	**MSE_{TRSS}**	**MSE_{SRS}**	**MSE_{RSS}**	**MSE_{TRSS}**	**MSE_{SRS}**	**MSE_{RSS}**	**MSE_{TRSS}**	**MSE_{SRS}**	**MSE_{RSS}**	**MSE_{TRSS}**
		1	1.320	0.975	1.113	0.962	1.282	0.988	1.698	1.136	1.276	1.009	1.320	0.975	1.113	0.962	1.282
	3	0.5	1.597	1.172	1.078	1.043	1.402	1.027	1.722	1.198	1.164	1.011	1.597	1.172	1.078	1.043	1.402
		0.25	1.541	1.047	1.208	1.024	1.354	1.018	1.608	1.114	1.298	1.069	1.541	1.047	1.208	1.024	1.354
		1	1.414	1.033	1.117	1.055	1.176	0.942	1.542	1.065	1.213	0.965	1.414	1.033	1.117	1.055	1.176
3	5	0.5	1.468	1.047	1.214	1.017	1.423	1.010	1.660	1.056	1.212	0.974	1.468	1.047	1.214	1.017	1.423
		0.25	1.443	1.050	1.256	1.180	1.419	0.989	1.603	1.079	1.336	1.000	1.443	1.050	1.256	1.180	1.419
		1	1.271	0.946	1.044	1.063	1.196	0.976	1.512	1.000	1.207	0.886	1.271	0.946	1.044	1.063	1.196
	10	0.5	1.453	1.095	1.103	1.050	1.309	1.001	1.556	1.023	1.223	1.063	1.453	1.095	1.103	1.050	1.309
		0.25	1.540	1.026	1.233	0.961	1.509	1.027	1.620	1.063	1.292	1.047	1.540	1.026	1.233	0.961	1.509
		1	1.406	1.027	1.076	1.018	1.322	1.092	1.612	1.159	1.192	0.967	1.406	1.027	1.076	1.018	1.322
	3	0.5	1.650	1.147	1.117	1.016	1.478	1.036	1.697	1.189	1.268	0.906	1.650	1.147	1.117	1.016	1.478
		0.25	1.797	1.123	1.261	0.998	1.480	0.999	1.750	1.141	1.205	0.889	1.797	1.123	1.261	0.998	1.480
		1	1.358	1.003	1.064	1.034	1.196	0.979	1.710	1.135	1.298	0.845	1.358	1.003	1.064	1.034	1.196
4	5	0.5	1.541	1.082	1.137	1.044	1.474	1.064	1.652	1.079	1.252	1.022	1.541	1.082	1.137	1.044	1.474
		0.25	1.619	1.075	1.208	1.065	1.452	1.131	1.660	1.079	1.326	1.030	1.619	1.075	1.208	1.065	1.452
		1	1.438	1.034	1.004	0.948	1.254	1.057	1.641	1.129	1.343	1.037	1.438	1.034	1.004	0.948	1.254
	10	0.5	1.616	1.091	1.111	0.916	1.351	1.018	1.701	1.104	1.332	1.041	1.616	1.091	1.111	0.916	1.351
		0.25	1.599	1.039	1.255	1.057	1.568	1.135	1.813	1.222	1.348	1.077	1.599	1.039	1.255	1.057	1.568

