DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCE

VIBRATION ANALYSES OF COMPLEX STRUCTURAL MEMBERS WITH VARIOUS TORSIONAL SPRING FOUNDATIONS AND SUSPENDED SPRING MASS SYSTEMS

by
Pelin SARSMAZ

September, 2016
İZMİR

VIBRATION ANALYSES OF COMPLEX STRUCTURAL MEMBERS WITH VARIOUS TORSIONAL SPRING FOUNDATIONS AND SUSPENDED SPRING MASS SYSTEMS

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M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"VIBRATION ANALYSES OF COMPLEX STRUCTURAL MEMBERS WITH VARIOUS TORSIONAL SPRING FOUNDATIONS AND SUSPENDED SPRING MASS SYSTEMS"** completed by **PELİN SARSMAZ** under supervision of **PROF.DR. MUSTAFA SABUNCU** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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DEĞİŞİK BURULMA SINIR ŞARTLARINA VE ASILI KÜTLE YAY SİSTEMLERİNE SAHİP KARMAŞIK YAPILARIN TİTREŞİM ANALİZİ

ÖZ

Karmaşık yapılar, mühendislik uygulamalarında önemli bir role sahiptir. Eğri çubuklar da bu karmaşık elemanlardan birisidir. Bu çalışmada, değişik burulma sınır şartlarına ve asılı kütle yay sistemlerine sahip bir eğri çubuğun düzlem içi doğal frekansları araştırılmıştır.Çubuk Euler kiriş kabul edilerek sonlu elemanlar metodu kullanılarak modellenmiştir. Asılı kütle yay sistemlerinin sadece düşey doğrultuda titreşim hareketi yaptıkları kabul edilmiştir. Değişik koşullar altında düzlem içi doğal frekanslar, *ANSYS Multiphysics 14.0* ve sonlu eleman programı geliştirilerek çözümünde *MATLAB* bilgisayar programı kullanılarak hesaplanmış bulunan sonuçlar birbiri ile karşılaştırılmıştır. Kesit alanının, eğrilik açısının, asılı kütle yay sistemlerinin ve burulma sınır şartlarının çubuğun düzlem içi doğal frekansları üzerine olan etkileri araştırılmıştır. Sonuçlar, tablo ve grafikler halinde verilmiş olup sonuçlar arasında iyi uyum olduğu gözlenmiştir.

Anahtar kelimeler: Karmaşık yapılar, asılı kütle yay, titreşim,eğri çubuk

VIBRATION ANALYSES OF COMPLEX STRUCTURAL MEMBERS WITH VARIOUS TORSIONAL SPRING FOUNDATIONS AND SUSPENDED SPRING MASS SYSTEMS

ABSTRACT

Complex structural members like curved beams have an important role in engineering applications. One of the complex members is a curved beam. In this study, in-plane natural frequencies of curved beams with various torsional spring foundations and suspended spring mass systems are investigated. The curved beam is assumed to be an Euler beam and modeled by using the Finite Element Method. The suspended spring mass system vibrating only in the vertical direction is considered. In plane natural frequencies of curved beam under different conditions are calculated with using *ANSYS Multiphysics 14.0* and *MATLAB* computer programmes. The results are compared with each other. The effects of cross sectional area, subtended angle, suspended spring-mass systems and rotational spring foundations on the in-plane frequencies of curved beams are investigated. The results of this investigation are given in the tables and graphics and good agreement is found.

Keywords: Complex structural members, suspended spring mass system, vibration, curved beam

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CHAPTER ONE
INTRODUCTION

1.1 Introduction

Complex structural members are used in many structures. One of them is a curved beam. Curved beams have significant importance in engineering applications and used in structural members such as bridges, industrial structures. Moreover curved beams are also widely used in turbine blade packets in aerospace industries. Thus, static and dynamic characteristics become a popular topic for many researchers. For this reason, the static and dynamic stability analysis of curved beams have been investigated in recent years. Vibration analysis of curved beams having torsional spring foundations and suspended spring mass systems is carried out in this study.

The vibration analyses of curved beams which have been studied by other investigators can be summarized as below:

Timoshenko and Gere (1961) used analytical method for buckling analysis of hinged-hinged Bernoulli-Euler curved beams. Rao and Sundararaian (1969) investigated the fundamental frequecies of the clamped circular arcs with various subtended angles and found lowest four natural frequencies of complete ring by using analytical method. Sabir and Aswell (1971) used different shape functions for the natural frequency analysis of circular ring. Petyt and Fleischer (1971) investigated free vibration of curved beam for different boundary conditions. Sabuncu (1978) has worked on the natural frequency analysis of thin curved beams by using different displacement functions and investigated the effects on natural frequencies. Bazalt and Cedolin (1991) worked on buckling analysis of curved beams by using analytical and energy methods.

Kawakami et al.(1995) investigated in- plane and out-of-plane free vibrations of curved beams. They studied the effect of arbitrary shapes and variable cross sections on free vibrations of curved beams. Wu and Chen (2011) analyzed the out-of plane free vibrations of horizontal circular curved beams carrying arbitrary set of concentrated elements. Wu, Lin and Shaw (2013) worked on free in-plane vibration analysis of a curved beam (arch) with arbitrary various concentrated elements. In those studies, the exact and FEM solutions are obtained for 'bare' (without any attachments) and 'loaded' (with radial, tangential, rotational springs and lumped masses) curved beams.

Guo et al. (2014) investigated out-of-plane buckling analysis of circular arches with elastic end restraints under other loading conditions with using approximate analytical solutions and the finite element method.

This study presents the in-plane vibration analysis of uniform curved beams with various torsional spring foundations and suspended spring-mass systems. The effects of the variation of the cross-section, subtended angle of curved beam, stiffness parameter of torsional spring foundation and mass parameter of suspended spring-mass systems on the natural frequencies are examined by using *ANSYS Multiphysics 14.0 (2011)* and *MATLAB (2011)* computer programmes. The results are compared with each other and given in tables and graphics.

CHAPTER TWO
THE FINITE ELEMENT METHOD

2.1 The Finite Element Method

The finite element method is a numerical method in order to deal with the problems of engineering and physics. This approach is a very practical analysis method for engineering challenges such as; complex geometry, loading type and material characteristics. It has developed for aerospace industry analyses at the beginning. Moreover, in following years the method is found out to be applicable to the solution of other applied sciences and engineering.

At the present time, it is realized that obtaining approximate numerical solutions to problems is more essential than exact closed-form solutions. For example, if we want to perform stress analysis of engine part that has complicated shape and material properties, we can find out the governing equations and boundary conditions for this problem, but we see immediately that no simple analytical solution can be found. In this point, the finite element method presents us the most effective solutions.

In the finite element method, we can obtain an approximate solution instead of exact solution due to the problem reduced to simpler main engineering one and results obtained are much closer to the exact results obtained by spending more effort to solve analytical equations. Lack of available conventional mathematical tools for solving complicated engineering problems shows us that the finite element method is the only method can be used in these problems.

In this method, the whole structure is replaced by several pieces or elements which is assumed to behave as a continious structural member called a finite element. The finite elements are assumed to be interrelated at certain points known as joints or nodes. For example, the circumference is calculated easily by dividing circle to polygonal elements by former mathematicians as shown in Figure 2.1. With today's statement, each side of polygon is called as a finite element. The connection between polygon sides is named as a node that used to compose algebraic equations. Stress analysis of systems is done by using the equations of equilibrium of these nodes exemplary. Depending on complexity of engineering structure and the increasing of the number of finite elements, too many equations are obtained. In this point, the digital computers must be used.

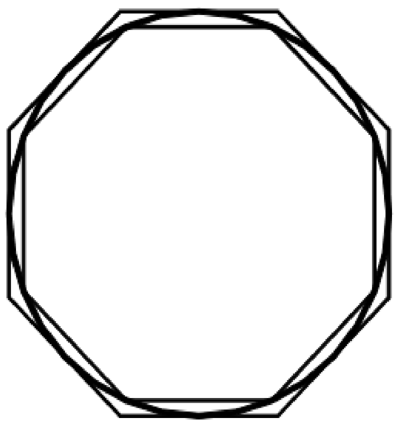

Figure 2.1 Circumference with the Finite Element Approach

2.2 Solution Steps of the Finite Element Method

The solution of a engineering problem by the finite element method follows a path as shown below.

(a) Discretization of a continuum region

The continuum region is divided into elements in this step. Figure 2.2 presents nodes and elements located on the model. These triangular and other type elements might be used to find out the stress distribution over the model. However, the selection of the number and types of elements must be performed with an intense care owing to the geometry, loading condition and etc.

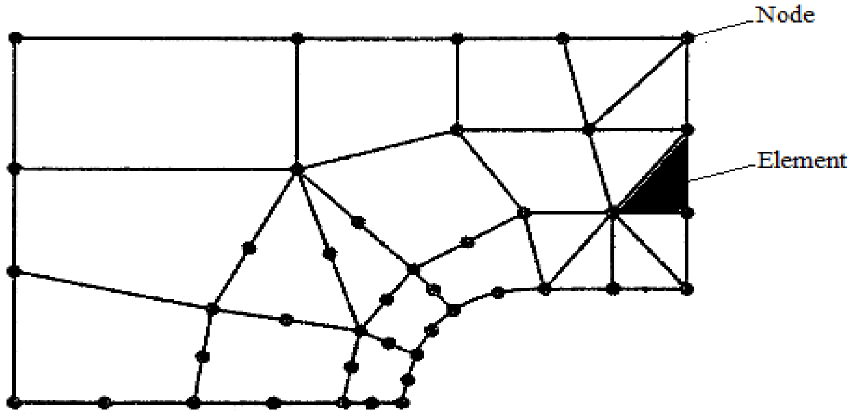

Figure 2.2 Nodes and elements in the Finite Element Method

(b) Interpolation function selection

In the following phase the elements must be assigned to the defined nodes and additionally selection of interpolation function which stands for the field variable over the element must be performed. Field variable might be scalar, vector or a higher order tensor. Interpolation functions for field variable are generally preferred as polynomials since it is straightforward to integrate and differentiate.

The degree of polynomials varies owing to some criteria and these are: assigned nodes, quantity of unknowns at each node and continuity requirements which are imposed at the nodes and the boundaries of element. The magnitude of the field variable and its derivatives might be unknowns at the nodes.

(c) Determination of element properties:

After elements and their interpolation functions are selected or in other words the FE model has been set up matrix equations which state the properties of the individual elements can be obtained. For this purpose it is possible to use one of the four approaches and these are: the direct approach, the variational approach, the weighted residual approach or the energy balance approach. The variational approach generally fits well with most of the challenges. However, the selection of approach is directly effected by problem characteristics.

(d) Constitue the element properties to obtain the system equations

Properties of whole system which is modeled by network of elements is a combination all elements properties. For this purpose matrix equations must be assembled and created which states the behavior of the overall region of system. The matrix equations of entire system have the similar form with an individual element but with a difference of including more terms due to existance of all nodes. Basically, assembly procedure comes from the equallity of field variable value for every element that shares the node in which elements are interconnected. Element equation assembly procedure is an easy challenge in the FE analysis and generally performed by a computer. Additionally, system equations must also consider the boundary conditions of the problem before the solution of system equation set.

(e) Solution of system equations

Assembly process results with an equation set which can be solved to obtain unknowns values at nodes.

2.3 Advantages of the Finite Element Method

The advantages of the finite element method is given below list respectively:

- The geometry of structures can be described by the way of diversity of shape and size of finite elements.
- The area which has difficulties like holes and corners can be analyzed easily.
- The boundary conditions of system is specified easily.
- The structures which have different material and shape can be examined easily.