Table 4.5 Biases of $\hat{\beta}_0$ for SRS, RSS and TRSS desings

			NORMAL			UNIFORM			EXPONENTIAL			GAMMA			LOGNORMAL		
k	m	σ_ε^2	$\hat{\beta}_{0_{SRS}}$	$\hat{\beta}_{0_{RSS}}$	$\hat{\beta}_{0_{TRSS}}$	$\hat{\beta}_{0_{SRS}}$	$\hat{\beta}_{0_{RSS}}$	$\hat{\beta}_{0_{TRSS}}$	$\hat{\beta}_{0_{SRS}}$	$\hat{\beta}_{0_{RSS}}$	$\hat{\beta}_{0_{TRSS}}$	$\hat{\beta}_{0_{SRS}}$	$\hat{\beta}_{0_{RSS}}$	$\hat{\beta}_{0_{TRSS}}$	$\hat{\beta}_{0_{SRS}}$	$\hat{\beta}_{0_{RSS}}$	$\hat{\beta}_{0_{TRSS}}$
		1	0.000	-0.005	0.002	0.017	-0.006	0.011	-0.007	0.001	-0.023	0.014	0.016	-0.021	-0.005	-0.003	-0.015
	3	0.5	-0.006	-0.005	0.000	-0.016	0.007	-0.006	0.001	0.011	0.020	0.033	0.004	0.017	-0.014	-0.006	-0.007
		0.25	-0.002	0.004	0.003	0.005	0.013	-0.004	0.003	0.005	0.002	0.008	-0.009	-0.005	-0.001	-0.002	0.002
		1	-0.002	-0.005	0.006	-0.001	-0.004	0.008	-0.014	-0.003	0.008	-0.019	-0.003	-0.031	-0.011	-0.022	-0.005
3	5	0.5	0.002	-0.001	-0.001	-0.016	-0.008	-0.013	0.005	0.005	0.007	0.007	0.002	-0.004	-0.007	0.001	0.000
		0.25	0.004	0.001	-0.001	-0.004	-0.002	0.000	0.006	-0.011	0.001	-0.003	0.001	0.005	0.003	0.005	0.006
		1	0.005	0.003	-0.006	0.014	-0.005	0.002	-0.004	-0.001	0.000	0.005	-0.003	-0.015	0.000	-0.020	-0.001
	10	0.5	0.002	0.003	-0.002	0.000	-0.006	0.013	0.001	0.007	0.003	0.012	0.003	0.000	-0.006	-0.004	-0.003
		0.25	0.003	0.002	-0.002	-0.003	0.001	0.008	0.001	0.000	-0.002	0.000	0.001	-0.010	0.003	0.000	0.001
		1	0.006	-0.001	-0.004	0.008	-0.025	-0.005	-0.002	0.007	0.017	-0.008	0.009	-0.021	-0.012	0.001	-0.001
	3	0.5	0.000	0.004	-0.005	0.001	-0.002	-0.008	0.005	0.003	0.001	0.002	-0.008	-0.001	-0.008	-0.004	-0.005
		0.25	0.003	-0.002	-0.001	-0.011	0.004	-0.009	0.008	0.003	-0.002	-0.001	0.001	0.007	-0.002	0.006	-0.004
		1	0.002	-0.005	0.000	0.006	-0.022	0.005	0.008	-0.001	-0.003	-0.014	0.025	0.006	-0.006	-0.017	-0.012
4	5	0.5	-0.003	-0.004	0.003	0.003	0.006	0.000	-0.002	0.000	-0.003	-0.004	0.011	0.005	-0.002	-0.002	-0.004
		0.25	0.003	0.001	-0.001	-0.005	0.002	-0.003	-0.001	0.006	-0.001	-0.008	0.008	-0.005	0.001	0.001	0.000
		1	-0.003	0.002	0.001	-0.014	0.004	-0.010	0.009	0.006	0.010	-0.001	0.015	-0.008	0.004	0.005	0.003
	10	0.5	-0.001	-0.003	0.001	-0.006	-0.001	-0.004	0.001	-0.003	0.000	0.003	0.005	-0.002	0.006	0.010	0.009
		0.25	-0.001	-0.001	-0.002	-0.005	0.001	-0.007	0.004	-0.002	-0.001	0.005	-0.004	0.001	-0.002	0.000	-0.002