2.4 The Finite Element Method Applications

The range of the finite element method extends to all engineering applications such as engineering of mechanical, aerospace, civil, automotive or etc. Generally the finite element method can be divided into three categories. These categories are arranged time-independent problems, eigenvalue problems and time-dependent problems respectively.

The majority of the finite element applications is time-independent problems or in other words equilibrium problems. The displacement or stress distribution under different loading conditions of mechanical systems can be found easily by using the finite element method.

The finite element method can be used to find natural frequencies and mode shapes of vibration of solids and fluids also. This type of analyses is called as eigenvalue problems.

The last category of the finite element method is time dependent problems. This category is consist of the problems that result when the time parameter is added to the problems.

CHAPTER THREE
DEFINITION OF THE PROBLEM AND THEORETICAL CONSIDERATION OF CURVED BEAMS

3.1 Definition of the Problem

The curved beam used in the study is shown in Figure 3.1. It carries different number of suspended spring-mass systems. All analyses of curved beams are performed by using "The Euler beam assumption" and "The Finite Element Method"

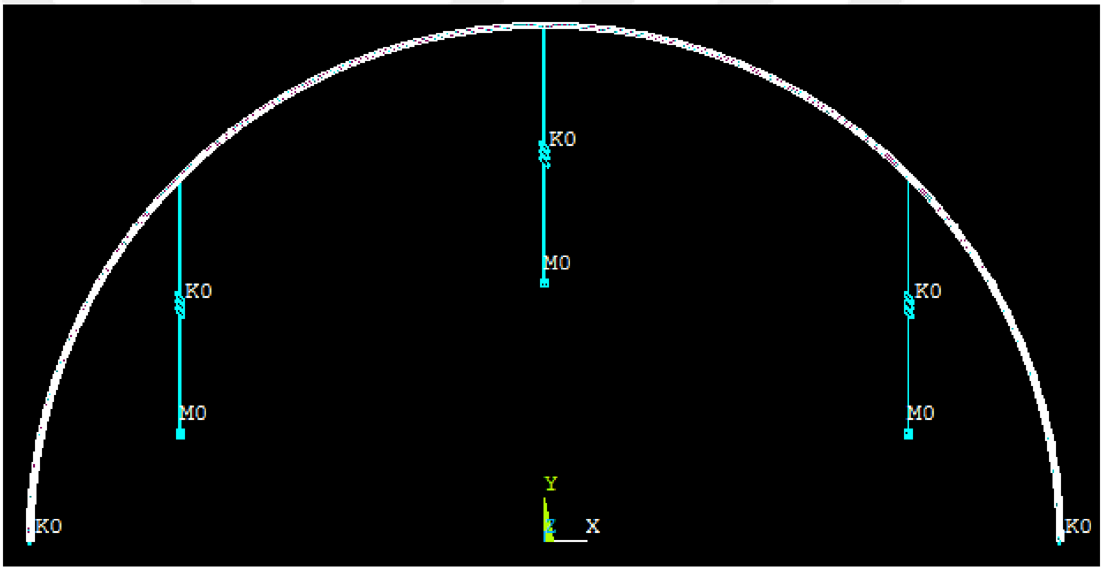

Figure 3.1 Curved beam with suspended mass-spring systems

3.2 Theoretical Analysis of Curved Beams

This part presents the theoretical consideration with geometrical shape of the curved beams considered in this thesis.

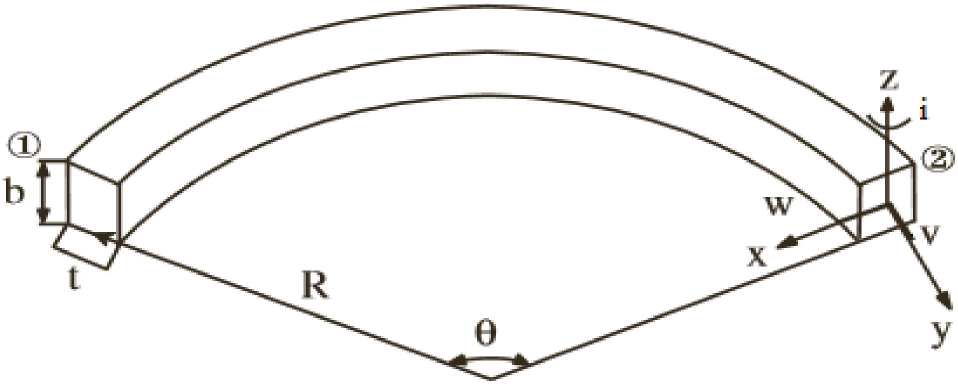

Figure 3.2 Displacements of a curved beam finite element

As shown in Figure 3.2 a curved finite element has six degrees of freedom. Nodal displacement functions used in the analysis of curved beam are given by Sabir & Ashwell (1971) as,

$$w = a_1 \cos(\phi) + a_2 \sin(\phi) + a_4 - a_6 \phi \tag{3.1}$$

$$v = -a_1 \sin(\phi) + a_2 \cos(\phi) + a_3 + a_5 \phi + \frac{1}{2} a_6 \phi^2 , \phi = \frac{y}{R} \tag{3.2}$$

$$i = \frac{\partial w}{\partial y} - \frac{v}{R} \tag{3.3}$$

Radial, axial displacements and rotation of a curved beam element are shown w, v and i respectively in Figure 3.2.

$$[q]^T = [v_i \quad w_i \quad i_i \quad v_{i+1} \quad w_{i+1} \quad i_{i+1}] \tag{3.4}$$

3.2.1 Kinetic Energy of In-Plane Vibration of a Curved Beam

The kinetic energy of a curved beam element is given by Belek, (1977) as below.

$$T = \frac{1}{2} \int_0^{l_{sh}} \rho A (\dot{v}^2 + \dot{w}^2) dy \tag{3.5}$$

where ρ is density and A is cross sectional area of curved beam.

The Eq.(3.5) can be written in a closed form as

$$T = \frac{1}{2} \dot{q}^T m_e \dot{q} \tag{3.6}$$

where,

$$m_e = [C]^T [m][C] \tag{3.7}$$

The mass matrix $[m]$ is;

$$[m] = \rho A R \begin{bmatrix} \beta & 0 & \cos\beta - 1 & \sin\beta & \beta\cos\beta - \sin\beta & C_0 \\ & \beta & \sin\beta & 1 - \cos\beta & \cos\beta + \beta\sin\beta - 1 & C_1 \\ & & \beta & 0 & \beta^2/2 & \beta^3/6 \\ & & & \beta & 0 & \beta^2/2 \\ & & & & \beta/3 & \beta^5/20 + \beta^3/3 \end{bmatrix} \tag{3.8}$$

where,

$$C_0 = \frac{(\beta^2 - 4)\cos\beta}{2} - 2\beta\sin\beta + 2, \; C_1 = \frac{(\beta^2 - 4)\sin\beta}{2} - 2\beta\cos\beta \tag{3.9}$$

Transformation matrix $[C]$ is;

$$[C]=\begin{bmatrix} -B_4 & B_1 & -RB_4 & -B_5 & -B_1 & -RB_5 \\ -B_3 & B_2 & -RB_3 & -B_1 & -B_2 & -RB_1 \\ B_1 & -B_2 & RB_3 & B_1 & B_2 & RB_1 \\ B_4 & -B_3 & RB_4 & B_5 & B_1 & RB_5 \\ B_6 & -B_7 & RB_8 & B_6 & B_7 & RB_9 \\ -B_1 & B_2 & -RB_1 & -B_1 & -B_2 & -RB_1 \end{bmatrix} \tag{3.10}$$

where,

$$D = 2\cos\beta - 2 + \beta\sin\beta \tag{3.11}$$

$$B_1 = \frac{(\cos\beta - 1)}{D},\ B_2 = \frac{\sin\beta}{D},\ B_3 = \frac{(1-\beta\sin\beta-\cos\beta)}{D} \tag{3.12}$$

$$B_4 = \frac{(\sin\beta - \beta\cos\beta)}{D}, B_5 = \frac{(\beta - \sin\beta)}{D}, B_6 = \frac{B_1\beta}{2} \tag{3.13}$$

$$B_7 = \frac{B_2\beta}{2},\ B_8 = B_6 + \frac{1}{\beta},\ B_9 = B_6 - \frac{1}{\beta} \tag{3.14}$$

3.2.2 Strain Energy of In-Plane Vibration of a Curved Beam

Elastic potential energy of a curved beam element in vibrating in its own plane is given by Belek, (1977) as

$$U = \frac{1}{2}\int_0^{l_{sh}} \left[\left(w'' - \frac{v'}{R} \right)^2 EI_{xx} + \left(v' + \frac{w}{R} \right)^2 EA \right] dy \tag{3.15}$$

where E is Young's modulus and I_{xx} is moment of inertia of area about x axis of a curved beam.

The Eq.(3.15) can be written in a closed form as

$$U = \frac{1}{2} q^T k_e q \tag{3.16}$$

where,

$$k_e = [C]^T [k][C] \tag{3.17}$$

where,

$$[k] = \frac{EI_{xx}}{R^4} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q\beta & Q\beta & 0 \\ 0 & 0 & 0 & Q\beta & \beta(1+Q) & \beta^2/2 \\ 0 & 0 & 0 & 0 & \beta^2/2 & \beta^3/3 \end{bmatrix}, \text{ where } Q = \frac{12R^2}{t^2} \tag{3.18}$$

Curved beam element elastic stiffness matrix $[k_e]$ and mass matrix $[m_e]$ are obtained by using the finite element method. Then the global elastic stiffness matrix $[K_e]$ and mass matrix $[M]$ are constituted by individual stiffness and mass matrices of each curved beam element. Stiffness matrices and mass matrices of suspended spring mass systems are placed into the nodal location of global elastic stiffness and global mass matrix individually. The natural frequencies of curved beams are obtained from the solution of the equation given below.

$$[[K_e] - \omega^2 [M]]\{q\} = 0 \tag{3.19}$$

3.2.3 Transfer Matrix For Suspended Spring Mass Systems

Each of the suspended spring mass system has three degree of freedom in our analysis. This coordinate system is called a local coordinate system and it is shown ***x, y*** and ***θ***. The curved beam coordinate system is also shown in Figure 3.2 as ***w, v*** and ***i*** are called global coordinate system. It is shown that ***w*** is axial, ***v*** is radial displacement and ***i*** is rotation. In order to organize to a single coordinate system, the transformation matrix is used. The local coordinate system can be converted into global coordinate system by using the Eq.(3.20).

$$\begin{Bmatrix} x_1 \\ y_1 \\ \theta_1 \\ x_2 \\ y_2 \\ \theta_2 \end{Bmatrix} = [L] \begin{Bmatrix} v_1 \\ w_1 \\ i_1 \\ v_2 \\ w_2 \\ i_2 \end{Bmatrix} \tag{3.20}$$

Here L is called transformation matrix and shown in Equation (3.20).

$$[L] = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & 0 & 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.21}$$

Global mass and stiffness matrices of suspended spring mass systems are,

$$[k_s] = [L]^T [k_s][L] \tag{3.22}$$

$$[m] = [L]^T [m][L] \tag{3.23}$$

CHAPTER FOUR
RESULTS AND DISCUSSIONS

In this chapter, the effects of cross sectional area, subtended angle, suspended spring-mass systems and rotational spring foundations on the in-plane frequencies of the curved beams are investigated. A curved beam without suspended spring mass systems is shown in Figure 4.1.