Table 4.6 Biases of $\hat{\beta}_1$ for SRS, RSS and TRSS desings

			NORMAL			UNIFORM			EXPONENTIAL			GAMMA			LOGNORMAL		
k	m	σ_ε^2	$\hat{\beta}_{1_{SRS}}$	$\hat{\beta}_{1_{RSS}}$	$\hat{\beta}_{1_{TRSS}}$	$\hat{\beta}_{1_{SRS}}$	$\hat{\beta}_{1_{RSS}}$	$\hat{\beta}_{1_{TRSS}}$	$\hat{\beta}_{1_{SRS}}$	$\hat{\beta}_{1_{RSS}}$	$\hat{\beta}_{1_{TRSS}}$	$\hat{\beta}_{1_{SRS}}$	$\hat{\beta}_{1_{RSS}}$	$\hat{\beta}_{1_{TRSS}}$	$\hat{\beta}_{1_{SRS}}$	$\hat{\beta}_{1_{RSS}}$	$\hat{\beta}_{1_{TRSS}}$
		1	-0.004	0.002	0.006	-0.003	-0.009	-0.011	0.013	0.015	0.012	-0.001	0.001	0.004	0.002	-0.006	0.000
	3	0.5	-0.002	-0.001	0.004	0.017	-0.002	0.008	-0.002	-0.012	-0.016	-0.005	-0.002	-0.002	0.002	-0.002	0.001
		0.25	0.003	0.004	-0.004	-0.003	-0.017	0.003	-0.005	-0.010	-0.004	-0.001	0.001	0.000	-0.002	-0.002	-0.002
		1	-0.003	-0.004	0.002	0.009	0.022	0.000	0.013	0.004	0.004	0.002	0.001	0.004	-0.002	0.001	0.000
3	5	0.5	-0.003	-0.004	0.003	0.019	0.005	-0.001	-0.004	0.002	-0.005	-0.002	0.002	0.001	-0.002	-0.002	-0.004
		0.25	0.000	-0.003	0.002	-0.001	0.005	0.005	-0.006	0.004	-0.002	0.000	0.000	-0.002	-0.003	-0.003	-0.003
		1	-0.002	-0.002	0.002	-0.007	0.016	0.011	0.006	0.006	0.005	0.001	0.001	0.002	-0.004	0.000	-0.003
	10	0.5	0.001	0.000	-0.004	0.003	0.009	-0.013	-0.003	0.001	0.002	-0.001	0.000	-0.001	0.001	0.000	-0.001
		0.25	-0.003	-0.001	-0.001	0.001	-0.003	-0.010	-0.002	0.001	-0.001	0.000	0.001	0.002	-0.001	-0.001	-0.001
		1	0.002	-0.005	0.001	-0.016	0.018	-0.002	0.014	0.003	0.002	0.003	0.002	0.008	-0.005	-0.006	-0.002
	3	0.5	0.008	0.006	0.001	-0.005	-0.007	0.000	0.000	-0.002	-0.002	0.000	0.003	0.002	0.000	-0.006	-0.001
		0.25	-0.002	0.001	-0.002	0.009	-0.004	0.006	-0.008	-0.003	-0.004	0.000	0.000	-0.001	-0.002	-0.001	0.001
		1	0.002	-0.003	0.003	-0.001	0.024	0.005	0.001	0.005	0.002	0.005	-0.001	0.001	-0.005	0.001	0.004
4	5	0.5	0.002	-0.003	0.000	-0.004	-0.005	0.005	0.001	-0.004	0.001	-0.001	-0.002	-0.001	-0.002	-0.001	-0.003
		0.25	0.003	0.000	0.000	0.006	0.007	0.007	-0.001	-0.004	-0.003	0.001	-0.001	0.001	0.000	-0.001	-0.001
		1	0.001	-0.003	0.001	0.028	0.000	0.006	-0.003	-0.002	0.000	0.001	-0.001	0.001	-0.001	-0.002	0.000
	10	0.5	-0.001	-0.004	-0.001	0.009	-0.002	0.003	-0.001	0.002	0.003	-0.001	-0.001	0.001	0.001	-0.002	-0.001
		0.25	-0.001	0.001	-0.001	0.008	0.000	0.007	-0.004	0.000	-0.001	-0.001	0.001	-0.001	0.001	0.000	0.001