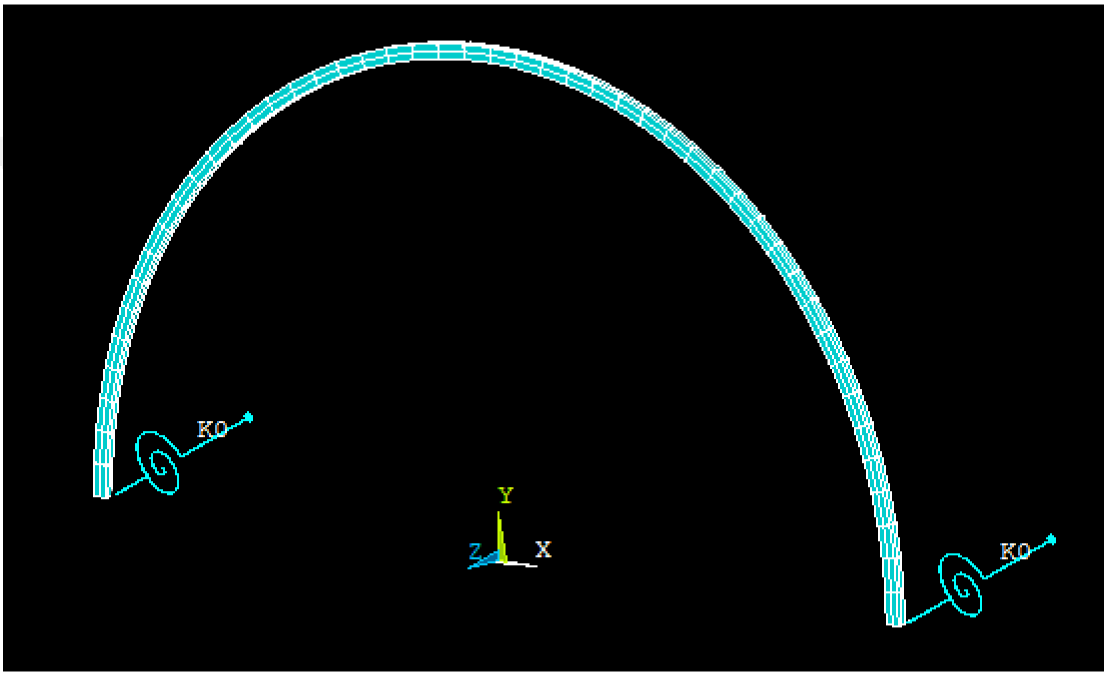

Figure 4.1 Curved beam with rotational spring ends.

The main physical properties of curved beam used throughout in this chapter are as follows: b=0.02 m, t=0.01 m, R=1 m, $E=2.069x10^{11}$ N/m², ν=0.3, ρ=7836.8 kg/m3 The foundation stiffnesses of rotational spring are $ki_1=5$ Nm/rad and $ki_2=5$ Nm/rad. Other parameters used in the modeling of the system are given in the table and figure legends. The numerical results are obtained by the Finite Element Method (FEM) using *ANSYS Multiphysics 14.0* and *MATLAB* computer programmes are compared.

In order to determine the sufficient number of elements, first three natural frequencies are found for a curved beam without suspended spring mass systems and obtained results are presented in Figures 4.2, 4.3 and 4.4.

It is clear from Figures 4.2, 4.3 and 4.4 that the first three natural frequencies do not vary significantly after the number of element $n_{el}=6$. Therefore, $n_{el}=48$ is selected for all of the cases to obtain a better approximation.

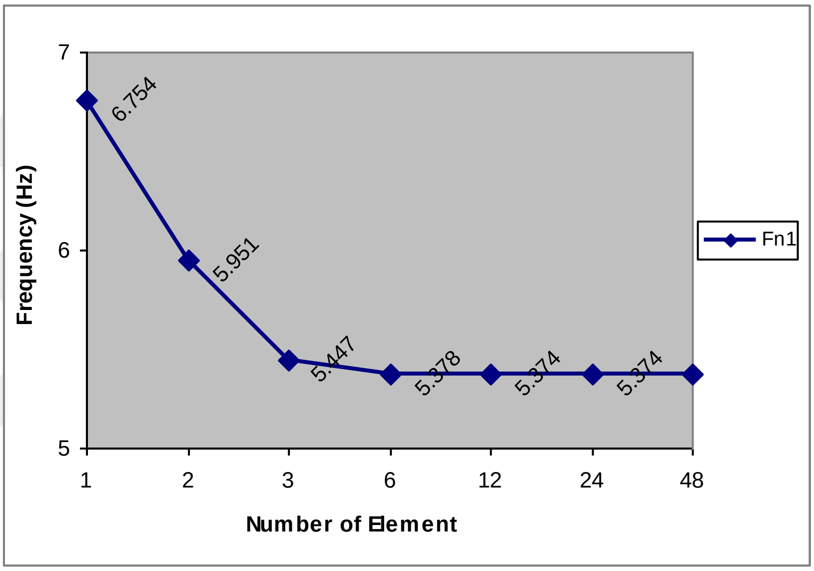

Figure 4.2 The effect of number of elements on the first natural frequency. (b=0.02 m, t=0.01 m, R=1 m, θ=180º, ρ=7836.8 kg/m³, E=2.069x10^{11} N/m², ν=0.3)

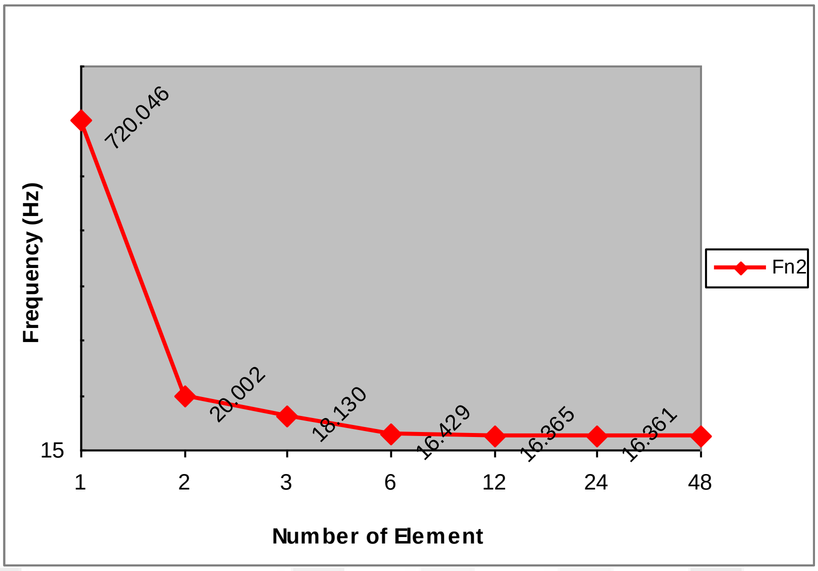

Figure 4.3 The effect of number of elements on the second natural frequency.(b=0.02 m, t=0.01 m, R=1 m, θ=180º, ρ=7836.8 kg/m³, E=2.069x10^{11} N/m², ν=0.3)

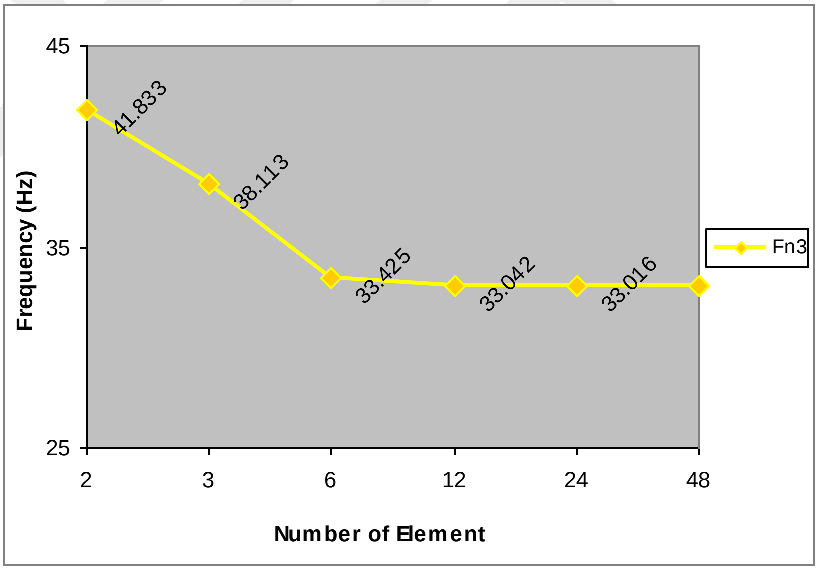

Figure 4.4 The effect of number of elements on the third natural frequency. (b=0.02 m, t=0.01 m, R=1 m, θ=180º, ρ=7836.8 kg/m³, E=2.069x10^{11} N/m², ν=0.3)

Figure 4.5 shows the variation of the first five mode shapes of a curved beam. The curved beam's physical properties are also given in Figure 4.2.

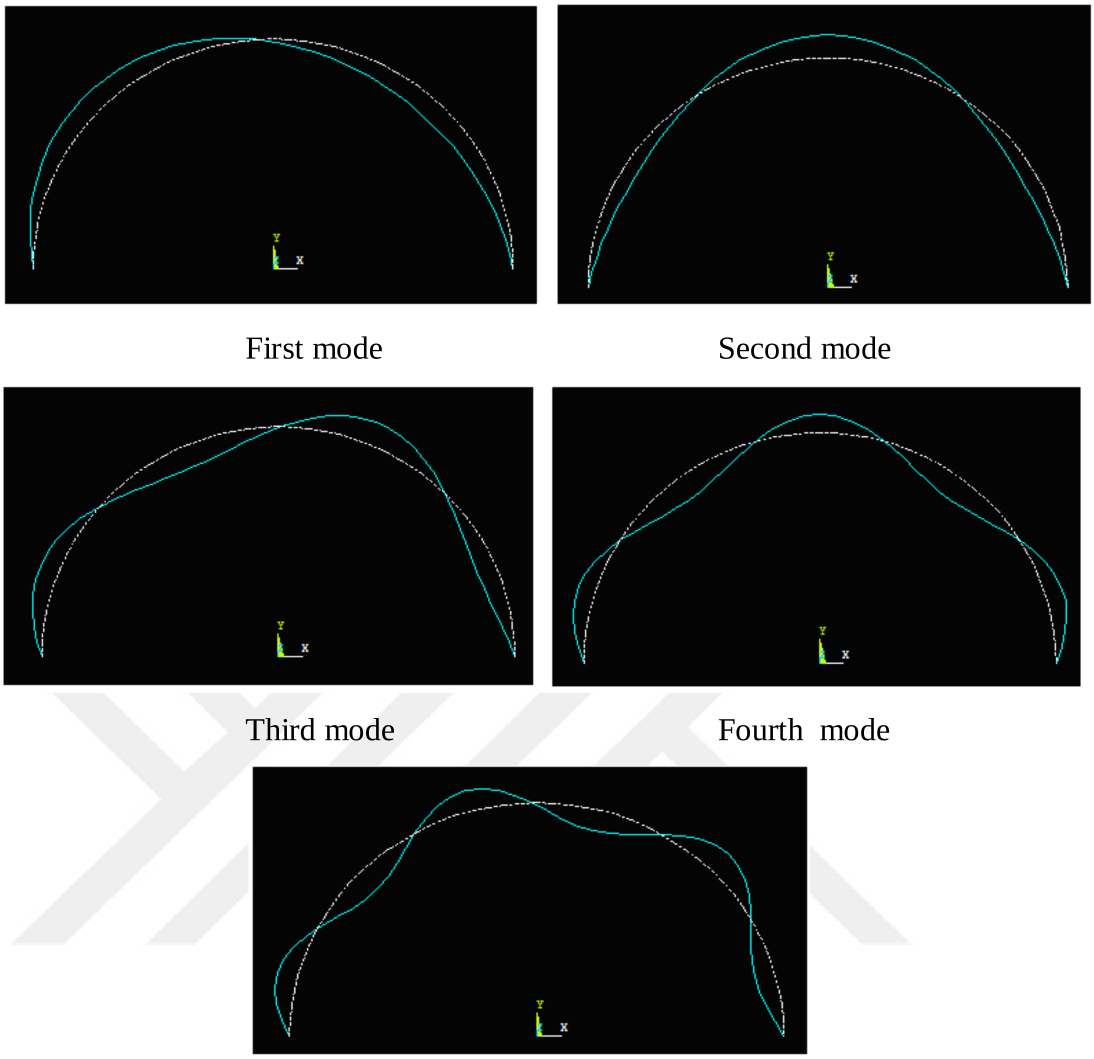

Figure 4.5 Mode shapes of in plane vibrations.

Table 4.1 shows the effect of foundation stiffness with rotational spring on the in-plane natural frequencies. As seen from the table that when the stiffness parameter increases the natural frequencies of the curved beam increases.

Table 4.2 shows the effect of thickness variation of curved beams on the in-plane natural frequencies. When the t/b ratio increases the natural frequencies of the curved beam increases this is due to increase in rigidity.

Table 4.3 shows the variation of subtended angle of a curved beam on the in-plane natural frequencies. When the subtended angle increases, the natural frequencies decrease this is because of the flexibility of the curved beam increases.

Table 4.1 The effect of stiffness of rotational spring foundations on the natural frequencies. (b=0.02 m, t=0.01 m, R=1 m, θ=180º, ρ=7836.8 kg/m³, E=2.069x10^{11} N/m², ν=0.3, nel=48)

Frequency (Hz)										
	f1		***f2***		***f3***		***f4***		***f5***	
Stiffness of Rotational Spring Foundation (Nm/rad)	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*
5	5.378	5.374	16.375	16.361	33.038	33.015	53.915	53.882	80.139	80.110
500	6.877	6.873	17.751	17.738	34.651	34.620	55.507	55.477	81.864	81.840
1000	7.673	7.669	18.650	18.638	35.813	35.794	56.728	56.701	83.244	83.224

Table 4.2 The effect of t/b ratio on the natural frequencies. (b=0.02 m, R=1 m, θ=180º, ρ=7836.8 kg/m³, E=2.069x10^{11} N/m², ν=0.3, nel=48)

Frequency (Hz)										
	f1		***f2***		***f3***		***f4***		***f5***	
t/b	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*
0,5	5.378	5.374	16.375	16.361	33.038	33.015	53.915	53.882	80.139	80.110
1	10.714	10.707	32.699	32.684	65.979	65.984	107.620	107.687	159.910	160.137
1,5	16.059	16.052	48.995	49.006	98.807	98.940	160.990	161.411	239.040	240.073
2	21.398	21.398	65.234	65.310	131.450	131.863	213.840	214.990	317.190	319.847

Table 4.3 Frequencies obtained for different subtended angles of curved beams. (b=0.02 m, t=0.01 m, R=1 m, ρ=7836.8 kg/m³, E=2.069x10^{11} N/m², ν=0.3, nel=48)

Frequency (Hz)										
	f1		*f2*		*f3*		*f4*		*f5*	
Subtended Angle(°)	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*
90	32.534	32.533	76.472	76.480	145.500	145.616	227.060	227.325	333.690	334.395
135	12.129	12.124	31.870	31.855	62.046	62.039	98.871	98.872	145.830	145.896
180	5.378	5.374	16.375	16.361	33.038	33.015	53.915	53.882	80.139	80.110
270	1.142	1.140	5.608	5.597	12.662	12.639	21.927	21.889	33.442	33.389

Figures 4.6, 4.7 and 4.8 show curved beams with three different suspended spring mass systems.

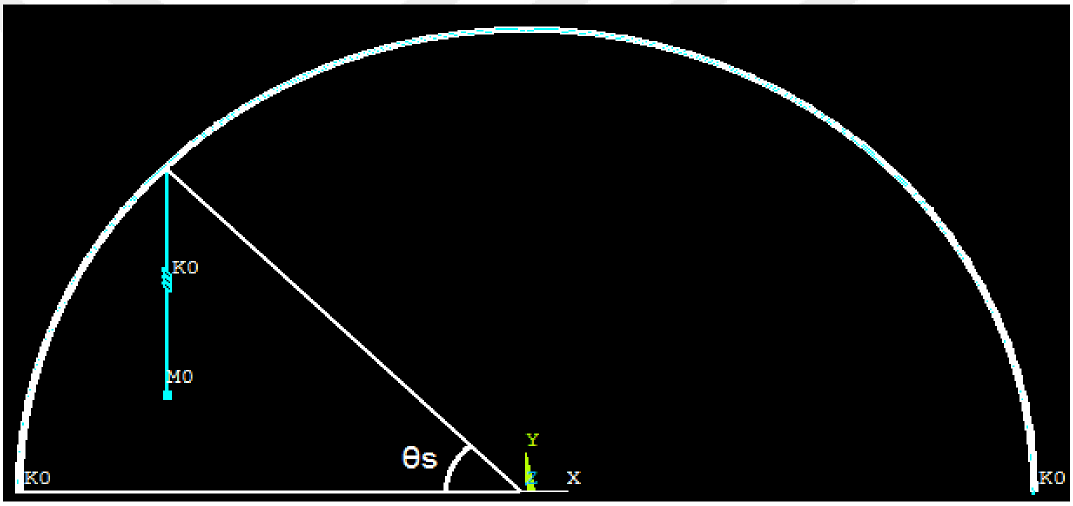

Figure 4.6 Curved beam carrying one suspended spring mass system(θ_{S1}=45º)

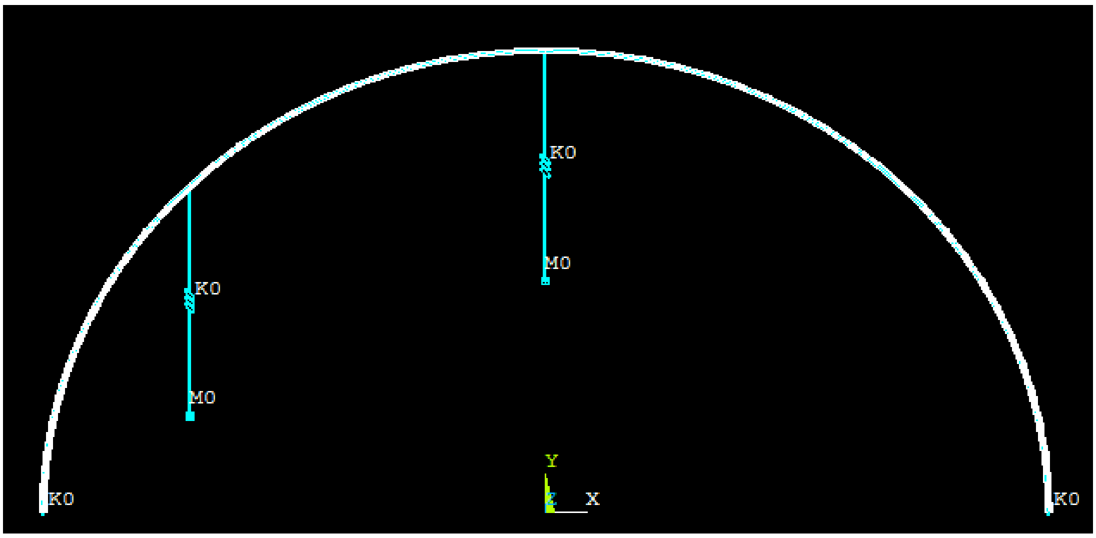

Figure 4.7 Curved beam carrying two suspended spring mass systems(θ_{S1}=45º, θ_{S2}=90º)

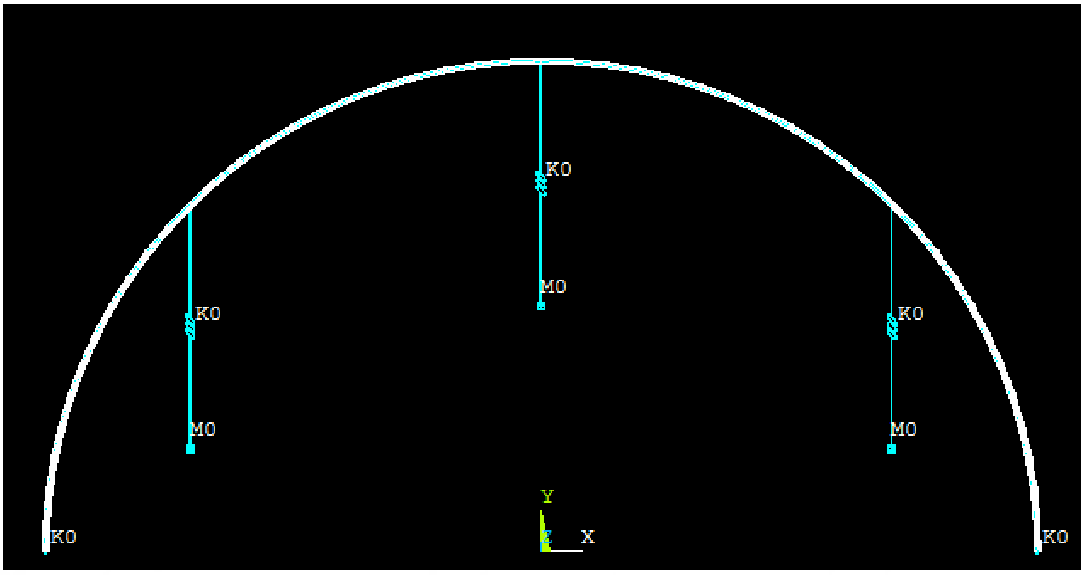

Figure 4.8 Curved beam carrying three suspended spring mass systems.(θ_{S1} =45º, θ_{S2} =90º, θ_{S3} =135)

Added suspended spring mass system is shown in Figure 4.9. The exact natural frequency of a fixed-free spring mass system that is placed to the different locations on the curved beam is found by using $\omega = \sqrt{\frac{k_s}{m}}$ formulation and for three different mass values (m=1 kg, m=3 kg and m=5 kg) and the same value of stiffness of spring (ks=5x10^3 N/m) gave 11.254 Hz, 6.497 Hz and 5.032 Hz values.

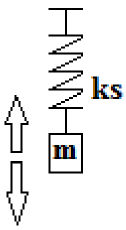

Figure 4.9 A suspended spring mass system

Tables 4.4, 4.5 and 4.6 show the effect of variation of suspended spring mass systems on the first five natural frequencies of curved beams for θ=180º. The exact natural frequency of spring mass systems appears in these tables clearly.

Curved beams having one, two and three suspended spring masses are seen in Figures 4.6., 4.7 and 4.8 respectively. As it can be seen from Tables 4.4, 4.5 and 4.6 that for a single suspended spring mass system, the first natural frequency of the system decreases, the second natural frequency is stable but the third natural frequency increases slightly. Secondly, the curved beam has two suspended spring masses as shown in Figure 4.7, the first and third natural frequencies of the system are stable, but the second natural frequency increases. Thirdly, the curved beam has three suspended spring masses as shown in Figure 4.8, the first natural frequency of system decreases again, the second natural frequency is stable but the third frequency also increases slightly.

All those phenomenon mentioned for subtended angle of 180° are related with the positions of spring-mass systems and mode shapes of curved beams. If the spring-mass system is suspended on the nodes of vibrational modes of curved beams, it has no effect on the frequencies. If the suspended spring mass is fixed on the vibrating portion of curved beams, it has a mass effect and decreases the frequency of fundamental mode. For the higher modes of curved beams, this type of positioning stiffens the curved beam as a result increases the frequency slightly.