Table 4.7 Biases of $\hat{\beta}_2$ for SRS, RSS and TRSS desings

			NORMAL			UNIFORM			EXPONENTIAL			GAMMA			LOGNORMAL		
k	m	σ_ε^2	$\hat{\beta}_{2_{SRS}}$	$\hat{\beta}_{2_{RSS}}$	$\hat{\beta}_{2_{TRSS}}$	$\hat{\beta}_{2_{SRS}}$	$\hat{\beta}_{2_{RSS}}$	$\hat{\beta}_{2_{TRSS}}$	$\hat{\beta}_{2_{SRS}}$	$\hat{\beta}_{2_{RSS}}$	$\hat{\beta}_{2_{TRSS}}$	$\hat{\beta}_{2_{SRS}}$	$\hat{\beta}_{2_{RSS}}$	$\hat{\beta}_{2_{TRSS}}$	$\hat{\beta}_{2_{SRS}}$	$\hat{\beta}_{2_{RSS}}$	$\hat{\beta}_{2_{TRSS}}$
		1	-0.004	0.002	0.006	-0.003	-0.009	-0.011	0.013	0.015	0.012	-0.001	0.001	0.004	0.002	-0.006	0.000
	3	0.5	-0.002	-0.001	0.004	0.017	-0.002	0.008	-0.002	-0.012	-0.016	-0.005	-0.002	-0.002	0.002	-0.002	0.001
		0.25	0.003	0.004	-0.004	-0.003	-0.017	0.003	-0.005	-0.010	-0.004	-0.001	0.001	0.000	-0.002	-0.002	-0.002
		1	-0.003	-0.004	0.002	0.009	0.022	0.000	0.013	0.004	0.004	0.002	0.001	0.004	-0.002	0.001	0.000
3	5	0.5	-0.003	-0.004	0.003	0.019	0.005	-0.001	-0.004	0.002	-0.005	-0.002	0.002	0.001	-0.002	-0.002	-0.004
		0.25	0.000	-0.003	0.002	-0.001	0.005	0.005	-0.006	0.004	-0.002	0.000	0.000	-0.002	-0.003	-0.003	-0.003
		1	-0.002	-0.002	0.002	-0.007	0.016	0.011	0.006	0.006	0.005	0.001	0.001	0.002	-0.004	0.000	-0.003
	10	0.5	0.001	0.000	-0.004	0.003	0.009	-0.013	-0.003	0.001	0.002	-0.001	0.000	-0.001	0.001	0.000	-0.001
		0.25	-0.003	-0.001	-0.001	0.001	-0.003	-0.010	-0.002	0.001	-0.001	0.000	0.001	0.002	-0.001	-0.001	-0.001
		1	0.002	-0.005	0.001	-0.016	0.018	-0.002	0.014	0.003	0.002	0.003	0.002	0.008	-0.005	-0.006	-0.002
	3	0.5	0.008	0.006	0.001	-0.005	-0.007	0.000	0.000	-0.002	-0.002	0.000	0.003	0.002	0.000	-0.006	-0.001
		0.25	-0.002	0.001	-0.002	0.009	-0.004	0.006	-0.008	-0.003	-0.004	0.000	0.000	-0.001	-0.002	-0.001	0.001
		1	0.002	-0.003	0.003	-0.001	0.024	0.005	0.001	0.005	0.002	0.005	-0.001	0.001	-0.005	0.001	0.004
4	5	0.5	0.002	-0.003	0.000	-0.004	-0.005	0.005	0.001	-0.004	0.001	-0.001	-0.002	-0.001	-0.002	-0.001	-0.003
		0.25	0.003	0.000	0.000	0.006	0.007	0.007	-0.001	-0.004	-0.003	0.001	-0.001	0.001	0.000	-0.001	-0.001
		1	0.001	-0.003	0.001	0.028	0.000	0.006	-0.003	-0.002	0.000	0.001	-0.001	0.001	-0.001	-0.002	0.000
	10	0.5	-0.001	-0.004	-0.001	0.009	-0.002	0.003	-0.001	0.002	0.003	-0.001	-0.001	0.001	0.001	-0.002	-0.001
		0.25	-0.001	0.001	-0.001	0.008	0.000	0.007	-0.004	0.000	-0.001	-0.001	0.001	-0.001	0.001	0.000	0.001

4.1.2 Simulation Result of Comparing TMRSS with MRSS

The results from simulation process of comparing TMRSS with MRSS are analyzed. The results of MSE of regression model and the relative efficieny of sample mean and the bias values of regression coefficients are given by Table 4.8-4.12.

The simulation results of MSE of the regression modelsare presented for MRSS and TMRSSdesings under five distributions by Table 4.8.RE_3 is calculated by Equaiton 4.5. In terms of this estimator, we can assume that MRSS and TMRSS designs have equivalent efficiency since mean square errors of bothdesings are very close to each other. In addition, when the variance of the residuals decreases, MSE of all methods also decrease for all distributions.

Table 4.9 shows that relative efficiencies of the sample mean estimator for MRSS and TMRSS. The table clearly shows that almost every value of relative efficiency is over than 1 and it indicates that the sample mean estimator in TMRSS design is more efficient than the sample mean estimators in MRSS design. Moreover, TMRSS is more effective under asymmetric distributions. In more detail, especially under asymmetric distributions, when other simulation parameters are fixed, going up set size k or cycle size m increases efficiency of TMRSS by MRSS.

Based on Table 4.10, Table 4.11 and Table 4.12 which are tables of biases value, the values on these tables are very close to 0. It indicates that the estimators for regression coefficients in MRSS and TMRSS designs are unbiased estimator.

Table 4.8 Mean square errors (MSE) of the regression models for MRSS and TMRSS desings

			NORMAL		UNIFORM		EXPONENTIAL		GAMMA		LOGNORMAL	
k	**m**	σ_ε^2	MSE_{MRSS}	MSE_{TMRSS}	MSE_{MRSS}	MSE_{TMRSS}	MSE_{MRSS}	MSE_{TMRSS}	MSE_{MRSS}	MSE_{TMRSS}	MSE_{MRSS}	MSE_{TMRSS}
		1	1.015	1.024	0.995	1.007	1.011	1.007	0.986	0.998	1.008	1.014
	3	0.5	0.496	0.495	0.501	0.503	0.495	0.498	0.502	0.504	0.497	0.501
		0.25	0.254	0.250	0.249	0.252	0.250	0.251	0.258	0.257	0.248	0.249
		1	1.018	1.007	0.991	0.998	1.013	1.011	0.975	0.977	1.024	1.016
3	5	0.5	0.499	0.502	0.505	0.501	0.494	0.497	0.498	0.502	0.502	0.507
		0.25	0.251	0.251	0.251	0.249	0.251	0.250	0.256	0.256	0.251	0.251
		1	1.027	1.017	0.996	0.999	1.016	1.007	0.976	0.971	1.020	1.017
	10	0.5	0.499	0.502	0.505	0.506	0.496	0.500	0.504	0.503	0.500	0.500
		0.25	0.252	0.250	0.250	0.249	0.249	0.251	0.257	0.257	0.250	0.252
		1	1.026	1.025	0.988	0.996	1.004	1.019	0.967	0.984	1.020	1.017
	3	0.5	0.501	0.504	0.502	0.506	0.499	0.495	0.508	0.504	0.498	0.507
		0.25	0.250	0.249	0.250	0.250	0.248	0.247	0.254	0.255	0.247	0.251
		1	1.027	1.025	0.993	1.002	1.013	1.019	0.969	0.975	1.021	1.025
4	5	0.5	0.497	0.503	0.504	0.508	0.495	0.500	0.503	0.501	0.496	0.502
		0.25	0.250	0.251	0.252	0.250	0.246	0.248	0.257	0.256	0.251	0.250
		1	1.025	1.019	1.001	1.001	1.011	1.010	0.975	0.979	1.014	1.018
	10	0.5	0.498	0.501	0.505	0.507	0.497	0.495	0.502	0.500	0.498	0.502
		0.25	0.249	0.249	0.249	0.249	0.247	0.249	0.255	0.254	0.250	0.250