Table 4.4 The effect of different number of suspended spring mass systems on the lowest five natural frequencies for m_n =1 kg. (θ=180º, ks_n =5x10^3 N/m)

Frequency(Hz)										
	f1		***f2***		***f3***		***f4***		***f5***	
Number of Spring Mass System	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*
0	5.378	5.374	16.375	16.361	33.038	33.015	53.915	53.882	80.139	80.110
1	5.281	5.277	11.312	11.310	16.375	16.361	33.164	33.143	54.204	54.172
2	5.281	5.277	9.503	9.500	11.312	11.310	19.176	19.166	33.164	33.143
3	5.191	5.187	9.503	9.499	11.115	11.114	11.501	11.499	19.177	19.167

Table 4.5 The effect of different number suspended spring mass systems on the lowest five natural frequencies for m_n =3 kg. (θ=180º, ks_n =5x10^3 N/m)

Frequency(Hz)										
	f1		***f2***		***f3***		***f4***		***f5***	
Number of Spring Mass System	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*
0	5.378	5.374	16.375	16.361	33.038	33.015	53.915	53.882	80.139	80.110
1	4.879	4.895	7.046	7.043	16.375	16.361	33.154	33.133	54.195	54.164
2	4.879	4.895	5.680	5.679	7.047	7.043	18.527	18.515	33.154	33.133
3	4.617	4.615	5.679	5.678	6.420	6.420	7.471	7.467	18.527	18.516

Table 4.6 The effect of different number suspended spring mass systems on the lowest five natural frequencies for m_n =5 kg. (θ=180º, ks_n =5x10^3 N/m)

Frequency(Hz)										
	f1		***f2***		***f3***		***f4***		***f5***	
Number of Spring Mass System	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*
0	5.378	5.374	16.375	16.361	33.038	33.015	53.915	53.882	80.139	80.110
1	4.296	4.296	6.222	6.217	16.375	16.361	33.153	33.131	54.194	54.162
2	4.296	4.295	4.426	4.425	6.222	6.217	18.423	18.411	33.153	33.131
3	3.996	3.995	4.424	4.423	4.973	4.973	6.688	6.682	18.424	18.412

Figure 4.10 shows the variation of the first five frequencies of a curved beam carrying different number of suspended spring mass systems for m_n =1 kg.

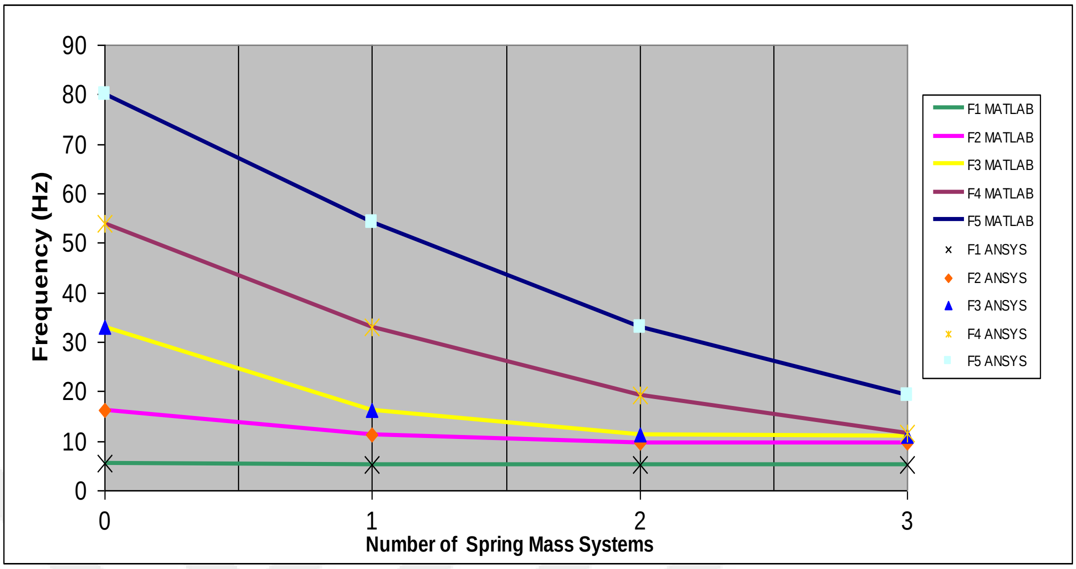

Figure 4.10 Comparisons of the lowest five natural frequencies of the curved beam carrying different number of suspended spring mass systems. (θ=180º, ki_1 =5 Nm/rad, ki_2 =5 Nm/rad, ks_n =5x10^3 N/m, m_n =1 kg)

Tables 4.7, 4.8 and 4.9 show the effect of subtended angle of curved beams carrying different number of spring mass systems on the first five natural frequencies for different subtended angle (θ=90º). In these tables, the curved beam having subtended angle θ=90º shows different behaviour than the one with subtended angle θ=180º. As mentioned before, if the subtended angle decreases, the natural frequencies of curved beam increases. This explains that the curved beam with θ=90º is more rigid than the one with θ=180º. When the suspended spring mass systems are fixed on the system in sequence, the first and second natural frequencies of curved beam increases because of stiffness effect of suspended spring mass system. However increasing of the value of masses, the raising ratio of first and second natural frequencies decreases because of the mass effect of the suspended masses. The third natural frequency increases slightly for the same amount with different spring mass systems.

Table 4.7 The effect of different number of suspended spring mass systems on the lowest five natural frequencies for θ=90º and m_n =1 kg.(ks_n =5x10^3 N/m)

Frequency(Hz)										
	f1		*f2*		*f3*		*f4*		*f5*	
Number of Spring Mass System	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*
0	32.534	32.533	76.472	76.480	145.500	145.616	227.060	227.325	333.690	334.395
1	10.890	10.890	33.541	33.539	76.538	76.546	145.510	145.623	227.230	227.493
2	10.882	10.883	11.099	11.099	33.541	33.539	77.565	77.573	145.510	145.623
3	10.598	10.598	11.066	11.066	11.238	11.238	34.510	34.508	77.628	77.646

Table 4.8 The effect of different number of suspended spring mass systems on the lowest five natural frequencies for θ=90º and m_n =3 kg.(ks_n =5x10^3 N/m)

Frequency(Hz)										
	f1		*f2*		*f3*		*f4*		*f5*	
Number of Spring Mass System	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*
0	32.534	32.533	76.472	76.480	145.500	145.616	227.060	227.325	333.690	334.395
1	6.302	6.302	33.464	33.462	76.537	76.545	145.510	145.623	227.230	227.493
2	6.297	6.297	6.410	6.410	33.464	33.463	77.550	77.558	145.510	145.623
3	6.144	6.144	6.391	6.391	6.488	6.488	34.370	34.368	77.612	77.620

Table 4.9 The effect of different number of suspended spring mass systems on the lowest five natural frequencies for θ=90º and m_n =5 kg.(ks_n =5x10^3 N/m)

Frequency(Hz)										
	f1		*f2*		*f3*		*f4*		*f5*	
Number of Spring Mass System	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*
0	32.534	32.533	76.472	76.480	145.500	145.616	227.060	227.325	333.690	334.395
1	4.884	4.884	33.450	33.448	76.537	76.545	145.510	145.623	227.230	227.493
2	4.880	4.880	4.965	4.965	33.450	33.448	77.547	77.555	145.510	145.623
3	4.762	4.763	4.950	4.950	5.026	5.026	34.344	34.342	77.609	77.616

Tables 4.10, 4.11 and 4.12 show the effect of subtended angle of curved beams carrying different number of spring mass systems on the first five natural frequencies for a subtended angle θ=135º. In these tables, the curved beam shows similar vibrational characteristics with the one having subtended angle θ=90º.

Table 4.10 The effect of different number suspended spring mass systems on the lowest five natural frequencies for θ=135º and m_n =1 kg.(ks_n =5x10^3 N/m)

Frequency(Hz)										
	f1		***f2***		***f3***		***f4***		***f5***	
Number of Spring Mass System	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*
0	12.129	12.124	31.870	31.855	62.046	62.039	98.871	98.872	145.830	145.896
1	9.656	9.655	14.050	14.044	31.905	31.890	62.086	62.079	99.104	99.107
2	9.652	9.651	10.653	10.653	14.052	14.047	33.561	33.548	62.086	62.079
3	8.981	8.981	10.618	10.618	11.202	11.201	15.132	15.126	33.589	33.576

Table 4.11 The effect of different number suspended spring mass systems on the lowest five natural frequencies for θ=135º and m_n =3 kg.(ks_n =5x10^3 N/m)

Frequency(Hz)										
	f1		***f2***		***f3***		***f4***		***f5***	
Number of Spring Mass System	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*
0	12.129	12.124	31.870	31.855	62.046	62.039	98.871	98.872	145.830	145.896
1	5.982	5.982	13.094	13.088	31.902	31.887	62.085	62.078	99.102	99.105
2	5.972	5.972	6.183	6.182	13.095	13.088	33.434	33.421	62.085	62.078
3	5.630	5.630	6.154	6.154	6.468	6.467	13.937	13.930	33.461	33.447

Table 4.12 The effect of different number suspended spring mass systems on the lowest five natural frequencies for θ=135º and m_n =5 kg.(ks_n =5x10^3 N/m)

Frequency(Hz)										
	f1		***f2***		***f3***		***f4***		***f5***	
Number of Spring Mass System	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*
0	12.129	12.124	31.870	31.855	62.046	62.039	98.871	98.872	145.830	145.896
1	4.671	4.671	12.990	12.984	31.901	31.886	62.085	62.078	99.102	99.105
2	4.661	4.661	4.795	4.794	12.991	12.984	33.411	33.397	62.085	62.078
3	4.413	4.413	4.770	4.770	5.010	5.010	13.775	13.767	33.437	33.767

Tables 4.13, 4.14 and 4.15 show the effect of subtended angle of curved beam carrying different number of spring mass systems on the first five natural frequencies for a subtended angle θ=270º. In these tables, the curved beam shows the different behaviour from the one with subtended angle θ=180º. When the suspended spring mass systems are fixed on the system, the first and second natural frequencies of curved beam decrease but the third natural frequency increases. This phenomenon explains the importance of mode shapes of curved beams and the positioning of spring mass systems.

Table 4.13 The effect of different number suspended spring mass systems on the lowest five natural frequencies for θ=270º and m_n =1 kg.(ks_n =5x10^3 N/m)

Frequency(Hz)										
	f1		***f2***		***f3***		***f4***		***f5***	
Number of Spring Mass System	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*	*Ansys*	*Matlab*
0	1.142	1.140	5.608	5.597	12.662	12.639	21.927	21.889	33.442	33.389
1	1.136	1.134	5.468	5.457	10.406	10.394	13.798	13.789	22.310	22.276
2	1.136	1.134	4.723	4.715	10.152	10.141	12.524	12.516	14.009	14.004
3	1.131	1.129	4.654	4.645	9.801	9.787	10.581	10.576	13.303	13.304

Table 4.14 The effect of different number suspended spring mass systems on the lowest five natural frequencies for θ=270º and m_n =3 kg.(ks_n =5x10^3 N/m)

Frequency(Hz)										
	f1		f2		f3		f4		f5	
Number of Spring Mass System	Ansys	Matlab	Ansys	Matlab	Ansys	Matlab	Ansys	Matlab	Ansys	Matlab
0	1.142	1.140	5.608	5.597	12.662	12.639	21.927	21.889	33.442	33.389
1	1.125	1.123	4.925	4.916	7.072	7.068	13.193	13.176	22.237	22.203
2	1.125	1.123	3.571	3.566	6.159	6.155	9.836	9.831	13.218	13.202
3	1.109	1.107	3.484	3.478	6.141	6.135	6.176	6.173	10.256	10.255

Table 4.15 The effect of different number suspended spring mass systems on the lowest five natural frequencies for θ=270º and m_n =5 kg.(ks_n =5x10^3 N/m)

Frequency(Hz)										
	f1		f2		f3		f4		f5	
Number of Spring Mass System	Ansys	Matlab	Ansys	Matlab	Ansys	Matlab	Ansys	Matlab	Ansys	Matlab
0	1.142	1.140	5.608	5.597	12.662	12.639	21.927	21.889	33.442	33.389
1	1.113	1.111	4.251	4.244	6.444	6.440	13.136	13.118	22.226	22.191
2	1.113	1.111	2.953	2.949	4.837	4.834	9.242	9.237	13.153	13.136
3	1.087	1.085	2.873	2.868	4.792	4.790	4.888	4.884	9.635	9.633

As we know that by increasing the subtended angle decreases the rigidity as a result decreases frequencies of curved beams but positioning and number of suspended spring mass systems cause variation in frequencies of curved beam. This effect depends on the modes of curved beams as discussed before.