Table 4.9 Relative efficiencies (RE) of the sample mean estimator for MRSS and TMRSS desings

			NORMAL	UNIFORM	EXPONENTIAL	GAMMA	LOGNORMAL
k	***m***	σ_{ε}^{2}	RE_3	RE_3	RE_3	RE_3	RE_3
		1	1.036	0.999	1.275	1.148	1.210
	3	0.5	1.008	1.056	1.143	1.170	1.258
		0.25	1.196	1.044	1.183	1.166	1.264
		1	1.091	0.931	1.131	1.160	1.272
3	5	0.5	1.088	0.885	1.278	1.135	1.283
		0.25	1.070	0.972	1.279	1.101	1.246
		1	1.029	0.905	1.290	1.209	1.361
	10	0.5	1.044	0.987	1.282	1.169	1.422
		0.25	1.057	1.040	1.454	1.166	1.390
		1	1.011	1.079	1.330	1.276	1.346
	3	0.5	1.118	1.022	1.364	1.253	1.308
		0.25	1.082	1.030	1.492	1.226	1.362
		1	1.040	1.027	1.445	1.400	1.538
4	5	0.5	1.091	1.098	1.522	1.329	1.357
		0.25	1.077	1.101	1.649	1.267	1.390
		1	1.137	1.006	1.492	1.647	1.642
	10	0.5	1.017	0.996	1.572	1.445	1.602
		0.25	1.048	1.082	1.763	1.382	1.616

Table 4.10 Biases of $\hat{\beta}_0$ for MRSS and TMRSS desings

			NORMAL		UNIFORM		EXPONENTIAL		GAMMA		LOGNORMAL	
k	m	σ_ε^2	$\hat{\beta}_{0_{MRSS}}$	$\hat{\beta}_{0_{TMRSS}}$	$\hat{\beta}_{0_{MRSS}}$	$\hat{\beta}_{0_{TMRSS}}$	$\hat{\beta}_{0_{MRSS}}$	$\hat{\beta}_{0_{TMRSS}}$	$\hat{\beta}_{0_{MRSS}}$	$\hat{\beta}_{0_{TMRSS}}$	$\hat{\beta}_{0_{MRSS}}$	$\hat{\beta}_{0_{TMRSS}}$
		1	-0.005	0.002	0.032	-0.017	-0.006	-0.014	0.052	-0.043	-0.005	0.011
	3	0.5	-0.004	-0.005	-0.015	-0.020	-0.016	-0.005	0.038	0.014	0.026	0.006
		0.25	-0.004	0.002	-0.005	0.003	0.007	0.018	-0.004	0.005	0.011	-0.008
		1	-0.006	-0.005	-0.018	-0.021	-0.004	-0.017	-0.010	-0.034	0.003	-0.003
3	5	0.5	-0.003	0.004	0.008	0.000	0.005	0.007	0.059	0.018	0.009	0.006
		0.25	0.004	0.000	-0.008	-0.006	0.021	0.011	-0.006	-0.006	0.004	0.011
		1	-0.005	-0.007	0.008	-0.001	-0.033	-0.008	0.012	-0.013	-0.007	-0.017
	10	0.5	-0.003	-0.001	-0.007	-0.010	-0.002	0.004	0.013	0.024	-0.003	-0.002
		0.25	0.002	0.002	0.004	-0.005	0.018	0.015	-0.019	0.001	-0.001	0.006
		1	-0.006	-0.008	-0.011	-0.018	-0.006	-0.011	0.059	-0.017	-0.019	-0.014
	3	0.5	0.006	-0.002	-0.031	-0.009	-0.003	-0.001	0.056	0.020	-0.017	0.010
		0.25	0.006	0.004	0.010	0.005	0.025	0.023	-0.019	0.004	-0.001	0.003
		1	-0.007	0.001	0.030	0.014	-0.017	-0.002	0.035	-0.020	-0.016	-0.007
4	5	0.5	0.001	0.000	-0.010	-0.020	-0.005	0.006	0.013	0.013	0.021	0.007
		0.25	0.000	0.002	0.004	0.000	0.024	0.009	-0.030	0.000	-0.006	-0.004
		1	-0.004	-0.003	0.007	-0.008	-0.023	-0.018	0.016	-0.031	-0.020	-0.005
	10	0.5	0.000	-0.001	-0.018	-0.005	-0.002	0.004	0.020	0.018	0.011	0.001
		0.25	0.005	0.001	0.007	0.001	0.019	0.012	-0.014	-0.004	0.001	0.006