Figures 4.11, 4.15 and 4.19 show variation in first three natural frequencies for different subtended angles of curved beams.

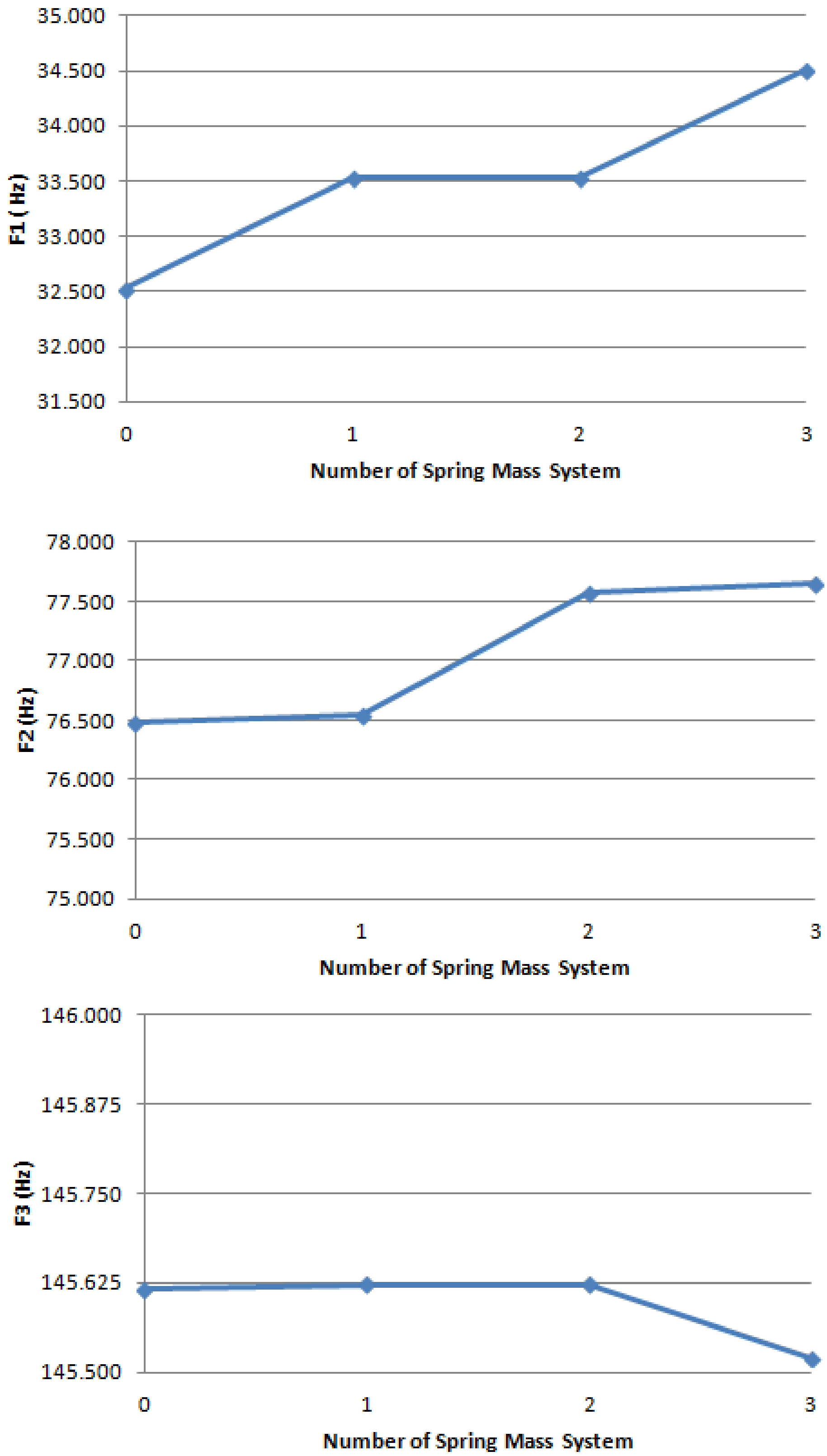

Figure 4.11 The effect of different number suspended spring mass systems on the lowest three natural frequencies for θ=90º and m_n =1 kg. (ks_n =5x10^3 N/m)

Figures 4.12, 4.13 and 4.14 show the first three mode shapes of a curved beam for θ=90º.

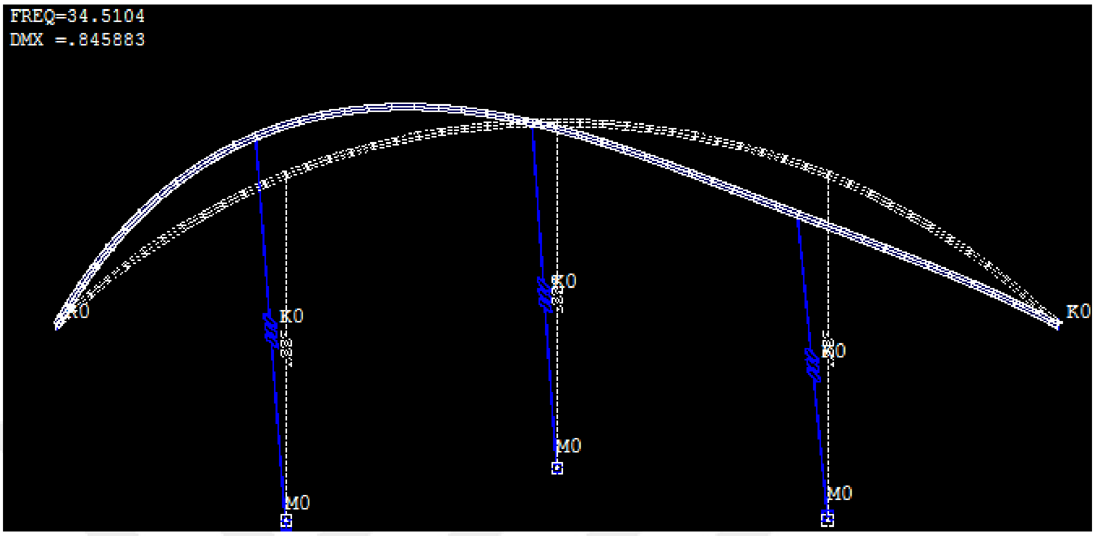

Figure 4.12 First mode shape of a curved beam. (θ=90º , ks_n =5x10^3 N/m, m_n =1 kg)

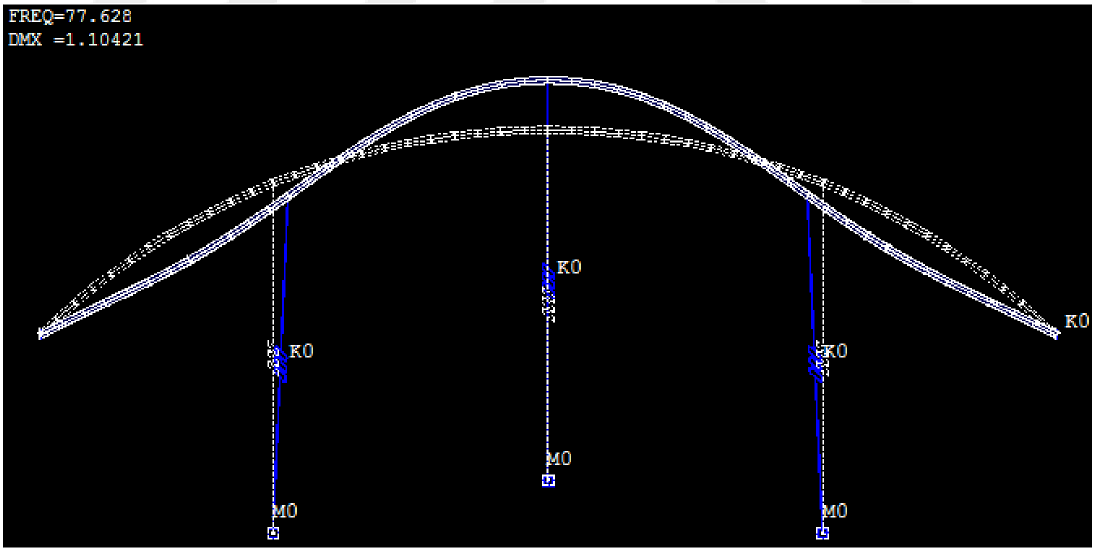

Figure 4.13 Second mode shape of a curved beam. (θ=90º , ks_n =5x10^3 N/m, m_n =1 kg)

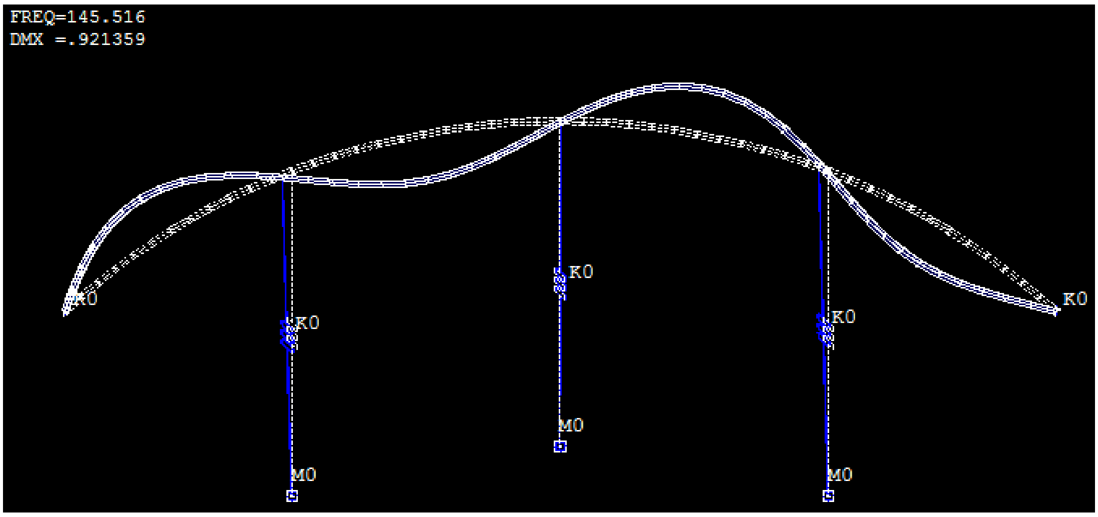

Figure 4.14 Third mode shape of a curved beam. (θ=90º, ks_n =5x10^3 N/m, m_n =1 kg)

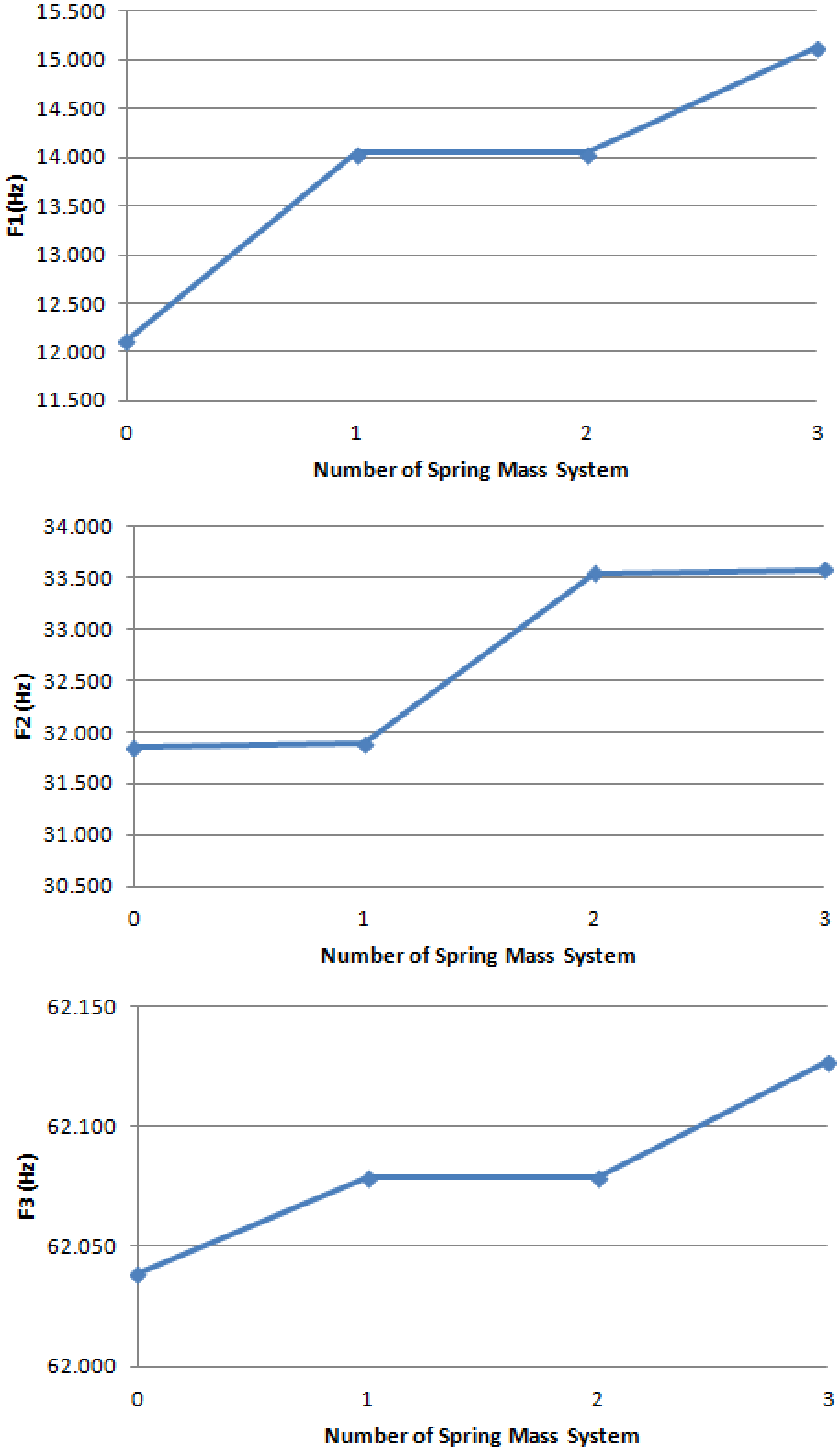

Figure 4.15 The effect of different number suspended spring mass systems on the lowest three natural frequencies for θ=135º and m_n =1 kg. (ks_n =5x10^3 N/m)

Figures 4.16, 4.17 and 4.18 show the first three mode shapes of a curved beam for θ=135º.

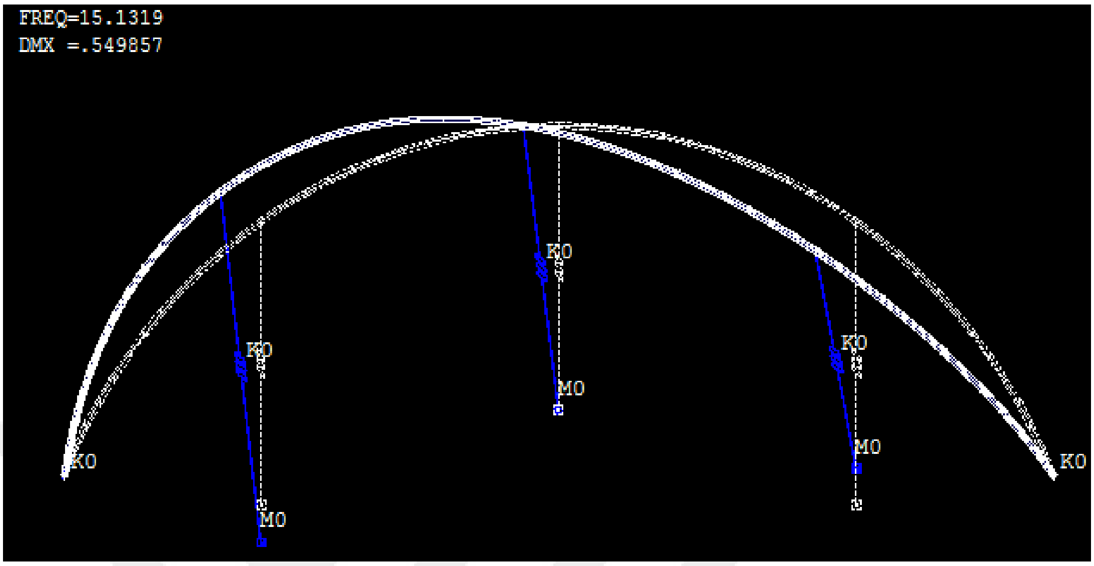

Figure 4.16 First mode shape of a curved beam. (θ=135º, ks_n =5x10^3 N/m, m_n =1 kg)

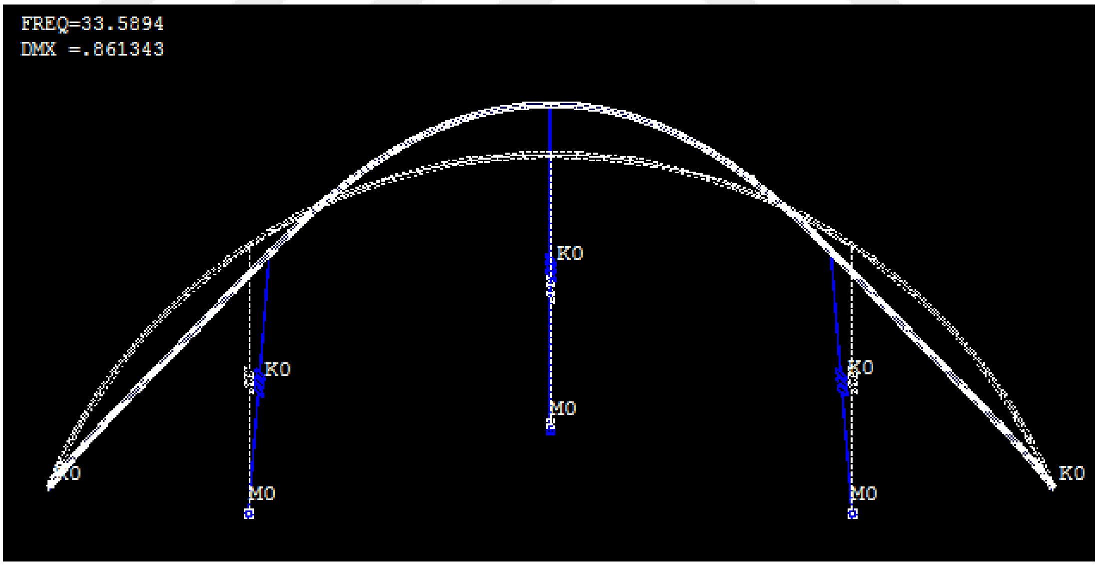

Figure 4.17 Second mode shape of a curved beam. (θ=135º, ks_n =5x10^3 N/m, m_n =1 kg)

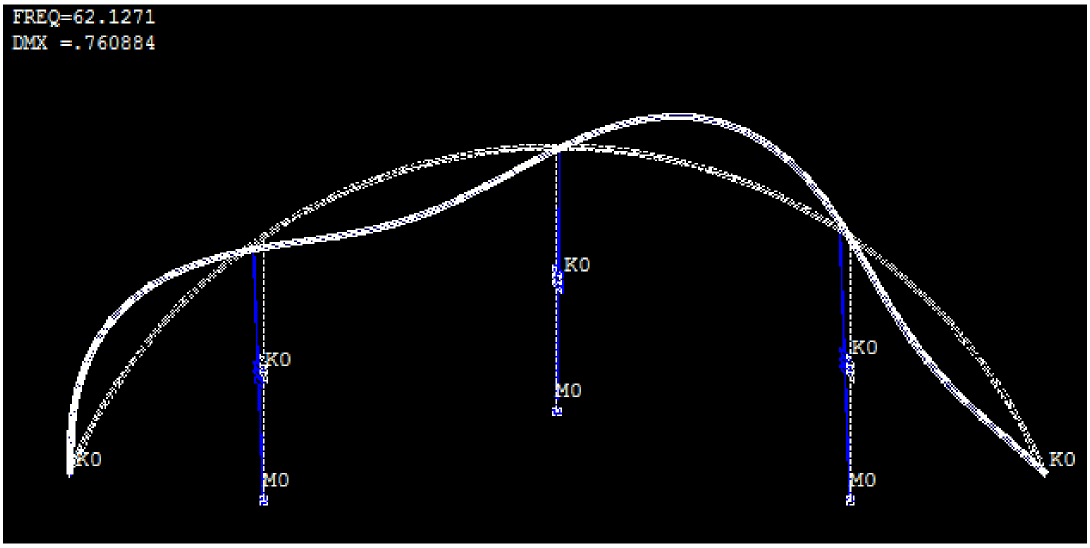

Figure 4.18 Third mode shape of a curved beam. (θ=135º, ks_n =5x10^3 N/m, m_n =1 kg)