Table 4.11 Biases of $\hat{\beta}_1$ for MRSS and TMRSS desings

			NORMAL		UNIFORM		EXPONENTIAL		GAMMA		LOGNORMAL	
k	***m***	σ_ε^2	$\hat{\beta}_{1_{MRSS}}$	$\hat{\beta}_{1_{TMRSS}}$	$\hat{\beta}_{1_{MRSS}}$	$\hat{\beta}_{1_{TMRSS}}$	$\hat{\beta}_{1_{MRSS}}$	$\hat{\beta}_{1_{TMRSS}}$	$\hat{\beta}_{1_{MRSS}}$	$\hat{\beta}_{1_{TMRSS}}$	$\hat{\beta}_{1_{MRSS}}$	$\hat{\beta}_{1_{TMRSS}}$
		1	0.022	0.006	-0.071	-0.026	0.019	0.028	-0.006	0.009	0.003	-0.014
	3	0.5	-0.009	-0.006	0.019	0.052	0.030	0.004	-0.005	0.001	-0.010	-0.005
		0.25	0.010	0.000	0.005	-0.009	-0.013	-0.014	0.000	-0.002	-0.001	0.002
		1	0.003	0.004	-0.028	0.006	0.024	0.031	0.004	0.008	-0.006	-0.008
3	5	0.5	-0.014	-0.013	-0.025	-0.029	-0.002	-0.009	-0.008	-0.002	-0.013	-0.005
		0.25	-0.002	0.003	0.011	0.016	-0.013	-0.014	0.001	0.001	0.001	-0.004
		1	-0.006	0.002	-0.049	-0.023	0.038	0.013	-0.001	0.004	-0.005	0.006
	10	0.5	-0.014	-0.013	0.000	-0.004	0.004	-0.003	0.000	-0.002	-0.005	-0.001
		0.25	0.001	0.003	0.002	0.007	-0.019	-0.017	0.004	0.001	0.001	-0.004
		1	0.003	-0.019	-0.031	0.006	0.017	0.019	-0.008	0.006	0.003	-0.001
	3	0.5	-0.015	-0.016	0.026	-0.002	0.017	0.004	-0.007	-0.001	0.007	-0.008
		0.25	0.005	0.009	-0.020	0.002	-0.031	-0.027	0.002	0.000	0.000	0.000
		1	0.012	-0.001	-0.073	-0.053	0.033	0.013	-0.002	0.004	0.001	-0.010
4	5	0.5	-0.007	-0.003	-0.007	0.025	0.012	-0.010	0.001	-0.002	-0.024	-0.004
		0.25	0.002	0.004	-0.010	0.000	-0.028	-0.010	0.004	-0.001	0.006	0.001
		1	0.018	0.006	-0.039	-0.012	0.033	0.025	-0.002	0.006	0.006	-0.001
	10	0.5	-0.011	-0.005	-0.010	-0.008	0.003	-0.003	0.000	-0.001	-0.011	-0.006
		0.25	0.004	0.001	-0.005	-0.001	-0.024	-0.015	0.002	0.001	0.000	-0.003