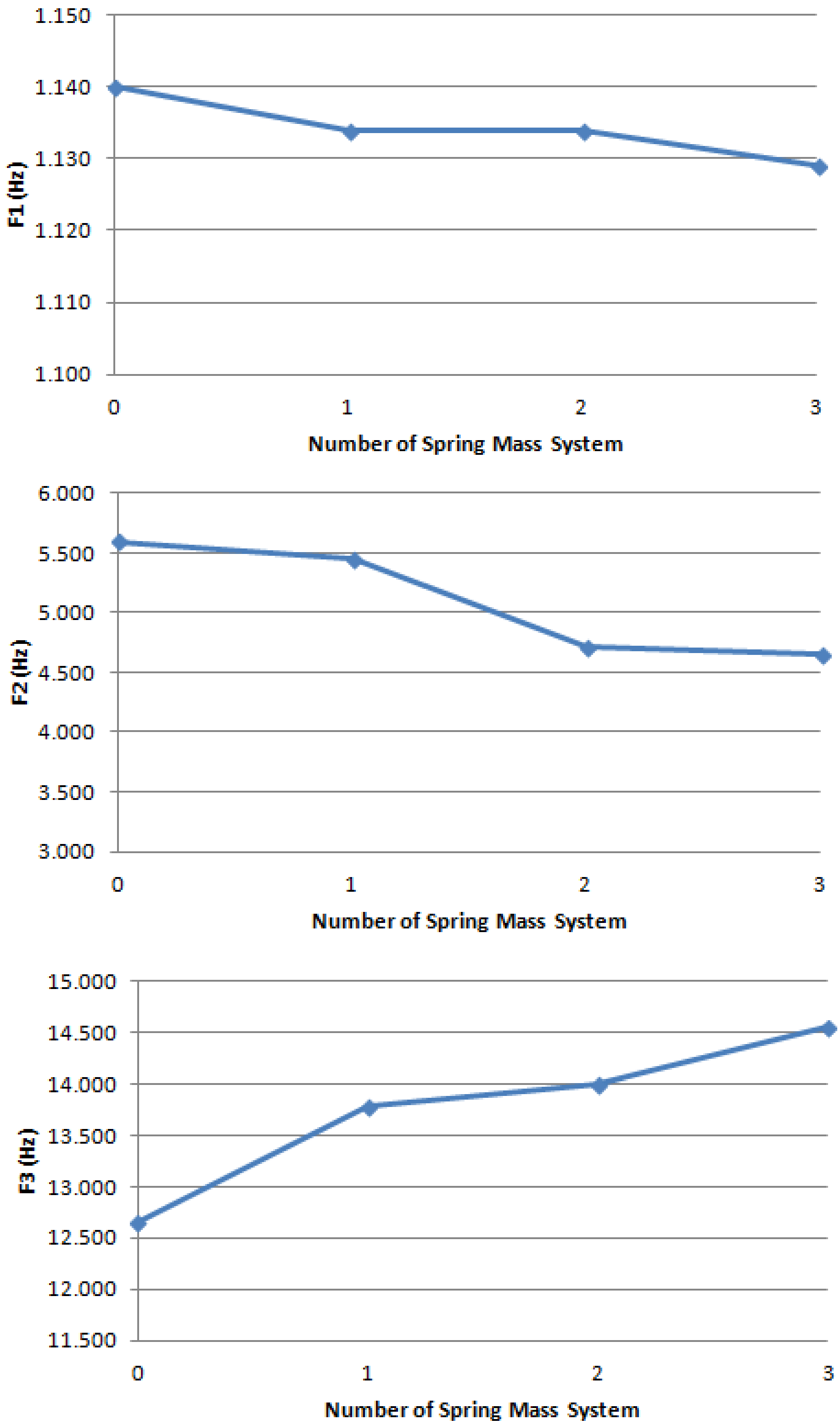

Figure 4.19 The effect of different number suspended spring mass systems on the lowest three natural frequencies for θ=270º and m_n =1 kg.(ks_n =5x10^3 N/m)

Figures 4.20, 4.21 and 4.22 show the first three mode shapes of curved beam for θ=270º.

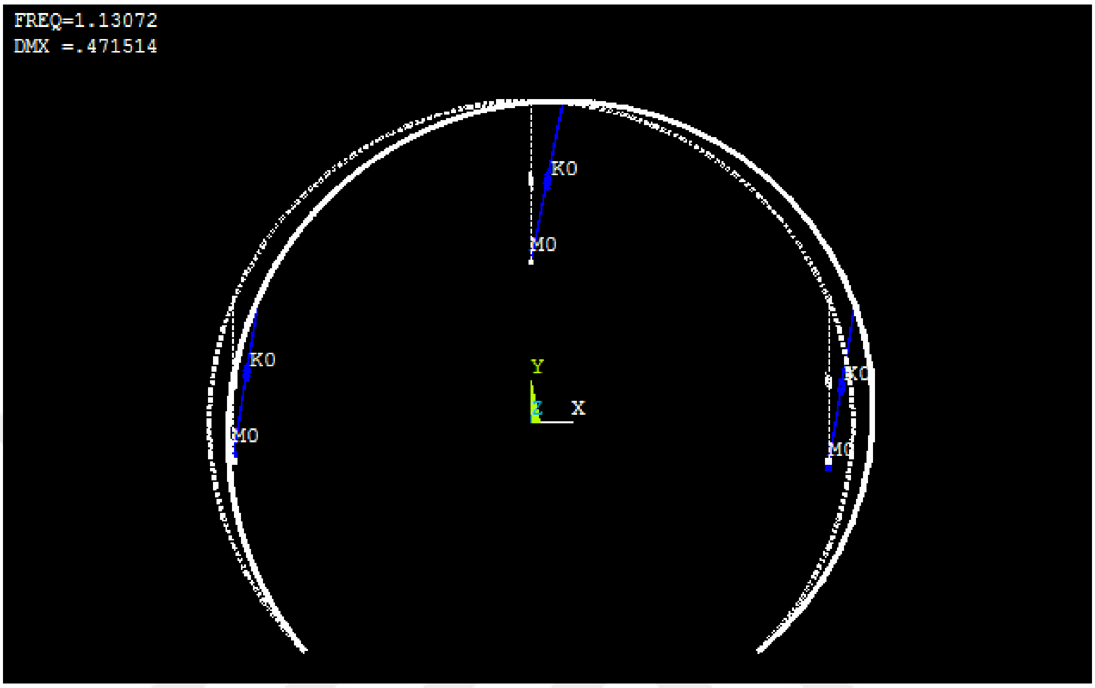

Figure 4.20 First mode shape of a curved beam. (θ=270º, ks_n =5x10^3 N/m, m_n =1 kg)

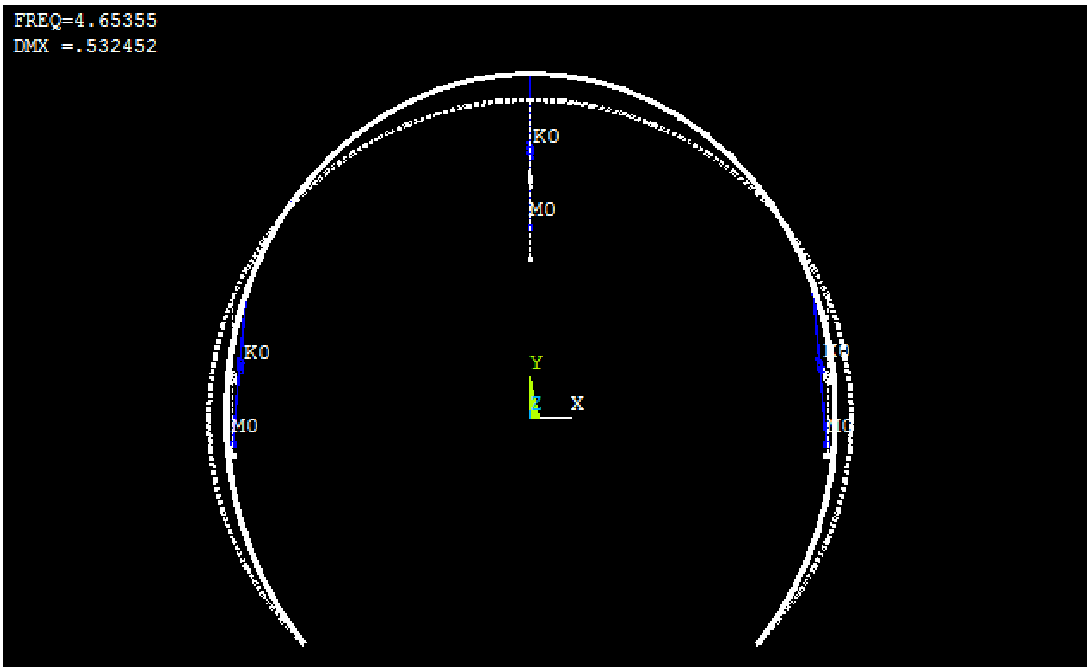

Figure 4.21 Second mode shape of a curved beam. (θ=270º, ks_n =5x10^3 N/m, m_n =1 kg)

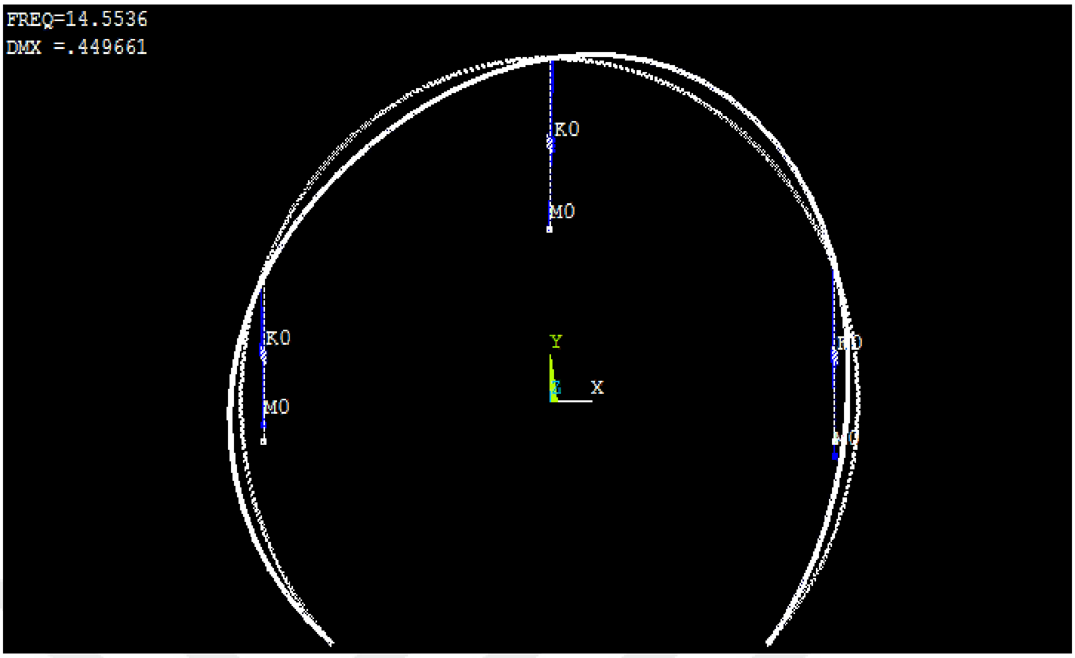

Figure 4.22 Third mode shape of a curved beam. (θ=270º, ks_n =5x10^3 N/m, m_n =1 kg)

CHAPTER FIVE
CONCLUSIONS

In this study the analysis of in-plane vibration of the curved beam having suspended spring mass systems and rotational spring foundation is carried out by using the Finite Element Method via ANSYS and MATLAB computer programmes.

The results obtained by both computer programmes using an equal number of elements gave very good accuracy when compared with each other.

The conclusions obtained from these analyses are summarized below:

1. When the stiffness of rotational spring foundation increases, the in-plane natural frequencies of the curved beam increase.

2. The increase of t/b ratio of curved beams also increases the natural frequencies of curved beams.

3. The subtended angle of curved beams increases, the in-plane natural frequencies decrease as the flexibility of the curved beam increases.

4 Number and position of suspended spring mass systems are also important parameters in our analyses. This is pointed out in our analyses for different subtended angles(θ=180º and θ=270º). When the spring mass system is suspended on the vibrational nodes of curved beams, there is no effect on the natural frequencies. It can be seen that when the first spring mass system is fixed on curved beams for θ=180º, the second natural frequency has no change. For the lower modes, the suspended spring mass system decreases the natural frequencies of curved beams because of mass effect whereas for higher modes, the natural frequencies increase because suspended spring mass system stiffens the curved beam. However, this phenomenon changes for subtended angle of θ=270º due to the change in mode shapes.

5. The variation of subtended angles of curved beams and masses of suspended systems show that the rigidity of curved beam is important parameter for the coupled vibration of spring mass systems. It can clearly seen that suspended spring mass systems increase the rigidity of curved beams for subtended angle θ=90º and θ=135º. As a result the natural frequencies of systems increase, the increase ratio of the natural frequencies decreases when the mass of suspended spring increases. This is due to the mass effect. However it can also be noticed that there is different behavior of the system for θ=180º and θ=270º subtended angles for the same conditions.

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