Table 4.12 Biases of $\hat{\beta}_2$ for MRSS and TMRSS desings

			NORMAL		UNIFORM		EXPONENTIAL		GAMMA		LOGNORMAL	
k	m	σ_ε^2	$\hat{\beta}_{2_{MRSS}}$	$\hat{\beta}_{2_{TMRSS}}$	$\hat{\beta}_{2_{MRSS}}$	$\hat{\beta}_{2_{TMRSS}}$	$\hat{\beta}_{2_{MRSS}}$	$\hat{\beta}_{2_{TMRSS}}$	$\hat{\beta}_{2_{MRSS}}$	$\hat{\beta}_{2_{TMRSS}}$	$\hat{\beta}_{2_{MRSS}}$	$\hat{\beta}_{2_{TMRSS}}$
		1	-0.002	0.002	0.000	0.018	0.000	-0.001	-0.004	0.001	0.000	0.001
	3	0.5	0.008	0.005	0.028	0.004	-0.004	-0.004	-0.003	-0.003	-0.005	-0.002
		0.25	0.003	0.005	-0.009	0.003	0.007	0.001	0.002	0.001	-0.007	0.001
		1	0.000	0.003	0.058	0.026	-0.005	0.008	-0.003	-0.002	0.000	0.002
3	5	0.5	0.005	0.009	0.016	0.023	-0.006	-0.002	-0.003	-0.001	0.003	0.000
		0.25	0.004	0.004	-0.002	-0.008	-0.001	0.004	0.000	0.001	-0.002	-0.003
		1	0.004	-0.001	0.029	0.009	0.006	0.000	-0.002	-0.001	0.003	0.001
	10	0.5	0.008	0.007	0.019	0.021	-0.002	0.000	-0.002	-0.003	0.003	0.000
		0.25	0.002	0.002	-0.017	-0.005	0.000	0.003	0.001	0.000	0.000	0.000
		1	0.002	0.005	0.032	0.014	-0.001	0.001	-0.003	-0.002	0.005	0.008
	3	0.5	0.008	0.007	0.043	0.016	-0.007	-0.001	-0.003	-0.004	0.004	0.001
		0.25	0.005	0.003	-0.011	-0.018	0.007	0.004	0.003	0.000	-0.002	-0.004
		1	0.003	-0.001	0.016	0.022	-0.007	-0.003	-0.005	-0.001	0.001	0.006
4	5	0.5	0.013	0.010	0.034	0.028	-0.004	0.002	-0.003	-0.001	0.003	0.001
		0.25	0.006	0.005	-0.002	-0.004	0.000	0.002	0.002	0.001	-0.001	0.002
		1	0.003	0.002	0.023	0.020	0.003	0.003	-0.001	0.000	0.003	0.001
	10	0.5	0.008	0.008	0.050	0.019	-0.003	-0.001	-0.003	-0.002	0.003	0.001
		0.25	0.004	0.002	-0.010	-0.005	0.002	0.003	0.001	0.000	0.000	-0.001

CHAPTER FIVE
CONCLUSIONS

In this study, the two-layer median RSS method is designed. By the help of simulation studies, comparison of SRS and RSS with TRSS and comparison of MRSS with TMRSS are evaluated in terms of MSE of regression models, relative efficiencies of sample mean estimators and bias values of regression coefficients.

Based on the results of simulations,

1. In terms of MSE of the regression model, we can conclude that SRS, RSS and TRSS designs and MRSS and TMRSS desings have equal efficiencies.
2. TRSS design is more efficient than SRS and RSS desings in the way of sample mean estimator. In addition, it is more effective under gamma distribution.
3. TMRSS design is more efficient than MRSS in respect to sample mean estimator. This efficiency is increased under asymmetric ditributions.
4. The regression coefficient estimators of all designs are assumed unbiased estimator for all distributions since all of the bias values are close or equal to zero.
5. The set size or cycle size of sample can increase the efficiency of TMRSS and TRSS.

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